Computer Algebra
Independent Integration Tests

Summer 2024

6-Hyperbolic-functions/6.4-Hyperbolic-cotangent/309-6.4.1

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**61**]. This is test number [309].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (61)	0.00 (0)
Mathematica	100.00 (61)	0.00 (0)
Fricas	100.00 (61)	0.00 (0)
Maple	95.08 (58)	4.92 (3)
Maxima	90.16 (55)	9.84 (6)
Giac	57.38 (35)	42.62 (26)
Mupad	45.90 (28)	54.10 (33)
Reduce	45.90 (28)	54.10 (33)
Sympy	45.90 (28)	54.10 (33)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

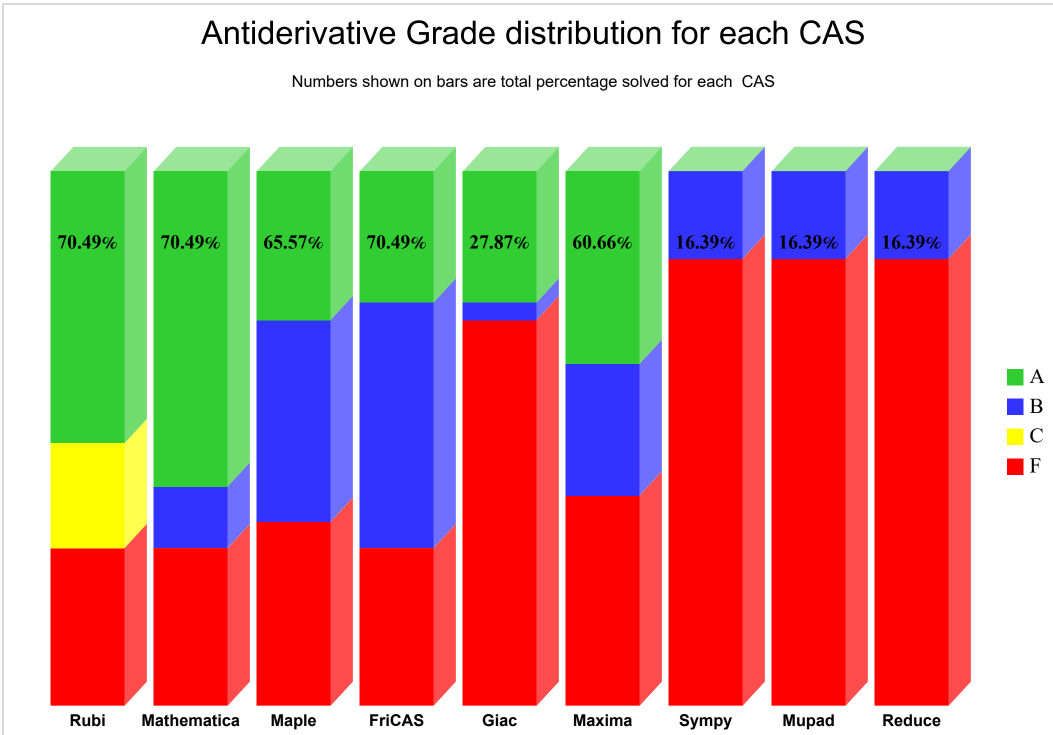
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

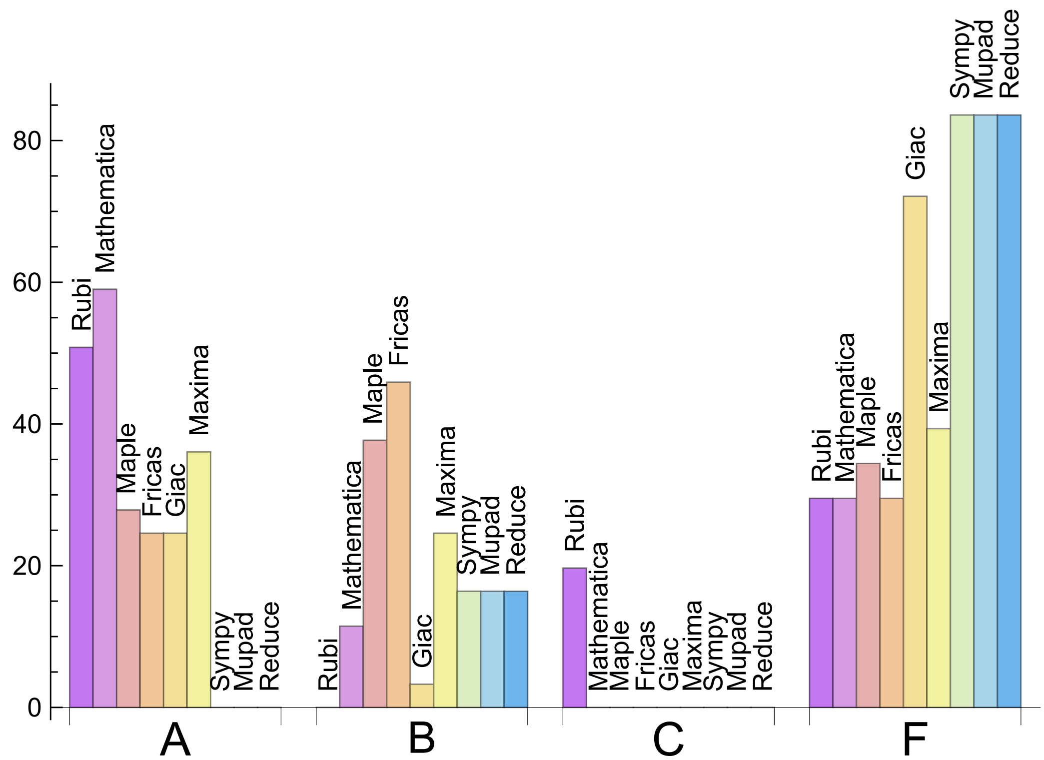
System	% A grade	% B grade	% C grade	% F grade
Mathematica	59.016	11.475	0.000	29.508
Rubi	50.820	0.000	19.672	29.508
Maxima	36.066	24.590	0.000	39.344
Maple	27.869	37.705	0.000	34.426
Fricas	24.590	45.902	0.000	29.508
Giac	24.590	3.279	0.000	72.131
Mupad	0.000	16.393	0.000	83.607
Reduce	0.000	16.393	0.000	83.607
Sympy	0.000	16.393	0.000	83.607

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	0	0.00	0.00	0.00
Maple	3	100.00	0.00	0.00
Maxima	6	100.00	0.00	0.00
Giac	26	100.00	0.00	0.00
Mupad	33	0.00	100.00	0.00
Reduce	33	100.00	0.00	0.00
Sympy	33	90.91	0.00	9.09

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.11
Giac	0.16
Maple	0.16
Reduce	0.69
Rubi	0.77
Maxima	0.91
Sympy	1.44
Mupad	2.58
Mathematica	5.76

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	75.86	1.12	22.00	1.11
Giac	132.83	1.14	22.00	1.11
Rubi	161.97	1.08	131.00	1.00
Mathematica	231.31	1.38	162.00	1.17
Maxima	302.78	5.93	170.00	2.18
Maple	351.26	1.88	159.50	1.00
Sympy	498.07	3.06	19.00	1.00
Fricas	887.82	4.05	216.00	2.35
Reduce	938.39	44.77	225.00	4.58

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

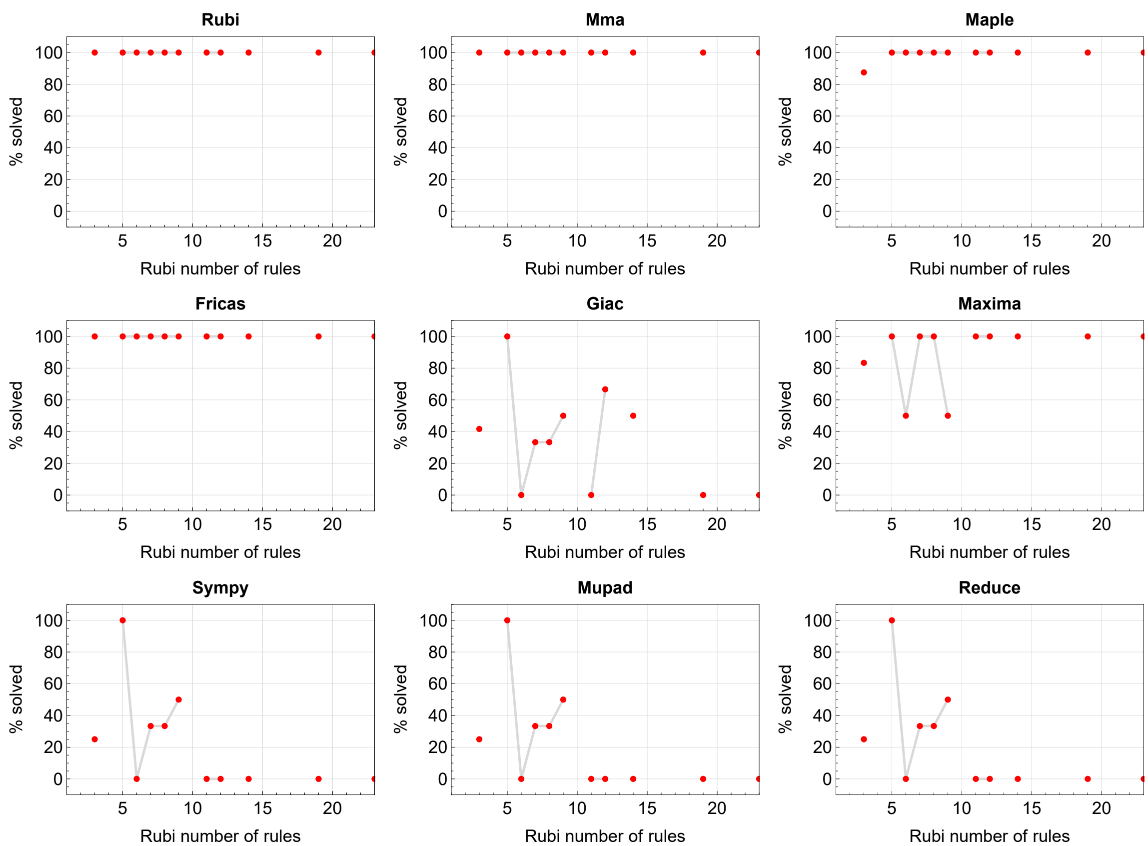


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

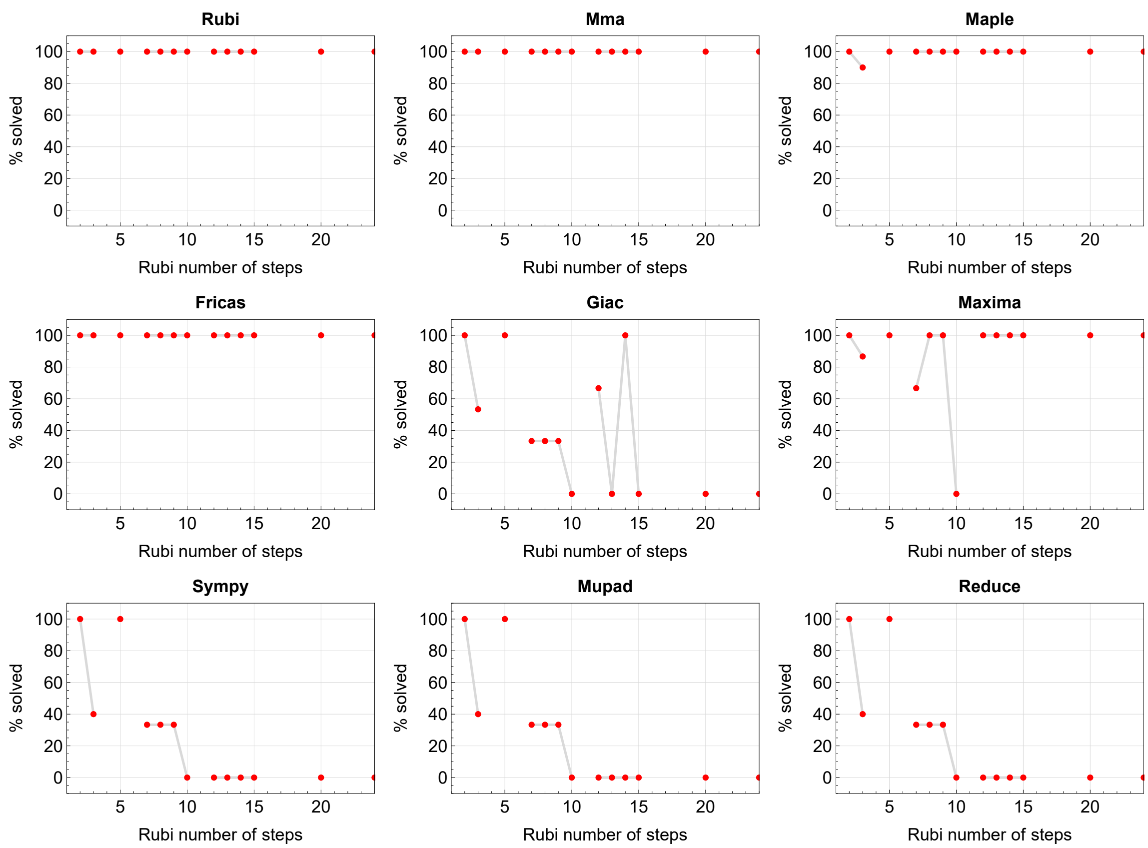


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

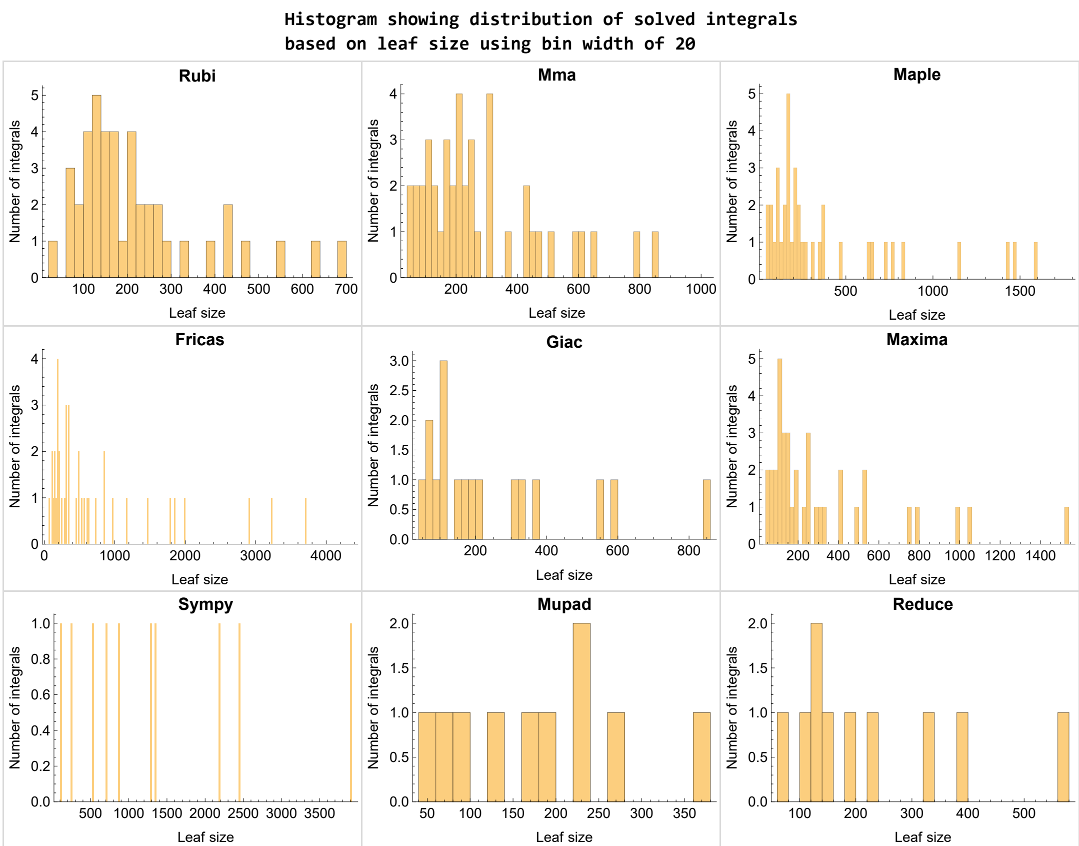


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

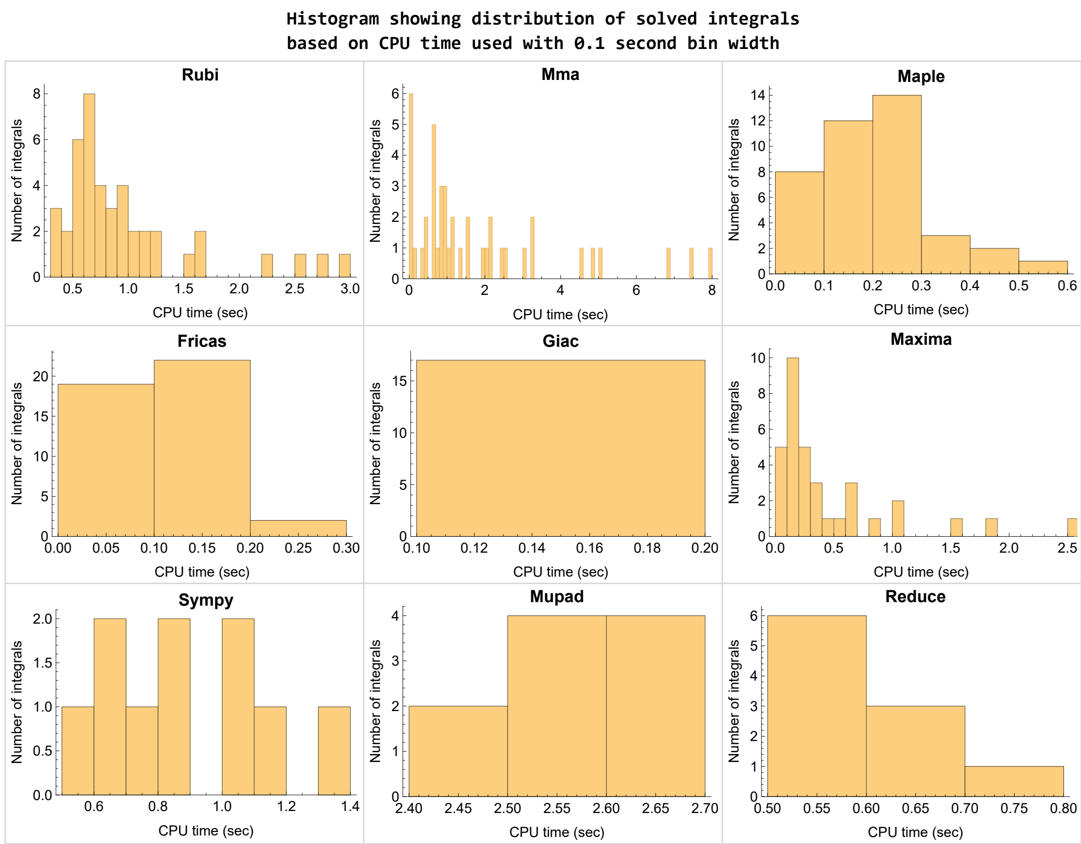


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

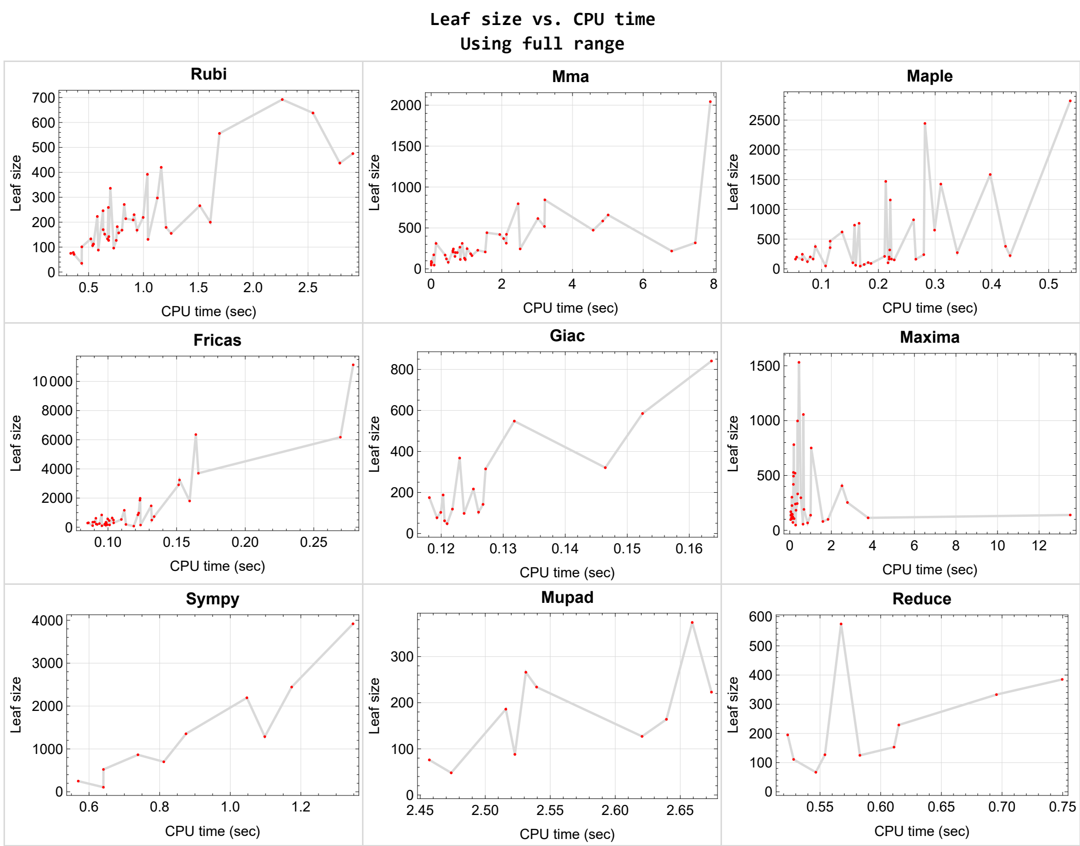


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{4, 5, 9, 10, 14, 15, 32, 33, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{57, 58\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

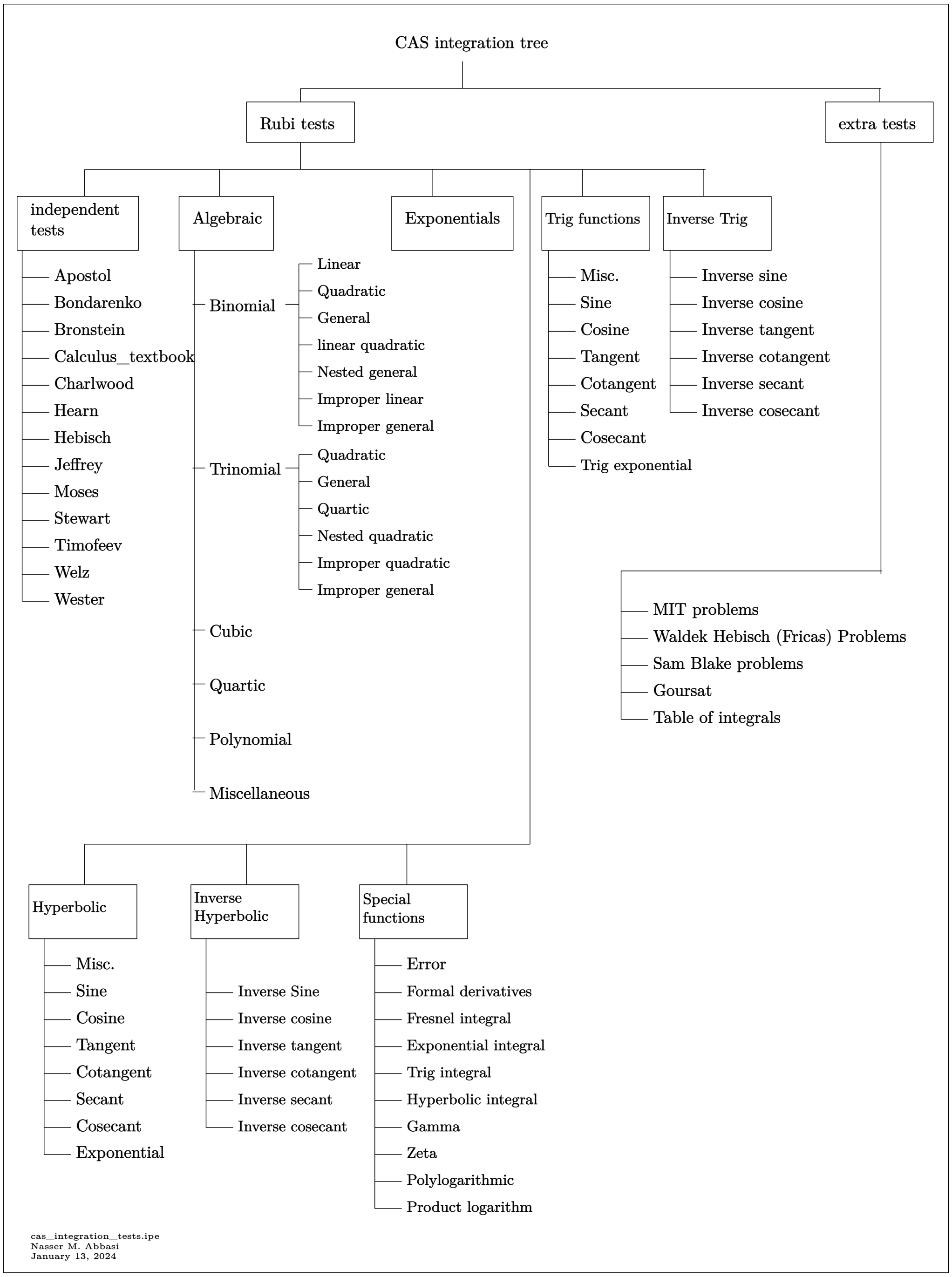
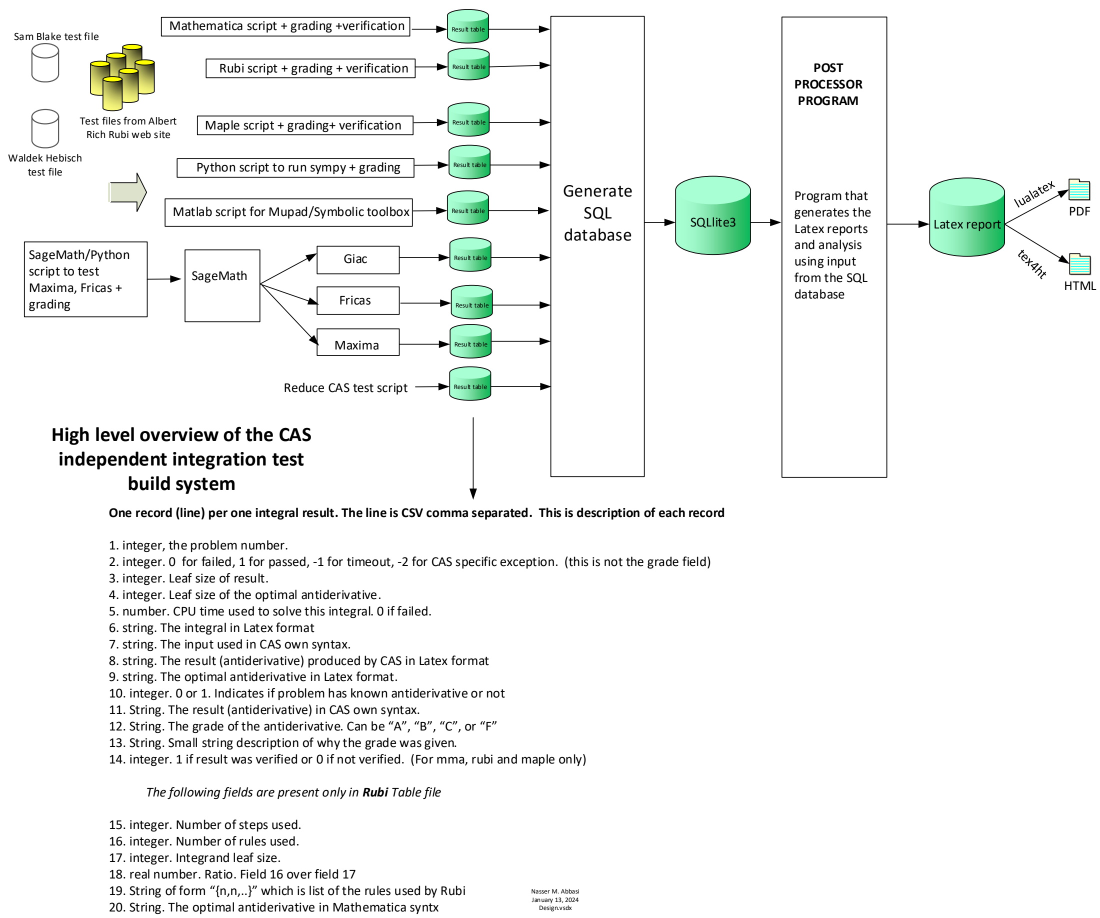


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 16, 17, 18, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 34, 35, 36, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

B grade { }

C grade { 1, 2, 3, 6, 7, 8, 11, 12, 13, 19, 20, 21 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 7, 8, 13, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 34, 35, 36, 38, 39, 44, 48, 49, 52, 53, 54, 57, 58, 59 }

B grade { 6, 11, 12, 37, 42, 43, 47 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 8, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31 }

B grade { 1, 2, 3, 6, 7, 11, 12, 13, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

C grade { }

F normal fail { 34, 35, 36 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 16, 17, 18, 19, 20, 21, 24, 25, 26, 29, 30, 31, 34, 35, 36 }

B grade { 1, 2, 3, 6, 7, 8, 11, 12, 13, 22, 23, 27, 28, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 6, 7, 11, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 44, 57, 58 }

B grade { 1, 2, 3, 8, 12, 13, 37, 38, 42, 43, 47, 48, 49, 52, 53 }

C grade { }

F normal fail { 34, 35, 36, 39, 54, 59 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31 }

B grade { 8, 20 }

C grade { }

F normal fail { 1, 2, 3, 6, 7, 11, 12, 13, 34, 35, 36, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 8, 16, 17, 18, 22, 23, 24, 27, 28, 29 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 6, 7, 11, 12, 13, 19, 20, 21, 25, 26, 30, 31, 34, 35, 36, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

F(-2) exception fail { }

Sympy

A grade { }

B grade { 8, 16, 17, 18, 22, 23, 24, 27, 28, 29 }

C grade { }

F normal fail { 1, 2, 3, 6, 7, 11, 12, 13, 19, 20, 21, 25, 26, 30, 31, 34, 35, 36, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54 }

F(-1) timedout fail { }

F(-2) exception fail { 57, 58, 59 }

Reduce

A grade { }

B grade { 8, 16, 17, 18, 22, 23, 24, 27, 28, 29 }

C grade { }

F normal fail { 1, 2, 3, 6, 7, 11, 12, 13, 19, 20, 21, 25, 26, 30, 31, 34, 35, 36, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	142	91	200	170	216	0	0	39	0
N.S.	1	1.63	1.05	2.30	1.95	2.48	0.00	0.00	0.45	0.00
time (sec)	N/A	0.684	0.005	0.219	0.050	0.091	0.000	0.000	0.612	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	106	66	166	138	168	0	0	39	0
N.S.	1	1.68	1.05	2.63	2.19	2.67	0.00	0.00	0.62	0.00
time (sec)	N/A	0.537	0.004	0.085	0.053	0.099	0.000	0.000	0.648	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	71	47	122	98	112	0	0	37	0
N.S.	1	1.58	1.04	2.71	2.18	2.49	0.00	0.00	0.82	0.00
time (sec)	N/A	0.367	0.003	0.075	0.042	0.089	0.000	0.000	0.685	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	35	12	8	12	25	12
N.S.	1	1.00	1.20	1.00	3.50	1.20	0.80	1.20	2.50	1.20
time (sec)	N/A	0.207	0.279	0.025	0.277	0.076	0.433	0.112	0.665	2.275

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	46	12	10	12	45	12
N.S.	1	1.00	1.20	1.00	4.60	1.20	1.00	1.20	4.50	1.20
time (sec)	N/A	0.211	0.700	0.018	0.188	0.087	0.439	0.112	0.638	2.308

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	131	222	198	146	632	0	0	307	0
N.S.	1	1.51	2.55	2.28	1.68	7.26	0.00	0.00	3.53	0.00
time (sec)	N/A	1.041	0.615	0.056	0.123	0.103	0.000	0.000	0.597	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	96	117	156	108	453	0	0	215	0
N.S.	1	1.48	1.80	2.40	1.66	6.97	0.00	0.00	3.31	0.00
time (sec)	N/A	0.729	0.962	0.066	0.119	0.101	0.000	0.000	0.570	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	35	46	50	115	189	107	98	127	48
N.S.	1	1.13	1.48	1.61	3.71	6.10	3.45	3.16	4.10	1.55
time (sec)	N/A	0.437	0.086	0.107	0.084	0.098	0.641	0.124	0.554	2.474

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	69	14	10	14	48	14
N.S.	1	1.00	1.17	1.00	5.75	1.17	0.83	1.17	4.00	1.17
time (sec)	N/A	0.367	0.130	0.023	0.136	0.078	0.500	0.123	0.705	2.465

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	91	14	12	14	221	14
N.S.	1	1.00	1.17	1.00	7.58	1.17	1.00	1.17	18.42	1.17
time (sec)	N/A	0.276	0.704	0.020	0.169	0.095	0.449	0.121	0.724	2.513

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	266	422	375	302	1985	0	0	776	0
N.S.	1	1.49	2.36	2.09	1.69	11.09	0.00	0.00	4.34	0.00
time (sec)	N/A	1.514	2.132	0.089	0.103	0.123	0.000	0.000	0.557	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	168	314	246	226	1467	0	0	583	0
N.S.	1	1.47	2.75	2.16	1.98	12.87	0.00	0.00	5.11	0.00
time (sec)	N/A	0.941	2.127	0.066	0.106	0.131	0.000	0.000	0.668	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	112	131	164	149	975	0	0	376	0
N.S.	1	1.37	1.60	2.00	1.82	11.89	0.00	0.00	4.59	0.00
time (sec)	N/A	0.544	0.943	0.054	0.091	0.122	0.000	0.000	0.757	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	144	14	10	14	111	14
N.S.	1	1.00	1.17	1.00	12.00	1.17	0.83	1.17	9.25	1.17
time (sec)	N/A	0.331	0.256	0.029	0.252	0.077	0.564	0.139	0.629	2.308

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	175	14	12	14	660	14
N.S.	1	1.00	1.17	1.00	14.58	1.17	1.00	1.17	55.00	1.17
time (sec)	N/A	0.367	0.201	0.024	0.201	0.087	0.525	0.139	0.678	2.272

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	179	244	165	183	304	864	188	195	223
N.S.	1	1.06	1.44	0.98	1.08	1.80	5.11	1.11	1.15	1.32
time (sec)	N/A	1.206	0.630	0.223	0.300	0.086	0.739	0.120	0.523	2.674

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	127	169	103	124	192	522	119	125	186
N.S.	1	1.04	1.39	0.84	1.02	1.57	4.28	0.98	1.02	1.52
time (sec)	N/A	0.752	0.396	0.155	0.141	0.092	0.641	0.122	0.583	2.516

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	78	81	46	72	101	250	62	67	76
N.S.	1	1.05	1.09	0.62	0.97	1.36	3.38	0.84	0.91	1.03
time (sec)	N/A	0.361	0.484	0.168	0.151	0.096	0.569	0.121	0.547	2.457

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	122	61	48	75	0	48	30	0
N.S.	1	1.00	0.78	0.39	0.31	0.48	0.00	0.31	0.19	0.00
time (sec)	N/A	0.774	0.433	0.160	0.282	0.119	0.000	0.121	0.562	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	168	206	91	56	216	0	321	129	0
N.S.	1	1.06	1.30	0.57	0.35	1.36	0.00	2.02	0.81	0.00
time (sec)	N/A	0.804	0.619	0.187	0.637	0.098	0.000	0.146	0.578	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	219	265	210	68	342	0	175	265	0
N.S.	1	1.04	1.26	1.00	0.32	1.62	0.00	0.83	1.26	0.00
time (sec)	N/A	0.997	0.814	0.211	0.859	0.099	0.000	0.118	0.634	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	420	273	297	571	2193	368	385	266
N.S.	1	1.00	1.83	1.19	1.29	2.48	9.53	1.60	1.67	1.16
time (sec)	N/A	0.915	1.942	0.339	0.533	0.100	1.047	0.123	0.750	2.531

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	207	163	191	359	1353	217	229	164
N.S.	1	1.00	1.22	0.96	1.12	2.11	7.96	1.28	1.35	0.96
time (sec)	N/A	0.633	1.531	0.266	0.681	0.090	0.875	0.125	0.615	2.639

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	114	74	107	190	700	103	111	88
N.S.	1	1.00	0.86	0.56	0.80	1.43	5.26	0.77	0.83	0.66
time (sec)	N/A	0.673	0.967	0.175	0.207	0.113	0.812	0.120	0.528	2.523

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	199	106	81	137	0	77	53	0
N.S.	1	1.00	0.67	0.36	0.27	0.46	0.00	0.26	0.18	0.00
time (sec)	N/A	1.127	0.740	0.182	1.594	0.098	0.000	0.119	0.626	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	442	164	100	615	0	585	249	0
N.S.	1	1.00	1.05	0.39	0.24	1.46	0.00	1.39	0.59	0.00
time (sec)	N/A	1.162	1.579	0.219	1.841	0.091	0.000	0.152	0.760	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	615	379	406	844	3918	548	575	374
N.S.	1	1.00	1.83	1.13	1.21	2.51	11.66	1.63	1.71	1.11
time (sec)	N/A	0.698	3.021	0.424	2.512	0.095	1.348	0.132	0.567	2.659

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	371	223	254	532	2443	315	333	234
N.S.	1	1.00	1.51	0.91	1.03	2.16	9.93	1.28	1.35	0.95
time (sec)	N/A	0.632	2.051	0.432	2.776	0.110	1.174	0.127	0.695	2.540

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	182	185	102	138	286	1287	142	153	127
N.S.	1	0.99	1.01	0.56	0.75	1.56	7.03	0.78	0.84	0.69
time (sec)	N/A	0.761	1.118	0.217	1.003	0.085	1.098	0.127	0.611	2.621

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	312	151	114	195	0	104	76	0
N.S.	1	1.00	0.71	0.35	0.26	0.45	0.00	0.24	0.17	0.00
time (sec)	N/A	2.790	0.879	0.228	3.771	0.099	0.000	0.126	0.636	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	692	692	796	239	140	1162	0	841	373	0
N.S.	1	1.00	1.15	0.35	0.20	1.68	0.00	1.22	0.54	0.00
time (sec)	N/A	2.265	2.462	0.280	13.502	0.112	0.000	0.164	0.694	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	122	37	44	22	5342	22
N.S.	1	1.00	1.10	1.00	6.10	1.85	2.20	1.10	267.10	1.10
time (sec)	N/A	0.375	27.681	0.061	0.396	0.080	4.346	0.152	0.882	2.695

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	68	20	24	20	84	20
N.S.	1	1.00	1.11	1.00	3.78	1.11	1.33	1.11	4.67	1.11
time (sec)	N/A	0.293	16.443	0.040	0.193	0.088	2.788	0.125	0.710	2.691

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	115	0	0	148	0	0	201	0
N.S.	1	1.00	1.31	0.00	0.00	1.68	0.00	0.00	2.28	0.00
time (sec)	N/A	0.589	0.825	0.000	0.000	0.124	0.000	0.000	0.490	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	162	0	0	248	0	0	352	0
N.S.	1	1.00	1.07	0.00	0.00	1.63	0.00	0.00	2.32	0.00
time (sec)	N/A	0.647	1.155	0.000	0.000	0.094	0.000	0.000	0.568	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	228	0	0	345	0	0	505	0
N.S.	1	1.00	1.02	0.00	0.00	1.55	0.00	0.00	2.26	0.00
time (sec)	N/A	0.580	1.319	0.000	0.000	0.089	0.000	0.000	0.637	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	311	766	419	488	0	0	231	0
N.S.	1	1.00	2.34	5.76	3.15	3.67	0.00	0.00	1.74	0.00
time (sec)	N/A	0.520	0.139	0.166	0.174	0.132	0.000	0.000	0.643	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	172	465	240	303	0	0	164	0
N.S.	1	1.00	1.70	4.60	2.38	3.00	0.00	0.00	1.62	0.00
time (sec)	N/A	0.438	0.074	0.115	0.270	0.099	0.000	0.000	0.656	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	79	201	0	156	0	0	95	0
N.S.	1	1.00	1.05	2.68	0.00	2.08	0.00	0.00	1.27	0.00
time (sec)	N/A	0.337	0.014	0.080	0.000	0.101	0.000	0.000	0.623	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	78	20	15	20	62	20
N.S.	1	1.00	1.11	1.00	4.33	1.11	0.83	1.11	3.44	1.11
time (sec)	N/A	0.215	3.761	0.043	0.287	0.074	0.919	0.121	0.588	2.604

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	131	31	17	20	168	20
N.S.	1	1.00	1.11	1.00	7.28	1.72	0.94	1.11	9.33	1.11
time (sec)	N/A	0.215	24.738	0.040	0.188	0.090	2.977	0.174	0.610	2.662

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	843	1425	781	3239	0	0	0	0
N.S.	1	1.00	3.11	5.26	2.88	11.95	0.00	0.00	0.00	0.00
time (sec)	N/A	0.825	3.219	0.310	0.195	0.152	0.000	0.000	0.742	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	473	825	494	1854	0	0	1288	0
N.S.	1	1.00	2.26	3.95	2.36	8.87	0.00	0.00	6.16	0.00
time (sec)	N/A	0.906	4.585	0.262	0.180	0.123	0.000	0.000	0.679	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	218	318	244	851	0	0	592	0
N.S.	1	1.00	1.72	2.50	1.92	6.70	0.00	0.00	4.66	0.00
time (sec)	N/A	0.682	6.807	0.220	0.358	0.122	0.000	0.000	0.739	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	226	36	17	22	258	22
N.S.	1	1.00	1.10	1.00	11.30	1.80	0.85	1.10	12.90	1.10
time (sec)	N/A	0.396	29.278	0.123	0.411	0.087	1.443	0.157	0.853	2.601

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	375	47	19	22	803	22
N.S.	1	1.00	1.10	1.00	18.75	2.35	0.95	1.10	40.15	1.10
time (sec)	N/A	0.331	22.584	0.116	0.812	0.107	2.443	0.271	0.782	2.945

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	556	556	2043	2823	1531	11137	0	0	0	0
N.S.	1	1.00	3.67	5.08	2.75	20.03	0.00	0.00	0.00	0.00
time (sec)	N/A	1.693	7.900	0.538	0.442	0.279	0.000	0.000	16.487	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	585	1586	997	6356	0	0	0	0
N.S.	1	1.00	1.49	4.05	2.54	16.21	0.00	0.00	0.00	0.00
time (sec)	N/A	1.036	4.855	0.397	0.374	0.164	0.000	0.000	0.659	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	318	651	528	2907	0	0	0	0
N.S.	1	1.00	1.23	2.51	2.04	11.22	0.00	0.00	0.00	0.00
time (sec)	N/A	0.680	7.475	0.299	0.175	0.151	0.000	0.000	0.728	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	644	52	17	22	793	22
N.S.	1	1.00	1.10	1.00	32.20	2.60	0.85	1.10	39.65	1.10
time (sec)	N/A	0.241	36.609	0.316	1.492	0.097	1.920	0.231	0.905	2.440

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	1144	63	19	22	2588	22
N.S.	1	1.00	1.10	1.00	57.20	3.15	0.95	1.10	129.40	1.10
time (sec)	N/A	0.239	40.489	0.241	1.455	0.107	3.311	0.553	1.023	2.636

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	200	247	1158	521	730	0	0	566	0
N.S.	1	0.95	1.18	5.51	2.48	3.48	0.00	0.00	2.70	0.00
time (sec)	N/A	1.609	1.018	0.221	0.252	0.134	0.000	0.000	0.766	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	155	200	735	332	492	0	0	389	0
N.S.	1	0.99	1.28	4.71	2.13	3.15	0.00	0.00	2.49	0.00
time (sec)	N/A	1.253	0.687	0.158	0.380	0.104	0.000	0.000	0.695	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	112	152	357	0	300	0	0	214	0
N.S.	1	1.04	1.41	3.31	0.00	2.78	0.00	0.00	1.98	0.00
time (sec)	N/A	0.544	0.674	0.115	0.000	0.104	0.000	0.000	0.625	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	117	27	17	22	271	22
N.S.	1	1.00	1.10	1.00	5.85	1.35	0.85	1.10	13.55	1.10
time (sec)	N/A	0.248	4.937	0.062	0.502	0.094	1.196	0.113	0.657	2.443

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	202	51	19	22	937	22
N.S.	1	1.00	1.10	1.00	10.10	2.55	0.95	1.10	46.85	1.10
time (sec)	N/A	0.247	8.795	0.056	0.758	0.099	1.908	0.189	0.753	2.514

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	638	638	659	2444	1056	6171	0	0	0	0
N.S.	1	1.00	1.03	3.83	1.66	9.67	0.00	0.00	0.00	0.00
time (sec)	N/A	2.545	5.008	0.282	0.655	0.269	0.000	0.000	0.716	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	475	475	519	1470	751	3702	0	0	0	0
N.S.	1	1.00	1.09	3.09	1.58	7.79	0.00	0.00	0.00	0.00
time (sec)	N/A	2.909	3.207	0.213	1.031	0.166	0.000	0.000	0.572	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	214	244	620	0	1797	0	0	0	0
N.S.	1	1.09	1.24	3.16	0.00	9.17	0.00	0.00	0.00	0.00
time (sec)	N/A	0.839	2.519	0.136	0.000	0.159	0.000	0.000	0.621	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	472	55	19	22	2595	22
N.S.	1	1.00	1.10	1.00	23.60	2.75	0.95	1.10	129.75	1.10
time (sec)	N/A	0.245	29.894	0.072	1.724	0.093	1.820	0.155	0.853	2.828

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	789	96	20	22	8964	22
N.S.	1	1.00	1.10	1.00	39.45	4.80	1.00	1.10	448.20	1.10
time (sec)	N/A	0.244	25.677	0.066	4.820	0.096	3.321	0.313	0.824	3.384

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [11] had the largest ratio of [1.9166700000000010]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	C	9	8	1.63	10	0.800
2	C	8	7	1.68	10	0.700
3	C	7	6	1.58	8	0.750
4	N/A	3	0	1.00	10	0.000
5	N/A	3	0	1.00	10	0.000
6	C	13	12	1.51	12	1.000
7	C	12	11	1.48	12	0.917
8	C	8	8	1.13	10	0.800
9	N/A	3	0	1.00	12	0.000
10	N/A	3	0	1.00	12	0.000
11	C	24	23	1.49	12	1.917
12	C	20	19	1.47	12	1.583
13	C	15	14	1.37	10	1.400
14	N/A	3	0	1.00	12	0.000
15	N/A	3	0	1.00	12	0.000
16	A	9	9	1.06	20	0.450
17	A	7	7	1.04	20	0.350
18	A	5	5	1.05	18	0.278
19	C	12	12	1.00	20	0.600
20	C	12	12	1.06	20	0.600

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	C	14	14	1.04	20	0.700
22	A	3	3	1.00	20	0.150
23	A	3	3	1.00	20	0.150
24	A	3	3	1.00	18	0.167
25	A	3	3	1.00	20	0.150
26	A	3	3	1.00	20	0.150
27	A	3	3	1.00	20	0.150
28	A	3	3	1.00	20	0.150
29	A	3	3	0.99	18	0.167
30	A	3	3	1.00	20	0.150
31	A	3	3	1.00	20	0.150
32	N/A	2	0	1.00	20	0.000
33	N/A	2	0	1.00	18	0.000
34	A	3	3	1.00	20	0.150
35	A	3	3	1.00	20	0.150
36	A	3	3	1.00	20	0.150
37	A	3	3	1.00	18	0.167
38	A	3	3	1.00	18	0.167
39	A	3	3	1.00	16	0.188
40	N/A	2	0	1.00	18	0.000
41	N/A	2	0	1.00	18	0.000
42	A	3	3	1.00	20	0.150
43	A	3	3	1.00	20	0.150
44	A	3	3	1.00	18	0.167
45	N/A	2	0	1.00	20	0.000
46	N/A	2	0	1.00	20	0.000
47	A	3	3	1.00	20	0.150
48	A	3	3	1.00	20	0.150
49	A	3	3	1.00	18	0.167
50	N/A	2	0	1.00	20	0.000
51	N/A	2	0	1.00	20	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
52	A	9	8	0.95	20	0.400
53	A	8	7	0.99	20	0.350
54	A	7	6	1.04	18	0.333
55	N/A	2	0	1.00	20	0.000
56	N/A	2	0	1.00	20	0.000
57	A	3	3	1.00	20	0.150
58	A	3	3	1.00	20	0.150
59	A	10	9	1.09	18	0.500
60	N/A	2	0	1.00	20	0.000
61	N/A	2	0	1.00	20	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^3 \coth(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 87

$$\int x^3 \coth(a+bx)\,dx = -\frac{x^4}{4} + \frac{x^3 \log\left(1-e^{2(a+bx)}\right)}{b} + \frac{3x^2 \operatorname{PolyLog}\left(2, e^{2(a+bx)}\right)}{2b^2} - \frac{3x \operatorname{PolyLog}\left(3, e^{2(a+bx)}\right)}{2b^3} + \frac{3 \operatorname{PolyLog}\left(4, e^{2(a+bx)}\right)}{4b^4}$$

output
```
-1/4*x^4+x^3*ln(1-exp(2*b*x+2*a))/b+3/2*x^2*polylog(2,exp(2*b*x+2*a))/b^2-
3/2*x*polylog(3,exp(2*b*x+2*a))/b^3+3/4*polylog(4,exp(2*b*x+2*a))/b^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$\int x^3 \coth(a+bx)\,dx = -\frac{x^4}{4} + \frac{x^3 \log\left(1-e^{2a+2bx}\right)}{b} + \frac{3x^2 \operatorname{PolyLog}\left(2, e^{2a+2bx}\right)}{2b^2} - \frac{3x \operatorname{PolyLog}\left(3, e^{2a+2bx}\right)}{2b^3} + \frac{3 \operatorname{PolyLog}\left(4, e^{2a+2bx}\right)}{4b^4}$$

input
```
Integrate[x^3*Coth[a + b*x],x]
```

output
```
-1/4*x^4 + (x^3*Log[1 - E^(2*a + 2*b*x)])/b + (3*x^2*PolyLog[2, E^(2*a + 2
*b*x)])/(2*b^2) - (3*x*PolyLog[3, E^(2*a + 2*b*x)])/(2*b^3) + (3*PolyLog[4
, E^(2*a + 2*b*x)])/(4*b^4)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.63, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 26, 4201, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -ix^3 \tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 26$$

$$-i\int x^3 \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx$$

$$\Big\downarrow \; 4201$$

$$-i\left(2i\int \frac{e^{2a+2bx-i\pi}x^3}{1+e^{2a+2bx-i\pi}}dx-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \; 2620$$

$$-i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\int x^2\log\left(1+e^{2a+2bx-i\pi}\right)dx}{2b}\right)-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \; 3011$$

$$-i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}(2,-e^{2a+2bx-i\pi})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-e^{2a+2bx-i\pi})}{2b}\right)}{2b}\right)-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \; 7163$$

$$-i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)\right.$$

$$\Big\downarrow \text{2720}$$

$$-i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}}{b}-\frac{x^2\,\text{PolyLo}}{}\right.}{2b}\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$-i\left(2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)}{2b}-\frac{\text{PolyLog}\left(4,-e^{2a+2bx-i\pi}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)-\right.$$

input
```
Int[x^3*Coth[a + b*x],x]
```

output
```
(-I)*((-1/4*I)*x^4 + (2*I)*((x^3*Log[1 + E^(2*a - I*Pi + 2*b*x)])/(2*b) -
(3*(-1/2*(x^2*PolyLog[2, -E^(2*a - I*Pi + 2*b*x)])/b + ((x*PolyLog[3, -E^(
2*a - I*Pi + 2*b*x)])/(2*b) - PolyLog[4, -E^(2*a - I*Pi + 2*b*x)]/(4*b^2))
/b))/(2*b)))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(79) = 158.

Time = 0.22 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.30

method	result
risch	$-\frac{x^4}{4}-\frac{3a^4}{2b^4}+\frac{6\,\mathrm{polylog}(4,-\mathrm{e}^{bx+a})}{b^4}+\frac{6\,\mathrm{polylog}(4,\mathrm{e}^{bx+a})}{b^4}-\frac{2a^3x}{b^3}+\frac{\ln(1+\mathrm{e}^{bx+a})x^3}{b}+\frac{3\,\mathrm{polylog}(2,-\mathrm{e}^{bx+a})x^2}{b^2}-\frac{6\,\mathrm{pol}}{}$

input
```
int(x^3*coth(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^4-3/2/b^4*a^4+6/b^4*polylog(4,-exp(b*x+a))+6/b^4*polylog(4,exp(b*x+
a))-2/b^3*a^3*x+1/b*ln(1+exp(b*x+a))*x^3+3/b^2*polylog(2,-exp(b*x+a))*x^2-
6/b^3*polylog(3,-exp(b*x+a))*x+1/b*ln(1-exp(b*x+a))*x^3+3/b^2*polylog(2,ex
p(b*x+a))*x^2-6/b^3*polylog(3,exp(b*x+a))*x-1/b^4*a^3*ln(exp(b*x+a)-1)+2/b
^4*a^3*ln(exp(b*x+a))+1/b^4*ln(1-exp(b*x+a))*a^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(78) = 156.

Time = 0.09 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.48

$$\int x^3\coth(a+bx)\,dx =$$
$$-\frac{b^4x^4-4\,b^3x^3\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)-12\,b^2x^2\mathrm{Li}_2(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))-12}{}$$

input
```
integrate(x^3*coth(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(b^4*x^4 - 4*b^3*x^3*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 12*b^2*
x^2*dilog(cosh(b*x + a) + sinh(b*x + a)) - 12*b^2*x^2*dilog(-cosh(b*x + a)
 - sinh(b*x + a)) + 4*a^3*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 24*b*x*
polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 24*b*x*polylog(3, -cosh(b*x +
a) - sinh(b*x + a)) - 4*(b^3*x^3 + a^3)*log(-cosh(b*x + a) - sinh(b*x + a)
 + 1) - 24*polylog(4, cosh(b*x + a) + sinh(b*x + a)) - 24*polylog(4, -cosh
(b*x + a) - sinh(b*x + a)))/b^4
```

Sympy [F]

$$\int x^3 \coth(a+bx)\,dx = \int x^3 \coth(a+bx)\,dx$$

input
```
integrate(x**3*coth(b*x+a),x)
```

output
```
Integral(x**3*coth(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(78) = 156$.

Time = 0.05 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.95

$$\int x^3 \coth(a+bx)\,dx = \frac{1}{4}x^4 \coth(bx+a) - \frac{1}{2}\left(\frac{x^4}{be^{(2\,bx+2\,a)}-b} + \frac{x^4}{b} - \frac{2\left(b^3x^3\log\left(e^{(bx+a)}+1\right) + 3\,b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 6\,bx\mathrm{Li}_3(-e^{(bx+a)}) + 6\,\mathrm{Li}_4(\right.}{b^5}$$

input
```
integrate(x^3*coth(b*x+a),x, algorithm="maxima")
```

output
```
1/4*x^4*coth(b*x + a) - 1/2*(x^4/(b*e^(2*b*x + 2*a) - b) + x^4/b - 2*(b^3*
x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3
, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^5 - 2*(b^3*x^3*log(-e^(b*x
 + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x + a))
+ 6*polylog(4, e^(b*x + a)))/b^5)*b
```

Giac [F]

$$\int x^3 \coth(a+bx)\,dx = \int x^3 \coth{(bx+a)}\,dx$$

input
```
integrate(x^3*coth(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^3*coth(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \coth(a+bx)\,dx = \int x^3\,\mathrm{coth}\left(a+b\,x\right)\,dx$$

input
```
int(x^3*coth(a + b*x),x)
```

output
```
int(x^3*coth(a + b*x), x)
```

Reduce [F]

$$\int x^3 \coth(a+bx)\,dx = 2e^{2a}\left(\int \frac{e^{2bx}x^3}{e^{2bx+2a}-1}dx\right) - \frac{x^4}{4}$$

input
```
int(x^3*coth(b*x+a),x)
```

output
```
(8*e**(2*a)*int((e**(2*b*x)*x**3)/(e**(2*a + 2*b*x) - 1),x) - x**4)/4
```

3.2 $\int x^2 \coth(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 63

$$\int x^2 \coth(a+bx)\,dx = -\frac{x^3}{3}+\frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{x\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2}-\frac{\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}$$

output
```
-1/3*x^3+x^2*ln(1-exp(2*b*x+2*a))/b+x*polylog(2,exp(2*b*x+2*a))/b^2-1/2*po
lylog(3,exp(2*b*x+2*a))/b^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int x^2 \coth(a+bx)\,dx = -\frac{x^3}{3}+\frac{x^2\log\left(1-e^{2a+2bx}\right)}{b}+\frac{x\,\text{PolyLog}\left(2,e^{2a+2bx}\right)}{b^2}-\frac{\text{PolyLog}\left(3,e^{2a+2bx}\right)}{2b^3}$$

input
```
Integrate[x^2*Coth[a + b*x],x]
```

output
```
-1/3*x^3 + (x^2*Log[1 - E^(2*a + 2*b*x)])/b + (x*PolyLog[2, E^(2*a + 2*b*x
)])/b^2 - PolyLog[3, E^(2*a + 2*b*x)]/(2*b^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.68, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -ix^2 \tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int x^2 \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx$$

$$\Big\downarrow \text{4201}$$

$$-i\left(2i\int \frac{e^{2a+2bx-i\pi}x^2}{1+e^{2a+2bx-i\pi}}dx-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{2620}$$

$$-i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int x\log\left(1+e^{2a+2bx-i\pi}\right)dx}{b}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{3011}$$

$$-i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int \text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{2720}$$

$$-i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\,\mathrm{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}-\frac{x\,\mathrm{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{7143}$$

$$-i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\mathrm{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)}{4b^2}-\frac{x\,\mathrm{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)$$

input
```
Int[x^2*Coth[a + b*x],x]
```

output
```
(-I)*((-1/3*I)*x^3 + (2*I)*((x^2*Log[1 + E^(2*a - I*Pi + 2*b*x)])/(2*b) -
(-1/2*(x*PolyLog[2, -E^(2*a - I*Pi + 2*b*x)])/b + PolyLog[3, -E^(2*a - I*P
i + 2*b*x)]/(4*b^2))/b))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(59) = 118$.

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.63

method	result
risch	$-\frac{x^3}{3}+\frac{a^2\ln\left(\mathrm{e}^{bx+a}-1\right)}{b^3}-\frac{2a^2\ln\left(\mathrm{e}^{bx+a}\right)}{b^3}-\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)a^2}{b^3}+\frac{4a^3}{3b^3}+\frac{2a^2x}{b^2}+\frac{\ln\left(1+\mathrm{e}^{bx+a}\right)x^2}{b}+\frac{2\,\mathrm{polylog}\left(2,-\mathrm{e}^{bx+a}\right)x}{b^2}$

input
```
int(x^2*coth(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3+1/b^3*a^2*ln(exp(b*x+a)-1)-2/b^3*a^2*ln(exp(b*x+a))-1/b^3*ln(1-ex
p(b*x+a))*a^2+4/3/b^3*a^3+2/b^2*a^2*x+1/b*ln(1+exp(b*x+a))*x^2+2/b^2*polyl
og(2,-exp(b*x+a))*x+1/b*ln(1-exp(b*x+a))*x^2+2/b^2*polylog(2,exp(b*x+a))*x
-2/b^3*polylog(3,-exp(b*x+a))-2/b^3*polylog(3,exp(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(58) = 116$.

Time = 0.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.67

$$\int x^2 \coth(a + bx)\, dx = -\frac{b^3x^3 - 3\,b^2x^2 \log\left(\cosh\left(bx + a\right) + \sinh\left(bx + a\right) + 1\right) - 6\,bx\mathrm{Li}_2(\cosh\left(bx + a\right) + \sinh\left(bx + a\right)) - 6\,bx\mathrm{L}$$

input
```
integrate(x^2*coth(b*x+a),x, algorithm="fricas")
```

output
```
-1/3*(b^3*x^3 - 3*b^2*x^2*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 6*b*x*d
ilog(cosh(b*x + a) + sinh(b*x + a)) - 6*b*x*dilog(-cosh(b*x + a) - sinh(b*
x + a)) - 3*a^2*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 3*(b^2*x^2 - a^2)
*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 6*polylog(3, cosh(b*x + a) + si
nh(b*x + a)) + 6*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/b^3
```

Sympy [F]

$$\int x^2 \coth(a + bx)\, dx = \int x^2 \coth\left(a + bx\right) dx$$

input
```
integrate(x**2*coth(b*x+a),x)
```

output
```
Integral(x**2*coth(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(58) = 116.

Time = 0.05 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.19

$$\int x^2 \coth(a+bx)\,dx = \frac{1}{3}x^3\coth(bx+a)$$
$$-\frac{1}{3}\left(\frac{2x^3}{be^{(2bx+2a)}-b}+\frac{2x^3}{b}-\frac{3\left(b^2x^2\log\left(e^{(bx+a)}+1\right)+2bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3\left(-e^{(bx+a)}\right)\right)}{b^4}-\frac{3\left(b^2x^2\right.}{}$$

input
```
integrate(x^2*coth(b*x+a),x, algorithm="maxima")
```

output
```
1/3*x^3*coth(b*x + a) - 1/3*(2*x^3/(b*e^(2*b*x + 2*a) - b) + 2*x^3/b - 3*(
b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e
^(b*x + a)))/b^4 - 3*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x +
 a)) - 2*polylog(3, e^(b*x + a)))/b^4)*b
```

Giac [F]

$$\int x^2 \coth(a+bx)\,dx = \int x^2 \coth(bx+a)\,dx$$

input
```
integrate(x^2*coth(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*coth(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth(a+bx)\,dx = \int x^2 \coth(a+b\,x)\ dx$$

input
```
int(x^2*coth(a + b*x),x)
```

output
```
int(x^2*coth(a + b*x), x)
```

Reduce [F]

$$\int x^2 \coth(a+bx)\,dx = 2e^{2a}\left(\int \frac{e^{2bx}x^2}{e^{2bx+2a}-1}dx\right) - \frac{x^3}{3}$$

input
```
int(x^2*coth(b*x+a),x)
```

output
```
(6*e**(2*a)*int((e**(2*b*x)*x**2)/(e**(2*a + 2*b*x) - 1),x) - x**3)/3
```

3.3 $\int x\coth(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 45

$$\int x\coth(a+bx)\,dx=-\frac{x^2}{2}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}$$

output
```
-1/2*x^2+x*ln(1-exp(2*b*x+2*a))/b+1/2*polylog(2,exp(2*b*x+2*a))/b^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int x\coth(a+bx)\,dx=-\frac{x^2}{2}+\frac{x\log\left(1-e^{2a+2bx}\right)}{b}+\frac{\text{PolyLog}\left(2,e^{2a+2bx}\right)}{2b^2}$$

input
```
Integrate[x*Coth[a + b*x],x]
```

output
```
-1/2*x^2 + (x*Log[1 - E^(2*a + 2*b*x)])/b + PolyLog[2, E^(2*a + 2*b*x)]/(2
*b^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.58, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -ix\tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx$$

$$\Big\downarrow \text{4201}$$

$$-i\left(2i\int \frac{e^{2a+2bx-i\pi}x}{1+e^{2a+2bx-i\pi}}dx-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$-i\left(2i\left(\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int \log\left(1+e^{2a+2bx-i\pi}\right)dx}{2b}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$-i\left(2i\left(\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int e^{-2a-2bx+i\pi}\log\left(1+e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2838}$$

$$-i\left(2i\left(\frac{\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{4b^2}+\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}\right)-\frac{ix^2}{2}\right)$$

input
```
Int[x*Coth[a + b*x],x]
```

output
```
(-I)*((-1/2*I)*x^2 + (2*I)*((x*Log[1 + E^(2*a - I*Pi + 2*b*x)])/(2*b) + Po
lyLog[2, -E^(2*a - I*Pi + 2*b*x)]/(4*b^2)))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(41) = 82.

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.71

method	result
risch	$-\frac{x^2}{2}-\frac{2ax}{b}-\frac{a^2}{b^2}+\frac{\ln(1+e^{bx+a})x}{b}+\frac{\text{polylog}(2,-e^{bx+a})}{b^2}+\frac{\ln(1-e^{bx+a})x}{b}+\frac{\ln(1-e^{bx+a})a}{b^2}+\frac{\text{polylog}(2,e^{bx+a})}{b^2}-a$

input
```
int(x*coth(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2-2/b*a*x-1/b^2*a^2+1/b*ln(1+exp(b*x+a))*x+1/b^2*polylog(2,-exp(b*x
+a))+1/b*ln(1-exp(b*x+a))*x+1/b^2*ln(1-exp(b*x+a))*a+1/b^2*polylog(2,exp(b
*x+a))-1/b^2*a*ln(exp(b*x+a)-1)+2/b^2*a*ln(exp(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(40) = 80.

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.49

$$\int x\coth(a+bx)\,dx = -\frac{b^2x^2 - 2\,bx\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+2\,a\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-1\right)-2\,(b}{}$$

input
```
integrate(x*coth(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(b^2*x^2 - 2*b*x*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*a*log(cos
h(b*x + a) + sinh(b*x + a) - 1) - 2*(b*x + a)*log(-cosh(b*x + a) - sinh(b*
x + a) + 1) - 2*dilog(cosh(b*x + a) + sinh(b*x + a)) - 2*dilog(-cosh(b*x +
 a) - sinh(b*x + a)))/b^2
```

Sympy [F]

$$\int x\coth(a+bx)\,dx=\int x\coth(a+bx)\,dx$$

input
```
integrate(x*coth(b*x+a),x)
```

output
```
Integral(x*coth(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(40) = 80$.

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.18

$$\int x\coth(a+bx)\,dx=\frac{1}{2}x^2\coth(bx+a)-b\left(\frac{x^2}{be^{(2\,bx+2\,a)}-b}+\frac{x^2}{b}-\frac{bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)}{b^3}-\frac{bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)}{b^3}\right)$$

input
```
integrate(x*coth(b*x+a),x, algorithm="maxima")
```

output
```
1/2*x^2*coth(b*x + a) - b*(x^2/(b*e^(2*b*x + 2*a) - b) + x^2/b - (b*x*log(
e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^3 - (b*x*log(-e^(b*x + a) + 1) +
 dilog(e^(b*x + a)))/b^3)
```

Giac [F]

$$\int x\coth(a+bx)\,dx=\int x\coth(bx+a)\,dx$$

input
```
integrate(x*coth(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*coth(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\coth(a+bx)\,dx=\int x\coth(a+b\,x)\;dx$$

input
```
int(x*coth(a + b*x),x)
```

output
```
int(x*coth(a + b*x), x)
```

Reduce [F]

$$\int x\coth(a+bx)\,dx=2e^{2a}\left(\int\frac{e^{2bx}x}{e^{2bx+2a}-1}dx\right)-\frac{x^2}{2}$$

input
```
int(x*coth(b*x+a),x)
```

output
```
(4*e**(2*a)*int((e**(2*b*x)*x)/(e**(2*a + 2*b*x) - 1),x) - x**2)/2
```

3.4 $\int \frac{\coth(a+bx)}{x} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\coth(a+bx)}{x} \, dx = \text{Int}\left(\frac{\coth(a+bx)}{x}, x\right)$$

output
```
Defer(Int)(coth(b*x+a)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\coth(a+bx)}{x} \, dx = \int \frac{\coth(a+bx)}{x} \, dx$$

input
```
Integrate[Coth[a + b*x]/x,x]
```

output
```
Integrate[Coth[a + b*x]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(a+bx)}{x}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i\tan\left(ia+ibx+\frac{\pi}{2}\right)}{x}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)}{x}dx$$

$$\Big\downarrow \; 4222$$

$$\int \frac{i\tan\left(\frac{1}{2}(-\pi-2ia)-ibx\right)}{x}dx$$

input
```
Int[Coth[a + b*x]/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\coth\left(bx+a\right)}{x}dx$$

input
```
int(coth(b*x+a)/x,x)
```

output
```
int(coth(b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\coth(a+bx)}{x}\,dx = \int \frac{\coth(bx+a)}{x}\,dx$$

input
```
integrate(coth(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(coth(b*x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\coth(a+bx)}{x}\,dx = \int \frac{\coth(a+bx)}{x}\,dx$$

input
```
integrate(coth(b*x+a)/x,x)
```

output
```
Integral(coth(a + b*x)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 3.50

$$\int \frac{\coth(a+bx)}{x}\,dx = \int \frac{\coth(bx+a)}{x}\,dx$$

input
```
integrate(coth(b*x+a)/x,x, algorithm="maxima")
```

output
```
-integrate(1/(x*e^(b*x + a) + x), x) + integrate(1/(x*e^(b*x + a) - x), x)
 + log(x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\coth(a+bx)}{x}\,dx = \int \frac{\coth(bx+a)}{x}\,dx$$

input
```
integrate(coth(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(coth(b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\coth(a+bx)}{x}\,dx = \int \frac{\coth(a+b\,x)}{x}\,dx$$

input
```
int(coth(a + b*x)/x,x)
```

output
```
int(coth(a + b*x)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.50

$$\int \frac{\coth(a+bx)}{x}\,dx = 2\left(\int \frac{1}{e^{2bx+2a}x - x}dx\right) + \log(x)$$

input
```
int(coth(b*x+a)/x,x)
```

output
```
2*int(1/(e**(2*a + 2*b*x)*x - x),x) + log(x)
```

3.5 $\int \frac{\coth(a+bx)}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\coth(a+bx)}{x^2}\,dx = \text{Int}\left(\frac{\coth(a+bx)}{x^2},x\right)$$

output
```
Defer(Int)(coth(b*x+a)/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\coth(a+bx)}{x^2}\,dx = \int \frac{\coth(a+bx)}{x^2}\,dx$$

input
```
Integrate[Coth[a + b*x]/x^2,x]
```

output
```
Integrate[Coth[a + b*x]/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(a+bx)}{x^2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i \tan\left(ia+ibx+\frac{\pi}{2}\right)}{x^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)}{x^2}dx$$

$$\Big\downarrow \text{4222}$$

$$\int \frac{i\tan\left(\frac{1}{2}(-\pi-2ia)-ibx\right)}{x^2}dx$$

input
```
Int[Coth[a + b*x]/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\coth(bx+a)}{x^2}dx$$

input
```
int(coth(b*x+a)/x^2,x)
```

output
```
int(coth(b*x+a)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\coth(a+bx)}{x^2}\,dx = \int \frac{\coth(bx+a)}{x^2}\,dx$$

input
```
integrate(coth(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
integral(coth(b*x + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\coth(a+bx)}{x^2}\,dx = \int \frac{\coth(a+bx)}{x^2}\,dx$$

input
```
integrate(coth(b*x+a)/x**2,x)
```

output
```
Integral(coth(a + b*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 4.60

$$\int \frac{\coth(a+bx)}{x^2}\,dx = \int \frac{\coth\left(bx+a\right)}{x^2}\,dx$$

input
```
integrate(coth(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
-1/x - integrate(1/(x^2*e^(b*x + a) + x^2), x) + integrate(1/(x^2*e^(b*x +
 a) - x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\coth(a+bx)}{x^2}\,dx = \int \frac{\coth\left(bx+a\right)}{x^2}\,dx$$

input
```
integrate(coth(b*x+a)/x^2,x, algorithm="giac")
```

output
```
integrate(coth(b*x + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\coth(a+bx)}{x^2}\,dx = \int \frac{\coth(a+b\,x)}{x^2}\,dx$$

input
```
int(coth(a + b*x)/x^2,x)
```

output
```
int(coth(a + b*x)/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 45, normalized size of antiderivative = 4.50

$$\int \frac{\coth(a+bx)}{x^2}\,dx = \frac{2e^{2a}\left(\int \frac{e^{2bx}}{e^{2bx+2a}x^2-x^2}dx\right)x+1}{x}$$

input
```
int(coth(b*x+a)/x^2,x)
```

output
```
(2*e**(2*a)*int(e**(2*b*x)/(e**(2*a + 2*b*x)*x**2 - x**2),x)*x + 1)/x
```

3.6 $\int x^3 \coth^2(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 87

$$\int x^3 \coth^2(a+bx)\,dx = -\frac{x^3}{b}+\frac{x^4}{4}-\frac{x^3\coth(a+bx)}{b}+\frac{3x^2\log\left(1-e^{2(a+bx)}\right)}{b^2} + \frac{3x\operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}-\frac{3\operatorname{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^4}$$

output
```
-x^3/b+1/4*x^4-x^3*coth(b*x+a)/b+3*x^2*ln(1-exp(2*b*x+2*a))/b^2+3*x*polylo
g(2,exp(2*b*x+2*a))/b^3-3/2*polylog(3,exp(2*b*x+2*a))/b^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 222 vs. 2(87) = 174.

Time = 0.61 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.55

$$\int x^3 \coth^2(a+bx)\,dx = \frac{x^4}{4} - \frac{e^{2a}\left(2b^3e^{-2a}x^3-3b^2\left(1-e^{-2a}\right)x^2\log\left(1-e^{-a-bx}\right)-3b^2\left(1-e^{-2a}\right)x^2\log\left(1+e^{-a-bx}\right)+6b\left(1-e^{-2a}\right)\cdots\right)}{\cdots} + \frac{x^3\operatorname{csch}(a)\operatorname{csch}(a+bx)\sinh(bx)}{b}$$

input
```
Integrate[x^3*Coth[a + b*x]^2,x]
```

output
```
x^4/4 - (E^(2*a)*((2*b^3*x^3)/E^(2*a) - 3*b^2*(1 - E^(-2*a))*x^2*Log[1 - E
^(-a - b*x)] - 3*b^2*(1 - E^(-2*a))*x^2*Log[1 + E^(-a - b*x)] + 6*b*(1 - E
^(-2*a))*x*PolyLog[2, -E^(-a - b*x)] + 6*b*(1 - E^(-2*a))*x*PolyLog[2, E^(
-a - b*x)] + 6*(1 - E^(-2*a))*PolyLog[3, -E^(-a - b*x)] + 6*(1 - E^(-2*a))
*PolyLog[3, E^(-a - b*x)]))/(b^4*(-1 + E^(2*a))) + (x^3*Csch[a]*Csch[a + b
*x]*Sinh[b*x])/b
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.51, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 25, 4203, 15, 26, 3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x^3 \tan\left(ia+ibx+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{25}$$

$$-\int x^3 \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2 dx$$

$$\Big\downarrow \text{4203}$$

$$-\frac{3i\int ix^2\coth(a+bx)dx}{b}+\int x^3dx-\frac{x^3\coth(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{3i\int ix^2\coth(a+bx)dx}{b}-\frac{x^3\coth(a+bx)}{b}+\frac{x^4}{4}$$

$$\Big\downarrow \text{26}$$

$$\frac{3\int x^2\coth(a+bx)dx}{b}-\frac{x^3\coth(a+bx)}{b}+\frac{x^4}{4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int -ix^2\tan\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}-\frac{x^3\coth(a+bx)}{b}+\frac{x^4}{4}$$

$$\Big\downarrow \text{26}$$

$$-\frac{3i\int x^2\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx}{b}-\frac{x^3\coth(a+bx)}{b}+\frac{x^4}{4}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{3i\left(2i\int\frac{e^{2a+2bx-i\pi}x^2}{1+e^{2a+2bx-i\pi}}dx-\frac{ix^3}{3}\right)}{b}-\frac{x^3\coth(a+bx)}{b}+\frac{x^4}{4}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{3i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int x\log\left(1+e^{2a+2bx-i\pi}\right)dx}{b}\right)-\frac{ix^3}{3}\right)}{b}-\frac{x^3\coth(a+bx)}{b}+\frac{x^4}{4}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{3i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int \text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)}{b}-\frac{x^3\coth(a+bx)}{b}+\frac{x^4}{4}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{3i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)}{b}-\frac{x^3\coth(a+bx)}{b}+\frac{x^4}{4}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{3i\left(2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)-\frac{ix^3}{3}\right)}{b}-\frac{x^3\coth(a+bx)}{b}+\frac{x^4}{4}$$

input
```
Int[x^3*Coth[a + b*x]^2,x]
```

output
```
x^4/4 - (x^3*Coth[a + b*x])/b - ((3*I)*((-1/3*I)*x^3 + (2*I)*((x^2*Log[1 +
 E^(2*a - I*Pi + 2*b*x)])/(2*b) - (-1/2*(x*PolyLog[2, -E^(2*a - I*Pi + 2*b
*x)])/b + PolyLog[3, -E^(2*a - I*Pi + 2*b*x)]/(4*b^2))/b)))/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(83) = 166$.

Time = 0.06 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.28

method	result
risch	$\frac{x^4}{4} - \frac{2x^3}{b(e^{2bx+2a}-1)} + \frac{3a^2\ln(e^{bx+a}-1)}{b^4} - \frac{6a^2\ln(e^{bx+a})}{b^4} - \frac{2x^3}{b} - \frac{3\ln(1-e^{bx+a})a^2}{b^4} + \frac{4a^3}{b^4} + \frac{6a^2x}{b^3} + \frac{3\ln(1+e^{bx+a})}{b^2}$

input
```
int(x^3*coth(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4-2*x^3/b/(exp(2*b*x+2*a)-1)+3/b^4*a^2*ln(exp(b*x+a)-1)-6/b^4*a^2*ln
(exp(b*x+a))-2*x^3/b-3/b^4*ln(1-exp(b*x+a))*a^2+4/b^4*a^3+6/b^3*a^2*x+3/b^
2*ln(1+exp(b*x+a))*x^2+6/b^3*polylog(2,-exp(b*x+a))*x+3/b^2*ln(1-exp(b*x+a
))*x^2+6/b^3*polylog(2,exp(b*x+a))*x-6/b^4*polylog(3,-exp(b*x+a))-6/b^4*po
lylog(3,exp(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(82) = 164$.

Time = 0.10 (sec) , antiderivative size = 632, normalized size of antiderivative = 7.26

$$\int x^3 \coth^2(a + bx)\, dx = \text{Too large to display}$$

input
```
integrate(x^3*coth(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(b^4*x^4 - 8*a^3 - (b^4*x^4 - 8*b^3*x^3 - 8*a^3)*cosh(b*x + a)^2 - 2*
(b^4*x^4 - 8*b^3*x^3 - 8*a^3)*cosh(b*x + a)*sinh(b*x + a) - (b^4*x^4 - 8*b
^3*x^3 - 8*a^3)*sinh(b*x + a)^2 - 24*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x
 + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*dilog(cosh(b*x + a) + sin
h(b*x + a)) - 24*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a)
+ b*x*sinh(b*x + a)^2 - b*x)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 12*(b
^2*x^2*cosh(b*x + a)^2 + 2*b^2*x^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*s
inh(b*x + a)^2 - b^2*x^2)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 12*(a^2
*cosh(b*x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x + a)^2
 - a^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 12*(b^2*x^2 - (b^2*x^2 -
a^2)*cosh(b*x + a)^2 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a) - (b^
2*x^2 - a^2)*sinh(b*x + a)^2 - a^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1
) + 24*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
- 1)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 24*(cosh(b*x + a)^2 + 2*c
osh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*polylog(3, -cosh(b*x + a
) - sinh(b*x + a)))/(b^4*cosh(b*x + a)^2 + 2*b^4*cosh(b*x + a)*sinh(b*x +
a) + b^4*sinh(b*x + a)^2 - b^4)
```

Sympy [F]

$$\int x^3 \coth^2(a+bx)\,dx = \int x^3 \coth^2{(a+bx)}\,dx$$

input
```
integrate(x**3*coth(b*x+a)**2,x)
```

output
```
Integral(x**3*coth(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.68

$$\begin{aligned}\int x^3 \coth^2(a+bx)\,dx &= -\frac{2\,x^3}{b} + \frac{bx^4e^{(2\,bx+2\,a)} - bx^4 - 8\,x^3}{4\,(be^{(2\,bx+2\,a)} - b)} \\ &\quad + \frac{3\left(b^2x^2\log\left(e^{(bx+a)}+1\right) + 2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 2\,\mathrm{Li}_3(-e^{(bx+a)})\right)}{b^4} \\ &\quad + \frac{3\left(b^2x^2\log\left(-e^{(bx+a)}+1\right) + 2\,bx\mathrm{Li}_2\left(e^{(bx+a)}\right) - 2\,\mathrm{Li}_3(e^{(bx+a)})\right)}{b^4}\end{aligned}$$

input
```
integrate(x^3*coth(b*x+a)^2,x, algorithm="maxima")
```

output
```
-2*x^3/b + 1/4*(b*x^4*e^(2*b*x + 2*a) - b*x^4 - 8*x^3)/(b*e^(2*b*x + 2*a)
- b) + 3*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*pol
ylog(3, -e^(b*x + a)))/b^4 + 3*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilo
g(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^4
```

Giac [F]

$$\int x^3 \coth^2(a+bx)\,dx = \int x^3 \coth{(bx+a)}^2\, dx$$

input
```
integrate(x^3*coth(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^3*coth(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \coth^2(a+bx)\,dx = \int x^3 \coth(a+b\,x)^2\,dx$$

input
```
int(x^3*coth(a + b*x)^2,x)
```

output
```
int(x^3*coth(a + b*x)^2, x)
```

Reduce [F]

$$\int x^3 \coth^2(a+bx)\,dx$$
$$= \frac{-24e^{2bx+2a}\left(\int \frac{x^2}{e^{4bx+4a}-2e^{2bx+2a}+1}dx\right)b^3 - 24e^{2bx+2a}\left(\int \frac{x}{e^{4bx+4a}-2e^{2bx+2a}+1}dx\right)b^2 + 6e^{2bx+2a}\log\left(e^{bx+a}-1\right) +}{}$$

input
```
int(x^3*coth(b*x+a)^2,x)
```

output
```
( - 24*e**(2*a + 2*b*x)*int(x**2/(e**(4*a + 4*b*x) - 2*e**(2*a + 2*b*x) +
1),x)*b**3 - 24*e**(2*a + 2*b*x)*int(x/(e**(4*a + 4*b*x) - 2*e**(2*a + 2*b
*x) + 1),x)*b**2 + 6*e**(2*a + 2*b*x)*log(e**(a + b*x) - 1) + 6*e**(2*a +
2*b*x)*log(e**(a + b*x) + 1) + e**(2*a + 2*b*x)*b**4*x**4 - 12*e**(2*a + 2
*b*x)*b*x + 24*int(x**2/(e**(4*a + 4*b*x) - 2*e**(2*a + 2*b*x) + 1),x)*b**
3 + 24*int(x/(e**(4*a + 4*b*x) - 2*e**(2*a + 2*b*x) + 1),x)*b**2 - 6*log(e
**(a + b*x) - 1) - 6*log(e**(a + b*x) + 1) - b**4*x**4 - 8*b**3*x**3 - 12*
b**2*x**2)/(4*b**4*(e**(2*a + 2*b*x) - 1))
```

3.7 $\int x^2 \coth^2(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 65

$$\int x^2 \coth^2(a+bx)\,dx = -\frac{x^2}{b}+\frac{x^3}{3}-\frac{x^2\coth(a+bx)}{b}+\frac{2x\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{\operatorname{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}$$

output
```
-x^2/b+1/3*x^3-x^2*coth(b*x+a)/b+2*x*ln(1-exp(2*b*x+2*a))/b^2+polylog(2,ex
p(2*b*x+2*a))/b^3
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.80

$$\int x^2 \coth^2(a+bx)\,dx = -\frac{2\operatorname{PolyLog}\left(2,-e^{-a-bx}\right)}{b^3}-\frac{2\operatorname{PolyLog}\left(2,e^{-a-bx}\right)}{b^3}+\frac{1}{3}x\left(\frac{6x}{b-be^{2a}}+x^2+\frac{6\log\left(1-e^{-a-bx}\right)}{b^2}+\frac{6\log\left(1+e^{-a-bx}\right)}{b^2}+\frac{3x\operatorname{csch}(a)\operatorname{csch}(a+bx)\sinh(bx)}{b}\right)$$

input
```
Integrate[x^2*Coth[a + b*x]^2,x]
```

output
```
(-2*PolyLog[2, -E^(-a - b*x)])/b^3 - (2*PolyLog[2, E^(-a - b*x)])/b^3 + (x
*((6*x)/(b - b*E^(2*a)) + x^2 + (6*Log[1 - E^(-a - b*x)])/b^2 + (6*Log[1 +
 E^(-a - b*x)])/b^2 + (3*x*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b))/3
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.48, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 25, 4203, 15, 26, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x^2 \tan\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int x^2 \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx$$

$$\Big\downarrow \text{4203}$$

$$-\frac{2i\int ix\coth(a+bx)dx}{b}+\int x^2dx-\frac{x^2\coth(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2i\int ix\coth(a+bx)dx}{b}-\frac{x^2\coth(a+bx)}{b}+\frac{x^3}{3}$$

$$\Big\downarrow \text{26}$$

$$\frac{2\int x\coth(a+bx)dx}{b}-\frac{x^2\coth(a+bx)}{b}+\frac{x^3}{3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int -ix\tan\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}-\frac{x^2\coth(a+bx)}{b}+\frac{x^3}{3}$$

$$\Big\downarrow \text{26}$$

$$-\frac{2i\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx}{b}-\frac{x^2\coth(a+bx)}{b}+\frac{x^3}{3}$$

$$\Big\downarrow \text{4201}$$

$$-\frac{2i\left(2i\int \frac{e^{2a+2bx-i\pi}x}{1+e^{2a+2bx-i\pi}}dx-\frac{ix^2}{2}\right)}{b}-\frac{x^2\coth(a+bx)}{b}+\frac{x^3}{3}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{2i\left(2i\left(\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int \log\left(1+e^{2a+2bx-i\pi}\right)dx}{2b}\right)-\frac{ix^2}{2}\right)}{b}-\frac{x^2\coth(a+bx)}{b}+\frac{x^3}{3}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{2i\left(2i\left(\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int e^{-2a-2bx+i\pi}\log\left(1+e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}\right)-\frac{ix^2}{2}\right)}{b}-\frac{x^2\coth(a+bx)}{b}+\frac{x^3}{3}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{2i\left(2i\left(\frac{\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{4b^2}+\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}\right)-\frac{ix^2}{2}\right)}{b}-\frac{x^2\coth(a+bx)}{b}+\frac{x^3}{3}$$

input
```
Int[x^2*Coth[a + b*x]^2,x]
```

output
```
x^3/3 - (x^2*Coth[a + b*x])/b - ((2*I)*((-1/2*I)*x^2 + (2*I)*((x*Log[1 + E
^(2*a - I*Pi + 2*b*x)])/(2*b) + PolyLog[2, -E^(2*a - I*Pi + 2*b*x)]/(4*b^2
))))/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(63) = 126.

Time = 0.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.40

method	result
risch	$\frac{x^3}{3} - \frac{2x^2}{b(e^{2bx+2a}-1)} - \frac{2x^2}{b} - \frac{4ax}{b^2} - \frac{2a^2}{b^3} + \frac{2\ln(1+e^{bx+a})x}{b^2} + \frac{2\operatorname{polylog}(2,-e^{bx+a})}{b^3} + \frac{2\ln(1-e^{bx+a})x}{b^2} + \frac{2\ln(1-e^{bx+}}{b^3}$

input
```
int(x^2*coth(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3-2*x^2/b/(exp(2*b*x+2*a)-1)-2*x^2/b-4/b^2*a*x-2/b^3*a^2+2/b^2*ln(1+
exp(b*x+a))*x+2/b^3*polylog(2,-exp(b*x+a))+2/b^2*ln(1-exp(b*x+a))*x+2/b^3*
ln(1-exp(b*x+a))*a+2/b^3*polylog(2,exp(b*x+a))-2/b^3*a*ln(exp(b*x+a)-1)+4/
b^3*a*ln(exp(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. 2(62) = 124.

Time = 0.10 (sec) , antiderivative size = 453, normalized size of antiderivative = 6.97

$$\int x^2 \coth^2(a+bx)\,dx = -\frac{b^3x^3 - (b^3x^3 - 6\,b^2x^2 + 6\,a^2)\cosh(bx+a)^2 - 2\,(b^3x^3 - 6\,b^2x^2 + 6\,a^2)\cosh(bx+a)\sinh(bx+a) - (b}{}$$

input
```
integrate(x^2*coth(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/3*(b^3*x^3 - (b^3*x^3 - 6*b^2*x^2 + 6*a^2)*cosh(b*x + a)^2 - 2*(b^3*x^3
 - 6*b^2*x^2 + 6*a^2)*cosh(b*x + a)*sinh(b*x + a) - (b^3*x^3 - 6*b^2*x^2 +
 6*a^2)*sinh(b*x + a)^2 + 6*a^2 - 6*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sin
h(b*x + a) + sinh(b*x + a)^2 - 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 6
*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*d
ilog(-cosh(b*x + a) - sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh
(b*x + a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*log(cosh(b*x + a) + s
inh(b*x + a) + 1) + 6*(a*cosh(b*x + a)^2 + 2*a*cosh(b*x + a)*sinh(b*x + a)
 + a*sinh(b*x + a)^2 - a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 6*((b*x
 + a)*cosh(b*x + a)^2 + 2*(b*x + a)*cosh(b*x + a)*sinh(b*x + a) + (b*x + a
)*sinh(b*x + a)^2 - b*x - a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1))/(b^3
*cosh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b*x + a)^2
 - b^3)
```

Sympy [F]

$$\int x^2 \coth^2(a+bx)\,dx = \int x^2 \coth^2{(a+bx)}\,dx$$

input
```
integrate(x**2*coth(b*x+a)**2,x)
```

output
```
Integral(x**2*coth(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.66

$$\int x^2 \coth^2(a+bx)\,dx = -\frac{2\,x^2}{b} + \frac{bx^3e^{(2\,bx+2\,a)} - bx^3 - 6\,x^2}{3\,(be^{(2\,bx+2\,a)} - b)} + \frac{2\,\left(bx\log\left(e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^3} + \frac{2\,\left(bx\log\left(-e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(e^{(bx+a)}\right)\right)}{b^3}$$

input
```
integrate(x^2*coth(b*x+a)^2,x, algorithm="maxima")
```

output
```
-2*x^2/b + 1/3*(b*x^3*e^(2*b*x + 2*a) - b*x^3 - 6*x^2)/(b*e^(2*b*x + 2*a)
- b) + 2*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^3 + 2*(b*x*log
(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^3
```

Giac [F]

$$\int x^2 \coth^2(a + bx)\, dx = \int x^2 \coth\left(bx + a\right)^2\, dx$$

input
```
integrate(x^2*coth(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^2*coth(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^2(a + bx)\, dx = \int x^2 \coth(a + b\, x)^2\, dx$$

input
```
int(x^2*coth(a + b*x)^2,x)
```

output
```
int(x^2*coth(a + b*x)^2, x)
```

Reduce [F]

$$\int x^2 \coth^2(a + bx)\, dx$$
$$= \frac{-12e^{2bx+2a}\left(\int \frac{x}{e^{4bx+4a}-2e^{2bx+2a}+1}dx\right)b^2 + 3e^{2bx+2a}\log\left(e^{bx+a}-1\right) + 3e^{2bx+2a}\log\left(e^{bx+a}+1\right) + e^{2bx+2a}b^3x^3 -}{3b^3\left(e^{2bx+2a}\right.}$$

input
```
int(x^2*coth(b*x+a)^2,x)
```

output
```
( - 12*e**(2*a + 2*b*x)*int(x/(e**(4*a + 4*b*x) - 2*e**(2*a + 2*b*x) + 1),
x)*b**2 + 3*e**(2*a + 2*b*x)*log(e**(a + b*x) - 1) + 3*e**(2*a + 2*b*x)*lo
g(e**(a + b*x) + 1) + e**(2*a + 2*b*x)*b**3*x**3 - 6*e**(2*a + 2*b*x)*b*x
+ 12*int(x/(e**(4*a + 4*b*x) - 2*e**(2*a + 2*b*x) + 1),x)*b**2 - 3*log(e**
(a + b*x) - 1) - 3*log(e**(a + b*x) + 1) - b**3*x**3 - 6*b**2*x**2)/(3*b**
3*(e**(2*a + 2*b*x) - 1))
```

3.8 $\int x \coth^2(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 31

$$\int x \coth^2(a+bx)\,dx = \frac{x^2}{2} - \frac{x\coth(a+bx)}{b} + \frac{\log(\sinh(a+bx))}{b^2}$$

output
```
1/2*x^2-x*coth(b*x+a)/b+ln(sinh(b*x+a))/b^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48

$$\int x \coth^2(a+bx)\,dx = \frac{b^2x^2 - 2bx\coth(a) + 2\log(\sinh(a+bx)) + 2bx\text{csch}(a)\text{csch}(a+bx)\sinh(bx)}{2b^2}$$

input
```
Integrate[x*Coth[a + b*x]^2,x]
```

output
```
(b^2*x^2 - 2*b*x*Coth[a] + 2*Log[Sinh[a + b*x]] + 2*b*x*Csch[a]*Csch[a + b
*x]*Sinh[b*x])/(2*b^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 25, 4203, 15, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -x\tan\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx$$

$$\Big\downarrow \text{ 4203}$$

$$-\frac{i\int i\coth(a+bx)dx}{b}+\int xdx-\frac{x\coth(a+bx)}{b}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{i\int i\coth(a+bx)dx}{b}-\frac{x\coth(a+bx)}{b}+\frac{x^2}{2}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\int\coth(a+bx)dx}{b}-\frac{x\coth(a+bx)}{b}+\frac{x^2}{2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int -i\tan\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}-\frac{x\coth(a+bx)}{b}+\frac{x^2}{2}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{i\int\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx}{b}-\frac{x\coth(a+bx)}{b}+\frac{x^2}{2}$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{\log(-i\sinh(a+bx))}{b^2} - \frac{x\coth(a+bx)}{b} + \frac{x^2}{2}$$

input
```
Int[x*Coth[a + b*x]^2,x]
```

output
```
x^2/2 - (x*Coth[a + b*x])/b + Log[(-I)*Sinh[a + b*x]]/b^2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.61

method	result	size
parallelrisch	$\frac{x^2b^2-2\coth(bx+a)bx-2bx+2\ln(\tanh(bx+a))-2\ln(1-\tanh(bx+a))}{2b^2}$	50
risch	$\frac{x^2}{2}-\frac{2x}{b}-\frac{2a}{b^2}-\frac{2x}{b\left(e^{2bx+2a}-1\right)}+\frac{\ln\left(e^{2bx+2a}-1\right)}{b^2}$	54

input
```
int(x*coth(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^2*b^2-2*coth(b*x+a)*b*x-2*b*x+2*ln(tanh(b*x+a))-2*ln(1-tanh(b*x+a))
)/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(29) = 58$.

