Computer Algebra
Independent Integration Tests

Summer 2024

6-Hyperbolic-functions/6.4-Hyperbolic-cotangent/310-6.4.2

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**229**]. This is test number [310].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (229)	0.00 (0)
Mathematica	99.56 (228)	0.44 (1)
Fricas	79.91 (183)	20.09 (46)
Maple	73.80 (169)	26.20 (60)
Mupad	58.95 (135)	41.05 (94)
Giac	56.33 (129)	43.67 (100)
Reduce	51.09 (117)	48.91 (112)
Maxima	47.16 (108)	52.84 (121)
Sympy	15.28 (35)	84.72 (194)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

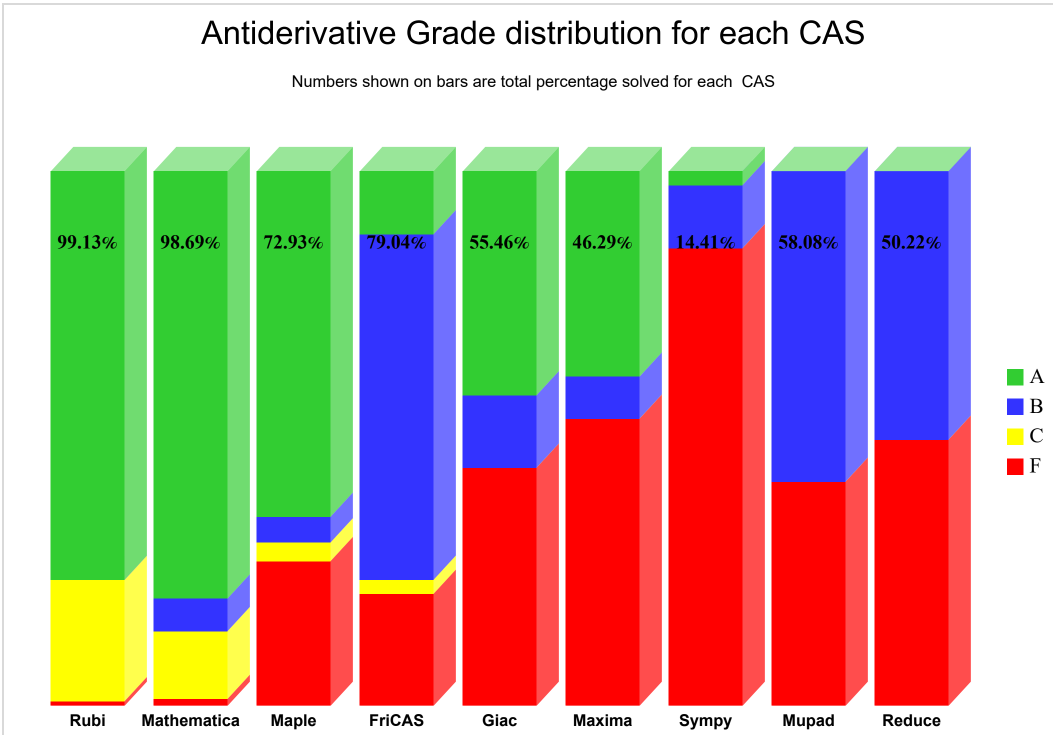
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

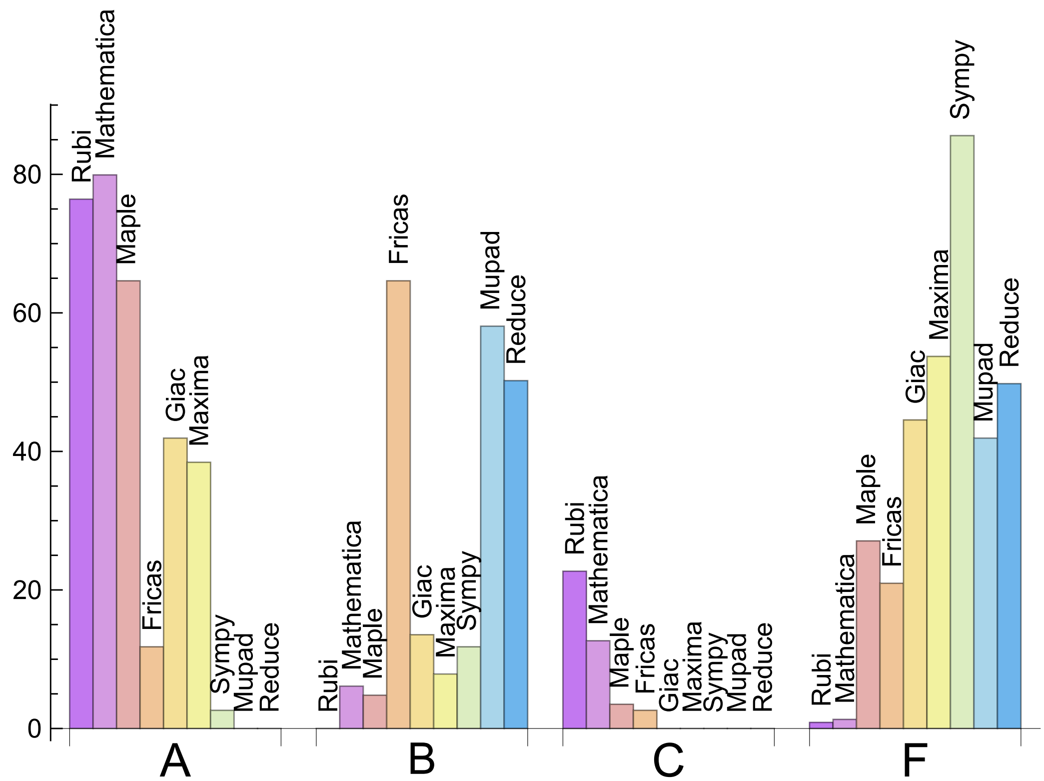
System	% A grade	% B grade	% C grade	% F grade
Mathematica	79.913	6.114	12.664	1.310
Rubi	76.419	0.000	22.707	0.873
Maple	64.629	4.803	3.493	27.074
Giac	41.921	13.537	0.000	44.541
Maxima	38.428	7.860	0.000	53.712
Fricas	11.790	64.629	2.620	20.961
Sympy	2.620	11.790	0.000	85.590
Mupad	0.000	58.079	0.000	41.921
Reduce	0.000	50.218	0.000	49.782

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	1	100.00	0.00	0.00
Fricas	46	78.26	0.00	21.74
Maple	60	100.00	0.00	0.00
Mupad	94	0.00	100.00	0.00
Giac	100	73.00	9.00	18.00
Reduce	112	100.00	0.00	0.00
Maxima	121	92.56	0.00	7.44
Sympy	194	90.72	6.19	3.09

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.13
Giac	0.14
Fricas	0.14
Reduce	0.25
Rubi	0.47
Maple	0.62
Mathematica	0.83
Mupad	2.16
Sympy	3.24

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	72.22	1.14	56.00	0.98
Mupad	76.15	1.33	42.00	1.00
Rubi	82.01	1.08	63.00	1.00
Giac	84.40	1.52	60.00	1.26
Maxima	86.55	1.56	47.50	1.14
Mathematica	92.48	1.24	62.50	1.00
Reduce	192.03	2.71	66.00	1.64
Sympy	204.80	3.93	136.00	3.70
Fricas	1045.77	9.47	292.00	5.02

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

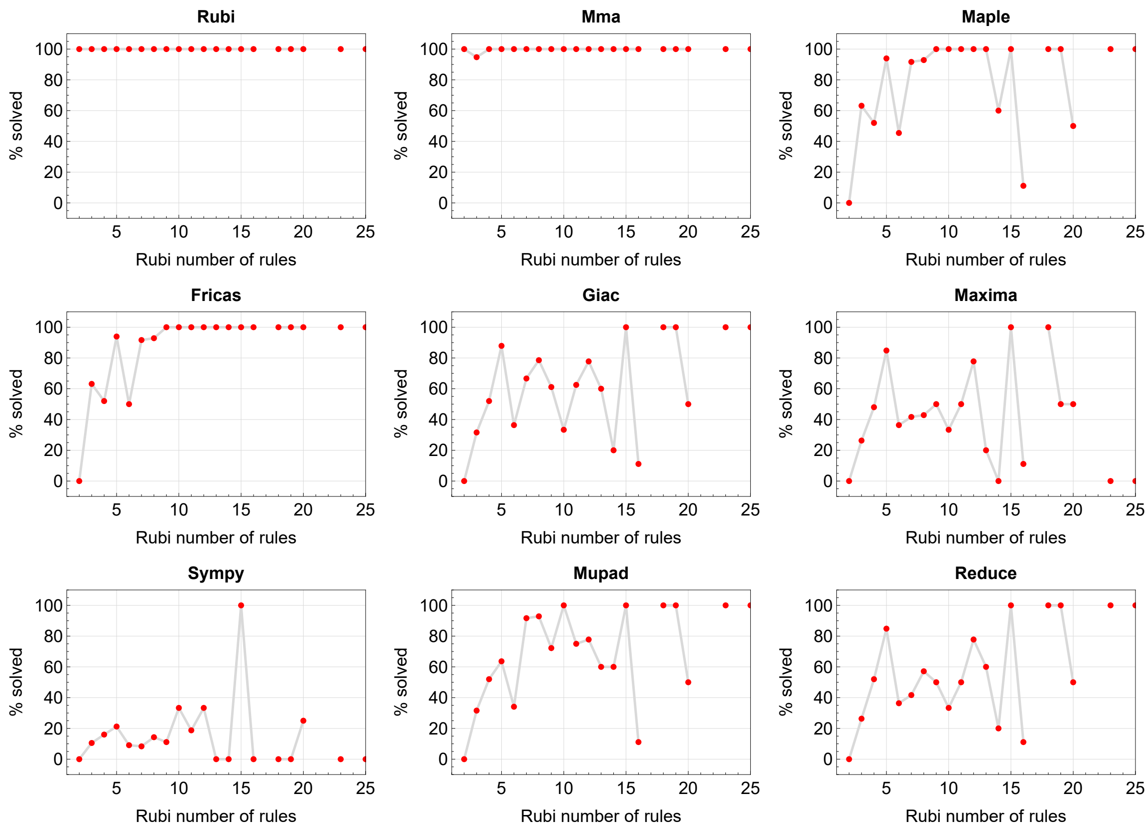


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

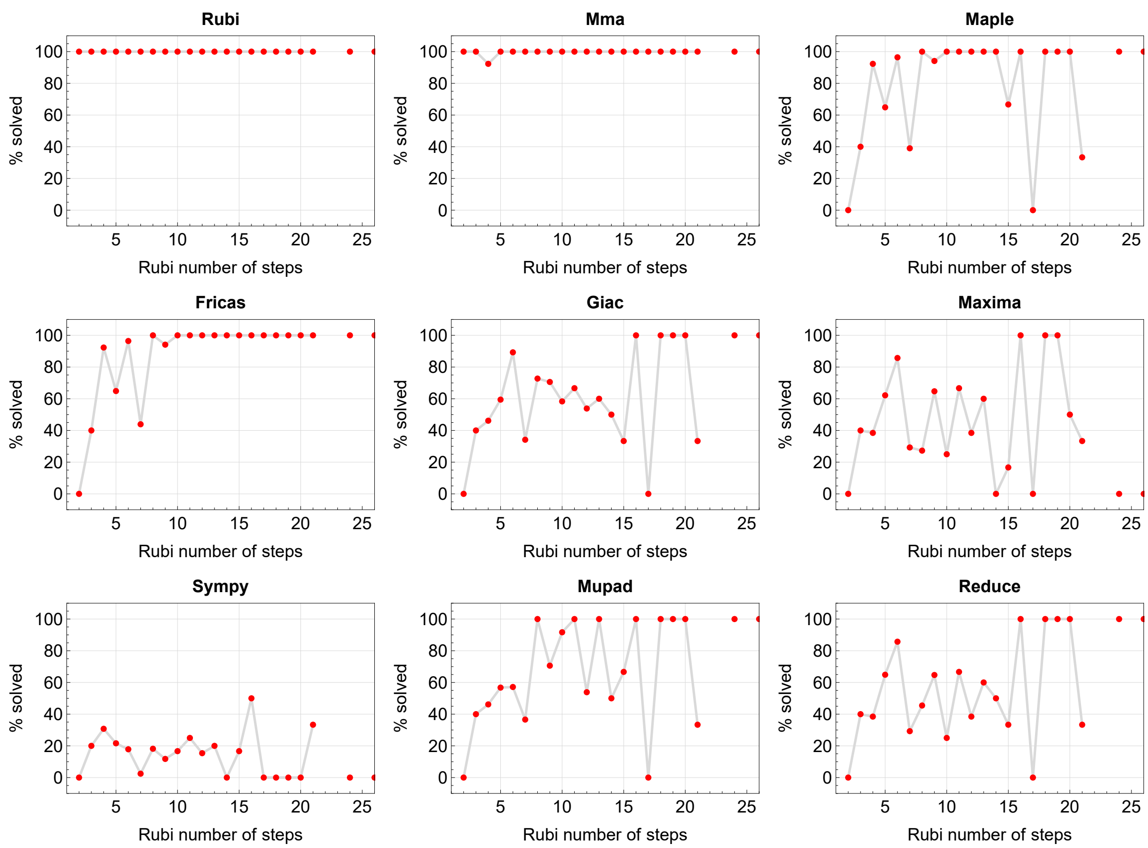


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

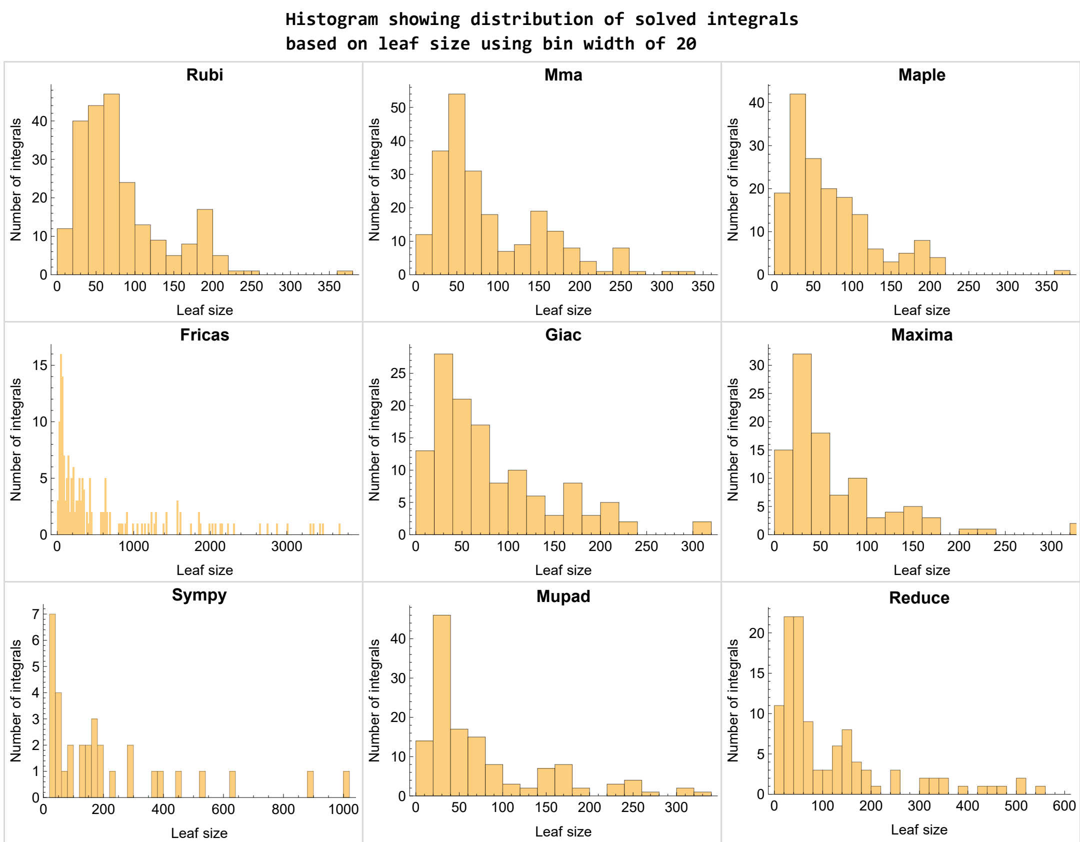


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

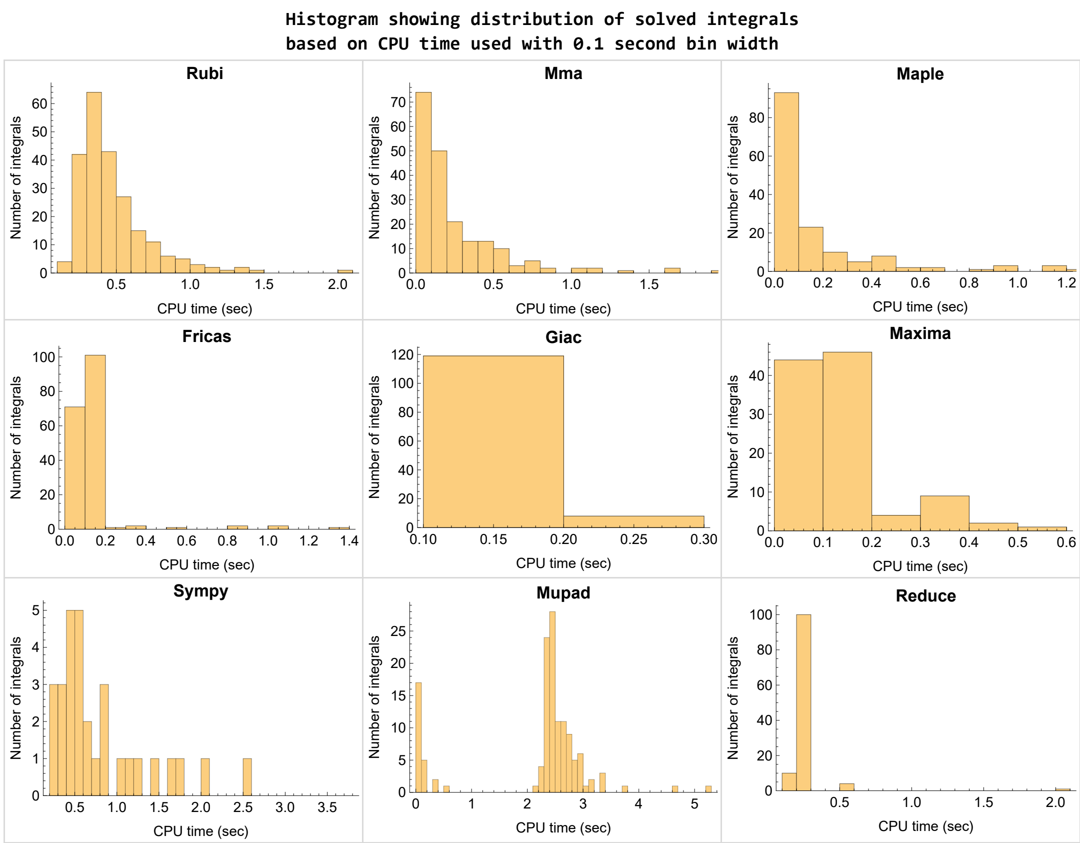


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

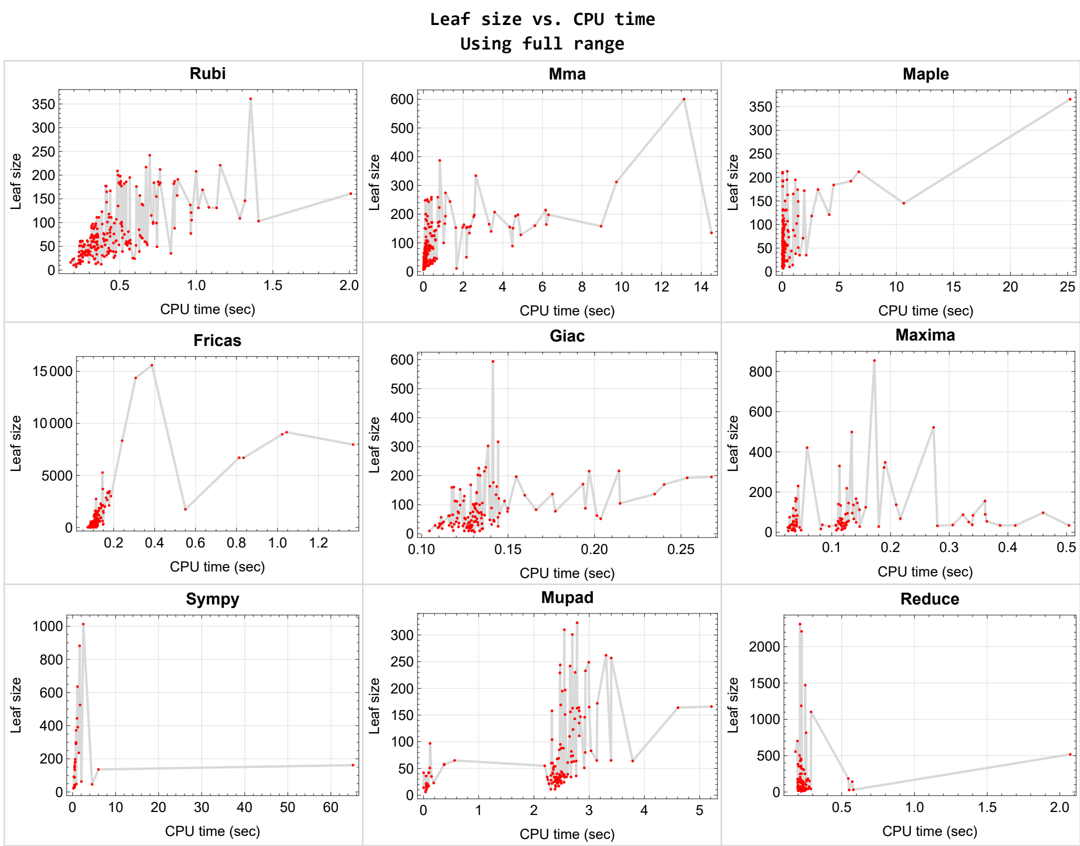


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{225, 229\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Reduce $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 210, 211, 212, 213, 214, 215\}$

Mathematica $\{173, 174, 177, 178, 179, 180, 181, 202, 203, 216, 217\}$

Maple $\{216, 217, 219, 220, 221\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

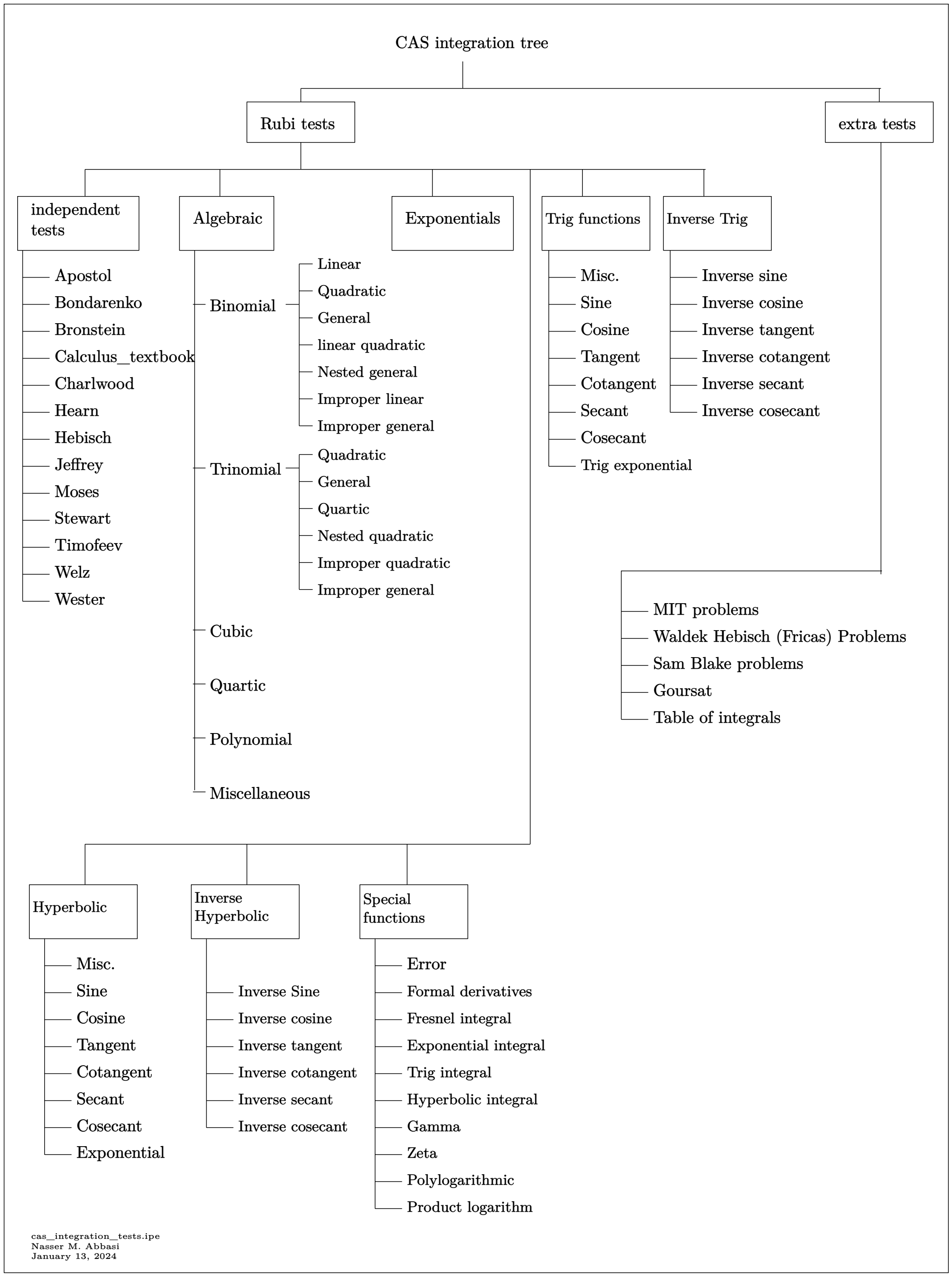
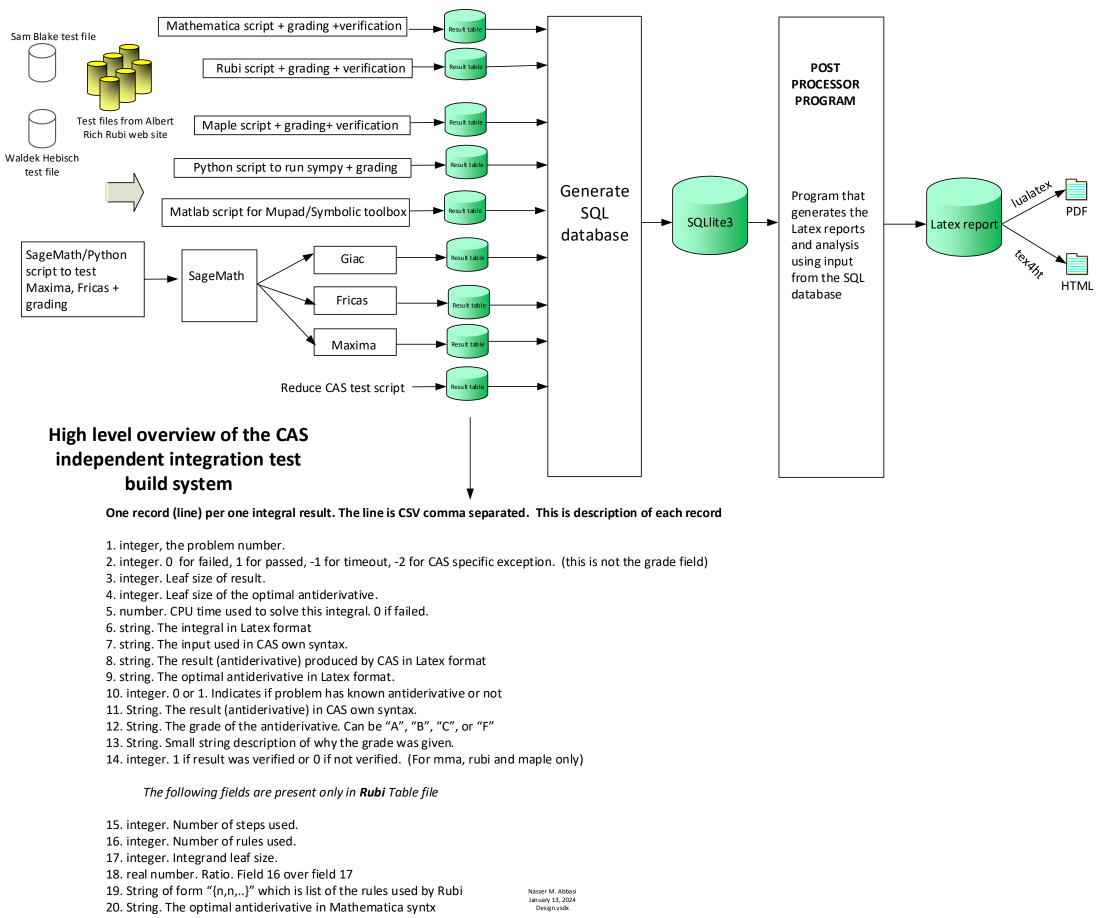


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 94, 95, 97, 98, 99, 100, 101, 102, 103, 104, 107, 110, 112, 115, 117, 118, 119, 121, 124, 126, 127, 132, 134, 141, 142, 143, 144, 149, 151, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 216, 217, 218, 219, 220, 221, 222, 223, 224, 226, 227, 228 }

B grade { }

C grade { 18, 19, 21, 35, 77, 78, 79, 80, 91, 93, 96, 105, 106, 108, 109, 111, 113, 114, 116, 120, 122, 123, 125, 128, 129, 130, 131, 133, 135, 136, 137, 138, 139, 140, 145, 146, 147, 148, 150, 152, 153, 154, 160, 186, 196, 198, 210, 211, 212, 213, 214, 215 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 65, 66, 67, 68, 69, 70, 71, 72, 73, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 90, 91, 92, 93, 94, 95, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 137, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 158, 160, 162, 163, 170, 171, 172, 174, 175, 176, 177, 178, 186, 189, 190, 191, 192, 194, 195, 196, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 221, 222, 223, 224, 226, 227, 228 }

B grade { 96, 109, 156, 173, 179, 180, 181, 182, 183, 184, 185, 187, 188, 202 }

C grade { 33, 34, 40, 41, 61, 62, 63, 64, 74, 75, 76, 87, 118, 134, 135, 136, 140, 157, 159, 161, 164, 165, 166, 167, 168, 169, 193, 197, 217 }

F normal fail { 89 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 18, 19, 20, 21, 29, 30, 31, 32, 33, 37, 38, 40, 41, 42, 43, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 97, 98, 99, 100, 101, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 160, 162, 163, 164, 165, 166, 167, 169, 186, 193, 196, 197, 198, 204, 205, 206, 207, 208, 209, 210, 211, 212, 215, 222, 223, 224, 226, 227, 228 }

B grade { 34, 35, 36, 96, 102, 103, 104, 118, 157, 159, 218 }

C grade { 116, 161, 168, 216, 217, 219, 220, 221 }

F normal fail { 15, 16, 17, 22, 23, 24, 25, 26, 27, 28, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 89, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 194, 195, 199, 200, 201, 202, 203, 213, 214 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 9, 46, 81, 85, 86, 92, 93, 94, 100, 107, 112, 113, 123, 127, 148, 149, 150, 151, 156, 157, 158, 160, 161, 163, 168, 218, 219 }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 47, 48, 49, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 87, 88, 90, 91, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 152, 153, 154, 155, 159, 162, 164, 165, 166, 167, 169, 186, 193, 196, 197, 198, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 220, 221 }

C grade { 222, 223, 224, 226, 227, 228 }

F normal fail { 15, 16, 17, 28, 39, 50, 89, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 194, 195, 199, 200, 201, 202, 203 }

F(-1) timedout fail { }

F(-2) exception fail { 51, 52, 53, 54, 55, 56, 57, 58, 59, 60 }

Maxima

A grade { 18, 19, 20, 21, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 64, 65, 66, 67, 68, 69, 80, 81, 82, 85, 86, 90, 91, 92, 93, 94, 95, 98, 99, 100, 101, 110, 111, 112, 113, 114, 115, 119, 121, 124, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 186, 193, 216, 217, 218, 219, 220, 221 }

B grade { 61, 62, 63, 77, 78, 79, 83, 84, 96, 97, 102, 103, 104, 116, 117, 196, 197, 198 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 70, 71, 72, 73, 74, 75, 76, 87, 88, 89, 118, 137, 138, 139, 140, 141, 142, 143, 144, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 194, 195, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 222, 223, 224, 226, 227, 228 }

F(-1) timedout fail { }

F(-2) exception fail { 105, 106, 107, 108, 109, 120, 122, 123, 125 }

Giac

A grade { 18, 19, 21, 40, 41, 42, 43, 61, 62, 63, 64, 65, 66, 67, 68, 69, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 90, 91, 92, 93, 94, 95, 97, 98, 99, 100, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 193, 197, 216, 217, 218, 219, 220, 221 }

B grade { 11, 12, 20, 70, 71, 72, 73, 74, 75, 76, 96, 101, 102, 103, 104, 118, 124, 126, 137, 138, 139, 140, 141, 142, 143, 144, 155, 160, 186, 196, 198 }

C grade { }

F normal fail { 15, 16, 17, 22, 23, 24, 25, 26, 27, 28, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 89, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 194, 195, 199, 200, 201, 202, 203, 210, 211, 215, 222, 223, 224, 226, 227, 228 }

F(-1) timedout fail { 204, 205, 206, 207, 208, 209, 212, 213, 214 }

F(-2) exception fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 14, 29, 30, 31, 32, 87, 88 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 186, 193, 196, 197, 198, 204, 205, 206, 207, 208, 209 }

C grade { }

F normal fail { }

F(-1) timedout fail { 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 89, 118, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 194, 195, 199, 200, 201, 202, 203, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 226, 227, 228 }

F(-2) exception fail { }

Sympy

A grade { 61, 62, 63, 64, 85, 86 }

B grade { 33, 34, 65, 66, 67, 68, 69, 77, 78, 79, 80, 81, 100, 132, 133, 134, 135, 136, 149, 150, 151, 152, 153, 154, 160, 167, 186 }

C grade { }

F normal fail { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 71, 72, 73, 74, 75, 76, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 199, 200, 201, 202, 206, 207, 210, 211, 212, 213, 214, 215, 218, 219, 222, 223, 224, 226, 227, 228 }

F(-1) timedout fail { 1, 55, 70, 203, 204, 205, 208, 209, 216, 217, 220, 221 }

F(-2) exception fail { 82, 83, 84, 196, 197, 198 }

Reduce

A grade { }

B grade { 18, 19, 20, 21, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 61, 62, 63, 64, 65, 66, 67, 68, 69, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 186, 193, 196, 197, 198, 216, 217, 218, 219, 220, 221 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 70, 71, 72, 73, 74, 75, 76, 87, 88, 89, 118, 137, 138, 139, 140, 141, 142, 143, 144, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 194, 195, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 222, 223, 224, 226, 227, 228 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	80	80	0	1567	0	0	58	83
N.S.	1	1.00	0.82	0.82	0.00	16.15	0.00	0.00	0.60	0.86
time (sec)	N/A	0.385	0.251	0.126	0.000	0.149	0.000	0.000	0.584	3.033

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	74	66	63	0	981	0	0	24	62
N.S.	1	0.95	0.85	0.81	0.00	12.58	0.00	0.00	0.31	0.79
time (sec)	N/A	0.319	0.108	0.058	0.000	0.136	0.000	0.000	0.581	2.681

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	72	66	62	0	631	0	0	37	61
N.S.	1	0.96	0.88	0.83	0.00	8.41	0.00	0.00	0.49	0.81
time (sec)	N/A	0.315	0.053	0.039	0.000	0.146	0.000	0.000	0.618	2.609

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	51	52	47	0	586	0	0	12	41
N.S.	1	0.88	0.90	0.81	0.00	10.10	0.00	0.00	0.21	0.71
time (sec)	N/A	0.236	0.027	0.081	0.000	0.133	0.000	0.000	0.628	2.517

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	51	49	46	0	592	0	0	24	38
N.S.	1	0.89	0.86	0.81	0.00	10.39	0.00	0.00	0.42	0.67
time (sec)	N/A	0.248	0.026	0.062	0.000	0.128	0.000	0.000	0.633	2.549

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	74	74	65	0	915	0	0	24	64
N.S.	1	0.95	0.95	0.83	0.00	11.73	0.00	0.00	0.31	0.82
time (sec)	N/A	0.316	0.062	0.039	0.000	0.139	0.000	0.000	0.680	2.765

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	76	78	64	0	1421	0	0	24	63
N.S.	1	0.96	0.99	0.81	0.00	17.99	0.00	0.00	0.30	0.80
time (sec)	N/A	0.352	0.078	0.041	0.000	0.150	0.000	0.000	0.668	2.720

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	101	90	83	0	2125	0	0	24	80
N.S.	1	1.01	0.90	0.83	0.00	21.25	0.00	0.00	0.24	0.80
time (sec)	N/A	0.419	0.187	0.040	0.000	0.177	0.000	0.000	0.753	2.928

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	198	193	209	0	292	0	0	30	249
N.S.	1	1.01	0.98	1.07	0.00	1.49	0.00	0.00	0.15	1.27
time (sec)	N/A	0.503	0.206	0.040	0.000	0.102	0.000	0.000	0.622	2.992

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	177	149	181	0	310	0	0	14	233
N.S.	1	0.99	0.84	1.02	0.00	1.74	0.00	0.00	0.08	1.31
time (sec)	N/A	0.411	0.125	0.056	0.000	0.106	0.000	0.000	0.471	2.932

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	111	108	109	0	291	0	217	14	146
N.S.	1	0.84	0.82	0.83	0.00	2.20	0.00	1.64	0.11	1.11
time (sec)	N/A	0.349	0.133	0.056	0.000	0.148	0.000	0.214	0.501	2.917

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	111	98	109	0	1187	0	216	14	147
N.S.	1	0.84	0.74	0.83	0.00	8.99	0.00	1.64	0.11	1.11
time (sec)	N/A	0.363	0.082	0.052	0.000	0.128	0.000	0.197	0.553	2.841

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	177	177	193	0	356	0	0	14	197
N.S.	1	0.99	0.99	1.08	0.00	2.00	0.00	0.00	0.08	1.11
time (sec)	N/A	0.407	0.158	0.052	0.000	0.149	0.000	0.000	0.528	2.565

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	200	243	211	0	3348	0	0	14	165
N.S.	1	1.01	1.23	1.07	0.00	16.91	0.00	0.00	0.07	0.83
time (sec)	N/A	0.489	0.262	0.036	0.000	0.169	0.000	0.000	0.512	2.999

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	0	0	10	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.341	0.101	0.000	0.000	0.000	0.000	0.000	0.586	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	49	0	0	0	0	0	14	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.319	0.091	0.000	0.000	0.000	0.000	0.000	0.616	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	0	0	0	0	0	16	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.458	0.075	0.000	0.000	0.000	0.000	0.000	0.536	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	53	97	823	0	90	185	0
N.S.	1	1.00	0.77	0.87	1.59	13.49	0.00	1.48	3.03	0.00
time (sec)	N/A	0.561	0.056	0.078	0.460	0.110	0.000	0.131	0.545	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	35	31	45	54	125	0	54	31	0
N.S.	1	1.13	1.00	1.45	1.74	4.03	0.00	1.74	1.00	0.00
time (sec)	N/A	0.295	0.027	0.044	0.364	0.117	0.000	0.121	0.580	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	56	34	128	0	60	27	30
N.S.	1	1.00	1.00	1.81	1.10	4.13	0.00	1.94	0.87	0.97
time (sec)	N/A	0.274	0.044	0.045	0.413	0.105	0.000	0.130	0.550	2.465

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	59	47	81	84	817	0	104	143	0
N.S.	1	0.91	0.72	1.25	1.29	12.57	0.00	1.60	2.20	0.00
time (sec)	N/A	0.336	0.079	0.046	0.341	0.114	0.000	0.131	0.572	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	243	187	166	0	0	1994	0	0	14	0
N.S.	1	0.77	0.68	0.00	0.00	8.21	0.00	0.00	0.06	0.00
time (sec)	N/A	0.507	0.237	0.000	0.000	0.157	0.000	0.000	0.619	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	236	182	200	0	0	2037	0	0	30	0
N.S.	1	0.77	0.85	0.00	0.00	8.63	0.00	0.00	0.13	0.00
time (sec)	N/A	0.528	0.116	0.000	0.000	0.156	0.000	0.000	0.592	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	211	167	151	0	0	1618	0	0	14	0
N.S.	1	0.79	0.72	0.00	0.00	7.67	0.00	0.00	0.07	0.00
time (sec)	N/A	0.437	0.072	0.000	0.000	0.127	0.000	0.000	0.624	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	211	165	182	0	0	8338	0	0	14	0
N.S.	1	0.78	0.86	0.00	0.00	39.52	0.00	0.00	0.07	0.00
time (sec)	N/A	0.414	0.118	0.000	0.000	0.241	0.000	0.000	0.562	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	236	184	248	0	0	2066	0	0	14	0
N.S.	1	0.78	1.05	0.00	0.00	8.75	0.00	0.00	0.06	0.00
time (sec)	N/A	0.764	0.191	0.000	0.000	0.142	0.000	0.000	0.341	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	253	187	251	0	0	14359	0	0	14	0
N.S.	1	0.74	0.99	0.00	0.00	56.75	0.00	0.00	0.06	0.00
time (sec)	N/A	0.858	0.390	0.000	0.000	0.307	0.000	0.000	0.259	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	0	0	0	0	0	16	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.538	0.071	0.000	0.000	0.000	0.000	0.000	0.229	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	97	82	107	0	2145	0	0	22	0
N.S.	1	0.72	0.61	0.80	0.00	16.01	0.00	0.00	0.16	0.00
time (sec)	N/A	0.509	0.365	0.092	0.000	0.147	0.000	0.000	0.220	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	77	63	86	0	627	0	0	36	0
N.S.	1	0.74	0.61	0.83	0.00	6.03	0.00	0.00	0.35	0.00
time (sec)	N/A	0.418	0.060	0.062	0.000	0.109	0.000	0.000	0.234	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	77	80	92	0	899	0	0	24	0
N.S.	1	0.73	0.76	0.88	0.00	8.56	0.00	0.00	0.23	0.00
time (sec)	N/A	0.393	0.060	0.052	0.000	0.118	0.000	0.000	0.194	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	99	90	106	0	3015	0	0	24	0
N.S.	1	0.70	0.64	0.75	0.00	21.38	0.00	0.00	0.17	0.00
time (sec)	N/A	0.464	0.132	0.050	0.000	0.187	0.000	0.000	0.235	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	50	43	130	87	1046	162	0	31	0
N.S.	1	0.68	0.58	1.76	1.18	14.14	2.19	0.00	0.42	0.00
time (sec)	N/A	0.334	0.062	0.076	0.323	0.109	65.144	0.000	0.247	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	38	41	119	34	392	136	0	19	0
N.S.	1	0.76	0.82	2.38	0.68	7.84	2.72	0.00	0.38	0.00
time (sec)	N/A	0.280	0.022	0.066	0.387	0.098	6.010	0.000	0.229	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	A	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	35	31	192	51	148	0	0	32	0
N.S.	1	1.13	1.00	6.19	1.65	4.77	0.00	0.00	1.03	0.00
time (sec)	N/A	0.260	0.017	0.066	0.333	0.119	0.000	0.000	0.236	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	192	32	187	0	0	25	0
N.S.	1	1.00	1.00	6.19	1.03	6.03	0.00	0.00	0.81	0.00
time (sec)	N/A	0.247	0.025	0.071	0.280	0.103	0.000	0.000	0.210	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	38	40	89	37	287	0	0	26	0
N.S.	1	0.76	0.80	1.78	0.74	5.74	0.00	0.00	0.52	0.00
time (sec)	N/A	0.263	0.045	0.068	0.339	0.106	0.000	0.000	0.252	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	52	51	132	89	1579	0	0	106	0
N.S.	1	0.65	0.64	1.65	1.11	19.74	0.00	0.00	1.32	0.00
time (sec)	N/A	0.548	0.064	0.080	0.361	0.116	0.000	0.000	0.230	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	53	0	0	0	0	0	16	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.541	0.077	0.000	0.000	0.000	0.000	0.000	0.226	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	69	43	77	137	3421	0	77	41	0
N.S.	1	0.63	0.39	0.70	1.25	31.10	0.00	0.70	0.37	0.00
time (sec)	N/A	0.641	0.051	0.085	0.210	0.175	0.000	0.150	0.254	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	38	41	55	34	415	0	27	18	0
N.S.	1	0.76	0.82	1.10	0.68	8.30	0.00	0.54	0.36	0.00
time (sec)	N/A	0.474	0.025	0.045	0.504	0.116	0.000	0.124	0.217	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	38	40	59	36	422	0	32	28	0
N.S.	1	0.76	0.80	1.18	0.72	8.44	0.00	0.64	0.56	0.00
time (sec)	N/A	0.420	0.045	0.044	0.306	0.120	0.000	0.141	0.220	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	71	68	84	155	3473	0	83	184	0
N.S.	1	0.60	0.58	0.71	1.31	29.43	0.00	0.70	1.56	0.00
time (sec)	N/A	0.437	0.169	0.046	0.361	0.180	0.000	0.166	0.212	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	299	217	244	0	0	2864	0	0	52	0
N.S.	1	0.73	0.82	0.00	0.00	9.58	0.00	0.00	0.17	0.00
time (sec)	N/A	0.671	1.338	0.000	0.000	0.163	0.000	0.000	0.265	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	238	186	166	0	0	618	0	0	14	0
N.S.	1	0.78	0.70	0.00	0.00	2.60	0.00	0.00	0.06	0.00
time (sec)	N/A	0.541	0.245	0.000	0.000	0.104	0.000	0.000	0.257	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	236	182	200	0	0	288	0	0	30	0
N.S.	1	0.77	0.85	0.00	0.00	1.22	0.00	0.00	0.13	0.00
time (sec)	N/A	0.500	0.110	0.000	0.000	0.111	0.000	0.000	0.221	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	236	184	248	0	0	3316	0	0	14	0
N.S.	1	0.78	1.05	0.00	0.00	14.05	0.00	0.00	0.06	0.00
time (sec)	N/A	0.493	0.083	0.000	0.000	0.176	0.000	0.000	0.266	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	238	184	251	0	0	1159	0	0	14	0
N.S.	1	0.77	1.05	0.00	0.00	4.87	0.00	0.00	0.06	0.00
time (sec)	N/A	0.722	0.191	0.000	0.000	0.118	0.000	0.000	0.218	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	313	221	274	0	0	15579	0	0	14	0
N.S.	1	0.71	0.88	0.00	0.00	49.77	0.00	0.00	0.04	0.00
time (sec)	N/A	1.156	1.107	0.000	0.000	0.387	0.000	0.000	0.253	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	0	0	16	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.555	0.079	0.000	0.000	0.000	0.000	0.000	0.253	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	0	0	0	0	0	16	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.415	0.087	0.000	0.000	0.000	0.000	0.000	0.224	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	68	54	0	0	0	0	0	15	0
N.S.	1	1.26	1.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.325	0.067	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	58	0	0	0	0	0	20	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.322	0.081	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	59	0	0	0	0	0	20	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.317	0.097	0.000	0.000	0.000	0.000	0.000	0.277	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	0	0	0	0	0	16	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.330	0.085	0.000	0.000	0.000	0.000	0.000	0.252	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	61	0	0	0	0	0	16	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.317	0.073	0.000	0.000	0.000	0.000	0.000	0.205	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	68	54	0	0	0	0	0	16	0
N.S.	1	1.26	1.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.308	0.068	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	58	0	0	0	0	0	18	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.317	0.075	0.000	0.000	0.000	0.000	0.000	0.211	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	61	0	0	0	0	0	18	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.351	0.085	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	61	0	0	0	0	0	18	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.317	0.091	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	52	86	31	140	448	48	41	177	88
N.S.	1	1.27	2.10	0.76	3.41	10.93	1.17	1.00	4.32	2.15
time (sec)	N/A	0.606	0.116	0.086	0.111	0.090	0.890	0.113	0.217	2.496

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	39	74	25	95	273	37	35	137	60
N.S.	1	1.26	2.39	0.81	3.06	8.81	1.19	1.13	4.42	1.94
time (sec)	N/A	0.627	0.104	0.049	0.135	0.091	0.539	0.110	0.204	2.329

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	26	48	19	55	142	31	29	95	36
N.S.	1	1.13	2.09	0.83	2.39	6.17	1.35	1.26	4.13	1.57
time (sec)	N/A	0.461	0.088	0.041	0.128	0.093	0.395	0.108	0.200	0.047

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	23	13	19	53	22	21	59	20
N.S.	1	1.00	1.77	1.00	1.46	4.08	1.69	1.62	4.54	1.54
time (sec)	N/A	0.343	0.009	0.023	0.122	0.094	0.230	0.110	0.232	0.051

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	11	10	26	27	10	19	14
N.S.	1	1.00	0.88	0.69	0.62	1.62	1.69	0.62	1.19	0.88
time (sec)	N/A	0.298	0.076	0.033	0.108	0.084	0.297	0.104	0.238	0.058

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	31	28	17	16	52	88	18	26	16
N.S.	1	1.19	1.08	0.65	0.62	2.00	3.38	0.69	1.00	0.62
time (sec)	N/A	0.386	0.452	0.048	0.081	0.085	0.498	0.112	0.217	0.050

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	46	33	23	22	86	182	24	33	22
N.S.	1	1.28	0.92	0.64	0.61	2.39	5.06	0.67	0.92	0.61
time (sec)	N/A	0.427	0.189	0.060	0.123	0.091	0.646	0.121	0.240	0.061

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	61	35	29	28	121	299	30	40	28
N.S.	1	1.33	0.76	0.63	0.61	2.63	6.50	0.65	0.87	0.61
time (sec)	N/A	0.397	0.227	0.074	0.147	0.078	0.819	0.111	0.220	2.311

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	76	41	35	34	159	444	36	47	34
N.S.	1	1.36	0.73	0.62	0.61	2.84	7.93	0.64	0.84	0.61
time (sec)	N/A	0.461	0.329	0.081	0.107	0.095	1.034	0.112	0.225	2.366

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	63	47	43	0	338	0	160	46	44
N.S.	1	1.11	0.82	0.75	0.00	5.93	0.00	2.81	0.81	0.77
time (sec)	N/A	0.407	0.542	0.095	0.000	0.093	0.000	0.118	0.216	2.495

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	48	39	35	0	210	0	112	32	54
N.S.	1	1.07	0.87	0.78	0.00	4.67	0.00	2.49	0.71	1.20
time (sec)	N/A	0.311	0.457	0.050	0.000	0.091	0.000	0.120	0.247	2.365

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	0	129	0	63	18	26
N.S.	1	1.00	1.00	0.82	0.00	3.91	0.00	1.91	0.55	0.79
time (sec)	N/A	0.255	0.361	0.035	0.000	0.096	0.000	0.116	0.234	2.389

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	0	101	0	37	7	16
N.S.	1	1.00	1.00	0.81	0.00	4.81	0.00	1.76	0.33	0.76
time (sec)	N/A	0.190	0.272	0.067	0.000	0.091	0.000	0.117	0.248	2.398

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	26	27	0	143	0	66	14	26
N.S.	1	1.00	0.81	0.84	0.00	4.47	0.00	2.06	0.44	0.81
time (sec)	N/A	0.252	0.296	0.053	0.000	0.095	0.000	0.118	0.255	2.491

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	28	35	0	246	0	113	20	32
N.S.	1	1.00	0.57	0.71	0.00	5.02	0.00	2.31	0.41	0.65
time (sec)	N/A	0.330	0.362	0.046	0.000	0.091	0.000	0.119	0.215	2.384

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	66	28	43	0	340	0	161	26	40
N.S.	1	1.08	0.46	0.70	0.00	5.57	0.00	2.64	0.43	0.66
time (sec)	N/A	0.392	0.402	0.046	0.000	0.101	0.000	0.119	0.198	2.344

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	146	141	150	348	2748	525	226	1471	244
N.S.	1	1.03	0.99	1.06	2.45	19.35	3.70	1.59	10.36	1.72
time (sec)	N/A	1.317	0.557	0.257	0.191	0.114	1.740	0.133	0.249	2.476

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	105	109	113	219	1396	391	153	816	158
N.S.	1	1.04	1.08	1.12	2.17	13.82	3.87	1.51	8.08	1.56
time (sec)	N/A	0.969	0.606	0.141	0.125	0.113	1.205	0.122	0.255	2.326

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	73	86	87	136	654	291	99	503	97
N.S.	1	1.06	1.25	1.26	1.97	9.48	4.22	1.43	7.29	1.41
time (sec)	N/A	0.632	0.275	0.106	0.137	0.102	0.783	0.122	0.210	0.119

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	65	53	49	205	177	57	178	51
N.S.	1	1.11	1.71	1.39	1.29	5.39	4.66	1.50	4.68	1.34
time (sec)	N/A	0.294	0.087	0.052	0.142	0.092	0.540	0.111	0.196	0.114

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	64	50	52	62	236	62	57	55
N.S.	1	1.00	1.28	1.00	1.04	1.24	4.72	1.24	1.14	1.10
time (sec)	N/A	0.380	0.069	0.089	0.111	0.094	1.446	0.116	0.241	2.195

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	98	100	93	124	426	0	130	428	104
N.S.	1	1.15	1.18	1.09	1.46	5.01	0.00	1.53	5.04	1.22
time (sec)	N/A	0.505	1.018	0.106	0.158	0.098	0.000	0.125	0.214	2.328

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	155	134	130	322	1431	0	203	1186	195
N.S.	1	1.20	1.04	1.01	2.50	11.09	0.00	1.57	9.19	1.51
time (sec)	N/A	0.739	2.315	0.267	0.189	0.121	0.000	0.133	0.221	2.513

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	208	214	163	522	3698	0	303	2312	310
N.S.	1	1.23	1.27	0.96	3.09	21.88	0.00	1.79	13.68	1.83
time (sec)	N/A	0.999	6.159	0.484	0.273	0.147	0.000	0.138	0.213	2.550

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	53	28	28	49	42	29	27	22
N.S.	1	1.00	1.71	0.90	0.90	1.58	1.35	0.94	0.87	0.71
time (sec)	N/A	0.288	0.032	0.079	0.180	0.096	0.523	0.116	0.235	0.050

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	53	28	29	48	42	24	24	22
N.S.	1	1.00	1.71	0.90	0.94	1.55	1.35	0.77	0.77	0.71
time (sec)	N/A	0.414	0.029	0.070	0.096	0.102	0.436	0.110	0.213	2.273

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	90	128	63	0	2231	0	0	13	151
N.S.	1	1.22	1.73	0.85	0.00	30.15	0.00	0.00	0.18	2.04
time (sec)	N/A	0.442	4.908	0.228	0.000	0.137	0.000	0.000	0.212	2.563

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	85	74	62	0	2307	0	0	26	242
N.S.	1	1.15	1.00	0.84	0.00	31.18	0.00	0.00	0.35	3.27
time (sec)	N/A	0.435	0.114	0.092	0.000	0.136	0.000	0.000	0.252	2.654

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	157	0	0	0	0	0	0	27	0
N.S.	1	1.03	0.00	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.631	0.000	0.000	0.000	0.000	0.000	0.000	0.247	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	62	42	35	36	93	0	42	47	34
N.S.	1	1.03	0.70	0.58	0.60	1.55	0.00	0.70	0.78	0.57
time (sec)	N/A	0.287	0.153	2.111	0.107	0.095	0.000	0.130	0.224	2.670

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	38	36	30	33	60	0	31	39	29
N.S.	1	1.31	1.24	1.03	1.14	2.07	0.00	1.07	1.34	1.00
time (sec)	N/A	0.295	0.150	0.655	0.029	0.077	0.000	0.129	0.225	2.442

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	40	30	23	22	50	0	30	33	22
N.S.	1	1.05	0.79	0.61	0.58	1.32	0.00	0.79	0.87	0.58
time (sec)	N/A	0.251	0.126	0.292	0.029	0.085	0.000	0.129	0.250	2.495

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	27	21	18	17	25	0	19	27	17
N.S.	1	1.42	1.11	0.95	0.89	1.32	0.00	1.00	1.42	0.89
time (sec)	N/A	0.257	0.105	0.116	0.036	0.089	0.000	0.128	0.241	0.094

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	7	7	6	9	0	6	7	6
N.S.	1	1.00	0.70	0.70	0.60	0.90	0.00	0.60	0.70	0.60
time (sec)	N/A	0.198	0.003	0.071	0.037	0.088	0.000	0.131	0.221	0.037

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	18	0	12	16	11
N.S.	1	1.00	1.00	1.14	1.00	2.57	0.00	1.71	2.29	1.57
time (sec)	N/A	0.215	0.001	0.112	0.026	0.080	0.000	0.130	0.225	2.380

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	18	21	23	31	77	0	26	55	29
N.S.	1	2.25	2.62	2.88	3.88	9.62	0.00	3.25	6.88	3.62
time (sec)	N/A	0.256	0.201	0.261	0.037	0.081	0.000	0.144	0.253	2.251

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	11	10	41	55	0	10	18	16
N.S.	1	1.00	0.92	0.83	3.42	4.58	0.00	0.83	1.50	1.33
time (sec)	N/A	0.212	1.667	0.647	0.033	0.080	0.000	0.129	0.244	0.078

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	195	156	192	166	1279	0	229	355	143
N.S.	1	1.16	0.93	1.14	0.99	7.61	0.00	1.36	2.11	0.85
time (sec)	N/A	0.565	0.385	6.016	0.040	0.106	0.000	0.137	0.204	2.739

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	118	75	107	83	331	0	114	150	85
N.S.	1	1.18	0.75	1.07	0.83	3.31	0.00	1.14	1.50	0.85
time (sec)	N/A	0.429	0.287	0.476	0.041	0.106	0.000	0.124	0.207	2.461

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	49	38	37	42	148	43	42	42
N.S.	1	1.00	1.26	0.97	0.95	1.08	3.79	1.10	1.08	1.08
time (sec)	N/A	0.517	0.137	0.073	0.040	0.097	0.448	0.131	0.268	0.100

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	20	13	12	43	0	46	39	51
N.S.	1	1.00	1.67	1.08	1.00	3.58	0.00	3.83	3.25	4.25
time (sec)	N/A	0.379	0.022	0.200	0.033	0.092	0.000	0.129	0.236	2.351

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	39	50	102	110	434	0	106	384	88
N.S.	1	0.98	1.25	2.55	2.75	10.85	0.00	2.65	9.60	2.20
time (sec)	N/A	0.457	2.167	1.270	0.039	0.116	0.000	0.135	0.199	2.534

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	89	212	230	1843	0	317	1103	169
N.S.	1	1.00	1.07	2.55	2.77	22.20	0.00	3.82	13.29	2.04
time (sec)	N/A	0.514	4.485	6.714	0.043	0.134	0.000	0.144	0.289	2.484

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	139	140	366	421	5283	0	594	2212	301
N.S.	1	0.99	1.00	2.61	3.01	37.74	0.00	4.24	15.80	2.15
time (sec)	N/A	0.645	3.409	25.209	0.058	0.145	0.000	0.141	0.224	2.696

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	132	171	172	0	1859	0	163	316	172
N.S.	1	0.99	1.28	1.28	0.00	13.87	0.00	1.22	2.36	1.28
time (sec)	N/A	1.082	0.737	1.938	0.000	0.137	0.000	0.143	0.231	3.146

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	75	80	93	0	431	0	72	171	156
N.S.	1	1.03	1.10	1.27	0.00	5.90	0.00	0.99	2.34	2.14
time (sec)	N/A	0.530	0.483	0.171	0.000	0.121	0.000	0.133	0.258	2.659

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	46	39	0	147	0	35	48	35
N.S.	1	1.00	1.21	1.03	0.00	3.87	0.00	0.92	1.26	0.92
time (sec)	N/A	0.234	0.029	0.095	0.000	0.103	0.000	0.135	0.247	0.145

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	68	75	85	0	384	0	85	143	230
N.S.	1	1.19	1.32	1.49	0.00	6.74	0.00	1.49	2.51	4.04
time (sec)	N/A	0.517	0.284	0.556	0.000	0.114	0.000	0.131	0.203	2.746

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	121	259	174	0	2647	0	177	702	257
N.S.	1	1.11	2.38	1.60	0.00	24.28	0.00	1.62	6.44	2.36
time (sec)	N/A	0.966	0.723	3.141	0.000	0.168	0.000	0.142	0.196	3.399

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	62	42	35	36	92	0	42	47	34
N.S.	1	1.03	0.70	0.58	0.60	1.53	0.00	0.70	0.78	0.57
time (sec)	N/A	0.476	0.144	1.533	0.034	0.093	0.000	0.120	0.246	2.589

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	35	34	24	27	56	0	25	32	23
N.S.	1	1.40	1.36	0.96	1.08	2.24	0.00	1.00	1.28	0.92
time (sec)	N/A	0.834	0.022	0.455	0.027	0.084	0.000	0.127	0.216	0.185

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	40	24	23	22	51	0	30	33	22
N.S.	1	1.05	0.63	0.61	0.58	1.34	0.00	0.79	0.87	0.58
time (sec)	N/A	0.441	0.015	0.216	0.027	0.086	0.000	0.133	0.208	2.429

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	25	19	12	11	23	0	11	18	11
N.S.	1	1.47	1.12	0.71	0.65	1.35	0.00	0.65	1.06	0.65
time (sec)	N/A	0.585	0.012	0.079	0.032	0.115	0.000	0.118	0.224	2.306

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	24	16	19	12	23	0	10	17	10
N.S.	1	2.40	1.60	1.90	1.20	2.30	0.00	1.00	1.70	1.00
time (sec)	N/A	0.596	0.072	0.131	0.117	0.082	0.000	0.127	0.239	0.049

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	13	10	22	18	78	0	27	51	21
N.S.	1	0.87	0.67	1.47	1.20	5.20	0.00	1.80	3.40	1.40
time (sec)	N/A	0.242	0.053	0.437	0.111	0.104	0.000	0.128	0.206	2.271

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	30	20	38	33	140	0	25	58	22
N.S.	1	1.50	1.00	1.90	1.65	7.00	0.00	1.25	2.90	1.10
time (sec)	N/A	0.468	0.203	1.182	0.121	0.095	0.000	0.129	0.196	2.328

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	14	14	75	84	0	18	36	18
N.S.	1	1.00	0.82	0.82	4.41	4.94	0.00	1.06	2.12	1.06
time (sec)	N/A	0.241	0.060	0.927	0.028	0.087	0.000	0.135	0.255	0.077

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	135	48	0	299	0	122	12	0
N.S.	1	1.00	6.43	2.29	0.00	14.24	0.00	5.81	0.57	0.00
time (sec)	N/A	0.233	14.517	0.072	0.000	0.095	0.000	0.145	0.212	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	242	144	184	154	1229	0	216	335	135
N.S.	1	1.65	0.98	1.25	1.05	8.36	0.00	1.47	2.28	0.92
time (sec)	N/A	0.695	0.622	4.513	0.038	0.121	0.000	0.136	0.218	2.818

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	157	167	174	0	1873	0	164	323	262
N.S.	1	1.16	1.24	1.29	0.00	13.87	0.00	1.21	2.39	1.94
time (sec)	N/A	0.874	1.074	1.359	0.000	0.128	0.000	0.139	0.225	3.305

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	143	73	100	80	334	0	104	152	82
N.S.	1	1.68	0.86	1.18	0.94	3.93	0.00	1.22	1.79	0.96
time (sec)	N/A	0.413	0.303	0.308	0.036	0.101	0.000	0.132	0.278	2.677

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	88	79	92	0	431	0	71	126	158
N.S.	1	1.22	1.10	1.28	0.00	5.99	0.00	0.99	1.75	2.19
time (sec)	N/A	0.858	0.213	0.125	0.000	0.116	0.000	0.128	0.219	2.819

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	60	60	54	0	200	0	48	71	164
N.S.	1	1.20	1.20	1.08	0.00	4.00	0.00	0.96	1.42	3.28
time (sec)	N/A	0.663	0.156	0.371	0.000	0.123	0.000	0.139	0.236	4.607

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	35	19	51	46	117	0	76	98	323
N.S.	1	1.21	0.66	1.76	1.59	4.03	0.00	2.62	3.38	11.14
time (sec)	N/A	0.426	0.098	1.417	0.123	0.107	0.000	0.121	0.255	2.780

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	98	85	121	0	856	0	102	250	166
N.S.	1	1.18	1.02	1.46	0.00	10.31	0.00	1.23	3.01	2.00
time (sec)	N/A	0.718	0.463	4.122	0.000	0.125	0.000	0.130	0.247	5.214

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	87	60	145	133	909	0	201	461	123
N.S.	1	1.10	0.76	1.84	1.68	11.51	0.00	2.54	5.84	1.56
time (sec)	N/A	0.352	0.202	10.655	0.115	0.108	0.000	0.134	0.218	2.707

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	33	38	41	38	41	0	26	66	65
N.S.	1	1.06	1.23	1.32	1.23	1.32	0.00	0.84	2.13	2.10
time (sec)	N/A	0.349	0.117	0.250	0.115	0.107	0.000	0.126	0.223	0.562

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	67	40	44	55	571	0	47	147	69
N.S.	1	1.56	0.93	1.02	1.28	13.28	0.00	1.09	3.42	1.60
time (sec)	N/A	0.647	0.163	0.144	0.111	0.088	0.000	0.142	0.256	2.447

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	60	32	30	43	354	0	39	108	35
N.S.	1	1.62	0.86	0.81	1.16	9.57	0.00	1.05	2.92	0.95
time (sec)	N/A	0.543	0.128	0.119	0.112	0.090	0.000	0.133	0.238	2.461

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	39	34	30	29	186	0	35	73	29
N.S.	1	1.34	1.17	1.03	1.00	6.41	0.00	1.21	2.52	1.00
time (sec)	N/A	0.426	0.123	0.099	0.117	0.112	0.000	0.125	0.197	2.412

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	32	31	18	17	73	0	17	34	17
N.S.	1	1.68	1.63	0.95	0.89	3.84	0.00	0.89	1.79	0.89
time (sec)	N/A	0.306	0.062	0.070	0.119	0.112	0.000	0.135	0.203	2.450

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	11	10	26	27	10	19	14
N.S.	1	1.00	0.88	0.69	0.62	1.62	1.69	0.62	1.19	0.88
time (sec)	N/A	0.178	0.007	0.000	0.048	0.109	0.275	0.125	0.199	0.002

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	24	18	11	10	26	27	10	19	12
N.S.	1	1.50	1.12	0.69	0.62	1.62	1.69	0.62	1.19	0.75
time (sec)	N/A	0.199	0.032	0.034	0.037	0.090	0.311	0.134	0.204	0.058

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	24	47	18	24	73	92	18	31	21
N.S.	1	1.26	2.47	0.95	1.26	3.84	4.84	0.95	1.63	1.11
time (sec)	N/A	0.337	0.121	0.077	0.045	0.091	0.321	0.128	0.214	0.069

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	44	47	28	38	196	160	36	40	21
N.S.	1	1.42	1.52	0.90	1.23	6.32	5.16	1.16	1.29	0.68
time (sec)	N/A	0.563	0.126	0.105	0.031	0.091	0.514	0.127	0.243	0.064

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	49	54	30	54	357	197	40	51	29
N.S.	1	1.32	1.46	0.81	1.46	9.65	5.32	1.08	1.38	0.78
time (sec)	N/A	0.743	0.140	0.126	0.034	0.086	0.627	0.128	0.211	2.396

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	56	39	35	0	210	0	135	23	34
N.S.	1	1.24	0.87	0.78	0.00	4.67	0.00	3.00	0.51	0.76
time (sec)	N/A	0.570	0.512	0.049	0.000	0.106	0.000	0.143	0.222	2.481

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	41	32	26	0	130	0	71	10	25
N.S.	1	1.28	1.00	0.81	0.00	4.06	0.00	2.22	0.31	0.78
time (sec)	N/A	0.434	0.373	0.050	0.000	0.101	0.000	0.145	0.207	2.450

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	41	30	25	0	143	0	64	16	24
N.S.	1	1.37	1.00	0.83	0.00	4.77	0.00	2.13	0.53	0.80
time (sec)	N/A	0.278	0.360	0.051	0.000	0.106	0.000	0.136	0.227	2.543

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	57	36	35	0	246	0	89	22	32
N.S.	1	1.16	0.73	0.71	0.00	5.02	0.00	1.82	0.45	0.65
time (sec)	N/A	0.348	0.428	0.051	0.000	0.112	0.000	0.140	0.284	2.427

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	35	0	338	0	197	25	34
N.S.	1	1.00	1.00	0.78	0.00	7.51	0.00	4.38	0.56	0.76
time (sec)	N/A	0.351	0.672	0.046	0.000	0.099	0.000	0.155	0.217	2.555

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	37	26	0	198	0	133	12	25
N.S.	1	1.00	1.09	0.76	0.00	5.82	0.00	3.91	0.35	0.74
time (sec)	N/A	0.272	0.537	0.053	0.000	0.108	0.000	0.160	0.209	2.400

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	37	35	0	163	0	88	18	36
N.S.	1	1.00	0.88	0.83	0.00	3.88	0.00	2.10	0.43	0.86
time (sec)	N/A	0.345	0.569	0.051	0.000	0.106	0.000	0.150	0.212	2.515

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	48	35	0	246	0	113	24	31
N.S.	1	1.00	0.98	0.71	0.00	5.02	0.00	2.31	0.49	0.63
time (sec)	N/A	0.383	0.564	0.052	0.000	0.087	0.000	0.148	0.247	2.427

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	131	112	107	146	1294	0	141	518	163
N.S.	1	1.35	1.15	1.10	1.51	13.34	0.00	1.45	5.34	1.68
time (sec)	N/A	1.133	0.600	0.159	0.144	0.120	0.000	0.132	2.072	2.783

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	109	91	78	94	637	0	97	359	111
N.S.	1	1.43	1.20	1.03	1.24	8.38	0.00	1.28	4.72	1.46
time (sec)	N/A	1.284	0.287	0.124	0.127	0.120	0.000	0.130	0.220	2.820

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	77	74	66	67	264	0	74	205	73
N.S.	1	1.28	1.23	1.10	1.12	4.40	0.00	1.23	3.42	1.22
time (sec)	N/A	0.964	0.274	0.096	0.118	0.100	0.000	0.134	0.214	2.676

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	63	66	44	50	73	0	57	76	58
N.S.	1	1.24	1.29	0.86	0.98	1.43	0.00	1.12	1.49	1.14
time (sec)	N/A	0.568	0.169	0.076	0.126	0.105	0.000	0.127	0.216	0.374

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	49	38	37	42	148	43	42	42
N.S.	1	1.00	1.26	0.97	0.95	1.08	3.79	1.10	1.08	1.08
time (sec)	N/A	0.304	0.015	0.000	0.032	0.087	0.498	0.131	0.288	0.002

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	48	50	39	36	43	134	43	43	42
N.S.	1	1.23	1.28	1.00	0.92	1.10	3.44	1.10	1.10	1.08
time (sec)	N/A	0.334	0.073	0.043	0.039	0.093	0.512	0.142	0.228	0.091

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	61	59	56	63	76	372	59	73	57
N.S.	1	0.97	0.94	0.89	1.00	1.21	5.90	0.94	1.16	0.90
time (sec)	N/A	0.474	0.096	0.089	0.040	0.098	0.824	0.131	0.234	0.375

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	81	65	67	82	271	636	76	92	74
N.S.	1	1.27	1.02	1.05	1.28	4.23	9.94	1.19	1.44	1.16
time (sec)	N/A	0.556	0.182	0.090	0.034	0.105	1.149	0.130	0.235	2.702

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	99	77	76	119	648	882	100	115	110
N.S.	1	1.30	1.01	1.00	1.57	8.53	11.61	1.32	1.51	1.45
time (sec)	N/A	0.739	0.286	0.125	0.034	0.131	1.651	0.133	0.245	2.644

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	131	92	96	169	1299	1013	143	139	164
N.S.	1	1.39	0.98	1.02	1.80	13.82	10.78	1.52	1.48	1.74
time (sec)	N/A	1.013	0.306	0.168	0.039	0.121	2.501	0.132	0.228	2.813

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	58	49	73	68	184	0	169	182	68
N.S.	1	1.07	0.91	1.35	1.26	3.41	0.00	3.13	3.37	1.26
time (sec)	N/A	0.674	0.111	0.587	0.217	0.111	0.000	0.128	0.267	2.420

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	28	64	24	36	28	0	24	55	23
N.S.	1	0.93	2.13	0.80	1.20	0.93	0.00	0.80	1.83	0.77
time (sec)	N/A	0.398	0.020	0.102	0.035	0.073	0.000	0.118	0.277	2.420

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	53	64	83	48	62	0	54	74	39
N.S.	1	1.18	1.42	1.84	1.07	1.38	0.00	1.20	1.64	0.87
time (sec)	N/A	0.403	0.168	0.075	0.118	0.091	0.000	0.125	0.275	2.467

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	26	37	36	33	0	37	47	25
N.S.	1	1.00	1.13	1.61	1.57	1.43	0.00	1.61	2.04	1.09
time (sec)	N/A	0.333	0.134	0.044	0.037	0.086	0.000	0.122	0.234	2.410

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	47	58	71	45	58	0	51	67	36
N.S.	1	1.18	1.45	1.78	1.12	1.45	0.00	1.28	1.68	0.90
time (sec)	N/A	0.359	0.133	0.036	0.119	0.100	0.000	0.125	0.213	2.468

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	16	12	11	10	18	27	21	41	18
N.S.	1	1.33	1.00	0.92	0.83	1.50	2.25	1.75	3.42	1.50
time (sec)	N/A	0.358	0.015	0.083	0.034	0.094	0.481	0.126	0.244	2.390

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	54	62	73	47	54	0	52	63	37
N.S.	1	1.32	1.51	1.78	1.15	1.32	0.00	1.27	1.54	0.90
time (sec)	N/A	0.267	0.131	0.062	0.116	0.089	0.000	0.132	0.226	2.306

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	27	35	30	38	0	33	59	25
N.S.	1	1.00	1.29	1.67	1.43	1.81	0.00	1.57	2.81	1.19
time (sec)	N/A	0.242	0.118	0.092	0.040	0.079	0.000	0.119	0.212	2.363

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	49	86	41	53	61	0	40	138	40
N.S.	1	1.04	1.83	0.87	1.13	1.30	0.00	0.85	2.94	0.85
time (sec)	N/A	0.262	0.081	0.087	0.034	0.085	0.000	0.128	0.235	2.418

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	89	154	100	66	104	0	72	160	60
N.S.	1	1.31	2.26	1.47	0.97	1.53	0.00	1.06	2.35	0.88
time (sec)	N/A	0.315	1.973	0.092	0.138	0.089	0.000	0.121	0.258	2.312

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	53	163	54	53	74	0	54	130	42
N.S.	1	1.29	3.98	1.32	1.29	1.80	0.00	1.32	3.17	1.02
time (sec)	N/A	0.270	2.035	0.067	0.036	0.085	0.000	0.135	0.197	2.336

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	153	86	60	97	0	66	152	54
N.S.	1	1.00	2.55	1.43	1.00	1.62	0.00	1.10	2.53	0.90
time (sec)	N/A	0.240	1.627	0.063	0.115	0.098	0.000	0.135	0.230	2.383

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	28	13	19	28	63	21	12	28
N.S.	1	1.00	2.00	0.93	1.36	2.00	4.50	1.50	0.86	2.00
time (sec)	N/A	0.227	0.052	0.062	0.034	0.079	2.061	0.127	0.218	2.397

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	93	153	101	69	97	0	77	148	60
N.S.	1	1.31	2.15	1.42	0.97	1.37	0.00	1.08	2.08	0.85
time (sec)	N/A	0.315	2.159	0.092	0.114	0.097	0.000	0.132	0.227	2.338

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	60	155	55	50	82	0	57	141	48
N.S.	1	1.40	3.60	1.28	1.16	1.91	0.00	1.33	3.28	1.12
time (sec)	N/A	0.289	2.226	0.115	0.040	0.091	0.000	0.144	0.220	2.351

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	46	0	0	0	0	0	58	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.98	0.00
time (sec)	N/A	0.243	0.092	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	97	77	0	0	0	0	0	567	0
N.S.	1	1.23	0.97	0.00	0.00	0.00	0.00	0.00	7.18	0.00
time (sec)	N/A	0.323	0.159	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	198	108	0	0	0	0	0	0	0
N.S.	1	1.57	0.86	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.500	0.192	0.000	0.000	0.000	0.000	0.000	0.262	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	259	0	0	0	0	0	59	0
N.S.	1	1.00	3.28	0.00	0.00	0.00	0.00	0.00	0.75	0.00
time (sec)	N/A	0.435	0.403	0.000	0.000	0.000	0.000	0.000	0.243	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	126	0	0	0	0	0	77	0
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.465	0.517	0.000	0.000	0.000	0.000	0.000	0.248	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	83	0	0	0	0	0	58	0
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	1.12	0.00
time (sec)	N/A	0.293	0.425	0.000	0.000	0.000	0.000	0.000	0.232	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	111	125	0	0	0	0	0	58	0
N.S.	1	1.03	1.16	0.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.429	0.544	0.000	0.000	0.000	0.000	0.000	0.225	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	177	142	0	0	0	0	0	58	0
N.S.	1	1.09	0.88	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.510	0.559	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	225	209	223	0	0	0	0	0	58	0
N.S.	1	0.93	0.99	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.484	0.793	0.000	0.000	0.000	0.000	0.000	0.253	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	171	0	0	0	0	0	47	0
N.S.	1	1.00	2.80	0.00	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	0.256	0.411	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	171	0	0	0	0	0	58	0
N.S.	1	1.00	2.80	0.00	0.00	0.00	0.00	0.00	0.95	0.00
time (sec)	N/A	0.245	0.418	0.000	0.000	0.000	0.000	0.000	0.229	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	171	0	0	0	0	0	58	0
N.S.	1	1.00	2.80	0.00	0.00	0.00	0.00	0.00	0.95	0.00
time (sec)	N/A	0.240	0.432	0.000	0.000	0.000	0.000	0.000	0.553	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	94	198	0	0	0	0	0	57	0
N.S.	1	1.62	3.41	0.00	0.00	0.00	0.00	0.00	0.98	0.00
time (sec)	N/A	0.341	4.761	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	98	207	0	0	0	0	0	57	0
N.S.	1	1.58	3.34	0.00	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	0.343	3.584	0.000	0.000	0.000	0.000	0.000	0.211	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	90	193	0	0	0	0	0	55	0
N.S.	1	1.67	3.57	0.00	0.00	0.00	0.00	0.00	1.02	0.00
time (sec)	N/A	0.332	4.640	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	87	198	0	0	0	0	0	52	0
N.S.	1	1.67	3.81	0.00	0.00	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.328	6.304	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	29	25	25	24	76	46	88	70	34
N.S.	1	1.16	1.00	1.00	0.96	3.04	1.84	3.52	2.80	1.36
time (sec)	N/A	0.233	0.039	0.181	0.025	0.099	4.542	0.195	0.219	2.240

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	92	197	0	0	0	0	0	62	0
N.S.	1	1.59	3.40	0.00	0.00	0.00	0.00	0.00	1.07	0.00
time (sec)	N/A	0.339	2.562	0.000	0.000	0.000	0.000	0.000	0.250	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	91	191	0	0	0	0	0	65	0
N.S.	1	1.65	3.47	0.00	0.00	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	0.336	2.531	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	182	155	0	0	0	0	0	78	0
N.S.	1	1.38	1.17	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.852	4.350	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	136	186	165	0	0	0	0	0	78	0
N.S.	1	1.37	1.21	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.750	3.309	0.000	0.000	0.000	0.000	0.000	0.218	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	180	151	0	0	0	0	0	76	0
N.S.	1	1.38	1.16	0.00	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.757	4.509	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	176	160	0	0	0	0	0	71	0
N.S.	1	1.40	1.27	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.608	5.618	0.000	0.000	0.000	0.000	0.000	0.252	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	33	49	33	37	72	0	52	36	34
N.S.	1	1.18	1.75	1.18	1.32	2.57	0.00	1.86	1.29	1.21
time (sec)	N/A	0.247	0.110	0.183	0.084	0.092	0.000	0.204	0.200	2.404

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	181	158	0	0	0	0	0	84	0
N.S.	1	1.35	1.18	0.00	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.499	2.375	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	182	156	0	0	0	0	0	87	0
N.S.	1	1.35	1.16	0.00	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.503	2.347	0.000	0.000	0.000	0.000	0.000	0.252	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	53	38	56	330	572	0	137	306	95
N.S.	1	1.23	0.88	1.30	7.67	13.30	0.00	3.19	7.12	2.21
time (sec)	N/A	0.308	0.123	0.471	0.113	0.105	0.000	0.176	0.240	2.478

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	47	44	44	499	171	0	78	46	163
N.S.	1	1.04	0.98	0.98	11.09	3.80	0.00	1.73	1.02	3.62
time (sec)	N/A	0.314	0.085	0.816	0.134	0.094	0.000	0.177	0.222	2.698

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	75	53	71	855	1576	0	171	556	229
N.S.	1	1.14	0.80	1.08	12.95	23.88	0.00	2.59	8.42	3.47
time (sec)	N/A	0.399	0.114	1.829	0.173	0.119	0.000	0.193	0.182	2.469

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	123	158	0	0	0	0	0	80	0
N.S.	1	1.41	1.82	0.00	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	0.383	8.950	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	212	312	0	0	0	0	0	158	0
N.S.	1	1.26	1.86	0.00	0.00	0.00	0.00	0.00	0.94	0.00
time (sec)	N/A	0.763	9.726	0.000	0.000	0.000	0.000	0.000	0.217	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	306	361	600	0	0	0	0	0	0	0
N.S.	1	1.18	1.96	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.355	13.135	0.000	0.000	0.000	0.000	0.000	0.419	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	387	0	0	0	0	0	98	0
N.S.	1	1.00	3.37	0.00	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.705	0.821	0.000	0.000	0.000	0.000	0.000	0.253	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	174	0	0	0	0	0	116	0
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.488	0.808	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	62	76	0	626	0	0	31	65
N.S.	1	1.00	0.85	1.04	0.00	8.58	0.00	0.00	0.42	0.89
time (sec)	N/A	0.356	0.220	1.117	0.000	0.117	0.000	0.000	0.211	3.396

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	71	57	74	0	334	0	0	56	51
N.S.	1	1.01	0.81	1.06	0.00	4.77	0.00	0.00	0.80	0.73
time (sec)	N/A	0.335	0.108	0.087	0.000	0.100	0.000	0.000	0.257	2.911

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	49	43	61	0	305	0	0	18	39
N.S.	1	1.02	0.90	1.27	0.00	6.35	0.00	0.00	0.38	0.81
time (sec)	N/A	0.262	0.063	0.095	0.000	0.101	0.000	0.000	0.228	2.612

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	49	47	37	0	303	0	0	31	36
N.S.	1	1.04	1.00	0.79	0.00	6.45	0.00	0.00	0.66	0.77
time (sec)	N/A	0.261	0.084	0.082	0.000	0.098	0.000	0.000	0.238	2.764

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	97	76	0	625	0	0	31	65
N.S.	1	1.00	1.37	1.07	0.00	8.80	0.00	0.00	0.44	0.92
time (sec)	N/A	0.342	0.117	0.079	0.000	0.110	0.000	0.000	0.195	3.137

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	73	101	74	0	1104	0	0	31	64
N.S.	1	1.01	1.40	1.03	0.00	15.33	0.00	0.00	0.43	0.89
time (sec)	N/A	0.341	0.172	0.079	0.000	0.103	0.000	0.000	0.242	3.787

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	92	158	149	0	8951	0	0	38	0
N.S.	1	0.68	1.17	1.10	0.00	66.30	0.00	0.00	0.28	0.00
time (sec)	N/A	0.514	0.765	1.401	0.000	1.023	0.000	0.000	0.270	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	53	121	90	0	6695	0	0	38	0
N.S.	1	0.50	1.15	0.86	0.00	63.76	0.00	0.00	0.36	0.00
time (sec)	N/A	0.680	0.126	0.154	0.000	0.835	0.000	0.000	0.248	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	29	66	52	0	1752	0	0	36	0
N.S.	1	0.50	1.14	0.90	0.00	30.21	0.00	0.00	0.62	0.00
time (sec)	N/A	0.322	0.037	0.162	0.000	0.550	0.000	0.000	0.230	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	111	159	0	0	6705	0	0	36	0
N.S.	1	1.05	1.50	0.00	0.00	63.25	0.00	0.00	0.34	0.00
time (sec)	N/A	0.444	0.774	0.000	0.000	0.812	0.000	0.000	0.228	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	191	193	0	0	9148	0	0	38	0
N.S.	1	1.35	1.36	0.00	0.00	64.42	0.00	0.00	0.27	0.00
time (sec)	N/A	0.879	1.104	0.000	0.000	1.045	0.000	0.000	0.219	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	85	155	165	0	7964	0	0	502	0
N.S.	1	0.64	1.17	1.25	0.00	60.33	0.00	0.00	3.80	0.00
time (sec)	N/A	0.625	0.282	0.975	0.000	1.370	0.000	0.000	0.348	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	319	169	164	195	167	1617	0	196	441	0
N.S.	1	0.53	0.51	0.61	0.52	5.07	0.00	0.61	1.38	0.00
time (sec)	N/A	1.041	6.193	1.159	0.141	0.109	0.000	0.268	0.221	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	197	107	334	131	112	613	0	170	253	0
N.S.	1	0.54	1.70	0.66	0.57	3.11	0.00	0.86	1.28	0.00
time (sec)	N/A	0.477	2.638	0.364	0.147	0.099	0.000	0.240	0.223	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	53	51	213	56	70	0	105	58	0
N.S.	1	0.64	0.61	2.57	0.67	0.84	0.00	1.27	0.70	0.00
time (sec)	N/A	0.353	0.047	0.454	0.129	0.090	0.000	0.215	0.237	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	83	53	51	48	35	53	0	63	31	0
N.S.	1	0.64	0.61	0.58	0.42	0.64	0.00	0.76	0.37	0.00
time (sec)	N/A	0.494	0.091	0.285	0.125	0.090	0.000	0.201	0.201	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	193	103	104	131	90	458	0	137	139	0
N.S.	1	0.53	0.54	0.68	0.47	2.37	0.00	0.71	0.72	0.00
time (sec)	N/A	1.407	0.227	0.392	0.123	0.101	0.000	0.235	0.198	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	311	161	133	195	145	1226	0	193	248	0
N.S.	1	0.52	0.43	0.63	0.47	3.94	0.00	0.62	0.80	0.00
time (sec)	N/A	2.010	0.342	0.418	0.128	0.108	0.000	0.254	0.213	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	137	124	118	0	696	0	0	11	0
N.S.	1	0.87	0.79	0.75	0.00	4.43	0.00	0.00	0.07	0.00
time (sec)	N/A	0.651	0.409	2.591	0.000	0.123	0.000	0.000	0.208	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	101	88	88	0	230	0	0	11	0
N.S.	1	0.88	0.77	0.77	0.00	2.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.512	0.338	0.973	0.000	0.105	0.000	0.000	0.213	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	69	59	58	0	216	0	0	9	0
N.S.	1	0.90	0.77	0.75	0.00	2.81	0.00	0.00	0.12	0.00
time (sec)	N/A	0.356	0.134	0.374	0.000	0.099	0.000	0.000	0.254	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	9	7	9	9	8	9	9	11
N.S.	1	1.00	1.29	1.00	1.29	1.29	1.14	1.29	1.29	1.57
time (sec)	N/A	0.527	2.046	0.174	0.214	0.083	9.868	0.212	0.235	3.057

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	137	124	118	0	698	0	0	11	0
N.S.	1	0.87	0.79	0.75	0.00	4.45	0.00	0.00	0.07	0.00
time (sec)	N/A	0.961	0.453	1.393	0.000	0.126	0.000	0.000	0.185	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	101	88	88	0	230	0	0	11	0
N.S.	1	0.88	0.77	0.77	0.00	2.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.718	0.371	0.496	0.000	0.112	0.000	0.000	0.227	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	69	62	58	0	216	0	0	9	0
N.S.	1	0.90	0.81	0.75	0.00	2.81	0.00	0.00	0.12	0.00
time (sec)	N/A	0.344	0.117	0.251	0.000	0.115	0.000	0.000	0.237	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	9	7	9	9	8	9	9	11
N.S.	1	1.00	1.29	1.00	1.29	1.29	1.14	1.29	1.29	1.57
time (sec)	N/A	0.320	4.567	0.160	0.428	0.110	3.701	0.216	0.200	3.055

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [61] had the largest ratio of [2]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	12	11	1.00	12	0.917
2	A	10	9	0.95	12	0.750
3	A	10	9	0.96	12	0.750
4	A	8	7	0.88	12	0.583
5	A	8	7	0.89	12	0.583
6	A	10	9	0.95	12	0.750
7	A	10	9	0.96	12	0.750
8	A	12	11	1.01	12	0.917
9	A	15	14	1.01	12	1.167
10	A	13	12	0.99	12	1.000
11	A	12	11	0.84	12	0.917
12	A	12	11	0.84	12	0.917
13	A	13	12	0.99	12	1.000
14	A	15	14	1.01	12	1.167
15	A	5	4	1.00	8	0.500
16	A	5	4	1.00	10	0.400
17	A	7	6	1.00	12	0.500
18	C	9	9	1.00	14	0.643
19	C	5	5	1.13	14	0.357
20	A	5	5	1.00	14	0.357
21	C	9	9	0.91	14	0.643

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	17	16	0.77	14	1.143
23	A	17	16	0.77	14	1.143
24	A	15	14	0.79	14	1.000
25	A	15	14	0.78	14	1.000
26	A	17	16	0.78	14	1.143
27	A	17	16	0.74	14	1.143
28	A	7	6	1.00	12	0.500
29	A	14	13	0.72	14	0.929
30	A	12	11	0.74	14	0.786
31	A	12	11	0.73	14	0.786
32	A	14	13	0.70	14	0.929
33	A	9	9	0.68	14	0.643
34	A	6	6	0.76	14	0.429
35	C	5	5	1.13	14	0.357
36	A	5	5	1.00	14	0.357
37	A	6	6	0.76	14	0.429
38	A	9	9	0.65	14	0.643
39	A	7	6	1.00	12	0.500
40	A	12	12	0.63	14	0.857
41	A	6	6	0.76	14	0.429
42	A	6	6	0.76	14	0.429
43	A	12	12	0.60	14	0.857
44	A	21	20	0.73	14	1.429
45	A	17	16	0.78	14	1.143
46	A	17	16	0.77	14	1.143
47	A	17	16	0.78	14	1.143
48	A	17	16	0.77	14	1.143
49	A	21	20	0.71	14	1.429
50	A	7	6	1.00	12	0.500
51	A	7	6	1.00	14	0.429
52	A	7	6	1.26	14	0.429
53	A	7	6	1.00	14	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	7	6	1.00	14	0.429
55	A	7	6	1.00	14	0.429
56	A	7	6	1.00	14	0.429
57	A	7	6	1.26	14	0.429
58	A	7	6	1.00	14	0.429
59	A	7	6	1.00	14	0.429
60	A	7	6	1.00	14	0.429
61	A	12	12	1.27	6	2.000
62	A	10	10	1.26	6	1.667
63	A	8	8	1.13	6	1.333
64	A	6	6	1.00	6	1.000
65	A	3	3	1.00	6	0.500
66	A	5	5	1.19	6	0.833
67	A	7	7	1.28	6	1.167
68	A	9	9	1.33	6	1.500
69	A	11	11	1.36	6	1.833
70	A	10	9	1.11	8	1.125
71	A	8	7	1.07	8	0.875
72	A	6	5	1.00	8	0.625
73	A	4	3	1.00	8	0.375
74	A	6	5	1.00	8	0.625
75	A	8	7	1.00	8	0.875
76	A	10	9	1.08	8	1.125
77	C	12	12	1.03	12	1.000
78	C	10	10	1.04	12	0.833
79	C	8	8	1.06	12	0.667
80	C	6	6	1.11	12	0.500
81	A	5	5	1.00	12	0.417
82	A	7	7	1.15	12	0.583
83	A	9	9	1.20	12	0.750
84	A	11	11	1.23	12	0.917
85	A	5	5	1.00	12	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	5	5	1.00	12	0.417
87	A	8	7	1.22	14	0.500
88	A	7	6	1.15	14	0.429
89	A	4	3	1.03	23	0.130
90	A	5	4	1.03	11	0.364
91	C	9	8	1.31	11	0.727
92	A	6	5	1.05	11	0.455
93	C	7	7	1.42	9	0.778
94	A	3	3	1.00	9	0.333
95	A	5	4	1.00	11	0.364
96	C	7	7	2.25	11	0.636
97	A	4	3	1.00	11	0.273
98	A	6	5	1.16	13	0.385
99	A	7	6	1.18	13	0.462
100	A	5	5	1.00	8	0.625
101	A	5	4	1.00	13	0.308
102	A	6	5	0.98	13	0.385
103	A	7	6	1.00	13	0.462
104	A	6	5	0.99	13	0.385
105	C	24	23	0.99	13	1.769
106	C	14	13	1.03	11	1.182
107	A	5	4	1.00	11	0.364
108	C	14	13	1.19	13	1.000
109	C	26	25	1.11	13	1.923
110	A	7	6	1.03	11	0.545
111	C	11	11	1.40	11	1.000
112	A	6	5	1.05	11	0.455
113	C	11	11	1.47	9	1.222
114	C	11	11	2.40	9	1.222
115	A	5	4	0.87	11	0.364
116	C	11	11	1.50	11	1.000
117	A	7	6	1.00	11	0.545

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	6	5	1.00	13	0.385
119	A	10	9	1.65	13	0.692
120	C	20	19	1.16	13	1.462
121	A	7	6	1.68	13	0.462
122	C	15	14	1.22	11	1.273
123	C	8	8	1.20	11	0.727
124	A	5	4	1.21	13	0.308
125	C	8	8	1.18	13	0.615
126	A	6	5	1.10	13	0.385
127	A	9	9	1.06	13	0.692
128	C	20	20	1.56	11	1.818
129	C	18	18	1.62	11	1.636
130	C	13	13	1.34	11	1.182
131	C	9	9	1.68	9	1.000
132	A	3	3	1.00	6	0.500
133	C	4	4	1.50	9	0.444
134	A	4	4	1.26	11	0.364
135	C	11	11	1.42	11	1.000
136	C	15	15	1.32	11	1.364
137	C	9	8	1.24	11	0.727
138	C	7	6	1.28	11	0.545
139	C	7	6	1.37	11	0.545
140	C	9	8	1.16	11	0.727
141	A	10	9	1.00	13	0.692
142	A	8	7	1.00	13	0.538
143	A	10	9	1.00	13	0.692
144	A	9	8	1.00	13	0.615
145	C	19	19	1.35	13	1.462
146	C	16	16	1.43	13	1.231
147	C	12	12	1.28	13	0.923
148	C	8	8	1.24	11	0.727
149	A	5	5	1.00	8	0.625

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	C	6	6	1.23	11	0.545
151	A	11	11	0.97	13	0.846
152	C	13	12	1.27	13	0.923
153	C	16	15	1.30	13	1.154
154	C	21	20	1.39	13	1.538
155	A	6	6	1.07	14	0.429
156	A	6	5	0.93	11	0.455
157	A	5	5	1.18	11	0.455
158	A	5	4	1.00	9	0.444
159	A	5	5	1.18	7	0.714
160	C	5	4	1.33	11	0.364
161	A	5	5	1.32	11	0.455
162	A	5	4	1.00	11	0.364
163	A	5	4	1.04	13	0.308
164	A	7	7	1.31	13	0.538
165	A	7	6	1.29	11	0.545
166	A	3	3	1.00	9	0.333
167	A	6	5	1.00	13	0.385
168	A	6	6	1.31	13	0.462
169	A	7	6	1.40	13	0.462
170	A	3	3	1.00	13	0.231
171	A	5	5	1.23	15	0.333
172	A	7	7	1.57	15	0.467
173	A	3	3	1.00	9	0.333
174	A	3	3	1.00	15	0.200
175	A	2	2	1.00	11	0.182
176	A	5	4	1.03	11	0.364
177	A	6	5	1.09	11	0.455
178	A	7	6	0.93	11	0.545
179	A	3	3	1.00	7	0.429
180	A	3	3	1.00	9	0.333
181	A	3	3	1.00	9	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	5	4	1.62	17	0.235
183	A	5	4	1.58	17	0.235
184	A	5	4	1.67	15	0.267
185	A	5	4	1.67	13	0.308
186	C	5	4	1.16	17	0.235
187	A	5	4	1.59	17	0.235
188	A	5	4	1.65	17	0.235
189	A	7	6	1.38	19	0.316
190	A	7	6	1.37	19	0.316
191	A	7	6	1.38	17	0.353
192	A	7	6	1.40	15	0.400
193	A	6	5	1.18	19	0.263
194	A	7	6	1.35	19	0.316
195	A	7	6	1.35	19	0.316
196	C	9	8	1.23	17	0.471
197	A	9	8	1.04	17	0.471
198	C	13	12	1.14	17	0.706
199	A	5	4	1.41	19	0.211
200	A	7	6	1.26	21	0.286
201	A	9	8	1.18	21	0.381
202	A	5	4	1.00	15	0.267
203	A	5	4	1.00	21	0.190
204	A	11	10	1.00	19	0.526
205	A	11	10	1.01	19	0.526
206	A	9	8	1.02	19	0.421
207	A	9	8	1.04	19	0.421
208	A	11	10	1.00	19	0.526
209	A	11	10	1.01	19	0.526
210	C	12	11	0.68	23	0.478
211	C	10	9	0.50	23	0.391
212	C	7	6	0.50	21	0.286
213	C	7	6	1.05	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	C	7	6	1.35	23	0.261
215	C	12	11	0.64	21	0.524
216	A	6	5	0.53	25	0.200
217	A	6	5	0.54	25	0.200
218	A	6	5	0.64	25	0.200
219	A	6	5	0.64	25	0.200
220	A	6	5	0.53	25	0.200
221	A	6	5	0.52	25	0.200
222	A	4	3	0.87	9	0.333
223	A	4	3	0.88	9	0.333
224	A	4	3	0.90	7	0.429
225	N/A	4	0	1.00	7	0.000
226	A	4	3	0.87	9	0.333
227	A	4	3	0.88	9	0.333
228	A	4	3	0.90	7	0.429
229	N/A	4	0	1.00	7	0.000

CHAPTER 3

Listing of integrals

3.1 $\int (b\coth(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 97

$$\int (b\coth(c+dx))^{7/2}\,dx = \frac{b^{7/2}\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{7/2}\text{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b^3\sqrt{b\coth(c+dx)}}{d} - \frac{2b(b\coth(c+dx))^{5/2}}{5d}$$

output
```
b^(7/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(7/2)*arctanh((b*coth(d*
x+c))^(1/2)/b^(1/2))/d-2*b^3*(b*coth(d*x+c))^(1/2)/d-2/5*b*(b*coth(d*x+c))
^(5/2)/d
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int (b\coth(c+dx))^{7/2}\,dx = -\frac{(b\coth(c+dx))^{7/2}\left(-\arctan\left(\sqrt{\coth(c+dx)}\right)-\text{arctanh}\left(\sqrt{\coth(c+dx)}\right)+2\sqrt{\coth(c+dx)}+\frac{2}{5}\text{cot}\right.}{d\coth^{\frac{7}{2}}(c+dx)}$$

input
```
Integrate[(b*Coth[c + d*x])^(7/2),x]
```

output
```
-(((b*Coth[c + d*x])^(7/2)*(-ArcTan[Sqrt[Coth[c + d*x]]] - ArcTanh[Sqrt[Co
th[c + d*x]]] + 2*Sqrt[Coth[c + d*x]] + (2*Coth[c + d*x]^(5/2))/5))/(d*Cot
h[c + d*x]^(7/2)))
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3954, 3042, 3954, 3042, 3957, 25, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\coth(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{7/2}dx$$

$$\Big\downarrow \text{3954}$$

$$b^2\int (b\coth(c+dx))^{3/2}dx-\frac{2b(b\coth(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b(b\coth(c+dx))^{5/2}}{5d}+b^2\int \left(-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3954}$$

$$b^2\left(b^2\int \frac{1}{\sqrt{b\coth(c+dx)}}dx-\frac{2b\sqrt{b\coth(c+dx)}}{d}\right)-\frac{2b(b\coth(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b(b\coth(c+dx))^{5/2}}{5d}+b^2\left(-\frac{2b\sqrt{b\coth(c+dx)}}{d}+b^2\int \frac{1}{\sqrt{-ib\tan\left(ic+idx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{3957}$$

$$b^2\left(-\frac{b^3\int -\frac{1}{\sqrt{b\coth(c+dx)}\left(b^2-b^2\coth^2(c+dx)\right)}d(b\coth(c+dx))}{d}-\frac{2b\sqrt{b\coth(c+dx)}}{d}\right)-\frac{2b(b\coth(c+dx))^{5/2}}{5d}$$

$\Big\downarrow$ 25

$$b^2\left(\frac{b^3\int \frac{1}{\sqrt{b\coth(c+dx)}\left(b^2-b^2\coth^2(c+dx)\right)}d(b\coth(c+dx))}{d}-\frac{2b\sqrt{b\coth(c+dx)}}{d}\right)-\frac{2b(b\coth(c+dx))^{5/2}}{5d}$$

$\Big\downarrow$ 266

$$b^2\left(\frac{2b^3\int \frac{1}{b^2-b^4\coth^4(c+dx)}d\sqrt{b\coth(c+dx)}}{d}-\frac{2b\sqrt{b\coth(c+dx)}}{d}\right)-\frac{2b(b\coth(c+dx))^{5/2}}{5d}$$

$\Big\downarrow$ 756

$$b^2\left(\frac{2b^3\left(\frac{\int \frac{1}{b-b^2\coth^2(c+dx)}d\sqrt{b\coth(c+dx)}}{2b}+\frac{\int \frac{1}{b^2\coth^2(c+dx)+b}d\sqrt{b\coth(c+dx)}}{2b}\right)}{d}-\frac{2b\sqrt{b\coth(c+dx)}}{d}\right)-\frac{2b(b\coth(c+dx))^{5/2}}{5d}$$

$\Big\downarrow$ 216

$$b^2\left(\frac{2b^3\left(\frac{\int \frac{1}{b-b^2\coth^2(c+dx)}d\sqrt{b\coth(c+dx)}}{2b}+\frac{\arctan\left(\sqrt{b}\coth(c+dx)\right)}{2b^{3/2}}\right)}{d}-\frac{2b\sqrt{b\coth(c+dx)}}{d}\right)-\frac{2b(b\coth(c+dx))^{5/2}}{5d}$$

$\Big\downarrow$ 219

$$b^2\left(\frac{2b^3\left(\frac{\arctan\left(\sqrt{b}\coth(c+dx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\coth(c+dx)\right)}{2b^{3/2}}\right)}{d}-\frac{2b\sqrt{b\coth(c+dx)}}{d}\right)-\frac{2b(b\coth(c+dx))^{5/2}}{5d}$$

input
```
Int[(b*Coth[c + d*x])^(7/2),x]
```

output
```
(-2*b*(b*Coth[c + d*x])^(5/2))/(5*d) + b^2*((2*b^3*(ArcTan[Sqrt[b]*Coth[c
+ d*x]]/(2*b^(3/2)) + ArcTanh[Sqrt[b]*Coth[c + d*x]]/(2*b^(3/2))))/d - (2*
b*Sqrt[b*Coth[c + d*x]])/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{b^{\frac{7}{2}} \arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d}+\frac{b^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d}-\frac{2b^{3}\sqrt{b\coth(dx+c)}}{d}-\frac{2b(b\coth(dx+c))^{\frac{5}{2}}}{5d}$	80
default	$\frac{b^{\frac{7}{2}} \arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d}+\frac{b^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d}-\frac{2b^{3}\sqrt{b\coth(dx+c)}}{d}-\frac{2b(b\coth(dx+c))^{\frac{5}{2}}}{5d}$	80

input
```
int((b*coth(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
b^(7/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(7/2)*arctanh((b*coth(d*
x+c))^(1/2)/b^(1/2))/d-2*b^3*(b*coth(d*x+c))^(1/2)/d-2/5*b*(b*coth(d*x+c))
^(5/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. 2(79) = 158.

Time = 0.15 (sec) , antiderivative size = 1567, normalized size of antiderivative = 16.15

$$\int (b\coth(c+dx))^{7/2}\,dx = \text{Too large to display}$$

input
```
integrate((b*coth(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
[-1/20*(10*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^
3*sinh(d*x + c)^4 - 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2
 - b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh
(d*x + c))*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c
) + sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*c
osh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2)) - 5
*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*sinh(d*x
 + c)^4 - 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 - b^3)*si
nh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh(d*x + c))
*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*
b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*
sinh(d*x + c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sin
h(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cos
h(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*
x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 16*(3*b^3
*cosh(d*x + c)^4 + 12*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + 3*b^3*sinh(d*x +
 c)^4 - 4*b^3*cosh(d*x + c)^2 + 3*b^3 + 2*(9*b^3*cosh(d*x + c)^2 - 2*b^3)*
sinh(d*x + c)^2 + 4*(3*b^3*cosh(d*x + c)^3 - 2*b^3*cosh(d*x + c))*sinh(d*x
 + c))*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(d*cosh(d*x + c)^4 + 4*d*cosh(
d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*...
```

Sympy [F(-1)]

Timed out.

$$\int (b\coth(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((b*coth(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\coth(c+dx))^{7/2}\,dx = \int (b\coth(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((b*coth(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c))^(7/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int (b\coth(c+dx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*coth(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 3.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int (b\coth(c+dx))^{7/2}\,dx = \frac{b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{d} - \frac{2\,b^3\,\sqrt{b\coth(c+d\,x)}}{d} - \frac{2\,b\,(b\coth(c+d\,x))^{5/2}}{5\,d} - \frac{b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}\,1\mathrm{i}}{\sqrt{b}}\right)\,1\mathrm{i}}{d}$$

input
```
int((b*coth(c + d*x))^(7/2),x)
```

output
```
(b^(7/2)*atan((b*coth(c + d*x))^(1/2)/b^(1/2)))/d - (2*b^3*(b*coth(c + d*x
))^(1/2))/d - (2*b*(b*coth(c + d*x))^(5/2))/(5*d) - (b^(7/2)*atan(((b*coth
(c + d*x))^(1/2)*1i)/b^(1/2))*1i)/d
```

Reduce [F]

$$\int (b\coth(c+dx))^{7/2}\,dx = \frac{\sqrt{b}\,b^3\left(-2\sqrt{\coth(dx+c)}\,\coth(dx+c)^2-10\sqrt{\coth(dx+c)}+5\left(\int\frac{\sqrt{\coth(dx+c)}}{\coth(dx+c)}dx\right)d\right)}{5d}$$

input
```
int((b*coth(d*x+c))^(7/2),x)
```

output
```
(sqrt(b)*b**3*( - 2*sqrt(coth(c + d*x))*coth(c + d*x)**2 - 10*sqrt(coth(c
+ d*x)) + 5*int(sqrt(coth(c + d*x))/coth(c + d*x),x)*d))/(5*d)
```

3.2 $\int (b\coth(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 78

$$\int (b\coth(c+dx))^{5/2}\,dx = -\frac{b^{5/2}\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b(b\coth(c+dx))^{3/2}}{3d}$$

output
```
-b^(5/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(5/2)*arctanh((b*coth(d
*x+c))^(1/2)/b^(1/2))/d-2/3*b*(b*coth(d*x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int (b\coth(c+dx))^{5/2}\,dx = -\frac{(b\coth(c+dx))^{5/2}\left(\arctan\left(\sqrt{\coth(c+dx)}\right) - \text{arctanh}\left(\sqrt{\coth(c+dx)}\right) + \frac{2}{3}\coth^{\frac{3}{2}}(c+dx)\right)}{d\coth^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(b*Coth[c + d*x])^(5/2),x]
```

output
```
-(((b*Coth[c + d*x])^(5/2)*(ArcTan[Sqrt[Coth[c + d*x]]] - ArcTanh[Sqrt[Cot
h[c + d*x]]] + (2*Coth[c + d*x]^(3/2))/3))/(d*Coth[c + d*x]^(5/2)))
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3954, 3042, 3957, 25, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\coth(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{3954}$$

$$b^2\int \sqrt{b\coth(c+dx)}dx-\frac{2b(b\coth(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b(b\coth(c+dx))^{3/2}}{3d}+b^2\int \sqrt{-ib\tan\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{b^3\int -\frac{\sqrt{b\coth(c+dx)}}{b^2-b^2\coth^2(c+dx)}d(b\coth(c+dx))}{d}-\frac{2b(b\coth(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^3\int \frac{\sqrt{b\coth(c+dx)}}{b^2-b^2\coth^2(c+dx)}d(b\coth(c+dx))}{d}-\frac{2b(b\coth(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{266}$$

$$\frac{2b^3\int \frac{b^2\coth^2(c+dx)}{b^2-b^4\coth^4(c+dx)}d\sqrt{b\coth(c+dx)}}{d}-\frac{2b(b\coth(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{827}$$

$$\frac{2b^3\left(\frac{1}{2}\int\frac{1}{b-b^2\coth^2(c+dx)}d\sqrt{b\coth(c+dx)}-\frac{1}{2}\int\frac{1}{b^2\coth^2(c+dx)+b}d\sqrt{b\coth(c+dx)}\right)}{d}-\frac{2b(b\coth(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{216}$$

$$\frac{2b^3\left(\frac{1}{2}\int\frac{1}{b-b^2\coth^2(c+dx)}d\sqrt{b\coth(c+dx)}-\frac{\arctan\left(\sqrt{b}\coth(c+dx)\right)}{2\sqrt{b}}\right)}{d}-\frac{2b(b\coth(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{219}$$

$$\frac{2b^3\left(\frac{\text{arctanh}\left(\sqrt{b}\coth(c+dx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\coth(c+dx)\right)}{2\sqrt{b}}\right)}{d}-\frac{2b(b\coth(c+dx))^{3/2}}{3d}$$

input
```
Int[(b*Coth[c + d*x])^(5/2),x]
```

output
```
(2*b^3*(-1/2*ArcTan[Sqrt[b]*Coth[c + d*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Coth[
c + d*x]]/(2*Sqrt[b])))/d - (2*b*(b*Coth[c + d*x])^(3/2))/(3*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{b^{\frac{5}{2}} \arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d}+\frac{b^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d}-\frac{2b(b\coth(dx+c))^{\frac{3}{2}}}{3d}$	63
default	$-\frac{b^{\frac{5}{2}} \arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d}+\frac{b^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d}-\frac{2b(b\coth(dx+c))^{\frac{3}{2}}}{3d}$	63

input
```
int((b*coth(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-b^(5/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(5/2)*arctanh((b*coth(d
*x+c))^(1/2)/b^(1/2))/d-2/3*b*(b*coth(d*x+c))^(3/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(62) = 124$.

Time = 0.14 (sec) , antiderivative size = 981, normalized size of antiderivative = 12.58

$$\int (b\coth(c+dx))^{5/2}\, dx = \text{Too large to display}$$

input
```
integrate((b*coth(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(6*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*s
inh(d*x + c)^2 - b^2)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*s
inh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x
 + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x +
 c)^2)) - 3*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2
*sinh(d*x + c)^2 - b^2)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x +
c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x +
c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x +
 c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sin
h(d*x + c)) - 2*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*
cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d
*x + c)^4)) + 8*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) +
 b^2*sinh(d*x + c)^2 + b^2)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(d*cosh(d
*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 - d), 1/12
*(6*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*
x + c)^2 - b^2)*sqrt(b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
 + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/sqrt(b))
+ 3*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*
x + c)^2 - b^2)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh
(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*si...
```

Sympy [F]

$$\int (b\coth(c+dx))^{5/2}\,dx = \int (b\coth(c+dx))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*coth(d*x+c))**(5/2),x)
```

output
```
Integral((b*coth(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int (b\coth(c+dx))^{5/2}\,dx = \int (b\coth(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*coth(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int (b\coth(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*coth(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

$$\int (b\coth(c+dx))^{5/2}\,dx = \frac{b^{5/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{d} - \frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{d} - \frac{2\,b\,(b\coth(c+d\,x))^{3/2}}{3\,d}$$

input
```
int((b*coth(c + d*x))^(5/2),x)
```

output
```
(b^(5/2)*atanh((b*coth(c + d*x))^(1/2)/b^(1/2)))/d - (b^(5/2)*atan((b*coth
(c + d*x))^(1/2)/b^(1/2)))/d - (2*b*(b*coth(c + d*x))^(3/2))/(3*d)
```

Reduce [F]

$$\int (b\coth(c+dx))^{5/2}\,dx = \sqrt{b}\left(\int \sqrt{\coth(dx+c)}\,\coth(dx+c)^2\,dx\right)b^2$$

input
```
int((b*coth(d*x+c))^(5/2),x)
```

output
```
sqrt(b)*int(sqrt(coth(c + d*x))*coth(c + d*x)**2,x)*b**2
```

3.3 $\int (b\coth(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 75

$$\int (b\coth(c+dx))^{3/2}\,dx = \frac{b^{3/2}\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b\sqrt{b\coth(c+dx)}}{d}$$

output
```
b^(3/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(3/2)*arctanh((b*coth(d*
x+c))^(1/2)/b^(1/2))/d-2*b*(b*coth(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

$$\int (b\coth(c+dx))^{3/2}\,dx = -\frac{\left(-\arctan\left(\sqrt{\coth(c+dx)}\right) - \text{arctanh}\left(\sqrt{\coth(c+dx)}\right) + 2\sqrt{\coth(c+dx)}\right)(b\coth(c+dx))^{3/2}}{d\coth^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(b*Coth[c + d*x])^(3/2),x]
```

output
```
-(((-ArcTan[Sqrt[Coth[c + d*x]]] - ArcTanh[Sqrt[Coth[c + d*x]]] + 2*Sqrt[C
oth[c + d*x]])*(b*Coth[c + d*x])^(3/2))/(d*Coth[c + d*x]^(3/2)))
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3954, 3042, 3957, 25, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\coth(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \ 3954$$

$$b^2\int \frac{1}{\sqrt{b\coth(c+dx)}}dx-\frac{2b\sqrt{b\coth(c+dx)}}{d}$$

$$\Big\downarrow \ 3042$$

$$-\frac{2b\sqrt{b\coth(c+dx)}}{d}+b^2\int \frac{1}{\sqrt{-ib\tan\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \ 3957$$

$$-\frac{b^3\int -\frac{1}{\sqrt{b\coth(c+dx)}\left(b^2-b^2\coth^2(c+dx)\right)}d(b\coth(c+dx))}{d}-\frac{2b\sqrt{b\coth(c+dx)}}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{b^3\int \frac{1}{\sqrt{b\coth(c+dx)}\left(b^2-b^2\coth^2(c+dx)\right)}d(b\coth(c+dx))}{d}-\frac{2b\sqrt{b\coth(c+dx)}}{d}$$

$$\Big\downarrow \ 266$$

$$\frac{2b^3\int \frac{1}{b^2-b^4\coth^4(c+dx)}d\sqrt{b\coth(c+dx)}}{d}-\frac{2b\sqrt{b\coth(c+dx)}}{d}$$

$$\downarrow 756$$

$$\frac{2b^3\left(\frac{\int \frac{1}{b-b^2\coth^2(c+dx)}d\sqrt{b\coth(c+dx)}}{2b}+\frac{\int \frac{1}{b^2\coth^2(c+dx)+b}d\sqrt{b\coth(c+dx)}}{2b}\right)}{d}-\frac{2b\sqrt{b\coth(c+dx)}}{d}$$

$$\downarrow 216$$

$$\frac{2b^3\left(\frac{\int \frac{1}{b-b^2\coth^2(c+dx)}d\sqrt{b\coth(c+dx)}}{2b}+\frac{\arctan\left(\sqrt{b}\coth(c+dx)\right)}{2b^{3/2}}\right)}{d}-\frac{2b\sqrt{b\coth(c+dx)}}{d}$$

$$\downarrow 219$$

$$\frac{2b^3\left(\frac{\arctan\left(\sqrt{b}\coth(c+dx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\coth(c+dx)\right)}{2b^{3/2}}\right)}{d}-\frac{2b\sqrt{b\coth(c+dx)}}{d}$$

input
```
Int[(b*Coth[c + d*x])^(3/2),x]
```

output
```
(2*b^3*(ArcTan[Sqrt[b]*Coth[c + d*x]]/(2*b^(3/2)) + ArcTanh[Sqrt[b]*Coth[c
 + d*x]]/(2*b^(3/2))))/d - (2*b*Sqrt[b*Coth[c + d*x]])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{b^{\frac{3}{2}} \arctan\left(\frac{\sqrt{b \coth(dx+c)}}{\sqrt{b}}\right)}{d}+\frac{b^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b \coth(dx+c)}}{\sqrt{b}}\right)}{d}-\frac{2b\sqrt{b \coth(dx+c)}}{d}$	62
default	$\frac{b^{\frac{3}{2}} \arctan\left(\frac{\sqrt{b \coth(dx+c)}}{\sqrt{b}}\right)}{d}+\frac{b^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b \coth(dx+c)}}{\sqrt{b}}\right)}{d}-\frac{2b\sqrt{b \coth(dx+c)}}{d}$	62

input
```
int((b*coth(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
b^(3/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(3/2)*arctanh((b*coth(d*
x+c))^(1/2)/b^(1/2))/d-2*b*(b*coth(d*x+c))^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(61) = 122$.

Time = 0.15 (sec) , antiderivative size = 631, normalized size of antiderivative = 8.41

$$\int (b\coth(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((b*coth(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(2*sqrt(-b)*b*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c
) + sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*c
osh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2)) - s
qrt(-b)*b*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*
b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*
sinh(d*x + c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sin
h(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cos
h(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*
x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*b*sqrt(
b*cosh(d*x + c)/sinh(d*x + c)))/d, -1/4*(2*b^(3/2)*arctan((cosh(d*x + c)^2
 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x +
c)/sinh(d*x + c))/sqrt(b)) - b^(3/2)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*
x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d
*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cos
h(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sin
h(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sin
h(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b) + 8*b*sqrt(b*
cosh(d*x + c)/sinh(d*x + c)))/d]
```

Sympy [F]

$$\int (b\coth(c+dx))^{3/2}\,dx = \int (b\coth{(c+dx)})^{\frac{3}{2}}\,dx$$

input
```
integrate((b*coth(d*x+c))**(3/2),x)
```

output
```
Integral((b*coth(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int (b\coth(c+dx))^{3/2}\,dx = \int (b\coth{(dx+c)})^{\frac{3}{2}}\,dx$$

input
```
integrate((b*coth(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int (b\coth(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*coth(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int (b\coth(c+dx))^{3/2}\,dx = \frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{d} - \frac{2\,b\,\sqrt{b\coth\left(c+d\,x\right)}}{d} + \frac{b^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{d}$$

input
```
int((b*coth(c + d*x))^(3/2),x)
```

output
```
(b^(3/2)*atan((b*coth(c + d*x))^(1/2)/b^(1/2)))/d - (2*b*(b*coth(c + d*x))
^(1/2))/d + (b^(3/2)*atanh((b*coth(c + d*x))^(1/2)/b^(1/2)))/d
```

Reduce [F]

$$\int (b\coth(c+dx))^{3/2}\,dx = \frac{\sqrt{b}\,b\left(-2\sqrt{\coth\left(dx+c\right)}+\left(\int \frac{\sqrt{\coth(dx+c)}}{\coth(dx+c)}dx\right)d\right)}{d}$$

input
```
int((b*coth(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*b*( - 2*sqrt(coth(c + d*x)) + int(sqrt(coth(c + d*x))/coth(c + d*
x),x)*d))/d
```

3.4 $\int \sqrt{b\coth(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \sqrt{b\coth(c+dx)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}$$

output
```
-b^(1/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(1/2)*arctanh((b*coth(d
*x+c))^(1/2)/b^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \sqrt{b\coth(c+dx)}\,dx = -\frac{\left(\arctan\left(\sqrt{\coth(c+dx)}\right) - \operatorname{arctanh}\left(\sqrt{\coth(c+dx)}\right)\right)\sqrt{b\coth(c+dx)}}{d\sqrt{\coth(c+dx)}}$$

input
```
Integrate[Sqrt[b*Coth[c + d*x]],x]
```

output
```
-(((ArcTan[Sqrt[Coth[c + d*x]]] - ArcTanh[Sqrt[Coth[c + d*x]]])*Sqrt[b*Cot
h[c + d*x]])/(d*Sqrt[Coth[c + d*x]]))
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 3957, 25, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\coth(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-ib\tan\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{b\int -\frac{\sqrt{b\coth(c+dx)}}{b^2-b^2\coth^2(c+dx)}d(b\coth(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\int \frac{\sqrt{b\coth(c+dx)}}{b^2-b^2\coth^2(c+dx)}d(b\coth(c+dx))}{d}$$

$$\Big\downarrow \text{266}$$

$$\frac{2b\int \frac{b^2\coth^2(c+dx)}{b^2-b^4\coth^4(c+dx)}d\sqrt{b\coth(c+dx)}}{d}$$

$$\Big\downarrow \text{827}$$

$$\frac{2b\left(\frac{1}{2}\int \frac{1}{b-b^2\coth^2(c+dx)}d\sqrt{b\coth(c+dx)}-\frac{1}{2}\int \frac{1}{b^2\coth^2(c+dx)+b}d\sqrt{b\coth(c+dx)}\right)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{2b\left(\frac{1}{2}\int \frac{1}{b-b^2\coth^2(c+dx)}d\sqrt{b\coth(c+dx)}-\frac{\arctan\left(\sqrt{b}\coth(c+dx)\right)}{2\sqrt{b}}\right)}{d}$$
$$\Big\downarrow \text{219}$$
$$\frac{2b\left(\frac{\operatorname{arctanh}\left(\sqrt{b}\coth(c+dx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\coth(c+dx)\right)}{2\sqrt{b}}\right)}{d}$$

input
```
Int[Sqrt[b*Coth[c + d*x]],x]
```

output
```
(2*b*(-1/2*ArcTan[Sqrt[b]*Coth[c + d*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Coth[c
+ d*x]]/(2*Sqrt[b])))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{\sqrt{b}\,\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d}+\frac{\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d}$	47
default	$-\frac{\sqrt{b}\,\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d}+\frac{\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d}$	47

input
```
int((b*coth(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-b^(1/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(1/2)*arctanh((b*coth(d
*x+c))^(1/2)/b^(1/2))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(46) = 92$.

Time = 0.13 (sec) , antiderivative size = 586, normalized size of antiderivative = 10.10

$$\int \sqrt{b\coth(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*coth(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*(2*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
+ sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cos
h(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2)) - sqr
t(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*co
sh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh
(d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*
x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*
x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x +
c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/d, 1/4*(2*sqrt
(b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c
)^2 - 1)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/sqrt(b)) + sqrt(b)*log(2*b*co
sh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*s
inh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 +
 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 +
(6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x
+ c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d
*x + c)) - b))/d]
```

Sympy [F]

$$\int \sqrt{b\coth(c+dx)}\,dx = \int \sqrt{b\coth(c+dx)}\,dx$$

input
```
integrate((b*coth(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*coth(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{b\coth(c+dx)}\,dx = \int \sqrt{b\coth\left(dx+c\right)}\,dx$$

input
```
integrate((b*coth(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*coth(d*x + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{b\coth(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*coth(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \sqrt{b\coth(c+dx)}\,dx = -\frac{\sqrt{b}\left(\operatorname{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)-\operatorname{atanh}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)\right)}{d}$$

input
```
int((b*coth(c + d*x))^(1/2),x)
```

output
```
-(b^(1/2)*(atan((b*coth(c + d*x))^(1/2)/b^(1/2)) - atanh((b*coth(c + d*x))
^(1/2)/b^(1/2))))/d
```

Reduce [F]

$$\int \sqrt{b\coth(c+dx)}\,dx = \sqrt{b}\left(\int \sqrt{\coth\left(dx+c\right)}dx\right)$$

input
```
int((b*coth(d*x+c))^(1/2),x)
```

output
```
sqrt(b)*int(sqrt(coth(c + d*x)),x)
```

3.5 $\int \frac{1}{\sqrt{b \coth(c+dx)}} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 57

$$\int \frac{1}{\sqrt{b \coth(c+dx)}} \, dx = \frac{\arctan\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b \coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}$$

output
```
arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(1/2)/d+arctanh((b*coth(d*x+c))^(1
/2)/b^(1/2))/b^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{b \coth(c+dx)}} \, dx = \frac{\left(\arctan\left(\sqrt{\coth(c+dx)}\right) + \operatorname{arctanh}\left(\sqrt{\coth(c+dx)}\right)\right) \sqrt{\coth(c+dx)}}{d\sqrt{b \coth(c+dx)}}$$

input
```
Integrate[1/Sqrt[b*Coth[c + d*x]],x]
```

output
```
((ArcTan[Sqrt[Coth[c + d*x]]] + ArcTanh[Sqrt[Coth[c + d*x]]])*Sqrt[Coth[c
+ d*x]])/(d*Sqrt[b*Coth[c + d*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3042, 3957, 25, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\coth(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-ib\tan\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{b\int -\frac{1}{\sqrt{b\coth(c+dx)}(b^2-b^2\coth^2(c+dx))}d(b\coth(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\int \frac{1}{\sqrt{b\coth(c+dx)}(b^2-b^2\coth^2(c+dx))}d(b\coth(c+dx))}{d}$$

$$\Big\downarrow \text{266}$$

$$\frac{2b\int \frac{1}{b^2-b^4\coth^4(c+dx)}d\sqrt{b\coth(c+dx)}}{d}$$

$$\Big\downarrow \text{756}$$

$$\frac{2b\left(\frac{\int \frac{1}{b-b^2\coth^2(c+dx)}d\sqrt{b\coth(c+dx)}}{2b}+\frac{\int \frac{1}{b^2\coth^2(c+dx)+b}d\sqrt{b\coth(c+dx)}}{2b}\right)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{2b\left(\frac{\int \frac{1}{b-b^2\coth^2(c+dx)}d\sqrt{b\coth(c+dx)}}{2b}+\frac{\arctan\left(\sqrt{b}\coth(c+dx)\right)}{2b^{3/2}}\right)}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2b\left(\frac{\arctan\left(\sqrt{b}\coth(c+dx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\coth(c+dx)\right)}{2b^{3/2}}\right)}{d}$$

input
```
Int[1/Sqrt[b*Coth[c + d*x]],x]
```

output
```
(2*b*(ArcTan[Sqrt[b]*Coth[c + d*x]]/(2*b^(3/2)) + ArcTanh[Sqrt[b]*Coth[c +
 d*x]]/(2*b^(3/2))))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}$	46
default	$\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}$	46

input
```
int(1/(b*coth(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(1/2)/d+arctanh((b*coth(d*x+c))^(1
/2)/b^(1/2))/b^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(45) = 90$.

Time = 0.13 (sec) , antiderivative size = 592, normalized size of antiderivative = 10.39

$$\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*coth(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*(2*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
+ sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cos
h(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2)) + sqr
t(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*co
sh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh
(d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*
x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*
x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x +
c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/(b*d), -1/4*(2
*sqrt(b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*
x + c)^2 - 1)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/sqrt(b)) - sqrt(b)*log(2
*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c
)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c
)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)
^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh
(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/s
inh(d*x + c)) - b))/(b*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\coth\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(b*coth(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\coth\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*coth(d*x + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*coth(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionDegree mismatch inside factorisation over extensionindex.c
c index_m
```

Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)+\operatorname{atanh}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}$$

input
```
int(1/(b*coth(c + d*x))^(1/2),x)
```

output
```
(atan((b*coth(c + d*x))^(1/2)/b^(1/2)) + atanh((b*coth(c + d*x))^(1/2)/b^(
1/2)))/(b^(1/2)*d)
```

Reduce [F]

$$\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\coth(dx+c)}}{\coth(dx+c)}dx\right)}{b}$$

input
```
int(1/(b*coth(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*int(sqrt(coth(c + d*x))/coth(c + d*x),x))/b
```

3.6 $\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 78

$$\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} - \frac{2}{bd\sqrt{b\coth(c+dx)}}$$

output
```
-arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d+arctanh((b*coth(d*x+c))^(
1/2)/b^(1/2))/b^(3/2)/d-2/b/d/(b*coth(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

$$\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx = \frac{-2-\arctan\left(\sqrt[4]{\coth^2(c+dx)}\right)\sqrt[4]{\coth^2(c+dx)}+\operatorname{arctanh}\left(\sqrt[4]{\coth^2(c+dx)}\right.}{bd\sqrt{b\coth(c+dx)}}$$

input
```
Integrate[(b*Coth[c + d*x])^(-3/2),x]
```

output
```
(-2 - ArcTan[(Coth[c + d*x]^2)^(1/4)]*(Coth[c + d*x]^2)^(1/4) + ArcTanh[(C
oth[c + d*x]^2)^(1/4)]*(Coth[c + d*x]^2)^(1/4))/(b*d*Sqrt[b*Coth[c + d*x]]
)
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3955, 3042, 3957, 25, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 3955$$

$$\frac{\int \sqrt{b\coth(c+dx)}dx}{b^2}-\frac{2}{bd\sqrt{b\coth(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2}{bd\sqrt{b\coth(c+dx)}}+\frac{\int \sqrt{-ib\tan\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}$$

$$\Big\downarrow \; 3957$$

$$-\frac{\int -\frac{\sqrt{b\coth(c+dx)}}{b^2-b^2\coth^2(c+dx)}d(b\coth(c+dx))}{bd}-\frac{2}{bd\sqrt{b\coth(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\sqrt{b\coth(c+dx)}}{b^2-b^2\coth^2(c+dx)}d(b\coth(c+dx))}{bd}-\frac{2}{bd\sqrt{b\coth(c+dx)}}$$

$$\Big\downarrow \; 266$$

$$\frac{2\int \frac{b^2\coth^2(c+dx)}{b^2-b^4\coth^4(c+dx)}d\sqrt{b\coth(c+dx)}}{bd}-\frac{2}{bd\sqrt{b\coth(c+dx)}}$$

$\Big\downarrow$ 827

$$\frac{2\left(\frac{1}{2}\int \frac{1}{b-b^2\coth^2(c+dx)}d\sqrt{b\coth(c+dx)}-\frac{1}{2}\int \frac{1}{b^2\coth^2(c+dx)+b}d\sqrt{b\coth(c+dx)}\right)}{bd}-\frac{2}{bd\sqrt{b\coth(c+dx)}}$$

$\Big\downarrow$ 216

$$\frac{2\left(\frac{1}{2}\int \frac{1}{b-b^2\coth^2(c+dx)}d\sqrt{b\coth(c+dx)}-\frac{\arctan\left(\sqrt{b}\coth(c+dx)\right)}{2\sqrt{b}}\right)}{bd}-\frac{2}{bd\sqrt{b\coth(c+dx)}}$$

$\Big\downarrow$ 219

$$\frac{2\left(\frac{\text{arctanh}\left(\sqrt{b}\coth(c+dx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\coth(c+dx)\right)}{2\sqrt{b}}\right)}{bd}-\frac{2}{bd\sqrt{b\coth(c+dx)}}$$

input
```
Int[(b*Coth[c + d*x])^(-3/2),x]
```

output
```
(2*(-1/2*ArcTan[Sqrt[b]*Coth[c + d*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Coth[c +
d*x]]/(2*Sqrt[b])))/(b*d) - 2/(b*d*Sqrt[b*Coth[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}d}-\frac{2}{bd\sqrt{b\coth(dx+c)}}$	65
default	$-\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}d}-\frac{2}{bd\sqrt{b\coth(dx+c)}}$	65

input
```
int(1/(b*coth(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d+arctanh((b*coth(d*x+c))^(
1/2)/b^(1/2))/b^(3/2)/d-2/b/d/(b*coth(d*x+c))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(64) = 128$.

Time = 0.14 (sec) , antiderivative size = 915, normalized size of antiderivative = 11.73

$$\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*coth(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^
2 + 1)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(
d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2)) + (cosh
(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-b
)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
 + c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x +
c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2
 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*(cosh(d*x + c)^
2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x +
 c)/sinh(d*x + c)))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*
x + c) + b^2*d*sinh(d*x + c)^2 + b^2*d), 1/4*(2*(cosh(d*x + c)^2 + 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(b)*arctan((cosh(d*x + c
)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x
 + c)/sinh(d*x + c))/sqrt(b)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*
x + c) + sinh(d*x + c)^2 + 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d
*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(
d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4...
```

Sympy [F]

$$\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx=\int \frac{1}{\left(b\coth\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c))**(3/2),x)
```

output
```
Integral((b*coth(c + d*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\coth(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c))^(-3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*coth(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.77 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{b^{3/2}\,d} - \frac{\operatorname{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{b^{3/2}\,d} - \frac{2}{b\,d\,\sqrt{b\coth(c+d\,x)}}$$

input
```
int(1/(b*coth(c + d*x))^(3/2),x)
```

output
```
atanh((b*coth(c + d*x))^(1/2)/b^(1/2))/(b^(3/2)*d) - atan((b*coth(c + d*x)
)^(1/2)/b^(1/2))/(b^(3/2)*d) - 2/(b*d*(b*coth(c + d*x))^(1/2))
```

Reduce [F]

$$\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\coth(dx+c)}}{\coth(dx+c)^2}dx\right)}{b^2}$$

input
```
int(1/(b*coth(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*int(sqrt(coth(c + d*x))/coth(c + d*x)**2,x))/b**2
```

3.7 $\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 79

$$\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d} - \frac{2}{3bd(b\coth(c+dx))^{3/2}}$$

output
```
arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d+arctanh((b*coth(d*x+c))^(1
/2)/b^(1/2))/b^(5/2)/d-2/3/b/d/(b*coth(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx = \frac{-2+3\arctan\left(\sqrt[4]{\coth^2(c+dx)}\right)\coth^2(c+dx)^{3/4}+3\operatorname{arctanh}\left(\sqrt[4]{\coth^2(c+\ldots}\right.}{3bd(b\coth(c+dx))^{3/2}}$$

input
```
Integrate[(b*Coth[c + d*x])^(-5/2),x]
```

output
```
(-2 + 3*ArcTan[(Coth[c + d*x]^2)^(1/4)]*(Coth[c + d*x]^2)^(3/4) + 3*ArcTan
h[(Coth[c + d*x]^2)^(1/4)]*(Coth[c + d*x]^2)^(3/4))/(3*b*d*(b*Coth[c + d*x
])^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3955, 3042, 3957, 25, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\coth(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3955}$$

$$\frac{\int \frac{1}{\sqrt{b\coth(c+dx)}}dx}{b^2}-\frac{2}{3bd(b\coth(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{3bd(b\coth(c+dx))^{3/2}}+\frac{\int \frac{1}{\sqrt{-ib\tan\left(ic+idx+\frac{\pi}{2}\right)}}dx}{b^2}$$

$$\Big\downarrow \text{3957}$$

$$-\frac{\int -\frac{1}{\sqrt{b\coth(c+dx)}\left(b^2-b^2\coth^2(c+dx)\right)}d(b\coth(c+dx))}{bd}-\frac{2}{3bd(b\coth(c+dx))^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{1}{\sqrt{b\coth(c+dx)}\left(b^2-b^2\coth^2(c+dx)\right)}d(b\coth(c+dx))}{bd}-\frac{2}{3bd(b\coth(c+dx))^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2\int \frac{1}{b^2-b^4\coth^4(c+dx)}d\sqrt{b\coth(c+dx)}}{bd}-\frac{2}{3bd(b\coth(c+dx))^{3/2}}$$

$$\Big\downarrow \textbf{756}$$

$$\frac{2\left(\frac{\int \frac{1}{b-b^2\coth^2(c+dx)}d\sqrt{b\coth(c+dx)}}{2b}+\frac{\int \frac{1}{b^2\coth^2(c+dx)+b}d\sqrt{b\coth(c+dx)}}{2b}\right)}{bd}-\frac{2}{3bd(b\coth(c+dx))^{3/2}}$$

$$\Big\downarrow \textbf{216}$$

$$\frac{2\left(\frac{\int \frac{1}{b-b^2\coth^2(c+dx)}d\sqrt{b\coth(c+dx)}}{2b}+\frac{\arctan\left(\sqrt{b}\coth(c+dx)\right)}{2b^{3/2}}\right)}{bd}-\frac{2}{3bd(b\coth(c+dx))^{3/2}}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{2\left(\frac{\arctan\left(\sqrt{b}\coth(c+dx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\coth(c+dx)\right)}{2b^{3/2}}\right)}{bd}-\frac{2}{3bd(b\coth(c+dx))^{3/2}}$$

input

```
Int[(b*Coth[c + d*x])^(-5/2),x]
```

output

```
(2*(ArcTan[Sqrt[b]*Coth[c + d*x]]/(2*b^(3/2)) + ArcTanh[Sqrt[b]*Coth[c + d
*x]]/(2*b^(3/2))))/(b*d) - 2/(3*b*d*(b*Coth[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}d}-\frac{2}{3bd(b\coth(dx+c))^{\frac{3}{2}}}$	64
default	$\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}d}-\frac{2}{3bd(b\coth(dx+c))^{\frac{3}{2}}}$	64

input
```
int(1/(b*coth(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d+arctanh((b*coth(d*x+c))^(1
/2)/b^(1/2))/b^(5/2)/d-2/3/b/d/(b*coth(d*x+c))^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(63) = 126$.

Time = 0.15 (sec) , antiderivative size = 1421, normalized size of antiderivative = 17.99

$$\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*coth(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(6*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x +
c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(
cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b)*arctan((cosh(
d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b)
*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c
)*sinh(d*x + c) + b*sinh(d*x + c)^2)) + 3*(cosh(d*x + c)^4 + 4*cosh(d*x +
c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x
+ c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x
+ c) + 1)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x
+ c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x +
c)^3 + b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
 c) + sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) -
2*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^
2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) +
8*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2
*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x
 + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(b*cosh(d*x + c)/sinh(d*x
+ c)))/(b^3*d*cosh(d*x + c)^4 + 4*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^
3*d*sinh(d*x + c)^4 + 2*b^3*d*cosh(d*x + c)^2 + b^3*d + 2*(3*b^3*d*cosh(d*
x + c)^2 + b^3*d)*sinh(d*x + c)^2 + 4*(b^3*d*cosh(d*x + c)^3 + b^3*d*co...
```

Sympy [F]

$$\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(b\coth\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c))**(5/2),x)
```

output
```
Integral((b*coth(c + d*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c))^(-5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*coth(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.80

$$\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{b^{5/2}\,d} - \frac{2}{3\,b\,d\,(b\coth\left(c+d\,x\right))^{3/2}} + \frac{\operatorname{atanh}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{b^{5/2}\,d}$$

input
```
int(1/(b*coth(c + d*x))^(5/2),x)
```

output
```
atan((b*coth(c + d*x))^(1/2)/b^(1/2))/(b^(5/2)*d) - 2/(3*b*d*(b*coth(c + d
*x))^(3/2)) + atanh((b*coth(c + d*x))^(1/2)/b^(1/2))/(b^(5/2)*d)
```

Reduce [F]

$$\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\coth(dx+c)}}{\coth(dx+c)^3}dx\right)}{b^3}$$

input
```
int(1/(b*coth(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*int(sqrt(coth(c + d*x))/coth(c + d*x)**3,x))/b**3
```

3.8 $\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 100

$$\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d} - \frac{2}{5bd(b\coth(c+dx))^{5/2}} - \frac{2}{b^3d\sqrt{b\coth(c+dx)}}$$

output
```
-arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d+arctanh((b*coth(d*x+c))^(
1/2)/b^(1/2))/b^(7/2)/d-2/5/b/d/(b*coth(d*x+c))^(5/2)-2/b^3/d/(b*coth(d*x+
c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx = \frac{5\operatorname{arctanh}\left(\sqrt[4]{\coth^2(c+dx)}\right)\sqrt[4]{\coth^2(c+dx)} - 5\left(2+\arctan\left(\sqrt[4]{\coth^2(c+d\right.\right.}{5b^3d\sqrt{b\coth(c+dx)}}$$

input
```
Integrate[(b*Coth[c + d*x])^(-7/2),x]
```

output
```
(5*ArcTanh[(Coth[c + d*x]^2)^(1/4)]*(Coth[c + d*x]^2)^(1/4) - 5*(2 + ArcTa
n[(Coth[c + d*x]^2)^(1/4)]*(Coth[c + d*x]^2)^(1/4)) - 2*Tanh[c + d*x]^2)/(
5*b^3*d*Sqrt[b*Coth[c + d*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3955, 3042, 3955, 3042, 3957, 25, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\coth(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3955}$$

$$\frac{\int \frac{1}{(b\coth(c+dx))^{3/2}}dx}{b^2}-\frac{2}{5bd(b\coth(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{5bd(b\coth(c+dx))^{5/2}}+\frac{\int \frac{1}{\left(-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b^2}$$

$$\Big\downarrow \text{3955}$$

$$\frac{\frac{\int \sqrt{b\coth(c+dx)}dx}{b^2}-\frac{2}{bd\sqrt{b\coth(c+dx)}}}{b^2}-\frac{2}{5bd(b\coth(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{5bd(b\coth(c+dx))^{5/2}}+\frac{-\frac{2}{bd\sqrt{b\coth(c+dx)}}+\frac{\int \sqrt{-ib\tan\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}}{b^2}$$

$$\Big\downarrow \text{3957}$$

$$-\frac{-\frac{\int -\frac{\sqrt{b\coth(c+dx)}}{b^2-b^2\coth^2(c+dx)}d(b\coth(c+dx))}{bd}-\frac{2}{bd\sqrt{b\coth(c+dx)}}}{b^2}-\frac{2}{5bd(b\coth(c+dx))^{5/2}}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{\sqrt{b\coth(c+dx)}}{b^2-b^2\coth^2(c+dx)}d(b\coth(c+dx))}{bd}-\frac{2}{bd\sqrt{b\coth(c+dx)}}}{b^2}-\frac{2}{5bd(b\coth(c+dx))^{5/2}}$$

$\Big\downarrow$ 266

$$\frac{\frac{2\int \frac{b^2\coth^2(c+dx)}{b^2-b^4\coth^4(c+dx)}d\sqrt{b\coth(c+dx)}}{bd}-\frac{2}{bd\sqrt{b\coth(c+dx)}}}{b^2}-\frac{2}{5bd(b\coth(c+dx))^{5/2}}$$

$\Big\downarrow$ 827

$$\frac{\frac{2\left(\frac{1}{2}\int \frac{1}{b-b^2\coth^2(c+dx)}d\sqrt{b\coth(c+dx)}-\frac{1}{2}\int \frac{1}{b^2\coth^2(c+dx)+b}d\sqrt{b\coth(c+dx)}\right)}{bd}-\frac{2}{bd\sqrt{b\coth(c+dx)}}}{b^2}-\frac{2}{5bd(b\coth(c+dx))^{5/2}}$$

$\Big\downarrow$ 216

$$\frac{\frac{2\left(\frac{1}{2}\int \frac{1}{b-b^2\coth^2(c+dx)}d\sqrt{b\coth(c+dx)}-\frac{\arctan\left(\sqrt{b}\coth(c+dx)\right)}{2\sqrt{b}}\right)}{bd}-\frac{2}{bd\sqrt{b\coth(c+dx)}}}{b^2}-\frac{2}{5bd(b\coth(c+dx))^{5/2}}$$

$\Big\downarrow$ 219

$$\frac{\frac{2\left(\frac{\operatorname{arctanh}\left(\sqrt{b}\coth(c+dx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\coth(c+dx)\right)}{2\sqrt{b}}\right)}{bd}-\frac{2}{bd\sqrt{b\coth(c+dx)}}}{b^2}-\frac{2}{5bd(b\coth(c+dx))^{5/2}}$$

input
```
Int[(b*Coth[c + d*x])^(-7/2),x]
```

output
```
-2/(5*b*d*(b*Coth[c + d*x])^(5/2)) + ((2*(-1/2*ArcTan[Sqrt[b]*Coth[c + d*x
]]/Sqrt[b] + ArcTanh[Sqrt[b]*Coth[c + d*x]]/(2*Sqrt[b])))/(b*d) - 2/(b*d*S
qrt[b*Coth[c + d*x]]))/b^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}d}-\frac{2}{5bd(b\coth(dx+c))^{\frac{5}{2}}}-\frac{2}{b^{3}d\sqrt{b\coth(dx+c)}}$	83
default	$-\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}d}-\frac{2}{5bd(b\coth(dx+c))^{\frac{5}{2}}}-\frac{2}{b^{3}d\sqrt{b\coth(dx+c)}}$	83

input
```
int(1/(b*coth(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d+arctanh((b*coth(d*x+c))^(
1/2)/b^(1/2))/b^(7/2)/d-2/5/b/d/(b*coth(d*x+c))^(5/2)-2/b^3/d/(b*coth(d*x+
c))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1034 vs. 2(82) = 164.

Time = 0.18 (sec) , antiderivative size = 2125, normalized size of antiderivative = 21.25

$$\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*coth(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
[-1/20*(10*(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x +
 c)^6 + 3*(5*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + 3*cosh(d*x + c)^4 + 4*
(5*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)
^4 + 6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(
d*x + c)^5 + 2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b
)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^
2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2
*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2)) + 5*(cosh(d*x + c)^6
+ 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6 + 3*(5*cosh(d*x + c)^2
 + 1)*sinh(d*x + c)^4 + 3*cosh(d*x + c)^4 + 4*(5*cosh(d*x + c)^3 + 3*cosh(
d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4 + 6*cosh(d*x + c)^2 + 1)*
sinh(d*x + c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(d*x + c)^5 + 2*cosh(d*x + c)
^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 +
4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 +
4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(cosh(d*x + c)^2
 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*co
sh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sin
h(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x
+ c)^3 + sinh(d*x + c)^4)) + 16*(3*cosh(d*x + c)^6 + 18*cosh(d*x + c)*sinh
(d*x + c)^5 + 3*sinh(d*x + c)^6 + (45*cosh(d*x + c)^2 + 1)*sinh(d*x + c...
```

Sympy [F]

$$\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx=\int \frac{1}{\left(b\coth\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c))**(7/2),x)
```

output
```
Integral((b*coth(c + d*x))**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\coth(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c))^(-7/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*coth(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{b^{7/2}\,d} - \frac{\operatorname{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{b^{7/2}\,d} - \frac{\frac{2}{5\,b}+\frac{2\coth(c+d\,x)^2}{b}}{d\,(b\coth(c+d\,x))^{5/2}}$$

input
```
int(1/(b*coth(c + d*x))^(7/2),x)
```

output
```
atanh((b*coth(c + d*x))^(1/2)/b^(1/2))/(b^(7/2)*d) - atan((b*coth(c + d*x)
)^(1/2)/b^(1/2))/(b^(7/2)*d) - (2/(5*b) + (2*coth(c + d*x)^2)/b)/(d*(b*cot
h(c + d*x))^(5/2))
```

Reduce [F]

$$\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\coth(dx+c)}}{\coth(dx+c)^4}dx\right)}{b^4}$$

input
```
int(1/(b*coth(d*x+c))^(7/2),x)
```

output
```
(sqrt(b)*int(sqrt(coth(c + d*x))/coth(c + d*x)**4,x))/b**4
```

3.9 $\int (b\coth(c+dx))^{4/3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 196

$$\int (b\coth(c+dx))^{4/3}\,dx = -\frac{\sqrt{3}b^{4/3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{b\coth(c+dx)}}{\sqrt{3}\sqrt[3]{b}}\right)}{2d} + \frac{\sqrt{3}b^{4/3}\arctan\left(\frac{\sqrt[3]{b}+2\sqrt[3]{b\coth(c+dx)}}{\sqrt{3}\sqrt[3]{b}}\right)}{2d} + \frac{b^{4/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} + \frac{b^{4/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}}{b^{2/3}+(b\coth(c+dx))^{2/3}}\right)}{2d} - \frac{3b\sqrt[3]{b\coth(c+dx)}}{d}$$

output
```
-1/2*3^(1/2)*b^(4/3)*arctan(1/3*(b^(1/3)-2*(b*coth(d*x+c))^(1/3))*3^(1/2)/
b^(1/3))/d+1/2*3^(1/2)*b^(4/3)*arctan(1/3*(b^(1/3)+2*(b*coth(d*x+c))^(1/3)
)*3^(1/2)/b^(1/3))/d+b^(4/3)*arctanh((b*coth(d*x+c))^(1/3)/b^(1/3))/d+1/2*
b^(4/3)*arctanh(b^(1/3)*(b*coth(d*x+c))^(1/3)/(b^(2/3)+(b*coth(d*x+c))^(2/
3)))/d-3*b*(b*coth(d*x+c))^(1/3)/d
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.98

$$\int (b\coth(c+dx))^{4/3}\,dx =$$

$$-\frac{b\sqrt[3]{b\coth(c+dx)}\left(6\sqrt[6]{\coth^2(c+dx)}+\log\left(1-\sqrt[6]{\coth^2(c+dx)}\right)-\log\left(1+\sqrt[6]{\coth^2(c+dx)}\right)-(-1\right.}{}$$

input
```
Integrate[(b*Coth[c + d*x])^(4/3),x]
```

output
```
-1/2*(b*(b*Coth[c + d*x])^(1/3)*(6*(Coth[c + d*x]^2)^(1/6) + Log[1 - (Coth
[c + d*x]^2)^(1/6)] - Log[1 + (Coth[c + d*x]^2)^(1/6)] - (-1)^(2/3)*Log[1
- (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)] + (-1)^(2/3)*Log[1 + (-1)^(1/3)*(Cot
h[c + d*x]^2)^(1/6)] - (-1)^(1/3)*Log[1 - (-1)^(2/3)*(Coth[c + d*x]^2)^(1/
6)] + (-1)^(1/3)*Log[1 + (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)]))/(d*(Coth[c
+ d*x]^2)^(1/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$, Rules used = {3042, 3954, 3042, 3957, 25, 266, 754, 27, 219, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\coth(c+dx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{4/3}dx$$

$$\Big\downarrow \text{3954}$$

$$b^2\int \frac{1}{(b\coth(c+dx))^{2/3}}dx-\frac{3b\sqrt[3]{b\coth(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3b\sqrt[3]{b\coth(c+dx)}}{d}+b^2\int \frac{1}{\left(-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{b^3\int -\frac{1}{(b\coth(c+dx))^{2/3}\left(b^2-b^2\coth^2(c+dx)\right)}d(b\coth(c+dx))}{d}-\frac{3b\sqrt[3]{b\coth(c+dx)}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^3\int \frac{1}{(b\coth(c+dx))^{2/3}\left(b^2-b^2\coth^2(c+dx)\right)}d(b\coth(c+dx))}{d}-\frac{3b\sqrt[3]{b\coth(c+dx)}}{d}$$

$$\Big\downarrow \text{266}$$

$$\frac{3b^3\int \frac{1}{b^2-b^6\coth^6(c+dx)}d\sqrt[3]{b\coth(c+dx)}}{d}-\frac{3b\sqrt[3]{b\coth(c+dx)}}{d}$$

$$\Big\downarrow \text{754}$$

$$\frac{3b^3\left(\frac{\int \frac{1}{b^{2/3}-b^2\coth^2(c+dx)}d\sqrt[3]{b\coth(c+dx)}}{3b^{4/3}}+\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{b\coth(c+dx)}}{2\left(b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}\right)}d\sqrt[3]{b\coth(c+dx)}}{3b^{5/3}}+\frac{\int \frac{2\sqrt[3]{b}+\sqrt[3]{b}}{2\left(b^2\coth^2(c+dx)-\right.}}{}\right.}{d}$$

$$\frac{3b\sqrt[3]{b\coth(c+dx)}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{3b^3\left(\frac{\int \frac{1}{b^{2/3}-b^2\coth^2(c+dx)}d\sqrt[3]{b\coth(c+dx)}}{3b^{4/3}}+\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6b^{5/3}}+\frac{\int \frac{2\sqrt[3]{b}+\sqrt[3]{b\cot}}{b^2\coth^2(c+dx)+b^{4/3}}}{}\right.}{d}$$

$$\frac{3b\sqrt[3]{b\coth(c+dx)}}{d}$$

$$\Big\downarrow \text{219}$$

$$3b^3\left(\frac{\int\frac{2\sqrt[3]{b}-\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6b^{5/3}}+\frac{\int\frac{2\sqrt[3]{b}+\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)+b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6b^{5/3}}+\text{arcta}\right.$$
$$\overline{\qquad\qquad\qquad d\qquad\qquad\qquad}$$

$$\frac{3b\sqrt[3]{b\coth(c+dx)}}{d}$$
$$\Big\downarrow 1142$$

$$3b^3\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}-\frac{1}{2}\int-\frac{\sqrt[3]{b}-2\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6b^{5/3}}+\right.$$

$$\frac{3b\sqrt[3]{b\coth(c+dx)}}{d}$$
$$\Big\downarrow 25$$

$$3b^3\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}+\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6b^{5/3}}+\right.$$

$$\frac{3b\sqrt[3]{b\coth(c+dx)}}{d}$$
$$\Big\downarrow 1082$$

$$3b^3\left(\frac{3\int\frac{1}{-b^2\coth^2(c+dx)-3}d(1-2b^{2/3}\coth(c+dx))+\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6b^{5/3}}+\frac{\frac{1}{2}\int\frac{\sqrt[3]{b}+2\sqrt[3]{b\,\mathrm{c}}}{b^2\coth^2(c+dx)+b^4}}{}\right.$$
$$\overline{\qquad\qquad\qquad d\qquad\qquad\qquad}$$

$$\frac{3b\sqrt[3]{b\coth(c+dx)}}{d}$$
$$\Big\downarrow 217$$

$$3b^3\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}-\sqrt{3}\arctan\left(\frac{1-2b^{2/3}\coth(c+dx)}{\sqrt{3}}\right)}{6b^{5/3}}+\frac{\frac{1}{2}\int\frac{\sqrt[3]{b}+2\sqrt[3]{b\coth(c+dx}}{b^2\coth^2(c+dx)+b^{4/3}\coth(c+dx)+}}{}\right.$$
$$\overline{\qquad\qquad\qquad d\qquad\qquad\qquad}$$

$$\frac{3b\sqrt[3]{b\coth(c+dx)}}{d}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3b^3\left(\frac{-\sqrt{3}\arctan\left(\frac{1-2b^{2/3}\coth(c+dx)}{\sqrt{3}}\right)-\frac{1}{2}\log\left(-b^{4/3}\coth(c+dx)+b^{2/3}+b^2\coth^2(c+dx)\right)}{6b^{5/3}}+\frac{\sqrt{3}\arctan\left(\frac{2b^{2/3}\coth(c+dx)+1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(b^{4/3}\cot\right.}{6b^{5/3}}\right.}{d}$$

$$\frac{3b\sqrt[3]{b\coth(c+dx)}}{d}$$

input
```
Int[(b*Coth[c + d*x])^(4/3),x]
```

output
```
(-3*b*(b*Coth[c + d*x])^(1/3))/d + (3*b^3*(ArcTanh[b^(2/3)*Coth[c + d*x]]/
(3*b^(5/3)) + (-(Sqrt[3]*ArcTan[(1 - 2*b^(2/3)*Coth[c + d*x])/Sqrt[3]]) -
Log[b^(2/3) - b^(4/3)*Coth[c + d*x] + b^2*Coth[c + d*x]^2]/2)/(6*b^(5/3))
+ (Sqrt[3]*ArcTan[(1 + 2*b^(2/3)*Coth[c + d*x])/Sqrt[3]] + Log[b^(2/3) + b
^(4/3)*Coth[c + d*x] + b^2*Coth[c + d*x]^2]/2)/(6*b^(5/3))))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.07

method	result
derivativedivides	$-\frac{3b(b\coth(dx+c))^{\frac{1}{3}}}{d}-\frac{b^{\frac{4}{3}}\ln\left((b\coth(dx+c))^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{2d}+\frac{b^{\frac{4}{3}}\ln\left((b\coth(dx+c))^{\frac{2}{3}}+b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{4d}+$
default	$-\frac{3b(b\coth(dx+c))^{\frac{1}{3}}}{d}-\frac{b^{\frac{4}{3}}\ln\left((b\coth(dx+c))^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{2d}+\frac{b^{\frac{4}{3}}\ln\left((b\coth(dx+c))^{\frac{2}{3}}+b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{4d}+$

input
```
int((b*coth(d*x+c))^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-3*b*(b*coth(d*x+c))^(1/3)/d-1/2/d*b^(4/3)*ln((b*coth(d*x+c))^(1/3)-b^(1/3
))+1/4/d*b^(4/3)*ln((b*coth(d*x+c))^(2/3)+b^(1/3)*(b*coth(d*x+c))^(1/3)+b^
(2/3))+1/2/d*b^(4/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*coth(d*x+c))^(1/3)/b
^(1/3)+1))+1/2/d*b^(4/3)*ln((b*coth(d*x+c))^(1/3)+b^(1/3))-1/4/d*b^(4/3)*l
n((b*coth(d*x+c))^(2/3)-b^(1/3)*(b*coth(d*x+c))^(1/3)+b^(2/3))+1/2/d*b^(4/
3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*coth(d*x+c))^(1/3)/b^(1/3)-1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.49

$$\int(b\coth(c+dx))^{4/3}\,dx=$$

$$-\frac{2\sqrt{3}(-b)^{\frac{1}{3}}\,b\arctan\left(\frac{\sqrt{3}b+2\sqrt{3}(-b)^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)-2\sqrt{3}b^{\frac{4}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}b^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)+(-b)^{\frac{1}{3}}\,b}{}$$

input
```
integrate((b*coth(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(3)*(-b)^(1/3)*b*arctan(1/3*(sqrt(3)*b + 2*sqrt(3)*(-b)^(2/3)*
(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) - 2*sqrt(3)*b^(4/3)*arctan(-1/3*
(sqrt(3)*b - 2*sqrt(3)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) +
 (-b)^(1/3)*b*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^
(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + b^(4/3)*log(b^(2/3) - b^(
1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c
))^(2/3)) - 2*(-b)^(1/3)*b*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3)) - 2*b^(4/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3))
+ 12*b*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/d
```

Sympy [F]

$$\int (b\coth(c+dx))^{4/3}\,dx = \int (b\coth(c+dx))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*coth(d*x+c))**(4/3),x)
```

output
```
Integral((b*coth(c + d*x))**(4/3), x)
```

Maxima [F]

$$\int (b\coth(c+dx))^{4/3}\,dx = \int (b\coth(dx+c))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*coth(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c))^(4/3), x)
```

Giac [F(-2)]

Exception generated.

$$\int (b\coth(c+dx))^{4/3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*coth(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Minimal poly. in rootof must be fra
ction free Error: Bad Argument ValueMinimal poly. in rootof must be fracti
on free E
```

Mupad [B] (verification not implemented)

Time = 2.99 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.27

$$\begin{aligned}\int (b\coth(c+dx))^{4/3}\,dx &= -\frac{3\,b\,(b\coth(c+d\,x))^{1/3}}{d} \\ &- \frac{b^{4/3}\,\mathrm{atan}\left(\frac{(b\coth(c+d\,x))^{1/3}\,1\mathrm{i}}{b^{1/3}}\right)\,1\mathrm{i}}{d} \\ &- \frac{b^{4/3}\,\ln\left(\frac{486\,b^{37/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{d^4}-\frac{486\,b^{12}\,(b\coth(c+d\,x))^{1/3}}{d^4}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d} \\ &- \frac{b^{4/3}\,\ln\left(\frac{486\,b^{37/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{d^4}-\frac{486\,b^{12}\,(b\coth(c+d\,x))^{1/3}}{d^4}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d} \\ &+ \frac{b^{4/3}\,\ln\left(\frac{972\,b^{37/3}\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d^4}+\frac{486\,b^{12}\,(b\coth(c+d\,x))^{1/3}}{d^4}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d} \\ &+ \frac{b^{4/3}\,\ln\left(\frac{972\,b^{37/3}\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d^4}+\frac{486\,b^{12}\,(b\coth(c+d\,x))^{1/3}}{d^4}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}\end{aligned}$$

input
```
int((b*coth(c + d*x))^(4/3),x)
```

output
```
(b^(4/3)*log((972*b^(37/3)*((3^(1/2)*1i)/4 - 1/4))/d^4 + (486*b^12*(b*coth
(c + d*x))^(1/3))/d^4)*((3^(1/2)*1i)/4 - 1/4))/d - (b^(4/3)*atan(((b*coth(
c + d*x))^(1/3)*1i)/b^(1/3))*1i)/d - (b^(4/3)*log((486*b^(37/3)*((3^(1/2)*
1i)/2 - 1/2))/d^4 - (486*b^12*(b*coth(c + d*x))^(1/3))/d^4)*((3^(1/2)*1i)/
2 - 1/2))/(2*d) - (b^(4/3)*log((486*b^(37/3)*((3^(1/2)*1i)/2 + 1/2))/d^4 -
 (486*b^12*(b*coth(c + d*x))^(1/3))/d^4)*((3^(1/2)*1i)/2 + 1/2))/(2*d) - (
3*b*(b*coth(c + d*x))^(1/3))/d + (b^(4/3)*log((972*b^(37/3)*((3^(1/2)*1i)/
4 + 1/4))/d^4 + (486*b^12*(b*coth(c + d*x))^(1/3))/d^4)*((3^(1/2)*1i)/4 +
1/4))/d
```

Reduce [F]

$$\int (b\coth(c+dx))^{4/3}\,dx = \frac{b^{\frac{4}{3}}\left(-3\coth\left(dx+c\right)^{\frac{1}{3}}+\left(\int \frac{1}{\coth(dx+c)^{\frac{2}{3}}}dx\right)d\right)}{d}$$

input
```
int((b*coth(d*x+c))^(4/3),x)
```

output
```
(b**(1/3)*b*( - 3*coth(c + d*x)**(1/3) + int(coth(c + d*x)**(1/3)/coth(c +
 d*x),x)*d))/d
```

3.10 $\int (b\coth(c+dx))^{2/3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 178

$$\int (b\coth(c+dx))^{2/3}\,dx = \frac{\sqrt{3}b^{2/3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{b\coth(c+dx)}}{\sqrt{3}\sqrt[3]{b}}\right)}{2d} - \frac{\sqrt{3}b^{2/3}\arctan\left(\frac{\sqrt[3]{b}+2\sqrt[3]{b\coth(c+dx)}}{\sqrt{3}\sqrt[3]{b}}\right)}{2d} + \frac{b^{2/3}\text{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} + \frac{b^{2/3}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}}{b^{2/3}+(b\coth(c+dx))^{2/3}}\right)}{2d}$$

output
```
1/2*3^(1/2)*b^(2/3)*arctan(1/3*(b^(1/3)-2*(b*coth(d*x+c))^(1/3))*3^(1/2)/b
^(1/3))/d-1/2*3^(1/2)*b^(2/3)*arctan(1/3*(b^(1/3)+2*(b*coth(d*x+c))^(1/3))
*3^(1/2)/b^(1/3))/d+b^(2/3)*arctanh((b*coth(d*x+c))^(1/3)/b^(1/3))/d+1/2*b
^(2/3)*arctanh(b^(1/3)*(b*coth(d*x+c))^(1/3)/(b^(2/3)+(b*coth(d*x+c))^(2/3
)))/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.84

$$\int (b\coth(c+dx))^{2/3}\,dx = \frac{(b\coth(c+dx))^{2/3}\left(2\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\right.}{}$$

input
```
Integrate[(b*Coth[c + d*x])^(2/3),x]
```

output
```
((b*Coth[c + d*x])^(2/3)*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqr
t[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] + 4*ArcTanh[
Coth[c + d*x]^(1/3)] - Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]
+ Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]))/(4*d*Coth[c + d*x]^
(2/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3957, 25, 266, 825, 27, 219, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\coth(c+dx))^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{2/3}dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{b\int -\frac{(b\coth(c+dx))^{2/3}}{b^2-b^2\coth^2(c+dx)}d(b\coth(c+dx))}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{b\int \frac{(b\coth(c+dx))^{2/3}}{b^2-b^2\coth^2(c+dx)}d(b\coth(c+dx))}{d}$$

$$\Big\downarrow \; 266$$

$$\frac{3b\int \frac{b^4\coth^4(c+dx)}{b^2-b^6\coth^6(c+dx)}d\sqrt[3]{b\coth(c+dx)}}{d}$$

$$\Big\downarrow \; 825$$

$$\frac{3b\left(\frac{1}{3}\int \frac{1}{b^{2/3}-b^2\coth^2(c+dx)}d\sqrt[3]{b\coth(c+dx)}+\frac{\int -\frac{\sqrt[3]{b}-\sqrt[3]{b\coth(c+dx)}}{2\left(b^2\coth^2(c+dx)+b^{4/3}\coth(c+dx)+b^{2/3}\right)}d\sqrt[3]{b\coth(c+dx)}}{3\sqrt[3]{b}}+\frac{\int -\frac{\sqrt[3]{b}}{2\left(b^2\coth^2\right.}}{}\right.}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{3b\left(\frac{1}{3}\int \frac{1}{b^{2/3}-b^2\coth^2(c+dx)}d\sqrt[3]{b\coth(c+dx)}-\frac{\int \frac{\sqrt[3]{b}-\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)+b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6\sqrt[3]{b}}-\frac{\int \frac{\sqrt[3]{b}+\sqrt[3]{b\,\mathrm{co}}}{b^2\coth^2(c+dx)-b^4}}{}\right.}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{3b\left(-\frac{\int \frac{\sqrt[3]{b}-\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)+b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6\sqrt[3]{b}}-\frac{\int \frac{\sqrt[3]{b}+\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6\sqrt[3]{b}}+\frac{\mathrm{arct}}{}\right.}{d}$$

$$\Big\downarrow \; 1142$$

$$3b\left(-\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}+\frac{1}{2}\int -\frac{\sqrt[3]{b}-2\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6\sqrt[3]{b}}\right.$$

$$\Big\downarrow \; 25$$

$$3b\left(-\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}-\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6\sqrt[3]{b}}-\right.$$

$\downarrow$ 1082

$$3b\left(-\frac{3\int \frac{1}{-b^2\coth^2(c+dx)-3}d(1-2b^{2/3}\coth(c+dx))-\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6\sqrt[3]{b}}-\frac{-3\int \frac{1}{-b^2\coth^2(c+dx)}}{}\right.$$
$$d$$

$\downarrow$ 217

$$3b\left(-\frac{-\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}-\sqrt{3}\arctan\left(\frac{1-2b^{2/3}\coth(c+dx)}{\sqrt{3}}\right)}{6\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{2b^{2/3}\coth(c+dx)+1}{\sqrt{3}}\right)}{}\right.$$
$$d$$

$\downarrow$ 1103

$$3b\left(-\frac{\frac{1}{2}\log\left(-b^{4/3}\coth(c+dx)+b^{2/3}+b^2\coth^2(c+dx)\right)-\sqrt{3}\arctan\left(\frac{1-2b^{2/3}\coth(c+dx)}{\sqrt{3}}\right)}{6\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{2b^{2/3}\coth(c+dx)+1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(b^{4/3}\coth\right.}{6\sqrt[3]{b}}\right.$$
$$d$$

input
```
Int[(b*Coth[c + d*x])^(2/3),x]
```

output
```
(3*b*(ArcTanh[b^(2/3)*Coth[c + d*x]]/(3*b^(1/3)) - (-(Sqrt[3]*ArcTan[(1 -
2*b^(2/3)*Coth[c + d*x])/Sqrt[3]]) + Log[b^(2/3) - b^(4/3)*Coth[c + d*x] +
 b^2*Coth[c + d*x]^2]/2)/(6*b^(1/3)) - (Sqrt[3]*ArcTan[(1 + 2*b^(2/3)*Coth
[c + d*x])/Sqrt[3]] - Log[b^(2/3) + b^(4/3)*Coth[c + d*x] + b^2*Coth[c + d
*x]^2]/2)/(6*b^(1/3))))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.02

method	result
derivativedivides	$-\frac{3b\left(-\frac{\ln\left((b\coth(dx+c))^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{6b^{\frac{1}{3}}}+\frac{\ln\left((b\coth(dx+c))^{\frac{2}{3}}-b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{12b^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\coth(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}-1\right)}{3}\right)}{6b^{\frac{1}{3}}}$
default	$-\frac{3b\left(-\frac{\ln\left((b\coth(dx+c))^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{6b^{\frac{1}{3}}}+\frac{\ln\left((b\coth(dx+c))^{\frac{2}{3}}-b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{12b^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\coth(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}-1\right)}{3}\right)}{6b^{\frac{1}{3}}}$

input
```
int((b*coth(d*x+c))^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-3/d*b*(-1/6/b^(1/3)*ln((b*coth(d*x+c))^(1/3)+b^(1/3))+1/12/b^(1/3)*ln((b*
coth(d*x+c))^(2/3)-b^(1/3)*(b*coth(d*x+c))^(1/3)+b^(2/3))+1/6*3^(1/2)/b^(1
/3)*arctan(1/3*3^(1/2)*(2*(b*coth(d*x+c))^(1/3)/b^(1/3)-1))+1/6/b^(1/3)*ln
((b*coth(d*x+c))^(1/3)-b^(1/3))-1/12/b^(1/3)*ln((b*coth(d*x+c))^(2/3)+b^(1
/3)*(b*coth(d*x+c))^(1/3)+b^(2/3))+1/6*3^(1/2)/b^(1/3)*arctan(1/3*3^(1/2)*
(2*(b*coth(d*x+c))^(1/3)/b^(1/3)+1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(136) = 272$.

Time = 0.11 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.74

$$\int (b\coth(c+dx))^{2/3}\,dx =$$

$$-\frac{2\sqrt{3}(-b^2)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}(-b^2)^{\frac{1}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)+2\sqrt{3}(b^2)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}(b^2)^{\frac{1}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)+}{}$$

input
```
integrate((b*coth(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(3)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*(-b^2)^(1/
3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + 2*sqrt(3)*(b^2)^(1/3)*arcta
n(-1/3*(sqrt(3)*b - 2*sqrt(3)*(b^2)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^
(1/3))/b) + (-b^2)^(1/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) - (-b
^2)^(1/3)*b + (-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + (b^2)^
(1/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) + (b^2)^(1/3)*b - (b^2)^
(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(-b^2)^(1/3)*log(b*(b*cos
h(d*x + c)/sinh(d*x + c))^(1/3) - (-b^2)^(2/3)) - 2*(b^2)^(1/3)*log(b*(b*c
osh(d*x + c)/sinh(d*x + c))^(1/3) + (b^2)^(2/3)))/d
```

Sympy [F]

$$\int (b\coth(c+dx))^{2/3}\,dx = \int (b\coth(c+dx))^{\frac{2}{3}}\,dx$$

input
```
integrate((b*coth(d*x+c))**(2/3),x)
```

output
```
Integral((b*coth(c + d*x))**(2/3), x)
```

Maxima [F]

$$\int (b\coth(c+dx))^{2/3}\,dx = \int (b\coth(dx+c))^{\frac{2}{3}}\,dx$$

input
```
integrate((b*coth(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c))^(2/3), x)
```

Giac [F(-2)]

Exception generated.

$$\int (b\coth(c+dx))^{2/3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*coth(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Minimal poly. in rootof must be fra
ction free Error: Bad Argument ValueMinimal poly. in rootof must be fracti
on free E
```

Mupad [B] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.31

$$\int (b\coth(c+dx))^{2/3}\,dx = -\frac{b^{2/3}\operatorname{atan}\left(\frac{(b\coth(c+dx))^{1/3}\,1\mathrm{i}}{b^{1/3}}\right)\,1\mathrm{i}}{d} - \frac{b^{2/3}\ln\left(\frac{972\,b^9}{d^3}-\frac{972\,b^{26/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(b\coth(c+dx))^{1/3}}{d^3}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d} - \frac{b^{2/3}\ln\left(\frac{972\,b^9}{d^3}-\frac{972\,b^{26/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(b\coth(c+dx))^{1/3}}{d^3}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d} + \frac{b^{2/3}\ln\left(\frac{972\,b^9}{d^3}+\frac{1944\,b^{26/3}\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)(b\coth(c+dx))^{1/3}}{d^3}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d} + \frac{b^{2/3}\ln\left(\frac{972\,b^9}{d^3}+\frac{1944\,b^{26/3}\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)(b\coth(c+dx))^{1/3}}{d^3}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}$$

input
```
int((b*coth(c + d*x))^(2/3),x)
```

output
```
(b^(2/3)*log((972*b^9)/d^3 + (1944*b^(26/3)*((3^(1/2)*1i)/4 - 1/4)*(b*coth
(c + d*x))^(1/3))/d^3)*((3^(1/2)*1i)/4 - 1/4))/d - (b^(2/3)*log((972*b^9)/
d^3 - (972*b^(26/3)*((3^(1/2)*1i)/2 - 1/2)*(b*coth(c + d*x))^(1/3))/d^3)*(
(3^(1/2)*1i)/2 - 1/2))/(2*d) - (b^(2/3)*log((972*b^9)/d^3 - (972*b^(26/3)*
((3^(1/2)*1i)/2 + 1/2)*(b*coth(c + d*x))^(1/3))/d^3)*((3^(1/2)*1i)/2 + 1/2
))/(2*d) - (b^(2/3)*atan(((b*coth(c + d*x))^(1/3)*1i)/b^(1/3))*1i)/d + (b^
(2/3)*log((972*b^9)/d^3 + (1944*b^(26/3)*((3^(1/2)*1i)/4 + 1/4)*(b*coth(c
+ d*x))^(1/3))/d^3)*((3^(1/2)*1i)/4 + 1/4))/d
```

Reduce [F]

$$\int (b\coth(c+dx))^{2/3}\,dx = b^{\frac{2}{3}}\left(\int \coth(dx+c)^{\frac{2}{3}}\,dx\right)$$

input
```
int((b*coth(d*x+c))^(2/3),x)
```

output
```
b**(2/3)*int(coth(c + d*x)**(2/3),x)
```

3.11 $\int \sqrt[3]{b\coth(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 132

$$\int \sqrt[3]{b\coth(c+dx)}\,dx = -\frac{\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{b^{2/3}+2(b\coth(c+dx))^{2/3}}{\sqrt{3}b^{2/3}}\right)}{2d} - \frac{\sqrt[3]{b}\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2d} + \frac{\sqrt[3]{b}\log\left(b^{4/3}+b^{2/3}(b\coth(c+dx))^{2/3}+(b\coth(c+dx))^{4/3}\right)}{4d}$$

output
```
-1/2*3^(1/2)*b^(1/3)*arctan(1/3*(b^(2/3)+2*(b*coth(d*x+c))^(2/3))*3^(1/2)/
b^(2/3))/d-1/2*b^(1/3)*ln(b^(2/3)-(b*coth(d*x+c))^(2/3))/d+1/4*b^(1/3)*ln(
b^(4/3)+b^(2/3)*(b*coth(d*x+c))^(2/3)+(b*coth(d*x+c))^(4/3))/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.82

$$\int \sqrt[3]{b\coth(c+dx)}\,dx = -\frac{(b\coth(c+dx))^{4/3}\left(\log\left(1-\sqrt[3]{\coth^2(c+dx)}\right)-\sqrt[3]{-1}\log\left(1+\sqrt[3]{-1}\sqrt[3]{\coth^2(c+dx)}\right)+(-1)^{2/3}\mathrm{lo}\right.}{2bd\coth^2(c+dx)^{2/3}}$$

input
```
Integrate[(b*Coth[c + d*x])^(1/3),x]
```

output
```
-1/2*((b*Coth[c + d*x])^(4/3)*(Log[1 - (Coth[c + d*x]^2)^(1/3)] - (-1)^(1/
3)*Log[1 + (-1)^(1/3)*(Coth[c + d*x]^2)^(1/3)] + (-1)^(2/3)*Log[1 - (-1)^(
2/3)*(Coth[c + d*x]^2)^(1/3)]))/(b*d*(Coth[c + d*x]^2)^(2/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.84, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3957, 25, 266, 807, 821, 16, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\coth(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{-ib\tan\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{b\int -\frac{\sqrt[3]{b\coth(c+dx)}}{b^2-b^2\coth^2(c+dx)}d(b\coth(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\int \frac{\sqrt[3]{b\coth(c+dx)}}{b^2-b^2\coth^2(c+dx)}d(b\coth(c+dx))}{d}$$

$$\Big\downarrow \text{266}$$

$$\frac{3b\int \frac{b^3\coth^3(c+dx)}{b^2-b^6\coth^6(c+dx)}d\sqrt[3]{b\coth(c+dx)}}{d}$$

$$\Big\downarrow \text{807}$$

$$\frac{3b\int \frac{b^2\coth^2(c+dx)}{b^2-b^3\coth^3(c+dx)}d\left(b^2\coth^2(c+dx)\right)}{2d}$$

$$\Big\downarrow \; 821$$

$$\frac{3b\left(\frac{\int \frac{1}{b^{2/3}-b^2\coth^2(c+dx)}d(b^2\coth^2(c+dx))}{3b^{2/3}}-\frac{\int \frac{b^{2/3}-b^2\coth^2(c+dx)}{b^2\coth^2(c+dx)+b^{5/3}\coth(c+dx)+b^{4/3}}d(b^2\coth^2(c+dx))}{3b^{2/3}}\right)}{2d}$$

$$\Big\downarrow \; 16$$

$$\frac{3b\left(-\frac{\int \frac{b^{2/3}-b^2\coth^2(c+dx)}{b^2\coth^2(c+dx)+b^{5/3}\coth(c+dx)+b^{4/3}}d(b^2\coth^2(c+dx))}{3b^{2/3}}-\frac{\log(b^{2/3}-b^2\coth^2(c+dx))}{3b^{2/3}}\right)}{2d}$$

$$\Big\downarrow \; 1142$$

$$\frac{3b\left(-\frac{\frac{3}{2}b^{2/3}\int \frac{1}{b^2\coth^2(c+dx)+b^{5/3}\coth(c+dx)+b^{4/3}}d(b^2\coth^2(c+dx))-\frac{1}{2}\int \frac{2b^2\coth^2(c+dx)+b^{2/3}}{b^2\coth^2(c+dx)+b^{5/3}\coth(c+dx)+b^{4/3}}d(b^2\coth^2(c+dx))}{3b^{2/3}}-\frac{\log(b^{2/3}}{}\right.}{2d}$$

$$\Big\downarrow \; 1082$$

$$\frac{3b\left(-\frac{-\frac{1}{2}\int \frac{2b^2\coth^2(c+dx)+b^{2/3}}{b^2\coth^2(c+dx)+b^{5/3}\coth(c+dx)+b^{4/3}}d(b^2\coth^2(c+dx))-3\int \frac{1}{-2\sqrt[3]{b}\coth(c+dx)-4}d\left(2\sqrt[3]{b}\coth(c+dx)+1\right)}{3b^{2/3}}-\frac{\log(b^{2/3}-b^2\coth^2(}{3b^{2/3}}\right.}{2d}$$

$$\Big\downarrow \; 217$$

$$\frac{3b\left(-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{b}\coth(c+dx)+1}{\sqrt{3}}\right)-\frac{1}{2}\int \frac{2b^2\coth^2(c+dx)+b^{2/3}}{b^2\coth^2(c+dx)+b^{5/3}\coth(c+dx)+b^{4/3}}d(b^2\coth^2(c+dx))}{3b^{2/3}}-\frac{\log(b^{2/3}-b^2\coth^2(c+dx))}{3b^{2/3}}\right)}{2d}$$

$$\Big\downarrow \; 1103$$

$$\frac{3b\left(-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{b}\coth(c+dx)+1}{\sqrt{3}}\right)-\frac{1}{2}\log(b^{5/3}\coth(c+dx)+b^{4/3}+b^2\coth^2(c+dx))}{3b^{2/3}}-\frac{\log(b^{2/3}-b^2\coth^2(c+dx))}{3b^{2/3}}\right)}{2d}$$

input
```
Int[(b*Coth[c + d*x])^(1/3),x]
```

output
```
(3*b*(-1/3*Log[b^(2/3) - b^2*Coth[c + d*x]^2]/b^(2/3) - (Sqrt[3]*ArcTan[(1
 + 2*b^(1/3)*Coth[c + d*x])/Sqrt[3]] - Log[b^(4/3) + b^(5/3)*Coth[c + d*x]
 + b^2*Coth[c + d*x]^2]/2)/(3*b^(2/3))))/(2*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.83

method	result
derivativedivides	$-\frac{3b\left(\frac{\ln\left((b\coth(dx+c))^{\frac{2}{3}}-\left(b^2\right)^{\frac{1}{3}}\right)}{6\left(b^2\right)^{\frac{1}{3}}}-\frac{\ln\left((b\coth(dx+c))^{\frac{4}{3}}+\left(b^2\right)^{\frac{1}{3}}(b\coth(dx+c))^{\frac{2}{3}}+\left(b^2\right)^{\frac{2}{3}}\right)}{12\left(b^2\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\coth(dx+}{\left(b^2\right)^{\frac{1}{3}}}\right.}{3}\right.}{6\left(b^2\right)^{\frac{1}{3}}}\right.}{d}$
default	$-\frac{3b\left(\frac{\ln\left((b\coth(dx+c))^{\frac{2}{3}}-\left(b^2\right)^{\frac{1}{3}}\right)}{6\left(b^2\right)^{\frac{1}{3}}}-\frac{\ln\left((b\coth(dx+c))^{\frac{4}{3}}+\left(b^2\right)^{\frac{1}{3}}(b\coth(dx+c))^{\frac{2}{3}}+\left(b^2\right)^{\frac{2}{3}}\right)}{12\left(b^2\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\coth(dx+}{\left(b^2\right)^{\frac{1}{3}}}\right.}{3}\right.}{6\left(b^2\right)^{\frac{1}{3}}}\right.}{d}$

input
```
int((b*coth(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/d*b*(1/6/(b^2)^(1/3)*ln((b*coth(d*x+c))^(2/3)-(b^2)^(1/3))-1/12/(b^2)^(
1/3)*ln((b*coth(d*x+c))^(4/3)+(b^2)^(1/3)*(b*coth(d*x+c))^(2/3)+(b^2)^(2/3
))+1/6*3^(1/2)/(b^2)^(1/3)*arctan(1/3*3^(1/2)*(2/(b^2)^(1/3)*(b*coth(d*x+c
))^(2/3)+1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(99) = 198$.

Time = 0.15 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.20

$$\int \sqrt[3]{b\coth(c+dx)}\,dx =$$

$$-\frac{2\sqrt{3}(-b)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}(-b)^{\frac{1}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{2}{3}}}{3b}\right)-2(-b)^{\frac{1}{3}}\log\left(-(-b)^{\frac{2}{3}}+\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{2}{3}}\right)+(-b)^{\frac{1}{3}}}{}$$

input
```
integrate((b*coth(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(3)*(-b)^(1/3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*(-b)^(1/3)*(
b*cosh(d*x + c)/sinh(d*x + c))^(2/3))/b) - 2*(-b)^(1/3)*log(-(-b)^(2/3) +
(b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + (-b)^(1/3)*log(((cosh(d*x + c)^2
+ 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(
d*x + c)/sinh(d*x + c))^(2/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sin
h(d*x + c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3) + (b*cosh(d*x + c)^2 + 2*b*
cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*(b*cosh(d*x + c)/sinh
(d*x + c))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(
d*x + c)^2 - 1)))/d
```

Sympy [F]

$$\int \sqrt[3]{b\coth(c+dx)}\,dx = \int \sqrt[3]{b\coth(c+dx)}\,dx$$

input
```
integrate((b*coth(d*x+c))**(1/3),x)
```

output
```
Integral((b*coth(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{b\coth(c+dx)}\,dx = \int (b\coth(dx+c))^{\frac{1}{3}}\,dx$$

input
```
integrate((b*coth(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c))^(1/3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(99) = 198$.

Time = 0.21 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.64

$$\int \sqrt[3]{b\coth(c+dx)}\,dx = -\frac{b\left(\frac{2\sqrt{3}|b|^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(\frac{be^{(2\,dx+2\,c)}+b}{e^{(2\,dx+2\,c)}-1}\right)^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{3\,|b|^{\frac{2}{3}}}\right)}{b^2} - \frac{|b|^{\frac{4}{3}}\log\left(\left(\frac{be^{(2\,dx+2\,c)}+b}{e^{(2\,dx+2\,c)}-1}\right)^{\frac{2}{3}}|b|^{\frac{2}{3}}+|b|^{\frac{4}{3}}+\frac{\left(be^{(2\,dx+2\,c)}+b\right)\left(\frac{be^{(2\,dx+2\,c)}+b}{e^{(2\,dx+2\,c)}-1}\right)^{\frac{1}{3}}}{e^{(2\,dx+2\,c)}-1}\right.}{b^2}}{4\,d}$$

input
```
integrate((b*coth(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
-1/4*b*(2*sqrt(3)*abs(b)^(4/3)*arctan(1/3*sqrt(3)*(2*((b*e^(2*d*x + 2*c) +
 b)/(e^(2*d*x + 2*c) - 1))^(2/3) + abs(b)^(2/3))/abs(b)^(2/3))/b^2 - abs(b
)^(4/3)*log(((b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(2/3)*abs(b)^(
2/3) + abs(b)^(4/3) + (b*e^(2*d*x + 2*c) + b)*((b*e^(2*d*x + 2*c) + b)/(e^
(2*d*x + 2*c) - 1))^(1/3)/(e^(2*d*x + 2*c) - 1))/b^2 + 2*abs(b)^(4/3)*log(
abs(((b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(2/3) - abs(b)^(2/3)))
/b^2)/d
```

Mupad [B] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

$$\int \sqrt[3]{b\coth(c+dx)}\,dx$$
$$= \frac{(-b)^{1/3}\ln\left(81\,(-b)^{16/3}\,(b\coth(c+d\,x))^{2/3}-81\,b^6\right)}{2\,d} - \frac{(-b)^{1/3}\ln\left(-\frac{81\,b^6}{d^4}-\frac{81\,(-b)^{16/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(b\coth(c+d\,x))^{2/3}}{d^4}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d} + \frac{(-b)^{1/3}\ln\left(-\frac{81\,b^6}{d^4}+\frac{162\,(-b)^{16/3}\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)(b\coth(c+d\,x))^{2/3}}{d^4}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}$$

input
```
int((b*coth(c + d*x))^(1/3),x)
```

output
```
((-b)^(1/3)*log(81*(-b)^(16/3)*(b*coth(c + d*x))^(2/3) - 81*b^6))/(2*d) -
((-b)^(1/3)*log(- (81*b^6)/d^4 - (81*(-b)^(16/3)*((3^(1/2)*1i)/2 + 1/2)*(b
*coth(c + d*x))^(2/3))/d^4)*((3^(1/2)*1i)/2 + 1/2))/(2*d) + ((-b)^(1/3)*lo
g((162*(-b)^(16/3)*((3^(1/2)*1i)/4 - 1/4)*(b*coth(c + d*x))^(2/3))/d^4 - (
81*b^6)/d^4)*((3^(1/2)*1i)/4 - 1/4))/d
```

Reduce [F]

$$\int \sqrt[3]{b\coth(c+dx)}\,dx = b^{\frac{1}{3}}\left(\int \coth(dx+c)^{\frac{1}{3}}\,dx\right)$$

input
```
int((b*coth(d*x+c))^(1/3),x)
```

output
```
b**(1/3)*int(coth(c + d*x)**(1/3),x)
```

3.12 $\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 132

$$\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{b^{2/3}+2(b\coth(c+dx))^{2/3}}{\sqrt{3}b^{2/3}}\right)}{2\sqrt[3]{b}d} - \frac{\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d} + \frac{\log\left(b^{4/3}+b^{2/3}(b\coth(c+dx))^{2/3}+(b\coth(c+dx))^{4/3}\right)}{4\sqrt[3]{b}d}$$

output
```
1/2*3^(1/2)*arctan(1/3*(b^(2/3)+2*(b*coth(d*x+c))^(2/3))*3^(1/2)/b^(2/3))/
b^(1/3)/d-1/2*ln(b^(2/3)-(b*coth(d*x+c))^(2/3))/b^(1/3)/d+1/4*ln(b^(4/3)+b
^(2/3)*(b*coth(d*x+c))^(2/3)+(b*coth(d*x+c))^(4/3))/b^(1/3)/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx = \frac{\sqrt[3]{\coth(c+dx)}\left(2\sqrt{3}\arctan\left(\frac{1+2\coth^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)-2\log\left(1-\coth^{\frac{2}{3}}(c+dx)\right)+\log\left(1+\coth^{\frac{2}{3}}(c+dx)+\right.\right.}{4d\sqrt[3]{b\coth(c+dx)}}$$

input
```
Integrate[(b*Coth[c + d*x])^(-1/3),x]
```

output
```
(Coth[c + d*x]^(1/3)*(2*Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(2/3))/Sqrt[3]
] - 2*Log[1 - Coth[c + d*x]^(2/3)] + Log[1 + Coth[c + d*x]^(2/3) + Coth[c
+ d*x]^(4/3)]))/(4*d*(b*Coth[c + d*x])^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.84, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3957, 25, 266, 807, 750, 16, 1142, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{-ib\tan\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{b\int -\frac{1}{\sqrt[3]{b\coth(c+dx)}(b^2-b^2\coth^2(c+dx))}d(b\coth(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\int \frac{1}{\sqrt[3]{b\coth(c+dx)}\left(b^2-b^2\coth^2(c+dx)\right)}d(b\coth(c+dx))}{d}$$

$\big\downarrow$ 266

$$\frac{3b\int \frac{\sqrt[3]{b\coth(c+dx)}}{b^2-b^6\coth^6(c+dx)}d\sqrt[3]{b\coth(c+dx)}}{d}$$

$\big\downarrow$ 807

$$\frac{3b\int \frac{1}{b^2-b^3\coth^3(c+dx)}d\left(b^2\coth^2(c+dx)\right)}{2d}$$

$\big\downarrow$ 750

$$\frac{3b\left(\frac{\int \frac{1}{b^{2/3}-b^2\coth^2(c+dx)}d\left(b^2\coth^2(c+dx)\right)}{3b^{4/3}}+\frac{\int \frac{b^2\coth^2(c+dx)+2b^{2/3}}{b^2\coth^2(c+dx)+b^{5/3}\coth(c+dx)+b^{4/3}}d\left(b^2\coth^2(c+dx)\right)}{3b^{4/3}}\right)}{2d}$$

$\big\downarrow$ 16

$$\frac{3b\left(\frac{\int \frac{b^2\coth^2(c+dx)+2b^{2/3}}{b^2\coth^2(c+dx)+b^{5/3}\coth(c+dx)+b^{4/3}}d\left(b^2\coth^2(c+dx)\right)}{3b^{4/3}}-\frac{\log\left(b^{2/3}-b^2\coth^2(c+dx)\right)}{3b^{4/3}}\right)}{2d}$$

$\big\downarrow$ 1142

$$\frac{3b\left(\frac{\frac{3}{2}b^{2/3}\int \frac{1}{b^2\coth^2(c+dx)+b^{5/3}\coth(c+dx)+b^{4/3}}d\left(b^2\coth^2(c+dx)\right)+\frac{1}{2}\int \frac{2b^2\coth^2(c+dx)+b^{2/3}}{b^2\coth^2(c+dx)+b^{5/3}\coth(c+dx)+b^{4/3}}d\left(b^2\coth^2(c+dx)\right)}{3b^{4/3}}-\frac{\log\left(b^{2/3}-\right.}{}\right.}{2d}$$

$\big\downarrow$ 1082

$$\frac{3b\left(\frac{\frac{1}{2}\int \frac{2b^2\coth^2(c+dx)+b^{2/3}}{b^2\coth^2(c+dx)+b^{5/3}\coth(c+dx)+b^{4/3}}d\left(b^2\coth^2(c+dx)\right)-3\int \frac{1}{-2\sqrt[3]{b}\coth(c+dx)-4}d\left(2\sqrt[3]{b}\coth(c+dx)+1\right)}{3b^{4/3}}-\frac{\log\left(b^{2/3}-b^2\coth^2(c+d\right.}{3b^{4/3}}\right.}{2d}$$

$\big\downarrow$ 217

$$\frac{3b\left(\frac{\frac{1}{2}\int \frac{2b^2\coth^2(c+dx)+b^{2/3}}{b^2\coth^2(c+dx)+b^{5/3}\coth(c+dx)+b^{4/3}}d(b^2\coth^2(c+dx))+\sqrt{3}\arctan\left(\frac{2\sqrt[3]{b}\coth(c+dx)+1}{\sqrt{3}}\right)}{3b^{4/3}}-\frac{\log(b^{2/3}-b^2\coth^2(c+dx))}{3b^{4/3}}\right)}{2d}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{3b\left(\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{b}\coth(c+dx)+1}{\sqrt{3}}\right)+\frac{1}{2}\log(b^{5/3}\coth(c+dx)+b^{4/3}+b^2\coth^2(c+dx))}{3b^{4/3}}-\frac{\log(b^{2/3}-b^2\coth^2(c+dx))}{3b^{4/3}}\right)}{2d}$$

input
```
Int[(b*Coth[c + d*x])^(-1/3),x]
```

output
```
(3*b*(-1/3*Log[b^(2/3) - b^2*Coth[c + d*x]^2]/b^(4/3) + (Sqrt[3]*ArcTan[(1
 + 2*b^(1/3)*Coth[c + d*x])/Sqrt[3]] + Log[b^(4/3) + b^(5/3)*Coth[c + d*x]
 + b^2*Coth[c + d*x]^2]/2)/(3*b^(4/3))))/(2*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.83

method	result
derivativedivides	$-\frac{3b\left(\frac{\ln\left((b\coth(dx+c))^{\frac{2}{3}}-(b^2)^{\frac{1}{3}}\right)}{6(b^2)^{\frac{2}{3}}}-\frac{\ln\left((b\coth(dx+c))^{\frac{4}{3}}+(b^2)^{\frac{1}{3}}(b\coth(dx+c))^{\frac{2}{3}}+(b^2)^{\frac{2}{3}}\right)}{12(b^2)^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\coth(dx+}{(b^2)^{\frac{1}{3}}}\right.}{3}\right.}{6(b^2)^{\frac{2}{3}}}\right.}{d}$
default	$-\frac{3b\left(\frac{\ln\left((b\coth(dx+c))^{\frac{2}{3}}-(b^2)^{\frac{1}{3}}\right)}{6(b^2)^{\frac{2}{3}}}-\frac{\ln\left((b\coth(dx+c))^{\frac{4}{3}}+(b^2)^{\frac{1}{3}}(b\coth(dx+c))^{\frac{2}{3}}+(b^2)^{\frac{2}{3}}\right)}{12(b^2)^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\coth(dx+}{(b^2)^{\frac{1}{3}}}\right.}{3}\right.}{6(b^2)^{\frac{2}{3}}}\right.}{d}$

input
```
int(1/(b*coth(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/d*b*(1/6/(b^2)^(2/3)*ln((b*coth(d*x+c))^(2/3)-(b^2)^(1/3))-1/12/(b^2)^(
2/3)*ln((b*coth(d*x+c))^(4/3)+(b^2)^(1/3)*(b*coth(d*x+c))^(2/3)+(b^2)^(2/3
))-1/6/(b^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(b^2)^(1/3)*(b*coth(d*x+c
))^(2/3)+1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(99) = 198.

Time = 0.13 (sec) , antiderivative size = 1187, normalized size of antiderivative = 8.99

$$\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*coth(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(3)*b*sqrt((-b)^(1/3)/b)*log((3*b*cosh(d*x + c)^4 + 12*b*cosh(d*
x + c)*sinh(d*x + c)^3 + 3*b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(9*
b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 3*(cosh(d*x + c)^4 + 4*cosh(d*x +
 c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x
 + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x
 + c) + 1)*(-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) - sqrt(3)*((co
sh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*c
osh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)
^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*
x + c))^(2/3) - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b
*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(
d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*(-
b)^(1/3) - 2*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*
cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*si
nh(d*x + c)^4 - b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))*sqrt((-b)^(1/3)/
b) + 4*(3*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + 3*b)/(cosh(
d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*(-b)^(2
/3)*log(-(-b)^(2/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + (-b)^(2/3)*
log(((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 -
1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) - (b*cosh(d*x + c)^...
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{b\coth\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c))**(1/3),x)
```

output
```
Integral((b*coth(c + d*x))**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c))^(-1/3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(99) = 198.

Time = 0.20 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.64

$$\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx = \frac{b\left(\frac{2\sqrt{3}|b|^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(\frac{be^{(2\,dx+2\,c)}+b}{e^{(2\,dx+2\,c)}-1}\right)^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{3\,|b|^{\frac{2}{3}}}\right)}{b^2}+\frac{|b|^{\frac{2}{3}}\log\left(\left(\frac{be^{(2\,dx+2\,c)}+b}{e^{(2\,dx+2\,c)}-1}\right)^{\frac{2}{3}}|b|^{\frac{2}{3}}+|b|^{\frac{4}{3}}+\frac{\left(be^{(2\,dx+2\,c)}+b\right)\left(\frac{be^{(2\,dx+2\,c)}+b}{e^{(2\,dx+2\,c)}-1}\right)^{\frac{1}{3}}}{e^{(2\,dx+2\,c)}-1}\right)}{b^2} - \cdots\right)}{4\,d}$$

input
```
integrate(1/(b*coth(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
1/4*b*(2*sqrt(3)*abs(b)^(2/3)*arctan(1/3*sqrt(3)*(2*((b*e^(2*d*x + 2*c) +
b)/(e^(2*d*x + 2*c) - 1))^(2/3) + abs(b)^(2/3))/abs(b)^(2/3))/b^2 + abs(b)
^(2/3)*log(((b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(2/3)*abs(b)^(2
/3) + abs(b)^(4/3) + (b*e^(2*d*x + 2*c) + b)*((b*e^(2*d*x + 2*c) + b)/(e^(
2*d*x + 2*c) - 1))^(1/3)/(e^(2*d*x + 2*c) - 1))/b^2 - 2*abs(b)^(2/3)*log(a
bs(((b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(2/3) - abs(b)^(2/3)))/
b^2)/d
```

Mupad [B] (verification not implemented)

Time = 2.84 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx = \frac{\ln\left(162\,(-b)^{11/3}+162\,b^3\,(b\coth(c+d\,x))^{2/3}\right)}{2\,(-b)^{1/3}\,d} + \frac{\ln\left(\frac{81\,(-b)^{11/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{d^3}+\frac{162\,b^3\,(b\coth(c+d\,x))^{2/3}}{d^3}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4\,(-b)^{1/3}\,d} - \frac{\ln\left(\frac{81\,(-b)^{11/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{d^3}-\frac{162\,b^3\,(b\coth(c+d\,x))^{2/3}}{d^3}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4\,(-b)^{1/3}\,d}$$

input
```
int(1/(b*coth(c + d*x))^(1/3),x)
```

output
```
log(162*(-b)^(11/3) + 162*b^3*(b*coth(c + d*x))^(2/3))/(2*(-b)^(1/3)*d) +
(log((81*(-b)^(11/3)*(3^(1/2)*1i - 1))/d^3 + (162*b^3*(b*coth(c + d*x))^(2
/3))/d^3)*(3^(1/2)*1i - 1))/(4*(-b)^(1/3)*d) - (log((81*(-b)^(11/3)*(3^(1/
2)*1i + 1))/d^3 - (162*b^3*(b*coth(c + d*x))^(2/3))/d^3)*(3^(1/2)*1i + 1))
/(4*(-b)^(1/3)*d)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx = \frac{\int \frac{1}{\coth(dx+c)^{\frac{1}{3}}}dx}{b^{\frac{1}{3}}}$$

input
```
int(1/(b*coth(d*x+c))^(1/3),x)
```

output
```
int(1/coth(c + d*x)**(1/3),x)/b**(1/3)
```

3.13 $\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 178

$$\begin{aligned}\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx &= -\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{b\coth(c+dx)}}{\sqrt{3}\sqrt[3]{b}}\right)}{2b^{2/3}d} \\ &+ \frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{b}+2\sqrt[3]{b\coth(c+dx)}}{\sqrt{3}\sqrt[3]{b}}\right)}{2b^{2/3}d} \\ &+ \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}}{b^{2/3}+(b\coth(c+dx))^{2/3}}\right)}{2b^{2/3}d}\end{aligned}$$

output
```
-1/2*3^(1/2)*arctan(1/3*(b^(1/3)-2*(b*coth(d*x+c))^(1/3))*3^(1/2)/b^(1/3))
/b^(2/3)/d+1/2*3^(1/2)*arctan(1/3*(b^(1/3)+2*(b*coth(d*x+c))^(1/3))*3^(1/2
)/b^(1/3))/b^(2/3)/d+arctanh((b*coth(d*x+c))^(1/3)/b^(1/3))/b^(2/3)/d+1/2*
arctanh(b^(1/3)*(b*coth(d*x+c))^(1/3)/(b^(2/3)+(b*coth(d*x+c))^(2/3)))/b^(
2/3)/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.99

$$\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx =$$

$$-\frac{\sqrt[3]{b\coth(c+dx)}\left(\log\left(1-\sqrt[6]{\coth^2(c+dx)}\right)-\log\left(1+\sqrt[6]{\coth^2(c+dx)}\right)+\sqrt[3]{-1}\left(-\sqrt[3]{-1}\log\left(1-\sqrt[3]{\right.\right.\right.}{}$$

input
```
Integrate[(b*Coth[c + d*x])^(-2/3),x]
```

output
```
-1/2*((b*Coth[c + d*x])^(1/3)*(Log[1 - (Coth[c + d*x]^2)^(1/6)] - Log[1 +
(Coth[c + d*x]^2)^(1/6)] + (-1)^(1/3)*(-((-1)^(1/3)*Log[1 - (-1)^(1/3)*(Co
th[c + d*x]^2)^(1/6)]) + (-1)^(1/3)*Log[1 + (-1)^(1/3)*(Coth[c + d*x]^2)^(
1/6)] - Log[1 - (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)] + Log[1 + (-1)^(2/3)*(
Coth[c + d*x]^2)^(1/6)])))/(b*d*(Coth[c + d*x]^2)^(1/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3957, 25, 266, 754, 27, 219, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{b\int -\frac{1}{(b\coth(c+dx))^{2/3}(b^2-b^2\coth^2(c+dx))}d(b\coth(c+dx))}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{b\int \frac{1}{(b\coth(c+dx))^{2/3}(b^2-b^2\coth^2(c+dx))}d(b\coth(c+dx))}{d}$$

$$\Big\downarrow \; 266$$

$$\frac{3b\int \frac{1}{b^2-b^6\coth^6(c+dx)}d\sqrt[3]{b\coth(c+dx)}}{d}$$

$$\Big\downarrow \; 754$$

$$\frac{3b\left(\frac{\int \frac{1}{b^{2/3}-b^2\coth^2(c+dx)}d\sqrt[3]{b\coth(c+dx)}}{3b^{4/3}}+\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{b\coth(c+dx)}}{2\left(b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}\right)}d\sqrt[3]{b\coth(c+dx)}}{3b^{5/3}}+\frac{\int \frac{2\sqrt[3]{b}+\sqrt[3]{b\coth}}{2\left(b^2\coth^2(c+dx)+b\right.}}{}\right.}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{3b\left(\frac{\int \frac{1}{b^{2/3}-b^2\coth^2(c+dx)}d\sqrt[3]{b\coth(c+dx)}}{3b^{4/3}}+\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6b^{5/3}}+\frac{\int \frac{2\sqrt[3]{b}+\sqrt[3]{b\coth}}{b^2\coth^2(c+dx)+b^{4/3}\mathrm{co}}}{}\right.}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{3b\left(\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6b^{5/3}}+\frac{\int \frac{2\sqrt[3]{b}+\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)+b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6b^{5/3}}+\frac{\mathrm{arctan}}{}\right.}{d}$$

$$\Big\downarrow \; 1142$$

$$3b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}-\frac{1}{2}\int -\frac{\sqrt[3]{b}-2\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6b^{5/3}}+\right.$$

$$\Big\downarrow \; 25$$

$$3b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}+\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6b^{5/3}}+\frac{\frac{3}{2}\cdots}{}\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{3b\left(\frac{3\int \frac{1}{-b^2\coth^2(c+dx)-3}d(1-2b^{2/3}\coth(c+dx))+\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6b^{5/3}}+\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}+2\sqrt[3]{b\,\mathrm{co}\cdots}}{b^2\coth^2(c+dx)+b^{4/3}\cdots}}{}\right.}{d}$$

$$\Big\downarrow \text{217}$$

$$\frac{3b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}-\sqrt{3}\arctan\left(\frac{1-2b^{2/3}\coth(c+dx)}{\sqrt{3}}\right)}{6b^{5/3}}+\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}+2\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)+b^{4/3}\coth(c+dx)+b\cdots}}{}\right.}{d}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3b\left(\frac{-\sqrt{3}\arctan\left(\frac{1-2b^{2/3}\coth(c+dx)}{\sqrt{3}}\right)-\frac{1}{2}\log\left(-b^{4/3}\coth(c+dx)+b^{2/3}+b^2\coth^2(c+dx)\right)}{6b^{5/3}}+\frac{\sqrt{3}\arctan\left(\frac{2b^{2/3}\coth(c+dx)+1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(b^{4/3}\coth\cdots\right.}{6b^{5/3}}\right.}{d}$$

input
```
Int[(b*Coth[c + d*x])^(-2/3),x]
```

output
```
(3*b*(ArcTanh[b^(2/3)*Coth[c + d*x]]/(3*b^(5/3)) + (-(Sqrt[3]*ArcTan[(1 - 
2*b^(2/3)*Coth[c + d*x])/Sqrt[3]]) - Log[b^(2/3) - b^(4/3)*Coth[c + d*x] + 
 b^2*Coth[c + d*x]^2]/2)/(6*b^(5/3)) + (Sqrt[3]*ArcTan[(1 + 2*b^(2/3)*Coth
[c + d*x])/Sqrt[3]] + Log[b^(2/3) + b^(4/3)*Coth[c + d*x] + b^2*Coth[c + d
*x]^2]/2)/(6*b^(5/3))))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{\ln\left(\left(b\coth(dx+c)\right)^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{2d\,b^{\frac{2}{3}}}-\frac{\ln\left(\left(b\coth(dx+c)\right)^{\frac{2}{3}}-b^{\frac{1}{3}}\left(b\coth(dx+c)\right)^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{4d\,b^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\coth(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right.}{3}\right.}{2d\,b^{\frac{2}{3}}}$
default	$\frac{\ln\left(\left(b\coth(dx+c)\right)^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{2d\,b^{\frac{2}{3}}}-\frac{\ln\left(\left(b\coth(dx+c)\right)^{\frac{2}{3}}-b^{\frac{1}{3}}\left(b\coth(dx+c)\right)^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{4d\,b^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\coth(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right.}{3}\right.}{2d\,b^{\frac{2}{3}}}$

input
```
int(1/(b*coth(d*x+c))^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/2/d/b^(2/3)*ln((b*coth(d*x+c))^(1/3)+b^(1/3))-1/4/d/b^(2/3)*ln((b*coth(d
*x+c))^(2/3)-b^(1/3)*(b*coth(d*x+c))^(1/3)+b^(2/3))+1/2/d/b^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2*(b*coth(d*x+c))^(1/3)/b^(1/3)-1))-1/2/d/b^(2/3)*ln((
b*coth(d*x+c))^(1/3)-b^(1/3))+1/4/d/b^(2/3)*ln((b*coth(d*x+c))^(2/3)+b^(1/
3)*(b*coth(d*x+c))^(1/3)+b^(2/3))+1/2/d/b^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)
*(2*(b*coth(d*x+c))^(1/3)/b^(1/3)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(136) = 272$.

Time = 0.15 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.00

$$\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx = \frac{2\sqrt{3}b\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}b\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}-2\sqrt{3}\left(-b^2\right)^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}}{3b^2}\right.}{}$$

input
```
integrate(1/(b*coth(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(3)*b*sqrt(-(-b^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*s
qrt(-(-b^2)^(1/3)) - 2*sqrt(3)*(-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3)*sqrt(-(-b^2)^(1/3)))/b^2) + 2*sqrt(3)*(b^2)^(1/6)*b*arctan(-1/3*sq
rt(3)*(b^2)^(1/6)*((b^2)^(1/3)*b - 2*(b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x
 + c))^(1/3))/b^2) + (-b^2)^(2/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2
/3) - (-b^2)^(1/3)*b + (-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))
 - (b^2)^(2/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) + (b^2)^(1/3)*b
 - (b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(-b^2)^(2/3)*log
(b*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - (-b^2)^(2/3)) + 2*(b^2)^(2/3)*l
og(b*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b^2)^(2/3)))/(b^2*d)
```

Sympy [F]

$$\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\coth{(c+dx)})^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c))**(2/3),x)
```

output
```
Integral((b*coth(c + d*x))**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\coth{(dx+c)})^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c))^(-2/3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*coth(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Minimal poly. in rootof must be fra
ction free Error: Bad Argument ValueMinimal poly. in rootof must be fracti
on free E
```

Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.11

$$\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx = \frac{\operatorname{atanh}\left(\frac{(b\coth(c+dx))^{1/3}}{b^{1/3}}\right)}{b^{2/3}\,d} - \frac{\operatorname{atan}\left(\frac{b^{10/3}\,(b\coth(c+dx))^{1/3}\,243\mathrm{i}}{-243\,b^{11/3}+\sqrt{3}\,b^{11/3}\,243\mathrm{i}} - \frac{243\,\sqrt{3}\,b^{10/3}\,(b\coth(c+dx))^{1/3}}{-243\,b^{11/3}+\sqrt{3}\,b^{11/3}\,243\mathrm{i}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b^{2/3}\,d} - \frac{\operatorname{atan}\left(\frac{b^{10/3}\,(b\coth(c+dx))^{1/3}\,243\mathrm{i}}{243\,b^{11/3}+\sqrt{3}\,b^{11/3}\,243\mathrm{i}} + \frac{243\,\sqrt{3}\,b^{10/3}\,(b\coth(c+dx))^{1/3}}{243\,b^{11/3}+\sqrt{3}\,b^{11/3}\,243\mathrm{i}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b^{2/3}\,d}$$

input
```
int(1/(b*coth(c + d*x))^(2/3),x)
```

output
```
atanh((b*coth(c + d*x))^(1/3)/b^(1/3))/(b^(2/3)*d) - (atan((b^(10/3)*(b*co
th(c + d*x))^(1/3)*243i)/(3^(1/2)*b^(11/3)*243i - 243*b^(11/3)) - (243*3^(
1/2)*b^(10/3)*(b*coth(c + d*x))^(1/3))/(3^(1/2)*b^(11/3)*243i - 243*b^(11/
3)))*(3^(1/2)*1i + 1)*1i)/(2*b^(2/3)*d) - (atan((b^(10/3)*(b*coth(c + d*x)
)^(1/3)*243i)/(3^(1/2)*b^(11/3)*243i + 243*b^(11/3)) + (243*3^(1/2)*b^(10/
3)*(b*coth(c + d*x))^(1/3))/(3^(1/2)*b^(11/3)*243i + 243*b^(11/3)))*(3^(1/
2)*1i - 1)*1i)/(2*b^(2/3)*d)
```

Reduce [F]

$$\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx = \frac{\int \frac{1}{\coth(dx+c)^{\frac{2}{3}}}dx}{b^{\frac{2}{3}}}$$

input
```
int(1/(b*coth(d*x+c))^(2/3),x)
```

output
```
int(1/coth(c + d*x)**(2/3),x)/b**(2/3)
```

3.14 $\int \frac{1}{(b\coth(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 198

$$\int \frac{1}{(b\coth(c+dx))^{4/3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{b\coth(c+dx)}}{\sqrt{3}\sqrt[3]{b}}\right)}{2b^{4/3}d} - \frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{b}+2\sqrt[3]{b\coth(c+dx)}}{\sqrt{3}\sqrt[3]{b}}\right)}{2b^{4/3}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}}{b^{2/3}+(b\coth(c+dx))^{2/3}}\right)}{2b^{4/3}d} - \frac{3}{bd\sqrt[3]{b\coth(c+dx)}}$$

output
```
1/2*3^(1/2)*arctan(1/3*(b^(1/3)-2*(b*coth(d*x+c))^(1/3))*3^(1/2)/b^(1/3))/
b^(4/3)/d-1/2*3^(1/2)*arctan(1/3*(b^(1/3)+2*(b*coth(d*x+c))^(1/3))*3^(1/2)
/b^(1/3))/b^(4/3)/d+arctanh((b*coth(d*x+c))^(1/3)/b^(1/3))/b^(4/3)/d+1/2*a
rctanh(b^(1/3)*(b*coth(d*x+c))^(1/3)/(b^(2/3)+(b*coth(d*x+c))^(2/3)))/b^(4
/3)/d-3/b/d/(b*coth(d*x+c))^(1/3)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.23

$$\int \frac{1}{(b\coth(c+dx))^{4/3}}\,dx =$$
$$-\frac{6+\sqrt[6]{\coth^2(c+dx)}\log\left(1-\sqrt[6]{\coth^2(c+dx)}\right)-\sqrt[6]{\coth^2(c+dx)}\log\left(1+\sqrt[6]{\coth^2(c+dx)}\right)+\sqrt[3]{-1}\,\sqrt[}{}$$

input
```
Integrate[(b*Coth[c + d*x])^(-4/3),x]
```

output
```
-1/2*(6 + (Coth[c + d*x]^2)^(1/6)*Log[1 - (Coth[c + d*x]^2)^(1/6)] - (Coth
[c + d*x]^2)^(1/6)*Log[1 + (Coth[c + d*x]^2)^(1/6)] + (-1)^(1/3)*(Coth[c +
 d*x]^2)^(1/6)*Log[1 - (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)] - (-1)^(1/3)*(C
oth[c + d*x]^2)^(1/6)*Log[1 + (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)] + (-1)^(
2/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 - (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)] -
 (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 + (-1)^(2/3)*(Coth[c + d*x]^2)^(
1/6)])/(b*d*(b*Coth[c + d*x])^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$, Rules used = {3042, 3955, 3042, 3957, 25, 266, 825, 27, 219, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\coth(c+dx))^{4/3}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\left(-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$
$$\Big\downarrow \text{3955}$$

$$\frac{\int (b\coth(c+dx))^{2/3}dx}{b^2}-\frac{3}{bd\sqrt[3]{b\coth(c+dx)}}$$

$\big\downarrow$ 3042

$$-\frac{3}{bd\sqrt[3]{b\coth(c+dx)}}+\frac{\int \left(-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b^2}$$

$\big\downarrow$ 3957

$$-\frac{\int -\frac{(b\coth(c+dx))^{2/3}}{b^2-b^2\coth^2(c+dx)}d(b\coth(c+dx))}{bd}-\frac{3}{bd\sqrt[3]{b\coth(c+dx)}}$$

$\big\downarrow$ 25

$$\frac{\int \frac{(b\coth(c+dx))^{2/3}}{b^2-b^2\coth^2(c+dx)}d(b\coth(c+dx))}{bd}-\frac{3}{bd\sqrt[3]{b\coth(c+dx)}}$$

$\big\downarrow$ 266

$$\frac{3\int \frac{b^4\coth^4(c+dx)}{b^2-b^6\coth^6(c+dx)}d\sqrt[3]{b\coth(c+dx)}}{bd}-\frac{3}{bd\sqrt[3]{b\coth(c+dx)}}$$

$\big\downarrow$ 825

$$\frac{3\left(\frac{1}{3}\int \frac{1}{b^{2/3}-b^2\coth^2(c+dx)}d\sqrt[3]{b\coth(c+dx)}+\frac{\int -\frac{\sqrt[3]{b}-\sqrt[3]{b\coth(c+dx)}}{2\left(b^2\coth^2(c+dx)+b^{4/3}\coth(c+dx)+b^{2/3}\right)}d\sqrt[3]{b\coth(c+dx)}}{3\sqrt[3]{b}}+\frac{\int -\frac{\sqrt[3]{b}+}{2\left(b^2\coth^2(c\right.}}{}\right.}{bd}$$

$$\frac{3}{bd\sqrt[3]{b\coth(c+dx)}}$$

$\big\downarrow$ 27

$$\frac{3\left(\frac{1}{3}\int \frac{1}{b^{2/3}-b^2\coth^2(c+dx)}d\sqrt[3]{b\coth(c+dx)}-\frac{\int \frac{\sqrt[3]{b}-\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)+b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6\sqrt[3]{b}}-\frac{\int \frac{\sqrt[3]{b}+\sqrt[3]{b\,\mathrm{co}}}{b^2\coth^2(c+dx)-b^{4/}}}{}\right.}{bd}$$

$$\frac{3}{bd\sqrt[3]{b\coth(c+dx)}}$$

$\big\downarrow$ 219

$$\frac{3\left(-\frac{\int \frac{\sqrt[3]{b}-\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)+b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6\sqrt[3]{b}}-\frac{\int \frac{\sqrt[3]{b}+\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6\sqrt[3]{b}}+\text{arcta}\right.}{bd}$$

$$\frac{3}{bd\sqrt[3]{b\coth(c+dx)}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{3\left(-\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}+\frac{1}{2}\int -\frac{\sqrt[3]{b}-2\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6\sqrt[3]{b}}-\right.}{}$$

$$\frac{3}{bd\sqrt[3]{b\coth(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\left(-\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}-\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6\sqrt[3]{b}}-\right.}{}$$

$$\frac{3}{bd\sqrt[3]{b\coth(c+dx)}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3\left(-\frac{3\int \frac{1}{-b^2\coth^2(c+dx)-3}d\left(1-2b^{2/3}\coth(c+dx)\right)-\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}}{6\sqrt[3]{b}}-\frac{-3\int \frac{1}{-b^2\coth^2(c+dx)-}}{}\right.}{bd}$$

$$\frac{3}{bd\sqrt[3]{b\coth(c+dx)}}$$

$$\Big\downarrow \text{217}$$

$$\frac{3\left(-\frac{-\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{b\coth(c+dx)}}{b^2\coth^2(c+dx)-b^{4/3}\coth(c+dx)+b^{2/3}}d\sqrt[3]{b\coth(c+dx)}-\sqrt{3}\arctan\left(\frac{1-2b^{2/3}\coth(c+dx)}{\sqrt{3}}\right)}{6\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{2b^{2/3}\coth(c+dx)+1}{\sqrt{3}}\right)-}{}\right.}{bd}$$

$$\frac{3}{bd\sqrt[3]{b\coth(c+dx)}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3\left(-\frac{\frac{1}{2}\log\left(-b^{4/3}\coth(c+dx)+b^{2/3}+b^2\coth^2(c+dx)\right)-\sqrt{3}\arctan\left(\frac{1-2b^{2/3}\coth(c+dx)}{\sqrt{3}}\right)}{6\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{2b^{2/3}\coth(c+dx)+1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(b^{4/3}\coth\right.}{6\sqrt[3]{b}}\right.}{bd}$$

$$\frac{3}{bd\sqrt[3]{b\coth(c+dx)}}$$

input
```
Int[(b*Coth[c + d*x])^(-4/3),x]
```

output
```
-3/(b*d*(b*Coth[c + d*x])^(1/3)) + (3*(ArcTanh[b^(2/3)*Coth[c + d*x]]/(3*b
^(1/3)) - (-(Sqrt[3]*ArcTan[(1 - 2*b^(2/3)*Coth[c + d*x])/Sqrt[3]]) + Log[
b^(2/3) - b^(4/3)*Coth[c + d*x] + b^2*Coth[c + d*x]^2]/2)/(6*b^(1/3)) - (S
qrt[3]*ArcTan[(1 + 2*b^(2/3)*Coth[c + d*x])/Sqrt[3]] - Log[b^(2/3) + b^(4/
3)*Coth[c + d*x] + b^2*Coth[c + d*x]^2]/2)/(6*b^(1/3))))/(b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{\ln\left((b\coth(dx+c))^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{2d\,b^{\frac{4}{3}}}-\frac{\ln\left((b\coth(dx+c))^{\frac{2}{3}}-b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{4d\,b^{\frac{4}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\coth(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right.}{3}\right.}{2d\,b^{\frac{4}{3}}}$
default	$\frac{\ln\left((b\coth(dx+c))^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{2d\,b^{\frac{4}{3}}}-\frac{\ln\left((b\coth(dx+c))^{\frac{2}{3}}-b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{4d\,b^{\frac{4}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\coth(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right.}{3}\right.}{2d\,b^{\frac{4}{3}}}$

input
```
int(1/(b*coth(d*x+c))^(4/3),x,method=_RETURNVERBOSE)
```

output
```
1/2/d/b^(4/3)*ln((b*coth(d*x+c))^(1/3)+b^(1/3))-1/4/d/b^(4/3)*ln((b*coth(d
*x+c))^(2/3)-b^(1/3)*(b*coth(d*x+c))^(1/3)+b^(2/3))-1/2/d/b^(4/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2*(b*coth(d*x+c))^(1/3)/b^(1/3)-1))-3/b/d/(b*coth(d*x+
c))^(1/3)-1/2/d/b^(4/3)*ln((b*coth(d*x+c))^(1/3)-b^(1/3))+1/4/d/b^(4/3)*ln
((b*coth(d*x+c))^(2/3)+b^(1/3)*(b*coth(d*x+c))^(1/3)+b^(2/3))-1/2/d/b^(4/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*coth(d*x+c))^(1/3)/b^(1/3)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 593 vs. 2(154) = 308.

Time = 0.17 (sec) , antiderivative size = 3348, normalized size of antiderivative = 16.91

$$\int \frac{1}{(b\coth(c+dx))^{4/3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*coth(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(b\coth(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\coth{(c+dx)})^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c))**(4/3),x)
```

output
```
Integral((b*coth(c + d*x))**(-4/3), x)
```

Maxima [F]

$$\int \frac{1}{(b\coth(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\coth{(dx+c)})^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c))^(-4/3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(b\coth(c+dx))^{4/3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*coth(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Minimal poly. in rootof must be fra
ction free Error: Bad Argument ValueMinimal poly. in rootof must be fracti
on free E
```

Mupad [B] (verification not implemented)

Time = 3.00 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.83

$$\int \frac{1}{(b\coth(c+dx))^{4/3}}\,dx = -\frac{3}{b\,d\,(b\coth(c+d\,x))^{1/3}} - \frac{\operatorname{atan}\left(\frac{(b\coth(c+d\,x))^{1/3}\,1\mathrm{i}}{b^{1/3}}\right)\,1\mathrm{i}}{b^{4/3}\,d} - \frac{\operatorname{atan}\left(\frac{b^9\,d^4\,(b\coth(c+d\,x))^{1/3}\,486\mathrm{i}}{243\,b^{28/3}\,d^4-\sqrt{3}\,b^{28/3}\,d^4\,243\mathrm{i}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b^{4/3}\,d} + \frac{\operatorname{atan}\left(\frac{b^9\,d^4\,(b\coth(c+d\,x))^{1/3}\,486\mathrm{i}}{243\,b^{28/3}\,d^4+\sqrt{3}\,b^{28/3}\,d^4\,243\mathrm{i}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b^{4/3}\,d}$$

input
```
int(1/(b*coth(c + d*x))^(4/3),x)
```

output
```
(atan((b^9*d^4*(b*coth(c + d*x))^(1/3)*486i)/(243*b^(28/3)*d^4 + 3^(1/2)*b
^(28/3)*d^4*243i))*(3^(1/2)*1i - 1)*1i)/(2*b^(4/3)*d) - (atan(((b*coth(c +
 d*x))^(1/3)*1i)/b^(1/3))*1i)/(b^(4/3)*d) - (atan((b^9*d^4*(b*coth(c + d*x
))^(1/3)*486i)/(243*b^(28/3)*d^4 - 3^(1/2)*b^(28/3)*d^4*243i))*(3^(1/2)*1i
 + 1)*1i)/(2*b^(4/3)*d) - 3/(b*d*(b*coth(c + d*x))^(1/3))
```

Reduce [F]

$$\int \frac{1}{(b\coth(c+dx))^{4/3}}\,dx = \frac{\int \frac{1}{\coth(dx+c)^{\frac{4}{3}}}dx}{b^{\frac{4}{3}}}$$

input
```
int(1/(b*coth(d*x+c))^(4/3),x)
```

output
```
int(1/(coth(c + d*x)**(1/3)*coth(c + d*x)),x)/(b**(1/3)*b)
```

3.15 $\int \coth^n(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 43

$$\int \coth^n(a+bx)\,dx = \frac{\coth^{1+n}(a+bx)\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\coth^2(a+bx)\right)}{b(1+n)}$$

output
```
coth(b*x+a)^(1+n)*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],coth(b*x+a)^2)/b/(1
+n)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \coth^n(a+bx)\,dx = \frac{\coth^{1+n}(a+bx)\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\coth^2(a+bx)\right)}{b(1+n)}$$

input
```
Integrate[Coth[a + b*x]^n,x]
```

output
```
(Coth[a + b*x]^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Coth[a +
 b*x]^2])/(b*(1 + n))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3957, 25, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^n(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-i\tan\left(ia+ibx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{\int -\frac{\coth^n(a+bx)}{1-\coth^2(a+bx)}d\coth(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\coth^n(a+bx)}{1-\coth^2(a+bx)}d\coth(a+bx)}{b}$$

$$\Big\downarrow \text{278}$$

$$\frac{\coth^{n+1}(a+bx)\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},\coth^2(a+bx)\right)}{b(n+1)}$$

input
```
Int[Coth[a + b*x]^n,x]
```

output
```
(Coth[a + b*x]^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Coth[a +
 b*x]^2])/(b*(1 + n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [F]

$$\int \coth \left(bx+a\right)^{n}dx$$

input
```
int(coth(b*x+a)^n,x)
```

output
```
int(coth(b*x+a)^n,x)
```

Fricas [F]

$$\int \coth^n(a+bx)\,dx=\int \coth \left(bx+a\right)^{n}\,dx$$

input
```
integrate(coth(b*x+a)^n,x, algorithm="fricas")
```

output
```
integral(coth(b*x + a)^n, x)
```

Sympy [F]

$$\int \coth^n(a+bx)\,dx = \int \coth^n(a+bx)\,dx$$

input
```
integrate(coth(b*x+a)**n,x)
```

output
```
Integral(coth(a + b*x)**n, x)
```

Maxima [F]

$$\int \coth^n(a+bx)\,dx = \int \coth(bx+a)^n\,dx$$

input
```
integrate(coth(b*x+a)^n,x, algorithm="maxima")
```

output
```
integrate(coth(b*x + a)^n, x)
```

Giac [F]

$$\int \coth^n(a+bx)\,dx = \int \coth(bx+a)^n\,dx$$

input
```
integrate(coth(b*x+a)^n,x, algorithm="giac")
```

output
```
integrate(coth(b*x + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^n(a+bx)\,dx=\int \coth(a+b\,x)^n\,dx$$

input
```
int(coth(a + b*x)^n,x)
```

output
```
int(coth(a + b*x)^n, x)
```

Reduce [F]

$$\int \coth^n(a+bx)\,dx=\int \coth\left(bx+a\right)^n dx$$

input
```
int(coth(b*x+a)^n,x)
```

output
```
int(coth(a + b*x)**n,x)
```

3.16 $\int (b\coth(c+dx))^n\,dx$

Optimal result

Integrand size = 10, antiderivative size = 48

$$\int (b\coth(c+dx))^n\,dx = \frac{(b\coth(c+dx))^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\coth^2(c+dx)\right)}{bd(1+n)}$$

output
```
(b*coth(d*x+c))^(1+n)*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],coth(d*x+c)^2)/
b/d/(1+n)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int (b\coth(c+dx))^n\,dx = \frac{\coth(c+dx)(b\coth(c+dx))^n\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\coth^2(c+dx)\right)}{d(1+n)}$$

input
```
Integrate[(b*Coth[c + d*x])^n,x]
```

output
```
(Coth[c + d*x]*(b*Coth[c + d*x])^n*Hypergeometric2F1[1, (1 + n)/2, (3 + n)
/2, Coth[c + d*x]^2])/(d*(1 + n))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3957, 25, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\coth(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{b\int -\frac{(b\coth(c+dx))^n}{b^2-b^2\coth^2(c+dx)}d(b\coth(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\int \frac{(b\coth(c+dx))^n}{b^2-b^2\coth^2(c+dx)}d(b\coth(c+dx))}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{(b\coth(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},\coth^2(c+dx)\right)}{bd(n+1)}$$

input
```
Int[(b*Coth[c + d*x])^n,x]
```

output
```
((b*Coth[c + d*x])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Coth
[c + d*x]^2])/(b*d*(1 + n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [F]

$$\int (b\coth (dx+c))^n dx$$

input
```
int((b*coth(d*x+c))^n,x)
```

output
```
int((b*coth(d*x+c))^n,x)
```

Fricas [F]

$$\int (b\coth(c+dx))^n dx = \int (b\coth (dx+c))^n dx$$

input
```
integrate((b*coth(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*coth(d*x + c))^n, x)
```

Sympy [F]

$$\int (b\coth(c+dx))^n\,dx = \int (b\coth(c+dx))^n\,dx$$

input
```
integrate((b*coth(d*x+c))**n,x)
```

output
```
Integral((b*coth(c + d*x))**n, x)
```

Maxima [F]

$$\int (b\coth(c+dx))^n\,dx = \int (b\coth(dx+c))^n\,dx$$

input
```
integrate((b*coth(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c))^n, x)
```

Giac [F]

$$\int (b\coth(c+dx))^n\,dx = \int (b\coth(dx+c))^n\,dx$$

input
```
integrate((b*coth(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\coth(c+dx))^n\,dx = \int (b\coth(c+d\,x))^n\,dx$$

input
```
int((b*coth(c + d*x))^n,x)
```

output
```
int((b*coth(c + d*x))^n, x)
```

Reduce [F]

$$\int (b\coth(c+dx))^n\,dx = b^n\left(\int \coth(dx+c)^n\,dx\right)$$

input
```
int((b*coth(d*x+c))^n,x)
```

output
```
b**n*int(coth(c + d*x)**n,x)
```

3.17 $\int \left(b \coth^2(c+dx)\right)^n \, dx$

Optimal result

Integrand size = 12, antiderivative size = 57

$$\int \left(b \coth^2(c+dx)\right)^n \, dx = \frac{\coth(c+dx) \left(b \coth^2(c+dx)\right)^n \text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+2n), \frac{1}{2}(3+2n), \coth^2(c+dx)\right)}{d(1+2n)}$$

output
```
coth(d*x+c)*(b*coth(d*x+c)^2)^n*hypergeom([1, 1/2+n],[3/2+n],coth(d*x+c)^2
)/d/(1+2*n)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \left(b \coth^2(c+dx)\right)^n \, dx = \frac{\coth(c+dx) \left(b \coth^2(c+dx)\right)^n \text{Hypergeometric2F1}\left(1, \frac{1}{2}+n, \frac{3}{2}+n, \coth^2(c+dx)\right)}{d(1+2n)}$$

input
```
Integrate[(b*Coth[c + d*x]^2)^n,x]
```

output
```
(Coth[c + d*x]*(b*Coth[c + d*x]^2)^n*Hypergeometric2F1[1, 1/2 + n, 3/2 + n
, Coth[c + d*x]^2])/(d*(1 + 2*n))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4141, 3042, 3957, 25, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\coth^2(c+dx)\right)^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-b\tan\left(ic+idx+\frac{\pi}{2}\right)^2\right)^n dx$$

$$\Big\downarrow \text{4141}$$

$$\coth^{-2n}(c+dx)\left(b\coth^2(c+dx)\right)^n\int \coth^{2n}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\coth^{-2n}(c+dx)\left(b\coth^2(c+dx)\right)^n\int \left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{2n} dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{\coth^{-2n}(c+dx)\left(b\coth^2(c+dx)\right)^n\int -\frac{\coth^{2n}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\coth^{-2n}(c+dx)\left(b\coth^2(c+dx)\right)^n\int \frac{\coth^{2n}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{\coth(c+dx)\left(b\coth^2(c+dx)\right)^n \text{Hypergeometric2F1}\left(1,\frac{1}{2}(2n+1),\frac{1}{2}(2n+3),\coth^2(c+dx)\right)}{d(2n+1)}$$

input
```
Int[(b*Coth[c + d*x]^2)^n,x]
```

output
```
(Coth[c + d*x]*(b*Coth[c + d*x]^2)^n*Hypergeometric2F1[1, (1 + 2*n)/2, (3
+ 2*n)/2, Coth[c + d*x]^2])/(d*(1 + 2*n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \left(b \coth \left(dx + c\right)^{2}\right)^{n} dx$$

input
```
int((b*coth(d*x+c)^2)^n,x)
```

output
```
int((b*coth(d*x+c)^2)^n,x)
```

Fricas [F]

$$\int \left(b \coth^2(c + dx)\right)^n \, dx = \int \left(b \coth \left(dx + c\right)^{2}\right)^{n} \, dx$$

input
```
integrate((b*coth(d*x+c)^2)^n,x, algorithm="fricas")
```

output
```
integral((b*coth(d*x + c)^2)^n, x)
```

Sympy [F]

$$\int \left(b \coth^2(c + dx)\right)^n \, dx = \int \left(b \coth^{2}\left(c + dx\right)\right)^{n} \, dx$$

input
```
integrate((b*coth(d*x+c)**2)**n,x)
```

output
```
Integral((b*coth(c + d*x)**2)**n, x)
```

Maxima [F]

$$\int \left(b\coth^2(c+dx)\right)^n\,dx = \int \left(b\coth\left(dx+c\right)^2\right)^n\,dx$$

input
```
integrate((b*coth(d*x+c)^2)^n,x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^2)^n, x)
```

Giac [F]

$$\int \left(b\coth^2(c+dx)\right)^n\,dx = \int \left(b\coth\left(dx+c\right)^2\right)^n\,dx$$

input
```
integrate((b*coth(d*x+c)^2)^n,x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^2)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\coth^2(c+dx)\right)^n\,dx = \int \left(b\coth(c+d\,x)^2\right)^n\,dx$$

input
```
int((b*coth(c + d*x)^2)^n,x)
```

output
```
int((b*coth(c + d*x)^2)^n, x)
```

Reduce [F]

$$\int \left(b\coth^2(c+dx)\right)^n\,dx = b^n\left(\int \coth\left(dx+c\right)^{2n}dx\right)$$

input
```
int((b*coth(d*x+c)^2)^n,x)
```

output
```
b**n*int(coth(c + d*x)**(2*n),x)
```

3.18 $\int \left(b\coth^2(c+dx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 61

$$\int \left(b\coth^2(c+dx)\right)^{3/2}\,dx=-\frac{b\coth(c+dx)\sqrt{b\coth^2(c+dx)}}{2d}+\frac{b\sqrt{b\coth^2(c+dx)}\log(\sinh(c+dx))\tanh(c+dx)}{d}$$

output
```
-1/2*b*coth(d*x+c)*(b*coth(d*x+c)^2)^(1/2)/d+b*(b*coth(d*x+c)^2)^(1/2)*ln(
sinh(d*x+c))*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \left(b\coth^2(c+dx)\right)^{3/2}\,dx=-\frac{\left(b\coth^2(c+dx)\right)^{3/2}\left(\operatorname{csch}^2(c+dx)-2\log(\sinh(c+dx))\right)\tanh^3(c+dx)}{2d}$$

input
```
Integrate[(b*Coth[c + d*x]^2)^(3/2),x]
```

output
```
-1/2*((b*Coth[c + d*x]^2)^(3/2)*(Csch[c + d*x]^2 - 2*Log[Sinh[c + d*x]])*T
anh[c + d*x]^3)/d
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3042, 4141, 3042, 26, 3954, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\coth^2(c+dx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-b\tan\left(ic+idx+\frac{\pi}{2}\right)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4141}$$

$$b\tanh(c+dx)\sqrt{b\coth^2(c+dx)}\int \coth^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$b\tanh(c+dx)\sqrt{b\coth^2(c+dx)}\int i\tan\left(ic+idx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{26}$$

$$ib\tanh(c+dx)\sqrt{b\coth^2(c+dx)}\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^3dx$$

$$\Big\downarrow \text{3954}$$

$$ib\tanh(c+dx)\sqrt{b\coth^2(c+dx)}\left(\frac{i\coth^2(c+dx)}{2d}-\int i\coth(c+dx)dx\right)$$

$$\Big\downarrow \text{26}$$

$$ib\tanh(c+dx)\sqrt{b\coth^2(c+dx)}\left(\frac{i\coth^2(c+dx)}{2d}-i\int \coth(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$ib\tanh(c+dx)\sqrt{b\coth^2(c+dx)}\left(\frac{i\coth^2(c+dx)}{2d}-i\int -i\tan\left(ic+idx+\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \text{26}$$

$$ib\tanh(c+dx)\sqrt{b\coth^2(c+dx)}\left(\frac{i\coth^2(c+dx)}{2d}-\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx\right)$$

$$\Big\downarrow \text{3956}$$

$$ib\tanh(c+dx)\sqrt{b\coth^2(c+dx)}\left(\frac{i\coth^2(c+dx)}{2d}-\frac{i\log(-i\sinh(c+dx))}{d}\right)$$

input
```
Int[(b*Coth[c + d*x]^2)^(3/2),x]
```

output
```
I*b*Sqrt[b*Coth[c + d*x]^2]*(((I/2)*Coth[c + d*x]^2)/d - (I*Log[(-I)*Sinh[
c + d*x]])/d)*Tanh[c + d*x]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

method	result
derivativedivides	$-\frac{\left(b\coth(dx+c)^{2}\right)^{\frac{3}{2}}\left(\coth(dx+c)^{2}+\ln(\coth(dx+c)-1)+\ln(\coth(dx+c)+1)\right)}{2d\coth(dx+c)^{3}}$
default	$-\frac{\left(b\coth(dx+c)^{2}\right)^{\frac{3}{2}}\left(\coth(dx+c)^{2}+\ln(\coth(dx+c)-1)+\ln(\coth(dx+c)+1)\right)}{2d\coth(dx+c)^{3}}$
risch	$\frac{b\sqrt{\frac{b\left(e^{2dx+2c}+1\right)^{2}}{\left(e^{2dx+2c}-1\right)^{2}}}\,(-e^{4dx+4c}dx+e^{4dx+4c}\ln\left(e^{2dx+2c}-1\right)-2\,e^{4dx+4c}c+2\,e^{2dx+2c}dx-2\,e^{2dx+2c}\ln\left(e^{2dx+2c}-1\right)+4\,e^{2c}}{\left(e^{2dx+2c}+1\right)\left(e^{2dx+2c}-1\right)d}$

input
```
int((b*coth(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*(b*coth(d*x+c)^2)^(3/2)*(coth(d*x+c)^2+ln(coth(d*x+c)-1)+ln(coth(d*
x+c)+1))/coth(d*x+c)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 823 vs. $2(55) = 110$.

Time = 0.11 (sec) , antiderivative size = 823, normalized size of antiderivative = 13.49

$$\int \left(b\coth^2(c+dx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((b*coth(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
(b*d*x*cosh(d*x + c)^4 - (b*d*x*e^(2*d*x + 2*c) - b*d*x)*sinh(d*x + c)^4 -
 4*(b*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) - b*d*x*cosh(d*x + c))*sinh(d*x +
c)^3 + b*d*x - 2*(b*d*x - b)*cosh(d*x + c)^2 + 2*(3*b*d*x*cosh(d*x + c)^2
- b*d*x - (3*b*d*x*cosh(d*x + c)^2 - b*d*x + b)*e^(2*d*x + 2*c) + b)*sinh(
d*x + c)^2 - (b*d*x*cosh(d*x + c)^4 + b*d*x - 2*(b*d*x - b)*cosh(d*x + c)^
2)*e^(2*d*x + 2*c) - (b*cosh(d*x + c)^4 - (b*e^(2*d*x + 2*c) - b)*sinh(d*x
 + c)^4 - 4*(b*cosh(d*x + c)*e^(2*d*x + 2*c) - b*cosh(d*x + c))*sinh(d*x +
 c)^3 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - (3*b*cosh(d*x + c)^
2 - b)*e^(2*d*x + 2*c) - b)*sinh(d*x + c)^2 - (b*cosh(d*x + c)^4 - 2*b*cos
h(d*x + c)^2 + b)*e^(2*d*x + 2*c) + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c)
 - (b*cosh(d*x + c)^3 - b*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c) +
b)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(b*d*x*cosh(d*
x + c)^3 - (b*d*x - b)*cosh(d*x + c) - (b*d*x*cosh(d*x + c)^3 - (b*d*x - b
)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c))*sqrt((b*e^(4*d*x + 4*c) +
 2*b*e^(2*d*x + 2*c) + b)/(e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c) + 1))/(d*co
sh(d*x + c)^4 + (d*e^(2*d*x + 2*c) + d)*sinh(d*x + c)^4 + 4*(d*cosh(d*x +
c)*e^(2*d*x + 2*c) + d*cosh(d*x + c))*sinh(d*x + c)^3 - 2*d*cosh(d*x + c)^
2 + 2*(3*d*cosh(d*x + c)^2 + (3*d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) - d
)*sinh(d*x + c)^2 + (d*cosh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + d)*e^(2*d*x
 + 2*c) + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c) + (d*cosh(d*x + c)^3 -...
```

Sympy [F]

$$\int \left(b\coth^2(c+dx)\right)^{3/2} dx = \int \left(b\coth^2\left(c+dx\right)\right)^{\frac{3}{2}} dx$$

input
```
integrate((b*coth(d*x+c)**2)**(3/2),x)
```

output
```
Integral((b*coth(c + d*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.59

$$\int \left(b\coth^2(c+dx)\right)^{3/2}\,dx = -\frac{(dx+c)b^{\frac{3}{2}}}{d} - \frac{b^{\frac{3}{2}}\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{b^{\frac{3}{2}}\log\left(e^{(-dx-c)}-1\right)}{d} - \frac{2\,b^{\frac{3}{2}}e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}$$

input
```
integrate((b*coth(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
-(d*x + c)*b^(3/2)/d - b^(3/2)*log(e^(-d*x - c) + 1)/d - b^(3/2)*log(e^(-d
*x - c) - 1)/d - 2*b^(3/2)*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4
*d*x - 4*c) - 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.48

$$\int \left(b\coth^2(c+dx)\right)^{3/2}\,dx = -\frac{\left((dx+c)\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)-\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)+\frac{2\,e^{(2\,dx+2\,c)}\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^2}\right)b^{\frac{3}{2}}}{d}$$

input
```
integrate((b*coth(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
-((d*x + c)*sgn(e^(4*d*x + 4*c) - 1) - log(abs(e^(2*d*x + 2*c) - 1))*sgn(e
^(4*d*x + 4*c) - 1) + 2*e^(2*d*x + 2*c)*sgn(e^(4*d*x + 4*c) - 1)/(e^(2*d*x
 + 2*c) - 1)^2)*b^(3/2)/d
```

Mupad [F(-1)]

Timed out.

$$\int \left(b \coth^2(c+dx)\right)^{3/2} dx = \int \left(b \coth(c+d\,x)^2\right)^{3/2} dx$$

input
```
int((b*coth(c + d*x)^2)^(3/2),x)
```

output
```
int((b*coth(c + d*x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.03

$$\int \left(b \coth^2(c+dx)\right)^{3/2} dx = \frac{\sqrt{b}\, b\left(e^{4dx+4c}\log\left(e^{dx+c}-1\right)+e^{4dx+4c}\log\left(e^{dx+c}+1\right)-e^{4dx+4c}dx-e^{4dx+4c}-2e^{2dx+2c}\log\left(e^{dx}\right.\right.}{d\left(e^{4dx+4c}-\right.}$$

input
```
int((b*coth(d*x+c)^2)^(3/2),x)
```

output
```
(sqrt(b)*b*(e**(4*c + 4*d*x)*log(e**(c + d*x) - 1) + e**(4*c + 4*d*x)*log(
e**(c + d*x) + 1) - e**(4*c + 4*d*x)*d*x - e**(4*c + 4*d*x) - 2*e**(2*c +
2*d*x)*log(e**(c + d*x) - 1) - 2*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1) +
2*e**(2*c + 2*d*x)*d*x + log(e**(c + d*x) - 1) + log(e**(c + d*x) + 1) - d
*x - 1))/(d*(e**(4*c + 4*d*x) - 2*e**(2*c + 2*d*x) + 1))
```

3.19 $\int \sqrt{b \coth^2(c+dx)}\, dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \sqrt{b \coth^2(c+dx)}\, dx = \frac{\sqrt{b \coth^2(c+dx)} \log(\sinh(c+dx)) \tanh(c+dx)}{d}$$

output
```
(b*coth(d*x+c)^2)^(1/2)*ln(sinh(d*x+c))*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \sqrt{b \coth^2(c+dx)}\, dx = \frac{\sqrt{b \coth^2(c+dx)} \log(\sinh(c+dx)) \tanh(c+dx)}{d}$$

input
```
Integrate[Sqrt[b*Coth[c + d*x]^2],x]
```

output
```
(Sqrt[b*Coth[c + d*x]^2]*Log[Sinh[c + d*x]]*Tanh[c + d*x])/d
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4141, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b \coth^2(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-b \tan\left(ic+idx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4141}$$

$$\tanh(c+dx)\sqrt{b \coth^2(c+dx)}\int \coth(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tanh(c+dx)\sqrt{b \coth^2(c+dx)}\int -i\tan\left(ic+idx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\tanh(c+dx)\sqrt{b \coth^2(c+dx)}\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx$$

$$\Big\downarrow \text{3956}$$

$$\frac{\tanh(c+dx)\sqrt{b \coth^2(c+dx)}\log(-i\sinh(c+dx))}{d}$$

input
```
Int[Sqrt[b*Coth[c + d*x]^2],x]
```

output
```
(Sqrt[b*Coth[c + d*x]^2]*Log[(-I)*Sinh[c + d*x]]*Tanh[c + d*x])/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

method	result
derivativedivides	$-\frac{\sqrt{b \coth(dx+c)^2}\,(\ln(\coth(dx+c)-1)+\ln(\coth(dx+c)+1))}{2d\coth(dx+c)}$
default	$-\frac{\sqrt{b \coth(dx+c)^2}\,(\ln(\coth(dx+c)-1)+\ln(\coth(dx+c)+1))}{2d\coth(dx+c)}$
risch	$\frac{\sqrt{\frac{b\left(e^{2dx+2c}+1\right)^2}{\left(e^{2dx+2c}-1\right)^2}}\,\left(e^{2dx+2c}-1\right)x}{e^{2dx+2c}+1} - \frac{2\sqrt{\frac{b\left(e^{2dx+2c}+1\right)^2}{\left(e^{2dx+2c}-1\right)^2}}\,\left(e^{2dx+2c}-1\right)(dx+c)}{\left(e^{2dx+2c}+1\right)d} + \frac{\sqrt{\frac{b\left(e^{2dx+2c}+1\right)^2}{\left(e^{2dx+2c}-1\right)^2}}\,\left(e^{2dx+2c}-1\right)\ln(\text{e}}{\left(e^{2dx+2c}+1\right)d}$

input
```
int((b*coth(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*(b*coth(d*x+c)^2)^(1/2)*(ln(coth(d*x+c)-1)+ln(coth(d*x+c)+1))/coth(
d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(29) = 58$.

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 4.03

$$\int \sqrt{b\coth^2(c+dx)}\,dx = -\frac{\left(dxe^{(2\,dx+2\,c)} - dx - \left(e^{(2\,dx+2\,c)} - 1\right)\log\left(\frac{2\,\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)\right)\sqrt{\frac{be^{(4\,dx+4\,c)}+2\,be^{(2\,dx+2\,c)}+b}{e^{(4\,dx+4\,c)}-2\,e^{(2\,dx+2\,c)}+1}}}{de^{(2\,dx+2\,c)} + d}$$

input
```
integrate((b*coth(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
-(d*x*e^(2*d*x + 2*c) - d*x - (e^(2*d*x + 2*c) - 1)*log(2*sinh(d*x + c)/(c
osh(d*x + c) - sinh(d*x + c))))*sqrt((b*e^(4*d*x + 4*c) + 2*b*e^(2*d*x + 2
*c) + b)/(e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c) + 1))/(d*e^(2*d*x + 2*c) + d
)
```

Sympy [F]

$$\int \sqrt{b\coth^2(c+dx)}\,dx = \int \sqrt{b\coth^2{(c+dx)}}\,dx$$

input
```
integrate((b*coth(d*x+c)**2)**(1/2),x)
```

output
```
Integral(sqrt(b*coth(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \sqrt{b\coth^2(c+dx)}\,dx = -\frac{(dx+c)\sqrt{b}}{d} - \frac{\sqrt{b}\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\sqrt{b}\log\left(e^{(-dx-c)}-1\right)}{d}$$

input
```
integrate((b*coth(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
-(d*x + c)*sqrt(b)/d - sqrt(b)*log(e^(-d*x - c) + 1)/d - sqrt(b)*log(e^(-d
*x - c) - 1)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \sqrt{b\coth^2(c+dx)}\,dx = -\frac{\left((dx+c)\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right) - \log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)\right)\sqrt{b}}{d}$$

input
```
integrate((b*coth(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
-((d*x + c)*sgn(e^(4*d*x + 4*c) - 1) - log(abs(e^(2*d*x + 2*c) - 1))*sgn(e
^(4*d*x + 4*c) - 1))*sqrt(b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\coth^2(c+dx)}\,dx = \int \sqrt{b\,{\mathrm{coth}\left(c+d\,x\right)}^2}\,dx$$

input
```
int((b*coth(c + d*x)^2)^(1/2),x)
```

output
```
int((b*coth(c + d*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \sqrt{b\coth^2(c+dx)}\,dx = \frac{\sqrt{b}\left(\log\left(e^{dx+c}-1\right)+\log\left(e^{dx+c}+1\right)-dx\right)}{d}$$

input
```
int((b*coth(d*x+c)^2)^(1/2),x)
```

output
```
(sqrt(b)*(log(e**(c + d*x) - 1) + log(e**(c + d*x) + 1) - d*x))/d
```

3.20 $\int \frac{1}{\sqrt{b\coth^2(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{1}{\sqrt{b\coth^2(c+dx)}}\,dx = \frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt{b\coth^2(c+dx)}}$$

output
```
coth(d*x+c)*ln(cosh(d*x+c))/d/(b*coth(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{b\coth^2(c+dx)}}\,dx = \frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt{b\coth^2(c+dx)}}$$

input
```
Integrate[1/Sqrt[b*Coth[c + d*x]^2],x]
```

output
```
(Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*Sqrt[b*Coth[c + d*x]^2])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4141, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\coth^2(c+dx)}}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{\sqrt{-b\tan\left(ic+idx+\frac{\pi}{2}\right)^2}}dx$$
$$\Big\downarrow \ 4141$$
$$\frac{\coth(c+dx)\int \tanh(c+dx)dx}{\sqrt{b\coth^2(c+dx)}}$$
$$\Big\downarrow \ 3042$$
$$\frac{\coth(c+dx)\int -i\tan(ic+idx)dx}{\sqrt{b\coth^2(c+dx)}}$$
$$\Big\downarrow \ 26$$
$$-\frac{i\coth(c+dx)\int \tan(ic+idx)dx}{\sqrt{b\coth^2(c+dx)}}$$
$$\Big\downarrow \ 3956$$
$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt{b\coth^2(c+dx)}}$$

input
```
Int[1/Sqrt[b*Coth[c + d*x]^2],x]
```

output
```
(Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*Sqrt[b*Coth[c + d*x]^2])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

method	result	size
derivativedivides	$\frac{\coth(dx+c)\left(2\ln(\coth(dx+c))-\ln(\coth(dx+c)+1)-\ln(\coth(dx+c)-1)\right)}{2d\sqrt{b\coth(dx+c)^{2}}}$	56
default	$\frac{\coth(dx+c)\left(2\ln(\coth(dx+c))-\ln(\coth(dx+c)+1)-\ln(\coth(dx+c)-1)\right)}{2d\sqrt{b\coth(dx+c)^{2}}}$	56
risch	$\frac{\left(e^{2dx+2c}+1\right)x}{\sqrt{\frac{b\left(e^{2dx+2c}+1\right)^{2}}{\left(e^{2dx+2c}-1\right)^{2}}}\left(e^{2dx+2c}-1\right)}-\frac{2\left(e^{2dx+2c}+1\right)(dx+c)}{\sqrt{\frac{b\left(e^{2dx+2c}+1\right)^{2}}{\left(e^{2dx+2c}-1\right)^{2}}}\left(e^{2dx+2c}-1\right)d}+\frac{\left(e^{2dx+2c}+1\right)\ln\left(e^{2dx+2c}+1\right)}{\sqrt{\frac{b\left(e^{2dx+2c}+1\right)^{2}}{\left(e^{2dx+2c}-1\right)^{2}}}\left(e^{2dx+2c}-1\right)d}$	192

input
```
int(1/(b*coth(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*coth(d*x+c)*(2*ln(coth(d*x+c))-ln(coth(d*x+c)+1)-ln(coth(d*x+c)-1))/
(b*coth(d*x+c)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(29) = 58$.

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 4.13

$$\int \frac{1}{\sqrt{b\coth^2(c+dx)}}\,dx = -\frac{\left(dxe^{(2\,dx+2\,c)} - dx - \left(e^{(2\,dx+2\,c)} - 1\right)\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)\right)\sqrt{\frac{be^{(4\,dx+4\,c)}+2\,be^{(2\,dx+2\,c)}+b}{e^{(4\,dx+4\,c)}-2\,e^{(2\,dx+2\,c)}+1}}}{bde^{(2\,dx+2\,c)} + bd}$$

input
```
integrate(1/(b*coth(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
-(d*x*e^(2*d*x + 2*c) - d*x - (e^(2*d*x + 2*c) - 1)*log(2*cosh(d*x + c)/(c
osh(d*x + c) - sinh(d*x + c))))*sqrt((b*e^(4*d*x + 4*c) + 2*b*e^(2*d*x + 2
*c) + b)/(e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c) + 1))/(b*d*e^(2*d*x + 2*c) +
 b*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\coth^2(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\coth^2{(c+dx)}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(b*coth(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{b\coth^2(c+dx)}}\,dx = -\frac{dx+c}{\sqrt{b}d} - \frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\sqrt{b}d}$$

input
```
integrate(1/(b*coth(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
-(d*x + c)/(sqrt(b)*d) - log(e^(-2*d*x - 2*c) + 1)/(sqrt(b)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(29) = 58.

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int \frac{1}{\sqrt{b\coth^2(c+dx)}}\,dx = -\frac{\frac{dx+c}{\sqrt{b}\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)} - \frac{\log\left(e^{(2\,dx+2\,c)}+1\right)}{\sqrt{b}\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}}{d}$$

input
```
integrate(1/(b*coth(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
-((d*x + c)/(sqrt(b)*sgn(e^(4*d*x + 4*c) - 1)) - log(e^(2*d*x + 2*c) + 1)/
(sqrt(b)*sgn(e^(4*d*x + 4*c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{b\coth^2(c+dx)}}\,dx = \frac{\mathrm{atanh}\left(\frac{\sqrt{b}\,\mathrm{coth}(c+d\,x)}{\sqrt{b\,{\mathrm{coth}(c+d\,x)}^2}}\right)}{\sqrt{b}\,d}$$

input
```
int(1/(b*coth(c + d*x)^2)^(1/2),x)
```

output
```
atanh((b^(1/2)*coth(c + d*x))/(b*coth(c + d*x)^2)^(1/2))/(b^(1/2)*d)
```

Reduce [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{b\coth^2(c+dx)}}\,dx=\frac{\sqrt{b}\left(\log\left(e^{2dx+2c}+1\right)-dx\right)}{bd}$$

input
```
int(1/(b*coth(d*x+c)^2)^(1/2),x)
```

output
```
(sqrt(b)*(log(e**(2*c + 2*d*x) + 1) - d*x))/(b*d)
```

3.21 $\int \frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 65

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx=\frac{\coth(c+dx)\log(\cosh(c+dx))}{bd\sqrt{b\coth^2(c+dx)}}-\frac{\tanh(c+dx)}{2bd\sqrt{b\coth^2(c+dx)}}$$

output
```
coth(d*x+c)*ln(cosh(d*x+c))/b/d/(b*coth(d*x+c)^2)^(1/2)-1/2*tanh(d*x+c)/b/
d/(b*coth(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx=\frac{\coth^3(c+dx)\left(2\log(\cosh(c+dx))+\operatorname{sech}^2(c+dx)\right)}{2d\left(b\coth^2(c+dx)\right)^{3/2}}$$

input
```
Integrate[(b*Coth[c + d*x]^2)^(-3/2),x]
```

output
```
(Coth[c + d*x]^3*(2*Log[Cosh[c + d*x]] + Sech[c + d*x]^2))/(2*d*(b*Coth[c
+ d*x]^2)^(3/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3042, 4141, 3042, 26, 3954, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\coth^2(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(-b\tan\left(ic+idx+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\coth(c+dx)\int \tanh^3(c+dx)dx}{b\sqrt{b\coth^2(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\coth(c+dx)\int i\tan(ic+idx)^3dx}{b\sqrt{b\coth^2(c+dx)}}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\coth(c+dx)\int \tan(ic+idx)^3dx}{b\sqrt{b\coth^2(c+dx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{i\coth(c+dx)\left(\frac{i\tanh^2(c+dx)}{2d}-\int i\tanh(c+dx)dx\right)}{b\sqrt{b\coth^2(c+dx)}}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\coth(c+dx)\left(\frac{i\tanh^2(c+dx)}{2d}-i\int \tanh(c+dx)dx\right)}{b\sqrt{b\coth^2(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{i\coth(c+dx)\left(\frac{i\tanh^2(c+dx)}{2d}-i\int -i\tan(ic+idx)dx\right)}{b\sqrt{b\coth^2(c+dx)}}$$

$$\Big\downarrow \; 26$$

$$\frac{i\coth(c+dx)\left(\frac{i\tanh^2(c+dx)}{2d}-\int \tan(ic+idx)dx\right)}{b\sqrt{b\coth^2(c+dx)}}$$

$$\Big\downarrow \; 3956$$

$$\frac{i\coth(c+dx)\left(\frac{i\tanh^2(c+dx)}{2d}-\frac{i\log(\cosh(c+dx))}{d}\right)}{b\sqrt{b\coth^2(c+dx)}}$$

input
```
Int[(b*Coth[c + d*x]^2)^(-3/2),x]
```

output
```
(I*Coth[c + d*x]*(((-I)*Log[Cosh[c + d*x]])/d + ((I/2)*Tanh[c + d*x]^2)/d)
)/(b*Sqrt[b*Coth[c + d*x]^2])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{\coth(dx+c)\left(2\ln(\coth(dx+c))\coth(dx+c)^2-\ln(\coth(dx+c)+1)\coth(dx+c)^2-\ln(\coth(dx+c)-1)\coth(dx+c)^2-1\right)}{2d\left(b\coth(dx+c)^2\right)^{\frac{3}{2}}}$
default	$\frac{\coth(dx+c)\left(2\ln(\coth(dx+c))\coth(dx+c)^2-\ln(\coth(dx+c)+1)\coth(dx+c)^2-\ln(\coth(dx+c)-1)\coth(dx+c)^2-1\right)}{2d\left(b\coth(dx+c)^2\right)^{\frac{3}{2}}}$
risch	$\frac{-e^{4dx+4c}dx+e^{4dx+4c}\ln\left(e^{2dx+2c}+1\right)-2\,e^{4dx+4c}c-2\,e^{2dx+2c}dx+2\,e^{2dx+2c}\ln\left(e^{2dx+2c}+1\right)-4\,e^{2dx+2c}c-dx+2\,e^{2dx+2}}{b\left(e^{2dx+2c}+1\right)\left(e^{2dx+2c}-1\right)\sqrt{\frac{b\left(e^{2dx+2c}+1\right)^2}{\left(e^{2dx+2c}-1\right)^2}}\,d}$

input
```
int(1/(b*coth(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*coth(d*x+c)*(2*ln(coth(d*x+c))*coth(d*x+c)^2-ln(coth(d*x+c)+1)*coth(
d*x+c)^2-ln(coth(d*x+c)-1)*coth(d*x+c)^2-1)/(b*coth(d*x+c)^2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 817 vs. $2(59) = 118$.

Time = 0.11 (sec) , antiderivative size = 817, normalized size of antiderivative = 12.57

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*coth(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
(d*x*cosh(d*x + c)^4 - (d*x*e^(2*d*x + 2*c) - d*x)*sinh(d*x + c)^4 - 4*(d*
x*cosh(d*x + c)*e^(2*d*x + 2*c) - d*x*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(
d*x - 1)*cosh(d*x + c)^2 + 2*(3*d*x*cosh(d*x + c)^2 + d*x - (3*d*x*cosh(d*
x + c)^2 + d*x - 1)*e^(2*d*x + 2*c) - 1)*sinh(d*x + c)^2 + d*x - (d*x*cosh
(d*x + c)^4 + 2*(d*x - 1)*cosh(d*x + c)^2 + d*x)*e^(2*d*x + 2*c) + ((e^(2*
d*x + 2*c) - 1)*sinh(d*x + c)^4 - cosh(d*x + c)^4 + 4*(cosh(d*x + c)*e^(2*
d*x + 2*c) - cosh(d*x + c))*sinh(d*x + c)^3 - 2*(3*cosh(d*x + c)^2 - (3*co
sh(d*x + c)^2 + 1)*e^(2*d*x + 2*c) + 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^
2 + (cosh(d*x + c)^4 + 2*cosh(d*x + c)^2 + 1)*e^(2*d*x + 2*c) - 4*(cosh(d*
x + c)^3 - (cosh(d*x + c)^3 + cosh(d*x + c))*e^(2*d*x + 2*c) + cosh(d*x +
c))*sinh(d*x + c) - 1)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))
) + 4*(d*x*cosh(d*x + c)^3 + (d*x - 1)*cosh(d*x + c) - (d*x*cosh(d*x + c)^
3 + (d*x - 1)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c))*sqrt((b*e^(4*
d*x + 4*c) + 2*b*e^(2*d*x + 2*c) + b)/(e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c)
 + 1))/(b^2*d*cosh(d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + (b^2*d*e^(2*d*x
+ 2*c) + b^2*d)*sinh(d*x + c)^4 + 4*(b^2*d*cosh(d*x + c)*e^(2*d*x + 2*c) +
 b^2*d*cosh(d*x + c))*sinh(d*x + c)^3 + b^2*d + 2*(3*b^2*d*cosh(d*x + c)^2
 + b^2*d + (3*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(2*d*x + 2*c))*sinh(d*x + c
)^2 + (b^2*d*cosh(d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(2*d*x +
 2*c) + 4*(b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c) + (b^2*d*cosh(d*...
```

Sympy [F]

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\coth^2{\left(c+dx\right)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)**2)**(3/2),x)
```

output
```
Integral((b*coth(c + d*x)**2)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.29

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx = -\frac{2\sqrt{b}e^{(-2\,dx-2\,c)}}{(2\,b^2e^{(-2\,dx-2\,c)}+b^2e^{(-4\,dx-4\,c)}+b^2)d} - \frac{dx+c}{b^{\frac{3}{2}}d} - \frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{b^{\frac{3}{2}}d}$$

input
```
integrate(1/(b*coth(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
-2*sqrt(b)*e^(-2*d*x - 2*c)/((2*b^2*e^(-2*d*x - 2*c) + b^2*e^(-4*d*x - 4*c
) + b^2)*d) - (d*x + c)/(b^(3/2)*d) - log(e^(-2*d*x - 2*c) + 1)/(b^(3/2)*d
)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.60

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx = -\frac{\frac{dx+c}{\sqrt{b}\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)} - \frac{\log\left(e^{(2\,dx+2\,c)}+1\right)}{\sqrt{b}\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)} - \frac{2\,e^{(2\,dx+2\,c)}}{\sqrt{b}\left(e^{(2\,dx+2\,c)}+1\right)^2\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}}{bd}$$

input
```
integrate(1/(b*coth(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
-((d*x + c)/(sqrt(b)*sgn(e^(4*d*x + 4*c) - 1)) - log(e^(2*d*x + 2*c) + 1)/
(sqrt(b)*sgn(e^(4*d*x + 4*c) - 1)) - 2*e^(2*d*x + 2*c)/(sqrt(b)*(e^(2*d*x
+ 2*c) + 1)^2*sgn(e^(4*d*x + 4*c) - 1)))/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\coth\left(c+d\,x\right)^2\right)^{3/2}}\,dx$$

input
```
int(1/(b*coth(c + d*x)^2)^(3/2),x)
```

output
```
int(1/(b*coth(c + d*x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.20

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx=\frac{\sqrt{b}\left(e^{4dx+4c}\log\left(e^{2dx+2c}+1\right)-e^{4dx+4c}dx-e^{4dx+4c}+2e^{2dx+2c}\log\left(e^{2dx+2c}+1\right)\right.}{b^2d\left(e^{4dx+4c}+2e^{2dx+2c}+1\right)}$$

input
```
int(1/(b*coth(d*x+c)^2)^(3/2),x)
```

output
```
(sqrt(b)*(e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1) - e**(4*c + 4*d*x)*d*
x - e**(4*c + 4*d*x) + 2*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1) - 2*e*
*(2*c + 2*d*x)*d*x + log(e**(2*c + 2*d*x) + 1) - d*x - 1))/(b**2*d*(e**(4*
c + 4*d*x) + 2*e**(2*c + 2*d*x) + 1))
```

3.22 $\int \left(b \coth^2(c+dx)\right)^{4/3} dx$

Optimal result

Integrand size = 14, antiderivative size = 243

$$\int \left(b \coth^2(c+dx)\right)^{4/3} dx = \frac{\sqrt{3} b \arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)} - \frac{\sqrt{3} b \arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)} + \frac{b \operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)} + \frac{b \operatorname{arctanh}\left(\frac{\sqrt[3]{\coth(c+dx)}}{1+\coth^{\frac{2}{3}}(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)} - \frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d}$$

output
```
1/2*3^(1/2)*b*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^2
)^(1/3)/d/coth(d*x+c)^(2/3)-1/2*3^(1/2)*b*arctan(1/3*(1+2*coth(d*x+c)^(1/3
))*3^(1/2))*(b*coth(d*x+c)^2)^(1/3)/d/coth(d*x+c)^(2/3)+b*arctanh(coth(d*x
+c)^(1/3))*(b*coth(d*x+c)^2)^(1/3)/d/coth(d*x+c)^(2/3)+1/2*b*arctanh(coth(
d*x+c)^(1/3)/(1+coth(d*x+c)^(2/3)))*(b*coth(d*x+c)^2)^(1/3)/d/coth(d*x+c)^
(2/3)-3/5*b*coth(d*x+c)*(b*coth(d*x+c)^2)^(1/3)/d
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.68

$$\int \left(b\coth^2(c+dx)\right)^{4/3}\,dx =$$

$$-\frac{\left(b\coth^2(c+dx)\right)^{4/3}\left(-20\text{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)+12\coth^{\frac{5}{3}}(c+dx)-5\left(2\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c}}{\sqrt{3}}\right.\right.\right.}{}$$

input
```
Integrate[(b*Coth[c + d*x]^2)^(4/3),x]
```

output
```
-1/20*((b*Coth[c + d*x]^2)^(4/3)*(-20*ArcTanh[Coth[c + d*x]^(1/3)] + 12*Co
th[c + d*x]^(5/3) - 5*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3
]] - 2*Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - Log[1 - Coth[
c + d*x]^(1/3) + Coth[c + d*x]^(2/3)] + Log[1 + Coth[c + d*x]^(1/3) + Coth
[c + d*x]^(2/3)])))/(d*Coth[c + d*x]^(8/3))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.77, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {3042, 4141, 3042, 3954, 3042, 3957, 25, 266, 825, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\coth^2(c+dx)\right)^{4/3} dx$$

$\Big\downarrow$ 3042

$$\int \left(-b\tan\left(ic+idx+\frac{\pi}{2}\right)^2\right)^{4/3} dx$$

$\Big\downarrow$ 4141

$$\frac{b\sqrt[3]{b\coth^2(c+dx)}\int \coth^{\frac{8}{3}}(c+dx)dx}{\coth^{\frac{2}{3}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{b\sqrt[3]{b\coth^2(c+dx)}\int \left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{8/3} dx}{\coth^{\frac{2}{3}}(c+dx)}$$

$\Big\downarrow$ 3954

$$\frac{b\sqrt[3]{b\coth^2(c+dx)}\left(\int \coth^{\frac{2}{3}}(c+dx)dx-\frac{3\coth^{\frac{5}{3}}(c+dx)}{5d}\right)}{\coth^{\frac{2}{3}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{b\sqrt[3]{b\coth^2(c+dx)}\left(-\frac{3\coth^{\frac{5}{3}}(c+dx)}{5d}+\int \left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{2/3} dx\right)}{\coth^{\frac{2}{3}}(c+dx)}$$

$\Big\downarrow$ 3957

$$\frac{b\sqrt[3]{b\coth^2(c+dx)}\left(-\frac{\int -\frac{\coth^{\frac{2}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}-\frac{3\coth^{\frac{5}{3}}(c+dx)}{5d}\right)}{\coth^{\frac{2}{3}}(c+dx)}$$

$\Big\downarrow$ 25

$$\frac{b\sqrt[3]{b\coth^2(c+dx)}\left(\frac{\int \frac{\coth^{\frac{2}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}-\frac{3\coth^{\frac{5}{3}}(c+dx)}{5d}\right)}{\coth^{\frac{2}{3}}(c+dx)}$$

$\Big\downarrow$ 266

$$\frac{b\sqrt[3]{b\coth^2(c+dx)}\left(\frac{3\int \frac{\coth^{\frac{4}{3}}(c+dx)}{1-\coth^2(c+dx)}d\sqrt[3]{\coth(c+dx)}}{d}-\frac{3\coth^{\frac{5}{3}}(c+dx)}{5d}\right)}{\coth^{\frac{2}{3}}(c+dx)}$$

$\downarrow$ 825

$$\frac{b\sqrt[3]{b\coth^2(c+dx)}\left(\frac{3\left(\frac{1}{3}\int \frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{3}\int -\frac{\sqrt[3]{\coth(c+dx)}+1}{2\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{3}\right.}{d}\right.}{\coth^{\frac{2}{3}}(c+dx)}$$

$\downarrow$ 27

$$\frac{b\sqrt[3]{b\coth^2(c+dx)}\left(\frac{3\left(\frac{1}{3}\int \frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}-\frac{1}{6}\int \frac{\sqrt[3]{\coth(c+dx)}+1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{6}\int \frac{}{\coth}\right.}{d}\right.}{\coth^{\frac{2}{3}}(c+dx)}$$

$\downarrow$ 219

$$\frac{b\sqrt[3]{b\coth^2(c+dx)}\left(\frac{3\left(-\frac{1}{6}\int \frac{\sqrt[3]{\coth(c+dx)}+1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{6}\int \frac{1-\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\mathrm{co}}\right.}{d}\right.}{\coth^{\frac{2}{3}}(c+dx)}$$

$\downarrow$ 1142

$$b\sqrt[3]{b\coth^2(c+dx)}\left(\frac{3\left(\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{2}\int -\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\right.\right.}{}\right.$$

$\downarrow$ 25

$$b\sqrt[3]{b\coth^2(c+dx)}\left(\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{3}{2}\int\frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{c\ldots}\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{ 1083}$$

$$b\sqrt[3]{b\coth^2(c+dx)}\left(\frac{3\left(\frac{1}{6}\left(3\int\frac{1}{-\coth^{\frac{2}{3}}(c+dx)-3}d\left(2\sqrt[3]{\coth(c+dx)}-1\right)+\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+d\ldots}\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{ 217}$$

$$b\sqrt[3]{b\coth^2(c+dx)}\left(\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(\ldots\right.\right.}{}\right.$$

$$\Big\downarrow \text{ 1103}$$

$$b\sqrt[3]{b\coth^2(c+dx)}\left(\frac{3\left(\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)\right)+\frac{1}{6}\left(\frac{1}{2}\log\left(\coth^{\frac{2}{3}}(\ldots\right.\right.\right.}{d\ldots}\right.$$

$$\overline{\coth^{\frac{2}{3}}(c+\ldots}$$

input
```
Int[(b*Coth[c + d*x]^2)^(4/3),x]
```

output
```
(b*(b*Coth[c + d*x]^2)^(1/3)*((-3*Coth[c + d*x]^(5/3))/(5*d) + (3*(ArcTanh
[Coth[c + d*x]^(1/3)]/3 + (-(Sqrt[3]*ArcTan[(-1 + 2*Coth[c + d*x]^(1/3))/S
qrt[3]]) - Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]/2)/6 + (-(Sq
rt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]) + Log[1 + Coth[c + d*x]
^(1/3) + Coth[c + d*x]^(2/3)]/2)/6))/d))/Coth[c + d*x]^(2/3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \left(b \coth \left(dx + c\right)^{2}\right)^{\frac{4}{3}} dx$$

input
```
int((b*coth(d*x+c)^2)^(4/3),x)
```

output
```
int((b*coth(d*x+c)^2)^(4/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1994 vs. 2(201) = 402.

Time = 0.16 (sec) , antiderivative size = 1994, normalized size of antiderivative = 8.21

$$\int \left(b\coth^2(c+dx)\right)^{4/3} dx = \text{Too large to display}$$

input
```
integrate((b*coth(d*x+c)^2)^(4/3),x, algorithm="fricas")
```

output
```
-1/20*(10*(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x
+ c) + sqrt(3)*b*sinh(d*x + c)^2 - sqrt(3)*b)*(-b)^(1/3)*arctan(1/3*(sqrt(
3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b
*sinh(d*x + c)^2 + 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*si
nh(d*x + c) + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(-b)^(2/3)*((b*cosh(d*x +
 c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1
/3) + sqrt(3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*
sinh(d*x + c)^2 + b)) - 10*(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d
*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh(d*x + c)^2 - sqrt(3)*b)*b^(1/3)*arc
tan(-1/3*(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x +
 c) + sqrt(3)*b*sinh(d*x + c)^2 - 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*c
osh(d*x + c)*sinh(d*x + c) + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*b^(2/3)*((
b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c
)^2 - 1))^(1/3) + sqrt(3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d
*x + c) + b*sinh(d*x + c)^2 + b)) + 5*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x +
c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3)*log(((cosh(d*x + c)^4
 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^
2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x
 + c))*sinh(d*x + c) + 1)*(-b)^(2/3) - (cosh(d*x + c)^4 + 4*cosh(d*x + c)^
3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*s...
```

Sympy [F]

$$\int \left(b \coth^2(c+dx)\right)^{4/3} dx = \int \left(b \coth^2{\left(c+dx \right)}\right)^{\frac{4}{3}} dx$$

input
```
integrate((b*coth(d*x+c)**2)**(4/3),x)
```

output
```
Integral((b*coth(c + d*x)**2)**(4/3), x)
```

Maxima [F]

$$\int \left(b \coth^2(c+dx)\right)^{4/3} dx = \int \left(b \coth\left(dx+c\right)^2\right)^{\frac{4}{3}} dx$$

input
```
integrate((b*coth(d*x+c)^2)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^2)^(4/3), x)
```

Giac [F]

$$\int \left(b \coth^2(c+dx)\right)^{4/3} dx = \int \left(b \coth\left(dx+c\right)^2\right)^{\frac{4}{3}} dx$$

input
```
integrate((b*coth(d*x+c)^2)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^2)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\coth^2(c+dx)\right)^{4/3}\,dx=\int \left(b\coth(c+d\,x)^2\right)^{4/3}\,dx$$

input
```
int((b*coth(c + d*x)^2)^(4/3),x)
```

output
```
int((b*coth(c + d*x)^2)^(4/3), x)
```

Reduce [F]

$$\int \left(b\coth^2(c+dx)\right)^{4/3}\,dx=b^{\frac{4}{3}}\left(\int \coth\left(dx+c\right)^{\frac{8}{3}}dx\right)$$

input
```
int((b*coth(d*x+c)^2)^(4/3),x)
```

output
```
b**(1/3)*int(coth(c + d*x)**(2/3)*coth(c + d*x)**2,x)*b
```

3.23 $\int \left(b\coth^2(c+dx)\right)^{2/3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 236

$$\int \left(b\coth^2(c+dx)\right)^{2/3}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{2d\coth^{\frac{4}{3}}(c+dx)} + \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{2d\coth^{\frac{4}{3}}(c+dx)} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{d\coth^{\frac{4}{3}}(c+dx)} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\coth(c+dx)}}{1+\coth^{\frac{2}{3}}(c+dx)}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{2d\coth^{\frac{4}{3}}(c+dx)} - \frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c+dx)}{d}$$

output
```
-1/2*3^(1/2)*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^2)
^(2/3)/d/coth(d*x+c)^(4/3)+1/2*3^(1/2)*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*
3^(1/2))*(b*coth(d*x+c)^2)^(2/3)/d/coth(d*x+c)^(4/3)+arctanh(coth(d*x+c)^(
1/3))*(b*coth(d*x+c)^2)^(2/3)/d/coth(d*x+c)^(4/3)+1/2*arctanh(coth(d*x+c)^
(1/3)/(1+coth(d*x+c)^(2/3)))*(b*coth(d*x+c)^2)^(2/3)/d/coth(d*x+c)^(4/3)-3
*(b*coth(d*x+c)^2)^(2/3)*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.85

$$\int \left(b\coth^2(c+dx)\right)^{2/3}\,dx =$$

$$-\frac{\left(b\coth^2(c+dx)\right)^{2/3}\left(6\sqrt[6]{\coth^2(c+dx)}+\log\left(1-\sqrt[6]{\coth^2(c+dx)}\right)-\log\left(1+\sqrt[6]{\coth^2(c+dx)}\right)-(\right.}{}$$

input
```
Integrate[(b*Coth[c + d*x]^2)^(2/3),x]
```

output
```
-1/2*((b*Coth[c + d*x]^2)^(2/3)*(6*(Coth[c + d*x]^2)^(1/6) + Log[1 - (Coth
[c + d*x]^2)^(1/6)] - Log[1 + (Coth[c + d*x]^2)^(1/6)] - (-1)^(2/3)*Log[1
- (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)] + (-1)^(2/3)*Log[1 + (-1)^(1/3)*(Cot
h[c + d*x]^2)^(1/6)] - (-1)^(1/3)*Log[1 - (-1)^(2/3)*(Coth[c + d*x]^2)^(1/
6)] + (-1)^(1/3)*Log[1 + (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)])*Tanh[c + d*x
])/(d*(Coth[c + d*x]^2)^(1/6))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.77, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {3042, 4141, 3042, 3954, 3042, 3957, 25, 266, 754, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\coth^2(c+dx)\right)^{2/3} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-b\tan\left(ic+idx+\frac{\pi}{2}\right)^2\right)^{2/3} dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\left(b\coth^2(c+dx)\right)^{2/3}\int \coth^{\frac{4}{3}}(c+dx)dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(b\coth^2(c+dx)\right)^{2/3}\int \left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{4/3}dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\left(b\coth^2(c+dx)\right)^{2/3}\left(\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}dx-\frac{3\sqrt[3]{\coth(c+dx)}}{d}\right)}{\coth^{\frac{4}{3}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(b\coth^2(c+dx)\right)^{2/3}\left(-\frac{3\sqrt[3]{\coth(c+dx)}}{d}+\int \frac{1}{\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{2/3}}dx\right)}{\coth^{\frac{4}{3}}(c+dx)}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\left(b\coth^2(c+dx)\right)^{2/3}\left(-\frac{\int -\frac{1}{\coth^{\frac{2}{3}}(c+dx)\left(1-\coth^2(c+dx)\right)}d\coth(c+dx)}{d}-\frac{3\sqrt[3]{\coth(c+dx)}}{d}\right)}{\coth^{\frac{4}{3}}(c+dx)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\left(b\coth^2(c+dx)\right)^{2/3}\left(\frac{\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)\left(1-\coth^2(c+dx)\right)}d\coth(c+dx)}{d}-\frac{3\sqrt[3]{\coth(c+dx)}}{d}\right)}{\coth^{\frac{4}{3}}(c+dx)}$$

$$\Big\downarrow \text{266}$$

$$\frac{\left(b\coth^2(c+dx)\right)^{2/3}\left(\frac{3\int \frac{1}{1-\coth^2(c+dx)}d\sqrt[3]{\coth(c+dx)}}{d}-\frac{3\sqrt[3]{\coth(c+dx)}}{d}\right)}{\coth^{\frac{4}{3}}(c+dx)}$$

$\downarrow$ 754

$$\frac{\left(b\coth^2(c+dx)\right)^{2/3}\left(\frac{3\left(\frac{1}{3}\int \frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{3}\int \frac{2-\sqrt[3]{\coth(c+dx)}}{2\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{3}\right.}{d}\right.}{\coth^{\frac{4}{3}}(c+dx)}$$

$\downarrow$ 27

$$\frac{\left(b\coth^2(c+dx)\right)^{2/3}\left(\frac{3\left(\frac{1}{3}\int \frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{6}\int \frac{2-\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\frac{1}{6}\int -\right.}{d}\right.}{\coth^{\frac{4}{3}}(c+dx)}$$

$\downarrow$ 219

$$\frac{\left(b\coth^2(c+dx)\right)^{2/3}\left(\frac{3\left(\frac{1}{6}\int \frac{2-\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\frac{1}{6}\int \frac{\sqrt[3]{\coth(c+dx)}+2}{\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\mathrm{co}\right.}{d}\right.}{\coth^{\frac{4}{3}}(c+dx)}$$

$\downarrow$ 1142

$$\frac{\left(b\coth^2(c+dx)\right)^{2/3}\left(\frac{3\left(\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{2}\int -\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\right.\right.}{}\right.}{}$$

$\downarrow$ 25

$$\left(b\coth^2(c+dx)\right)^{2/3}\left(\frac{3\left(\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\right.\right.}{}\right.$$

$$\Big\downarrow \textbf{1083}$$

$$\left(b\coth^2(c+dx)\right)^{2/3}\left(\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-3\int\frac{1}{-\coth^{\frac{2}{3}}(c+dx)-3}d\left(2\sqrt[3]{\coth(c+dx}\right.\right.\right.}{}\right.$$

$$\Big\downarrow \textbf{217}$$

$$\left(b\coth^2(c+dx)\right)^{2/3}\left(\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\right.}{}\right.$$

$$\Big\downarrow \textbf{1103}$$

$$\frac{\left(b\coth^2(c+dx)\right)^{2/3}\left(\frac{3\left(\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)\right)+\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{2}{}\right.\right.\right.}{}\right.}{\coth^{\frac{4}{3}}(c}$$

input
```
Int[(b*Coth[c + d*x]^2)^(2/3),x]
```

output
```
((b*Coth[c + d*x]^2)^(2/3)*((-3*Coth[c + d*x]^(1/3))/d + (3*(ArcTanh[Coth[
c + d*x]^(1/3)]/3 + (Sqrt[3]*ArcTan[(-1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]
- Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]/2)/6 + (Sqrt[3]*ArcTa
n[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] + Log[1 + Coth[c + d*x]^(1/3) + Cot
h[c + d*x]^(2/3)]/2)/6))/d))/Coth[c + d*x]^(4/3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \left(b \coth \left(dx+c\right)^{2}\right)^{\frac{2}{3}}dx$$

input
```
int((b*coth(d*x+c)^2)^(2/3),x)
```

output
```
int((b*coth(d*x+c)^2)^(2/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2037 vs. $2(196) = 392$.

Time = 0.16 (sec) , antiderivative size = 2037, normalized size of antiderivative = 8.63

$$\int \left(b\coth^2(c+dx)\right)^{2/3} dx = \text{Too large to display}$$

input
```
integrate((b*coth(d*x+c)^2)^(2/3),x, algorithm="fricas")
```

output
```
-1/4*(2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c) +
 sqrt(3)*sinh(d*x + c)^2 + sqrt(3))*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b*co
sh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh(d
*x + c)^2 - 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x
+ c) + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(-b^2)^(1/3)*((b*cosh(d*x + c)^2
 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) +
 sqrt(3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(
d*x + c)^2 + b)) + 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*si
nh(d*x + c) + sqrt(3)*sinh(d*x + c)^2 + sqrt(3))*(b^2)^(1/3)*arctan(-1/3*(
sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt
(3)*b*sinh(d*x + c)^2 - 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x +
c)*sinh(d*x + c) + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(b^2)^(1/3)*((b*cosh
(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 -
1))^(1/3) + sqrt(3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c
) + b*sinh(d*x + c)^2 + b)) + (-b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x +
 c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*log(((cosh(d*x + c)^4 + 4*cosh(d*
x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x +
c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*(-b^2)^(2/3)*((b*cosh(d*x + c)^2
 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) +
 (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + ...
```

Sympy [F]

$$\int \left(b\coth^2(c+dx)\right)^{2/3} dx = \int \left(b\coth^2\left(c+dx\right)\right)^{\frac{2}{3}} dx$$

input
```
integrate((b*coth(d*x+c)**2)**(2/3),x)
```

output
```
Integral((b*coth(c + d*x)**2)**(2/3), x)
```

Maxima [F]

$$\int \left(b\coth^2(c+dx)\right)^{2/3} dx = \int \left(b\coth\left(dx+c\right)^2\right)^{\frac{2}{3}} dx$$

input
```
integrate((b*coth(d*x+c)^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^2)^(2/3), x)
```

Giac [F]

$$\int \left(b\coth^2(c+dx)\right)^{2/3} dx = \int \left(b\coth\left(dx+c\right)^2\right)^{\frac{2}{3}} dx$$

input
```
integrate((b*coth(d*x+c)^2)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^2)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\coth^2(c+dx)\right)^{2/3}\,dx=\int \left(b\coth(c+d\,x)^2\right)^{2/3}\,dx$$

input
```
int((b*coth(c + d*x)^2)^(2/3),x)
```

output
```
int((b*coth(c + d*x)^2)^(2/3), x)
```

Reduce [F]

$$\int \left(b\coth^2(c+dx)\right)^{2/3}\,dx=\frac{b^{\frac{2}{3}}\left(-3\coth\left(dx+c\right)^{\frac{1}{3}}+\left(\int \frac{1}{\coth(dx+c)^{\frac{2}{3}}}dx\right)d\right)}{d}$$

input
```
int((b*coth(d*x+c)^2)^(2/3),x)
```

output
```
(b**(2/3)*( - 3*coth(c + d*x)**(1/3) + int(coth(c + d*x)**(1/3)/coth(c + d
*x),x)*d))/d
```

3.24 $\int \sqrt[3]{b\coth^2(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 211

$$\int \sqrt[3]{b\coth^2(c+dx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)} - \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\coth(c+dx)}}{1+\coth^{\frac{2}{3}}(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)}$$

output
```
1/2*3^(1/2)*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^2)^
(1/3)/d/coth(d*x+c)^(2/3)-1/2*3^(1/2)*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3
^(1/2))*(b*coth(d*x+c)^2)^(1/3)/d/coth(d*x+c)^(2/3)+arctanh(coth(d*x+c)^(1
/3))*(b*coth(d*x+c)^2)^(1/3)/d/coth(d*x+c)^(2/3)+1/2*arctanh(coth(d*x+c)^(
1/3)/(1+coth(d*x+c)^(2/3)))*(b*coth(d*x+c)^2)^(1/3)/d/coth(d*x+c)^(2/3)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.72

$$\int \sqrt[3]{b\coth^2(c+dx)}\,dx$$
$$= \frac{\sqrt[3]{b\coth^2(c+dx)}\left(2\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)+4\text{arctanh}(\ldots}{4d\,\text{co}\ldots}$$

input
```
Integrate[(b*Coth[c + d*x]^2)^(1/3),x]
```

output
```
((b*Coth[c + d*x]^2)^(1/3)*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/S
qrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] + 4*ArcTan
h[Coth[c + d*x]^(1/3)] - Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)
] + Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]))/(4*d*Coth[c + d*x
]^(2/3))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.79, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4141, 3042, 3957, 25, 266, 825, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\coth^2(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt[3]{-b\tan\left(ic+idx+\frac{\pi}{2}\right)^2}dx$$
$$\Big\downarrow \text{4141}$$
$$\frac{\sqrt[3]{b\coth^2(c+dx)}\int \coth^{\frac{2}{3}}(c+dx)dx}{\coth^{\frac{2}{3}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{\sqrt[3]{b\coth^2(c+dx)}\int\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{2/3}dx}{\coth^{\frac{2}{3}}(c+dx)}$$

$\Big\downarrow$ 3957

$$-\frac{\sqrt[3]{b\coth^2(c+dx)}\int-\frac{\coth^{\frac{2}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d\coth^{\frac{2}{3}}(c+dx)}$$

$\Big\downarrow$ 25

$$\frac{\sqrt[3]{b\coth^2(c+dx)}\int\frac{\coth^{\frac{2}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d\coth^{\frac{2}{3}}(c+dx)}$$

$\Big\downarrow$ 266

$$\frac{3\sqrt[3]{b\coth^2(c+dx)}\int\frac{\coth^{\frac{4}{3}}(c+dx)}{1-\coth^2(c+dx)}d\sqrt[3]{\coth(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)}$$

$\Big\downarrow$ 825

$$\frac{3\sqrt[3]{b\coth^2(c+dx)}\left(\frac{1}{3}\int\frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{3}\int-\frac{\sqrt[3]{\coth(c+dx)}+1}{2\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}d\sqrt[3]{\coth(c+da}{d\coth^{\frac{2}{3}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{3\sqrt[3]{b\coth^2(c+dx)}\left(\frac{1}{3}\int\frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}-\frac{1}{6}\int\frac{\sqrt[3]{\coth(c+dx)}+1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{6}}{d\coth^{\frac{2}{3}}(c+dx)}$$

$\Big\downarrow$ 219

$$\frac{3\sqrt[3]{b\coth^2(c+dx)}\left(-\frac{1}{6}\int\frac{\sqrt[3]{\coth(c+dx)}+1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{6}\int\frac{1-\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{}}{d\coth^{\frac{2}{3}}(c+dx)}$$

$\Big\downarrow$ 1142

$$3\sqrt[3]{b\coth^2(c+dx)}\left(\frac{1}{6}\left(-\frac{3}{2}\int\frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{2}\int-\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}\right.\right.$$

$$\Big\downarrow \text{25}$$

$$3\sqrt[3]{b\coth^2(c+dx)}\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{3}{2}\int\frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\right.\right.$$

$$\Big\downarrow \text{1083}$$

$$3\sqrt[3]{b\coth^2(c+dx)}\left(\frac{1}{6}\left(3\int\frac{1}{-\coth^{\frac{2}{3}}(c+dx)-3}d\left(2\sqrt[3]{\coth(c+dx)}-1\right)+\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth}\right.\right.$$

$$\Big\downarrow \text{217}$$

$$3\sqrt[3]{b\coth^2(c+dx)}\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)\right)\right.$$

$$\Big\downarrow \text{1103}$$

$$3\sqrt[3]{b\coth^2(c+dx)}\left(\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)\right)+\frac{1}{6}\right.$$
$$d\coth$$

input
```
Int[(b*Coth[c + d*x]^2)^(1/3),x]
```

output
```
(3*(b*Coth[c + d*x]^2)^(1/3)*(ArcTanh[Coth[c + d*x]^(1/3)]/3 + (-(Sqrt[3]*
ArcTan[(-1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]) - Log[1 - Coth[c + d*x]^(1/3
) + Coth[c + d*x]^(2/3)]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/
3))/Sqrt[3]]) + Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]/2)/6))/
(d*Coth[c + d*x]^(2/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \left(b \coth \left(dx+c\right)^{2}\right)^{\frac{1}{3}}dx$$

input
```
int((b*coth(d*x+c)^2)^(1/3),x)
```

output
```
int((b*coth(d*x+c)^2)^(1/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1618 vs. $2(173) = 346$.

Time = 0.13 (sec) , antiderivative size = 1618, normalized size of antiderivative = 7.67

$$\int \sqrt[3]{b\coth^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*coth(d*x+c)^2)^(1/3),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(3)*(-b)^(1/3)*arctan(1/3*(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(
3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh(d*x + c)^2 + 2*(sqrt(3)*
cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*sinh(d*x
 + c)^2 - sqrt(3))*(-b)^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)
/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + sqrt(3)*b)/(b*cosh(d*x +
 c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 2*sqrt
(3)*b^(1/3)*arctan(-1/3*(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x
+ c)*sinh(d*x + c) + sqrt(3)*b*sinh(d*x + c)^2 - 2*(sqrt(3)*cosh(d*x + c)^
2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*sinh(d*x + c)^2 - sqrt
(3))*b^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2
 + sinh(d*x + c)^2 - 1))^(1/3) + sqrt(3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(
d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + (-b)^(1/3)*log(((cosh(d
*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(
d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 +
 cosh(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3) - (cosh(d*x + c)^4 + 4*cosh(
d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x
+ c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*(-b)^(1/3)*((b*cosh(d*x + c)^2
 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) +
 (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*
(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d...
```

Sympy [F]

$$\int \sqrt[3]{b\coth^2(c+dx)}\,dx = \int \sqrt[3]{b\coth^2{(c+dx)}}\,dx$$

input
```
integrate((b*coth(d*x+c)**2)**(1/3),x)
```

output
```
Integral((b*coth(c + d*x)**2)**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{b\coth^2(c+dx)}\,dx = \int \left(b\coth\left(dx+c\right)^2\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*coth(d*x+c)^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^2)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{b\coth^2(c+dx)}\,dx = \int \left(b\coth\left(dx+c\right)^2\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*coth(d*x+c)^2)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^2)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b \coth^2(c+dx)}\, dx = \int \left(b \coth(c+d\,x)^2\right)^{1/3} dx$$

input
```
int((b*coth(c + d*x)^2)^(1/3),x)
```

output
```
int((b*coth(c + d*x)^2)^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{b \coth^2(c+dx)}\, dx = b^{\frac{1}{3}} \left(\int \coth (dx+c)^{\frac{2}{3}} dx\right)$$

input
```
int((b*coth(d*x+c)^2)^(1/3),x)
```

output
```
b**(1/3)*int(coth(c + d*x)**(2/3),x)
```

3.25 $\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 211

$$\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\coth(c+dx)}}{1+\coth^{\frac{2}{3}}(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{2d\sqrt[3]{b\coth^2(c+dx)}}$$

output
```
-1/2*3^(1/2)*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(2/3)
/d/(b*coth(d*x+c)^2)^(1/3)+1/2*3^(1/2)*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*
3^(1/2))*coth(d*x+c)^(2/3)/d/(b*coth(d*x+c)^2)^(1/3)+arctanh(coth(d*x+c)^(
1/3))*coth(d*x+c)^(2/3)/d/(b*coth(d*x+c)^2)^(1/3)+1/2*arctanh(coth(d*x+c)^
(1/3)/(1+coth(d*x+c)^(2/3)))*coth(d*x+c)^(2/3)/d/(b*coth(d*x+c)^2)^(1/3)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx =$$

$$-\frac{\coth(c+dx)\left(\log\left(1-\sqrt[6]{\coth^2(c+dx)}\right)-\log\left(1+\sqrt[6]{\coth^2(c+dx)}\right)+\sqrt[3]{-1}\left(-\sqrt[3]{-1}\log\left(1-\sqrt[3]{\right.\right.\right.}}{}$$

input
```
Integrate[(b*Coth[c + d*x]^2)^(-1/3),x]
```

output
```
-1/2*(Coth[c + d*x]*(Log[1 - (Coth[c + d*x]^2)^(1/6)] - Log[1 + (Coth[c +
d*x]^2)^(1/6)] + (-1)^(1/3)*(-((-1)^(1/3)*Log[1 - (-1)^(1/3)*(Coth[c + d*x
]^2)^(1/6)]) + (-1)^(1/3)*Log[1 + (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)] - Lo
g[1 - (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)] + Log[1 + (-1)^(2/3)*(Coth[c + d
*x]^2)^(1/6)])))/(d*(Coth[c + d*x]^2)^(1/6)*(b*Coth[c + d*x]^2)^(1/3))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.78, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4141, 3042, 3957, 25, 266, 754, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt[3]{-b\tan\left(ic+idx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \; 4141$$

$$\frac{\coth^{\frac{2}{3}}(c+dx)\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}dx}{\sqrt[3]{b\coth^2(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\coth^{\frac{2}{3}}(c+dx)\int \frac{1}{\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{\sqrt[3]{b\coth^2(c+dx)}}$$

$$\Big\downarrow \; 3957$$

$$-\frac{\coth^{\frac{2}{3}}(c+dx)\int -\frac{1}{\coth^{\frac{2}{3}}(c+dx)\left(1-\coth^2(c+dx)\right)}d\coth(c+dx)}{d\sqrt[3]{b\coth^2(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\coth^{\frac{2}{3}}(c+dx)\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)\left(1-\coth^2(c+dx)\right)}d\coth(c+dx)}{d\sqrt[3]{b\coth^2(c+dx)}}$$

$$\Big\downarrow \; 266$$

$$\frac{3\coth^{\frac{2}{3}}(c+dx)\int \frac{1}{1-\coth^2(c+dx)}d\sqrt[3]{\coth(c+dx)}}{d\sqrt[3]{b\coth^2(c+dx)}}$$

$$\Big\downarrow \; 754$$

$$\frac{3\coth^{\frac{2}{3}}(c+dx)\left(\frac{1}{3}\int \frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{3}\int \frac{2-\sqrt[3]{\coth(c+dx)}}{2\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}d\sqrt[3]{\coth(c+dx)}+\right.}{d\sqrt[3]{b\coth^2(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{3\coth^{\frac{2}{3}}(c+dx)\left(\frac{1}{3}\int \frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{6}\int \frac{2-\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\frac{1}{6}\int\right.}{d\sqrt[3]{b\coth^2(c+dx)}}$$

$$\Big\downarrow \; 219$$

$$\frac{3\coth^{\frac{2}{3}}(c+dx)\left(\frac{1}{6}\int\frac{2-\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\frac{1}{6}\int\frac{\sqrt[3]{\coth(c+dx)}+2}{\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\cot}\right.}{d\sqrt[3]{b\coth^2(c+dx)}}$$

$$\Big\downarrow \text{1142}$$

$$3\coth^{\frac{2}{3}}(c+dx)\left(\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{2}\int-\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\right.\right.$$

$$\Big\downarrow \text{25}$$

$$3\coth^{\frac{2}{3}}(c+dx)\left(\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{}\right.\right.$$

$$\Big\downarrow \text{1083}$$

$$3\coth^{\frac{2}{3}}(c+dx)\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-3\int\frac{1}{-\coth^{\frac{2}{3}}(c+dx)-3}d\left(2\sqrt[3]{\coth(c+dx)}\right.\right.\right.$$

$$\Big\downarrow \text{217}$$

$$\frac{3\coth^{\frac{2}{3}}(c+dx)\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)\right)+\right.}{d\sqrt[3]{}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3\coth^{\frac{2}{3}}(c+dx)\left(\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)\right)+\frac{1}{6}\left(\sqrt{3}\right.\right.}{d\sqrt[3]{b\coth}}$$

input
```
Int[(b*Coth[c + d*x]^2)^(-1/3),x]
```

output
```
(3*Coth[c + d*x]^(2/3)*(ArcTanh[Coth[c + d*x]^(1/3)]/3 + (Sqrt[3]*ArcTan[(
-1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - Log[1 - Coth[c + d*x]^(1/3) + Coth[
c + d*x]^(2/3)]/2)/6 + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]
] + Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]/2)/6))/(d*(b*Coth[c
 + d*x]^2)^(1/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \frac{1}{\left(b \coth \left(dx+c\right)^{2}\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(b*coth(d*x+c)^2)^(1/3),x)
```

output
```
int(1/(b*coth(d*x+c)^2)^(1/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1594 vs. 2(173) = 346.

Time = 0.24 (sec) , antiderivative size = 8338, normalized size of antiderivative = 39.52

$$\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*coth(d*x+c)^2)^(1/3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{b\coth^2{(c+dx)}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)**2)**(1/3),x)
```

output
```
Integral((b*coth(c + d*x)**2)**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^2\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^2)^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^2\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^2)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^2)^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx = \int \frac{1}{\left(b\coth\left(c+d\,x\right)^2\right)^{1/3}}\,dx$$

input
```
int(1/(b*coth(c + d*x)^2)^(1/3),x)
```

output
```
int(1/(b*coth(c + d*x)^2)^(1/3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx = \frac{\int \frac{1}{\coth(dx+c)^{\frac{2}{3}}}dx}{b^{\frac{1}{3}}}$$

input
```
int(1/(b*coth(d*x+c)^2)^(1/3),x)
```

output
```
int(1/coth(c + d*x)**(2/3),x)/b**(1/3)
```

3.26 $\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 236

$$\begin{aligned}\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx = &-\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}}\\ &+\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2d\left(b\coth^2(c+dx)\right)^{2/3}}\\ &-\frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2d\left(b\coth^2(c+dx)\right)^{2/3}}\\ &+\frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}}\\ &+\frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\coth(c+dx)}}{1+\coth^{\frac{2}{3}}(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{2d\left(b\coth^2(c+dx)\right)^{2/3}}\end{aligned}$$

output
```
-3*coth(d*x+c)/d/(b*coth(d*x+c)^2)^(2/3)+1/2*3^(1/2)*arctan(1/3*(1-2*coth(
d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(4/3)/d/(b*coth(d*x+c)^2)^(2/3)-1/2*3^(
1/2)*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(4/3)/d/(b*co
th(d*x+c)^2)^(2/3)+arctanh(coth(d*x+c)^(1/3))*coth(d*x+c)^(4/3)/d/(b*coth(
d*x+c)^2)^(2/3)+1/2*arctanh(coth(d*x+c)^(1/3)/(1+coth(d*x+c)^(2/3)))*coth(
d*x+c)^(4/3)/d/(b*coth(d*x+c)^2)^(2/3)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx =$$

$$-\frac{\coth(c+dx)\left(6+\sqrt[6]{\coth^2(c+dx)}\log\left(1-\sqrt[6]{\coth^2(c+dx)}\right)-\sqrt[6]{\coth^2(c+dx)}\log\left(1+\sqrt[6]{\coth^2(c+\right.\right.}{}$$

input
```
Integrate[(b*Coth[c + d*x]^2)^(-2/3),x]
```

output
```
-1/2*(Coth[c + d*x]*(6 + (Coth[c + d*x]^2)^(1/6)*Log[1 - (Coth[c + d*x]^2)
^(1/6)] - (Coth[c + d*x]^2)^(1/6)*Log[1 + (Coth[c + d*x]^2)^(1/6)] + (-1)^
(1/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 - (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)]
- (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 + (-1)^(1/3)*(Coth[c + d*x]^2)^
(1/6)] + (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 - (-1)^(2/3)*(Coth[c + d
*x]^2)^(1/6)] - (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 + (-1)^(2/3)*(Cot
h[c + d*x]^2)^(1/6)]))/(d*(b*Coth[c + d*x]^2)^(2/3))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.78, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {3042, 4141, 3042, 3955, 3042, 3957, 25, 266, 825, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}dx$$

$\Big\downarrow$ 3042

$$\int \frac{1}{\left(-b\tan\left(ic+idx+\frac{\pi}{2}\right)^2\right)^{2/3}}dx$$

$\Big\downarrow$ 4141

$$\frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\coth^{\frac{4}{3}}(c+dx)}dx}{\left(b\coth^2(c+dx)\right)^{2/3}}$$

$\Big\downarrow$ 3042

$$\frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{4/3}}dx}{\left(b\coth^2(c+dx)\right)^{2/3}}$$

$\Big\downarrow$ 3955

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\int \coth^{\frac{2}{3}}(c+dx)dx-\frac{3}{d\sqrt[3]{\coth(c+dx)}}\right)}{\left(b\coth^2(c+dx)\right)^{2/3}}$$

$\Big\downarrow$ 3042

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(-\frac{3}{d\sqrt[3]{\coth(c+dx)}}+\int \left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{2/3}dx\right)}{\left(b\coth^2(c+dx)\right)^{2/3}}$$

$\Big\downarrow$ 3957

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(-\frac{\int -\frac{\coth^{\frac{2}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}-\frac{3}{d\sqrt[3]{\coth(c+dx)}}\right)}{\left(b\coth^2(c+dx)\right)^{2/3}}$$

$\Big\downarrow$ 25

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\frac{\int \frac{\coth^{\frac{2}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}-\frac{3}{d\sqrt[3]{\coth(c+dx)}}\right)}{\left(b\coth^2(c+dx)\right)^{2/3}}$$

$\Big\downarrow$ 266

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\int \frac{\coth^{\frac{4}{3}}(c+dx)}{1-\coth^2(c+dx)}d\sqrt[3]{\coth(c+dx)}}{d}-\frac{3}{d\sqrt[3]{\coth(c+dx)}}\right)}{\left(b\coth^2(c+dx)\right)^{2/3}}$$

$\Big\downarrow$ 825

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(\frac{1}{3}\int \frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{3}\int -\frac{\sqrt[3]{\coth(c+dx)}+1}{2\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{3}\int -\frac{}{2}\right.}{d}\right.}{\left(b\coth^2(c+dx)\right)^{2/3}}$$

$\Big\downarrow$ 27

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(\frac{1}{3}\int \frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}-\frac{1}{6}\int \frac{\sqrt[3]{\coth(c+dx)}+1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{6}\int \frac{1-}{\coth^{\frac{2}{3}}(c+}\right.}{d}\right.}{\left(b\coth^2(c+dx)\right)^{2/3}}$$

$\Big\downarrow$ 219

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(-\frac{1}{6}\int \frac{\sqrt[3]{\coth(c+dx)}+1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{6}\int \frac{1-\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c\right.}{d}\right.}{\left(b\coth^2(c+dx)\right)^{2/3}}$$

$\Big\downarrow$ 1142

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{2}\int -\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\text{co}\right.\right.}{}\right.}{}$$

$\Big\downarrow$ 25

$$\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{3}{2}\int\frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(\right.\right.\right.}{}\right.}{}$$

$\downarrow$ 1083

$$\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(3\int\frac{1}{-\coth^{\frac{2}{3}}(c+dx)-3}d\left(2\sqrt[3]{\coth(c+dx)}-1\right)+\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}\right)\right.}{}\right.}{}$$

$\downarrow$ 217

$$\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(\frac{1}{2}\int -\right.\right.}{}\right.}{(b}$$

$\downarrow$ 1103

$$\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)\right)+\frac{1}{6}\left(\frac{1}{2}\log\left(\coth^{\frac{2}{3}}(c+dx\right.\right.\right.}{d}\right.}{\left(b\coth^2(c+\right.}$$

input
```
Int[(b*Coth[c + d*x]^2)^(-2/3),x]
```

output
```
(Coth[c + d*x]^(4/3)*(-3/(d*Coth[c + d*x]^(1/3)) + (3*(ArcTanh[Coth[c + d*
x]^(1/3)]/3 + (-(Sqrt[3]*ArcTan[(-1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]) - L
og[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]/2)/6 + (-(Sqrt[3]*ArcTan
[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]) + Log[1 + Coth[c + d*x]^(1/3) + Cot
h[c + d*x]^(2/3)]/2)/6))/d))/(b*Coth[c + d*x]^2)^(2/3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \frac{1}{\left(b \coth \left(dx+c\right)^{2}\right)^{\frac{2}{3}}}dx$$

input
```
int(1/(b*coth(d*x+c)^2)^(2/3),x)
```

output
```
int(1/(b*coth(d*x+c)^2)^(2/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2066 vs. 2(196) = 392.

Time = 0.14 (sec) , antiderivative size = 2066, normalized size of antiderivative = 8.75

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*coth(d*x+c)^2)^(2/3),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*si
nh(d*x + c)^2 + b)*sqrt(-(-b^2)^(1/3))*arctan(1/3*(2*sqrt(3)*(-b^2)^(2/3)*
(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sq
rt(-(-b^2)^(1/3))*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x +
 c)^2 + sinh(d*x + c)^2 - 1))^(1/3) - sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cos
h(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*(-b^2)^(1/3)*sqrt(-(-b^2
)^(1/3)))/(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*s
inh(d*x + c)^2 + b^2)) + 2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*
sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*(b^2)^(1/6)*arctan(1/3*sqrt(3)*(b^2
)^(1/6)*(2*(b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*
x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x
 + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*(b^2)^(1/3))/(b^2*cosh(d*x +
c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b^2)) + (
-b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
c)^2 + 1)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh
(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x +
 c)^4 - 1)*(-b^2)^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh
(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (b*cosh(d*x + c)^4 + 4*b*cosh(
d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*...
```

Sympy [F]

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx = \int \frac{1}{\left(b\coth^2\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)**2)**(2/3),x)
```

output
```
Integral((b*coth(c + d*x)**2)**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^2\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^2)^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^2\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^2)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^2)^(-2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx = \int \frac{1}{\left(b\coth\left(c+d\,x\right)^2\right)^{2/3}}\,dx$$

input
```
int(1/(b*coth(c + d*x)^2)^(2/3),x)
```

output
```
int(1/(b*coth(c + d*x)^2)^(2/3), x)
```

Reduce [F]

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx = \frac{\int \frac{1}{\coth(dx+c)^{\frac{4}{3}}}dx}{b^{\frac{2}{3}}}$$

input
```
int(1/(b*coth(d*x+c)^2)^(2/3),x)
```

output
```
int(1/(coth(c + d*x)**(1/3)*coth(c + d*x)),x)/b**(2/3)
```

3.27 $\int \frac{1}{\left(b\coth^2(c+dx)\right)^{4/3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 253

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{4/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^2(c+dx)}} + \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^2(c+dx)}} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{bd\sqrt[3]{b\coth^2(c+dx)}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\coth(c+dx)}}{1+\coth^{\frac{2}{3}}(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^2(c+dx)}} - \frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}}$$

output
```
-1/2*3^(1/2)*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(2/3)
/b/d/(b*coth(d*x+c)^2)^(1/3)+1/2*3^(1/2)*arctan(1/3*(1+2*coth(d*x+c)^(1/3)
)*3^(1/2))*coth(d*x+c)^(2/3)/b/d/(b*coth(d*x+c)^2)^(1/3)+arctanh(coth(d*x+
c)^(1/3))*coth(d*x+c)^(2/3)/b/d/(b*coth(d*x+c)^2)^(1/3)+1/2*arctanh(coth(d
*x+c)^(1/3)/(1+coth(d*x+c)^(2/3)))*coth(d*x+c)^(2/3)/b/d/(b*coth(d*x+c)^2)
^(1/3)-3/5*tanh(d*x+c)/b/d/(b*coth(d*x+c)^2)^(1/3)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{4/3}}\,dx =$$

$$-\frac{\coth(c+dx)\left(6+5\coth^2(c+dx)^{5/6}\log\left(1-\sqrt[6]{\coth^2(c+dx)}\right)-5\coth^2(c+dx)^{5/6}\log\left(1+\sqrt[6]{\coth^2(\right.\right.}{}$$

input
```
Integrate[(b*Coth[c + d*x]^2)^(-4/3),x]
```

output
```
-1/10*(Coth[c + d*x]*(6 + 5*(Coth[c + d*x]^2)^(5/6)*Log[1 - (Coth[c + d*x]
^2)^(1/6)] - 5*(Coth[c + d*x]^2)^(5/6)*Log[1 + (Coth[c + d*x]^2)^(1/6)] -
5*(-1)^(2/3)*(Coth[c + d*x]^2)^(5/6)*Log[1 - (-1)^(1/3)*(Coth[c + d*x]^2)^
(1/6)] + 5*(-1)^(2/3)*(Coth[c + d*x]^2)^(5/6)*Log[1 + (-1)^(1/3)*(Coth[c +
 d*x]^2)^(1/6)] - 5*(-1)^(1/3)*(Coth[c + d*x]^2)^(5/6)*Log[1 - (-1)^(2/3)*
(Coth[c + d*x]^2)^(1/6)] + 5*(-1)^(1/3)*(Coth[c + d*x]^2)^(5/6)*Log[1 + (-
1)^(2/3)*(Coth[c + d*x]^2)^(1/6)]))/(d*(b*Coth[c + d*x]^2)^(4/3))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.74, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {3042, 4141, 3042, 3955, 3042, 3957, 25, 266, 754, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{4/3}}dx$$

$\Big\downarrow$ 3042

$$\int \frac{1}{\left(-b\tan\left(ic+idx+\frac{\pi}{2}\right)^2\right)^{4/3}}dx$$

$\Big\downarrow$ 4141

$$\frac{\coth^{\frac{2}{3}}(c+dx)\int \frac{1}{\coth^{\frac{8}{3}}(c+dx)}dx}{b\sqrt[3]{b\coth^2(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\coth^{\frac{2}{3}}(c+dx)\int \frac{1}{\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{8/3}}dx}{b\sqrt[3]{b\coth^2(c+dx)}}$$

$\Big\downarrow$ 3955

$$\frac{\coth^{\frac{2}{3}}(c+dx)\left(\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}dx-\frac{3}{5d\coth^{\frac{5}{3}}(c+dx)}\right)}{b\sqrt[3]{b\coth^2(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\coth^{\frac{2}{3}}(c+dx)\left(-\frac{3}{5d\coth^{\frac{5}{3}}(c+dx)}+\int \frac{1}{\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{2/3}}dx\right)}{b\sqrt[3]{b\coth^2(c+dx)}}$$

$\Big\downarrow$ 3957

$$\frac{\coth^{\frac{2}{3}}(c+dx)\left(-\frac{\int -\frac{1}{\coth^{\frac{2}{3}}(c+dx)\left(1-\coth^2(c+dx)\right)}d\coth(c+dx)}{d}-\frac{3}{5d\coth^{\frac{5}{3}}(c+dx)}\right)}{b\sqrt[3]{b\coth^2(c+dx)}}$$

$\Big\downarrow$ 25

$$\frac{\coth^{\frac{2}{3}}(c+dx)\left(\frac{\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)\left(1-\coth^2(c+dx)\right)}d\coth(c+dx)}{d}-\frac{3}{5d\coth^{\frac{5}{3}}(c+dx)}\right)}{b\sqrt[3]{b\coth^2(c+dx)}}$$

$$\downarrow 266$$

$$\frac{\coth^{\frac{2}{3}}(c+dx)\left(\frac{3\int\frac{1}{1-\coth^2(c+dx)}d\sqrt[3]{\coth(c+dx)}}{d}-\frac{3}{5d\coth^{\frac{5}{3}}(c+dx)}\right)}{b\sqrt[3]{b\coth^2(c+dx)}}$$

$$\downarrow 754$$

$$\frac{\coth^{\frac{2}{3}}(c+dx)\left(\frac{3\left(\frac{1}{3}\int\frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{3}\int\frac{2-\sqrt[3]{\coth(c+dx)}}{2\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{3}\int\frac{}{2\left(\coth\right.}\right.}{d}\right.}{b\sqrt[3]{b\coth^2(c+dx)}}$$

$$\downarrow 27$$

$$\frac{\coth^{\frac{2}{3}}(c+dx)\left(\frac{3\left(\frac{1}{3}\int\frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{6}\int\frac{2-\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\frac{1}{6}\int\frac{\sqrt[3]{}}{\coth^{\frac{2}{3}}(c+}\right.}{d}\right.}{b\sqrt[3]{b\coth^2(c+dx)}}$$

$$\downarrow 219$$

$$\frac{\coth^{\frac{2}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\int\frac{2-\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\frac{1}{6}\int\frac{\sqrt[3]{\coth(c+dx)}+2}{\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+\right.}{d}\right.}{b\sqrt[3]{b\coth^2(c+dx)}}$$

$$\downarrow 1142$$

$$\frac{\coth^{\frac{2}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{2}\int-\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\cot\right.\right.}{}\right.}{}$$

$\downarrow$ 25

$$\frac{\coth^{\frac{2}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(\right.\right.}{}\right.}{}$$

$\downarrow$ 1083

$$\frac{\coth^{\frac{2}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-3\int\frac{1}{-\coth^{\frac{2}{3}}(c+dx)-3}d\left(2\sqrt[3]{\coth(c+dx)}-1\right)\right)\right.}{}\right.}{}$$

$\downarrow$ 217

$$\frac{\coth^{\frac{2}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(\frac{1}{2}\int\right.\right.}{}\right.}{b\sqrt[3]{}}$$

$\downarrow$ 1103

$$\frac{\coth^{\frac{2}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)\right)+\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\cot}}{}\right.\right.\right.}{d}\right.}{b\sqrt[3]{b\coth^2(c+}}$$

input
```
Int[(b*Coth[c + d*x]^2)^(-4/3),x]
```

output
```
(Coth[c + d*x]^(2/3)*(-3/(5*d*Coth[c + d*x]^(5/3)) + (3*(ArcTanh[Coth[c +
d*x]^(1/3)]/3 + (Sqrt[3]*ArcTan[(-1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - Lo
g[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]/2)/6 + (Sqrt[3]*ArcTan[(1
 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] + Log[1 + Coth[c + d*x]^(1/3) + Coth[c
+ d*x]^(2/3)]/2)/6))/d))/(b*(b*Coth[c + d*x]^2)^(1/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \frac{1}{\left(b \coth \left(d x+c\right)^{2}\right)^{\frac{4}{3}}}d x$$

input
```
int(1/(b*coth(d*x+c)^2)^(4/3),x)
```

output
```
int(1/(b*coth(d*x+c)^2)^(4/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3099 vs. $2(211) = 422$.

Time = 0.31 (sec) , antiderivative size = 14359, normalized size of antiderivative = 56.75

$$\int \frac{1}{\left(b \coth^2(c+dx)\right)^{4/3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*coth(d*x+c)^2)^(4/3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{\left(b \coth^2(c+dx)\right)^{4/3}}\,dx = \int \frac{1}{\left(b \coth^{2}{\left(c + d x \right)}\right)^{\frac{4}{3}}}\, dx$$

input
```
integrate(1/(b*coth(d*x+c)**2)**(4/3),x)
```

output
```
Integral((b*coth(c + d*x)**2)**(-4/3), x)
```

Maxima [F]

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{4/3}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^2\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^2)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^2)^(-4/3), x)
```

Giac [F]

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{4/3}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^2\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^2)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^2)^(-4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{4/3}}\,dx = \int \frac{1}{\left(b\coth\left(c+d\,x\right)^2\right)^{4/3}}\,dx$$

input
```
int(1/(b*coth(c + d*x)^2)^(4/3),x)
```

output
```
int(1/(b*coth(c + d*x)^2)^(4/3), x)
```

Reduce [F]

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{4/3}}\,dx = \frac{\int \frac{1}{\coth(dx+c)^{\frac{8}{3}}}dx}{b^{\frac{4}{3}}}$$

input
```
int(1/(b*coth(d*x+c)^2)^(4/3),x)
```

output
```
int(1/(coth(c + d*x)**(2/3)*coth(c + d*x)**2),x)/(b**(1/3)*b)
```

3.28 $\int \left(b \coth^3(c+dx)\right)^n \, dx$

Optimal result

Integrand size = 12, antiderivative size = 55

$$\int \left(b \coth^3(c+dx)\right)^n \, dx = \frac{\coth(c+dx) \left(b \coth^3(c+dx)\right)^n \text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+3n), \frac{3(1+n)}{2}, \coth^2(c+dx)\right)}{d(1+3n)}$$

output
```
coth(d*x+c)*(b*coth(d*x+c)^3)^n*hypergeom([1, 1/2+3/2*n],[3/2+3/2*n],coth(
d*x+c)^2)/d/(1+3*n)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \left(b \coth^3(c+dx)\right)^n \, dx = \frac{\coth(c+dx) \left(b \coth^3(c+dx)\right)^n \text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+3n), \frac{3(1+n)}{2}, \coth^2(c+dx)\right)}{d(1+3n)}$$

input
```
Integrate[(b*Coth[c + d*x]^3)^n,x]
```

output
```
(Coth[c + d*x]*(b*Coth[c + d*x]^3)^n*Hypergeometric2F1[1, (1 + 3*n)/2, (3*
(1 + n))/2, Coth[c + d*x]^2])/(d*(1 + 3*n))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4141, 3042, 3957, 25, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\coth^3(c+dx)\right)^n dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(ib\tan\left(ic+idx+\frac{\pi}{2}\right)^3\right)^n dx$$

$$\Big\downarrow \text{4141}$$

$$\coth^{-3n}(c+dx)\left(b\coth^3(c+dx)\right)^n\int \coth^{3n}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\coth^{-3n}(c+dx)\left(b\coth^3(c+dx)\right)^n\int \left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{3n}dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{\coth^{-3n}(c+dx)\left(b\coth^3(c+dx)\right)^n\int -\frac{\coth^{3n}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\coth^{-3n}(c+dx)\left(b\coth^3(c+dx)\right)^n\int \frac{\coth^{3n}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{\coth(c+dx)\left(b\coth^3(c+dx)\right)^n\text{Hypergeometric2F1}\left(1,\frac{1}{2}(3n+1),\frac{3(n+1)}{2},\coth^2(c+dx)\right)}{d(3n+1)}$$

input
```
Int[(b*Coth[c + d*x]^3)^n,x]
```

output
```
(Coth[c + d*x]*(b*Coth[c + d*x]^3)^n*Hypergeometric2F1[1, (1 + 3*n)/2, (3*
(1 + n))/2, Coth[c + d*x]^2])/(d*(1 + 3*n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \left(b \coth \left(dx+c\right)^{3}\right)^{n}dx$$

input
```
int((b*coth(d*x+c)^3)^n,x)
```

output
```
int((b*coth(d*x+c)^3)^n,x)
```

Fricas [F]

$$\int \left(b\coth^3(c+dx)\right)^n \, dx = \int \left(b \coth \left(dx+c\right)^{3}\right)^{n} \, dx$$

input
```
integrate((b*coth(d*x+c)^3)^n,x, algorithm="fricas")
```

output
```
integral((b*coth(d*x + c)^3)^n, x)
```

Sympy [F]

$$\int \left(b\coth^3(c+dx)\right)^n \, dx = \int \left(b \coth^{3}\left(c+dx\right)\right)^{n}\, dx$$

input
```
integrate((b*coth(d*x+c)**3)**n,x)
```

output
```
Integral((b*coth(c + d*x)**3)**n, x)
```

Maxima [F]

$$\int \left(b\coth^3(c+dx)\right)^n\,dx=\int \left(b\coth\left(dx+c\right)^3\right)^n\,dx$$

input
```
integrate((b*coth(d*x+c)^3)^n,x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^3)^n, x)
```

Giac [F]

$$\int \left(b\coth^3(c+dx)\right)^n\,dx=\int \left(b\coth\left(dx+c\right)^3\right)^n\,dx$$

input
```
integrate((b*coth(d*x+c)^3)^n,x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^3)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\coth^3(c+dx)\right)^n\,dx=\int \left(b\coth(c+d\,x)^3\right)^n\,dx$$

input
```
int((b*coth(c + d*x)^3)^n,x)
```

output
```
int((b*coth(c + d*x)^3)^n, x)
```

Reduce [F]

$$\int \left(b\coth^3(c+dx)\right)^n\,dx = b^n\left(\int \coth\left(dx+c\right)^{3n}dx\right)$$

input
```
int((b*coth(d*x+c)^3)^n,x)
```

output
```
b**n*int(coth(c + d*x)**(3*n),x)
```

3.29 $\int \left(b\coth^3(c+dx)\right)^{3/2} dx$

Optimal result

Integrand size = 14, antiderivative size = 134

$$\begin{aligned}\int \left(b\coth^3(c+dx)\right)^{3/2} dx = &-\frac{2b\sqrt{b\coth^3(c+dx)}}{3d}\\ &-\frac{b\arctan\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)}\\ &+\frac{b\text{arctanh}\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)}\\ &-\frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d}\end{aligned}$$

output
```
-2/3*b*(b*coth(d*x+c)^3)^(1/2)/d-b*arctan(coth(d*x+c)^(1/2))*(b*coth(d*x+c
)^3)^(1/2)/d/coth(d*x+c)^(3/2)+b*arctanh(coth(d*x+c)^(1/2))*(b*coth(d*x+c)
^3)^(1/2)/d/coth(d*x+c)^(3/2)-2/7*b*coth(d*x+c)^2*(b*coth(d*x+c)^3)^(1/2)/
d
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.61

$$\int \left(b\coth^3(c+dx)\right)^{3/2}\,dx =$$
$$-\frac{\left(b\coth^3(c+dx)\right)^{3/2}\left(\arctan\left(\sqrt{\coth(c+dx)}\right)-\operatorname{arctanh}\left(\sqrt{\coth(c+dx)}\right)+\frac{2}{3}\coth^{\frac{3}{2}}(c+dx)+\frac{2}{7}\coth\right.}{d\coth^{\frac{9}{2}}(c+dx)}$$

input
```
Integrate[(b*Coth[c + d*x]^3)^(3/2),x]
```

output
```
-(((b*Coth[c + d*x]^3)^(3/2)*(ArcTan[Sqrt[Coth[c + d*x]]] - ArcTanh[Sqrt[C
oth[c + d*x]]] + (2*Coth[c + d*x]^(3/2))/3 + (2*Coth[c + d*x]^(7/2))/7))/(
d*Coth[c + d*x]^(9/2)))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.72, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {3042, 4141, 3042, 3954, 3042, 3954, 3042, 3957, 25, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\coth^3(c+dx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(ib\tan\left(ic+idx+\frac{\pi}{2}\right)^3\right)^{3/2}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{b\sqrt{b\coth^3(c+dx)}\int \coth^{\frac{9}{2}}(c+dx)dx}{\coth^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\coth^3(c+dx)}\int\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{9/2}dx}{\coth^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3954}$$

$$\frac{b\sqrt{b\coth^3(c+dx)}\left(\int\coth^{\frac{5}{2}}(c+dx)dx-\frac{2\coth^{\frac{7}{2}}(c+dx)}{7d}\right)}{\coth^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\coth^3(c+dx)}\left(-\frac{2\coth^{\frac{7}{2}}(c+dx)}{7d}+\int\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{5/2}dx\right)}{\coth^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3954}$$

$$\frac{b\sqrt{b\coth^3(c+dx)}\left(\int\sqrt{\coth(c+dx)}dx-\frac{2\coth^{\frac{7}{2}}(c+dx)}{7d}-\frac{2\coth^{\frac{3}{2}}(c+dx)}{3d}\right)}{\coth^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\coth^3(c+dx)}\left(\int\sqrt{-i\tan\left(ic+idx+\frac{\pi}{2}\right)}dx-\frac{2\coth^{\frac{7}{2}}(c+dx)}{7d}-\frac{2\coth^{\frac{3}{2}}(c+dx)}{3d}\right)}{\coth^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3957}$$

$$\frac{b\sqrt{b\coth^3(c+dx)}\left(-\frac{\int-\frac{\sqrt{\coth(c+dx)}}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}-\frac{2\coth^{\frac{7}{2}}(c+dx)}{7d}-\frac{2\coth^{\frac{3}{2}}(c+dx)}{3d}\right)}{\coth^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\sqrt{b\coth^3(c+dx)}\left(\frac{\int\frac{\sqrt{\coth(c+dx)}}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}-\frac{2\coth^{\frac{7}{2}}(c+dx)}{7d}-\frac{2\coth^{\frac{3}{2}}(c+dx)}{3d}\right)}{\coth^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{266}$$

$$\frac{b\sqrt{b\coth^3(c+dx)}\left(\frac{2\int\frac{\coth(c+dx)}{1-\coth^2(c+dx)}d\sqrt{\coth(c+dx)}}{d}-\frac{2\coth^{\frac{7}{2}}(c+dx)}{7d}-\frac{2\coth^{\frac{3}{2}}(c+dx)}{3d}\right)}{\coth^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{827}$$

$$\frac{b\sqrt{b\coth^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(c+dx)}d\sqrt{\coth(c+dx)}-\frac{1}{2}\int \frac{1}{\coth(c+dx)+1}d\sqrt{\coth(c+dx)}\right)}{d}-\frac{2\coth^{\frac{7}{2}}(c+dx)}{7d}-\frac{2\coth^{\frac{3}{2}}(c+dx)}{3d}\right)}{\coth^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{216}$$

$$\frac{b\sqrt{b\coth^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(c+dx)}d\sqrt{\coth(c+dx)}-\frac{1}{2}\arctan\left(\sqrt{\coth(c+dx)}\right)\right)}{d}-\frac{2\coth^{\frac{7}{2}}(c+dx)}{7d}-\frac{2\coth^{\frac{3}{2}}(c+dx)}{3d}\right)}{\coth^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\sqrt{b\coth^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\coth(c+dx)}\right)-\frac{1}{2}\arctan\left(\sqrt{\coth(c+dx)}\right)\right)}{d}-\frac{2\coth^{\frac{7}{2}}(c+dx)}{7d}-\frac{2\coth^{\frac{3}{2}}(c+dx)}{3d}\right)}{\coth^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(b*Coth[c + d*x]^3)^(3/2),x]
```

output
```
(b*Sqrt[b*Coth[c + d*x]^3]*((2*(-1/2*ArcTan[Sqrt[Coth[c + d*x]]] + ArcTanh
[Sqrt[Coth[c + d*x]]]/2))/d - (2*Coth[c + d*x]^(3/2))/(3*d) - (2*Coth[c +
d*x]^(7/2))/(7*d)))/Coth[c + d*x]^(3/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{\left(b\coth(dx+c)^3\right)^{\frac{3}{2}}\left(21b^{\frac{7}{2}}\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)-21b^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)+6(b\coth(dx+c))^{\frac{7}{2}}+14b^2(b\cot\right.}{21d\coth(dx+c)^3(b\coth(dx+c))^{\frac{3}{2}}b^2}$
default	$-\frac{\left(b\coth(dx+c)^3\right)^{\frac{3}{2}}\left(21b^{\frac{7}{2}}\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)-21b^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)+6(b\coth(dx+c))^{\frac{7}{2}}+14b^2(b\cot\right.}{21d\coth(dx+c)^3(b\coth(dx+c))^{\frac{3}{2}}b^2}$

input
```
int((b*coth(d*x+c)^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21/d*(b*coth(d*x+c)^3)^(3/2)*(21*b^(7/2)*arctan((b*coth(d*x+c))^(1/2)/b
^(1/2))-21*b^(7/2)*arctanh((b*coth(d*x+c))^(1/2)/b^(1/2))+6*(b*coth(d*x+c)
)^(7/2)+14*b^2*(b*coth(d*x+c))^(3/2))/coth(d*x+c)^3/(b*coth(d*x+c))^(3/2)/
b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1044 vs. 2(114) = 228.

Time = 0.15 (sec) , antiderivative size = 2145, normalized size of antiderivative = 16.01

$$\int \left(b\coth^3(c+dx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((b*coth(d*x+c)^3)^(3/2),x, algorithm="fricas")
```

output
```
[-1/84*(42*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh
(d*x + c)^6 - 3*b*cosh(d*x + c)^4 + 3*(5*b*cosh(d*x + c)^2 - b)*sinh(d*x +
 c)^4 + 4*(5*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b*
cosh(d*x + c)^2 + 3*(5*b*cosh(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)*sinh(d
*x + c)^2 + 6*(b*cosh(d*x + c)^5 - 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*
sinh(d*x + c) - b)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh
(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x +
c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)
^2)) - 21*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(
d*x + c)^6 - 3*b*cosh(d*x + c)^4 + 3*(5*b*cosh(d*x + c)^2 - b)*sinh(d*x +
c)^4 + 4*(5*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b*c
osh(d*x + c)^2 + 3*(5*b*cosh(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*
x + c)^2 + 6*(b*cosh(d*x + c)^5 - 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*s
inh(d*x + c) - b)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*s
inh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sin
h(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*si
nh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x
+ c)) - 2*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c
)^4)) + 16*(5*b*cosh(d*x + c)^6 + 30*b*cosh(d*x + c)*sinh(d*x + c)^5 + ...
```

Sympy [F]

$$\int \left(b\coth^3(c+dx)\right)^{3/2}\,dx=\int \left(b\coth^3\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*coth(d*x+c)**3)**(3/2),x)
```

output
```
Integral((b*coth(c + d*x)**3)**(3/2), x)
```

Maxima [F]

$$\int \left(b\coth^3(c+dx)\right)^{3/2} dx = \int \left(b\coth\left(dx+c\right)^3\right)^{\frac{3}{2}} dx$$

input
```
integrate((b*coth(d*x+c)^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^3)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(b\coth^3(c+dx)\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*coth(d*x+c)^3)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\coth^3(c+dx)\right)^{3/2} dx = \int \left(b\coth(c+d\,x)^3\right)^{3/2} dx$$

input
```
int((b*coth(c + d*x)^3)^(3/2),x)
```

output
```
int((b*coth(c + d*x)^3)^(3/2), x)
```

Reduce [F]

$$\int \left(b\coth^3(c+dx)\right)^{3/2}\,dx=\sqrt{b}\left(\int\sqrt{\coth\left(dx+c\right)}\,\coth\left(dx+c\right)^4dx\right)b$$

input
```
int((b*coth(d*x+c)^3)^(3/2),x)
```

output
```
sqrt(b)*int(sqrt(coth(c + d*x))*coth(c + d*x)**4,x)*b
```

3.30 $\int \sqrt{b\coth^3(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 104

$$\int \sqrt{b\coth^3(c+dx)}\,dx = \frac{\arctan\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)} - \frac{2\sqrt{b\coth^3(c+dx)}\tanh(c+dx)}{d}$$

output
```
arctan(coth(d*x+c)^(1/2))*(b*coth(d*x+c)^3)^(1/2)/d/coth(d*x+c)^(3/2)+arct
anh(coth(d*x+c)^(1/2))*(b*coth(d*x+c)^3)^(1/2)/d/coth(d*x+c)^(3/2)-2*(b*co
th(d*x+c)^3)^(1/2)*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.61

$$\int \sqrt{b \coth^3(c+dx)}\,dx = \frac{\left(\arctan\left(\sqrt{\coth(c+dx)}\right)+\operatorname{arctanh}\left(\sqrt{\coth(c+dx)}\right)-2\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Sqrt[b*Coth[c + d*x]^3],x]
```

output
```
((ArcTan[Sqrt[Coth[c + d*x]]] + ArcTanh[Sqrt[Coth[c + d*x]]] - 2*Sqrt[Coth
[c + d*x]])*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.74, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {3042, 4141, 3042, 3954, 3042, 3957, 25, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b \coth^3(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{ib \tan\left(ic+idx+\frac{\pi}{2}\right)^3}\,dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\sqrt{b\coth^3(c+dx)}\int \coth^{\frac{3}{2}}(c+dx)dx}{\coth^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\coth^3(c+dx)}\int\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}dx}{\coth^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\sqrt{b\coth^3(c+dx)}\left(\int\frac{1}{\sqrt{\coth(c+dx)}}dx-\frac{2\sqrt{\coth(c+dx)}}{d}\right)}{\coth^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\coth^3(c+dx)}\left(-\frac{2\sqrt{\coth(c+dx)}}{d}+\int\frac{1}{\sqrt{-i\tan\left(ic+idx+\frac{\pi}{2}\right)}}dx\right)}{\coth^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\sqrt{b\coth^3(c+dx)}\left(-\frac{\int-\frac{1}{\sqrt{\coth(c+dx)}\left(1-\coth^2(c+dx)\right)}d\coth(c+dx)}{d}-\frac{2\sqrt{\coth(c+dx)}}{d}\right)}{\coth^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{b\coth^3(c+dx)}\left(\frac{\int\frac{1}{\sqrt{\coth(c+dx)}\left(1-\coth^2(c+dx)\right)}d\coth(c+dx)}{d}-\frac{2\sqrt{\coth(c+dx)}}{d}\right)}{\coth^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{266}$$

$$\frac{\sqrt{b\coth^3(c+dx)}\left(\frac{2\int\frac{1}{1-\coth^2(c+dx)}d\sqrt{\coth(c+dx)}}{d}-\frac{2\sqrt{\coth(c+dx)}}{d}\right)}{\coth^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{756}$$

$$\frac{\sqrt{b\coth^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\int\frac{1}{1-\coth(c+dx)}d\sqrt{\coth(c+dx)}+\frac{1}{2}\int\frac{1}{\coth(c+dx)+1}d\sqrt{\coth(c+dx)}\right)}{d}-\frac{2\sqrt{\coth(c+dx)}}{d}\right)}{\coth^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{b\coth^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\int\frac{1}{1-\coth(c+dx)}d\sqrt{\coth(c+dx)}+\frac{1}{2}\arctan\left(\sqrt{\coth(c+dx)}\right)\right)}{d}-\frac{2\sqrt{\coth(c+dx)}}{d}\right)}{\coth^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{b\coth^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\arctan\left(\sqrt{\coth(c+dx)}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt{\coth(c+dx)}\right)\right)}{d}-\frac{2\sqrt{\coth(c+dx)}}{d}\right)}{\coth^{\frac{3}{2}}(c+dx)}$$

input
```
Int[Sqrt[b*Coth[c + d*x]^3],x]
```

output
```
(((2*(ArcTan[Sqrt[Coth[c + d*x]]]/2 + ArcTanh[Sqrt[Coth[c + d*x]]]/2))/d -
 (2*Sqrt[Coth[c + d*x]])/d)*Sqrt[b*Coth[c + d*x]^3])/Coth[c + d*x]^(3/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\sqrt{b\coth(dx+c)^{3}}\left(-2\sqrt{b\coth(dx+c)}+\sqrt{b}\,\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)+\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)\right)}{d\coth(dx+c)\sqrt{b\coth(dx+c)}}$	86
default	$\frac{\sqrt{b\coth(dx+c)^{3}}\left(-2\sqrt{b\coth(dx+c)}+\sqrt{b}\,\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)+\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)\right)}{d\coth(dx+c)\sqrt{b\coth(dx+c)}}$	86

input
```
int((b*coth(d*x+c)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b*coth(d*x+c)^3)^(1/2)/coth(d*x+c)/(b*coth(d*x+c))^(1/2)*(-2*(b*coth(
d*x+c))^(1/2)+b^(1/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))+b^(1/2)*arctan
h((b*coth(d*x+c))^(1/2)/b^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(90) = 180$.

Time = 0.11 (sec) , antiderivative size = 627, normalized size of antiderivative = 6.03

$$\int \sqrt{b\coth^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*coth(d*x+c)^3)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*(2*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
+ sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cos
h(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2)) - sqr
t(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*co
sh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh
(d*x + c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*
x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*
x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x +
c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*sqrt(b*cosh
(d*x + c)/sinh(d*x + c)))/d, -1/4*(2*sqrt(b)*arctan((cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x + c)/sin
h(d*x + c))/sqrt(b)) - sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)
^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c
)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x
+ c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x
+ c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x
+ c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b) + 8*sqrt(b*cosh(d*x
 + c)/sinh(d*x + c)))/d]
```

Sympy [F]

$$\int \sqrt{b\coth^3(c+dx)}\,dx = \int \sqrt{b\coth^3{(c+dx)}}\,dx$$

input
```
integrate((b*coth(d*x+c)**3)**(1/2),x)
```

output
```
Integral(sqrt(b*coth(c + d*x)**3), x)
```

Maxima [F]

$$\int \sqrt{b \coth^3(c+dx)}\,dx = \int \sqrt{b \coth{(dx+c)}^3}\,dx$$

input
```
integrate((b*coth(d*x+c)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*coth(d*x + c)^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{b \coth^3(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*coth(d*x+c)^3)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b \coth^3(c+dx)}\,dx = \int \sqrt{b \coth{(c+d\,x)}^3}\,dx$$

input
```
int((b*coth(c + d*x)^3)^(1/2),x)
```

output
```
int((b*coth(c + d*x)^3)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{b \coth^3(c+dx)}\,dx = \frac{\sqrt{b}\left(-2\sqrt{\coth(dx+c)}+\left(\int \frac{\sqrt{\coth(dx+c)}}{\coth(dx+c)}dx\right)d\right)}{d}$$

input
```
int((b*coth(d*x+c)^3)^(1/2),x)
```

output
```
(sqrt(b)*( - 2*sqrt(coth(c + d*x)) + int(sqrt(coth(c + d*x))/coth(c + d*x)
,x)*d))/d
```

3.31 $\int \frac{1}{\sqrt{b \coth^3(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 105

$$\int \frac{1}{\sqrt{b \coth^3(c+dx)}}\,dx = -\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}} - \frac{\arctan\left(\sqrt{\coth(c+dx)}\right)\coth^{\frac{3}{2}}(c+dx)}{d\sqrt{b\coth^3(c+dx)}} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(c+dx)}\right)\coth^{\frac{3}{2}}(c+dx)}{d\sqrt{b\coth^3(c+dx)}}$$

output
```
-2*coth(d*x+c)/d/(b*coth(d*x+c)^3)^(1/2)-arctan(coth(d*x+c)^(1/2))*coth(d*
x+c)^(3/2)/d/(b*coth(d*x+c)^3)^(1/2)+arctanh(coth(d*x+c)^(1/2))*coth(d*x+c
)^(3/2)/d/(b*coth(d*x+c)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{b\coth^3(c+dx)}}\,dx = -\frac{\coth(c+dx)\left(2+\arctan\left(\sqrt[4]{\coth^2(c+dx)}\right)\sqrt[4]{\coth^2(c+dx)}-\operatorname{arctanh}\left(\sqrt[4]{\coth^2(c+dx)}\right)\sqrt[4]{\coth^2\right.}{d\sqrt{b\coth^3(c+dx)}}$$

input
```
Integrate[1/Sqrt[b*Coth[c + d*x]^3],x]
```

output
```
-((Coth[c + d*x]*(2 + ArcTan[(Coth[c + d*x]^2)^(1/4)]*(Coth[c + d*x]^2)^(1
/4) - ArcTanh[(Coth[c + d*x]^2)^(1/4)]*(Coth[c + d*x]^2)^(1/4)))/(d*Sqrt[b
*Coth[c + d*x]^3]))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.73, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {3042, 4141, 3042, 3955, 3042, 3957, 25, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\coth^3(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{ib\tan\left(ic+idx+\frac{\pi}{2}\right)^3}}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\coth^{\frac{3}{2}}(c+dx)\int \frac{1}{\coth^{\frac{3}{2}}(c+dx)}dx}{\sqrt{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\coth^{\frac{3}{2}}(c+dx)\int \frac{1}{\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{\sqrt{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{3955}$$

$$\frac{\coth^{\frac{3}{2}}(c+dx)\left(\int \sqrt{\coth(c+dx)}dx-\frac{2}{d\sqrt{\coth(c+dx)}}\right)}{\sqrt{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\coth^{\frac{3}{2}}(c+dx)\left(-\frac{2}{d\sqrt{\coth(c+dx)}}+\int \sqrt{-i\tan\left(ic+idx+\frac{\pi}{2}\right)}dx\right)}{\sqrt{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\coth^{\frac{3}{2}}(c+dx)\left(-\frac{\int -\frac{\sqrt{\coth(c+dx)}}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}-\frac{2}{d\sqrt{\coth(c+dx)}}\right)}{\sqrt{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\coth^{\frac{3}{2}}(c+dx)\left(\frac{\int \frac{\sqrt{\coth(c+dx)}}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}-\frac{2}{d\sqrt{\coth(c+dx)}}\right)}{\sqrt{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\coth^{\frac{3}{2}}(c+dx)\left(\frac{2\int \frac{\coth(c+dx)}{1-\coth^2(c+dx)}d\sqrt{\coth(c+dx)}}{d}-\frac{2}{d\sqrt{\coth(c+dx)}}\right)}{\sqrt{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{827}$$

$$\frac{\coth^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(c+dx)}d\sqrt{\coth(c+dx)}-\frac{1}{2}\int \frac{1}{\coth(c+dx)+1}d\sqrt{\coth(c+dx)}\right)}{d}-\frac{2}{d\sqrt{\coth(c+dx)}}\right)}{\sqrt{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\coth^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\int\frac{1}{1-\coth(c+dx)}d\sqrt{\coth(c+dx)}-\frac{1}{2}\arctan\left(\sqrt{\coth(c+dx)}\right)\right)}{d}-\frac{2}{d\sqrt{\coth(c+dx)}}\right)}{\sqrt{b\coth^3(c+dx)}}$$

$$\Big\downarrow \; 219$$

$$\frac{\coth^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\coth(c+dx)}\right)-\frac{1}{2}\arctan\left(\sqrt{\coth(c+dx)}\right)\right)}{d}-\frac{2}{d\sqrt{\coth(c+dx)}}\right)}{\sqrt{b\coth^3(c+dx)}}$$

input
```
Int[1/Sqrt[b*Coth[c + d*x]^3],x]
```

output
```
(((2*(-1/2*ArcTan[Sqrt[Coth[c + d*x]]] + ArcTanh[Sqrt[Coth[c + d*x]]]/2))/
d - 2/(d*Sqrt[Coth[c + d*x]]))*Coth[c + d*x]^(3/2))/Sqrt[b*Coth[c + d*x]^3
]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

method	result	siz
derivativedivides	$-\frac{\coth(dx+c)\left(2b^{\frac{5}{2}}+\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)b^{2}\sqrt{b\coth(dx+c)}-\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)b^{2}\sqrt{b\coth(dx+c)}\right)}{d\sqrt{b\coth(dx+c)^{3}}\,b^{\frac{5}{2}}}$	92
default	$-\frac{\coth(dx+c)\left(2b^{\frac{5}{2}}+\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)b^{2}\sqrt{b\coth(dx+c)}-\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)b^{2}\sqrt{b\coth(dx+c)}\right)}{d\sqrt{b\coth(dx+c)^{3}}\,b^{\frac{5}{2}}}$	92

input
```
int(1/(b*coth(d*x+c)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*coth(d*x+c)*(2*b^(5/2)+arctan((b*coth(d*x+c))^(1/2)/b^(1/2))*b^2*(b*c
oth(d*x+c))^(1/2)-arctanh((b*coth(d*x+c))^(1/2)/b^(1/2))*b^2*(b*coth(d*x+c
))^(1/2))/(b*coth(d*x+c)^3)^(1/2)/b^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(91) = 182$.

Time = 0.12 (sec) , antiderivative size = 899, normalized size of antiderivative = 8.56

$$\int \frac{1}{\sqrt{b\coth^3(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*coth(d*x+c)^3)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^
2 + 1)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(
d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2)) + (cosh
(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-b
)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
 + c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x +
c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2
 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*(cosh(d*x + c)^
2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x +
 c)/sinh(d*x + c)))/(b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x +
c) + b*d*sinh(d*x + c)^2 + b*d), 1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(b)*arctan((cosh(d*x + c)^2 + 2*
cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x + c)/si
nh(d*x + c))/sqrt(b)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2 + 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^
3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)
*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*...
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\coth^3(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\coth^3{(c+dx)}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)**3)**(1/2),x)
```

output
```
Integral(1/sqrt(b*coth(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\coth^3(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\coth{(dx+c)}^3}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*coth(d*x + c)^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{b\coth^3(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*coth(d*x+c)^3)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\coth^3(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\coth\left(c+d\,x\right)^3}}\,dx$$

input
```
int(1/(b*coth(c + d*x)^3)^(1/2),x)
```

output
```
int(1/(b*coth(c + d*x)^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{b\coth^3(c+dx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\coth(dx+c)}}{\coth(dx+c)^2}dx\right)}{b}$$

input
```
int(1/(b*coth(d*x+c)^3)^(1/2),x)
```

output
```
(sqrt(b)*int(sqrt(coth(c + d*x))/coth(c + d*x)**2,x))/b
```

3.32 $\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 141

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}\,dx = -\frac{2}{3bd\sqrt{b\coth^3(c+dx)}} + \frac{\arctan\left(\sqrt{\coth(c+dx)}\right)\coth^{\frac{3}{2}}(c+dx)}{bd\sqrt{b\coth^3(c+dx)}} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(c+dx)}\right)\coth^{\frac{3}{2}}(c+dx)}{bd\sqrt{b\coth^3(c+dx)}} - \frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}}$$

output
```
-2/3/b/d/(b*coth(d*x+c)^3)^(1/2)+arctan(coth(d*x+c)^(1/2))*coth(d*x+c)^(3/
2)/b/d/(b*coth(d*x+c)^3)^(1/2)+arctanh(coth(d*x+c)^(1/2))*coth(d*x+c)^(3/2
)/b/d/(b*coth(d*x+c)^3)^(1/2)-2/7*tanh(d*x+c)^2/b/d/(b*coth(d*x+c)^3)^(1/2
)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.64

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}\,dx = \frac{-14+21\arctan\left(\sqrt[4]{\coth^2(c+dx)}\right)\coth^2(c+dx)^{3/4}+21\text{arctanh}\left(\sqrt[4]{\coth^2(}\right.}{21bd\sqrt{b\coth^3(c+dx)}}$$

input
```
Integrate[(b*Coth[c + d*x]^3)^(-3/2),x]
```

output
```
(-14 + 21*ArcTan[(Coth[c + d*x]^2)^(1/4)]*(Coth[c + d*x]^2)^(3/4) + 21*Arc
Tanh[(Coth[c + d*x]^2)^(1/4)]*(Coth[c + d*x]^2)^(3/4) - 6*Tanh[c + d*x]^2)
/(21*b*d*Sqrt[b*Coth[c + d*x]^3])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.70, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {3042, 4141, 3042, 3955, 3042, 3955, 3042, 3957, 25, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(ib\tan\left(ic+idx+\frac{\pi}{2}\right)^3\right)^{3/2}}dx$$

$$\Big\downarrow \; 4141$$

$$\frac{\coth^{\frac{3}{2}}(c+dx)\int \frac{1}{\coth^{\frac{9}{2}}(c+dx)}dx}{b\sqrt{b\coth^3(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\coth^{\frac{3}{2}}(c+dx)\int\frac{1}{\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{9/2}}dx}{b\sqrt{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{3955}$$

$$\frac{\coth^{\frac{3}{2}}(c+dx)\left(\int\frac{1}{\coth^{\frac{5}{2}}(c+dx)}dx-\frac{2}{7d\coth^{\frac{7}{2}}(c+dx)}\right)}{b\sqrt{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\coth^{\frac{3}{2}}(c+dx)\left(-\frac{2}{7d\coth^{\frac{7}{2}}(c+dx)}+\int\frac{1}{\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{5/2}}dx\right)}{b\sqrt{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{3955}$$

$$\frac{\coth^{\frac{3}{2}}(c+dx)\left(\int\frac{1}{\sqrt{\coth(c+dx)}}dx-\frac{2}{3d\coth^{\frac{3}{2}}(c+dx)}-\frac{2}{7d\coth^{\frac{7}{2}}(c+dx)}\right)}{b\sqrt{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\coth^{\frac{3}{2}}(c+dx)\left(\int\frac{1}{\sqrt{-i\tan\left(ic+idx+\frac{\pi}{2}\right)}}dx-\frac{2}{3d\coth^{\frac{3}{2}}(c+dx)}-\frac{2}{7d\coth^{\frac{7}{2}}(c+dx)}\right)}{b\sqrt{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\coth^{\frac{3}{2}}(c+dx)\left(-\frac{\int-\frac{1}{\sqrt{\coth(c+dx)}\left(1-\coth^2(c+dx)\right)}d\coth(c+dx)}{d}-\frac{2}{3d\coth^{\frac{3}{2}}(c+dx)}-\frac{2}{7d\coth^{\frac{7}{2}}(c+dx)}\right)}{b\sqrt{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\coth^{\frac{3}{2}}(c+dx)\left(\frac{\int\frac{1}{\sqrt{\coth(c+dx)}\left(1-\coth^2(c+dx)\right)}d\coth(c+dx)}{d}-\frac{2}{3d\coth^{\frac{3}{2}}(c+dx)}-\frac{2}{7d\coth^{\frac{7}{2}}(c+dx)}\right)}{b\sqrt{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\coth^{\frac{3}{2}}(c+dx)\left(\frac{2\int\frac{1}{1-\coth^2(c+dx)}d\sqrt{\coth(c+dx)}}{d}-\frac{2}{3d\coth^{\frac{3}{2}}(c+dx)}-\frac{2}{7d\coth^{\frac{7}{2}}(c+dx)}\right)}{b\sqrt{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{756}$$

$$\frac{\coth^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\int\frac{1}{1-\coth(c+dx)}d\sqrt{\coth(c+dx)}+\frac{1}{2}\int\frac{1}{\coth(c+dx)+1}d\sqrt{\coth(c+dx)}\right)}{d}-\frac{2}{3d\coth^{\frac{3}{2}}(c+dx)}-\frac{2}{7d\coth^{\frac{7}{2}}(c+dx)}\right)}{b\sqrt{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\coth^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\int\frac{1}{1-\coth(c+dx)}d\sqrt{\coth(c+dx)}+\frac{1}{2}\arctan\left(\sqrt{\coth(c+dx)}\right)\right)}{d}-\frac{2}{3d\coth^{\frac{3}{2}}(c+dx)}-\frac{2}{7d\coth^{\frac{7}{2}}(c+dx)}\right)}{b\sqrt{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\coth^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\arctan\left(\sqrt{\coth(c+dx)}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt{\coth(c+dx)}\right)\right)}{d}-\frac{2}{3d\coth^{\frac{3}{2}}(c+dx)}-\frac{2}{7d\coth^{\frac{7}{2}}(c+dx)}\right)}{b\sqrt{b\coth^3(c+dx)}}$$

input
```
Int[(b*Coth[c + d*x]^3)^(-3/2),x]
```

output
```
(((2*(ArcTan[Sqrt[Coth[c + d*x]]]/2 + ArcTanh[Sqrt[Coth[c + d*x]]]/2))/d -
 2/(7*d*Coth[c + d*x]^(7/2)) - 2/(3*d*Coth[c + d*x]^(3/2)))*Coth[c + d*x]^
(3/2))/(b*Sqrt[b*Coth[c + d*x]^3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.75

method	result
derivativedivides	$-\frac{\coth(dx+c)\left(14b^{\frac{15}{2}}\coth(dx+c)^{2}+6b^{\frac{15}{2}}-21\,\mathrm{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)b^{4}(b\coth(dx+c))^{\frac{7}{2}}-21\arctan\left(\frac{\sqrt{b\coth(dx+c}}{\sqrt{b}}\right.\right.}{21d\,b^{\frac{15}{2}}\left(b\coth(dx+c)^{3}\right)^{\frac{3}{2}}}$
default	$-\frac{\coth(dx+c)\left(14b^{\frac{15}{2}}\coth(dx+c)^{2}+6b^{\frac{15}{2}}-21\,\mathrm{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)b^{4}(b\coth(dx+c))^{\frac{7}{2}}-21\arctan\left(\frac{\sqrt{b\coth(dx+c}}{\sqrt{b}}\right.\right.}{21d\,b^{\frac{15}{2}}\left(b\coth(dx+c)^{3}\right)^{\frac{3}{2}}}$

input
```
int(1/(b*coth(d*x+c)^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21/d*coth(d*x+c)/b^(15/2)*(14*b^(15/2)*coth(d*x+c)^2+6*b^(15/2)-21*arct
anh((b*coth(d*x+c))^(1/2)/b^(1/2))*b^4*(b*coth(d*x+c))^(7/2)-21*arctan((b*
coth(d*x+c))^(1/2)/b^(1/2))*b^4*(b*coth(d*x+c))^(7/2))/(b*coth(d*x+c)^3)^(
3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1479 vs. 2(121) = 242.

Time = 0.19 (sec) , antiderivative size = 3015, normalized size of antiderivative = 21.38

$$\int \frac{1}{\left(b\coth^{3}(c+dx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*coth(d*x+c)^3)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\coth^3\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)**3)**(3/2),x)
```

output
```
Integral((b*coth(c + d*x)**3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^3)^(-3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*coth(d*x+c)^3)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\coth\left(c+d\,x\right)^3\right)^{3/2}}\,dx$$

input
```
int(1/(b*coth(c + d*x)^3)^(3/2),x)
```

output
```
int(1/(b*coth(c + d*x)^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\coth(dx+c)}}{\coth(dx+c)^5}dx\right)}{b^2}$$

input
```
int(1/(b*coth(d*x+c)^3)^(3/2),x)
```

output
```
(sqrt(b)*int(sqrt(coth(c + d*x))/coth(c + d*x)**5,x))/b**2
```

3.33 $\int \left(b\coth^3(c+dx)\right)^{4/3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 74

$$\int \left(b\coth^3(c+dx)\right)^{4/3}\,dx = -\frac{b\sqrt[3]{b\coth^3(c+dx)}}{d} - \frac{b\coth^2(c+dx)\sqrt[3]{b\coth^3(c+dx)}}{3d} + bx\sqrt[3]{b\coth^3(c+dx)}\tanh(c+dx)$$

output
```
-b*(b*coth(d*x+c)^3)^(1/3)/d-1/3*b*coth(d*x+c)^2*(b*coth(d*x+c)^3)^(1/3)/d
+b*x*(b*coth(d*x+c)^3)^(1/3)*tanh(d*x+c)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.58

$$\int \left(b\coth^3(c+dx)\right)^{4/3}\,dx = -\frac{\left(b\coth^3(c+dx)\right)^{4/3}\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\tanh^2(c+dx)\right)\tanh(c+dx)}{3d}$$

input
```
Integrate[(b*Coth[c + d*x]^3)^(4/3),x]
```

output
```
-1/3*((b*Coth[c + d*x]^3)^(4/3)*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[c +
d*x]^2]*Tanh[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3042, 4141, 3042, 3954, 25, 3042, 25, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\coth^3(c+dx)\right)^{4/3} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(ib\tan\left(ic+idx+\frac{\pi}{2}\right)^3\right)^{4/3} dx$$

$$\Big\downarrow \text{4141}$$

$$b\tanh(c+dx)\sqrt[3]{b\coth^3(c+dx)}\int \coth^4(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$b\tanh(c+dx)\sqrt[3]{b\coth^3(c+dx)}\int \tan\left(ic+idx+\frac{\pi}{2}\right)^4 dx$$

$$\Big\downarrow \text{3954}$$

$$b\tanh(c+dx)\sqrt[3]{b\coth^3(c+dx)}\left(-\int -\coth^2(c+dx)dx-\frac{\coth^3(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{25}$$

$$b\tanh(c+dx)\sqrt[3]{b\coth^3(c+dx)}\left(\int \coth^2(c+dx)dx-\frac{\coth^3(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\tanh(c+dx)\sqrt[3]{b\coth^3(c+dx)}\left(-\frac{\coth^3(c+dx)}{3d}+\int -\tan\left(ic+idx+\frac{\pi}{2}\right)^2dx\right)$$

$$\Big\downarrow \text{25}$$

$$b\tanh(c+dx)\sqrt[3]{b\coth^3(c+dx)}\left(-\frac{\coth^3(c+dx)}{3d}-\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx\right)$$

$$\Big\downarrow \text{3954}$$

$$b\tanh(c+dx)\sqrt[3]{b\coth^3(c+dx)}\left(\int 1dx-\frac{\coth^3(c+dx)}{3d}-\frac{\coth(c+dx)}{d}\right)$$

$$\Big\downarrow \text{24}$$

$$b\tanh(c+dx)\left(-\frac{\coth^3(c+dx)}{3d}-\frac{\coth(c+dx)}{d}+x\right)\sqrt[3]{b\coth^3(c+dx)}$$

input
```
Int[(b*Coth[c + d*x]^3)^(4/3),x]
```

output
```
b*(b*Coth[c + d*x]^3)^(1/3)*(x - Coth[c + d*x]/d - Coth[c + d*x]^3/(3*d))*
Tanh[c + d*x]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.76

method	result	size
risch	$-\frac{b\left(\frac{b\left(\mathrm{e}^{2dx+2c}+1\right)^{3}}{\left(\mathrm{e}^{2dx+2c}-1\right)^{3}}\right)^{\frac{1}{3}}\left(-3\,\mathrm{e}^{6dx+6c}dx+9\,\mathrm{e}^{4dx+4c}dx-9\,\mathrm{e}^{2dx+2c}dx+3dx+12\,\mathrm{e}^{4dx+4c}-12\,\mathrm{e}^{2dx+2c}+8\right)}{3\left(\mathrm{e}^{2dx+2c}+1\right)\left(\mathrm{e}^{2dx+2c}-1\right)^{2}d}$	130

input
```
int((b*coth(d*x+c)^3)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-1/3*b*(b*(exp(2*d*x+2*c)+1)^3/(exp(2*d*x+2*c)-1)^3)^(1/3)*(-3*exp(6*d*x+6
*c)*d*x+9*exp(4*d*x+4*c)*d*x-9*exp(2*d*x+2*c)*d*x+3*d*x+12*exp(4*d*x+4*c)-
12*exp(2*d*x+2*c)+8)/(exp(2*d*x+2*c)+1)/(exp(2*d*x+2*c)-1)^2/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. 2(66) = 132.

Time = 0.11 (sec) , antiderivative size = 1046, normalized size of antiderivative = 14.14

$$\int \left(b\coth^3(c+dx)\right)^{4/3}\,dx = \text{Too large to display}$$

input
```
integrate((b*coth(d*x+c)^3)^(4/3),x, algorithm="fricas")
```

output
```
-1/3*(3*b*d*x*cosh(d*x + c)^6 - 3*(b*d*x*e^(2*d*x + 2*c) - b*d*x)*sinh(d*x
 + c)^6 - 18*(b*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) - b*d*x*cosh(d*x + c))*s
inh(d*x + c)^5 - 3*(3*b*d*x + 4*b)*cosh(d*x + c)^4 + 3*(15*b*d*x*cosh(d*x
+ c)^2 - 3*b*d*x - (15*b*d*x*cosh(d*x + c)^2 - 3*b*d*x - 4*b)*e^(2*d*x + 2
*c) - 4*b)*sinh(d*x + c)^4 + 12*(5*b*d*x*cosh(d*x + c)^3 - (3*b*d*x + 4*b)
*cosh(d*x + c) - (5*b*d*x*cosh(d*x + c)^3 - (3*b*d*x + 4*b)*cosh(d*x + c))
*e^(2*d*x + 2*c))*sinh(d*x + c)^3 - 3*b*d*x + 3*(3*b*d*x + 4*b)*cosh(d*x +
 c)^2 + 3*(15*b*d*x*cosh(d*x + c)^4 + 3*b*d*x - 6*(3*b*d*x + 4*b)*cosh(d*x
 + c)^2 - (15*b*d*x*cosh(d*x + c)^4 + 3*b*d*x - 6*(3*b*d*x + 4*b)*cosh(d*x
 + c)^2 + 4*b)*e^(2*d*x + 2*c) + 4*b)*sinh(d*x + c)^2 - (3*b*d*x*cosh(d*x
+ c)^6 - 3*(3*b*d*x + 4*b)*cosh(d*x + c)^4 - 3*b*d*x + 3*(3*b*d*x + 4*b)*c
osh(d*x + c)^2 - 8*b)*e^(2*d*x + 2*c) + 6*(3*b*d*x*cosh(d*x + c)^5 - 2*(3*
b*d*x + 4*b)*cosh(d*x + c)^3 + (3*b*d*x + 4*b)*cosh(d*x + c) - (3*b*d*x*co
sh(d*x + c)^5 - 2*(3*b*d*x + 4*b)*cosh(d*x + c)^3 + (3*b*d*x + 4*b)*cosh(d
*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c) - 8*b)*((b*e^(6*d*x + 6*c) + 3*b*e
^(4*d*x + 4*c) + 3*b*e^(2*d*x + 2*c) + b)/(e^(6*d*x + 6*c) - 3*e^(4*d*x +
4*c) + 3*e^(2*d*x + 2*c) - 1))^(1/3)/(d*cosh(d*x + c)^6 + (d*e^(2*d*x + 2*
c) + d)*sinh(d*x + c)^6 + 6*(d*cosh(d*x + c)*e^(2*d*x + 2*c) + d*cosh(d*x
+ c))*sinh(d*x + c)^5 - 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + (5*
d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) - d)*sinh(d*x + c)^4 + 4*(5*d*co...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(66) = 132$.

Time = 65.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.19

$$\int \left(b\coth^3(c+dx)\right)^{4/3}\,dx = \begin{cases} x\left(b\coth^3(c)\right)^{\frac{4}{3}} & \text{for } d \\ -\frac{\left(b\coth^3\left(dx+\log\left(-e^{-dx}\right)\right)\right)^{\frac{4}{3}}\log\left(-e^{-dx}\right)}{d} & \text{for } c \\ -\frac{\left(b\coth^3\left(dx+\log\left(e^{-dx}\right)\right)\right)^{\frac{4}{3}}\log\left(e^{-dx}\right)}{d} & \text{for } c \\ x\left(\frac{b}{\tanh^3(c+dx)}\right)^{\frac{4}{3}}\tanh^4(c+dx) - \frac{\left(\frac{b}{\tanh^3(c+dx)}\right)^{\frac{4}{3}}\tanh^3(c+dx)}{d} - \frac{\left(\frac{b}{\tanh^3(c+dx)}\right)^{\frac{4}{3}}\tanh(c+dx)}{3d} & \text{other} \end{cases}$$

input
```
integrate((b*coth(d*x+c)**3)**(4/3),x)
```

output
```
Piecewise((x*(b*coth(c)**3)**(4/3), Eq(d, 0)), (-(b*coth(d*x + log(-exp(-d
*x)))**3)**(4/3)*log(-exp(-d*x))/d, Eq(c, log(-exp(-d*x)))), (-(b*coth(d*x
 + log(exp(-d*x)))**3)**(4/3)*log(exp(-d*x))/d, Eq(c, log(exp(-d*x)))), (x
*(b/tanh(c + d*x)**3)**(4/3)*tanh(c + d*x)**4 - (b/tanh(c + d*x)**3)**(4/3
)*tanh(c + d*x)**3/d - (b/tanh(c + d*x)**3)**(4/3)*tanh(c + d*x)/(3*d), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.18

$$\int \left(b\coth^3(c+dx)\right)^{4/3}\,dx = \frac{(dx+c)b^{\frac{4}{3}}}{d} - \frac{4\left(3\,b^{\frac{4}{3}}e^{(-2\,dx-2\,c)} - 3\,b^{\frac{4}{3}}e^{(-4\,dx-4\,c)} - 2\,b^{\frac{4}{3}}\right)}{3\,d(3\,e^{(-2\,dx-2\,c)} - 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} - 1)}$$

input
```
integrate((b*coth(d*x+c)^3)^(4/3),x, algorithm="maxima")
```

output
```
(d*x + c)*b^(4/3)/d - 4/3*(3*b^(4/3)*e^(-2*d*x - 2*c) - 3*b^(4/3)*e^(-4*d*
x - 4*c) - 2*b^(4/3))/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*
d*x - 6*c) - 1))
```

Giac [F]

$$\int \left(b\coth^3(c+dx)\right)^{4/3}\,dx = \int \left(b\coth\left(dx+c\right)^3\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((b*coth(d*x+c)^3)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^3)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\coth^3(c+dx)\right)^{4/3}\,dx=\int \left(b\coth(c+d\,x)^3\right)^{4/3}\,dx$$

input
```
int((b*coth(c + d*x)^3)^(4/3),x)
```

output
```
int((b*coth(c + d*x)^3)^(4/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.42

$$\int \left(b\coth^3(c+dx)\right)^{4/3}\,dx=\frac{b^{\frac{4}{3}}\left(-\coth\left(dx+c\right)^3-3\coth\left(dx+c\right)+3dx\right)}{3d}$$

input
```
int((b*coth(d*x+c)^3)^(4/3),x)
```

output
```
(b**(1/3)*b*( - coth(c + d*x)**3 - 3*coth(c + d*x) + 3*d*x))/(3*d)
```

3.34 $\int \left(b\coth^3(c+dx)\right)^{2/3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \left(b\coth^3(c+dx)\right)^{2/3}\,dx=-\frac{\left(b\coth^3(c+dx)\right)^{2/3}\tanh(c+dx)}{d}+x\left(b\coth^3(c+dx)\right)^{2/3}\tanh^2(c+dx)$$

output
```
-(b*coth(d*x+c)^3)^(2/3)*tanh(d*x+c)/d+x*(b*coth(d*x+c)^3)^(2/3)*tanh(d*x+
c)^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \left(b\coth^3(c+dx)\right)^{2/3}\,dx=-\frac{\left(b\coth^3(c+dx)\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(c+dx)\right)\tanh(c+dx)}{d}$$

input
```
Integrate[(b*Coth[c + d*x]^3)^(2/3),x]
```

output
```
-(((b*Coth[c + d*x]^3)^(2/3)*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]
^2]*Tanh[c + d*x])/d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4141, 3042, 25, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\coth^3(c+dx)\right)^{2/3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(ib\tan\left(ic+idx+\frac{\pi}{2}\right)^3\right)^{2/3}dx$$

$$\Big\downarrow \text{4141}$$

$$\tanh^2(c+dx)\left(b\coth^3(c+dx)\right)^{2/3}\int \coth^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tanh^2(c+dx)\left(b\coth^3(c+dx)\right)^{2/3}\int -\tan\left(ic+idx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$\tanh^2(c+dx)\left(-\left(b\coth^3(c+dx)\right)^{2/3}\right)\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx$$

$$\Big\downarrow \text{3954}$$

$$\tanh^2(c+dx)\left(-\left(b\coth^3(c+dx)\right)^{2/3}\right)\left(\frac{\coth(c+dx)}{d}-\int 1dx\right)$$

$$\Big\downarrow \text{24}$$

$$\tanh^2(c+dx)\left(\frac{\coth(c+dx)}{d}-x\right)\left(-\left(b\coth^3(c+dx)\right)^{2/3}\right)$$

input
```
Int[(b*Coth[c + d*x]^3)^(2/3),x]
```

output
```
-((b*Coth[c + d*x]^3)^(2/3)*(-x + Coth[c + d*x]/d)*Tanh[c + d*x]^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(46) = 92$.

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.38

method	result	size
risch	$\frac{\left(\frac{b\left(e^{2dx+2c}+1\right)^{3}}{\left(e^{2dx+2c}-1\right)^{3}}\right)^{\frac{2}{3}}\left(e^{2dx+2c}-1\right)^{2}x}{\left(e^{2dx+2c}+1\right)^{2}}-\frac{2\left(\frac{b\left(e^{2dx+2c}+1\right)^{3}}{\left(e^{2dx+2c}-1\right)^{3}}\right)^{\frac{2}{3}}\left(e^{2dx+2c}-1\right)}{\left(e^{2dx+2c}+1\right)^{2}d}$	119

input
```
int((b*coth(d*x+c)^3)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
(b*(exp(2*d*x+2*c)+1)^3/(exp(2*d*x+2*c)-1)^3)^(2/3)/(exp(2*d*x+2*c)+1)^2*(
exp(2*d*x+2*c)-1)^2*x-2*(b*(exp(2*d*x+2*c)+1)^3/(exp(2*d*x+2*c)-1)^3)^(2/3
)/(exp(2*d*x+2*c)+1)^2*(exp(2*d*x+2*c)-1)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. 2(46) = 92.

Time = 0.10 (sec) , antiderivative size = 392, normalized size of antiderivative = 7.84

$$\int \left(b\coth^3(c+dx)\right)^{2/3}\,dx = \frac{\left(dx\cosh\left(dx+c\right)\right)^{2}+\left(dxe^{(4\,dx+4\,c)}-2\,dxe^{(2\,dx+2\,c)}+dx\right)\sinh\left(dx+c\right)^{2}-dx+\left(dx\cosh\left(dx\right.\right.}{d\cosh\left(dx+c\right)^{2}+\left(de^{(4\,dx+4\,c)}+2\,de^{(2\,dx+2\,c)}+d\right)\sinh\left(dx\right.}$$

input
```
integrate((b*coth(d*x+c)^3)^(2/3),x, algorithm="fricas")
```

output
```
(d*x*cosh(d*x + c)^2 + (d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c) + d*x)
*sinh(d*x + c)^2 - d*x + (d*x*cosh(d*x + c)^2 - d*x - 2)*e^(4*d*x + 4*c) -
 2*(d*x*cosh(d*x + c)^2 - d*x - 2)*e^(2*d*x + 2*c) + 2*(d*x*cosh(d*x + c)*
e^(4*d*x + 4*c) - 2*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))
*sinh(d*x + c) - 2)*((b*e^(6*d*x + 6*c) + 3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d
*x + 2*c) + b)/(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) -
1))^(2/3)/(d*cosh(d*x + c)^2 + (d*e^(4*d*x + 4*c) + 2*d*e^(2*d*x + 2*c) +
d)*sinh(d*x + c)^2 + (d*cosh(d*x + c)^2 - d)*e^(4*d*x + 4*c) + 2*(d*cosh(d
*x + c)^2 - d)*e^(2*d*x + 2*c) + 2*(d*cosh(d*x + c)*e^(4*d*x + 4*c) + 2*d*
cosh(d*x + c)*e^(2*d*x + 2*c) + d*cosh(d*x + c))*sinh(d*x + c) - d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(44) = 88$.

Time = 6.01 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.72

$$\int \left(b\coth^3(c+dx)\right)^{2/3}\,dx = \begin{cases} x\left(b\coth^3(c)\right)^{\frac{2}{3}} & \text{for } d=0 \\ -\frac{\left(b\coth^3\left(dx+\log\left(-e^{-dx}\right)\right)\right)^{\frac{2}{3}}\log\left(-e^{-dx}\right)}{d} & \text{for } c=\log\left(-e^{-dx}\right) \\ -\frac{\left(b\coth^3\left(dx+\log\left(e^{-dx}\right)\right)\right)^{\frac{2}{3}}\log\left(e^{-dx}\right)}{d} & \text{for } c=\log\left(e^{-dx}\right) \\ x\left(\frac{b}{\tanh^3(c+dx)}\right)^{\frac{2}{3}}\tanh^2(c+dx)-\frac{\left(\frac{b}{\tanh^3(c+dx)}\right)^{\frac{2}{3}}\tanh(c+dx)}{d} & \text{otherwise} \end{cases}$$

input
```
integrate((b*coth(d*x+c)**3)**(2/3),x)
```

output
```
Piecewise((x*(b*coth(c)**3)**(2/3), Eq(d, 0)), (-(b*coth(d*x + log(-exp(-d
*x)))**3)**(2/3)*log(-exp(-d*x))/d, Eq(c, log(-exp(-d*x)))), (-(b*coth(d*x
 + log(exp(-d*x)))**3)**(2/3)*log(exp(-d*x))/d, Eq(c, log(exp(-d*x)))), (x
*(b/tanh(c + d*x)**3)**(2/3)*tanh(c + d*x)**2 - (b/tanh(c + d*x)**3)**(2/3
)*tanh(c + d*x)/d, True))
```

Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int \left(b\coth^3(c+dx)\right)^{2/3}\,dx = \frac{(dx+c)b^{\frac{2}{3}}}{d} + \frac{2\,b^{\frac{2}{3}}}{d\left(e^{(-2\,dx-2\,c)}-1\right)}$$

input
```
integrate((b*coth(d*x+c)^3)^(2/3),x, algorithm="maxima")
```

output
```
(d*x + c)*b^(2/3)/d + 2*b^(2/3)/(d*(e^(-2*d*x - 2*c) - 1))
```

Giac [F]

$$\int \left(b\coth^3(c+dx)\right)^{2/3} dx = \int \left(b\coth\left(dx+c\right)^3\right)^{\frac{2}{3}} dx$$

input
```
integrate((b*coth(d*x+c)^3)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^3)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\coth^3(c+dx)\right)^{2/3} dx = \int \left(b\coth(c+d\,x)^3\right)^{2/3} dx$$

input
```
int((b*coth(c + d*x)^3)^(2/3),x)
```

output
```
int((b*coth(c + d*x)^3)^(2/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.38

$$\int \left(b\coth^3(c+dx)\right)^{2/3} dx = \frac{b^{\frac{2}{3}}\left(-\coth\left(dx+c\right)+dx\right)}{d}$$

input
```
int((b*coth(d*x+c)^3)^(2/3),x)
```

output
```
(b**(2/3)*( - coth(c + d*x) + d*x))/d
```

3.35 $\int \sqrt[3]{b \coth^3(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \sqrt[3]{b \coth^3(c+dx)}\,dx = \frac{\sqrt[3]{b \coth^3(c+dx)} \log(\sinh(c+dx)) \tanh(c+dx)}{d}$$

output
```
(b*coth(d*x+c)^3)^(1/3)*ln(sinh(d*x+c))*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{b \coth^3(c+dx)}\,dx = \frac{\sqrt[3]{b \coth^3(c+dx)} \log(\sinh(c+dx)) \tanh(c+dx)}{d}$$

input
```
Integrate[(b*Coth[c + d*x]^3)^(1/3),x]
```

output
```
((b*Coth[c + d*x]^3)^(1/3)*Log[Sinh[c + d*x]]*Tanh[c + d*x])/d
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4141, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\coth^3(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{ib\tan\left(ic+idx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{4141}$$

$$\tanh(c+dx)\sqrt[3]{b\coth^3(c+dx)}\int \coth(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tanh(c+dx)\sqrt[3]{b\coth^3(c+dx)}\int -i\tan\left(ic+idx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\tanh(c+dx)\sqrt[3]{b\coth^3(c+dx)}\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx$$

$$\Big\downarrow \text{3956}$$

$$\frac{\tanh(c+dx)\sqrt[3]{b\coth^3(c+dx)}\log(-i\sinh(c+dx))}{d}$$

input
```
Int[(b*Coth[c + d*x]^3)^(1/3),x]
```

output
```
((b*Coth[c + d*x]^3)^(1/3)*Log[(-I)*Sinh[c + d*x]]*Tanh[c + d*x])/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(29) = 58$.

Time = 0.07 (sec) , antiderivative size = 192, normalized size of antiderivative = 6.19

method	result
risch	$\frac{\left(\frac{b\left(e^{2dx+2c}+1\right)^{3}}{\left(e^{2dx+2c}-1\right)^{3}}\right)^{\frac{1}{3}}\left(e^{2dx+2c}-1\right)x}{e^{2dx+2c}+1}-\frac{2\left(\frac{b\left(e^{2dx+2c}+1\right)^{3}}{\left(e^{2dx+2c}-1\right)^{3}}\right)^{\frac{1}{3}}\left(e^{2dx+2c}-1\right)(dx+c)}{\left(e^{2dx+2c}+1\right)d}+\frac{\left(\frac{b\left(e^{2dx+2c}+1\right)^{3}}{\left(e^{2dx+2c}-1\right)^{3}}\right)^{\frac{1}{3}}\left(e^{2dx+2c}-1\right)\ln\left(e^{2dx+\right.}{\left(e^{2dx+2c}+1\right)d}$

input
```
int((b*coth(d*x+c)^3)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
(b*(exp(2*d*x+2*c)+1)^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(exp(2*d*x+2*c)+1)*(ex
p(2*d*x+2*c)-1)*x-2*(b*(exp(2*d*x+2*c)+1)^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(e
xp(2*d*x+2*c)+1)*(exp(2*d*x+2*c)-1)/d*(d*x+c)+(b*(exp(2*d*x+2*c)+1)^3/(exp
(2*d*x+2*c)-1)^3)^(1/3)/(exp(2*d*x+2*c)+1)*(exp(2*d*x+2*c)-1)/d*ln(exp(2*d
*x+2*c)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(29) = 58$.

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.77

$$\int \sqrt[3]{b\coth^3(c+dx)}\,dx = -\frac{\left(dxe^{(2\,dx+2\,c)} - dx - \left(e^{(2\,dx+2\,c)} - 1\right)\log\left(\frac{2\,\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)\right)\left(\frac{be^{(6\,dx+6\,c)}+3\,be^{(4\,dx+4\,c)}+3\,be^{(2\,dx+2\,c)}+b}{e^{(6\,dx+6\,c)}-3\,e^{(4\,dx+4\,c)}+3\,e^{(2\,dx+2\,c)}-1}\right)^{\frac{1}{3}}}{de^{(2\,dx+2\,c)}+d}$$

input
```
integrate((b*coth(d*x+c)^3)^(1/3),x, algorithm="fricas")
```

output
```
-(d*x*e^(2*d*x + 2*c) - d*x - (e^(2*d*x + 2*c) - 1)*log(2*sinh(d*x + c)/(c
osh(d*x + c) - sinh(d*x + c))))*((b*e^(6*d*x + 6*c) + 3*b*e^(4*d*x + 4*c)
+ 3*b*e^(2*d*x + 2*c) + b)/(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d
*x + 2*c) - 1))^(1/3)/(d*e^(2*d*x + 2*c) + d)
```

Sympy [F]

$$\int \sqrt[3]{b\coth^3(c+dx)}\,dx = \int \sqrt[3]{b\coth^3(c+dx)}\,dx$$

input
```
integrate((b*coth(d*x+c)**3)**(1/3),x)
```

output
```
Integral((b*coth(c + d*x)**3)**(1/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \sqrt[3]{b\coth^3(c+dx)}\,dx = \frac{(dx+c)b^{\frac{1}{3}}}{d} + \frac{b^{\frac{1}{3}}\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{b^{\frac{1}{3}}\log\left(e^{(-dx-c)}-1\right)}{d}$$

input
```
integrate((b*coth(d*x+c)^3)^(1/3),x, algorithm="maxima")
```

output
```
(d*x + c)*b^(1/3)/d + b^(1/3)*log(e^(-d*x - c) + 1)/d + b^(1/3)*log(e^(-d*
x - c) - 1)/d
```

Giac [F]

$$\int \sqrt[3]{b\coth^3(c+dx)}\,dx = \int \left(b\coth\left(dx+c\right)^3\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*coth(d*x+c)^3)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^3)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\coth^3(c+dx)}\,dx = \int \left(b\coth(c+d\,x)^3\right)^{1/3}\,dx$$

input
```
int((b*coth(c + d*x)^3)^(1/3),x)
```

output
```
int((b*coth(c + d*x)^3)^(1/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \sqrt[3]{b\coth^3(c+dx)}\,dx = \frac{b^{\frac{1}{3}}\left(\log\left(e^{dx+c}-1\right)+\log\left(e^{dx+c}+1\right)-dx\right)}{d}$$

input
```
int((b*coth(d*x+c)^3)^(1/3),x)
```

output
```
(b**(1/3)*(log(e**(c + d*x) - 1) + log(e**(c + d*x) + 1) - d*x))/d
```

3.36 $\int \frac{1}{\sqrt[3]{b\coth^3(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{1}{\sqrt[3]{b\coth^3(c+dx)}}\,dx = \frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt[3]{b\coth^3(c+dx)}}$$

output
```
coth(d*x+c)*ln(cosh(d*x+c))/d/(b*coth(d*x+c)^3)^(1/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[3]{b\coth^3(c+dx)}}\,dx = \frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt[3]{b\coth^3(c+dx)}}$$

input
```
Integrate[(b*Coth[c + d*x]^3)^(-1/3),x]
```

output
```
(Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*(b*Coth[c + d*x]^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4141, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{b\coth^3(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt[3]{ib\tan\left(ic+idx+\frac{\pi}{2}\right)^3}}dx$$

$$\Big\downarrow \text{ 4141}$$

$$\frac{\coth(c+dx)\int \tanh(c+dx)dx}{\sqrt[3]{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\coth(c+dx)\int -i\tan(ic+idx)dx}{\sqrt[3]{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{i\coth(c+dx)\int \tan(ic+idx)dx}{\sqrt[3]{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt[3]{b\coth^3(c+dx)}}$$

input
```
Int[(b*Coth[c + d*x]^3)^(-1/3),x]
```

output
```
(Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*(b*Coth[c + d*x]^3)^(1/3))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(29) = 58$.

Time = 0.07 (sec) , antiderivative size = 192, normalized size of antiderivative = 6.19

method	result	size
risch	$\frac{(e^{2dx+2c}+1)x}{\left(\frac{b(e^{2dx+2c}+1)^3}{(e^{2dx+2c}-1)^3}\right)^{\frac{1}{3}}(e^{2dx+2c}-1)}-\frac{2(e^{2dx+2c}+1)(dx+c)}{\left(\frac{b(e^{2dx+2c}+1)^3}{(e^{2dx+2c}-1)^3}\right)^{\frac{1}{3}}(e^{2dx+2c}-1)d}+\frac{(e^{2dx+2c}+1)\ln(e^{2dx+2c}+1)}{\left(\frac{b(e^{2dx+2c}+1)^3}{(e^{2dx+2c}-1)^3}\right)^{\frac{1}{3}}(e^{2dx+2c}-1)d}$	192

input
```
int(1/(b*coth(d*x+c)^3)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/(b*(exp(2*d*x+2*c)+1)^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(exp(2*d*x+2*c)-1)*(
exp(2*d*x+2*c)+1)*x-2/(b*(exp(2*d*x+2*c)+1)^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/
(exp(2*d*x+2*c)-1)*(exp(2*d*x+2*c)+1)/d*(d*x+c)+1/(b*(exp(2*d*x+2*c)+1)^3/
(exp(2*d*x+2*c)-1)^3)^(1/3)/(exp(2*d*x+2*c)-1)*(exp(2*d*x+2*c)+1)/d*ln(exp
(2*d*x+2*c)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(29) = 58$.

Time = 0.10 (sec) , antiderivative size = 187, normalized size of antiderivative = 6.03

$$\int \frac{1}{\sqrt[3]{b\coth^3(c+dx)}}\,dx = -\frac{\left(dxe^{(4\,dx+4\,c)}-2\,dxe^{(2\,dx+2\,c)}+dx-\left(e^{(4\,dx+4\,c)}-2\,e^{(2\,dx+2\,c)}+1\right)\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)\right)\left(\frac{be^{(6\,dx+6\,c}}{e^{(6\,dx+6\,c}}\right.}{bde^{(4\,dx+4\,c)}+2\,bde^{(2\,dx+2\,c)}+bd}$$

input
```
integrate(1/(b*coth(d*x+c)^3)^(1/3),x, algorithm="fricas")
```

output
```
-(d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c) + d*x - (e^(4*d*x + 4*c) - 2
*e^(2*d*x + 2*c) + 1)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c)))
)*((b*e^(6*d*x + 6*c) + 3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x + 2*c) + b)/(e^
(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1))^(2/3)/(b*d*e^(
4*d*x + 4*c) + 2*b*d*e^(2*d*x + 2*c) + b*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{b\coth^3(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{b\coth^3{(c+dx)}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)**3)**(1/3),x)
```

output
```
Integral((b*coth(c + d*x)**3)**(-1/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt[3]{b\coth^3(c+dx)}}\,dx=\frac{dx+c}{b^{\frac{1}{3}}d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{b^{\frac{1}{3}}d}$$

input
```
integrate(1/(b*coth(d*x+c)^3)^(1/3),x, algorithm="maxima")
```

output
```
(d*x + c)/(b^(1/3)*d) + log(e^(-2*d*x - 2*c) + 1)/(b^(1/3)*d)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{b\coth^3(c+dx)}}\,dx=\int \frac{1}{\left(b\coth\left(dx+c\right)^3\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^3)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^3)^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{b\coth^3(c+dx)}}\,dx=\int \frac{1}{\left(b\coth\left(c+d\,x\right)^3\right)^{1/3}}\,dx$$

input
```
int(1/(b*coth(c + d*x)^3)^(1/3),x)
```

output
```
int(1/(b*coth(c + d*x)^3)^(1/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt[3]{b \coth^3(c+dx)}}\,dx = \frac{\log\left(e^{2dx+2c}+1\right)-dx}{b^{\frac{1}{3}}d}$$

input
```
int(1/(b*coth(d*x+c)^3)^(1/3),x)
```

output
```
(log(e**(2*c + 2*d*x) + 1) - d*x)/(b**(1/3)*d)
```

3.37 $\int \frac{1}{\left(b\coth^3(c+dx)\right)^{2/3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{2/3}}\,dx = -\frac{\coth(c+dx)}{d\left(b\coth^3(c+dx)\right)^{2/3}}+\frac{x\coth^2(c+dx)}{\left(b\coth^3(c+dx)\right)^{2/3}}$$

output
```
-coth(d*x+c)/d/(b*coth(d*x+c)^3)^(2/3)+x*coth(d*x+c)^2/(b*coth(d*x+c)^3)^(
2/3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{2/3}}\,dx = \frac{\coth(c+dx)(-1+\operatorname{arctanh}(\tanh(c+dx))\coth(c+dx))}{d\left(b\coth^3(c+dx)\right)^{2/3}}$$

input
```
Integrate[(b*Coth[c + d*x]^3)^(-2/3),x]
```

output
```
(Coth[c + d*x]*(-1 + ArcTanh[Tanh[c + d*x]]*Coth[c + d*x]))/(d*(b*Coth[c +
 d*x]^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4141, 3042, 25, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b \coth^3(c+dx)\right)^{2/3}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\left(ib \tan\left(ic+idx+\frac{\pi}{2}\right)^3\right)^{2/3}}dx$$
$$\Big\downarrow \text{4141}$$
$$\frac{\coth^2(c+dx)\int \tanh^2(c+dx)dx}{\left(b \coth^3(c+dx)\right)^{2/3}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\coth^2(c+dx)\int -\tan(ic+idx)^2dx}{\left(b \coth^3(c+dx)\right)^{2/3}}$$
$$\Big\downarrow \text{25}$$
$$-\frac{\coth^2(c+dx)\int \tan(ic+idx)^2dx}{\left(b \coth^3(c+dx)\right)^{2/3}}$$
$$\Big\downarrow \text{3954}$$
$$-\frac{\coth^2(c+dx)\left(\frac{\tanh(c+dx)}{d}-\int 1dx\right)}{\left(b \coth^3(c+dx)\right)^{2/3}}$$
$$\Big\downarrow \text{24}$$
$$-\frac{\left(\frac{\tanh(c+dx)}{d}-x\right)\coth^2(c+dx)}{\left(b \coth^3(c+dx)\right)^{2/3}}$$

input
```
Int[(b*Coth[c + d*x]^3)^(-2/3),x]
```

output
```
-((Coth[c + d*x]^2*(-x + Tanh[c + d*x]/d))/(b*Coth[c + d*x]^3)^(2/3))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.78

method	result	size
risch	$\frac{e^{4dx+4c}dx+2\,e^{2dx+2c}dx+dx+2\,e^{2dx+2c}+2}{\left(\frac{b\left(e^{2dx+2c}+1\right)^{3}}{\left(e^{2dx+2c}-1\right)^{3}}\right)^{\frac{2}{3}}\left(e^{2dx+2c}-1\right)^{2}d}$	89

input
```
int(1/(b*coth(d*x+c)^3)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
(exp(4*d*x+4*c)*d*x+2*exp(2*d*x+2*c)*d*x+d*x+2*exp(2*d*x+2*c)+2)/(b*(exp(2
*d*x+2*c)+1)^3/(exp(2*d*x+2*c)-1)^3)^(2/3)/(exp(2*d*x+2*c)-1)^2/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(46) = 92$.

Time = 0.11 (sec) , antiderivative size = 287, normalized size of antiderivative = 5.74

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{2/3}}\,dx =$$

$$-\frac{\left(dx\cosh\left(dx+c\right)^2-\left(dxe^{(2\,dx+2\,c)}-dx\right)\sinh\left(dx+c\right)^2+dx-\left(dx\cosh\left(dx+c\right)^2+dx+2\right)e^{(2\,dx+2\,c)}-2}{bd\cosh\left(dx+c\right)^2+\left(bde^{(2\,dx+2\,c)}+bd\right)\sinh\left(dx+c\right)^2+bd+\left(bd\cosh\left(dx+c\right)^2+\right.}$$

input
```
integrate(1/(b*coth(d*x+c)^3)^(2/3),x, algorithm="fricas")
```

output
```
-(d*x*cosh(d*x + c)^2 - (d*x*e^(2*d*x + 2*c) - d*x)*sinh(d*x + c)^2 + d*x
- (d*x*cosh(d*x + c)^2 + d*x + 2)*e^(2*d*x + 2*c) - 2*(d*x*cosh(d*x + c)*e
^(2*d*x + 2*c) - d*x*cosh(d*x + c))*sinh(d*x + c) + 2)*((b*e^(6*d*x + 6*c)
 + 3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x + 2*c) + b)/(e^(6*d*x + 6*c) - 3*e^(
4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1))^(1/3)/(b*d*cosh(d*x + c)^2 + (b*d*e
^(2*d*x + 2*c) + b*d)*sinh(d*x + c)^2 + b*d + (b*d*cosh(d*x + c)^2 + b*d)*
e^(2*d*x + 2*c) + 2*(b*d*cosh(d*x + c)*e^(2*d*x + 2*c) + b*d*cosh(d*x + c)
)*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{2/3}}\,dx = \int \frac{1}{\left(b\coth^3\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)**3)**(2/3),x)
```

output
```
Integral((b*coth(c + d*x)**3)**(-2/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{2/3}}\,dx = \frac{dx+c}{b^{\frac{2}{3}}d} - \frac{2}{\left(b^{\frac{2}{3}}e^{(-2\,dx-2\,c)}+b^{\frac{2}{3}}\right)d}$$

input
```
integrate(1/(b*coth(d*x+c)^3)^(2/3),x, algorithm="maxima")
```

output
```
(d*x + c)/(b^(2/3)*d) - 2/((b^(2/3)*e^(-2*d*x - 2*c) + b^(2/3))*d)
```

Giac [F]

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{2/3}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^3\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^3)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^3)^(-2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{2/3}}\,dx = \int \frac{1}{\left(b\coth\left(c+d\,x\right)^3\right)^{2/3}}\,dx$$

input
```
int(1/(b*coth(c + d*x)^3)^(2/3),x)
```

output
```
int(1/(b*coth(c + d*x)^3)^(2/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.52

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{2/3}}\,dx=\frac{\coth\left(dx+c\right)dx-1}{b^{\frac{2}{3}}\coth\left(dx+c\right)d}$$

input
```
int(1/(b*coth(d*x+c)^3)^(2/3),x)
```

output
```
(coth(c + d*x)*d*x - 1)/(b**(2/3)*coth(c + d*x)*d)
```

3.38 $\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 80

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx=-\frac{1}{bd\sqrt[3]{b\coth^3(c+dx)}}+\frac{x\coth(c+dx)}{b\sqrt[3]{b\coth^3(c+dx)}}-\frac{\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}}$$

output
```
-1/b/d/(b*coth(d*x+c)^3)^(1/3)+x*coth(d*x+c)/b/(b*coth(d*x+c)^3)^(1/3)-1/3
*tanh(d*x+c)^2/b/d/(b*coth(d*x+c)^3)^(1/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.64

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx=\frac{-3+3\text{arctanh}(\tanh(c+dx))\coth(c+dx)-\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}}$$

input
```
Integrate[(b*Coth[c + d*x]^3)^(-4/3),x]
```

output
```
(-3 + 3*ArcTanh[Tanh[c + d*x]]*Coth[c + d*x] - Tanh[c + d*x]^2)/(3*b*d*(b*
Coth[c + d*x]^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.65, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3042, 4141, 3042, 3954, 25, 3042, 25, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(ib\tan\left(ic+idx+\frac{\pi}{2}\right)^3\right)^{4/3}}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\coth(c+dx)\int\tanh^4(c+dx)dx}{b\sqrt[3]{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\coth(c+dx)\int\tan(ic+idx)^4dx}{b\sqrt[3]{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\coth(c+dx)\left(-\int-\tanh^2(c+dx)dx-\frac{\tanh^3(c+dx)}{3d}\right)}{b\sqrt[3]{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\coth(c+dx)\left(\int\tanh^2(c+dx)dx-\frac{\tanh^3(c+dx)}{3d}\right)}{b\sqrt[3]{b\coth^3(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\coth(c+dx)\left(-\frac{\tanh^3(c+dx)}{3d}+\int -\tan(ic+idx)^2dx\right)}{b\sqrt[3]{b\coth^3(c+dx)}}$$

$$\Big\downarrow \ 25$$

$$\frac{\coth(c+dx)\left(-\frac{\tanh^3(c+dx)}{3d}-\int \tan(ic+idx)^2dx\right)}{b\sqrt[3]{b\coth^3(c+dx)}}$$

$$\Big\downarrow \ 3954$$

$$\frac{\coth(c+dx)\left(\int 1dx-\frac{\tanh^3(c+dx)}{3d}-\frac{\tanh(c+dx)}{d}\right)}{b\sqrt[3]{b\coth^3(c+dx)}}$$

$$\Big\downarrow \ 24$$

$$\frac{\left(-\frac{\tanh^3(c+dx)}{3d}-\frac{\tanh(c+dx)}{d}+x\right)\coth(c+dx)}{b\sqrt[3]{b\coth^3(c+dx)}}$$

input
```
Int[(b*Coth[c + d*x]^3)^(-4/3),x]
```

output
```
(Coth[c + d*x]*(x - Tanh[c + d*x]/d - Tanh[c + d*x]^3/(3*d)))/(b*(b*Coth[c
 + d*x]^3)^(1/3))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.65

method	result	size
risch	$\frac{3\,\mathrm{e}^{6dx+6c}dx+9\,\mathrm{e}^{4dx+4c}dx+9\,\mathrm{e}^{2dx+2c}dx+3dx+12\,\mathrm{e}^{4dx+4c}+12\,\mathrm{e}^{2dx+2c}+8}{3b\left(\mathrm{e}^{2dx+2c}+1\right)^{2}\left(\mathrm{e}^{2dx+2c}-1\right)\left(\frac{b\left(\mathrm{e}^{2dx+2c}+1\right)^{3}}{\left(\mathrm{e}^{2dx+2c}-1\right)^{3}}\right)^{\frac{1}{3}}d}$	132

input
```
int(1/(b*coth(d*x+c)^3)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*exp(6*d*x+6*c)*d*x+9*exp(4*d*x+4*c)*d*x+9*exp(2*d*x+2*c)*d*x+3*d*x+
12*exp(4*d*x+4*c)+12*exp(2*d*x+2*c)+8)/b/(exp(2*d*x+2*c)+1)^2/(exp(2*d*x+2
*c)-1)/(b*(exp(2*d*x+2*c)+1)^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1579 vs. $2(72) = 144$.

Time = 0.12 (sec) , antiderivative size = 1579, normalized size of antiderivative = 19.74

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*coth(d*x+c)^3)^(4/3),x, algorithm="fricas")
```

output
```
1/3*(3*d*x*cosh(d*x + c)^6 + 3*(d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c
) + d*x)*sinh(d*x + c)^6 + 18*(d*x*cosh(d*x + c)*e^(4*d*x + 4*c) - 2*d*x*c
osh(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(3*d
*x + 4)*cosh(d*x + c)^4 + 3*(15*d*x*cosh(d*x + c)^2 + 3*d*x + (15*d*x*cosh
(d*x + c)^2 + 3*d*x + 4)*e^(4*d*x + 4*c) - 2*(15*d*x*cosh(d*x + c)^2 + 3*d
*x + 4)*e^(2*d*x + 2*c) + 4)*sinh(d*x + c)^4 + 12*(5*d*x*cosh(d*x + c)^3 +
 (3*d*x + 4)*cosh(d*x + c) + (5*d*x*cosh(d*x + c)^3 + (3*d*x + 4)*cosh(d*x
 + c))*e^(4*d*x + 4*c) - 2*(5*d*x*cosh(d*x + c)^3 + (3*d*x + 4)*cosh(d*x +
 c))*e^(2*d*x + 2*c))*sinh(d*x + c)^3 + 3*(3*d*x + 4)*cosh(d*x + c)^2 + 3*
(15*d*x*cosh(d*x + c)^4 + 6*(3*d*x + 4)*cosh(d*x + c)^2 + 3*d*x + (15*d*x*
cosh(d*x + c)^4 + 6*(3*d*x + 4)*cosh(d*x + c)^2 + 3*d*x + 4)*e^(4*d*x + 4*
c) - 2*(15*d*x*cosh(d*x + c)^4 + 6*(3*d*x + 4)*cosh(d*x + c)^2 + 3*d*x + 4
)*e^(2*d*x + 2*c) + 4)*sinh(d*x + c)^2 + 3*d*x + (3*d*x*cosh(d*x + c)^6 +
3*(3*d*x + 4)*cosh(d*x + c)^4 + 3*(3*d*x + 4)*cosh(d*x + c)^2 + 3*d*x + 8)
*e^(4*d*x + 4*c) - 2*(3*d*x*cosh(d*x + c)^6 + 3*(3*d*x + 4)*cosh(d*x + c)^
4 + 3*(3*d*x + 4)*cosh(d*x + c)^2 + 3*d*x + 8)*e^(2*d*x + 2*c) + 6*(3*d*x*
cosh(d*x + c)^5 + 2*(3*d*x + 4)*cosh(d*x + c)^3 + (3*d*x + 4)*cosh(d*x + c
) + (3*d*x*cosh(d*x + c)^5 + 2*(3*d*x + 4)*cosh(d*x + c)^3 + (3*d*x + 4)*c
osh(d*x + c))*e^(4*d*x + 4*c) - 2*(3*d*x*cosh(d*x + c)^5 + 2*(3*d*x + 4)*c
osh(d*x + c)^3 + (3*d*x + 4)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x +...
```

Sympy [F]

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx=\int \frac{1}{\left(b\coth^3{\left(c+dx\right)}\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)**3)**(4/3),x)
```

output
```
Integral((b*coth(c + d*x)**3)**(-4/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx = -\frac{4\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+2\right)}{3\left(3\,b^{\frac{4}{3}}e^{(-2\,dx-2\,c)}+3\,b^{\frac{4}{3}}e^{(-4\,dx-4\,c)}+b^{\frac{4}{3}}e^{(-6\,dx-6\,c)}+b^{\frac{4}{3}}\right)d}+\frac{dx+c}{b^{\frac{4}{3}}d}$$

input
```
integrate(1/(b*coth(d*x+c)^3)^(4/3),x, algorithm="maxima")
```

output
```
-4/3*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/((3*b^(4/3)*e^(-2*d*x -
 2*c) + 3*b^(4/3)*e^(-4*d*x - 4*c) + b^(4/3)*e^(-6*d*x - 6*c) + b^(4/3))*d
) + (d*x + c)/(b^(4/3)*d)
```

Giac [F]

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^3\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^3)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^3)^(-4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx = \int \frac{1}{\left(b\coth\left(c+d\,x\right)^3\right)^{4/3}}\,dx$$

input
```
int(1/(b*coth(c + d*x)^3)^(4/3),x)
```

output
```
int(1/(b*coth(c + d*x)^3)^(4/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.32

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx = \frac{3e^{6dx+6c}dx-4e^{6dx+6c}+9e^{4dx+4c}dx+9e^{2dx+2c}dx+3dx+4}{3b^{\frac{4}{3}}d\left(e^{6dx+6c}+3e^{4dx+4c}+3e^{2dx+2c}+1\right)}$$

input
```
int(1/(b*coth(d*x+c)^3)^(4/3),x)
```

output
```
(3*e**(6*c + 6*d*x)*d*x - 4*e**(6*c + 6*d*x) + 9*e**(4*c + 4*d*x)*d*x + 9*
e**(2*c + 2*d*x)*d*x + 3*d*x + 4)/(3*b**(1/3)*b*d*(e**(6*c + 6*d*x) + 3*e*
*(4*c + 4*d*x) + 3*e**(2*c + 2*d*x) + 1))
```

3.39 $\int \left(b\coth^4(c+dx)\right)^n\,dx$

Optimal result

Integrand size = 12, antiderivative size = 57

$$\int \left(b\coth^4(c+dx)\right)^n\,dx = \frac{\coth(c+dx)\left(b\coth^4(c+dx)\right)^n\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+4n),\frac{1}{2}(3+4n),\coth^2(c+dx)\right)}{d(1+4n)}$$

output
```
coth(d*x+c)*(b*coth(d*x+c)^4)^n*hypergeom([1, 1/2+2*n],[3/2+2*n],coth(d*x+
c)^2)/d/(1+4*n)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \left(b\coth^4(c+dx)\right)^n\,dx = \frac{\coth(c+dx)\left(b\coth^4(c+dx)\right)^n\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+2n,\frac{3}{2}+2n,\coth^2(c+dx)\right)}{d(1+4n)}$$

input
```
Integrate[(b*Coth[c + d*x]^4)^n,x]
```

output
```
(Coth[c + d*x]*(b*Coth[c + d*x]^4)^n*Hypergeometric2F1[1, 1/2 + 2*n, 3/2 +
 2*n, Coth[c + d*x]^2])/(d*(1 + 4*n))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4141, 3042, 3957, 25, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b \coth^4(c+dx)\right)^n dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(b \tan\left(ic+idx+\frac{\pi}{2}\right)^4\right)^n dx$$

$$\Big\downarrow \text{ 4141}$$

$$\coth^{-4n}(c+dx) \left(b \coth^4(c+dx)\right)^n \int \coth^{4n}(c+dx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\coth^{-4n}(c+dx) \left(b \coth^4(c+dx)\right)^n \int \left(-i \tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{4n} dx$$

$$\Big\downarrow \text{ 3957}$$

$$-\frac{\coth^{-4n}(c+dx) \left(b \coth^4(c+dx)\right)^n \int -\frac{\coth^{4n}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\coth^{-4n}(c+dx) \left(b \coth^4(c+dx)\right)^n \int \frac{\coth^{4n}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{\coth(c+dx) \left(b \coth^4(c+dx)\right)^n \text{Hypergeometric2F1}\left(1,\frac{1}{2}(4n+1),\frac{1}{2}(4n+3),\coth^2(c+dx)\right)}{d(4n+1)}$$

input
```
Int[(b*Coth[c + d*x]^4)^n,x]
```

output
```
(Coth[c + d*x]*(b*Coth[c + d*x]^4)^n*Hypergeometric2F1[1, (1 + 4*n)/2, (3
+ 4*n)/2, Coth[c + d*x]^2])/(d*(1 + 4*n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \left(b \coth \left(dx + c\right)^{4}\right)^{n} dx$$

input
```
int((b*coth(d*x+c)^4)^n,x)
```

output
```
int((b*coth(d*x+c)^4)^n,x)
```

Fricas [F]

$$\int \left(b \coth^4(c+dx)\right)^n \, dx = \int \left(b \coth \left(dx + c\right)^{4}\right)^{n} \, dx$$

input
```
integrate((b*coth(d*x+c)^4)^n,x, algorithm="fricas")
```

output
```
integral((b*coth(d*x + c)^4)^n, x)
```

Sympy [F]

$$\int \left(b \coth^4(c+dx)\right)^n \, dx = \int \left(b \coth^{4}\left(c + dx\right)\right)^{n} \, dx$$

input
```
integrate((b*coth(d*x+c)**4)**n,x)
```

output
```
Integral((b*coth(c + d*x)**4)**n, x)
```

Maxima [F]

$$\int \left(b\coth^4(c+dx)\right)^n\,dx = \int \left(b\coth\left(dx+c\right)^4\right)^n\,dx$$

input
```
integrate((b*coth(d*x+c)^4)^n,x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^4)^n, x)
```

Giac [F]

$$\int \left(b\coth^4(c+dx)\right)^n\,dx = \int \left(b\coth\left(dx+c\right)^4\right)^n\,dx$$

input
```
integrate((b*coth(d*x+c)^4)^n,x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^4)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\coth^4(c+dx)\right)^n\,dx = \int \left(b\coth(c+d\,x)^4\right)^n\,dx$$

input
```
int((b*coth(c + d*x)^4)^n,x)
```

output
```
int((b*coth(c + d*x)^4)^n, x)
```

Reduce [F]

$$\int \left(b\coth^4(c+dx)\right)^n \, dx = b^n \left(\int \coth(dx+c)^{4n}\, dx\right)$$

input
```
int((b*coth(d*x+c)^4)^n,x)
```

output
```
b**n*int(coth(c + d*x)**(4*n),x)
```

3.40 $\int \left(b\coth^4(c+dx)\right)^{3/2} dx$

Optimal result

Integrand size = 14, antiderivative size = 110

$$\int \left(b\coth^4(c+dx)\right)^{3/2} dx = -\frac{b\coth(c+dx)\sqrt{b\coth^4(c+dx)}}{3d} - \frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d} - \frac{b\sqrt{b\coth^4(c+dx)}\tanh(c+dx)}{d} + bx\sqrt{b\coth^4(c+dx)}\tanh^2(c+dx)$$

output
```
-1/3*b*coth(d*x+c)*(b*coth(d*x+c)^4)^(1/2)/d-1/5*b*coth(d*x+c)^3*(b*coth(d
*x+c)^4)^(1/2)/d-b*(b*coth(d*x+c)^4)^(1/2)*tanh(d*x+c)/d+b*x*(b*coth(d*x+c
)^4)^(1/2)*tanh(d*x+c)^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.39

$$\int \left(b\coth^4(c+dx)\right)^{3/2} dx = -\frac{\left(b\coth^4(c+dx)\right)^{3/2}\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\tanh^2(c+dx)\right)\tanh(c+dx)}{5d}$$

input
```
Integrate[(b*Coth[c + d*x]^4)^(3/2),x]
```

output
```
-1/5*((b*Coth[c + d*x]^4)^(3/2)*Hypergeometric2F1[-5/2, 1, -3/2, Tanh[c +
d*x]^2]*Tanh[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.63, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 4141, 3042, 25, 3954, 3042, 3954, 25, 3042, 25, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\coth^4(c+dx)\right)^{3/2}\,dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \left(b\tan\left(ic+idx+\frac{\pi}{2}\right)^4\right)^{3/2}dx$$

$$\Big\downarrow \textbf{4141}$$

$$b\tanh^2(c+dx)\sqrt{b\coth^4(c+dx)}\int \coth^6(c+dx)dx$$

$$\Big\downarrow \textbf{3042}$$

$$b\tanh^2(c+dx)\sqrt{b\coth^4(c+dx)}\int -\tan\left(ic+idx+\frac{\pi}{2}\right)^6dx$$

$$\Big\downarrow \textbf{25}$$

$$-b\tanh^2(c+dx)\sqrt{b\coth^4(c+dx)}\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^6dx$$

$$\Big\downarrow \textbf{3954}$$

$$-b\tanh^2(c+dx)\sqrt{b\coth^4(c+dx)}\left(\frac{\coth^5(c+dx)}{5d}-\int \coth^4(c+dx)dx\right)$$

$$\Big\downarrow \textbf{3042}$$

$$-b\tanh^2(c+dx)\sqrt{b\coth^4(c+dx)}\left(\frac{\coth^5(c+dx)}{5d}-\int\tan\left(ic+idx+\frac{\pi}{2}\right)^4dx\right)$$

$$\Big\downarrow \text{3954}$$

$$-b\tanh^2(c+dx)\sqrt{b\coth^4(c+dx)}\left(\int-\coth^2(c+dx)dx+\frac{\coth^5(c+dx)}{5d}+\frac{\coth^3(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{25}$$

$$-b\tanh^2(c+dx)\sqrt{b\coth^4(c+dx)}\left(-\int\coth^2(c+dx)dx+\frac{\coth^5(c+dx)}{5d}+\frac{\coth^3(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$-b\tanh^2(c+dx)\sqrt{b\coth^4(c+dx)}\left(-\int-\tan\left(ic+idx+\frac{\pi}{2}\right)^2dx+\frac{\coth^5(c+dx)}{5d}+\frac{\coth^3(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{25}$$

$$-b\tanh^2(c+dx)\sqrt{b\coth^4(c+dx)}\left(\int\tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx+\frac{\coth^5(c+dx)}{5d}+\frac{\coth^3(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3954}$$

$$-b\tanh^2(c+dx)\sqrt{b\coth^4(c+dx)}\left(-\int1dx+\frac{\coth^5(c+dx)}{5d}+\frac{\coth^3(c+dx)}{3d}+\frac{\coth(c+dx)}{d}\right)$$

$$\Big\downarrow \text{24}$$

$$-b\tanh^2(c+dx)\left(\frac{\coth^5(c+dx)}{5d}+\frac{\coth^3(c+dx)}{3d}+\frac{\coth(c+dx)}{d}-x\right)\sqrt{b\coth^4(c+dx)}$$

input
```
Int[(b*Coth[c + d*x]^4)^(3/2),x]
```

output
```
-(b*Sqrt[b*Coth[c + d*x]^4]*(-x + Coth[c + d*x]/d + Coth[c + d*x]^3/(3*d)
+ Coth[c + d*x]^5/(5*d))*Tanh[c + d*x]^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{\left(b \coth(dx+c)^4\right)^{\frac{3}{2}}\left(6\coth(dx+c)^5+10\coth(dx+c)^3+15\ln(\coth(dx+c)-1)-15\ln(\coth(dx+c)+1)+30\coth(dx+c)\right.}{30d\coth(dx+c)^6}$
default	$-\frac{\left(b \coth(dx+c)^4\right)^{\frac{3}{2}}\left(6\coth(dx+c)^5+10\coth(dx+c)^3+15\ln(\coth(dx+c)-1)-15\ln(\coth(dx+c)+1)+30\coth(dx+c)\right.}{30d\coth(dx+c)^6}$
risch	$\frac{b\left(e^{2dx+2c}-1\right)^2\sqrt{\frac{b\left(e^{2dx+2c}+1\right)^4}{\left(e^{2dx+2c}-1\right)^4}}\,x}{\left(e^{2dx+2c}+1\right)^2}-\frac{2b\sqrt{\frac{b\left(e^{2dx+2c}+1\right)^4}{\left(e^{2dx+2c}-1\right)^4}}\left(45\,e^{8dx+8c}-90\,e^{6dx+6c}+140\,e^{4dx+4c}-70\,e^{2dx+2c}+23\right)}{15\left(e^{2dx+2c}+1\right)^2\left(e^{2dx+2c}-1\right)^3d}$

input
```
int((b*coth(d*x+c)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30/d*(b*coth(d*x+c)^4)^(3/2)*(6*coth(d*x+c)^5+10*coth(d*x+c)^3+15*ln(co
th(d*x+c)-1)-15*ln(coth(d*x+c)+1)+30*coth(d*x+c))/coth(d*x+c)^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3421 vs. $2(98) = 196$.

Time = 0.17 (sec) , antiderivative size = 3421, normalized size of antiderivative = 31.10

$$\int \left(b\coth^4(c+dx)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((b*coth(d*x+c)^4)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \left(b\coth^4(c+dx)\right)^{3/2} dx = \int \left(b\coth^4{(c+dx)}\right)^{\frac{3}{2}} dx$$

input
```
integrate((b*coth(d*x+c)**4)**(3/2),x)
```

output
```
Integral((b*coth(c + d*x)**4)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.25

$$\int \left(b\coth^4(c+dx)\right)^{3/2} dx = \frac{(dx+c)b^{\frac{3}{2}}}{d} - \frac{2\left(70\,b^{\frac{3}{2}}e^{(-2\,dx-2\,c)} - 140\,b^{\frac{3}{2}}e^{(-4\,dx-4\,c)} + 90\,b^{\frac{3}{2}}e^{(-6\,dx-6\,c)} - 45\,b^{\frac{3}{2}}e^{(-8\,dx-8\,c)} - 23\,b^{\frac{3}{2}}\right)}{15\,d(5\,e^{(-2\,dx-2\,c)} - 10\,e^{(-4\,dx-4\,c)} + 10\,e^{(-6\,dx-6\,c)} - 5\,e^{(-8\,dx-8\,c)} + e^{(-10\,dx-10\,c)} - 1)}$$

input
```
integrate((b*coth(d*x+c)^4)^(3/2),x, algorithm="maxima")
```

output
```
(d*x + c)*b^(3/2)/d - 2/15*(70*b^(3/2)*e^(-2*d*x - 2*c) - 140*b^(3/2)*e^(-
4*d*x - 4*c) + 90*b^(3/2)*e^(-6*d*x - 6*c) - 45*b^(3/2)*e^(-8*d*x - 8*c) -
 23*b^(3/2))/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.70

$$\int \left(b\coth^4(c+dx)\right)^{3/2} dx = \frac{\left(15\,dx + 15\,c - \frac{2\left(45\,e^{(8\,dx+8\,c)} - 90\,e^{(6\,dx+6\,c)} + 140\,e^{(4\,dx+4\,c)} - 70\,e^{(2\,dx+2\,c)} + 23\right)}{\left(e^{(2\,dx+2\,c)} - 1\right)^5}\right)b^{\frac{3}{2}}}{15\,d}$$

input
```
integrate((b*coth(d*x+c)^4)^(3/2),x, algorithm="giac")
```

output
```
1/15*(15*d*x + 15*c - 2*(45*e^(8*d*x + 8*c) - 90*e^(6*d*x + 6*c) + 140*e^(
4*d*x + 4*c) - 70*e^(2*d*x + 2*c) + 23)/(e^(2*d*x + 2*c) - 1)^5)*b^(3/2)/d
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\coth^4(c+dx)\right)^{3/2} dx = \int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^4\right)}^{3/2} \,d x$$

input
```
int((b*coth(c + d*x)^4)^(3/2),x)
```

output
```
int((b*coth(c + d*x)^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.37

$$\int \left(b \coth^4(c + dx)\right)^{3/2} dx = \frac{\sqrt{b}\, b\left(-3 \coth(dx + c)^5 - 5 \coth(dx + c)^3 - 15 \coth(dx + c) + 15dx\right)}{15d}$$

input
```
int((b*coth(d*x+c)^4)^(3/2),x)
```

output
```
(sqrt(b)*b*( - 3*coth(c + d*x)**5 - 5*coth(c + d*x)**3 - 15*coth(c + d*x)
+ 15*d*x))/(15*d)
```

3.41 $\int \sqrt{b\coth^4(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \sqrt{b\coth^4(c+dx)}\,dx = -\frac{\sqrt{b\coth^4(c+dx)}\tanh(c+dx)}{d} + x\sqrt{b\coth^4(c+dx)}\tanh^2(c+dx)$$

output
```
-(b*coth(d*x+c)^4)^(1/2)*tanh(d*x+c)/d+x*(b*coth(d*x+c)^4)^(1/2)*tanh(d*x+
c)^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \sqrt{b\coth^4(c+dx)}\,dx = -\frac{\sqrt{b\coth^4(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(c+dx)\right)\tanh(c+dx)}{d}$$

input
```
Integrate[Sqrt[b*Coth[c + d*x]^4],x]
```

output
```
-((Sqrt[b*Coth[c + d*x]^4]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2
]*Tanh[c + d*x])/d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4141, 3042, 25, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\coth^4(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\tan\left(ic+idx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{4141}$$

$$\tanh^2(c+dx)\sqrt{b\coth^4(c+dx)}\int \coth^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tanh^2(c+dx)\sqrt{b\coth^4(c+dx)}\int -\tan\left(ic+idx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$\tanh^2(c+dx)\left(-\sqrt{b\coth^4(c+dx)}\right)\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx$$

$$\Big\downarrow \text{3954}$$

$$\tanh^2(c+dx)\left(-\sqrt{b\coth^4(c+dx)}\right)\left(\frac{\coth(c+dx)}{d}-\int 1dx\right)$$

$$\Big\downarrow \text{24}$$

$$\tanh^2(c+dx)\left(\frac{\coth(c+dx)}{d}-x\right)\left(-\sqrt{b\coth^4(c+dx)}\right)$$

input
```
Int[Sqrt[b*Coth[c + d*x]^4],x]
```

output
```
-(Sqrt[b*Coth[c + d*x]^4]*(-x + Coth[c + d*x]/d)*Tanh[c + d*x]^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$-\frac{\sqrt{b\coth(dx+c)^4}\,(2\coth(dx+c)+\ln(\coth(dx+c)-1)-\ln(\coth(dx+c)+1))}{2d\coth(dx+c)^2}$	55
default	$-\frac{\sqrt{b\coth(dx+c)^4}\,(2\coth(dx+c)+\ln(\coth(dx+c)-1)-\ln(\coth(dx+c)+1))}{2d\coth(dx+c)^2}$	55
risch	$\frac{\sqrt{\frac{b(e^{2dx+2c}+1)^4}{(e^{2dx+2c}-1)^4}}\,(e^{2dx+2c}-1)^2x}{(e^{2dx+2c}+1)^2}-\frac{2\sqrt{\frac{b(e^{2dx+2c}+1)^4}{(e^{2dx+2c}-1)^4}}\,(e^{2dx+2c}-1)}{(e^{2dx+2c}+1)^2d}$	119

input
```
int((b*coth(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*(b*coth(d*x+c)^4)^(1/2)*(2*coth(d*x+c)+ln(coth(d*x+c)-1)-ln(coth(d*
x+c)+1))/coth(d*x+c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(46) = 92.

Time = 0.12 (sec) , antiderivative size = 415, normalized size of antiderivative = 8.30

$$\int\sqrt{b\coth^4(c+dx)}\,dx$$
$$=\frac{\left(dx\cosh\left(dx+c\right)^2+\left(dxe^{(4\,dx+4\,c)}-2\,dxe^{(2\,dx+2\,c)}+dx\right)\sinh\left(dx+c\right)^2-dx+\left(dx\cosh\left(dx+c\right)^2-dx\right.}{d\cosh\left(dx+c\right)^2+\left(de^{(4\,dx+4\,c)}+2\,de^{(2\,dx+2\,c)}+d\right)\sinh\left(dx+c\right)^2+}$$

input
```
integrate((b*coth(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
(d*x*cosh(d*x + c)^2 + (d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c) + d*x)
*sinh(d*x + c)^2 - d*x + (d*x*cosh(d*x + c)^2 - d*x - 2)*e^(4*d*x + 4*c) -
 2*(d*x*cosh(d*x + c)^2 - d*x - 2)*e^(2*d*x + 2*c) + 2*(d*x*cosh(d*x + c)*
e^(4*d*x + 4*c) - 2*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))
*sinh(d*x + c) - 2)*sqrt((b*e^(8*d*x + 8*c) + 4*b*e^(6*d*x + 6*c) + 6*b*e^
(4*d*x + 4*c) + 4*b*e^(2*d*x + 2*c) + b)/(e^(8*d*x + 8*c) - 4*e^(6*d*x + 6
*c) + 6*e^(4*d*x + 4*c) - 4*e^(2*d*x + 2*c) + 1))/(d*cosh(d*x + c)^2 + (d*
e^(4*d*x + 4*c) + 2*d*e^(2*d*x + 2*c) + d)*sinh(d*x + c)^2 + (d*cosh(d*x +
 c)^2 - d)*e^(4*d*x + 4*c) + 2*(d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) + 2
*(d*cosh(d*x + c)*e^(4*d*x + 4*c) + 2*d*cosh(d*x + c)*e^(2*d*x + 2*c) + d*
cosh(d*x + c))*sinh(d*x + c) - d)
```

Sympy [F]

$$\int \sqrt{b\coth^4(c+dx)}\,dx = \int \sqrt{b\coth^4{(c+dx)}}\,dx$$

input
```
integrate((b*coth(d*x+c)**4)**(1/2),x)
```

output
```
Integral(sqrt(b*coth(c + d*x)**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int \sqrt{b\coth^4(c+dx)}\,dx = \frac{(dx+c)\sqrt{b}}{d} + \frac{2\sqrt{b}}{d(e^{(-2\,dx-2\,c)}-1)}$$

input
```
integrate((b*coth(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
(d*x + c)*sqrt(b)/d + 2*sqrt(b)/(d*(e^(-2*d*x - 2*c) - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.54

$$\int \sqrt{b\coth^4(c+dx)}\,dx = \frac{\left(dx+c-\frac{2}{e^{(2\,dx+2\,c)}-1}\right)\sqrt{b}}{d}$$

input
```
integrate((b*coth(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
(d*x + c - 2/(e^(2*d*x + 2*c) - 1))*sqrt(b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\coth^4(c+dx)}\,dx = \int \sqrt{b\coth\left(c+d\,x\right)^4}\,dx$$

input
```
int((b*coth(c + d*x)^4)^(1/2),x)
```

output
```
int((b*coth(c + d*x)^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.36

$$\int \sqrt{b\coth^4(c+dx)}\,dx = \frac{\sqrt{b}\,(-\coth\left(dx+c\right)+dx)}{d}$$

input
```
int((b*coth(d*x+c)^4)^(1/2),x)
```

output
```
(sqrt(b)*( - coth(c + d*x) + d*x))/d
```

3.42 $\int \frac{1}{\sqrt{b\coth^4(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \frac{1}{\sqrt{b\coth^4(c+dx)}}\,dx=-\frac{\coth(c+dx)}{d\sqrt{b\coth^4(c+dx)}}+\frac{x\coth^2(c+dx)}{\sqrt{b\coth^4(c+dx)}}$$

output
```
-coth(d*x+c)/d/(b*coth(d*x+c)^4)^(1/2)+x*coth(d*x+c)^2/(b*coth(d*x+c)^4)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{b\coth^4(c+dx)}}\,dx=\frac{\coth(c+dx)(-1+\operatorname{arctanh}(\tanh(c+dx))\coth(c+dx))}{d\sqrt{b\coth^4(c+dx)}}$$

input
```
Integrate[1/Sqrt[b*Coth[c + d*x]^4],x]
```

output
```
(Coth[c + d*x]*(-1 + ArcTanh[Tanh[c + d*x]]*Coth[c + d*x]))/(d*Sqrt[b*Coth
[c + d*x]^4])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4141, 3042, 25, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\coth^4(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{b\tan\left(ic+idx+\frac{\pi}{2}\right)^4}}dx$$
$$\Big\downarrow \text{4141}$$
$$\frac{\coth^2(c+dx)\int \tanh^2(c+dx)dx}{\sqrt{b\coth^4(c+dx)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\coth^2(c+dx)\int -\tan(ic+idx)^2dx}{\sqrt{b\coth^4(c+dx)}}$$
$$\Big\downarrow \text{25}$$
$$-\frac{\coth^2(c+dx)\int \tan(ic+idx)^2dx}{\sqrt{b\coth^4(c+dx)}}$$
$$\Big\downarrow \text{3954}$$
$$-\frac{\coth^2(c+dx)\left(\frac{\tanh(c+dx)}{d}-\int 1dx\right)}{\sqrt{b\coth^4(c+dx)}}$$
$$\Big\downarrow \text{24}$$
$$-\frac{\left(\frac{\tanh(c+dx)}{d}-x\right)\coth^2(c+dx)}{\sqrt{b\coth^4(c+dx)}}$$

input
```
Int[1/Sqrt[b*Coth[c + d*x]^4],x]
```

output
```
-((Coth[c + d*x]^2*(-x + Tanh[c + d*x]/d))/Sqrt[b*Coth[c + d*x]^4])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

method	result	size
derivativedivides	$\frac{\coth(dx+c)(\ln(\coth(dx+c)+1)\coth(dx+c)-\ln(\coth(dx+c)-1)\coth(dx+c)-2)}{2d\sqrt{b\coth(dx+c)^4}}$	59
default	$\frac{\coth(dx+c)(\ln(\coth(dx+c)+1)\coth(dx+c)-\ln(\coth(dx+c)-1)\coth(dx+c)-2)}{2d\sqrt{b\coth(dx+c)^4}}$	59
risch	$\frac{e^{4dx+4c}dx+2\,e^{2dx+2c}dx+dx+2\,e^{2dx+2c}+2}{\sqrt{\frac{b\left(e^{2dx+2c}+1\right)^4}{\left(e^{2dx+2c}-1\right)^4}}\,\left(e^{2dx+2c}-1\right)^2 d}$	89

input
```
int(1/(b*coth(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*coth(d*x+c)*(ln(coth(d*x+c)+1)*coth(d*x+c)-ln(coth(d*x+c)-1)*coth(d*
x+c)-2)/(b*coth(d*x+c)^4)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(46) = 92.

Time = 0.12 (sec) , antiderivative size = 422, normalized size of antiderivative = 8.44

$$\int \frac{1}{\sqrt{b\coth^4(c+dx)}}\,dx$$
$$= \frac{\left(dx\cosh\left(dx+c\right)\right)^2+\left(dxe^{(4\,dx+4\,c)}-2\,dxe^{(2\,dx+2\,c)}+dx\right)\sinh\left(dx+c\right)^2+dx+\left(dx\cosh\left(dx+c\right)^2+dx-\right.}{bd\cosh\left(dx+c\right)^2+\left(bde^{(4\,dx+4\,c)}+2\,bde^{(2\,dx+2\,c)}+bd\right)\sinh\left(dx+c\right)^2+b}$$

input
```
integrate(1/(b*coth(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
(d*x*cosh(d*x + c)^2 + (d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c) + d*x)
*sinh(d*x + c)^2 + d*x + (d*x*cosh(d*x + c)^2 + d*x + 2)*e^(4*d*x + 4*c) -
 2*(d*x*cosh(d*x + c)^2 + d*x + 2)*e^(2*d*x + 2*c) + 2*(d*x*cosh(d*x + c)*
e^(4*d*x + 4*c) - 2*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))
*sinh(d*x + c) + 2)*sqrt((b*e^(8*d*x + 8*c) + 4*b*e^(6*d*x + 6*c) + 6*b*e^
(4*d*x + 4*c) + 4*b*e^(2*d*x + 2*c) + b)/(e^(8*d*x + 8*c) - 4*e^(6*d*x + 6
*c) + 6*e^(4*d*x + 4*c) - 4*e^(2*d*x + 2*c) + 1))/(b*d*cosh(d*x + c)^2 + (
b*d*e^(4*d*x + 4*c) + 2*b*d*e^(2*d*x + 2*c) + b*d)*sinh(d*x + c)^2 + b*d +
 (b*d*cosh(d*x + c)^2 + b*d)*e^(4*d*x + 4*c) + 2*(b*d*cosh(d*x + c)^2 + b*
d)*e^(2*d*x + 2*c) + 2*(b*d*cosh(d*x + c)*e^(4*d*x + 4*c) + 2*b*d*cosh(d*x
 + c)*e^(2*d*x + 2*c) + b*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\coth^4(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\coth^4{(c+dx)}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)**4)**(1/2),x)
```

output
```
Integral(1/sqrt(b*coth(c + d*x)**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt{b\coth^4(c+dx)}}\,dx = \frac{dx+c}{\sqrt{b}d} - \frac{2\sqrt{b}}{(be^{(-2\,dx-2\,c)}+b)d}$$

input
```
integrate(1/(b*coth(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
(d*x + c)/(sqrt(b)*d) - 2*sqrt(b)/((b*e^(-2*d*x - 2*c) + b)*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{b\coth^4(c+dx)}}\,dx = \frac{\frac{dx+c}{\sqrt{b}} + \frac{2}{\sqrt{b}\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

input
```
integrate(1/(b*coth(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
((d*x + c)/sqrt(b) + 2/(sqrt(b)*(e^(2*d*x + 2*c) + 1)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\coth^4(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\coth\left(c+d\,x\right)^4}}\,dx$$

input
```
int(1/(b*coth(c + d*x)^4)^(1/2),x)
```

output
```
int(1/(b*coth(c + d*x)^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{b\coth^4(c+dx)}}\,dx = \frac{\sqrt{b}\,(\coth(dx+c)\,dx-1)}{\coth(dx+c)\,bd}$$

input
```
int(1/(b*coth(d*x+c)^4)^(1/2),x)
```

output
```
(sqrt(b)*(coth(c + d*x)*d*x - 1))/(coth(c + d*x)*b*d)
```

3.43 $\int \frac{1}{\left(b \coth^4(c+dx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 118

$$\int \frac{1}{\left(b \coth^4(c+dx)\right)^{3/2}}\,dx = -\frac{\coth(c+dx)}{bd\sqrt{b\coth^4(c+dx)}} + \frac{x\coth^2(c+dx)}{b\sqrt{b\coth^4(c+dx)}} - \frac{\tanh(c+dx)}{3bd\sqrt{b\coth^4(c+dx)}} - \frac{\tanh^3(c+dx)}{5bd\sqrt{b\coth^4(c+dx)}}$$

output
```
-coth(d*x+c)/b/d/(b*coth(d*x+c)^4)^(1/2)+x*coth(d*x+c)^2/b/(b*coth(d*x+c)^
4)^(1/2)-1/3*tanh(d*x+c)/b/d/(b*coth(d*x+c)^4)^(1/2)-1/5*tanh(d*x+c)^3/b/d
/(b*coth(d*x+c)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.58

$$\int \frac{1}{\left(b \coth^4(c+dx)\right)^{3/2}}\,dx = \frac{-15\coth(c+dx) + 15\text{arctanh}(\tanh(c+dx))\coth^2(c+dx) - 5\tanh(c+dx}{15bd\sqrt{b\coth^4(c+dx)}}$$

input
```
Integrate[(b*Coth[c + d*x]^4)^(-3/2),x]
```

output
```
(-15*Coth[c + d*x] + 15*ArcTanh[Tanh[c + d*x]]*Coth[c + d*x]^2 - 5*Tanh[c
+ d*x] - 3*Tanh[c + d*x]^3)/(15*b*d*Sqrt[b*Coth[c + d*x]^4])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.60, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 4141, 3042, 25, 3954, 3042, 3954, 25, 3042, 25, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(b\tan\left(ic+idx+\frac{\pi}{2}\right)^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\coth^2(c+dx)\int \tanh^6(c+dx)dx}{b\sqrt{b\coth^4(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\coth^2(c+dx)\int -\tan(ic+idx)^6dx}{b\sqrt{b\coth^4(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\coth^2(c+dx)\int \tan(ic+idx)^6dx}{b\sqrt{b\coth^4(c+dx)}}$$

$$\Big\downarrow \text{3954}$$

$$-\frac{\coth^2(c+dx)\left(\frac{\tanh^5(c+dx)}{5d}-\int \tanh^4(c+dx)dx\right)}{b\sqrt{b\coth^4(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\coth^2(c+dx)\left(\frac{\tanh^5(c+dx)}{5d}-\int \tan(ic+idx)^4dx\right)}{b\sqrt{b\coth^4(c+dx)}}$$

$\Big\downarrow$ 3954

$$-\frac{\coth^2(c+dx)\left(\int -\tanh^2(c+dx)dx+\frac{\tanh^5(c+dx)}{5d}+\frac{\tanh^3(c+dx)}{3d}\right)}{b\sqrt{b\coth^4(c+dx)}}$$

$\Big\downarrow$ 25

$$-\frac{\coth^2(c+dx)\left(-\int \tanh^2(c+dx)dx+\frac{\tanh^5(c+dx)}{5d}+\frac{\tanh^3(c+dx)}{3d}\right)}{b\sqrt{b\coth^4(c+dx)}}$$

$\Big\downarrow$ 3042

$$-\frac{\coth^2(c+dx)\left(-\int -\tan(ic+idx)^2dx+\frac{\tanh^5(c+dx)}{5d}+\frac{\tanh^3(c+dx)}{3d}\right)}{b\sqrt{b\coth^4(c+dx)}}$$

$\Big\downarrow$ 25

$$-\frac{\coth^2(c+dx)\left(\int \tan(ic+idx)^2dx+\frac{\tanh^5(c+dx)}{5d}+\frac{\tanh^3(c+dx)}{3d}\right)}{b\sqrt{b\coth^4(c+dx)}}$$

$\Big\downarrow$ 3954

$$-\frac{\coth^2(c+dx)\left(-\int 1dx+\frac{\tanh^5(c+dx)}{5d}+\frac{\tanh^3(c+dx)}{3d}+\frac{\tanh(c+dx)}{d}\right)}{b\sqrt{b\coth^4(c+dx)}}$$

$\Big\downarrow$ 24

$$-\frac{\left(\frac{\tanh^5(c+dx)}{5d}+\frac{\tanh^3(c+dx)}{3d}+\frac{\tanh(c+dx)}{d}-x\right)\coth^2(c+dx)}{b\sqrt{b\coth^4(c+dx)}}$$

input
```
Int[(b*Coth[c + d*x]^4)^(-3/2),x]
```

output
```
-((Coth[c + d*x]^2*(-x + Tanh[c + d*x]/d + Tanh[c + d*x]^3/(3*d) + Tanh[c
+ d*x]^5/(5*d)))/(b*Sqrt[b*Coth[c + d*x]^4]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{\coth(dx+c)\left(15\ln(\coth(dx+c)+1)\coth(dx+c)^5-15\ln(\coth(dx+c)-1)\coth(dx+c)^5-30\coth(dx+c)^4-10\coth(dx+c)\right.}{30d\left(b\coth(dx+c)^4\right)^{\frac{3}{2}}}$
default	$\frac{\coth(dx+c)\left(15\ln(\coth(dx+c)+1)\coth(dx+c)^5-15\ln(\coth(dx+c)-1)\coth(dx+c)^5-30\coth(dx+c)^4-10\coth(dx+c)\right.}{30d\left(b\coth(dx+c)^4\right)^{\frac{3}{2}}}$
risch	$\frac{\left(e^{2dx+2c}+1\right)^2x}{b\left(e^{2dx+2c}-1\right)^2\sqrt{\frac{b\left(e^{2dx+2c}+1\right)^4}{\left(e^{2dx+2c}-1\right)^4}}}+\frac{6\,e^{8dx+8c}+12\,e^{6dx+6c}+\frac{56\,e^{4dx+4c}}{3}+\frac{28\,e^{2dx+2c}}{3}+\frac{46}{15}}{b\left(e^{2dx+2c}+1\right)^3\left(e^{2dx+2c}-1\right)^2\sqrt{\frac{b\left(e^{2dx+2c}+1\right)^4}{\left(e^{2dx+2c}-1\right)^4}}\,d}$

input
```
int(1/(b*coth(d*x+c)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/30/d*coth(d*x+c)*(15*ln(coth(d*x+c)+1)*coth(d*x+c)^5-15*ln(coth(d*x+c)-1
)*coth(d*x+c)^5-30*coth(d*x+c)^4-10*coth(d*x+c)^2-6)/(b*coth(d*x+c)^4)^(3/
2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3473 vs. $2(106) = 212$.

Time = 0.18 (sec) , antiderivative size = 3473, normalized size of antiderivative = 29.43

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*coth(d*x+c)^4)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\coth^4{(c+dx)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)**4)**(3/2),x)
```

output
```
Integral((b*coth(c + d*x)**4)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.31

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{3/2}}\,dx = -\frac{2\left(70\sqrt{b}e^{(-2\,dx-2\,c)}+140\sqrt{b}e^{(-4\,dx-4\,c)}+90\sqrt{b}e^{(-6\,dx-6\,c)}+45\sqrt{b}e^{(-8\,dx-8\,c)}+23\sqrt{b}\right)}{15\left(5\,b^2e^{(-2\,dx-2\,c)}+10\,b^2e^{(-4\,dx-4\,c)}+10\,b^2e^{(-6\,dx-6\,c)}+5\,b^2e^{(-8\,dx-8\,c)}+b^2e^{(-10\,dx-10\,c)}+b^2\right)d} + \frac{dx+c}{b^{\frac{3}{2}}d}$$

input
```
integrate(1/(b*coth(d*x+c)^4)^(3/2),x, algorithm="maxima")
```

output
```
-2/15*(70*sqrt(b)*e^(-2*d*x - 2*c) + 140*sqrt(b)*e^(-4*d*x - 4*c) + 90*sqr
t(b)*e^(-6*d*x - 6*c) + 45*sqrt(b)*e^(-8*d*x - 8*c) + 23*sqrt(b))/((5*b^2*
e^(-2*d*x - 2*c) + 10*b^2*e^(-4*d*x - 4*c) + 10*b^2*e^(-6*d*x - 6*c) + 5*b
^2*e^(-8*d*x - 8*c) + b^2*e^(-10*d*x - 10*c) + b^2)*d) + (d*x + c)/(b^(3/2
)*d)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{3/2}}\,dx = \frac{\frac{15\,(dx+c)}{\sqrt{b}} + \frac{2\left(45\,e^{(8\,dx+8\,c)}+90\,e^{(6\,dx+6\,c)}+140\,e^{(4\,dx+4\,c)}+70\,e^{(2\,dx+2\,c)}+23\right)}{\sqrt{b}\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,bd}$$

input
```
integrate(1/(b*coth(d*x+c)^4)^(3/2),x, algorithm="giac")
```

output
```
1/15*(15*(d*x + c)/sqrt(b) + 2*(45*e^(8*d*x + 8*c) + 90*e^(6*d*x + 6*c) +
140*e^(4*d*x + 4*c) + 70*e^(2*d*x + 2*c) + 23)/(sqrt(b)*(e^(2*d*x + 2*c) +
 1)^5))/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\coth\left(c+d\,x\right)^4\right)^{3/2}}\,dx$$

input
```
int(1/(b*coth(c + d*x)^4)^(3/2),x)
```

output
```
int(1/(b*coth(c + d*x)^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.56

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{3/2}}\,dx=\frac{\sqrt{b}\left(15e^{10dx+10c}dx-18e^{10dx+10c}+75e^{8dx+8c}dx+150e^{6dx+6c}dx+150e^{4dx+4c}d\right.}{15b^2d\left(e^{10dx+10c}+5e^{8dx+8c}+10e^{6dx+6c}+10\right.}$$

input
```
int(1/(b*coth(d*x+c)^4)^(3/2),x)
```

output
```
(sqrt(b)*(15*e**(10*c + 10*d*x)*d*x - 18*e**(10*c + 10*d*x) + 75*e**(8*c +
 8*d*x)*d*x + 150*e**(6*c + 6*d*x)*d*x + 150*e**(4*c + 4*d*x)*d*x + 100*e*
*(4*c + 4*d*x) + 75*e**(2*c + 2*d*x)*d*x + 50*e**(2*c + 2*d*x) + 15*d*x +
28))/(15*b**2*d*(e**(10*c + 10*d*x) + 5*e**(8*c + 8*d*x) + 10*e**(6*c + 6*
d*x) + 10*e**(4*c + 4*d*x) + 5*e**(2*c + 2*d*x) + 1))
```

3.44 $\int \left(b \coth^4(c+dx)\right)^{4/3} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 299

$$\begin{aligned}
\int \left(b \coth^4(c+dx)\right)^{4/3} \, dx = & -\frac{\sqrt{3} b \arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \sqrt[3]{b \coth^4(c+dx)}}{2d \coth^{\frac{4}{3}}(c+dx)} \\
& + \frac{\sqrt{3} b \arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \sqrt[3]{b \coth^4(c+dx)}}{2d \coth^{\frac{4}{3}}(c+dx)} \\
& + \frac{b \operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right) \sqrt[3]{b \coth^4(c+dx)}}{d \coth^{\frac{4}{3}}(c+dx)} \\
& + \frac{b \operatorname{arctanh}\left(\frac{\sqrt[3]{\coth(c+dx)}}{1+\coth^{\frac{2}{3}}(c+dx)}\right) \sqrt[3]{b \coth^4(c+dx)}}{2d \coth^{\frac{4}{3}}(c+dx)} \\
& - \frac{3b \coth(c+dx) \sqrt[3]{b \coth^4(c+dx)}}{7d} - \frac{3b \coth^3(c+dx) \sqrt[3]{b \coth^4(c+dx)}}{13d} \\
& - \frac{3b \sqrt[3]{b \coth^4(c+dx)} \tanh(c+dx)}{d}
\end{aligned}$$

output
```
-1/2*3^(1/2)*b*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^
4)^(1/3)/d/coth(d*x+c)^(4/3)+1/2*3^(1/2)*b*arctan(1/3*(1+2*coth(d*x+c)^(1/
3))*3^(1/2))*(b*coth(d*x+c)^4)^(1/3)/d/coth(d*x+c)^(4/3)+b*arctanh(coth(d*
x+c)^(1/3))*(b*coth(d*x+c)^4)^(1/3)/d/coth(d*x+c)^(4/3)+1/2*b*arctanh(coth
(d*x+c)^(1/3)/(1+coth(d*x+c)^(2/3)))*(b*coth(d*x+c)^4)^(1/3)/d/coth(d*x+c)
^(4/3)-3/7*b*coth(d*x+c)*(b*coth(d*x+c)^4)^(1/3)/d-3/13*b*coth(d*x+c)^3*(b
*coth(d*x+c)^4)^(1/3)/d-3*b*(b*coth(d*x+c)^4)^(1/3)*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.82

$$\int \left(b\coth^4(c+dx)\right)^{4/3}\,dx =$$

$$-\frac{b\sqrt[3]{b\coth^4(c+dx)}\left(42\coth^4(c+dx)\sqrt[6]{\coth^2(c+dx)}+13\left(42\sqrt[6]{\coth^2(c+dx)}+6\coth^2(c+dx)^{7/6}+7\right.\right.}{}$$

input
```
Integrate[(b*Coth[c + d*x]^4)^(4/3),x]
```

output
```
-1/182*(b*(b*Coth[c + d*x]^4)^(1/3)*(42*Coth[c + d*x]^4*(Coth[c + d*x]^2)^
(1/6) + 13*(42*(Coth[c + d*x]^2)^(1/6) + 6*(Coth[c + d*x]^2)^(7/6) + 7*Log
[1 - (Coth[c + d*x]^2)^(1/6)] - 7*Log[1 + (Coth[c + d*x]^2)^(1/6)] - 7*(-1
)^(2/3)*Log[1 - (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)] + 7*(-1)^(2/3)*Log[1 +
 (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)] - 7*(-1)^(1/3)*Log[1 - (-1)^(2/3)*(Co
th[c + d*x]^2)^(1/6)] + 7*(-1)^(1/3)*Log[1 + (-1)^(2/3)*(Coth[c + d*x]^2)^
(1/6)]))*Tanh[c + d*x])/(d*(Coth[c + d*x]^2)^(1/6))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.73, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 1.429$, Rules used = {3042, 4141, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3957, 25, 266, 754, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\coth^4(c+dx)\right)^{4/3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan\left(ic+idx+\frac{\pi}{2}\right)^4\right)^{4/3}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{b\sqrt[3]{b\coth^4(c+dx)}\int\coth^{\frac{16}{3}}(c+dx)dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt[3]{b\coth^4(c+dx)}\int\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{16/3}dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$\Big\downarrow \text{3954}$$

$$\frac{b\sqrt[3]{b\coth^4(c+dx)}\left(\int\coth^{\frac{10}{3}}(c+dx)dx-\frac{3\coth^{\frac{13}{3}}(c+dx)}{13d}\right)}{\coth^{\frac{4}{3}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt[3]{b\coth^4(c+dx)}\left(-\frac{3\coth^{\frac{13}{3}}(c+dx)}{13d}+\int\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{10/3}dx\right)}{\coth^{\frac{4}{3}}(c+dx)}$$

$$\Big\downarrow \text{3954}$$

$$\frac{b\sqrt[3]{b\coth^4(c+dx)}\left(\int\coth^{\frac{4}{3}}(c+dx)dx-\frac{3\coth^{\frac{13}{3}}(c+dx)}{13d}-\frac{3\coth^{\frac{7}{3}}(c+dx)}{7d}\right)}{\coth^{\frac{4}{3}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt[3]{b\coth^4(c+dx)}\left(\int\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{4/3}dx-\frac{3\coth^{\frac{13}{3}}(c+dx)}{13d}-\frac{3\coth^{\frac{7}{3}}(c+dx)}{7d}\right)}{\coth^{\frac{4}{3}}(c+dx)}$$

$$\Big\downarrow \text{3954}$$

$$\frac{b\sqrt[3]{b\coth^4(c+dx)}\left(\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}dx-\frac{3\coth^{\frac{13}{3}}(c+dx)}{13d}-\frac{3\coth^{\frac{7}{3}}(c+dx)}{7d}-\frac{3\sqrt[3]{\coth(c+dx)}}{d}\right)}{\coth^{\frac{4}{3}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{b\sqrt[3]{b\coth^4(c+dx)}\left(\int \frac{1}{\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{2/3}}dx-\frac{3\coth^{\frac{13}{3}}(c+dx)}{13d}-\frac{3\coth^{\frac{7}{3}}(c+dx)}{7d}-\frac{3\sqrt[3]{\coth(c+dx)}}{d}\right)}{\coth^{\frac{4}{3}}(c+dx)}$$

$\downarrow$ 3957

$$\frac{b\sqrt[3]{b\coth^4(c+dx)}\left(-\frac{\int -\frac{1}{\coth^{\frac{2}{3}}(c+dx)\left(1-\coth^2(c+dx)\right)}d\coth(c+dx)}{d}-\frac{3\coth^{\frac{13}{3}}(c+dx)}{13d}-\frac{3\coth^{\frac{7}{3}}(c+dx)}{7d}-\frac{3\sqrt[3]{\coth(c+dx)}}{d}\right)}{\coth^{\frac{4}{3}}(c+dx)}$$

$\downarrow$ 25

$$\frac{b\sqrt[3]{b\coth^4(c+dx)}\left(\frac{\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)\left(1-\coth^2(c+dx)\right)}d\coth(c+dx)}{d}-\frac{3\coth^{\frac{13}{3}}(c+dx)}{13d}-\frac{3\coth^{\frac{7}{3}}(c+dx)}{7d}-\frac{3\sqrt[3]{\coth(c+dx)}}{d}\right)}{\coth^{\frac{4}{3}}(c+dx)}$$

$\downarrow$ 266

$$\frac{b\sqrt[3]{b\coth^4(c+dx)}\left(\frac{3\int \frac{1}{1-\coth^2(c+dx)}d\sqrt[3]{\coth(c+dx)}}{d}-\frac{3\coth^{\frac{13}{3}}(c+dx)}{13d}-\frac{3\coth^{\frac{7}{3}}(c+dx)}{7d}-\frac{3\sqrt[3]{\coth(c+dx)}}{d}\right)}{\coth^{\frac{4}{3}}(c+dx)}$$

$\downarrow$ 754

$$\frac{b\sqrt[3]{b\coth^4(c+dx)}\left(\frac{3\left(\frac{1}{3}\int \frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{3}\int \frac{2-\sqrt[3]{\coth(c+dx)}}{2\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{3}\int\right.}{d}\right.}{\coth^{\frac{4}{3}}(c+d}$$

$\downarrow$ 27

$$\frac{b\sqrt[3]{b\coth^4(c+dx)}\left(\frac{3\left(\frac{1}{3}\int\frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{6}\int\frac{2-\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\frac{1}{6}\int\frac{}{\coth}\right.}{d}\right.}{\coth^{\frac{4}{3}}(c+dx)}$$

$\downarrow$ 219

$$\frac{b\sqrt[3]{b\coth^4(c+dx)}\left(\frac{3\left(\frac{1}{6}\int\frac{2-\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\frac{1}{6}\int\frac{\sqrt[3]{\coth(c+dx)}+2}{\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth}\right.}{d}\right.}{\coth^{\frac{4}{3}}(c+dx)}$$

$\downarrow$ 1142

$$b\sqrt[3]{b\coth^4(c+dx)}\left(3\left(\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{2}\int-\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{}\right.\right.\right.$$

$\downarrow$ 25

$$b\sqrt[3]{b\coth^4(c+dx)}\left(3\left(\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{c}\right.\right.\right.$$

$\downarrow$ 1083

$$b\sqrt[3]{b\coth^4(c+dx)}\left(3\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-3\int\frac{1}{-\coth^{\frac{2}{3}}(c+dx)-3}d\left(2\sqrt[3]{\coth(c+dx)}-\right.\right.\right.\right.$$

$\downarrow$ 217

$$\frac{b\sqrt[3]{b\coth^4(c+dx)}\left(\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(\right.}{}\right.}{}$$

$\downarrow$ 1103

$$\frac{b\sqrt[3]{b\coth^4(c+dx)}\left(\frac{3\left(\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)\right)+\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{}}{}\right.\right.}{d}\right.}{}$$

input
```
Int[(b*Coth[c + d*x]^4)^(4/3),x]
```

output
```
(b*(b*Coth[c + d*x]^4)^(1/3)*((-3*Coth[c + d*x]^(1/3))/d - (3*Coth[c + d*x
]^(7/3))/(7*d) - (3*Coth[c + d*x]^(13/3))/(13*d) + (3*(ArcTanh[Coth[c + d*
x]^(1/3)]/3 + (Sqrt[3]*ArcTan[(-1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - Log[
1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]/2)/6 + (Sqrt[3]*ArcTan[(1 +
 2*Coth[c + d*x]^(1/3))/Sqrt[3]] + Log[1 + Coth[c + d*x]^(1/3) + Coth[c +
d*x]^(2/3)]/2)/6))/d))/Coth[c + d*x]^(4/3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \left(b \coth \left(d x+c\right)^{4}\right)^{\frac{4}{3}}d x$$

input
```
int((b*coth(d*x+c)^4)^(4/3),x)
```

output
```
int((b*coth(d*x+c)^4)^(4/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2864 vs. $2(251) = 502$.

Time = 0.16 (sec) , antiderivative size = 2864, normalized size of antiderivative = 9.58

$$\int \left(b \coth^4(c+dx)\right)^{4/3} dx = \text{Too large to display}$$

input
```
integrate((b*coth(d*x+c)^4)^(4/3),x, algorithm="fricas")
```

output
```
-1/364*(182*(sqrt(3)*b*cosh(d*x + c)^8 + 8*sqrt(3)*b*cosh(d*x + c)*sinh(d*
x + c)^7 + sqrt(3)*b*sinh(d*x + c)^8 - 4*sqrt(3)*b*cosh(d*x + c)^6 + 4*(7*
sqrt(3)*b*cosh(d*x + c)^2 - sqrt(3)*b)*sinh(d*x + c)^6 + 8*(7*sqrt(3)*b*co
sh(d*x + c)^3 - 3*sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c)^5 + 6*sqrt(3)*b*c
osh(d*x + c)^4 + 2*(35*sqrt(3)*b*cosh(d*x + c)^4 - 30*sqrt(3)*b*cosh(d*x +
 c)^2 + 3*sqrt(3)*b)*sinh(d*x + c)^4 + 8*(7*sqrt(3)*b*cosh(d*x + c)^5 - 10
*sqrt(3)*b*cosh(d*x + c)^3 + 3*sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c)^3 -
4*sqrt(3)*b*cosh(d*x + c)^2 + 4*(7*sqrt(3)*b*cosh(d*x + c)^6 - 15*sqrt(3)*
b*cosh(d*x + c)^4 + 9*sqrt(3)*b*cosh(d*x + c)^2 - sqrt(3)*b)*sinh(d*x + c)
^2 + 8*(sqrt(3)*b*cosh(d*x + c)^7 - 3*sqrt(3)*b*cosh(d*x + c)^5 + 3*sqrt(3
)*b*cosh(d*x + c)^3 - sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c) + sqrt(3)*b)*
(-b)^(1/3)*arctan(1/3*(sqrt(3)*b + 2*sqrt(3)*(-b)^(2/3)*(b*cosh(d*x + c)/s
inh(d*x + c))^(1/3))/b) - 182*(sqrt(3)*b*cosh(d*x + c)^8 + 8*sqrt(3)*b*cos
h(d*x + c)*sinh(d*x + c)^7 + sqrt(3)*b*sinh(d*x + c)^8 - 4*sqrt(3)*b*cosh(
d*x + c)^6 + 4*(7*sqrt(3)*b*cosh(d*x + c)^2 - sqrt(3)*b)*sinh(d*x + c)^6 +
 8*(7*sqrt(3)*b*cosh(d*x + c)^3 - 3*sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c)
^5 + 6*sqrt(3)*b*cosh(d*x + c)^4 + 2*(35*sqrt(3)*b*cosh(d*x + c)^4 - 30*sq
rt(3)*b*cosh(d*x + c)^2 + 3*sqrt(3)*b)*sinh(d*x + c)^4 + 8*(7*sqrt(3)*b*co
sh(d*x + c)^5 - 10*sqrt(3)*b*cosh(d*x + c)^3 + 3*sqrt(3)*b*cosh(d*x + c))*
sinh(d*x + c)^3 - 4*sqrt(3)*b*cosh(d*x + c)^2 + 4*(7*sqrt(3)*b*cosh(d*x...
```

Sympy [F]

$$\int \left(b\coth^4(c+dx)\right)^{4/3}\,dx=\int \left(b\coth^4{(c+dx)}\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((b*coth(d*x+c)**4)**(4/3),x)
```

output
```
Integral((b*coth(c + d*x)**4)**(4/3), x)
```

Maxima [F]

$$\int \left(b\coth^4(c+dx)\right)^{4/3} dx = \int \left(b\coth\left(dx+c\right)^4\right)^{\frac{4}{3}} dx$$

input
```
integrate((b*coth(d*x+c)^4)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^4)^(4/3), x)
```

Giac [F]

$$\int \left(b\coth^4(c+dx)\right)^{4/3} dx = \int \left(b\coth\left(dx+c\right)^4\right)^{\frac{4}{3}} dx$$

input
```
integrate((b*coth(d*x+c)^4)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^4)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\coth^4(c+dx)\right)^{4/3} dx = \int \left(b\coth(c+d\,x)^4\right)^{4/3} dx$$

input
```
int((b*coth(c + d*x)^4)^(4/3),x)
```

output
```
int((b*coth(c + d*x)^4)^(4/3), x)
```

Reduce [F]

$$\int \left(b\coth^4(c+dx)\right)^{4/3}\,dx = \frac{b^{\frac{4}{3}}\left(-21\coth(dx+c)^{\frac{13}{3}}-39\coth(dx+c)^{\frac{7}{3}}-273\coth(dx+c)^{\frac{1}{3}}+91\left(\int \frac{1}{\coth(dx+c)^{\frac{2}{3}}}dx\right)d$$
$$91d$$

input
```
int((b*coth(d*x+c)^4)^(4/3),x)
```

output
```
(b**(1/3)*b*( - 21*coth(c + d*x)**(1/3)*coth(c + d*x)**4 - 39*coth(c + d*x
)**(1/3)*coth(c + d*x)**2 - 273*coth(c + d*x)**(1/3) + 91*int(coth(c + d*x
)**(1/3)/coth(c + d*x),x)*d))/(91*d)
```

3.45 $\int \left(b\coth^4(c+dx)\right)^{2/3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 238

$$\begin{aligned}\int \left(b\coth^4(c+dx)\right)^{2/3}\,dx &= \frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{2d\coth^{\frac{8}{3}}(c+dx)} \\ &- \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{2d\coth^{\frac{8}{3}}(c+dx)} \\ &+ \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{d\coth^{\frac{8}{3}}(c+dx)} \\ &+ \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\coth(c+dx)}}{1+\coth^{\frac{2}{3}}(c+dx)}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{2d\coth^{\frac{8}{3}}(c+dx)} \\ &- \frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c+dx)}{5d}\end{aligned}$$

output
```
1/2*3^(1/2)*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^4)^
(2/3)/d/coth(d*x+c)^(8/3)-1/2*3^(1/2)*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3
^(1/2))*(b*coth(d*x+c)^4)^(2/3)/d/coth(d*x+c)^(8/3)+arctanh(coth(d*x+c)^(1
/3))*(b*coth(d*x+c)^4)^(2/3)/d/coth(d*x+c)^(8/3)+1/2*arctanh(coth(d*x+c)^(
1/3)/(1+coth(d*x+c)^(2/3)))*(b*coth(d*x+c)^4)^(2/3)/d/coth(d*x+c)^(8/3)-3/
5*(b*coth(d*x+c)^4)^(2/3)*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.70

$$\int \left(b\coth^4(c+dx)\right)^{2/3}\,dx=\frac{\left(b\coth^4(c+dx)\right)^{2/3}\left(20\text{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)-12\coth^{\frac{5}{3}}(c+dx)+5\left(2\sqrt{3}\arctan\left(\frac{1\cdots}{}\right.\right.\right.}{}$$

input
```
Integrate[(b*Coth[c + d*x]^4)^(2/3),x]
```

output
```
((b*Coth[c + d*x]^4)^(2/3)*(20*ArcTanh[Coth[c + d*x]^(1/3)] - 12*Coth[c +
d*x]^(5/3) + 5*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - 2*
Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - Log[1 - Coth[c + d*x
]^(1/3) + Coth[c + d*x]^(2/3)] + Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*
x]^(2/3)])))/(20*d*Coth[c + d*x]^(8/3))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.78, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {3042, 4141, 3042, 3954, 3042, 3957, 25, 266, 825, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\coth^4(c+dx)\right)^{2/3}\,dx$$

$\Big\downarrow$ 3042

$$\int \left(b\tan\left(ic+idx+\frac{\pi}{2}\right)^4\right)^{2/3}\,dx$$

$\Big\downarrow$ 4141

$$\frac{\left(b\coth^4(c+dx)\right)^{2/3}\int \coth^{\frac{8}{3}}(c+dx)dx}{\coth^{\frac{8}{3}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{\left(b\coth^4(c+dx)\right)^{2/3}\int \left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{8/3}dx}{\coth^{\frac{8}{3}}(c+dx)}$$

$\Big\downarrow$ 3954

$$\frac{\left(b\coth^4(c+dx)\right)^{2/3}\left(\int \coth^{\frac{2}{3}}(c+dx)dx-\frac{3\coth^{\frac{5}{3}}(c+dx)}{5d}\right)}{\coth^{\frac{8}{3}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{\left(b\coth^4(c+dx)\right)^{2/3}\left(-\frac{3\coth^{\frac{5}{3}}(c+dx)}{5d}+\int \left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{2/3}dx\right)}{\coth^{\frac{8}{3}}(c+dx)}$$

$\Big\downarrow$ 3957

$$\frac{\left(b\coth^4(c+dx)\right)^{2/3}\left(-\frac{\int -\frac{\coth^{\frac{2}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}-\frac{3\coth^{\frac{5}{3}}(c+dx)}{5d}\right)}{\coth^{\frac{8}{3}}(c+dx)}$$

$\Big\downarrow$ 25

$$\frac{\left(b\coth^4(c+dx)\right)^{2/3}\left(\frac{\int \frac{\coth^{\frac{2}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}-\frac{3\coth^{\frac{5}{3}}(c+dx)}{5d}\right)}{\coth^{\frac{8}{3}}(c+dx)}$$

$\Big\downarrow$ 266

$$\frac{\left(b\coth^4(c+dx)\right)^{2/3}\left(\frac{3\int\frac{\coth^{\frac{4}{3}}(c+dx)}{1-\coth^2(c+dx)}d\sqrt[3]{\coth(c+dx)}}{d}-\frac{3\coth^{\frac{5}{3}}(c+dx)}{5d}\right)}{\coth^{\frac{8}{3}}(c+dx)}$$

$\big\downarrow$ 825

$$\frac{\left(b\coth^4(c+dx)\right)^{2/3}\left(\frac{3\left(\frac{1}{3}\int\frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{3}\int-\frac{\sqrt[3]{\coth(c+dx)}+1}{2\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}d\sqrt[3]{\coth(c+dx)}+}{d}\right.}{\coth^{\frac{8}{3}}(c+dx)}$$

$\big\downarrow$ 27

$$\frac{\left(b\coth^4(c+dx)\right)^{2/3}\left(\frac{3\left(\frac{1}{3}\int\frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}-\frac{1}{6}\int\frac{\sqrt[3]{\coth(c+dx)}+1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{6}\int-}{d}\right.}{\coth^{\frac{8}{3}}(c+dx)}$$

$\big\downarrow$ 219

$$\frac{\left(b\coth^4(c+dx)\right)^{2/3}\left(\frac{3\left(-\frac{1}{6}\int\frac{\sqrt[3]{\coth(c+dx)}+1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{6}\int\frac{1-\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{c}\right.}{d}\right.}{\coth^{\frac{8}{3}}(c+dx)}$$

$\big\downarrow$ 1142

$$\frac{\left(b\coth^4(c+dx)\right)^{2/3}\left(\frac{3\left(\frac{1}{6}\left(-\frac{3}{2}\int\frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{2}\int-\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}\right.\right.}{}\right.}{}$$

$\big\downarrow$ 25

$$\left(b\coth^4(c+dx)\right)^{2/3}\left(\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{3}{2}\int\frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\right.\right.}{}\right.$$

$\downarrow$ 1083

$$\left(b\coth^4(c+dx)\right)^{2/3}\left(\frac{3\left(\frac{1}{6}\left(3\int\frac{1}{-\coth^{\frac{2}{3}}(c+dx)-3}d\left(2\sqrt[3]{\coth(c+dx)}-1\right)+\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+\right.\right.}{}\right.$$

$\downarrow$ 217

$$\left(b\coth^4(c+dx)\right)^{2/3}\left(\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\right.}{}\right.$$

$\downarrow$ 1103

$$\left(b\coth^4(c+dx)\right)^{2/3}\left(\frac{3\left(\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)\right)+\frac{1}{6}\left(\frac{1}{2}\log\left(\coth\right.\right.\right.}{\coth^{\frac{8}{3}}(c-}\right.$$

input
```
Int[(b*Coth[c + d*x]^4)^(2/3),x]
```

output
```
((b*Coth[c + d*x]^4)^(2/3)*((-3*Coth[c + d*x]^(5/3))/(5*d) + (3*(ArcTanh[C
oth[c + d*x]^(1/3)]/3 + (-(Sqrt[3]*ArcTan[(-1 + 2*Coth[c + d*x]^(1/3))/Sqr
t[3]]) - Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]/2)/6 + (-(Sqrt
[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]) + Log[1 + Coth[c + d*x]^(
1/3) + Coth[c + d*x]^(2/3)]/2)/6))/d))/Coth[c + d*x]^(8/3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \left(b \coth \left(dx+c\right)^{4}\right)^{\frac{2}{3}}dx$$

input
```
int((b*coth(d*x+c)^4)^(2/3),x)
```

output
```
int((b*coth(d*x+c)^4)^(2/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(196) = 392$.