Time = 0.10 (sec) , antiderivative size = 189, normalized size of antiderivative = 6.10

$$\int x\coth^2(a+bx)\,dx = -\frac{b^2x^2-(b^2x^2-4\,bx)\cosh\left(bx+a\right)^2-2\left(b^2x^2-4\,bx\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)-\left(b^2x^2-4\,bx\right)\sinh(}{2\left(b^2\cosh\left(bx+a\right)^2+2\,b^2\cosh\left(bx+\right.\right.}$$

input
```
integrate(x*coth(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(b^2*x^2 - (b^2*x^2 - 4*b*x)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 4*b*x)*co
sh(b*x + a)*sinh(b*x + a) - (b^2*x^2 - 4*b*x)*sinh(b*x + a)^2 - 2*(cosh(b*
x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(2*sinh
(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b^2*cosh(b*x + a)^2 + 2*b^2*c
osh(b*x + a)*sinh(b*x + a) + b^2*sinh(b*x + a)^2 - b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(26) = 52.

Time = 0.64 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.45

$$\int x \coth^2(a+bx)\,dx = \begin{cases} \frac{x^2\coth^2(a)}{2} & \text{for } b=0 \\ \frac{x^2\coth^2(bx+\log(-e^{-bx}))}{2} & \text{for } a=\log(-e^{-bx}) \\ \frac{x^2\coth^2(bx+\log(e^{-bx}))}{2} & \text{for } a=\log(e^{-bx}) \\ \frac{x^2}{2}+\frac{x}{b}-\frac{x}{b\tanh(a+bx)}-\frac{\log(\tanh(a+bx)+1)}{b^2}+\frac{\log(\tanh(a+bx))}{b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*coth(b*x+a)**2,x)
```

output
```
Piecewise((x**2*coth(a)**2/2, Eq(b, 0)), (x**2*coth(b*x + log(-exp(-b*x)))
**2/2, Eq(a, log(-exp(-b*x)))), (x**2*coth(b*x + log(exp(-b*x)))**2/2, Eq(
a, log(exp(-b*x)))), (x**2/2 + x/b - x/(b*tanh(a + b*x)) - log(tanh(a + b*
x) + 1)/b**2 + log(tanh(a + b*x))/b**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(29) = 58.

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.71

$$\int x\coth^2(a+bx)\,dx = -\frac{xe^{(2\,bx+2\,a)}}{be^{(2\,bx+2\,a)}-b} - \frac{bx^2-\left(bx^2e^{(2\,a)}-2\,xe^{(2\,a)}\right)e^{(2\,bx)}}{2\left(be^{(2\,bx+2\,a)}-b\right)} + \frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2} + \frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}$$

input
```
integrate(x*coth(b*x+a)^2,x, algorithm="maxima")
```

output
```
-x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) - b) - 1/2*(b*x^2 - (b*x^2*e^(2*a) -
 2*x*e^(2*a))*e^(2*b*x))/(b*e^(2*b*x + 2*a) - b) + log((e^(b*x + a) + 1)*e
^(-a))/b^2 + log((e^(b*x + a) - 1)*e^(-a))/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(29) = 58$.

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.16

$$\int x\coth^2(a+bx)\,dx = \frac{b^2x^2e^{(2\,bx+2\,a)} - b^2x^2 - 4\,bxe^{(2\,bx+2\,a)} + 2\,e^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)} - 1\right) - 2\,\log\left(e^{(2\,bx+2\,a)} - 1\right)}{2\left(b^2e^{(2\,bx+2\,a)} - b^2\right)}$$

input
```
integrate(x*coth(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*(b^2*x^2*e^(2*b*x + 2*a) - b^2*x^2 - 4*b*x*e^(2*b*x + 2*a) + 2*e^(2*b*
x + 2*a)*log(e^(2*b*x + 2*a) - 1) - 2*log(e^(2*b*x + 2*a) - 1))/(b^2*e^(2*
b*x + 2*a) - b^2)
```

Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int x\coth^2(a+bx)\,dx = \frac{\ln\left(e^{2\,a}\,e^{2\,b\,x} - 1\right)}{b^2} - \frac{2\,x}{b} + \frac{x^2}{2} - \frac{2\,x}{b\,\left(e^{2\,a+2\,b\,x} - 1\right)}$$

input
```
int(x*coth(a + b*x)^2,x)
```

output
```
log(exp(2*a)*exp(2*b*x) - 1)/b^2 - (2*x)/b + x^2/2 - (2*x)/(b*(exp(2*a + 2
*b*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 127, normalized size of antiderivative = 4.10

$$\int x\coth^2(a+bx)\,dx$$
$$= \frac{2e^{2bx+2a}\log\left(e^{bx+a}-1\right)+2e^{2bx+2a}\log\left(e^{bx+a}+1\right)+e^{2bx+2a}b^2x^2-4e^{2bx+2a}bx-2\log\left(e^{bx+a}-1\right)-2\log\left(e\right.}{2b^2\left(e^{2bx+2a}-1\right)}$$

input
```
int(x*coth(b*x+a)^2,x)
```

output
```
(2*e**(2*a + 2*b*x)*log(e**(a + b*x) - 1) + 2*e**(2*a + 2*b*x)*log(e**(a +
 b*x) + 1) + e**(2*a + 2*b*x)*b**2*x**2 - 4*e**(2*a + 2*b*x)*b*x - 2*log(e
**(a + b*x) - 1) - 2*log(e**(a + b*x) + 1) - b**2*x**2)/(2*b**2*(e**(2*a +
 2*b*x) - 1))
```

3.9 $\int \frac{\coth^2(a+bx)}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\coth^2(a+bx)}{x}\,dx = \text{Int}\left(\frac{\coth^2(a+bx)}{x},x\right)$$

output
```
Defer(Int)(coth(b*x+a)^2/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^2(a+bx)}{x}\,dx = \int \frac{\coth^2(a+bx)}{x}\,dx$$

input
```
Integrate[Coth[a + b*x]^2/x,x]
```

output
```
Integrate[Coth[a + b*x]^2/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(a+bx)}{x} \, dx$$
$$\Big\downarrow \; 3042$$
$$\int -\frac{\tan\left(ia+ibx+\frac{\pi}{2}\right)^2}{x} dx$$
$$\Big\downarrow \; 25$$
$$-\int \frac{\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2}{x} dx$$
$$\Big\downarrow \; 4222$$
$$\int -\frac{\tan^2\left(\frac{1}{2}(-\pi-2ia)-ibx\right)}{x} dx$$

input
```
Int[Coth[a + b*x]^2/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\coth(bx+a)^2}{x} dx$$

input
```
int(coth(b*x+a)^2/x,x)
```

output
```
int(coth(b*x+a)^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^2(a+bx)}{x}\,dx=\int \frac{\coth{(bx+a)}^2}{x}\,dx$$

input
```
integrate(coth(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
integral(coth(b*x + a)^2/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\coth^2(a+bx)}{x}\,dx=\int \frac{\coth^2{(a+bx)}}{x}\,dx$$

input
```
integrate(coth(b*x+a)**2/x,x)
```

output
```
Integral(coth(a + b*x)**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 5.75

$$\int \frac{\coth^2(a+bx)}{x}\,dx=\int \frac{\coth{(bx+a)}^2}{x}\,dx$$

input
```
integrate(coth(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
-2/(b*x*e^(2*b*x + 2*a) - b*x) + integrate(1/(b*x^2*e^(b*x + a) + b*x^2),
x) - integrate(1/(b*x^2*e^(b*x + a) - b*x^2), x) + log(x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^2(a+bx)}{x}\,dx=\int \frac{\coth{(bx+a)}^2}{x}\,dx$$

input
```
integrate(coth(b*x+a)^2/x,x, algorithm="giac")
```

output
```
integrate(coth(b*x + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.46 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^2(a+bx)}{x}\,dx=\int \frac{\coth(a+b\,x)^2}{x}\,dx$$

input
```
int(coth(a + b*x)^2/x,x)
```

output
```
int(coth(a + b*x)^2/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 48, normalized size of antiderivative = 4.00

$$\int \frac{\coth^2(a+bx)}{x}\,dx = 4e^{2a}\left(\int \frac{e^{2bx}}{e^{4bx+4a}x-2e^{2bx+2a}x+x}dx\right)+\log(x)$$

input
```
int(coth(b*x+a)^2/x,x)
```

output
```
4*e**(2*a)*int(e**(2*b*x)/(e**(4*a + 4*b*x)*x - 2*e**(2*a + 2*b*x)*x + x),
x) + log(x)
```

3.10 $\int \frac{\coth^2(a+bx)}{x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\coth^2(a+bx)}{x^2} \, dx = \text{Int}\left(\frac{\coth^2(a+bx)}{x^2}, x\right)$$

output
```
Defer(Int)(coth(b*x+a)^2/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^2(a+bx)}{x^2} \, dx = \int \frac{\coth^2(a+bx)}{x^2} \, dx$$

input
```
Integrate[Coth[a + b*x]^2/x^2,x]
```

output
```
Integrate[Coth[a + b*x]^2/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(a+bx)}{x^2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(ia+ibx+\frac{\pi}{2}\right)^2}{x^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2}{x^2}dx$$

$$\Big\downarrow \text{4222}$$

$$\int -\frac{\tan^2\left(\frac{1}{2}(-\pi-2ia)-ibx\right)}{x^2}dx$$

input
```
Int[Coth[a + b*x]^2/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\coth\left(bx+a\right)^2}{x^2}dx$$

input
```
int(coth(b*x+a)^2/x^2,x)
```

output
```
int(coth(b*x+a)^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^2(a+bx)}{x^2}\,dx = \int \frac{\coth\left(bx+a\right)^2}{x^2}\,dx$$

input
```
integrate(coth(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
integral(coth(b*x + a)^2/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\coth^2(a+bx)}{x^2}\,dx = \int \frac{\coth^2\left(a+bx\right)}{x^2}\,dx$$

input
```
integrate(coth(b*x+a)**2/x**2,x)
```

output
```
Integral(coth(a + b*x)**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 7.58

$$\int \frac{\coth^2(a+bx)}{x^2}\,dx = \int \frac{\coth{(bx+a)}^2}{x^2}\,dx$$

input
```
integrate(coth(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
-(b*x*e^(2*b*x + 2*a) - b*x + 2)/(b*x^2*e^(2*b*x + 2*a) - b*x^2) + 2*integ
rate(1/(b*x^3*e^(b*x + a) + b*x^3), x) - 2*integrate(1/(b*x^3*e^(b*x + a)
- b*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^2(a+bx)}{x^2}\,dx = \int \frac{\coth{(bx+a)}^2}{x^2}\,dx$$

input
```
integrate(coth(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
integrate(coth(b*x + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.51 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^2(a+bx)}{x^2}\,dx = \int \frac{\coth(a+b\,x)^2}{x^2}\,dx$$

input
```
int(coth(a + b*x)^2/x^2,x)
```

output
```
int(coth(a + b*x)^2/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 221, normalized size of antiderivative = 18.42

$$\int \frac{\coth^2(a+bx)}{x^2}\,dx$$
$$= \frac{-8e^{2bx+4a}\left(\int \frac{e^{2bx}}{e^{4bx+4a}x-2e^{2bx+2a}x+x}dx\right)bx+4e^{2bx+2a}\left(\int \frac{1}{e^{4bx+4a}x^2-2e^{2bx+2a}x^2+x^2}dx\right)x-e^{2bx+2a}+8e^{2a}\left(\int \frac{}{e^{4bx}}\right.}{x\left(e^{2bx+2a}-1\right)}$$

input
```
int(coth(b*x+a)^2/x^2,x)
```

output
```
( - 8*e**(4*a + 2*b*x)*int(e**(2*b*x)/(e**(4*a + 4*b*x)*x - 2*e**(2*a + 2*
b*x)*x + x),x)*b*x + 4*e**(2*a + 2*b*x)*int(1/(e**(4*a + 4*b*x)*x**2 - 2*e
**(2*a + 2*b*x)*x**2 + x**2),x)*x - e**(2*a + 2*b*x) + 8*e**(2*a)*int(e**(
2*b*x)/(e**(4*a + 4*b*x)*x - 2*e**(2*a + 2*b*x)*x + x),x)*b*x - 4*int(1/(e
**(4*a + 4*b*x)*x**2 - 2*e**(2*a + 2*b*x)*x**2 + x**2),x)*x - 3)/(x*(e**(2
*a + 2*b*x) - 1))
```

3.11 $\int x^3 \coth^3(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 179

$$\int x^3 \coth^3(a+bx)\,dx = -\frac{3x^2}{2b^2}+\frac{x^3}{2b}-\frac{x^4}{4}-\frac{3x^2\coth(a+bx)}{2b^2}-\frac{x^3\coth^2(a+bx)}{2b} + \frac{3x\log\left(1-e^{2(a+bx)}\right)}{b^3}+\frac{x^3\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{3\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^4}+\frac{3x^2\,\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2} - \frac{3x\,\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}+\frac{3\,\text{PolyLog}\left(4,e^{2(a+bx)}\right)}{4b^4}$$

output
```
-3/2*x^2/b^2+1/2*x^3/b-1/4*x^4-3/2*x^2*coth(b*x+a)/b^2-1/2*x^3*coth(b*x+a)
^2/b+3*x*ln(1-exp(2*b*x+2*a))/b^3+x^3*ln(1-exp(2*b*x+2*a))/b+3/2*polylog(2
,exp(2*b*x+2*a))/b^4+3/2*x^2*polylog(2,exp(2*b*x+2*a))/b^2-3/2*x*polylog(3
,exp(2*b*x+2*a))/b^3+3/4*polylog(4,exp(2*b*x+2*a))/b^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 422 vs. 2(179) = 358.

Time = 2.13 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.36

$$\int x^3 \coth^3(a+bx)\,dx = \frac{1}{4}\left(x^4\coth(a) - \frac{2x^3\operatorname{csch}^2(a+bx)}{b}\right.$$
$$- \frac{2e^{2a}\left(6b^2e^{-2a}x^2 + b^4e^{-2a}x^4 - 6b(1-e^{-2a})\,x\log\left(1-e^{-a-bx}\right) - 2b^3e^{-2a}(-1+e^{2a})\,x^3\log\left(1-e^{-a-bx}\right)\right.}{}$$
$$\left. + \frac{6x^2\operatorname{csch}(a)\operatorname{csch}(a+bx)\sinh(bx)}{b^2}\right)$$

input
```
Integrate[x^3*Coth[a + b*x]^3,x]
```

output
```
(x^4*Coth[a] - (2*x^3*Csch[a + b*x]^2)/b - (2*E^(2*a)*((6*b^2*x^2)/E^(2*a)
 + (b^4*x^4)/E^(2*a) - 6*b*(1 - E^(-2*a))*x*Log[1 - E^(-a - b*x)] - (2*b^3
*(-1 + E^(2*a))*x^3*Log[1 - E^(-a - b*x)])/E^(2*a) - 6*b*(1 - E^(-2*a))*x*
Log[1 + E^(-a - b*x)] - (2*b^3*(-1 + E^(2*a))*x^3*Log[1 + E^(-a - b*x)])/E
^(2*a) + 6*(1 - E^(-2*a))*PolyLog[2, -E^(-a - b*x)] + 6*b^2*(1 - E^(-2*a))
*x^2*PolyLog[2, -E^(-a - b*x)] + 6*(1 - E^(-2*a))*PolyLog[2, E^(-a - b*x)]
 + 6*b^2*(1 - E^(-2*a))*x^2*PolyLog[2, E^(-a - b*x)] + 12*b*(1 - E^(-2*a))
*x*PolyLog[3, -E^(-a - b*x)] + 12*b*(1 - E^(-2*a))*x*PolyLog[3, E^(-a - b*
x)] + 12*(1 - E^(-2*a))*PolyLog[4, -E^(-a - b*x)] + 12*(1 - E^(-2*a))*Poly
Log[4, E^(-a - b*x)]))/(b^4*(-1 + E^(2*a))) + (6*x^2*Csch[a]*Csch[a + b*x]
*Sinh[b*x])/b^2)/4
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.49, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.917, Rules used = {3042, 26, 4203, 25, 26, 3042, 25, 26, 4201, 2620, 3011, 4203, 15, 26, 3042, 26, 4201, 2620, 2715, 2838, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^3(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int ix^3 \tan\left(ia+ibx+\frac{\pi}{2}\right)^3 dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int x^3 \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^3 dx$$

$$\Big\downarrow \text{ 4203}$$

$$i\left(-\int ix^3 \coth(a+bx)dx+\frac{3i\int -x^2\coth^2(a+bx)dx}{2b}+\frac{ix^3\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{ 25}$$

$$i\left(-\int ix^3 \coth(a+bx)dx-\frac{3i\int x^2\coth^2(a+bx)dx}{2b}+\frac{ix^3\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{ 26}$$

$$i\left(-i\int x^3 \coth(a+bx)dx-\frac{3i\int x^2\coth^2(a+bx)dx}{2b}+\frac{ix^3\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$i\left(-i\int -ix^3 \tan\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{3i\int -x^2\tan\left(ia+ibx+\frac{\pi}{2}\right)^2dx}{2b}+\frac{ix^3\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{ 25}$$

$$i\left(-i\int -ix^3 \tan\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{3i\int x^2\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx}{2b}+\frac{ix^3\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{ 26}$$

$$i\left(-\int x^3 \tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx+\frac{3i\int x^2\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx}{2b}+\frac{ix^3\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{ 4201}$$

$$i\left(-2i\int \frac{e^{2a+2bx-i\pi}x^3}{1+e^{2a+2bx-i\pi}}dx+\frac{3i\int x^2\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx}{2b}+\frac{ix^3\coth^2(a+bx)}{2b}+\frac{ix^4}{4}\right)$$

$$\Big\downarrow \; 2620$$

$$i\left(\frac{3i\int x^2\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx}{2b}-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\int x^2\log\left(1+e^{2a+2bx-i\pi}\right)dx}{2b}\right)+\frac{ix^3\operatorname{coth}$$

$$\Big\downarrow \; 3011$$

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\operatorname{PolyLog}(2,-e^{2a+2bx-i\pi})dx}{b}-\frac{x^2\operatorname{PolyLog}(2,-e^{2a+2bx-i\pi})}{2b}\right)}{2b}\right)+\frac{3i\int x^2\tan\left(\frac{1}{2}(2i\right.$$

$$\Big\downarrow \; 4203$$

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\operatorname{PolyLog}(2,-e^{2a+2bx-i\pi})dx}{b}-\frac{x^2\operatorname{PolyLog}(2,-e^{2a+2bx-i\pi})}{2b}\right)}{2b}\right)+\frac{3i\left(\frac{2i\int ix\coth(a+b\right.}{b}$$

$$\Big\downarrow \; 15$$

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\operatorname{PolyLog}(2,-e^{2a+2bx-i\pi})dx}{b}-\frac{x^2\operatorname{PolyLog}(2,-e^{2a+2bx-i\pi})}{2b}\right)}{2b}\right)+\frac{3i\left(\frac{2i\int ix\coth(a+b\right.}{b}$$

$$\Big\downarrow \; 26$$

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\operatorname{PolyLog}(2,-e^{2a+2bx-i\pi})dx}{b}-\frac{x^2\operatorname{PolyLog}(2,-e^{2a+2bx-i\pi})}{2b}\right)}{2b}\right)+\frac{3i\left(-\frac{2\int x\coth(a+i\right.}{b}$$

$$\Big\downarrow \; 3042$$

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\operatorname{PolyLog}(2,-e^{2a+2bx-i\pi})dx}{b}-\frac{x^2\operatorname{PolyLog}(2,-e^{2a+2bx-i\pi})}{2b}\right)}{2b}\right)+\frac{3i\left(-\frac{2\int -ix\tan(ia\right.}{b}$$

$$\Big\downarrow \; 26$$

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\operatorname{PolyLog}(2,-e^{2a+2bx-i\pi})dx}{b}-\frac{x^2\operatorname{PolyLog}(2,-e^{2a+2bx-i\pi})}{2b}\right)}{2b}\right)+\frac{3i\left(\frac{2i\int x\tan\left(\frac{1}{2}(2ia\right.}{b}$$

$$\Big\downarrow \; 4201$$

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)+\frac{3i\left(\frac{2i\left(2i\int\frac{e^{2a+2bx}}{1+e^{2a+2}}\right.}{b}\right.}{}\right.$$

$\downarrow$ 2620

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)+\frac{3i\left(\frac{2i\left(2i\left(\frac{x\log\left(1+\right.}{}\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 2715

$$i\left(\frac{3i\left(\frac{2i\left(2i\left(\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int e^{-2a-2bx+i\pi}\log\left(1+e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}\right)-\frac{ix^2}{2}\right)}{b}+\frac{x^2\coth(a+bx)}{b}-\frac{x^3}{3}\right)}{2b}-2i\left(\frac{x^3\log\left(\right.}{}\right.\right.$$

$\downarrow$ 2838

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)+\frac{3i\left(\frac{2i\left(2i\left(\frac{\text{PolyLog}}{}\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 7163

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)\right.$$

$$\Big\downarrow \; 2720$$

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}}{b}-\frac{x^2\,\text{PolyLo}}{}\right.}{2b}\right.\right.$$

$$\Big\downarrow \; 7143$$

$$i\left(-2i\left(\frac{x^3\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)}{2b}-\frac{\text{PolyLog}\left(4,-e^{2a+2bx-i\pi}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}\right)}{2b}\right)+\right.$$

input
```
Int[x^3*Coth[a + b*x]^3,x]
```

output
```
I*((I/4)*x^4 + ((I/2)*x^3*Coth[a + b*x]^2)/b + (((3*I)/2)*(-1/3*x^3 + (x^2
*Coth[a + b*x])/b + ((2*I)*((-1/2*I)*x^2 + (2*I)*((x*Log[1 + E^(2*a - I*Pi
 + 2*b*x)])/(2*b) + PolyLog[2, -E^(2*a - I*Pi + 2*b*x)]/(4*b^2))))/b))/b -
 (2*I)*((x^3*Log[1 + E^(2*a - I*Pi + 2*b*x)])/(2*b) - (3*(-1/2*(x^2*PolyLo
g[2, -E^(2*a - I*Pi + 2*b*x)])/b + ((x*PolyLog[3, -E^(2*a - I*Pi + 2*b*x)]
)/(2*b) - PolyLog[4, -E^(2*a - I*Pi + 2*b*x)]/(4*b^2))/b))/(2*b)))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(161) = 322.

Time = 0.09 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.09

method	result
risch	$-\frac{3x^2}{b^2}-\frac{x^4}{4}-\frac{x^2\left(2x\,e^{2bx+2a}b+3\,e^{2bx+2a}-3\right)}{b^2\left(e^{2bx+2a}-1\right)^2}+\frac{3\ln\left(1-e^{bx+a}\right)a}{b^4}-\frac{3a\ln\left(e^{bx+a}-1\right)}{b^4}+\frac{6a\ln\left(e^{bx+a}\right)}{b^4}-\frac{6ax}{b^3}+\frac{3\ln(1+e\ldots}{b^3}$

input
```
int(x^3*coth(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3*x^2/b^2-1/4*x^4-x^2*(2*x*exp(2*b*x+2*a)*b+3*exp(2*b*x+2*a)-3)/b^2/(exp(
2*b*x+2*a)-1)^2+3/b^4*ln(1-exp(b*x+a))*a-3/b^4*a*ln(exp(b*x+a)-1)+6/b^4*a*
ln(exp(b*x+a))-6/b^3*a*x+3/b^3*ln(1+exp(b*x+a))*x+3/b^3*ln(1-exp(b*x+a))*x
-2/b^3*a^3*x+1/b*ln(1+exp(b*x+a))*x^3+3/b^2*polylog(2,-exp(b*x+a))*x^2-6/b
^3*polylog(3,-exp(b*x+a))*x+1/b*ln(1-exp(b*x+a))*x^3+3/b^2*polylog(2,exp(b
*x+a))*x^2-6/b^3*polylog(3,exp(b*x+a))*x-1/b^4*a^3*ln(exp(b*x+a)-1)+2/b^4*
a^3*ln(exp(b*x+a))+1/b^4*ln(1-exp(b*x+a))*a^3-3/2/b^4*a^4+6/b^4*polylog(4,
-exp(b*x+a))+6/b^4*polylog(4,exp(b*x+a))-3/b^4*a^2+3/b^4*polylog(2,exp(b*x
+a))+3/b^4*polylog(2,-exp(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1985 vs. 2(159) = 318.

Time = 0.12 (sec) , antiderivative size = 1985, normalized size of antiderivative = 11.09

$$\int x^3\coth^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*coth(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(b^4*x^4 + (b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)^4 +
4*(b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)*sinh(b*x + a)^3 +
(b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*sinh(b*x + a)^4 - 2*a^4 - 2*(b^4*x
^4 - 4*b^3*x^3 - 2*a^4 + 6*b^2*x^2 - 12*a^2)*cosh(b*x + a)^2 - 2*(b^4*x^4
- 4*b^3*x^3 - 2*a^4 + 6*b^2*x^2 - 3*(b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2
)*cosh(b*x + a)^2 - 12*a^2)*sinh(b*x + a)^2 - 12*a^2 - 12*((b^2*x^2 + 1)*c
osh(b*x + a)^4 + 4*(b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2
+ 1)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 + 1)*cosh(b*x + a)^2 - 2*(b^2*
x^2 - 3*(b^2*x^2 + 1)*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 +
 1)*cosh(b*x + a)^3 - (b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*dilo
g(cosh(b*x + a) + sinh(b*x + a)) - 12*((b^2*x^2 + 1)*cosh(b*x + a)^4 + 4*(
b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 1)*sinh(b*x + a)^4
 + b^2*x^2 - 2*(b^2*x^2 + 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 1
)*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 1)*cosh(b*x + a)^3
- (b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(-cosh(b*x + a) - s
inh(b*x + a)) - 4*(b^3*x^3 + (b^3*x^3 + 3*b*x)*cosh(b*x + a)^4 + 4*(b^3*x^
3 + 3*b*x)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + 3*b*x)*sinh(b*x + a)
^4 - 2*(b^3*x^3 + 3*b*x)*cosh(b*x + a)^2 - 2*(b^3*x^3 - 3*(b^3*x^3 + 3*b*x
)*cosh(b*x + a)^2 + 3*b*x)*sinh(b*x + a)^2 + 3*b*x + 4*((b^3*x^3 + 3*b*x)*
cosh(b*x + a)^3 - (b^3*x^3 + 3*b*x)*cosh(b*x + a))*sinh(b*x + a))*log(c...
```

Sympy [F]

$$\int x^3 \coth^3(a+bx)\,dx = \int x^3 \coth^3{(a+bx)}\,dx$$

input
```
integrate(x**3*coth(b*x+a)**3,x)
```

output
```
Integral(x**3*coth(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.69

$$\int x^3 \coth^3(a+bx)\,dx = \frac{b^2x^4e^{(4\,bx+4\,a)} + b^2x^4 + 12\,x^2 - 2\,\left(b^2x^4e^{(2\,a)} + 4\,bx^3e^{(2\,a)} + 6\,x^2e^{(2\,a)}\right)e^{(2\,bx)}}{4\,\left(b^2e^{(4\,bx+4\,a)} - 2\,b^2e^{(2\,bx+2\,a)} + b^2\right)} - \frac{b^4x^4 + 6\,b^2x^2}{2\,b^4} + \frac{b^3x^3\log\left(e^{(bx+a)}+1\right) + 3\,b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 6\,bx\mathrm{Li}_3\left(-e^{(bx+a)}\right) + 6\,\mathrm{Li}_4\left(-e^{(bx+a)}\right)}{b^4} + \frac{b^3x^3\log\left(-e^{(bx+a)}+1\right) + 3\,b^2x^2\mathrm{Li}_2\left(e^{(bx+a)}\right) - 6\,bx\mathrm{Li}_3\left(e^{(bx+a)}\right) + 6\,\mathrm{Li}_4\left(e^{(bx+a)}\right)}{b^4} + \frac{3\,\left(bx\log\left(e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)}{b^4} + \frac{3\,\left(bx\log\left(-e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(e^{(bx+a)}\right)\right)}{b^4}$$

input
```
integrate(x^3*coth(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*(b^2*x^4*e^(4*b*x + 4*a) + b^2*x^4 + 12*x^2 - 2*(b^2*x^4*e^(2*a) + 4*b
*x^3*e^(2*a) + 6*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2
*b*x + 2*a) + b^2) - 1/2*(b^4*x^4 + 6*b^2*x^2)/b^4 + (b^3*x^3*log(e^(b*x +
 a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a))
+ 6*polylog(4, -e^(b*x + a)))/b^4 + (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2
*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(
b*x + a)))/b^4 + 3*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^4 +
3*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^4
```

Giac [F]

$$\int x^3 \coth^3(a+bx)\,dx = \int x^3 \coth\left(bx+a\right)^3\,dx$$

input
```
integrate(x^3*coth(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^3*coth(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \coth^3(a+bx)\,dx = \int x^3 \coth(a+b\,x)^3\,dx$$

input
```
int(x^3*coth(a + b*x)^3,x)
```

output
```
int(x^3*coth(a + b*x)^3, x)
```

Reduce [F]

$$\int x^3 \coth^3(a+bx)\,dx$$
$$= \frac{-45-288e^{2bx+2a}b^2x^2+720e^{4bx+4a}\left(\int \frac{x}{e^{6bx+6a}-3e^{4bx+4a}+3e^{2bx+2a}-1}dx\right)b^2-256e^{2bx+2a}\left(\int \frac{x^3}{e^{6bx+6a}-3e^{4bx+4a}+3e^{2}\cdots}\right.\cdots}{\cdots}$$

input
```
int(x^3*coth(b*x+a)^3,x)
```

output
```
(128*e**(4*a + 4*b*x)*int(x**3/(e**(6*a + 6*b*x) - 3*e**(4*a + 4*b*x) + 3*
e**(2*a + 2*b*x) - 1),x)*b**4 + 288*e**(4*a + 4*b*x)*int(x**2/(e**(6*a + 6
*b*x) - 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) - 1),x)*b**3 + 720*e**(4*a
 + 4*b*x)*int(x/(e**(6*a + 6*b*x) - 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x
) - 1),x)*b**2 + 234*e**(4*a + 4*b*x)*log(e**(a + b*x) - 1) + 234*e**(4*a
+ 4*b*x)*log(e**(a + b*x) + 1) + 16*e**(4*a + 4*b*x)*b**4*x**4 - 468*e**(4
*a + 4*b*x)*b*x + 45*e**(4*a + 4*b*x) - 256*e**(2*a + 2*b*x)*int(x**3/(e**
(6*a + 6*b*x) - 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) - 1),x)*b**4 - 576
*e**(2*a + 2*b*x)*int(x**2/(e**(6*a + 6*b*x) - 3*e**(4*a + 4*b*x) + 3*e**(
2*a + 2*b*x) - 1),x)*b**3 - 1440*e**(2*a + 2*b*x)*int(x/(e**(6*a + 6*b*x)
- 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) - 1),x)*b**2 - 468*e**(2*a + 2*b
*x)*log(e**(a + b*x) - 1) - 468*e**(2*a + 2*b*x)*log(e**(a + b*x) + 1) - 3
2*e**(2*a + 2*b*x)*b**4*x**4 - 192*e**(2*a + 2*b*x)*b**3*x**3 - 288*e**(2*
a + 2*b*x)*b**2*x**2 + 648*e**(2*a + 2*b*x)*b*x + 128*int(x**3/(e**(6*a +
6*b*x) - 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) - 1),x)*b**4 + 288*int(x*
*2/(e**(6*a + 6*b*x) - 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) - 1),x)*b**
3 + 720*int(x/(e**(6*a + 6*b*x) - 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x)
- 1),x)*b**2 + 234*log(e**(a + b*x) - 1) + 234*log(e**(a + b*x) + 1) + 16*
b**4*x**4 + 96*b**3*x**3 + 360*b**2*x**2 - 45)/(64*b**4*(e**(4*a + 4*b*x)
- 2*e**(2*a + 2*b*x) + 1))
```

3.12 $\int x^2 \coth^3(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 114

$$\int x^2 \coth^3(a+bx)\,dx = \frac{x^2}{2b} - \frac{x^3}{3} - \frac{x\coth(a+bx)}{b^2} - \frac{x^2\coth^2(a+bx)}{2b} + \frac{x^2\log\left(1-e^{2(a+bx)}\right)}{b} + \frac{\log(\sinh(a+bx))}{b^3} + \frac{x\,\mathrm{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^2} - \frac{\mathrm{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^3}$$

output
```
1/2*x^2/b-1/3*x^3-x*coth(b*x+a)/b^2-1/2*x^2*coth(b*x+a)^2/b+x^2*ln(1-exp(2
*b*x+2*a))/b+ln(sinh(b*x+a))/b^3+x*polylog(2,exp(2*b*x+2*a))/b^2-1/2*polyl
og(3,exp(2*b*x+2*a))/b^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 314 vs. 2(114) = 228.

Time = 2.13 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.75

$$\int x^2 \coth^3(a+bx)\,dx = \frac{1}{3}x^3\coth(a) - \frac{x^2\text{csch}^2(a+bx)}{2b}$$
$$-\frac{e^{2a}\left(6be^{-2a}x + 6b(1-e^{-2a})\,x + 2b^3e^{-2a}x^3 - 3b^2e^{-2a}(-1+e^{2a})\,x^2\log\left(1-e^{-a-bx}\right) - 3b^2e^{-2a}(-1+e^2\right.}{}$$
$$+\frac{x\text{csch}(a)\text{csch}(a+bx)\sinh(bx)}{b^2}$$

input
```
Integrate[x^2*Coth[a + b*x]^3,x]
```

output
```
(x^3*Coth[a])/3 - (x^2*Csch[a + b*x]^2)/(2*b) - (E^(2*a)*((6*b*x)/E^(2*a)
+ 6*b*(1 - E^(-2*a))*x + (2*b^3*x^3)/E^(2*a) - (3*b^2*(-1 + E^(2*a))*x^2*L
og[1 - E^(-a - b*x)])/E^(2*a) - (3*b^2*(-1 + E^(2*a))*x^2*Log[1 + E^(-a -
b*x)])/E^(2*a) - 3*(1 - E^(-2*a))*Log[1 - E^(a + b*x)] - 3*(1 - E^(-2*a))*
Log[1 + E^(a + b*x)] + 6*b*(1 - E^(-2*a))*x*PolyLog[2, -E^(-a - b*x)] + 6*
b*(1 - E^(-2*a))*x*PolyLog[2, E^(-a - b*x)] + 6*(1 - E^(-2*a))*PolyLog[3,
-E^(-a - b*x)] + 6*(1 - E^(-2*a))*PolyLog[3, E^(-a - b*x)]))/(3*b^3*(-1 +
E^(2*a))) + (x*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b^2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.47, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.583, Rules used = {3042, 26, 4203, 25, 26, 3042, 25, 26, 4201, 2620, 3011, 2720, 4203, 15, 26, 3042, 26, 3956, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int ix^2 \tan\left(ia+ibx+\frac{\pi}{2}\right)^3 dx$$

$$\Big\downarrow \text{26}$$

$$i\int x^2\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^3dx$$

$$\Big\downarrow \text{4203}$$

$$i\left(-\int ix^2\coth(a+bx)dx+\frac{i\int -x\coth^2(a+bx)dx}{b}+\frac{ix^2\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-\int ix^2\coth(a+bx)dx-\frac{i\int x\coth^2(a+bx)dx}{b}+\frac{ix^2\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-i\int x^2\coth(a+bx)dx-\frac{i\int x\coth^2(a+bx)dx}{b}+\frac{ix^2\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-i\int -ix^2\tan\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{i\int -x\tan\left(ia+ibx+\frac{\pi}{2}\right)^2dx}{b}+\frac{ix^2\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-i\int -ix^2\tan\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{i\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx}{b}+\frac{ix^2\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\int x^2\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx+\frac{i\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx}{b}+\frac{ix^2\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{4201}$$

$$i\left(-2i\int\frac{e^{2a+2bx-i\pi}x^2}{1+e^{2a+2bx-i\pi}}dx+\frac{i\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx}{b}+\frac{ix^2\coth^2(a+bx)}{2b}+\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{2620}$$

$$i\left(-2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int x\log\left(1+e^{2a+2bx-i\pi}\right)dx}{b}\right)+\frac{i\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx}{b}+\frac{ix^2\coth^2(\ldots}{2b}\right.$$

$$\Big\downarrow \text{3011}$$

$$i\left(-2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int \text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)+\frac{i\int x\tan\left(\frac{1}{2}(2ia+\pi)+\right.}{b}\right.$$

$\downarrow$ 2720

$$i\left(-2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)+\frac{i\int x}{}\right.$$

$\downarrow$ 4203

$$i\left(-2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)+\frac{i\left(\frac{i\int}{}\right.}{}\right.$$

$\downarrow$ 15

$$i\left(-2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)+\frac{i\left(\frac{i\int}{}\right.}{}\right.$$

$\downarrow$ 26

$$i\left(-2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)+\frac{i\left(-\right.}{}\right.$$

$\downarrow$ 3042

$$i\left(-2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)+\frac{i\left(-\right.}{}\right.$$

$\downarrow$ 26

$$i\left(-2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)+\frac{i\left(\frac{i\int}{}\right.}{}\right.$$

$\downarrow$ 3956

$$i\left(-2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\int e^{-2a-2bx+i\pi}\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)+\frac{i\left(-\right.}{}\right.$$

$\downarrow$ 7143

$$i\left(-2i\left(\frac{x^2\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\frac{\text{PolyLog}\left(3,-e^{2a+2bx-i\pi}\right)}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{2b}}{b}\right)+\frac{i\left(-\frac{\log(-i\sinh(a+bx))}{b^2}+\frac{x\cot}{}\right.}{b}\right.$$

input
```
Int[x^2*Coth[a + b*x]^3,x]
```

output
```
I*((I/3)*x^3 + ((I/2)*x^2*Coth[a + b*x]^2)/b + (I*(-1/2*x^2 + (x*Coth[a +
b*x])/b - Log[(-I)*Sinh[a + b*x]]/b^2))/b - (2*I)*((x^2*Log[1 + E^(2*a - I
*Pi + 2*b*x)])/(2*b) - (-1/2*(x*PolyLog[2, -E^(2*a - I*Pi + 2*b*x)])/b + P
olyLog[3, -E^(2*a - I*Pi + 2*b*x)]/(4*b^2))/b))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(106) = 212$.

Time = 0.07 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.16

method	result
risch	$-\frac{x^3}{3} - \frac{2x\left(x\,\mathrm{e}^{2bx+2a}b+\mathrm{e}^{2bx+2a}-1\right)}{b^2\left(\mathrm{e}^{2bx+2a}-1\right)^2} + \frac{a^2\ln\left(\mathrm{e}^{bx+a}-1\right)}{b^3} - \frac{2a^2\ln\left(\mathrm{e}^{bx+a}\right)}{b^3} - \frac{\ln\left(1-\mathrm{e}^{bx+a}\right)a^2}{b^3} + \frac{4a^3}{3b^3} + \frac{2\,\mathrm{polylog}\left(2,-\mathrm{e}^{bx+a}\right.}{b^2}$

input
```
int(x^2*coth(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3-2*x*(x*exp(2*b*x+2*a)*b+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1)^
2+1/b^3*a^2*ln(exp(b*x+a)-1)-2/b^3*a^2*ln(exp(b*x+a))-1/b^3*ln(1-exp(b*x+a
))*a^2+4/3/b^3*a^3+2/b^2*polylog(2,-exp(b*x+a))*x+1/b*ln(1-exp(b*x+a))*x^2
+2/b^2*polylog(2,exp(b*x+a))*x+2/b^2*a^2*x+1/b*ln(1+exp(b*x+a))*x^2+1/b^3*
ln(exp(b*x+a)-1)-2/b^3*ln(exp(b*x+a))+1/b^3*ln(1+exp(b*x+a))-2/b^3*polylog
(3,-exp(b*x+a))-2/b^3*polylog(3,exp(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1467 vs. $2(105) = 210$.

Time = 0.13 (sec) , antiderivative size = 1467, normalized size of antiderivative = 12.87

$$\int x^2 \coth^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*coth(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/3*(b^3*x^3 + (b^3*x^3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^4 + 4*(b^3*x
^3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + 2*a^3
 + 6*b*x + 6*a)*sinh(b*x + a)^4 + 2*a^3 - 2*(b^3*x^3 - 3*b^2*x^2 + 2*a^3 +
 3*b*x + 6*a)*cosh(b*x + a)^2 - 2*(b^3*x^3 - 3*b^2*x^2 + 2*a^3 - 3*(b^3*x^
3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^2 + 3*b*x + 6*a)*sinh(b*x + a)^2 -
6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*
x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*
x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a)
)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^4 + 4*b*x*co
sh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2
+ 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x
+ a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x
 + a)) - 3*((b^2*x^2 + 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 + 1)*cosh(b*x + a)*
sinh(b*x + a)^3 + (b^2*x^2 + 1)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 + 1
)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 1)*cosh(b*x + a)^2 + 1)*sinh
(b*x + a)^2 + 4*((b^2*x^2 + 1)*cosh(b*x + a)^3 - (b^2*x^2 + 1)*cosh(b*x +
a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 3*((a^2 +
1)*cosh(b*x + a)^4 + 4*(a^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^2 + 1)
*sinh(b*x + a)^4 - 2*(a^2 + 1)*cosh(b*x + a)^2 + 2*(3*(a^2 + 1)*cosh(b*x +
 a)^2 - a^2 - 1)*sinh(b*x + a)^2 + a^2 + 4*((a^2 + 1)*cosh(b*x + a)^3 -...
```

Sympy [F]

$$\int x^2 \coth^3(a+bx)\,dx=\int x^2 \coth^3{\left(a+bx\right)}\,dx$$

input
```
integrate(x**2*coth(b*x+a)**3,x)
```

output
```
Integral(x**2*coth(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(105) = 210$.

Time = 0.11 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.98

$$\int x^2 \coth^3(a+bx)\,dx$$
$$= -\frac{2}{3}x^3 + \frac{b^2x^3e^{(4\,bx+4\,a)} + b^2x^3 - 2\left(b^2x^3e^{(2\,a)} + 3\,bx^2e^{(2\,a)} + 3\,xe^{(2\,a)}\right)e^{(2\,bx)} + 6\,x}{3\left(b^2e^{(4\,bx+4\,a)} - 2\,b^2e^{(2\,bx+2\,a)} + b^2\right)}$$
$$- \frac{2\,x}{b^2} + \frac{b^2x^2\log\left(e^{(bx+a)}+1\right) + 2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right) - 2\,\mathrm{Li}_3(-e^{(bx+a)})}{b^3}$$
$$+ \frac{b^2x^2\log\left(-e^{(bx+a)}+1\right) + 2\,bx\mathrm{Li}_2\left(e^{(bx+a)}\right) - 2\,\mathrm{Li}_3(e^{(bx+a)})}{b^3}$$
$$+ \frac{\log\left(e^{(bx+a)}+1\right)}{b^3} + \frac{\log\left(e^{(bx+a)}-1\right)}{b^3}$$

input

```
integrate(x^2*coth(b*x+a)^3,x, algorithm="maxima")
```

output

```
-2/3*x^3 + 1/3*(b^2*x^3*e^(4*b*x + 4*a) + b^2*x^3 - 2*(b^2*x^3*e^(2*a) + 3
*b*x^2*e^(2*a) + 3*x*e^(2*a))*e^(2*b*x) + 6*x)/(b^2*e^(4*b*x + 4*a) - 2*b^
2*e^(2*b*x + 2*a) + b^2) - 2*x/b^2 + (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x
*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 + (b^2*x^2*log(-e^(
b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3
+ log(e^(b*x + a) + 1)/b^3 + log(e^(b*x + a) - 1)/b^3
```

Giac [F]

$$\int x^2 \coth^3(a+bx)\,dx = \int x^2 \coth\left(bx+a\right)^3\,dx$$

input

```
integrate(x^2*coth(b*x+a)^3,x, algorithm="giac")
```

output

```
integrate(x^2*coth(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^3(a+bx)\,dx = \int x^2 \coth(a+b\,x)^3\,dx$$

input
```
int(x^2*coth(a + b*x)^3,x)
```

output
```
int(x^2*coth(a + b*x)^3, x)
```

Reduce [F]

$$\int x^2 \coth^3(a+bx)\,dx$$
$$= \frac{96e^{4bx+4a}\left(\int \frac{x^2}{e^{6bx+6a}-3e^{4bx+4a}+3e^{2bx+2a}-1}dx\right)b^3 + 144e^{4bx+4a}\left(\int \frac{x}{e^{6bx+6a}-3e^{4bx+4a}+3e^{2bx+2a}-1}dx\right)b^2 + 90e^{4bx+4a}}{}$$

input
```
int(x^2*coth(b*x+a)^3,x)
```

output
```
(96*e**(4*a + 4*b*x)*int(x**2/(e**(6*a + 6*b*x) - 3*e**(4*a + 4*b*x) + 3*e
**(2*a + 2*b*x) - 1),x)*b**3 + 144*e**(4*a + 4*b*x)*int(x/(e**(6*a + 6*b*x
) - 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) - 1),x)*b**2 + 90*e**(4*a + 4*
b*x)*log(e**(a + b*x) - 1) + 90*e**(4*a + 4*b*x)*log(e**(a + b*x) + 1) + 1
6*e**(4*a + 4*b*x)*b**3*x**3 - 180*e**(4*a + 4*b*x)*b*x + 9*e**(4*a + 4*b*
x) - 192*e**(2*a + 2*b*x)*int(x**2/(e**(6*a + 6*b*x) - 3*e**(4*a + 4*b*x)
 + 3*e**(2*a + 2*b*x) - 1),x)*b**3 - 288*e**(2*a + 2*b*x)*int(x/(e**(6*a +
 6*b*x) - 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) - 1),x)*b**2 - 180*e**(2*
a + 2*b*x)*log(e**(a + b*x) - 1) - 180*e**(2*a + 2*b*x)*log(e**(a + b*x) +
 1) - 32*e**(2*a + 2*b*x)*b**3*x**3 - 144*e**(2*a + 2*b*x)*b**2*x**2 + 216
*e**(2*a + 2*b*x)*b*x + 96*int(x**2/(e**(6*a + 6*b*x) - 3*e**(4*a + 4*b*x)
 + 3*e**(2*a + 2*b*x) - 1),x)*b**3 + 144*int(x/(e**(6*a + 6*b*x) - 3*e**(4
*a + 4*b*x) + 3*e**(2*a + 2*b*x) - 1),x)*b**2 + 90*log(e**(a + b*x) - 1) +
 90*log(e**(a + b*x) + 1) + 16*b**3*x**3 + 72*b**2*x**2 - 9)/(48*b**3*(e**
(4*a + 4*b*x) - 2*e**(2*a + 2*b*x) + 1))
```

3.13 $\int x \coth^3(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 82

$$\int x \coth^3(a+bx)\,dx = \frac{x}{2b}-\frac{x^2}{2}-\frac{\coth(a+bx)}{2b^2}-\frac{x\coth^2(a+bx)}{2b}+\frac{x\log\left(1-e^{2(a+bx)}\right)}{b}+\frac{\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{2b^2}$$

output
```
1/2*x/b-1/2*x^2-1/2*coth(b*x+a)/b^2-1/2*x*coth(b*x+a)^2/b+x*ln(1-exp(2*b*x
+2*a))/b+1/2*polylog(2,exp(2*b*x+2*a))/b^2
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.60

$$\int x \coth^3(a+bx)\,dx = \frac{1}{2}\left(-\frac{2x^2}{-1+e^{2a}}+x^2\coth(a)-\frac{x\text{csch}^2(a+bx)}{b}+\frac{2x\log\left(1-e^{-a-bx}\right)}{b}+\frac{2x\log\left(1+e^{-a-bx}\right)}{b}-\frac{2\,\text{PolyLog}\left(2,-e^{-a-bx}\right)}{b^2}-\frac{2\,\text{PolyLog}\left(2,e^{-a-bx}\right)}{b^2}+\frac{\text{csch}(a)\text{csch}(a+bx)\sinh(bx)}{b^2}\right)$$

input
```
Integrate[x*Coth[a + b*x]^3,x]
```

output
```
((-2*x^2)/(-1 + E^(2*a)) + x^2*Coth[a] - (x*Csch[a + b*x]^2)/b + (2*x*Log[
1 - E^(-a - b*x)])/b + (2*x*Log[1 + E^(-a - b*x)])/b - (2*PolyLog[2, -E^(-
a - b*x)])/b^2 - (2*PolyLog[2, E^(-a - b*x)])/b^2 + (Csch[a]*Csch[a + b*x]
*Sinh[b*x])/b^2)/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.37, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.400, Rules used = {3042, 26, 4203, 25, 26, 3042, 25, 26, 3954, 24, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int ix\tan\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^3dx$$

$$\Big\downarrow \text{4203}$$

$$i\left(\frac{i\int -\coth^2(a+bx)dx}{2b}-\int ix\coth(a+bx)dx+\frac{ix\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-\frac{i\int \coth^2(a+bx)dx}{2b}-\int ix\coth(a+bx)dx+\frac{ix\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{i\int\coth^2(a+bx)dx}{2b}-i\int x\coth(a+bx)dx+\frac{ix\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \ 3042$$

$$i\left(-i\int -ix\tan\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{i\int -\tan\left(ia+ibx+\frac{\pi}{2}\right)^2dx}{2b}+\frac{ix\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \ 25$$

$$i\left(-i\int -ix\tan\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{i\int\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx}{2b}+\frac{ix\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \ 26$$

$$i\left(-\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx+\frac{i\int\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx}{2b}+\frac{ix\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \ 3954$$

$$i\left(-\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx+\frac{i\left(\frac{\coth(a+bx)}{b}-\int 1dx\right)}{2b}+\frac{ix\coth^2(a+bx)}{2b}\right)$$

$$\Big\downarrow \ 24$$

$$i\left(-\int x\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx+\frac{ix\coth^2(a+bx)}{2b}+\frac{i\left(\frac{\coth(a+bx)}{b}-x\right)}{2b}\right)$$

$$\Big\downarrow \ 4201$$

$$i\left(-2i\int\frac{e^{2a+2bx-i\pi}x}{1+e^{2a+2bx-i\pi}}dx+\frac{ix\coth^2(a+bx)}{2b}+\frac{i\left(\frac{\coth(a+bx)}{b}-x\right)}{2b}+\frac{ix^2}{2}\right)$$

$$\Big\downarrow \ 2620$$

$$i\left(-2i\left(\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int\log\left(1+e^{2a+2bx-i\pi}\right)dx}{2b}\right)+\frac{ix\coth^2(a+bx)}{2b}+\frac{i\left(\frac{\coth(a+bx)}{b}-x\right)}{2b}+\frac{ix^2}{2}\right)$$

$$\Big\downarrow \ 2715$$

$$i\left(-2i\left(\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}-\frac{\int e^{-2a-2bx+i\pi}\log\left(1+e^{2a+2bx-i\pi}\right)de^{2a+2bx-i\pi}}{4b^2}\right)+\frac{ix\coth^2(a+bx)}{2b}+\right.$$

$$\Big\downarrow \text{2838}$$

$$i\left(-2i\left(\frac{\text{PolyLog}\left(2,-e^{2a+2bx-i\pi}\right)}{4b^2}+\frac{x\log\left(1+e^{2a+2bx-i\pi}\right)}{2b}\right)+\frac{ix\coth^2(a+bx)}{2b}+\frac{i\left(\frac{\coth(a+bx)}{b}-x\right)}{2b}+\frac{ix^2}{2}\right)$$

input
```
Int[x*Coth[a + b*x]^3,x]
```

output
```
I*((I/2)*x^2 + ((I/2)*x*Coth[a + b*x]^2)/b + ((I/2)*(-x + Coth[a + b*x]/b)
)/b - (2*I)*((x*Log[1 + E^(2*a - I*Pi + 2*b*x)])/(2*b) + PolyLog[2, -E^(2*
a - I*Pi + 2*b*x)]/(4*b^2)))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(72) = 144$.

Time = 0.05 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.00

method	result
risch	$-\frac{x^2}{2}-\frac{2x\,\mathrm{e}^{2bx+2a}b+\mathrm{e}^{2bx+2a}-1}{b^2\left(\mathrm{e}^{2bx+2a}-1\right)^2}-\frac{2ax}{b}-\frac{a^2}{b^2}+\frac{\ln\left(1+\mathrm{e}^{bx+a}\right)x}{b}+\frac{\mathrm{polylog}\left(2,-\mathrm{e}^{bx+a}\right)}{b^2}+\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)x}{b}+\frac{\ln\left(1-\mathrm{e}^{bx+a}\right)}{b^2}$

input
```
int(x*coth(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2-(2*x*exp(2*b*x+2*a)*b+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1)^2-
2/b*a*x-1/b^2*a^2+1/b*ln(1+exp(b*x+a))*x+1/b^2*polylog(2,-exp(b*x+a))+1/b*
ln(1-exp(b*x+a))*x+1/b^2*ln(1-exp(b*x+a))*a+1/b^2*polylog(2,exp(b*x+a))-1/
b^2*a*ln(exp(b*x+a)-1)+2/b^2*a*ln(exp(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 975 vs. $2(71) = 142$.

Time = 0.12 (sec) , antiderivative size = 975, normalized size of antiderivative = 11.89

$$\int x\coth^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x*coth(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*((b^2*x^2 - 2*a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 2*a^2)*cosh(b*x + a
)*sinh(b*x + a)^3 + (b^2*x^2 - 2*a^2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x
^2 - 2*a^2 - 2*b*x - 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - 2*a^2)
*cosh(b*x + a)^2 - 2*a^2 - 2*b*x - 1)*sinh(b*x + a)^2 - 2*a^2 - 2*(cosh(b*
x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b
*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 -
cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 2
*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*
(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x
+ a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x
 + a)) - 2*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*
x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x
)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh
(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*(a*cosh(b*x + a)^4 +
 4*a*cosh(b*x + a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 - 2*a*cosh(b*x + a)
^2 + 2*(3*a*cosh(b*x + a)^2 - a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 -
a*cosh(b*x + a))*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - 1)
 - 2*((b*x + a)*cosh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^
3 + (b*x + a)*sinh(b*x + a)^4 - 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x +
a)*cosh(b*x + a)^2 - b*x - a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh...
```

Sympy [F]

$$\int x\coth^3(a+bx)\,dx = \int x\coth^3(a+bx)\,dx$$

input
```
integrate(x*coth(b*x+a)**3,x)
```

output
```
Integral(x*coth(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(71) = 142.

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.82

$$\int x\coth^3(a+bx)\,dx = -x^2 + \frac{b^2x^2e^{(4\,bx+4\,a)} + b^2x^2 - 2\left(b^2x^2e^{(2\,a)} + 2\,bxe^{(2\,a)} + e^{(2\,a)}\right)e^{(2\,bx)} + 2}{2\left(b^2e^{(4\,bx+4\,a)} - 2\,b^2e^{(2\,bx+2\,a)} + b^2\right)} + \frac{bx\log\left(e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(-e^{(bx+a)}\right)}{b^2} + \frac{bx\log\left(-e^{(bx+a)}+1\right) + \mathrm{Li}_2\left(e^{(bx+a)}\right)}{b^2}$$

input
```
integrate(x*coth(b*x+a)^3,x, algorithm="maxima")
```

output
```
-x^2 + 1/2*(b^2*x^2*e^(4*b*x + 4*a) + b^2*x^2 - 2*(b^2*x^2*e^(2*a) + 2*b*x
*e^(2*a) + e^(2*a))*e^(2*b*x) + 2)/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x +
 2*a) + b^2) + (b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 + (b*x
*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
```

Giac [F]

$$\int x \coth^3(a+bx)\,dx=\int x \coth\left(bx+a\right)^3\,dx$$

input
```
integrate(x*coth(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x*coth(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x \coth^3(a+bx)\,dx=\int x\,\coth(a+b\,x)^3\,dx$$

input
```
int(x*coth(a + b*x)^3,x)
```

output
```
int(x*coth(a + b*x)^3, x)
```

Reduce [F]

$$\int x \coth^3(a+bx)\,dx$$
$$=\frac{16e^{4bx+4a}\left(\int \frac{x}{e^{6bx+6a}-3e^{4bx+4a}+3e^{2bx+2a}-1}dx\right)b^2+6e^{4bx+4a}\log\left(e^{bx+a}-1\right)+6e^{4bx+4a}\log\left(e^{bx+a}+1\right)+4e^{4bx+}}{}$$

input
```
int(x*coth(b*x+a)^3,x)
```

output
```
(16*e**(4*a + 4*b*x)*int(x/(e**(6*a + 6*b*x) - 3*e**(4*a + 4*b*x) + 3*e**(
2*a + 2*b*x) - 1),x)*b**2 + 6*e**(4*a + 4*b*x)*log(e**(a + b*x) - 1) + 6*e
**(4*a + 4*b*x)*log(e**(a + b*x) + 1) + 4*e**(4*a + 4*b*x)*b**2*x**2 - 12*
e**(4*a + 4*b*x)*b*x - 3*e**(4*a + 4*b*x) - 32*e**(2*a + 2*b*x)*int(x/(e**
(6*a + 6*b*x) - 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) - 1),x)*b**2 - 12*
e**(2*a + 2*b*x)*log(e**(a + b*x) - 1) - 12*e**(2*a + 2*b*x)*log(e**(a + b
*x) + 1) - 8*e**(2*a + 2*b*x)*b**2*x**2 + 16*int(x/(e**(6*a + 6*b*x) - 3*e
**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) - 1),x)*b**2 + 6*log(e**(a + b*x) - 1
) + 6*log(e**(a + b*x) + 1) + 4*b**2*x**2 + 3)/(8*b**2*(e**(4*a + 4*b*x) -
 2*e**(2*a + 2*b*x) + 1))
```

3.14 $\int \frac{\coth^3(a+bx)}{x} dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\coth^3(a+bx)}{x} dx = \text{Int}\left(\frac{\coth^3(a+bx)}{x}, x\right)$$

output
```
Defer(Int)(coth(b*x+a)^3/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^3(a+bx)}{x} dx = \int \frac{\coth^3(a+bx)}{x} dx$$

input
```
Integrate[Coth[a + b*x]^3/x,x]
```

output
```
Integrate[Coth[a + b*x]^3/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(a+bx)}{x} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i \tan \left(ia+ibx+\frac{\pi}{2}\right)^3}{x}dx$$

$$\Big\downarrow \text{ 26}$$

$$i \int \frac{\tan \left(\frac{1}{2}(2ia+\pi)+ibx\right)^3}{x}dx$$

$$\Big\downarrow \text{ 4222}$$

$$\int -\frac{i \tan^3\left(\frac{1}{2}(-\pi-2ia)-ibx\right)}{x}dx$$

input
```
Int[Coth[a + b*x]^3/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\coth (bx+a)^3}{x}dx$$

input
```
int(coth(b*x+a)^3/x,x)
```

output
```
int(coth(b*x+a)^3/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^3(a+bx)}{x}\,dx=\int \frac{\coth\left(bx+a\right)^3}{x}\,dx$$

input
```
integrate(coth(b*x+a)^3/x,x, algorithm="fricas")
```

output
```
integral(coth(b*x + a)^3/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\coth^3(a+bx)}{x}\,dx=\int \frac{\coth^3\left(a+bx\right)}{x}\,dx$$

input
```
integrate(coth(b*x+a)**3/x,x)
```

output
```
Integral(coth(a + b*x)**3/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 12.00

$$\int \frac{\coth^3(a+bx)}{x}\,dx = \int \frac{\coth(bx+a)^3}{x}\,dx$$

input
```
integrate(coth(b*x+a)^3/x,x, algorithm="maxima")
```

output
```
-((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) + 1)/(b^2*x^2*e^(4*b*x + 4*a) - 2*b^
2*x^2*e^(2*b*x + 2*a) + b^2*x^2) - integrate((b^2*x^2 + 1)/(b^2*x^3*e^(b*x
 + a) + b^2*x^3), x) + integrate((b^2*x^2 + 1)/(b^2*x^3*e^(b*x + a) - b^2*
x^3), x) + log(x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^3(a+bx)}{x}\,dx = \int \frac{\coth(bx+a)^3}{x}\,dx$$

input
```
integrate(coth(b*x+a)^3/x,x, algorithm="giac")
```

output
```
integrate(coth(b*x + a)^3/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^3(a+bx)}{x}\,dx = \int \frac{\coth(a+b\,x)^3}{x}\,dx$$

input
```
int(coth(a + b*x)^3/x,x)
```

output
```
int(coth(a + b*x)^3/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 111, normalized size of antiderivative = 9.25

$$\int \frac{\coth^3(a+bx)}{x}\,dx = 6e^{4a}\left(\int \frac{e^{4bx}}{e^{6bx+6a}x - 3e^{4bx+4a}x + 3e^{2bx+2a}x - x}dx\right) + 2\left(\int \frac{1}{e^{6bx+6a}x - 3e^{4bx+4a}x + 3e^{2bx+2a}x - x}dx\right) + \log(x)$$

input
```
int(coth(b*x+a)^3/x,x)
```

output
```
6*e**(4*a)*int(e**(4*b*x)/(e**(6*a + 6*b*x)*x - 3*e**(4*a + 4*b*x)*x + 3*e
**(2*a + 2*b*x)*x - x),x) + 2*int(1/(e**(6*a + 6*b*x)*x - 3*e**(4*a + 4*b*
x)*x + 3*e**(2*a + 2*b*x)*x - x),x) + log(x)
```

3.15 $\int \frac{\coth^3(a+bx)}{x^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\coth^3(a+bx)}{x^2} dx = \text{Int}\left(\frac{\coth^3(a+bx)}{x^2}, x\right)$$

output
```
Defer(Int)(coth(b*x+a)^3/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^3(a+bx)}{x^2} dx = \int \frac{\coth^3(a+bx)}{x^2} dx$$

input
```
Integrate[Coth[a + b*x]^3/x^2,x]
```

output
```
Integrate[Coth[a + b*x]^3/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(a+bx)}{x^2} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i \tan \left(ia+ibx+\frac{\pi}{2}\right)^3}{x^2}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\tan \left(\frac{1}{2}(2ia+\pi)+ibx\right)^3}{x^2}dx$$

$$\Big\downarrow \text{ 4222}$$

$$\int -\frac{i \tan^3 \left(\frac{1}{2}(-\pi-2ia)-ibx\right)}{x^2}dx$$

input
```
Int[Coth[a + b*x]^3/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\coth \left(bx+a\right)^3}{x^2}dx$$

input
```
int(coth(b*x+a)^3/x^2,x)
```

output
```
int(coth(b*x+a)^3/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^3(a+bx)}{x^2}\,dx = \int \frac{\coth{(bx+a)}^3}{x^2}\,dx$$

input
```
integrate(coth(b*x+a)^3/x^2,x, algorithm="fricas")
```

output
```
integral(coth(b*x + a)^3/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\coth^3(a+bx)}{x^2}\,dx = \int \frac{\coth^3{(a+bx)}}{x^2}\,dx$$

input
```
integrate(coth(b*x+a)**3/x**2,x)
```

output
```
Integral(coth(a + b*x)**3/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 14.58

$$\int \frac{\coth^3(a+bx)}{x^2}\,dx = \int \frac{\coth{(bx+a)}^3}{x^2}\,dx$$

input
```
integrate(coth(b*x+a)^3/x^2,x, algorithm="maxima")
```

output
```
-(b^2*x^2*e^(4*b*x + 4*a) + b^2*x^2 - 2*(b^2*x^2*e^(2*a) - b*x*e^(2*a) + e
^(2*a))*e^(2*b*x) + 2)/(b^2*x^3*e^(4*b*x + 4*a) - 2*b^2*x^3*e^(2*b*x + 2*a
) + b^2*x^3) - integrate((b^2*x^2 + 3)/(b^2*x^4*e^(b*x + a) + b^2*x^4), x)
 + integrate((b^2*x^2 + 3)/(b^2*x^4*e^(b*x + a) - b^2*x^4), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^3(a+bx)}{x^2}\,dx = \int \frac{\coth{(bx+a)}^3}{x^2}\,dx$$

input
```
integrate(coth(b*x+a)^3/x^2,x, algorithm="giac")
```

output
```
integrate(coth(b*x + a)^3/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\coth^3(a+bx)}{x^2}\,dx = \int \frac{\coth(a+b\,x)^3}{x^2}\,dx$$

input
```
int(coth(a + b*x)^3/x^2,x)
```

output
```
int(coth(a + b*x)^3/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 660, normalized size of antiderivative = 55.00

$$\int \frac{\coth^3(a+bx)}{x^2}\,dx = \frac{-12e^{4bx+8a}\left(\int \frac{e^{4bx}}{e^{6bx+6a}x-3e^{4bx+4a}x+3e^{2bx+2a}x-x}dx\right)bx+9e^{4bx+6a}\left(\int \frac{e^{2bx}}{e^{6bx+6a}x^2-3e^{4bx+4a}x^2+3e^{2bx+2a}x^2-x^2}dx\right)x-e}{}$$

input
```
int(coth(b*x+a)^3/x^2,x)
```

output
```
( - 12*e**(8*a + 4*b*x)*int(e**(4*b*x)/(e**(6*a + 6*b*x)*x - 3*e**(4*a + 4
*b*x)*x + 3*e**(2*a + 2*b*x)*x - x),x)*b*x + 9*e**(6*a + 4*b*x)*int(e**(2*
b*x)/(e**(6*a + 6*b*x)*x**2 - 3*e**(4*a + 4*b*x)*x**2 + 3*e**(2*a + 2*b*x)
*x**2 - x**2),x)*x - e**(4*a + 4*b*x)*int(1/(e**(6*a + 6*b*x)*x**2 - 3*e**
(4*a + 4*b*x)*x**2 + 3*e**(2*a + 2*b*x)*x**2 - x**2),x)*x - e**(4*a + 4*b*
x) + 24*e**(6*a + 2*b*x)*int(e**(4*b*x)/(e**(6*a + 6*b*x)*x - 3*e**(4*a +
4*b*x)*x + 3*e**(2*a + 2*b*x)*x - x),x)*b*x - 18*e**(4*a + 2*b*x)*int(e**(
2*b*x)/(e**(6*a + 6*b*x)*x**2 - 3*e**(4*a + 4*b*x)*x**2 + 3*e**(2*a + 2*b*
x)*x**2 - x**2),x)*x + 2*e**(2*a + 2*b*x)*int(1/(e**(6*a + 6*b*x)*x**2 - 3
*e**(4*a + 4*b*x)*x**2 + 3*e**(2*a + 2*b*x)*x**2 - x**2),x)*x - 4*e**(2*a
+ 2*b*x) - 12*e**(4*a)*int(e**(4*b*x)/(e**(6*a + 6*b*x)*x - 3*e**(4*a + 4*
b*x)*x + 3*e**(2*a + 2*b*x)*x - x),x)*b*x + 9*e**(2*a)*int(e**(2*b*x)/(e**
(6*a + 6*b*x)*x**2 - 3*e**(4*a + 4*b*x)*x**2 + 3*e**(2*a + 2*b*x)*x**2 - x
**2),x)*x - int(1/(e**(6*a + 6*b*x)*x**2 - 3*e**(4*a + 4*b*x)*x**2 + 3*e**
(2*a + 2*b*x)*x**2 - x**2),x)*x + 2)/(x*(e**(4*a + 4*b*x) - 2*e**(2*a + 2*
b*x) + 1))
```

3.16 $\int \frac{(c+dx)^3}{a+a\coth(e+fx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 169

$$\begin{aligned}\int \frac{(c+dx)^3}{a+a\coth(e+fx)}\,dx &= \frac{3d^3x}{8af^3}+\frac{3d(c+dx)^2}{8af^2}+\frac{(c+dx)^3}{4af}+\frac{(c+dx)^4}{8ad}\\ &\quad-\frac{3d^3}{8f^4(a+a\coth(e+fx))}-\frac{3d^2(c+dx)}{4f^3(a+a\coth(e+fx))}\\ &\quad-\frac{3d(c+dx)^2}{4f^2(a+a\coth(e+fx))}-\frac{(c+dx)^3}{2f(a+a\coth(e+fx))}\end{aligned}$$

output

```
3/8*d^3*x/a/f^3+3/8*d*(d*x+c)^2/a/f^2+1/4*(d*x+c)^3/a/f+1/8*(d*x+c)^4/a/d-
3/8*d^3/f^4/(a+a*coth(f*x+e))-3/4*d^2*(d*x+c)/f^3/(a+a*coth(f*x+e))-3/4*d*
(d*x+c)^2/f^2/(a+a*coth(f*x+e))-1/2*(d*x+c)^3/f/(a+a*coth(f*x+e))
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.44

$$\int \frac{(c+dx)^3}{a+a\coth(e+fx)}\,dx$$

$$= \frac{\operatorname{csch}(e+fx)(\cosh(fx)+\sinh(fx))\left((4c^3f^3+6c^2df^2(1+2fx)+6cd^2f(1+2fx+2f^2x^2)+d^3(3+6fx\right.}{}$$

input
```
Integrate[(c + d*x)^3/(a + a*Coth[e + f*x]),x]
```

output
```
(Csch[e + f*x]*(Cosh[f*x] + Sinh[f*x])*((4*c^3*f^3 + 6*c^2*d*f^2*(1 + 2*f*
x) + 6*c*d^2*f*(1 + 2*f*x + 2*f^2*x^2) + d^3*(3 + 6*f*x + 6*f^2*x^2 + 4*f^
3*x^3))*Cosh[2*f*x]*(Cosh[e] - Sinh[e]) + 2*f^4*x*(4*c^3 + 6*c^2*d*x + 4*c
*d^2*x^2 + d^3*x^3)*(Cosh[e] + Sinh[e]) + (4*c^3*f^3 + 6*c^2*d*f^2*(1 + 2*
f*x) + 6*c*d^2*f*(1 + 2*f*x + 2*f^2*x^2) + d^3*(3 + 6*f*x + 6*f^2*x^2 + 4*
f^3*x^3))*(-Cosh[e] + Sinh[e])*Sinh[2*f*x]))/(16*a*f^4*(1 + Coth[e + f*x])
)
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3042, 4206, 3042, 4206, 3042, 4206, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{a\coth(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^3}{a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4206}$$

$$\frac{3d\int \frac{(c+dx)^2}{\coth(e+fx)a+a}dx}{2f}-\frac{(c+dx)^3}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^4}{8ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d\int \frac{(c+dx)^2}{a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)}dx}{2f}-\frac{(c+dx)^3}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^4}{8ad}$$

$$\Big\downarrow \text{4206}$$

$$\frac{3d\left(\frac{d\int \frac{c+dx}{\coth(e+fx)a+a}dx}{f}-\frac{(c+dx)^2}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^3}{6ad}\right)}{2f}-\frac{(c+dx)^3}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^4}{8ad}$$

$$\Big\downarrow \ 3042$$

$$\frac{3d\left(\frac{d\int \frac{c+dx}{a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)}dx}{f}-\frac{(c+dx)^2}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^3}{6ad}\right)}{2f}-\frac{(c+dx)^3}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^4}{8ad}$$

$$\Big\downarrow \ 4206$$

$$\frac{3d\left(\frac{d\left(\frac{d\int \frac{1}{\coth(e+fx)a+a}dx}{2f}-\frac{c+dx}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^2}{4ad}\right)}{f}-\frac{(c+dx)^2}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^3}{6ad}\right)}{2f}-\frac{(c+dx)^3}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^4}{8ad}$$

$$\Big\downarrow \ 3042$$

$$\frac{3d\left(\frac{d\left(\frac{d\int \frac{1}{a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)}dx}{2f}-\frac{c+dx}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^2}{4ad}\right)}{f}-\frac{(c+dx)^2}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^3}{6ad}\right)}{2f}-\frac{(c+dx)^3}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^4}{8ad}$$

$$\Big\downarrow \ 3960$$

$$\frac{3d\left(\frac{d\left(\frac{d\left(\frac{\int 1dx}{2a}-\frac{1}{2f(a\coth(e+fx)+a)}\right)}{2f}-\frac{c+dx}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^2}{4ad}\right)}{f}-\frac{(c+dx)^2}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^3}{6ad}\right)}{2f}-\frac{(c+dx)^3}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^4}{8ad}$$

$$\Big\downarrow \ 24$$

$$-\frac{(c+dx)^3}{2f(a\coth(e+fx)+a)}+\frac{3d\left(-\frac{(c+dx)^2}{2f(a\coth(e+fx)+a)}+\frac{d\left(-\frac{c+dx}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^2}{4ad}+\frac{d\left(\frac{x}{2a}-\frac{1}{2f(a\coth(e+fx)+a)}\right)}{2f}\right)}{f}+\frac{(c+dx)^3}{6ad}\right)}{2f}+\frac{(c+dx)^4}{8ad}$$

input
```
Int[(c + d*x)^3/(a + a*Coth[e + f*x]),x]
```

output
```
(c + d*x)^4/(8*a*d) - (c + d*x)^3/(2*f*(a + a*Coth[e + f*x])) + (3*d*((c +
 d*x)^3/(6*a*d) - (c + d*x)^2/(2*f*(a + a*Coth[e + f*x])) + (d*((c + d*x)^
2/(4*a*d) - (c + d*x)/(2*f*(a + a*Coth[e + f*x])) + (d*(x/(2*a) - 1/(2*f*(
a + a*Coth[e + f*x]))))/(2*f)))/f))/(2*f)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 4206
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Simp[a*d*(m/(2*b*f))
   Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[a*((c + d*x)^m
/(2*b*f*(a + b*Tan[e + f*x]))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
a^2 + b^2, 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.98

method	result
risch	$\frac{d^3x^4}{8a}+\frac{d^2c\,x^3}{2a}+\frac{3d\,c^2x^2}{4a}+\frac{c^3x}{2a}+\frac{c^4}{8ad}+\frac{(4d^3x^3f^3+12c\,d^2f^3x^2+12c^2d\,f^3x+6d^3f^2x^2+4c^3f^3+12c\,d^2f^2x+6c}{16f^4a}$
parallelrisch	$\frac{((d^3x^4+4c\,d^2x^3+6c^2d\,x^2+4c^3x)f^4+(-2d^3x^3-6c\,d^2x^2-6c^2dx-4c^3)f^3+(-3d^3x^2-6c\,d^2x-6c^2d)f^2+(-3d^3x-6c\,d^2}{8f^4a(\tanh(fx+}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^3/(a+a*coth(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/8/a*d^3*x^4+1/2/a*d^2*c*x^3+3/4/a*d*c^2*x^2+1/2/a*c^3*x+1/8/a/d*c^4+1/16
*(4*d^3*f^3*x^3+12*c*d^2*f^3*x^2+12*c^2*d*f^3*x+6*d^3*f^2*x^2+4*c^3*f^3+12
*c*d^2*f^2*x+6*c^2*d*f^2+6*d^3*f*x+6*c*d^2*f+3*d^3)/f^4/a*exp(-2*f*x-2*e)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.80

$$\int \frac{(c+dx)^3}{a+a\coth(e+fx)}\,dx$$
$$=\frac{(2\,d^3f^4x^4+4\,c^3f^3+6\,c^2df^2+6\,cd^2f+4\,(2\,cd^2f^4+d^3f^3)x^3+3\,d^3+6\,(2\,c^2df^4+2\,cd^2f^3+d^3f^2)x^2+2}{}$$

input
```
integrate((d*x+c)^3/(a+a*coth(f*x+e)),x, algorithm="fricas")
```

output
```
1/16*((2*d^3*f^4*x^4 + 4*c^3*f^3 + 6*c^2*d*f^2 + 6*c*d^2*f + 4*(2*c*d^2*f^
4 + d^3*f^3)*x^3 + 3*d^3 + 6*(2*c^2*d*f^4 + 2*c*d^2*f^3 + d^3*f^2)*x^2 + 2
*(4*c^3*f^4 + 6*c^2*d*f^3 + 6*c*d^2*f^2 + 3*d^3*f)*x)*cosh(f*x + e) + (2*d
^3*f^4*x^4 - 4*c^3*f^3 - 6*c^2*d*f^2 - 6*c*d^2*f + 4*(2*c*d^2*f^4 - d^3*f^
3)*x^3 - 3*d^3 + 6*(2*c^2*d*f^4 - 2*c*d^2*f^3 - d^3*f^2)*x^2 + 2*(4*c^3*f^
4 - 6*c^2*d*f^3 - 6*c*d^2*f^2 - 3*d^3*f)*x)*sinh(f*x + e))/(a*f^4*cosh(f*x
 + e) + a*f^4*sinh(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 864 vs. 2(144) = 288.

Time = 0.74 (sec) , antiderivative size = 864, normalized size of antiderivative = 5.11

$$\int \frac{(c+dx)^3}{a+a\coth(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3/(a+a*coth(f*x+e)),x)
```

output
```
Piecewise((4*c**3*f**4*x*tanh(e + f*x)/(8*a*f**4*tanh(e + f*x) + 8*a*f**4)
 + 4*c**3*f**4*x/(8*a*f**4*tanh(e + f*x) + 8*a*f**4) + 4*c**3*f**3/(8*a*f*
*4*tanh(e + f*x) + 8*a*f**4) + 6*c**2*d*f**4*x**2*tanh(e + f*x)/(8*a*f**4*
tanh(e + f*x) + 8*a*f**4) + 6*c**2*d*f**4*x**2/(8*a*f**4*tanh(e + f*x) + 8
*a*f**4) - 6*c**2*d*f**3*x*tanh(e + f*x)/(8*a*f**4*tanh(e + f*x) + 8*a*f**
4) + 6*c**2*d*f**3*x/(8*a*f**4*tanh(e + f*x) + 8*a*f**4) + 6*c**2*d*f**2/(
8*a*f**4*tanh(e + f*x) + 8*a*f**4) + 4*c*d**2*f**4*x**3*tanh(e + f*x)/(8*a
*f**4*tanh(e + f*x) + 8*a*f**4) + 4*c*d**2*f**4*x**3/(8*a*f**4*tanh(e + f*
x) + 8*a*f**4) - 6*c*d**2*f**3*x**2*tanh(e + f*x)/(8*a*f**4*tanh(e + f*x)
+ 8*a*f**4) + 6*c*d**2*f**3*x**2/(8*a*f**4*tanh(e + f*x) + 8*a*f**4) - 6*c
*d**2*f**2*x*tanh(e + f*x)/(8*a*f**4*tanh(e + f*x) + 8*a*f**4) + 6*c*d**2*
f**2*x/(8*a*f**4*tanh(e + f*x) + 8*a*f**4) + 6*c*d**2*f/(8*a*f**4*tanh(e +
 f*x) + 8*a*f**4) + d**3*f**4*x**4*tanh(e + f*x)/(8*a*f**4*tanh(e + f*x) +
 8*a*f**4) + d**3*f**4*x**4/(8*a*f**4*tanh(e + f*x) + 8*a*f**4) - 2*d**3*f
**3*x**3*tanh(e + f*x)/(8*a*f**4*tanh(e + f*x) + 8*a*f**4) + 2*d**3*f**3*x
**3/(8*a*f**4*tanh(e + f*x) + 8*a*f**4) - 3*d**3*f**2*x**2*tanh(e + f*x)/(
8*a*f**4*tanh(e + f*x) + 8*a*f**4) + 3*d**3*f**2*x**2/(8*a*f**4*tanh(e + f
*x) + 8*a*f**4) - 3*d**3*f*x*tanh(e + f*x)/(8*a*f**4*tanh(e + f*x) + 8*a*f
**4) + 3*d**3*f*x/(8*a*f**4*tanh(e + f*x) + 8*a*f**4) + 3*d**3/(8*a*f**4*t
anh(e + f*x) + 8*a*f**4), Ne(f, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**...
```

Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^3}{a+a\coth(e+fx)}\,dx = \frac{1}{4}c^3\left(\frac{2(fx+e)}{af}+\frac{e^{(-2fx-2e)}}{af}\right)+\frac{3\left(2f^2x^2e^{(2e)}+(2fx+1)e^{(-2fx)}\right)c^2de^{(-2e)}}{8af^2}+\frac{\left(4f^3x^3e^{(2e)}+3(2f^2x^2+2fx+1)e^{(-2fx)}\right)cd^2e^{(-2e)}}{8af^3}+\frac{\left(2f^4x^4e^{(2e)}+(4f^3x^3+6f^2x^2+6fx+3)e^{(-2fx)}\right)d^3e^{(-2e)}}{16af^4}$$

input
```
integrate((d*x+c)^3/(a+a*coth(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*c^3*(2*(f*x + e)/(a*f) + e^(-2*f*x - 2*e)/(a*f)) + 3/8*(2*f^2*x^2*e^(2
*e) + (2*f*x + 1)*e^(-2*f*x))*c^2*d*e^(-2*e)/(a*f^2) + 1/8*(4*f^3*x^3*e^(2
*e) + 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x))*c*d^2*e^(-2*e)/(a*f^3) + 1/16*
(2*f^4*x^4*e^(2*e) + (4*f^3*x^3 + 6*f^2*x^2 + 6*f*x + 3)*e^(-2*f*x))*d^3*e
^(-2*e)/(a*f^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.11

$$\int \frac{(c+dx)^3}{a+a\coth(e+fx)}\,dx = \frac{\left(2d^3f^4x^4e^{(2fx+2e)}+8cd^2f^4x^3e^{(2fx+2e)}+12c^2df^4x^2e^{(2fx+2e)}+4d^3f^3x^3+8c^3f^4xe^{(2fx+2e)}+12cd^2f^3\right.}{16af^4}$$

input
```
integrate((d*x+c)^3/(a+a*coth(f*x+e)),x, algorithm="giac")
```

output
```
1/16*(2*d^3*f^4*x^4*e^(2*f*x + 2*e) + 8*c*d^2*f^4*x^3*e^(2*f*x + 2*e) + 12
*c^2*d*f^4*x^2*e^(2*f*x + 2*e) + 4*d^3*f^3*x^3 + 8*c^3*f^4*x*e^(2*f*x + 2*
e) + 12*c*d^2*f^3*x^2 + 12*c^2*d*f^3*x + 6*d^3*f^2*x^2 + 4*c^3*f^3 + 12*c*
d^2*f^2*x + 6*c^2*d*f^2 + 6*d^3*f*x + 6*c*d^2*f + 3*d^3)*e^(-2*f*x - 2*e)/
(a*f^4)
```

Mupad [B] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.32

$$\int \frac{(c+dx)^3}{a+a\coth(e+fx)}\,dx$$
$$= \frac{4\,c^3\,f^4\,x+6\,c^2\,d\,f^4\,x^2+6\,c^2\,d\,f^3\,x+4\,c\,d^2\,f^4\,x^3+6\,c\,d^2\,f^3\,x^2+6\,c\,d^2\,f^2\,x+d^3\,f^4\,x^4+2\,d^3\,f^3\,x^3+3\,d^3}{8\,a\,f^4}$$
$$-\frac{4\,c^3\,f^3+12\,c^2\,d\,f^3\,x+6\,c^2\,d\,f^2+12\,c\,d^2\,f^3\,x^2+12\,c\,d^2\,f^2\,x+6\,c\,d^2\,f+4\,d^3\,f^3\,x^3+6\,d^3\,f^2\,x^2+6\,d^3}{8\,a\,f^4\,\left(\coth\left(e+f\,x\right)+1\right)}$$

input
```
int((c + d*x)^3/(a + a*coth(e + f*x)),x)
```

output
```
(3*d^3*f*x + 4*c^3*f^4*x + 3*d^3*f^2*x^2 + 2*d^3*f^3*x^3 + d^3*f^4*x^4 + 6
*c*d^2*f^3*x^2 + 6*c^2*d*f^4*x^2 + 4*c*d^2*f^4*x^3 + 6*c*d^2*f^2*x + 6*c^2
*d*f^3*x)/(8*a*f^4) - (3*d^3 + 4*c^3*f^3 + 6*d^3*f*x + 6*c^2*d*f^2 + 6*d^3
*f^2*x^2 + 4*d^3*f^3*x^3 + 6*c*d^2*f + 12*c*d^2*f^3*x^2 + 12*c*d^2*f^2*x +
 12*c^2*d*f^3*x)/(8*a*f^4*(coth(e + f*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.15

$$\int \frac{(c+dx)^3}{a+a\coth(e+fx)}\,dx$$
$$= \frac{8e^{2fx+2e}c^3f^4x+12e^{2fx+2e}c^2d\,f^4x^2+8e^{2fx+2e}c\,d^2f^4x^3+2e^{2fx+2e}d^3f^4x^4+4c^3f^3+12c^2d\,f^3x+6c^2d\,f^2}{16e^{2fx+2e}a\,f^4}$$

input
```
int((d*x+c)^3/(a+a*coth(f*x+e)),x)
```

output
```
(8*e**(2*e + 2*f*x)*c**3*f**4*x + 12*e**(2*e + 2*f*x)*c**2*d*f**4*x**2 + 8
*e**(2*e + 2*f*x)*c*d**2*f**4*x**3 + 2*e**(2*e + 2*f*x)*d**3*f**4*x**4 + 4
*c**3*f**3 + 12*c**2*d*f**3*x + 6*c**2*d*f**2 + 12*c*d**2*f**3*x**2 + 12*c
*d**2*f**2*x + 6*c*d**2*f + 4*d**3*f**3*x**3 + 6*d**3*f**2*x**2 + 6*d**3*f
*x + 3*d**3)/(16*e**(2*e + 2*f*x)*a*f**4)
```

3.17 $\int \frac{(c+dx)^2}{a+a\coth(e+fx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 122

$$\int \frac{(c+dx)^2}{a+a\coth(e+fx)}\,dx = \frac{d^2x}{4af^2}+\frac{(c+dx)^2}{4af}+\frac{(c+dx)^3}{6ad}-\frac{d^2}{4f^3(a+a\coth(e+fx))}-\frac{d(c+dx)}{2f^2(a+a\coth(e+fx))}-\frac{(c+dx)^2}{2f(a+a\coth(e+fx))}$$

output
```
1/4*d^2*x/a/f^2+1/4*(d*x+c)^2/a/f+1/6*(d*x+c)^3/a/d-1/4*d^2/f^3/(a+a*coth(
f*x+e))-1/2*d*(d*x+c)/f^2/(a+a*coth(f*x+e))-1/2*(d*x+c)^2/f/(a+a*coth(f*x+
e))
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.39

$$\int \frac{(c+dx)^2}{a+a\coth(e+fx)}\,dx = \frac{\operatorname{csch}(e+fx)(\cosh(fx)+\sinh(fx))\left((2c^2f^2+2cdf(1+2fx)+d^2(1+2fx+2f^2x^2))\cosh(2fx)(\cosh(e\right.}{}$$

input
```
Integrate[(c + d*x)^2/(a + a*Coth[e + f*x]),x]
```

output
```
(Csch[e + f*x]*(Cosh[f*x] + Sinh[f*x])*((2*c^2*f^2 + 2*c*d*f*(1 + 2*f*x) +
 d^2*(1 + 2*f*x + 2*f^2*x^2))*Cosh[2*f*x]*(Cosh[e] - Sinh[e]) + (4*f^3*x*(
3*c^2 + 3*c*d*x + d^2*x^2)*(Cosh[e] + Sinh[e]))/3 + (2*c^2*f^2 + 2*c*d*f*(
1 + 2*f*x) + d^2*(1 + 2*f*x + 2*f^2*x^2))*(-Cosh[e] + Sinh[e])*Sinh[2*f*x]
))/(8*a*f^3*(1 + Coth[e + f*x]))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3042, 4206, 3042, 4206, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{a\coth(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^2}{a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4206}$$

$$\frac{d\int \frac{c+dx}{\coth(e+fx)a+a}dx}{f}-\frac{(c+dx)^2}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^3}{6ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d\int \frac{c+dx}{a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)}dx}{f}-\frac{(c+dx)^2}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^3}{6ad}$$

$$\Big\downarrow \text{4206}$$

$$\frac{d\left(\frac{d\int \frac{1}{\coth(e+fx)a+a}dx}{2f}-\frac{c+dx}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^2}{4ad}\right)}{f}-\frac{(c+dx)^2}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^3}{6ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d\left(\frac{d\int \frac{1}{a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)}dx}{2f}-\frac{c+dx}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^2}{4ad}\right)}{f}-\frac{(c+dx)^2}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^3}{6ad}$$

$$\Big\downarrow \; 3960$$

$$\frac{d\left(\frac{d\left(\frac{\int 1dx}{2a}-\frac{1}{2f(a\coth(e+fx)+a)}\right)}{2f}-\frac{c+dx}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^2}{4ad}\right)}{f}-\frac{(c+dx)^2}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^3}{6ad}$$

$$\Big\downarrow \; 24$$

$$-\frac{(c+dx)^2}{2f(a\coth(e+fx)+a)}+\frac{d\left(-\frac{c+dx}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^2}{4ad}+\frac{d\left(\frac{x}{2a}-\frac{1}{2f(a\coth(e+fx)+a)}\right)}{2f}\right)}{f}+\frac{(c+dx)^3}{6ad}$$

input
```
Int[(c + d*x)^2/(a + a*Coth[e + f*x]),x]
```

output
```
(c + d*x)^3/(6*a*d) - (c + d*x)^2/(2*f*(a + a*Coth[e + f*x])) + (d*((c + d
*x)^2/(4*a*d) - (c + d*x)/(2*f*(a + a*Coth[e + f*x])) + (d*(x/(2*a) - 1/(2
*f*(a + a*Coth[e + f*x]))))/(2*f)))/f
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 4206
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Simp[a*d*(m/(2*b*f))
   Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[a*((c + d*x)^m
/(2*b*f*(a + b*Tan[e + f*x]))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
a^2 + b^2, 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.84

method	result
risch	$\frac{d^2x^3}{6a}+\frac{dc\,x^2}{2a}+\frac{c^2x}{2a}+\frac{c^3}{6ad}+\frac{(2d^2x^2f^2+4cd\,f^2x+2c^2f^2+2d^2fx+2cdf+d^2)\,\mathrm{e}^{-2fx-2e}}{8f^3a}$
parallelrisch	$\frac{((2d^2x^3+6cd\,x^2+6c^2x)f^3+(-3d^2x^2-6cdx-6c^2)f^2+(-3d^2x-6cd)f-3d^2)\tanh(fx+e)+6f\left(\left(\frac{1}{3}d^2x^2+cdx+c^2\right)f^2+\right.}{12f^3a(\tanh(fx+e)+1)}$
derivativedivides	$\frac{-c^2f^2\left(\frac{\sinh(fx+e)\cosh(fx+e)}{2}-\frac{fx}{2}-\frac{e}{2}\right)+2decf\left(\frac{\sinh(fx+e)\cosh(fx+e)}{2}-\frac{fx}{2}-\frac{e}{2}\right)-2dcf\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}\right.}{}$
default	$\frac{-c^2f^2\left(\frac{\sinh(fx+e)\cosh(fx+e)}{2}-\frac{fx}{2}-\frac{e}{2}\right)+2decf\left(\frac{\sinh(fx+e)\cosh(fx+e)}{2}-\frac{fx}{2}-\frac{e}{2}\right)-2dcf\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}\right.}{}$

input
```
int((d*x+c)^2/(a+a*coth(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/6/a*d^2*x^3+1/2/a*d*c*x^2+1/2/a*c^2*x+1/6/a/d*c^3+1/8*(2*d^2*f^2*x^2+4*c
*d*f^2*x+2*c^2*f^2+2*d^2*f*x+2*c*d*f+d^2)/f^3/a*exp(-2*f*x-2*e)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.57

$$\int \frac{(c+dx)^2}{a+a\coth(e+fx)}\,dx = \frac{(4\,d^2f^3x^3+6\,c^2f^2+6\,cdf+6\,(2\,cdf^3+d^2f^2)x^2+3\,d^2+6\,(2\,c^2f^3+2\,cdf^2+d^2f)x)\cosh\left(fx+e\right)+(4}{24\,(af^3\cosh\left(fx+e\right)+a}$$

input
```
integrate((d*x+c)^2/(a+a*coth(f*x+e)),x, algorithm="fricas")
```

output
```
1/24*((4*d^2*f^3*x^3 + 6*c^2*f^2 + 6*c*d*f + 6*(2*c*d*f^3 + d^2*f^2)*x^2 +
 3*d^2 + 6*(2*c^2*f^3 + 2*c*d*f^2 + d^2*f)*x)*cosh(f*x + e) + (4*d^2*f^3*x
^3 - 6*c^2*f^2 - 6*c*d*f + 6*(2*c*d*f^3 - d^2*f^2)*x^2 - 3*d^2 + 6*(2*c^2*
f^3 - 2*c*d*f^2 - d^2*f)*x)*sinh(f*x + e))/(a*f^3*cosh(f*x + e) + a*f^3*si
nh(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(95) = 190$.