Time = 0.10 (sec) , antiderivative size = 618, normalized size of antiderivative = 2.60

$$\int \left(b\coth^4(c+dx)\right)^{2/3} \, dx = \text{Too large to display}$$

input
```
integrate((b*coth(d*x+c)^4)^(2/3),x, algorithm="fricas")
```

output
```
-1/20*(10*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c)
 + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b
- 2*sqrt(3)*(-b^2)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + 10*(s
qrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*s
inh(d*x + c)^2 - sqrt(3))*(b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*(
b^2)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + 5*(-b^2)^(1/3)*(cos
h(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*log(b*
(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) - (-b^2)^(1/3)*b + (-b^2)^(2/3)*(b*c
osh(d*x + c)/sinh(d*x + c))^(1/3)) + 5*(b^2)^(1/3)*(cosh(d*x + c)^2 + 2*co
sh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*log(b*(b*cosh(d*x + c)/si
nh(d*x + c))^(2/3) + (b^2)^(1/3)*b - (b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x
 + c))^(1/3)) - 10*(-b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*
x + c) + sinh(d*x + c)^2 - 1)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)
- (-b^2)^(2/3)) - 10*(b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2 - 1)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)
 + (b^2)^(2/3)) + 12*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2 + 1)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3))/(d*cosh(d*x + c)
^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 - d)
```

Sympy [F]

$$\int \left(b\coth^4(c+dx)\right)^{2/3} dx = \int \left(b\coth^4{\left(c+dx\right)}\right)^{\frac{2}{3}} dx$$

input
```
integrate((b*coth(d*x+c)**4)**(2/3),x)
```

output
```
Integral((b*coth(c + d*x)**4)**(2/3), x)
```

Maxima [F]

$$\int \left(b\coth^4(c+dx)\right)^{2/3} dx = \int \left(b\coth\left(dx+c\right)^4\right)^{\frac{2}{3}} dx$$

input
```
integrate((b*coth(d*x+c)^4)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^4)^(2/3), x)
```

Giac [F]

$$\int \left(b\coth^4(c+dx)\right)^{2/3} dx = \int \left(b\coth\left(dx+c\right)^4\right)^{\frac{2}{3}} dx$$

input
```
integrate((b*coth(d*x+c)^4)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^4)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\coth^4(c+dx)\right)^{2/3} \,dx = \int \left(b\coth(c+d\,x)^4\right)^{2/3} \,dx$$

input
```
int((b*coth(c + d*x)^4)^(2/3),x)
```

output
```
int((b*coth(c + d*x)^4)^(2/3), x)
```

Reduce [F]

$$\int \left(b\coth^4(c+dx)\right)^{2/3} \,dx = b^{\frac{2}{3}}\left(\int \coth\left(dx+c\right)^{\frac{8}{3}}dx\right)$$

input
```
int((b*coth(d*x+c)^4)^(2/3),x)
```

output
```
b**(2/3)*int(coth(c + d*x)**(2/3)*coth(c + d*x)**2,x)
```

3.46 $\int \sqrt[3]{b\coth^4(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 236

$$\int \sqrt[3]{b\coth^4(c+dx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)} + \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{d\coth^{\frac{4}{3}}(c+dx)} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\coth(c+dx)}}{1+\coth^{\frac{2}{3}}(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)} - \frac{3\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d}$$

output
```
-1/2*3^(1/2)*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^4)
^(1/3)/d/coth(d*x+c)^(4/3)+1/2*3^(1/2)*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*
3^(1/2))*(b*coth(d*x+c)^4)^(1/3)/d/coth(d*x+c)^(4/3)+arctanh(coth(d*x+c)^(
1/3))*(b*coth(d*x+c)^4)^(1/3)/d/coth(d*x+c)^(4/3)+1/2*arctanh(coth(d*x+c)^
(1/3)/(1+coth(d*x+c)^(2/3)))*(b*coth(d*x+c)^4)^(1/3)/d/coth(d*x+c)^(4/3)-3
*(b*coth(d*x+c)^4)^(1/3)*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.85

$$\int \sqrt[3]{b \coth^4(c+dx)}\, dx =$$
$$-\frac{\sqrt[3]{b \coth^4(c+dx)}\left(6\sqrt[6]{\coth^2(c+dx)} + \log\left(1-\sqrt[6]{\coth^2(c+dx)}\right) - \log\left(1+\sqrt[6]{\coth^2(c+dx)}\right) - (\right.}{}$$

input
```
Integrate[(b*Coth[c + d*x]^4)^(1/3),x]
```

output
```
-1/2*((b*Coth[c + d*x]^4)^(1/3)*(6*(Coth[c + d*x]^2)^(1/6) + Log[1 - (Coth
[c + d*x]^2)^(1/6)] - Log[1 + (Coth[c + d*x]^2)^(1/6)] - (-1)^(2/3)*Log[1
- (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)] + (-1)^(2/3)*Log[1 + (-1)^(1/3)*(Cot
h[c + d*x]^2)^(1/6)] - (-1)^(1/3)*Log[1 - (-1)^(2/3)*(Coth[c + d*x]^2)^(1/
6)] + (-1)^(1/3)*Log[1 + (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)])*Tanh[c + d*x
])/(d*(Coth[c + d*x]^2)^(1/6))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.77, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {3042, 4141, 3042, 3954, 3042, 3957, 25, 266, 754, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\coth^4(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt[3]{b\tan\left(ic+idx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \; 4141$$

$$\frac{\sqrt[3]{b\coth^4(c+dx)}\int \coth^{\frac{4}{3}}(c+dx)dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt[3]{b\coth^4(c+dx)}\int \left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{4/3}dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$\Big\downarrow \; 3954$$

$$\frac{\sqrt[3]{b\coth^4(c+dx)}\left(\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}dx-\frac{3\sqrt[3]{\coth(c+dx)}}{d}\right)}{\coth^{\frac{4}{3}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt[3]{b\coth^4(c+dx)}\left(-\frac{3\sqrt[3]{\coth(c+dx)}}{d}+\int \frac{1}{\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{2/3}}dx\right)}{\coth^{\frac{4}{3}}(c+dx)}$$

$$\Big\downarrow \; 3957$$

$$\frac{\sqrt[3]{b\coth^4(c+dx)}\left(-\frac{\int -\frac{1}{\coth^{\frac{2}{3}}(c+dx)\left(1-\coth^2(c+dx)\right)}d\coth(c+dx)}{d}-\frac{3\sqrt[3]{\coth(c+dx)}}{d}\right)}{\coth^{\frac{4}{3}}(c+dx)}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt[3]{b\coth^4(c+dx)}\left(\frac{\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)\left(1-\coth^2(c+dx)\right)}d\coth(c+dx)}{d}-\frac{3\sqrt[3]{\coth(c+dx)}}{d}\right)}{\coth^{\frac{4}{3}}(c+dx)}$$

$$\Big\downarrow \; 266$$

$$\frac{\sqrt[3]{b\coth^4(c+dx)}\left(\frac{3\int\frac{1}{1-\coth^2(c+dx)}d\sqrt[3]{\coth(c+dx)}}{d}-\frac{3\sqrt[3]{\coth(c+dx)}}{d}\right)}{\coth^{\frac{4}{3}}(c+dx)}$$

$\downarrow$ 754

$$\frac{\sqrt[3]{b\coth^4(c+dx)}\left(\frac{3\left(\frac{1}{3}\int\frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{3}\int\frac{2-\sqrt[3]{\coth(c+dx)}}{2\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{3}\int\right.}{d}\right.}{\coth^{\frac{4}{3}}(c+dx)}$$

$\downarrow$ 27

$$\frac{\sqrt[3]{b\coth^4(c+dx)}\left(\frac{3\left(\frac{1}{3}\int\frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{6}\int\frac{2-\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\frac{1}{6}\int\right.}{d}\right.}{\coth^{\frac{4}{3}}(c+dx)}$$

$\downarrow$ 219

$$\frac{\sqrt[3]{b\coth^4(c+dx)}\left(\frac{3\left(\frac{1}{6}\int\frac{2-\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\frac{1}{6}\int\frac{\sqrt[3]{\coth(c+dx)}+2}{\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(\right.}{d}\right.}{\coth^{\frac{4}{3}}(c+dx)}$$

$\downarrow$ 1142

$$\frac{\sqrt[3]{b\coth^4(c+dx)}\left(\frac{3\left(\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{2}\int-\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\right.\right.}{}\right.}{}$$

$\downarrow$ 25

$$\frac{\sqrt[3]{b\coth^4(c+dx)}\left(3\left(\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\co\ldots}\right.\right.\right.}{}$$

$\Big\downarrow$ 1083

$$\frac{\sqrt[3]{b\coth^4(c+dx)}\left(3\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-3\int\frac{1}{-\coth^{\frac{2}{3}}(c+dx)-3}d\left(2\sqrt[3]{\coth(c+dx)}-1\ldots\right.\right.\right.\right.}{}$$

$\Big\downarrow$ 217

$$\frac{\sqrt[3]{b\coth^4(c+dx)}\left(3\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(\frac{1}{2}\ldots\right.\right.\right.}{}$$

$\Big\downarrow$ 1103

$$\frac{\sqrt[3]{b\coth^4(c+dx)}\left(\frac{3\left(\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)\right)+\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\ldots}}{}\right.\right.\ldots}{d}\right.}{\coth^{\frac{4}{3}}(c\ldots}$$

input
```
Int[(b*Coth[c + d*x]^4)^(1/3),x]
```

output
```
((b*Coth[c + d*x]^4)^(1/3)*((-3*Coth[c + d*x]^(1/3))/d + (3*(ArcTanh[Coth[
c + d*x]^(1/3)]/3 + (Sqrt[3]*ArcTan[(-1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]
- Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]/2)/6 + (Sqrt[3]*ArcTa
n[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] + Log[1 + Coth[c + d*x]^(1/3) + Cot
h[c + d*x]^(2/3)]/2)/6))/d))/Coth[c + d*x]^(4/3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \left(b \coth \left(dx+c\right)^{4}\right)^{\frac{1}{3}} dx$$

input
```
int((b*coth(d*x+c)^4)^(1/3),x)
```

output
```
int((b*coth(d*x+c)^4)^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.22

$$\int \sqrt[3]{b\coth^4(c+dx)}\,dx =$$

$$-\frac{2\sqrt{3}(-b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}b+2\sqrt{3}(-b)^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)-2\sqrt{3}b^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}b^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)+(-b)^{\frac{1}{3}}}{}$$

input
```
integrate((b*coth(d*x+c)^4)^(1/3),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(3)*(-b)^(1/3)*arctan(1/3*(sqrt(3)*b + 2*sqrt(3)*(-b)^(2/3)*(b
*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) - 2*sqrt(3)*b^(1/3)*arctan(-1/3*(s
qrt(3)*b - 2*sqrt(3)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + (
-b)^(1/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3
) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + b^(1/3)*log(b^(2/3) - b^(1/3)
*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(
2/3)) - 2*(-b)^(1/3)*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3
)) - 2*b^(1/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + 12*(
b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/d
```

Sympy [F]

$$\int \sqrt[3]{b\coth^4(c+dx)}\,dx = \int \sqrt[3]{b\coth^4{(c+dx)}}\,dx$$

input
```
integrate((b*coth(d*x+c)**4)**(1/3),x)
```

output
```
Integral((b*coth(c + d*x)**4)**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{b\coth^4(c+dx)}\,dx = \int \left(b\coth\left(dx+c\right)^4\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*coth(d*x+c)^4)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^4)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{b\coth^4(c+dx)}\,dx = \int \left(b\coth\left(dx+c\right)^4\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*coth(d*x+c)^4)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^4)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\coth^4(c+dx)}\,dx = \int \left(b\coth(c+d\,x)^4\right)^{1/3}\,dx$$

input
```
int((b*coth(c + d*x)^4)^(1/3),x)
```

output
```
int((b*coth(c + d*x)^4)^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{b\coth^4(c+dx)}\,dx = \frac{b^{\frac{1}{3}}\left(-3\coth(dx+c)^{\frac{1}{3}}+\left(\int \frac{1}{\coth(dx+c)^{\frac{2}{3}}}dx\right)d\right)}{d}$$

input
```
int((b*coth(d*x+c)^4)^(1/3),x)
```

output
```
(b**(1/3)*( - 3*coth(c + d*x)**(1/3) + int(coth(c + d*x)**(1/3)/coth(c + d
*x),x)*d))/d
```

3.47 $\int \frac{1}{\sqrt[3]{b \coth^4(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 236

$$\int \frac{1}{\sqrt[3]{b\coth^4(c+dx)}}\,dx = -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2d\sqrt[3]{b\coth^4(c+dx)}} - \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\coth(c+dx)}}{1+\coth^{\frac{2}{3}}(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{2d\sqrt[3]{b\coth^4(c+dx)}}$$

output
```
-3*coth(d*x+c)/d/(b*coth(d*x+c)^4)^(1/3)+1/2*3^(1/2)*arctan(1/3*(1-2*coth(
d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(4/3)/d/(b*coth(d*x+c)^4)^(1/3)-1/2*3^(
1/2)*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(4/3)/d/(b*co
th(d*x+c)^4)^(1/3)+arctanh(coth(d*x+c)^(1/3))*coth(d*x+c)^(4/3)/d/(b*coth(
d*x+c)^4)^(1/3)+1/2*arctanh(coth(d*x+c)^(1/3)/(1+coth(d*x+c)^(2/3)))*coth(
d*x+c)^(4/3)/d/(b*coth(d*x+c)^4)^(1/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt[3]{b\coth^4(c+dx)}}\,dx =$$

$$-\frac{\coth(c+dx)\left(6+\sqrt[6]{\coth^2(c+dx)}\log\left(1-\sqrt[6]{\coth^2(c+dx)}\right)-\sqrt[6]{\coth^2(c+dx)}\log\left(1+\sqrt[6]{\coth^2(c+dx)}\right.\right.}{}$$

input
```
Integrate[(b*Coth[c + d*x]^4)^(-1/3),x]
```

output
```
-1/2*(Coth[c + d*x]*(6 + (Coth[c + d*x]^2)^(1/6)*Log[1 - (Coth[c + d*x]^2)
^(1/6)] - (Coth[c + d*x]^2)^(1/6)*Log[1 + (Coth[c + d*x]^2)^(1/6)] + (-1)^
(1/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 - (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)]
 - (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 + (-1)^(1/3)*(Coth[c + d*x]^2)^
(1/6)] + (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 - (-1)^(2/3)*(Coth[c + d
*x]^2)^(1/6)] - (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 + (-1)^(2/3)*(Cot
h[c + d*x]^2)^(1/6)]))/(d*(b*Coth[c + d*x]^4)^(1/3))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.78, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {3042, 4141, 3042, 3955, 3042, 3957, 25, 266, 825, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{b\coth^4(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{b\tan\left(ic+idx+\frac{\pi}{2}\right)^4}}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\coth^{\frac{4}{3}}(c+dx)}dx}{\sqrt[3]{b\coth^4(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{4/3}}dx}{\sqrt[3]{b\coth^4(c+dx)}}$$

$$\Big\downarrow \text{3955}$$

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\int \coth^{\frac{2}{3}}(c+dx)dx-\frac{3}{d\sqrt[3]{\coth(c+dx)}}\right)}{\sqrt[3]{b\coth^4(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(-\frac{3}{d\sqrt[3]{\coth(c+dx)}}+\int \left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{2/3}dx\right)}{\sqrt[3]{b\coth^4(c+dx)}}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(-\frac{\int -\frac{\coth^{\frac{2}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}-\frac{3}{d\sqrt[3]{\coth(c+dx)}}\right)}{\sqrt[3]{b\coth^4(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\frac{\int \frac{\coth^{\frac{2}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}-\frac{3}{d\sqrt[3]{\coth(c+dx)}}\right)}{\sqrt[3]{b\coth^4(c+dx)}}$$

$\downarrow$ 266

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\int \frac{\coth^{\frac{4}{3}}(c+dx)}{1-\coth^2(c+dx)}d\sqrt[3]{\coth(c+dx)}}{d}-\frac{3}{d\sqrt[3]{\coth(c+dx)}}\right)}{\sqrt[3]{b\coth^4(c+dx)}}$$

$\downarrow$ 825

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(\frac{1}{3}\int \frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{3}\int -\frac{\sqrt[3]{\coth(c+dx)}+1}{2\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{3}\int -\frac{}{2}\right.}{d}\right.}{\sqrt[3]{b\coth^4(c+dx)}}$$

$\downarrow$ 27

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(\frac{1}{3}\int \frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}-\frac{1}{6}\int \frac{\sqrt[3]{\coth(c+dx)}+1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{6}\int \frac{1-}{\coth^{\frac{2}{3}}(c+}\right.}{d}\right.}{\sqrt[3]{b\coth^4(c+dx)}}$$

$\downarrow$ 219

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(-\frac{1}{6}\int \frac{\sqrt[3]{\coth(c+dx)}+1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{6}\int \frac{1-\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c\right.}{d}\right.}{\sqrt[3]{b\coth^4(c+dx)}}$$

$\downarrow$ 1142

$$\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(-\frac{3}{2}\int\frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{2}\int-\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{co\right.\right.\right.}{}\right.$$

$$\downarrow 25$$

$$\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{3}{2}\int\frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(\right.\right.\right.}{}\right.$$

$$\downarrow 1083$$

$$\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(3\int\frac{1}{-\coth^{\frac{2}{3}}(c+dx)-3}d\left(2\sqrt[3]{\coth(c+dx)}-1\right)+\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}\right)\right.\right.}{}\right.$$

$$\downarrow 217$$

$$\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(\frac{1}{2}\int-\right.\right.}{}\right.$$

$$\downarrow 1103$$

$$\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)\right)+\frac{1}{6}\left(\frac{1}{2}\log\left(\coth^{\frac{2}{3}}(c+dx\right.\right.\right.}{d}\right.$$

$$\sqrt[3]{b\coth^4(c}$$

input
```
Int[(b*Coth[c + d*x]^4)^(-1/3),x]
```

output
```
(Coth[c + d*x]^(4/3)*(-3/(d*Coth[c + d*x]^(1/3)) + (3*(ArcTanh[Coth[c + d*
x]^(1/3)]/3 + (-(Sqrt[3]*ArcTan[(-1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]) - L
og[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]/2)/6 + (-(Sqrt[3]*ArcTan
[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]) + Log[1 + Coth[c + d*x]^(1/3) + Cot
h[c + d*x]^(2/3)]/2)/6))/d))/(b*Coth[c + d*x]^4)^(1/3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \frac{1}{\left(b \coth \left(dx+c\right)^{4}\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(b*coth(d*x+c)^4)^(1/3),x)
```

output
```
int(1/(b*coth(d*x+c)^4)^(1/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(196) = 392$.

Time = 0.18 (sec) , antiderivative size = 3316, normalized size of antiderivative = 14.05

$$\int \frac{1}{\sqrt[3]{b \coth^4(c+dx)}}\, dx = \text{Too large to display}$$

input
```
integrate(1/(b*coth(d*x+c)^4)^(1/3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{b\coth^4(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{b\coth^4{(c+dx)}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)**4)**(1/3),x)
```

output
```
Integral((b*coth(c + d*x)**4)**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{b\coth^4(c+dx)}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^4\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^4)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^4)^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{b\coth^4(c+dx)}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^4\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^4)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^4)^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{b \coth^4(c+dx)}}\,dx = \int \frac{1}{\left(b \coth\left(c+d\,x\right)^4\right)^{1/3}}\,dx$$

input
```
int(1/(b*coth(c + d*x)^4)^(1/3),x)
```

output
```
int(1/(b*coth(c + d*x)^4)^(1/3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{b \coth^4(c+dx)}}\,dx = \frac{\int \frac{1}{\coth(dx+c)^{\frac{4}{3}}}dx}{b^{\frac{1}{3}}}$$

input
```
int(1/(b*coth(d*x+c)^4)^(1/3),x)
```

output
```
int(1/(coth(c + d*x)**(1/3)*coth(c + d*x)),x)/b**(1/3)
```

3.48 $\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 238

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}\,dx = -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} - \frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{8}{3}}(c+dx)}{2d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{8}{3}}(c+dx)}{2d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{8}{3}}(c+dx)}{d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\coth(c+dx)}}{1+\coth^{\frac{2}{3}}(c+dx)}\right)\coth^{\frac{8}{3}}(c+dx)}{2d\left(b\coth^4(c+dx)\right)^{2/3}}$$

output
```
-3/5*coth(d*x+c)/d/(b*coth(d*x+c)^4)^(2/3)-1/2*3^(1/2)*arctan(1/3*(1-2*cot
h(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(8/3)/d/(b*coth(d*x+c)^4)^(2/3)+1/2*3
^(1/2)*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(8/3)/d/(b*
coth(d*x+c)^4)^(2/3)+arctanh(coth(d*x+c)^(1/3))*coth(d*x+c)^(8/3)/d/(b*cot
h(d*x+c)^4)^(2/3)+1/2*arctanh(coth(d*x+c)^(1/3)/(1+coth(d*x+c)^(2/3)))*cot
h(d*x+c)^(8/3)/d/(b*coth(d*x+c)^4)^(2/3)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}\,dx =$$

$$-\frac{\coth(c+dx)\left(6+5\coth^2(c+dx)^{5/6}\log\left(1-\sqrt[6]{\coth^2(c+dx)}\right)-5\coth^2(c+dx)^{5/6}\log\left(1+\sqrt[6]{\coth^2(\right.\right.}{}$$

input
```
Integrate[(b*Coth[c + d*x]^4)^(-2/3),x]
```

output
```
-1/10*(Coth[c + d*x]*(6 + 5*(Coth[c + d*x]^2)^(5/6)*Log[1 - (Coth[c + d*x]
^2)^(1/6)] - 5*(Coth[c + d*x]^2)^(5/6)*Log[1 + (Coth[c + d*x]^2)^(1/6)] -
5*(-1)^(2/3)*(Coth[c + d*x]^2)^(5/6)*Log[1 - (-1)^(1/3)*(Coth[c + d*x]^2)^
(1/6)] + 5*(-1)^(2/3)*(Coth[c + d*x]^2)^(5/6)*Log[1 + (-1)^(1/3)*(Coth[c +
 d*x]^2)^(1/6)] - 5*(-1)^(1/3)*(Coth[c + d*x]^2)^(5/6)*Log[1 - (-1)^(2/3)*
(Coth[c + d*x]^2)^(1/6)] + 5*(-1)^(1/3)*(Coth[c + d*x]^2)^(5/6)*Log[1 + (-
1)^(2/3)*(Coth[c + d*x]^2)^(1/6)]))/(d*(b*Coth[c + d*x]^4)^(2/3))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.77, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {3042, 4141, 3042, 3955, 3042, 3957, 25, 266, 754, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}dx$$

$\Big\downarrow$ 3042

$$\int \frac{1}{\left(b\tan\left(ic+idx+\frac{\pi}{2}\right)^4\right)^{2/3}}dx$$

$\Big\downarrow$ 4141

$$\frac{\coth^{\frac{8}{3}}(c+dx)\int \frac{1}{\coth^{\frac{8}{3}}(c+dx)}dx}{\left(b\coth^4(c+dx)\right)^{2/3}}$$

$\Big\downarrow$ 3042

$$\frac{\coth^{\frac{8}{3}}(c+dx)\int \frac{1}{\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{8/3}}dx}{\left(b\coth^4(c+dx)\right)^{2/3}}$$

$\Big\downarrow$ 3955

$$\frac{\coth^{\frac{8}{3}}(c+dx)\left(\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}dx-\frac{3}{5d\coth^{\frac{5}{3}}(c+dx)}\right)}{\left(b\coth^4(c+dx)\right)^{2/3}}$$

$\Big\downarrow$ 3042

$$\frac{\coth^{\frac{8}{3}}(c+dx)\left(-\frac{3}{5d\coth^{\frac{5}{3}}(c+dx)}+\int \frac{1}{\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{2/3}}dx\right)}{\left(b\coth^4(c+dx)\right)^{2/3}}$$

$\Big\downarrow$ 3957

$$\frac{\coth^{\frac{8}{3}}(c+dx)\left(-\frac{\int -\frac{1}{\coth^{\frac{2}{3}}(c+dx)\left(1-\coth^2(c+dx)\right)}d\coth(c+dx)}{d}-\frac{3}{5d\coth^{\frac{5}{3}}(c+dx)}\right)}{\left(b\coth^4(c+dx)\right)^{2/3}}$$

$\Big\downarrow$ 25

$$\frac{\coth^{\frac{8}{3}}(c+dx)\left(\frac{\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)\left(1-\coth^2(c+dx)\right)}d\coth(c+dx)}{d}-\frac{3}{5d\coth^{\frac{5}{3}}(c+dx)}\right)}{\left(b\coth^4(c+dx)\right)^{2/3}}$$

$\Big\downarrow$ 266

$$\frac{\coth^{\frac{8}{3}}(c+dx)\left(\frac{3\int \frac{1}{1-\coth^2(c+dx)}d\sqrt[3]{\coth(c+dx)}}{d}-\frac{3}{5d\coth^{\frac{5}{3}}(c+dx)}\right)}{\left(b\coth^4(c+dx)\right)^{2/3}}$$

$\downarrow$ 754

$$\frac{\coth^{\frac{8}{3}}(c+dx)\left(\frac{3\left(\frac{1}{3}\int \frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{3}\int \frac{2-\sqrt[3]{\coth(c+dx)}}{2\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{3}\int \frac{\cdots}{2\left(\cot\cdots\right.}\right.}{d}\right.}{\left(b\coth^4(c+dx)\right)^{2/3}}$$

$\downarrow$ 27

$$\frac{\coth^{\frac{8}{3}}(c+dx)\left(\frac{3\left(\frac{1}{3}\int \frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{6}\int \frac{2-\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\frac{1}{6}\int \frac{\sqrt[3]{\cdots}}{\coth^{\frac{2}{3}}(c+\cdots}\right.}{d}\right.}{\left(b\coth^4(c+dx)\right)^{2/3}}$$

$\downarrow$ 219

$$\frac{\coth^{\frac{8}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\int \frac{2-\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\frac{1}{6}\int \frac{\sqrt[3]{\coth(c+dx)}+2}{\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+\cdots}\right.}{d}\right.}{\left(b\coth^4(c+dx)\right)^{2/3}}$$

$\downarrow$ 1142

$$\frac{\coth^{\frac{8}{3}}(c+dx)\left(3\left(\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{2}\int -\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\cot\cdots}\right.\right.\right.}{\cdots}$$

$\downarrow$ 25

$$\coth^{\frac{8}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\frac{1}{2}\int \frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}\right.\right.\right.}{}\right.$$

$\downarrow$ 1083

$$\coth^{\frac{8}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-3\int \frac{1}{-\coth^{\frac{2}{3}}(c+dx)-3}d\left(2\sqrt[3]{\coth(c+dx)}-1\right)\right)\right.}{}\right.$$

$\downarrow$ 217

$$\coth^{\frac{8}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}+\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(\frac{1}{2}\int\right.\right.}{}\right.$$

$\downarrow$ 1103

$$\coth^{\frac{8}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)\right)+\frac{1}{6}\left(\sqrt{3}\arctan\left(\right.\right.\right.}{d}\right.$$

input
```
Int[(b*Coth[c + d*x]^4)^(-2/3),x]
```

output
```
(Coth[c + d*x]^(8/3)*(-3/(5*d*Coth[c + d*x]^(5/3)) + (3*(ArcTanh[Coth[c +
d*x]^(1/3)]/3 + (Sqrt[3]*ArcTan[(-1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - Lo
g[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]/2)/6 + (Sqrt[3]*ArcTan[(1
 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] + Log[1 + Coth[c + d*x]^(1/3) + Coth[c
+ d*x]^(2/3)]/2)/6))/d))/(b*Coth[c + d*x]^4)^(2/3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \frac{1}{\left(b \coth \left(dx+c\right)^{4}\right)^{\frac{2}{3}}}dx$$

input
```
int(1/(b*coth(d*x+c)^4)^(2/3),x)
```

output
```
int(1/(b*coth(d*x+c)^4)^(2/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1159 vs. $2(196) = 392$.

Time = 0.12 (sec) , antiderivative size = 1159, normalized size of antiderivative = 4.87

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*coth(d*x+c)^4)^(2/3),x, algorithm="fricas")
```

output
```
1/20*(10*sqrt(3)*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 +
b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh
(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b)*s
qrt(-(-b^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*sqrt(-(-b^2)^(1/3))
 - 2*sqrt(3)*(-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)*sqrt(-(-b^
2)^(1/3)))/b^2) + 10*sqrt(3)*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d
*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)
^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x
 + c) + b)*(b^2)^(1/6)*arctan(-1/3*sqrt(3)*(b^2)^(1/6)*((b^2)^(1/3)*b - 2*
(b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b^2) + 5*(cosh(d*x + c)
^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c
)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d
*x + c))*sinh(d*x + c) + 1)*(-b^2)^(2/3)*log(b*(b*cosh(d*x + c)/sinh(d*x +
 c))^(2/3) - (-b^2)^(1/3)*b + (-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))
^(1/3)) - 5*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x
+ c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4
*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*(b^2)^(2/3)*log(b*(b
*cosh(d*x + c)/sinh(d*x + c))^(2/3) + (b^2)^(1/3)*b - (b^2)^(2/3)*(b*cosh(
d*x + c)/sinh(d*x + c))^(1/3)) - 10*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sin
h(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c...
```

Sympy [F]

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}\,dx = \int \frac{1}{\left(b\coth^4\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)**4)**(2/3),x)
```

output
```
Integral((b*coth(c + d*x)**4)**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^4\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^4)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^4)^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^4\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^4)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^4)^(-2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}\,dx = \int \frac{1}{\left(b\coth\left(c+d\,x\right)^4\right)^{2/3}}\,dx$$

input
```
int(1/(b*coth(c + d*x)^4)^(2/3),x)
```

output
```
int(1/(b*coth(c + d*x)^4)^(2/3), x)
```

Reduce [F]

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}\,dx = \frac{\int \frac{1}{\coth(dx+c)^{\frac{8}{3}}}dx}{b^{\frac{2}{3}}}$$

input
```
int(1/(b*coth(d*x+c)^4)^(2/3),x)
```

output
```
int(1/(coth(c + d*x)**(2/3)*coth(c + d*x)**2),x)/b**(2/3)
```

3.49 $\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 313

$$\begin{aligned}\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx = & -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} \\ & +\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^4(c+dx)}} \\ & -\frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^4(c+dx)}} \\ & +\frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} \\ & +\frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\coth(c+dx)}}{1+\coth^{\frac{2}{3}}(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^4(c+dx)}} \\ & -\frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}}-\frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}}\end{aligned}$$

output
```
-3*coth(d*x+c)/b/d/(b*coth(d*x+c)^4)^(1/3)+1/2*3^(1/2)*arctan(1/3*(1-2*cot
h(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(4/3)/b/d/(b*coth(d*x+c)^4)^(1/3)-1/2
*3^(1/2)*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(4/3)/b/d
/(b*coth(d*x+c)^4)^(1/3)+arctanh(coth(d*x+c)^(1/3))*coth(d*x+c)^(4/3)/b/d/
(b*coth(d*x+c)^4)^(1/3)+1/2*arctanh(coth(d*x+c)^(1/3)/(1+coth(d*x+c)^(2/3)
))*coth(d*x+c)^(4/3)/b/d/(b*coth(d*x+c)^4)^(1/3)-3/7*tanh(d*x+c)/b/d/(b*co
th(d*x+c)^4)^(1/3)-3/13*tanh(d*x+c)^3/b/d/(b*coth(d*x+c)^4)^(1/3)
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx = \frac{-91\coth(c+dx)\left(6+\sqrt[6]{\coth^2(c+dx)}\log\left(1-\sqrt[6]{\coth^2(c+dx)}\right)-\sqrt[6]{\coth}\right.}{}$$

input
```
Integrate[(b*Coth[c + d*x]^4)^(-4/3),x]
```

output
```
(-91*Coth[c + d*x]*(6 + (Coth[c + d*x]^2)^(1/6)*Log[1 - (Coth[c + d*x]^2)^
(1/6)] - (Coth[c + d*x]^2)^(1/6)*Log[1 + (Coth[c + d*x]^2)^(1/6)] + (-1)^(
1/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 - (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)] -
 (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 + (-1)^(1/3)*(Coth[c + d*x]^2)^(
1/6)] + (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 - (-1)^(2/3)*(Coth[c + d*
x]^2)^(1/6)] - (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 + (-1)^(2/3)*(Coth
[c + d*x]^2)^(1/6)]) - 6*Tanh[c + d*x]*(13 + 7*Tanh[c + d*x]^2))/(182*b*d*
(b*Coth[c + d*x]^4)^(1/3))
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.71, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 1.429$, Rules used = {3042, 4141, 3042, 3955, 3042, 3955, 3042, 3955, 3042, 3957, 25, 266, 825, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx$$

$\Big\downarrow$ 3042

$$\int \frac{1}{\left(b\tan\left(ic+idx+\frac{\pi}{2}\right)^4\right)^{4/3}}dx$$

$\Big\downarrow$ 4141

$$\frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\coth^{\frac{16}{3}}(c+dx)}dx}{b\sqrt[3]{b\coth^4(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{16/3}}dx}{b\sqrt[3]{b\coth^4(c+dx)}}$$

$\Big\downarrow$ 3955

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\int \frac{1}{\coth^{\frac{10}{3}}(c+dx)}dx-\frac{3}{13d\coth^{\frac{13}{3}}(c+dx)}\right)}{b\sqrt[3]{b\coth^4(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(-\frac{3}{13d\coth^{\frac{13}{3}}(c+dx)}+\int \frac{1}{\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{10/3}}dx\right)}{b\sqrt[3]{b\coth^4(c+dx)}}$$

$\Big\downarrow$ 3955

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\int \frac{1}{\coth^{\frac{4}{3}}(c+dx)}dx-\frac{3}{7d\coth^{\frac{7}{3}}(c+dx)}-\frac{3}{13d\coth^{\frac{13}{3}}(c+dx)}\right)}{b\sqrt[3]{b\coth^4(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\int \frac{1}{\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{4/3}}dx-\frac{3}{7d\coth^{\frac{7}{3}}(c+dx)}-\frac{3}{13d\coth^{\frac{13}{3}}(c+dx)}\right)}{b\sqrt[3]{b\coth^4(c+dx)}}$$

$\Big\downarrow$ 3955

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\int \coth^{\frac{2}{3}}(c+dx)dx-\frac{3}{7d\coth^{\frac{7}{3}}(c+dx)}-\frac{3}{13d\coth^{\frac{13}{3}}(c+dx)}-\frac{3}{d\sqrt[3]{\coth(c+dx)}}\right)}{b\sqrt[3]{b\coth^4(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\int \left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{2/3}dx-\frac{3}{7d\coth^{\frac{7}{3}}(c+dx)}-\frac{3}{13d\coth^{\frac{13}{3}}(c+dx)}-\frac{3}{d\sqrt[3]{\coth(c+dx)}}\right)}{b\sqrt[3]{b\coth^4(c+dx)}}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(-\frac{\int -\frac{\coth^{\frac{2}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}-\frac{3}{7d\coth^{\frac{7}{3}}(c+dx)}-\frac{3}{13d\coth^{\frac{13}{3}}(c+dx)}-\frac{3}{d\sqrt[3]{\coth(c+dx)}}\right)}{b\sqrt[3]{b\coth^4(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\frac{\int \frac{\coth^{\frac{2}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}-\frac{3}{7d\coth^{\frac{7}{3}}(c+dx)}-\frac{3}{13d\coth^{\frac{13}{3}}(c+dx)}-\frac{3}{d\sqrt[3]{\coth(c+dx)}}\right)}{b\sqrt[3]{b\coth^4(c+dx)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\int \frac{\coth^{\frac{4}{3}}(c+dx)}{1-\coth^2(c+dx)}d\sqrt[3]{\coth(c+dx)}}{d}-\frac{3}{7d\coth^{\frac{7}{3}}(c+dx)}-\frac{3}{13d\coth^{\frac{13}{3}}(c+dx)}-\frac{3}{d\sqrt[3]{\coth(c+dx)}}\right)}{b\sqrt[3]{b\coth^4(c+dx)}}$$

$$\Big\downarrow \text{825}$$

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(\frac{1}{3}\int \frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{3}\int -\frac{\sqrt[3]{\coth(c+dx)}+1}{2\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}d\sqrt[3]{\coth(c+dx)}+\frac{1}{3}\int --\cdots\right.}{d}\right.}{b\sqrt[3]{b\coth^4(c\cdots}}$$

$$\Big\downarrow \ 27$$

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(\frac{1}{3}\int \frac{1}{1-\coth^{\frac{2}{3}}(c+dx)}d\sqrt[3]{\coth(c+dx)}-\frac{1}{6}\int \frac{\sqrt[3]{\coth(c+dx)}+1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{6}\int \frac{1-}{\coth^{\frac{2}{3}}(c+}\right.}{d}\right.}{b\sqrt[3]{b\coth^4(c+dx}}$$

$$\Big\downarrow \ 219$$

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(-\frac{1}{6}\int \frac{\sqrt[3]{\coth(c+dx)}+1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{6}\int \frac{1-\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c\right.}{d}\right.}{b\sqrt[3]{b\coth^4(c+dx)}}$$

$$\Big\downarrow \ 1142$$

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{1}{2}\int -\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\mathrm{co}\right.\right.}{}\right.}{}$$

$$\Big\downarrow \ 25$$

$$\frac{\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\frac{3}{2}\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(\right.\right.}{}\right.}{}$$

$$\Big\downarrow \ 1083$$

$$\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(3\int\frac{1}{-\coth^{\frac{2}{3}}(c+dx)-3}d\left(2\sqrt[3]{\coth(c+dx)}-1\right)+\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}\right)\cdots}{\cdots}\right.$$

$\Big\downarrow$ 217

$$\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{\coth(c+dx)}}{\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1}d\sqrt[3]{\coth(c+dx)}-\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(\frac{1}{2}\int\cdots\right.\right.}{\cdots}\right.$$

$\Big\downarrow$ 1103

$$\coth^{\frac{4}{3}}(c+dx)\left(\frac{3\left(\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)\right)+\frac{1}{6}\left(\frac{1}{2}\log\left(\coth^{\frac{2}{3}}(c+dx\cdots\right.\right.\right.}{d}\cdots\right.$$

input
```
Int[(b*Coth[c + d*x]^4)^(-4/3),x]
```

output
```
(Coth[c + d*x]^(4/3)*(-3/(13*d*Coth[c + d*x]^(13/3)) - 3/(7*d*Coth[c + d*x
]^(7/3)) - 3/(d*Coth[c + d*x]^(1/3)) + (3*(ArcTanh[Coth[c + d*x]^(1/3)]/3
+ (-(Sqrt[3]*ArcTan[(-1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]) - Log[1 - Coth[
c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + 2*Coth
[c + d*x]^(1/3))/Sqrt[3]]) + Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(
2/3)]/2)/6))/d))/(b*(b*Coth[c + d*x]^4)^(1/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \frac{1}{\left(b \coth \left(dx+c\right)^{4}\right)^{\frac{4}{3}}}dx$$

input
```
int(1/(b*coth(d*x+c)^4)^(4/3),x)
```

output
```
int(1/(b*coth(d*x+c)^4)^(4/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3650 vs. 2(265) = 530.

Time = 0.39 (sec) , antiderivative size = 15579, normalized size of antiderivative = 49.77

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*coth(d*x+c)^4)^(4/3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx = \int \frac{1}{\left(b\coth^4\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)**4)**(4/3),x)
```

output
```
Integral((b*coth(c + d*x)**4)**(-4/3), x)
```

Maxima [F]

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^4\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^4)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^4)^(-4/3), x)
```

Giac [F]

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^4\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^4)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^4)^(-4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx = \int \frac{1}{\left(b\coth\left(c+d\,x\right)^4\right)^{4/3}}\,dx$$

input
```
int(1/(b*coth(c + d*x)^4)^(4/3),x)
```

output
```
int(1/(b*coth(c + d*x)^4)^(4/3), x)
```

Reduce [F]

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx = \frac{\int \frac{1}{\coth(dx+c)^{\frac{16}{3}}}dx}{b^{\frac{4}{3}}}$$

input
```
int(1/(b*coth(d*x+c)^4)^(4/3),x)
```

output
```
int(1/(coth(c + d*x)**(1/3)*coth(c + d*x)**5),x)/(b**(1/3)*b)
```

3.50 $\int (b\coth^m(c+dx))^n \, dx$

Optimal result

Integrand size = 12, antiderivative size = 57

$$\int (b\coth^m(c+dx))^n \, dx = \frac{\coth(c+dx)\,(b\coth^m(c+dx))^n \operatorname{Hypergeometric2F1}\left(1,\frac{1}{2}(1+mn),\frac{1}{2}(3+mn),\coth^2(c+dx)\right)}{d(1+mn)}$$

output
```
coth(d*x+c)*(b*coth(d*x+c)^m)^n*hypergeom([1, 1/2*m*n+1/2],[1/2*m*n+3/2],c
oth(d*x+c)^2)/d/(m*n+1)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int (b\coth^m(c+dx))^n \, dx = \frac{\coth(c+dx)\,(b\coth^m(c+dx))^n \operatorname{Hypergeometric2F1}\left(1,\frac{1}{2}(1+mn),\frac{1}{2}(3+mn),\coth^2(c+dx)\right)}{d(1+mn)}$$

input
```
Integrate[(b*Coth[c + d*x]^m)^n,x]
```

output
```
(Coth[c + d*x]*(b*Coth[c + d*x]^m)^n*Hypergeometric2F1[1, (1 + m*n)/2, (3
+ m*n)/2, Coth[c + d*x]^2])/(d*(1 + m*n))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4142, 3042, 3957, 25, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\coth^m(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^m\right)^n dx$$

$$\Big\downarrow \text{4142}$$

$$\coth^{-mn}(c+dx)\,(b\coth^m(c+dx))^n\int \coth^{mn}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\coth^{-mn}(c+dx)\,(b\coth^m(c+dx))^n\int \left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{mn} dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{\coth^{-mn}(c+dx)\,(b\coth^m(c+dx))^n\int -\frac{\coth^{mn}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\coth^{-mn}(c+dx)\,(b\coth^m(c+dx))^n\int \frac{\coth^{mn}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{\coth(c+dx)\,(b\coth^m(c+dx))^n\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(mn+1),\frac{1}{2}(mn+3),\coth^2(c+dx)\right)}{d(mn+1)}$$

input
```
Int[(b*Coth[c + d*x]^m)^n,x]
```

output
```
(Coth[c + d*x]*(b*Coth[c + d*x]^m)^n*Hypergeometric2F1[1, (1 + m*n)/2, (3
+ m*n)/2, Coth[c + d*x]^2])/(d*(1 + m*n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int (b\coth (dx+c)^m)^n\,dx$$

input
```
int((b*coth(d*x+c)^m)^n,x)
```

output
```
int((b*coth(d*x+c)^m)^n,x)
```

Fricas [F]

$$\int (b\coth^m(c+dx))^n\ dx = \int (b\coth (dx+c)^m)^n\ dx$$

input
```
integrate((b*coth(d*x+c)^m)^n,x, algorithm="fricas")
```

output
```
integral((b*coth(d*x + c)^m)^n, x)
```

Sympy [F]

$$\int (b\coth^m(c+dx))^n\ dx = \int (b\coth^m (c+dx))^n\ dx$$

input
```
integrate((b*coth(d*x+c)**m)**n,x)
```

output
```
Integral((b*coth(c + d*x)**m)**n, x)
```

Maxima [F]

$$\int (b\coth^m(c+dx))^n\,dx = \int (b\coth(dx+c)^m)^n\,dx$$

input
```
integrate((b*coth(d*x+c)^m)^n,x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^m)^n, x)
```

Giac [F]

$$\int (b\coth^m(c+dx))^n\,dx = \int (b\coth(dx+c)^m)^n\,dx$$

input
```
integrate((b*coth(d*x+c)^m)^n,x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^m)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\coth^m(c+dx))^n\,dx = \int (b\coth(c+d\,x)^m)^n\,dx$$

input
```
int((b*coth(c + d*x)^m)^n,x)
```

output
```
int((b*coth(c + d*x)^m)^n, x)
```

Reduce [F]

$$\int (b \coth^m (c + dx))^n \, dx = b^n \left(\int \coth (dx + c)^{mn} \, dx \right)$$

input
```
int((b*coth(d*x+c)^m)^n,x)
```

output
```
b**n*int(coth(c + d*x)**(m*n),x)
```

3.51 $\int (b\coth^m(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 63

$$\int (b\coth^m(c+dx))^{3/2}\,dx = \frac{2b\coth^{1+m}(c+dx)\sqrt{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2+3m),\frac{3(2+m)}{4},\coth^2(c+\right.}{d(2+3m)}$$

output
```
2*b*coth(d*x+c)^(1+m)*(b*coth(d*x+c)^m)^(1/2)*hypergeom([1, 1/2+3/4*m],[3/
2+3/4*m],coth(d*x+c)^2)/d/(2+3*m)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int (b\coth^m(c+dx))^{3/2}\,dx = \frac{\coth(c+dx)\,(b\coth^m(c+dx))^{3/2}\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2+3m),\frac{3(2+m)}{4},\coth^2(c+dx)\right.}{d\left(1+\frac{3m}{2}\right)}$$

input
```
Integrate[(b*Coth[c + d*x]^m)^(3/2),x]
```

output
```
(Coth[c + d*x]*(b*Coth[c + d*x]^m)^(3/2)*Hypergeometric2F1[1, (2 + 3*m)/4,
 (3*(2 + m))/4, Coth[c + d*x]^2])/(d*(1 + (3*m)/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4142, 3042, 3957, 25, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\coth^m(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^m\right)^{3/2}dx$$

$$\Big\downarrow \text{4142}$$

$$b\coth^{-\frac{m}{2}}(c+dx)\sqrt{b\coth^m(c+dx)}\int \coth^{\frac{3m}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$b\coth^{-\frac{m}{2}}(c+dx)\sqrt{b\coth^m(c+dx)}\int \left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{3m/2}dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{b\coth^{-\frac{m}{2}}(c+dx)\sqrt{b\coth^m(c+dx)}\int -\frac{\coth^{\frac{3m}{2}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\coth^{-\frac{m}{2}}(c+dx)\sqrt{b\coth^m(c+dx)}\int \frac{\coth^{\frac{3m}{2}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{2b\coth^{m+1}(c+dx)\sqrt{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(3m+2),\frac{3(m+2)}{4},\coth^2(c+dx)\right)}{d(3m+2)}$$

input
```
Int[(b*Coth[c + d*x]^m)^(3/2),x]
```

output
```
(2*b*Coth[c + d*x]^(1 + m)*Sqrt[b*Coth[c + d*x]^m]*Hypergeometric2F1[1, (2
 + 3*m)/4, (3*(2 + m))/4, Coth[c + d*x]^2])/(d*(2 + 3*m))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int (b\coth (dx+c)^{m})^{\frac{3}{2}} dx$$

input
```
int((b*coth(d*x+c)^m)^(3/2),x)
```

output
```
int((b*coth(d*x+c)^m)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (b\coth^m(c+dx))^{3/2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*coth(d*x+c)^m)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (b\coth^m(c+dx))^{3/2}\, dx = \int (b\coth^{m}(c+dx))^{\frac{3}{2}}\, dx$$

input
```
integrate((b*coth(d*x+c)**m)**(3/2),x)
```

output
```
Integral((b*coth(c + d*x)**m)**(3/2), x)
```

Maxima [F]

$$\int (b\coth^m(c+dx))^{3/2}\,dx = \int (b\coth{(dx+c)}^m)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*coth(d*x+c)^m)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^m)^(3/2), x)
```

Giac [F]

$$\int (b\coth^m(c+dx))^{3/2}\,dx = \int (b\coth{(dx+c)}^m)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*coth(d*x+c)^m)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^m)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\coth^m(c+dx))^{3/2}\,dx = \int (b\coth(c+d\,x)^m)^{3/2}\,dx$$

input
```
int((b*coth(c + d*x)^m)^(3/2),x)
```

output
```
int((b*coth(c + d*x)^m)^(3/2), x)
```

Reduce [F]

$$\int (b\coth^m(c+dx))^{3/2}\,dx = \sqrt{b}\left(\int \coth(dx+c)^{\frac{3m}{2}}\,dx\right)b$$

input
```
int((b*coth(d*x+c)^m)^(3/2),x)
```

output
```
sqrt(b)*int(coth(c + d*x)**((3*m)/2),x)*b
```

3.52 $\int \sqrt{b \coth^m(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int \sqrt{b \coth^m(c+dx)}\,dx = \frac{2\coth(c+dx)\sqrt{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{4},\frac{6+m}{4},\coth^2(c+dx)\right)}{d(2+m)}$$

output
```
2*coth(d*x+c)*(b*coth(d*x+c)^m)^(1/2)*hypergeom([1, 1/2+1/4*m],[3/2+1/4*m]
,coth(d*x+c)^2)/d/(2+m)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \sqrt{b \coth^m(c+dx)}\,dx = \frac{2\coth(c+dx)\sqrt{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{4},\frac{6+m}{4},\coth^2(c+dx)\right)}{d(2+m)}$$

input
```
Integrate[Sqrt[b*Coth[c + d*x]^m],x]
```

output
```
(2*Coth[c + d*x]*Sqrt[b*Coth[c + d*x]^m]*Hypergeometric2F1[1, (2 + m)/4, (
6 + m)/4, Coth[c + d*x]^2])/(d*(2 + m))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4142, 3042, 3957, 25, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\coth^m(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^m}dx$$

$$\Big\downarrow \text{4142}$$

$$\coth^{-\frac{m}{2}}(c+dx)\sqrt{b\coth^m(c+dx)}\int \coth^{\frac{m}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\coth^{-\frac{m}{2}}(c+dx)\sqrt{b\coth^m(c+dx)}\int \left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{m/2}dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{\coth^{-\frac{m}{2}}(c+dx)\sqrt{b\coth^m(c+dx)}\int -\frac{\coth^{\frac{m}{2}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\coth^{-\frac{m}{2}}(c+dx)\sqrt{b\coth^m(c+dx)}\int \frac{\coth^{\frac{m}{2}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{2\coth^{\frac{m+2}{2}-\frac{m}{2}}(c+dx)\sqrt{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{4},\frac{m+6}{4},\coth^2(c+dx)\right)}{d(m+2)}$$

input
```
Int[Sqrt[b*Coth[c + d*x]^m],x]
```

output
```
(2*Coth[c + d*x]^(-1/2*m + (2 + m)/2)*Sqrt[b*Coth[c + d*x]^m]*Hypergeometr
ic2F1[1, (2 + m)/4, (6 + m)/4, Coth[c + d*x]^2])/(d*(2 + m))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \sqrt{b \coth (dx+c)^m}dx$$

input
```
int((b*coth(d*x+c)^m)^(1/2),x)
```

output
```
int((b*coth(d*x+c)^m)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{b \coth^m(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*coth(d*x+c)^m)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{b \coth^m(c+dx)}\,dx = \int \sqrt{b \coth^m{(c+dx)}}\,dx$$

input
```
integrate((b*coth(d*x+c)**m)**(1/2),x)
```

output
```
Integral(sqrt(b*coth(c + d*x)**m), x)
```

Maxima [F]

$$\int \sqrt{b \coth^m(c+dx)}\,dx = \int \sqrt{b \coth(dx+c)^m}\,dx$$

input
```
integrate((b*coth(d*x+c)^m)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*coth(d*x + c)^m), x)
```

Giac [F]

$$\int \sqrt{b \coth^m(c+dx)}\,dx = \int \sqrt{b \coth(dx+c)^m}\,dx$$

input
```
integrate((b*coth(d*x+c)^m)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*coth(d*x + c)^m), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b \coth^m(c+dx)}\,dx = \int \sqrt{b \coth(c+d\,x)^m}\,dx$$

input
```
int((b*coth(c + d*x)^m)^(1/2),x)
```

output
```
int((b*coth(c + d*x)^m)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{b\coth^m(c+dx)}\,dx = \sqrt{b}\left(\int \coth(dx+c)^{\frac{m}{2}}\,dx\right)$$

input
```
int((b*coth(d*x+c)^m)^(1/2),x)
```

output
```
sqrt(b)*int(coth(c + d*x)**(m/2),x)
```

3.53 $\int \frac{1}{\sqrt{b\coth^m(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 60

$$\int \frac{1}{\sqrt{b\coth^m(c+dx)}}\,dx = \frac{2\coth(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{2-m}{4},\frac{6-m}{4},\coth^2(c+dx)\right)}{d(2-m)\sqrt{b\coth^m(c+dx)}}$$

output
```
2*coth(d*x+c)*hypergeom([1, 1/2-1/4*m],[3/2-1/4*m],coth(d*x+c)^2)/d/(2-m)/
(b*coth(d*x+c)^m)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{b\coth^m(c+dx)}}\,dx = -\frac{2\coth(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{2-m}{4},\frac{6-m}{4},\coth^2(c+dx)\right)}{d(-2+m)\sqrt{b\coth^m(c+dx)}}$$

input
```
Integrate[1/Sqrt[b*Coth[c + d*x]^m],x]
```

output
```
(-2*Coth[c + d*x]*Hypergeometric2F1[1, (2 - m)/4, (6 - m)/4, Coth[c + d*x]
^2])/(d*(-2 + m)*Sqrt[b*Coth[c + d*x]^m])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4142, 3042, 3957, 25, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\coth^m(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{b\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^m}}dx$$

$$\Big\downarrow \text{4142}$$

$$\frac{\coth^{\frac{m}{2}}(c+dx)\int \coth^{-\frac{m}{2}}(c+dx)dx}{\sqrt{b\coth^m(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\coth^{\frac{m}{2}}(c+dx)\int \left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{-m/2}dx}{\sqrt{b\coth^m(c+dx)}}$$

$$\Big\downarrow \text{3957}$$

$$-\frac{\coth^{\frac{m}{2}}(c+dx)\int -\frac{\coth^{-\frac{m}{2}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d\sqrt{b\coth^m(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\coth^{\frac{m}{2}}(c+dx)\int \frac{\coth^{-\frac{m}{2}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d\sqrt{b\coth^m(c+dx)}}$$

$$\Big\downarrow \text{278}$$

$$\frac{2\coth(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{2-m}{4},\frac{6-m}{4},\coth^2(c+dx)\right)}{d(2-m)\sqrt{b\coth^m(c+dx)}}$$

input
```
Int[1/Sqrt[b*Coth[c + d*x]^m],x]
```

output
```
(2*Coth[c + d*x]*Hypergeometric2F1[1, (2 - m)/4, (6 - m)/4, Coth[c + d*x]^
2])/(d*(2 - m)*Sqrt[b*Coth[c + d*x]^m])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \frac{1}{\sqrt{b \coth \left(dx+c\right)^{m}}}dx$$

input
```
int(1/(b*coth(d*x+c)^m)^(1/2),x)
```

output
```
int(1/(b*coth(d*x+c)^m)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{b \coth^m(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*coth(d*x+c)^m)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{\sqrt{b \coth^m(c+dx)}}\,dx = \int \frac{1}{\sqrt{b \coth^{m}\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)**m)**(1/2),x)
```

output
```
Integral(1/sqrt(b*coth(c + d*x)**m), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\coth^m(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\coth(dx+c)^m}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^m)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*coth(d*x + c)^m), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{b\coth^m(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\coth(dx+c)^m}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^m)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*coth(d*x + c)^m), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\coth^m(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\coth(c+d\,x)^m}}\,dx$$

input
```
int(1/(b*coth(c + d*x)^m)^(1/2),x)
```

output
```
int(1/(b*coth(c + d*x)^m)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{b \coth^m(c+dx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{1}{\coth(dx+c)^{\frac{m}{2}}}dx\right)}{b}$$

input
```
int(1/(b*coth(d*x+c)^m)^(1/2),x)
```

output
```
(sqrt(b)*int(1/coth(c + d*x)**(m/2),x))/b
```

3.54 $\int \frac{1}{(b\coth^m(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 69

$$\int \frac{1}{(b\coth^m(c+dx))^{3/2}}\,dx = \frac{2\coth^{1-m}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-3m),\frac{3(2-m)}{4},\coth^2(c+dx)\right)}{bd(2-3m)\sqrt{b\coth^m(c+dx)}}$$

output
```
2*coth(d*x+c)^(1-m)*hypergeom([1, 1/2-3/4*m],[3/2-3/4*m],coth(d*x+c)^2)/b/
d/(2-3*m)/(b*coth(d*x+c)^m)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{1}{(b\coth^m(c+dx))^{3/2}}\,dx = \frac{\coth(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-3m),-\frac{3}{4}(-2+m),\coth^2(c+dx)\right)}{d\left(1-\frac{3m}{2}\right)(b\coth^m(c+dx))^{3/2}}$$

input
```
Integrate[(b*Coth[c + d*x]^m)^(-3/2),x]
```

output
```
(Coth[c + d*x]*Hypergeometric2F1[1, (2 - 3*m)/4, (-3*(-2 + m))/4, Coth[c +
 d*x]^2])/(d*(1 - (3*m)/2)*(b*Coth[c + d*x]^m)^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4142, 3042, 3957, 25, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\coth^m(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(b\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^m\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4142}$$

$$\frac{\coth^{\frac{m}{2}}(c+dx)\int \coth^{-\frac{3m}{2}}(c+dx)dx}{b\sqrt{b\coth^m(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\coth^{\frac{m}{2}}(c+dx)\int \left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{-3m/2}dx}{b\sqrt{b\coth^m(c+dx)}}$$

$$\Big\downarrow \text{ 3957}$$

$$-\frac{\coth^{\frac{m}{2}}(c+dx)\int -\frac{\coth^{-\frac{3m}{2}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{bd\sqrt{b\coth^m(c+dx)}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\coth^{\frac{m}{2}}(c+dx)\int \frac{\coth^{-\frac{3m}{2}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{bd\sqrt{b\coth^m(c+dx)}}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{2\coth^{1-m}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-3m),\frac{3(2-m)}{4},\coth^2(c+dx)\right)}{bd(2-3m)\sqrt{b\coth^m(c+dx)}}$$

input
```
Int[(b*Coth[c + d*x]^m)^(-3/2),x]
```

output
```
(2*Coth[c + d*x]^(1 - m)*Hypergeometric2F1[1, (2 - 3*m)/4, (3*(2 - m))/4,
Coth[c + d*x]^2])/(b*d*(2 - 3*m)*Sqrt[b*Coth[c + d*x]^m])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \frac{1}{\left(b \coth \left(d x +c \right)^{m}\right)^{\frac{3}{2}}}d x$$

input
```
int(1/(b*coth(d*x+c)^m)^(3/2),x)
```

output
```
int(1/(b*coth(d*x+c)^m)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(b \coth^m(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*coth(d*x+c)^m)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{(b \coth^m(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(b \coth^{m}\left(c + d x\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)**m)**(3/2),x)
```

output
```
Integral((b*coth(c + d*x)**m)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(b\coth^m(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^m\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^m)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^m)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(b\coth^m(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^m\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^m)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^m)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\coth^m(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\coth\left(c+d\,x\right)^m\right)^{3/2}}\,dx$$

input
```
int(1/(b*coth(c + d*x)^m)^(3/2),x)
```

output
```
int(1/(b*coth(c + d*x)^m)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(b\coth^m(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{1}{\coth(dx+c)^{\frac{3m}{2}}}dx\right)}{b^2}$$

input
```
int(1/(b*coth(d*x+c)^m)^(3/2),x)
```

output
```
(sqrt(b)*int(1/coth(c + d*x)**((3*m)/2),x))/b**2
```

3.55 $\int (b\coth^m(c+dx))^{4/3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 65

$$\int (b\coth^m(c+dx))^{4/3}\,dx = \frac{3b\coth^{1+m}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{6}(3+4m),\frac{1}{6}(9+4m),\coth^2(\ldots\right.}{d(3+4m)}$$

output
```
3*b*coth(d*x+c)^(1+m)*(b*coth(d*x+c)^m)^(1/3)*hypergeom([1, 1/2+2/3*m],[3/
2+2/3*m],coth(d*x+c)^2)/d/(3+4*m)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int (b\coth^m(c+dx))^{4/3}\,dx = \frac{\coth(c+dx)\,(b\coth^m(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(1,\frac{1}{6}(3+4m),\frac{1}{6}(9+4m),\coth^2(c+\ldots\right.}{d\left(1+\frac{4m}{3}\right)}$$

input
```
Integrate[(b*Coth[c + d*x]^m)^(4/3),x]
```

output
```
(Coth[c + d*x]*(b*Coth[c + d*x]^m)^(4/3)*Hypergeometric2F1[1, (3 + 4*m)/6,
 (9 + 4*m)/6, Coth[c + d*x]^2])/(d*(1 + (4*m)/3))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4142, 3042, 3957, 25, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\coth^m(c+dx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^m\right)^{4/3}dx$$

$$\Big\downarrow \text{4142}$$

$$b\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\int \coth^{\frac{4m}{3}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$b\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\int \left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{4m/3}dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{b\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\int -\frac{\coth^{\frac{4m}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\int \frac{\coth^{\frac{4m}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{3b\coth^{m+1}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{6}(4m+3),\frac{1}{6}(4m+9),\coth^2(c+dx)\right)}{d(4m+3)}$$

input
```
Int[(b*Coth[c + d*x]^m)^(4/3),x]
```

output
```
(3*b*Coth[c + d*x]^(1 + m)*(b*Coth[c + d*x]^m)^(1/3)*Hypergeometric2F1[1,
(3 + 4*m)/6, (9 + 4*m)/6, Coth[c + d*x]^2])/(d*(3 + 4*m))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \left(b \coth \left(d x +c \right)^{m}\right)^{\frac{4}{3}}d x$$

input
```
int((b*coth(d*x+c)^m)^(4/3),x)
```

output
```
int((b*coth(d*x+c)^m)^(4/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (b\coth^m(c+dx))^{4/3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*coth(d*x+c)^m)^(4/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int (b\coth^m(c+dx))^{4/3}\,dx = \text{Timed out}$$

input
```
integrate((b*coth(d*x+c)**m)**(4/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\coth^m(c+dx))^{4/3}\,dx = \int (b\coth{(dx+c)}^m)^{\frac{4}{3}}\,dx$$

input
```
integrate((b*coth(d*x+c)^m)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^m)^(4/3), x)
```

Giac [F]

$$\int (b\coth^m(c+dx))^{4/3}\,dx = \int (b\coth{(dx+c)}^m)^{\frac{4}{3}}\,dx$$

input
```
integrate((b*coth(d*x+c)^m)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^m)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\coth^m(c+dx))^{4/3}\,dx = \int (b\coth(c+d\,x)^m)^{4/3}\,dx$$

input
```
int((b*coth(c + d*x)^m)^(4/3),x)
```

output
```
int((b*coth(c + d*x)^m)^(4/3), x)
```

Reduce [F]

$$\int (b \coth^m(c+dx))^{4/3} \, dx = b^{\frac{4}{3}} \left(\int \coth(dx+c)^{\frac{4m}{3}} \, dx \right)$$

input
```
int((b*coth(d*x+c)^m)^(4/3),x)
```

output
```
b**(1/3)*int(coth(c + d*x)**((4*m)/3),x)*b
```

3.56 $\int (b \coth^m(c+dx))^{2/3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 60

$$\int (b \coth^m(c+dx))^{2/3}\,dx = \frac{3\coth(c+dx)\,(b\coth^m(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(1,\frac{1}{6}(3+2m),\frac{1}{6}(9+2m),\coth^2(c+dx)\right)}{d(3+2m)}$$

output
```
3*coth(d*x+c)*(b*coth(d*x+c)^m)^(2/3)*hypergeom([1, 1/2+1/3*m],[3/2+1/3*m]
,coth(d*x+c)^2)/d/(3+2*m)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int (b \coth^m(c+dx))^{2/3}\,dx = \frac{\coth(c+dx)\,(b\coth^m(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(1,\frac{1}{6}(3+2m),\frac{1}{6}(9+2m),\coth^2(c+dx)\right)}{d\left(1+\frac{2m}{3}\right)}$$

input
```
Integrate[(b*Coth[c + d*x]^m)^(2/3),x]
```

output
```
(Coth[c + d*x]*(b*Coth[c + d*x]^m)^(2/3)*Hypergeometric2F1[1, (3 + 2*m)/6,
 (9 + 2*m)/6, Coth[c + d*x]^2])/(d*(1 + (2*m)/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4142, 3042, 3957, 25, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\coth^m(c+dx))^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^m\right)^{2/3}dx$$

$$\Big\downarrow \text{4142}$$

$$\coth^{-\frac{2m}{3}}(c+dx)\,(b\coth^m(c+dx))^{2/3}\int \coth^{\frac{2m}{3}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\coth^{-\frac{2m}{3}}(c+dx)\,(b\coth^m(c+dx))^{2/3}\int \left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{2m/3}dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{\coth^{-\frac{2m}{3}}(c+dx)\,(b\coth^m(c+dx))^{2/3}\int -\frac{\coth^{\frac{2m}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\coth^{-\frac{2m}{3}}(c+dx)\,(b\coth^m(c+dx))^{2/3}\int \frac{\coth^{\frac{2m}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{3\coth(c+dx)\,(b\coth^m(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(1,\frac{1}{6}(2m+3),\frac{1}{6}(2m+9),\coth^2(c+dx)\right)}{d(2m+3)}$$

input
```
Int[(b*Coth[c + d*x]^m)^(2/3),x]
```

output
```
(3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(2/3)*Hypergeometric2F1[1, (3 + 2*m)/
6, (9 + 2*m)/6, Coth[c + d*x]^2])/(d*(3 + 2*m))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \left(b \coth \left(dx+c\right)^{m}\right)^{\frac{2}{3}}dx$$

input
```
int((b*coth(d*x+c)^m)^(2/3),x)
```

output
```
int((b*coth(d*x+c)^m)^(2/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (b\coth^m(c+dx))^{2/3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*coth(d*x+c)^m)^(2/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (b\coth^m(c+dx))^{2/3}\,dx = \int \left(b \coth^{m}\left(c+dx\right)\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((b*coth(d*x+c)**m)**(2/3),x)
```

output
```
Integral((b*coth(c + d*x)**m)**(2/3), x)
```

Maxima [F]

$$\int (b\coth^m(c+dx))^{2/3}\,dx = \int (b\coth(dx+c)^m)^{\frac{2}{3}}\,dx$$

input
```
integrate((b*coth(d*x+c)^m)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^m)^(2/3), x)
```

Giac [F]

$$\int (b\coth^m(c+dx))^{2/3}\,dx = \int (b\coth(dx+c)^m)^{\frac{2}{3}}\,dx$$

input
```
integrate((b*coth(d*x+c)^m)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^m)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\coth^m(c+dx))^{2/3}\,dx = \int (b\coth(c+d\,x)^m)^{2/3}\,dx$$

input
```
int((b*coth(c + d*x)^m)^(2/3),x)
```

output
```
int((b*coth(c + d*x)^m)^(2/3), x)
```

Reduce [F]

$$\int (b\coth^m(c+dx))^{2/3}\,dx = b^{\frac{2}{3}}\left(\int \coth(dx+c)^{\frac{2m}{3}}\,dx\right)$$

input
```
int((b*coth(d*x+c)^m)^(2/3),x)
```

output
```
b**(2/3)*int(coth(c + d*x)**((2*m)/3),x)
```

3.57 $\int \sqrt[3]{b\coth^m(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int \sqrt[3]{b\coth^m(c+dx)}\,dx = \frac{3\coth(c+dx)\sqrt[3]{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{6},\frac{9+m}{6},\coth^2(c+dx)\right)}{d(3+m)}$$

output
```
3*coth(d*x+c)*(b*coth(d*x+c)^m)^(1/3)*hypergeom([1, 1/2+1/6*m],[3/2+1/6*m]
,coth(d*x+c)^2)/d/(3+m)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{b\coth^m(c+dx)}\,dx = \frac{3\coth(c+dx)\sqrt[3]{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{6},\frac{9+m}{6},\coth^2(c+dx)\right)}{d(3+m)}$$

input
```
Integrate[(b*Coth[c + d*x]^m)^(1/3),x]
```

output
```
(3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(1/3)*Hypergeometric2F1[1, (3 + m)/6,
 (9 + m)/6, Coth[c + d*x]^2])/(d*(3 + m))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4142, 3042, 3957, 25, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\coth^m(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{b\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^m}dx$$

$$\Big\downarrow \text{4142}$$

$$\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\int \coth^{\frac{m}{3}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\int \left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{m/3}dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\int -\frac{\coth^{\frac{m}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\int \frac{\coth^{\frac{m}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{3\coth^{\frac{m+3}{3}-\frac{m}{3}}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{m+3}{6},\frac{m+9}{6},\coth^2(c+dx)\right)}{d(m+3)}$$

input
```
Int[(b*Coth[c + d*x]^m)^(1/3),x]
```

output
```
(3*Coth[c + d*x]^(-1/3*m + (3 + m)/3)*(b*Coth[c + d*x]^m)^(1/3)*Hypergeome
tric2F1[1, (3 + m)/6, (9 + m)/6, Coth[c + d*x]^2])/(d*(3 + m))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int (b\coth (dx+c)^m)^{\frac{1}{3}}\,dx$$

input
```
int((b*coth(d*x+c)^m)^(1/3),x)
```

output
```
int((b*coth(d*x+c)^m)^(1/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt[3]{b\coth^m(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*coth(d*x+c)^m)^(1/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt[3]{b\coth^m(c+dx)}\,dx = \int \sqrt[3]{b\coth^m{(c+dx)}}\,dx$$

input
```
integrate((b*coth(d*x+c)**m)**(1/3),x)
```

output
```
Integral((b*coth(c + d*x)**m)**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{b\coth^m(c+dx)}\,dx = \int (b\coth(dx+c)^m)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*coth(d*x+c)^m)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^m)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{b\coth^m(c+dx)}\,dx = \int (b\coth(dx+c)^m)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*coth(d*x+c)^m)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^m)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\coth^m(c+dx)}\,dx = \int (b\coth(c+d\,x)^m)^{1/3}\,dx$$

input
```
int((b*coth(c + d*x)^m)^(1/3),x)
```

output
```
int((b*coth(c + d*x)^m)^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{b\coth^m(c+dx)}\,dx = b^{\frac{1}{3}}\left(\int \coth\left(dx+c\right)^{\frac{m}{3}}dx\right)$$

input
```
int((b*coth(d*x+c)^m)^(1/3),x)
```

output
```
b**(1/3)*int(coth(c + d*x)**(m/3),x)
```

3.58 $\int \frac{1}{\sqrt[3]{b \coth^m(c+dx)}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 60

$$\int \frac{1}{\sqrt[3]{b \coth^m(c+dx)}} \, dx = \frac{3 \coth(c+dx) \text{Hypergeometric2F1}\left(1, \frac{3-m}{6}, \frac{9-m}{6}, \coth^2(c+dx)\right)}{d(3-m) \sqrt[3]{b \coth^m(c+dx)}}$$

output
```
3*coth(d*x+c)*hypergeom([1, 1/2-1/6*m],[3/2-1/6*m],coth(d*x+c)^2)/d/(3-m)/
(b*coth(d*x+c)^m)^(1/3)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt[3]{b \coth^m(c+dx)}} \, dx = -\frac{3 \coth(c+dx) \text{Hypergeometric2F1}\left(1, \frac{3-m}{6}, \frac{9-m}{6}, \coth^2(c+dx)\right)}{d(-3+m) \sqrt[3]{b \coth^m(c+dx)}}$$

input
```
Integrate[(b*Coth[c + d*x]^m)^(-1/3),x]
```

output
```
(-3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - m)/6, (9 - m)/6, Coth[c + d*x]
^2])/(d*(-3 + m)*(b*Coth[c + d*x]^m)^(1/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4142, 3042, 3957, 25, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{b \coth^m(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{b \left(-i \tan \left(ic+idx+\frac{\pi}{2}\right)\right)^m}}dx$$

$$\Big\downarrow \text{4142}$$

$$\frac{\coth^{\frac{m}{3}}(c+dx)\int \coth^{-\frac{m}{3}}(c+dx)dx}{\sqrt[3]{b \coth^m(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\coth^{\frac{m}{3}}(c+dx)\int \left(-i \tan \left(ic+idx+\frac{\pi}{2}\right)\right)^{-m/3}dx}{\sqrt[3]{b \coth^m(c+dx)}}$$

$$\Big\downarrow \text{3957}$$

$$-\frac{\coth^{\frac{m}{3}}(c+dx)\int -\frac{\coth^{-\frac{m}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d\sqrt[3]{b \coth^m(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\coth^{\frac{m}{3}}(c+dx)\int \frac{\coth^{-\frac{m}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d\sqrt[3]{b \coth^m(c+dx)}}$$

$$\Big\downarrow \text{278}$$

$$\frac{3\coth(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{3-m}{6},\frac{9-m}{6},\coth^2(c+dx)\right)}{d(3-m)\sqrt[3]{b\coth^m(c+dx)}}$$

input
```
Int[(b*Coth[c + d*x]^m)^(-1/3),x]
```

output
```
(3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - m)/6, (9 - m)/6, Coth[c + d*x]^
2])/(d*(3 - m)*(b*Coth[c + d*x]^m)^(1/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \frac{1}{\left(b \coth \left(d x +c \right)^{m}\right)^{\frac{1}{3}}}d x$$

input
```
int(1/(b*coth(d*x+c)^m)^(1/3),x)
```

output
```
int(1/(b*coth(d*x+c)^m)^(1/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt[3]{b \coth^m(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*coth(d*x+c)^m)^(1/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{b \coth^m(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{b \coth^{m}{\left(c + dx \right)}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)**m)**(1/3),x)
```

output
```
Integral((b*coth(c + d*x)**m)**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{b\coth^m(c+dx)}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^m\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^m)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^m)^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{b\coth^m(c+dx)}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^m\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^m)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^m)^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{b\coth^m(c+dx)}}\,dx = \int \frac{1}{\left(b\coth\left(c+d\,x\right)^m\right)^{1/3}}\,dx$$

input
```
int(1/(b*coth(c + d*x)^m)^(1/3),x)
```

output
```
int(1/(b*coth(c + d*x)^m)^(1/3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{b \coth^m(c+dx)}}\,dx = \frac{\int \frac{1}{\coth(dx+c)^{\frac{m}{3}}}dx}{b^{\frac{1}{3}}}$$

input
```
int(1/(b*coth(d*x+c)^m)^(1/3),x)
```

output
```
int(1/coth(c + d*x)**(m/3),x)/b**(1/3)
```

3.59 $\int \frac{1}{(b\coth^m(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 60

$$\int \frac{1}{(b\coth^m(c+dx))^{2/3}}\,dx = \frac{3\coth(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{6}(3-2m),\frac{1}{6}(9-2m),\coth^2(c+dx)\right)}{d(3-2m)\,(b\coth^m(c+dx))^{2/3}}$$

output
```
3*coth(d*x+c)*hypergeom([1, 1/2-1/3*m],[3/2-1/3*m],coth(d*x+c)^2)/d/(3-2*m
)/(b*coth(d*x+c)^m)^(2/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \frac{1}{(b\coth^m(c+dx))^{2/3}}\,dx = \frac{\coth(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{6}(3-2m),\frac{1}{6}(9-2m),\coth^2(c+dx)\right)}{d\left(1-\frac{2m}{3}\right)(b\coth^m(c+dx))^{2/3}}$$

input
```
Integrate[(b*Coth[c + d*x]^m)^(-2/3),x]
```

output
```
(Coth[c + d*x]*Hypergeometric2F1[1, (3 - 2*m)/6, (9 - 2*m)/6, Coth[c + d*x
]^2])/(d*(1 - (2*m)/3)*(b*Coth[c + d*x]^m)^(2/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4142, 3042, 3957, 25, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\coth^m(c+dx))^{2/3}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\left(b\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^m\right)^{2/3}}dx$$
$$\Big\downarrow \text{4142}$$
$$\frac{\coth^{\frac{2m}{3}}(c+dx)\int \coth^{-\frac{2m}{3}}(c+dx)dx}{(b\coth^m(c+dx))^{2/3}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\coth^{\frac{2m}{3}}(c+dx)\int \left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{-2m/3}dx}{(b\coth^m(c+dx))^{2/3}}$$
$$\Big\downarrow \text{3957}$$
$$-\frac{\coth^{\frac{2m}{3}}(c+dx)\int -\frac{\coth^{-\frac{2m}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d(b\coth^m(c+dx))^{2/3}}$$
$$\Big\downarrow \text{25}$$
$$\frac{\coth^{\frac{2m}{3}}(c+dx)\int \frac{\coth^{-\frac{2m}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{d(b\coth^m(c+dx))^{2/3}}$$
$$\Big\downarrow \text{278}$$
$$\frac{3\coth(c+dx)\text{Hypergeometric2F1}\left(1,\frac{1}{6}(3-2m),\frac{1}{6}(9-2m),\coth^2(c+dx)\right)}{d(3-2m)(b\coth^m(c+dx))^{2/3}}$$

input
```
Int[(b*Coth[c + d*x]^m)^(-2/3),x]
```

output
```
(3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - 2*m)/6, (9 - 2*m)/6, Coth[c + d
*x]^2])/(d*(3 - 2*m)*(b*Coth[c + d*x]^m)^(2/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \frac{1}{\left(b \coth \left(d x+c\right)^{m}\right)^{\frac{2}{3}}}d x$$

input
```
int(1/(b*coth(d*x+c)^m)^(2/3),x)
```

output
```
int(1/(b*coth(d*x+c)^m)^(2/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(b \coth^m(c+dx))^{2/3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*coth(d*x+c)^m)^(2/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{(b \coth^m(c+dx))^{2/3}}\,dx = \int \frac{1}{\left(b \coth^{m}\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)**m)**(2/3),x)
```

output
```
Integral((b*coth(c + d*x)**m)**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{(b\coth^m(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\coth(dx+c)^m)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^m)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^m)^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{(b\coth^m(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\coth(dx+c)^m)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^m)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^m)^(-2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\coth^m(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\coth(c+d\,x)^m)^{2/3}}\,dx$$

input
```
int(1/(b*coth(c + d*x)^m)^(2/3),x)
```

output
```
int(1/(b*coth(c + d*x)^m)^(2/3), x)
```

Reduce [F]

$$\int \frac{1}{\left(b \coth^m(c+dx)\right)^{2/3}}\,dx = \frac{\int \frac{1}{\coth(dx+c)^{\frac{2m}{3}}}dx}{b^{\frac{2}{3}}}$$

input
```
int(1/(b*coth(d*x+c)^m)^(2/3),x)
```

output
```
int(1/coth(c + d*x)**((2*m)/3),x)/b**(2/3)
```

3.60 $\int \frac{1}{(b\coth^m(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 69

$$\int \frac{1}{(b\coth^m(c+dx))^{4/3}}\,dx = \frac{3\coth^{1-m}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{6}(3-4m),\frac{1}{6}(9-4m),\coth^2(c+\right.}{bd(3-4m)\sqrt[3]{b\coth^m(c+dx)}}$$

output
```
3*coth(d*x+c)^(1-m)*hypergeom([1, 1/2-2/3*m],[3/2-2/3*m],coth(d*x+c)^2)/b/
d/(3-4*m)/(b*coth(d*x+c)^m)^(1/3)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{1}{(b\coth^m(c+dx))^{4/3}}\,dx = \frac{\coth(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{6}(3-4m),\frac{1}{6}(9-4m),\coth^2(c+dx)\right)}{d\left(1-\frac{4m}{3}\right)(b\coth^m(c+dx))^{4/3}}$$

input
```
Integrate[(b*Coth[c + d*x]^m)^(-4/3),x]
```

output
```
(Coth[c + d*x]*Hypergeometric2F1[1, (3 - 4*m)/6, (9 - 4*m)/6, Coth[c + d*x
]^2])/(d*(1 - (4*m)/3)*(b*Coth[c + d*x]^m)^(4/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4142, 3042, 3957, 25, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\coth^m(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(b\left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^m\right)^{4/3}}dx$$

$$\Big\downarrow \text{ 4142}$$

$$\frac{\coth^{\frac{m}{3}}(c+dx)\int \coth^{-\frac{4m}{3}}(c+dx)dx}{b\sqrt[3]{b\coth^m(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\coth^{\frac{m}{3}}(c+dx)\int \left(-i\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^{-4m/3}dx}{b\sqrt[3]{b\coth^m(c+dx)}}$$

$$\Big\downarrow \text{ 3957}$$

$$-\frac{\coth^{\frac{m}{3}}(c+dx)\int -\frac{\coth^{-\frac{4m}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{bd\sqrt[3]{b\coth^m(c+dx)}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\coth^{\frac{m}{3}}(c+dx)\int \frac{\coth^{-\frac{4m}{3}}(c+dx)}{1-\coth^2(c+dx)}d\coth(c+dx)}{bd\sqrt[3]{b\coth^m(c+dx)}}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{3\coth^{1-m}(c+dx)\text{Hypergeometric2F1}\left(1,\frac{1}{6}(3-4m),\frac{1}{6}(9-4m),\coth^2(c+dx)\right)}{bd(3-4m)\sqrt[3]{b\coth^m(c+dx)}}$$

input
```
Int[(b*Coth[c + d*x]^m)^(-4/3),x]
```

output
```
(3*Coth[c + d*x]^(1 - m)*Hypergeometric2F1[1, (3 - 4*m)/6, (9 - 4*m)/6, Co
th[c + d*x]^2])/(b*d*(3 - 4*m)*(b*Coth[c + d*x]^m)^(1/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \frac{1}{\left(b \coth \left(d x+c \right)^{m}\right)^{\frac{4}{3}}}d x$$

input
```
int(1/(b*coth(d*x+c)^m)^(4/3),x)
```

output
```
int(1/(b*coth(d*x+c)^m)^(4/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(b \coth^m(c+dx))^{4/3}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*coth(d*x+c)^m)^(4/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{(b \coth^m(c+dx))^{4/3}}\, dx = \int \frac{1}{\left(b \coth^{m}{\left(c + d x \right)}\right)^{\frac{4}{3}}}\, dx$$

input
```
integrate(1/(b*coth(d*x+c)**m)**(4/3),x)
```

output
```
Integral((b*coth(c + d*x)**m)**(-4/3), x)
```

Maxima [F]

$$\int \frac{1}{(b\coth^m(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\coth(dx+c)^m)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^m)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*coth(d*x + c)^m)^(-4/3), x)
```

Giac [F]

$$\int \frac{1}{(b\coth^m(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\coth(dx+c)^m)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*coth(d*x+c)^m)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*coth(d*x + c)^m)^(-4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\coth^m(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\coth(c+d\,x)^m)^{4/3}}\,dx$$

input
```
int(1/(b*coth(c + d*x)^m)^(4/3),x)
```

output
```
int(1/(b*coth(c + d*x)^m)^(4/3), x)
```

Reduce [F]

$$\int \frac{1}{(b\coth^m(c+dx))^{4/3}}\,dx = \frac{\int \frac{1}{\coth(dx+c)^{\frac{4m}{3}}}dx}{b^{\frac{4}{3}}}$$

input
```
int(1/(b*coth(d*x+c)^m)^(4/3),x)
```

output
```
int(1/coth(c + d*x)**((4*m)/3),x)/(b**(1/3)*b)
```

3.61 $\int (1+\coth(x))^5\,dx$

Optimal result

Integrand size = 6, antiderivative size = 41

$$\int (1+\coth(x))^5\,dx = 16x - 8\coth(x) - 2(1+\coth(x))^2 - \frac{2}{3}(1+\coth(x))^3 - \frac{1}{4}(1+\coth(x))^4 + 16\log(\sinh(x))$$

output
```
16*x-8*coth(x)-2*(1+coth(x))^2-2/3*(1+coth(x))^3-1/4*(1+coth(x))^4+16*ln(s
inh(x))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.10

$$\int (1+\coth(x))^5\,dx = \frac{(1+\coth(x))^5\sinh(x)\left(-20\cosh^3(x)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\tanh^2(x)\right)\sinh(x)+3(-1-24\,\text{si}}{12(\text{cosh}}$$

input
```
Integrate[(1 + Coth[x])^5,x]
```

output
```
((1 + Coth[x])^5*Sinh[x]*(-20*Cosh[x]^3*Hypergeometric2F1[-3/2, 1, -1/2, T
anh[x]^2]*Sinh[x] + 3*(-1 - 24*Sinh[x]^2 - 40*Cosh[x]*Hypergeometric2F1[-1
/2, 1, 1/2, Tanh[x]^2]*Sinh[x]^3 + 4*x*Sinh[x]^4 + 64*Log[Sinh[x]]*Sinh[x]
^4)))/(12*(Cosh[x] + Sinh[x])^5)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.000, Rules used = {3042, 3959, 3042, 3959, 3042, 3959, 3042, 3958, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\coth(x)+1)^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)^5 dx$$

$$\Big\downarrow \text{3959}$$

$$2\int (\coth(x)+1)^4dx-\frac{1}{4}(\coth(x)+1)^4$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{4}(\coth(x)+1)^4+2\int \left(1-i\tan\left(ix+\frac{\pi}{2}\right)\right)^4 dx$$

$$\Big\downarrow \text{3959}$$

$$2\left(2\int (\coth(x)+1)^3dx-\frac{1}{3}(\coth(x)+1)^3\right)-\frac{1}{4}(\coth(x)+1)^4$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{4}(\coth(x)+1)^4+2\left(-\frac{1}{3}(\coth(x)+1)^3+2\int \left(1-i\tan\left(ix+\frac{\pi}{2}\right)\right)^3 dx\right)$$

$$\Big\downarrow \text{3959}$$

$$2\left(2\left(2\int (\coth(x)+1)^2dx-\frac{1}{2}(\coth(x)+1)^2\right)-\frac{1}{3}(\coth(x)+1)^3\right)-\frac{1}{4}(\coth(x)+1)^4$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{4}(\coth(x)+1)^4+2\left(-\frac{1}{3}(\coth(x)+1)^3+2\left(-\frac{1}{2}(\coth(x)+1)^2+2\int\left(1-i\tan\left(ix+\frac{\pi}{2}\right)\right)^2dx\right)\right)$$

$$\Big\downarrow \text{3958}$$

$$-\frac{1}{4}(\coth(x)+1)^4+2\left(-\frac{1}{3}(\coth(x)+1)^3+2\left(-\frac{1}{2}(\coth(x)+1)^2+2(-2i\int i\coth(x)dx+2x-\coth(x))\right)\right)$$

$$\Big\downarrow \text{26}$$

$$2\left(2\left(2(2\int\coth(x)dx+2x-\coth(x))-\frac{1}{2}(\coth(x)+1)^2\right)-\frac{1}{3}(\coth(x)+1)^3\right)-\frac{1}{4}(\coth(x)+1)^4$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{4}(\coth(x)+1)^4+2\left(-\frac{1}{3}(\coth(x)+1)^3+2\left(-\frac{1}{2}(\coth(x)+1)^2+2\left(2\int-i\tan\left(ix+\frac{\pi}{2}\right)dx+2x-\coth(x)\right)\right)\right)$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{4}(\coth(x)+1)^4+2\left(-\frac{1}{3}(\coth(x)+1)^3+2\left(-\frac{1}{2}(\coth(x)+1)^2+2\left(-2i\int\tan\left(ix+\frac{\pi}{2}\right)dx+2x-\coth(x)\right)\right)\right)$$

$$\Big\downarrow \text{3956}$$

$$2\left(2\left(2(2x-\coth(x)+2\log(\sinh(x)))-\frac{1}{2}(\coth(x)+1)^2\right)-\frac{1}{3}(\coth(x)+1)^3\right)-\frac{1}{4}(\coth(x)+1)^4$$

input
```
Int[(1 + Coth[x])^5,x]
```

output
```
-1/4*(1 + Coth[x])^4 + 2*(-1/3*(1 + Coth[x])^3 + 2*(-1/2*(1 + Coth[x])^2 +
 2*(2*x - Coth[x] + 2*Log[Sinh[x]])))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3959
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[2*a   Int[(a + b*Tan[c + d*
x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n
, 1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result
derivativedivides	$-\frac{\coth(x)^4}{4}-\frac{5\coth(x)^3}{3}-\frac{11\coth(x)^2}{2}-15\coth(x)-16\ln(\coth(x)-1)$
default	$-\frac{\coth(x)^4}{4}-\frac{5\coth(x)^3}{3}-\frac{11\coth(x)^2}{2}-15\coth(x)-16\ln(\coth(x)-1)$
parallelrisch	$-\frac{\coth(x)^4}{4}+16\ln(\tanh(x))-16\ln(1-\tanh(x))-15\coth(x)-\frac{11\coth(x)^2}{2}-\frac{5\coth(x)^3}{3}$
risch	$-\frac{4(48\,e^{6x}-108\,e^{4x}+88\,e^{2x}-25)}{3(e^{2x}-1)^4}+16\ln(e^{2x}-1)$
parts	$x-\frac{\coth(x)^4}{4}-\frac{11\coth(x)^2}{2}-13\ln(\coth(x)-1)+2\ln(1+\coth(x))-15\coth(x)-\frac{5\,\mathrm{co}}{}$

input
```
int((1+coth(x))^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*coth(x)^4-5/3*coth(x)^3-11/2*coth(x)^2-15*coth(x)-16*ln(coth(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 448 vs. 2(37) = 74.

Time = 0.09 (sec) , antiderivative size = 448, normalized size of antiderivative = 10.93

$$\int (1+\coth(x))^5 \, dx = \text{Too large to display}$$

input
```
integrate((1+coth(x))^5,x, algorithm="fricas")
```

output
```
-4/3*(48*cosh(x)^6 + 288*cosh(x)*sinh(x)^5 + 48*sinh(x)^6 + 36*(20*cosh(x)
^2 - 3)*sinh(x)^4 - 108*cosh(x)^4 + 48*(20*cosh(x)^3 - 9*cosh(x))*sinh(x)^
3 + 8*(90*cosh(x)^4 - 81*cosh(x)^2 + 11)*sinh(x)^2 + 88*cosh(x)^2 - 12*(co
sh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6
- 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 -
30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3
+ 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*
sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(
x))*sinh(x) + 1)*log(2*sinh(x)/(cosh(x) - sinh(x))) + 16*(18*cosh(x)^5 - 2
7*cosh(x)^3 + 11*cosh(x))*sinh(x) - 25)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 +
 sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3
- 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6
*cosh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*c
osh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(co
sh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int (1+\coth(x))^5 \, dx = 32x - 16\log(\tanh(x)+1) + 16\log(\tanh(x)) - \frac{15}{\tanh(x)} - \frac{11}{2\tanh^2(x)} - \frac{5}{3\tanh^3(x)} - \frac{1}{4\tanh^4(x)}$$

input
```
integrate((1+coth(x))**5,x)
```

output
```
32*x - 16*log(tanh(x) + 1) + 16*log(tanh(x)) - 15/tanh(x) - 11/(2*tanh(x)*
*2) - 5/(3*tanh(x)**3) - 1/(4*tanh(x)**4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(37) = 74.

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.41

$$\int (1+\coth(x))^5\,dx = 27\,x - \frac{20\,\left(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} - 2\right)}{3\,\left(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1\right)} + \frac{4\,\left(e^{(-2\,x)} - e^{(-4\,x)} + e^{(-6\,x)}\right)}{4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1} + \frac{20\,e^{(-2\,x)}}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} + \frac{20}{e^{(-2\,x)} - 1} + 11\,\log\left(e^{(-x)} + 1\right) + 11\,\log\left(e^{(-x)} - 1\right) + 5\,\log\left(\sinh\left(x\right)\right)$$

input
```
integrate((1+coth(x))^5,x, algorithm="maxima")
```

output
```
27*x - 20/3*(3*e^(-2*x) - 3*e^(-4*x) - 2)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6
*x) - 1) + 4*(e^(-2*x) - e^(-4*x) + e^(-6*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4
*e^(-6*x) - e^(-8*x) - 1) + 20*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + 20/(
e^(-2*x) - 1) + 11*log(e^(-x) + 1) + 11*log(e^(-x) - 1) + 5*log(sinh(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int (1+\coth(x))^5\,dx = -\frac{4\,\left(48\,e^{(6\,x)} - 108\,e^{(4\,x)} + 88\,e^{(2\,x)} - 25\right)}{3\,\left(e^{(2\,x)} - 1\right)^4} + 16\,\log\left(\left|e^{(2\,x)} - 1\right|\right)$$

input
```
integrate((1+coth(x))^5,x, algorithm="giac")
```

output
```
-4/3*(48*e^(6*x) - 108*e^(4*x) + 88*e^(2*x) - 25)/(e^(2*x) - 1)^4 + 16*log
(abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.15

$$\int (1+\coth(x))^5\,dx = 16\,\ln\left(\mathrm{e}^{2\,x}-1\right)-\frac{64}{3\,\left(3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1\right)} -\frac{48}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1}-\frac{4}{6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1}-\frac{64}{\mathrm{e}^{2\,x}-1}$$

input
```
int((coth(x) + 1)^5,x)
```

output
```
16*log(exp(2*x) - 1) - 64/(3*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - 4
8/(exp(4*x) - 2*exp(2*x) + 1) - 4/(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) +
exp(8*x) + 1) - 64/(exp(2*x) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 177, normalized size of antiderivative = 4.32

$$\int (1+\coth(x))^5\,dx = \frac{48e^{8x}\log(e^x-1)+48e^{8x}\log(e^x+1)-48e^{8x}-192e^{6x}\log(e^x-1)-192e^{6x}\log(e^x+1)+288e^{4x}\log(e^x-}{3e^{8x}-}$$

input
```
int((1+coth(x))^5,x)
```

output
```
(4*(12*e**(8*x)*log(e**x - 1) + 12*e**(8*x)*log(e**x + 1) - 12*e**(8*x) -
48*e**(6*x)*log(e**x - 1) - 48*e**(6*x)*log(e**x + 1) + 72*e**(4*x)*log(e*
*x - 1) + 72*e**(4*x)*log(e**x + 1) + 36*e**(4*x) - 48*e**(2*x)*log(e**x -
 1) - 48*e**(2*x)*log(e**x + 1) - 40*e**(2*x) + 12*log(e**x - 1) + 12*log(
e**x + 1) + 13))/(3*(e**(8*x) - 4*e**(6*x) + 6*e**(4*x) - 4*e**(2*x) + 1))
```

3.62 $\int (1+\coth(x))^4 \, dx$

Optimal result

Integrand size = 6, antiderivative size = 31

$$\int (1+\coth(x))^4 \, dx = 8x - 4\coth(x) - (1+\coth(x))^2 - \frac{1}{3}(1+\coth(x))^3 + 8\log(\sinh(x))$$

output
```
8*x-4*coth(x)-(1+coth(x))^2-1/3*(1+coth(x))^3+8*ln(sinh(x))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.39

$$\int (1+\coth(x))^4 \, dx = \frac{(1+\coth(x))^4 \sinh(x) \left(-\cosh^3(x)\, \text{Hypergeometric2F1}\left(-\frac{3}{2}, 1, -\frac{1}{2}, \tanh^2(x)\right) + 3\sinh(x) \left(-2 - 6\cosh(\right.\right.}{3(\cosh(x) + \sinh(x)}$$

input
```
Integrate[(1 + Coth[x])^4,x]
```

output
```
((1 + Coth[x])^4*Sinh[x]*(-(Cosh[x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tan
h[x]^2]) + 3*Sinh[x]*(-2 - 6*Cosh[x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[
x]^2]*Sinh[x] + (x + 8*Log[Sinh[x]])*Sinh[x]^2)))/(3*(Cosh[x] + Sinh[x])^4
)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.667$, Rules used = {3042, 3959, 3042, 3959, 3042, 3958, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\coth(x)+1)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)^4dx$$

$$\Big\downarrow \text{3959}$$

$$2\int (\coth(x)+1)^3dx-\frac{1}{3}(\coth(x)+1)^3$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}(\coth(x)+1)^3+2\int \left(1-i\tan\left(ix+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{3959}$$

$$2\left(2\int (\coth(x)+1)^2dx-\frac{1}{2}(\coth(x)+1)^2\right)-\frac{1}{3}(\coth(x)+1)^3$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}(\coth(x)+1)^3+2\left(-\frac{1}{2}(\coth(x)+1)^2+2\int \left(1-i\tan\left(ix+\frac{\pi}{2}\right)\right)^2dx\right)$$

$$\Big\downarrow \text{3958}$$

$$-\frac{1}{3}(\coth(x)+1)^3+2\left(-\frac{1}{2}(\coth(x)+1)^2+2\left(-2i\int i\coth(x)dx+2x-\coth(x)\right)\right)$$

$$\Big\downarrow \text{26}$$

$$2\left(2\left(2\int \coth(x)dx+2x-\coth(x)\right)-\frac{1}{2}(\coth(x)+1)^2\right)-\frac{1}{3}(\coth(x)+1)^3$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}(\coth(x)+1)^3+2\left(-\frac{1}{2}(\coth(x)+1)^2+2\left(2\int -i\tan\left(ix+\frac{\pi}{2}\right)dx+2x-\coth(x)\right)\right)$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{3}(\coth(x)+1)^3+2\left(-\frac{1}{2}(\coth(x)+1)^2+2\left(-2i\int \tan\left(ix+\frac{\pi}{2}\right)dx+2x-\coth(x)\right)\right)$$

$$\Big\downarrow \text{3956}$$

$$2\left(2(2x-\coth(x)+2\log(\sinh(x)))-\frac{1}{2}(\coth(x)+1)^2\right)-\frac{1}{3}(\coth(x)+1)^3$$

input
```
Int[(1 + Coth[x])^4,x]
```

output
```
-1/3*(1 + Coth[x])^3 + 2*(-1/2*(1 + Coth[x])^2 + 2*(2*x - Coth[x] + 2*Log[
Sinh[x]]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3959
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[2*a   Int[(a + b*Tan[c + d*
x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n
, 1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

method	result
derivativedivides	$-\frac{\coth(x)^3}{3}-2\coth(x)^2-7\coth(x)-8\ln(\coth(x)-1)$
default	$-\frac{\coth(x)^3}{3}-2\coth(x)^2-7\coth(x)-8\ln(\coth(x)-1)$
parallelrisch	$-\frac{\coth(x)^3}{3}+8\ln(\tanh(x))-8\ln(1-\tanh(x))-7\coth(x)-2\coth(x)^2$
risch	$-\frac{4(18\,e^{4x}-27\,e^{2x}+11)}{3(e^{2x}-1)^3}+8\ln(e^{2x}-1)$
parts	$x-\frac{\coth(x)^3}{3}-7\coth(x)-\frac{11\ln(\coth(x)-1)}{2}+\frac{3\ln(1+\coth(x))}{2}-2\coth(x)^2+4\ln(\sinh(x))$

input
```
int((1+coth(x))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*coth(x)^3-2*coth(x)^2-7*coth(x)-8*ln(coth(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(29) = 58$.

Time = 0.09 (sec) , antiderivative size = 273, normalized size of antiderivative = 8.81

$$\int (1+\coth(x))^4\,dx =$$
$$-\frac{4\left(18\cosh(x)^4+72\cosh(x)\sinh(x)^3+18\sinh(x)^4+27\left(4\cosh(x)^2-1\right)\sinh(x)^2-27\cosh(x)^2\right.}{3\left(\cosh(x)^6+6\right.}$$

input
```
integrate((1+coth(x))^4,x, algorithm="fricas")
```

output
```
-4/3*(18*cosh(x)^4 + 72*cosh(x)*sinh(x)^3 + 18*sinh(x)^4 + 27*(4*cosh(x)^2
 - 1)*sinh(x)^2 - 27*cosh(x)^2 - 6*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh
(x)^6 + 3*(5*cosh(x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*c
osh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 - 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x
)^2 + 6*(cosh(x)^5 - 2*cosh(x)^3 + cosh(x))*sinh(x) - 1)*log(2*sinh(x)/(co
sh(x) - sinh(x))) + 18*(4*cosh(x)^3 - 3*cosh(x))*sinh(x) + 11)/(cosh(x)^6
+ 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 - 1)*sinh(x)^4 - 3*cosh
(x)^4 + 4*(5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 - 6*cosh(x)
^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 - 2*cosh(x)^3 + cosh(x))*si
nh(x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int (1+\coth(x))^4\,dx = 16x - 8\log(\tanh(x)+1) + 8\log(\tanh(x)) - \frac{7}{\tanh(x)} - \frac{2}{\tanh^2(x)} - \frac{1}{3\tanh^3(x)}$$

input
```
integrate((1+coth(x))**4,x)
```

output
```
16*x - 8*log(tanh(x) + 1) + 8*log(tanh(x)) - 7/tanh(x) - 2/tanh(x)**2 - 1/
(3*tanh(x)**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(29) = 58$.

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.06

$$\int (1+\coth(x))^4\,dx = 12\,x - \frac{4\,\left(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} - 2\right)}{3\,\left(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1\right)} + \frac{8\,e^{(-2\,x)}}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} + \frac{12}{e^{(-2\,x)} - 1} + 4\,\log\left(e^{(-x)} + 1\right) + 4\,\log\left(e^{(-x)} - 1\right) + 4\,\log\left(\sinh\left(x\right)\right)$$

input
```
integrate((1+coth(x))^4,x, algorithm="maxima")
```

output
```
12*x - 4/3*(3*e^(-2*x) - 3*e^(-4*x) - 2)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*
x) - 1) + 8*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + 12/(e^(-2*x) - 1) + 4*l
og(e^(-x) + 1) + 4*log(e^(-x) - 1) + 4*log(sinh(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int (1+\coth(x))^4\,dx = -\frac{4\,\left(18\,e^{(4\,x)} - 27\,e^{(2\,x)} + 11\right)}{3\,\left(e^{(2\,x)} - 1\right)^3} + 8\,\log\left(\left|e^{(2\,x)} - 1\right|\right)$$

input
```
integrate((1+coth(x))^4,x, algorithm="giac")
```

output
```
-4/3*(18*e^(4*x) - 27*e^(2*x) + 11)/(e^(2*x) - 1)^3 + 8*log(abs(e^(2*x) -
1))
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int (1+\coth(x))^4\,dx = 8\,\ln\left(\mathrm{e}^{2\,x}-1\right)-\frac{8}{3\,\left(3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1\right)}-\frac{12}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1}-\frac{24}{\mathrm{e}^{2\,x}-1}$$

input
```
int((coth(x) + 1)^4,x)
```

output
```
8*log(exp(2*x) - 1) - 8/(3*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - 12/
(exp(4*x) - 2*exp(2*x) + 1) - 24/(exp(2*x) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 4.42

$$\int (1+\coth(x))^4\,dx = \frac{24e^{6x}\log(e^x-1)+24e^{6x}\log(e^x+1)-24e^{6x}-72e^{4x}\log(e^x-1)-72e^{4x}\log(e^x+1)+72e^{2x}\log(e^x-1)}{3e^{6x}-9e^{4x}+9e^{2x}-3}$$

input
```
int((1+coth(x))^4,x)
```

output
```
(4*(6*e**(6*x)*log(e**x - 1) + 6*e**(6*x)*log(e**x + 1) - 6*e**(6*x) - 18*
e**(4*x)*log(e**x - 1) - 18*e**(4*x)*log(e**x + 1) + 18*e**(2*x)*log(e**x
- 1) + 18*e**(2*x)*log(e**x + 1) + 9*e**(2*x) - 6*log(e**x - 1) - 6*log(e*
*x + 1) - 5))/(3*(e**(6*x) - 3*e**(4*x) + 3*e**(2*x) - 1))
```

3.63 $\int (1+\coth(x))^3 \, dx$

Optimal result

Integrand size = 6, antiderivative size = 23

$$\int (1+\coth(x))^3 \, dx = 4x - 2\coth(x) - \frac{1}{2}(1+\coth(x))^2 + 4\log(\sinh(x))$$

output
```
4*x-2*coth(x)-1/2*(1+coth(x))^2+4*ln(sinh(x))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.09

$$\int (1+\coth(x))^3 \, dx = \frac{1}{2}\text{csch}^2(x) \left(-1-x-4\log(\sinh(x))+\cosh(2x)(x+4\log(\sinh(x))) - 6\cosh(x) \, \text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(x)\right)\sinh(x)\right)$$

input
```
Integrate[(1 + Coth[x])^3,x]
```

output
```
(Csch[x]^2*(-1 - x - 4*Log[Sinh[x]] + Cosh[2*x]*(x + 4*Log[Sinh[x]]) - 6*C
osh[x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[x]^2]*Sinh[x]))/2
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {3042, 3959, 3042, 3958, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\coth(x)+1)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)^3dx$$

$$\Big\downarrow \text{3959}$$

$$2\int (\coth(x)+1)^2dx-\frac{1}{2}(\coth(x)+1)^2$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}(\coth(x)+1)^2+2\int \left(1-i\tan\left(ix+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3958}$$

$$-\frac{1}{2}(\coth(x)+1)^2+2(-2i\int i\coth(x)dx+2x-\coth(x))$$

$$\Big\downarrow \text{26}$$

$$2(2\int \coth(x)dx+2x-\coth(x))-\frac{1}{2}(\coth(x)+1)^2$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}(\coth(x)+1)^2+2\left(2\int -i\tan\left(ix+\frac{\pi}{2}\right)dx+2x-\coth(x)\right)$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{2}(\coth(x)+1)^2+2\left(-2i\int \tan\left(ix+\frac{\pi}{2}\right)dx+2x-\coth(x)\right)$$

$$\Big\downarrow \text{3956}$$

$$2(2x-\coth(x)+2\log(\sinh(x)))-\frac{1}{2}(\coth(x)+1)^2$$

input
```
Int[(1 + Coth[x])^3,x]
```

output
```
-1/2*(1 + Coth[x])^2 + 2*(2*x - Coth[x] + 2*Log[Sinh[x]])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3959
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[2*a   Int[(a + b*Tan[c + d*
x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n
, 1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	siz
derivativedivides	$-\frac{\coth(x)^2}{2}-3\coth\left(x\right)-4\ln\left(\coth\left(x\right)-1\right)$	19
default	$-\frac{\coth(x)^2}{2}-3\coth\left(x\right)-4\ln\left(\coth\left(x\right)-1\right)$	19
parallelrisch	$4\ln\left(\tanh\left(x\right)\right)-4\ln\left(1-\tanh\left(x\right)\right)-3\coth\left(x\right)-\frac{\coth(x)^2}{2}$	26
risch	$-\frac{2(4\,\mathrm{e}^{2x}-3)}{(\mathrm{e}^{2x}-1)^2}+4\ln\left(\mathrm{e}^{2x}-1\right)$	29
parts	$x-\frac{\coth(x)^2}{2}-2\ln\left(\coth\left(x\right)-1\right)+\ln\left(1+\coth\left(x\right)\right)-3\coth\left(x\right)+3\ln\left(\sinh\left(x\right)\right)$	30

input
```
int((1+coth(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*coth(x)^2-3*coth(x)-4*ln(coth(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(21) = 42.

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 6.17

$$\int (1+\coth(x))^3\,dx =$$
$$-\frac{2\left(4\cosh\left(x\right)^2-2\left(\cosh\left(x\right)^4+4\cosh\left(x\right)\sinh\left(x\right)^3+\sinh\left(x\right)^4+2\left(3\cosh\left(x\right)^2-1\right)\sinh\left(x\right)^2-2\cos\right.\right.}{\cosh\left(x\right)^4+4\cosh\left(x\right)\sinh\left(x\right)^3+\sinh\left(x\right)^4+2\left(3\cosh\left(x\right)^2-\right.}$$

input
```
integrate((1+coth(x))^3,x, algorithm="fricas")
```

output
```
-2*(4*cosh(x)^2 - 2*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*co
sh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1
)*log(2*sinh(x)/(cosh(x) - sinh(x))) + 8*cosh(x)*sinh(x) + 4*sinh(x)^2 - 3
)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(
x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int (1+\coth(x))^3\,dx = 8x - 4\log(\tanh(x)+1) + 4\log(\tanh(x)) - \frac{3}{\tanh(x)} - \frac{1}{2\tanh^2(x)}$$

input
```
integrate((1+coth(x))**3,x)
```

output
```
8*x - 4*log(tanh(x) + 1) + 4*log(tanh(x)) - 3/tanh(x) - 1/(2*tanh(x)**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(21) = 42.

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.39

$$\int (1+\coth(x))^3\,dx = 5\,x + \frac{2\,e^{(-2\,x)}}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} + \frac{6}{e^{(-2\,x)} - 1} + \log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right) + 3\,\log(\sinh(x))$$

input
```
integrate((1+coth(x))^3,x, algorithm="maxima")
```

output
```
5*x + 2*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + 6/(e^(-2*x) - 1) + log(e^(-
x) + 1) + log(e^(-x) - 1) + 3*log(sinh(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int (1+\coth(x))^3\,dx = -\frac{2\left(4\,e^{(2\,x)} - 3\right)}{\left(e^{(2\,x)} - 1\right)^2} + 4\,\log\left(\left|e^{(2\,x)} - 1\right|\right)$$

input
```
integrate((1+coth(x))^3,x, algorithm="giac")
```

output
```
-2*(4*e^(2*x) - 3)/(e^(2*x) - 1)^2 + 4*log(abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int (1+\coth(x))^3\,dx = 4\,\ln\left(\mathrm{e}^{2\,x}-1\right)-\frac{2}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1}-\frac{8}{\mathrm{e}^{2\,x}-1}$$

input
```
int((coth(x) + 1)^3,x)
```

output
```
4*log(exp(2*x) - 1) - 2/(exp(4*x) - 2*exp(2*x) + 1) - 8/(exp(2*x) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 4.13

$$\int (1+\coth(x))^3\,dx = \frac{4e^{4x}\log(e^x-1)+4e^{4x}\log(e^x+1)-4e^{4x}-8e^{2x}\log(e^x-1)-8e^{2x}\log(e^x+1)+4\log(e^x-1)+4\log(e^x}{e^{4x}-2e^{2x}+1}$$

input
```
int((1+coth(x))^3,x)
```

output
```
(2*(2*e**(4*x)*log(e**x - 1) + 2*e**(4*x)*log(e**x + 1) - 2*e**(4*x) - 4*e
**(2*x)*log(e**x - 1) - 4*e**(2*x)*log(e**x + 1) + 2*log(e**x - 1) + 2*log
(e**x + 1) + 1))/(e**(4*x) - 2*e**(2*x) + 1)
```

3.64 $\int (1+\coth(x))^2\,dx$

Optimal result

Integrand size = 6, antiderivative size = 13

$$\int (1+\coth(x))^2\,dx = 2x-\coth(x)+2\log(\sinh(x))$$

output
```
2*x-coth(x)+2*ln(sinh(x))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int (1+\coth(x))^2\,dx = x-\coth(x)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(x)\right) + 2\log(\sinh(x))$$

input
```
Integrate[(1 + Coth[x])^2,x]
```

output
```
x - Coth[x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[x]^2] + 2*Log[Sinh[x]]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3958, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\coth(x)+1)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)^2 dx$$

$$\Big\downarrow \text{3958}$$

$$-2i\int i\coth(x)dx+2x-\coth(x)$$

$$\Big\downarrow \text{26}$$

$$2\int \coth(x)dx+2x-\coth(x)$$

$$\Big\downarrow \text{3042}$$

$$2\int -i\tan\left(ix+\frac{\pi}{2}\right)dx+2x-\coth(x)$$

$$\Big\downarrow \text{26}$$

$$-2i\int \tan\left(ix+\frac{\pi}{2}\right)dx+2x-\coth(x)$$

$$\Big\downarrow \text{3956}$$

$$2x-\coth(x)+2\log(\sinh(x))$$

input
```
Int[(1 + Coth[x])^2,x]
```

output
```
2*x - Coth[x] + 2*Log[Sinh[x]]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\coth(x)-2\ln(\coth(x)-1)$	13
default	$-\coth(x)-2\ln(\coth(x)-1)$	13
parallelrisch	$2\ln(\tanh(x))-2\ln(1-\tanh(x))-\coth(x)$	20
risch	$-\frac{2}{e^{2x}-1}+2\ln(e^{2x}-1)$	21
parts	$x-\coth(x)-\frac{\ln(\coth(x)-1)}{2}+\frac{\ln(1+\coth(x))}{2}+2\ln(\sinh(x))$	26

input
```
int((1+coth(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-coth(x)-2*ln(coth(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(13) = 26$.

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 4.08

$$\int (1+\coth(x))^2\,dx = \frac{2\left(\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1\right)\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right) - 1\right)}{\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1}$$

input
```
integrate((1+coth(x))^2,x, algorithm="fricas")
```

output
```
2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(2*sinh(x)/(cosh(x)
- sinh(x))) - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int (1+\coth(x))^2\,dx = 4x - 2\log(\tanh(x)+1) + 2\log(\tanh(x)) - \frac{1}{\tanh(x)}$$

input
```
integrate((1+coth(x))**2,x)
```

output
```
4*x - 2*log(tanh(x) + 1) + 2*log(tanh(x)) - 1/tanh(x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int (1+\coth(x))^2\,dx = 2x + \frac{2}{e^{(-2x)} - 1} + 2\log(\sinh(x))$$

input
```
integrate((1+coth(x))^2,x, algorithm="maxima")
```

output
```
2*x + 2/(e^(-2*x) - 1) + 2*log(sinh(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int (1+\coth(x))^2\,dx = -\frac{2}{e^{(2x)}-1} + 2\log\left(\left|e^{(2x)}-1\right|\right)$$

input
```
integrate((1+coth(x))^2,x, algorithm="giac")
```

output
```
-2/(e^(2*x) - 1) + 2*log(abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int (1+\coth(x))^2\,dx = 2\ln\left(e^{2x}-1\right) - \frac{2}{e^{2x}-1}$$

input
```
int((coth(x) + 1)^2,x)
```

output
```
2*log(exp(2*x) - 1) - 2/(exp(2*x) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 4.54

$$\int (1+\coth(x))^2\,dx = \frac{2e^{2x}\log(e^x-1)+2e^{2x}\log(e^x+1)-2e^{2x}-2\log(e^x-1)-2\log(e^x+1)}{e^{2x}-1}$$

input
```
int((1+coth(x))^2,x)
```

output

```
(2*(e**(2*x)*log(e**x - 1) + e**(2*x)*log(e**x + 1) - e**(2*x) - log(e**x
- 1) - log(e**x + 1)))/(e**(2*x) - 1)
```

3.65 $\int \frac{1}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 16

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{x}{2} - \frac{1}{2(1+\coth(x))}$$

output
```
1/2*x-1/(2+2*coth(x))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{1}{2}\left(\operatorname{arctanh}(\tanh(x)) + \frac{1}{1+\tanh(x)}\right)$$

input
```
Integrate[(1 + Coth[x])^(-1),x]
```

output
```
(ArcTanh[Tanh[x]] + (1 + Tanh[x])^(-1))/2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\coth(x)+1} dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{1-i\tan\left(\frac{\pi}{2}+ix\right)}dx$$
$$\Big\downarrow \text{3960}$$
$$\frac{\int 1dx}{2}-\frac{1}{2(\coth(x)+1)}$$
$$\Big\downarrow \text{24}$$
$$\frac{x}{2}-\frac{1}{2(\coth(x)+1)}$$

input
```
Int[(1 + Coth[x])^(-1),x]
```

output
```
x/2 - 1/(2*(1 + Coth[x]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{x}{2}+\frac{{\mathrm e}^{-2 x}}{4}$	11
parallelrisch	$\frac{x}{2}+\frac{\cosh\left(2 x\right)}{4}+\frac{1}{4}-\frac{\sinh\left(2 x\right)}{4}$	18
derivativedivides	$-\frac{\ln\left(\coth\left(x\right)-1\right)}{4}-\frac{1}{2\left(1+\coth\left(x\right)\right)}+\frac{\ln\left(1+\coth\left(x\right)\right)}{4}$	24
default	$-\frac{\ln\left(\coth\left(x\right)-1\right)}{4}-\frac{1}{2\left(1+\coth\left(x\right)\right)}+\frac{\ln\left(1+\coth\left(x\right)\right)}{4}$	24

input
```
int(1/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/4*exp(-2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{1}{1+\coth(x)}\,dx=\frac{(2\,x+1)\cosh\left(x\right)+(2\,x-1)\sinh\left(x\right)}{4\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

input
```
integrate(1/(1+coth(x)),x, algorithm="fricas")
```

output
```
1/4*((2*x + 1)*cosh(x) + (2*x - 1)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{x\tanh{(x)}}{2\tanh{(x)}+2} + \frac{x}{2\tanh{(x)}+2} + \frac{1}{2\tanh{(x)}+2}$$

input
```
integrate(1/(1+coth(x)),x)
```

output
```
x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) + 1/(2*tanh(x) + 2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{1}{2}\,x + \frac{1}{4}\,e^{(-2\,x)}$$

input
```
integrate(1/(1+coth(x)),x, algorithm="maxima")
```

output
```
1/2*x + 1/4*e^(-2*x)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{1}{2}\,x + \frac{1}{4}\,e^{(-2\,x)}$$

input
```
integrate(1/(1+coth(x)),x, algorithm="giac")
```

output
```
1/2*x + 1/4*e^(-2*x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{x}{2} - \frac{1}{2\left(\coth\left(x\right)+1\right)}$$

input
```
int(1/(coth(x) + 1),x)
```

output
```
x/2 - 1/(2*(coth(x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{2e^{2x}x+1}{4e^{2x}}$$

input
```
int(1/(1+coth(x)),x)
```

output
```
(2*e**(2*x)*x + 1)/(4*e**(2*x))
```

3.66 $\int \frac{1}{(1+\coth(x))^2}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 26

$$\int \frac{1}{(1+\coth(x))^2}\,dx = \frac{x}{4} - \frac{1}{4(1+\coth(x))^2} - \frac{1}{4(1+\coth(x))}$$

output
```
1/4*x-1/4/(1+coth(x))^2-1/(4+4*coth(x))
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1+\coth(x))^2}\,dx = \frac{1}{4}\text{arctanh}(\tanh(x)) - \frac{1}{4(1+\tanh(x))^2} + \frac{3}{4(1+\tanh(x))}$$

input
```
Integrate[(1 + Coth[x])^(-2),x]
```

output
```
ArcTanh[Tanh[x]]/4 - 1/(4*(1 + Tanh[x])^2) + 3/(4*(1 + Tanh[x]))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 3960, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\coth(x)+1)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{3960}$$

$$\frac{1}{2}\int \frac{1}{\coth(x)+1}dx-\frac{1}{4(\coth(x)+1)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{4(\coth(x)+1)^2}+\frac{1}{2}\int \frac{1}{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3960}$$

$$\frac{1}{2}\left(\frac{\int 1dx}{2}-\frac{1}{2(\coth(x)+1)}\right)-\frac{1}{4(\coth(x)+1)^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}\left(\frac{x}{2}-\frac{1}{2(\coth(x)+1)}\right)-\frac{1}{4(\coth(x)+1)^2}$$

input
```
Int[(1 + Coth[x])^(-2),x]
```

output
```
-1/4*1/(1 + Coth[x])^2 + (x/2 - 1/(2*(1 + Coth[x])))/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{x}{4}+\frac{{\rm e}^{-2x}}{4}-\frac{{\rm e}^{-4x}}{16}$	17
parallelrisch	$\frac{\tanh(x)^{2}x+(2x+3)\tanh(x)+x+2}{4(\tanh(x)+1)^{2}}$	26
derivativedivides	$-\frac{\ln(\coth(x)-1)}{8}-\frac{1}{4(1+\coth(x))^{2}}-\frac{1}{4(1+\coth(x))}+\frac{\ln(1+\coth(x))}{8}$	32
default	$-\frac{\ln(\coth(x)-1)}{8}-\frac{1}{4(1+\coth(x))^{2}}-\frac{1}{4(1+\coth(x))}+\frac{\ln(1+\coth(x))}{8}$	32

input
```
int(1/(1+coth(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x+1/4*exp(-2*x)-1/16*exp(-4*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(20) = 40$.

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.00

$$\int \frac{1}{(1+\coth(x))^2}\,dx = \frac{(4\,x-1)\cosh\left(x\right)^2+2\,(4\,x+1)\cosh\left(x\right)\sinh\left(x\right)+(4\,x-1)\sinh\left(x\right)^2+4}{16\,\left(\cosh\left(x\right)^2+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^2\right)}$$

input
```
integrate(1/(1+coth(x))^2,x, algorithm="fricas")
```

output
```
1/16*((4*x - 1)*cosh(x)^2 + 2*(4*x + 1)*cosh(x)*sinh(x) + (4*x - 1)*sinh(x
)^2 + 4)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(20) = 40$.

Time = 0.50 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.38

$$\int \frac{1}{(1+\coth(x))^2}\,dx = \frac{x\tanh^2\left(x\right)}{4\tanh^2\left(x\right)+8\tanh\left(x\right)+4} + \frac{2x\tanh\left(x\right)}{4\tanh^2\left(x\right)+8\tanh\left(x\right)+4} + \frac{x}{4\tanh^2\left(x\right)+8\tanh\left(x\right)+4} + \frac{3\tanh\left(x\right)}{4\tanh^2\left(x\right)+8\tanh\left(x\right)+4} + \frac{2}{4\tanh^2\left(x\right)+8\tanh\left(x\right)+4}$$

input
```
integrate(1/(1+coth(x))**2,x)
```

output
```
x*tanh(x)**2/(4*tanh(x)**2 + 8*tanh(x) + 4) + 2*x*tanh(x)/(4*tanh(x)**2 +
8*tanh(x) + 4) + x/(4*tanh(x)**2 + 8*tanh(x) + 4) + 3*tanh(x)/(4*tanh(x)**
2 + 8*tanh(x) + 4) + 2/(4*tanh(x)**2 + 8*tanh(x) + 4)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{1}{(1+\coth(x))^2}\,dx = \frac{1}{4}\,x + \frac{1}{4}\,e^{(-2\,x)} - \frac{1}{16}\,e^{(-4\,x)}$$

input
```
integrate(1/(1+coth(x))^2,x, algorithm="maxima")
```

output
```
1/4*x + 1/4*e^(-2*x) - 1/16*e^(-4*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{1}{(1+\coth(x))^2}\,dx = \frac{1}{16}\,\left(4\,e^{(2\,x)} - 1\right)e^{(-4\,x)} + \frac{1}{4}\,x$$

input
```
integrate(1/(1+coth(x))^2,x, algorithm="giac")
```

output
```
1/16*(4*e^(2*x) - 1)*e^(-4*x) + 1/4*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{1}{(1+\coth(x))^2}\,dx = \frac{x}{4} + \frac{\mathrm{e}^{-2\,x}}{4} - \frac{\mathrm{e}^{-4\,x}}{16}$$

input
```
int(1/(coth(x) + 1)^2,x)
```

output
```
x/4 + exp(-2*x)/4 - exp(-4*x)/16
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+\coth(x))^2}\,dx = \frac{4e^{4x}x + 4e^{2x} - 1}{16e^{4x}}$$

input
```
int(1/(1+coth(x))^2,x)
```

output
```
(4*e**(4*x)*x + 4*e**(2*x) - 1)/(16*e**(4*x))
```

3.67 $\int \frac{1}{(1+\coth(x))^3}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 36

$$\int \frac{1}{(1+\coth(x))^3}\,dx = \frac{x}{8} - \frac{1}{6(1+\coth(x))^3} - \frac{1}{8(1+\coth(x))^2} - \frac{1}{8(1+\coth(x))}$$

output
```
1/8*x-1/6/(1+coth(x))^3-1/8/(1+coth(x))^2-1/(8+8*coth(x))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{1}{(1+\coth(x))^3}\,dx = \frac{10+27\tanh(x)+21\tanh^2(x)+3\text{arctanh}(\tanh(x))(1+\tanh(x))^3}{24(1+\tanh(x))^3}$$

input
```
Integrate[(1 + Coth[x])^(-3),x]
```

output
```
(10 + 27*Tanh[x] + 21*Tanh[x]^2 + 3*ArcTanh[Tanh[x]]*(1 + Tanh[x])^3)/(24*
(1 + Tanh[x])^3)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {3042, 3960, 3042, 3960, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\coth(x)+1)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)^3}dx$$

$$\Big\downarrow \text{3960}$$

$$\frac{1}{2}\int \frac{1}{(\coth(x)+1)^2}dx-\frac{1}{6(\coth(x)+1)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{6(\coth(x)+1)^3}+\frac{1}{2}\int \frac{1}{\left(1-i\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3960}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\coth(x)+1}dx-\frac{1}{4(\coth(x)+1)^2}\right)-\frac{1}{6(\coth(x)+1)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{6(\coth(x)+1)^3}+\frac{1}{2}\left(-\frac{1}{4(\coth(x)+1)^2}+\frac{1}{2}\int \frac{1}{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3960}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int 1dx}{2}-\frac{1}{2(\coth(x)+1)}\right)-\frac{1}{4(\coth(x)+1)^2}\right)-\frac{1}{6(\coth(x)+1)^3}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{x}{2}-\frac{1}{2(\coth(x)+1)}\right)-\frac{1}{4(\coth(x)+1)^2}\right)-\frac{1}{6(\coth(x)+1)^3}$$

input
```
Int[(1 + Coth[x])^(-3),x]
```

output
```
-1/6*1/(1 + Coth[x])^3 + (-1/4*1/(1 + Coth[x])^2 + (x/2 - 1/(2*(1 + Coth[x
])))/2)/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{x}{8}+\frac{3\,{\mathrm e}^{-2 x}}{16}-\frac{3\,{\mathrm e}^{-4 x}}{32}+\frac{{\mathrm e}^{-6 x}}{48}$	23
parallelrisch	$\frac{3 \tanh \left(x \right)^{3} x +\left(9 x +21\right) \tanh \left(x \right)^{2}+\left(9 x +27\right) \tanh \left(x \right)+3 x +10}{24 \left(\tanh \left(x \right)+1\right)^{3}}$	39
derivativedivides	$-\frac{\ln \left(\coth \left(x \right)-1\right)}{16}-\frac{1}{6 \left(1+\coth \left(x \right)\right)^{3}}-\frac{1}{8 \left(1+\coth \left(x \right)\right)^{2}}-\frac{1}{8 \left(1+\coth \left(x \right)\right)}+\frac{\ln \left(1+\coth \left(x \right)\right)}{16}$	40
default	$-\frac{\ln \left(\coth \left(x \right)-1\right)}{16}-\frac{1}{6 \left(1+\coth \left(x \right)\right)^{3}}-\frac{1}{8 \left(1+\coth \left(x \right)\right)^{2}}-\frac{1}{8 \left(1+\coth \left(x \right)\right)}+\frac{\ln \left(1+\coth \left(x \right)\right)}{16}$	40

input
```
int(1/(1+coth(x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*x+3/16*exp(-2*x)-3/32*exp(-4*x)+1/48*exp(-6*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(28) = 56$.

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.39

$$\int \frac{1}{(1+\coth(x))^3}\,dx = \frac{2\,(6\,x+1)\cosh\left(x\right)^{3}+6\,(6\,x+1)\cosh\left(x\right)\sinh\left(x\right)^{2}+2\,(6\,x-1)\sinh\left(x\right)^{3}+3\,\left(2\,(6\,x-1)\cosh\left(x\right)^{2}+9\right)}{96\,\left(\cosh\left(x\right)^{3}+3\,\cosh\left(x\right)^{2}\sinh\left(x\right)+3\,\cosh\left(x\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}\right)}$$

input
```
integrate(1/(1+coth(x))^3,x, algorithm="fricas")
```

output
```
1/96*(2*(6*x + 1)*cosh(x)^3 + 6*(6*x + 1)*cosh(x)*sinh(x)^2 + 2*(6*x - 1)*
sinh(x)^3 + 3*(2*(6*x - 1)*cosh(x)^2 + 9)*sinh(x) + 9*cosh(x))/(cosh(x)^3
+ 3*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(31) = 62$.

Time = 0.65 (sec) , antiderivative size = 182, normalized size of antiderivative = 5.06

$$\begin{aligned}\int \frac{1}{(1+\coth(x))^3}\,dx &= \frac{3x\tanh^{3}(x)}{24\tanh^{3}(x)+72\tanh^{2}(x)+72\tanh(x)+24} \\ &+ \frac{9x\tanh^{2}(x)}{24\tanh^{3}(x)+72\tanh^{2}(x)+72\tanh(x)+24} \\ &+ \frac{9x\tanh(x)}{24\tanh^{3}(x)+72\tanh^{2}(x)+72\tanh(x)+24} \\ &+ \frac{3x}{24\tanh^{3}(x)+72\tanh^{2}(x)+72\tanh(x)+24} \\ &+ \frac{21\tanh^{2}(x)}{24\tanh^{3}(x)+72\tanh^{2}(x)+72\tanh(x)+24} \\ &+ \frac{27\tanh(x)}{24\tanh^{3}(x)+72\tanh^{2}(x)+72\tanh(x)+24} \\ &+ \frac{10}{24\tanh^{3}(x)+72\tanh^{2}(x)+72\tanh(x)+24}\end{aligned}$$

input
```
integrate(1/(1+coth(x))**3,x)
```

output
```
3*x*tanh(x)**3/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24) + 9*x*tan
h(x)**2/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24) + 9*x*tanh(x)/(2
4*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24) + 3*x/(24*tanh(x)**3 + 72*
tanh(x)**2 + 72*tanh(x) + 24) + 21*tanh(x)**2/(24*tanh(x)**3 + 72*tanh(x)*
*2 + 72*tanh(x) + 24) + 27*tanh(x)/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tan
h(x) + 24) + 10/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \frac{1}{(1+\coth(x))^3}\,dx = \frac{1}{8}\,x + \frac{3}{16}\,e^{(-2\,x)} - \frac{3}{32}\,e^{(-4\,x)} + \frac{1}{48}\,e^{(-6\,x)}$$

input
```
integrate(1/(1+coth(x))^3,x, algorithm="maxima")
```

output
```
1/8*x + 3/16*e^(-2*x) - 3/32*e^(-4*x) + 1/48*e^(-6*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{1}{(1+\coth(x))^3}\,dx = \frac{1}{96}\,\left(18\,e^{(4\,x)} - 9\,e^{(2\,x)} + 2\right)e^{(-6\,x)} + \frac{1}{8}\,x$$

input
```
integrate(1/(1+coth(x))^3,x, algorithm="giac")
```

output
```
1/96*(18*e^(4*x) - 9*e^(2*x) + 2)*e^(-6*x) + 1/8*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \frac{1}{(1+\coth(x))^3}\,dx = \frac{x}{8} + \frac{3\,\mathrm{e}^{-2\,x}}{16} - \frac{3\,\mathrm{e}^{-4\,x}}{32} + \frac{\mathrm{e}^{-6\,x}}{48}$$

input
```
int(1/(coth(x) + 1)^3,x)
```

output
```
x/8 + (3*exp(-2*x))/16 - (3*exp(-4*x))/32 + exp(-6*x)/48
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{1}{(1+\coth(x))^3}\,dx = \frac{12e^{6x}x + 18e^{4x} - 9e^{2x} + 2}{96e^{6x}}$$

input
```
int(1/(1+coth(x))^3,x)
```

output
```
(12*e**(6*x)*x + 18*e**(4*x) - 9*e**(2*x) + 2)/(96*e**(6*x))
```

3.68 $\int \frac{1}{(1+\coth(x))^4} \, dx$

Optimal result

Integrand size = 6, antiderivative size = 46

$$\int \frac{1}{(1+\coth(x))^4} \, dx = \frac{x}{16} - \frac{1}{8(1+\coth(x))^4} - \frac{1}{12(1+\coth(x))^3} - \frac{1}{16(1+\coth(x))^2} - \frac{1}{16(1+\coth(x))}$$

output
```
1/16*x-1/8/(1+coth(x))^4-1/12/(1+coth(x))^3-1/16/(1+coth(x))^2-1/(16+16*co
th(x))
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1+\coth(x))^4} \, dx = \frac{1}{48}\left(3\text{arctanh}(\tanh(x)) + \frac{16+61\tanh(x)+84\tanh^2(x)+45\tanh^3(x)}{(1+\tanh(x))^4}\right)$$

input
```
Integrate[(1 + Coth[x])^(-4),x]
```

output
```
(3*ArcTanh[Tanh[x]] + (16 + 61*Tanh[x] + 84*Tanh[x]^2 + 45*Tanh[x]^3)/(1 +
 Tanh[x])^4)/48
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 3960, 3042, 3960, 3042, 3960, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\coth (x)+1)^4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\left(1-i \tan \left(\frac{\pi }{2}+i x\right)\right)^4}dx$$
$$\Big\downarrow \text{3960}$$
$$\frac{1}{2} \int \frac{1}{(\coth (x)+1)^3}dx-\frac{1}{8 (\coth (x)+1)^4}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{1}{8 (\coth (x)+1)^4}+\frac{1}{2} \int \frac{1}{\left(1-i \tan \left(i x+\frac{\pi }{2}\right)\right)^3}dx$$
$$\Big\downarrow \text{3960}$$
$$\frac{1}{2} \left(\frac{1}{2} \int \frac{1}{(\coth (x)+1)^2}dx-\frac{1}{6 (\coth (x)+1)^3}\right)-\frac{1}{8 (\coth (x)+1)^4}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{1}{8 (\coth (x)+1)^4}+\frac{1}{2} \left(-\frac{1}{6 (\coth (x)+1)^3}+\frac{1}{2} \int \frac{1}{\left(1-i \tan \left(i x+\frac{\pi }{2}\right)\right)^2}dx\right)$$
$$\Big\downarrow \text{3960}$$
$$\frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \int \frac{1}{\coth (x)+1}dx-\frac{1}{4 (\coth (x)+1)^2}\right)-\frac{1}{6 (\coth (x)+1)^3}\right)-\frac{1}{8 (\coth (x)+1)^4}$$
$$\Big\downarrow \text{3042}$$

$$-\frac{1}{8(\coth(x)+1)^4}+\frac{1}{2}\left(-\frac{1}{6(\coth(x)+1)^3}+\frac{1}{2}\left(-\frac{1}{4(\coth(x)+1)^2}+\frac{1}{2}\int\frac{1}{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx\right)\right)$$

$$\Big\downarrow \text{3960}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int 1dx}{2}-\frac{1}{2(\coth(x)+1)}\right)-\frac{1}{4(\coth(x)+1)^2}\right)-\frac{1}{6(\coth(x)+1)^3}\right)-\frac{1}{8(\coth(x)+1)^4}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{x}{2}-\frac{1}{2(\coth(x)+1)}\right)-\frac{1}{4(\coth(x)+1)^2}\right)-\frac{1}{6(\coth(x)+1)^3}\right)-\frac{1}{8(\coth(x)+1)^4}$$

input
```
Int[(1 + Coth[x])^(-4),x]
```

output
```
-1/8*1/(1 + Coth[x])^4 + (-1/6*1/(1 + Coth[x])^3 + (-1/4*1/(1 + Coth[x])^2
 + (x/2 - 1/(2*(1 + Coth[x])))/2)/2)/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

method	result
risch	$\frac{x}{16}+\frac{\mathrm{e}^{-2x}}{8}-\frac{3\,\mathrm{e}^{-4x}}{32}+\frac{\mathrm{e}^{-6x}}{24}-\frac{\mathrm{e}^{-8x}}{128}$
derivativedivides	$-\frac{\ln(\coth(x)-1)}{32}-\frac{1}{8(1+\coth(x))^4}-\frac{1}{12(1+\coth(x))^3}-\frac{1}{16(1+\coth(x))^2}-\frac{1}{16(1+\coth(x))}+\frac{\ln(1+\coth(x))}{32}$
default	$-\frac{\ln(\coth(x)-1)}{32}-\frac{1}{8(1+\coth(x))^4}-\frac{1}{12(1+\coth(x))^3}-\frac{1}{16(1+\coth(x))^2}-\frac{1}{16(1+\coth(x))}+\frac{\ln(1+\coth(x))}{32}$
parallelrisch	$\frac{3\tanh(x)^4x+(12x+45)\tanh(x)^3+(18x+84)\tanh(x)^2+(12x+61)\tanh(x)+3x+16}{48(\tanh(x)+1)^4}$

input
```
int(1/(1+coth(x))^4,x,method=_RETURNVERBOSE)
```

output
```
1/16*x+1/8*exp(-2*x)-3/32*exp(-4*x)+1/24*exp(-6*x)-1/128*exp(-8*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(36) = 72.

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.63

$$\int \frac{1}{(1+\coth(x))^4}\,dx = \frac{3\,(8\,x-1)\cosh(x)^4+12\,(8\,x+1)\cosh(x)\sinh(x)^3+3\,(8\,x-1)\sinh(x)^4+2\,(9\,(8\,x-1)\cosh(x)^2+3}{384\,(\cosh(x)^4+4\,\cosh(x)^3\sinh(x)+6\,\cosh(x)^2\sinh(x)}$$

input
```
integrate(1/(1+coth(x))^4,x, algorithm="fricas")
```

output
```
1/384*(3*(8*x - 1)*cosh(x)^4 + 12*(8*x + 1)*cosh(x)*sinh(x)^3 + 3*(8*x - 1
)*sinh(x)^4 + 2*(9*(8*x - 1)*cosh(x)^2 + 32)*sinh(x)^2 + 64*cosh(x)^2 + 4*
(3*(8*x + 1)*cosh(x)^3 + 16*cosh(x))*sinh(x) - 36)/(cosh(x)^4 + 4*cosh(x)^
3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(41) = 82$.

Time = 0.82 (sec) , antiderivative size = 299, normalized size of antiderivative = 6.50

$$
\begin{aligned}
\int \frac{1}{(1+\coth(x))^4}\,dx
&= \frac{3x\tanh^4(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48} \\
&\quad+ \frac{12x\tanh^3(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48} \\
&\quad+ \frac{18x\tanh^2(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48} \\
&\quad+ \frac{12x\tanh(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48} \\
&\quad+ \frac{3x}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48} \\
&\quad+ \frac{45\tanh^3(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48} \\
&\quad+ \frac{84\tanh^2(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48} \\
&\quad+ \frac{61\tanh(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48} \\
&\quad+ \frac{16}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48}
\end{aligned}
$$

input
```
integrate(1/(1+coth(x))**4,x)
```

output
```
3*x*tanh(x)**4/(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh
(x) + 48) + 12*x*tanh(x)**3/(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)*
*2 + 192*tanh(x) + 48) + 18*x*tanh(x)**2/(48*tanh(x)**4 + 192*tanh(x)**3 +
 288*tanh(x)**2 + 192*tanh(x) + 48) + 12*x*tanh(x)/(48*tanh(x)**4 + 192*ta
nh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 3*x/(48*tanh(x)**4 + 192*t
anh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 45*tanh(x)**3/(48*tanh(x)
**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 84*tanh(x)**2/
(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 61*
tanh(x)/(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 4
8) + 16/(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 4
8)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int \frac{1}{(1+\coth(x))^4}\,dx = \frac{1}{16}\,x + \frac{1}{8}\,e^{(-2\,x)} - \frac{3}{32}\,e^{(-4\,x)} + \frac{1}{24}\,e^{(-6\,x)} - \frac{1}{128}\,e^{(-8\,x)}$$

input
```
integrate(1/(1+coth(x))^4,x, algorithm="maxima")
```

output
```
1/16*x + 1/8*e^(-2*x) - 3/32*e^(-4*x) + 1/24*e^(-6*x) - 1/128*e^(-8*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

$$\int \frac{1}{(1+\coth(x))^4}\,dx = \frac{1}{384}\left(48\,e^{(6\,x)} - 36\,e^{(4\,x)} + 16\,e^{(2\,x)} - 3\right)e^{(-8\,x)} + \frac{1}{16}\,x$$

input
```
integrate(1/(1+coth(x))^4,x, algorithm="giac")
```

output
```
1/384*(48*e^(6*x) - 36*e^(4*x) + 16*e^(2*x) - 3)*e^(-8*x) + 1/16*x
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int \frac{1}{(1+\coth(x))^4}\,dx = \frac{x}{16} + \frac{\mathrm{e}^{-2\,x}}{8} - \frac{3\,\mathrm{e}^{-4\,x}}{32} + \frac{\mathrm{e}^{-6\,x}}{24} - \frac{\mathrm{e}^{-8\,x}}{128}$$

input
```
int(1/(coth(x) + 1)^4,x)
```

output
```
x/16 + exp(-2*x)/8 - (3*exp(-4*x))/32 + exp(-6*x)/24 - exp(-8*x)/128
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{(1+\coth(x))^4}\,dx = \frac{24e^{8x}x + 48e^{6x} - 36e^{4x} + 16e^{2x} - 3}{384e^{8x}}$$

input
```
int(1/(1+coth(x))^4,x)
```

output
```
(24*e**(8*x)*x + 48*e**(6*x) - 36*e**(4*x) + 16*e**(2*x) - 3)/(384*e**(8*x
))
```

3.69 $\int \frac{1}{(1+\coth(x))^5}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 56

$$\int \frac{1}{(1+\coth(x))^5}\,dx = \frac{x}{32} - \frac{1}{10(1+\coth(x))^5} - \frac{1}{16(1+\coth(x))^4} - \frac{1}{24(1+\coth(x))^3} - \frac{1}{32(1+\coth(x))^2} - \frac{1}{32(1+\coth(x))}$$

output
```
1/32*x-1/10/(1+coth(x))^5-1/16/(1+coth(x))^4-1/24/(1+coth(x))^3-1/32/(1+co
th(x))^2-1/(32+32*coth(x))
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.73

$$\int \frac{1}{(1+\coth(x))^5}\,dx = \frac{1}{480}\left(15\text{arctanh}(\tanh(x)) + \frac{128+625\tanh(x)+1205\tanh^2(x)+1125\tanh^3(x)+465\tanh^4(x)}{(1+\tanh(x))^5}\right)$$

input
```
Integrate[(1 + Coth[x])^(-5),x]
```

output
```
(15*ArcTanh[Tanh[x]] + (128 + 625*Tanh[x] + 1205*Tanh[x]^2 + 1125*Tanh[x]^
3 + 465*Tanh[x]^4)/(1 + Tanh[x])^5)/480
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.833$, Rules used = {3042, 3960, 3042, 3960, 3042, 3960, 3042, 3960, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\coth(x)+1)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)^5}dx$$

$$\Big\downarrow \text{3960}$$

$$\frac{1}{2}\int \frac{1}{(\coth(x)+1)^4}dx-\frac{1}{10(\coth(x)+1)^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{10(\coth(x)+1)^5}+\frac{1}{2}\int \frac{1}{\left(1-i\tan\left(ix+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{3960}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{(\coth(x)+1)^3}dx-\frac{1}{8(\coth(x)+1)^4}\right)-\frac{1}{10(\coth(x)+1)^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{10(\coth(x)+1)^5}+\frac{1}{2}\left(-\frac{1}{8(\coth(x)+1)^4}+\frac{1}{2}\int \frac{1}{\left(1-i\tan\left(ix+\frac{\pi}{2}\right)\right)^3}dx\right)$$

$$\Big\downarrow \text{3960}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{(\coth(x)+1)^2}dx-\frac{1}{6(\coth(x)+1)^3}\right)-\frac{1}{8(\coth(x)+1)^4}\right)-\frac{1}{10(\coth(x)+1)^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{10(\coth(x)+1)^5}+\frac{1}{2}\left(-\frac{1}{8(\coth(x)+1)^4}+\frac{1}{2}\left(-\frac{1}{6(\coth(x)+1)^3}+\frac{1}{2}\int\frac{1}{\left(1-i\tan\left(ix+\frac{\pi}{2}\right)\right)^2}dx\right)\right)$$

$$\Big\downarrow \text{3960}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\coth(x)+1}dx-\frac{1}{4(\coth(x)+1)^2}\right)-\frac{1}{6(\coth(x)+1)^3}\right)-\frac{1}{8(\coth(x)+1)^4}\right)-\frac{1}{10(\coth(x)+1)^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{10(\coth(x)+1)^5}+\frac{1}{2}\left(-\frac{1}{8(\coth(x)+1)^4}+\frac{1}{2}\left(-\frac{1}{6(\coth(x)+1)^3}+\frac{1}{2}\left(-\frac{1}{4(\coth(x)+1)^2}+\frac{1}{2}\int\frac{1}{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx\right)\right)\right)$$

$$\Big\downarrow \text{3960}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int 1dx}{2}-\frac{1}{2(\coth(x)+1)}\right)-\frac{1}{4(\coth(x)+1)^2}\right)-\frac{1}{6(\coth(x)+1)^3}\right)-\frac{1}{8(\coth(x)+1)^4}\right)-\frac{1}{10(\coth(x)+1)^5}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{x}{2}-\frac{1}{2(\coth(x)+1)}\right)-\frac{1}{4(\coth(x)+1)^2}\right)-\frac{1}{6(\coth(x)+1)^3}\right)-\frac{1}{8(\coth(x)+1)^4}\right)-\frac{1}{10(\coth(x)+1)^5}$$

input
```
Int[(1 + Coth[x])^(-5),x]
```

output
```
-1/10*1/(1 + Coth[x])^5 + (-1/8*1/(1 + Coth[x])^4 + (-1/6*1/(1 + Coth[x])^
3 + (-1/4*1/(1 + Coth[x])^2 + (x/2 - 1/(2*(1 + Coth[x])))/2)/2)/2)/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.62

method	result
risch	$\frac{x}{32}+\frac{5\,{\mathrm e}^{-2x}}{64}-\frac{5\,{\mathrm e}^{-4x}}{64}+\frac{5\,{\mathrm e}^{-6x}}{96}-\frac{5\,{\mathrm e}^{-8x}}{256}+\frac{{\mathrm e}^{-10x}}{320}$
derivativedivides	$-\frac{\ln(\coth(x)-1)}{64}-\frac{1}{10(1+\coth(x))^{5}}-\frac{1}{16(1+\coth(x))^{4}}-\frac{1}{24(1+\coth(x))^{3}}-\frac{1}{32(1+\coth(x))^{2}}-\frac{1}{32(1+\coth(x}$
default	$-\frac{\ln(\coth(x)-1)}{64}-\frac{1}{10(1+\coth(x))^{5}}-\frac{1}{16(1+\coth(x))^{4}}-\frac{1}{24(1+\coth(x))^{3}}-\frac{1}{32(1+\coth(x))^{2}}-\frac{1}{32(1+\coth(x}$
parallelrisch	$\frac{15\tanh(x)^{5}x+(75x+465)\tanh(x)^{4}+(150x+1125)\tanh(x)^{3}+(150x+1205)\tanh(x)^{2}+(75x+625)\tanh(x)+15x+128}{480(\tanh(x)+1)^{5}}$

input
```
int(1/(1+coth(x))^5,x,method=_RETURNVERBOSE)
```

output
```
1/32*x+5/64*exp(-2*x)-5/64*exp(-4*x)+5/96*exp(-6*x)-5/256*exp(-8*x)+1/320*
exp(-10*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(44) = 88$.

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.84

$$\int \frac{1}{(1+\coth(x))^5}\,dx = \frac{12\,(10\,x+1)\cosh(x)^5 + 60\,(10\,x+1)\cosh(x)\sinh(x)^4 + 12\,(10\,x-1)\sinh(x)^5 + 15\,(8\,(10\,x-1)\cosh}{3840\,(\cosh(x)^5 + 5\,\cosh(x)^4\,\mathrm{s}}$$

input
```
integrate(1/(1+coth(x))^5,x, algorithm="fricas")
```

output
```
1/3840*(12*(10*x + 1)*cosh(x)^5 + 60*(10*x + 1)*cosh(x)*sinh(x)^4 + 12*(10
*x - 1)*sinh(x)^5 + 15*(8*(10*x - 1)*cosh(x)^2 + 25)*sinh(x)^3 + 225*cosh(
x)^3 + 15*(8*(10*x + 1)*cosh(x)^3 + 45*cosh(x))*sinh(x)^2 + 5*(12*(10*x -
1)*cosh(x)^4 + 225*cosh(x)^2 - 100)*sinh(x) - 100*cosh(x))/(cosh(x)^5 + 5*
cosh(x)^4*sinh(x) + 10*cosh(x)^3*sinh(x)^2 + 10*cosh(x)^2*sinh(x)^3 + 5*co
sh(x)*sinh(x)^4 + sinh(x)^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(51) = 102$.

Time = 1.03 (sec) , antiderivative size = 444, normalized size of antiderivative = 7.93

$$\int \frac{1}{(1+\coth(x))^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1+coth(x))**5,x)
```

output
```
15*x*tanh(x)**5/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800
*tanh(x)**2 + 2400*tanh(x) + 480) + 75*x*tanh(x)**4/(480*tanh(x)**5 + 2400
*tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480) + 15
0*x*tanh(x)**3/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*
tanh(x)**2 + 2400*tanh(x) + 480) + 150*x*tanh(x)**2/(480*tanh(x)**5 + 2400
*tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480) + 75
*x*tanh(x)/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh
(x)**2 + 2400*tanh(x) + 480) + 15*x/(480*tanh(x)**5 + 2400*tanh(x)**4 + 48
00*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480) + 465*tanh(x)**4/(48
0*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2 + 2400*
tanh(x) + 480) + 1125*tanh(x)**3/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*
tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480) + 1205*tanh(x)**2/(480*
tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2 + 2400*ta
nh(x) + 480) + 625*tanh(x)/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x
)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480) + 128/(480*tanh(x)**5 + 2400*
tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.61

$$\int \frac{1}{(1+\coth(x))^5}\,dx = \frac{1}{32}\,x + \frac{5}{64}\,e^{(-2\,x)} - \frac{5}{64}\,e^{(-4\,x)} + \frac{5}{96}\,e^{(-6\,x)} - \frac{5}{256}\,e^{(-8\,x)} + \frac{1}{320}\,e^{(-10\,x)}$$

input
```
integrate(1/(1+coth(x))^5,x, algorithm="maxima")
```

output
```
1/32*x + 5/64*e^(-2*x) - 5/64*e^(-4*x) + 5/96*e^(-6*x) - 5/256*e^(-8*x) +
1/320*e^(-10*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.64

$$\int \frac{1}{(1+\coth(x))^5}\,dx = \frac{1}{3840}\left(300\,e^{(8\,x)} - 300\,e^{(6\,x)} + 200\,e^{(4\,x)} - 75\,e^{(2\,x)} + 12\right)e^{(-10\,x)} + \frac{1}{32}\,x$$

input
```
integrate(1/(1+coth(x))^5,x, algorithm="giac")
```

output
```
1/3840*(300*e^(8*x) - 300*e^(6*x) + 200*e^(4*x) - 75*e^(2*x) + 12)*e^(-10*
x) + 1/32*x
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.61

$$\int \frac{1}{(1+\coth(x))^5}\,dx = \frac{x}{32} + \frac{5\,\mathrm{e}^{-2\,x}}{64} - \frac{5\,\mathrm{e}^{-4\,x}}{64} + \frac{5\,\mathrm{e}^{-6\,x}}{96} - \frac{5\,\mathrm{e}^{-8\,x}}{256} + \frac{\mathrm{e}^{-10\,x}}{320}$$

input
```
int(1/(coth(x) + 1)^5,x)
```

output
```
x/32 + (5*exp(-2*x))/64 - (5*exp(-4*x))/64 + (5*exp(-6*x))/96 - (5*exp(-8*
x))/256 + exp(-10*x)/320
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

$$\int \frac{1}{(1+\coth(x))^5}\,dx = \frac{120e^{10x}x + 300e^{8x} - 300e^{6x} + 200e^{4x} - 75e^{2x} + 12}{3840e^{10x}}$$

input
```
int(1/(1+coth(x))^5,x)
```

output
```
(120*e**(10*x)*x + 300*e**(8*x) - 300*e**(6*x) + 200*e**(4*x) - 75*e**(2*x
) + 12)/(3840*e**(10*x))
```

3.70 $\int (1+\coth(x))^{7/2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 57

$$\int (1+\coth(x))^{7/2}\,dx = 8\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 8\sqrt{1+\coth(x)} - \frac{4}{3}(1+\coth(x))^{3/2} - \frac{2}{5}(1+\coth(x))^{5/2}$$

output
```
8*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-8*(1+coth(x))^(1/2)-4/3*(
1+coth(x))^(3/2)-2/5*(1+coth(x))^(5/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int (1+\coth(x))^{7/2}\,dx = 8\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - \frac{2}{15}\sqrt{1+\coth(x)}\left(73+16\coth(x)+3\coth^2(x)\right)$$

input
```
Integrate[(1 + Coth[x])^(7/2),x]
```

output
```
8*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - (2*Sqrt[1 + Coth[x]]*(73 +
16*Coth[x] + 3*Coth[x]^2))/15
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {3042, 3959, 3042, 3959, 3042, 3959, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\coth(x)+1)^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)^{7/2}dx$$

$$\Big\downarrow \text{3959}$$

$$2\int (\coth(x)+1)^{5/2}dx-\frac{2}{5}(\coth(x)+1)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{5}(\coth(x)+1)^{5/2}+2\int \left(1-i\tan\left(ix+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{3959}$$

$$2\left(2\int (\coth(x)+1)^{3/2}dx-\frac{2}{3}(\coth(x)+1)^{3/2}\right)-\frac{2}{5}(\coth(x)+1)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{5}(\coth(x)+1)^{5/2}+2\left(-\frac{2}{3}(\coth(x)+1)^{3/2}+2\int \left(1-i\tan\left(ix+\frac{\pi}{2}\right)\right)^{3/2}dx\right)$$

$$\Big\downarrow \text{3959}$$

$$2\left(2\left(2\int \sqrt{\coth(x)+1}dx-2\sqrt{\coth(x)+1}\right)-\frac{2}{3}(\coth(x)+1)^{3/2}\right)-\frac{2}{5}(\coth(x)+1)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{5}(\coth(x)+1)^{5/2}+2\left(-\frac{2}{3}(\coth(x)+1)^{3/2}+2\left(-2\sqrt{\coth(x)+1}+2\int\sqrt{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx\right)\right)$$

$$\Big\downarrow \text{3961}$$

$$2\left(2\left(4\int\frac{1}{1-\coth(x)}d\sqrt{\coth(x)+1}-2\sqrt{\coth(x)+1}\right)-\frac{2}{3}(\coth(x)+1)^{3/2}\right)-\frac{2}{5}(\coth(x)+1)^{5/2}$$

$$\Big\downarrow \text{219}$$

$$2\left(2\left(2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)-2\sqrt{\coth(x)+1}\right)-\frac{2}{3}(\coth(x)+1)^{3/2}\right)-\frac{2}{5}(\coth(x)+1)^{5/2}$$

input
```
Int[(1 + Coth[x])^(7/2),x]
```

output
```
(-2*(1 + Coth[x])^(5/2))/5 + 2*((-2*(1 + Coth[x])^(3/2))/3 + 2*(2*Sqrt[2]*
ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]]))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3959
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[2*a   Int[(a + b*Tan[c + d*
x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n
, 1]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$8\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)-8\sqrt{1+\coth(x)}-\frac{4(1+\coth(x))^{\frac{3}{2}}}{3}-\frac{2(1+\coth(x))^{\frac{5}{2}}}{5}$	43
default	$8\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)-8\sqrt{1+\coth(x)}-\frac{4(1+\coth(x))^{\frac{3}{2}}}{3}-\frac{2(1+\coth(x))^{\frac{5}{2}}}{5}$	43

input
```
int((1+coth(x))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
8*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-8*(1+coth(x))^(1/2)-4/3*(
1+coth(x))^(3/2)-2/5*(1+coth(x))^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(42) = 84$.

Time = 0.09 (sec) , antiderivative size = 338, normalized size of antiderivative = 5.93

$$\int (1+\coth(x))^{7/2}\,dx = \frac{4\left(15\left(\sqrt{2}\cosh(x)^4+4\sqrt{2}\cosh(x)\sinh(x)^3+\sqrt{2}\sinh(x)^4+2\left(3\sqrt{2}\cosh(x)^2-\sqrt{2}\right)\right.\right.}{}$$

input
```
integrate((1+coth(x))^(7/2),x, algorithm="fricas")
```

output
```
4/15*(15*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x
)^4 + 2*(3*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^2 - 2*sqrt(2)*cosh(x)^2 +
4*(sqrt(2)*cosh(x)^3 - sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(2*cosh(x)^2
 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + sqrt(2)*(sqrt(2)*cosh(x)^3 + 3*sqrt(2
)*cosh(x)*sinh(x)^2 + sqrt(2)*sinh(x)^3 + (3*sqrt(2)*cosh(x)^2 - sqrt(2))*
sinh(x) - sqrt(2)*cosh(x))/sqrt(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
- 1) - 1) - 2*sqrt(2)*(23*cosh(x)^5 + 115*cosh(x)*sinh(x)^4 + 23*sinh(x)^5
 + 5*(46*cosh(x)^2 - 7)*sinh(x)^3 - 35*cosh(x)^3 + 5*(46*cosh(x)^3 - 21*co
sh(x))*sinh(x)^2 + 5*(23*cosh(x)^4 - 21*cosh(x)^2 + 3)*sinh(x) + 15*cosh(x
))/sqrt(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1))/(cosh(x)^4 + 4*cos
h(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 +
 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int (1 + \coth(x))^{7/2}\, dx = \text{Timed out}$$

input
```
integrate((1+coth(x))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (1 + \coth(x))^{7/2}\, dx = \int (\coth(x) + 1)^{\frac{7}{2}}\, dx$$

input
```
integrate((1+coth(x))^(7/2),x, algorithm="maxima")
```

output
```
integrate((coth(x) + 1)^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(42) = 84$.

Time = 0.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.81

$$\int (1+\coth(x))^{7/2}\,dx = -\frac{4}{15}\sqrt{2}\left(\frac{2\left(45\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^4+135\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^3+170\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)\right.}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}+1\right)^5}\right.$$
$$-1)$$

input
```
integrate((1+coth(x))^(7/2),x, algorithm="giac")
```

output
```
-4/15*sqrt(2)*(2*(45*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^4 + 135*(sqrt(e^(
4*x) - e^(2*x)) - e^(2*x))^3 + 170*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2 +
 100*sqrt(e^(4*x) - e^(2*x)) - 100*e^(2*x) + 23)/(sqrt(e^(4*x) - e^(2*x))
- e^(2*x) + 1)^5 + 15*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1)))
*sgn(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int (1+\coth(x))^{7/2}\,dx = -8\sqrt{\coth(x)+1} - \frac{4(\coth(x)+1)^{3/2}}{3} - \frac{2(\coth(x)+1)^{5/2}}{5} - \sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{\coth(x)+1}\,1\mathrm{i}}{2}\right)8\mathrm{i}$$

input
```
int((coth(x) + 1)^(7/2),x)
```

output
```
- 2^(1/2)*atan((2^(1/2)*(coth(x) + 1)^(1/2)*1i)/2)*8i - 8*(coth(x) + 1)^(1
/2) - (4*(coth(x) + 1)^(3/2))/3 - (2*(coth(x) + 1)^(5/2))/5
```

Reduce [F]

$$\int (1+\coth(x))^{7/2}\,dx = \int \sqrt{\coth(x)+1}dx + 3\left(\int \sqrt{\coth(x)+1}\,\coth(x)\,dx\right) + \int \sqrt{\coth(x)+1}\,\coth(x)^3\,dx + 3\left(\int \sqrt{\coth(x)+1}\,\coth(x)^2\,dx\right)$$

input
```
int((1+coth(x))^(7/2),x)
```

output
```
int(sqrt(coth(x) + 1),x) + 3*int(sqrt(coth(x) + 1)*coth(x),x) + int(sqrt(c
oth(x) + 1)*coth(x)**3,x) + 3*int(sqrt(coth(x) + 1)*coth(x)**2,x)
```

3.71 $\int (1+\coth(x))^{5/2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 45

$$\int (1+\coth(x))^{5/2}\,dx = 4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 4\sqrt{1+\coth(x)} - \frac{2}{3}(1+\coth(x))^{3/2}$$

output
```
4*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-4*(1+coth(x))^(1/2)-2/3*(
1+coth(x))^(3/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int (1+\coth(x))^{5/2}\,dx = 4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - \frac{2}{3}\sqrt{1+\coth(x)}(7+\coth(x))$$

input
```
Integrate[(1 + Coth[x])^(5/2),x]
```

output
```
4*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - (2*Sqrt[1 + Coth[x]]*(7 + C
oth[x]))/3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {3042, 3959, 3042, 3959, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\coth(x)+1)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{3959}$$

$$2\int (\coth(x)+1)^{3/2}dx-\frac{2}{3}(\coth(x)+1)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{3}(\coth(x)+1)^{3/2}+2\int \left(1-i\tan\left(ix+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3959}$$

$$2\left(2\int \sqrt{\coth(x)+1}dx-2\sqrt{\coth(x)+1}\right)-\frac{2}{3}(\coth(x)+1)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{3}(\coth(x)+1)^{3/2}+2\left(-2\sqrt{\coth(x)+1}+2\int \sqrt{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3961}$$

$$2\left(4\int \frac{1}{1-\coth(x)}d\sqrt{\coth(x)+1}-2\sqrt{\coth(x)+1}\right)-\frac{2}{3}(\coth(x)+1)^{3/2}$$

$$\Big\downarrow \text{219}$$

$$2\left(2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)-2\sqrt{\coth(x)+1}\right)-\frac{2}{3}(\coth(x)+1)^{3/2}$$

input
```
Int[(1 + Coth[x])^(5/2),x]
```

output
```
(-2*(1 + Coth[x])^(3/2))/3 + 2*(2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2
]] - 2*Sqrt[1 + Coth[x]])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3959
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[2*a   Int[(a + b*Tan[c + d*
x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n
, 1]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$4\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)-4\sqrt{1+\coth\left(x\right)}-\frac{2(1+\coth(x))^{\frac{3}{2}}}{3}$	35
default	$4\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)-4\sqrt{1+\coth\left(x\right)}-\frac{2(1+\coth(x))^{\frac{3}{2}}}{3}$	35

input
```
int((1+coth(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
4*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-4*(1+coth(x))^(1/2)-2/3*(
1+coth(x))^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(34) = 68$.

Time = 0.09 (sec) , antiderivative size = 210, normalized size of antiderivative = 4.67

$$\int (1+\coth(x))^{5/2}\,dx = \frac{2\left(3\left(\sqrt{2}\cosh(x)^2 + 2\sqrt{2}\cosh(x)\sinh(x) + \sqrt{2}\sinh(x)^2 - \sqrt{2}\right)\log\left(2\cosh(x)^2 + 4\,\mathrm{c}\right.}{}$$

input
```
integrate((1+coth(x))^(5/2),x, algorithm="fricas")
```

output
```
2/3*(3*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2
- sqrt(2))*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + sqrt(2)*(sq
rt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)*sinh(x)^3 + (3*sqr
t(2)*cosh(x)^2 - sqrt(2))*sinh(x) - sqrt(2)*cosh(x))/sqrt(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2 - 1) - 1) - 2*sqrt(2)*(4*cosh(x)^3 + 12*cosh(x)*
sinh(x)^2 + 4*sinh(x)^3 + 3*(4*cosh(x)^2 - 1)*sinh(x) - 3*cosh(x))/sqrt(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1))/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2 - 1)
```

Sympy [F]

$$\int (1+\coth(x))^{5/2}\,dx = \int (\coth(x)+1)^{\frac{5}{2}}\,dx$$

input
```
integrate((1+coth(x))**(5/2),x)
```

output
```
Integral((coth(x) + 1)**(5/2), x)
```

Maxima [F]

$$\int (1+\coth(x))^{5/2}\,dx = \int (\coth(x)+1)^{\frac{5}{2}}\,dx$$

input
```
integrate((1+coth(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((coth(x) + 1)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(34) = 68$.

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.49

$$\int (1+\coth(x))^{5/2}\,dx =$$

$$-\frac{2}{3}\sqrt{2}\left(\frac{2\left(6\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^2+9\sqrt{e^{(4x)}-e^{(2x)}}-9e^{(2x)}+4\right)}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}+1\right)^3}+3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e\right.\right.\right.$$

$$-1)$$

input
```
integrate((1+coth(x))^(5/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(2)*(2*(6*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2 + 9*sqrt(e^(4*x)
- e^(2*x)) - 9*e^(2*x) + 4)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1)^3 + 3*
log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1)))*sgn(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int (1+\coth(x))^{5/2}\,dx = \sqrt{8}\,\ln\left(-2\sqrt{8}\,\sqrt{\coth(x)+1}-8\right) - \frac{2(\coth(x)+1)^{3/2}}{3} - 2\sqrt{2}\,\ln\left(4\sqrt{2}\,\sqrt{\coth(x)+1}-8\right) - 4\sqrt{\coth(x)+1}$$

input
```
int((coth(x) + 1)^(5/2),x)
```

output
```
8^(1/2)*log(- 2*8^(1/2)*(coth(x) + 1)^(1/2) - 8) - (2*(coth(x) + 1)^(3/2))
/3 - 2*2^(1/2)*log(4*2^(1/2)*(coth(x) + 1)^(1/2) - 8) - 4*(coth(x) + 1)^(1
/2)
```

Reduce [F]

$$\int (1+\coth(x))^{5/2}\,dx = \int \sqrt{\coth(x)+1}dx + 2\left(\int \sqrt{\coth(x)+1}\,\coth(x)\,dx\right) + \int \sqrt{\coth(x)+1}\,\coth(x)^2\,dx$$

input
```
int((1+coth(x))^(5/2),x)
```

output
```
int(sqrt(coth(x) + 1),x) + 2*int(sqrt(coth(x) + 1)*coth(x),x) + int(sqrt(c
oth(x) + 1)*coth(x)**2,x)
```

3.72 $\int (1+\coth(x))^{3/2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 33

$$\int (1+\coth(x))^{3/2}\,dx = 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\coth(x)}$$

output
```
2*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-2*(1+coth(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int (1+\coth(x))^{3/2}\,dx = 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\coth(x)}$$

input
```
Integrate[(1 + Coth[x])^(3/2),x]
```

output
```
2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 3959, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\coth(x)+1)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3959}$$

$$2\int \sqrt{\coth(x)+1}dx-2\sqrt{\coth(x)+1}$$

$$\Big\downarrow \text{3042}$$

$$-2\sqrt{\coth(x)+1}+2\int \sqrt{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3961}$$

$$4\int \frac{1}{1-\coth(x)}d\sqrt{\coth(x)+1}-2\sqrt{\coth(x)+1}$$

$$\Big\downarrow \text{219}$$

$$2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)-2\sqrt{\coth(x)+1}$$

input
```
Int[(1 + Coth[x])^(3/2),x]
```

output
```
2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3959
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[2*a   Int[(a + b*Tan[c + d*
x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n
, 1]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$2\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)-2\sqrt{1+\coth\left(x\right)}$	27
default	$2\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)-2\sqrt{1+\coth\left(x\right)}$	27

input
```
int((1+coth(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-2*(1+coth(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(26) = 52$.

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.91

$$\int (1+\coth(x))^{3/2}\,dx = \sqrt{2}\log\left(2\cosh(x)^2 + 4\cosh(x)\sinh(x) + 2\sinh(x)^2 + \frac{\sqrt{2}\left(\sqrt{2}\cosh(x)^3 + 3\sqrt{2}\cosh(x)\sinh(x)^2 + \sqrt{2}\sinh(x)^3 + \left(3\sqrt{2}\cosh(x)^2 - \sqrt{2}\right)\sinh(x) - \sqrt{2}\cosh(x)\right)}{\sqrt{\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1}} - 1\right) - \frac{2\sqrt{2}(\cosh(x)+\sinh(x))}{\sqrt{\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1}}$$

input
```
integrate((1+coth(x))^(3/2),x, algorithm="fricas")
```

output
```
sqrt(2)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + sqrt(2)*(sqrt(
2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)*sinh(x)^3 + (3*sqrt(2
)*cosh(x)^2 - sqrt(2))*sinh(x) - sqrt(2)*cosh(x))/sqrt(cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2 - 1) - 1) - 2*sqrt(2)*(cosh(x) + sinh(x))/sqrt(cosh
(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [F]

$$\int (1+\coth(x))^{3/2}\,dx = \int (\coth(x)+1)^{\frac{3}{2}}\,dx$$

input
```
integrate((1+coth(x))**(3/2),x)
```

output
```
Integral((coth(x) + 1)**(3/2), x)
```

Maxima [F]

$$\int (1+\coth(x))^{3/2}\,dx = \int (\coth(x)+1)^{\frac{3}{2}}\,dx$$

input
```
integrate((1+coth(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((coth(x) + 1)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(26) = 52$.

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.91

$$\int (1+\coth(x))^{3/2}\,dx = -\sqrt{2}\left(\frac{2}{\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}+1}+\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\right)\operatorname{sgn}\left(e^{(2x)}-1\right)$$

input
```
integrate((1+coth(x))^(3/2),x, algorithm="giac")
```

output
```
-sqrt(2)*(2/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1) + log(abs(2*sqrt(e^(4*
x) - e^(2*x)) - 2*e^(2*x) + 1)))*sgn(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int (1+\coth(x))^{3/2}\,dx = 2\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{\coth(x)+1}}{2}\right)-2\sqrt{\coth(x)+1}$$

input
```
int((coth(x) + 1)^(3/2),x)
```

output
```
2*2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2) - 2*(coth(x) + 1)^(1/2)
```

Reduce [F]

$$\int (1+\coth(x))^{3/2}\,dx=\int \sqrt{\coth\left(x\right)+1}dx+\int \sqrt{\coth\left(x\right)+1}\,\coth\left(x\right)dx$$

input
```
int((1+coth(x))^(3/2),x)
```

output
```
int(sqrt(coth(x) + 1),x) + int(sqrt(coth(x) + 1)*coth(x),x)
```

3.73 $\int \sqrt{1+\coth(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 21

$$\int \sqrt{1+\coth(x)}\,dx = \sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)$$

output
```
2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \sqrt{1+\coth(x)}\,dx = \sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)$$

input
```
Integrate[Sqrt[1 + Coth[x]],x]
```

output
```
Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\coth(x)+1}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{1-i\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \; 3961$$

$$2\int \frac{1}{1-\coth(x)}d\sqrt{\coth(x)+1}$$

$$\Big\downarrow \; 219$$

$$\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

input
```
Int[Sqrt[1 + Coth[x]],x]
```

output
```
Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\, \sqrt{2}}{2}\right)$	17
default	$\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\, \sqrt{2}}{2}\right)$	17

input
```
int((1+coth(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(16) = 32.

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 4.81

$$\int \sqrt{1+\coth(x)}\, dx = \frac{1}{2}\sqrt{2}\log\left(2\cosh(x)^2 + 4\cosh(x)\sinh(x) + 2\sinh(x)^2 + \frac{\sqrt{2}\left(\sqrt{2}\cosh(x)^3 + 3\sqrt{2}\cosh(x)\sinh(x)^2 + \sqrt{2}\sinh(x)^3 + \left(3\sqrt{2}\cosh(x)^2 - \sqrt{2}\right)\sinh(x) - \sqrt{2}\cos\right.}{\sqrt{\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1}} - 1\right)$$

input
```
integrate((1+coth(x))^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + sqrt(2)*(s
qrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)*sinh(x)^3 + (3*sq
rt(2)*cosh(x)^2 - sqrt(2))*sinh(x) - sqrt(2)*cosh(x))/sqrt(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2 - 1) - 1)
```

Sympy [F]

$$\int \sqrt{1+\coth(x)}\,dx = \int \sqrt{\coth(x)+1}\,dx$$

input
```
integrate((1+coth(x))**(1/2),x)
```

output
```
Integral(sqrt(coth(x) + 1), x)
```

Maxima [F]

$$\int \sqrt{1+\coth(x)}\,dx = \int \sqrt{\coth(x)+1}\,dx$$

input
```
integrate((1+coth(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(coth(x) + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(16) = 32$.

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \sqrt{1+\coth(x)}\,dx = -\frac{1}{2}\sqrt{2}\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\operatorname{sgn}\left(e^{(2x)}-1\right)$$

input
```
integrate((1+coth(x))^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(2*
x) - 1)
```

Mupad [B] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \sqrt{1+\coth(x)}\,dx = \sqrt{2}\,\text{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth\left(x\right)+1}}{2}\right)$$

input
```
int((coth(x) + 1)^(1/2),x)
```

output
```
2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2)
```

Reduce [F]

$$\int \sqrt{1+\coth(x)}\,dx = \int \sqrt{\coth\left(x\right)+1}dx$$

input
```
int((1+coth(x))^(1/2),x)
```

output
```
int(sqrt(coth(x) + 1),x)
```

3.74 $\int \frac{1}{\sqrt{1+\coth(x)}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 32

$$\int \frac{1}{\sqrt{1+\coth(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{1+\coth(x)}}$$

output
```
1/2*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-1/(1+coth(x))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{1+\coth(x)}}\,dx=-\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1+\coth(x))\right)}{\sqrt{1+\coth(x)}}$$

input
```
Integrate[1/Sqrt[1 + Coth[x]],x]
```

output
```
-(Hypergeometric2F1[-1/2, 1, 1/2, (1 + Coth[x])/2]/Sqrt[1 + Coth[x]])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3042, 3960, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\coth(x)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{1-i\tan\left(\frac{\pi}{2}+ix\right)}}dx$$

$$\Big\downarrow \text{3960}$$

$$\frac{1}{2}\int \sqrt{\coth(x)+1}dx-\frac{1}{\sqrt{\coth(x)+1}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{\sqrt{\coth(x)+1}}+\frac{1}{2}\int \sqrt{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3961}$$

$$\int \frac{1}{1-\coth(x)}d\sqrt{\coth(x)+1}-\frac{1}{\sqrt{\coth(x)+1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{\coth(x)+1}}$$

input
```
Int[1/Sqrt[1 + Coth[x]],x]
```

output
```
ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Coth[x]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)}{2}-\frac{1}{\sqrt{1+\coth(x)}}$	27
default	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)}{2}-\frac{1}{\sqrt{1+\coth(x)}}$	27

input
```
int(1/(1+coth(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-1/(1+coth(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(26) = 52$.

Time = 0.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 4.47

$$\int \frac{1}{\sqrt{1+\coth(x)}}\,dx = \frac{\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\log\left(2\cosh\left(x\right)^{2}+4\cosh\left(x\right)\sinh\left(x\right)+2\sinh\left(x\right)^{2}+\frac{\sqrt{2}\left(\sqrt{2}\cosh(x)^{3}+3\sqrt{2}\cosh(\cdots\right.}{\cdots}\right.}{4\left(\cos\cdots\right.}$$

input
```
integrate(1/(1+coth(x))^(1/2),x, algorithm="fricas")
```

output
```
1/4*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log(2*cosh(x)^2 + 4*cosh(x)*sinh(
x) + 2*sinh(x)^2 + sqrt(2)*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)*sinh(x)^
2 + sqrt(2)*sinh(x)^3 + (3*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x) - sqrt(2)*
cosh(x))/sqrt(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1) - 1) - 2*sqrt
(2)*sqrt(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1))/(cosh(x) + sinh(x
))
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+\coth(x)}}\,dx = \int \frac{1}{\sqrt{\coth\left(x\right)+1}}\,dx$$

input
```
integrate(1/(1+coth(x))**(1/2),x)
```

output
```
Integral(1/sqrt(coth(x) + 1), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+\coth(x)}}\,dx = \int \frac{1}{\sqrt{\coth(x)+1}}\,dx$$

input
```
integrate(1/(1+coth(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(coth(x) + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(26) = 52$.

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.06

$$\int \frac{1}{\sqrt{1+\coth(x)}}\,dx = \frac{\sqrt{2}\left(\frac{2}{\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}} - \log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\right)}{4\,\mathrm{sgn}\left(e^{(2x)}-1\right)}$$

input
```
integrate(1/(1+coth(x))^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(2/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x)) - log(abs(2*sqrt(e^(4*x
) - e^(2*x)) - 2*e^(2*x) + 1)))/sgn(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{1+\coth(x)}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{\coth(x)+1}}{2}\right)}{2} - \frac{1}{\sqrt{\coth(x)+1}}$$

input
```
int(1/(coth(x) + 1)^(1/2),x)
```

output
```
(2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/2 - 1/(coth(x) + 1)^(1/2)
```

Reduce [F]

$$\int \frac{1}{\sqrt{1+\coth(x)}}\,dx = \int \frac{\sqrt{\coth(x)+1}}{\coth(x)+1}dx$$

input
```
int(1/(1+coth(x))^(1/2),x)
```

output
```
int(sqrt(coth(x) + 1)/(coth(x) + 1),x)
```

3.75 $\int \frac{1}{(1+\coth(x))^{3/2}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 49

$$\int \frac{1}{(1+\coth(x))^{3/2}}\,dx = \frac{\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{1}{3(1+\coth(x))^{3/2}} - \frac{1}{2\sqrt{1+\coth(x)}}$$

output
```
1/4*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-1/3/(1+coth(x))^(3/2)-1
/2/(1+coth(x))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{1}{(1+\coth(x))^{3/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(1+\coth(x))\right)}{3(1+\coth(x))^{3/2}}$$

input
```
Integrate[(1 + Coth[x])^(-3/2),x]
```

output
```
-1/3*Hypergeometric2F1[-3/2, 1, -1/2, (1 + Coth[x])/2]/(1 + Coth[x])^(3/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {3042, 3960, 3042, 3960, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\coth(x)+1)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3960}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\coth(x)+1}}dx-\frac{1}{3(\coth(x)+1)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3(\coth(x)+1)^{3/2}}+\frac{1}{2}\int \frac{1}{\sqrt{1-i\tan\left(ix+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3960}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \sqrt{\coth(x)+1}dx-\frac{1}{\sqrt{\coth(x)+1}}\right)-\frac{1}{3(\coth(x)+1)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3(\coth(x)+1)^{3/2}}+\frac{1}{2}\left(-\frac{1}{\sqrt{\coth(x)+1}}+\frac{1}{2}\int \sqrt{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3961}$$

$$\frac{1}{2}\left(\int \frac{1}{1-\coth(x)}d\sqrt{\coth(x)+1}-\frac{1}{\sqrt{\coth(x)+1}}\right)-\frac{1}{3(\coth(x)+1)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{\coth(x)+1}}\right)-\frac{1}{3(\coth(x)+1)^{3/2}}$$

input
```
Int[(1 + Coth[x])^(-3/2),x]
```

output
```
-1/3*1/(1 + Coth[x])^(3/2) + (ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] -
 1/Sqrt[1 + Coth[x]])/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)}{4}-\frac{1}{3(1+\coth(x))^{\frac{3}{2}}}-\frac{1}{2\sqrt{1+\coth(x)}}$	35
default	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)}{4}-\frac{1}{3(1+\coth(x))^{\frac{3}{2}}}-\frac{1}{2\sqrt{1+\coth(x)}}$	35

input
```
int(1/(1+coth(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-1/3/(1+coth(x))^(3/2)-1
/2/(1+coth(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(34) = 68.

Time = 0.09 (sec) , antiderivative size = 246, normalized size of antiderivative = 5.02

$$\int \frac{1}{(1+\coth(x))^{3/2}}\,dx = \frac{3\left(\sqrt{2}\cosh(x)^3+3\sqrt{2}\cosh(x)^2\sinh(x)+3\sqrt{2}\cosh(x)\sinh(x)^2+\sqrt{2}\sinh(x)^3\right.}{}$$

input
```
integrate(1/(1+coth(x))^(3/2),x, algorithm="fricas")
```

output
```
1/24*(3*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2)*cosh(
x)*sinh(x)^2 + sqrt(2)*sinh(x)^3)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*
sinh(x)^2 + sqrt(2)*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqr
t(2)*sinh(x)^3 + (3*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x) - sqrt(2)*cosh(x)
)/sqrt(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1) - 1) - 2*sqrt(2)*(4*
cosh(x)^4 + 16*cosh(x)*sinh(x)^3 + 4*sinh(x)^4 + (24*cosh(x)^2 - 5)*sinh(x
)^2 - 5*cosh(x)^2 + 2*(8*cosh(x)^3 - 5*cosh(x))*sinh(x) + 1)/sqrt(cosh(x)^
2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1))/(cosh(x)^3 + 3*cosh(x)^2*sinh(x) +
 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [F]

$$\int \frac{1}{(1+\coth(x))^{3/2}}\,dx = \int \frac{1}{(\coth{(x)}+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(1+coth(x))**(3/2),x)
```

output
```
Integral((coth(x) + 1)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(1+\coth(x))^{3/2}}\,dx = \int \frac{1}{(\coth{(x)}+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(1+coth(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((coth(x) + 1)^(-3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(34) = 68$.

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.31

$$\int \frac{1}{(1+\coth(x))^{3/2}}\,dx = \frac{\sqrt{2}\left(\frac{2\left(6\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^{2}+3\sqrt{e^{(4x)}-e^{(2x)}}-3e^{(2x)}+1\right)}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^{3}} - 3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}} - \right.\right.}{24\,\mathrm{sgn}\left(e^{(2x)}-1\right)}$$

input
```
integrate(1/(1+coth(x))^(3/2),x, algorithm="giac")
```

output
```
1/24*sqrt(2)*(2*(6*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2 + 3*sqrt(e^(4*x)
- e^(2*x)) - 3*e^(2*x) + 1)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^3 - 3*log(
abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1)))/sgn(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.65

$$\int \frac{1}{(1+\coth(x))^{3/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)}{4} - \frac{\frac{\coth(x)}{2}+\frac{5}{6}}{(\coth(x)+1)^{3/2}}$$

input
```
int(1/(coth(x) + 1)^(3/2),x)
```

output
```
(2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/4 - (coth(x)/2 + 5/6)/(co
th(x) + 1)^(3/2)
```

Reduce [F]

$$\int \frac{1}{(1+\coth(x))^{3/2}}\,dx = \int \frac{\sqrt{\coth(x)+1}}{\coth(x)^2+2\coth(x)+1}dx$$

input
```
int(1/(1+coth(x))^(3/2),x)
```

output
```
int(sqrt(coth(x) + 1)/(coth(x)**2 + 2*coth(x) + 1),x)
```

3.76 $\int \frac{1}{(1+\coth(x))^{5/2}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 61

$$\int \frac{1}{(1+\coth(x))^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{4\sqrt{2}} - \frac{1}{5(1+\coth(x))^{5/2}} - \frac{1}{6(1+\coth(x))^{3/2}} - \frac{1}{4\sqrt{1+\coth(x)}}$$

output
```
1/8*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-1/5/(1+coth(x))^(5/2)-1
/6/(1+coth(x))^(3/2)-1/4/(1+coth(x))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.46

$$\int \frac{1}{(1+\coth(x))^{5/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{1}{2}(1+\coth(x))\right)}{5(1+\coth(x))^{5/2}}$$

input
```
Integrate[(1 + Coth[x])^(-5/2),x]
```

output
```
-1/5*Hypergeometric2F1[-5/2, 1, -3/2, (1 + Coth[x])/2]/(1 + Coth[x])^(5/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {3042, 3960, 3042, 3960, 3042, 3960, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\coth(x)+1)^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3960}$$

$$\frac{1}{2}\int \frac{1}{(\coth(x)+1)^{3/2}}dx-\frac{1}{5(\coth(x)+1)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{5(\coth(x)+1)^{5/2}}+\frac{1}{2}\int \frac{1}{\left(1-i\tan\left(ix+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3960}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{\coth(x)+1}}dx-\frac{1}{3(\coth(x)+1)^{3/2}}\right)-\frac{1}{5(\coth(x)+1)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{5(\coth(x)+1)^{5/2}}+\frac{1}{2}\left(-\frac{1}{3(\coth(x)+1)^{3/2}}+\frac{1}{2}\int \frac{1}{\sqrt{1-i\tan\left(ix+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{3960}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \sqrt{\coth(x)+1}dx-\frac{1}{\sqrt{\coth(x)+1}}\right)-\frac{1}{3(\coth(x)+1)^{3/2}}\right)-\frac{1}{5(\coth(x)+1)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{5(\coth(x)+1)^{5/2}}+\frac{1}{2}\left(-\frac{1}{3(\coth(x)+1)^{3/2}}+\frac{1}{2}\left(-\frac{1}{\sqrt{\coth(x)+1}}+\frac{1}{2}\int\sqrt{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx\right)\right)$$

$$\Big\downarrow \text{3961}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\int\frac{1}{1-\coth(x)}d\sqrt{\coth(x)+1}-\frac{1}{\sqrt{\coth(x)+1}}\right)-\frac{1}{3(\coth(x)+1)^{3/2}}\right)-\frac{1}{5(\coth(x)+1)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{\coth(x)+1}}\right)-\frac{1}{3(\coth(x)+1)^{3/2}}\right)-\frac{1}{5(\coth(x)+1)^{5/2}}$$

input
```
Int[(1 + Coth[x])^(-5/2),x]
```

output
```
-1/5*1/(1 + Coth[x])^(5/2) + (-1/3*1/(1 + Coth[x])^(3/2) + (ArcTanh[Sqrt[1
 + Coth[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Coth[x]])/2)/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)}{8}-\frac{1}{5(1+\coth(x))^{\frac{5}{2}}}-\frac{1}{6(1+\coth(x))^{\frac{3}{2}}}-\frac{1}{4\sqrt{1+\coth(x)}}$	43
default	$\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)}{8}-\frac{1}{5(1+\coth(x))^{\frac{5}{2}}}-\frac{1}{6(1+\coth(x))^{\frac{3}{2}}}-\frac{1}{4\sqrt{1+\coth(x)}}$	43

input
```
int(1/(1+coth(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-1/5/(1+coth(x))^(5/2)-1
/6/(1+coth(x))^(3/2)-1/4/(1+coth(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(42) = 84$.

Time = 0.10 (sec) , antiderivative size = 340, normalized size of antiderivative = 5.57

$$\int \frac{1}{(1+\coth(x))^{5/2}}\,dx = \frac{15\left(\sqrt{2}\cosh(x)^5+5\sqrt{2}\cosh(x)^4\sinh(x)+10\sqrt{2}\cosh(x)^3\sinh(x)^2+10\sqrt{2}\,\mathrm{co}\right.}{}$$

input
```
integrate(1/(1+coth(x))^(5/2),x, algorithm="fricas")
```

output
```
1/240*(15*(sqrt(2)*cosh(x)^5 + 5*sqrt(2)*cosh(x)^4*sinh(x) + 10*sqrt(2)*co
sh(x)^3*sinh(x)^2 + 10*sqrt(2)*cosh(x)^2*sinh(x)^3 + 5*sqrt(2)*cosh(x)*sin
h(x)^4 + sqrt(2)*sinh(x)^5)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x
)^2 + sqrt(2)*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)*s
inh(x)^3 + (3*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x) - sqrt(2)*cosh(x))/sqrt
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1) - 1) - 2*sqrt(2)*(23*cosh(
x)^6 + 138*cosh(x)*sinh(x)^5 + 23*sinh(x)^6 + (345*cosh(x)^2 - 34)*sinh(x)
^4 - 34*cosh(x)^4 + 4*(115*cosh(x)^3 - 34*cosh(x))*sinh(x)^3 + (345*cosh(x
)^4 - 204*cosh(x)^2 + 14)*sinh(x)^2 + 14*cosh(x)^2 + 2*(69*cosh(x)^5 - 68*
cosh(x)^3 + 14*cosh(x))*sinh(x) - 3)/sqrt(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 - 1))/(cosh(x)^5 + 5*cosh(x)^4*sinh(x) + 10*cosh(x)^3*sinh(x)^2
+ 10*cosh(x)^2*sinh(x)^3 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5)
```

Sympy [F]

$$\int \frac{1}{(1+\coth(x))^{5/2}}\,dx = \int \frac{1}{(\coth(x)+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(1+coth(x))**(5/2),x)
```

output
```
Integral((coth(x) + 1)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(1+\coth(x))^{5/2}}\,dx = \int \frac{1}{(\coth(x)+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(1+coth(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((coth(x) + 1)^(-5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(42) = 84$.

Time = 0.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.64

$$\int \frac{1}{(1+\coth(x))^{5/2}}\,dx = \frac{\sqrt{2}\left(\frac{2\left(45\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^4+45\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^3+35\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^2+15\sqrt{e^{(4x)}\cdots}\right.}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^5}\cdots}{240\,\mathrm{sgn}\left(e^{(2x)}-1\right)\cdots}$$

input
```
integrate(1/(1+coth(x))^(5/2),x, algorithm="giac")
```

output
```
1/240*sqrt(2)*(2*(45*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^4 + 45*(sqrt(e^(4
*x) - e^(2*x)) - e^(2*x))^3 + 35*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2 + 1
5*sqrt(e^(4*x) - e^(2*x)) - 15*e^(2*x) + 3)/(sqrt(e^(4*x) - e^(2*x)) - e^(
2*x))^5 - 15*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1)))/sgn(e^(2
*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

$$\int \frac{1}{(1+\coth(x))^{5/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{\coth(x)+1}}{2}\right)}{8} - \frac{\frac{\coth(x)}{6}+\frac{(\coth(x)+1)^2}{4}+\frac{11}{30}}{(\coth(x)+1)^{5/2}}$$

input
```
int(1/(coth(x) + 1)^(5/2),x)
```

output
```
(2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/8 - (coth(x)/6 + (coth(x)
 + 1)^2/4 + 11/30)/(coth(x) + 1)^(5/2)
```

Reduce [F]

$$\int \frac{1}{(1+\coth(x))^{5/2}}\,dx = \int \frac{\sqrt{\coth(x)+1}}{\coth(x)^3+3\coth(x)^2+3\coth(x)+1}dx$$

input
```
int(1/(1+coth(x))^(5/2),x)
```

output
```
int(sqrt(coth(x) + 1)/(coth(x)**3 + 3*coth(x)**2 + 3*coth(x) + 1),x)
```

3.77 $\int (a+b\coth(c+dx))^5\,dx$

Optimal result

Integrand size = 12, antiderivative size = 142

$$\begin{aligned}\int (a+b\coth(c+dx))^5\,dx &= a\left(a^4+10a^2b^2+5b^4\right)x-\frac{4ab^2(a^2+b^2)\coth(c+dx)}{d}\\&\quad-\frac{b(3a^2+b^2)\,(a+b\coth(c+dx))^2}{2d}\\&\quad-\frac{2ab(a+b\coth(c+dx))^3}{3d}-\frac{b(a+b\coth(c+dx))^4}{4d}\\&\quad+\frac{b(5a^4+10a^2b^2+b^4)\log(\sinh(c+dx))}{d}\end{aligned}$$

output
```
a*(a^4+10*a^2*b^2+5*b^4)*x-4*a*b^2*(a^2+b^2)*coth(d*x+c)/d-1/2*b*(3*a^2+b^
2)*(a+b*coth(d*x+c))^2/d-2/3*a*b*(a+b*coth(d*x+c))^3/d-1/4*b*(a+b*coth(d*x
+c))^4/d+b*(5*a^4+10*a^2*b^2+b^4)*ln(sinh(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\int (a+b\coth(c+dx))^5\,dx =$$
$$-\frac{60ab^2(2a^2+b^2)\coth(c+dx)+6b^3(10a^2+b^2)\coth^2(c+dx)+20ab^4\coth^3(c+dx)+3b^5\coth^4(c+d$$

input
```
Integrate[(a + b*Coth[c + d*x])^5,x]
```

output
```
-1/12*(60*a*b^2*(2*a^2 + b^2)*Coth[c + d*x] + 6*b^3*(10*a^2 + b^2)*Coth[c
+ d*x]^2 + 20*a*b^4*Coth[c + d*x]^3 + 3*b^5*Coth[c + d*x]^4 + 6*(a + b)^5*
Log[1 - Tanh[c + d*x]] - 12*b*(5*a^4 + 10*a^2*b^2 + b^4)*Log[Tanh[c + d*x]
] - 6*(a - b)^5*Log[1 + Tanh[c + d*x]])/d
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3963, 3042, 4011, 3042, 4011, 3042, 4008, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\coth(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^5dx$$

$$\Big\downarrow \text{3963}$$

$$\int (a+b\coth(c+dx))^3\left(a^2+2b\coth(c+dx)a+b^2\right)dx-\frac{b(a+b\coth(c+dx))^4}{4d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b(a+b\coth(c+dx))^4}{4d}+\int\left(a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^3\left(a^2-2ib\tan\left(ic+idx+\frac{\pi}{2}\right)a+b^2\right)dx$$

$$\Big\downarrow \text{ 4011}$$

$$\int(a+b\coth(c+dx))^2\left(a\left(a^2+3b^2\right)+b\left(3a^2+b^2\right)\coth(c+dx)\right)dx-\frac{b(a+b\coth(c+dx))^4}{4d}-\frac{2ab(a+b\coth(c+dx))^3}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$\int\left(a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^2\left(a\left(a^2+3b^2\right)-ib\left(3a^2+b^2\right)\tan\left(ic+idx+\frac{\pi}{2}\right)\right)dx-\frac{b(a+b\coth(c+dx))^4}{4d}-\frac{2ab(a+b\coth(c+dx))^3}{3d}$$

$$\Big\downarrow \text{ 4011}$$

$$\int(a+b\coth(c+dx))\left(a^4+6b^2a^2+4b\left(a^2+b^2\right)\coth(c+dx)a+b^4\right)dx-\frac{b\left(3a^2+b^2\right)(a+b\coth(c+dx))^2}{2d}-\frac{b(a+b\coth(c+dx))^4}{4d}-\frac{2ab(a+b\coth(c+dx))^3}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$\int\left(a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)\left(a^4+6b^2a^2-4ib\left(a^2+b^2\right)\tan\left(ic+idx+\frac{\pi}{2}\right)a+b^4\right)dx-\frac{b\left(3a^2+b^2\right)(a+b\coth(c+dx))^2}{2d}-\frac{b(a+b\coth(c+dx))^4}{4d}-\frac{2ab(a+b\coth(c+dx))^3}{3d}$$

$$\Big\downarrow \text{ 4008}$$

$$-ib\left(5a^4+10a^2b^2+b^4\right)\int i\coth(c+dx)dx-\frac{b\left(3a^2+b^2\right)(a+b\coth(c+dx))^2}{2d}-\frac{4ab^2\left(a^2+b^2\right)\coth(c+dx)}{d}+ax\left(a^4+10a^2b^2+5b^4\right)-\frac{b(a+b\coth(c+dx))^4}{4d}-\frac{2ab(a+b\coth(c+dx))^3}{3d}$$

$$\Big\downarrow \text{ 26}$$

$$b\left(5a^4+10a^2b^2+b^4\right)\int\coth(c+dx)dx-\frac{b\left(3a^2+b^2\right)(a+b\coth(c+dx))^2}{2d}-\frac{4ab^2\left(a^2+b^2\right)\coth(c+dx)}{d}+ax\left(a^4+10a^2b^2+5b^4\right)-\frac{b(a+b\coth(c+dx))^4}{4d}-\frac{2ab(a+b\coth(c+dx))^3}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$b(5a^4+10a^2b^2+b^4)\int -i\tan\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{b(3a^2+b^2)(a+b\coth(c+dx))^2}{2d}-\frac{4ab^2(a^2+b^2)\coth(c+dx)}{d}+ax(a^4+10a^2b^2+5b^4)-\frac{b(a+b\coth(c+dx))^4}{4d}-\frac{2ab(a+b\coth(c+dx))^3}{3d}$$

$\Big\downarrow$ 26

$$-ib(5a^4+10a^2b^2+b^4)\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx-\frac{b(3a^2+b^2)(a+b\coth(c+dx))^2}{2d}-\frac{4ab^2(a^2+b^2)\coth(c+dx)}{d}+ax(a^4+10a^2b^2+5b^4)-\frac{b(a+b\coth(c+dx))^4}{4d}-\frac{2ab(a+b\coth(c+dx))^3}{3d}$$

$\Big\downarrow$ 3956

$$-\frac{b(3a^2+b^2)(a+b\coth(c+dx))^2}{2d}-\frac{4ab^2(a^2+b^2)\coth(c+dx)}{d}+\frac{b(5a^4+10a^2b^2+b^4)\log(-i\sinh(c+dx))}{d}+ax(a^4+10a^2b^2+5b^4)-\frac{b(a+b\coth(c+dx))^4}{4d}-\frac{2ab(a+b\coth(c+dx))^3}{3d}$$

input
```
Int[(a + b*Coth[c + d*x])^5,x]
```

output
```
a*(a^4 + 10*a^2*b^2 + 5*b^4)*x - (4*a*b^2*(a^2 + b^2)*Coth[c + d*x])/d - (
b*(3*a^2 + b^2)*(a + b*Coth[c + d*x])^2)/(2*d) - (2*a*b*(a + b*Coth[c + d*
x])^3)/(3*d) - (b*(a + b*Coth[c + d*x])^4)/(4*d) + (b*(5*a^4 + 10*a^2*b^2
+ b^4)*Log[(-I)*Sinh[c + d*x]])/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3963
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06

method	result
parallelrisch	$\frac{\left(-60a^4b-120a^2b^3-12b^5\right)\ln\left(1-\tanh(dx+c)\right)+\left(60a^4b+120a^2b^3+12b^5\right)\ln\left(\tanh(dx+c)\right)-3b^5\coth(dx+c)^4-20a\,b^4\operatorname{cot}\ldots}{12d}$
derivativedivides	$\frac{-\frac{b^5\coth(dx+c)^4}{4}-5a^2b^3\coth(dx+c)^2-\frac{5a\,b^4\coth(dx+c)^3}{3}+\frac{\left(a^5-5a^4b+10a^3b^2-10a^2b^3+5b^4a-b^5\right)\ln\left(\coth(dx+c)+1\right)}{2}-10a^3\ldots}{d}$
default	$\frac{-\frac{b^5\coth(dx+c)^4}{4}-5a^2b^3\coth(dx+c)^2-\frac{5a\,b^4\coth(dx+c)^3}{3}+\frac{\left(a^5-5a^4b+10a^3b^2-10a^2b^3+5b^4a-b^5\right)\ln\left(\coth(dx+c)+1\right)}{2}-10a^3\ldots}{d}$
parts	$a^5x+\frac{b^5\left(-\frac{\coth(dx+c)^4}{4}-\frac{\coth(dx+c)^2}{2}-\frac{\ln\left(\coth(dx+c)-1\right)}{2}-\frac{\ln\left(\coth(dx+c)+1\right)}{2}\right)}{d}+\frac{10a^2b^3\left(-\frac{\coth(dx+c)^2}{2}-\frac{\ln(\coth(d\ldots}{2}\ldots\right.}{d}$
risch	$a^5x-5b\,a^4x+10a^3b^2x-10b^3a^2x+5a\,b^4x-b^5x-\frac{10b\,a^4c}{d}-\frac{20b^3a^2c}{d}-\frac{2b^5c}{d}-\frac{4b^2\left(15a^3\mathrm{e}^{6}\ldots\right.}{\ldots}$

input
```
int((a+b*coth(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/12*((-60*a^4*b-120*a^2*b^3-12*b^5)*ln(1-tanh(d*x+c))+(60*a^4*b+120*a^2*b
^3+12*b^5)*ln(tanh(d*x+c))-3*b^5*coth(d*x+c)^4-20*a*b^4*coth(d*x+c)^3+(-60
*a^2*b^3-6*b^5)*coth(d*x+c)^2+(-120*a^3*b^2-60*a*b^4)*coth(d*x+c)+12*d*x*(
a-b)^5)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2748 vs. $2(136) = 272$.

Time = 0.11 (sec) , antiderivative size = 2748, normalized size of antiderivative = 19.35

$$\int (a + b\coth(c + dx))^5 \, dx = \text{Too large to display}$$

input
```
integrate((a+b*coth(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/3*(3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(
d*x + c)^8 + 24*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*
d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2
*b^3 + 5*a*b^4 - b^5)*d*x*sinh(d*x + c)^8 - 12*(5*a^3*b^2 + 5*a^2*b^3 + 5*
a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*
x)*cosh(d*x + c)^6 - 12*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5 - 7*(a^5 -
5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)^2 + (
a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*sinh(d*x + c
)^6 + 24*(7*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*
cosh(d*x + c)^3 - 3*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*
b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c))*sinh(d*x
+ c)^5 + 60*a^3*b^2 + 40*a*b^4 + 6*(30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2
*b^5 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*co
sh(d*x + c)^4 + 6*(35*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 -
 b^5)*d*x*cosh(d*x + c)^4 + 30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3
*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x - 30*(5*a^3
*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^
3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 24*(7*(a^5 - 5*
a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)^5 - 10*
(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(133) = 266$.

Time = 1.74 (sec) , antiderivative size = 525, normalized size of antiderivative = 3.70

$$\int (a + b\coth(c + dx))^5\,dx$$

$$= \begin{cases} x(a + b\coth(c))^5 \\ a^5x + 5a^4bx\coth\left(dx + \log\left(-e^{-dx}\right)\right) + 10a^3b^2x\coth^2\left(dx + \log\left(-e^{-dx}\right)\right) + 10a^2b^3x\coth^3\left(dx + \log\left(\right.\right. \\ a^5x + 5a^4bx\coth\left(dx + \log\left(e^{-dx}\right)\right) + 10a^3b^2x\coth^2\left(dx + \log\left(e^{-dx}\right)\right) + 10a^2b^3x\coth^3\left(dx + \log\left(e^{-dx}\right.\right. \\ a^5x + 5a^4bx - \frac{5a^4b\log(\tanh(c+dx)+1)}{d} + \frac{5a^4b\log(\tanh(c+dx))}{d} + 10a^3b^2x - \frac{10a^3b^2}{d\tanh(c+dx)} + 10a^2b^3x - \frac{10a^2b^3\log(\mathrm{ta}}{} \end{cases}$$

input
```
integrate((a+b*coth(d*x+c))**5,x)
```

output
```
Piecewise((x*(a + b*coth(c))**5, Eq(d, 0)), (a**5*x + 5*a**4*b*x*coth(d*x
+ log(-exp(-d*x))) + 10*a**3*b**2*x*coth(d*x + log(-exp(-d*x)))**2 + 10*a*
*2*b**3*x*coth(d*x + log(-exp(-d*x)))**3 + 5*a*b**4*x*coth(d*x + log(-exp(
-d*x)))**4 + b**5*x*coth(d*x + log(-exp(-d*x)))**5, Eq(c, log(-exp(-d*x)))
), (a**5*x + 5*a**4*b*x*coth(d*x + log(exp(-d*x))) + 10*a**3*b**2*x*coth(d
*x + log(exp(-d*x)))**2 + 10*a**2*b**3*x*coth(d*x + log(exp(-d*x)))**3 + 5
*a*b**4*x*coth(d*x + log(exp(-d*x)))**4 + b**5*x*coth(d*x + log(exp(-d*x))
)**5, Eq(c, log(exp(-d*x)))), (a**5*x + 5*a**4*b*x - 5*a**4*b*log(tanh(c +
 d*x) + 1)/d + 5*a**4*b*log(tanh(c + d*x))/d + 10*a**3*b**2*x - 10*a**3*b*
*2/(d*tanh(c + d*x)) + 10*a**2*b**3*x - 10*a**2*b**3*log(tanh(c + d*x) + 1
)/d + 10*a**2*b**3*log(tanh(c + d*x))/d - 5*a**2*b**3/(d*tanh(c + d*x)**2)
 + 5*a*b**4*x - 5*a*b**4/(d*tanh(c + d*x)) - 5*a*b**4/(3*d*tanh(c + d*x)**
3) + b**5*x - b**5*log(tanh(c + d*x) + 1)/d + b**5*log(tanh(c + d*x))/d -
b**5/(2*d*tanh(c + d*x)**2) - b**5/(4*d*tanh(c + d*x)**4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(136) = 272$.

Time = 0.19 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.45

$$\int (a + b\coth(c + dx))^5 \, dx$$

$$= \frac{5}{3}ab^4\left(3\,x + \frac{3\,c}{d} - \frac{4\,\left(3\,e^{(-2\,dx-2\,c)} - 3\,e^{(-4\,dx-4\,c)} - 2\right)}{d(3\,e^{(-2\,dx-2\,c)} - 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} - 1)}\right)$$
$$+ b^5\left(x + \frac{c}{d} + \frac{\log\left(e^{(-dx-c)} + 1\right)}{d} + \frac{\log\left(e^{(-dx-c)} - 1\right)}{d} + \frac{4\,\left(e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} + e^{(-6\,dx-}\right.}{d(4\,e^{(-2\,dx-2\,c)} - 6\,e^{(-4\,dx-4\,c)} + 4\,e^{(-6\,dx-6\,c)} - e}\right.$$
$$+ 10\,a^2b^3\left(x + \frac{c}{d} + \frac{\log\left(e^{(-dx-c)} + 1\right)}{d} + \frac{\log\left(e^{(-dx-c)} - 1\right)}{d} + \frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} - 1)}\right)$$
$$+ 10\,a^3b^2\left(x + \frac{c}{d} + \frac{2}{d(e^{(-2\,dx-2\,c)} - 1)}\right) + a^5x + \frac{5\,a^4b\log\left(\sinh\left(dx + c\right)\right)}{d}$$

input
```
integrate((a+b*coth(d*x+c))^5,x, algorithm="maxima")
```

output
```
5/3*a*b^4*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(
d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + b^5
*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2*d
*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) -
6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 10*a^2
*b^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-
2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + 10*a^3*b^2
*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + a^5*x + 5*a^4*b*log(sinh(d*x +
 c))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.59

$$\int (a+b\coth(c+dx))^5\,dx$$
$$= \frac{3\,(a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5)(dx+c)+3\,(5\,a^4b+10\,a^2b^3+b^5)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+}{}$$

input
```
integrate((a+b*coth(d*x+c))^5,x, algorithm="giac")
```

output
```
1/3*(3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*(d*x + c)
 + 3*(5*a^4*b + 10*a^2*b^3 + b^5)*log(abs(e^(2*d*x + 2*c) - 1)) + 4*(15*a^
3*b^2 + 10*a*b^4 - 3*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5)*e^(6*d*x + 6*
c) + 3*(15*a^3*b^2 + 10*a^2*b^3 + 10*a*b^4 + b^5)*e^(4*d*x + 4*c) - (45*a^
3*b^2 + 15*a^2*b^3 + 25*a*b^4 + 3*b^5)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c) -
 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.72

$$\begin{aligned}\int (a+b\coth(c+dx))^5\,dx &= x\,(a-b)^5-\frac{4\,(5\,a^3\,b^2+5\,a^2\,b^3+5\,a\,b^4+b^5)}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}\\ &+\frac{\ln\left(e^{2\,c}\,e^{2\,d\,x}-1\right)\,\left(5\,a^4\,b+10\,a^2\,b^3+b^5\right)}{d}\\ &-\frac{4\,(5\,a^2\,b^3+5\,a\,b^4+2\,b^5)}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}\\ &-\frac{4\,b^5}{d\,\left(6\,e^{4\,c+4\,d\,x}-4\,e^{2\,c+2\,d\,x}-4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}\\ &-\frac{8\,(3\,b^5+5\,a\,b^4)}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1\right)}\end{aligned}$$

input
```
int((a + b*coth(c + d*x))^5,x)
```

output
```
x*(a - b)^5 - (4*(5*a*b^4 + b^5 + 5*a^2*b^3 + 5*a^3*b^2))/(d*(exp(2*c + 2*
d*x) - 1)) + (log(exp(2*c)*exp(2*d*x) - 1)*(5*a^4*b + b^5 + 10*a^2*b^3))/d
 - (4*(5*a*b^4 + 2*b^5 + 5*a^2*b^3))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*
d*x) + 1)) - (4*b^5)/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6
*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (8*(5*a*b^4 + 3*b^5))/(3*d*(3*exp(2
*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1471, normalized size of antiderivative = 10.36

$$\int (a + b\coth(c + dx))^5\,dx = \text{Too large to display}$$

input
```
int((a+b*coth(d*x+c))^5,x)
```

output
```
(15*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*a**4*b + 30*e**(8*c + 8*d*x)*lo
g(e**(c + d*x) - 1)*a**2*b**3 + 3*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*b
**5 + 15*e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)*a**4*b + 30*e**(8*c + 8*d*
x)*log(e**(c + d*x) + 1)*a**2*b**3 + 3*e**(8*c + 8*d*x)*log(e**(c + d*x) +
 1)*b**5 + 3*e**(8*c + 8*d*x)*a**5*d*x - 15*e**(8*c + 8*d*x)*a**4*b*d*x +
30*e**(8*c + 8*d*x)*a**3*b**2*d*x - 15*e**(8*c + 8*d*x)*a**3*b**2 - 30*e**
(8*c + 8*d*x)*a**2*b**3*d*x - 15*e**(8*c + 8*d*x)*a**2*b**3 + 15*e**(8*c +
 8*d*x)*a*b**4*d*x - 15*e**(8*c + 8*d*x)*a*b**4 - 3*e**(8*c + 8*d*x)*b**5*
d*x - 3*e**(8*c + 8*d*x)*b**5 - 60*e**(6*c + 6*d*x)*log(e**(c + d*x) - 1)*
a**4*b - 120*e**(6*c + 6*d*x)*log(e**(c + d*x) - 1)*a**2*b**3 - 12*e**(6*c
 + 6*d*x)*log(e**(c + d*x) - 1)*b**5 - 60*e**(6*c + 6*d*x)*log(e**(c + d*x
) + 1)*a**4*b - 120*e**(6*c + 6*d*x)*log(e**(c + d*x) + 1)*a**2*b**3 - 12*
e**(6*c + 6*d*x)*log(e**(c + d*x) + 1)*b**5 - 12*e**(6*c + 6*d*x)*a**5*d*x
 + 60*e**(6*c + 6*d*x)*a**4*b*d*x - 120*e**(6*c + 6*d*x)*a**3*b**2*d*x + 1
20*e**(6*c + 6*d*x)*a**2*b**3*d*x - 60*e**(6*c + 6*d*x)*a*b**4*d*x + 12*e*
*(6*c + 6*d*x)*b**5*d*x + 90*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**4*b
 + 180*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**2*b**3 + 18*e**(4*c + 4*d
*x)*log(e**(c + d*x) - 1)*b**5 + 90*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)
*a**4*b + 180*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**2*b**3 + 18*e**(4*
c + 4*d*x)*log(e**(c + d*x) + 1)*b**5 + 18*e**(4*c + 4*d*x)*a**5*d*x - ...
```

3.78 $\int (a+b\coth(c+dx))^4\,dx$

Optimal result

Integrand size = 12, antiderivative size = 101

$$\int (a+b\coth(c+dx))^4\,dx = \left(a^4+6a^2b^2+b^4\right)x - \frac{b^2(3a^2+b^2)\coth(c+dx)}{d} - \frac{ab(a+b\coth(c+dx))^2}{d} - \frac{b(a+b\coth(c+dx))^3}{3d} + \frac{4ab(a^2+b^2)\log(\sinh(c+dx))}{d}$$

output
```
(a^4+6*a^2*b^2+b^4)*x-b^2*(3*a^2+b^2)*coth(d*x+c)/d-a*b*(a+b*coth(d*x+c))^
2/d-1/3*b*(a+b*coth(d*x+c))^3/d+4*a*b*(a^2+b^2)*ln(sinh(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.08

$$\int (a+b\coth(c+dx))^4\,dx = -\frac{6b^2(6a^2+b^2)\coth(c+dx)+12ab^3\coth^2(c+dx)+2b^4\coth^3(c+dx)+3(a+b)^4\log(1-\tanh(c+dx}{6d}$$

input
```
Integrate[(a + b*Coth[c + d*x])^4,x]
```

output
```
-1/6*(6*b^2*(6*a^2 + b^2)*Coth[c + d*x] + 12*a*b^3*Coth[c + d*x]^2 + 2*b^4
*Coth[c + d*x]^3 + 3*(a + b)^4*Log[1 - Tanh[c + d*x]] - 24*a*b*(a^2 + b^2)
*Log[Tanh[c + d*x]] - 3*(a - b)^4*Log[1 + Tanh[c + d*x]])/d
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 3963, 3042, 4011, 3042, 4008, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\coth(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^4dx$$

$$\Big\downarrow \text{3963}$$

$$\int (a+b\coth(c+dx))^2\left(a^2+2b\coth(c+dx)a+b^2\right)dx-\frac{b(a+b\coth(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b(a+b\coth(c+dx))^3}{3d}+\int \left(a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^2\left(a^2-2ib\tan\left(ic+idx+\frac{\pi}{2}\right)a+b^2\right)dx$$

$$\Big\downarrow \text{4011}$$

$$\int (a+b\coth(c+dx))\left(a\left(a^2+3b^2\right)+b\left(3a^2+b^2\right)\coth(c+dx)\right)dx-\frac{b(a+b\coth(c+dx))^3}{3d}-\frac{ab(a+b\coth(c+dx))^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)\left(a(a^2+3b^2)-ib(3a^2+b^2)\tan\left(ic+idx+\frac{\pi}{2}\right)\right)dx-\frac{b(a+b\coth(c+dx))^3}{3d}-\frac{ab(a+b\coth(c+dx))^2}{d}$$

$$\Big\downarrow \text{4008}$$

$$-4iab(a^2+b^2)\int i\coth(c+dx)dx-\frac{b^2(3a^2+b^2)\coth(c+dx)}{d}+x(a^4+6a^2b^2+b^4)-\frac{b(a+b\coth(c+dx))^3}{3d}-\frac{ab(a+b\coth(c+dx))^2}{d}$$

$$\Big\downarrow \text{26}$$

$$4ab(a^2+b^2)\int \coth(c+dx)dx-\frac{b^2(3a^2+b^2)\coth(c+dx)}{d}+x(a^4+6a^2b^2+b^4)-\frac{b(a+b\coth(c+dx))^3}{3d}-\frac{ab(a+b\coth(c+dx))^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$4ab(a^2+b^2)\int -i\tan\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{b^2(3a^2+b^2)\coth(c+dx)}{d}+x(a^4+6a^2b^2+b^4)-\frac{b(a+b\coth(c+dx))^3}{3d}-\frac{ab(a+b\coth(c+dx))^2}{d}$$

$$\Big\downarrow \text{26}$$

$$-4iab(a^2+b^2)\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx-\frac{b^2(3a^2+b^2)\coth(c+dx)}{d}+x(a^4+6a^2b^2+b^4)-\frac{b(a+b\coth(c+dx))^3}{3d}-\frac{ab(a+b\coth(c+dx))^2}{d}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{b^2(3a^2+b^2)\coth(c+dx)}{d}+\frac{4ab(a^2+b^2)\log(-i\sinh(c+dx))}{d}+x(a^4+6a^2b^2+b^4)-\frac{b(a+b\coth(c+dx))^3}{3d}-\frac{ab(a+b\coth(c+dx))^2}{d}$$

input
```
Int[(a + b*Coth[c + d*x])^4,x]
```

output
```
(a^4 + 6*a^2*b^2 + b^4)*x - (b^2*(3*a^2 + b^2)*Coth[c + d*x])/d - (a*b*(a
+ b*Coth[c + d*x])^2)/d - (b*(a + b*Coth[c + d*x])^3)/(3*d) + (4*a*b*(a^2
+ b^2)*Log[(-I)*Sinh[c + d*x]])/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3963
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.12

method	result
parallelrisch	$\frac{12(-a^3b-a\,b^3)\ln(1-\tanh(dx+c))+12(a^3b+a\,b^3)\ln(\tanh(dx+c))-b^4\coth(dx+c)^3-6a\,b^3\coth(dx+c)^2+3(-6a^2b^2-\cdots}{3d}$
derivativedivides	$\frac{-\frac{b^4\coth(dx+c)^3}{3}-2a\,b^3\coth(dx+c)^2-6a^2b^2\coth(dx+c)-b^4\coth(dx+c)-\frac{(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4)\ln(\coth(dx+c)-1)\cdots}{2}}{d}$
default	$\frac{-\frac{b^4\coth(dx+c)^3}{3}-2a\,b^3\coth(dx+c)^2-6a^2b^2\coth(dx+c)-b^4\coth(dx+c)-\frac{(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4)\ln(\coth(dx+c)-1)\cdots}{2}}{d}$
parts	$a^4x+\frac{b^4\left(-\frac{\coth(dx+c)^3}{3}-\coth(dx+c)-\frac{\ln(\coth(dx+c)-1)}{2}+\frac{\ln(\coth(dx+c)+1)}{2}\right)}{d}+\frac{4a\,b^3\left(-\frac{\coth(dx+c)^2}{2}-\frac{\ln(\coth(dx+\cdots}{2}\right.}{d}$
risch	$a^4x-4b\,a^3x+6a^2b^2x-4b^3ax+b^4x-\frac{8b\,a^3c}{d}-\frac{8b^3ac}{d}-\frac{4b^2(9a^2e^{4dx+4c}+6ab\,e^{4dx+4c}+3b^2e^{4dx+\cdots}}{3\cdots}$

input
```
int((a+b*coth(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*(12*(-a^3*b-a*b^3)*ln(1-tanh(d*x+c))+12*(a^3*b+a*b^3)*ln(tanh(d*x+c))-
b^4*coth(d*x+c)^3-6*a*b^3*coth(d*x+c)^2+3*(-6*a^2*b^2-b^4)*coth(d*x+c)+3*d
*x*(a-b)^4)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1396 vs. 2(99) = 198.

Time = 0.11 (sec) , antiderivative size = 1396, normalized size of antiderivative = 13.82

$$\int (a+b\coth(c+dx))^4\,dx = \text{Too large to display}$$

input
```
integrate((a+b*coth(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/3*(3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)^6 + 1
8*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)*sinh(d*x +
 c)^5 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*sinh(d*x + c)^6
- 3*(12*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3
 + b^4)*d*x)*cosh(d*x + c)^4 + 3*(15*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3
+ b^4)*d*x*cosh(d*x + c)^2 - 12*a^2*b^2 - 8*a*b^3 - 4*b^4 - 3*(a^4 - 4*a^3
*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*sinh(d*x + c)^4 - 36*a^2*b^2 - 8*b^4
+ 12*(5*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)^3 -
(12*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b
^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4
*a*b^3 + b^4)*d*x + 3*(24*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6
*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^2 + 3*(15*(a^4 - 4*a^3*b + 6*
a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)^4 + 24*a^2*b^2 + 8*a*b^3 + 4*b^
4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x - 6*(12*a^2*b^2 + 8*
a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*cosh(d*
x + c)^2)*sinh(d*x + c)^2 + 12*((a^3*b + a*b^3)*cosh(d*x + c)^6 + 6*(a^3*b
 + a*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^3*b + a*b^3)*sinh(d*x + c)^6
- 3*(a^3*b + a*b^3)*cosh(d*x + c)^4 - 3*(a^3*b + a*b^3 - 5*(a^3*b + a*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^4 - a^3*b - a*b^3 + 4*(5*(a^3*b + a*b^3)*c
osh(d*x + c)^3 - 3*(a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(92) = 184$.

Time = 1.20 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.87

$$\int (a + b\coth(c + dx))^4\,dx$$

$$= \begin{cases} x(a + b\coth(c))^4 \\ a^4x + 4a^3bx\coth\left(dx + \log\left(-e^{-dx}\right)\right) + 6a^2b^2x\coth^2\left(dx + \log\left(-e^{-dx}\right)\right) + 4ab^3x\coth^3\left(dx + \log\left(-e\right.\right.\right. \\ a^4x + 4a^3bx\coth\left(dx + \log\left(e^{-dx}\right)\right) + 6a^2b^2x\coth^2\left(dx + \log\left(e^{-dx}\right)\right) + 4ab^3x\coth^3\left(dx + \log\left(e^{-dx}\right)\right) \\ a^4x + 4a^3bx - \frac{4a^3b\log(\tanh(c+dx)+1)}{d} + \frac{4a^3b\log(\tanh(c+dx))}{d} + 6a^2b^2x - \frac{6a^2b^2}{d\tanh(c+dx)} + 4ab^3x - \frac{4ab^3\log(\tanh(c+\right.}{d} \end{cases}$$

input
```
integrate((a+b*coth(d*x+c))**4,x)
```

output
```
Piecewise((x*(a + b*coth(c))**4, Eq(d, 0)), (a**4*x + 4*a**3*b*x*coth(d*x
+ log(-exp(-d*x))) + 6*a**2*b**2*x*coth(d*x + log(-exp(-d*x)))**2 + 4*a*b*
*3*x*coth(d*x + log(-exp(-d*x)))**3 + b**4*x*coth(d*x + log(-exp(-d*x)))**
4, Eq(c, log(-exp(-d*x)))), (a**4*x + 4*a**3*b*x*coth(d*x + log(exp(-d*x))
) + 6*a**2*b**2*x*coth(d*x + log(exp(-d*x)))**2 + 4*a*b**3*x*coth(d*x + lo
g(exp(-d*x)))**3 + b**4*x*coth(d*x + log(exp(-d*x)))**4, Eq(c, log(exp(-d*
x)))), (a**4*x + 4*a**3*b*x - 4*a**3*b*log(tanh(c + d*x) + 1)/d + 4*a**3*b
*log(tanh(c + d*x))/d + 6*a**2*b**2*x - 6*a**2*b**2/(d*tanh(c + d*x)) + 4*
a*b**3*x - 4*a*b**3*log(tanh(c + d*x) + 1)/d + 4*a*b**3*log(tanh(c + d*x))
/d - 2*a*b**3/(d*tanh(c + d*x)**2) + b**4*x - b**4/(d*tanh(c + d*x)) - b**
4/(3*d*tanh(c + d*x)**3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(99) = 198$.

Time = 0.13 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.17

$$\int (a + b\coth(c + dx))^4\,dx$$

$$= \frac{1}{3}b^4\left(3x + \frac{3c}{d} - \frac{4\left(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} - 2\right)}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)}\right) + 4ab^3\left(x + \frac{c}{d} + \frac{\log\left(e^{(-dx-c)} + 1\right)}{d} + \frac{\log\left(e^{(-dx-c)} - 1\right)}{d} + \frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1)}\right) + 6a^2b^2\left(x + \frac{c}{d} + \frac{2}{d(e^{(-2dx-2c)} - 1)}\right) + a^4x + \frac{4a^3b\log\left(\sinh\left(dx + c\right)\right)}{d}$$

input
```
integrate((a+b*coth(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/3*b^4*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*
(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 4*a*b
^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*
d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + 6*a^2*b^2*(x
 + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + a^4*x + 4*a^3*b*log(sinh(d*x + c)
)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.51

$$\int (a+b\coth(c+dx))^4\,dx$$
$$= \frac{3\,(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4)(dx+c)+12\,(a^3b+ab^3)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)-\frac{4\,(9\,a^2b^2+2\,b^4+3\,(3\,a^2b^2+2\cdots}{\cdots}}{3\,d}$$

input
```
integrate((a+b*coth(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(d*x + c) + 12*(a^3*b +
 a*b^3)*log(abs(e^(2*d*x + 2*c) - 1)) - 4*(9*a^2*b^2 + 2*b^4 + 3*(3*a^2*b^
2 + 2*a*b^3 + b^4)*e^(4*d*x + 4*c) - 3*(6*a^2*b^2 + 2*a*b^3 + b^4)*e^(2*d*
x + 2*c))/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.56

$$\begin{aligned}\int (a+b\coth(c+dx))^4\,dx &= x\,(a-b)^4-\frac{4\,(3\,a^2\,b^2+2\,a\,b^3+b^4)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}\\ &\quad-\frac{4\,(b^4+2\,a\,b^3)}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}\\ &\quad+\frac{\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-1\right)\,\left(4\,a^3\,b+4\,a\,b^3\right)}{d}\\ &\quad-\frac{8\,b^4}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}\end{aligned}$$

input
```
int((a + b*coth(c + d*x))^4,x)
```

output
```
x*(a - b)^4 - (4*(2*a*b^3 + b^4 + 3*a^2*b^2))/(d*(exp(2*c + 2*d*x) - 1)) -
 (4*(2*a*b^3 + b^4))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) + (lo
g(exp(2*c)*exp(2*d*x) - 1)*(4*a*b^3 + 4*a^3*b))/d - (8*b^4)/(3*d*(3*exp(2*
c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 816, normalized size of antiderivative = 8.08

$$\int (a + b\coth(c + dx))^4 \, dx$$
$$= \frac{-3a^4dx - 3b^4dx - 4b^4 - 12e^{6dx+6c}a^2b^2 - 8e^{6dx+6c}a\,b^3 + 36e^{2dx+2c}a^2b^2 + 12e^{6dx+6c}\log\left(e^{dx+c} - 1\right)a^3b + 1}{}$$

input
```
int((a+b*coth(d*x+c))^4,x)
```

output
```
(12*e**(6*c + 6*d*x)*log(e**(c + d*x) - 1)*a**3*b + 12*e**(6*c + 6*d*x)*lo
g(e**(c + d*x) - 1)*a*b**3 + 12*e**(6*c + 6*d*x)*log(e**(c + d*x) + 1)*a**
3*b + 12*e**(6*c + 6*d*x)*log(e**(c + d*x) + 1)*a*b**3 + 3*e**(6*c + 6*d*x
)*a**4*d*x - 12*e**(6*c + 6*d*x)*a**3*b*d*x + 18*e**(6*c + 6*d*x)*a**2*b**
2*d*x - 12*e**(6*c + 6*d*x)*a**2*b**2 - 12*e**(6*c + 6*d*x)*a*b**3*d*x - 8
*e**(6*c + 6*d*x)*a*b**3 + 3*e**(6*c + 6*d*x)*b**4*d*x - 4*e**(6*c + 6*d*x
)*b**4 - 36*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**3*b - 36*e**(4*c + 4
*d*x)*log(e**(c + d*x) - 1)*a*b**3 - 36*e**(4*c + 4*d*x)*log(e**(c + d*x)
+ 1)*a**3*b - 36*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a*b**3 - 9*e**(4*c
 + 4*d*x)*a**4*d*x + 36*e**(4*c + 4*d*x)*a**3*b*d*x - 54*e**(4*c + 4*d*x)*
a**2*b**2*d*x + 36*e**(4*c + 4*d*x)*a*b**3*d*x - 9*e**(4*c + 4*d*x)*b**4*d
*x + 36*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**3*b + 36*e**(2*c + 2*d*x
)*log(e**(c + d*x) - 1)*a*b**3 + 36*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)
*a**3*b + 36*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a*b**3 + 9*e**(2*c + 2
*d*x)*a**4*d*x - 36*e**(2*c + 2*d*x)*a**3*b*d*x + 54*e**(2*c + 2*d*x)*a**2
*b**2*d*x + 36*e**(2*c + 2*d*x)*a**2*b**2 - 36*e**(2*c + 2*d*x)*a*b**3*d*x
 + 9*e**(2*c + 2*d*x)*b**4*d*x - 12*log(e**(c + d*x) - 1)*a**3*b - 12*log(
e**(c + d*x) - 1)*a*b**3 - 12*log(e**(c + d*x) + 1)*a**3*b - 12*log(e**(c
+ d*x) + 1)*a*b**3 - 3*a**4*d*x + 12*a**3*b*d*x - 18*a**2*b**2*d*x - 24*a*
*2*b**2 + 12*a*b**3*d*x + 8*a*b**3 - 3*b**4*d*x - 4*b**4)/(3*d*(e**(6*c...
```

3.79 $\int (a+b\coth(c+dx))^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 69

$$\int (a+b\coth(c+dx))^3\,dx = a\left(a^2+3b^2\right)x-\frac{2ab^2\coth(c+dx)}{d}-\frac{b(a+b\coth(c+dx))^2}{2d}+\frac{b(3a^2+b^2)\log(\sinh(c+dx))}{d}$$

output
```
a*(a^2+3*b^2)*x-2*a*b^2*coth(d*x+c)/d-1/2*b*(a+b*coth(d*x+c))^2/d+b*(3*a^2
+b^2)*ln(sinh(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.25

$$\int (a+b\coth(c+dx))^3\,dx = -\frac{6ab^2\coth(c+dx)+b^3\coth^2(c+dx)+(a+b)^3\log(1-\tanh(c+dx))-2b(3a^2+b^2)\log(\tanh(c+dx}{2d}$$

input
```
Integrate[(a + b*Coth[c + d*x])^3,x]
```

output
```
-1/2*(6*a*b^2*Coth[c + d*x] + b^3*Coth[c + d*x]^2 + (a + b)^3*Log[1 - Tanh
[c + d*x]] - 2*b*(3*a^2 + b^2)*Log[Tanh[c + d*x]] - (a - b)^3*Log[1 + Tanh
[c + d*x]])/d
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3963, 3042, 4008, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\coth(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{3963}$$

$$\int (a+b\coth(c+dx))\left(a^2+2b\coth(c+dx)a+b^2\right)dx-\frac{b(a+b\coth(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b(a+b\coth(c+dx))^2}{2d}+\int \left(a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)\left(a^2-2ib\tan\left(ic+idx+\frac{\pi}{2}\right)a+b^2\right)dx$$

$$\Big\downarrow \text{4008}$$

$$-ib\left(3a^2+b^2\right)\int i\coth(c+dx)dx+ax\left(a^2+3b^2\right)-\frac{2ab^2\coth(c+dx)}{d}-\frac{b(a+b\coth(c+dx))^2}{2d}$$

$$\Big\downarrow \text{26}$$

$$b\left(3a^2+b^2\right)\int \coth(c+dx)dx+ax\left(a^2+3b^2\right)-\frac{2ab^2\coth(c+dx)}{d}-\frac{b(a+b\coth(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$b(3a^2+b^2)\int -i\tan\left(ic+idx+\frac{\pi}{2}\right)dx+ax(a^2+3b^2)-\frac{2ab^2\coth(c+dx)}{d}-\frac{b(a+b\coth(c+dx))^2}{2d}$$

$\big\downarrow$ 26

$$-ib(3a^2+b^2)\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx+ax(a^2+3b^2)-\frac{2ab^2\coth(c+dx)}{d}-\frac{b(a+b\coth(c+dx))^2}{2d}$$

$\big\downarrow$ 3956

$$\frac{b(3a^2+b^2)\log(-i\sinh(c+dx))}{d}+ax(a^2+3b^2)-\frac{2ab^2\coth(c+dx)}{d}-\frac{b(a+b\coth(c+dx))^2}{2d}$$

input
```
Int[(a + b*Coth[c + d*x])^3,x]
```

output
```
a*(a^2 + 3*b^2)*x - (2*a*b^2*Coth[c + d*x])/d - (b*(a + b*Coth[c + d*x])^2
)/(2*d) + (b*(3*a^2 + b^2)*Log[(-I)*Sinh[c + d*x]])/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3963
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

method	result
parallelrisch	$\frac{\left(-6a^2b-2b^3\right)\ln(1-\tanh(dx+c))+\left(6a^2b+2b^3\right)\ln(\tanh(dx+c))-b^3\coth(dx+c)^2-6a\,b^2\coth(dx+c)+2dx(a-b)^3}{2d}$
derivativedivides	$\frac{-\frac{b^3\coth(dx+c)^2}{2}-3a\,b^2\coth(dx+c)-\frac{\left(a^3+3a^2b+3a\,b^2+b^3\right)\ln(\coth(dx+c)-1)}{2}+\frac{\left(a^3-3a^2b+3a\,b^2-b^3\right)\ln(\coth(dx+c)+1)}{2}}{d}$
default	$\frac{-\frac{b^3\coth(dx+c)^2}{2}-3a\,b^2\coth(dx+c)-\frac{\left(a^3+3a^2b+3a\,b^2+b^3\right)\ln(\coth(dx+c)-1)}{2}+\frac{\left(a^3-3a^2b+3a\,b^2-b^3\right)\ln(\coth(dx+c)+1)}{2}}{d}$
parts	$a^3x+\frac{b^3\left(-\frac{\coth(dx+c)^2}{2}-\frac{\ln(\coth(dx+c)-1)}{2}-\frac{\ln(\coth(dx+c)+1)}{2}\right)}{d}+\frac{3a^2b\ln(\sinh(dx+c))}{d}+\frac{3a\,b^2\left(-\coth(dx+c)-\right.}{}$
risch	$a^3x-3b\,a^2x+3a\,b^2x-b^3x-\frac{6bc\,a^2}{d}-\frac{2b^3c}{d}-\frac{2b^2\left(3a\,e^{2dx+2c}+b\,e^{2dx+2c}-3a\right)}{d\left(e^{2dx+2c}-1\right)^2}+\frac{3b\ln\left(e^{2dx+2c}-1\right)a}{d}$

input
```
int((a+b*coth(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*((-6*a^2*b-2*b^3)*ln(1-tanh(d*x+c))+(6*a^2*b+2*b^3)*ln(tanh(d*x+c))-b^
3*coth(d*x+c)^2-6*a*b^2*coth(d*x+c)+2*d*x*(a-b)^3)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. 2(67) = 134.

Time = 0.10 (sec) , antiderivative size = 654, normalized size of antiderivative = 9.48

$$\int (a+b\coth(c+dx))^3\,dx = \text{Too large to display}$$

input
```
integrate((a+b*coth(d*x+c))^3,x, algorithm="fricas")
```

output
```
((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^4 + 4*(a^3 - 3*a^2*b +
3*a*b^2 - b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 - 3*a^2*b + 3*a*b^
2 - b^3)*d*x*sinh(d*x + c)^4 + 6*a*b^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d
*x - 2*(3*a*b^2 + b^3 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x)*cosh(d*x + c)
^2 + 2*(3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^2 - 3*a*b^2 -
b^3 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x)*sinh(d*x + c)^2 + ((3*a^2*b + b
^3)*cosh(d*x + c)^4 + 4*(3*a^2*b + b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (3
*a^2*b + b^3)*sinh(d*x + c)^4 + 3*a^2*b + b^3 - 2*(3*a^2*b + b^3)*cosh(d*x
 + c)^2 - 2*(3*a^2*b + b^3 - 3*(3*a^2*b + b^3)*cosh(d*x + c)^2)*sinh(d*x +
 c)^2 + 4*((3*a^2*b + b^3)*cosh(d*x + c)^3 - (3*a^2*b + b^3)*cosh(d*x + c)
)*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*
((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^3 - (3*a*b^2 + b^3 + (a
^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d
*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*co
sh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*
x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(63) = 126$.

Time = 0.78 (sec) , antiderivative size = 291, normalized size of antiderivative = 4.22

$$\int (a+b\coth(c+dx))^3\,dx$$
$$= \begin{cases} x(a+b\coth(c))^3 \\ a^3x+3a^2bx\coth\left(dx+\log\left(-e^{-dx}\right)\right)+3ab^2x\coth^2\left(dx+\log\left(-e^{-dx}\right)\right)+b^3x\coth^3\left(dx+\log\left(-e^{-dx}\right.\right. \\ a^3x+3a^2bx\coth\left(dx+\log\left(e^{-dx}\right)\right)+3ab^2x\coth^2\left(dx+\log\left(e^{-dx}\right)\right)+b^3x\coth^3\left(dx+\log\left(e^{-dx}\right)\right) \\ a^3x+3a^2bx-\frac{3a^2b\log(\tanh(c+dx)+1)}{d}+\frac{3a^2b\log(\tanh(c+dx))}{d}+3ab^2x-\frac{3ab^2}{d\tanh(c+dx)}+b^3x-\frac{b^3\log(\tanh(c+dx)+1}{d} \end{cases}$$

input
```
integrate((a+b*coth(d*x+c))**3,x)
```

output
```
Piecewise((x*(a + b*coth(c))**3, Eq(d, 0)), (a**3*x + 3*a**2*b*x*coth(d*x
+ log(-exp(-d*x))) + 3*a*b**2*x*coth(d*x + log(-exp(-d*x)))**2 + b**3*x*co
th(d*x + log(-exp(-d*x)))**3, Eq(c, log(-exp(-d*x)))), (a**3*x + 3*a**2*b*
x*coth(d*x + log(exp(-d*x))) + 3*a*b**2*x*coth(d*x + log(exp(-d*x)))**2 +
b**3*x*coth(d*x + log(exp(-d*x)))**3, Eq(c, log(exp(-d*x)))), (a**3*x + 3*
a**2*b*x - 3*a**2*b*log(tanh(c + d*x) + 1)/d + 3*a**2*b*log(tanh(c + d*x))
/d + 3*a*b**2*x - 3*a*b**2/(d*tanh(c + d*x)) + b**3*x - b**3*log(tanh(c +
d*x) + 1)/d + b**3*log(tanh(c + d*x))/d - b**3/(2*d*tanh(c + d*x)**2), Tru
e))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(67) = 134$.

Time = 0.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.97

$$\int (a + b\coth(c + dx))^3 \, dx$$
$$= b^3 \left(x + \frac{c}{d} + \frac{\log\left(e^{(-dx-c)} + 1\right)}{d} + \frac{\log\left(e^{(-dx-c)} - 1\right)}{d} + \frac{2\, e^{(-2\, dx-2\, c)}}{d(2\, e^{(-2\, dx-2\, c)} - e^{(-4\, dx-4\, c)} - 1)} \right) + 3\, ab^2 \left(x + \frac{c}{d} + \frac{2}{d(e^{(-2\, dx-2\, c)} - 1)} \right) + a^3 x + \frac{3\, a^2 b \log\left(\sinh\left(dx + c\right)\right)}{d}$$

input
```
integrate((a+b*coth(d*x+c))^3,x, algorithm="maxima")
```

output
```
b^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2
*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + 3*a*b^2*(x
+ c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + a^3*x + 3*a^2*b*log(sinh(d*x + c))
/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.43

$$\int (a+b\coth(c+dx))^3\,dx$$
$$=\frac{(a^3-3\,a^2b+3\,ab^2-b^3)(dx+c)+(3\,a^2b+b^3)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{2\,\left(3\,ab^2-(3\,ab^2+b^3)e^{(2\,dx+2\,c)}\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^2}}{d}$$

input
```
integrate((a+b*coth(d*x+c))^3,x, algorithm="giac")
```

output
```
((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(d*x + c) + (3*a^2*b + b^3)*log(abs(e^(2*
d*x + 2*c) - 1)) + 2*(3*a*b^2 - (3*a*b^2 + b^3)*e^(2*d*x + 2*c))/(e^(2*d*x
 + 2*c) - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.41

$$\int (a+b\coth(c+dx))^3\,dx=x\,(a-b)^3-\frac{2\,(b^3+3\,a\,b^2)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{2\,b^3}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}+\frac{\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-1\right)\,\left(3\,a^2\,b+b^3\right)}{d}$$

input
```
int((a + b*coth(c + d*x))^3,x)
```

output
```
x*(a - b)^3 - (2*(3*a*b^2 + b^3))/(d*(exp(2*c + 2*d*x) - 1)) - (2*b^3)/(d*
(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) + (log(exp(2*c)*exp(2*d*x) -
1)*(3*a^2*b + b^3))/d
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 503, normalized size of antiderivative = 7.29

$$\int (a + b\coth(c + dx))^3 \, dx$$
$$= \frac{3\log(e^{dx+c} - 1)\, a^2 b + 3\log(e^{dx+c} + 1)\, a^2 b - 3e^{4dx+4c} a\, b^2 - b^3 + 3a\, b^2 - e^{4dx+4c} b^3 + e^{4dx+4c} \log(e^{dx+c} - 1}{}$$

input
```
int((a+b*coth(d*x+c))^3,x)
```

output
```
(3*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**2*b + e**(4*c + 4*d*x)*log(e*
*(c + d*x) - 1)*b**3 + 3*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**2*b + e
**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*b**3 + e**(4*c + 4*d*x)*a**3*d*x - 3
*e**(4*c + 4*d*x)*a**2*b*d*x + 3*e**(4*c + 4*d*x)*a*b**2*d*x - 3*e**(4*c +
 4*d*x)*a*b**2 - e**(4*c + 4*d*x)*b**3*d*x - e**(4*c + 4*d*x)*b**3 - 6*e**
(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**2*b - 2*e**(2*c + 2*d*x)*log(e**(c
+ d*x) - 1)*b**3 - 6*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a**2*b - 2*e**
(2*c + 2*d*x)*log(e**(c + d*x) + 1)*b**3 - 2*e**(2*c + 2*d*x)*a**3*d*x + 6
*e**(2*c + 2*d*x)*a**2*b*d*x - 6*e**(2*c + 2*d*x)*a*b**2*d*x + 2*e**(2*c +
 2*d*x)*b**3*d*x + 3*log(e**(c + d*x) - 1)*a**2*b + log(e**(c + d*x) - 1)*
b**3 + 3*log(e**(c + d*x) + 1)*a**2*b + log(e**(c + d*x) + 1)*b**3 + a**3*
d*x - 3*a**2*b*d*x + 3*a*b**2*d*x + 3*a*b**2 - b**3*d*x - b**3)/(d*(e**(4*
c + 4*d*x) - 2*e**(2*c + 2*d*x) + 1))
```

3.80 $\int (a+b\coth(c+dx))^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 38

$$\int (a+b\coth(c+dx))^2\,dx = \left(a^2+b^2\right)x - \frac{b^2\coth(c+dx)}{d} + \frac{2ab\log(\sinh(c+dx))}{d}$$

output
```
(a^2+b^2)*x-b^2*coth(d*x+c)/d+2*a*b*ln(sinh(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int (a+b\coth(c+dx))^2\,dx$$
$$= \frac{-2b^2\coth(c+dx)-(a+b)^2\log(1-\tanh(c+dx))+4ab\log(\tanh(c+dx))+(a-b)^2\log(1+\tanh(c}{2d}$$

input
```
Integrate[(a + b*Coth[c + d*x])^2,x]
```

output
```
(-2*b^2*Coth[c + d*x] - (a + b)^2*Log[1 - Tanh[c + d*x]] + 4*a*b*Log[Tanh[
c + d*x]] + (a - b)^2*Log[1 + Tanh[c + d*x]])/(2*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3958, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\coth(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3958}$$

$$-2iab\int i\coth(c+dx)dx+x\left(a^2+b^2\right)-\frac{b^2\coth(c+dx)}{d}$$

$$\Big\downarrow \text{26}$$

$$2ab\int \coth(c+dx)dx+x\left(a^2+b^2\right)-\frac{b^2\coth(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$2ab\int -i\tan\left(ic+idx+\frac{\pi}{2}\right)dx+x\left(a^2+b^2\right)-\frac{b^2\coth(c+dx)}{d}$$

$$\Big\downarrow \text{26}$$

$$-2iab\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx+x\left(a^2+b^2\right)-\frac{b^2\coth(c+dx)}{d}$$

$$\Big\downarrow \text{3956}$$

$$x\left(a^2+b^2\right)+\frac{2ab\log(-i\sinh(c+dx))}{d}-\frac{b^2\coth(c+dx)}{d}$$

input
```
Int[(a + b*Coth[c + d*x])^2,x]
```

output
```
(a^2 + b^2)*x - (b^2*Coth[c + d*x])/d + (2*a*b*Log[(-I)*Sinh[c + d*x]])/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.39

method	result	size
parallelrisch	$\frac{-2\ln(1-\tanh(dx+c))ab+2ab\ln(\tanh(dx+c))-b^2\coth(dx+c)+dx(a-b)^2}{d}$	53
parts	$a^2x+\frac{b^2\left(-\coth(dx+c)-\frac{\ln(\coth(dx+c)-1)}{2}+\frac{\ln(\coth(dx+c)+1)}{2}\right)}{d}+\frac{2ab\ln(\sinh(dx+c))}{d}$	59
derivativedivides	$\frac{-b^2\coth(dx+c)-\frac{(a^2+2ab+b^2)\ln(\coth(dx+c)-1)}{2}+\frac{(a^2-2ab+b^2)\ln(\coth(dx+c)+1)}{2}}{d}$	61
default	$\frac{-b^2\coth(dx+c)-\frac{(a^2+2ab+b^2)\ln(\coth(dx+c)-1)}{2}+\frac{(a^2-2ab+b^2)\ln(\coth(dx+c)+1)}{2}}{d}$	61
risch	$a^2x-2abx+b^2x-\frac{4abc}{d}-\frac{2b^2}{d(e^{2dx+2c}-1)}+\frac{2ab\ln(e^{2dx+2c}-1)}{d}$	65

input
```
int((a+b*coth(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
(-2*ln(1-tanh(d*x+c))*a*b+2*a*b*ln(tanh(d*x+c))-b^2*coth(d*x+c)+d*x*(a-b)^
2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(38) = 76$.

Time = 0.09 (sec) , antiderivative size = 205, normalized size of antiderivative = 5.39

$$\int (a + b\coth(c + dx))^2 \, dx$$
$$= \frac{(a^2 - 2\,ab + b^2)dx\cosh\left(dx + c\right)^2 + 2\left(a^2 - 2\,ab + b^2\right)dx\cosh\left(dx + c\right)\sinh\left(dx + c\right) + \left(a^2 - 2\,ab + b^2\right)dx}{d\cosh\left(dx + \right.}$$

input
```
integrate((a+b*coth(d*x+c))^2,x, algorithm="fricas")
```

output
```
((a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - 2*a*b + b^2)*d*x*cosh(
d*x + c)*sinh(d*x + c) + (a^2 - 2*a*b + b^2)*d*x*sinh(d*x + c)^2 - (a^2 -
2*a*b + b^2)*d*x - 2*b^2 + 2*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*si
nh(d*x + c) + a*b*sinh(d*x + c)^2 - a*b)*log(2*sinh(d*x + c)/(cosh(d*x + c
) - sinh(d*x + c))))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c)
+ d*sinh(d*x + c)^2 - d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(34) = 68$.

Time = 0.54 (sec) , antiderivative size = 177, normalized size of antiderivative = 4.66

$$\int (a + b\coth(c + dx))^2 \, dx$$
$$= \begin{cases} x(a + b\coth(c))^2 & \text{for } d = 0 \\ a^2x + 2abx\coth\left(dx + \log\left(-e^{-dx}\right)\right) + b^2x\coth^2\left(dx + \log\left(-e^{-dx}\right)\right) & \text{for } c = \log\left(-e^{-dx}\right) \\ a^2x + 2abx\coth\left(dx + \log\left(e^{-dx}\right)\right) + b^2x\coth^2\left(dx + \log\left(e^{-dx}\right)\right) & \text{for } c = \log\left(e^{-dx}\right) \\ a^2x + 2abx - \frac{2ab\log(\tanh(c+dx)+1)}{d} + \frac{2ab\log(\tanh(c+dx))}{d} + b^2x - \frac{b^2}{d\tanh(c+dx)} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*coth(d*x+c))**2,x)
```

output
```
Piecewise((x*(a + b*coth(c))**2, Eq(d, 0)), (a**2*x + 2*a*b*x*coth(d*x + l
og(-exp(-d*x))) + b**2*x*coth(d*x + log(-exp(-d*x)))**2, Eq(c, log(-exp(-d
*x)))), (a**2*x + 2*a*b*x*coth(d*x + log(exp(-d*x))) + b**2*x*coth(d*x + l
og(exp(-d*x)))**2, Eq(c, log(exp(-d*x)))), (a**2*x + 2*a*b*x - 2*a*b*log(t
anh(c + d*x) + 1)/d + 2*a*b*log(tanh(c + d*x))/d + b**2*x - b**2/(d*tanh(c
 + d*x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int (a+b\coth(c+dx))^2\,dx = b^2\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2\,dx-2\,c)}-1)}\right)+a^2x+\frac{2\,ab\log\left(\sinh\left(dx+c\right)\right)}{d}$$

input
```
integrate((a+b*coth(d*x+c))^2,x, algorithm="maxima")
```

output
```
b^2*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + a^2*x + 2*a*b*log(sinh(d*x
+ c))/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50

$$\int (a+b\coth(c+dx))^2\,dx = \frac{2\,ab\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+(a^2-2\,ab+b^2)(dx+c)-\frac{2\,b^2}{e^{(2\,dx+2\,c)}-1}}{d}$$

input
```
integrate((a+b*coth(d*x+c))^2,x, algorithm="giac")
```

output
```
(2*a*b*log(abs(e^(2*d*x + 2*c) - 1)) + (a^2 - 2*a*b + b^2)*(d*x + c) - 2*b
^2/(e^(2*d*x + 2*c) - 1))/d
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int (a + b\coth(c + dx))^2\,dx = x\,(a - b)^2 - \frac{2\,b^2}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x} - 1\right)} + \frac{2\,a\,b\,\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x} - 1\right)}{d}$$

input
```
int((a + b*coth(c + d*x))^2,x)
```

output
```
x*(a - b)^2 - (2*b^2)/(d*(exp(2*c + 2*d*x) - 1)) + (2*a*b*log(exp(2*c)*exp
(2*d*x) - 1))/d
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 178, normalized size of antiderivative = 4.68

$$\int (a + b\coth(c + dx))^2\,dx$$
$$= \frac{2e^{2dx+2c}\log\left(e^{dx+c} - 1\right)ab + 2e^{2dx+2c}\log\left(e^{dx+c} + 1\right)ab + e^{2dx+2c}a^2dx - 2e^{2dx+2c}abdx + e^{2dx+2c}b^2dx - 2e^{2}}{d\left(e^{2dx+2c} - 1\right)}$$

input
```
int((a+b*coth(d*x+c))^2,x)
```

output
```
(2*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a*b + 2*e**(2*c + 2*d*x)*log(e**
(c + d*x) + 1)*a*b + e**(2*c + 2*d*x)*a**2*d*x - 2*e**(2*c + 2*d*x)*a*b*d*
x + e**(2*c + 2*d*x)*b**2*d*x - 2*e**(2*c + 2*d*x)*b**2 - 2*log(e**(c + d*
x) - 1)*a*b - 2*log(e**(c + d*x) + 1)*a*b - a**2*d*x + 2*a*b*d*x - b**2*d*
x)/(d*(e**(2*c + 2*d*x) - 1))
```

3.81 $\int \frac{1}{a+b\coth(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 50

$$\int \frac{1}{a+b\coth(c+dx)}\,dx=\frac{ax}{a^2-b^2}-\frac{b\log(b\cosh(c+dx)+a\sinh(c+dx))}{(a^2-b^2)\,d}$$

output
```
a*x/(a^2-b^2)-b*ln(b*cosh(d*x+c)+a*sinh(d*x+c))/(a^2-b^2)/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.28

$$\int \frac{1}{a+b\coth(c+dx)}\,dx =\frac{(-a+b)\log(1-\tanh(c+dx))+(a+b)\log(1+\tanh(c+dx))-2b\log(b+a\tanh(c+dx))}{2(a-b)(a+b)d}$$

input
```
Integrate[(a + b*Coth[c + d*x])^(-1),x]
```

output
```
((-a + b)*Log[1 - Tanh[c + d*x]] + (a + b)*Log[1 + Tanh[c + d*x]] - 2*b*Lo
g[b + a*Tanh[c + d*x]])/(2*(a - b)*(a + b)*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 3965, 26, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\coth(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3965}$$

$$\frac{ax}{a^2-b^2}-\frac{ib\int -\frac{i(b+a\coth(c+dx))}{a+b\coth(c+dx)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{ax}{a^2-b^2}-\frac{b\int \frac{b+a\coth(c+dx)}{a+b\coth(c+dx)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{ax}{a^2-b^2}-\frac{b\int \frac{b-ia\tan\left(ic+idx+\frac{\pi}{2}\right)}{a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{4013}$$

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\sinh(c+dx)+b\cosh(c+dx))}{d\left(a^2-b^2\right)}$$

input
```
Int[(a + b*Coth[c + d*x])^(-1),x]
```

output
```
(a*x)/(a^2 - b^2) - (b*Log[b*Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2
)*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3965
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Simp[b/(a^2 + b^2)   Int[(b - a*Tan[c + d*x])/(a + b*Tan[c
+ d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{-b\ln(a\tanh(dx+c)+b)+\ln(1-\tanh(dx+c))b+dx(a+b)}{d(a^2-b^2)}$	50
derivativedivides	$\frac{\frac{\ln(\coth(dx+c)+1)}{2a-2b}-\frac{\ln(\coth(dx+c)-1)}{2a+2b}-\frac{b\ln(a+b\coth(dx+c))}{(a+b)(a-b)}}{d}$	71
default	$\frac{\frac{\ln(\coth(dx+c)+1)}{2a-2b}-\frac{\ln(\coth(dx+c)-1)}{2a+2b}-\frac{b\ln(a+b\coth(dx+c))}{(a+b)(a-b)}}{d}$	71
risch	$\frac{x}{a+b}+\frac{2bx}{a^2-b^2}+\frac{2bc}{d(a^2-b^2)}-\frac{b\ln\left(e^{2dx+2c}-\frac{a-b}{a+b}\right)}{d(a^2-b^2)}$	82

input
```
int(1/(a+b*coth(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(-b*ln(a*tanh(d*x+c)+b)+ln(1-tanh(d*x+c))*b+d*x*(a+b))/d/(a^2-b^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.24

$$\int \frac{1}{a+b\coth(c+dx)}\,dx = \frac{(a+b)dx - b\log\left(\frac{2(b\cosh(dx+c)+a\sinh(dx+c))}{\cosh(dx+c)-\sinh(dx+c)}\right)}{(a^2-b^2)d}$$

input
```
integrate(1/(a+b*coth(d*x+c)),x, algorithm="fricas")
```

output
```
((a + b)*d*x - b*log(2*(b*cosh(d*x + c) + a*sinh(d*x + c))/(cosh(d*x + c)
- sinh(d*x + c))))/((a^2 - b^2)*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(37) = 74$.

Time = 1.45 (sec) , antiderivative size = 236, normalized size of antiderivative = 4.72

$$\int \frac{1}{a+b\coth(c+dx)}\,dx$$

$$= \begin{cases} \frac{\tilde{\infty}x}{\coth(c)} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{x-\frac{\log(\tanh(c+dx)+1)}{d}}{b} & \text{for } a=0 \\ -\frac{dx\tanh(c+dx)}{2bd\tanh(c+dx)-2bd} + \frac{dx}{2bd\tanh(c+dx)-2bd} - \frac{1}{2bd\tanh(c+dx)-2bd} & \text{for } a=-b \\ \frac{dx\tanh(c+dx)}{2bd\tanh(c+dx)+2bd} + \frac{dx}{2bd\tanh(c+dx)+2bd} + \frac{1}{2bd\tanh(c+dx)+2bd} & \text{for } a=b \\ \frac{x}{a+b\coth(c)} & \text{for } d=0 \\ \frac{adx}{a^2d-b^2d} - \frac{bdx}{a^2d-b^2d} + \frac{b\log(\tanh(c+dx)+1)}{a^2d-b^2d} - \frac{b\log\left(\tanh(c+dx)+\frac{b}{a}\right)}{a^2d-b^2d} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*coth(d*x+c)),x)
```

output
```
Piecewise((zoo*x/coth(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - log(tanh(
c + d*x) + 1)/d)/b, Eq(a, 0)), (-d*x*tanh(c + d*x)/(2*b*d*tanh(c + d*x) -
2*b*d) + d*x/(2*b*d*tanh(c + d*x) - 2*b*d) - 1/(2*b*d*tanh(c + d*x) - 2*b*
d), Eq(a, -b)), (d*x*tanh(c + d*x)/(2*b*d*tanh(c + d*x) + 2*b*d) + d*x/(2*
b*d*tanh(c + d*x) + 2*b*d) + 1/(2*b*d*tanh(c + d*x) + 2*b*d), Eq(a, b)), (
x/(a + b*coth(c)), Eq(d, 0)), (a*d*x/(a**2*d - b**2*d) - b*d*x/(a**2*d - b
**2*d) + b*log(tanh(c + d*x) + 1)/(a**2*d - b**2*d) - b*log(tanh(c + d*x)
+ b/a)/(a**2*d - b**2*d), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \frac{1}{a+b\coth(c+dx)}\,dx = -\frac{b\log\left(-(a-b)e^{(-2\,dx-2\,c)}+a+b\right)}{(a^2-b^2)d}+\frac{dx+c}{(a+b)d}$$

input
```
integrate(1/(a+b*coth(d*x+c)),x, algorithm="maxima")
```

output
```
-b*log(-(a - b)*e^(-2*d*x - 2*c) + a + b)/((a^2 - b^2)*d) + (d*x + c)/((a
+ b)*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.24

$$\int \frac{1}{a+b\coth(c+dx)}\,dx = -\frac{\frac{b\log\left(\left|ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right|\right)}{a^2-b^2}-\frac{dx+c}{a-b}}{d}$$

input
```
integrate(1/(a+b*coth(d*x+c)),x, algorithm="giac")
```

output
```
-(b*log(abs(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a + b))/(a^2 - b^2) -
(d*x + c)/(a - b))/d
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10

$$\int \frac{1}{a+b\coth(c+dx)}\,dx = \frac{x}{a-b} - \frac{b\ln\left(b-a+a\,e^{2\,c}\,e^{2\,d\,x}+b\,e^{2\,c}\,e^{2\,d\,x}\right)}{a^2\,d-b^2\,d}$$

input
```
int(1/(a + b*coth(c + d*x)),x)
```

output
```
x/(a - b) - (b*log(b - a + a*exp(2*c)*exp(2*d*x) + b*exp(2*c)*exp(2*d*x)))
/(a^2*d - b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\coth(c+dx)}\,dx = \frac{-\log\left(e^{2dx+2c}a+e^{2dx+2c}b-a+b\right)b+adx+bdx}{d\left(a^2-b^2\right)}$$

input
```
int(1/(a+b*coth(d*x+c)),x)
```

output
```
( - log(e**(2*c + 2*d*x)*a + e**(2*c + 2*d*x)*b - a + b)*b + a*d*x + b*d*x
)/(d*(a**2 - b**2))
```

3.82 $\int \frac{1}{(a+b\coth(c+dx))^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 85

$$\int \frac{1}{(a+b\coth(c+dx))^2}\,dx = \frac{(a^2+b^2)\,x}{(a^2-b^2)^2} + \frac{b}{(a^2-b^2)\,d(a+b\coth(c+dx))} - \frac{2ab\log(b\cosh(c+dx)+a\sinh(c+dx))}{(a^2-b^2)^2\,d}$$

output
```
(a^2+b^2)*x/(a^2-b^2)^2+b/(a^2-b^2)/d/(a+b*coth(d*x+c))-2*a*b*ln(b*cosh(d*
x+c)+a*sinh(d*x+c))/(a^2-b^2)^2/d
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18

$$\int \frac{1}{(a+b\coth(c+dx))^2}\,dx = \frac{-\frac{\log(1-\tanh(c+dx))}{(a+b)^2} + \frac{\log(1+\tanh(c+dx))}{(a-b)^2} + \frac{2b\left(-2a^2\log(b+a\tanh(c+dx))+\frac{-a^2b+b^3}{b+a\tanh(c+dx)}\right)}{a(a^2-b^2)^2}}{2d}$$

input
```
Integrate[(a + b*Coth[c + d*x])^(-2),x]
```

output
```
(-(Log[1 - Tanh[c + d*x]]/(a + b)^2) + Log[1 + Tanh[c + d*x]]/(a - b)^2 +
(2*b*(-2*a^2*Log[b + a*Tanh[c + d*x]] + (-(a^2*b) + b^3)/(b + a*Tanh[c + d
*x])))/(a*(a^2 - b^2)^2))/(2*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 3964, 3042, 4014, 26, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\coth(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3964}$$

$$\frac{\int \frac{a-b\coth(c+dx)}{a+b\coth(c+dx)}dx}{a^2-b^2}+\frac{b}{d(a^2-b^2)(a+b\coth(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b}{d(a^2-b^2)(a+b\coth(c+dx))}+\frac{\int \frac{a+ib\tan\left(ic+idx+\frac{\pi}{2}\right)}{a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{4014}$$

$$\frac{b}{d(a^2-b^2)(a+b\coth(c+dx))}+\frac{\frac{x(a^2+b^2)}{a^2-b^2}-\frac{2iab\int -\frac{i(b+a\coth(c+dx))}{a+b\coth(c+dx)}dx}{a^2-b^2}}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{x(a^2+b^2)}{a^2-b^2}-\frac{2ab\int \frac{b+a\coth(c+dx)}{a+b\coth(c+dx)}dx}{a^2-b^2}}{a^2-b^2}+\frac{b}{d(a^2-b^2)(a+b\coth(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b}{d(a^2-b^2)(a+b\coth(c+dx))}+\frac{\frac{x(a^2+b^2)}{a^2-b^2}-\frac{2ab\int\frac{b-ia\tan\left(ic+idx+\frac{\pi}{2}\right)}{a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{a^2-b^2}$$

$$\Big\downarrow \text{4013}$$

$$\frac{b}{d(a^2-b^2)(a+b\coth(c+dx))}+\frac{\frac{x(a^2+b^2)}{a^2-b^2}-\frac{2ab\log(a\sinh(c+dx)+b\cosh(c+dx))}{d(a^2-b^2)}}{a^2-b^2}$$

input
```
Int[(a + b*Coth[c + d*x])^(-2),x]
```

output
```
b/((a^2 - b^2)*d*(a + b*Coth[c + d*x])) + (((a^2 + b^2)*x)/(a^2 - b^2) - (
2*a*b*Log[b*Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)*d))/(a^2 - b^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3964
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)
 Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{\frac{\ln(\coth(dx+c)+1)}{2(a-b)^2}-\frac{\ln(\coth(dx+c)-1)}{2(a+b)^2}+\frac{b}{(a+b)(a-b)(a+b\coth(dx+c))}-\frac{2ab\ln(a+b\coth(dx+c))}{(a+b)^2(a-b)^2}}{d}$
default	$\frac{\frac{\ln(\coth(dx+c)+1)}{2(a-b)^2}-\frac{\ln(\coth(dx+c)-1)}{2(a+b)^2}+\frac{b}{(a+b)(a-b)(a+b\coth(dx+c))}-\frac{2ab\ln(a+b\coth(dx+c))}{(a+b)^2(a-b)^2}}{d}$
parallelrisch	$\frac{(-2\tanh(dx+c)a^3b-2a^2b^2)\ln(a\tanh(dx+c)+b)+(2\tanh(dx+c)a^3b+2a^2b^2)\ln(1-\tanh(dx+c))+(a^2dx(a+b)\tanh(}{(a-b)^2(a+b)^2(a\tanh(dx+c)+b)ad}$
risch	$\frac{x}{a^2+2ab+b^2}+\frac{4abx}{a^4-2a^2b^2+b^4}+\frac{4abc}{d(a^4-2a^2b^2+b^4)}-\frac{2b^2}{(a-b)d(a^2+2ab+b^2)(a\,e^{2dx+2c}+b\,e^{2dx+2c}-a+b)}-\frac{2ab\ln}{d(a}$

input
```
int(1/(a+b*coth(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/(a-b)^2*ln(coth(d*x+c)+1)-1/2/(a+b)^2*ln(coth(d*x+c)-1)+b/(a+b)/(
a-b)/(a+b*coth(d*x+c))-2*a*b/(a+b)^2/(a-b)^2*ln(a+b*coth(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. 2(85) = 170.

Time = 0.10 (sec) , antiderivative size = 426, normalized size of antiderivative = 5.01

$$\int \frac{1}{(a+b\coth(c+dx))^2}\,dx$$
$$= \frac{(a^3+3a^2b+3ab^2+b^3)dx\cosh(dx+c)^2+2(a^3+3a^2b+3ab^2+b^3)dx\cosh(dx+c)\sinh(dx+c)+(}{(a^5+a^4b-2a^3b^2-2a^2b^3+ab^4+b^5)d\cosh(dx+}$$

input
```
integrate(1/(a+b*coth(d*x+c))^2,x, algorithm="fricas")
```

output
```
((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 + 2*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c) + (a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*d*x*sinh(d*x + c)^2 - 2*a*b^2 + 2*b^3 - (a^3 + a^2*b - a*b^2 - b^3)
*d*x + 2*(a^2*b - a*b^2 - (a^2*b + a*b^2)*cosh(d*x + c)^2 - 2*(a^2*b + a*b
^2)*cosh(d*x + c)*sinh(d*x + c) - (a^2*b + a*b^2)*sinh(d*x + c)^2)*log(2*(
b*cosh(d*x + c) + a*sinh(d*x + c))/(cosh(d*x + c) - sinh(d*x + c))))/((a^5
 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*cosh(d*x + c)^2 + 2*(a^5
 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x +
 c) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*sinh(d*x + c)^
2 - (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\coth(c+dx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*coth(d*x+c))**2,x)
```

output
```
Exception raised: TypeError >> Invalid NaN comparison
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.46

$$\int \frac{1}{(a+b\coth(c+dx))^2}\,dx = -\frac{2\,ab\log\left(-(a-b)e^{(-2\,dx-2\,c)}+a+b\right)}{(a^4-2\,a^2b^2+b^4)d} - \frac{2\,b^2}{\left(a^4-2\,a^2b^2+b^4-(a^4-2\,a^3b+2\,ab^3-b^4)e^{(-2\,dx-2\,c)}\right)d} + \frac{dx+c}{(a^2+2\,ab+b^2)d}$$

input
```
integrate(1/(a+b*coth(d*x+c))^2,x, algorithm="maxima")
```

output
```
-2*a*b*log(-(a - b)*e^(-2*d*x - 2*c) + a + b)/((a^4 - 2*a^2*b^2 + b^4)*d)
- 2*b^2/((a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*d*
x - 2*c))*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.53

$$\int \frac{1}{(a+b\coth(c+dx))^2}\,dx = -\frac{\frac{2\,ab\log\left(\left|ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right|\right)}{a^4-2\,a^2b^2+b^4}-\frac{dx+c}{a^2-2\,ab+b^2}+\frac{2\,(ab^2-b^3)}{(ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b)(a+b)^2(a-b)^2}}{d}$$

input
```
integrate(1/(a+b*coth(d*x+c))^2,x, algorithm="giac")
```

output
```
-(2*a*b*log(abs(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a + b))/(a^4 - 2*a
^2*b^2 + b^4) - (d*x + c)/(a^2 - 2*a*b + b^2) + 2*(a*b^2 - b^3)/((a*e^(2*d
*x + 2*c) + b*e^(2*d*x + 2*c) - a + b)*(a + b)^2*(a - b)^2))/d
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+b\coth(c+dx))^2}\,dx = \frac{x}{(a-b)^2}-\frac{2\,a\,b\,\ln\left(b-a+a\,e^{2\,c}\,e^{2\,d\,x}+b\,e^{2\,c}\,e^{2\,d\,x}\right)}{d\,a^4-2\,d\,a^2\,b^2+d\,b^4} - \frac{2\,b^2}{d\,(a+b)^2\,(a-b)\,\left(b-a+e^{2\,c+2\,d\,x}\,(a+b)\right)}$$

input
```
int(1/(a + b*coth(c + d*x))^2,x)
```

output
```
x/(a - b)^2 - (2*a*b*log(b - a + a*exp(2*c)*exp(2*d*x) + b*exp(2*c)*exp(2*
d*x)))/(a^4*d + b^4*d - 2*a^2*b^2*d) - (2*b^2)/(d*(a + b)^2*(a - b)*(b - a
 + exp(2*c + 2*d*x)*(a + b)))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 428, normalized size of antiderivative = 5.04

$$\int \frac{1}{(a+b\coth(c+dx))^2}\,dx = \frac{-2e^{2dx+2c}\log\left(e^{2dx+2c}a+e^{2dx+2c}b-a+b\right)a^2b-2e^{2dx+2c}\log\left(e^{2dx+2c}a+e^{2dx+2c}b-a+b\right)a\,b^2+e^{2dx+2c}a}{d\left(e^{2dx+2c}a^5+e\right.}$$

input
```
int(1/(a+b*coth(d*x+c))^2,x)
```

output
```
( - 2*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x)*a + e**(2*c + 2*d*x)*b - a + b
)*a**2*b - 2*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x)*a + e**(2*c + 2*d*x)*b
- a + b)*a*b**2 + e**(2*c + 2*d*x)*a**3*d*x + 3*e**(2*c + 2*d*x)*a**2*b*d*
x + 3*e**(2*c + 2*d*x)*a*b**2*d*x - 2*e**(2*c + 2*d*x)*a*b**2 + e**(2*c +
2*d*x)*b**3*d*x - 2*e**(2*c + 2*d*x)*b**3 + 2*log(e**(2*c + 2*d*x)*a + e**
(2*c + 2*d*x)*b - a + b)*a**2*b - 2*log(e**(2*c + 2*d*x)*a + e**(2*c + 2*d
*x)*b - a + b)*a*b**2 - a**3*d*x - a**2*b*d*x + a*b**2*d*x + b**3*d*x)/(d*
(e**(2*c + 2*d*x)*a**5 + e**(2*c + 2*d*x)*a**4*b - 2*e**(2*c + 2*d*x)*a**3
*b**2 - 2*e**(2*c + 2*d*x)*a**2*b**3 + e**(2*c + 2*d*x)*a*b**4 + e**(2*c +
 2*d*x)*b**5 - a**5 + a**4*b + 2*a**3*b**2 - 2*a**2*b**3 - a*b**4 + b**5))
```

3.83 $\int \frac{1}{(a+b\coth(c+dx))^3} dx$

Optimal result

Integrand size = 12, antiderivative size = 129

$$\int \frac{1}{(a+b\coth(c+dx))^3} dx = \frac{a(a^2+3b^2)\,x}{(a^2-b^2)^3} + \frac{b}{2(a^2-b^2)\,d(a+b\coth(c+dx))^2} + \frac{2ab}{(a^2-b^2)^2\,d(a+b\coth(c+dx))} - \frac{b(3a^2+b^2)\log(b\cosh(c+dx)+a\sinh(c+dx))}{(a^2-b^2)^3\,d}$$

output
```
a*(a^2+3*b^2)*x/(a^2-b^2)^3+1/2*b/(a^2-b^2)/d/(a+b*coth(d*x+c))^2+2*a*b/(a
^2-b^2)^2/d/(a+b*coth(d*x+c))-b*(3*a^2+b^2)*ln(b*cosh(d*x+c)+a*sinh(d*x+c)
)/(a^2-b^2)^3/d
```

Mathematica [A] (verified)

Time = 2.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.04

$$\int \frac{1}{(a+b\coth(c+dx))^3} dx = -\frac{\frac{\log(1-\tanh(c+dx))}{(a+b)^3} - \frac{\log(1+\tanh(c+dx))}{(a-b)^3} + \frac{b\left(2(3a^2+b^2)\log(b+a\tanh(c+dx)) + \frac{b(-a^2+b^2)\left(-5a^2b+b^3+(-6a^3+2ab^2)\tanh(c+dx)\right)}{a^2(b+a\tanh(c+dx))^2}\right)}{(a^2-b^2)^3}}{2d}$$

input
```
Integrate[(a + b*Coth[c + d*x])^(-3),x]
```

output
```
-1/2*(Log[1 - Tanh[c + d*x]]/(a + b)^3 - Log[1 + Tanh[c + d*x]]/(a - b)^3
+ (b*(2*(3*a^2 + b^2)*Log[b + a*Tanh[c + d*x]] + (b*(-a^2 + b^2)*(-5*a^2*b
 + b^3 + (-6*a^3 + 2*a*b^2)*Tanh[c + d*x]))/(a^2*(b + a*Tanh[c + d*x])^2))
)/(a^2 - b^2)^3)/d
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3964, 3042, 4012, 3042, 4014, 26, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\coth(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3964}$$

$$\frac{\int \frac{a-b\coth(c+dx)}{(a+b\coth(c+dx))^2}dx}{a^2-b^2}+\frac{b}{2d\left(a^2-b^2\right)(a+b\coth(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b}{2d\left(a^2-b^2\right)(a+b\coth(c+dx))^2}+\frac{\int \frac{a+ib\tan\left(ic+idx+\frac{\pi}{2}\right)}{\left(a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\frac{\int \frac{a^2-2b\coth(c+dx)a+b^2}{a+b\coth(c+dx)}dx}{a^2-b^2}+\frac{2ab}{d(a^2-b^2)(a+b\coth(c+dx))}}{a^2-b^2}+\frac{b}{2d\left(a^2-b^2\right)(a+b\coth(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b}{2d\left(a^2-b^2\right)(a+b\coth(c+dx))^2}+\frac{\frac{2ab}{d(a^2-b^2)(a+b\coth(c+dx))}+\frac{\int \frac{a^2+2ib\tan\left(ic+idx+\frac{\pi}{2}\right)a+b^2}{a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{a^2-b^2}$$

$\Big\downarrow$ 4014

$$\frac{b}{2d\left(a^2-b^2\right)(a+b\coth(c+dx))^2}+\frac{\frac{2ab}{d(a^2-b^2)(a+b\coth(c+dx))}+\frac{\frac{ax\left(a^2+3b^2\right)}{a^2-b^2}-\frac{ib\left(3a^2+b^2\right)\int -\frac{i(b+a\coth(c+dx))}{a+b\coth(c+dx)}dx}{a^2-b^2}}{a^2-b^2}}{a^2-b^2}$$

$\Big\downarrow$ 26

$$\frac{\frac{\frac{ax\left(a^2+3b^2\right)}{a^2-b^2}-\frac{b\left(3a^2+b^2\right)\int \frac{b+a\coth(c+dx)}{a+b\coth(c+dx)}dx}{a^2-b^2}}{a^2-b^2}+\frac{2ab}{d(a^2-b^2)(a+b\coth(c+dx))}}{a^2-b^2}+\frac{b}{2d\left(a^2-b^2\right)(a+b\coth(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{b}{2d\left(a^2-b^2\right)(a+b\coth(c+dx))^2}+\frac{\frac{2ab}{d(a^2-b^2)(a+b\coth(c+dx))}+\frac{\frac{ax\left(a^2+3b^2\right)}{a^2-b^2}-\frac{b\left(3a^2+b^2\right)\int \frac{b-ia\tan\left(ic+idx+\frac{\pi}{2}\right)}{a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{a^2-b^2}}{a^2-b^2}$$

$\Big\downarrow$ 4013

$$\frac{b}{2d\left(a^2-b^2\right)(a+b\coth(c+dx))^2}+\frac{\frac{2ab}{d(a^2-b^2)(a+b\coth(c+dx))}+\frac{\frac{ax\left(a^2+3b^2\right)}{a^2-b^2}-\frac{b\left(3a^2+b^2\right)\log(a\sinh(c+dx)+b\cosh(c+dx))}{d\left(a^2-b^2\right)}}{a^2-b^2}}{a^2-b^2}$$

input
```
Int[(a + b*Coth[c + d*x])^(-3),x]
```

output
```
b/(2*(a^2 - b^2)*d*(a + b*Coth[c + d*x])^2) + ((2*a*b)/((a^2 - b^2)*d*(a +
 b*Coth[c + d*x])) + ((a*(a^2 + 3*b^2)*x)/(a^2 - b^2) - (b*(3*a^2 + b^2)*L
og[b*Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)*d))/(a^2 - b^2))/(a^2
- b^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3964
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)
 Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{\ln(\coth(dx+c)+1)}{2(a-b)^3}-\frac{\ln(\coth(dx+c)-1)}{2(a+b)^3}+\frac{b}{2(a+b)(a-b)(a+b\coth(dx+c))^2}+\frac{2ab}{(a+b)^2(a-b)^2(a+b\coth(dx+c))}-\frac{b\left(3a^2+b^2\right)\ln(a+b\,c}{(a+b)^3(a-}}{d}$
default	$\frac{\frac{\ln(\coth(dx+c)+1)}{2(a-b)^3}-\frac{\ln(\coth(dx+c)-1)}{2(a+b)^3}+\frac{b}{2(a+b)(a-b)(a+b\coth(dx+c))^2}+\frac{2ab}{(a+b)^2(a-b)^2(a+b\coth(dx+c))}-\frac{b\left(3a^2+b^2\right)\ln(a+b\,c}{(a+b)^3(a-}}{d}$
parallelrisch	$\frac{-3b\left(a^2+\frac{b^2}{3}\right)a^2(a\tanh(dx+c)+b)^2\ln(a\tanh(dx+c)+b)+3b\left(a^2+\frac{b^2}{3}\right)a^2(a\tanh(dx+c)+b)^2\ln(1-\tanh(dx+c))+\left(a^4\right.}{(a-}$
risch	$\frac{x}{a^3+3a^2b+3a\,b^2+b^3}+\frac{6b\,a^2x}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{2b^3x}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{6bc\,a^2}{d(a^6-3a^4b^2+3a^2b^4-b^6)}+\frac{2}{d(a^6-3a^4b}$

input
```
int(1/(a+b*coth(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/(a-b)^3*ln(coth(d*x+c)+1)-1/2/(a+b)^3*ln(coth(d*x+c)-1)+1/2*b/(a+
b)/(a-b)/(a+b*coth(d*x+c))^2+2*a*b/(a+b)^2/(a-b)^2/(a+b*coth(d*x+c))-b*(3*
a^2+b^2)/(a+b)^3/(a-b)^3*ln(a+b*coth(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1431 vs. 2(127) = 254.

Time = 0.12 (sec) , antiderivative size = 1431, normalized size of antiderivative = 11.09

$$\int \frac{1}{(a+b\coth(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*coth(d*x+c))^3,x, algorithm="fricas")
```

output
```
((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(d*x +
c)^4 + 4*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cos
h(d*x + c)*sinh(d*x + c)^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*
a*b^4 + b^5)*d*x*sinh(d*x + c)^4 + 6*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 + (a^5
 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*x - 2*(3*a^3*b^2 - a^2*b
^3 - 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^
5)*d*x)*cosh(d*x + c)^2 - 2*(3*a^3*b^2 - a^2*b^3 - 3*a*b^4 + b^5 - 3*(a^5
+ 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(d*x + c)^2 +
 (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*x)*sinh(d*x + c
)^2 - (3*a^4*b - 6*a^3*b^2 + 4*a^2*b^3 - 2*a*b^4 + b^5 + (3*a^4*b + 6*a^3*
b^2 + 4*a^2*b^3 + 2*a*b^4 + b^5)*cosh(d*x + c)^4 + 4*(3*a^4*b + 6*a^3*b^2
+ 4*a^2*b^3 + 2*a*b^4 + b^5)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^4*b + 6*
a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + b^5)*sinh(d*x + c)^4 - 2*(3*a^4*b - 2*a^2*
b^3 - b^5)*cosh(d*x + c)^2 - 2*(3*a^4*b - 2*a^2*b^3 - b^5 - 3*(3*a^4*b + 6
*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4
*((3*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + b^5)*cosh(d*x + c)^3 - (3*a
^4*b - 2*a^2*b^3 - b^5)*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*cosh(d*x +
c) + a*sinh(d*x + c))/(cosh(d*x + c) - sinh(d*x + c))) + 4*((a^5 + 5*a^4*b
 + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(d*x + c)^3 - (3*a^3*b
^2 - a^2*b^3 - 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 -...
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\coth(c+dx))^3} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*coth(d*x+c))**3,x)
```

output
```
Exception raised: TypeError >> Invalid NaN comparison
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(127) = 254$.

Time = 0.19 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.50

$$\int \frac{1}{(a+b\coth(c+dx))^3}\,dx = -\frac{(3a^2b+b^3)\log\left(-(a-b)e^{(-2dx-2c)}+a+b\right)}{(a^6-3a^4b^2+3a^2b^4-b^6)d} - \frac{2\left(3a^2b^2+3ab^3-(3a^2b^2-\right.}{(a^7+a^6b-3a^5b^2-3a^4b^3+3a^3b^4+3a^2b^5-ab^6-b^7-2(a^7-a^6b-3a^5b^2+3a^4b^3+3a^3b^4-3a^2} + \frac{dx+c}{(a^3+3a^2b+3ab^2+b^3)d}$$

input
```
integrate(1/(a+b*coth(d*x+c))^3,x, algorithm="maxima")
```

output
```
-(3*a^2*b + b^3)*log(-(a - b)*e^(-2*d*x - 2*c) + a + b)/((a^6 - 3*a^4*b^2
+ 3*a^2*b^4 - b^6)*d) - 2*(3*a^2*b^2 + 3*a*b^3 - (3*a^2*b^2 - 2*a*b^3 - b^
4)*e^(-2*d*x - 2*c))/((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3
*a^2*b^5 - a*b^6 - b^7 - 2*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^
4 - 3*a^2*b^5 - a*b^6 + b^7)*e^(-2*d*x - 2*c) + (a^7 - 3*a^6*b + a^5*b^2 +
 5*a^4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*e^(-4*d*x - 4*c))*d) + (
d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.57

$$\int \frac{1}{(a+b\coth(c+dx))^3}\,dx = -\frac{\frac{(3a^2b+b^3)\log\left(\left|ae^{(2dx+2c)}+be^{(2dx+2c)}-a+b\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6} - \frac{dx+c}{a^3-3a^2b+3ab^2-b^3} + \frac{2\left((3a^2b^2-4ab^3+b^4)e^{(2dx+2c)}-\frac{3(a^3b^2-2a^2b^3+ab^4)}{a+b}\right)}{\left(ae^{(2dx+2c)}+be^{(2dx+2c)}-a+b\right)^2(a+b)^2(a-b)^3}}{d}$$

input
```
integrate(1/(a+b*coth(d*x+c))^3,x, algorithm="giac")
```

output
```
-((3*a^2*b + b^3)*log(abs(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a + b))/
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) - (d*x + c)/(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3) + 2*((3*a^2*b^2 - 4*a*b^3 + b^4)*e^(2*d*x + 2*c) - 3*(a^3*b^2 - 2*a^
2*b^3 + a*b^4)/(a + b))/((a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a + b)^2
*(a + b)^2*(a - b)^3))/d
```

Mupad [B] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.51

$$\int \frac{1}{(a+b\coth(c+dx))^3}\,dx = \frac{x}{(a-b)^3} - \frac{\ln\left(b-a+a\,e^{2\,c}\,e^{2\,d\,x}+b\,e^{2\,c}\,e^{2\,d\,x}\right)\,\left(3\,a^2\,b+b^3\right)}{d\,a^6-3\,d\,a^4\,b^2+3\,d\,a^2\,b^4-d\,b^6} + \frac{2\,b^3}{d\,(a+b)^3\,(a-b)\,\left(e^{4\,c+4\,d\,x}\,(a+b)^2+(a-b)^2-2\,e^{2\,c+2\,d\,x}\,(a+b)\,(a-b)\right)} - \frac{2\,(3\,a\,b^2-b^3)}{d\,(a+b)^3\,(a-b)^2\,\left(b-a+e^{2\,c+2\,d\,x}\,(a+b)\right)}$$

input
```
int(1/(a + b*coth(c + d*x))^3,x)
```

output
```
x/(a - b)^3 - (log(b - a + a*exp(2*c)*exp(2*d*x) + b*exp(2*c)*exp(2*d*x))*
(3*a^2*b + b^3))/(a^6*d - b^6*d + 3*a^2*b^4*d - 3*a^4*b^2*d) + (2*b^3)/(d*
(a + b)^3*(a - b)*(exp(4*c + 4*d*x)*(a + b)^2 + (a - b)^2 - 2*exp(2*c + 2*
d*x)*(a + b)*(a - b))) - (2*(3*a*b^2 - b^3))/(d*(a + b)^3*(a - b)^2*(b - a
 + exp(2*c + 2*d*x)*(a + b)))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1186, normalized size of antiderivative = 9.19

$$\int \frac{1}{(a+b\coth(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*coth(d*x+c))^3,x)
```

output

```
( - 3*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*a + e**(2*c + 2*d*x)*b - a + b
)*a**4*b - 6*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*a + e**(2*c + 2*d*x)*b
- a + b)*a**3*b**2 - 4*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*a + e**(2*c +
 2*d*x)*b - a + b)*a**2*b**3 - 2*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*a +
 e**(2*c + 2*d*x)*b - a + b)*a*b**4 - e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x
)*a + e**(2*c + 2*d*x)*b - a + b)*b**5 + e**(4*c + 4*d*x)*a**5*d*x + 5*e**
(4*c + 4*d*x)*a**4*b*d*x + 10*e**(4*c + 4*d*x)*a**3*b**2*d*x - 3*e**(4*c +
 4*d*x)*a**3*b**2 + 10*e**(4*c + 4*d*x)*a**2*b**3*d*x - 5*e**(4*c + 4*d*x)
*a**2*b**3 + 5*e**(4*c + 4*d*x)*a*b**4*d*x - e**(4*c + 4*d*x)*a*b**4 + e**
(4*c + 4*d*x)*b**5*d*x + e**(4*c + 4*d*x)*b**5 + 6*e**(2*c + 2*d*x)*log(e*
*(2*c + 2*d*x)*a + e**(2*c + 2*d*x)*b - a + b)*a**4*b - 4*e**(2*c + 2*d*x)
*log(e**(2*c + 2*d*x)*a + e**(2*c + 2*d*x)*b - a + b)*a**2*b**3 - 2*e**(2*
c + 2*d*x)*log(e**(2*c + 2*d*x)*a + e**(2*c + 2*d*x)*b - a + b)*b**5 - 2*e
**(2*c + 2*d*x)*a**5*d*x - 6*e**(2*c + 2*d*x)*a**4*b*d*x - 4*e**(2*c + 2*d
*x)*a**3*b**2*d*x + 4*e**(2*c + 2*d*x)*a**2*b**3*d*x + 6*e**(2*c + 2*d*x)*
a*b**4*d*x + 2*e**(2*c + 2*d*x)*b**5*d*x - 3*log(e**(2*c + 2*d*x)*a + e**(
2*c + 2*d*x)*b - a + b)*a**4*b + 6*log(e**(2*c + 2*d*x)*a + e**(2*c + 2*d*
x)*b - a + b)*a**3*b**2 - 4*log(e**(2*c + 2*d*x)*a + e**(2*c + 2*d*x)*b -
a + b)*a**2*b**3 + 2*log(e**(2*c + 2*d*x)*a + e**(2*c + 2*d*x)*b - a + b)*
a*b**4 - log(e**(2*c + 2*d*x)*a + e**(2*c + 2*d*x)*b - a + b)*b**5 + a*...
```

3.84 $\int \frac{1}{(a+b\coth(c+dx))^4} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 169

$$\int \frac{1}{(a+b\coth(c+dx))^4} \, dx = \frac{(a^4+6a^2b^2+b^4)\,x}{(a^2-b^2)^4} + \frac{b}{3\,(a^2-b^2)\,d(a+b\coth(c+dx))^3} + \frac{ab}{(a^2-b^2)^2\,d(a+b\coth(c+dx))^2} + \frac{b(3a^2+b^2)}{(a^2-b^2)^3\,d(a+b\coth(c+dx))} - \frac{4ab(a^2+b^2)\log(b\cosh(c+dx)+a\sinh(c+dx))}{(a^2-b^2)^4\,d}$$

output
```
(a^4+6*a^2*b^2+b^4)*x/(a^2-b^2)^4+1/3*b/(a^2-b^2)/d/(a+b*coth(d*x+c))^3+a*
b/(a^2-b^2)^2/d/(a+b*coth(d*x+c))^2+b*(3*a^2+b^2)/(a^2-b^2)^3/d/(a+b*coth(
d*x+c))-4*a*b*(a^2+b^2)*ln(b*cosh(d*x+c)+a*sinh(d*x+c))/(a^2-b^2)^4/d
```

Mathematica [A] (verified)

Time = 6.16 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.27

$$\int \frac{1}{(a+b\coth(c+dx))^4}\,dx = -\frac{\log(1-\tanh(c+dx))}{2(a+b)^4d}+\frac{\log(1+\tanh(c+dx))}{2(a-b)^4d} - \frac{4ab(a^2+b^2)\log(b+a\tanh(c+dx))}{(a^2-b^2)^4\,d} - \frac{b^4}{3a^3(a^2-b^2)\,d(b+a\tanh(c+dx))^3} + \frac{b^3(2a^2-b^2)}{a^3(a^2-b^2)^2\,d(b+a\tanh(c+dx))^2} - \frac{b^2(6a^4-3a^2b^2+b^4)}{a^3(a^2-b^2)^3\,d(b+a\tanh(c+dx))}$$

input
```
Integrate[(a + b*Coth[c + d*x])^(-4),x]
```

output
```
-1/2*Log[1 - Tanh[c + d*x]]/((a + b)^4*d) + Log[1 + Tanh[c + d*x]]/(2*(a -
 b)^4*d) - (4*a*b*(a^2 + b^2)*Log[b + a*Tanh[c + d*x]])/((a^2 - b^2)^4*d)
- b^4/(3*a^3*(a^2 - b^2)*d*(b + a*Tanh[c + d*x])^3) + (b^3*(2*a^2 - b^2))/
(a^3*(a^2 - b^2)^2*d*(b + a*Tanh[c + d*x])^2) - (b^2*(6*a^4 - 3*a^2*b^2 +
b^4))/(a^3*(a^2 - b^2)^3*d*(b + a*Tanh[c + d*x]))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3964, 3042, 4012, 3042, 4012, 3042, 4014, 26, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\coth(c+dx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{ 3964}$$

$$\frac{\int \frac{a-b\coth(c+dx)}{(a+b\coth(c+dx))^3}dx}{a^2-b^2}+\frac{b}{3d\left(a^2-b^2\right)(a+b\coth(c+dx))^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b}{3d\left(a^2-b^2\right)(a+b\coth(c+dx))^3}+\frac{\int \frac{a+ib\tan\left(ic+idx+\frac{\pi}{2}\right)}{\left(a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^3}dx}{a^2-b^2}$$

$$\Big\downarrow \text{ 4012}$$

$$\frac{\frac{\int \frac{a^2-2b\coth(c+dx)a+b^2}{(a+b\coth(c+dx))^2}dx}{a^2-b^2}+\frac{ab}{d(a^2-b^2)(a+b\coth(c+dx))^2}}{a^2-b^2}+\frac{b}{3d\left(a^2-b^2\right)(a+b\coth(c+dx))^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b}{3d\left(a^2-b^2\right)(a+b\coth(c+dx))^3}+\frac{\frac{ab}{d(a^2-b^2)(a+b\coth(c+dx))^2}+\frac{\int \frac{a^2+2ib\tan\left(ic+idx+\frac{\pi}{2}\right)a+b^2}{\left(a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)\right)^2}dx}{a^2-b^2}}{a^2-b^2}$$

$$\Big\downarrow \text{ 4012}$$

$$\frac{\frac{\frac{\int \frac{a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\coth(c+dx)}{a+b\coth(c+dx)}dx}{a^2-b^2}+\frac{b\left(3a^2+b^2\right)}{d\left(a^2-b^2\right)(a+b\coth(c+dx))}}{a^2-b^2}+\frac{ab}{d(a^2-b^2)(a+b\coth(c+dx))^2}}{a^2-b^2}+\frac{b}{3d\left(a^2-b^2\right)(a+b\coth(c+dx))^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b}{3d\left(a^2-b^2\right)(a+b\coth(c+dx))^3}+\frac{\frac{ab}{d(a^2-b^2)(a+b\coth(c+dx))^2}+\frac{\frac{b\left(3a^2+b^2\right)}{d\left(a^2-b^2\right)(a+b\coth(c+dx))}+\frac{\int \frac{a\left(a^2+3b^2\right)+ib\left(3a^2+b^2\right)\tan\left(ic+idx+\frac{\pi}{2}\right)}{a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{a^2-b^2}}{a^2-b^2}$$

$$\Big\downarrow \text{ 4014}$$

$$\frac{b}{3d\left(a^2-b^2\right)(a+b\coth(c+dx))^3}+\frac{\frac{ab}{d(a^2-b^2)(a+b\coth(c+dx))^2}+\frac{\frac{b\left(3a^2+b^2\right)}{d\left(a^2-b^2\right)(a+b\coth(c+dx))}+\frac{\frac{x\left(a^4+6a^2b^2+b^4\right)}{a^2-b^2}-\frac{4iab\left(a^2+b^2\right)\int -\frac{i(b+a\coth(c+dx))}{a+b\coth(c+dx)}dx}{a^2-b^2}}{a^2-b^2}}{a^2-b^2}}{a^2-b^2}$$

$\Big\downarrow$ 26

$$\frac{\frac{\frac{\frac{x\left(a^4+6a^2b^2+b^4\right)}{a^2-b^2}-\frac{4ab\left(a^2+b^2\right)\int \frac{b+a\coth(c+dx)}{a+b\coth(c+dx)}dx}{a^2-b^2}}{a^2-b^2}+\frac{b\left(3a^2+b^2\right)}{d\left(a^2-b^2\right)(a+b\coth(c+dx))}}{a^2-b^2}+\frac{ab}{d(a^2-b^2)(a+b\coth(c+dx))^2}}{a^2-b^2}+\frac{b}{3d\left(a^2-b^2\right)(a+b\coth(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{b}{3d\left(a^2-b^2\right)(a+b\coth(c+dx))^3}+\frac{\frac{ab}{d(a^2-b^2)(a+b\coth(c+dx))^2}+\frac{\frac{b\left(3a^2+b^2\right)}{d\left(a^2-b^2\right)(a+b\coth(c+dx))}+\frac{\frac{x\left(a^4+6a^2b^2+b^4\right)}{a^2-b^2}-\frac{4ab\left(a^2+b^2\right)\int \frac{b-ia\tan\left(ic+idx+\frac{\pi}{2}\right)}{a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{a^2-b^2}}{a^2-b^2}}{a^2-b^2}$$

$\Big\downarrow$ 4013

$$\frac{b}{3d\left(a^2-b^2\right)(a+b\coth(c+dx))^3}+\frac{\frac{ab}{d(a^2-b^2)(a+b\coth(c+dx))^2}+\frac{\frac{b\left(3a^2+b^2\right)}{d\left(a^2-b^2\right)(a+b\coth(c+dx))}+\frac{\frac{x\left(a^4+6a^2b^2+b^4\right)}{a^2-b^2}-\frac{4ab\left(a^2+b^2\right)\log(a\sinh(c+dx)+b\cosh(c+dx))}{d\left(a^2-b^2\right)}}{a^2-b^2}}{a^2-b^2}}{a^2-b^2}$$

input

```
Int[(a + b*Coth[c + d*x])^(-4),x]
```

output

```
b/(3*(a^2 - b^2)*d*(a + b*Coth[c + d*x])^3) + ((a*b)/((a^2 - b^2)*d*(a + b
*Coth[c + d*x])^2) + ((b*(3*a^2 + b^2))/((a^2 - b^2)*d*(a + b*Coth[c + d*x
])) + (((a^4 + 6*a^2*b^2 + b^4)*x)/(a^2 - b^2) - (4*a*b*(a^2 + b^2)*Log[b*
Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)*d))/(a^2 - b^2))/(a^2 - b^2
))/(a^2 - b^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3964
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)
 Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{\ln(\coth(dx+c)+1)}{2(a-b)^4}-\frac{\ln(\coth(dx+c)-1)}{2(a+b)^4}+\frac{b}{3(a+b)(a-b)(a+b\coth(dx+c))^3}+\frac{ab}{(a+b)^2(a-b)^2(a+b\coth(dx+c))^2}+\frac{b\left(3a^2+b\right.}{(a+b)^3(a-b)^3(a+b}}{d}$
default	$\frac{\frac{\ln(\coth(dx+c)+1)}{2(a-b)^4}-\frac{\ln(\coth(dx+c)-1)}{2(a+b)^4}+\frac{b}{3(a+b)(a-b)(a+b\coth(dx+c))^3}+\frac{ab}{(a+b)^2(a-b)^2(a+b\coth(dx+c))^2}+\frac{b\left(3a^2+b\right.}{(a+b)^3(a-b)^3(a+b}}{d}$
parallelrisch	$\frac{-4ba^2(a^2+b^2)(a\tanh(dx+c)+b)^3\ln(a\tanh(dx+c)+b)+4ba^2(a^2+b^2)(a\tanh(dx+c)+b)^3\ln(1-\tanh(dx+c))+(a+b)}{}$
risch	$\frac{x}{a^4+4a^3b+6a^2b^2+4a\,b^3+b^4}+\frac{8b\,a^3x}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}+\frac{8b^3ax}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}+\frac{8b\,a^3}{d(a^8-4a^6b^2+6a^4}$

input
```
int(1/(a+b*coth(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/(a-b)^4*ln(coth(d*x+c)+1)-1/2/(a+b)^4*ln(coth(d*x+c)-1)+1/3*b/(a+
b)/(a-b)/(a+b*coth(d*x+c))^3+a*b/(a+b)^2/(a-b)^2/(a+b*coth(d*x+c))^2+b*(3*
a^2+b^2)/(a+b)^3/(a-b)^3/(a+b*coth(d*x+c))-4*b*a*(a^2+b^2)/(a+b)^4/(a-b)^4
*ln(a+b*coth(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3698 vs. 2(167) = 334.

Time = 0.15 (sec) , antiderivative size = 3698, normalized size of antiderivative = 21.88

$$\int \frac{1}{(a+b\coth(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*coth(d*x+c))^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\coth(c+dx))^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*coth(d*x+c))**4,x)
```

output
```
Exception raised: TypeError >> Invalid NaN comparison
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 522 vs. 2(167) = 334.

Time = 0.27 (sec) , antiderivative size = 522, normalized size of antiderivative = 3.09

$$\int \frac{1}{(a+b\coth(c+dx))^4}\,dx = -\frac{4\,(a^3b+ab^3)\log\left(-(a-b)e^{(-2\,dx-2\,c)}+a+b\right)}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)d}$$
$$-\frac{}{3\,(a^{10}+2\,a^9b-3\,a^8b^2-8\,a^7b^3+2\,a^6b^4+12\,a^5b^5+2\,a^4b^6-8\,a^3b^7-3\,a^2b^8+2\,ab^9+b^{10}-3\,(a^{10}-5}$$
$$+\frac{dx+c}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)d}$$

input
```
integrate(1/(a+b*coth(d*x+c))^4,x, algorithm="maxima")
```

output
```
-4*(a^3*b + a*b^3)*log(-(a - b)*e^(-2*d*x - 2*c) + a + b)/((a^8 - 4*a^6*b^
2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d) - 4/3*(9*a^4*b^2 + 18*a^3*b^3 + 11*a^2
*b^4 + 4*a*b^5 + 2*b^6 - 3*(6*a^4*b^2 + 2*a^3*b^3 - 5*a^2*b^4 - 2*a*b^5 -
b^6)*e^(-2*d*x - 2*c) + 3*(3*a^4*b^2 - 4*a^3*b^3 + b^6)*e^(-4*d*x - 4*c))/
((a^10 + 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*
b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10 - 3*(a^10 - 5*a^8*b^2 + 10*a^
6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*e^(-2*d*x - 2*c) + 3*(a^10 - 2*a^9*
b - 3*a^8*b^2 + 8*a^7*b^3 + 2*a^6*b^4 - 12*a^5*b^5 + 2*a^4*b^6 + 8*a^3*b^7
 - 3*a^2*b^8 - 2*a*b^9 + b^10)*e^(-4*d*x - 4*c) - (a^10 - 4*a^9*b + 3*a^8*
b^2 + 8*a^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 4*a*b^
9 - b^10)*e^(-6*d*x - 6*c))*d) + (d*x + c)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4
*a*b^3 + b^4)*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.79

$$\int \frac{1}{(a+b\coth(c+dx))^4}\,dx = -\frac{\frac{12\,(a^3b+ab^3)\log\left(\left|ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right|\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}-\frac{3\,(dx+c)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}+\frac{4\left(3\left(3\,a^4b^2-2\,a^3b^3-2\,a^2b^4+2\,ab^5-b^6\right)e^{(4\,dx+4\,c)}\ldots}{(\ldots}}{3\,d}$$

input

```
integrate(1/(a+b*coth(d*x+c))^4,x, algorithm="giac")
```

output

```
-1/3*(12*(a^3*b + a*b^3)*log(abs(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a
 + b))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - 3*(d*x + c)/(a^4
- 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + 4*(3*(3*a^4*b^2 - 2*a^3*b^3 - 2*a
^2*b^4 + 2*a*b^5 - b^6)*e^(4*d*x + 4*c) - 3*(6*a^4*b^2 - 14*a^3*b^3 + 11*a
^2*b^4 - 4*a*b^5 + b^6)*e^(2*d*x + 2*c) + (9*a^5*b^2 - 27*a^4*b^3 + 29*a^3
*b^4 - 15*a^2*b^5 + 6*a*b^6 - 2*b^7)/(a + b))/((a*e^(2*d*x + 2*c) + b*e^(2
*d*x + 2*c) - a + b)^3*(a + b)^3*(a - b)^4))/d
```

Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.83

$$\begin{aligned}
&\int \frac{1}{(a+b\coth(c+dx))^4}\,dx \\
&= \frac{x}{(a-b)^4} - \frac{\ln\left(b-a+a\,e^{2\,c}\,e^{2\,d\,x}+b\,e^{2\,c}\,e^{2\,d\,x}\right)\,\left(4\,a^3\,b+4\,a\,b^3\right)}{d\,a^8-4\,d\,a^6\,b^2+6\,d\,a^4\,b^4-4\,d\,a^2\,b^6+d\,b^8} \\
&\quad - \frac{4\,\left(3\,a^2\,b^2-2\,a\,b^3+b^4\right)}{d\,(a+b)^4\,(a-b)^3\,\left(b-a+e^{2\,c+2\,d\,x}\,(a+b)\right)} \\
&\quad - \frac{8\,b^4}{3\,d\,(a+b)^4\,(a-b)\,\left(e^{6\,c+6\,d\,x}\,(a+b)^3-(a-b)^3+3\,e^{2\,c+2\,d\,x}\,(a+b)\,(a-b)^2-3\,e^{4\,c+4\,d\,x}\,(a+b)^2\,(a-\ldots\right)} \\
&\quad + \frac{4\,\left(2\,a\,b^3-b^4\right)}{d\,(a+b)^4\,(a-b)^2\,\left(e^{4\,c+4\,d\,x}\,(a+b)^2+(a-b)^2-2\,e^{2\,c+2\,d\,x}\,(a+b)\,(a-b)\right)}
\end{aligned}$$

input

```
int(1/(a + b*coth(c + d*x))^4,x)
```

output
```
x/(a - b)^4 - (log(b - a + a*exp(2*c)*exp(2*d*x) + b*exp(2*c)*exp(2*d*x))*
(4*a*b^3 + 4*a^3*b))/(a^8*d + b^8*d - 4*a^2*b^6*d + 6*a^4*b^4*d - 4*a^6*b^
2*d) - (4*(b^4 - 2*a*b^3 + 3*a^2*b^2))/(d*(a + b)^4*(a - b)^3*(b - a + exp
(2*c + 2*d*x)*(a + b))) - (8*b^4)/(3*d*(a + b)^4*(a - b)*(exp(6*c + 6*d*x)
*(a + b)^3 - (a - b)^3 + 3*exp(2*c + 2*d*x)*(a + b)*(a - b)^2 - 3*exp(4*c
+ 4*d*x)*(a + b)^2*(a - b))) + (4*(2*a*b^3 - b^4))/(d*(a + b)^4*(a - b)^2*
(exp(4*c + 4*d*x)*(a + b)^2 + (a - b)^2 - 2*exp(2*c + 2*d*x)*(a + b)*(a -
b)))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 2312, normalized size of antiderivative = 13.68

$$\int \frac{1}{(a+b\coth(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*coth(d*x+c))^4,x)
```

output
```
( - 12*e**(6*c + 6*d*x)*log(e**(2*c + 2*d*x)*a + e**(2*c + 2*d*x)*b - a +
b)*a**6*b - 36*e**(6*c + 6*d*x)*log(e**(2*c + 2*d*x)*a + e**(2*c + 2*d*x)*
b - a + b)*a**5*b**2 - 48*e**(6*c + 6*d*x)*log(e**(2*c + 2*d*x)*a + e**(2*
c + 2*d*x)*b - a + b)*a**4*b**3 - 48*e**(6*c + 6*d*x)*log(e**(2*c + 2*d*x)
*a + e**(2*c + 2*d*x)*b - a + b)*a**3*b**4 - 36*e**(6*c + 6*d*x)*log(e**(2
*c + 2*d*x)*a + e**(2*c + 2*d*x)*b - a + b)*a**2*b**5 - 12*e**(6*c + 6*d*x
)*log(e**(2*c + 2*d*x)*a + e**(2*c + 2*d*x)*b - a + b)*a*b**6 + 3*e**(6*c
+ 6*d*x)*a**7*d*x + 21*e**(6*c + 6*d*x)*a**6*b*d*x + 63*e**(6*c + 6*d*x)*a
**5*b**2*d*x - 12*e**(6*c + 6*d*x)*a**5*b**2 + 105*e**(6*c + 6*d*x)*a**4*b
**3*d*x - 28*e**(6*c + 6*d*x)*a**4*b**3 + 105*e**(6*c + 6*d*x)*a**3*b**4*d
*x - 16*e**(6*c + 6*d*x)*a**3*b**4 + 63*e**(6*c + 6*d*x)*a**2*b**5*d*x + 2
1*e**(6*c + 6*d*x)*a*b**6*d*x - 4*e**(6*c + 6*d*x)*a*b**6 + 3*e**(6*c + 6*
d*x)*b**7*d*x - 4*e**(6*c + 6*d*x)*b**7 + 36*e**(4*c + 4*d*x)*log(e**(2*c
+ 2*d*x)*a + e**(2*c + 2*d*x)*b - a + b)*a**6*b + 36*e**(4*c + 4*d*x)*log(
e**(2*c + 2*d*x)*a + e**(2*c + 2*d*x)*b - a + b)*a**5*b**2 - 36*e**(4*c +
4*d*x)*log(e**(2*c + 2*d*x)*a + e**(2*c + 2*d*x)*b - a + b)*a**2*b**5 - 36
*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*a + e**(2*c + 2*d*x)*b - a + b)*a*b
**6 - 9*e**(4*c + 4*d*x)*a**7*d*x - 45*e**(4*c + 4*d*x)*a**6*b*d*x - 81*e*
*(4*c + 4*d*x)*a**5*b**2*d*x - 45*e**(4*c + 4*d*x)*a**4*b**3*d*x + 45*e**(
4*c + 4*d*x)*a**3*b**4*d*x + 81*e**(4*c + 4*d*x)*a**2*b**5*d*x + 45*e**...
```

3.85 $\int \frac{1}{4+6\coth(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{1}{4+6\coth(c+dx)}\,dx = -\frac{x}{5}+\frac{3\log(3\cosh(c+dx)+2\sinh(c+dx))}{10d}$$

output
```
-1/5*x+3/10*ln(3*cosh(d*x+c)+2*sinh(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \frac{1}{4+6\coth(c+dx)}\,dx = -\frac{\log(1-\tanh(c+dx))}{20d}-\frac{\log(1+\tanh(c+dx))}{4d}+\frac{3\log(3+2\tanh(c+dx))}{10d}$$

input
```
Integrate[(4 + 6*Coth[c + d*x])^(-1),x]
```

output
```
-1/20*Log[1 - Tanh[c + d*x]]/d - Log[1 + Tanh[c + d*x]]/(4*d) + (3*Log[3 +
 2*Tanh[c + d*x]])/(10*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 3965, 26, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{6\coth(c+dx)+4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{4-6i\tan\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3965}$$

$$-\frac{x}{5}+\frac{3}{10}i\int -\frac{i(2\coth(c+dx)+3)}{3\coth(c+dx)+2}dx$$

$$\Big\downarrow \text{26}$$

$$\frac{3}{10}\int \frac{2\coth(c+dx)+3}{3\coth(c+dx)+2}dx-\frac{x}{5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{x}{5}+\frac{3}{10}\int \frac{3-2i\tan\left(ic+idx+\frac{\pi}{2}\right)}{2-3i\tan\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4013}$$

$$\frac{3\log(2\sinh(c+dx)+3\cosh(c+dx))}{10d}-\frac{x}{5}$$

input
```
Int[(4 + 6*Coth[c + d*x])^(-1),x]
```

output
```
-1/5*x + (3*Log[3*Cosh[c + d*x] + 2*Sinh[c + d*x]])/(10*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3965
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Simp[b/(a^2 + b^2)   Int[(b - a*Tan[c + d*x])/(a + b*Tan[c
+ d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
risch	$-\frac{x}{2}-\frac{3c}{5d}+\frac{3\ln\left(\mathrm{e}^{2dx+2c}+\frac{1}{5}\right)}{10d}$	28
parallelrisch	$\frac{3\ln(2\tanh(dx+c)+3)-3\ln(1-\tanh(dx+c))+\ln\left(\frac{1}{8}\right)-5dx}{10d}$	39
derivativedivides	$\frac{-\frac{\ln(\coth(dx+c)+1)}{2}-\frac{\ln(\coth(dx+c)-1)}{10}+\frac{3\ln(2+3\coth(dx+c))}{5}}{2d}$	42
default	$\frac{-\frac{\ln(\coth(dx+c)+1)}{2}-\frac{\ln(\coth(dx+c)-1)}{10}+\frac{3\ln(2+3\coth(dx+c))}{5}}{2d}$	42

input
```
int(1/(4+6*coth(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x-3/5*c/d+3/10/d*ln(exp(2*d*x+2*c)+1/5)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \frac{1}{4+6\coth(c+dx)}\,dx = -\frac{5\,dx - 3\,\log\left(\frac{2\,(3\,\cosh(dx+c)+2\,\sinh(dx+c))}{\cosh(dx+c)-\sinh(dx+c)}\right)}{10\,d}$$

input
```
integrate(1/(4+6*coth(d*x+c)),x, algorithm="fricas")
```

output
```
-1/10*(5*d*x - 3*log(2*(3*cosh(d*x + c) + 2*sinh(d*x + c))/(cosh(d*x + c)
- sinh(d*x + c))))/d
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{1}{4+6\coth(c+dx)}\,dx = \begin{cases} \frac{x}{10} - \frac{3\log(\tanh(c+dx)+1)}{10d} + \frac{3\log(2\tanh(c+dx)+3)}{10d} & \text{for}\ d \neq 0 \\ \frac{x}{6\coth(c)+4} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(4+6*coth(d*x+c)),x)
```

output
```
Piecewise((x/10 - 3*log(tanh(c + d*x) + 1)/(10*d) + 3*log(2*tanh(c + d*x)
+ 3)/(10*d), Ne(d, 0)), (x/(6*coth(c) + 4), True))
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1}{4+6\coth(c+dx)}\,dx = \frac{dx+c}{10\,d} + \frac{3\,\log\left(e^{(-2\,dx-2\,c)}+5\right)}{10\,d}$$

input
```
integrate(1/(4+6*coth(d*x+c)),x, algorithm="maxima")
```

output
```
1/10*(d*x + c)/d + 3/10*log(e^(-2*d*x - 2*c) + 5)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{1}{4+6\coth(c+dx)}\,dx = -\frac{5\,dx+5\,c-3\,\log\left(5\,e^{(2\,dx+2\,c)}+1\right)}{10\,d}$$

input
```
integrate(1/(4+6*coth(d*x+c)),x, algorithm="giac")
```

output
```
-1/10*(5*d*x + 5*c - 3*log(5*e^(2*d*x + 2*c) + 1))/d
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{1}{4+6\coth(c+dx)}\,dx = \frac{3\,\ln\left(e^{2\,c}\,e^{2\,d\,x}+\frac{1}{5}\right)}{10\,d}-\frac{x}{2}$$

input
```
int(1/(6*coth(c + d*x) + 4),x)
```

output
```
(3*log(exp(2*c)*exp(2*d*x) + 1/5))/(10*d) - x/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{4+6\coth(c+dx)}\,dx = \frac{3\log\left(5e^{2dx+2c}+1\right)-5dx}{10d}$$

input
```
int(1/(4+6*coth(d*x+c)),x)
```

output
```
(3*log(5*e**(2*c + 2*d*x) + 1) - 5*d*x)/(10*d)
```

3.86 $\int \frac{1}{4-6\coth(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{1}{4-6\coth(c+dx)}\,dx = -\frac{x}{5} - \frac{3\log(3\cosh(c+dx)-2\sinh(c+dx))}{10d}$$

output
```
-1/5*x-3/10*ln(3*cosh(d*x+c)-2*sinh(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \frac{1}{4-6\coth(c+dx)}\,dx = -\frac{3\log(3-2\tanh(c+dx))}{10d} + \frac{\log(1-\tanh(c+dx))}{4d} + \frac{\log(1+\tanh(c+dx))}{20d}$$

input
```
Integrate[(4 - 6*Coth[c + d*x])^(-1),x]
```

output
```
(-3*Log[3 - 2*Tanh[c + d*x]])/(10*d) + Log[1 - Tanh[c + d*x]]/(4*d) + Log[
1 + Tanh[c + d*x]]/(20*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3965, 26, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4-6\coth(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{4+6i\tan\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3965}$$

$$-\frac{x}{5}-\frac{3}{10}i\int \frac{i(3-2\coth(c+dx))}{2-3\coth(c+dx)}dx$$

$$\Big\downarrow \text{ 26}$$

$$\frac{3}{10}\int \frac{3-2\coth(c+dx)}{2-3\coth(c+dx)}dx-\frac{x}{5}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{x}{5}+\frac{3}{10}\int \frac{2i\tan\left(ic+idx+\frac{\pi}{2}\right)+3}{3i\tan\left(ic+idx+\frac{\pi}{2}\right)+2}dx$$

$$\Big\downarrow \text{ 4013}$$

$$-\frac{3\log(3\cosh(c+dx)-2\sinh(c+dx))}{10d}-\frac{x}{5}$$

input
```
Int[(4 - 6*Coth[c + d*x])^(-1),x]
```

output
```
-1/5*x - (3*Log[3*Cosh[c + d*x] - 2*Sinh[c + d*x]])/(10*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3965
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Simp[b/(a^2 + b^2)   Int[(b - a*Tan[c + d*x])/(a + b*Tan[c
+ d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
risch	$\frac{x}{10}+\frac{3c}{5d}-\frac{3\ln\left(\mathrm{e}^{2dx+2c}+5\right)}{10d}$	28
parallelrisch	$\frac{-3\ln(2\tanh(dx+c)-3)+3\ln(1-\tanh(dx+c))-\ln\left(\frac{1}{8}\right)+dx}{10d}$	40
derivativedivides	$\frac{-\frac{3\ln(-2+3\coth(dx+c))}{5}+\frac{\ln(\coth(dx+c)+1)}{10}+\frac{\ln(\coth(dx+c)-1)}{2}}{2d}$	42
default	$\frac{-\frac{3\ln(-2+3\coth(dx+c))}{5}+\frac{\ln(\coth(dx+c)+1)}{10}+\frac{\ln(\coth(dx+c)-1)}{2}}{2d}$	42

input
```
int(1/(4-6*coth(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/10*x+3/5*c/d-3/10/d*ln(exp(2*d*x+2*c)+5)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \frac{1}{4-6\coth(c+dx)}\,dx = \frac{dx - 3\,\log\left(\frac{2\,(3\,\cosh(dx+c)-2\,\sinh(dx+c))}{\cosh(dx+c)-\sinh(dx+c)}\right)}{10\,d}$$

input
```
integrate(1/(4-6*coth(d*x+c)),x, algorithm="fricas")
```

output
```
1/10*(d*x - 3*log(2*(3*cosh(d*x + c) - 2*sinh(d*x + c))/(cosh(d*x + c) - s
inh(d*x + c))))/d
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{1}{4-6\coth(c+dx)}\,dx = \begin{cases} -\frac{x}{2} + \frac{3\log\left(\tanh\left(c+dx\right)+1\right)}{10d} - \frac{3\log\left(2\tanh\left(c+dx\right)-3\right)}{10d} & \text{for}\ d \neq 0 \\ \frac{x}{4-6\coth\left(c\right)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(4-6*coth(d*x+c)),x)
```

output
```
Piecewise((-x/2 + 3*log(tanh(c + d*x) + 1)/(10*d) - 3*log(2*tanh(c + d*x)
- 3)/(10*d), Ne(d, 0)), (x/(4 - 6*coth(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{1}{4-6\coth(c+dx)}\,dx = -\frac{1}{2}\,x - \frac{c}{2\,d} - \frac{3\,\log\left(5\,e^{(-2\,dx-2\,c)}+1\right)}{10\,d}$$

input
```
integrate(1/(4-6*coth(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*x - 1/2*c/d - 3/10*log(5*e^(-2*d*x - 2*c) + 1)/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1}{4-6\coth(c+dx)}\,dx = \frac{dx+c-3\,\log\left(e^{(2\,dx+2\,c)}+5\right)}{10\,d}$$

input
```
integrate(1/(4-6*coth(d*x+c)),x, algorithm="giac")
```

output
```
1/10*(d*x + c - 3*log(e^(2*d*x + 2*c) + 5))/d
```

Mupad [B] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{1}{4-6\coth(c+dx)}\,dx = \frac{x}{10} - \frac{3\,\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+5\right)}{10\,d}$$

input
```
int(-1/(6*coth(c + d*x) - 4),x)
```

output
```
x/10 - (3*log(exp(2*c)*exp(2*d*x) + 5))/(10*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1}{4-6\coth(c+dx)}\,dx = \frac{-3\log\left(e^{2dx+2c}+5\right)+dx}{10d}$$

input
```
int(1/(4-6*coth(d*x+c)),x)
```

output
```
( - 3*log(e**(2*c + 2*d*x) + 5) + d*x)/(10*d)
```

3.87 $\int \sqrt{a+b\coth(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 74

$$\int \sqrt{a+b\coth(c+dx)}\,dx=-\frac{\sqrt{a-b}\,\text{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{d}+\frac{\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

output
```
-(a-b)^(1/2)*arctanh((a+b*coth(d*x+c))^(1/2)/(a-b)^(1/2))/d+(a+b)^(1/2)*ar
ctanh((a+b*coth(d*x+c))^(1/2)/(a+b)^(1/2))/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.91 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.73

$$\int \sqrt{a+b\coth(c+dx)}\,dx=\frac{\left(-\sqrt{i(a-b)}\,\text{arctanh}\left(\frac{\sqrt{i(a+b\coth(c+dx))}}{\sqrt{i(a-b)}}\right)+\sqrt{i(a+b)}\,\text{arctanh}\left(\frac{\sqrt{i(a+b\coth(c+dx))}}{\sqrt{i(a+b)}}\right)\right)\sqrt{a+b\coth(c+dx)}}{d\sqrt{i(a+b\coth(c+dx))}}$$

input
```
Integrate[Sqrt[a + b*Coth[c + d*x]],x]
```

output
```
((-(Sqrt[I*(a - b)]*ArcTanh[Sqrt[I*(a + b*Coth[c + d*x])]/Sqrt[I*(a - b)]]
) + Sqrt[I*(a + b)]*ArcTanh[Sqrt[I*(a + b*Coth[c + d*x])]/Sqrt[I*(a + b)]]
)*Sqrt[a + b*Coth[c + d*x]])/(d*Sqrt[I*(a + b*Coth[c + d*x])])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3966, 25, 483, 25, 1450, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\coth(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3966}$$

$$-\frac{b\int -\frac{\sqrt{a+b\coth(c+dx)}}{b^2-b^2\coth^2(c+dx)}d(b\coth(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\int \frac{\sqrt{a+b\coth(c+dx)}}{b^2-b^2\coth^2(c+dx)}d(b\coth(c+dx))}{d}$$

$$\Big\downarrow \text{483}$$

$$\frac{2b\int -\frac{b^2\coth^2(c+dx)}{b^4\coth^4(c+dx)-2ab^2\coth^2(c+dx)+a^2-b^2}d\sqrt{a+b\coth(c+dx)}}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2b\int \frac{b^2\coth^2(c+dx)}{b^4\coth^4(c+dx)-2ab^2\coth^2(c+dx)+a^2-b^2}d\sqrt{a+b\coth(c+dx)}}{d}$$

$$\Big\downarrow \ 1450$$

$$\frac{2b\left(-\frac{(a+b)\int \frac{1}{b^2\coth^2(c+dx)-a-b}d\sqrt{a+b\coth(c+dx)}}{2b}-\frac{1}{2}\left(1-\frac{a}{b}\right)\int \frac{1}{b^2\coth^2(c+dx)-a+b}d\sqrt{a+b\coth(c+dx)}\right)}{d}$$

$$\Big\downarrow \ 220$$

$$\frac{2b\left(\frac{\left(1-\frac{a}{b}\right)\text{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{2\sqrt{a-b}}+\frac{\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{2b}\right)}{d}$$

input
```
Int[Sqrt[a + b*Coth[c + d*x]],x]
```

output
```
(2*b*(((1 - a/b)*ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a - b]])/(2*Sqrt[a
 - b]) + (Sqrt[a + b]*ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a + b]])/(2*b
)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 483
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[2*d
Subst[Int[x^2/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]], x
] /; FreeQ[{a, b, c, d}, x]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3966
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Su
bst[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && NeQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{\sqrt{-a+b}\,\arctan\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{-a+b}}\right)}{d}+\frac{\sqrt{a+b}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{a+b}}\right)}{d}$	63
default	$-\frac{\sqrt{-a+b}\,\arctan\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{-a+b}}\right)}{d}+\frac{\sqrt{a+b}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{a+b}}\right)}{d}$	63

input
```
int((a+b*coth(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-a+b)^(1/2)*arctan((a+b*coth(d*x+c))^(1/2)/(-a+b)^(1/2))+(a+b)^(1/2)
*arctanh((a+b*coth(d*x+c))^(1/2)/(a+b)^(1/2))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(62) = 124$.

Time = 0.14 (sec) , antiderivative size = 2231, normalized size of antiderivative = 30.15

$$\int \sqrt{a + b \coth(c + dx)}\, dx = \text{Too large to display}$$

input
```
integrate((a+b*coth(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a + b)*log(2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 8*(a^2 + 2*a
*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*sinh(d*x +
 c)^4 - 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^2 - a^2 - a*b)*sinh(d*x + c)^2 + 2*a^2 - b^2 + 2*((a + b)*cosh(d*x +
c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 -
 (2*a + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x + c)^2 - 2*a - b)*sinh(d*
x + c)^2 + 2*(2*(a + b)*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*
x + c) + a)*sqrt(a + b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x
+ c)) + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^2 + a*b)*cosh(d*x + c)
)*sinh(d*x + c)) + sqrt(a - b)*log(((2*a^2 - b^2)*cosh(d*x + c)^4 + 4*(2*a
^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 - b^2)*sinh(d*x + c)^4 -
4*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(2*a^2 - b^2)*cosh(d*x + c)^2 - 2*a^2
 + 2*a*b)*sinh(d*x + c)^2 + 2*a^2 - 4*a*b + 2*b^2 - 2*(a*cosh(d*x + c)^4 +
 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - (2*a - b)*cosh(d*
x + c)^2 + (6*a*cosh(d*x + c)^2 - 2*a + b)*sinh(d*x + c)^2 + 2*(2*a*cosh(d
*x + c)^3 - (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*sqrt(a - b)*sq
rt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c)) + 4*((2*a^2 - b^2)*c
osh(d*x + c)^3 - 2*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c
)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2
+ 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/d, -1/4*(2*sqrt(...
```

Sympy [F]

$$\int \sqrt{a+b\coth(c+dx)}\,dx = \int \sqrt{a+b\coth{(c+dx)}}\,dx$$

input
```
integrate((a+b*coth(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*coth(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{a+b\coth(c+dx)}\,dx = \int \sqrt{b\coth{(dx+c)}+a}\,dx$$

input
```
integrate((a+b*coth(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*coth(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+b\coth(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*coth(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.04

$$\int \sqrt{a+b\coth(c+dx)}\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{b^2\sqrt{a-b}\sqrt{a+b\coth(c+dx)}\,1\mathrm{i}+a\,b\sqrt{a-b}\sqrt{a+b\coth(c+dx)}\,1\mathrm{i}}{a^2\,b-b^3}\right)\sqrt{a-b}\,1\mathrm{i}}{d}$$
$$+\frac{\operatorname{atan}\left(\frac{b^2\sqrt{a+b}\sqrt{a+b\coth(c+dx)}\,1\mathrm{i}-a\,b\sqrt{a+b}\sqrt{a+b\coth(c+dx)}\,1\mathrm{i}}{a^2\,b-b^3}\right)\sqrt{a+b}\,1\mathrm{i}}{d}$$

input
```
int((a + b*coth(c + d*x))^(1/2),x)
```

output
```
(atan((b^2*(a - b)^(1/2)*(a + b*coth(c + d*x))^(1/2)*1i + a*b*(a - b)^(1/2
)*(a + b*coth(c + d*x))^(1/2)*1i)/(a^2*b - b^3))*(a - b)^(1/2)*1i)/d + (at
an((b^2*(a + b)^(1/2)*(a + b*coth(c + d*x))^(1/2)*1i - a*b*(a + b)^(1/2)*(
a + b*coth(c + d*x))^(1/2)*1i)/(a^2*b - b^3))*(a + b)^(1/2)*1i)/d
```

Reduce [F]

$$\int \sqrt{a+b\coth(c+dx)}\,dx = \int \sqrt{\coth(dx+c)\,b+a}dx$$

input
```
int((a+b*coth(d*x+c))^(1/2),x)
```

output
```
int(sqrt(coth(c + d*x)*b + a),x)
```

3.88 $\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 74

$$\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d}$$

output
```
-arctanh((a+b*coth(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)/d+arctanh((a+b*c
oth(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d}$$

input
```
Integrate[1/Sqrt[a + b*Coth[c + d*x]],x]
```

output
```
-(ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d)) + ArcTan
h[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3966, 25, 484, 1406, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\coth(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a-ib\tan\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3966}$$

$$-\frac{b\int -\frac{1}{\sqrt{a+b\coth(c+dx)}(b^2-b^2\coth^2(c+dx))}d(b\coth(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\int \frac{1}{\sqrt{a+b\coth(c+dx)}(b^2-b^2\coth^2(c+dx))}d(b\coth(c+dx))}{d}$$

$$\Big\downarrow \text{484}$$

$$\frac{2b\int \frac{1}{-b^4\coth^4(c+dx)+2ab^2\coth^2(c+dx)-a^2+b^2}d\sqrt{a+b\coth(c+dx)}}{d}$$

$$\Big\downarrow \text{1406}$$

$$\frac{2b\left(\frac{\int \frac{1}{-b^2\coth^2(c+dx)+a+b}d\sqrt{a+b\coth(c+dx)}}{2b}-\frac{\int \frac{1}{-b^2\coth^2(c+dx)+a-b}d\sqrt{a+b\coth(c+dx)}}{2b}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{2b\left(\frac{\text{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}-\frac{\text{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{2b\sqrt{a-b}}\right)}{d}$$

input
```
Int[1/Sqrt[a + b*Coth[c + d*x]],x]
```

output
```
(2*b*(-1/2*ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*b)
+ ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a + b]]/(2*b*Sqrt[a + b])))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 484
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[2*
d   Subst[Int[1/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3966
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Su
bst[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && NeQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{-a+b}}\right)}{d\sqrt{-a+b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,d}$	62
default	$\frac{\arctan\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{-a+b}}\right)}{d\sqrt{-a+b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,d}$	62

input
```
int(1/(a+b*coth(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(-a+b)^(1/2)*arctan((a+b*coth(d*x+c))^(1/2)/(-a+b)^(1/2))+arctanh((a+b
*coth(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(62) = 124$.

Time = 0.14 (sec) , antiderivative size = 2307, normalized size of antiderivative = 31.18

$$\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*coth(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a + b)*(a - b)*log(2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 8*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*si
nh(d*x + c)^4 - 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)^2 + 2*a^2 - b^2 + 2*((a + b)*cos
h(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x
+ c)^4 - (2*a + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x + c)^2 - 2*a - b)
*sinh(d*x + c)^2 + 2*(2*(a + b)*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))
*sinh(d*x + c) + a)*sqrt(a + b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/s
inh(d*x + c)) + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^2 + a*b)*cosh(
d*x + c))*sinh(d*x + c)) + (a + b)*sqrt(a - b)*log(((2*a^2 - b^2)*cosh(d*x
 + c)^4 + 4*(2*a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 - b^2)*si
nh(d*x + c)^4 - 4*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(2*a^2 - b^2)*cosh(d*
x + c)^2 - 2*a^2 + 2*a*b)*sinh(d*x + c)^2 + 2*a^2 - 4*a*b + 2*b^2 - 2*(a*c
osh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - (
2*a - b)*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 - 2*a + b)*sinh(d*x + c)^2
 + 2*(2*a*cosh(d*x + c)^3 - (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b
)*sqrt(a - b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c)) + 4*
((2*a^2 - b^2)*cosh(d*x + c)^3 - 2*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c
))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*
sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/...
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\coth{(c+dx)}}}\,dx$$

input
```
integrate(1/(a+b*coth(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*coth(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\coth\left(dx+c\right)+a}}\,dx$$

input
```
integrate(1/(a+b*coth(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*coth(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*coth(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 2.65 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.27

$$\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx = \frac{\mathrm{atanh}\left(\frac{16\,a\,b^2\,\sqrt{a+b\,\mathrm{coth}(c+d\,x)}}{\left(\frac{16\,b^4\,d^3}{a\,d^3-b\,d^3}-\frac{16\,a\,b^3\,d^3}{a\,d^3-b\,d^3}\right)\,\sqrt{a-b}}+\frac{\left(a\,d^3-b\,d^3\right)\,\sqrt{a+b\,\mathrm{coth}(c+d\,x)}}{b\,d^3\,\sqrt{a-b}}\right)}{d\,\sqrt{a-b}} - \frac{\mathrm{atanh}\left(\frac{16\,a\,b^2\,\sqrt{a+b\,\mathrm{coth}(c+d\,x)}}{\left(\frac{16\,b^4\,d^3}{a\,d^3+b\,d^3}+\frac{16\,a\,b^3\,d^3}{a\,d^3+b\,d^3}\right)\,\sqrt{a+b}}-\frac{\left(a\,d^3+b\,d^3\right)\,\sqrt{a+b\,\mathrm{coth}(c+d\,x)}}{b\,d^3\,\sqrt{a+b}}\right)}{d\,\sqrt{a+b}}$$

input
```
int(1/(a + b*coth(c + d*x))^(1/2),x)
```

output
```
atanh((16*a*b^2*(a + b*coth(c + d*x))^(1/2))/(((16*b^4*d^3)/(a*d^3 - b*d^3
) - (16*a*b^3*d^3)/(a*d^3 - b*d^3))*(a - b)^(1/2)) + ((a*d^3 - b*d^3)*(a +
 b*coth(c + d*x))^(1/2))/(b*d^3*(a - b)^(1/2)))/(d*(a - b)^(1/2)) - atanh(
(16*a*b^2*(a + b*coth(c + d*x))^(1/2))/(((16*b^4*d^3)/(a*d^3 + b*d^3) + (1
6*a*b^3*d^3)/(a*d^3 + b*d^3))*(a + b)^(1/2)) - ((a*d^3 + b*d^3)*(a + b*cot
h(c + d*x))^(1/2))/(b*d^3*(a + b)^(1/2)))/(d*(a + b)^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx=\int \frac{\sqrt{\coth(dx+c)\,b+a}}{\coth(dx+c)\,b+a}dx$$

input
```
int(1/(a+b*coth(d*x+c))^(1/2),x)
```

output
```
int(sqrt(coth(c + d*x)*b + a)/(coth(c + d*x)*b + a),x)
```

3.89 $\int (a+b\coth(e+fx))^n(d\text{csch}(e+fx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 153

$$\int (a+b\coth(e+fx))^n(d\text{csch}(e+fx))^m\,dx =$$
$$-\frac{b\,\text{AppellF1}\left(1+n,1-\frac{m}{2},1-\frac{m}{2},2+n,\frac{a+b\coth(e+fx)}{a-b},\frac{a+b\coth(e+fx)}{a+b}\right)(a+b\coth(e+fx))^{1+n}\left(1-\frac{a+b}{}\right.}{(a^2-b^2)\,f(1+n)}$$

output
```
-b*AppellF1(1+n,1-1/2*m,1-1/2*m,2+n,(a+b*coth(f*x+e))/(a-b),(a+b*coth(f*x+
e))/(a+b))*(a+b*coth(f*x+e))^(1+n)*(d*csch(f*x+e))^m/(a^2-b^2)/f/(1+n)/((1
-(a+b*coth(f*x+e))/(a-b))^(1/2*m))/((1-(a+b*coth(f*x+e))/(a+b))^(1/2*m))
```

Mathematica [F]

$$\int (a+b\coth(e+fx))^n(d\text{csch}(e+fx))^m\,dx = \int (a+b\coth(e+fx))^n(d\text{csch}(e+fx))^m\,dx$$

input
```
Integrate[(a + b*Coth[e + f*x])^n*(d*Csch[e + f*x])^m,x]
```

output
```
Integrate[(a + b*Coth[e + f*x])^n*(d*Csch[e + f*x])^m, x]
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 3995, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\operatorname{csch}(e+fx))^m(a+b\coth(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(id\sec\left(ie+ifx-\frac{\pi}{2}\right)\right)^m\left(a-ib\tan\left(ie+ifx-\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{3995}$$

$$-\frac{b(d\operatorname{csch}(e+fx))^m\left(1-\frac{a+b\coth(e+fx)}{a-b}\right)^{-m/2}\left(1-\frac{a+b\coth(e+fx)}{a+b}\right)^{-m/2}\int(a+b\coth(e+fx))^n\left(1-\frac{a+b\coth(e+f}{a-b}\right.}{f\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{150}$$

$$-\frac{b(d\operatorname{csch}(e+fx))^m\left(1-\frac{a+b\coth(e+fx)}{a-b}\right)^{-m/2}\left(1-\frac{a+b\coth(e+fx)}{a+b}\right)^{-m/2}(a+b\coth(e+fx))^{n+1}\operatorname{AppellF1}\left(n+\right.}{f(n+1)\left(a^2-b^2\right)}$$

input
```
Int[(a + b*Coth[e + f*x])^n*(d*Csch[e + f*x])^m,x]
```

output
```
-((b*AppellF1[1 + n, (2 - m)/2, (2 - m)/2, 2 + n, (a + b*Coth[e + f*x])/(a
 - b), (a + b*Coth[e + f*x])/(a + b)]*(a + b*Coth[e + f*x])^(1 + n)*(d*Csc
h[e + f*x])^m)/((a^2 - b^2)*f*(1 + n)*(1 - (a + b*Coth[e + f*x])/(a - b))^
(m/2)*(1 - (a + b*Coth[e + f*x])/(a + b))^(m/2)))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3995
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*(a^2 + b^2)^(IntPart[m/2]
- 1)*((d*Sec[e + f*x])^(2*FracPart[m/2])/(f*b^(2*IntPart[m/2] - 1)*(1 - (a
+ b*Tan[e + f*x])/(a - Rt[-b^2, 2]))^FracPart[m/2]*(1 - (a + b*Tan[e + f*x]
)/(a + Rt[-b^2, 2]))^FracPart[m/2]))   Subst[Int[x^n*(1 - x/(a - Rt[-b^2, 2
]))^(m/2 - 1)*(1 - x/(a + Rt[-b^2, 2]))^(m/2 - 1), x], x, a + b*Tan[e + f*x
]], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 + b^2, 0] &&  !Integer
Q[m] &&  !IntegerQ[n]
```

Maple [F]

$$\int (a+b\coth(fx+e))^n (d \operatorname{csch}(fx+e))^m \, dx$$

input
```
int((a+b*coth(f*x+e))^n*(d*csch(f*x+e))^m,x)
```

output
```
int((a+b*coth(f*x+e))^n*(d*csch(f*x+e))^m,x)
```

Fricas [F]

$$\int (a+b\coth(e+fx))^n(d\text{csch}(e+fx))^m\,dx = \int (b\coth(fx+e)+a)^n(d\,\text{csch}(fx+e))^m\,dx$$

input
```
integrate((a+b*coth(f*x+e))^n*(d*csch(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*coth(f*x + e) + a)^n*(d*csch(f*x + e))^m, x)
```

Sympy [F]

$$\int (a+b\coth(e+fx))^n(d\text{csch}(e+fx))^m\,dx = \int (d\,\text{csch}(e+fx))^m\,(a+b\coth(e+fx))^n\,dx$$

input
```
integrate((a+b*coth(f*x+e))**n*(d*csch(f*x+e))**m,x)
```

output
```
Integral((d*csch(e + f*x))**m*(a + b*coth(e + f*x))**n, x)
```

Maxima [F]

$$\int (a+b\coth(e+fx))^n(d\text{csch}(e+fx))^m\,dx = \int (b\coth(fx+e)+a)^n(d\,\text{csch}(fx+e))^m\,dx$$

input
```
integrate((a+b*coth(f*x+e))^n*(d*csch(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*coth(f*x + e) + a)^n*(d*csch(f*x + e))^m, x)
```

Giac [F]

$$\int (a+b\coth(e+fx))^n(d\operatorname{csch}(e+fx))^m\,dx = \int (b\coth(fx+e)+a)^n(d\operatorname{csch}(fx+e))^m\,dx$$

input
```
integrate((a+b*coth(f*x+e))^n*(d*csch(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*coth(f*x + e) + a)^n*(d*csch(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\coth(e+fx))^n(d\operatorname{csch}(e+fx))^m\,dx = \int \left(\frac{d}{\sinh(e+f\,x)}\right)^m (a+b\coth(e+f\,x))^n\,dx$$

input
```
int((d/sinh(e + f*x))^m*(a + b*coth(e + f*x))^n,x)
```

output
```
int((d/sinh(e + f*x))^m*(a + b*coth(e + f*x))^n, x)
```

Reduce [F]

$$\int (a+b\coth(e+fx))^n(d\operatorname{csch}(e+fx))^m\,dx = d^m\left(\int (\coth(fx+e)\,b+a)^n\operatorname{csch}(fx+e)^m\,dx\right)$$

input
```
int((a+b*coth(f*x+e))^n*(d*csch(f*x+e))^m,x)
```

output
```
d**m*int((coth(e + f*x)*b + a)**n*csch(e + f*x)**m,x)
```

3.90 $\int \frac{\sinh^4(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 60

$$\int \frac{\sinh^4(x)}{1+\coth(x)}\,dx = \frac{5x}{16} + \frac{1}{32(1-\coth(x))^2} + \frac{1}{8(1-\coth(x))} - \frac{1}{24(1+\coth(x))^3} - \frac{3}{32(1+\coth(x))^2} - \frac{3}{16(1+\coth(x))}$$

output
```
5/16*x+1/32/(1-coth(x))^2+1/(8-8*coth(x))-1/24/(1+coth(x))^3-3/32/(1+coth(
x))^2-3/(16+16*coth(x))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int \frac{\sinh^4(x)}{1+\coth(x)}\,dx = \frac{1}{192}(60x + 15\cosh(2x) - 6\cosh(4x) + \cosh(6x) - 45\sinh(2x) + 9\sinh(4x) - \sinh(6x))$$

input
```
Integrate[Sinh[x]^4/(1 + Coth[x]),x]
```

output
```
(60*x + 15*Cosh[2*x] - 6*Cosh[4*x] + Cosh[6*x] - 45*Sinh[2*x] + 9*Sinh[4*x
] - Sinh[6*x])/192
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(x)}{\coth(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1-i\tan\left(-\frac{\pi}{2}+ix\right)\right)\sec\left(-\frac{\pi}{2}+ix\right)^4}dx$$

$$\Big\downarrow \text{3968}$$

$$\int \frac{1}{(1-\coth(x))^3(\coth(x)+1)^4}d\coth(x)$$

$$\Big\downarrow \text{54}$$

$$\int \left(-\frac{5}{16\left(\coth^2(x)-1\right)}+\frac{1}{8(\coth(x)-1)^2}+\frac{3}{16(\coth(x)+1)^2}-\frac{1}{16(\coth(x)-1)^3}+\frac{3}{16(\coth(x)+1)^3}+\frac{}{8(\text{co}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{16}\text{arctanh}(\coth(x))+\frac{1}{8(1-\coth(x))}-\frac{3}{16(\coth(x)+1)}+\frac{1}{32(1-\coth(x))^2}-\frac{3}{32(\coth(x)+1)^2}-\frac{1}{24(\coth(x)+1)^3}$$

input
```
Int[Sinh[x]^4/(1 + Coth[x]),x]
```

output
```
(5*ArcTanh[Coth[x]])/16 + 1/(32*(1 - Coth[x])^2) + 1/(8*(1 - Coth[x])) - 1
/(24*(1 + Coth[x])^3) - 3/(32*(1 + Coth[x])^2) - 3/(16*(1 + Coth[x]))
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.58

method	result
risch	$\frac{5x}{16}+\frac{\mathrm{e}^{4x}}{128}-\frac{5\,\mathrm{e}^{2x}}{64}+\frac{5\,\mathrm{e}^{-2x}}{32}-\frac{5\,\mathrm{e}^{-4x}}{128}+\frac{\mathrm{e}^{-6x}}{192}$
parallelrisch	$\frac{(-60\cosh(x)-60\sinh(x))\ln(1-\tanh(x))+(60\cosh(x)+60\sinh(x))\ln(\tanh(x)+1)+136\cosh(x)+16\sinh(x)-45\cosh(3x)+5}{384\sinh(x)+384\cosh(x)}$
default	$\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{6}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{5}}+\frac{5}{8\left(\tanh\left(\frac{x}{2}\right)+1\right)^{4}}+\frac{5}{12\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{3}{8\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{5\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{16}+$

input
```
int(sinh(x)^4/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
5/16*x+1/128*exp(4*x)-5/64*exp(2*x)+5/32*exp(-2*x)-5/128*exp(-4*x)+1/192*e
xp(-6*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(44) = 88.

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.55

$$\int \frac{\sinh^4(x)}{1+\coth(x)}\,dx = \frac{5\cosh(x)^5 + 25\cosh(x)\sinh(x)^4 + \sinh(x)^5 + 5\left(2\cosh(x)^2 - 3\right)\sinh(x)^3 - 45\cosh(x)^3 + 5\,(10\cos}{384\,(\cosh(x}$$

input
```
integrate(sinh(x)^4/(1+coth(x)),x, algorithm="fricas")
```

output
```
1/384*(5*cosh(x)^5 + 25*cosh(x)*sinh(x)^4 + sinh(x)^5 + 5*(2*cosh(x)^2 - 3
)*sinh(x)^3 - 45*cosh(x)^3 + 5*(10*cosh(x)^3 - 27*cosh(x))*sinh(x)^2 + 60*
(2*x + 1)*cosh(x) + 5*(cosh(x)^4 - 9*cosh(x)^2 + 24*x - 12)*sinh(x))/(cosh
(x) + sinh(x))
```

Sympy [F]

$$\int \frac{\sinh^4(x)}{1+\coth(x)}\,dx = \int \frac{\sinh^4(x)}{\coth(x)+1}\,dx$$

input
```
integrate(sinh(x)**4/(1+coth(x)),x)
```

output
```
Integral(sinh(x)**4/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.60

$$\int \frac{\sinh^4(x)}{1+\coth(x)}\,dx = -\frac{1}{128}\left(10\,e^{(-2\,x)}-1\right)e^{(4\,x)}+\frac{5}{16}\,x+\frac{5}{32}\,e^{(-2\,x)}-\frac{5}{128}\,e^{(-4\,x)}+\frac{1}{192}\,e^{(-6\,x)}$$

input
```
integrate(sinh(x)^4/(1+coth(x)),x, algorithm="maxima")
```

output
```
-1/128*(10*e^(-2*x) - 1)*e^(4*x) + 5/16*x + 5/32*e^(-2*x) - 5/128*e^(-4*x)
 + 1/192*e^(-6*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int \frac{\sinh^4(x)}{1+\coth(x)}\,dx = -\frac{1}{384}\left(110\,e^{(6\,x)}-60\,e^{(4\,x)}+15\,e^{(2\,x)}-2\right)e^{(-6\,x)}+\frac{5}{16}\,x+\frac{1}{128}\,e^{(4\,x)}-\frac{5}{64}\,e^{(2\,x)}$$

input
```
integrate(sinh(x)^4/(1+coth(x)),x, algorithm="giac")
```

output
```
-1/384*(110*e^(6*x) - 60*e^(4*x) + 15*e^(2*x) - 2)*e^(-6*x) + 5/16*x + 1/1
28*e^(4*x) - 5/64*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \frac{\sinh^4(x)}{1+\coth(x)}\,dx = \frac{5\,x}{16}+\frac{5\,\mathrm{e}^{-2\,x}}{32}-\frac{5\,\mathrm{e}^{2\,x}}{64}-\frac{5\,\mathrm{e}^{-4\,x}}{128}+\frac{\mathrm{e}^{4\,x}}{128}+\frac{\mathrm{e}^{-6\,x}}{192}$$

input
```
int(sinh(x)^4/(coth(x) + 1),x)
```

output
```
(5*x)/16 + (5*exp(-2*x))/32 - (5*exp(2*x))/64 - (5*exp(-4*x))/128 + exp(4*
x)/128 + exp(-6*x)/192
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

$$\int \frac{\sinh^4(x)}{1+\coth(x)}\,dx = \frac{3e^{10x}-30e^{8x}+120e^{6x}x+60e^{4x}-15e^{2x}+2}{384e^{6x}}$$

input
```
int(sinh(x)^4/(1+coth(x)),x)
```

output
```
(3*e**(10*x) - 30*e**(8*x) + 120*e**(6*x)*x + 60*e**(4*x) - 15*e**(2*x) +
2)/(384*e**(6*x))
```

3.91 $\int \frac{\sinh^3(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \frac{\sinh^3(x)}{1+\coth(x)}\,dx = -\frac{4\cosh(x)}{5}+\frac{4\cosh^3(x)}{15}-\frac{\sinh^3(x)}{5(1+\coth(x))}$$

output
```
-4/5*cosh(x)+4/15*cosh(x)^3-sinh(x)^3/(5+5*coth(x))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \frac{\sinh^3(x)}{1+\coth(x)}\,dx = \frac{\operatorname{csch}(x)(-45-20\cosh(2x)+\cosh(4x)-40\sinh(2x)+4\sinh(4x))}{120(1+\coth(x))}$$

input
```
Integrate[Sinh[x]^3/(1 + Coth[x]),x]
```

output
```
(Csch[x]*(-45 - 20*Cosh[2*x] + Cosh[4*x] - 40*Sinh[2*x] + 4*Sinh[4*x]))/(1
20*(1 + Coth[x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.31, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {3042, 26, 3983, 26, 3042, 26, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{\coth(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\left(1-i\tan\left(-\frac{\pi}{2}+ix\right)\right)\sec\left(-\frac{\pi}{2}+ix\right)^3}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sec\left(ix-\frac{\pi}{2}\right)^3\left(1-i\tan\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3983}$$

$$i\left(\frac{4}{5}\int -i\sinh^3(x)dx+\frac{i\sinh^3(x)}{5(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\sinh^3(x)}{5(\coth(x)+1)}-\frac{4}{5}i\int \sinh^3(x)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\sinh^3(x)}{5(\coth(x)+1)}-\frac{4}{5}i\int i\sin(ix)^3dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{4}{5}\int \sin(ix)^3dx+\frac{i\sinh^3(x)}{5(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{3113}$$

$$i\left(\frac{4}{5}i\int \left(1-\cosh^2(x)\right)d\cosh(x)+\frac{i\sinh^3(x)}{5(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$i\left(\frac{4}{5}i\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)+\frac{i\sinh^3(x)}{5(\coth(x)+1)}\right)$$

input
```
Int[Sinh[x]^3/(1 + Coth[x]),x]
```

output
```
I*(((4*I)/5)*(Cosh[x] - Cosh[x]^3/3) + ((I/5)*Sinh[x]^3)/(1 + Coth[x]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result
risch	$\frac{e^{3x}}{48}-\frac{e^{x}}{4}-\frac{3\,e^{-x}}{8}+\frac{e^{-3x}}{12}-\frac{e^{-5x}}{80}$
parallelrisch	$\frac{-45-40\sinh(2x)+96\sinh(x)+4\sinh(4x)+\cosh(4x)-20\cosh(2x)+96\cosh(x)}{120\sinh(x)+120\cosh(x)}$
default	$-\frac{2}{5(\tanh(\frac{x}{2})+1)^5}+\frac{1}{(\tanh(\frac{x}{2})+1)^4}-\frac{1}{3(\tanh(\frac{x}{2})+1)^3}-\frac{1}{2(\tanh(\frac{x}{2})+1)^2}-\frac{3}{8(\tanh(\frac{x}{2})+1)}-\frac{1}{6(\tanh(\frac{x}{2})-1)^3}-$

input
```
int(sinh(x)^3/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
1/48*exp(3*x)-1/4*exp(x)-3/8*exp(-x)+1/12*exp(-3*x)-1/80*exp(-5*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(23) = 46.

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.07

$$\int \frac{\sinh^3(x)}{1+\coth(x)}\,dx = \frac{\cosh(x)^4+16\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh(x)^2-10\right)\sinh(x)^2-20\cosh(x)^2+16\left(\cosh(\right.}{120\left(\cosh(x)+\sinh(x)\right)}$$

input
```
integrate(sinh(x)^3/(1+coth(x)),x, algorithm="fricas")
```

output
```
1/120*(cosh(x)^4 + 16*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 10)
*sinh(x)^2 - 20*cosh(x)^2 + 16*(cosh(x)^3 - 5*cosh(x))*sinh(x) - 45)/(cosh
(x) + sinh(x))
```

Sympy [F]

$$\int \frac{\sinh^3(x)}{1+\coth(x)}\,dx = \int \frac{\sinh^3(x)}{\coth(x)+1}\,dx$$

input
```
integrate(sinh(x)**3/(1+coth(x)),x)
```

output
```
Integral(sinh(x)**3/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{\sinh^3(x)}{1+\coth(x)}\,dx = -\frac{1}{48}\left(12\,e^{(-2\,x)}-1\right)e^{(3\,x)}-\frac{3}{8}\,e^{(-x)}+\frac{1}{12}\,e^{(-3\,x)}-\frac{1}{80}\,e^{(-5\,x)}$$

input
```
integrate(sinh(x)^3/(1+coth(x)),x, algorithm="maxima")
```

output
```
-1/48*(12*e^(-2*x) - 1)*e^(3*x) - 3/8*e^(-x) + 1/12*e^(-3*x) - 1/80*e^(-5*
x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sinh^3(x)}{1+\coth(x)}\,dx = -\frac{1}{240}\left(90\,e^{(4\,x)}-20\,e^{(2\,x)}+3\right)e^{(-5\,x)}+\frac{1}{48}\,e^{(3\,x)}-\frac{1}{4}\,e^{x}$$

input
```
integrate(sinh(x)^3/(1+coth(x)),x, algorithm="giac")
```

output
```
-1/240*(90*e^(4*x) - 20*e^(2*x) + 3)*e^(-5*x) + 1/48*e^(3*x) - 1/4*e^x
```

Mupad [B] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(x)}{1+\coth(x)}\,dx = \frac{e^{-3x}}{12} - \frac{3\,e^{-x}}{8} + \frac{e^{3x}}{48} - \frac{e^{-5x}}{80} - \frac{e^{x}}{4}$$

input
```
int(sinh(x)^3/(coth(x) + 1),x)
```

output
```
exp(-3*x)/12 - (3*exp(-x))/8 + exp(3*x)/48 - exp(-5*x)/80 - exp(x)/4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{\sinh^3(x)}{1+\coth(x)}\,dx = \frac{5e^{8x}-60e^{6x}-90e^{4x}+20e^{2x}-3}{240e^{5x}}$$

input
```
int(sinh(x)^3/(1+coth(x)),x)
```

output
```
(5*e**(8*x) - 60*e**(6*x) - 90*e**(4*x) + 20*e**(2*x) - 3)/(240*e**(5*x))
```

3.92 $\int \frac{\sinh^2(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \frac{\sinh^2(x)}{1+\coth(x)}\,dx = -\frac{3x}{8} - \frac{1}{8(1-\coth(x))} + \frac{1}{8(1+\coth(x))^2} + \frac{1}{4(1+\coth(x))}$$

output
```
-3/8*x-1/(8-8*coth(x))+1/8/(1+coth(x))^2+1/(4+4*coth(x))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{\sinh^2(x)}{1+\coth(x)}\,dx = \frac{1}{32}(-12x - 4\cosh(2x) + \cosh(4x) + 8\sinh(2x) - \sinh(4x))$$

input
```
Integrate[Sinh[x]^2/(1 + Coth[x]),x]
```

output
```
(-12*x - 4*Cosh[2*x] + Cosh[4*x] + 8*Sinh[2*x] - Sinh[4*x])/32
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 25, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{\coth(x)+1}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{1}{\left(1-i\tan\left(-\frac{\pi}{2}+ix\right)\right)\sec\left(-\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{1}{\sec\left(ix-\frac{\pi}{2}\right)^2\left(1-i\tan\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 3968}$$

$$-\int \frac{1}{(1-\coth(x))^2(\coth(x)+1)^3}d\coth(x)$$

$$\Big\downarrow \text{ 54}$$

$$-\int \left(\frac{1}{8(\coth(x)-1)^2}+\frac{1}{4(\coth(x)+1)^2}+\frac{1}{4(\coth(x)+1)^3}-\frac{3}{8\left(\coth^2(x)-1\right)}\right)d\coth(x)$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{3}{8}\text{arctanh}(\coth(x))-\frac{1}{8(1-\coth(x))}+\frac{1}{4(\coth(x)+1)}+\frac{1}{8(\coth(x)+1)^2}$$

input
```
Int[Sinh[x]^2/(1 + Coth[x]),x]
```

output
```
(-3*ArcTanh[Coth[x]])/8 - 1/(8*(1 - Coth[x])) + 1/(8*(1 + Coth[x])^2) + 1/
(4*(1 + Coth[x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result
risch	$-\frac{3x}{8}+\frac{{\mathrm e}^{2x}}{16}-\frac{3\,{\mathrm e}^{-2x}}{16}+\frac{{\mathrm e}^{-4x}}{32}$
parallelrisch	$\frac{3\cosh\left(3x\right)+\sinh\left(3x\right)+\left(-12x-3\right)\cosh\left(x\right)+\left(-12x+9\right)\sinh\left(x\right)}{32\sinh\left(x\right)+32\cosh\left(x\right)}$
default	$\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{4}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{1}{2\tanh\left(\frac{x}{2}\right)+2}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{1}{4\tanh\left(\frac{x}{2}\right)-4}+$ 3 ln(

input
```
int(sinh(x)^2/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
-3/8*x+1/16*exp(2*x)-3/16*exp(-2*x)+1/32*exp(-4*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{\sinh^2(x)}{1+\coth(x)}\,dx = \frac{3\cosh(x)^3+9\cosh(x)\sinh(x)^2+\sinh(x)^3-6(2x+1)\cosh(x)+3\left(\cosh(x)^2-4x+2\right)\sinh(x)}{32(\cosh(x)+\sinh(x))}$$

input
```
integrate(sinh(x)^2/(1+coth(x)),x, algorithm="fricas")
```

output
```
1/32*(3*cosh(x)^3 + 9*cosh(x)*sinh(x)^2 + sinh(x)^3 - 6*(2*x + 1)*cosh(x)
+ 3*(cosh(x)^2 - 4*x + 2)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \frac{\sinh^2(x)}{1+\coth(x)}\,dx = \int \frac{\sinh^2(x)}{\coth(x)+1}\,dx$$

input
```
integrate(sinh(x)**2/(1+coth(x)),x)
```

output
```
Integral(sinh(x)**2/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\sinh^2(x)}{1+\coth(x)}\,dx = -\frac{3}{8}x+\frac{1}{16}e^{(2x)}-\frac{3}{16}e^{(-2x)}+\frac{1}{32}e^{(-4x)}$$

input
```
integrate(sinh(x)^2/(1+coth(x)),x, algorithm="maxima")
```

output
```
-3/8*x + 1/16*e^(2*x) - 3/16*e^(-2*x) + 1/32*e^(-4*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{\sinh^2(x)}{1+\coth(x)}\,dx = \frac{1}{32}\left(9\,e^{(4\,x)} - 6\,e^{(2\,x)} + 1\right)e^{(-4\,x)} - \frac{3}{8}\,x + \frac{1}{16}\,e^{(2\,x)}$$

input
```
integrate(sinh(x)^2/(1+coth(x)),x, algorithm="giac")
```

output
```
1/32*(9*e^(4*x) - 6*e^(2*x) + 1)*e^(-4*x) - 3/8*x + 1/16*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\sinh^2(x)}{1+\coth(x)}\,dx = \frac{e^{2\,x}}{16} - \frac{3\,e^{-2\,x}}{16} - \frac{3\,x}{8} + \frac{e^{-4\,x}}{32}$$

input
```
int(sinh(x)^2/(coth(x) + 1),x)
```

output
```
exp(2*x)/16 - (3*exp(-2*x))/16 - (3*x)/8 + exp(-4*x)/32
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{\sinh^2(x)}{1+\coth(x)}\,dx = \frac{2e^{6x} - 12e^{4x}x - 6e^{2x} + 1}{32e^{4x}}$$

input
```
int(sinh(x)^2/(1+coth(x)),x)
```

output
```
(2*e**(6*x) - 12*e**(4*x)*x - 6*e**(2*x) + 1)/(32*e**(4*x))
```

3.93 $\int \frac{\sinh(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 19

$$\int \frac{\sinh(x)}{1+\coth(x)}\,dx = \frac{2\cosh(x)}{3} - \frac{\sinh(x)}{3(1+\coth(x))}$$

output
```
2/3*cosh(x)-sinh(x)/(3+3*coth(x))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\sinh(x)}{1+\coth(x)}\,dx = \frac{1}{12}\left(9\cosh(x) - \cosh(3x) + 4\sinh^3(x)\right)$$

input
```
Integrate[Sinh[x]/(1 + Coth[x]),x]
```

output
```
(9*Cosh[x] - Cosh[3*x] + 4*Sinh[x]^3)/12
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {3042, 26, 3983, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{\coth(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\left(1-i\tan\left(-\frac{\pi}{2}+ix\right)\right)\sec\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\sec\left(ix-\frac{\pi}{2}\right)\left(1-i\tan\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3983}$$

$$-i\left(\frac{2}{3}\int i\sinh(x)dx-\frac{i\sinh(x)}{3(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{2}{3}i\int \sinh(x)dx-\frac{i\sinh(x)}{3(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{2}{3}i\int -i\sin(ix)dx-\frac{i\sinh(x)}{3(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{2}{3}\int \sin(ix)dx-\frac{i\sinh(x)}{3(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{3118}$$

$$-i\left(\frac{2}{3}i\cosh(x)-\frac{i\sinh(x)}{3(\coth(x)+1)}\right)$$

input
```
Int[Sinh[x]/(1 + Coth[x]),x]
```

output
```
(-I)*(((2*I)/3)*Cosh[x] - ((I/3)*Sinh[x])/(1 + Coth[x]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{{\mathrm e}^{x}}{4}+\frac{{\mathrm e}^{-x}}{2}-\frac{{\mathrm e}^{-3 x}}{12}$	18
parallelrisch	$\frac{3+2 \sinh \left(2 x \right)+2 \sinh \left(x \right)+\cosh \left(2 x \right)+2 \cosh \left(x \right)}{6 \sinh \left(x \right)+6 \cosh \left(x \right)}$	33
default	$-\frac{2}{3 \left(\tanh \left(\frac{x}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tanh \left(\frac{x}{2}\right)+1\right)^{2}}+\frac{1}{2 \tanh \left(\frac{x}{2}\right)+2}-\frac{1}{2 \left(\tanh \left(\frac{x}{2}\right)-1\right)}$	40

input
```
int(sinh(x)/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(x)+1/2*exp(-x)-1/12*exp(-3*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{\sinh(x)}{1+\coth(x)}\,dx = \frac{\cosh\left(x\right)^{2}+4\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+3}{6\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

input
```
integrate(sinh(x)/(1+coth(x)),x, algorithm="fricas")
```

output
```
1/6*(cosh(x)^2 + 4*cosh(x)*sinh(x) + sinh(x)^2 + 3)/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \frac{\sinh(x)}{1+\coth(x)}\,dx = \int \frac{\sinh\left(x\right)}{\coth\left(x\right)+1}\,dx$$

input
```
integrate(sinh(x)/(1+coth(x)),x)
```

output
```
Integral(sinh(x)/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\sinh(x)}{1+\coth(x)}\,dx = \frac{1}{2}\,e^{(-x)} - \frac{1}{12}\,e^{(-3\,x)} + \frac{1}{4}\,e^{x}$$

input
```
integrate(sinh(x)/(1+coth(x)),x, algorithm="maxima")
```

output
```
1/2*e^(-x) - 1/12*e^(-3*x) + 1/4*e^x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{1+\coth(x)}\,dx = \frac{1}{12}\left(6\,e^{(2\,x)} - 1\right)e^{(-3\,x)} + \frac{1}{4}\,e^{x}$$

input
```
integrate(sinh(x)/(1+coth(x)),x, algorithm="giac")
```

output
```
1/12*(6*e^(2*x) - 1)*e^(-3*x) + 1/4*e^x
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\sinh(x)}{1+\coth(x)}\,dx = \frac{\mathrm{e}^{-x}}{2} - \frac{\mathrm{e}^{-3\,x}}{12} + \frac{\mathrm{e}^{x}}{4}$$

input
```
int(sinh(x)/(coth(x) + 1),x)
```

output
```
exp(-x)/2 - exp(-3*x)/12 + exp(x)/4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{\sinh(x)}{1+\coth(x)}\,dx = \frac{12e^{3x}\cosh(x) - 3e^{4x} - 1}{12e^{3x}}$$

input
```
int(sinh(x)/(1+coth(x)),x)
```

output
```
(12*e**(3*x)*cosh(x) - 3*e**(4*x) - 1)/(12*e**(3*x))
```

3.94 $\int \frac{\text{csch}(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \frac{\text{csch}(x)}{1+\coth(x)}\,dx = -\frac{\text{csch}(x)}{1+\coth(x)}$$

output
```
-csch(x)/(1+coth(x))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{\text{csch}(x)}{1+\coth(x)}\,dx = -\cosh(x)+\sinh(x)$$

input
```
Integrate[Csch[x]/(1 + Coth[x]),x]
```

output
```
-Cosh[x] + Sinh[x]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(x)}{\coth(x)+1}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\sec\left(-\frac{\pi}{2}+ix\right)}{1-i\tan\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\sec\left(ix-\frac{\pi}{2}\right)}{1-i\tan\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3969}$$

$$-\frac{\operatorname{csch}(x)}{\coth(x)+1}$$

input
```
Int[Csch[x]/(1 + Coth[x]),x]
```

output
```
-(Csch[x]/(1 + Coth[x]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
risch	$-{\mathrm e}^{-x}$	7
gosper	$-\frac{\operatorname{csch}\left(x \right)}{1+\coth \left(x \right)}$	11
default	$-\frac{2}{\tanh \left(\frac{x}{2}\right)+1}$	11
orering	$-\frac{\operatorname{csch}\left(x \right)}{1+\coth \left(x \right)}$	11

input
```
int(csch(x)/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
-exp(-x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{csch}(x)}{1+\coth(x)} \, dx = -\frac{1}{\cosh\left(x\right) + \sinh\left(x\right)}$$

input
```
integrate(csch(x)/(1+coth(x)),x, algorithm="fricas")
```

output
```
-1/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{1+\coth(x)}\,dx = \int \frac{\operatorname{csch}(x)}{\coth(x)+1}\,dx$$

input
```
integrate(csch(x)/(1+coth(x)),x)
```

output
```
Integral(csch(x)/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\operatorname{csch}(x)}{1+\coth(x)}\,dx = -e^{(-x)}$$

input
```
integrate(csch(x)/(1+coth(x)),x, algorithm="maxima")
```

output
```
-e^(-x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\operatorname{csch}(x)}{1+\coth(x)}\,dx = -e^{(-x)}$$

input
```
integrate(csch(x)/(1+coth(x)),x, algorithm="giac")
```

output
```
-e^(-x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\operatorname{csch}(x)}{1+\coth(x)}\,dx = -e^{-x}$$

input
```
int(1/(sinh(x)*(coth(x) + 1)),x)
```

output
```
-exp(-x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{\operatorname{csch}(x)}{1+\coth(x)}\,dx = -\frac{1}{e^x}$$

input
```
int(csch(x)/(1+coth(x)),x)
```

output
```
( - 1)/e**x
```

3.95 $\int \frac{\operatorname{csch}^2(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 7

$$\int \frac{\operatorname{csch}^2(x)}{1+\coth(x)}\,dx = -\log(1+\coth(x))$$

output
```
-ln(1+coth(x))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(x)}{1+\coth(x)}\,dx = -x+\log(\sinh(x))$$

input
```
Integrate[Csch[x]^2/(1 + Coth[x]),x]
```

output
```
-x + Log[Sinh[x]]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 25, 3968, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(x)}{\coth(x)+1}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\sec\left(-\frac{\pi}{2}+ix\right)^2}{1-i\tan\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\sec\left(ix-\frac{\pi}{2}\right)^2}{1-i\tan\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3968}$$

$$-\int \frac{1}{\coth(x)+1}d\coth(x)$$

$$\Big\downarrow \text{ 16}$$

$$-\log(\coth(x)+1)$$

input
```
Int[Csch[x]^2/(1 + Coth[x]),x]
```

output
```
-Log[1 + Coth[x]]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$-\ln \left(1+\coth \left(x\right)\right)$	8
default	$-\ln \left(1+\coth \left(x\right)\right)$	8
risch	$-2x+\ln \left({\mathrm e}^{2x}-1\right)$	12

input
```
int(csch(x)^2/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(1+coth(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(7) = 14.

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.57

$$\int \frac{\operatorname{csch}^2(x)}{1+\coth(x)}\,dx = -2\,x + \log\left(\frac{2\,\sinh(x)}{\cosh(x) - \sinh(x)}\right)$$

input
```
integrate(csch(x)^2/(1+coth(x)),x, algorithm="fricas")
```

output
```
-2*x + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{1+\coth(x)}\,dx = \int \frac{\operatorname{csch}^2(x)}{\coth(x)+1}\,dx$$

input
```
integrate(csch(x)**2/(1+coth(x)),x)
```

output
```
Integral(csch(x)**2/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(x)}{1+\coth(x)}\,dx = -\log(\coth(x)+1)$$

input
```
integrate(csch(x)^2/(1+coth(x)),x, algorithm="maxima")
```

output
```
-log(coth(x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

$$\int \frac{\operatorname{csch}^2(x)}{1+\coth(x)}\,dx = -2\,x + \log\left(\left|e^{(2\,x)} - 1\right|\right)$$

input
```
integrate(csch(x)^2/(1+coth(x)),x, algorithm="giac")
```

output
```
-2*x + log(abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

$$\int \frac{\operatorname{csch}^2(x)}{1+\coth(x)}\,dx = \ln\left(e^{2\,x} - 1\right) - 2\,x$$

input
```
int(1/(sinh(x)^2*(coth(x) + 1)),x)
```

output
```
log(exp(2*x) - 1) - 2*x
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.29

$$\int \frac{\operatorname{csch}^2(x)}{1+\coth(x)}\,dx = \log(e^x - 1) + \log(e^x + 1) - 2x$$

input
```
int(csch(x)^2/(1+coth(x)),x)
```

output
```
log(e**x - 1) + log(e**x + 1) - 2*x
```

3.96 $\int \frac{\operatorname{csch}^3(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 8

$$\int \frac{\operatorname{csch}^3(x)}{1+\coth(x)}\,dx = \operatorname{arctanh}(\cosh(x)) - \operatorname{csch}(x)$$

output
```
arctanh(cosh(x))-csch(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(8) = 16.

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.62

$$\int \frac{\operatorname{csch}^3(x)}{1+\coth(x)}\,dx = -\operatorname{csch}(x) + \log\left(\cosh\left(\frac{x}{2}\right)\right) - \log\left(\sinh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Csch[x]^3/(1 + Coth[x]),x]
```

output
```
-Csch[x] + Log[Cosh[x/2]] - Log[Sinh[x/2]]
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 26, 3982, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(x)}{\coth(x)+1} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i \sec\left(-\frac{\pi}{2}+ix\right)^3}{1-i\tan\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{\sec\left(ix-\frac{\pi}{2}\right)^3}{1-i\tan\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3982$$

$$-i\left(\int -i\operatorname{csch}(x)dx - i\operatorname{csch}(x)\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(-i\int \operatorname{csch}(x)dx - i\operatorname{csch}(x)\right)$$

$$\Big\downarrow \; 3042$$

$$-i\left(-i\int i\csc(ix)dx - i\operatorname{csch}(x)\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(\int \csc(ix)dx - i\operatorname{csch}(x)\right)$$

$$\Big\downarrow \; 4257$$

$$-i(i\operatorname{arctanh}(\cosh(x)) - i\operatorname{csch}(x))$$

input
```
Int[Csch[x]^3/(1 + Coth[x]),x]
```

output
```
(-I)*(I*ArcTanh[Cosh[x]] - I*Csch[x])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(8) = 16$.

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.88

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)}{2}-\frac{1}{2\tanh\left(\frac{x}{2}\right)}-\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	23
risch	$-\frac{2\,{\mathrm e}^{x}}{{\mathrm e}^{2x}-1}+\ln\left(1+{\mathrm e}^{x}\right)-\ln\left({\mathrm e}^{x}-1\right)$	26

input
```
int(csch(x)^3/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*tanh(1/2*x)-1/2/tanh(1/2*x)-ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(8) = 16$.

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 9.62

$$\int \frac{\operatorname{csch}^3(x)}{1+\coth(x)}\,dx = \frac{\left(\cosh\left(x\right)^2+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^2-1\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-\left(\cosh\left(x\right)^2+2\cosh\left(x\right)\text{s}\right.}{\cosh\left(x\right)^2+2\cosh\left(x\right)\sinh\left(x\right)+\text{sin}}$$

input
```
integrate(csch(x)^3/(1+coth(x)),x, algorithm="fricas")
```

output
```
((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(cosh(x) + sinh(x) + 1
) - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(cosh(x) + sinh(x)
- 1) - 2*cosh(x) - 2*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 -
 1)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{1+\coth(x)}\,dx = \int \frac{\operatorname{csch}^3\left(x\right)}{\coth\left(x\right)+1}\,dx$$

input
```
integrate(csch(x)**3/(1+coth(x)),x)
```

output
```
Integral(csch(x)**3/(coth(x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(8) = 16$.

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 3.88

$$\int \frac{\operatorname{csch}^3(x)}{1+\coth(x)}\,dx = \frac{2\,e^{(-x)}}{e^{(-2\,x)}-1} + \log\left(e^{(-x)}+1\right) - \log\left(e^{(-x)}-1\right)$$

input
```
integrate(csch(x)^3/(1+coth(x)),x, algorithm="maxima")
```

output
```
2*e^(-x)/(e^(-2*x) - 1) + log(e^(-x) + 1) - log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(8) = 16$.

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 3.25

$$\int \frac{\operatorname{csch}^3(x)}{1+\coth(x)}\,dx = -\frac{2\,e^{x}}{e^{(2\,x)}-1} + \log\left(e^{x}+1\right) - \log\left(|e^{x}-1|\right)$$

input
```
integrate(csch(x)^3/(1+coth(x)),x, algorithm="giac")
```

output
```
-2*e^x/(e^(2*x) - 1) + log(e^x + 1) - log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 3.62

$$\int \frac{\operatorname{csch}^3(x)}{1+\coth(x)}\,dx = \ln\left(2\,\mathrm{e}^{x}+2\right) - \ln\left(2\,\mathrm{e}^{x}-2\right) - \frac{2\,\mathrm{e}^{x}}{\mathrm{e}^{2\,x}-1}$$

input
```
int(1/(sinh(x)^3*(coth(x) + 1)),x)
```

output
```
log(2*exp(x) + 2) - log(2*exp(x) - 2) - (2*exp(x))/(exp(2*x) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 6.88

$$\int \frac{\operatorname{csch}^3(x)}{1+\coth(x)}\,dx = \frac{-e^{2x}\log(e^x-1)+e^{2x}\log(e^x+1)-2e^x+\log(e^x-1)-\log(e^x+1)}{e^{2x}-1}$$

input
```
int(csch(x)^3/(1+coth(x)),x)
```

output
```
( - e**(2*x)*log(e**x - 1) + e**(2*x)*log(e**x + 1) - 2*e**x + log(e**x -
1) - log(e**x + 1))/(e**(2*x) - 1)
```

3.97 $\int \frac{\text{csch}^4(x)}{1+\text{coth}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{\text{csch}^4(x)}{1+\coth(x)}\,dx=-\frac{1}{2}(1-\coth(x))^2$$

output
```
-1/2*(1-coth(x))^2
```

Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{\text{csch}^4(x)}{1+\coth(x)}\,dx=\coth(x)-\frac{\text{csch}^2(x)}{2}$$

input
```
Integrate[Csch[x]^4/(1 + Coth[x]),x]
```

output
```
Coth[x] - Csch[x]^2/2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(x)}{\coth(x)+1} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec\left(-\frac{\pi}{2}+ix\right)^4}{1-i\tan\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3968}$$

$$\int (1-\coth(x))d\coth(x)$$

$$\Big\downarrow \text{17}$$

$$-\frac{1}{2}(1-\coth(x))^2$$

input
```
Int[Csch[x]^4/(1 + Coth[x]),x]
```

output
```
-1/2*(1 - Coth[x])^2
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\coth(x)-\frac{\coth(x)^{2}}{2}$	10
default	$\coth(x)-\frac{\coth(x)^{2}}{2}$	10
risch	$-\frac{2}{\left(e^{2x}-1\right)^{2}}$	11
parallelrisch	$\frac{\left(-3\operatorname{sech}\left(\frac{x}{2}\right)^{2}+14\right)\operatorname{csch}\left(\frac{x}{2}\right)^{2}}{24}-\frac{\operatorname{sech}\left(\frac{x}{2}\right)\operatorname{csch}\left(\frac{x}{2}\right)}{2}-\frac{7\coth\left(\frac{x}{2}\right)^{2}}{12}+\coth\left(\frac{x}{2}\right)$	42

input
```
int(csch(x)^4/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
coth(x)-1/2*coth(x)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(8)=16$.

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 4.58

$$\int \frac{\operatorname{csch}^4(x)}{1+\coth(x)}\,dx =$$
$$-\frac{2}{\cosh(x)^{4}+4\cosh(x)\sinh(x)^{3}+\sinh(x)^{4}+2\left(3\cosh(x)^{2}-1\right)\sinh(x)^{2}-2\cosh(x)^{2}+4\left(\cosh(x)\right.}$$

input
```
integrate(csch(x)^4/(1+coth(x)),x, algorithm="fricas")
```

output
```
-2/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh
(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{1+\coth(x)}\,dx = \int \frac{\operatorname{csch}^4(x)}{\coth(x)+1}\,dx$$

input
```
integrate(csch(x)**4/(1+coth(x)),x)
```

output
```
Integral(csch(x)**4/(coth(x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(8) = 16$.

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.42

$$\int \frac{\operatorname{csch}^4(x)}{1+\coth(x)}\,dx = \frac{4\,e^{(-2\,x)}}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} - \frac{2}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1}$$

input
```
integrate(csch(x)^4/(1+coth(x)),x, algorithm="maxima")
```

output
```
4*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) - 2/(2*e^(-2*x) - e^(-4*x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{csch}^4(x)}{1+\coth(x)}\,dx = -\frac{2}{\left(e^{(2x)}-1\right)^2}$$

input
```
integrate(csch(x)^4/(1+coth(x)),x, algorithm="giac")
```

output
```
-2/(e^(2*x) - 1)^2
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{\operatorname{csch}^4(x)}{1+\coth(x)}\,dx = -\frac{2}{e^{4x}-2e^{2x}+1}$$

input
```
int(1/(sinh(x)^4*(coth(x) + 1)),x)
```

output
```
-2/(exp(4*x) - 2*exp(2*x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{\operatorname{csch}^4(x)}{1+\coth(x)}\,dx = -\frac{2}{e^{4x}-2e^{2x}+1}$$

input
```
int(csch(x)^4/(1+coth(x)),x)
```

output
```
( - 2)/(e**(4*x) - 2*e**(2*x) + 1)
```

3.98 $\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 168

$$\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx = \frac{1}{16(a+b)(1-\coth(x))^2}+\frac{3a+5b}{16(a+b)^2(1-\coth(x))} - \frac{1}{16(a-b)(1+\coth(x))^2}-\frac{3a-5b}{16(a-b)^2(1+\coth(x))} - \frac{(3a^2+9ab+8b^2)\log(1-\coth(x))}{16(a+b)^3} + \frac{(3a^2-9ab+8b^2)\log(1+\coth(x))}{16(a-b)^3}-\frac{b^5\log(a+b\coth(x))}{(a^2-b^2)^3}$$

output
```
1/16/(a+b)/(1-coth(x))^2+1/16*(3*a+5*b)/(a+b)^2/(1-coth(x))-1/16/(a-b)/(1+
coth(x))^2-1/16*(3*a-5*b)/(a-b)^2/(1+coth(x))-1/16*(3*a^2+9*a*b+8*b^2)*ln(
1-coth(x))/(a+b)^3+1/16*(3*a^2-9*a*b+8*b^2)*ln(1+coth(x))/(a-b)^3-b^5*ln(a
+b*coth(x))/(a^2-b^2)^3
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.93

$$\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx$$
$$= \frac{12a^5x-40a^3b^2x+60ab^4x+4b(a^4-4a^2b^2+3b^4)\cosh(2x)-b(a^2-b^2)^2\cosh(4x)-32b^5\log(b\cosh(x)}{32(a-}$$

input
```
Integrate[Sinh[x]^4/(a + b*Coth[x]),x]
```

output
```
(12*a^5*x - 40*a^3*b^2*x + 60*a*b^4*x + 4*b*(a^4 - 4*a^2*b^2 + 3*b^4)*Cosh
[2*x] - b*(a^2 - b^2)^2*Cosh[4*x] - 32*b^5*Log[b*Cosh[x] + a*Sinh[x]] - 8*
a^5*Sinh[2*x] + 24*a^3*b^2*Sinh[2*x] - 16*a*b^4*Sinh[2*x] + a^5*Sinh[4*x]
- 2*a^3*b^2*Sinh[4*x] + a*b^4*Sinh[4*x])/(32*(a - b)^3*(a + b)^3)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3987, 27, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sec\left(-\frac{\pi}{2}+ix\right)^4\left(a-ib\tan\left(-\frac{\pi}{2}+ix\right)\right)}dx$$
$$\Big\downarrow \; 3987$$
$$\frac{\int \frac{b^6}{(a+b\coth(x))\left(b^2-b^2\coth^2(x)\right)^3}d(b\coth(x))}{b}$$
$$\Big\downarrow \; 27$$

$$b^5 \int \frac{1}{(a+b\coth(x))\left(b^2-b^2\coth^2(x)\right)^3}d(b\coth(x))$$

$$\Big\downarrow \text{477}$$

$$\frac{\int \left(-\frac{b^6}{(a^2-b^2)^3(a+b\coth(x))}+\frac{b^3}{8(a+b)(b-b\coth(x))^3}+\frac{b^3}{8(a-b)(\coth(x)b+b)^3}+\frac{(3a+5b)b^2}{16(a+b)^2(b-b\coth(x))^2}+\frac{(3a-5b)b^2}{16(a-b)^2(\coth(x)b+b)^2}+\right.}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b(3a^2+9ab+8b^2)\log(b-b\coth(x))}{16(a+b)^3}+\frac{b(3a^2-9ab+8b^2)\log(b\coth(x)+b)}{16(a-b)^3}-\frac{b^6\log(a+b\coth(x))}{(a^2-b^2)^3}+\frac{b^3}{16(a+b)(b-b\coth(x))^2}-\frac{b^3}{16(a-b)(b\,\mathrm{co}}}{b}$$

input
```
Int[Sinh[x]^4/(a + b*Coth[x]),x]
```

output
```
(b^3/(16*(a + b)*(b - b*Coth[x])^2) + (b^2*(3*a + 5*b))/(16*(a + b)^2*(b -
 b*Coth[x])) - b^3/(16*(a - b)*(b + b*Coth[x])^2) - ((3*a - 5*b)*b^2)/(16*
(a - b)^2*(b + b*Coth[x])) - (b*(3*a^2 + 9*a*b + 8*b^2)*Log[b - b*Coth[x]]
)/(16*(a + b)^3) - (b^6*Log[a + b*Coth[x]])/(a^2 - b^2)^3 + (b*(3*a^2 - 9*
a*b + 8*b^2)*Log[b + b*Coth[x]])/(16*(a - b)^3))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 6.02 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.14

method	result
risch	$\frac{3x\,a^{2}}{8(a+b)^{3}}+\frac{9xab}{8(a+b)^{3}}+\frac{x\,b^{2}}{(a+b)^{3}}+\frac{{\mathrm e}^{4x}}{64a+64b}-\frac{{\mathrm e}^{2x}a}{8(a+b)^{2}}-\frac{3\,{\mathrm e}^{2x}b}{16(a+b)^{2}}+\frac{{\mathrm e}^{-2x}a}{8(a-b)^{2}}-\frac{3\,{\mathrm e}^{-2x}b}{16(a-b)^{2}}-\frac{{\mathrm e}^{-4x}}{64(a-b)}+\frac{2b^{5}}{a^{6}-3a^{4}b^{2}+}$
default	$-\frac{16}{(64a-64b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^{4}}+\frac{64}{(128a-128b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{-a+3b}{8(a-b)^{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{3a-5b}{8(a-b)^{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{(3a^{2}-9ab}{}$

input
```
int(sinh(x)^4/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
3/8*x/(a+b)^3*a^2+9/8*x/(a+b)^3*a*b+x/(a+b)^3*b^2+1/64/(a+b)*exp(4*x)-1/8/
(a+b)^2*exp(2*x)*a-3/16/(a+b)^2*exp(2*x)*b+1/8/(a-b)^2*exp(-2*x)*a-3/16/(a
-b)^2*exp(-2*x)*b-1/64/(a-b)*exp(-4*x)+2*b^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)
*x-b^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*ln(exp(2*x)-(a-b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1279 vs. $2(152) = 304$.

Time = 0.11 (sec) , antiderivative size = 1279, normalized size of antiderivative = 7.61

$$\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
1/64*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^8 + 8*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^7 + (a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^8 - 4*(2*a^5 - a^4
*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)^6 - 4*(2*a^5 - a^4*b
 - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5 - 7*(a^5 - a^4*b - 2*a^3*b^2 +
2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^6 + 8*(3*a^5 - 10*a^3*b^2 + 15
*a*b^4 + 8*b^5)*x*cosh(x)^4 + 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 +
a*b^4 - b^5)*cosh(x)^3 - 3*(2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^
4 - 3*b^5)*cosh(x))*sinh(x)^5 - a^5 - a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^
4 - b^5 + 2*(35*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x
)^4 - 30*(2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)
^2 + 4*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*x)*sinh(x)^4 + 8*(7*(a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 - 10*(2*a^5 - a^4*b
 - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)^3 + 4*(3*a^5 - 10*a^3*
b^2 + 15*a*b^4 + 8*b^5)*x*cosh(x))*sinh(x)^3 + 4*(2*a^5 + a^4*b - 6*a^3*b^
2 - 4*a^2*b^3 + 4*a*b^4 + 3*b^5)*cosh(x)^2 + 4*(7*(a^5 - a^4*b - 2*a^3*b^2
 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 2*a^5 + a^4*b - 6*a^3*b^2 - 4*a^2*
b^3 + 4*a*b^4 + 3*b^5 - 15*(2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^
4 - 3*b^5)*cosh(x)^4 + 12*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*x*cosh(x
)^2)*sinh(x)^2 - 64*(b^5*cosh(x)^4 + 4*b^5*cosh(x)^3*sinh(x) + 6*b^5*co...
```

Sympy [F]

$$\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx=\int \frac{\sinh^4{(x)}}{a+b\coth{(x)}}\,dx$$

input
```
integrate(sinh(x)**4/(a+b*coth(x)),x)
```

output
```
Integral(sinh(x)**4/(a + b*coth(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99

$$\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx = -\frac{b^5\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{(3a^2+9ab+8b^2)x}{8(a^3+3a^2b+3ab^2+b^3)} - \frac{\left(4(2a+3b)e^{(-2x)}-a-b\right)e^{(4x)}}{64(a^2+2ab+b^2)} + \frac{4(2a-3b)e^{(-2x)}-(a-b)e^{(-4x)}}{64(a^2-2ab+b^2)}$$

input
```
integrate(sinh(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
-b^5*log(-(a - b)*e^(-2*x) + a + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) +
1/8*(3*a^2 + 9*a*b + 8*b^2)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/64*(4*(2
*a + 3*b)*e^(-2*x) - a - b)*e^(4*x)/(a^2 + 2*a*b + b^2) + 1/64*(4*(2*a - 3
*b)*e^(-2*x) - (a - b)*e^(-4*x))/(a^2 - 2*a*b + b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.36

$$\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx = -\frac{b^5\log\left(\left|-ae^{(2x)}-be^{(2x)}+a-b\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{(3a^2-9ab+8b^2)x}{8(a^3-3a^2b+3ab^2-b^3)} - \frac{\left(18a^2e^{(4x)}-54abe^{(4x)}+48b^2e^{(4x)}-8a^2e^{(2x)}+20abe^{(2x)}-12b^2e^{(2x)}+a^2-2ab+b^2\right)e^{(-4x)}}{64(a^3-3a^2b+3ab^2-b^3)} + \frac{ae^{(4x)}+be^{(4x)}-8ae^{(2x)}-12be^{(2x)}}{64(a^2+2ab+b^2)}$$

input
```
integrate(sinh(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

output
```
-b^5*log(abs(-a*e^(2*x) - b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6) + 1/8*(3*a^2 - 9*a*b + 8*b^2)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) -
1/64*(18*a^2*e^(4*x) - 54*a*b*e^(4*x) + 48*b^2*e^(4*x) - 8*a^2*e^(2*x) + 2
0*a*b*e^(2*x) - 12*b^2*e^(2*x) + a^2 - 2*a*b + b^2)*e^(-4*x)/(a^3 - 3*a^2*
b + 3*a*b^2 - b^3) + 1/64*(a*e^(4*x) + b*e^(4*x) - 8*a*e^(2*x) - 12*b*e^(2
*x))/(a^2 + 2*a*b + b^2)
```

Mupad [B] (verification not implemented)

Time = 2.74 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.85

$$\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx = \frac{e^{4x}}{64a+64b} - \frac{e^{-4x}}{64a-64b} + \frac{e^{-2x}(2a-3b)}{16(a-b)^2} - \frac{b^5\ln(b-a+a\,e^{2x}+b\,e^{2x})}{a^6-3a^4b^2+3a^2b^4-b^6} + \frac{x(3a^2-9ab+8b^2)}{8(a-b)^3} - \frac{e^{2x}(2a+3b)}{16(a+b)^2}$$

input
```
int(sinh(x)^4/(a + b*coth(x)),x)
```

output
```
exp(4*x)/(64*a + 64*b) - exp(-4*x)/(64*a - 64*b) + (exp(-2*x)*(2*a - 3*b))
/(16*(a - b)^2) - (b^5*log(b - a + a*exp(2*x) + b*exp(2*x)))/(a^6 - b^6 +
3*a^2*b^4 - 3*a^4*b^2) + (x*(3*a^2 - 9*a*b + 8*b^2))/(8*(a - b)^3) - (exp(
2*x)*(2*a + 3*b))/(16*(a + b)^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.11

$$\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx = \frac{-2e^{8x}a^3b^2+2e^{8x}a^2b^3+4e^{6x}a^4b+24e^{6x}a^3b^2-16e^{6x}a^2b^3-16e^{6x}ab^4-64e^{4x}\log(e^{2x}a+e^{2x}b-a+b)\,b^5}{}$$

input
```
int(sinh(x)^4/(a+b*coth(x)),x)
```

output

```
(e**(8*x)*a**5 - e**(8*x)*a**4*b - 2*e**(8*x)*a**3*b**2 + 2*e**(8*x)*a**2*
b**3 + e**(8*x)*a*b**4 - e**(8*x)*b**5 - 8*e**(6*x)*a**5 + 4*e**(6*x)*a**4
*b + 24*e**(6*x)*a**3*b**2 - 16*e**(6*x)*a**2*b**3 - 16*e**(6*x)*a*b**4 +
12*e**(6*x)*b**5 - 64*e**(4*x)*log(e**(2*x)*a + e**(2*x)*b - a + b)*b**5 +
 24*e**(4*x)*a**5*x - 80*e**(4*x)*a**3*b**2*x + 120*e**(4*x)*a*b**4*x + 64
*e**(4*x)*b**5*x + 8*e**(2*x)*a**5 + 4*e**(2*x)*a**4*b - 24*e**(2*x)*a**3*
b**2 - 16*e**(2*x)*a**2*b**3 + 16*e**(2*x)*a*b**4 + 12*e**(2*x)*b**5 - a**
5 - a**4*b + 2*a**3*b**2 + 2*a**2*b**3 - a*b**4 - b**5)/(64*e**(4*x)*(a**6
 - 3*a**4*b**2 + 3*a**2*b**4 - b**6))
```

3.99 $\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 100

$$\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx = -\frac{1}{4(a+b)(1-\coth(x))}+\frac{1}{4(a-b)(1+\coth(x))}+\frac{(a+2b)\log(1-\coth(x))}{4(a+b)^2}-\frac{(a-2b)\log(1+\coth(x))}{4(a-b)^2}-\frac{b^3\log(a+b\coth(x))}{(a^2-b^2)^2}$$

output
```
-1/4/(a+b)/(1-coth(x))+1/4/(a-b)/(1+coth(x))+1/4*(a+2*b)*ln(1-coth(x))/(a+
b)^2-1/4*(a-2*b)*ln(1+coth(x))/(a-b)^2-b^3*ln(a+b*coth(x))/(a^2-b^2)^2
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.75

$$\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx = \frac{-2a^3x+6ab^2x+(-a^2b+b^3)\cosh(2x)-4b^3\log(b\cosh(x)+a\sinh(x))+a(a^2-b^2)\sinh(2x)}{4(a-b)^2(a+b)^2}$$

input
```
Integrate[Sinh[x]^2/(a + b*Coth[x]),x]
```

output
```
(-2*a^3*x + 6*a*b^2*x + (-(a^2*b) + b^3)*Cosh[2*x] - 4*b^3*Log[b*Cosh[x] +
 a*Sinh[x]] + a*(a^2 - b^2)*Sinh[2*x])/(4*(a - b)^2*(a + b)^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 25, 3987, 27, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{a+b\coth(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\sec\left(-\frac{\pi}{2}+ix\right)^2\left(a-ib\tan\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sec\left(ix-\frac{\pi}{2}\right)^2\left(a-ib\tan\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3987}$$

$$-\frac{\int \frac{b^4}{(a+b\coth(x))(b^2-b^2\coth^2(x))^2}d(b\coth(x))}{b}$$

$$\Big\downarrow \text{27}$$

$$-b^3\int \frac{1}{(a+b\coth(x))\left(b^2-b^2\coth^2(x)\right)^2}d(b\coth(x))$$

$$\Big\downarrow \text{477}$$

$$-\frac{\int \left(\frac{b^4}{(a^2-b^2)^2(a+b\coth(x))}+\frac{b^2}{4(a+b)(b-b\coth(x))^2}+\frac{b^2}{4(a-b)(\coth(x)b+b)^2}+\frac{(a+2b)b}{4(a+b)^2(b-b\coth(x))}+\frac{(a-2b)b}{4(a-b)^2(\coth(x)b+b)}\right)d(b\,\text{co}}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{b^4\log(a+b\coth(x))}{(a^2-b^2)^2}+\frac{b^2}{4(a+b)(b-b\coth(x))}-\frac{b^2}{4(a-b)(b\coth(x)+b)}-\frac{b(a+2b)\log(b-b\coth(x))}{4(a+b)^2}+\frac{b(a-2b)\log(b\coth(x)+b)}{4(a-b)^2}}{b}$$

input
```
Int[Sinh[x]^2/(a + b*Coth[x]),x]
```

output
```
-((b^2/(4*(a + b)*(b - b*Coth[x])) - b^2/(4*(a - b)*(b + b*Coth[x])) - (b*
(a + 2*b)*Log[b - b*Coth[x]])/(4*(a + b)^2) + (b^4*Log[a + b*Coth[x]])/(a^
2 - b^2)^2 + ((a - 2*b)*b*Log[b + b*Coth[x]])/(4*(a - b)^2))/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{xa}{2(a+b)^2}-\frac{xb}{(a+b)^2}+\frac{e^{2x}}{8a+8b}-\frac{e^{-2x}}{8(a-b)}+\frac{2b^3x}{a^4-2a^2b^2+b^4}-\frac{b^3\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{a^4-2a^2b^2+b^4}$
default	$-\frac{8}{(16a-16b)(\tanh(\frac{x}{2})+1)^2}+\frac{16}{(32a-32b)(\tanh(\frac{x}{2})+1)}+\frac{(-a+2b)\ln(\tanh(\frac{x}{2})+1)}{2(a-b)^2}-\frac{b^3\ln\left(b\tanh(\frac{x}{2})^2+2a\tanh(\frac{x}{2})+b\right)}{(a-b)^2(a+b)^2}+$

input
```
int(sinh(x)^2/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/(a+b)^2*a-x/(a+b)^2*b+1/8/(a+b)*exp(2*x)-1/8/(a-b)*exp(-2*x)+2*b^3/
(a^4-2*a^2*b^2+b^4)*x-b^3/(a^4-2*a^2*b^2+b^4)*ln(exp(2*x)-(a-b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(90) = 180.

Time = 0.11 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.31

$$\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx = \frac{(a^3-a^2b-ab^2+b^3)\cosh(x)^4+4(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)^3+(a^3-a^2b-ab^2+b^3)\sinh(x)}{}$$

input
```
integrate(sinh(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)
*cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 - 4*(a^3 - 3*a*
b^2 - 2*b^3)*x*cosh(x)^2 - a^3 - a^2*b + a*b^2 + b^3 + 2*(3*(a^3 - a^2*b -
 a*b^2 + b^3)*cosh(x)^2 - 2*(a^3 - 3*a*b^2 - 2*b^3)*x)*sinh(x)^2 - 8*(b^3*
cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2)*log(2*(b*cosh(x) + a*si
nh(x))/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 - 2
*(a^3 - 3*a*b^2 - 2*b^3)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh
(x)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4
)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx = \int \frac{\sinh^2{(x)}}{a+b\coth{(x)}}\,dx$$

input
```
integrate(sinh(x)**2/(a+b*coth(x)),x)
```

output
```
Integral(sinh(x)**2/(a + b*coth(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.83

$$\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx = -\frac{b^3\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^4-2a^2b^2+b^4} - \frac{(a+2b)x}{2(a^2+2ab+b^2)} + \frac{e^{(2x)}}{8(a+b)} - \frac{e^{(-2x)}}{8(a-b)}$$

input
```
integrate(sinh(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
-b^3*log(-(a - b)*e^(-2*x) + a + b)/(a^4 - 2*a^2*b^2 + b^4) - 1/2*(a + 2*b
)*x/(a^2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) - 1/8*e^(-2*x)/(a - b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.14

$$\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx = -\frac{b^3\log\left(\left|-ae^{(2x)}-be^{(2x)}+a-b\right|\right)}{a^4-2a^2b^2+b^4}-\frac{(a-2b)x}{2(a^2-2ab+b^2)}+\frac{\left(2ae^{(2x)}-4be^{(2x)}-a+b\right)e^{(-2x)}}{8(a^2-2ab+b^2)}+\frac{e^{(2x)}}{8(a+b)}$$

input
```
integrate(sinh(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

output
```
-b^3*log(abs(-a*e^(2*x) - b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) - 1/
2*(a - 2*b)*x/(a^2 - 2*a*b + b^2) + 1/8*(2*a*e^(2*x) - 4*b*e^(2*x) - a + b
)*e^(-2*x)/(a^2 - 2*a*b + b^2) + 1/8*e^(2*x)/(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.85

$$\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx = \frac{e^{2x}}{8a+8b}-\frac{e^{-2x}}{8a-8b}-\frac{b^3\ln\left(b-a+a\,e^{2x}+b\,e^{2x}\right)}{a^4-2a^2b^2+b^4}-\frac{x(a-2b)}{2(a-b)^2}$$

input
```
int(sinh(x)^2/(a + b*coth(x)),x)
```

output
```
exp(2*x)/(8*a + 8*b) - exp(-2*x)/(8*a - 8*b) - (b^3*log(b - a + a*exp(2*x)
 + b*exp(2*x)))/(a^4 + b^4 - 2*a^2*b^2) - (x*(a - 2*b))/(2*(a - b)^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.50

$$\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx$$
$$= \frac{e^{4x}a^3 - e^{4x}a^2b - e^{4x}a\,b^2 + e^{4x}b^3 - 8e^{2x}\log(e^{2x}a + e^{2x}b - a + b)\,b^3 - 4e^{2x}a^3x + 12e^{2x}a\,b^2x + 8e^{2x}b^3x - a}{8e^{2x}\,(a^4 - 2a^2b^2 + b^4)}$$

input
```
int(sinh(x)^2/(a+b*coth(x)),x)
```

output
```
(e**(4*x)*a**3 - e**(4*x)*a**2*b - e**(4*x)*a*b**2 + e**(4*x)*b**3 - 8*e**
(2*x)*log(e**(2*x)*a + e**(2*x)*b - a + b)*b**3 - 4*e**(2*x)*a**3*x + 12*e
**(2*x)*a*b**2*x + 8*e**(2*x)*b**3*x - a**3 - a**2*b + a*b**2 + b**3)/(8*e
**(2*x)*(a**4 - 2*a**2*b**2 + b**4))
```

3.100 $\int \frac{1}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 39

$$\int \frac{1}{a+b\coth(x)}\,dx = \frac{ax}{a^2-b^2} - \frac{b\log(b\cosh(x)+a\sinh(x))}{a^2-b^2}$$

output
```
a*x/(a^2-b^2)-b*ln(b*cosh(x)+a*sinh(x))/(a^2-b^2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{1}{a+b\coth(x)}\,dx = \frac{(-a+b)\log(1-\tanh(x))+(a+b)\log(1+\tanh(x))-2b\log(b+a\tanh(x))}{2(a-b)(a+b)}$$

input
```
Integrate[(a + b*Coth[x])^(-1),x]
```

output
```
((-a + b)*Log[1 - Tanh[x]] + (a + b)*Log[1 + Tanh[x]] - 2*b*Log[b + a*Tanh
[x]])/(2*(a - b)*(a + b))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3042, 3965, 26, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\coth(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-ib\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3965}$$

$$\frac{ax}{a^2-b^2}-\frac{ib\int -\frac{i(b+a\coth(x))}{a+b\coth(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{ax}{a^2-b^2}-\frac{b\int \frac{b+a\coth(x)}{a+b\coth(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{ax}{a^2-b^2}-\frac{b\int \frac{b-ia\tan\left(ix+\frac{\pi}{2}\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{4013}$$

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}$$

input
```
Int[(a + b*Coth[x])^(-1),x]
```

output
```
(a*x)/(a^2 - b^2) - (b*Log[b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3965
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Simp[b/(a^2 + b^2)   Int[(b - a*Tan[c + d*x])/(a + b*Tan[c
+ d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

method	result	size
parallelrisch	$\frac{-b\ln(\tanh(x)a+b)+\ln(1-\tanh(x))b+x(a+b)}{a^2-b^2}$	38
derivativedivides	$-\frac{b\ln(a+b\coth(x))}{(a+b)(a-b)}-\frac{\ln(\coth(x)-1)}{2a+2b}+\frac{\ln(1+\coth(x))}{2a-2b}$	55
default	$-\frac{b\ln(a+b\coth(x))}{(a+b)(a-b)}-\frac{\ln(\coth(x)-1)}{2a+2b}+\frac{\ln(1+\coth(x))}{2a-2b}$	55
risch	$\frac{x}{a+b}+\frac{2bx}{a^2-b^2}-\frac{b\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{a^2-b^2}$	56

input
```
int(1/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
(-b*ln(tanh(x)*a+b)+ln(1-tanh(x))*b+x*(a+b))/(a^2-b^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{1}{a+b\coth(x)}\,dx = \frac{(a+b)x - b\log\left(\frac{2(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

input
```
integrate(1/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
((a + b)*x - b*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))))/(a^2 -
b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(29) = 58.

Time = 0.45 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.79

$$\int \frac{1}{a+b\coth(x)}\,dx = \begin{cases} \tilde{\infty}(x-\log(\tanh(x)+1)) & \text{for } a=0 \wedge b=0 \\ \frac{x-\log(\tanh(x)+1)}{b} & \text{for } a=0 \\ -\frac{x\tanh(x)}{2b\tanh(x)-2b} + \frac{x}{2b\tanh(x)-2b} - \frac{1}{2b\tanh(x)-2b} & \text{for } a=-b \\ \frac{x\tanh(x)}{2b\tanh(x)+2b} + \frac{x}{2b\tanh(x)+2b} + \frac{1}{2b\tanh(x)+2b} & \text{for } a=b \\ \frac{ax}{a^2-b^2} - \frac{bx}{a^2-b^2} + \frac{b\log(\tanh(x)+1)}{a^2-b^2} - \frac{b\log\left(\tanh(x)+\frac{b}{a}\right)}{a^2-b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*coth(x)),x)
```

output
```
Piecewise((zoo*(x - log(tanh(x) + 1)), Eq(a, 0) & Eq(b, 0)), ((x - log(tan
h(x) + 1))/b, Eq(a, 0)), (-x*tanh(x)/(2*b*tanh(x) - 2*b) + x/(2*b*tanh(x)
- 2*b) - 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b)
 + x/(2*b*tanh(x) + 2*b) + 1/(2*b*tanh(x) + 2*b), Eq(a, b)), (a*x/(a**2 -
b**2) - b*x/(a**2 - b**2) + b*log(tanh(x) + 1)/(a**2 - b**2) - b*log(tanh(
x) + b/a)/(a**2 - b**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{1}{a+b\coth(x)}\,dx = -\frac{b\log\left(-(a-b)e^{(-2\,x)}+a+b\right)}{a^2-b^2}+\frac{x}{a+b}$$

input
```
integrate(1/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
-b*log(-(a - b)*e^(-2*x) + a + b)/(a^2 - b^2) + x/(a + b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{1}{a+b\coth(x)}\,dx = -\frac{b\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}-a+b\right|\right)}{a^2-b^2}+\frac{x}{a-b}$$

input
```
integrate(1/(a+b*coth(x)),x, algorithm="giac")
```

output
```
-b*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2 - b^2) + x/(a - b)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{1}{a+b\coth(x)}\,dx = \frac{x}{a-b}-\frac{b\,\ln\left(b-a+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x}\right)}{a^2-b^2}$$

input
```
int(1/(a + b*coth(x)),x)
```

output
```
x/(a - b) - (b*log(b - a + a*exp(2*x) + b*exp(2*x)))/(a^2 - b^2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{1}{a+b\coth(x)}\,dx = \frac{-\log(e^{2x}a+e^{2x}b-a+b)\,b+ax+bx}{a^2-b^2}$$

input
```
int(1/(a+b*coth(x)),x)
```

output
```
( - log(e**(2*x)*a + e**(2*x)*b - a + b)*b + a*x + b*x)/(a**2 - b**2)
```

3.101 $\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx=-\frac{\log(a+b\coth(x))}{b}$$

output
```
-ln(a+b*coth(x))/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx=\frac{\log(\sinh(x))-\log(b\cosh(x)+a\sinh(x))}{b}$$

input
```
Integrate[Csch[x]^2/(a + b*Coth[x]),x]
```

output
```
(Log[Sinh[x]] - Log[b*Cosh[x] + a*Sinh[x]])/b
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 25, 3987, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(x)}{a+b\coth(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{\sec\left(-\frac{\pi}{2}+ix\right)^2}{a-ib\tan\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\sec\left(ix-\frac{\pi}{2}\right)^2}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 3987$$

$$-\frac{\int \frac{1}{a+b\coth(x)}d(b\coth(x))}{b}$$

$$\Big\downarrow \ 16$$

$$-\frac{\log(a+b\coth(x))}{b}$$

input
```
Int[Csch[x]^2/(a + b*Coth[x]),x]
```

output
```
-(Log[a + b*Coth[x]]/b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{\ln(a+b\coth(x))}{b}$	13
default	$-\frac{\ln(a+b\coth(x))}{b}$	13
risch	$-\frac{\ln\left(\mathrm{e}^{2x}-\frac{a-b}{a+b}\right)}{b}+\frac{\ln\left(\mathrm{e}^{2x}-1\right)}{b}$	36

input
```
int(csch(x)^2/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(a+b*coth(x))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(12) = 24$.

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.58

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx = -\frac{\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\log\left(\frac{2\,\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{b}$$

input
```
integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
-(log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - log(2*sinh(x)/(cosh
(x) - sinh(x))))/b
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx = \int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx$$

input
```
integrate(csch(x)**2/(a+b*coth(x)),x)
```

output
```
Integral(csch(x)**2/(a + b*coth(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx = -\frac{\log\left(b\coth(x)+a\right)}{b}$$

input
```
integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
-log(b*coth(x) + a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(12) = 24$.

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.83

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx = -\frac{(a+b)\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{ab+b^2}+\frac{\log\left(\left|e^{(2x)}-1\right|\right)}{b}$$

input
```
integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

output
```
-(a + b)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a*b + b^2) + log(abs(e^(
2*x) - 1))/b
```

Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 51, normalized size of antiderivative = 4.25

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{a\,e^{2\,x}\,\sqrt{-b^2}-a\,\sqrt{-b^2}+b\,e^{2\,x}\,\sqrt{-b^2}}{b^2}\right)}{\sqrt{-b^2}}$$

input
```
int(1/(sinh(x)^2*(a + b*coth(x))),x)
```

output
```
-(2*atan((a*exp(2*x)*(-b^2)^(1/2) - a*(-b^2)^(1/2) + b*exp(2*x)*(-b^2)^(1/
2))/b^2))/(-b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 3.25

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx = \frac{\log(e^x-1)+\log(e^x+1)-\log(e^{2x}a+e^{2x}b-a+b)}{b}$$

input
```
int(csch(x)^2/(a+b*coth(x)),x)
```

output

```
(log(e**x - 1) + log(e**x + 1) - log(e**(2*x)*a + e**(2*x)*b - a + b))/b
```

3.102 $\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}\,dx=\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}-\frac{(a^2-b^2)\log(a+b\coth(x))}{b^3}$$

output
```
a*coth(x)/b^2-1/2*coth(x)^2/b-(a^2-b^2)*ln(a+b*coth(x))/b^3
```

Mathematica [A] (verified)

Time = 2.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}\,dx = \frac{2ab\coth(x)-b^2\operatorname{csch}^2(x)+2(a^2-b^2)\left(\log(\sinh(x))-\log(b\cosh(x)+a\sinh(x))\right)}{2b^3}$$

input
```
Integrate[Csch[x]^4/(a + b*Coth[x]),x]
```

output
```
(2*a*b*Coth[x] - b^2*Csch[x]^2 + 2*(a^2 - b^2)*(Log[Sinh[x]] - Log[b*Cosh[
x] + a*Sinh[x]]))/(2*b^3)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec\left(-\frac{\pi}{2}+ix\right)^4}{a-ib\tan\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{b^2-b^2\coth^2(x)}{b^2(a+b\coth(x))}d(b\coth(x))}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b^2-b^2\coth^2(x)}{a+b\coth(x)}d(b\coth(x))}{b^3}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left(a-b\coth(x)+\frac{b^2-a^2}{a+b\coth(x)}\right)d(b\coth(x))}{b^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\left(a^2-b^2\right)\log(a+b\coth(x))+ab\coth(x)-\frac{1}{2}b^2\coth^2(x)}{b^3}$$

input
```
Int[Csch[x]^4/(a + b*Coth[x]),x]
```

output
```
(a*b*Coth[x] - (b^2*Coth[x]^2)/2 - (a^2 - b^2)*Log[a + b*Coth[x]])/b^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(38) = 76$.

Time = 1.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.55

method	result
default	$\frac{-\frac{b\tanh\left(\frac{x}{2}\right)^{2}}{2}+2a\tanh\left(\frac{x}{2}\right)}{4b^{2}}+\frac{\left(-4a^{2}+4b^{2}\right)\ln\left(b\tanh\left(\frac{x}{2}\right)^{2}+2a\tanh\left(\frac{x}{2}\right)+b\right)}{4b^{3}}-\frac{1}{8b\tanh\left(\frac{x}{2}\right)^{2}}+\frac{\left(4a^{2}-4b^{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4b^{3}}+\frac{}{2b^{2}}$
risch	$\frac{2\,\mathrm{e}^{2x}a-2\,\mathrm{e}^{2x}b-2a}{\left(\mathrm{e}^{2x}-1\right)^{2}b^{2}}+\frac{\ln\left(\mathrm{e}^{2x}-1\right)a^{2}}{b^{3}}-\frac{\ln\left(\mathrm{e}^{2x}-1\right)}{b}-\frac{\ln\left(\mathrm{e}^{2x}-\frac{a-b}{a+b}\right)a^{2}}{b^{3}}+\frac{\ln\left(\mathrm{e}^{2x}-\frac{a-b}{a+b}\right)}{b}$

input
```
int(csch(x)^4/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
1/4/b^2*(-1/2*b*tanh(1/2*x)^2+2*a*tanh(1/2*x))+1/4/b^3*(-4*a^2+4*b^2)*ln(b
*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b)-1/8/b/tanh(1/2*x)^2+1/4/b^3*(4*a^2-4*b^2
)*ln(tanh(1/2*x))+1/2/b^2*a/tanh(1/2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(38) = 76$.

Time = 0.12 (sec) , antiderivative size = 434, normalized size of antiderivative = 10.85

$$\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}\,dx$$
$$= \frac{2\,(ab-b^2)\cosh{(x)}^2+4\,(ab-b^2)\cosh{(x)}\sinh{(x)}+2\,(ab-b^2)\sinh{(x)}^2-2\,ab-\left((a^2-b^2)\cosh{(x)}^4+\right.}{}$$

input
```
integrate(csch(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
(2*(a*b - b^2)*cosh(x)^2 + 4*(a*b - b^2)*cosh(x)*sinh(x) + 2*(a*b - b^2)*s
inh(x)^2 - 2*a*b - ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^
3 + (a^2 - b^2)*sinh(x)^4 - 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cos
h(x)^2 - a^2 + b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 - (a^
2 - b^2)*cosh(x))*sinh(x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x
))) + ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^2 - b^
2)*sinh(x)^4 - 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)^2 - a^2
+ b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 - (a^2 - b^2)*cosh
(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))))/(b^3*cosh(x)^4 + 4*b^3*c
osh(x)*sinh(x)^3 + b^3*sinh(x)^4 - 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x
)^2 - b^3)*sinh(x)^2 + 4*(b^3*cosh(x)^3 - b^3*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}\,dx=\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}\,dx$$

input
```
integrate(csch(x)**4/(a+b*coth(x)),x)
```

output
```
Integral(csch(x)**4/(a + b*coth(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(38) = 76$.

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.75

$$\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}\,dx=\frac{2\left((a+b)e^{(-2x)}-a\right)}{2b^2e^{(-2x)}-b^2e^{(-4x)}-b^2}-\frac{(a^2-b^2)\log\left(-(a-b)e^{(-2x)}+a+b\right)}{b^3}+\frac{(a^2-b^2)\log\left(e^{(-x)}+1\right)}{b^3}+\frac{(a^2-b^2)\log\left(e^{(-x)}-1\right)}{b^3}$$

input
```
integrate(csch(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
2*((a + b)*e^(-2*x) - a)/(2*b^2*e^(-2*x) - b^2*e^(-4*x) - b^2) - (a^2 - b^
2)*log(-(a - b)*e^(-2*x) + a + b)/b^3 + (a^2 - b^2)*log(e^(-x) + 1)/b^3 +
(a^2 - b^2)*log(e^(-x) - 1)/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(38) = 76$.

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.65

$$\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}\,dx=-\frac{(a^3+a^2b-ab^2-b^3)\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{ab^3+b^4}+\frac{(a^2-b^2)\log\left(\left|e^{(2x)}-1\right|\right)}{b^3}-\frac{2\left(ab-(ab-b^2)e^{(2x)}\right)}{b^3\left(e^{(2x)}-1\right)^2}$$

input
```
integrate(csch(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

output
```
-(a^3 + a^2*b - a*b^2 - b^3)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a*b^
3 + b^4) + (a^2 - b^2)*log(abs(e^(2*x) - 1))/b^3 - 2*(a*b - (a*b - b^2)*e^
(2*x))/(b^3*(e^(2*x) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.20

$$\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}\,dx = \frac{2\,(a-b)}{b^2\,\left(\mathrm{e}^{2\,x}-1\right)} - \frac{2}{b\,\left(\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1\right)} - \frac{\ln\left(b-a+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x}\right)\,\left(a+b\right)\,\left(a-b\right)}{b^3} + \frac{\ln\left(\mathrm{e}^{2\,x}-1\right)\,\left(a+b\right)\,\left(a-b\right)}{b^3}$$

input
```
int(1/(sinh(x)^4*(a + b*coth(x))),x)
```

output
```
(2*(a - b))/(b^2*(exp(2*x) - 1)) - 2/(b*(exp(4*x) - 2*exp(2*x) + 1)) - (lo
g(b - a + a*exp(2*x) + b*exp(2*x))*(a + b)*(a - b))/b^3 + (log(exp(2*x) -
1)*(a + b)*(a - b))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 384, normalized size of antiderivative = 9.60

$$\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}\,dx = \frac{e^{4x}\log(e^x-1)\,a^2 - e^{4x}\log(e^x-1)\,b^2 + e^{4x}\log(e^x+1)\,a^2 - e^{4x}\log(e^x+1)\,b^2 - e^{4x}\log(e^{2x}a+e^{2x}b-a+}{}$$

input
```
int(csch(x)^4/(a+b*coth(x)),x)
```

output
```
(e**(4*x)*log(e**x - 1)*a**2 - e**(4*x)*log(e**x - 1)*b**2 + e**(4*x)*log(
e**x + 1)*a**2 - e**(4*x)*log(e**x + 1)*b**2 - e**(4*x)*log(e**(2*x)*a + e
**(2*x)*b - a + b)*a**2 + e**(4*x)*log(e**(2*x)*a + e**(2*x)*b - a + b)*b*
*2 + e**(4*x)*a*b - e**(4*x)*b**2 - 2*e**(2*x)*log(e**x - 1)*a**2 + 2*e**(
2*x)*log(e**x - 1)*b**2 - 2*e**(2*x)*log(e**x + 1)*a**2 + 2*e**(2*x)*log(e
**x + 1)*b**2 + 2*e**(2*x)*log(e**(2*x)*a + e**(2*x)*b - a + b)*a**2 - 2*e
**(2*x)*log(e**(2*x)*a + e**(2*x)*b - a + b)*b**2 + log(e**x - 1)*a**2 - l
og(e**x - 1)*b**2 + log(e**x + 1)*a**2 - log(e**x + 1)*b**2 - log(e**(2*x)
*a + e**(2*x)*b - a + b)*a**2 + log(e**(2*x)*a + e**(2*x)*b - a + b)*b**2
 - a*b - b**2)/(b**3*(e**(4*x) - 2*e**(2*x) + 1))
```

3.103 $\int \frac{\text{csch}^6(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 83

$$\int \frac{\text{csch}^6(x)}{a+b\coth(x)}\,dx=\frac{a(a^2-2b^2)\coth(x)}{b^4}-\frac{(a^2-2b^2)\coth^2(x)}{2b^3}+\frac{a\coth^3(x)}{3b^2}-\frac{\coth^4(x)}{4b}-\frac{(a^2-b^2)^2\log(a+b\coth(x))}{b^5}$$

output
```
a*(a^2-2*b^2)*coth(x)/b^4-1/2*(a^2-2*b^2)*coth(x)^2/b^3+1/3*a*coth(x)^3/b^
2-1/4*coth(x)^4/b-(a^2-b^2)^2*ln(a+b*coth(x))/b^5
```

Mathematica [A] (verified)

Time = 4.49 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07

$$\int \frac{\text{csch}^6(x)}{a+b\coth(x)}\,dx$$
$$=\frac{6b^2(-a^2+b^2)\,\text{csch}^2(x)-3b^4\text{csch}^4(x)+4ab\coth(x)\left(3a^2-5b^2+b^2\text{csch}^2(x)\right)+12(a^2-b^2)^2\left(\log(\sinh(x\right.}{12b^5}$$

input
```
Integrate[Csch[x]^6/(a + b*Coth[x]),x]
```

output
```
(6*b^2*(-a^2 + b^2)*Csch[x]^2 - 3*b^4*Csch[x]^4 + 4*a*b*Coth[x]*(3*a^2 - 5
*b^2 + b^2*Csch[x]^2) + 12*(a^2 - b^2)^2*(Log[Sinh[x]] - Log[b*Cosh[x] + a
*Sinh[x]]))/(12*b^5)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 25, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^6(x)}{a+b\coth(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\sec\left(-\frac{\pi}{2}+ix\right)^6}{a-ib\tan\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\sec\left(ix-\frac{\pi}{2}\right)^6}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3987}$$

$$-\frac{\int \frac{\left(b^2-b^2\coth^2(x)\right)^2}{b^4(a+b\coth(x))}d(b\coth(x))}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{\left(b^2-b^2\coth^2(x)\right)^2}{a+b\coth(x)}d(b\coth(x))}{b^5}$$

$$\Big\downarrow \text{ 476}$$

$$-\frac{\int \left(-\left(\left(1-\frac{2b^2}{a^2}\right)a^3\right)-b^2\coth^2(x)a+b^3\coth^3(x)+b\left(a^2-2b^2\right)\coth(x)+\frac{\left(a^2-b^2\right)^2}{a+b\coth(x)}\right)d(b\coth(x))}{b^5}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{1}{2}b^2(a^2-2b^2)\coth^2(x)-ab(a^2-2b^2)\coth(x)+(a^2-b^2)^2\log(a+b\coth(x))-\frac{1}{3}ab^3\coth^3(x)+\frac{1}{4}b^4\coth^4(x)}{b^5}$$

input
```
Int[Csch[x]^6/(a + b*Coth[x]),x]
```

output
```
-((-(a*b*(a^2 - 2*b^2)*Coth[x]) + (b^2*(a^2 - 2*b^2)*Coth[x]^2)/2 - (a*b^3
*Coth[x]^3)/3 + (b^4*Coth[x]^4)/4 + (a^2 - b^2)^2*Log[a + b*Coth[x]])/b^5)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(77) = 154$.

Time = 6.71 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.55

method	result
default	$\frac{-\frac{\tanh\left(\frac{x}{2}\right)^{4} b^{3}}{4}+\frac{2 a \tanh\left(\frac{x}{2}\right)^{3} b^{2}}{3}-2 a^{2} b \tanh\left(\frac{x}{2}\right)^{2}+3 b^{3} \tanh\left(\frac{x}{2}\right)^{2}+8 \tanh\left(\frac{x}{2}\right) a^{3}-14 a \, b^{2} \tanh\left(\frac{x}{2}\right)}{16 b^{4}}+\frac{\left(-16 a^{4}+32 a^{2} b^{2}-16 b^{4}\right) \ln\left(b \tanh\ldots\right.}{16 b^{5}}$
risch	$\frac{2 a^{3} e^{6 x}-2 a^{2} b \, e^{6 x}-2 a \, b^{2} e^{6 x}+2 b^{3} e^{6 x}-6 a^{3} e^{4 x}+4 a^{2} b \, e^{4 x}+10 a \, b^{2} e^{4 x}-8 b^{3} e^{4 x}+6 a^{3} e^{2 x}-2 a^{2} b \, e^{2 x}-\frac{34 a \, b^{2} e^{2 x}}{3}+2 b^{3} e^{2 x}-2 a^{3}+\frac{10 a \, b^{2}}{3}}{b^{4} \left(e^{2 x}-1\right)^{4}}+$

input
```
int(csch(x)^6/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
1/16/b^4*(-1/4*tanh(1/2*x)^4*b^3+2/3*a*tanh(1/2*x)^3*b^2-2*a^2*b*tanh(1/2*
x)^2+3*b^3*tanh(1/2*x)^2+8*tanh(1/2*x)*a^3-14*a*b^2*tanh(1/2*x))+1/16/b^5*
(-16*a^4+32*a^2*b^2-16*b^4)*ln(b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b)-1/64/b/t
anh(1/2*x)^4-1/32*(4*a^2-6*b^2)/b^3/tanh(1/2*x)^2+1/16/b^5*(16*a^4-32*a^2*
b^2+16*b^4)*ln(tanh(1/2*x))+1/24/b^2*a/tanh(1/2*x)^3+1/8*a*(4*a^2-7*b^2)/b
^4/tanh(1/2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1843 vs. $2(77) = 154$.

Time = 0.13 (sec) , antiderivative size = 1843, normalized size of antiderivative = 22.20

$$\int \frac{\operatorname{csch}^6(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(x)^6/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
1/3*(6*(a^3*b - a^2*b^2 - a*b^3 + b^4)*cosh(x)^6 + 36*(a^3*b - a^2*b^2 - a
*b^3 + b^4)*cosh(x)*sinh(x)^5 + 6*(a^3*b - a^2*b^2 - a*b^3 + b^4)*sinh(x)^
6 - 6*(3*a^3*b - 2*a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(x)^4 - 6*(3*a^3*b - 2*a
^2*b^2 - 5*a*b^3 + 4*b^4 - 15*(a^3*b - a^2*b^2 - a*b^3 + b^4)*cosh(x)^2)*s
inh(x)^4 - 6*a^3*b + 10*a*b^3 + 24*(5*(a^3*b - a^2*b^2 - a*b^3 + b^4)*cosh
(x)^3 - (3*a^3*b - 2*a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(x))*sinh(x)^3 + 2*(9*
a^3*b - 3*a^2*b^2 - 17*a*b^3 + 3*b^4)*cosh(x)^2 + 2*(45*(a^3*b - a^2*b^2 -
 a*b^3 + b^4)*cosh(x)^4 + 9*a^3*b - 3*a^2*b^2 - 17*a*b^3 + 3*b^4 - 18*(3*a
^3*b - 2*a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(x)^2)*sinh(x)^2 - 3*((a^4 - 2*a^2
*b^2 + b^4)*cosh(x)^8 + 8*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^7 + (a^4
 - 2*a^2*b^2 + b^4)*sinh(x)^8 - 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^6 - 4*(a
^4 - 2*a^2*b^2 + b^4 - 7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^6 + 8*
(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 - 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x))*
sinh(x)^5 + 6*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 2*(35*(a^4 - 2*a^2*b^2 +
 b^4)*cosh(x)^4 + 3*a^4 - 6*a^2*b^2 + 3*b^4 - 30*(a^4 - 2*a^2*b^2 + b^4)*c
osh(x)^2)*sinh(x)^4 + a^4 - 2*a^2*b^2 + b^4 + 8*(7*(a^4 - 2*a^2*b^2 + b^4)
*cosh(x)^5 - 10*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b
^4)*cosh(x))*sinh(x)^3 - 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 + 4*(7*(a^4 -
 2*a^2*b^2 + b^4)*cosh(x)^6 - 15*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 - a^4 +
 2*a^2*b^2 - b^4 + 9*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 8*(...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^6(x)}{a+b\coth(x)}\,dx=\int \frac{\operatorname{csch}^6(x)}{a+b\coth(x)}\,dx$$

input
```
integrate(csch(x)**6/(a+b*coth(x)),x)
```

output
```
Integral(csch(x)**6/(a + b*coth(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(77) = 154.

Time = 0.04 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.77

$$\int \frac{\operatorname{csch}^6(x)}{a+b\coth(x)}\,dx = -\frac{2\left(3a^3-5ab^2-(9a^3+3a^2b-17ab^2-3b^3)e^{(-2x)}+3(3a^3+2a^2b-5ab^2-4b^3)e^{(-4x)}-3(a^3+a\right.}{3\left(4b^4e^{(-2x)}-6b^4e^{(-4x)}+4b^4e^{(-6x)}-b^4e^{(-8x)}-b^4\right)} - \frac{(a^4-2a^2b^2+b^4)\log\left(-(a-b)e^{(-2x)}+a+b\right)}{b^5} + \frac{(a^4-2a^2b^2+b^4)\log\left(e^{(-x)}+1\right)}{b^5} + \frac{(a^4-2a^2b^2+b^4)\log\left(e^{(-x)}-1\right)}{b^5}$$

input
```
integrate(csch(x)^6/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
-2/3*(3*a^3 - 5*a*b^2 - (9*a^3 + 3*a^2*b - 17*a*b^2 - 3*b^3)*e^(-2*x) + 3*
(3*a^3 + 2*a^2*b - 5*a*b^2 - 4*b^3)*e^(-4*x) - 3*(a^3 + a^2*b - a*b^2 - b^
3)*e^(-6*x))/(4*b^4*e^(-2*x) - 6*b^4*e^(-4*x) + 4*b^4*e^(-6*x) - b^4*e^(-8
*x) - b^4) - (a^4 - 2*a^2*b^2 + b^4)*log(-(a - b)*e^(-2*x) + a + b)/b^5 +
(a^4 - 2*a^2*b^2 + b^4)*log(e^(-x) + 1)/b^5 + (a^4 - 2*a^2*b^2 + b^4)*log(
e^(-x) - 1)/b^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(77) = 154.

Time = 0.14 (sec) , antiderivative size = 317, normalized size of antiderivative = 3.82

$$\int \frac{\operatorname{csch}^6(x)}{a+b\coth(x)}\,dx = -\frac{(a^5+a^4b-2a^3b^2-2a^2b^3+ab^4+b^5)\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{ab^5+b^6} + \frac{(a^4-2a^2b^2+b^4)\log\left(\left|e^{(2x)}-1\right|\right)}{b^5} - \frac{25a^4e^{(8x)}-50a^2b^2e^{(8x)}+25b^4e^{(8x)}-100a^4e^{(6x)}-24a^3be^{(6x)}+224a^2b^2e^{(6x)}+24ab^3e^{(6x)}-124b^4}{}$$

input
```
integrate(csch(x)^6/(a+b*coth(x)),x, algorithm="giac")
```

output
```
-(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*log(abs(a*e^(2*x) + b
*e^(2*x) - a + b))/(a*b^5 + b^6) + (a^4 - 2*a^2*b^2 + b^4)*log(abs(e^(2*x)
 - 1))/b^5 - 1/12*(25*a^4*e^(8*x) - 50*a^2*b^2*e^(8*x) + 25*b^4*e^(8*x) -
100*a^4*e^(6*x) - 24*a^3*b*e^(6*x) + 224*a^2*b^2*e^(6*x) + 24*a*b^3*e^(6*x
) - 124*b^4*e^(6*x) + 150*a^4*e^(4*x) + 72*a^3*b*e^(4*x) - 348*a^2*b^2*e^(
4*x) - 120*a*b^3*e^(4*x) + 246*b^4*e^(4*x) - 100*a^4*e^(2*x) - 72*a^3*b*e^
(2*x) + 224*a^2*b^2*e^(2*x) + 136*a*b^3*e^(2*x) - 124*b^4*e^(2*x) + 25*a^4
 + 24*a^3*b - 50*a^2*b^2 - 40*a*b^3 + 25*b^4)/(b^5*(e^(2*x) - 1)^4)
```

Mupad [B] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.04

$$\int \frac{\operatorname{csch}^6(x)}{a+b\coth(x)}\,dx = \frac{8\,(a-3\,b)}{3\,b^2\,\left(3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1\right)}-\frac{2\,{(a-b)}^2}{b^3\,\left(\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1\right)}-\frac{4}{b\,\left(6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}+\frac{2\,(a+b)\,{(a-b)}^2}{b^4\,\left(\mathrm{e}^{2\,x}-1\right)}-\frac{\ln\left(b-a+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x}\right)\,{(a+b)}^2\,{(a-b)}^2}{b^5}+\frac{\ln\left(\mathrm{e}^{2\,x}-1\right)\,{(a+b)}^2\,{(a-b)}^2}{b^5}$$

input
```
int(1/(sinh(x)^6*(a + b*coth(x))),x)
```

output
```
(8*(a - 3*b))/(3*b^2*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - (2*(a - b
)^2)/(b^3*(exp(4*x) - 2*exp(2*x) + 1)) - 4/(b*(6*exp(4*x) - 4*exp(2*x) - 4
*exp(6*x) + exp(8*x) + 1)) + (2*(a + b)*(a - b)^2)/(b^4*(exp(2*x) - 1)) -
(log(b - a + a*exp(2*x) + b*exp(2*x))*(a + b)^2*(a - b)^2)/b^5 + (log(exp(
2*x) - 1)*(a + b)^2*(a - b)^2)/b^5
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1103, normalized size of antiderivative = 13.29

$$\int \frac{\operatorname{csch}^6(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

input
```
int(csch(x)^6/(a+b*coth(x)),x)
```

output
```
(6*e**(8*x)*log(e**x - 1)*a**4 - 12*e**(8*x)*log(e**x - 1)*a**2*b**2 + 6*e
**(8*x)*log(e**x - 1)*b**4 + 6*e**(8*x)*log(e**x + 1)*a**4 - 12*e**(8*x)*l
og(e**x + 1)*a**2*b**2 + 6*e**(8*x)*log(e**x + 1)*b**4 - 6*e**(8*x)*log(e*
*(2*x)*a + e**(2*x)*b - a + b)*a**4 + 12*e**(8*x)*log(e**(2*x)*a + e**(2*x
)*b - a + b)*a**2*b**2 - 6*e**(8*x)*log(e**(2*x)*a + e**(2*x)*b - a + b)*b
**4 + 3*e**(8*x)*a**3*b - 3*e**(8*x)*a**2*b**2 - 3*e**(8*x)*a*b**3 + 3*e**
(8*x)*b**4 - 24*e**(6*x)*log(e**x - 1)*a**4 + 48*e**(6*x)*log(e**x - 1)*a*
*2*b**2 - 24*e**(6*x)*log(e**x - 1)*b**4 - 24*e**(6*x)*log(e**x + 1)*a**4
+ 48*e**(6*x)*log(e**x + 1)*a**2*b**2 - 24*e**(6*x)*log(e**x + 1)*b**4 + 2
4*e**(6*x)*log(e**(2*x)*a + e**(2*x)*b - a + b)*a**4 - 48*e**(6*x)*log(e**
(2*x)*a + e**(2*x)*b - a + b)*a**2*b**2 + 24*e**(6*x)*log(e**(2*x)*a + e**
(2*x)*b - a + b)*b**4 + 36*e**(4*x)*log(e**x - 1)*a**4 - 72*e**(4*x)*log(e
**x - 1)*a**2*b**2 + 36*e**(4*x)*log(e**x - 1)*b**4 + 36*e**(4*x)*log(e**x
 + 1)*a**4 - 72*e**(4*x)*log(e**x + 1)*a**2*b**2 + 36*e**(4*x)*log(e**x +
1)*b**4 - 36*e**(4*x)*log(e**(2*x)*a + e**(2*x)*b - a + b)*a**4 + 72*e**(4
*x)*log(e**(2*x)*a + e**(2*x)*b - a + b)*a**2*b**2 - 36*e**(4*x)*log(e**(2
*x)*a + e**(2*x)*b - a + b)*b**4 - 18*e**(4*x)*a**3*b + 6*e**(4*x)*a**2*b*
*2 + 42*e**(4*x)*a*b**3 - 30*e**(4*x)*b**4 - 24*e**(2*x)*log(e**x - 1)*a**
4 + 48*e**(2*x)*log(e**x - 1)*a**2*b**2 - 24*e**(2*x)*log(e**x - 1)*b**4 -
 24*e**(2*x)*log(e**x + 1)*a**4 + 48*e**(2*x)*log(e**x + 1)*a**2*b**2 -...
```

3.104 $\int \frac{\operatorname{csch}^8(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 140

$$\int \frac{\operatorname{csch}^8(x)}{a+b\coth(x)}\,dx = \frac{a(a^4-3a^2b^2+3b^4)\coth(x)}{b^6} - \frac{(a^4-3a^2b^2+3b^4)\coth^2(x)}{2b^5} + \frac{a(a^2-3b^2)\coth^3(x)}{3b^4} - \frac{(a^2-3b^2)\coth^4(x)}{4b^3} + \frac{a\coth^5(x)}{5b^2} - \frac{\coth^6(x)}{6b} - \frac{(a^2-b^2)^3\log(a+b\coth(x))}{b^7}$$

output
```
a*(a^4-3*a^2*b^2+3*b^4)*coth(x)/b^6-1/2*(a^4-3*a^2*b^2+3*b^4)*coth(x)^2/b^
5+1/3*a*(a^2-3*b^2)*coth(x)^3/b^4-1/4*(a^2-3*b^2)*coth(x)^4/b^3+1/5*a*coth
(x)^5/b^2-1/6*coth(x)^6/b-(a^2-b^2)^3*ln(a+b*coth(x))/b^7
```

Mathematica [A] (verified)

Time = 3.41 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^8(x)}{a+b\coth(x)}\,dx = \frac{4ab\coth(x)\left(15a^4-40a^2b^2+33b^4+b^2(5a^2-9b^2)\operatorname{csch}^2(x)+3b^4\operatorname{csch}^4(x)\right)+5\left(-6b^2(a^2-b^2)^2\operatorname{csch}^2(x)\right.}{60b^7}$$

input
```
Integrate[Csch[x]^8/(a + b*Coth[x]),x]
```

output
```
(4*a*b*Coth[x]*(15*a^4 - 40*a^2*b^2 + 33*b^4 + b^2*(5*a^2 - 9*b^2)*Csch[x]
^2 + 3*b^4*Csch[x]^4) + 5*(-6*b^2*(a^2 - b^2)^2*Csch[x]^2 + 3*b^4*(-a^2 +
b^2)*Csch[x]^4 - 2*b^6*Csch[x]^6 + 12*(a^2 - b^2)^3*(Log[Sinh[x]] - Log[b*
Cosh[x] + a*Sinh[x]])))/(60*b^7)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^8(x)}{a+b\coth(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec\left(-\frac{\pi}{2}+ix\right)^8}{a-ib\tan\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{\left(b^2-b^2\coth^2(x)\right)^3}{b^6(a+b\coth(x))}d(b\coth(x))}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(b^2-b^2\coth^2(x)\right)^3}{a+b\coth(x)}d(b\coth(x))}{b^7}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left(\left(\frac{3(b^2-a^2)b^2}{a^4}+1\right)a^5+b^4\coth^4(x)a+b^2\left(a^2-3b^2\right)\coth^2(x)a-b^5\coth^5(x)-b^3\left(a^2-3b^2\right)\coth^3(x)-b(a^4-}{b^7}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-(a^2-b^2)^3\log(a+b\coth(x))-\frac{1}{4}b^4(a^2-3b^2)\coth^4(x)+\frac{1}{3}ab^3(a^2-3b^2)\coth^3(x)-\frac{1}{2}b^2(a^4-3a^2b^2+3b^4)\cot}{b^7}$$

input
```
Int[Csch[x]^8/(a + b*Coth[x]),x]
```

output
```
(a*b*(a^4 - 3*a^2*b^2 + 3*b^4)*Coth[x] - (b^2*(a^4 - 3*a^2*b^2 + 3*b^4)*Co
th[x]^2)/2 + (a*b^3*(a^2 - 3*b^2)*Coth[x]^3)/3 - (b^4*(a^2 - 3*b^2)*Coth[x
]^4)/4 + (a*b^5*Coth[x]^5)/5 - (b^6*Coth[x]^6)/6 - (a^2 - b^2)^3*Log[a + b
*Coth[x]])/b^7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(130) = 260$.

Time = 25.21 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.61

method	result
default	$\frac{-\frac{\tanh\left(\frac{x}{2}\right)^{6} b^{5}}{6}+\frac{2 a \tanh\left(\frac{x}{2}\right)^{5} b^{4}}{5}-a^{2} b^{3} \tanh\left(\frac{x}{2}\right)^{4}+2 \tanh\left(\frac{x}{2}\right)^{4} b^{5}+\frac{8 a^{3} b^{2} \tanh\left(\frac{x}{2}\right)^{3}}{3}-6 a\, b^{4} \tanh\left(\frac{x}{2}\right)^{3}-8 a^{4} b \tanh\left(\frac{x}{2}\right)^{2}+20 a^{2} b^{3} \tanh\left(\frac{x}{2}\right.}{64 b^{6}}$
risch	$32 a^{2} b^{3} {\mathrm e}^{6 x}+\frac{16 a^{3} b^{2}}{3}-\frac{160 a^{3} b^{2} {\mathrm e}^{6 x}}{3}+24 a^{3} b^{2} {\mathrm e}^{8 x}-20 a^{2} b^{3} {\mathrm e}^{8 x}-14 a\, b^{4} {\mathrm e}^{8 x}-12 a^{4} b\, {\mathrm e}^{6 x}-4 a^{3} b^{2} {\mathrm e}^{10 x}+4 a^{2} b^{3} {\mathrm e}^{10 x}-20 a^{2} b^{3} {\mathrm e}^{4 x}-52 a\, b^{4} {\mathrm e}^{4 x}-2$

input
```
int(csch(x)^8/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
1/64/b^6*(-1/6*tanh(1/2*x)^6*b^5+2/5*a*tanh(1/2*x)^5*b^4-a^2*b^3*tanh(1/2*
x)^4+2*tanh(1/2*x)^4*b^5+8/3*a^3*b^2*tanh(1/2*x)^3-6*a*b^4*tanh(1/2*x)^3-8
*a^4*b*tanh(1/2*x)^2+20*a^2*b^3*tanh(1/2*x)^2-29/2*b^5*tanh(1/2*x)^2+32*ta
nh(1/2*x)*a^5-88*tanh(1/2*x)*a^3*b^2+76*b^4*a*tanh(1/2*x))+1/64/b^7*(-64*a
^6+192*a^4*b^2-192*a^2*b^4+64*b^6)*ln(b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b)-1
/384/b/tanh(1/2*x)^6-1/256*(4*a^2-8*b^2)/b^3/tanh(1/2*x)^4-1/128/b^5*(16*a
^4-40*a^2*b^2+29*b^4)/tanh(1/2*x)^2+1/64/b^7*(64*a^6-192*a^4*b^2+192*a^2*b
^4-64*b^6)*ln(tanh(1/2*x))+1/160/b^2*a/tanh(1/2*x)^5+1/96*a/b^4*(4*a^2-9*b
^2)/tanh(1/2*x)^3+1/16*a*(8*a^4-22*a^2*b^2+19*b^4)/b^6/tanh(1/2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5283 vs. $2(130) = 260$.

Time = 0.15 (sec) , antiderivative size = 5283, normalized size of antiderivative = 37.74

$$\int \frac{\operatorname{csch}^8(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(x)^8/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{csch}^8(x)}{a+b\coth(x)}\,dx = \int \frac{\operatorname{csch}^8(x)}{a+b\coth(x)}\,dx$$

input
```
integrate(csch(x)**8/(a+b*coth(x)),x)
```

output
```
Integral(csch(x)**8/(a + b*coth(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. 2(130) = 260.

Time = 0.06 (sec) , antiderivative size = 421, normalized size of antiderivative = 3.01

$$\int \frac{\operatorname{csch}^8(x)}{a+b\coth(x)}\,dx = -\frac{2\,(15\,a^5 - 40\,a^3b^2 + 33\,ab^4 - 3\,(25\,a^5 + 5\,a^4b - 70\,a^3b^2 - 10\,a^2b^3 + 61\,ab^4 + 5\,b^5)e^{(-2\,x)} + 30\,(5\,a^5 + 2}{}$$
$$- \frac{(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6)\log\left(-(a-b)e^{(-2\,x)} + a + b\right)}{b^7}$$
$$+ \frac{(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6)\log\left(e^{(-x)} + 1\right)}{b^7}$$
$$+ \frac{(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6)\log\left(e^{(-x)} - 1\right)}{b^7}$$

input
```
integrate(csch(x)^8/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
-2/15*(15*a^5 - 40*a^3*b^2 + 33*a*b^4 - 3*(25*a^5 + 5*a^4*b - 70*a^3*b^2 -
 10*a^2*b^3 + 61*a*b^4 + 5*b^5)*e^(-2*x) + 30*(5*a^5 + 2*a^4*b - 14*a^3*b^
2 - 5*a^2*b^3 + 13*a*b^4 + 3*b^5)*e^(-4*x) - 10*(15*a^5 + 9*a^4*b - 40*a^3
*b^2 - 24*a^2*b^3 + 33*a*b^4 + 23*b^5)*e^(-6*x) + 15*(5*a^5 + 4*a^4*b - 12
*a^3*b^2 - 10*a^2*b^3 + 7*a*b^4 + 6*b^5)*e^(-8*x) - 15*(a^5 + a^4*b - 2*a^
3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*e^(-10*x))/(6*b^6*e^(-2*x) - 15*b^6*e^(-4
*x) + 20*b^6*e^(-6*x) - 15*b^6*e^(-8*x) + 6*b^6*e^(-10*x) - b^6*e^(-12*x)
- b^6) - (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*log(-(a - b)*e^(-2*x) + a + b
)/b^7 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*log(e^(-x) + 1)/b^7 + (a^6 - 3
*a^4*b^2 + 3*a^2*b^4 - b^6)*log(e^(-x) - 1)/b^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(130) = 260$.

Time = 0.14 (sec) , antiderivative size = 594, normalized size of antiderivative = 4.24

$$\int \frac{\operatorname{csch}^8(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(x)^8/(a+b*coth(x)),x, algorithm="giac")
```

output
```
-(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^
7)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a*b^7 + b^8) + (a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6)*log(abs(e^(2*x) - 1))/b^7 - 1/60*(147*a^6*e^(12*x) -
441*a^4*b^2*e^(12*x) + 441*a^2*b^4*e^(12*x) - 147*b^6*e^(12*x) - 882*a^6*e
^(10*x) - 120*a^5*b*e^(10*x) + 2766*a^4*b^2*e^(10*x) + 240*a^3*b^3*e^(10*x
) - 2886*a^2*b^4*e^(10*x) - 120*a*b^5*e^(10*x) + 1002*b^6*e^(10*x) + 2205*
a^6*e^(8*x) + 600*a^5*b*e^(8*x) - 7095*a^4*b^2*e^(8*x) - 1440*a^3*b^3*e^(8
*x) + 7815*a^2*b^4*e^(8*x) + 840*a*b^5*e^(8*x) - 2925*b^6*e^(8*x) - 2940*a
^6*e^(6*x) - 1200*a^5*b*e^(6*x) + 9540*a^4*b^2*e^(6*x) + 3200*a^3*b^3*e^(6
*x) - 10740*a^2*b^4*e^(6*x) - 2640*a*b^5*e^(6*x) + 4780*b^6*e^(6*x) + 2205
*a^6*e^(4*x) + 1200*a^5*b*e^(4*x) - 7095*a^4*b^2*e^(4*x) - 3360*a^3*b^3*e^
(4*x) + 7815*a^2*b^4*e^(4*x) + 3120*a*b^5*e^(4*x) - 2925*b^6*e^(4*x) - 882
*a^6*e^(2*x) - 600*a^5*b*e^(2*x) + 2766*a^4*b^2*e^(2*x) + 1680*a^3*b^3*e^(
2*x) - 2886*a^2*b^4*e^(2*x) - 1464*a*b^5*e^(2*x) + 1002*b^6*e^(2*x) + 147*
a^6 + 120*a^5*b - 441*a^4*b^2 - 320*a^3*b^3 + 441*a^2*b^4 + 264*a*b^5 - 14
7*b^6)/(b^7*(e^(2*x) - 1)^6)
```

Mupad [B] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.15

$$\int \frac{\operatorname{csch}^8(x)}{a+b\coth(x)}\,dx = \frac{32\,(a-5\,b)}{5\,b^2\,\left(5\,e^{2\,x}-10\,e^{4\,x}+10\,e^{6\,x}-5\,e^{8\,x}+e^{10\,x}-1\right)} - \frac{32}{3\,b\,\left(15\,e^{4\,x}-6\,e^{2\,x}-20\,e^{6\,x}+15\,e^{8\,x}-6\,e^{10\,x}+e^{12\,x}+1\right)} - \frac{4\,(a^2-4\,a\,b+7\,b^2)}{b^3\,\left(6\,e^{4\,x}-4\,e^{2\,x}-4\,e^{6\,x}+e^{8\,x}+1\right)} - \frac{\ln\left(b-a+a\,e^{2\,x}+b\,e^{2\,x}\right)\,(a+b)^3\,(a-b)^3}{b^7} + \frac{\ln\left(e^{2\,x}-1\right)\,(a+b)^3\,(a-b)^3}{b^7} + \frac{8\,(a-b)\,(a^2-2\,a\,b+b^2)}{3\,b^4\,\left(3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1\right)} + \frac{2\,(a+b)^2\,(a-b)\,(a^2-2\,a\,b+b^2)}{b^6\,\left(e^{2\,x}-1\right)} - \frac{2\,(a+b)\,(a-b)\,(a^2-2\,a\,b+b^2)}{b^5\,\left(e^{4\,x}-2\,e^{2\,x}+1\right)}$$

input
```
int(1/(sinh(x)^8*(a + b*coth(x))),x)
```

output
```
(32*(a - 5*b))/(5*b^2*(5*exp(2*x) - 10*exp(4*x) + 10*exp(6*x) - 5*exp(8*x)
 + exp(10*x) - 1)) - 32/(3*b*(15*exp(4*x) - 6*exp(2*x) - 20*exp(6*x) + 15*
exp(8*x) - 6*exp(10*x) + exp(12*x) + 1)) - (4*(a^2 - 4*a*b + 7*b^2))/(b^3*
(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1)) - (log(b - a + a*ex
p(2*x) + b*exp(2*x))*(a + b)^3*(a - b)^3)/b^7 + (log(exp(2*x) - 1)*(a + b)
^3*(a - b)^3)/b^7 + (8*(a - b)*(a^2 - 2*a*b + b^2))/(3*b^4*(3*exp(2*x) - 3
*exp(4*x) + exp(6*x) - 1)) + (2*(a + b)^2*(a - b)*(a^2 - 2*a*b + b^2))/(b^
6*(exp(2*x) - 1)) - (2*(a + b)*(a - b)*(a^2 - 2*a*b + b^2))/(b^5*(exp(4*x)
 - 2*exp(2*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 2212, normalized size of antiderivative = 15.80

$$\int \frac{\operatorname{csch}^8(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

input
```
int(csch(x)^8/(a+b*coth(x)),x)
```

output
```
(15*e**(12*x)*log(e**x - 1)*a**6 - 45*e**(12*x)*log(e**x - 1)*a**4*b**2 +
45*e**(12*x)*log(e**x - 1)*a**2*b**4 - 15*e**(12*x)*log(e**x - 1)*b**6 + 1
5*e**(12*x)*log(e**x + 1)*a**6 - 45*e**(12*x)*log(e**x + 1)*a**4*b**2 + 45
*e**(12*x)*log(e**x + 1)*a**2*b**4 - 15*e**(12*x)*log(e**x + 1)*b**6 - 15*
e**(12*x)*log(e**(2*x)*a + e**(2*x)*b - a + b)*a**6 + 45*e**(12*x)*log(e**
(2*x)*a + e**(2*x)*b - a + b)*a**4*b**2 - 45*e**(12*x)*log(e**(2*x)*a + e*
*(2*x)*b - a + b)*a**2*b**4 + 15*e**(12*x)*log(e**(2*x)*a + e**(2*x)*b - a
 + b)*b**6 + 5*e**(12*x)*a**5*b - 5*e**(12*x)*a**4*b**2 - 10*e**(12*x)*a**
3*b**3 + 10*e**(12*x)*a**2*b**4 + 5*e**(12*x)*a*b**5 - 5*e**(12*x)*b**6 -
90*e**(10*x)*log(e**x - 1)*a**6 + 270*e**(10*x)*log(e**x - 1)*a**4*b**2 -
270*e**(10*x)*log(e**x - 1)*a**2*b**4 + 90*e**(10*x)*log(e**x - 1)*b**6 -
90*e**(10*x)*log(e**x + 1)*a**6 + 270*e**(10*x)*log(e**x + 1)*a**4*b**2 -
270*e**(10*x)*log(e**x + 1)*a**2*b**4 + 90*e**(10*x)*log(e**x + 1)*b**6 +
90*e**(10*x)*log(e**(2*x)*a + e**(2*x)*b - a + b)*a**6 - 270*e**(10*x)*log
(e**(2*x)*a + e**(2*x)*b - a + b)*a**4*b**2 + 270*e**(10*x)*log(e**(2*x)*a
 + e**(2*x)*b - a + b)*a**2*b**4 - 90*e**(10*x)*log(e**(2*x)*a + e**(2*x)*
b - a + b)*b**6 + 225*e**(8*x)*log(e**x - 1)*a**6 - 675*e**(8*x)*log(e**x
- 1)*a**4*b**2 + 675*e**(8*x)*log(e**x - 1)*a**2*b**4 - 225*e**(8*x)*log(e
**x - 1)*b**6 + 225*e**(8*x)*log(e**x + 1)*a**6 - 675*e**(8*x)*log(e**x +
1)*a**4*b**2 + 675*e**(8*x)*log(e**x + 1)*a**2*b**4 - 225*e**(8*x)*log(...
```

3.105 $\int \frac{\sinh^3(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 134

$$\int \frac{\sinh^3(x)}{a+b\coth(x)}\,dx = -\frac{b^4\operatorname{arctanh}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{ab^2\cosh(x)}{(a^2-b^2)^2} - \frac{a\cosh(x)}{a^2-b^2} + \frac{a\cosh^3(x)}{3(a^2-b^2)} - \frac{b^3\sinh(x)}{(a^2-b^2)^2} - \frac{b\sinh^3(x)}{3(a^2-b^2)}$$

output
```
-b^4*arctanh((b+a*coth(x))*sinh(x)/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+a*b^2*
cosh(x)/(a^2-b^2)^2-a*cosh(x)/(a^2-b^2)+a*cosh(x)^3/(3*a^2-3*b^2)-b^3*sinh
(x)/(a^2-b^2)^2-b*sinh(x)^3/(3*a^2-3*b^2)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.28

$$\int \frac{\sinh^3(x)}{a+b\coth(x)}\,dx = \frac{24b^4\sqrt{a+b}\arctan\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right) - 3a\sqrt{-a+b}(3a^3+3a^2b-7ab^2-7b^3)\cosh(x) - a(-a+b)^{3/2}(a+b)^{\cdots}}{12(-a+b)^{5/2}(a+\cdots}$$

input
```
Integrate[Sinh[x]^3/(a + b*Coth[x]),x]
```

output
```
(24*b^4*Sqrt[a + b]*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])] -
 3*a*Sqrt[-a + b]*(3*a^3 + 3*a^2*b - 7*a*b^2 - 7*b^3)*Cosh[x] - a*(-a + b)
^(3/2)*(a + b)^2*Cosh[3*x] + 3*b*Sqrt[-a + b]*(a^3 + a^2*b - 5*a*b^2 - 5*b
^3)*Sinh[x] + b*(-a + b)^(3/2)*(a + b)^2*Sinh[3*x])/(12*(-a + b)^(5/2)*(a
+ b)^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.99, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.769$, Rules used = {3042, 26, 3990, 26, 3042, 26, 3967, 26, 3042, 26, 3113, 2009, 3990, 26, 3042, 26, 3967, 26, 3042, 26, 3118, 3988, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{a+b\coth(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sec\left(-\frac{\pi}{2}+ix\right)^3\left(a-ib\tan\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sec\left(ix-\frac{\pi}{2}\right)^3\left(a-ib\tan\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3990}$$

$$i\left(\frac{\int -i(a-b\coth(x))\sinh^3(x)dx}{a^2-b^2}-\frac{b^2\int \frac{i\sinh(x)}{a+b\coth(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{i\int(a-b\coth(x))\sinh^3(x)dx}{a^2-b^2}-\frac{ib^2\int\frac{\sinh(x)}{a+b\coth(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-\frac{ib^2\int-\frac{i}{\sec\left(ix-\frac{\pi}{2}\right)\left(a-ib\tan\left(ix-\frac{\pi}{2}\right)\right)}dx}{a^2-b^2}-\frac{i\int\frac{i\left(a+ib\tan\left(ix-\frac{\pi}{2}\right)\right)}{\sec\left(ix-\frac{\pi}{2}\right)^3}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{\int\frac{a+ib\tan\left(ix-\frac{\pi}{2}\right)}{\sec\left(ix-\frac{\pi}{2}\right)^3}dx}{a^2-b^2}-\frac{b^2\int\frac{1}{\sec\left(ix-\frac{\pi}{2}\right)\left(a-ib\tan\left(ix-\frac{\pi}{2}\right)\right)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3967}$$

$$i\left(\frac{a\int-i\sinh^3(x)dx+\frac{1}{3}ib\sinh^3(x)}{a^2-b^2}-\frac{b^2\int\frac{1}{\sec\left(ix-\frac{\pi}{2}\right)\left(a-ib\tan\left(ix-\frac{\pi}{2}\right)\right)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{\frac{1}{3}ib\sinh^3(x)-ia\int\sinh^3(x)dx}{a^2-b^2}-\frac{b^2\int\frac{1}{\sec\left(ix-\frac{\pi}{2}\right)\left(a-ib\tan\left(ix-\frac{\pi}{2}\right)\right)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{\frac{1}{3}ib\sinh^3(x)-ia\int i\sin(ix)^3dx}{a^2-b^2}-\frac{b^2\int\frac{1}{\sec\left(ix-\frac{\pi}{2}\right)\left(a-ib\tan\left(ix-\frac{\pi}{2}\right)\right)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{a\int\sin(ix)^3dx+\frac{1}{3}ib\sinh^3(x)}{a^2-b^2}-\frac{b^2\int\frac{1}{\sec\left(ix-\frac{\pi}{2}\right)\left(a-ib\tan\left(ix-\frac{\pi}{2}\right)\right)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3113}$$

$$i\left(\frac{ia\int\left(1-\cosh^2(x)\right)d\cosh(x)+\frac{1}{3}ib\sinh^3(x)}{a^2-b^2}-\frac{b^2\int\frac{1}{\sec\left(ix-\frac{\pi}{2}\right)\left(a-ib\tan\left(ix-\frac{\pi}{2}\right)\right)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)+\frac{1}{3}ib\sinh^3(x)}{a^2-b^2}-\frac{b^2\int\frac{1}{\sec\left(ix-\frac{\pi}{2}\right)\left(a-ib\tan\left(ix-\frac{\pi}{2}\right)\right)}dx}{a^2-b^2}\right)$$

$\Big\downarrow$ 3990

$$i\left(\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)+\frac{1}{3}ib\sinh^3(x)}{a^2-b^2}-\frac{b^2\left(\frac{\int i(a-b\coth(x))\sinh(x)dx}{a^2-b^2}-\frac{b^2\int-\frac{i\operatorname{csch}(x)}{a+b\coth(x)}dx}{a^2-b^2}\right)}{a^2-b^2}\right)$$

$\Big\downarrow$ 26

$$i\left(\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)+\frac{1}{3}ib\sinh^3(x)}{a^2-b^2}-\frac{b^2\left(\frac{i\int(a-b\coth(x))\sinh(x)dx}{a^2-b^2}+\frac{ib^2\int\frac{\operatorname{csch}(x)}{a+b\coth(x)}dx}{a^2-b^2}\right)}{a^2-b^2}\right)$$

$\Big\downarrow$ 3042

$$i\left(\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)+\frac{1}{3}ib\sinh^3(x)}{a^2-b^2}-\frac{b^2\left(\frac{ib^2\int\frac{i\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{i\int-\frac{i\left(a+ib\tan\left(ix-\frac{\pi}{2}\right)\right)}{\sec\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}\right)}{a^2-b^2}\right)$$

$\Big\downarrow$ 26

$$i\left(\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)+\frac{1}{3}ib\sinh^3(x)}{a^2-b^2}-\frac{b^2\left(\frac{\int\frac{a+ib\tan\left(ix-\frac{\pi}{2}\right)}{\sec\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{b^2\int\frac{\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}\right)}{a^2-b^2}\right)$$

$\Big\downarrow$ 3967

$$i\left(\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)+\frac{1}{3}ib\sinh^3(x)}{a^2-b^2}-\frac{b^2\left(\frac{a\int i\sinh(x)dx-ib\sinh(x)}{a^2-b^2}-\frac{b^2\int\frac{\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}\right)}{a^2-b^2}\right)$$

$\downarrow$ 26

$$i\left(\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)+\frac{1}{3}ib\sinh^3(x)}{a^2-b^2}-\frac{b^2\left(\frac{ia\int\sinh(x)dx-ib\sinh(x)}{a^2-b^2}-\frac{b^2\int\frac{\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}\right)}{a^2-b^2}\right)$$

$\downarrow$ 3042

$$i\left(\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)+\frac{1}{3}ib\sinh^3(x)}{a^2-b^2}-\frac{b^2\left(\frac{ia\int-i\sin(ix)dx-ib\sinh(x)}{a^2-b^2}-\frac{b^2\int\frac{\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}\right)}{a^2-b^2}\right)$$

$\downarrow$ 26

$$i\left(\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)+\frac{1}{3}ib\sinh^3(x)}{a^2-b^2}-\frac{b^2\left(\frac{a\int\sin(ix)dx-ib\sinh(x)}{a^2-b^2}-\frac{b^2\int\frac{\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}\right)}{a^2-b^2}\right)$$

$\downarrow$ 3118

$$i\left(\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)+\frac{1}{3}ib\sinh^3(x)}{a^2-b^2}-\frac{b^2\left(\frac{ia\cosh(x)-ib\sinh(x)}{a^2-b^2}-\frac{b^2\int\frac{\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}\right)}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3988}$$

$$i\left(\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)+\frac{1}{3}ib\sinh^3(x)}{a^2-b^2}-\frac{b^2\left(\frac{ia\cosh(x)-ib\sinh(x)}{a^2-b^2}-\frac{ib^2\int\frac{1}{a^2-b^2-(b+a\coth(x))^2\sinh^2(x)}d((b+a\coth(x))\sinh(x)}{a^2-b^2}\right.}{a^2-b^2}\right.$$

$$\Big\downarrow \text{219}$$

$$i\left(\frac{ia\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)+\frac{1}{3}ib\sinh^3(x)}{a^2-b^2}-\frac{b^2\left(\frac{ia\cosh(x)-ib\sinh(x)}{a^2-b^2}-\frac{ib^2\text{arctanh}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}\right)}{a^2-b^2}\right)$$

input
```
Int[Sinh[x]^3/(a + b*Coth[x]),x]
```

output
```
I*((I*a*(Cosh[x] - Cosh[x]^3/3) + (I/3)*b*Sinh[x]^3)/(a^2 - b^2) - (b^2*((
(-I)*b^2*ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(
3/2) + (I*a*Cosh[x] - I*b*Sinh[x])/(a^2 - b^2)))/(a^2 - b^2))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3988
```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Simp[-f^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f
*x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

rule 3990
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_
Symbol] :> Simp[1/(a^2 + b^2)   Int[Sec[e + f*x]^m*(a - b*Tan[e + f*x]), x]
, x] + Simp[b^2/(a^2 + b^2)   Int[Sec[e + f*x]^(m + 2)/(a + b*Tan[e + f*x])
, x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[(m - 1)/2,
0]
```

Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.28

method	result
risch	$\frac{e^{3x}}{24a+24b}-\frac{3\,e^{x}a}{8(a+b)^{2}}-\frac{5\,e^{x}b}{8(a+b)^{2}}-\frac{3\,e^{-x}a}{8(a-b)^{2}}+\frac{5\,e^{-x}b}{8(a-b)^{2}}+\frac{e^{-3x}}{24a-24b}+\frac{b^{4}\ln\left(e^{x}-\frac{a-b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)^{2}(a-b)^{2}}-\frac{b^{4}\ln\left(e^{x}+\frac{a-b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)^{2}(a-b)^{2}}$
default	$-\frac{16}{(32a-32b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{32}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}(32a-32b)}-\frac{a-2b}{2(a-b)^{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{2b^{4}\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^{2}+b^{2}}}\right)}{(a-b)^{2}(a+b)^{2}\sqrt{-a^{2}+b^{2}}}-\frac{}{3(\tan}$

input
```
int(sinh(x)^3/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
1/24/(a+b)*exp(x)^3-3/8/(a+b)^2*exp(x)*a-5/8/(a+b)^2*exp(x)*b-3/8/(a-b)^2/
exp(x)*a+5/8/(a-b)^2/exp(x)*b+1/24/(a-b)/exp(x)^3+1/(a^2-b^2)^(1/2)*b^4/(a
+b)^2/(a-b)^2*ln(exp(x)-(a-b)/(a^2-b^2)^(1/2))-1/(a^2-b^2)^(1/2)*b^4/(a+b)
^2/(a-b)^2*ln(exp(x)+(a-b)/(a^2-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(126) = 252$.

Time = 0.14 (sec) , antiderivative size = 1859, normalized size of antiderivative = 13.87

$$\int \frac{\sinh^3(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
[1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(
a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b -
 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a
^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x)^4 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^
2*b^3 + 7*a*b^4 - 5*b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
 b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a
*b^4 - b^5)*cosh(x)^3 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^
4 - 5*b^5)*cosh(x))*sinh(x)^3 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3
+ 7*a*b^4 + 5*b^5)*cosh(x)^2 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 +
 7*a*b^4 + 5*b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*c
osh(x)^4 + 6*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*co
sh(x)^2)*sinh(x)^2 + 24*(b^4*cosh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*c
osh(x)*sinh(x)^2 + b^4*sinh(x)^3)*sqrt(a^2 - b^2)*log(((a + b)*cosh(x)^2 +
 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(a^2 - b^2)*(cosh(x
) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
+ b)*sinh(x)^2 - a + b)) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4
 - b^5)*cosh(x)^5 - 2*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 -
5*b^5)*cosh(x)^3 - (3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b
^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + ...
```

Sympy [F]

$$\int \frac{\sinh^3(x)}{a + b\coth(x)}\,dx = \int \frac{\sinh^3(x)}{a + b\coth(x)}\,dx$$

input
```
integrate(sinh(x)**3/(a+b*coth(x)),x)
```

output
```
Integral(sinh(x)**3/(a + b*coth(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^3(x)}{a + b\coth(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.22

$$\int \frac{\sinh^3(x)}{a + b\coth(x)}\,dx = -\frac{2\,b^4\arctan\left(-\frac{ae^x + be^x}{\sqrt{-a^2+b^2}}\right)}{(a^4 - 2\,a^2b^2 + b^4)\sqrt{-a^2+b^2}} - \frac{\left(9\,ae^{(2\,x)} - 15\,be^{(2\,x)} - a + b\right)e^{(-3\,x)}}{24\,(a^2 - 2\,ab + b^2)} + \frac{a^2e^{(3\,x)} + 2\,abe^{(3\,x)} + b^2e^{(3\,x)} - 9\,a^2e^x - 24\,abe^x - 15\,b^2e^x}{24\,(a^3 + 3\,a^2b + 3\,ab^2 + b^3)}$$

input
```
integrate(sinh(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

output
```
-2*b^4*arctan(-(a*e^x + b*e^x)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*b^2 + b^4)*
sqrt(-a^2 + b^2)) - 1/24*(9*a*e^(2*x) - 15*b*e^(2*x) - a + b)*e^(-3*x)/(a^
2 - 2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) - 9*a^2
*e^x - 24*a*b*e^x - 15*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

Mupad [B] (verification not implemented)

Time = 3.15 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.28

$$\int \frac{\sinh^3(x)}{a+b\coth(x)}\,dx = \frac{e^{-3x}}{24a-24b} + \frac{e^{3x}}{24a+24b} - \frac{e^{-x}(3a-5b)}{8(a-b)^2} - \frac{e^{x}(3a+5b)}{8(a+b)^2} - \frac{b^4\ln\left(2a^3b-2ab^3+a^4-b^4+e^{x}(a+b)^{7/2}\sqrt{a-b}\right)}{(a+b)^{5/2}(a-b)^{5/2}} + \frac{b^4\ln\left(2ab^3-2a^3b-a^4+b^4+e^{x}(a+b)^{7/2}\sqrt{a-b}\right)}{(a+b)^{5/2}(a-b)^{5/2}}$$

input
```
int(sinh(x)^3/(a + b*coth(x)),x)
```

output
```
exp(-3*x)/(24*a - 24*b) + exp(3*x)/(24*a + 24*b) - (exp(-x)*(3*a - 5*b))/(
8*(a - b)^2) - (exp(x)*(3*a + 5*b))/(8*(a + b)^2) - (b^4*log(2*a^3*b - 2*a
*b^3 + a^4 - b^4 + exp(x)*(a + b)^(7/2)*(a - b)^(1/2)))/((a + b)^(5/2)*(a
- b)^(5/2)) + (b^4*log(2*a*b^3 - 2*a^3*b - a^4 + b^4 + exp(x)*(a + b)^(7/2
)*(a - b)^(1/2)))/((a + b)^(5/2)*(a - b)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.36

$$\int \frac{\sinh^3(x)}{a+b\coth(x)}\,dx$$
$$= \frac{-e^{6x}a^4b-2e^{6x}a^3b^2+2e^{6x}a^2b^3+e^{6x}a\,b^4-3e^{2x}a^4b+30e^{2x}a^3b^2+18e^{2x}a^2b^3-21e^{2x}a\,b^4+3e^{4x}a^4b+30e}{}$$

input
```
int(sinh(x)^3/(a+b*coth(x)),x)
```

output
```
( - 48*e**(3*x)*sqrt( - a**2 + b**2)*atan((e**x*a + e**x*b)/sqrt( - a**2 +
 b**2))*b**4 + e**(6*x)*a**5 - e**(6*x)*a**4*b - 2*e**(6*x)*a**3*b**2 + 2*
e**(6*x)*a**2*b**3 + e**(6*x)*a*b**4 - e**(6*x)*b**5 - 9*e**(4*x)*a**5 + 3
*e**(4*x)*a**4*b + 30*e**(4*x)*a**3*b**2 - 18*e**(4*x)*a**2*b**3 - 21*e**(
4*x)*a*b**4 + 15*e**(4*x)*b**5 - 9*e**(2*x)*a**5 - 3*e**(2*x)*a**4*b + 30*
e**(2*x)*a**3*b**2 + 18*e**(2*x)*a**2*b**3 - 21*e**(2*x)*a*b**4 - 15*e**(2
*x)*b**5 + a**5 + a**4*b - 2*a**3*b**2 - 2*a**2*b**3 + a*b**4 + b**5)/(24*
e**(3*x)*(a**6 - 3*a**4*b**2 + 3*a**2*b**4 - b**6))
```

3.106 $\int \frac{\sinh(x)}{a+b\coth(x)} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 73

$$\int \frac{\sinh(x)}{a+b\coth(x)} \, dx = -\frac{b^2 \text{arctanh}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + \frac{a\cosh(x)}{a^2-b^2} - \frac{b\sinh(x)}{a^2-b^2}$$

output
```
-b^2*arctanh((b+a*coth(x))*sinh(x)/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)+a*cosh
(x)/(a^2-b^2)-b*sinh(x)/(a^2-b^2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int \frac{\sinh(x)}{a+b\coth(x)} \, dx = \frac{a\cosh(x)}{a^2-b^2} + b\left(-\frac{2b\arctan\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)}{(-a+b)^{3/2}(a+b)^{3/2}} + \frac{\sinh(x)}{-a^2+b^2}\right)$$

input
```
Integrate[Sinh[x]/(a + b*Coth[x]),x]
```

output
```
(a*Cosh[x])/(a^2 - b^2) + b*((-2*b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*
Sqrt[a + b])])/((-a + b)^(3/2)*(a + b)^(3/2)) + Sinh[x]/(-a^2 + b^2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.182$, Rules used = {3042, 26, 3990, 26, 3042, 26, 3967, 26, 3042, 26, 3118, 3988, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{a+b\coth(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\sec\left(-\frac{\pi}{2}+ix\right)\left(a-ib\tan\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\sec\left(ix-\frac{\pi}{2}\right)\left(a-ib\tan\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3990}$$

$$-i\left(\frac{\int i(a-b\coth(x))\sinh(x)dx}{a^2-b^2}-\frac{b^2\int -\frac{i\text{csch}(x)}{a+b\coth(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\int (a-b\coth(x))\sinh(x)dx}{a^2-b^2}+\frac{ib^2\int \frac{\text{csch}(x)}{a+b\coth(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{ib^2\int \frac{i\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{i\int -\frac{i\left(a+ib\tan\left(ix-\frac{\pi}{2}\right)\right)}{\sec\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{\int \frac{a+ib\tan\left(ix-\frac{\pi}{2}\right)}{\sec\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{b^2\int \frac{\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3967}$$

$$-i\left(\frac{a\int i\sinh(x)dx-ib\sinh(x)}{a^2-b^2}-\frac{b^2\int \frac{\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{ia\int \sinh(x)dx-ib\sinh(x)}{a^2-b^2}-\frac{b^2\int \frac{\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{ia\int -i\sin(ix)dx-ib\sinh(x)}{a^2-b^2}-\frac{b^2\int \frac{\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{a\int \sin(ix)dx-ib\sinh(x)}{a^2-b^2}-\frac{b^2\int \frac{\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3118}$$

$$-i\left(\frac{ia\cosh(x)-ib\sinh(x)}{a^2-b^2}-\frac{b^2\int \frac{\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3988}$$

$$-i\left(\frac{ia\cosh(x)-ib\sinh(x)}{a^2-b^2}-\frac{ib^2\int \frac{1}{a^2-b^2-(b+a\coth(x))^2\sinh^2(x)}d((b+a\coth(x))\sinh(x))}{a^2-b^2}\right)$$

$$\Big\downarrow \text{219}$$

$$-i\left(\frac{ia\cosh(x)-ib\sinh(x)}{a^2-b^2}-\frac{ib^2\text{arctanh}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}\right)$$

input
```
Int[Sinh[x]/(a + b*Coth[x]),x]
```

output
```
(-I)*(((-I)*b^2*ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 -
 b^2)^(3/2) + (I*a*Cosh[x] - I*b*Sinh[x])/(a^2 - b^2))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3988
```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Simp[-f^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f
*x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

rule 3990
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_
Symbol] :> Simp[1/(a^2 + b^2)   Int[Sec[e + f*x]^m*(a - b*Tan[e + f*x]), x]
, x] + Simp[b^2/(a^2 + b^2)   Int[Sec[e + f*x]^(m + 2)/(a + b*Tan[e + f*x])
, x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[(m - 1)/2,
0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

method	result	size
default	$\frac{8}{(8a-8b)\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{2b^{2}\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^{2}+b^{2}}}\right)}{(a-b)(a+b)\sqrt{-a^{2}+b^{2}}}-\frac{8}{(8a+8b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}$	93
risch	$\frac{e^{x}}{2a+2b}+\frac{e^{-x}}{2a-2b}+\frac{b^{2}\ln\left(e^{x}-\frac{a-b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)(a-b)}-\frac{b^{2}\ln\left(e^{x}+\frac{a-b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)(a-b)}$	122

input
```
int(sinh(x)/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
8/(8*a-8*b)/(tanh(1/2*x)+1)+2*b^2/(a-b)/(a+b)/(-a^2+b^2)^(1/2)*arctan(1/2*
(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))-8/(8*a+8*b)/(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(69) = 138$.

Time = 0.12 (sec) , antiderivative size = 431, normalized size of antiderivative = 5.90

$$\int \frac{\sinh(x)}{a+b\coth(x)}\,dx$$
$$= \left[\frac{a^{3}+a^{2}b-ab^{2}-b^{3}+\left(a^{3}-a^{2}b-ab^{2}+b^{3}\right)\cosh(x)^{2}+2\left(a^{3}-a^{2}b-ab^{2}+b^{3}\right)\cosh(x)\sinh(x)+\left(a^{3}\right.}{2\left(\left(a^{4}-2\right.\right.}\right.$$

input
```
integrate(sinh(x)/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
[1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 +
2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) + (a^3 - a^2*b - a*b^2 + b^3
)*sinh(x)^2 - 2*(b^2*cosh(x) + b^2*sinh(x))*sqrt(a^2 - b^2)*log(((a + b)*c
osh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(a^2 - b^
2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sin
h(x) + (a + b)*sinh(x)^2 - a + b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^
4 - 2*a^2*b^2 + b^4)*sinh(x)), 1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a^2
*b - a*b^2 + b^3)*cosh(x)^2 + 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x
) + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^2 + 4*(b^2*cosh(x) + b^2*sinh(x))*
sqrt(-a^2 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x
))))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]
```

Sympy [F]

$$\int \frac{\sinh(x)}{a + b\coth(x)}\,dx = \int \frac{\sinh(x)}{a + b\coth(x)}\,dx$$

input
```
integrate(sinh(x)/(a+b*coth(x)),x)
```

output
```
Integral(sinh(x)/(a + b*coth(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh(x)}{a + b\coth(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(x)/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int \frac{\sinh(x)}{a+b\coth(x)}\,dx = \frac{2\,b^2\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{(a^2-b^2)\sqrt{-a^2+b^2}} + \frac{e^{(-x)}}{2\,(a-b)} + \frac{e^x}{2\,(a+b)}$$

input
```
integrate(sinh(x)/(a+b*coth(x)),x, algorithm="giac")
```

output
```
2*b^2*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^
2)) + 1/2*e^(-x)/(a - b) + 1/2*e^x/(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.66 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.14

$$\int \frac{\sinh(x)}{a+b\coth(x)}\,dx = \frac{e^x}{2\,a+2\,b} + \frac{e^{-x}}{2\,a-2\,b} - \frac{b^2\,\ln\left(\frac{2\,b^2\,e^x}{-a^3-a^2\,b+a\,b^2+b^3} - \frac{2\,b^2}{(a+b)^{5/2}\,\sqrt{a-b}}\right)}{(a+b)^{3/2}\,(a-b)^{3/2}} + \frac{b^2\,\ln\left(\frac{2\,b^2}{(a+b)^{5/2}\,\sqrt{a-b}} + \frac{2\,b^2\,e^x}{-a^3-a^2\,b+a\,b^2+b^3}\right)}{(a+b)^{3/2}\,(a-b)^{3/2}}$$

input
```
int(sinh(x)/(a + b*coth(x)),x)
```

output
```
exp(x)/(2*a + 2*b) + exp(-x)/(2*a - 2*b) - (b^2*log((2*b^2*exp(x))/(a*b^2
- a^2*b - a^3 + b^3) - (2*b^2)/((a + b)^(5/2)*(a - b)^(1/2))))/((a + b)^(3
/2)*(a - b)^(3/2)) + (b^2*log((2*b^2)/((a + b)^(5/2)*(a - b)^(1/2)) + (2*b
^2*exp(x))/(a*b^2 - a^2*b - a^3 + b^3)))/((a + b)^(3/2)*(a - b)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.34

$$\int \frac{\sinh(x)}{a+b\coth(x)}\,dx$$
$$= \frac{-4e^{x}\sqrt{-a^{2}+b^{2}}\,atan\left(\frac{e^{x}a+e^{x}b}{\sqrt{-a^{2}+b^{2}}}\right)a\,b^{2}+2e^{x}\cosh\left(x\right)a^{4}-4e^{x}\cosh\left(x\right)a^{2}b^{2}+2e^{x}\cosh\left(x\right)b^{4}-e^{2x}a^{3}b+e^{2}}{2e^{x}a\left(a^{4}-2a^{2}b^{2}+b^{4}\right)}$$

input
```
int(sinh(x)/(a+b*coth(x)),x)
```

output
```
( - 4*e**x*sqrt( - a**2 + b**2)*atan((e**x*a + e**x*b)/sqrt( - a**2 + b**2
))*a*b**2 + 2*e**x*cosh(x)*a**4 - 4*e**x*cosh(x)*a**2*b**2 + 2*e**x*cosh(x
)*b**4 - e**(2*x)*a**3*b + e**(2*x)*a**2*b**2 + e**(2*x)*a*b**3 - e**(2*x)
*b**4 + a**3*b + a**2*b**2 - a*b**3 - b**4)/(2*e**x*a*(a**4 - 2*a**2*b**2
+ b**4))
```

3.107 $\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx=-\frac{\operatorname{arctanh}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

output
```
-arctanh((b+a*coth(x))*sinh(x)/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx=\frac{2\arctan\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)}{\sqrt{-a+b}\sqrt{a+b}}$$

input
```
Integrate[Csch[x]/(a + b*Coth[x]),x]
```

output
```
(2*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/(Sqrt[-a + b]*Sqr
t[a + b])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 26, 3988, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sec\left(-\frac{\pi}{2}+ix\right)}{a-ib\tan\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3988}$$

$$-\int \frac{1}{a^2-b^2-(b+a\coth(x))^2\sinh^2(x)}d((b+a\coth(x))\sinh(x))$$

$$\Big\downarrow \text{219}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

input
```
Int[Csch[x]/(a + b*Coth[x]),x]
```

output
```
-(ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]]/Sqrt[a^2 - b^2])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3988
```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Simp[-f^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f
*x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{2 \arctan\left(\frac{2 \tanh\left(\frac{x}{2}\right) b+2 a}{2 \sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}}$	39
risch	$\frac{\ln\left(\mathrm{e}^{x}-\frac{a-b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}}-\frac{\ln\left(\mathrm{e}^{x}+\frac{a-b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}}$	70

input
```
int(csch(x)/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.87

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx$$
$$= \left[\frac{\log\left(\frac{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-2\,\sqrt{a^2-b^2}(\cosh(x)+\sinh(x))+a-b}{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-a+b}\right)}{\sqrt{a^2-b^2}}, \frac{2\,\sqrt{-a^2+b^2}\arctan\left(\frac{\cdots}{(a+b)\cdots}\right)}{a^2-b^2}\right.$$

input
```
integrate(csch(x)/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
[log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 -
2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a +
 b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b))/sqrt(a^2 - b^2), 2*sqrt(
-a^2 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x)))/(
a^2 - b^2)]
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx = \int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx$$

input
```
integrate(csch(x)/(a+b*coth(x)),x)
```

output
```
Integral(csch(x)/(a + b*coth(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csch(x)/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx = \frac{2\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

input
```
integrate(csch(x)/(a+b*coth(x)),x, algorithm="giac")
```

output
```
2*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^x\,\sqrt{b^2-a^2}}{a-b}\right)}{\sqrt{b^2-a^2}}$$

input
```
int(1/(sinh(x)*(a + b*coth(x))),x)
```

output
```
-(2*atan((exp(x)*(b^2 - a^2)^(1/2))/(a - b)))/(b^2 - a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx = -\frac{2\sqrt{-a^2+b^2}\,atan\left(\frac{e^x a+e^x b}{\sqrt{-a^2+b^2}}\right)}{a^2-b^2}$$

input
```
int(csch(x)/(a+b*coth(x)),x)
```

output
```
( - 2*sqrt( - a**2 + b**2)*atan((e**x*a + e**x*b)/sqrt( - a**2 + b**2)))/(
a**2 - b**2)
```

3.108 $\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 57

$$\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx = \frac{a\operatorname{arctanh}(\cosh(x))}{b^2} - \frac{\sqrt{a^2-b^2}\operatorname{arctanh}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b^2} - \frac{\operatorname{csch}(x)}{b}$$

output
```
a*arctanh(cosh(x))/b^2-(a^2-b^2)^(1/2)*arctanh((b+a*coth(x))*sinh(x)/(a^2-
b^2)^(1/2))/b^2-csch(x)/b
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx = -\frac{2\sqrt{-a+b}\sqrt{a+b}\arctan\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right) + b\operatorname{csch}(x) + a\left(-\log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right)\right)}{b^2}$$

input
```
Integrate[Csch[x]^3/(a + b*Coth[x]),x]
```

output
```
-((2*Sqrt[-a + b]*Sqrt[a + b]*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[
a + b])] + b*Csch[x] + a*(-Log[Cosh[x/2]] + Log[Sinh[x/2]]))/b^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 26, 3989, 26, 3042, 26, 3967, 26, 3042, 26, 3988, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\sec\left(-\frac{\pi}{2}+ix\right)^3}{a-ib\tan\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\sec\left(ix-\frac{\pi}{2}\right)^3}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3989}$$

$$-i\left(\frac{\int -i(a-b\coth(x))\operatorname{csch}(x)dx}{b^2}-\frac{(a^2-b^2)\int -\frac{i\operatorname{csch}(x)}{a+b\coth(x)}dx}{b^2}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(\frac{i(a^2-b^2)\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}dx}{b^2}-\frac{i\int (a-b\coth(x))\operatorname{csch}(x)dx}{b^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(\frac{i(a^2-b^2)\int \frac{i\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{b^2}-\frac{i\int i\sec\left(ix-\frac{\pi}{2}\right)\left(a+ib\tan\left(ix-\frac{\pi}{2}\right)\right)dx}{b^2}\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(\frac{\int \sec\left(ix-\frac{\pi}{2}\right)\left(a+ib\tan\left(ix-\frac{\pi}{2}\right)\right)dx}{b^2}-\frac{(a^2-b^2)\int \frac{\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{b^2}\right)$$

$$\Big\downarrow \; 3967$$

$$-i\left(\frac{a\int -i\text{csch}(x)dx-ib\text{csch}(x)}{b^2}-\frac{(a^2-b^2)\int \frac{\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{b^2}\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(\frac{-ia\int \text{csch}(x)dx-ib\text{csch}(x)}{b^2}-\frac{(a^2-b^2)\int \frac{\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{b^2}\right)$$

$$\Big\downarrow \; 3042$$

$$-i\left(\frac{-ia\int i\csc(ix)dx-ib\text{csch}(x)}{b^2}-\frac{(a^2-b^2)\int \frac{\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{b^2}\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(\frac{a\int \csc(ix)dx-ib\text{csch}(x)}{b^2}-\frac{(a^2-b^2)\int \frac{\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{b^2}\right)$$

$$\Big\downarrow \; 3988$$

$$-i\left(\frac{a\int \csc(ix)dx-ib\text{csch}(x)}{b^2}-\frac{i(a^2-b^2)\int \frac{1}{a^2-b^2-(b+a\coth(x))^2\sinh^2(x)}d((b+a\coth(x))\sinh(x))}{b^2}\right)$$

$$\Big\downarrow \; 219$$

$$-i\left(\frac{a\int \csc(ix)dx-ib\text{csch}(x)}{b^2}-\frac{i\sqrt{a^2-b^2}\text{arctanh}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{b^2}\right)$$

$$\Big\downarrow \; 4257$$

$$-i\left(\frac{ia\text{arctanh}(\cosh(x))-ib\text{csch}(x)}{b^2}-\frac{i\sqrt{a^2-b^2}\text{arctanh}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{b^2}\right)$$

input
```
Int[Csch[x]^3/(a + b*Coth[x]),x]
```

output
```
(-I)*(((-I)*Sqrt[a^2 - b^2]*ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b
^2]])/b^2 + (I*a*ArcTanh[Cosh[x]] - I*b*Csch[x])/b^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3988
```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Simp[-f^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f
*x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

rule 3989
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_
Symbol] :> Simp[-(b^2)^(-1)   Int[Sec[e + f*x]^(m - 2)*(a - b*Tan[e + f*x])
, x], x] + Simp[(a^2 + b^2)/b^2   Int[Sec[e + f*x]^(m - 2)/(a + b*Tan[e + f
*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0] && IGtQ[(m - 1)
/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.49

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)}{2b}+\frac{\left(4a^{2}-4b^{2}\right)\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^{2}+b^{2}}}\right)}{2b^{2}\sqrt{-a^{2}+b^{2}}}-\frac{1}{2b\tanh\left(\frac{x}{2}\right)}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b^{2}}$	85
risch	$-\frac{2\,\mathrm{e}^{x}}{b\left(\mathrm{e}^{2x}-1\right)}+\frac{a\ln\left(1+\mathrm{e}^{x}\right)}{b^{2}}-\frac{a\ln\left(\mathrm{e}^{x}-1\right)}{b^{2}}+\frac{\sqrt{a^{2}-b^{2}}\,\ln\left(\mathrm{e}^{x}-\frac{\sqrt{a^{2}-b^{2}}}{a+b}\right)}{b^{2}}-\frac{\sqrt{a^{2}-b^{2}}\,\ln\left(\mathrm{e}^{x}+\frac{\sqrt{a^{2}-b^{2}}}{a+b}\right)}{b^{2}}$	112

input
```
int(csch(x)^3/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*tanh(1/2*x)/b+1/2/b^2*(4*a^2-4*b^2)/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tan
h(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))-1/2/b/tanh(1/2*x)-1/b^2*a*ln(tanh(1/2*x)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(53) = 106.