Time = 0.64 (sec) , antiderivative size = 522, normalized size of antiderivative = 4.28

$$\int \frac{(c+dx)^2}{a+a\coth(e+fx)}\,dx$$
$$= \begin{cases} \frac{6c^2f^3x\tanh(e+fx)}{12af^3\tanh(e+fx)+12af^3} + \frac{6c^2f^3x}{12af^3\tanh(e+fx)+12af^3} + \frac{6c^2f^2}{12af^3\tanh(e+fx)+12af^3} + \frac{6cdf^3x^2\tanh(e+fx)}{12af^3\tanh(e+fx)+12af^3} + \frac{6cd\ldots}{12af^3\tanh(\ldots} \\ \frac{c^2x+cdx^2+\frac{d^2x^3}{3}}{a\coth(e)+a} \end{cases}$$

input
```
integrate((d*x+c)**2/(a+a*coth(f*x+e)),x)
```

output
```
Piecewise((6*c**2*f**3*x*tanh(e + f*x)/(12*a*f**3*tanh(e + f*x) + 12*a*f**
3) + 6*c**2*f**3*x/(12*a*f**3*tanh(e + f*x) + 12*a*f**3) + 6*c**2*f**2/(12
*a*f**3*tanh(e + f*x) + 12*a*f**3) + 6*c*d*f**3*x**2*tanh(e + f*x)/(12*a*f
**3*tanh(e + f*x) + 12*a*f**3) + 6*c*d*f**3*x**2/(12*a*f**3*tanh(e + f*x)
+ 12*a*f**3) - 6*c*d*f**2*x*tanh(e + f*x)/(12*a*f**3*tanh(e + f*x) + 12*a*
f**3) + 6*c*d*f**2*x/(12*a*f**3*tanh(e + f*x) + 12*a*f**3) + 6*c*d*f/(12*a
*f**3*tanh(e + f*x) + 12*a*f**3) + 2*d**2*f**3*x**3*tanh(e + f*x)/(12*a*f*
*3*tanh(e + f*x) + 12*a*f**3) + 2*d**2*f**3*x**3/(12*a*f**3*tanh(e + f*x)
+ 12*a*f**3) - 3*d**2*f**2*x**2*tanh(e + f*x)/(12*a*f**3*tanh(e + f*x) + 1
2*a*f**3) + 3*d**2*f**2*x**2/(12*a*f**3*tanh(e + f*x) + 12*a*f**3) - 3*d**
2*f*x*tanh(e + f*x)/(12*a*f**3*tanh(e + f*x) + 12*a*f**3) + 3*d**2*f*x/(12
*a*f**3*tanh(e + f*x) + 12*a*f**3) + 3*d**2/(12*a*f**3*tanh(e + f*x) + 12*
a*f**3), Ne(f, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)/(a*coth(e) + a), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx)^2}{a+a\coth(e+fx)}\,dx = \frac{1}{4}\,c^2\left(\frac{2\,(fx+e)}{af}+\frac{e^{(-2\,fx-2\,e)}}{af}\right) + \frac{\left(2\,f^2x^2e^{(2\,e)}+(2\,fx+1)e^{(-2\,fx)}\right)cde^{(-2\,e)}}{4\,af^2} + \frac{\left(4\,f^3x^3e^{(2\,e)}+3\,(2\,f^2x^2+2\,fx+1)e^{(-2\,fx)}\right)d^2e^{(-2\,e)}}{24\,af^3}$$

input
```
integrate((d*x+c)^2/(a+a*coth(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*c^2*(2*(f*x + e)/(a*f) + e^(-2*f*x - 2*e)/(a*f)) + 1/4*(2*f^2*x^2*e^(2
*e) + (2*f*x + 1)*e^(-2*f*x))*c*d*e^(-2*e)/(a*f^2) + 1/24*(4*f^3*x^3*e^(2*
e) + 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x))*d^2*e^(-2*e)/(a*f^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx)^2}{a+a\coth(e+fx)}\,dx = \frac{\left(4\,d^2f^3x^3e^{(2\,fx+2\,e)}+12\,cdf^3x^2e^{(2\,fx+2\,e)}+12\,c^2f^3xe^{(2\,fx+2\,e)}+6\,d^2f^2x^2+12\,cdf^2x+6\,c^2f^2+6\,d^2fx+\right.}{24\,af^3}$$

input
```
integrate((d*x+c)^2/(a+a*coth(f*x+e)),x, algorithm="giac")
```

output
```
1/24*(4*d^2*f^3*x^3*e^(2*f*x + 2*e) + 12*c*d*f^3*x^2*e^(2*f*x + 2*e) + 12*
c^2*f^3*x*e^(2*f*x + 2*e) + 6*d^2*f^2*x^2 + 12*c*d*f^2*x + 6*c^2*f^2 + 6*d
^2*f*x + 6*c*d*f + 3*d^2)*e^(-2*f*x - 2*e)/(a*f^3)
```

Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.52

$$\int \frac{(c+dx)^2}{a+a\coth(e+fx)}\,dx = \frac{\mathrm{e}^{-2\,e-2\,f\,x}\,\left(12\,c^2\,x\,\mathrm{e}^{2\,e+2\,f\,x}+4\,d^2\,x^3\,\mathrm{e}^{2\,e+2\,f\,x}+12\,c\,d\,x^2\,\mathrm{e}^{2\,e+2\,f\,x}\right)}{24\,a} + \frac{\frac{\mathrm{e}^{-2\,e-2\,f\,x}\,\left(3\,d^2+3\,d^2\,\mathrm{e}^{2\,e+2\,f\,x}\right)}{24}+\frac{f\,\mathrm{e}^{-2\,e-2\,f\,x}\,\left(6\,c\,d+6\,d^2\,x+6\,c\,d\,\mathrm{e}^{2\,e+2\,f\,x}\right)}{24}+\frac{f^2\,\mathrm{e}^{-2\,e-2\,f\,x}\,\left(6\,c^2+6\,c^2\,\mathrm{e}^{2\,e+2\,f\,x}+6\,d^2\,x^2+12\,c\,d\,x\right)}{24}}{a\,f^3}$$

input
```
int((c + d*x)^2/(a + a*coth(e + f*x)),x)
```

output
```
(exp(- 2*e - 2*f*x)*(12*c^2*x*exp(2*e + 2*f*x) + 4*d^2*x^3*exp(2*e + 2*f*x
) + 12*c*d*x^2*exp(2*e + 2*f*x)))/(24*a) + ((exp(- 2*e - 2*f*x)*(3*d^2 + 3
*d^2*exp(2*e + 2*f*x)))/24 + (f*exp(- 2*e - 2*f*x)*(6*c*d + 6*d^2*x + 6*c*
d*exp(2*e + 2*f*x)))/24 + (f^2*exp(- 2*e - 2*f*x)*(6*c^2 + 6*c^2*exp(2*e +
 2*f*x) + 6*d^2*x^2 + 12*c*d*x))/24)/(a*f^3)
```

Reduce [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx)^2}{a+a\coth(e+fx)}\,dx = \frac{12e^{2fx+2e}c^2f^3x+12e^{2fx+2e}cd\,f^3x^2+4e^{2fx+2e}d^2f^3x^3+6c^2f^2+12cd\,f^2x+6cdf+6d^2f^2x^2+6d^2fx+3}{24e^{2fx+2e}a\,f^3}$$

input
```
int((d*x+c)^2/(a+a*coth(f*x+e)),x)
```

output
```
(12*e**(2*e + 2*f*x)*c**2*f**3*x + 12*e**(2*e + 2*f*x)*c*d*f**3*x**2 + 4*e
**(2*e + 2*f*x)*d**2*f**3*x**3 + 6*c**2*f**2 + 12*c*d*f**2*x + 6*c*d*f + 6
*d**2*f**2*x**2 + 6*d**2*f*x + 3*d**2)/(24*e**(2*e + 2*f*x)*a*f**3)
```

3.18 $\int \frac{c+dx}{a+a\coth(e+fx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 74

$$\int \frac{c+dx}{a+a\coth(e+fx)}\,dx = \frac{dx}{4af}+\frac{(c+dx)^2}{4ad}-\frac{d}{4f^2(a+a\coth(e+fx))} - \frac{c+dx}{2f(a+a\coth(e+fx))}$$

output
```
1/4*d*x/a/f+1/4*(d*x+c)^2/a/d-1/4*d/f^2/(a+a*coth(f*x+e))-1/2*(d*x+c)/f/(a
+a*coth(f*x+e))
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.09

$$\int \frac{c+dx}{a+a\coth(e+fx)}\,dx = \frac{2cf(-1+2fx)+d(-1-2fx+2f^2x^2)+(2cf(1+2fx)+d(1+2fx+2f^2x^2))\coth(e+fx)}{8af^2(1+\coth(e+fx))}$$

input
```
Integrate[(c + d*x)/(a + a*Coth[e + f*x]),x]
```

output
```
(2*c*f*(-1 + 2*f*x) + d*(-1 - 2*f*x + 2*f^2*x^2) + (2*c*f*(1 + 2*f*x) + d*
(1 + 2*f*x + 2*f^2*x^2))*Coth[e + f*x])/(8*a*f^2*(1 + Coth[e + f*x]))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 4206, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a\coth(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+dx}{a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4206}$$

$$\frac{d\int \frac{1}{\coth(e+fx)a+a}dx}{2f}-\frac{c+dx}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^2}{4ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d\int \frac{1}{a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)}dx}{2f}-\frac{c+dx}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^2}{4ad}$$

$$\Big\downarrow \text{3960}$$

$$\frac{d\left(\frac{\int 1dx}{2a}-\frac{1}{2f(a\coth(e+fx)+a)}\right)}{2f}-\frac{c+dx}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^2}{4ad}$$

$$\Big\downarrow \text{24}$$

$$-\frac{c+dx}{2f(a\coth(e+fx)+a)}+\frac{(c+dx)^2}{4ad}+\frac{d\left(\frac{x}{2a}-\frac{1}{2f(a\coth(e+fx)+a)}\right)}{2f}$$

input
```
Int[(c + d*x)/(a + a*Coth[e + f*x]),x]
```

output
```
(c + d*x)^2/(4*a*d) - (c + d*x)/(2*f*(a + a*Coth[e + f*x])) + (d*(x/(2*a)
- 1/(2*f*(a + a*Coth[e + f*x]))))/(2*f)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 4206
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Simp[a*d*(m/(2*b*f))
   Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[a*((c + d*x)^m
/(2*b*f*(a + b*Tan[e + f*x]))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
a^2 + b^2, 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.62

method	result	size
risch	$\frac{d\,x^{2}}{4a}+\frac{xc}{2a}+\frac{(2dfx+2cf+d)\,\mathrm{e}^{-2fx-2e}}{8f^{2}a}$	46
parallelrisch	$\frac{((2dx+2c)f+d)\cosh(2fx+2e)+((-2dx-2c)f-d)\sinh(2fx+2e)+(2d\,x^{2}+4xc)f^{2}-2cf-d}{8f^{2}a}$	78

input
```
int((d*x+c)/(a+a*coth(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*d*x^2+1/2/a*x*c+1/8*(2*d*f*x+2*c*f+d)/f^2/a*exp(-2*f*x-2*e)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.36

$$\int \frac{c+dx}{a+a\coth(e+fx)}\,dx = \frac{(2\,df^2x^2+2\,cf+2\,(2\,cf^2+df)x+d)\cosh\left(fx+e\right)+(2\,df^2x^2-2\,cf+2\,(2\,cf^2-df)x-d)\sinh\left(fx+\right.}{8\left(af^2\cosh\left(fx+e\right)+af^2\sinh\left(fx+e\right)\right)}$$

input
```
integrate((d*x+c)/(a+a*coth(f*x+e)),x, algorithm="fricas")
```

output
```
1/8*((2*d*f^2*x^2 + 2*c*f + 2*(2*c*f^2 + d*f)*x + d)*cosh(f*x + e) + (2*d*
f^2*x^2 - 2*c*f + 2*(2*c*f^2 - d*f)*x - d)*sinh(f*x + e))/(a*f^2*cosh(f*x
+ e) + a*f^2*sinh(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(56) = 112.

Time = 0.57 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.38

$$\int \frac{c+dx}{a+a\coth(e+fx)}\,dx = \begin{cases} \frac{2cf^2x\tanh\left(e+fx\right)}{4af^2\tanh\left(e+fx\right)+4af^2}+\frac{2cf^2x}{4af^2\tanh\left(e+fx\right)+4af^2}+\frac{2cf}{4af^2\tanh\left(e+fx\right)+4af^2}+\frac{df^2x^2\tanh\left(e+fx\right)}{4af^2\tanh\left(e+fx\right)+4af^2}+\frac{df^2x^2}{4af^2\tanh\left(e+fx\right)+4a\ldots} \\ \frac{cx+\frac{dx^2}{2}}{a\coth\left(e\right)+a} \end{cases}$$

input
```
integrate((d*x+c)/(a+a*coth(f*x+e)),x)
```

output
```
Piecewise(((2*c*f**2*x*tanh(e + f*x)/(4*a*f**2*tanh(e + f*x) + 4*a*f**2) +
2*c*f**2*x/(4*a*f**2*tanh(e + f*x) + 4*a*f**2) + 2*c*f/(4*a*f**2*tanh(e +
f*x) + 4*a*f**2) + d*f**2*x**2*tanh(e + f*x)/(4*a*f**2*tanh(e + f*x) + 4*a
*f**2) + d*f**2*x**2/(4*a*f**2*tanh(e + f*x) + 4*a*f**2) - d*f*x*tanh(e +
f*x)/(4*a*f**2*tanh(e + f*x) + 4*a*f**2) + d*f*x/(4*a*f**2*tanh(e + f*x) +
 4*a*f**2) + d/(4*a*f**2*tanh(e + f*x) + 4*a*f**2), Ne(f, 0)), ((c*x + d*x
**2/2)/(a*coth(e) + a), True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \frac{c+dx}{a+a\coth(e+fx)}\,dx = \frac{1}{4}\,c\left(\frac{2\,(fx+e)}{af}+\frac{e^{(-2\,fx-2\,e)}}{af}\right) + \frac{\left(2\,f^2x^2e^{(2\,e)}+(2\,fx+1)e^{(-2\,fx)}\right)de^{(-2\,e)}}{8\,af^2}$$

input
```
integrate((d*x+c)/(a+a*coth(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*c*(2*(f*x + e)/(a*f) + e^(-2*f*x - 2*e)/(a*f)) + 1/8*(2*f^2*x^2*e^(2*e
) + (2*f*x + 1)*e^(-2*f*x))*d*e^(-2*e)/(a*f^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{c+dx}{a+a\coth(e+fx)}\,dx = \frac{\left(2\,df^2x^2e^{(2\,fx+2\,e)}+4\,cf^2xe^{(2\,fx+2\,e)}+2\,dfx+2\,cf+d\right)e^{(-2\,fx-2\,e)}}{8\,af^2}$$

input
```
integrate((d*x+c)/(a+a*coth(f*x+e)),x, algorithm="giac")
```

output
```
1/8*(2*d*f^2*x^2*e^(2*f*x + 2*e) + 4*c*f^2*x*e^(2*f*x + 2*e) + 2*d*f*x + 2
*c*f + d)*e^(-2*f*x - 2*e)/(a*f^2)
```

Mupad [B] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03

$$\int \frac{c+dx}{a+a\coth(e+fx)}\,dx = \frac{\frac{d\,x^2}{4}+\left(\frac{c}{2}+\frac{d}{4\,f}\right)\,x}{a}-\frac{\frac{\frac{d}{4}+\frac{c\,f}{2}}{f^2}-x\,\left(\frac{c}{2}-\frac{d}{4\,f}\right)+x\,\left(\frac{c}{2}+\frac{d}{4\,f}\right)}{a+a\coth\left(e+f\,x\right)}$$

input
```
int((c + d*x)/(a + a*coth(e + f*x)),x)
```

output
```
(x*(c/2 + d/(4*f)) + (d*x^2)/4)/a - ((d/4 + (c*f)/2)/f^2 - x*(c/2 - d/(4*f
)) + x*(c/2 + d/(4*f)))/(a + a*coth(e + f*x))
```

Reduce [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

$$\int \frac{c+dx}{a+a\coth(e+fx)}\,dx = \frac{4e^{2fx+2e}c\,f^2x+2e^{2fx+2e}d\,f^2x^2+2cf+2dfx+d}{8e^{2fx+2e}a\,f^2}$$

input
```
int((d*x+c)/(a+a*coth(f*x+e)),x)
```

output
```
(4*e**(2*e + 2*f*x)*c*f**2*x + 2*e**(2*e + 2*f*x)*d*f**2*x**2 + 2*c*f + 2*
d*f*x + d)/(8*e**(2*e + 2*f*x)*a*f**2)
```

3.19 $\int \frac{1}{(c+dx)(a+a\coth(e+fx))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 157

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))}\,dx = -\frac{\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)}{2ad}+\frac{\log(c+dx)}{2ad}+\frac{\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{2ad}+\frac{\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(\frac{2cf}{d}+2fx\right)}{2ad}-\frac{\sinh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(\frac{2cf}{d}+2fx\right)}{2ad}$$

output
```
-1/2*cosh(-2*e+2*c*f/d)*Chi(2*c*f/d+2*f*x)/a/d+1/2*ln(d*x+c)/a/d-1/2*Chi(2
*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a/d+1/2*cosh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*
f*x)/a/d+1/2*sinh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a/d
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.78

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))}\,dx$$
$$= \frac{\operatorname{csch}(e+fx)(\cosh(fx)+\sinh(fx))\left(\log(f(c+dx))(\cosh(e)+\sinh(e))+\operatorname{Chi}\left(\frac{2f(c+dx)}{d}\right)\left(-\cosh\left(e-\frac{2c}{d}\right.\right.\right.}{2ad(1+\coth(e+fx))}$$

input
```
Integrate[1/((c + d*x)*(a + a*Coth[e + f*x])),x]
```

output
```
(Csch[e + f*x]*(Cosh[f*x] + Sinh[f*x])*(Log[f*(c + d*x)]*(Cosh[e] + Sinh[e
]) + CoshIntegral[(2*f*(c + d*x))/d]*(-Cosh[e - (2*c*f)/d] + Sinh[e - (2*c
*f)/d]) + (Cosh[e - (2*c*f)/d] - Sinh[e - (2*c*f)/d])*SinhIntegral[(2*f*(c
 + d*x))/d]))/(2*a*d*(1 + Coth[e + f*x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4209, 25, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a\coth(e+fx)+a)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{(c+dx)\left(a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)}dx$$
$$\Big\downarrow \text{ 4209}$$
$$\frac{i\int -\frac{i\sinh(2e+2fx)}{c+dx}dx}{2a}+\frac{\int -\frac{\cosh(2e+2fx)}{c+dx}dx}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\Big\downarrow \; 25$$

$$\frac{i\int -\frac{i\sinh(2e+2fx)}{c+dx}dx}{2a}-\frac{\int \frac{\cosh(2e+2fx)}{c+dx}dx}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\Big\downarrow \; 26$$

$$\frac{\int \frac{\sinh(2e+2fx)}{c+dx}dx}{2a}-\frac{\int \frac{\cosh(2e+2fx)}{c+dx}dx}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int -\frac{i\sin(2ie+2ifx)}{c+dx}dx}{2a}-\frac{\int \frac{\sin\left(2ie+2ifx+\frac{\pi}{2}\right)}{c+dx}dx}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\Big\downarrow \; 26$$

$$-\frac{i\int \frac{\sin(2ie+2ifx)}{c+dx}dx}{2a}-\frac{\int \frac{\sin\left(2ie+2ifx+\frac{\pi}{2}\right)}{c+dx}dx}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\Big\downarrow \; 3784$$

$$-\frac{i\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{i\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{2a}-\frac{\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx-i\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{i\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\Big\downarrow \; 26$$

$$-\frac{i\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+i\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{2a}-\frac{\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\sinh\left(2e-\frac{2cf}{d}\right)\int -\frac{i\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx}{2a}-\frac{i\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+i\cosh\left(2e-\frac{2cf}{d}\right)\int -\frac{i\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx\right)}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\Big\downarrow 26$$

$$-\frac{\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx - i\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx}{2a} - \frac{i\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx + \cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx\right)}{2a} + \frac{\log(c+dx)}{2ad}$$

$$\Big\downarrow 3779$$

$$-\frac{\frac{\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d} + \cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx}{2a} - \frac{i\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx + \frac{i\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}\right)}{2a} + \frac{\log(c+dx)}{2ad}$$

$$\Big\downarrow 3782$$

$$-\frac{i\left(\frac{i\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d} + \frac{i\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}\right)}{2a} - \frac{\frac{\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{d} + \frac{\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}}{2a} + \frac{\log(c+dx)}{2ad}$$

input
```
Int[1/((c + d*x)*(a + a*Coth[e + f*x])),x]
```

output
```
Log[c + d*x]/(2*a*d) - ((I/2)*((I*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e
 - (2*c*f)/d])/d + (I*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x
])/d))/a - ((Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/d + (S
inh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d)/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4209
```
Int[1/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])), x_Symb
ol] :> Simp[Log[c + d*x]/(2*a*d), x] + (Simp[1/(2*a)   Int[Cos[2*e + 2*f*x]
/(c + d*x), x], x] + Simp[1/(2*b)   Int[Sin[2*e + 2*f*x]/(c + d*x), x], x])
 /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.39

method	result	size
risch	$\frac{\ln(dx+c)}{2ad}+\frac{\mathrm{e}^{\frac{2cf-2de}{d}}\,\mathrm{expIntegral}_1\left(2fx+2e+\frac{2cf-2de}{d}\right)}{2ad}$	61

input
```
int(1/(d*x+c)/(a+a*coth(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(d*x+c)/a/d+1/2/a/d*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.48

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))}\,dx = -\frac{\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\cosh\left(-\frac{2\,(de-cf)}{d}\right)+\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\sinh\left(-\frac{2\,(de-cf)}{d}\right)-\log\left(dx+c\right)}{2\,ad}$$

input
```
integrate(1/(d*x+c)/(a+a*coth(f*x+e)),x, algorithm="fricas")
```

output
```
-1/2*(Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) + Ei(-2*(d*f*x + c*f)/
d)*sinh(-2*(d*e - c*f)/d) - log(d*x + c))/(a*d)
```

Sympy [F]

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))}\,dx = \frac{\int \frac{1}{c\coth(e+fx)+c+dx\coth(e+fx)+dx}\,dx}{a}$$

input
```
integrate(1/(d*x+c)/(a+a*coth(f*x+e)),x)
```

output
```
Integral(1/(c*coth(e + f*x) + c + d*x*coth(e + f*x) + d*x), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.31

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))}\,dx = \frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_1\left(\frac{2(dx+c)f}{d}\right)}{2ad} + \frac{\log(dx+c)}{2ad}$$

input
```
integrate(1/(d*x+c)/(a+a*coth(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*e^(-2*e + 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/(a*d) + 1/2*log(
d*x + c)/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.31

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))}\,dx = -\frac{\left(\mathrm{Ei}\left(-\frac{2(dfx+cf)}{d}\right)e^{\left(\frac{2cf}{d}\right)} - e^{(2e)}\log(dx+c)\right)e^{(-2e)}}{2ad}$$

input
```
integrate(1/(d*x+c)/(a+a*coth(f*x+e)),x, algorithm="giac")
```

output
```
-1/2*(Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d) - e^(2*e)*log(d*x + c))*e^(-2*e)/
(a*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))}\,dx = \int \frac{1}{(a+a\coth(e+f\,x))\,(c+d\,x)}\,dx$$

input
```
int(1/((a + a*coth(e + f*x))*(c + d*x)),x)
```

output
```
int(1/((a + a*coth(e + f*x))*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))}\,dx = \frac{\int \frac{1}{\coth(fx+e)c+\coth(fx+e)dx+c+dx}dx}{a}$$

input
```
int(1/(d*x+c)/(a+a*coth(f*x+e)),x)
```

output
```
int(1/(coth(e + f*x)*c + coth(e + f*x)*d*x + c + d*x),x)/a
```

3.20 $\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 159

$$\begin{aligned}\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))}\,dx &= \frac{f\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{ad^2} \\ &- \frac{1}{d(c+dx)(a+a\coth(e+fx))} \\ &- \frac{f\text{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{ad^2} \\ &- \frac{f\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{ad^2} \\ &+ \frac{f\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{ad^2}\end{aligned}$$

output

```
f*cosh(-2*e+2*c*f/d)*Chi(2*c*f/d+2*f*x)/a/d^2-1/d/(d*x+c)/(a+a*coth(f*x+e)
)+f*Chi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a/d^2-f*cosh(-2*e+2*c*f/d)*Shi(2
*c*f/d+2*f*x)/a/d^2-f*sinh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a/d^2
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.30

$$\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))}\,dx =$$
$$-\frac{\operatorname{csch}(e+fx)\left(\cosh\left(\frac{cf}{d}\right)+\sinh\left(\frac{cf}{d}\right)\right)\left(d\left(\cosh\left(e+f\left(-\frac{c}{d}+x\right)\right)-\cosh\left(e+f\left(\frac{c}{d}+x\right)\right)+\sinh\left(e+f(\right.\right.\right.}{}$$

input
```
Integrate[1/((c + d*x)^2*(a + a*Coth[e + f*x])),x]
```

output
```
-1/2*(Csch[e + f*x]*(Cosh[(c*f)/d] + Sinh[(c*f)/d])*(d*(Cosh[e + f*(-(c/d)
 + x)] - Cosh[e + f*(c/d + x)] + Sinh[e + f*(-(c/d) + x)] + Sinh[e + f*(c/
d + x)]) + 2*f*(c + d*x)*CoshIntegral[(2*f*(c + d*x))/d]*(-Cosh[e - (f*(c
+ d*x))/d] + Sinh[e - (f*(c + d*x))/d]) + 2*f*(c + d*x)*(Cosh[e - (f*(c +
d*x))/d] - Sinh[e - (f*(c + d*x))/d])*SinhIntegral[(2*f*(c + d*x))/d]))/(a
*d^2*(c + d*x)*(1 + Coth[e + f*x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4207, 25, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a\coth(e+fx)+a)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c+dx)^2\left(a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4207}$$

$$-\frac{if\int -\frac{i\sinh(2e+2fx)}{c+dx}dx}{ad}-\frac{f\int -\frac{\cosh(2e+2fx)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\coth(e+fx)+a)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{if\int -\frac{i\sinh(2e+2fx)}{c+dx}dx}{ad}+\frac{f\int \frac{\cosh(2e+2fx)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\coth(e+fx)+a)}$$

$$\Big\downarrow \text{26}$$

$$-\frac{f\int \frac{\sinh(2e+2fx)}{c+dx}dx}{ad}+\frac{f\int \frac{\cosh(2e+2fx)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\coth(e+fx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{f\int -\frac{i\sin(2ie+2ifx)}{c+dx}dx}{ad}+\frac{f\int \frac{\sin\left(2ie+2ifx+\frac{\pi}{2}\right)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\coth(e+fx)+a)}$$

$$\Big\downarrow \text{26}$$

$$\frac{if\int \frac{\sin(2ie+2ifx)}{c+dx}dx}{ad}+\frac{f\int \frac{\sin\left(2ie+2ifx+\frac{\pi}{2}\right)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\coth(e+fx)+a)}$$

$$\Big\downarrow \text{3784}$$

$$\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{i\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{ad}+\frac{f\left(\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx-i\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{i\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{ad}-\frac{1}{d(c+dx)(a\coth(e+fx)+a)}$$

$$\Big\downarrow \text{26}$$

$$\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+i\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{ad}+\frac{f\left(\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{ad}-\frac{1}{d(c+dx)(a\coth(e+fx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{f\left(\sinh\left(2e-\frac{2cf}{d}\right)\int -\frac{i\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx\right)}{ad}+\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+i\cosh\left(2e-\frac{2cf}{d}\right)\int -\frac{i\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx\right)}{ad}-\frac{1}{d(c+dx)(a\coth(e+fx)+a)}$$

$$\Big\downarrow \; 26$$

$$\frac{f\left(\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx-i\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx\right)}{ad}+\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx\right)}{ad}-\frac{1}{d(c+dx)(a\coth(e+fx)+a)}$$

$$\Big\downarrow \; 3779$$

$$\frac{f\left(\frac{\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}+\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx\right)}{ad}+\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+\frac{i\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}\right)}{ad}-\frac{1}{d(c+dx)(a\coth(e+fx)+a)}$$

$$\Big\downarrow \; 3782$$

$$\frac{if\left(\frac{i\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d}+\frac{i\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}\right)}{ad}+\frac{f\left(\frac{\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{d}+\frac{\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}\right)}{ad}-\frac{1}{d(c+dx)(a\coth(e+fx)+a)}$$

input
```
Int[1/((c + d*x)^2*(a + a*Coth[e + f*x])),x]
```

output
```
-(1/(d*(c + d*x)*(a + a*Coth[e + f*x]))) + (I*f*((I*CoshIntegral[(2*c*f)/d
 + 2*f*x]*Sinh[2*e - (2*c*f)/d])/d + (I*Cosh[2*e - (2*c*f)/d]*SinhIntegral
[(2*c*f)/d + 2*f*x])/d))/(a*d) + (f*((Cosh[2*e - (2*c*f)/d]*CoshIntegral[(
2*c*f)/d + 2*f*x])/d + (Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f
*x])/d))/(a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4207
```
Int[1/(((c_.) + (d_.)*(x_))^2*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])), x_Sy
mbol] :> -Simp[(d*(c + d*x)*(a + b*Tan[e + f*x]))^(-1), x] + (-Simp[f/(a*d)
   Int[Sin[2*e + 2*f*x]/(c + d*x), x], x] + Simp[f/(b*d)   Int[Cos[2*e + 2*
f*x]/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.57

method	result	size
risch	$-\frac{1}{2da(dx+c)}+\frac{f\,\mathrm{e}^{-2fx-2e}}{2ad(dfx+cf)}-\frac{f\,\mathrm{e}^{\frac{2cf-2de}{d}}\,\mathrm{expIntegral}_1\left(2fx+2e+\frac{2cf-2de}{d}\right)}{a\,d^2}$	91

input
```
int(1/(d*x+c)^2/(a+a*coth(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/a/(d*x+c)+1/2*f/a*exp(-2*f*x-2*e)/d/(d*f*x+c*f)-f/a/d^2*exp(2*(c*f-
d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.36

$$\int\frac{1}{(c+dx)^2(a+a\coth(e+fx))}\,dx$$
$$=\frac{(dfx+cf)\mathrm{Ei}\left(-\frac{2(dfx+cf)}{d}\right)\cosh(fx+e)\cosh\left(-\frac{2(de-cf)}{d}\right)+(dfx+cf)\mathrm{Ei}\left(-\frac{2(dfx+cf)}{d}\right)\cosh(fx+e)\sin}{(ad^3x+acd^2)\cosh}$$

input
```
integrate(1/(d*x+c)^2/(a+a*coth(f*x+e)),x, algorithm="fricas")
```

output
```
((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(f*x + e)*cosh(-2*(d*e - c*f)/d)
 + (d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(f*x + e)*sinh(-2*(d*e - c*f)/
d) + ((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) + (d*f*x
 + c*f)*Ei(-2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)/d) - d)*sinh(f*x + e))/
((a*d^3*x + a*c*d^2)*cosh(f*x + e) + (a*d^3*x + a*c*d^2)*sinh(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))}\,dx = \frac{\int \frac{1}{c^2\coth(e+fx)+c^2+2cdx\coth(e+fx)+2cdx+d^2x^2\coth(e+fx)+d^2x^2}\,dx}{a}$$

input
```
integrate(1/(d*x+c)**2/(a+a*coth(f*x+e)),x)
```

output
```
Integral(1/(c**2*coth(e + f*x) + c**2 + 2*c*d*x*coth(e + f*x) + 2*c*d*x +
d**2*x**2*coth(e + f*x) + d**2*x**2), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.35

$$\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))}\,dx = -\frac{1}{2\,(ad^2x+acd)} + \frac{e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_2\left(\frac{2\,(dx+c)f}{d}\right)}{2\,(dx+c)ad}$$

input
```
integrate(1/(d*x+c)^2/(a+a*coth(f*x+e)),x, algorithm="maxima")
```

output
```
-1/2/(a*d^2*x + a*c*d) + 1/2*e^(-2*e + 2*c*f/d)*exp_integral_e(2, 2*(d*x +
 c)*f/d)/((d*x + c)*a*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(159) = 318.

Time = 0.15 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.02

$$\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))}\,dx = \frac{\left(2\,(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)f^2\mathrm{Ei}\left(-\frac{2\left((dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf\right)}{d}\right)e^{\left(-\frac{2\,(de-cf)}{d}\right)} - 2\,def^2\mathrm{Ei}\left(-\frac{2\left((dx+c)\right.}{}\right.\right.}{2\left((dx+c)a\right.}$$

input
```
integrate(1/(d*x+c)^2/(a+a*coth(f*x+e)),x, algorithm="giac")
```

output
```
1/2*(2*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-2*((d*x + c)*
(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) -
 2*d*e*f^2*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*
f)/d)*e^(-2*(d*e - c*f)/d) + 2*c*f^3*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f
/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) + d*f^2*e^(-2*(d*x +
c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d) - d*f^2)*d^2/(((d*x + c)*a*d^4*(
d*e/(d*x + c) - c*f/(d*x + c) + f) - a*d^5*e + a*c*d^4*f)*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))}\,dx=\int \frac{1}{\left(a+a\coth\left(e+f\,x\right)\right)\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(1/((a + a*coth(e + f*x))*(c + d*x)^2),x)
```

output
```
int(1/((a + a*coth(e + f*x))*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))}\,dx = \frac{-\left(\int \frac{1}{e^{2fx+2e}c^2+2e^{2fx+2e}cdx+e^{2fx+2e}d^2x^2}dx\right)c^2-\left(\int \frac{1}{e^{2fx+2e}c^2+2e^{2fx+2e}cdx+e^{2fx+2e}d^2x^2}dx\right)cdx+x}{2ac\,(dx+c)}$$

input
```
int(1/(d*x+c)^2/(a+a*coth(f*x+e)),x)
```

output
```
( - int(1/(e**(2*e + 2*f*x)*c**2 + 2*e**(2*e + 2*f*x)*c*d*x + e**(2*e + 2*
f*x)*d**2*x**2),x)*c**2 - int(1/(e**(2*e + 2*f*x)*c**2 + 2*e**(2*e + 2*f*x
)*c*d*x + e**(2*e + 2*f*x)*d**2*x**2),x)*c*d*x + x)/(2*a*c*(c + d*x))
```

3.21 $\int \frac{1}{(c+dx)^3(a+a\coth(e+fx))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 211

$$\int \frac{1}{(c+dx)^3(a+a\coth(e+fx))}\,dx = -\frac{f}{2ad^2(c+dx)} - \frac{f^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{ad^3} - \frac{1}{2d(c+dx)^2(a+a\coth(e+fx))} + \frac{f}{d^2(c+dx)(a+a\coth(e+fx))} + \frac{f^2\text{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{ad^3} + \frac{f^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{ad^3} - \frac{f^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{ad^3}$$

output

```
-1/2*f/a/d^2/(d*x+c)-f^2*cosh(-2*e+2*c*f/d)*Chi(2*c*f/d+2*f*x)/a/d^3-1/2/d
/(d*x+c)^2/(a+a*coth(f*x+e))+f/d^2/(d*x+c)/(a+a*coth(f*x+e))-f^2*Chi(2*c*f
/d+2*f*x)*sinh(-2*e+2*c*f/d)/a/d^3+f^2*cosh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*
x)/a/d^3+f^2*sinh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a/d^3
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.26

$$\int \frac{1}{(c+dx)^3(a+a\coth(e+fx))}\,dx =$$
$$-\frac{\operatorname{csch}(e+fx)\left(\cosh\left(\frac{cf}{d}\right)+\sinh\left(\frac{cf}{d}\right)\right)\left(d\left(d\cosh\left(e+f\left(-\frac{c}{d}+x\right)\right)+(-d+2cf+2dfx)\cosh\left(e+f\left(\frac{c}{d}\right.\right.\right.\right.}{}$$

input
```
Integrate[1/((c + d*x)^3*(a + a*Coth[e + f*x])),x]
```

output
```
-1/4*(Csch[e + f*x]*(Cosh[(c*f)/d] + Sinh[(c*f)/d])*(d*(d*Cosh[e + f*(-(c/
d) + x)] + (-d + 2*c*f + 2*d*f*x)*Cosh[e + f*(c/d + x)] + d*Sinh[e + f*(-(
c/d) + x)] + d*Sinh[e + f*(c/d + x)] - 2*c*f*Sinh[e + f*(c/d + x)] - 2*d*f
*x*Sinh[e + f*(c/d + x)]) + 4*f^2*(c + d*x)^2*CoshIntegral[(2*f*(c + d*x))
/d]*(Cosh[e - (f*(c + d*x))/d] - Sinh[e - (f*(c + d*x))/d]) + 4*f^2*(c + d
*x)^2*(-Cosh[e - (f*(c + d*x))/d] + Sinh[e - (f*(c + d*x))/d])*SinhIntegra
l[(2*f*(c + d*x))/d]))/(a*d^3*(c + d*x)^2*(1 + Coth[e + f*x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3042, 4208, 3042, 4207, 25, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^3(a\coth(e+fx)+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c+dx)^3\left(a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4208}$$

$$-\frac{f\int \frac{1}{(c+dx)^2(\coth(e+fx)a+a)}dx}{d}-\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\coth(e+fx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{f\int \frac{1}{(c+dx)^2\left(a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)}dx}{d}-\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\coth(e+fx)+a)}$$

$$\Big\downarrow \text{4207}$$

$$-\frac{f\left(-\frac{if\int -\frac{i\sinh(2e+2fx)}{c+dx}dx}{ad}-\frac{f\int -\frac{\cosh(2e+2fx)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\coth(e+fx)+a)}\right)}{d}-\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\coth(e+fx)+a)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{f\left(-\frac{if\int -\frac{i\sinh(2e+2fx)}{c+dx}dx}{ad}+\frac{f\int \frac{\cosh(2e+2fx)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\coth(e+fx)+a)}\right)}{d}-\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\coth(e+fx)+a)}$$

$$\Big\downarrow \text{26}$$

$$-\frac{f\left(-\frac{f\int \frac{\sinh(2e+2fx)}{c+dx}dx}{ad}+\frac{f\int \frac{\cosh(2e+2fx)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\coth(e+fx)+a)}\right)}{d}-\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\coth(e+fx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{f\left(-\frac{f\int -\frac{i\sin(2ie+2ifx)}{c+dx}dx}{ad}+\frac{f\int \frac{\sin\left(2ie+2ifx+\frac{\pi}{2}\right)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\coth(e+fx)+a)}\right)}{d}-\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\coth(e+fx)+a)}$$

$$\Big\downarrow \text{26}$$

$$-\frac{f\left(\frac{if\int \frac{\sin(2ie+2ifx)}{c+dx}dx}{ad}+\frac{f\int \frac{\sin\left(2ie+2ifx+\frac{\pi}{2}\right)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\coth(e+fx)+a)}\right)}{d}-\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\coth(e+fx)+a)}$$

$\Big\downarrow$ 3784

$$-\frac{f\left(\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{i\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{ad}+\frac{f\left(\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx-i\sinh\left(2e-\right.\right.}{ad}}{d}$$

$$\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\coth(e+fx)+a)}$$

$\Big\downarrow$ 26

$$-\frac{f\left(\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+i\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{ad}+\frac{f\left(\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+\cosh\left(2e-\right.\right.}{ad}}{d}$$

$$\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\coth(e+fx)+a)}$$

$\Big\downarrow$ 3042

$$-\frac{f\left(\frac{f\left(\sinh\left(2e-\frac{2cf}{d}\right)\int-\frac{i\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx\right)}{ad}+\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+i\,c\right.}{ad}}{d}$$

$$\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\coth(e+fx)+a)}$$

$\Big\downarrow$ 26

$$-\frac{f\left(\frac{f\left(\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx-i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx\right)}{ad}+\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+\cosh\right.}{ad}}{d}$$

$$\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\coth(e+fx)+a)}$$

$\Big\downarrow$ 3779

$$-\frac{f\left(\frac{f\left(\frac{\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}+\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx\right)}{ad}+\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+\frac{i\cosh\left(2e-2\right.}{}\right.}{ad}\right.}{d}$$

$$\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\coth(e+fx)+a)}$$

$\Big\downarrow$ 3782

$$-\frac{f\left(\frac{if\left(\frac{i\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d}+\frac{i\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}\right)}{ad}+\frac{f\left(\frac{\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{d}+\frac{\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right.}{d}\right.}{ad}\right.}{d}$$

$$\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\coth(e+fx)+a)}$$

input
```
Int[1/((c + d*x)^3*(a + a*Coth[e + f*x])),x]
```

output
```
-1/2*f/(a*d^2*(c + d*x)) - 1/(2*d*(c + d*x)^2*(a + a*Coth[e + f*x])) - (f*
(-(1/(d*(c + d*x)*(a + a*Coth[e + f*x]))) + (I*f*((I*CoshIntegral[(2*c*f)/
d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/d + (I*Cosh[2*e - (2*c*f)/d]*SinhIntegra
l[(2*c*f)/d + 2*f*x])/d))/(a*d) + (f*((Cosh[2*e - (2*c*f)/d]*CoshIntegral[
(2*c*f)/d + 2*f*x])/d + (Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*
f*x])/d))/(a*d)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4207
```
Int[1/(((c_.) + (d_.)*(x_))^2*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])), x_Sy
mbol] :> -Simp[(d*(c + d*x)*(a + b*Tan[e + f*x]))^(-1), x] + (-Simp[f/(a*d)
   Int[Sin[2*e + 2*f*x]/(c + d*x), x], x] + Simp[f/(b*d)   Int[Cos[2*e + 2*
f*x]/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 4208
```
Int[((c_.) + (d_.)*(x_))^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sym
bol] :> Simp[f*((c + d*x)^(m + 2)/(b*d^2*(m + 1)*(m + 2))), x] + (Simp[2*b*
(f/(a*d*(m + 1)))   Int[(c + d*x)^(m + 1)/(a + b*Tan[e + f*x]), x], x] + Si
mp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + b*Tan[e + f*x])), x]) /; FreeQ[{a, b,
c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{1}{4da(dx+c)^{2}}-\frac{f^{3}e^{-2fx-2e}x}{2ad(d^{2}x^{2}f^{2}+2cd\,f^{2}x+c^{2}f^{2})}-\frac{f^{3}e^{-2fx-2e}c}{2a\,d^{2}(d^{2}x^{2}f^{2}+2cd\,f^{2}x+c^{2}f^{2})}+\frac{f^{2}e^{-2fx-2e}}{4ad(d^{2}x^{2}f^{2}+2cd\,f^{2}x+c^{2}f^{2})}+\frac{f^{2}e^{\frac{2cf-}{d}}}{}$

input
```
int(1/(d*x+c)^3/(a+a*coth(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/a/(d*x+c)^2-1/2*f^3/a*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^
2*f^2)*x-1/2*f^3/a*exp(-2*f*x-2*e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c
+1/4*f^2/a*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)+f^2/a/d^3*e
xp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.62

$$\int \frac{1}{(c+dx)^3(a+a\coth(e+fx))}\,dx = -\frac{2\left(d^2f^2x^2+2\,cdf^2x+c^2f^2\right)\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\cosh\left(fx+e\right)\sinh\left(-\frac{2\,(de-cf)}{d}\right)+\left(d^2fx+cdf+2\left(d^2f^2x\right.\right.}{}$$

input
```
integrate(1/(d*x+c)^3/(a+a*coth(f*x+e)),x, algorithm="fricas")
```

output
```
-1/2*(2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*Ei(-2*(d*f*x + c*f)/d)*cosh(
f*x + e)*sinh(-2*(d*e - c*f)/d) + (d^2*f*x + c*d*f + 2*(d^2*f^2*x^2 + 2*c*
d*f^2*x + c^2*f^2)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d))*cosh(f*x
 + e) - (d^2*f*x + c*d*f - 2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*Ei(-2*(
d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) - 2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^
2*f^2)*Ei(-2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)/d) - d^2)*sinh(f*x + e))
/((a*d^5*x^2 + 2*a*c*d^4*x + a*c^2*d^3)*cosh(f*x + e) + (a*d^5*x^2 + 2*a*c
*d^4*x + a*c^2*d^3)*sinh(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(c+dx)^3(a+a\coth(e+fx))}\,dx = \frac{\int \frac{1}{c^3\coth(e+fx)+c^3+3c^2dx\coth(e+fx)+3c^2dx+3cd^2x^2\coth(e+fx)+3cd^2x^2+d^3x^3\coth(e+fx)+d^3x^3}\,dx}{a}$$

input
```
integrate(1/(d*x+c)**3/(a+a*coth(f*x+e)),x)
```

output
```
Integral(1/(c**3*coth(e + f*x) + c**3 + 3*c**2*d*x*coth(e + f*x) + 3*c**2*
d*x + 3*c*d**2*x**2*coth(e + f*x) + 3*c*d**2*x**2 + d**3*x**3*coth(e + f*x
) + d**3*x**3), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.32

$$\int \frac{1}{(c+dx)^3(a+a\coth(e+fx))}\,dx = -\frac{1}{4\,(ad^3x^2+2\,acd^2x+ac^2d)} + \frac{e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_3\left(\frac{2\,(dx+c)f}{d}\right)}{2\,(dx+c)^2ad}$$

input
```
integrate(1/(d*x+c)^3/(a+a*coth(f*x+e)),x, algorithm="maxima")
```

output
```
-1/4/(a*d^3*x^2 + 2*a*c*d^2*x + a*c^2*d) + 1/2*e^(-2*e + 2*c*f/d)*exp_inte
gral_e(3, 2*(d*x + c)*f/d)/((d*x + c)^2*a*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.83

$$\int \frac{1}{(c+dx)^3(a+a\coth(e+fx))}\,dx = -\frac{4\,d^2f^2x^2\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(\frac{2\,cf}{d}\right)}+8\,cdf^2x\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(\frac{2\,cf}{d}\right)}+4\,c^2f^2\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(\frac{2\,cf}{d}\right)}+2\,d^2\ldots}{4\,(ad^5x^2e^{(2\,e)}+2\,acd^4xe^{(2\,e)}+ac^2d^3e^{(2\,e)})}$$

input
```
integrate(1/(d*x+c)^3/(a+a*coth(f*x+e)),x, algorithm="giac")
```

output
```
-1/4*(4*d^2*f^2*x^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d) + 8*c*d*f^2*x*Ei(-2
*(d*f*x + c*f)/d)*e^(2*c*f/d) + 4*c^2*f^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/
d) + 2*d^2*f*x*e^(-2*f*x) + 2*c*d*f*e^(-2*f*x) - d^2*e^(-2*f*x) + d^2*e^(2
*e))/(a*d^5*x^2*e^(2*e) + 2*a*c*d^4*x*e^(2*e) + a*c^2*d^3*e^(2*e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^3(a+a\coth(e+fx))}\,dx = \int \frac{1}{(a+a\coth(e+f\,x))\,(c+d\,x)^3}\,dx$$

input
```
int(1/((a + a*coth(e + f*x))*(c + d*x)^3),x)
```

output
```
int(1/((a + a*coth(e + f*x))*(c + d*x)^3), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^3(a+a\coth(e+fx))}\,dx = \frac{-2\left(\int \frac{1}{e^{2fx+2e}c^3+3e^{2fx+2e}c^2dx+3e^{2fx+2e}c\,d^2x^2+e^{2fx+2e}d^3x^3}dx\right)c^2d-4\left(\int \frac{1}{e^{2fx+2e}c^3+3e^{2fx+2e}c^2dx+3e^{2fx+2e}c\,d^2x^2+e^{2fx+2e}}\right.}{4ad\,(d^2x^2+2cdx+c^2)}$$

input
```
int(1/(d*x+c)^3/(a+a*coth(f*x+e)),x)
```

output
```
( - 2*int(1/(e**(2*e + 2*f*x)*c**3 + 3*e**(2*e + 2*f*x)*c**2*d*x + 3*e**(2
*e + 2*f*x)*c*d**2*x**2 + e**(2*e + 2*f*x)*d**3*x**3),x)*c**2*d - 4*int(1/
(e**(2*e + 2*f*x)*c**3 + 3*e**(2*e + 2*f*x)*c**2*d*x + 3*e**(2*e + 2*f*x)*
c*d**2*x**2 + e**(2*e + 2*f*x)*d**3*x**3),x)*c*d**2*x - 2*int(1/(e**(2*e +
 2*f*x)*c**3 + 3*e**(2*e + 2*f*x)*c**2*d*x + 3*e**(2*e + 2*f*x)*c*d**2*x**
2 + e**(2*e + 2*f*x)*d**3*x**3),x)*d**3*x**2 - 1)/(4*a*d*(c**2 + 2*c*d*x +
 d**2*x**2))
```

3.22 $\int \frac{(c+dx)^3}{(a+a\coth(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 230

$$\int \frac{(c+dx)^3}{(a+a\coth(e+fx))^2}\,dx = -\frac{3d^3e^{-4e-4fx}}{512a^2f^4}+\frac{3d^3e^{-2e-2fx}}{16a^2f^4}-\frac{3d^2e^{-4e-4fx}(c+dx)}{128a^2f^3}+\frac{3d^2e^{-2e-2fx}(c+dx)}{8a^2f^3}-\frac{3de^{-4e-4fx}(c+dx)^2}{64a^2f^2}+\frac{3de^{-2e-2fx}(c+dx)^2}{8a^2f^2}-\frac{e^{-4e-4fx}(c+dx)^3}{16a^2f}+\frac{e^{-2e-2fx}(c+dx)^3}{4a^2f}+\frac{(c+dx)^4}{16a^2d}$$

output
```
-3/512*d^3*exp(-4*f*x-4*e)/a^2/f^4+3/16*d^3*exp(-2*f*x-2*e)/a^2/f^4-3/128*
d^2*exp(-4*f*x-4*e)*(d*x+c)/a^2/f^3+3/8*d^2*exp(-2*f*x-2*e)*(d*x+c)/a^2/f^
3-3/64*d*exp(-4*f*x-4*e)*(d*x+c)^2/a^2/f^2+3/8*d*exp(-2*f*x-2*e)*(d*x+c)^2
/a^2/f^2-1/16*exp(-4*f*x-4*e)*(d*x+c)^3/a^2/f+1/4*exp(-2*f*x-2*e)*(d*x+c)^
3/a^2/f+1/16*(d*x+c)^4/a^2/d
```

Mathematica [A] (verified)

Time = 1.94 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.83

$$\int \frac{(c+dx)^3}{(a+a\coth(e+fx))^2}\,dx$$
$$= \frac{\operatorname{csch}^2(e+fx)(\cosh(fx)+\sinh(fx))^2\left((4c^3f^3+6c^2df^2(1+2fx)+6cd^2f(1+2fx+2f^2x^2)+d^3(3+6\right.}{}$$

input
```
Integrate[(c + d*x)^3/(a + a*Coth[e + f*x])^2,x]
```

output
```
(Csch[e + f*x]^2*(Cosh[f*x] + Sinh[f*x])^2*((4*c^3*f^3 + 6*c^2*d*f^2*(1 +
2*f*x) + 6*c*d^2*f*(1 + 2*f*x + 2*f^2*x^2) + d^3*(3 + 6*f*x + 6*f^2*x^2 +
4*f^3*x^3))*Cosh[2*f*x] + ((32*c^3*f^3 + 24*c^2*d*f^2*(1 + 4*f*x) + 12*c*d
^2*f*(1 + 4*f*x + 8*f^2*x^2) + d^3*(3 + 12*f*x + 24*f^2*x^2 + 32*f^3*x^3))
*Cosh[4*f*x]*(-Cosh[2*e] + Sinh[2*e]))/32 + f^4*x*(4*c^3 + 6*c^2*d*x + 4*c
*d^2*x^2 + d^3*x^3)*(Cosh[2*e] + Sinh[2*e]) - (4*c^3*f^3 + 6*c^2*d*f^2*(1
+ 2*f*x) + 6*c*d^2*f*(1 + 2*f*x + 2*f^2*x^2) + d^3*(3 + 6*f*x + 6*f^2*x^2
+ 4*f^3*x^3))*Sinh[2*f*x] + ((32*c^3*f^3 + 24*c^2*d*f^2*(1 + 4*f*x) + 12*c
*d^2*f*(1 + 4*f*x + 8*f^2*x^2) + d^3*(3 + 12*f*x + 24*f^2*x^2 + 32*f^3*x^3
))*(Cosh[2*e] - Sinh[2*e])*Sinh[4*f*x])/32))/(16*a^2*f^4*(1 + Coth[e + f*x
])^2)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4212, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a\coth(e+fx)+a)^2}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{(c+dx)^3}{\left(a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{ 4212}$$

$$\int \left(\frac{(c+dx)^3e^{-4e-4fx}}{4a^2}-\frac{(c+dx)^3e^{-2e-2fx}}{2a^2}+\frac{(c+dx)^3}{4a^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{3d^2(c+dx)e^{-4e-4fx}}{128a^2f^3}+\frac{3d^2(c+dx)e^{-2e-2fx}}{8a^2f^3}-\frac{3d(c+dx)^2e^{-4e-4fx}}{64a^2f^2}+\frac{3d(c+dx)^2e^{-2e-2fx}}{8a^2f^2}-\frac{(c+dx)^3e^{-4e-4fx}}{16a^2f}+\frac{(c+dx)^3e^{-2e-2fx}}{4a^2f}+\frac{(c+dx)^4}{16a^2d}-\frac{3d^3e^{-4e-4fx}}{512a^2f^4}+\frac{3d^3e^{-2e-2fx}}{16a^2f^4}$$

input
```
Int[(c + d*x)^3/(a + a*Coth[e + f*x])^2,x]
```

output
```
(-3*d^3*E^(-4*e - 4*f*x))/(512*a^2*f^4) + (3*d^3*E^(-2*e - 2*f*x))/(16*a^2
*f^4) - (3*d^2*E^(-4*e - 4*f*x)*(c + d*x))/(128*a^2*f^3) + (3*d^2*E^(-2*e
- 2*f*x)*(c + d*x))/(8*a^2*f^3) - (3*d*E^(-4*e - 4*f*x)*(c + d*x)^2)/(64*a
^2*f^2) + (3*d*E^(-2*e - 2*f*x)*(c + d*x)^2)/(8*a^2*f^2) - (E^(-4*e - 4*f*
x)*(c + d*x)^3)/(16*a^2*f) + (E^(-2*e - 2*f*x)*(c + d*x)^3)/(4*a^2*f) + (c
 + d*x)^4/(16*a^2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4212
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.19

method	result
risch	$\frac{d^3x^4}{16a^2}+\frac{d^2c\,x^3}{4a^2}+\frac{3d\,c^2x^2}{8a^2}+\frac{c^3x}{4a^2}+\frac{c^4}{16a^2d}+\frac{(4d^3x^3f^3+12c\,d^2f^3x^2+12c^2d\,f^3x+6d^3f^2x^2+4c^3f^3+12c\,d^2f^2x+6c^2d\,f}{16a^2f^4}$
parallelrisch	$\frac{32f\left(\left(\frac{1}{4}d^3x^3+c\,d^2x^2+\frac{3}{2}c^2dx+c^3\right)f^3-\frac{15\left(\frac{1}{3}d^2x^2+cdx+c^2\right)d\,f^2}{4}-\frac{27\left(\frac{dx}{2}+c\right)d^2f}{8}-\frac{51d^3}{32}\right)x\tanh(fx+e)^2+\left(\left(16d^3x^4+64c\,d^2x^3\right.\right.}{}$

input
```
int((d*x+c)^3/(a+a*coth(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/16/a^2*d^3*x^4+1/4/a^2*d^2*c*x^3+3/8/a^2*d*c^2*x^2+1/4/a^2*c^3*x+1/16/a^
2/d*c^4+1/16*(4*d^3*f^3*x^3+12*c*d^2*f^3*x^2+12*c^2*d*f^3*x+6*d^3*f^2*x^2+
4*c^3*f^3+12*c*d^2*f^2*x+6*c^2*d*f^2+6*d^3*f*x+6*c*d^2*f+3*d^3)/a^2/f^4*ex
p(-2*f*x-2*e)-1/512*(32*d^3*f^3*x^3+96*c*d^2*f^3*x^2+96*c^2*d*f^3*x+24*d^3
*f^2*x^2+32*c^3*f^3+48*c*d^2*f^2*x+24*c^2*d*f^2+12*d^3*f*x+12*c*d^2*f+3*d^
3)/a^2/f^4*exp(-4*f*x-4*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(204) = 408.

Time = 0.10 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.48

$$\int \frac{(c+dx)^3}{(a+a\coth(e+fx))^2}\,dx$$
$$=\frac{128\,d^3f^3x^3+128\,c^3f^3+192\,c^2df^2+192\,cd^2f+96\,d^3+192\,(2\,cd^2f^3+d^3f^2)x^2+(32\,d^3f^4x^4-32\,c^3f^3}{}$$

input
```
integrate((d*x+c)^3/(a+a*coth(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/512*(128*d^3*f^3*x^3 + 128*c^3*f^3 + 192*c^2*d*f^2 + 192*c*d^2*f + 96*d^
3 + 192*(2*c*d^2*f^3 + d^3*f^2)*x^2 + (32*d^3*f^4*x^4 - 32*c^3*f^3 - 24*c^
2*d*f^2 - 12*c*d^2*f + 32*(4*c*d^2*f^4 - d^3*f^3)*x^3 - 3*d^3 + 24*(8*c^2*
d*f^4 - 4*c*d^2*f^3 - d^3*f^2)*x^2 + 4*(32*c^3*f^4 - 24*c^2*d*f^3 - 12*c*d
^2*f^2 - 3*d^3*f)*x)*cosh(f*x + e)^2 + 2*(32*d^3*f^4*x^4 + 32*c^3*f^3 + 24
*c^2*d*f^2 + 12*c*d^2*f + 32*(4*c*d^2*f^4 + d^3*f^3)*x^3 + 3*d^3 + 24*(8*c
^2*d*f^4 + 4*c*d^2*f^3 + d^3*f^2)*x^2 + 4*(32*c^3*f^4 + 24*c^2*d*f^3 + 12*
c*d^2*f^2 + 3*d^3*f)*x)*cosh(f*x + e)*sinh(f*x + e) + (32*d^3*f^4*x^4 - 32
*c^3*f^3 - 24*c^2*d*f^2 - 12*c*d^2*f + 32*(4*c*d^2*f^4 - d^3*f^3)*x^3 - 3*
d^3 + 24*(8*c^2*d*f^4 - 4*c*d^2*f^3 - d^3*f^2)*x^2 + 4*(32*c^3*f^4 - 24*c^
2*d*f^3 - 12*c*d^2*f^2 - 3*d^3*f)*x)*sinh(f*x + e)^2 + 192*(2*c^2*d*f^3 +
2*c*d^2*f^2 + d^3*f)*x)/(a^2*f^4*cosh(f*x + e)^2 + 2*a^2*f^4*cosh(f*x + e)
*sinh(f*x + e) + a^2*f^4*sinh(f*x + e)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2193 vs. $2(236) = 472$.

Time = 1.05 (sec) , antiderivative size = 2193, normalized size of antiderivative = 9.53

$$\int \frac{(c+dx)^3}{(a+a\coth(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3/(a+a*coth(f*x+e))**2,x)
```

output
```
Piecewise((32*c**3*f**4*x*tanh(e + f*x)**2/(128*a**2*f**4*tanh(e + f*x)**2
 + 256*a**2*f**4*tanh(e + f*x) + 128*a**2*f**4) + 64*c**3*f**4*x*tanh(e +
f*x)/(128*a**2*f**4*tanh(e + f*x)**2 + 256*a**2*f**4*tanh(e + f*x) + 128*a
**2*f**4) + 32*c**3*f**4*x/(128*a**2*f**4*tanh(e + f*x)**2 + 256*a**2*f**4
*tanh(e + f*x) + 128*a**2*f**4) + 96*c**3*f**3*tanh(e + f*x)/(128*a**2*f**
4*tanh(e + f*x)**2 + 256*a**2*f**4*tanh(e + f*x) + 128*a**2*f**4) + 64*c**
3*f**3/(128*a**2*f**4*tanh(e + f*x)**2 + 256*a**2*f**4*tanh(e + f*x) + 128
*a**2*f**4) + 48*c**2*d*f**4*x**2*tanh(e + f*x)**2/(128*a**2*f**4*tanh(e +
 f*x)**2 + 256*a**2*f**4*tanh(e + f*x) + 128*a**2*f**4) + 96*c**2*d*f**4*x
**2*tanh(e + f*x)/(128*a**2*f**4*tanh(e + f*x)**2 + 256*a**2*f**4*tanh(e +
 f*x) + 128*a**2*f**4) + 48*c**2*d*f**4*x**2/(128*a**2*f**4*tanh(e + f*x)*
*2 + 256*a**2*f**4*tanh(e + f*x) + 128*a**2*f**4) - 120*c**2*d*f**3*x*tanh
(e + f*x)**2/(128*a**2*f**4*tanh(e + f*x)**2 + 256*a**2*f**4*tanh(e + f*x)
 + 128*a**2*f**4) + 48*c**2*d*f**3*x*tanh(e + f*x)/(128*a**2*f**4*tanh(e +
 f*x)**2 + 256*a**2*f**4*tanh(e + f*x) + 128*a**2*f**4) + 72*c**2*d*f**3*x
/(128*a**2*f**4*tanh(e + f*x)**2 + 256*a**2*f**4*tanh(e + f*x) + 128*a**2*
f**4) + 120*c**2*d*f**2*tanh(e + f*x)/(128*a**2*f**4*tanh(e + f*x)**2 + 25
6*a**2*f**4*tanh(e + f*x) + 128*a**2*f**4) + 96*c**2*d*f**2/(128*a**2*f**4
*tanh(e + f*x)**2 + 256*a**2*f**4*tanh(e + f*x) + 128*a**2*f**4) + 32*c*d*
*2*f**4*x**3*tanh(e + f*x)**2/(128*a**2*f**4*tanh(e + f*x)**2 + 256*a**...
```

Maxima [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx)^3}{(a+a\coth(e+fx))^2}\,dx = \frac{1}{16}\,c^3\left(\frac{4\,(fx+e)}{a^2f} + \frac{4\,e^{(-2\,fx-2\,e)} - e^{(-4\,fx-4\,e)}}{a^2f}\right) + \frac{3\left(8\,f^2x^2e^{(4\,e)} + 8\left(2\,fxe^{(2\,e)} + e^{(2\,e)}\right)e^{(-2\,fx)} - (4\,fx+1)e^{(-4\,fx)}\right)c^2de^{(-4\,e)}}{64\,a^2f^2} + \frac{\left(32\,f^3x^3e^{(4\,e)} + 48\left(2\,f^2x^2e^{(2\,e)} + 2\,fxe^{(2\,e)} + e^{(2\,e)}\right)e^{(-2\,fx)} - 3\left(8\,f^2x^2 + 4\,fx + 1\right)e^{(-4\,fx)}\right)cd^2e^{(-4\,e)}}{128\,a^2f^3} + \frac{\left(32\,f^4x^4e^{(4\,e)} + 32\left(4\,f^3x^3e^{(2\,e)} + 6\,f^2x^2e^{(2\,e)} + 6\,fxe^{(2\,e)} + 3\,e^{(2\,e)}\right)e^{(-2\,fx)} - (32\,f^3x^3 + 24\,f^2x^2 + 12\,f}{512\,a^2f^4}$$

input
```
integrate((d*x+c)^3/(a+a*coth(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/16*c^3*(4*(f*x + e)/(a^2*f) + (4*e^(-2*f*x - 2*e) - e^(-4*f*x - 4*e))/(a
^2*f)) + 3/64*(8*f^2*x^2*e^(4*e) + 8*(2*f*x*e^(2*e) + e^(2*e))*e^(-2*f*x)
- (4*f*x + 1)*e^(-4*f*x))*c^2*d*e^(-4*e)/(a^2*f^2) + 1/128*(32*f^3*x^3*e^(
4*e) + 48*(2*f^2*x^2*e^(2*e) + 2*f*x*e^(2*e) + e^(2*e))*e^(-2*f*x) - 3*(8*
f^2*x^2 + 4*f*x + 1)*e^(-4*f*x))*c*d^2*e^(-4*e)/(a^2*f^3) + 1/512*(32*f^4*
x^4*e^(4*e) + 32*(4*f^3*x^3*e^(2*e) + 6*f^2*x^2*e^(2*e) + 6*f*x*e^(2*e) +
3*e^(2*e))*e^(-2*f*x) - (32*f^3*x^3 + 24*f^2*x^2 + 12*f*x + 3)*e^(-4*f*x))
*d^3*e^(-4*e)/(a^2*f^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.60

$$\int \frac{(c+dx)^3}{(a+a\coth(e+fx))^2}\,dx = \frac{\left(32\,d^3f^4x^4e^{(4\,fx+4\,e)}+128\,cd^2f^4x^3e^{(4\,fx+4\,e)}+192\,c^2df^4x^2e^{(4\,fx+4\,e)}+128\,d^3f^3x^3e^{(2\,fx+2\,e)}-32\,d^3f^3x^3+\right.}{}$$

input
```
integrate((d*x+c)^3/(a+a*coth(f*x+e))^2,x, algorithm="giac")
```

output
```
1/512*(32*d^3*f^4*x^4*e^(4*f*x + 4*e) + 128*c*d^2*f^4*x^3*e^(4*f*x + 4*e)
+ 192*c^2*d*f^4*x^2*e^(4*f*x + 4*e) + 128*d^3*f^3*x^3*e^(2*f*x + 2*e) - 32
*d^3*f^3*x^3 + 128*c^3*f^4*x*e^(4*f*x + 4*e) + 384*c*d^2*f^3*x^2*e^(2*f*x
+ 2*e) - 96*c*d^2*f^3*x^2 + 384*c^2*d*f^3*x*e^(2*f*x + 2*e) + 192*d^3*f^2*
x^2*e^(2*f*x + 2*e) - 96*c^2*d*f^3*x - 24*d^3*f^2*x^2 + 128*c^3*f^3*e^(2*f
*x + 2*e) + 384*c*d^2*f^2*x*e^(2*f*x + 2*e) - 32*c^3*f^3 - 48*c*d^2*f^2*x
+ 192*c^2*d*f^2*e^(2*f*x + 2*e) + 192*d^3*f*x*e^(2*f*x + 2*e) - 24*c^2*d*f
^2 - 12*d^3*f*x + 192*c*d^2*f*e^(2*f*x + 2*e) - 12*c*d^2*f + 96*d^3*e^(2*f
*x + 2*e) - 3*d^3)*e^(-4*f*x - 4*e)/(a^2*f^4)
```

Mupad [B] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx)^3}{(a+a\coth(e+fx))^2}\,dx = \mathrm{e}^{-2\,e-2\,f\,x}\,\left(\frac{4\,c^3\,f^3+6\,c^2\,d\,f^2+6\,c\,d^2\,f+3\,d^3}{16\,a^2\,f^4}+\frac{d^3\,x^3}{4\,a^2\,f}+\frac{3\,d\,x\,\left(2\,c^2\,f^2+2\,c\,d\,f+d^2\right)}{8\,a^2\,f^3}+\frac{3\,d^2\,x^2\,\left(d+2\,c\,f\right)}{8\,a^2\,f^2}\right)-\mathrm{e}^{-4\,e-4\,f\,x}\,\left(\frac{32\,c^3\,f^3+24\,c^2\,d\,f^2+12\,c\,d^2\,f+3\,d^3}{512\,a^2\,f^4}+\frac{d^3\,x^3}{16\,a^2\,f}+\frac{3\,d\,x\,\left(8\,c^2\,f^2+4\,c\,d\,f+d^2\right)}{128\,a^2\,f^3}+\frac{3\,d^2\,x^2\,\left(d+4\,c\,f\right)}{64\,a^2\,f^2}\right)+\frac{c^3\,x}{4\,a^2}+\frac{d^3\,x^4}{16\,a^2}+\frac{3\,c^2\,d\,x^2}{8\,a^2}+\frac{c\,d^2\,x^3}{4\,a^2}$$

input
```
int((c + d*x)^3/(a + a*coth(e + f*x))^2,x)
```

output
```
exp(- 2*e - 2*f*x)*((3*d^3 + 4*c^3*f^3 + 6*c^2*d*f^2 + 6*c*d^2*f)/(16*a^2*
f^4) + (d^3*x^3)/(4*a^2*f) + (3*d*x*(d^2 + 2*c^2*f^2 + 2*c*d*f))/(8*a^2*f^
3) + (3*d^2*x^2*(d + 2*c*f))/(8*a^2*f^2)) - exp(- 4*e - 4*f*x)*((3*d^3 + 3
2*c^3*f^3 + 24*c^2*d*f^2 + 12*c*d^2*f)/(512*a^2*f^4) + (d^3*x^3)/(16*a^2*f
) + (3*d*x*(d^2 + 8*c^2*f^2 + 4*c*d*f))/(128*a^2*f^3) + (3*d^2*x^2*(d + 4*
c*f))/(64*a^2*f^2)) + (c^3*x)/(4*a^2) + (d^3*x^4)/(16*a^2) + (3*c^2*d*x^2)
/(8*a^2) + (c*d^2*x^3)/(4*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.67

$$\int \frac{(c+dx)^3}{(a+a\coth(e+fx))^2}\,dx = \frac{128e^{4fx+4e}c^3f^4x+192e^{4fx+4e}c^2d\,f^4x^2+128e^{4fx+4e}c\,d^2f^4x^3+32e^{4fx+4e}d^3f^4x^4+128e^{2fx+2e}c^3f^3+384e}{}$$

input
```
int((d*x+c)^3/(a+a*coth(f*x+e))^2,x)
```

output

```
(128*e**(4*e + 4*f*x)*c**3*f**4*x + 192*e**(4*e + 4*f*x)*c**2*d*f**4*x**2
+ 128*e**(4*e + 4*f*x)*c*d**2*f**4*x**3 + 32*e**(4*e + 4*f*x)*d**3*f**4*x*
*4 + 128*e**(2*e + 2*f*x)*c**3*f**3 + 384*e**(2*e + 2*f*x)*c**2*d*f**3*x +
 192*e**(2*e + 2*f*x)*c**2*d*f**2 + 384*e**(2*e + 2*f*x)*c*d**2*f**3*x**2
+ 384*e**(2*e + 2*f*x)*c*d**2*f**2*x + 192*e**(2*e + 2*f*x)*c*d**2*f + 128
*e**(2*e + 2*f*x)*d**3*f**3*x**3 + 192*e**(2*e + 2*f*x)*d**3*f**2*x**2 + 1
92*e**(2*e + 2*f*x)*d**3*f*x + 96*e**(2*e + 2*f*x)*d**3 - 32*c**3*f**3 - 9
6*c**2*d*f**3*x - 24*c**2*d*f**2 - 96*c*d**2*f**3*x**2 - 48*c*d**2*f**2*x
- 12*c*d**2*f - 32*d**3*f**3*x**3 - 24*d**3*f**2*x**2 - 12*d**3*f*x - 3*d*
*3)/(512*e**(4*e + 4*f*x)*a**2*f**4)
```

3.23 $\int \frac{(c+dx)^2}{(a+a\coth(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 170

$$\int \frac{(c+dx)^2}{(a+a\coth(e+fx))^2}\,dx = -\frac{d^2e^{-4e-4fx}}{128a^2f^3}+\frac{d^2e^{-2e-2fx}}{8a^2f^3}-\frac{de^{-4e-4fx}(c+dx)}{32a^2f^2}+\frac{de^{-2e-2fx}(c+dx)}{4a^2f^2}-\frac{e^{-4e-4fx}(c+dx)^2}{16a^2f}+\frac{e^{-2e-2fx}(c+dx)^2}{4a^2f}+\frac{(c+dx)^3}{12a^2d}$$

output

```
-1/128*d^2*exp(-4*f*x-4*e)/a^2/f^3+1/8*d^2*exp(-2*f*x-2*e)/a^2/f^3-1/32*d*
exp(-4*f*x-4*e)*(d*x+c)/a^2/f^2+1/4*d*exp(-2*f*x-2*e)*(d*x+c)/a^2/f^2-1/16
*exp(-4*f*x-4*e)*(d*x+c)^2/a^2/f+1/4*exp(-2*f*x-2*e)*(d*x+c)^2/a^2/f+1/12*
(d*x+c)^3/a^2/d
```

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx)^2}{(a+a\coth(e+fx))^2}\,dx$$
$$= \frac{\operatorname{csch}^2(e+fx)\,(48(2c^2f^2+2cdf(1+2fx)+d^2(1+2fx+2f^2x^2))+(24c^2f^2(-1+4fx)+12cdf(-1-$$

input
```
Integrate[(c + d*x)^2/(a + a*Coth[e + f*x])^2,x]
```

output
```
(Csch[e + f*x]^2*(48*(2*c^2*f^2 + 2*c*d*f*(1 + 2*f*x) + d^2*(1 + 2*f*x + 2
*f^2*x^2)) + (24*c^2*f^2*(-1 + 4*f*x) + 12*c*d*f*(-1 - 4*f*x + 8*f^2*x^2)
+ d^2*(-3 - 12*f*x - 24*f^2*x^2 + 32*f^3*x^3))*Cosh[2*(e + f*x)] + (24*c^2
*f^2*(1 + 4*f*x) + 12*c*d*f*(1 + 4*f*x + 8*f^2*x^2) + d^2*(3 + 12*f*x + 24
*f^2*x^2 + 32*f^3*x^3))*Sinh[2*(e + f*x)]))/(384*a^2*f^3*(1 + Coth[e + f*x
])^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4212, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a\coth(e+fx)+a)^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c+dx)^2}{\left(a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{4212}$$
$$\int \left(\frac{(c+dx)^2e^{-4e-4fx}}{4a^2}-\frac{(c+dx)^2e^{-2e-2fx}}{2a^2}+\frac{(c+dx)^2}{4a^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d(c+dx)e^{-4e-4fx}}{32a^2f^2}+\frac{d(c+dx)e^{-2e-2fx}}{4a^2f^2}-\frac{(c+dx)^2e^{-4e-4fx}}{16a^2f}+\frac{(c+dx)^2e^{-2e-2fx}}{4a^2f}+\frac{(c+dx)^3}{12a^2d}-\frac{d^2e^{-4e-4fx}}{128a^2f^3}+\frac{d^2e^{-2e-2fx}}{8a^2f^3}$$

input
```
Int[(c + d*x)^2/(a + a*Coth[e + f*x])^2,x]
```

output
```
-1/128*(d^2*E^(-4*e - 4*f*x))/(a^2*f^3) + (d^2*E^(-2*e - 2*f*x))/(8*a^2*f^
3) - (d*E^(-4*e - 4*f*x)*(c + d*x))/(32*a^2*f^2) + (d*E^(-2*e - 2*f*x)*(c
+ d*x))/(4*a^2*f^2) - (E^(-4*e - 4*f*x)*(c + d*x)^2)/(16*a^2*f) + (E^(-2*e
 - 2*f*x)*(c + d*x)^2)/(4*a^2*f) + (c + d*x)^3/(12*a^2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4212
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.96

method	result
risch	$\frac{d^2x^3}{12a^2}+\frac{dc\,x^2}{4a^2}+\frac{c^2x}{4a^2}+\frac{c^3}{12a^2d}+\frac{(2d^2x^2f^2+4cd\,f^2x+2c^2f^2+2d^2fx+2cdf+d^2)\,\mathrm{e}^{-2fx-2e}}{8a^2f^3}-\frac{(8d^2x^2f^2+16cd\,f^2x+8c$
parallelrisch	$\frac{24f\left(\left(\frac{1}{3}d^2x^2+cdx+c^2\right)f^2-\frac{5\left(\frac{dx}{2}+c\right)df}{2}-\frac{9d^2}{8}\right)x\tanh(fx+e)^2+(16(d^2x^3+3cd\,x^2+3c^2x)f^3+12(d^2x^2+2cdx+6c^2)f^2+6(d^2}{96f^3a^2(\tanh(fx+e)}$

input
```
int((d*x+c)^2/(a+a*coth(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/12/a^2*d^2*x^3+1/4/a^2*d*c*x^2+1/4/a^2*c^2*x+1/12/a^2/d*c^3+1/8*(2*d^2*f
^2*x^2+4*c*d*f^2*x+2*c^2*f^2+2*d^2*f*x+2*c*d*f+d^2)/a^2/f^3*exp(-2*f*x-2*e
)-1/128*(8*d^2*f^2*x^2+16*c*d*f^2*x+8*c^2*f^2+4*d^2*f*x+4*c*d*f+d^2)/a^2/f
^3*exp(-4*f*x-4*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(150) = 300$.

Time = 0.09 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.11

$$\int \frac{(c+dx)^2}{(a+a\coth(e+fx))^2}\,dx$$
$$= \frac{96\,d^2f^2x^2+96\,c^2f^2+96\,cdf+(32\,d^2f^3x^3-24\,c^2f^2-12\,cdf+24\,(4\,cdf^3-d^2f^2)x^2-3\,d^2+12\,(8\,c^2f^3}{}$$

input
```
integrate((d*x+c)^2/(a+a*coth(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/384*(96*d^2*f^2*x^2 + 96*c^2*f^2 + 96*c*d*f + (32*d^2*f^3*x^3 - 24*c^2*f
^2 - 12*c*d*f + 24*(4*c*d*f^3 - d^2*f^2)*x^2 - 3*d^2 + 12*(8*c^2*f^3 - 4*c
*d*f^2 - d^2*f)*x)*cosh(f*x + e)^2 + 2*(32*d^2*f^3*x^3 + 24*c^2*f^2 + 12*c
*d*f + 24*(4*c*d*f^3 + d^2*f^2)*x^2 + 3*d^2 + 12*(8*c^2*f^3 + 4*c*d*f^2 +
d^2*f)*x)*cosh(f*x + e)*sinh(f*x + e) + (32*d^2*f^3*x^3 - 24*c^2*f^2 - 12*
c*d*f + 24*(4*c*d*f^3 - d^2*f^2)*x^2 - 3*d^2 + 12*(8*c^2*f^3 - 4*c*d*f^2 -
 d^2*f)*x)*sinh(f*x + e)^2 + 48*d^2 + 96*(2*c*d*f^2 + d^2*f)*x)/(a^2*f^3*c
osh(f*x + e)^2 + 2*a^2*f^3*cosh(f*x + e)*sinh(f*x + e) + a^2*f^3*sinh(f*x
+ e)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1353 vs. $2(165) = 330$.

Time = 0.87 (sec) , antiderivative size = 1353, normalized size of antiderivative = 7.96

$$\int \frac{(c+dx)^2}{(a+a\coth(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2/(a+a*coth(f*x+e))**2,x)
```

output
```
Piecewise((24*c**2*f**3*x*tanh(e + f*x)**2/(96*a**2*f**3*tanh(e + f*x)**2
+ 192*a**2*f**3*tanh(e + f*x) + 96*a**2*f**3) + 48*c**2*f**3*x*tanh(e + f*
x)/(96*a**2*f**3*tanh(e + f*x)**2 + 192*a**2*f**3*tanh(e + f*x) + 96*a**2*
f**3) + 24*c**2*f**3*x/(96*a**2*f**3*tanh(e + f*x)**2 + 192*a**2*f**3*tanh
(e + f*x) + 96*a**2*f**3) + 72*c**2*f**2*tanh(e + f*x)/(96*a**2*f**3*tanh(
e + f*x)**2 + 192*a**2*f**3*tanh(e + f*x) + 96*a**2*f**3) + 48*c**2*f**2/(
96*a**2*f**3*tanh(e + f*x)**2 + 192*a**2*f**3*tanh(e + f*x) + 96*a**2*f**3
) + 24*c*d*f**3*x**2*tanh(e + f*x)**2/(96*a**2*f**3*tanh(e + f*x)**2 + 192
*a**2*f**3*tanh(e + f*x) + 96*a**2*f**3) + 48*c*d*f**3*x**2*tanh(e + f*x)/
(96*a**2*f**3*tanh(e + f*x)**2 + 192*a**2*f**3*tanh(e + f*x) + 96*a**2*f**
3) + 24*c*d*f**3*x**2/(96*a**2*f**3*tanh(e + f*x)**2 + 192*a**2*f**3*tanh(
e + f*x) + 96*a**2*f**3) - 60*c*d*f**2*x*tanh(e + f*x)**2/(96*a**2*f**3*ta
nh(e + f*x)**2 + 192*a**2*f**3*tanh(e + f*x) + 96*a**2*f**3) + 24*c*d*f**2
*x*tanh(e + f*x)/(96*a**2*f**3*tanh(e + f*x)**2 + 192*a**2*f**3*tanh(e + f
*x) + 96*a**2*f**3) + 36*c*d*f**2*x/(96*a**2*f**3*tanh(e + f*x)**2 + 192*a
**2*f**3*tanh(e + f*x) + 96*a**2*f**3) + 60*c*d*f*tanh(e + f*x)/(96*a**2*f
**3*tanh(e + f*x)**2 + 192*a**2*f**3*tanh(e + f*x) + 96*a**2*f**3) + 48*c*
d*f/(96*a**2*f**3*tanh(e + f*x)**2 + 192*a**2*f**3*tanh(e + f*x) + 96*a**2
*f**3) + 8*d**2*f**3*x**3*tanh(e + f*x)**2/(96*a**2*f**3*tanh(e + f*x)**2
+ 192*a**2*f**3*tanh(e + f*x) + 96*a**2*f**3) + 16*d**2*f**3*x**3*tanh(...
```

Maxima [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx)^2}{(a+a\coth(e+fx))^2}\,dx = \frac{1}{16}\,c^2\left(\frac{4\,(fx+e)}{a^2f}+\frac{4\,e^{(-2\,fx-2\,e)}-e^{(-4\,fx-4\,e)}}{a^2f}\right) + \frac{\left(8\,f^2x^2e^{(4\,e)}+8\,\left(2\,fxe^{(2\,e)}+e^{(2\,e)}\right)e^{(-2\,fx)}-(4\,fx+1)e^{(-4\,fx)}\right)cde^{(-4\,e)}}{32\,a^2f^2} + \frac{\left(32\,f^3x^3e^{(4\,e)}+48\,\left(2\,f^2x^2e^{(2\,e)}+2\,fxe^{(2\,e)}+e^{(2\,e)}\right)e^{(-2\,fx)}-3\,(8\,f^2x^2+4\,fx+1)e^{(-4\,fx)}\right)d^2e^{(-4\,e)}}{384\,a^2f^3}$$

input
```
integrate((d*x+c)^2/(a+a*coth(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/16*c^2*(4*(f*x + e)/(a^2*f) + (4*e^(-2*f*x - 2*e) - e^(-4*f*x - 4*e))/(a
^2*f)) + 1/32*(8*f^2*x^2*e^(4*e) + 8*(2*f*x*e^(2*e) + e^(2*e))*e^(-2*f*x)
- (4*f*x + 1)*e^(-4*f*x))*c*d*e^(-4*e)/(a^2*f^2) + 1/384*(32*f^3*x^3*e^(4*
e) + 48*(2*f^2*x^2*e^(2*e) + 2*f*x*e^(2*e) + e^(2*e))*e^(-2*f*x) - 3*(8*f^
2*x^2 + 4*f*x + 1)*e^(-4*f*x))*d^2*e^(-4*e)/(a^2*f^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx)^2}{(a+a\coth(e+fx))^2}\,dx = \frac{\left(32\,d^2f^3x^3e^{(4\,fx+4\,e)}+96\,cdf^3x^2e^{(4\,fx+4\,e)}+96\,c^2f^3xe^{(4\,fx+4\,e)}+96\,d^2f^2x^2e^{(2\,fx+2\,e)}-24\,d^2f^2x^2+192\,c\right.}{}$$

input
```
integrate((d*x+c)^2/(a+a*coth(f*x+e))^2,x, algorithm="giac")
```

output
```
1/384*(32*d^2*f^3*x^3*e^(4*f*x + 4*e) + 96*c*d*f^3*x^2*e^(4*f*x + 4*e) + 9
6*c^2*f^3*x*e^(4*f*x + 4*e) + 96*d^2*f^2*x^2*e^(2*f*x + 2*e) - 24*d^2*f^2*
x^2 + 192*c*d*f^2*x*e^(2*f*x + 2*e) - 48*c*d*f^2*x + 96*c^2*f^2*e^(2*f*x +
 2*e) + 96*d^2*f*x*e^(2*f*x + 2*e) - 24*c^2*f^2 - 12*d^2*f*x + 96*c*d*f*e^
(2*f*x + 2*e) - 12*c*d*f + 48*d^2*e^(2*f*x + 2*e) - 3*d^2)*e^(-4*f*x - 4*e
)/(a^2*f^3)
```

Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx)^2}{(a+a\coth(e+fx))^2}\,dx = \mathrm{e}^{-2e-2fx}\left(\frac{2c^2f^2+2cdf+d^2}{8a^2f^3}+\frac{d^2x^2}{4a^2f}+\frac{dx(d+2cf)}{4a^2f^2}\right)-\mathrm{e}^{-4e-4fx}\left(\frac{8c^2f^2+4cdf+d^2}{128a^2f^3}+\frac{d^2x^2}{16a^2f}+\frac{dx(d+4cf)}{32a^2f^2}\right)+\frac{c^2x}{4a^2}+\frac{d^2x^3}{12a^2}+\frac{cdx^2}{4a^2}$$

input
```
int((c + d*x)^2/(a + a*coth(e + f*x))^2,x)
```

output
```
exp(- 2*e - 2*f*x)*((d^2 + 2*c^2*f^2 + 2*c*d*f)/(8*a^2*f^3) + (d^2*x^2)/(4
*a^2*f) + (d*x*(d + 2*c*f))/(4*a^2*f^2)) - exp(- 4*e - 4*f*x)*((d^2 + 8*c^
2*f^2 + 4*c*d*f)/(128*a^2*f^3) + (d^2*x^2)/(16*a^2*f) + (d*x*(d + 4*c*f))/
(32*a^2*f^2)) + (c^2*x)/(4*a^2) + (d^2*x^3)/(12*a^2) + (c*d*x^2)/(4*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.35

$$\int \frac{(c+dx)^2}{(a+a\coth(e+fx))^2}\,dx = \frac{96e^{4fx+4e}c^2f^3x+96e^{4fx+4e}cd\,f^3x^2+32e^{4fx+4e}d^2f^3x^3+96e^{2fx+2e}c^2f^2+192e^{2fx+2e}cd\,f^2x+96e^{2fx+2e}c\cdots}{384\cdots}$$

input
```
int((d*x+c)^2/(a+a*coth(f*x+e))^2,x)
```

output
```
(96*e**(4*e + 4*f*x)*c**2*f**3*x + 96*e**(4*e + 4*f*x)*c*d*f**3*x**2 + 32*
e**(4*e + 4*f*x)*d**2*f**3*x**3 + 96*e**(2*e + 2*f*x)*c**2*f**2 + 192*e**(
2*e + 2*f*x)*c*d*f**2*x + 96*e**(2*e + 2*f*x)*c*d*f + 96*e**(2*e + 2*f*x)*
d**2*f**2*x**2 + 96*e**(2*e + 2*f*x)*d**2*f*x + 48*e**(2*e + 2*f*x)*d**2 -
 24*c**2*f**2 - 48*c*d*f**2*x - 12*c*d*f - 24*d**2*f**2*x**2 - 12*d**2*f*x
 - 3*d**2)/(384*e**(4*e + 4*f*x)*a**2*f**3)
```

3.24 $\int \frac{c+dx}{(a+a\coth(e+fx))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 133

$$\int \frac{c+dx}{(a+a\coth(e+fx))^2}\,dx = \frac{3dx}{16a^2f}-\frac{dx^2}{8a^2}+\frac{x(c+dx)}{4a^2}-\frac{d}{16f^2(a+a\coth(e+fx))^2}-\frac{c+dx}{4f(a+a\coth(e+fx))^2}-\frac{3d}{16f^2\left(a^2+a^2\coth(e+fx)\right)}-\frac{c+dx}{4f\left(a^2+a^2\coth(e+fx)\right)}$$

output
```
3/16*d*x/a^2/f-1/8*d*x^2/a^2+1/4*x*(d*x+c)/a^2-1/16*d/f^2/(a+a*coth(f*x+e)
)^2-1/4*(d*x+c)/f/(a+a*coth(f*x+e))^2-3/16*d/f^2/(a^2+a^2*coth(f*x+e))-1/4
*(d*x+c)/f/(a^2+a^2*coth(f*x+e))
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.86

$$\int \frac{c+dx}{(a+a\coth(e+fx))^2}\,dx = \frac{\operatorname{csch}^2(e+fx)\left(8(d+2cf+2dfx)+(4cf(-1+4fx)+d(-1-4fx+8f^2x^2))\cosh(2(e+fx))+(4cf(\right.}{64a^2f^2(1+\coth(e+fx))^2}$$

input
```
Integrate[(c + d*x)/(a + a*Coth[e + f*x])^2,x]
```

output
```
(Csch[e + f*x]^2*(8*(d + 2*c*f + 2*d*f*x) + (4*c*f*(-1 + 4*f*x) + d*(-1 -
4*f*x + 8*f^2*x^2))*Cosh[2*(e + f*x)] + (4*c*f*(1 + 4*f*x) + d*(1 + 4*f*x
+ 8*f^2*x^2))*Sinh[2*(e + f*x)]))/(64*a^2*f^2*(1 + Coth[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4213, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a\coth(e+fx)+a)^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{c+dx}{\left(a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{4213}$$
$$-d\int \left(\frac{x}{4a^2}-\frac{1}{4f\left(\coth(e+fx)a^2+a^2\right)}-\frac{1}{4f(\coth(e+fx)a+a)^2}\right)dx-\frac{c+dx}{4f\left(a^2\coth(e+fx)+a^2\right)}+\frac{x(c+dx)}{4a^2}-\frac{c+dx}{4f(a\coth(e+fx)+a)^2}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{c+dx}{4f\left(a^2\coth(e+fx)+a^2\right)}+\frac{x(c+dx)}{4a^2}-d\left(\frac{3}{16f^2\left(a^2\coth(e+fx)+a^2\right)}-\frac{3x}{16a^2f}+\frac{x^2}{8a^2}+\frac{1}{16f^2(a\coth(e+fx)+a)^2}\right)-\frac{c+dx}{4f(a\coth(e+fx)+a)^2}$$

input
```
Int[(c + d*x)/(a + a*Coth[e + f*x])^2,x]
```

output
```
(x*(c + d*x))/(4*a^2) - (c + d*x)/(4*f*(a + a*Coth[e + f*x])^2) - (c + d*x
)/(4*f*(a^2 + a^2*Coth[e + f*x])) - d*((-3*x)/(16*a^2*f) + x^2/(8*a^2) + 1
/(16*f^2*(a + a*Coth[e + f*x])^2) + 3/(16*f^2*(a^2 + a^2*Coth[e + f*x])))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4213
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> With[{u = IntHide[(a + b*Tan[e + f*x])^n, x]}, Simp[(c + d*x)
^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)   u, x], x]] /; FreeQ[{a, b,
c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.56

method	result	si
risch	$\frac{d\,x^{2}}{8a^{2}}+\frac{xc}{4a^{2}}+\frac{\left(2dfx+2cf+d\right){\mathrm e}^{-2fx-2e}}{8a^{2}f^{2}}-\frac{\left(4dfx+4cf+d\right){\mathrm e}^{-4fx-4e}}{64a^{2}f^{2}}$	7
parallelrisch	$\frac{4fx\left(\left(\frac{dx}{2}+c\right)f-\frac{5d}{4}\right)\tanh\left(fx+e\right)^{2}+\left(4\left(d\,x^{2}+2xc\right)f^{2}+2\left(dx+6c\right)f+5d\right)\tanh\left(fx+e\right)+2\left(d\,x^{2}+2xc\right)f^{2}+\left(3dx+8c\right)f+4d}{16f^{2}a^{2}\left(\tanh\left(fx+e\right)+1\right)^{2}}$	1

input
```
int((d*x+c)/(a+a*coth(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*d*x^2/a^2+1/4/a^2*x*c+1/8*(2*d*f*x+2*c*f+d)/a^2/f^2*exp(-2*f*x-2*e)-1/
64*(4*d*f*x+4*c*f+d)/a^2/f^2*exp(-4*f*x-4*e)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.43

$$\int \frac{c+dx}{(a+a\coth(e+fx))^2}\,dx$$
$$= \frac{16\,dfx + (8\,df^2x^2 - 4\,cf + 4\,(4\,cf^2 - df)x - d)\cosh\left(fx+e\right)^2 + 2\,(8\,df^2x^2 + 4\,cf + 4\,(4\,cf^2 + df)x + d}{64\,\left(a^2f^2\cosh\left(fx+e\right)^2 + 2\,a^2f^2\cosh\left(fx+e\right.\right.}$$

input
```
integrate((d*x+c)/(a+a*coth(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/64*(16*d*f*x + (8*d*f^2*x^2 - 4*c*f + 4*(4*c*f^2 - d*f)*x - d)*cosh(f*x
+ e)^2 + 2*(8*d*f^2*x^2 + 4*c*f + 4*(4*c*f^2 + d*f)*x + d)*cosh(f*x + e)*s
inh(f*x + e) + (8*d*f^2*x^2 - 4*c*f + 4*(4*c*f^2 - d*f)*x - d)*sinh(f*x +
e)^2 + 16*c*f + 8*d)/(a^2*f^2*cosh(f*x + e)^2 + 2*a^2*f^2*cosh(f*x + e)*si
nh(f*x + e) + a^2*f^2*sinh(f*x + e)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(119) = 238$.

Time = 0.81 (sec) , antiderivative size = 700, normalized size of antiderivative = 5.26

$$\int \frac{c+dx}{(a+a\coth(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(a+a*coth(f*x+e))**2,x)
```

output
```
Piecewise((4*c*f**2*x*tanh(e + f*x)**2/(16*a**2*f**2*tanh(e + f*x)**2 + 32
*a**2*f**2*tanh(e + f*x) + 16*a**2*f**2) + 8*c*f**2*x*tanh(e + f*x)/(16*a*
*2*f**2*tanh(e + f*x)**2 + 32*a**2*f**2*tanh(e + f*x) + 16*a**2*f**2) + 4*
c*f**2*x/(16*a**2*f**2*tanh(e + f*x)**2 + 32*a**2*f**2*tanh(e + f*x) + 16*
a**2*f**2) + 12*c*f*tanh(e + f*x)/(16*a**2*f**2*tanh(e + f*x)**2 + 32*a**2
*f**2*tanh(e + f*x) + 16*a**2*f**2) + 8*c*f/(16*a**2*f**2*tanh(e + f*x)**2
 + 32*a**2*f**2*tanh(e + f*x) + 16*a**2*f**2) + 2*d*f**2*x**2*tanh(e + f*x
)**2/(16*a**2*f**2*tanh(e + f*x)**2 + 32*a**2*f**2*tanh(e + f*x) + 16*a**2
*f**2) + 4*d*f**2*x**2*tanh(e + f*x)/(16*a**2*f**2*tanh(e + f*x)**2 + 32*a
**2*f**2*tanh(e + f*x) + 16*a**2*f**2) + 2*d*f**2*x**2/(16*a**2*f**2*tanh(
e + f*x)**2 + 32*a**2*f**2*tanh(e + f*x) + 16*a**2*f**2) - 5*d*f*x*tanh(e
+ f*x)**2/(16*a**2*f**2*tanh(e + f*x)**2 + 32*a**2*f**2*tanh(e + f*x) + 16
*a**2*f**2) + 2*d*f*x*tanh(e + f*x)/(16*a**2*f**2*tanh(e + f*x)**2 + 32*a*
*2*f**2*tanh(e + f*x) + 16*a**2*f**2) + 3*d*f*x/(16*a**2*f**2*tanh(e + f*x
)**2 + 32*a**2*f**2*tanh(e + f*x) + 16*a**2*f**2) + 5*d*tanh(e + f*x)/(16*
a**2*f**2*tanh(e + f*x)**2 + 32*a**2*f**2*tanh(e + f*x) + 16*a**2*f**2) +
4*d/(16*a**2*f**2*tanh(e + f*x)**2 + 32*a**2*f**2*tanh(e + f*x) + 16*a**2*
f**2), Ne(f, 0)), ((c*x + d*x**2/2)/(a*coth(e) + a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.80

$$\int \frac{c+dx}{(a+a\coth(e+fx))^2}\,dx = \frac{1}{16}\,c\left(\frac{4\,(fx+e)}{a^2f}+\frac{4\,e^{(-2\,fx-2\,e)}-e^{(-4\,fx-4\,e)}}{a^2f}\right) + \frac{\left(8\,f^2x^2e^{(4\,e)}+8\,\left(2\,fxe^{(2\,e)}+e^{(2\,e)}\right)e^{(-2\,fx)}-(4\,fx+1)e^{(-4\,fx)}\right)de^{(-4\,e)}}{64\,a^2f^2}$$

input
```
integrate((d*x+c)/(a+a*coth(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/16*c*(4*(f*x + e)/(a^2*f) + (4*e^(-2*f*x - 2*e) - e^(-4*f*x - 4*e))/(a^2
*f)) + 1/64*(8*f^2*x^2*e^(4*e) + 8*(2*f*x*e^(2*e) + e^(2*e))*e^(-2*f*x) -
(4*f*x + 1)*e^(-4*f*x))*d*e^(-4*e)/(a^2*f^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.77

$$\int \frac{c+dx}{(a+a\coth(e+fx))^2}\,dx = \frac{(8\,df^2x^2e^{(4\,fx+4\,e)}+16\,cf^2xe^{(4\,fx+4\,e)}+16\,dfxe^{(2\,fx+2\,e)}-4\,dfx+16\,cfe^{(2\,fx+2\,e)}-4\,cf+8\,de^{(2\,fx+2\,e)}-}{64\,a^2f^2}$$

input
```
integrate((d*x+c)/(a+a*coth(f*x+e))^2,x, algorithm="giac")
```

output
```
1/64*(8*d*f^2*x^2*e^(4*f*x + 4*e) + 16*c*f^2*x*e^(4*f*x + 4*e) + 16*d*f*x*
e^(2*f*x + 2*e) - 4*d*f*x + 16*c*f*e^(2*f*x + 2*e) - 4*c*f + 8*d*e^(2*f*x
+ 2*e) - d)*e^(-4*f*x - 4*e)/(a^2*f^2)
```

Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.66

$$\int \frac{c+dx}{(a+a\coth(e+fx))^2}\,dx = \mathrm{e}^{-2\,e-2\,f\,x}\left(\frac{d+2\,c\,f}{8\,a^2\,f^2}+\frac{d\,x}{4\,a^2\,f}\right) - \mathrm{e}^{-4\,e-4\,f\,x}\left(\frac{d+4\,c\,f}{64\,a^2\,f^2}+\frac{d\,x}{16\,a^2\,f}\right)+\frac{d\,x^2}{8\,a^2}+\frac{c\,x}{4\,a^2}$$

input
```
int((c + d*x)/(a + a*coth(e + f*x))^2,x)
```

output
```
exp(- 2*e - 2*f*x)*((d + 2*c*f)/(8*a^2*f^2) + (d*x)/(4*a^2*f)) - exp(- 4*e
 - 4*f*x)*((d + 4*c*f)/(64*a^2*f^2) + (d*x)/(16*a^2*f)) + (d*x^2)/(8*a^2)
+ (c*x)/(4*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

$$\int \frac{c+dx}{(a+a\coth(e+fx))^2}\,dx$$
$$=\frac{16e^{4fx+4e}cf^2x+8e^{4fx+4e}df^2x^2+16e^{2fx+2e}cf+16e^{2fx+2e}dfx+8e^{2fx+2e}d-4cf-4dfx-d}{64e^{4fx+4e}a^2f^2}$$

input
```
int((d*x+c)/(a+a*coth(f*x+e))^2,x)
```

output
```
(16*e**(4*e + 4*f*x)*c*f**2*x + 8*e**(4*e + 4*f*x)*d*f**2*x**2 + 16*e**(2*
e + 2*f*x)*c*f + 16*e**(2*e + 2*f*x)*d*f*x + 8*e**(2*e + 2*f*x)*d - 4*c*f
- 4*d*f*x - d)/(64*e**(4*e + 4*f*x)*a**2*f**2)
```

3.25 $\int \frac{1}{(c+dx)(a+a\coth(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 297

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))^2}\,dx = -\frac{\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)}{2a^2d} + \frac{\cosh\left(4e-\frac{4cf}{d}\right)\operatorname{Chi}\left(\frac{4cf}{d}+4fx\right)}{4a^2d} + \frac{\log(c+dx)}{4a^2d} - \frac{\operatorname{Chi}\left(\frac{4cf}{d}+4fx\right)\sinh\left(4e-\frac{4cf}{d}\right)}{4a^2d} + \frac{\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{2a^2d} + \frac{\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{2a^2d} - \frac{\sinh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{2a^2d} - \frac{\cosh\left(4e-\frac{4cf}{d}\right)\operatorname{Shi}\left(\frac{4cf}{d}+4fx\right)}{4a^2d} + \frac{\sinh\left(4e-\frac{4cf}{d}\right)\operatorname{Shi}\left(\frac{4cf}{d}+4fx\right)}{4a^2d}$$

output
```
-1/2*cosh(-2*e+2*c*f/d)*Chi(2*c*f/d+2*f*x)/a^2/d+1/4*cosh(-4*e+4*c*f/d)*Ch
i(4*c*f/d+4*f*x)/a^2/d+1/4*ln(d*x+c)/a^2/d+1/4*Chi(4*c*f/d+4*f*x)*sinh(-4*
e+4*c*f/d)/a^2/d-1/2*Chi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a^2/d+1/2*cosh(
-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a^2/d+1/2*sinh(-2*e+2*c*f/d)*Shi(2*c*f/d+
2*f*x)/a^2/d-1/4*cosh(-4*e+4*c*f/d)*Shi(4*c*f/d+4*f*x)/a^2/d-1/4*sinh(-4*e
+4*c*f/d)*Shi(4*c*f/d+4*f*x)/a^2/d
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.67

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))^2}\,dx = \frac{\left(\cosh\left(2e-\frac{2cf}{d}\right)-\sinh\left(2e-\frac{2cf}{d}\right)\right)\left(-2\text{Chi}\left(\frac{2f(c+dx)}{d}\right)+\cosh\left(2e-\frac{2cf}{d}\right)\log(f(c+dx))+\text{Chi}\left(\frac{4f(c+dx)}{d}\right.\right.}{}$$

input
```
Integrate[1/((c + d*x)*(a + a*Coth[e + f*x])^2),x]
```

output
```
((Cosh[2*e - (2*c*f)/d] - Sinh[2*e - (2*c*f)/d])*(-2*CoshIntegral[(2*f*(c
+ d*x))/d] + Cosh[2*e - (2*c*f)/d]*Log[f*(c + d*x)] + CoshIntegral[(4*f*(c
 + d*x))/d]*(Cosh[2*e - (2*c*f)/d] - Sinh[2*e - (2*c*f)/d]) + Log[f*(c + d
*x)]*Sinh[2*e - (2*c*f)/d] + 2*SinhIntegral[(2*f*(c + d*x))/d] - Cosh[2*e
- (2*c*f)/d]*SinhIntegral[(4*f*(c + d*x))/d] + Sinh[2*e - (2*c*f)/d]*SinhI
ntegral[(4*f*(c + d*x))/d]))/(4*a^2*d)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4211, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a\coth(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c+dx)\left(a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4211}$$

$$\int \left(\frac{\sinh^2(2e+2fx)}{4a^2(c+dx)}+\frac{\sinh(2e+2fx)}{2a^2(c+dx)}-\frac{\sinh(4e+4fx)}{4a^2(c+dx)}+\frac{\cosh^2(2e+2fx)}{4a^2(c+dx)}-\frac{\cosh(2e+2fx)}{2a^2(c+dx)}+\frac{1}{4a^2(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{2a^2d}-\frac{\text{Chi}\left(4xf+\frac{4cf}{d}\right)\sinh\left(4e-\frac{4cf}{d}\right)}{4a^2d}-\frac{\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{2a^2d}+\frac{\text{Chi}\left(4xf+\frac{4cf}{d}\right)\cosh\left(4e-\frac{4cf}{d}\right)}{4a^2d}-\frac{\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{2a^2d}+\frac{\sinh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(4xf+\frac{4cf}{d}\right)}{4a^2d}+\frac{\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{2a^2d}-\frac{\cosh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(4xf+\frac{4cf}{d}\right)}{4a^2d}+\frac{\log(c+dx)}{4a^2d}$$

input

```
Int[1/((c + d*x)*(a + a*Coth[e + f*x])^2),x]
```

output

```
-1/2*(Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(a^2*d) + (Co
sh[4*e - (4*c*f)/d]*CoshIntegral[(4*c*f)/d + 4*f*x])/(4*a^2*d) + Log[c + d
*x]/(4*a^2*d) - (CoshIntegral[(4*c*f)/d + 4*f*x]*Sinh[4*e - (4*c*f)/d])/(4
*a^2*d) + (CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/(2*a^2*d
) + (Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2*a^2*d) - (S
inh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2*a^2*d) - (Cosh[4*
e - (4*c*f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(4*a^2*d) + (Sinh[4*e - (4
*c*f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(4*a^2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4211
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.36

method	result	size
risch	$\frac{\ln(dx+c)}{4a^2d}-\frac{\mathrm{e}^{\frac{4cf-4de}{d}}\operatorname{expIntegral}_1\left(4fx+4e+\frac{4cf-4de}{d}\right)}{4a^2d}+\frac{\mathrm{e}^{\frac{2cf-2de}{d}}\operatorname{expIntegral}_1\left(2fx+2e+\frac{2cf-2de}{d}\right)}{2a^2d}$	106

input
```
int(1/(d*x+c)/(a+a*coth(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(d*x+c)/a^2/d-1/4/a^2/d*exp(4*(c*f-d*e)/d)*Ei(1,4*f*x+4*e+4*(c*f-d*e
)/d)+1/2/a^2/d*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.46

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))^2}\,dx = -\frac{2\,\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\cosh\left(-\frac{2\,(de-cf)}{d}\right)-\mathrm{Ei}\left(-\frac{4\,(dfx+cf)}{d}\right)\cosh\left(-\frac{4\,(de-cf)}{d}\right)+2\,\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\sinh\left(-\frac{2}{}\right.}{4\,a^2d}$$

input
```
integrate(1/(d*x+c)/(a+a*coth(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/4*(2*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) - Ei(-4*(d*f*x + c*f
)/d)*cosh(-4*(d*e - c*f)/d) + 2*Ei(-2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)
/d) - Ei(-4*(d*f*x + c*f)/d)*sinh(-4*(d*e - c*f)/d) - log(d*x + c))/(a^2*d
)
```

Sympy [F]

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))^2}\,dx = \frac{\int \frac{1}{c\coth^2(e+fx)+2c\coth(e+fx)+c+dx\coth^2(e+fx)+2dx\coth(e+fx)+dx}\,dx}{a^2}$$

input
```
integrate(1/(d*x+c)/(a+a*coth(f*x+e))**2,x)
```

output
```
Integral(1/(c*coth(e + f*x)**2 + 2*c*coth(e + f*x) + c + d*x*coth(e + f*x)
**2 + 2*d*x*coth(e + f*x) + d*x), x)/a**2
```

Maxima [A] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.27

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))^2}\,dx = -\frac{e^{\left(-4e+\frac{4cf}{d}\right)}E_1\left(\frac{4(dx+c)f}{d}\right)}{4a^2d} + \frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_1\left(\frac{2(dx+c)f}{d}\right)}{2a^2d} + \frac{\log(dx+c)}{4a^2d}$$

input
```
integrate(1/(d*x+c)/(a+a*coth(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/4*e^(-4*e + 4*c*f/d)*exp_integral_e(1, 4*(d*x + c)*f/d)/(a^2*d) + 1/2*e
^(-2*e + 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/(a^2*d) + 1/4*log(d*x
 + c)/(a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.26

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))^2}\,dx = -\frac{\left(2\,\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(2\,e+\frac{2\,cf}{d}\right)} - \mathrm{Ei}\left(-\frac{4\,(dfx+cf)}{d}\right)e^{\left(\frac{4\,cf}{d}\right)} - e^{(4\,e)}\log\left(dx+c\right)\right)e^{(-4\,e)}}{4\,a^2d}$$

input
```
integrate(1/(d*x+c)/(a+a*coth(f*x+e))^2,x, algorithm="giac")
```

output
```
-1/4*(2*Ei(-2*(d*f*x + c*f)/d)*e^(2*e + 2*c*f/d) - Ei(-4*(d*f*x + c*f)/d)*
e^(4*c*f/d) - e^(4*e)*log(d*x + c))*e^(-4*e)/(a^2*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))^2}\,dx = \int \frac{1}{\left(a+a\coth\left(e+f\,x\right)\right)^2\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((a + a*coth(e + f*x))^2*(c + d*x)),x)
```

output
```
int(1/((a + a*coth(e + f*x))^2*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))^2}\,dx$$
$$= \frac{\int \frac{1}{\coth(fx+e)^2c+\coth(fx+e)^2dx+2\coth(fx+e)c+2\coth(fx+e)dx+c+dx}dx}{a^2}$$

input
```
int(1/(d*x+c)/(a+a*coth(f*x+e))^2,x)
```

output
```
int(1/(coth(e + f*x)**2*c + coth(e + f*x)**2*d*x + 2*coth(e + f*x)*c + 2*c
oth(e + f*x)*d*x + c + d*x),x)/a**2
```

3.26 $\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 420

$$\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))^2}\,dx = -\frac{1}{4a^2d(c+dx)}+\frac{\cosh(2e+2fx)}{2a^2d(c+dx)} - \frac{\cosh^2(2e+2fx)}{4a^2d(c+dx)} + \frac{f\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{a^2d^2} - \frac{f\cosh\left(4e-\frac{4cf}{d}\right)\text{Chi}\left(\frac{4cf}{d}+4fx\right)}{a^2d^2} + \frac{f\text{Chi}\left(\frac{4cf}{d}+4fx\right)\sinh\left(4e-\frac{4cf}{d}\right)}{a^2d^2} - \frac{f\text{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{a^2d^2} - \frac{\sinh(2e+2fx)}{2a^2d(c+dx)} - \frac{\sinh^2(2e+2fx)}{4a^2d(c+dx)} + \frac{\sinh(4e+4fx)}{4a^2d(c+dx)} - \frac{f\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{a^2d^2} + \frac{f\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{a^2d^2} + \frac{f\cosh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(\frac{4cf}{d}+4fx\right)}{a^2d^2} - \frac{f\sinh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(\frac{4cf}{d}+4fx\right)}{a^2d^2}$$

output
```
-1/4/a^2/d/(d*x+c)+1/2*cosh(2*f*x+2*e)/a^2/d/(d*x+c)-1/4*cosh(2*f*x+2*e)^2
/a^2/d/(d*x+c)+f*cosh(-2*e+2*c*f/d)*Chi(2*c*f/d+2*f*x)/a^2/d^2-f*cosh(-4*e
+4*c*f/d)*Chi(4*c*f/d+4*f*x)/a^2/d^2-f*Chi(4*c*f/d+4*f*x)*sinh(-4*e+4*c*f/
d)/a^2/d^2+f*Chi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a^2/d^2-1/2*sinh(2*f*x+
2*e)/a^2/d/(d*x+c)-1/4*sinh(2*f*x+2*e)^2/a^2/d/(d*x+c)+1/4*sinh(4*f*x+4*e)
/a^2/d/(d*x+c)-f*cosh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a^2/d^2-f*sinh(-2*e
+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a^2/d^2+f*cosh(-4*e+4*c*f/d)*Shi(4*c*f/d+4*f*
x)/a^2/d^2+f*sinh(-4*e+4*c*f/d)*Shi(4*c*f/d+4*f*x)/a^2/d^2
```

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.05

$$\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))^2}\,dx$$
$$= \frac{\left(-\cosh\left(2\left(e+f\left(-\frac{c}{d}+x\right)\right)\right)+\sinh\left(2\left(e+f\left(-\frac{c}{d}+x\right)\right)\right)\right)\left(-2d\cosh\left(\frac{2cf}{d}\right)+d\cosh\left(2\left(e+f\left(-\frac{c}{d}+x\right)\right)\right.\right.}{}$$

input
```
Integrate[1/((c + d*x)^2*(a + a*Coth[e + f*x])^2),x]
```

output
```
((-Cosh[2*(e + f*(-(c/d) + x))] + Sinh[2*(e + f*(-(c/d) + x))])*(-2*d*Cosh
[(2*c*f)/d] + d*Cosh[2*(e + f*(-(c/d) + x))] + d*Cosh[2*(e + f*(c/d + x))]
 + 2*d*Sinh[(2*c*f)/d] - 4*f*(c + d*x)*CoshIntegral[(2*f*(c + d*x))/d]*(Co
sh[2*f*x] + Sinh[2*f*x]) + d*Sinh[2*(e + f*(-(c/d) + x))] - d*Sinh[2*(e +
f*(c/d + x))] + 4*f*(c + d*x)*CoshIntegral[(4*f*(c + d*x))/d]*(Cosh[2*e -
(2*f*(c + d*x))/d] - Sinh[2*e - (2*f*(c + d*x))/d]) + 4*c*f*Cosh[2*f*x]*Si
nhIntegral[(2*f*(c + d*x))/d] + 4*d*f*x*Cosh[2*f*x]*SinhIntegral[(2*f*(c +
 d*x))/d] + 4*c*f*Sinh[2*f*x]*SinhIntegral[(2*f*(c + d*x))/d] + 4*d*f*x*Si
nh[2*f*x]*SinhIntegral[(2*f*(c + d*x))/d] - 4*c*f*Cosh[2*e - (2*f*(c + d*x
))/d]*SinhIntegral[(4*f*(c + d*x))/d] - 4*d*f*x*Cosh[2*e - (2*f*(c + d*x))
/d]*SinhIntegral[(4*f*(c + d*x))/d] + 4*c*f*Sinh[2*e - (2*f*(c + d*x))/d]*
SinhIntegral[(4*f*(c + d*x))/d] + 4*d*f*x*Sinh[2*e - (2*f*(c + d*x))/d]*Si
nhIntegral[(4*f*(c + d*x))/d]))/(4*a^2*d^2*(c + d*x))
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4211, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a\coth(e+fx)+a)^2}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c+dx)^2\left(a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}dx$$

$\Big\downarrow$ 4211

$$\int \left(\frac{\sinh^2(2e+2fx)}{4a^2(c+dx)^2}+\frac{\sinh(2e+2fx)}{2a^2(c+dx)^2}-\frac{\sinh(4e+4fx)}{4a^2(c+dx)^2}+\frac{\cosh^2(2e+2fx)}{4a^2(c+dx)^2}-\frac{\cosh(2e+2fx)}{2a^2(c+dx)^2}+\frac{1}{4a^2(c+dx)^2}\right.$$

$\Big\downarrow$ 2009

$$\frac{f\text{Chi}\left(4xf+\frac{4cf}{d}\right)\sinh\left(4e-\frac{4cf}{d}\right)}{a^2d^2}-\frac{f\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{a^2d^2}+\frac{f\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{a^2d^2}-\frac{f\text{Chi}\left(4xf+\frac{4cf}{d}\right)\cosh\left(4e-\frac{4cf}{d}\right)}{a^2d^2}+\frac{f\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{a^2d^2}-\frac{f\sinh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(4xf+\frac{4cf}{d}\right)}{a^2d^2}-\frac{f\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{a^2d^2}+\frac{f\cosh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(4xf+\frac{4cf}{d}\right)}{a^2d^2}-\frac{\sinh^2(2e+2fx)}{4a^2d(c+dx)}-\frac{\sinh(2e+2fx)}{2a^2d(c+dx)}+\frac{\sinh(4e+4fx)}{4a^2d(c+dx)}-\frac{\cosh^2(2e+2fx)}{4a^2d(c+dx)}+\frac{\cosh(2e+2fx)}{2a^2d(c+dx)}-\frac{1}{4a^2d(c+dx)}$$

input
```
Int[1/((c + d*x)^2*(a + a*Coth[e + f*x])^2),x]
```

output
```
-1/4*1/(a^2*d*(c + d*x)) + Cosh[2*e + 2*f*x]/(2*a^2*d*(c + d*x)) - Cosh[2*
e + 2*f*x]^2/(4*a^2*d*(c + d*x)) + (f*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(
2*c*f)/d + 2*f*x])/(a^2*d^2) - (f*Cosh[4*e - (4*c*f)/d]*CoshIntegral[(4*c*
f)/d + 4*f*x])/(a^2*d^2) + (f*CoshIntegral[(4*c*f)/d + 4*f*x]*Sinh[4*e - (
4*c*f)/d])/(a^2*d^2) - (f*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*
f)/d])/(a^2*d^2) - Sinh[2*e + 2*f*x]/(2*a^2*d*(c + d*x)) - Sinh[2*e + 2*f*
x]^2/(4*a^2*d*(c + d*x)) + Sinh[4*e + 4*f*x]/(4*a^2*d*(c + d*x)) - (f*Cosh
[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(a^2*d^2) + (f*Sinh[2*e
 - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(a^2*d^2) + (f*Cosh[4*e - (
4*c*f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(a^2*d^2) - (f*Sinh[4*e - (4*c*
f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(a^2*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4211
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.39

method	result
risch	$-\frac{1}{4a^2d(dx+c)}-\frac{f\,\mathrm{e}^{-4fx-4e}}{4a^2d(dfx+cf)}+\frac{f\,\mathrm{e}^{\frac{4cf-4de}{d}}\operatorname{expIntegral}_1\left(4fx+4e+\frac{4cf-4de}{d}\right)}{a^2d^2}+\frac{f\,\mathrm{e}^{-2fx-2e}}{2a^2d(dfx+cf)}-\frac{f\,\mathrm{e}^{\frac{2cf-2de}{d}}\operatorname{expIntegra}}{a^2}$

input
```
int(1/(d*x+c)^2/(a+a*coth(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^2/d/(d*x+c)-1/4*f/a^2*exp(-4*f*x-4*e)/d/(d*f*x+c*f)+f/a^2/d^2*exp(4
*(c*f-d*e)/d)*Ei(1,4*f*x+4*e+4*(c*f-d*e)/d)+1/2*f/a^2*exp(-2*f*x-2*e)/d/(d
*f*x+c*f)-f/a^2/d^2*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.46

$$\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))^2}\,dx=\text{Too large to display}$$

input
```
integrate(1/(d*x+c)^2/(a+a*coth(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/2*(2*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(f*x + e)^2*sinh(-2*(d*e -
 c*f)/d) - 2*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(f*x + e)^2*sinh(-4*
(d*e - c*f)/d) + (2*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*
f)/d) - 2*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(-4*(d*e - c*f)/d) - d)
*cosh(f*x + e)^2 + (2*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e -
c*f)/d) - 2*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(-4*(d*e - c*f)/d) +
2*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)/d) - 2*(d*f*x +
 c*f)*Ei(-4*(d*f*x + c*f)/d)*sinh(-4*(d*e - c*f)/d) - d)*sinh(f*x + e)^2 +
 4*((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(f*x + e)*sinh(-2*(d*e - c*f)
/d) - (d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(f*x + e)*sinh(-4*(d*e - c*
f)/d) + ((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) - (d*
f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(-4*(d*e - c*f)/d))*cosh(f*x + e))*s
inh(f*x + e) + d)/((a^2*d^3*x + a^2*c*d^2)*cosh(f*x + e)^2 + 2*(a^2*d^3*x
+ a^2*c*d^2)*cosh(f*x + e)*sinh(f*x + e) + (a^2*d^3*x + a^2*c*d^2)*sinh(f*
x + e)^2)
```

Sympy [F]

$$\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))^2}\,dx = \frac{\int \frac{1}{c^2\coth^2(e+fx)+2c^2\coth(e+fx)+c^2+2cdx\coth^2(e+fx)+4cdx\coth(e+fx)+2cdx+d^2x^2\coth^2(e+fx)+2d^2x^2\coth(e+fx)+d^2x^2}\,dx}{a^2}$$

input
```
integrate(1/(d*x+c)**2/(a+a*coth(f*x+e))**2,x)
```

output
```
Integral(1/(c**2*coth(e + f*x)**2 + 2*c**2*coth(e + f*x) + c**2 + 2*c*d*x*
coth(e + f*x)**2 + 4*c*d*x*coth(e + f*x) + 2*c*d*x + d**2*x**2*coth(e + f*
x)**2 + 2*d**2*x**2*coth(e + f*x) + d**2*x**2), x)/a**2
```

Maxima [A] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.24

$$\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))^2}\,dx = -\frac{1}{4\left(a^2d^2x+a^2cd\right)} - \frac{e^{\left(-4e+\frac{4cf}{d}\right)}E_2\left(\frac{4(dx+c)f}{d}\right)}{4(dx+c)a^2d} + \frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_2\left(\frac{2(dx+c)f}{d}\right)}{2(dx+c)a^2d}$$

input
```
integrate(1/(d*x+c)^2/(a+a*coth(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/4/(a^2*d^2*x + a^2*c*d) - 1/4*e^(-4*e + 4*c*f/d)*exp_integral_e(2, 4*(d
*x + c)*f/d)/((d*x + c)*a^2*d) + 1/2*e^(-2*e + 2*c*f/d)*exp_integral_e(2,
2*(d*x + c)*f/d)/((d*x + c)*a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.39

$$\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))^2}\,dx$$

$$= \Bigg(4(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)f^2\text{Ei}\left(-\frac{2\left((dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf\right)}{d}\right)e^{\left(-\frac{2(de-cf)}{d}\right)} - 4def^2\text{Ei}\Bigg(-\frac{2\big((dx+c)}{}$$

input
```
integrate(1/(d*x+c)^2/(a+a*coth(f*x+e))^2,x, algorithm="giac")
```

output
```
1/4*(4*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-2*((d*x + c)*
(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) -
 4*d*e*f^2*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*
f)/d)*e^(-2*(d*e - c*f)/d) + 4*c*f^3*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f
/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) - 4*(d*x + c)*(d*e/(d
*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-4*((d*x + c)*(d*e/(d*x + c) - c*f/(d*
x + c) + f) - d*e + c*f)/d)*e^(-4*(d*e - c*f)/d) + 4*d*e*f^2*Ei(-4*((d*x +
 c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-4*(d*e - c*f)/
d) - 4*c*f^3*Ei(-4*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e +
c*f)/d)*e^(-4*(d*e - c*f)/d) + 2*d*f^2*e^(-2*(d*x + c)*(d*e/(d*x + c) - c*
f/(d*x + c) + f)/d) - d*f^2*e^(-4*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c)
 + f)/d) - d*f^2)*d^2/(((d*x + c)*a^2*d^4*(d*e/(d*x + c) - c*f/(d*x + c) +
 f) - a^2*d^5*e + a^2*c*d^4*f)*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))^2}\,dx = \int \frac{1}{(a+a\coth(e+f\,x))^2\,(c+d\,x)^2}\,dx$$

input
```
int(1/((a + a*coth(e + f*x))^2*(c + d*x)^2),x)
```

output
```
int(1/((a + a*coth(e + f*x))^2*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))^2}\,dx = \frac{-2e^{2e}\left(\int \frac{1}{e^{2fx+4e}c^2+2e^{2fx+4e}cdx+e^{2fx+4e}d^2x^2}dx\right)c^2-2e^{2e}\left(\int \frac{1}{e^{2fx+4e}c^2+2e^{2fx+4e}cdx+e^{2fx+4e}d^2x^2}dx\right)cdx+\left(\int \frac{1}{e^{4fx+}\cdots}\right.}{4a^2c\,(dx+c)}$$

input
```
int(1/(d*x+c)^2/(a+a*coth(f*x+e))^2,x)
```

output
```
( - 2*e**(2*e)*int(1/(e**(4*e + 2*f*x)*c**2 + 2*e**(4*e + 2*f*x)*c*d*x + e
**(4*e + 2*f*x)*d**2*x**2),x)*c**2 - 2*e**(2*e)*int(1/(e**(4*e + 2*f*x)*c*
*2 + 2*e**(4*e + 2*f*x)*c*d*x + e**(4*e + 2*f*x)*d**2*x**2),x)*c*d*x + int
(1/(e**(4*e + 4*f*x)*c**2 + 2*e**(4*e + 4*f*x)*c*d*x + e**(4*e + 4*f*x)*d*
*2*x**2),x)*c**2 + int(1/(e**(4*e + 4*f*x)*c**2 + 2*e**(4*e + 4*f*x)*c*d*x
 + e**(4*e + 4*f*x)*d**2*x**2),x)*c*d*x + x)/(4*a**2*c*(c + d*x))
```

3.27 $\int \frac{(c+dx)^3}{(a+a\coth(e+fx))^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 336

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+a\coth(e+fx))^3}\,dx &= \frac{d^3e^{-6e-6fx}}{1728a^3f^4} - \frac{9d^3e^{-4e-4fx}}{1024a^3f^4} + \frac{9d^3e^{-2e-2fx}}{64a^3f^4} \\ &+ \frac{d^2e^{-6e-6fx}(c+dx)}{288a^3f^3} - \frac{9d^2e^{-4e-4fx}(c+dx)}{256a^3f^3} \\ &+ \frac{9d^2e^{-2e-2fx}(c+dx)}{32a^3f^3} + \frac{de^{-6e-6fx}(c+dx)^2}{96a^3f^2} \\ &- \frac{9de^{-4e-4fx}(c+dx)^2}{128a^3f^2} + \frac{9de^{-2e-2fx}(c+dx)^2}{32a^3f^2} \\ &+ \frac{e^{-6e-6fx}(c+dx)^3}{48a^3f} - \frac{3e^{-4e-4fx}(c+dx)^3}{32a^3f} \\ &+ \frac{3e^{-2e-2fx}(c+dx)^3}{16a^3f} + \frac{(c+dx)^4}{32a^3d}\end{aligned}$$

output
```
1/1728*d^3*exp(-6*f*x-6*e)/a^3/f^4-9/1024*d^3*exp(-4*f*x-4*e)/a^3/f^4+9/64
*d^3*exp(-2*f*x-2*e)/a^3/f^4+1/288*d^2*exp(-6*f*x-6*e)*(d*x+c)/a^3/f^3-9/2
56*d^2*exp(-4*f*x-4*e)*(d*x+c)/a^3/f^3+9/32*d^2*exp(-2*f*x-2*e)*(d*x+c)/a^
3/f^3+1/96*d*exp(-6*f*x-6*e)*(d*x+c)^2/a^3/f^2-9/128*d*exp(-4*f*x-4*e)*(d*
x+c)^2/a^3/f^2+9/32*d*exp(-2*f*x-2*e)*(d*x+c)^2/a^3/f^2+1/48*exp(-6*f*x-6*
e)*(d*x+c)^3/a^3/f-3/32*exp(-4*f*x-4*e)*(d*x+c)^3/a^3/f+3/16*exp(-2*f*x-2*
e)*(d*x+c)^3/a^3/f+1/32*(d*x+c)^4/a^3/d
```

Mathematica [A] (verified)

Time = 3.02 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.83

$$\int \frac{(c+dx)^3}{(a+a\coth(e+fx))^3}\,dx$$
$$= \frac{\operatorname{csch}^3(e+fx)\,(81(32c^3f^3+24c^2df^2(3+4fx)+12cd^2f(7+12fx+8f^2x^2)+d^3(45+84fx+72f^2x^2+$$

input
```
Integrate[(c + d*x)^3/(a + a*Coth[e + f*x])^3,x]
```

output
```
(Csch[e + f*x]^3*(81*(32*c^3*f^3 + 24*c^2*d*f^2*(3 + 4*f*x) + 12*c*d^2*f*(
7 + 12*f*x + 8*f^2*x^2) + d^3*(45 + 84*f*x + 72*f^2*x^2 + 32*f^3*x^3))*Cos
h[e + f*x] + 16*(36*c^3*f^3*(1 + 6*f*x) + 18*c^2*d*f^2*(1 + 6*f*x + 18*f^2
*x^2) + 6*c*d^2*f*(1 + 6*f*x + 18*f^2*x^2 + 36*f^3*x^3) + d^3*(1 + 6*f*x +
 18*f^2*x^2 + 36*f^3*x^3 + 54*f^4*x^4))*Cosh[3*(e + f*x)] + 4131*d^3*Sinh[
e + f*x] + 8748*c*d^2*f*Sinh[e + f*x] + 9720*c^2*d*f^2*Sinh[e + f*x] + 777
6*c^3*f^3*Sinh[e + f*x] + 8748*d^3*f*x*Sinh[e + f*x] + 19440*c*d^2*f^2*x*S
inh[e + f*x] + 23328*c^2*d*f^3*x*Sinh[e + f*x] + 9720*d^3*f^2*x^2*Sinh[e +
 f*x] + 23328*c*d^2*f^3*x^2*Sinh[e + f*x] + 7776*d^3*f^3*x^3*Sinh[e + f*x]
 - 16*d^3*Sinh[3*(e + f*x)] - 96*c*d^2*f*Sinh[3*(e + f*x)] - 288*c^2*d*f^2
*Sinh[3*(e + f*x)] - 576*c^3*f^3*Sinh[3*(e + f*x)] - 96*d^3*f*x*Sinh[3*(e
+ f*x)] - 576*c*d^2*f^2*x*Sinh[3*(e + f*x)] - 1728*c^2*d*f^3*x*Sinh[3*(e +
 f*x)] + 3456*c^3*f^4*x*Sinh[3*(e + f*x)] - 288*d^3*f^2*x^2*Sinh[3*(e + f*
x)] - 1728*c*d^2*f^3*x^2*Sinh[3*(e + f*x)] + 5184*c^2*d*f^4*x^2*Sinh[3*(e
+ f*x)] - 576*d^3*f^3*x^3*Sinh[3*(e + f*x)] + 3456*c*d^2*f^4*x^3*Sinh[3*(e
 + f*x)] + 864*d^3*f^4*x^4*Sinh[3*(e + f*x)]))/(27648*a^3*f^4*(1 + Coth[e
+ f*x])^3)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4212, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a\coth(e+fx)+a)^3}\,dx$$

$\Big\downarrow$ 3042

$$\int \frac{(c+dx)^3}{\left(a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^3}dx$$

$\Big\downarrow$ 4212

$$\int \left(-\frac{(c+dx)^3e^{-6e-6fx}}{8a^3}+\frac{3(c+dx)^3e^{-4e-4fx}}{8a^3}-\frac{3(c+dx)^3e^{-2e-2fx}}{8a^3}+\frac{(c+dx)^3}{8a^3}\right)dx$$

$\Big\downarrow$ 2009

$$\frac{d^2(c+dx)e^{-6e-6fx}}{288a^3f^3}-\frac{9d^2(c+dx)e^{-4e-4fx}}{256a^3f^3}+\frac{9d^2(c+dx)e^{-2e-2fx}}{32a^3f^3}+\frac{d(c+dx)^2e^{-6e-6fx}}{96a^3f^2}-\frac{9d(c+dx)^2e^{-4e-4fx}}{128a^3f^2}+\frac{9d(c+dx)^2e^{-2e-2fx}}{32a^3f^2}+\frac{(c+dx)^3e^{-6e-6fx}}{48a^3f}-\frac{3(c+dx)^3e^{-4e-4fx}}{32a^3f}+\frac{3(c+dx)^3e^{-2e-2fx}}{16a^3f}+\frac{(c+dx)^4}{32a^3d}+\frac{d^3e^{-6e-6fx}}{1728a^3f^4}-\frac{9d^3e^{-4e-4fx}}{1024a^3f^4}+\frac{9d^3e^{-2e-2fx}}{64a^3f^4}$$

input
```
Int[(c + d*x)^3/(a + a*Coth[e + f*x])^3,x]
```

output
```
(d^3*E^(-6*e - 6*f*x))/(1728*a^3*f^4) - (9*d^3*E^(-4*e - 4*f*x))/(1024*a^3
*f^4) + (9*d^3*E^(-2*e - 2*f*x))/(64*a^3*f^4) + (d^2*E^(-6*e - 6*f*x)*(c +
 d*x))/(288*a^3*f^3) - (9*d^2*E^(-4*e - 4*f*x)*(c + d*x))/(256*a^3*f^3) +
(9*d^2*E^(-2*e - 2*f*x)*(c + d*x))/(32*a^3*f^3) + (d*E^(-6*e - 6*f*x)*(c +
 d*x)^2)/(96*a^3*f^2) - (9*d*E^(-4*e - 4*f*x)*(c + d*x)^2)/(128*a^3*f^2) +
 (9*d*E^(-2*e - 2*f*x)*(c + d*x)^2)/(32*a^3*f^2) + (E^(-6*e - 6*f*x)*(c +
d*x)^3)/(48*a^3*f) - (3*E^(-4*e - 4*f*x)*(c + d*x)^3)/(32*a^3*f) + (3*E^(-
2*e - 2*f*x)*(c + d*x)^3)/(16*a^3*f) + (c + d*x)^4/(32*a^3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4212
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.13

method	result
risch	$\frac{d^3x^4}{32a^3}+\frac{d^2c\,x^3}{8a^3}+\frac{3d\,c^2x^2}{16a^3}+\frac{c^3x}{8a^3}+\frac{c^4}{32a^3d}+\frac{3\left(4d^3x^3f^3+12c\,d^2f^3x^2+12c^2d\,f^3x+6d^3f^2x^2+4c^3f^3+12c\,d^2f^2x+6c^2d\right.}{64a^3f^4}$
parallelrisch	$\frac{864f\left(\left(\frac{1}{4}d^3x^3+c\,d^2x^2+\frac{3}{2}c^2dx+c^3\right)f^3-\frac{29\left(\frac{1}{3}d^2x^2+cdx+c^2\right)d\,f^2}{4}-\frac{139\left(\frac{dx}{2}+c\right)d^2f}{24}-\frac{737d^3}{288}\right)x\tanh(fx+e)^3+\left(\left(648d^3x^4+2592\right.\right.}{}$

input
```
int((d*x+c)^3/(a+a*coth(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/32/a^3*d^3*x^4+1/8/a^3*d^2*c*x^3+3/16/a^3*d*c^2*x^2+1/8/a^3*c^3*x+1/32/a
^3/d*c^4+3/64*(4*d^3*f^3*x^3+12*c*d^2*f^3*x^2+12*c^2*d*f^3*x+6*d^3*f^2*x^2
+4*c^3*f^3+12*c*d^2*f^2*x+6*c^2*d*f^2+6*d^3*f*x+6*c*d^2*f+3*d^3)/a^3/f^4*e
xp(-2*f*x-2*e)-3/1024*(32*d^3*f^3*x^3+96*c*d^2*f^3*x^2+96*c^2*d*f^3*x+24*d
^3*f^2*x^2+32*c^3*f^3+48*c*d^2*f^2*x+24*c^2*d*f^2+12*d^3*f*x+12*c*d^2*f+3*
d^3)/a^3/f^4*exp(-4*f*x-4*e)+1/1728*(36*d^3*f^3*x^3+108*c*d^2*f^3*x^2+108*
c^2*d*f^3*x+18*d^3*f^2*x^2+36*c^3*f^3+36*c*d^2*f^2*x+18*c^2*d*f^2+6*d^3*f*
x+6*c*d^2*f+d^3)/a^3/f^4*exp(-6*f*x-6*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 844 vs. 2(298) = 596.

Time = 0.10 (sec) , antiderivative size = 844, normalized size of antiderivative = 2.51

$$\int\frac{(c+dx)^3}{(a+a\coth(e+fx))^3}\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+a*coth(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/27648*(16*(54*d^3*f^4*x^4 + 36*c^3*f^3 + 18*c^2*d*f^2 + 6*c*d^2*f + 36*(
6*c*d^2*f^4 + d^3*f^3)*x^3 + d^3 + 18*(18*c^2*d*f^4 + 6*c*d^2*f^3 + d^3*f^
2)*x^2 + 6*(36*c^3*f^4 + 18*c^2*d*f^3 + 6*c*d^2*f^2 + d^3*f)*x)*cosh(f*x +
 e)^3 + 48*(54*d^3*f^4*x^4 + 36*c^3*f^3 + 18*c^2*d*f^2 + 6*c*d^2*f + 36*(6
*c*d^2*f^4 + d^3*f^3)*x^3 + d^3 + 18*(18*c^2*d*f^4 + 6*c*d^2*f^3 + d^3*f^2
)*x^2 + 6*(36*c^3*f^4 + 18*c^2*d*f^3 + 6*c*d^2*f^2 + d^3*f)*x)*cosh(f*x +
e)*sinh(f*x + e)^2 + 16*(54*d^3*f^4*x^4 - 36*c^3*f^3 - 18*c^2*d*f^2 - 6*c*
d^2*f + 36*(6*c*d^2*f^4 - d^3*f^3)*x^3 - d^3 + 18*(18*c^2*d*f^4 - 6*c*d^2*
f^3 - d^3*f^2)*x^2 + 6*(36*c^3*f^4 - 18*c^2*d*f^3 - 6*c*d^2*f^2 - d^3*f)*x
)*sinh(f*x + e)^3 + 81*(32*d^3*f^3*x^3 + 32*c^3*f^3 + 72*c^2*d*f^2 + 84*c*
d^2*f + 45*d^3 + 24*(4*c*d^2*f^3 + 3*d^3*f^2)*x^2 + 12*(8*c^2*d*f^3 + 12*c
*d^2*f^2 + 7*d^3*f)*x)*cosh(f*x + e) + 3*(2592*d^3*f^3*x^3 + 2592*c^3*f^3
+ 3240*c^2*d*f^2 + 2916*c*d^2*f + 1377*d^3 + 648*(12*c*d^2*f^3 + 5*d^3*f^2
)*x^2 + 16*(54*d^3*f^4*x^4 - 36*c^3*f^3 - 18*c^2*d*f^2 - 6*c*d^2*f + 36*(6
*c*d^2*f^4 - d^3*f^3)*x^3 - d^3 + 18*(18*c^2*d*f^4 - 6*c*d^2*f^3 - d^3*f^2
)*x^2 + 6*(36*c^3*f^4 - 18*c^2*d*f^3 - 6*c*d^2*f^2 - d^3*f)*x)*cosh(f*x +
e)^2 + 324*(24*c^2*d*f^3 + 20*c*d^2*f^2 + 9*d^3*f)*x)*sinh(f*x + e))/(a^3*
f^4*cosh(f*x + e)^3 + 3*a^3*f^4*cosh(f*x + e)^2*sinh(f*x + e) + 3*a^3*f^4*
cosh(f*x + e)*sinh(f*x + e)^2 + a^3*f^4*sinh(f*x + e)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3918 vs. $2(347) = 694$.

Time = 1.35 (sec) , antiderivative size = 3918, normalized size of antiderivative = 11.66

$$\int \frac{(c+dx)^3}{(a+a\coth(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3/(a+a*coth(f*x+e))**3,x)
```

output
```
Piecewise((864*c**3*f**4*x*tanh(e + f*x)**3/(6912*a**3*f**4*tanh(e + f*x)*
*3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x) + 69
12*a**3*f**4) + 2592*c**3*f**4*x*tanh(e + f*x)**2/(6912*a**3*f**4*tanh(e +
 f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x
) + 6912*a**3*f**4) + 2592*c**3*f**4*x*tanh(e + f*x)/(6912*a**3*f**4*tanh(
e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e +
f*x) + 6912*a**3*f**4) + 864*c**3*f**4*x/(6912*a**3*f**4*tanh(e + f*x)**3
+ 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x) + 6912*
a**3*f**4) + 6048*c**3*f**3*tanh(e + f*x)**2/(6912*a**3*f**4*tanh(e + f*x)
**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x) + 6
912*a**3*f**4) + 7776*c**3*f**3*tanh(e + f*x)/(6912*a**3*f**4*tanh(e + f*x
)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x) +
6912*a**3*f**4) + 2880*c**3*f**3/(6912*a**3*f**4*tanh(e + f*x)**3 + 20736*
a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x) + 6912*a**3*f**
4) + 1296*c**2*d*f**4*x**2*tanh(e + f*x)**3/(6912*a**3*f**4*tanh(e + f*x)*
*3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x) + 69
12*a**3*f**4) + 3888*c**2*d*f**4*x**2*tanh(e + f*x)**2/(6912*a**3*f**4*tan
h(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e
+ f*x) + 6912*a**3*f**4) + 3888*c**2*d*f**4*x**2*tanh(e + f*x)/(6912*a**3*
f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f...
```

Maxima [A] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.21

$$\int \frac{(c+dx)^3}{(a+a\coth(e+fx))^3}\,dx$$
$$= \frac{1}{96}c^3\left(\frac{12\,(fx+e)}{a^3 f} + \frac{18\,e^{(-2\,fx-2\,e)} - 9\,e^{(-4\,fx-4\,e)} + 2\,e^{(-6\,fx-6\,e)}}{a^3 f}\right)$$
$$+ \frac{\left(72\,f^2x^2e^{(6\,e)} + 108\,\left(2\,fxe^{(4\,e)} + e^{(4\,e)}\right)e^{(-2\,fx)} - 27\,\left(4\,fxe^{(2\,e)} + e^{(2\,e)}\right)e^{(-4\,fx)} + 4\,(6\,fx+1)e^{(-6\,fx)}\right)c^2}{384\,a^3 f^2}$$
$$+ \frac{\left(288\,f^3x^3e^{(6\,e)} + 648\,\left(2\,f^2x^2e^{(4\,e)} + 2\,fxe^{(4\,e)} + e^{(4\,e)}\right)e^{(-2\,fx)} - 81\,\left(8\,f^2x^2e^{(2\,e)} + 4\,fxe^{(2\,e)} + e^{(2\,e)}\right)e^{(-4}}{2304\,a^3 f^3}$$
$$+ \frac{\left(864\,f^4x^4e^{(6\,e)} + 1296\,\left(4\,f^3x^3e^{(4\,e)} + 6\,f^2x^2e^{(4\,e)} + 6\,fxe^{(4\,e)} + 3\,e^{(4\,e)}\right)e^{(-2\,fx)} - 81\,\left(32\,f^3x^3e^{(2\,e)} + 24\,\right.}{27648\,a^3 f^4}$$

input
```
integrate((d*x+c)^3/(a+a*coth(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/96*c^3*(12*(f*x + e)/(a^3*f) + (18*e^(-2*f*x - 2*e) - 9*e^(-4*f*x - 4*e)
 + 2*e^(-6*f*x - 6*e))/(a^3*f)) + 1/384*(72*f^2*x^2*e^(6*e) + 108*(2*f*x*e
^(4*e) + e^(4*e))*e^(-2*f*x) - 27*(4*f*x*e^(2*e) + e^(2*e))*e^(-4*f*x) + 4
*(6*f*x + 1)*e^(-6*f*x))*c^2*d*e^(-6*e)/(a^3*f^2) + 1/2304*(288*f^3*x^3*e^
(6*e) + 648*(2*f^2*x^2*e^(4*e) + 2*f*x*e^(4*e) + e^(4*e))*e^(-2*f*x) - 81*
(8*f^2*x^2*e^(2*e) + 4*f*x*e^(2*e) + e^(2*e))*e^(-4*f*x) + 8*(18*f^2*x^2 +
 6*f*x + 1)*e^(-6*f*x))*c*d^2*e^(-6*e)/(a^3*f^3) + 1/27648*(864*f^4*x^4*e^
(6*e) + 1296*(4*f^3*x^3*e^(4*e) + 6*f^2*x^2*e^(4*e) + 6*f*x*e^(4*e) + 3*e^
(4*e))*e^(-2*f*x) - 81*(32*f^3*x^3*e^(2*e) + 24*f^2*x^2*e^(2*e) + 12*f*x*e
^(2*e) + 3*e^(2*e))*e^(-4*f*x) + 16*(36*f^3*x^3 + 18*f^2*x^2 + 6*f*x + 1)*
e^(-6*f*x))*d^3*e^(-6*e)/(a^3*f^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.63

$$\int \frac{(c+dx)^3}{(a+a\coth(e+fx))^3}\,dx = \frac{(864\,d^3f^4x^4e^{(6\,fx+6\,e)} + 3456\,cd^2f^4x^3e^{(6\,fx+6\,e)} + 5184\,c^2df^4x^2e^{(6\,fx+6\,e)} + 5184\,d^3f^3x^3e^{(4\,fx+4\,e)} - 2592\,d^3}{}$$

input
```
integrate((d*x+c)^3/(a+a*coth(f*x+e))^3,x, algorithm="giac")
```

output
```
1/27648*(864*d^3*f^4*x^4*e^(6*f*x + 6*e) + 3456*c*d^2*f^4*x^3*e^(6*f*x + 6
*e) + 5184*c^2*d*f^4*x^2*e^(6*f*x + 6*e) + 5184*d^3*f^3*x^3*e^(4*f*x + 4*e
) - 2592*d^3*f^3*x^3*e^(2*f*x + 2*e) + 576*d^3*f^3*x^3 + 3456*c^3*f^4*x*e^
(6*f*x + 6*e) + 15552*c*d^2*f^3*x^2*e^(4*f*x + 4*e) - 7776*c*d^2*f^3*x^2*e
^(2*f*x + 2*e) + 1728*c*d^2*f^3*x^2 + 15552*c^2*d*f^3*x*e^(4*f*x + 4*e) +
7776*d^3*f^2*x^2*e^(4*f*x + 4*e) - 7776*c^2*d*f^3*x*e^(2*f*x + 2*e) - 1944
*d^3*f^2*x^2*e^(2*f*x + 2*e) + 1728*c^2*d*f^3*x + 288*d^3*f^2*x^2 + 5184*c
^3*f^3*e^(4*f*x + 4*e) + 15552*c*d^2*f^2*x*e^(4*f*x + 4*e) - 2592*c^3*f^3*
e^(2*f*x + 2*e) - 3888*c*d^2*f^2*x*e^(2*f*x + 2*e) + 576*c^3*f^3 + 576*c*d
^2*f^2*x + 7776*c^2*d*f^2*e^(4*f*x + 4*e) + 7776*d^3*f*x*e^(4*f*x + 4*e) -
 1944*c^2*d*f^2*e^(2*f*x + 2*e) - 972*d^3*f*x*e^(2*f*x + 2*e) + 288*c^2*d*
f^2 + 96*d^3*f*x + 7776*c*d^2*f*e^(4*f*x + 4*e) - 972*c*d^2*f*e^(2*f*x + 2
*e) + 96*c*d^2*f + 3888*d^3*e^(4*f*x + 4*e) - 243*d^3*e^(2*f*x + 2*e) + 16
*d^3)*e^(-6*f*x - 6*e)/(a^3*f^4)
```

Mupad [B] (verification not implemented)

Time = 2.66 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.11

$$\int \frac{(c+dx)^3}{(a+a\coth(e+fx))^3}\,dx = e^{-2e-2fx}\left(\frac{12c^3f^3+18c^2df^2+18cd^2f+9d^3}{64a^3f^4} + \frac{3d^3x^3}{16a^3f} + \frac{9dx(2c^2f^2+2cdf+d^2)}{32a^3f^3} + \frac{9d^2x^2(d+2cf)}{32a^3f^2}\right) - e^{-4e-4fx}\left(\frac{96c^3f^3+72c^2df^2+36cd^2f+9d^3}{1024a^3f^4} + \frac{3d^3x^3}{32a^3f} + \frac{9dx(8c^2f^2+4cdf+d^2)}{256a^3f^3} + \frac{9d^2x^2(d+4cf)}{128a^3f^2}\right) + e^{-6e-6fx}\left(\frac{36c^3f^3+18c^2df^2+6cd^2f+d^3}{1728a^3f^4} + \frac{d^3x^3}{48a^3f} + \frac{dx(18c^2f^2+6cdf+d^2)}{288a^3f^3} + \frac{d^2x^2(d+6cf)}{96a^3f^2}\right) + \frac{c^3x}{8a^3} + \frac{d^3x^4}{32a^3} + \frac{3c^2dx^2}{16a^3} + \frac{cd^2x^3}{8a^3}$$

input
```
int((c + d*x)^3/(a + a*coth(e + f*x))^3,x)
```

output
```
exp(- 2*e - 2*f*x)*((9*d^3 + 12*c^3*f^3 + 18*c^2*d*f^2 + 18*c*d^2*f)/(64*a
^3*f^4) + (3*d^3*x^3)/(16*a^3*f) + (9*d*x*(d^2 + 2*c^2*f^2 + 2*c*d*f))/(32
*a^3*f^3) + (9*d^2*x^2*(d + 2*c*f))/(32*a^3*f^2)) - exp(- 4*e - 4*f*x)*((9
*d^3 + 96*c^3*f^3 + 72*c^2*d*f^2 + 36*c*d^2*f)/(1024*a^3*f^4) + (3*d^3*x^3
)/(32*a^3*f) + (9*d*x*(d^2 + 8*c^2*f^2 + 4*c*d*f))/(256*a^3*f^3) + (9*d^2*
x^2*(d + 4*c*f))/(128*a^3*f^2)) + exp(- 6*e - 6*f*x)*((d^3 + 36*c^3*f^3 +
18*c^2*d*f^2 + 6*c*d^2*f)/(1728*a^3*f^4) + (d^3*x^3)/(48*a^3*f) + (d*x*(d^
2 + 18*c^2*f^2 + 6*c*d*f))/(288*a^3*f^3) + (d^2*x^2*(d + 6*c*f))/(96*a^3*f
^2)) + (c^3*x)/(8*a^3) + (d^3*x^4)/(32*a^3) + (3*c^2*d*x^2)/(16*a^3) + (c*
d^2*x^3)/(8*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.71

$$\int \frac{(c+dx)^3}{(a+a\coth(e+fx))^3}\,dx$$
$$= \frac{288c^2d\,f^2+96c\,d^2f+96d^3fx-2592e^{2fx+2e}c^3f^3+576d^3f^3x^3+288d^3f^2x^2+3456e^{6fx+6e}c^3f^4x+864e}{}$$

input
```
int((d*x+c)^3/(a+a*coth(f*x+e))^3,x)
```

output
```
(3456*e**(6*e + 6*f*x)*c**3*f**4*x + 5184*e**(6*e + 6*f*x)*c**2*d*f**4*x**
2 + 3456*e**(6*e + 6*f*x)*c*d**2*f**4*x**3 + 864*e**(6*e + 6*f*x)*d**3*f**
4*x**4 + 5184*e**(4*e + 4*f*x)*c**3*f**3 + 15552*e**(4*e + 4*f*x)*c**2*d*f
**3*x + 7776*e**(4*e + 4*f*x)*c**2*d*f**2 + 15552*e**(4*e + 4*f*x)*c*d**2*
f**3*x**2 + 15552*e**(4*e + 4*f*x)*c*d**2*f**2*x + 7776*e**(4*e + 4*f*x)*c
*d**2*f + 5184*e**(4*e + 4*f*x)*d**3*f**3*x**3 + 7776*e**(4*e + 4*f*x)*d**
3*f**2*x**2 + 7776*e**(4*e + 4*f*x)*d**3*f*x + 3888*e**(4*e + 4*f*x)*d**3
- 2592*e**(2*e + 2*f*x)*c**3*f**3 - 7776*e**(2*e + 2*f*x)*c**2*d*f**3*x -
1944*e**(2*e + 2*f*x)*c**2*d*f**2 - 7776*e**(2*e + 2*f*x)*c*d**2*f**3*x**2
 - 3888*e**(2*e + 2*f*x)*c*d**2*f**2*x - 972*e**(2*e + 2*f*x)*c*d**2*f - 2
592*e**(2*e + 2*f*x)*d**3*f**3*x**3 - 1944*e**(2*e + 2*f*x)*d**3*f**2*x**2
 - 972*e**(2*e + 2*f*x)*d**3*f*x - 243*e**(2*e + 2*f*x)*d**3 + 576*c**3*f*
*3 + 1728*c**2*d*f**3*x + 288*c**2*d*f**2 + 1728*c*d**2*f**3*x**2 + 576*c*
d**2*f**2*x + 96*c*d**2*f + 576*d**3*f**3*x**3 + 288*d**3*f**2*x**2 + 96*d
**3*f*x + 16*d**3)/(27648*e**(6*e + 6*f*x)*a**3*f**4)
```

3.28 $\int \frac{(c+dx)^2}{(a+a\coth(e+fx))^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 246

$$\int \frac{(c+dx)^2}{(a+a\coth(e+fx))^3}\,dx = \frac{d^2e^{-6e-6fx}}{864a^3f^3} - \frac{3d^2e^{-4e-4fx}}{256a^3f^3} + \frac{3d^2e^{-2e-2fx}}{32a^3f^3} + \frac{de^{-6e-6fx}(c+dx)}{144a^3f^2} - \frac{3de^{-4e-4fx}(c+dx)}{64a^3f^2} + \frac{3de^{-2e-2fx}(c+dx)}{16a^3f^2} + \frac{e^{-6e-6fx}(c+dx)^2}{48a^3f} - \frac{3e^{-4e-4fx}(c+dx)^2}{32a^3f} + \frac{3e^{-2e-2fx}(c+dx)^2}{16a^3f} + \frac{(c+dx)^3}{24a^3d}$$

output
```
1/864*d^2*exp(-6*f*x-6*e)/a^3/f^3-3/256*d^2*exp(-4*f*x-4*e)/a^3/f^3+3/32*d
^2*exp(-2*f*x-2*e)/a^3/f^3+1/144*d*exp(-6*f*x-6*e)*(d*x+c)/a^3/f^2-3/64*d*
exp(-4*f*x-4*e)*(d*x+c)/a^3/f^2+3/16*d*exp(-2*f*x-2*e)*(d*x+c)/a^3/f^2+1/4
8*exp(-6*f*x-6*e)*(d*x+c)^2/a^3/f-3/32*exp(-4*f*x-4*e)*(d*x+c)^2/a^3/f+3/1
6*exp(-2*f*x-2*e)*(d*x+c)^2/a^3/f+1/24*(d*x+c)^3/a^3/d
```

Mathematica [A] (verified)

Time = 2.05 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.51

$$\int \frac{(c+dx)^2}{(a+a\coth(e+fx))^3}\,dx$$
$$= \frac{\operatorname{csch}^3(e+fx)\left(81(8c^2f^2+4cdf(3+4fx)+d^2(7+12fx+8f^2x^2))\cosh(e+fx)+8(18c^2f^2(1+6fx)+\right.}{}$$

input
```
Integrate[(c + d*x)^2/(a + a*Coth[e + f*x])^3,x]
```

output
```
(Csch[e + f*x]^3*(81*(8*c^2*f^2 + 4*c*d*f*(3 + 4*f*x) + d^2*(7 + 12*f*x + 
8*f^2*x^2))*Cosh[e + f*x] + 8*(18*c^2*f^2*(1 + 6*f*x) + 6*c*d*f*(1 + 6*f*x
 + 18*f^2*x^2) + d^2*(1 + 6*f*x + 18*f^2*x^2 + 36*f^3*x^3))*Cosh[3*(e + f*
x)] + 729*d^2*Sinh[e + f*x] + 1620*c*d*f*Sinh[e + f*x] + 1944*c^2*f^2*Sinh
[e + f*x] + 1620*d^2*f*x*Sinh[e + f*x] + 3888*c*d*f^2*x*Sinh[e + f*x] + 19
44*d^2*f^2*x^2*Sinh[e + f*x] - 8*d^2*Sinh[3*(e + f*x)] - 48*c*d*f*Sinh[3*(
e + f*x)] - 144*c^2*f^2*Sinh[3*(e + f*x)] - 48*d^2*f*x*Sinh[3*(e + f*x)] -
 288*c*d*f^2*x*Sinh[3*(e + f*x)] + 864*c^2*f^3*x*Sinh[3*(e + f*x)] - 144*d
^2*f^2*x^2*Sinh[3*(e + f*x)] + 864*c*d*f^3*x^2*Sinh[3*(e + f*x)] + 288*d^2
*f^3*x^3*Sinh[3*(e + f*x)]))/(6912*a^3*f^3*(1 + Coth[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4212, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a\coth(e+fx)+a)^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c+dx)^2}{\left(a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4212}$$

$$\int \left(-\frac{(c+dx)^2 e^{-6e-6fx}}{8a^3}+\frac{3(c+dx)^2 e^{-4e-4fx}}{8a^3}-\frac{3(c+dx)^2 e^{-2e-2fx}}{8a^3}+\frac{(c+dx)^2}{8a^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d(c+dx)e^{-6e-6fx}}{144a^3f^2}-\frac{3d(c+dx)e^{-4e-4fx}}{64a^3f^2}+\frac{3d(c+dx)e^{-2e-2fx}}{16a^3f^2}+\frac{(c+dx)^2e^{-6e-6fx}}{48a^3f}-\frac{3(c+dx)^2e^{-4e-4fx}}{32a^3f}+\frac{3(c+dx)^2e^{-2e-2fx}}{16a^3f}+\frac{(c+dx)^3}{24a^3d}+\frac{d^2e^{-6e-6fx}}{864a^3f^3}-\frac{3d^2e^{-4e-4fx}}{256a^3f^3}+\frac{3d^2e^{-2e-2fx}}{32a^3f^3}$$

input
```
Int[(c + d*x)^2/(a + a*Coth[e + f*x])^3,x]
```

output
```
(d^2*E^(-6*e - 6*f*x))/(864*a^3*f^3) - (3*d^2*E^(-4*e - 4*f*x))/(256*a^3*f
^3) + (3*d^2*E^(-2*e - 2*f*x))/(32*a^3*f^3) + (d*E^(-6*e - 6*f*x)*(c + d*x
))/(144*a^3*f^2) - (3*d*E^(-4*e - 4*f*x)*(c + d*x))/(64*a^3*f^2) + (3*d*E^
(-2*e - 2*f*x)*(c + d*x))/(16*a^3*f^2) + (E^(-6*e - 6*f*x)*(c + d*x)^2)/(4
8*a^3*f) - (3*E^(-4*e - 4*f*x)*(c + d*x)^2)/(32*a^3*f) + (3*E^(-2*e - 2*f*
x)*(c + d*x)^2)/(16*a^3*f) + (c + d*x)^3/(24*a^3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4212
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.91

method	result
risch	$\frac{d^2x^3}{24a^3}+\frac{dc\,x^2}{8a^3}+\frac{c^2x}{8a^3}+\frac{c^3}{24a^3d}+\frac{3(2d^2x^2f^2+4cd\,f^2x+2c^2f^2+2d^2fx+2cdf+d^2)\mathrm{e}^{-2fx-2e}}{32a^3f^3}-\frac{3(8d^2x^2f^2+16cd\,f^2x+$
parallelrisch	$\frac{((288d^2x^3+864cd\,x^2+864c^2x)f^3+(144d^2x^2+288cdx+216c^2)f^2+(48d^2x+324cd)f+189d^2)\cosh(3fx+3e)+((288d^2x^3+86}{}$

input
```
int((d*x+c)^2/(a+a*coth(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/24/a^3*d^2*x^3+1/8/a^3*d*c*x^2+1/8/a^3*c^2*x+1/24/a^3/d*c^3+3/32*(2*d^2*
f^2*x^2+4*c*d*f^2*x+2*c^2*f^2+2*d^2*f*x+2*c*d*f+d^2)/a^3/f^3*exp(-2*f*x-2*
e)-3/256*(8*d^2*f^2*x^2+16*c*d*f^2*x+8*c^2*f^2+4*d^2*f*x+4*c*d*f+d^2)/a^3/
f^3*exp(-4*f*x-4*e)+1/864*(18*d^2*f^2*x^2+36*c*d*f^2*x+18*c^2*f^2+6*d^2*f*
x+6*c*d*f+d^2)/a^3/f^3*exp(-6*f*x-6*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(217) = 434$.

Time = 0.11 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.16

$$\int \frac{(c+dx)^2}{(a+a\coth(e+fx))^3}\,dx$$
$$= \frac{8\,(36\,d^2f^3x^3+18\,c^2f^2+6\,cdf+18\,(6\,cdf^3+d^2f^2)x^2+d^2+6\,(18\,c^2f^3+6\,cdf^2+d^2f)x)\cosh\,(fx+e)^3}{}$$

input
```
integrate((d*x+c)^2/(a+a*coth(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/6912*(8*(36*d^2*f^3*x^3 + 18*c^2*f^2 + 6*c*d*f + 18*(6*c*d*f^3 + d^2*f^2
)*x^2 + d^2 + 6*(18*c^2*f^3 + 6*c*d*f^2 + d^2*f)*x)*cosh(f*x + e)^3 + 24*(
36*d^2*f^3*x^3 + 18*c^2*f^2 + 6*c*d*f + 18*(6*c*d*f^3 + d^2*f^2)*x^2 + d^2
 + 6*(18*c^2*f^3 + 6*c*d*f^2 + d^2*f)*x)*cosh(f*x + e)*sinh(f*x + e)^2 + 8
*(36*d^2*f^3*x^3 - 18*c^2*f^2 - 6*c*d*f + 18*(6*c*d*f^3 - d^2*f^2)*x^2 - d
^2 + 6*(18*c^2*f^3 - 6*c*d*f^2 - d^2*f)*x)*sinh(f*x + e)^3 + 81*(8*d^2*f^2
*x^2 + 8*c^2*f^2 + 12*c*d*f + 7*d^2 + 4*(4*c*d*f^2 + 3*d^2*f)*x)*cosh(f*x
+ e) + 3*(648*d^2*f^2*x^2 + 648*c^2*f^2 + 540*c*d*f + 8*(36*d^2*f^3*x^3 -
18*c^2*f^2 - 6*c*d*f + 18*(6*c*d*f^3 - d^2*f^2)*x^2 - d^2 + 6*(18*c^2*f^3
- 6*c*d*f^2 - d^2*f)*x)*cosh(f*x + e)^2 + 243*d^2 + 108*(12*c*d*f^2 + 5*d^
2*f)*x)*sinh(f*x + e))/(a^3*f^3*cosh(f*x + e)^3 + 3*a^3*f^3*cosh(f*x + e)^
2*sinh(f*x + e) + 3*a^3*f^3*cosh(f*x + e)*sinh(f*x + e)^2 + a^3*f^3*sinh(f
*x + e)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2443 vs. $2(252) = 504$.

Time = 1.17 (sec) , antiderivative size = 2443, normalized size of antiderivative = 9.93

$$\int \frac{(c+dx)^2}{(a+a\coth(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2/(a+a*coth(f*x+e))**3,x)
```

output
```
Piecewise((216*c**2*f**3*x*tanh(e + f*x)**3/(1728*a**3*f**3*tanh(e + f*x)*
*3 + 5184*a**3*f**3*tanh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728
*a**3*f**3) + 648*c**2*f**3*x*tanh(e + f*x)**2/(1728*a**3*f**3*tanh(e + f*
x)**3 + 5184*a**3*f**3*tanh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1
728*a**3*f**3) + 648*c**2*f**3*x*tanh(e + f*x)/(1728*a**3*f**3*tanh(e + f*
x)**3 + 5184*a**3*f**3*tanh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1
728*a**3*f**3) + 216*c**2*f**3*x/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a
**3*f**3*tanh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3)
 + 1512*c**2*f**2*tanh(e + f*x)**2/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184
*a**3*f**3*tanh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**
3) + 1944*c**2*f**2*tanh(e + f*x)/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*
a**3*f**3*tanh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3
) + 720*c**2*f**2/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*tanh(e
 + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) + 216*c*d*f**3
*x**2*tanh(e + f*x)**3/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*t
anh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) + 648*c*d
*f**3*x**2*tanh(e + f*x)**2/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f
**3*tanh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) + 64
8*c*d*f**3*x**2*tanh(e + f*x)/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3
*f**3*tanh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3)...
```

Maxima [A] (verification not implemented)

Time = 2.78 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx)^2}{(a+a\coth(e+fx))^3}\,dx$$
$$= \frac{1}{96}c^2\left(\frac{12\,(fx+e)}{a^3f}+\frac{18\,e^{(-2\,fx-2\,e)}-9\,e^{(-4\,fx-4\,e)}+2\,e^{(-6\,fx-6\,e)}}{a^3f}\right)$$
$$+\frac{\left(72\,f^2x^2e^{(6\,e)}+108\,\left(2\,fxe^{(4\,e)}+e^{(4\,e)}\right)e^{(-2\,fx)}-27\,\left(4\,fxe^{(2\,e)}+e^{(2\,e)}\right)e^{(-4\,fx)}+4\,(6\,fx+1)e^{(-6\,fx)}\right)cd}{576\,a^3f^2}$$
$$+\frac{\left(288\,f^3x^3e^{(6\,e)}+648\,\left(2\,f^2x^2e^{(4\,e)}+2\,fxe^{(4\,e)}+e^{(4\,e)}\right)e^{(-2\,fx)}-81\,\left(8\,f^2x^2e^{(2\,e)}+4\,fxe^{(2\,e)}+e^{(2\,e)}\right)e^{(-4}\right.}{6912\,a^3f^3}$$

input
```
integrate((d*x+c)^2/(a+a*coth(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/96*c^2*(12*(f*x + e)/(a^3*f) + (18*e^(-2*f*x - 2*e) - 9*e^(-4*f*x - 4*e)
 + 2*e^(-6*f*x - 6*e))/(a^3*f)) + 1/576*(72*f^2*x^2*e^(6*e) + 108*(2*f*x*e
^(4*e) + e^(4*e))*e^(-2*f*x) - 27*(4*f*x*e^(2*e) + e^(2*e))*e^(-4*f*x) + 4
*(6*f*x + 1)*e^(-6*f*x))*c*d*e^(-6*e)/(a^3*f^2) + 1/6912*(288*f^3*x^3*e^(6
*e) + 648*(2*f^2*x^2*e^(4*e) + 2*f*x*e^(4*e) + e^(4*e))*e^(-2*f*x) - 81*(8
*f^2*x^2*e^(2*e) + 4*f*x*e^(2*e) + e^(2*e))*e^(-4*f*x) + 8*(18*f^2*x^2 + 6
*f*x + 1)*e^(-6*f*x))*d^2*e^(-6*e)/(a^3*f^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx)^2}{(a+a\coth(e+fx))^3}\,dx$$
$$= \frac{\left(288\,d^2f^3x^3e^{(6\,fx+6\,e)} + 864\,cdf^3x^2e^{(6\,fx+6\,e)} + 864\,c^2f^3xe^{(6\,fx+6\,e)} + 1296\,d^2f^2x^2e^{(4\,fx+4\,e)} - 648\,d^2f^2x^2e^{(\cdots}\right.}{\cdots}$$

input
```
integrate((d*x+c)^2/(a+a*coth(f*x+e))^3,x, algorithm="giac")
```

output
```
1/6912*(288*d^2*f^3*x^3*e^(6*f*x + 6*e) + 864*c*d*f^3*x^2*e^(6*f*x + 6*e)
+ 864*c^2*f^3*x*e^(6*f*x + 6*e) + 1296*d^2*f^2*x^2*e^(4*f*x + 4*e) - 648*d
^2*f^2*x^2*e^(2*f*x + 2*e) + 144*d^2*f^2*x^2 + 2592*c*d*f^2*x*e^(4*f*x + 4
*e) - 1296*c*d*f^2*x*e^(2*f*x + 2*e) + 288*c*d*f^2*x + 1296*c^2*f^2*e^(4*f
*x + 4*e) + 1296*d^2*f*x*e^(4*f*x + 4*e) - 648*c^2*f^2*e^(2*f*x + 2*e) - 3
24*d^2*f*x*e^(2*f*x + 2*e) + 144*c^2*f^2 + 48*d^2*f*x + 1296*c*d*f*e^(4*f*
x + 4*e) - 324*c*d*f*e^(2*f*x + 2*e) + 48*c*d*f + 648*d^2*e^(4*f*x + 4*e)
- 81*d^2*e^(2*f*x + 2*e) + 8*d^2)*e^(-6*f*x - 6*e)/(a^3*f^3)
```

Mupad [B] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx)^2}{(a+a\coth(e+fx))^3}\,dx = \mathrm{e}^{-6\,e-6\,f\,x}\left(\frac{18\,c^2\,f^2+6\,c\,d\,f+d^2}{864\,a^3\,f^3}+\frac{d^2\,x^2}{48\,a^3\,f}+\frac{d\,x\,(d+6\,c\,f)}{144\,a^3\,f^2}\right)+\mathrm{e}^{-2\,e-2\,f\,x}\left(\frac{6\,c^2\,f^2+6\,c\,d\,f+3\,d^2}{32\,a^3\,f^3}+\frac{3\,d^2\,x^2}{16\,a^3\,f}+\frac{3\,d\,x\,(d+2\,c\,f)}{16\,a^3\,f^2}\right)-\mathrm{e}^{-4\,e-4\,f\,x}\left(\frac{24\,c^2\,f^2+12\,c\,d\,f+3\,d^2}{256\,a^3\,f^3}+\frac{3\,d^2\,x^2}{32\,a^3\,f}+\frac{3\,d\,x\,(d+4\,c\,f)}{64\,a^3\,f^2}\right)+\frac{c^2\,x}{8\,a^3}+\frac{d^2\,x^3}{24\,a^3}+\frac{c\,d\,x^2}{8\,a^3}$$

input
```
int((c + d*x)^2/(a + a*coth(e + f*x))^3,x)
```

output
```
exp(- 6*e - 6*f*x)*((d^2 + 18*c^2*f^2 + 6*c*d*f)/(864*a^3*f^3) + (d^2*x^2)
/(48*a^3*f) + (d*x*(d + 6*c*f))/(144*a^3*f^2)) + exp(- 2*e - 2*f*x)*((3*d^
2 + 6*c^2*f^2 + 6*c*d*f)/(32*a^3*f^3) + (3*d^2*x^2)/(16*a^3*f) + (3*d*x*(d
 + 2*c*f))/(16*a^3*f^2)) - exp(- 4*e - 4*f*x)*((3*d^2 + 24*c^2*f^2 + 12*c*
d*f)/(256*a^3*f^3) + (3*d^2*x^2)/(32*a^3*f) + (3*d*x*(d + 4*c*f))/(64*a^3*
f^2)) + (c^2*x)/(8*a^3) + (d^2*x^3)/(24*a^3) + (c*d*x^2)/(8*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.35

$$\int \frac{(c+dx)^2}{(a+a\coth(e+fx))^3}\,dx = \frac{864e^{6fx+6e}c^2f^3x+864e^{6fx+6e}cd\,f^3x^2+288e^{6fx+6e}d^2f^3x^3+1296e^{4fx+4e}c^2f^2+2592e^{4fx+4e}cd\,f^2x+129}{}$$

input
```
int((d*x+c)^2/(a+a*coth(f*x+e))^3,x)
```

output

```
(864*e**(6*e + 6*f*x)*c**2*f**3*x + 864*e**(6*e + 6*f*x)*c*d*f**3*x**2 + 2
88*e**(6*e + 6*f*x)*d**2*f**3*x**3 + 1296*e**(4*e + 4*f*x)*c**2*f**2 + 259
2*e**(4*e + 4*f*x)*c*d*f**2*x + 1296*e**(4*e + 4*f*x)*c*d*f + 1296*e**(4*e
 + 4*f*x)*d**2*f**2*x**2 + 1296*e**(4*e + 4*f*x)*d**2*f*x + 648*e**(4*e +
4*f*x)*d**2 - 648*e**(2*e + 2*f*x)*c**2*f**2 - 1296*e**(2*e + 2*f*x)*c*d*f
**2*x - 324*e**(2*e + 2*f*x)*c*d*f - 648*e**(2*e + 2*f*x)*d**2*f**2*x**2 -
 324*e**(2*e + 2*f*x)*d**2*f*x - 81*e**(2*e + 2*f*x)*d**2 + 144*c**2*f**2
+ 288*c*d*f**2*x + 48*c*d*f + 144*d**2*f**2*x**2 + 48*d**2*f*x + 8*d**2)/(
6912*e**(6*e + 6*f*x)*a**3*f**3)
```

3.29 $\int \frac{c+dx}{(a+a\coth(e+fx))^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 183

$$\begin{aligned}\int \frac{c+dx}{(a+a\coth(e+fx))^3}\,dx &= \frac{11dx}{96a^3f}-\frac{dx^2}{16a^3}+\frac{x(c+dx)}{8a^3}-\frac{d}{36f^2(a+a\coth(e+fx))^3}\\ &\quad-\frac{c+dx}{6f(a+a\coth(e+fx))^3}-\frac{5d}{96af^2(a+a\coth(e+fx))^2}\\ &\quad-\frac{c+dx}{8af(a+a\coth(e+fx))^2}\\ &\quad-\frac{11d}{96f^2\left(a^3+a^3\coth(e+fx)\right)}\\ &\quad-\frac{c+dx}{8f\left(a^3+a^3\coth(e+fx)\right)}\end{aligned}$$

output
```
11/96*d*x/a^3/f-1/16*d*x^2/a^3+1/8*x*(d*x+c)/a^3-1/36*d/f^2/(a+a*coth(f*x+
e))^3-1/6*(d*x+c)/f/(a+a*coth(f*x+e))^3-5/96*d/a/f^2/(a+a*coth(f*x+e))^2-1
/8*(d*x+c)/a/f/(a+a*coth(f*x+e))^2-11/96*d/f^2/(a^3+a^3*coth(f*x+e))-1/8*(
d*x+c)/f/(a^3+a^3*coth(f*x+e))
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.01

$$\int \frac{c+dx}{(a+a\coth(e+fx))^3}\,dx$$
$$= \frac{\operatorname{csch}^3(e+fx)\,(27(4cf+d(3+4fx))\cosh(e+fx)+4(6cf(1+6fx)+d(1+6fx+18f^2x^2))\cosh(3(e$$

input
```
Integrate[(c + d*x)/(a + a*Coth[e + f*x])^3,x]
```

output
```
(Csch[e + f*x]^3*(27*(4*c*f + d*(3 + 4*f*x))*Cosh[e + f*x] + 4*(6*c*f*(1 +
 6*f*x) + d*(1 + 6*f*x + 18*f^2*x^2))*Cosh[3*(e + f*x)] + 135*d*Sinh[e + f
*x] + 324*c*f*Sinh[e + f*x] + 324*d*f*x*Sinh[e + f*x] - 4*d*Sinh[3*(e + f*
x)] - 24*c*f*Sinh[3*(e + f*x)] - 24*d*f*x*Sinh[3*(e + f*x)] + 144*c*f^2*x*
Sinh[3*(e + f*x)] + 72*d*f^2*x^2*Sinh[3*(e + f*x)]))/(1152*a^3*f^2*(1 + Co
th[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4213, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a\coth(e+fx)+a)^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{c+dx}{\left(a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \; 4213$$

$$-d\int \left(\frac{x}{8a^3}-\frac{1}{8f\left(\coth(e+fx)a^3+a^3\right)}-\frac{1}{8af(\coth(e+fx)a+a)^2}-\frac{1}{6f(\coth(e+fx)a+a)^3}\right)dx-\frac{c+dx}{8f\left(a^3\coth(e+fx)+a^3\right)}+\frac{x(c+dx)}{8a^3}-\frac{c+dx}{8af(a\coth(e+fx)+a)^2}-\frac{c+dx}{6f(a\coth(e+fx)+a)^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c+dx}{8f\left(a^3\coth(e+fx)+a^3\right)}+\frac{x(c+dx)}{8a^3}-d\left(\frac{11}{96f^2\left(a^3\coth(e+fx)+a^3\right)}-\frac{11x}{96a^3f}+\frac{x^2}{16a^3}+\frac{5}{96af^2(a\coth(e+fx)+a)^2}+\frac{1}{36f^2(a\coth(e+fx)+a)^3}\right)-\frac{c+dx}{8af(a\coth(e+fx)+a)^2}-\frac{c+dx}{6f(a\coth(e+fx)+a)^3}$$

input
```
Int[(c + d*x)/(a + a*Coth[e + f*x])^3,x]
```

output
```
(x*(c + d*x))/(8*a^3) - (c + d*x)/(6*f*(a + a*Coth[e + f*x])^3) - (c + d*x
)/(8*a*f*(a + a*Coth[e + f*x])^2) - (c + d*x)/(8*f*(a^3 + a^3*Coth[e + f*x
])) - d*((-11*x)/(96*a^3*f) + x^2/(16*a^3) + 1/(36*f^2*(a + a*Coth[e + f*x
])^3) + 5/(96*a*f^2*(a + a*Coth[e + f*x])^2) + 11/(96*f^2*(a^3 + a^3*Coth[
e + f*x])))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4213
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> With[{u = IntHide[(a + b*Tan[e + f*x])^n, x]}, Simp[(c + d*x)
^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)   u, x], x]] /; FreeQ[{a, b,
c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.56

method	result
risch	$\frac{d x^2}{16a^3}+\frac{xc}{8a^3}+\frac{3(2dfx+2cf+d)\mathrm{e}^{-2fx-2e}}{32a^3f^2}-\frac{3(4dfx+4cf+d)\mathrm{e}^{-4fx-4e}}{128a^3f^2}+\frac{(6dfx+6cf+d)\mathrm{e}^{-6fx-6e}}{288a^3f^2}$
parallelrisch	$\frac{36\left(\left(\frac{dx}{2}+c\right)f-\frac{29d}{12}\right)fx\tanh(fx+e)^3+\left((54d\,x^2+108xc)f^2+(-9dx+252c)f+87d\right)\tanh(fx+e)^2+\left((54d\,x^2+108xc)f^2+(63c\right.}{288f^2a^3(\tanh(fx+e)+1)^3}$

input
```
int((d*x+c)/(a+a*coth(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*d*x^2/a^3+1/8/a^3*x*c+3/32*(2*d*f*x+2*c*f+d)/a^3/f^2*exp(-2*f*x-2*e)-
3/128*(4*d*f*x+4*c*f+d)/a^3/f^2*exp(-4*f*x-4*e)+1/288*(6*d*f*x+6*c*f+d)/a^
3/f^2*exp(-6*f*x-6*e)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.56

$$\int \frac{c+dx}{(a+a\coth(e+fx))^3}\,dx = \frac{4\,(18\,df^2x^2+6\,cf+6\,(6\,cf^2+df)x+d)\cosh{(fx+e)}^3+12\,(18\,df^2x^2+6\,cf+6\,(6\,cf^2+df)x+d)\cos}{1}$$

input
```
integrate((d*x+c)/(a+a*coth(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/1152*(4*(18*d*f^2*x^2 + 6*c*f + 6*(6*c*f^2 + d*f)*x + d)*cosh(f*x + e)^3
 + 12*(18*d*f^2*x^2 + 6*c*f + 6*(6*c*f^2 + d*f)*x + d)*cosh(f*x + e)*sinh(
f*x + e)^2 + 4*(18*d*f^2*x^2 - 6*c*f + 6*(6*c*f^2 - d*f)*x - d)*sinh(f*x +
 e)^3 + 27*(4*d*f*x + 4*c*f + 3*d)*cosh(f*x + e) + 3*(108*d*f*x + 4*(18*d*
f^2*x^2 - 6*c*f + 6*(6*c*f^2 - d*f)*x - d)*cosh(f*x + e)^2 + 108*c*f + 45*
d)*sinh(f*x + e))/(a^3*f^2*cosh(f*x + e)^3 + 3*a^3*f^2*cosh(f*x + e)^2*sin
h(f*x + e) + 3*a^3*f^2*cosh(f*x + e)*sinh(f*x + e)^2 + a^3*f^2*sinh(f*x +
e)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1287 vs. $2(165) = 330$.

Time = 1.10 (sec) , antiderivative size = 1287, normalized size of antiderivative = 7.03

$$\int \frac{c+dx}{(a+a\coth(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(a+a*coth(f*x+e))**3,x)
```

output
```
Piecewise((36*c*f**2*x*tanh(e + f*x)**3/(288*a**3*f**2*tanh(e + f*x)**3 +
864*a**3*f**2*tanh(e + f*x)**2 + 864*a**3*f**2*tanh(e + f*x) + 288*a**3*f*
*2) + 108*c*f**2*x*tanh(e + f*x)**2/(288*a**3*f**2*tanh(e + f*x)**3 + 864*
a**3*f**2*tanh(e + f*x)**2 + 864*a**3*f**2*tanh(e + f*x) + 288*a**3*f**2)
+ 108*c*f**2*x*tanh(e + f*x)/(288*a**3*f**2*tanh(e + f*x)**3 + 864*a**3*f*
*2*tanh(e + f*x)**2 + 864*a**3*f**2*tanh(e + f*x) + 288*a**3*f**2) + 36*c*
f**2*x/(288*a**3*f**2*tanh(e + f*x)**3 + 864*a**3*f**2*tanh(e + f*x)**2 +
864*a**3*f**2*tanh(e + f*x) + 288*a**3*f**2) + 252*c*f*tanh(e + f*x)**2/(2
88*a**3*f**2*tanh(e + f*x)**3 + 864*a**3*f**2*tanh(e + f*x)**2 + 864*a**3*
f**2*tanh(e + f*x) + 288*a**3*f**2) + 324*c*f*tanh(e + f*x)/(288*a**3*f**2
*tanh(e + f*x)**3 + 864*a**3*f**2*tanh(e + f*x)**2 + 864*a**3*f**2*tanh(e
+ f*x) + 288*a**3*f**2) + 120*c*f/(288*a**3*f**2*tanh(e + f*x)**3 + 864*a*
*3*f**2*tanh(e + f*x)**2 + 864*a**3*f**2*tanh(e + f*x) + 288*a**3*f**2) +
18*d*f**2*x**2*tanh(e + f*x)**3/(288*a**3*f**2*tanh(e + f*x)**3 + 864*a**3
*f**2*tanh(e + f*x)**2 + 864*a**3*f**2*tanh(e + f*x) + 288*a**3*f**2) + 54
*d*f**2*x**2*tanh(e + f*x)**2/(288*a**3*f**2*tanh(e + f*x)**3 + 864*a**3*f
**2*tanh(e + f*x)**2 + 864*a**3*f**2*tanh(e + f*x) + 288*a**3*f**2) + 54*d
*f**2*x**2*tanh(e + f*x)/(288*a**3*f**2*tanh(e + f*x)**3 + 864*a**3*f**2*t
anh(e + f*x)**2 + 864*a**3*f**2*tanh(e + f*x) + 288*a**3*f**2) + 18*d*f**2
*x**2/(288*a**3*f**2*tanh(e + f*x)**3 + 864*a**3*f**2*tanh(e + f*x)**2 ...
```

Maxima [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.75

$$\int \frac{c+dx}{(a+a\coth(e+fx))^3}\,dx$$
$$= \frac{1}{96}c\left(\frac{12\,(fx+e)}{a^3f}+\frac{18\,e^{(-2\,fx-2\,e)}-9\,e^{(-4\,fx-4\,e)}+2\,e^{(-6\,fx-6\,e)}}{a^3f}\right)$$
$$+\frac{\left(72\,f^2x^2e^{(6\,e)}+108\,\left(2\,fxe^{(4\,e)}+e^{(4\,e)}\right)e^{(-2\,fx)}-27\,\left(4\,fxe^{(2\,e)}+e^{(2\,e)}\right)e^{(-4\,fx)}+4\,(6\,fx+1)e^{(-6\,fx)}\right)de}{1152\,a^3f^2}$$

input
```
integrate((d*x+c)/(a+a*coth(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/96*c*(12*(f*x + e)/(a^3*f) + (18*e^(-2*f*x - 2*e) - 9*e^(-4*f*x - 4*e) +
 2*e^(-6*f*x - 6*e))/(a^3*f)) + 1/1152*(72*f^2*x^2*e^(6*e) + 108*(2*f*x*e^
(4*e) + e^(4*e))*e^(-2*f*x) - 27*(4*f*x*e^(2*e) + e^(2*e))*e^(-4*f*x) + 4*
(6*f*x + 1)*e^(-6*f*x))*d*e^(-6*e)/(a^3*f^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.78

$$\int \frac{c+dx}{(a+a\coth(e+fx))^3}\,dx$$
$$= \frac{\left(72\,df^2x^2e^{(6\,fx+6\,e)}+144\,cf^2xe^{(6\,fx+6\,e)}+216\,dfxe^{(4\,fx+4\,e)}-108\,dfxe^{(2\,fx+2\,e)}+24\,dfx+216\,cfe^{(4\,fx+4\,e)}\right.}{1152\,a^3f^2}$$

input
```
integrate((d*x+c)/(a+a*coth(f*x+e))^3,x, algorithm="giac")
```

output
```
1/1152*(72*d*f^2*x^2*e^(6*f*x + 6*e) + 144*c*f^2*x*e^(6*f*x + 6*e) + 216*d
*f*x*e^(4*f*x + 4*e) - 108*d*f*x*e^(2*f*x + 2*e) + 24*d*f*x + 216*c*f*e^(4
*f*x + 4*e) - 108*c*f*e^(2*f*x + 2*e) + 24*c*f + 108*d*e^(4*f*x + 4*e) - 2
7*d*e^(2*f*x + 2*e) + 4*d)*e^(-6*f*x - 6*e)/(a^3*f^2)
```

Mupad [B] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

$$\int \frac{c+dx}{(a+a\coth(e+fx))^3}\,dx = \mathrm{e}^{-6\,e-6\,f\,x}\left(\frac{d+6\,c\,f}{288\,a^3\,f^2}+\frac{d\,x}{48\,a^3\,f}\right) + \mathrm{e}^{-2\,e-2\,f\,x}\left(\frac{3\,d+6\,c\,f}{32\,a^3\,f^2}+\frac{3\,d\,x}{16\,a^3\,f}\right) - \mathrm{e}^{-4\,e-4\,f\,x}\left(\frac{3\,d+12\,c\,f}{128\,a^3\,f^2}+\frac{3\,d\,x}{32\,a^3\,f}\right)+\frac{d\,x^2}{16\,a^3}+\frac{c\,x}{8\,a^3}$$

input
```
int((c + d*x)/(a + a*coth(e + f*x))^3,x)
```

output
```
exp(- 6*e - 6*f*x)*((d + 6*c*f)/(288*a^3*f^2) + (d*x)/(48*a^3*f)) + exp(-
2*e - 2*f*x)*((3*d + 6*c*f)/(32*a^3*f^2) + (3*d*x)/(16*a^3*f)) - exp(- 4*e
 - 4*f*x)*((3*d + 12*c*f)/(128*a^3*f^2) + (3*d*x)/(32*a^3*f)) + (d*x^2)/(1
6*a^3) + (c*x)/(8*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.84

$$\int \frac{c+dx}{(a+a\coth(e+fx))^3}\,dx = \frac{144e^{6fx+6e}cf^2x+72e^{6fx+6e}df^2x^2+216e^{4fx+4e}cf+216e^{4fx+4e}dfx+108e^{4fx+4e}d-108e^{2fx+2e}cf-108}{1152e^{6fx+6e}a^3f^2}$$

input
```
int((d*x+c)/(a+a*coth(f*x+e))^3,x)
```

output
```
(144*e**(6*e + 6*f*x)*c*f**2*x + 72*e**(6*e + 6*f*x)*d*f**2*x**2 + 216*e**
(4*e + 4*f*x)*c*f + 216*e**(4*e + 4*f*x)*d*f*x + 108*e**(4*e + 4*f*x)*d -
108*e**(2*e + 2*f*x)*c*f - 108*e**(2*e + 2*f*x)*d*f*x - 27*e**(2*e + 2*f*x
)*d + 24*c*f + 24*d*f*x + 4*d)/(1152*e**(6*e + 6*f*x)*a**3*f**2)
```

3.30 $\int \frac{1}{(c+dx)(a+a\coth(e+fx))^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 437

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))^3}\,dx = -\frac{3\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{8a^3d} + \frac{3\cosh\left(4e-\frac{4cf}{d}\right)\text{Chi}\left(\frac{4cf}{d}+4fx\right)}{8a^3d} - \frac{\cosh\left(6e-\frac{6cf}{d}\right)\text{Chi}\left(\frac{6cf}{d}+6fx\right)}{8a^3d} + \frac{\log(c+dx)}{8a^3d} + \frac{\text{Chi}\left(\frac{6cf}{d}+6fx\right)\sinh\left(6e-\frac{6cf}{d}\right)}{8a^3d} - \frac{3\text{Chi}\left(\frac{4cf}{d}+4fx\right)\sinh\left(4e-\frac{4cf}{d}\right)}{8a^3d} + \frac{3\text{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{8a^3d} + \frac{3\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{8a^3d} - \frac{3\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{8a^3d} - \frac{3\cosh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(\frac{4cf}{d}+4fx\right)}{8a^3d} + \frac{3\sinh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(\frac{4cf}{d}+4fx\right)}{8a^3d} + \frac{\cosh\left(6e-\frac{6cf}{d}\right)\text{Shi}\left(\frac{6cf}{d}+6fx\right)}{8a^3d} - \frac{\sinh\left(6e-\frac{6cf}{d}\right)\text{Shi}\left(\frac{6cf}{d}+6fx\right)}{8a^3d}$$

output

```
-3/8*cosh(-2*e+2*c*f/d)*Chi(2*c*f/d+2*f*x)/a^3/d+3/8*cosh(-4*e+4*c*f/d)*Ch
i(4*c*f/d+4*f*x)/a^3/d-1/8*cosh(-6*e+6*c*f/d)*Chi(6*c*f/d+6*f*x)/a^3/d+1/8
*ln(d*x+c)/a^3/d-1/8*Chi(6*c*f/d+6*f*x)*sinh(-6*e+6*c*f/d)/a^3/d+3/8*Chi(4
*c*f/d+4*f*x)*sinh(-4*e+4*c*f/d)/a^3/d-3/8*Chi(2*c*f/d+2*f*x)*sinh(-2*e+2*
c*f/d)/a^3/d+3/8*cosh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a^3/d+3/8*sinh(-2*e
+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a^3/d-3/8*cosh(-4*e+4*c*f/d)*Shi(4*c*f/d+4*f*
x)/a^3/d-3/8*sinh(-4*e+4*c*f/d)*Shi(4*c*f/d+4*f*x)/a^3/d+1/8*cosh(-6*e+6*c
*f/d)*Shi(6*c*f/d+6*f*x)/a^3/d+1/8*sinh(-6*e+6*c*f/d)*Shi(6*c*f/d+6*f*x)/a
^3/d
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.71

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))^3} dx$$
$$= \frac{\operatorname{csch}^3(e+fx)(\cosh(fx)+\sinh(fx))^3 \left(\cosh(3e)\log(f(c+dx))+\log(f(c+dx))\sinh(3e)+(-\cosh\left(e\right.\right.}{}$$

input
```
Integrate[1/((c + d*x)*(a + a*Coth[e + f*x])^3),x]
```

output
```
(Csch[e + f*x]^3*(Cosh[f*x] + Sinh[f*x])^3*(Cosh[3*e]*Log[f*(c + d*x)] + L
og[f*(c + d*x)]*Sinh[3*e] + (-Cosh[e - (4*c*f)/d] + Sinh[e - (4*c*f)/d])*(
-3*CoshIntegral[(4*f*(c + d*x))/d] + Cosh[2*e - (2*c*f)/d]*CoshIntegral[(6
*f*(c + d*x))/d] - CoshIntegral[(6*f*(c + d*x))/d]*Sinh[2*e - (2*c*f)/d] +
 3*CoshIntegral[(2*f*(c + d*x))/d]*(Cosh[2*e - (2*c*f)/d] + Sinh[2*e - (2*
c*f)/d]) - 3*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d] - 3*Sin
h[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d] + 3*SinhIntegral[(4*f*(
c + d*x))/d] - Cosh[2*e - (2*c*f)/d]*SinhIntegral[(6*f*(c + d*x))/d] + Sin
h[2*e - (2*c*f)/d]*SinhIntegral[(6*f*(c + d*x))/d])))/(8*a^3*d*(1 + Coth[e
 + f*x])^3)
```

Rubi [A] (verified)

Time = 2.79 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4211, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a\coth(e+fx)+a)^3} dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)\left(a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^3} dx$$

$$\Big\downarrow \text{4211}$$

$$\int \left(\frac{\sinh^3(2e+2fx)}{8a^3(c+dx)}+\frac{3\sinh^2(2e+2fx)}{8a^3(c+dx)}+\frac{3\sinh(2e+2fx)}{8a^3(c+dx)}-\frac{3\sinh(2e+2fx)\sinh(4e+4fx)}{16a^3(c+dx)}-\frac{3\sinh(4e+}{8a^3(c+}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{8a^3d}+\frac{\text{Chi}\left(6xf+\frac{6cf}{d}\right)\sinh\left(6e-\frac{6cf}{d}\right)}{8a^3d}-\frac{3\text{Chi}\left(4xf+\frac{4cf}{d}\right)\sinh\left(4e-\frac{4cf}{d}\right)}{8a^3d}-\frac{3\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{8a^3d}+\frac{3\text{Chi}\left(4xf+\frac{4cf}{d}\right)\cosh\left(4e-\frac{4cf}{d}\right)}{8a^3d}-\frac{\text{Chi}\left(6xf+\frac{6cf}{d}\right)\cosh\left(6e-\frac{6cf}{d}\right)}{8a^3d}-\frac{3\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{8a^3d}+\frac{3\sinh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(4xf+\frac{4cf}{d}\right)}{8a^3d}-\frac{\sinh\left(6e-\frac{6cf}{d}\right)\text{Shi}\left(6xf+\frac{6cf}{d}\right)}{8a^3d}+\frac{3\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{8a^3d}-\frac{3\cosh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(4xf+\frac{4cf}{d}\right)}{8a^3d}+\frac{\cosh\left(6e-\frac{6cf}{d}\right)\text{Shi}\left(6xf+\frac{6cf}{d}\right)}{8a^3d}+\frac{\log(c+dx)}{8a^3d}$$

input
```
Int[1/((c + d*x)*(a + a*Coth[e + f*x])^3),x]
```

output
```
(-3*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) + (3*
Cosh[4*e - (4*c*f)/d]*CoshIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) - (Cosh[6
*e - (6*c*f)/d]*CoshIntegral[(6*c*f)/d + 6*f*x])/(8*a^3*d) + Log[c + d*x]/
(8*a^3*d) + (CoshIntegral[(6*c*f)/d + 6*f*x]*Sinh[6*e - (6*c*f)/d])/(8*a^3
*d) - (3*CoshIntegral[(4*c*f)/d + 4*f*x]*Sinh[4*e - (4*c*f)/d])/(8*a^3*d)
+ (3*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/(8*a^3*d) + (3
*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) - (3*Sin
h[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) - (3*Cosh[4*
e - (4*c*f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) + (3*Sinh[4*e -
(4*c*f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) + (Cosh[6*e - (6*c*f
)/d]*SinhIntegral[(6*c*f)/d + 6*f*x])/(8*a^3*d) - (Sinh[6*e - (6*c*f)/d]*S
inhIntegral[(6*c*f)/d + 6*f*x])/(8*a^3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4211
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.35

method	result
risch	$\frac{\ln(dx+c)}{8a^3d}+\frac{\mathrm{e}^{\frac{6cf-6de}{d}}\,\mathrm{expIntegral}_1\left(6fx+6e+\frac{6cf-6de}{d}\right)}{8a^3d}-\frac{3\,\mathrm{e}^{\frac{4cf-4de}{d}}\,\mathrm{expIntegral}_1\left(4fx+4e+\frac{4cf-4de}{d}\right)}{8a^3d}+\frac{3\,\mathrm{e}^{\frac{2cf-2de}{d}}\,\mathrm{expIn}}{}$

input
```
int(1/(d*x+c)/(a+a*coth(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(d*x+c)/a^3/d+1/8/a^3/d*exp(6*(c*f-d*e)/d)*Ei(1,6*f*x+6*e+6*(c*f-d*e
)/d)-3/8/a^3/d*exp(4*(c*f-d*e)/d)*Ei(1,4*f*x+4*e+4*(c*f-d*e)/d)+3/8/a^3/d*
exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.45

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))^3}\,dx = -\frac{3\,\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\cosh\left(-\frac{2\,(de-cf)}{d}\right)-3\,\mathrm{Ei}\left(-\frac{4\,(dfx+cf)}{d}\right)\cosh\left(-\frac{4\,(de-cf)}{d}\right)+\mathrm{Ei}\left(-\frac{6\,(dfx+cf)}{d}\right)\cosh\left(-\frac{6}{}\right.}{}$$

input
```
integrate(1/(d*x+c)/(a+a*coth(f*x+e))^3,x, algorithm="fricas")
```

output
```
-1/8*(3*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) - 3*Ei(-4*(d*f*x + c
*f)/d)*cosh(-4*(d*e - c*f)/d) + Ei(-6*(d*f*x + c*f)/d)*cosh(-6*(d*e - c*f)
/d) + 3*Ei(-2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)/d) - 3*Ei(-4*(d*f*x + c
*f)/d)*sinh(-4*(d*e - c*f)/d) + Ei(-6*(d*f*x + c*f)/d)*sinh(-6*(d*e - c*f)
/d) - log(d*x + c))/(a^3*d)
```

Sympy [F]

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))^3}\,dx = \frac{\int \frac{1}{c\coth^3(e+fx)+3c\coth^2(e+fx)+3c\coth(e+fx)+c+dx\coth^3(e+fx)+3dx\coth^2(e+fx)+3dx\coth(e+fx)+dx}\,dx}{a^3}$$

input
```
integrate(1/(d*x+c)/(a+a*coth(f*x+e))**3,x)
```

output
```
Integral(1/(c*coth(e + f*x)**3 + 3*c*coth(e + f*x)**2 + 3*c*coth(e + f*x)
+ c + d*x*coth(e + f*x)**3 + 3*d*x*coth(e + f*x)**2 + 3*d*x*coth(e + f*x)
+ d*x), x)/a**3
```

Maxima [A] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.26

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))^3}\,dx = \frac{e^{\left(-6e+\frac{6cf}{d}\right)}E_1\left(\frac{6(dx+c)f}{d}\right)}{8a^3d} - \frac{3e^{\left(-4e+\frac{4cf}{d}\right)}E_1\left(\frac{4(dx+c)f}{d}\right)}{8a^3d} + \frac{3e^{\left(-2e+\frac{2cf}{d}\right)}E_1\left(\frac{2(dx+c)f}{d}\right)}{8a^3d} + \frac{\log(dx+c)}{8a^3d}$$

input
```
integrate(1/(d*x+c)/(a+a*coth(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/8*e^(-6*e + 6*c*f/d)*exp_integral_e(1, 6*(d*x + c)*f/d)/(a^3*d) - 3/8*e^
(-4*e + 4*c*f/d)*exp_integral_e(1, 4*(d*x + c)*f/d)/(a^3*d) + 3/8*e^(-2*e
+ 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/(a^3*d) + 1/8*log(d*x + c)/(
a^3*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.24