Time = 0.11 (sec) , antiderivative size = 384, normalized size of antiderivative = 6.74

$$\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx$$

$$=\left[\frac{\sqrt{a^2-b^2}\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)\log\left(\frac{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x}{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x}\right.}{}\right.$$

input
```
integrate(csch(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
[(sqrt(a^2 - b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(((a
+ b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(a^
2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(
x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) - 2*b*cosh(x) + (a*cosh(x)^2 + 2*
a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x) + sinh(x) + 1) - (a*cosh(
x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x) + sinh(x) - 1) -
 2*b*sinh(x))/(b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 - b^2
), (2*sqrt(-a^2 + b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*arc
tan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - 2*b*cosh(x) +
(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x) + sinh(x
) + 1) - (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x)
 + sinh(x) - 1) - 2*b*sinh(x))/(b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^
2*sinh(x)^2 - b^2)]
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx = \int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx$$

input
```
integrate(csch(x)**3/(a+b*coth(x)),x)
```

output
```
Integral(csch(x)**3/(a + b*coth(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csch(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.49

$$\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx = \frac{a\log\left(e^x+1\right)}{b^2} - \frac{a\log\left(\left|e^x-1\right|\right)}{b^2} + \frac{2\left(a^2-b^2\right)\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b^2} - \frac{2\,e^x}{b(e^{(2\,x)}-1)}$$

input
```
integrate(csch(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

output
```
a*log(e^x + 1)/b^2 - a*log(abs(e^x - 1))/b^2 + 2*(a^2 - b^2)*arctan((a*e^x
 + b*e^x)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2) - 2*e^x/(b*(e^(2*x) - 1
))
```

Mupad [B] (verification not implemented)

Time = 2.75 (sec) , antiderivative size = 230, normalized size of antiderivative = 4.04

$$\begin{aligned}\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx &= \frac{2\,\mathrm{e}^x}{b-b\,\mathrm{e}^{2\,x}} - \frac{a\,\ln\left(32\,a\,b^2-64\,a^2\,b+32\,a^3-32\,a^3\,\mathrm{e}^x-32\,a\,b^2\,\mathrm{e}^x+64\,a^2\,b\,\mathrm{e}^x\right)}{b^2}\\ &\quad+ \frac{a\,\ln\left(32\,a\,b^2-64\,a^2\,b+32\,a^3+32\,a^3\,\mathrm{e}^x+32\,a\,b^2\,\mathrm{e}^x-64\,a^2\,b\,\mathrm{e}^x\right)}{b^2}\\ &\quad+ \frac{\ln\left(32\,a\,\sqrt{a^2-b^2}-32\,b\,\sqrt{a^2-b^2}-32\,a^2\,\mathrm{e}^x+32\,b^2\,\mathrm{e}^x\right)\,\sqrt{a^2-b^2}}{b^2}\\ &\quad- \frac{\ln\left(32\,a\,\sqrt{a^2-b^2}-32\,b\,\sqrt{a^2-b^2}+32\,a^2\,\mathrm{e}^x-32\,b^2\,\mathrm{e}^x\right)\,\sqrt{a^2-b^2}}{b^2}\end{aligned}$$

input
```
int(1/(sinh(x)^3*(a + b*coth(x))),x)
```

output
```
(2*exp(x))/(b - b*exp(2*x)) - (a*log(32*a*b^2 - 64*a^2*b + 32*a^3 - 32*a^3
*exp(x) - 32*a*b^2*exp(x) + 64*a^2*b*exp(x)))/b^2 + (a*log(32*a*b^2 - 64*a
^2*b + 32*a^3 + 32*a^3*exp(x) + 32*a*b^2*exp(x) - 64*a^2*b*exp(x)))/b^2 +
(log(32*a*(a^2 - b^2)^(1/2) - 32*b*(a^2 - b^2)^(1/2) - 32*a^2*exp(x) + 32*
b^2*exp(x))*(a^2 - b^2)^(1/2))/b^2 - (log(32*a*(a^2 - b^2)^(1/2) - 32*b*(a
^2 - b^2)^(1/2) + 32*a^2*exp(x) - 32*b^2*exp(x))*(a^2 - b^2)^(1/2))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.51

$$\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx = \frac{-2e^{2x}\sqrt{-a^2+b^2}\,atan\left(\frac{e^x a+e^x b}{\sqrt{-a^2+b^2}}\right)+2\sqrt{-a^2+b^2}\,atan\left(\frac{e^x a+e^x b}{\sqrt{-a^2+b^2}}\right)-e^{2x}\log(e^x-1)\,a+e^{2x}\log(e^x+1)\,a-}{b^2\left(e^{2x}-1\right)}$$

input
```
int(csch(x)^3/(a+b*coth(x)),x)
```

output
```
( - 2*e**(2*x)*sqrt( - a**2 + b**2)*atan((e**x*a + e**x*b)/sqrt( - a**2 +
b**2)) + 2*sqrt( - a**2 + b**2)*atan((e**x*a + e**x*b)/sqrt( - a**2 + b**2
)) - e**(2*x)*log(e**x - 1)*a + e**(2*x)*log(e**x + 1)*a - 2*e**x*b + log(
e**x - 1)*a - log(e**x + 1)*a)/(b**2*(e**(2*x) - 1))
```

3.109 $\int \frac{\operatorname{csch}^5(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 109

$$\int \frac{\operatorname{csch}^5(x)}{a+b\coth(x)}\,dx = -\frac{a\operatorname{arctanh}(\cosh(x))}{2b^2}+\frac{a(a^2-b^2)\operatorname{arctanh}(\cosh(x))}{b^4} - \frac{(a^2-b^2)^{3/2}\operatorname{arctanh}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b^4} - \frac{(a^2-b^2)\operatorname{csch}(x)}{b^3}+\frac{a\coth(x)\operatorname{csch}(x)}{2b^2}-\frac{\operatorname{csch}^3(x)}{3b}$$

output
```
-1/2*a*arctanh(cosh(x))/b^2+a*(a^2-b^2)*arctanh(cosh(x))/b^4-(a^2-b^2)^(3/
2)*arctanh((b+a*coth(x))*sinh(x)/(a^2-b^2)^(1/2))/b^4-(a^2-b^2)*csch(x)/b^
3+1/2*a*coth(x)*csch(x)/b^2-1/3*csch(x)^3/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 259 vs. 2(109) = 218.

Time = 0.72 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.38

$$\int \frac{\operatorname{csch}^5(x)}{a+b\coth(x)}\,dx = \frac{-96a^2\sqrt{-a+b}\sqrt{a+b}\arctan\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)+96b^2\sqrt{-a+b}\sqrt{a+b}\arctan\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)+4b(-6a^2+7}{}$$

input
```
Integrate[Csch[x]^5/(a + b*Coth[x]),x]
```

output
```
(-96*a^2*Sqrt[-a + b]*Sqrt[a + b]*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*S
qrt[a + b])] + 96*b^2*Sqrt[-a + b]*Sqrt[a + b]*ArcTan[(a + b*Tanh[x/2])/(S
qrt[-a + b]*Sqrt[a + b])] + 4*b*(-6*a^2 + 7*b^2)*Coth[x/2] + 6*a*b^2*Csch[
x/2]^2 + 48*a^3*Log[Cosh[x/2]] - 72*a*b^2*Log[Cosh[x/2]] - 48*a^3*Log[Sinh
[x/2]] + 72*a*b^2*Log[Sinh[x/2]] + 6*a*b^2*Sech[x/2]^2 - 16*b^3*Csch[x]^3*
Sinh[x/2]^4 - b^3*Csch[x/2]^4*Sinh[x] + 24*a^2*b*Tanh[x/2] - 28*b^3*Tanh[x
/2])/(48*b^4)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.11, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.923$, Rules used = {3042, 26, 3989, 26, 3042, 26, 3967, 26, 3042, 26, 3989, 26, 3042, 26, 3967, 26, 3042, 26, 3988, 219, 4255, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^5(x)}{a+b\coth(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sec\left(-\frac{\pi}{2}+ix\right)^5}{a-ib\tan\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sec\left(ix-\frac{\pi}{2}\right)^5}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3989}$$

$$i\left(\frac{\int i(a-b\coth(x))\operatorname{csch}^3(x)dx}{b^2}-\frac{\left(a^2-b^2\right)\int \frac{i\operatorname{csch}^3(x)}{a+b\coth(x)}dx}{b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\int (a-b\coth(x))\text{csch}^3(x)dx}{b^2}-\frac{i(a^2-b^2)\int \frac{\text{csch}^3(x)}{a+b\coth(x)}dx}{b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\int -i\sec\left(ix-\frac{\pi}{2}\right)^3\left(a+ib\tan\left(ix-\frac{\pi}{2}\right)\right)dx}{b^2}-\frac{i(a^2-b^2)\int -\frac{i\sec\left(ix-\frac{\pi}{2}\right)^3}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^3\left(a+ib\tan\left(ix-\frac{\pi}{2}\right)\right)dx}{b^2}-\frac{(a^2-b^2)\int \frac{\sec\left(ix-\frac{\pi}{2}\right)^3}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{b^2}\right)$$

$$\Big\downarrow \text{3967}$$

$$i\left(\frac{a\int i\text{csch}^3(x)dx+\frac{1}{3}ib\text{csch}^3(x)}{b^2}-\frac{(a^2-b^2)\int \frac{\sec\left(ix-\frac{\pi}{2}\right)^3}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{ia\int \text{csch}^3(x)dx+\frac{1}{3}ib\text{csch}^3(x)}{b^2}-\frac{(a^2-b^2)\int \frac{\sec\left(ix-\frac{\pi}{2}\right)^3}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{ia\int -i\csc(ix)^3dx+\frac{1}{3}ib\text{csch}^3(x)}{b^2}-\frac{(a^2-b^2)\int \frac{\sec\left(ix-\frac{\pi}{2}\right)^3}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{a\int \csc(ix)^3dx+\frac{1}{3}ib\text{csch}^3(x)}{b^2}-\frac{(a^2-b^2)\int \frac{\sec\left(ix-\frac{\pi}{2}\right)^3}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{b^2}\right)$$

$$\Big\downarrow \text{3989}$$

$$i\left(\frac{a\int \csc(ix)^3dx+\frac{1}{3}ib\text{csch}^3(x)}{b^2}-\frac{(a^2-b^2)\left(\frac{\int -i(a-b\coth(x))\text{csch}(x)dx}{b^2}-\frac{(a^2-b^2)\int -\frac{i\text{csch}(x)}{a+b\coth(x)}dx}{b^2}\right)}{b^2}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{a\int \csc(ix)^3dx+\frac{1}{3}ib\text{csch}^3(x)}{b^2}-\frac{(a^2-b^2)\left(\frac{i(a^2-b^2)\int \frac{\text{csch}(x)}{a+b\coth(x)}dx}{b^2}-\frac{i\int (a-b\coth(x))\text{csch}(x)dx}{b^2}\right)}{b^2}\right)$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{a\int \csc(ix)^3dx+\frac{1}{3}ib\text{csch}^3(x)}{b^2}-\frac{(a^2-b^2)\left(\frac{i(a^2-b^2)\int \frac{i\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{b^2}-\frac{i\int i\sec\left(ix-\frac{\pi}{2}\right)\left(a+ib\tan\left(ix-\frac{\pi}{2}\right)\right)dx}{b^2}\right)}{b^2}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{a\int \csc(ix)^3dx+\frac{1}{3}ib\text{csch}^3(x)}{b^2}-\frac{(a^2-b^2)\left(\frac{\int \sec\left(ix-\frac{\pi}{2}\right)\left(a+ib\tan\left(ix-\frac{\pi}{2}\right)\right)dx}{b^2}-\frac{(a^2-b^2)\int \frac{\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{b^2}\right)}{b^2}\right)$$

$$\Big\downarrow \; 3967$$

$$i\left(\frac{a\int \csc(ix)^3dx+\frac{1}{3}ib\text{csch}^3(x)}{b^2}-\frac{(a^2-b^2)\left(\frac{a\int -i\text{csch}(x)dx-ib\text{csch}(x)}{b^2}-\frac{(a^2-b^2)\int \frac{\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{b^2}\right)}{b^2}\right)$$

$$\Big\downarrow \ 26$$

$$i\left(\frac{a\int \csc(ix)^3dx+\frac{1}{3}ib\text{csch}^3(x)}{b^2}-\frac{\left(a^2-b^2\right)\left(\frac{-ia\int \text{csch}(x)dx-ib\text{csch}(x)}{b^2}-\frac{\left(a^2-b^2\right)\int \frac{\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{b^2}\right)}{b^2}\right)$$

$$\Big\downarrow \ 3042$$

$$i\left(\frac{a\int \csc(ix)^3dx+\frac{1}{3}ib\text{csch}^3(x)}{b^2}-\frac{\left(a^2-b^2\right)\left(\frac{-ia\int i\csc(ix)dx-ib\text{csch}(x)}{b^2}-\frac{\left(a^2-b^2\right)\int \frac{\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{b^2}\right)}{b^2}\right)$$

$$\Big\downarrow \ 26$$

$$i\left(\frac{a\int \csc(ix)^3dx+\frac{1}{3}ib\text{csch}^3(x)}{b^2}-\frac{\left(a^2-b^2\right)\left(\frac{a\int \csc(ix)dx-ib\text{csch}(x)}{b^2}-\frac{\left(a^2-b^2\right)\int \frac{\sec\left(ix-\frac{\pi}{2}\right)}{a-ib\tan\left(ix-\frac{\pi}{2}\right)}dx}{b^2}\right)}{b^2}\right)$$

$$\Big\downarrow \ 3988$$

$$i\left(\frac{a\int \csc(ix)^3dx+\frac{1}{3}ib\text{csch}^3(x)}{b^2}-\frac{\left(a^2-b^2\right)\left(\frac{a\int \csc(ix)dx-ib\text{csch}(x)}{b^2}-\frac{i\left(a^2-b^2\right)\int \frac{1}{a^2-b^2-(b+a\coth(x))^2\sinh^2(x)}d((b+a\coth(\right.}{b^2}}{b^2}\right.$$

$$\Big\downarrow \ 219$$

$$i\left(\frac{a\int \csc(ix)^3dx+\frac{1}{3}ib\text{csch}^3(x)}{b^2}-\frac{(a^2-b^2)\left(\frac{a\int \csc(ix)dx-ib\text{csch}(x)}{b^2}-\frac{i\sqrt{a^2-b^2}\text{arctanh}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{b^2}\right)}{b^2}\right)$$

$\big\downarrow$ 4255

$$i\left(\frac{a\left(\frac{1}{2}\int -i\text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+\frac{1}{3}ib\text{csch}^3(x)}{b^2}-\frac{(a^2-b^2)\left(\frac{a\int \csc(ix)dx-ib\text{csch}(x)}{b^2}-\frac{i\sqrt{a^2-b^2}\text{arctanh}(\cdots}{\cdots}\cdots\right.}{b^2}\cdots\right.$$

$\big\downarrow$ 26

$$i\left(\frac{a\left(-\frac{1}{2}i\int \text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+\frac{1}{3}ib\text{csch}^3(x)}{b^2}-\frac{(a^2-b^2)\left(\frac{a\int \csc(ix)dx-ib\text{csch}(x)}{b^2}-\frac{i\sqrt{a^2-b^2}\text{arctanh}(\cdots}{\cdots}\cdots\right.}{b^2}\cdots\right.$$

$\big\downarrow$ 3042

$$i\left(\frac{a\left(-\frac{1}{2}i\int i\csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+\frac{1}{3}ib\text{csch}^3(x)}{b^2}-\frac{(a^2-b^2)\left(\frac{a\int \csc(ix)dx-ib\text{csch}(x)}{b^2}-\frac{i\sqrt{a^2-b^2}\text{arctanh}\cdots}{\cdots}\cdots\right.}{b^2}\cdots\right.$$

$\big\downarrow$ 26

$$i\left(\frac{a\left(\frac{1}{2}\int \csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+\frac{1}{3}ib\text{csch}^3(x)}{b^2}-\frac{(a^2-b^2)\left(\frac{a\int \csc(ix)dx-ib\text{csch}(x)}{b^2}-\frac{i\sqrt{a^2-b^2}\text{arctanh}\left(\frac{\sin\cdots}{\cdots}\right.}{b^2}\cdots\right.}{b^2}\cdots\right.$$

$\big\downarrow$ 4257

$$i\left(\frac{a\left(\frac{1}{2}i\text{arctanh}(\cosh(x))-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+\frac{1}{3}ib\text{csch}^3(x)}{b^2}-\frac{(a^2-b^2)\left(\frac{ia\text{arctanh}(\cosh(x))-ib\text{csch}(x)}{b^2}-\frac{i\sqrt{a^2-b^2}\cdots}{\cdots}\right.}{b^2}\right.$$

input
```
Int[Csch[x]^5/(a + b*Coth[x]),x]
```

output
```
I*(-(((a^2 - b^2)*(((-I)*Sqrt[a^2 - b^2]*ArcTanh[((b + a*Coth[x])*Sinh[x])
/Sqrt[a^2 - b^2]])/b^2 + (I*a*ArcTanh[Cosh[x]] - I*b*Csch[x])/b^2))/b^2) +
 ((I/3)*b*Csch[x]^3 + a*((I/2)*ArcTanh[Cosh[x]] - (I/2)*Coth[x]*Csch[x]))/
b^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3988
```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Simp[-f^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f
*x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

rule 3989
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_
Symbol] :> Simp[-(b^2)^(-1)   Int[Sec[e + f*x]^(m - 2)*(a - b*Tan[e + f*x])
, x], x] + Simp[(a^2 + b^2)/b^2   Int[Sec[e + f*x]^(m - 2)/(a + b*Tan[e + f
*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0] && IGtQ[(m - 1)
/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.60

method	result
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^{3}b^{2}}{3}-\tanh\left(\frac{x}{2}\right)^{2}ab+4\tanh\left(\frac{x}{2}\right)a^{2}-5\tanh\left(\frac{x}{2}\right)b^{2}}{8b^{3}}+\frac{\left(16a^{4}-32a^{2}b^{2}+16b^{4}\right)\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^{2}+b^{2}}}\right)}{8b^{4}\sqrt{-a^{2}+b^{2}}}-\frac{1}{24b\tanh\left(\frac{x}{2}\right)^{3}}-$
risch	$-\frac{e^{x}\left(6\,e^{4x}a^{2}-3\,e^{4x}ab-6b^{2}e^{4x}-12\,e^{2x}a^{2}+20b^{2}e^{2x}+6a^{2}+3ab-6b^{2}\right)}{3b^{3}\left(e^{2x}-1\right)^{3}}+\frac{\sqrt{a^{2}-b^{2}}\,\ln\left(e^{x}-\frac{\sqrt{a^{2}-b^{2}}}{a+b}\right)a^{2}}{b^{4}}-\frac{\sqrt{a^{2}-b^{2}}\,\ln\left(e^{x}-\frac{\sqrt{a^{2}-}}{a+b}\right.}{b^{2}}$

input
```
int(csch(x)^5/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
1/8/b^3*(1/3*tanh(1/2*x)^3*b^2-tanh(1/2*x)^2*a*b+4*tanh(1/2*x)*a^2-5*tanh(
1/2*x)*b^2)+1/8/b^4*(16*a^4-32*a^2*b^2+16*b^4)/(-a^2+b^2)^(1/2)*arctan(1/2
*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))-1/24/b/tanh(1/2*x)^3-1/8*(4*a^2-5
*b^2)/b^3/tanh(1/2*x)+1/8/b^2*a/tanh(1/2*x)^2-1/2/b^4*a*(2*a^2-3*b^2)*ln(t
anh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1296 vs. $2(99) = 198$.

Time = 0.17 (sec) , antiderivative size = 2647, normalized size of antiderivative = 24.28

$$\int \frac{\operatorname{csch}^5(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(x)^5/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
[-1/6*(6*(2*a^2*b - a*b^2 - 2*b^3)*cosh(x)^5 + 30*(2*a^2*b - a*b^2 - 2*b^3
)*cosh(x)*sinh(x)^4 + 6*(2*a^2*b - a*b^2 - 2*b^3)*sinh(x)^5 - 8*(3*a^2*b -
 5*b^3)*cosh(x)^3 - 4*(6*a^2*b - 10*b^3 - 15*(2*a^2*b - a*b^2 - 2*b^3)*cos
h(x)^2)*sinh(x)^3 + 12*(5*(2*a^2*b - a*b^2 - 2*b^3)*cosh(x)^3 - 2*(3*a^2*b
 - 5*b^3)*cosh(x))*sinh(x)^2 + 6*((a^2 - b^2)*cosh(x)^6 + 6*(a^2 - b^2)*co
sh(x)*sinh(x)^5 + (a^2 - b^2)*sinh(x)^6 - 3*(a^2 - b^2)*cosh(x)^4 + 3*(5*(
a^2 - b^2)*cosh(x)^2 - a^2 + b^2)*sinh(x)^4 + 4*(5*(a^2 - b^2)*cosh(x)^3 -
 3*(a^2 - b^2)*cosh(x))*sinh(x)^3 + 3*(a^2 - b^2)*cosh(x)^2 + 3*(5*(a^2 -
b^2)*cosh(x)^4 - 6*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 - a^2 + b^
2 + 6*((a^2 - b^2)*cosh(x)^5 - 2*(a^2 - b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(
x))*sinh(x))*sqrt(a^2 - b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*si
nh(x) + (a + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)
/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a +
b)) + 6*(2*a^2*b + a*b^2 - 2*b^3)*cosh(x) - 3*((2*a^3 - 3*a*b^2)*cosh(x)^6
 + 6*(2*a^3 - 3*a*b^2)*cosh(x)*sinh(x)^5 + (2*a^3 - 3*a*b^2)*sinh(x)^6 - 3
*(2*a^3 - 3*a*b^2)*cosh(x)^4 - 3*(2*a^3 - 3*a*b^2 - 5*(2*a^3 - 3*a*b^2)*co
sh(x)^2)*sinh(x)^4 + 4*(5*(2*a^3 - 3*a*b^2)*cosh(x)^3 - 3*(2*a^3 - 3*a*b^2
)*cosh(x))*sinh(x)^3 - 2*a^3 + 3*a*b^2 + 3*(2*a^3 - 3*a*b^2)*cosh(x)^2 + 3
*(5*(2*a^3 - 3*a*b^2)*cosh(x)^4 + 2*a^3 - 3*a*b^2 - 6*(2*a^3 - 3*a*b^2)*co
sh(x)^2)*sinh(x)^2 + 6*((2*a^3 - 3*a*b^2)*cosh(x)^5 - 2*(2*a^3 - 3*a*b^...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^5(x)}{a+b\coth(x)}\,dx=\int \frac{\operatorname{csch}^5(x)}{a+b\coth(x)}\,dx$$

input
```
integrate(csch(x)**5/(a+b*coth(x)),x)
```

output
```
Integral(csch(x)**5/(a + b*coth(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^5(x)}{a+b\coth(x)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(csch(x)^5/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.62

$$\begin{aligned}\int \frac{\operatorname{csch}^5(x)}{a+b\coth(x)}\,dx&=\frac{(2a^3-3ab^2)\log(e^x+1)}{2b^4}-\frac{(2a^3-3ab^2)\log(|e^x-1|)}{2b^4}\\&+\frac{2(a^4-2a^2b^2+b^4)\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b^4}\\&-\frac{6a^2e^{(5x)}-3abe^{(5x)}-6b^2e^{(5x)}-12a^2e^{(3x)}+20b^2e^{(3x)}+6a^2e^x+3abe^x-6b^2e^x}{3b^3(e^{(2x)}-1)^3}\end{aligned}$$

input
```
integrate(csch(x)^5/(a+b*coth(x)),x, algorithm="giac")
```

output
```
1/2*(2*a^3 - 3*a*b^2)*log(e^x + 1)/b^4 - 1/2*(2*a^3 - 3*a*b^2)*log(abs(e^x
 - 1))/b^4 + 2*(a^4 - 2*a^2*b^2 + b^4)*arctan((a*e^x + b*e^x)/sqrt(-a^2 +
b^2))/(sqrt(-a^2 + b^2)*b^4) - 1/3*(6*a^2*e^(5*x) - 3*a*b*e^(5*x) - 6*b^2*
e^(5*x) - 12*a^2*e^(3*x) + 20*b^2*e^(3*x) + 6*a^2*e^x + 3*a*b*e^x - 6*b^2*
e^x)/(b^3*(e^(2*x) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.36

$$\int \frac{\operatorname{csch}^5(x)}{a+b\coth(x)}\,dx = \frac{\ln(e^x-1)\,(3\,a\,b^2-2\,a^3)}{2\,b^4} - \frac{8\,e^x}{3\,b\,(3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1)} - \frac{\ln(e^x+1)\,(3\,a\,b^2-2\,a^3)}{2\,b^4} - \frac{\ln\left(\sqrt{(a+b)^3\,(a-b)^3}+a^3\,e^x-b^3\,e^x-a\,b^2\,e^x+a^2\,b\,e^x\right)\sqrt{(a+b)^3\,(a-b)^3}}{b^4} + \frac{\ln\left(\sqrt{(a+b)^3\,(a-b)^3}-a^3\,e^x+b^3\,e^x+a\,b^2\,e^x-a^2\,b\,e^x\right)\sqrt{(a+b)^3\,(a-b)^3}}{b^4} + \frac{2\,e^x\,(3\,a-4\,b)}{3\,b^2\,(e^{4\,x}-2\,e^{2\,x}+1)} + \frac{e^x\,(-2\,a^2+a\,b+2\,b^2)}{b^3\,(e^{2\,x}-1)}$$

input
```
int(1/(sinh(x)^5*(a + b*coth(x))),x)
```

output
```
(log(exp(x) - 1)*(3*a*b^2 - 2*a^3))/(2*b^4) - (8*exp(x))/(3*b*(3*exp(2*x)
- 3*exp(4*x) + exp(6*x) - 1)) - (log(exp(x) + 1)*(3*a*b^2 - 2*a^3))/(2*b^4
) - (log(((a + b)^3*(a - b)^3)^(1/2) + a^3*exp(x) - b^3*exp(x) - a*b^2*exp
(x) + a^2*b*exp(x))*((a + b)^3*(a - b)^3)^(1/2))/b^4 + (log(((a + b)^3*(a
- b)^3)^(1/2) - a^3*exp(x) + b^3*exp(x) + a*b^2*exp(x) - a^2*b*exp(x))*((a
 + b)^3*(a - b)^3)^(1/2))/b^4 + (2*exp(x)*(3*a - 4*b))/(3*b^2*(exp(4*x) -
2*exp(2*x) + 1)) + (exp(x)*(a*b - 2*a^2 + 2*b^2))/(b^3*(exp(2*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 702, normalized size of antiderivative = 6.44

$$\int \frac{\operatorname{csch}^5(x)}{a+b\coth(x)}\,dx$$
$$= \frac{12\sqrt{-a^2+b^2}\,atan\left(\frac{e^x a+e^x b}{\sqrt{-a^2+b^2}}\right)a^2-12\sqrt{-a^2+b^2}\,atan\left(\frac{e^x a+e^x b}{\sqrt{-a^2+b^2}}\right)b^2-6e^{6x}\log(e^x-1)\,a^3+6e^{6x}\log(e^x+1}{}$$

input
```
int(csch(x)^5/(a+b*coth(x)),x)
```

output
```
( - 12*e**(6*x)*sqrt( - a**2 + b**2)*atan((e**x*a + e**x*b)/sqrt( - a**2 +
 b**2))*a**2 + 12*e**(6*x)*sqrt( - a**2 + b**2)*atan((e**x*a + e**x*b)/sqr
t( - a**2 + b**2))*b**2 + 36*e**(4*x)*sqrt( - a**2 + b**2)*atan((e**x*a +
e**x*b)/sqrt( - a**2 + b**2))*a**2 - 36*e**(4*x)*sqrt( - a**2 + b**2)*atan
((e**x*a + e**x*b)/sqrt( - a**2 + b**2))*b**2 - 36*e**(2*x)*sqrt( - a**2 +
 b**2)*atan((e**x*a + e**x*b)/sqrt( - a**2 + b**2))*a**2 + 36*e**(2*x)*sqr
t( - a**2 + b**2)*atan((e**x*a + e**x*b)/sqrt( - a**2 + b**2))*b**2 + 12*s
qrt( - a**2 + b**2)*atan((e**x*a + e**x*b)/sqrt( - a**2 + b**2))*a**2 - 12
*sqrt( - a**2 + b**2)*atan((e**x*a + e**x*b)/sqrt( - a**2 + b**2))*b**2 -
6*e**(6*x)*log(e**x - 1)*a**3 + 9*e**(6*x)*log(e**x - 1)*a*b**2 + 6*e**(6*
x)*log(e**x + 1)*a**3 - 9*e**(6*x)*log(e**x + 1)*a*b**2 - 12*e**(5*x)*a**2
*b + 6*e**(5*x)*a*b**2 + 12*e**(5*x)*b**3 + 18*e**(4*x)*log(e**x - 1)*a**3
 - 27*e**(4*x)*log(e**x - 1)*a*b**2 - 18*e**(4*x)*log(e**x + 1)*a**3 + 27*
e**(4*x)*log(e**x + 1)*a*b**2 + 24*e**(3*x)*a**2*b - 40*e**(3*x)*b**3 - 18
*e**(2*x)*log(e**x - 1)*a**3 + 27*e**(2*x)*log(e**x - 1)*a*b**2 + 18*e**(2
*x)*log(e**x + 1)*a**3 - 27*e**(2*x)*log(e**x + 1)*a*b**2 - 12*e**x*a**2*b
 - 6*e**x*a*b**2 + 12*e**x*b**3 + 6*log(e**x - 1)*a**3 - 9*log(e**x - 1)*a
*b**2 - 6*log(e**x + 1)*a**3 + 9*log(e**x + 1)*a*b**2)/(6*b**4*(e**(6*x) -
 3*e**(4*x) + 3*e**(2*x) - 1))
```

3.110 $\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 60

$$\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx = \frac{x}{16}+\frac{1}{32(1-\coth(x))^2}-\frac{1}{8(1-\coth(x))}-\frac{1}{24(1+\coth(x))^3}+\frac{5}{32(1+\coth(x))^2}-\frac{3}{16(1+\coth(x))}$$

output
```
1/16*x+1/32/(1-coth(x))^2-1/(8-8*coth(x))-1/24/(1+coth(x))^3+5/32/(1+coth(
x))^2-3/(16+16*coth(x))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx = \frac{1}{192}(12x+15\cosh(2x)+6\cosh(4x)+\cosh(6x)+3\sinh(2x)-3\sinh(4x)-\sinh(6x))$$

input
```
Integrate[Cosh[x]^4/(1 + Coth[x]),x]
```

output
```
(12*x + 15*Cosh[2*x] + 6*Cosh[4*x] + Cosh[6*x] + 3*Sinh[2*x] - 3*Sinh[4*x]
 - Sinh[6*x])/192
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 3999, 25, 516, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(x)}{\coth(x)+1}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^4}{1-i\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \ 3999$$

$$-\int -\frac{\coth^4(x)}{(\coth(x)+1)\left(1-\coth^2(x)\right)^3}d\coth(x)$$

$$\Big\downarrow \ 25$$

$$\int \frac{\coth^4(x)}{(\coth(x)+1)\left(1-\coth^2(x)\right)^3}d\coth(x)$$

$$\Big\downarrow \ 516$$

$$\int \frac{\coth^4(x)}{(1-\coth(x))^3(\coth(x)+1)^4}d\coth(x)$$

$$\Big\downarrow \ 99$$

$$\int \left(-\frac{1}{16\left(\coth^2(x)-1\right)}-\frac{1}{8(\coth(x)-1)^2}+\frac{3}{16(\coth(x)+1)^2}-\frac{1}{16(\coth(x)-1)^3}-\frac{5}{16(\coth(x)+1)^3}+\frac{}{8(\text{co}}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{16}\operatorname{arctanh}(\coth(x)) - \frac{1}{8(1-\coth(x))} - \frac{3}{16(\coth(x)+1)} + \frac{1}{32(1-\coth(x))^2} + \frac{5}{32(\coth(x)+1)^2} - \frac{1}{24(\coth(x)+1)^3}$$

input
```
Int[Cosh[x]^4/(1 + Coth[x]),x]
```

output
```
ArcTanh[Coth[x]]/16 + 1/(32*(1 - Coth[x])^2) - 1/(8*(1 - Coth[x])) - 1/(24
*(1 + Coth[x])^3) + 5/(32*(1 + Coth[x])^2) - 3/(16*(1 + Coth[x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 516
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.58

method	result
risch	$\frac{x}{16}+\frac{e^{4x}}{128}+\frac{3\,e^{2x}}{64}+\frac{e^{-2x}}{32}+\frac{3\,e^{-4x}}{128}+\frac{e^{-6x}}{192}$
parallelrisch	$\frac{(-12\cosh(x)-12\sinh(x))\ln(1-\tanh(x))+(12\cosh(x)+12\sinh(x))\ln(\tanh(x)+1)+64\cosh(x)+40\sinh(x)+27\cosh(3x)+5\cosh}{384\sinh(x)+384\cosh(x)}$
default	$\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{6}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{5}}+\frac{13}{8\left(\tanh\left(\frac{x}{2}\right)+1\right)^{4}}-\frac{19}{12\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{3}{8\left(\tanh\left(\frac{x}{2}\right)+1\right)}+$

input
```
int(cosh(x)^4/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
1/16*x+1/128*exp(4*x)+3/64*exp(2*x)+1/32*exp(-2*x)+3/128*exp(-4*x)+1/192*e
xp(-6*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(44) = 88.

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.53

$$\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx$$
$$=\frac{5\cosh(x)^5+25\cosh(x)\sinh(x)^4+\sinh(x)^5+\left(10\cosh(x)^2+9\right)\sinh(x)^3+27\cosh(x)^3+\left(50\cosh\right.}{384\,(\cosh(x)}$$

input
```
integrate(cosh(x)^4/(1+coth(x)),x, algorithm="fricas")
```

output
```
1/384*(5*cosh(x)^5 + 25*cosh(x)*sinh(x)^4 + sinh(x)^5 + (10*cosh(x)^2 + 9)
*sinh(x)^3 + 27*cosh(x)^3 + (50*cosh(x)^3 + 81*cosh(x))*sinh(x)^2 + 12*(2*
x + 1)*cosh(x) + (5*cosh(x)^4 + 27*cosh(x)^2 + 24*x - 12)*sinh(x))/(cosh(x
) + sinh(x))
```

Sympy [F]

$$\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx = \int \frac{\cosh^4(x)}{\coth(x)+1}\,dx$$

input
```
integrate(cosh(x)**4/(1+coth(x)),x)
```

output
```
Integral(cosh(x)**4/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.60

$$\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx = \frac{1}{128}\left(6\,e^{(-2\,x)}+1\right)e^{(4\,x)} + \frac{1}{16}\,x + \frac{1}{32}\,e^{(-2\,x)} + \frac{3}{128}\,e^{(-4\,x)} + \frac{1}{192}\,e^{(-6\,x)}$$

input
```
integrate(cosh(x)^4/(1+coth(x)),x, algorithm="maxima")
```

output
```
1/128*(6*e^(-2*x) + 1)*e^(4*x) + 1/16*x + 1/32*e^(-2*x) + 3/128*e^(-4*x) +
 1/192*e^(-6*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx = -\frac{1}{384}\left(22\,e^{(6\,x)} - 12\,e^{(4\,x)} - 9\,e^{(2\,x)} - 2\right)e^{(-6\,x)} + \frac{1}{16}\,x + \frac{1}{128}\,e^{(4\,x)} + \frac{3}{64}\,e^{(2\,x)}$$

input
```
integrate(cosh(x)^4/(1+coth(x)),x, algorithm="giac")
```

output
```
-1/384*(22*e^(6*x) - 12*e^(4*x) - 9*e^(2*x) - 2)*e^(-6*x) + 1/16*x + 1/128
*e^(4*x) + 3/64*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 2.59 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx = \frac{x}{16}+\frac{e^{-2\,x}}{32}+\frac{3\,e^{2\,x}}{64}+\frac{3\,e^{-4\,x}}{128}+\frac{e^{4\,x}}{128}+\frac{e^{-6\,x}}{192}$$

input
```
int(cosh(x)^4/(coth(x) + 1),x)
```

output
```
x/16 + exp(-2*x)/32 + (3*exp(2*x))/64 + (3*exp(-4*x))/128 + exp(4*x)/128 +
 exp(-6*x)/192
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

$$\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx = \frac{3e^{10x}+18e^{8x}+24e^{6x}x+12e^{4x}+9e^{2x}+2}{384e^{6x}}$$

input
```
int(cosh(x)^4/(1+coth(x)),x)
```

output
```
(3*e**(10*x) + 18*e**(8*x) + 24*e**(6*x)*x + 12*e**(4*x) + 9*e**(2*x) + 2)
/(384*e**(6*x))
```

3.111 $\int \frac{\cosh^3(x)}{1+\coth(x)} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \frac{\cosh^3(x)}{1+\coth(x)} \, dx = \frac{\cosh^5(x)}{5} - \frac{\sinh^3(x)}{3} - \frac{\sinh^5(x)}{5}$$

output
```
1/5*cosh(x)^5-1/3*sinh(x)^3-1/5*sinh(x)^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{\cosh^3(x)}{1+\coth(x)} \, dx = \frac{1}{120}(\cosh(x)-\sinh(x))(20\cosh(2x)+4\cosh(4x)+10\sinh(2x) + \sinh(4x))$$

input
```
Integrate[Cosh[x]^3/(1 + Coth[x]),x]
```

output
```
((Cosh[x] - Sinh[x])*(20*Cosh[2*x] + 4*Cosh[4*x] + 10*Sinh[2*x] + Sinh[4*x
]))/120
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4001, 26, 26, 3042, 26, 3587, 3042, 26, 3586, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{\coth(x)+1} dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^3}{1-i\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \; 4001$$

$$\int -\frac{i\sinh(x)\cosh^3(x)}{-i\sinh(x)-i\cosh(x)}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{i\cosh^3(x)\sinh(x)}{\cosh(x)+\sinh(x)}dx$$

$$\Big\downarrow \; 26$$

$$\int \frac{\sinh(x)\cosh^3(x)}{\sinh(x)+\cosh(x)}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i\sin(ix)\cos(ix)^3}{\cos(ix)-i\sin(ix)}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{\cos(ix)^3\sin(ix)}{\cos(ix)-i\sin(ix)}dx$$

$$\Big\downarrow \; 3587$$

$$\int \sinh(x)\cosh^3(x)(\cosh(x)-\sinh(x))dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ix)\cos(ix)^3(i\sin(ix)+\cos(ix))dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \cos(ix)^3(\cos(ix)+i\sin(ix))\sin(ix)dx$$

$$\Big\downarrow \text{3586}$$

$$-i\int \left(i\cosh^4(x)\sinh(x)-i\cosh^3(x)\sinh^2(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(-\frac{1}{5}i\sinh^5(x)-\frac{1}{3}i\sinh^3(x)+\frac{1}{5}i\cosh^5(x)\right)$$

input
```
Int[Cosh[x]^3/(1 + Coth[x]),x]
```

output
```
(-I)*((I/5)*Cosh[x]^5 - (I/3)*Sinh[x]^3 - (I/5)*Sinh[x]^5)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3586
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

rule 3587
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[a^p*b^p   Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
 d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && IL
tQ[p, 0]
```

rule 4001
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result
risch	$\frac{e^{3x}}{48}+\frac{e^{x}}{8}+\frac{e^{-3x}}{24}+\frac{e^{-5x}}{80}$
parallelrisch	$\frac{\sinh(4x)+10\sinh(2x)+20\cosh(2x)+4\cosh(4x)+64\sinh(x)+64\cosh(x)}{120\sinh(x)+120\cosh(x)}$
default	$-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{4}}+\frac{2}{5\left(\tanh\left(\frac{x}{2}\right)+1\right)^{5}}+\frac{4}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{3}{8\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{1}{6\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-$

input
```
int(cosh(x)^3/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
1/48*exp(3*x)+1/8*exp(x)+1/24*exp(-3*x)+1/80*exp(-5*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(19) = 38.

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.24

$$\int \frac{\cosh^3(x)}{1+\coth(x)}\,dx$$
$$=\frac{\cosh(x)^4+\cosh(x)\sinh(x)^3+\sinh(x)^4+\left(6\cosh(x)^2+5\right)\sinh(x)^2+5\cosh(x)^2+\left(\cosh(x)^3+5\,\text{co}\right.}{30\left(\cosh(x)+\sinh(x)\right)}$$

input
```
integrate(cosh(x)^3/(1+coth(x)),x, algorithm="fricas")
```

output
```
1/30*(cosh(x)^4 + cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 5)*sinh(x
)^2 + 5*cosh(x)^2 + (cosh(x)^3 + 5*cosh(x))*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \frac{\cosh^3(x)}{1+\coth(x)}\,dx = \int \frac{\cosh^3(x)}{\coth(x)+1}\,dx$$

input
```
integrate(cosh(x)**3/(1+coth(x)),x)
```

output
```
Integral(cosh(x)**3/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\cosh^3(x)}{1+\coth(x)}\,dx = \frac{1}{48}\left(6\,e^{(-2\,x)}+1\right)e^{(3\,x)} + \frac{1}{24}\,e^{(-3\,x)} + \frac{1}{80}\,e^{(-5\,x)}$$

input
```
integrate(cosh(x)^3/(1+coth(x)),x, algorithm="maxima")
```

output
```
1/48*(6*e^(-2*x) + 1)*e^(3*x) + 1/24*e^(-3*x) + 1/80*e^(-5*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^3(x)}{1+\coth(x)}\,dx = \frac{1}{240}\left(10\,e^{(2\,x)}+3\right)e^{(-5\,x)}+\frac{1}{48}\,e^{(3\,x)}+\frac{1}{8}\,e^{x}$$

input
```
integrate(cosh(x)^3/(1+coth(x)),x, algorithm="giac")
```

output
```
1/240*(10*e^(2*x) + 3)*e^(-5*x) + 1/48*e^(3*x) + 1/8*e^x
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cosh^3(x)}{1+\coth(x)}\,dx = \frac{e^{-3\,x}}{24}+\frac{e^{3\,x}}{48}+\frac{e^{-5\,x}}{80}+\frac{e^{x}}{8}$$

input
```
int(cosh(x)^3/(coth(x) + 1),x)
```

output
```
exp(-3*x)/24 + exp(3*x)/48 + exp(-5*x)/80 + exp(x)/8
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{\cosh^3(x)}{1+\coth(x)}\,dx = \frac{5e^{8x}+30e^{6x}+10e^{2x}+3}{240e^{5x}}$$

input
```
int(cosh(x)^3/(1+coth(x)),x)
```

output
```
(5*e**(8*x) + 30*e**(6*x) + 10*e**(2*x) + 3)/(240*e**(5*x))
```

3.112 $\int \frac{\cosh^2(x)}{1+\coth(x)} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \frac{\cosh^2(x)}{1+\coth(x)} \, dx = \frac{x}{8} - \frac{1}{8(1-\coth(x))} + \frac{1}{8(1+\coth(x))^2} - \frac{1}{4(1+\coth(x))}$$

output
```
1/8*x-1/(8-8*coth(x))+1/8/(1+coth(x))^2-1/(4+4*coth(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{\cosh^2(x)}{1+\coth(x)} \, dx = \frac{1}{32}(4x + 4\cosh(2x) + \cosh(4x) - \sinh(4x))$$

input
```
Integrate[Cosh[x]^2/(1 + Coth[x]),x]
```

output
```
(4*x + 4*Cosh[2*x] + Cosh[4*x] - Sinh[4*x])/32
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3999, 516, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{\coth(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^2}{1-i\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3999}$$

$$-\int \frac{\coth^2(x)}{(\coth(x)+1)\left(1-\coth^2(x)\right)^2}d\coth(x)$$

$$\Big\downarrow \text{516}$$

$$-\int \frac{\coth^2(x)}{(1-\coth(x))^2(\coth(x)+1)^3}d\coth(x)$$

$$\Big\downarrow \text{99}$$

$$-\int \left(\frac{1}{8(\coth(x)-1)^2}-\frac{1}{4(\coth(x)+1)^2}+\frac{1}{4(\coth(x)+1)^3}+\frac{1}{8\left(\coth^2(x)-1\right)}\right)d\coth(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}\text{arctanh}(\coth(x))-\frac{1}{8(1-\coth(x))}-\frac{1}{4(\coth(x)+1)}+\frac{1}{8(\coth(x)+1)^2}$$

input
```
Int[Cosh[x]^2/(1 + Coth[x]),x]
```

output
```
ArcTanh[Coth[x]]/8 - 1/(8*(1 - Coth[x])) + 1/(8*(1 + Coth[x])^2) - 1/(4*(1
 + Coth[x]))
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 516
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result
risch	$\frac{x}{8}+\frac{{\mathrm e}^{2 x}}{16}+\frac{{\mathrm e}^{-2 x}}{16}+\frac{{\mathrm e}^{-4 x}}{32}$
parallelrisch	$\frac{3 \cosh \left(3 x \right)+\sinh \left(3 x \right)+\left(4 x -3\right) \cosh \left(x \right)+\left(4 x -7\right) \sinh \left(x \right)}{32 \sinh \left(x \right)+32 \cosh \left(x \right)}$
default	$\frac{1}{2 \left(\tanh \left(\frac{x}{2}\right)+1\right)^{4}}-\frac{1}{\left(\tanh \left(\frac{x}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tanh \left(\frac{x}{2}\right)+1\right)^{2}}-\frac{1}{2 \left(\tanh \left(\frac{x}{2}\right)+1\right)}+\frac{\ln \left(\tanh \left(\frac{x}{2}\right)+1\right)}{8}+\frac{1}{4 \left(\tanh \left(\frac{x}{2}\right)-1\right)^{2}}+\frac{}{4 \mathrm{t}}$

input
```
int(cosh(x)^2/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
1/8*x+1/16*exp(2*x)+1/16*exp(-2*x)+1/32*exp(-4*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \frac{\cosh^2(x)}{1+\coth(x)}\,dx = \frac{3\cosh(x)^3+9\cosh(x)\sinh(x)^2+\sinh(x)^3+2(2x+1)\cosh(x)+\left(3\cosh(x)^2+4x-2\right)\sinh(x)}{32(\cosh(x)+\sinh(x))}$$

input
```
integrate(cosh(x)^2/(1+coth(x)),x, algorithm="fricas")
```

output
```
1/32*(3*cosh(x)^3 + 9*cosh(x)*sinh(x)^2 + sinh(x)^3 + 2*(2*x + 1)*cosh(x)
+ (3*cosh(x)^2 + 4*x - 2)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \frac{\cosh^2(x)}{1+\coth(x)}\,dx = \int \frac{\cosh^2(x)}{\coth(x)+1}\,dx$$

input
```
integrate(cosh(x)**2/(1+coth(x)),x)
```

output
```
Integral(cosh(x)**2/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\cosh^2(x)}{1+\coth(x)}\,dx = \frac{1}{8}\,x + \frac{1}{16}\,e^{(2\,x)} + \frac{1}{16}\,e^{(-2\,x)} + \frac{1}{32}\,e^{(-4\,x)}$$

input
```
integrate(cosh(x)^2/(1+coth(x)),x, algorithm="maxima")
```

output
```
1/8*x + 1/16*e^(2*x) + 1/16*e^(-2*x) + 1/32*e^(-4*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{\cosh^2(x)}{1+\coth(x)}\,dx = -\frac{1}{32}\,\left(3\,e^{(4\,x)} - 2\,e^{(2\,x)} - 1\right)e^{(-4\,x)} + \frac{1}{8}\,x + \frac{1}{16}\,e^{(2\,x)}$$

input
```
integrate(cosh(x)^2/(1+coth(x)),x, algorithm="giac")
```

output
```
-1/32*(3*e^(4*x) - 2*e^(2*x) - 1)*e^(-4*x) + 1/8*x + 1/16*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\cosh^2(x)}{1+\coth(x)}\,dx = \frac{x}{8} + \frac{\mathrm{e}^{-2\,x}}{16} + \frac{\mathrm{e}^{2\,x}}{16} + \frac{\mathrm{e}^{-4\,x}}{32}$$

input
```
int(cosh(x)^2/(coth(x) + 1),x)
```

output
```
x/8 + exp(-2*x)/16 + exp(2*x)/16 + exp(-4*x)/32
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{\cosh^2(x)}{1+\coth(x)}\,dx = \frac{2e^{6x}+4e^{4x}x+2e^{2x}+1}{32e^{4x}}$$

input
```
int(cosh(x)^2/(1+coth(x)),x)
```

output
```
(2*e**(6*x) + 4*e**(4*x)*x + 2*e**(2*x) + 1)/(32*e**(4*x))
```

3.113 $\int \frac{\cosh(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \frac{\cosh(x)}{1+\coth(x)}\,dx = \frac{\cosh^3(x)}{3} - \frac{\sinh^3(x)}{3}$$

output
```
1/3*cosh(x)^3-1/3*sinh(x)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(x)}{1+\coth(x)}\,dx = \frac{1}{12}\left(3\cosh(x)+\cosh(3x)-4\sinh^3(x)\right)$$

input
```
Integrate[Cosh[x]/(1 + Coth[x]),x]
```

output
```
(3*Cosh[x] + Cosh[3*x] - 4*Sinh[x]^3)/12
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.222, Rules used = {3042, 4001, 26, 26, 3042, 26, 3587, 3042, 26, 3586, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{\coth(x)+1}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)}{1-i\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \ 4001$$

$$\int -\frac{i\sinh(x)\cosh(x)}{-i\sinh(x)-i\cosh(x)}dx$$

$$\Big\downarrow \ 26$$

$$-i\int \frac{i\cosh(x)\sinh(x)}{\cosh(x)+\sinh(x)}dx$$

$$\Big\downarrow \ 26$$

$$\int \frac{\sinh(x)\cosh(x)}{\sinh(x)+\cosh(x)}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{i\sin(ix)\cos(ix)}{\cos(ix)-i\sin(ix)}dx$$

$$\Big\downarrow \ 26$$

$$-i\int \frac{\cos(ix)\sin(ix)}{\cos(ix)-i\sin(ix)}dx$$

$$\Big\downarrow \ 3587$$

$$\int \sinh(x)\cosh(x)(\cosh(x)-\sinh(x))dx$$

$$\Big\downarrow \ 3042$$

$$\int -i\sin(ix)\cos(ix)(i\sin(ix)+\cos(ix))dx$$

$$\Big\downarrow \ 26$$

$$-i\int \cos(ix)(\cos(ix)+i\sin(ix))\sin(ix)dx$$

$$\Big\downarrow \ 3586$$

$$-i\int \left(i\cosh^2(x)\sinh(x)-i\cosh(x)\sinh^2(x)\right)dx$$

$$\Big\downarrow \ 2009$$

$$-i\left(\frac{1}{3}i\cosh^3(x)-\frac{1}{3}i\sinh^3(x)\right)$$

input
```
Int[Cosh[x]/(1 + Coth[x]),x]
```

output
```
(-I)*((I/3)*Cosh[x]^3 - (I/3)*Sinh[x]^3)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3586
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

rule 3587
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[a^p*b^p   Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
 d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && IL
tQ[p, 0]
```

rule 4001
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{{\mathrm e}^{x}}{4}+\frac{{\mathrm e}^{-3 x}}{12}$	12
parallelrisch	$\frac{\cosh \left(3 x \right)}{12}+\frac{\cosh \left(x \right)}{4}-\frac{\sinh \left(3 x \right)}{12}+\frac{\sinh \left(x \right)}{4}-\frac{1}{3}$	23
default	$\frac{2}{3 \left(\tanh \left(\frac{x}{2}\right)+1\right)^{3}}-\frac{1}{\left(\tanh \left(\frac{x}{2}\right)+1\right)^{2}}+\frac{1}{2 \tanh \left(\frac{x}{2}\right)+2}-\frac{1}{2 \left(\tanh \left(\frac{x}{2}\right)-1\right)}$	42

input
```
int(cosh(x)/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(x)+1/12*exp(-3*x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \frac{\cosh(x)}{1+\coth(x)}\,dx=\frac{\cosh \left(x\right)^{2}+\cosh \left(x\right) \sinh \left(x\right)+\sinh \left(x\right)^{2}}{3 \left(\cosh \left(x\right)+\sinh \left(x\right)\right)}$$

input
```
integrate(cosh(x)/(1+coth(x)),x, algorithm="fricas")
```

output
```
1/3*(cosh(x)^2 + cosh(x)*sinh(x) + sinh(x)^2)/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \frac{\cosh(x)}{1+\coth(x)}\,dx = \int \frac{\cosh(x)}{\coth(x)+1}\,dx$$

input
```
integrate(cosh(x)/(1+coth(x)),x)
```

output
```
Integral(cosh(x)/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\cosh(x)}{1+\coth(x)}\,dx = \frac{1}{12}\,e^{(-3\,x)} + \frac{1}{4}\,e^{x}$$

input
```
integrate(cosh(x)/(1+coth(x)),x, algorithm="maxima")
```

output
```
1/12*e^(-3*x) + 1/4*e^x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\cosh(x)}{1+\coth(x)}\,dx = \frac{1}{12}\,e^{(-3\,x)} + \frac{1}{4}\,e^{x}$$

input
```
integrate(cosh(x)/(1+coth(x)),x, algorithm="giac")
```

output
```
1/12*e^(-3*x) + 1/4*e^x
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\cosh(x)}{1+\coth(x)}\,dx = \frac{\mathrm{e}^{-3\,x}}{12} + \frac{\mathrm{e}^{x}}{4}$$

input
```
int(cosh(x)/(coth(x) + 1),x)
```

output
```
exp(-3*x)/12 + exp(x)/4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(x)}{1+\coth(x)}\,dx = \frac{3e^{4x}+1}{12e^{3x}}$$

input
```
int(cosh(x)/(1+coth(x)),x)
```

output
```
(3*e**(4*x) + 1)/(12*e**(3*x))
```

3.114 $\int \frac{\operatorname{sech}(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \frac{\operatorname{sech}(x)}{1+\coth(x)}\,dx = \arctan(\sinh(x)) + \cosh(x) - \sinh(x)$$

output
```
arctan(sinh(x))+cosh(x)-sinh(x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{\operatorname{sech}(x)}{1+\coth(x)}\,dx = 2\arctan\left(\tanh\left(\frac{x}{2}\right)\right) + \cosh(x) - \sinh(x)$$

input
```
Integrate[Sech[x]/(1 + Coth[x]),x]
```

output
```
2*ArcTan[Tanh[x/2]] + Cosh[x] - Sinh[x]
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.40, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.222, Rules used = {3042, 4001, 26, 26, 3042, 26, 3587, 3042, 26, 3586, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(x)}{\coth(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)\left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{4001}$$

$$\int -\frac{i\tanh(x)}{-i\sinh(x)-i\cosh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{i\tanh(x)}{\cosh(x)+\sinh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh(x)}{\sinh(x)+\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)}{\cos(ix)(\cos(ix)-i\sin(ix))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ix)}{\cos(ix)(\cos(ix)-i\sin(ix))}dx$$

$$\Big\downarrow \text{3587}$$

$$\int \tanh(x)(\cosh(x)-\sinh(x))dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)(i\sin(ix)+\cos(ix))}{\cos(ix)}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{(\cos(ix)+i\sin(ix))\sin(ix)}{\cos(ix)}dx$$

$$\Big\downarrow \; 3586$$

$$-i\int (i\sinh(x)-i\sinh(x)\tanh(x))dx$$

$$\Big\downarrow \; 2009$$

$$-i(i\arctan(\sinh(x))-i\sinh(x)+i\cosh(x))$$

input
```
Int[Sech[x]/(1 + Coth[x]),x]
```

output
```
(-I)*(I*ArcTan[Sinh[x]] + I*Cosh[x] - I*Sinh[x])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3586
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

rule 3587
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[a^p*b^p   Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
 d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && IL
tQ[p, 0]
```

rule 4001
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

method	result	size
default	$2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{2}{\tanh\left(\frac{x}{2}\right)+1}$	19
risch	$\mathrm{e}^{-x}+i\ln\left(\mathrm{e}^{x}+i\right)-i\ln\left(\mathrm{e}^{x}-i\right)$	24

input
```
int(sech(x)/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
2*arctan(tanh(1/2*x))+2/(tanh(1/2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.30

$$\int \frac{\operatorname{sech}(x)}{1+\coth(x)}\,dx=\frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+1}{\cosh\left(x\right)+\sinh\left(x\right)}$$

input
```
integrate(sech(x)/(1+coth(x)),x, algorithm="fricas")
```

output
```
(2*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x)) + 1)/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{1+\coth(x)}\,dx = \int \frac{\operatorname{sech}(x)}{\coth(x)+1}\,dx$$

input
```
integrate(sech(x)/(1+coth(x)),x)
```

output
```
Integral(sech(x)/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{sech}(x)}{1+\coth(x)}\,dx = -2\arctan\left(e^{(-x)}\right) + e^{(-x)}$$

input
```
integrate(sech(x)/(1+coth(x)),x, algorithm="maxima")
```

output
```
-2*arctan(e^(-x)) + e^(-x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(x)}{1+\coth(x)}\,dx = 2\arctan\left(e^{x}\right) + e^{(-x)}$$

input
```
integrate(sech(x)/(1+coth(x)),x, algorithm="giac")
```

output
```
2*arctan(e^x) + e^(-x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(x)}{1+\coth(x)}\,dx = \mathrm{e}^{-x} + 2\operatorname{atan}(\mathrm{e}^{x})$$

input
```
int(1/(cosh(x)*(coth(x) + 1)),x)
```

output
```
exp(-x) + 2*atan(exp(x))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{\operatorname{sech}(x)}{1+\coth(x)}\,dx = \frac{2e^{x}\,atan(e^{x})+1}{e^{x}}$$

input
```
int(sech(x)/(1+coth(x)),x)
```

output
```
(2*e**x*atan(e**x) + 1)/e**x
```

3.115 $\int \frac{\operatorname{sech}^2(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{\operatorname{sech}^2(x)}{1+\coth(x)}\,dx = -\log(1+\coth(x)) - \log(\tanh(x)) + \tanh(x)$$

output
```
-ln(1+coth(x))-ln(tanh(x))+tanh(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{\operatorname{sech}^2(x)}{1+\coth(x)}\,dx = -\log(1+\tanh(x)) + \tanh(x)$$

input
```
Integrate[Sech[x]^2/(1 + Coth[x]),x]
```

output
```
-Log[1 + Tanh[x]] + Tanh[x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 3999, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(x)}{\coth(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)^2\left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3999}$$

$$-\int \frac{\tanh^2(x)}{\coth(x)+1}d\coth(x)$$

$$\Big\downarrow \text{54}$$

$$-\int \left(\tanh^2(x)-\tanh(x)+\frac{1}{\coth(x)+1}\right)d\coth(x)$$

$$\Big\downarrow \text{2009}$$

$$\tanh(x)+\log(\coth(x))-\log(\coth(x)+1)$$

input
```
Int[Sech[x]^2/(1 + Coth[x]),x]
```

output
```
Log[Coth[x]] - Log[1 + Coth[x]] + Tanh[x]
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

method	result	size
risch	$-2x-\frac{2}{{\mathrm e}^{2x}+1}+\ln\left({\mathrm e}^{2x}+1\right)$	22
default	$-2\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{2\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^{2}+1}+\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)$	36

input
```
int(sech(x)^2/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*x-2/(exp(2*x)+1)+ln(exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(15) = 30$.

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 5.20

$$\int \frac{\operatorname{sech}^2(x)}{1+\coth(x)}\,dx =$$
$$-\frac{2\,x\cosh\left(x\right)^{2}+4\,x\cosh\left(x\right)\sinh\left(x\right)+2\,x\sinh\left(x\right)^{2}-\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+1\right)}{\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+1}$$

input
```
integrate(sech(x)^2/(1+coth(x)),x, algorithm="fricas")
```

output
```
-(2*x*cosh(x)^2 + 4*x*cosh(x)*sinh(x) + 2*x*sinh(x)^2 - (cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 + 1)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*x + 2
)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{1+\coth(x)}\,dx = \int \frac{\operatorname{sech}^{2}\left(x\right)}{\coth\left(x\right)+1}\,dx$$

input
```
integrate(sech(x)**2/(1+coth(x)),x)
```

output
```
Integral(sech(x)**2/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{sech}^2(x)}{1+\coth(x)}\,dx = \frac{2}{e^{(-2\,x)}+1} + \log\left(e^{(-2\,x)}+1\right)$$

input
```
integrate(sech(x)^2/(1+coth(x)),x, algorithm="maxima")
```

output
```
2/(e^(-2*x) + 1) + log(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \frac{\operatorname{sech}^2(x)}{1+\coth(x)}\,dx = -2\,x - \frac{e^{(2\,x)}+3}{e^{(2\,x)}+1} + \log\left(e^{(2\,x)}+1\right)$$

input
```
integrate(sech(x)^2/(1+coth(x)),x, algorithm="giac")
```

output
```
-2*x - (e^(2*x) + 3)/(e^(2*x) + 1) + log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int \frac{\operatorname{sech}^2(x)}{1+\coth(x)}\,dx = \ln\left(\mathrm{e}^{2\,x}+1\right) - 2\,x - \frac{2}{\mathrm{e}^{2\,x}+1}$$

input
```
int(1/(cosh(x)^2*(coth(x) + 1)),x)
```

output
```
log(exp(2*x) + 1) - 2*x - 2/(exp(2*x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.40

$$\int \frac{\mathrm{sech}^2(x)}{1+\coth(x)}\,dx = \frac{e^{2x}\log(e^{2x}+1)-2e^{2x}x+2e^{2x}+\log(e^{2x}+1)-2x}{e^{2x}+1}$$

input
```
int(sech(x)^2/(1+coth(x)),x)
```

output
```
(e**(2*x)*log(e**(2*x) + 1) - 2*e**(2*x)*x + 2*e**(2*x) + log(e**(2*x) + 1
) - 2*x)/(e**(2*x) + 1)
```

3.116 $\int \frac{\operatorname{sech}^3(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{\operatorname{sech}^3(x)}{1+\coth(x)}\,dx = -\frac{1}{2}\arctan(\sinh(x)) - \operatorname{sech}(x) + \frac{1}{2}\operatorname{sech}(x)\tanh(x)$$

output
```
-1/2*arctan(sinh(x))-sech(x)+1/2*sech(x)*tanh(x)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^3(x)}{1+\coth(x)}\,dx = \frac{1}{2}\left(2\cot^{-1}(\sinh(x)) + \arctan(\sinh(x)) + \operatorname{sech}(x)(-2+\tanh(x))\right)$$

input
```
Integrate[Sech[x]^3/(1 + Coth[x]),x]
```

output
```
(2*ArcCot[Sinh[x]] + ArcTan[Sinh[x]] + Sech[x]*(-2 + Tanh[x]))/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4001, 26, 26, 3042, 26, 3587, 3042, 26, 3586, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^3(x)}{\coth(x)+1}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)^3\left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{ 4001}$$

$$\int -\frac{i\tanh(x)\text{sech}^2(x)}{-i\sinh(x)-i\cosh(x)}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{i\text{sech}^2(x)\tanh(x)}{\cosh(x)+\sinh(x)}dx$$

$$\Big\downarrow \text{ 26}$$

$$\int \frac{\tanh(x)\text{sech}^2(x)}{\sinh(x)+\cosh(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\sin(ix)}{\cos(ix)^3(\cos(ix)-i\sin(ix))}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\sin(ix)}{\cos(ix)^3(\cos(ix)-i\sin(ix))}dx$$

$$\Big\downarrow \text{ 3587}$$

$$\int \tanh(x)\text{sech}^2(x)(\cosh(x)-\sinh(x))dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)(i\sin(ix)+\cos(ix))}{\cos(ix)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{(\cos(ix)+i\sin(ix))\sin(ix)}{\cos(ix)^3}dx$$

$$\Big\downarrow \text{3586}$$

$$-i\int \left(i\text{sech}(x)\tanh(x)-i\text{sech}(x)\tanh^2(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(-\frac{1}{2}i\arctan(\sinh(x))-i\text{sech}(x)+\frac{1}{2}i\tanh(x)\text{sech}(x)\right)$$

input
```
Int[Sech[x]^3/(1 + Coth[x]),x]
```

output
```
(-I)*((-1/2*I)*ArcTan[Sinh[x]] - I*Sech[x] + (I/2)*Sech[x]*Tanh[x])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3586
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

rule 3587
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[a^p*b^p   Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
 d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && IL
tQ[p, 0]
```

rule 4001
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

method	result	size
risch	$-\frac{e^{x}\left(e^{2x}+3\right)}{\left(e^{2x}+1\right)^{2}}+\frac{i\ln\left(e^{x}-i\right)}{2}-\frac{i\ln\left(e^{x}+i\right)}{2}$	38
default	$\frac{-\tanh\left(\frac{x}{2}\right)^{3}-2\tanh\left(\frac{x}{2}\right)^{2}+\tanh\left(\frac{x}{2}\right)-2}{\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{2}}-\arctan\left(\tanh\left(\frac{x}{2}\right)\right)$	45

input
```
int(sech(x)^3/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
-exp(x)*(exp(2*x)+3)/(exp(2*x)+1)^2+1/2*I*ln(exp(x)-I)-1/2*I*ln(exp(x)+I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(16) = 32.

Time = 0.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 7.00

$$\int \frac{\operatorname{sech}^3(x)}{1+\coth(x)}\,dx =$$
$$-\frac{\cosh\left(x\right)^{3}+3\,\cosh\left(x\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}+\left(\cosh\left(x\right)^{4}+4\,\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}+2\,\left(3\,\cosh\right.\right.}{\cosh\left(x\right)^{4}+4\,\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}}$$

input
```
integrate(sech(x)^3/(1+coth(x)),x, algorithm="fricas")
```

output
```
-(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (cosh(x)^4 + 4*cosh(x)*sin
h(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh
(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 3*(cosh(x)^2 + 1
)*sinh(x) + 3*cosh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3
*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x)
+ 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{1+\coth(x)}\,dx = \int \frac{\operatorname{sech}^3(x)}{\coth(x)+1}\,dx$$

input
```
integrate(sech(x)**3/(1+coth(x)),x)
```

output
```
Integral(sech(x)**3/(coth(x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(16) = 32$.

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\operatorname{sech}^3(x)}{1+\coth(x)}\,dx = -\frac{e^{(-x)} + 3\,e^{(-3\,x)}}{2\,e^{(-2\,x)} + e^{(-4\,x)} + 1} + \arctan\left(e^{(-x)}\right)$$

input
```
integrate(sech(x)^3/(1+coth(x)),x, algorithm="maxima")
```

output
```
-(e^(-x) + 3*e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1) + arctan(e^(-x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{sech}^3(x)}{1+\coth(x)}\,dx = -\frac{e^{(3x)}+3e^x}{\left(e^{(2x)}+1\right)^2} - \arctan\left(e^x\right)$$

input
```
integrate(sech(x)^3/(1+coth(x)),x, algorithm="giac")
```

output
```
-(e^(3*x) + 3*e^x)/(e^(2*x) + 1)^2 - arctan(e^x)
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{sech}^3(x)}{1+\coth(x)}\,dx = -\mathrm{atan}(\mathrm{e}^x) - \frac{1}{2\cosh(x)} - \frac{\mathrm{e}^{-x}}{2\cosh(x)^2}$$

input
```
int(1/(cosh(x)^3*(coth(x) + 1)),x)
```

output
```
- atan(exp(x)) - 1/(2*cosh(x)) - exp(-x)/(2*cosh(x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.90

$$\int \frac{\operatorname{sech}^3(x)}{1+\coth(x)}\,dx = \frac{-e^{4x}atan(e^x) - 2e^{2x}atan(e^x) - atan(e^x) - e^{3x} - 3e^x}{e^{4x}+2e^{2x}+1}$$

input
```
int(sech(x)^3/(1+coth(x)),x)
```

output
```
( - e**(4*x)*atan(e**x) - 2*e**(2*x)*atan(e**x) - atan(e**x) - e**(3*x) -
3*e**x)/(e**(4*x) + 2*e**(2*x) + 1)
```

3.117 $\int \frac{\text{sech}^4(x)}{1+\coth(x)} dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{\text{sech}^4(x)}{1+\coth(x)}\,dx = \frac{\tanh^2(x)}{2} - \frac{\tanh^3(x)}{3}$$

output
```
1/2*tanh(x)^2-1/3*tanh(x)^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{\text{sech}^4(x)}{1+\coth(x)}\,dx = \frac{1}{6}(-2+3\coth(x))\tanh^3(x)$$

input
```
Integrate[Sech[x]^4/(1 + Coth[x]),x]
```

output
```
((-2 + 3*Coth[x])*Tanh[x]^3)/6
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 3999, 25, 516, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^4(x)}{\coth(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)^4\left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3999}$$

$$-\int -\frac{\left(1-\coth^2(x)\right)\tanh^4(x)}{\coth(x)+1}d\coth(x)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\tanh^4(x)\left(1-\coth^2(x)\right)}{\coth(x)+1}d\coth(x)$$

$$\Big\downarrow \text{516}$$

$$\int \tanh^4(x)(1-\coth(x))d\coth(x)$$

$$\Big\downarrow \text{53}$$

$$\int \left(\tanh^4(x)-\tanh^3(x)\right)d\coth(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\tanh^2(x)}{2}-\frac{\tanh^3(x)}{3}$$

input
```
Int[Sech[x]^4/(1 + Coth[x]),x]
```

output
```
Tanh[x]^2/2 - Tanh[x]^3/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 516
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{1}{2\coth(x)^{2}}-\frac{1}{3\coth(x)^{3}}$	14
default	$\frac{1}{2\coth(x)^{2}}-\frac{1}{3\coth(x)^{3}}$	14
risch	$-\frac{2(3\,\mathrm{e}^{2x}-1)}{3(\mathrm{e}^{2x}+1)^{3}}$	19
parallelrisch	$\frac{-3\cosh(x)+18\sinh(x)-6\sinh(3x)+11\cosh(3x)}{18\cosh(3x)+54\cosh(x)}$	36

input
```
int(sech(x)^4/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2/coth(x)^2-1/3/coth(x)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(13) = 26$.

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 4.94

$$\int \frac{\operatorname{sech}^4(x)}{1+\coth(x)}\,dx = -\frac{4\left(\cosh\left(x\right)+2\,\sinh\right.}{3\left(\cosh\left(x\right)^5+5\,\cosh\left(x\right)\sinh\left(x\right)^4+\sinh\left(x\right)^5+\left(10\,\cosh\left(x\right)^2+3\right)\sinh\left(x\right)^3+3\,\cosh\left(x\right)^3+\left(10\,\cosh\right.\right.}$$

input
```
integrate(sech(x)^4/(1+coth(x)),x, algorithm="fricas")
```

output
```
-4/3*(cosh(x) + 2*sinh(x))/(cosh(x)^5 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5 +
(10*cosh(x)^2 + 3)*sinh(x)^3 + 3*cosh(x)^3 + (10*cosh(x)^3 + 9*cosh(x))*si
nh(x)^2 + (5*cosh(x)^4 + 9*cosh(x)^2 + 2)*sinh(x) + 4*cosh(x))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{1+\coth(x)}\,dx = \int \frac{\operatorname{sech}^4\left(x\right)}{\coth\left(x\right)+1}\,dx$$

input
```
integrate(sech(x)**4/(1+coth(x)),x)
```

output
```
Integral(sech(x)**4/(coth(x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(13) = 26$.

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 4.41

$$\int \frac{\operatorname{sech}^4(x)}{1+\coth(x)}\,dx = -\frac{2\,e^{(-2\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1} - \frac{4\,e^{(-4\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1} - \frac{2}{3\,(3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1)}$$

input
```
integrate(sech(x)^4/(1+coth(x)),x, algorithm="maxima")
```

output
```
-2*e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) - 4*e^(-4*x)/(3*e^(-2
*x) + 3*e^(-4*x) + e^(-6*x) + 1) - 2/3/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x)
 + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{sech}^4(x)}{1+\coth(x)}\,dx = -\frac{2\,(3\,e^{(2\,x)}-1)}{3\,(e^{(2\,x)}+1)^3}$$

input
```
integrate(sech(x)^4/(1+coth(x)),x, algorithm="giac")
```

output
```
-2/3*(3*e^(2*x) - 1)/(e^(2*x) + 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{sech}^4(x)}{1+\coth(x)}\,dx = -\frac{2\,(3\,e^{2\,x}-1)}{3\,(e^{2\,x}+1)^3}$$

input
```
int(1/(cosh(x)^4*(coth(x) + 1)),x)
```

output
```
-(2*(3*exp(2*x) - 1))/(3*(exp(2*x) + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.12

$$\int \frac{\operatorname{sech}^4(x)}{1+\coth(x)}\,dx = \frac{-6e^{2x}+2}{3e^{6x}+9e^{4x}+9e^{2x}+3}$$

input
```
int(sech(x)^4/(1+coth(x)),x)
```

output
```
(2*( - 3*e**(2*x) + 1))/(3*(e**(6*x) + 3*e**(4*x) + 3*e**(2*x) + 1))
```

3.118 $\int \sqrt{1+\coth(x)}\,\mathbf{sech}^2(x)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \sqrt{1+\coth(x)}\,\mathrm{sech}^2(x)\,dx = \operatorname{arctanh}\left(\sqrt{1+\coth(x)}\right) + \sqrt{1+\coth(x)}\tanh(x)$$

output
```
arctanh((1+coth(x))^(1/2))+(1+coth(x))^(1/2)*tanh(x)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.52 (sec) , antiderivative size = 135, normalized size of antiderivative = 6.43

$$\int \sqrt{1+\coth(x)}\operatorname{sech}^2(x)\,dx$$
$$= \frac{1}{2}\sqrt{1+\coth(x)}\left(\frac{(1-i)\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(1+\coth(x))}\right)}{\sqrt{i(1+\coth(x))}} + \frac{2\left(-2\operatorname{arctanh}\left(\sqrt{\tanh\left(\frac{x}{2}\right)}\right)+\sqrt{2}\operatorname{arctanh}\left(\frac{1+\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{\tanh\left(\frac{x}{2}\right)}}\right)\right)\cosh^2\left(\frac{x}{2}\right)\operatorname{csch}(x)\sqrt{\tanh\left(\frac{x}{2}\right)}\left(1+\tanh\left(\frac{x}{2}\right)\right)}{1+\coth(x)} + 2\tanh(x)\right)$$

input
```
Integrate[Sqrt[1 + Coth[x]]*Sech[x]^2,x]
```

output
```
(Sqrt[1 + Coth[x]]*(((1 - I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]])/Sq
rt[I*(1 + Coth[x])] + (2*(-2*ArcTanh[Sqrt[Tanh[x/2]]] + Sqrt[2]*ArcTanh[(1
 + Tanh[x/2])/(Sqrt[2]*Sqrt[Tanh[x/2]])])*Cosh[x/2]^2*Csch[x]*Sqrt[Tanh[x/
2]]*(1 + Tanh[x/2]))/(1 + Coth[x]) + 2*Tanh[x]))/2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3999, 51, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\coth(x)+1}\operatorname{sech}^2(x)\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{1-i\tan\left(\frac{\pi}{2}+ix\right)}}{\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \; 3999$$

$$-\int \sqrt{\coth(x)+1}\tanh^2(x)d\coth(x)$$

$$\Big\downarrow \; 51$$

$$\tanh(x)\sqrt{\coth(x)+1}-\frac{1}{2}\int \frac{\tanh(x)}{\sqrt{\coth(x)+1}}d\coth(x)$$

$$\Big\downarrow \; 73$$

$$\tanh(x)\sqrt{\coth(x)+1}-\int \tanh(x)d\sqrt{\coth(x)+1}$$

$$\Big\downarrow \; 220$$

$$\text{arctanh}\left(\sqrt{\coth(x)+1}\right)+\tanh(x)\sqrt{\coth(x)+1}$$

input
```
Int[Sqrt[1 + Coth[x]]*Sech[x]^2,x]
```

output
```
ArcTanh[Sqrt[1 + Coth[x]]] + Sqrt[1 + Coth[x]]*Tanh[x]
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.29

method	result	size
derivativedivides	$\frac{1}{2\sqrt{1+\coth(x)}+2}+\frac{\ln\left(\sqrt{1+\coth(x)}+1\right)}{2}+\frac{1}{2\sqrt{1+\coth(x)}-2}-\frac{\ln\left(\sqrt{1+\coth(x)}-1\right)}{2}$	48
default	$\frac{1}{2\sqrt{1+\coth(x)}+2}+\frac{\ln\left(\sqrt{1+\coth(x)}+1\right)}{2}+\frac{1}{2\sqrt{1+\coth(x)}-2}-\frac{\ln\left(\sqrt{1+\coth(x)}-1\right)}{2}$	48

input
```
int((1+coth(x))^(1/2)*sech(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/((1+coth(x))^(1/2)+1)+1/2*ln((1+coth(x))^(1/2)+1)+1/2/((1+coth(x))^(1/
2)-1)-1/2*ln((1+coth(x))^(1/2)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(17) = 34$.

Time = 0.10 (sec) , antiderivative size = 299, normalized size of antiderivative = 14.24

$$\int \sqrt{1+\coth(x)}\operatorname{sech}^2(x)\,dx$$
$$= \frac{\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)\log\left(\frac{3\cosh(x)^2+6\cosh(x)\sinh(x)+3\sinh(x)^2+\frac{2\sqrt{2}\left(\cosh(x)^3+3\cosh(x)\ldots\right)}{\sqrt{\cosh(\ldots}}}{\cosh(x)^2+2\cosh(x)\sinh(x\ldots}\right.}{\ldots}$$

input
```
integrate((1+coth(x))^(1/2)*sech(x)^2,x, algorithm="fricas")
```

output
```
1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log((3*cosh(x)^2 + 6*
cosh(x)*sinh(x) + 3*sinh(x)^2 + 2*sqrt(2)*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2
 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x))/sqrt(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 - 1) - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x
)^2)) - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log((3*cosh(x)^2 +
 6*cosh(x)*sinh(x) + 3*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^3 + 3*cosh(x)*sinh(x
)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x))/sqrt(cosh(x)^2 + 2*
cosh(x)*sinh(x) + sinh(x)^2 - 1) - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sin
h(x)^2)) + 4*sqrt(2)*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cos
h(x)^2 - 1)*sinh(x) - cosh(x))/sqrt(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x
)^2 - 1))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [F]

$$\int \sqrt{1+\coth(x)}\operatorname{sech}^2(x)\,dx = \int \sqrt{\coth(x)+1}\operatorname{sech}^2(x)\,dx$$

input
```
integrate((1+coth(x))**(1/2)*sech(x)**2,x)
```

output
```
Integral(sqrt(coth(x) + 1)*sech(x)**2, x)
```

Maxima [F]

$$\int \sqrt{1+\coth(x)}\text{sech}^2(x)\,dx = \int \sqrt{\coth(x)+1}\,\text{sech}(x)^2\,dx$$

input
```
integrate((1+coth(x))^(1/2)*sech(x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(coth(x) + 1)*sech(x)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(17) = 34$.

Time = 0.14 (sec) , antiderivative size = 122, normalized size of antiderivative = 5.81

$$\int \sqrt{1+\coth(x)}\text{sech}^2(x)\,dx = -\frac{1}{4}\sqrt{2}\left(\sqrt{2}\log\left(\frac{\left(\sqrt{e^{(2x)}-1}-e^x\right)^2-2\sqrt{2}+3}{\left(\sqrt{e^{(2x)}-1}-e^x\right)^2+2\sqrt{2}+3}\right)-\frac{8\left(3\left(\sqrt{e^{(2x)}-1}-e^x\right)^2+1\right)}{\left(\sqrt{e^{(2x)}-1}-e^x\right)^4+6\left(\sqrt{e^{(2x)}-1}-e^x\right)^2+1}\right)\text{sgn}\left(e^{(2x)}-1\right)$$

input
```
integrate((1+coth(x))^(1/2)*sech(x)^2,x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(sqrt(2)*log(((sqrt(e^(2*x) - 1) - e^x)^2 - 2*sqrt(2) + 3)/((
sqrt(e^(2*x) - 1) - e^x)^2 + 2*sqrt(2) + 3)) - 8*(3*(sqrt(e^(2*x) - 1) - e
^x)^2 + 1)/((sqrt(e^(2*x) - 1) - e^x)^4 + 6*(sqrt(e^(2*x) - 1) - e^x)^2 +
1))*sgn(e^(2*x) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\coth(x)}\text{sech}^2(x)\,dx = \int \frac{\sqrt{\coth(x)+1}}{\cosh(x)^2}\,dx$$

input
```
int((coth(x) + 1)^(1/2)/cosh(x)^2,x)
```

output
```
int((coth(x) + 1)^(1/2)/cosh(x)^2, x)
```

Reduce [F]

$$\int \sqrt{1+\coth(x)}\text{sech}^2(x)\,dx = \int \sqrt{\coth(x)+1}\,\text{sech}(x)^2\,dx$$

input
```
int((1+coth(x))^(1/2)*sech(x)^2,x)
```

output
```
int(sqrt(coth(x) + 1)*sech(x)**2,x)
```

3.119 $\int \frac{\cosh^4(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 147

$$\int \frac{\cosh^4(x)}{a+b\coth(x)}\,dx = -\frac{a(3a+b)\log(1-\coth(x))}{16(a+b)^3} + \frac{a(3a-b)\log(1+\coth(x))}{16(a-b)^3} - \frac{a^4b\log(a+b\coth(x))}{(a^2-b^2)^3} - \frac{(4b(2a^2-b^2)-a(5a^2-b^2)\coth(x))\sinh^2(x)}{8(a^2-b^2)^2} - \frac{(b-a\coth(x))\sinh^4(x)}{4(a^2-b^2)}$$

output
```
-1/16*a*(3*a+b)*ln(1-coth(x))/(a+b)^3+1/16*a*(3*a-b)*ln(1+coth(x))/(a-b)^3
-a^4*b*ln(a+b*coth(x))/(a^2-b^2)^3-1/8*(4*b*(2*a^2-b^2)-a*(5*a^2-b^2)*coth
(x))*sinh(x)^2/(a^2-b^2)^2-(b-a*coth(x))*sinh(x)^4/(4*a^2-4*b^2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.98

$$\int \frac{\cosh^4(x)}{a+b\coth(x)}\,dx$$
$$= \frac{12a^5x+24a^3b^2x-4ab^4x-4b(3a^4-4a^2b^2+b^4)\cosh(2x)-b(a^2-b^2)^2\cosh(4x)-32a^4b\log(b\cosh(x)}{32(a-b)^3(a+b)^3}$$

input
```
Integrate[Cosh[x]^4/(a + b*Coth[x]),x]
```

output
```
(12*a^5*x + 24*a^3*b^2*x - 4*a*b^4*x - 4*b*(3*a^4 - 4*a^2*b^2 + b^4)*Cosh[
2*x] - b*(a^2 - b^2)^2*Cosh[4*x] - 32*a^4*b*Log[b*Cosh[x] + a*Sinh[x]] + 8
*a^3*(a^2 - b^2)*Sinh[2*x] + a^5*Sinh[4*x] - 2*a^3*b^2*Sinh[4*x] + a*b^4*S
inh[4*x])/(32*(a - b)^3*(a + b)^3)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.65, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {3042, 3999, 25, 601, 2178, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(x)}{a+b\coth(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^4}{a-ib\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \; 3999$$

$$-b\int -\frac{b^4\coth^4(x)}{(a+b\coth(x))\left(b^2-b^2\coth^2(x)\right)^3}d(b\coth(x))$$

$$\Big\downarrow \; 25$$

$$b\int \frac{b^4\coth^4(x)}{(a+b\coth(x))\left(b^2-b^2\coth^2(x)\right)^3}d(b\coth(x))$$

$$\Big\downarrow \ 601$$

$$-b\left(\frac{\int \frac{-\frac{3a\coth(x)b^5}{a^2-b^2}+4\coth^2(x)b^4+\frac{a^2b^4}{a^2-b^2}}{(a+b\coth(x))\left(b^2-b^2\coth^2(x)\right)^2}d(b\coth(x))}{4b^2}+\frac{b^2\left(\frac{b^2}{a^2-b^2}-\frac{ab\coth(x)}{a^2-b^2}\right)}{4\left(b^2-b^2\coth^2(x)\right)^2}\right)$$

$$\Big\downarrow \ 2178$$

$$-b\left(\frac{\frac{\int -\frac{ab^4\left(a\left(3a^2+b^2\right)-b\left(5a^2-b^2\right)\coth(x)\right)}{\left(a^2-b^2\right)^2(a+b\coth(x))\left(b^2-b^2\coth^2(x)\right)}d(b\coth(x))}{2b^2}-\frac{b^2\left(4b^2\left(2a^2-b^2\right)-ab\left(5a^2-b^2\right)\coth(x)\right)}{2\left(a^2-b^2\right)^2\left(b^2-b^2\coth^2(x)\right)}}{4b^2}+\frac{b^2\left(\frac{b^2}{a^2-b^2}-\frac{ab\coth(x)}{a^2-b^2}\right)}{4\left(b^2-b^2\coth^2(x)\right)^2}\right)$$

$$\Big\downarrow \ 25$$

$$-b\left(\frac{-\frac{\int \frac{ab^4\left(a\left(3a^2+b^2\right)-b\left(5a^2-b^2\right)\coth(x)\right)}{\left(a^2-b^2\right)^2(a+b\coth(x))\left(b^2-b^2\coth^2(x)\right)}d(b\coth(x))}{2b^2}-\frac{b^2\left(4b^2\left(2a^2-b^2\right)-ab\left(5a^2-b^2\right)\coth(x)\right)}{2\left(a^2-b^2\right)^2\left(b^2-b^2\coth^2(x)\right)}}{4b^2}+\frac{b^2\left(\frac{b^2}{a^2-b^2}-\frac{ab\coth(x)}{a^2-b^2}\right)}{4\left(b^2-b^2\coth^2(x)\right)^2}\right)$$

$$\Big\downarrow \ 27$$

$$-b\left(\frac{-\frac{ab^2\int \frac{a\left(3a^2+b^2\right)-b\left(5a^2-b^2\right)\coth(x)}{(a+b\coth(x))\left(b^2-b^2\coth^2(x)\right)}d(b\coth(x))}{2\left(a^2-b^2\right)^2}-\frac{b^2\left(4b^2\left(2a^2-b^2\right)-ab\left(5a^2-b^2\right)\coth(x)\right)}{2\left(a^2-b^2\right)^2\left(b^2-b^2\coth^2(x)\right)}}{4b^2}+\frac{b^2\left(\frac{b^2}{a^2-b^2}-\frac{ab\coth(x)}{a^2-b^2}\right)}{4\left(b^2-b^2\coth^2(x)\right)^2}\right)$$

$$\Big\downarrow \ 657$$

$$-b\left(\frac{-\frac{ab^2\int\left(-\frac{8a^3}{(a-b)(a+b)(a+b\coth(x))}+\frac{(a-b)^2(3a+b)}{2b(a+b)(b-b\coth(x))}+\frac{(3a-b)(a+b)^2}{2(a-b)b(\coth(x)b+b)}\right)d(b\coth(x))}{2\left(a^2-b^2\right)^2}-\frac{b^2\left(4b^2\left(2a^2-b^2\right)-ab\left(5a^2-b^2\right)\coth(x)\right)}{2\left(a^2-b^2\right)^2\left(b^2-b^2\coth^2(x)\right)}}{4b^2}+\right.$$

$$\Big\downarrow \ 2009$$

$$-b\left(\frac{b^2\left(\frac{b^2}{a^2-b^2}-\frac{ab\coth(x)}{a^2-b^2}\right)}{4\left(b^2-b^2\coth^2(x)\right)^2}+\frac{-\frac{b^2\left(4b^2(2a^2-b^2)-ab(5a^2-b^2)\coth(x)\right)}{2(a^2-b^2)^2\left(b^2-b^2\coth^2(x)\right)}-\frac{ab^2\left(-\frac{8a^3\log(a+b\coth(x))}{a^2-b^2}-\frac{(a-b)^2(3a+b)\log(b-b\coth(x))}{2b(a+b)}+\right.}{2(a^2-b^2)^2}}{4b^2}\right.$$

input
```
Int[Cosh[x]^4/(a + b*Coth[x]),x]
```

output
```
-(b*((b^2*(b^2/(a^2 - b^2) - (a*b*Coth[x])/(a^2 - b^2)))/(4*(b^2 - b^2*Cot
h[x]^2)^2) + (-1/2*(b^2*(4*b^2*(2*a^2 - b^2) - a*b*(5*a^2 - b^2)*Coth[x]))
/((a^2 - b^2)^2*(b^2 - b^2*Coth[x]^2)) - (a*b^2*(-1/2*((a - b)^2*(3*a + b)
*Log[b - b*Coth[x]])/(b*(a + b)) - (8*a^3*Log[a + b*Coth[x]])/(a^2 - b^2)
+ ((3*a - b)*(a + b)^2*Log[b + b*Coth[x]])/(2*(a - b)*b)))/(2*(a^2 - b^2)^
2))/(4*b^2)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 4.51 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.25

method	result
risch	$\frac{3x\,a^2}{8(a+b)^3}+\frac{xab}{8(a+b)^3}+\frac{e^{4x}}{64a+64b}+\frac{e^{2x}a}{8(a+b)^2}+\frac{e^{2x}b}{16(a+b)^2}-\frac{e^{-2x}a}{8(a-b)^2}+\frac{e^{-2x}b}{16(a-b)^2}-\frac{e^{-4x}}{64(a-b)}+\frac{2a^4bx}{a^6-3a^4b^2+3a^2b^4-b^6}$ -
default	$-\frac{1}{(4a-4b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}+\frac{4}{(8a-8b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}-\frac{-5a+3b}{8(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{7a-5b}{8(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{a(3a-b)\ln(\tanh}{8(a-b)^3}$

input
```
int(cosh(x)^4/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
3/8*x/(a+b)^3*a^2+1/8*x/(a+b)^3*a*b+1/64/(a+b)*exp(4*x)+1/8/(a+b)^2*exp(2*
x)*a+1/16/(a+b)^2*exp(2*x)*b-1/8/(a-b)^2*exp(-2*x)*a+1/16/(a-b)^2*exp(-2*x
)*b-1/64/(a-b)*exp(-4*x)+2*a^4*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*x-a^4*b/(a^
6-3*a^4*b^2+3*a^2*b^4-b^6)*ln(exp(2*x)-(a-b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1229 vs. $2(139) = 278$.

Time = 0.12 (sec) , antiderivative size = 1229, normalized size of antiderivative = 8.36

$$\int \frac{\cosh^4(x)}{a + b\coth(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
1/64*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^8 + 8*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^7 + (a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^8 + 4*(2*a^5 - 3*a
^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^6 + 4*(2*a^5 - 3*a^4*b - 2*a^3
*b^2 + 4*a^2*b^3 - b^5 + 7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
b^5)*cosh(x)^2)*sinh(x)^6 + 8*(3*a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh
(x)^4 + 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3
 + 3*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x))*sinh(x)^5 -
a^5 - a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 + 2*(35*(a^5 - a^4*b - 2
*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 30*(2*a^5 - 3*a^4*b - 2*a^
3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^2 + 4*(3*a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^
4)*x)*sinh(x)^4 + 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)
*cosh(x)^5 + 10*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^3
+ 4*(3*a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x))*sinh(x)^3 - 4*(2*a^5
+ 3*a^4*b - 2*a^3*b^2 - 4*a^2*b^3 + b^5)*cosh(x)^2 + 4*(7*(a^5 - a^4*b - 2
*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 - 2*a^5 - 3*a^4*b + 2*a^3*b^
2 + 4*a^2*b^3 - b^5 + 15*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*c
osh(x)^4 + 12*(3*a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x)^2)*sinh(x)^2
 - 64*(a^4*b*cosh(x)^4 + 4*a^4*b*cosh(x)^3*sinh(x) + 6*a^4*b*cosh(x)^2*sin
h(x)^2 + 4*a^4*b*cosh(x)*sinh(x)^3 + a^4*b*sinh(x)^4)*log(2*(b*cosh(x) ...
```

Sympy [F]

$$\int \frac{\cosh^4(x)}{a + b\coth(x)}\,dx = \int \frac{\cosh^4{(x)}}{a + b\coth{(x)}}\,dx$$

input
```
integrate(cosh(x)**4/(a+b*coth(x)),x)
```

output
```
Integral(cosh(x)**4/(a + b*coth(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05

$$\int \frac{\cosh^4(x)}{a + b\coth(x)}\,dx = -\frac{a^4 b \log\left(-(a-b)e^{(-2x)} + a + b\right)}{a^6 - 3a^4b^2 + 3a^2b^4 - b^6} + \frac{(3a^2 + ab)x}{8(a^3 + 3a^2b + 3ab^2 + b^3)} + \frac{\left(4(2a+b)e^{(-2x)} + a + b\right)e^{(4x)}}{64(a^2 + 2ab + b^2)} - \frac{4(2a-b)e^{(-2x)} + (a-b)e^{(-4x)}}{64(a^2 - 2ab + b^2)}$$

input
```
integrate(cosh(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
-a^4*b*log(-(a - b)*e^(-2*x) + a + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
+ 1/8*(3*a^2 + a*b)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/64*(4*(2*a + b)*
e^(-2*x) + a + b)*e^(4*x)/(a^2 + 2*a*b + b^2) - 1/64*(4*(2*a - b)*e^(-2*x)
 + (a - b)*e^(-4*x))/(a^2 - 2*a*b + b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.47

$$\int \frac{\cosh^4(x)}{a+b\coth(x)}\,dx = -\frac{a^4b\log\left(\left|-ae^{(2x)}-be^{(2x)}+a-b\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{(3a^2-ab)x}{8(a^3-3a^2b+3ab^2-b^3)} - \frac{(18a^2e^{(4x)}-6abe^{(4x)}+8a^2e^{(2x)}-12abe^{(2x)}+4b^2e^{(2x)}+a^2-2ab+b^2)e^{(-4x)}}{64(a^3-3a^2b+3ab^2-b^3)} + \frac{ae^{(4x)}+be^{(4x)}+8ae^{(2x)}+4be^{(2x)}}{64(a^2+2ab+b^2)}$$

input
```
integrate(cosh(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

output
```
-a^4*b*log(abs(-a*e^(2*x) - b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6) + 1/8*(3*a^2 - a*b)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 1/64*(18
*a^2*e^(4*x) - 6*a*b*e^(4*x) + 8*a^2*e^(2*x) - 12*a*b*e^(2*x) + 4*b^2*e^(2
*x) + a^2 - 2*a*b + b^2)*e^(-4*x)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/64*(
a*e^(4*x) + b*e^(4*x) + 8*a*e^(2*x) + 4*b*e^(2*x))/(a^2 + 2*a*b + b^2)
```

Mupad [B] (verification not implemented)

Time = 2.82 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

$$\int \frac{\cosh^4(x)}{a+b\coth(x)}\,dx = \frac{e^{4x}}{64a+64b}-\frac{e^{-4x}}{64a-64b}-\frac{e^{-2x}(2a-b)}{16(a-b)^2}+\frac{e^{2x}(2a+b)}{16(a+b)^2} - \frac{a^4b\ln\left(b-a+ae^{2x}+be^{2x}\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{ax(3a-b)}{8(a-b)^3}$$

input
```
int(cosh(x)^4/(a + b*coth(x)),x)
```

output
```
exp(4*x)/(64*a + 64*b) - exp(-4*x)/(64*a - 64*b) - (exp(-2*x)*(2*a - b))/(
16*(a - b)^2) + (exp(2*x)*(2*a + b))/(16*(a + b)^2) - (a^4*b*log(b - a + a
*exp(2*x) + b*exp(2*x)))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (a*x*(3*a -
 b))/(8*(a - b)^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.28

$$\int \frac{\cosh^4(x)}{a+b\coth(x)}\,dx$$
$$= \frac{-2e^{8x}a^3b^2 + 2e^{8x}a^2b^3 - 12e^{6x}a^4b - 8e^{6x}a^3b^2 + 16e^{6x}a^2b^3 + 24e^{4x}a^5x - 12e^{2x}a^4b + 8e^{2x}a^3b^2 + 16e^{2x}a^2b^3}{}$$

input
```
int(cosh(x)^4/(a+b*coth(x)),x)
```

output
```
(e**(8*x)*a**5 - e**(8*x)*a**4*b - 2*e**(8*x)*a**3*b**2 + 2*e**(8*x)*a**2*
b**3 + e**(8*x)*a*b**4 - e**(8*x)*b**5 + 8*e**(6*x)*a**5 - 12*e**(6*x)*a**
4*b - 8*e**(6*x)*a**3*b**2 + 16*e**(6*x)*a**2*b**3 - 4*e**(6*x)*b**5 - 64*
e**(4*x)*log(e**(2*x)*a + e**(2*x)*b - a + b)*a**4*b + 24*e**(4*x)*a**5*x
+ 64*e**(4*x)*a**4*b*x + 48*e**(4*x)*a**3*b**2*x - 8*e**(4*x)*a*b**4*x - 8
*e**(2*x)*a**5 - 12*e**(2*x)*a**4*b + 8*e**(2*x)*a**3*b**2 + 16*e**(2*x)*a
**2*b**3 - 4*e**(2*x)*b**5 - a**5 - a**4*b + 2*a**3*b**2 + 2*a**2*b**3 - a
*b**4 - b**5)/(64*e**(4*x)*(a**6 - 3*a**4*b**2 + 3*a**2*b**4 - b**6))
```

3.120 $\int \frac{\cosh^3(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 135

$$\int \frac{\cosh^3(x)}{a+b\coth(x)}\,dx = \frac{a^3 b\,\text{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{a^2 b\cosh(x)}{(a^2-b^2)^2} - \frac{b\cosh^3(x)}{3(a^2-b^2)} + \frac{ab^2\sinh(x)}{(a^2-b^2)^2} + \frac{a\sinh(x)}{a^2-b^2} + \frac{a\sinh^3(x)}{3(a^2-b^2)}$$

output
```
a^3*b*arctanh((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)-a^2*b
*cosh(x)/(a^2-b^2)^2-b*cosh(x)^3/(3*a^2-3*b^2)+a*b^2*sinh(x)/(a^2-b^2)^2+a
*sinh(x)/(a^2-b^2)+a*sinh(x)^3/(3*a^2-3*b^2)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.24

$$\int \frac{\cosh^3(x)}{a+b\coth(x)}\,dx = \frac{1}{12}\left(-\frac{24a^3b\arctan\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)}{(-a+b)^{5/2}(a+b)^{5/2}}+\frac{3b(-5a^2+b^2)\cosh(x)}{(a-b)^2(a+b)^2}+\frac{b\cosh(3x)}{(-a+b)(a+b)}+\frac{3a(3a^2+b^2)\sinh(x)}{(a-b)^2(a+b)^2}+\frac{a^3\sinh(3x)}{(a-b)^2(a+b)^2}-\frac{ab^2\sinh(3x)}{(a-b)^2(a+b)^2}\right)$$

input
```
Integrate[Cosh[x]^3/(a + b*Coth[x]),x]
```

output
```
((-24*a^3*b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/((-a + b
)^(5/2)*(a + b)^(5/2)) + (3*b*(-5*a^2 + b^2)*Cosh[x])/((a - b)^2*(a + b)^2
) + (b*Cosh[3*x])/((-a + b)*(a + b)) + (3*a*(3*a^2 + b^2)*Sinh[x])/((a - b
)^2*(a + b)^2) + (a^3*Sinh[3*x])/((a - b)^2*(a + b)^2) - (a*b^2*Sinh[3*x])
/((a - b)^2*(a + b)^2))/12
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.16, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.462, Rules used = {3042, 4001, 26, 26, 3042, 26, 3588, 26, 3042, 26, 3045, 15, 3113, 2009, 3579, 3042, 3117, 3553, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{a+b\coth(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^3}{a-ib\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{4001}$$

$$\int -\frac{i\sinh(x)\cosh^3(x)}{-ia\sinh(x)-ib\cosh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{i\cosh^3(x)\sinh(x)}{b\cosh(x)+a\sinh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\sinh(x)\cosh^3(x)}{a\sinh(x)+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)\cos(ix)^3}{b\cos(ix)-ia\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ix)^3\sin(ix)}{b\cos(ix)-ia\sin(ix)}dx$$

$$\Big\downarrow \text{3588}$$

$$-i\left(\frac{ia\int\cosh^3(x)dx}{a^2-b^2}-\frac{b\int i\cosh^2(x)\sinh(x)dx}{a^2-b^2}-\frac{iab\int\frac{\cosh^2(x)}{b\cosh(x)+a\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{ia\int\cosh^3(x)dx}{a^2-b^2}-\frac{ib\int\cosh^2(x)\sinh(x)dx}{a^2-b^2}-\frac{iab\int\frac{\cosh^2(x)}{b\cosh(x)+a\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{ia\int\sin\left(ix+\frac{\pi}{2}\right)^3dx}{a^2-b^2}-\frac{ib\int -i\cos(ix)^2\sin(ix)dx}{a^2-b^2}-\frac{iab\int\frac{\cos(ix)^2}{b\cos(ix)-ia\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{ia\int\sin\left(ix+\frac{\pi}{2}\right)^3dx}{a^2-b^2}-\frac{b\int\cos(ix)^2\sin(ix)dx}{a^2-b^2}-\frac{iab\int\frac{\cos(ix)^2}{b\cos(ix)-ia\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3045}$$

$$-i\left(\frac{ia\int \sin\left(ix+\frac{\pi}{2}\right)^3dx}{a^2-b^2}-\frac{ib\int \cosh^2(x)d\cosh(x)}{a^2-b^2}-\frac{iab\int \frac{\cos(ix)^2}{b\cos(ix)-ia\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{15}$$

$$-i\left(\frac{ia\int \sin\left(ix+\frac{\pi}{2}\right)^3dx}{a^2-b^2}-\frac{iab\int \frac{\cos(ix)^2}{b\cos(ix)-ia\sin(ix)}dx}{a^2-b^2}-\frac{ib\cosh^3(x)}{3\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \text{3113}$$

$$-i\left(-\frac{a\int \left(\sinh^2(x)+1\right)d(-i\sinh(x))}{a^2-b^2}-\frac{iab\int \frac{\cos(ix)^2}{b\cos(ix)-ia\sin(ix)}dx}{a^2-b^2}-\frac{ib\cosh^3(x)}{3\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$-i\left(-\frac{iab\int \frac{\cos(ix)^2}{b\cos(ix)-ia\sin(ix)}dx}{a^2-b^2}-\frac{a\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2-b^2}-\frac{ib\cosh^3(x)}{3\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \text{3579}$$

$$-i\left(-\frac{iab\left(-\frac{b\int \cosh(x)dx}{a^2-b^2}+\frac{a^2\int \frac{1}{b\cosh(x)+a\sinh(x)}dx}{a^2-b^2}+\frac{a\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2-b^2}-\frac{ib\cosh^3(x)}{3\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{iab\left(-\frac{b\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}+\frac{a^2\int \frac{1}{b\cos(ix)-ia\sin(ix)}dx}{a^2-b^2}+\frac{a\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2-b^2}-\frac{ib\cosh^3(x)}{3\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \text{3117}$$

$$-i\left(-\frac{iab\left(\frac{a^2\int \frac{1}{b\cos(ix)-ia\sin(ix)}dx}{a^2-b^2}-\frac{b\sinh(x)}{a^2-b^2}+\frac{a\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2-b^2}-\frac{ib\cosh^3(x)}{3\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \text{3553}$$

$$-i\left(-\frac{iab\left(\frac{ia^2\int \frac{1}{-a^2+b^2-(-ia\cosh(x)-ib\sinh(x))^2}d(-ia\cosh(x)-ib\sinh(x))}{a^2-b^2}-\frac{b\sinh(x)}{a^2-b^2}+\frac{a\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(\right.}{a^2-b^2}\right.$$

$$\Big\downarrow 217$$

$$-i\left(-\frac{iab\left(-\frac{ia^2\arctan\left(\frac{-ia\cosh(x)-ib\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{b\sinh(x)}{a^2-b^2}+\frac{a\cosh(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2-b^2}-\frac{ib\cosh^3(x)}{3\left(a^2-b^2\right)}\right)$$

input
```
Int[Cosh[x]^3/(a + b*Coth[x]),x]
```

output
```
(-I)*(((-1/3*I)*b*Cosh[x]^3)/(a^2 - b^2) - (I*a*b*(((-I)*a^2*ArcTan[((-I)*
a*Cosh[x] - I*b*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + (a*Cosh[x])
/(a^2 - b^2) - (b*Sinh[x])/(a^2 - b^2)))/(a^2 - b^2) - (a*((-I)*Sinh[x] -
(I/3)*Sinh[x]^3))/(a^2 - b^2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3579
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 1), x], x]
 + Simp[b^2/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[
c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
 1]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 4001
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.29

method	result
risch	$\frac{e^{3x}}{24a+24b}+\frac{3\,e^{x}a}{8(a+b)^{2}}+\frac{e^{x}b}{8(a+b)^{2}}-\frac{3\,e^{-x}a}{8(a-b)^{2}}+\frac{e^{-x}b}{8(a-b)^{2}}-\frac{e^{-3x}}{24(a-b)}+\frac{b\,a^{3}\ln\left(e^{x}+\frac{a-b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)^{2}(a-b)^{2}}-\frac{b\,a^{3}\ln\left(e^{x}-\frac{a-b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)^{2}(a-b)^{2}}$
default	$-\frac{4}{3(\tanh(\frac{x}{2})+1)^{3}(4a-4b)}+\frac{2}{(4a-4b)(\tanh(\frac{x}{2})+1)^{2}}-\frac{2a-b}{2(a-b)^{2}(\tanh(\frac{x}{2})+1)}-\frac{2a^{3}b\arctan\left(\frac{2\tanh(\frac{x}{2})b+2a}{2\sqrt{-a^{2}+b^{2}}}\right)}{(a-b)^{2}(a+b)^{2}\sqrt{-a^{2}+b^{2}}}-$ 3(tanh(

input
```
int(cosh(x)^3/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
1/24/(a+b)*exp(x)^3+3/8/(a+b)^2*exp(x)*a+1/8/(a+b)^2*exp(x)*b-3/8/(a-b)^2/
exp(x)*a+1/8/(a-b)^2/exp(x)*b-1/24/(a-b)/exp(x)^3+1/(a^2-b^2)^(1/2)*b*a^3/
(a+b)^2/(a-b)^2*ln(exp(x)+(a-b)/(a^2-b^2)^(1/2))-1/(a^2-b^2)^(1/2)*b*a^3/(
a+b)^2/(a-b)^2*ln(exp(x)-(a-b)/(a^2-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 909 vs. 2(127) = 254.

Time = 0.13 (sec) , antiderivative size = 1873, normalized size of antiderivative = 13.87

$$\int \frac{\cosh^3(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
[1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(
a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b +
 2*a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*
a^2*b^3 - a*b^4 - b^5)*cosh(x)^4 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*
b^3 - a*b^4 - b^5 + 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*
cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
 b^5)*cosh(x)^3 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5
)*cosh(x))*sinh(x)^3 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4
+ b^5)*cosh(x)^2 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^
5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 6*(3
*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2
 + 24*(a^3*b*cosh(x)^3 + 3*a^3*b*cosh(x)^2*sinh(x) + 3*a^3*b*cosh(x)*sinh(
x)^2 + a^3*b*sinh(x)^3)*sqrt(a^2 - b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)
*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x
)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(
x)^2 - a + b)) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*co
sh(x)^5 + 2*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x
)^3 - (3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x))*sin
h(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b...
```

Sympy [F]

$$\int \frac{\cosh^3(x)}{a + b\coth(x)}\,dx = \int \frac{\cosh^3(x)}{a + b\coth(x)}\,dx$$

input
```
integrate(cosh(x)**3/(a+b*coth(x)),x)
```

output
```
Integral(cosh(x)**3/(a + b*coth(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(x)}{a+b\coth(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.21

$$\int \frac{\cosh^3(x)}{a+b\coth(x)}\,dx = \frac{2\,a^3b\arctan\left(-\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{-a^2+b^2}} - \frac{\left(9\,ae^{(2\,x)}-3\,be^{(2\,x)}+a-b\right)e^{(-3\,x)}}{24\,(a^2-2\,ab+b^2)} + \frac{a^2e^{(3\,x)}+2\,abe^{(3\,x)}+b^2e^{(3\,x)}+9\,a^2e^x+12\,abe^x+3\,b^2e^x}{24\,(a^3+3\,a^2b+3\,ab^2+b^3)}$$

input
```
integrate(cosh(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

output
```
2*a^3*b*arctan(-(a*e^x + b*e^x)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*b^2 + b^4)
*sqrt(-a^2 + b^2)) - 1/24*(9*a*e^(2*x) - 3*b*e^(2*x) + a - b)*e^(-3*x)/(a^
2 - 2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) + 9*a^2
*e^x + 12*a*b*e^x + 3*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

Mupad [B] (verification not implemented)

Time = 3.30 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.94

$$\int \frac{\cosh^3(x)}{a+b\coth(x)}\,dx$$
$$= \frac{e^{3x}}{24a+24b} - \frac{e^{-3x}}{24a-24b} + \frac{e^{x}(3a+b)}{8(a+b)^2} - \frac{e^{-x}(3a-b)}{8(a-b)^2} + \frac{2\,\mathrm{atan}\left(\frac{a^3\,b\,e^x\,\sqrt{-a^{10}+5\,a^8\,b^2-10\,a^6\,b^4+10\,a^4\,b^6-5\,a^2\,b^8+b^{10}}}{a^5\,\sqrt{a^6\,b^2}-b^5\,\sqrt{a^6\,b^2}+2\,a^2\,b^3\,\sqrt{a^6\,b^2}-2\,a^3\,b^2\,\sqrt{a^6\,b^2}+a\,b^4\,\sqrt{a^6\,b^2}-a^4\,b\,\sqrt{a^6\,b^2}}\right)\sqrt{a^6\,b^2}}{\sqrt{-a^{10}+5\,a^8\,b^2-10\,a^6\,b^4+10\,a^4\,b^6-5\,a^2\,b^8+b^{10}}}$$

input
```
int(cosh(x)^3/(a + b*coth(x)),x)
```

output
```
exp(3*x)/(24*a + 24*b) - exp(-3*x)/(24*a - 24*b) + (exp(x)*(3*a + b))/(8*(
a + b)^2) - (exp(-x)*(3*a - b))/(8*(a - b)^2) + (2*atan((a^3*b*exp(x)*(b^1
0 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2))/(a^5*(a
^6*b^2)^(1/2) - b^5*(a^6*b^2)^(1/2) + 2*a^2*b^3*(a^6*b^2)^(1/2) - 2*a^3*b^
2*(a^6*b^2)^(1/2) + a*b^4*(a^6*b^2)^(1/2) - a^4*b*(a^6*b^2)^(1/2)))*(a^6*b
^2)^(1/2))/(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)
^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.39

$$\int \frac{\cosh^3(x)}{a+b\coth(x)}\,dx$$
$$= \frac{-e^{6x}a^4b - 2e^{6x}a^3b^2 + 2e^{6x}a^2b^3 + e^{6x}a\,b^4 - 15e^{2x}a^4b + 6e^{2x}a^3b^2 + 18e^{2x}a^2b^3 + 3e^{2x}a\,b^4 - 15e^{4x}a^4b - 6e^{4}}{}$$

input
```
int(cosh(x)^3/(a+b*coth(x)),x)
```

output
```
(48*e**(3*x)*sqrt( - a**2 + b**2)*atan((e**x*a + e**x*b)/sqrt( - a**2 + b*
*2))*a**3*b + e**(6*x)*a**5 - e**(6*x)*a**4*b - 2*e**(6*x)*a**3*b**2 + 2*e
**(6*x)*a**2*b**3 + e**(6*x)*a*b**4 - e**(6*x)*b**5 + 9*e**(4*x)*a**5 - 15
*e**(4*x)*a**4*b - 6*e**(4*x)*a**3*b**2 + 18*e**(4*x)*a**2*b**3 - 3*e**(4*
x)*a*b**4 - 3*e**(4*x)*b**5 - 9*e**(2*x)*a**5 - 15*e**(2*x)*a**4*b + 6*e**
(2*x)*a**3*b**2 + 18*e**(2*x)*a**2*b**3 + 3*e**(2*x)*a*b**4 - 3*e**(2*x)*b
**5 - a**5 - a**4*b + 2*a**3*b**2 + 2*a**2*b**3 - a*b**4 - b**5)/(24*e**(3
*x)*(a**6 - 3*a**4*b**2 + 3*a**2*b**4 - b**6))
```

3.121 $\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 85

$$\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx = -\frac{a\log(1-\coth(x))}{4(a+b)^2}+\frac{a\log(1+\coth(x))}{4(a-b)^2} -\frac{a^2b\log(a+b\coth(x))}{(a^2-b^2)^2}-\frac{(b-a\coth(x))\sinh^2(x)}{2(a^2-b^2)}$$

output
```
-1/4*a*ln(1-coth(x))/(a+b)^2+1/4*a*ln(1+coth(x))/(a-b)^2-a^2*b*ln(a+b*coth
(x))/(a^2-b^2)^2-(b-a*coth(x))*sinh(x)^2/(2*a^2-2*b^2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx = \frac{(-a^2b+b^3)\cosh(2x)+a(2(a^2+b^2)x-4ab\log(b\cosh(x)+a\sinh(x))+(a^2-b^2)\sinh(2x))}{4(a-b)^2(a+b)^2}$$

input
```
Integrate[Cosh[x]^2/(a + b*Coth[x]),x]
```

output
```
((-(a^2*b) + b^3)*Cosh[2*x] + a*(2*(a^2 + b^2)*x - 4*a*b*Log[b*Cosh[x] + a
*Sinh[x]] + (a^2 - b^2)*Sinh[2*x]))/(4*(a - b)^2*(a + b)^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.68, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 3999, 601, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^2}{a-ib\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 3999}$$

$$-b\int \frac{b^2\coth^2(x)}{(a+b\coth(x))\left(b^2-b^2\coth^2(x)\right)^2}d(b\coth(x))$$

$$\Big\downarrow \text{ 601}$$

$$-b\left(-\frac{\int \frac{ab^2(a-b\coth(x))}{(a^2-b^2)(a+b\coth(x))(b^2-b^2\coth^2(x))}d(b\coth(x))}{2b^2}-\frac{\frac{b^2}{a^2-b^2}-\frac{ab\coth(x)}{a^2-b^2}}{2\left(b^2-b^2\coth^2(x)\right)}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-b\left(-\frac{a\int \frac{a-b\coth(x)}{(a+b\coth(x))(b^2-b^2\coth^2(x))}d(b\coth(x))}{2\left(a^2-b^2\right)}-\frac{\frac{b^2}{a^2-b^2}-\frac{ab\coth(x)}{a^2-b^2}}{2\left(b^2-b^2\coth^2(x)\right)}\right)$$

$$\Big\downarrow \text{ 657}$$

$$-b\left(-\frac{a\int \left(-\frac{2a}{(a-b)(a+b)(a+b\coth(x))}+\frac{a-b}{2b(a+b)(b-b\coth(x))}+\frac{a+b}{2(a-b)b(\coth(x)b+b)}\right)d(b\coth(x))}{2\left(a^2-b^2\right)}-\frac{\frac{b^2}{a^2-b^2}-\frac{ab\coth(x)}{a^2-b^2}}{2\left(b^2-b^2\coth^2(x)\right)}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$-b\left(-\frac{\frac{b^2}{a^2-b^2}-\frac{ab\coth(x)}{a^2-b^2}}{2\left(b^2-b^2\coth^2(x)\right)}-\frac{a\left(-\frac{2a\log(a+b\coth(x))}{a^2-b^2}-\frac{(a-b)\log(b-b\coth(x))}{2b(a+b)}+\frac{(a+b)\log(b\coth(x)+b)}{2b(a-b)}\right)}{2\left(a^2-b^2\right)}\right)$$

input
```
Int[Cosh[x]^2/(a + b*Coth[x]),x]
```

output
```
-(b*(-1/2*(b^2/(a^2 - b^2) - (a*b*Coth[x])/(a^2 - b^2))/(b^2 - b^2*Coth[x]
^2) - (a*(-1/2*((a - b)*Log[b - b*Coth[x]])/(b*(a + b)) - (2*a*Log[a + b*C
oth[x]])/(a^2 - b^2) + ((a + b)*Log[b + b*Coth[x]])/(2*(a - b)*b)))/(2*(a^
2 - b^2))))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18

method	result
risch	$\frac{xa}{2(a+b)^2}+\frac{e^{2x}}{8a+8b}-\frac{e^{-2x}}{8(a-b)}+\frac{2a^2bx}{a^4-2a^2b^2+b^4}-\frac{a^2b\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{a^4-2a^2b^2+b^4}$
default	$-\frac{2}{(4a-4b)(\tanh(\frac{x}{2})+1)^2}+\frac{4}{(8a-8b)(\tanh(\frac{x}{2})+1)}+\frac{a\ln(\tanh(\frac{x}{2})+1)}{2(a-b)^2}-\frac{a^2b\ln\left(b\tanh(\frac{x}{2})^2+2a\tanh(\frac{x}{2})+b\right)}{(a-b)^2(a+b)^2}+\frac{}{(4a+4b)(}$

input
```
int(cosh(x)^2/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/(a+b)^2*a+1/8/(a+b)*exp(2*x)-1/8/(a-b)*exp(-2*x)+2*a^2*b/(a^4-2*a^2*
b^2+b^4)*x-a^2*b/(a^4-2*a^2*b^2+b^4)*ln(exp(2*x)-(a-b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(80) = 160.

Time = 0.10 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.93

$$\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx$$
$$=\frac{(a^3-a^2b-ab^2+b^3)\cosh(x)^4+4(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)^3+(a^3-a^2b-ab^2+b^3)\sinh(}{}$$

input
```
integrate(cosh(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)
*cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 + 4*(a^3 + 2*a^
2*b + a*b^2)*x*cosh(x)^2 - a^3 - a^2*b + a*b^2 + b^3 + 2*(3*(a^3 - a^2*b -
 a*b^2 + b^3)*cosh(x)^2 + 2*(a^3 + 2*a^2*b + a*b^2)*x)*sinh(x)^2 - 8*(a^2*
b*cosh(x)^2 + 2*a^2*b*cosh(x)*sinh(x) + a^2*b*sinh(x)^2)*log(2*(b*cosh(x)
+ a*sinh(x))/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)
^3 + 2*(a^3 + 2*a^2*b + a*b^2)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4
)*cosh(x)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2
 + b^4)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx=\int \frac{\cosh^2{(x)}}{a+b\coth{(x)}}\,dx$$

input
```
integrate(cosh(x)**2/(a+b*coth(x)),x)
```

output
```
Integral(cosh(x)**2/(a + b*coth(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx=-\frac{a^2b\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^4-2a^2b^2+b^4}+\frac{ax}{2(a^2+2ab+b^2)}+\frac{e^{(2x)}}{8(a+b)}-\frac{e^{(-2x)}}{8(a-b)}$$

input
```
integrate(cosh(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
-a^2*b*log(-(a - b)*e^(-2*x) + a + b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*a*x/(a
^2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) - 1/8*e^(-2*x)/(a - b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx = -\frac{a^2 b\log\left(\left|-ae^{(2x)}-be^{(2x)}+a-b\right|\right)}{a^4-2a^2b^2+b^4}+\frac{ax}{2(a^2-2ab+b^2)} - \frac{\left(2ae^{(2x)}+a-b\right)e^{(-2x)}}{8(a^2-2ab+b^2)}+\frac{e^{(2x)}}{8(a+b)}$$

input
```
integrate(cosh(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

output
```
-a^2*b*log(abs(-a*e^(2*x) - b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) +
1/2*a*x/(a^2 - 2*a*b + b^2) - 1/8*(2*a*e^(2*x) + a - b)*e^(-2*x)/(a^2 - 2*
a*b + b^2) + 1/8*e^(2*x)/(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx = \frac{e^{2x}}{8a+8b}-\frac{e^{-2x}}{8a-8b}+\frac{ax}{2(a-b)^2}-\frac{a^2 b\ln\left(b-a+a e^{2x}+b e^{2x}\right)}{a^4-2a^2b^2+b^4}$$

input
```
int(cosh(x)^2/(a + b*coth(x)),x)
```

output
```
exp(2*x)/(8*a + 8*b) - exp(-2*x)/(8*a - 8*b) + (a*x)/(2*(a - b)^2) - (a^2*
b*log(b - a + a*exp(2*x) + b*exp(2*x)))/(a^4 + b^4 - 2*a^2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.79

$$\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx$$
$$= \frac{e^{4x}a^3 - e^{4x}a^2b - e^{4x}a\,b^2 + e^{4x}b^3 - 8e^{2x}\log(e^{2x}a + e^{2x}b - a + b)\,a^2b + 4e^{2x}a^3x + 8e^{2x}a^2bx + 4e^{2x}a\,b^2x -}{8e^{2x}\,(a^4 - 2a^2b^2 + b^4)}$$

input
```
int(cosh(x)^2/(a+b*coth(x)),x)
```

output
```
(e**(4*x)*a**3 - e**(4*x)*a**2*b - e**(4*x)*a*b**2 + e**(4*x)*b**3 - 8*e**
(2*x)*log(e**(2*x)*a + e**(2*x)*b - a + b)*a**2*b + 4*e**(2*x)*a**3*x + 8*
e**(2*x)*a**2*b*x + 4*e**(2*x)*a*b**2*x - a**3 - a**2*b + a*b**2 + b**3)/(
8*e**(2*x)*(a**4 - 2*a**2*b**2 + b**4))
```

3.122 $\int \frac{\cosh(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 72

$$\int \frac{\cosh(x)}{a+b\coth(x)}\,dx=\frac{ab\operatorname{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}-\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}$$

output
```
a*b*arctanh((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)-b*cosh(
x)/(a^2-b^2)+a*sinh(x)/(a^2-b^2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int \frac{\cosh(x)}{a+b\coth(x)}\,dx=\frac{2ab\arctan\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)}{(-a+b)^{3/2}(a+b)^{3/2}}+\frac{b\cosh(x)}{-a^2+b^2}+\frac{a\sinh(x)}{a^2-b^2}$$

input
```
Integrate[Cosh[x]/(a + b*Coth[x]),x]
```

output
```
(2*a*b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/((-a + b)^(3/
2)*(a + b)^(3/2)) + (b*Cosh[x])/(-a^2 + b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.273$, Rules used = {3042, 4001, 26, 26, 3042, 26, 3588, 26, 3042, 26, 3117, 3118, 3553, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{a+b\coth(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)}{a-ib\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{4001}$$

$$\int -\frac{i\sinh(x)\cosh(x)}{-ia\sinh(x)-ib\cosh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{i\cosh(x)\sinh(x)}{b\cosh(x)+a\sinh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\sinh(x)\cosh(x)}{a\sinh(x)+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)\cos(ix)}{b\cos(ix)-ia\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ix)\sin(ix)}{b\cos(ix)-ia\sin(ix)}dx$$

$$\Big\downarrow \text{3588}$$

$$-i\left(-\frac{b\int i\sinh(x)dx}{a^2-b^2}+\frac{ia\int \cosh(x)dx}{a^2-b^2}-\frac{iab\int \frac{1}{b\cosh(x)+a\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \ 26$$

$$-i\left(-\frac{ib\int \sinh(x)dx}{a^2-b^2}+\frac{ia\int \cosh(x)dx}{a^2-b^2}-\frac{iab\int \frac{1}{b\cosh(x)+a\sinh(x)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \ 3042$$

$$-i\left(\frac{ia\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}-\frac{ib\int -i\sin(ix)dx}{a^2-b^2}-\frac{iab\int \frac{1}{b\cos(ix)-ia\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \ 26$$

$$-i\left(\frac{ia\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}-\frac{b\int \sin(ix)dx}{a^2-b^2}-\frac{iab\int \frac{1}{b\cos(ix)-ia\sin(ix)}dx}{a^2-b^2}\right)$$

$$\Big\downarrow \ 3117$$

$$-i\left(-\frac{b\int \sin(ix)dx}{a^2-b^2}-\frac{iab\int \frac{1}{b\cos(ix)-ia\sin(ix)}dx}{a^2-b^2}+\frac{ia\sinh(x)}{a^2-b^2}\right)$$

$$\Big\downarrow \ 3118$$

$$-i\left(-\frac{iab\int \frac{1}{b\cos(ix)-ia\sin(ix)}dx}{a^2-b^2}+\frac{ia\sinh(x)}{a^2-b^2}-\frac{ib\cosh(x)}{a^2-b^2}\right)$$

$$\Big\downarrow \ 3553$$

$$-i\left(\frac{ab\int \frac{1}{-a^2+b^2-(-ia\cosh(x)-ib\sinh(x))^2}d(-ia\cosh(x)-ib\sinh(x))}{a^2-b^2}+\frac{ia\sinh(x)}{a^2-b^2}-\frac{ib\cosh(x)}{a^2-b^2}\right)$$

$$\Big\downarrow \ 217$$

$$-i\left(-\frac{ab\arctan\left(\frac{-ia\cosh(x)-ib\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{ia\sinh(x)}{a^2-b^2}-\frac{ib\cosh(x)}{a^2-b^2}\right)$$

input
```
Int[Cosh[x]/(a + b*Coth[x]),x]
```

output
```
(-I)*(-((a*b*ArcTan[((-I)*a*Cosh[x] - I*b*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2
- b^2)^(3/2)) - (I*b*Cosh[x])/(a^2 - b^2) + (I*a*Sinh[x])/(a^2 - b^2))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 4001
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

method	result	size
default	$-\frac{2ab\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^{2}+b^{2}}}\right)}{(a-b)(a+b)\sqrt{-a^{2}+b^{2}}}-\frac{4}{(4a+4b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{4}{(4a-4b)\left(\tanh\left(\frac{x}{2}\right)+1\right)}$	92
risch	$\frac{e^{x}}{2a+2b}-\frac{e^{-x}}{2(a-b)}+\frac{ba\ln\left(e^{x}+\frac{a-b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)(a-b)}-\frac{ba\ln\left(e^{x}-\frac{a-b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)(a-b)}$	120

input
```
int(cosh(x)/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*a*b/(a-b)/(a+b)/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2
+b^2)^(1/2))-4/(4*a+4*b)/(tanh(1/2*x)-1)-4/(4*a-4*b)/(tanh(1/2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(68) = 136.

Time = 0.12 (sec) , antiderivative size = 431, normalized size of antiderivative = 5.99

$$\int \frac{\cosh(x)}{a+b\coth(x)}\,dx$$

$$= \left[-\frac{a^{3}+a^{2}b-ab^{2}-b^{3}-\left(a^{3}-a^{2}b-ab^{2}+b^{3}\right)\cosh\left(x\right)^{2}-2\left(a^{3}-a^{2}b-ab^{2}+b^{3}\right)\cosh\left(x\right)\sinh\left(x\right)-(a}{2\left((a^{4}-\right.}\right.$$

$$-\frac{a^{3}+a^{2}b-ab^{2}-b^{3}-\left(a^{3}-a^{2}b-ab^{2}+b^{3}\right)\cosh\left(x\right)^{2}-2\left(a^{3}-a^{2}b-ab^{2}+b^{3}\right)\cosh\left(x\right)\sinh\left(x\right)-(a^{3}}{2\left((a^{4}-2\,a^{2}b^{2}+b^{4})\cosh\left(x\right)\right.}$$

input
```
integrate(cosh(x)/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
[-1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 -
 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) - (a^3 - a^2*b - a*b^2 + b^
3)*sinh(x)^2 + 2*(a*b*cosh(x) + a*b*sinh(x))*sqrt(a^2 - b^2)*log(((a + b)*
cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(a^2 - b
^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*si
nh(x) + (a + b)*sinh(x)^2 - a + b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a
^4 - 2*a^2*b^2 + b^4)*sinh(x)), -1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a
^2*b - a*b^2 + b^3)*cosh(x)^2 - 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh
(x) - (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^2 + 4*(a*b*cosh(x) + a*b*sinh(x)
)*sqrt(-a^2 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh
(x))))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))
]
```

Sympy [F]

$$\int \frac{\cosh(x)}{a+b\coth(x)}\,dx = \int \frac{\cosh{(x)}}{a+b\coth{(x)}}\,dx$$

input
```
integrate(cosh(x)/(a+b*coth(x)),x)
```

output
```
Integral(cosh(x)/(a + b*coth(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh(x)}{a+b\coth(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{\cosh(x)}{a+b\coth(x)}\,dx = -\frac{2\,ab\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{(a^2-b^2)\sqrt{-a^2+b^2}} - \frac{e^{(-x)}}{2\,(a-b)} + \frac{e^x}{2\,(a+b)}$$

input
```
integrate(cosh(x)/(a+b*coth(x)),x, algorithm="giac")
```

output
```
-2*a*b*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b
^2)) - 1/2*e^(-x)/(a - b) + 1/2*e^x/(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.82 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.19

$$\int \frac{\cosh(x)}{a+b\coth(x)}\,dx = \frac{e^x}{2\,a+2\,b} - \frac{e^{-x}}{2\,a-2\,b} + \frac{2\,\mathrm{atan}\left(\frac{a\,b\,e^x\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}{a^3\,\sqrt{a^2\,b^2}+b^3\,\sqrt{a^2\,b^2}-a\,b^2\,\sqrt{a^2\,b^2}-a^2\,b\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}$$

input
```
int(cosh(x)/(a + b*coth(x)),x)
```

output
```
exp(x)/(2*a + 2*b) - exp(-x)/(2*a - 2*b) + (2*atan((a*b*exp(x)*(b^6 - a^6
- 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(a^3*(a^2*b^2)^(1/2) + b^3*(a^2*b^2)^(1/2)
 - a*b^2*(a^2*b^2)^(1/2) - a^2*b*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(b^6 -
 a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.75

$$\int \frac{\cosh(x)}{a+b\coth(x)}\,dx = \frac{4e^{x}\sqrt{-a^{2}+b^{2}}\,atan\left(\frac{e^{x}a+e^{x}b}{\sqrt{-a^{2}+b^{2}}}\right)ab+e^{2x}a^{3}-e^{2x}a^{2}b-e^{2x}a\,b^{2}+e^{2x}b^{3}-a^{3}-a^{2}b+a\,b^{2}+b^{3}}{2e^{x}\left(a^{4}-2a^{2}b^{2}+b^{4}\right)}$$

input
```
int(cosh(x)/(a+b*coth(x)),x)
```

output
```
(4*e**x*sqrt( - a**2 + b**2)*atan((e**x*a + e**x*b)/sqrt( - a**2 + b**2))*
a*b + e**(2*x)*a**3 - e**(2*x)*a**2*b - e**(2*x)*a*b**2 + e**(2*x)*b**3 -
a**3 - a**2*b + a*b**2 + b**3)/(2*e**x*(a**4 - 2*a**2*b**2 + b**4))
```

3.123 $\int \frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 50

$$\int \frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx=\frac{\arctan(\sinh(x))}{a}+\frac{b\operatorname{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

output
```
arctan(sinh(x))/a+b*arctanh((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))/a/(a^2-
b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx=\frac{2\left(\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{b\arctan\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)}{a}$$

input
```
Integrate[Sech[x]/(a + b*Coth[x]),x]
```

output
```
(2*(ArcTan[Tanh[x/2]] - (b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a +
 b])])/(Sqrt[-a + b]*Sqrt[a + b])))/a
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {3042, 4001, 26, 26, 3042, 26, 3589, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(x)}{a+b\coth(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)\left(a-ib\tan\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{ 4001}$$

$$\int -\frac{i\tanh(x)}{-ia\sinh(x)-ib\cosh(x)}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{i\tanh(x)}{b\cosh(x)+a\sinh(x)}dx$$

$$\Big\downarrow \text{ 26}$$

$$\int \frac{\tanh(x)}{a\sinh(x)+b\cosh(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\sin(ix)}{\cos(ix)(b\cos(ix)-ia\sin(ix))}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\sin(ix)}{\cos(ix)(b\cos(ix)-ia\sin(ix))}dx$$

$$\Big\downarrow \text{3589}$$

$$-i\int \left(\frac{i\text{sech}(x)}{a}-\frac{ib}{a(b\cosh(x)+a\sinh(x))}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{ib\text{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}+\frac{i\arctan(\sinh(x))}{a}\right)$$

input
```
Int[Sech[x]/(a + b*Coth[x]),x]
```

output
```
(-I)*((I*ArcTan[Sinh[x]])/a + (I*b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^
2 - b^2]])/(a*Sqrt[a^2 - b^2]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3589
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

rule 4001
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{2b\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^{2}+b^{2}}}\right)}{a\sqrt{-a^{2}+b^{2}}}+\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	54
risch	$\frac{i\ln\left(e^{x}+i\right)}{a}-\frac{i\ln\left(e^{x}-i\right)}{a}+\frac{b\ln\left(e^{x}+\frac{a-b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,a}-\frac{b\ln\left(e^{x}-\frac{a-b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,a}$	102

input
```
int(sech(x)/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/a*b/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))
+2/a*arctan(tanh(1/2*x))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 200, normalized size of antiderivative = 4.00

$$\int \frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx$$
$$=\left[\frac{\sqrt{a^{2}-b^{2}}b\log\left(\frac{(a+b)\cosh(x)^{2}+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^{2}+2\,\sqrt{a^{2}-b^{2}}(\cosh(x)+\sinh(x))+a-b}{(a+b)\cosh(x)^{2}+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^{2}-a+b}\right)+2\,(a^{2}-b^{2})\,\mathrm{arc}}{a^{3}-ab^{2}}\right.$$
$$\left.-\frac{2\left(\sqrt{-a^{2}+b^{2}}b\arctan\left(\frac{\sqrt{-a^{2}+b^{2}}}{(a+b)\cosh(x)+(a+b)\sinh(x)}\right)-(a^{2}-b^{2})\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\right)}{a^{3}-ab^{2}}\right]$$

input
```
integrate(sech(x)/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
[(sqrt(a^2 - b^2)*b*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (
a + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)
*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) + 2*(
a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^3 - a*b^2), -2*(sqrt(-a^2 + b^2)*
b*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - (a^2 - b^
2)*arctan(cosh(x) + sinh(x)))/(a^3 - a*b^2)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx = \int \frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx$$

input
```
integrate(sech(x)/(a+b*coth(x)),x)
```

output
```
Integral(sech(x)/(a + b*coth(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sech(x)/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx = -\frac{2\,b\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}a} + \frac{2\,\arctan\left(e^x\right)}{a}$$

input
```
integrate(sech(x)/(a+b*coth(x)),x, algorithm="giac")
```

output
```
-2*b*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a) + 2*arc
tan(e^x)/a
```

Mupad [B] (verification not implemented)

Time = 4.61 (sec) , antiderivative size = 164, normalized size of antiderivative = 3.28

$$\int \frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx = \frac{b\ln\left(32\,a\,b^2\,e^x+32\,a^2\,b\,e^x+32\,a\,b\,\sqrt{a^2-b^2}\right)}{a\,\sqrt{a^2-b^2}} - \frac{b\ln\left(32\,a\,b^2\,e^x+32\,a^2\,b\,e^x-32\,a\,b\,\sqrt{a^2-b^2}\right)}{a\,\sqrt{a^2-b^2}} + \frac{\ln\left(32\,a\,b\,e^x-32\,a^2\,e^x+a\,b\,32\mathrm{i}-a^2\,32\mathrm{i}\right)\,1\mathrm{i}}{a} - \frac{\ln\left(32\,a^2\,e^x-32\,a\,b\,e^x+a\,b\,32\mathrm{i}-a^2\,32\mathrm{i}\right)\,1\mathrm{i}}{a}$$

input
```
int(1/(cosh(x)*(a + b*coth(x))),x)
```

output
```
(log(a*b*32i - a^2*32i - 32*a^2*exp(x) + 32*a*b*exp(x))*1i)/a - (log(a*b*3
2i - a^2*32i + 32*a^2*exp(x) - 32*a*b*exp(x))*1i)/a - (b*log(32*a*b^2*exp(
x) + 32*a^2*b*exp(x) - 32*a*b*(a^2 - b^2)^(1/2)))/(a*(a^2 - b^2)^(1/2)) +
(b*log(32*a*b^2*exp(x) + 32*a^2*b*exp(x) + 32*a*b*(a^2 - b^2)^(1/2)))/(a*(
a^2 - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.42

$$\int \frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx = \frac{2atan(e^x)\,a^2-2atan(e^x)\,b^2+2\sqrt{-a^2+b^2}\,atan\left(\frac{e^x a+e^x b}{\sqrt{-a^2+b^2}}\right)b}{a\,(a^2-b^2)}$$

input
```
int(sech(x)/(a+b*coth(x)),x)
```

output
```
(2*(atan(e**x)*a**2 - atan(e**x)*b**2 + sqrt( - a**2 + b**2)*atan((e**x*a
+ e**x*b)/sqrt( - a**2 + b**2))*b))/(a*(a**2 - b**2))
```

3.124 $\int \frac{\operatorname{sech}^2(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \frac{\operatorname{sech}^2(x)}{a+b\coth(x)}\,dx = -\frac{b\log(a+b\coth(x))}{a^2}-\frac{b\log(\tanh(x))}{a^2}+\frac{\tanh(x)}{a}$$

output
```
-b*ln(a+b*coth(x))/a^2-b*ln(tanh(x))/a^2+tanh(x)/a
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{\operatorname{sech}^2(x)}{a+b\coth(x)}\,dx = \frac{-b\log(b+a\tanh(x))+a\tanh(x)}{a^2}$$

input
```
Integrate[Sech[x]^2/(a + b*Coth[x]),x]
```

output
```
(-(b*Log[b + a*Tanh[x]]) + a*Tanh[x])/a^2
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3999, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(x)}{a+b\coth(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)^2\left(a-ib\tan\left(\frac{\pi}{2}+ix\right)\right)}dx$$
$$\Big\downarrow \text{3999}$$
$$-b\int \frac{\tanh^2(x)}{b^2(a+b\coth(x))}d(b\coth(x))$$
$$\Big\downarrow \text{54}$$
$$-b\int \left(\frac{\tanh^2(x)}{ab^2}-\frac{\tanh(x)}{a^2b}+\frac{1}{a^2(a+b\coth(x))}\right)d(b\coth(x))$$
$$\Big\downarrow \text{2009}$$
$$-b\left(-\frac{\log(b\coth(x))}{a^2}+\frac{\log(a+b\coth(x))}{a^2}-\frac{\tanh(x)}{ab}\right)$$

input
```
Int[Sech[x]^2/(a + b*Coth[x]),x]
```

output
```
-(b*(-(Log[b*Coth[x]]/a^2) + Log[a + b*Coth[x]]/a^2 - Tanh[x]/(a*b)))
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.76

method	result	size
risch	$-\frac{2}{a\left(e^{2x}+1\right)}-\frac{b\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{a^{2}}+\frac{b\ln\left(e^{2x}+1\right)}{a^{2}}$	51
default	$-\frac{b\ln\left(b\tanh\left(\frac{x}{2}\right)^{2}+2a\tanh\left(\frac{x}{2}\right)+b\right)}{a^{2}}-\frac{2\left(-\frac{a\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^{2}+1}-\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)}{2}\right)}{a^{2}}$	61

input
```
int(sech(x)^2/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/a/(exp(2*x)+1)-b/a^2*ln(exp(2*x)-(a-b)/(a+b))+b/a^2*ln(exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(29) = 58$.

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 4.03

$$\int \frac{\operatorname{sech}^2(x)}{a + b\coth(x)}\,dx =$$

$$-\frac{\left(b\cosh\left(x\right)^{2} + 2\,b\cosh\left(x\right)\sinh\left(x\right) + b\sinh\left(x\right)^{2} + b\right)\log\left(\frac{2\,\left(b\cosh(x)+a\sinh(x)\right)}{\cosh(x)-\sinh(x)}\right) - \left(b\cosh\left(x\right)^{2} + 2\,b\cosh}{a^{2}\cosh\left(x\right)^{2} + 2\,a^{2}\cosh\left(x\right)\sinh\left(x\right) + a^{2}\sinh\left(x\right)^{2}}$$

input
```
integrate(sech(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
-((b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*log(2*(b*cosh(x) +
 a*sinh(x))/(cosh(x) - sinh(x))) - (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*
sinh(x)^2 + b)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*a)/(a^2*cosh(x)^2 +
2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{a + b\coth(x)}\,dx = \int \frac{\operatorname{sech}^2\left(x\right)}{a + b\coth\left(x\right)}\,dx$$

input
```
integrate(sech(x)**2/(a+b*coth(x)),x)
```

output
```
Integral(sech(x)**2/(a + b*coth(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.59

$$\int \frac{\operatorname{sech}^2(x)}{a+b\coth(x)}\,dx = -\frac{b\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^2}+\frac{b\log\left(e^{(-2x)}+1\right)}{a^2}+\frac{2}{ae^{(-2x)}+a}$$

input
```
integrate(sech(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
-b*log(-(a - b)*e^(-2*x) + a + b)/a^2 + b*log(e^(-2*x) + 1)/a^2 + 2/(a*e^(
-2*x) + a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(29) = 58.

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.62

$$\int \frac{\operatorname{sech}^2(x)}{a+b\coth(x)}\,dx = -\frac{(ab+b^2)\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^3+a^2b}+\frac{b\log\left(e^{(2x)}+1\right)}{a^2}-\frac{be^{(2x)}+2a+b}{a^2(e^{(2x)}+1)}$$

input
```
integrate(sech(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

output
```
-(a*b + b^2)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^3 + a^2*b) + b*log
(e^(2*x) + 1)/a^2 - (b*e^(2*x) + 2*a + b)/(a^2*(e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 2.78 (sec) , antiderivative size = 323, normalized size of antiderivative = 11.14

$$\int \frac{\operatorname{sech}^2(x)}{a+b\coth(x)}\,dx$$
$$= \frac{2\,\mathrm{atan}\left(\frac{b\left(a^4\,{\left(b^2\right)}^{3/2}-a^6\,\sqrt{b^2}\right)\left(b^6\,\sqrt{-a^4}-a\,b^5\,\sqrt{-a^4}-a^2\,b^4\,\sqrt{-a^4}+a^3\,b^3\,\sqrt{-a^4}+b^6\,\mathrm{e}^{2\,x}\,\sqrt{-a^4}-2\,a^2\,b^4\,\mathrm{e}^{2\,x}\,\sqrt{-a^4}+a^4\,b^2\,\mathrm{e}^{2\,x}\,\sqrt{-a^4}\right)+b^2\,\left(\cdots\right.}{-a^{12}\,b^4+3\,a^{10}\,b^6-3\,\cdots}\right.}{\sqrt{-a^4}\,\cdots}$$
$$-\frac{2}{a\,\left(\mathrm{e}^{2\,x}+1\right)}$$

input
```
int(1/(cosh(x)^2*(a + b*coth(x))),x)
```

output
```
(2*atan((b*(a^4*(b^2)^(3/2) - a^6*(b^2)^(1/2))*(b^6*(-a^4)^(1/2) - a*b^5*(
-a^4)^(1/2) - a^2*b^4*(-a^4)^(1/2) + a^3*b^3*(-a^4)^(1/2) + b^6*exp(2*x)*(
-a^4)^(1/2) - 2*a^2*b^4*exp(2*x)*(-a^4)^(1/2) + a^4*b^2*exp(2*x)*(-a^4)^(1
/2)) + b^2*(a^3*(b^2)^(3/2) - a^5*(b^2)^(1/2))*(b^6*(-a^4)^(1/2) - a*b^5*(
-a^4)^(1/2) - a^2*b^4*(-a^4)^(1/2) + a^3*b^3*(-a^4)^(1/2) + b^6*exp(2*x)*(
-a^4)^(1/2) - 2*a^2*b^4*exp(2*x)*(-a^4)^(1/2) + a^4*b^2*exp(2*x)*(-a^4)^(1
/2)))/(a^6*b^10 - 3*a^8*b^8 + 3*a^10*b^6 - a^12*b^4))*(b^2)^(1/2))/(-a^4)^
(1/2) - 2/(a*(exp(2*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.38

$$\int \frac{\operatorname{sech}^2(x)}{a+b\coth(x)}\,dx$$
$$= \frac{e^{2x}\log(e^{2x}+1)\,b-e^{2x}\log(e^{2x}a+e^{2x}b-a+b)\,b+2e^{2x}a+\log(e^{2x}+1)\,b-\log(e^{2x}a+e^{2x}b-a+b)\,b}{a^2\,(e^{2x}+1)}$$

input
```
int(sech(x)^2/(a+b*coth(x)),x)
```

output
```
(e**(2*x)*log(e**(2*x) + 1)*b - e**(2*x)*log(e**(2*x)*a + e**(2*x)*b - a +
 b)*b + 2*e**(2*x)*a + log(e**(2*x) + 1)*b - log(e**(2*x)*a + e**(2*x)*b -
 a + b)*b)/(a**2*(e**(2*x) + 1))
```

3.125 $\int \frac{\operatorname{sech}^3(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 83

$$\int \frac{\operatorname{sech}^3(x)}{a+b\coth(x)}\,dx = \frac{\arctan(\sinh(x))}{2a} - \frac{b^2\arctan(\sinh(x))}{a^3} + \frac{b\sqrt{a^2-b^2}\operatorname{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a^3} - \frac{b\operatorname{sech}(x)}{a^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2a}$$

output
```
1/2*arctan(sinh(x))/a-b^2*arctan(sinh(x))/a^3+b*(a^2-b^2)^(1/2)*arctanh((a
*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))/a^3-b*sech(x)/a^2+1/2*sech(x)*tanh(x)
/a
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{sech}^3(x)}{a+b\coth(x)}\,dx = \frac{2(a^2-2b^2)\arctan\left(\tanh\left(\frac{x}{2}\right)\right) + 4b\sqrt{-a+b}\sqrt{a+b}\arctan\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right) + a\operatorname{sech}(x)(-2b+a\tanh(x))}{2a^3}$$

input
```
Integrate[Sech[x]^3/(a + b*Coth[x]),x]
```

output
```
(2*(a^2 - 2*b^2)*ArcTan[Tanh[x/2]] + 4*b*Sqrt[-a + b]*Sqrt[a + b]*ArcTan[(
a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])] + a*Sech[x]*(-2*b + a*Tanh[x]
))/(2*a^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {3042, 4001, 26, 26, 3042, 26, 3589, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^3(x)}{a+b\coth(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)^3\left(a-ib\tan\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{4001}$$

$$\int -\frac{i\tanh(x)\text{sech}^2(x)}{-ia\sinh(x)-ib\cosh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{i\text{sech}^2(x)\tanh(x)}{b\cosh(x)+a\sinh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh(x)\text{sech}^2(x)}{a\sinh(x)+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)}{\cos(ix)^3(b\cos(ix)-ia\sin(ix))}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{\sin(ix)}{\cos(ix)^3(b\cos(ix)-ia\sin(ix))}dx$$

$$\Big\downarrow \; 3589$$

$$-i\int \left(\frac{i\text{sech}^3(x)}{a}+\frac{b\text{sech}^2(x)}{a(ib\cosh(x)+ia\sinh(x))}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-i\left(-\frac{ib^2\arctan(\sinh(x))}{a^3}-\frac{ib\text{sech}(x)}{a^2}+\frac{ib\sqrt{a^2-b^2}\text{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a^3}+\frac{i\arctan(\sinh(x))}{2a}+\frac{i\tanh(x)}{2c}\right.$$

input
```
Int[Sech[x]^3/(a + b*Coth[x]),x]
```

output
```
(-I)*(((I/2)*ArcTan[Sinh[x]])/a - (I*b^2*ArcTan[Sinh[x]])/a^3 + (I*b*Sqrt[
a^2 - b^2]*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/a^3 - (I*b*Se
ch[x])/a^2 + ((I/2)*Sech[x]*Tanh[x])/a)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3589
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

rule 4001
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.46

method	result
default	$-\frac{2b(a^2-b^2)\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^2+b^2}}\right)}{a^3\sqrt{-a^2+b^2}}+\frac{\frac{2\left(-\frac{\tanh\left(\frac{x}{2}\right)^3a^2}{2}-\tanh\left(\frac{x}{2}\right)^2ab+\frac{\tanh\left(\frac{x}{2}\right)a^2}{2}-ab\right)}{\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^2}+(a^2-2b^2)\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a^3}$
risch	$\frac{e^x(e^{2x}a-2e^{2x}b-a-2b)}{(e^{2x}+1)^2a^2}+\frac{i\ln(e^x+i)}{2a}-\frac{i\ln(e^x+i)b^2}{a^3}-\frac{i\ln(e^x-i)}{2a}+\frac{i\ln(e^x-i)b^2}{a^3}+\frac{\sqrt{a^2-b^2}\,b\ln\left(e^x+\frac{\sqrt{a^2-b^2}}{a+b}\right)}{a^3}-\sqrt{a^2-}$

input
```
int(sech(x)^3/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(a^2-b^2)/a^3/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2
+b^2)^(1/2))+2/a^3*((-1/2*tanh(1/2*x)^3*a^2-tanh(1/2*x)^2*a*b+1/2*tanh(1/2
*x)*a^2-a*b)/(tanh(1/2*x)^2+1)^2+1/2*(a^2-2*b^2)*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. 2(75) = 150.

Time = 0.13 (sec) , antiderivative size = 856, normalized size of antiderivative = 10.31

$$\int \frac{\operatorname{sech}^3(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
[((a^2 - 2*a*b)*cosh(x)^3 + 3*(a^2 - 2*a*b)*cosh(x)*sinh(x)^2 + (a^2 - 2*a
*b)*sinh(x)^3 + (b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*c
osh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*s
inh(x) + b)*sqrt(a^2 - b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sin
h(x) + (a + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/
((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b
)) + ((a^2 - 2*b^2)*cosh(x)^4 + 4*(a^2 - 2*b^2)*cosh(x)*sinh(x)^3 + (a^2 -
 2*b^2)*sinh(x)^4 + 2*(a^2 - 2*b^2)*cosh(x)^2 + 2*(3*(a^2 - 2*b^2)*cosh(x)
^2 + a^2 - 2*b^2)*sinh(x)^2 + a^2 - 2*b^2 + 4*((a^2 - 2*b^2)*cosh(x)^3 + (
a^2 - 2*b^2)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2 + 2*a*b)*c
osh(x) + (3*(a^2 - 2*a*b)*cosh(x)^2 - a^2 - 2*a*b)*sinh(x))/(a^3*cosh(x)^4
 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3*
a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x)),
 ((a^2 - 2*a*b)*cosh(x)^3 + 3*(a^2 - 2*a*b)*cosh(x)*sinh(x)^2 + (a^2 - 2*a
*b)*sinh(x)^3 - 2*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b
*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))
*sinh(x) + b)*sqrt(-a^2 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) +
(a + b)*sinh(x))) + ((a^2 - 2*b^2)*cosh(x)^4 + 4*(a^2 - 2*b^2)*cosh(x)*sin
h(x)^3 + (a^2 - 2*b^2)*sinh(x)^4 + 2*(a^2 - 2*b^2)*cosh(x)^2 + 2*(3*(a^2 -
 2*b^2)*cosh(x)^2 + a^2 - 2*b^2)*sinh(x)^2 + a^2 - 2*b^2 + 4*((a^2 - 2*...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{a+b\coth(x)}\,dx=\int \frac{\operatorname{sech}^3(x)}{a+b\coth(x)}\,dx$$

input
```
integrate(sech(x)**3/(a+b*coth(x)),x)
```

output
```
Integral(sech(x)**3/(a + b*coth(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^3(x)}{a+b\coth(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sech(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23

$$\int \frac{\operatorname{sech}^3(x)}{a+b\coth(x)}\,dx = \frac{(a^2-2\,b^2)\arctan\left(e^x\right)}{a^3} - \frac{2\,(a^2b-b^3)\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}a^3} + \frac{ae^{(3\,x)}-2\,be^{(3\,x)}-ae^x-2\,be^x}{a^2(e^{(2\,x)}+1)^2}$$

input
```
integrate(sech(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

output
```
(a^2 - 2*b^2)*arctan(e^x)/a^3 - 2*(a^2*b - b^3)*arctan((a*e^x + b*e^x)/sqr
t(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^3) + (a*e^(3*x) - 2*b*e^(3*x) - a*e^x -
 2*b*e^x)/(a^2*(e^(2*x) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 5.21 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.00

$$\int \frac{\operatorname{sech}^3(x)}{a+b\coth(x)}\,dx = \frac{e^x\,(a-2\,b)}{a^2\,\left(e^{2\,x}+1\right)}+\frac{\ln\left(e^x+1i\right)\,\left(a^2\,1i-b^2\,2i\right)}{2\,a^3} - \frac{2\,e^x}{a\,\left(2\,e^{2\,x}+e^{4\,x}+1\right)}-\frac{\ln\left(e^x-i\right)\,\left(a^2\,1i-b^2\,2i\right)}{2\,a^3} + \frac{b\,\ln\left(a\,e^x+b\,e^x+\sqrt{a^2-b^2}\right)\,\sqrt{(a+b)\,(a-b)}}{a^3} - \frac{b\,\ln\left(a\,e^x+b\,e^x-\sqrt{a^2-b^2}\right)\,\sqrt{(a+b)\,(a-b)}}{a^3}$$

input
```
int(1/(cosh(x)^3*(a + b*coth(x))),x)
```

output
```
(log(exp(x) + 1i)*(a^2*1i - b^2*2i))/(2*a^3) - (log(exp(x) - 1i)*(a^2*1i -
 b^2*2i))/(2*a^3) - (2*exp(x))/(a*(2*exp(2*x) + exp(4*x) + 1)) + (exp(x)*(
a - 2*b))/(a^2*(exp(2*x) + 1)) + (b*log(a*exp(x) + b*exp(x) + (a^2 - b^2)^
(1/2))*((a + b)*(a - b))^(1/2))/a^3 - (b*log(a*exp(x) + b*exp(x) - (a^2 -
b^2)^(1/2))*((a + b)*(a - b))^(1/2))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.01

$$\int \frac{\operatorname{sech}^3(x)}{a+b\coth(x)}\,dx = \frac{e^{4x}atan(e^x)\,a^2-2e^{4x}atan(e^x)\,b^2+2e^{2x}atan(e^x)\,a^2-4e^{2x}atan(e^x)\,b^2+atan(e^x)\,a^2-2atan(e^x)\,b^2+2e^{\cdots}}{\cdots}$$

input
```
int(sech(x)^3/(a+b*coth(x)),x)
```

output
```
(e**(4*x)*atan(e**x)*a**2 - 2*e**(4*x)*atan(e**x)*b**2 + 2*e**(2*x)*atan(e
**x)*a**2 - 4*e**(2*x)*atan(e**x)*b**2 + atan(e**x)*a**2 - 2*atan(e**x)*b*
*2 + 2*e**(4*x)*sqrt( - a**2 + b**2)*atan((e**x*a + e**x*b)/sqrt( - a**2 +
 b**2))*b + 4*e**(2*x)*sqrt( - a**2 + b**2)*atan((e**x*a + e**x*b)/sqrt( -
 a**2 + b**2))*b + 2*sqrt( - a**2 + b**2)*atan((e**x*a + e**x*b)/sqrt( - a
**2 + b**2))*b + e**(3*x)*a**2 - 2*e**(3*x)*a*b - e**x*a**2 - 2*e**x*a*b)/
(a**3*(e**(4*x) + 2*e**(2*x) + 1))
```

3.126 $\int \frac{\operatorname{sech}^4(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 79

$$\int \frac{\operatorname{sech}^4(x)}{a+b\coth(x)}\,dx = -\frac{b(a^2-b^2)\log(a+b\coth(x))}{a^4} - \frac{b(a^2-b^2)\log(\tanh(x))}{a^4} + \frac{(a^2-b^2)\tanh(x)}{a^3} + \frac{b\tanh^2(x)}{2a^2} - \frac{\tanh^3(x)}{3a}$$

output
```
-b*(a^2-b^2)*ln(a+b*coth(x))/a^4-b*(a^2-b^2)*ln(tanh(x))/a^4+(a^2-b^2)*tan
h(x)/a^3+1/2*b*tanh(x)^2/a^2-1/3*tanh(x)^3/a
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.76

$$\int \frac{\operatorname{sech}^4(x)}{a+b\coth(x)}\,dx = \frac{6b(-a^2+b^2)\log(b+a\tanh(x)) + 6a(a^2-b^2)\tanh(x) + 3a^2b\tanh^2(x) - 2a^3\tanh^3(x)}{6a^4}$$

input
```
Integrate[Sech[x]^4/(a + b*Coth[x]),x]
```

output
```
(6*b*(-a^2 + b^2)*Log[b + a*Tanh[x]] + 6*a*(a^2 - b^2)*Tanh[x] + 3*a^2*b*T
anh[x]^2 - 2*a^3*Tanh[x]^3)/(6*a^4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3999, 25, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^4(x)}{a+b\coth(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)^4\left(a-ib\tan\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3999}$$

$$-b\int -\frac{\left(b^2-b^2\coth^2(x)\right)\tanh^4(x)}{b^4(a+b\coth(x))}d(b\coth(x))$$

$$\Big\downarrow \text{25}$$

$$b\int \frac{\left(b^2-b^2\coth^2(x)\right)\tanh^4(x)}{b^4(a+b\coth(x))}d(b\coth(x))$$

$$\Big\downarrow \text{522}$$

$$b\int \left(\frac{\tanh^4(x)}{ab^2}-\frac{\tanh^3(x)}{a^2b}+\frac{\left(b^2-a^2\right)\tanh^2(x)}{a^3b^2}+\frac{\left(a^2-b^2\right)\tanh(x)}{a^4b}+\frac{b^2-a^2}{a^4(a+b\coth(x))}\right)d(b\coth(x))$$

$$\Big\downarrow \text{2009}$$

$$-b\left(-\frac{\tanh^2(x)}{2a^2}-\frac{\left(a^2-b^2\right)\log(b\coth(x))}{a^4}+\frac{\left(a^2-b^2\right)\log(a+b\coth(x))}{a^4}-\frac{\left(a^2-b^2\right)\tanh(x)}{a^3b}+\frac{\tanh^3(x)}{3ab}\right)$$

input
```
Int[Sech[x]^4/(a + b*Coth[x]),x]
```

output
```
-(b*(-(((a^2 - b^2)*Log[b*Coth[x]])/a^4) + ((a^2 - b^2)*Log[a + b*Coth[x]]
)/a^4 - ((a^2 - b^2)*Tanh[x])/(a^3*b) - Tanh[x]^2/(2*a^2) + Tanh[x]^3/(3*a
*b)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 10.66 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.84

method	result
risch	$-\frac{2\left(3\,\mathrm{e}^{4x} a b-3 b^{2} \mathrm{e}^{4x}+6\,\mathrm{e}^{2x} a^{2}+3\,\mathrm{e}^{2x} a b-6 b^{2} \mathrm{e}^{2x}+2 a^{2}-3 b^{2}\right)}{3 a^{3} \left(\mathrm{e}^{2x}+1\right)^{3}}-\frac{b \ln\left(\mathrm{e}^{2x}-\frac{a-b}{a+b}\right)}{a^{2}}+\frac{b^{3} \ln\left(\mathrm{e}^{2x}-\frac{a-b}{a+b}\right)}{a^{4}}+\frac{b \ln\left(\mathrm{e}^{2x}+1\right)}{a^{2}}-\frac{b^{3} \ln(\mathrm{e}^{2}}{a^{4}}$
default	$-\frac{b\left(a^{2}-b^{2}\right) \ln\left(b \tanh\left(\frac{x}{2}\right)^{2}+2 a \tanh\left(\frac{x}{2}\right)+b\right)}{a^{4}}-\frac{2\left(\frac{\left(-a^{3}+a\,b^{2}\right) \tanh\left(\frac{x}{2}\right)^{5}-a^{2} b \tanh\left(\frac{x}{2}\right)^{4}+\left(-\frac{2}{3} a^{3}+2 a\,b^{2}\right) \tanh\left(\frac{x}{2}\right)^{3}-a^{2} b \tanh\left(\frac{x}{2}\right)^{2}+}{\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{3}}\right.}{a^{4}}$

input
```
int(sech(x)^4/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3*exp(4*x)*a*b-3*b^2*exp(4*x)+6*exp(2*x)*a^2+3*exp(2*x)*a*b-6*b^2*ex
p(2*x)+2*a^2-3*b^2)/a^3/(exp(2*x)+1)^3-b/a^2*ln(exp(2*x)-(a-b)/(a+b))+b^3/
a^4*ln(exp(2*x)-(a-b)/(a+b))+b/a^2*ln(exp(2*x)+1)-b^3/a^4*ln(exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 909 vs. $2(75) = 150$.

Time = 0.11 (sec) , antiderivative size = 909, normalized size of antiderivative = 11.51

$$\int \frac{\operatorname{sech}^4(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
-1/3*(6*(a^2*b - a*b^2)*cosh(x)^4 + 24*(a^2*b - a*b^2)*cosh(x)*sinh(x)^3 +
 6*(a^2*b - a*b^2)*sinh(x)^4 + 4*a^3 - 6*a*b^2 + 6*(2*a^3 + a^2*b - 2*a*b^
2)*cosh(x)^2 + 6*(2*a^3 + a^2*b - 2*a*b^2 + 6*(a^2*b - a*b^2)*cosh(x)^2)*s
inh(x)^2 + 3*((a^2*b - b^3)*cosh(x)^6 + 6*(a^2*b - b^3)*cosh(x)*sinh(x)^5
+ (a^2*b - b^3)*sinh(x)^6 + 3*(a^2*b - b^3)*cosh(x)^4 + 3*(a^2*b - b^3 + 5
*(a^2*b - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^2*b - b^3)*cosh(x)^3 + 3*(a^
2*b - b^3)*cosh(x))*sinh(x)^3 + a^2*b - b^3 + 3*(a^2*b - b^3)*cosh(x)^2 +
3*(5*(a^2*b - b^3)*cosh(x)^4 + a^2*b - b^3 + 6*(a^2*b - b^3)*cosh(x)^2)*si
nh(x)^2 + 6*((a^2*b - b^3)*cosh(x)^5 + 2*(a^2*b - b^3)*cosh(x)^3 + (a^2*b
- b^3)*cosh(x))*sinh(x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))
) - 3*((a^2*b - b^3)*cosh(x)^6 + 6*(a^2*b - b^3)*cosh(x)*sinh(x)^5 + (a^2*
b - b^3)*sinh(x)^6 + 3*(a^2*b - b^3)*cosh(x)^4 + 3*(a^2*b - b^3 + 5*(a^2*b
 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^2*b - b^3)*cosh(x)^3 + 3*(a^2*b - b
^3)*cosh(x))*sinh(x)^3 + a^2*b - b^3 + 3*(a^2*b - b^3)*cosh(x)^2 + 3*(5*(a
^2*b - b^3)*cosh(x)^4 + a^2*b - b^3 + 6*(a^2*b - b^3)*cosh(x)^2)*sinh(x)^2
 + 6*((a^2*b - b^3)*cosh(x)^5 + 2*(a^2*b - b^3)*cosh(x)^3 + (a^2*b - b^3)*
cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + 12*(2*(a^2*b - a*b^
2)*cosh(x)^3 + (2*a^3 + a^2*b - 2*a*b^2)*cosh(x))*sinh(x))/(a^4*cosh(x)^6
+ 6*a^4*cosh(x)*sinh(x)^5 + a^4*sinh(x)^6 + 3*a^4*cosh(x)^4 + 3*a^4*cosh(x
)^2 + 3*(5*a^4*cosh(x)^2 + a^4)*sinh(x)^4 + a^4 + 4*(5*a^4*cosh(x)^3 + ...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{a+b\coth(x)}\,dx=\int \frac{\operatorname{sech}^4(x)}{a+b\coth(x)}\,dx$$

input
```
integrate(sech(x)**4/(a+b*coth(x)),x)
```

output
```
Integral(sech(x)**4/(a + b*coth(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.68

$$\int \frac{\operatorname{sech}^4(x)}{a+b\coth(x)}\,dx=\frac{2\left(2a^2-3b^2+3(2a^2-ab-2b^2)e^{(-2x)}-3(ab+b^2)e^{(-4x)}\right)}{3\left(3a^3e^{(-2x)}+3a^3e^{(-4x)}+a^3e^{(-6x)}+a^3\right)}-\frac{(a^2b-b^3)\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^4}+\frac{(a^2b-b^3)\log\left(e^{(-2x)}+1\right)}{a^4}$$

input
```
integrate(sech(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
2/3*(2*a^2 - 3*b^2 + 3*(2*a^2 - a*b - 2*b^2)*e^(-2*x) - 3*(a*b + b^2)*e^(-
4*x))/(3*a^3*e^(-2*x) + 3*a^3*e^(-4*x) + a^3*e^(-6*x) + a^3) - (a^2*b - b^
3)*log(-(a - b)*e^(-2*x) + a + b)/a^4 + (a^2*b - b^3)*log(e^(-2*x) + 1)/a^
4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(75) = 150$.

Time = 0.13 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.54

$$\int \frac{\operatorname{sech}^4(x)}{a+b\coth(x)}\,dx$$
$$= -\frac{(a^3b+a^2b^2-ab^3-b^4)\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^5+a^4b} + \frac{(a^2b-b^3)\log\left(e^{(2x)}+1\right)}{a^4}$$
$$- \frac{11\,a^2be^{(6x)} - 11\,b^3e^{(6x)} + 45\,a^2be^{(4x)} - 12\,ab^2e^{(4x)} - 33\,b^3e^{(4x)} + 24\,a^3e^{(2x)} + 45\,a^2be^{(2x)} - 24\,ab^2e^{(2x)}}{6\,a^4\left(e^{(2x)}+1\right)^3}$$

input
```
integrate(sech(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

output
```
-(a^3*b + a^2*b^2 - a*b^3 - b^4)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(
a^5 + a^4*b) + (a^2*b - b^3)*log(e^(2*x) + 1)/a^4 - 1/6*(11*a^2*b*e^(6*x)
- 11*b^3*e^(6*x) + 45*a^2*b*e^(4*x) - 12*a*b^2*e^(4*x) - 33*b^3*e^(4*x) +
24*a^3*e^(2*x) + 45*a^2*b*e^(2*x) - 24*a*b^2*e^(2*x) - 33*b^3*e^(2*x) + 8*
a^3 + 11*a^2*b - 12*a*b^2 - 11*b^3)/(a^4*(e^(2*x) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.56

$$\int \frac{\operatorname{sech}^4(x)}{a+b\coth(x)}\,dx = \frac{8}{3\,a\,\left(3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1\right)} - \frac{2\,(2\,a-b)}{a^2\,\left(2\,e^{2\,x}+e^{4\,x}+1\right)}$$
$$- \frac{2\,b\,(a-b)}{a^3\,\left(e^{2\,x}+1\right)} - \frac{b\,\ln\left(b-a+a\,e^{2\,x}+b\,e^{2\,x}\right)\,(a+b)\,(a-b)}{a^4}$$
$$+ \frac{b\,\ln\left(e^{2\,x}+1\right)\,(a+b)\,(a-b)}{a^4}$$

input
```
int(1/(cosh(x)^4*(a + b*coth(x))),x)
```

output
```
8/(3*a*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) - (2*(2*a - b))/(a^2*(2*e
xp(2*x) + exp(4*x) + 1)) - (2*b*(a - b))/(a^3*(exp(2*x) + 1)) - (b*log(b -
 a + a*exp(2*x) + b*exp(2*x))*(a + b)*(a - b))/a^4 + (b*log(exp(2*x) + 1)*
(a + b)*(a - b))/a^4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 461, normalized size of antiderivative = 5.84

$$\int \frac{\operatorname{sech}^4(x)}{a + b\coth(x)}\,dx$$
$$= \frac{3e^{6x}\log(e^{2x}+1)\,a^2b - 3e^{6x}\log(e^{2x}+1)\,b^3 - 3e^{6x}\log(e^{2x}a + e^{2x}b - a + b)\,a^2b + 3e^{6x}\log(e^{2x}a + e^{2x}b - a +}{}$$

input
```
int(sech(x)^4/(a+b*coth(x)),x)
```

output
```
(3*e**(6*x)*log(e**(2*x) + 1)*a**2*b - 3*e**(6*x)*log(e**(2*x) + 1)*b**3 -
 3*e**(6*x)*log(e**(2*x)*a + e**(2*x)*b - a + b)*a**2*b + 3*e**(6*x)*log(e
**(2*x)*a + e**(2*x)*b - a + b)*b**3 + 2*e**(6*x)*a**2*b - 2*e**(6*x)*a*b*
*2 + 9*e**(4*x)*log(e**(2*x) + 1)*a**2*b - 9*e**(4*x)*log(e**(2*x) + 1)*b*
*3 - 9*e**(4*x)*log(e**(2*x)*a + e**(2*x)*b - a + b)*a**2*b + 9*e**(4*x)*l
og(e**(2*x)*a + e**(2*x)*b - a + b)*b**3 + 9*e**(2*x)*log(e**(2*x) + 1)*a*
*2*b - 9*e**(2*x)*log(e**(2*x) + 1)*b**3 - 9*e**(2*x)*log(e**(2*x)*a + e**
(2*x)*b - a + b)*a**2*b + 9*e**(2*x)*log(e**(2*x)*a + e**(2*x)*b - a + b)*
b**3 - 12*e**(2*x)*a**3 + 6*e**(2*x)*a*b**2 + 3*log(e**(2*x) + 1)*a**2*b -
 3*log(e**(2*x) + 1)*b**3 - 3*log(e**(2*x)*a + e**(2*x)*b - a + b)*a**2*b
+ 3*log(e**(2*x)*a + e**(2*x)*b - a + b)*b**3 - 4*a**3 + 2*a**2*b + 4*a*b*
*2)/(3*a**4*(e**(6*x) + 3*e**(4*x) + 3*e**(2*x) + 1))
```

3.127 $\int \frac{\text{sech}(x)}{i+2\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\text{sech}(x)}{i+2\coth(x)}\,dx = -i\arctan(\sinh(x)) - \frac{2\text{arctanh}\left(\frac{\cosh(x)-2i\sinh(x)}{\sqrt{5}}\right)}{\sqrt{5}}$$

output
```
-I*arctan(sinh(x))-2/5*arctanh(1/5*(cosh(x)-2*I*sinh(x))*5^(1/2))*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{\text{sech}(x)}{i+2\coth(x)}\,dx = -2i\arctan\left(\tanh\left(\frac{x}{2}\right)\right) - \frac{4\text{arctanh}\left(\frac{1-2i\tanh\left(\frac{x}{2}\right)}{\sqrt{5}}\right)}{\sqrt{5}}$$

input
```
Integrate[Sech[x]/(I + 2*Coth[x]),x]
```

output
```
(-2*I)*ArcTan[Tanh[x/2]] - (4*ArcTanh[(1 - (2*I)*Tanh[x/2])/Sqrt[5]])/Sqrt
[5]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3042, 4001, 26, 25, 3042, 26, 26, 3589, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(x)}{2\coth(x)+i}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)\left(i-2i\tan\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \ 4001$$

$$\int -\frac{i\tanh(x)}{\sinh(x)-2i\cosh(x)}dx$$

$$\Big\downarrow \ 26$$

$$-i\int -\frac{\tanh(x)}{2i\cosh(x)-\sinh(x)}dx$$

$$\Big\downarrow \ 25$$

$$i\int \frac{\tanh(x)}{2i\cosh(x)-\sinh(x)}dx$$

$$\Big\downarrow \ 3042$$

$$i\int -\frac{i\sin(ix)}{\cos(ix)(2i\cos(ix)+i\sin(ix))}dx$$

$$\Big\downarrow \ 26$$

$$\int -\frac{i\sin(ix)}{\cos(ix)(\sin(ix)+2\cos(ix))}dx$$

$$\Big\downarrow \ 26$$

$$-i\int \frac{\sin(ix)}{\cos(ix)(2\cos(ix)+\sin(ix))}dx$$

$$\Big\downarrow \ 3589$$

$$-i\int \left(\operatorname{sech}(x)-\frac{2}{2\cosh(x)+i\sinh(x)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-i\left(\arctan(\sinh(x))-\frac{2i\text{arctanh}\left(\frac{\cosh(x)-2i\sinh(x)}{\sqrt{5}}\right)}{\sqrt{5}}\right)$$

input
```
Int[Sech[x]/(I + 2*Coth[x]),x]
```

output
```
(-I)*(ArcTan[Sinh[x]] - ((2*I)*ArcTanh[(Cosh[x] - (2*I)*Sinh[x])/Sqrt[5]])
/Sqrt[5])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3589
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

rule 4001
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

method	result	size
default	$\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)+\frac{4 i \sqrt{5}\, \arctan\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+i\right)\sqrt{5}}{5}\right)}{5}-\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)$	41
risch	$-\ln\left(e^{x}-i\right)+\frac{2\sqrt{5}\,\ln\left(e^{x}-\frac{2 i \sqrt{5}}{5}-\frac{\sqrt{5}}{5}\right)}{5}-\frac{2\sqrt{5}\,\ln\left(e^{x}+\frac{2 i \sqrt{5}}{5}+\frac{\sqrt{5}}{5}\right)}{5}+\ln\left(e^{x}+i\right)$	56

input
```
int(sech(x)/(I+2*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(tanh(1/2*x)+I)+4/5*I*5^(1/2)*arctan(1/5*(2*tanh(1/2*x)+I)*5^(1/2))-ln(t
anh(1/2*x)-I)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{sech}(x)}{i+2\coth(x)}\,dx = -\frac{2}{5}\sqrt{5}\log\left(\left(\frac{2}{5}i+\frac{1}{5}\right)\sqrt{5}+e^{x}\right) + \frac{2}{5}\sqrt{5}\log\left(-\left(\frac{2}{5}i+\frac{1}{5}\right)\sqrt{5}+e^{x}\right)+\log\left(e^{x}+i\right)-\log\left(e^{x}-i\right)$$

input
```
integrate(sech(x)/(I+2*coth(x)),x, algorithm="fricas")
```

output
```
-2/5*sqrt(5)*log((2/5*I + 1/5)*sqrt(5) + e^x) + 2/5*sqrt(5)*log(-(2/5*I +
1/5)*sqrt(5) + e^x) + log(e^x + I) - log(e^x - I)
```

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{i+2\coth(x)}\,dx = \int \frac{\operatorname{sech}(x)}{2\coth(x)+i}\,dx$$

input
```
integrate(sech(x)/(I+2*coth(x)),x)
```

output
```
Integral(sech(x)/(2*coth(x) + I), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{\operatorname{sech}(x)}{i+2\coth(x)}\,dx = \frac{2}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-(2i+1)\,e^{(-x)}}{\sqrt{5}+(2i+1)\,e^{(-x)}}\right) + 2i\arctan\left(e^{(-x)}\right)$$

input
```
integrate(sech(x)/(I+2*coth(x)),x, algorithm="maxima")
```

output
```
2/5*sqrt(5)*log(-(sqrt(5) - (2*I + 1)*e^(-x))/(sqrt(5) + (2*I + 1)*e^(-x))
) + 2*I*arctan(e^(-x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{\operatorname{sech}(x)}{i+2\coth(x)}\,dx = \frac{4}{5}i\sqrt{5}\arctan\left(\left(\frac{1}{5}i+\frac{2}{5}\right)\sqrt{5}e^{x}\right) + \log\left(e^{x}+i\right) - \log\left(e^{x}-i\right)$$

input
```
integrate(sech(x)/(I+2*coth(x)),x, algorithm="giac")
```

output
```
4/5*I*sqrt(5)*arctan((1/5*I + 2/5)*sqrt(5)*e^x) + log(e^x + I) - log(e^x -
 I)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.10

$$\int \frac{\operatorname{sech}(x)}{i+2\coth(x)}\,dx = \ln\left(e^x\,(32+64i)-64+32i\right)-\ln\left(e^x\,(32+64i)+64-32i\right) - \frac{2\sqrt{5}\,\ln\left(e^x\left(-\frac{256}{5}+\frac{192}{5}i\right)+\sqrt{5}\left(-\frac{128}{5}-\frac{64}{5}i\right)\right)}{5} + \frac{2\sqrt{5}\,\ln\left(e^x\left(-\frac{256}{5}+\frac{192}{5}i\right)+\sqrt{5}\left(\frac{128}{5}+\frac{64}{5}i\right)\right)}{5}$$

input
```
int(1/(cosh(x)*(2*coth(x) + 1i)),x)
```

output
```
log(exp(x)*(32 + 64i) - (64 - 32i)) - log(exp(x)*(32 + 64i) + (64 - 32i))
- (2*5^(1/2)*log(- exp(x)*(256/5 - 192i/5) - 5^(1/2)*(128/5 + 64i/5)))/5 +
 (2*5^(1/2)*log(5^(1/2)*(128/5 + 64i/5) - exp(x)*(256/5 - 192i/5)))/5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.13

$$\int \frac{\operatorname{sech}(x)}{i+2\coth(x)}\,dx = \frac{10atan(e^x)\,i+20atan(e^x)-4\sqrt{5}\,atan\left(\frac{e^x i+2e^x}{\sqrt{5}}\right)i-8\sqrt{5}\,atan\left(\frac{e^x i+2e^x}{\sqrt{5}}\right)}{10i-5}$$

input
```
int(sech(x)/(I+2*coth(x)),x)
```

output
```
(2*(5*atan(e**x)*i + 10*atan(e**x) - 2*sqrt(5)*atan((e**x*i + 2*e**x)/sqrt
(5))*i - 4*sqrt(5)*atan((e**x*i + 2*e**x)/sqrt(5))))/(5*(2*i - 1))
```

3.128 $\int \frac{\tanh^4(x)}{1+\coth(x)} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 43

$$\int \frac{\tanh^4(x)}{1+\coth(x)} \, dx = \frac{5x}{2} - 2\log(\cosh(x)) - \frac{5\tanh(x)}{2} + \tanh^2(x) - \frac{5\tanh^3(x)}{6} + \frac{\tanh^3(x)}{2(1+\coth(x))}$$

output
```
5/2*x-2*ln(cosh(x))-5/2*tanh(x)+tanh(x)^2-5/6*tanh(x)^3+tanh(x)^3/(2+2*cot
h(x))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{\tanh^4(x)}{1+\coth(x)} \, dx = \frac{5}{2}\operatorname{arctanh}(\tanh(x)) - 2\log(\cosh(x)) - \frac{5\tanh(x)}{2} + \tanh^2(x) + \left(-\frac{5}{6} + \frac{1}{2+2\coth(x)}\right)\tanh^3(x)$$

input
```
Integrate[Tanh[x]^4/(1 + Coth[x]),x]
```

output
```
(5*ArcTanh[Tanh[x]])/2 - 2*Log[Cosh[x]] - (5*Tanh[x])/2 + Tanh[x]^2 + (-5/
6 + (2 + 2*Coth[x])^(-1))*Tanh[x]^3
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.56, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 1.818$, Rules used = {3042, 4035, 25, 3042, 4012, 25, 3042, 26, 4012, 26, 3042, 25, 4012, 3042, 26, 4014, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(x)}{\coth(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)\tan\left(\frac{\pi}{2}+ix\right)^4}dx$$

$$\Big\downarrow \text{4035}$$

$$\frac{\tanh^3(x)}{2(\coth(x)+1)}-\frac{1}{2}\int -((5-4\coth(x))\tanh^4(x))dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int (5-4\coth(x))\tanh^4(x)dx+\frac{\tanh^3(x)}{2(\coth(x)+1)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh^3(x)}{2(\coth(x)+1)}+\frac{1}{2}\int \frac{4i\tan\left(ix+\frac{\pi}{2}\right)+5}{\tan\left(ix+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{4012}$$

$$\frac{1}{2}\left(\int -((4-5\coth(x))\tanh^3(x))dx-\frac{5\tanh^3(x)}{3}\right)+\frac{\tanh^3(x)}{2(\coth(x)+1)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\int (4-5\coth(x))\tanh^3(x)dx-\frac{5}{3}\tanh^3(x)\right)+\frac{\tanh^3(x)}{2(\coth(x)+1)}$$

$$\Big\downarrow \ \color{blue}{3042}$$

$$\frac{\tanh^3(x)}{2(\coth(x)+1)}+\frac{1}{2}\left(-\frac{5}{3}\tanh^3(x)-\int -\frac{i\left(5i\tan\left(ix+\frac{\pi}{2}\right)+4\right)}{\tan\left(ix+\frac{\pi}{2}\right)^3}dx\right)$$

$$\Big\downarrow \ \color{blue}{26}$$

$$\frac{\tanh^3(x)}{2(\coth(x)+1)}+\frac{1}{2}\left(-\frac{5\tanh^3(x)}{3}+i\int \frac{5i\tan\left(ix+\frac{\pi}{2}\right)+4}{\tan\left(ix+\frac{\pi}{2}\right)^3}dx\right)$$

$$\Big\downarrow \ \color{blue}{4012}$$

$$\frac{\tanh^3(x)}{2(\coth(x)+1)}+\frac{1}{2}\left(-\frac{5\tanh^3(x)}{3}+i\left(\int -i(5-4\coth(x))\tanh^2(x)dx-2i\tanh^2(x)\right)\right)$$

$$\Big\downarrow \ \color{blue}{26}$$

$$\frac{\tanh^3(x)}{2(\coth(x)+1)}+\frac{1}{2}\left(-\frac{5\tanh^3(x)}{3}+i\left(-i\int (5-4\coth(x))\tanh^2(x)dx-2i\tanh^2(x)\right)\right)$$

$$\Big\downarrow \ \color{blue}{3042}$$

$$\frac{\tanh^3(x)}{2(\coth(x)+1)}+\frac{1}{2}\left(-\frac{5\tanh^3(x)}{3}+i\left(-i\int -\frac{4i\tan\left(ix+\frac{\pi}{2}\right)+5}{\tan\left(ix+\frac{\pi}{2}\right)^2}dx-2i\tanh^2(x)\right)\right)$$

$$\Big\downarrow \ \color{blue}{25}$$

$$\frac{\tanh^3(x)}{2(\coth(x)+1)}+\frac{1}{2}\left(-\frac{5\tanh^3(x)}{3}+i\left(i\int \frac{4i\tan\left(ix+\frac{\pi}{2}\right)+5}{\tan\left(ix+\frac{\pi}{2}\right)^2}dx-2i\tanh^2(x)\right)\right)$$

$$\Big\downarrow \ \color{blue}{4012}$$

$$\frac{\tanh^3(x)}{2(\coth(x)+1)}+$$
$$\frac{1}{2}\left(-\frac{5\tanh^3(x)}{3}+i\left(i(\int (4-5\coth(x))\tanh(x)dx+5\tanh(x))-2i\tanh^2(x)\right)\right)$$

$$\Big\downarrow \ \color{blue}{3042}$$

$$\frac{\tanh^3(x)}{2(\coth(x)+1)}+$$
$$\frac{1}{2}\left(-\frac{5\tanh^3(x)}{3}+i\left(i\left(5\tanh(x)+\int \frac{i\left(5i\tan\left(ix+\frac{\pi}{2}\right)+4\right)}{\tan\left(ix+\frac{\pi}{2}\right)}dx\right)-2i\tanh^2(x)\right)\right)$$

$$\Big\downarrow \ \color{blue}{26}$$

$$\frac{\tanh^3(x)}{2(\coth(x)+1)}+\frac{1}{2}\left(-\frac{5\tanh^3(x)}{3}+i\left(i\left(5\tanh(x)+i\int\frac{5i\tan\left(ix+\frac{\pi}{2}\right)+4}{\tan\left(ix+\frac{\pi}{2}\right)}dx\right)-2i\tanh^2(x)\right)\right)$$

$$\Big\downarrow \text{4014}$$

$$\frac{\tanh^3(x)}{2(\coth(x)+1)}+\frac{1}{2}\left(-\frac{5\tanh^3(x)}{3}+i\left(i(5\tanh(x)+i(4\int-i\tanh(x)dx+5ix))-2i\tanh^2(x)\right)\right)$$

$$\Big\downarrow \text{26}$$

$$\frac{\tanh^3(x)}{2(\coth(x)+1)}+\frac{1}{2}\left(-\frac{5\tanh^3(x)}{3}+i\left(i(5\tanh(x)+i(5ix-4i\int\tanh(x)dx))-2i\tanh^2(x)\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh^3(x)}{2(\coth(x)+1)}+\frac{1}{2}\left(-\frac{5\tanh^3(x)}{3}+i\left(i(5\tanh(x)+i(5ix-4i\int-i\tan(ix)dx))-2i\tanh^2(x)\right)\right)$$

$$\Big\downarrow \text{26}$$

$$\frac{\tanh^3(x)}{2(\coth(x)+1)}+\frac{1}{2}\left(-\frac{5\tanh^3(x)}{3}+i\left(i(5\tanh(x)+i(5ix-4\int\tan(ix)dx))-2i\tanh^2(x)\right)\right)$$

$$\Big\downarrow \text{3956}$$

$$\frac{\tanh^3(x)}{2(\coth(x)+1)}+\frac{1}{2}\left(-\frac{5\tanh^3(x)}{3}+i(i(5\tanh(x)+i(5ix-4i\log(\cosh(x))))-2i\tanh^2(x))\right)$$

input
```
Int[Tanh[x]^4/(1 + Coth[x]),x]
```

output
```
Tanh[x]^3/(2*(1 + Coth[x])) + ((-5*Tanh[x]^3)/3 + I*((-2*I)*Tanh[x]^2 + I*
(I*((5*I)*x - (4*I)*Log[Cosh[x]]) + 5*Tanh[x])))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4035
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Simp[1/(2*a*(b*c - a*d))   Int[(c + d
*Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

method	result
risch	$\frac{9x}{2}+\frac{{\mathrm e}^{-2x}}{4}+\frac{4\,{\mathrm e}^{4x}+6\,{\mathrm e}^{2x}+\frac{14}{3}}{\left({\mathrm e}^{2x}+1\right)^{3}}-2\ln\left({\mathrm e}^{2x}+1\right)$
parallelrisch	$\frac{\left(12\tanh\left(x\right)+12\right)\ln\left(1-\tanh\left(x\right)\right)-2\tanh\left(x\right)^{4}+\tanh\left(x\right)^{3}+27\tanh\left(x\right)x-9\tanh\left(x\right)^{2}+27x+15}{6\tanh\left(x\right)+6}$
default	$\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{1}{\tanh\left(\frac{x}{2}\right)+1}+\frac{9\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}-\frac{4\left(\tanh\left(\frac{x}{2}\right)^{5}-\frac{\tanh\left(\frac{x}{2}\right)^{4}}{2}+\frac{8\tanh\left(\frac{x}{2}\right)^{3}}{3}-\frac{\tanh\left(\frac{x}{2}\right)^{2}}{2}+\tanh\left(\frac{x}{2}\right)\right)}{\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{3}}-$

input
```
int(tanh(x)^4/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
9/2*x+1/4*exp(-2*x)+2/3*(6*exp(4*x)+9*exp(2*x)+7)/(exp(2*x)+1)^3-2*ln(exp(
2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(35) = 70$.

Time = 0.09 (sec) , antiderivative size = 571, normalized size of antiderivative = 13.28

$$\int \frac{\tanh^4(x)}{1+\coth(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^4/(1+coth(x)),x, algorithm="fricas")
```

output
```
1/12*(54*x*cosh(x)^8 + 432*x*cosh(x)*sinh(x)^7 + 54*x*sinh(x)^8 + 3*(54*x
+ 17)*cosh(x)^6 + 3*(504*x*cosh(x)^2 + 54*x + 17)*sinh(x)^6 + 18*(168*x*co
sh(x)^3 + (54*x + 17)*cosh(x))*sinh(x)^5 + 81*(2*x + 1)*cosh(x)^4 + 9*(420
*x*cosh(x)^4 + 5*(54*x + 17)*cosh(x)^2 + 18*x + 9)*sinh(x)^4 + 12*(252*x*c
osh(x)^5 + 5*(54*x + 17)*cosh(x)^3 + 27*(2*x + 1)*cosh(x))*sinh(x)^3 + (54
*x + 65)*cosh(x)^2 + (1512*x*cosh(x)^6 + 45*(54*x + 17)*cosh(x)^4 + 486*(2
*x + 1)*cosh(x)^2 + 54*x + 65)*sinh(x)^2 - 24*(cosh(x)^8 + 8*cosh(x)*sinh(
x)^7 + sinh(x)^8 + (28*cosh(x)^2 + 3)*sinh(x)^6 + 3*cosh(x)^6 + 2*(28*cosh
(x)^3 + 9*cosh(x))*sinh(x)^5 + (70*cosh(x)^4 + 45*cosh(x)^2 + 3)*sinh(x)^4
 + 3*cosh(x)^4 + 4*(14*cosh(x)^5 + 15*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + (
28*cosh(x)^6 + 45*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*
(4*cosh(x)^7 + 9*cosh(x)^5 + 6*cosh(x)^3 + cosh(x))*sinh(x))*log(2*cosh(x)
/(cosh(x) - sinh(x))) + 2*(216*x*cosh(x)^7 + 9*(54*x + 17)*cosh(x)^5 + 162
*(2*x + 1)*cosh(x)^3 + (54*x + 65)*cosh(x))*sinh(x) + 3)/(cosh(x)^8 + 8*co
sh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(x)^2 + 3)*sinh(x)^6 + 3*cosh(x)^6 +
 2*(28*cosh(x)^3 + 9*cosh(x))*sinh(x)^5 + (70*cosh(x)^4 + 45*cosh(x)^2 + 3
)*sinh(x)^4 + 3*cosh(x)^4 + 4*(14*cosh(x)^5 + 15*cosh(x)^3 + 3*cosh(x))*si
nh(x)^3 + (28*cosh(x)^6 + 45*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^2 + cos
h(x)^2 + 2*(4*cosh(x)^7 + 9*cosh(x)^5 + 6*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{1+\coth(x)}\,dx = \int \frac{\tanh^4{(x)}}{\coth{(x)}+1}\,dx$$

input
```
integrate(tanh(x)**4/(1+coth(x)),x)
```

output
```
Integral(tanh(x)**4/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.28

$$\int \frac{\tanh^4(x)}{1+\coth(x)}\,dx = \frac{1}{2}\,x - \frac{2\,\left(15\,e^{(-2\,x)} + 12\,e^{(-4\,x)} + 7\right)}{3\,\left(3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1\right)} + \frac{1}{4}\,e^{(-2\,x)} - 2\,\log\left(e^{(-2\,x)} + 1\right)$$

input
```
integrate(tanh(x)^4/(1+coth(x)),x, algorithm="maxima")
```

output
```
1/2*x - 2/3*(15*e^(-2*x) + 12*e^(-4*x) + 7)/(3*e^(-2*x) + 3*e^(-4*x) + e^(
-6*x) + 1) + 1/4*e^(-2*x) - 2*log(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \frac{\tanh^4(x)}{1+\coth(x)}\,dx = \frac{9}{2}\,x + \frac{\left(51\,e^{(6\,x)} + 81\,e^{(4\,x)} + 65\,e^{(2\,x)} + 3\right)e^{(-2\,x)}}{12\,\left(e^{(2\,x)} + 1\right)^3} - 2\,\log\left(e^{(2\,x)} + 1\right)$$

input
```
integrate(tanh(x)^4/(1+coth(x)),x, algorithm="giac")
```

output
```
9/2*x + 1/12*(51*e^(6*x) + 81*e^(4*x) + 65*e^(2*x) + 3)*e^(-2*x)/(e^(2*x)
+ 1)^3 - 2*log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

$$\int \frac{\tanh^4(x)}{1+\coth(x)}\,dx = \frac{9\,x}{2} - 2\,\ln\left(\mathrm{e}^{2\,x} + 1\right) + \frac{\mathrm{e}^{-2\,x}}{4} + \frac{8}{3\,\left(3\,\mathrm{e}^{2\,x} + 3\,\mathrm{e}^{4\,x} + \mathrm{e}^{6\,x} + 1\right)} - \frac{2}{2\,\mathrm{e}^{2\,x} + \mathrm{e}^{4\,x} + 1} + \frac{4}{\mathrm{e}^{2\,x} + 1}$$

input
```
int(tanh(x)^4/(coth(x) + 1),x)
```

output
```
(9*x)/2 - 2*log(exp(2*x) + 1) + exp(-2*x)/4 + 8/(3*(3*exp(2*x) + 3*exp(4*x
) + exp(6*x) + 1)) - 2/(2*exp(2*x) + exp(4*x) + 1) + 4/(exp(2*x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.42

$$\int \frac{\tanh^4(x)}{1+\coth(x)}\,dx$$
$$= \frac{-24e^{8x}\log(e^{2x}+1)+54e^{8x}x-17e^{8x}-72e^{6x}\log(e^{2x}+1)+162e^{6x}x-72e^{4x}\log(e^{2x}+1)+162e^{4x}x+3}{12e^{2x}\left(e^{6x}+3e^{4x}+3e^{2x}+1\right)}$$

input
```
int(tanh(x)^4/(1+coth(x)),x)
```

output
```
( - 24*e**(8*x)*log(e**(2*x) + 1) + 54*e**(8*x)*x - 17*e**(8*x) - 72*e**(6
*x)*log(e**(2*x) + 1) + 162*e**(6*x)*x - 72*e**(4*x)*log(e**(2*x) + 1) + 1
62*e**(4*x)*x + 30*e**(4*x) - 24*e**(2*x)*log(e**(2*x) + 1) + 54*e**(2*x)*
x + 48*e**(2*x) + 3)/(12*e**(2*x)*(e**(6*x) + 3*e**(4*x) + 3*e**(2*x) + 1)
)
```

3.129 $\int \frac{\tanh^3(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{\tanh^3(x)}{1+\coth(x)}\,dx=-\frac{3x}{2}+2\log(\cosh(x))+\frac{3\tanh(x)}{2}-\tanh^2(x)+\frac{\tanh^2(x)}{2(1+\coth(x))}$$

output
```
-3/2*x+2*ln(cosh(x))+3/2*tanh(x)-tanh(x)^2+tanh(x)^2/(2+2*coth(x))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{\tanh^3(x)}{1+\coth(x)}\,dx=\frac{1}{2}\left(-3\text{arctanh}(\tanh(x))+4\log(\cosh(x))+3\tanh(x)+\left(-2+\frac{1}{1+\coth(x)}\right)\tanh^2(x)\right)$$

input
```
Integrate[Tanh[x]^3/(1 + Coth[x]),x]
```

output
```
(-3*ArcTanh[Tanh[x]] + 4*Log[Cosh[x]] + 3*Tanh[x] + (-2 + (1 + Coth[x])^(-
1))*Tanh[x]^2)/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.636$, Rules used = {3042, 26, 4035, 26, 3042, 26, 4012, 26, 3042, 25, 4012, 3042, 26, 4014, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{\coth(x)+1}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i}{\left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)\tan\left(\frac{\pi}{2}+ix\right)^3}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{1}{\left(1-i\tan\left(ix+\frac{\pi}{2}\right)\right)\tan\left(ix+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{ 4035}$$

$$-i\left(\frac{i\tanh^2(x)}{2(\coth(x)+1)}-\frac{1}{2}\int -i(4-3\coth(x))\tanh^3(x)dx\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(\frac{1}{2}i\int (4-3\coth(x))\tanh^3(x)dx+\frac{i\tanh^2(x)}{2(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(\frac{1}{2}i\int -\frac{i\left(3i\tan\left(ix+\frac{\pi}{2}\right)+4\right)}{\tan\left(ix+\frac{\pi}{2}\right)^3}dx+\frac{i\tanh^2(x)}{2(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(\frac{1}{2}\int \frac{3i\tan\left(ix+\frac{\pi}{2}\right)+4}{\tan\left(ix+\frac{\pi}{2}\right)^3}dx+\frac{i\tanh^2(x)}{2(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{ 4012}$$

$$-i\left(\frac{1}{2}\left(\int -i(3-4\coth(x))\tanh^2(x)dx-2i\tanh^2(x)\right)+\frac{i\tanh^2(x)}{2(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(\frac{1}{2}\left(-i\int (3-4\coth(x))\tanh^2(x)dx-2i\tanh^2(x)\right)+\frac{i\tanh^2(x)}{2(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(\frac{1}{2}\left(-i\int -\frac{4i\tan\left(ix+\frac{\pi}{2}\right)+3}{\tan\left(ix+\frac{\pi}{2}\right)^2}dx-2i\tanh^2(x)\right)+\frac{i\tanh^2(x)}{2(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-i\left(\frac{1}{2}\left(i\int \frac{4i\tan\left(ix+\frac{\pi}{2}\right)+3}{\tan\left(ix+\frac{\pi}{2}\right)^2}dx-2i\tanh^2(x)\right)+\frac{i\tanh^2(x)}{2(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{ 4012}$$

$$-i\left(\frac{1}{2}\left(i(\int (4-3\coth(x))\tanh(x)dx+3\tanh(x))-2i\tanh^2(x)\right)+\frac{i\tanh^2(x)}{2(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(\frac{1}{2}\left(i\left(3\tanh(x)+\int \frac{i\left(3i\tan\left(ix+\frac{\pi}{2}\right)+4\right)}{\tan\left(ix+\frac{\pi}{2}\right)}dx\right)-2i\tanh^2(x)\right)+\frac{i\tanh^2(x)}{2(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(\frac{1}{2}\left(i\left(3\tanh(x)+i\int \frac{3i\tan\left(ix+\frac{\pi}{2}\right)+4}{\tan\left(ix+\frac{\pi}{2}\right)}dx\right)-2i\tanh^2(x)\right)+\frac{i\tanh^2(x)}{2(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{ 4014}$$

$$-i\left(\frac{1}{2}\left(i(3\tanh(x)+i(4\int -i\tanh(x)dx+3ix))-2i\tanh^2(x)\right)+\frac{i\tanh^2(x)}{2(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(\frac{1}{2}\left(i(3\tanh(x)+i(3ix-4i\int \tanh(x)dx))-2i\tanh^2(x)\right)+\frac{i\tanh^2(x)}{2(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(\frac{1}{2}\left(i(3\tanh(x)+i(3ix-4i\int -i\tan(ix)dx))-2i\tanh^2(x)\right)+\frac{i\tanh^2(x)}{2(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(\frac{1}{2}\left(i(3\tanh(x)+i(3ix-4\int\tan(ix)dx))-2i\tanh^2(x)\right)+\frac{i\tanh^2(x)}{2(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{3956}$$

$$-i\left(\frac{i\tanh^2(x)}{2(\coth(x)+1)}+\frac{1}{2}\left(i(3\tanh(x)+i(3ix-4i\log(\cosh(x))))-2i\tanh^2(x)\right)\right)$$

input
```
Int[Tanh[x]^3/(1 + Coth[x]),x]
```

output
```
(-I)*(((I/2)*Tanh[x]^2)/(1 + Coth[x]) + ((-2*I)*Tanh[x]^2 + I*(I*((3*I)*x
- (4*I)*Log[Cosh[x]]) + 3*Tanh[x]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4035
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Simp[1/(2*a*(b*c - a*d))   Int[(c + d
*Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{7x}{2}-\frac{\mathrm{e}^{-2x}}{4}-\frac{2}{\left(\mathrm{e}^{2x}+1\right)^{2}}+2\ln\left(\mathrm{e}^{2x}+1\right)$
parallelrisch	$\frac{\left(-4\tanh(x)-4\right)\ln\left(1-\tanh(x)\right)-\tanh(x)^{3}-7\tanh(x)x+\tanh(x)^{2}-7x-3}{2\tanh(x)+2}$
default	$-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{1}{\tanh\left(\frac{x}{2}\right)+1}-\frac{7\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}+\frac{2\tanh\left(\frac{x}{2}\right)^{3}-2\tanh\left(\frac{x}{2}\right)^{2}+2\tanh\left(\frac{x}{2}\right)}{\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{2}}+2\ln\left(\tanh\left(\frac{x}{2}\right)^{2}\right.$

input
```
int(tanh(x)^3/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
-7/2*x-1/4*exp(-2*x)-2/(exp(2*x)+1)^2+2*ln(exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(31) = 62$.

Time = 0.09 (sec) , antiderivative size = 354, normalized size of antiderivative = 9.57

$$\int \frac{\tanh^3(x)}{1+\coth(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^3/(1+coth(x)),x, algorithm="fricas")
```

output
```
-1/4*(14*x*cosh(x)^6 + 84*x*cosh(x)*sinh(x)^5 + 14*x*sinh(x)^6 + (28*x + 1
)*cosh(x)^4 + (210*x*cosh(x)^2 + 28*x + 1)*sinh(x)^4 + 4*(70*x*cosh(x)^3 +
 (28*x + 1)*cosh(x))*sinh(x)^3 + 2*(7*x + 5)*cosh(x)^2 + 2*(105*x*cosh(x)^
4 + 3*(28*x + 1)*cosh(x)^2 + 7*x + 5)*sinh(x)^2 - 8*(cosh(x)^6 + 6*cosh(x)
*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^2 + 2)*sinh(x)^4 + 2*cosh(x)^4 + 4*(5
*cosh(x)^3 + 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 + 12*cosh(x)^2 + 1)*sinh
(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 + 4*cosh(x)^3 + cosh(x))*sinh(x))*log(2
*cosh(x)/(cosh(x) - sinh(x))) + 4*(21*x*cosh(x)^5 + (28*x + 1)*cosh(x)^3 +
 (7*x + 5)*cosh(x))*sinh(x) + 1)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x
)^6 + (15*cosh(x)^2 + 2)*sinh(x)^4 + 2*cosh(x)^4 + 4*(5*cosh(x)^3 + 2*cosh
(x))*sinh(x)^3 + (15*cosh(x)^4 + 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 +
 2*(3*cosh(x)^5 + 4*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{1+\coth(x)}\,dx = \int \frac{\tanh^3{(x)}}{\coth{(x)}+1}\,dx$$

input
```
integrate(tanh(x)**3/(1+coth(x)),x)
```

output
```
Integral(tanh(x)**3/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

$$\int \frac{\tanh^3(x)}{1+\coth(x)}\,dx = \frac{1}{2}\,x + \frac{2\left(2\,e^{(-2\,x)}+1\right)}{2\,e^{(-2\,x)}+e^{(-4\,x)}+1} - \frac{1}{4}\,e^{(-2\,x)} + 2\,\log\left(e^{(-2\,x)}+1\right)$$

input
```
integrate(tanh(x)^3/(1+coth(x)),x, algorithm="maxima")
```

output
```
1/2*x + 2*(2*e^(-2*x) + 1)/(2*e^(-2*x) + e^(-4*x) + 1) - 1/4*e^(-2*x) + 2*
log(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \frac{\tanh^3(x)}{1+\coth(x)}\,dx = -\frac{7}{2}\,x - \frac{\left(e^{(4\,x)}+10\,e^{(2\,x)}+1\right)e^{(-2\,x)}}{4\left(e^{(2\,x)}+1\right)^2} + 2\,\log\left(e^{(2\,x)}+1\right)$$

input
```
integrate(tanh(x)^3/(1+coth(x)),x, algorithm="giac")
```

output
```
-7/2*x - 1/4*(e^(4*x) + 10*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) + 1)^2 + 2*log(e
^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{\tanh^3(x)}{1+\coth(x)}\,dx = 2\,\ln\left(\mathrm{e}^{2\,x}+1\right) - \frac{7\,x}{2} - \frac{\mathrm{e}^{-2\,x}}{4} - \frac{2}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1}$$

input
```
int(tanh(x)^3/(coth(x) + 1),x)
```

output
```
2*log(exp(2*x) + 1) - (7*x)/2 - exp(-2*x)/4 - 2/(2*exp(2*x) + exp(4*x) + 1
)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.92

$$\int \frac{\tanh^3(x)}{1+\coth(x)}\,dx = \frac{16e^{6x}\log(e^{2x}+1)-28e^{6x}x+e^{6x}+32e^{4x}\log(e^{2x}+1)-56e^{4x}x+16e^{2x}\log(e^{2x}+1)-28e^{2x}x-19e^{2x}-}{8e^{2x}\left(e^{4x}+2e^{2x}+1\right)}$$

input
```
int(tanh(x)^3/(1+coth(x)),x)
```

output
```
(16*e**(6*x)*log(e**(2*x) + 1) - 28*e**(6*x)*x + e**(6*x) + 32*e**(4*x)*lo
g(e**(2*x) + 1) - 56*e**(4*x)*x + 16*e**(2*x)*log(e**(2*x) + 1) - 28*e**(2
*x)*x - 19*e**(2*x) - 2)/(8*e**(2*x)*(e**(4*x) + 2*e**(2*x) + 1))
```

3.130 $\int \frac{\tanh^2(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \frac{\tanh^2(x)}{1+\coth(x)}\,dx=\frac{3x}{2}-\log(\cosh(x))-\frac{3\tanh(x)}{2}+\frac{\tanh(x)}{2(1+\coth(x))}$$

output
```
3/2*x-ln(cosh(x))-3/2*tanh(x)+tanh(x)/(2+2*coth(x))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{\tanh^2(x)}{1+\coth(x)}\,dx=\frac{1}{4}\left(-\log(1-\tanh(x))+5\log(1+\tanh(x))+\left(-6+\frac{2}{1+\coth(x)}\right)\tanh(x)\right)$$

input
```
Integrate[Tanh[x]^2/(1 + Coth[x]),x]
```

output
```
(-Log[1 - Tanh[x]] + 5*Log[1 + Tanh[x]] + (-6 + 2/(1 + Coth[x]))*Tanh[x])/
4
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.182, Rules used = {3042, 25, 4035, 3042, 25, 4012, 3042, 26, 4014, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)}{\coth(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)\tan\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\left(1-i\tan\left(ix+\frac{\pi}{2}\right)\right)\tan\left(ix+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4035}$$

$$\frac{1}{2}\int (3-2\coth(x))\tanh^2(x)dx+\frac{\tanh(x)}{2(\coth(x)+1)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)}{2(\coth(x)+1)}+\frac{1}{2}\int -\frac{2i\tan\left(ix+\frac{\pi}{2}\right)+3}{\tan\left(ix+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{\tanh(x)}{2(\coth(x)+1)}-\frac{1}{2}\int \frac{2i\tan\left(ix+\frac{\pi}{2}\right)+3}{\tan\left(ix+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4012}$$

$$\frac{1}{2}(-\int (2-3\coth(x))\tanh(x)dx-3\tanh(x))+\frac{\tanh(x)}{2(\coth(x)+1)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)}{2(\coth(x)+1)}+\frac{1}{2}\left(-3\tanh(x)-\int \frac{i\left(3i\tan\left(ix+\frac{\pi}{2}\right)+2\right)}{\tan\left(ix+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{26}$$

$$\frac{\tanh(x)}{2(\coth(x)+1)}+\frac{1}{2}\left(-3\tanh(x)-i\int\frac{3i\tan\left(ix+\frac{\pi}{2}\right)+2}{\tan\left(ix+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{4014}$$

$$\frac{\tanh(x)}{2(\coth(x)+1)}+\frac{1}{2}(-3\tanh(x)-i(2\int-i\tanh(x)dx+3ix))$$

$$\Big\downarrow \text{26}$$

$$\frac{\tanh(x)}{2(\coth(x)+1)}+\frac{1}{2}(-3\tanh(x)-i(3ix-2i\int\tanh(x)dx))$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)}{2(\coth(x)+1)}+\frac{1}{2}(-3\tanh(x)-i(3ix-2i\int-i\tan(ix)dx))$$

$$\Big\downarrow \text{26}$$

$$\frac{\tanh(x)}{2(\coth(x)+1)}+\frac{1}{2}(-3\tanh(x)-i(3ix-2\int\tan(ix)dx))$$

$$\Big\downarrow \text{3956}$$

$$\frac{\tanh(x)}{2(\coth(x)+1)}+\frac{1}{2}(-3\tanh(x)-i(3ix-2i\log(\cosh(x))))$$

input
```
Int[Tanh[x]^2/(1 + Coth[x]),x]
```

output
```
((-I)*((3*I)*x - (2*I)*Log[Cosh[x]]) - 3*Tanh[x])/2 + Tanh[x]/(2*(1 + Coth
[x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4035
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Simp[1/(2*a*(b*c - a*d))   Int[(c + d
*Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result
risch	$\frac{5x}{2}+\frac{{\mathrm e}^{-2x}}{4}+\frac{2}{{\mathrm e}^{2x}+1}-\ln\left({\mathrm e}^{2x}+1\right)$
parallelrisch	$\frac{(2\tanh(x)+2)\ln(1-\tanh(x))+5\tanh(x)x-2\tanh(x)^{2}+5x+3}{2\tanh(x)+2}$
default	$\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{1}{\tanh\left(\frac{x}{2}\right)+1}+\frac{5\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}-\frac{2\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^{2}+1}-\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}$

input
```
int(tanh(x)^2/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
5/2*x+1/4*exp(-2*x)+2/(exp(2*x)+1)-ln(exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(23) = 46$.

Time = 0.11 (sec) , antiderivative size = 186, normalized size of antiderivative = 6.41

$$\int \frac{\tanh^2(x)}{1+\coth(x)}\,dx$$
$$= \frac{10\,x\cosh\left(x\right)^{4}+40\,x\cosh\left(x\right)\sinh\left(x\right)^{3}+10\,x\sinh\left(x\right)^{4}+\left(10\,x+9\right)\cosh\left(x\right)^{2}+\left(60\,x\cosh\left(x\right)^{2}+10\,x+\right.}{4\left(\cosh\right.}$$

input
```
integrate(tanh(x)^2/(1+coth(x)),x, algorithm="fricas")
```

output
```
1/4*(10*x*cosh(x)^4 + 40*x*cosh(x)*sinh(x)^3 + 10*x*sinh(x)^4 + (10*x + 9)
*cosh(x)^2 + (60*x*cosh(x)^2 + 10*x + 9)*sinh(x)^2 - 4*(cosh(x)^4 + 4*cosh
(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2
*cosh(x)^3 + cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*(20*
x*cosh(x)^3 + (10*x + 9)*cosh(x))*sinh(x) + 1)/(cosh(x)^4 + 4*cosh(x)*sinh
(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*cosh(x)
^3 + cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{1+\coth(x)}\,dx = \int \frac{\tanh^2(x)}{\coth(x)+1}\,dx$$

input
```
integrate(tanh(x)**2/(1+coth(x)),x)
```

output
```
Integral(tanh(x)**2/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^2(x)}{1+\coth(x)}\,dx = \frac{1}{2}x - \frac{2}{e^{(-2x)}+1} + \frac{1}{4}e^{(-2x)} - \log\left(e^{(-2x)}+1\right)$$

input
```
integrate(tanh(x)^2/(1+coth(x)),x, algorithm="maxima")
```

output
```
1/2*x - 2/(e^(-2*x) + 1) + 1/4*e^(-2*x) - log(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{\tanh^2(x)}{1+\coth(x)}\,dx = \frac{5}{2}x + \frac{\left(9e^{(2x)}+1\right)e^{(-2x)}}{4\left(e^{(2x)}+1\right)} - \log\left(e^{(2x)}+1\right)$$

input
```
integrate(tanh(x)^2/(1+coth(x)),x, algorithm="giac")
```

output
```
5/2*x + 1/4*(9*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) + 1) - log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^2(x)}{1+\coth(x)}\,dx = \frac{5\,x}{2} - \ln\left(\mathrm{e}^{2\,x}+1\right) + \frac{\mathrm{e}^{-2\,x}}{4} + \frac{2}{\mathrm{e}^{2\,x}+1}$$

input
```
int(tanh(x)^2/(coth(x) + 1),x)
```

output
```
(5*x)/2 - log(exp(2*x) + 1) + exp(-2*x)/4 + 2/(exp(2*x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.52

$$\int \frac{\tanh^2(x)}{1+\coth(x)}\,dx = \frac{-4e^{4x}\log(e^{2x}+1)+10e^{4x}x-9e^{4x}-4e^{2x}\log(e^{2x}+1)+10e^{2x}x+1}{4e^{2x}\left(e^{2x}+1\right)}$$

input
```
int(tanh(x)^2/(1+coth(x)),x)
```

output
```
( - 4*e**(4*x)*log(e**(2*x) + 1) + 10*e**(4*x)*x - 9*e**(4*x) - 4*e**(2*x)
*log(e**(2*x) + 1) + 10*e**(2*x)*x + 1)/(4*e**(2*x)*(e**(2*x) + 1))
```

3.131 $\int \frac{\tanh(x)}{1+\coth(x)} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 19

$$\int \frac{\tanh(x)}{1+\coth(x)} \, dx = -\frac{x}{2} + \frac{1}{2(1+\coth(x))} + \log(\cosh(x))$$

output
```
-1/2*x+1/(2+2*coth(x))+ln(cosh(x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \frac{\tanh(x)}{1+\coth(x)} \, dx = \frac{1}{2(1+\coth(x))} - \frac{1}{4}\log(1-\tanh(x)) - \frac{3}{4}\log(1+\tanh(x))$$

input
```
Integrate[Tanh[x]/(1 + Coth[x]),x]
```

output
```
1/(2*(1 + Coth[x])) - Log[1 - Tanh[x]]/4 - (3*Log[1 + Tanh[x]])/4
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 26, 4034, 26, 3042, 26, 3956, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\coth(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\left(1-i\tan\left(ix+\frac{\pi}{2}\right)\right)\tan\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4034}$$

$$i\left(\int -i\tanh(x)dx+i\int \frac{1}{\coth(x)+1}dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(i\int \frac{1}{\coth(x)+1}dx-i\int \tanh(x)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(i\int \frac{1}{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx-i\int -i\tan(ix)dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(i\int \frac{1}{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx-\int \tan(ix)dx\right)$$

$$\Big\downarrow \text{3956}$$

$$i\left(i\int \frac{1}{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx-i\log(\cosh(x))\right)$$

$$\Big\downarrow \text{3960}$$

$$i\left(i\left(\frac{\int 1dx}{2}-\frac{1}{2(\coth(x)+1)}\right)-i\log(\cosh(x))\right)$$

$$\Big\downarrow \text{24}$$

$$i\left(i\left(\frac{x}{2}-\frac{1}{2(\coth(x)+1)}\right)-i\log(\cosh(x))\right)$$

input
```
Int[Tanh[x]/(1 + Coth[x]),x]
```

output
```
I*(I*(x/2 - 1/(2*(1 + Coth[x]))) - I*Log[Cosh[x]])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 4034
```
Int[1/(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Tan[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2
 + d^2, 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
risch	$-\frac{3x}{2}-\frac{e^{-2x}}{4}+\ln\left(e^{2x}+1\right)$	18
parallelrisch	$\frac{(-2\tanh(x)-2)\ln(1-\tanh(x))-3\tanh(x)x-3x-1}{2\tanh(x)+2}$	34
default	$-\frac{1}{\left(\tanh(\frac{x}{2})+1\right)^2}+\frac{1}{\tanh(\frac{x}{2})+1}-\frac{3\ln(\tanh(\frac{x}{2})+1)}{2}+\ln\left(\tanh\left(\frac{x}{2}\right)^2+1\right)-\frac{\ln(\tanh(\frac{x}{2})-1)}{2}$	47

input
```
int(tanh(x)/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
-3/2*x-1/4*exp(-2*x)+ln(exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(15) = 30$.

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.84

$$\int \frac{\tanh(x)}{1+\coth(x)}\,dx =$$
$$-\frac{6\,x\cosh\left(x\right)^2+12\,x\cosh\left(x\right)\sinh\left(x\right)+6\,x\sinh\left(x\right)^2-4\left(\cosh\left(x\right)^2+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^2\right)\mathrm{lo}}{4\left(\cosh\left(x\right)^2+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^2\right)}$$

input
```
integrate(tanh(x)/(1+coth(x)),x, algorithm="fricas")
```

output
```
-1/4*(6*x*cosh(x)^2 + 12*x*cosh(x)*sinh(x) + 6*x*sinh(x)^2 - 4*(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 1)/(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [F]

$$\int \frac{\tanh(x)}{1+\coth(x)}\,dx = \int \frac{\tanh{(x)}}{\coth{(x)}+1}\,dx$$

input
```
integrate(tanh(x)/(1+coth(x)),x)
```

output
```
Integral(tanh(x)/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\tanh(x)}{1+\coth(x)}\,dx = \frac{1}{2}\,x - \frac{1}{4}\,e^{(-2\,x)} + \log\left(e^{(-2\,x)}+1\right)$$

input
```
integrate(tanh(x)/(1+coth(x)),x, algorithm="maxima")
```

output
```
1/2*x - 1/4*e^(-2*x) + log(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\tanh(x)}{1+\coth(x)}\,dx = -\frac{3}{2}\,x - \frac{1}{4}\,e^{(-2\,x)} + \log\left(e^{(2\,x)}+1\right)$$

input
```
integrate(tanh(x)/(1+coth(x)),x, algorithm="giac")
```

output
```
-3/2*x - 1/4*e^(-2*x) + log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\tanh(x)}{1+\coth(x)}\,dx = \ln\left(\mathrm{e}^{2\,x}+1\right)-\frac{3\,x}{2}-\frac{\mathrm{e}^{-2\,x}}{4}$$

input
```
int(tanh(x)/(coth(x) + 1),x)
```

output
```
log(exp(2*x) + 1) - (3*x)/2 - exp(-2*x)/4
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{\tanh(x)}{1+\coth(x)}\,dx = \frac{4e^{2x}\log(e^{2x}+1)-6e^{2x}x-1}{4e^{2x}}$$

input
```
int(tanh(x)/(1+coth(x)),x)
```

output
```
(4*e**(2*x)*log(e**(2*x) + 1) - 6*e**(2*x)*x - 1)/(4*e**(2*x))
```

3.132 $\int \frac{1}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 16

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{x}{2} - \frac{1}{2(1+\coth(x))}$$

output
```
1/2*x-1/(2+2*coth(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{1}{2}\left(\operatorname{arctanh}(\tanh(x)) + \frac{1}{1+\tanh(x)}\right)$$

input
```
Integrate[(1 + Coth[x])^(-1),x]
```

output
```
(ArcTanh[Tanh[x]] + (1 + Tanh[x])^(-1))/2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\coth(x)+1}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{1-i\tan\left(\frac{\pi}{2}+ix\right)}dx$$
$$\Big\downarrow \text{ 3960}$$
$$\frac{\int 1dx}{2}-\frac{1}{2(\coth(x)+1)}$$
$$\Big\downarrow \text{ 24}$$
$$\frac{x}{2}-\frac{1}{2(\coth(x)+1)}$$

input
```
Int[(1 + Coth[x])^(-1),x]
```

output
```
x/2 - 1/(2*(1 + Coth[x]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{x}{2}+\frac{{\mathrm e}^{-2 x}}{4}$	11
parallelrisch	$\frac{x}{2}+\frac{\cosh \left(2 x \right)}{4}+\frac{1}{4}-\frac{\sinh \left(2 x \right)}{4}$	18
derivativedivides	$-\frac{\ln \left(\coth \left(x \right)-1\right)}{4}-\frac{1}{2 \left(1+\coth \left(x \right)\right)}+\frac{\ln \left(1+\coth \left(x \right)\right)}{4}$	24
default	$-\frac{\ln \left(\coth \left(x \right)-1\right)}{4}-\frac{1}{2 \left(1+\coth \left(x \right)\right)}+\frac{\ln \left(1+\coth \left(x \right)\right)}{4}$	24

input
```
int(1/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/4*exp(-2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{1}{1+\coth (x)} \, dx = \frac{{\left(2 \, x + 1\right)} \cosh \left(x\right) + {\left(2 \, x - 1\right)} \sinh \left(x\right)}{4 \, {\left(\cosh \left(x\right) + \sinh \left(x\right)\right)}}$$

input
```
integrate(1/(1+coth(x)),x, algorithm="fricas")
```

output
```
1/4*((2*x + 1)*cosh(x) + (2*x - 1)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{x\tanh{(x)}}{2\tanh{(x)}+2} + \frac{x}{2\tanh{(x)}+2} + \frac{1}{2\tanh{(x)}+2}$$

input
```
integrate(1/(1+coth(x)),x)
```

output
```
x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) + 1/(2*tanh(x) + 2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{1}{2}\,x + \frac{1}{4}\,e^{(-2\,x)}$$

input
```
integrate(1/(1+coth(x)),x, algorithm="maxima")
```

output
```
1/2*x + 1/4*e^(-2*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{1}{2}\,x + \frac{1}{4}\,e^{(-2\,x)}$$

input
```
integrate(1/(1+coth(x)),x, algorithm="giac")
```

output
```
1/2*x + 1/4*e^(-2*x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{x}{2} - \frac{1}{2\,(\coth(x)+1)}$$

input
```
int(1/(coth(x) + 1),x)
```

output
```
x/2 - 1/(2*(coth(x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{2e^{2x}x+1}{4e^{2x}}$$

input
```
int(1/(1+coth(x)),x)
```

output
```
(2*e**(2*x)*x + 1)/(4*e**(2*x))
```

3.133 $\int \frac{\coth(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 16

$$\int \frac{\coth(x)}{1+\coth(x)}\,dx = \frac{x}{2}+\frac{1}{2(1+\coth(x))}$$

output
```
1/2*x+1/(2+2*coth(x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth(x)}{1+\coth(x)}\,dx = \frac{1}{2}\text{arctanh}(\tanh(x))-\frac{1}{2(1+\tanh(x))}$$

input
```
Integrate[Coth[x]/(1 + Coth[x]),x]
```

output
```
ArcTanh[Tanh[x]]/2 - 1/(2*(1 + Tanh[x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 26, 4009, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{\coth(x)+1} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan\left(\frac{\pi}{2}+ix\right)}{1-i\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan\left(ix+\frac{\pi}{2}\right)}{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4009}$$

$$-i\left(\frac{i\int 1dx}{2}+\frac{i}{2(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{24}$$

$$-i\left(\frac{ix}{2}+\frac{i}{2(\coth(x)+1)}\right)$$

input
```
Int[Coth[x]/(1 + Coth[x]),x]
```

output
```
(-I)*((I/2)*x + (I/2)/(1 + Coth[x]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{x}{2}-\frac{{\mathrm e}^{-2 x}}{4}$	11
parallelrisch	$\frac{x}{2}-\frac{\cosh \left(2 x \right)}{4}-\frac{1}{4}+\frac{\sinh \left(2 x \right)}{4}$	18
derivativedivides	$-\frac{\ln \left(\coth \left(x \right)-1\right)}{4}+\frac{1}{2+2 \coth \left(x \right)}+\frac{\ln \left(1+\coth \left(x \right)\right)}{4}$	24
default	$-\frac{\ln \left(\coth \left(x \right)-1\right)}{4}+\frac{1}{2+2 \coth \left(x \right)}+\frac{\ln \left(1+\coth \left(x \right)\right)}{4}$	24

input
```
int(coth(x)/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x-1/4*exp(-2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{\coth(x)}{1+\coth(x)}\,dx = \frac{(2\,x-1)\cosh\left(x\right)+(2\,x+1)\sinh\left(x\right)}{4\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

input
```
integrate(coth(x)/(1+coth(x)),x, algorithm="fricas")
```

output
```
1/4*((2*x - 1)*cosh(x) + (2*x + 1)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \frac{\coth(x)}{1+\coth(x)}\,dx = \frac{x\tanh\left(x\right)}{2\tanh\left(x\right)+2}+\frac{x}{2\tanh\left(x\right)+2}-\frac{1}{2\tanh\left(x\right)+2}$$

input
```
integrate(coth(x)/(1+coth(x)),x)
```

output
```
x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) - 1/(2*tanh(x) + 2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\coth(x)}{1+\coth(x)}\,dx = \frac{1}{2}\,x-\frac{1}{4}\,e^{(-2\,x)}$$

input
```
integrate(coth(x)/(1+coth(x)),x, algorithm="maxima")
```

output
```
1/2*x - 1/4*e^(-2*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\coth(x)}{1+\coth(x)}\,dx = \frac{1}{2}\,x - \frac{1}{4}\,e^{(-2\,x)}$$

input
```
integrate(coth(x)/(1+coth(x)),x, algorithm="giac")
```

output
```
1/2*x - 1/4*e^(-2*x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\coth(x)}{1+\coth(x)}\,dx = \frac{x}{2} + \frac{1}{2\,(\coth{(x)}+1)}$$

input
```
int(coth(x)/(coth(x) + 1),x)
```

output
```
x/2 + 1/(2*(coth(x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{\coth(x)}{1+\coth(x)}\,dx = \frac{2e^{2x}x-1}{4e^{2x}}$$

input
```
int(coth(x)/(1+coth(x)),x)
```

output
```
(2*e**(2*x)*x - 1)/(4*e**(2*x))
```

3.134 $\int \frac{\coth^2(x)}{1+\coth(x)} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{\coth^2(x)}{1+\coth(x)} \, dx = -\frac{x}{2} - \frac{1}{2(1+\coth(x))} + \log(\sinh(x))$$

output
```
-1/2*x-1/(2+2*coth(x))+ln(sinh(x))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.47

$$\begin{aligned}\int \frac{\coth^2(x)}{1+\coth(x)} \, dx = &-\frac{1}{2}\coth^2(x) + \frac{\coth^3(x)}{2(1+\coth(x))} \\ &+ \frac{1}{2}\coth(x)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(x)\right) \\ &+ \log(\cosh(x)) + \log(\tanh(x))\end{aligned}$$

input
```
Integrate[Coth[x]^2/(1 + Coth[x]),x]
```

output
```
-1/2*Coth[x]^2 + Coth[x]^3/(2*(1 + Coth[x])) + (Coth[x]*Hypergeometric2F1[
-1/2, 1, 1/2, Tanh[x]^2])/2 + Log[Cosh[x]] + Log[Tanh[x]]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 25, 4023, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{\coth(x)+1}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{\tan\left(\frac{\pi}{2}+ix\right)^2}{1-i\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\tan\left(ix+\frac{\pi}{2}\right)^2}{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 4023$$

$$-\frac{1}{2}\int (1-2\coth(x))dx-\frac{1}{2(\coth(x)+1)}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}(2\log(\sinh(x))-x)-\frac{1}{2(\coth(x)+1)}$$

input
```
Int[Coth[x]^2/(1 + Coth[x]),x]
```

output
```
-1/2*1/(1 + Coth[x]) + (-x + 2*Log[Sinh[x]])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4023
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(a*c + b*d)^2*((a + b*Tan[e + f*x])^
m/(2*a^3*f*m)), x] + Simp[1/(2*a^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp
[a*c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
risch	$-\frac{3x}{2}+\frac{{\mathrm e}^{-2x}}{4}+\ln\left({\mathrm e}^{2x}-1\right)$	18
derivativedivides	$-\frac{\ln(\coth(x)-1)}{4}-\frac{1}{2(1+\coth(x))}-\frac{3\ln(1+\coth(x))}{4}$	24
default	$-\frac{\ln(\coth(x)-1)}{4}-\frac{1}{2(1+\coth(x))}-\frac{3\ln(1+\coth(x))}{4}$	24
parallelrisch	$\frac{(-2\tanh(x)-2)\ln(1-\tanh(x))+(2\tanh(x)+2)\ln(\tanh(x))-3\tanh(x)x-3x+1}{2\tanh(x)+2}$	44

input
```
int(coth(x)^2/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
-3/2*x+1/4*exp(-2*x)+ln(exp(2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(15) = 30$.

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.84

$$\int \frac{\coth^2(x)}{1+\coth(x)}\,dx = -\frac{6\,x\cosh(x)^2+12\,x\cosh(x)\sinh(x)+6\,x\sinh(x)^2-4\left(\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x)^2\right)\mathrm{lo}}{4\left(\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x)^2\right)}$$

input
```
integrate(coth(x)^2/(1+coth(x)),x, algorithm="fricas")
```

output
```
-1/4*(6*x*cosh(x)^2 + 12*x*cosh(x)*sinh(x) + 6*x*sinh(x)^2 - 4*(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2)*log(2*sinh(x)/(cosh(x) - sinh(x))) - 1)/(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(15) = 30$.

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.84

$$\int \frac{\coth^2(x)}{1+\coth(x)}\,dx = \frac{x\tanh(x)}{2\tanh(x)+2}+\frac{x}{2\tanh(x)+2}-\frac{2\log(\tanh(x)+1)\tanh(x)}{2\tanh(x)+2}-\frac{2\log(\tanh(x)+1)}{2\tanh(x)+2}+\frac{2\log(\tanh(x))\tanh(x)}{2\tanh(x)+2}+\frac{2\log(\tanh(x))}{2\tanh(x)+2}+\frac{1}{2\tanh(x)+2}$$

input
```
integrate(coth(x)**2/(1+coth(x)),x)
```

output
```
x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) - 2*log(tanh(x) + 1)*tanh(x)
/(2*tanh(x) + 2) - 2*log(tanh(x) + 1)/(2*tanh(x) + 2) + 2*log(tanh(x))*tan
h(x)/(2*tanh(x) + 2) + 2*log(tanh(x))/(2*tanh(x) + 2) + 1/(2*tanh(x) + 2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{\coth^2(x)}{1+\coth(x)}\,dx = \frac{1}{2}\,x + \frac{1}{4}\,e^{(-2\,x)} + \log\left(e^{(-x)}+1\right) + \log\left(e^{(-x)}-1\right)$$

input
```
integrate(coth(x)^2/(1+coth(x)),x, algorithm="maxima")
```

output
```
1/2*x + 1/4*e^(-2*x) + log(e^(-x) + 1) + log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\coth^2(x)}{1+\coth(x)}\,dx = -\frac{3}{2}\,x + \frac{1}{4}\,e^{(-2\,x)} + \log\left(\left|e^{(2\,x)}-1\right|\right)$$

input
```
integrate(coth(x)^2/(1+coth(x)),x, algorithm="giac")
```

output
```
-3/2*x + 1/4*e^(-2*x) + log(abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\coth^2(x)}{1+\coth(x)}\,dx = \frac{x}{2} - \ln\left(\coth(x)+1\right) - \frac{1}{2\,\left(\coth\left(x\right)+1\right)}$$

input
```
int(coth(x)^2/(coth(x) + 1),x)
```

output
```
x/2 - log(coth(x) + 1) - 1/(2*(coth(x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \frac{\coth^2(x)}{1+\coth(x)}\,dx = \frac{-4e^{2x}\log(\coth(x)+1)+2e^{2x}x+1}{4e^{2x}}$$

input
```
int(coth(x)^2/(1+coth(x)),x)
```

output
```
( - 4*e**(2*x)*log(coth(x) + 1) + 2*e**(2*x)*x + 1)/(4*e**(2*x))
```

3.135 $\int \frac{\coth^3(x)}{1+\coth(x)} dx$

Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{\coth^3(x)}{1+\coth(x)} dx = \frac{3x}{2} - \frac{3\coth(x)}{2} + \frac{\coth^2(x)}{2(1+\coth(x))} - \log(\sinh(x))$$

output
```
3/2*x-3/2*coth(x)+coth(x)^2/(2+2*coth(x))-ln(sinh(x))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \frac{\coth^3(x)}{1+\coth(x)} dx = \frac{1}{2}\left(\coth^2(x) + \frac{\coth^4(x)}{1+\coth(x)} - \coth^3(x)\,\text{Hypergeometric2F1}\left(-\frac{3}{2}, 1, -\frac{1}{2}, \tanh^2(x)\right) - 2(\log(\cosh(x)) + \log(\tanh(x)))\right)$$

input
```
Integrate[Coth[x]^3/(1 + Coth[x]),x]
```

output
```
(Coth[x]^2 + Coth[x]^4/(1 + Coth[x]) - Coth[x]^3*Hypergeometric2F1[-3/2, 1
, -1/2, Tanh[x]^2] - 2*(Log[Cosh[x]] + Log[Tanh[x]]))/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 26, 4033, 26, 3042, 26, 4008, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(x)}{\coth(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan\left(\frac{\pi}{2}+ix\right)^3}{1-i\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\tan\left(ix+\frac{\pi}{2}\right)^3}{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4033}$$

$$i\left(\frac{1}{2}\int i(2-3\coth(x))\coth(x)dx-\frac{i\coth^2(x)}{2(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{1}{2}i\int (2-3\coth(x))\coth(x)dx-\frac{i\coth^2(x)}{2(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{1}{2}i\int -i\left(3i\tan\left(ix+\frac{\pi}{2}\right)+2\right)\tan\left(ix+\frac{\pi}{2}\right)dx-\frac{i\coth^2(x)}{2(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{1}{2}\int \left(3i\tan\left(ix+\frac{\pi}{2}\right)+2\right)\tan\left(ix+\frac{\pi}{2}\right)dx-\frac{i\coth^2(x)}{2(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{4008}$$

$$i\left(\frac{1}{2}(2\int i\coth(x)dx-3ix+3i\coth(x))-\frac{i\coth^2(x)}{2(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{1}{2}(2i\int \coth(x)dx-3ix+3i\coth(x))-\frac{i\coth^2(x)}{2(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{1}{2}\left(2i\int -i\tan\left(ix+\frac{\pi}{2}\right)dx-3ix+3i\coth(x)\right)-\frac{i\coth^2(x)}{2(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{1}{2}\left(2\int \tan\left(ix+\frac{\pi}{2}\right)dx-3ix+3i\coth(x)\right)-\frac{i\coth^2(x)}{2(\coth(x)+1)}\right)$$

$$\Big\downarrow \text{3956}$$

$$i\left(\frac{1}{2}(-3ix+3i\coth(x)+2i\log(\sinh(x)))-\frac{i\coth^2(x)}{2(\coth(x)+1)}\right)$$

input
```
Int[Coth[x]^3/(1 + Coth[x]),x]
```

output
```
I*(((-1/2*I)*Coth[x]^2)/(1 + Coth[x]) + ((-3*I)*x + (3*I)*Coth[x] + (2*I)*
Log[Sinh[x]])/2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4033
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(
2*a*f*(a + b*Tan[e + f*x]))), x] + Simp[1/(2*a^2)   Int[(c + d*Tan[e + f*x]
)^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-\coth\left(x\right)-\frac{\ln\left(\coth\left(x\right)-1\right)}{4}+\frac{1}{2+2\coth\left(x\right)}+\frac{5\ln\left(1+\coth\left(x\right)\right)}{4}$	28
default	$-\coth\left(x\right)-\frac{\ln\left(\coth\left(x\right)-1\right)}{4}+\frac{1}{2+2\coth\left(x\right)}+\frac{5\ln\left(1+\coth\left(x\right)\right)}{4}$	28
risch	$\frac{5x}{2}-\frac{{\mathrm e}^{-2x}}{4}-\frac{2}{{\mathrm e}^{2x}-1}-\ln\left({\mathrm e}^{2x}-1\right)$	30
parallelrisch	$\frac{\left(2\tanh\left(x\right)+2\right)\ln\left(1-\tanh\left(x\right)\right)+\left(-2\tanh\left(x\right)-2\right)\ln\left(\tanh\left(x\right)\right)+5\tanh\left(x\right)x+5x-2\coth\left(x\right)-3}{2\tanh\left(x\right)+2}$	48

input
```
int(coth(x)^3/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
-coth(x)-1/4*ln(coth(x)-1)+1/2/(1+coth(x))+5/4*ln(1+coth(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(25) = 50$.

Time = 0.09 (sec) , antiderivative size = 196, normalized size of antiderivative = 6.32

$$\int \frac{\coth^3(x)}{1+\coth(x)}\,dx = \frac{10\,x\cosh\left(x\right)^4 + 40\,x\cosh\left(x\right)\sinh\left(x\right)^3 + 10\,x\sinh\left(x\right)^4 - \left(10\,x+9\right)\cosh\left(x\right)^2 + \left(60\,x\cosh\left(x\right)^2 - 10\,x -}{4\left(\cosh\right.}$$

input
```
integrate(coth(x)^3/(1+coth(x)),x, algorithm="fricas")
```

output
```
1/4*(10*x*cosh(x)^4 + 40*x*cosh(x)*sinh(x)^3 + 10*x*sinh(x)^4 - (10*x + 9)
*cosh(x)^2 + (60*x*cosh(x)^2 - 10*x - 9)*sinh(x)^2 - 4*(cosh(x)^4 + 4*cosh
(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2
*cosh(x)^3 - cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) + 2*(20*
x*cosh(x)^3 - (10*x + 9)*cosh(x))*sinh(x) + 1)/(cosh(x)^4 + 4*cosh(x)*sinh
(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)
^3 - cosh(x))*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(27) = 54$.

Time = 0.51 (sec) , antiderivative size = 160, normalized size of antiderivative = 5.16

$$\begin{aligned}\int \frac{\coth^3(x)}{1+\coth(x)}\,dx &= \frac{x\tanh^2\left(x\right)}{2\tanh^2\left(x\right)+2\tanh\left(x\right)} + \frac{x\tanh\left(x\right)}{2\tanh^2\left(x\right)+2\tanh\left(x\right)} \\ &+ \frac{2\log\left(\tanh\left(x\right)+1\right)\tanh^2\left(x\right)}{2\tanh^2\left(x\right)+2\tanh\left(x\right)} + \frac{2\log\left(\tanh\left(x\right)+1\right)\tanh\left(x\right)}{2\tanh^2\left(x\right)+2\tanh\left(x\right)} \\ &- \frac{2\log\left(\tanh\left(x\right)\right)\tanh^2\left(x\right)}{2\tanh^2\left(x\right)+2\tanh\left(x\right)} - \frac{2\log\left(\tanh\left(x\right)\right)\tanh\left(x\right)}{2\tanh^2\left(x\right)+2\tanh\left(x\right)} \\ &- \frac{3\tanh\left(x\right)}{2\tanh^2\left(x\right)+2\tanh\left(x\right)} - \frac{2}{2\tanh^2\left(x\right)+2\tanh\left(x\right)}\end{aligned}$$

input
```
integrate(coth(x)**3/(1+coth(x)),x)
```

output
```
x*tanh(x)**2/(2*tanh(x)**2 + 2*tanh(x)) + x*tanh(x)/(2*tanh(x)**2 + 2*tanh
(x)) + 2*log(tanh(x) + 1)*tanh(x)**2/(2*tanh(x)**2 + 2*tanh(x)) + 2*log(ta
nh(x) + 1)*tanh(x)/(2*tanh(x)**2 + 2*tanh(x)) - 2*log(tanh(x))*tanh(x)**2/
(2*tanh(x)**2 + 2*tanh(x)) - 2*log(tanh(x))*tanh(x)/(2*tanh(x)**2 + 2*tanh
(x)) - 3*tanh(x)/(2*tanh(x)**2 + 2*tanh(x)) - 2/(2*tanh(x)**2 + 2*tanh(x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{\coth^3(x)}{1+\coth(x)}\,dx = \frac{1}{2}\,x + \frac{2}{e^{(-2\,x)}-1} - \frac{1}{4}\,e^{(-2\,x)} - \log\left(e^{(-x)}+1\right) - \log\left(e^{(-x)}-1\right)$$

input
```
integrate(coth(x)^3/(1+coth(x)),x, algorithm="maxima")
```

output
```
1/2*x + 2/(e^(-2*x) - 1) - 1/4*e^(-2*x) - log(e^(-x) + 1) - log(e^(-x) - 1
)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{\coth^3(x)}{1+\coth(x)}\,dx = \frac{5}{2}\,x - \frac{\left(9\,e^{(2\,x)}-1\right)e^{(-2\,x)}}{4\,\left(e^{(2\,x)}-1\right)} - \log\left(\left|e^{(2\,x)}-1\right|\right)$$

input
```
integrate(coth(x)^3/(1+coth(x)),x, algorithm="giac")
```

output
```
5/2*x - 1/4*(9*e^(2*x) - 1)*e^(-2*x)/(e^(2*x) - 1) - log(abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{\coth^3(x)}{1+\coth(x)}\,dx=\frac{x}{2}+\ln\left(\coth(x)+1\right)-\coth(x)+\frac{1}{2\,\left(\coth\left(x\right)+1\right)}$$

input
```
int(coth(x)^3/(coth(x) + 1),x)
```

output
```
x/2 + log(coth(x) + 1) - coth(x) + 1/(2*(coth(x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{\coth^3(x)}{1+\coth(x)}\,dx=\frac{-4e^{2x}\coth\left(x\right)+4e^{2x}\log(\coth\left(x\right)+1)+2e^{2x}x-1}{4e^{2x}}$$

input
```
int(coth(x)^3/(1+coth(x)),x)
```

output
```
( - 4*e**(2*x)*coth(x) + 4*e**(2*x)*log(coth(x) + 1) + 2*e**(2*x)*x - 1)/(
4*e**(2*x))
```

3.136 $\int \frac{\coth^4(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{\coth^4(x)}{1+\coth(x)}\,dx = -\frac{3x}{2}+\frac{3\coth(x)}{2}-\coth^2(x)+\frac{\coth^3(x)}{2(1+\coth(x))}+2\log(\sinh(x))$$

output
```
-3/2*x+3/2*coth(x)-coth(x)^2+coth(x)^3/(2+2*coth(x))+2*ln(sinh(x))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int \frac{\coth^4(x)}{1+\coth(x)}\,dx = \frac{1}{2}\left(-2\coth^2(x)-\coth^4(x)+\frac{\coth^5(x)}{1+\coth(x)}\right.$$
$$+\coth^3(x)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\tanh^2(x)\right)$$
$$\left.+4(\log(\cosh(x))+\log(\tanh(x)))\right)$$

input
```
Integrate[Coth[x]^4/(1 + Coth[x]),x]
```

output
```
(-2*Coth[x]^2 - Coth[x]^4 + Coth[x]^5/(1 + Coth[x]) + Coth[x]^3*Hypergeome
tric2F1[-3/2, 1, -1/2, Tanh[x]^2] + 4*(Log[Cosh[x]] + Log[Tanh[x]]))/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.364$, Rules used = {3042, 4033, 25, 3042, 25, 4011, 26, 26, 3042, 26, 4008, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(x)}{\coth(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan\left(\frac{\pi}{2}+ix\right)^4}{1-i\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{4033}$$

$$\frac{1}{2}\int -\left((3-4\coth(x))\coth^2(x)\right)dx+\frac{\coth^3(x)}{2(\coth(x)+1)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\coth^3(x)}{2(\coth(x)+1)}-\frac{1}{2}\int (3-4\coth(x))\coth^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\coth^3(x)}{2(\coth(x)+1)}-\frac{1}{2}\int -\left(\left(4i\tan\left(ix+\frac{\pi}{2}\right)+3\right)\tan\left(ix+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{25}$$

$$\frac{\coth^3(x)}{2(\coth(x)+1)}+\frac{1}{2}\int \left(4i\tan\left(ix+\frac{\pi}{2}\right)+3\right)\tan\left(ix+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{4011}$$

$$\frac{\coth^3(x)}{2(\coth(x)+1)}+\frac{1}{2}\left(-2\coth^2(x)+\int i(3i\coth(x)-4i)\coth(x)dx\right)$$

$$\Big\downarrow \; 26$$

$$\frac{\coth^3(x)}{2(\coth(x)+1)}+\frac{1}{2}\left(-2\coth^2(x)+i\int -i(4-3\coth(x))\coth(x)dx\right)$$

$$\Big\downarrow \; 26$$

$$\frac{1}{2}\left(\int (4-3\coth(x))\coth(x)dx-2\coth^2(x)\right)+\frac{\coth^3(x)}{2(\coth(x)+1)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\coth^3(x)}{2(\coth(x)+1)}+\frac{1}{2}\left(-2\coth^2(x)+\int -i\left(3i\tan\left(ix+\frac{\pi}{2}\right)+4\right)\tan\left(ix+\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \; 26$$

$$\frac{\coth^3(x)}{2(\coth(x)+1)}+\frac{1}{2}\left(-2\coth^2(x)-i\int \left(3i\tan\left(ix+\frac{\pi}{2}\right)+4\right)\tan\left(ix+\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \; 4008$$

$$\frac{\coth^3(x)}{2(\coth(x)+1)}+\frac{1}{2}\left(-2\coth^2(x)-i(4\int i\coth(x)dx-3ix+3i\coth(x))\right)$$

$$\Big\downarrow \; 26$$

$$\frac{\coth^3(x)}{2(\coth(x)+1)}+\frac{1}{2}\left(-2\coth^2(x)-i(4i\int \coth(x)dx-3ix+3i\coth(x))\right)$$

$$\Big\downarrow \; 3042$$

$$\frac{\coth^3(x)}{2(\coth(x)+1)}+\frac{1}{2}\left(-2\coth^2(x)-i\left(4i\int -i\tan\left(ix+\frac{\pi}{2}\right)dx-3ix+3i\coth(x)\right)\right)$$

$$\Big\downarrow \; 26$$

$$\frac{\coth^3(x)}{2(\coth(x)+1)}+\frac{1}{2}\left(-2\coth^2(x)-i\left(4\int \tan\left(ix+\frac{\pi}{2}\right)dx-3ix+3i\coth(x)\right)\right)$$

$$\Big\downarrow \; 3956$$

$$\frac{\coth^3(x)}{2(\coth(x)+1)}+\frac{1}{2}(-2\coth^2(x)-i(-3ix+3i\coth(x)+4i\log(\sinh(x))))$$

input
```
Int[Coth[x]^4/(1 + Coth[x]),x]
```

output
```
Coth[x]^3/(2*(1 + Coth[x])) + (-2*Coth[x]^2 - I*((-3*I)*x + (3*I)*Coth[x]
+ (4*I)*Log[Sinh[x]]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4033
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(
2*a*f*(a + b*Tan[e + f*x]))), x] + Simp[1/(2*a^2)   Int[(c + d*Tan[e + f*x]
)^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
risch	$-\frac{7x}{2}+\frac{{\mathrm e}^{-2x}}{4}-\frac{2}{\left({\mathrm e}^{2x}-1\right)^{2}}+2\ln\left({\mathrm e}^{2x}-1\right)$	30
derivativedivides	$\coth\left(x\right)-\frac{\coth\left(x\right)^{2}}{2}-\frac{\ln\left(\coth\left(x\right)-1\right)}{4}-\frac{1}{2\left(1+\coth\left(x\right)\right)}-\frac{7\ln\left(1+\coth\left(x\right)\right)}{4}$	32
default	$\coth\left(x\right)-\frac{\coth\left(x\right)^{2}}{2}-\frac{\ln\left(\coth\left(x\right)-1\right)}{4}-\frac{1}{2\left(1+\coth\left(x\right)\right)}-\frac{7\ln\left(1+\coth\left(x\right)\right)}{4}$	32
parallelrisch	$\frac{\left(-4\tanh\left(x\right)-4\right)\ln\left(1-\tanh\left(x\right)\right)+\left(4\tanh\left(x\right)+4\right)\ln\left(\tanh\left(x\right)\right)-7\tanh\left(x\right)x-\coth\left(x\right)^{2}-7x+\coth\left(x\right)+3}{2\tanh\left(x\right)+2}$	52

input
```
int(coth(x)^4/(1+coth(x)),x,method=_RETURNVERBOSE)
```

output
```
-7/2*x+1/4*exp(-2*x)-2/(exp(2*x)-1)^2+2*ln(exp(2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(31) = 62$.

Time = 0.09 (sec) , antiderivative size = 357, normalized size of antiderivative = 9.65

$$\int \frac{\coth^4(x)}{1+\coth(x)}\,dx=\text{Too large to display}$$

input
```
integrate(coth(x)^4/(1+coth(x)),x, algorithm="fricas")
```

output
```
-1/4*(14*x*cosh(x)^6 + 84*x*cosh(x)*sinh(x)^5 + 14*x*sinh(x)^6 - (28*x + 1
)*cosh(x)^4 + (210*x*cosh(x)^2 - 28*x - 1)*sinh(x)^4 + 4*(70*x*cosh(x)^3 -
 (28*x + 1)*cosh(x))*sinh(x)^3 + 2*(7*x + 5)*cosh(x)^2 + 2*(105*x*cosh(x)^
4 - 3*(28*x + 1)*cosh(x)^2 + 7*x + 5)*sinh(x)^2 - 8*(cosh(x)^6 + 6*cosh(x)
*sinh(x)^5 + sinh(x)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5
*cosh(x)^3 - 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh
(x)^2 + cosh(x)^2 + 2*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*log(2
*sinh(x)/(cosh(x) - sinh(x))) + 4*(21*x*cosh(x)^5 - (28*x + 1)*cosh(x)^3 +
 (7*x + 5)*cosh(x))*sinh(x) - 1)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x
)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh
(x))*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 +
 2*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(34) = 68$.

Time = 0.63 (sec) , antiderivative size = 197, normalized size of antiderivative = 5.32

$$\begin{aligned}\int \frac{\coth^4(x)}{1+\coth(x)}\,dx = {} & \frac{x\tanh^3(x)}{2\tanh^3(x)+2\tanh^2(x)} + \frac{x\tanh^2(x)}{2\tanh^3(x)+2\tanh^2(x)} \\ & - \frac{4\log(\tanh(x)+1)\tanh^3(x)}{2\tanh^3(x)+2\tanh^2(x)} \\ & - \frac{4\log(\tanh(x)+1)\tanh^2(x)}{2\tanh^3(x)+2\tanh^2(x)} + \frac{4\log(\tanh(x))\tanh^3(x)}{2\tanh^3(x)+2\tanh^2(x)} \\ & + \frac{4\log(\tanh(x))\tanh^2(x)}{2\tanh^3(x)+2\tanh^2(x)} + \frac{3\tanh^2(x)}{2\tanh^3(x)+2\tanh^2(x)} \\ & + \frac{\tanh(x)}{2\tanh^3(x)+2\tanh^2(x)} - \frac{1}{2\tanh^3(x)+2\tanh^2(x)}\end{aligned}$$

input
```
integrate(coth(x)**4/(1+coth(x)),x)
```

output
```
x*tanh(x)**3/(2*tanh(x)**3 + 2*tanh(x)**2) + x*tanh(x)**2/(2*tanh(x)**3 +
2*tanh(x)**2) - 4*log(tanh(x) + 1)*tanh(x)**3/(2*tanh(x)**3 + 2*tanh(x)**2
) - 4*log(tanh(x) + 1)*tanh(x)**2/(2*tanh(x)**3 + 2*tanh(x)**2) + 4*log(ta
nh(x))*tanh(x)**3/(2*tanh(x)**3 + 2*tanh(x)**2) + 4*log(tanh(x))*tanh(x)**
2/(2*tanh(x)**3 + 2*tanh(x)**2) + 3*tanh(x)**2/(2*tanh(x)**3 + 2*tanh(x)**
2) + tanh(x)/(2*tanh(x)**3 + 2*tanh(x)**2) - 1/(2*tanh(x)**3 + 2*tanh(x)**
2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int \frac{\coth^4(x)}{1+\coth(x)}\,dx = \frac{1}{2}\,x + \frac{2\left(2\,e^{(-2\,x)} - 1\right)}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} + \frac{1}{4}\,e^{(-2\,x)} + 2\,\log\left(e^{(-x)} + 1\right) + 2\,\log\left(e^{(-x)} - 1\right)$$

input
```
integrate(coth(x)^4/(1+coth(x)),x, algorithm="maxima")
```

output
```
1/2*x + 2*(2*e^(-2*x) - 1)/(2*e^(-2*x) - e^(-4*x) - 1) + 1/4*e^(-2*x) + 2*
log(e^(-x) + 1) + 2*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{\coth^4(x)}{1+\coth(x)}\,dx = -\frac{7}{2}\,x + \frac{\left(e^{(4\,x)} - 10\,e^{(2\,x)} + 1\right)e^{(-2\,x)}}{4\,\left(e^{(2\,x)} - 1\right)^2} + 2\,\log\left(\left|e^{(2\,x)} - 1\right|\right)$$

input
```
integrate(coth(x)^4/(1+coth(x)),x, algorithm="giac")
```

output
```
-7/2*x + 1/4*(e^(4*x) - 10*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) - 1)^2 + 2*log(a
bs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\coth^4(x)}{1+\coth(x)}\,dx = \frac{x}{2} - 2\,\ln\left(\coth(x)+1\right) + \coth(x) - \frac{\coth(x)^2}{2} - \frac{1}{2\,\left(\coth\left(x\right)+1\right)}$$

input
```
int(coth(x)^4/(coth(x) + 1),x)
```

output
```
x/2 - 2*log(coth(x) + 1) + coth(x) - coth(x)^2/2 - 1/(2*(coth(x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

$$\int \frac{\coth^4(x)}{1+\coth(x)}\,dx = \frac{-2e^{2x}\coth\left(x\right)^2 + 4e^{2x}\coth\left(x\right) - 8e^{2x}\log(\coth\left(x\right)+1) + 2e^{2x}x + 1}{4e^{2x}}$$

input
```
int(coth(x)^4/(1+coth(x)),x)
```

output
```
( - 2*e**(2*x)*coth(x)**2 + 4*e**(2*x)*coth(x) - 8*e**(2*x)*log(coth(x) +
1) + 2*e**(2*x)*x + 1)/(4*e**(2*x))
```

3.137 $\int \coth(x)(1+\coth(x))^{3/2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 45

$$\int \coth(x)(1+\coth(x))^{3/2}\,dx = 2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\coth(x)} - \frac{2}{3}(1+\coth(x))^{3/2}$$

output
```
2*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-2*(1+coth(x))^(1/2)-2/3*(
1+coth(x))^(3/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \coth(x)(1+\coth(x))^{3/2}\,dx = 2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - \frac{2}{3}\sqrt{1+\coth(x)}(4+\coth(x))$$

input
```
Integrate[Coth[x]*(1 + Coth[x])^(3/2),x]
```

output
```
2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - (2*Sqrt[1 + Coth[x]]*(4 + C
oth[x]))/3
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {3042, 26, 4010, 3042, 3959, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(x)(\coth(x)+1)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)^{3/2}\tan\left(\frac{\pi}{2}+ix\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \left(1-i\tan\left(ix+\frac{\pi}{2}\right)\right)^{3/2}\tan\left(ix+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4010}$$

$$-i\left(i\int (\coth(x)+1)^{3/2}dx-\frac{2}{3}i(\coth(x)+1)^{3/2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(i\int \left(1-i\tan\left(ix+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{2}{3}i(\coth(x)+1)^{3/2}\right)$$

$$\Big\downarrow \text{3959}$$

$$-i\left(i\left(2\int \sqrt{\coth(x)+1}dx-2\sqrt{\coth(x)+1}\right)-\frac{2}{3}i(\coth(x)+1)^{3/2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(i\left(-2\sqrt{\coth(x)+1}+2\int \sqrt{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx\right)-\frac{2}{3}i(\coth(x)+1)^{3/2}\right)$$

$$\Big\downarrow \text{3961}$$

$$-i\left(i\left(4\int \frac{1}{1-\coth(x)}d\sqrt{\coth(x)+1}-2\sqrt{\coth(x)+1}\right)-\frac{2}{3}i(\coth(x)+1)^{3/2}\right)$$

$$\Big\downarrow \text{219}$$

$$-i\left(i\left(2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)-2\sqrt{\coth(x)+1}\right)-\frac{2}{3}i(\coth(x)+1)^{3/2}\right)$$

input
```
Int[Coth[x]*(1 + Coth[x])^(3/2),x]
```

output
```
(-I)*(((-2*I)/3)*(1 + Coth[x])^(3/2) + I*(2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[
x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3959
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[2*a   Int[(a + b*Tan[c + d*
x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n
, 1]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)-2\sqrt{1+\coth\left(x\right)}-\frac{2(1+\coth(x))^{\frac{3}{2}}}{3}$	35
default	$2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)-2\sqrt{1+\coth\left(x\right)}-\frac{2(1+\coth(x))^{\frac{3}{2}}}{3}$	35

input
```
int(coth(x)*(1+coth(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-2*(1+coth(x))^(1/2)-2/3*(
1+coth(x))^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(34) = 68$.

Time = 0.11 (sec) , antiderivative size = 210, normalized size of antiderivative = 4.67

$$\int \coth(x)(1+\coth(x))^{3/2}\,dx = \frac{3\left(\sqrt{2}\cosh\left(x\right)^{2}+2\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)+\sqrt{2}\sinh\left(x\right)^{2}-\sqrt{2}\right)\log\left(2\cosh\left(x\right)^{2}+4\cosh\right.}{}$$

input
```
integrate(coth(x)*(1+coth(x))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(3*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2
- sqrt(2))*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + sqrt(2)*(sq
rt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)*sinh(x)^3 + (3*sqr
t(2)*cosh(x)^2 - sqrt(2))*sinh(x) - sqrt(2)*cosh(x))/sqrt(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2 - 1) - 1) - 2*sqrt(2)*(5*cosh(x)^3 + 15*cosh(x)*
sinh(x)^2 + 5*sinh(x)^3 + 3*(5*cosh(x)^2 - 1)*sinh(x) - 3*cosh(x))/sqrt(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1))/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2 - 1)
```

Sympy [F]

$$\int \coth(x)(1+\coth(x))^{3/2}\,dx = \int (\coth{(x)} + 1)^{\frac{3}{2}} \coth{(x)}\,dx$$

input
```
integrate(coth(x)*(1+coth(x))**(3/2),x)
```

output
```
Integral((coth(x) + 1)**(3/2)*coth(x), x)
```

Maxima [F]

$$\int \coth(x)(1+\coth(x))^{3/2}\,dx = \int (\coth{(x)} + 1)^{\frac{3}{2}} \coth{(x)}\,dx$$

input
```
integrate(coth(x)*(1+coth(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((coth(x) + 1)^(3/2)*coth(x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(34) = 68.

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.00

$$\int \coth(x)(1+\coth(x))^{3/2}\,dx =$$

$$-\frac{1}{3}\sqrt{2}\left(3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\operatorname{sgn}\left(e^{(2x)}-1\right)+\frac{2\left(9\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^2\operatorname{sgn}\left(e^{(2x)}\right.\right.}{}\right.$$

input
```
integrate(coth(x)*(1+coth(x))^(3/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(2)*(3*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^
(2*x) - 1) + 2*(9*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2*sgn(e^(2*x) - 1) +
 12*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))*sgn(e^(2*x) - 1) + 5*sgn(e^(2*x) -
 1))/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \coth(x)(1+\coth(x))^{3/2}\,dx = 2\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{\coth(x)+1}}{2}\right) - 2\sqrt{\coth(x)+1} - \frac{2(\coth(x)+1)^{3/2}}{3}$$

input
```
int(coth(x)*(coth(x) + 1)^(3/2),x)
```

output
```
2*2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2) - 2*(coth(x) + 1)^(1/2) -
 (2*(coth(x) + 1)^(3/2))/3
```

Reduce [F]

$$\int \coth(x)(1+\coth(x))^{3/2}\,dx = \int \sqrt{\coth(x)+1}\,\coth(x)\,dx + \int \sqrt{\coth(x)+1}\,\coth(x)^2\,dx$$

input
```
int(coth(x)*(1+coth(x))^(3/2),x)
```

output
```
int(sqrt(coth(x) + 1)*coth(x),x) + int(sqrt(coth(x) + 1)*coth(x)**2,x)
```

3.138 $\int \coth(x)\sqrt{1+\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \coth(x)\sqrt{1+\coth(x)}\,dx = \sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-2\sqrt{1+\coth(x)}$$

output
```
2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-2*(1+coth(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \coth(x)\sqrt{1+\coth(x)}\,dx = \sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-2\sqrt{1+\coth(x)}$$

input
```
Integrate[Coth[x]*Sqrt[1 + Coth[x]],x]
```

output
```
Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]]
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 26, 4010, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(x)\sqrt{\coth(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sqrt{1-i\tan\left(\frac{\pi}{2}+ix\right)}\tan\left(\frac{\pi}{2}+ix\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sqrt{1-i\tan\left(ix+\frac{\pi}{2}\right)}\tan\left(ix+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4010}$$

$$-i\left(i\int \sqrt{\coth(x)+1}dx-2i\sqrt{\coth(x)+1}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(i\int \sqrt{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx-2i\sqrt{\coth(x)+1}\right)$$

$$\Big\downarrow \text{3961}$$

$$-i\left(2i\int \frac{1}{1-\coth(x)}d\sqrt{\coth(x)+1}-2i\sqrt{\coth(x)+1}\right)$$

$$\Big\downarrow \text{219}$$

$$-i\left(i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)-2i\sqrt{\coth(x)+1}\right)$$

input
```
Int[Coth[x]*Sqrt[1 + Coth[x]],x]
```

output
```
(-I)*(I*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - (2*I)*Sqrt[1 + Coth[x
]])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{1+\coth \left(x \right)}\, \sqrt{2}}{2}\right)-2 \sqrt{1+\coth \left(x \right)}$	26
default	$\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{1+\coth \left(x \right)}\, \sqrt{2}}{2}\right)-2 \sqrt{1+\coth \left(x \right)}$	26

input
```
int(coth(x)*(1+coth(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-2*(1+coth(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(25) = 50$.

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 4.06

$$\int \coth(x)\sqrt{1+\coth(x)}\,dx = \frac{1}{2}\sqrt{2}\log\left(2\cosh(x)^2 + 4\cosh(x)\sinh(x) + 2\sinh(x)^2 + \frac{\sqrt{2}\left(\sqrt{2}\cosh(x)^3 + 3\sqrt{2}\cosh(x)\sinh(x)^2 + \sqrt{2}\sinh(x)^3 + \left(3\sqrt{2}\cosh(x)^2 - \sqrt{2}\right)\sinh(x) - \sqrt{2}\,\mathrm{cos}}{\sqrt{\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1}} - 1\right) - \frac{2\sqrt{2}(\cosh(x) + \sinh(x))}{\sqrt{\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1}}$$

input
```
integrate(coth(x)*(1+coth(x))^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + sqrt(2)*(s
qrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)*sinh(x)^3 + (3*sq
rt(2)*cosh(x)^2 - sqrt(2))*sinh(x) - sqrt(2)*cosh(x))/sqrt(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2 - 1) - 1) - 2*sqrt(2)*(cosh(x) + sinh(x))/sqrt(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [F]

$$\int \coth(x)\sqrt{1+\coth(x)}\,dx = \int \sqrt{\coth(x)+1}\coth(x)\,dx$$

input
```
integrate(coth(x)*(1+coth(x))**(1/2),x)
```

output
```
Integral(sqrt(coth(x) + 1)*coth(x), x)
```

Maxima [F]

$$\int \coth(x)\sqrt{1+\coth(x)}\,dx = \int \sqrt{\coth(x)+1}\coth(x)\,dx$$

input
```
integrate(coth(x)*(1+coth(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(coth(x) + 1)*coth(x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(25) = 50$.

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.22

$$\int \coth(x)\sqrt{1+\coth(x)}\,dx = -\frac{1}{2}\sqrt{2}\left(\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\mathrm{sgn}\left(e^{(2x)}-1\right)+\frac{4\,\mathrm{sgn}\left(e^{(2x)}-1\right)}{\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}+1}\right)$$

input
```
integrate(coth(x)*(1+coth(x))^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(2
*x) - 1) + 4*sgn(e^(2*x) - 1)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \coth(x)\sqrt{1+\coth(x)}\,dx = \sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right) - 2\,\sqrt{\coth(x)+1}$$

input
```
int(coth(x)*(coth(x) + 1)^(1/2),x)
```

output
```
2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2) - 2*(coth(x) + 1)^(1/2)
```

Reduce [F]

$$\int \coth(x)\sqrt{1+\coth(x)}\,dx = \int \sqrt{\coth(x)+1}\,\coth(x)\,dx$$

input
```
int(coth(x)*(1+coth(x))^(1/2),x)
```

output
```
int(sqrt(coth(x) + 1)*coth(x),x)
```

3.139 $\int \frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{1}{\sqrt{1+\coth(x)}}$$

output
```
1/2*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))+1/(1+coth(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{1}{\sqrt{1+\coth(x)}}$$

input
```
Integrate[Coth[x]/Sqrt[1 + Coth[x]],x]
```

output
```
ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] + 1/Sqrt[1 + Coth[x]]
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 26, 4009, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{\sqrt{\coth(x)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan\left(\frac{\pi}{2}+ix\right)}{\sqrt{1-i\tan\left(\frac{\pi}{2}+ix\right)}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan\left(ix+\frac{\pi}{2}\right)}{\sqrt{1-i\tan\left(ix+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4009}$$

$$-i\left(\frac{1}{2}i\int \sqrt{\coth(x)+1}dx+\frac{i}{\sqrt{\coth(x)+1}}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{1}{2}i\int \sqrt{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx+\frac{i}{\sqrt{\coth(x)+1}}\right)$$

$$\Big\downarrow \text{3961}$$

$$-i\left(i\int \frac{1}{1-\coth(x)}d\sqrt{\coth(x)+1}+\frac{i}{\sqrt{\coth(x)+1}}\right)$$

$$\Big\downarrow \text{219}$$

$$-i\left(\frac{i\text{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{i}{\sqrt{\coth(x)+1}}\right)$$

input
```
Int[Coth[x]/Sqrt[1 + Coth[x]],x]
```

output
```
(-I)*((I*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]])/Sqrt[2] + I/Sqrt[1 + Coth[x]]
)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)}{2}+\frac{1}{\sqrt{1+\coth(x)}}$	25
default	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)}{2}+\frac{1}{\sqrt{1+\coth(x)}}$	25

input
```
int(coth(x)/(1+coth(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))+1/(1+coth(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(24) = 48.

Time = 0.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 4.77

$$\int \frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx$$
$$= \frac{\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\log\left(2\cosh(x)^2+4\cosh(x)\sinh(x)+2\sinh(x)^2+\frac{\sqrt{2}\left(\sqrt{2}\cosh(x)^3+3\sqrt{2}\cosh(\ldots}{\ldots}\right.}{4(\cosh\ldots}$$

input
```
integrate(coth(x)/(1+coth(x))^(1/2),x, algorithm="fricas")
```

output
```
1/4*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log(2*cosh(x)^2 + 4*cosh(x)*sinh(
x) + 2*sinh(x)^2 + sqrt(2)*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)*sinh(x)^
2 + sqrt(2)*sinh(x)^3 + (3*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x) - sqrt(2)*
cosh(x))/sqrt(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1) - 1) + 2*sqrt
(2)*sqrt(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1))/(cosh(x) + sinh(x
))
```

Sympy [F]

$$\int \frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{\coth(x)+1}}\,dx$$

input
```
integrate(coth(x)/(1+coth(x))**(1/2),x)
```

output
```
Integral(coth(x)/sqrt(coth(x) + 1), x)
```

Maxima [F]

$$\int \frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{\coth(x)+1}}\,dx$$

input
```
integrate(coth(x)/(1+coth(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)/sqrt(coth(x) + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(24) = 48.

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.13

$$\int \frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx = -\frac{\sqrt{2}\left(\frac{2}{\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}} + \log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}} - 2e^{(2x)} + 1\right|\right)\right)}{4\,\mathrm{sgn}\left(e^{(2x)}-1\right)}$$

input
```
integrate(coth(x)/(1+coth(x))^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(2/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x)) + log(abs(2*sqrt(e^(4*
x) - e^(2*x)) - 2*e^(2*x) + 1)))/sgn(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)}{2} + \frac{1}{\sqrt{\coth(x)+1}}$$

input
```
int(coth(x)/(coth(x) + 1)^(1/2),x)
```

output
```
(2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/2 + 1/(coth(x) + 1)^(1/2)
```

Reduce [F]

$$\int \frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx = \int \frac{\sqrt{\coth(x)+1}\,\coth(x)}{\coth(x)+1}dx$$

input
```
int(coth(x)/(1+coth(x))^(1/2),x)
```

output
```
int((sqrt(coth(x) + 1)*coth(x))/(coth(x) + 1),x)
```

3.140 $\int \frac{\coth(x)}{(1+\coth(x))^{3/2}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{\coth(x)}{(1+\coth(x))^{3/2}} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{2\sqrt{2}} + \frac{1}{3(1+\coth(x))^{3/2}} - \frac{1}{2\sqrt{1+\coth(x)}}$$

output
```
1/4*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))+1/3/(1+coth(x))^(3/2)-1
/2/(1+coth(x))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \frac{\coth(x)}{(1+\coth(x))^{3/2}} \, dx = \frac{2 - 3(1+\coth(x)) \operatorname{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, \frac{1}{2}(1+\coth(x))\right)}{6(1+\coth(x))^{3/2}}$$

input
```
Integrate[Coth[x]/(1 + Coth[x])^(3/2),x]
```

output
```
(2 - 3*(1 + Coth[x])*Hypergeometric2F1[-1/2, 1, 1/2, (1 + Coth[x])/2])/(6*
(1 + Coth[x])^(3/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {3042, 26, 4009, 3042, 3960, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{(\coth(x)+1)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan\left(\frac{\pi}{2}+ix\right)}{\left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan\left(ix+\frac{\pi}{2}\right)}{\left(1-i\tan\left(ix+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4009}$$

$$-i\left(\frac{1}{2}i\int \frac{1}{\sqrt{\coth(x)+1}}dx+\frac{i}{3(\coth(x)+1)^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{1}{2}i\int \frac{1}{\sqrt{1-i\tan\left(ix+\frac{\pi}{2}\right)}}dx+\frac{i}{3(\coth(x)+1)^{3/2}}\right)$$

$$\Big\downarrow \text{3960}$$

$$-i\left(\frac{1}{2}i\left(\frac{1}{2}\int \sqrt{\coth(x)+1}dx-\frac{1}{\sqrt{\coth(x)+1}}\right)+\frac{i}{3(\coth(x)+1)^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{1}{2}i\left(-\frac{1}{\sqrt{\coth(x)+1}}+\frac{1}{2}\int \sqrt{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx\right)+\frac{i}{3(\coth(x)+1)^{3/2}}\right)$$

$$\Big\downarrow \text{3961}$$

$$-i\left(\frac{1}{2}i\left(\int \frac{1}{1-\coth(x)}d\sqrt{\coth(x)+1}-\frac{1}{\sqrt{\coth(x)+1}}\right)+\frac{i}{3(\coth(x)+1)^{3/2}}\right)$$

$$\Big\downarrow \text{219}$$

$$-i\left(\frac{1}{2}i\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{\coth(x)+1}}\right)+\frac{i}{3(\coth(x)+1)^{3/2}}\right)$$

input
```
Int[Coth[x]/(1 + Coth[x])^(3/2),x]
```

output
```
(-I)*((I/3)/(1 + Coth[x])^(3/2) + (I/2)*(ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]
]/Sqrt[2] - 1/Sqrt[1 + Coth[x]]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\, \sqrt{2}}{2}\right)}{4}+\frac{1}{3(1+\coth(x))^{\frac{3}{2}}}-\frac{1}{2\sqrt{1+\coth(x)}}$	35
default	$\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\, \sqrt{2}}{2}\right)}{4}+\frac{1}{3(1+\coth(x))^{\frac{3}{2}}}-\frac{1}{2\sqrt{1+\coth(x)}}$	35

input
```
int(coth(x)/(1+coth(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))+1/3/(1+coth(x))^(3/2)-1
/2/(1+coth(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(34) = 68$.

Time = 0.11 (sec) , antiderivative size = 246, normalized size of antiderivative = 5.02

$$\int \frac{\coth(x)}{(1+\coth(x))^{3/2}}\,dx = \frac{3\left(\sqrt{2}\cosh(x)^3+3\sqrt{2}\cosh(x)^2\sinh(x)+3\sqrt{2}\cosh(x)\sinh(x)^2+\sqrt{2}\sinh(x)^3\right.}{}$$

input
```
integrate(coth(x)/(1+coth(x))^(3/2),x, algorithm="fricas")
```

output
```
1/24*(3*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2)*cosh(
x)*sinh(x)^2 + sqrt(2)*sinh(x)^3)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*
sinh(x)^2 + sqrt(2)*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqr
t(2)*sinh(x)^3 + (3*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x) - sqrt(2)*cosh(x)
)/sqrt(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1) - 1) - 2*sqrt(2)*(2*
cosh(x)^4 + 8*cosh(x)*sinh(x)^3 + 2*sinh(x)^4 + (12*cosh(x)^2 - 1)*sinh(x)
^2 - cosh(x)^2 + 2*(4*cosh(x)^3 - cosh(x))*sinh(x) - 1)/sqrt(cosh(x)^2 + 2
*cosh(x)*sinh(x) + sinh(x)^2 - 1))/(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*co
sh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [F]

$$\int \frac{\coth(x)}{(1+\coth(x))^{3/2}}\, dx = \int \frac{\coth(x)}{(\coth(x)+1)^{\frac{3}{2}}}\, dx$$

input
```
integrate(coth(x)/(1+coth(x))**(3/2),x)
```

output
```
Integral(coth(x)/(coth(x) + 1)**(3/2), x)
```

Maxima [F]

$$\int \frac{\coth(x)}{(1+\coth(x))^{3/2}}\, dx = \int \frac{\coth(x)}{(\coth(x)+1)^{\frac{3}{2}}}\, dx$$

input
```
integrate(coth(x)/(1+coth(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)/(coth(x) + 1)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(34) = 68$.

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.82

$$\int \frac{\coth(x)}{(1+\coth(x))^{3/2}}\,dx = -\frac{\sqrt{2}\left(\frac{2\left(3\sqrt{e^{(4x)}-e^{(2x)}}-3e^{(2x)}+1\right)}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^3}+3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\right)}{24\,\mathrm{sgn}\left(e^{(2x)}-1\right)}$$

input
```
integrate(coth(x)/(1+coth(x))^(3/2),x, algorithm="giac")
```

output
```
-1/24*sqrt(2)*(2*(3*sqrt(e^(4*x) - e^(2*x)) - 3*e^(2*x) + 1)/(sqrt(e^(4*x)
 - e^(2*x)) - e^(2*x))^3 + 3*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x)
 + 1)))/sgn(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.65

$$\int \frac{\coth(x)}{(1+\coth(x))^{3/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)}{4} - \frac{\frac{\coth(x)}{2}+\frac{1}{6}}{\left(\coth(x)+1\right)^{3/2}}$$

input
```
int(coth(x)/(coth(x) + 1)^(3/2),x)
```

output
```
(2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/4 - (coth(x)/2 + 1/6)/(co
th(x) + 1)^(3/2)
```

Reduce [F]

$$\int \frac{\coth(x)}{(1+\coth(x))^{3/2}}\,dx = \int \frac{\sqrt{\coth(x)+1}\,\coth(x)}{\coth(x)^2+2\coth(x)+1}dx$$

input
```
int(coth(x)/(1+coth(x))^(3/2),x)
```

output
```
int((sqrt(coth(x) + 1)*coth(x))/(coth(x)**2 + 2*coth(x) + 1),x)
```

3.141 $\int \coth^2(x)(1+\coth(x))^{3/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 45

$$\int \coth^2(x)(1+\coth(x))^{3/2}\,dx = 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\coth(x)} - \frac{2}{5}(1+\coth(x))^{5/2}$$

output
```
2*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-2*(1+coth(x))^(1/2)-2/5*(
1+coth(x))^(5/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \coth^2(x)(1+\coth(x))^{3/2}\,dx = 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\coth(x)} - \frac{2}{5}(1+\coth(x))^{5/2}$$

input
```
Integrate[Coth[x]^2*(1 + Coth[x])^(3/2),x]
```

output
```
2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]] - (2*(1
 + Coth[x])^(5/2))/5
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3042, 25, 4026, 25, 3042, 3959, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(x)(\coth(x)+1)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)^{3/2}\tan\left(\frac{\pi}{2}+ix\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int \left(1-i\tan\left(ix+\frac{\pi}{2}\right)\right)^{3/2}\tan\left(ix+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{4026}$$

$$-\int -(\coth(x)+1)^{3/2}dx-\frac{2}{5}(\coth(x)+1)^{5/2}$$

$$\Big\downarrow \text{25}$$

$$\int (\coth(x)+1)^{3/2}dx-\frac{2}{5}(\coth(x)+1)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{5}(\coth(x)+1)^{5/2}+\int \left(1-i\tan\left(ix+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3959}$$

$$2\int \sqrt{\coth(x)+1}dx-\frac{2}{5}(\coth(x)+1)^{5/2}-2\sqrt{\coth(x)+1}$$

$$\Big\downarrow \text{3042}$$

$$2\int \sqrt{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx-\frac{2}{5}(\coth(x)+1)^{5/2}-2\sqrt{\coth(x)+1}$$

$$\Big\downarrow \; 3961$$

$$4\int \frac{1}{1-\coth(x)}d\sqrt{\coth(x)+1}-\frac{2}{5}(\coth(x)+1)^{5/2}-2\sqrt{\coth(x)+1}$$

$$\Big\downarrow \; 219$$

$$2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)-\frac{2}{5}(\coth(x)+1)^{5/2}-2\sqrt{\coth(x)+1}$$

input
```
Int[Coth[x]^2*(1 + Coth[x])^(3/2),x]
```

output
```
2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]] - (2*(1
 + Coth[x])^(5/2))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3959
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[2*a   Int[(a + b*Tan[c + d*
x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n
, 1]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4026
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)-2\sqrt{1+\coth\left(x\right)}-\frac{2(1+\coth(x))^{\frac{5}{2}}}{5}$	35
default	$2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)-2\sqrt{1+\coth\left(x\right)}-\frac{2(1+\coth(x))^{\frac{5}{2}}}{5}$	35

input
```
int(coth(x)^2*(1+coth(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-2*(1+coth(x))^(1/2)-2/5*(
1+coth(x))^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(34) = 68.

Time = 0.10 (sec) , antiderivative size = 338, normalized size of antiderivative = 7.51

$$\int \coth^2(x)(1+\coth(x))^{3/2}\,dx = \frac{5\left(\sqrt{2}\cosh\left(x\right)^{4}+4\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)^{3}+\sqrt{2}\sinh\left(x\right)^{4}+2\left(3\sqrt{2}\cosh\left(x\right)^{2}-\sqrt{2}\right)\sinh}{}$$

input
```
integrate(coth(x)^2*(1+coth(x))^(3/2),x, algorithm="fricas")
```

output
```
1/5*(5*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^
4 + 2*(3*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^2 - 2*sqrt(2)*cosh(x)^2 + 4*
(sqrt(2)*cosh(x)^3 - sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(2*cosh(x)^2 +
 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + sqrt(2)*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*
cosh(x)*sinh(x)^2 + sqrt(2)*sinh(x)^3 + (3*sqrt(2)*cosh(x)^2 - sqrt(2))*si
nh(x) - sqrt(2)*cosh(x))/sqrt(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 -
1) - 1) - 2*sqrt(2)*(9*cosh(x)^5 + 45*cosh(x)*sinh(x)^4 + 9*sinh(x)^5 + 10
*(9*cosh(x)^2 - 1)*sinh(x)^3 - 10*cosh(x)^3 + 30*(3*cosh(x)^3 - cosh(x))*s
inh(x)^2 + 5*(9*cosh(x)^4 - 6*cosh(x)^2 + 1)*sinh(x) + 5*cosh(x))/sqrt(cos
h(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1))/(cosh(x)^4 + 4*cosh(x)*sinh(x
)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)
^3 - cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \coth^2(x)(1+\coth(x))^{3/2}\,dx = \int (\coth(x)+1)^{\frac{3}{2}}\coth^2(x)\,dx$$

input
```
integrate(coth(x)**2*(1+coth(x))**(3/2),x)
```

output
```
Integral((coth(x) + 1)**(3/2)*coth(x)**2, x)
```

Maxima [F]

$$\int \coth^2(x)(1+\coth(x))^{3/2}\,dx = \int (\coth(x)+1)^{\frac{3}{2}}\coth(x)^2\,dx$$

input
```
integrate(coth(x)^2*(1+coth(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((coth(x) + 1)^(3/2)*coth(x)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(34) = 68$.

Time = 0.15 (sec) , antiderivative size = 197, normalized size of antiderivative = 4.38

$$\int \coth^2(x)(1+\coth(x))^{3/2}\,dx =$$

$$-\frac{1}{5}\sqrt{2}\left(5\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\operatorname{sgn}\left(e^{(2x)}-1\right)+\frac{2\left(25\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^4\operatorname{sgn}\left(e^{(2x}\right.\right.}{}\right.$$

input
```
integrate(coth(x)^2*(1+coth(x))^(3/2),x, algorithm="giac")
```

output
```
-1/5*sqrt(2)*(5*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^
(2*x) - 1) + 2*(25*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^4*sgn(e^(2*x) - 1)
+ 60*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^3*sgn(e^(2*x) - 1) + 70*(sqrt(e^(
4*x) - e^(2*x)) - e^(2*x))^2*sgn(e^(2*x) - 1) + 40*(sqrt(e^(4*x) - e^(2*x)
) - e^(2*x))*sgn(e^(2*x) - 1) + 9*sgn(e^(2*x) - 1))/(sqrt(e^(4*x) - e^(2*x
)) - e^(2*x) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \coth^2(x)(1+\coth(x))^{3/2}\,dx = 2\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{\coth(x)+1}}{2}\right) - 2\sqrt{\coth(x)+1} - \frac{2(\coth(x)+1)^{5/2}}{5}$$

input
```
int(coth(x)^2*(coth(x) + 1)^(3/2),x)
```

output
```
2*2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2) - 2*(coth(x) + 1)^(1/2) -
 (2*(coth(x) + 1)^(5/2))/5
```

Reduce [F]

$$\int \coth^2(x)(1+\coth(x))^{3/2}\,dx = \int \sqrt{\coth(x)+1}\,\coth(x)^3\,dx + \int \sqrt{\coth(x)+1}\,\coth(x)^2\,dx$$

input
```
int(coth(x)^2*(1+coth(x))^(3/2),x)
```

output
```
int(sqrt(coth(x) + 1)*coth(x)**3,x) + int(sqrt(coth(x) + 1)*coth(x)**2,x)
```

3.142 $\int \coth^2(x)\sqrt{1+\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \coth^2(x)\sqrt{1+\coth(x)}\,dx=\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-\frac{2}{3}(1+\coth(x))^{3/2}$$

output
```
2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-2/3*(1+coth(x))^(3/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \coth^2(x)\sqrt{1+\coth(x)}\,dx=-2\left(-\frac{\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{3}(1+\coth(x))^{3/2}\right)$$

input
```
Integrate[Coth[x]^2*Sqrt[1 + Coth[x]],x]
```

output
```
-2*(-(ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2]) + (1 + Coth[x])^(3/2)/3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3042, 25, 4026, 25, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(x)\sqrt{\coth(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sqrt{1-i\tan\left(\frac{\pi}{2}+ix\right)}\tan\left(\frac{\pi}{2}+ix\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sqrt{1-i\tan\left(ix+\frac{\pi}{2}\right)}\tan\left(ix+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{4026}$$

$$-\int -\sqrt{\coth(x)+1}dx-\frac{2}{3}(\coth(x)+1)^{3/2}$$

$$\Big\downarrow \text{25}$$

$$\int \sqrt{\coth(x)+1}dx-\frac{2}{3}(\coth(x)+1)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{3}(\coth(x)+1)^{3/2}+\int \sqrt{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3961}$$

$$2\int \frac{1}{1-\coth(x)}d\sqrt{\coth(x)+1}-\frac{2}{3}(\coth(x)+1)^{3/2}$$

$$\Big\downarrow \text{219}$$

$$\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)-\frac{2}{3}(\coth(x)+1)^{3/2}$$

input
```
Int[Coth[x]^2*Sqrt[1 + Coth[x]],x]
```

output
```
Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - (2*(1 + Coth[x])^(3/2))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4026
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)-\frac{2(1+\coth(x))^{\frac{3}{2}}}{3}$	26
default	$\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)-\frac{2(1+\coth(x))^{\frac{3}{2}}}{3}$	26

input
```
int(coth(x)^2*(1+coth(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-2/3*(1+coth(x))^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(25) = 50.

Time = 0.11 (sec) , antiderivative size = 198, normalized size of antiderivative = 5.82

$$\int \coth^2(x)\sqrt{1+\coth(x)}\,dx$$
$$= \frac{3\left(\sqrt{2}\cosh(x)^2+2\sqrt{2}\cosh(x)\sinh(x)+\sqrt{2}\sinh(x)^2-\sqrt{2}\right)\log\left(2\cosh(x)^2+4\cosh(x)\sinh(x)+2\right.}{6\,(\mathrm{co}}$$

input
```
integrate(coth(x)^2*(1+coth(x))^(1/2),x, algorithm="fricas")
```

output
```
1/6*(3*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2
- sqrt(2))*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + sqrt(2)*(sq
rt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)*sinh(x)^3 + (3*sqr
t(2)*cosh(x)^2 - sqrt(2))*sinh(x) - sqrt(2)*cosh(x))/sqrt(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2 - 1) - 1) - 8*sqrt(2)*(cosh(x)^3 + 3*cosh(x)^2*s
inh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)/sqrt(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2 - 1))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [F]

$$\int \coth^2(x)\sqrt{1+\coth(x)}\,dx = \int \sqrt{\coth(x)+1}\coth^2(x)\,dx$$

input
```
integrate(coth(x)**2*(1+coth(x))**(1/2),x)
```

output
```
Integral(sqrt(coth(x) + 1)*coth(x)**2, x)
```

Maxima [F]

$$\int \coth^2(x)\sqrt{1+\coth(x)}\,dx = \int \sqrt{\coth(x)+1}\coth(x)^2\,dx$$

input
```
integrate(coth(x)^2*(1+coth(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(coth(x) + 1)*coth(x)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(25) = 50.

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.91

$$\int \coth^2(x)\sqrt{1+\coth(x)}\,dx =$$

$$-\frac{1}{6}\sqrt{2}\left(3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\mathrm{sgn}\left(e^{(2x)}-1\right)+\frac{8\left(3\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^2\mathrm{sgn}\left(e^{(\right.\right.}{}\right.$$

input
```
integrate(coth(x)^2*(1+coth(x))^(1/2),x, algorithm="giac")
```

output
```
-1/6*sqrt(2)*(3*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^
(2*x) - 1) + 8*(3*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2*sgn(e^(2*x) - 1) +
 3*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))*sgn(e^(2*x) - 1) + sgn(e^(2*x) - 1)
)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \coth^2(x)\sqrt{1+\coth(x)}\,dx = \sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth\left(x\right)+1}}{2}\right) - \frac{2\left(\coth(x)+1\right)^{3/2}}{3}$$

input
```
int(coth(x)^2*(coth(x) + 1)^(1/2),x)
```

output
```
2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2) - (2*(coth(x) + 1)^(3/2))/3
```

Reduce [F]

$$\int \coth^2(x)\sqrt{1+\coth(x)}\,dx = \int \sqrt{\coth\left(x\right)+1}\,\coth\left(x\right)^2 dx$$

input
```
int(coth(x)^2*(1+coth(x))^(1/2),x)
```

output
```
int(sqrt(coth(x) + 1)*coth(x)**2,x)
```

3.143 $\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 42

$$\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{\sqrt{1+\coth(x)}} - 2\sqrt{1+\coth(x)}$$

output
```
1/2*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-1/(1+coth(x))^(1/2)-2*(
1+coth(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{-3-2\coth(x)}{\sqrt{1+\coth(x)}}$$

input
```
Integrate[Coth[x]^2/Sqrt[1 + Coth[x]],x]
```

output
```
ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] + (-3 - 2*Coth[x])/Sqrt[1 + Cot
h[x]]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3042, 25, 4026, 25, 3042, 3960, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{\sqrt{\coth(x)+1}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\tan\left(\frac{\pi}{2}+ix\right)^2}{\sqrt{1-i\tan\left(\frac{\pi}{2}+ix\right)}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\tan\left(ix+\frac{\pi}{2}\right)^2}{\sqrt{1-i\tan\left(ix+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4026}$$

$$-\int -\frac{1}{\sqrt{\coth(x)+1}}dx-2\sqrt{\coth(x)+1}$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{1}{\sqrt{\coth(x)+1}}dx-2\sqrt{\coth(x)+1}$$

$$\Big\downarrow \text{ 3042}$$

$$-2\sqrt{\coth(x)+1}+\int \frac{1}{\sqrt{1-i\tan\left(ix+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3960}$$

$$\frac{1}{2}\int \sqrt{\coth(x)+1}dx-2\sqrt{\coth(x)+1}-\frac{1}{\sqrt{\coth(x)+1}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int \sqrt{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx-2\sqrt{\coth(x)+1}-\frac{1}{\sqrt{\coth(x)+1}}$$

$$\Big\downarrow \text{3961}$$

$$\int \frac{1}{1-\coth(x)}d\sqrt{\coth(x)+1}-2\sqrt{\coth(x)+1}-\frac{1}{\sqrt{\coth(x)+1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}-2\sqrt{\coth(x)+1}-\frac{1}{\sqrt{\coth(x)+1}}$$

input
```
Int[Coth[x]^2/Sqrt[1 + Coth[x]],x]
```

output
```
ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Coth[x]] - 2*Sqrt[
1 + Coth[x]]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4026
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\, \sqrt{2}}{2}\right)}{2}-\frac{1}{\sqrt{1+\coth(x)}}-2\sqrt{1+\coth\left(x\right)}$	35
default	$\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\, \sqrt{2}}{2}\right)}{2}-\frac{1}{\sqrt{1+\coth(x)}}-2\sqrt{1+\coth\left(x\right)}$	35

input
```
int(coth(x)^2/(1+coth(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-1/(1+coth(x))^(1/2)-2*(
1+coth(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(34) = 68.

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.88

$$\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx = \frac{\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\log\left(2\,\cosh\left(x\right)^2+4\,\cosh\left(x\right)\sinh\left(x\right)+2\,\sinh\left(x\right)^2+\frac{\sqrt{2}\left(\sqrt{2}\cosh(x)^3+3\sqrt{2}\cosh(\right.}{\ }\right.}{4\left(\cosh\left(x\right)+\right.}$$

input
```
integrate(coth(x)^2/(1+coth(x))^(1/2),x, algorithm="fricas")
```

output
```
1/4*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log(2*cosh(x)^2 + 4*cosh(x)*sinh(
x) + 2*sinh(x)^2 + sqrt(2)*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)*sinh(x)^
2 + sqrt(2)*sinh(x)^3 + (3*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x) - sqrt(2)*
cosh(x))/sqrt(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1) - 1) - 2*sqrt
(2)*(5*cosh(x)^2 + 10*cosh(x)*sinh(x) + 5*sinh(x)^2 - 1)/sqrt(cosh(x)^2 +
2*cosh(x)*sinh(x) + sinh(x)^2 - 1))/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx = \int \frac{\coth^2(x)}{\sqrt{\coth(x)+1}}\,dx$$

input
```
integrate(coth(x)**2/(1+coth(x))**(1/2),x)
```

output
```
Integral(coth(x)**2/sqrt(coth(x) + 1), x)
```

Maxima [F]

$$\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx = \int \frac{\coth(x)^2}{\sqrt{\coth(x)+1}}\,dx$$

input
```
integrate(coth(x)^2/(1+coth(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)^2/sqrt(coth(x) + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(34) = 68$.

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.10

$$\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx = -\frac{\frac{5\sqrt{2}e^{(2x)}}{\operatorname{sgn}(e^{(2x)}-1)} - \frac{\sqrt{2}}{\operatorname{sgn}(e^{(2x)}-1)}}{2\sqrt{e^{(4x)}-e^{(2x)}}} - \frac{\sqrt{2}\log\left(\left|4\sqrt{e^{(4x)}-e^{(2x)}} - 4e^{(2x)} + 2\right|\right)}{4\operatorname{sgn}\left(e^{(2x)}-1\right)}$$

input
```
integrate(coth(x)^2/(1+coth(x))^(1/2),x, algorithm="giac")
```

output
```
-1/2*(5*sqrt(2)*e^(2*x)/sgn(e^(2*x) - 1) - sqrt(2)/sgn(e^(2*x) - 1))/sqrt(
e^(4*x) - e^(2*x)) - 1/4*sqrt(2)*log(abs(4*sqrt(e^(4*x) - e^(2*x)) - 4*e^(
2*x) + 2))/sgn(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx = \frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{\coth(x)+1}}{2}\right)}{2} - \frac{3}{\sqrt{\coth(x)+1}} - \frac{2\coth(x)}{\sqrt{\coth(x)+1}}$$

input
```
int(coth(x)^2/(coth(x) + 1)^(1/2),x)
```

output
```
(2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/2 - 3/(coth(x) + 1)^(1/2)
 - (2*coth(x))/(coth(x) + 1)^(1/2)
```

Reduce [F]

$$\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx = \int \frac{\sqrt{\coth(x)+1}\,\coth(x)^2}{\coth(x)+1}dx$$

input
```
int(coth(x)^2/(1+coth(x))^(1/2),x)
```

output
```
int((sqrt(coth(x) + 1)*coth(x)**2)/(coth(x) + 1),x)
```

3.144 $\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}} dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{1}{3(1+\coth(x))^{3/2}} + \frac{3}{2\sqrt{1+\coth(x)}}$$

output
```
1/4*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-1/3/(1+coth(x))^(3/2)+3
/2/(1+coth(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}} dx = \frac{14 + 18\coth(x) + 3\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)(1+\coth(x))^{3/2}}{12(1+\coth(x))^{3/2}}$$

input
```
Integrate[Coth[x]^2/(1 + Coth[x])^(3/2),x]
```

output
```
(14 + 18*Coth[x] + 3*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]*(1 + Coth[
x])^(3/2))/(12*(1 + Coth[x])^(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {3042, 25, 4023, 3042, 4009, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{(\coth(x)+1)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(\frac{\pi}{2}+ix\right)^2}{\left(1-i\tan\left(\frac{\pi}{2}+ix\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(ix+\frac{\pi}{2}\right)^2}{\left(1-i\tan\left(ix+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4023}$$

$$-\frac{1}{2}\int \frac{1-2\coth(x)}{\sqrt{\coth(x)+1}}dx-\frac{1}{3(\coth(x)+1)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3(\coth(x)+1)^{3/2}}-\frac{1}{2}\int \frac{2i\tan\left(ix+\frac{\pi}{2}\right)+1}{\sqrt{1-i\tan\left(ix+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4009}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \sqrt{\coth(x)+1}dx+\frac{3}{\sqrt{\coth(x)+1}}\right)-\frac{1}{3(\coth(x)+1)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3(\coth(x)+1)^{3/2}}+\frac{1}{2}\left(\frac{3}{\sqrt{\coth(x)+1}}+\frac{1}{2}\int \sqrt{1-i\tan\left(ix+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3961}$$

$$\frac{1}{2}\left(\int \frac{1}{1-\coth(x)}d\sqrt{\coth(x)+1}+\frac{3}{\sqrt{\coth(x)+1}}\right)-\frac{1}{3(\coth(x)+1)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{3}{\sqrt{\coth(x)+1}}\right)-\frac{1}{3(\coth(x)+1)^{3/2}}$$

input
```
Int[Coth[x]^2/(1 + Coth[x])^(3/2),x]
```

output
```
-1/3*1/(1 + Coth[x])^(3/2) + (ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] +
 3/Sqrt[1 + Coth[x]])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

rule 4023
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(a*c + b*d)^2*((a + b*Tan[e + f*x])^
m/(2*a^3*f*m)), x] + Simp[1/(2*a^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp
[a*c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)}{4}-\frac{1}{3(1+\coth(x))^{\frac{3}{2}}}+\frac{3}{2\sqrt{1+\coth(x)}}$	35
default	$\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)}{4}-\frac{1}{3(1+\coth(x))^{\frac{3}{2}}}+\frac{3}{2\sqrt{1+\coth(x)}}$	35

input
```
int(coth(x)^2/(1+coth(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))-1/3/(1+coth(x))^(3/2)+3
/2/(1+coth(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(34) = 68$.

Time = 0.09 (sec) , antiderivative size = 246, normalized size of antiderivative = 5.02

$$\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}}\,dx = \frac{3\left(\sqrt{2}\cosh(x)^3+3\sqrt{2}\cosh(x)^2\sinh(x)+3\sqrt{2}\cosh(x)\sinh(x)^2+\sqrt{2}\sinh(x)^3\right.}{}$$

input
```
integrate(coth(x)^2/(1+coth(x))^(3/2),x, algorithm="fricas")
```

output
```
1/24*(3*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2)*cosh(
x)*sinh(x)^2 + sqrt(2)*sinh(x)^3)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*
sinh(x)^2 + sqrt(2)*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqr
t(2)*sinh(x)^3 + (3*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x) - sqrt(2)*cosh(x)
)/sqrt(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1) - 1) + 2*sqrt(2)*(8*
cosh(x)^4 + 32*cosh(x)*sinh(x)^3 + 8*sinh(x)^4 + (48*cosh(x)^2 - 7)*sinh(x
)^2 - 7*cosh(x)^2 + 2*(16*cosh(x)^3 - 7*cosh(x))*sinh(x) - 1)/sqrt(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1))/(cosh(x)^3 + 3*cosh(x)^2*sinh(x)
+ 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [F]

$$\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}}\,dx = \int \frac{\coth^2{(x)}}{(\coth{(x)}+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(x)**2/(1+coth(x))**(3/2),x)
```

output
```
Integral(coth(x)**2/(coth(x) + 1)**(3/2), x)
```

Maxima [F]

$$\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}}\,dx = \int \frac{\coth{(x)}^2}{(\coth{(x)}+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(x)^2/(1+coth(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)^2/(coth(x) + 1)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(34) = 68$.

Time = 0.15 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.31

$$\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}}\,dx = -\frac{\sqrt{2}\left(\frac{2\left(6\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^2-3\sqrt{e^{(4x)}-e^{(2x)}}+3e^{(2x)}-1\right)}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^3}+3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\right)}{24\,\mathrm{sgn}\left(e^{(2x)}-1\right)}$$

input
```
integrate(coth(x)^2/(1+coth(x))^(3/2),x, algorithm="giac")
```

output
```
-1/24*sqrt(2)*(2*(6*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2 - 3*sqrt(e^(4*x)
 - e^(2*x)) + 3*e^(2*x) - 1)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^3 + 3*log
(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1)))/sgn(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{\coth(x)+1}}{2}\right)}{4} + \frac{\frac{3\coth(x)}{2}+\frac{7}{6}}{(\coth(x)+1)^{3/2}}$$

input
```
int(coth(x)^2/(coth(x) + 1)^(3/2),x)
```

output
```
(2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/4 + ((3*coth(x))/2 + 7/6)
/(coth(x) + 1)^(3/2)
```

Reduce [F]

$$\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}}\,dx = \int \frac{\sqrt{\coth(x)+1}\,\coth(x)^2}{\coth(x)^2+2\coth(x)+1}dx$$

input
```
int(coth(x)^2/(1+coth(x))^(3/2),x)
```

output
```
int((sqrt(coth(x) + 1)*coth(x)**2)/(coth(x)**2 + 2*coth(x) + 1),x)
```

3.145 $\int \frac{\tanh^4(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 97

$$\int \frac{\tanh^4(x)}{a+b\coth(x)}\,dx=\frac{ax}{a^2-b^2}-\frac{b(a^2+b^2)\log(\cosh(x))}{a^4}-\frac{b^5\log(b\cosh(x)+a\sinh(x))}{a^4(a^2-b^2)}-\frac{(a^2+b^2)\tanh(x)}{a^3}+\frac{b\tanh^2(x)}{2a^2}-\frac{\tanh^3(x)}{3a}$$

output
```
a*x/(a^2-b^2)-b*(a^2+b^2)*ln(cosh(x))/a^4-b^5*ln(b*cosh(x)+a*sinh(x))/a^4/
(a^2-b^2)-(a^2+b^2)*tanh(x)/a^3+1/2*b*tanh(x)^2/a^2-1/3*tanh(x)^3/a
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.15

$$\int \frac{\tanh^4(x)}{a+b\coth(x)}\,dx=\frac{1}{6}\left(-\frac{3\log(1-\coth(x))}{a+b}+\frac{3\log(1+\coth(x))}{a-b}+\frac{6b^5\log(a+b\coth(x))}{a^4(-a^2+b^2)}+\frac{6b\log(\tanh(x))}{a^2}+\frac{6b^3\log(\tanh(x))}{a^4}-\frac{6(a^2+b^2)\tanh(x)}{a^3}+\frac{3b\tanh^2(x)}{a^2}-\frac{2\tanh^3(x)}{a}\right)$$

input
```
Integrate[Tanh[x]^4/(a + b*Coth[x]),x]
```

output
```
((-3*Log[1 - Coth[x]])/(a + b) + (3*Log[1 + Coth[x]])/(a - b) + (6*b^5*Log
[a + b*Coth[x]])/(a^4*(-a^2 + b^2)) + (6*b*Log[Tanh[x]])/a^2 + (6*b^3*Log[
Tanh[x]])/a^4 - (6*(a^2 + b^2)*Tanh[x])/a^3 + (3*b*Tanh[x]^2)/a^2 - (2*Tan
h[x]^3)/a)/6
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.35, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.462, Rules used = {3042, 4052, 27, 3042, 26, 4132, 27, 3042, 25, 4133, 25, 3042, 26, 4134, 26, 3042, 26, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(x)}{a+b\coth(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan\left(\frac{\pi}{2}+ix\right)^4\left(a-ib\tan\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{4052}$$

$$-\frac{\int \frac{3\left(-b\coth^2(x)-a\coth(x)+b\right)\tanh^3(x)}{a+b\coth(x)}dx}{3a}-\frac{\tanh^3(x)}{3a}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\left(-b\coth^2(x)-a\coth(x)+b\right)\tanh^3(x)}{a+b\coth(x)}dx}{a}-\frac{\tanh^3(x)}{3a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh^3(x)}{3a}-\frac{\int -\frac{i\left(b\tan\left(ix+\frac{\pi}{2}\right)^2+ia\tan\left(ix+\frac{\pi}{2}\right)+b\right)}{\tan\left(ix+\frac{\pi}{2}\right)^3\left(a-ib\tan\left(ix+\frac{\pi}{2}\right)\right)}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\tanh^3(x)}{3a}+\frac{i\int \frac{b\tan\left(ix+\frac{\pi}{2}\right)^2+ia\tan\left(ix+\frac{\pi}{2}\right)+b}{\tan\left(ix+\frac{\pi}{2}\right)^3\left(a-ib\tan\left(ix+\frac{\pi}{2}\right)\right)}dx}{a}$$

$$\Big\downarrow \text{4132}$$

$$-\frac{\tanh^3(x)}{3a}+\frac{i\left(-\frac{\int \frac{2i\left(a^2+b^2-b^2\coth^2(x)\right)\tanh^2(x)}{a+b\coth(x)}dx}{2a}-\frac{ib\tanh^2(x)}{2a}\right)}{a}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\tanh^3(x)}{3a}+\frac{i\left(-\frac{i\int \frac{\left(a^2+b^2-b^2\coth^2(x)\right)\tanh^2(x)}{a+b\coth(x)}dx}{a}-\frac{ib\tanh^2(x)}{2a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh^3(x)}{3a}+\frac{i\left(-\frac{i\int -\frac{a^2+b^2+b^2\tan\left(ix+\frac{\pi}{2}\right)^2}{\tan\left(ix+\frac{\pi}{2}\right)^2\left(a-ib\tan\left(ix+\frac{\pi}{2}\right)\right)}dx}{a}-\frac{ib\tanh^2(x)}{2a}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tanh^3(x)}{3a}+\frac{i\left(\frac{i\int \frac{a^2+b^2+b^2\tan\left(ix+\frac{\pi}{2}\right)^2}{\tan\left(ix+\frac{\pi}{2}\right)^2\left(a-ib\tan\left(ix+\frac{\pi}{2}\right)\right)}dx}{a}-\frac{ib\tanh^2(x)}{2a}\right)}{a}$$

$$\Big\downarrow \text{4133}$$

$$-\frac{\tanh^3(x)}{3a}+\frac{i\left(\frac{i\left(\frac{\left(a^2+b^2\right)\tanh(x)}{a}-\frac{\int -\frac{\left(-\coth(x)a^3-b\left(a^2+b^2\right)\coth^2(x)+b\left(a^2+b^2\right)\right)\tanh(x)}{a+b\coth(x)}dx}{a}\right)}{a}-\frac{ib\tanh^2(x)}{2a}\right)}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tanh^3(x)}{3a}+\frac{i\left(\frac{i\left(\frac{\int \frac{\left(-\coth(x)a^3-b\left(a^2+b^2\right)\coth^2(x)+b\left(a^2+b^2\right)\right)\tanh(x)}{a+b\coth(x)}dx}{a}+\frac{\left(a^2+b^2\right)\tanh(x)}{a}\right)}{a}-\frac{ib\tanh^2(x)}{2a}\right)}{a}$$

$\downarrow$ 3042

$$-\frac{\tanh^3(x)}{3a}+\frac{i\left(\frac{i\left(\frac{\left(a^2+b^2\right)\tanh(x)}{a}+\frac{\int \frac{i\left(i\tan\left(ix+\frac{\pi}{2}\right)a^3+b\left(a^2+b^2\right)\tan\left(ix+\frac{\pi}{2}\right)^2+b\left(a^2+b^2\right)\right)}{\tan\left(ix+\frac{\pi}{2}\right)\left(a-ib\tan\left(ix+\frac{\pi}{2}\right)\right)}dx}{a}\right)}{a}-\frac{ib\tanh^2(x)}{2a}\right)}{a}$$

$\downarrow$ 26

$$-\frac{\tanh^3(x)}{3a}+\frac{i\left(\frac{i\left(\frac{\left(a^2+b^2\right)\tanh(x)}{a}+\frac{i\int \frac{i\tan\left(ix+\frac{\pi}{2}\right)a^3+b\left(a^2+b^2\right)\tan\left(ix+\frac{\pi}{2}\right)^2+b\left(a^2+b^2\right)}{\tan\left(ix+\frac{\pi}{2}\right)\left(a-ib\tan\left(ix+\frac{\pi}{2}\right)\right)}dx}{a}\right)}{a}-\frac{ib\tanh^2(x)}{2a}\right)}{a}$$

$\downarrow$ 4134

$$-\frac{\tanh^3(x)}{3a}+\frac{i\left(\frac{i\left(\frac{\left(a^2+b^2\right)\tanh(x)}{a}+\frac{i\left(\frac{b\left(a^2+b^2\right)\int -i\tanh(x)dx}{a}+\frac{b^5\int -\frac{i(b+a\coth(x))}{a+b\coth(x)}dx}{a\left(a^2-b^2\right)}+\frac{ia^4x}{a^2-b^2}\right)}{a}\right)}{a}-\frac{ib\tanh^2(x)}{2a}\right)}{a}$$

$\downarrow$ 26

$$-\frac{\tanh^3(x)}{3a}+\frac{i\left(\frac{i\left(\frac{(a^2+b^2)\tanh(x)}{a}+\frac{i\left(-\frac{ib(a^2+b^2)\int \tanh(x)dx}{a}-\frac{ib^5\int \frac{b+a\coth(x)}{a+b\coth(x)}dx}{a(a^2-b^2)}+\frac{ia^4x}{a^2-b^2}\right)}{a}\right)}{a}-\frac{ib\tanh^2(x)}{2a}\right)}{a}$$

$\downarrow$ 3042

$$-\frac{\tanh^3(x)}{3a}+\frac{i\left(\frac{i\left(\frac{(a^2+b^2)\tanh(x)}{a}+\frac{i\left(-\frac{ib(a^2+b^2)\int -i\tan(ix)dx}{a}-\frac{ib^5\int \frac{b-ia\tan\left(ix+\frac{\pi}{2}\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a(a^2-b^2)}+\frac{ia^4x}{a^2-b^2}\right)}{a}\right)}{a}-\frac{ib\tanh^2(x)}{2a}\right)}{a}$$

$\downarrow$ 26

$$-\frac{\tanh^3(x)}{3a}+\frac{i\left(\frac{i\left(\frac{(a^2+b^2)\tanh(x)}{a}+\frac{i\left(-\frac{b(a^2+b^2)\int \tan(ix)dx}{a}-\frac{ib^5\int \frac{b-ia\tan\left(ix+\frac{\pi}{2}\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a(a^2-b^2)}+\frac{ia^4x}{a^2-b^2}\right)}{a}\right)}{a}-\frac{ib\tanh^2(x)}{2a}\right)}{a}$$

$$\Big\downarrow \ 3956$$

$$-\frac{\tanh^3(x)}{3a}+\frac{i\left(\frac{i\left(\frac{(a^2+b^2)\tanh(x)}{a}+\frac{i\left(-\frac{ib^5\int\frac{b-ia\tan\left(ix+\frac{\pi}{2}\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a(a^2-b^2)}-\frac{ib(a^2+b^2)\log(\cosh(x))}{a}+\frac{ia^4x}{a^2-b^2}\right)}{a}\right)}{a}-\frac{ib\tanh^2(x)}{2a}\right)}{a}$$

$$\Big\downarrow \ 4013$$

$$-\frac{\tanh^3(x)}{3a}+\frac{i\left(\frac{i\left(\frac{(a^2+b^2)\tanh(x)}{a}+\frac{i\left(-\frac{ib(a^2+b^2)\log(\cosh(x))}{a}-\frac{ib^5\log(a\sinh(x)+b\cosh(x))}{a(a^2-b^2)}+\frac{ia^4x}{a^2-b^2}\right)}{a}\right)}{a}-\frac{ib\tanh^2(x)}{2a}\right)}{a}$$

input

```
Int[Tanh[x]^4/(a + b*Coth[x]),x]
```

output

```
-1/3*Tanh[x]^3/a + (I*(((-1/2*I)*b*Tanh[x]^2)/a + (I*((I*((I*a^4*x)/(a^2 -
 b^2) - (I*b*(a^2 + b^2)*Log[Cosh[x]])/a - (I*b^5*Log[b*Cosh[x] + a*Sinh[x
]])/(a*(a^2 - b^2))))/a + ((a^2 + b^2)*Tanh[x])/a))/a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4052
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Simp[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
 a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Integ
erQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4133
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n
 + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Sim
p[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
 + 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(
m + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m
, -1] &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10

method	result
parallelrisch	$\frac{-6b^5\ln(\tanh(x)a+b)+6\ln(1-\tanh(x))a^4b+(-2a^5+2a^3b^2)\tanh(x)^3+(3b\,a^4-3b^3a^2)\tanh(x)^2+(-6a^5+6b^4a)\tanh(}{6a^6-6a^4b^2}$
derivativedivides	$-\frac{b^5\ln(a+b\coth(x))}{a^4(a+b)(a-b)}+\frac{b}{2a^2\coth(x)^2}+\frac{-a^2-b^2}{a^3\coth(x)}-\frac{(a^2+b^2)b\ln(\coth(x))}{a^4}-\frac{1}{3a\coth(x)^3}-\frac{\ln(\coth(x)-1)}{2a+2b}$
default	$-\frac{b^5\ln(a+b\coth(x))}{a^4(a+b)(a-b)}+\frac{b}{2a^2\coth(x)^2}+\frac{-a^2-b^2}{a^3\coth(x)}-\frac{(a^2+b^2)b\ln(\coth(x))}{a^4}-\frac{1}{3a\coth(x)^3}-\frac{\ln(\coth(x)-1)}{2a+2b}$
risch	$\frac{x}{a+b}+\frac{2xb}{a^2}+\frac{2x\,b^3}{a^4}+\frac{2x\,b^5}{a^4(a^2-b^2)}+\frac{4\,e^{4x}a^2-2\,e^{4x}ab+2b^2e^{4x}+4\,e^{2x}a^2-2\,e^{2x}ab+4b^2e^{2x}+\frac{8a^2}{3}+2b^2}{a^3(e^{2x}+1)^3}-\frac{b\ln(e^{2x}}{a^2}$

input
```
int(tanh(x)^4/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
(-6*b^5*ln(tanh(x)*a+b)+6*ln(1-tanh(x))*a^4*b+(-2*a^5+2*a^3*b^2)*tanh(x)^3
+(3*a^4*b-3*a^2*b^3)*tanh(x)^2+(-6*a^5+6*a*b^4)*tanh(x)+6*a^4*x*(a+b))/(6*
a^6-6*a^4*b^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1294 vs. 2(93) = 186.

Time = 0.12 (sec) , antiderivative size = 1294, normalized size of antiderivative = 13.34

$$\int \frac{\tanh^4(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
1/3*(3*(a^5 + a^4*b)*x*cosh(x)^6 + 18*(a^5 + a^4*b)*x*cosh(x)*sinh(x)^5 +
3*(a^5 + a^4*b)*x*sinh(x)^6 + 8*a^5 - 2*a^3*b^2 - 6*a*b^4 + 3*(4*a^5 - 2*a
^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x)^4 + 3*
(4*a^5 - 2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 15*(a^5 + a^4*b)*x*co
sh(x)^2 + 3*(a^5 + a^4*b)*x)*sinh(x)^4 + 12*(5*(a^5 + a^4*b)*x*cosh(x)^3 +
 (4*a^5 - 2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 + a^4*b)*x)*c
osh(x))*sinh(x)^3 + 3*(4*a^5 - 2*a^4*b + 2*a^2*b^3 - 4*a*b^4 + 3*(a^5 + a^
4*b)*x)*cosh(x)^2 + 3*(15*(a^5 + a^4*b)*x*cosh(x)^4 + 4*a^5 - 2*a^4*b + 2*
a^2*b^3 - 4*a*b^4 + 6*(4*a^5 - 2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 +
 3*(a^5 + a^4*b)*x)*cosh(x)^2 + 3*(a^5 + a^4*b)*x)*sinh(x)^2 + 3*(a^5 + a^
4*b)*x - 3*(b^5*cosh(x)^6 + 6*b^5*cosh(x)*sinh(x)^5 + b^5*sinh(x)^6 + 3*b^
5*cosh(x)^4 + 3*b^5*cosh(x)^2 + b^5 + 3*(5*b^5*cosh(x)^2 + b^5)*sinh(x)^4
+ 4*(5*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3 + 3*(5*b^5*cosh(x)^4 + 6*b
^5*cosh(x)^2 + b^5)*sinh(x)^2 + 6*(b^5*cosh(x)^5 + 2*b^5*cosh(x)^3 + b^5*c
osh(x))*sinh(x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - 3*((
a^4*b - b^5)*cosh(x)^6 + 6*(a^4*b - b^5)*cosh(x)*sinh(x)^5 + (a^4*b - b^5)
*sinh(x)^6 + a^4*b - b^5 + 3*(a^4*b - b^5)*cosh(x)^4 + 3*(a^4*b - b^5 + 5*
(a^4*b - b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^4*b - b^5)*cosh(x)^3 + 3*(a^4
*b - b^5)*cosh(x))*sinh(x)^3 + 3*(a^4*b - b^5)*cosh(x)^2 + 3*(a^4*b - b^5
+ 5*(a^4*b - b^5)*cosh(x)^4 + 6*(a^4*b - b^5)*cosh(x)^2)*sinh(x)^2 + 6*...
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{a+b\coth(x)}\,dx=\int \frac{\tanh^4{(x)}}{a+b\coth{(x)}}\,dx$$

input
```
integrate(tanh(x)**4/(a+b*coth(x)),x)
```

output
```
Integral(tanh(x)**4/(a + b*coth(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.51

$$\int \frac{\tanh^4(x)}{a+b\coth(x)}\,dx = -\frac{b^5\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^6-a^4b^2} - \frac{2\left(4a^2+3b^2+3\left(2a^2+ab+2b^2\right)e^{(-2x)}+3\left(2a^2+ab+b^2\right)e^{(-4x)}\right)}{3\left(3a^3e^{(-2x)}+3a^3e^{(-4x)}+a^3e^{(-6x)}+a^3\right)} + \frac{x}{a+b} - \frac{\left(a^2b+b^3\right)\log\left(e^{(-2x)}+1\right)}{a^4}$$

input
```
integrate(tanh(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
-b^5*log(-(a - b)*e^(-2*x) + a + b)/(a^6 - a^4*b^2) - 2/3*(4*a^2 + 3*b^2 +
 3*(2*a^2 + a*b + 2*b^2)*e^(-2*x) + 3*(2*a^2 + a*b + b^2)*e^(-4*x))/(3*a^3
*e^(-2*x) + 3*a^3*e^(-4*x) + a^3*e^(-6*x) + a^3) + x/(a + b) - (a^2*b + b^
3)*log(e^(-2*x) + 1)/a^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.45

$$\int \frac{\tanh^4(x)}{a+b\coth(x)}\,dx = -\frac{b^5\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^6-a^4b^2} + \frac{x}{a-b} - \frac{\left(a^2b+b^3\right)\log\left(e^{(2x)}+1\right)}{a^4} + \frac{2\left(4a^3+3ab^2+3\left(2a^3-a^2b+ab^2\right)e^{(4x)}+3\left(2a^3-a^2b+2ab^2\right)e^{(2x)}\right)}{3a^4\left(e^{(2x)}+1\right)^3}$$

input
```
integrate(tanh(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

output
```
-b^5*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^6 - a^4*b^2) + x/(a - b) -
 (a^2*b + b^3)*log(e^(2*x) + 1)/a^4 + 2/3*(4*a^3 + 3*a*b^2 + 3*(2*a^3 - a^
2*b + a*b^2)*e^(4*x) + 3*(2*a^3 - a^2*b + 2*a*b^2)*e^(2*x))/(a^4*(e^(2*x)
+ 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.78 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.68

$$\int \frac{\tanh^4(x)}{a+b\coth(x)}\,dx = \frac{8}{3\,a\,\left(3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1\right)}+\frac{x}{a-b} - \frac{b^5\,\ln\left(b-a+a\,e^{2\,x}+b\,e^{2\,x}\right)}{a^6-a^4\,b^2}-\frac{\ln\left(e^{2\,x}+1\right)\,\left(a^2\,b+b^3\right)}{a^4} + \frac{2\,\left(2\,a^3+a^2\,b+b^3\right)}{a^3\,\left(a+b\right)\,\left(e^{2\,x}+1\right)}-\frac{2\,\left(2\,a^2+a\,b-b^2\right)}{a^2\,\left(a+b\right)\,\left(2\,e^{2\,x}+e^{4\,x}+1\right)}$$

input
```
int(tanh(x)^4/(a + b*coth(x)),x)
```

output
```
8/(3*a*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) + x/(a - b) - (b^5*log(b
- a + a*exp(2*x) + b*exp(2*x)))/(a^6 - a^4*b^2) - (log(exp(2*x) + 1)*(a^2*
b + b^3))/a^4 + (2*(a^2*b + 2*a^3 + b^3))/(a^3*(a + b)*(exp(2*x) + 1)) - (
2*(a*b + 2*a^2 - b^2))/(a^2*(a + b)*(2*exp(2*x) + exp(4*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 518, normalized size of antiderivative = 5.34

$$\int \frac{\tanh^4(x)}{a+b\coth(x)}\,dx = \frac{2e^{6x}a^4b+2e^{6x}a^3b^2-2e^{6x}a^2b^3+2e^{6x}a\,b^4-9e^{4x}\log(e^{2x}a+e^{2x}b-a+b)\,b^5+9e^{4x}a^5x+6e^{2x}a^3b^2-6e^{2x}\cdots}{\cdots}$$

input
```
int(tanh(x)^4/(a+b*coth(x)),x)
```

output

```
( - 3*e**(6*x)*log(e**(2*x) + 1)*a**4*b + 3*e**(6*x)*log(e**(2*x) + 1)*b**
5 - 3*e**(6*x)*log(e**(2*x)*a + e**(2*x)*b - a + b)*b**5 + 3*e**(6*x)*a**5
*x - 4*e**(6*x)*a**5 + 3*e**(6*x)*a**4*b*x + 2*e**(6*x)*a**4*b + 2*e**(6*x
)*a**3*b**2 - 2*e**(6*x)*a**2*b**3 + 2*e**(6*x)*a*b**4 - 9*e**(4*x)*log(e*
*(2*x) + 1)*a**4*b + 9*e**(4*x)*log(e**(2*x) + 1)*b**5 - 9*e**(4*x)*log(e*
*(2*x)*a + e**(2*x)*b - a + b)*b**5 + 9*e**(4*x)*a**5*x + 9*e**(4*x)*a**4*
b*x - 9*e**(2*x)*log(e**(2*x) + 1)*a**4*b + 9*e**(2*x)*log(e**(2*x) + 1)*b
**5 - 9*e**(2*x)*log(e**(2*x)*a + e**(2*x)*b - a + b)*b**5 + 9*e**(2*x)*a*
*5*x + 9*e**(2*x)*a**4*b*x + 6*e**(2*x)*a**3*b**2 - 6*e**(2*x)*a*b**4 - 3*
log(e**(2*x) + 1)*a**4*b + 3*log(e**(2*x) + 1)*b**5 - 3*log(e**(2*x)*a + e
**(2*x)*b - a + b)*b**5 + 3*a**5*x + 4*a**5 + 3*a**4*b*x + 2*a**4*b - 2*a*
*2*b**3 - 4*a*b**4)/(3*a**4*(e**(6*x)*a**2 - e**(6*x)*b**2 + 3*e**(4*x)*a*
*2 - 3*e**(4*x)*b**2 + 3*e**(2*x)*a**2 - 3*e**(2*x)*b**2 + a**2 - b**2))
```

3.146 $\int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 76

$$\int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx = -\frac{bx}{a^2-b^2}+\frac{(a^2+b^2)\log(\cosh(x))}{a^3}+\frac{b^4\log(b\cosh(x)+a\sinh(x))}{a^3(a^2-b^2)}+\frac{b\tanh(x)}{a^2}-\frac{\tanh^2(x)}{2a}$$

output
```
-b*x/(a^2-b^2)+(a^2+b^2)*ln(cosh(x))/a^3+b^4*ln(b*cosh(x)+a*sinh(x))/a^3/(
a^2-b^2)+b*tanh(x)/a^2-1/2*tanh(x)^2/a
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.20

$$\int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx = -\frac{\log(1-\coth(x))}{2(a+b)}-\frac{\log(1+\coth(x))}{2(a-b)}+\frac{b^4\log(a+b\coth(x))}{a^3(a^2-b^2)}-\frac{(a^2+b^2)\log(\tanh(x))}{a^3}+\frac{b\tanh(x)}{a^2}-\frac{\tanh^2(x)}{2a}$$

input
```
Integrate[Tanh[x]^3/(a + b*Coth[x]),x]
```

output
```
-1/2*Log[1 - Coth[x]]/(a + b) - Log[1 + Coth[x]]/(2*(a - b)) + (b^4*Log[a
+ b*Coth[x]])/(a^3*(a^2 - b^2)) - ((a^2 + b^2)*Log[Tanh[x]])/a^3 + (b*Tanh
[x])/a^2 - Tanh[x]^2/(2*a)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.43, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.231$, Rules used = {3042, 26, 4052, 27, 3042, 25, 4132, 25, 3042, 26, 4135, 26, 3042, 26, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{a+b\coth(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\tan\left(\frac{\pi}{2}+ix\right)^3\left(a-ib\tan\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\tan\left(ix+\frac{\pi}{2}\right)^3\left(a-ib\tan\left(ix+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4052}$$

$$-i\left(-\frac{\int \frac{2i(-b\coth^2(x)-a\coth(x)+b)\tanh^2(x)}{a+b\coth(x)}dx}{2a}-\frac{i\tanh^2(x)}{2a}\right)$$

$$\Big\downarrow \text{27}$$

$$-i\left(-\frac{i\int \frac{(-b\coth^2(x)-a\coth(x)+b)\tanh^2(x)}{a+b\coth(x)}dx}{a}-\frac{i\tanh^2(x)}{2a}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{i\int -\frac{b\tan\left(ix+\frac{\pi}{2}\right)^2+ia\tan\left(ix+\frac{\pi}{2}\right)+b}{\tan\left(ix+\frac{\pi}{2}\right)^2\left(a-ib\tan\left(ix+\frac{\pi}{2}\right)\right)}dx}{a}-\frac{i\tanh^2(x)}{2a}\right)$$

$$\Big\downarrow \ 25$$

$$-i\left(\frac{i\int \frac{b\tan\left(ix+\frac{\pi}{2}\right)^2+ia\tan\left(ix+\frac{\pi}{2}\right)+b}{\tan\left(ix+\frac{\pi}{2}\right)^2\left(a-ib\tan\left(ix+\frac{\pi}{2}\right)\right)}dx}{a}-\frac{i\tanh^2(x)}{2a}\right)$$

$$\Big\downarrow \ 4132$$

$$-i\left(\frac{i\left(\frac{b\tanh(x)}{a}-\frac{\int -\frac{\left(a^2+b^2-b^2\coth^2(x)\right)\tanh(x)}{a+b\coth(x)}dx}{a}\right)}{a}-\frac{i\tanh^2(x)}{2a}\right)$$

$$\Big\downarrow \ 25$$

$$-i\left(\frac{i\left(\frac{\int \frac{\left(a^2+b^2-b^2\coth^2(x)\right)\tanh(x)}{a+b\coth(x)}dx}{a}+\frac{b\tanh(x)}{a}\right)}{a}-\frac{i\tanh^2(x)}{2a}\right)$$

$$\Big\downarrow \ 3042$$

$$-i\left(\frac{i\left(\frac{b\tanh(x)}{a}+\frac{\int \frac{i\left(a^2+b^2+b^2\tan\left(ix+\frac{\pi}{2}\right)^2\right)}{\tan\left(ix+\frac{\pi}{2}\right)\left(a-ib\tan\left(ix+\frac{\pi}{2}\right)\right)}dx}{a}\right)}{a}-\frac{i\tanh^2(x)}{2a}\right)$$

$$\Big\downarrow \ 26$$

$$-i\left(\frac{i\left(\frac{b\tanh(x)}{a}+\frac{i\int\frac{a^2+b^2+b^2\tan\left(ix+\frac{\pi}{2}\right)^2}{\tan\left(ix+\frac{\pi}{2}\right)\left(a-ib\tan\left(ix+\frac{\pi}{2}\right)\right)}dx}{a}\right)}{a}-\frac{i\tanh^2(x)}{2a}\right)$$

$$\Big\downarrow \text{4135}$$

$$-i\left(\frac{i\left(\frac{b\tanh(x)}{a}+\frac{i\left(\frac{\left(a^2+b^2\right)\int -i\tanh(x)dx}{a}+\frac{b^4\int -\frac{i(b+a\coth(x))}{a+b\coth(x)}dx}{a\left(a^2-b^2\right)}+\frac{ia^2bx}{a^2-b^2}\right)}{a}\right)}{a}-\frac{i\tanh^2(x)}{2a}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\left(\frac{b\tanh(x)}{a}+\frac{i\left(-\frac{i\left(a^2+b^2\right)\int\tanh(x)dx}{a}-\frac{ib^4\int\frac{b+a\coth(x)}{a+b\coth(x)}dx}{a\left(a^2-b^2\right)}+\frac{ia^2bx}{a^2-b^2}\right)}{a}\right)}{a}-\frac{i\tanh^2(x)}{2a}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\left(\frac{b\tanh(x)}{a}+\frac{i\left(-\frac{i\left(a^2+b^2\right)\int -i\tan(ix)dx}{a}-\frac{ib^4\int\frac{b-ia\tan\left(ix+\frac{\pi}{2}\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a\left(a^2-b^2\right)}+\frac{ia^2bx}{a^2-b^2}\right)}{a}\right)}{a}-\frac{i\tanh^2(x)}{2a}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\left(\frac{b\tanh(x)}{a}+\frac{i\left(-\frac{(a^2+b^2)\int\tan(ix)dx}{a}-\frac{ib^4\int\frac{b-ia\tan\left(ix+\frac{\pi}{2}\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a(a^2-b^2)}+\frac{ia^2bx}{a^2-b^2}\right)}{a}\right)}{a}-\frac{i\tanh^2(x)}{2a}\right)$$

$$\Big\downarrow \text{3956}$$

$$-i\left(\frac{i\left(\frac{b\tanh(x)}{a}+\frac{i\left(-\frac{ib^4\int\frac{b-ia\tan\left(ix+\frac{\pi}{2}\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a(a^2-b^2)}+\frac{ia^2bx}{a^2-b^2}-\frac{i(a^2+b^2)\log(\cosh(x))}{a}\right)}{a}\right)}{a}-\frac{i\tanh^2(x)}{2a}\right)$$

$$\Big\downarrow \text{4013}$$

$$-i\left(\frac{i\left(\frac{b\tanh(x)}{a}+\frac{i\left(\frac{ia^2bx}{a^2-b^2}-\frac{i(a^2+b^2)\log(\cosh(x))}{a}-\frac{ib^4\log(a\sinh(x)+b\cosh(x))}{a(a^2-b^2)}\right)}{a}\right)}{a}-\frac{i\tanh^2(x)}{2a}\right)$$

input
```
Int[Tanh[x]^3/(a + b*Coth[x]),x]
```

output
```
(-I)*(((-1/2*I)*Tanh[x]^2)/a + (I*((I*((I*a^2*b*x)/(a^2 - b^2) - (I*(a^2 +
 b^2)*Log[Cosh[x]])/a - (I*b^4*Log[b*Cosh[x] + a*Sinh[x]])/(a*(a^2 - b^2))
))/a + (b*Tanh[x])/a))/a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4052
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Simp[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
 a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Integ
erQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4135
```
Int[((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/(((a_) + (b_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(a*(
A*c - c*C) - b*(A*d - C*d))*(x/((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^
2 + a^2*C)/((b*c - a*d)*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[
e + f*x]), x], x] - Simp[(c^2*C + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d
 - c*Tan[e + f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

method	result
parallelrisch	$\frac{2b^{4}\ln(\tanh(x)a+b)-2\left(a^{3}\ln(1-\tanh(x))+(a+b)\left(\frac{a(a-b)\tanh(x)^{2}}{2}-b(a-b)\tanh(x)+a^{2}x\right)\right)a}{2a^{5}-2a^{3}b^{2}}$
derivativedivides	$\frac{b^{4}\ln(a+b\coth(x))}{a^{3}(a+b)(a-b)}+\frac{b}{a^{2}\coth(x)}-\frac{(-a^{2}-b^{2})\ln(\coth(x))}{a^{3}}-\frac{1}{2a\coth(x)^{2}}-\frac{\ln(\coth(x)-1)}{2a+2b}-\frac{\ln(1+\coth(x))}{2a-2b}$
default	$\frac{b^{4}\ln(a+b\coth(x))}{a^{3}(a+b)(a-b)}+\frac{b}{a^{2}\coth(x)}-\frac{(-a^{2}-b^{2})\ln(\coth(x))}{a^{3}}-\frac{1}{2a\coth(x)^{2}}-\frac{\ln(\coth(x)-1)}{2a+2b}-\frac{\ln(1+\coth(x))}{2a-2b}$
risch	$\frac{x}{a+b}-\frac{2x}{a}-\frac{2x\,b^{2}}{a^{3}}-\frac{2x\,b^{4}}{a^{3}(a^{2}-b^{2})}+\frac{2\,e^{2x}a-2\,e^{2x}b-2b}{(e^{2x}+1)^{2}a^{2}}+\frac{\ln(e^{2x}+1)}{a}+\frac{\ln(e^{2x}+1)\,b^{2}}{a^{3}}+\frac{b^{4}\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{a^{3}(a^{2}-b^{2})}$

input
```
int(tanh(x)^3/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
(2*b^4*ln(tanh(x)*a+b)-2*(a^3*ln(1-tanh(x))+(a+b)*(1/2*a*(a-b)*tanh(x)^2-b
*(a-b)*tanh(x)+a^2*x))*a)/(2*a^5-2*a^3*b^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. 2(74) = 148.

Time = 0.12 (sec) , antiderivative size = 637, normalized size of antiderivative = 8.38

$$\int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
-((a^4 + a^3*b)*x*cosh(x)^4 + 4*(a^4 + a^3*b)*x*cosh(x)*sinh(x)^3 + (a^4 +
 a^3*b)*x*sinh(x)^4 + 2*a^3*b - 2*a*b^3 - 2*(a^4 - a^3*b - a^2*b^2 + a*b^3
 - (a^4 + a^3*b)*x)*cosh(x)^2 - 2*(a^4 - a^3*b - a^2*b^2 + a*b^3 - 3*(a^4
+ a^3*b)*x*cosh(x)^2 - (a^4 + a^3*b)*x)*sinh(x)^2 + (a^4 + a^3*b)*x - (b^4
*cosh(x)^4 + 4*b^4*cosh(x)*sinh(x)^3 + b^4*sinh(x)^4 + 2*b^4*cosh(x)^2 + b
^4 + 2*(3*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 + b^4*cosh(x))
*sinh(x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - ((a^4 - b^4
)*cosh(x)^4 + 4*(a^4 - b^4)*cosh(x)*sinh(x)^3 + (a^4 - b^4)*sinh(x)^4 + a^
4 - b^4 + 2*(a^4 - b^4)*cosh(x)^2 + 2*(a^4 - b^4 + 3*(a^4 - b^4)*cosh(x)^2
)*sinh(x)^2 + 4*((a^4 - b^4)*cosh(x)^3 + (a^4 - b^4)*cosh(x))*sinh(x))*log
(2*cosh(x)/(cosh(x) - sinh(x))) + 4*((a^4 + a^3*b)*x*cosh(x)^3 - (a^4 - a^
3*b - a^2*b^2 + a*b^3 - (a^4 + a^3*b)*x)*cosh(x))*sinh(x))/(a^5 - a^3*b^2
+ (a^5 - a^3*b^2)*cosh(x)^4 + 4*(a^5 - a^3*b^2)*cosh(x)*sinh(x)^3 + (a^5 -
 a^3*b^2)*sinh(x)^4 + 2*(a^5 - a^3*b^2)*cosh(x)^2 + 2*(a^5 - a^3*b^2 + 3*(
a^5 - a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 - a^3*b^2)*cosh(x)^3 + (a^5
- a^3*b^2)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx=\int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx$$

input
```
integrate(tanh(x)**3/(a+b*coth(x)),x)
```

output
```
Integral(tanh(x)**3/(a + b*coth(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.24

$$\int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx=\frac{b^4\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^5-a^3b^2}+\frac{2\left((a+b)e^{(-2x)}+b\right)}{2a^2e^{(-2x)}+a^2e^{(-4x)}+a^2}+\frac{x}{a+b}+\frac{(a^2+b^2)\log\left(e^{(-2x)}+1\right)}{a^3}$$

input
```
integrate(tanh(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
b^4*log(-(a - b)*e^(-2*x) + a + b)/(a^5 - a^3*b^2) + 2*((a + b)*e^(-2*x) +
 b)/(2*a^2*e^(-2*x) + a^2*e^(-4*x) + a^2) + x/(a + b) + (a^2 + b^2)*log(e^
(-2*x) + 1)/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.28

$$\int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx = \frac{b^4\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^5-a^3b^2} - \frac{x}{a-b} + \frac{(a^2+b^2)\log\left(e^{(2x)}+1\right)}{a^3} - \frac{2\left(ab-(a^2-ab)e^{(2x)}\right)}{a^3\left(e^{(2x)}+1\right)^2}$$

input
```
integrate(tanh(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

output
```
b^4*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^5 - a^3*b^2) - x/(a - b) +
(a^2 + b^2)*log(e^(2*x) + 1)/a^3 - 2*(a*b - (a^2 - a*b)*e^(2*x))/(a^3*(e^(
2*x) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 2.82 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.46

$$\int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx = \frac{\ln\left(e^{2x}+1\right)\,\left(a^2+b^2\right)}{a^3} - \frac{x}{a-b} - \frac{2}{a\,\left(2\,e^{2x}+e^{4x}+1\right)} + \frac{b^4\,\ln\left(b-a+a\,e^{2x}+b\,e^{2x}\right)}{a^5-a^3\,b^2} + \frac{2\,\left(a^2-b^2\right)}{a^2\,\left(a+b\right)\,\left(e^{2x}+1\right)}$$

input
```
int(tanh(x)^3/(a + b*coth(x)),x)
```

output
```
(log(exp(2*x) + 1)*(a^2 + b^2))/a^3 - x/(a - b) - 2/(a*(2*exp(2*x) + exp(4
*x) + 1)) + (b^4*log(b - a + a*exp(2*x) + b*exp(2*x)))/(a^5 - a^3*b^2) + (
2*(a^2 - b^2))/(a^2*(a + b)*(exp(2*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 359, normalized size of antiderivative = 4.72

$$\int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx = \frac{e^{4x}\log(e^{2x}+1)\,a^4 - e^{4x}\log(e^{2x}+1)\,b^4 + e^{4x}\log(e^{2x}a+e^{2x}b-a+b)\,b^4 - e^{4x}a^4x - e^{4x}a^4 - e^{4x}a^3bx + e^{4x}\cdots}{\cdots}$$

input
```
int(tanh(x)^3/(a+b*coth(x)),x)
```

output
```
(e**(4*x)*log(e**(2*x) + 1)*a**4 - e**(4*x)*log(e**(2*x) + 1)*b**4 + e**(4
*x)*log(e**(2*x)*a + e**(2*x)*b - a + b)*b**4 - e**(4*x)*a**4*x - e**(4*x)
*a**4 - e**(4*x)*a**3*b*x + e**(4*x)*a**3*b + e**(4*x)*a**2*b**2 - e**(4*x
)*a*b**3 + 2*e**(2*x)*log(e**(2*x) + 1)*a**4 - 2*e**(2*x)*log(e**(2*x) + 1
)*b**4 + 2*e**(2*x)*log(e**(2*x)*a + e**(2*x)*b - a + b)*b**4 - 2*e**(2*x)
*a**4*x - 2*e**(2*x)*a**3*b*x + log(e**(2*x) + 1)*a**4 - log(e**(2*x) + 1)
*b**4 + log(e**(2*x)*a + e**(2*x)*b - a + b)*b**4 - a**4*x - a**4 - a**3*b
*x - a**3*b + a**2*b**2 + a*b**3)/(a**3*(e**(4*x)*a**2 - e**(4*x)*b**2 + 2
*e**(2*x)*a**2 - 2*e**(2*x)*b**2 + a**2 - b**2))
```

3.147 $\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx=\frac{ax}{a^2-b^2}-\frac{b\log(\cosh(x))}{a^2}-\frac{b^3\log(b\cosh(x)+a\sinh(x))}{a^2(a^2-b^2)}-\frac{\tanh(x)}{a}$$

output
```
a*x/(a^2-b^2)-b*ln(cosh(x))/a^2-b^3*ln(b*cosh(x)+a*sinh(x))/a^2/(a^2-b^2)-
tanh(x)/a
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23

$$\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx=-\frac{\log(1-\coth(x))}{2(a+b)}+\frac{\log(1+\coth(x))}{2(a-b)}-\frac{b^3\log(a+b\coth(x))}{a^2(a^2-b^2)}+\frac{b\log(\tanh(x))}{a^2}-\frac{\tanh(x)}{a}$$

input
```
Integrate[Tanh[x]^2/(a + b*Coth[x]),x]
```

output
```
-1/2*Log[1 - Coth[x]]/(a + b) + Log[1 + Coth[x]]/(2*(a - b)) - (b^3*Log[a
+ b*Coth[x]])/(a^2*(a^2 - b^2)) + (b*Log[Tanh[x]])/a^2 - Tanh[x]/a
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.28, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$, Rules used = {3042, 25, 4052, 25, 3042, 26, 4134, 26, 3042, 26, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)}{a+b\coth(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan\left(\frac{\pi}{2}+ix\right)^2\left(a-ib\tan\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\tan\left(ix+\frac{\pi}{2}\right)^2\left(a-ib\tan\left(ix+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4052}$$

$$\frac{\int -\frac{(-b\coth^2(x)-a\coth(x)+b)\tanh(x)}{a+b\coth(x)}dx}{a}-\frac{\tanh(x)}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{(-b\coth^2(x)-a\coth(x)+b)\tanh(x)}{a+b\coth(x)}dx}{a}-\frac{\tanh(x)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(x)}{a}-\frac{\int \frac{i\left(b\tan\left(ix+\frac{\pi}{2}\right)^2+ia\tan\left(ix+\frac{\pi}{2}\right)+b\right)}{\tan\left(ix+\frac{\pi}{2}\right)\left(a-ib\tan\left(ix+\frac{\pi}{2}\right)\right)}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\tanh(x)}{a}-\frac{i\int \frac{b\tan\left(ix+\frac{\pi}{2}\right)^2+ia\tan\left(ix+\frac{\pi}{2}\right)+b}{\tan\left(ix+\frac{\pi}{2}\right)\left(a-ib\tan\left(ix+\frac{\pi}{2}\right)\right)}dx}{a}$$

$$\Big\downarrow \text{4134}$$

$$-\frac{\tanh(x)}{a}-\frac{i\left(\frac{b^3\int -\frac{i(b+a\coth(x))}{a+b\coth(x)}dx}{a(a^2-b^2)}+\frac{b\int -i\tanh(x)dx}{a}+\frac{ia^2x}{a^2-b^2}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\tanh(x)}{a}-\frac{i\left(-\frac{ib^3\int \frac{b+a\coth(x)}{a+b\coth(x)}dx}{a(a^2-b^2)}-\frac{ib\int \tanh(x)dx}{a}+\frac{ia^2x}{a^2-b^2}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(x)}{a}-\frac{i\left(-\frac{ib^3\int \frac{b-ia\tan\left(ix+\frac{\pi}{2}\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a(a^2-b^2)}-\frac{ib\int -i\tan(ix)dx}{a}+\frac{ia^2x}{a^2-b^2}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\tanh(x)}{a}-\frac{i\left(-\frac{ib^3\int \frac{b-ia\tan\left(ix+\frac{\pi}{2}\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a(a^2-b^2)}-\frac{b\int \tan(ix)dx}{a}+\frac{ia^2x}{a^2-b^2}\right)}{a}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{\tanh(x)}{a}-\frac{i\left(-\frac{ib^3\int \frac{b-ia\tan\left(ix+\frac{\pi}{2}\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a(a^2-b^2)}+\frac{ia^2x}{a^2-b^2}-\frac{ib\log(\cosh(x))}{a}\right)}{a}$$

$$\Big\downarrow \text{4013}$$

$$-\frac{\tanh(x)}{a}-\frac{i\left(\frac{ia^2x}{a^2-b^2}-\frac{ib^3\log(a\sinh(x)+b\cosh(x))}{a(a^2-b^2)}-\frac{ib\log(\cosh(x))}{a}\right)}{a}$$

input
```
Int[Tanh[x]^2/(a + b*Coth[x]),x]
```

output
```
((-I)*((I*a^2*x)/(a^2 - b^2) - (I*b*Log[Cosh[x]])/a - (I*b^3*Log[b*Cosh[x]
 + a*Sinh[x]])/(a*(a^2 - b^2))))/a - Tanh[x]/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4052
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Simp[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
 a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Integ
erQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

method	result	size
parallelrisch	$\frac{\ln(1-\tanh(x))a^2b-b^3\ln(\tanh(x)a+b)+a^3x+b\,a^2x-\tanh(x)a^3+\tanh(x)a\,b^2}{a^2(a^2-b^2)}$	66
derivativedivides	$-\frac{b^3\ln(a+b\coth(x))}{a^2(a+b)(a-b)}-\frac{b\ln(\coth(x))}{a^2}-\frac{1}{a\coth(x)}-\frac{\ln(\coth(x)-1)}{2a+2b}+\frac{\ln(1+\coth(x))}{2a-2b}$	78
default	$-\frac{b^3\ln(a+b\coth(x))}{a^2(a+b)(a-b)}-\frac{b\ln(\coth(x))}{a^2}-\frac{1}{a\coth(x)}-\frac{\ln(\coth(x)-1)}{2a+2b}+\frac{\ln(1+\coth(x))}{2a-2b}$	78
risch	$\frac{x}{a+b}+\frac{2xb}{a^2}+\frac{2x\,b^3}{a^2(a^2-b^2)}+\frac{2}{a(e^{2x}+1)}-\frac{b\ln(e^{2x}+1)}{a^2}-\frac{b^3\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{a^2(a^2-b^2)}$	99

input
```
int(tanh(x)^2/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
(ln(1-tanh(x))*a^2*b-b^3*ln(tanh(x)*a+b)+a^3*x+b*a^2*x-tanh(x)*a^3+tanh(x)
*a*b^2)/a^2/(a^2-b^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(60) = 120$.

Time = 0.10 (sec) , antiderivative size = 264, normalized size of antiderivative = 4.40

$$\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx$$
$$=\frac{(a^3+a^2b)x\cosh(x)^2+2(a^3+a^2b)x\cosh(x)\sinh(x)+(a^3+a^2b)x\sinh(x)^2+2a^3-2ab^2+(a^3+a^2b}{}$$

input
```
integrate(tanh(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
((a^3 + a^2*b)*x*cosh(x)^2 + 2*(a^3 + a^2*b)*x*cosh(x)*sinh(x) + (a^3 + a^
2*b)*x*sinh(x)^2 + 2*a^3 - 2*a*b^2 + (a^3 + a^2*b)*x - (b^3*cosh(x)^2 + 2*
b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2 + b^3)*log(2*(b*cosh(x) + a*sinh(x))/(
cosh(x) - sinh(x))) - (a^2*b - b^3 + (a^2*b - b^3)*cosh(x)^2 + 2*(a^2*b -
b^3)*cosh(x)*sinh(x) + (a^2*b - b^3)*sinh(x)^2)*log(2*cosh(x)/(cosh(x) - s
inh(x))))/(a^4 - a^2*b^2 + (a^4 - a^2*b^2)*cosh(x)^2 + 2*(a^4 - a^2*b^2)*c
osh(x)*sinh(x) + (a^4 - a^2*b^2)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx = \int \frac{\tanh^2{(x)}}{a+b\coth{(x)}}\,dx$$

input
```
integrate(tanh(x)**2/(a+b*coth(x)),x)
```

output
```
Integral(tanh(x)**2/(a + b*coth(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx = -\frac{b^3\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^4-a^2b^2} + \frac{x}{a+b} - \frac{b\log\left(e^{(-2x)}+1\right)}{a^2} - \frac{2}{ae^{(-2x)}+a}$$

input
```
integrate(tanh(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
-b^3*log(-(a - b)*e^(-2*x) + a + b)/(a^4 - a^2*b^2) + x/(a + b) - b*log(e^
(-2*x) + 1)/a^2 - 2/(a*e^(-2*x) + a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23

$$\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx = -\frac{b^3\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^4-a^2b^2} + \frac{x}{a-b} - \frac{b\log\left(e^{(2x)}+1\right)}{a^2} + \frac{2}{a(e^{(2x)}+1)}$$

input
```
integrate(tanh(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

output
```
-b^3*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^4 - a^2*b^2) + x/(a - b) -
 b*log(e^(2*x) + 1)/a^2 + 2/(a*(e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

$$\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx = \frac{2}{a\,\left(\mathrm{e}^{2x}+1\right)} + \frac{x}{a-b} - \frac{b^3\,\ln\left(b-a+a\,\mathrm{e}^{2x}+b\,\mathrm{e}^{2x}\right)}{a^4-a^2\,b^2} - \frac{b\,\ln\left(\mathrm{e}^{2x}+1\right)}{a^2}$$

input
```
int(tanh(x)^2/(a + b*coth(x)),x)
```

output
```
2/(a*(exp(2*x) + 1)) + x/(a - b) - (b^3*log(b - a + a*exp(2*x) + b*exp(2*x
)))/(a^4 - a^2*b^2) - (b*log(exp(2*x) + 1))/a^2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 205, normalized size of antiderivative = 3.42

$$\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx = \frac{-e^{2x}\log(e^{2x}+1)\,a^2b + e^{2x}\log(e^{2x}+1)\,b^3 - e^{2x}\log(e^{2x}a+e^{2x}b-a+b)\,b^3 + e^{2x}a^3x - 2e^{2x}a^3 + e^{2x}a^2bx -}{a^2\left(e^{2x}a^2 - e^{2x}b^2 + a^2\right.}$$

input
```
int(tanh(x)^2/(a+b*coth(x)),x)
```

output
```
( - e**(2*x)*log(e**(2*x) + 1)*a**2*b + e**(2*x)*log(e**(2*x) + 1)*b**3 -
e**(2*x)*log(e**(2*x)*a + e**(2*x)*b - a + b)*b**3 + e**(2*x)*a**3*x - 2*e
**(2*x)*a**3 + e**(2*x)*a**2*b*x + 2*e**(2*x)*a*b**2 - log(e**(2*x) + 1)*a
**2*b + log(e**(2*x) + 1)*b**3 - log(e**(2*x)*a + e**(2*x)*b - a + b)*b**3
 + a**3*x + a**2*b*x)/(a**2*(e**(2*x)*a**2 - e**(2*x)*b**2 + a**2 - b**2))
```

3.148 $\int \frac{\tanh(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 51

$$\int \frac{\tanh(x)}{a+b\coth(x)}\,dx = -\frac{bx}{a^2-b^2}+\frac{\log(\cosh(x))}{a}+\frac{b^2\log(b\cosh(x)+a\sinh(x))}{a\,(a^2-b^2)}$$

output
```
-b*x/(a^2-b^2)+ln(cosh(x))/a+b^2*ln(b*cosh(x)+a*sinh(x))/a/(a^2-b^2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{\tanh(x)}{a+b\coth(x)}\,dx = -\frac{\log(1-\coth(x))}{2(a+b)}-\frac{\log(1+\coth(x))}{2(a-b)}+\frac{b^2\log(a+b\coth(x))}{a\,(a^2-b^2)}-\frac{\log(\tanh(x))}{a}$$

input
```
Integrate[Tanh[x]/(a + b*Coth[x]),x]
```

output
```
-1/2*Log[1 - Coth[x]]/(a + b) - Log[1 + Coth[x]]/(2*(a - b)) + (b^2*Log[a
+ b*Coth[x]])/(a*(a^2 - b^2)) - Log[Tanh[x]]/a
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {3042, 26, 4054, 26, 3042, 26, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{a+b\coth(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan\left(\frac{\pi}{2}+ix\right)\left(a-ib\tan\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\tan\left(ix+\frac{\pi}{2}\right)\left(a-ib\tan\left(ix+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4054}$$

$$i\left(\frac{b^2\int -\frac{i(b+a\coth(x))}{a+b\coth(x)}dx}{a\left(a^2-b^2\right)}+\frac{\int -i\tanh(x)dx}{a}+\frac{ibx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{ib^2\int \frac{b+a\coth(x)}{a+b\coth(x)}dx}{a\left(a^2-b^2\right)}-\frac{i\int \tanh(x)dx}{a}+\frac{ibx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-\frac{ib^2\int \frac{b-ia\tan\left(ix+\frac{\pi}{2}\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a\left(a^2-b^2\right)}-\frac{i\int -i\tan(ix)dx}{a}+\frac{ibx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{ib^2\int \frac{b-ia\tan\left(ix+\frac{\pi}{2}\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a\left(a^2-b^2\right)}-\frac{\int \tan(ix)dx}{a}+\frac{ibx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3956}$$

$$i\left(-\frac{ib^2\int\frac{b-ia\tan\left(ix+\frac{\pi}{2}\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a\left(a^2-b^2\right)}+\frac{ibx}{a^2-b^2}-\frac{i\log(\cosh(x))}{a}\right)$$

$$\Big\downarrow \text{4013}$$

$$i\left(\frac{ibx}{a^2-b^2}-\frac{ib^2\log(a\sinh(x)+b\cosh(x))}{a\left(a^2-b^2\right)}-\frac{i\log(\cosh(x))}{a}\right)$$

input
```
Int[Tanh[x]/(a + b*Coth[x]),x]
```

output
```
I*((I*b*x)/(a^2 - b^2) - (I*Log[Cosh[x]])/a - (I*b^2*Log[b*Cosh[x] + a*Sin
h[x]])/(a*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4054
```
Int[1/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f
_.)*(x_)])), x_Symbol] :> Simp[(a*c - b*d)*(x/((a^2 + b^2)*(c^2 + d^2))), x
] + (Simp[b^2/((b*c - a*d)*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*T
an[e + f*x]), x], x] - Simp[d^2/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[
e + f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

method	result	size
parallelrisch	$\frac{b^2 \ln(\tanh(x) a+b)-(a \ln(1-\tanh(x))+x(a+b))a}{a^3-a\,b^2}$	44
derivativedivides	$\frac{b^2 \ln(a+b\coth(x))}{a(a+b)(a-b)}+\frac{\ln(\coth(x))}{a}-\frac{\ln(\coth(x)-1)}{2a+2b}-\frac{\ln(1+\coth(x))}{2a-2b}$	67
default	$\frac{b^2 \ln(a+b\coth(x))}{a(a+b)(a-b)}+\frac{\ln(\coth(x))}{a}-\frac{\ln(\coth(x)-1)}{2a+2b}-\frac{\ln(1+\coth(x))}{2a-2b}$	67
risch	$\frac{x}{a+b}-\frac{2x}{a}-\frac{2x\,b^2}{a(a^2-b^2)}+\frac{\ln(e^{2x}+1)}{a}+\frac{b^2 \ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{a(a^2-b^2)}$	82

input
```
int(tanh(x)/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
(b^2*ln(tanh(x)*a+b)-(a*ln(1-tanh(x))+x*(a+b))*a)/(a^3-a*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.43

$$\int \frac{\tanh(x)}{a+b\coth(x)}\,dx = \frac{b^2\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)-(a^2+ab)x+(a^2-b^2)\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^3-ab^2}$$

input
```
integrate(tanh(x)/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
(b^2*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - (a^2 + a*b)*x +
(a^2 - b^2)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^3 - a*b^2)
```

Sympy [F]

$$\int \frac{\tanh(x)}{a + b\coth(x)}\, dx = \int \frac{\tanh{(x)}}{a + b\coth{(x)}}\, dx$$

input
```
integrate(tanh(x)/(a+b*coth(x)),x)
```

output
```
Integral(tanh(x)/(a + b*coth(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{\tanh(x)}{a + b\coth(x)}\, dx = \frac{b^2 \log\left(-(a-b)e^{(-2x)} + a + b\right)}{a^3 - ab^2} + \frac{x}{a+b} + \frac{\log\left(e^{(-2x)} + 1\right)}{a}$$

input
```
integrate(tanh(x)/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
b^2*log(-(a - b)*e^(-2*x) + a + b)/(a^3 - a*b^2) + x/(a + b) + log(e^(-2*x
) + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12

$$\int \frac{\tanh(x)}{a + b\coth(x)}\, dx = \frac{b^2 \log\left(\left|ae^{(2x)} + be^{(2x)} - a + b\right|\right)}{a^3 - ab^2} - \frac{x}{a-b} + \frac{\log\left(e^{(2x)} + 1\right)}{a}$$

input
```
integrate(tanh(x)/(a+b*coth(x)),x, algorithm="giac")
```

output
```
b^2*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^3 - a*b^2) - x/(a - b) + lo
g(e^(2*x) + 1)/a
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \frac{\tanh(x)}{a+b\coth(x)}\,dx = \frac{\ln\left(e^{2\,x}+1\right)}{a} - \frac{x}{a-b} - \frac{b^2\,\ln\left(b-a+a\,e^{2\,x}+b\,e^{2\,x}\right)}{a\,b^2-a^3}$$

input
```
int(tanh(x)/(a + b*coth(x)),x)
```

output
```
log(exp(2*x) + 1)/a - x/(a - b) - (b^2*log(b - a + a*exp(2*x) + b*exp(2*x)
))/(a*b^2 - a^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.49

$$\int \frac{\tanh(x)}{a+b\coth(x)}\,dx = \frac{\log(e^{2x}+1)\,a^2-\log(e^{2x}+1)\,b^2+\log(e^{2x}a+e^{2x}b-a+b)\,b^2-a^2x-abx}{a\,(a^2-b^2)}$$

input
```
int(tanh(x)/(a+b*coth(x)),x)
```

output
```
(log(e**(2*x) + 1)*a**2 - log(e**(2*x) + 1)*b**2 + log(e**(2*x)*a + e**(2*
x)*b - a + b)*b**2 - a**2*x - a*b*x)/(a*(a**2 - b**2))
```

3.149 $\int \frac{1}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 39

$$\int \frac{1}{a+b\coth(x)}\,dx = \frac{ax}{a^2-b^2} - \frac{b\log(b\cosh(x)+a\sinh(x))}{a^2-b^2}$$

output
```
a*x/(a^2-b^2)-b*ln(b*cosh(x)+a*sinh(x))/(a^2-b^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{1}{a+b\coth(x)}\,dx = \frac{(-a+b)\log(1-\tanh(x))+(a+b)\log(1+\tanh(x))-2b\log(b+a\tanh(x))}{2(a-b)(a+b)}$$

input
```
Integrate[(a + b*Coth[x])^(-1),x]
```

output
```
((-a + b)*Log[1 - Tanh[x]] + (a + b)*Log[1 + Tanh[x]] - 2*b*Log[b + a*Tanh
[x]])/(2*(a - b)*(a + b))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 3965, 26, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\coth(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-ib\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3965}$$

$$\frac{ax}{a^2-b^2}-\frac{ib\int -\frac{i(b+a\coth(x))}{a+b\coth(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{ax}{a^2-b^2}-\frac{b\int \frac{b+a\coth(x)}{a+b\coth(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{ax}{a^2-b^2}-\frac{b\int \frac{b-ia\tan\left(ix+\frac{\pi}{2}\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{4013}$$

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}$$

input
```
Int[(a + b*Coth[x])^(-1),x]
```

output
```
(a*x)/(a^2 - b^2) - (b*Log[b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3965
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Simp[b/(a^2 + b^2)   Int[(b - a*Tan[c + d*x])/(a + b*Tan[c
+ d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

method	result	size
parallelrisch	$\frac{-b\ln(\tanh(x)a+b)+\ln(1-\tanh(x))b+x(a+b)}{a^2-b^2}$	38
derivativedivides	$-\frac{b\ln(a+b\coth(x))}{(a+b)(a-b)}-\frac{\ln(\coth(x)-1)}{2a+2b}+\frac{\ln(1+\coth(x))}{2a-2b}$	55
default	$-\frac{b\ln(a+b\coth(x))}{(a+b)(a-b)}-\frac{\ln(\coth(x)-1)}{2a+2b}+\frac{\ln(1+\coth(x))}{2a-2b}$	55
risch	$\frac{x}{a+b}+\frac{2bx}{a^2-b^2}-\frac{b\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{a^2-b^2}$	56

input
```
int(1/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
(-b*ln(tanh(x)*a+b)+ln(1-tanh(x))*b+x*(a+b))/(a^2-b^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{1}{a+b\coth(x)}\,dx = \frac{(a+b)x - b\log\left(\frac{2(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

input
```
integrate(1/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
((a + b)*x - b*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))))/(a^2 -
b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(29) = 58.

Time = 0.50 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.79

$$\int \frac{1}{a+b\coth(x)}\,dx = \begin{cases} \tilde{\infty}(x-\log(\tanh(x)+1)) & \text{for } a=0 \wedge b=0 \\ \frac{x-\log(\tanh(x)+1)}{b} & \text{for } a=0 \\ -\frac{x\tanh(x)}{2b\tanh(x)-2b} + \frac{x}{2b\tanh(x)-2b} - \frac{1}{2b\tanh(x)-2b} & \text{for } a=-b \\ \frac{x\tanh(x)}{2b\tanh(x)+2b} + \frac{x}{2b\tanh(x)+2b} + \frac{1}{2b\tanh(x)+2b} & \text{for } a=b \\ \frac{ax}{a^2-b^2} - \frac{bx}{a^2-b^2} + \frac{b\log(\tanh(x)+1)}{a^2-b^2} - \frac{b\log\left(\tanh(x)+\frac{b}{a}\right)}{a^2-b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*coth(x)),x)
```

output
```
Piecewise((zoo*(x - log(tanh(x) + 1)), Eq(a, 0) & Eq(b, 0)), ((x - log(tan
h(x) + 1))/b, Eq(a, 0)), (-x*tanh(x)/(2*b*tanh(x) - 2*b) + x/(2*b*tanh(x)
- 2*b) - 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b)
 + x/(2*b*tanh(x) + 2*b) + 1/(2*b*tanh(x) + 2*b), Eq(a, b)), (a*x/(a**2 -
b**2) - b*x/(a**2 - b**2) + b*log(tanh(x) + 1)/(a**2 - b**2) - b*log(tanh(
x) + b/a)/(a**2 - b**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{1}{a+b\coth(x)}\,dx = -\frac{b\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^2-b^2}+\frac{x}{a+b}$$

input
```
integrate(1/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
-b*log(-(a - b)*e^(-2*x) + a + b)/(a^2 - b^2) + x/(a + b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{1}{a+b\coth(x)}\,dx = -\frac{b\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^2-b^2}+\frac{x}{a-b}$$

input
```
integrate(1/(a+b*coth(x)),x, algorithm="giac")
```

output
```
-b*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2 - b^2) + x/(a - b)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{1}{a+b\coth(x)}\,dx = \frac{x}{a-b}-\frac{b\,\ln\left(b-a+a\,e^{2x}+b\,e^{2x}\right)}{a^2-b^2}$$

input
```
int(1/(a + b*coth(x)),x)
```

output
```
x/(a - b) - (b*log(b - a + a*exp(2*x) + b*exp(2*x)))/(a^2 - b^2)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{1}{a+b\coth(x)}\,dx=\frac{-\log(e^{2x}a+e^{2x}b-a+b)\,b+ax+bx}{a^2-b^2}$$

input
```
int(1/(a+b*coth(x)),x)
```

output
```
( - log(e**(2*x)*a + e**(2*x)*b - a + b)*b + a*x + b*x)/(a**2 - b**2)
```

3.150 $\int \frac{\coth(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \frac{\coth(x)}{a+b\coth(x)}\,dx=-\frac{bx}{a^2-b^2}+\frac{a\log(b\cosh(x)+a\sinh(x))}{a^2-b^2}$$

output
```
-b*x/(a^2-b^2)+a*ln(b*cosh(x)+a*sinh(x))/(a^2-b^2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int \frac{\coth(x)}{a+b\coth(x)}\,dx = \frac{(-a+b)\log(1-\tanh(x))-(a+b)\log(1+\tanh(x))+2a\log(b+a\tanh(x))}{2(a-b)(a+b)}$$

input
```
Integrate[Coth[x]/(a + b*Coth[x]),x]
```

output
```
((-a + b)*Log[1 - Tanh[x]] - (a + b)*Log[1 + Tanh[x]] + 2*a*Log[b + a*Tanh
[x]])/(2*(a - b)*(a + b))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 26, 4014, 26, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{a+b\coth(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan\left(\frac{\pi}{2}+ix\right)}{a-ib\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan\left(ix+\frac{\pi}{2}\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4014}$$

$$-i\left(-\frac{a\int -\frac{i(b+a\coth(x))}{a+b\coth(x)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{ia\int \frac{b+a\coth(x)}{a+b\coth(x)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{ia\int \frac{b-ia\tan\left(ix+\frac{\pi}{2}\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)$$

$$\Big\downarrow \text{4013}$$

$$-i\left(\frac{ia\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}-\frac{ibx}{a^2-b^2}\right)$$

input
```
Int[Coth[x]/(a + b*Coth[x]),x]
```

output
```
(-I)*(((-I)*b*x)/(a^2 - b^2) + (I*a*Log[b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2
))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{a\ln(\tanh(x)a+b)-a\ln(1-\tanh(x))-x(a+b)}{a^2-b^2}$	39
derivativedivides	$\frac{a\ln(a+b\coth(x))}{(a+b)(a-b)}-\frac{\ln(\coth(x)-1)}{2a+2b}-\frac{\ln(1+\coth(x))}{2a-2b}$	55
default	$\frac{a\ln(a+b\coth(x))}{(a+b)(a-b)}-\frac{\ln(\coth(x)-1)}{2a+2b}-\frac{\ln(1+\coth(x))}{2a-2b}$	55
risch	$\frac{x}{a+b}-\frac{2ax}{a^2-b^2}+\frac{a\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{a^2-b^2}$	55

input
```
int(coth(x)/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
(a*ln(tanh(x)*a+b)-a*ln(1-tanh(x))-x*(a+b))/(a^2-b^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{\coth(x)}{a+b\coth(x)}\,dx = -\frac{(a+b)x-a\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

input
```
integrate(coth(x)/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
-((a + b)*x - a*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))))/(a^2 -
b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(29) = 58$.

Time = 0.51 (sec) , antiderivative size = 134, normalized size of antiderivative = 3.44

$$\int \frac{\coth(x)}{a + b\coth(x)}\, dx = \begin{cases} \tilde{\infty}x & \text{for } a = 0 \wedge b = 0 \\ \frac{x}{b} & \text{for } a = 0 \\ \frac{x\tanh(x)}{2b\tanh(x) - 2b} - \frac{x}{2b\tanh(x) - 2b} - \frac{1}{2b\tanh(x) - 2b} & \text{for } a = -b \\ \frac{x\tanh(x)}{2b\tanh(x) + 2b} + \frac{x}{2b\tanh(x) + 2b} - \frac{1}{2b\tanh(x) + 2b} & \text{for } a = b \\ \frac{ax}{a^2 - b^2} - \frac{a\log(\tanh(x) + 1)}{a^2 - b^2} + \frac{a\log\left(\tanh(x) + \frac{b}{a}\right)}{a^2 - b^2} - \frac{bx}{a^2 - b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(coth(x)/(a+b*coth(x)),x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b, Eq(a, 0)), (x*tanh(x)/(2*b*t
anh(x) - 2*b) - x/(2*b*tanh(x) - 2*b) - 1/(2*b*tanh(x) - 2*b), Eq(a, -b)),
 (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b*tanh(x) + 2*b) - 1/(2*b*tanh(x) +
 2*b), Eq(a, b)), (a*x/(a**2 - b**2) - a*log(tanh(x) + 1)/(a**2 - b**2) +
a*log(tanh(x) + b/a)/(a**2 - b**2) - b*x/(a**2 - b**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{\coth(x)}{a + b\coth(x)}\, dx = \frac{a\log\left(-(a-b)e^{(-2\,x)} + a + b\right)}{a^2 - b^2} + \frac{x}{a + b}$$

input
```
integrate(coth(x)/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
a*log(-(a - b)*e^(-2*x) + a + b)/(a^2 - b^2) + x/(a + b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{\coth(x)}{a+b\coth(x)}\,dx = \frac{a\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}-a+b\right|\right)}{a^2-b^2}-\frac{x}{a-b}$$

input
```
integrate(coth(x)/(a+b*coth(x)),x, algorithm="giac")
```

output
```
a*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2 - b^2) - x/(a - b)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{\coth(x)}{a+b\coth(x)}\,dx = \frac{a\,\ln\left(b-a+a\,e^{2\,x}+b\,e^{2\,x}\right)}{a^2-b^2}-\frac{x}{a-b}$$

input
```
int(coth(x)/(a + b*coth(x)),x)
```

output
```
(a*log(b - a + a*exp(2*x) + b*exp(2*x)))/(a^2 - b^2) - x/(a - b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{\coth(x)}{a+b\coth(x)}\,dx = \frac{\log(e^{2x}a+e^{2x}b-a+b)\,a-ax-bx}{a^2-b^2}$$

input
```
int(coth(x)/(a+b*coth(x)),x)
```

output
```
(log(e**(2*x)*a + e**(2*x)*b - a + b)*a - a*x - b*x)/(a**2 - b**2)
```

3.151 $\int \frac{\coth^2(x)}{a+b\coth(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 63

$$\int \frac{\coth^2(x)}{a+b\coth(x)} \, dx = -\frac{ax}{b^2} + \frac{a^3x}{b^2(a^2-b^2)} + \frac{\log(\sinh(x))}{b} - \frac{a^2\log(b\cosh(x)+a\sinh(x))}{b(a^2-b^2)}$$

output
```
-a*x/b^2+a^3*x/b^2/(a^2-b^2)+ln(sinh(x))/b-a^2*ln(b*cosh(x)+a*sinh(x))/b/(
a^2-b^2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{\coth^2(x)}{a+b\coth(x)} \, dx = -\frac{\log(1-\coth(x))}{2(a+b)} + \frac{\log(1+\coth(x))}{2(a-b)} - \frac{a^2\log(a+b\coth(x))}{b(a^2-b^2)}$$

input
```
Integrate[Coth[x]^2/(a + b*Coth[x]),x]
```

output
```
-1/2*Log[1 - Coth[x]]/(a + b) + Log[1 + Coth[x]]/(2*(a - b)) - (a^2*Log[a
+ b*Coth[x]])/(b*(a^2 - b^2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.846, Rules used = {3042, 25, 4024, 26, 3042, 26, 3956, 3965, 26, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{a+b\coth(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\tan\left(\frac{\pi}{2}+ix\right)^2}{a-ib\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\tan\left(ix+\frac{\pi}{2}\right)^2}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4024}$$

$$\frac{a^2\int \frac{1}{a+b\coth(x)}dx}{b^2}-\frac{i\int i\coth(x)dx}{b}-\frac{ax}{b^2}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{a^2\int \frac{1}{a+b\coth(x)}dx}{b^2}+\frac{\int \coth(x)dx}{b}-\frac{ax}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a^2\int \frac{1}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\int -i\tan\left(ix+\frac{\pi}{2}\right)dx}{b}-\frac{ax}{b^2}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{a^2\int \frac{1}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{b^2}-\frac{i\int \tan\left(ix+\frac{\pi}{2}\right)dx}{b}-\frac{ax}{b^2}$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{a^2\int \frac{1}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{b^2}-\frac{ax}{b^2}+\frac{\log(\sinh(x))}{b}$$

$$\Big\downarrow \text{ 3965}$$

$$\frac{a^2\left(\frac{ax}{a^2-b^2}-\frac{ib\int -\frac{i(b+a\coth(x))}{a+b\coth(x)}dx}{a^2-b^2}\right)}{b^2}-\frac{ax}{b^2}+\frac{\log(\sinh(x))}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{a^2\left(\frac{ax}{a^2-b^2}-\frac{b\int \frac{b+a\coth(x)}{a+b\coth(x)}dx}{a^2-b^2}\right)}{b^2}-\frac{ax}{b^2}+\frac{\log(\sinh(x))}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{ax}{a^2-b^2}-\frac{b\int \frac{b-ia\tan\left(ix+\frac{\pi}{2}\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}\right)}{b^2}-\frac{ax}{b^2}+\frac{\log(\sinh(x))}{b}$$

$$\Big\downarrow \text{4013}$$

$$\frac{a^2\left(\frac{ax}{a^2-b^2}-\frac{b\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}\right)}{b^2}-\frac{ax}{b^2}+\frac{\log(\sinh(x))}{b}$$

input
```
Int[Coth[x]^2/(a + b*Coth[x]),x]
```

output
```
-((a*x)/b^2) + Log[Sinh[x]]/b + (a^2*((a*x)/(a^2 - b^2) - (b*Log[b*Cosh[x]
 + a*Sinh[x]])/(a^2 - b^2)))/b^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3965
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Simp[b/(a^2 + b^2)   Int[(b - a*Tan[c + d*x])/(a + b*Tan[c
+ d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4024
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^2/((a_.) + (b_.)*tan[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[d*(2*b*c - a*d)*(x/b^2), x] + (Simp[d^2/b   I
nt[Tan[e + f*x], x], x] + Simp[(b*c - a*d)^2/b^2   Int[1/(a + b*Tan[e + f*x
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$\frac{-a^{2} \ln(\tanh(x) a+b)+\ln(1-\tanh(x)) b^{2}+(a+b)((a-b)\ln(\tanh(x))+b x)}{a^{2} b-b^{3}}$	56
derivativedivides	$-\frac{a^{2} \ln(a+b\coth(x))}{(a+b)(a-b) b}-\frac{\ln(\coth(x)-1)}{2a+2b}+\frac{\ln(1+\coth(x))}{2a-2b}$	60
default	$-\frac{a^{2} \ln(a+b\coth(x))}{(a+b)(a-b) b}-\frac{\ln(\coth(x)-1)}{2a+2b}+\frac{\ln(1+\coth(x))}{2a-2b}$	60
risch	$\frac{x}{a+b}+\frac{2x a^{2}}{b(a^{2}-b^{2})}-\frac{2x}{b}-\frac{a^{2} \ln\left(\mathrm{e}^{2x}-\frac{a-b}{a+b}\right)}{b(a^{2}-b^{2})}+\frac{\ln(\mathrm{e}^{2x}-1)}{b}$	83

input
```
int(coth(x)^2/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
(-a^2*ln(tanh(x)*a+b)+ln(1-tanh(x))*b^2+(a+b)*((a-b)*ln(tanh(x))+b*x))/(a^
2*b-b^3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21

$$\int \frac{\coth^2(x)}{a+b\coth(x)}\,dx = -\frac{a^2\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)-(ab+b^2)x-(a^2-b^2)\log\left(\frac{2\,\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{a^2b-b^3}$$

input
```
integrate(coth(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
-(a^2*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - (a*b + b^2)*x -
 (a^2 - b^2)*log(2*sinh(x)/(cosh(x) - sinh(x))))/(a^2*b - b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(51) = 102$.

Time = 0.82 (sec) , antiderivative size = 372, normalized size of antiderivative = 5.90

$$\int \frac{\coth^2(x)}{a+b\coth(x)}\,dx = \begin{cases} \tilde{\infty}(x-\log(\tanh(x)+1)+\log(\tanh(x))) \\ \frac{x-\log(\tanh(x)+1)+\log(\tanh(x))}{b} \\ \frac{3x\tanh(x)}{2b\tanh(x)-2b}-\frac{3x}{2b\tanh(x)-2b}-\frac{2\log(\tanh(x)+1)\tanh(x)}{2b\tanh(x)-2b}+\frac{2\log(\tanh(x)+1)}{2b\tanh(x)-2b}+\frac{2\log(\tanh(x))\tanh(x)}{2b\tanh(x)-2b}-\frac{2\log(\tanh(x))}{2b\tanh(x)-2b}- \\ \frac{x\tanh(x)}{2b\tanh(x)+2b}+\frac{x}{2b\tanh(x)+2b}-\frac{2\log(\tanh(x)+1)\tanh(x)}{2b\tanh(x)+2b}-\frac{2\log(\tanh(x)+1)}{2b\tanh(x)+2b}+\frac{2\log(\tanh(x))\tanh(x)}{2b\tanh(x)+2b}+\frac{2\log(\tanh(x))}{2b\tanh(x)+2b}+ \\ \frac{x-\frac{1}{\tanh(x)}}{a} \\ -\frac{a^2\log\left(\tanh(x)+\frac{b}{a}\right)}{a^2b-b^3}+\frac{a^2\log(\tanh(x))}{a^2b-b^3}+\frac{abx}{a^2b-b^3}-\frac{b^2x}{a^2b-b^3}+\frac{b^2\log(\tanh(x)+1)}{a^2b-b^3}-\frac{b^2\log(\tanh(x))}{a^2b-b^3} \end{cases}$$

input
```
integrate(coth(x)**2/(a+b*coth(x)),x)
```

output
```
Piecewise((zoo*(x - log(tanh(x) + 1) + log(tanh(x))), Eq(a, 0) & Eq(b, 0))
, ((x - log(tanh(x) + 1) + log(tanh(x)))/b, Eq(a, 0)), (3*x*tanh(x)/(2*b*t
anh(x) - 2*b) - 3*x/(2*b*tanh(x) - 2*b) - 2*log(tanh(x) + 1)*tanh(x)/(2*b*
tanh(x) - 2*b) + 2*log(tanh(x) + 1)/(2*b*tanh(x) - 2*b) + 2*log(tanh(x))*t
anh(x)/(2*b*tanh(x) - 2*b) - 2*log(tanh(x))/(2*b*tanh(x) - 2*b) - 1/(2*b*t
anh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b*tanh(x)
 + 2*b) - 2*log(tanh(x) + 1)*tanh(x)/(2*b*tanh(x) + 2*b) - 2*log(tanh(x) +
 1)/(2*b*tanh(x) + 2*b) + 2*log(tanh(x))*tanh(x)/(2*b*tanh(x) + 2*b) + 2*l
og(tanh(x))/(2*b*tanh(x) + 2*b) + 1/(2*b*tanh(x) + 2*b), Eq(a, b)), ((x -
1/tanh(x))/a, Eq(b, 0)), (-a**2*log(tanh(x) + b/a)/(a**2*b - b**3) + a**2*
log(tanh(x))/(a**2*b - b**3) + a*b*x/(a**2*b - b**3) - b**2*x/(a**2*b - b*
*3) + b**2*log(tanh(x) + 1)/(a**2*b - b**3) - b**2*log(tanh(x))/(a**2*b -
b**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{\coth^2(x)}{a+b\coth(x)}\,dx = -\frac{a^2\log\left(-(a-b)e^{(-2\,x)}+a+b\right)}{a^2b-b^3}+\frac{x}{a+b}+\frac{\log\left(e^{(-x)}+1\right)}{b}+\frac{\log\left(e^{(-x)}-1\right)}{b}$$

input
```
integrate(coth(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
-a^2*log(-(a - b)*e^(-2*x) + a + b)/(a^2*b - b^3) + x/(a + b) + log(e^(-x)
 + 1)/b + log(e^(-x) - 1)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{\coth^2(x)}{a+b\coth(x)}\,dx = -\frac{a^2\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}-a+b\right|\right)}{a^2b-b^3}+\frac{x}{a-b}+\frac{\log\left(\left|e^{(2\,x)}-1\right|\right)}{b}$$

input
```
integrate(coth(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

output
```
-a^2*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b - b^3) + x/(a - b) + l
og(abs(e^(2*x) - 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{\coth^2(x)}{a+b\coth(x)}\,dx = \frac{\ln\left(e^{2x}-1\right)}{b}+\frac{x}{a-b}-\frac{a^2\,\ln\left(b-a+a\,e^{2x}+b\,e^{2x}\right)}{a^2\,b-b^3}$$

input
```
int(coth(x)^2/(a + b*coth(x)),x)
```

output
```
log(exp(2*x) - 1)/b + x/(a - b) - (a^2*log(b - a + a*exp(2*x) + b*exp(2*x)
))/(a^2*b - b^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16

$$\int \frac{\coth^2(x)}{a+b\coth(x)}\,dx$$
$$= \frac{-\log(\coth(x)\,b+a)\,a^2+\log(\coth(x)\,b+a)\,b^2-\log(e^{2x}a+e^{2x}b-a+b)\,b^2+abx+b^2x}{b\,(a^2-b^2)}$$

input
```
int(coth(x)^2/(a+b*coth(x)),x)
```

output
```
( - log(coth(x)*b + a)*a**2 + log(coth(x)*b + a)*b**2 - log(e**(2*x)*a + e
**(2*x)*b - a + b)*b**2 + a*b*x + b**2*x)/(b*(a**2 - b**2))
```

3.152 $\int \frac{\coth^3(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{\coth^3(x)}{a+b\coth(x)}\,dx = -\frac{bx}{a^2-b^2}-\frac{\coth(x)}{b}+\frac{a^3\log(a+b\coth(x))}{b^2(a^2-b^2)}+\frac{a\log(\sinh(x))}{a^2-b^2}$$

output
```
-b*x/(a^2-b^2)-coth(x)/b+a^3*ln(a+b*coth(x))/b^2/(a^2-b^2)+a*ln(sinh(x))/(
a^2-b^2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{\coth^3(x)}{a+b\coth(x)}\,dx = -\frac{\coth(x)}{b}-\frac{\log(1-\coth(x))}{2(a+b)} - \frac{\log(1+\coth(x))}{2(a-b)}+\frac{a^3\log(a+b\coth(x))}{b^2(a^2-b^2)}$$

input
```
Integrate[Coth[x]^3/(a + b*Coth[x]),x]
```

output
```
-(Coth[x]/b) - Log[1 - Coth[x]]/(2*(a + b)) - Log[1 + Coth[x]]/(2*(a - b))
 + (a^3*Log[a + b*Coth[x]])/(b^2*(a^2 - b^2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.27, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923, Rules used = {3042, 26, 4049, 25, 3042, 4109, 26, 3042, 26, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(x)}{a+b\coth(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan\left(\frac{\pi}{2}+ix\right)^3}{a-ib\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\tan\left(ix+\frac{\pi}{2}\right)^3}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4049}$$

$$i\left(\frac{i\int -\frac{-a\coth^2(x)+b\coth(x)+a}{a+b\coth(x)}dx}{b}+\frac{i\coth(x)}{b}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(\frac{i\coth(x)}{b}-\frac{i\int \frac{-a\coth^2(x)+b\coth(x)+a}{a+b\coth(x)}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\coth(x)}{b}-\frac{i\int \frac{a\tan\left(ix+\frac{\pi}{2}\right)^2-ib\tan\left(ix+\frac{\pi}{2}\right)+a}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{b}\right)$$

$$\Big\downarrow \text{4109}$$

$$i\left(\frac{i\coth(x)}{b}-\frac{i\left(-\frac{iab\int i\coth(x)dx}{a^2-b^2}+\frac{a^3\int\frac{1-\coth^2(x)}{a+b\coth(x)}dx}{a^2-b^2}-\frac{b^2x}{a^2-b^2}\right)}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\coth(x)}{b}-\frac{i\left(\frac{ab\int\coth(x)dx}{a^2-b^2}+\frac{a^3\int\frac{1-\coth^2(x)}{a+b\coth(x)}dx}{a^2-b^2}-\frac{b^2x}{a^2-b^2}\right)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\coth(x)}{b}-\frac{i\left(\frac{ab\int -i\tan\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}+\frac{a^3\int\frac{\tan\left(ix+\frac{\pi}{2}\right)^2+1}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{b^2x}{a^2-b^2}\right)}{b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\coth(x)}{b}-\frac{i\left(-\frac{iab\int\tan\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}+\frac{a^3\int\frac{\tan\left(ix+\frac{\pi}{2}\right)^2+1}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{b^2x}{a^2-b^2}\right)}{b}\right)$$

$$\Big\downarrow \text{3956}$$

$$i\left(\frac{i\coth(x)}{b}-\frac{i\left(\frac{a^3\int\frac{\tan\left(ix+\frac{\pi}{2}\right)^2+1}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{b^2x}{a^2-b^2}+\frac{ab\log(\sinh(x))}{a^2-b^2}\right)}{b}\right)$$

$$\Big\downarrow \text{4100}$$

$$i\left(\frac{i\coth(x)}{b}-\frac{i\left(\frac{a^3\int\frac{1}{a+b\coth(x)}d(b\coth(x))}{b(a^2-b^2)}-\frac{b^2x}{a^2-b^2}+\frac{ab\log(\sinh(x))}{a^2-b^2}\right)}{b}\right)$$

$$\Big\downarrow \text{16}$$

$$i\left(\frac{i\coth(x)}{b}-\frac{i\left(-\frac{b^2x}{a^2-b^2}+\frac{ab\log(\sinh(x))}{a^2-b^2}+\frac{a^3\log(a+b\coth(x))}{b(a^2-b^2)}\right)}{b}\right)$$

input
```
Int[Coth[x]^3/(a + b*Coth[x]),x]
```

output
```
I*((I*Coth[x])/b - (I*(-((b^2*x)/(a^2 - b^2)) + (a^3*Log[a + b*Coth[x]])/(
b*(a^2 - b^2)) + (a*b*Log[Sinh[x]])/(a^2 - b^2)))/b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4049
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))
 Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n
- 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[
e + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2
, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || I
ntegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0]))
)
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$-\frac{\coth(x)}{b}+\frac{a^3\ln(a+b\coth(x))}{b^2(a+b)(a-b)}-\frac{\ln(\coth(x)-1)}{2a+2b}-\frac{\ln(1+\coth(x))}{2a-2b}$	67
default	$-\frac{\coth(x)}{b}+\frac{a^3\ln(a+b\coth(x))}{b^2(a+b)(a-b)}-\frac{\ln(\coth(x)-1)}{2a+2b}-\frac{\ln(1+\coth(x))}{2a-2b}$	67
parallelrisch	$\frac{a^3\ln(\tanh(x)a+b)-\ln(1-\tanh(x))a\,b^2-(a+b)(a(a-b)\ln(\tanh(x))+((a-b)\coth(x)+bx)b)}{a^2b^2-b^4}$	72
risch	$\frac{x}{a+b}+\frac{2ax}{b^2}-\frac{2a^3x}{b^2(a^2-b^2)}-\frac{2}{b(e^{2x}-1)}-\frac{a\ln(e^{2x}-1)}{b^2}+\frac{a^3\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{b^2(a^2-b^2)}$	98

input
```
int(coth(x)^3/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
-coth(x)/b+1/b^2*a^3/(a+b)/(a-b)*ln(a+b*coth(x))-1/(2*a+2*b)*ln(coth(x)-1)
-1/(2*a-2*b)*ln(1+coth(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(64) = 128$.

Time = 0.10 (sec) , antiderivative size = 271, normalized size of antiderivative = 4.23

$$\int \frac{\coth^3(x)}{a+b\coth(x)}\,dx = \frac{(ab^2+b^3)x\cosh(x)^2 + 2(ab^2+b^3)x\cosh(x)\sinh(x) + (ab^2+b^3)x\sinh(x)^2 + 2a^2b - 2b^3 - (ab^2+b^3)}{}$$

input
```
integrate(coth(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
((a*b^2 + b^3)*x*cosh(x)^2 + 2*(a*b^2 + b^3)*x*cosh(x)*sinh(x) + (a*b^2 +
b^3)*x*sinh(x)^2 + 2*a^2*b - 2*b^3 - (a*b^2 + b^3)*x - (a^3*cosh(x)^2 + 2*
a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2 - a^3)*log(2*(b*cosh(x) + a*sinh(x))/(
cosh(x) - sinh(x))) - (a^3 - a*b^2 - (a^3 - a*b^2)*cosh(x)^2 - 2*(a^3 - a*
b^2)*cosh(x)*sinh(x) - (a^3 - a*b^2)*sinh(x)^2)*log(2*sinh(x)/(cosh(x) - s
inh(x))))/(a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cosh(x)^2 - 2*(a^2*b^2 - b^4)*c
osh(x)*sinh(x) - (a^2*b^2 - b^4)*sinh(x)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 636 vs. $2(49) = 98$.

Time = 1.15 (sec) , antiderivative size = 636, normalized size of antiderivative = 9.94

$$\int \frac{\coth^3(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)**3/(a+b*coth(x)),x)
```

output
```
Piecewise((zoo*(x - 1/tanh(x)), Eq(a, 0) & Eq(b, 0)), ((x - 1/tanh(x))/b,
Eq(a, 0)), (5*x*tanh(x)**2/(2*b*tanh(x)**2 - 2*b*tanh(x)) - 5*x*tanh(x)/(2
*b*tanh(x)**2 - 2*b*tanh(x)) - 2*log(tanh(x) + 1)*tanh(x)**2/(2*b*tanh(x)*
*2 - 2*b*tanh(x)) + 2*log(tanh(x) + 1)*tanh(x)/(2*b*tanh(x)**2 - 2*b*tanh(
x)) + 2*log(tanh(x))*tanh(x)**2/(2*b*tanh(x)**2 - 2*b*tanh(x)) - 2*log(tan
h(x))*tanh(x)/(2*b*tanh(x)**2 - 2*b*tanh(x)) - 3*tanh(x)/(2*b*tanh(x)**2 -
 2*b*tanh(x)) + 2/(2*b*tanh(x)**2 - 2*b*tanh(x)), Eq(a, -b)), (x*tanh(x)**
2/(2*b*tanh(x)**2 + 2*b*tanh(x)) + x*tanh(x)/(2*b*tanh(x)**2 + 2*b*tanh(x)
) + 2*log(tanh(x) + 1)*tanh(x)**2/(2*b*tanh(x)**2 + 2*b*tanh(x)) + 2*log(t
anh(x) + 1)*tanh(x)/(2*b*tanh(x)**2 + 2*b*tanh(x)) - 2*log(tanh(x))*tanh(x
)**2/(2*b*tanh(x)**2 + 2*b*tanh(x)) - 2*log(tanh(x))*tanh(x)/(2*b*tanh(x)*
*2 + 2*b*tanh(x)) - 3*tanh(x)/(2*b*tanh(x)**2 + 2*b*tanh(x)) - 2/(2*b*tanh
(x)**2 + 2*b*tanh(x)), Eq(a, b)), ((x - log(tanh(x) + 1) + log(tanh(x)) -
1/(2*tanh(x)**2))/a, Eq(b, 0)), (a**3*log(tanh(x) + b/a)*tanh(x)/(a**2*b**
2*tanh(x) - b**4*tanh(x)) - a**3*log(tanh(x))*tanh(x)/(a**2*b**2*tanh(x) -
 b**4*tanh(x)) - a**2*b/(a**2*b**2*tanh(x) - b**4*tanh(x)) + a*b**2*x*tanh
(x)/(a**2*b**2*tanh(x) - b**4*tanh(x)) - a*b**2*log(tanh(x) + 1)*tanh(x)/(
a**2*b**2*tanh(x) - b**4*tanh(x)) + a*b**2*log(tanh(x))*tanh(x)/(a**2*b**2
*tanh(x) - b**4*tanh(x)) - b**3*x*tanh(x)/(a**2*b**2*tanh(x) - b**4*tanh(x
)) + b**3/(a**2*b**2*tanh(x) - b**4*tanh(x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.28

$$\int \frac{\coth^3(x)}{a+b\coth(x)}\,dx = \frac{a^3\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^2b^2-b^4} + \frac{x}{a+b} - \frac{a\log\left(e^{(-x)}+1\right)}{b^2} - \frac{a\log\left(e^{(-x)}-1\right)}{b^2} + \frac{2}{be^{(-2x)}-b}$$

input
```
integrate(coth(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
a^3*log(-(a - b)*e^(-2*x) + a + b)/(a^2*b^2 - b^4) + x/(a + b) - a*log(e^(
-x) + 1)/b^2 - a*log(e^(-x) - 1)/b^2 + 2/(b*e^(-2*x) - b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.19

$$\int \frac{\coth^3(x)}{a+b\coth(x)}\,dx = \frac{a^3\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^2b^2-b^4} - \frac{x}{a-b} - \frac{a\log\left(\left|e^{(2x)}-1\right|\right)}{b^2} - \frac{2}{b(e^{(2x)}-1)}$$

input
```
integrate(coth(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

output
```
a^3*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b^2 - b^4) - x/(a - b) -
a*log(abs(e^(2*x) - 1))/b^2 - 2/(b*(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.16

$$\int \frac{\coth^3(x)}{a+b\coth(x)}\,dx = -\frac{2}{b\left(e^{2x}-1\right)} - \frac{x}{a-b} - \frac{a^3\ln\left(b-a+a\,e^{2x}+b\,e^{2x}\right)}{b^4-a^2\,b^2} - \frac{a\ln\left(e^{2x}-1\right)}{b^2}$$

input
```
int(coth(x)^3/(a + b*coth(x)),x)
```

output
```
- 2/(b*(exp(2*x) - 1)) - x/(a - b) - (a^3*log(b - a + a*exp(2*x) + b*exp(2
*x)))/(b^4 - a^2*b^2) - (a*log(exp(2*x) - 1))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.44

$$\int \frac{\coth^3(x)}{a+b\coth(x)}\,dx = \frac{-\coth(x)\,a^2b+\coth(x)\,b^3+\log(\coth(x)\,b+a)\,a^3-\log(\coth(x)\,b+a)\,a\,b^2+\log(e^{2x}a+e^{2x}b-a+b)}{b^2\,(a^2-b^2)}$$

input
```
int(coth(x)^3/(a+b*coth(x)),x)
```

output
```
( - coth(x)*a**2*b + coth(x)*b**3 + log(coth(x)*b + a)*a**3 - log(coth(x)*
b + a)*a*b**2 + log(e**(2*x)*a + e**(2*x)*b - a + b)*a*b**2 - a*b**2*x - b
**3*x)/(b**2*(a**2 - b**2))
```

3.153 $\int \frac{\coth^4(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 76

$$\int \frac{\coth^4(x)}{a+b\coth(x)}\,dx = \frac{ax}{a^2-b^2}+\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}-\frac{a^4\log(a+b\coth(x))}{b^3(a^2-b^2)}-\frac{b\log(\sinh(x))}{a^2-b^2}$$

output
```
a*x/(a^2-b^2)+a*coth(x)/b^2-1/2*coth(x)^2/b-a^4*ln(a+b*coth(x))/b^3/(a^2-b
^2)-b*ln(sinh(x))/(a^2-b^2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int \frac{\coth^4(x)}{a+b\coth(x)}\,dx = \frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}-\frac{\log(1-\coth(x))}{2(a+b)}+\frac{\log(1+\coth(x))}{2(a-b)}-\frac{a^4\log(a+b\coth(x))}{b^3(a^2-b^2)}$$

input
```
Integrate[Coth[x]^4/(a + b*Coth[x]),x]
```

output
```
(a*Coth[x])/b^2 - Coth[x]^2/(2*b) - Log[1 - Coth[x]]/(2*(a + b)) + Log[1 +
 Coth[x]]/(2*(a - b)) - (a^4*Log[a + b*Coth[x]])/(b^3*(a^2 - b^2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.30, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.154$, Rules used = {3042, 4049, 27, 3042, 26, 4130, 25, 3042, 4110, 26, 3042, 26, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(x)}{a+b\coth(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan\left(\frac{\pi}{2}+ix\right)^4}{a-ib\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{4049}$$

$$-\frac{\coth^2(x)}{2b}+\frac{i\int -\frac{2i\coth(x)(-a\coth^2(x)+b\coth(x)+a)}{a+b\coth(x)}dx}{2b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\coth(x)(-a\coth^2(x)+b\coth(x)+a)}{a+b\coth(x)}dx}{b}-\frac{\coth^2(x)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\coth^2(x)}{2b}+\frac{\int -\frac{i\tan\left(ix+\frac{\pi}{2}\right)\left(a\tan\left(ix+\frac{\pi}{2}\right)^2-ib\tan\left(ix+\frac{\pi}{2}\right)+a\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\coth^2(x)}{2b}-\frac{i\int \frac{\tan\left(ix+\frac{\pi}{2}\right)\left(a\tan\left(ix+\frac{\pi}{2}\right)^2-ib\tan\left(ix+\frac{\pi}{2}\right)+a\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{4130}$$

$$-\frac{\coth^2(x)}{2b}-\frac{i\left(\frac{i\int -\frac{a^2-\left(a^2+b^2\right)\coth^2(x)}{a+b\coth(x)}dx}{b}+\frac{ia\coth(x)}{b}\right)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\coth^2(x)}{2b}-\frac{i\left(\frac{ia\coth(x)}{b}-\frac{i\int \frac{a^2-\left(a^2+b^2\right)\coth^2(x)}{a+b\coth(x)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\coth^2(x)}{2b}-\frac{i\left(\frac{ia\coth(x)}{b}-\frac{i\int \frac{a^2+\left(a^2+b^2\right)\tan\left(ix+\frac{\pi}{2}\right)^2}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{4110}$$

$$-\frac{\coth^2(x)}{2b}-\frac{i\left(\frac{ia\coth(x)}{b}-\frac{i\left(-\frac{ib^3\int i\coth(x)dx}{a^2-b^2}+\frac{a^4\int \frac{1-\coth^2(x)}{a+b\coth(x)}dx}{a^2-b^2}-\frac{ab^2x}{a^2-b^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\coth^2(x)}{2b}-\frac{i\left(\frac{ia\coth(x)}{b}-\frac{i\left(\frac{b^3\int \coth(x)dx}{a^2-b^2}+\frac{a^4\int \frac{1-\coth^2(x)}{a+b\coth(x)}dx}{a^2-b^2}-\frac{ab^2x}{a^2-b^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\coth^2(x)}{2b}-\frac{i\left(\frac{ia\coth(x)}{b}-\frac{i\left(\frac{b^3\int -i\tan\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}+\frac{a^4\int \frac{\tan\left(ix+\frac{\pi}{2}\right)^2+1}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{ab^2x}{a^2-b^2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\coth^2(x)}{2b}-\frac{i\left(\frac{ia\coth(x)}{b}-\frac{i\left(-\frac{ib^3\int\tan\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}+\frac{a^4\int\frac{\tan\left(ix+\frac{\pi}{2}\right)^2+1}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{ab^2x}{a^2-b^2}\right)}{b}\right)}{b}$$

$\downarrow$ 3956

$$-\frac{\coth^2(x)}{2b}-\frac{i\left(\frac{ia\coth(x)}{b}-\frac{i\left(\frac{a^4\int\frac{\tan\left(ix+\frac{\pi}{2}\right)^2+1}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{ab^2x}{a^2-b^2}+\frac{b^3\log(\sinh(x))}{a^2-b^2}\right)}{b}\right)}{b}$$

$\downarrow$ 4100

$$-\frac{\coth^2(x)}{2b}-\frac{i\left(\frac{ia\coth(x)}{b}-\frac{i\left(\frac{a^4\int\frac{1}{a+b\coth(x)}d(b\coth(x))}{b\left(a^2-b^2\right)}-\frac{ab^2x}{a^2-b^2}+\frac{b^3\log(\sinh(x))}{a^2-b^2}\right)}{b}\right)}{b}$$

$\downarrow$ 16

$$-\frac{\coth^2(x)}{2b}-\frac{i\left(\frac{ia\coth(x)}{b}-\frac{i\left(-\frac{ab^2x}{a^2-b^2}+\frac{b^3\log(\sinh(x))}{a^2-b^2}+\frac{a^4\log(a+b\coth(x))}{b\left(a^2-b^2\right)}\right)}{b}\right)}{b}$$

input
```
Int[Coth[x]^4/(a + b*Coth[x]),x]
```

output
```
-1/2*Coth[x]^2/b - (I*((I*a*Coth[x])/b - (I*(-((a*b^2*x)/(a^2 - b^2)) + (a
^4*Log[a + b*Coth[x]])/(b*(a^2 - b^2)) + (b^3*Log[Sinh[x]])/(a^2 - b^2)))/
b))/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4049
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))
 Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n
- 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[
e + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2
, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || I
ntegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0]))
)
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4110
```
Int[((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[a*(A - C)*(x/(a^2 + b^2)), x] + (Simp[(a^2*C +
A*b^2)/(a^2 + b^2)   Int[(1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x]
- Simp[b*((A - C)/(a^2 + b^2))   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b,
e, f, A, C}, x] && NeQ[a^2*C + A*b^2, 0] && NeQ[a^2 + b^2, 0] && NeQ[A, C]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

method	result
derivativedivides	$-\frac{\coth(x)^2}{2b}+\frac{a\coth(x)}{b^2}-\frac{a^4\ln(a+b\coth(x))}{b^3(a+b)(a-b)}-\frac{\ln(\coth(x)-1)}{2a+2b}+\frac{\ln(1+\coth(x))}{2a-2b}$
default	$-\frac{\coth(x)^2}{2b}+\frac{a\coth(x)}{b^2}-\frac{a^4\ln(a+b\coth(x))}{b^3(a+b)(a-b)}-\frac{\ln(\coth(x)-1)}{2a+2b}+\frac{\ln(1+\coth(x))}{2a-2b}$
parallelrisch	$\frac{-2\ln(\tanh(x)a+b)a^4+2\ln(1-\tanh(x))b^4+(2a^4-2b^4)\ln(\tanh(x))+2b\left(-\frac{b\coth(x)^2(a-b)}{2}+\coth(x)a(a-b)+b^2x\right)(a+}{2b^3a^2-2b^5}$
risch	$\frac{x}{a+b}-\frac{2a^2x}{b^3}-\frac{2x}{b}+\frac{2xa^4}{b^3(a^2-b^2)}+\frac{2e^{2x}a-2e^{2x}b-2a}{(e^{2x}-1)^2b^2}+\frac{\ln(e^{2x}-1)a^2}{b^3}+\frac{\ln(e^{2x}-1)}{b}-\frac{a^4\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{b^3(a^2-b^2)}$

input
```
int(coth(x)^4/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*coth(x)^2/b+a*coth(x)/b^2-1/b^3*a^4/(a+b)/(a-b)*ln(a+b*coth(x))-1/(2*
a+2*b)*ln(coth(x)-1)+1/(2*a-2*b)*ln(1+coth(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(74) = 148$.

Time = 0.13 (sec) , antiderivative size = 648, normalized size of antiderivative = 8.53

$$\int \frac{\coth^4(x)}{a + b\coth(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
((a*b^3 + b^4)*x*cosh(x)^4 + 4*(a*b^3 + b^4)*x*cosh(x)*sinh(x)^3 + (a*b^3
+ b^4)*x*sinh(x)^4 - 2*a^3*b + 2*a*b^3 + 2*(a^3*b - a^2*b^2 - a*b^3 + b^4
- (a*b^3 + b^4)*x)*cosh(x)^2 + 2*(a^3*b - a^2*b^2 - a*b^3 + b^4 + 3*(a*b^3
 + b^4)*x*cosh(x)^2 - (a*b^3 + b^4)*x)*sinh(x)^2 + (a*b^3 + b^4)*x - (a^4*
cosh(x)^4 + 4*a^4*cosh(x)*sinh(x)^3 + a^4*sinh(x)^4 - 2*a^4*cosh(x)^2 + a^
4 + 2*(3*a^4*cosh(x)^2 - a^4)*sinh(x)^2 + 4*(a^4*cosh(x)^3 - a^4*cosh(x))*
sinh(x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) + ((a^4 - b^4)
*cosh(x)^4 + 4*(a^4 - b^4)*cosh(x)*sinh(x)^3 + (a^4 - b^4)*sinh(x)^4 + a^4
 - b^4 - 2*(a^4 - b^4)*cosh(x)^2 - 2*(a^4 - b^4 - 3*(a^4 - b^4)*cosh(x)^2)
*sinh(x)^2 + 4*((a^4 - b^4)*cosh(x)^3 - (a^4 - b^4)*cosh(x))*sinh(x))*log(
2*sinh(x)/(cosh(x) - sinh(x))) + 4*((a*b^3 + b^4)*x*cosh(x)^3 + (a^3*b - a
^2*b^2 - a*b^3 + b^4 - (a*b^3 + b^4)*x)*cosh(x))*sinh(x))/(a^2*b^3 - b^5 +
 (a^2*b^3 - b^5)*cosh(x)^4 + 4*(a^2*b^3 - b^5)*cosh(x)*sinh(x)^3 + (a^2*b^
3 - b^5)*sinh(x)^4 - 2*(a^2*b^3 - b^5)*cosh(x)^2 - 2*(a^2*b^3 - b^5 - 3*(a
^2*b^3 - b^5)*cosh(x)^2)*sinh(x)^2 + 4*((a^2*b^3 - b^5)*cosh(x)^3 - (a^2*b
^3 - b^5)*cosh(x))*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(61) = 122$.

Time = 1.65 (sec) , antiderivative size = 882, normalized size of antiderivative = 11.61

$$\int \frac{\coth^4(x)}{a + b\coth(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)**4/(a+b*coth(x)),x)
```

output
```
Piecewise((zoo*(x - log(tanh(x) + 1) + log(tanh(x)) - 1/(2*tanh(x)**2)), E
q(a, 0) & Eq(b, 0)), ((x - log(tanh(x) + 1) + log(tanh(x)) - 1/(2*tanh(x)*
*2))/b, Eq(a, 0)), (7*x*tanh(x)**3/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) - 7*x
*tanh(x)**2/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) - 4*log(tanh(x) + 1)*tanh(x)
**3/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) + 4*log(tanh(x) + 1)*tanh(x)**2/(2*b
*tanh(x)**3 - 2*b*tanh(x)**2) + 4*log(tanh(x))*tanh(x)**3/(2*b*tanh(x)**3
- 2*b*tanh(x)**2) - 4*log(tanh(x))*tanh(x)**2/(2*b*tanh(x)**3 - 2*b*tanh(x
)**2) - 3*tanh(x)**2/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) + tanh(x)/(2*b*tanh
(x)**3 - 2*b*tanh(x)**2) + 1/(2*b*tanh(x)**3 - 2*b*tanh(x)**2), Eq(a, -b))
, (x*tanh(x)**3/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + x*tanh(x)**2/(2*b*tanh
(x)**3 + 2*b*tanh(x)**2) - 4*log(tanh(x) + 1)*tanh(x)**3/(2*b*tanh(x)**3 +
 2*b*tanh(x)**2) - 4*log(tanh(x) + 1)*tanh(x)**2/(2*b*tanh(x)**3 + 2*b*tan
h(x)**2) + 4*log(tanh(x))*tanh(x)**3/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + 4
*log(tanh(x))*tanh(x)**2/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + 3*tanh(x)**2/
(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + tanh(x)/(2*b*tanh(x)**3 + 2*b*tanh(x)*
*2) - 1/(2*b*tanh(x)**3 + 2*b*tanh(x)**2), Eq(a, b)), ((x - 1/tanh(x) - 1/
(3*tanh(x)**3))/a, Eq(b, 0)), (-2*a**4*log(tanh(x) + b/a)*tanh(x)**2/(2*a*
*2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) + 2*a**4*log(tanh(x))*tanh(x)**2/(
2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) + 2*a**3*b*tanh(x)/(2*a**2*b**
3*tanh(x)**2 - 2*b**5*tanh(x)**2) - a**2*b**2/(2*a**2*b**3*tanh(x)**2 -...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.57

$$\int \frac{\coth^4(x)}{a+b\coth(x)}\,dx = -\frac{a^4\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^2b^3-b^5} + \frac{2\left((a+b)e^{(-2x)}-a\right)}{2\,b^2e^{(-2x)}-b^2e^{(-4x)}-b^2} + \frac{x}{a+b} + \frac{(a^2+b^2)\log\left(e^{(-x)}+1\right)}{b^3} + \frac{(a^2+b^2)\log\left(e^{(-x)}-1\right)}{b^3}$$

input
```
integrate(coth(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
-a^4*log(-(a - b)*e^(-2*x) + a + b)/(a^2*b^3 - b^5) + 2*((a + b)*e^(-2*x)
- a)/(2*b^2*e^(-2*x) - b^2*e^(-4*x) - b^2) + x/(a + b) + (a^2 + b^2)*log(e
^(-x) + 1)/b^3 + (a^2 + b^2)*log(e^(-x) - 1)/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.32

$$\int \frac{\coth^4(x)}{a+b\coth(x)}\,dx = -\frac{a^4\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^2b^3-b^5}+\frac{x}{a-b}+\frac{(a^2+b^2)\log\left(\left|e^{(2x)}-1\right|\right)}{b^3}-\frac{2\left(ab-(ab-b^2)e^{(2x)}\right)}{b^3(e^{(2x)}-1)^2}$$

input
```
integrate(coth(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

output
```
-a^4*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b^3 - b^5) + x/(a - b) +
 (a^2 + b^2)*log(abs(e^(2*x) - 1))/b^3 - 2*(a*b - (a*b - b^2)*e^(2*x))/(b^
3*(e^(2*x) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.45

$$\int \frac{\coth^4(x)}{a+b\coth(x)}\,dx = \frac{x}{a-b}-\frac{2}{b\,\left(e^{4x}-2\,e^{2x}+1\right)}+\frac{\ln\left(e^{2x}-1\right)\,\left(a^2+b^2\right)}{b^3}+\frac{a^4\,\ln\left(b-a+a\,e^{2x}+b\,e^{2x}\right)}{b^5-a^2\,b^3}+\frac{2\,\left(a^2-b^2\right)}{b^2\,\left(a+b\right)\,\left(e^{2x}-1\right)}$$

input
```
int(coth(x)^4/(a + b*coth(x)),x)
```

output
```
x/(a - b) - 2/(b*(exp(4*x) - 2*exp(2*x) + 1)) + (log(exp(2*x) - 1)*(a^2 +
b^2))/b^3 + (a^4*log(b - a + a*exp(2*x) + b*exp(2*x)))/(b^5 - a^2*b^3) + (
2*(a^2 - b^2))/(b^2*(a + b)*(exp(2*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.51

$$\int \frac{\coth^4(x)}{a+b\coth(x)}\,dx$$
$$= \frac{-\coth(x)^2 a^2b^2 + \coth(x)^2 b^4 + 2\coth(x)\,a^3b - 2\coth(x)\,a\,b^3 - 2\log(\coth(x)\,b+a)\,a^4 + 2\log(\coth(x}{2b^3(a^2-b^2)}$$

input
```
int(coth(x)^4/(a+b*coth(x)),x)
```

output
```
( - coth(x)**2*a**2*b**2 + coth(x)**2*b**4 + 2*coth(x)*a**3*b - 2*coth(x)*
a*b**3 - 2*log(coth(x)*b + a)*a**4 + 2*log(coth(x)*b + a)*b**4 - 2*log(e**
(2*x)*a + e**(2*x)*b - a + b)*b**4 + 2*a*b**3*x + 2*b**4*x)/(2*b**3*(a**2
- b**2))
```

3.154 $\int \frac{\coth^5(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 94

$$\int \frac{\coth^5(x)}{a+b\coth(x)}\,dx = -\frac{bx}{a^2-b^2}-\frac{(a^2+b^2)\coth(x)}{b^3}+\frac{a\coth^2(x)}{2b^2}-\frac{\coth^3(x)}{3b}+\frac{a^5\log(a+b\coth(x))}{b^4(a^2-b^2)}+\frac{a\log(\sinh(x))}{a^2-b^2}$$

output
```
-b*x/(a^2-b^2)-(a^2+b^2)*coth(x)/b^3+1/2*a*coth(x)^2/b^2-1/3*coth(x)^3/b+a
^5*ln(a+b*coth(x))/b^4/(a^2-b^2)+a*ln(sinh(x))/(a^2-b^2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int \frac{\coth^5(x)}{a+b\coth(x)}\,dx = \frac{1}{6}\left(-\frac{6(a^2+b^2)\coth(x)}{b^3}+\frac{3a\coth^2(x)}{b^2}-\frac{2\coth^3(x)}{b}-\frac{3\log(1-\coth(x))}{a+b}-\frac{3\log(1+\coth(x))}{a-b}+\frac{6a^5\log(a+b\coth(x))}{b^4(a^2-b^2)}\right)$$

input
```
Integrate[Coth[x]^5/(a + b*Coth[x]),x]
```

output
```
((-6*(a^2 + b^2)*Coth[x])/b^3 + (3*a*Coth[x]^2)/b^2 - (2*Coth[x]^3)/b - (3
*Log[1 - Coth[x]])/(a + b) - (3*Log[1 + Coth[x]])/(a - b) + (6*a^5*Log[a +
 b*Coth[x]])/(b^4*(a^2 - b^2)))/6
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.39, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 1.538$, Rules used = {3042, 26, 4049, 27, 3042, 25, 4130, 27, 3042, 26, 4131, 25, 3042, 4109, 26, 3042, 26, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^5(x)}{a+b\coth(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan\left(\frac{\pi}{2}+ix\right)^5}{a-ib\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan\left(ix+\frac{\pi}{2}\right)^5}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4049}$$

$$-i\left(\frac{i\int \frac{3\coth^2(x)(-a\coth^2(x)+b\coth(x)+a)}{a+b\coth(x)}dx}{3b}-\frac{i\coth^3(x)}{3b}\right)$$

$$\Big\downarrow \text{27}$$

$$-i\left(\frac{i\int \frac{\coth^2(x)(-a\coth^2(x)+b\coth(x)+a)}{a+b\coth(x)}dx}{b}-\frac{i\coth^3(x)}{3b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\int -\frac{\tan\left(ix+\frac{\pi}{2}\right)^2\left(a\tan\left(ix+\frac{\pi}{2}\right)^2-ib\tan\left(ix+\frac{\pi}{2}\right)+a\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{b}-\frac{i\coth^3(x)}{3b}\right)$$

$$\Big\downarrow \text{25}$$

$$-i\left(-\frac{i\int \frac{\tan\left(ix+\frac{\pi}{2}\right)^2\left(a\tan\left(ix+\frac{\pi}{2}\right)^2-ib\tan\left(ix+\frac{\pi}{2}\right)+a\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{b}-\frac{i\coth^3(x)}{3b}\right)$$

$$\Big\downarrow \text{4130}$$

$$-i\left(-\frac{i\left(-\frac{a\coth^2(x)}{2b}+\frac{i\int -\frac{2i\coth(x)\left(a^2-\left(a^2+b^2\right)\coth^2(x)\right)}{a+b\coth(x)}dx}{2b}\right)}{b}-\frac{i\coth^3(x)}{3b}\right)$$

$$\Big\downarrow \text{27}$$

$$-i\left(-\frac{i\left(\frac{\int \frac{\coth(x)\left(a^2-\left(a^2+b^2\right)\coth^2(x)\right)}{a+b\coth(x)}dx}{b}-\frac{a\coth^2(x)}{2b}\right)}{b}-\frac{i\coth^3(x)}{3b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{i\left(-\frac{a\coth^2(x)}{2b}+\frac{\int -\frac{i\tan\left(ix+\frac{\pi}{2}\right)\left(a^2+\left(a^2+b^2\right)\tan\left(ix+\frac{\pi}{2}\right)^2\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{b}\right)}{b}-\frac{i\coth^3(x)}{3b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{i\left(-\frac{a\coth^2(x)}{2b}-\frac{i\int\frac{\tan\left(ix+\frac{\pi}{2}\right)\left(a^2+\left(a^2+b^2\right)\tan\left(ix+\frac{\pi}{2}\right)^2\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{b}\right)}{b}-\frac{i\coth^3(x)}{3b}\right)$$

$\downarrow$ 4131

$$-i\left(-\frac{i\left(-\frac{a\coth^2(x)}{2b}-\frac{i\left(\frac{i\int-\frac{\coth(x)b^3-a\left(a^2+b^2\right)\coth^2(x)+a\left(a^2+b^2\right)}{a+b\coth(x)}dx}{b}+\frac{i\left(a^2+b^2\right)\coth(x)}{b}\right)}{b}\right)}{b}-\frac{i\coth^3(x)}{3b}\right)$$

$\downarrow$ 25

$$-i\left(-\frac{i\left(-\frac{a\coth^2(x)}{2b}-\frac{i\left(\frac{i\left(a^2+b^2\right)\coth(x)}{b}-\frac{i\int\frac{\coth(x)b^3-a\left(a^2+b^2\right)\coth^2(x)+a\left(a^2+b^2\right)}{a+b\coth(x)}dx}{b}\right)}{b}\right)}{b}-\frac{i\coth^3(x)}{3b}\right)$$

$\downarrow$ 3042

$$-i\left(-\frac{i\left(-\frac{a\coth^2(x)}{2b}-\frac{i\left(\frac{i\left(a^2+b^2\right)\coth(x)}{b}-\frac{i\int\frac{-i\tan\left(ix+\frac{\pi}{2}\right)b^3+a\left(a^2+b^2\right)\tan\left(ix+\frac{\pi}{2}\right)^2+a\left(a^2+b^2\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{b}\right)}{b}\right)}{b}-\frac{i\coth^3(x)}{3b}\right)$$

$$\Big\downarrow \text{ 4109}$$

$$-i\left(-\frac{i\left(-\frac{a\coth^2(x)}{2b}-\frac{i\left(\frac{i\left(a^2+b^2\right)\coth(x)}{b}-\frac{i\left(-\frac{iab^3\int i\coth(x)dx}{a^2-b^2}+\frac{a^5\int\frac{1-\coth^2(x)}{a+b\coth(x)}dx}{a^2-b^2}-\frac{b^4x}{a^2-b^2}\right)}{b}\right)}{b}\right)}{b}-\frac{i\coth^3(x)}{3b}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(-\frac{i\left(-\frac{a\coth^2(x)}{2b}-\frac{i\left(\frac{i\left(a^2+b^2\right)\coth(x)}{b}-\frac{i\left(\frac{ab^3\int\coth(x)dx}{a^2-b^2}+\frac{a^5\int\frac{1-\coth^2(x)}{a+b\coth(x)}dx}{a^2-b^2}-\frac{b^4x}{a^2-b^2}\right)}{b}\right)}{b}\right)}{b}-\frac{i\coth^3(x)}{3b}\right)$$

$$\downarrow 3042$$

$$-i\left(-\frac{i\left(-\frac{a\coth^2(x)}{2b}-\frac{i\left(\frac{i\left(a^2+b^2\right)\coth(x)}{b}-\frac{i\left(\frac{ab^3\int -i\tan\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}+\frac{a^5\int \frac{\tan\left(ix+\frac{\pi}{2}\right)^2+1}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{b^4x}{a^2-b^2}\right)}{b}\right)}{b}\right)}{b}-\frac{i\coth^3(x)}{3b}\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(-\frac{i\left(-\frac{a\coth^2(x)}{2b}-\frac{i\left(\frac{i(a^2+b^2)\coth(x)}{b}-\frac{i\left(-\frac{iab^3\int\tan\left(ix+\frac{\pi}{2}\right)dx}{a^2-b^2}+\frac{a^5\int\frac{\tan\left(ix+\frac{\pi}{2}\right)^2+1}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{b^4x}{a^2-b^2}\right)}{b}\right)}{b}\right)}{b}-\frac{i\coth^3(x)}{3b}\right)$$

$\downarrow$ 3956

$$-i\left(-\frac{i\left(-\frac{a\coth^2(x)}{2b}-\frac{i\left(\frac{i(a^2+b^2)\coth(x)}{b}-\frac{i\left(\frac{a^5\int\frac{\tan\left(ix+\frac{\pi}{2}\right)^2+1}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{b^4x}{a^2-b^2}+\frac{ab^3\log(\sinh(x))}{a^2-b^2}\right)}{b}\right)}{b}\right)}{b}-\frac{i\coth^3(x)}{3b}\right)$$

$\downarrow$ 4100

$$-i\left(-\frac{i\left(-\frac{a\coth^2(x)}{2b}-\frac{i\left(\frac{i\left(a^2+b^2\right)\coth(x)}{b}-\frac{i\left(\frac{a^5\int\frac{1}{a+b\coth(x)}d(b\coth(x))}{b\left(a^2-b^2\right)}-\frac{b^4x}{a^2-b^2}+\frac{ab^3\log(\sinh(x))}{a^2-b^2}\right)}{b}\right)}{b}\right)}{b}-\frac{i\coth^3(x)}{3b}\right)$$

$$\Big\downarrow 16$$

$$-i\left(-\frac{i\left(-\frac{a\coth^2(x)}{2b}-\frac{i\left(\frac{i\left(a^2+b^2\right)\coth(x)}{b}-\frac{i\left(-\frac{b^4x}{a^2-b^2}+\frac{ab^3\log(\sinh(x))}{a^2-b^2}+\frac{a^5\log(a+b\coth(x))}{b\left(a^2-b^2\right)}\right)}{b}\right)}{b}\right)}{b}-\frac{i\coth^3(x)}{3b}\right)$$

input
```
Int[Coth[x]^5/(a + b*Coth[x]),x]
```

output
```
(-I)*(((-1/3*I)*Coth[x]^3)/b - (I*(-1/2*(a*Coth[x]^2)/b - (I*((I*(a^2 + b^
2)*Coth[x])/b - (I*(-((b^4*x)/(a^2 - b^2)) + (a^5*Log[a + b*Coth[x]])/(b*(
a^2 - b^2)) + (a*b^3*Log[Sinh[x]])/(a^2 - b^2)))/b))/b))/b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4049
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))
 Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n
- 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[
e + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2
, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || I
ntegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0]))
)
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4131
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n +
1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d
*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*(b*c*m + a*d*(n + 1)) + d*(A*b
- b*C)*(m + n + 1)*Tan[e + f*x] - C*m*(b*c - a*d)*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
+ b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ
[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

method	result
derivativedivides	$-\frac{\coth(x)^{3}}{3b}+\frac{a\coth(x)^{2}}{2b^{2}}-\frac{\coth(x)a^{2}}{b^{3}}-\frac{\coth(x)}{b}+\frac{a^{5}\ln(a+b\coth(x))}{b^{4}(a+b)(a-b)}-\frac{\ln(\coth(x)-1)}{2a+2b}-\frac{\ln(1+\coth(x))}{2a-2b}$
default	$-\frac{\coth(x)^{3}}{3b}+\frac{a\coth(x)^{2}}{2b^{2}}-\frac{\coth(x)a^{2}}{b^{3}}-\frac{\coth(x)}{b}+\frac{a^{5}\ln(a+b\coth(x))}{b^{4}(a+b)(a-b)}-\frac{\ln(\coth(x)-1)}{2a+2b}-\frac{\ln(1+\coth(x))}{2a-2b}$
parallelrisch	$\frac{6a^{5}\ln(\tanh(x)a+b)-6\ln(1-\tanh(x))a\,b^{4}+(-6a^{5}+6b^{4}a)\ln(\tanh(x))+(-2b^{3}a^{2}+2b^{5})\coth(x)^{3}+(3a^{3}b^{2}-3b^{4}a)\coth(}{6a^{2}b^{4}-6b^{6}}$
risch	$\frac{x}{a+b}+\frac{2x\,a^{3}}{b^{4}}+\frac{2ax}{b^{2}}-\frac{2x\,a^{5}}{b^{4}(a^{2}-b^{2})}-\frac{2(3\,e^{4x}a^{2}-3\,e^{4x}ab+6b^{2}e^{4x}-6\,e^{2x}a^{2}+3\,e^{2x}ab-6b^{2}e^{2x}+3a^{2}+4b^{2})}{3b^{3}(e^{2x}-1)^{3}}-\frac{a^{3}\ln}{}$

input

```
int(coth(x)^5/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

output

```
-1/3*coth(x)^3/b+1/2*a*coth(x)^2/b^2-1/b^3*coth(x)*a^2-coth(x)/b+1/b^4*a^5
/(a+b)/(a-b)*ln(a+b*coth(x))-1/(2*a+2*b)*ln(coth(x)-1)-1/(2*a-2*b)*ln(1+co
th(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1299 vs. 2(90) = 180.

Time = 0.12 (sec) , antiderivative size = 1299, normalized size of antiderivative = 13.82

$$\int \frac{\coth^5(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

input

```
integrate(coth(x)^5/(a+b*coth(x)),x, algorithm="fricas")
```

output
```
-1/3*(3*(a*b^4 + b^5)*x*cosh(x)^6 + 18*(a*b^4 + b^5)*x*cosh(x)*sinh(x)^5 +
 3*(a*b^4 + b^5)*x*sinh(x)^6 + 6*a^4*b + 2*a^2*b^3 - 8*b^5 + 3*(2*a^4*b -
2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x)^4 + 3
*(2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 + 15*(a*b^4 + b^5)*x*c
osh(x)^2 - 3*(a*b^4 + b^5)*x)*sinh(x)^4 + 12*(5*(a*b^4 + b^5)*x*cosh(x)^3
+ (2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*
cosh(x))*sinh(x)^3 - 3*(4*a^4*b - 2*a^3*b^2 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 +
 b^5)*x)*cosh(x)^2 + 3*(15*(a*b^4 + b^5)*x*cosh(x)^4 - 4*a^4*b + 2*a^3*b^2
 - 2*a*b^4 + 4*b^5 + 6*(2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5
- 3*(a*b^4 + b^5)*x)*cosh(x)^2 + 3*(a*b^4 + b^5)*x)*sinh(x)^2 - 3*(a*b^4 +
 b^5)*x - 3*(a^5*cosh(x)^6 + 6*a^5*cosh(x)*sinh(x)^5 + a^5*sinh(x)^6 - 3*a
^5*cosh(x)^4 + 3*a^5*cosh(x)^2 - a^5 + 3*(5*a^5*cosh(x)^2 - a^5)*sinh(x)^4
 + 4*(5*a^5*cosh(x)^3 - 3*a^5*cosh(x))*sinh(x)^3 + 3*(5*a^5*cosh(x)^4 - 6*
a^5*cosh(x)^2 + a^5)*sinh(x)^2 + 6*(a^5*cosh(x)^5 - 2*a^5*cosh(x)^3 + a^5*
cosh(x))*sinh(x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) + 3*(
(a^5 - a*b^4)*cosh(x)^6 + 6*(a^5 - a*b^4)*cosh(x)*sinh(x)^5 + (a^5 - a*b^4
)*sinh(x)^6 - a^5 + a*b^4 - 3*(a^5 - a*b^4)*cosh(x)^4 - 3*(a^5 - a*b^4 - 5
*(a^5 - a*b^4)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5 - a*b^4)*cosh(x)^3 - 3*(a^
5 - a*b^4)*cosh(x))*sinh(x)^3 + 3*(a^5 - a*b^4)*cosh(x)^2 + 3*(a^5 - a*b^4
 + 5*(a^5 - a*b^4)*cosh(x)^4 - 6*(a^5 - a*b^4)*cosh(x)^2)*sinh(x)^2 + 6...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1013 vs. $2(78) = 156$.

Time = 2.50 (sec) , antiderivative size = 1013, normalized size of antiderivative = 10.78

$$\int \frac{\coth^5(x)}{a + b\coth(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)**5/(a+b*coth(x)),x)
```

output
```
Piecewise((zoo*(x - 1/tanh(x) - 1/(3*tanh(x)**3)), Eq(a, 0) & Eq(b, 0)), (
(x - 1/tanh(x) - 1/(3*tanh(x)**3))/b, Eq(a, 0)), (27*x*tanh(x)**4/(6*b*tan
h(x)**4 - 6*b*tanh(x)**3) - 27*x*tanh(x)**3/(6*b*tanh(x)**4 - 6*b*tanh(x)*
*3) - 12*log(tanh(x) + 1)*tanh(x)**4/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) + 1
2*log(tanh(x) + 1)*tanh(x)**3/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) + 12*log(t
anh(x))*tanh(x)**4/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) - 12*log(tanh(x))*tan
h(x)**3/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) - 15*tanh(x)**3/(6*b*tanh(x)**4
- 6*b*tanh(x)**3) + 9*tanh(x)**2/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) + tanh(
x)/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) + 2/(6*b*tanh(x)**4 - 6*b*tanh(x)**3)
, Eq(a, -b)), (3*x*tanh(x)**4/(6*b*tanh(x)**4 + 6*b*tanh(x)**3) + 3*x*tanh
(x)**3/(6*b*tanh(x)**4 + 6*b*tanh(x)**3) + 12*log(tanh(x) + 1)*tanh(x)**4/
(6*b*tanh(x)**4 + 6*b*tanh(x)**3) + 12*log(tanh(x) + 1)*tanh(x)**3/(6*b*ta
nh(x)**4 + 6*b*tanh(x)**3) - 12*log(tanh(x))*tanh(x)**4/(6*b*tanh(x)**4 +
6*b*tanh(x)**3) - 12*log(tanh(x))*tanh(x)**3/(6*b*tanh(x)**4 + 6*b*tanh(x)
**3) - 15*tanh(x)**3/(6*b*tanh(x)**4 + 6*b*tanh(x)**3) - 9*tanh(x)**2/(6*b
*tanh(x)**4 + 6*b*tanh(x)**3) + tanh(x)/(6*b*tanh(x)**4 + 6*b*tanh(x)**3)
- 2/(6*b*tanh(x)**4 + 6*b*tanh(x)**3), Eq(a, b)), ((x - log(tanh(x) + 1) +
 log(tanh(x)) - 1/(2*tanh(x)**2) - 1/(4*tanh(x)**4))/a, Eq(b, 0)), (6*a**5
*log(tanh(x) + b/a)*tanh(x)**3/(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3
) - 6*a**5*log(tanh(x))*tanh(x)**3/(6*a**2*b**4*tanh(x)**3 - 6*b**6*tan...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.80

$$\int \frac{\coth^5(x)}{a+b\coth(x)}\,dx = \frac{a^5\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^2b^4-b^6} + \frac{2\left(3a^2+4b^2-3(2a^2+ab+2b^2)e^{(-2x)}+3(a^2+ab+2b^2)e^{(-4x)}\right)}{3\left(3b^3e^{(-2x)}-3b^3e^{(-4x)}+b^3e^{(-6x)}-b^3\right)} + \frac{x}{a+b} - \frac{(a^3+ab^2)\log\left(e^{(-x)}+1\right)}{b^4} - \frac{(a^3+ab^2)\log\left(e^{(-x)}-1\right)}{b^4}$$

input
```
integrate(coth(x)^5/(a+b*coth(x)),x, algorithm="maxima")
```

output
```
a^5*log(-(a - b)*e^(-2*x) + a + b)/(a^2*b^4 - b^6) + 2/3*(3*a^2 + 4*b^2 -
3*(2*a^2 + a*b + 2*b^2)*e^(-2*x) + 3*(a^2 + a*b + 2*b^2)*e^(-4*x))/(3*b^3*
e^(-2*x) - 3*b^3*e^(-4*x) + b^3*e^(-6*x) - b^3) + x/(a + b) - (a^3 + a*b^2
)*log(e^(-x) + 1)/b^4 - (a^3 + a*b^2)*log(e^(-x) - 1)/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.52

$$\int \frac{\coth^5(x)}{a+b\coth(x)}\,dx = \frac{a^5\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^2b^4-b^6} - \frac{x}{a-b} - \frac{(a^3+ab^2)\log\left(\left|e^{(2x)}-1\right|\right)}{b^4} - \frac{2\left(3a^2b+4b^3+3\,(a^2b-ab^2+2b^3)e^{(4x)}-3\,(2a^2b-ab^2+2b^3)e^{(2x)}\right)}{3b^4(e^{(2x)}-1)^3}$$

input
```
integrate(coth(x)^5/(a+b*coth(x)),x, algorithm="giac")
```

output
```
a^5*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b^4 - b^6) - x/(a - b) -
(a^3 + a*b^2)*log(abs(e^(2*x) - 1))/b^4 - 2/3*(3*a^2*b + 4*b^3 + 3*(a^2*b
- a*b^2 + 2*b^3)*e^(4*x) - 3*(2*a^2*b - a*b^2 + 2*b^3)*e^(2*x))/(b^4*(e^(2
*x) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.81 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.74

$$\int \frac{\coth^5(x)}{a+b\coth(x)}\,dx = -\frac{8}{3b\,\left(3\,e^{2x}-3\,e^{4x}+e^{6x}-1\right)} - \frac{x}{a-b} - \frac{a^5\,\ln\left(b-a+a\,e^{2x}+b\,e^{2x}\right)}{b^6-a^2\,b^4} - \frac{\ln\left(e^{2x}-1\right)\,\left(a^3+a\,b^2\right)}{b^4} - \frac{2\,\left(a^3+a\,b^2+2\,b^3\right)}{b^3\,\left(a+b\right)\,\left(e^{2x}-1\right)} - \frac{2\,\left(-a^2+a\,b+2\,b^2\right)}{b^2\,\left(a+b\right)\,\left(e^{4x}-2\,e^{2x}+1\right)}$$

input
```
int(coth(x)^5/(a + b*coth(x)),x)
```

output
```
- 8/(3*b*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - x/(a - b) - (a^5*log(
b - a + a*exp(2*x) + b*exp(2*x)))/(b^6 - a^2*b^4) - (log(exp(2*x) - 1)*(a*
b^2 + a^3))/b^4 - (2*(a*b^2 + a^3 + 2*b^3))/(b^3*(a + b)*(exp(2*x) - 1)) -
 (2*(a*b - a^2 + 2*b^2))/(b^2*(a + b)*(exp(4*x) - 2*exp(2*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.48

$$\int \frac{\coth^5(x)}{a+b\coth(x)}\,dx$$
$$= \frac{-2\coth(x)^3 a^2b^3 + 2\coth(x)^3 b^5 + 3\coth(x)^2 a^3b^2 - 3\coth(x)^2 a\,b^4 - 6\coth(x)\,a^4b + 6\coth(x)\,b^5 + 6}{6b^4(a^2 -}$$

input
```
int(coth(x)^5/(a+b*coth(x)),x)
```

output
```
( - 2*coth(x)**3*a**2*b**3 + 2*coth(x)**3*b**5 + 3*coth(x)**2*a**3*b**2 -
3*coth(x)**2*a*b**4 - 6*coth(x)*a**4*b + 6*coth(x)*b**5 + 6*log(coth(x)*b
+ a)*a**5 - 6*log(coth(x)*b + a)*a*b**4 + 6*log(e**(2*x)*a + e**(2*x)*b -
a + b)*a*b**4 - 6*a*b**4*x - 6*b**5*x)/(6*b**4*(a**2 - b**2))
```

3.155 $\int \frac{x\text{csch}^2(x)}{(a+b\coth(x))^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int \frac{x\text{csch}^2(x)}{(a+b\coth(x))^2}\,dx=-\frac{ax}{b\,(a^2-b^2)}+\frac{x}{b(a+b\coth(x))}+\frac{\log(b\cosh(x)+a\sinh(x))}{a^2-b^2}$$

output
```
-a*x/b/(a^2-b^2)+x/b/(a+b*coth(x))+ln(b*cosh(x)+a*sinh(x))/(a^2-b^2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{x\text{csch}^2(x)}{(a+b\coth(x))^2}\,dx=\frac{ax-b\log(b\cosh(x)+a\sinh(x))}{-a^2b+b^3}+\frac{x\sinh(x)}{b^2\cosh(x)+ab\sinh(x)}$$

input
```
Integrate[(x*Csch[x]^2)/(a + b*Coth[x])^2,x]
```

output
```
(a*x - b*Log[b*Cosh[x] + a*Sinh[x]])/(-(a^2*b) + b^3) + (x*Sinh[x])/(b^2*C
osh[x] + a*b*Sinh[x])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5990, 3042, 3965, 26, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\text{csch}^2(x)}{(a+b\coth(x))^2}dx$$

$$\Big\downarrow \text{ 5990}$$

$$\frac{x}{b(a+b\coth(x))}-\frac{\int \frac{1}{a+b\coth(x)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x}{b(a+b\coth(x))}-\frac{\int \frac{1}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{ 3965}$$

$$\frac{x}{b(a+b\coth(x))}-\frac{\frac{ax}{a^2-b^2}-\frac{ib\int -\frac{i(b+a\coth(x))}{a+b\coth(x)}dx}{a^2-b^2}}{b}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{x}{b(a+b\coth(x))}-\frac{\frac{ax}{a^2-b^2}-\frac{b\int \frac{b+a\coth(x)}{a+b\coth(x)}dx}{a^2-b^2}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x}{b(a+b\coth(x))}-\frac{\frac{ax}{a^2-b^2}-\frac{b\int \frac{b-ia\tan\left(ix+\frac{\pi}{2}\right)}{a-ib\tan\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{b}$$

$$\Big\downarrow \text{ 4013}$$

$$\frac{x}{b(a+b\coth(x))}-\frac{\frac{ax}{a^2-b^2}-\frac{b\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}}{b}$$

input
```
Int[(x*Csch[x]^2)/(a + b*Coth[x])^2,x]
```

output
```
x/(b*(a + b*Coth[x])) - ((a*x)/(a^2 - b^2) - (b*Log[b*Cosh[x] + a*Sinh[x]]
)/(a^2 - b^2))/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3965
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Simp[b/(a^2 + b^2)   Int[(b - a*Tan[c + d*x])/(a + b*Tan[c
+ d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 5990
```
Int[Csch[(c_.) + (d_.)*(x_)]^2*(Coth[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.
)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e + f*x)^m)*((a + b*Coth
[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b*d*(n + 1)))   Int[(e +
 f*x)^(m - 1)*(a + b*Coth[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

method	result	size
risch	$-\frac{2x}{a^{2}-b^{2}}-\frac{2x}{\left(\mathrm{e}^{2x} a+\mathrm{e}^{2x} b-a+b\right)\left(a+b\right)}+\frac{\ln\left(\mathrm{e}^{2x}-\frac{a-b}{a+b}\right)}{a^{2}-b^{2}}$	73

input
```
int(x*csch(x)^2/(a+b*coth(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/(a^2-b^2)*x-2*x/(exp(2*x)*a+exp(2*x)*b-a+b)/(a+b)+1/(a^2-b^2)*ln(exp(2*
x)-(a-b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(54) = 108$.

Time = 0.11 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.41

$$\int \frac{x\mathrm{csch}^2(x)}{(a+b\coth(x))^2}\,dx = \frac{2\,(a+b)x\cosh\left(x\right)^{2}+4\,(a+b)x\cosh\left(x\right)\sinh\left(x\right)+2\,(a+b)x\sinh\left(x\right)^{2}-\left((a+b)\cosh\left(x\right)^{2}+2\,(a+b)\,c\right.}{a^3-a^2b-ab^2+b^3-(a^3+a^2b-ab^2-b^3)\cosh\left(x\right)^{2}-2\,(a^3+a^2b-ab^2-b^3)\cos}$$

input
```
integrate(x*csch(x)^2/(a+b*coth(x))^2,x, algorithm="fricas")
```

output
```
(2*(a + b)*x*cosh(x)^2 + 4*(a + b)*x*cosh(x)*sinh(x) + 2*(a + b)*x*sinh(x)
^2 - ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 -
a + b)*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))))/(a^3 - a^2*b -
a*b^2 + b^3 - (a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 - 2*(a^3 + a^2*b - a*b
^2 - b^3)*cosh(x)*sinh(x) - (a^3 + a^2*b - a*b^2 - b^3)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{x\text{csch}^2(x)}{(a+b\coth(x))^2}\,dx = \int \frac{x\,\text{csch}^2\,(x)}{(a+b\coth\,(x))^2}\,dx$$

input
```
integrate(x*csch(x)**2/(a+b*coth(x))**2,x)
```

output
```
Integral(x*csch(x)**2/(a + b*coth(x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \frac{x\text{csch}^2(x)}{(a+b\coth(x))^2}\,dx = \frac{2\,xe^{(2\,x)}}{a^2-2\,ab+b^2-(a^2-b^2)e^{(2\,x)}} + \frac{\log\left(\frac{(a+b)e^{(2\,x)}-a+b}{a+b}\right)}{a^2-b^2}$$

input
```
integrate(x*csch(x)^2/(a+b*coth(x))^2,x, algorithm="maxima")
```

output
```
2*x*e^(2*x)/(a^2 - 2*a*b + b^2 - (a^2 - b^2)*e^(2*x)) + log(((a + b)*e^(2*
x) - a + b)/(a + b))/(a^2 - b^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(54) = 108.

Time = 0.13 (sec) , antiderivative size = 169, normalized size of antiderivative = 3.13

$$\int \frac{x\text{csch}^2(x)}{(a+b\coth(x))^2}\,dx = -\frac{2\,axe^{(2\,x)}+2\,bxe^{(2\,x)}-ae^{(2\,x)}\log\left(ae^{(2\,x)}+be^{(2\,x)}-a+b\right)-be^{(2\,x)}\log\left(ae^{(2\,x)}+be^{(2\,x)}-a+b\right)+a\,\text{lo}}{a^3e^{(2\,x)}+a^2be^{(2\,x)}-ab^2e^{(2\,x)}-b^3e^{(2\,x)}-a^3+a^2b+}$$

input
```
integrate(x*csch(x)^2/(a+b*coth(x))^2,x, algorithm="giac")
```

output
```
-(2*a*x*e^(2*x) + 2*b*x*e^(2*x) - a*e^(2*x)*log(a*e^(2*x) + b*e^(2*x) - a
+ b) - b*e^(2*x)*log(a*e^(2*x) + b*e^(2*x) - a + b) + a*log(a*e^(2*x) + b*
e^(2*x) - a + b) - b*log(a*e^(2*x) + b*e^(2*x) - a + b))/(a^3*e^(2*x) + a^
2*b*e^(2*x) - a*b^2*e^(2*x) - b^3*e^(2*x) - a^3 + a^2*b + a*b^2 - b^3)
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \frac{x\mathrm{csch}^2(x)}{(a+b\coth(x))^2}\,dx = \frac{\ln\left(b-a+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x}\right)}{a^2-b^2}-\frac{2\,x}{a^2-b^2} - \frac{2\,x}{(a+b)\,\left(b-a+\mathrm{e}^{2\,x}\,\left(a+b\right)\right)}$$

input
```
int(x/(sinh(x)^2*(a + b*coth(x))^2),x)
```

output
```
log(b - a + a*exp(2*x) + b*exp(2*x))/(a^2 - b^2) - (2*x)/(a^2 - b^2) - (2*
x)/((a + b)*(b - a + exp(2*x)*(a + b)))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.37

$$\int \frac{x\mathrm{csch}^2(x)}{(a+b\coth(x))^2}\,dx = \frac{e^{2x}\log(e^{2x}a+e^{2x}b-a+b)\,a+e^{2x}\log(e^{2x}a+e^{2x}b-a+b)\,b-2e^{2x}ax-2e^{2x}bx-\log(e^{2x}a+e^{2x}b-a+}{e^{2x}a^3+e^{2x}a^2b-e^{2x}a\,b^2-e^{2x}b^3-a^3+a^2b+a\,b^2-b^3}$$

input
```
int(x*csch(x)^2/(a+b*coth(x))^2,x)
```

output
```
(e**(2*x)*log(e**(2*x)*a + e**(2*x)*b - a + b)*a + e**(2*x)*log(e**(2*x)*a
 + e**(2*x)*b - a + b)*b - 2*e**(2*x)*a*x - 2*e**(2*x)*b*x - log(e**(2*x)*
a + e**(2*x)*b - a + b)*a + log(e**(2*x)*a + e**(2*x)*b - a + b)*b)/(e**(2
*x)*a**3 + e**(2*x)*a**2*b - e**(2*x)*a*b**2 - e**(2*x)*b**3 - a**3 + a**2
*b + a*b**2 - b**3)
```

3.156 $\int x^3 \coth(a+2\log(x))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int x^3 \coth(a+2\log(x))\,dx = \frac{x^4}{4} + \frac{1}{2}e^{-2a}\log\left(1-e^{2a}x^4\right)$$

output
```
1/4*x^4+1/2*ln(1-exp(2*a)*x^4)/exp(2*a)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 64 vs. 2(30) = 60.

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.13

$$\int x^3 \coth(a+2\log(x))\,dx = \frac{x^4}{4} + \frac{1}{2}\cosh(2a)\log\left(-\cosh(a)+x^4\cosh(a)+\sinh(a)+x^4\sinh(a)\right) - \frac{1}{2}\log\left(-\cosh(a)+x^4\cosh(a)+\sinh(a)+x^4\sinh(a)\right)\sinh(2a)$$

input
```
Integrate[x^3*Coth[a + 2*Log[x]],x]
```

output
```
x^4/4 + (Cosh[2*a]*Log[-Cosh[a] + x^4*Cosh[a] + Sinh[a] + x^4*Sinh[a]])/2
- (Log[-Cosh[a] + x^4*Cosh[a] + Sinh[a] + x^4*Sinh[a]]*Sinh[2*a])/2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {6072, 946, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth(a+2\log(x))\,dx$$

$$\Big\downarrow \text{6072}$$

$$\int \frac{x^3\left(-e^{2a}x^4-1\right)}{1-e^{2a}x^4}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{4}\int -\frac{e^{2a}x^4+1}{1-e^{2a}x^4}dx^4$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{4}\int \frac{e^{2a}x^4+1}{1-e^{2a}x^4}dx^4$$

$$\Big\downarrow \text{49}$$

$$-\frac{1}{4}\int \left(-1-\frac{2}{e^{2a}x^4-1}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(2e^{-2a}\log\left(1-e^{2a}x^4\right)+x^4\right)$$

input
```
Int[x^3*Coth[a + 2*Log[x]],x]
```

output
```
(x^4 + (2*Log[1 - E^(2*a)*x^4])/E^(2*a))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{x^4}{4}+\frac{\mathrm{e}^{-2a}\ln\left(\mathrm{e}^{2a}x^4-1\right)}{2}$	24

input
```
int(x^3*coth(a+2*ln(x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4+1/2*exp(-2*a)*ln(exp(2*a)*x^4-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int x^3 \coth(a+2\log(x))\,dx = \frac{1}{4}\left(x^4e^{(2a)} + 2\log\left(x^4e^{(2a)} - 1\right)\right)e^{(-2a)}$$

input
```
integrate(x^3*coth(a+2*log(x)),x, algorithm="fricas")
```

output
```
1/4*(x^4*e^(2*a) + 2*log(x^4*e^(2*a) - 1))*e^(-2*a)
```

Sympy [F]

$$\int x^3 \coth(a+2\log(x))\,dx = \int x^3 \coth\left(a+2\log\left(x\right)\right)\,dx$$

input
```
integrate(x**3*coth(a+2*ln(x)),x)
```

output
```
Integral(x**3*coth(a + 2*log(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int x^3 \coth(a+2\log(x))\,dx = \frac{1}{4}x^4 + \frac{1}{2}e^{(-2a)}\log\left(x^2e^a+1\right) + \frac{1}{2}e^{(-2a)}\log\left(x^2e^a-1\right)$$

input
```
integrate(x^3*coth(a+2*log(x)),x, algorithm="maxima")
```

output
```
1/4*x^4 + 1/2*e^(-2*a)*log(x^2*e^a + 1) + 1/2*e^(-2*a)*log(x^2*e^a - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^3 \coth(a+2\log(x))\,dx = \frac{1}{4}x^4 + \frac{1}{2}e^{(-2a)}\log\left(\left|x^4e^{(2a)} - 1\right|\right)$$

input
```
integrate(x^3*coth(a+2*log(x)),x, algorithm="giac")
```

output
```
1/4*x^4 + 1/2*e^(-2*a)*log(abs(x^4*e^(2*a) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int x^3 \coth(a+2\log(x))\,dx = \frac{\ln\left(x^4 - \mathrm{e}^{-2a}\right)\mathrm{e}^{-2a}}{2} + \frac{x^4}{4}$$

input
```
int(x^3*coth(a + 2*log(x)),x)
```

output
```
(log(x^4 - exp(-2*a))*exp(-2*a))/2 + x^4/4
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.83

$$\int x^3 \coth(a+2\log(x))\,dx = \frac{e^{2a}x^4 + 2\log\left(e^{\frac{a}{2}}x - 1\right) + 2\log\left(e^{\frac{a}{2}}x + 1\right) + 2\log(e^a x^2 + 1)}{4e^{2a}}$$

input
```
int(x^3*coth(a+2*log(x)),x)
```

output
```
(e**(2*a)*x**4 + 2*log(e**(a/2)*x - 1) + 2*log(e**(a/2)*x + 1) + 2*log(e**
a*x**2 + 1))/(4*e**(2*a))
```

3.157 $\int x^2 \coth(a+2\log(x))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 45

$$\int x^2 \coth(a+2\log(x))\,dx = \frac{x^3}{3} + e^{-3a/2}\arctan\left(e^{a/2}x\right) - e^{-3a/2}\text{arctanh}\left(e^{a/2}x\right)$$

output
```
1/3*x^3+arctan(exp(1/2*a)*x)/exp(3/2*a)-arctanh(exp(1/2*a)*x)/exp(3/2*a)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.42

$$\int x^2 \coth(a+2\log(x))\,dx = \frac{1}{6}\left(2x^3 + 3\text{RootSum}\left[-\cosh(a)+\sinh(a)+\cosh(a)\#1^4 + \sinh(a)\#1^4\&, \frac{\log(x)-\log(x-\#1)}{\#1}\&\right](-\cosh(2a) + \sinh(2a))\right)$$

input
```
Integrate[x^2*Coth[a + 2*Log[x]],x]
```

output
```
(2*x^3 + 3*RootSum[-Cosh[a] + Sinh[a] + Cosh[a]*#1^4 + Sinh[a]*#1^4 & , (L
og[x] - Log[x - #1])/#1 & ]*(-Cosh[2*a] + Sinh[2*a]))/6
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {6072, 959, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth(a+2\log(x))\,dx$$

$$\Big\downarrow \text{6072}$$

$$\int \frac{x^2\left(-e^{2a}x^4-1\right)}{1-e^{2a}x^4}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{x^3}{3}-2\int \frac{x^2}{1-e^{2a}x^4}dx$$

$$\Big\downarrow \text{827}$$

$$\frac{x^3}{3}-2\left(\frac{1}{2}e^{-a}\int \frac{1}{1-e^ax^2}dx-\frac{1}{2}e^{-a}\int \frac{1}{e^ax^2+1}dx\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{x^3}{3}-2\left(\frac{1}{2}e^{-a}\int \frac{1}{1-e^ax^2}dx-\frac{1}{2}e^{-3a/2}\arctan\left(e^{a/2}x\right)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{x^3}{3}-2\left(\frac{1}{2}e^{-3a/2}\text{arctanh}\left(e^{a/2}x\right)-\frac{1}{2}e^{-3a/2}\arctan\left(e^{a/2}x\right)\right)$$

input
```
Int[x^2*Coth[a + 2*Log[x]],x]
```

output
```
x^3/3 - 2*(-1/2*ArcTan[E^(a/2)*x]/E^((3*a)/2) + ArcTanh[E^(a/2)*x]/(2*E^((
3*a)/2)))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(35) = 70$.

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.84

method	result	size
risch	$\frac{x^3}{3}+\frac{\ln\left(\left(-{\mathrm e}^{a}\right)^{\frac{3}{2}}-{\mathrm e}^{2 a} x\right)}{2 \left(-{\mathrm e}^{a}\right)^{\frac{3}{2}}}-\frac{\ln\left(\left(-{\mathrm e}^{a}\right)^{\frac{3}{2}}+{\mathrm e}^{2 a} x\right)}{2 \left(-{\mathrm e}^{a}\right)^{\frac{3}{2}}}+\frac{\ln\left(-x \sqrt{{\mathrm e}^{a}}+1\right)}{2 \left({\mathrm e}^{a}\right)^{\frac{3}{2}}}-\frac{\ln\left(x \sqrt{{\mathrm e}^{a}}+1\right)}{2 \left({\mathrm e}^{a}\right)^{\frac{3}{2}}}$	83

input
```
int(x^2*coth(a+2*ln(x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3+1/2/(-exp(a))^(3/2)*ln((-exp(a))^(3/2)-exp(2*a)*x)-1/2/(-exp(a))^(
3/2)*ln((-exp(a))^(3/2)+exp(2*a)*x)+1/2/exp(a)^(3/2)*ln(-x*exp(a)^(1/2)+1)
-1/2/exp(a)^(3/2)*ln(x*exp(a)^(1/2)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.38

$$\int x^2 \coth(a+2\log(x))\,dx$$
$$= \frac{1}{6}\left(2\,x^3 e^{(2\,a)} + 6\,\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(\frac{1}{2}\,a\right)} + 3\,e^{\left(\frac{1}{2}\,a\right)}\log\left(\frac{x^2e^a - 2\,xe^{\left(\frac{1}{2}\,a\right)} + 1}{x^2e^a - 1}\right)\right)e^{(-2\,a)}$$

input
```
integrate(x^2*coth(a+2*log(x)),x, algorithm="fricas")
```

output
```
1/6*(2*x^3*e^(2*a) + 6*arctan(x*e^(1/2*a))*e^(1/2*a) + 3*e^(1/2*a)*log((x^
2*e^a - 2*x*e^(1/2*a) + 1)/(x^2*e^a - 1)))*e^(-2*a)
```

Sympy [F]

$$\int x^2 \coth(a + 2\log(x))\,dx = \int x^2 \coth\left(a + 2\log\left(x\right)\right)dx$$

input
```
integrate(x**2*coth(a+2*ln(x)),x)
```

output
```
Integral(x**2*coth(a + 2*log(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07

$$\int x^2 \coth(a+2\log(x))\,dx = \frac{1}{3}\,x^3 + \arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{3}{2}\,a\right)} + \frac{1}{2}\,e^{\left(-\frac{3}{2}\,a\right)}\log\left(\frac{xe^a - e^{\left(\frac{1}{2}\,a\right)}}{xe^a + e^{\left(\frac{1}{2}\,a\right)}}\right)$$

input
```
integrate(x^2*coth(a+2*log(x)),x, algorithm="maxima")
```

output
```
1/3*x^3 + arctan(x*e^(1/2*a))*e^(-3/2*a) + 1/2*e^(-3/2*a)*log((x*e^a - e^(
1/2*a))/(x*e^a + e^(1/2*a)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int x^2 \coth(a + 2\log(x))\,dx = \frac{1}{3}\,x^3 + \arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{3}{2}\,a\right)} + \frac{1}{2}\,e^{\left(-\frac{3}{2}\,a\right)}\log\left(\frac{\left|2\,xe^a - 2\,e^{\left(\frac{1}{2}\,a\right)}\right|}{\left|2\,xe^a + 2\,e^{\left(\frac{1}{2}\,a\right)}\right|}\right)$$

input
```
integrate(x^2*coth(a+2*log(x)),x, algorithm="giac")
```

output
```
1/3*x^3 + arctan(x*e^(1/2*a))*e^(-3/2*a) + 1/2*e^(-3/2*a)*log(abs(2*x*e^a
- 2*e^(1/2*a))/abs(2*x*e^a + 2*e^(1/2*a)))
```

Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int x^2 \coth(a+2\log(x))\,dx = \frac{\operatorname{atan}\left(x\,\left(e^{2\,a}\right)^{1/4}\right)}{\left(e^{2\,a}\right)^{3/4}} - \frac{\operatorname{atanh}\left(x\,\left(e^{2\,a}\right)^{1/4}\right)}{\left(e^{2\,a}\right)^{3/4}} + \frac{x^3}{3}$$

input
```
int(x^2*coth(a + 2*log(x)),x)
```

output
```
atan(x*exp(2*a)^(1/4))/exp(2*a)^(3/4) - atanh(x*exp(2*a)^(1/4))/exp(2*a)^(
3/4) + x^3/3
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.64

$$\int x^2 \coth(a+2\log(x))\,dx$$
$$= \frac{6e^{\frac{a}{2}}\,atan\left(\frac{e^a x}{e^{\frac{a}{2}}}\right) + 3e^{\frac{a}{2}}\log\left(e^{\frac{a}{2}}x-1\right) - 3e^{\frac{a}{2}}\log\left(e^{\frac{a}{2}}x+1\right) + 2e^{2a}x^3}{6e^{2a}}$$

input
```
int(x^2*coth(a+2*log(x)),x)
```

output
```
(6*e**(a/2)*atan((e**a*x)/e**(a/2)) + 3*e**(a/2)*log(e**(a/2)*x - 1) - 3*e
**(a/2)*log(e**(a/2)*x + 1) + 2*e**(2*a)*x**3)/(6*e**(2*a))
```

3.158 $\int x \coth(a+2\log(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 23

$$\int x \coth(a+2\log(x))\,dx = \frac{x^2}{2} - e^{-a}\text{arctanh}\left(e^a x^2\right)$$

output
```
1/2*x^2-arctanh(exp(a)*x^2)/exp(a)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int x \coth(a+2\log(x))\,dx = \frac{x^2}{2} + \text{arctanh}\left(x^2(\cosh(a)+\sinh(a))\right)(-\cosh(a)+\sinh(a))$$

input
```
Integrate[x*Coth[a + 2*Log[x]],x]
```

output
```
x^2/2 + ArcTanh[x^2*(Cosh[a] + Sinh[a])]*(-Cosh[a] + Sinh[a])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6072, 959, 807, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth(a + 2\log(x)) \, dx$$

$$\Big\downarrow \text{ 6072}$$

$$\int \frac{x\left(-e^{2a}x^4 - 1\right)}{1 - e^{2a}x^4}dx$$

$$\Big\downarrow \text{ 959}$$

$$\frac{x^2}{2} - 2\int \frac{x}{1 - e^{2a}x^4}dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{x^2}{2} - \int \frac{1}{1 - e^{2a}x^4}dx^2$$

$$\Big\downarrow \text{ 219}$$

$$\frac{x^2}{2} - e^{-a}\text{arctanh}\left(e^a x^2\right)$$

input
```
Int[x*Coth[a + 2*Log[x]],x]
```

output
```
x^2/2 - ArcTanh[E^a*x^2]/E^a
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

method	result	size
risch	$\frac{x^2}{2}+\frac{\mathrm{e}^{-a}\ln\left(\mathrm{e}^{a}x^2-1\right)}{2}-\frac{\mathrm{e}^{-a}\ln\left(\mathrm{e}^{a}x^2+1\right)}{2}$	37

input
```
int(x*coth(a+2*ln(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+1/2/exp(a)*ln(exp(a)*x^2-1)-1/2/exp(a)*ln(exp(a)*x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int x\coth(a+2\log(x))\,dx=\frac{1}{2}\left(x^2e^a-\log\left(x^2e^a+1\right)+\log\left(x^2e^a-1\right)\right)e^{(-a)}$$

input
```
integrate(x*coth(a+2*log(x)),x, algorithm="fricas")
```

output
```
1/2*(x^2*e^a - log(x^2*e^a + 1) + log(x^2*e^a - 1))*e^(-a)
```

Sympy [F]

$$\int x\coth(a+2\log(x))\,dx=\int x\coth\left(a+2\log\left(x\right)\right)\,dx$$

input
```
integrate(x*coth(a+2*ln(x)),x)
```

output
```
Integral(x*coth(a + 2*log(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int x\coth(a+2\log(x))\,dx=\frac{1}{2}x^2-\frac{1}{2}e^{(-a)}\log\left(x^2e^a+1\right)+\frac{1}{2}e^{(-a)}\log\left(x^2e^a-1\right)$$

input
```
integrate(x*coth(a+2*log(x)),x, algorithm="maxima")
```

output
```
1/2*x^2 - 1/2*e^(-a)*log(x^2*e^a + 1) + 1/2*e^(-a)*log(x^2*e^a - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int x \coth(a + 2\log(x))\, dx = \frac{1}{2}x^2 - \frac{1}{2}e^{(-a)}\log\left(x^2e^a + 1\right) + \frac{1}{2}e^{(-a)}\log\left(\left|x^2e^a - 1\right|\right)$$

input
```
integrate(x*coth(a+2*log(x)),x, algorithm="giac")
```

output
```
1/2*x^2 - 1/2*e^(-a)*log(x^2*e^a + 1) + 1/2*e^(-a)*log(abs(x^2*e^a - 1))
```

Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x \coth(a + 2\log(x))\, dx = \frac{x^2}{2} - \frac{\operatorname{atanh}\left(x^2\sqrt{e^{2a}}\right)}{\sqrt{e^{2a}}}$$

input
```
int(x*coth(a + 2*log(x)),x)
```

output
```
x^2/2 - atanh(x^2*exp(2*a)^(1/2))/exp(2*a)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int x \coth(a + 2\log(x))\, dx = \frac{e^a x^2 + \log\left(e^{\frac{a}{2}}x - 1\right) + \log\left(e^{\frac{a}{2}}x + 1\right) - \log\left(e^a x^2 + 1\right)}{2e^a}$$

input
```
int(x*coth(a+2*log(x)),x)
```

output
```
(e**a*x**2 + log(e**(a/2)*x - 1) + log(e**(a/2)*x + 1) - log(e**a*x**2 + 1
))/(2*e**a)
```

3.159 $\int \coth(a+2\log(x))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 40

$$\int \coth(a+2\log(x))\,dx = x-e^{-a/2}\arctan\left(e^{a/2}x\right)-e^{-a/2}\text{arctanh}\left(e^{a/2}x\right)$$

output
```
x-arctan(exp(1/2*a)*x)/exp(1/2*a)-arctanh(exp(1/2*a)*x)/exp(1/2*a)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \coth(a+2\log(x))\,dx = x+\frac{1}{2}\text{RootSum}\left[-\cosh(a)+\sinh(a)+\cosh(a)\#1^4+\sinh(a)\#1^4\&,\frac{\log(x)-\log(x-\#1)}{\#1^3}\&\right](-\cosh(2a)+\sinh(2a))$$

input
```
Integrate[Coth[a + 2*Log[x]],x]
```

output
```
x + (RootSum[-Cosh[a] + Sinh[a] + Cosh[a]*#1^4 + Sinh[a]*#1^4 & , (Log[x]
- Log[x - #1])/#1^3 & ]*(-Cosh[2*a] + Sinh[2*a]))/2
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6068, 913, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(a+2\log(x))\,dx$$

$$\Big\downarrow \text{6068}$$

$$\int \frac{-e^{2a}x^4-1}{1-e^{2a}x^4}dx$$

$$\Big\downarrow \text{913}$$

$$x-2\int \frac{1}{1-e^{2a}x^4}dx$$

$$\Big\downarrow \text{756}$$

$$x-2\left(\frac{1}{2}\int \frac{1}{1-e^a x^2}dx+\frac{1}{2}\int \frac{1}{e^a x^2+1}dx\right)$$

$$\Big\downarrow \text{216}$$

$$x-2\left(\frac{1}{2}\int \frac{1}{1-e^a x^2}dx+\frac{1}{2}e^{-a/2}\arctan\left(e^{a/2}x\right)\right)$$

$$\Big\downarrow \text{219}$$

$$x-2\left(\frac{1}{2}e^{-a/2}\arctan\left(e^{a/2}x\right)+\frac{1}{2}e^{-a/2}\text{arctanh}\left(e^{a/2}x\right)\right)$$

input
```
Int[Coth[a + 2*Log[x]],x]
```

output
```
x - 2*(ArcTan[E^(a/2)*x]/(2*E^(a/2)) + ArcTanh[E^(a/2)*x]/(2*E^(a/2)))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 6068
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(32) = 64$.

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.78

method	result	size
risch	$x-\frac{\ln\left(x\sqrt{-{\mathrm e}^{a}}+1\right)}{2\sqrt{-{\mathrm e}^{a}}}+\frac{\ln\left(x\sqrt{-{\mathrm e}^{a}}-1\right)}{2\sqrt{-{\mathrm e}^{a}}}+\frac{\ln\left(x\sqrt{{\mathrm e}^{a}}-1\right)}{2\sqrt{{\mathrm e}^{a}}}-\frac{\ln\left(x\sqrt{{\mathrm e}^{a}}+1\right)}{2\sqrt{{\mathrm e}^{a}}}$	71

input
```
int(coth(a+2*ln(x)),x,method=_RETURNVERBOSE)
```

output
```
x-1/2/(-exp(a))^(1/2)*ln(x*(-exp(a))^(1/2)+1)+1/2/(-exp(a))^(1/2)*ln(x*(-e
xp(a))^(1/2)-1)+1/2/exp(a)^(1/2)*ln(x*exp(a)^(1/2)-1)-1/2/exp(a)^(1/2)*ln(
x*exp(a)^(1/2)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(28) = 56$.

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \coth(a + 2\log(x))\,dx$$
$$= -\frac{1}{2}\left(2\,\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(\frac{1}{2}\,a\right)} - 2\,xe^{a} - e^{\left(\frac{1}{2}\,a\right)}\log\left(\frac{x^{2}e^{a} - 2\,xe^{\left(\frac{1}{2}\,a\right)} + 1}{x^{2}e^{a} - 1}\right)\right)e^{(-a)}$$

input
```
integrate(coth(a+2*log(x)),x, algorithm="fricas")
```

output
```
-1/2*(2*arctan(x*e^(1/2*a))*e^(1/2*a) - 2*x*e^a - e^(1/2*a)*log((x^2*e^a -
 2*x*e^(1/2*a) + 1)/(x^2*e^a - 1)))*e^(-a)
```

Sympy [F]

$$\int \coth(a + 2\log(x))\,dx = \int \coth\left(a + 2\log\left(x\right)\right)dx$$

input
```
integrate(coth(a+2*ln(x)),x)
```

output
```
Integral(coth(a + 2*log(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \coth(a+2\log(x))\,dx = -\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{1}{2}\,a\right)} + \frac{1}{2}\,e^{\left(-\frac{1}{2}\,a\right)}\log\left(\frac{xe^a - e^{\left(\frac{1}{2}\,a\right)}}{xe^a + e^{\left(\frac{1}{2}\,a\right)}}\right) + x$$

input
```
integrate(coth(a+2*log(x)),x, algorithm="maxima")
```

output
```
-arctan(x*e^(1/2*a))*e^(-1/2*a) + 1/2*e^(-1/2*a)*log((x*e^a - e^(1/2*a))/(
x*e^a + e^(1/2*a))) + x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \coth(a+2\log(x))\,dx = -\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{1}{2}\,a\right)} + \frac{1}{2}\,e^{\left(-\frac{1}{2}\,a\right)}\log\left(\frac{\left|2\,xe^a - 2\,e^{\left(\frac{1}{2}\,a\right)}\right|}{\left|2\,xe^a + 2\,e^{\left(\frac{1}{2}\,a\right)}\right|}\right) + x$$

input
```
integrate(coth(a+2*log(x)),x, algorithm="giac")
```

output
```
-arctan(x*e^(1/2*a))*e^(-1/2*a) + 1/2*e^(-1/2*a)*log(abs(2*x*e^a - 2*e^(1/
2*a))/abs(2*x*e^a + 2*e^(1/2*a))) + x
```

Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \coth(a+2\log(x))\,dx = x - \frac{\mathrm{atan}\left(x\,{\left(\mathrm{e}^{2\,a}\right)}^{1/4}\right)}{{\left(\mathrm{e}^{2\,a}\right)}^{1/4}} - \frac{\mathrm{atanh}\left(x\,{\left(\mathrm{e}^{2\,a}\right)}^{1/4}\right)}{{\left(\mathrm{e}^{2\,a}\right)}^{1/4}}$$

input
```
int(coth(a + 2*log(x)),x)
```

output
```
x - atan(x*exp(2*a)^(1/4))/exp(2*a)^(1/4) - atanh(x*exp(2*a)^(1/4))/exp(2*
a)^(1/4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.68

$$\int \coth(a+2\log(x))\,dx = \frac{-2e^{\frac{a}{2}}\,atan\left(\frac{e^{a}x}{e^{\frac{a}{2}}}\right)+e^{\frac{a}{2}}\log\left(e^{\frac{a}{2}}x-1\right)-e^{\frac{a}{2}}\log\left(e^{\frac{a}{2}}x+1\right)+2e^{a}x}{2e^{a}}$$

input
```
int(coth(a+2*log(x)),x)
```

output
```
( - 2*e**(a/2)*atan((e**a*x)/e**(a/2)) + e**(a/2)*log(e**(a/2)*x - 1) - e*
*(a/2)*log(e**(a/2)*x + 1) + 2*e**a*x)/(2*e**a)
```

3.160 $\int \frac{\coth(a+2\log(x))}{x} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{\coth(a+2\log(x))}{x} \, dx = \frac{1}{2}\log(\sinh(a+2\log(x)))$$

output
```
1/2*ln(sinh(a+2*ln(x)))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\coth(a+2\log(x))}{x} \, dx = \frac{1}{2}\log(\sinh(a+2\log(x)))$$

input
```
Integrate[Coth[a + 2*Log[x]]/x,x]
```

output
```
Log[Sinh[a + 2*Log[x]]]/2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3039, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(a+2\log(x))}{x} \, dx$$

$$\Big\downarrow \text{3039}$$

$$\int \coth(a+2\log(x))d\log(x)$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan\left(ia+2i\log(x)+\frac{\pi}{2}\right)d\log(x)$$

$$\Big\downarrow \text{26}$$

$$-i\int \tan\left(\frac{1}{2}(2ia+\pi)+2i\log(x)\right)d\log(x)$$

$$\Big\downarrow \text{3956}$$

$$\frac{1}{2}\log(-i\sinh(a+2\log(x)))$$

input
```
Int[Coth[a + 2*Log[x]]/x,x]
```

output
```
Log[(-I)*Sinh[a + 2*Log[x]]]/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\ln(\sinh(a+2\ln(x)))}{2}$	11
default	$\frac{\ln(\sinh(a+2\ln(x)))}{2}$	11
risch	$-\ln\left(x\right)+\frac{\ln(1-\mathrm{e}^{2a}x^{4})}{2}$	20
parallelrisch	$-\ln\left(x\right)+\ln\left(\sqrt{\tanh\left(a+2\ln\left(x\right)\right)}\right)+\ln\left(\frac{1}{\sqrt{1-\tanh(a+2\ln(x))}}\right)$	30

input
```
int(coth(a+2*ln(x))/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(sinh(a+2*ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{\coth(a+2\log(x))}{x}\,dx = \frac{1}{2}\log\left(x^4e^{(2\,a)}-1\right)-\log\left(x\right)$$

input
```
integrate(coth(a+2*log(x))/x,x, algorithm="fricas")
```

output
```
1/2*log(x^4*e^(2*a) - 1) - log(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(10) = 20.