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))^3}\,dx = -\frac{\left(3\,\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(4\,e+\frac{2\,cf}{d}\right)} - 3\,\mathrm{Ei}\left(-\frac{4\,(dfx+cf)}{d}\right)e^{\left(2\,e+\frac{4\,cf}{d}\right)} + \mathrm{Ei}\left(-\frac{6\,(dfx+cf)}{d}\right)e^{\left(\frac{6\,cf}{d}\right)} - e^{(6\,e)}\log\left(dx+\right.\right.}{8\,a^3d}$$

input
```
integrate(1/(d*x+c)/(a+a*coth(f*x+e))^3,x, algorithm="giac")
```

output
```
-1/8*(3*Ei(-2*(d*f*x + c*f)/d)*e^(4*e + 2*c*f/d) - 3*Ei(-4*(d*f*x + c*f)/d
)*e^(2*e + 4*c*f/d) + Ei(-6*(d*f*x + c*f)/d)*e^(6*c*f/d) - e^(6*e)*log(d*x
 + c))*e^(-6*e)/(a^3*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))^3}\,dx = \int \frac{1}{{\left(a+a\coth\left(e+f\,x\right)\right)}^3\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((a + a*coth(e + f*x))^3*(c + d*x)),x)
```

output
```
int(1/((a + a*coth(e + f*x))^3*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)(a+a\coth(e+fx))^3}\,dx$$
$$=\frac{\int \frac{1}{\coth(fx+e)^3c+\coth(fx+e)^3dx+3\coth(fx+e)^2c+3\coth(fx+e)^2dx+3\coth(fx+e)c+3\coth(fx+e)dx+c+dx}dx}{a^3}$$

input
```
int(1/(d*x+c)/(a+a*coth(f*x+e))^3,x)
```

output
```
int(1/(coth(e + f*x)**3*c + coth(e + f*x)**3*d*x + 3*coth(e + f*x)**2*c +
3*coth(e + f*x)**2*d*x + 3*coth(e + f*x)*c + 3*coth(e + f*x)*d*x + c + d*x
),x)/a**3
```

3.31 $\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 692

$$\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))^3}\,dx = -\frac{1}{8a^3d(c+dx)}+\frac{9\cosh(2e+2fx)}{32a^3d(c+dx)} - \frac{3\cosh^2(2e+2fx)}{8a^3d(c+dx)} + \frac{\cosh^3(2e+2fx)}{8a^3d(c+dx)}+\frac{3\cosh(6e+6fx)}{32a^3d(c+dx)} + \frac{3f\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{4a^3d^2} - \frac{3f\cosh\left(4e-\frac{4cf}{d}\right)\text{Chi}\left(\frac{4cf}{d}+4fx\right)}{2a^3d^2} + \frac{3f\cosh\left(6e-\frac{6cf}{d}\right)\text{Chi}\left(\frac{6cf}{d}+6fx\right)}{4a^3d^2} - \frac{3f\text{Chi}\left(\frac{6cf}{d}+6fx\right)\sinh\left(6e-\frac{6cf}{d}\right)}{4a^3d^2} + \frac{3f\text{Chi}\left(\frac{4cf}{d}+4fx\right)\sinh\left(4e-\frac{4cf}{d}\right)}{2a^3d^2} - \frac{3f\text{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{4a^3d^2} - \frac{15\sinh(2e+2fx)}{32a^3d(c+dx)} - \frac{3\sinh^2(2e+2fx)}{8a^3d(c+dx)}-\frac{\sinh^3(2e+2fx)}{8a^3d(c+dx)} + \frac{3\sinh(4e+4fx)}{8a^3d(c+dx)}-\frac{3\sinh(6e+6fx)}{32a^3d(c+dx)} - \frac{3f\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{4a^3d^2} + \frac{3f\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{4a^3d^2} + \frac{3f\cosh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(\frac{4cf}{d}+4fx\right)}{2a^3d^2} - \frac{3f\sinh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(\frac{4cf}{d}+4fx\right)}{2a^3d^2} - \frac{3f\cosh\left(6e-\frac{6cf}{d}\right)\text{Shi}\left(\frac{6cf}{d}+6fx\right)}{4a^3d^2} + \frac{3f\sinh\left(6e-\frac{6cf}{d}\right)\text{Shi}\left(\frac{6cf}{d}+6fx\right)}{4a^3d^2}$$

output
```
-1/8/a^3/d/(d*x+c)+9/32*cosh(2*f*x+2*e)/a^3/d/(d*x+c)-3/8*cosh(2*f*x+2*e)^
2/a^3/d/(d*x+c)+1/8*cosh(2*f*x+2*e)^3/a^3/d/(d*x+c)+3/32*cosh(6*f*x+6*e)/a
^3/d/(d*x+c)+3/4*f*cosh(-2*e+2*c*f/d)*Chi(2*c*f/d+2*f*x)/a^3/d^2-3/2*f*cos
h(-4*e+4*c*f/d)*Chi(4*c*f/d+4*f*x)/a^3/d^2+3/4*f*cosh(-6*e+6*c*f/d)*Chi(6*
c*f/d+6*f*x)/a^3/d^2+3/4*f*Chi(6*c*f/d+6*f*x)*sinh(-6*e+6*c*f/d)/a^3/d^2-3
/2*f*Chi(4*c*f/d+4*f*x)*sinh(-4*e+4*c*f/d)/a^3/d^2+3/4*f*Chi(2*c*f/d+2*f*x
)*sinh(-2*e+2*c*f/d)/a^3/d^2-15/32*sinh(2*f*x+2*e)/a^3/d/(d*x+c)-3/8*sinh(
2*f*x+2*e)^2/a^3/d/(d*x+c)-1/8*sinh(2*f*x+2*e)^3/a^3/d/(d*x+c)+3/8*sinh(4*
f*x+4*e)/a^3/d/(d*x+c)-3/32*sinh(6*f*x+6*e)/a^3/d/(d*x+c)-3/4*f*cosh(-2*e+
2*c*f/d)*Shi(2*c*f/d+2*f*x)/a^3/d^2-3/4*f*sinh(-2*e+2*c*f/d)*Shi(2*c*f/d+2
*f*x)/a^3/d^2+3/2*f*cosh(-4*e+4*c*f/d)*Shi(4*c*f/d+4*f*x)/a^3/d^2+3/2*f*si
nh(-4*e+4*c*f/d)*Shi(4*c*f/d+4*f*x)/a^3/d^2-3/4*f*cosh(-6*e+6*c*f/d)*Shi(6
*c*f/d+6*f*x)/a^3/d^2-3/4*f*sinh(-6*e+6*c*f/d)*Shi(6*c*f/d+6*f*x)/a^3/d^2
```

Mathematica [A] (verified)

Time = 2.46 (sec) , antiderivative size = 796, normalized size of antiderivative = 1.15

$$\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((c + d*x)^2*(a + a*Coth[e + f*x])^3),x]
```

output
```
(Csch[e + f*x]^3*(Cosh[(3*c*f)/d] + Sinh[(3*c*f)/d])*(3*d*Cosh[e + f*((-3*
c)/d + x)] - d*Cosh[3*(e + f*(-(c/d) + x))] + d*Cosh[3*(e + f*(c/d + x))]
- 3*d*Cosh[e + f*((3*c)/d + x)] + 6*c*f*Cosh[3*e - (3*f*(c + d*x))/d]*Cosh
Integral[(6*f*(c + d*x))/d] + 6*d*f*x*Cosh[3*e - (3*f*(c + d*x))/d]*CoshIn
tegral[(6*f*(c + d*x))/d] + 6*f*(c + d*x)*CoshIntegral[(2*f*(c + d*x))/d]*
(Cosh[e - (c*f)/d + 3*f*x] + Sinh[e - (c*f)/d + 3*f*x]) + 3*d*Sinh[e + f*(
(-3*c)/d + x)] - d*Sinh[3*(e + f*(-(c/d) + x))] - d*Sinh[3*(e + f*(c/d + x
))] + 3*d*Sinh[e + f*((3*c)/d + x)] - 6*c*f*CoshIntegral[(6*f*(c + d*x))/d
]*Sinh[3*e - (3*f*(c + d*x))/d] - 6*d*f*x*CoshIntegral[(6*f*(c + d*x))/d]*
Sinh[3*e - (3*f*(c + d*x))/d] + 12*f*(c + d*x)*CoshIntegral[(4*f*(c + d*x)
)/d]*(-Cosh[e - (f*(c + 3*d*x))/d] + Sinh[e - (f*(c + 3*d*x))/d]) - 6*c*f*
Cosh[e - (c*f)/d + 3*f*x]*SinhIntegral[(2*f*(c + d*x))/d] - 6*d*f*x*Cosh[e
 - (c*f)/d + 3*f*x]*SinhIntegral[(2*f*(c + d*x))/d] - 6*c*f*Sinh[e - (c*f)
/d + 3*f*x]*SinhIntegral[(2*f*(c + d*x))/d] - 6*d*f*x*Sinh[e - (c*f)/d + 3
*f*x]*SinhIntegral[(2*f*(c + d*x))/d] + 12*c*f*Cosh[e - (f*(c + 3*d*x))/d]
*SinhIntegral[(4*f*(c + d*x))/d] + 12*d*f*x*Cosh[e - (f*(c + 3*d*x))/d]*Si
nhIntegral[(4*f*(c + d*x))/d] - 12*c*f*Sinh[e - (f*(c + 3*d*x))/d]*SinhInt
egral[(4*f*(c + d*x))/d] - 12*d*f*x*Sinh[e - (f*(c + 3*d*x))/d]*SinhIntegr
al[(4*f*(c + d*x))/d] - 6*c*f*Cosh[3*e - (3*f*(c + d*x))/d]*SinhIntegral[(
6*f*(c + d*x))/d] - 6*d*f*x*Cosh[3*e - (3*f*(c + d*x))/d]*SinhIntegral[...
```

Rubi [A] (verified)

Time = 2.27 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4211, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a\coth(e+fx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c+dx)^2\left(a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4211}$$

$$\int \left(\frac{\sinh^3(2e+2fx)}{8a^3(c+dx)^2}+\frac{3\sinh^2(2e+2fx)}{8a^3(c+dx)^2}+\frac{3\sinh(2e+2fx)}{8a^3(c+dx)^2}-\frac{3\sinh(2e+2fx)\sinh(4e+4fx)}{16a^3(c+dx)^2}-\frac{3\sinh(4e+}{8a^3(c+d}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3f\text{Chi}\left(6xf+\frac{6cf}{d}\right)\sinh\left(6e-\frac{6cf}{d}\right)}{4a^3d^2}+\frac{3f\text{Chi}\left(4xf+\frac{4cf}{d}\right)\sinh\left(4e-\frac{4cf}{d}\right)}{2a^3d^2}-\frac{3f\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{4a^3d^2}+\frac{3f\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{4a^3d^2}-\frac{3f\text{Chi}\left(4xf+\frac{4cf}{d}\right)\cosh\left(4e-\frac{4cf}{d}\right)}{2a^3d^2}+\frac{3f\text{Chi}\left(6xf+\frac{6cf}{d}\right)\cosh\left(6e-\frac{6cf}{d}\right)}{4a^3d^2}+\frac{3f\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{4a^3d^2}-\frac{3f\sinh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(4xf+\frac{4cf}{d}\right)}{2a^3d^2}+\frac{3f\sinh\left(6e-\frac{6cf}{d}\right)\text{Shi}\left(6xf+\frac{6cf}{d}\right)}{4a^3d^2}-\frac{3f\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{4a^3d^2}+\frac{3f\cosh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(4xf+\frac{4cf}{d}\right)}{2a^3d^2}-\frac{3f\cosh\left(6e-\frac{6cf}{d}\right)\text{Shi}\left(6xf+\frac{6cf}{d}\right)}{4a^3d^2}-\frac{\sinh^3(2e+2fx)}{8a^3d(c+dx)}-\frac{3\sinh^2(2e+2fx)}{8a^3d(c+dx)}-\frac{15\sinh(2e+2fx)}{32a^3d(c+dx)}+\frac{3\sinh(4e+4fx)}{8a^3d(c+dx)}-\frac{3\sinh(6e+6fx)}{32a^3d(c+dx)}+\frac{\cosh^3(2e+2fx)}{8a^3d(c+dx)}-\frac{3\cosh^2(2e+2fx)}{8a^3d(c+dx)}+\frac{9\cosh(2e+2fx)}{32a^3d(c+dx)}+\frac{3\cosh(6e+6fx)}{32a^3d(c+dx)}-\frac{1}{8a^3d(c+dx)}$$

input
```
Int[1/((c + d*x)^2*(a + a*Coth[e + f*x])^3),x]
```

output
```
-1/8*1/(a^3*d*(c + d*x)) + (9*Cosh[2*e + 2*f*x])/(32*a^3*d*(c + d*x)) - (3
*Cosh[2*e + 2*f*x]^2)/(8*a^3*d*(c + d*x)) + Cosh[2*e + 2*f*x]^3/(8*a^3*d*(
c + d*x)) + (3*Cosh[6*e + 6*f*x])/(32*a^3*d*(c + d*x)) + (3*f*Cosh[2*e - (
2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(4*a^3*d^2) - (3*f*Cosh[4*e - (
4*c*f)/d]*CoshIntegral[(4*c*f)/d + 4*f*x])/(2*a^3*d^2) + (3*f*Cosh[6*e - (
6*c*f)/d]*CoshIntegral[(6*c*f)/d + 6*f*x])/(4*a^3*d^2) - (3*f*CoshIntegral
[(6*c*f)/d + 6*f*x]*Sinh[6*e - (6*c*f)/d])/(4*a^3*d^2) + (3*f*CoshIntegral
[(4*c*f)/d + 4*f*x]*Sinh[4*e - (4*c*f)/d])/(2*a^3*d^2) - (3*f*CoshIntegral
[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/(4*a^3*d^2) - (15*Sinh[2*e + 2*
f*x])/(32*a^3*d*(c + d*x)) - (3*Sinh[2*e + 2*f*x]^2)/(8*a^3*d*(c + d*x)) -
 Sinh[2*e + 2*f*x]^3/(8*a^3*d*(c + d*x)) + (3*Sinh[4*e + 4*f*x])/(8*a^3*d*
(c + d*x)) - (3*Sinh[6*e + 6*f*x])/(32*a^3*d*(c + d*x)) - (3*f*Cosh[2*e -
(2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(4*a^3*d^2) + (3*f*Sinh[2*e -
(2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(4*a^3*d^2) + (3*f*Cosh[4*e -
(4*c*f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(2*a^3*d^2) - (3*f*Sinh[4*e -
(4*c*f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(2*a^3*d^2) - (3*f*Cosh[6*e -
(6*c*f)/d]*SinhIntegral[(6*c*f)/d + 6*f*x])/(4*a^3*d^2) + (3*f*Sinh[6*e -
(6*c*f)/d]*SinhIntegral[(6*c*f)/d + 6*f*x])/(4*a^3*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4211
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.35

method	result
risch	$-\frac{1}{8a^3d(dx+c)}+\frac{f\,\mathrm{e}^{-6fx-6e}}{8a^3d(dfx+cf)}-\frac{3f\,\mathrm{e}^{\frac{6cf-6de}{d}}\,\mathrm{expIntegral}_1\left(6fx+6e+\frac{6cf-6de}{d}\right)}{4a^3d^2}-\frac{3f\,\mathrm{e}^{-4fx-4e}}{8a^3d(dfx+cf)}+\frac{3f\,\mathrm{e}^{\frac{4cf-4de}{d}}\,\mathrm{expInteg}}{2}$

input
```
int(1/(d*x+c)^2/(a+a*coth(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8/a^3/d/(d*x+c)+1/8*f/a^3*exp(-6*f*x-6*e)/d/(d*f*x+c*f)-3/4*f/a^3/d^2*e
xp(6*(c*f-d*e)/d)*Ei(1,6*f*x+6*e+6*(c*f-d*e)/d)-3/8*f/a^3*exp(-4*f*x-4*e)/
d/(d*f*x+c*f)+3/2*f/a^3/d^2*exp(4*(c*f-d*e)/d)*Ei(1,4*f*x+4*e+4*(c*f-d*e)/
d)+3/8*f/a^3*exp(-2*f*x-2*e)/d/(d*f*x+c*f)-3/4*f/a^3/d^2*exp(2*(c*f-d*e)/d
)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 1162, normalized size of antiderivative = 1.68

$$\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^2/(a+a*coth(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/4*(3*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(f*x + e)^3*sinh(-2*(d*e -
 c*f)/d) - 6*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(f*x + e)^3*sinh(-4*
(d*e - c*f)/d) + 3*(d*f*x + c*f)*Ei(-6*(d*f*x + c*f)/d)*cosh(f*x + e)^3*si
nh(-6*(d*e - c*f)/d) + 3*((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*
e - c*f)/d) - 2*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(-4*(d*e - c*f)/d
) + (d*f*x + c*f)*Ei(-6*(d*f*x + c*f)/d)*cosh(-6*(d*e - c*f)/d))*cosh(f*x
+ e)^3 + (3*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) -
6*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(-4*(d*e - c*f)/d) + 3*(d*f*x +
 c*f)*Ei(-6*(d*f*x + c*f)/d)*cosh(-6*(d*e - c*f)/d) + 3*(d*f*x + c*f)*Ei(-
2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)/d) - 6*(d*f*x + c*f)*Ei(-4*(d*f*x +
 c*f)/d)*sinh(-4*(d*e - c*f)/d) + 3*(d*f*x + c*f)*Ei(-6*(d*f*x + c*f)/d)*s
inh(-6*(d*e - c*f)/d) - d)*sinh(f*x + e)^3 + 9*((d*f*x + c*f)*Ei(-2*(d*f*x
 + c*f)/d)*cosh(f*x + e)*sinh(-2*(d*e - c*f)/d) - 2*(d*f*x + c*f)*Ei(-4*(d
*f*x + c*f)/d)*cosh(f*x + e)*sinh(-4*(d*e - c*f)/d) + (d*f*x + c*f)*Ei(-6*
(d*f*x + c*f)/d)*cosh(f*x + e)*sinh(-6*(d*e - c*f)/d) + ((d*f*x + c*f)*Ei(
-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) - 2*(d*f*x + c*f)*Ei(-4*(d*f*x
+ c*f)/d)*cosh(-4*(d*e - c*f)/d) + (d*f*x + c*f)*Ei(-6*(d*f*x + c*f)/d)*co
sh(-6*(d*e - c*f)/d))*cosh(f*x + e))*sinh(f*x + e)^2 + 3*(3*(d*f*x + c*f)*
Ei(-2*(d*f*x + c*f)/d)*cosh(f*x + e)^2*sinh(-2*(d*e - c*f)/d) - 6*(d*f*x +
 c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(f*x + e)^2*sinh(-4*(d*e - c*f)/d) + 3...
```

Sympy [F]

$$\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))^3}\,dx$$
$$= \frac{\int \frac{1}{c^2\coth^3(e+fx)+3c^2\coth^2(e+fx)+3c^2\coth(e+fx)+c^2+2cdx\coth^3(e+fx)+6cdx\coth^2(e+fx)+6cdx\coth(e+fx)+2cdx+d^2x^2\coth^3(e+}}{a^3}$$

input
```
integrate(1/(d*x+c)**2/(a+a*coth(f*x+e))**3,x)
```

output
```
Integral(1/(c**2*coth(e + f*x)**3 + 3*c**2*coth(e + f*x)**2 + 3*c**2*coth(
e + f*x) + c**2 + 2*c*d*x*coth(e + f*x)**3 + 6*c*d*x*coth(e + f*x)**2 + 6*
c*d*x*coth(e + f*x) + 2*c*d*x + d**2*x**2*coth(e + f*x)**3 + 3*d**2*x**2*c
oth(e + f*x)**2 + 3*d**2*x**2*coth(e + f*x) + d**2*x**2), x)/a**3
```

Maxima [A] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.20

$$\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))^3}\,dx = -\frac{1}{8\,(a^3d^2x+a^3cd)} + \frac{e^{\left(-6\,e+\frac{6\,cf}{d}\right)}E_2\left(\frac{6\,(dx+c)f}{d}\right)}{8\,(dx+c)a^3d} - \frac{3\,e^{\left(-4\,e+\frac{4\,cf}{d}\right)}E_2\left(\frac{4\,(dx+c)f}{d}\right)}{8\,(dx+c)a^3d} + \frac{3\,e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_2\left(\frac{2\,(dx+c)f}{d}\right)}{8\,(dx+c)a^3d}$$

input
```
integrate(1/(d*x+c)^2/(a+a*coth(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/8/(a^3*d^2*x + a^3*c*d) + 1/8*e^(-6*e + 6*c*f/d)*exp_integral_e(2, 6*(d
*x + c)*f/d)/((d*x + c)*a^3*d) - 3/8*e^(-4*e + 4*c*f/d)*exp_integral_e(2,
4*(d*x + c)*f/d)/((d*x + c)*a^3*d) + 3/8*e^(-2*e + 2*c*f/d)*exp_integral_e
(2, 2*(d*x + c)*f/d)/((d*x + c)*a^3*d)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 841, normalized size of antiderivative = 1.22

$$\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^2/(a+a*coth(f*x+e))^3,x, algorithm="giac")
```

output
```
1/8*(6*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-2*((d*x + c)*
(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) -
 6*d*e*f^2*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*
f)/d)*e^(-2*(d*e - c*f)/d) + 6*c*f^3*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f
/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) - 12*(d*x + c)*(d*e/(
d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-4*((d*x + c)*(d*e/(d*x + c) - c*f/(d
*x + c) + f) - d*e + c*f)/d)*e^(-4*(d*e - c*f)/d) + 12*d*e*f^2*Ei(-4*((d*x
 + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-4*(d*e - c*f
)/d) - 12*c*f^3*Ei(-4*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e
 + c*f)/d)*e^(-4*(d*e - c*f)/d) + 6*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x +
c) + f)*f^2*Ei(-6*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c
*f)/d)*e^(-6*(d*e - c*f)/d) - 6*d*e*f^2*Ei(-6*((d*x + c)*(d*e/(d*x + c) -
c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-6*(d*e - c*f)/d) + 6*c*f^3*Ei(-6*((
d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-6*(d*e -
c*f)/d) + 3*d*f^2*e^(-2*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d) -
 3*d*f^2*e^(-4*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d) + d*f^2*e^
(-6*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d) - d*f^2)*d^2/(((d*x +
 c)*a^3*d^4*(d*e/(d*x + c) - c*f/(d*x + c) + f) - a^3*d^5*e + a^3*c*d^4*f)
*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))^3}\,dx=\int \frac{1}{\left(a+a\coth\left(e+f\,x\right)\right)^3\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/((a + a*coth(e + f*x))^3*(c + d*x)^2),x)
```

output
```
int(1/((a + a*coth(e + f*x))^3*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))^3}\,dx$$
$$= \frac{-3e^{4e}\left(\int \frac{1}{e^{2fx+6e}c^2+2e^{2fx+6e}cdx+e^{2fx+6e}d^2x^2}dx\right)c^2-3e^{4e}\left(\int \frac{1}{e^{2fx+6e}c^2+2e^{2fx+6e}cdx+e^{2fx+6e}d^2x^2}dx\right)cdx+3e^{2e}\left(\int\right.}{}$$

input
```
int(1/(d*x+c)^2/(a+a*coth(f*x+e))^3,x)
```

output
```
( - 3*e**(4*e)*int(1/(e**(6*e + 2*f*x)*c**2 + 2*e**(6*e + 2*f*x)*c*d*x + e
**(6*e + 2*f*x)*d**2*x**2),x)*c**2 - 3*e**(4*e)*int(1/(e**(6*e + 2*f*x)*c*
*2 + 2*e**(6*e + 2*f*x)*c*d*x + e**(6*e + 2*f*x)*d**2*x**2),x)*c*d*x + 3*e
**(2*e)*int(1/(e**(6*e + 4*f*x)*c**2 + 2*e**(6*e + 4*f*x)*c*d*x + e**(6*e
+ 4*f*x)*d**2*x**2),x)*c**2 + 3*e**(2*e)*int(1/(e**(6*e + 4*f*x)*c**2 + 2*
e**(6*e + 4*f*x)*c*d*x + e**(6*e + 4*f*x)*d**2*x**2),x)*c*d*x - int(1/(e**
(6*e + 6*f*x)*c**2 + 2*e**(6*e + 6*f*x)*c*d*x + e**(6*e + 6*f*x)*d**2*x**2
),x)*c**2 - int(1/(e**(6*e + 6*f*x)*c**2 + 2*e**(6*e + 6*f*x)*c*d*x + e**(
6*e + 6*f*x)*d**2*x**2),x)*c*d*x + x)/(8*a**3*c*(c + d*x))
```

3.32 $\int (c+dx)^m(a+a\coth(e+fx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (c+dx)^m(a+a\coth(e+fx))^2\,dx=\operatorname{Int}\left((c+dx)^m(a+a\coth(e+fx))^2,x\right)$$

output
```
Defer(Int)((d*x+c)^m*(a+a*coth(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 27.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+a\coth(e+fx))^2\,dx=\int (c+dx)^m(a+a\coth(e+fx))^2\,dx$$

input
```
Integrate[(c + d*x)^m*(a + a*Coth[e + f*x])^2,x]
```

output
```
Integrate[(c + d*x)^m*(a + a*Coth[e + f*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a\coth(e+fx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m\left(a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4223}$$

$$\int (c+dx)^m(a\coth(e+fx)+a)^2dx$$

input
```
Int[(c + d*x)^m*(a + a*Coth[e + f*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (dx+c)^m\left(a+a\coth\left(fx+e\right)\right)^2dx$$

input
```
int((d*x+c)^m*(a+a*coth(f*x+e))^2,x)
```

output
```
int((d*x+c)^m*(a+a*coth(f*x+e))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int (c+dx)^m(a+a\coth(e+fx))^2\,dx = \int (a\coth(fx+e)+a)^2(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*coth(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((a^2*coth(f*x + e)^2 + 2*a^2*coth(f*x + e) + a^2)*(d*x + c)^m, x)
```

Sympy [N/A]

Not integrable

Time = 4.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int (c+dx)^m(a+a\coth(e+fx))^2\,dx = a^2\left(\int 2(c+dx)^m\coth(e+fx)\,dx + \int (c+dx)^m\coth^2(e+fx)\,dx + \int (c+dx)^m\,dx\right)$$

input
```
integrate((d*x+c)**m*(a+a*coth(f*x+e))**2,x)
```

output
```
a**2*(Integral(2*(c + d*x)**m*coth(e + f*x), x) + Integral((c + d*x)**m*co
th(e + f*x)**2, x) + Integral((c + d*x)**m, x))
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 122, normalized size of antiderivative = 6.10

$$\int (c+dx)^m(a+a\coth(e+fx))^2\,dx=\int (a\coth\left(fx+e\right)+a)^2(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*coth(f*x+e))^2,x, algorithm="maxima")
```

output
```
(d*x + c)^(m + 1)*a^2/(d*(m + 1)) + integrate((d*x + c)^m*a^2*(e^(f*x + e)
 + e^(-f*x - e))^2/(e^(f*x + e) - e^(-f*x - e))^2 + 2*(d*x + c)^m*a^2*(e^(
f*x + e) + e^(-f*x - e))/(e^(f*x + e) - e^(-f*x - e)), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+a\coth(e+fx))^2\,dx=\int (a\coth\left(fx+e\right)+a)^2(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*coth(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((a*coth(f*x + e) + a)^2*(d*x + c)^m, x)
```

Mupad [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+a\coth(e+fx))^2\,dx=\int {\left(a+a\,\mathrm{coth}\left(e+f\,x\right)\right)}^2\,{\left(c+d\,x\right)}^m\,dx$$

input
```
int((a + a*coth(e + f*x))^2*(c + d*x)^m,x)
```

output
```
int((a + a*coth(e + f*x))^2*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 5342, normalized size of antiderivative = 267.10

$$\int (c+dx)^m (a+a\coth(e+fx))^2\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^m*(a+a*coth(f*x+e))^2,x)
```

output
```
(4*a**2*(2*e**(2*e + 2*f*x)*(c + d*x)**m*c**2*f + 2*e**(2*e + 2*f*x)*(c +
d*x)**m*c*d*f*x - e**(2*e + 2*f*x)*(c + d*x)**m*c*d*m - e**(2*e + 2*f*x)*(
c + d*x)**m*d**2*m*x - 2*(c + d*x)**m*c**2*f - 2*(c + d*x)**m*c*d*f*x - (c
 + d*x)**m*c*d*m - 2*(c + d*x)**m*c*d + (c + d*x)**m*d**2*m*x - 4*e**(4*e
+ 2*f*x)*int((e**(2*f*x)*(c + d*x)**m*x)/(2*e**(4*e + 4*f*x)*c**2*f + 2*e*
*(4*e + 4*f*x)*c*d*f*x - e**(4*e + 4*f*x)*c*d*m - e**(4*e + 4*f*x)*d**2*m*
x - 4*e**(2*e + 2*f*x)*c**2*f - 4*e**(2*e + 2*f*x)*c*d*f*x + 2*e**(2*e + 2
*f*x)*c*d*m + 2*e**(2*e + 2*f*x)*d**2*m*x + 2*c**2*f + 2*c*d*f*x - c*d*m -
 d**2*m*x),x)*c*d**3*f*m**2 - 4*e**(4*e + 2*f*x)*int((e**(2*f*x)*(c + d*x)
**m*x)/(2*e**(4*e + 4*f*x)*c**2*f + 2*e**(4*e + 4*f*x)*c*d*f*x - e**(4*e +
 4*f*x)*c*d*m - e**(4*e + 4*f*x)*d**2*m*x - 4*e**(2*e + 2*f*x)*c**2*f - 4*
e**(2*e + 2*f*x)*c*d*f*x + 2*e**(2*e + 2*f*x)*c*d*m + 2*e**(2*e + 2*f*x)*d
**2*m*x + 2*c**2*f + 2*c*d*f*x - c*d*m - d**2*m*x),x)*c*d**3*f*m + 2*e**(4
*e + 2*f*x)*int((e**(2*f*x)*(c + d*x)**m*x)/(2*e**(4*e + 4*f*x)*c**2*f + 2
*e**(4*e + 4*f*x)*c*d*f*x - e**(4*e + 4*f*x)*c*d*m - e**(4*e + 4*f*x)*d**2
*m*x - 4*e**(2*e + 2*f*x)*c**2*f - 4*e**(2*e + 2*f*x)*c*d*f*x + 2*e**(2*e
+ 2*f*x)*c*d*m + 2*e**(2*e + 2*f*x)*d**2*m*x + 2*c**2*f + 2*c*d*f*x - c*d*
m - d**2*m*x),x)*d**4*m**3 + 2*e**(4*e + 2*f*x)*int((e**(2*f*x)*(c + d*x)*
*m*x)/(2*e**(4*e + 4*f*x)*c**2*f + 2*e**(4*e + 4*f*x)*c*d*f*x - e**(4*e +
4*f*x)*c*d*m - e**(4*e + 4*f*x)*d**2*m*x - 4*e**(2*e + 2*f*x)*c**2*f - ...
```

3.33 $\int (c+dx)^m(a+a\coth(e+fx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (c+dx)^m(a+a\coth(e+fx))\,dx=\text{Int}((c+dx)^m(a+a\coth(e+fx)),x)$$

output
```
Defer(Int)((d*x+c)^m*(a+a*coth(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 16.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(a+a\coth(e+fx))\,dx=\int (c+dx)^m(a+a\coth(e+fx))\,dx$$

input
```
Integrate[(c + d*x)^m*(a + a*Coth[e + f*x]),x]
```

output
```
Integrate[(c + d*x)^m*(a + a*Coth[e + f*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m (a\coth(e+fx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m \left(a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4223}$$

$$\int (c+dx)^m (a\coth(e+fx)+a)dx$$

input
```
Int[(c + d*x)^m*(a + a*Coth[e + f*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx+c)^m (a+a\coth(fx+e))\,dx$$

input
```
int((d*x+c)^m*(a+a*coth(f*x+e)),x)
```

output
```
int((d*x+c)^m*(a+a*coth(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(a+a\coth(e+fx))\,dx = \int (a\coth(fx+e)+a)(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*coth(f*x+e)),x, algorithm="fricas")
```

output
```
integral((a*coth(f*x + e) + a)*(d*x + c)^m, x)
```

Sympy [N/A]

Not integrable

Time = 2.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int (c+dx)^m(a+a\coth(e+fx))\,dx = a\left(\int (c+dx)^m\coth(e+fx)\,dx + \int (c+dx)^m\,dx\right)$$

input
```
integrate((d*x+c)**m*(a+a*coth(f*x+e)),x)
```

output
```
a*(Integral((c + d*x)**m*coth(e + f*x), x) + Integral((c + d*x)**m, x))
```

Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.78

$$\int (c+dx)^m(a+a\coth(e+fx))\,dx = \int (a\coth(fx+e)+a)(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*coth(f*x+e)),x, algorithm="maxima")
```

output
```
a*integrate((d*x + c)^m*(e^(f*x + e) + e^(-f*x - e))/(e^(f*x + e) - e^(-f*
x - e)), x) + (d*x + c)^(m + 1)*a/(d*(m + 1))
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(a+a\coth(e+fx))\,dx=\int (a\coth(fx+e)+a)(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*coth(f*x+e)),x, algorithm="giac")
```

output
```
integrate((a*coth(f*x + e) + a)*(d*x + c)^m, x)
```

Mupad [N/A]

Not integrable

Time = 2.69 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(a+a\coth(e+fx))\,dx=\int (a+a\coth(e+f\,x))\,\left(c+d\,x\right)^m\,dx$$

input
```
int((a + a*coth(e + f*x))*(c + d*x)^m,x)
```

output
```
int((a + a*coth(e + f*x))*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 84, normalized size of antiderivative = 4.67

$$\int (c+dx)^m(a+a\coth(e+fx))\,dx$$
$$=\frac{2a\left((dx+c)^m c+(dx+c)^m dx+\left(\int \frac{(dx+c)^m}{e^{2fx+2e}-1}dx\right)dm+\left(\int \frac{(dx+c)^m}{e^{2fx+2e}-1}dx\right)d\right)}{d\,(m+1)}$$

input
```
int((d*x+c)^m*(a+a*coth(f*x+e)),x)
```

output
```
(2*a*((c + d*x)**m*c + (c + d*x)**m*d*x + int((c + d*x)**m/(e**(2*e + 2*f*
x) - 1),x)*d*m + int((c + d*x)**m/(e**(2*e + 2*f*x) - 1),x)*d))/(d*(m + 1)
)
```

3.34 $\int \frac{(c+dx)^m}{a+a\coth(e+fx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{(c+dx)^m}{a+a\coth(e+fx)}\,dx = \frac{(c+dx)^{1+m}}{2ad(1+m)} + \frac{2^{-2-m}e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{af}$$

output
```
1/2*(d*x+c)^(1+m)/a/d/(1+m)+2^(-2-m)*exp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA(1+m
,2*f*(d*x+c)/d)/a/f/((f*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.31

$$\int \frac{(c+dx)^m}{a+a\coth(e+fx)}\,dx = \frac{\operatorname{csch}(e+fx)\left(\frac{e^e f(c+dx)^{1+m}}{d(1+m)} + 2^{-1-m}e^{-e+\frac{2cf}{d}}(c+dx)^m\left(\frac{cf}{d}+fx\right)^{-m}\Gamma\left(1+m,2\left(\frac{cf}{d}+fx\right)\right)\right)(\cosh(fx)+}{2f(a+a\coth(e+fx))}$$

input
```
Integrate[(c + d*x)^m/(a + a*Coth[e + f*x]),x]
```

output
```
(Csch[e + f*x]*((E^e*f*(c + d*x)^(1 + m))/(d*(1 + m)) + (2^(-1 - m)*E^(-e
+ (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, 2*((c*f)/d + f*x)])/((c*f)/d + f*x)^
m)*(Cosh[f*x] + Sinh[f*x]))/(2*f*(a + a*Coth[e + f*x]))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4210, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{a\coth(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^m}{a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4210}$$

$$\frac{(c+dx)^{m+1}}{2ad(m+1)}+\frac{\int e^{i(2ie+2ifx+\pi)}(c+dx)^m dx}{2a}$$

$$\Big\downarrow \text{2612}$$

$$\frac{2^{-m-2}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2f(c+dx)}{d}\right)}{af}+\frac{(c+dx)^{m+1}}{2ad(m+1)}$$

input
```
Int[(c + d*x)^m/(a + a*Coth[e + f*x]),x]
```

output
```
(c + d*x)^(1 + m)/(2*a*d*(1 + m)) + (2^(-2 - m)*E^(-2*e + (2*c*f)/d)*(c +
d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(a*f*((f*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4210
```
Int[((c_.) + (d_.)*(x_))^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sym
bol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + Simp[1/(2*a)   Int[(c
+ d*x)^m*E^(2*(a/b)*(e + f*x)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[a^2 + b^2, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(dx+c)^m}{a+a\coth(fx+e)}dx$$

input
```
int((d*x+c)^m/(a+a*coth(f*x+e)),x)
```

output
```
int((d*x+c)^m/(a+a*coth(f*x+e)),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.68

$$\int \frac{(c+dx)^m}{a+a\coth(e+fx)}\,dx = \frac{(dm+d)\cosh\left(\frac{dm\log\left(\frac{2f}{d}\right)+2de-2cf}{d}\right)\Gamma\left(m+1,\frac{2(dfx+cf)}{d}\right)-(dm+d)\Gamma\left(m+1,\frac{2(dfx+cf)}{d}\right)\sinh\left(\frac{dm\log\left(\frac{2}{d}\right)}{}\right)}{4(adfm+adf)}$$

input
```
integrate((d*x+c)^m/(a+a*coth(f*x+e)),x, algorithm="fricas")
```

output
```
1/4*((d*m + d)*cosh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d)*gamma(m + 1, 2*(d*
f*x + c*f)/d) - (d*m + d)*gamma(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*
f/d) + 2*d*e - 2*c*f)/d) + 2*(d*f*x + c*f)*cosh(m*log(d*x + c)) + 2*(d*f*x
 + c*f)*sinh(m*log(d*x + c)))/(a*d*f*m + a*d*f)
```

Sympy [F]

$$\int \frac{(c+dx)^m}{a+a\coth(e+fx)}\,dx = \frac{\int \frac{(c+dx)^m}{\coth(e+fx)+1}\,dx}{a}$$

input
```
integrate((d*x+c)**m/(a+a*coth(f*x+e)),x)
```

output
```
Integral((c + d*x)**m/(coth(e + f*x) + 1), x)/a
```

Maxima [F]

$$\int \frac{(c+dx)^m}{a+a\coth(e+fx)}\,dx = \int \frac{(dx+c)^m}{a\coth(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*coth(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(a*coth(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(c+dx)^m}{a+a\coth(e+fx)}\,dx=\int \frac{(dx+c)^m}{a\coth\left(fx+e\right)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*coth(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(a*coth(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^m}{a+a\coth(e+fx)}\,dx=\int \frac{(c+d\,x)^m}{a+a\coth\left(e+f\,x\right)}\,dx$$

input
```
int((c + d*x)^m/(a + a*coth(e + f*x)),x)
```

output
```
int((c + d*x)^m/(a + a*coth(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(c+dx)^m}{a+a\coth(e+fx)}\,dx$$
$$=\frac{2e^{2fx+2e}(dx+c)^m cf+2e^{2fx+2e}(dx+c)^m dfx+(dx+c)^m dm+(dx+c)^m d-e^{2fx+2e}\left(\int \frac{(dx+c)^m}{e^{2fx+2e}c+e^{2fx+2}}\right.}{4e^{2fx+2e}adf\left(m+1\right)}$$

input
```
int((d*x+c)^m/(a+a*coth(f*x+e)),x)
```

output

```
(2*e**(2*e + 2*f*x)*(c + d*x)**m*c*f + 2*e**(2*e + 2*f*x)*(c + d*x)**m*d*f
*x + (c + d*x)**m*d*m + (c + d*x)**m*d - e**(2*e + 2*f*x)*int((c + d*x)**m
/(e**(2*e + 2*f*x)*c + e**(2*e + 2*f*x)*d*x),x)*d**2*m**2 - e**(2*e + 2*f*
x)*int((c + d*x)**m/(e**(2*e + 2*f*x)*c + e**(2*e + 2*f*x)*d*x),x)*d**2*m)
/(4*e**(2*e + 2*f*x)*a*d*f*(m + 1))
```

3.35 $\int \frac{(c+dx)^m}{(a+a\coth(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 152

$$\int \frac{(c+dx)^m}{(a+a\coth(e+fx))^2}\,dx = \frac{(c+dx)^{1+m}}{4a^2d(1+m)} + \frac{2^{-2-m}e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{a^2f} - \frac{4^{-2-m}e^{-4e+\frac{4cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{4f(c+dx)}{d}\right)}{a^2f}$$

output
```
1/4*(d*x+c)^(1+m)/a^2/d/(1+m)+2^(-2-m)*exp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA(1
+m,2*f*(d*x+c)/d)/a^2/f/((f*(d*x+c)/d)^m)-4^(-2-m)*exp(-4*e+4*c*f/d)*(d*x+
c)^m*GAMMA(1+m,4*f*(d*x+c)/d)/a^2/f/((f*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx)^m}{(a+a\coth(e+fx))^2}\,dx$$
$$=\frac{(c+dx)^m\mathrm{csch}^2(e+fx)\left(\frac{4e^{2e}f(c+dx)}{d(1+m)}+2^{2-m}e^{\frac{2cf}{d}}\left(f\left(\frac{c}{d}+x\right)\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)-4^{-m}e^{-2e+\frac{4cf}{d}}\left(\frac{f(c+d\cdots}{d}\right.\right.}{16a^2f(1+\coth(e+fx))^2}$$

input
```
Integrate[(c + d*x)^m/(a + a*Coth[e + f*x])^2,x]
```

output
```
((c + d*x)^m*Csch[e + f*x]^2*((4*E^(2*e)*f*(c + d*x))/(d*(1 + m)) + (2^(2
- m)*E^((2*c*f)/d)*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*(c/d + x))^m - (E^(
-2*e + (4*c*f)/d)*Gamma[1 + m, (4*f*(c + d*x))/d])/(4^m*((f*(c + d*x))/d)^
m))*(Cosh[f*x] + Sinh[f*x])^2)/(16*a^2*f*(1 + Coth[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 4212, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{(a\coth(e+fx)+a)^2}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{(c+dx)^m}{\left(a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \ 4212$$
$$\int \left(\frac{e^{-4e-4fx}(c+dx)^m}{4a^2}-\frac{e^{-2e-2fx}(c+dx)^m}{2a^2}+\frac{(c+dx)^m}{4a^2}\right)dx$$
$$\Big\downarrow \ 2009$$

$$\frac{2^{-m-2}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2f(c+dx)}{d}\right)}{a^2f}-\frac{4^{-m-2}e^{\frac{4cf}{d}-4e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{4f(c+dx)}{d}\right)}{a^2f}+\frac{(c+dx)^{m+1}}{4a^2d(m+1)}$$

input
```
Int[(c + d*x)^m/(a + a*Coth[e + f*x])^2,x]
```

output
```
(c + d*x)^(1 + m)/(4*a^2*d*(1 + m)) + (2^(-2 - m)*E^(-2*e + (2*c*f)/d)*(c
+ d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(a^2*f*((f*(c + d*x))/d)^m) - (4
^(-2 - m)*E^(-4*e + (4*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (4*f*(c + d*x))/d]
)/(a^2*f*((f*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4212
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Maple [F]

$$\int \frac{(dx+c)^m}{(a+a\coth(fx+e))^2}dx$$

input
```
int((d*x+c)^m/(a+a*coth(f*x+e))^2,x)
```

output
```
int((d*x+c)^m/(a+a*coth(f*x+e))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.63

$$\int \frac{(c+dx)^m}{(a+a\coth(e+fx))^2}\,dx =$$
$$-\frac{(dm+d)\cosh\left(\frac{dm\log\left(\frac{4f}{d}\right)+4\,de-4\,cf}{d}\right)\Gamma\left(m+1,\frac{4\,(dfx+cf)}{d}\right)-4\,(dm+d)\cosh\left(\frac{dm\log\left(\frac{2f}{d}\right)+2\,de-2\,cf}{d}\right)\Gamma\left(\right.}{}$$

input
```
integrate((d*x+c)^m/(a+a*coth(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/16*((d*m + d)*cosh((d*m*log(4*f/d) + 4*d*e - 4*c*f)/d)*gamma(m + 1, 4*(
d*f*x + c*f)/d) - 4*(d*m + d)*cosh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d)*gam
ma(m + 1, 2*(d*f*x + c*f)/d) - (d*m + d)*gamma(m + 1, 4*(d*f*x + c*f)/d)*s
inh((d*m*log(4*f/d) + 4*d*e - 4*c*f)/d) + 4*(d*m + d)*gamma(m + 1, 2*(d*f*
x + c*f)/d)*sinh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d) - 4*(d*f*x + c*f)*cos
h(m*log(d*x + c)) - 4*(d*f*x + c*f)*sinh(m*log(d*x + c)))/(a^2*d*f*m + a^2
*d*f)
```

Sympy [F]

$$\int \frac{(c+dx)^m}{(a+a\coth(e+fx))^2}\,dx = \frac{\int \frac{(c+dx)^m}{\coth^2(e+fx)+2\coth(e+fx)+1}\,dx}{a^2}$$

input
```
integrate((d*x+c)**m/(a+a*coth(f*x+e))**2,x)
```

output
```
Integral((c + d*x)**m/(coth(e + f*x)**2 + 2*coth(e + f*x) + 1), x)/a**2
```

Maxima [F]

$$\int \frac{(c+dx)^m}{(a+a\coth(e+fx))^2}\,dx=\int \frac{(dx+c)^m}{(a\coth(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*coth(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(a*coth(f*x + e) + a)^2, x)
```

Giac [F]

$$\int \frac{(c+dx)^m}{(a+a\coth(e+fx))^2}\,dx=\int \frac{(dx+c)^m}{(a\coth(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*coth(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(a*coth(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^m}{(a+a\coth(e+fx))^2}\,dx=\int \frac{(c+d\,x)^m}{(a+a\coth(e+f\,x))^2}\,dx$$

input
```
int((c + d*x)^m/(a + a*coth(e + f*x))^2,x)
```

output
```
int((c + d*x)^m/(a + a*coth(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(c+dx)^m}{(a+a\coth(e+fx))^2}\,dx$$
$$= \frac{4e^{4fx+4e}(dx+c)^m cf + 4e^{4fx+4e}(dx+c)^m dfx + 4e^{2fx+2e}(dx+c)^m dm + 4e^{2fx+2e}(dx+c)^m d - (dx+c}{}$$

input
```
int((d*x+c)^m/(a+a*coth(f*x+e))^2,x)
```

output
```
(4*e**(4*e + 4*f*x)*(c + d*x)**m*c*f + 4*e**(4*e + 4*f*x)*(c + d*x)**m*d*f
*x + 4*e**(2*e + 2*f*x)*(c + d*x)**m*d*m + 4*e**(2*e + 2*f*x)*(c + d*x)**m
*d - (c + d*x)**m*d*m - (c + d*x)**m*d - 4*e**(6*e + 4*f*x)*int((c + d*x)*
*m/(e**(4*e + 2*f*x)*c + e**(4*e + 2*f*x)*d*x),x)*d**2*m**2 - 4*e**(6*e +
4*f*x)*int((c + d*x)**m/(e**(4*e + 2*f*x)*c + e**(4*e + 2*f*x)*d*x),x)*d**
2*m + e**(4*e + 4*f*x)*int((c + d*x)**m/(e**(4*e + 4*f*x)*c + e**(4*e + 4*
f*x)*d*x),x)*d**2*m**2 + e**(4*e + 4*f*x)*int((c + d*x)**m/(e**(4*e + 4*f*
x)*c + e**(4*e + 4*f*x)*d*x),x)*d**2*m)/(16*e**(4*e + 4*f*x)*a**2*d*f*(m +
 1))
```

3.36 $\int \frac{(c+dx)^m}{(a+a\coth(e+fx))^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 223

$$\int \frac{(c+dx)^m}{(a+a\coth(e+fx))^3}\,dx = \frac{(c+dx)^{1+m}}{8a^3d(1+m)} + \frac{3\,2^{-4-m}e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{a^3f} - \frac{3\,2^{-5-2m}e^{-4e+\frac{4cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{4f(c+dx)}{d}\right)}{a^3f} + \frac{2^{-4-m}3^{-1-m}e^{-6e+\frac{6cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{6f(c+dx)}{d}\right)}{a^3f}$$

output
```
1/8*(d*x+c)^(1+m)/a^3/d/(1+m)+3*2^(-4-m)*exp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA
(1+m,2*f*(d*x+c)/d)/a^3/f/((f*(d*x+c)/d)^m)-3*2^(-5-2*m)*exp(-4*e+4*c*f/d)
*(d*x+c)^m*GAMMA(1+m,4*f*(d*x+c)/d)/a^3/f/((f*(d*x+c)/d)^m)+2^(-4-m)*3^(-1
-m)*exp(-6*e+6*c*f/d)*(d*x+c)^m*GAMMA(1+m,6*f*(d*x+c)/d)/a^3/f/((f*(d*x+c)
/d)^m)
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx)^m}{(a+a\coth(e+fx))^3}\,dx$$
$$= \frac{2^{-5-2m}3^{-1-m}e^{-3e}\left(f\left(\frac{c}{d}+x\right)\right)^{-m}(c+dx)^m\operatorname{csch}^3(e+fx)\left(12^{1+m}e^{6e}f\left(f\left(\frac{c}{d}+x\right)\right)^m(c+dx)+2^{1+m}3^{2+m}c\right.}{}$$

input
```
Integrate[(c + d*x)^m/(a + a*Coth[e + f*x])^3,x]
```

output
```
(2^(-5 - 2*m)*3^(-1 - m)*(c + d*x)^m*Csch[e + f*x]^3*(12^(1 + m)*E^(6*e)*f
*(f*(c/d + x))^m*(c + d*x) + 2^(1 + m)*3^(2 + m)*d*E^(4*e + (2*c*f)/d)*(1
+ m)*Gamma[1 + m, (2*f*(c + d*x))/d] - 3^(2 + m)*d*E^(2*e + (4*c*f)/d)*(1
+ m)*Gamma[1 + m, (4*f*(c + d*x))/d] + 2^(1 + m)*d*E^((6*c*f)/d)*(1 + m)*G
amma[1 + m, (6*f*(c + d*x))/d])*(Cosh[f*x] + Sinh[f*x])^3)/(a^3*d*E^(3*e)*
f*(1 + m)*(f*(c/d + x))^m*(1 + Coth[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4212, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{(a\coth(e+fx)+a)^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c+dx)^m}{\left(a-ia\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \text{4212}$$
$$\int \left(-\frac{e^{-6e-6fx}(c+dx)^m}{8a^3}+\frac{3e^{-4e-4fx}(c+dx)^m}{8a^3}-\frac{3e^{-2e-2fx}(c+dx)^m}{8a^3}+\frac{(c+dx)^m}{8a^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\,2^{-m-4}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2f(c+dx)}{d}\right)}{a^3f}-\frac{3\,2^{-2m-5}e^{\frac{4cf}{d}-4e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{4f(c+dx)}{d}\right)}{a^3f}+\frac{2^{-m-4}3^{-m-1}e^{\frac{6cf}{d}-6e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{6f(c+dx)}{d}\right)}{a^3f}+\frac{(c+dx)^{m+1}}{8a^3d(m+1)}$$

input
```
Int[(c + d*x)^m/(a + a*Coth[e + f*x])^3,x]
```

output
```
(c + d*x)^(1 + m)/(8*a^3*d*(1 + m)) + (3*2^(-4 - m)*E^(-2*e + (2*c*f)/d)*(
c + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(a^3*f*((f*(c + d*x))/d)^m) -
(3*2^(-5 - 2*m)*E^(-4*e + (4*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (4*f*(c + d*
x))/d])/(a^3*f*((f*(c + d*x))/d)^m) + (2^(-4 - m)*3^(-1 - m)*E^(-6*e + (6*
c*f)/d)*(c + d*x)^m*Gamma[1 + m, (6*f*(c + d*x))/d])/(a^3*f*((f*(c + d*x))
/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4212
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Maple [F]

$$\int \frac{(dx+c)^m}{(a+a\coth(fx+e))^3}dx$$

input
```
int((d*x+c)^m/(a+a*coth(f*x+e))^3,x)
```

output
```
int((d*x+c)^m/(a+a*coth(f*x+e))^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.55

$$\int \frac{(c+dx)^m}{(a+a\coth(e+fx))^3}\,dx = \frac{2\,(dm+d)\cosh\left(\frac{dm\log\left(\frac{6\,f}{d}\right)+6\,de-6\,cf}{d}\right)\Gamma\left(m+1,\frac{6\,(dfx+cf)}{d}\right)-9\,(dm+d)\cosh\left(\frac{dm\log\left(\frac{4\,f}{d}\right)+4\,de-4\,cf}{d}\right)\Gamma\left(m\right.}{}$$

input
```
integrate((d*x+c)^m/(a+a*coth(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/96*(2*(d*m + d)*cosh((d*m*log(6*f/d) + 6*d*e - 6*c*f)/d)*gamma(m + 1, 6*
(d*f*x + c*f)/d) - 9*(d*m + d)*cosh((d*m*log(4*f/d) + 4*d*e - 4*c*f)/d)*ga
mma(m + 1, 4*(d*f*x + c*f)/d) + 18*(d*m + d)*cosh((d*m*log(2*f/d) + 2*d*e
- 2*c*f)/d)*gamma(m + 1, 2*(d*f*x + c*f)/d) - 2*(d*m + d)*gamma(m + 1, 6*(
d*f*x + c*f)/d)*sinh((d*m*log(6*f/d) + 6*d*e - 6*c*f)/d) + 9*(d*m + d)*gam
ma(m + 1, 4*(d*f*x + c*f)/d)*sinh((d*m*log(4*f/d) + 4*d*e - 4*c*f)/d) - 18
*(d*m + d)*gamma(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) + 2*d*e -
2*c*f)/d) + 12*(d*f*x + c*f)*cosh(m*log(d*x + c)) + 12*(d*f*x + c*f)*sinh(
m*log(d*x + c)))/(a^3*d*f*m + a^3*d*f)
```

Sympy [F]

$$\int \frac{(c+dx)^m}{(a+a\coth(e+fx))^3}\,dx = \frac{\int \frac{(c+dx)^m}{\coth^3(e+fx)+3\coth^2(e+fx)+3\coth(e+fx)+1}\,dx}{a^3}$$

input
```
integrate((d*x+c)**m/(a+a*coth(f*x+e))**3,x)
```

output
```
Integral((c + d*x)**m/(coth(e + f*x)**3 + 3*coth(e + f*x)**2 + 3*coth(e +
f*x) + 1), x)/a**3
```

Maxima [F]

$$\int \frac{(c+dx)^m}{(a+a\coth(e+fx))^3}\,dx = \int \frac{(dx+c)^m}{(a\coth(fx+e)+a)^3}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*coth(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(a*coth(f*x + e) + a)^3, x)
```

Giac [F]

$$\int \frac{(c+dx)^m}{(a+a\coth(e+fx))^3}\,dx = \int \frac{(dx+c)^m}{(a\coth(fx+e)+a)^3}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*coth(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(a*coth(f*x + e) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^m}{(a+a\coth(e+fx))^3}\,dx = \int \frac{(c+d\,x)^m}{(a+a\coth\left(e+f\,x\right))^3}\,dx$$

input
```
int((c + d*x)^m/(a + a*coth(e + f*x))^3,x)
```

output
```
int((c + d*x)^m/(a + a*coth(e + f*x))^3, x)
```

Reduce [F]

$$\int \frac{(c+dx)^m}{(a+a\coth(e+fx))^3}\,dx = \frac{12e^{6fx+6e}(dx+c)^m\,cf+12e^{6fx+6e}(dx+c)^m\,dfx+18e^{4fx+4e}(dx+c)^m\,dm+18e^{4fx+4e}(dx+c)^m\,d-9e^{2}}{}$$

input
```
int((d*x+c)^m/(a+a*coth(f*x+e))^3,x)
```

output
```
(12*e**(6*e + 6*f*x)*(c + d*x)**m*c*f + 12*e**(6*e + 6*f*x)*(c + d*x)**m*d
*f*x + 18*e**(4*e + 4*f*x)*(c + d*x)**m*d*m + 18*e**(4*e + 4*f*x)*(c + d*x
)**m*d - 9*e**(2*e + 2*f*x)*(c + d*x)**m*d*m - 9*e**(2*e + 2*f*x)*(c + d*x
)**m*d + 2*(c + d*x)**m*d*m + 2*(c + d*x)**m*d - 18*e**(10*e + 6*f*x)*int(
(c + d*x)**m/(e**(6*e + 2*f*x)*c + e**(6*e + 2*f*x)*d*x),x)*d**2*m**2 - 18
*e**(10*e + 6*f*x)*int((c + d*x)**m/(e**(6*e + 2*f*x)*c + e**(6*e + 2*f*x)
*d*x),x)*d**2*m + 9*e**(8*e + 6*f*x)*int((c + d*x)**m/(e**(6*e + 4*f*x)*c
+ e**(6*e + 4*f*x)*d*x),x)*d**2*m**2 + 9*e**(8*e + 6*f*x)*int((c + d*x)**m
/(e**(6*e + 4*f*x)*c + e**(6*e + 4*f*x)*d*x),x)*d**2*m - 2*e**(6*e + 6*f*x
)*int((c + d*x)**m/(e**(6*e + 6*f*x)*c + e**(6*e + 6*f*x)*d*x),x)*d**2*m**
2 - 2*e**(6*e + 6*f*x)*int((c + d*x)**m/(e**(6*e + 6*f*x)*c + e**(6*e + 6*
f*x)*d*x),x)*d**2*m)/(96*e**(6*e + 6*f*x)*a**3*d*f*(m + 1))
```

3.37 $\int (c+dx)^3(a+b\coth(e+fx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 133

$$\int (c+dx)^3(a+b\coth(e+fx))\,dx = \frac{a(c+dx)^4}{4d}-\frac{b(c+dx)^4}{4d}+\frac{b(c+dx)^3\log\left(1-e^{2(e+fx)}\right)}{f}+\frac{3bd(c+dx)^2\operatorname{PolyLog}\left(2,e^{2(e+fx)}\right)}{2f^2}-\frac{3bd^2(c+dx)\operatorname{PolyLog}\left(3,e^{2(e+fx)}\right)}{2f^3}+\frac{3bd^3\operatorname{PolyLog}\left(4,e^{2(e+fx)}\right)}{4f^4}$$

output
```
1/4*a*(d*x+c)^4/d-1/4*b*(d*x+c)^4/d+b*(d*x+c)^3*ln(1-exp(2*f*x+2*e))/f+3/2
*b*d*(d*x+c)^2*polylog(2,exp(2*f*x+2*e))/f^2-3/2*b*d^2*(d*x+c)*polylog(3,e
xp(2*f*x+2*e))/f^3+3/4*b*d^3*polylog(4,exp(2*f*x+2*e))/f^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 311 vs. $2(133) = 266$.

Time = 0.14 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.34

$$\int (c+dx)^3(a+b\coth(e+fx))\,dx = ac^3x+\frac{3}{2}ac^2dx^2-\frac{3}{2}bc^2dx^2+acd^2x^3-bcd^2x^3+\frac{1}{4}ad^3x^4-\frac{1}{4}bd^3x^4+\frac{3bc^2dx\log\left(1-e^{2e+2fx}\right)}{f}+\frac{3bcd^2x^2\log\left(1-e^{2e+2fx}\right)}{f}+\frac{bd^3x^3\log\left(1-e^{2e+2fx}\right)}{f}+\frac{bc^3\log(\sinh(e+fx))}{f}+\frac{3bc^2d\operatorname{PolyLog}\left(2,e^{2e+2fx}\right)}{2f^2}+\frac{3bcd^2x\operatorname{PolyLog}\left(2,e^{2e+2fx}\right)}{f^2}+\frac{3bd^3x^2\operatorname{PolyLog}\left(2,e^{2e+2fx}\right)}{2f^2}-\frac{3bcd^2\operatorname{PolyLog}\left(3,e^{2e+2fx}\right)}{2f^3}-\frac{3bd^3x\operatorname{PolyLog}\left(3,e^{2e+2fx}\right)}{2f^3}+\frac{3bd^3\operatorname{PolyLog}\left(4,e^{2e+2fx}\right)}{4f^4}$$

input
```
Integrate[(c + d*x)^3*(a + b*Coth[e + f*x]),x]
```

output
```
a*c^3*x + (3*a*c^2*d*x^2)/2 - (3*b*c^2*d*x^2)/2 + a*c*d^2*x^3 - b*c*d^2*x^
3 + (a*d^3*x^4)/4 - (b*d^3*x^4)/4 + (3*b*c^2*d*x*Log[1 - E^(2*e + 2*f*x)])
/f + (3*b*c*d^2*x^2*Log[1 - E^(2*e + 2*f*x)])/f + (b*d^3*x^3*Log[1 - E^(2*
e + 2*f*x)])/f + (b*c^3*Log[Sinh[e + f*x]])/f + (3*b*c^2*d*PolyLog[2, E^(2
*e + 2*f*x)])/(2*f^2) + (3*b*c*d^2*x*PolyLog[2, E^(2*e + 2*f*x)])/f^2 + (3
*b*d^3*x^2*PolyLog[2, E^(2*e + 2*f*x)])/(2*f^2) - (3*b*c*d^2*PolyLog[3, E^
(2*e + 2*f*x)])/(2*f^3) - (3*b*d^3*x*PolyLog[3, E^(2*e + 2*f*x)])/(2*f^3)
+ (3*b*d^3*PolyLog[4, E^(2*e + 2*f*x)])/(4*f^4)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a+b\coth(e+fx))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (c+dx)^3\left(a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \ 4205$$

$$\int \left(a(c+dx)^3+b(c+dx)^3\coth(e+fx)\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a(c+dx)^4}{4d}-\frac{3bd^2(c+dx)\operatorname{PolyLog}\left(3,e^{2(e+fx)}\right)}{2f^3}+\frac{3bd(c+dx)^2\operatorname{PolyLog}\left(2,e^{2(e+fx)}\right)}{2f^2}+\frac{b(c+dx)^3\log\left(1-e^{2(e+fx)}\right)}{f}-\frac{b(c+dx)^4}{4d}+\frac{3bd^3\operatorname{PolyLog}\left(4,e^{2(e+fx)}\right)}{4f^4}$$

input
```
Int[(c + d*x)^3*(a + b*Coth[e + f*x]),x]
```

output
```
(a*(c + d*x)^4)/(4*d) - (b*(c + d*x)^4)/(4*d) + (b*(c + d*x)^3*Log[1 - E^(
2*(e + f*x))])/f + (3*b*d*(c + d*x)^2*PolyLog[2, E^(2*(e + f*x))])/(2*f^2)
 - (3*b*d^2*(c + d*x)*PolyLog[3, E^(2*(e + f*x))])/(2*f^3) + (3*b*d^3*Poly
Log[4, E^(2*(e + f*x))])/(4*f^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(123) = 246$.

Time $= 0.17$ (sec) , antiderivative size $= 766$, normalized size of antiderivative $= 5.76$

method	result
risch	$\frac{d^3 a x^4}{4} - \frac{d^3 b x^4}{4} + \frac{a c^4}{4d} + \frac{b c^4}{4d} - \frac{6bd c^2 ex}{f} + \frac{6b d^2 c e^2 x}{f^2} - \frac{3b c^2 de \ln\left(e^{fx+e}-1\right)}{f^2} + \frac{6b c^2 de \ln\left(e^{fx+e}\right)}{f^2} + \frac{3bd c^2 \ln\left(1-e^{fx}\right.}{f}$

input
```
int((d*x+c)^3*(a+b*coth(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/4*d^3*a*x^4-1/4*d^3*b*x^4+1/4/d*a*c^4+1/4/d*b*c^4-6/f*b*d*c^2*e*x+6/f^2*
b*d^2*c*e^2*x-3/f^2*b*c^2*d*e*ln(exp(f*x+e)-1)+6/f^2*b*c^2*d*e*ln(exp(f*x+
e))+3/f*b*d*c^2*ln(1-exp(f*x+e))*x+3/f^2*b*d*c^2*ln(1-exp(f*x+e))*e+3/f*b*
d*c^2*ln(exp(f*x+e)+1)*x+6/f^2*b*d^2*c*polylog(2,exp(f*x+e))*x+3/f*b*d^2*c
*ln(exp(f*x+e)+1)*x^2+6/f^2*b*d^2*c*polylog(2,-exp(f*x+e))*x+3/f*b*d^2*c*l
n(1-exp(f*x+e))*x^2-3/f^3*b*d^2*c*ln(1-exp(f*x+e))*e^2+3/f^3*b*c*d^2*e^2*l
n(exp(f*x+e)-1)-6/f^3*b*c*d^2*e^2*ln(exp(f*x+e))-2/f^3*b*d^3*e^3*x-3/f^2*b
*d*c^2*e^2+4/f^3*b*d^2*c*e^3+1/f^4*b*d^3*ln(1-exp(f*x+e))*e^3+3/f^2*b*d^3*
polylog(2,-exp(f*x+e))*x^2-6/f^3*b*d^3*polylog(3,-exp(f*x+e))*x+3/f^2*b*d*
c^2*polylog(2,exp(f*x+e))+3/f^2*b*d*c^2*polylog(2,-exp(f*x+e))-6/f^3*b*d^2
*c*polylog(3,exp(f*x+e))-6/f^3*b*d^2*c*polylog(3,-exp(f*x+e))+1/f*b*d^3*ln
(1-exp(f*x+e))*x^3+3/f^2*b*d^3*polylog(2,exp(f*x+e))*x^2-6/f^3*b*d^3*polyl
og(3,exp(f*x+e))*x+1/f*b*d^3*ln(exp(f*x+e)+1)*x^3-1/f^4*b*d^3*e^3*ln(exp(f
*x+e)-1)+2/f^4*b*d^3*e^3*ln(exp(f*x+e))+d^2*a*c*x^3-d^2*b*c*x^3+3/2*d*a*c^
2*x^2-3/2*d*b*c^2*x^2+a*c^3*x+b*c^3*x-3/2/f^4*b*d^3*e^4+1/f*b*c^3*ln(exp(f
*x+e)-1)-2/f*b*c^3*ln(exp(f*x+e))+1/f*b*c^3*ln(exp(f*x+e)+1)+6/f^4*b*d^3*p
olylog(4,exp(f*x+e))+6/f^4*b*d^3*polylog(4,-exp(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(122) = 244$.

Time = 0.13 (sec) , antiderivative size = 488, normalized size of antiderivative = 3.67

$$\int (c+dx)^3(a+b\coth(e+fx))\,dx$$
$$= \frac{(a-b)d^3f^4x^4 + 4\,(a-b)cd^2f^4x^3 + 6\,(a-b)c^2df^4x^2 + 4\,(a-b)c^3f^4x + 24\,bd^3\mathrm{polylog}(4, \cosh\,(fx+e)\,}{}$$

input
```
integrate((d*x+c)^3*(a+b*coth(f*x+e)),x, algorithm="fricas")
```

output
```
1/4*((a - b)*d^3*f^4*x^4 + 4*(a - b)*c*d^2*f^4*x^3 + 6*(a - b)*c^2*d*f^4*x
^2 + 4*(a - b)*c^3*f^4*x + 24*b*d^3*polylog(4, cosh(f*x + e) + sinh(f*x +
e)) + 24*b*d^3*polylog(4, -cosh(f*x + e) - sinh(f*x + e)) + 12*(b*d^3*f^2*
x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*dilog(cosh(f*x + e) + sinh(f*x + e))
+ 12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*dilog(-cosh(f*x + e)
- sinh(f*x + e)) + 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x
+ b*c^3*f^3)*log(cosh(f*x + e) + sinh(f*x + e) + 1) - 4*(b*d^3*e^3 - 3*b*c
*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(cosh(f*x + e) + sinh(f*x + e
) - 1) + 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^
3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*log(-cosh(f*x + e) - sinh(f*x + e)
+ 1) - 24*(b*d^3*f*x + b*c*d^2*f)*polylog(3, cosh(f*x + e) + sinh(f*x + e)
) - 24*(b*d^3*f*x + b*c*d^2*f)*polylog(3, -cosh(f*x + e) - sinh(f*x + e)))
/f^4
```

Sympy [F]

$$\int (c+dx)^3(a+b\coth(e+fx))\,dx=\int (a+b\coth{(e+fx)})\,(c+dx)^3\,dx$$

input
```
integrate((d*x+c)**3*(a+b*coth(f*x+e)),x)
```

output
```
Integral((a + b*coth(e + f*x))*(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(122) = 244$.

Time = 0.17 (sec) , antiderivative size = 419, normalized size of antiderivative = 3.15

$$\int (c+dx)^3(a+b\coth(e+fx))\,dx$$
$$= \frac{1}{4}ad^3x^4 + \frac{1}{4}bd^3x^4 + acd^2x^3 + bcd^2x^3 + \frac{3}{2}ac^2dx^2 + \frac{3}{2}bc^2dx^2 + ac^3x + \frac{bc^3\log(\sinh(fx+e))}{f} + \frac{3\left(fx\log\left(e^{(fx+e)}+1\right) + \mathrm{Li}_2\left(-e^{(fx+e)}\right)\right)bc^2d}{f^2} + \frac{3\left(fx\log\left(-e^{(fx+e)}+1\right) + \mathrm{Li}_2\left(e^{(fx+e)}\right)\right)bc^2d}{f^2} + \frac{3\left(f^2x^2\log\left(e^{(fx+e)}+1\right) + 2\,fx\mathrm{Li}_2\left(-e^{(fx+e)}\right) - 2\,\mathrm{Li}_3\left(-e^{(fx+e)}\right)\right)bcd^2}{f^3} + \frac{3\left(f^2x^2\log\left(-e^{(fx+e)}+1\right) + 2\,fx\mathrm{Li}_2\left(e^{(fx+e)}\right) - 2\,\mathrm{Li}_3\left(e^{(fx+e)}\right)\right)bcd^2}{f^3} + \frac{\left(f^3x^3\log\left(e^{(fx+e)}+1\right) + 3\,f^2x^2\mathrm{Li}_2\left(-e^{(fx+e)}\right) - 6\,fx\mathrm{Li}_3\left(-e^{(fx+e)}\right) + 6\,\mathrm{Li}_4\left(-e^{(fx+e)}\right)\right)bd^3}{f^4} + \frac{\left(f^3x^3\log\left(-e^{(fx+e)}+1\right) + 3\,f^2x^2\mathrm{Li}_2\left(e^{(fx+e)}\right) - 6\,fx\mathrm{Li}_3\left(e^{(fx+e)}\right) + 6\,\mathrm{Li}_4\left(e^{(fx+e)}\right)\right)bd^3}{f^4} - \frac{bd^3f^4x^4 + 4\,bcd^2f^4x^3 + 6\,bc^2df^4x^2}{2\,f^4}$$

input
```
integrate((d*x+c)^3*(a+b*coth(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*a*d^3*x^4 + 1/4*b*d^3*x^4 + a*c*d^2*x^3 + b*c*d^2*x^3 + 3/2*a*c^2*d*x^
2 + 3/2*b*c^2*d*x^2 + a*c^3*x + b*c^3*log(sinh(f*x + e))/f + 3*(f*x*log(e^
(f*x + e) + 1) + dilog(-e^(f*x + e)))*b*c^2*d/f^2 + 3*(f*x*log(-e^(f*x + e
) + 1) + dilog(e^(f*x + e)))*b*c^2*d/f^2 + 3*(f^2*x^2*log(e^(f*x + e) + 1)
 + 2*f*x*dilog(-e^(f*x + e)) - 2*polylog(3, -e^(f*x + e)))*b*c*d^2/f^3 + 3
*(f^2*x^2*log(-e^(f*x + e) + 1) + 2*f*x*dilog(e^(f*x + e)) - 2*polylog(3,
e^(f*x + e)))*b*c*d^2/f^3 + (f^3*x^3*log(e^(f*x + e) + 1) + 3*f^2*x^2*dilo
g(-e^(f*x + e)) - 6*f*x*polylog(3, -e^(f*x + e)) + 6*polylog(4, -e^(f*x +
e)))*b*d^3/f^4 + (f^3*x^3*log(-e^(f*x + e) + 1) + 3*f^2*x^2*dilog(e^(f*x +
 e)) - 6*f*x*polylog(3, e^(f*x + e)) + 6*polylog(4, e^(f*x + e)))*b*d^3/f^
4 - 1/2*(b*d^3*f^4*x^4 + 4*b*c*d^2*f^4*x^3 + 6*b*c^2*d*f^4*x^2)/f^4
```

Giac [F]

$$\int (c+dx)^3(a+b\coth(e+fx))\,dx=\int (dx+c)^3(b\coth(fx+e)+a)\,dx$$

input
```
integrate((d*x+c)^3*(a+b*coth(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*(b*coth(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3(a+b\coth(e+fx))\,dx=\int (a+b\coth(e+f\,x))\,(c+d\,x)^3\,dx$$

input
```
int((a + b*coth(e + f*x))*(c + d*x)^3,x)
```

output
```
int((a + b*coth(e + f*x))*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3(a+b\coth(e+fx))\,dx$$
$$=\frac{8e^{2e}\left(\int\frac{e^{2fx}x^3}{e^{2fx+2e}-1}dx\right)b\,d^3f+24e^{2e}\left(\int\frac{e^{2fx}x^2}{e^{2fx+2e}-1}dx\right)bc\,d^2f+24e^{2e}\left(\int\frac{e^{2fx}x}{e^{2fx+2e}-1}dx\right)b\,c^2df+4\log\left(e^{fx+e}-\right.}{}$$

input
```
int((d*x+c)^3*(a+b*coth(f*x+e)),x)
```

output
```
(8*e**(2*e)*int((e**(2*f*x)*x**3)/(e**(2*e + 2*f*x) - 1),x)*b*d**3*f + 24*
e**(2*e)*int((e**(2*f*x)*x**2)/(e**(2*e + 2*f*x) - 1),x)*b*c*d**2*f + 24*e
**(2*e)*int((e**(2*f*x)*x)/(e**(2*e + 2*f*x) - 1),x)*b*c**2*d*f + 4*log(e*
*(e + f*x) - 1)*b*c**3 + 4*log(e**(e + f*x) + 1)*b*c**3 + 4*a*c**3*f*x + 6
*a*c**2*d*f*x**2 + 4*a*c*d**2*f*x**3 + a*d**3*f*x**4 - 4*b*c**3*f*x - 6*b*
c**2*d*f*x**2 - 4*b*c*d**2*f*x**3 - b*d**3*f*x**4)/(4*f)
```

3.38 $\int (c+dx)^2(a+b\coth(e+fx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 101

$$\int (c+dx)^2(a+b\coth(e+fx))\,dx = \frac{a(c+dx)^3}{3d} - \frac{b(c+dx)^3}{3d} + \frac{b(c+dx)^2\log\left(1-e^{2(e+fx)}\right)}{f} + \frac{bd(c+dx)\operatorname{PolyLog}\left(2,e^{2(e+fx)}\right)}{f^2} - \frac{bd^2\operatorname{PolyLog}\left(3,e^{2(e+fx)}\right)}{2f^3}$$

output
```
1/3*a*(d*x+c)^3/d-1/3*b*(d*x+c)^3/d+b*(d*x+c)^2*ln(1-exp(2*f*x+2*e))/f+b*d
*(d*x+c)*polylog(2,exp(2*f*x+2*e))/f^2-1/2*b*d^2*polylog(3,exp(2*f*x+2*e))
/f^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.70

$$\int (c+dx)^2(a+b\coth(e+fx))\,dx = ac^2x+acdx^2-bcdx^2+\frac{1}{3}ad^2x^3 -\frac{1}{3}bd^2x^3+\frac{2bcdx\log\left(1-e^{2e+2fx}\right)}{f} +\frac{bd^2x^2\log\left(1-e^{2e+2fx}\right)}{f}+\frac{bc^2\log(\sinh(e+fx))}{f} +\frac{bcd\,\text{PolyLog}\left(2,e^{2e+2fx}\right)}{f^2} +\frac{bd^2x\,\text{PolyLog}\left(2,e^{2e+2fx}\right)}{f^2} -\frac{bd^2\,\text{PolyLog}\left(3,e^{2e+2fx}\right)}{2f^3}$$

input
```
Integrate[(c + d*x)^2*(a + b*Coth[e + f*x]),x]
```

output
```
a*c^2*x + a*c*d*x^2 - b*c*d*x^2 + (a*d^2*x^3)/3 - (b*d^2*x^3)/3 + (2*b*c*d
*x*Log[1 - E^(2*e + 2*f*x)])/f + (b*d^2*x^2*Log[1 - E^(2*e + 2*f*x)])/f +
(b*c^2*Log[Sinh[e + f*x]])/f + (b*c*d*PolyLog[2, E^(2*e + 2*f*x)])/f^2 + (
b*d^2*x*PolyLog[2, E^(2*e + 2*f*x)])/f^2 - (b*d^2*PolyLog[3, E^(2*e + 2*f*
x)])/(2*f^3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a+b\coth(e+fx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^2 \left(a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4205}$$

$$\int \left(a(c+dx)^2+b(c+dx)^2\coth(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(c+dx)^3}{3d}+\frac{bd(c+dx)\,\text{PolyLog}\left(2,e^{2(e+fx)}\right)}{f^2}+\frac{b(c+dx)^2\log\left(1-e^{2(e+fx)}\right)}{f}-\frac{b(c+dx)^3}{3d}-\frac{bd^2\,\text{PolyLog}\left(3,e^{2(e+fx)}\right)}{2f^3}$$

input
```
Int[(c + d*x)^2*(a + b*Coth[e + f*x]),x]
```

output
```
(a*(c + d*x)^3)/(3*d) - (b*(c + d*x)^3)/(3*d) + (b*(c + d*x)^2*Log[1 - E^(
2*(e + f*x))])/f + (b*d*(c + d*x)*PolyLog[2, E^(2*(e + f*x))])/f^2 - (b*d^
2*PolyLog[3, E^(2*(e + f*x))])/(2*f^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(95) = 190$.

Time = 0.12 (sec) , antiderivative size = 465, normalized size of antiderivative = 4.60

method	result
risch	$\frac{b\,d^2 e^2 \ln(\mathrm{e}^{fx+e}-1)}{f^3} - \frac{2b\,d^2 e^2 \ln(\mathrm{e}^{fx+e})}{f^3} + \frac{b\,d^2 \ln(1-\mathrm{e}^{fx+e})x^2}{f} - \frac{2bdc\,e^2}{f^2} - \frac{b\,d^2 \ln(1-\mathrm{e}^{fx+e})e^2}{f^3} + \frac{2b\,d^2 \operatorname{polylog}(2,\mathrm{e}^{fx+e})}{f^2}$

input
```
int((d*x+c)^2*(a+b*coth(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/f^3*b*d^2*e^2*ln(exp(f*x+e)-1)-2/f^3*b*d^2*e^2*ln(exp(f*x+e))+1/f*b*d^2*
ln(1-exp(f*x+e))*x^2-2/f^2*b*d*c*e^2-1/f^3*b*d^2*ln(1-exp(f*x+e))*e^2+2/f^
2*b*d^2*polylog(2,exp(f*x+e))*x+1/f*b*d^2*ln(exp(f*x+e)+1)*x^2+2/f^2*b*d^2
*polylog(2,-exp(f*x+e))*x+2/f^2*b*d*c*polylog(2,exp(f*x+e))+2/f^2*b*d*c*po
lylog(2,-exp(f*x+e))+2/f^2*b*d^2*e^2*x+1/3*d^2*a*x^3-1/3*d^2*b*x^3+1/3/d*a
*c^3+1/3/d*b*c^3-4/f*b*d*c*e*x-2/f^2*b*c*d*e*ln(exp(f*x+e)-1)+4/f^2*b*c*d*
e*ln(exp(f*x+e))+2/f*b*d*c*ln(1-exp(f*x+e))*x+2/f^2*b*d*c*ln(1-exp(f*x+e))
*e+2/f*b*d*c*ln(exp(f*x+e)+1)*x+4/3/f^3*b*d^2*e^3-2/f^3*b*d^2*polylog(3,ex
p(f*x+e))-2/f^3*b*d^2*polylog(3,-exp(f*x+e))+1/f*b*c^2*ln(exp(f*x+e)-1)-2/
f*b*c^2*ln(exp(f*x+e))+1/f*b*c^2*ln(exp(f*x+e)+1)+d*a*c*x^2-d*b*c*x^2+a*c^
2*x+b*c^2*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(94) = 188$.

Time = 0.10 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.00

$$\int (c+dx)^2(a+b\coth(e+fx))\,dx$$
$$= \frac{(a-b)d^2f^3x^3 + 3\,(a-b)cdf^3x^2 + 3\,(a-b)c^2f^3x - 6\,bd^2\mathrm{polylog}(3, \cosh{(fx+e)} + \sinh{(fx+e)}) - 6\,b}{}$$

input
```
integrate((d*x+c)^2*(a+b*coth(f*x+e)),x, algorithm="fricas")
```

output
```
1/3*((a - b)*d^2*f^3*x^3 + 3*(a - b)*c*d*f^3*x^2 + 3*(a - b)*c^2*f^3*x - 6
*b*d^2*polylog(3, cosh(f*x + e) + sinh(f*x + e)) - 6*b*d^2*polylog(3, -cos
h(f*x + e) - sinh(f*x + e)) + 6*(b*d^2*f*x + b*c*d*f)*dilog(cosh(f*x + e)
+ sinh(f*x + e)) + 6*(b*d^2*f*x + b*c*d*f)*dilog(-cosh(f*x + e) - sinh(f*x
 + e)) + 3*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*log(cosh(f*x + e) +
 sinh(f*x + e) + 1) + 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(cosh(f*x
 + e) + sinh(f*x + e) - 1) + 3*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2
+ 2*b*c*d*e*f)*log(-cosh(f*x + e) - sinh(f*x + e) + 1))/f^3
```

Sympy [F]

$$\int (c+dx)^2(a+b\coth(e+fx))\,dx=\int(a+b\coth(e+fx))\,(c+dx)^2\,dx$$

input
```
integrate((d*x+c)**2*(a+b*coth(f*x+e)),x)
```

output
```
Integral((a + b*coth(e + f*x))*(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(94) = 188$.

Time = 0.27 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.38

$$\int (c+dx)^2(a+b\coth(e+fx))\,dx$$
$$=\frac{1}{3}ad^2x^3+\frac{1}{3}bd^2x^3+acdx^2+bcdx^2+ac^2x+\frac{bc^2\log(\sinh(fx+e))}{f}$$
$$+\frac{2\left(fx\log\left(e^{(fx+e)}+1\right)+\mathrm{Li}_2\left(-e^{(fx+e)}\right)\right)bcd}{f^2}$$
$$+\frac{2\left(fx\log\left(-e^{(fx+e)}+1\right)+\mathrm{Li}_2\left(e^{(fx+e)}\right)\right)bcd}{f^2}$$
$$+\frac{\left(f^2x^2\log\left(e^{(fx+e)}+1\right)+2\,fx\mathrm{Li}_2\left(-e^{(fx+e)}\right)-2\,\mathrm{Li}_3(-e^{(fx+e)})\right)bd^2}{f^3}$$
$$+\frac{\left(f^2x^2\log\left(-e^{(fx+e)}+1\right)+2\,fx\mathrm{Li}_2\left(e^{(fx+e)}\right)-2\,\mathrm{Li}_3(e^{(fx+e)})\right)bd^2}{f^3}$$
$$-\frac{2\left(bd^2f^3x^3+3\,bcdf^3x^2\right)}{3\,f^3}$$

input
```
integrate((d*x+c)^2*(a+b*coth(f*x+e)),x, algorithm="maxima")
```

output
```
1/3*a*d^2*x^3 + 1/3*b*d^2*x^3 + a*c*d*x^2 + b*c*d*x^2 + a*c^2*x + b*c^2*lo
g(sinh(f*x + e))/f + 2*(f*x*log(e^(f*x + e) + 1) + dilog(-e^(f*x + e)))*b*
c*d/f^2 + 2*(f*x*log(-e^(f*x + e) + 1) + dilog(e^(f*x + e)))*b*c*d/f^2 + (
f^2*x^2*log(e^(f*x + e) + 1) + 2*f*x*dilog(-e^(f*x + e)) - 2*polylog(3, -e
^(f*x + e)))*b*d^2/f^3 + (f^2*x^2*log(-e^(f*x + e) + 1) + 2*f*x*dilog(e^(f
*x + e)) - 2*polylog(3, e^(f*x + e)))*b*d^2/f^3 - 2/3*(b*d^2*f^3*x^3 + 3*b
*c*d*f^3*x^2)/f^3
```

Giac [F]

$$\int (c+dx)^2(a+b\coth(e+fx))\,dx=\int (dx+c)^2(b\coth(fx+e)+a)\,dx$$

input
```
integrate((d*x+c)^2*(a+b*coth(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*(b*coth(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2(a+b\coth(e+fx))\,dx=\int (a+b\coth(e+f\,x))\,\left(c+d\,x\right)^2\,dx$$

input
```
int((a + b*coth(e + f*x))*(c + d*x)^2,x)
```

output
```
int((a + b*coth(e + f*x))*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2(a+b\coth(e+fx))\,dx$$
$$=\frac{6e^{2e}\left(\int \frac{e^{2fx}x^2}{e^{2fx+2e}-1}dx\right)b\,d^2f+12e^{2e}\left(\int \frac{e^{2fx}x}{e^{2fx+2e}-1}dx\right)bcdf+3\log\left(e^{fx+e}-1\right)b\,c^2+3\log\left(e^{fx+e}+1\right)b\,c^2+3}{3f}$$

input
```
int((d*x+c)^2*(a+b*coth(f*x+e)),x)
```

output
```
(6*e**(2*e)*int((e**(2*f*x)*x**2)/(e**(2*e + 2*f*x) - 1),x)*b*d**2*f + 12*
e**(2*e)*int((e**(2*f*x)*x)/(e**(2*e + 2*f*x) - 1),x)*b*c*d*f + 3*log(e**(
e + f*x) - 1)*b*c**2 + 3*log(e**(e + f*x) + 1)*b*c**2 + 3*a*c**2*f*x + 3*a
*c*d*f*x**2 + a*d**2*f*x**3 - 3*b*c**2*f*x - 3*b*c*d*f*x**2 - b*d**2*f*x**
3)/(3*f)
```

3.39 $\int (c+dx)(a+b\coth(e+fx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int (c+dx)(a+b\coth(e+fx))\,dx=\frac{a(c+dx)^2}{2d}-\frac{b(c+dx)^2}{2d}+\frac{b(c+dx)\log\left(1-e^{2(e+fx)}\right)}{f}+\frac{bd\,\text{PolyLog}\left(2,e^{2(e+fx)}\right)}{2f^2}$$

output
```
1/2*a*(d*x+c)^2/d-1/2*b*(d*x+c)^2/d+b*(d*x+c)*ln(1-exp(2*f*x+2*e))/f+1/2*b
*d*polylog(2,exp(2*f*x+2*e))/f^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int (c+dx)(a+b\coth(e+fx))\,dx=acx+\frac{1}{2}adx^2-\frac{1}{2}bdx^2+\frac{bdx\log\left(1-e^{2e+2fx}\right)}{f}+\frac{bc\log(\sinh(e+fx))}{f}+\frac{bd\,\text{PolyLog}\left(2,e^{2e+2fx}\right)}{2f^2}$$

input
```
Integrate[(c + d*x)*(a + b*Coth[e + f*x]),x]
```

output
```
a*c*x + (a*d*x^2)/2 - (b*d*x^2)/2 + (b*d*x*Log[1 - E^(2*e + 2*f*x)])/f + (
b*c*Log[Sinh[e + f*x]])/f + (b*d*PolyLog[2, E^(2*e + 2*f*x)])/(2*f^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a+b\coth(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\left(a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4205}$$

$$\int (a(c+dx)+b(c+dx)\coth(e+fx))dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\log\left(1-e^{2(e+fx)}\right)}{f}-\frac{b(c+dx)^2}{2d}+\frac{bd\,\text{PolyLog}\left(2,e^{2(e+fx)}\right)}{2f^2}$$

input
```
Int[(c + d*x)*(a + b*Coth[e + f*x]),x]
```

output
```
(a*(c + d*x)^2)/(2*d) - (b*(c + d*x)^2)/(2*d) + (b*(c + d*x)*Log[1 - E^(2*
(e + f*x))])/f + (b*d*PolyLog[2, E^(2*(e + f*x))])/(2*f^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(69) = 138$.