Time = 0.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int \frac{\coth(a+2\log(x))}{x}\,dx = \log\left(x\right)-\frac{\log\left(\tanh\left(a+2\log\left(x\right)\right)+1\right)}{2}+\frac{\log\left(\tanh\left(a+2\log\left(x\right)\right)\right)}{2}$$

input
```
integrate(coth(a+2*ln(x))/x,x)
```

output
```
log(x) - log(tanh(a + 2*log(x)) + 1)/2 + log(tanh(a + 2*log(x)))/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\coth(a+2\log(x))}{x}\,dx = \frac{1}{2}\log\left(\sinh\left(a+2\,\log\left(x\right)\right)\right)$$

input
```
integrate(coth(a+2*log(x))/x,x, algorithm="maxima")
```

output
```
1/2*log(sinh(a + 2*log(x)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75

$$\int \frac{\coth(a + 2\log(x))}{x}\, dx = -\frac{1}{4} \log\left(x^4\right) + \frac{1}{2} \log\left(\left|x^4 e^{(2\,a)} - 1\right|\right)$$

input
```
integrate(coth(a+2*log(x))/x,x, algorithm="giac")
```

output
```
-1/4*log(x^4) + 1/2*log(abs(x^4*e^(2*a) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{\coth(a + 2\log(x))}{x}\, dx = \frac{\ln\left(x^4 - \mathrm{e}^{-2\,a}\right)}{2} - \ln\left(x\right)$$

input
```
int(coth(a + 2*log(x))/x,x)
```

output
```
log(x^4 - exp(-2*a))/2 - log(x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.42

$$\int \frac{\coth(a + 2\log(x))}{x}\, dx = \frac{\log\left(e^{\frac{a}{2}} x - 1\right)}{2} + \frac{\log\left(e^{\frac{a}{2}} x + 1\right)}{2} + \frac{\log(e^{a} x^{2} + 1)}{2} - \log(x)$$

input
```
int(coth(a+2*log(x))/x,x)
```

output
```
(log(e**(a/2)*x - 1) + log(e**(a/2)*x + 1) + log(e**a*x**2 + 1) - 2*log(x)
)/2
```

3.161 $\int \frac{\coth(a+2\log(x))}{x^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 41

$$\int \frac{\coth(a+2\log(x))}{x^2} \, dx = \frac{1}{x} + e^{a/2} \arctan\left(e^{a/2} x\right) - e^{a/2} \text{arctanh}\left(e^{a/2} x\right)$$

output
```
1/x+exp(1/2*a)*arctan(exp(1/2*a)*x)-exp(1/2*a)*arctanh(exp(1/2*a)*x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.51

$$\int \frac{\coth(a+2\log(x))}{x^2} \, dx = \frac{2 + x\text{RootSum}\left[-\cosh(a) - \sinh(a) + \cosh(a)\#1^4 - \sinh(a)\#1^4\&, \frac{\log(x)+\log\left(\frac{1}{x}-\#1\right)}{\#1^3}\&\right] (\cosh(a) + \sinh}{2x}$$

input
```
Integrate[Coth[a + 2*Log[x]]/x^2,x]
```

output
```
(2 + x*RootSum[-Cosh[a] - Sinh[a] + Cosh[a]*#1^4 - Sinh[a]*#1^4 & , (Log[x
] + Log[x^(-1) - #1])/#1^3 & ]*(Cosh[a] + Sinh[a])^2)/(2*x)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {6072, 955, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(a+2\log(x))}{x^2}\,dx$$
$$\Big\downarrow \text{6072}$$
$$\int \frac{-e^{2a}x^4-1}{x^2\left(1-e^{2a}x^4\right)}dx$$
$$\Big\downarrow \text{955}$$
$$\frac{1}{x}-2e^{2a}\int \frac{x^2}{1-e^{2a}x^4}dx$$
$$\Big\downarrow \text{827}$$
$$\frac{1}{x}-2e^{2a}\left(\frac{1}{2}e^{-a}\int \frac{1}{1-e^a x^2}dx-\frac{1}{2}e^{-a}\int \frac{1}{e^a x^2+1}dx\right)$$
$$\Big\downarrow \text{216}$$
$$\frac{1}{x}-2e^{2a}\left(\frac{1}{2}e^{-a}\int \frac{1}{1-e^a x^2}dx-\frac{1}{2}e^{-3a/2}\arctan\left(e^{a/2}x\right)\right)$$
$$\Big\downarrow \text{219}$$
$$\frac{1}{x}-2e^{2a}\left(\frac{1}{2}e^{-3a/2}\text{arctanh}\left(e^{a/2}x\right)-\frac{1}{2}e^{-3a/2}\arctan\left(e^{a/2}x\right)\right)$$

input
```
Int[Coth[a + 2*Log[x]]/x^2,x]
```

output
```
x^(-1) - 2*E^(2*a)*(-1/2*ArcTan[E^(a/2)*x]/E^((3*a)/2) + ArcTanh[E^(a/2)*x
]/(2*E^((3*a)/2)))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.78

method	result
risch	$\frac{1}{x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{2}-\mathrm{e}^{a}\right)}\_R\ln\left(\left(-5\_R^{4}+4\,\mathrm{e}^{2a}\right)x+\_R^{3}\right)\right)}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{2}+\mathrm{e}^{a}\right)}\_R\ln\left(\left(-5\_R^{4}+4\,\mathrm{e}^{2a}\right)x+\right.\right.}{2}$

input
```
int(coth(a+2*ln(x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x+1/2*sum(_R*ln((-5*_R^4+4*exp(2*a))*x+_R^3),_R=RootOf(_Z^2-exp(a)))+1/2
*sum(_R*ln((-5*_R^4+4*exp(2*a))*x+_R^3),_R=RootOf(_Z^2+exp(a)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int \frac{\coth(a+2\log(x))}{x^2}\,dx = \frac{2\,x\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(\frac{1}{2}\,a\right)} + xe^{\left(\frac{1}{2}\,a\right)}\log\left(\frac{x^2e^a-2\,xe^{\left(\frac{1}{2}\,a\right)}+1}{x^2e^a-1}\right)+2}{2\,x}$$

input
```
integrate(coth(a+2*log(x))/x^2,x, algorithm="fricas")
```

output
```
1/2*(2*x*arctan(x*e^(1/2*a))*e^(1/2*a) + x*e^(1/2*a)*log((x^2*e^a - 2*x*e^
(1/2*a) + 1)/(x^2*e^a - 1)) + 2)/x
```

Sympy [F]

$$\int \frac{\coth(a+2\log(x))}{x^2}\,dx = \int \frac{\coth\left(a+2\log\left(x\right)\right)}{x^2}\,dx$$

input
```
integrate(coth(a+2*ln(x))/x**2,x)
```

output
```
Integral(coth(a + 2*log(x))/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

$$\int \frac{\coth(a+2\log(x))}{x^2}\,dx = -\arctan\left(\frac{e^{\left(-\frac{1}{2}a\right)}}{x}\right)e^{\left(\frac{1}{2}a\right)} + \frac{1}{2}e^{\left(\frac{1}{2}a\right)}\log\left(\frac{\frac{1}{x}-e^{\left(\frac{1}{2}a\right)}}{\frac{1}{x}+e^{\left(\frac{1}{2}a\right)}}\right) + \frac{1}{x}$$

input
```
integrate(coth(a+2*log(x))/x^2,x, algorithm="maxima")
```

output
```
-arctan(e^(-1/2*a)/x)*e^(1/2*a) + 1/2*e^(1/2*a)*log((1/x - e^(1/2*a))/(1/x
 + e^(1/2*a))) + 1/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27

$$\int \frac{\coth(a+2\log(x))}{x^2}\,dx = \arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)} + \frac{1}{2}e^{\left(\frac{1}{2}a\right)}\log\left(\frac{\left|2\,xe^{a}-2\,e^{\left(\frac{1}{2}a\right)}\right|}{\left|2\,xe^{a}+2\,e^{\left(\frac{1}{2}a\right)}\right|}\right) + \frac{1}{x}$$

input
```
integrate(coth(a+2*log(x))/x^2,x, algorithm="giac")
```

output
```
arctan(x*e^(1/2*a))*e^(1/2*a) + 1/2*e^(1/2*a)*log(abs(2*x*e^a - 2*e^(1/2*a
))/abs(2*x*e^a + 2*e^(1/2*a))) + 1/x
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{\coth(a+2\log(x))}{x^2}\,dx = \left(e^{2a}\right)^{1/4}\operatorname{atan}\left(x\left(e^{2a}\right)^{1/4}\right) - \left(e^{2a}\right)^{1/4}\operatorname{atanh}\left(x\left(e^{2a}\right)^{1/4}\right) + \frac{1}{x}$$

input
```
int(coth(a + 2*log(x))/x^2,x)
```

output
```
exp(2*a)^(1/4)*atan(x*exp(2*a)^(1/4)) - exp(2*a)^(1/4)*atanh(x*exp(2*a)^(1
/4)) + 1/x
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.54

$$\int \frac{\coth(a+2\log(x))}{x^2}\,dx = \frac{2e^{\frac{a}{2}}atan\left(\frac{e^a x}{e^{\frac{a}{2}}}\right)x + e^{\frac{a}{2}}\log\left(e^{\frac{a}{2}}x-1\right)x - e^{\frac{a}{2}}\log\left(e^{\frac{a}{2}}x+1\right)x + 2}{2x}$$

input
```
int(coth(a+2*log(x))/x^2,x)
```

output
```
(2*e**(a/2)*atan((e**a*x)/e**(a/2))*x + e**(a/2)*log(e**(a/2)*x - 1)*x - e
**(a/2)*log(e**(a/2)*x + 1)*x + 2)/(2*x)
```

3.162 $\int \frac{\coth(a+2\log(x))}{x^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \frac{\coth(a+2\log(x))}{x^3}\,dx = \frac{1}{2x^2} - e^a \operatorname{arctanh}\left(e^a x^2\right)$$

output
```
1/2/x^2-exp(a)*arctanh(exp(a)*x^2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{\coth(a+2\log(x))}{x^3}\,dx = \frac{1}{2x^2} - \operatorname{arctanh}\left(\frac{\cosh(a)-\sinh(a)}{x^2}\right)(\cosh(a)+\sinh(a))$$

input
```
Integrate[Coth[a + 2*Log[x]]/x^3,x]
```

output
```
1/(2*x^2) - ArcTanh[(Cosh[a] - Sinh[a])/x^2]*(Cosh[a] + Sinh[a])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6072, 955, 807, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(a+2\log(x))}{x^3}\,dx$$

$$\Big\downarrow \text{ 6072}$$

$$\int \frac{-e^{2a}x^4-1}{x^3\left(1-e^{2a}x^4\right)}dx$$

$$\Big\downarrow \text{ 955}$$

$$\frac{1}{2x^2}-2e^{2a}\int \frac{x}{1-e^{2a}x^4}dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2x^2}-e^{2a}\int \frac{1}{1-e^{2a}x^4}dx^2$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2x^2}-e^a\text{arctanh}\left(e^ax^2\right)$$

input
```
Int[Coth[a + 2*Log[x]]/x^3,x]
```

output
```
1/(2*x^2) - E^a*ArcTanh[E^a*x^2]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.67

method	result	size
risch	$\frac{1}{2x^2} - \frac{e^a \ln(-e^a x^2-1)}{2} + \frac{e^a \ln(-e^a x^2+1)}{2}$	35

input
```
int(coth(a+2*ln(x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/x^2-1/2*exp(a)*ln(-exp(a)*x^2-1)+1/2*exp(a)*ln(-exp(a)*x^2+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.81

$$\int \frac{\coth(a+2\log(x))}{x^3}\,dx = -\frac{x^2e^a\log\left(x^2e^a+1\right)-x^2e^a\log\left(x^2e^a-1\right)-1}{2\,x^2}$$

input
```
integrate(coth(a+2*log(x))/x^3,x, algorithm="fricas")
```

output
```
-1/2*(x^2*e^a*log(x^2*e^a + 1) - x^2*e^a*log(x^2*e^a - 1) - 1)/x^2
```

Sympy [F]

$$\int \frac{\coth(a+2\log(x))}{x^3}\,dx = \int \frac{\coth\left(a+2\log\left(x\right)\right)}{x^3}\,dx$$

input
```
integrate(coth(a+2*ln(x))/x**3,x)
```

output
```
Integral(coth(a + 2*log(x))/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

$$\int \frac{\coth(a+2\log(x))}{x^3}\,dx = -\frac{1}{2}\,e^a\log\left(\frac{1}{x^2}+e^a\right)+\frac{1}{2}\,e^a\log\left(\frac{1}{x^2}-e^a\right)+\frac{1}{2\,x^2}$$

input
```
integrate(coth(a+2*log(x))/x^3,x, algorithm="maxima")
```

output
```
-1/2*e^a*log(1/x^2 + e^a) + 1/2*e^a*log(1/x^2 - e^a) + 1/2/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

$$\int \frac{\coth(a+2\log(x))}{x^3}\,dx = -\frac{1}{2}\,e^a\log\left(x^2e^a+1\right) + \frac{1}{2}\,e^a\log\left(\left|x^2e^a-1\right|\right) + \frac{1}{2\,x^2}$$

input
```
integrate(coth(a+2*log(x))/x^3,x, algorithm="giac")
```

output
```
-1/2*e^a*log(x^2*e^a + 1) + 1/2*e^a*log(abs(x^2*e^a - 1)) + 1/2/x^2
```

Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{\coth(a+2\log(x))}{x^3}\,dx = \frac{1}{2\,x^2} - \operatorname{atanh}\left(x^2\,\sqrt{\mathrm{e}^{2\,a}}\right)\,\sqrt{\mathrm{e}^{2\,a}}$$

input
```
int(coth(a + 2*log(x))/x^3,x)
```

output
```
1/(2*x^2) - atanh(x^2*exp(2*a)^(1/2))*exp(2*a)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.81

$$\int \frac{\coth(a+2\log(x))}{x^3}\,dx = \frac{e^a\log\left(e^{\frac{a}{2}}x-1\right)x^2 + e^a\log\left(e^{\frac{a}{2}}x+1\right)x^2 - e^a\log(e^ax^2+1)\,x^2+1}{2x^2}$$

input
```
int(coth(a+2*log(x))/x^3,x)
```

output
```
(e**a*log(e**(a/2)*x - 1)*x**2 + e**a*log(e**(a/2)*x + 1)*x**2 - e**a*log(
e**a*x**2 + 1)*x**2 + 1)/(2*x**2)
```

3.163 $\int x^3 \coth^2(a+2\log(x))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int x^3 \coth^2(a+2\log(x))\,dx=\frac{x^4}{4}+\frac{e^{-2a}}{1-e^{2a}x^4}+e^{-2a}\log\left(1-e^{2a}x^4\right)$$

output
```
1/4*x^4+1/exp(2*a)/(1-exp(2*a)*x^4)+ln(1-exp(2*a)*x^4)/exp(2*a)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.83

$$\begin{aligned}\int x^3 \coth^2(a+2\log(x))\,dx=&\frac{x^4}{4}+\cosh(2a)\log\left(\left(-1+x^4\right)\cosh(a)+\left(1+x^4\right)\sinh(a)\right)\\&-\log\left(\left(-1+x^4\right)\cosh(a)+\left(1+x^4\right)\sinh(a)\right)\sinh(2a)\\&+\frac{-\cosh(3a)+\sinh(3a)}{(-1+x^4)\cosh(a)+(1+x^4)\sinh(a)}\end{aligned}$$

input
```
Integrate[x^3*Coth[a + 2*Log[x]]^2,x]
```

output
```
x^4/4 + Cosh[2*a]*Log[(-1 + x^4)*Cosh[a] + (1 + x^4)*Sinh[a]] - Log[(-1 +
x^4)*Cosh[a] + (1 + x^4)*Sinh[a]]*Sinh[2*a] + (-Cosh[3*a] + Sinh[3*a])/((-
1 + x^4)*Cosh[a] + (1 + x^4)*Sinh[a])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6072, 946, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^2(a + 2\log(x))\,dx$$

$$\Big\downarrow \text{6072}$$

$$\int \frac{x^3 \left(-e^{2a}x^4 - 1\right)^2}{\left(1 - e^{2a}x^4\right)^2}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{4}\int \frac{\left(e^{2a}x^4 + 1\right)^2}{\left(1 - e^{2a}x^4\right)^2}dx^4$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4}\int \left(1 + \frac{4}{e^{2a}x^4 - 1} + \frac{4}{\left(e^{2a}x^4 - 1\right)^2}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{4e^{-2a}}{1 - e^{2a}x^4} + 4e^{-2a}\log\left(1 - e^{2a}x^4\right) + x^4\right)$$

input
```
Int[x^3*Coth[a + 2*Log[x]]^2,x]
```

output
```
(x^4 + 4/(E^(2*a)*(1 - E^(2*a)*x^4)) + (4*Log[1 - E^(2*a)*x^4])/E^(2*a))/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

method	result	size
risch	$\frac{x^4}{4}-\frac{\mathrm{e}^{-2a}}{\mathrm{e}^{2a}x^4-1}+\mathrm{e}^{-2a}\ln\left(\mathrm{e}^{2a}x^4-1\right)$	41

input
```
int(x^3*coth(a+2*ln(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4-exp(-2*a)/(exp(2*a)*x^4-1)+exp(-2*a)*ln(exp(2*a)*x^4-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.30

$$\int x^3 \coth^2(a+2\log(x))\,dx = \frac{x^8e^{(4a)} - x^4e^{(2a)} + 4\left(x^4e^{(2a)} - 1\right)\log\left(x^4e^{(2a)} - 1\right) - 4}{4\left(x^4e^{(4a)} - e^{(2a)}\right)}$$

input
```
integrate(x^3*coth(a+2*log(x))^2,x, algorithm="fricas")
```

output
```
1/4*(x^8*e^(4*a) - x^4*e^(2*a) + 4*(x^4*e^(2*a) - 1)*log(x^4*e^(2*a) - 1)
- 4)/(x^4*e^(4*a) - e^(2*a))
```

Sympy [F]

$$\int x^3 \coth^2(a+2\log(x))\,dx = \int x^3 \coth^2\left(a+2\log\left(x\right)\right)dx$$

input
```
integrate(x**3*coth(a+2*ln(x))**2,x)
```

output
```
Integral(x**3*coth(a + 2*log(x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int x^3 \coth^2(a+2\log(x))\,dx = \frac{1}{4}x^4 + e^{(-2a)}\log\left(x^2e^a + 1\right) + e^{(-2a)}\log\left(x^2e^a - 1\right) - \frac{1}{x^4e^{(4a)} - e^{(2a)}}$$

input
```
integrate(x^3*coth(a+2*log(x))^2,x, algorithm="maxima")
```

output
```
1/4*x^4 + e^(-2*a)*log(x^2*e^a + 1) + e^(-2*a)*log(x^2*e^a - 1) - 1/(x^4*e
^(4*a) - e^(2*a))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int x^3 \coth^2(a+2\log(x))\,dx = \frac{1}{4}\,x^4 - \frac{x^4}{x^4e^{(2\,a)}-1} + e^{(-2\,a)}\log\left(\left|x^4e^{(2\,a)}-1\right|\right)$$

input
```
integrate(x^3*coth(a+2*log(x))^2,x, algorithm="giac")
```

output
```
1/4*x^4 - x^4/(x^4*e^(2*a) - 1) + e^(-2*a)*log(abs(x^4*e^(2*a) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int x^3 \coth^2(a+2\log(x))\,dx = \ln\left(x^4-\mathrm{e}^{-2\,a}\right)\,\mathrm{e}^{-2\,a} - \frac{\mathrm{e}^{-2\,a}}{x^4\,\mathrm{e}^{2\,a}-1} + \frac{x^4}{4}$$

input
```
int(x^3*coth(a + 2*log(x))^2,x)
```

output
```
log(x^4 - exp(-2*a))*exp(-2*a) - exp(-2*a)/(x^4*exp(2*a) - 1) + x^4/4
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.94

$$\int x^3 \coth^2(a+2\log(x))\,dx$$
$$= \frac{e^{4a}x^8 + 4e^{2a}\log\left(e^{\frac{a}{2}}x-1\right)x^4 + 4e^{2a}\log\left(e^{\frac{a}{2}}x+1\right)x^4 + 4e^{2a}\log\left(e^{a}x^2+1\right)x^4 - 5e^{2a}x^4 - 4\log\left(e^{\frac{a}{2}}x-1\right) -}{4e^{2a}\left(e^{2a}x^4-1\right)}$$

input
```
int(x^3*coth(a+2*log(x))^2,x)
```

output
```
(e**(4*a)*x**8 + 4*e**(2*a)*log(e**(a/2)*x - 1)*x**4 + 4*e**(2*a)*log(e**(
a/2)*x + 1)*x**4 + 4*e**(2*a)*log(e**a*x**2 + 1)*x**4 - 5*e**(2*a)*x**4 -
4*log(e**(a/2)*x - 1) - 4*log(e**(a/2)*x + 1) - 4*log(e**a*x**2 + 1))/(4*e
**(2*a)*(e**(2*a)*x**4 - 1))
```

3.164 $\int x^2 \coth^2(a+2\log(x))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 68

$$\int x^2 \coth^2(a+2\log(x))\,dx = \frac{x^3}{3} + \frac{x^3}{1-e^{2a}x^4} + \frac{3}{2}e^{-3a/2}\arctan\left(e^{a/2}x\right) - \frac{3}{2}e^{-3a/2}\text{arctanh}\left(e^{a/2}x\right)$$

output
```
1/3*x^3+x^3/(1-exp(2*a)*x^4)+3/2*arctan(exp(1/2*a)*x)/exp(3/2*a)-3/2*arcta
nh(exp(1/2*a)*x)/exp(3/2*a)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.97 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.26

$$\int x^2 \coth^2(a+2\log(x))\,dx = \frac{e^{-4a}\left(-9317-17825e^{2a}x^4-4787e^{4a}x^8+1481e^{6a}x^{12}+7(1331+1976e^{2a}x^4-398e^{4a}x^8-632e^{6a}x^{12}+2\right.}{2688x^5} + \frac{16e^{2a}x^7\left(1+e^{2a}x^4\right)^2 {}_4F_3\left(\frac{7}{4},2,2,2;1,1,\frac{19}{4};e^{2a}x^4\right)}{1155}$$

input
```
Integrate[x^2*Coth[a + 2*Log[x]]^2,x]
```

output
```
(-9317 - 17825*E^(2*a)*x^4 - 4787*E^(4*a)*x^8 + 1481*E^(6*a)*x^12 + 7*(133
1 + 1976*E^(2*a)*x^4 - 398*E^(4*a)*x^8 - 632*E^(6*a)*x^12 + 27*E^(8*a)*x^1
6)*Hypergeometric2F1[3/4, 1, 7/4, E^(2*a)*x^4])/(2688*E^(4*a)*x^5) + (16*E
^(2*a)*x^7*(1 + E^(2*a)*x^4)^2*HypergeometricPFQ[{7/4, 2, 2, 2}, {1, 1, 19
/4}, E^(2*a)*x^4])/1155
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {6072, 963, 27, 959, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^2(a+2\log(x))\,dx$$

$$\Big\downarrow \text{6072}$$

$$\int \frac{x^2\left(-e^{2a}x^4-1\right)^2}{\left(1-e^{2a}x^4\right)^2}dx$$

$$\Big\downarrow \text{963}$$

$$\frac{x^3}{1-e^{2a}x^4}-\frac{1}{4}e^{-4a}\int \frac{4x^2\left(e^{6a}x^4+2e^{4a}\right)}{1-e^{2a}x^4}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3}{1-e^{2a}x^4}-e^{-4a}\int \frac{x^2\left(e^{6a}x^4+2e^{4a}\right)}{1-e^{2a}x^4}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{x^3}{1-e^{2a}x^4}-e^{-4a}\left(3e^{4a}\int \frac{x^2}{1-e^{2a}x^4}dx-\frac{1}{3}e^{4a}x^3\right)$$

$$\Big\downarrow \text{827}$$

$$\frac{x^3}{1-e^{2a}x^4}-e^{-4a}\left(3e^{4a}\left(\frac{1}{2}e^{-a}\int\frac{1}{1-e^ax^2}dx-\frac{1}{2}e^{-a}\int\frac{1}{e^ax^2+1}dx\right)-\frac{1}{3}e^{4a}x^3\right)$$

$$\Big\downarrow \; 216$$

$$\frac{x^3}{1-e^{2a}x^4}-e^{-4a}\left(3e^{4a}\left(\frac{1}{2}e^{-a}\int\frac{1}{1-e^ax^2}dx-\frac{1}{2}e^{-3a/2}\arctan\left(e^{a/2}x\right)\right)-\frac{1}{3}e^{4a}x^3\right)$$

$$\Big\downarrow \; 219$$

$$\frac{x^3}{1-e^{2a}x^4}-e^{-4a}\left(3e^{4a}\left(\frac{1}{2}e^{-3a/2}\text{arctanh}\left(e^{a/2}x\right)-\frac{1}{2}e^{-3a/2}\arctan\left(e^{a/2}x\right)\right)-\frac{1}{3}e^{4a}x^3\right)$$

input
```
Int[x^2*Coth[a + 2*Log[x]]^2,x]
```

output
```
x^3/(1 - E^(2*a)*x^4) - (-1/3*(E^(4*a)*x^3) + 3*E^(4*a)*(-1/2*ArcTan[E^(a/
2)*x]/E^((3*a)/2) + ArcTanh[E^(a/2)*x]/(2*E^((3*a)/2))))/E^(4*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 963
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Simp[1/(a*b^2*n*(p + 1))   Int[(e*x)^m*(a + b*x
^n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.47

method	result	size
risch	$\frac{x^{3}}{3}-\frac{x^{3}}{{\mathrm e}^{2 a} x^{4}-1}+\frac{3 \ln \left(\left(-{\mathrm e}^{a}\right)^{\frac{3}{2}}-{\mathrm e}^{2 a} x \right)}{4 \left(-{\mathrm e}^{a}\right)^{\frac{3}{2}}}-\frac{3 \ln \left(\left(-{\mathrm e}^{a}\right)^{\frac{3}{2}}+{\mathrm e}^{2 a} x \right)}{4 \left(-{\mathrm e}^{a}\right)^{\frac{3}{2}}}+\frac{3 \ln \left(-x \sqrt{{\mathrm e}^{a}}+1\right)}{4 \left({\mathrm e}^{a}\right)^{\frac{3}{2}}}-\frac{3 \ln \left(x \sqrt{{\mathrm e}^{a}}+1\right)}{4 \left({\mathrm e}^{a}\right)^{\frac{3}{2}}}$	100

input
```
int(x^2*coth(a+2*ln(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3-x^3/(exp(2*a)*x^4-1)+3/4/(-exp(a))^(3/2)*ln((-exp(a))^(3/2)-exp(2*
a)*x)-3/4/(-exp(a))^(3/2)*ln((-exp(a))^(3/2)+exp(2*a)*x)+3/4/exp(a)^(3/2)*
ln(-x*exp(a)^(1/2)+1)-3/4/exp(a)^(3/2)*ln(x*exp(a)^(1/2)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(49) = 98$.

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.53

$$\int x^2 \coth^2(a + 2\log(x))\, dx$$
$$= \frac{4\,x^7 e^{(4\,a)} - 16\,x^3 e^{(2\,a)} + 18\,\left(x^4 e^{(2\,a)} - 1\right) \arctan\left(x e^{\left(\frac{1}{2}\,a\right)}\right) e^{\left(\frac{1}{2}\,a\right)} + 9\,\left(x^4 e^{(2\,a)} - 1\right) e^{\left(\frac{1}{2}\,a\right)} \log\left(\frac{x^2 e^a - 2\,x e^{\left(\frac{1}{2}\,a\right)} +}{x^2 e^a - 1}\right.}{12\,\left(x^4 e^{(4\,a)} - e^{(2\,a)}\right)}$$

input
```
integrate(x^2*coth(a+2*log(x))^2,x, algorithm="fricas")
```

output
```
1/12*(4*x^7*e^(4*a) - 16*x^3*e^(2*a) + 18*(x^4*e^(2*a) - 1)*arctan(x*e^(1/
2*a))*e^(1/2*a) + 9*(x^4*e^(2*a) - 1)*e^(1/2*a)*log((x^2*e^a - 2*x*e^(1/2*
a) + 1)/(x^2*e^a - 1)))/(x^4*e^(4*a) - e^(2*a))
```

Sympy [F]

$$\int x^2 \coth^2(a + 2\log(x))\, dx = \int x^2 \coth^2\left(a + 2\log\left(x\right)\right) dx$$

input
```
integrate(x**2*coth(a+2*ln(x))**2,x)
```

output
```
Integral(x**2*coth(a + 2*log(x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int x^2 \coth^2(a+2\log(x))\,dx = \frac{1}{3}\,x^3 - \frac{x^3}{x^4e^{(2\,a)}-1} + \frac{3}{2}\,\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{3}{2}\,a\right)} + \frac{3}{4}\,e^{\left(-\frac{3}{2}\,a\right)}\log\left(\frac{xe^a-e^{\left(\frac{1}{2}\,a\right)}}{xe^a+e^{\left(\frac{1}{2}\,a\right)}}\right)$$

input
```
integrate(x^2*coth(a+2*log(x))^2,x, algorithm="maxima")
```

output
```
1/3*x^3 - x^3/(x^4*e^(2*a) - 1) + 3/2*arctan(x*e^(1/2*a))*e^(-3/2*a) + 3/4
*e^(-3/2*a)*log((x*e^a - e^(1/2*a))/(x*e^a + e^(1/2*a)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.06

$$\int x^2 \coth^2(a+2\log(x))\,dx = \frac{1}{3}\,x^3 - \frac{x^3}{x^4e^{(2\,a)}-1} + \frac{3}{2}\,\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{3}{2}\,a\right)} + \frac{3}{4}\,e^{\left(-\frac{3}{2}\,a\right)}\log\left(\frac{\left|2\,xe^a-2\,e^{\left(\frac{1}{2}\,a\right)}\right|}{\left|2\,xe^a+2\,e^{\left(\frac{1}{2}\,a\right)}\right|}\right)$$

input
```
integrate(x^2*coth(a+2*log(x))^2,x, algorithm="giac")
```

output
```
1/3*x^3 - x^3/(x^4*e^(2*a) - 1) + 3/2*arctan(x*e^(1/2*a))*e^(-3/2*a) + 3/4
*e^(-3/2*a)*log(abs(2*x*e^a - 2*e^(1/2*a))/abs(2*x*e^a + 2*e^(1/2*a)))
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int x^2 \coth^2(a+2\log(x))\,dx = \frac{3\,\mathrm{atan}\left(x\,\left(\mathrm{e}^{2\,a}\right)^{1/4}\right)}{2\,\left(\mathrm{e}^{2\,a}\right)^{3/4}} - \frac{x^3}{x^4\,\mathrm{e}^{2\,a}-1} + \frac{x^3}{3} + \frac{\mathrm{atan}\left(x\,\left(\mathrm{e}^{2\,a}\right)^{1/4}\,1\mathrm{i}\right)\,3\mathrm{i}}{2\,\left(\mathrm{e}^{2\,a}\right)^{3/4}}$$

input
```
int(x^2*coth(a + 2*log(x))^2,x)
```

output
```
(3*atan(x*exp(2*a)^(1/4)))/(2*exp(2*a)^(3/4)) - x^3/(x^4*exp(2*a) - 1) + (
atan(x*exp(2*a)^(1/4)*1i)*3i)/(2*exp(2*a)^(3/4)) + x^3/3
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.35

$$\int x^2 \coth^2(a+2\log(x))\,dx = \frac{18e^{\frac{5a}{2}}atan\left(\frac{e^a x}{e^{\frac{a}{2}}}\right)x^4 - 18e^{\frac{a}{2}}atan\left(\frac{e^a x}{e^{\frac{a}{2}}}\right) + 9e^{\frac{5a}{2}}\log\left(e^{\frac{a}{2}}x-1\right)x^4 - 9e^{\frac{5a}{2}}\log\left(e^{\frac{a}{2}}x+1\right)x^4 - 9e^{\frac{a}{2}}\log\left(e^{\frac{a}{2}}x-1\right)}{12e^{2a}\left(e^{2a}x^4-1\right)}$$

input
```
int(x^2*coth(a+2*log(x))^2,x)
```

output
```
(18*e**((5*a)/2)*atan((e**a*x)/e**(a/2))*x**4 - 18*e**(a/2)*atan((e**a*x)/
e**(a/2)) + 9*e**((5*a)/2)*log(e**(a/2)*x - 1)*x**4 - 9*e**((5*a)/2)*log(e
**(a/2)*x + 1)*x**4 - 9*e**(a/2)*log(e**(a/2)*x - 1) + 9*e**(a/2)*log(e**(
a/2)*x + 1) + 4*e**(4*a)*x**7 - 16*e**(2*a)*x**3)/(12*e**(2*a)*(e**(2*a)*x
**4 - 1))
```

3.165 $\int x\coth^2(a+2\log(x))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 41

$$\int x\coth^2(a+2\log(x))\,dx=\frac{x^2}{2}+\frac{x^2}{1-e^{2a}x^4}-e^{-a}\text{arctanh}\left(e^ax^2\right)$$

output
```
1/2*x^2+x^2/(1-exp(2*a)*x^4)-arctanh(exp(a)*x^2)/exp(a)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.98

$$\int x\coth^2(a+2\log(x))\,dx =\frac{e^{-4a}\left(-375-713e^{2a}x^4-181e^{4a}x^8+61e^{6a}x^{12}+\frac{3(125+196e^{2a}x^4-14e^{4a}x^8-52e^{6a}x^{12}+e^{8a}x^{16})\text{arctanh}\left(\sqrt{e^{2a}x^4}\right)}{\sqrt{e^{2a}x^4}}\right)}{96x^6} +\frac{2}{105}e^{2a}x^6\left(1+e^{2a}x^4\right)^2{}_4F_3\left(\frac{3}{2},2,2,2;1,1,\frac{9}{2};e^{2a}x^4\right)$$

input
```
Integrate[x*Coth[a + 2*Log[x]]^2,x]
```

output
```
(-375 - 713*E^(2*a)*x^4 - 181*E^(4*a)*x^8 + 61*E^(6*a)*x^12 + (3*(125 + 19
6*E^(2*a)*x^4 - 14*E^(4*a)*x^8 - 52*E^(6*a)*x^12 + E^(8*a)*x^16)*ArcTanh[S
qrt[E^(2*a)*x^4]])/Sqrt[E^(2*a)*x^4])/(96*E^(4*a)*x^6) + (2*E^(2*a)*x^6*(1
 + E^(2*a)*x^4)^2*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, E^(2*a)*x
^4])/105
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {6072, 963, 27, 959, 807, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth^2(a+2\log(x))\,dx$$

$$\Big\downarrow \text{6072}$$

$$\int \frac{x\left(-e^{2a}x^4-1\right)^2}{\left(1-e^{2a}x^4\right)^2}dx$$

$$\Big\downarrow \text{963}$$

$$\frac{x^2}{1-e^{2a}x^4}-\frac{1}{4}e^{-4a}\int \frac{4x\left(e^{6a}x^4+e^{4a}\right)}{1-e^{2a}x^4}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2}{1-e^{2a}x^4}-e^{-4a}\int \frac{x\left(e^{6a}x^4+e^{4a}\right)}{1-e^{2a}x^4}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{x^2}{1-e^{2a}x^4}-e^{-4a}\left(2e^{4a}\int \frac{x}{1-e^{2a}x^4}dx-\frac{1}{2}e^{4a}x^2\right)$$

$$\Big\downarrow \text{807}$$

$$\frac{x^2}{1-e^{2a}x^4}-e^{-4a}\left(e^{4a}\int \frac{1}{1-e^{2a}x^4}dx^2-\frac{1}{2}e^{4a}x^2\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{x^2}{1-e^{2a}x^4} - e^{-4a}\left(e^{3a}\text{arctanh}\left(e^a x^2\right) - \frac{1}{2}e^{4a}x^2\right)$$

input
```
Int[x*Coth[a + 2*Log[x]]^2,x]
```

output
```
x^2/(1 - E^(2*a)*x^4) - (-1/2*(E^(4*a)*x^2) + E^(3*a)*ArcTanh[E^a*x^2])/E^
(4*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 963
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Simp[1/(a*b^2*n*(p + 1))   Int[(e*x)^m*(a + b*x
^n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

method	result	size
risch	$\frac{x^{2}}{2}-\frac{x^{2}}{{\mathrm e}^{2 a} x^{4}-1}+\frac{{\mathrm e}^{-a} \ln \left({\mathrm e}^{a} x^{2}-1\right)}{2}-\frac{{\mathrm e}^{-a} \ln \left({\mathrm e}^{a} x^{2}+1\right)}{2}$	54

input
```
int(x*coth(a+2*ln(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2-x^2/(exp(a)^2*x^4-1)+1/2/exp(a)*ln(exp(a)*x^2-1)-1/2/exp(a)*ln(exp
(a)*x^2+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(36) = 72.

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.80

$$\int x \coth^2(a+2\log(x))\,dx = \frac{x^{6} e^{(3\,a)} - 3\,x^{2} e^{a} - \left(x^{4} e^{(2\,a)} - 1\right) \log\left(x^{2} e^{a} + 1\right) + \left(x^{4} e^{(2\,a)} - 1\right) \log\left(x^{2} e^{a} - 1\right)}{2\,\left(x^{4} e^{(3\,a)} - e^{a}\right)}$$

input
```
integrate(x*coth(a+2*log(x))^2,x, algorithm="fricas")
```

output
```
1/2*(x^6*e^(3*a) - 3*x^2*e^a - (x^4*e^(2*a) - 1)*log(x^2*e^a + 1) + (x^4*e
^(2*a) - 1)*log(x^2*e^a - 1))/(x^4*e^(3*a) - e^a)
```

Sympy [F]

$$\int x\coth^2(a+2\log(x))\,dx = \int x\coth^2\left(a+2\log\left(x\right)\right)dx$$

input
```
integrate(x*coth(a+2*ln(x))**2,x)
```

output
```
Integral(x*coth(a + 2*log(x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.29

$$\int x\coth^2(a+2\log(x))\,dx = \frac{1}{2}\,x^2 - \frac{1}{2}\,e^{(-a)}\log\left(x^2e^a+1\right) + \frac{1}{2}\,e^{(-a)}\log\left(x^2e^a-1\right) - \frac{x^2}{x^4e^{(2\,a)}-1}$$

input
```
integrate(x*coth(a+2*log(x))^2,x, algorithm="maxima")
```

output
```
1/2*x^2 - 1/2*e^(-a)*log(x^2*e^a + 1) + 1/2*e^(-a)*log(x^2*e^a - 1) - x^2/
(x^4*e^(2*a) - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int x\coth^2(a+2\log(x))\,dx = \frac{1}{2}\,x^2 - \frac{1}{2}\,e^{(-a)}\log\left(x^2e^a+1\right) + \frac{1}{2}\,e^{(-a)}\log\left(\left|x^2e^a-1\right|\right) - \frac{x^2}{x^4e^{(2\,a)}-1}$$

input
```
integrate(x*coth(a+2*log(x))^2,x, algorithm="giac")
```

output
```
1/2*x^2 - 1/2*e^(-a)*log(x^2*e^a + 1) + 1/2*e^(-a)*log(abs(x^2*e^a - 1)) -
 x^2/(x^4*e^(2*a) - 1)
```

Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int x\coth^2(a+2\log(x))\,dx=\frac{x^2}{2}-\frac{x^2}{x^4\,\mathrm{e}^{2\,a}-1}-\frac{\mathrm{atanh}\left(x^2\,\sqrt{\mathrm{e}^{2\,a}}\right)}{\sqrt{\mathrm{e}^{2\,a}}}$$

input
```
int(x*coth(a + 2*log(x))^2,x)
```

output
```
x^2/2 - x^2/(x^4*exp(2*a) - 1) - atanh(x^2*exp(2*a)^(1/2))/exp(2*a)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 130, normalized size of antiderivative = 3.17

$$\int x\coth^2(a+2\log(x))\,dx$$
$$=\frac{e^{3a}x^6+e^{2a}\log\left(e^{\frac{a}{2}}x-1\right)x^4+e^{2a}\log\left(e^{\frac{a}{2}}x+1\right)x^4-e^{2a}\log(e^ax^2+1)\,x^4-3e^ax^2-\log\left(e^{\frac{a}{2}}x-1\right)-\log(e}{2e^a\left(e^{2a}x^4-1\right)}$$

input
```
int(x*coth(a+2*log(x))^2,x)
```

output
```
(e**(3*a)*x**6 + e**(2*a)*log(e**(a/2)*x - 1)*x**4 + e**(2*a)*log(e**(a/2)
*x + 1)*x**4 - e**(2*a)*log(e**a*x**2 + 1)*x**4 - 3*e**a*x**2 - log(e**(a/
2)*x - 1) - log(e**(a/2)*x + 1) + log(e**a*x**2 + 1))/(2*e**a*(e**(2*a)*x*
*4 - 1))
```

3.166 $\int \coth^2(a+2\log(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 60

$$\int \coth^2(a+2\log(x))\,dx = x+\frac{x}{1-e^{2a}x^4}-\frac{1}{2}e^{-a/2}\arctan\left(e^{a/2}x\right)-\frac{1}{2}e^{-a/2}\text{arctanh}\left(e^{a/2}x\right)$$

output
```
x+x/(1-exp(2*a)*x^4)-1/2*arctan(exp(1/2*a)*x)/exp(1/2*a)-1/2*arctanh(exp(1
/2*a)*x)/exp(1/2*a)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.63 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.55

$$\int \coth^2(a+2\log(x))\,dx$$
$$= \frac{e^{-4a}\left(-3645-6769e^{2a}x^4-1483e^{4a}x^8+681e^{6a}x^{12}+5(729+1208e^{2a}x^4+102e^{4a}x^8-248e^{6a}x^{12}+e^{8a}x\right.}{640x^7}$$
$$+\frac{16}{585}e^{2a}x^5\left(1+e^{2a}x^4\right)^2\,{}_4F_3\left(\frac{5}{4},2,2,2;1,1,\frac{17}{4};e^{2a}x^4\right)$$

input
```
Integrate[Coth[a + 2*Log[x]]^2,x]
```

output
```
(-3645 - 6769*E^(2*a)*x^4 - 1483*E^(4*a)*x^8 + 681*E^(6*a)*x^12 + 5*(729 +
 1208*E^(2*a)*x^4 + 102*E^(4*a)*x^8 - 248*E^(6*a)*x^12 + E^(8*a)*x^16)*Hyp
ergeometric2F1[1/4, 1, 5/4, E^(2*a)*x^4])/(640*E^(4*a)*x^7) + (16*E^(2*a)*
x^5*(1 + E^(2*a)*x^4)^2*HypergeometricPFQ[{5/4, 2, 2, 2}, {1, 1, 17/4}, E^
(2*a)*x^4])/585
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6068, 915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(a+2\log(x))\,dx$$

$$\Big\downarrow \text{6068}$$

$$\int \frac{(-e^{2a}x^4-1)^2}{(1-e^{2a}x^4)^2}dx$$

$$\Big\downarrow \text{915}$$

$$\int \left(\frac{4e^{2a}x^4}{(1-e^{2a}x^4)^2}+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}e^{-a/2}\arctan\left(e^{a/2}x\right)-\frac{1}{2}e^{-a/2}\text{arctanh}\left(e^{a/2}x\right)+\frac{x}{1-e^{2a}x^4}+x$$

input
```
Int[Coth[a + 2*Log[x]]^2,x]
```

output
```
x + x/(1 - E^(2*a)*x^4) - ArcTan[E^(a/2)*x]/(2*E^(a/2)) - ArcTanh[E^(a/2)*
x]/(2*E^(a/2))
```

Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6068
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

method	result	size
risch	$x-\frac{x}{\mathrm{e}^{2a}x^4-1}+\frac{\ln\left(x\sqrt{\mathrm{e}^a}-1\right)}{4\sqrt{\mathrm{e}^a}}-\frac{\ln\left(x\sqrt{\mathrm{e}^a}+1\right)}{4\sqrt{\mathrm{e}^a}}-\frac{\ln\left(x\sqrt{-\mathrm{e}^a}+1\right)}{4\sqrt{-\mathrm{e}^a}}+\frac{\ln\left(x\sqrt{-\mathrm{e}^a}-1\right)}{4\sqrt{-\mathrm{e}^a}}$	86

input
```
int(coth(a+2*ln(x))^2,x,method=_RETURNVERBOSE)
```

output
```
x-x/(exp(a)^2*x^4-1)+1/4/exp(a)^(1/2)*ln(x*exp(a)^(1/2)-1)-1/4/exp(a)^(1/2
)*ln(x*exp(a)^(1/2)+1)-1/4/(-exp(a))^(1/2)*ln(x*(-exp(a))^(1/2)+1)+1/4/(-e
xp(a))^(1/2)*ln(x*(-exp(a))^(1/2)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(43) = 86.

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

$$\int \coth^2(a+2\log(x))\,dx$$
$$=\frac{4\,x^5e^{(3\,a)}-2\,\left(x^4e^{(2\,a)}-1\right)\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(\frac{1}{2}\,a\right)}+\left(x^4e^{(2\,a)}-1\right)e^{\left(\frac{1}{2}\,a\right)}\log\left(\frac{x^2e^a-2\,xe^{\left(\frac{1}{2}\,a\right)}+1}{x^2e^a-1}\right)-8\,xe^a}{4\,\left(x^4e^{(3\,a)}-e^a\right)}$$

input
```
integrate(coth(a+2*log(x))^2,x, algorithm="fricas")
```

output
```
1/4*(4*x^5*e^(3*a) - 2*(x^4*e^(2*a) - 1)*arctan(x*e^(1/2*a))*e^(1/2*a) + (
x^4*e^(2*a) - 1)*e^(1/2*a)*log((x^2*e^a - 2*x*e^(1/2*a) + 1)/(x^2*e^a - 1)
) - 8*x*e^a)/(x^4*e^(3*a) - e^a)
```

Sympy [F]

$$\int \coth^2(a+2\log(x))\,dx = \int \coth^2\left(a+2\log\left(x\right)\right)dx$$

input
```
integrate(coth(a+2*ln(x))**2,x)
```

output
```
Integral(coth(a + 2*log(x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \coth^2(a+2\log(x))\,dx = -\frac{1}{2}\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)} + \frac{1}{4}e^{\left(-\frac{1}{2}a\right)}\log\left(\frac{xe^{a}-e^{\left(\frac{1}{2}a\right)}}{xe^{a}+e^{\left(\frac{1}{2}a\right)}}\right) + x - \frac{x}{x^{4}e^{\left(2a\right)}-1}$$

input
```
integrate(coth(a+2*log(x))^2,x, algorithm="maxima")
```

output
```
-1/2*arctan(x*e^(1/2*a))*e^(-1/2*a) + 1/4*e^(-1/2*a)*log((x*e^a - e^(1/2*a
))/(x*e^a + e^(1/2*a))) + x - x/(x^4*e^(2*a) - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int \coth^2(a+2\log(x))\,dx = -\frac{1}{2}\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)} + \frac{1}{4}e^{\left(-\frac{1}{2}a\right)}\log\left(\frac{\left|2\,xe^{a}-2\,e^{\left(\frac{1}{2}a\right)}\right|}{\left|2\,xe^{a}+2\,e^{\left(\frac{1}{2}a\right)}\right|}\right) + x - \frac{x}{x^4e^{(2\,a)}-1}$$

input
```
integrate(coth(a+2*log(x))^2,x, algorithm="giac")
```

output
```
-1/2*arctan(x*e^(1/2*a))*e^(-1/2*a) + 1/4*e^(-1/2*a)*log(abs(2*x*e^a - 2*e
^(1/2*a))/abs(2*x*e^a + 2*e^(1/2*a))) + x - x/(x^4*e^(2*a) - 1)
```

Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \coth^2(a+2\log(x))\,dx = x - \frac{\mathrm{atan}\left(x\,{\left(\mathrm{e}^{2\,a}\right)}^{1/4}\right)}{2\,{\left(\mathrm{e}^{2\,a}\right)}^{1/4}} - \frac{x}{x^4\,\mathrm{e}^{2\,a}-1} + \frac{\mathrm{atan}\left(x\,{\left(\mathrm{e}^{2\,a}\right)}^{1/4}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,{\left(\mathrm{e}^{2\,a}\right)}^{1/4}}$$

input
```
int(coth(a + 2*log(x))^2,x)
```

output
```
x - atan(x*exp(2*a)^(1/4))/(2*exp(2*a)^(1/4)) + (atan(x*exp(2*a)^(1/4)*1i)
*1i)/(2*exp(2*a)^(1/4)) - x/(x^4*exp(2*a) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.53

$$\int \coth^2(a + 2\log(x))\,dx$$
$$= \frac{-2e^{\frac{5a}{2}}\,atan\left(\frac{e^{a}x}{e^{\frac{a}{2}}}\right)x^4 + 2e^{\frac{a}{2}}\,atan\left(\frac{e^{a}x}{e^{\frac{a}{2}}}\right) + e^{\frac{5a}{2}}\log\left(e^{\frac{a}{2}}x - 1\right)x^4 - e^{\frac{5a}{2}}\log\left(e^{\frac{a}{2}}x + 1\right)x^4 - e^{\frac{a}{2}}\log\left(e^{\frac{a}{2}}x - 1\right) + e}{4e^{a}\left(e^{2a}x^4 - 1\right)}$$

input
```
int(coth(a+2*log(x))^2,x)
```

output
```
( - 2*e**((5*a)/2)*atan((e**a*x)/e**(a/2))*x**4 + 2*e**(a/2)*atan((e**a*x)
/e**(a/2)) + e**((5*a)/2)*log(e**(a/2)*x - 1)*x**4 - e**((5*a)/2)*log(e**(
a/2)*x + 1)*x**4 - e**(a/2)*log(e**(a/2)*x - 1) + e**(a/2)*log(e**(a/2)*x
+ 1) + 4*e**(3*a)*x**5 - 8*e**a*x)/(4*e**a*(e**(2*a)*x**4 - 1))
```

3.167 $\int \frac{\coth^2(a+2\log(x))}{x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{\coth^2(a+2\log(x))}{x}\,dx=-\frac{1}{2}\coth(a+2\log(x))+\log(x)$$

output
```
-1/2*coth(a+2*ln(x))+ln(x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \frac{\coth^2(a+2\log(x))}{x}\,dx=-\frac{1}{2}\coth(a+2\log(x))\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(a+2\log(x))\right)$$

input
```
Integrate[Coth[a + 2*Log[x]]^2/x,x]
```

output
```
-1/2*(Coth[a + 2*Log[x]]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[a + 2*Log[x]
]^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3039, 3042, 25, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(a+2\log(x))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\int \coth^2(a+2\log(x))d\log(x)$$

$$\Big\downarrow \text{3042}$$

$$\int -\tan\left(ia+2i\log(x)+\frac{\pi}{2}\right)^2d\log(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \tan\left(\frac{1}{2}(2ia+\pi)+2i\log(x)\right)^2d\log(x)$$

$$\Big\downarrow \text{3954}$$

$$\int 1d\log(x)-\frac{1}{2}\coth(a+2\log(x))$$

$$\Big\downarrow \text{24}$$

$$\log(x)-\frac{1}{2}\coth(a+2\log(x))$$

input
```
Int[Coth[a + 2*Log[x]]^2/x,x]
```

output
```
-1/2*Coth[a + 2*Log[x]] + Log[x]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$-\frac{\coth(a+2\ln(x))}{2}+\ln(x)$	13
risch	$-\frac{1}{e^{2a}x^4-1}+\ln(x)$	18
derivativedivides	$-\frac{\coth(a+2\ln(x))}{2}-\frac{\ln(\coth(a+2\ln(x))-1)}{4}+\frac{\ln(\coth(a+2\ln(x))+1)}{4}$	35
default	$-\frac{\coth(a+2\ln(x))}{2}-\frac{\ln(\coth(a+2\ln(x))-1)}{4}+\frac{\ln(\coth(a+2\ln(x))+1)}{4}$	35

input
```
int(coth(a+2*ln(x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*coth(a+2*ln(x))+ln(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(12) = 24$.

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \frac{\coth^2(a+2\log(x))}{x}\,dx = \frac{\left(x^4e^{(2\,a)}-1\right)\log\left(x\right)-1}{x^4e^{(2\,a)}-1}$$

input
```
integrate(coth(a+2*log(x))^2/x,x, algorithm="fricas")
```

output
```
((x^4*e^(2*a) - 1)*log(x) - 1)/(x^4*e^(2*a) - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(12) = 24$.

Time = 2.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 4.50

$$\int \frac{\coth^2(a+2\log(x))}{x}\,dx = \begin{cases} \frac{\log\left(x\right)}{\tanh^2\left(\log\left(-\frac{1}{x^2}\right)+2\log\left(x\right)\right)} & \text{for}\ a=\log\left(-\frac{1}{x^2}\right) \\ \frac{\log\left(x\right)}{\tanh^2\left(\log\left(\frac{1}{x^2}\right)+2\log\left(x\right)\right)} & \text{for}\ a=\log\left(\frac{1}{x^2}\right) \\ \log\left(x\right)-\frac{1}{2\tanh\left(a+2\log\left(x\right)\right)} & \text{otherwise} \end{cases}$$

input
```
integrate(coth(a+2*ln(x))**2/x,x)
```

output
```
Piecewise((log(x)/tanh(log(-1/x**2) + 2*log(x))**2, Eq(a, log(-1/x**2))),
(log(x)/tanh(log(x**(-2)) + 2*log(x))**2, Eq(a, log(x**(-2)))), (log(x) -
1/(2*tanh(a + 2*log(x))), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \frac{\coth^2(a+2\log(x))}{x}\,dx = \frac{1}{2}\,a + \frac{1}{e^{(-2\,a-4\,\log(x))}-1} + \log{(x)}$$

input
```
integrate(coth(a+2*log(x))^2/x,x, algorithm="maxima")
```

output
```
1/2*a + 1/(e^(-2*a - 4*log(x)) - 1) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{\coth^2(a+2\log(x))}{x}\,dx = -\frac{1}{x^4e^{(2\,a)}-1} + \frac{1}{4}\,\log{(x^4)}$$

input
```
integrate(coth(a+2*log(x))^2/x,x, algorithm="giac")
```

output
```
-1/(x^4*e^(2*a) - 1) + 1/4*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \frac{\coth^2(a+2\log(x))}{x}\,dx = \ln{(x)} - \frac{e^{2\,a}\,x^4+1}{2\,(x^4\,e^{2\,a}-1)}$$

input
```
int(coth(a + 2*log(x))^2/x,x)
```

output
```
log(x) - (x^4*exp(2*a) + 1)/(2*(x^4*exp(2*a) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\coth^2(a+2\log(x))}{x}\,dx = -\frac{\coth(2\log(x)+a)}{2} + \log(x)$$

input
```
int(coth(a+2*log(x))^2/x,x)
```

output
```
( - coth(2*log(x) + a) + 2*log(x))/2
```

3.168 $\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 71

$$\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx = -\frac{1}{x}+\frac{e^{2a}x^3}{1-e^{2a}x^4}-\frac{1}{2}e^{a/2}\arctan\left(e^{a/2}x\right)+\frac{1}{2}e^{a/2}\text{arctanh}\left(e^{a/2}x\right)$$

output
```
-1/x+exp(2*a)*x^3/(1-exp(2*a)*x^4)-1/2*exp(1/2*a)*arctan(exp(1/2*a)*x)+1/2
*exp(1/2*a)*arctanh(exp(1/2*a)*x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.16 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.15

$$\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx = \frac{e^{-2a}\left(-343-1163e^{2a}x^4-241e^{4a}x^8+3e^{6a}x^{12}+\left(343+632e^{2a}x^4+362e^{4a}x^8-56e^{6a}x^{12}-e^{8a}x^{16}\right)\text{Hype}}{384x^5} + \frac{16}{231}e^{2a}x^3\left(1+e^{2a}x^4\right)^2\,{}_4F_3\left(\frac{3}{4},2,2,2;1,1,\frac{15}{4};e^{2a}x^4\right)$$

input
```
Integrate[Coth[a + 2*Log[x]]^2/x^2,x]
```

output
```
(-343 - 1163*E^(2*a)*x^4 - 241*E^(4*a)*x^8 + 3*E^(6*a)*x^12 + (343 + 632*E
^(2*a)*x^4 + 362*E^(4*a)*x^8 - 56*E^(6*a)*x^12 - E^(8*a)*x^16)*Hypergeomet
ric2F1[3/4, 1, 7/4, E^(2*a)*x^4])/(384*E^(2*a)*x^5) + (16*E^(2*a)*x^3*(1 +
 E^(2*a)*x^4)^2*HypergeometricPFQ[{3/4, 2, 2, 2}, {1, 1, 15/4}, E^(2*a)*x^
4])/231
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.31, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {6072, 962, 957, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(a+2\log(x))}{x^2}dx$$

$$\Big\downarrow \text{6072}$$

$$\int \frac{(-e^{2a}x^4-1)^2}{x^2(1-e^{2a}x^4)^2}dx$$

$$\Big\downarrow \text{962}$$

$$\int \frac{x^2(e^{4a}x^4+7e^{2a})}{(1-e^{2a}x^4)^2}dx-\frac{1}{x(1-e^{2a}x^4)}$$

$$\Big\downarrow \text{957}$$

$$e^{2a}\int \frac{x^2}{1-e^{2a}x^4}dx-\frac{1}{x(1-e^{2a}x^4)}+\frac{2e^{2a}x^3}{1-e^{2a}x^4}$$

$$\Big\downarrow \text{827}$$

$$e^{2a}\left(\frac{1}{2}e^{-a}\int \frac{1}{1-e^ax^2}dx-\frac{1}{2}e^{-a}\int \frac{1}{e^ax^2+1}dx\right)-\frac{1}{x(1-e^{2a}x^4)}+\frac{2e^{2a}x^3}{1-e^{2a}x^4}$$

$$\Big\downarrow \text{216}$$

$$e^{2a}\left(\frac{1}{2}e^{-a}\int \frac{1}{1-e^ax^2}dx-\frac{1}{2}e^{-3a/2}\arctan\left(e^{a/2}x\right)\right)-\frac{1}{x(1-e^{2a}x^4)}+\frac{2e^{2a}x^3}{1-e^{2a}x^4}$$

$$\Big\downarrow \text{219}$$

$$e^{2a}\left(\frac{1}{2}e^{-3a/2}\text{arctanh}\left(e^{a/2}x\right)-\frac{1}{2}e^{-3a/2}\arctan\left(e^{a/2}x\right)\right)-\frac{1}{x\left(1-e^{2a}x^4\right)}+\frac{2e^{2a}x^3}{1-e^{2a}x^4}$$

input
```
Int[Coth[a + 2*Log[x]]^2/x^2,x]
```

output
```
-(1/(x*(1 - E^(2*a)*x^4))) + (2*E^(2*a)*x^3)/(1 - E^(2*a)*x^4) + E^(2*a)*(
-1/2*ArcTan[E^(a/2)*x]/E^((3*a)/2) + ArcTanh[E^(a/2)*x]/(2*E^((3*a)/2)))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 962
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2
*n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1]
&& GtQ[n, 0]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.42

method	result
risch	$\frac{-2\,\mathrm{e}^{2a}x^4+1}{x\left(\mathrm{e}^{2a}x^4-1\right)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^2+\mathrm{e}^{a}\right)}\_R\ln\left(\left(-5\_R^4+4\,\mathrm{e}^{2a}\right)x-\_R^3\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^2-\mathrm{e}^{a}\right)}\_R\ln\left(\left(-5\_R^4\right.\right.\right.}{4}$

input
```
int(coth(a+2*ln(x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
(-2*exp(2*a)*x^4+1)/x/(exp(2*a)*x^4-1)+1/4*sum(_R*ln((-5*_R^4+4*exp(2*a))*
x-_R^3),_R=RootOf(_Z^2+exp(a)))+1/4*sum(_R*ln((-5*_R^4+4*exp(2*a))*x-_R^3)
,_R=RootOf(_Z^2-exp(a)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.37

$$\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx = -\frac{8\,x^4e^{(2\,a)}+2\,\left(x^5e^{(2\,a)}-x\right)\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(\frac{1}{2}\,a\right)}-\left(x^5e^{(2\,a)}-x\right)e^{\left(\frac{1}{2}\,a\right)}\log\left(\frac{x^2e^a+2\,xe^{\left(\frac{1}{2}\,a\right)}+1}{x^2e^a-1}\right)-4}{4\,\left(x^5e^{(2\,a)}-x\right)}$$

input
```
integrate(coth(a+2*log(x))^2/x^2,x, algorithm="fricas")
```

output
```
-1/4*(8*x^4*e^(2*a) + 2*(x^5*e^(2*a) - x)*arctan(x*e^(1/2*a))*e^(1/2*a) -
(x^5*e^(2*a) - x)*e^(1/2*a)*log((x^2*e^a + 2*x*e^(1/2*a) + 1)/(x^2*e^a - 1
)) - 4)/(x^5*e^(2*a) - x)
```

Sympy [F]

$$\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx = \int \frac{\coth^2\left(a+2\log\left(x\right)\right)}{x^2}\,dx$$

input
```
integrate(coth(a+2*ln(x))**2/x**2,x)
```

output
```
Integral(coth(a + 2*log(x))**2/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx = \frac{1}{2}\arctan\left(\frac{e^{\left(-\frac{1}{2}\,a\right)}}{x}\right)e^{\left(\frac{1}{2}\,a\right)} - \frac{1}{4}\,e^{\left(\frac{1}{2}\,a\right)}\log\left(\frac{\frac{1}{x}-e^{\left(\frac{1}{2}\,a\right)}}{\frac{1}{x}+e^{\left(\frac{1}{2}\,a\right)}}\right)-\frac{1}{x}+\frac{e^{(2\,a)}}{x\left(\frac{1}{x^4}-e^{(2\,a)}\right)}$$

input
```
integrate(coth(a+2*log(x))^2/x^2,x, algorithm="maxima")
```

output
```
1/2*arctan(e^(-1/2*a)/x)*e^(1/2*a) - 1/4*e^(1/2*a)*log((1/x - e^(1/2*a))/(
1/x + e^(1/2*a))) - 1/x + e^(2*a)/(x*(1/x^4 - e^(2*a)))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08

$$\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx = -\frac{1}{2}\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)} - \frac{1}{4}e^{\left(\frac{1}{2}a\right)}\log\left(\frac{\left|2\,xe^{a}-2\,e^{\left(\frac{1}{2}a\right)}\right|}{\left|2\,xe^{a}+2\,e^{\left(\frac{1}{2}a\right)}\right|}\right) - \frac{2\,x^4e^{(2\,a)}-1}{x^5e^{(2\,a)}-x}$$

input
```
integrate(coth(a+2*log(x))^2/x^2,x, algorithm="giac")
```

output
```
-1/2*arctan(x*e^(1/2*a))*e^(1/2*a) - 1/4*e^(1/2*a)*log(abs(2*x*e^a - 2*e^(
1/2*a))/abs(2*x*e^a + 2*e^(1/2*a))) - (2*x^4*e^(2*a) - 1)/(x^5*e^(2*a) - x
)
```

Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.85

$$\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx = \frac{\left(\mathrm{e}^{2\,a}\right)^{1/4}\,\mathrm{atanh}\left(x\,\left(\mathrm{e}^{2\,a}\right)^{1/4}\right)}{2} - \frac{\left(\mathrm{e}^{2\,a}\right)^{1/4}\,\mathrm{atan}\left(x\,\left(\mathrm{e}^{2\,a}\right)^{1/4}\right)}{2} + \frac{2\,x^4\,\mathrm{e}^{2\,a}-1}{x-x^5\,\mathrm{e}^{2\,a}}$$

input
```
int(coth(a + 2*log(x))^2/x^2,x)
```

output
```
(exp(2*a)^(1/4)*atanh(x*exp(2*a)^(1/4)))/2 - (exp(2*a)^(1/4)*atan(x*exp(2*
a)^(1/4)))/2 + (2*x^4*exp(2*a) - 1)/(x - x^5*exp(2*a))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.08

$$\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx = \frac{-2e^{\frac{5a}{2}} atan\left(\frac{e^a x}{e^{\frac{a}{2}}}\right)x^5 + 2e^{\frac{a}{2}} atan\left(\frac{e^a x}{e^{\frac{a}{2}}}\right)x - e^{\frac{5a}{2}}\log\left(e^{\frac{a}{2}}x-1\right)x^5 + e^{\frac{5a}{2}}\log\left(e^{\frac{a}{2}}x+1\right)x^5 + e^{\frac{a}{2}}\log\left(e^{\frac{a}{2}}x-1\right)x}{4x\left(e^{2a}x^4-1\right)}$$

input
```
int(coth(a+2*log(x))^2/x^2,x)
```

output
```
( - 2*e**((5*a)/2)*atan((e**a*x)/e**(a/2))*x**5 + 2*e**(a/2)*atan((e**a*x)
/e**(a/2))*x - e**((5*a)/2)*log(e**(a/2)*x - 1)*x**5 + e**((5*a)/2)*log(e*
*(a/2)*x + 1)*x**5 + e**(a/2)*log(e**(a/2)*x - 1)*x - e**(a/2)*log(e**(a/2
)*x + 1)*x - 8*e**(2*a)*x**4 + 4)/(4*x*(e**(2*a)*x**4 - 1))
```

3.169 $\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx = -\frac{1}{2x^2}+\frac{e^{2a}x^2}{1-e^{2a}x^4}+e^a\text{arctanh}\left(e^ax^2\right)$$

output
```
-1/2/x^2+exp(2*a)*x^2/(1-exp(2*a)*x^4)+exp(a)*arctanh(exp(a)*x^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.23 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.60

$$\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx$$
$$= \frac{15\left(-77-\frac{27e^{-2a}}{x^4}-17e^{2a}x^4+e^{4a}x^8\right)-\frac{15\left(-27-52e^{2a}x^4-54e^{4a}x^8+4e^{6a}x^{12}+e^{8a}x^{16}\right)\text{arctanh}\left(\sqrt{e^{2a}x^4}\right)}{\left(e^{2a}x^4\right)^{3/2}}+64(e^ax^2+}{480x^2}$$

input
```
Integrate[Coth[a + 2*Log[x]]^2/x^3,x]
```

output
```
(15*(-77 - 27/(E^(2*a)*x^4) - 17*E^(2*a)*x^4 + E^(4*a)*x^8) - (15*(-27 - 5
2*E^(2*a)*x^4 - 54*E^(4*a)*x^8 + 4*E^(6*a)*x^12 + E^(8*a)*x^16)*ArcTanh[Sq
rt[E^(2*a)*x^4]])/(E^(2*a)*x^4)^(3/2) + 64*(E^a*x^2 + E^(3*a)*x^6)^2*Hyper
geometricPFQ[{1/2, 2, 2, 2}, {1, 1, 7/2}, E^(2*a)*x^4])/(480*x^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {6072, 962, 27, 957, 807, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx$$

$$\Big\downarrow \text{ 6072}$$

$$\int \frac{\left(-e^{2a}x^4-1\right)^2}{x^3\left(1-e^{2a}x^4\right)^2}dx$$

$$\Big\downarrow \text{ 962}$$

$$\frac{1}{2}\int \frac{2x\left(e^{4a}x^4+5e^{2a}\right)}{\left(1-e^{2a}x^4\right)^2}dx-\frac{1}{2x^2\left(1-e^{2a}x^4\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x\left(e^{4a}x^4+5e^{2a}\right)}{\left(1-e^{2a}x^4\right)^2}dx-\frac{1}{2x^2\left(1-e^{2a}x^4\right)}$$

$$\Big\downarrow \text{ 957}$$

$$2e^{2a}\int \frac{x}{1-e^{2a}x^4}dx+\frac{3e^{2a}x^2}{2\left(1-e^{2a}x^4\right)}-\frac{1}{2x^2\left(1-e^{2a}x^4\right)}$$

$$\Big\downarrow \text{ 807}$$

$$e^{2a}\int \frac{1}{1-e^{2a}x^4}dx^2+\frac{3e^{2a}x^2}{2\left(1-e^{2a}x^4\right)}-\frac{1}{2x^2\left(1-e^{2a}x^4\right)}$$

$$\Big\downarrow \text{ 219}$$

$$e^a\text{arctanh}\left(e^ax^2\right)+\frac{3e^{2a}x^2}{2\left(1-e^{2a}x^4\right)}-\frac{1}{2x^2\left(1-e^{2a}x^4\right)}$$

input
```
Int[Coth[a + 2*Log[x]]^2/x^3,x]
```

output
```
-1/2*1/(x^2*(1 - E^(2*a)*x^4)) + (3*E^(2*a)*x^2)/(2*(1 - E^(2*a)*x^4)) + E
^a*ArcTanh[E^a*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 962
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2
*n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1]
&& GtQ[n, 0]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.28

method	result	size
risch	$\frac{-\frac{3\,\mathrm{e}^{2a} x^{4}}{2}+\frac{1}{2}}{x^{2} \left(\mathrm{e}^{2a} x^{4}-1\right)}+\frac{\mathrm{e}^{a} \ln \left(\mathrm{e}^{a} x^{2}+1\right)}{2}-\frac{\mathrm{e}^{a} \ln \left(\mathrm{e}^{a} x^{2}-1\right)}{2}$	55

input
```
int(coth(a+2*ln(x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
(-3/2*exp(a)^2*x^4+1/2)/x^2/(exp(a)^2*x^4-1)+1/2*exp(a)*ln(exp(a)*x^2+1)-1
/2*exp(a)*ln(exp(a)*x^2-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(37) = 74$.

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.91

$$\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx = -\frac{3\,x^4e^{(2\,a)} - \left(x^6e^{(3\,a)} - x^2e^a\right)\log\left(x^2e^a+1\right) + \left(x^6e^{(3\,a)} - x^2e^a\right)\log\left(x^2e^a-1\right) - 1}{2\left(x^6e^{(2\,a)} - x^2\right)}$$

input
```
integrate(coth(a+2*log(x))^2/x^3,x, algorithm="fricas")
```

output
```
-1/2*(3*x^4*e^(2*a) - (x^6*e^(3*a) - x^2*e^a)*log(x^2*e^a + 1) + (x^6*e^(3
*a) - x^2*e^a)*log(x^2*e^a - 1) - 1)/(x^6*e^(2*a) - x^2)
```

Sympy [F]

$$\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx = \int \frac{\coth^2\left(a+2\log\left(x\right)\right)}{x^3}\,dx$$

input
```
integrate(coth(a+2*ln(x))**2/x**3,x)
```

output
```
Integral(coth(a + 2*log(x))**2/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16

$$\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx = \frac{1}{2}\,e^a\log\left(\frac{1}{x^2}+e^a\right) - \frac{1}{2}\,e^a\log\left(\frac{1}{x^2}-e^a\right) - \frac{1}{2\,x^2} + \frac{e^{(2\,a)}}{x^2\left(\frac{1}{x^4}-e^{(2\,a)}\right)}$$

input
```
integrate(coth(a+2*log(x))^2/x^3,x, algorithm="maxima")
```

output
```
1/2*e^a*log(1/x^2 + e^a) - 1/2*e^a*log(1/x^2 - e^a) - 1/2/x^2 + e^(2*a)/(x
^2*(1/x^4 - e^(2*a)))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.33

$$\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx = \frac{1}{2}\,e^a\log\left(x^2e^a+1\right) - \frac{1}{2}\,e^a\log\left(\left|x^2e^a-1\right|\right) - \frac{3\,x^4e^{(2\,a)}-1}{2\,\left(x^6e^{(2\,a)}-x^2\right)}$$

input
```
integrate(coth(a+2*log(x))^2/x^3,x, algorithm="giac")
```

output
```
1/2*e^a*log(x^2*e^a + 1) - 1/2*e^a*log(abs(x^2*e^a - 1)) - 1/2*(3*x^4*e^(2
*a) - 1)/(x^6*e^(2*a) - x^2)
```

Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx = \operatorname{atanh}\left(x^2\,\sqrt{\mathrm{e}^{2\,a}}\right)\,\sqrt{\mathrm{e}^{2\,a}} - \frac{\frac{3\,x^4\,\mathrm{e}^{2\,a}}{2}-\frac{1}{2}}{x^6\,\mathrm{e}^{2\,a}-x^2}$$

input
```
int(coth(a + 2*log(x))^2/x^3,x)
```

output
```
atanh(x^2*exp(2*a)^(1/2))*exp(2*a)^(1/2) - ((3*x^4*exp(2*a))/2 - 1/2)/(x^6
*exp(2*a) - x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.28

$$\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx$$
$$= \frac{-e^{3a}\log\left(e^{\frac{a}{2}}x-1\right)x^6 - e^{3a}\log\left(e^{\frac{a}{2}}x+1\right)x^6 + e^{3a}\log\left(e^ax^2+1\right)x^6 - 3e^{2a}x^4 + e^a\log\left(e^{\frac{a}{2}}x-1\right)x^2 + e^a\log(}{2x^2\left(e^{2a}x^4-1\right)}$$

input
```
int(coth(a+2*log(x))^2/x^3,x)
```

output
```
( - e**(3*a)*log(e**(a/2)*x - 1)*x**6 - e**(3*a)*log(e**(a/2)*x + 1)*x**6
+ e**(3*a)*log(e**a*x**2 + 1)*x**6 - 3*e**(2*a)*x**4 + e**a*log(e**(a/2)*x
 - 1)*x**2 + e**a*log(e**(a/2)*x + 1)*x**2 - e**a*log(e**a*x**2 + 1)*x**2
+ 1)/(2*x**2*(e**(2*a)*x**4 - 1))
```

3.170 $\int (ex)^m \coth(a+2\log(x))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 59

$$\int (ex)^m \coth(a+2\log(x))\,dx = \frac{(ex)^{1+m}}{e(1+m)} - \frac{2(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},e^{2a}x^4\right)}{e(1+m)}$$

output
```
(e*x)^(1+m)/e/(1+m)-2*(e*x)^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],exp
(2*a)*x^4)/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int (ex)^m \coth(a+2\log(x))\,dx = -\frac{x(ex)^m\left(-1+2\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},x^4(\cosh(2a)+\sinh(2a))\right)\right)}{1+m}$$

input
```
Integrate[(e*x)^m*Coth[a + 2*Log[x]],x]
```

output
```
-((x*(e*x)^m*(-1 + 2*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, x^4*(Cosh[
2*a] + Sinh[2*a])]))/(1 + m))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6072, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \coth(a+2\log(x))\,dx$$

$$\Big\downarrow \text{6072}$$

$$\int \frac{\left(-e^{2a}x^4-1\right)(ex)^m}{1-e^{2a}x^4}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(ex)^{m+1}}{e(m+1)}-2\int \frac{(ex)^m}{1-e^{2a}x^4}dx$$

$$\Big\downarrow \text{888}$$

$$\frac{(ex)^{m+1}}{e(m+1)}-\frac{2(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},e^{2a}x^4\right)}{e(m+1)}$$

input
```
Int[(e*x)^m*Coth[a + 2*Log[x]],x]
```

output
```
(e*x)^(1 + m)/(e*(1 + m)) - (2*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/
4, (5 + m)/4, E^(2*a)*x^4])/(e*(1 + m))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [F]

$$\int (ex)^m \coth (a+2\ln (x))\, dx$$

input
```
int((e*x)^m*coth(a+2*ln(x)),x)
```

output
```
int((e*x)^m*coth(a+2*ln(x)),x)
```

Fricas [F]

$$\int (ex)^m \coth(a+2\log(x))\, dx = \int (ex)^m \coth \left(a+2\, \log \left(x\right)\right)\, dx$$

input
```
integrate((e*x)^m*coth(a+2*log(x)),x, algorithm="fricas")
```

output
```
integral((e*x)^m*coth(a + 2*log(x)), x)
```

Sympy [F]

$$\int (ex)^m \coth(a+2\log(x))\,dx = \int (ex)^m \coth\left(a+2\log\left(x\right)\right)dx$$

input
```
integrate((e*x)**m*coth(a+2*ln(x)),x)
```

output
```
Integral((e*x)**m*coth(a + 2*log(x)), x)
```

Maxima [F]

$$\int (ex)^m \coth(a+2\log(x))\,dx = \int (ex)^m \coth\left(a+2\,\log\left(x\right)\right)\,dx$$

input
```
integrate((e*x)^m*coth(a+2*log(x)),x, algorithm="maxima")
```

output
```
integrate((e*x)^m*coth(a + 2*log(x)), x)
```

Giac [F]

$$\int (ex)^m \coth(a+2\log(x))\,dx = \int (ex)^m \coth\left(a+2\,\log\left(x\right)\right)\,dx$$

input
```
integrate((e*x)^m*coth(a+2*log(x)),x, algorithm="giac")
```

output
```
integrate((e*x)^m*coth(a + 2*log(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \coth(a+2\log(x))\,dx = \int \coth(a+2\,\ln(x))\,(e\,x)^m\,dx$$

input
```
int(coth(a + 2*log(x))*(e*x)^m,x)
```

output
```
int(coth(a + 2*log(x))*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \coth(a+2\log(x))\,dx = \frac{e^m\left(x^m x+2\left(\int \frac{x^m}{e^{2a}x^4-1}dx\right)m+2\left(\int \frac{x^m}{e^{2a}x^4-1}dx\right)\right)}{m+1}$$

input
```
int((e*x)^m*coth(a+2*log(x)),x)
```

output
```
(e**m*(x**m*x + 2*int(x**m/(e**(2*a)*x**4 - 1),x)*m + 2*int(x**m/(e**(2*a)
*x**4 - 1),x)))/(m + 1)
```

3.171 $\int (ex)^m \coth^2(a+2\log(x))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int (ex)^m \coth^2(a+2\log(x))\,dx = \frac{(ex)^{1+m}}{e(1+m)} + \frac{(ex)^{1+m}}{e\,(1-e^{2a}x^4)} - \frac{(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},e^{2a}x^4\right)}{e}$$

output
```
(e*x)^(1+m)/e/(1+m)+(e*x)^(1+m)/e/(1-exp(2*a)*x^4)-(e*x)^(1+m)*hypergeom([
1, 1/4+1/4*m],[5/4+1/4*m],exp(2*a)*x^4)/e
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\int (ex)^m \coth^2(a+2\log(x))\,dx = -\frac{x(ex)^m\left(-1+4\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},x^4(\cosh(2a)+\sinh(2a))\right)-4\,\text{Hypergeometric2F1}\right.}{1+m}$$

input
```
Integrate[(e*x)^m*Coth[a + 2*Log[x]]^2,x]
```

output
```
-((x*(e*x)^m*(-1 + 4*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, x^4*(Cosh[
2*a] + Sinh[2*a])] - 4*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, x^4*(Cos
h[2*a] + Sinh[2*a])]))/(1 + m))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6072, 963, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \coth^2(a+2\log(x))\,dx$$

$$\Big\downarrow \text{6072}$$

$$\int \frac{\left(-e^{2a}x^4-1\right)^2(ex)^m}{\left(1-e^{2a}x^4\right)^2}dx$$

$$\Big\downarrow \text{963}$$

$$\frac{(ex)^{m+1}}{e\left(1-e^{2a}x^4\right)}-\frac{1}{4}e^{-4a}\int \frac{4(ex)^m\left(e^{6a}x^4+e^{4a}m\right)}{1-e^{2a}x^4}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{(ex)^{m+1}}{e\left(1-e^{2a}x^4\right)}-e^{-4a}\int \frac{(ex)^m\left(e^{6a}x^4+e^{4a}m\right)}{1-e^{2a}x^4}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(ex)^{m+1}}{e\left(1-e^{2a}x^4\right)}-e^{-4a}\left(e^{4a}(m+1)\int \frac{(ex)^m}{1-e^{2a}x^4}dx-\frac{e^{4a}(ex)^{m+1}}{e(m+1)}\right)$$

$$\Big\downarrow \text{888}$$

$$\frac{(ex)^{m+1}}{e\left(1-e^{2a}x^4\right)}-e^{-4a}\left(\frac{e^{4a}(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},e^{2a}x^4\right)}{e}-\frac{e^{4a}(ex)^{m+1}}{e(m+1)}\right)$$

input
```
Int[(e*x)^m*Coth[a + 2*Log[x]]^2,x]
```

output
```
(e*x)^(1 + m)/(e*(1 - E^(2*a)*x^4)) - (-((E^(4*a)*(e*x)^(1 + m))/(e*(1 + m
))) + (E^(4*a)*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, E^
(2*a)*x^4])/e)/E^(4*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 963
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Simp[1/(a*b^2*n*(p + 1))   Int[(e*x)^m*(a + b*x
^n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [F]

$$\int (ex)^m \coth (a+2\ln (x))^2 \,dx$$

input
```
int((e*x)^m*coth(a+2*ln(x))^2,x)
```

output
```
int((e*x)^m*coth(a+2*ln(x))^2,x)
```

Fricas [F]

$$\int (ex)^m \coth^2(a+2\log(x))\,dx=\int (ex)^m \coth \left(a+2\,\log \left(x\right)\right)^2\,dx$$

input
```
integrate((e*x)^m*coth(a+2*log(x))^2,x, algorithm="fricas")
```

output
```
integral((e*x)^m*coth(a + 2*log(x))^2, x)
```

Sympy [F]

$$\int (ex)^m \coth^2(a+2\log(x))\,dx=\int (ex)^m \coth^2 \left(a+2\log \left(x\right)\right)dx$$

input
```
integrate((e*x)**m*coth(a+2*ln(x))**2,x)
```

output
```
Integral((e*x)**m*coth(a + 2*log(x))**2, x)
```

Maxima [F]

$$\int (ex)^m \coth^2(a+2\log(x))\,dx = \int (ex)^m \coth\left(a+2\,\log\left(x\right)\right)^2\,dx$$

input
```
integrate((e*x)^m*coth(a+2*log(x))^2,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*coth(a + 2*log(x))^2, x)
```

Giac [F]

$$\int (ex)^m \coth^2(a+2\log(x))\,dx = \int (ex)^m \coth\left(a+2\,\log\left(x\right)\right)^2\,dx$$

input
```
integrate((e*x)^m*coth(a+2*log(x))^2,x, algorithm="giac")
```

output
```
integrate((e*x)^m*coth(a + 2*log(x))^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \coth^2(a+2\log(x))\,dx = \int \coth(a+2\,\ln\left(x\right))^2\,(e\,x)^m\,dx$$

input
```
int(coth(a + 2*log(x))^2*(e*x)^m,x)
```

output
```
int(coth(a + 2*log(x))^2*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \coth^2(a+2\log(x))\,dx$$
$$= \frac{e^m \left(x^m e^{2a} m\, x^5 - 3x^m e^{2a} x^5 + 4e^{2a} \left(\int \frac{x^m}{e^{4a} m\, x^8 - 3e^{4a} x^8 - 2e^{2a} m\, x^4 + 6e^{2a} x^4 + m - 3} dx\right) m^3 x^4 - 4e^{2a} \left(\int \frac{}{e^{4a} m\, x^8 - 3e^{4a} x^8 -}\right.\right.}{}$$

input
```
int((e*x)^m*coth(a+2*log(x))^2,x)
```

output
```
(e**m*(x**m*e**(2*a)*m*x**5 - 3*x**m*e**(2*a)*x**5 + 4*e**(2*a)*int(x**m/(
e**(4*a)*m*x**8 - 3*e**(4*a)*x**8 - 2*e**(2*a)*m*x**4 + 6*e**(2*a)*x**4 +
m - 3),x)*m**3*x**4 - 4*e**(2*a)*int(x**m/(e**(4*a)*m*x**8 - 3*e**(4*a)*x*
*8 - 2*e**(2*a)*m*x**4 + 6*e**(2*a)*x**4 + m - 3),x)*m**2*x**4 - 20*e**(2*
a)*int(x**m/(e**(4*a)*m*x**8 - 3*e**(4*a)*x**8 - 2*e**(2*a)*m*x**4 + 6*e**
(2*a)*x**4 + m - 3),x)*m*x**4 - 12*e**(2*a)*int(x**m/(e**(4*a)*m*x**8 - 3*
e**(4*a)*x**8 - 2*e**(2*a)*m*x**4 + 6*e**(2*a)*x**4 + m - 3),x)*x**4 + 3*x
**m*m*x + 7*x**m*x - 4*int(x**m/(e**(4*a)*m*x**8 - 3*e**(4*a)*x**8 - 2*e**
(2*a)*m*x**4 + 6*e**(2*a)*x**4 + m - 3),x)*m**3 + 4*int(x**m/(e**(4*a)*m*x
**8 - 3*e**(4*a)*x**8 - 2*e**(2*a)*m*x**4 + 6*e**(2*a)*x**4 + m - 3),x)*m*
*2 + 20*int(x**m/(e**(4*a)*m*x**8 - 3*e**(4*a)*x**8 - 2*e**(2*a)*m*x**4 +
6*e**(2*a)*x**4 + m - 3),x)*m + 12*int(x**m/(e**(4*a)*m*x**8 - 3*e**(4*a)*
x**8 - 2*e**(2*a)*m*x**4 + 6*e**(2*a)*x**4 + m - 3),x)))/(e**(2*a)*m**2*x*
*4 - 2*e**(2*a)*m*x**4 - 3*e**(2*a)*x**4 - m**2 + 2*m + 3)
```

3.172 $\int (ex)^m \coth^3(a+2\log(x))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 126

$$
\begin{aligned}
\int (ex)^m \coth^3(a+2\log(x))\,dx
&= \frac{(ex)^{1+m}}{e(1+m)} - \frac{(ex)^{1+m}}{e\left(1-e^{2a}x^4\right)^2} + \frac{(5+m)(ex)^{1+m}}{4e\left(1-e^{2a}x^4\right)} \\
&\quad - \frac{\left(9+2m+m^2\right)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},e^{2a}x^4\right)}{4e(1+m)}
\end{aligned}
$$

output

```
(e*x)^(1+m)/e/(1+m)-(e*x)^(1+m)/e/(1-exp(2*a)*x^4)^2+1/4*(5+m)*(e*x)^(1+m)
/e/(1-exp(2*a)*x^4)-1/4*(m^2+2*m+9)*(e*x)^(1+m)*hypergeom([1, 1/4+1/4*m],[
5/4+1/4*m],exp(2*a)*x^4)/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

$$
\int (ex)^m \coth^3(a+2\log(x))\,dx =
$$

$$
-\frac{x(ex)^m\left(-1+6\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},x^4(\cosh(2a)+\sinh(2a))\right)-12\,\text{Hypergeometric2F}\cdots\right.}{1\cdots}
$$

input
```
Integrate[(e*x)^m*Coth[a + 2*Log[x]]^3,x]
```

output
```
-((x*(e*x)^m*(-1 + 6*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, x^4*(Cosh[
2*a] + Sinh[2*a])] - 12*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, x^4*(Co
sh[2*a] + Sinh[2*a])] + 8*Hypergeometric2F1[3, (1 + m)/4, (5 + m)/4, x^4*(
Cosh[2*a] + Sinh[2*a])]))/(1 + m))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.57, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {6072, 968, 27, 1047, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \coth^3(a+2\log(x))\,dx$$

$$\Big\downarrow \text{6072}$$

$$\int \frac{\left(-e^{2a}x^4-1\right)^3 (ex)^m}{\left(1-e^{2a}x^4\right)^3}dx$$

$$\Big\downarrow \text{968}$$

$$\frac{1}{8}e^{-2a}\int -\frac{2(ex)^m \left(e^{2a}x^4+1\right)\left(e^{2a}(3-m)-e^{4a}(m+5)x^4\right)}{\left(1-e^{2a}x^4\right)^2}dx-\frac{\left(e^{2a}x^4+1\right)^2 (ex)^{m+1}}{4e\left(1-e^{2a}x^4\right)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{4}e^{-2a}\int \frac{(ex)^m \left(e^{2a}x^4+1\right)\left(e^{2a}(3-m)-e^{4a}(m+5)x^4\right)}{\left(1-e^{2a}x^4\right)^2}dx-\frac{\left(e^{2a}x^4+1\right)^2 (ex)^{m+1}}{4e\left(1-e^{2a}x^4\right)^2}$$

$$\Big\downarrow \text{1047}$$

$$-\frac{1}{4}e^{-2a}\left(\frac{1}{4}e^{-2a}\int \frac{2(ex)^m \left(e^{6a}(m+3)(m+5)x^4+e^{4a}(1-m)(3-m)\right)}{1-e^{2a}x^4}dx+\frac{\left(e^{2a}(3-m)-e^{4a}(m+5)x^4\right)(ex}{2e\left(1-e^{2a}x^4\right)}\right.$$
$$\frac{\left(e^{2a}x^4+1\right)^2 (ex)^{m+1}}{4e\left(1-e^{2a}x^4\right)^2}$$

$$\downarrow \textbf{27}$$

$$-\frac{1}{4}e^{-2a}\left(\frac{1}{2}e^{-2a}\int \frac{(ex)^m\left(e^{6a}(m+3)(m+5)x^4+e^{4a}(1-m)(3-m)\right)}{1-e^{2a}x^4}dx+\frac{\left(e^{2a}(3-m)-e^{4a}(m+5)x^4\right)(ex)^{m}}{2e\left(1-e^{2a}x^4\right)}\right.$$
$$\frac{\left(e^{2a}x^4+1\right)^2(ex)^{m+1}}{4e\left(1-e^{2a}x^4\right)^2}$$

$$\downarrow \textbf{959}$$

$$-\frac{1}{4}e^{-2a}\left(\frac{1}{2}e^{-2a}\left(2e^{4a}\left(m^2+2m+9\right)\int \frac{(ex)^m}{1-e^{2a}x^4}dx-\frac{e^{4a}(m+3)(m+5)(ex)^{m+1}}{e(m+1)}\right)+\frac{\left(e^{2a}(3-m)-e^{4a}(m+\right.}{2e\left(1-e^{2a}\right.}\right.$$
$$\frac{\left(e^{2a}x^4+1\right)^2(ex)^{m+1}}{4e\left(1-e^{2a}x^4\right)^2}$$

$$\downarrow \textbf{888}$$

$$-\frac{1}{4}e^{-2a}\left(\frac{1}{2}e^{-2a}\left(\frac{2e^{4a}\left(m^2+2m+9\right)(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},e^{2a}x^4\right)}{e(m+1)}-\frac{e^{4a}(m+3)(m+5)(e}{e(m+1)}\right.\right.$$
$$\frac{\left(e^{2a}x^4+1\right)^2(ex)^{m+1}}{4e\left(1-e^{2a}x^4\right)^2}$$

input
```
Int[(e*x)^m*Coth[a + 2*Log[x]]^3,x]
```

output
```
-1/4*((e*x)^(1 + m)*(1 + E^(2*a)*x^4)^2)/(e*(1 - E^(2*a)*x^4)^2) - (((e*x)
^(1 + m)*(E^(2*a)*(3 - m) - E^(4*a)*(5 + m)*x^4))/(2*e*(1 - E^(2*a)*x^4))
+ (-((E^(4*a)*(3 + m)*(5 + m)*(e*x)^(1 + m))/(e*(1 + m))) + (2*E^(4*a)*(9
+ 2*m + m^2)*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, E^(2
*a)*x^4])/(e*(1 + m)))/(2*E^(2*a)))/(4*E^(2*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 968
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
 x]
```

rule 1047
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Simp[1/(
a*b*n*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c
*(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m
+ n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e
- a*f])
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [F]

$$\int (ex)^m \coth (a + 2\ln (x))^3 \, dx$$

input
```
int((e*x)^m*coth(a+2*ln(x))^3,x)
```

output
```
int((e*x)^m*coth(a+2*ln(x))^3,x)
```

Fricas [F]

$$\int (ex)^m \coth^3(a+2\log(x))\,dx = \int (ex)^m \coth (a + 2\,\log (x))^3 \, dx$$

input
```
integrate((e*x)^m*coth(a+2*log(x))^3,x, algorithm="fricas")
```

output
```
integral((e*x)^m*coth(a + 2*log(x))^3, x)
```

Sympy [F]

$$\int (ex)^m \coth^3(a+2\log(x))\,dx = \int (ex)^m \coth^3 (a + 2\log (x))\,dx$$

input
```
integrate((e*x)**m*coth(a+2*ln(x))**3,x)
```

output
```
Integral((e*x)**m*coth(a + 2*log(x))**3, x)
```

Maxima [F]

$$\int (ex)^m \coth^3(a+2\log(x))\,dx = \int (ex)^m \coth\left(a+2\,\log\left(x\right)\right)^3\,dx$$

input
```
integrate((e*x)^m*coth(a+2*log(x))^3,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*coth(a + 2*log(x))^3, x)
```

Giac [F]

$$\int (ex)^m \coth^3(a+2\log(x))\,dx = \int (ex)^m \coth\left(a+2\,\log\left(x\right)\right)^3\,dx$$

input
```
integrate((e*x)^m*coth(a+2*log(x))^3,x, algorithm="giac")
```

output
```
integrate((e*x)^m*coth(a + 2*log(x))^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \coth^3(a+2\log(x))\,dx = \int \coth(a+2\,\ln\left(x\right))^3\,(e\,x)^m\,dx$$

input
```
int(coth(a + 2*log(x))^3*(e*x)^m,x)
```

output
```
int(coth(a + 2*log(x))^3*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \coth^3(a+2\log(x))\,dx = \text{too large to display}$$

input
```
int((e*x)^m*coth(a+2*log(x))^3,x)
```

output
```
(e**m*(x**m*e**(4*a)*m**2*x**9 - 10*x**m*e**(4*a)*m*x**9 + 21*x**m*e**(4*a
)*x**9 + 8*e**(4*a)*int(x**m/(e**(6*a)*m**2*x**12 - 10*e**(6*a)*m*x**12 +
21*e**(6*a)*x**12 - 3*e**(4*a)*m**2*x**8 + 30*e**(4*a)*m*x**8 - 63*e**(4*a
)*x**8 + 3*e**(2*a)*m**2*x**4 - 30*e**(2*a)*m*x**4 + 63*e**(2*a)*x**4 - m*
*2 + 10*m - 21),x)*m**5*x**8 - 56*e**(4*a)*int(x**m/(e**(6*a)*m**2*x**12 -
 10*e**(6*a)*m*x**12 + 21*e**(6*a)*x**12 - 3*e**(4*a)*m**2*x**8 + 30*e**(4
*a)*m*x**8 - 63*e**(4*a)*x**8 + 3*e**(2*a)*m**2*x**4 - 30*e**(2*a)*m*x**4
+ 63*e**(2*a)*x**4 - m**2 + 10*m - 21),x)*m**4*x**8 + 16*e**(4*a)*int(x**m
/(e**(6*a)*m**2*x**12 - 10*e**(6*a)*m*x**12 + 21*e**(6*a)*x**12 - 3*e**(4*
a)*m**2*x**8 + 30*e**(4*a)*m*x**8 - 63*e**(4*a)*x**8 + 3*e**(2*a)*m**2*x**
4 - 30*e**(2*a)*m*x**4 + 63*e**(2*a)*x**4 - m**2 + 10*m - 21),x)*m**3*x**8
 - 304*e**(4*a)*int(x**m/(e**(6*a)*m**2*x**12 - 10*e**(6*a)*m*x**12 + 21*e
**(6*a)*x**12 - 3*e**(4*a)*m**2*x**8 + 30*e**(4*a)*m*x**8 - 63*e**(4*a)*x*
*8 + 3*e**(2*a)*m**2*x**4 - 30*e**(2*a)*m*x**4 + 63*e**(2*a)*x**4 - m**2 +
 10*m - 21),x)*m**2*x**8 + 1128*e**(4*a)*int(x**m/(e**(6*a)*m**2*x**12 - 1
0*e**(6*a)*m*x**12 + 21*e**(6*a)*x**12 - 3*e**(4*a)*m**2*x**8 + 30*e**(4*a
)*m*x**8 - 63*e**(4*a)*x**8 + 3*e**(2*a)*m**2*x**4 - 30*e**(2*a)*m*x**4 +
63*e**(2*a)*x**4 - m**2 + 10*m - 21),x)*m*x**8 + 1512*e**(4*a)*int(x**m/(e
**(6*a)*m**2*x**12 - 10*e**(6*a)*m*x**12 + 21*e**(6*a)*x**12 - 3*e**(4*a)*
m**2*x**8 + 30*e**(4*a)*m*x**8 - 63*e**(4*a)*x**8 + 3*e**(2*a)*m**2*x**...
```

3.173 $\int \coth^p(a+b\log(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 79

$$\int \coth^p(a+b\log(x))\,dx = x\left(-1-e^{2a}x^{2b}\right)^p\left(1+e^{2a}x^{2b}\right)^{-p}\text{AppellF1}\left(\frac{1}{2b},p,-p,\frac{1}{2}\left(2+\frac{1}{b}\right),e^{2a}x^{2b},-e^{2a}x^{2b}\right)$$

output
```
x*(-1-exp(2*a)*x^(2*b))^p*AppellF1(1/2/b,p,-p,1+1/2/b,exp(2*a)*x^(2*b),-ex
p(2*a)*x^(2*b))/((1+exp(2*a)*x^(2*b))^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 259 vs. 2(79) = 158.

Time = 0.40 (sec) , antiderivative size = 259, normalized size of antiderivative = 3.28

$$\int \coth^p(a+b\log(x))\,dx$$

$$= \frac{(1+2b)x\left(\frac{1+e^{2a}x^{2b}}{-1+e^{2a}x^{2b}}\right)^p\text{AppellF1}\left(\frac{1}{2b},p,-p,1+\right.}{2be^{2a}px^{2b}\,\text{AppellF1}\left(1+\frac{1}{2b},p,1-p,2+\frac{1}{2b},e^{2a}x^{2b},-e^{2a}x^{2b}\right)+2be^{2a}px^{2b}\,\text{AppellF1}\left(1+\frac{1}{2b},1+p,-p,2\right.}$$

input
```
Integrate[Coth[a + b*Log[x]]^p,x]
```

output
```
((1 + 2*b)*x*((1 + E^(2*a)*x^(2*b))/(-1 + E^(2*a)*x^(2*b)))^p*AppellF1[1/(
2*b), p, -p, 1 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))])/(2*b*E^(2*
a)*p*x^(2*b)*AppellF1[1 + 1/(2*b), p, 1 - p, 2 + 1/(2*b), E^(2*a)*x^(2*b),
 -(E^(2*a)*x^(2*b))] + 2*b*E^(2*a)*p*x^(2*b)*AppellF1[1 + 1/(2*b), 1 + p,
-p, 2 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))] + (1 + 2*b)*AppellF1
[1/(2*b), p, -p, 1 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6068, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^p(a + b\log(x))\,dx$$
$$\Big\downarrow \text{6068}$$
$$\int \left(-e^{2a}x^{2b}-1\right)^p \left(1-e^{2a}x^{2b}\right)^{-p}dx$$
$$\Big\downarrow \text{937}$$
$$\left(-e^{2a}x^{2b}-1\right)^p \left(e^{2a}x^{2b}+1\right)^{-p}\int \left(1-e^{2a}x^{2b}\right)^{-p}\left(e^{2a}x^{2b}+1\right)^p dx$$
$$\Big\downarrow \text{936}$$
$$x\left(-e^{2a}x^{2b}-1\right)^p \left(e^{2a}x^{2b}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2b},p,-p,\frac{1}{2}\left(2+\frac{1}{b}\right),e^{2a}x^{2b},-e^{2a}x^{2b}\right)$$

input
```
Int[Coth[a + b*Log[x]]^p,x]
```

output
```
(x*(-1 - E^(2*a)*x^(2*b))^p*AppellF1[1/(2*b), p, -p, (2 + b^(-1))/2, E^(2*
a)*x^(2*b), -(E^(2*a)*x^(2*b))])/(1 + E^(2*a)*x^(2*b))^p
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 6068
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Maple [F]

$$\int \coth\left(a+b\ln\left(x\right)\right)^{p}dx$$

input
```
int(coth(a+b*ln(x))^p,x)
```

output
```
int(coth(a+b*ln(x))^p,x)
```

Fricas [F]

$$\int \coth^p(a+b\log(x))\,dx=\int \coth\left(b\log\left(x\right)+a\right)^{p}\,dx$$

input
```
integrate(coth(a+b*log(x))^p,x, algorithm="fricas")
```

output
```
integral(coth(b*log(x) + a)^p, x)
```

Sympy [F]

$$\int \coth^p(a+b\log(x))\,dx = \int \coth^p\left(a+b\log\left(x\right)\right)dx$$

input
```
integrate(coth(a+b*ln(x))**p,x)
```

output
```
Integral(coth(a + b*log(x))**p, x)
```

Maxima [F]

$$\int \coth^p(a+b\log(x))\,dx = \int \coth\left(b\log\left(x\right)+a\right)^p\,dx$$

input
```
integrate(coth(a+b*log(x))^p,x, algorithm="maxima")
```

output
```
integrate(coth(b*log(x) + a)^p, x)
```

Giac [F]

$$\int \coth^p(a+b\log(x))\,dx = \int \coth\left(b\log\left(x\right)+a\right)^p\,dx$$

input
```
integrate(coth(a+b*log(x))^p,x, algorithm="giac")
```

output
```
integrate(coth(b*log(x) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^p(a+b\log(x))\,dx=\int \coth(a+b\ \ln(x))^p\,dx$$

input
```
int(coth(a + b*log(x))^p,x)
```

output
```
int(coth(a + b*log(x))^p, x)
```

Reduce [F]

$$\int \coth^p(a+b\log(x))\,dx=\coth\left(\log(x)\,b+a\right)^p x-\left(\int \frac{\coth\left(\log(x)\,b+a\right)^p}{\coth\left(\log\left(x\right)b+a\right)}dx\right)bp+\left(\int \coth\left(\log(x)\,b+a\right)^p\coth\left(\log(x)\,b+a\right)dx\right)bp$$

input
```
int(coth(a+b*log(x))^p,x)
```

output
```
coth(log(x)*b + a)**p*x - int(coth(log(x)*b + a)**p/coth(log(x)*b + a),x)*
b*p + int(coth(log(x)*b + a)**p*coth(log(x)*b + a),x)*b*p
```

3.174 $\int (ex)^m \coth^p(a+b\log(x))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 99

$$\int (ex)^m \coth^p(a+b\log(x))\,dx = \frac{(ex)^{1+m}\left(-1-e^{2a}x^{2b}\right)^p\left(1+e^{2a}x^{2b}\right)^{-p}\operatorname{AppellF1}\left(\frac{1+m}{2b},p,-p,1+\frac{1+m}{2b},e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{e(1+m)}$$

output
```
(e*x)^(1+m)*(-1-exp(2*a)*x^(2*b))^p*AppellF1(1/2*(1+m)/b,p,-p,1+1/2*(1+m)/
b,exp(2*a)*x^(2*b),-exp(2*a)*x^(2*b))/e/(1+m)/((1+exp(2*a)*x^(2*b))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.27

$$\int (ex)^m \coth^p(a+b\log(x))\,dx = \frac{x(ex)^m\left(1-e^{2a}x^{2b}\right)^p\left(1+e^{2a}x^{2b}\right)^{-p}\left(\frac{1+e^{2a}x^{2b}}{-1+e^{2a}x^{2b}}\right)^p\operatorname{AppellF1}\left(\frac{1+m}{2b},p,-p,1+\frac{1+m}{2b},e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{1+m}$$

input
```
Integrate[(e*x)^m*Coth[a + b*Log[x]]^p,x]
```

output
```
(x*(e*x)^m*(1 - E^(2*a)*x^(2*b))^p*((1 + E^(2*a)*x^(2*b))/(-1 + E^(2*a)*x^
(2*b)))^p*AppellF1[(1 + m)/(2*b), p, -p, 1 + (1 + m)/(2*b), E^(2*a)*x^(2*b
), -(E^(2*a)*x^(2*b))])/((1 + m)*(1 + E^(2*a)*x^(2*b))^p)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6072, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \coth^p(a+b\log(x))\,dx$$

$$\Big\downarrow \text{6072}$$

$$\int (ex)^m \left(-e^{2a}x^{2b}-1\right)^p \left(1-e^{2a}x^{2b}\right)^{-p}dx$$

$$\Big\downarrow \text{1013}$$

$$\left(-e^{2a}x^{2b}-1\right)^p \left(e^{2a}x^{2b}+1\right)^{-p}\int (ex)^m \left(1-e^{2a}x^{2b}\right)^{-p}\left(e^{2a}x^{2b}+1\right)^p dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{(ex)^{m+1}\left(-e^{2a}x^{2b}-1\right)^p\left(e^{2a}x^{2b}+1\right)^{-p}\text{AppellF1}\left(\frac{m+1}{2b},p,-p,\frac{m+1}{2b}+1,e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{e(m+1)}$$

input
```
Int[(e*x)^m*Coth[a + b*Log[x]]^p,x]
```

output
```
((e*x)^(1 + m)*(-1 - E^(2*a)*x^(2*b))^p*AppellF1[(1 + m)/(2*b), p, -p, 1 +
 (1 + m)/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))])/(e*(1 + m)*(1 + E^(2
*a)*x^(2*b))^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [F]

$$\int (ex)^m \coth (a+b\ln (x))^p\,dx$$

input
```
int((e*x)^m*coth(a+b*ln(x))^p,x)
```

output
```
int((e*x)^m*coth(a+b*ln(x))^p,x)
```

Fricas [F]

$$\int (ex)^m \coth^p(a+b\log(x))\,dx = \int (ex)^m \coth (b\log (x)+a)^p\,dx$$

input
```
integrate((e*x)^m*coth(a+b*log(x))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^m*coth(b*log(x) + a)^p, x)
```

Sympy [F]

$$\int (ex)^m \coth^p(a + b\log(x))\, dx = \int (ex)^m \coth^p(a + b\log(x))\, dx$$

input
```
integrate((e*x)**m*coth(a+b*ln(x))**p,x)
```

output
```
Integral((e*x)**m*coth(a + b*log(x))**p, x)
```

Maxima [F]

$$\int (ex)^m \coth^p(a + b\log(x))\, dx = \int (ex)^m \coth(b\log(x) + a)^p\, dx$$

input
```
integrate((e*x)^m*coth(a+b*log(x))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*coth(b*log(x) + a)^p, x)
```

Giac [F]

$$\int (ex)^m \coth^p(a + b\log(x))\, dx = \int (ex)^m \coth(b\log(x) + a)^p\, dx$$

input
```
integrate((e*x)^m*coth(a+b*log(x))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^m*coth(b*log(x) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \coth^p(a+b\log(x))\,dx = \int \coth(a+b\,\ln(x))^p\,(e\,x)^m\,dx$$

input
```
int(coth(a + b*log(x))^p*(e*x)^m,x)
```

output
```
int(coth(a + b*log(x))^p*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \coth^p(a+b\log(x))\,dx$$
$$= \frac{e^m \left(x^m \coth(\log(x)\,b+a)^p\,x - \left(\int \frac{x^m \coth(\log(x)b+a)^p}{\coth(\log(x)b+a)}dx\right) bp + \left(\int x^m \coth(\log(x)\,b+a)^p \coth(\log(x)\,b+\right.}{m+1}$$

input
```
int((e*x)^m*coth(a+b*log(x))^p,x)
```

output
```
(e**m*(x**m*coth(log(x)*b + a)**p*x - int((x**m*coth(log(x)*b + a)**p)/cot
h(log(x)*b + a),x)*b*p + int(x**m*coth(log(x)*b + a)**p*coth(log(x)*b + a)
,x)*b*p))/(m + 1)
```

3.175 $\int \coth^p\left(a+\frac{\log(x)}{2}\right) dx$

Optimal result

Integrand size = 11, antiderivative size = 52

$$\int \coth^p\left(a+\frac{\log(x)}{2}\right) dx = -\frac{2^{-p}e^{-2a}(-1-e^{2a}x)^{1+p}\text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{1}{2}(1+e^{2a}x)\right)}{1+p}$$

output
```
-(-1-exp(2*a)*x)^(p+1)*hypergeom([p, p+1],[2+p],1/2+1/2*exp(2*a)*x)/(2^p)/
exp(2*a)/(p+1)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.60

$$\int \coth^p\left(a+\frac{\log(x)}{2}\right) dx = -\frac{2^pe^{-2a}(1+e^{2a}x)^{1-p}\left(\frac{1+e^{2a}x}{-1+e^{2a}x}\right)^{-1+p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,\frac{1}{2}-\frac{1}{2}e^{2a}x\right)}{-1+p}$$

input
```
Integrate[Coth[a + Log[x]/2]^p,x]
```

output
```
-((2^p*(1 + E^(2*a)*x)^(1 - p)*((1 + E^(2*a)*x)/(-1 + E^(2*a)*x))^(-1 + p)
*Hypergeometric2F1[1 - p, -p, 2 - p, 1/2 - (E^(2*a)*x)/2])/(E^(2*a)*(-1 +
p)))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = $\{6068, 79\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^p\left(a+\frac{\log(x)}{2}\right)\,dx$$

$$\Big\downarrow \; 6068$$

$$\int \left(-e^{2a}x-1\right)^p\left(1-e^{2a}x\right)^{-p}dx$$

$$\Big\downarrow \; 79$$

$$-\frac{e^{-2a}2^{-p}\left(-e^{2a}x-1\right)^{p+1}\text{Hypergeometric2F1}\left(p,p+1,p+2,\frac{1}{2}\left(e^{2a}x+1\right)\right)}{p+1}$$

input
```
Int[Coth[a + Log[x]/2]^p,x]
```

output
```
-(((-1 - E^(2*a)*x)^(1 + p)*Hypergeometric2F1[p, 1 + p, 2 + p, (1 + E^(2*a
)*x)/2])/(2^p*E^(2*a)*(1 + p)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 6068
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Maple [F]

$$\int \coth\left(a+\frac{\ln(x)}{2}\right)^{p} dx$$

input
```
int(coth(a+1/2*ln(x))^p,x)
```

output
```
int(coth(a+1/2*ln(x))^p,x)
```

Fricas [F]

$$\int \coth^p\left(a+\frac{\log(x)}{2}\right) dx = \int \coth\left(a+\frac{1}{2}\log(x)\right)^{p} dx$$

input
```
integrate(coth(a+1/2*log(x))^p,x, algorithm="fricas")
```

output
```
integral(coth(a + 1/2*log(x))^p, x)
```

Sympy [F]

$$\int \coth^p\left(a+\frac{\log(x)}{2}\right)\,dx=\int \coth^p\left(a+\frac{\log\left(x\right)}{2}\right)\,dx$$

input
```
integrate(coth(a+1/2*ln(x))**p,x)
```

output
```
Integral(coth(a + log(x)/2)**p, x)
```

Maxima [F]

$$\int \coth^p\left(a+\frac{\log(x)}{2}\right)\,dx=\int \coth\left(a+\frac{1}{2}\,\log\left(x\right)\right)^p\,dx$$

input
```
integrate(coth(a+1/2*log(x))^p,x, algorithm="maxima")
```

output
```
integrate(coth(a + 1/2*log(x))^p, x)
```

Giac [F]

$$\int \coth^p\left(a+\frac{\log(x)}{2}\right)\,dx=\int \coth\left(a+\frac{1}{2}\,\log\left(x\right)\right)^p\,dx$$

input
```
integrate(coth(a+1/2*log(x))^p,x, algorithm="giac")
```

output
```
integrate(coth(a + 1/2*log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^p\left(a+\frac{\log(x)}{2}\right)\,dx=\int \coth\left(a+\frac{\ln(x)}{2}\right)^p\,dx$$

input
```
int(coth(a + log(x)/2)^p,x)
```

output
```
int(coth(a + log(x)/2)^p, x)
```

Reduce [F]

$$\int \coth^p\left(a+\frac{\log(x)}{2}\right)\,dx=\coth\left(\frac{\log(x)}{2}+a\right)^p x-\frac{\left(\int \frac{\coth\left(\frac{\log(x)}{2}+a\right)^p}{\coth\left(\frac{\log(x)}{2}+a\right)}dx\right)p}{2}+\frac{\left(\int \coth\left(\frac{\log(x)}{2}+a\right)^p\coth\left(\frac{\log(x)}{2}+a\right)dx\right)p}{2}$$

input
```
int(coth(a+1/2*log(x))^p,x)
```

output
```
(2*coth((log(x) + 2*a)/2)**p*x - int(coth((log(x) + 2*a)/2)**p/coth((log(x
) + 2*a)/2),x)*p + int(coth((log(x) + 2*a)/2)**p*coth((log(x) + 2*a)/2),x)
*p)/2
```

3.176 $\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 108

$$\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx = e^{-4a}\left(-1-e^{2a}\sqrt{x}\right)^{1+p}\left(1-e^{2a}\sqrt{x}\right)^{1-p} - \frac{2^{1-p}e^{-4a}p\left(-1-e^{2a}\sqrt{x}\right)^{1+p}\text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{1}{2}\left(1+e^{2a}\sqrt{x}\right)\right)}{1+p}$$

output
```
(-1-exp(2*a)*x^(1/2))^(p+1)*(1-exp(2*a)*x^(1/2))^(1-p)/exp(4*a)-2^(1-p)*p*
(-1-exp(2*a)*x^(1/2))^(p+1)*hypergeom([p, p+1],[2+p],1/2+1/2*exp(2*a)*x^(1
/2))/exp(4*a)/(p+1)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.16

$$\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx = \frac{e^{-4a}\left(1+e^{2a}\sqrt{x}\right)^{1-p}\left(\frac{1+e^{2a}\sqrt{x}}{-1+e^{2a}\sqrt{x}}\right)^{-1+p}\left(\left(-1+p\right)\left(1+e^{2a}\sqrt{x}\right)^{1+p}-2^{1+p}p\,\text{Hypergeometric2F1}\left(1-p,-p\right.\right.}{-1+p}$$

input
```
Integrate[Coth[a + Log[x]/4]^p,x]
```

output
```
((1 + E^(2*a)*Sqrt[x])^(1 - p)*((1 + E^(2*a)*Sqrt[x])/(-1 + E^(2*a)*Sqrt[x
]))^(-1 + p)*((-1 + p)*(1 + E^(2*a)*Sqrt[x])^(1 + p) - 2^(1 + p)*p*Hyperge
ometric2F1[1 - p, -p, 2 - p, 1/2 - (E^(2*a)*Sqrt[x])/2]))/(E^(4*a)*(-1 + p
))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6068, 900, 90, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^p\left(a+\frac{\log(x)}{4}\right)dx$$

$$\Big\downarrow \text{6068}$$

$$\int \left(-e^{2a}\sqrt{x}-1\right)^p\left(1-e^{2a}\sqrt{x}\right)^{-p}dx$$

$$\Big\downarrow \text{900}$$

$$2\int \left(-e^{2a}\sqrt{x}-1\right)^p\left(1-e^{2a}\sqrt{x}\right)^{-p}\sqrt{x}d\sqrt{x}$$

$$\Big\downarrow \text{90}$$

$$2\left(e^{-2a}p\int \left(-e^{2a}\sqrt{x}-1\right)^p\left(1-e^{2a}\sqrt{x}\right)^{-p}d\sqrt{x}+\frac{1}{2}e^{-4a}\left(-e^{2a}\sqrt{x}-1\right)^{p+1}\left(1-e^{2a}\sqrt{x}\right)^{1-p}\right)$$

$$\Big\downarrow \text{79}$$

$$2\left(\frac{1}{2}e^{-4a}\left(-e^{2a}\sqrt{x}-1\right)^{p+1}\left(1-e^{2a}\sqrt{x}\right)^{1-p}-\frac{e^{-4a}2^{-p}p\left(-e^{2a}\sqrt{x}-1\right)^{p+1}\text{Hypergeometric2F1}\left(p,p+1,p+2,\frac{1}{2}\right.}{p+1}\right.$$

input
```
Int[Coth[a + Log[x]/4]^p,x]
```

output
```
2*(((-1 - E^(2*a)*Sqrt[x])^(1 + p)*(1 - E^(2*a)*Sqrt[x])^(1 - p))/(2*E^(4*
a)) - (p*(-1 - E^(2*a)*Sqrt[x])^(1 + p)*Hypergeometric2F1[p, 1 + p, 2 + p,
 (1 + E^(2*a)*Sqrt[x])/2])/(2^p*E^(4*a)*(1 + p)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 6068
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Maple [F]

$$\int \coth\left(a+\frac{\ln(x)}{4}\right)^{p} dx$$

input
```
int(coth(a+1/4*ln(x))^p,x)
```

output
```
int(coth(a+1/4*ln(x))^p,x)
```

Fricas [F]

$$\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx=\int \coth\left(a+\frac{1}{4}\log\left(x\right)\right)^p\,dx$$

input
```
integrate(coth(a+1/4*log(x))^p,x, algorithm="fricas")
```

output
```
integral(coth(a + 1/4*log(x))^p, x)
```

Sympy [F]

$$\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx=\int \coth^p\left(a+\frac{\log\left(x\right)}{4}\right)\,dx$$

input
```
integrate(coth(a+1/4*ln(x))**p,x)
```

output
```
Integral(coth(a + log(x)/4)**p, x)
```

Maxima [F]

$$\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx=\int \coth\left(a+\frac{1}{4}\log\left(x\right)\right)^p\,dx$$

input
```
integrate(coth(a+1/4*log(x))^p,x, algorithm="maxima")
```

output
```
integrate(coth(a + 1/4*log(x))^p, x)
```

Giac [F]

$$\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx=\int \coth\left(a+\frac{1}{4}\,\log\left(x\right)\right)^p\,dx$$

input
```
integrate(coth(a+1/4*log(x))^p,x, algorithm="giac")
```

output
```
integrate(coth(a + 1/4*log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx=\int \coth\left(a+\frac{\ln\left(x\right)}{4}\right)^p\,dx$$

input
```
int(coth(a + log(x)/4)^p,x)
```

output
```
int(coth(a + log(x)/4)^p, x)
```

Reduce [F]

$$\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx=\coth\left(\frac{\log(x)}{4}+a\right)^p x-\frac{\left(\int \frac{\coth\left(\frac{\log(x)}{4}+a\right)^p}{\coth\left(\frac{\log(x)}{4}+a\right)}dx\right)p}{4}+\frac{\left(\int \coth\left(\frac{\log(x)}{4}+a\right)^p\coth\left(\frac{\log(x)}{4}+a\right)dx\right)p}{4}$$

input
```
int(coth(a+1/4*log(x))^p,x)
```

output
```
(4*coth((log(x) + 4*a)/4)**p*x - int(coth((log(x) + 4*a)/4)**p/coth((log(x
) + 4*a)/4),x)*p + int(coth((log(x) + 4*a)/4)**p*coth((log(x) + 4*a)/4),x)
*p)/4
```

3.177 $\int \coth^p\left(a+\frac{\log(x)}{6}\right)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 162

$$\int \coth^p\left(a+\frac{\log(x)}{6}\right)\,dx = e^{-6a}p\left(-1-e^{2a}\sqrt[3]{x}\right)^{1+p}\left(1-e^{2a}\sqrt[3]{x}\right)^{1-p}+e^{-4a}\left(-1-e^{2a}\sqrt[3]{x}\right)^{1+p}\left(1-e^{2a}\sqrt[3]{x}\right)^{1-p}\sqrt[3]{x} - \frac{2^{-p}e^{-6a}(1+2p^2)\left(-1-e^{2a}\sqrt[3]{x}\right)^{1+p}\text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{1}{2}\left(1+e^{2a}\sqrt[3]{x}\right)\right)}{1+p}$$

output
```
p*(-1-exp(2*a)*x^(1/3))^(p+1)*(1-exp(2*a)*x^(1/3))^(1-p)/exp(6*a)+(-1-exp(
2*a)*x^(1/3))^(p+1)*(1-exp(2*a)*x^(1/3))^(1-p)*x^(1/3)/exp(4*a)-(2*p^2+1)*
(-1-exp(2*a)*x^(1/3))^(p+1)*hypergeom([p, p+1],[2+p],1/2+1/2*exp(2*a)*x^(1
/3))/(2^p)/exp(6*a)/(p+1)
```

Mathematica [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.88

$$\int \coth^p\left(a+\frac{\log(x)}{6}\right)\,dx = \frac{e^{-6a}\left(1+e^{2a}\sqrt[3]{x}\right)^{1-p}\left(\frac{1+e^{2a}\sqrt[3]{x}}{-1+e^{2a}\sqrt[3]{x}}\right)^{-1+p}\left(\left(-1+p\right)\left(1+e^{2a}\sqrt[3]{x}\right)^{1+p}\left(p+e^{2a}\sqrt[3]{x}\right)-2^p(1+2p^2)\,\text{Hypergeor}\right.}{-1+p}$$

input
```
Integrate[Coth[a + Log[x]/6]^p,x]
```

output
```
((1 + E^(2*a)*x^(1/3))^(1 - p)*((1 + E^(2*a)*x^(1/3))/(-1 + E^(2*a)*x^(1/3
)))^(-1 + p)*((-1 + p)*(1 + E^(2*a)*x^(1/3))^(1 + p)*(p + E^(2*a)*x^(1/3))
 - 2^p*(1 + 2*p^2)*Hypergeometric2F1[1 - p, -p, 2 - p, 1/2 - (E^(2*a)*x^(1
/3))/2]))/(E^(6*a)*(-1 + p))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {6068, 900, 101, 90, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^p\left(a+\frac{\log(x)}{6}\right)dx$$

$$\Big\downarrow \text{6068}$$

$$\int \left(-e^{2a}\sqrt[3]{x}-1\right)^p\left(1-e^{2a}\sqrt[3]{x}\right)^{-p}dx$$

$$\Big\downarrow \text{900}$$

$$3\int \left(-e^{2a}\sqrt[3]{x}-1\right)^p\left(1-e^{2a}\sqrt[3]{x}\right)^{-p}x^{2/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{101}$$

$$3\left(\frac{1}{3}e^{-4a}\int \left(-e^{2a}\sqrt[3]{x}-1\right)^p\left(1-e^{2a}\sqrt[3]{x}\right)^{-p}\left(2e^{2a}\sqrt[3]{x}p+1\right)d\sqrt[3]{x}+\frac{1}{3}e^{-4a}\sqrt[3]{x}\left(-e^{2a}\sqrt[3]{x}-1\right)^{p+1}\left(1-e^{2a}\sqrt[3]{x}\right)^{1-p}\right)$$

$$\Big\downarrow \text{90}$$

$$3\left(\frac{1}{3}e^{-4a}\left(\left(2p^2+1\right)\int \left(-e^{2a}\sqrt[3]{x}-1\right)^p\left(1-e^{2a}\sqrt[3]{x}\right)^{-p}d\sqrt[3]{x}+e^{-2a}p\left(-e^{2a}\sqrt[3]{x}-1\right)^{p+1}\left(1-e^{2a}\sqrt[3]{x}\right)^{1-p}\right)+\frac{1}{3}e^{-4}\right.$$

$$\Big\downarrow \text{79}$$

$$3\left(\frac{1}{3}e^{-4a}\left(e^{-2a}p\left(-e^{2a}\sqrt[3]{x}-1\right)^{p+1}\left(1-e^{2a}\sqrt[3]{x}\right)^{1-p}-\frac{e^{-2a}2^{-p}\left(2p^2+1\right)\left(-e^{2a}\sqrt[3]{x}-1\right)^{p+1}\text{Hypergeometric2F1}\left(\right.}{p+1}\right.\right.$$

input
```
Int[Coth[a + Log[x]/6]^p,x]
```

output
```
3*(((-1 - E^(2*a)*x^(1/3))^(1 + p)*(1 - E^(2*a)*x^(1/3))^(1 - p)*x^(1/3))/
(3*E^(4*a)) + ((p*(-1 - E^(2*a)*x^(1/3))^(1 + p)*(1 - E^(2*a)*x^(1/3))^(1
- p))/E^(2*a) - ((1 + 2*p^2)*(-1 - E^(2*a)*x^(1/3))^(1 + p)*Hypergeometric
2F1[p, 1 + p, 2 + p, (1 + E^(2*a)*x^(1/3))/2])/(2^p*E^(2*a)*(1 + p)))/(3*E
^(4*a)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 6068
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Maple [F]

$$\int \coth\left(a+\frac{\ln\left(x\right)}{6}\right)^{p} dx$$

input
```
int(coth(a+1/6*ln(x))^p,x)
```

output
```
int(coth(a+1/6*ln(x))^p,x)
```

Fricas [F]

$$\int \coth^{p}\left(a+\frac{\log(x)}{6}\right) dx = \int \coth\left(a+\frac{1}{6}\log\left(x\right)\right)^{p} dx$$

input
```
integrate(coth(a+1/6*log(x))^p,x, algorithm="fricas")
```

output
```
integral(coth(a + 1/6*log(x))^p, x)
```

Sympy [F]

$$\int \coth^p\left(a+\frac{\log(x)}{6}\right)\,dx=\int \coth^p\left(a+\frac{\log\left(x\right)}{6}\right)\,dx$$

input
```
integrate(coth(a+1/6*ln(x))**p,x)
```

output
```
Integral(coth(a + log(x)/6)**p, x)
```

Maxima [F]

$$\int \coth^p\left(a+\frac{\log(x)}{6}\right)\,dx=\int \coth\left(a+\frac{1}{6}\,\log\left(x\right)\right)^p\,dx$$

input
```
integrate(coth(a+1/6*log(x))^p,x, algorithm="maxima")
```

output
```
integrate(coth(a + 1/6*log(x))^p, x)
```

Giac [F]

$$\int \coth^p\left(a+\frac{\log(x)}{6}\right)\,dx=\int \coth\left(a+\frac{1}{6}\,\log\left(x\right)\right)^p\,dx$$

input
```
integrate(coth(a+1/6*log(x))^p,x, algorithm="giac")
```

output
```
integrate(coth(a + 1/6*log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^p\left(a+\frac{\log(x)}{6}\right)\,dx=\int \coth\left(a+\frac{\ln(x)}{6}\right)^p\,dx$$

input
```
int(coth(a + log(x)/6)^p,x)
```

output
```
int(coth(a + log(x)/6)^p, x)
```

Reduce [F]

$$\int \coth^p\left(a+\frac{\log(x)}{6}\right)\,dx=\coth\left(\frac{\log(x)}{6}+a\right)^p x-\frac{\left(\int \frac{\coth\left(\frac{\log(x)}{6}+a\right)^p}{\coth\left(\frac{\log(x)}{6}+a\right)}dx\right)p}{6}+\frac{\left(\int \coth\left(\frac{\log(x)}{6}+a\right)^p\coth\left(\frac{\log(x)}{6}+a\right)dx\right)p}{6}$$

input
```
int(coth(a+1/6*log(x))^p,x)
```

output
```
(6*coth((log(x) + 6*a)/6)**p*x - int(coth((log(x) + 6*a)/6)**p/coth((log(x
) + 6*a)/6),x)*p + int(coth((log(x) + 6*a)/6)**p*coth((log(x) + 6*a)/6),x)
*p)/6
```

3.178 $\int \coth^p\left(a+\frac{\log(x)}{8}\right)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 225

$$\int \coth^p\left(a+\frac{\log(x)}{8}\right)\,dx=\frac{1}{3}e^{-8a}(3-2p+2p^2)\left(-1-e^{2a}\sqrt[4]{x}\right)^{1+p}\left(1-e^{2a}\sqrt[4]{x}\right)^{1-p}-\frac{2}{3}e^{-8a}p\left(-1-e^{2a}\sqrt[4]{x}\right)^{2+p}\left(1-e^{2a}\sqrt[4]{x}\right)^{1-p}+e^{-4a}\left(-1-e^{2a}\sqrt[4]{x}\right)^{1+p}\left(1-e^{2a}\sqrt[4]{x}\right)^{1-p}\sqrt{x}-\frac{2^{2-p}e^{-8a}p(2+p^2)\left(-1-e^{2a}\sqrt[4]{x}\right)^{1+p}\text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{1}{2}\left(1+e^{2a}\sqrt[4]{x}\right)\right)}{3(1+p)}$$

output

```
1/3*(2*p^2-2*p+3)*(-1-exp(2*a)*x^(1/4))^(p+1)*(1-exp(2*a)*x^(1/4))^(1-p)/e
xp(8*a)-2/3*p*(-1-exp(2*a)*x^(1/4))^(2+p)*(1-exp(2*a)*x^(1/4))^(1-p)/exp(8
*a)+(-1-exp(2*a)*x^(1/4))^(p+1)*(1-exp(2*a)*x^(1/4))^(1-p)*x^(1/2)/exp(4*a
)-1/3*2^(2-p)*p*(p^2+2)*(-1-exp(2*a)*x^(1/4))^(p+1)*hypergeom([p, p+1],[2+
p],1/2+1/2*exp(2*a)*x^(1/4))/exp(8*a)/(p+1)
```

Mathematica [A] (warning: unable to verify)

Time = 0.79 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.99

$$\int \coth^p\left(a+\frac{\log(x)}{8}\right)\,dx$$
$$=\frac{e^{-8a}\left(1+e^{2a}\sqrt[4]{x}\right)^{1-p}\left(\frac{1+e^{2a}\sqrt[4]{x}}{-1+e^{2a}\sqrt[4]{x}}\right)^{-1+p}\left(-2^{3+p}p\,\text{Hypergeometric2F1}\left(-2-p,1-p,2-p,\frac{1}{2}-\frac{1}{2}e^{2a}\sqrt[4]{x}\right)\right.}{}$$

input
```
Integrate[Coth[a + Log[x]/8]^p,x]
```

output
```
((1 + E^(2*a)*x^(1/4))^(1 - p)*((1 + E^(2*a)*x^(1/4))/(-1 + E^(2*a)*x^(1/4
)))^(-1 + p)*(-(2^(3 + p)*p*Hypergeometric2F1[-2 - p, 1 - p, 2 - p, 1/2 -
(E^(2*a)*x^(1/4))/2]) + 2^(2 + p)*(-1 + 2*p)*Hypergeometric2F1[-1 - p, 1 -
 p, 2 - p, 1/2 - (E^(2*a)*x^(1/4))/2] + (-1 + p)*(E^(4*a)*(1 + E^(2*a)*x^(
1/4))^(1 + p)*Sqrt[x] - 2^(1 + p)*Hypergeometric2F1[1 - p, -p, 2 - p, 1/2
- (E^(2*a)*x^(1/4))/2])))/(E^(8*a)*(-1 + p))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {6068, 900, 111, 27, 164, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^p\left(a+\frac{\log(x)}{8}\right)\,dx$$
$$\Big\downarrow \text{6068}$$
$$\int \left(-e^{2a}\sqrt[4]{x}-1\right)^p\left(1-e^{2a}\sqrt[4]{x}\right)^{-p}dx$$
$$\Big\downarrow \text{900}$$
$$4\int \left(-e^{2a}\sqrt[4]{x}-1\right)^p\left(1-e^{2a}\sqrt[4]{x}\right)^{-p}x^{3/4}d\sqrt[4]{x}$$

$$\Big\downarrow \text{111}$$

$$4\left(\frac{1}{4}e^{-4a}\int 2\left(-e^{2a}\sqrt[4]{x}-1\right)^p\left(1-e^{2a}\sqrt[4]{x}\right)^{-p}\left(e^{2a}\sqrt[4]{x}p+1\right)\sqrt[4]{x}d\sqrt[4]{x}+\frac{1}{4}e^{-4a}\sqrt{x}\left(-e^{2a}\sqrt[4]{x}-1\right)^{p+1}\left(1-e^{2a}\sqrt[4]{x}\right)^{1-}$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{1}{2}e^{-4a}\int\left(-e^{2a}\sqrt[4]{x}-1\right)^p\left(1-e^{2a}\sqrt[4]{x}\right)^{-p}\left(e^{2a}\sqrt[4]{x}p+1\right)\sqrt[4]{x}d\sqrt[4]{x}+\frac{1}{4}e^{-4a}\sqrt{x}\left(-e^{2a}\sqrt[4]{x}-1\right)^{p+1}\left(1-e^{2a}\sqrt[4]{x}\right)^{1-}$$

$$\Big\downarrow \text{164}$$

$$4\left(\frac{1}{2}e^{-4a}\left(\frac{2}{3}e^{-2a}p\left(p^2+2\right)\int\left(-e^{2a}\sqrt[4]{x}-1\right)^p\left(1-e^{2a}\sqrt[4]{x}\right)^{-p}d\sqrt[4]{x}+\frac{1}{6}e^{-8a}\left(-e^{2a}\sqrt[4]{x}-1\right)^{p+1}\left(e^{4a}\left(2p^2+3\right)+2e\right.\right.\right.$$

$$\Big\downarrow \text{79}$$

$$4\left(\frac{1}{2}e^{-4a}\left(\frac{1}{6}e^{-8a}\left(-e^{2a}\sqrt[4]{x}-1\right)^{p+1}\left(1-e^{2a}\sqrt[4]{x}\right)^{1-p}\left(e^{4a}\left(2p^2+3\right)+2e^{6a}p\sqrt[4]{x}\right)-\frac{e^{-4a}2^{1-p}p\left(p^2+2\right)\left(-e^{2a}\sqrt[4]{x}-\right.}{}\right.\right.$$

input
```
Int[Coth[a + Log[x]/8]^p,x]
```

output
```
4*(((-1 - E^(2*a)*x^(1/4))^(1 + p)*(1 - E^(2*a)*x^(1/4))^(1 - p)*Sqrt[x])/
(4*E^(4*a)) + (((-1 - E^(2*a)*x^(1/4))^(1 + p)*(1 - E^(2*a)*x^(1/4))^(1 -
p)*(E^(4*a)*(3 + 2*p^2) + 2*E^(6*a)*p*x^(1/4)))/(6*E^(8*a)) - (2^(1 - p)*p
*(2 + p^2)*(-1 - E^(2*a)*x^(1/4))^(1 + p)*Hypergeometric2F1[p, 1 + p, 2 +
p, (1 + E^(2*a)*x^(1/4))/2])/(3*E^(4*a)*(1 + p)))/(2*E^(4*a)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 6068
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Maple [F]

$$\int \coth\left(a+\frac{\ln\left(x\right)}{8}\right)^{p} dx$$

input
```
int(coth(a+1/8*ln(x))^p,x)
```

output
```
int(coth(a+1/8*ln(x))^p,x)
```

Fricas [F]

$$\int \coth^p\left(a+\frac{\log(x)}{8}\right)\,dx=\int \coth\left(a+\frac{1}{8}\log\left(x\right)\right)^{p}\,dx$$

input
```
integrate(coth(a+1/8*log(x))^p,x, algorithm="fricas")
```

output
```
integral(coth(a + 1/8*log(x))^p, x)
```

Sympy [F]

$$\int \coth^p\left(a+\frac{\log(x)}{8}\right)\,dx=\int \coth^{p}\left(a+\frac{\log\left(x\right)}{8}\right)\,dx$$

input
```
integrate(coth(a+1/8*ln(x))**p,x)
```

output
```
Integral(coth(a + log(x)/8)**p, x)
```

Maxima [F]

$$\int \coth^p\left(a+\frac{\log(x)}{8}\right)\,dx=\int \coth\left(a+\frac{1}{8}\log\left(x\right)\right)^p\,dx$$

input
```
integrate(coth(a+1/8*log(x))^p,x, algorithm="maxima")
```

output
```
integrate(coth(a + 1/8*log(x))^p, x)
```

Giac [F]

$$\int \coth^p\left(a+\frac{\log(x)}{8}\right)\,dx=\int \coth\left(a+\frac{1}{8}\log\left(x\right)\right)^p\,dx$$

input
```
integrate(coth(a+1/8*log(x))^p,x, algorithm="giac")
```

output
```
integrate(coth(a + 1/8*log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^p\left(a+\frac{\log(x)}{8}\right)\,dx=\int \coth\left(a+\frac{\ln\left(x\right)}{8}\right)^p\,dx$$

input
```
int(coth(a + log(x)/8)^p,x)
```

output
```
int(coth(a + log(x)/8)^p, x)
```

Reduce [F]

$$\int \coth^p\left(a+\frac{\log(x)}{8}\right)dx = \coth\left(\frac{\log(x)}{8}+a\right)^p x - \frac{\left(\int \frac{\coth\left(\frac{\log(x)}{8}+a\right)^p}{\coth\left(\frac{\log(x)}{8}+a\right)}dx\right)p}{8} + \frac{\left(\int \coth\left(\frac{\log(x)}{8}+a\right)^p \coth\left(\frac{\log(x)}{8}+a\right)dx\right)p}{8}$$

input
```
int(coth(a+1/8*log(x))^p,x)
```

output
```
(8*coth((log(x) + 8*a)/8)**p*x - int(coth((log(x) + 8*a)/8)**p/coth((log(x
) + 8*a)/8),x)*p + int(coth((log(x) + 8*a)/8)**p*coth((log(x) + 8*a)/8),x)
*p)/8
```

3.179 $\int \coth^p(a + \log(x))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 61

$$\int \coth^p(a+\log(x))\,dx = x\left(-1-e^{2a}x^2\right)^p\left(1+e^{2a}x^2\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},p,-p,\frac{3}{2},e^{2a}x^2,-e^{2a}x^2\right)$$

output
```
x*(-1-exp(2*a)*x^2)^p*AppellF1(1/2,p,-p,3/2,exp(2*a)*x^2,-exp(2*a)*x^2)/((
1+exp(2*a)*x^2)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 171 vs. $2(61) = 122$.

Time = 0.41 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.80

$$\int \coth^p(a+\log(x))\,dx$$

$$= \frac{3x\left(\frac{1+e^{2a}x^2}{-1+e^{2a}x^2}\right)^p\operatorname{AppellF1}\left(\frac{1}{2},p,-p,\frac{3}{2},e^{2a}x^2,-e^{2a}x^2\right)}{3\operatorname{AppellF1}\left(\frac{1}{2},p,-p,\frac{3}{2},e^{2a}x^2,-e^{2a}x^2\right)+2e^{2a}px^2\left(\operatorname{AppellF1}\left(\frac{3}{2},p,1-p,\frac{5}{2},e^{2a}x^2,-e^{2a}x^2\right)+\operatorname{AppellF1}\left(\right.\right.}$$

input
```
Integrate[Coth[a + Log[x]]^p,x]
```

output
```
(3*x*((1 + E^(2*a)*x^2)/(-1 + E^(2*a)*x^2))^p*AppellF1[1/2, p, -p, 3/2, E^
(2*a)*x^2, -(E^(2*a)*x^2)])/(3*AppellF1[1/2, p, -p, 3/2, E^(2*a)*x^2, -(E^
(2*a)*x^2)] + 2*E^(2*a)*p*x^2*(AppellF1[3/2, p, 1 - p, 5/2, E^(2*a)*x^2, -
(E^(2*a)*x^2)] + AppellF1[3/2, 1 + p, -p, 5/2, E^(2*a)*x^2, -(E^(2*a)*x^2)
]))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6068, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^p(a + \log(x))\,dx$$

$$\Big\downarrow \text{6068}$$

$$\int \left(-e^{2a}x^2 - 1\right)^p \left(1 - e^{2a}x^2\right)^{-p} dx$$

$$\Big\downarrow \text{334}$$

$$\left(-e^{2a}x^2 - 1\right)^p \left(e^{2a}x^2 + 1\right)^{-p} \int \left(1 - e^{2a}x^2\right)^{-p} \left(e^{2a}x^2 + 1\right)^p dx$$

$$\Big\downarrow \text{333}$$

$$x\left(-e^{2a}x^2 - 1\right)^p \left(e^{2a}x^2 + 1\right)^{-p} \text{AppellF1}\left(\frac{1}{2}, p, -p, \frac{3}{2}, e^{2a}x^2, -e^{2a}x^2\right)$$

input
```
Int[Coth[a + Log[x]]^p,x]
```

output
```
(x*(-1 - E^(2*a)*x^2)^p*AppellF1[1/2, p, -p, 3/2, E^(2*a)*x^2, -(E^(2*a)*x
^2)])/(1 + E^(2*a)*x^2)^p
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 6068
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Maple [F]

$$\int \coth\left(a+\ln\left(x\right)\right)^{p} dx$$

input
```
int(coth(a+ln(x))^p,x)
```

output
```
int(coth(a+ln(x))^p,x)
```

Fricas [F]

$$\int \coth^p(a+\log(x))\,dx=\int \coth\left(a+\log\left(x\right)\right)^{p}\,dx$$

input
```
integrate(coth(a+log(x))^p,x, algorithm="fricas")
```

output
```
integral(coth(a + log(x))^p, x)
```

Sympy [F]

$$\int \coth^p(a+\log(x))\,dx = \int \coth^p\left(a+\log\left(x\right)\right)dx$$

input
```
integrate(coth(a+ln(x))**p,x)
```

output
```
Integral(coth(a + log(x))**p, x)
```

Maxima [F]

$$\int \coth^p(a+\log(x))\,dx = \int \coth\left(a+\log\left(x\right)\right)^p\,dx$$

input
```
integrate(coth(a+log(x))^p,x, algorithm="maxima")
```

output
```
integrate(coth(a + log(x))^p, x)
```

Giac [F]

$$\int \coth^p(a+\log(x))\,dx = \int \coth\left(a+\log\left(x\right)\right)^p\,dx$$

input
```
integrate(coth(a+log(x))^p,x, algorithm="giac")
```

output
```
integrate(coth(a + log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^p(a+\log(x))\,dx = \int \coth(a+\ln(x))^p\,dx$$

input
```
int(coth(a + log(x))^p,x)
```

output
```
int(coth(a + log(x))^p, x)
```

Reduce [F]

$$\int \coth^p(a+\log(x))\,dx = \coth(\log(x)+a)^p\,x - \left(\int \frac{\coth(\log(x)+a)^p}{\coth(\log(x)+a)}dx\right)p + \left(\int \coth(\log(x)+a)^p\coth(\log(x)+a)\,dx\right)p$$

input
```
int(coth(a+log(x))^p,x)
```

output
```
coth(log(x) + a)**p*x - int(coth(log(x) + a)**p/coth(log(x) + a),x)*p + in
t(coth(log(x) + a)**p*coth(log(x) + a),x)*p
```

3.180 $\int \coth^p(a+2\log(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 61

$$\int \coth^p(a+2\log(x))\,dx = x\left(-1-e^{2a}x^4\right)^p\left(1+e^{2a}x^4\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{4},p,-p,\frac{5}{4},e^{2a}x^4,-e^{2a}x^4\right)$$

output
```
x*(-1-exp(2*a)*x^4)^p*AppellF1(1/4,p,-p,5/4,exp(2*a)*x^4,-exp(2*a)*x^4)/((
1+exp(2*a)*x^4)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 171 vs. 2(61) = 122.

Time = 0.42 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.80

$$\int \coth^p(a+2\log(x))\,dx = \frac{5x\left(\frac{1+e^{2a}x^4}{-1+e^{2a}x^4}\right)^p\operatorname{AppellF1}\left(\frac{1}{4},p,-p,\frac{5}{4},e^{2a}x^4,-e^{2a}x^4\right)}{5\operatorname{AppellF1}\left(\frac{1}{4},p,-p,\frac{5}{4},e^{2a}x^4,-e^{2a}x^4\right)+4e^{2a}px^4\left(\operatorname{AppellF1}\left(\frac{5}{4},p,1-p,\frac{9}{4},e^{2a}x^4,-e^{2a}x^4\right)+\operatorname{AppellF1}\left(\right.\right.}$$

input
```
Integrate[Coth[a + 2*Log[x]]^p,x]
```

output
```
(5*x*((1 + E^(2*a)*x^4)/(-1 + E^(2*a)*x^4))^p*AppellF1[1/4, p, -p, 5/4, E^
(2*a)*x^4, -(E^(2*a)*x^4)])/(5*AppellF1[1/4, p, -p, 5/4, E^(2*a)*x^4, -(E^
(2*a)*x^4)] + 4*E^(2*a)*p*x^4*(AppellF1[5/4, p, 1 - p, 9/4, E^(2*a)*x^4, -
(E^(2*a)*x^4)] + AppellF1[5/4, 1 + p, -p, 9/4, E^(2*a)*x^4, -(E^(2*a)*x^4)
]))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6068, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^p(a+2\log(x))\,dx$$

$$\Big\downarrow \text{6068}$$

$$\int \left(-e^{2a}x^4-1\right)^p \left(1-e^{2a}x^4\right)^{-p}dx$$

$$\Big\downarrow \text{937}$$

$$\left(-e^{2a}x^4-1\right)^p \left(e^{2a}x^4+1\right)^{-p}\int \left(1-e^{2a}x^4\right)^{-p} \left(e^{2a}x^4+1\right)^p dx$$

$$\Big\downarrow \text{936}$$

$$x\left(-e^{2a}x^4-1\right)^p \left(e^{2a}x^4+1\right)^{-p}\text{AppellF1}\left(\frac{1}{4},p,-p,\frac{5}{4},e^{2a}x^4,-e^{2a}x^4\right)$$

input
```
Int[Coth[a + 2*Log[x]]^p,x]
```

output
```
(x*(-1 - E^(2*a)*x^4)^p*AppellF1[1/4, p, -p, 5/4, E^(2*a)*x^4, -(E^(2*a)*x
^4)])/(1 + E^(2*a)*x^4)^p
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 6068
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Maple [F]

$$\int \coth\left(a+2\ln\left(x\right)\right)^{p}dx$$

input
```
int(coth(a+2*ln(x))^p,x)
```

output
```
int(coth(a+2*ln(x))^p,x)
```

Fricas [F]

$$\int \coth^p(a+2\log(x))\,dx = \int \coth\left(a+2\,\log\left(x\right)\right)^{p}\,dx$$

input
```
integrate(coth(a+2*log(x))^p,x, algorithm="fricas")
```

output
```
integral(coth(a + 2*log(x))^p, x)
```

Sympy [F]

$$\int \coth^p(a+2\log(x))\,dx = \int \coth^p\left(a+2\log\left(x\right)\right)dx$$

input
```
integrate(coth(a+2*ln(x))**p,x)
```

output
```
Integral(coth(a + 2*log(x))**p, x)
```

Maxima [F]

$$\int \coth^p(a+2\log(x))\,dx = \int \coth\left(a+2\,\log\left(x\right)\right)^p\,dx$$

input
```
integrate(coth(a+2*log(x))^p,x, algorithm="maxima")
```

output
```
integrate(coth(a + 2*log(x))^p, x)
```

Giac [F]

$$\int \coth^p(a+2\log(x))\,dx = \int \coth\left(a+2\,\log\left(x\right)\right)^p\,dx$$

input
```
integrate(coth(a+2*log(x))^p,x, algorithm="giac")
```

output
```
integrate(coth(a + 2*log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^p(a+2\log(x))\,dx=\int \coth(a+2\,\ln(x))^p\,dx$$

input
```
int(coth(a + 2*log(x))^p,x)
```

output
```
int(coth(a + 2*log(x))^p, x)
```

Reduce [F]

$$\int \coth^p(a+2\log(x))\,dx=\coth(2\log(x)+a)^p\,x-2\left(\int \frac{\coth(2\log(x)+a)^p}{\coth(2\log(x)+a)}dx\right)p+2\left(\int \coth(2\log(x)+a)^p\coth(2\log(x)+a)\,dx\right)p$$

input
```
int(coth(a+2*log(x))^p,x)
```

output
```
coth(2*log(x) + a)**p*x - 2*int(coth(2*log(x) + a)**p/coth(2*log(x) + a),x
)*p + 2*int(coth(2*log(x) + a)**p*coth(2*log(x) + a),x)*p
```

3.181 $\int \coth^p(a+3\log(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 61

$$\int \coth^p(a+3\log(x))\,dx = x\left(-1-e^{2a}x^6\right)^p\left(1+e^{2a}x^6\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{6},p,-p,\frac{7}{6},e^{2a}x^6,-e^{2a}x^6\right)$$

output
```
x*(-1-exp(2*a)*x^6)^p*AppellF1(1/6,p,-p,7/6,exp(2*a)*x^6,-exp(2*a)*x^6)/((
1+exp(2*a)*x^6)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 171 vs. $2(61)=122$.

Time = 0.43 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.80

$$\int \coth^p(a+3\log(x))\,dx$$

$$= \frac{7x\left(\frac{1+e^{2a}x^6}{-1+e^{2a}x^6}\right)^p\operatorname{AppellF1}\left(\frac{1}{6},p,-p,\frac{7}{6},e^{2a}x^6,-e^{2a}x^6\right)}{7\operatorname{AppellF1}\left(\frac{1}{6},p,-p,\frac{7}{6},e^{2a}x^6,-e^{2a}x^6\right)+6e^{2a}px^6\left(\operatorname{AppellF1}\left(\frac{7}{6},p,1-p,\frac{13}{6},e^{2a}x^6,-e^{2a}x^6\right)+\operatorname{AppellF1}\left(\right.\right.}$$

input
```
Integrate[Coth[a + 3*Log[x]]^p,x]
```

output
```
(7*x*((1 + E^(2*a)*x^6)/(-1 + E^(2*a)*x^6))^p*AppellF1[1/6, p, -p, 7/6, E^
(2*a)*x^6, -(E^(2*a)*x^6)])/(7*AppellF1[1/6, p, -p, 7/6, E^(2*a)*x^6, -(E^
(2*a)*x^6)] + 6*E^(2*a)*p*x^6*(AppellF1[7/6, p, 1 - p, 13/6, E^(2*a)*x^6,
-(E^(2*a)*x^6)] + AppellF1[7/6, 1 + p, -p, 13/6, E^(2*a)*x^6, -(E^(2*a)*x^
6)]))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6068, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^p(a+3\log(x))\,dx$$

$$\Big\downarrow \text{6068}$$

$$\int \left(-e^{2a}x^6-1\right)^p\left(1-e^{2a}x^6\right)^{-p}dx$$

$$\Big\downarrow \text{937}$$

$$\left(-e^{2a}x^6-1\right)^p\left(e^{2a}x^6+1\right)^{-p}\int \left(1-e^{2a}x^6\right)^{-p}\left(e^{2a}x^6+1\right)^p dx$$

$$\Big\downarrow \text{936}$$

$$x\left(-e^{2a}x^6-1\right)^p\left(e^{2a}x^6+1\right)^{-p}\text{AppellF1}\left(\frac{1}{6},p,-p,\frac{7}{6},e^{2a}x^6,-e^{2a}x^6\right)$$

input
```
Int[Coth[a + 3*Log[x]]^p,x]
```

output
```
(x*(-1 - E^(2*a)*x^6)^p*AppellF1[1/6, p, -p, 7/6, E^(2*a)*x^6, -(E^(2*a)*x
^6)])/(1 + E^(2*a)*x^6)^p
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 6068
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Maple [F]

$$\int \coth\left(a+3\ln\left(x\right)\right)^{p}dx$$

input
```
int(coth(a+3*ln(x))^p,x)
```

output
```
int(coth(a+3*ln(x))^p,x)
```

Fricas [F]

$$\int \coth^p(a+3\log(x))\,dx=\int \coth\left(a+3\,\log\left(x\right)\right)^{p}\,dx$$

input
```
integrate(coth(a+3*log(x))^p,x, algorithm="fricas")
```

output
```
integral(coth(a + 3*log(x))^p, x)
```

Sympy [F]

$$\int \coth^p(a+3\log(x))\,dx = \int \coth^p\left(a+3\log\left(x\right)\right)dx$$

input
```
integrate(coth(a+3*ln(x))**p,x)
```

output
```
Integral(coth(a + 3*log(x))**p, x)
```

Maxima [F]

$$\int \coth^p(a+3\log(x))\,dx = \int \coth\left(a+3\,\log\left(x\right)\right)^p\,dx$$

input
```
integrate(coth(a+3*log(x))^p,x, algorithm="maxima")
```

output
```
integrate(coth(a + 3*log(x))^p, x)
```

Giac [F]

$$\int \coth^p(a+3\log(x))\,dx = \int \coth\left(a+3\,\log\left(x\right)\right)^p\,dx$$

input
```
integrate(coth(a+3*log(x))^p,x, algorithm="giac")
```

output
```
integrate(coth(a + 3*log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^p(a+3\log(x))\,dx = \int \coth(a+3\,\ln(x))^p\,dx$$

input
```
int(coth(a + 3*log(x))^p,x)
```

output
```
int(coth(a + 3*log(x))^p, x)
```

Reduce [F]

$$\int \coth^p(a+3\log(x))\,dx = \coth(3\log(x)+a)^p\,x - 3\left(\int \frac{\coth(3\log(x)+a)^p}{\coth(3\log(x)+a)}dx\right)p + 3\left(\int \coth(3\log(x)+a)^p\coth(3\log(x)+a)\,dx\right)p$$

input
```
int(coth(a+3*log(x))^p,x)
```

output
```
coth(3*log(x) + a)**p*x - 3*int(coth(3*log(x) + a)**p/coth(3*log(x) + a),x
)*p + 3*int(coth(3*log(x) + a)**p*coth(3*log(x) + a),x)*p
```

3.182 $\int x^3 \coth(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int x^3 \coth(d(a+b\log(cx^n)))\,dx=\frac{x^4}{4}-\frac{1}{2}x^4\,\text{Hypergeometric2F1}\left(1,\frac{2}{bdn},1+\frac{2}{bdn},e^{2ad}(cx^n)^{2bd}\right)$$

output
```
1/4*x^4-1/2*x^4*hypergeom([1, 2/b/d/n],[1+2/b/d/n],exp(2*a*d)*(c*x^n)^(2*b
*d))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 198 vs. 2(58) = 116.

Time = 4.76 (sec) , antiderivative size = 198, normalized size of antiderivative = 3.41

$$\int x^3 \coth(d(a+b\log(cx^n)))\,dx=$$
$$-\frac{x^4\left(2e^{2d(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1+\frac{2}{bdn},2+\frac{2}{bdn},e^{2d(a+b\log(cx^n))}\right)+(2+bdn)\left(\coth(d(a+b\right.\right.}{}$$

input
```
Integrate[x^3*Coth[d*(a + b*Log[c*x^n])],x]
```

output
```
-((x^4*(2*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 2/(b*d*n), 2
 + 2/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (2 + b*d*n)*(Coth[d*(a + b*Log
[c*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] + Hypergeometric2F1[1,
 2/(b*d*n), 1 + 2/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a + b*Log
[c*x^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])))/(8
 + 4*b*d*n))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.62, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {6074, 6072, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth (d(a+b\log (cx^n))) \, dx$$

$$\Big\downarrow \text{ 6074}$$

$$\frac{x^4(cx^n)^{-4/n}\int (cx^n)^{\frac{4}{n}-1}\coth (d(a+b\log (cx^n)))d(cx^n)}{n}$$

$$\Big\downarrow \text{ 6072}$$

$$\frac{x^4(cx^n)^{-4/n}\int \frac{(cx^n)^{\frac{4}{n}-1}\left(-e^{2ad}(cx^n)^{2bd}-1\right)}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{n}$$

$$\Big\downarrow \text{ 959}$$

$$\frac{x^4(cx^n)^{-4/n}\left(\frac{1}{4}n(cx^n)^{4/n}-2\int \frac{(cx^n)^{\frac{4}{n}-1}}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)\right)}{n}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^4(cx^n)^{-4/n}\left(\frac{1}{4}n(cx^n)^{4/n}-\frac{1}{2}n(cx^n)^{4/n}\text{Hypergeometric2F1}\left(1,\frac{2}{bdn},1+\frac{2}{bdn},e^{2ad}(cx^n)^{2bd}\right)\right)}{n}$$

input
```
Int[x^3*Coth[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^4*((n*(c*x^n)^(4/n))/4 - (n*(c*x^n)^(4/n)*Hypergeometric2F1[1, 2/(b*d*n
), 1 + 2/(b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/2))/(n*(c*x^n)^(4/n))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 6074
```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^3 \coth \left(d(a + b \ln \left(c\, x^n\right))\right) dx$$

input
```
int(x^3*coth(d*(a+b*ln(c*x^n))),x)
```

output
```
int(x^3*coth(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int x^3 \coth(d(a + b\log(cx^n)))\,dx = \int x^3 \coth((b\log(cx^n) + a)d)\,dx$$

input
```
integrate(x^3*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral(x^3*coth(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int x^3 \coth(d(a + b\log(cx^n)))\,dx = \int x^3 \coth(ad + bd\log(cx^n))\,dx$$

input
```
integrate(x**3*coth(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(x**3*coth(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x^3 \coth(d(a + b\log(cx^n)))\,dx = \int x^3 \coth((b\log(cx^n) + a)d)\,dx$$

input
```
integrate(x^3*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
1/4*x^4 - integrate(x^3/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integra
te(x^3/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)
```

Giac [F]

$$\int x^3 \coth (d(a + b \log (cx^n)))\,dx = \int x^3 \coth ((b \log (cx^n) + a)d)\,dx$$

input
```
integrate(x^3*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(x^3*coth((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \coth (d(a + b \log (cx^n)))\,dx = \int x^3 \coth(d\,(a + b\,\ln (c\,x^n)))\,dx$$

input
```
int(x^3*coth(d*(a + b*log(c*x^n))),x)
```

output
```
int(x^3*coth(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int x^3 \coth (d(a + b \log (cx^n)))\,dx = 2e^{2ad}c^{2bd}\left(\int \frac{x^{2bdn}x^3}{x^{2bdn}e^{2ad}c^{2bd} - 1}dx\right) - \frac{x^4}{4}$$

input
```
int(x^3*coth(d*(a+b*log(c*x^n))),x)
```

output
```
(8*e**(2*a*d)*c**(2*b*d)*int((x**(2*b*d*n)*x**3)/(x**(2*b*d*n)*e**(2*a*d)*
c**(2*b*d) - 1),x) - x**4)/4
```

3.183 $\int x^2 \coth (d(a+b\log (cx^n)))\, dx$

Optimal result

Integrand size = 17, antiderivative size = 62

$$\int x^2 \coth (d(a+b\log (cx^n)))\, dx=\frac{x^3}{3}-\frac{2}{3}x^3 \text{Hypergeometric2F1}\left(1,\frac{3}{2bdn},1+\frac{3}{2bdn},e^{2ad}(cx^n)^{2bd}\right)$$

output
```
1/3*x^3-2/3*x^3*hypergeom([1, 3/2/b/d/n],[1+3/2/b/d/n],exp(2*a*d)*(c*x^n)^
(2*b*d))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 207 vs. 2(62) = 124.

Time = 3.58 (sec) , antiderivative size = 207, normalized size of antiderivative = 3.34

$$\int x^2 \coth (d(a+b\log (cx^n)))\, dx = -\frac{x^3\left(3e^{2d(a+b\log(cx^n))}\text{Hypergeometric2F1}\left(1,1+\frac{3}{2bdn},2+\frac{3}{2bdn},e^{2d(a+b\log(cx^n))}\right)+(3+2bdn)\left(\coth (d(a\right.\right.}{}$$

input
```
Integrate[x^2*Coth[d*(a + b*Log[c*x^n])],x]
```

output
```
-((x^3*(3*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 3/(2*b*d*n),
 2 + 3/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (3 + 2*b*d*n)*(Coth[d*(a +
 b*Log[c*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] + Hypergeometric
2F1[1, 3/(2*b*d*n), 1 + 3/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*
(a + b*Log[c*x^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log
[x]])))/(9 + 6*b*d*n))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.58, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {6074, 6072, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth (d(a+b\log (cx^n)))\,dx$$

$$\Big\downarrow \text{6074}$$

$$\frac{x^3(cx^n)^{-3/n}\int (cx^n)^{\frac{3}{n}-1}\coth (d(a+b\log (cx^n)))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{6072}$$

$$\frac{x^3(cx^n)^{-3/n}\int \frac{(cx^n)^{\frac{3}{n}-1}\left(-e^{2ad}(cx^n)^{2bd}-1\right)}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{n}$$

$$\Big\downarrow \text{959}$$

$$\frac{x^3(cx^n)^{-3/n}\left(\frac{1}{3}n(cx^n)^{3/n}-2\int \frac{(cx^n)^{\frac{3}{n}-1}}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)\right)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^3(cx^n)^{-3/n}\left(\frac{1}{3}n(cx^n)^{3/n}-\frac{2}{3}n(cx^n)^{3/n}\text{Hypergeometric2F1}\left(1,\frac{3}{2bdn},1+\frac{3}{2bdn},e^{2ad}(cx^n)^{2bd}\right)\right)}{n}$$

input
```
Int[x^2*Coth[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^3*((n*(c*x^n)^(3/n))/3 - (2*n*(c*x^n)^(3/n)*Hypergeometric2F1[1, 3/(2*b
*d*n), 1 + 3/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/3))/(n*(c*x^n)^(3/n))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 6074
```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^2 \coth \left(d(a+b\ln \left(c\,x^n\right))\right) dx$$

input
```
int(x^2*coth(d*(a+b*ln(c*x^n))),x)
```

output
```
int(x^2*coth(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int x^2 \coth (d(a + b \log (cx^n))) \, dx = \int x^2 \coth ((b \log (cx^n) + a)d) \, dx$$

input
```
integrate(x^2*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral(x^2*coth(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int x^2 \coth (d(a + b \log (cx^n))) \, dx = \int x^2 \coth (ad + bd \log (cx^n)) \, dx$$

input
```
integrate(x**2*coth(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(x**2*coth(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x^2 \coth (d(a + b \log (cx^n))) \, dx = \int x^2 \coth ((b \log (cx^n) + a)d) \, dx$$

input
```
integrate(x^2*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
1/3*x^3 - integrate(x^2/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integra
te(x^2/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)
```

Giac [F]

$$\int x^2 \coth \left(d(a+b\log \left(cx^n\right))\right) \, dx = \int x^2 \coth \left((b\log \left(cx^n\right)+a)d\right) \, dx$$

input
```
integrate(x^2*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(x^2*coth((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth \left(d(a+b\log \left(cx^n\right))\right) \, dx = \int x^2 \coth\left(d\left(a+b\,\ln\left(c\,x^n\right)\right)\right) \, dx$$

input
```
int(x^2*coth(d*(a + b*log(c*x^n))),x)
```

output
```
int(x^2*coth(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int x^2 \coth \left(d(a+b\log \left(cx^n\right))\right) \, dx = 2e^{2ad}c^{2bd}\left(\int \frac{x^{2bdn}x^2}{x^{2bdn}e^{2ad}c^{2bd}-1}dx\right)-\frac{x^3}{3}$$

input
```
int(x^2*coth(d*(a+b*log(c*x^n))),x)
```

output
```
(6*e**(2*a*d)*c**(2*b*d)*int((x**(2*b*d*n)*x**2)/(x**(2*b*d*n)*e**(2*a*d)*
c**(2*b*d) - 1),x) - x**3)/3
```

3.184 $\int x \coth(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int x \coth(d(a+b\log(cx^n)))\,dx = \frac{x^2}{2} - x^2 \text{Hypergeometric2F1}\left(1, \frac{1}{bdn}, 1 + \frac{1}{bdn}, e^{2ad}(cx^n)^{2bd}\right)$$

output
```
1/2*x^2-x^2*hypergeom([1, 1/b/d/n],[1+1/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 193 vs. 2(54) = 108.

Time = 4.64 (sec) , antiderivative size = 193, normalized size of antiderivative = 3.57

$$\int x \coth(d(a+b\log(cx^n)))\,dx = -\frac{x^2\left(e^{2d(a+b\log(cx^n))}\text{Hypergeometric2F1}\left(1, 1+\frac{1}{bdn}, 2+\frac{1}{bdn}, e^{2d(a+b\log(cx^n))}\right) + (1+bdn)\left(\coth(d(a+b\log\right.\right.}{}$$

input
```
Integrate[x*Coth[d*(a + b*Log[c*x^n])],x]
```

output
```
-((x^2*(E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 1/(b*d*n), 2 +
 1/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (1 + b*d*n)*(Coth[d*(a + b*Log[c
*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] + Hypergeometric2F1[1, 1
/(b*d*n), 1 + 1/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a + b*Log[c
*x^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])))/(2 +
 2*b*d*n))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6074, 6072, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth(d(a + b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{6074}$$

$$\frac{x^2(cx^n)^{-2/n}\int (cx^n)^{\frac{2}{n}-1}\coth(d(a+b\log(cx^n)))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{6072}$$

$$\frac{x^2(cx^n)^{-2/n}\int \frac{(cx^n)^{\frac{2}{n}-1}\left(-e^{2ad}(cx^n)^{2bd}-1\right)}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{n}$$

$$\Big\downarrow \text{959}$$

$$\frac{x^2(cx^n)^{-2/n}\left(\frac{1}{2}n(cx^n)^{2/n}-2\int \frac{(cx^n)^{\frac{2}{n}-1}}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)\right)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2(cx^n)^{-2/n}\left(\frac{1}{2}n(cx^n)^{2/n}-n(cx^n)^{2/n}\text{Hypergeometric2F1}\left(1,\frac{1}{bdn},1+\frac{1}{bdn},e^{2ad}(cx^n)^{2bd}\right)\right)}{n}$$

input
```
Int[x*Coth[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^2*((n*(c*x^n)^(2/n))/2 - n*(c*x^n)^(2/n)*Hypergeometric2F1[1, 1/(b*d*n)
, 1 + 1/(b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)]))/(n*(c*x^n)^(2/n))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 6074
```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x \coth \left(d(a+b\ln \left(c\,x^{n}\right))\right) dx$$

input
```
int(x*coth(d*(a+b*ln(c*x^n))),x)
```

output
```
int(x*coth(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int x \coth (d(a + b \log (cx^n))) \, dx = \int x \coth ((b \log (cx^n) + a)d) \, dx$$

input
```
integrate(x*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral(x*coth(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int x \coth (d(a + b \log (cx^n))) \, dx = \int x \coth (ad + bd \log (cx^n)) \, dx$$

input
```
integrate(x*coth(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(x*coth(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x \coth (d(a + b \log (cx^n))) \, dx = \int x \coth ((b \log (cx^n) + a)d) \, dx$$

input
```
integrate(x*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
1/2*x^2 - integrate(x/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integrate
(x/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)
```

Giac [F]

$$\int x\coth\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\coth\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

input
```
integrate(x*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(x*coth((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\coth\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\coth\left(d\left(a+b\,\ln\left(c\,x^n\right)\right)\right)\,dx$$

input
```
int(x*coth(d*(a + b*log(c*x^n))),x)
```

output
```
int(x*coth(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int x\coth\left(d(a+b\log\left(cx^n\right))\right)\,dx=2e^{2ad}c^{2bd}\left(\int\frac{x^{2bdn}x}{x^{2bdn}e^{2ad}c^{2bd}-1}dx\right)-\frac{x^2}{2}$$

input
```
int(x*coth(d*(a+b*log(c*x^n))),x)
```

output
```
(4*e**(2*a*d)*c**(2*b*d)*int((x**(2*b*d*n)*x)/(x**(2*b*d*n)*e**(2*a*d)*c**
(2*b*d) - 1),x) - x**2)/2
```

3.185 $\int \coth(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \coth(d(a+b\log(cx^n)))\,dx = x - 2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2bdn},1+\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd}\right)$$

output
```
x-2*x*hypergeom([1, 1/2/b/d/n],[1+1/2/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 198 vs. $2(52) = 104$.

Time = 6.30 (sec) , antiderivative size = 198, normalized size of antiderivative = 3.81

$$\int \coth\left(d(a+b\log\left(cx^n\right))\right)\,dx = -\frac{e^{2d(a+b\log(cx^n))}x\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{2bdn},2+\frac{1}{2bdn},e^{2d(a+b\log(cx^n))}\right)}{1+2bdn} - x\left(\coth\left(d(a+b\log\left(cx^n\right))\right)-\coth\left(d(a-bn\log(x)+b\log\left(cx^n\right))\right) + \text{Hypergeometric2F1}\left(1,\frac{1}{2bdn},1+\frac{1}{2bdn},e^{2d(a+b\log(cx^n))}\right) + \text{csch}(d(a+b\log\left(cx^n\right)))\,\text{csch}(d(a-bn\log(x)+b\log\left(cx^n\right)))\sinh(bdn\log(x))\right)$$

input
```
Integrate[Coth[d*(a + b*Log[c*x^n])],x]
```

output
```
-((E^(2*d*(a + b*Log[c*x^n]))*x*Hypergeometric2F1[1, 1 + 1/(2*b*d*n), 2 +
1/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))])/(1 + 2*b*d*n)) - x*(Coth[d*(a +
b*Log[c*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] + Hypergeometric2
F1[1, 1/(2*b*d*n), 1 + 1/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(
a + b*Log[c*x^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[
x]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6070, 6072, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

$$\Big\downarrow \text{6070}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\coth\left(d(a+b\log(cx^n))\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{6072}$$

$$\frac{x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}-1}\left(-e^{2ad}(cx^n)^{2bd}-1\right)}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{n}$$

$$\Big\downarrow \text{959}$$

$$\frac{x(cx^n)^{-1/n}\left(n(cx^n)^{\frac{1}{n}}-2\int \frac{(cx^n)^{\frac{1}{n}-1}}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)\right)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x(cx^n)^{-1/n}\left(n(cx^n)^{\frac{1}{n}}-2n(cx^n)^{\frac{1}{n}}\text{Hypergeometric2F1}\left(1,\frac{1}{2bdn},1+\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd}\right)\right)}{n}$$

input
```
Int[Coth[d*(a + b*Log[c*x^n])],x]
```

output
```
(x*(n*(c*x^n)^n^(-1) - 2*n*(c*x^n)^n^(-1)*Hypergeometric2F1[1, 1/(2*b*d*n)
, 1 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)]))/(n*(c*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6070
```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Coth[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [F]

$$\int \coth \left(d(a+b\ln \left(c\,x^{n}\right))\right)dx$$

input
```
int(coth(d*(a+b*ln(c*x^n))),x)
```

output
```
int(coth(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int \coth \left(d(a+b\log \left(cx^n\right))\right)\,dx = \int \coth \left((b\log \left(cx^n\right)+a)d\right)\,dx$$

input
```
integrate(coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral(coth(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int \coth \left(d(a + b\log \left(cx^n\right))\right) \, dx = \int \coth \left(d(a + b\log \left(cx^n\right))\right) dx$$

input
```
integrate(coth(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(coth(d*(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \coth \left(d(a + b\log \left(cx^n\right))\right) \, dx = \int \coth \left((b\log \left(cx^n\right) + a)d\right) \, dx$$

input
```
integrate(coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
x - integrate(1/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integrate(1/(c^
(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)
```

Giac [F]

$$\int \coth \left(d(a + b\log \left(cx^n\right))\right) \, dx = \int \coth \left((b\log \left(cx^n\right) + a)d\right) \, dx$$

input
```
integrate(coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(coth((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int \coth(d\left(a+b\,\ln\left(c\,x^n\right)\right))\,dx$$

input
```
int(coth(d*(a + b*log(c*x^n))),x)
```

output
```
int(coth(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int \coth\left(d(a+b\log\left(cx^n\right))\right)\,dx = 2e^{2ad}c^{2bd}\left(\int \frac{x^{2bdn}}{x^{2bdn}e^{2ad}c^{2bd}-1}dx\right)-x$$

input
```
int(coth(d*(a+b*log(c*x^n))),x)
```

output
```
2*e**(2*a*d)*c**(2*b*d)*int(x**(2*b*d*n)/(x**(2*b*d*n)*e**(2*a*d)*c**(2*b*
d) - 1),x) - x
```

3.186 $\int \frac{\coth(d(a+b\log(cx^n)))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 25

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x}\,dx = \frac{\log(\sinh(ad+bd\log(cx^n)))}{bdn}$$

output
```
ln(sinh(a*d+b*d*ln(c*x^n)))/b/d/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x}\,dx = \frac{\log(\sinh(ad+bd\log(cx^n)))}{bdn}$$

input
```
Integrate[Coth[d*(a + b*Log[c*x^n])]/x,x]
```

output
```
Log[Sinh[a*d + b*d*Log[c*x^n]]]/(b*d*n)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3039, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth (d(a+b\log (cx^n)))}{x}dx$$

$$\Big\downarrow \text{ 3039}$$

$$\frac{\int \coth (d(a+b\log (cx^n)))d\log (cx^n)}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int -i\tan \left(iad+ib\log (cx^n)d+\frac{\pi}{2}\right)d\log (cx^n)}{n}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{i\int \tan \left(\frac{1}{2}(2iad+\pi)+ibd\log (cx^n)\right)d\log (cx^n)}{n}$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{\log (-i\sinh (ad+bd\log (cx^n)))}{bdn}$$

input
```
Int[Coth[d*(a + b*Log[c*x^n])]/x,x]
```

output
```
Log[(-I)*Sinh[a*d + b*d*Log[c*x^n]]]/(b*d*n)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\ln(\sinh(d(a+b\ln(c\,x^n))))}{nbd}$
default	$\frac{\ln(\sinh(d(a+b\ln(c\,x^n))))}{nbd}$
parallelrisch	$\frac{-bd\ln(c\,x^n)+\ln(\tanh(d(a+b\ln(c\,x^n))))-\ln(1-\tanh(d(a+b\ln(c\,x^n))))}{ndb}$
risch	$\ln(x)-\frac{2a}{nb}-\frac{2\ln(c)}{n}-\frac{2\ln(x^n)}{n}-\frac{i\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}{n}+\frac{i\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)}{n}+\frac{i\pi\ \mathrm{csg}$

input
```
int(coth(d*(a+b*ln(c*x^n)))/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b/d*ln(sinh(d*(a+b*ln(c*x^n))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(25) = 50$.

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.04

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x}\,dx = -\frac{bdn\log(x) - \log\left(\frac{2\sinh(bdn\log(x)+bd\log(c)+ad)}{\cosh(bdn\log(x)+bd\log(c)+ad)-\sinh(bdn\log(x)+bd\log(c)+ad)}\right)}{bdn}$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

output
```
-(b*d*n*log(x) - log(2*sinh(b*d*n*log(x) + b*d*log(c) + a*d)/(cosh(b*d*n*l
og(x) + b*d*log(c) + a*d) - sinh(b*d*n*log(x) + b*d*log(c) + a*d))))/(b*d*
n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(20) = 40$.

Time = 4.54 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x}\,dx = \begin{cases} \log(x)\coth(ad) & \text{for } b=0 \\ \tilde{\infty}\log(x) & \text{for } d=0 \\ \log(x)\coth(ad+bd\log(c)) & \text{for } n=0 \\ \frac{\log(\sinh(ad+bd\log(cx^n)))}{bdn} & \text{otherwise} \end{cases}$$

input
```
integrate(coth(d*(a+b*ln(c*x**n)))/x,x)
```

output
```
Piecewise((log(x)*coth(a*d), Eq(b, 0)), (zoo*log(x), Eq(d, 0)), (log(x)*co
th(a*d + b*d*log(c)), Eq(n, 0)), (log(sinh(a*d + b*d*log(c*x**n)))/(b*d*n)
, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\coth\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = \frac{\log\left(\sinh\left((b\log\left(cx^n\right)+a)d\right)\right)}{bdn}$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

output
```
log(sinh((b*log(c*x^n) + a)*d))/(b*d*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(25) = 50.

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.52

$$\int \frac{\coth\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = -\frac{\log\left(x^{bdn}\right)}{bdn} + \frac{\log\left(\sqrt{-2\,x^{2\,bdn}|c|^{2\,bd}\cos\left(\pi bd\mathrm{sgn}\left(c\right)-\pi bd\right)e^{(2\,ad)}+x^{4\,bdn}|c|^{4\,bd}e^{(4\,ad)}+1}\right)}{bdn}$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

output
```
-log(x^(b*d*n))/(b*d*n) + log(sqrt(-2*x^(2*b*d*n)*abs(c)^(2*b*d)*cos(pi*b*
d*sgn(c) - pi*b*d)*e^(2*a*d) + x^(4*b*d*n)*abs(c)^(4*b*d)*e^(4*a*d) + 1))/
(b*d*n)
```

Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{\coth (d(a+b\log (cx^n)))}{x}\,dx=\frac{\ln \left(\mathrm{e}^{2\,a\,d}\,{\left(c\,x^n\right)}^{2\,b\,d}-1\right)}{b\,d\,n}-\ln (x)$$

input
```
int(coth(d*(a + b*log(c*x^n)))/x,x)
```

output
```
log(exp(2*a*d)*(c*x^n)^(2*b*d) - 1)/(b*d*n) - log(x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.80

$$\int \frac{\coth (d(a+b\log (cx^n)))}{x}\,dx=\frac{\log\left(x^{bdn}e^{ad}c^{2bd}+c^{bd}\right)+\log\left(x^{bdn}e^{ad}c^{2bd}-c^{bd}\right)-\log(x)\,bdn}{bdn}$$

input
```
int(coth(d*(a+b*log(c*x^n)))/x,x)
```

output
```
(log(x**(b*d*n)*e**(a*d)*c**(2*b*d) + c**(b*d)) + log(x**(b*d*n)*e**(a*d)*
c**(2*b*d) - c**(b*d)) - log(x)*b*d*n)/(b*d*n)
```

3.187 $\int \frac{\coth(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^2}\,dx = -\frac{1}{x}+\frac{2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2bdn},1-\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd}\right)}{x}$$

output
```
-1/x+2*hypergeom([1, -1/2/b/d/n],[1-1/2/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))
/x
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 197 vs. 2(58) = 116.

Time = 2.56 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.40

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^2}\,dx = \frac{\coth(d(a+b\log(cx^n)))-\coth(d(a-bn\log(x)+b\log(cx^n)))-\frac{e^{2d(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1-\frac{1}{2bdn},2-\right.}{-1+2bdn}}{}$$

input
```
Integrate[Coth[d*(a + b*Log[c*x^n])]/x^2,x]
```

output
```
(Coth[d*(a + b*Log[c*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] - (E
^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(2*b*d*n), 2 - 1/(2*b
*d*n), E^(2*d*(a + b*Log[c*x^n]))])/(-1 + 2*b*d*n) + Hypergeometric2F1[1,
-1/2*1/(b*d*n), 1 - 1/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a +
 b*Log[c*x^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]]
)/x
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.59, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6074, 6072, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth (d(a+b\log (cx^n)))}{x^2} \, dx$$

$$\Big\downarrow \text{6074}$$

$$\frac{(cx^n)^{\frac{1}{n}} \int (cx^n)^{-1-\frac{1}{n}} \coth (d(a+b\log (cx^n)))d(cx^n)}{nx}$$

$$\Big\downarrow \text{6072}$$

$$\frac{(cx^n)^{\frac{1}{n}} \int \frac{(cx^n)^{-1-\frac{1}{n}}\left(-e^{2ad}(cx^n)^{2bd}-1\right)}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{nx}$$

$$\Big\downarrow \text{959}$$

$$\frac{(cx^n)^{\frac{1}{n}} \left(-2\int \frac{(cx^n)^{-1-\frac{1}{n}}}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)-n(cx^n)^{-1/n}\right)}{nx}$$

$$\Big\downarrow \text{888}$$

$$\frac{(cx^n)^{\frac{1}{n}} \left(2n(cx^n)^{-1/n} \text{Hypergeometric2F1}\left(1,-\frac{1}{2bdn},1-\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd}\right)-n(cx^n)^{-1/n}\right)}{nx}$$

input
```
Int[Coth[d*(a + b*Log[c*x^n])]/x^2,x]
```

output
```
((c*x^n)^n^(-1)*(-(n/(c*x^n)^n^(-1)) + (2*n*Hypergeometric2F1[1, -1/2*1/(b
*d*n), 1 - 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(c*x^n)^n^(-1)))/(n*x)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 6074
```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\coth \left(d(a+b\ln \left(c\,x^{n}\right))\right)}{x^{2}}dx$$

input
```
int(coth(d*(a+b*ln(c*x^n)))/x^2,x)
```

output
```
int(coth(d*(a+b*ln(c*x^n)))/x^2,x)
```

Fricas [F]

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\coth((b\log(cx^n)+a)d)}{x^2}\,dx$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

output
```
integral(coth(b*d*log(c*x^n) + a*d)/x^2, x)
```

Sympy [F]

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\coth(ad+bd\log(cx^n))}{x^2}\,dx$$

input
```
integrate(coth(d*(a+b*ln(c*x**n)))/x**2,x)
```

output
```
Integral(coth(a*d + b*d*log(c*x**n))/x**2, x)
```

Maxima [F]

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\coth((b\log(cx^n)+a)d)}{x^2}\,dx$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

output
```
-1/x - integrate(1/(c^(b*d)*x^2*e^(b*d*log(x^n) + a*d) + x^2), x) + integr
ate(1/(c^(b*d)*x^2*e^(b*d*log(x^n) + a*d) - x^2), x)
```

Giac [F]

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\coth((b\log(cx^n)+a)d)}{x^2}\,dx$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

output
```
integrate(coth((b*log(c*x^n) + a)*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\coth(d\,(a+b\,\ln(c\,x^n)))}{x^2}\,dx$$

input
```
int(coth(d*(a + b*log(c*x^n)))/x^2,x)
```

output
```
int(coth(d*(a + b*log(c*x^n)))/x^2, x)
```

Reduce [F]

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^2}\,dx = \frac{2e^{2ad}c^{2bd}\left(\int \frac{x^{2bdn}}{x^{2bdn}e^{2ad}c^{2bd}x^2-x^2}dx\right)x+1}{x}$$

input
```
int(coth(d*(a+b*log(c*x^n)))/x^2,x)
```

output
```
(2*e**(2*a*d)*c**(2*b*d)*int(x**(2*b*d*n)/(x**(2*b*d*n)*e**(2*a*d)*c**(2*b
*d)*x**2 - x**2),x)*x + 1)/x
```

3.188 $\int \frac{\coth(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^3}\,dx = -\frac{1}{2x^2} + \frac{\text{Hypergeometric2F1}\left(1,-\frac{1}{bdn},1-\frac{1}{bdn},e^{2ad}(cx^n)^{2bd}\right)}{x^2}$$

output
```
-1/2/x^2+hypergeom([1, -1/b/d/n],[1-1/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))/x
^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 191 vs. 2(55) = 110.

Time = 2.53 (sec) , antiderivative size = 191, normalized size of antiderivative = 3.47

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^3}\,dx = \frac{\coth(d(a+b\log(cx^n))) - \coth(d(a-bn\log(x)+b\log(cx^n))) - \frac{e^{2d(a+b\log(cx^n))}\text{Hypergeometric2F1}\left(1,1-\frac{1}{bdn},2-\right.}{-1+bdn}}{}$$

input
```
Integrate[Coth[d*(a + b*Log[c*x^n])]/x^3,x]
```

output
```
(Coth[d*(a + b*Log[c*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] - (E
^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(b*d*n), 2 - 1/(b*d*n
), E^(2*d*(a + b*Log[c*x^n]))])/(-1 + b*d*n) + Hypergeometric2F1[1, -(1/(b
*d*n)), 1 - 1/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a + b*Log[c*x
^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])/(2*x^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.65, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {6074, 6072, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^3} \, dx$$

$$\Big\downarrow \text{6074}$$

$$\frac{(cx^n)^{2/n} \int (cx^n)^{-1-\frac{2}{n}} \coth(d(a+b\log(cx^n))) d(cx^n)}{nx^2}$$

$$\Big\downarrow \text{6072}$$

$$\frac{(cx^n)^{2/n} \int \frac{(cx^n)^{-1-\frac{2}{n}}\left(-e^{2ad}(cx^n)^{2bd}-1\right)}{1-e^{2ad}(cx^n)^{2bd}} d(cx^n)}{nx^2}$$

$$\Big\downarrow \text{959}$$

$$\frac{(cx^n)^{2/n} \left(-2\int \frac{(cx^n)^{-1-\frac{2}{n}}}{1-e^{2ad}(cx^n)^{2bd}} d(cx^n) - \frac{1}{2}n(cx^n)^{-2/n}\right)}{nx^2}$$

$$\Big\downarrow \text{888}$$

$$\frac{(cx^n)^{2/n} \left(n(cx^n)^{-2/n} \text{Hypergeometric2F1}\left(1,-\frac{1}{bdn},1-\frac{1}{bdn},e^{2ad}(cx^n)^{2bd}\right) - \frac{1}{2}n(cx^n)^{-2/n}\right)}{nx^2}$$

input
```
Int[Coth[d*(a + b*Log[c*x^n])]/x^3,x]
```

output
```
((c*x^n)^(2/n)*(-1/2*n/(c*x^n)^(2/n) + (n*Hypergeometric2F1[1, -(1/(b*d*n)
), 1 - 1/(b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(c*x^n)^(2/n)))/(n*x^2)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 6074
```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\coth\left(d(a+b\ln\left(c\,x^n\right))\right)}{x^3}dx$$

input
```
int(coth(d*(a+b*ln(c*x^n)))/x^3,x)
```

output
```
int(coth(d*(a+b*ln(c*x^n)))/x^3,x)
```

Fricas [F]

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\coth((b\log(cx^n)+a)d)}{x^3}\,dx$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

output
```
integral(coth(b*d*log(c*x^n) + a*d)/x^3, x)
```

Sympy [F]

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\coth(ad+bd\log(cx^n))}{x^3}\,dx$$

input
```
integrate(coth(d*(a+b*ln(c*x**n)))/x**3,x)
```

output
```
Integral(coth(a*d + b*d*log(c*x**n))/x**3, x)
```

Maxima [F]

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\coth((b\log(cx^n)+a)d)}{x^3}\,dx$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

output
```
-1/2/x^2 - integrate(1/(c^(b*d)*x^3*e^(b*d*log(x^n) + a*d) + x^3), x) + in
tegrate(1/(c^(b*d)*x^3*e^(b*d*log(x^n) + a*d) - x^3), x)
```

Giac [F]

$$\int \frac{\coth\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx=\int \frac{\coth\left((b\log\left(cx^n\right)+a)d\right)}{x^3}\,dx$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

output
```
integrate(coth((b*log(c*x^n) + a)*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx=\int \frac{\coth(d\left(a+b\,\ln\left(c\,x^n\right)\right))}{x^3}\,dx$$

input
```
int(coth(d*(a + b*log(c*x^n)))/x^3,x)
```

output
```
int(coth(d*(a + b*log(c*x^n)))/x^3, x)
```

Reduce [F]

$$\int \frac{\coth\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx=\frac{4e^{2ad}c^{2bd}\left(\int \frac{x^{2bdn}}{x^{2bdn}e^{2ad}c^{2bd}x^3-x^3}dx\right)x^2+1}{2x^2}$$

input
```
int(coth(d*(a+b*log(c*x^n)))/x^3,x)
```

output
```
(4*e**(2*a*d)*c**(2*b*d)*int(x**(2*b*d*n)/(x**(2*b*d*n)*e**(2*a*d)*c**(2*b
*d)*x**3 - x**3),x)*x**2 + 1)/(2*x**2)
```

3.189 $\int x^3 \coth^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 132

$$\int x^3 \coth^2(d(a+b\log(cx^n)))\,dx$$
$$=\frac{1}{4}\left(1+\frac{4}{bdn}\right)x^4+\frac{x^4\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1-e^{2ad}(cx^n)^{2bd}\right)}$$
$$-\frac{2x^4\,\text{Hypergeometric2F1}\left(1,\frac{2}{bdn},1+\frac{2}{bdn},e^{2ad}(cx^n)^{2bd}\right)}{bdn}$$

output

```
1/4*(1+4/b/d/n)*x^4+x^4*(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/(1-exp(2*a*d)
*(c*x^n)^(2*b*d))-2*x^4*hypergeom([1, 2/b/d/n],[1+2/b/d/n],exp(2*a*d)*(c*x
^n)^(2*b*d))/b/d/n
```

Mathematica [A] (verified)

Time = 4.35 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.17

$$\int x^3 \coth^2(d(a+b\log(cx^n)))\,dx$$
$$= \frac{x^4\left(-8e^{2d(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1+\frac{2}{bdn},2+\frac{2}{bdn},e^{2d(a+b\log(cx^n))}\right)+(2+bdn)\left(bdn-4\coth\left(\right.\right.\right.}{4bdn(2+bdn)}$$

input
```
Integrate[x^3*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
(x^4*(-8*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 2/(b*d*n), 2
+ 2/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (2 + b*d*n)*(b*d*n - 4*Coth[d*(
a + b*Log[c*x^n])] - 4*Hypergeometric2F1[1, 2/(b*d*n), 1 + 2/(b*d*n), E^(2
*d*(a + b*Log[c*x^n]))])))/(4*b*d*n*(2 + b*d*n))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.38, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6074, 6072, 1004, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \coth^2(d(a+b\log(cx^n)))\,dx$$
$$\Big\downarrow \text{ 6074}$$
$$\frac{x^4(cx^n)^{-4/n}\int (cx^n)^{\frac{4}{n}-1}\coth^2(d(a+b\log(cx^n)))\,d(cx^n)}{n}$$
$$\Big\downarrow \text{ 6072}$$
$$\frac{x^4(cx^n)^{-4/n}\int \frac{(cx^n)^{\frac{4}{n}-1}\left(-e^{2ad}(cx^n)^{2bd}-1\right)^2}{\left(1-e^{2ad}(cx^n)^{2bd}\right)^2}d(cx^n)}{n}$$
$$\Big\downarrow \text{ 1004}$$

$$\frac{x^4 (cx^n)^{-4/n}\left(\frac{e^{-2ad}\int -\frac{2(cx^n)^{\frac{4}{n}-1}\left(\frac{e^{4ad}(bdn+4)(cx^n)^{2bd}}{n}+\frac{e^{2ad}(4-bdn)}{n}\right)}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{2bd}+\frac{(cx^n)^{4/n}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}\right)}{n}$$

$$\Big\downarrow \; 27$$

$$\frac{x^4 (cx^n)^{-4/n}\left(\frac{(cx^n)^{4/n}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}-\frac{e^{-2ad}\int \frac{(cx^n)^{\frac{4}{n}-1}\left(\frac{e^{4ad}(bdn+4)(cx^n)^{2bd}}{n}+\frac{e^{2ad}(4-bdn)}{n}\right)}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{bd}\right)}{n}$$

$$\Big\downarrow \; 959$$

$$\frac{x^4 (cx^n)^{-4/n}\left(\frac{(cx^n)^{4/n}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}-\frac{e^{-2ad}\left(\frac{8e^{2ad}\int \frac{(cx^n)^{\frac{4}{n}-1}}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{n}-\frac{1}{4}e^{2ad}(bdn+4)(cx^n)^{4/n}\right)}{bd}\right)}{n}$$

$$\Big\downarrow \; 888$$

$$\frac{x^4 (cx^n)^{-4/n}\left(\frac{(cx^n)^{4/n}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}-\frac{e^{-2ad}\left(2e^{2ad}(cx^n)^{4/n}\,\text{Hypergeometric2F1}\left(1,\frac{2}{bdn},1+\frac{2}{bdn},e^{2ad}(cx^n)^{2bd}\right)-\frac{1}{4}e^{2ad}(bdn+4)(cx\cdots\right)}{bd}\right)}{n}$$

input
```
Int[x^3*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
(x^4*(((c*x^n)^(4/n)*(1 + E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*(1 - E^(2*a*d)*
(c*x^n)^(2*b*d))) - (-1/4*(E^(2*a*d)*(4 + b*d*n)*(c*x^n)^(4/n)) + 2*E^(2*a
*d)*(c*x^n)^(4/n)*Hypergeometric2F1[1, 2/(b*d*n), 1 + 2/(b*d*n), E^(2*a*d)
*(c*x^n)^(2*b*d)])/(b*d*E^(2*a*d))))/(n*(c*x^n)^(4/n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 6074
```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^3 \coth\left(d(a+b\ln\left(c\,x^n\right))\right)^2 dx$$

input
```
int(x^3*coth(d*(a+b*ln(c*x^n)))^2,x)
```

output
```
int(x^3*coth(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int x^3 \coth^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int x^3 \coth\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

input
```
integrate(x^3*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

output
```
integral(x^3*coth(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int x^3 \coth^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int x^3 \coth^2\left(ad+bd\log\left(cx^n\right)\right)dx$$

input
```
integrate(x**3*coth(d*(a+b*ln(c*x**n)))**2,x)
```

output
```
Integral(x**3*coth(a*d + b*d*log(c*x**n))**2, x)
```

Maxima [F]

$$\int x^3 \coth^2(d(a+b\log(cx^n)))\,dx = \int x^3 \coth((b\log(cx^n)+a)d)^2\,dx$$

input
```
integrate(x^3*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

output
```
1/4*(b*c^(2*b*d)*d*n*x^4*e^(2*b*d*log(x^n) + 2*a*d) - (b*d*n + 8)*x^4)/(b*
c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) - 4*integrate(x^3/(b*c^(
b*d)*d*n*e^(b*d*log(x^n) + a*d) + b*d*n), x) + 4*integrate(x^3/(b*c^(b*d)*
d*n*e^(b*d*log(x^n) + a*d) - b*d*n), x)
```

Giac [F]

$$\int x^3 \coth^2(d(a+b\log(cx^n)))\,dx = \int x^3 \coth((b\log(cx^n)+a)d)^2\,dx$$

input
```
integrate(x^3*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

output
```
integrate(x^3*coth((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \coth^2(d(a+b\log(cx^n)))\,dx = \int x^3 \coth(d\,(a+b\,\ln(c\,x^n)))^2\,dx$$

input
```
int(x^3*coth(d*(a + b*log(c*x^n)))^2,x)
```

output
```
int(x^3*coth(d*(a + b*log(c*x^n)))^2, x)
```

Reduce [F]

$$\int x^3 \coth^2(d(a+b\log(cx^n)))\,dx = 4e^{2ad}c^{2bd}\left(\int \frac{x^{2bdn}x^3}{x^{4bdn}e^{4ad}c^{4bd} - 2x^{2bdn}e^{2ad}c^{2bd} + 1}dx\right) + \frac{x^4}{4}$$

input
```
int(x^3*coth(d*(a+b*log(c*x^n)))^2,x)
```

output
```
(16*e**(2*a*d)*c**(2*b*d)*int((x**(2*b*d*n)*x**3)/(x**(4*b*d*n)*e**(4*a*d)
*c**(4*b*d) - 2*x**(2*b*d*n)*e**(2*a*d)*c**(2*b*d) + 1),x) + x**4)/4
```

3.190 $\int x^2 \coth^2(d(a + b\log(cx^n)))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 136

$$\int x^2 \coth^2(d(a + b\log(cx^n)))\,dx = \frac{1}{3}\left(1 + \frac{3}{bdn}\right)x^3 + \frac{x^3\left(1 + e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1 - e^{2ad}(cx^n)^{2bd}\right)} - \frac{2x^3\,\text{Hypergeometric2F1}\left(1, \frac{3}{2bdn}, 1 + \frac{3}{2bdn}, e^{2ad}(cx^n)^{2bd}\right)}{bdn}$$

output
```
1/3*(1+3/b/d/n)*x^3+x^3*(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/(1-exp(2*a*d)
*(c*x^n)^(2*b*d))-2*x^3*hypergeom([1, 3/2/b/d/n],[1+3/2/b/d/n],exp(2*a*d)*
(c*x^n)^(2*b*d))/b/d/n
```

Mathematica [A] (verified)

Time = 3.31 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.21

$$\int x^2 \coth^2(d(a+b\log(cx^n)))\,dx$$
$$= \frac{x^3\left(-9e^{2d(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1+\frac{3}{2bdn},2+\frac{3}{2bdn},e^{2d(a+b\log(cx^n))}\right)+(3+2bdn)\left(bdn-3\,\text{co}\right.}{3bdn(3+2bdn)}$$

input
```
Integrate[x^2*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
(x^3*(-9*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 3/(2*b*d*n),
2 + 3/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (3 + 2*b*d*n)*(b*d*n - 3*Co
th[d*(a + b*Log[c*x^n])] - 3*Hypergeometric2F1[1, 3/(2*b*d*n), 1 + 3/(2*b*
d*n), E^(2*d*(a + b*Log[c*x^n]))])))/(3*b*d*n*(3 + 2*b*d*n))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.37, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6074, 6072, 1004, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \coth^2(d(a+b\log(cx^n)))\,dx$$
$$\Big\downarrow \text{6074}$$
$$\frac{x^3(cx^n)^{-3/n}\int (cx^n)^{\frac{3}{n}-1}\coth^2(d(a+b\log(cx^n)))\,d(cx^n)}{n}$$
$$\Big\downarrow \text{6072}$$
$$\frac{x^3(cx^n)^{-3/n}\int \frac{(cx^n)^{\frac{3}{n}-1}\left(-e^{2ad}(cx^n)^{2bd}-1\right)^2}{\left(1-e^{2ad}(cx^n)^{2bd}\right)^2}d(cx^n)}{n}$$
$$\Big\downarrow \text{1004}$$

$$\frac{x^3 (cx^n)^{-3/n} \left(\frac{e^{-2ad}\int -\frac{2(cx^n)^{\frac{3}{n}-1}\left(\frac{e^{4ad}(bdn+3)(cx^n)^{2bd}}{n}+\frac{e^{2ad}(3-bdn)}{n}\right)}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{2bd}+\frac{(cx^n)^{3/n}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}\right)}{n}$$

$$\Big\downarrow \; 27$$

$$\frac{x^3 (cx^n)^{-3/n} \left(\frac{(cx^n)^{3/n}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}-\frac{e^{-2ad}\int \frac{(cx^n)^{\frac{3}{n}-1}\left(\frac{e^{4ad}(bdn+3)(cx^n)^{2bd}}{n}+\frac{e^{2ad}(3-bdn)}{n}\right)}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{bd}\right)}{n}$$

$$\Big\downarrow \; 959$$

$$\frac{x^3 (cx^n)^{-3/n} \left(\frac{(cx^n)^{3/n}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}-\frac{e^{-2ad}\left(\frac{6e^{2ad}\int \frac{(cx^n)^{\frac{3}{n}-1}}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{n}-\frac{1}{3}e^{2ad}(bdn+3)(cx^n)^{3/n}\right)}{bd}\right)}{n}$$

$$\Big\downarrow \; 888$$

$$\frac{x^3 (cx^n)^{-3/n} \left(\frac{(cx^n)^{3/n}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}-\frac{e^{-2ad}\left(2e^{2ad}(cx^n)^{3/n}\text{Hypergeometric2F1}\left(1,\frac{3}{2bdn},1+\frac{3}{2bdn},e^{2ad}(cx^n)^{2bd}\right)-\frac{1}{3}e^{2ad}(bdn+3)(c\right.}{bd}\right.}{n}$$

input
```
Int[x^2*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
(x^3*(((c*x^n)^(3/n)*(1 + E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*(1 - E^(2*a*d)*
(c*x^n)^(2*b*d))) - (-1/3*(E^(2*a*d)*(3 + b*d*n)*(c*x^n)^(3/n)) + 2*E^(2*a
*d)*(c*x^n)^(3/n)*Hypergeometric2F1[1, 3/(2*b*d*n), 1 + 3/(2*b*d*n), E^(2*
a*d)*(c*x^n)^(2*b*d)])/(b*d*E^(2*a*d))))/(n*(c*x^n)^(3/n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 6074
```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^2 \coth\left(d(a+b\ln\left(c\,x^n\right))\right)^2 dx$$

input
```
int(x^2*coth(d*(a+b*ln(c*x^n)))^2,x)
```

output
```
int(x^2*coth(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int x^2 \coth^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int x^2 \coth\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

input
```
integrate(x^2*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

output
```
integral(x^2*coth(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int x^2 \coth^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int x^2 \coth^2\left(ad+bd\log\left(cx^n\right)\right)dx$$

input
```
integrate(x**2*coth(d*(a+b*ln(c*x**n)))**2,x)
```

output
```
Integral(x**2*coth(a*d + b*d*log(c*x**n))**2, x)
```

Maxima [F]

$$\int x^2 \coth^2(d(a + b\log(cx^n)))\,dx = \int x^2 \coth\left((b\log(cx^n) + a)d\right)^2\,dx$$

input
```
integrate(x^2*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

output
```
1/3*(b*c^(2*b*d)*d*n*x^3*e^(2*b*d*log(x^n) + 2*a*d) - (b*d*n + 6)*x^3)/(b*
c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) - 3*integrate(x^2/(b*c^(
b*d)*d*n*e^(b*d*log(x^n) + a*d) + b*d*n), x) + 3*integrate(x^2/(b*c^(b*d)*
d*n*e^(b*d*log(x^n) + a*d) - b*d*n), x)
```

Giac [F]

$$\int x^2 \coth^2(d(a + b\log(cx^n)))\,dx = \int x^2 \coth\left((b\log(cx^n) + a)d\right)^2\,dx$$

input
```
integrate(x^2*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

output
```
integrate(x^2*coth((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^2(d(a + b\log(cx^n)))\,dx = \int x^2 \coth(d\,(a + b\,\ln(c\,x^n)))^2\,dx$$

input
```
int(x^2*coth(d*(a + b*log(c*x^n)))^2,x)
```

output
```
int(x^2*coth(d*(a + b*log(c*x^n)))^2, x)
```

Reduce [F]

$$\int x^2 \coth^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = 4e^{2ad}c^{2bd}\left(\int \frac{x^{2bdn}x^2}{x^{4bdn}e^{4ad}c^{4bd}-2x^{2bdn}e^{2ad}c^{2bd}+1}dx\right)+\frac{x^3}{3}$$

input
```
int(x^2*coth(d*(a+b*log(c*x^n)))^2,x)
```

output
```
(12*e**(2*a*d)*c**(2*b*d)*int((x**(2*b*d*n)*x**2)/(x**(4*b*d*n)*e**(4*a*d)
*c**(4*b*d) - 2*x**(2*b*d*n)*e**(2*a*d)*c**(2*b*d) + 1),x) + x**3)/3
```

3.191 $\int x \coth^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 130

$$\int x \coth^2(d(a+b\log(cx^n)))\,dx = \frac{1}{2}\left(1+\frac{2}{bdn}\right)x^2 + \frac{x^2\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1-e^{2ad}(cx^n)^{2bd}\right)} - \frac{2x^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{bdn},1+\frac{1}{bdn},e^{2ad}(cx^n)^{2bd}\right)}{bdn}$$

output
```
1/2*(1+2/b/d/n)*x^2+x^2*(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/(1-exp(2*a*d)
*(c*x^n)^(2*b*d))-2*x^2*hypergeom([1, 1/b/d/n],[1+1/b/d/n],exp(2*a*d)*(c*x
^n)^(2*b*d))/b/d/n
```

Mathematica [A] (verified)

Time = 4.51 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.16

$$\int x \coth^2(d(a+b\log(cx^n)))\,dx$$
$$= \frac{x^2\left(-2e^{2d(a+b\log(cx^n))}\text{Hypergeometric2F1}\left(1,1+\frac{1}{bdn},2+\frac{1}{bdn},e^{2d(a+b\log(cx^n))}\right)+(1+bdn)\left(bdn-2\coth\left(\right.\right.\right.}{2bdn(1+bdn)}$$

input
```
Integrate[x*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
(x^2*(-2*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 1/(b*d*n), 2
+ 1/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (1 + b*d*n)*(b*d*n - 2*Coth[d*(
a + b*Log[c*x^n])] - 2*Hypergeometric2F1[1, 1/(b*d*n), 1 + 1/(b*d*n), E^(2
*d*(a + b*Log[c*x^n]))])))/(2*b*d*n*(1 + b*d*n))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.38, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6074, 6072, 1004, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \coth^2(d(a+b\log(cx^n)))\,dx$$
$$\Big\downarrow \text{6074}$$
$$\frac{x^2(cx^n)^{-2/n}\int (cx^n)^{\frac{2}{n}-1}\coth^2(d(a+b\log(cx^n)))\,d(cx^n)}{n}$$
$$\Big\downarrow \text{6072}$$
$$\frac{x^2(cx^n)^{-2/n}\int \frac{(cx^n)^{\frac{2}{n}-1}\left(-e^{2ad}(cx^n)^{2bd}-1\right)^2}{\left(1-e^{2ad}(cx^n)^{2bd}\right)^2}d(cx^n)}{n}$$
$$\Big\downarrow \text{1004}$$

$$\frac{x^2(cx^n)^{-2/n}\left(\frac{e^{-2ad}\int -\frac{2(cx^n)^{\frac{2}{n}-1}\left(\frac{e^{4ad}(bdn+2)(cx^n)^{2bd}}{n}+\frac{e^{2ad}(2-bdn)}{n}\right)}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{2bd}+\frac{(cx^n)^{2/n}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}\right)}{n}$$

$$\Big\downarrow 27$$

$$\frac{x^2(cx^n)^{-2/n}\left(\frac{(cx^n)^{2/n}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}-\frac{e^{-2ad}\int \frac{(cx^n)^{\frac{2}{n}-1}\left(\frac{e^{4ad}(bdn+2)(cx^n)^{2bd}}{n}+\frac{e^{2ad}(2-bdn)}{n}\right)}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{bd}\right)}{n}$$

$$\Big\downarrow 959$$

$$\frac{x^2(cx^n)^{-2/n}\left(\frac{(cx^n)^{2/n}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}-\frac{e^{-2ad}\left(\frac{4e^{2ad}\int \frac{(cx^n)^{\frac{2}{n}-1}}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{n}-\frac{1}{2}e^{2ad}(bdn+2)(cx^n)^{2/n}\right)}{bd}\right)}{n}$$

$$\Big\downarrow 888$$

$$\frac{x^2(cx^n)^{-2/n}\left(\frac{(cx^n)^{2/n}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}-\frac{e^{-2ad}\left(2e^{2ad}(cx^n)^{2/n}\text{Hypergeometric2F1}\left(1,\frac{1}{bdn},1+\frac{1}{bdn},e^{2ad}(cx^n)^{2bd}\right)-\frac{1}{2}e^{2ad}(bdn+2)(cx^n)^{2/n}\right)}{bd}\right)}{n}$$

input
```
Int[x*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
(x^2*(((c*x^n)^(2/n)*(1 + E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*(1 - E^(2*a*d)*
(c*x^n)^(2*b*d))) - (-1/2*(E^(2*a*d)*(2 + b*d*n)*(c*x^n)^(2/n)) + 2*E^(2*a
*d)*(c*x^n)^(2/n)*Hypergeometric2F1[1, 1/(b*d*n), 1 + 1/(b*d*n), E^(2*a*d)
*(c*x^n)^(2*b*d)])/(b*d*E^(2*a*d))))/(n*(c*x^n)^(2/n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 6074
```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x\coth\left(d(a+b\ln\left(c\,x^{n}\right))\right)^{2}dx$$

input
```
int(x*coth(d*(a+b*ln(c*x^n)))^2,x)
```

output
```
int(x*coth(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int x\coth^2\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\coth\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

input
```
integrate(x*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

output
```
integral(x*coth(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int x\coth^2\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\coth^2\left(ad+bd\log\left(cx^n\right)\right)dx$$

input
```
integrate(x*coth(d*(a+b*ln(c*x**n)))**2,x)
```

output
```
Integral(x*coth(a*d + b*d*log(c*x**n))**2, x)
```

Maxima [F]

$$\int x\coth^2\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\coth\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

input
```
integrate(x*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

output
```
1/2*(b*c^(2*b*d)*d*n*x^2*e^(2*b*d*log(x^n) + 2*a*d) - (b*d*n + 4)*x^2)/(b*
c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) - 2*integrate(x/(b*c^(b*
d)*d*n*e^(b*d*log(x^n) + a*d) + b*d*n), x) + 2*integrate(x/(b*c^(b*d)*d*n*
e^(b*d*log(x^n) + a*d) - b*d*n), x)
```

Giac [F]

$$\int x\coth^2\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\coth\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

input
```
integrate(x*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

output
```
integrate(x*coth((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\coth^2\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\,\mathrm{coth}\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)^2\,dx$$

input
```
int(x*coth(d*(a + b*log(c*x^n)))^2,x)
```

output
```
int(x*coth(d*(a + b*log(c*x^n)))^2, x)
```

Reduce [F]

$$\int x \coth^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = 4e^{2ad}c^{2bd}\left(\int \frac{x^{2bdn}x}{x^{4bdn}e^{4ad}c^{4bd}-2x^{2bdn}e^{2ad}c^{2bd}+1}dx\right) + \frac{x^2}{2}$$

input
```
int(x*coth(d*(a+b*log(c*x^n)))^2,x)
```

output
```
(8*e**(2*a*d)*c**(2*b*d)*int((x**(2*b*d*n)*x)/(x**(4*b*d*n)*e**(4*a*d)*c**
(4*b*d) - 2*x**(2*b*d*n)*e**(2*a*d)*c**(2*b*d) + 1),x) + x**2)/2
```

3.192 $\int \coth^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 126

$$\int \coth^2(d(a+b\log(cx^n)))\,dx = \left(1+\frac{1}{bdn}\right)x+\frac{x\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1-e^{2ad}(cx^n)^{2bd}\right)} - \frac{2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2bdn},1+\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd}\right)}{bdn}$$

output

```
(1+1/b/d/n)*x+x*(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/(1-exp(2*a*d)*(c*x^n)
^(2*b*d))-2*x*hypergeom([1, 1/2/b/d/n],[1+1/2/b/d/n],exp(2*a*d)*(c*x^n)^(2
*b*d))/b/d/n
```

Mathematica [A] (verified)

Time = 5.62 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.27

$$\int \coth^2(d(a+b\log(cx^n)))\,dx$$
$$= \frac{x\left(-e^{2d(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{2bdn},2+\frac{1}{2bdn},e^{2d(a+b\log(cx^n))}\right)+(1+2bdn)\left(bdn-\coth(\right.\right.}{bdn(1+2bdn)}$$

input
```
Integrate[Coth[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
(x*(-(E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 1/(2*b*d*n), 2 +
 1/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))]) + (1 + 2*b*d*n)*(b*d*n - Coth[d
*(a + b*Log[c*x^n])] - Hypergeometric2F1[1, 1/(2*b*d*n), 1 + 1/(2*b*d*n),
E^(2*d*(a + b*Log[c*x^n]))])))/(b*d*n*(1 + 2*b*d*n))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.40, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6070, 6072, 1004, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(d(a+b\log(cx^n)))\,dx$$
$$\Big\downarrow \text{6070}$$
$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\coth^2(d(a+b\log(cx^n)))\,d(cx^n)}{n}$$
$$\Big\downarrow \text{6072}$$
$$\frac{x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}-1}\left(-e^{2ad}(cx^n)^{2bd}-1\right)^2}{\left(1-e^{2ad}(cx^n)^{2bd}\right)^2}d(cx^n)}{n}$$
$$\Big\downarrow \text{1004}$$

$$\frac{x(cx^n)^{-1/n}\left(\frac{e^{-2ad}\int -\frac{2(cx^n)^{\frac{1}{n}-1}\left(e^{4ad}\left(bd+\frac{1}{n}\right)(cx^n)^{2bd}+\frac{e^{2ad}(1-bdn)}{n}\right)}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{2bd}+\frac{(cx^n)^{\frac{1}{n}}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}\right)}{n}$$

$$\Big\downarrow \text{27}$$

$$\frac{x(cx^n)^{-1/n}\left(\frac{(cx^n)^{\frac{1}{n}}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}-\frac{e^{-2ad}\int \frac{(cx^n)^{\frac{1}{n}-1}\left(e^{4ad}\left(bd+\frac{1}{n}\right)(cx^n)^{2bd}+\frac{e^{2ad}(1-bdn)}{n}\right)}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{bd}\right)}{n}$$

$$\Big\downarrow \text{959}$$

$$\frac{x(cx^n)^{-1/n}\left(\frac{(cx^n)^{\frac{1}{n}}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}-\frac{e^{-2ad}\left(\frac{2e^{2ad}\int \frac{(cx^n)^{\frac{1}{n}-1}}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{n}-e^{2ad}(bdn+1)(cx^n)^{\frac{1}{n}}\right)}{bd}\right)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x(cx^n)^{-1/n}\left(\frac{(cx^n)^{\frac{1}{n}}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}-\frac{e^{-2ad}\left(2e^{2ad}(cx^n)^{\frac{1}{n}}\text{Hypergeometric2F1}\left(1,\frac{1}{2bdn},1+\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd}\right)-e^{2ad}(bdn+1)(cx^n)^{\frac{1}{n}}\right.}{bd}}{n}$$

input
```
Int[Coth[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
(x*(((c*x^n)^n^(-1)*(1 + E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*(1 - E^(2*a*d)*(
c*x^n)^(2*b*d))) - (-(E^(2*a*d)*(1 + b*d*n)*(c*x^n)^n^(-1)) + 2*E^(2*a*d)*
(c*x^n)^n^(-1)*Hypergeometric2F1[1, 1/(2*b*d*n), 1 + 1/(2*b*d*n), E^(2*a*d
)*(c*x^n)^(2*b*d)])/(b*d*E^(2*a*d))))/(n*(c*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 6070
```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Coth[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [F]

$$\int \coth \left(d(a+b\ln \left(c\,x^{n}\right))\right)^{2}dx$$

input
```
int(coth(d*(a+b*ln(c*x^n)))^2,x)
```

output
```
int(coth(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int \coth^2 \left(d(a+b\log \left(cx^n\right))\right)\,dx=\int \coth \left((b\log \left(cx^n\right)+a)d\right)^2\,dx$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

output
```
integral(coth(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int \coth^2 \left(d(a+b\log \left(cx^n\right))\right)\,dx=\int \coth^2 \left(d(a+b\log \left(cx^n\right))\right)dx$$

input
```
integrate(coth(d*(a+b*ln(c*x**n)))**2,x)
```

output
```
Integral(coth(d*(a + b*log(c*x**n)))**2, x)
```

Maxima [F]

$$\int \coth^2(d(a+b\log(cx^n)))\,dx = \int \coth\left((b\log(cx^n)+a)d\right)^2\,dx$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

output
```
(b*c^(2*b*d)*d*n*x*e^(2*b*d*log(x^n) + 2*a*d) - (b*d*n + 2)*x)/(b*c^(2*b*d
)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) - integrate(1/(b*c^(b*d)*d*n*e^(
b*d*log(x^n) + a*d) + b*d*n), x) + integrate(1/(b*c^(b*d)*d*n*e^(b*d*log(x
^n) + a*d) - b*d*n), x)
```

Giac [F]

$$\int \coth^2(d(a+b\log(cx^n)))\,dx = \int \coth\left((b\log(cx^n)+a)d\right)^2\,dx$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

output
```
integrate(coth((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^2(d(a+b\log(cx^n)))\,dx = \int \coth(d\,(a+b\,\ln(c\,x^n)))^2\,dx$$

input
```
int(coth(d*(a + b*log(c*x^n)))^2,x)
```

output
```
int(coth(d*(a + b*log(c*x^n)))^2, x)
```

Reduce [F]

$$\int \coth^2(d(a+b\log(cx^n)))\,dx = 4e^{2ad}c^{2bd}\left(\int \frac{x^{2bdn}}{x^{4bdn}e^{4ad}c^{4bd}-2x^{2bdn}e^{2ad}c^{2bd}+1}dx\right)+x$$

input
```
int(coth(d*(a+b*log(c*x^n)))^2,x)
```

output
```
4*e**(2*a*d)*c**(2*b*d)*int(x**(2*b*d*n)/(x**(4*b*d*n)*e**(4*a*d)*c**(4*b*
d) - 2*x**(2*b*d*n)*e**(2*a*d)*c**(2*b*d) + 1),x) + x
```

3.193 $\int \frac{\coth^2(d(a+b\log(cx^n)))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x}\,dx = -\frac{\coth(ad+bd\log(cx^n))}{bdn}+\log(x)$$

output
```
-coth(a*d+b*d*ln(c*x^n))/b/d/n+ln(x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x}\,dx = -\frac{\coth(ad+bd\log(cx^n))\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(ad+bd\log(cx^n))\right)}{bdn}$$

input
```
Integrate[Coth[d*(a + b*Log[c*x^n])]^2/x,x]
```

output
```
-((Coth[a*d + b*d*Log[c*x^n]]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[a*d + b
*d*Log[c*x^n]]^2])/(b*d*n))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3039, 3042, 25, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \coth^2(d(a+b\log(cx^n)))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\tan\left(iad+ib\log(cx^n)d+\frac{\pi}{2}\right)^2d\log(cx^n)}{n}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \tan\left(\frac{1}{2}(2iad+\pi)+ibd\log(cx^n)\right)^2d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\int 1d\log(cx^n)-\frac{\coth(ad+bd\log(cx^n))}{bd}}{n}$$

$$\Big\downarrow \text{24}$$

$$\frac{\log(cx^n)-\frac{\coth(ad+bd\log(cx^n))}{bd}}{n}$$

input
```
Int[Coth[d*(a + b*Log[c*x^n])]^2/x,x]
```

output
```
(-(Coth[a*d + b*d*Log[c*x^n]]/(b*d)) + Log[c*x^n])/n
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

method	result
parallelrisch	$\frac{\ln(x) d b n - \coth(d(a+b\ln(c\,x^n)))}{b d n}$
derivativedivides	$\frac{-\coth(d(a+b\ln(c\,x^n))) - \frac{\ln(\coth(d(a+b\ln(c\,x^n)))-1)}{2} + \frac{\ln(\coth(d(a+b\ln(c\,x^n)))+1)}{2}}{n b d}$
default	$\frac{-\coth(d(a+b\ln(c\,x^n))) - \frac{\ln(\coth(d(a+b\ln(c\,x^n)))-1)}{2} + \frac{\ln(\coth(d(a+b\ln(c\,x^n)))+1)}{2}}{n b d}$
risch	$\ln(x) - \frac{2}{d b n \left(c^{2bd} (x^n)^{2bd} e^{d\left(i b\pi\ \mathrm{csgn}(i x^n)\ \mathrm{csgn}(i c\,x^n)^2 - i b\pi\ \mathrm{csgn}(i x^n)\ \mathrm{csgn}(i c\,x^n)\ \mathrm{csgn}(i c) - i b\pi\ \mathrm{csgn}(i c\,x^n)^3 + i b\pi\ \mathrm{csgn}(i c\,x\right.}\right.}$

input
```
int(coth(d*(a+b*ln(c*x^n)))^2/x,x,method=_RETURNVERBOSE)
```

output
```
(ln(x)*d*b*n-coth(d*(a+b*ln(c*x^n))))/b/d/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(28) = 56$.

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.57

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x}\,dx = \frac{(bdn\log(x)+1)\sinh(bdn\log(x)+bd\log(c)+ad) - \cosh(bdn\log(x)+bd\log(c)+ad)}{bdn\sinh(bdn\log(x)+bd\log(c)+ad)}$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))^2/x,x, algorithm="fricas")
```

output
```
((b*d*n*log(x) + 1)*sinh(b*d*n*log(x) + b*d*log(c) + a*d) - cosh(b*d*n*log
(x) + b*d*log(c) + a*d))/(b*d*n*sinh(b*d*n*log(x) + b*d*log(c) + a*d))
```

Sympy [F]

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x}\,dx = \int \frac{\coth^2(ad+bd\log(cx^n))}{x}\,dx$$

input
```
integrate(coth(d*(a+b*ln(c*x**n)))**2/x,x)
```

output
```
Integral(coth(a*d + b*d*log(c*x**n))**2/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x}\,dx = -\frac{2}{bc^{2\,bd}dne^{(2\,bd\log(x^n)+2\,ad)} - bdn} + \log(x)$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))^2/x,x, algorithm="maxima")
```

output
```
-2/(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x}\,dx = \frac{\log(x^{bdn})}{bdn} - \frac{2}{(c^{2bd}x^{2bdn}e^{(2ad)}-1)bdn}$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))^2/x,x, algorithm="giac")
```

output
```
log(x^(b*d*n))/(b*d*n) - 2/((c^(2*b*d)*x^(2*b*d*n)*e^(2*a*d) - 1)*b*d*n)
```

Mupad [B] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x}\,dx = \ln(x) - \frac{2}{b\,d\,n\,\left(e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d}-1\right)}$$

input
```
int(coth(d*(a + b*log(c*x^n)))^2/x,x)
```

output
```
log(x) - 2/(b*d*n*(exp(2*a*d)*(c*x^n)^(2*b*d) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x}\,dx = \frac{-\coth(\log(x^n c)\,bd+ad)+\log(x^n c)\,bd}{bdn}$$

input
```
int(coth(d*(a+b*log(c*x^n)))^2/x,x)
```

output
```
( - coth(log(x**n*c)*b*d + a*d) + log(x**n*c)*b*d)/(b*d*n)
```

3.194 $\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 134

$$\int \frac{\coth^2\left(d(a+b\log\left(cx^n\right))\right)}{x^2}\,dx = -\frac{1-\frac{1}{bdn}}{x}+\frac{1+e^{2ad}(cx^n)^{2bd}}{bdnx\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)} - \frac{2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2bdn},1-\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd}\right)}{bdnx}$$

output
```
-(1-1/b/d/n)/x+(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/x/(1-exp(2*a*d)*(c*x^n
)^(2*b*d))-2*hypergeom([1, -1/2/b/d/n],[1-1/2/b/d/n],exp(2*a*d)*(c*x^n)^(2
*b*d))/b/d/n/x
```

Mathematica [A] (verified)

Time = 2.37 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.18

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^2}\,dx$$
$$= \frac{e^{2d(a+b\log(cx^n))}\text{Hypergeometric2F1}\left(1,1-\frac{1}{2bdn},2-\frac{1}{2bdn},e^{2d(a+b\log(cx^n))}\right)-(-1+2bdn)\left(bdn+\coth\left(d(a\right.\right.}{bdn(-1+2bdn)x}$$

input
```
Integrate[Coth[d*(a + b*Log[c*x^n])]^2/x^2,x]
```

output
```
(E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(2*b*d*n), 2 - 1/(2
*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] - (-1 + 2*b*d*n)*(b*d*n + Coth[d*(a +
 b*Log[c*x^n])] + Hypergeometric2F1[1, -1/2*1/(b*d*n), 1 - 1/(2*b*d*n), E^
(2*d*(a + b*Log[c*x^n]))]))/(b*d*n*(-1 + 2*b*d*n)*x)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.35, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6074, 6072, 1004, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^2}\,dx$$

$$\Big\downarrow \ 6074$$

$$\frac{(cx^n)^{\frac{1}{n}}\int (cx^n)^{-1-\frac{1}{n}}\coth^2(d(a+b\log(cx^n)))\,d(cx^n)}{nx}$$

$$\Big\downarrow \ 6072$$

$$\frac{(cx^n)^{\frac{1}{n}}\int \frac{(cx^n)^{-1-\frac{1}{n}}\left(-e^{2ad}(cx^n)^{2bd}-1\right)^2}{\left(1-e^{2ad}(cx^n)^{2bd}\right)^2}d(cx^n)}{nx}$$

$$\Big\downarrow \ 1004$$

$$\frac{(cx^n)^{\frac{1}{n}}\left(\frac{e^{-2ad}\int\frac{2(cx^n)^{-1-\frac{1}{n}}\left(\frac{e^{4ad}(1-bdn)(cx^n)^{2bd}}{n}+\frac{e^{2ad}(bdn+1)}{n}\right)}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{2bd}+\frac{(cx^n)^{-1/n}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}\right)}{nx}$$

$$\Big\downarrow \text{27}$$

$$\frac{(cx^n)^{\frac{1}{n}}\left(\frac{e^{-2ad}\int\frac{(cx^n)^{-1-\frac{1}{n}}\left(\frac{e^{4ad}(1-bdn)(cx^n)^{2bd}}{n}+\frac{e^{2ad}(bdn+1)}{n}\right)}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{bd}+\frac{(cx^n)^{-1/n}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}\right)}{nx}$$

$$\Big\downarrow \text{959}$$

$$\frac{(cx^n)^{\frac{1}{n}}\left(\frac{e^{-2ad}\left(\frac{2e^{2ad}\int\frac{(cx^n)^{-1-\frac{1}{n}}}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{n}+e^{2ad}(1-bdn)(cx^n)^{-1/n}\right)}{bd}+\frac{(cx^n)^{-1/n}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}\right)}{nx}$$

$$\Big\downarrow \text{888}$$

$$\frac{(cx^n)^{\frac{1}{n}}\left(\frac{e^{-2ad}\left(e^{2ad}(1-bdn)(cx^n)^{-1/n}-2e^{2ad}(cx^n)^{-1/n}\text{Hypergeometric2F1}\left(1,-\frac{1}{2bdn},1-\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd}\right)\right)}{bd}+\frac{(cx^n)^{-1/n}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}\right)}{nx}$$

input
```
Int[Coth[d*(a + b*Log[c*x^n])]^2/x^2,x]
```

output
```
((c*x^n)^n^(-1)*((1 + E^(2*a*d)*(c*x^n)^(2*b*d))/(b*d*(c*x^n)^n^(-1)*(1 -
E^(2*a*d)*(c*x^n)^(2*b*d))) + ((E^(2*a*d)*(1 - b*d*n))/(c*x^n)^n^(-1) - (2
*E^(2*a*d)*Hypergeometric2F1[1, -1/2*1/(b*d*n), 1 - 1/(2*b*d*n), E^(2*a*d)
*(c*x^n)^(2*b*d)])/(c*x^n)^n^(-1))/(b*d*E^(2*a*d))))/(n*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 6074
```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\coth\left(d(a+b\ln\left(c\,x^{n}\right))\right)^{2}}{x^{2}}dx$$

input
```
int(coth(d*(a+b*ln(c*x^n)))^2/x^2,x)
```

output
```
int(coth(d*(a+b*ln(c*x^n)))^2/x^2,x)
```

Fricas [F]

$$\int \frac{\coth^2\left(d(a+b\log\left(cx^n\right))\right)}{x^2}\,dx=\int \frac{\coth\left((b\log\left(cx^n\right)+a)d\right)^2}{x^2}\,dx$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="fricas")
```

output
```
integral(coth(b*d*log(c*x^n) + a*d)^2/x^2, x)
```

Sympy [F]

$$\int \frac{\coth^2\left(d(a+b\log\left(cx^n\right))\right)}{x^2}\,dx=\int \frac{\coth^2\left(ad+bd\log\left(cx^n\right)\right)}{x^2}\,dx$$

input
```
integrate(coth(d*(a+b*ln(c*x**n)))**2/x**2,x)
```

output
```
Integral(coth(a*d + b*d*log(c*x**n))**2/x**2, x)
```

Maxima [F]

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\coth\left((b\log(cx^n)+a)d\right)^2}{x^2}\,dx$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="maxima")
```

output
```
-(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n + 2)/(b*c^(2*b*d)*d*n
*x*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n*x) + integrate(1/(b*c^(b*d)*d*n*x^2*
e^(b*d*log(x^n) + a*d) + b*d*n*x^2), x) - integrate(1/(b*c^(b*d)*d*n*x^2*e
^(b*d*log(x^n) + a*d) - b*d*n*x^2), x)
```

Giac [F]

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\coth\left((b\log(cx^n)+a)d\right)^2}{x^2}\,dx$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="giac")
```

output
```
integrate(coth((b*log(c*x^n) + a)*d)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\coth\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)^2}{x^2}\,dx$$

input
```
int(coth(d*(a + b*log(c*x^n)))^2/x^2,x)
```

output
```
int(coth(d*(a + b*log(c*x^n)))^2/x^2, x)
```

Reduce [F]

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^2}\,dx = \frac{4e^{2ad}c^{2bd}\left(\int \frac{x^{2bdn}}{x^{4bdn}e^{4ad}c^{4bd}x^2-2x^{2bdn}e^{2ad}c^{2bd}x^2+x^2}dx\right)x-1}{x}$$

input
```
int(coth(d*(a+b*log(c*x^n)))^2/x^2,x)
```

output
```
(4*e**(2*a*d)*c**(2*b*d)*int(x**(2*b*d*n)/(x**(4*b*d*n)*e**(4*a*d)*c**(4*b
*d)*x**2 - 2*x**(2*b*d*n)*e**(2*a*d)*c**(2*b*d)*x**2 + x**2),x)*x - 1)/x
```

3.195 $\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 135

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^3}\,dx = \frac{2-bdn}{2bdnx^2}+\frac{1+e^{2ad}(cx^n)^{2bd}}{bdnx^2\left(1-e^{2ad}(cx^n)^{2bd}\right)} - \frac{2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{bdn},1-\frac{1}{bdn},e^{2ad}(cx^n)^{2bd}\right)}{bdnx^2}$$

output
```
1/2*(-b*d*n+2)/b/d/n/x^2+(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/x^2/(1-exp(2
*a*d)*(c*x^n)^(2*b*d))-2*hypergeom([1, -1/b/d/n],[1-1/b/d/n],exp(2*a*d)*(c
*x^n)^(2*b*d))/b/d/n/x^2
```

Mathematica [A] (verified)

Time = 2.35 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.16

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^3}\,dx = \frac{2e^{2d(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1-\frac{1}{bdn},2-\frac{1}{bdn},e^{2d(a+b\log(cx^n))}\right)-(-1+bdn)\left(bdn+2\coth(d(a$$
$$2bdn(-1+bdn)x^2$$

input
```
Integrate[Coth[d*(a + b*Log[c*x^n])]^2/x^3,x]
```

output
```
(2*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(b*d*n), 2 - 1/(b
*d*n), E^(2*d*(a + b*Log[c*x^n]))] - (-1 + b*d*n)*(b*d*n + 2*Coth[d*(a + b
*Log[c*x^n])] + 2*Hypergeometric2F1[1, -(1/(b*d*n)), 1 - 1/(b*d*n), E^(2*d
*(a + b*Log[c*x^n]))]))/(2*b*d*n*(-1 + b*d*n)*x^2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.35, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6074, 6072, 1004, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^3}\,dx$$

$$\Big\downarrow \text{ 6074}$$

$$\frac{(cx^n)^{2/n}\int (cx^n)^{-1-\frac{2}{n}}\coth^2(d(a+b\log(cx^n)))\,d(cx^n)}{nx^2}$$

$$\Big\downarrow \text{ 6072}$$

$$\frac{(cx^n)^{2/n}\int \frac{(cx^n)^{-1-\frac{2}{n}}\left(-e^{2ad}(cx^n)^{2bd}-1\right)^2}{\left(1-e^{2ad}(cx^n)^{2bd}\right)^2}d(cx^n)}{nx^2}$$

$$\Big\downarrow \text{ 1004}$$

$$\frac{(cx^n)^{2/n}\left(\frac{e^{-2ad}\int \frac{2(cx^n)^{-1-\frac{2}{n}}\left(\frac{e^{4ad}(2-bdn)(cx^n)^{2bd}}{n}+\frac{e^{2ad}(bdn+2)}{n}\right)}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{2bd}+\frac{(cx^n)^{-2/n}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}\right)}{nx^2}$$

$$\Big\downarrow \; 27$$

$$\frac{(cx^n)^{2/n}\left(\frac{e^{-2ad}\int \frac{(cx^n)^{-1-\frac{2}{n}}\left(\frac{e^{4ad}(2-bdn)(cx^n)^{2bd}}{n}+\frac{e^{2ad}(bdn+2)}{n}\right)}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{bd}+\frac{(cx^n)^{-2/n}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}\right)}{nx^2}$$

$$\Big\downarrow \; 959$$

$$\frac{(cx^n)^{2/n}\left(\frac{e^{-2ad}\left(\frac{4e^{2ad}\int \frac{(cx^n)^{-1-\frac{2}{n}}}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{n}+\frac{1}{2}e^{2ad}(2-bdn)(cx^n)^{-2/n}\right)}{bd}+\frac{(cx^n)^{-2/n}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}\right)}{nx^2}$$

$$\Big\downarrow \; 888$$

$$\frac{(cx^n)^{2/n}\left(\frac{e^{-2ad}\left(\frac{1}{2}e^{2ad}(2-bdn)(cx^n)^{-2/n}-2e^{2ad}(cx^n)^{-2/n}\text{Hypergeometric2F1}\left(1,-\frac{1}{bdn},1-\frac{1}{bdn},e^{2ad}(cx^n)^{2bd}\right)\right)}{bd}+\frac{(cx^n)^{-2/n}\left(e^{2ad}(cx^n\right.}{bd\left(1-e^{2ad}(cx^n)\right.}\right.}{nx^2}$$

input
```
Int[Coth[d*(a + b*Log[c*x^n])]^2/x^3,x]
```

output
```
((c*x^n)^(2/n)*((1 + E^(2*a*d)*(c*x^n)^(2*b*d))/(b*d*(c*x^n)^(2/n)*(1 - E^
(2*a*d)*(c*x^n)^(2*b*d))) + ((E^(2*a*d)*(2 - b*d*n))/(2*(c*x^n)^(2/n)) - (
2*E^(2*a*d)*Hypergeometric2F1[1, -(1/(b*d*n)), 1 - 1/(b*d*n), E^(2*a*d)*(c
*x^n)^(2*b*d)])/(c*x^n)^(2/n))/(b*d*E^(2*a*d))))/(n*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 6074
```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\coth\left(d(a+b\ln\left(c\,x^{n}\right))\right)^{2}}{x^{3}}dx$$

input
```
int(coth(d*(a+b*ln(c*x^n)))^2/x^3,x)
```

output
```
int(coth(d*(a+b*ln(c*x^n)))^2/x^3,x)
```

Fricas [F]

$$\int \frac{\coth^{2}\left(d(a+b\log\left(cx^{n}\right))\right)}{x^{3}}\,dx = \int \frac{\coth\left((b\log\left(cx^{n}\right)+a)d\right)^{2}}{x^{3}}\,dx$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="fricas")
```

output
```
integral(coth(b*d*log(c*x^n) + a*d)^2/x^3, x)
```

Sympy [F]

$$\int \frac{\coth^{2}\left(d(a+b\log\left(cx^{n}\right))\right)}{x^{3}}\,dx = \int \frac{\coth^{2}\left(ad+bd\log\left(cx^{n}\right)\right)}{x^{3}}\,dx$$

input
```
integrate(coth(d*(a+b*ln(c*x**n)))**2/x**3,x)
```

output
```
Integral(coth(a*d + b*d*log(c*x**n))**2/x**3, x)
```

Maxima [F]