Time = 0.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.68

method	result
risch	$\frac{ad\,x^2}{2}+axc-\frac{bd\,x^2}{2}+bxc+\frac{bc\ln(\mathrm{e}^{fx+e}-1)}{f}-\frac{2bc\ln(\mathrm{e}^{fx+e})}{f}+\frac{bc\ln(\mathrm{e}^{fx+e}+1)}{f}-\frac{2bdex}{f}-\frac{bd\,e^2}{f^2}+\frac{bd\ln(1-\mathrm{e}^{fx+}}{f}$

input
```
int((d*x+c)*(a+b*coth(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*d*x^2+a*x*c-1/2*b*d*x^2+b*x*c+1/f*b*c*ln(exp(f*x+e)-1)-2/f*b*c*ln(ex
p(f*x+e))+1/f*b*c*ln(exp(f*x+e)+1)-2/f*b*d*e*x-1/f^2*b*d*e^2+1/f*b*d*ln(1-
exp(f*x+e))*x+1/f^2*b*d*ln(1-exp(f*x+e))*e+1/f^2*b*d*polylog(2,exp(f*x+e))
+1/f*b*d*ln(exp(f*x+e)+1)*x+1/f^2*b*d*polylog(2,-exp(f*x+e))-1/f^2*b*d*e*l
n(exp(f*x+e)-1)+2/f^2*b*d*e*ln(exp(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(68) = 136$.

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.08

$$\int (c+dx)(a+b\coth(e+fx))\,dx = \frac{(a-b)df^2x^2+2(a-b)cf^2x+2bd\text{Li}_2(\cosh(fx+e)+\sinh(fx+e))+2bd\text{Li}_2(-\cosh(fx+e)-\sinh$$

input
```
integrate((d*x+c)*(a+b*coth(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*((a - b)*d*f^2*x^2 + 2*(a - b)*c*f^2*x + 2*b*d*dilog(cosh(f*x + e) + s
inh(f*x + e)) + 2*b*d*dilog(-cosh(f*x + e) - sinh(f*x + e)) + 2*(b*d*f*x +
 b*c*f)*log(cosh(f*x + e) + sinh(f*x + e) + 1) - 2*(b*d*e - b*c*f)*log(cos
h(f*x + e) + sinh(f*x + e) - 1) + 2*(b*d*f*x + b*d*e)*log(-cosh(f*x + e) -
 sinh(f*x + e) + 1))/f^2
```

Sympy [F]

$$\int (c+dx)(a+b\coth(e+fx))\,dx = \int (a+b\coth(e+fx))\,(c+dx)\,dx$$

input
```
integrate((d*x+c)*(a+b*coth(f*x+e)),x)
```

output
```
Integral((a + b*coth(e + f*x))*(c + d*x), x)
```

Maxima [F]

$$\int (c+dx)(a+b\coth(e+fx))\,dx=\int (dx+c)(b\coth(fx+e)+a)\,dx$$

input
```
integrate((d*x+c)*(a+b*coth(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*a*d*x^2 + 1/2*(x^2 - 2*integrate(x/(e^(f*x + e) + 1), x) + 2*integrate
(x/(e^(f*x + e) - 1), x))*b*d + a*c*x + b*c*log(sinh(f*x + e))/f
```

Giac [F]

$$\int (c+dx)(a+b\coth(e+fx))\,dx=\int (dx+c)(b\coth(fx+e)+a)\,dx$$

input
```
integrate((d*x+c)*(a+b*coth(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)*(b*coth(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)(a+b\coth(e+fx))\,dx=\int (a+b\coth(e+f\,x))\,(c+d\,x)\,dx$$

input
```
int((a + b*coth(e + f*x))*(c + d*x),x)
```

output
```
int((a + b*coth(e + f*x))*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)(a+b\coth(e+fx))\,dx$$
$$= \frac{4e^{2e}\left(\int \frac{e^{2fx}x}{e^{2fx+2e}-1}dx\right)bdf + 2\log\left(e^{fx+e}-1\right)bc + 2\log\left(e^{fx+e}+1\right)bc + 2acfx + adf\,x^2 - 2bcfx - bdf\,x^2}{2f}$$

input
```
int((d*x+c)*(a+b*coth(f*x+e)),x)
```

output
```
(4*e**(2*e)*int((e**(2*f*x)*x)/(e**(2*e + 2*f*x) - 1),x)*b*d*f + 2*log(e**
(e + f*x) - 1)*b*c + 2*log(e**(e + f*x) + 1)*b*c + 2*a*c*f*x + a*d*f*x**2
- 2*b*c*f*x - b*d*f*x**2)/(2*f)
```

3.40 $\int \frac{a+b\coth(e+fx)}{c+dx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a+b\coth(e+fx)}{c+dx}\,dx=\text{Int}\left(\frac{a+b\coth(e+fx)}{c+dx},x\right)$$

output
```
Defer(Int)((a+b*coth(f*x+e))/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 3.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\coth(e+fx)}{c+dx}\,dx=\int \frac{a+b\coth(e+fx)}{c+dx}\,dx$$

input
```
Integrate[(a + b*Coth[e + f*x])/(c + d*x),x]
```

output
```
Integrate[(a + b*Coth[e + f*x])/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\coth(e+fx)}{c+dx}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)}{c+dx}dx$$

$$\Big\downarrow \text{4223}$$

$$\int \frac{a+b\coth(e+fx)}{c+dx}dx$$

input
```
Int[(a + b*Coth[e + f*x])/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\coth\left(fx+e\right)}{dx+c}dx$$

input
```
int((a+b*coth(f*x+e))/(d*x+c),x)
```

output
```
int((a+b*coth(f*x+e))/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\coth(e+fx)}{c+dx}\,dx = \int \frac{b\coth(fx+e)+a}{dx+c}\,dx$$

input
```
integrate((a+b*coth(f*x+e))/(d*x+c),x, algorithm="fricas")
```

output
```
integral((b*coth(f*x + e) + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{a+b\coth(e+fx)}{c+dx}\,dx = \int \frac{a+b\coth(e+fx)}{c+dx}\,dx$$

input
```
integrate((a+b*coth(f*x+e))/(d*x+c),x)
```

output
```
Integral((a + b*coth(e + f*x))/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 4.33

$$\int \frac{a+b\coth(e+fx)}{c+dx}\,dx = \int \frac{b\coth(fx+e)+a}{dx+c}\,dx$$

input
```
integrate((a+b*coth(f*x+e))/(d*x+c),x, algorithm="maxima")
```

output
```
b*(log(d*x + c)/d - integrate(1/(d*x + (d*x*e^e + c*e^e)*e^(f*x) + c), x)
+ integrate(-1/(d*x - (d*x*e^e + c*e^e)*e^(f*x) + c), x)) + a*log(d*x + c)
/d
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b\coth(e + fx)}{c + dx}\,dx = \int \frac{b\coth\left(fx + e\right) + a}{dx + c}\,dx$$

input
```
integrate((a+b*coth(f*x+e))/(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*coth(f*x + e) + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b\coth(e + fx)}{c + dx}\,dx = \int \frac{a + b\coth(e + f\,x)}{c + d\,x}\,dx$$

input
```
int((a + b*coth(e + f*x))/(c + d*x),x)
```

output
```
int((a + b*coth(e + f*x))/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 62, normalized size of antiderivative = 3.44

$$\int \frac{a+b\coth(e+fx)}{c+dx}\,dx$$
$$=\frac{2\left(\int \frac{1}{e^{2fx+2e}c+e^{2fx+2e}dx-c-dx}dx\right)bd+\log(dx+c)\,a+\log(dx+c)\,b}{d}$$

input
```
int((a+b*coth(f*x+e))/(d*x+c),x)
```

output
```
(2*int(1/(e**(2*e + 2*f*x)*c + e**(2*e + 2*f*x)*d*x - c - d*x),x)*b*d + lo
g(c + d*x)*a + log(c + d*x)*b)/d
```

3.41 $\int \frac{a+b\coth(e+fx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a+b\coth(e+fx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{a+b\coth(e+fx)}{(c+dx)^2},x\right)$$

output
```
Defer(Int)((a+b*coth(f*x+e))/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 24.74 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\coth(e+fx)}{(c+dx)^2}\,dx = \int \frac{a+b\coth(e+fx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(a + b*Coth[e + f*x])/(c + d*x)^2,x]
```

output
```
Integrate[(a + b*Coth[e + f*x])/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\coth(e+fx)}{(c+dx)^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)}{(c+dx)^2}dx$$
$$\Big\downarrow \text{4223}$$
$$\int \frac{a+b\coth(e+fx)}{(c+dx)^2}dx$$

input
```
Int[(a + b*Coth[e + f*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\coth\left(fx+e\right)}{\left(dx+c\right)^{2}}dx$$

input
```
int((a+b*coth(f*x+e))/(d*x+c)^2,x)
```

output
```
int((a+b*coth(f*x+e))/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{a+b\coth(e+fx)}{(c+dx)^2}\,dx=\int \frac{b\coth\left(fx+e\right)+a}{\left(dx+c\right)^2}\,dx$$

input
```
integrate((a+b*coth(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((b*coth(f*x + e) + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.98 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{a+b\coth(e+fx)}{(c+dx)^2}\,dx=\int \frac{a+b\coth\left(e+fx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate((a+b*coth(f*x+e))/(d*x+c)**2,x)
```

output
```
Integral((a + b*coth(e + f*x))/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 7.28

$$\int \frac{a+b\coth(e+fx)}{(c+dx)^2}\,dx=\int \frac{b\coth\left(fx+e\right)+a}{\left(dx+c\right)^2}\,dx$$

input
```
integrate((a+b*coth(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-b*(1/(d^2*x + c*d) + integrate(1/(d^2*x^2 + 2*c*d*x + c^2 + (d^2*x^2*e^e
+ 2*c*d*x*e^e + c^2*e^e)*e^(f*x)), x) - integrate(-1/(d^2*x^2 + 2*c*d*x +
c^2 - (d^2*x^2*e^e + 2*c*d*x*e^e + c^2*e^e)*e^(f*x)), x)) - a/(d^2*x + c*d
)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\coth(e+fx)}{(c+dx)^2}\,dx = \int \frac{b\coth\left(fx+e\right)+a}{\left(dx+c\right)^2}\,dx$$

input
```
integrate((a+b*coth(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*coth(f*x + e) + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\coth(e+fx)}{(c+dx)^2}\,dx = \int \frac{a+b\coth(e+f\,x)}{\left(c+d\,x\right)^2}\,dx$$

input
```
int((a + b*coth(e + f*x))/(c + d*x)^2,x)
```

output
```
int((a + b*coth(e + f*x))/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 168, normalized size of antiderivative = 9.33

$$\int \frac{a+b\coth(e+fx)}{(c+dx)^2}\,dx = \frac{2\left(\int \frac{1}{e^{2fx+2e}c^2+2e^{2fx+2e}cdx+e^{2fx+2e}d^2x^2-c^2-2cdx-d^2x^2}dx\right) b\,c^2 + 2\left(\int \frac{1}{e^{2fx+2e}c^2+2e^{2fx+2e}cdx+e^{2fx+2e}d^2x^2-c^2-2cdx-d^2x^2}\right.}{c\,(dx+c)}$$

input
```
int((a+b*coth(f*x+e))/(d*x+c)^2,x)
```

output
```
(2*int(1/(e**(2*e + 2*f*x)*c**2 + 2*e**(2*e + 2*f*x)*c*d*x + e**(2*e + 2*f
*x)*d**2*x**2 - c**2 - 2*c*d*x - d**2*x**2),x)*b*c**2 + 2*int(1/(e**(2*e +
 2*f*x)*c**2 + 2*e**(2*e + 2*f*x)*c*d*x + e**(2*e + 2*f*x)*d**2*x**2 - c**
2 - 2*c*d*x - d**2*x**2),x)*b*c*d*x + a*x + b*x)/(c*(c + d*x))
```

3.42 $\int (c+dx)^3(a+b\coth(e+fx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 271

$$\begin{aligned}\int (c+dx)^3(a+b\coth(e+fx))^2\,dx =& -\frac{b^2(c+dx)^3}{f}+\frac{a^2(c+dx)^4}{4d}-\frac{ab(c+dx)^4}{2d} \\ &+\frac{b^2(c+dx)^4}{4d}-\frac{b^2(c+dx)^3\coth(e+fx)}{f} \\ &+\frac{3b^2d(c+dx)^2\log\left(1-e^{2(e+fx)}\right)}{f^2} \\ &+\frac{2ab(c+dx)^3\log\left(1-e^{2(e+fx)}\right)}{f} \\ &+\frac{3b^2d^2(c+dx)\operatorname{PolyLog}\left(2,e^{2(e+fx)}\right)}{f^3} \\ &+\frac{3abd(c+dx)^2\operatorname{PolyLog}\left(2,e^{2(e+fx)}\right)}{f^2} \\ &-\frac{3b^2d^3\operatorname{PolyLog}\left(3,e^{2(e+fx)}\right)}{2f^4} \\ &-\frac{3abd^2(c+dx)\operatorname{PolyLog}\left(3,e^{2(e+fx)}\right)}{f^3} \\ &+\frac{3abd^3\operatorname{PolyLog}\left(4,e^{2(e+fx)}\right)}{2f^4}\end{aligned}$$

output
```
-b^2*(d*x+c)^3/f+1/4*a^2*(d*x+c)^4/d-1/2*a*b*(d*x+c)^4/d+1/4*b^2*(d*x+c)^4
/d-b^2*(d*x+c)^3*coth(f*x+e)/f+3*b^2*d*(d*x+c)^2*ln(1-exp(2*f*x+2*e))/f^2+
2*a*b*(d*x+c)^3*ln(1-exp(2*f*x+2*e))/f+3*b^2*d^2*(d*x+c)*polylog(2,exp(2*f
*x+2*e))/f^3+3*a*b*d*(d*x+c)^2*polylog(2,exp(2*f*x+2*e))/f^2-3/2*b^2*d^3*p
olylog(3,exp(2*f*x+2*e))/f^4-3*a*b*d^2*(d*x+c)*polylog(3,exp(2*f*x+2*e))/f
^3+3/2*a*b*d^3*polylog(4,exp(2*f*x+2*e))/f^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 843 vs. $2(271) = 542$.

Time = 3.22 (sec) , antiderivative size = 843, normalized size of antiderivative = 3.11

$$\int (c+dx)^3(a+b\coth(e+fx))^2\,dx = \text{Too large to display}$$

input
```
Integrate[(c + d*x)^3*(a + b*Coth[e + f*x])^2,x]
```

output
```
(-2*b*c^2*(3*b*d + 2*a*c*f)*x)/f - (2*b^2*(c + d*x)^3)/((-1 + E^(2*e))*f)
- (a*b*(c + d*x)^4)/(d*(-1 + E^(2*e))) + (6*b*c*d*(b*d + a*c*f)*x*Log[1 -
E^(-e - f*x)])/f^2 + (3*b*d^2*(b*d + 2*a*c*f)*x^2*Log[1 - E^(-e - f*x)])/f
^2 + (2*a*b*d^3*x^3*Log[1 - E^(-e - f*x)])/f + (6*b*c*d*(b*d + a*c*f)*x*Lo
g[1 + E^(-e - f*x)])/f^2 + (3*b*d^2*(b*d + 2*a*c*f)*x^2*Log[1 + E^(-e - f*
x)])/f^2 + (2*a*b*d^3*x^3*Log[1 + E^(-e - f*x)])/f + (b*c^2*(3*b*d + 2*a*c
*f)*Log[1 - E^(e + f*x)])/f^2 + (b*c^2*(3*b*d + 2*a*c*f)*Log[1 + E^(e + f*
x)])/f^2 - (6*b*c*d*(b*d + a*c*f)*PolyLog[2, -E^(-e - f*x)])/f^3 - (6*b*d^
2*(b*d + 2*a*c*f)*x*PolyLog[2, -E^(-e - f*x)])/f^3 - (6*a*b*d^3*x^2*PolyLo
g[2, -E^(-e - f*x)])/f^2 - (6*b*c*d*(b*d + a*c*f)*PolyLog[2, E^(-e - f*x)]
)/f^3 - (6*b*d^2*(b*d + 2*a*c*f)*x*PolyLog[2, E^(-e - f*x)])/f^3 - (6*a*b*
d^3*x^2*PolyLog[2, E^(-e - f*x)])/f^2 - (6*b*d^2*(b*d + 2*a*c*f)*PolyLog[3
, -E^(-e - f*x)])/f^4 - (12*a*b*d^3*x*PolyLog[3, -E^(-e - f*x)])/f^3 - (6*
b*d^2*(b*d + 2*a*c*f)*PolyLog[3, E^(-e - f*x)])/f^4 - (12*a*b*d^3*x*PolyLo
g[3, E^(-e - f*x)])/f^3 - (12*a*b*d^3*PolyLog[4, -E^(-e - f*x)])/f^4 - (12
*a*b*d^3*PolyLog[4, E^(-e - f*x)])/f^4 + (Csch[e]*Csch[e + f*x]*(-((a^2 +
b^2)*f*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Cosh[f*x]) + (a^2 + b
^2)*f*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Cosh[2*e + f*x] + 2*b*
((4*b*(c + d*x)^3 + a*f*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))*Sin
h[f*x] + a*f*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Sinh[2*e + f...
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a+b\coth(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3\left(a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4205}$$

$$\int \left(a^2(c+dx)^3+2ab(c+dx)^3\coth(e+fx)+b^2(c+dx)^3\coth^2(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(c+dx)^4}{4d}-\frac{3abd^2(c+dx)\text{PolyLog}\left(3,e^{2(e+fx)}\right)}{f^3}+\frac{3abd(c+dx)^2\text{PolyLog}\left(2,e^{2(e+fx)}\right)}{f^2}+\frac{2ab(c+dx)^3\log\left(1-e^{2(e+fx)}\right)}{f}-\frac{ab(c+dx)^4}{2d}+\frac{3abd^3\text{PolyLog}\left(4,e^{2(e+fx)}\right)}{2f^4}+\frac{3b^2d^2(c+dx)\text{PolyLog}\left(2,e^{2(e+fx)}\right)}{f^3}+\frac{3b^2d(c+dx)^2\log\left(1-e^{2(e+fx)}\right)}{f^2}-\frac{b^2(c+dx)^3\coth(e+fx)}{f}-\frac{b^2(c+dx)^3}{f}+\frac{b^2(c+dx)^4}{4d}-\frac{3b^2d^3\text{PolyLog}\left(3,e^{2(e+fx)}\right)}{2f^4}$$

input
```
Int[(c + d*x)^3*(a + b*Coth[e + f*x])^2,x]
```

output
```
-((b^2*(c + d*x)^3)/f) + (a^2*(c + d*x)^4)/(4*d) - (a*b*(c + d*x)^4)/(2*d)
 + (b^2*(c + d*x)^4)/(4*d) - (b^2*(c + d*x)^3*Coth[e + f*x])/f + (3*b^2*d*
(c + d*x)^2*Log[1 - E^(2*(e + f*x))])/f^2 + (2*a*b*(c + d*x)^3*Log[1 - E^(
2*(e + f*x))])/f + (3*b^2*d^2*(c + d*x)*PolyLog[2, E^(2*(e + f*x))])/f^3 +
 (3*a*b*d*(c + d*x)^2*PolyLog[2, E^(2*(e + f*x))])/f^2 - (3*b^2*d^3*PolyLo
g[3, E^(2*(e + f*x))])/(2*f^4) - (3*a*b*d^2*(c + d*x)*PolyLog[3, E^(2*(e +
 f*x))])/f^3 + (3*a*b*d^3*PolyLog[4, E^(2*(e + f*x))])/(2*f^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1424 vs. $2(261) = 522$.

Time = 0.31 (sec) , antiderivative size = 1425, normalized size of antiderivative = 5.26

method	result	size
risch	Expression too large to display	1425

input
```
int((d*x+c)^3*(a+b*coth(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*d^3*a*b*x^4+d^2*a^2*c*x^3+d^2*b^2*c*x^3+3/2*d*a^2*c^2*x^2+3/2*d*b^2*c
^2*x^2+a^2*c^3*x+b^2*c^3*x+1/2/d*a*b*c^4-2/f*b^2*d^3*x^3+4/f^4*b^2*d^3*e^3
-6/f^4*b^2*d^3*polylog(3,exp(f*x+e))-6/f^4*b^2*d^3*polylog(3,-exp(f*x+e))+
1/4*d^3*a^2*x^4+1/4*d^3*b^2*x^4+1/4/d*a^2*c^4+1/4/d*b^2*c^4+8/f^3*b*a*c*d^
2*e^3-6/f^2*b*a*c^2*d*e^2-12/f^2*b^2*c*d^2*e*x-4/f^3*b*a*d^3*e^3*x-12/f^3*
b*a*c*d^2*polylog(3,exp(f*x+e))-2*d^2*a*b*c*x^3-3*d*a*b*c^2*x^2+2*a*b*c^3*
x+6/f^3*b^2*d^3*e^2*x-3/f^4*b*a*d^3*e^4-6/f*b^2*c*d^2*x^2-6/f^3*b^2*c*d^2*
e^2+3/f^2*b^2*d^3*ln(1-exp(f*x+e))*x^2-3/f^4*b^2*d^3*ln(1-exp(f*x+e))*e^2+
6/f^3*b^2*d^3*polylog(2,exp(f*x+e))*x+3/f^2*b^2*d^3*ln(exp(f*x+e)+1)*x^2+6
/f^3*b^2*d^3*polylog(2,-exp(f*x+e))*x+3/f^2*b^2*c^2*d*ln(exp(f*x+e)-1)-6/f
^2*b^2*c^2*d*ln(exp(f*x+e))+3/f^2*b^2*c^2*d*ln(exp(f*x+e)+1)+12/f^4*b*a*d^
3*polylog(4,exp(f*x+e))+12/f^4*b*a*d^3*polylog(4,-exp(f*x+e))+2/f*b*a*c^3*
ln(exp(f*x+e)-1)-4/f*b*a*c^3*ln(exp(f*x+e))+2/f*b*a*c^3*ln(exp(f*x+e)+1)+3
/f^4*b^2*e^2*d^3*ln(exp(f*x+e)-1)-6/f^4*b^2*e^2*d^3*ln(exp(f*x+e))+6/f^3*b
^2*c*d^2*polylog(2,exp(f*x+e))+6/f^3*b^2*c*d^2*polylog(2,-exp(f*x+e))-12/f
^3*b*a*c*d^2*polylog(3,-exp(f*x+e))+6/f^2*b*a*c^2*d*polylog(2,exp(f*x+e))+
6/f^2*b*a*c^2*d*polylog(2,-exp(f*x+e))-6/f^3*b^2*e*c*d^2*ln(exp(f*x+e)-1)+
12/f^3*b^2*e*c*d^2*ln(exp(f*x+e))+6/f^2*b^2*c*d^2*ln(1-exp(f*x+e))*x+6/f^3
*b^2*c*d^2*ln(1-exp(f*x+e))*e+6/f^2*b^2*c*d^2*ln(exp(f*x+e)+1)*x+2/f^4*b*a
*d^3*ln(1-exp(f*x+e))*e^3+2/f*b*a*d^3*ln(1-exp(f*x+e))*x^3+6/f^2*b*a*d^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3239 vs. $2(259) = 518$.

Time = 0.15 (sec) , antiderivative size = 3239, normalized size of antiderivative = 11.95

$$\int (c+dx)^3(a+b\coth(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(a+b*coth(f*x+e))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int (c+dx)^3(a+b\coth(e+fx))^2\,dx=\int (a+b\coth{(e+fx)})^2\,(c+dx)^3\,dx$$

input
```
integrate((d*x+c)**3*(a+b*coth(f*x+e))**2,x)
```

output
```
Integral((a + b*coth(e + f*x))**2*(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 781 vs. $2(259) = 518$.

Time = 0.20 (sec) , antiderivative size = 781, normalized size of antiderivative = 2.88

$$\int (c+dx)^3(a+b\coth(e+fx))^2\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)^3*(a+b*coth(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 + a^2*c^3*x - 6*b^2*c^
2*d*x/f + 2*a*b*c^3*log(sinh(f*x + e))/f + 3*b^2*c^2*d*log(e^(f*x + e) + 1
)/f^2 + 3*b^2*c^2*d*log(e^(f*x + e) - 1)/f^2 + 2*(f^3*x^3*log(e^(f*x + e)
+ 1) + 3*f^2*x^2*dilog(-e^(f*x + e)) - 6*f*x*polylog(3, -e^(f*x + e)) + 6*
polylog(4, -e^(f*x + e)))*a*b*d^3/f^4 + 2*(f^3*x^3*log(-e^(f*x + e) + 1) +
 3*f^2*x^2*dilog(e^(f*x + e)) - 6*f*x*polylog(3, e^(f*x + e)) + 6*polylog(
4, e^(f*x + e)))*a*b*d^3/f^4 - 1/4*(8*b^2*c^3 + (2*a*b*d^3*f + b^2*d^3*f)*
x^4 + 4*(c^3*f + 6*c^2*d)*b^2*x + 4*(2*a*b*c*d^2*f + (c*d^2*f + 2*d^3)*b^2
)*x^3 + 6*(2*a*b*c^2*d*f + (c^2*d*f + 4*c*d^2)*b^2)*x^2 - (4*b^2*c^3*f*x*e
^(2*e) + (2*a*b*d^3*f*e^(2*e) + b^2*d^3*f*e^(2*e))*x^4 + 4*(2*a*b*c*d^2*f*
e^(2*e) + b^2*c*d^2*f*e^(2*e))*x^3 + 6*(2*a*b*c^2*d*f*e^(2*e) + b^2*c^2*d*
f*e^(2*e))*x^2)*e^(2*f*x))/(f*e^(2*f*x + 2*e) - f) + 6*(a*b*c^2*d*f + b^2*
c*d^2)*(f*x*log(e^(f*x + e) + 1) + dilog(-e^(f*x + e)))/f^3 + 6*(a*b*c^2*d
*f + b^2*c*d^2)*(f*x*log(-e^(f*x + e) + 1) + dilog(e^(f*x + e)))/f^3 + 3*(
2*a*b*c*d^2*f + b^2*d^3)*(f^2*x^2*log(e^(f*x + e) + 1) + 2*f*x*dilog(-e^(f
*x + e)) - 2*polylog(3, -e^(f*x + e)))/f^4 + 3*(2*a*b*c*d^2*f + b^2*d^3)*(
f^2*x^2*log(-e^(f*x + e) + 1) + 2*f*x*dilog(e^(f*x + e)) - 2*polylog(3, e^
(f*x + e)))/f^4 - (a*b*d^3*f^4*x^4 + 2*(2*a*b*c*d^2*f + b^2*d^3)*f^3*x^3 +
 6*(a*b*c^2*d*f^2 + b^2*c*d^2*f)*f^2*x^2)/f^4
```

Giac [F]

$$\int (c+dx)^3(a+b\coth(e+fx))^2\,dx=\int (dx+c)^3(b\coth(fx+e)+a)^2\,dx$$

input
```
integrate((d*x+c)^3*(a+b*coth(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*(b*coth(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3(a+b\coth(e+fx))^2\,dx=\int \left(a+b\coth(e+f\,x)\right)^2\left(c+d\,x\right)^3\,dx$$

input
```
int((a + b*coth(e + f*x))^2*(c + d*x)^3,x)
```

output
```
int((a + b*coth(e + f*x))^2*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3(a+b\coth(e+fx))^2\,dx=\text{too large to display}$$

input
```
int((d*x+c)^3*(a+b*coth(f*x+e))^2,x)
```

output
```
( - 16*e**(2*e + 2*f*x)*int(x**3/(e**(4*e + 4*f*x) - 2*e**(2*e + 2*f*x) +
1),x)*a*b*d**3*f**4 - 48*e**(2*e + 2*f*x)*int(x**2/(e**(4*e + 4*f*x) - 2*e
**(2*e + 2*f*x) + 1),x)*a*b*c*d**2*f**4 - 24*e**(2*e + 2*f*x)*int(x**2/(e*
*(4*e + 4*f*x) - 2*e**(2*e + 2*f*x) + 1),x)*a*b*d**3*f**3 - 24*e**(2*e + 2
*f*x)*int(x**2/(e**(4*e + 4*f*x) - 2*e**(2*e + 2*f*x) + 1),x)*b**2*d**3*f*
*3 - 48*e**(2*e + 2*f*x)*int(x/(e**(4*e + 4*f*x) - 2*e**(2*e + 2*f*x) + 1)
,x)*a*b*c**2*d*f**4 - 48*e**(2*e + 2*f*x)*int(x/(e**(4*e + 4*f*x) - 2*e**(
2*e + 2*f*x) + 1),x)*a*b*c*d**2*f**3 - 24*e**(2*e + 2*f*x)*int(x/(e**(4*e
+ 4*f*x) - 2*e**(2*e + 2*f*x) + 1),x)*a*b*d**3*f**2 - 48*e**(2*e + 2*f*x)*
int(x/(e**(4*e + 4*f*x) - 2*e**(2*e + 2*f*x) + 1),x)*b**2*c*d**2*f**3 - 24
*e**(2*e + 2*f*x)*int(x/(e**(4*e + 4*f*x) - 2*e**(2*e + 2*f*x) + 1),x)*b**
2*d**3*f**2 + 8*e**(2*e + 2*f*x)*log(e**(e + f*x) - 1)*a*b*c**3*f**3 + 12*
e**(2*e + 2*f*x)*log(e**(e + f*x) - 1)*a*b*c**2*d*f**2 + 12*e**(2*e + 2*f*
x)*log(e**(e + f*x) - 1)*a*b*c*d**2*f + 6*e**(2*e + 2*f*x)*log(e**(e + f*x
) - 1)*a*b*d**3 + 12*e**(2*e + 2*f*x)*log(e**(e + f*x) - 1)*b**2*c**2*d*f*
*2 + 12*e**(2*e + 2*f*x)*log(e**(e + f*x) - 1)*b**2*c*d**2*f + 6*e**(2*e +
 2*f*x)*log(e**(e + f*x) - 1)*b**2*d**3 + 8*e**(2*e + 2*f*x)*log(e**(e + f
*x) + 1)*a*b*c**3*f**3 + 12*e**(2*e + 2*f*x)*log(e**(e + f*x) + 1)*a*b*c**
2*d*f**2 + 12*e**(2*e + 2*f*x)*log(e**(e + f*x) + 1)*a*b*c*d**2*f + 6*e**(
2*e + 2*f*x)*log(e**(e + f*x) + 1)*a*b*d**3 + 12*e**(2*e + 2*f*x)*log(e...
```

3.43 $\int (c+dx)^2(a+b\coth(e+fx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 209

$$\begin{aligned}\int (c+dx)^2(a+b\coth(e+fx))^2\,dx = &-\frac{b^2(c+dx)^2}{f}+\frac{a^2(c+dx)^3}{3d}-\frac{2ab(c+dx)^3}{3d}\\&+\frac{b^2(c+dx)^3}{3d}-\frac{b^2(c+dx)^2\coth(e+fx)}{f}\\&+\frac{2b^2d(c+dx)\log\left(1-e^{2(e+fx)}\right)}{f^2}\\&+\frac{2ab(c+dx)^2\log\left(1-e^{2(e+fx)}\right)}{f}\\&+\frac{b^2d^2\operatorname{PolyLog}\left(2,e^{2(e+fx)}\right)}{f^3}\\&+\frac{2abd(c+dx)\operatorname{PolyLog}\left(2,e^{2(e+fx)}\right)}{f^2}\\&-\frac{abd^2\operatorname{PolyLog}\left(3,e^{2(e+fx)}\right)}{f^3}\end{aligned}$$

output
```
-b^2*(d*x+c)^2/f+1/3*a^2*(d*x+c)^3/d-2/3*a*b*(d*x+c)^3/d+1/3*b^2*(d*x+c)^3
/d-b^2*(d*x+c)^2*coth(f*x+e)/f+2*b^2*d*(d*x+c)*ln(1-exp(2*f*x+2*e))/f^2+2*
a*b*(d*x+c)^2*ln(1-exp(2*f*x+2*e))/f+b^2*d^2*polylog(2,exp(2*f*x+2*e))/f^3
+2*a*b*d*(d*x+c)*polylog(2,exp(2*f*x+2*e))/f^2-a*b*d^2*polylog(3,exp(2*f*x
+2*e))/f^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 473 vs. 2(209) = 418.

Time = 4.59 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.26

$$\int (c+dx)^2(a+b\coth(e+fx))^2\,dx = -\frac{4bc(bd+acf)x}{f}-\frac{4bc(bd+acf)x}{(-1+e^{2e})\,f} -\frac{2bd(bd+2acf)x^2}{(-1+e^{2e})\,f}+\frac{4abd^2x^3}{3-3e^{2e}} +\frac{1}{3}x\left(3c^2+3cdx+d^2x^2\right)\left(a^2+b^2+2ab\coth(e)\right) +\frac{2bd(bd+2acf)x\log\left(1-e^{-e-fx}\right)}{f^2} +\frac{2abd^2x^2\log\left(1-e^{-e-fx}\right)}{f} +\frac{2bd(bd+2acf)x\log\left(1+e^{-e-fx}\right)}{f^2} +\frac{2abd^2x^2\log\left(1+e^{-e-fx}\right)}{f} +\frac{2bc(bd+acf)\log\left(1-e^{e+fx}\right)}{f^2} +\frac{2bc(bd+acf)\log\left(1+e^{e+fx}\right)}{f^2} -\frac{2bd(bd+2acf)\,\text{PolyLog}\left(2,-e^{-e-fx}\right)}{f^3} -\frac{4abd^2x\,\text{PolyLog}\left(2,-e^{-e-fx}\right)}{f^2} -\frac{2bd(bd+2acf)\,\text{PolyLog}\left(2,e^{-e-fx}\right)}{f^3} -\frac{4abd^2x\,\text{PolyLog}\left(2,e^{-e-fx}\right)}{f^2} -\frac{4abd^2\,\text{PolyLog}\left(3,-e^{-e-fx}\right)}{f^3} -\frac{4abd^2\,\text{PolyLog}\left(3,e^{-e-fx}\right)}{f^3} +\frac{b^2(c+dx)^2\text{csch}(e)\text{csch}(e+fx)\sinh(fx)}{f}$$

input
```
Integrate[(c + d*x)^2*(a + b*Coth[e + f*x])^2,x]
```

output
```
(-4*b*c*(b*d + a*c*f)*x)/f - (4*b*c*(b*d + a*c*f)*x)/((-1 + E^(2*e))*f) -
(2*b*d*(b*d + 2*a*c*f)*x^2)/((-1 + E^(2*e))*f) + (4*a*b*d^2*x^3)/(3 - 3*E^
(2*e)) + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*(a^2 + b^2 + 2*a*b*Coth[e]))/3 + (
2*b*d*(b*d + 2*a*c*f)*x*Log[1 - E^(-e - f*x)])/f^2 + (2*a*b*d^2*x^2*Log[1
- E^(-e - f*x)])/f + (2*b*d*(b*d + 2*a*c*f)*x*Log[1 + E^(-e - f*x)])/f^2 +
 (2*a*b*d^2*x^2*Log[1 + E^(-e - f*x)])/f + (2*b*c*(b*d + a*c*f)*Log[1 - E^
(e + f*x)])/f^2 + (2*b*c*(b*d + a*c*f)*Log[1 + E^(e + f*x)])/f^2 - (2*b*d*
(b*d + 2*a*c*f)*PolyLog[2, -E^(-e - f*x)])/f^3 - (4*a*b*d^2*x*PolyLog[2, -
E^(-e - f*x)])/f^2 - (2*b*d*(b*d + 2*a*c*f)*PolyLog[2, E^(-e - f*x)])/f^3
- (4*a*b*d^2*x*PolyLog[2, E^(-e - f*x)])/f^2 - (4*a*b*d^2*PolyLog[3, -E^(-
e - f*x)])/f^3 - (4*a*b*d^2*PolyLog[3, E^(-e - f*x)])/f^3 + (b^2*(c + d*x)
^2*Csch[e]*Csch[e + f*x]*Sinh[f*x])/f
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a+b\coth(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2\left(a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4205}$$

$$\int \left(a^2(c+dx)^2+2ab(c+dx)^2\coth(e+fx)+b^2(c+dx)^2\coth^2(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(c+dx)^3}{3d}+\frac{2abd(c+dx)\operatorname{PolyLog}\left(2,e^{2(e+fx)}\right)}{f^2}+\frac{2ab(c+dx)^2\log\left(1-e^{2(e+fx)}\right)}{f}-\frac{2ab(c+dx)^3}{3d}-\frac{abd^2\operatorname{PolyLog}\left(3,e^{2(e+fx)}\right)}{f^3}+\frac{2b^2d(c+dx)\log\left(1-e^{2(e+fx)}\right)}{f^2}-\frac{b^2(c+dx)^2\coth(e+fx)}{f}-\frac{b^2(c+dx)^2}{f}+\frac{b^2(c+dx)^3}{3d}+\frac{b^2d^2\operatorname{PolyLog}\left(2,e^{2(e+fx)}\right)}{f^3}$$

input
```
Int[(c + d*x)^2*(a + b*Coth[e + f*x])^2,x]
```

output
```
-((b^2*(c + d*x)^2)/f) + (a^2*(c + d*x)^3)/(3*d) - (2*a*b*(c + d*x)^3)/(3*
d) + (b^2*(c + d*x)^3)/(3*d) - (b^2*(c + d*x)^2*Coth[e + f*x])/f + (2*b^2*
d*(c + d*x)*Log[1 - E^(2*(e + f*x))])/f^2 + (2*a*b*(c + d*x)^2*Log[1 - E^(
2*(e + f*x))])/f + (b^2*d^2*PolyLog[2, E^(2*(e + f*x))])/f^3 + (2*a*b*d*(c
 + d*x)*PolyLog[2, E^(2*(e + f*x))])/f^2 - (a*b*d^2*PolyLog[3, E^(2*(e + f
*x))])/f^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 824 vs. 2(203) = 406.

Time = 0.26 (sec) , antiderivative size = 825, normalized size of antiderivative = 3.95

method	result
risch	$\frac{d^2a^2x^3}{3} + \frac{d^2x^3b^2}{3} + \frac{c^3a^2}{3d} + \frac{c^3b^2}{3d} + \frac{8ba\,d^2e^3}{3f^3} - \frac{4b^2d^2ex}{f^2} + \frac{2ba\,c^2\ln(e^{fx+e}-1)}{f} - \frac{4ba\,c^2\ln(e^{fx+e})}{f} + \frac{2ba\,c^2\ln(e^{fx+e}+}{f}$

input
```
int((d*x+c)^2*(a+b*coth(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*d^2*a^2*x^3+1/3*d^2*x^3*b^2+1/3/d*c^3*a^2+1/3/d*c^3*b^2+8/3/f^3*b*a*d^
2*e^3-4/f^2*b^2*d^2*e*x+2/f*b*a*c^2*ln(exp(f*x+e)-1)-4/f*b*a*c^2*ln(exp(f*
x+e))+2/f*b*a*c^2*ln(exp(f*x+e)+1)+2/f^2*b^2*c*d*ln(exp(f*x+e)-1)-4/f^2*b^
2*c*d*ln(exp(f*x+e))+2/f^2*b^2*c*d*ln(exp(f*x+e)+1)-4/f^3*b*a*d^2*polylog(
3,exp(f*x+e))-4/f^3*b*a*d^2*polylog(3,-exp(f*x+e))-2/f^3*b^2*e*d^2*ln(exp(
f*x+e)-1)+4/f^3*b^2*e*d^2*ln(exp(f*x+e))+2/f^2*b^2*d^2*ln(1-exp(f*x+e))*x+
2/f^3*b^2*d^2*ln(1-exp(f*x+e))*e+2/f^2*b^2*d^2*ln(exp(f*x+e)+1)*x+4/f^2*b*
a*d^2*e^2*x-4/f^3*b*e^2*a*d^2*ln(exp(f*x+e))+2/f*b*a*d^2*ln(1-exp(f*x+e))*
x^2-2/f^3*b*a*d^2*ln(1-exp(f*x+e))*e^2+4/f^2*b*a*d^2*polylog(2,exp(f*x+e))
*x+2/f*b*a*d^2*ln(exp(f*x+e)+1)*x^2+4/f^2*b*a*d^2*polylog(2,-exp(f*x+e))*x
+4/f^2*b*a*c*d*polylog(2,exp(f*x+e))+4/f^2*b*a*c*d*polylog(2,-exp(f*x+e))+
2/f^3*b*e^2*a*d^2*ln(exp(f*x+e)-1)-2/f*b^2*d^2*x^2-2/f^3*b^2*d^2*e^2-4/f^2
*b*a*c*d*e^2+2/f^3*b^2*d^2*polylog(2,exp(f*x+e))+2/f^3*b^2*d^2*polylog(2,-
exp(f*x+e))-2/3*d^2*a*b*x^3+d*a^2*c*x^2+d*x^2*c*b^2+a^2*c^2*x+x*c^2*b^2+2/
3/d*c^3*a*b-2/f*b^2*(d^2*x^2+2*c*d*x+c^2)/(exp(2*f*x+2*e)-1)-8/f*b*a*c*d*e
*x+4/f*b*a*c*d*ln(1-exp(f*x+e))*x+4/f*b*a*c*d*ln(exp(f*x+e)+1)*x-4/f^2*b*e
*a*c*d*ln(exp(f*x+e)-1)+8/f^2*b*e*a*c*d*ln(exp(f*x+e))+4/f^2*b*a*c*d*ln(1-
exp(f*x+e))*e-2*d*a*b*c*x^2+2*a*b*c^2*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1854 vs. 2(201) = 402.

Time = 0.12 (sec) , antiderivative size = 1854, normalized size of antiderivative = 8.87

$$\int (c+dx)^2(a+b\coth(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(a+b*coth(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/3*((a^2 - 2*a*b + b^2)*d^2*f^3*x^3 + 3*(a^2 - 2*a*b + b^2)*c*d*f^3*x^2
- 4*a*b*d^2*e^3 + 3*(a^2 - 2*a*b + b^2)*c^2*f^3*x + 6*b^2*d^2*e^2 - 6*(2*a
*b*c^2*e - b^2*c^2)*f^2 - ((a^2 - 2*a*b + b^2)*d^2*f^3*x^3 - 4*a*b*d^2*e^3
 - 12*a*b*c^2*e*f^2 + 6*b^2*d^2*e^2 - 3*(2*b^2*d^2*f^2 - (a^2 - 2*a*b + b^
2)*c*d*f^3)*x^2 + 12*(a*b*c*d*e^2 - b^2*c*d*e)*f - 3*(4*b^2*c*d*f^2 - (a^2
 - 2*a*b + b^2)*c^2*f^3)*x)*cosh(f*x + e)^2 - 2*((a^2 - 2*a*b + b^2)*d^2*f
^3*x^3 - 4*a*b*d^2*e^3 - 12*a*b*c^2*e*f^2 + 6*b^2*d^2*e^2 - 3*(2*b^2*d^2*f
^2 - (a^2 - 2*a*b + b^2)*c*d*f^3)*x^2 + 12*(a*b*c*d*e^2 - b^2*c*d*e)*f - 3
*(4*b^2*c*d*f^2 - (a^2 - 2*a*b + b^2)*c^2*f^3)*x)*cosh(f*x + e)*sinh(f*x +
 e) - ((a^2 - 2*a*b + b^2)*d^2*f^3*x^3 - 4*a*b*d^2*e^3 - 12*a*b*c^2*e*f^2
+ 6*b^2*d^2*e^2 - 3*(2*b^2*d^2*f^2 - (a^2 - 2*a*b + b^2)*c*d*f^3)*x^2 + 12
*(a*b*c*d*e^2 - b^2*c*d*e)*f - 3*(4*b^2*c*d*f^2 - (a^2 - 2*a*b + b^2)*c^2*
f^3)*x)*sinh(f*x + e)^2 + 12*(a*b*c*d*e^2 - b^2*c*d*e)*f + 6*(2*a*b*d^2*f*
x + 2*a*b*c*d*f + b^2*d^2 - (2*a*b*d^2*f*x + 2*a*b*c*d*f + b^2*d^2)*cosh(f
*x + e)^2 - 2*(2*a*b*d^2*f*x + 2*a*b*c*d*f + b^2*d^2)*cosh(f*x + e)*sinh(f
*x + e) - (2*a*b*d^2*f*x + 2*a*b*c*d*f + b^2*d^2)*sinh(f*x + e)^2)*dilog(c
osh(f*x + e) + sinh(f*x + e)) + 6*(2*a*b*d^2*f*x + 2*a*b*c*d*f + b^2*d^2 -
 (2*a*b*d^2*f*x + 2*a*b*c*d*f + b^2*d^2)*cosh(f*x + e)^2 - 2*(2*a*b*d^2*f*
x + 2*a*b*c*d*f + b^2*d^2)*cosh(f*x + e)*sinh(f*x + e) - (2*a*b*d^2*f*x +
2*a*b*c*d*f + b^2*d^2)*sinh(f*x + e)^2)*dilog(-cosh(f*x + e) - sinh(f*x...
```

Sympy [F]

$$\int (c+dx)^2(a+b\coth(e+fx))^2\,dx = \int (a+b\coth(e+fx))^2\,(c+dx)^2\,dx$$

input
```
integrate((d*x+c)**2*(a+b*coth(f*x+e))**2,x)
```

output
```
Integral((a + b*coth(e + f*x))**2*(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(201) = 402$.

Time = 0.18 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.36

$$\int (c+dx)^2(a+b\coth(e+fx))^2\,dx = \frac{1}{3}a^2d^2x^3 + a^2cdx^2 + a^2c^2x - \frac{4\,b^2cdx}{f} + \frac{2\,abc^2\log(\sinh(fx+e))}{f} + \frac{2\,b^2cd\log\left(e^{(fx+e)}+1\right)}{f^2} + \frac{2\,b^2cd\log\left(e^{(fx+e)}-1\right)}{f^2} + \frac{2\left(f^2x^2\log\left(e^{(fx+e)}+1\right) + 2\,fx\mathrm{Li}_2\left(-e^{(fx+e)}\right) - 2\,\mathrm{Li}_3(-e^{(fx+e)})\right)abd^2}{f^3} + \frac{2\left(f^2x^2\log\left(-e^{(fx+e)}+1\right) + 2\,fx\mathrm{Li}_2\left(e^{(fx+e)}\right) - 2\,\mathrm{Li}_3(e^{(fx+e)})\right)abd^2}{f^3} - \frac{6\,b^2c^2 + 3\,(c^2f+4\,cd)b^2x + (2\,abd^2f + b^2d^2f)x^3 + 3\,(2\,abcdf + (cdf+2\,d^2)b^2)x^2 - \left(3\,b^2c^2fxe^{(2\,e)} + \left(\right.\right.}{3\,(fe^{(2\,fx+2\,e)} - f)} + \frac{2\,(2\,abcdf + b^2d^2)\left(fx\log\left(e^{(fx+e)}+1\right) + \mathrm{Li}_2\left(-e^{(fx+e)}\right)\right)}{f^3} + \frac{2\,(2\,abcdf + b^2d^2)\left(fx\log\left(-e^{(fx+e)}+1\right) + \mathrm{Li}_2\left(e^{(fx+e)}\right)\right)}{f^3} - \frac{2\,(2\,abd^2f^3x^3 + 3\,(2\,abcdf + b^2d^2)f^2x^2)}{3\,f^3}$$

input
```
integrate((d*x+c)^2*(a+b*coth(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*d^2*x^3 + a^2*c*d*x^2 + a^2*c^2*x - 4*b^2*c*d*x/f + 2*a*b*c^2*log(
sinh(f*x + e))/f + 2*b^2*c*d*log(e^(f*x + e) + 1)/f^2 + 2*b^2*c*d*log(e^(f
*x + e) - 1)/f^2 + 2*(f^2*x^2*log(e^(f*x + e) + 1) + 2*f*x*dilog(-e^(f*x +
 e)) - 2*polylog(3, -e^(f*x + e)))*a*b*d^2/f^3 + 2*(f^2*x^2*log(-e^(f*x +
e) + 1) + 2*f*x*dilog(e^(f*x + e)) - 2*polylog(3, e^(f*x + e)))*a*b*d^2/f^
3 - 1/3*(6*b^2*c^2 + 3*(c^2*f + 4*c*d)*b^2*x + (2*a*b*d^2*f + b^2*d^2*f)*x
^3 + 3*(2*a*b*c*d*f + (c*d*f + 2*d^2)*b^2)*x^2 - (3*b^2*c^2*f*x*e^(2*e) +
(2*a*b*d^2*f*e^(2*e) + b^2*d^2*f*e^(2*e))*x^3 + 3*(2*a*b*c*d*f*e^(2*e) + b
^2*c*d*f*e^(2*e))*x^2)*e^(2*f*x))/(f*e^(2*f*x + 2*e) - f) + 2*(2*a*b*c*d*f
 + b^2*d^2)*(f*x*log(e^(f*x + e) + 1) + dilog(-e^(f*x + e)))/f^3 + 2*(2*a*
b*c*d*f + b^2*d^2)*(f*x*log(-e^(f*x + e) + 1) + dilog(e^(f*x + e)))/f^3 -
2/3*(2*a*b*d^2*f^3*x^3 + 3*(2*a*b*c*d*f + b^2*d^2)*f^2*x^2)/f^3
```

Giac [F]

$$\int (c+dx)^2(a+b\coth(e+fx))^2\,dx=\int (dx+c)^2(b\coth(fx+e)+a)^2\,dx$$

input
```
integrate((d*x+c)^2*(a+b*coth(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*(b*coth(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2(a+b\coth(e+fx))^2\,dx=\int (a+b\coth(e+f\,x))^2\,(c+d\,x)^2\,dx$$

input
```
int((a + b*coth(e + f*x))^2*(c + d*x)^2,x)
```

output
```
int((a + b*coth(e + f*x))^2*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2(a+b\coth(e+fx))^2\,dx=\text{Too large to display}$$

input
```
int((d*x+c)^2*(a+b*coth(f*x+e))^2,x)
```

output
```
( - 12*e**(2*e + 2*f*x)*int(x**2/(e**(4*e + 4*f*x) - 2*e**(2*e + 2*f*x) +
1),x)*a*b*d**2*f**3 - 24*e**(2*e + 2*f*x)*int(x/(e**(4*e + 4*f*x) - 2*e**(
2*e + 2*f*x) + 1),x)*a*b*c*d*f**3 - 12*e**(2*e + 2*f*x)*int(x/(e**(4*e + 4
*f*x) - 2*e**(2*e + 2*f*x) + 1),x)*a*b*d**2*f**2 - 12*e**(2*e + 2*f*x)*int
(x/(e**(4*e + 4*f*x) - 2*e**(2*e + 2*f*x) + 1),x)*b**2*d**2*f**2 + 6*e**(2
*e + 2*f*x)*log(e**(e + f*x) - 1)*a*b*c**2*f**2 + 6*e**(2*e + 2*f*x)*log(e
**(e + f*x) - 1)*a*b*c*d*f + 3*e**(2*e + 2*f*x)*log(e**(e + f*x) - 1)*a*b*
d**2 + 6*e**(2*e + 2*f*x)*log(e**(e + f*x) - 1)*b**2*c*d*f + 3*e**(2*e + 2
*f*x)*log(e**(e + f*x) - 1)*b**2*d**2 + 6*e**(2*e + 2*f*x)*log(e**(e + f*x
) + 1)*a*b*c**2*f**2 + 6*e**(2*e + 2*f*x)*log(e**(e + f*x) + 1)*a*b*c*d*f
+ 3*e**(2*e + 2*f*x)*log(e**(e + f*x) + 1)*a*b*d**2 + 6*e**(2*e + 2*f*x)*l
og(e**(e + f*x) + 1)*b**2*c*d*f + 3*e**(2*e + 2*f*x)*log(e**(e + f*x) + 1)
*b**2*d**2 + 3*e**(2*e + 2*f*x)*a**2*c**2*f**3*x + 3*e**(2*e + 2*f*x)*a**2
*c*d*f**3*x**2 + e**(2*e + 2*f*x)*a**2*d**2*f**3*x**3 - 6*e**(2*e + 2*f*x)
*a*b*c**2*f**3*x + 6*e**(2*e + 2*f*x)*a*b*c*d*f**3*x**2 - 12*e**(2*e + 2*f
*x)*a*b*c*d*f**2*x + 2*e**(2*e + 2*f*x)*a*b*d**2*f**3*x**3 - 6*e**(2*e + 2
*f*x)*a*b*d**2*f*x + 3*e**(2*e + 2*f*x)*b**2*c**2*f**3*x - 6*e**(2*e + 2*f
*x)*b**2*c**2*f**2 + 3*e**(2*e + 2*f*x)*b**2*c*d*f**3*x**2 - 12*e**(2*e +
2*f*x)*b**2*c*d*f**2*x + e**(2*e + 2*f*x)*b**2*d**2*f**3*x**3 - 6*e**(2*e
+ 2*f*x)*b**2*d**2*f*x + 12*int(x**2/(e**(4*e + 4*f*x) - 2*e**(2*e + 2*...
```

3.44 $\int (c+dx)(a+b\coth(e+fx))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 127

$$\int (c+dx)(a+b\coth(e+fx))^2\,dx = \frac{a^2(c+dx)^2}{2d}-\frac{ab(c+dx)^2}{d}+\frac{b^2(c+dx)^2}{2d}-\frac{b^2(c+dx)\coth(e+fx)}{f}+\frac{2ab(c+dx)\log\left(1-e^{2(e+fx)}\right)}{f}+\frac{b^2d\log(\sinh(e+fx))}{f^2}+\frac{abd\,\text{PolyLog}\left(2,e^{2(e+fx)}\right)}{f^2}$$

output
```
1/2*a^2*(d*x+c)^2/d-a*b*(d*x+c)^2/d+1/2*b^2*(d*x+c)^2/d-b^2*(d*x+c)*coth(f
*x+e)/f+2*a*b*(d*x+c)*ln(1-exp(2*f*x+2*e))/f+b^2*d*ln(sinh(f*x+e))/f^2+a*b
*d*polylog(2,exp(2*f*x+2*e))/f^2
```

Mathematica [A] (verified)

Time = 6.81 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.72

$$\int (c+dx)(a+b\coth(e+fx))^2\,dx$$

$$= \frac{(a+b\coth(e+fx))^2\sinh(e+fx)\left(-2b^2f(c+dx)\cosh(e+fx)-(a^2+b^2)(e+fx)(-2cf+d(e-fx\right.}{}$$

input
```
Integrate[(c + d*x)*(a + b*Coth[e + f*x])^2,x]
```

output
```
((a + b*Coth[e + f*x])^2*Sinh[e + f*x]*(-2*b^2*f*(c + d*x)*Cosh[e + f*x] -
 (a^2 + b^2)*(e + f*x)*(-2*c*f + d*(e - f*x))*Sinh[e + f*x] + 2*b*((a*f^2*
(c + d*x)^2)/d + b*d*(e + f*x) + (b*d + 2*a*f*(c + d*x))*Log[1 - E^(-e - f
*x)] + (b*d + 2*a*f*(c + d*x))*Log[1 + E^(-e - f*x)] - 2*a*d*PolyLog[2, -E
^(-e - f*x)] - 2*a*d*PolyLog[2, E^(-e - f*x)])*Sinh[e + f*x]))/(2*f^2*(b*C
osh[e + f*x] + a*Sinh[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a+b\coth(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\left(a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4205}$$

$$\int \left(a^2(c+dx)+2ab(c+dx)\coth(e+fx)+b^2(c+dx)\coth^2(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(c+dx)^2}{2d}+\frac{2ab(c+dx)\log\left(1-e^{2(e+fx)}\right)}{f}-\frac{ab(c+dx)^2}{d}+\frac{abd\operatorname{PolyLog}\left(2,e^{2(e+fx)}\right)}{f^2}-\frac{b^2(c+dx)\coth(e+fx)}{f}+\frac{b^2(c+dx)^2}{2d}+\frac{b^2d\log(\sinh(e+fx))}{f^2}$$

input
```
Int[(c + d*x)*(a + b*Coth[e + f*x])^2,x]
```

output
```
(a^2*(c + d*x)^2)/(2*d) - (a*b*(c + d*x)^2)/d + (b^2*(c + d*x)^2)/(2*d) -
(b^2*(c + d*x)*Coth[e + f*x])/f + (2*a*b*(c + d*x)*Log[1 - E^(2*(e + f*x))
])/f + (b^2*d*Log[Sinh[e + f*x]])/f^2 + (a*b*d*PolyLog[2, E^(2*(e + f*x))]
)/f^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(123) = 246$.

Time = 0.22 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.50

method	result
risch	$\frac{a^2 d\,x^2}{2}-abd\,x^2+\frac{b^2 d\,x^2}{2}+a^2cx+2abcx+b^2cx-\frac{2b^2(dx+c)}{f\left(e^{2fx+2e}-1\right)}+\frac{b^2 d\ln\left(e^{fx+e}-1\right)}{f^2}-\frac{2b^2 d\ln\left(e^{fx+e}\right)}{f^2}+\frac{b^2}{}$

input
```
int((d*x+c)*(a+b*coth(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*d*x^2-a*b*d*x^2+1/2*b^2*d*x^2+a^2*c*x+2*a*b*c*x+b^2*c*x-2/f*b^2*(d
*x+c)/(exp(2*f*x+2*e)-1)+1/f^2*b^2*d*ln(exp(f*x+e)-1)-2/f^2*b^2*d*ln(exp(f
*x+e))+1/f^2*b^2*d*ln(exp(f*x+e)+1)+2/f*b*a*c*ln(exp(f*x+e)-1)-4/f*b*a*c*l
n(exp(f*x+e))+2/f*b*a*c*ln(exp(f*x+e)+1)-2/f^2*b*e*a*d*ln(exp(f*x+e)-1)+4/
f^2*b*e*a*d*ln(exp(f*x+e))-4/f*b*a*d*e*x-2/f^2*b*a*d*e^2+2/f*b*a*d*ln(1-ex
p(f*x+e))*x+2/f^2*b*a*d*ln(1-exp(f*x+e))*e+2/f^2*b*a*d*polylog(2,exp(f*x+e
))+2/f*b*a*d*ln(exp(f*x+e)+1)*x+2/f^2*b*a*d*polylog(2,-exp(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. 2(122) = 244.

Time = 0.12 (sec) , antiderivative size = 851, normalized size of antiderivative = 6.70

$$\int (c+dx)(a+b\coth(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(a+b*coth(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/2*((a^2 - 2*a*b + b^2)*d*f^2*x^2 + 4*a*b*d*e^2 + 2*(a^2 - 2*a*b + b^2)*
c*f^2*x - 4*b^2*d*e - ((a^2 - 2*a*b + b^2)*d*f^2*x^2 + 4*a*b*d*e^2 - 8*a*b
*c*e*f - 4*b^2*d*e - 2*(2*b^2*d*f - (a^2 - 2*a*b + b^2)*c*f^2)*x)*cosh(f*x
 + e)^2 - 2*((a^2 - 2*a*b + b^2)*d*f^2*x^2 + 4*a*b*d*e^2 - 8*a*b*c*e*f - 4
*b^2*d*e - 2*(2*b^2*d*f - (a^2 - 2*a*b + b^2)*c*f^2)*x)*cosh(f*x + e)*sinh
(f*x + e) - ((a^2 - 2*a*b + b^2)*d*f^2*x^2 + 4*a*b*d*e^2 - 8*a*b*c*e*f - 4
*b^2*d*e - 2*(2*b^2*d*f - (a^2 - 2*a*b + b^2)*c*f^2)*x)*sinh(f*x + e)^2 -
4*(2*a*b*c*e - b^2*c)*f - 4*(a*b*d*cosh(f*x + e)^2 + 2*a*b*d*cosh(f*x + e)
*sinh(f*x + e) + a*b*d*sinh(f*x + e)^2 - a*b*d)*dilog(cosh(f*x + e) + sinh
(f*x + e)) - 4*(a*b*d*cosh(f*x + e)^2 + 2*a*b*d*cosh(f*x + e)*sinh(f*x + e
) + a*b*d*sinh(f*x + e)^2 - a*b*d)*dilog(-cosh(f*x + e) - sinh(f*x + e)) +
 2*(2*a*b*d*f*x + 2*a*b*c*f + b^2*d - (2*a*b*d*f*x + 2*a*b*c*f + b^2*d)*co
sh(f*x + e)^2 - 2*(2*a*b*d*f*x + 2*a*b*c*f + b^2*d)*cosh(f*x + e)*sinh(f*x
 + e) - (2*a*b*d*f*x + 2*a*b*c*f + b^2*d)*sinh(f*x + e)^2)*log(cosh(f*x +
e) + sinh(f*x + e) + 1) - 2*(2*a*b*d*e - 2*a*b*c*f - b^2*d - (2*a*b*d*e -
2*a*b*c*f - b^2*d)*cosh(f*x + e)^2 - 2*(2*a*b*d*e - 2*a*b*c*f - b^2*d)*cos
h(f*x + e)*sinh(f*x + e) - (2*a*b*d*e - 2*a*b*c*f - b^2*d)*sinh(f*x + e)^2
)*log(cosh(f*x + e) + sinh(f*x + e) - 1) + 4*(a*b*d*f*x + a*b*d*e - (a*b*d
*f*x + a*b*d*e)*cosh(f*x + e)^2 - 2*(a*b*d*f*x + a*b*d*e)*cosh(f*x + e)*si
nh(f*x + e) - (a*b*d*f*x + a*b*d*e)*sinh(f*x + e)^2)*log(-cosh(f*x + e)...
```

Sympy [F]

$$\int (c+dx)(a+b\coth(e+fx))^2\,dx=\int (a+b\coth{(e+fx)})^2\,(c+dx)\;dx$$

input
```
integrate((d*x+c)*(a+b*coth(f*x+e))**2,x)
```

output
```
Integral((a + b*coth(e + f*x))**2*(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.92

$$\int (c+dx)(a+b\coth(e+fx))^2\,dx = \frac{1}{2}\,a^2dx^2 - 2\,abdx^2 + a^2cx - \frac{2\,b^2dx}{f} + \frac{2\,abc\log\left(\sinh\left(fx+e\right)\right)}{f} + \frac{2\,\left(fx\log\left(e^{(fx+e)}+1\right) + \mathrm{Li}_2\left(-e^{(fx+e)}\right)\right)abd}{f^2} + \frac{2\,\left(fx\log\left(-e^{(fx+e)}+1\right) + \mathrm{Li}_2\left(e^{(fx+e)}\right)\right)abd}{f^2} + \frac{b^2d\log\left(e^{(fx+e)}+1\right)}{f^2} + \frac{b^2d\log\left(e^{(fx+e)}-1\right)}{f^2} - \frac{2\,(cf+2\,d)b^2x + 4\,b^2c + (2\,abdf + b^2df)x^2 - \left(2\,b^2cfxe^{(2\,e)} + \left(2\,abdfe^{(2\,e)} + b^2dfe^{(2\,e)}\right)x^2\right)e^{(2\,fx)}}{2\,\left(fe^{(2\,fx+2\,e)} - f\right)}$$

input
```
integrate((d*x+c)*(a+b*coth(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*d*x^2 - 2*a*b*d*x^2 + a^2*c*x - 2*b^2*d*x/f + 2*a*b*c*log(sinh(f*x
 + e))/f + 2*(f*x*log(e^(f*x + e) + 1) + dilog(-e^(f*x + e)))*a*b*d/f^2 +
2*(f*x*log(-e^(f*x + e) + 1) + dilog(e^(f*x + e)))*a*b*d/f^2 + b^2*d*log(e
^(f*x + e) + 1)/f^2 + b^2*d*log(e^(f*x + e) - 1)/f^2 - 1/2*(2*(c*f + 2*d)*
b^2*x + 4*b^2*c + (2*a*b*d*f + b^2*d*f)*x^2 - (2*b^2*c*f*x*e^(2*e) + (2*a*
b*d*f*e^(2*e) + b^2*d*f*e^(2*e))*x^2)*e^(2*f*x))/(f*e^(2*f*x + 2*e) - f)
```

Giac [F]

$$\int (c+dx)(a+b\coth(e+fx))^2\,dx = \int (dx+c)(b\coth\left(fx+e\right)+a)^2\,dx$$

input
```
integrate((d*x+c)*(a+b*coth(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)*(b*coth(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)(a+b\coth(e+fx))^2\,dx=\int (a+b\coth(e+f\,x))^2\,(c+d\,x)\,dx$$

input
```
int((a + b*coth(e + f*x))^2*(c + d*x),x)
```

output
```
int((a + b*coth(e + f*x))^2*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)(a+b\coth(e+fx))^2\,dx$$
$$=\frac{-2b^2c\,f^2x-a^2d\,f^2x^2-b^2d\,f^2x^2+2e^{2fx+2e}\log\left(e^{fx+e}-1\right)b^2d+2e^{2fx+2e}\log\left(e^{fx+e}+1\right)b^2d-4e^{2fx+2e}b}{}$$

input
```
int((d*x+c)*(a+b*coth(f*x+e))^2,x)
```

output
```
( - 8*e**(2*e + 2*f*x)*int(x/(e**(4*e + 4*f*x) - 2*e**(2*e + 2*f*x) + 1),x
)*a*b*d*f**2 + 4*e**(2*e + 2*f*x)*log(e**(e + f*x) - 1)*a*b*c*f + 2*e**(2*
e + 2*f*x)*log(e**(e + f*x) - 1)*a*b*d + 2*e**(2*e + 2*f*x)*log(e**(e + f*
x) - 1)*b**2*d + 4*e**(2*e + 2*f*x)*log(e**(e + f*x) + 1)*a*b*c*f + 2*e**(
2*e + 2*f*x)*log(e**(e + f*x) + 1)*a*b*d + 2*e**(2*e + 2*f*x)*log(e**(e +
f*x) + 1)*b**2*d + 2*e**(2*e + 2*f*x)*a**2*c*f**2*x + e**(2*e + 2*f*x)*a**
2*d*f**2*x**2 - 4*e**(2*e + 2*f*x)*a*b*c*f**2*x + 2*e**(2*e + 2*f*x)*a*b*d
*f**2*x**2 - 4*e**(2*e + 2*f*x)*a*b*d*f*x + 2*e**(2*e + 2*f*x)*b**2*c*f**2
*x - 4*e**(2*e + 2*f*x)*b**2*c*f + e**(2*e + 2*f*x)*b**2*d*f**2*x**2 - 4*e
**(2*e + 2*f*x)*b**2*d*f*x + 8*int(x/(e**(4*e + 4*f*x) - 2*e**(2*e + 2*f*x
) + 1),x)*a*b*d*f**2 - 4*log(e**(e + f*x) - 1)*a*b*c*f - 2*log(e**(e + f*x
) - 1)*a*b*d - 2*log(e**(e + f*x) - 1)*b**2*d - 4*log(e**(e + f*x) + 1)*a*
b*c*f - 2*log(e**(e + f*x) + 1)*a*b*d - 2*log(e**(e + f*x) + 1)*b**2*d - 2
*a**2*c*f**2*x - a**2*d*f**2*x**2 + 4*a*b*c*f**2*x - 2*a*b*d*f**2*x**2 - 2
*b**2*c*f**2*x - b**2*d*f**2*x**2)/(2*f**2*(e**(2*e + 2*f*x) - 1))
```

3.45 $\int \frac{(a+b\coth(e+fx))^2}{c+dx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(a+b\coth(e+fx))^2}{c+dx}\,dx=\text{Int}\left(\frac{(a+b\coth(e+fx))^2}{c+dx},x\right)$$

output
```
Defer(Int)((a+b*coth(f*x+e))^2/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 29.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\coth(e+fx))^2}{c+dx}\,dx=\int \frac{(a+b\coth(e+fx))^2}{c+dx}\,dx$$

input
```
Integrate[(a + b*Coth[e + f*x])^2/(c + d*x),x]
```

output
```
Integrate[(a + b*Coth[e + f*x])^2/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\coth(e+fx))^2}{c+dx}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}{c+dx}dx$$

$$\Big\downarrow \; 4223$$

$$\int \frac{(a+b\coth(e+fx))^2}{c+dx}dx$$

input
```
Int[(a + b*Coth[e + f*x])^2/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\coth(fx+e))^2}{dx+c}dx$$

input
```
int((a+b*coth(f*x+e))^2/(d*x+c),x)
```

output
```
int((a+b*coth(f*x+e))^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{(a+b\coth(e+fx))^2}{c+dx}\,dx = \int \frac{(b\coth(fx+e)+a)^2}{dx+c}\,dx$$

input
```
integrate((a+b*coth(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral((b^2*coth(f*x + e)^2 + 2*a*b*coth(f*x + e) + a^2)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(a+b\coth(e+fx))^2}{c+dx}\,dx = \int \frac{(a+b\coth(e+fx))^2}{c+dx}\,dx$$

input
```
integrate((a+b*coth(f*x+e))**2/(d*x+c),x)
```

output
```
Integral((a + b*coth(e + f*x))**2/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 226, normalized size of antiderivative = 11.30

$$\int \frac{(a+b\coth(e+fx))^2}{c+dx}\,dx = \int \frac{(b\coth(fx+e)+a)^2}{dx+c}\,dx$$

input
```
integrate((a+b*coth(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

output
```
a^2*log(d*x + c)/d + 2*b^2/(d*f*x + c*f - (d*f*x*e^(2*e) + c*f*e^(2*e))*e^
(2*f*x)) + (2*a*b + b^2)*log(d*x + c)/d - integrate((2*a*b*d*f*x + 2*a*b*c
*f - b^2*d)/(d^2*f*x^2 + 2*c*d*f*x + c^2*f + (d^2*f*x^2*e^e + 2*c*d*f*x*e^
e + c^2*f*e^e)*e^(f*x)), x) + integrate(-(2*a*b*d*f*x + 2*a*b*c*f - b^2*d)
/(d^2*f*x^2 + 2*c*d*f*x + c^2*f - (d^2*f*x^2*e^e + 2*c*d*f*x*e^e + c^2*f*e
^e)*e^(f*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b\coth(e + fx))^2}{c + dx}\,dx = \int \frac{(b\coth(fx + e) + a)^2}{dx + c}\,dx$$

input
```
integrate((a+b*coth(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*coth(f*x + e) + a)^2/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b\coth(e + fx))^2}{c + dx}\,dx = \int \frac{(a + b\coth(e + f\,x))^2}{c + d\,x}\,dx$$

input
```
int((a + b*coth(e + f*x))^2/(c + d*x),x)
```

output
```
int((a + b*coth(e + f*x))^2/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 258, normalized size of antiderivative = 12.90

$$\int \frac{(a + b\coth(e + fx))^2}{c + dx}\,dx$$
$$= \frac{4e^{2e}\left(\int \frac{e^{2fx}}{e^{4fx+4e}c+e^{4fx+4e}dx-2e^{2fx+2e}c-2e^{2fx+2e}dx+c+dx}dx\right)abd + 4e^{2e}\left(\int \frac{e^{2fx}}{e^{4fx+4e}c+e^{4fx+4e}dx-2e^{2fx+2e}c-2e^{2fx+2e}dx+c+}\right.}{}$$

input
```
int((a+b*coth(f*x+e))^2/(d*x+c),x)
```

output
```
(4*e**(2*e)*int(e**(2*f*x)/(e**(4*e + 4*f*x)*c + e**(4*e + 4*f*x)*d*x - 2*
e**(2*e + 2*f*x)*c - 2*e**(2*e + 2*f*x)*d*x + c + d*x),x)*a*b*d + 4*e**(2*
e)*int(e**(2*f*x)/(e**(4*e + 4*f*x)*c + e**(4*e + 4*f*x)*d*x - 2*e**(2*e +
 2*f*x)*c - 2*e**(2*e + 2*f*x)*d*x + c + d*x),x)*b**2*d - 4*int(1/(e**(4*e
 + 4*f*x)*c + e**(4*e + 4*f*x)*d*x - 2*e**(2*e + 2*f*x)*c - 2*e**(2*e + 2*
f*x)*d*x + c + d*x),x)*a*b*d + log(c + d*x)*a**2 + 2*log(c + d*x)*a*b + lo
g(c + d*x)*b**2)/d
```

3.46 $\int \frac{(a+b\coth(e+fx))^2}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(a+b\coth(e+fx))^2}{(c+dx)^2}\,dx = \operatorname{Int}\left(\frac{(a+b\coth(e+fx))^2}{(c+dx)^2},x\right)$$

output
```
Defer(Int)((a+b*coth(f*x+e))^2/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 22.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\coth(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{(a+b\coth(e+fx))^2}{(c+dx)^2}\,dx$$

input
```
Integrate[(a + b*Coth[e + f*x])^2/(c + d*x)^2,x]
```

output
```
Integrate[(a + b*Coth[e + f*x])^2/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\coth(e+fx))^2}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}{(c+dx)^2}dx$$

$$\Big\downarrow \text{4223}$$

$$\int \frac{(a+b\coth(e+fx))^2}{(c+dx)^2}dx$$

input
```
Int[(a + b*Coth[e + f*x])^2/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\coth(fx+e))^2}{(dx+c)^2}dx$$

input
```
int((a+b*coth(f*x+e))^2/(d*x+c)^2,x)
```

output
```
int((a+b*coth(f*x+e))^2/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.35

$$\int \frac{(a+b\coth(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{(b\coth(fx+e)+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*coth(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((b^2*coth(f*x + e)^2 + 2*a*b*coth(f*x + e) + a^2)/(d^2*x^2 + 2*c*
d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\coth(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{(a+b\coth(e+fx))^2}{(c+dx)^2}\,dx$$

input
```
integrate((a+b*coth(f*x+e))**2/(d*x+c)**2,x)
```

output
```
Integral((a + b*coth(e + f*x))**2/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 375, normalized size of antiderivative = 18.75

$$\int \frac{(a+b\coth(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{(b\coth(fx+e)+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*coth(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-a^2/(d^2*x + c*d) - (2*a*b*c*f + (c*f - 2*d)*b^2 + (2*a*b*d*f + b^2*d*f)*
x - (2*a*b*c*f*e^(2*e) + b^2*c*f*e^(2*e) + (2*a*b*d*f*e^(2*e) + b^2*d*f*e^
(2*e))*x)*e^(2*f*x))/(d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f - (d^3*f*x^2*e^(2*
e) + 2*c*d^2*f*x*e^(2*e) + c^2*d*f*e^(2*e))*e^(2*f*x)) - integrate(2*(a*b*
d*f*x + a*b*c*f - b^2*d)/(d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f
+ (d^3*f*x^3*e^e + 3*c*d^2*f*x^2*e^e + 3*c^2*d*f*x*e^e + c^3*f*e^e)*e^(f*x
)), x) + integrate(-2*(a*b*d*f*x + a*b*c*f - b^2*d)/(d^3*f*x^3 + 3*c*d^2*f
*x^2 + 3*c^2*d*f*x + c^3*f - (d^3*f*x^3*e^e + 3*c*d^2*f*x^2*e^e + 3*c^2*d*
f*x*e^e + c^3*f*e^e)*e^(f*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\coth(e+fx))^2}{(c+dx)^2}\,dx=\int \frac{(b\coth{(fx+e)}+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*coth(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*coth(f*x + e) + a)^2/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\coth(e+fx))^2}{(c+dx)^2}\,dx=\int \frac{(a+b\coth(e+f\,x))^2}{(c+d\,x)^2}\,dx$$

input
```
int((a + b*coth(e + f*x))^2/(c + d*x)^2,x)
```

output
```
int((a + b*coth(e + f*x))^2/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 803, normalized size of antiderivative = 40.15

$$\int \frac{(a+b\coth(e+fx))^2}{(c+dx)^2}\,dx$$
$$= \frac{4e^{2e}\left(\int \frac{e^{2fx}}{e^{4fx+4e}c^2+2e^{4fx+4e}cdx+e^{4fx+4e}d^2x^2-2e^{2fx+2e}c^2-4e^{2fx+2e}cdx-2e^{2fx+2e}d^2x^2+c^2+2cdx+d^2x^2}dx\right)ab\,c^2+4e^{2e}\left(\int \frac{}{e^{4f}}\right.}{}$$

input
```
int((a+b*coth(f*x+e))^2/(d*x+c)^2,x)
```

output
```
(4*e**(2*e)*int(e**(2*f*x)/(e**(4*e + 4*f*x)*c**2 + 2*e**(4*e + 4*f*x)*c*d
*x + e**(4*e + 4*f*x)*d**2*x**2 - 2*e**(2*e + 2*f*x)*c**2 - 4*e**(2*e + 2*
f*x)*c*d*x - 2*e**(2*e + 2*f*x)*d**2*x**2 + c**2 + 2*c*d*x + d**2*x**2),x)
*a*b*c**2 + 4*e**(2*e)*int(e**(2*f*x)/(e**(4*e + 4*f*x)*c**2 + 2*e**(4*e +
 4*f*x)*c*d*x + e**(4*e + 4*f*x)*d**2*x**2 - 2*e**(2*e + 2*f*x)*c**2 - 4*e
**(2*e + 2*f*x)*c*d*x - 2*e**(2*e + 2*f*x)*d**2*x**2 + c**2 + 2*c*d*x + d*
*2*x**2),x)*a*b*c*d*x + 4*e**(2*e)*int(e**(2*f*x)/(e**(4*e + 4*f*x)*c**2 +
 2*e**(4*e + 4*f*x)*c*d*x + e**(4*e + 4*f*x)*d**2*x**2 - 2*e**(2*e + 2*f*x
)*c**2 - 4*e**(2*e + 2*f*x)*c*d*x - 2*e**(2*e + 2*f*x)*d**2*x**2 + c**2 +
2*c*d*x + d**2*x**2),x)*b**2*c**2 + 4*e**(2*e)*int(e**(2*f*x)/(e**(4*e + 4
*f*x)*c**2 + 2*e**(4*e + 4*f*x)*c*d*x + e**(4*e + 4*f*x)*d**2*x**2 - 2*e**
(2*e + 2*f*x)*c**2 - 4*e**(2*e + 2*f*x)*c*d*x - 2*e**(2*e + 2*f*x)*d**2*x*
*2 + c**2 + 2*c*d*x + d**2*x**2),x)*b**2*c*d*x - 4*int(1/(e**(4*e + 4*f*x)
*c**2 + 2*e**(4*e + 4*f*x)*c*d*x + e**(4*e + 4*f*x)*d**2*x**2 - 2*e**(2*e
+ 2*f*x)*c**2 - 4*e**(2*e + 2*f*x)*c*d*x - 2*e**(2*e + 2*f*x)*d**2*x**2 +
c**2 + 2*c*d*x + d**2*x**2),x)*a*b*c**2 - 4*int(1/(e**(4*e + 4*f*x)*c**2 +
 2*e**(4*e + 4*f*x)*c*d*x + e**(4*e + 4*f*x)*d**2*x**2 - 2*e**(2*e + 2*f*x
)*c**2 - 4*e**(2*e + 2*f*x)*c*d*x - 2*e**(2*e + 2*f*x)*d**2*x**2 + c**2 +
2*c*d*x + d**2*x**2),x)*a*b*c*d*x + a**2*x + 2*a*b*x + b**2*x)/(c*(c + d*x
))
```

3.47 $\int (c+dx)^3(a+b\coth(e+fx))^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 556

$$\int (c+dx)^3(a+b\coth(e+fx))^3\,dx = -\frac{3b^3d(c+dx)^2}{2f^2}-\frac{3ab^2(c+dx)^3}{f}+\frac{b^3(c+dx)^3}{2f}+\frac{a^3(c+dx)^4}{4d}-\frac{3a^2b(c+dx)^4}{4d}+\frac{3ab^2(c+dx)^4}{4d}-\frac{b^3(c+dx)^4}{4d}-\frac{3b^3d(c+dx)^2\coth(e+fx)}{2f^2}-\frac{3ab^2(c+dx)^3\coth(e+fx)}{f}-\frac{b^3(c+dx)^3\coth^2(e+fx)}{2f}+\frac{3b^3d^2(c+dx)\log\left(1-e^{2(e+fx)}\right)}{f^3}+\frac{9ab^2d(c+dx)^2\log\left(1-e^{2(e+fx)}\right)}{f^2}+\frac{3a^2b(c+dx)^3\log\left(1-e^{2(e+fx)}\right)}{f}+\frac{b^3(c+dx)^3\log\left(1-e^{2(e+fx)}\right)}{f}+\frac{3b^3d^3\,\text{PolyLog}\left(2,e^{2(e+fx)}\right)}{2f^4}+\frac{9ab^2d^2(c+dx)\,\text{PolyLog}\left(2,e^{2(e+fx)}\right)}{f^3}+\frac{9a^2bd(c+dx)^2\,\text{PolyLog}\left(2,e^{2(e+fx)}\right)}{2f^2}+\frac{3b^3d(c+dx)^2\,\text{PolyLog}\left(2,e^{2(e+fx)}\right)}{2f^2}-\frac{9ab^2d^3\,\text{PolyLog}\left(3,e^{2(e+fx)}\right)}{2f^4}-\frac{9a^2bd^2(c+dx)\,\text{PolyLog}\left(3,e^{2(e+fx)}\right)}{2f^3}-\frac{3b^3d^2(c+dx)\,\text{PolyLog}\left(3,e^{2(e+fx)}\right)}{2f^3}+\frac{9a^2bd^3\,\text{PolyLog}\left(4,e^{2(e+fx)}\right)}{4f^4}+\frac{3b^3d^3\,\text{PolyLog}\left(4,e^{2(e+fx)}\right)}{4f^4}$$

output
```
-3/2*b^3*d*(d*x+c)^2/f^2-3*a*b^2*(d*x+c)^3/f+1/2*b^3*(d*x+c)^3/f+1/4*a^3*(
d*x+c)^4/d-3/4*a^2*b*(d*x+c)^4/d+3/4*a*b^2*(d*x+c)^4/d-1/4*b^3*(d*x+c)^4/d
-3/2*b^3*d*(d*x+c)^2*coth(f*x+e)/f^2-3*a*b^2*(d*x+c)^3*coth(f*x+e)/f-1/2*b
^3*(d*x+c)^3*coth(f*x+e)^2/f+3*b^3*d^2*(d*x+c)*ln(1-exp(2*f*x+2*e))/f^3+9*
a*b^2*d*(d*x+c)^2*ln(1-exp(2*f*x+2*e))/f^2+3*a^2*b*(d*x+c)^3*ln(1-exp(2*f*
x+2*e))/f+b^3*(d*x+c)^3*ln(1-exp(2*f*x+2*e))/f+3/2*b^3*d^3*polylog(2,exp(2
*f*x+2*e))/f^4+9*a*b^2*d^2*(d*x+c)*polylog(2,exp(2*f*x+2*e))/f^3+9/2*a^2*b
*d*(d*x+c)^2*polylog(2,exp(2*f*x+2*e))/f^2+3/2*b^3*d*(d*x+c)^2*polylog(2,e
xp(2*f*x+2*e))/f^2-9/2*a*b^2*d^3*polylog(3,exp(2*f*x+2*e))/f^4-9/2*a^2*b*d
^2*(d*x+c)*polylog(3,exp(2*f*x+2*e))/f^3-3/2*b^3*d^2*(d*x+c)*polylog(3,exp
(2*f*x+2*e))/f^3+9/4*a^2*b*d^3*polylog(4,exp(2*f*x+2*e))/f^4+3/4*b^3*d^3*p
olylog(4,exp(2*f*x+2*e))/f^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2043 vs. $2(556) = 1112$.

Time = 7.90 (sec) , antiderivative size = 2043, normalized size of antiderivative = 3.67

$$\int (c+dx)^3(a+b\coth(e+fx))^3\,dx = \text{Result too large to show}$$

input
```
Integrate[(c + d*x)^3*(a + b*Coth[e + f*x])^3,x]
```

output
```
((-(b^3*c^3) - 3*b^3*c^2*d*x - 3*b^3*c*d^2*x^2 - b^3*d^3*x^3)*Csch[e + f*x
]^2)/(2*f) - (b*E^(2*e)*(24*b^2*c*d^2*x + 72*a*b*c^2*d*f*x + 24*a^2*c^3*f^
2*x + 8*b^2*c^3*f^2*x + 12*b^2*d^3*x^2 + 72*a*b*c*d^2*f*x^2 + 36*a^2*c^2*d
*f^2*x^2 + 12*b^2*c^2*d*f^2*x^2 + 24*a*b*d^3*f*x^3 + 24*a^2*c*d^2*f^2*x^3
+ 8*b^2*c*d^2*f^2*x^3 + 6*a^2*d^3*f^2*x^4 + 2*b^2*d^3*f^2*x^4 - 36*a*b*c^2
*d*Log[1 - E^(2*(e + f*x))] + (36*a*b*c^2*d*Log[1 - E^(2*(e + f*x))])/E^(2
*e) - (12*b^2*c*d^2*Log[1 - E^(2*(e + f*x))])/f + (12*b^2*c*d^2*Log[1 - E^
(2*(e + f*x))])/(E^(2*e)*f) - 12*a^2*c^3*f*Log[1 - E^(2*(e + f*x))] - 4*b^
2*c^3*f*Log[1 - E^(2*(e + f*x))] + (12*a^2*c^3*f*Log[1 - E^(2*(e + f*x))])
/E^(2*e) + (4*b^2*c^3*f*Log[1 - E^(2*(e + f*x))])/E^(2*e) - 72*a*b*c*d^2*x
*Log[1 - E^(2*(e + f*x))] + (72*a*b*c*d^2*x*Log[1 - E^(2*(e + f*x))])/E^(2
*e) - (12*b^2*d^3*x*Log[1 - E^(2*(e + f*x))])/f + (12*b^2*d^3*x*Log[1 - E^
(2*(e + f*x))])/(E^(2*e)*f) - 36*a^2*c^2*d*f*x*Log[1 - E^(2*(e + f*x))] -
12*b^2*c^2*d*f*x*Log[1 - E^(2*(e + f*x))] + (36*a^2*c^2*d*f*x*Log[1 - E^(2
*(e + f*x))])/E^(2*e) + (12*b^2*c^2*d*f*x*Log[1 - E^(2*(e + f*x))])/E^(2*e
) - 36*a*b*d^3*x^2*Log[1 - E^(2*(e + f*x))] + (36*a*b*d^3*x^2*Log[1 - E^(2
*(e + f*x))])/E^(2*e) - 36*a^2*c*d^2*f*x^2*Log[1 - E^(2*(e + f*x))] - 12*b
^2*c*d^2*f*x^2*Log[1 - E^(2*(e + f*x))] + (36*a^2*c*d^2*f*x^2*Log[1 - E^(2
*(e + f*x))])/E^(2*e) + (12*b^2*c*d^2*f*x^2*Log[1 - E^(2*(e + f*x))])/E^(2
*e) - 12*a^2*d^3*f*x^3*Log[1 - E^(2*(e + f*x))] - 4*b^2*d^3*f*x^3*Log[1...
```

Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a+b\coth(e+fx))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^3\left(a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{ 4205}$$

$$\int \left(a^3(c+dx)^3+3a^2b(c+dx)^3\coth(e+fx)+3ab^2(c+dx)^3\coth^2(e+fx)+b^3(c+dx)^3\coth^3(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3(c+dx)^4}{4d}-\frac{9a^2bd^2(c+dx)\operatorname{PolyLog}\left(3,e^{2(e+fx)}\right)}{2f^3}+\frac{9a^2bd(c+dx)^2\operatorname{PolyLog}\left(2,e^{2(e+fx)}\right)}{2f^2}+\frac{3a^2b(c+dx)^3\log\left(1-e^{2(e+fx)}\right)}{f}-\frac{3a^2b(c+dx)^4}{4d}+\frac{9a^2bd^3\operatorname{PolyLog}\left(4,e^{2(e+fx)}\right)}{4f^4}+\frac{9ab^2d^2(c+dx)\operatorname{PolyLog}\left(2,e^{2(e+fx)}\right)}{f^3}+\frac{9ab^2d(c+dx)^2\log\left(1-e^{2(e+fx)}\right)}{f^2}-\frac{3ab^2(c+dx)^3\coth(e+fx)}{f}-\frac{3ab^2(c+dx)^3}{f}+\frac{3ab^2(c+dx)^4}{4d}-\frac{9ab^2d^3\operatorname{PolyLog}\left(3,e^{2(e+fx)}\right)}{2f^4}-\frac{3b^3d^2(c+dx)\operatorname{PolyLog}\left(3,e^{2(e+fx)}\right)}{2f^3}+\frac{3b^3d^2(c+dx)\log\left(1-e^{2(e+fx)}\right)}{f^3}+\frac{3b^3d(c+dx)^2\operatorname{PolyLog}\left(2,e^{2(e+fx)}\right)}{2f^2}-\frac{3b^3d(c+dx)^2\coth(e+fx)}{2f^2}+\frac{b^3(c+dx)^3\log\left(1-e^{2(e+fx)}\right)}{f}-\frac{b^3(c+dx)^3\coth^2(e+fx)}{2f}-\frac{3b^3d(c+dx)^2}{2f^2}+\frac{b^3(c+dx)^3}{2f}-\frac{b^3(c+dx)^4}{4d}+\frac{3b^3d^3\operatorname{PolyLog}\left(2,e^{2(e+fx)}\right)}{2f^4}+\frac{3b^3d^3\operatorname{PolyLog}\left(4,e^{2(e+fx)}\right)}{4f^4}$$

input
```
Int[(c + d*x)^3*(a + b*Coth[e + f*x])^3,x]
```

output
```
(-3*b^3*d*(c + d*x)^2)/(2*f^2) - (3*a*b^2*(c + d*x)^3)/f + (b^3*(c + d*x)^
3)/(2*f) + (a^3*(c + d*x)^4)/(4*d) - (3*a^2*b*(c + d*x)^4)/(4*d) + (3*a*b^
2*(c + d*x)^4)/(4*d) - (b^3*(c + d*x)^4)/(4*d) - (3*b^3*d*(c + d*x)^2*Coth
[e + f*x])/(2*f^2) - (3*a*b^2*(c + d*x)^3*Coth[e + f*x])/f - (b^3*(c + d*x
)^3*Coth[e + f*x]^2)/(2*f) + (3*b^3*d^2*(c + d*x)*Log[1 - E^(2*(e + f*x))]
)/f^3 + (9*a*b^2*d*(c + d*x)^2*Log[1 - E^(2*(e + f*x))])/f^2 + (3*a^2*b*(c
 + d*x)^3*Log[1 - E^(2*(e + f*x))])/f + (b^3*(c + d*x)^3*Log[1 - E^(2*(e +
 f*x))])/f + (3*b^3*d^3*PolyLog[2, E^(2*(e + f*x))])/(2*f^4) + (9*a*b^2*d^
2*(c + d*x)*PolyLog[2, E^(2*(e + f*x))])/f^3 + (9*a^2*b*d*(c + d*x)^2*Poly
Log[2, E^(2*(e + f*x))])/(2*f^2) + (3*b^3*d*(c + d*x)^2*PolyLog[2, E^(2*(e
 + f*x))])/(2*f^2) - (9*a*b^2*d^3*PolyLog[3, E^(2*(e + f*x))])/(2*f^4) - (
9*a^2*b*d^2*(c + d*x)*PolyLog[3, E^(2*(e + f*x))])/(2*f^3) - (3*b^3*d^2*(c
 + d*x)*PolyLog[3, E^(2*(e + f*x))])/(2*f^3) + (9*a^2*b*d^3*PolyLog[4, E^(
2*(e + f*x))])/(4*f^4) + (3*b^3*d^3*PolyLog[4, E^(2*(e + f*x))])/(4*f^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2822 vs. $2(524) = 1048$.

Time = 0.54 (sec) , antiderivative size = 2823, normalized size of antiderivative = 5.08

method	result	size
risch	Expression too large to display	2823

input
```
int((d*x+c)^3*(a+b*coth(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*d^3*a^3*x^4-1/4*d^3*b^3*x^4+1/4/d*c^4*a^3+1/4/d*c^4*b^3+6/f^4*b^3*e*d^
3*ln(exp(f*x+e))+3/f^3*b^3*d^3*ln(1-exp(f*x+e))*x+3/f^4*b^3*d^3*ln(1-exp(f
*x+e))*e+3/f^3*b^3*d^3*ln(exp(f*x+e)+1)*x+1/f*b^3*d^3*ln(1-exp(f*x+e))*x^3
+3/f^2*b^3*d^3*polylog(2,exp(f*x+e))*x^2-6/f^3*b^3*d^3*polylog(3,exp(f*x+e
))*x+1/f*b^3*d^3*ln(exp(f*x+e)+1)*x^3+3/f^2*b^3*d^3*polylog(2,-exp(f*x+e))
*x^2-6/f^3*b^3*d^3*polylog(3,-exp(f*x+e))*x+1/f^4*b^3*d^3*ln(1-exp(f*x+e))
*e^3+18/f^4*b*a^2*d^3*polylog(4,exp(f*x+e))+18/f^4*b*a^2*d^3*polylog(4,-ex
p(f*x+e))-6/f^3*b^3*c*d^2*polylog(3,exp(f*x+e))-6/f^3*b^3*c*d^2*polylog(3,
-exp(f*x+e))+3/f*b*a^2*c^3*ln(exp(f*x+e)-1)-6/f*b*a^2*c^3*ln(exp(f*x+e))+3
/f*b*a^2*c^3*ln(exp(f*x+e)+1)+3/f^3*b^3*c*d^2*ln(exp(f*x+e)-1)-6/f^3*b^3*c
*d^2*ln(exp(f*x+e))+3/f^3*b^3*c*d^2*ln(exp(f*x+e)+1)-6/f*b^2*a*d^3*x^3-3/f
^2*b^3*d*c^2*e^2-6/f^3*b^3*d^3*e*x-2/f^3*b^3*d^3*e^3*x-9/2/f^4*b*a^2*d^3*e
^4+4/f^3*b^3*c*d^2*e^3+12/f^4*b^2*e^3*a*d^3-18/f^4*b^2*a*d^3*polylog(3,exp
(f*x+e))-18/f^4*b^2*a*d^3*polylog(3,-exp(f*x+e))+3/f^2*b^3*d*c^2*polylog(2
,exp(f*x+e))+3/f^2*b^3*d*c^2*polylog(2,-exp(f*x+e))-1/f^4*b^3*e^3*d^3*ln(e
xp(f*x+e)-1)+2/f^4*b^3*e^3*d^3*ln(exp(f*x+e))-3/f^4*b^3*e*d^3*ln(exp(f*x+e
)-1)-9/f^2*b*c^2*a^2*d*e^2+12/f^3*b*c*a^2*d^2*e^3+9/f^2*b*a^2*d^3*polylog(
2,-exp(f*x+e))*x^2-18/f^3*b*a^2*d^3*polylog(3,-exp(f*x+e))*x+3/f^4*b*a^2*d
^3*ln(1-exp(f*x+e))*e^3-3/f^3*b^3*c*d^2*ln(1-exp(f*x+e))*e^2+9/f^2*b*c^2*a
^2*d*polylog(2,exp(f*x+e))+9/f^2*b*c^2*a^2*d*polylog(2,-exp(f*x+e))-18/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11137 vs. $2(520) = 1040$.

Time = 0.28 (sec) , antiderivative size = 11137, normalized size of antiderivative = 20.03

$$\int (c + dx)^3 (a + b \coth(e + fx))^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(a+b*coth(f*x+e))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int (c+dx)^3(a+b\coth(e+fx))^3\,dx=\int (a+b\coth{(e+fx)})^3\,(c+dx)^3\,dx$$

input
```
integrate((d*x+c)**3*(a+b*coth(f*x+e))**3,x)
```

output
```
Integral((a + b*coth(e + f*x))**3*(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1531 vs. 2(520) = 1040.

Time = 0.44 (sec) , antiderivative size = 1531, normalized size of antiderivative = 2.75

$$\int (c+dx)^3(a+b\coth(e+fx))^3\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)^3*(a+b*coth(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/4*a^3*d^3*x^4 + a^3*c*d^2*x^3 + 3/2*a^3*c^2*d*x^2 + a^3*c^3*x + 3*a^2*b*
c^3*log(sinh(f*x + e))/f + 1/4*(24*a*b^2*c^3*f + 12*b^3*c^2*d + (3*a^2*b*d
^3*f^2 + 3*a*b^2*d^3*f^2 + b^3*d^3*f^2)*x^4 + 4*(3*a^2*b*c*d^2*f^2 + b^3*c
*d^2*f^2 + 3*(c*d^2*f^2 + 2*d^3*f)*a*b^2)*x^3 + 6*(3*a^2*b*c^2*d*f^2 + 3*(
c^2*d*f^2 + 4*c*d^2*f)*a*b^2 + (c^2*d*f^2 + 2*d^3)*b^3)*x^2 + 4*(3*(c^3*f^
2 + 6*c^2*d*f)*a*b^2 + (c^3*f^2 + 6*c*d^2)*b^3)*x + ((3*a^2*b*d^3*f^2*e^(4
*e) + 3*a*b^2*d^3*f^2*e^(4*e) + b^3*d^3*f^2*e^(4*e))*x^4 + 4*(3*a^2*b*c*d^
2*f^2*e^(4*e) + 3*a*b^2*c*d^2*f^2*e^(4*e) + b^3*c*d^2*f^2*e^(4*e))*x^3 + 6
*(3*a^2*b*c^2*d*f^2*e^(4*e) + 3*a*b^2*c^2*d*f^2*e^(4*e) + b^3*c^2*d*f^2*e^
(4*e))*x^2 + 4*(3*a*b^2*c^3*f^2*e^(4*e) + b^3*c^3*f^2*e^(4*e))*x)*e^(4*f*x
) - 2*(12*a*b^2*c^3*f*e^(2*e) + (3*a^2*b*d^3*f^2*e^(2*e) + 3*a*b^2*d^3*f^2
*e^(2*e) + b^3*d^3*f^2*e^(2*e))*x^4 + 2*(2*c^3*f*e^(2*e) + 3*c^2*d*e^(2*e)
)*b^3 + 4*(3*a^2*b*c*d^2*f^2*e^(2*e) + 3*(c*d^2*f^2*e^(2*e) + d^3*f*e^(2*e
))*a*b^2 + (c*d^2*f^2*e^(2*e) + d^3*f*e^(2*e))*b^3)*x^3 + 6*(3*a^2*b*c^2*d
*f^2*e^(2*e) + 3*(c^2*d*f^2*e^(2*e) + 2*c*d^2*f*e^(2*e))*a*b^2 + (c^2*d*f^
2*e^(2*e) + 2*c*d^2*f*e^(2*e) + d^3*e^(2*e))*b^3)*x^2 + 4*(3*(c^3*f^2*e^(2
*e) + 3*c^2*d*f*e^(2*e))*a*b^2 + (c^3*f^2*e^(2*e) + 3*c^2*d*f*e^(2*e) + 3*
c*d^2*e^(2*e))*b^3)*x)*e^(2*f*x))/(f^2*e^(4*f*x + 4*e) - 2*f^2*e^(2*f*x +
2*e) + f^2) - 2*(9*a*b^2*c^2*d*f + (c^3*f^2 + 3*c*d^2)*b^3)*x/f^2 + (9*a*b
^2*c^2*d*f + (c^3*f^2 + 3*c*d^2)*b^3)*log(e^(f*x + e) + 1)/f^3 + (9*a*b...
```

Giac [F]

$$\int (c+dx)^3(a+b\coth(e+fx))^3\,dx=\int (dx+c)^3(b\coth(fx+e)+a)^3\,dx$$

input
```
integrate((d*x+c)^3*(a+b*coth(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*(b*coth(f*x + e) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3(a+b\coth(e+fx))^3\,dx=\int {\left(a+b\,\mathrm{coth}\left(e+f\,x\right)\right)}^3\,{\left(c+d\,x\right)}^3 \,d x$$

input
```
int((a + b*coth(e + f*x))^3*(c + d*x)^3,x)
```

output
```
int((a + b*coth(e + f*x))^3*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3(a+b\coth(e+fx))^3\,dx=\text{too large to display}$$

input
```
int((d*x+c)^3*(a+b*coth(f*x+e))^3,x)
```

output
```
(384*e**(4*e + 4*f*x)*int(x**3/(e**(6*e + 6*f*x) - 3*e**(4*e + 4*f*x) + 3*
e**(2*e + 2*f*x) - 1),x)*a**2*b*d**3*f**4 + 128*e**(4*e + 4*f*x)*int(x**3/
(e**(6*e + 6*f*x) - 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) - 1),x)*b**3*d
**3*f**4 + 1152*e**(4*e + 4*f*x)*int(x**2/(e**(6*e + 6*f*x) - 3*e**(4*e +
4*f*x) + 3*e**(2*e + 2*f*x) - 1),x)*a**2*b*c*d**2*f**4 + 864*e**(4*e + 4*f
*x)*int(x**2/(e**(6*e + 6*f*x) - 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) -
 1),x)*a**2*b*d**3*f**3 + 1152*e**(4*e + 4*f*x)*int(x**2/(e**(6*e + 6*f*x)
 - 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) - 1),x)*a*b**2*d**3*f**3 + 384*
e**(4*e + 4*f*x)*int(x**2/(e**(6*e + 6*f*x) - 3*e**(4*e + 4*f*x) + 3*e**(2
*e + 2*f*x) - 1),x)*b**3*c*d**2*f**4 + 288*e**(4*e + 4*f*x)*int(x**2/(e**(
6*e + 6*f*x) - 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) - 1),x)*b**3*d**3*f
**3 + 1152*e**(4*e + 4*f*x)*int(x/(e**(6*e + 6*f*x) - 3*e**(4*e + 4*f*x) +
 3*e**(2*e + 2*f*x) - 1),x)*a**2*b*c**2*d*f**4 + 1728*e**(4*e + 4*f*x)*int
(x/(e**(6*e + 6*f*x) - 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) - 1),x)*a**
2*b*c*d**2*f**3 + 1008*e**(4*e + 4*f*x)*int(x/(e**(6*e + 6*f*x) - 3*e**(4*
e + 4*f*x) + 3*e**(2*e + 2*f*x) - 1),x)*a**2*b*d**3*f**2 + 2304*e**(4*e +
4*f*x)*int(x/(e**(6*e + 6*f*x) - 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) -
 1),x)*a*b**2*c*d**2*f**3 + 1728*e**(4*e + 4*f*x)*int(x/(e**(6*e + 6*f*x)
- 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) - 1),x)*a*b**2*d**3*f**2 + 384*e
**(4*e + 4*f*x)*int(x/(e**(6*e + 6*f*x) - 3*e**(4*e + 4*f*x) + 3*e**(2*...
```

3.48 $\int (c+dx)^2(a+b\coth(e+fx))^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 392

$$\int (c+dx)^2(a+b\coth(e+fx))^3\,dx = -\frac{3ab^2(c+dx)^2}{f}+\frac{b^3(c+dx)^2}{2f}+\frac{a^3(c+dx)^3}{3d}-\frac{a^2b(c+dx)^3}{d}+\frac{ab^2(c+dx)^3}{d}-\frac{b^3(c+dx)^3}{3d}-\frac{b^3d(c+dx)\coth(e+fx)}{f^2}-\frac{3ab^2(c+dx)^2\coth(e+fx)}{f}-\frac{b^3(c+dx)^2\coth^2(e+fx)}{2f}+\frac{6ab^2d(c+dx)\log\left(1-e^{2(e+fx)}\right)}{f^2}+\frac{3a^2b(c+dx)^2\log\left(1-e^{2(e+fx)}\right)}{f}+\frac{b^3(c+dx)^2\log\left(1-e^{2(e+fx)}\right)}{f}+\frac{b^3d^2\log(\sinh(e+fx))}{f^3}+\frac{3ab^2d^2\,\text{PolyLog}\left(2,e^{2(e+fx)}\right)}{f^3}+\frac{3a^2bd(c+dx)\,\text{PolyLog}\left(2,e^{2(e+fx)}\right)}{f^2}+\frac{b^3d(c+dx)\,\text{PolyLog}\left(2,e^{2(e+fx)}\right)}{f^2}-\frac{3a^2bd^2\,\text{PolyLog}\left(3,e^{2(e+fx)}\right)}{2f^3}-\frac{b^3d^2\,\text{PolyLog}\left(3,e^{2(e+fx)}\right)}{2f^3}$$

output
```
-3*a*b^2*(d*x+c)^2/f+1/2*b^3*(d*x+c)^2/f+1/3*a^3*(d*x+c)^3/d-a^2*b*(d*x+c)
^3/d+a*b^2*(d*x+c)^3/d-1/3*b^3*(d*x+c)^3/d-b^3*d*(d*x+c)*coth(f*x+e)/f^2-3
*a*b^2*(d*x+c)^2*coth(f*x+e)/f-1/2*b^3*(d*x+c)^2*coth(f*x+e)^2/f+6*a*b^2*d
*(d*x+c)*ln(1-exp(2*f*x+2*e))/f^2+3*a^2*b*(d*x+c)^2*ln(1-exp(2*f*x+2*e))/f
+b^3*(d*x+c)^2*ln(1-exp(2*f*x+2*e))/f+b^3*d^2*ln(sinh(f*x+e))/f^3+3*a*b^2*
d^2*polylog(2,exp(2*f*x+2*e))/f^3+3*a^2*b*d*(d*x+c)*polylog(2,exp(2*f*x+2*
e))/f^2+b^3*d*(d*x+c)*polylog(2,exp(2*f*x+2*e))/f^2-3/2*a^2*b*d^2*polylog(
3,exp(2*f*x+2*e))/f^3-1/2*b^3*d^2*polylog(3,exp(2*f*x+2*e))/f^3
```

Mathematica [A] (verified)

Time = 4.86 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.49

$$\int (c+dx)^2(a+b\coth(e+fx))^3\,dx$$
$$= \frac{-\frac{8be^{2e}fx(9abdf(2c+dx)+3a^2f^2(3c^2+3cdx+d^2x^2)+b^2(3c^2f^2+3cdf^2x+d^2(3+f^2x^2)))}{-1+e^{2e}} + 12b(6abdf(c+dx)+3a^2f^2(c+dx)^2}{}$$

input
```
Integrate[(c + d*x)^2*(a + b*Coth[e + f*x])^3,x]
```

output
```
((-8*b*E^(2*e)*f*x*(9*a*b*d*f*(2*c + d*x) + 3*a^2*f^2*(3*c^2 + 3*c*d*x + d
^2*x^2) + b^2*(3*c^2*f^2 + 3*c*d*f^2*x + d^2*(3 + f^2*x^2))))/(-1 + E^(2*e
)) + 12*b*(6*a*b*d*f*(c + d*x) + 3*a^2*f^2*(c + d*x)^2 + b^2*(c^2*f^2 + 2*
c*d*f^2*x + d^2*(1 + f^2*x^2)))*Log[1 - E^(2*(e + f*x))] + 12*b*d*(3*a*b*d
 + 3*a^2*f*(c + d*x) + b^2*f*(c + d*x))*PolyLog[2, E^(2*(e + f*x))] - 6*b*
(3*a^2 + b^2)*d^2*PolyLog[3, E^(2*(e + f*x))] + f*Csch[e]*Csch[e + f*x]^2*
(-2*b*(9*a*b*f*(c + d*x)^2 + 3*a^2*f^2*x*(3*c^2 + 3*c*d*x + d^2*x^2) + b^2
*(3*c^2*f^2*x + d^2*x*(3 + f^2*x^2) + 3*c*(d + d*f^2*x^2)))*Cosh[e] + b*(1
8*a*b*f*(c + d*x)^2 + 3*a^2*f^2*x*(3*c^2 + 3*c*d*x + d^2*x^2) + b^2*(3*c^2
*f^2*x + 3*c*d*(2 + f^2*x^2) + d^2*x*(6 + f^2*x^2)))*Cosh[e + 2*f*x] + f*(
b*(3*a^2 + b^2)*f*x*(3*c^2 + 3*c*d*x + d^2*x^2)*Cosh[3*e + 2*f*x] - 2*(3*b
^3*(c + d*x)^2 + a^3*f*x*(3*c^2 + 3*c*d*x + d^2*x^2) + 3*a*b^2*f*x*(3*c^2
+ 3*c*d*x + d^2*x^2) - a*(a^2 + 3*b^2)*f*x*(3*c^2 + 3*c*d*x + d^2*x^2)*Cos
h[2*(e + f*x)])*Sinh[e])))/(12*f^3)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a+b\coth(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2\left(a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4205}$$

$$\int \left(a^3(c+dx)^2+3a^2b(c+dx)^2\coth(e+fx)+3ab^2(c+dx)^2\coth^2(e+fx)+b^3(c+dx)^2\coth^3(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3(c+dx)^3}{3d}+\frac{3a^2bd(c+dx)\text{PolyLog}\left(2,e^{2(e+fx)}\right)}{f^2}+\frac{3a^2b(c+dx)^2\log\left(1-e^{2(e+fx)}\right)}{f}-\frac{a^2b(c+dx)^3}{d}-\frac{3a^2bd^2\text{PolyLog}\left(3,e^{2(e+fx)}\right)}{2f^3}+\frac{6ab^2d(c+dx)\log\left(1-e^{2(e+fx)}\right)}{f^2}-\frac{3ab^2(c+dx)^2\coth(e+fx)}{f}-\frac{3ab^2(c+dx)^2}{f}+\frac{ab^2(c+dx)^3}{d}+\frac{3ab^2d^2\text{PolyLog}\left(2,e^{2(e+fx)}\right)}{f^3}+\frac{b^3d(c+dx)\text{PolyLog}\left(2,e^{2(e+fx)}\right)}{f^2}-\frac{b^3d(c+dx)\coth(e+fx)}{f^2}+\frac{b^3(c+dx)^2\log\left(1-e^{2(e+fx)}\right)}{f}-\frac{b^3(c+dx)^2\coth^2(e+fx)}{2f}+\frac{b^3(c+dx)^2}{2f}-\frac{b^3(c+dx)^3}{3d}-\frac{b^3d^2\text{PolyLog}\left(3,e^{2(e+fx)}\right)}{2f^3}+\frac{b^3d^2\log(\sinh(e+fx))}{f^3}$$

input
```
Int[(c + d*x)^2*(a + b*Coth[e + f*x])^3,x]
```

output
```
(-3*a*b^2*(c + d*x)^2)/f + (b^3*(c + d*x)^2)/(2*f) + (a^3*(c + d*x)^3)/(3*
d) - (a^2*b*(c + d*x)^3)/d + (a*b^2*(c + d*x)^3)/d - (b^3*(c + d*x)^3)/(3*
d) - (b^3*d*(c + d*x)*Coth[e + f*x])/f^2 - (3*a*b^2*(c + d*x)^2*Coth[e + f
*x])/f - (b^3*(c + d*x)^2*Coth[e + f*x]^2)/(2*f) + (6*a*b^2*d*(c + d*x)*Lo
g[1 - E^(2*(e + f*x))])/f^2 + (3*a^2*b*(c + d*x)^2*Log[1 - E^(2*(e + f*x))
])/f + (b^3*(c + d*x)^2*Log[1 - E^(2*(e + f*x))])/f + (b^3*d^2*Log[Sinh[e
+ f*x]])/f^3 + (3*a*b^2*d^2*PolyLog[2, E^(2*(e + f*x))])/f^3 + (3*a^2*b*d*
(c + d*x)*PolyLog[2, E^(2*(e + f*x))])/f^2 + (b^3*d*(c + d*x)*PolyLog[2, E
^(2*(e + f*x))])/f^2 - (3*a^2*b*d^2*PolyLog[3, E^(2*(e + f*x))])/(2*f^3) -
 (b^3*d^2*PolyLog[3, E^(2*(e + f*x))])/(2*f^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1585 vs. $2(380) = 760$.

Time = 0.40 (sec) , antiderivative size = 1586, normalized size of antiderivative = 4.05

method	result	size
risch	Expression too large to display	1586

input
```
int((d*x+c)^2*(a+b*coth(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-d^2*a^2*b*x^3+d^2*a*b^2*x^3+d*a^3*c*x^2-d*b^3*c*x^2+a^3*c^2*x+b^3*c^2*x+1
/d*a^2*b*c^3+1/d*a*b^2*c^3+4/3/f^3*b^3*d^2*e^3+1/f*b^3*c^2*ln(exp(f*x+e)-1
)-2/f*b^3*c^2*ln(exp(f*x+e))+1/f*b^3*c^2*ln(exp(f*x+e)+1)+1/f^3*b^3*d^2*ln
(exp(f*x+e)-1)-2/f^3*b^3*d^2*ln(exp(f*x+e))+1/f^3*b^3*d^2*ln(exp(f*x+e)+1)
-2/f^3*b^3*d^2*polylog(3,exp(f*x+e))-2/f^3*b^3*d^2*polylog(3,-exp(f*x+e))+
6/f*b*c*a^2*d*ln(1-exp(f*x+e))*x+6/f^2*b*c*a^2*d*ln(1-exp(f*x+e))*e+6/f*b*
c*a^2*d*ln(exp(f*x+e)+1)*x-6/f^2*b*e*c*a^2*d*ln(exp(f*x+e)-1)+12/f^2*b*e*c
*a^2*d*ln(exp(f*x+e))-2*b^2*(3*a*d^2*f*x^2*exp(2*f*x+2*e)+b*d^2*f*x^2*exp(
2*f*x+2*e)+6*a*c*d*f*x*exp(2*f*x+2*e)+2*b*c*d*f*x*exp(2*f*x+2*e)+3*a*c^2*f
*exp(2*f*x+2*e)-3*a*d^2*f*x^2+b*c^2*f*exp(2*f*x+2*e)+b*d^2*x*exp(2*f*x+2*e
)-6*a*c*d*f*x+b*c*d*exp(2*f*x+2*e)-3*a*c^2*f-b*d^2*x-b*c*d)/f^2/(exp(2*f*x
+2*e)-1)^2+4/f^3*b*a^2*d^2*e^3-2/f^2*b^3*d*c*e^2+2/f^2*b^3*d^2*e^2*x-6/f*b
^2*a*d^2*x^2+6/f^3*b^2*a*d^2*polylog(2,-exp(f*x+e))+1/f^3*b^3*e^2*d^2*ln(e
xp(f*x+e)-1)-2/f^3*b^3*e^2*d^2*ln(exp(f*x+e))-1/f^3*b^3*d^2*ln(1-exp(f*x+e
))*e^2+3/f*b*a^2*c^2*ln(exp(f*x+e)-1)-6/f*b*a^2*c^2*ln(exp(f*x+e))+3/f*b*a
^2*c^2*ln(exp(f*x+e)+1)-6/f^3*b*a^2*d^2*polylog(3,exp(f*x+e))-6/f^3*b*a^2*
d^2*polylog(3,-exp(f*x+e))+2/f^2*b^3*d*c*polylog(2,exp(f*x+e))+2/f^2*b^3*d
*c*polylog(2,-exp(f*x+e))+1/f*b^3*d^2*ln(1-exp(f*x+e))*x^2+2/f^2*b^3*d^2*p
olylog(2,exp(f*x+e))*x+1/f*b^3*d^2*ln(exp(f*x+e)+1)*x^2+2/f^2*b^3*d^2*poly
log(2,-exp(f*x+e))*x+6/f^3*b^2*a*d^2*polylog(2,exp(f*x+e))-6/f^3*b^2*a*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6356 vs. $2(377) = 754$.

Time = 0.16 (sec) , antiderivative size = 6356, normalized size of antiderivative = 16.21

$$\int (c+dx)^2(a+b\coth(e+fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(a+b*coth(f*x+e))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int (c+dx)^2(a+b\coth(e+fx))^3\,dx = \int (a+b\coth{(e+fx)})^3\,(c+dx)^2\,dx$$

input
```
integrate((d*x+c)**2*(a+b*coth(f*x+e))**3,x)
```

output
```
Integral((a + b*coth(e + f*x))**3*(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 997 vs. $2(377) = 754$.

Time = 0.37 (sec) , antiderivative size = 997, normalized size of antiderivative = 2.54

$$\int (c+dx)^2(a+b\coth(e+fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(a+b*coth(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/3*a^3*d^2*x^3 + a^3*c*d*x^2 + a^3*c^2*x + 3*a^2*b*c^2*log(sinh(f*x + e))
/f + 1/3*(18*a*b^2*c^2*f + 6*b^3*c*d + (3*a^2*b*d^2*f^2 + 3*a*b^2*d^2*f^2
+ b^3*d^2*f^2)*x^3 + 3*(3*a^2*b*c*d*f^2 + b^3*c*d*f^2 + 3*(c*d*f^2 + 2*d^2
*f)*a*b^2)*x^2 + 3*(3*(c^2*f^2 + 4*c*d*f)*a*b^2 + (c^2*f^2 + 2*d^2)*b^3)*x
 + ((3*a^2*b*d^2*f^2*e^(4*e) + 3*a*b^2*d^2*f^2*e^(4*e) + b^3*d^2*f^2*e^(4*
e))*x^3 + 3*(3*a^2*b*c*d*f^2*e^(4*e) + 3*a*b^2*c*d*f^2*e^(4*e) + b^3*c*d*f
^2*e^(4*e))*x^2 + 3*(3*a*b^2*c^2*f^2*e^(4*e) + b^3*c^2*f^2*e^(4*e))*x)*e^(
4*f*x) - 2*(9*a*b^2*c^2*f*e^(2*e) + 3*(c^2*f*e^(2*e) + c*d*e^(2*e))*b^3 +
(3*a^2*b*d^2*f^2*e^(2*e) + 3*a*b^2*d^2*f^2*e^(2*e) + b^3*d^2*f^2*e^(2*e))*
x^3 + 3*(3*a^2*b*c*d*f^2*e^(2*e) + 3*(c*d*f^2*e^(2*e) + d^2*f*e^(2*e))*a*b
^2 + (c*d*f^2*e^(2*e) + d^2*f*e^(2*e))*b^3)*x^2 + 3*(3*(c^2*f^2*e^(2*e) +
2*c*d*f*e^(2*e))*a*b^2 + (c^2*f^2*e^(2*e) + 2*c*d*f*e^(2*e) + d^2*e^(2*e))
*b^3)*x)*e^(2*f*x))/(f^2*e^(4*f*x + 4*e) - 2*f^2*e^(2*f*x + 2*e) + f^2) -
2*(6*a*b^2*c*d*f + (c^2*f^2 + d^2)*b^3)*x/f^2 + (3*a^2*b*d^2 + b^3*d^2)*(f
^2*x^2*log(e^(f*x + e) + 1) + 2*f*x*dilog(-e^(f*x + e)) - 2*polylog(3, -e^
(f*x + e)))/f^3 + (3*a^2*b*d^2 + b^3*d^2)*(f^2*x^2*log(-e^(f*x + e) + 1) +
 2*f*x*dilog(e^(f*x + e)) - 2*polylog(3, e^(f*x + e)))/f^3 + 2*(3*a^2*b*c*
d*f + b^3*c*d*f + 3*a*b^2*d^2)*(f*x*log(e^(f*x + e) + 1) + dilog(-e^(f*x +
 e)))/f^3 + 2*(3*a^2*b*c*d*f + b^3*c*d*f + 3*a*b^2*d^2)*(f*x*log(-e^(f*x +
 e) + 1) + dilog(e^(f*x + e)))/f^3 + (6*a*b^2*c*d*f + (c^2*f^2 + d^2)*b...
```

Giac [F]

$$\int (c+dx)^2(a+b\coth(e+fx))^3\,dx=\int (dx+c)^2(b\coth(fx+e)+a)^3\,dx$$

input
```
integrate((d*x+c)^2*(a+b*coth(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*(b*coth(f*x + e) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2(a+b\coth(e+fx))^3\,dx=\int \left(a+b\coth(e+f\,x)\right)^3\,\left(c+d\,x\right)^2\,dx$$

input
```
int((a + b*coth(e + f*x))^3*(c + d*x)^2,x)
```

output
```
int((a + b*coth(e + f*x))^3*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2(a+b\coth(e+fx))^3\,dx=\text{too large to display}$$

input
```
int((d*x+c)^2*(a+b*coth(f*x+e))^3,x)
```

output
```
(288*e**(4*e + 4*f*x)*int(x**2/(e**(6*e + 6*f*x) - 3*e**(4*e + 4*f*x) + 3*
e**(2*e + 2*f*x) - 1),x)*a**2*b*d**2*f**3 + 96*e**(4*e + 4*f*x)*int(x**2/(
e**(6*e + 6*f*x) - 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) - 1),x)*b**3*d*
*2*f**3 + 576*e**(4*e + 4*f*x)*int(x/(e**(6*e + 6*f*x) - 3*e**(4*e + 4*f*x
) + 3*e**(2*e + 2*f*x) - 1),x)*a**2*b*c*d*f**3 + 432*e**(4*e + 4*f*x)*int(
x/(e**(6*e + 6*f*x) - 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) - 1),x)*a**2
*b*d**2*f**2 + 576*e**(4*e + 4*f*x)*int(x/(e**(6*e + 6*f*x) - 3*e**(4*e +
4*f*x) + 3*e**(2*e + 2*f*x) - 1),x)*a*b**2*d**2*f**2 + 192*e**(4*e + 4*f*x
)*int(x/(e**(6*e + 6*f*x) - 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) - 1),x
)*b**3*c*d*f**3 + 144*e**(4*e + 4*f*x)*int(x/(e**(6*e + 6*f*x) - 3*e**(4*e
 + 4*f*x) + 3*e**(2*e + 2*f*x) - 1),x)*b**3*d**2*f**2 + 144*e**(4*e + 4*f*
x)*log(e**(e + f*x) - 1)*a**2*b*c**2*f**2 + 216*e**(4*e + 4*f*x)*log(e**(e
 + f*x) - 1)*a**2*b*c*d*f + 126*e**(4*e + 4*f*x)*log(e**(e + f*x) - 1)*a**
2*b*d**2 + 288*e**(4*e + 4*f*x)*log(e**(e + f*x) - 1)*a*b**2*c*d*f + 216*e
**(4*e + 4*f*x)*log(e**(e + f*x) - 1)*a*b**2*d**2 + 48*e**(4*e + 4*f*x)*lo
g(e**(e + f*x) - 1)*b**3*c**2*f**2 + 72*e**(4*e + 4*f*x)*log(e**(e + f*x)
- 1)*b**3*c*d*f + 90*e**(4*e + 4*f*x)*log(e**(e + f*x) - 1)*b**3*d**2 + 14
4*e**(4*e + 4*f*x)*log(e**(e + f*x) + 1)*a**2*b*c**2*f**2 + 216*e**(4*e +
4*f*x)*log(e**(e + f*x) + 1)*a**2*b*c*d*f + 126*e**(4*e + 4*f*x)*log(e**(e
 + f*x) + 1)*a**2*b*d**2 + 288*e**(4*e + 4*f*x)*log(e**(e + f*x) + 1)*a...
```

3.49 $\int (c+dx)(a+b\coth(e+fx))^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 259

$$\begin{aligned}\int (c+dx)(a+b\coth(e+fx))^3\,dx &= \frac{b^3dx}{2f}+\frac{a^3(c+dx)^2}{2d}-\frac{3a^2b(c+dx)^2}{2d} \\ &+\frac{3ab^2(c+dx)^2}{2d}-\frac{b^3(c+dx)^2}{2d} \\ &-\frac{b^3d\coth(e+fx)}{2f^2}-\frac{3ab^2(c+dx)\coth(e+fx)}{f} \\ &-\frac{b^3(c+dx)\coth^2(e+fx)}{2f} \\ &+\frac{3a^2b(c+dx)\log\left(1-e^{2(e+fx)}\right)}{f} \\ &+\frac{b^3(c+dx)\log\left(1-e^{2(e+fx)}\right)}{f} \\ &+\frac{3ab^2d\log(\sinh(e+fx))}{f^2} \\ &+\frac{3a^2bd\,\text{PolyLog}\left(2,e^{2(e+fx)}\right)}{2f^2} \\ &+\frac{b^3d\,\text{PolyLog}\left(2,e^{2(e+fx)}\right)}{2f^2}\end{aligned}$$

output
```
1/2*b^3*d*x/f+1/2*a^3*(d*x+c)^2/d-3/2*a^2*b*(d*x+c)^2/d+3/2*a*b^2*(d*x+c)^
2/d-1/2*b^3*(d*x+c)^2/d-1/2*b^3*d*coth(f*x+e)/f^2-3*a*b^2*(d*x+c)*coth(f*x
+e)/f-1/2*b^3*(d*x+c)*coth(f*x+e)^2/f+3*a^2*b*(d*x+c)*ln(1-exp(2*f*x+2*e))
/f+b^3*(d*x+c)*ln(1-exp(2*f*x+2*e))/f+3*a*b^2*d*ln(sinh(f*x+e))/f^2+3/2*a^
2*b*d*polylog(2,exp(2*f*x+2*e))/f^2+1/2*b^3*d*polylog(2,exp(2*f*x+2*e))/f^
2
```

Mathematica [A] (verified)

Time = 7.47 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.23

$$\int (c+dx)(a+b\coth(e+fx))^3\,dx$$
$$= \frac{(a+b\coth(e+fx))^3\sinh(e+fx)\left(-b^3f(c+dx)-a(a^2+3b^2)(e+fx)(-2cf+d(e-fx))\sinh^2(e+\right.}{}$$

input
```
Integrate[(c + d*x)*(a + b*Coth[e + f*x])^3,x]
```

output
```
((a + b*Coth[e + f*x])^3*Sinh[e + f*x]*(-(b^3*f*(c + d*x)) - a*(a^2 + 3*b^
2)*(e + f*x)*(-2*c*f + d*(e - f*x))*Sinh[e + f*x]^2 + 2*b*((3*a^2*f^2*(c +
 d*x)^2)/(2*d) + (b^2*f^2*(c + d*x)^2)/(2*d) + 3*a*b*d*(e + f*x) + (3*a*b*
d + 3*a^2*f*(c + d*x) + b^2*f*(c + d*x))*Log[1 - E^(-e - f*x)] + (3*a*b*d
+ 3*a^2*f*(c + d*x) + b^2*f*(c + d*x))*Log[1 + E^(-e - f*x)] - (3*a^2 + b^
2)*d*PolyLog[2, -E^(-e - f*x)] - (3*a^2 + b^2)*d*PolyLog[2, E^(-e - f*x)])
*Sinh[e + f*x]^2 - (b^2*(b*d + 6*a*f*(c + d*x))*Sinh[2*(e + f*x)])/2))/(2*
f^2*(b*Cosh[e + f*x] + a*Sinh[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a+b\coth(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\left(a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4205}$$

$$\int \left(a^3(c+dx)+3a^2b(c+dx)\coth(e+fx)+3ab^2(c+dx)\coth^2(e+fx)+b^3(c+dx)\coth^3(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3(c+dx)^2}{2d}+\frac{3a^2b(c+dx)\log\left(1-e^{2(e+fx)}\right)}{f}-\frac{3a^2b(c+dx)^2}{2d}+\frac{3a^2bd\,\text{PolyLog}\left(2,e^{2(e+fx)}\right)}{2f^2}-\frac{3ab^2(c+dx)\coth(e+fx)}{f}+\frac{3ab^2(c+dx)^2}{2d}+\frac{3ab^2d\log(\sinh(e+fx))}{f^2}+\frac{b^3(c+dx)\log\left(1-e^{2(e+fx)}\right)}{f}-\frac{b^3(c+dx)\coth^2(e+fx)}{2f}-\frac{b^3(c+dx)^2}{2d}+\frac{b^3d\,\text{PolyLog}\left(2,e^{2(e+fx)}\right)}{2f^2}-\frac{b^3d\coth(e+fx)}{2f^2}+\frac{b^3dx}{2f}$$

input
```
Int[(c + d*x)*(a + b*Coth[e + f*x])^3,x]
```

output
```
(b^3*d*x)/(2*f) + (a^3*(c + d*x)^2)/(2*d) - (3*a^2*b*(c + d*x)^2)/(2*d) +
(3*a*b^2*(c + d*x)^2)/(2*d) - (b^3*(c + d*x)^2)/(2*d) - (b^3*d*Coth[e + f*
x])/(2*f^2) - (3*a*b^2*(c + d*x)*Coth[e + f*x])/f - (b^3*(c + d*x)*Coth[e
+ f*x]^2)/(2*f) + (3*a^2*b*(c + d*x)*Log[1 - E^(2*(e + f*x))])/f + (b^3*(c
 + d*x)*Log[1 - E^(2*(e + f*x))])/f + (3*a*b^2*d*Log[Sinh[e + f*x]])/f^2 +
 (3*a^2*b*d*PolyLog[2, E^(2*(e + f*x))])/(2*f^2) + (b^3*d*PolyLog[2, E^(2*
(e + f*x))])/(2*f^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 650 vs. $2(241) = 482$.

Time = 0.30 (sec) , antiderivative size = 651, normalized size of antiderivative = 2.51

method	result
risch	$-\frac{b^3 d e^2}{f^2}+\frac{b^3 c \ln(e^{fx+e}-1)}{f}-\frac{2b^3 c \ln(e^{fx+e})}{f}+\frac{b^3 c \ln(e^{fx+e}+1)}{f}+\frac{b^3 d \operatorname{polylog}(2,e^{fx+e})}{f^2}+\frac{b^3 d \operatorname{polylog}(2,-e^{fx+e})}{f^2}-$

input
```
int((d*x+c)*(a+b*coth(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/f^2*b^3*d*e^2+1/f*b^3*c*ln(exp(f*x+e)-1)-2/f*b^3*c*ln(exp(f*x+e))+1/f*b
^3*c*ln(exp(f*x+e)+1)+1/f^2*b^3*d*polylog(2,exp(f*x+e))+1/f^2*b^3*d*polylo
g(2,-exp(f*x+e))-6/f*b*a^2*d*e*x-3/f^2*b*e*a^2*d*ln(exp(f*x+e)-1)+6/f^2*b*
e*a^2*d*ln(exp(f*x+e))+3/f^2*b*a^2*d*ln(1-exp(f*x+e))*e+3/f*b*a^2*d*ln(1-e
xp(f*x+e))*x+3/f*b*a^2*d*ln(exp(f*x+e)+1)*x-b^2*(6*a*d*f*x*exp(2*f*x+2*e)+
2*b*d*f*x*exp(2*f*x+2*e)+6*a*c*f*exp(2*f*x+2*e)+2*b*c*f*exp(2*f*x+2*e)-6*a
*d*f*x+b*d*exp(2*f*x+2*e)-6*a*c*f-b*d)/f^2/(exp(2*f*x+2*e)-1)^2+1/2*a^3*d*
x^2-1/2*b^3*d*x^2+a^3*c*x+b^3*c*x-3/2*a^2*b*d*x^2+3/2*a*b^2*d*x^2+3*a^2*b*
c*x+3*a*b^2*c*x-3/f^2*b*a^2*d*e^2-2/f*b^3*d*e*x+1/f^2*b^3*d*ln(1-exp(f*x+e
))*e+3/f^2*b^2*a*d*ln(exp(f*x+e)-1)-6/f^2*b^2*a*d*ln(exp(f*x+e))+3/f^2*b^2
*a*d*ln(exp(f*x+e)+1)+3/f*b*a^2*c*ln(exp(f*x+e)-1)-6/f*b*a^2*c*ln(exp(f*x+
e))+3/f*b*a^2*c*ln(exp(f*x+e)+1)-1/f^2*b^3*e*d*ln(exp(f*x+e)-1)+2/f^2*b^3*
e*d*ln(exp(f*x+e))+3/f^2*b*a^2*d*polylog(2,exp(f*x+e))+3/f^2*b*a^2*d*polyl
og(2,-exp(f*x+e))+1/f*b^3*d*ln(1-exp(f*x+e))*x+1/f*b^3*d*ln(exp(f*x+e)+1)*
x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2907 vs. $2(239) = 478$.

Time = 0.15 (sec) , antiderivative size = 2907, normalized size of antiderivative = 11.22

$$\int (c+dx)(a+b\coth(e+fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(a+b*coth(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/2*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + 2*(a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*c*f^2*x + ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*f^2*x
^2 - 12*a*b^2*d*e + 2*(3*a^2*b + b^3)*d*e^2 - 4*(3*a^2*b + b^3)*c*e*f - 2*
(6*a*b^2*d*f - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*f^2)*x)*cosh(f*x + e)^4 +
 4*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + 2*(3*a^2*b
+ b^3)*d*e^2 - 4*(3*a^2*b + b^3)*c*e*f - 2*(6*a*b^2*d*f - (a^3 - 3*a^2*b +
 3*a*b^2 - b^3)*c*f^2)*x)*cosh(f*x + e)*sinh(f*x + e)^3 + ((a^3 - 3*a^2*b
+ 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + 2*(3*a^2*b + b^3)*d*e^2 - 4*(3
*a^2*b + b^3)*c*e*f - 2*(6*a*b^2*d*f - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*f
^2)*x)*sinh(f*x + e)^4 + 2*b^3*d + 2*(3*a^2*b + b^3)*d*e^2 - 2*((a^3 - 3*a
^2*b + 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + b^3*d + 2*(3*a^2*b + b^3)
*d*e^2 - 2*(2*(3*a^2*b + b^3)*c*e - (3*a*b^2 + b^3)*c)*f + 2*((a^3 - 3*a^2
*b + 3*a*b^2 - b^3)*c*f^2 - (3*a*b^2 - b^3)*d*f)*x)*cosh(f*x + e)^2 - 2*((
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + b^3*d + 2*(3*a^2
*b + b^3)*d*e^2 - 3*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*
d*e + 2*(3*a^2*b + b^3)*d*e^2 - 4*(3*a^2*b + b^3)*c*e*f - 2*(6*a*b^2*d*f -
 (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*f^2)*x)*cosh(f*x + e)^2 - 2*(2*(3*a^2*b
 + b^3)*c*e - (3*a*b^2 + b^3)*c)*f + 2*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*
f^2 - (3*a*b^2 - b^3)*d*f)*x)*sinh(f*x + e)^2 + 4*(3*a*b^2*c - (3*a^2*b +
b^3)*c*e)*f + 2*((3*a^2*b + b^3)*d*cosh(f*x + e)^4 + 4*(3*a^2*b + b^3)*...
```

Sympy [F]

$$\int (c+dx)(a+b\coth(e+fx))^3\,dx=\int \left(a+b\coth\left(e+fx\right)\right)^3\left(c+dx\right)\,dx$$

input
```
integrate((d*x+c)*(a+b*coth(f*x+e))**3,x)
```

output
```
Integral((a + b*coth(e + f*x))**3*(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(239) = 478$.

Time = 0.18 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.04

$$\int (c+dx)(a+b\coth(e+fx))^3\,dx$$
$$= \frac{1}{2}a^3dx^2 + a^3cx + \frac{3a^2bc\log(\sinh(fx+e))}{f} - (3a^2bd + b^3d)x^2 - \frac{2(b^3cf + 3ab^2d)x}{f}$$
$$+ \frac{12ab^2cf + 2b^3d + (3a^2bdf^2 + 3ab^2df^2 + b^3df^2)x^2 + 2(b^3cf^2 + 3(cf^2 + 2df)ab^2)x + ((3a^2bdf^2e^{(4e)}}{}$$
$$+ \frac{(3a^2bd + b^3d)\left(fx\log\left(e^{(fx+e)} + 1\right) + \mathrm{Li}_2\left(-e^{(fx+e)}\right)\right)}{f^2}$$
$$+ \frac{(3a^2bd + b^3d)\left(fx\log\left(-e^{(fx+e)} + 1\right) + \mathrm{Li}_2\left(e^{(fx+e)}\right)\right)}{f^2}$$
$$+ \frac{(b^3cf + 3ab^2d)\log\left(e^{(fx+e)} + 1\right)}{f^2} + \frac{(b^3cf + 3ab^2d)\log\left(e^{(fx+e)} - 1\right)}{f^2}$$

input
```
integrate((d*x+c)*(a+b*coth(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/2*a^3*d*x^2 + a^3*c*x + 3*a^2*b*c*log(sinh(f*x + e))/f - (3*a^2*b*d + b^
3*d)*x^2 - 2*(b^3*c*f + 3*a*b^2*d)*x/f + 1/2*(12*a*b^2*c*f + 2*b^3*d + (3*
a^2*b*d*f^2 + 3*a*b^2*d*f^2 + b^3*d*f^2)*x^2 + 2*(b^3*c*f^2 + 3*(c*f^2 + 2
*d*f)*a*b^2)*x + ((3*a^2*b*d*f^2*e^(4*e) + 3*a*b^2*d*f^2*e^(4*e) + b^3*d*f
^2*e^(4*e))*x^2 + 2*(3*a*b^2*c*f^2*e^(4*e) + b^3*c*f^2*e^(4*e))*x)*e^(4*f*
x) - 2*(6*a*b^2*c*f*e^(2*e) + (2*c*f*e^(2*e) + d*e^(2*e))*b^3 + (3*a^2*b*d
*f^2*e^(2*e) + 3*a*b^2*d*f^2*e^(2*e) + b^3*d*f^2*e^(2*e))*x^2 + 2*(3*(c*f^
2*e^(2*e) + d*f*e^(2*e))*a*b^2 + (c*f^2*e^(2*e) + d*f*e^(2*e))*b^3)*x)*e^(
2*f*x))/(f^2*e^(4*f*x + 4*e) - 2*f^2*e^(2*f*x + 2*e) + f^2) + (3*a^2*b*d +
 b^3*d)*(f*x*log(e^(f*x + e) + 1) + dilog(-e^(f*x + e)))/f^2 + (3*a^2*b*d
+ b^3*d)*(f*x*log(-e^(f*x + e) + 1) + dilog(e^(f*x + e)))/f^2 + (b^3*c*f +
 3*a*b^2*d)*log(e^(f*x + e) + 1)/f^2 + (b^3*c*f + 3*a*b^2*d)*log(e^(f*x +
e) - 1)/f^2
```

Giac [F]

$$\int (c+dx)(a+b\coth(e+fx))^3\,dx=\int (dx+c)(b\coth\left(fx+e\right)+a)^3\,dx$$

input
```
integrate((d*x+c)*(a+b*coth(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)*(b*coth(f*x + e) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)(a+b\coth(e+fx))^3\,dx=\int {\left(a+b\,\mathrm{coth}\left(e+f\,x\right)\right)}^3\,\left(c+d\,x\right)\,dx$$

input
```
int((a + b*coth(e + f*x))^3*(c + d*x),x)
```

output
```
int((a + b*coth(e + f*x))^3*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)(a+b\coth(e+fx))^3\,dx=\text{too large to display}$$

input
```
int((d*x+c)*(a+b*coth(f*x+e))^3,x)
```

output
```
(48*e**(4*e + 4*f*x)*int(x/(e**(6*e + 6*f*x) - 3*e**(4*e + 4*f*x) + 3*e**(
2*e + 2*f*x) - 1),x)*a**2*b*d*f**2 + 16*e**(4*e + 4*f*x)*int(x/(e**(6*e +
6*f*x) - 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) - 1),x)*b**3*d*f**2 + 24*
e**(4*e + 4*f*x)*log(e**(e + f*x) - 1)*a**2*b*c*f + 18*e**(4*e + 4*f*x)*lo
g(e**(e + f*x) - 1)*a**2*b*d + 24*e**(4*e + 4*f*x)*log(e**(e + f*x) - 1)*a
*b**2*d + 8*e**(4*e + 4*f*x)*log(e**(e + f*x) - 1)*b**3*c*f + 6*e**(4*e +
4*f*x)*log(e**(e + f*x) - 1)*b**3*d + 24*e**(4*e + 4*f*x)*log(e**(e + f*x)
 + 1)*a**2*b*c*f + 18*e**(4*e + 4*f*x)*log(e**(e + f*x) + 1)*a**2*b*d + 24
*e**(4*e + 4*f*x)*log(e**(e + f*x) + 1)*a*b**2*d + 8*e**(4*e + 4*f*x)*log(
e**(e + f*x) + 1)*b**3*c*f + 6*e**(4*e + 4*f*x)*log(e**(e + f*x) + 1)*b**3
*d + 8*e**(4*e + 4*f*x)*a**3*c*f**2*x + 4*e**(4*e + 4*f*x)*a**3*d*f**2*x**
2 - 24*e**(4*e + 4*f*x)*a**2*b*c*f**2*x + 12*e**(4*e + 4*f*x)*a**2*b*d*f**
2*x**2 - 36*e**(4*e + 4*f*x)*a**2*b*d*f*x + 3*e**(4*e + 4*f*x)*a**2*b*d +
24*e**(4*e + 4*f*x)*a*b**2*c*f**2*x - 24*e**(4*e + 4*f*x)*a*b**2*c*f + 12*
e**(4*e + 4*f*x)*a*b**2*d*f**2*x**2 - 48*e**(4*e + 4*f*x)*a*b**2*d*f*x - 8
*e**(4*e + 4*f*x)*b**3*c*f**2*x - 8*e**(4*e + 4*f*x)*b**3*c*f + 4*e**(4*e
+ 4*f*x)*b**3*d*f**2*x**2 - 12*e**(4*e + 4*f*x)*b**3*d*f*x - 3*e**(4*e + 4
*f*x)*b**3*d - 96*e**(2*e + 2*f*x)*int(x/(e**(6*e + 6*f*x) - 3*e**(4*e + 4
*f*x) + 3*e**(2*e + 2*f*x) - 1),x)*a**2*b*d*f**2 - 32*e**(2*e + 2*f*x)*int
(x/(e**(6*e + 6*f*x) - 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) - 1),x)*...
```

3.50 $\int \frac{(a+b\coth(e+fx))^3}{c+dx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(a+b\coth(e+fx))^3}{c+dx}\,dx = \text{Int}\left(\frac{(a+b\coth(e+fx))^3}{c+dx},x\right)$$

output
```
Defer(Int)((a+b*coth(f*x+e))^3/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 36.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\coth(e+fx))^3}{c+dx}\,dx = \int \frac{(a+b\coth(e+fx))^3}{c+dx}\,dx$$

input
```
Integrate[(a + b*Coth[e + f*x])^3/(c + d*x),x]
```

output
```
Integrate[(a + b*Coth[e + f*x])^3/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\coth(e+fx))^3}{c+dx}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^3}{c+dx}dx$$

$$\Big\downarrow \; 4223$$

$$\int \frac{(a+b\coth(e+fx))^3}{c+dx}dx$$

input
```
Int[(a + b*Coth[e + f*x])^3/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\coth(fx+e))^3}{dx+c}dx$$

input
```
int((a+b*coth(f*x+e))^3/(d*x+c),x)
```

output
```
int((a+b*coth(f*x+e))^3/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \frac{(a+b\coth(e+fx))^3}{c+dx}\,dx=\int \frac{(b\coth(fx+e)+a)^3}{dx+c}\,dx$$

input
```
integrate((a+b*coth(f*x+e))^3/(d*x+c),x, algorithm="fricas")
```

output
```
integral((b^3*coth(f*x + e)^3 + 3*a*b^2*coth(f*x + e)^2 + 3*a^2*b*coth(f*x
 + e) + a^3)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 1.92 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(a+b\coth(e+fx))^3}{c+dx}\,dx=\int \frac{(a+b\coth(e+fx))^3}{c+dx}\,dx$$

input
```
integrate((a+b*coth(f*x+e))**3/(d*x+c),x)
```

output
```
Integral((a + b*coth(e + f*x))**3/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 644, normalized size of antiderivative = 32.20

$$\int \frac{(a+b\coth(e+fx))^3}{c+dx}\,dx=\int \frac{(b\coth(fx+e)+a)^3}{dx+c}\,dx$$

input
```
integrate((a+b*coth(f*x+e))^3/(d*x+c),x, algorithm="maxima")
```

output
```
a^3*log(d*x + c)/d + (3*a^2*b + 3*a*b^2 + b^3)*log(d*x + c)/d + (6*a*b^2*d
*f*x + 6*a*b^2*c*f - b^3*d - (6*a*b^2*c*f*e^(2*e) + (2*c*f*e^(2*e) - d*e^(
2*e))*b^3 + 2*(3*a*b^2*d*f*e^(2*e) + b^3*d*f*e^(2*e))*x)*e^(2*f*x))/(d^2*f
^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + (d^2*f^2*x^2*e^(4*e) + 2*c*d*f^2*x*e^(4*e
) + c^2*f^2*e^(4*e))*e^(4*f*x) - 2*(d^2*f^2*x^2*e^(2*e) + 2*c*d*f^2*x*e^(2
*e) + c^2*f^2*e^(2*e))*e^(2*f*x)) - integrate((3*a^2*b*c^2*f^2 - 3*a*b^2*c
*d*f + (c^2*f^2 + d^2)*b^3 + (3*a^2*b*d^2*f^2 + b^3*d^2*f^2)*x^2 + (6*a^2*
b*c*d*f^2 + 2*b^3*c*d*f^2 - 3*a*b^2*d^2*f)*x)/(d^3*f^2*x^3 + 3*c*d^2*f^2*x
^2 + 3*c^2*d*f^2*x + c^3*f^2 + (d^3*f^2*x^3*e^e + 3*c*d^2*f^2*x^2*e^e + 3*
c^2*d*f^2*x*e^e + c^3*f^2*e^e)*e^(f*x)), x) + integrate(-(3*a^2*b*c^2*f^2
- 3*a*b^2*c*d*f + (c^2*f^2 + d^2)*b^3 + (3*a^2*b*d^2*f^2 + b^3*d^2*f^2)*x^
2 + (6*a^2*b*c*d*f^2 + 2*b^3*c*d*f^2 - 3*a*b^2*d^2*f)*x)/(d^3*f^2*x^3 + 3*
c*d^2*f^2*x^2 + 3*c^2*d*f^2*x + c^3*f^2 - (d^3*f^2*x^3*e^e + 3*c*d^2*f^2*x
^2*e^e + 3*c^2*d*f^2*x*e^e + c^3*f^2*e^e)*e^(f*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\coth(e+fx))^3}{c+dx}\,dx=\int \frac{(b\coth{(fx+e)}+a)^3}{dx+c}\,dx$$

input
```
integrate((a+b*coth(f*x+e))^3/(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*coth(f*x + e) + a)^3/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\coth(e+fx))^3}{c+dx}\,dx=\int \frac{(a+b\coth(e+f\,x))^3}{c+d\,x}\,dx$$

input
```
int((a + b*coth(e + f*x))^3/(c + d*x),x)
```

output
```
int((a + b*coth(e + f*x))^3/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 793, normalized size of antiderivative = 39.65

$$\int \frac{(a+b\coth(e+fx))^3}{c+dx}\,dx$$
$$= \frac{6e^{4e}\left(\int \frac{e^{4fx}}{e^{6fx+6e}c+e^{6fx+6e}dx-3e^{4fx+4e}c-3e^{4fx+4e}dx+3e^{2fx+2e}c+3e^{2fx+2e}dx-c-dx}dx\right)a^2bd+12e^{4e}\left(\int \frac{}{e^{6fx+6e}c+e^{6fx+6e}dx-}\right.}{}$$

input
```
int((a+b*coth(f*x+e))^3/(d*x+c),x)
```

output
```
(6*e**(4*e)*int(e**(4*f*x)/(e**(6*e + 6*f*x)*c + e**(6*e + 6*f*x)*d*x - 3*
e**(4*e + 4*f*x)*c - 3*e**(4*e + 4*f*x)*d*x + 3*e**(2*e + 2*f*x)*c + 3*e**
(2*e + 2*f*x)*d*x - c - d*x),x)*a**2*b*d + 12*e**(4*e)*int(e**(4*f*x)/(e**
(6*e + 6*f*x)*c + e**(6*e + 6*f*x)*d*x - 3*e**(4*e + 4*f*x)*c - 3*e**(4*e
+ 4*f*x)*d*x + 3*e**(2*e + 2*f*x)*c + 3*e**(2*e + 2*f*x)*d*x - c - d*x),x)
*a*b**2*d + 6*e**(4*e)*int(e**(4*f*x)/(e**(6*e + 6*f*x)*c + e**(6*e + 6*f*
x)*d*x - 3*e**(4*e + 4*f*x)*c - 3*e**(4*e + 4*f*x)*d*x + 3*e**(2*e + 2*f*x
)*c + 3*e**(2*e + 2*f*x)*d*x - c - d*x),x)*b**3*d - 12*e**(2*e)*int(e**(2*
f*x)/(e**(6*e + 6*f*x)*c + e**(6*e + 6*f*x)*d*x - 3*e**(4*e + 4*f*x)*c - 3
*e**(4*e + 4*f*x)*d*x + 3*e**(2*e + 2*f*x)*c + 3*e**(2*e + 2*f*x)*d*x - c
- d*x),x)*a**2*b*d - 12*e**(2*e)*int(e**(2*f*x)/(e**(6*e + 6*f*x)*c + e**(
6*e + 6*f*x)*d*x - 3*e**(4*e + 4*f*x)*c - 3*e**(4*e + 4*f*x)*d*x + 3*e**(2
*e + 2*f*x)*c + 3*e**(2*e + 2*f*x)*d*x - c - d*x),x)*a*b**2*d + 6*int(1/(e
**(6*e + 6*f*x)*c + e**(6*e + 6*f*x)*d*x - 3*e**(4*e + 4*f*x)*c - 3*e**(4*
e + 4*f*x)*d*x + 3*e**(2*e + 2*f*x)*c + 3*e**(2*e + 2*f*x)*d*x - c - d*x),
x)*a**2*b*d + 2*int(1/(e**(6*e + 6*f*x)*c + e**(6*e + 6*f*x)*d*x - 3*e**(4
*e + 4*f*x)*c - 3*e**(4*e + 4*f*x)*d*x + 3*e**(2*e + 2*f*x)*c + 3*e**(2*e
+ 2*f*x)*d*x - c - d*x),x)*b**3*d + log(c + d*x)*a**3 + 3*log(c + d*x)*a**
2*b + 3*log(c + d*x)*a*b**2 + log(c + d*x)*b**3)/d
```

3.51 $\int \frac{(a+b\coth(e+fx))^3}{(c+dx)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(a+b\coth(e+fx))^3}{(c+dx)^2} dx = \text{Int}\left(\frac{(a+b\coth(e+fx))^3}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)((a+b*coth(f*x+e))^3/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 40.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\coth(e+fx))^3}{(c+dx)^2} dx = \int \frac{(a+b\coth(e+fx))^3}{(c+dx)^2} dx$$

input
```
Integrate[(a + b*Coth[e + f*x])^3/(c + d*x)^2,x]
```

output
```
Integrate[(a + b*Coth[e + f*x])^3/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\coth(e+fx))^3}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^3}{(c+dx)^2}dx$$

$$\Big\downarrow \text{ 4223}$$

$$\int \frac{(a+b\coth(e+fx))^3}{(c+dx)^2}dx$$

input
```
Int[(a + b*Coth[e + f*x])^3/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\coth(fx+e))^3}{(dx+c)^2}dx$$

input
```
int((a+b*coth(f*x+e))^3/(d*x+c)^2,x)
```

output
```
int((a+b*coth(f*x+e))^3/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.15

$$\int \frac{(a+b\coth(e+fx))^3}{(c+dx)^2}\,dx=\int \frac{(b\coth(fx+e)+a)^3}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*coth(f*x+e))^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((b^3*coth(f*x + e)^3 + 3*a*b^2*coth(f*x + e)^2 + 3*a^2*b*coth(f*x
 + e) + a^3)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 3.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\coth(e+fx))^3}{(c+dx)^2}\,dx=\int \frac{(a+b\coth(e+fx))^3}{(c+dx)^2}\,dx$$

input
```
integrate((a+b*coth(f*x+e))**3/(d*x+c)**2,x)
```

output
```
Integral((a + b*coth(e + f*x))**3/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 1144, normalized size of antiderivative = 57.20

$$\int \frac{(a+b\coth(e+fx))^3}{(c+dx)^2}\,dx=\int \frac{(b\coth(fx+e)+a)^3}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*coth(f*x+e))^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-a^3/(d^2*x + c*d) - (3*a^2*b*c^2*f^2 + 3*(c^2*f^2 - 2*c*d*f)*a*b^2 + (c^2
*f^2 + 2*d^2)*b^3 + (3*a^2*b*d^2*f^2 + 3*a*b^2*d^2*f^2 + b^3*d^2*f^2)*x^2
+ 2*(3*a^2*b*c*d*f^2 + b^3*c*d*f^2 + 3*(c*d*f^2 - d^2*f)*a*b^2)*x + (3*a^2
*b*c^2*f^2*e^(4*e) + 3*a*b^2*c^2*f^2*e^(4*e) + b^3*c^2*f^2*e^(4*e) + (3*a^
2*b*d^2*f^2*e^(4*e) + 3*a*b^2*d^2*f^2*e^(4*e) + b^3*d^2*f^2*e^(4*e))*x^2 +
 2*(3*a^2*b*c*d*f^2*e^(4*e) + 3*a*b^2*c*d*f^2*e^(4*e) + b^3*c*d*f^2*e^(4*e
))*x)*e^(4*f*x) - 2*(3*a^2*b*c^2*f^2*e^(2*e) + 3*(c^2*f^2*e^(2*e) - c*d*f*
e^(2*e))*a*b^2 + (c^2*f^2*e^(2*e) - c*d*f*e^(2*e) + d^2*e^(2*e))*b^3 + (3*
a^2*b*d^2*f^2*e^(2*e) + 3*a*b^2*d^2*f^2*e^(2*e) + b^3*d^2*f^2*e^(2*e))*x^2
 + (6*a^2*b*c*d*f^2*e^(2*e) + 3*(2*c*d*f^2*e^(2*e) - d^2*f*e^(2*e))*a*b^2
+ (2*c*d*f^2*e^(2*e) - d^2*f*e^(2*e))*b^3)*x)*e^(2*f*x))/(d^4*f^2*x^3 + 3*
c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2 + (d^4*f^2*x^3*e^(4*e) + 3*c*d
^3*f^2*x^2*e^(4*e) + 3*c^2*d^2*f^2*x*e^(4*e) + c^3*d*f^2*e^(4*e))*e^(4*f*x
) - 2*(d^4*f^2*x^3*e^(2*e) + 3*c*d^3*f^2*x^2*e^(2*e) + 3*c^2*d^2*f^2*x*e^(
2*e) + c^3*d*f^2*e^(2*e))*e^(2*f*x)) - integrate((3*a^2*b*c^2*f^2 - 6*a*b^
2*c*d*f + (c^2*f^2 + 3*d^2)*b^3 + (3*a^2*b*d^2*f^2 + b^3*d^2*f^2)*x^2 + 2*
(3*a^2*b*c*d*f^2 + b^3*c*d*f^2 - 3*a*b^2*d^2*f)*x)/(d^4*f^2*x^4 + 4*c*d^3*
f^2*x^3 + 6*c^2*d^2*f^2*x^2 + 4*c^3*d*f^2*x + c^4*f^2 + (d^4*f^2*x^4*e^e +
 4*c*d^3*f^2*x^3*e^e + 6*c^2*d^2*f^2*x^2*e^e + 4*c^3*d*f^2*x*e^e + c^4*f^2
*e^e)*e^(f*x)), x) + integrate(-(3*a^2*b*c^2*f^2 - 6*a*b^2*c*d*f + (c^2...
```

Giac [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\coth(e+fx))^3}{(c+dx)^2}\,dx=\int \frac{(b\coth(fx+e)+a)^3}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*coth(f*x+e))^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*coth(f*x + e) + a)^3/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\coth(e+fx))^3}{(c+dx)^2}\,dx=\int \frac{{\left(a+b\,\mathrm{coth}\left(e+f\,x\right)\right)}^3}{{\left(c+d\,x\right)}^2} \,d x$$

input
```
int((a + b*coth(e + f*x))^3/(c + d*x)^2,x)
```

output
```
int((a + b*coth(e + f*x))^3/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 2588, normalized size of antiderivative = 129.40

$$\int \frac{(a+b\coth(e+fx))^3}{(c+dx)^2}\,dx=\text{Too large to display}$$

input
```
int((a+b*coth(f*x+e))^3/(d*x+c)^2,x)
```

output

```
(6*e**(4*e)*int(e**(4*f*x)/(e**(6*e + 6*f*x)*c**2 + 2*e**(6*e + 6*f*x)*c*d
*x + e**(6*e + 6*f*x)*d**2*x**2 - 3*e**(4*e + 4*f*x)*c**2 - 6*e**(4*e + 4*
f*x)*c*d*x - 3*e**(4*e + 4*f*x)*d**2*x**2 + 3*e**(2*e + 2*f*x)*c**2 + 6*e*
*(2*e + 2*f*x)*c*d*x + 3*e**(2*e + 2*f*x)*d**2*x**2 - c**2 - 2*c*d*x - d**
2*x**2),x)*a**2*b*c**2 + 6*e**(4*e)*int(e**(4*f*x)/(e**(6*e + 6*f*x)*c**2
+ 2*e**(6*e + 6*f*x)*c*d*x + e**(6*e + 6*f*x)*d**2*x**2 - 3*e**(4*e + 4*f*
x)*c**2 - 6*e**(4*e + 4*f*x)*c*d*x - 3*e**(4*e + 4*f*x)*d**2*x**2 + 3*e**(
2*e + 2*f*x)*c**2 + 6*e**(2*e + 2*f*x)*c*d*x + 3*e**(2*e + 2*f*x)*d**2*x**
2 - c**2 - 2*c*d*x - d**2*x**2),x)*a**2*b*c*d*x + 12*e**(4*e)*int(e**(4*f*
x)/(e**(6*e + 6*f*x)*c**2 + 2*e**(6*e + 6*f*x)*c*d*x + e**(6*e + 6*f*x)*d*
*2*x**2 - 3*e**(4*e + 4*f*x)*c**2 - 6*e**(4*e + 4*f*x)*c*d*x - 3*e**(4*e +
 4*f*x)*d**2*x**2 + 3*e**(2*e + 2*f*x)*c**2 + 6*e**(2*e + 2*f*x)*c*d*x + 3
*e**(2*e + 2*f*x)*d**2*x**2 - c**2 - 2*c*d*x - d**2*x**2),x)*a*b**2*c**2 +
 12*e**(4*e)*int(e**(4*f*x)/(e**(6*e + 6*f*x)*c**2 + 2*e**(6*e + 6*f*x)*c*
d*x + e**(6*e + 6*f*x)*d**2*x**2 - 3*e**(4*e + 4*f*x)*c**2 - 6*e**(4*e + 4
*f*x)*c*d*x - 3*e**(4*e + 4*f*x)*d**2*x**2 + 3*e**(2*e + 2*f*x)*c**2 + 6*e
**(2*e + 2*f*x)*c*d*x + 3*e**(2*e + 2*f*x)*d**2*x**2 - c**2 - 2*c*d*x - d*
*2*x**2),x)*a*b**2*c*d*x + 6*e**(4*e)*int(e**(4*f*x)/(e**(6*e + 6*f*x)*c**
2 + 2*e**(6*e + 6*f*x)*c*d*x + e**(6*e + 6*f*x)*d**2*x**2 - 3*e**(4*e + 4*
f*x)*c**2 - 6*e**(4*e + 4*f*x)*c*d*x - 3*e**(4*e + 4*f*x)*d**2*x**2 + 3...
```

3.52 $\int \frac{(c+dx)^3}{a+b\coth(e+fx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 210

$$\int \frac{(c+dx)^3}{a+b\coth(e+fx)}\,dx = \frac{(c+dx)^4}{4(a+b)d} - \frac{b(c+dx)^3\log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)\,f} + \frac{3bd(c+dx)^2\operatorname{PolyLog}\left(2,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2\,(a^2-b^2)\,f^2} + \frac{3bd^2(c+dx)\operatorname{PolyLog}\left(3,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2\,(a^2-b^2)\,f^3} + \frac{3bd^3\operatorname{PolyLog}\left(4,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{4\,(a^2-b^2)\,f^4}$$

output
```
1/4*(d*x+c)^4/(a+b)/d-b*(d*x+c)^3*ln(1-(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^
2)/f+3/2*b*d*(d*x+c)^2*polylog(2,(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^2
+3/2*b*d^2*(d*x+c)*polylog(3,(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^3+3/4
*b*d^3*polylog(4,(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^4
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.18

$$\int \frac{(c+dx)^3}{a+b\coth(e+fx)}\,dx$$
$$=\frac{1}{4}\left(\frac{2b(c+dx)^4}{(a+b)d\left(a\left(-1+e^{2e}\right)+b\left(1+e^{2e}\right)\right)}-\frac{4b(c+dx)^3\log\left(1+\frac{(-a+b)e^{-2(e+fx)}}{a+b}\right)}{(a-b)(a+b)f}\right.$$
$$+\frac{3bd\left(2f^2(c+dx)^2\operatorname{PolyLog}\left(2,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)+d\left(2f(c+dx)\operatorname{PolyLog}\left(3,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)+d\operatorname{PolyLog}\right.\right.}{(a-b)(a+b)f^4}$$
$$\left.+\frac{x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)\sinh(e)}{b\cosh(e)+a\sinh(e)}\right)$$

input
```
Integrate[(c + d*x)^3/(a + b*Coth[e + f*x]),x]
```

output
```
((2*b*(c + d*x)^4)/((a + b)*d*(a*(-1 + E^(2*e)) + b*(1 + E^(2*e)))) - (4*b
*(c + d*x)^3*Log[1 + (-a + b)/((a + b)*E^(2*(e + f*x)))])/((a - b)*(a + b)
*f) + (3*b*d*(2*f^2*(c + d*x)^2*PolyLog[2, (a - b)/((a + b)*E^(2*(e + f*x)
))] + d*(2*f*(c + d*x)*PolyLog[3, (a - b)/((a + b)*E^(2*(e + f*x)))] + d*P
olyLog[4, (a - b)/((a + b)*E^(2*(e + f*x)))])))/((a - b)*(a + b)*f^4) + (x
*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Sinh[e])/(b*Cosh[e] + a*Sinh[
e]))/4
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4214, 25, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{a+b\coth(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(c+dx)^3}{a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4214$$

$$2b\int -\frac{e^{-2(e+fx)}(c+dx)^3}{(a+b)^2-(a^2-b^2)\,e^{-2(e+fx)}}dx+\frac{(c+dx)^4}{4d(a+b)}$$

$$\Big\downarrow \; 25$$

$$\frac{(c+dx)^4}{4d(a+b)}-2b\int \frac{e^{-2(e+fx)}(c+dx)^3}{(a+b)^2-(a^2-b^2)\,e^{-2(e+fx)}}dx$$

$$\Big\downarrow \; 2620$$

$$\frac{(c+dx)^4}{4d(a+b)}-2b\left(\frac{(c+dx)^3\log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f(a^2-b^2)}-\frac{3d\int(c+dx)^2\log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)dx}{2f(a^2-b^2)}\right)$$

$$\Big\downarrow \; 3011$$

$$\frac{(c+dx)^4}{4d(a+b)}-2b\left(\frac{(c+dx)^3\log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f(a^2-b^2)}-\frac{3d\left(\frac{(c+dx)^2\,\text{PolyLog}\left(2,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f}-\frac{d\int(c+dx)\,\text{PolyLog}\left(2,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)dx}{f}\right.}{2f(a^2-b^2)}\right.$$

$$\Big\downarrow \; 7163$$

$$\frac{(c+dx)^4}{4d(a+b)}-2b\left(\frac{(c+dx)^3\log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f(a^2-b^2)}-\frac{3d\left(\frac{(c+dx)^2\,\text{PolyLog}\left(2,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f}-\frac{d\left(\frac{d\int\text{PolyLog}\left(3,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)dx}{2f}-\frac{(c+}{}\right.}{f}\right.}{2f(a^2-b^2)}\right.$$

$\downarrow$ 2720

$$\frac{(c+dx)^4}{4d(a+b)} - 2b\left(\frac{(c+dx)^3\log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f(a^2-b^2)} - \frac{3d\left(\frac{(c+dx)^2\operatorname{PolyLog}\left(2,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f} - \frac{d\left(-\frac{d\int e^{2(e+fx)}\operatorname{PolyLog}\left(3,\frac{(a-b)e^{-2(e+fx}}{a+b}\right.}{4f^2}\right.}{}\right.}{2f(a^2-b^2)}\right.$$

$\downarrow$ 7143

$$\frac{(c+dx)^4}{4d(a+b)} - 2b\left(\frac{(c+dx)^3\log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f(a^2-b^2)} - \frac{3d\left(\frac{(c+dx)^2\operatorname{PolyLog}\left(2,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f} - \frac{d\left(-\frac{(c+dx)\operatorname{PolyLog}\left(3,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f} - \right.}{f}\right.}{2f(a^2-b^2)}\right.$$

input
```
Int[(c + d*x)^3/(a + b*Coth[e + f*x]),x]
```

output
```
(c + d*x)^4/(4*(a + b)*d) - 2*b*(((c + d*x)^3*Log[1 - (a - b)/((a + b)*E^(
2*(e + f*x)))])/(2*(a^2 - b^2)*f) - (3*d*(((c + d*x)^2*PolyLog[2, (a - b)/
((a + b)*E^(2*(e + f*x)))])/(2*f) - (d*(-1/2*((c + d*x)*PolyLog[3, (a - b)
/((a + b)*E^(2*(e + f*x)))])/f - (d*PolyLog[4, (a - b)/((a + b)*E^(2*(e +
f*x)))])/(4*f^2)))/f))/(2*(a^2 - b^2)*f))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4214
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Simp
[2*I*b   Int[(c + d*x)^m*E^(2*I*k*Pi)*(E^Simp[2*I*(e + f*x), x]/((a + I*b)^
2 + (a^2 + b^2)*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x])), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && IntegerQ[4*k] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1157 vs. $2(210) = 420$.

Time = 0.22 (sec) , antiderivative size = 1158, normalized size of antiderivative = 5.51

method	result	size
risch	Expression too large to display	1158

input
```
int((d*x+c)^3/(a+b*coth(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-4/f^3*b/(a+b)/(a-b)*d^2*c*e^3+3*b/(a+b)/(a-b)*d*c^2*x^2+3/f^2*b/(a+b)/(a-
b)*d*c^2*e^2+2*b/(a+b)/(a-b)*d^2*c*x^3+2/f^3*b/(a+b)/(a-b)*d^3*e^3*x+3/2/f
^3*b/(a+b)/(a-b)*d^2*c*polylog(3,(a+b)*exp(2*f*x+2*e)/(a-b))-3/2/f^2*b/(a+
b)/(a-b)*d*c^2*polylog(2,(a+b)*exp(2*f*x+2*e)/(a-b))-1/f^4*b/(a+b)/(a-b)*d
^3*ln(1-(a+b)*exp(2*f*x+2*e)/(a-b))*e^3+3/2/f^3*b/(a+b)/(a-b)*d^3*polylog(
3,(a+b)*exp(2*f*x+2*e)/(a-b))*x+1/f^4*b/(a+b)*d^3*e^3/(a-b)*ln(exp(2*f*x+2
*e)*a+exp(2*f*x+2*e)*b-a+b)-2/f^4*b/(a+b)*d^3*e^3/(a-b)*ln(exp(f*x+e))-1/f
*b/(a+b)/(a-b)*d^3*ln(1-(a+b)*exp(2*f*x+2*e)/(a-b))*x^3-3/2/f^2*b/(a+b)/(a
-b)*d^3*polylog(2,(a+b)*exp(2*f*x+2*e)/(a-b))*x^2+1/2*b/(a+b)/(a-b)*d^3*x^
4+3/2/f^4*b/(a+b)/(a-b)*d^3*e^4-1/f*b/(a+b)*c^3/(a-b)*ln(exp(2*f*x+2*e)*a+
exp(2*f*x+2*e)*b-a+b)+2/f*b/(a+b)*c^3/(a-b)*ln(exp(f*x+e))-3/4/f^4*b/(a+b)
/(a-b)*d^3*polylog(4,(a+b)*exp(2*f*x+2*e)/(a-b))+1/4/(a+b)*d^3*x^4+1/4/(a+
b)/d*c^4+6/f*b/(a+b)/(a-b)*d*c^2*e*x-6/f^2*b/(a+b)/(a-b)*d^2*c*e^2*x+3/f^2
*b/(a+b)*c^2*d*e/(a-b)*ln(exp(2*f*x+2*e)*a+exp(2*f*x+2*e)*b-a+b)-6/f^2*b/(
a+b)*c^2*d*e/(a-b)*ln(exp(f*x+e))-3/f*b/(a+b)/(a-b)*d*c^2*ln(1-(a+b)*exp(2
*f*x+2*e)/(a-b))*x-3/f^2*b/(a+b)/(a-b)*d*c^2*ln(1-(a+b)*exp(2*f*x+2*e)/(a-
b))*e-3/f^3*b/(a+b)*c*d^2*e^2/(a-b)*ln(exp(2*f*x+2*e)*a+exp(2*f*x+2*e)*b-a
+b)+6/f^3*b/(a+b)*c*d^2*e^2/(a-b)*ln(exp(f*x+e))-3/f*b/(a+b)/(a-b)*d^2*c*l
n(1-(a+b)*exp(2*f*x+2*e)/(a-b))*x^2+3/f^3*b/(a+b)/(a-b)*d^2*c*ln(1-(a+b)*e
xp(2*f*x+2*e)/(a-b))*e^2-3/f^2*b/(a+b)/(a-b)*d^2*c*polylog(2,(a+b)*exp(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(201) = 402$.

Time = 0.13 (sec) , antiderivative size = 730, normalized size of antiderivative = 3.48

$$\int \frac{(c+dx)^3}{a+b\coth(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+b*coth(f*x+e)),x, algorithm="fricas")
```

output
```
1/4*((a + b)*d^3*f^4*x^4 + 4*(a + b)*c*d^2*f^4*x^3 + 6*(a + b)*c^2*d*f^4*x
^2 + 4*(a + b)*c^3*f^4*x - 24*b*d^3*polylog(4, sqrt((a + b)/(a - b))*(cosh
(f*x + e) + sinh(f*x + e))) - 24*b*d^3*polylog(4, -sqrt((a + b)/(a - b))*(
cosh(f*x + e) + sinh(f*x + e))) - 12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*
c^2*d*f^2)*dilog(sqrt((a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))) -
12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*dilog(-sqrt((a + b)/(a
- b))*(cosh(f*x + e) + sinh(f*x + e))) + 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*f +
3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(2*(a + b)*cosh(f*x + e) + 2*(a + b)*sinh(
f*x + e) + 2*(a - b)*sqrt((a + b)/(a - b))) + 4*(b*d^3*e^3 - 3*b*c*d^2*e^2
*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(2*(a + b)*cosh(f*x + e) + 2*(a + b)*
sinh(f*x + e) - 2*(a - b)*sqrt((a + b)/(a - b))) - 4*(b*d^3*f^3*x^3 + 3*b*
c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*
e*f^2)*log(sqrt((a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e)) + 1) - 4*
(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3 - 3*b*c*d
^2*e^2*f + 3*b*c^2*d*e*f^2)*log(-sqrt((a + b)/(a - b))*(cosh(f*x + e) + si
nh(f*x + e)) + 1) + 24*(b*d^3*f*x + b*c*d^2*f)*polylog(3, sqrt((a + b)/(a
- b))*(cosh(f*x + e) + sinh(f*x + e))) + 24*(b*d^3*f*x + b*c*d^2*f)*polylo
g(3, -sqrt((a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))))/((a^2 - b^2)
*f^4)
```

Sympy [F]

$$\int \frac{(c+dx)^3}{a+b\coth(e+fx)}\,dx = \int \frac{(c+dx)^3}{a+b\coth(e+fx)}\,dx$$

input
```
integrate((d*x+c)**3/(a+b*coth(f*x+e)),x)
```

output
```
Integral((c + d*x)**3/(a + b*coth(e + f*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(201) = 402.

Time = 0.25 (sec) , antiderivative size = 521, normalized size of antiderivative = 2.48

$$\int \frac{(c+dx)^3}{a+b\coth(e+fx)}\,dx$$
$$= -\frac{3\left(2\,fx\log\left(-\frac{\left(ae^{(2\,e)}+be^{(2\,e)}\right)e^{(2\,fx)}}{a-b}+1\right)+\mathrm{Li}_2\left(\frac{\left(ae^{(2\,e)}+be^{(2\,e)}\right)e^{(2\,fx)}}{a-b}\right)\right)bc^2d}{2\,(a^2f^2-b^2f^2)}$$
$$-\frac{3\left(2\,f^2x^2\log\left(-\frac{\left(ae^{(2\,e)}+be^{(2\,e)}\right)e^{(2\,fx)}}{a-b}+1\right)+2\,fx\mathrm{Li}_2\left(\frac{\left(ae^{(2\,e)}+be^{(2\,e)}\right)e^{(2\,fx)}}{a-b}\right)-\mathrm{Li}_3\left(\frac{\left(ae^{(2\,e)}+be^{(2\,e)}\right)e^{(2\,fx)}}{a-b}\right)\right)bcd^2}{2\,(a^2f^3-b^2f^3)}$$
$$-\frac{\left(4\,f^3x^3\log\left(-\frac{\left(ae^{(2\,e)}+be^{(2\,e)}\right)e^{(2\,fx)}}{a-b}+1\right)+6\,f^2x^2\mathrm{Li}_2\left(\frac{\left(ae^{(2\,e)}+be^{(2\,e)}\right)e^{(2\,fx)}}{a-b}\right)-6\,fx\mathrm{Li}_3\left(\frac{\left(ae^{(2\,e)}+be^{(2\,e)}\right)e^{(2\,fx)}}{a-b}\right)+}{3\,(a^2f^4-b^2f^4)}$$
$$-c^3\left(\frac{b\log\left(-(a-b)e^{(-2\,fx-2\,e)}+a+b\right)}{(a^2-b^2)f}-\frac{fx+e}{(a+b)f}\right)$$
$$+\frac{bd^3f^4x^4+4\,bcd^2f^4x^3+6\,bc^2df^4x^2}{2\,(a^2f^4-b^2f^4)}+\frac{d^3x^4+4\,cd^2x^3+6\,c^2dx^2}{4\,(a+b)}$$

input
```
integrate((d*x+c)^3/(a+b*coth(f*x+e)),x, algorithm="maxima")
```

output
```
-3/2*(2*f*x*log(-(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + dilog((a
*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)))*b*c^2*d/(a^2*f^2 - b^2*f^2) - 3/
2*(2*f^2*x^2*log(-(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + 2*f*x*d
ilog((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)) - polylog(3, (a*e^(2*e) +
b*e^(2*e))*e^(2*f*x)/(a - b)))*b*c*d^2/(a^2*f^3 - b^2*f^3) - 1/3*(4*f^3*x^
3*log(-(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + 6*f^2*x^2*dilog((a
*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)) - 6*f*x*polylog(3, (a*e^(2*e) + b
*e^(2*e))*e^(2*f*x)/(a - b)) + 3*polylog(4, (a*e^(2*e) + b*e^(2*e))*e^(2*f
*x)/(a - b)))*b*d^3/(a^2*f^4 - b^2*f^4) - c^3*(b*log(-(a - b)*e^(-2*f*x -
2*e) + a + b)/((a^2 - b^2)*f) - (f*x + e)/((a + b)*f)) + 1/2*(b*d^3*f^4*x^
4 + 4*b*c*d^2*f^4*x^3 + 6*b*c^2*d*f^4*x^2)/(a^2*f^4 - b^2*f^4) + 1/4*(d^3*
x^4 + 4*c*d^2*x^3 + 6*c^2*d*x^2)/(a + b)
```

Giac [F]

$$\int \frac{(c+dx)^3}{a+b\coth(e+fx)}\,dx=\int \frac{(dx+c)^3}{b\coth(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^3/(a+b*coth(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(b*coth(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{a+b\coth(e+fx)}\,dx=\int \frac{(c+d\,x)^3}{a+b\coth(e+f\,x)}\,dx$$

input
```
int((c + d*x)^3/(a + b*coth(e + f*x)),x)
```

output
```
int((c + d*x)^3/(a + b*coth(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(c+dx)^3}{a+b\coth(e+fx)}\,dx$$
$$=\frac{-8e^{2e}\left(\int \frac{e^{2fx}x^3}{e^{2fx+2e}a^2-e^{2fx+2e}b^2-a^2+2ab-b^2}dx\right)a^2b\,d^3f+8e^{2e}\left(\int \frac{e^{2fx}x^3}{e^{2fx+2e}a^2-e^{2fx+2e}b^2-a^2+2ab-b^2}dx\right)b^3d^3f-24e^{2e}\left(\right.}{}$$

input
```
int((d*x+c)^3/(a+b*coth(f*x+e)),x)
```

output

```
( - 8*e**(2*e)*int((e**(2*f*x)*x**3)/(e**(2*e + 2*f*x)*a**2 - e**(2*e + 2*
f*x)*b**2 - a**2 + 2*a*b - b**2),x)*a**2*b*d**3*f + 8*e**(2*e)*int((e**(2*
f*x)*x**3)/(e**(2*e + 2*f*x)*a**2 - e**(2*e + 2*f*x)*b**2 - a**2 + 2*a*b -
 b**2),x)*b**3*d**3*f - 24*e**(2*e)*int((e**(2*f*x)*x**2)/(e**(2*e + 2*f*x
)*a**2 - e**(2*e + 2*f*x)*b**2 - a**2 + 2*a*b - b**2),x)*a**2*b*c*d**2*f +
 24*e**(2*e)*int((e**(2*f*x)*x**2)/(e**(2*e + 2*f*x)*a**2 - e**(2*e + 2*f*
x)*b**2 - a**2 + 2*a*b - b**2),x)*b**3*c*d**2*f - 24*e**(2*e)*int((e**(2*f
*x)*x)/(e**(2*e + 2*f*x)*a**2 - e**(2*e + 2*f*x)*b**2 - a**2 + 2*a*b - b**
2),x)*a**2*b*c**2*d*f + 24*e**(2*e)*int((e**(2*f*x)*x)/(e**(2*e + 2*f*x)*a
**2 - e**(2*e + 2*f*x)*b**2 - a**2 + 2*a*b - b**2),x)*b**3*c**2*d*f - 4*lo
g(e**(2*e + 2*f*x)*a + e**(2*e + 2*f*x)*b - a + b)*b*c**3 + 4*a*c**3*f*x +
 6*a*c**2*d*f*x**2 + 4*a*c*d**2*f*x**3 + a*d**3*f*x**4 + 4*b*c**3*f*x + 6*
b*c**2*d*f*x**2 + 4*b*c*d**2*f*x**3 + b*d**3*f*x**4)/(4*f*(a**2 - b**2))
```

3.53 $\int \frac{(c+dx)^2}{a+b\coth(e+fx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 156

$$\int \frac{(c+dx)^2}{a+b\coth(e+fx)}\,dx = \frac{(c+dx)^3}{3(a+b)d} - \frac{b(c+dx)^2\log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)\,f} + \frac{bd(c+dx)\operatorname{PolyLog}\left(2,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)\,f^2} + \frac{bd^2\operatorname{PolyLog}\left(3,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2\,(a^2-b^2)\,f^3}$$

output
```
1/3*(d*x+c)^3/(a+b)/d-b*(d*x+c)^2*ln(1-(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^
2)/f+b*d*(d*x+c)*polylog(2,(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^2+1/2*b
*d^2*polylog(3,(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^3
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx)^2}{a+b\coth(e+fx)}\,dx = \frac{1}{6}\left(\frac{4b(c+dx)^3}{(a+b)d\left(a\left(-1+e^{2e}\right)+b\left(1+e^{2e}\right)\right)} - \frac{6b(c+dx)^2\log\left(1+\frac{(-a+b)e^{-2(e+fx)}}{a+b}\right)}{(a-b)(a+b)f} + \frac{3bd\left(2f(c+dx)\operatorname{PolyLog}\left(2,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)+d\operatorname{PolyLog}\left(3,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)\right)}{(a-b)(a+b)f^3} + \frac{2x(3c^2+3cdx+d^2x^2)\sinh(e)}{b\cosh(e)+a\sinh(e)}\right)$$

input
```
Integrate[(c + d*x)^2/(a + b*Coth[e + f*x]),x]
```

output
```
((4*b*(c + d*x)^3)/((a + b)*d*(a*(-1 + E^(2*e)) + b*(1 + E^(2*e)))) - (6*b
*(c + d*x)^2*Log[1 + (-a + b)/((a + b)*E^(2*(e + f*x)))])/((a - b)*(a + b)
*f) + (3*b*d*(2*f*(c + d*x)*PolyLog[2, (a - b)/((a + b)*E^(2*(e + f*x)))]
+ d*PolyLog[3, (a - b)/((a + b)*E^(2*(e + f*x)))]))/((a - b)*(a + b)*f^3)
+ (2*x*(3*c^2 + 3*c*d*x + d^2*x^2)*Sinh[e])/(b*Cosh[e] + a*Sinh[e]))/6
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3042, 4214, 25, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{a+b\coth(e+fx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(c+dx)^2}{a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4214}$$

$$2b\int -\frac{e^{-2(e+fx)}(c+dx)^2}{(a+b)^2-(a^2-b^2)\,e^{-2(e+fx)}}dx+\frac{(c+dx)^3}{3d(a+b)}$$

$$\Big\downarrow \text{25}$$

$$\frac{(c+dx)^3}{3d(a+b)}-2b\int \frac{e^{-2(e+fx)}(c+dx)^2}{(a+b)^2-(a^2-b^2)\,e^{-2(e+fx)}}dx$$

$$\Big\downarrow \text{2620}$$

$$\frac{(c+dx)^3}{3d(a+b)}-2b\left(\frac{(c+dx)^2\log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f\,(a^2-b^2)}-\frac{d\int (c+dx)\log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)dx}{f\,(a^2-b^2)}\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{(c+dx)^3}{3d(a+b)}-2b\left(\frac{(c+dx)^2\log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f\,(a^2-b^2)}-\frac{d\left(\frac{(c+dx)\operatorname{PolyLog}\left(2,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f}-\frac{d\int \operatorname{PolyLog}\left(2,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)dx}{2f}\right)}{f\,(a^2-b^2)}\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{(c+dx)^3}{3d(a+b)}-2b\left(\frac{(c+dx)^2\log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f\,(a^2-b^2)}-\frac{d\left(\frac{d\int e^{2(e+fx)}\operatorname{PolyLog}\left(2,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)de^{-2(e+fx)}}{4f^2}+\frac{(c+dx)\operatorname{PolyLog}\left(2,\frac{(a-b)e^{-2}}{a+}\right.}{2f}\right.}{f\,(a^2-b^2)}\right.$$

$$\Big\downarrow \text{7143}$$

$$\frac{(c+dx)^3}{3d(a+b)} - 2b\left(\frac{(c+dx)^2\log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f(a^2-b^2)} - \frac{d\left(\frac{(c+dx)\operatorname{PolyLog}\left(2,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f} + \frac{d\operatorname{PolyLog}\left(3,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{4f^2}\right)}{f(a^2-b^2)}\right)$$

input
```
Int[(c + d*x)^2/(a + b*Coth[e + f*x]),x]
```

output
```
(c + d*x)^3/(3*(a + b)*d) - 2*b*(((c + d*x)^2*Log[1 - (a - b)/((a + b)*E^(
2*(e + f*x)))])/(2*(a^2 - b^2)*f) - (d*(((c + d*x)*PolyLog[2, (a - b)/((a
+ b)*E^(2*(e + f*x)))])/(2*f) + (d*PolyLog[3, (a - b)/((a + b)*E^(2*(e + f
*x)))])/(4*f^2)))/((a^2 - b^2)*f))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4214
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Simp
[2*I*b   Int[(c + d*x)^m*E^(2*I*k*Pi)*(E^Simp[2*I*(e + f*x), x]/((a + I*b)^
2 + (a^2 + b^2)*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x])), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && IntegerQ[4*k] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 734 vs. $2(158) = 316$.

Time = 0.16 (sec) , antiderivative size = 735, normalized size of antiderivative = 4.71

method	result
risch	$\frac{d^2x^3}{3a+3b}+\frac{dc\,x^2}{a+b}+\frac{c^2x}{a+b}+\frac{c^3}{3(a+b)d}-\frac{b\,d^2e^2\ln\left(e^{2fx+2e}a+e^{2fx+2e}b-a+b\right)}{f^3(a+b)(a-b)}+\frac{2b\,d^2e^2\ln\left(e^{fx+e}\right)}{f^3(a+b)(a-b)}-\frac{2b\,d^2e^2x}{f^2(a+b)(a-b)}+\frac{b\,d^2}{}$

input
```
int((d*x+c)^2/(a+b*coth(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/3/(a+b)*d^2*x^3+1/(a+b)*d*c*x^2+1/(a+b)*c^2*x+1/3/(a+b)/d*c^3-1/f^3*b/(a
+b)*d^2*e^2/(a-b)*ln(exp(2*f*x+2*e)*a+exp(2*f*x+2*e)*b-a+b)+2/f^3*b/(a+b)*
d^2*e^2/(a-b)*ln(exp(f*x+e))-2/f^2*b/(a+b)/(a-b)*d^2*e^2*x+1/f^3*b/(a+b)/(
a-b)*d^2*ln(1-(a+b)*exp(2*f*x+2*e)/(a-b))*e^2-1/f^2*b/(a+b)/(a-b)*d^2*poly
log(2,(a+b)*exp(2*f*x+2*e)/(a-b))*x+2/f*b/(a+b)*c^2/(a-b)*ln(exp(f*x+e))-1
/f*b/(a+b)*c^2/(a-b)*ln(exp(2*f*x+2*e)*a+exp(2*f*x+2*e)*b-a+b)+2/f^2*b/(a+
b)*c*d*e/(a-b)*ln(exp(2*f*x+2*e)*a+exp(2*f*x+2*e)*b-a+b)-4/f^2*b/(a+b)*c*d
*e/(a-b)*ln(exp(f*x+e))+4/f*b/(a+b)/(a-b)*d*c*e*x-2/f^2*b/(a+b)/(a-b)*d*c*
ln(1-(a+b)*exp(2*f*x+2*e)/(a-b))*e+2/3*b/(a+b)/(a-b)*d^2*x^3-4/3/f^3*b/(a+
b)/(a-b)*d^2*e^3-1/f*b/(a+b)/(a-b)*d^2*ln(1-(a+b)*exp(2*f*x+2*e)/(a-b))*x^
2+1/2/f^3*b/(a+b)/(a-b)*d^2*polylog(3,(a+b)*exp(2*f*x+2*e)/(a-b))+2*b/(a+b
)/(a-b)*d*c*x^2+2/f^2*b/(a+b)/(a-b)*d*c*e^2-2/f*b/(a+b)/(a-b)*d*c*ln(1-(a+
b)*exp(2*f*x+2*e)/(a-b))*x-1/f^2*b/(a+b)/(a-b)*d*c*polylog(2,(a+b)*exp(2*f
*x+2*e)/(a-b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(151) = 302$.

Time = 0.10 (sec) , antiderivative size = 492, normalized size of antiderivative = 3.15

$$\int \frac{(c+dx)^2}{a+b\coth(e+fx)}\,dx$$
$$= \frac{(a+b)d^2f^3x^3 + 3\,(a+b)cdf^3x^2 + 3\,(a+b)c^2f^3x + 6\,bd^2\mathrm{polylog}\Big(3,\ \sqrt{\tfrac{a+b}{a-b}}(\cosh\left(fx+e\right)+\sinh\left(fx+e\right)}{}$$

input
```
integrate((d*x+c)^2/(a+b*coth(f*x+e)),x, algorithm="fricas")
```

output
```
1/3*((a + b)*d^2*f^3*x^3 + 3*(a + b)*c*d*f^3*x^2 + 3*(a + b)*c^2*f^3*x + 6
*b*d^2*polylog(3, sqrt((a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))) +
 6*b*d^2*polylog(3, -sqrt((a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))
) - 6*(b*d^2*f*x + b*c*d*f)*dilog(sqrt((a + b)/(a - b))*(cosh(f*x + e) + s
inh(f*x + e))) - 6*(b*d^2*f*x + b*c*d*f)*dilog(-sqrt((a + b)/(a - b))*(cos
h(f*x + e) + sinh(f*x + e))) - 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log
(2*(a + b)*cosh(f*x + e) + 2*(a + b)*sinh(f*x + e) + 2*(a - b)*sqrt((a + b
)/(a - b))) - 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(2*(a + b)*cosh(f
*x + e) + 2*(a + b)*sinh(f*x + e) - 2*(a - b)*sqrt((a + b)/(a - b))) - 3*(
b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*log(sqrt((a + b)/
(a - b))*(cosh(f*x + e) + sinh(f*x + e)) + 1) - 3*(b*d^2*f^2*x^2 + 2*b*c*d
*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*log(-sqrt((a + b)/(a - b))*(cosh(f*x + e
) + sinh(f*x + e)) + 1))/((a^2 - b^2)*f^3)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{a+b\coth(e+fx)}\,dx = \int \frac{(c+dx)^2}{a+b\coth{(e+fx)}}\,dx$$

input
```
integrate((d*x+c)**2/(a+b*coth(f*x+e)),x)
```

output
```
Integral((c + d*x)**2/(a + b*coth(e + f*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(151) = 302$.

Time = 0.38 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.13

$$\int \frac{(c+dx)^2}{a+b\coth(e+fx)}\,dx = -\frac{\left(2\,fx\log\left(-\frac{\left(ae^{(2\,e)}+be^{(2\,e)}\right)e^{(2\,fx)}}{a-b}+1\right)+\mathrm{Li}_2\left(\frac{\left(ae^{(2\,e)}+be^{(2\,e)}\right)e^{(2\,fx)}}{a-b}\right)\right)bcd}{a^2f^2-b^2f^2} - \frac{\left(2\,f^2x^2\log\left(-\frac{\left(ae^{(2\,e)}+be^{(2\,e)}\right)e^{(2\,fx)}}{a-b}+1\right)+2\,fx\mathrm{Li}_2\left(\frac{\left(ae^{(2\,e)}+be^{(2\,e)}\right)e^{(2\,fx)}}{a-b}\right)-\mathrm{Li}_3\left(\frac{\left(ae^{(2\,e)}+be^{(2\,e)}\right)e^{(2\,fx)}}{a-b}\right)\right)bd^2}{2\left(a^2f^3-b^2f^3\right)} - c^2\left(\frac{b\log\left(-(a-b)e^{(-2\,fx-2\,e)}+a+b\right)}{(a^2-b^2)f}-\frac{fx+e}{(a+b)f}\right) + \frac{2\left(bd^2f^3x^3+3\,bcdf^3x^2\right)}{3\left(a^2f^3-b^2f^3\right)}+\frac{d^2x^3+3\,cdx^2}{3\,(a+b)}$$

input
```
integrate((d*x+c)^2/(a+b*coth(f*x+e)),x, algorithm="maxima")
```

output
```
-(2*f*x*log(-(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + dilog((a*e^(
2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)))*b*c*d/(a^2*f^2 - b^2*f^2) - 1/2*(2*f
^2*x^2*log(-(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + 2*f*x*dilog((
a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)) - polylog(3, (a*e^(2*e) + b*e^(2
*e))*e^(2*f*x)/(a - b)))*b*d^2/(a^2*f^3 - b^2*f^3) - c^2*(b*log(-(a - b)*e
^(-2*f*x - 2*e) + a + b)/((a^2 - b^2)*f) - (f*x + e)/((a + b)*f)) + 2/3*(b
*d^2*f^3*x^3 + 3*b*c*d*f^3*x^2)/(a^2*f^3 - b^2*f^3) + 1/3*(d^2*x^3 + 3*c*d
*x^2)/(a + b)
```

Giac [F]

$$\int \frac{(c+dx)^2}{a+b\coth(e+fx)}\,dx = \int \frac{(dx+c)^2}{b\coth(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^2/(a+b*coth(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(b*coth(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{a+b\coth(e+fx)}\,dx=\int \frac{{\left(c+d\,x\right)}^2}{a+b\,\mathrm{coth}\left(e+f\,x\right)} \,d x$$

input
```
int((c + d*x)^2/(a + b*coth(e + f*x)),x)
```

output
```
int((c + d*x)^2/(a + b*coth(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{a+b\coth(e+fx)}\,dx = \frac{-6e^{2e}\left(\int \frac{e^{2fx}x^2}{e^{2fx+2e}a^2-e^{2fx+2e}b^2-a^2+2ab-b^2}dx\right)a^2b\,d^2f+6e^{2e}\left(\int \frac{e^{2fx}x^2}{e^{2fx+2e}a^2-e^{2fx+2e}b^2-a^2+2ab-b^2}dx\right)b^3d^2f-12e^{2e}(}{}$$

input
```
int((d*x+c)^2/(a+b*coth(f*x+e)),x)
```

output
```
( - 6*e**(2*e)*int((e**(2*f*x)*x**2)/(e**(2*e + 2*f*x)*a**2 - e**(2*e + 2*
f*x)*b**2 - a**2 + 2*a*b - b**2),x)*a**2*b*d**2*f + 6*e**(2*e)*int((e**(2*
f*x)*x**2)/(e**(2*e + 2*f*x)*a**2 - e**(2*e + 2*f*x)*b**2 - a**2 + 2*a*b -
 b**2),x)*b**3*d**2*f - 12*e**(2*e)*int((e**(2*f*x)*x)/(e**(2*e + 2*f*x)*a
**2 - e**(2*e + 2*f*x)*b**2 - a**2 + 2*a*b - b**2),x)*a**2*b*c*d*f + 12*e*
*(2*e)*int((e**(2*f*x)*x)/(e**(2*e + 2*f*x)*a**2 - e**(2*e + 2*f*x)*b**2 -
 a**2 + 2*a*b - b**2),x)*b**3*c*d*f - 3*log(e**(2*e + 2*f*x)*a + e**(2*e +
 2*f*x)*b - a + b)*b*c**2 + 3*a*c**2*f*x + 3*a*c*d*f*x**2 + a*d**2*f*x**3
+ 3*b*c**2*f*x + 3*b*c*d*f*x**2 + b*d**2*f*x**3)/(3*f*(a**2 - b**2))
```

3.54 $\int \frac{c+dx}{a+b\coth(e+fx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 108

$$\int \frac{c+dx}{a+b\coth(e+fx)}\,dx=\frac{(c+dx)^2}{2(a+b)d}-\frac{b(c+dx)\log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)\,f}+\frac{bd\,\mathrm{PolyLog}\left(2,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2\,(a^2-b^2)\,f^2}$$

output
```
1/2*(d*x+c)^2/(a+b)/d-b*(d*x+c)*ln(1-(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)
/f+1/2*b*d*polylog(2,(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^2
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.41

$$\int \frac{c+dx}{a+b\coth(e+fx)}\,dx=\frac{1}{2}\left(\frac{2b(c+dx)^2}{(a+b)d\,(a\,(-1+e^{2e})+b\,(1+e^{2e}))}-\frac{2b(c+dx)\log\left(1+\frac{(-a+b)e^{-2(e+fx)}}{a+b}\right)}{(a-b)(a+b)f}+\frac{bd\,\mathrm{PolyLog}\left(2,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a-b)(a+b)f^2}+\frac{x(2c+dx)\sinh(e)}{b\cosh(e)+a\sinh(e)}\right)$$

input
```
Integrate[(c + d*x)/(a + b*Coth[e + f*x]),x]
```

output
```
((2*b*(c + d*x)^2)/((a + b)*d*(a*(-1 + E^(2*e)) + b*(1 + E^(2*e)))) - (2*b
*(c + d*x)*Log[1 + (-a + b)/((a + b)*E^(2*(e + f*x)))])/((a - b)*(a + b)*f
) + (b*d*PolyLog[2, (a - b)/((a + b)*E^(2*(e + f*x)))])/((a - b)*(a + b)*f
^2) + (x*(2*c + d*x)*Sinh[e])/(b*Cosh[e] + a*Sinh[e]))/2
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4214, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a+b\coth(e+fx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{c+dx}{a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4214$$

$$2b\int -\frac{e^{-2(e+fx)}(c+dx)}{(a+b)^2-(a^2-b^2)\,e^{-2(e+fx)}}dx+\frac{(c+dx)^2}{2d(a+b)}$$

$$\Big\downarrow \; 25$$

$$\frac{(c+dx)^2}{2d(a+b)}-2b\int \frac{e^{-2(e+fx)}(c+dx)}{(a+b)^2-(a^2-b^2)\,e^{-2(e+fx)}}dx$$

$$\Big\downarrow \; 2620$$

$$\frac{(c+dx)^2}{2d(a+b)}-2b\left(\frac{(c+dx)\log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f\,(a^2-b^2)}-\frac{d\int \log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)dx}{2f\,(a^2-b^2)}\right)$$

$$\Big\downarrow \; 2715$$

$$\frac{(c+dx)^2}{2d(a+b)}-$$
$$2b\left(\frac{d\int e^{2(e+fx)}\log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)de^{-2(e+fx)}}{4f^2\left(a^2-b^2\right)}+\frac{(c+dx)\log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{(c+dx)^2}{2d(a+b)}-2b\left(\frac{(c+dx)\log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f\left(a^2-b^2\right)}-\frac{d\,\text{PolyLog}\left(2,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{4f^2\left(a^2-b^2\right)}\right)$$

input
```
Int[(c + d*x)/(a + b*Coth[e + f*x]),x]
```

output
```
(c + d*x)^2/(2*(a + b)*d) - 2*b*(((c + d*x)*Log[1 - (a - b)/((a + b)*E^(2*
(e + f*x)))])/(2*(a^2 - b^2)*f) - (d*PolyLog[2, (a - b)/((a + b)*E^(2*(e +
 f*x)))])/(4*(a^2 - b^2)*f^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4214
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Simp
[2*I*b   Int[(c + d*x)^m*E^(2*I*k*Pi)*(E^Simp[2*I*(e + f*x), x]/((a + I*b)^
2 + (a^2 + b^2)*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x])), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && IntegerQ[4*k] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(108) = 216$.

Time = 0.12 (sec) , antiderivative size = 357, normalized size of antiderivative = 3.31

method	result
risch	$\frac{d\,x^2}{2a+2b}+\frac{xc}{a+b}-\frac{bc\ln\left(e^{2fx+2e}a+e^{2fx+2e}b-a+b\right)}{f(a+b)(a-b)}+\frac{2bc\ln\left(e^{fx+e}\right)}{f(a+b)(a-b)}+\frac{bd\,x^2}{(a+b)(a-b)}-\frac{bd\ln\left(1-\frac{(a+b)e^{2fx+2e}}{a-b}\right)x}{f(a+b)(a-b)}+\frac{2bc}{f(a+b}$

input
```
int((d*x+c)/(a+b*coth(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a+b)*d*x^2+1/(a+b)*x*c-1/f*b/(a+b)*c/(a-b)*ln(exp(2*f*x+2*e)*a+exp(2*
f*x+2*e)*b-a+b)+2/f*b/(a+b)*c/(a-b)*ln(exp(f*x+e))+b/(a+b)/(a-b)*d*x^2-1/f
*b/(a+b)/(a-b)*d*ln(1-(a+b)*exp(2*f*x+2*e)/(a-b))*x+2/f*b/(a+b)/(a-b)*d*e*
x-1/f^2*b/(a+b)/(a-b)*d*ln(1-(a+b)*exp(2*f*x+2*e)/(a-b))*e+1/f^2*b/(a+b)/(
a-b)*d*e^2-1/2/f^2*b/(a+b)/(a-b)*d*polylog(2,(a+b)*exp(2*f*x+2*e)/(a-b))+1
/f^2*b/(a+b)*d*e/(a-b)*ln(exp(2*f*x+2*e)*a+exp(2*f*x+2*e)*b-a+b)-2/f^2*b/(
a+b)*d*e/(a-b)*ln(exp(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(103) = 206$.

Time = 0.10 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.78

$$\int \frac{c+dx}{a+b\coth(e+fx)}\,dx = \frac{(a+b)df^2x^2+2\,(a+b)cf^2x-2\,bd\mathrm{Li}_2\left(\sqrt{\frac{a+b}{a-b}}(\cosh\left(fx+e\right)+\sinh\left(fx+e\right))\right)-2\,bd\mathrm{Li}_2\left(-\sqrt{\frac{a+b}{a-b}}(\cosh\right.}{}$$

input
```
integrate((d*x+c)/(a+b*coth(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*((a + b)*d*f^2*x^2 + 2*(a + b)*c*f^2*x - 2*b*d*dilog(sqrt((a + b)/(a -
 b))*(cosh(f*x + e) + sinh(f*x + e))) - 2*b*d*dilog(-sqrt((a + b)/(a - b))
*(cosh(f*x + e) + sinh(f*x + e))) + 2*(b*d*e - b*c*f)*log(2*(a + b)*cosh(f
*x + e) + 2*(a + b)*sinh(f*x + e) + 2*(a - b)*sqrt((a + b)/(a - b))) + 2*(
b*d*e - b*c*f)*log(2*(a + b)*cosh(f*x + e) + 2*(a + b)*sinh(f*x + e) - 2*(
a - b)*sqrt((a + b)/(a - b))) - 2*(b*d*f*x + b*d*e)*log(sqrt((a + b)/(a -
b))*(cosh(f*x + e) + sinh(f*x + e)) + 1) - 2*(b*d*f*x + b*d*e)*log(-sqrt((
a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e)) + 1))/((a^2 - b^2)*f^2)
```

Sympy [F]

$$\int \frac{c+dx}{a+b\coth(e+fx)}\,dx = \int \frac{c+dx}{a+b\coth\left(e+fx\right)}\,dx$$

input
```
integrate((d*x+c)/(a+b*coth(f*x+e)),x)
```

output
```
Integral((c + d*x)/(a + b*coth(e + f*x)), x)
```

Maxima [F]

$$\int \frac{c+dx}{a+b\coth(e+fx)}\,dx = \int \frac{dx+c}{b\coth(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)/(a+b*coth(f*x+e)),x, algorithm="maxima")
```

output
```
-1/2*(4*b*integrate(-x/(a^2 - b^2 - (a^2*e^(2*e) + 2*a*b*e^(2*e) + b^2*e^(
2*e))*e^(2*f*x)), x) - x^2/(a + b))*d - c*(b*log(-(a - b)*e^(-2*f*x - 2*e)
 + a + b)/((a^2 - b^2)*f) - (f*x + e)/((a + b)*f))
```

Giac [F]

$$\int \frac{c+dx}{a+b\coth(e+fx)}\,dx = \int \frac{dx+c}{b\coth(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)/(a+b*coth(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)/(b*coth(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{a+b\coth(e+fx)}\,dx = \int \frac{c+d\,x}{a+b\coth(e+f\,x)}\,dx$$

input
```
int((c + d*x)/(a + b*coth(e + f*x)),x)
```

output
```
int((c + d*x)/(a + b*coth(e + f*x)), x)
```

Reduce [F]

$$\int \frac{c+dx}{a+b\coth(e+fx)}\,dx$$
$$= \frac{-4e^{2e}\left(\int \frac{e^{2fx}x}{e^{2fx+2e}a^2-e^{2fx+2e}b^2-a^2+2ab-b^2}\,dx\right)a^2bdf+4e^{2e}\left(\int \frac{e^{2fx}x}{e^{2fx+2e}a^2-e^{2fx+2e}b^2-a^2+2ab-b^2}\,dx\right)b^3df-2\log\left(e^{2f\cdots}\right.}{2f\left(a^2-b^2\right)}$$

input
```
int((d*x+c)/(a+b*coth(f*x+e)),x)
```

output
```
( - 4*e**(2*e)*int((e**(2*f*x)*x)/(e**(2*e + 2*f*x)*a**2 - e**(2*e + 2*f*x
)*b**2 - a**2 + 2*a*b - b**2),x)*a**2*b*d*f + 4*e**(2*e)*int((e**(2*f*x)*x
)/(e**(2*e + 2*f*x)*a**2 - e**(2*e + 2*f*x)*b**2 - a**2 + 2*a*b - b**2),x)
*b**3*d*f - 2*log(e**(2*e + 2*f*x)*a + e**(2*e + 2*f*x)*b - a + b)*b*c + 2
*a*c*f*x + a*d*f*x**2 + 2*b*c*f*x + b*d*f*x**2)/(2*f*(a**2 - b**2))
```

3.55 $\int \frac{1}{(c+dx)(a+b\coth(e+fx))} dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+b\coth(e+fx))}\,dx=\text{Int}\left(\frac{1}{(c+dx)(a+b\coth(e+fx))},x\right)$$

output
```
Defer(Int)(1/(d*x+c)/(a+b*coth(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 4.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\coth(e+fx))}\,dx=\int \frac{1}{(c+dx)(a+b\coth(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + b*Coth[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)*(a + b*Coth[e + f*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a+b\coth(e+fx))}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)\left(a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)}dx$$
$$\Big\downarrow \text{4223}$$
$$\int \frac{1}{(c+dx)(a+b\coth(e+fx))}dx$$

input
```
Int[1/((c + d*x)*(a + b*Coth[e + f*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\left(a+b\coth\left(fx+e\right)\right)}dx$$

input
```
int(1/(d*x+c)/(a+b*coth(f*x+e)),x)
```

output
```
int(1/(d*x+c)/(a+b*coth(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{(c+dx)(a+b\coth(e+fx))}\,dx=\int \frac{1}{(dx+c)(b\coth\left(fx+e\right)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*coth(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d*x + a*c + (b*d*x + b*c)*coth(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{(c+dx)(a+b\coth(e+fx))}\,dx=\int \frac{1}{(a+b\coth\left(e+fx\right))\left(c+dx\right)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*coth(f*x+e)),x)
```

output
```
Integral(1/((a + b*coth(e + f*x))*(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 117, normalized size of antiderivative = 5.85

$$\int \frac{1}{(c+dx)(a+b\coth(e+fx))}\,dx=\int \frac{1}{(dx+c)(b\coth\left(fx+e\right)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*coth(f*x+e)),x, algorithm="maxima")
```

output
```
-2*b*integrate(-1/(a^2*c - b^2*c + (a^2*d - b^2*d)*x - (a^2*c*e^(2*e) + 2*
a*b*c*e^(2*e) + b^2*c*e^(2*e) + (a^2*d*e^(2*e) + 2*a*b*d*e^(2*e) + b^2*d*e
^(2*e))*x)*e^(2*f*x)), x) + log(d*x + c)/(a*d + b*d)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\coth(e+fx))}\,dx = \int \frac{1}{(dx+c)(b\coth{(fx+e)}+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*coth(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(b*coth(f*x + e) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\coth(e+fx))}\,dx = \int \frac{1}{(a+b\coth{(e+f\,x)})\,(c+d\,x)}\,dx$$

input
```
int(1/((a + b*coth(e + f*x))*(c + d*x)),x)
```

output
```
int(1/((a + b*coth(e + f*x))*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 271, normalized size of antiderivative = 13.55

$$\int \frac{1}{(c+dx)(a+b\coth(e+fx))}\,dx$$
$$= \frac{-2\left(\int \frac{1}{e^{2fx+2e}a^2c+e^{2fx+2e}a^2dx+2e^{2fx+2e}abc+2e^{2fx+2e}abdx+e^{2fx+2e}b^2c+e^{2fx+2e}b^2dx-a^2c-a^2dx+b^2c+b^2dx}dx\right)abd-2\left(\int \frac{}{e^{2f}}\right.}{d\,(a+}$$

input
```
int(1/(d*x+c)/(a+b*coth(f*x+e)),x)
```

output
```
( - 2*int(1/(e**(2*e + 2*f*x)*a**2*c + e**(2*e + 2*f*x)*a**2*d*x + 2*e**(2
*e + 2*f*x)*a*b*c + 2*e**(2*e + 2*f*x)*a*b*d*x + e**(2*e + 2*f*x)*b**2*c +
 e**(2*e + 2*f*x)*b**2*d*x - a**2*c - a**2*d*x + b**2*c + b**2*d*x),x)*a*b
*d - 2*int(1/(e**(2*e + 2*f*x)*a**2*c + e**(2*e + 2*f*x)*a**2*d*x + 2*e**(
2*e + 2*f*x)*a*b*c + 2*e**(2*e + 2*f*x)*a*b*d*x + e**(2*e + 2*f*x)*b**2*c
+ e**(2*e + 2*f*x)*b**2*d*x - a**2*c - a**2*d*x + b**2*c + b**2*d*x),x)*b*
*2*d + log(c + d*x))/(d*(a + b))
```

3.56 $\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))}\,dx=\text{Int}\left(\frac{1}{(c+dx)^2(a+b\coth(e+fx))},x\right)$$

output
```
Defer(Int)(1/(d*x+c)^2/(a+b*coth(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 8.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))}\,dx=\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)^2*(a + b*Coth[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + b*Coth[e + f*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{(c+dx)^2\left(a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)}dx$$
$$\Big\downarrow \; 4223$$
$$\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))}dx$$

input
```
Int[1/((c + d*x)^2*(a + b*Coth[e + f*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)^2\left(a+b\coth\left(fx+e\right)\right)}dx$$

input
```
int(1/(d*x+c)^2/(a+b*coth(f*x+e)),x)
```

output
```
int(1/(d*x+c)^2/(a+b*coth(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(b\coth(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*coth(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2
)*coth(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 1.91 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))}\,dx = \int \frac{1}{(a+b\coth(e+fx))\,(c+dx)^2}\,dx$$

input
```
integrate(1/(d*x+c)**2/(a+b*coth(f*x+e)),x)
```

output
```
Integral(1/((a + b*coth(e + f*x))*(c + d*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 202, normalized size of antiderivative = 10.10

$$\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(b\coth(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*coth(f*x+e)),x, algorithm="maxima")
```

output
```
-2*b*integrate(-1/(a^2*c^2 - b^2*c^2 + (a^2*d^2 - b^2*d^2)*x^2 + 2*(a^2*c*
d - b^2*c*d)*x - (a^2*c^2*e^(2*e) + 2*a*b*c^2*e^(2*e) + b^2*c^2*e^(2*e) +
(a^2*d^2*e^(2*e) + 2*a*b*d^2*e^(2*e) + b^2*d^2*e^(2*e))*x^2 + 2*(a^2*c*d*e
^(2*e) + 2*a*b*c*d*e^(2*e) + b^2*c*d*e^(2*e))*x)*e^(2*f*x)), x) - 1/(a*c*d
 + b*c*d + (a*d^2 + b*d^2)*x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))}\,dx=\int \frac{1}{(dx+c)^2(b\coth(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*coth(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(b*coth(f*x + e) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))}\,dx=\int \frac{1}{(a+b\coth(e+f\,x))\,(c+d\,x)^2}\,dx$$

input
```
int(1/((a + b*coth(e + f*x))*(c + d*x)^2),x)
```

output
```
int(1/((a + b*coth(e + f*x))*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 937, normalized size of antiderivative = 46.85

$$\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))}\,dx$$
$$= \frac{-2\left(\int \frac{1}{e^{2fx+2e}a^2c^2+2e^{2fx+2e}a^2cdx+e^{2fx+2e}a^2d^2x^2+2e^{2fx+2e}abc^2+4e^{2fx+2e}abcdx+2e^{2fx+2e}abd^2x^2+e^{2fx+2e}b^2c^2+2e^{2fx+2e}b^2cdx+e^{2}}\right.}{}$$

input
```
int(1/(d*x+c)^2/(a+b*coth(f*x+e)),x)
```

output
```
( - 2*int(1/(e**(2*e + 2*f*x)*a**2*c**2 + 2*e**(2*e + 2*f*x)*a**2*c*d*x +
e**(2*e + 2*f*x)*a**2*d**2*x**2 + 2*e**(2*e + 2*f*x)*a*b*c**2 + 4*e**(2*e
+ 2*f*x)*a*b*c*d*x + 2*e**(2*e + 2*f*x)*a*b*d**2*x**2 + e**(2*e + 2*f*x)*b
**2*c**2 + 2*e**(2*e + 2*f*x)*b**2*c*d*x + e**(2*e + 2*f*x)*b**2*d**2*x**2
 - a**2*c**2 - 2*a**2*c*d*x - a**2*d**2*x**2 + b**2*c**2 + 2*b**2*c*d*x +
b**2*d**2*x**2),x)*a*b*c**2 - 2*int(1/(e**(2*e + 2*f*x)*a**2*c**2 + 2*e**(
2*e + 2*f*x)*a**2*c*d*x + e**(2*e + 2*f*x)*a**2*d**2*x**2 + 2*e**(2*e + 2*
f*x)*a*b*c**2 + 4*e**(2*e + 2*f*x)*a*b*c*d*x + 2*e**(2*e + 2*f*x)*a*b*d**2
*x**2 + e**(2*e + 2*f*x)*b**2*c**2 + 2*e**(2*e + 2*f*x)*b**2*c*d*x + e**(2
*e + 2*f*x)*b**2*d**2*x**2 - a**2*c**2 - 2*a**2*c*d*x - a**2*d**2*x**2 + b
**2*c**2 + 2*b**2*c*d*x + b**2*d**2*x**2),x)*a*b*c*d*x - 2*int(1/(e**(2*e
+ 2*f*x)*a**2*c**2 + 2*e**(2*e + 2*f*x)*a**2*c*d*x + e**(2*e + 2*f*x)*a**2
*d**2*x**2 + 2*e**(2*e + 2*f*x)*a*b*c**2 + 4*e**(2*e + 2*f*x)*a*b*c*d*x +
2*e**(2*e + 2*f*x)*a*b*d**2*x**2 + e**(2*e + 2*f*x)*b**2*c**2 + 2*e**(2*e
+ 2*f*x)*b**2*c*d*x + e**(2*e + 2*f*x)*b**2*d**2*x**2 - a**2*c**2 - 2*a**2
*c*d*x - a**2*d**2*x**2 + b**2*c**2 + 2*b**2*c*d*x + b**2*d**2*x**2),x)*b*
*2*c**2 - 2*int(1/(e**(2*e + 2*f*x)*a**2*c**2 + 2*e**(2*e + 2*f*x)*a**2*c*
d*x + e**(2*e + 2*f*x)*a**2*d**2*x**2 + 2*e**(2*e + 2*f*x)*a*b*c**2 + 4*e*
*(2*e + 2*f*x)*a*b*c*d*x + 2*e**(2*e + 2*f*x)*a*b*d**2*x**2 + e**(2*e + 2*
f*x)*b**2*c**2 + 2*e**(2*e + 2*f*x)*b**2*c*d*x + e**(2*e + 2*f*x)*b**2*...
```

3.57 $\int \frac{(c+dx)^3}{(a+b\coth(e+fx))^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 638

$$\int \frac{(c+dx)^3}{(a+b\coth(e+fx))^2}\,dx = -\frac{2b^2(c+dx)^3}{(a^2-b^2)^2 f} + \frac{2b^2(c+dx)^3}{(a-b)(a+b)^2\left(a-b-(a+b)e^{2e+2fx}\right)f} + \frac{(c+dx)^4}{4(a-b)^2 d} + \frac{3b^2d(c+dx)^2\log\left(1-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a^2-b^2)^2 f^2} - \frac{2b(c+dx)^3\log\left(1-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f} + \frac{2b^2(c+dx)^3\log\left(1-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a^2-b^2)^2 f} + \frac{3b^2d^2(c+dx)\operatorname{PolyLog}\left(2,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a^2-b^2)^2 f^3} - \frac{3bd(c+dx)^2\operatorname{PolyLog}\left(2,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f^2} + \frac{3b^2d(c+dx)^2\operatorname{PolyLog}\left(2,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a^2-b^2)^2 f^2} - \frac{3b^2d^3\operatorname{PolyLog}\left(3,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{2(a^2-b^2)^2 f^4} + \frac{3bd^2(c+dx)\operatorname{PolyLog}\left(3,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f^3} - \frac{3b^2d^2(c+dx)\operatorname{PolyLog}\left(3,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a^2-b^2)^2 f^3} - \frac{3bd^3\operatorname{PolyLog}\left(4,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{2(a-b)^2(a+b)f^4} + \frac{3b^2d^3\operatorname{PolyLog}\left(4,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{2(a^2-b^2)^2 f^4}$$

output
```
-2*b^2*(d*x+c)^3/(a^2-b^2)^2/f+2*b^2*(d*x+c)^3/(a-b)/(a+b)^2/(a-b-(a+b)*ex
p(2*f*x+2*e))/f+1/4*(d*x+c)^4/(a-b)^2/d+3*b^2*d*(d*x+c)^2*ln(1-(a+b)*exp(2
*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^2-2*b*(d*x+c)^3*ln(1-(a+b)*exp(2*f*x+2*e)/(
a-b))/(a-b)^2/(a+b)/f+2*b^2*(d*x+c)^3*ln(1-(a+b)*exp(2*f*x+2*e)/(a-b))/(a^
2-b^2)^2/f+3*b^2*d^2*(d*x+c)*polylog(2,(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^
2)^2/f^3-3*b*d*(d*x+c)^2*polylog(2,(a+b)*exp(2*f*x+2*e)/(a-b))/(a-b)^2/(a+
b)/f^2+3*b^2*d*(d*x+c)^2*polylog(2,(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2
/f^2-3/2*b^2*d^3*polylog(3,(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^4+3*b
*d^2*(d*x+c)*polylog(3,(a+b)*exp(2*f*x+2*e)/(a-b))/(a-b)^2/(a+b)/f^3-3*b^2
*d^2*(d*x+c)*polylog(3,(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^3-3/2*b*d
^3*polylog(4,(a+b)*exp(2*f*x+2*e)/(a-b))/(a-b)^2/(a+b)/f^4+3/2*b^2*d^3*pol
ylog(4,(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^4
```

Mathematica [A] (warning: unable to verify)

Time = 5.01 (sec) , antiderivative size = 659, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx)^3}{(a+b\coth(e+fx))^2}\,dx = \frac{16bc^2f^3(-3bd+2acf)x - \frac{16(a-b)b^2f^3(c+dx)^3}{a(-1+e^{2e})+b(1+e^{2e})} + \frac{8a(a-b)bf^4(c+dx)^4}{d(a(-1+e^{2e})+b(1+e^{2e}))} + 48bcdf^2(bd-acf)x\log\left(1+\frac{(-a+b)e}{a-}\right.}{}$$

input
```
Integrate[(c + d*x)^3/(a + b*Coth[e + f*x])^2,x]
```

output
```
(16*b*c^2*f^3*(-3*b*d + 2*a*c*f)*x - (16*(a - b)*b^2*f^3*(c + d*x)^3)/(a*(
-1 + E^(2*e)) + b*(1 + E^(2*e))) + (8*a*(a - b)*b*f^4*(c + d*x)^4)/(d*(a*(
-1 + E^(2*e)) + b*(1 + E^(2*e)))) + 48*b*c*d*f^2*(b*d - a*c*f)*x*Log[1 + (
-a + b)/((a + b)*E^(2*(e + f*x)))] + 24*b*d^2*f^2*(b*d - 2*a*c*f)*x^2*Log[
1 + (-a + b)/((a + b)*E^(2*(e + f*x)))] - 16*a*b*d^3*f^3*x^3*Log[1 + (-a +
 b)/((a + b)*E^(2*(e + f*x)))] + 8*b*c^2*f^2*(3*b*d - 2*a*c*f)*Log[a - b -
 (a + b)*E^(2*(e + f*x))] + 24*b*c*d*f*(-(b*d) + a*c*f)*PolyLog[2, (a - b)
/((a + b)*E^(2*(e + f*x)))] - 12*b*d^2*(b*d - 2*a*c*f)*(2*f*x*PolyLog[2, (
a - b)/((a + b)*E^(2*(e + f*x)))] + PolyLog[3, (a - b)/((a + b)*E^(2*(e +
f*x)))]) + 12*a*b*d^3*(2*f^2*x^2*PolyLog[2, (a - b)/((a + b)*E^(2*(e + f*x
)))] + 2*f*x*PolyLog[3, (a - b)/((a + b)*E^(2*(e + f*x)))] + PolyLog[4, (a
 - b)/((a + b)*E^(2*(e + f*x)))]) - ((a - b)*(a + b)*f^3*((a^2 + b^2)*f*x*
(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Cosh[f*x] - (a^2 - b^2)*f*x*(4
*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Cosh[2*e + f*x] + 2*b*(-4*b*(c +
 d*x)^3 + a*f*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))*Sinh[f*x]))/(
(b*Cosh[e] + a*Sinh[e])*(b*Cosh[e + f*x] + a*Sinh[e + f*x])))/(8*(a - b)^2
*(a + b)^2*f^4)
```

Rubi [A] (verified)

Time = 2.55 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4217, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a+b\coth(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^3}{\left(a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4217}$$

$$\int \left(\frac{4b^2(c+dx)^3e^{4e+4fx}}{(a-b)^2\left(a\left(1-\frac{b}{a}\right)-a\left(\frac{b}{a}+1\right)e^{2e+2fx}\right)^2}+\frac{4b(c+dx)^3e^{2e+2fx}}{(a-b)^2\left(a\left(1-\frac{b}{a}\right)-a\left(\frac{b}{a}+1\right)e^{2e+2fx}\right)}+\frac{(c+dx)^3}{(a-b)^2}\right)dx$$

$\downarrow$ 2009

$$\frac{3b^2d^2(c+dx)\operatorname{PolyLog}\left(2,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3(a^2-b^2)^2}-\frac{3b^2d^2(c+dx)\operatorname{PolyLog}\left(3,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3(a^2-b^2)^2}+\frac{3b^2d(c+dx)^2\operatorname{PolyLog}\left(2,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^2(a^2-b^2)^2}+\frac{3b^2d(c+dx)^2\log\left(1-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^2(a^2-b^2)^2}+\frac{2b^2(c+dx)^3\log\left(1-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f(a^2-b^2)^2}-\frac{2b^2(c+dx)^3}{f(a^2-b^2)^2}-\frac{3b^2d^3\operatorname{PolyLog}\left(3,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{2f^4(a^2-b^2)^2}+\frac{3b^2d^3\operatorname{PolyLog}\left(4,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{2f^4(a^2-b^2)^2}+\frac{2b^2(c+dx)^3}{f(a-b)(a+b)^2(-(a+b)e^{2e+2fx}+a-b)}+\frac{3bd^2(c+dx)\operatorname{PolyLog}\left(3,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3(a-b)^2(a+b)}-\frac{3bd(c+dx)^2\operatorname{PolyLog}\left(2,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^2(a-b)^2(a+b)}-\frac{2b(c+dx)^3\log\left(1-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f(a-b)^2(a+b)}+\frac{(c+dx)^4}{4d(a-b)^2}-\frac{3bd^3\operatorname{PolyLog}\left(4,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{2f^4(a-b)^2(a+b)}$$

input
```
Int[(c + d*x)^3/(a + b*Coth[e + f*x])^2,x]
```

output
```
(-2*b^2*(c + d*x)^3)/((a^2 - b^2)^2*f) + (2*b^2*(c + d*x)^3)/((a - b)*(a +
 b)^2*(a - b - (a + b)*E^(2*e + 2*f*x))*f) + (c + d*x)^4/(4*(a - b)^2*d) +
 (3*b^2*d*(c + d*x)^2*Log[1 - ((a + b)*E^(2*e + 2*f*x))/(a - b)])/((a^2 -
b^2)^2*f^2) - (2*b*(c + d*x)^3*Log[1 - ((a + b)*E^(2*e + 2*f*x))/(a - b)])
/((a - b)^2*(a + b)*f) + (2*b^2*(c + d*x)^3*Log[1 - ((a + b)*E^(2*e + 2*f*
x))/(a - b)])/((a^2 - b^2)^2*f) + (3*b^2*d^2*(c + d*x)*PolyLog[2, ((a + b)
*E^(2*e + 2*f*x))/(a - b)])/((a^2 - b^2)^2*f^3) - (3*b*d*(c + d*x)^2*PolyL
og[2, ((a + b)*E^(2*e + 2*f*x))/(a - b)])/((a - b)^2*(a + b)*f^2) + (3*b^2
*d*(c + d*x)^2*PolyLog[2, ((a + b)*E^(2*e + 2*f*x))/(a - b)])/((a^2 - b^2)
^2*f^2) - (3*b^2*d^3*PolyLog[3, ((a + b)*E^(2*e + 2*f*x))/(a - b)])/(2*(a^
2 - b^2)^2*f^4) + (3*b*d^2*(c + d*x)*PolyLog[3, ((a + b)*E^(2*e + 2*f*x))/
(a - b)])/((a - b)^2*(a + b)*f^3) - (3*b^2*d^2*(c + d*x)*PolyLog[3, ((a +
b)*E^(2*e + 2*f*x))/(a - b)])/((a^2 - b^2)^2*f^3) - (3*b*d^3*PolyLog[4, ((
a + b)*E^(2*e + 2*f*x))/(a - b)])/(2*(a - b)^2*(a + b)*f^4) + (3*b^2*d^3*P
olyLog[4, ((a + b)*E^(2*e + 2*f*x))/(a - b)])/(2*(a^2 - b^2)^2*f^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4217
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(a - I*b) - 2*I*(b/(a^2 +
 b^2 + (a - I*b)^2*E^(2*I*(e + f*x)))))^(-n), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[n, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2443 vs. $2(618) = 1236$.

Time = 0.28 (sec) , antiderivative size = 2444, normalized size of antiderivative = 3.83

method	result	size
risch	Expression too large to display	2444

input
```
int((d*x+c)^3/(a+b*coth(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-6/(a^2+2*a*b+b^2)/f^2/(a-b)^2*b*a*c*d^2*polylog(2,(a+b)*exp(2*f*x+2*e)/(a
-b))*x+6/(a^2+2*a*b+b^2)/f^2/(a-b)^2*b*e*a*c^2*d*ln(exp(2*f*x+2*e)*a+exp(2
*f*x+2*e)*b-a+b)-12/(a^2+2*a*b+b^2)/f^2/(a-b)^2*b*e*a*c^2*d*ln(exp(f*x+e))
-6/(a^2+2*a*b+b^2)/f^2/(a-b)^2*b*a*c^2*d*ln(1-(a+b)*exp(2*f*x+2*e)/(a-b))*
e+12/(a^2+2*a*b+b^2)/f/(a-b)^2*b*a*c^2*d*e*x-12/(a^2+2*a*b+b^2)/f^2/(a-b)^
2*b*a*c*d^2*e^2*x-6/(a^2+2*a*b+b^2)/f/(a-b)^2*b*a*c^2*d*ln(1-(a+b)*exp(2*f
*x+2*e)/(a-b))*x-6/(a^2+2*a*b+b^2)/f^3/(a-b)^2*b*e^2*a*c*d^2*ln(exp(2*f*x+
2*e)*a+exp(2*f*x+2*e)*b-a+b)+12/(a^2+2*a*b+b^2)/f^3/(a-b)^2*b*e^2*a*c*d^2*
ln(exp(f*x+e))-6/(a^2+2*a*b+b^2)/f/(a-b)^2*b*a*c*d^2*ln(1-(a+b)*exp(2*f*x+
2*e)/(a-b))*x^2+6/(a^2+2*a*b+b^2)/f^3/(a-b)^2*b*a*c*d^2*ln(1-(a+b)*exp(2*f
*x+2*e)/(a-b))*e^2+1/4*d^3/(a^2+2*a*b+b^2)*x^4+1/4/d/(a^2+2*a*b+b^2)*c^4+4
/(a^2+2*a*b+b^2)/(a-b)^2*b*a*c*d^2*x^3-8/(a^2+2*a*b+b^2)/f^3/(a-b)^2*b*a*c
*d^2*e^3+6/(a^2+2*a*b+b^2)/f^2/(a-b)^2*b*a*c^2*d*e^2+4/(a^2+2*a*b+b^2)/f^3
/(a-b)^2*b*a*d^3*e^3*x-12/(a^2+2*a*b+b^2)/f^2/(a-b)^2*b^2*d^2*c*e*x+6/(a^2
+2*a*b+b^2)/(a-b)^2*b*a*c^2*d*x^2+3/(a^2+2*a*b+b^2)/f^3/(a-b)^2*b*a*c*d^2*
polylog(3,(a+b)*exp(2*f*x+2*e)/(a-b))-3/(a^2+2*a*b+b^2)/f^2/(a-b)^2*b*a*c^
2*d*polylog(2,(a+b)*exp(2*f*x+2*e)/(a-b))+2/(a^2+2*a*b+b^2)/f^4/(a-b)^2*b*
e^3*a*d^3*ln(exp(2*f*x+2*e)*a+exp(2*f*x+2*e)*b-a+b)-4/(a^2+2*a*b+b^2)/f^4/
(a-b)^2*b*e^3*a*d^3*ln(exp(f*x+e))-2/(a^2+2*a*b+b^2)/f/(a-b)^2*b*a*d^3*ln(
1-(a+b)*exp(2*f*x+2*e)/(a-b))*x^3-3/(a^2+2*a*b+b^2)/f^2/(a-b)^2*b*a*d^3...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6171 vs. $2(614) = 1228$.

Time = 0.27 (sec) , antiderivative size = 6171, normalized size of antiderivative = 9.67

$$\int \frac{(c+dx)^3}{(a+b\coth(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+b*coth(f*x+e))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^3}{(a+b\coth(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**3/(a+b*coth(f*x+e))**2,x)
```

output
```
Exception raised: TypeError >> Invalid NaN comparison
```

Maxima [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 1056, normalized size of antiderivative = 1.66

$$\int \frac{(c+dx)^3}{(a+b\coth(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+b*coth(f*x+e))^2,x, algorithm="maxima")
```

output
```
-6*b^2*c^2*d*f*x/(a^4*f^2 - 2*a^2*b^2*f^2 + b^4*f^2) - 2/3*(4*f^3*x^3*log(
-(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + 6*f^2*x^2*dilog((a*e^(2*
e) + b*e^(2*e))*e^(2*f*x)/(a - b)) - 6*f*x*polylog(3, (a*e^(2*e) + b*e^(2*
e))*e^(2*f*x)/(a - b)) + 3*polylog(4, (a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a
 - b)))*a*b*d^3/(a^4*f^4 - 2*a^2*b^2*f^4 + b^4*f^4) + 3*b^2*c^2*d*log((a*e
^(2*e) + b*e^(2*e))*e^(2*f*x) - a + b)/(a^4*f^2 - 2*a^2*b^2*f^2 + b^4*f^2)
 - c^3*(2*a*b*log(-(a - b)*e^(-2*f*x - 2*e) + a + b)/((a^4 - 2*a^2*b^2 + b
^4)*f) + 2*b^2/((a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e
^(-2*f*x - 2*e))*f) - (f*x + e)/((a^2 + 2*a*b + b^2)*f)) - 3/2*(2*a*b*c*d^
2*f - b^2*d^3)*(2*f^2*x^2*log(-(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) +
 1) + 2*f*x*dilog((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)) - polylog(3,
(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)))/(a^4*f^4 - 2*a^2*b^2*f^4 + b^4
*f^4) - 3*(a*b*c^2*d*f - b^2*c*d^2)*(2*f*x*log(-(a*e^(2*e) + b*e^(2*e))*e^
(2*f*x)/(a - b) + 1) + dilog((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)))/(
a^4*f^3 - 2*a^2*b^2*f^3 + b^4*f^3) + (a*b*d^3*f^4*x^4 + 2*(2*a*b*c*d^2*f -
 b^2*d^3)*f^3*x^3 + 6*(a*b*c^2*d*f^2 - b^2*c*d^2*f)*f^2*x^2)/(a^4*f^4 - 2*
a^2*b^2*f^4 + b^4*f^4) + 1/4*(24*b^2*c^2*d*x + (a^2*d^3*f - 2*a*b*d^3*f +
b^2*d^3*f)*x^4 + 4*(a^2*c*d^2*f - 2*a*b*c*d^2*f + (c*d^2*f + 2*d^3)*b^2)*x
^3 + 6*(a^2*c^2*d*f - 2*a*b*c^2*d*f + (c^2*d*f + 4*c*d^2)*b^2)*x^2 - ((a^2
*d^3*f*e^(2*e) - b^2*d^3*f*e^(2*e))*x^4 + 4*(a^2*c*d^2*f*e^(2*e) - b^2*...
```

Giac [F]

$$\int \frac{(c+dx)^3}{(a+b\coth(e+fx))^2}\,dx = \int \frac{(dx+c)^3}{(b\coth(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^3/(a+b*coth(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(b*coth(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{(a+b\coth(e+fx))^2}\,dx = \int \frac{(c+d\,x)^3}{(a+b\coth\left(e+f\,x\right))^2}\,dx$$

input
```
int((c + d*x)^3/(a + b*coth(e + f*x))^2,x)
```

output
```
int((c + d*x)^3/(a + b*coth(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(c+dx)^3}{(a+b\coth(e+fx))^2}\,dx = \text{too large to display}$$

input
```
int((d*x+c)^3/(a+b*coth(f*x+e))^2,x)
```

output
```
(16*e**(2*e + 2*f*x)*int(x**3/(e**(4*e + 4*f*x)*a**4 + 4*e**(4*e + 4*f*x)*
a**3*b + 6*e**(4*e + 4*f*x)*a**2*b**2 + 4*e**(4*e + 4*f*x)*a*b**3 + e**(4*
e + 4*f*x)*b**4 - 2*e**(2*e + 2*f*x)*a**4 - 4*e**(2*e + 2*f*x)*a**3*b + 4*
e**(2*e + 2*f*x)*a*b**3 + 2*e**(2*e + 2*f*x)*b**4 + a**4 - 2*a**2*b**2 + b
**4),x)*a**6*b*d**3*f**4 + 16*e**(2*e + 2*f*x)*int(x**3/(e**(4*e + 4*f*x)*
a**4 + 4*e**(4*e + 4*f*x)*a**3*b + 6*e**(4*e + 4*f*x)*a**2*b**2 + 4*e**(4*
e + 4*f*x)*a*b**3 + e**(4*e + 4*f*x)*b**4 - 2*e**(2*e + 2*f*x)*a**4 - 4*e*
*(2*e + 2*f*x)*a**3*b + 4*e**(2*e + 2*f*x)*a*b**3 + 2*e**(2*e + 2*f*x)*b**
4 + a**4 - 2*a**2*b**2 + b**4),x)*a**5*b**2*d**3*f**4 - 32*e**(2*e + 2*f*x
)*int(x**3/(e**(4*e + 4*f*x)*a**4 + 4*e**(4*e + 4*f*x)*a**3*b + 6*e**(4*e
+ 4*f*x)*a**2*b**2 + 4*e**(4*e + 4*f*x)*a*b**3 + e**(4*e + 4*f*x)*b**4 - 2
*e**(2*e + 2*f*x)*a**4 - 4*e**(2*e + 2*f*x)*a**3*b + 4*e**(2*e + 2*f*x)*a*
b**3 + 2*e**(2*e + 2*f*x)*b**4 + a**4 - 2*a**2*b**2 + b**4),x)*a**4*b**3*d
**3*f**4 - 32*e**(2*e + 2*f*x)*int(x**3/(e**(4*e + 4*f*x)*a**4 + 4*e**(4*e
 + 4*f*x)*a**3*b + 6*e**(4*e + 4*f*x)*a**2*b**2 + 4*e**(4*e + 4*f*x)*a*b**
3 + e**(4*e + 4*f*x)*b**4 - 2*e**(2*e + 2*f*x)*a**4 - 4*e**(2*e + 2*f*x)*a
**3*b + 4*e**(2*e + 2*f*x)*a*b**3 + 2*e**(2*e + 2*f*x)*b**4 + a**4 - 2*a**
2*b**2 + b**4),x)*a**3*b**4*d**3*f**4 + 16*e**(2*e + 2*f*x)*int(x**3/(e**(
4*e + 4*f*x)*a**4 + 4*e**(4*e + 4*f*x)*a**3*b + 6*e**(4*e + 4*f*x)*a**2*b*
*2 + 4*e**(4*e + 4*f*x)*a*b**3 + e**(4*e + 4*f*x)*b**4 - 2*e**(2*e + 2*...
```

3.58 $\int \frac{(c+dx)^2}{(a+b\coth(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 475

$$\int \frac{(c+dx)^2}{(a+b\coth(e+fx))^2}\,dx = -\frac{2b^2(c+dx)^2}{(a^2-b^2)^2 f} + \frac{2b^2(c+dx)^2}{(a-b)(a+b)^2\left(a-b-(a+b)e^{2e+2fx}\right) f} + \frac{(c+dx)^3}{3(a-b)^2 d} + \frac{2b^2 d(c+dx)\log\left(1-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a^2-b^2)^2 f^2} - \frac{2b(c+dx)^2\log\left(1-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f} + \frac{2b^2(c+dx)^2\log\left(1-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a^2-b^2)^2 f} + \frac{b^2d^2\,\text{PolyLog}\left(2,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a^2-b^2)^2 f^3} - \frac{2bd(c+dx)\,\text{PolyLog}\left(2,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f^2} + \frac{2b^2d(c+dx)\,\text{PolyLog}\left(2,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a^2-b^2)^2 f^2} + \frac{bd^2\,\text{PolyLog}\left(3,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f^3} - \frac{b^2d^2\,\text{PolyLog}\left(3,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a^2-b^2)^2 f^3}$$

output
```
-2*b^2*(d*x+c)^2/(a^2-b^2)^2/f+2*b^2*(d*x+c)^2/(a-b)/(a+b)^2/(a-b-(a+b)*ex
p(2*f*x+2*e))/f+1/3*(d*x+c)^3/(a-b)^2/d+2*b^2*d*(d*x+c)*ln(1-(a+b)*exp(2*f
*x+2*e)/(a-b))/(a^2-b^2)^2/f^2-2*b*(d*x+c)^2*ln(1-(a+b)*exp(2*f*x+2*e)/(a-
b))/(a-b)^2/(a+b)/f+2*b^2*(d*x+c)^2*ln(1-(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-
b^2)^2/f+b^2*d^2*polylog(2,(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^3-2*b
*d*(d*x+c)*polylog(2,(a+b)*exp(2*f*x+2*e)/(a-b))/(a-b)^2/(a+b)/f^2+2*b^2*d
*(d*x+c)*polylog(2,(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^2+b*d^2*polyl
og(3,(a+b)*exp(2*f*x+2*e)/(a-b))/(a-b)^2/(a+b)/f^3-b^2*d^2*polylog(3,(a+b)
*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^3
```

Mathematica [A] (warning: unable to verify)

Time = 3.21 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx)^2}{(a+b\coth(e+fx))^2}\,dx = \frac{24bcf^2(-bd+acf)x + \frac{24(a-b)bcf^2(-bd+acf)x}{a(-1+e^{2e})+b(1+e^{2e})} + \frac{12(a-b)bdf^2(-bd+2acf)x^2}{a(-1+e^{2e})+b(1+e^{2e})} + \frac{8a(a-b)bd^2f^3x^3}{a(-1+e^{2e})+b(1+e^{2e})} + 12bdf(bd-2ac}{}$$

input
```
Integrate[(c + d*x)^2/(a + b*Coth[e + f*x])^2,x]
```

output
```
(24*b*c*f^2*(-(b*d) + a*c*f)*x + (24*(a - b)*b*c*f^2*(-(b*d) + a*c*f)*x)/(
a*(-1 + E^(2*e)) + b*(1 + E^(2*e))) + (12*(a - b)*b*d*f^2*(-(b*d) + 2*a*c*
f)*x^2)/(a*(-1 + E^(2*e)) + b*(1 + E^(2*e))) + (8*a*(a - b)*b*d^2*f^3*x^3)
/(a*(-1 + E^(2*e)) + b*(1 + E^(2*e))) + 12*b*d*f*(b*d - 2*a*c*f)*x*Log[1 +
 (-a + b)/((a + b)*E^(2*(e + f*x)))] - 12*a*b*d^2*f^2*x^2*Log[1 + (-a + b)
/((a + b)*E^(2*(e + f*x)))] + 12*b*c*f*(b*d - a*c*f)*Log[a - b - (a + b)*E
^(2*(e + f*x))] - 6*b*d*(b*d - 2*a*c*f)*PolyLog[2, (a - b)/((a + b)*E^(2*(
e + f*x)))] + 6*a*b*d^2*(2*f*x*PolyLog[2, (a - b)/((a + b)*E^(2*(e + f*x))
)] + PolyLog[3, (a - b)/((a + b)*E^(2*(e + f*x)))]) - ((a - b)*(a + b)*f^2
*((a^2 + b^2)*f*x*(3*c^2 + 3*c*d*x + d^2*x^2)*Cosh[f*x] - (a^2 - b^2)*f*x*
(3*c^2 + 3*c*d*x + d^2*x^2)*Cosh[2*e + f*x] + 2*b*(-3*b*(c + d*x)^2 + a*f*
x*(3*c^2 + 3*c*d*x + d^2*x^2))*Sinh[f*x]))/((b*Cosh[e] + a*Sinh[e])*(b*Cos
h[e + f*x] + a*Sinh[e + f*x])))/(6*(a - b)^2*(a + b)^2*f^3)
```

Rubi [A] (verified)

Time = 2.91 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4217, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a+b\coth(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^2}{\left(a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4217}$$

$$\int \left(\frac{4b^2(c+dx)^2e^{4e+4fx}}{(a-b)^2\left(a\left(1-\frac{b}{a}\right)-a\left(\frac{b}{a}+1\right)e^{2e+2fx}\right)^2}+\frac{4b(c+dx)^2e^{2e+2fx}}{(a-b)^2\left(a\left(1-\frac{b}{a}\right)-a\left(\frac{b}{a}+1\right)e^{2e+2fx}\right)}+\frac{(c+dx)^2}{(a-b)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2b^2d(c+dx)\operatorname{PolyLog}\left(2,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^2\left(a^2-b^2\right)^2}+\frac{2b^2d(c+dx)\log\left(1-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^2\left(a^2-b^2\right)^2}+\frac{2b^2(c+dx)^2\log\left(1-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f\left(a^2-b^2\right)^2}-\frac{2b^2(c+dx)^2}{f\left(a^2-b^2\right)^2}+\frac{b^2d^2\operatorname{PolyLog}\left(2,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3\left(a^2-b^2\right)^2}-\frac{b^2d^2\operatorname{PolyLog}\left(3,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3\left(a^2-b^2\right)^2}+\frac{2b^2(c+dx)^2}{f(a-b)(a+b)^2\left(-(a+b)e^{2e+2fx}+a-b\right)}-\frac{2bd(c+dx)\operatorname{PolyLog}\left(2,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^2(a-b)^2(a+b)}-\frac{2b(c+dx)^2\log\left(1-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f(a-b)^2(a+b)}+\frac{(c+dx)^3}{3d(a-b)^2}+\frac{bd^2\operatorname{PolyLog}\left(3,\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3(a-b)^2(a+b)}$$

input
```
Int[(c + d*x)^2/(a + b*Coth[e + f*x])^2,x]
```

output
```
(-2*b^2*(c + d*x)^2)/((a^2 - b^2)^2*f) + (2*b^2*(c + d*x)^2)/((a - b)*(a +
 b)^2*(a - b - (a + b)*E^(2*e + 2*f*x))*f) + (c + d*x)^3/(3*(a - b)^2*d) +
 (2*b^2*d*(c + d*x)*Log[1 - ((a + b)*E^(2*e + 2*f*x))/(a - b)])/((a^2 - b^
2)^2*f^2) - (2*b*(c + d*x)^2*Log[1 - ((a + b)*E^(2*e + 2*f*x))/(a - b)])/(
(a - b)^2*(a + b)*f) + (2*b^2*(c + d*x)^2*Log[1 - ((a + b)*E^(2*e + 2*f*x)
)/(a - b)])/((a^2 - b^2)^2*f) + (b^2*d^2*PolyLog[2, ((a + b)*E^(2*e + 2*f*
x))/(a - b)])/((a^2 - b^2)^2*f^3) - (2*b*d*(c + d*x)*PolyLog[2, ((a + b)*E
^(2*e + 2*f*x))/(a - b)])/((a - b)^2*(a + b)*f^2) + (2*b^2*d*(c + d*x)*Pol
yLog[2, ((a + b)*E^(2*e + 2*f*x))/(a - b)])/((a^2 - b^2)^2*f^2) + (b*d^2*P
olyLog[3, ((a + b)*E^(2*e + 2*f*x))/(a - b)])/((a - b)^2*(a + b)*f^3) - (b
^2*d^2*PolyLog[3, ((a + b)*E^(2*e + 2*f*x))/(a - b)])/((a^2 - b^2)^2*f^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4217
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(a - I*b) - 2*I*(b/(a^2 +
 b^2 + (a - I*b)^2*E^(2*I*(e + f*x)))))^(-n), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[n, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1469 vs. $2(464) = 928$.

Time = 0.21 (sec) , antiderivative size = 1470, normalized size of antiderivative = 3.09

method	result	size
risch	Expression too large to display	1470

input
```
int((d*x+c)^2/(a+b*coth(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-4/(a^2+2*a*b+b^2)/f^2/(a-b)^2*b*a*c*d*ln(1-(a+b)*exp(2*f*x+2*e)/(a-b))*e-
4/(a^2+2*a*b+b^2)/f/(a-b)^2*b*a*c*d*ln(1-(a+b)*exp(2*f*x+2*e)/(a-b))*x+4/(
a^2+2*a*b+b^2)/f^2/(a-b)^2*b*e*a*c*d*ln(exp(2*f*x+2*e)*a+exp(2*f*x+2*e)*b-
a+b)-8/(a^2+2*a*b+b^2)/f^2/(a-b)^2*b*e*a*c*d*ln(exp(f*x+e))+8/(a^2+2*a*b+b
^2)/f/(a-b)^2*b*a*c*d*e*x-2/(a^2+2*a*b+b^2)/f/(a-b)^2*b^2*d^2*x^2-2/(a^2+2
*a*b+b^2)/f^3/(a-b)^2*b^2*d^2*e^2+1/(a^2+2*a*b+b^2)/f^3/(a-b)^2*b^2*d^2*po
lylog(2,(a+b)*exp(2*f*x+2*e)/(a-b))-2/(a-b)/f/(a^2+2*a*b+b^2)*(d^2*x^2+2*c
*d*x+c^2)*b^2/(exp(2*f*x+2*e)*a+exp(2*f*x+2*e)*b-a+b)+d/(a^2+2*a*b+b^2)*c*
x^2+1/(a^2+2*a*b+b^2)*c^2*x+1/3*d^2/(a^2+2*a*b+b^2)*x^3+1/3/d/(a^2+2*a*b+b
^2)*c^3-4/(a^2+2*a*b+b^2)/f^2/(a-b)^2*b*a*d^2*e^2*x-2/(a^2+2*a*b+b^2)/f^2/
(a-b)^2*b*a*c*d*polylog(2,(a+b)*exp(2*f*x+2*e)/(a-b))-2/(a^2+2*a*b+b^2)/f/
(a-b)^2*b*a*d^2*ln(1-(a+b)*exp(2*f*x+2*e)/(a-b))*x^2+2/(a^2+2*a*b+b^2)/f^3
/(a-b)^2*b*a*d^2*ln(1-(a+b)*exp(2*f*x+2*e)/(a-b))*e^2-2/(a^2+2*a*b+b^2)/f^
2/(a-b)^2*b*a*d^2*polylog(2,(a+b)*exp(2*f*x+2*e)/(a-b))*x-2/(a^2+2*a*b+b^2
)/f^3/(a-b)^2*b*e^2*a*d^2*ln(exp(2*f*x+2*e)*a+exp(2*f*x+2*e)*b-a+b)+4/(a^2
+2*a*b+b^2)/f^3/(a-b)^2*b*e^2*a*d^2*ln(exp(f*x+e))+4/(a^2+2*a*b+b^2)/(a-b)
^2*b*a*c*d*x^2+4/(a^2+2*a*b+b^2)/f^2/(a-b)^2*b*a*c*d*e^2+4/(a^2+2*a*b+b^2)
/f^3/(a-b)^2*b^2*e*d^2*ln(exp(f*x+e))+2/(a^2+2*a*b+b^2)/f^2/(a-b)^2*b^2*c*
d*ln(exp(2*f*x+2*e)*a+exp(2*f*x+2*e)*b-a+b)-4/(a^2+2*a*b+b^2)/f^2/(a-b)^2*
b^2*c*d*ln(exp(f*x+e))-4/(a^2+2*a*b+b^2)/f^2/(a-b)^2*b^2*d^2*e*x+4/3/(a...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3702 vs. $2(460) = 920$.

Time = 0.17 (sec) , antiderivative size = 3702, normalized size of antiderivative = 7.79

$$\int \frac{(c+dx)^2}{(a+b\coth(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2/(a+b*coth(f*x+e))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2}{(a+b\coth(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**2/(a+b*coth(f*x+e))**2,x)
```

output
```
Exception raised: TypeError >> Invalid NaN comparison
```

Maxima [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 751, normalized size of antiderivative = 1.58

$$\int \frac{(c+dx)^2}{(a+b\coth(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2/(a+b*coth(f*x+e))^2,x, algorithm="maxima")
```

output
```
-4*b^2*c*d*f*x/(a^4*f^2 - 2*a^2*b^2*f^2 + b^4*f^2) - (2*f^2*x^2*log(-(a*e^
(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + 2*f*x*dilog((a*e^(2*e) + b*e^(
2*e))*e^(2*f*x)/(a - b)) - polylog(3, (a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a
 - b)))*a*b*d^2/(a^4*f^3 - 2*a^2*b^2*f^3 + b^4*f^3) + 2*b^2*c*d*log((a*e^(
2*e) + b*e^(2*e))*e^(2*f*x) - a + b)/(a^4*f^2 - 2*a^2*b^2*f^2 + b^4*f^2) -
 c^2*(2*a*b*log(-(a - b)*e^(-2*f*x - 2*e) + a + b)/((a^4 - 2*a^2*b^2 + b^4
)*f) + 2*b^2/((a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(
-2*f*x - 2*e))*f) - (f*x + e)/((a^2 + 2*a*b + b^2)*f)) - (2*a*b*c*d*f - b^
2*d^2)*(2*f*x*log(-(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + dilog(
(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)))/(a^4*f^3 - 2*a^2*b^2*f^3 + b^4
*f^3) + 2/3*(2*a*b*d^2*f^3*x^3 + 3*(2*a*b*c*d*f - b^2*d^2)*f^2*x^2)/(a^4*f
^3 - 2*a^2*b^2*f^3 + b^4*f^3) + 1/3*(12*b^2*c*d*x + (a^2*d^2*f - 2*a*b*d^2
*f + b^2*d^2*f)*x^3 + 3*(a^2*c*d*f - 2*a*b*c*d*f + (c*d*f + 2*d^2)*b^2)*x^
2 - ((a^2*d^2*f*e^(2*e) - b^2*d^2*f*e^(2*e))*x^3 + 3*(a^2*c*d*f*e^(2*e) -
b^2*c*d*f*e^(2*e))*x^2)*e^(2*f*x))/(a^4*f - 2*a^2*b^2*f + b^4*f - (a^4*f*e
^(2*e) + 2*a^3*b*f*e^(2*e) - 2*a*b^3*f*e^(2*e) - b^4*f*e^(2*e))*e^(2*f*x))
```

Giac [F]

$$\int \frac{(c+dx)^2}{(a+b\coth(e+fx))^2}\,dx = \int \frac{(dx+c)^2}{(b\coth(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^2/(a+b*coth(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(b*coth(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+b\coth(e+fx))^2}\,dx = \int \frac{(c+d\,x)^2}{(a+b\coth(e+f\,x))^2}\,dx$$

input
```
int((c + d*x)^2/(a + b*coth(e + f*x))^2,x)
```

output
```
int((c + d*x)^2/(a + b*coth(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{(a+b\coth(e+fx))^2}\,dx = \text{too large to display}$$

input
```
int((d*x+c)^2/(a+b*coth(f*x+e))^2,x)
```

output
```
(12*e**(2*e + 2*f*x)*int(x**2/(e**(4*e + 4*f*x)*a**4 + 4*e**(4*e + 4*f*x)*
a**3*b + 6*e**(4*e + 4*f*x)*a**2*b**2 + 4*e**(4*e + 4*f*x)*a*b**3 + e**(4*
e + 4*f*x)*b**4 - 2*e**(2*e + 2*f*x)*a**4 - 4*e**(2*e + 2*f*x)*a**3*b + 4*
e**(2*e + 2*f*x)*a*b**3 + 2*e**(2*e + 2*f*x)*b**4 + a**4 - 2*a**2*b**2 + b
**4),x)*a**6*b*d**2*f**3 + 12*e**(2*e + 2*f*x)*int(x**2/(e**(4*e + 4*f*x)*
a**4 + 4*e**(4*e + 4*f*x)*a**3*b + 6*e**(4*e + 4*f*x)*a**2*b**2 + 4*e**(4*
e + 4*f*x)*a*b**3 + e**(4*e + 4*f*x)*b**4 - 2*e**(2*e + 2*f*x)*a**4 - 4*e*
*(2*e + 2*f*x)*a**3*b + 4*e**(2*e + 2*f*x)*a*b**3 + 2*e**(2*e + 2*f*x)*b**
4 + a**4 - 2*a**2*b**2 + b**4),x)*a**5*b**2*d**2*f**3 - 24*e**(2*e + 2*f*x
)*int(x**2/(e**(4*e + 4*f*x)*a**4 + 4*e**(4*e + 4*f*x)*a**3*b + 6*e**(4*e
+ 4*f*x)*a**2*b**2 + 4*e**(4*e + 4*f*x)*a*b**3 + e**(4*e + 4*f*x)*b**4 - 2
*e**(2*e + 2*f*x)*a**4 - 4*e**(2*e + 2*f*x)*a**3*b + 4*e**(2*e + 2*f*x)*a*
b**3 + 2*e**(2*e + 2*f*x)*b**4 + a**4 - 2*a**2*b**2 + b**4),x)*a**4*b**3*d
**2*f**3 - 24*e**(2*e + 2*f*x)*int(x**2/(e**(4*e + 4*f*x)*a**4 + 4*e**(4*e
 + 4*f*x)*a**3*b + 6*e**(4*e + 4*f*x)*a**2*b**2 + 4*e**(4*e + 4*f*x)*a*b**
3 + e**(4*e + 4*f*x)*b**4 - 2*e**(2*e + 2*f*x)*a**4 - 4*e**(2*e + 2*f*x)*a
**3*b + 4*e**(2*e + 2*f*x)*a*b**3 + 2*e**(2*e + 2*f*x)*b**4 + a**4 - 2*a**
2*b**2 + b**4),x)*a**3*b**4*d**2*f**3 + 12*e**(2*e + 2*f*x)*int(x**2/(e**(
4*e + 4*f*x)*a**4 + 4*e**(4*e + 4*f*x)*a**3*b + 6*e**(4*e + 4*f*x)*a**2*b*
*2 + 4*e**(4*e + 4*f*x)*a*b**3 + e**(4*e + 4*f*x)*b**4 - 2*e**(2*e + 2*...
```

3.59 $\int \frac{c+dx}{(a+b\coth(e+fx))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 196

$$\int \frac{c+dx}{(a+b\coth(e+fx))^2}\,dx = -\frac{(c+dx)^2}{2(a^2-b^2)\,d}+\frac{(bd-2acf-2adfx)^2}{4a(a-b)(a+b)^2df^2}+\frac{b(c+dx)}{(a^2-b^2)\,f(a+b\coth(e+fx))}+\frac{b(bd-2acf-2adfx)\log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)^2\,f^2}+\frac{abd\,\mathrm{PolyLog}\left(2,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)^2\,f^2}$$

output
```
-1/2*(d*x+c)^2/(a^2-b^2)/d+1/4*(-2*a*d*f*x-2*a*c*f+b*d)^2/a/(a-b)/(a+b)^2/
d/f^2+b*(d*x+c)/(a^2-b^2)/f/(a+b*coth(f*x+e))+b*(-2*a*d*f*x-2*a*c*f+b*d)*l
n(1-(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)^2/f^2+a*b*d*polylog(2,(a-b)/(a+b
)/exp(2*f*x+2*e))/(a^2-b^2)^2/f^2
```

Mathematica [A] (verified)

Time = 2.52 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.24

$$\int \frac{c+dx}{(a+b\coth(e+fx))^2}\,dx$$

$$= \frac{\operatorname{csch}^2(e+fx)(b\cosh(e+fx)+a\sinh(e+fx))\left(4(a-b)bf(c+dx)\sinh(e+fx)+2(a-b)(e+fx)(-\right.}{}$$

input
```
Integrate[(c + d*x)/(a + b*Coth[e + f*x])^2,x]
```

output
```
(Csch[e + f*x]^2*(b*Cosh[e + f*x] + a*Sinh[e + f*x])*(4*(a - b)*b*f*(c + d
*x)*Sinh[e + f*x] + 2*(a - b)*(e + f*x)*(-2*c*f + d*(e - f*x))*(b*Cosh[e +
 f*x] + a*Sinh[e + f*x]) + (((-(b*d) + 2*a*f*(c + d*x))*((a - b)*(-(b*d) +
 2*a*f*(c + d*x)) - 4*a*b*d*Log[1 + (-a + b)/((a + b)*E^(2*(e + f*x)))]) +
 4*a^2*b*d^2*PolyLog[2, (a - b)/((a + b)*E^(2*(e + f*x)))])*(b*Cosh[e + f*
x] + a*Sinh[e + f*x]))/(a*(a + b)*d)))/(4*(a - b)^2*(a + b)*f^2*(a + b*Cot
h[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4216, 26, 3042, 4214, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+b\coth(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+dx}{\left(a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4216}$$

$$-\frac{i\int -\frac{i(bd-2afxd-2acf)}{a+b\coth(e+fx)}dx}{f\left(a^2-b^2\right)}+\frac{b(c+dx)}{f\left(a^2-b^2\right)(a+b\coth(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{\int \frac{bd-2afxd-2acf}{a+b\coth(e+fx)}dx}{f\left(a^2-b^2\right)}+\frac{b(c+dx)}{f\left(a^2-b^2\right)(a+b\coth(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \frac{bd-2afxd-2acf}{a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)}dx}{f\left(a^2-b^2\right)}+\frac{b(c+dx)}{f\left(a^2-b^2\right)(a+b\coth(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 4214}$$

$$-\frac{2b\int -\frac{e^{-2(e+fx)}(bd-2afxd-2acf)}{(a+b)^2-\left(a^2-b^2\right)e^{-2(e+fx)}}dx-\frac{(-2acf-2adfx+bd)^2}{4adf(a+b)}}{f\left(a^2-b^2\right)}+\frac{b(c+dx)}{f\left(a^2-b^2\right)(a+b\coth(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{-2b\int \frac{e^{-2(e+fx)}(bd-2afxd-2acf)}{(a+b)^2-\left(a^2-b^2\right)e^{-2(e+fx)}}dx-\frac{(-2acf-2adfx+bd)^2}{4adf(a+b)}}{f\left(a^2-b^2\right)}+\frac{b(c+dx)}{f\left(a^2-b^2\right)(a+b\coth(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{-2b\left(\frac{ad\int \log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)dx}{a^2-b^2}+\frac{(-2acf-2adfx+bd)\log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f\left(a^2-b^2\right)}\right)-\frac{(-2acf-2adfx+bd)^2}{4adf(a+b)}}{f\left(a^2-b^2\right)}+\frac{b(c+dx)}{f\left(a^2-b^2\right)(a+b\coth(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 2715}$$

$$-\frac{-2b\left(\frac{(-2acf-2adfx+bd)\log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f\left(a^2-b^2\right)}-\frac{ad\int e^{2(e+fx)}\log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)de^{-2(e+fx)}}{2f\left(a^2-b^2\right)}\right)-\frac{(-2acf-2adfx+bd)^2}{4adf(a+b)}}{f\left(a^2-b^2\right)}+\frac{b(c+dx)}{f\left(a^2-b^2\right)(a+b\coth(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{-2b\left(\frac{(-2acf-2adfx+bd)\log\left(1-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f(a^2-b^2)}+\frac{ad\,\text{PolyLog}\left(2,\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f(a^2-b^2)}\right)-\frac{(-2acf-2adfx+bd)^2}{4adf(a+b)}}{f\left(a^2-b^2\right)}+\frac{b(c+dx)}{f\left(a^2-b^2\right)(a+b\coth(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2-b^2\right)}$$

input
```
Int[(c + d*x)/(a + b*Coth[e + f*x])^2,x]
```

output
```
-1/2*(c + d*x)^2/((a^2 - b^2)*d) + (b*(c + d*x))/((a^2 - b^2)*f*(a + b*Cot
h[e + f*x])) - (-1/4*(b*d - 2*a*c*f - 2*a*d*f*x)^2/(a*(a + b)*d*f) - 2*b*(
((b*d - 2*a*c*f - 2*a*d*f*x)*Log[1 - (a - b)/((a + b)*E^(2*(e + f*x)))])/(
2*(a^2 - b^2)*f) + (a*d*PolyLog[2, (a - b)/((a + b)*E^(2*(e + f*x)))])/(2*
(a^2 - b^2)*f)))/((a^2 - b^2)*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4214
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Simp
[2*I*b   Int[(c + d*x)^m*E^(2*I*k*Pi)*(E^Simp[2*I*(e + f*x), x]/((a + I*b)^
2 + (a^2 + b^2)*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x])), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && IntegerQ[4*k] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

rule 4216
```
Int[((c_.) + (d_.)*(x_))/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol
] :> Simp[-(c + d*x)^2/(2*d*(a^2 + b^2)), x] + (Simp[1/(f*(a^2 + b^2))   In
t[(b*d + 2*a*c*f + 2*a*d*f*x)/(a + b*Tan[e + f*x]), x], x] - Simp[b*((c + d
*x)/(f*(a^2 + b^2)*(a + b*Tan[e + f*x]))), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(196) = 392$.

Time = 0.14 (sec) , antiderivative size = 620, normalized size of antiderivative = 3.16

method	result
risch	$\frac{d\,x^2}{2a^2+4ab+2b^2}+\frac{xc}{a^2+2ab+b^2}-\frac{2(dx+c)b^2}{(a-b)f(a^2+2ab+b^2)\left(e^{2fx+2e}a+e^{2fx+2e}b-a+b\right)}+\frac{b^2 d\ln\left(e^{2fx+2e}a+e^{2fx+2e}b-a+b\right)}{(a^2+2ab+b^2)f^2(a-b)^2}-$

input
```
int((d*x+c)/(a+b*coth(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(a^2+2*a*b+b^2)*d*x^2+1/(a^2+2*a*b+b^2)*x*c-2/(a-b)/f/(a^2+2*a*b+b^2)*
(d*x+c)*b^2/(exp(2*f*x+2*e)*a+exp(2*f*x+2*e)*b-a+b)+1/(a^2+2*a*b+b^2)/f^2/
(a-b)^2*b^2*d*ln(exp(2*f*x+2*e)*a+exp(2*f*x+2*e)*b-a+b)-2/(a^2+2*a*b+b^2)/
f^2/(a-b)^2*b^2*d*ln(exp(f*x+e))-2/(a^2+2*a*b+b^2)/f/(a-b)^2*b*a*c*ln(exp(
2*f*x+2*e)*a+exp(2*f*x+2*e)*b-a+b)+4/(a^2+2*a*b+b^2)/f/(a-b)^2*b*a*c*ln(ex
p(f*x+e))+2/(a^2+2*a*b+b^2)/f^2/(a-b)^2*b*a*d*e*ln(exp(2*f*x+2*e)*a+exp(2*
f*x+2*e)*b-a+b)-4/(a^2+2*a*b+b^2)/f^2/(a-b)^2*b*a*d*e*ln(exp(f*x+e))+2/(a^
2+2*a*b+b^2)/(a-b)^2*b*a*d*x^2-2/(a^2+2*a*b+b^2)/f/(a-b)^2*b*a*d*ln(1-(a+b
)*exp(2*f*x+2*e)/(a-b))*x+4/(a^2+2*a*b+b^2)/f/(a-b)^2*b*a*d*e*x-2/(a^2+2*a
*b+b^2)/f^2/(a-b)^2*b*a*d*ln(1-(a+b)*exp(2*f*x+2*e)/(a-b))*e+2/(a^2+2*a*b+
b^2)/f^2/(a-b)^2*b*a*d*e^2-1/(a^2+2*a*b+b^2)/f^2/(a-b)^2*b*a*d*polylog(2,(
a+b)*exp(2*f*x+2*e)/(a-b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1797 vs. $2(194) = 388$.

Time = 0.16 (sec) , antiderivative size = 1797, normalized size of antiderivative = 9.17

$$\int \frac{c+dx}{(a+b\coth(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(a+b*coth(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/2*((a^3 + a^2*b - a*b^2 - b^3)*d*f^2*x^2 + 2*(a^3 + a^2*b - a*b^2 - b^3
)*c*f^2*x - 4*(a^2*b - a*b^2)*d*e^2 - 4*(a*b^2 - b^3)*d*e - ((a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*d*f^2*x^2 - 4*(a^2*b + a*b^2)*d*e^2 + 8*(a^2*b + a*b^2)
*c*e*f - 4*(a*b^2 + b^3)*d*e + 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c*f^2 -
2*(a*b^2 + b^3)*d*f)*x)*cosh(f*x + e)^2 - 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*d*f^2*x^2 - 4*(a^2*b + a*b^2)*d*e^2 + 8*(a^2*b + a*b^2)*c*e*f - 4*(a*b^
2 + b^3)*d*e + 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c*f^2 - 2*(a*b^2 + b^3)*
d*f)*x)*cosh(f*x + e)*sinh(f*x + e) - ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*f
^2*x^2 - 4*(a^2*b + a*b^2)*d*e^2 + 8*(a^2*b + a*b^2)*c*e*f - 4*(a*b^2 + b^
3)*d*e + 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c*f^2 - 2*(a*b^2 + b^3)*d*f)*x
)*sinh(f*x + e)^2 + 4*(2*(a^2*b - a*b^2)*c*e + (a*b^2 - b^3)*c)*f + 4*((a^
2*b + a*b^2)*d*cosh(f*x + e)^2 + 2*(a^2*b + a*b^2)*d*cosh(f*x + e)*sinh(f*
x + e) + (a^2*b + a*b^2)*d*sinh(f*x + e)^2 - (a^2*b - a*b^2)*d)*dilog(sqrt
((a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))) + 4*((a^2*b + a*b^2)*d*
cosh(f*x + e)^2 + 2*(a^2*b + a*b^2)*d*cosh(f*x + e)*sinh(f*x + e) + (a^2*b
 + a*b^2)*d*sinh(f*x + e)^2 - (a^2*b - a*b^2)*d)*dilog(-sqrt((a + b)/(a -
b))*(cosh(f*x + e) + sinh(f*x + e))) + 2*(2*(a^2*b - a*b^2)*d*e - 2*(a^2*b
 - a*b^2)*c*f - (2*(a^2*b + a*b^2)*d*e - 2*(a^2*b + a*b^2)*c*f + (a*b^2 +
b^3)*d)*cosh(f*x + e)^2 - 2*(2*(a^2*b + a*b^2)*d*e - 2*(a^2*b + a*b^2)*c*f
 + (a*b^2 + b^3)*d)*cosh(f*x + e)*sinh(f*x + e) - (2*(a^2*b + a*b^2)*d*...
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{c+dx}{(a+b\coth(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)/(a+b*coth(f*x+e))**2,x)
```

output
```
Exception raised: TypeError >> Invalid NaN comparison
```

Maxima [F]

$$\int \frac{c+dx}{(a+b\coth(e+fx))^2}\,dx=\int \frac{dx+c}{(b\coth(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)/(a+b*coth(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/2*(8*a*b*f*integrate(x/(a^4*f*e^(2*f*x + 2*e) + 2*a^3*b*f*e^(2*f*x + 2*
e) - 2*a*b^3*f*e^(2*f*x + 2*e) - b^4*f*e^(2*f*x + 2*e) - a^4*f + 2*a^2*b^2
*f - b^4*f), x) + 2*b^2*(2*(f*x + e)/((a^4 - 2*a^2*b^2 + b^4)*f^2) - log((
a + b)*e^(2*f*x + 2*e) - a + b)/((a^4 - 2*a^2*b^2 + b^4)*f^2)) + ((a^2*f*e
^(2*e) - b^2*f*e^(2*e))*x^2*e^(2*f*x) - 4*b^2*x - (a^2*f - 2*a*b*f + b^2*f
)*x^2)/(a^4*f - 2*a^2*b^2*f + b^4*f - (a^4*f*e^(2*e) + 2*a^3*b*f*e^(2*e) -
 2*a*b^3*f*e^(2*e) - b^4*f*e^(2*e))*e^(2*f*x)))*d - c*(2*a*b*log(-(a - b)*
e^(-2*f*x - 2*e) + a + b)/((a^4 - 2*a^2*b^2 + b^4)*f) + 2*b^2/((a^4 - 2*a^
2*b^2 + b^4 - (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*f*x - 2*e))*f) - (f*x
+ e)/((a^2 + 2*a*b + b^2)*f))
```

Giac [F]

$$\int \frac{c+dx}{(a+b\coth(e+fx))^2}\,dx=\int \frac{dx+c}{(b\coth(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)/(a+b*coth(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)/(b*coth(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{(a+b\coth(e+fx))^2}\,dx=\int \frac{c+d\,x}{\left(a+b\coth\left(e+f\,x\right)\right)^2}\,dx$$

input
```
int((c + d*x)/(a + b*coth(e + f*x))^2,x)
```

output
```
int((c + d*x)/(a + b*coth(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{c+dx}{(a+b\coth(e+fx))^2}\,dx=\text{too large to display}$$

input
```
int((d*x+c)/(a+b*coth(f*x+e))^2,x)
```

output
```
(8*e**(2*e + 2*f*x)*int(x/(e**(4*e + 4*f*x)*a**4 + 4*e**(4*e + 4*f*x)*a**3
*b + 6*e**(4*e + 4*f*x)*a**2*b**2 + 4*e**(4*e + 4*f*x)*a*b**3 + e**(4*e +
4*f*x)*b**4 - 2*e**(2*e + 2*f*x)*a**4 - 4*e**(2*e + 2*f*x)*a**3*b + 4*e**(
2*e + 2*f*x)*a*b**3 + 2*e**(2*e + 2*f*x)*b**4 + a**4 - 2*a**2*b**2 + b**4)
,x)*a**6*b*d*f**2 + 8*e**(2*e + 2*f*x)*int(x/(e**(4*e + 4*f*x)*a**4 + 4*e*
*(4*e + 4*f*x)*a**3*b + 6*e**(4*e + 4*f*x)*a**2*b**2 + 4*e**(4*e + 4*f*x)*
a*b**3 + e**(4*e + 4*f*x)*b**4 - 2*e**(2*e + 2*f*x)*a**4 - 4*e**(2*e + 2*f
*x)*a**3*b + 4*e**(2*e + 2*f*x)*a*b**3 + 2*e**(2*e + 2*f*x)*b**4 + a**4 -
2*a**2*b**2 + b**4),x)*a**5*b**2*d*f**2 - 16*e**(2*e + 2*f*x)*int(x/(e**(4
*e + 4*f*x)*a**4 + 4*e**(4*e + 4*f*x)*a**3*b + 6*e**(4*e + 4*f*x)*a**2*b**
2 + 4*e**(4*e + 4*f*x)*a*b**3 + e**(4*e + 4*f*x)*b**4 - 2*e**(2*e + 2*f*x)
*a**4 - 4*e**(2*e + 2*f*x)*a**3*b + 4*e**(2*e + 2*f*x)*a*b**3 + 2*e**(2*e
+ 2*f*x)*b**4 + a**4 - 2*a**2*b**2 + b**4),x)*a**4*b**3*d*f**2 - 16*e**(2*
e + 2*f*x)*int(x/(e**(4*e + 4*f*x)*a**4 + 4*e**(4*e + 4*f*x)*a**3*b + 6*e*
*(4*e + 4*f*x)*a**2*b**2 + 4*e**(4*e + 4*f*x)*a*b**3 + e**(4*e + 4*f*x)*b*
*4 - 2*e**(2*e + 2*f*x)*a**4 - 4*e**(2*e + 2*f*x)*a**3*b + 4*e**(2*e + 2*f
*x)*a*b**3 + 2*e**(2*e + 2*f*x)*b**4 + a**4 - 2*a**2*b**2 + b**4),x)*a**3*
b**4*d*f**2 + 8*e**(2*e + 2*f*x)*int(x/(e**(4*e + 4*f*x)*a**4 + 4*e**(4*e
+ 4*f*x)*a**3*b + 6*e**(4*e + 4*f*x)*a**2*b**2 + 4*e**(4*e + 4*f*x)*a*b**3
 + e**(4*e + 4*f*x)*b**4 - 2*e**(2*e + 2*f*x)*a**4 - 4*e**(2*e + 2*f*x)...
```

3.60 $\int \frac{1}{(c+dx)(a+b\coth(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+b\coth(e+fx))^2}\,dx = \text{Int}\left(\frac{1}{(c+dx)(a+b\coth(e+fx))^2},x\right)$$

output
```
Defer(Int)(1/(d*x+c)/(a+b*coth(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 29.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\coth(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+b\coth(e+fx))^2}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + b*Coth[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)*(a + b*Coth[e + f*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a+b\coth(e+fx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)\left(a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{4223}$$
$$\int \frac{1}{(c+dx)(a+b\coth(e+fx))^2}dx$$

input
```
Int[1/((c + d*x)*(a + b*Coth[e + f*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\left(a+b\coth\left(fx+e\right)\right)^2}dx$$

input
```
int(1/(d*x+c)/(a+b*coth(f*x+e))^2,x)
```

output
```
int(1/(d*x+c)/(a+b*coth(f*x+e))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.75

$$\int \frac{1}{(c+dx)(a+b\coth(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(b\coth(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*coth(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*d*x + a^2*c + (b^2*d*x + b^2*c)*coth(f*x + e)^2 + 2*(a*b*d
*x + a*b*c)*coth(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c+dx)(a+b\coth(e+fx))^2}\,dx = \int \frac{1}{(a+b\coth(e+fx))^2(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*coth(f*x+e))**2,x)
```

output
```
Integral(1/((a + b*coth(e + f*x))**2*(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 472, normalized size of antiderivative = 23.60

$$\int \frac{1}{(c+dx)(a+b\coth(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(b\coth(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*coth(f*x+e))^2,x, algorithm="maxima")
```

output
```
2*b^2/(a^4*c*f - 2*a^2*b^2*c*f + b^4*c*f + (a^4*d*f - 2*a^2*b^2*d*f + b^4*
d*f)*x - (a^4*c*f*e^(2*e) + 2*a^3*b*c*f*e^(2*e) - 2*a*b^3*c*f*e^(2*e) - b^
4*c*f*e^(2*e) + (a^4*d*f*e^(2*e) + 2*a^3*b*d*f*e^(2*e) - 2*a*b^3*d*f*e^(2*
e) - b^4*d*f*e^(2*e))*x)*e^(2*f*x)) + log(d*x + c)/(a^2*d + 2*a*b*d + b^2*
d) - integrate(-2*(2*a*b*d*f*x + 2*a*b*c*f + b^2*d)/(a^4*c^2*f - 2*a^2*b^2
*c^2*f + b^4*c^2*f + (a^4*d^2*f - 2*a^2*b^2*d^2*f + b^4*d^2*f)*x^2 + 2*(a^
4*c*d*f - 2*a^2*b^2*c*d*f + b^4*c*d*f)*x - (a^4*c^2*f*e^(2*e) + 2*a^3*b*c^
2*f*e^(2*e) - 2*a*b^3*c^2*f*e^(2*e) - b^4*c^2*f*e^(2*e) + (a^4*d^2*f*e^(2*
e) + 2*a^3*b*d^2*f*e^(2*e) - 2*a*b^3*d^2*f*e^(2*e) - b^4*d^2*f*e^(2*e))*x^
2 + 2*(a^4*c*d*f*e^(2*e) + 2*a^3*b*c*d*f*e^(2*e) - 2*a*b^3*c*d*f*e^(2*e) -
 b^4*c*d*f*e^(2*e))*x)*e^(2*f*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\coth(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(b\coth(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*coth(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(b*coth(f*x + e) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\coth(e+fx))^2}\,dx = \int \frac{1}{(a+b\coth(e+f\,x))^2\,(c+d\,x)}\,dx$$

input
```
int(1/((a + b*coth(e + f*x))^2*(c + d*x)),x)
```

output
```
int(1/((a + b*coth(e + f*x))^2*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 2595, normalized size of antiderivative = 129.75

$$\int \frac{1}{(c+dx)(a+b\coth(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)/(a+b*coth(f*x+e))^2,x)
```

output
```
( - 4*e**(2*e)*int(e**(2*f*x)/(e**(4*e + 4*f*x)*a**4*c + e**(4*e + 4*f*x)*
a**4*d*x + 4*e**(4*e + 4*f*x)*a**3*b*c + 4*e**(4*e + 4*f*x)*a**3*b*d*x + 6
*e**(4*e + 4*f*x)*a**2*b**2*c + 6*e**(4*e + 4*f*x)*a**2*b**2*d*x + 4*e**(4
*e + 4*f*x)*a*b**3*c + 4*e**(4*e + 4*f*x)*a*b**3*d*x + e**(4*e + 4*f*x)*b*
*4*c + e**(4*e + 4*f*x)*b**4*d*x - 2*e**(2*e + 2*f*x)*a**4*c - 2*e**(2*e +
 2*f*x)*a**4*d*x - 4*e**(2*e + 2*f*x)*a**3*b*c - 4*e**(2*e + 2*f*x)*a**3*b
*d*x + 4*e**(2*e + 2*f*x)*a*b**3*c + 4*e**(2*e + 2*f*x)*a*b**3*d*x + 2*e**
(2*e + 2*f*x)*b**4*c + 2*e**(2*e + 2*f*x)*b**4*d*x + a**4*c + a**4*d*x - 2
*a**2*b**2*c - 2*a**2*b**2*d*x + b**4*c + b**4*d*x),x)*a**3*b*d - 12*e**(2
*e)*int(e**(2*f*x)/(e**(4*e + 4*f*x)*a**4*c + e**(4*e + 4*f*x)*a**4*d*x +
4*e**(4*e + 4*f*x)*a**3*b*c + 4*e**(4*e + 4*f*x)*a**3*b*d*x + 6*e**(4*e +
4*f*x)*a**2*b**2*c + 6*e**(4*e + 4*f*x)*a**2*b**2*d*x + 4*e**(4*e + 4*f*x)
*a*b**3*c + 4*e**(4*e + 4*f*x)*a*b**3*d*x + e**(4*e + 4*f*x)*b**4*c + e**(
4*e + 4*f*x)*b**4*d*x - 2*e**(2*e + 2*f*x)*a**4*c - 2*e**(2*e + 2*f*x)*a**
4*d*x - 4*e**(2*e + 2*f*x)*a**3*b*c - 4*e**(2*e + 2*f*x)*a**3*b*d*x + 4*e*
*(2*e + 2*f*x)*a*b**3*c + 4*e**(2*e + 2*f*x)*a*b**3*d*x + 2*e**(2*e + 2*f*
x)*b**4*c + 2*e**(2*e + 2*f*x)*b**4*d*x + a**4*c + a**4*d*x - 2*a**2*b**2*
c - 2*a**2*b**2*d*x + b**4*c + b**4*d*x),x)*a**2*b**2*d - 12*e**(2*e)*int(
e**(2*f*x)/(e**(4*e + 4*f*x)*a**4*c + e**(4*e + 4*f*x)*a**4*d*x + 4*e**(4*
e + 4*f*x)*a**3*b*c + 4*e**(4*e + 4*f*x)*a**3*b*d*x + 6*e**(4*e + 4*f*x...
```

3.61 $\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))^2}\,dx=\text{Int}\left(\frac{1}{(c+dx)^2(a+b\coth(e+fx))^2},x\right)$$

output
```
Defer(Int)(1/(d*x+c)^2/(a+b*coth(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 25.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))^2}\,dx=\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))^2}\,dx$$

input
```
Integrate[1/((c + d*x)^2*(a + b*Coth[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + b*Coth[e + f*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)^2\left(a-ib\tan\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{4223}$$
$$\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))^2}dx$$

input
```
Int[1/((c + d*x)^2*(a + b*Coth[e + f*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)^2\,(a+b\coth(fx+e))^2}dx$$

input
```
int(1/(d*x+c)^2/(a+b*coth(f*x+e))^2,x)
```

output
```
int(1/(d*x+c)^2/(a+b*coth(f*x+e))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 4.80

$$\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(b\coth(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*coth(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2)*coth(f*x + e)^2 + 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*co
th(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 3.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))^2}\,dx = \int \frac{1}{(a+b\coth(e+fx))^2\,(c+dx)^2}\,dx$$

input
```
integrate(1/(d*x+c)**2/(a+b*coth(f*x+e))**2,x)
```

output
```
Integral(1/((a + b*coth(e + f*x))**2*(c + d*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 4.82 (sec) , antiderivative size = 789, normalized size of antiderivative = 39.45

$$\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(b\coth(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*coth(f*x+e))^2,x, algorithm="maxima")
```

output
```
-(a^2*c*f - 2*a*b*c*f + (c*f - 2*d)*b^2 + (a^2*d*f - 2*a*b*d*f + b^2*d*f)*
x - (a^2*c*f*e^(2*e) - b^2*c*f*e^(2*e) + (a^2*d*f*e^(2*e) - b^2*d*f*e^(2*e
))*x)*e^(2*f*x))/(a^4*c^2*d*f - 2*a^2*b^2*c^2*d*f + b^4*c^2*d*f + (a^4*d^3
*f - 2*a^2*b^2*d^3*f + b^4*d^3*f)*x^2 + 2*(a^4*c*d^2*f - 2*a^2*b^2*c*d^2*f
 + b^4*c*d^2*f)*x - (a^4*c^2*d*f*e^(2*e) + 2*a^3*b*c^2*d*f*e^(2*e) - 2*a*b
^3*c^2*d*f*e^(2*e) - b^4*c^2*d*f*e^(2*e) + (a^4*d^3*f*e^(2*e) + 2*a^3*b*d^
3*f*e^(2*e) - 2*a*b^3*d^3*f*e^(2*e) - b^4*d^3*f*e^(2*e))*x^2 + 2*(a^4*c*d^
2*f*e^(2*e) + 2*a^3*b*c*d^2*f*e^(2*e) - 2*a*b^3*c*d^2*f*e^(2*e) - b^4*c*d^
2*f*e^(2*e))*x)*e^(2*f*x)) - integrate(-4*(a*b*d*f*x + a*b*c*f + b^2*d)/(a
^4*c^3*f - 2*a^2*b^2*c^3*f + b^4*c^3*f + (a^4*d^3*f - 2*a^2*b^2*d^3*f + b^
4*d^3*f)*x^3 + 3*(a^4*c*d^2*f - 2*a^2*b^2*c*d^2*f + b^4*c*d^2*f)*x^2 + 3*(
a^4*c^2*d*f - 2*a^2*b^2*c^2*d*f + b^4*c^2*d*f)*x - (a^4*c^3*f*e^(2*e) + 2*
a^3*b*c^3*f*e^(2*e) - 2*a*b^3*c^3*f*e^(2*e) - b^4*c^3*f*e^(2*e) + (a^4*d^3
*f*e^(2*e) + 2*a^3*b*d^3*f*e^(2*e) - 2*a*b^3*d^3*f*e^(2*e) - b^4*d^3*f*e^(
2*e))*x^3 + 3*(a^4*c*d^2*f*e^(2*e) + 2*a^3*b*c*d^2*f*e^(2*e) - 2*a*b^3*c*d
^2*f*e^(2*e) - b^4*c*d^2*f*e^(2*e))*x^2 + 3*(a^4*c^2*d*f*e^(2*e) + 2*a^3*b
*c^2*d*f*e^(2*e) - 2*a*b^3*c^2*d*f*e^(2*e) - b^4*c^2*d*f*e^(2*e))*x)*e^(2*
f*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(b\coth(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*coth(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(b*coth(f*x + e) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 3.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))^2}\,dx = \int \frac{1}{\left(a+b\coth\left(e+f\,x\right)\right)^2\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/((a + b*coth(e + f*x))^2*(c + d*x)^2),x)
```

output
```
int(1/((a + b*coth(e + f*x))^2*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 8964, normalized size of antiderivative = 448.20

$$\int \frac{1}{(c+dx)^2(a+b\coth(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)^2/(a+b*coth(f*x+e))^2,x)
```

output

```
( - 4*e**(2*e)*int(e**(2*f*x)/(e**(4*e + 4*f*x)*a**4*c**2 + 2*e**(4*e + 4*
f*x)*a**4*c*d*x + e**(4*e + 4*f*x)*a**4*d**2*x**2 + 4*e**(4*e + 4*f*x)*a**
3*b*c**2 + 8*e**(4*e + 4*f*x)*a**3*b*c*d*x + 4*e**(4*e + 4*f*x)*a**3*b*d**
2*x**2 + 6*e**(4*e + 4*f*x)*a**2*b**2*c**2 + 12*e**(4*e + 4*f*x)*a**2*b**2
*c*d*x + 6*e**(4*e + 4*f*x)*a**2*b**2*d**2*x**2 + 4*e**(4*e + 4*f*x)*a*b**
3*c**2 + 8*e**(4*e + 4*f*x)*a*b**3*c*d*x + 4*e**(4*e + 4*f*x)*a*b**3*d**2*
x**2 + e**(4*e + 4*f*x)*b**4*c**2 + 2*e**(4*e + 4*f*x)*b**4*c*d*x + e**(4*
e + 4*f*x)*b**4*d**2*x**2 - 2*e**(2*e + 2*f*x)*a**4*c**2 - 4*e**(2*e + 2*f
*x)*a**4*c*d*x - 2*e**(2*e + 2*f*x)*a**4*d**2*x**2 - 4*e**(2*e + 2*f*x)*a*
*3*b*c**2 - 8*e**(2*e + 2*f*x)*a**3*b*c*d*x - 4*e**(2*e + 2*f*x)*a**3*b*d*
*2*x**2 + 4*e**(2*e + 2*f*x)*a*b**3*c**2 + 8*e**(2*e + 2*f*x)*a*b**3*c*d*x
 + 4*e**(2*e + 2*f*x)*a*b**3*d**2*x**2 + 2*e**(2*e + 2*f*x)*b**4*c**2 + 4*
e**(2*e + 2*f*x)*b**4*c*d*x + 2*e**(2*e + 2*f*x)*b**4*d**2*x**2 + a**4*c**
2 + 2*a**4*c*d*x + a**4*d**2*x**2 - 2*a**2*b**2*c**2 - 4*a**2*b**2*c*d*x -
 2*a**2*b**2*d**2*x**2 + b**4*c**2 + 2*b**4*c*d*x + b**4*d**2*x**2),x)*a**
3*b*c**2 - 4*e**(2*e)*int(e**(2*f*x)/(e**(4*e + 4*f*x)*a**4*c**2 + 2*e**(4
*e + 4*f*x)*a**4*c*d*x + e**(4*e + 4*f*x)*a**4*d**2*x**2 + 4*e**(4*e + 4*f
*x)*a**3*b*c**2 + 8*e**(4*e + 4*f*x)*a**3*b*c*d*x + 4*e**(4*e + 4*f*x)*a**
3*b*d**2*x**2 + 6*e**(4*e + 4*f*x)*a**2*b**2*c**2 + 12*e**(4*e + 4*f*x)*a*
*2*b**2*c*d*x + 6*e**(4*e + 4*f*x)*a**2*b**2*d**2*x**2 + 4*e**(4*e + 4*...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
         'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
         'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
         'elliptic_pi','exp_integral_e','log_integral',
         'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
         'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file