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\coth((b\log(cx^n)+a)d)^2}{x^3}\,dx$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="maxima")
```

output
```
-1/2*(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n + 4)/(b*c^(2*b*d)
*d*n*x^2*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n*x^2) + 2*integrate(1/(b*c^(b*d
)*d*n*x^3*e^(b*d*log(x^n) + a*d) + b*d*n*x^3), x) - 2*integrate(1/(b*c^(b*
d)*d*n*x^3*e^(b*d*log(x^n) + a*d) - b*d*n*x^3), x)
```

Giac [F]

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\coth((b\log(cx^n)+a)d)^2}{x^3}\,dx$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="giac")
```

output
```
integrate(coth((b*log(c*x^n) + a)*d)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\coth(d\,(a+b\,\ln(c\,x^n)))^2}{x^3}\,dx$$

input
```
int(coth(d*(a + b*log(c*x^n)))^2/x^3,x)
```

output
```
int(coth(d*(a + b*log(c*x^n)))^2/x^3, x)
```

Reduce [F]

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^3}\,dx = \frac{8e^{2ad}c^{2bd}\left(\int \frac{x^{2bdn}}{x^{4bdn}e^{4ad}c^{4bd}x^3-2x^{2bdn}e^{2ad}c^{2bd}x^3+x^3}\,dx\right)x^2-1}{2x^2}$$

input
```
int(coth(d*(a+b*log(c*x^n)))^2/x^3,x)
```

output
```
(8*e**(2*a*d)*c**(2*b*d)*int(x**(2*b*d*n)/(x**(4*b*d*n)*e**(4*a*d)*c**(4*b
*d)*x**3 - 2*x**(2*b*d*n)*e**(2*a*d)*c**(2*b*d)*x**3 + x**3),x)*x**2 - 1)/
(2*x**2)
```

3.196 $\int \frac{\coth^3(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{\coth^3(a+b\log(cx^n))}{x}\,dx=-\frac{\coth^2(a+b\log(cx^n))}{2bn}+\frac{\log(\sinh(a+b\log(cx^n)))}{bn}$$

output
```
-1/2*coth(a+b*ln(c*x^n))^2/b/n+ln(sinh(a+b*ln(c*x^n)))/b/n
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{\coth^3(a+b\log(cx^n))}{x}\,dx=-\frac{\operatorname{csch}^2(a+b\log(cx^n))-2\log(\sinh(a+b\log(cx^n)))}{2bn}$$

input
```
Integrate[Coth[a + b*Log[c*x^n]]^3/x,x]
```

output
```
-1/2*(Csch[a + b*Log[c*x^n]]^2 - 2*Log[Sinh[a + b*Log[c*x^n]]])/(b*n)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {3039, 3042, 26, 3954, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \coth^3(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int i\tan\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^3d\log(cx^n)}{n}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int \tan\left(\frac{1}{2}(2ia+\pi)+ib\log(cx^n)\right)^3d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3954}$$

$$\frac{i\left(\frac{i\coth^2(a+b\log(cx^n))}{2b}-\int i\coth(a+b\log(cx^n))d\log(cx^n)\right)}{n}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(\frac{i\coth^2(a+b\log(cx^n))}{2b}-i\int \coth(a+b\log(cx^n))d\log(cx^n)\right)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(\frac{i\coth^2(a+b\log(cx^n))}{2b}-i\int -i\tan\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)d\log(cx^n)\right)}{n}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(\frac{i\coth^2(a+b\log(cx^n))}{2b}-\int \tan\left(\frac{1}{2}(2ia+\pi)+ib\log(cx^n)\right)d\log(cx^n)\right)}{n}$$

$$\Big\downarrow \text{3956}$$

$$\frac{i\left(\frac{i\coth^2(a+b\log(cx^n))}{2b}-\frac{i\log(-i\sinh(a+b\log(cx^n)))}{b}\right)}{n}$$

input
```
Int[Coth[a + b*Log[c*x^n]]^3/x,x]
```

output
```
(I*(((I/2)*Coth[a + b*Log[c*x^n]]^2)/b - (I*Log[(-I)*Sinh[a + b*Log[c*x^n]
]])/b))/n
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{-\frac{\coth(a+b\ln(c\,x^n))^2}{2}-\frac{\ln(\coth(a+b\ln(c\,x^n))-1)}{2}-\frac{\ln(\coth(a+b\ln(c\,x^n))+1)}{2}}{nb}$
default	$\frac{-\frac{\coth(a+b\ln(c\,x^n))^2}{2}-\frac{\ln(\coth(a+b\ln(c\,x^n))-1)}{2}-\frac{\ln(\coth(a+b\ln(c\,x^n))+1)}{2}}{nb}$
parallelrisch	$\frac{-2\ln(x)bn+2\ln(\tanh(a+b\ln(c\,x^n)))-2\ln(1-\tanh(a+b\ln(c\,x^n)))-\coth(a+b\ln(c\,x^n))^2}{2bn}$
risch	$\ln(x)-\frac{2a}{nb}-\frac{2\ln(c)}{n}-\frac{2\ln(x^n)}{n}-\frac{i\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}{n}+\frac{i\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)}{n}+\frac{i\pi\,\mathrm{csg}}{}$

input
```
int(coth(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(-1/2*coth(a+b*ln(c*x^n))^2-1/2*ln(coth(a+b*ln(c*x^n))-1)-1/2*ln(cot
h(a+b*ln(c*x^n))+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. 2(41) = 82.

Time = 0.11 (sec) , antiderivative size = 572, normalized size of antiderivative = 13.30

$$\int \frac{\coth^3(a+b\log(cx^n))}{x}\,dx = \text{Too large to display}$$

input
```
integrate(coth(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

output
```
-(b*n*cosh(b*n*log(x) + b*log(c) + a)^4*log(x) + 4*b*n*cosh(b*n*log(x) + b
*log(c) + a)*log(x)*sinh(b*n*log(x) + b*log(c) + a)^3 + b*n*log(x)*sinh(b*
n*log(x) + b*log(c) + a)^4 - 2*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c)
 + a)^2 + b*n*log(x) + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2*log(x) -
 b*n*log(x) + 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - (cosh(b*n*log(x) + b*
log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(
c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*log(x) + b*l
og(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) +
b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b*n*log(x) +
 b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(2*sinh(b*n*log(x)
 + b*log(c) + a)/(cosh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*lo
g(c) + a))) + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3*log(x) - (b*n*log(x
) - 1)*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))/(
b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 4*b*n*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a)^3 + b*n*sinh(b*n*log(x) + b*log(c) + a
)^4 - 2*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(3*b*n*cosh(b*n*log(x) +
 b*log(c) + a)^2 - b*n)*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n + 4*(b*n*c
osh(b*n*log(x) + b*log(c) + a)^3 - b*n*cosh(b*n*log(x) + b*log(c) + a))*si
nh(b*n*log(x) + b*log(c) + a))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\coth^3(a+b\log(cx^n))}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(a+b*ln(c*x**n))**3/x,x)
```

output
```
Exception raised: TypeError >> Invalid NaN comparison
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(41) = 82$.

Time = 0.11 (sec) , antiderivative size = 330, normalized size of antiderivative = 7.67

$$\int \frac{\coth^3(a+b\log(cx^n))}{x}\,dx = -\frac{4\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)}-3}{4\left(bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}-2\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn\right)} - \frac{3\left(2\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)}-1\right)}{4\left(bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}-2\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn\right)} + \frac{2\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)}-3}{4\left(bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}-2\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn\right)} - \frac{3}{4\left(bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}-2\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn\right)} + \frac{\log\left(\frac{\left(c^be^{(b\log(x^n)+a)}+1\right)e^{(-a)}}{c^b}\right)}{bn} + \frac{\log\left(\frac{\left(c^be^{(b\log(x^n)+a)}-1\right)e^{(-a)}}{c^b}\right)}{bn} - \log(x)$$

input
```
integrate(coth(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

output
```
-1/4*(4*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 3)/(b*c^(4*b)*n*e^(4*b*log(x^n) +
 4*a) - 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) - 3/4*(2*c^(2*b)*e^(2*
b*log(x^n) + 2*a) - 1)/(b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 2*b*c^(2*b)*n
*e^(2*b*log(x^n) + 2*a) + b*n) + 1/4*(2*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 3
)/(b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*
a) + b*n) - 3/4/(b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 2*b*c^(2*b)*n*e^(2*b
*log(x^n) + 2*a) + b*n) + log((c^b*e^(b*log(x^n) + a) + 1)*e^(-a)/c^b)/(b*
n) + log((c^b*e^(b*log(x^n) + a) - 1)*e^(-a)/c^b)/(b*n) - log(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(41) = 82$.

Time = 0.18 (sec) , antiderivative size = 137, normalized size of antiderivative = 3.19

$$\int \frac{\coth^3(a + b\log(cx^n))}{x}\,dx$$
$$= -\frac{\log(x^{bn})}{bn} + \frac{\log\left(\sqrt{-2\,x^{2\,bn}|c|^{2\,b}\cos(\pi b\mathrm{sgn}(c) - \pi b)\,e^{(2\,a)} + x^{4\,bn}|c|^{4\,b}e^{(4\,a)} + 1}\right)}{bn}$$
$$- \frac{3\,c^{4\,b}x^{4\,bn}e^{(4\,a)} - 2\,c^{2\,b}x^{2\,bn}e^{(2\,a)} + 3}{2\,(c^{2\,b}x^{2\,bn}e^{(2\,a)} - 1)^2 bn}$$

input
```
integrate(coth(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

output
```
-log(x^(b*n))/(b*n) + log(sqrt(-2*x^(2*b*n)*abs(c)^(2*b)*cos(pi*b*sgn(c) -
 pi*b)*e^(2*a) + x^(4*b*n)*abs(c)^(4*b)*e^(4*a) + 1))/(b*n) - 1/2*(3*c^(4*
b)*x^(4*b*n)*e^(4*a) - 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 3)/((c^(2*b)*x^(2*b*n
)*e^(2*a) - 1)^2*b*n)
```

Mupad [B] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.21

$$\int \frac{\coth^3(a + b\log(cx^n))}{x}\,dx = \frac{2}{b\,n - b\,n\,\mathrm{e}^{2\,a}\,(c\,x^n)^{2\,b}} - \ln(x)$$
$$- \frac{2}{b\,n - 2\,b\,n\,\mathrm{e}^{2\,a}\,(c\,x^n)^{2\,b} + b\,n\,\mathrm{e}^{4\,a}\,(c\,x^n)^{4\,b}}$$
$$+ \frac{\ln\left(\mathrm{e}^{2\,a}\,(c\,x^n)^{2\,b} - 1\right)}{b\,n}$$

input
```
int(coth(a + b*log(c*x^n))^3/x,x)
```

output
```
2/(b*n - b*n*exp(2*a)*(c*x^n)^(2*b)) - log(x) - 2/(b*n - 2*b*n*exp(2*a)*(c
*x^n)^(2*b) + b*n*exp(4*a)*(c*x^n)^(4*b)) + log(exp(2*a)*(c*x^n)^(2*b) - 1
)/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 306, normalized size of antiderivative = 7.12

$$\int \frac{\coth^3(a+b\log(cx^n))}{x}\,dx$$
$$= \frac{x^{4bn}e^{4a}c^{4b}\log\left(x^{bn}e^{a}c^{2b}+c^{b}\right)+x^{4bn}e^{4a}c^{4b}\log\left(x^{bn}e^{a}c^{2b}-c^{b}\right)-x^{4bn}e^{4a}c^{4b}\log(x)\,bn-x^{4bn}e^{4a}c^{4b}-2x^{2bn}e^{2a}}{b}$$

input
```
int(coth(a+b*log(c*x^n))^3/x,x)
```

output
```
(x**(4*b*n)*e**(4*a)*c**(4*b)*log(x**(b*n)*e**a*c**(2*b) + c**b) + x**(4*b
*n)*e**(4*a)*c**(4*b)*log(x**(b*n)*e**a*c**(2*b) - c**b) - x**(4*b*n)*e**(
4*a)*c**(4*b)*log(x)*b*n - x**(4*b*n)*e**(4*a)*c**(4*b) - 2*x**(2*b*n)*e**
(2*a)*c**(2*b)*log(x**(b*n)*e**a*c**(2*b) + c**b) - 2*x**(2*b*n)*e**(2*a)*
c**(2*b)*log(x**(b*n)*e**a*c**(2*b) - c**b) + 2*x**(2*b*n)*e**(2*a)*c**(2*
b)*log(x)*b*n + log(x**(b*n)*e**a*c**(2*b) + c**b) + log(x**(b*n)*e**a*c**
(2*b) - c**b) - log(x)*b*n - 1)/(b*n*(x**(4*b*n)*e**(4*a)*c**(4*b) - 2*x**
(2*b*n)*e**(2*a)*c**(2*b) + 1))
```

3.197 $\int \frac{\coth^4(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \frac{\coth^4(a+b\log(cx^n))}{x}\,dx = -\frac{\coth(a+b\log(cx^n))}{bn}-\frac{\coth^3(a+b\log(cx^n))}{3bn}+\log(x)$$

output
```
-coth(a+b*ln(c*x^n))/b/n-1/3*coth(a+b*ln(c*x^n))^3/b/n+ln(x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{\coth^4(a+b\log(cx^n))}{x}\,dx = -\frac{\coth^3(a+b\log(cx^n))\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\tanh^2(a+b\log(cx^n))\right)}{3bn}$$

input
```
Integrate[Coth[a + b*Log[c*x^n]]^4/x,x]
```

output
```
-1/3*(Coth[a + b*Log[c*x^n]]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[a + b
*Log[c*x^n]]^2])/(b*n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3039, 3042, 3954, 25, 3042, 25, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(a+b\log(cx^n))}{x}\,dx$$

$$\Big\downarrow \text{ 3039}$$

$$\frac{\int \coth^4(a+b\log(cx^n))\,d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \tan\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^4 d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 3954}$$

$$\frac{-\int -\coth^2(a+b\log(cx^n))\,d\log(cx^n)-\frac{\coth^3(a+b\log(cx^n))}{3b}}{n}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \coth^2(a+b\log(cx^n))\,d\log(cx^n)-\frac{\coth^3(a+b\log(cx^n))}{3b}}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{\coth^3(a+b\log(cx^n))}{3b}+\int -\tan\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^2 d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{-\frac{\coth^3(a+b\log(cx^n))}{3b}-\int \tan\left(\frac{1}{2}(2ia+\pi)+ib\log(cx^n)\right)^2 d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 3954}$$

$$\frac{\int 1d\log(cx^n)-\frac{\coth^3(a+b\log(cx^n))}{3b}-\frac{\coth(a+b\log(cx^n))}{b}}{n}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{-\frac{\coth^3(a+b\log(cx^n))}{3b}-\frac{\coth(a+b\log(cx^n))}{b}+\log(cx^n)}{n}$$

input
```
Int[Coth[a + b*Log[c*x^n]]^4/x,x]
```

output
```
(-(Coth[a + b*Log[c*x^n]]/b) - Coth[a + b*Log[c*x^n]]^3/(3*b) + Log[c*x^n]
)/n
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

method	result
parallelrisch	$\frac{-\coth(a+b\ln(c\,x^{n}))^{3}+3\ln(x)bn-3\coth(a+b\ln(c\,x^{n}))}{3bn}$
derivativedivides	$\frac{-\frac{\coth(a+b\ln(c\,x^{n}))^{3}}{3}-\coth(a+b\ln(c\,x^{n}))-\frac{\ln(\coth(a+b\ln(c\,x^{n}))-1)}{2}+\frac{\ln(\coth(a+b\ln(c\,x^{n}))+1)}{2}}{nb}$
default	$\frac{-\frac{\coth(a+b\ln(c\,x^{n}))^{3}}{3}-\coth(a+b\ln(c\,x^{n}))-\frac{\ln(\coth(a+b\ln(c\,x^{n}))-1)}{2}+\frac{\ln(\coth(a+b\ln(c\,x^{n}))+1)}{2}}{nb}$
risch	$\ln(x)-\frac{4\left(3(x^{n})^{4b}c^{4b}e^{4a}e^{2ib\pi\,\mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})^{2}}e^{-2ib\pi\,\mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})\,\mathrm{csgn}(ic)}e^{-2ib\pi\,\mathrm{csgn}(ic\,x^{n})^{3}}e^{2ib\pi\,\mathrm{csgn}}\right.}{3bn\left((x^{n})^{2b}c^{2b}e^{2a}e^{ib\pi\,\mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})^{2}}e^{-}\right.}$

input
```
int(coth(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-coth(a+b*ln(c*x^n))^3+3*ln(x)*b*n-3*coth(a+b*ln(c*x^n)))/b/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(43) = 86.

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 3.80

$$\int \frac{\coth^4(a+b\log(cx^n))}{x}\,dx$$
$$= \frac{(3\,bn\log(x)+4)\sinh(bn\log(x)+b\log(c)+a)^3-4\,\cosh(bn\log(x)+b\log(c)+a)^3-12\,\cosh(bn\log}{3\,(bn\sinh(bn\log(x)+b\log(c)}$$

input
```
integrate(coth(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

output
```
1/3*((3*b*n*log(x) + 4)*sinh(b*n*log(x) + b*log(c) + a)^3 - 4*cosh(b*n*log
(x) + b*log(c) + a)^3 - 12*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x)
 + b*log(c) + a)^2 + 3*((3*b*n*log(x) + 4)*cosh(b*n*log(x) + b*log(c) + a)
^2 - 3*b*n*log(x) - 4)*sinh(b*n*log(x) + b*log(c) + a))/(b*n*sinh(b*n*log(
x) + b*log(c) + a)^3 + 3*(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sin
h(b*n*log(x) + b*log(c) + a))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\coth^4(a+b\log(cx^n))}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(a+b*ln(c*x**n))**4/x,x)
```

output
```
Exception raised: TypeError >> Invalid NaN comparison
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. 2(43) = 86.

Time = 0.13 (sec) , antiderivative size = 499, normalized size of antiderivative = 11.09

$$\begin{aligned}
&\int \frac{\coth^4(a+b\log(cx^n))}{x}\,dx \\
&= -\frac{18\,c^{4\,b}e^{(4\,b\log(x^n)+4\,a)} - 27\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)} + 11}{12\,(bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)} - 3\,bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)} + 3\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)} - bn)} \\
&\quad - \frac{6\,c^{4\,b}e^{(4\,b\log(x^n)+4\,a)} - 15\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)} + 11}{12\,(bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)} - 3\,bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)} + 3\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)} - bn)} \\
&\quad - \frac{2\left(3\,c^{4\,b}e^{(4\,b\log(x^n)+4\,a)} - 3\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)} + 1\right)}{3\,(bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)} - 3\,bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)} + 3\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)} - bn)} \\
&\quad - \frac{3\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)} - 1}{2\,(bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)} - 3\,bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)} + 3\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)} - bn)} \\
&\quad - \frac{2}{3\,(bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)} - 3\,bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)} + 3\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)} - bn)} \\
&\quad + \log(x)
\end{aligned}$$

input
```
integrate(coth(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

output
```
-1/12*(18*c^(4*b)*e^(4*b*log(x^n) + 4*a) - 27*c^(2*b)*e^(2*b*log(x^n) + 2*
a) + 11)/(b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) - 3*b*c^(4*b)*n*e^(4*b*log(x^
n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) - b*n) - 1/12*(6*c^(4*b)*
e^(4*b*log(x^n) + 4*a) - 15*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 11)/(b*c^(6*b
)*n*e^(6*b*log(x^n) + 6*a) - 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^
(2*b)*n*e^(2*b*log(x^n) + 2*a) - b*n) - 2/3*(3*c^(4*b)*e^(4*b*log(x^n) + 4
*a) - 3*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)/(b*c^(6*b)*n*e^(6*b*log(x^n) +
 6*a) - 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^
n) + 2*a) - b*n) - 1/2*(3*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 1)/(b*c^(6*b)*n
*e^(6*b*log(x^n) + 6*a) - 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*
b)*n*e^(2*b*log(x^n) + 2*a) - b*n) - 2/3/(b*c^(6*b)*n*e^(6*b*log(x^n) + 6*
a) - 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n)
+ 2*a) - b*n) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.73

$$\int \frac{\coth^4(a+b\log(cx^n))}{x}\,dx = \frac{\log(x^{bn})}{bn} - \frac{4\left(3\,c^{4\,b}x^{4\,bn}e^{(4\,a)} - 3\,c^{2\,b}x^{2\,bn}e^{(2\,a)} + 2\right)}{3\left(c^{2\,b}x^{2\,bn}e^{(2\,a)} - 1\right)^3 bn}$$

input
```
integrate(coth(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

output
```
log(x^(b*n))/(b*n) - 4/3*(3*c^(4*b)*x^(4*b*n)*e^(4*a) - 3*c^(2*b)*x^(2*b*n
)*e^(2*a) + 2)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^3*b*n)
```

Mupad [B] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.62

$$\int \frac{\coth^4(a+b\log(cx^n))}{x}\,dx = \ln(x) - \frac{\frac{4}{3\,b\,n} + \frac{4\,e^{4\,a}\,(c\,x^n)^{4\,b}}{3\,b\,n}}{3\,e^{2\,a}\,(c\,x^n)^{2\,b} - 3\,e^{4\,a}\,(c\,x^n)^{4\,b} + e^{6\,a}\,(c\,x^n)^{6\,b} - 1} - \frac{4}{3\,b\,n\,\left(e^{2\,a}\,(c\,x^n)^{2\,b} - 1\right)} - \frac{4\,e^{2\,a}\,(c\,x^n)^{2\,b}}{3\,b\,n\,\left(e^{4\,a}\,(c\,x^n)^{4\,b} - 2\,e^{2\,a}\,(c\,x^n)^{2\,b} + 1\right)}$$

input
```
int(coth(a + b*log(c*x^n))^4/x,x)
```

output
```
log(x) - (4/(3*b*n) + (4*exp(4*a)*(c*x^n)^(4*b))/(3*b*n))/(3*exp(2*a)*(c*x
^n)^(2*b) - 3*exp(4*a)*(c*x^n)^(4*b) + exp(6*a)*(c*x^n)^(6*b) - 1) - 4/(3*
b*n*(exp(2*a)*(c*x^n)^(2*b) - 1)) - (4*exp(2*a)*(c*x^n)^(2*b))/(3*b*n*(exp
(4*a)*(c*x^n)^(4*b) - 2*exp(2*a)*(c*x^n)^(2*b) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \frac{\coth^4(a+b\log(cx^n))}{x}\,dx = \frac{-\coth(\log(x^nc)\,b+a)^3-3\coth(\log(x^nc)\,b+a)+3\log(x^nc)\,b}{3bn}$$

input
```
int(coth(a+b*log(c*x^n))^4/x,x)
```

output
```
( - coth(log(x**n*c)*b + a)**3 - 3*coth(log(x**n*c)*b + a) + 3*log(x**n*c)
*b)/(3*b*n)
```

3.198 $\int \frac{\coth^5(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \frac{\coth^5(a+b\log(cx^n))}{x}\,dx = -\frac{\coth^2(a+b\log(cx^n))}{2bn} - \frac{\coth^4(a+b\log(cx^n))}{4bn} + \frac{\log(\sinh(a+b\log(cx^n)))}{bn}$$

output
```
-1/2*coth(a+b*ln(c*x^n))^2/b/n-1/4*coth(a+b*ln(c*x^n))^4/b/n+ln(sinh(a+b*l
n(c*x^n)))/b/n
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int \frac{\coth^5(a+b\log(cx^n))}{x}\,dx = -\frac{4\text{csch}^2(a+b\log(cx^n)) + \text{csch}^4(a+b\log(cx^n)) - 4\log(\sinh(a+b\log(cx^n)))}{4bn}$$

input
```
Integrate[Coth[a + b*Log[c*x^n]]^5/x,x]
```

output
```
-1/4*(4*Csch[a + b*Log[c*x^n]]^2 + Csch[a + b*Log[c*x^n]]^4 - 4*Log[Sinh[a
 + b*Log[c*x^n]]])/(b*n)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {3039, 3042, 26, 3954, 26, 3042, 26, 3954, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^5(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \coth^5(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -i\tan\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^5d\log(cx^n)}{n}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int \tan\left(\frac{1}{2}(2ia+\pi)+ib\log(cx^n)\right)^5d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3954}$$

$$-\frac{i\left(-\int -i\coth^3(a+b\log(cx^n))d\log(cx^n)-\frac{i\coth^4(a+b\log(cx^n))}{4b}\right)}{n}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\left(i\int \coth^3(a+b\log(cx^n))d\log(cx^n)-\frac{i\coth^4(a+b\log(cx^n))}{4b}\right)}{n}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\left(i\int i\tan\left(ia+ib\log\left(cx^n\right)+\frac{\pi}{2}\right)^3d\log\left(cx^n\right)-\frac{i\coth^4(a+b\log(cx^n))}{4b}\right)}{n}$$

$$\Big\downarrow \ 26$$

$$-\frac{i\left(-\int\tan\left(\frac{1}{2}(2ia+\pi)+ib\log\left(cx^n\right)\right)^3d\log\left(cx^n\right)-\frac{i\coth^4(a+b\log(cx^n))}{4b}\right)}{n}$$

$$\Big\downarrow \ 3954$$

$$-\frac{i\left(\int i\coth\left(a+b\log\left(cx^n\right)\right)d\log\left(cx^n\right)-\frac{i\coth^4(a+b\log(cx^n))}{4b}-\frac{i\coth^2(a+b\log(cx^n))}{2b}\right)}{n}$$

$$\Big\downarrow \ 26$$

$$-\frac{i\left(i\int\coth\left(a+b\log\left(cx^n\right)\right)d\log\left(cx^n\right)-\frac{i\coth^4(a+b\log(cx^n))}{4b}-\frac{i\coth^2(a+b\log(cx^n))}{2b}\right)}{n}$$

$$\Big\downarrow \ 3042$$

$$-\frac{i\left(i\int-i\tan\left(ia+ib\log\left(cx^n\right)+\frac{\pi}{2}\right)d\log\left(cx^n\right)-\frac{i\coth^4(a+b\log(cx^n))}{4b}-\frac{i\coth^2(a+b\log(cx^n))}{2b}\right)}{n}$$

$$\Big\downarrow \ 26$$

$$-\frac{i\left(\int\tan\left(\frac{1}{2}(2ia+\pi)+ib\log\left(cx^n\right)\right)d\log\left(cx^n\right)-\frac{i\coth^4(a+b\log(cx^n))}{4b}-\frac{i\coth^2(a+b\log(cx^n))}{2b}\right)}{n}$$

$$\Big\downarrow \ 3956$$

$$-\frac{i\left(\frac{i\log(-i\sinh(a+b\log(cx^n)))}{b}-\frac{i\coth^4(a+b\log(cx^n))}{4b}-\frac{i\coth^2(a+b\log(cx^n))}{2b}\right)}{n}$$

input
```
Int[Coth[a + b*Log[c*x^n]]^5/x,x]
```

output
```
((-I)*(((-1/2*I)*Coth[a + b*Log[c*x^n]]^2)/b - ((I/4)*Coth[a + b*Log[c*x^n
]]^4)/b + (I*Log[(-I)*Sinh[a + b*Log[c*x^n]]])/b))/n
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{-\frac{\coth(a+b\ln(c\,x^n))^4}{4}-\frac{\coth(a+b\ln(c\,x^n))^2}{2}-\frac{\ln(\coth(a+b\ln(c\,x^n))-1)}{2}-\frac{\ln(\coth(a+b\ln(c\,x^n))+1)}{2}}{nb}$
default	$\frac{-\frac{\coth(a+b\ln(c\,x^n))^4}{4}-\frac{\coth(a+b\ln(c\,x^n))^2}{2}-\frac{\ln(\coth(a+b\ln(c\,x^n))-1)}{2}-\frac{\ln(\coth(a+b\ln(c\,x^n))+1)}{2}}{nb}$
parallelrisch	$\frac{-\coth(a+b\ln(c\,x^n))^4-4\ln(x)bn+4\ln(\tanh(a+b\ln(c\,x^n)))-4\ln(1-\tanh(a+b\ln(c\,x^n)))-2\coth(a+b\ln(c\,x^n))^2}{4bn}$
risch	$\ln(x)-\frac{2a}{nb}-\frac{2\ln(c)}{n}-\frac{2\ln(x^n)}{n}-\frac{i\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}{n}+\frac{i\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)}{n}+\frac{i\pi\,\mathrm{csg}}{}$

input
```
int(coth(a+b*ln(c*x^n))^5/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(-1/4*coth(a+b*ln(c*x^n))^4-1/2*coth(a+b*ln(c*x^n))^2-1/2*ln(coth(a+
b*ln(c*x^n))-1)-1/2*ln(coth(a+b*ln(c*x^n))+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1576 vs. $2(62) = 124$.

Time = 0.12 (sec) , antiderivative size = 1576, normalized size of antiderivative = 23.88

$$\int \frac{\coth^5(a + b\log(cx^n))}{x} \, dx = \text{Too large to display}$$

input
```
integrate(coth(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

output
```
-(b*n*cosh(b*n*log(x) + b*log(c) + a)^8*log(x) + 8*b*n*cosh(b*n*log(x) + b
*log(c) + a)*log(x)*sinh(b*n*log(x) + b*log(c) + a)^7 + b*n*log(x)*sinh(b*
n*log(x) + b*log(c) + a)^8 - 4*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c)
 + a)^6 + 4*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^2*log(x) - b*n*log(x) +
 1)*sinh(b*n*log(x) + b*log(c) + a)^6 + 8*(7*b*n*cosh(b*n*log(x) + b*log(c
) + a)^3*log(x) - 3*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a))*sinh
(b*n*log(x) + b*log(c) + a)^5 + 2*(3*b*n*log(x) - 2)*cosh(b*n*log(x) + b*l
og(c) + a)^4 + 2*(35*b*n*cosh(b*n*log(x) + b*log(c) + a)^4*log(x) - 30*(b*
n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*b*n*log(x) - 2)*sinh(b
*n*log(x) + b*log(c) + a)^4 + 8*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^5*l
og(x) - 10*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^3 + (3*b*n*log
(x) - 2)*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^
3 - 4*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^2 + b*n*log(x) + 4*
(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^6*log(x) - 15*(b*n*log(x) - 1)*cosh
(b*n*log(x) + b*log(c) + a)^4 + 3*(3*b*n*log(x) - 2)*cosh(b*n*log(x) + b*l
og(c) + a)^2 - b*n*log(x) + 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - (cosh(b
*n*log(x) + b*log(c) + a)^8 + 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*l
og(x) + b*log(c) + a)^7 + sinh(b*n*log(x) + b*log(c) + a)^8 + 4*(7*cosh(b*
n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^6 - 4*cosh
(b*n*log(x) + b*log(c) + a)^6 + 8*(7*cosh(b*n*log(x) + b*log(c) + a)^3 ...
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\coth^5(a+b\log(cx^n))}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(a+b*ln(c*x**n))**5/x,x)
```

output
```
Exception raised: TypeError >> Invalid NaN comparison
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 855 vs. 2(62) = 124.

Time = 0.17 (sec) , antiderivative size = 855, normalized size of antiderivative = 12.95

$$\int \frac{\coth^5(a+b\log(cx^n))}{x}\,dx = \text{Too large to display}$$

input
```
integrate(coth(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

output
```
-1/24*(48*c^(6*b)*e^(6*b*log(x^n) + 6*a) - 108*c^(4*b)*e^(4*b*log(x^n) + 4
*a) + 88*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 25)/(b*c^(8*b)*n*e^(8*b*log(x^n)
 + 8*a) - 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(
x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) + 1/24*(12*c^(6*
b)*e^(6*b*log(x^n) + 6*a) - 42*c^(4*b)*e^(4*b*log(x^n) + 4*a) + 52*c^(2*b)
*e^(2*b*log(x^n) + 2*a) - 25)/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^
(6*b)*n*e^(6*b*log(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*
b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) - 5/8*(4*c^(6*b)*e^(6*b*log(x^n)
 + 6*a) - 6*c^(4*b)*e^(4*b*log(x^n) + 4*a) + 4*c^(2*b)*e^(2*b*log(x^n) + 2
*a) - 1)/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*n*e^(6*b*log(x^
n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2*b*lo
g(x^n) + 2*a) + b*n) - 5/12*(6*c^(4*b)*e^(4*b*log(x^n) + 4*a) - 4*c^(2*b)*
e^(2*b*log(x^n) + 2*a) + 1)/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6
*b)*n*e^(6*b*log(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*
c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) - 5/12*(4*c^(2*b)*e^(2*b*log(x^n)
+ 2*a) - 1)/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*n*e^(6*b*log
(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2*b
*log(x^n) + 2*a) + b*n) - 5/8/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^
(6*b)*n*e^(6*b*log(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*
b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) + log((c^b*e^(b*log(x^n) + a)...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(62) = 124$.

Time = 0.19 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.59

$$\int \frac{\coth^5(a+b\log(cx^n))}{x}\,dx$$
$$= -\frac{\log(x^{bn})}{bn} + \frac{\log\left(\sqrt{-2\,x^{2\,bn}|c|^{2\,b}\cos(\pi b\mathrm{sgn}(c) - \pi b)\,e^{(2\,a)} + x^{4\,bn}|c|^{4\,b}e^{(4\,a)} + 1}\right)}{bn} - \frac{25\,c^{8\,b}x^{8\,bn}e^{(8\,a)} - 52\,c^{6\,b}x^{6\,bn}e^{(6\,a)} + 102\,c^{4\,b}x^{4\,bn}e^{(4\,a)} - 52\,c^{2\,b}x^{2\,bn}e^{(2\,a)} + 25}{12\,(c^{2\,b}x^{2\,bn}e^{(2\,a)} - 1)^4 bn}$$

input
```
integrate(coth(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

output
```
-log(x^(b*n))/(b*n) + log(sqrt(-2*x^(2*b*n)*abs(c)^(2*b)*cos(pi*b*sgn(c) -
 pi*b)*e^(2*a) + x^(4*b*n)*abs(c)^(4*b)*e^(4*a) + 1))/(b*n) - 1/12*(25*c^(
8*b)*x^(8*b*n)*e^(8*a) - 52*c^(6*b)*x^(6*b*n)*e^(6*a) + 102*c^(4*b)*x^(4*b
*n)*e^(4*a) - 52*c^(2*b)*x^(2*b*n)*e^(2*a) + 25)/((c^(2*b)*x^(2*b*n)*e^(2*
a) - 1)^4*b*n)
```

Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 229, normalized size of antiderivative = 3.47

$$\int \frac{\coth^5(a+b\log(cx^n))}{x}\,dx = \frac{8}{bn-3\,b\,n\,\mathrm{e}^{2\,a}\,{(c\,x^n)}^{2\,b}+3\,b\,n\,\mathrm{e}^{4\,a}\,{(c\,x^n)}^{4\,b}-b\,n\,\mathrm{e}^{6\,a}\,{(c\,x^n)}^{6\,b}} - \ln(x) + \frac{4}{b\,n-b\,n\,\mathrm{e}^{2\,a}\,{(c\,x^n)}^{2\,b}} - \frac{4}{b\,n-4\,b\,n\,\mathrm{e}^{2\,a}\,{(c\,x^n)}^{2\,b}+6\,b\,n\,\mathrm{e}^{4\,a}\,{(c\,x^n)}^{4\,b}-4\,b\,n\,\mathrm{e}^{6\,a}\,{(c\,x^n)}^{6\,b}+b\,n\,\mathrm{e}^{8\,a}\,{(c\,x^n)}^{8\,b}} - \frac{8}{b\,n-2\,b\,n\,\mathrm{e}^{2\,a}\,{(c\,x^n)}^{2\,b}+b\,n\,\mathrm{e}^{4\,a}\,{(c\,x^n)}^{4\,b}} + \frac{\ln\left(\mathrm{e}^{2\,a}\,{(c\,x^n)}^{2\,b}-1\right)}{b\,n}$$

input
```
int(coth(a + b*log(c*x^n))^5/x,x)
```

output
```
8/(b*n - 3*b*n*exp(2*a)*(c*x^n)^(2*b) + 3*b*n*exp(4*a)*(c*x^n)^(4*b) - b*n
*exp(6*a)*(c*x^n)^(6*b)) - log(x) + 4/(b*n - b*n*exp(2*a)*(c*x^n)^(2*b)) -
 4/(b*n - 4*b*n*exp(2*a)*(c*x^n)^(2*b) + 6*b*n*exp(4*a)*(c*x^n)^(4*b) - 4*
b*n*exp(6*a)*(c*x^n)^(6*b) + b*n*exp(8*a)*(c*x^n)^(8*b)) - 8/(b*n - 2*b*n*
exp(2*a)*(c*x^n)^(2*b) + b*n*exp(4*a)*(c*x^n)^(4*b)) + log(exp(2*a)*(c*x^n
)^(2*b) - 1)/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 556, normalized size of antiderivative = 8.42

$$\int \frac{\coth^5(a+b\log(cx^n))}{x}\,dx = \frac{x^{8bn}e^{8a}c^{8b}\log(x^{bn}e^a c^{2b}+c^b)+x^{8bn}e^{8a}c^{8b}\log(x^{bn}e^a c^{2b}-c^b)-x^{8bn}e^{8a}c^{8b}\log(x)\,bn-x^{8bn}e^{8a}c^{8b}-4x^{6bn}e^{6a}}{}$$

input
```
int(coth(a+b*log(c*x^n))^5/x,x)
```

output
```
(x**(8*b*n)*e**(8*a)*c**(8*b)*log(x**(b*n)*e**a*c**(2*b) + c**b) + x**(8*b
*n)*e**(8*a)*c**(8*b)*log(x**(b*n)*e**a*c**(2*b) - c**b) - x**(8*b*n)*e**(
8*a)*c**(8*b)*log(x)*b*n - x**(8*b*n)*e**(8*a)*c**(8*b) - 4*x**(6*b*n)*e**
(6*a)*c**(6*b)*log(x**(b*n)*e**a*c**(2*b) + c**b) - 4*x**(6*b*n)*e**(6*a)*
c**(6*b)*log(x**(b*n)*e**a*c**(2*b) - c**b) + 4*x**(6*b*n)*e**(6*a)*c**(6*
b)*log(x)*b*n + 6*x**(4*b*n)*e**(4*a)*c**(4*b)*log(x**(b*n)*e**a*c**(2*b)
+ c**b) + 6*x**(4*b*n)*e**(4*a)*c**(4*b)*log(x**(b*n)*e**a*c**(2*b) - c**b
) - 6*x**(4*b*n)*e**(4*a)*c**(4*b)*log(x)*b*n - 2*x**(4*b*n)*e**(4*a)*c**(
4*b) - 4*x**(2*b*n)*e**(2*a)*c**(2*b)*log(x**(b*n)*e**a*c**(2*b) + c**b) -
 4*x**(2*b*n)*e**(2*a)*c**(2*b)*log(x**(b*n)*e**a*c**(2*b) - c**b) + 4*x**
(2*b*n)*e**(2*a)*c**(2*b)*log(x)*b*n + log(x**(b*n)*e**a*c**(2*b) + c**b)
+ log(x**(b*n)*e**a*c**(2*b) - c**b) - log(x)*b*n - 1)/(b*n*(x**(8*b*n)*e*
*(8*a)*c**(8*b) - 4*x**(6*b*n)*e**(6*a)*c**(6*b) + 6*x**(4*b*n)*e**(4*a)*c
**(4*b) - 4*x**(2*b*n)*e**(2*a)*c**(2*b) + 1))
```

3.199 $\int (ex)^m \coth (d(a + b \log (cx^n))) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 87

$$\int (ex)^m \coth (d(a + b \log (cx^n))) \, dx = \frac{(ex)^{1+m}}{e(1+m)} - \frac{2(ex)^{1+m} \operatorname{Hypergeometric2F1}\left(1, \frac{1+m}{2bdn}, 1 + \frac{1+m}{2bdn}, e^{2ad}(cx^n)^{2bd}\right)}{e(1+m)}$$

output
```
(e*x)^(1+m)/e/(1+m)-2*(e*x)^(1+m)*hypergeom([1, 1/2*(1+m)/b/d/n],[1+1/2*(1
+m)/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))/e/(1+m)
```

Mathematica [A] (verified)

Time = 8.95 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.82

$$\int (ex)^m \coth (d(a + b \log (cx^n))) \, dx = \frac{x(ex)^m \left(-\operatorname{Hypergeometric2F1}\left(1, \frac{1+m}{2bdn}, 1 + \frac{1+m}{2bdn}, e^{2d(a+b\log(cx^n))}\right) - \frac{e^{2ad}(1+m)(cx^n)^{2bd} \operatorname{Hypergeometric2F1}\left(1, \frac{1+m}{2b\ldots}\right.}{1+m+2bdn}\right.}{1+m}$$

input
```
Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])],x]
```

output
```
(x*(e*x)^m*(-Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n)
, E^(2*d*(a + b*Log[c*x^n]))] - (E^(2*a*d)*(1 + m)*(c*x^n)^(2*b*d)*Hyperge
ometric2F1[1, (1 + m + 2*b*d*n)/(2*b*d*n), (1 + m + 4*b*d*n)/(2*b*d*n), E^
(2*a*d)*(c*x^n)^(2*b*d)])/(1 + m + 2*b*d*n)))/(1 + m)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.41, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {6074, 6072, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \coth (d(a+b\log (cx^n)))\, dx$$

$$\Big\downarrow \text{6074}$$

$$\frac{(ex)^{m+1} (cx^n)^{-\frac{m+1}{n}} \int (cx^n)^{\frac{m+1}{n}-1} \coth (d(a+b\log (cx^n)))\, d(cx^n)}{en}$$

$$\Big\downarrow \text{6072}$$

$$\frac{(ex)^{m+1} (cx^n)^{-\frac{m+1}{n}} \int \frac{(cx^n)^{\frac{m+1}{n}-1} \left(-e^{2ad}(cx^n)^{2bd}-1\right)}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{en}$$

$$\Big\downarrow \text{959}$$

$$\frac{(ex)^{m+1} (cx^n)^{-\frac{m+1}{n}} \left(\frac{n(cx^n)^{\frac{m+1}{n}}}{m+1}-2\int \frac{(cx^n)^{\frac{m+1}{n}-1}}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)\right)}{en}$$

$$\Big\downarrow \text{888}$$

$$\frac{(ex)^{m+1} (cx^n)^{-\frac{m+1}{n}} \left(\frac{n(cx^n)^{\frac{m+1}{n}}}{m+1}-\frac{2n(cx^n)^{\frac{m+1}{n}} \text{Hypergeometric2F1}\left(1,\frac{m+1}{2bdn},\frac{m+1}{2bdn}+1,e^{2ad}(cx^n)^{2bd}\right)}{m+1}\right)}{en}$$

input
```
Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])],x]
```

output
```
((e*x)^(1 + m)*((n*(c*x^n)^((1 + m)/n))/(1 + m) - (2*n*(c*x^n)^((1 + m)/n)
*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n), E^(2*a*d)*
(c*x^n)^(2*b*d)])/(1 + m)))/(e*n*(c*x^n)^((1 + m)/n))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 6074
```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int (ex)^m \coth (d(a+b\ln (c\,x^n)))\,dx$$

input
```
int((e*x)^m*coth(d*(a+b*ln(c*x^n))),x)
```

output
```
int((e*x)^m*coth(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int (ex)^m \coth (d(a+b\log (cx^n)))\, dx=\int (ex)^m \coth ((b\log (cx^n)+a)d)\, dx$$

input
```
integrate((e*x)^m*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral((e*x)^m*coth(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int (ex)^m \coth (d(a+b\log (cx^n)))\, dx=\int (ex)^m \coth (ad+bd\log (cx^n))\, dx$$

input
```
integrate((e*x)**m*coth(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral((e*x)**m*coth(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int (ex)^m \coth (d(a+b\log (cx^n)))\, dx=\int (ex)^m \coth ((b\log (cx^n)+a)d)\, dx$$

input
```
integrate((e*x)^m*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
e^m*x*x^m/(m + 1) - e^m*integrate(x^m/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1)
, x) + e^m*integrate(x^m/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)
```

Giac [F]

$$\int (ex)^m \coth (d(a+b\log (cx^n)))\,dx = \int (ex)^m \coth ((b\log (cx^n)+a)d)\,dx$$

input
```
integrate((e*x)^m*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate((e*x)^m*coth((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \coth (d(a+b\log (cx^n)))\,dx = \int \coth(d\,(a+b\,\ln (c\,x^n)))\,(e\,x)^m\,dx$$

input
```
int(coth(d*(a + b*log(c*x^n)))*(e*x)^m,x)
```

output
```
int(coth(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \coth (d(a+b\log (cx^n)))\,dx = \frac{e^m\left(x^m x+2\left(\int \frac{x^m}{x^{2bdn}e^{2ad}c^{2bd}-1}dx\right)m+2\left(\int \frac{x^m}{x^{2bdn}e^{2ad}c^{2bd}-1}dx\right)\right)}{m+1}$$

input
```
int((e*x)^m*coth(d*(a+b*log(c*x^n))),x)
```

output
```
(e**m*(x**m*x + 2*int(x**m/(x**(2*b*d*n)*e**(2*a*d)*c**(2*b*d) - 1),x)*m +
 2*int(x**m/(x**(2*b*d*n)*e**(2*a*d)*c**(2*b*d) - 1),x)))/(m + 1)
```

3.200 $\int (ex)^m \coth^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 168

$$\int (ex)^m \coth^2(d(a+b\log(cx^n)))\,dx = \frac{(1+m+bdn)(ex)^{1+m}}{bde(1+m)n} + \frac{(ex)^{1+m}\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bden\left(1-e^{2ad}(cx^n)^{2bd}\right)} - \frac{2(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2bdn},1+\frac{1+m}{2bdn},e^{2ad}(cx^n)^{2bd}\right)}{bden}$$

output

```
(b*d*n+m+1)*(e*x)^(1+m)/b/d/e/(1+m)/n+(e*x)^(1+m)*(1+exp(2*a*d)*(c*x^n)^(2
*b*d))/b/d/e/n/(1-exp(2*a*d)*(c*x^n)^(2*b*d))-2*(e*x)^(1+m)*hypergeom([1,
1/2*(1+m)/b/d/n],[1+1/2*(1+m)/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/e/n
```

Mathematica [A] (verified)

Time = 9.73 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.86

$$\int (ex)^m \coth^2(d(a+b\log(cx^n)))\,dx = (ex)^m \left(\frac{x}{1+m} - \frac{e^{-\frac{(1+2m)(a-bn\log(x)+b\log(cx^n))}{bn}}x^{-2m}\left(e^{\frac{(1+2m)(a+b\log(cx^n))}{bn}}(1+m+2bdn)\coth(d(a+b\log(cx^n)))+e^{\frac{(1+2m)(a+b}{bn}}\right.}{}\right.$$

input
```
Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
(e*x)^m*(x/(1 + m) - (E^(((1 + 2*m)*(a + b*Log[c*x^n]))/(b*n))*(1 + m + 2*
b*d*n)*Coth[d*(a + b*Log[c*x^n])] + E^(((1 + 2*m)*(a + b*Log[c*x^n]))/(b*n
))*(1 + m + 2*b*d*n)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(
2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + E^(((1 + 2*m + 2*b*d*n)*(a - b*n*L
og[x] + b*Log[c*x^n]))/(b*n))*(1 + m)*x^(1 + 2*m + 2*b*d*n)*Hypergeometric
2F1[1, (1 + m + 2*b*d*n)/(2*b*d*n), (1 + m + 4*b*d*n)/(2*b*d*n), E^(2*d*(a
 + b*Log[c*x^n]))])/(b*d*E^(((1 + 2*m)*(a - b*n*Log[x] + b*Log[c*x^n]))/(b
*n))*n*(1 + m + 2*b*d*n)*x^(2*m)))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6074, 6072, 1004, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \coth^2(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{6074}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\coth^2(d(a+b\log(cx^n)))\,d(cx^n)}{en}$$

$\Big\downarrow$ 6072

$$\frac{(ex)^{m+1}\,(cx^n)^{-\frac{m+1}{n}}\int \frac{(cx^n)^{\frac{m+1}{n}-1}\left(-e^{2ad}(cx^n)^{2bd}-1\right)^2}{\left(1-e^{2ad}(cx^n)^{2bd}\right)^2}d(cx^n)}{en}$$

$\Big\downarrow$ 1004

$$\frac{(ex)^{m+1}\,(cx^n)^{-\frac{m+1}{n}}\left(\frac{e^{-2ad}\int -\frac{2(cx^n)^{\frac{m+1}{n}-1}\left(\frac{e^{4ad}(m+bdn+1)(cx^n)^{2bd}}{n}+\frac{e^{2ad}(m-bdn+1)}{n}\right)}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{2bd}+\frac{(cx^n)^{\frac{m+1}{n}}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}\right)}{en}$$

$\Big\downarrow$ 27

$$\frac{(ex)^{m+1}\,(cx^n)^{-\frac{m+1}{n}}\left(\frac{(cx^n)^{\frac{m+1}{n}}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}-\frac{e^{-2ad}\int \frac{(cx^n)^{\frac{m+1}{n}-1}\left(\frac{e^{4ad}(m+bdn+1)(cx^n)^{2bd}}{n}+\frac{e^{2ad}(m-bdn+1)}{n}\right)}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{bd}\right)}{en}$$

$\Big\downarrow$ 959

$$\frac{(ex)^{m+1}\,(cx^n)^{-\frac{m+1}{n}}\left(\frac{(cx^n)^{\frac{m+1}{n}}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}-\frac{e^{-2ad}\left(\frac{2(m+1)e^{2ad}\int \frac{(cx^n)^{\frac{m+1}{n}-1}}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{n}-\frac{e^{2ad}(bdn+m+1)(cx^n)^{\frac{m+1}{n}}}{m+1}\right)}{bd}\right)}{en}$$

$\Big\downarrow$ 888

$$\frac{(ex)^{m+1}\,(cx^n)^{-\frac{m+1}{n}}\left(\frac{(cx^n)^{\frac{m+1}{n}}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}-\frac{e^{-2ad}\left(2e^{2ad}(cx^n)^{\frac{m+1}{n}}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2bdn},\frac{m+1}{2bdn}+1,e^{2ad}(cx^n)^{2bd}\right)-\frac{e^{2}}{}\right.}{bd}}{en}$$

input
```
Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
((e*x)^(1 + m)*(((c*x^n)^((1 + m)/n)*(1 + E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d
*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))) - (-((E^(2*a*d)*(1 + m + b*d*n)*(c*x^n)^
((1 + m)/n))/(1 + m)) + 2*E^(2*a*d)*(c*x^n)^((1 + m)/n)*Hypergeometric2F1[
1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(
b*d*E^(2*a*d))))/(e*n*(c*x^n)^((1 + m)/n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 6074
```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int (ex)^m \coth \left(d(a+b\ln \left(c\, x^n\right))\right)^2 dx$$

input
```
int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^2,x)
```

output
```
int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int (ex)^m \coth^2\left(d(a+b\log\left(cx^n\right))\right)\, dx = \int (ex)^m \coth\left((b\log\left(cx^n\right)+a)d\right)^2\, dx$$

input
```
integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

output
```
integral((e*x)^m*coth(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int (ex)^m \coth^2\left(d(a+b\log\left(cx^n\right))\right)\, dx = \int (ex)^m \coth^2\left(ad+bd\log\left(cx^n\right)\right) dx$$

input
```
integrate((e*x)**m*coth(d*(a+b*ln(c*x**n)))**2,x)
```

output
```
Integral((e*x)**m*coth(a*d + b*d*log(c*x**n))**2, x)
```

Maxima [F]

$$\int (ex)^m \coth^2(d(a+b\log(cx^n)))\,dx = \int (ex)^m \coth((b\log(cx^n)+a)d)^2\,dx$$

input
```
integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

output
```
-e^m*(m + 1)*integrate(x^m/(b*c^(b*d)*d*n*e^(b*d*log(x^n) + a*d) + b*d*n),
 x) + e^m*(m + 1)*integrate(x^m/(b*c^(b*d)*d*n*e^(b*d*log(x^n) + a*d) - b*
d*n), x) + (b*c^(2*b*d)*d*e^m*n*x*e^(2*b*d*log(x^n) + 2*a*d + m*log(x)) -
(b*d*e^m*n + 2*e^m*(m + 1))*x*x^m)/((m*n + n)*b*c^(2*b*d)*d*e^(2*b*d*log(x
^n) + 2*a*d) - (m*n + n)*b*d)
```

Giac [F]

$$\int (ex)^m \coth^2(d(a+b\log(cx^n)))\,dx = \int (ex)^m \coth((b\log(cx^n)+a)d)^2\,dx$$

input
```
integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

output
```
integrate((e*x)^m*coth((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \coth^2(d(a+b\log(cx^n)))\,dx = \int \coth(d\,(a+b\,\ln(c\,x^n)))^2\,(e\,x)^m\,dx$$

input
```
int(coth(d*(a + b*log(c*x^n)))^2*(e*x)^m,x)
```

output
```
int(coth(d*(a + b*log(c*x^n)))^2*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \coth^2 (d(a + b\log (cx^n)))\, dx$$
$$= \frac{e^m \left(4e^{2ad}c^{2bd}\left(\int \frac{x^{2bdn+m}}{x^{4bdn}e^{4ad}c^{4bd}-2x^{2bdn}e^{2ad}c^{2bd}+1}dx\right) m + 4e^{2ad}c^{2bd}\left(\int \frac{x^{2bdn+m}}{x^{4bdn}e^{4ad}c^{4bd}-2x^{2bdn}e^{2ad}c^{2bd}+1}dx\right) + x^m x\right)}{m+1}$$

input
```
int((e*x)^m*coth(d*(a+b*log(c*x^n)))^2,x)
```

output
```
(e**m*(4*e**(2*a*d)*c**(2*b*d)*int(x**(2*b*d*n + m)/(x**(4*b*d*n)*e**(4*a*
d)*c**(4*b*d) - 2*x**(2*b*d*n)*e**(2*a*d)*c**(2*b*d) + 1),x)*m + 4*e**(2*a
*d)*c**(2*b*d)*int(x**(2*b*d*n + m)/(x**(4*b*d*n)*e**(4*a*d)*c**(4*b*d) -
2*x**(2*b*d*n)*e**(2*a*d)*c**(2*b*d) + 1),x) + x**m*x))/(m + 1)
```

3.201 $\int (ex)^m \coth^3(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 306

$$\int (ex)^m \coth^3(d(a+b\log(cx^n)))\,dx = \frac{(1+m+bdn)(1+m+2bdn)(ex)^{1+m}}{2b^2d^2e(1+m)n^2} - \frac{(ex)^{1+m}\left(1+e^{2ad}(cx^n)^{2bd}\right)^2}{2bden\left(1-e^{2ad}(cx^n)^{2bd}\right)^2} + \frac{e^{-2ad}(ex)^{1+m}\left(\frac{e^{2ad}(1+m-2bdn)}{n}+\frac{e^{4ad}(1+m+2bdn)(cx^n)^{2bd}}{n}\right)}{2b^2d^2en\left(1-e^{2ad}(cx^n)^{2bd}\right)} - \frac{(1+2m+m^2+2b^2d^2n^2)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2bdn},1+\frac{1+m}{2bdn},e^{2ad}(cx^n)^{2bd}\right)}{b^2d^2e(1+m)n^2}$$

output

```
1/2*(b*d*n+m+1)*(2*b*d*n+m+1)*(e*x)^(1+m)/b^2/d^2/e/(1+m)/n^2-1/2*(e*x)^(1
+m)*(1+exp(2*a*d)*(c*x^n)^(2*b*d))^2/b/d/e/n/(1-exp(2*a*d)*(c*x^n)^(2*b*d)
)^2+1/2*(e*x)^(1+m)*(exp(2*a*d)*(-2*b*d*n+m+1)/n+exp(4*a*d)*(2*b*d*n+m+1)*
(c*x^n)^(2*b*d)/n)/b^2/d^2/e/exp(2*a*d)/n/(1-exp(2*a*d)*(c*x^n)^(2*b*d))-(
2*b^2*d^2*n^2+m^2+2*m+1)*(e*x)^(1+m)*hypergeom([1, 1/2*(1+m)/b/d/n],[1+1/2
*(1+m)/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))/b^2/d^2/e/(1+m)/n^2
```

Mathematica [A] (verified)

Time = 13.14 (sec) , antiderivative size = 600, normalized size of antiderivative = 1.96

$$\int (ex)^m \coth^3(d(a+b\log(cx^n)))\,dx = \frac{x(ex)^m \coth(d(a+b(-n\log(x)+\log(cx^n))))}{1+m}$$
$$-\frac{x(ex)^m \text{csch}^2(bdn\log(x)+d(a+b(-n\log(x)+\log(cx^n))))}{2bdn}$$
$$+\frac{(1+m)x(ex)^m \text{csch}(d(a+b(-n\log(x)+\log(cx^n))))\,\text{csch}(bdn\log(x)+d(a+b(-n\log(x)+\log(cx^n)$$
$$2b^2d^2n^2$$
$$-\frac{(1+2m+m^2+2b^2d^2n^2)\,x^{-m}(ex)^m \text{csch}(d(a+b(-n\log(x)+\log(cx^n))))\left(\frac{x^{1+m}\text{csch}(d(a+b\log(cx^n)))\sinh}{1+m}\right.}{}$$

input
```
Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^3,x]
```

output
```
(x*(e*x)^m*Coth[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))])/(1 + m) - (x*(e*x)^
m*Csch[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]^2)/(2*b*d*n) +
 ((1 + m)*x*(e*x)^m*Csch[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Csch[b*d*n*
Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sinh[b*d*n*Log[x]])/(2*b^2*
d^2*n^2) - ((1 + 2*m + m^2 + 2*b^2*d^2*n^2)*(e*x)^m*Csch[d*(a + b*(-(n*Log
[x]) + Log[c*x^n]))]*((x^(1 + m)*Csch[d*(a + b*Log[c*x^n])]*Sinh[b*d*n*Log
[x]])/(1 + m) + ((E^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Lo
g[x]) + Log[c*x^n]))/(b*n))*(1 + m + 2*b*d*n)*Coth[d*(a + b*Log[c*x^n])] +
 E^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n
]))/(b*n))*(1 + m + 2*b*d*n)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (
1 + m)/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + E^((a*(1 + 2*m + 2*b*d*n))
/(b*n) + (1 + m + 2*b*d*n)*Log[x] + ((1 + 2*m + 2*b*d*n)*(-(n*Log[x]) + Lo
g[c*x^n]))/n)*(1 + m)*Hypergeometric2F1[1, (1 + m + 2*b*d*n)/(2*b*d*n), (1
 + m + 4*b*d*n)/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))])*Sinh[d*(a + b*(-(n
*Log[x]) + Log[c*x^n]))])/(E^(((1 + 2*m)*(a + b*(-(n*Log[x]) + Log[c*x^n])
))/(b*n))*(1 + m)*(1 + m + 2*b*d*n))))/(2*b^2*d^2*n^2*x^m)
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {6074, 6072, 1004, 27, 1064, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \coth^3(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{6074}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\coth^3(d(a+b\log(cx^n)))\,d(cx^n)}{en}$$

$$\Big\downarrow \text{6072}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int \frac{(cx^n)^{\frac{m+1}{n}-1}\left(-e^{2ad}(cx^n)^{2bd}-1\right)^3}{\left(1-e^{2ad}(cx^n)^{2bd}\right)^3}d(cx^n)}{en}$$

$$\Big\downarrow \text{1004}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{e^{-2ad}\int \frac{2(cx^n)^{\frac{m+1}{n}-1}\left(e^{2ad}(cx^n)^{2bd}+1\right)\left(\frac{e^{4ad}(m+2bdn+1)(cx^n)^{2bd}}{n}+\frac{e^{2ad}(m-2bdn+1)}{n}\right)}{\left(1-e^{2ad}(cx^n)^{2bd}\right)^2}d(cx^n)}{4bd}-\frac{(cx^n)^{\frac{m+1}{n}}\left(e^{2ad}\cdots\right.}{2bd\left(1-e^{2ad}(\cdots\right.}\cdots\right.}{en}$$

$$\Big\downarrow \text{27}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{e^{-2ad}\int \frac{(cx^n)^{\frac{m+1}{n}-1}\left(e^{2ad}(cx^n)^{2bd}+1\right)\left(\frac{e^{4ad}(m+2bdn+1)(cx^n)^{2bd}}{n}+\frac{e^{2ad}(m-2bdn+1)}{n}\right)}{\left(1-e^{2ad}(cx^n)^{2bd}\right)^2}d(cx^n)}{2bd}-\frac{(cx^n)^{\frac{m+1}{n}}\left(e^{2ad}(\cdots\right.}{2bd\left(1-e^{2ad}(c\cdots\right.}\cdots\right.}{en}$$

$$\Big\downarrow \text{1064}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{e^{-2ad}\left(\frac{e^{-2ad}\int -\frac{2(cx^n)^{\frac{m+1}{n}-1}\left(\frac{e^{6ad}(m+bdn+1)(m+2bdn+1)(cx^n)^{2bd}}{n^2}+\frac{e^{4ad}(m-2bdn+1)(m-bdn+1)}{n^2}\right)}{1-e^{2ad}(cx^n)^{2bd}}d(cx^n)}{2bd}+\frac{(cx^n)}{}\right.}{2bd}\right.}{en}$$

$\downarrow$ 27

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{e^{-2ad}\left(\frac{(cx^n)^{\frac{m+1}{n}}\left(\frac{e^{4ad}(2bdn+m+1)(cx^n)^{2bd}}{n}+\frac{e^{2ad}(-2bdn+m+1)}{n}\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}-\frac{e^{-2ad}\int \frac{(cx^n)^{\frac{m+1}{n}-1}\left(\frac{e^{6ad}(m+bdn+1)(m+2bd}{n^2}\right.}{1-e^2}}{}\right.}{2bd}\right.}{en}$$

$\downarrow$ 959

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{e^{-2ad}\left(\frac{(cx^n)^{\frac{m+1}{n}}\left(\frac{e^{4ad}(2bdn+m+1)(cx^n)^{2bd}}{n}+\frac{e^{2ad}(-2bdn+m+1)}{n}\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}-\frac{e^{-2ad}\left(\frac{2e^{4ad}\left(2b^2d^2n^2+m^2+2m+1\right)\int \frac{(cx^n)^{\frac{m}{}}}{1-e^{2ad}(c}}{n^2}\right.}{}\right.}{2bd}\right.}{en}$$

$\downarrow$ 888

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{e^{-2ad}\left(\frac{(cx^n)^{\frac{m+1}{n}}\left(\frac{e^{4ad}(2bdn+m+1)(cx^n)^{2bd}}{n}+\frac{e^{2ad}(-2bdn+m+1)}{n}\right)}{bd\left(1-e^{2ad}(cx^n)^{2bd}\right)}-\frac{e^{-2ad}\left(\frac{2e^{4ad}\left(2b^2d^2n^2+m^2+2m+1\right)(cx^n)^{\frac{m+1}{n}}\cdots}{\cdots}\right.}{\cdots}\right)}{2bd}\cdots\right.}{en}$$

input
```
Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^3,x]
```

output
```
((e*x)^(1 + m)*(-1/2*((c*x^n)^((1 + m)/n)*(1 + E^(2*a*d)*(c*x^n)^(2*b*d))^
2)/(b*d*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))^2) + (((c*x^n)^((1 + m)/n)*((E^(2*
a*d)*(1 + m - 2*b*d*n))/n + (E^(4*a*d)*(1 + m + 2*b*d*n)*(c*x^n)^(2*b*d))/
n))/(b*d*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))) - (-((E^(4*a*d)*(1 + m + b*d*n)*
(1 + m + 2*b*d*n)*(c*x^n)^((1 + m)/n))/((1 + m)*n)) + (2*E^(4*a*d)*(1 + 2*
m + m^2 + 2*b^2*d^2*n^2)*(c*x^n)^((1 + m)/n)*Hypergeometric2F1[1, (1 + m)/
(2*b*d*n), 1 + (1 + m)/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/((1 + m)*n))
/(b*d*E^(2*a*d)))/(2*b*d*E^(2*a*d))))/(e*n*(c*x^n)^((1 + m)/n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1064
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Simp[1/(
a*b*n*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c
*(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m
+ n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && LtQ
[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 6074
```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int (ex)^m \coth\left(d(a+b\ln\left(c\,x^n\right))\right)^3 dx$$

input
```
int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^3,x)
```

output
```
int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^3,x)
```

Fricas [F]

$$\int (ex)^m \coth^3\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int (ex)^m \coth\left((b\log\left(cx^n\right)+a)d\right)^3\,dx$$

input
```
integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^3,x, algorithm="fricas")
```

output
```
integral((e*x)^m*coth(b*d*log(c*x^n) + a*d)^3, x)
```

Sympy [F]

$$\int (ex)^m \coth^3\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int (ex)^m \coth^3\left(ad+bd\log\left(cx^n\right)\right)dx$$

input
```
integrate((e*x)**m*coth(d*(a+b*ln(c*x**n)))**3,x)
```

output
```
Integral((e*x)**m*coth(a*d + b*d*log(c*x**n))**3, x)
```

Maxima [F]

$$\int (ex)^m \coth^3 (d(a + b\log (cx^n)))\, dx = \int (ex)^m \coth \left((b\log (cx^n) + a)d\right)^3\, dx$$

input
```
integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^3,x, algorithm="maxima")
```

output
```
-(2*b^2*d^2*e^m*n^2 + (m^2 + 2*m + 1)*e^m)*integrate(1/2*x^m/(b^2*c^(b*d)*
d^2*n^2*e^(b*d*log(x^n) + a*d) + b^2*d^2*n^2), x) + (2*b^2*d^2*e^m*n^2 + (
m^2 + 2*m + 1)*e^m)*integrate(1/2*x^m/(b^2*c^(b*d)*d^2*n^2*e^(b*d*log(x^n)
 + a*d) - b^2*d^2*n^2), x) + (b^2*c^(4*b*d)*d^2*e^m*n^2*x*e^(4*b*d*log(x^n
) + 4*a*d + m*log(x)) + (b^2*d^2*e^m*n^2 + (m^2 + 2*m + 1)*e^m)*x*x^m - (2
*b^2*c^(2*b*d)*d^2*e^m*n^2*e^(2*a*d) + 2*(m*n + n)*b*c^(2*b*d)*d*e^m*e^(2*
a*d) + (m^2 + 2*m + 1)*c^(2*b*d)*e^m*e^(2*a*d))*x*e^(2*b*d*log(x^n) + m*lo
g(x)))/((m*n^2 + n^2)*b^2*c^(4*b*d)*d^2*e^(4*b*d*log(x^n) + 4*a*d) - 2*(m*
n^2 + n^2)*b^2*c^(2*b*d)*d^2*e^(2*b*d*log(x^n) + 2*a*d) + (m*n^2 + n^2)*b^
2*d^2)
```

Giac [F]

$$\int (ex)^m \coth^3 (d(a + b\log (cx^n)))\, dx = \int (ex)^m \coth \left((b\log (cx^n) + a)d\right)^3\, dx$$

input
```
integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^3,x, algorithm="giac")
```

output
```
integrate((e*x)^m*coth((b*log(c*x^n) + a)*d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \coth^3(d(a+b\log(cx^n)))\,dx = \int \coth(d\,(a+b\,\ln(c\,x^n)))^3\,(e\,x)^m\,dx$$

input
```
int(coth(d*(a + b*log(c*x^n)))^3*(e*x)^m,x)
```

output
```
int(coth(d*(a + b*log(c*x^n)))^3*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \coth^3(d(a+b\log(cx^n)))\,dx = \text{too large to display}$$

input
```
int((e*x)^m*coth(d*(a+b*log(c*x^n)))^3,x)
```

output
```
(e**m*(8*x**(4*b*d*n + m)*e**(4*a*d)*c**(4*b*d)*b**2*d**2*n**2*x - 6*x**(4
*b*d*n + m)*e**(4*a*d)*c**(4*b*d)*b*d*m*n*x - 6*x**(4*b*d*n + m)*e**(4*a*d
)*c**(4*b*d)*b*d*n*x + x**(4*b*d*n + m)*e**(4*a*d)*c**(4*b*d)*m**2*x + 2*x
**(4*b*d*n + m)*e**(4*a*d)*c**(4*b*d)*m*x + x**(4*b*d*n + m)*e**(4*a*d)*c*
*(4*b*d)*x + 128*x**(4*b*d*n)*e**(4*a*d)*c**(4*b*d)*int(x**m/(8*x**(6*b*d*
n)*e**(6*a*d)*c**(6*b*d)*b**2*d**2*n**2 - 6*x**(6*b*d*n)*e**(6*a*d)*c**(6*
b*d)*b*d*m*n - 6*x**(6*b*d*n)*e**(6*a*d)*c**(6*b*d)*b*d*n + x**(6*b*d*n)*e
**(6*a*d)*c**(6*b*d)*m**2 + 2*x**(6*b*d*n)*e**(6*a*d)*c**(6*b*d)*m + x**(6
*b*d*n)*e**(6*a*d)*c**(6*b*d) - 24*x**(4*b*d*n)*e**(4*a*d)*c**(4*b*d)*b**2
*d**2*n**2 + 18*x**(4*b*d*n)*e**(4*a*d)*c**(4*b*d)*b*d*m*n + 18*x**(4*b*d*
n)*e**(4*a*d)*c**(4*b*d)*b*d*n - 3*x**(4*b*d*n)*e**(4*a*d)*c**(4*b*d)*m**2
 - 6*x**(4*b*d*n)*e**(4*a*d)*c**(4*b*d)*m - 3*x**(4*b*d*n)*e**(4*a*d)*c**(
4*b*d) + 24*x**(2*b*d*n)*e**(2*a*d)*c**(2*b*d)*b**2*d**2*n**2 - 18*x**(2*b
*d*n)*e**(2*a*d)*c**(2*b*d)*b*d*m*n - 18*x**(2*b*d*n)*e**(2*a*d)*c**(2*b*d
)*b*d*n + 3*x**(2*b*d*n)*e**(2*a*d)*c**(2*b*d)*m**2 + 6*x**(2*b*d*n)*e**(2
*a*d)*c**(2*b*d)*m + 3*x**(2*b*d*n)*e**(2*a*d)*c**(2*b*d) - 8*b**2*d**2*n*
*2 + 6*b*d*m*n + 6*b*d*n - m**2 - 2*m - 1),x)*b**4*d**4*m*n**4 + 128*x**(4
*b*d*n)*e**(4*a*d)*c**(4*b*d)*int(x**m/(8*x**(6*b*d*n)*e**(6*a*d)*c**(6*b*
d)*b**2*d**2*n**2 - 6*x**(6*b*d*n)*e**(6*a*d)*c**(6*b*d)*b*d*m*n - 6*x**(6
*b*d*n)*e**(6*a*d)*c**(6*b*d)*b*d*n + x**(6*b*d*n)*e**(6*a*d)*c**(6*b*d...
```

3.202 $\int \coth^p(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 115

$$\int \coth^p(d(a+b\log(cx^n)))\,dx = x\left(-1-e^{2ad}(cx^n)^{2bd}\right)^p\left(1+e^{2ad}(cx^n)^{2bd}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2bdn},p,-p,1+\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)$$

output
```
x*(-1-exp(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1(1/2/b/d/n,p,-p,1+1/2/b/d/n,ex
p(2*a*d)*(c*x^n)^(2*b*d),-exp(2*a*d)*(c*x^n)^(2*b*d))/((1+exp(2*a*d)*(c*x^
n)^(2*b*d))^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 387 vs. 2(115) = 230.

Time = 0.82 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.37

$$\int \coth^p(d(a+b\log(cx^n)))\,dx$$

$$= \frac{(1+2bdn)x\left(\frac{1+e^{2ad}(cx^n}{-1+e^{2ad}(cx}\right.}{2bde^{2ad}np(cx^n)^{2bd}\operatorname{AppellF1}\left(1+\frac{1}{2bdn},p,1-p,2+\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)+2bde^{2ad}np(cx^n}$$

input
```
Integrate[Coth[d*(a + b*Log[c*x^n])]^p,x]
```

output
```
((1 + 2*b*d*n)*x*((1 + E^(2*a*d)*(c*x^n)^(2*b*d))/(-1 + E^(2*a*d)*(c*x^n)^
(2*b*d)))^p*AppellF1[1/(2*b*d*n), p, -p, 1 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n
)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(2*b*d*E^(2*a*d)*n*p*(c*x^n)^(2*
b*d)*AppellF1[1 + 1/(2*b*d*n), p, 1 - p, 2 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n
)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))] + 2*b*d*E^(2*a*d)*n*p*(c*x^n)^(2*
b*d)*AppellF1[1 + 1/(2*b*d*n), 1 + p, -p, 2 + 1/(2*b*d*n), E^(2*a*d)*(c*x^
n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))] + (1 + 2*b*d*n)*AppellF1[1/(2*b*
d*n), p, -p, 1 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^
n)^(2*b*d))])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6070, 6072, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^p\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

$$\Big\downarrow \text{ 6070}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\coth^p\left(d(a+b\log\left(cx^n\right))\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{ 6072}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\left(-e^{2ad}(cx^n)^{2bd}-1\right)^p\left(1-e^{2ad}(cx^n)^{2bd}\right)^{-p}d(cx^n)}{n}$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{x(cx^n)^{-1/n}\left(-e^{2ad}(cx^n)^{2bd}-1\right)^p\left(e^{2ad}(cx^n)^{2bd}+1\right)^{-p}\int (cx^n)^{\frac{1}{n}-1}\left(1-e^{2ad}(cx^n)^{2bd}\right)^{-p}\left(e^{2ad}(cx^n)^{2bd}+1\right)^p d(}{n}$$

$$\Big\downarrow \text{ 1012}$$

$$x\left(-e^{2ad}(cx^n)^{2bd}-1\right)^p\left(e^{2ad}(cx^n)^{2bd}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2bdn},p,-p,1+\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)$$

input
```
Int[Coth[d*(a + b*Log[c*x^n])]^p,x]
```

output
```
(x*(-1 - E^(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1[1/(2*b*d*n), p, -p, 1 + 1/(
2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(1 + E
^(2*a*d)*(c*x^n)^(2*b*d))^p
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 6070
```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Coth[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [F]

$$\int \coth\left(d(a + b\ln\left(c\,x^{n}\right))\right)^{p}dx$$

input
```
int(coth(d*(a+b*ln(c*x^n)))^p,x)
```

output
```
int(coth(d*(a+b*ln(c*x^n)))^p,x)
```

Fricas [F]

$$\int \coth^{p}\left(d(a + b\log\left(cx^{n}\right))\right)\,dx = \int \coth\left((b\log\left(cx^{n}\right) + a)d\right)^{p}\,dx$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

output
```
integral(coth(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F]

$$\int \coth^{p}\left(d(a + b\log\left(cx^{n}\right))\right)\,dx = \int \coth^{p}\left(d(a + b\log\left(cx^{n}\right))\right)dx$$

input
```
integrate(coth(d*(a+b*ln(c*x**n)))**p,x)
```

output
```
Integral(coth(d*(a + b*log(c*x**n)))**p, x)
```

Maxima [F]

$$\int \coth^p(d(a+b\log(cx^n)))\,dx = \int \coth((b\log(cx^n)+a)d)^p\,dx$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
```

output
```
integrate(coth((b*log(c*x^n) + a)*d)^p, x)
```

Giac [F]

$$\int \coth^p(d(a+b\log(cx^n)))\,dx = \int \coth((b\log(cx^n)+a)d)^p\,dx$$

input
```
integrate(coth(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

output
```
integrate(coth((b*log(c*x^n) + a)*d)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^p(d(a+b\log(cx^n)))\,dx = \int \coth(d(a+b\ln(cx^n)))^p\,dx$$

input
```
int(coth(d*(a + b*log(c*x^n)))^p,x)
```

output
```
int(coth(d*(a + b*log(c*x^n)))^p, x)
```

Reduce [F]

$$\int \coth^p(d(a+b\log(cx^n)))\,dx = \coth(\log(x^n c)\,bd+ad)^p x - \left(\int \frac{\coth(\log(x^n c)\,bd+ad)^p}{\coth(\log(x^n c)\,bd+ad)}dx\right) bdnp + \left(\int \coth(\log(x^n c)\,bd+ad)^p \coth(\log(x^n c)\,bd + ad)\,dx\right) bdnp$$

input
```
int(coth(d*(a+b*log(c*x^n)))^p,x)
```

output
```
coth(log(x**n*c)*b*d + a*d)**p*x - int(coth(log(x**n*c)*b*d + a*d)**p/coth
(log(x**n*c)*b*d + a*d),x)*b*d*n*p + int(coth(log(x**n*c)*b*d + a*d)**p*co
th(log(x**n*c)*b*d + a*d),x)*b*d*n*p
```

3.203 $\int (ex)^m \coth^p(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 135

$$\int (ex)^m \coth^p(d(a+b\log(cx^n)))\,dx = \frac{(ex)^{1+m}\left(-1-e^{2ad}(cx^n)^{2bd}\right)^p\left(1+e^{2ad}(cx^n)^{2bd}\right)^{-p}\operatorname{AppellF1}\left(\frac{1+m}{2bdn},p,-p,1+\frac{1+m}{2bdn},e^{2ad}(cx^n)^{2bd},-e^{2ad}(\right.}{e(1+m)}$$

output
```
(e*x)^(1+m)*(-1-exp(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1(1/2*(1+m)/b/d/n,p,-
p,1+1/2*(1+m)/b/d/n,exp(2*a*d)*(c*x^n)^(2*b*d),-exp(2*a*d)*(c*x^n)^(2*b*d)
)/e/(1+m)/((1+exp(2*a*d)*(c*x^n)^(2*b*d))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.81 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.29

$$\int (ex)^m \coth^p(d(a+b\log(cx^n)))\,dx = \frac{x(ex)^m\left(1-e^{2ad}(cx^n)^{2bd}\right)^p\left(1+e^{2ad}(cx^n)^{2bd}\right)^{-p}\left(\frac{1+e^{2ad}(cx^n)^{2bd}}{-1+e^{2ad}(cx^n)^{2bd}}\right)^p\operatorname{AppellF1}\left(\frac{1+m}{2bdn},p,-p,1+\frac{1+m}{2bdn},e^{2ad}(\right.}{1+m}$$

input
```
Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^p,x]
```

output
```
(x*(e*x)^m*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))^p*((1 + E^(2*a*d)*(c*x^n)^(2*b*
d))/(-1 + E^(2*a*d)*(c*x^n)^(2*b*d)))^p*AppellF1[(1 + m)/(2*b*d*n), p, -p,
 1 + (1 + m)/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*
b*d))])/((1 + m)*(1 + E^(2*a*d)*(c*x^n)^(2*b*d))^p)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6074, 6072, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \coth^p(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{6074}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\coth^p(d(a+b\log(cx^n)))d(cx^n)}{en}$$

$$\Big\downarrow \text{6072}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\left(-e^{2ad}(cx^n)^{2bd}-1\right)^p\left(1-e^{2ad}(cx^n)^{2bd}\right)^{-p}d(cx^n)}{en}$$

$$\Big\downarrow \text{1013}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(-e^{2ad}(cx^n)^{2bd}-1\right)^p\left(e^{2ad}(cx^n)^{2bd}+1\right)^{-p}\int (cx^n)^{\frac{m+1}{n}-1}\left(1-e^{2ad}(cx^n)^{2bd}\right)^{-p}\left(e^{2ad}(cx^n)^{2}\right.}{en}$$

$$\Big\downarrow \text{1012}$$

$$\frac{(ex)^{m+1}\left(-e^{2ad}(cx^n)^{2bd}-1\right)^p\left(e^{2ad}(cx^n)^{2bd}+1\right)^{-p}\text{AppellF1}\left(\frac{m+1}{2bdn},p,-p,\frac{m+1}{2bdn}+1,e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2b}\right.}{e(m+1)}$$

input
```
Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^p,x]
```

output
```
((e*x)^(1 + m)*(-1 - E^(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1[(1 + m)/(2*b*d*
n), p, -p, 1 + (1 + m)/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(
c*x^n)^(2*b*d))])/(e*(1 + m)*(1 + E^(2*a*d)*(c*x^n)^(2*b*d))^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 6072
```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 6074
```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int (ex)^m \coth \left(d(a+b\ln \left(c\,x^{n}\right))\right)^{p}dx$$

input
```
int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^p,x)
```

output
```
int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^p,x)
```

Fricas [F]

$$\int (ex)^m \coth^p\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int (ex)^m \coth\left((b\log\left(cx^n\right)+a)d\right)^p\,dx$$

input
```
integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^m*coth(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \coth^p\left(d(a+b\log\left(cx^n\right))\right)\,dx=\text{Timed out}$$

input
```
integrate((e*x)**m*coth(d*(a+b*ln(c*x**n)))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m \coth^p(d(a+b\log(cx^n)))\,dx = \int (ex)^m \coth((b\log(cx^n)+a)d)^p\,dx$$

input
```
integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*coth((b*log(c*x^n) + a)*d)^p, x)
```

Giac [F]

$$\int (ex)^m \coth^p(d(a+b\log(cx^n)))\,dx = \int (ex)^m \coth((b\log(cx^n)+a)d)^p\,dx$$

input
```
integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^m*coth((b*log(c*x^n) + a)*d)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \coth^p(d(a+b\log(cx^n)))\,dx = \int \coth(d\,(a+b\,\ln(c\,x^n)))^p\,(e\,x)^m\,dx$$

input
```
int(coth(d*(a + b*log(c*x^n)))^p*(e*x)^m,x)
```

output
```
int(coth(d*(a + b*log(c*x^n)))^p*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \coth^p (d(a+b\log (cx^n)))\,dx$$
$$= \frac{e^m \left(x^m \coth \left(\log(x^n c)\,bd+ad\right)^p x - \left(\int \frac{x^m \coth(\log(x^n c)bd+ad)^p}{\coth(\log(x^n c)bd+ad)}dx\right) bdnp + \left(\int x^m \coth \left(\log(x^n c)\,bd+ad\right)^p \coth \right.}{m+1}$$

input
```
int((e*x)^m*coth(d*(a+b*log(c*x^n)))^p,x)
```

output
```
(e**m*(x**m*coth(log(x**n*c)*b*d + a*d)**p*x - int((x**m*coth(log(x**n*c)*
b*d + a*d)**p)/coth(log(x**n*c)*b*d + a*d),x)*b*d*n*p + int(x**m*coth(log(
x**n*c)*b*d + a*d)**p*coth(log(x**n*c)*b*d + a*d),x)*b*d*n*p))/(m + 1)
```

3.204 $\int \frac{\coth^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \frac{\coth^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3bn}$$

output
```
-arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/
b/n-2/3*coth(a+b*ln(c*x^n))^(3/2)/b/n
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \frac{\coth^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right) - \operatorname{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right) + \frac{2}{3}\coth^{\frac{3}{2}}(a+b\log(cx^n))}{bn}$$

input
```
Integrate[Coth[a + b*Log[c*x^n]]^(5/2)/x,x]
```

output
```
-((ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]] - ArcTanh[Sqrt[Coth[a + b*Log[c*x^
n]]]] + (2*Coth[a + b*Log[c*x^n]]^(3/2))/3)/(b*n))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3039, 3042, 3954, 3042, 3957, 25, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{\frac{5}{2}}(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \coth^{\frac{5}{2}}(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(-i\tan\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)\right)^{5/2}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\int \sqrt{\coth(a+b\log(cx^n))}d\log(cx^n)-\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3b}+\int \sqrt{-i\tan\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3957}$$

$$\frac{-\frac{\int -\frac{\sqrt{\coth(a+b\log(cx^n))}}{1-\coth^2(a+b\log(cx^n))}d\coth(a+b\log(cx^n))}{b}-\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3b}}{n}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\sqrt{\coth(a+b\log(cx^n))}}{1-\coth^2(a+b\log(cx^n))}d\coth(a+b\log(cx^n))}{b}-\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3b}}{n}$$

$$\Big\downarrow \; 266$$

$$\frac{\frac{2\int \frac{\coth(a+b\log(cx^n))}{1-\coth^2(a+b\log(cx^n))}d\sqrt{\coth(a+b\log(cx^n))}}{b}-\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3b}}{n}$$

$$\Big\downarrow \; 827$$

$$\frac{\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+b\log(cx^n))}d\sqrt{\coth(a+b\log(cx^n))}-\frac{1}{2}\int \frac{1}{\coth(a+b\log(cx^n))+1}d\sqrt{\coth(a+b\log(cx^n))}\right)}{b}-\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3b}}{n}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+b\log(cx^n))}d\sqrt{\coth(a+b\log(cx^n))}-\frac{1}{2}\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)\right)}{b}-\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3b}}{n}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{2\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)-\frac{1}{2}\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)\right)}{b}-\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3b}}{n}$$

input
```
Int[Coth[a + b*Log[c*x^n]]^(5/2)/x,x]
```

output
```
((2*(-1/2*ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]] + ArcTanh[Sqrt[Coth[a + b*L
og[c*x^n]]]]/2))/b - (2*Coth[a + b*Log[c*x^n]]^(3/2))/(3*b))/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

method	result	siz
derivativedivides	$\frac{-\frac{2\coth\left(a+b\ln\left(c\,x^{n}\right)\right)^{\frac{3}{2}}}{3}-\frac{\ln\left(\sqrt{\coth\left(a+b\ln\left(c\,x^{n}\right)\right)}-1\right)}{2}+\frac{\ln\left(\sqrt{\coth\left(a+b\ln\left(c\,x^{n}\right)\right)}+1\right)}{2}-\arctan\left(\sqrt{\coth\left(a+b\ln\left(c\,x^{n}\right)\right)}\right)}{nb}$	76
default	$\frac{-\frac{2\coth\left(a+b\ln\left(c\,x^{n}\right)\right)^{\frac{3}{2}}}{3}-\frac{\ln\left(\sqrt{\coth\left(a+b\ln\left(c\,x^{n}\right)\right)}-1\right)}{2}+\frac{\ln\left(\sqrt{\coth\left(a+b\ln\left(c\,x^{n}\right)\right)}+1\right)}{2}-\arctan\left(\sqrt{\coth\left(a+b\ln\left(c\,x^{n}\right)\right)}\right)}{nb}$	76

input
```
int(coth(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(-2/3*coth(a+b*ln(c*x^n))^(3/2)-1/2*ln(coth(a+b*ln(c*x^n))^(1/2)-1)+
1/2*ln(coth(a+b*ln(c*x^n))^(1/2)+1)-arctan(coth(a+b*ln(c*x^n))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 626 vs. 2(65) = 130.

Time = 0.12 (sec) , antiderivative size = 626, normalized size of antiderivative = 8.58

$$\int \frac{\coth^{\frac{5}{2}}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx = \text{Too large to display}$$

input
```
integrate(coth(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

output
```
1/6*(6*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 -
1)*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c
) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2
 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*
sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*s
qrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) - 4*
cosh(b*n*log(x) + b*log(c) + a)^2 - 3*(cosh(b*n*log(x) + b*log(c) + a)^2 +
 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(
b*n*log(x) + b*log(c) + a)^2 - 1)*log(-cosh(b*n*log(x) + b*log(c) + a)^2 -
 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(
b*n*log(x) + b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh
(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log
(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*l
og(x) + b*log(c) + a))) - 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x
) + b*log(c) + a) - 4*sinh(b*n*log(x) + b*log(c) + a)^2 - 4*(cosh(b*n*log(
x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) +
 b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)*sqrt(cosh(b*n*log(
x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a)) + 4)/(b*n*cosh(b*n*log
(x) + b*log(c) + a)^2 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\coth^{\frac{5}{2}}(a + b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(coth(a+b*ln(c*x**n))**(5/2)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\coth^{\frac{5}{2}}(a + b\log(cx^n))}{x}\,dx = \int \frac{\coth(b\log(cx^n) + a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate(coth(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

output
```
integrate(coth(b*log(c*x^n) + a)^(5/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\coth^{\frac{5}{2}}(a + b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(coth(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{\coth^{\frac{5}{2}}(a + b\log(cx^n))}{x}\,dx = \frac{\operatorname{atanh}\left(\sqrt{\coth(a + b\,\ln(c\,x^n))}\right)}{b\,n} - \frac{\operatorname{atan}\left(\sqrt{\coth(a + b\,\ln(c\,x^n))}\right)}{b\,n} - \frac{2\coth(a + b\,\ln(c\,x^n))^{3/2}}{3\,b\,n}$$

input
```
int(coth(a + b*log(c*x^n))^(5/2)/x,x)
```

output
```
atanh(coth(a + b*log(c*x^n))^(1/2))/(b*n) - atan(coth(a + b*log(c*x^n))^(1
/2))/(b*n) - (2*coth(a + b*log(c*x^n))^(3/2))/(3*b*n)
```

Reduce [F]

$$\int \frac{\coth^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx=\int \frac{\sqrt{\coth(\log(x^n c)\,b+a)}\,\coth(\log(x^n c)\,b+a)^2}{x}dx$$

input
```
int(coth(a+b*log(c*x^n))^(5/2)/x,x)
```

output
```
int((sqrt(coth(log(x**n*c)*b + a))*coth(log(x**n*c)*b + a)**2)/x,x)
```

3.205 $\int \frac{\coth^{\frac{3}{2}}(a+b\log(cx^n))}{x} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 70

$$\int \frac{\coth^{\frac{3}{2}}(a+b\log(cx^n))}{x} \, dx = \frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{2\sqrt{\coth(a+b\log(cx^n))}}{bn}$$

output
```
arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b
/n-2*coth(a+b*ln(c*x^n))^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.81

$$\int \frac{\coth^{\frac{3}{2}}(a+b\log(cx^n))}{x} \, dx = \frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right) + \operatorname{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right) - 2\sqrt{\coth(a+b\log(cx^n))}}{bn}$$

input
```
Integrate[Coth[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]] + ArcTanh[Sqrt[Coth[a + b*Log[c*x^n]
]]] - 2*Sqrt[Coth[a + b*Log[c*x^n]]])/(b*n)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3039, 3042, 3954, 3042, 3957, 25, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^{\frac{3}{2}}(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \coth^{\frac{3}{2}}(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(-i\tan\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)\right)^{3/2}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\int \frac{1}{\sqrt{\coth(a+b\log(cx^n))}}d\log(cx^n)-\frac{2\sqrt{\coth(a+b\log(cx^n))}}{b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{2\sqrt{\coth(a+b\log(cx^n))}}{b}+\int \frac{1}{\sqrt{-i\tan\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3957}$$

$$\frac{-\frac{\int -\frac{1}{\sqrt{\coth(a+b\log(cx^n))}\left(1-\coth^2(a+b\log(cx^n))\right)}d\coth(a+b\log(cx^n))}{b}-\frac{2\sqrt{\coth(a+b\log(cx^n))}}{b}}{n}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{1}{\sqrt{\coth(a+b\log(cx^n))}\left(1-\coth^2(a+b\log(cx^n))\right)}d\coth(a+b\log(cx^n))}{b}-\frac{2\sqrt{\coth(a+b\log(cx^n))}}{b}}{n}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{2\int \frac{1}{1-\coth^2(a+b\log(cx^n))}d\sqrt{\coth(a+b\log(cx^n))}}{b}-\frac{2\sqrt{\coth(a+b\log(cx^n))}}{b}}{n}$$

$$\Big\downarrow \text{756}$$

$$\frac{\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+b\log(cx^n))}d\sqrt{\coth(a+b\log(cx^n))}+\frac{1}{2}\int \frac{1}{\coth(a+b\log(cx^n))+1}d\sqrt{\coth(a+b\log(cx^n))}\right)}{b}-\frac{2\sqrt{\coth(a+b\log(cx^n))}}{b}}{n}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+b\log(cx^n))}d\sqrt{\coth(a+b\log(cx^n))}+\frac{1}{2}\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)\right)}{b}-\frac{2\sqrt{\coth(a+b\log(cx^n))}}{b}}{n}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{2\left(\frac{1}{2}\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)\right)}{b}-\frac{2\sqrt{\coth(a+b\log(cx^n))}}{b}}{n}$$

input
```
Int[Coth[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
((2*(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/2 + ArcTanh[Sqrt[Coth[a + b*Log[
c*x^n]]]]/2))/b - (2*Sqrt[Coth[a + b*Log[c*x^n]]])/b)/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

method	result	si
derivativedivides	$\frac{-2\sqrt{\coth(a+b\ln(c\,x^n))}-\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}-1\right)}{2}+\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}+1\right)}{2}+\arctan\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)}{nb}$	7
default	$\frac{-2\sqrt{\coth(a+b\ln(c\,x^n))}-\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}-1\right)}{2}+\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}+1\right)}{2}+\arctan\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)}{nb}$	7

input
```
int(coth(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(-2*coth(a+b*ln(c*x^n))^(1/2)-1/2*ln(coth(a+b*ln(c*x^n))^(1/2)-1)+1/
2*ln(coth(a+b*ln(c*x^n))^(1/2)+1)+arctan(coth(a+b*ln(c*x^n))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(64) = 128$.

Time = 0.10 (sec) , antiderivative size = 334, normalized size of antiderivative = 4.77

$$\int \frac{\coth^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx =$$
$$-\frac{4\sqrt{\frac{\cosh(bn\log(x)+b\log(c)+a)}{\sinh(bn\log(x)+b\log(c)+a)}}+2\arctan\left(-\cosh(bn\log(x)+b\log(c)+a)^2-2\cosh(bn\log(x)+b\log(\right.}{}$$

input
```
integrate(coth(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

output
```
-1/2*(4*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) +
a)) + 2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*
log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) +
 a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c)
+ a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 -
 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a)))
 + log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 +
(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sin
h(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt
(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\coth^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(coth(a+b*ln(c*x**n))**(3/2)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\coth^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \int \frac{\coth(b\log(cx^n) + a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(coth(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(coth(b*log(c*x^n) + a)^(3/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\coth^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(coth(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.73

$$\int \frac{\coth^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\operatorname{atan}\left(\sqrt{\coth\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)+\operatorname{atanh}\left(\sqrt{\coth\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)-2\,\sqrt{\coth\left(a+b\,\ln\left(c\,x^n\right)\right)}}{b\,n}$$

input
```
int(coth(a + b*log(c*x^n))^(3/2)/x,x)
```

output
```
(atan(coth(a + b*log(c*x^n))^(1/2)) + atanh(coth(a + b*log(c*x^n))^(1/2))
- 2*coth(a + b*log(c*x^n))^(1/2))/(b*n)
```

Reduce [F]

$$\int \frac{\coth^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{-2\sqrt{\coth(\log(x^nc)\,b+a)}+\left(\int \frac{\sqrt{\coth(\log(x^nc)b+a)}}{\coth(\log(x^nc)b+a)x}dx\right)bn}{bn}$$

input
```
int(coth(a+b*log(c*x^n))^(3/2)/x,x)
```

output
```
( - 2*sqrt(coth(log(x**n*c)*b + a)) + int(sqrt(coth(log(x**n*c)*b + a))/(c
oth(log(x**n*c)*b + a)*x),x)*b*n)/(b*n)
```

3.206 $\int \frac{\sqrt{\coth(a+b\log(cx^n))}}{x} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \frac{\sqrt{\coth(a+b\log(cx^n))}}{x} \, dx = -\frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

output
```
-arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/
b/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{\coth(a+b\log(cx^n))}}{x} \, dx = -\frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right) - \operatorname{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

input
```
Integrate[Sqrt[Coth[a + b*Log[c*x^n]]]/x,x]
```

output
```
-((ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]] - ArcTanh[Sqrt[Coth[a + b*Log[c*x^
n]]]])/(b*n))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3039, 3042, 3957, 25, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\coth(a+b\log(cx^n))}}{x}dx$$

$$\Big\downarrow \textbf{3039}$$

$$\frac{\int \sqrt{\coth(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\int \sqrt{-i\tan\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}d\log(cx^n)}{n}$$

$$\Big\downarrow \textbf{3957}$$

$$-\frac{\int -\frac{\sqrt{\coth(a+b\log(cx^n))}}{1-\coth^2(a+b\log(cx^n))}d\coth(a+b\log(cx^n))}{bn}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\int \frac{\sqrt{\coth(a+b\log(cx^n))}}{1-\coth^2(a+b\log(cx^n))}d\coth(a+b\log(cx^n))}{bn}$$

$$\Big\downarrow \textbf{266}$$

$$\frac{2\int \frac{\coth(a+b\log(cx^n))}{1-\coth^2(a+b\log(cx^n))}d\sqrt{\coth(a+b\log(cx^n))}}{bn}$$

$$\Big\downarrow \textbf{827}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+b\log(cx^n))}d\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}-\frac{1}{2}\int \frac{1}{\coth(a+b\log(cx^n))+1}d\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+b\log(cx^n))}d\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}-\frac{1}{2}\arctan\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)\right)}{bn}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)-\frac{1}{2}\arctan\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)\right)}{bn}$$

input
```
Int[Sqrt[Coth[a + b*Log[c*x^n]]]/x,x]
```

output
```
(2*(-1/2*ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]] + ArcTanh[Sqrt[Coth[a + b*Lo
g[c*x^n]]]]/2))/(b*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^{n}))}-1\right)}{2}+\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^{n}))}+1\right)}{2}-\arctan\left(\sqrt{\coth(a+b\ln(c\,x^{n}))}\right)}{nb}$	61
default	$\frac{-\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^{n}))}-1\right)}{2}+\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^{n}))}+1\right)}{2}-\arctan\left(\sqrt{\coth(a+b\ln(c\,x^{n}))}\right)}{nb}$	61

input
```
int(coth(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(-1/2*ln(coth(a+b*ln(c*x^n))^(1/2)-1)+1/2*ln(coth(a+b*ln(c*x^n))^(1/
2)+1)-arctan(coth(a+b*ln(c*x^n))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(44) = 88$.

Time = 0.10 (sec) , antiderivative size = 305, normalized size of antiderivative = 6.35

$$\int \frac{\sqrt{\coth(a + b\log(cx^n))}}{x} dx$$
$$= \frac{2 \arctan\left(-\cosh(bn\log(x) + b\log(c) + a)^2 - 2\cosh(bn\log(x) + b\log(c) + a)\sinh(bn\log(x) + b\log\right.}{}$$

input
```
integrate(coth(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

output
```
1/2*(2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*l
og(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) +
a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 -
1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a)))
- log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) +
a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (
cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh
(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(
cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F]

$$\int \frac{\sqrt{\coth(a + b\log(cx^n))}}{x} dx = \int \frac{\sqrt{\coth(a + b\log(cx^n))}}{x} dx$$

input
```
integrate(coth(a+b*ln(c*x**n))**(1/2)/x,x)
```

output
```
Integral(sqrt(coth(a + b*log(c*x**n)))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{\coth(a+b\log(cx^n))}}{x}\,dx = \int \frac{\sqrt{\coth(b\log(cx^n)+a)}}{x}\,dx$$

input
```
integrate(coth(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(coth(b*log(c*x^n) + a))/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\coth(a+b\log(cx^n))}}{x}\,dx = \text{Timed out}$$

input
```
integrate(coth(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{\coth(a+b\log(cx^n))}}{x}\,dx = -\frac{\operatorname{atan}\left(\sqrt{\coth(a+b\,\ln(c\,x^n))}\right)-\operatorname{atanh}\left(\sqrt{\coth(a+b\,\ln(c\,x^n))}\right)}{b\,n}$$

input
```
int(coth(a + b*log(c*x^n))^(1/2)/x,x)
```

output
```
-(atan(coth(a + b*log(c*x^n))^(1/2)) - atanh(coth(a + b*log(c*x^n))^(1/2))
)/(b*n)
```

Reduce [F]

$$\int \frac{\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx=\int \frac{\sqrt{\coth\left(\log\left(x^n c\right)b+a\right)}}{x}dx$$

input
```
int(coth(a+b*log(c*x^n))^(1/2)/x,x)
```

output
```
int(sqrt(coth(log(x**n*c)*b + a))/x,x)
```

3.207 $\int \frac{1}{x\sqrt{\coth(a+b\log(cx^n))}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{1}{x\sqrt{\coth(a+b\log(cx^n))}}\,dx = \frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

output
```
arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b
/n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\coth(a+b\log(cx^n))}}\,dx = \frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

input
```
Integrate[1/(x*Sqrt[Coth[a + b*Log[c*x^n]]]),x]
```

output
```
ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c
*x^n]]]]/(b*n)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3039, 3042, 3957, 25, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\sqrt{\coth(a+b\log(cx^n))}}d\log\left(cx^n\right)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{-i\tan\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}}d\log\left(cx^n\right)}{n}$$

$$\Big\downarrow \text{3957}$$

$$-\frac{\int -\frac{1}{\sqrt{\coth(a+b\log(cx^n))}\left(1-\coth^2(a+b\log(cx^n))\right)}d\coth\left(a+b\log\left(cx^n\right)\right)}{bn}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{1}{\sqrt{\coth(a+b\log(cx^n))}\left(1-\coth^2(a+b\log(cx^n))\right)}d\coth\left(a+b\log\left(cx^n\right)\right)}{bn}$$

$$\Big\downarrow \text{266}$$

$$\frac{2\int \frac{1}{1-\coth^2(a+b\log(cx^n))}d\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}{bn}$$

$$\Big\downarrow \text{756}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+b\log(cx^n))}d\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}+\frac{1}{2}\int \frac{1}{\coth(a+b\log(cx^n))+1}d\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+b\log(cx^n))}d\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}+\frac{1}{2}\arctan\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)\right)}{bn}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\left(\frac{1}{2}\arctan\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)\right)}{bn}$$

input
```
Int[1/(x*Sqrt[Coth[a + b*Log[c*x^n]]]),x]
```

output
```
(2*(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/2 + ArcTanh[Sqrt[Coth[a + b*Log[c
*x^n]]]]/2))/(b*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)+\arctan\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)}{nb}$	37
default	$\frac{\operatorname{arctanh}\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)+\arctan\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)}{nb}$	37

input
```
int(1/x/coth(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(arctanh(coth(a+b*ln(c*x^n))^(1/2))+arctan(coth(a+b*ln(c*x^n))^(1/2)
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(43) = 86$.

Time = 0.10 (sec) , antiderivative size = 303, normalized size of antiderivative = 6.45

$$\int \frac{1}{x\sqrt{\coth(a+b\log(cx^n))}}\,dx =$$
$$-\frac{2 \arctan\left(-\cosh(bn\log(x)+b\log(c)+a)^2 - 2\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log\right.}{}$$

input
```
integrate(1/x/coth(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*
log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) +
 a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c)
+ a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 -
 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a)))
 + log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 +
(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sin
h(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt
(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{\coth(a+b\log(cx^n))}}\,dx = \int \frac{1}{x\sqrt{\coth(a+b\log(cx^n))}}\,dx$$

input
```
integrate(1/x/coth(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(coth(a + b*log(c*x**n)))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{\coth\left(a+b\log\left(cx^{n}\right)\right)}}\,dx=\int \frac{1}{x\sqrt{\coth\left(b\log\left(cx^{n}\right)+a\right)}}\,dx$$

input
```
integrate(1/x/coth(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sqrt(coth(b*log(c*x^n) + a))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\coth\left(a+b\log\left(cx^{n}\right)\right)}}\,dx=\text{Timed out}$$

input
```
integrate(1/x/coth(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{1}{x\sqrt{\coth\left(a+b\log\left(cx^{n}\right)\right)}}\,dx=\frac{\operatorname{atan}\left(\sqrt{\coth\left(a+b\,\ln\left(c\,x^{n}\right)\right)}\right)+\operatorname{atanh}\left(\sqrt{\coth\left(a+b\,\ln\left(c\,x^{n}\right)\right)}\right)}{b\,n}$$

input
```
int(1/(x*coth(a + b*log(c*x^n))^(1/2)),x)
```

output
```
(atan(coth(a + b*log(c*x^n))^(1/2)) + atanh(coth(a + b*log(c*x^n))^(1/2)))
/(b*n)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{\sqrt{\coth\left(\log\left(x^nc\right)b+a\right)}}{\coth\left(\log\left(x^nc\right)b+a\right)x}dx$$

input
```
int(1/x/coth(a+b*log(c*x^n))^(1/2),x)
```

output
```
int(sqrt(coth(log(x**n*c)*b + a))/(coth(log(x**n*c)*b + a)*x),x)
```

3.208 $\int \frac{1}{x \coth^{\frac{3}{2}}(a+b \log(cx^n))} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 71

$$\int \frac{1}{x \coth^{\frac{3}{2}}(a+b \log(cx^n))} \, dx = -\frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{2}{bn\sqrt{\coth(a+b\log(cx^n))}}$$

output
```
-arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/
b/n-2/b/n/coth(a+b*ln(c*x^n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.37

$$\int \frac{1}{x \coth^{\frac{3}{2}}(a+b \log(cx^n))} \, dx = \frac{-2 - \arctan\left(\sqrt[4]{\coth^2(a+b\log(cx^n))}\right)\sqrt[4]{\coth^2(a+b\log(cx^n))} + \operatorname{arctanh}\left(\sqrt[4]{\coth^2(a+b\log(cx^n))}\right)}{bn\sqrt{\coth(a+b\log(cx^n))}}$$

input
```
Integrate[1/(x*Coth[a + b*Log[c*x^n]]^(3/2)),x]
```

output
```
(-2 - ArcTan[(Coth[a + b*Log[c*x^n]]^2)^(1/4)]*(Coth[a + b*Log[c*x^n]]^2)^
(1/4) + ArcTanh[(Coth[a + b*Log[c*x^n]]^2)^(1/4)]*(Coth[a + b*Log[c*x^n]]^
2)^(1/4))/(b*n*Sqrt[Coth[a + b*Log[c*x^n]]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3039, 3042, 3955, 3042, 3957, 25, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\coth^{\frac{3}{2}}(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\coth^{\frac{3}{2}}(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\left(-i\tan\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)\right)^{3/2}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3955}$$

$$\frac{\int \sqrt{\coth(a+b\log(cx^n))}d\log(cx^n)-\frac{2}{b\sqrt{\coth(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{2}{b\sqrt{\coth(a+b\log(cx^n))}}+\int \sqrt{-i\tan\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3957}$$

$$\frac{-\frac{\int -\frac{\sqrt{\coth(a+b\log(cx^n))}}{1-\coth^2(a+b\log(cx^n))}d\coth(a+b\log(cx^n))}{b}-\frac{2}{b\sqrt{\coth(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\sqrt{\coth(a+b\log(cx^n))}}{1-\coth^2(a+b\log(cx^n))}d\coth(a+b\log(cx^n))}{b}-\frac{2}{b\sqrt{\coth(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{2\int \frac{\coth(a+b\log(cx^n))}{1-\coth^2(a+b\log(cx^n))}d\sqrt{\coth(a+b\log(cx^n))}}{b}-\frac{2}{b\sqrt{\coth(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{827}$$

$$\frac{\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+b\log(cx^n))}d\sqrt{\coth(a+b\log(cx^n))}-\frac{1}{2}\int \frac{1}{\coth(a+b\log(cx^n))+1}d\sqrt{\coth(a+b\log(cx^n))}\right)}{b}-\frac{2}{b\sqrt{\coth(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+b\log(cx^n))}d\sqrt{\coth(a+b\log(cx^n))}-\frac{1}{2}\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)\right)}{b}-\frac{2}{b\sqrt{\coth(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{2\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)-\frac{1}{2}\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)\right)}{b}-\frac{2}{b\sqrt{\coth(a+b\log(cx^n))}}}{n}$$

input

```
Int[1/(x*Coth[a + b*Log[c*x^n]]^(3/2)),x]
```

output

```
((2*(-1/2*ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]] + ArcTanh[Sqrt[Coth[a + b*L
og[c*x^n]]]]/2))/b - 2/(b*Sqrt[Coth[a + b*Log[c*x^n]]]))/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{-\frac{2}{\sqrt{\coth(a+b\ln(c\,x^n))}}-\arctan\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)+\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}+1\right)}{2}-\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}-1\right)}{2}}{nb}$	76
default	$\frac{-\frac{2}{\sqrt{\coth(a+b\ln(c\,x^n))}}-\arctan\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)+\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}+1\right)}{2}-\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}-1\right)}{2}}{nb}$	76

input
```
int(1/x/coth(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(-2/coth(a+b*ln(c*x^n))^(1/2)-arctan(coth(a+b*ln(c*x^n))^(1/2))+1/2*
ln(coth(a+b*ln(c*x^n))^(1/2)+1)-1/2*ln(coth(a+b*ln(c*x^n))^(1/2)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. 2(65) = 130.

Time = 0.11 (sec) , antiderivative size = 625, normalized size of antiderivative = 8.80

$$\int \frac{1}{x\coth^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/coth(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
1/2*(2*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 +
1)*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c
) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2
 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*
sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*s
qrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) - 4*
cosh(b*n*log(x) + b*log(c) + a)^2 - (cosh(b*n*log(x) + b*log(c) + a)^2 + 2
*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*
n*log(x) + b*log(c) + a)^2 + 1)*log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2
*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*
n*log(x) + b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b
*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x
) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log
(x) + b*log(c) + a))) - 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x)
+ b*log(c) + a) - 4*sinh(b*n*log(x) + b*log(c) + a)^2 - 4*(cosh(b*n*log(x)
 + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b
*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n*log(x)
 + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a)) - 4)/(b*n*cosh(b*n*log(x
) + b*log(c) + a)^2 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*lo...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x \coth^{\frac{3}{2}}(a + b \log(cx^n))} \, dx = \text{Timed out}$$

input
```
integrate(1/x/coth(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x\coth^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\coth(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/coth(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*coth(b*log(c*x^n) + a)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\coth^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/x/coth(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 3.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\coth^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\operatorname{atanh}\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)}{b\,n} - \frac{\operatorname{atan}\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)}{b\,n} - \frac{2}{b\,n\,\sqrt{\coth(a+b\ln(c\,x^n))}}$$

input
```
int(1/(x*coth(a + b*log(c*x^n))^(3/2)),x)
```

output
```
atanh(coth(a + b*log(c*x^n))^(1/2))/(b*n) - atan(coth(a + b*log(c*x^n))^(1
/2))/(b*n) - 2/(b*n*coth(a + b*log(c*x^n))^(1/2))
```

Reduce [F]

$$\int \frac{1}{x\coth^{\frac{3}{2}}(a+b\log(cx^n))}\,dx=\int \frac{\sqrt{\coth(\log(x^nc)\,b+a)}}{\coth(\log(x^nc)\,b+a)^2x}dx$$

input
```
int(1/x/coth(a+b*log(c*x^n))^(3/2),x)
```

output
```
int(sqrt(coth(log(x**n*c)*b + a))/(coth(log(x**n*c)*b + a)**2*x),x)
```

3.209 $\int \frac{1}{x\coth^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{1}{x\coth^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{2}{3bn\coth^{\frac{3}{2}}(a+b\log(cx^n))}$$

output
```
arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b
/n-2/3/b/n/coth(a+b*ln(c*x^n))^(3/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.40

$$\int \frac{1}{x\coth^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{-2+3\arctan\left(\sqrt[4]{\coth^2(a+b\log(cx^n))}\right)\coth^2(a+b\log(cx^n))^{3/4}+3\operatorname{arctanh}\left(\sqrt[4]{\coth^2(a+b\log(cx^n))}\right)\ldots}{3bn\coth^{\frac{3}{2}}(a+b\log(cx^n))}$$

input
```
Integrate[1/(x*Coth[a + b*Log[c*x^n]]^(5/2)),x]
```

output
```
(-2 + 3*ArcTan[(Coth[a + b*Log[c*x^n]]^2)^(1/4)]*(Coth[a + b*Log[c*x^n]]^2
)^(3/4) + 3*ArcTanh[(Coth[a + b*Log[c*x^n]]^2)^(1/4)]*(Coth[a + b*Log[c*x^
n]]^2)^(3/4))/(3*b*n*Coth[a + b*Log[c*x^n]]^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3039, 3042, 3955, 3042, 3957, 25, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \coth^{\frac{5}{2}}(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\coth^{\frac{5}{2}}(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\left(-i\tan\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)\right)^{5/2}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3955}$$

$$\frac{\int \frac{1}{\sqrt{\coth(a+b\log(cx^n))}}d\log(cx^n) - \frac{2}{3b\coth^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{2}{3b\coth^{\frac{3}{2}}(a+b\log(cx^n))} + \int \frac{1}{\sqrt{-i\tan\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3957}$$

$$-\frac{\frac{\int -\frac{1}{\sqrt{\coth(a+b\log(cx^n))}\left(1-\coth^2(a+b\log(cx^n))\right)}d\coth(a+b\log(cx^n))}{b}-\frac{2}{3b\coth^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{1}{\sqrt{\coth(a+b\log(cx^n))}\left(1-\coth^2(a+b\log(cx^n))\right)}d\coth(a+b\log(cx^n))}{b}-\frac{2}{3b\coth^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$\Big\downarrow$ 266

$$\frac{\frac{2\int \frac{1}{1-\coth^2(a+b\log(cx^n))}d\sqrt{\coth(a+b\log(cx^n))}}{b}-\frac{2}{3b\coth^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$\Big\downarrow$ 756

$$\frac{\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+b\log(cx^n))}d\sqrt{\coth(a+b\log(cx^n))}+\frac{1}{2}\int \frac{1}{\coth(a+b\log(cx^n))+1}d\sqrt{\coth(a+b\log(cx^n))}\right)}{b}-\frac{2}{3b\coth^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$\Big\downarrow$ 216

$$\frac{\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+b\log(cx^n))}d\sqrt{\coth(a+b\log(cx^n))}+\frac{1}{2}\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)\right)}{b}-\frac{2}{3b\coth^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$\Big\downarrow$ 219

$$\frac{\frac{2\left(\frac{1}{2}\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)\right)}{b}-\frac{2}{3b\coth^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

input

```
Int[1/(x*Coth[a + b*Log[c*x^n]]^(5/2)),x]
```

output

```
((2*(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/2 + ArcTanh[Sqrt[Coth[a + b*Log[
c*x^n]]]]/2))/b - 2/(3*b*Coth[a + b*Log[c*x^n]]^(3/2)))/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{-\frac{2}{3\coth(a+b\ln(c\,x^n))^{\frac{3}{2}}}+\arctan\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)+\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}+1\right)}{2}-\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}-1\right)}{2}}{nb}$	74
default	$\frac{-\frac{2}{3\coth(a+b\ln(c\,x^n))^{\frac{3}{2}}}+\arctan\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)+\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}+1\right)}{2}-\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}-1\right)}{2}}{nb}$	74

input
```
int(1/x/coth(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(-2/3/coth(a+b*ln(c*x^n))^(3/2)+arctan(coth(a+b*ln(c*x^n))^(1/2))+1/
2*ln(coth(a+b*ln(c*x^n))^(1/2)+1)-1/2*ln(coth(a+b*ln(c*x^n))^(1/2)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1104 vs. 2(64) = 128.

Time = 0.10 (sec) , antiderivative size = 1104, normalized size of antiderivative = 15.33

$$\int \frac{1}{x\coth^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/coth(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(4*cosh(b*n*log(x) + b*log(c) + a)^4 + 16*cosh(b*n*log(x) + b*log(c)
+ a)*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*sinh(b*n*log(x) + b*log(c) + a)
^4 + 8*(3*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*log(c
) + a)^2 + 6*(cosh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) +
 a)^4 + 2*(3*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*lo
g(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b
*log(c) + a)^3 + cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(
c) + a) + 1)*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x)
 + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log
(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a
)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) +
 a))) + 8*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*(cosh(b*n*log(x) + b*log(c
) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) +
a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*log(x) + b*log(c)
 + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log
(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 + cosh(b*n*log(x) + b*lo
g(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(-cosh(b*n*log(x) + b*l
og(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*lo...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x \coth^{\frac{5}{2}}(a + b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/x/coth(a+b*ln(c*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x\coth^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\coth(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/coth(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*coth(b*log(c*x^n) + a)^(5/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\coth^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/x/coth(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\coth^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\operatorname{atan}\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)}{b\,n} + \frac{\operatorname{atanh}\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)}{b\,n} - \frac{2}{3\,b\,n\coth(a+b\ln(c\,x^n))^{3/2}}$$

input
```
int(1/(x*coth(a + b*log(c*x^n))^(5/2)),x)
```

output
```
atan(coth(a + b*log(c*x^n))^(1/2))/(b*n) + atanh(coth(a + b*log(c*x^n))^(1
/2))/(b*n) - 2/(3*b*n*coth(a + b*log(c*x^n))^(3/2))
```

Reduce [F]

$$\int \frac{1}{x\coth^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{\sqrt{\coth(\log(x^n c)\,b+a)}}{\coth(\log(x^n c)\,b+a)^3 x}dx$$

input
```
int(1/x/coth(a+b*log(c*x^n))^(5/2),x)
```

output
```
int(sqrt(coth(log(x**n*c)*b + a))/(coth(log(x**n*c)*b + a)**3*x),x)
```

3.210 $\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \frac{(b-2c)\operatorname{arctanh}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4c^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}} - \frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2c}$$

output
```
1/4*(b-2*c)*arctanh(1/2*(b+2*c*coth(x)^2)/c^(1/2)/(a+b*coth(x)^2+c*coth(x)
^4)^(1/2))/c^(3/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2)/(a+b+c)^(1/2)
/(a+b*coth(x)^2+c*coth(x)^4)^(1/2))/(a+b+c)^(1/2)-1/2*(a+b*coth(x)^2+c*cot
h(x)^4)^(1/2)/c
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.17

$$\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \frac{(b-2c)\operatorname{arctanh}\left(\frac{b+2c+2c\operatorname{csch}^2(x)}{2\sqrt{c}\sqrt{a+b+c+(b+2c)\operatorname{csch}^2(x)+c\operatorname{csch}^4(x)}}\right)}{4c^{3/2}} + \frac{1}{2}\left(\frac{\operatorname{arctanh}\left(\frac{2(a+b+c)+(b+2c)\operatorname{csch}^2(x)}{2\sqrt{a+b+c}\sqrt{a+b+c+(b+2c)\operatorname{csch}^2(x)+c\operatorname{csch}^4(x)}}\right)}{\sqrt{a+b+c}} - \frac{\sqrt{a+b+c+(b+2c)\operatorname{csch}^2(x)+c\operatorname{csch}^4(x)}}{c}\right)$$

input
```
Integrate[Coth[x]^5/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

output
```
((b - 2*c)*ArcTanh[(b + 2*c + 2*c*Csch[x]^2)/(2*Sqrt[c]*Sqrt[a + b + c + (
b + 2*c)*Csch[x]^2 + c*Csch[x]^4])])/(4*c^(3/2)) + (ArcTanh[(2*(a + b + c)
 + (b + 2*c)*Csch[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b + c + (b + 2*c)*Csch
[x]^2 + c*Csch[x]^4])]/Sqrt[a + b + c] - Sqrt[a + b + c + (b + 2*c)*Csch[x
]^2 + c*Csch[x]^4]/c)/2
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.68, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3042, 26, 4184, 1578, 1267, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cot(ix)^5}{\sqrt{a-b\cot(ix)^2+c\cot(ix)^4}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cot(ix)^5}{\sqrt{c\cot(ix)^4-b\cot(ix)^2+a}}dx$$

$$\Big\downarrow \text{4184}$$

$$-\int -\frac{i\coth^5(x)}{(1-\coth^2(x))\sqrt{c\coth^4(x)+b\coth^2(x)+a}}d(-i\coth(x))$$

$$\Big\downarrow \text{1578}$$

$$-\frac{1}{2}\int -\frac{\coth^2(x)}{(1-\coth^2(x))\sqrt{-c\coth^2(x)+ib\coth(x)+a}}d(-\coth^2(x))$$

$$\Big\downarrow \text{1267}$$

$$\frac{1}{2}\left(-\frac{\int \frac{b-(b-2c)\coth^2(x)}{2(1-\coth^2(x))\sqrt{-c\coth^2(x)+ib\coth(x)+a}}d(-\coth^2(x))}{c}-\frac{\sqrt{a+ib\coth(x)-c\coth^2(x)}}{c}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{b-(b-2c)\coth^2(x)}{(1-\coth^2(x))\sqrt{-c\coth^2(x)+ib\coth(x)+a}}d(-\coth^2(x))}{2c}-\frac{\sqrt{a+ib\coth(x)-c\coth^2(x)}}{c}\right)$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(-\frac{(b-2c)\int \frac{1}{\sqrt{-c\coth^2(x)+ib\coth(x)+a}}d(-\coth^2(x))+2c\int \frac{1}{(1-\coth^2(x))\sqrt{-c\coth^2(x)+ib\coth(x)+a}}d(-\coth^2(x))}{2c}-\right.$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(-\frac{2(b-2c)\int \frac{1}{\coth^2(x)+4c}d\left(-\frac{b+2ic\coth(x)}{\sqrt{-c\coth^2(x)+ib\coth(x)+a}}\right)+2c\int \frac{1}{(1-\coth^2(x))\sqrt{-c\coth^2(x)+ib\coth(x)+a}}d(-\coth^2(x)}{2c}\right.$$

$$\Big\downarrow \textbf{219}$$

$$\frac{1}{2}\left(-\frac{2c\int \frac{1}{(1-\coth^2(x))\sqrt{-c\coth^2(x)+ib\coth(x)+a}}d(-\coth^2(x))-\frac{i(b-2c)\arctan\left(\frac{\coth(x)}{2\sqrt{c}}\right)}{\sqrt{c}}}{2c}-\frac{\sqrt{a+ib\coth(x)-c\coth^2(x}}{c}\right.$$

$$\Big\downarrow \textbf{1154}$$

$$\frac{1}{2}\left(-\frac{-4c\int \frac{1}{\coth^2(x)+4(a+b+c)}d\frac{2a+b+i(b+2c)\coth(x)}{\sqrt{-c\coth^2(x)+ib\coth(x)+a}}-\frac{i(b-2c)\arctan\left(\frac{\coth(x)}{2\sqrt{c}}\right)}{\sqrt{c}}}{2c}-\frac{\sqrt{a+ib\coth(x)-c\coth^2(x)}}{c}\right)$$

$$\Big\downarrow \textbf{219}$$

$$\frac{1}{2}\left(-\frac{\frac{2ic\arctan\left(\frac{\coth(x)}{2\sqrt{a+b+c}}\right)}{\sqrt{a+b+c}}-\frac{i(b-2c)\arctan\left(\frac{\coth(x)}{2\sqrt{c}}\right)}{\sqrt{c}}}{2c}-\frac{\sqrt{a+ib\coth(x)-c\coth^2(x)}}{c}\right)$$

input
```
Int[Coth[x]^5/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

output
```
(-1/2*(((-I)*(b - 2*c)*ArcTan[Coth[x]/(2*Sqrt[c])])/Sqrt[c] + ((2*I)*c*Arc
Tan[Coth[x]/(2*Sqrt[a + b + c])])/Sqrt[a + b + c])/c - Sqrt[a + I*b*Coth[x
] - c*Coth[x]^2]/c)/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4184
```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> Simp[-f/e   Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)),
x], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[
n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.10

method	result
derivativedivides	$-\frac{\ln\left(\frac{\frac{b}{2}+c\coth(x)^2}{\sqrt{c}}+\sqrt{a+b\coth(x)^2+c\coth(x)^4}\right)}{2\sqrt{c}}-\frac{\sqrt{a+b\coth(x)^2+c\coth(x)^4}}{2c}+\frac{b\ln\left(\frac{\frac{b}{2}+c\coth(x)^2}{\sqrt{c}}+\sqrt{a+b\cot\right.}{4c^{\frac{3}{2}}}$
default	$-\frac{\ln\left(\frac{\frac{b}{2}+c\coth(x)^2}{\sqrt{c}}+\sqrt{a+b\coth(x)^2+c\coth(x)^4}\right)}{2\sqrt{c}}-\frac{\sqrt{a+b\coth(x)^2+c\coth(x)^4}}{2c}+\frac{b\ln\left(\frac{\frac{b}{2}+c\coth(x)^2}{\sqrt{c}}+\sqrt{a+b\cot\right.}{4c^{\frac{3}{2}}}$

input
```
int(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln((1/2*b+c*coth(x)^2)/c^(1/2)+(a+b*coth(x)^2+c*coth(x)^4)^(1/2))/c^(
1/2)-1/2*(a+b*coth(x)^2+c*coth(x)^4)^(1/2)/c+1/4*b/c^(3/2)*ln((1/2*b+c*cot
h(x)^2)/c^(1/2)+(a+b*coth(x)^2+c*coth(x)^4)^(1/2))+1/2/(a+b+c)^(1/2)*arcta
nh(1/2*(b*coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+c)^(1/2)/(a+b*coth(x)^2+c*co
th(x)^4)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2086 vs. $2(111) = 222$.

Time = 1.02 (sec) , antiderivative size = 8951, normalized size of antiderivative = 66.30

$$\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$$

input
```
integrate(coth(x)**5/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
```

output
```
Integral(coth(x)**5/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)
```

Maxima [F]

$$\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \int \frac{\coth(x)^5}{\sqrt{c\coth(x)^4+b\coth(x)^2+a}}\,dx$$

input
```
integrate(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(coth(x)^5/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Giac [F]

$$\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \int \frac{\coth(x)^5}{\sqrt{c\coth(x)^4+b\coth(x)^2+a}}\,dx$$

input
```
integrate(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(coth(x)^5/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \int \frac{\coth(x)^5}{\sqrt{c\coth(x)^4+b\coth(x)^2+a}}\,dx$$

input
```
int(coth(x)^5/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
```

output
```
int(coth(x)^5/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \int \frac{\sqrt{\coth(x)^4c+\coth(x)^2b+a}\,\coth(x)^5}{\coth(x)^4c+\coth(x)^2b+a}\,dx$$

input
```
int(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

output
```
int((sqrt(coth(x)**4*c + coth(x)**2*b + a)*coth(x)**5)/(coth(x)**4*c + cot
h(x)**2*b + a),x)
```

3.211 $\int \frac{\coth^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \frac{\coth^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{c}} + \frac{\operatorname{arctanh}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

output
```
-1/2*arctanh(1/2*(b+2*c*coth(x)^2)/c^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/
2))/c^(1/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2)/(a+b+c)^(1/2)/(a+b*c
oth(x)^2+c*coth(x)^4)^(1/2))/(a+b+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.15

$$\int \frac{\coth^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \frac{1}{2}\left(-\frac{\operatorname{arctanh}\left(\frac{b+2c+2c\operatorname{csch}^2(x)}{2\sqrt{c}\sqrt{a+b+c+(b+2c)\operatorname{csch}^2(x)+c\operatorname{csch}^4(x)}}\right)}{\sqrt{c}} + \frac{\operatorname{arctanh}\left(\frac{2(a+b+c)+(b+2c)\operatorname{csch}^2(x)}{2\sqrt{a+b+c}\sqrt{a+b+c+(b+2c)\operatorname{csch}^2(x)+c\operatorname{csch}^4(x)}}\right)}{\sqrt{a+b+c}}\right)$$

input
```
Integrate[Coth[x]^3/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

output
```
(-(ArcTanh[(b + 2*c + 2*c*Csch[x]^2)/(2*Sqrt[c]*Sqrt[a + b + c + (b + 2*c)
*Csch[x]^2 + c*Csch[x]^4])]/Sqrt[c]) + ArcTanh[(2*(a + b + c) + (b + 2*c)*
Csch[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b + c + (b + 2*c)*Csch[x]^2 + c*Csc
h[x]^4])]/Sqrt[a + b + c])/2
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.50, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 26, 4184, 1578, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cot(ix)^3}{\sqrt{a-b\cot(ix)^2+c\cot(ix)^4}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cot(ix)^3}{\sqrt{c\cot(ix)^4-b\cot(ix)^2+a}}dx$$

$$\Big\downarrow \text{4184}$$

$$\int \frac{i\coth^3(x)}{(1-\coth^2(x))\sqrt{a+b\coth^2(x)+c\coth^4(x)}}d(-i\coth(x))$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int -\frac{\coth^2(x)}{(1-i\coth(x))\sqrt{-c\coth^2(x)+ib\coth(x)+a}}d(-\coth^2(x))$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(\int \frac{1}{\sqrt{-c\coth^2(x)+ib\coth(x)+a}}d(-\coth^2(x))-\int \frac{1}{(1-\coth^2(x))\sqrt{-c\coth^2(x)+ib\coth(x)+a}}d(-\cot$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(2\int \frac{1}{\coth^2(x)+4c}d\left(-\frac{b+2ic\coth(x)}{\sqrt{-c\coth^2(x)+ib\coth(x)+a}}\right)-\int \frac{1}{(1-\coth^2(x))\sqrt{-c\coth^2(x)+ib\coth(x)+}}\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\int \frac{1}{(1-\coth^2(x))\sqrt{-c\coth^2(x)+ib\coth(x)+a}}d(-\coth^2(x))-\frac{i\arctan\left(\frac{\coth(x)}{2\sqrt{c}}\right)}{\sqrt{c}}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(2\int \frac{1}{\coth^2(x)+4(a+b+c)}d\frac{2a+b+i(b+2c)\coth(x)}{\sqrt{-c\coth^2(x)+ib\coth(x)+a}}-\frac{i\arctan\left(\frac{\coth(x)}{2\sqrt{c}}\right)}{\sqrt{c}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\frac{i\arctan\left(\frac{\coth(x)}{2\sqrt{a+b+c}}\right)}{\sqrt{a+b+c}}-\frac{i\arctan\left(\frac{\coth(x)}{2\sqrt{c}}\right)}{\sqrt{c}}\right)$$

input
```
Int[Coth[x]^3/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

output
```
(((-I)*ArcTan[Coth[x]/(2*Sqrt[c])])/Sqrt[c] - (I*ArcTan[Coth[x]/(2*Sqrt[a
+ b + c])])/Sqrt[a + b + c])/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4184
```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> Simp[-f/e   Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)),
x], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[
n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{\ln\left(\frac{\frac{b}{2}+c\coth(x)^2}{\sqrt{c}}+\sqrt{a+b\coth(x)^2+c\coth(x)^4}\right)}{2\sqrt{c}}+\frac{\operatorname{arctanh}\left(\frac{b\coth(x)^2+2c\coth(x)^2+2a+b}{2\sqrt{a+b+c}\,\sqrt{a+b\coth(x)^2+c\coth(x)^4}}\right)}{2\sqrt{a+b+c}}$	90
default	$-\frac{\ln\left(\frac{\frac{b}{2}+c\coth(x)^2}{\sqrt{c}}+\sqrt{a+b\coth(x)^2+c\coth(x)^4}\right)}{2\sqrt{c}}+\frac{\operatorname{arctanh}\left(\frac{b\coth(x)^2+2c\coth(x)^2+2a+b}{2\sqrt{a+b+c}\,\sqrt{a+b\coth(x)^2+c\coth(x)^4}}\right)}{2\sqrt{a+b+c}}$	90

input
```
int(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln((1/2*b+c*coth(x)^2)/c^(1/2)+(a+b*coth(x)^2+c*coth(x)^4)^(1/2))/c^(
1/2)+1/2/(a+b+c)^(1/2)*arctanh(1/2*(b*coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+
c)^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1522 vs. $2(85) = 170$.

Time = 0.83 (sec) , antiderivative size = 6695, normalized size of antiderivative = 63.76

$$\int \frac{\coth^3(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth^3(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\,dx = \int \frac{\coth^3(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\,dx$$

input
```
integrate(coth(x)**3/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
```

output
```
Integral(coth(x)**3/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)
```

Maxima [F]

$$\int \frac{\coth^3(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\,dx = \int \frac{\coth(x)^3}{\sqrt{c\coth(x)^4 + b\coth(x)^2 + a}}\,dx$$

input
```
integrate(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(coth(x)^3/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Giac [F]

$$\int \frac{\coth^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx=\int \frac{\coth(x)^3}{\sqrt{c\coth(x)^4+b\coth(x)^2+a}}\,dx$$

input
```
integrate(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(coth(x)^3/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx=\int \frac{\coth(x)^3}{\sqrt{c\coth(x)^4+b\coth(x)^2+a}}\,dx$$

input
```
int(coth(x)^3/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
```

output
```
int(coth(x)^3/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{\coth^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx=\int \frac{\sqrt{\coth(x)^4c+\coth(x)^2b+a}\,\coth(x)^3}{\coth(x)^4c+\coth(x)^2b+a}dx$$

input
```
int(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

output
```
int((sqrt(coth(x)**4*c + coth(x)**2*b + a)*coth(x)**3)/(coth(x)**4*c + cot
h(x)**2*b + a),x)
```

3.212 $\int \frac{\coth(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\coth(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

output
```
1/2*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2)/(a+b+c)^(1/2)/(a+b*coth(x)^2+c*c
oth(x)^4)^(1/2))/(a+b+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int \frac{\coth(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{2(a+b+c)+(b+2c)\operatorname{csch}^2(x)}{2\sqrt{a+b+c}\sqrt{a+b+c+(b+2c)\operatorname{csch}^2(x)+c\operatorname{csch}^4(x)}}\right)}{2\sqrt{a+b+c}}$$

input
```
Integrate[Coth[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

output
```
ArcTanh[(2*(a + b + c) + (b + 2*c)*Csch[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a +
b + c + (b + 2*c)*Csch[x]^2 + c*Csch[x]^4])]/(2*Sqrt[a + b + c])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.50, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 26, 4184, 1576, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cot(ix)}{\sqrt{a-b\cot(ix)^2+c\cot(ix)^4}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cot(ix)}{\sqrt{c\cot(ix)^4-b\cot(ix)^2+a}}dx$$

$$\Big\downarrow \text{4184}$$

$$-\int -\frac{i\coth(x)}{(1-\coth^2(x))\sqrt{c\coth^4(x)+b\coth^2(x)+a}}d(-i\coth(x))$$

$$\Big\downarrow \text{1576}$$

$$-\frac{1}{2}\int \frac{1}{(1-\coth^2(x))\sqrt{-c\coth^2(x)+ib\coth(x)+a}}d(-\coth^2(x))$$

$$\Big\downarrow \text{1154}$$

$$\int \frac{1}{4(a+b+c)+\coth^2(x)}d\frac{2a+i(b+2c)\coth(x)+b}{\sqrt{a+ib\coth(x)-c\coth^2(x)}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{i \arctan \left(\frac{\coth(x)}{2\sqrt{a+b+c}}\right)}{2\sqrt{a+b+c}}$$

input
```
Int[Coth[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

output
```
((-1/2*I)*ArcTan[Coth[x]/(2*Sqrt[a + b + c])])/Sqrt[a + b + c]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4184
```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> Simp[-f/e   Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)),
x], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[
n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{b\coth(x)^{2}+2c\coth(x)^{2}+2a+b}{2\sqrt{a+b+c}\,\sqrt{a+b\coth(x)^{2}+c\coth(x)^{4}}}\right)}{2\sqrt{a+b+c}}$	52
default	$\frac{\operatorname{arctanh}\left(\frac{b\coth(x)^{2}+2c\coth(x)^{2}+2a+b}{2\sqrt{a+b+c}\,\sqrt{a+b\coth(x)^{2}+c\coth(x)^{4}}}\right)}{2\sqrt{a+b+c}}$	52

input
```
int(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a+b+c)^(1/2)*arctanh(1/2*(b*coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+c)^(1
/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 884 vs. $2(48) = 96$.

Time = 0.55 (sec) , antiderivative size = 1752, normalized size of antiderivative = 30.21

$$\int \frac{\coth(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a
*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(
a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^
2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*si
nh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2
+ a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3
*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4
- 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c
^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 1
0*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2
)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2
*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cos
h(x)^4 + 3*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b
*c + c^2)*sinh(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(
x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b +
 c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh
(x))*sinh(x) + a + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a
 + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2
*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*
cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b +...
```

Sympy [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx=\int \frac{\coth{(x)}}{\sqrt{a+b\coth^2{(x)}+c\coth^4{(x)}}}\,dx$$

input
```
integrate(coth(x)/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
```

output
```
Integral(coth(x)/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)
```

Maxima [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \int \frac{\coth\left(x\right)}{\sqrt{c\coth\left(x\right)^4+b\coth\left(x\right)^2+a}}\,dx$$

input
```
integrate(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\coth(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \text{Timed out}$$

input
```
integrate(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{c\coth\left(x\right)^4+b\coth\left(x\right)^2+a}}\,dx$$

input
```
int(coth(x)/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
```

output
```
int(coth(x)/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \int \frac{\sqrt{\coth(x)^4 c+\coth(x)^2 b+a}\,\coth(x)}{\coth(x)^4 c+\coth(x)^2 b+a}dx$$

input
```
int(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

output
```
int((sqrt(coth(x)**4*c + coth(x)**2*b + a)*coth(x))/(coth(x)**4*c + coth(x
)**2*b + a),x)
```

3.213 $\int \frac{\tanh(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\tanh(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}} + \frac{\operatorname{arctanh}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

output
```
-1/2*arctanh(1/2*(2*a+b*coth(x)^2)/a^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/
2))/a^(1/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2)/(a+b+c)^(1/2)/(a+b*c
oth(x)^2+c*coth(x)^4)^(1/2))/(a+b+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.50

$$\int \frac{\tanh(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx =$$

$$-\frac{\left((a+b+c)\text{arctanh}\left(\frac{b+2a\tanh^2(x)}{2\sqrt{a}\sqrt{c+b\tanh^2(x)+a\tanh^4(x)}}\right)-\sqrt{a}\sqrt{a+b+c}\text{arctanh}\left(\frac{b+2c+(2a+b)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{c+b\tanh^2(x)+a\tanh^4(x)}}\right)\right.}{2\sqrt{a}(a+b+c)\sqrt{a+b\coth^2(x)+c\coth^4(x)}}$$

input
```
Integrate[Tanh[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

output
```
-1/2*(((a + b + c)*ArcTanh[(b + 2*a*Tanh[x]^2)/(2*Sqrt[a]*Sqrt[c + b*Tanh[
x]^2 + a*Tanh[x]^4])] - Sqrt[a]*Sqrt[a + b + c]*ArcTanh[(b + 2*c + (2*a +
b)*Tanh[x]^2)/(2*Sqrt[a + b + c]*Sqrt[c + b*Tanh[x]^2 + a*Tanh[x]^4])])*Co
th[x]^2*Sqrt[c + b*Tanh[x]^2 + a*Tanh[x]^4])/(Sqrt[a]*(a + b + c)*Sqrt[a +
 b*Coth[x]^2 + c*Coth[x]^4])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 26, 4184, 1578, 1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\cot(ix)\sqrt{a-b\cot(ix)^2+c\cot(ix)^4}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\cot(ix)\sqrt{c\cot(ix)^4-b\cot(ix)^2+a}}dx$$

$$\Big\downarrow \text{4184}$$

$$\int \frac{i\tanh(x)}{\left(1-\coth^2(x)\right)\sqrt{a+b\coth^2(x)+c\coth^4(x)}}d(-i\coth(x))$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{i\tanh(x)}{\left(1-\coth^2(x)\right)\sqrt{-c\coth^2(x)+ib\coth(x)+a}}d\left(-\coth^2(x)\right)$$

$$\Big\downarrow \text{1289}$$

$$\frac{1}{2}\int \left(\frac{i\tanh(x)}{\sqrt{-c\coth^2(x)+ib\coth(x)+a}}+\frac{1}{(i\coth(x)-1)\sqrt{-c\coth^2(x)+ib\coth(x)+a}}\right)d\left(-\coth^2(x)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\operatorname{arctanh}\left(\frac{2a+i(b+2c)\coth(x)+b}{2\sqrt{a+b+c}\sqrt{a+ib\coth(x)-c\coth^2(x)}}\right)}{\sqrt{a+b+c}}-\frac{\operatorname{arctanh}\left(\frac{2a+ib\coth(x)}{2\sqrt{a}\sqrt{a+ib\coth(x)-c\coth^2(x)}}\right)}{\sqrt{a}}\right)$$

input
```
Int[Tanh[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

output
```
(-(ArcTanh[(2*a + I*b*Coth[x])/(2*Sqrt[a]*Sqrt[a + I*b*Coth[x] - c*Coth[x]
^2])]/Sqrt[a]) + ArcTanh[(2*a + b + I*(b + 2*c)*Coth[x])/(2*Sqrt[a + b + c
]*Sqrt[a + I*b*Coth[x] - c*Coth[x]^2])]/Sqrt[a + b + c])/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4184
```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> Simp[-f/e   Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)),
x], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[
n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{\tanh \left(x\right)}{\sqrt{a+b \coth \left(x\right)^{2}+c \coth \left(x\right)^{4}}}dx$$

input
```
int(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

output
```
int(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1524 vs. 2(86) = 172.

Time = 0.81 (sec) , antiderivative size = 6705, normalized size of antiderivative = 63.25

$$\int \frac{\tanh(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \int \frac{\tanh{(x)}}{\sqrt{a+b\coth^2{(x)}+c\coth^4{(x)}}}\,dx$$

input
```
integrate(tanh(x)/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
```

output
```
Integral(tanh(x)/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)
```

Maxima [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \int \frac{\tanh{(x)}}{\sqrt{c\coth{(x)}^4+b\coth{(x)}^2+a}}\,dx$$

input
```
integrate(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tanh(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx=\text{Timed out}$$

input
```
integrate(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx=\int \frac{\tanh(x)}{\sqrt{c\coth\left(x\right)^4+b\coth\left(x\right)^2+a}}\,dx$$

input
```
int(tanh(x)/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
```

output
```
int(tanh(x)/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx=\int \frac{\sqrt{\coth\left(x\right)^4c+\coth\left(x\right)^2b+a}\,\tanh\left(x\right)}{\coth\left(x\right)^4c+\coth\left(x\right)^2b+a}dx$$

input
```
int(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

output
```
int((sqrt(coth(x)**4*c + coth(x)**2*b + a)*tanh(x))/(coth(x)**4*c + coth(x
)**2*b + a),x)
```

3.214 $\int \frac{\tanh^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 142

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = -\frac{(2a-b)\text{arctanh}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4a^{3/2}} + \frac{\text{arctanh}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}} - \frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}\tanh^2(x)}{2a}$$

output
```
-1/4*(2*a-b)*arctanh(1/2*(2*a+b*coth(x)^2)/a^(1/2)/(a+b*coth(x)^2+c*coth(x
)^4)^(1/2))/a^(3/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2)/(a+b+c)^(1/2
)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2))/(a+b+c)^(1/2)-1/2*(a+b*coth(x)^2+c*co
th(x)^4)^(1/2)*tanh(x)^2/a
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.36

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = -\frac{\coth^2(x)\sqrt{c+b\tanh^2(x)+a\tanh^4(x)}\left((2a-b)(a+b+c)\operatorname{arctanh}\left(\frac{b+2a\tanh^2(x)}{2\sqrt{a}\sqrt{c+b\tanh^2(x)+a\tanh^4(x)}}\right)+2\sqrt{\ }\right.}{4a^{3/2}(a+b+c)\sqrt{a+}}$$

input
```
Integrate[Tanh[x]^3/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

output
```
-1/4*(Coth[x]^2*Sqrt[c + b*Tanh[x]^2 + a*Tanh[x]^4]*((2*a - b)*(a + b + c)
*ArcTanh[(b + 2*a*Tanh[x]^2)/(2*Sqrt[a]*Sqrt[c + b*Tanh[x]^2 + a*Tanh[x]^4
])] + 2*Sqrt[a]*(-(a*Sqrt[a + b + c]*ArcTanh[(b + 2*c + (2*a + b)*Tanh[x]^
2)/(2*Sqrt[a + b + c]*Sqrt[c + b*Tanh[x]^2 + a*Tanh[x]^4])]) + (a + b + c)
*Sqrt[c + b*Tanh[x]^2 + a*Tanh[x]^4])))/(a^(3/2)*(a + b + c)*Sqrt[a + b*Co
th[x]^2 + c*Coth[x]^4])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.35, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 26, 4184, 1578, 1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\cot(ix)^3\sqrt{a-b\cot(ix)^2+c\cot(ix)^4}}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{1}{\cot(ix)^3\sqrt{c\cot(ix)^4-b\cot(ix)^2+a}}dx$$

$$\Big\downarrow \; 4184$$

$$-\int -\frac{i\tanh^3(x)}{\left(1-\coth^2(x)\right)\sqrt{c\coth^4(x)+b\coth^2(x)+a}}d(-i\coth(x))$$

$$\Big\downarrow \; 1578$$

$$-\frac{1}{2}\int -\frac{\tanh^2(x)}{\left(1-\coth^2(x)\right)\sqrt{-c\coth^2(x)+ib\coth(x)+a}}d(-\coth^2(x))$$

$$\Big\downarrow \; 1289$$

$$-\frac{1}{2}\int \left(-\frac{\tanh^2(x)}{\sqrt{-c\coth^2(x)+ib\coth(x)+a}}-\frac{i\tanh(x)}{\sqrt{-c\coth^2(x)+ib\coth(x)+a}}+\frac{1}{\left(1-\coth^2(x)\right)\sqrt{-c\coth^2(x)+\cdots}}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{b\,\text{arctanh}\left(\frac{2a+ib\coth(x)}{2\sqrt{a}\sqrt{a+ib\coth(x)-c\coth^2(x)}}\right)}{2a^{3/2}}-\frac{\text{arctanh}\left(\frac{2a+ib\coth(x)}{2\sqrt{a}\sqrt{a+ib\coth(x)-c\coth^2(x)}}\right)}{\sqrt{a}}+\frac{\text{arctanh}\left(\frac{2a+i(b+2c)\,\text{co}\cdots}{2\sqrt{a+b+c}\sqrt{a+ib\,\text{coth}\cdots}}\right)}{\sqrt{a+b+c}}\right.$$

input
```
Int[Tanh[x]^3/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

output
```
(-(ArcTanh[(2*a + I*b*Coth[x])/(2*Sqrt[a]*Sqrt[a + I*b*Coth[x] - c*Coth[x]
^2])]/Sqrt[a]) + (b*ArcTanh[(2*a + I*b*Coth[x])/(2*Sqrt[a]*Sqrt[a + I*b*Co
th[x] - c*Coth[x]^2])])/(2*a^(3/2)) + ArcTanh[(2*a + b + I*(b + 2*c)*Coth[
x])/(2*Sqrt[a + b + c]*Sqrt[a + I*b*Coth[x] - c*Coth[x]^2])]/Sqrt[a + b +
c] + (I*Sqrt[a + I*b*Coth[x] - c*Coth[x]^2]*Tanh[x])/a)/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4184
```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> Simp[-f/e   Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)),
x], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[
n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{\tanh(x)^3}{\sqrt{a + b\coth(x)^2 + c\coth(x)^4}}dx$$

input
```
int(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

output
```
int(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2135 vs. $2(118) = 236$.

Time = 1.04 (sec) , antiderivative size = 9148, normalized size of antiderivative = 64.42

$$\int \frac{\tanh^3(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\,dx = \int \frac{\tanh^3(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\,dx$$

input
```
integrate(tanh(x)**3/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
```

output
```
Integral(tanh(x)**3/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)
```

Maxima [F]

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \int \frac{\tanh(x)^3}{\sqrt{c\coth(x)^4+b\coth(x)^2+a}}\,dx$$

input
```
integrate(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(tanh(x)^3/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \text{Timed out}$$

input
```
integrate(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \int \frac{\tanh(x)^3}{\sqrt{c\coth(x)^4+b\coth(x)^2+a}}\,dx$$

input
```
int(tanh(x)^3/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
```

output
```
int(tanh(x)^3/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx=\int \frac{\sqrt{\coth (x)^4 c+\coth (x)^2 b+a}\,\tanh (x)^3}{\coth (x)^4 c+\coth (x)^2 b+a}dx$$

input
```
int(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

output
```
int((sqrt(coth(x)**4*c + coth(x)**2*b + a)*tanh(x)**3)/(coth(x)**4*c + cot
h(x)**2*b + a),x)
```

3.215 $\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 132

$$\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx = -\frac{(b+2c)\operatorname{arctanh}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4\sqrt{c}} + \frac{1}{2}\sqrt{a+b+c}\operatorname{arctanh}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right) - \frac{1}{2}\sqrt{a+b\coth^2(x)+c\coth^4(x)}$$

output
```
-1/4*(b+2*c)*arctanh(1/2*(b+2*c*coth(x)^2)/c^(1/2)/(a+b*coth(x)^2+c*coth(x
)^4)^(1/2))/c^(1/2)+1/2*(a+b+c)^(1/2)*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2
)/(a+b+c)^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2))-1/2*(a+b*coth(x)^2+c*co
th(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.17

$$\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx$$
$$= -\frac{(b+2c)\operatorname{arctanh}\left(\frac{b+2c+2c\operatorname{csch}^2(x)}{2\sqrt{c}\sqrt{a+b+c+(b+2c)\operatorname{csch}^2(x)+c\operatorname{csch}^4(x)}}\right)}{4\sqrt{c}} + \frac{1}{2}\left(\sqrt{a+b+c}\operatorname{arctanh}\left(\frac{2(a+b+c)+(b+2c)\operatorname{csch}^2(x)}{2\sqrt{a+b+c}\sqrt{a+b+c+(b+2c)\operatorname{csch}^2(x)+c\operatorname{csch}^4(x)}}\right) - \sqrt{a+b+c+(b+2c)\operatorname{csch}^2(x)+c\operatorname{csch}^4(x)}\right)$$

input
```
Integrate[Coth[x]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

output
```
-1/4*((b + 2*c)*ArcTanh[(b + 2*c + 2*c*Csch[x]^2)/(2*Sqrt[c]*Sqrt[a + b +
c + (b + 2*c)*Csch[x]^2 + c*Csch[x]^4])])/Sqrt[c] + (Sqrt[a + b + c]*ArcTa
nh[(2*(a + b + c) + (b + 2*c)*Csch[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b + c
 + (b + 2*c)*Csch[x]^2 + c*Csch[x]^4])] - Sqrt[a + b + c + (b + 2*c)*Csch[
x]^2 + c*Csch[x]^4])/2
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.64, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 26, 4184, 1576, 1162, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int i\cot(ix)\sqrt{a-b\cot(ix)^2+c\cot(ix)^4}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \cot(ix)\sqrt{c\cot(ix)^4-b\cot(ix)^2+a}dx$$

$$\Big\downarrow \text{4184}$$

$$-\int -\frac{i\coth(x)\sqrt{c\coth^4(x)+b\coth^2(x)+a}}{1-\coth^2(x)}d(-i\coth(x))$$

$$\Big\downarrow \text{1576}$$

$$-\frac{1}{2}\int \frac{\sqrt{-c\coth^2(x)+ib\coth(x)+a}}{1-\coth^2(x)}d(-\coth^2(x))$$

$$\Big\downarrow \text{1162}$$

$$\frac{1}{2}\left(\frac{1}{2}\int -\frac{2a+b+i(b+2c)\coth(x)}{(1-\coth^2(x))\sqrt{-c\coth^2(x)+ib\coth(x)+a}}d(-\coth^2(x))-\sqrt{a+ib\coth(x)-c\coth^2(x)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{2a+b+i(b+2c)\coth(x)}{(1-\coth^2(x))\sqrt{-c\coth^2(x)+ib\coth(x)+a}}d(-\coth^2(x))-\sqrt{a+ib\coth(x)-c\coth^2(x)}\right)$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(\frac{1}{2}\left((b+2c)\int \frac{1}{\sqrt{-c\coth^2(x)+ib\coth(x)+a}}d(-\coth^2(x))-2(a+b+c)\int \frac{1}{(1-\coth^2(x))\sqrt{-c\coth^2(x)}}\right.\right.$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(2(b+2c)\int \frac{1}{\coth^2(x)+4c}d\left(-\frac{b+2ic\coth(x)}{\sqrt{-c\coth^2(x)+ib\coth(x)+a}}\right)-2(a+b+c)\int \frac{}{(1-\coth^2(x))\sqrt{-c}}\right.\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-2(a+b+c)\int \frac{1}{(1-\coth^2(x))\sqrt{-c\coth^2(x)+ib\coth(x)+a}}d(-\coth^2(x))-\frac{i(b+2c)\arctan\left(\frac{\coth(x)}{2\sqrt{c}}\right)}{\sqrt{c}}\right.\right.$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(4(a+b+c)\int \frac{1}{\coth^2(x)+4(a+b+c)}d\frac{2a+b+i(b+2c)\coth(x)}{\sqrt{-c\coth^2(x)+ib\coth(x)+a}}-\frac{i(b+2c)\arctan\left(\frac{\coth(x)}{2\sqrt{c}}\right)}{\sqrt{c}}\right)-\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-2i\sqrt{a+b+c}\arctan\left(\frac{\coth(x)}{2\sqrt{a+b+c}}\right)-\frac{i(b+2c)\arctan\left(\frac{\coth(x)}{2\sqrt{c}}\right)}{\sqrt{c}}\right)-\sqrt{a+ib\coth(x)-c\coth^2(x)}\right)$$

input
```
Int[Coth[x]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

output
```
((((-I)*(b + 2*c)*ArcTan[Coth[x]/(2*Sqrt[c])])/Sqrt[c] - (2*I)*Sqrt[a + b
+ c]*ArcTan[Coth[x]/(2*Sqrt[a + b + c])])/2 - Sqrt[a + I*b*Coth[x] - c*Cot
h[x]^2])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4184
```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> Simp[-f/e   Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)),
x], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[
n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.25

method	result
derivativedivides	$-\frac{\sqrt{c\left(-1+\coth\left(x\right)^{2}\right)^{2}+\left(b+2c\right)\left(-1+\coth\left(x\right)^{2}\right)+a+b+c}}{2}-\frac{\left(b+2c\right)\ln\left(\frac{\frac{b}{2}+c+c\left(-1+\coth\left(x\right)^{2}\right)}{\sqrt{c}}+\sqrt{c\left(-1+\coth\left(x\right)^{2}\right)^{2}}\right.}{4\sqrt{c}}$
default	$-\frac{\sqrt{c\left(-1+\coth\left(x\right)^{2}\right)^{2}+\left(b+2c\right)\left(-1+\coth\left(x\right)^{2}\right)+a+b+c}}{2}-\frac{\left(b+2c\right)\ln\left(\frac{\frac{b}{2}+c+c\left(-1+\coth\left(x\right)^{2}\right)}{\sqrt{c}}+\sqrt{c\left(-1+\coth\left(x\right)^{2}\right)^{2}}\right.}{4\sqrt{c}}$

input
```
int(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*(-1+coth(x)^2)^2+(b+2*c)*(-1+coth(x)^2)+a+b+c)^(1/2)-1/4*(b+2*c)*l
n((1/2*b+c+c*(-1+coth(x)^2))/c^(1/2)+(c*(-1+coth(x)^2)^2+(b+2*c)*(-1+coth(
x)^2)+a+b+c)^(1/2))/c^(1/2)+1/2*(a+b+c)^(1/2)*ln((2*a+2*b+2*c+(b+2*c)*(-1+
coth(x)^2)+2*(a+b+c)^(1/2)*(c*(-1+coth(x)^2)^2+(b+2*c)*(-1+coth(x)^2)+a+b+
c)^(1/2))/(-1+coth(x)^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1840 vs. 2(108) = 216.

Time = 1.37 (sec) , antiderivative size = 7964, normalized size of antiderivative = 60.33

$$\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx$$
$$=\int \sqrt{a+b\coth^2(x)+c\coth^4(x)}\coth(x)\,dx$$

input
```
integrate(coth(x)*(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
```

output
```
Integral(sqrt(a + b*coth(x)**2 + c*coth(x)**4)*coth(x), x)
```

Maxima [F]

$$\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx$$
$$=\int \sqrt{c\coth(x)^4+b\coth(x)^2+a}\coth(x)\,dx$$

input
```
integrate(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*coth(x)^4 + b*coth(x)^2 + a)*coth(x), x)
```

Giac [F]

$$\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx$$
$$=\int \sqrt{c\coth(x)^4+b\coth(x)^2+a}\coth(x)\,dx$$

input
```
integrate(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*coth(x)^4 + b*coth(x)^2 + a)*coth(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx$$
$$=\int \coth(x)\ \sqrt{c\coth{(x)}^4+b\coth{(x)}^2+a}\,dx$$

input
```
int(coth(x)*(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
```

output
```
int(coth(x)*(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

Reduce [F]

$$\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx$$
$$=\frac{-\sqrt{\coth{(x)}^4c+\coth{(x)}^2b+a}\,b+\left(\int\frac{\sqrt{\coth(x)^4c+\coth(x)^2b+a}\,\coth(x)}{\coth(x)^4bc-2\coth(x)^4c^2+\coth(x)^2b^2-2\coth(x)^2bc+ab-2ac}dx\right)a\,b^2-4\left(\int\frac{}{\mathrm{co}}\right.}{}$$

input
```
int(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

output
```
( - sqrt(coth(x)**4*c + coth(x)**2*b + a)*b + int((sqrt(coth(x)**4*c + cot
h(x)**2*b + a)*coth(x))/(coth(x)**4*b*c - 2*coth(x)**4*c**2 + coth(x)**2*b
**2 - 2*coth(x)**2*b*c + a*b - 2*a*c),x)*a*b**2 - 4*int((sqrt(coth(x)**4*c
 + coth(x)**2*b + a)*coth(x))/(coth(x)**4*b*c - 2*coth(x)**4*c**2 + coth(x
)**2*b**2 - 2*coth(x)**2*b*c + a*b - 2*a*c),x)*a*b*c + 4*int((sqrt(coth(x)
**4*c + coth(x)**2*b + a)*coth(x))/(coth(x)**4*b*c - 2*coth(x)**4*c**2 + c
oth(x)**2*b**2 - 2*coth(x)**2*b*c + a*b - 2*a*c),x)*a*c**2 + int((sqrt(cot
h(x)**4*c + coth(x)**2*b + a)*coth(x))/(coth(x)**4*b*c - 2*coth(x)**4*c**2
 + coth(x)**2*b**2 - 2*coth(x)**2*b*c + a*b - 2*a*c),x)*b**3 - 2*int((sqrt
(coth(x)**4*c + coth(x)**2*b + a)*coth(x))/(coth(x)**4*b*c - 2*coth(x)**4*
c**2 + coth(x)**2*b**2 - 2*coth(x)**2*b*c + a*b - 2*a*c),x)*b**2*c - int((
sqrt(coth(x)**4*c + coth(x)**2*b + a)*coth(x)**5)/(coth(x)**4*b*c - 2*coth
(x)**4*c**2 + coth(x)**2*b**2 - 2*coth(x)**2*b*c + a*b - 2*a*c),x)*b**2*c
+ 4*int((sqrt(coth(x)**4*c + coth(x)**2*b + a)*coth(x)**5)/(coth(x)**4*b*c
 - 2*coth(x)**4*c**2 + coth(x)**2*b**2 - 2*coth(x)**2*b*c + a*b - 2*a*c),x
)*c**3)/(b - 2*c)
```

3.216 $\int e^{c(a+bx)} \coth^2(ac+bcx)^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 319

$$\begin{aligned}\int e^{c(a+bx)} \coth^2(ac+bcx)^{5/2}\,dx &= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} \\ &- \frac{4e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4} \\ &+ \frac{26e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3} \\ &- \frac{55e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{6bc\left(1-e^{2c(a+bx)}\right)^2} \\ &+ \frac{25e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{4bc\left(1-e^{2c(a+bx)}\right)} \\ &- \frac{15\operatorname{arctanh}\left(e^{c(a+bx)}\right)\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{4bc}\end{aligned}$$

output
```
exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c-4*exp(c*(b*x+
a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^4+2
6/3*exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c/(1-exp(2*
c*(b*x+a)))^3-55/6*exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c
)/b/c/(1-exp(2*c*(b*x+a)))^2+25/4*exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)
*tanh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))-15/4*arctanh(exp(c*(b*x+a)))*(co
th(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c
```

Mathematica [A] (warning: unable to verify)

Time = 6.19 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.51

$$\int e^{c(a+bx)}\coth^2(ac+bcx)^{5/2}\,dx = \frac{\sqrt{\coth^2(c(a+bx))}\left(66e^{c(a+bx)}-314e^{3c(a+bx)}+374e^{5c(a+bx)}-246e^{7c(a+bx)}+24e^{9c(a+bx)}+45\right.}{24bc\,(-1+}$$

input
```
Integrate[E^(c*(a + b*x))*(Coth[a*c + b*c*x]^2)^(5/2),x]
```

output
```
(Sqrt[Coth[c*(a + b*x)]^2]*(66*E^(c*(a + b*x)) - 314*E^(3*c*(a + b*x)) + 3
74*E^(5*c*(a + b*x)) - 246*E^(7*c*(a + b*x)) + 24*E^(9*c*(a + b*x)) + 45*(
-1 + E^(2*c*(a + b*x)))^4*Log[1 - E^(c*(a + b*x))] - 45*(-1 + E^(2*c*(a +
b*x)))^4*Log[1 + E^(c*(a + b*x))])*Tanh[c*(a + b*x)])/(24*b*c*(-1 + E^(2*c
*(a + b*x)))^4)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.53, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7271, 2720, 25, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)} \coth^2(ac+bcx)^{5/2}\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}\int e^{c(a+bx)}\coth^5(ac+bxc)dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}\int -\frac{\left(1+e^{2c(a+bx)}\right)^5}{\left(1-e^{2c(a+bx)}\right)^5}de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}\int \frac{\left(1+e^{2c(a+bx)}\right)^5}{\left(1-e^{2c(a+bx)}\right)^5}de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{ 300}$$

$$-\frac{\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}\int \left(\frac{2\left(1+10e^{4c(a+bx)}+5e^{8c(a+bx)}\right)}{\left(1-e^{2c(a+bx)}\right)^5}-1\right)de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(-\frac{15}{4}\text{arctanh}\left(e^{c(a+bx)}\right)+e^{c(a+bx)}+\frac{25e^{c(a+bx)}}{4\left(1-e^{2c(a+bx)}\right)}-\frac{55e^{c(a+bx)}}{6\left(1-e^{2c(a+bx)}\right)^2}+\frac{26e^{c(a+bx)}}{3\left(1-e^{2c(a+bx)}\right)^3}-\frac{4e^{c(a+bx)}}{\left(1-e^{2c(a+bx)}\right)^4}\right)\tanh(ac+b}{bc}$$

input
```
Int[E^(c*(a + b*x))*(Coth[a*c + b*c*x]^2)^(5/2),x]
```

output
```
((E^(c*(a + b*x)) - (4*E^(c*(a + b*x)))/(1 - E^(2*c*(a + b*x)))^4 + (26*E^
(c*(a + b*x)))/(3*(1 - E^(2*c*(a + b*x)))^3) - (55*E^(c*(a + b*x)))/(6*(1
- E^(2*c*(a + b*x)))^2) + (25*E^(c*(a + b*x)))/(4*(1 - E^(2*c*(a + b*x))))
 - (15*ArcTanh[E^(c*(a + b*x))])/4)*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b
*c*x])/(b*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.16 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.61

method	result
default	$\frac{\operatorname{csgn}(\coth(c(bx+a)))\left(\frac{\cosh(bxc+ac)^4}{\sinh(bxc+ac)^3}-\frac{4\cosh(bxc+ac)^2}{\sinh(bxc+ac)^3}+\frac{8}{3\sinh(bxc+ac)^3}+\frac{\cosh(bxc+ac)^5}{\sinh(bxc+ac)^4}-\frac{5\cosh(bxc+ac)^3}{\sinh(bxc+ac)^4}+\frac{5\cosh(bxc+ac)}{\sinh(bxc+ac)^4}+5\left(-\operatorname{csch}\right.\right.}{cb}$
risch	$\frac{\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\, e^{c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)cb}-\frac{\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\, e^{c(bx+a)}\left(75\,e^{6c(bx+a)}-115\,e^{4c(bx+a)}+109\,e^{2c(bx+a)}-21\right)}{12\left(1+e^{2c(bx+a)}\right)\left(e^{2c(bx+a)}-1\right)^3cb}+-\frac{15}{}$

input
```
int(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(coth(c*(b*x+a)))/c/b*(1/sinh(b*c*x+a*c)^3*cosh(b*c*x+a*c)^4-4/sinh(b*
c*x+a*c)^3*cosh(b*c*x+a*c)^2+8/3/sinh(b*c*x+a*c)^3+cosh(b*c*x+a*c)^5/sinh(
b*c*x+a*c)^4-5/sinh(b*c*x+a*c)^4*cosh(b*c*x+a*c)^3+5/sinh(b*c*x+a*c)^4*cos
h(b*c*x+a*c)+5*(-1/4*csch(b*c*x+a*c)^3+3/8*csch(b*c*x+a*c))*coth(b*c*x+a*c
)-15/4*arctanh(exp(b*c*x+a*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1617 vs. $2(281) = 562$.

Time = 0.11 (sec) , antiderivative size = 1617, normalized size of antiderivative = 5.07

$$\int e^{c(a+bx)} \coth^2(ac+bcx)^{5/2} \, dx = \text{Too large to display}$$

input
```
integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/24*(24*cosh(b*c*x + a*c)^9 + 216*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^8 +
 24*sinh(b*c*x + a*c)^9 + 6*(144*cosh(b*c*x + a*c)^2 - 41)*sinh(b*c*x + a*
c)^7 - 246*cosh(b*c*x + a*c)^7 + 42*(48*cosh(b*c*x + a*c)^3 - 41*cosh(b*c*
x + a*c))*sinh(b*c*x + a*c)^6 + 2*(1512*cosh(b*c*x + a*c)^4 - 2583*cosh(b*
c*x + a*c)^2 + 187)*sinh(b*c*x + a*c)^5 + 374*cosh(b*c*x + a*c)^5 + 2*(151
2*cosh(b*c*x + a*c)^5 - 4305*cosh(b*c*x + a*c)^3 + 935*cosh(b*c*x + a*c))*
sinh(b*c*x + a*c)^4 + 2*(1008*cosh(b*c*x + a*c)^6 - 4305*cosh(b*c*x + a*c)
^4 + 1870*cosh(b*c*x + a*c)^2 - 157)*sinh(b*c*x + a*c)^3 - 314*cosh(b*c*x
+ a*c)^3 + 2*(432*cosh(b*c*x + a*c)^7 - 2583*cosh(b*c*x + a*c)^5 + 1870*co
sh(b*c*x + a*c)^3 - 471*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 - 45*(cosh(
b*c*x + a*c)^8 + 8*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^7 + sinh(b*c*x + a*
c)^8 + 4*(7*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^6 - 4*cosh(b*c*x +
a*c)^6 + 8*(7*cosh(b*c*x + a*c)^3 - 3*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)
^5 + 2*(35*cosh(b*c*x + a*c)^4 - 30*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x +
a*c)^4 + 6*cosh(b*c*x + a*c)^4 + 8*(7*cosh(b*c*x + a*c)^5 - 10*cosh(b*c*x
+ a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^3 + 4*(7*cosh(b*c*x + a*
c)^6 - 15*cosh(b*c*x + a*c)^4 + 9*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*
c)^2 - 4*cosh(b*c*x + a*c)^2 + 8*(cosh(b*c*x + a*c)^7 - 3*cosh(b*c*x + a*c
)^5 + 3*cosh(b*c*x + a*c)^3 - cosh(b*c*x + a*c))*sinh(b*c*x + a*c) + 1)*lo
g(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) + 1) + 45*(cosh(b*c*x + a*c)^8 ...
```

Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\coth^2(ac+bcx)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.52

$$\int e^{c(a+bx)} \coth^2(ac+bcx)^{5/2}\,dx = -\frac{15\,\log\left(e^{(bcx+ac)}+1\right)}{8\,bc} + \frac{15\,\log\left(e^{(bcx+ac)}-1\right)}{8\,bc} + \frac{12\,e^{(9\,bcx+9\,ac)} - 123\,e^{(7\,bcx+7\,ac)} + 187\,e^{(5\,bcx+5\,ac)} - 157\,e^{(3\,bcx+3\,ac)} + 33\,e^{(bcx+ac)}}{12\,bc(e^{(8\,bcx+8\,ac)} - 4\,e^{(6\,bcx+6\,ac)} + 6\,e^{(4\,bcx+4\,ac)} - 4\,e^{(2\,bcx+2\,ac)} + 1)}$$

input
```
integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

output
```
-15/8*log(e^(b*c*x + a*c) + 1)/(b*c) + 15/8*log(e^(b*c*x + a*c) - 1)/(b*c)
 + 1/12*(12*e^(9*b*c*x + 9*a*c) - 123*e^(7*b*c*x + 7*a*c) + 187*e^(5*b*c*x
 + 5*a*c) - 157*e^(3*b*c*x + 3*a*c) + 33*e^(b*c*x + a*c))/(b*c*(e^(8*b*c*x
 + 8*a*c) - 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x +
 2*a*c) + 1))
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.61

$$\int e^{c(a+bx)} \coth^2(ac+bcx)^{5/2}\,dx = \frac{e^{(bcx+ac)}}{bc\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)} - \frac{15\,\log\left(e^{(bcx+ac)}+1\right)}{8\,bc\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)} + \frac{15\,\log\left(\left|e^{(bcx+ac)}-1\right|\right)}{8\,bc\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)} - \frac{75\,e^{(7\,bcx+7\,ac)} - 115\,e^{(5\,bcx+5\,ac)} + 109\,e^{(3\,bcx+3\,ac)} - 21\,e^{(bcx+ac)}}{12\,bc(e^{(2\,bcx+2\,ac)}-1)^4\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}$$

input
```
integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

output
```
e^(b*c*x + a*c)/(b*c*sgn(e^(2*b*c*x + 2*a*c) - 1)) - 15/8*log(e^(b*c*x + a
*c) + 1)/(b*c*sgn(e^(2*b*c*x + 2*a*c) - 1)) + 15/8*log(abs(e^(b*c*x + a*c)
 - 1))/(b*c*sgn(e^(2*b*c*x + 2*a*c) - 1)) - 1/12*(75*e^(7*b*c*x + 7*a*c) -
 115*e^(5*b*c*x + 5*a*c) + 109*e^(3*b*c*x + 3*a*c) - 21*e^(b*c*x + a*c))/(
b*c*(e^(2*b*c*x + 2*a*c) - 1)^4*sgn(e^(2*b*c*x + 2*a*c) - 1))
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)} \coth^2(ac+bcx)^{5/2}\,dx = \int e^{c\,(a+b\,x)}\,\left(\coth(a\,c+b\,c\,x)^2\right)^{5/2}\,dx$$

input
```
int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(5/2),x)
```

output
```
int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.38

$$\int e^{c(a+bx)} \coth^2(ac+bcx)^{5/2}\,dx = \frac{24e^{9bcx+9ac} - 45e^{8bcx+8ac}\log\left(e^{bcx+2ac}+e^{ac}\right) + 45e^{8bcx+8ac}\log\left(e^{bcx+2ac}-e^{ac}\right) - 246e^{7bcx+7ac} + \cdots}{\cdots}$$

input
```
int(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2),x)
```

output
```
(24*e**(9*a*c + 9*b*c*x) - 45*e**(8*a*c + 8*b*c*x)*log(e**(2*a*c + b*c*x)
+ e**(a*c)) + 45*e**(8*a*c + 8*b*c*x)*log(e**(2*a*c + b*c*x) - e**(a*c)) -
 246*e**(7*a*c + 7*b*c*x) + 180*e**(6*a*c + 6*b*c*x)*log(e**(2*a*c + b*c*x
) + e**(a*c)) - 180*e**(6*a*c + 6*b*c*x)*log(e**(2*a*c + b*c*x) - e**(a*c)
) + 374*e**(5*a*c + 5*b*c*x) - 270*e**(4*a*c + 4*b*c*x)*log(e**(2*a*c + b*
c*x) + e**(a*c)) + 270*e**(4*a*c + 4*b*c*x)*log(e**(2*a*c + b*c*x) - e**(a
*c)) - 314*e**(3*a*c + 3*b*c*x) + 180*e**(2*a*c + 2*b*c*x)*log(e**(2*a*c +
 b*c*x) + e**(a*c)) - 180*e**(2*a*c + 2*b*c*x)*log(e**(2*a*c + b*c*x) - e*
*(a*c)) + 66*e**(a*c + b*c*x) - 45*log(e**(2*a*c + b*c*x) + e**(a*c)) + 45
*log(e**(2*a*c + b*c*x) - e**(a*c)))/(24*b*c*(e**(8*a*c + 8*b*c*x) - 4*e**
(6*a*c + 6*b*c*x) + 6*e**(4*a*c + 4*b*c*x) - 4*e**(2*a*c + 2*b*c*x) + 1))
```

3.217 $\int e^{c(a+bx)} \coth^2(ac+bcx)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 197

$$\begin{aligned}\int e^{c(a+bx)} \coth^2(ac+bcx)^{3/2}\,dx &= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} \\ &- \frac{2e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2} \\ &+ \frac{3e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)} \\ &- \frac{3\text{arctanh}\left(e^{c(a+bx)}\right)\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc}\end{aligned}$$

output

```
exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c-2*exp(c*(b*x+
a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^2+3
*exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c/(1-exp(2*c*(
b*x+a)))-3*arctanh(exp(c*(b*x+a)))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*
c)/b/c
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.64 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.70

$$\int e^{c(a+bx)}\coth^2(ac+bcx)^{3/2}\,dx =$$

$$-\frac{e^{-5c(a+bx)}\coth^2(c(a+bx))^{3/2}\left(-21\left(252105+507305e^{2c(a+bx)}+173916e^{4c(a+bx)}-154296e^{6c(a+bx)}-738\right.\right.}{}$$

input
```
Integrate[E^(c*(a + b*x))*(Coth[a*c + b*c*x]^2)^(3/2),x]
```

output
```
-1/60480*((Coth[c*(a + b*x)]^2)^(3/2)*(-21*(252105 + 507305*E^(2*c*(a + b*
x)) + 173916*E^(4*c*(a + b*x)) - 154296*E^(6*c*(a + b*x)) - 73885*E^(8*c*(
a + b*x)) + 4887*E^(10*c*(a + b*x))) - (315*(-16807 - 28218*E^(2*c*(a + b*
x)) + 1173*E^(4*c*(a + b*x)) + 17748*E^(6*c*(a + b*x)) + 4299*E^(8*c*(a +
b*x)) - 1434*E^(10*c*(a + b*x)) + 7*E^(12*c*(a + b*x)))*ArcTanh[Sqrt[E^(2*
c*(a + b*x))]])/Sqrt[E^(2*c*(a + b*x))] + 384*E^(8*c*(a + b*x))*(1 + E^(2*
c*(a + b*x)))^2*(7 + 5*E^(2*c*(a + b*x)))*HypergeometricPFQ[{3/2, 2, 2, 2,
 2}, {1, 1, 1, 11/2}, E^(2*c*(a + b*x))] + 256*E^(8*c*(a + b*x))*(1 + E^(2
*c*(a + b*x)))^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2
}, E^(2*c*(a + b*x))])*Tanh[c*(a + b*x)]^3)/(b*c*E^(5*c*(a + b*x)))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.54, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7271, 2720, 25, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\coth^2(ac+bcx)^{3/2}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}\int e^{c(a+bx)}\coth^3(ac+bxc)dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}\int -\frac{\left(1+e^{2c(a+bx)}\right)^3}{\left(1-e^{2c(a+bx)}\right)^3}de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \; 25$$

$$-\frac{\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}\int \frac{\left(1+e^{2c(a+bx)}\right)^3}{\left(1-e^{2c(a+bx)}\right)^3}de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \; 300$$

$$-\frac{\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}\int \left(\frac{2\left(1+3e^{4c(a+bx)}\right)}{\left(1-e^{2c(a+bx)}\right)^3}-1\right)de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \; 2009$$

$$\frac{\left(-3\text{arctanh}\left(e^{c(a+bx)}\right)+e^{c(a+bx)}+\frac{3e^{c(a+bx)}}{1-e^{2c(a+bx)}}-\frac{2e^{c(a+bx)}}{\left(1-e^{2c(a+bx)}\right)^2}\right)\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}$$

input
```
Int[E^(c*(a + b*x))*(Coth[a*c + b*c*x]^2)^(3/2),x]
```

output
```
((E^(c*(a + b*x)) - (2*E^(c*(a + b*x)))/(1 - E^(2*c*(a + b*x)))^2 + (3*E^(
c*(a + b*x)))/(1 - E^(2*c*(a + b*x))) - 3*ArcTanh[E^(c*(a + b*x))])*Sqrt[C
oth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.36 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.66

method	result
default	$\frac{\operatorname{csgn}(\coth(c(bx+a)))\left(\frac{\cosh(bxc+ac)^2}{\sinh(bxc+ac)}-\frac{2}{\sinh(bxc+ac)}+\frac{\cosh(bxc+ac)^3}{\sinh(bxc+ac)^2}-\frac{3\cosh(bxc+ac)}{\sinh(bxc+ac)^2}+\frac{3\operatorname{csch}(bxc+ac)\coth(bxc+ac)}{2}-3\operatorname{arctanh}(e^{bxc+c}\right.}{cb}$
risch	$\frac{\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\,\left(2\,e^{5c(bx+a)}-3\ln\left(e^{c(bx+a)}+1\right)e^{4c(bx+a)}+3\ln\left(e^{c(bx+a)}-1\right)e^{4c(bx+a)}-10\,e^{3c(bx+a)}+6\ln\left(e^{c(bx+a)}+1\right)e^{2c(bx+a)}\right.}{2\left(1+e^{2c(bx+a)}\right)\left(e^{2c(bx+a)}-1\right)cb}$

input
```
int(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(coth(c*(b*x+a)))/c/b*(1/sinh(b*c*x+a*c)*cosh(b*c*x+a*c)^2-2/sinh(b*c*
x+a*c)+cosh(b*c*x+a*c)^3/sinh(b*c*x+a*c)^2-3/sinh(b*c*x+a*c)^2*cosh(b*c*x+
a*c)+3/2*csch(b*c*x+a*c)*coth(b*c*x+a*c)-3*arctanh(exp(b*c*x+a*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(179) = 358$.

Time = 0.10 (sec) , antiderivative size = 613, normalized size of antiderivative = 3.11

$$\int e^{c(a+bx)} \coth^2(ac+bcx)^{3/2} \, dx = \text{Too large to display}$$

input
```
integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(2*cosh(b*c*x + a*c)^5 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 + 2*
sinh(b*c*x + a*c)^5 + 10*(2*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^3 -
 10*cosh(b*c*x + a*c)^3 + 10*(2*cosh(b*c*x + a*c)^3 - 3*cosh(b*c*x + a*c))
*sinh(b*c*x + a*c)^2 - 3*(cosh(b*c*x + a*c)^4 + 4*cosh(b*c*x + a*c)*sinh(b
*c*x + a*c)^3 + sinh(b*c*x + a*c)^4 + 2*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b
*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)^2 + 4*(cosh(b*c*x + a*c)^3 - cosh(b*c*
x + a*c))*sinh(b*c*x + a*c) + 1)*log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c)
 + 1) + 3*(cosh(b*c*x + a*c)^4 + 4*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^3 +
 sinh(b*c*x + a*c)^4 + 2*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^2 -
 2*cosh(b*c*x + a*c)^2 + 4*(cosh(b*c*x + a*c)^3 - cosh(b*c*x + a*c))*sinh(
b*c*x + a*c) + 1)*log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) - 1) + 2*(5*co
sh(b*c*x + a*c)^4 - 15*cosh(b*c*x + a*c)^2 + 2)*sinh(b*c*x + a*c) + 4*cosh
(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^4 + 4*b*c*cosh(b*c*x + a*c)*sinh(b*c
*x + a*c)^3 + b*c*sinh(b*c*x + a*c)^4 - 2*b*c*cosh(b*c*x + a*c)^2 + 2*(3*b
*c*cosh(b*c*x + a*c)^2 - b*c)*sinh(b*c*x + a*c)^2 + b*c + 4*(b*c*cosh(b*c*
x + a*c)^3 - b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)} \coth^2(ac+bcx)^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.57

$$\int e^{c(a+bx)}\coth^2(ac+bcx)^{3/2}\,dx = -\frac{3\,\log\left(e^{(bcx+ac)}+1\right)}{2\,bc} + \frac{3\,\log\left(e^{(bcx+ac)}-1\right)}{2\,bc} + \frac{e^{(5\,bcx+5\,ac)}-5\,e^{(3\,bcx+3\,ac)}+2\,e^{(bcx+ac)}}{bc(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1)}$$

input
```
integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

output
```
-3/2*log(e^(b*c*x + a*c) + 1)/(b*c) + 3/2*log(e^(b*c*x + a*c) - 1)/(b*c) +
 (e^(5*b*c*x + 5*a*c) - 5*e^(3*b*c*x + 3*a*c) + 2*e^(b*c*x + a*c))/(b*c*(e
^(4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.86

$$\int e^{c(a+bx)}\coth^2(ac+bcx)^{3/2}\,dx = \frac{e^{(bcx+ac)}}{bc\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)} - \frac{3\,\log\left(e^{(bcx+ac)}+1\right)}{2\,bc\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)} + \frac{3\,\log\left(\left|e^{(bcx+ac)}-1\right|\right)}{2\,bc\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)} - \frac{3\,e^{(3\,bcx+3\,ac)}-e^{(bcx+ac)}}{bc(e^{(2\,bcx+2\,ac)}-1)^2\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}$$

input
```
integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

output
```
e^(b*c*x + a*c)/(b*c*sgn(e^(2*b*c*x + 2*a*c) - 1)) - 3/2*log(e^(b*c*x + a*
c) + 1)/(b*c*sgn(e^(2*b*c*x + 2*a*c) - 1)) + 3/2*log(abs(e^(b*c*x + a*c) -
 1))/(b*c*sgn(e^(2*b*c*x + 2*a*c) - 1)) - (3*e^(3*b*c*x + 3*a*c) - e^(b*c*
x + a*c))/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^2*sgn(e^(2*b*c*x + 2*a*c) - 1))
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\coth^2(ac+bcx)^{3/2}\,dx = \int e^{c\,(a+b\,x)}\,\left(\coth(a\,c+b\,c\,x)^2\right)^{3/2}\,dx$$

input
```
int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(3/2),x)
```

output
```
int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.28

$$\int e^{c(a+bx)}\coth^2(ac+bcx)^{3/2}\,dx = \frac{2e^{5bcx+5ac}-3e^{4bcx+4ac}\log\left(e^{bcx+2ac}+e^{ac}\right)+3e^{4bcx+4ac}\log\left(e^{bcx+2ac}-e^{ac}\right)-10e^{3bcx+3ac}+6e^{2b}}{2bc\,(e}$$

input
```
int(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2),x)
```

output
```
(2*e**(5*a*c + 5*b*c*x) - 3*e**(4*a*c + 4*b*c*x)*log(e**(2*a*c + b*c*x) +
e**(a*c)) + 3*e**(4*a*c + 4*b*c*x)*log(e**(2*a*c + b*c*x) - e**(a*c)) - 10
*e**(3*a*c + 3*b*c*x) + 6*e**(2*a*c + 2*b*c*x)*log(e**(2*a*c + b*c*x) + e*
*(a*c)) - 6*e**(2*a*c + 2*b*c*x)*log(e**(2*a*c + b*c*x) - e**(a*c)) + 4*e*
*(a*c + b*c*x) - 3*log(e**(2*a*c + b*c*x) + e**(a*c)) + 3*log(e**(2*a*c +
b*c*x) - e**(a*c)))/(2*b*c*(e**(4*a*c + 4*b*c*x) - 2*e**(2*a*c + 2*b*c*x)
+ 1))
```

3.218 $\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 83

$$\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx=\frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc}-\frac{2\text{arctanh}\left(e^{c(a+bx)}\right)\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc}$$

output
```
exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c-2*arctanh(exp
(c*(b*x+a)))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.61

$$\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx=\frac{\left(e^{c(a+bx)}-2\text{arctanh}\left(e^{c(a+bx)}\right)\right)\sqrt{\coth^2(c(a+bx))}\tanh(c(a+bx))}{bc}$$

input
```
Integrate[E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2],x]
```

output
```
((E^(c*(a + b*x)) - 2*ArcTanh[E^(c*(a + b*x))])*Sqrt[Coth[c*(a + b*x)]^2]*
Tanh[c*(a + b*x)])/(b*c)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.64, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7271, 2720, 25, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}\int e^{c(a+bx)}\coth(ac+bxc)dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}\int -\frac{1+e^{2c(a+bx)}}{1-e^{2c(a+bx)}}de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}\int \frac{1+e^{2c(a+bx)}}{1-e^{2c(a+bx)}}de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{299}$$

$$\frac{\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}\left(e^{c(a+bx)}-2\int \frac{1}{1-e^{2c(a+bx)}}de^{c(a+bx)}\right)}{bc}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(e^{c(a+bx)}-2\text{arctanh}\left(e^{c(a+bx)}\right)\right)\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}$$

input
```
Int[E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2],x]
```

output
```
((E^(c*(a + b*x)) - 2*ArcTanh[E^(c*(a + b*x))])*Sqrt[Coth[a*c + b*c*x]^2]*
Tanh[a*c + b*c*x])/(b*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(77) = 154.

Time = 0.45 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.57

method	result
risch	$\frac{\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^{2}}{\left(e^{2c(bx+a)}-1\right)^{2}}}\, e^{c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)cb}+\frac{\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^{2}}{\left(e^{2c(bx+a)}-1\right)^{2}}}\,\ln\left(e^{c(bx+a)}-1\right)}{\left(1+e^{2c(bx+a)}\right)cb}-\frac{\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+e^{2c(bx+}\right.}{\left(e^{2c(bx+a)}-\right.}}}{\left(1+e^{2c(bx+a}\right.}$

input
```
int(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2
*c*(b*x+a))-1)^2)^(1/2)*exp(c*(b*x+a))/c/b+1/(1+exp(2*c*(b*x+a)))*(exp(2*c
*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(
exp(c*(b*x+a))-1)-1/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*
(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp(c*(b*x+a))+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx = \frac{\cosh(bcx+ac)-\log(\cosh(bcx+ac)+\sinh(bcx+ac)+1)+\log(\cosh(bcx+ac)+\sinh(bcx+ac)-}{bc}$$

input
```
integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

output
```
(cosh(b*c*x + a*c) - log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) + 1) + log(
cosh(b*c*x + a*c) + sinh(b*c*x + a*c) - 1) + sinh(b*c*x + a*c))/(b*c)
```

Sympy [F]

$$\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx = e^{ac}\int\sqrt{\coth^2\left(ac+bcx\right)}e^{bcx}\,dx$$

input
```
integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)**2)**(1/2),x)
```

output
```
exp(a*c)*Integral(sqrt(coth(a*c + b*c*x)**2)*exp(b*c*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

$$\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx = \frac{e^{(bcx+ac)}}{bc} - \frac{\log\left(e^{(bcx+ac)}+1\right)}{bc} + \frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

output
```
e^(b*c*x + a*c)/(b*c) - log(e^(b*c*x + a*c) + 1)/(b*c) + log(e^(b*c*x + a*
c) - 1)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.27

$$\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx = \frac{e^{(bcx+ac)}}{bc\,\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)} - \frac{\log\left(e^{(bcx+ac)}+1\right)}{bc\,\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)} + \frac{\log\left(\left|e^{(bcx+ac)}-1\right|\right)}{bc\,\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}$$

input
```
integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

output
```
e^(b*c*x + a*c)/(b*c*sgn(e^(2*b*c*x + 2*a*c) - 1)) - log(e^(b*c*x + a*c) +
 1)/(b*c*sgn(e^(2*b*c*x + 2*a*c) - 1)) + log(abs(e^(b*c*x + a*c) - 1))/(b*
c*sgn(e^(2*b*c*x + 2*a*c) - 1))
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx=\int e^{c\,(a+b\,x)}\,\sqrt{\coth\left(a\,c+b\,c\,x\right)^2}\,dx$$

input
```
int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(1/2),x)
```

output
```
int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx=\frac{e^{bcx+ac}-\log\left(e^{bcx+2ac}+e^{ac}\right)+\log\left(e^{bcx+2ac}-e^{ac}\right)}{bc}$$

input
```
int(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x)
```

output
```
(e**(a*c + b*c*x) - log(e**(2*a*c + b*c*x) + e**(a*c)) + log(e**(2*a*c + b
*c*x) - e**(a*c)))/(b*c)
```

3.219 $\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 83

$$\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}\,dx=\frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}-\frac{2\arctan\left(e^{c(a+bx)}\right)\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}$$

output
```
exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(coth(b*c*x+a*c)^2)^(1/2)-2*arctan(exp(
c*(b*x+a)))*coth(b*c*x+a*c)/b/c/(coth(b*c*x+a*c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.61

$$\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}\,dx=\frac{\left(e^{c(a+bx)}-2\arctan\left(e^{c(a+bx)}\right)\right)\coth(c(a+bx))}{bc\sqrt{\coth^2(c(a+bx))}}$$

input
```
Integrate[E^(c*(a + b*x))/Sqrt[Coth[a*c + b*c*x]^2],x]
```

output
```
((E^(c*(a + b*x)) - 2*ArcTan[E^(c*(a + b*x))])*Coth[c*(a + b*x)])/(b*c*Sqr
t[Coth[c*(a + b*x)]^2])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.64, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7271, 2720, 25, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\coth(ac+bcx)\int e^{c(a+bx)}\tanh(ac+bxc)dx}{\sqrt{\coth^2(ac+bcx)}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\coth(ac+bcx)\int -\frac{1-e^{2c(a+bx)}}{1+e^{2c(a+bx)}}de^{c(a+bx)}}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\coth(ac+bcx)\int \frac{1-e^{2c(a+bx)}}{1+e^{2c(a+bx)}}de^{c(a+bx)}}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\coth(ac+bcx)\left(e^{c(a+bx)}-2\int \frac{1}{1+e^{2c(a+bx)}}de^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\left(e^{c(a+bx)}-2\arctan\left(e^{c(a+bx)}\right)\right)\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}$$

input
```
Int[E^(c*(a + b*x))/Sqrt[Coth[a*c + b*c*x]^2],x]
```

output
```
((E^(c*(a + b*x)) - 2*ArcTan[E^(c*(a + b*x))])*Coth[a*c + b*c*x])/(b*c*Sqr
t[Coth[a*c + b*c*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{\operatorname{csgn}(\coth(c(bx+a)))(\cosh(bxc+ac)+\sinh(bxc+ac)-2\arctan(\mathrm{e}^{bxc+ac}))}{cb}$	48
risch	$\frac{(1+\mathrm{e}^{2c(bx+a)})\mathrm{e}^{c(bx+a)}}{\sqrt{\frac{(1+\mathrm{e}^{2c(bx+a)})^2}{(\mathrm{e}^{2c(bx+a)}-1)^2}}\,(\mathrm{e}^{2c(bx+a)}-1)cb}+\frac{i(1+\mathrm{e}^{2c(bx+a)})\ln(\mathrm{e}^{c(bx+a)}-i)}{\sqrt{\frac{(1+\mathrm{e}^{2c(bx+a)})^2}{(\mathrm{e}^{2c(bx+a)}-1)^2}}\,(\mathrm{e}^{2c(bx+a)}-1)cb}-\frac{i(1+\mathrm{e}^{2c(bx+a)})\ln(\mathrm{e}^{c(bx+a)}+i)}{\sqrt{\frac{(1+\mathrm{e}^{2c(bx+a)})^2}{(\mathrm{e}^{2c(bx+a)}-1)^2}}\,(\mathrm{e}^{2c(bx+a)}-1)cb}$	218

input
```
int(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(coth(c*(b*x+a)))/c/b*(cosh(b*c*x+a*c)+sinh(b*c*x+a*c)-2*arctan(exp(b*
c*x+a*c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.64

$$\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}\,dx = -\frac{2\arctan\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)-\cosh\left(bcx+ac\right)-\sinh\left(bcx+ac\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

output
```
-(2*arctan(cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) - cosh(b*c*x + a*c) - si
nh(b*c*x + a*c))/(b*c)
```

Sympy [F]

$$\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}\,dx = e^{ac}\int \frac{e^{bcx}}{\sqrt{\coth^2(ac+bcx)}}\,dx$$

input
```
integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)**2)**(1/2),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)/sqrt(coth(a*c + b*c*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.42

$$\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}\,dx = -\frac{2\arctan\left(e^{(bcx+ac)}\right)}{bc} + \frac{e^{(bcx+ac)}}{bc}$$

input
```
integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

output
```
-2*arctan(e^(b*c*x + a*c))/(b*c) + e^(b*c*x + a*c)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.76

$$\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}\,dx = -\frac{2\arctan\left(e^{(bcx+ac)}\right)\operatorname{sgn}\left(e^{(2bcx+2ac)}-1\right)}{bc} + \frac{e^{(bcx+ac)}\operatorname{sgn}\left(e^{(2bcx+2ac)}-1\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

output
```
-2*arctan(e^(b*c*x + a*c))*sgn(e^(2*b*c*x + 2*a*c) - 1)/(b*c) + e^(b*c*x +
 a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}\,dx=\int \frac{e^{c\,(a+b\,x)}}{\sqrt{\coth\left(a\,c+b\,c\,x\right)^2}}\,dx$$

input
```
int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(1/2),x)
```

output
```
int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.37

$$\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}\,dx=\frac{-2atan\left(e^{bcx+ac}\right)+e^{bcx+ac}}{bc}$$

input
```
int(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2),x)
```

output
```
( - 2*atan(e**(a*c + b*c*x)) + e**(a*c + b*c*x))/(b*c)
```

3.220 $\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 193

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}}\,dx=\frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}-\frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}}+\frac{3e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)\sqrt{\coth^2(ac+bcx)}}-\frac{3\arctan\left(e^{c(a+bx)}\right)\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}$$

output

```
exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(coth(b*c*x+a*c)^2)^(1/2)-2*exp(c*(b*x+
a))*coth(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^2/(coth(b*c*x+a*c)^2)^(1/2)+3
*exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))/(coth(b*c*x+a*c)^
2)^(1/2)-3*arctan(exp(c*(b*x+a)))*coth(b*c*x+a*c)/b/c/(coth(b*c*x+a*c)^2)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.54

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}}\,dx = \frac{\left(e^{c(a+bx)}\left(2+5e^{2c(a+bx)}+e^{4c(a+bx)}\right)-3\left(1+e^{2c(a+bx)}\right)^2\arctan\left(e^{c(a+bx)}\right)\right)\coth(}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(c(a+bx))}}$$

input
```
Integrate[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(3/2),x]
```

output
```
((E^(c*(a + b*x))*(2 + 5*E^(2*c*(a + b*x)) + E^(4*c*(a + b*x))) - 3*(1 + E
^(2*c*(a + b*x)))^2*ArcTan[E^(c*(a + b*x))])*Coth[c*(a + b*x)])/(b*c*(1 +
E^(2*c*(a + b*x)))^2*Sqrt[Coth[c*(a + b*x)]^2])
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.53, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7271, 2720, 25, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\coth(ac+bcx)\int e^{c(a+bx)}\tanh^3(ac+bxc)dx}{\sqrt{\coth^2(ac+bcx)}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\coth(ac+bcx)\int -\frac{\left(1-e^{2c(a+bx)}\right)^3}{\left(1+e^{2c(a+bx)}\right)^3}de^{c(a+bx)}}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\coth(ac+bcx)\int\frac{\left(1-e^{2c(a+bx)}\right)^3}{\left(1+e^{2c(a+bx)}\right)^3}de^{c(a+bx)}}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$\Big\downarrow \text{300}$$

$$-\frac{\coth(ac+bcx)\int\left(\frac{2\left(1+3e^{4c(a+bx)}\right)}{\left(1+e^{2c(a+bx)}\right)^3}-1\right)de^{c(a+bx)}}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-3\arctan\left(e^{c(a+bx)}\right)+e^{c(a+bx)}+\frac{3e^{c(a+bx)}}{e^{2c(a+bx)}+1}-\frac{2e^{c(a+bx)}}{\left(e^{2c(a+bx)}+1\right)^2}\right)\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}$$

input
```
Int[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(3/2),x]
```

output
```
((E^(c*(a + b*x)) - (2*E^(c*(a + b*x)))/(1 + E^(2*c*(a + b*x)))^2 + (3*E^(
c*(a + b*x)))/(1 + E^(2*c*(a + b*x))) - 3*ArcTan[E^(c*(a + b*x))])*Coth[a*
c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.39 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.68

method	result
default	$\frac{\operatorname{csgn}(\coth(c(bx+a)))\left(\frac{\sinh(bxc+ac)^3}{\cosh(bxc+ac)^2}+\frac{3\sinh(bxc+ac)}{\cosh(bxc+ac)^2}-\frac{3\tanh(bxc+ac)\operatorname{sech}(bxc+ac)}{2}-3\arctan\left(\mathrm{e}^{bxc+ac}\right)+\frac{\sinh(bxc+ac)^2}{\cosh(bxc+ac)}+\frac{2}{\cosh(bxc+ac)}\right.}{cb}$
risch	$\frac{3i\mathrm{e}^{4c(bx+a)}\ln\left(\mathrm{e}^{c(bx+a)}-i\right)-3i\mathrm{e}^{4c(bx+a)}\ln\left(\mathrm{e}^{c(bx+a)}+i\right)+2\,\mathrm{e}^{5c(bx+a)}+6i\mathrm{e}^{2c(bx+a)}\ln\left(\mathrm{e}^{c(bx+a)}-i\right)-6i\mathrm{e}^{2c(bx+a)}\ln\left(\mathrm{e}^{c(bx+a)}+i\right)+1}{2\left(1+\mathrm{e}^{2c(bx+a)}\right)\left(\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^2}}\,cb}$

input
```
int(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(coth(c*(b*x+a)))/c/b*(sinh(b*c*x+a*c)^3/cosh(b*c*x+a*c)^2+3*sinh(b*c*
x+a*c)/cosh(b*c*x+a*c)^2-3/2*tanh(b*c*x+a*c)*sech(b*c*x+a*c)-3*arctan(exp(
b*c*x+a*c))+sinh(b*c*x+a*c)^2/cosh(b*c*x+a*c)+2/cosh(b*c*x+a*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. 2(179) = 358.

Time = 0.10 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.37

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}}\,dx = \frac{\cosh(bcx+ac)^5 + 5\cosh(bcx+ac)\sinh(bcx+ac)^4 + \sinh(bcx+ac)^5 + 5\,(2$$

input
```
integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

output
```
(cosh(b*c*x + a*c)^5 + 5*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 + sinh(b*c*
x + a*c)^5 + 5*(2*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^3 + 5*cosh(b*
c*x + a*c)^3 + 5*(2*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x
+ a*c)^2 - 3*(cosh(b*c*x + a*c)^4 + 4*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^
3 + sinh(b*c*x + a*c)^4 + 2*(3*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^
2 + 2*cosh(b*c*x + a*c)^2 + 4*(cosh(b*c*x + a*c)^3 + cosh(b*c*x + a*c))*si
nh(b*c*x + a*c) + 1)*arctan(cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) + (5*co
sh(b*c*x + a*c)^4 + 15*cosh(b*c*x + a*c)^2 + 2)*sinh(b*c*x + a*c) + 2*cosh
(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^4 + 4*b*c*cosh(b*c*x + a*c)*sinh(b*c
*x + a*c)^3 + b*c*sinh(b*c*x + a*c)^4 + 2*b*c*cosh(b*c*x + a*c)^2 + 2*(3*b
*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^2 + b*c + 4*(b*c*cosh(b*c*
x + a*c)^3 + b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.47

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}}\,dx = -\frac{3\arctan\left(e^{(bcx+ac)}\right)}{bc} + \frac{e^{(5\,bcx+5\,ac)} + 5\,e^{(3\,bcx+3\,ac)} + 2\,e^{(bcx+ac)}}{bc\left(e^{(4\,bcx+4\,ac)} + 2\,e^{(2\,bcx+2\,ac)} + 1\right)}$$

input
```
integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

output
```
-3*arctan(e^(b*c*x + a*c))/(b*c) + (e^(5*b*c*x + 5*a*c) + 5*e^(3*b*c*x + 3
*a*c) + 2*e^(b*c*x + a*c))/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*
c) + 1))
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.71

$$\begin{aligned}\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}}\,dx = & -\frac{3\arctan\left(e^{(bcx+ac)}\right)\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}{bc} \\ & + \frac{e^{(bcx+ac)}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}{bc} \\ & + \frac{3\,e^{(3\,bcx+3\,ac)}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right) + e^{(bcx+ac)}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}{bc\left(e^{(2\,bcx+2\,ac)}+1\right)^2}\end{aligned}$$

input
```
integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

output
```
-3*arctan(e^(b*c*x + a*c))*sgn(e^(2*b*c*x + 2*a*c) - 1)/(b*c) + e^(b*c*x +
 a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1)/(b*c) + (3*e^(3*b*c*x + 3*a*c)*sgn(e^(2
*b*c*x + 2*a*c) - 1) + e^(b*c*x + a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1))/(b*c*
(e^(2*b*c*x + 2*a*c) + 1)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}}\,dx = \int \frac{e^{c\,(a+b\,x)}}{\left(\coth\left(a\,c+b\,c\,x\right)^2\right)^{3/2}}\,dx$$

input
```
int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(3/2),x)
```

output
```
int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.72

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}}\,dx = \frac{-3e^{4bcx+4ac}atan\left(e^{bcx+ac}\right)-6e^{2bcx+2ac}atan\left(e^{bcx+ac}\right)-3atan\left(e^{bcx+ac}\right)+e^{5bcx+5a}}{bc\left(e^{4bcx+4ac}+2e^{2bcx+2ac}+1\right)}$$

input
```
int(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(3/2),x)
```

output
```
( - 3*e**(4*a*c + 4*b*c*x)*atan(e**(a*c + b*c*x)) - 6*e**(2*a*c + 2*b*c*x)
*atan(e**(a*c + b*c*x)) - 3*atan(e**(a*c + b*c*x)) + e**(5*a*c + 5*b*c*x)
+ 5*e**(3*a*c + 3*b*c*x) + 2*e**(a*c + b*c*x))/(b*c*(e**(4*a*c + 4*b*c*x)
+ 2*e**(2*a*c + 2*b*c*x) + 1))
```

3.221 $\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 311

$$\begin{aligned}\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}\,dx &= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} \\ &- \frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+bx)}\coth(ac+bcx)}{3bc\left(1+e^{2c(a+bx)}\right)^3\sqrt{\coth^2(ac+bcx)}} \\ &- \frac{55e^{c(a+bx)}\coth(ac+bcx)}{6bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}} \\ &+ \frac{25e^{c(a+bx)}\coth(ac+bcx)}{4bc\left(1+e^{2c(a+bx)}\right)\sqrt{\coth^2(ac+bcx)}} - \frac{15\arctan\left(e^{c(a+bx)}\right)\coth(ac+bcx)}{4bc\sqrt{\coth^2(ac+bcx)}}\end{aligned}$$

output
```
exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(coth(b*c*x+a*c)^2)^(1/2)-4*exp(c*(b*x+
a))*coth(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^4/(coth(b*c*x+a*c)^2)^(1/2)+2
6/3*exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^3/(coth(b*c*x+
a*c)^2)^(1/2)-55/6*exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))
^2/(coth(b*c*x+a*c)^2)^(1/2)+25/4*exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(1+ex
p(2*c*(b*x+a)))/(coth(b*c*x+a*c)^2)^(1/2)-15/4*arctan(exp(c*(b*x+a)))*coth
(b*c*x+a*c)/b/c/(coth(b*c*x+a*c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.43

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}\,dx = \frac{\left(e^{c(a+bx)}\left(33+157e^{2c(a+bx)}+187e^{4c(a+bx)}+123e^{6c(a+bx)}+12e^{8c(a+bx)}\right)-45(1\right.}{12bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(c(a+b}}$$

input
```
Integrate[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(5/2),x]
```

output
```
((E^(c*(a + b*x))*(33 + 157*E^(2*c*(a + b*x)) + 187*E^(4*c*(a + b*x)) + 12
3*E^(6*c*(a + b*x)) + 12*E^(8*c*(a + b*x))) - 45*(1 + E^(2*c*(a + b*x)))^4
*ArcTan[E^(c*(a + b*x))])*Coth[c*(a + b*x)])/(12*b*c*(1 + E^(2*c*(a + b*x)
))^4*Sqrt[Coth[c*(a + b*x)]^2])
```

Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.52, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7271, 2720, 25, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\coth(ac+bcx)\int e^{c(a+bx)}\tanh^5(ac+bxc)dx}{\sqrt{\coth^2(ac+bcx)}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\coth(ac+bcx)\int -\frac{\left(1-e^{2c(a+bx)}\right)^5}{\left(1+e^{2c(a+bx)}\right)^5}de^{c(a+bx)}}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\coth(ac+bcx)\int\frac{\left(1-e^{2c(a+bx)}\right)^5}{\left(1+e^{2c(a+bx)}\right)^5}de^{c(a+bx)}}{bc\sqrt{\coth^2(ac+bcx)}}$$

$\big\downarrow$ 300

$$-\frac{\coth(ac+bcx)\int\left(\frac{2\left(1+10e^{4c(a+bx)}+5e^{8c(a+bx)}\right)}{\left(1+e^{2c(a+bx)}\right)^5}-1\right)de^{c(a+bx)}}{bc\sqrt{\coth^2(ac+bcx)}}$$

$\big\downarrow$ 2009

$$\frac{\left(-\frac{15}{4}\arctan\left(e^{c(a+bx)}\right)+e^{c(a+bx)}+\frac{25e^{c(a+bx)}}{4\left(e^{2c(a+bx)}+1\right)}-\frac{55e^{c(a+bx)}}{6\left(e^{2c(a+bx)}+1\right)^2}+\frac{26e^{c(a+bx)}}{3\left(e^{2c(a+bx)}+1\right)^3}-\frac{4e^{c(a+bx)}}{\left(e^{2c(a+bx)}+1\right)^4}\right)\coth(ac+b}{bc\sqrt{\coth^2(ac+bcx)}}$$

input
```
Int[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(5/2),x]
```

output
```
((E^(c*(a + b*x)) - (4*E^(c*(a + b*x)))/(1 + E^(2*c*(a + b*x)))^4 + (26*E^
(c*(a + b*x)))/(3*(1 + E^(2*c*(a + b*x)))^3) - (55*E^(c*(a + b*x)))/(6*(1
+ E^(2*c*(a + b*x)))^2) + (25*E^(c*(a + b*x)))/(4*(1 + E^(2*c*(a + b*x))))
 - (15*ArcTan[E^(c*(a + b*x))])/4)*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c +
 b*c*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.42 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.63

method	result
default	$\frac{\operatorname{csgn}(\coth(c(bx+a)))\left(\frac{\sinh(bxc+ac)^5}{\cosh(bxc+ac)^4}+\frac{5\sinh(bxc+ac)^3}{\cosh(bxc+ac)^4}+\frac{5\sinh(bxc+ac)}{\cosh(bxc+ac)^4}-5\left(\frac{\operatorname{sech}(bxc+ac)^3}{4}+\frac{3\operatorname{sech}(bxc+ac)}{8}\right)\tanh(bxc+ac)-\frac{15\arctan}{}\right.}{cb}$
risch	$\frac{(1+e^{2c(bx+a)})e^{c(bx+a)}}{\sqrt{\frac{(1+e^{2c(bx+a)})^2}{(e^{2c(bx+a)}-1)^2}}\,(e^{2c(bx+a)}-1)cb}+\frac{e^{c(bx+a)}(75\,e^{6c(bx+a)}+115\,e^{4c(bx+a)}+109\,e^{2c(bx+a)}+21)}{12(1+e^{2c(bx+a)})^3(e^{2c(bx+a)}-1)\sqrt{\frac{(1+e^{2c(bx+a)})^2}{(e^{2c(bx+a)}-1)^2}}\,cb}+\frac{15i(1+e^{2c(bx+a)})\ln(e^{c(bx}}{8\sqrt{\frac{(1+e^{2c(bx+a)})^2}{(e^{2c(bx+a)}-1)^2}}\,(e^{2c(bx}}$

input
```
int(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(coth(c*(b*x+a)))/c/b*(sinh(b*c*x+a*c)^5/cosh(b*c*x+a*c)^4+5*sinh(b*c*
x+a*c)^3/cosh(b*c*x+a*c)^4+5*sinh(b*c*x+a*c)/cosh(b*c*x+a*c)^4-5*(1/4*sech
(b*c*x+a*c)^3+3/8*sech(b*c*x+a*c))*tanh(b*c*x+a*c)-15/4*arctan(exp(b*c*x+a
*c))+sinh(b*c*x+a*c)^4/cosh(b*c*x+a*c)^3+4*sinh(b*c*x+a*c)^2/cosh(b*c*x+a*
c)^3+8/3/cosh(b*c*x+a*c)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1226 vs. $2(281) = 562$.

Time = 0.11 (sec) , antiderivative size = 1226, normalized size of antiderivative = 3.94

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(12*cosh(b*c*x + a*c)^9 + 108*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^8 +
 12*sinh(b*c*x + a*c)^9 + 3*(144*cosh(b*c*x + a*c)^2 + 41)*sinh(b*c*x + a*
c)^7 + 123*cosh(b*c*x + a*c)^7 + 21*(48*cosh(b*c*x + a*c)^3 + 41*cosh(b*c*
x + a*c))*sinh(b*c*x + a*c)^6 + (1512*cosh(b*c*x + a*c)^4 + 2583*cosh(b*c*
x + a*c)^2 + 187)*sinh(b*c*x + a*c)^5 + 187*cosh(b*c*x + a*c)^5 + (1512*co
sh(b*c*x + a*c)^5 + 4305*cosh(b*c*x + a*c)^3 + 935*cosh(b*c*x + a*c))*sinh
(b*c*x + a*c)^4 + (1008*cosh(b*c*x + a*c)^6 + 4305*cosh(b*c*x + a*c)^4 + 1
870*cosh(b*c*x + a*c)^2 + 157)*sinh(b*c*x + a*c)^3 + 157*cosh(b*c*x + a*c)
^3 + (432*cosh(b*c*x + a*c)^7 + 2583*cosh(b*c*x + a*c)^5 + 1870*cosh(b*c*x
 + a*c)^3 + 471*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 - 45*(cosh(b*c*x +
a*c)^8 + 8*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^7 + sinh(b*c*x + a*c)^8 + 4
*(7*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^6 + 4*cosh(b*c*x + a*c)^6 +
 8*(7*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(
35*cosh(b*c*x + a*c)^4 + 30*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)^4 +
 6*cosh(b*c*x + a*c)^4 + 8*(7*cosh(b*c*x + a*c)^5 + 10*cosh(b*c*x + a*c)^3
 + 3*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^3 + 4*(7*cosh(b*c*x + a*c)^6 + 1
5*cosh(b*c*x + a*c)^4 + 9*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^2 + 4
*cosh(b*c*x + a*c)^2 + 8*(cosh(b*c*x + a*c)^7 + 3*cosh(b*c*x + a*c)^5 + 3*
cosh(b*c*x + a*c)^3 + cosh(b*c*x + a*c))*sinh(b*c*x + a*c) + 1)*arctan(cos
h(b*c*x + a*c) + sinh(b*c*x + a*c)) + (108*cosh(b*c*x + a*c)^8 + 861*co...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.47

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}\,dx = -\frac{15\arctan\left(e^{(bcx+ac)}\right)}{4\,bc} + \frac{12\,e^{(9\,bcx+9\,ac)}+123\,e^{(7\,bcx+7\,ac)}+187\,e^{(5\,bcx+5\,ac)}+157\,e^{(3\,bcx+3\,ac)}+33\,e^{(bcx+ac)}}{12\,bc(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1)}$$

input
```
integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

output
```
-15/4*arctan(e^(b*c*x + a*c))/(b*c) + 1/12*(12*e^(9*b*c*x + 9*a*c) + 123*e
^(7*b*c*x + 7*a*c) + 187*e^(5*b*c*x + 5*a*c) + 157*e^(3*b*c*x + 3*a*c) + 3
3*e^(b*c*x + a*c))/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) + 6*e
^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.62

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}\,dx = -\frac{15\arctan\left(e^{(bcx+ac)}\right)\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}{4\,bc} + \frac{e^{(bcx+ac)}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}{bc} + \frac{75\,e^{(7\,bcx+7\,ac)}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)+115\,e^{(5\,bcx+5\,ac)}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)+109\,e^{(3\,bcx+3\,ac)}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-\right.}{12\,bc(e^{(2\,bcx+2\,ac)}+1)^4}$$

input
```
integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

output
```
-15/4*arctan(e^(b*c*x + a*c))*sgn(e^(2*b*c*x + 2*a*c) - 1)/(b*c) + e^(b*c*
x + a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1)/(b*c) + 1/12*(75*e^(7*b*c*x + 7*a*c)
*sgn(e^(2*b*c*x + 2*a*c) - 1) + 115*e^(5*b*c*x + 5*a*c)*sgn(e^(2*b*c*x + 2
*a*c) - 1) + 109*e^(3*b*c*x + 3*a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) + 21*e^(
b*c*x + a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1))/(b*c*(e^(2*b*c*x + 2*a*c) + 1)^
4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}\,dx = \int \frac{e^{c\,(a+b\,x)}}{\left(\coth\left(a\,c+b\,c\,x\right)^2\right)^{5/2}}\,dx$$

input
```
int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(5/2),x)
```

output
```
int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.80

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}\,dx = \frac{-45e^{8bcx+8ac}atan\left(e^{bcx+ac}\right) - 180e^{6bcx+6ac}atan\left(e^{bcx+ac}\right) - 270e^{4bcx+4ac}atan\left(e^{bcx}\right.}{12bc\left(e\right.}$$

input
```
int(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2),x)
```

output
```
( - 45*e**(8*a*c + 8*b*c*x)*atan(e**(a*c + b*c*x)) - 180*e**(6*a*c + 6*b*c
*x)*atan(e**(a*c + b*c*x)) - 270*e**(4*a*c + 4*b*c*x)*atan(e**(a*c + b*c*x
)) - 180*e**(2*a*c + 2*b*c*x)*atan(e**(a*c + b*c*x)) - 45*atan(e**(a*c + b
*c*x)) + 12*e**(9*a*c + 9*b*c*x) + 123*e**(7*a*c + 7*b*c*x) + 187*e**(5*a*
c + 5*b*c*x) + 157*e**(3*a*c + 3*b*c*x) + 33*e**(a*c + b*c*x))/(12*b*c*(e*
*(8*a*c + 8*b*c*x) + 4*e**(6*a*c + 6*b*c*x) + 6*e**(4*a*c + 4*b*c*x) + 4*e
**(2*a*c + 2*b*c*x) + 1))
```

3.222 $\int \sin^3(\coth(a+bx))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 157

$$\int \sin^3(\coth(a+bx))\,dx = -\frac{3\,\text{CosIntegral}(1-\coth(a+bx))\sin(1)}{8b} - \frac{3\,\text{CosIntegral}(1+\coth(a+bx))\sin(1)}{8b} + \frac{\text{CosIntegral}(3-3\coth(a+bx))\sin(3)}{8b} + \frac{\text{CosIntegral}(3+3\coth(a+bx))\sin(3)}{8b} - \frac{\cos(3)\text{Si}(3-3\coth(a+bx))}{8b} + \frac{3\cos(1)\text{Si}(1-\coth(a+bx))}{8b} + \frac{3\cos(1)\text{Si}(1+\coth(a+bx))}{8b} - \frac{\cos(3)\text{Si}(3+3\coth(a+bx))}{8b}$$

output
```
-3/8*Ci(1-coth(b*x+a))*sin(1)/b-3/8*Ci(1+coth(b*x+a))*sin(1)/b+1/8*Ci(3-3*
coth(b*x+a))*sin(3)/b+1/8*Ci(3+3*coth(b*x+a))*sin(3)/b+1/8*cos(3)*Si(-3+3*
coth(b*x+a))/b-3/8*cos(1)*Si(-1+coth(b*x+a))/b+3/8*cos(1)*Si(1+coth(b*x+a)
)/b-1/8*cos(3)*Si(3+3*coth(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

$$\int \sin^3(\coth(a+bx))\,dx$$
$$= \frac{-6\,\text{CosIntegral}(1-\coth(a+bx))\sin(1)-6\,\text{CosIntegral}(1+\coth(a+bx))\sin(1)+2\,\text{CosIntegral}(3-3}{}$$

input
```
Integrate[Sin[Coth[a + b*x]]^3,x]
```

output
```
(-6*CosIntegral[1 - Coth[a + b*x]]*Sin[1] - 6*CosIntegral[1 + Coth[a + b*x
]]*Sin[1] + 2*CosIntegral[3 - 3*Coth[a + b*x]]*Sin[3] + 2*CosIntegral[3 +
3*Coth[a + b*x]]*Sin[3] - 2*Cos[3]*SinIntegral[3 - 3*Coth[a + b*x]] + 6*Co
s[1]*SinIntegral[1 - Coth[a + b*x]] + 6*Cos[1]*SinIntegral[1 + Coth[a + b*
x]] - 2*Cos[3]*SinIntegral[3 + 3*Coth[a + b*x]])/(16*b)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4852, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(\coth(a+bx))\,dx$$
$$\Big\downarrow \text{4852}$$
$$\frac{\int \frac{\sin^3(\coth(a+bx))}{1-\coth^2(a+bx)}d\coth(a+bx)}{b}$$
$$\Big\downarrow \text{7276}$$
$$\frac{\int \left(\frac{\sin^3(\coth(a+bx))}{2(\coth(a+bx)+1)}-\frac{\sin^3(\coth(a+bx))}{2(\coth(a+bx)-1)}\right)d\coth(a+bx)}{b}$$
$$\Big\downarrow \text{2009}$$

$\frac{1}{8}\sin(3)\,\text{CosIntegral}(3-3\coth(a+bx))+\frac{1}{8}\sin(3)\,\text{CosIntegral}(3\coth(a+bx)+3)-\frac{3}{8}\sin(1)\,\text{CosIntegral}(1-\text{c}$

input
```
Int[Sin[Coth[a + b*x]]^3,x]
```

output
```
((-3*CosIntegral[1 - Coth[a + b*x]]*Sin[1])/8 - (3*CosIntegral[1 + Coth[a
+ b*x]]*Sin[1])/8 + (CosIntegral[3 - 3*Coth[a + b*x]]*Sin[3])/8 + (CosInte
gral[3 + 3*Coth[a + b*x]]*Sin[3])/8 - (Cos[3]*SinIntegral[3 - 3*Coth[a + b
*x]])/8 + (3*Cos[1]*SinIntegral[1 - Coth[a + b*x]])/8 + (3*Cos[1]*SinInteg
ral[1 + Coth[a + b*x]])/8 - (Cos[3]*SinIntegral[3 + 3*Coth[a + b*x]])/8)/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4852
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cot[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*
x^2), Cot[v]/d, u, x], x], x, Cot[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[
NonfreeFactors[Cot[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u],
x]]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [A] (verified)

Time = 2.59 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{\frac{\mathrm{Si}(-3+3\coth(bx+a))\cos(3)}{8}+\frac{\mathrm{Ci}(-3+3\coth(bx+a))\sin(3)}{8}-\frac{\mathrm{Si}(3+3\coth(bx+a))\cos(3)}{8}+\frac{\mathrm{Ci}(3+3\coth(bx+a))\sin(3)}{8}-\frac{3\ \mathrm{Si}(-1+\mathrm{co}}{}}{b}$
default	$\frac{\frac{\mathrm{Si}(-3+3\coth(bx+a))\cos(3)}{8}+\frac{\mathrm{Ci}(-3+3\coth(bx+a))\sin(3)}{8}-\frac{\mathrm{Si}(3+3\coth(bx+a))\cos(3)}{8}+\frac{\mathrm{Ci}(3+3\coth(bx+a))\sin(3)}{8}-\frac{3\ \mathrm{Si}(-1+\mathrm{co}}{}}{b}$
risch	$-\frac{i e^{-3i}\,\mathrm{expIntegral}_1\left(-\frac{6i e^{-a}}{e^{2bx+a}-e^{-a}}-6i\right)}{16b}+\frac{i e^{3i}\,\mathrm{expIntegral}_1\left(-\frac{6i e^{-a}}{e^{2bx+a}-e^{-a}}\right)}{16b}+\frac{i e^{3i}\,\mathrm{expIntegral}_1\left(\frac{6i e^{-a}}{e^{2bx+a}-e^{-a}}\right.}{16b}$

input
```
int(sin(coth(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*Si(-3+3*coth(b*x+a))*cos(3)+1/8*Ci(-3+3*coth(b*x+a))*sin(3)-1/8*S
i(3+3*coth(b*x+a))*cos(3)+1/8*Ci(3+3*coth(b*x+a))*sin(3)-3/8*Si(-1+coth(b*
x+a))*cos(1)-3/8*Ci(-1+coth(b*x+a))*sin(1)+3/8*Si(1+coth(b*x+a))*cos(1)-3/
8*Ci(1+coth(b*x+a))*sin(1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 696, normalized size of antiderivative = 4.43

$$\int \sin^3(\coth(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate(sin(coth(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/16*((-I*cos(3)^2*cos(1) - (-I*cos(1) + sin(1))*sin(3)^2 - 2*I*(I*cos(3)*
cos(1) - cos(3)*sin(1))*sin(3) + I*(-I*cos(3)^2 + I)*sin(1) + I*cos(1))*co
s_integral(3*(cosh(b*x + a) + sinh(b*x + a))/sinh(b*x + a)) - 3*(2*cos(3)*
cos(1)*sin(1) + I*cos(3)*sin(1)^2 + (-I*cos(1)^2 + I)*cos(3) + I*(-I*cos(1
)^2 + 2*cos(1)*sin(1) + I*sin(1)^2 + I)*sin(3))*cos_integral((cosh(b*x + a
) + sinh(b*x + a))/sinh(b*x + a)) + (-I*cos(3)^2*cos(1) - (-I*cos(1) + sin
(1))*sin(3)^2 - 2*I*(I*cos(3)*cos(1) - cos(3)*sin(1))*sin(3) + I*(-I*cos(3
)^2 + I)*sin(1) + I*cos(1))*cos_integral(6/(cosh(b*x + a)^2 + 2*cosh(b*x +
 a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) - 3*(2*cos(3)*cos(1)*sin(1) + I*
cos(3)*sin(1)^2 + (-I*cos(1)^2 + I)*cos(3) + I*(-I*cos(1)^2 + 2*cos(1)*sin
(1) + I*sin(1)^2 + I)*sin(3))*cos_integral(2/(cosh(b*x + a)^2 + 2*cosh(b*x
 + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) - (cos(3)^2*cos(1) - (cos(1) +
 I*sin(1))*sin(3)^2 + 2*I*(cos(3)*cos(1) + I*cos(3)*sin(1))*sin(3) + I*(co
s(3)^2 + 1)*sin(1) + cos(1))*sin_integral(3*(cosh(b*x + a) + sinh(b*x + a)
)/sinh(b*x + a)) - 3*(-2*I*cos(3)*cos(1)*sin(1) + cos(3)*sin(1)^2 - (cos(1
)^2 + 1)*cos(3) - I*(cos(1)^2 + 2*I*cos(1)*sin(1) - sin(1)^2 + 1)*sin(3))*
sin_integral((cosh(b*x + a) + sinh(b*x + a))/sinh(b*x + a)) + (cos(3)^2*co
s(1) - (cos(1) + I*sin(1))*sin(3)^2 + 2*I*(cos(3)*cos(1) + I*cos(3)*sin(1)
)*sin(3) + I*(cos(3)^2 + 1)*sin(1) + cos(1))*sin_integral(6/(cosh(b*x + a)
^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) - 3*(2*I*cos...
```

Sympy [F]

$$\int \sin^3(\coth(a+bx))\,dx = \int \sin^3\left(\coth\left(a+bx\right)\right)dx$$

input
```
integrate(sin(coth(b*x+a))**3,x)
```

output
```
Integral(sin(coth(a + b*x))**3, x)
```

Maxima [F]

$$\int \sin^3(\coth(a+bx))\,dx = \int \sin\left(\coth\left(bx+a\right)\right)^3\,dx$$

input
```
integrate(sin(coth(b*x+a))^3,x, algorithm="maxima")
```

output
```
integrate(sin(coth(b*x + a))^3, x)
```

Giac [F]

$$\int \sin^3(\coth(a+bx))\,dx = \int \sin\left(\coth\left(bx+a\right)\right)^3\,dx$$

input
```
integrate(sin(coth(b*x+a))^3,x, algorithm="giac")
```

output
```
integrate(sin(coth(b*x + a))^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^3(\coth(a+bx))\,dx = \int \sin\left(\coth(a+b\,x)\right)^3\,dx$$

input
```
int(sin(coth(a + b*x))^3,x)
```

output
```
int(sin(coth(a + b*x))^3, x)
```

Reduce [F]

$$\int \sin^3(\coth(a+bx))\,dx = \int \sin\left(\coth\left(bx+a\right)\right)^3 dx$$

input
```
int(sin(coth(b*x+a))^3,x)
```

output
```
int(sin(coth(a + b*x))**3,x)
```

3.223 $\int \sin^2(\coth(a+bx))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 115

$$\int \sin^2(\coth(a+bx))\,dx = \frac{\cos(2)\,\mathrm{CosIntegral}(2-2\coth(a+bx))}{4b} - \frac{\cos(2)\,\mathrm{CosIntegral}(2+2\coth(a+bx))}{4b} - \frac{\log(1-\coth(a+bx))}{4b} + \frac{\log(1+\coth(a+bx))}{4b} + \frac{\sin(2)\mathrm{Si}(2-2\coth(a+bx))}{4b} - \frac{\sin(2)\mathrm{Si}(2+2\coth(a+bx))}{4b}$$

output

```
1/4*cos(2)*Ci(2-2*coth(b*x+a))/b-1/4*cos(2)*Ci(2+2*coth(b*x+a))/b-1/4*ln(1
-coth(b*x+a))/b+1/4*ln(1+coth(b*x+a))/b-1/4*sin(2)*Si(-2+2*coth(b*x+a))/b-
1/4*sin(2)*Si(2+2*coth(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$\int \sin^2(\coth(a+bx))\,dx$$
$$= \frac{\cos(2)\,\mathrm{CosIntegral}(2-2\coth(a+bx)) - \cos(2)\,\mathrm{CosIntegral}(2(1+\coth(a+bx))) - \log(1-\coth(a+b}{4b}$$

input
```
Integrate[Sin[Coth[a + b*x]]^2,x]
```

output
```
(Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]] - Cos[2]*CosIntegral[2*(1 + Coth[
a + b*x])] - Log[1 - Coth[a + b*x]] + Log[1 + Coth[a + b*x]] + Sin[2]*SinI
ntegral[2 - 2*Coth[a + b*x]] - Sin[2]*SinIntegral[2*(1 + Coth[a + b*x])])/
(4*b)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4852, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(\coth(a+bx))\,dx$$
$$\Big\downarrow \text{ 4852}$$
$$\frac{\int \frac{\sin^2(\coth(a+bx))}{1-\coth^2(a+bx)}d\coth(a+bx)}{b}$$
$$\Big\downarrow \text{ 7276}$$
$$\frac{\int \left(\frac{\sin^2(\coth(a+bx))}{2(\coth(a+bx)+1)}-\frac{\sin^2(\coth(a+bx))}{2(\coth(a+bx)-1)}\right)d\coth(a+bx)}{b}$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{4}\cos(2)\,\text{CosIntegral}(2-2\coth(a+bx))-\frac{1}{4}\cos(2)\,\text{CosIntegral}(2\coth(a+bx)+2)+\frac{1}{4}\sin(2)\text{Si}(2-2\coth(a+}{b}$$

input
```
Int[Sin[Coth[a + b*x]]^2,x]
```

output
```
((Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]])/4 - (Cos[2]*CosIntegral[2 + 2*C
oth[a + b*x]])/4 - Log[1 - Coth[a + b*x]]/4 + Log[1 + Coth[a + b*x]]/4 + (
Sin[2]*SinIntegral[2 - 2*Coth[a + b*x]])/4 - (Sin[2]*SinIntegral[2 + 2*Cot
h[a + b*x]])/4)/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4852
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cot[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*
x^2), Cot[v]/d, u, x], x], x, Cot[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[
NonfreeFactors[Cot[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u],
x]]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{-\frac{\ln(-1+\coth(bx+a))}{4}+\frac{\ln(1+\coth(bx+a))}{4}-\frac{\mathrm{Si}(-2+2\coth(bx+a))\sin(2)}{4}+\frac{\mathrm{Ci}(-2+2\coth(bx+a))\cos(2)}{4}-\frac{\mathrm{Si}(2+2\coth(bx+a))\sin(}{4}}{b}$
default	$\frac{-\frac{\ln(-1+\coth(bx+a))}{4}+\frac{\ln(1+\coth(bx+a))}{4}-\frac{\mathrm{Si}(-2+2\coth(bx+a))\sin(2)}{4}+\frac{\mathrm{Ci}(-2+2\coth(bx+a))\cos(2)}{4}-\frac{\mathrm{Si}(2+2\coth(bx+a))\sin(}{4}}{b}$
risch	$\frac{e^{2i}\,\mathrm{expIntegral}_1\left(\frac{4ie^{-a}}{e^{2bx+a}-e^{-a}}+4i\right)}{8b}-\frac{e^{-2i}\,\mathrm{expIntegral}_1\left(\frac{4ie^{-a}}{e^{2bx+a}-e^{-a}}\right)}{8b}+\frac{e^{-2i}\,\mathrm{expIntegral}_1\left(-\frac{4ie^{-a}}{e^{2bx+a}-e^{-a}}-4i\right)}{8b}$

input
```
int(sin(coth(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4*ln(-1+coth(b*x+a))+1/4*ln(1+coth(b*x+a))-1/4*Si(-2+2*coth(b*x+a)
)*sin(2)+1/4*Ci(-2+2*coth(b*x+a))*cos(2)-1/4*Si(2+2*coth(b*x+a))*sin(2)-1/
4*Ci(2+2*coth(b*x+a))*cos(2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.00

$$\int \sin^2(\coth(a+bx))\,dx$$
$$= \frac{4\,bx\cos(2)+4i\,bx\sin(2)-\left(\cos(2)^2+2i\,\cos(2)\sin(2)-\sin(2)^2+1\right)\mathrm{Ci}\left(\frac{2(\cosh(bx+a)+\sinh(bx+a))}{\sinh(bx+a)}\right)+(\mathrm{c}}{}$$

input
```
integrate(sin(coth(b*x+a))^2,x, algorithm="fricas")
```

output
```
1/8*(4*b*x*cos(2) + 4*I*b*x*sin(2) - (cos(2)^2 + 2*I*cos(2)*sin(2) - sin(2
)^2 + 1)*cos_integral(2*(cosh(b*x + a) + sinh(b*x + a))/sinh(b*x + a)) + (
cos(2)^2 + 2*I*cos(2)*sin(2) - sin(2)^2 + 1)*cos_integral(4/(cosh(b*x + a)
^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + (I*cos(2)^2 -
 2*cos(2)*sin(2) - I*sin(2)^2 - I)*sin_integral(2*(cosh(b*x + a) + sinh(b*
x + a))/sinh(b*x + a)) + (I*cos(2)^2 - 2*cos(2)*sin(2) - I*sin(2)^2 - I)*s
in_integral(4/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x
+ a)^2 - 1)))/(b*cos(2) + I*b*sin(2))
```

Sympy [F]

$$\int \sin^2(\coth(a+bx))\,dx = \int \sin^2\left(\coth\left(a+bx\right)\right)dx$$

input
```
integrate(sin(coth(b*x+a))**2,x)
```

output
```
Integral(sin(coth(a + b*x))**2, x)
```

Maxima [F]

$$\int \sin^2(\coth(a+bx))\,dx = \int \sin\left(\coth\left(bx+a\right)\right)^2\,dx$$

input
```
integrate(sin(coth(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/2*x - 1/2*integrate(cos(2*(e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1)),
x)
```

Giac [F]

$$\int \sin^2(\coth(a+bx))\,dx = \int \sin\left(\coth\left(bx+a\right)\right)^2\,dx$$

input
```
integrate(sin(coth(b*x+a))^2,x, algorithm="giac")
```

output
```
integrate(sin(coth(b*x + a))^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(\coth(a+bx))\,dx = \int \sin\left(\coth(a+b\,x)\right)^2\,dx$$

input
```
int(sin(coth(a + b*x))^2,x)
```

output
```
int(sin(coth(a + b*x))^2, x)
```

Reduce [F]

$$\int \sin^2(\coth(a+bx))\,dx = \int \sin\left(\coth\left(bx+a\right)\right)^2\,dx$$

input
```
int(sin(coth(b*x+a))^2,x)
```

output
```
int(sin(coth(a + b*x))**2,x)
```

3.224 $\int \sin(\coth(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 77

$$\int \sin(\coth(a+bx))\,dx = -\frac{\text{CosIntegral}(1-\coth(a+bx))\sin(1)}{2b} - \frac{\text{CosIntegral}(1+\coth(a+bx))\sin(1)}{2b} + \frac{\cos(1)\text{Si}(1-\coth(a+bx))}{2b} + \frac{\cos(1)\text{Si}(1+\coth(a+bx))}{2b}$$

output
```
-1/2*Ci(1-coth(b*x+a))*sin(1)/b-1/2*Ci(1+coth(b*x+a))*sin(1)/b-1/2*cos(1)*
Si(-1+coth(b*x+a))/b+1/2*cos(1)*Si(1+coth(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.77

$$\int \sin(\coth(a+bx))\,dx = -\frac{\text{CosIntegral}(1-\coth(a+bx))\sin(1)+\text{CosIntegral}(1+\coth(a+bx))\sin(1)-\cos(1)(\text{Si}(1-\coth(a}{2b}$$

input
```
Integrate[Sin[Coth[a + b*x]],x]
```

output
```
-1/2*(CosIntegral[1 - Coth[a + b*x]]*Sin[1] + CosIntegral[1 + Coth[a + b*x
]]*Sin[1] - Cos[1]*(SinIntegral[1 - Coth[a + b*x]] + SinIntegral[1 + Coth[
a + b*x]]))/b
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4852, 3814, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(\coth(a+bx))\,dx$$

$$\Big\downarrow \; 4852$$

$$\frac{\int \frac{\sin(\coth(a+bx))}{1-\coth^2(a+bx)}d\coth(a+bx)}{b}$$

$$\Big\downarrow \; 3814$$

$$\frac{\int \left(\frac{\sin(\coth(a+bx))}{2(1-\coth(a+bx))}+\frac{\sin(\coth(a+bx))}{2(\coth(a+bx)+1)}\right)d\coth(a+bx)}{b}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{1}{2}\sin(1)\,\text{CosIntegral}(1-\coth(a+bx))-\frac{1}{2}\sin(1)\,\text{CosIntegral}(\coth(a+bx)+1)+\frac{1}{2}\cos(1)\text{Si}(1-\coth(a+bx)}{b}$$

input
```
Int[Sin[Coth[a + b*x]],x]
```

output
```
(-1/2*(CosIntegral[1 - Coth[a + b*x]]*Sin[1]) - (CosIntegral[1 + Coth[a +
b*x]]*Sin[1])/2 + (Cos[1]*SinIntegral[1 - Coth[a + b*x]])/2 + (Cos[1]*SinI
ntegral[1 + Coth[a + b*x]])/2)/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3814
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 4852
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cot[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*
x^2), Cot[v]/d, u, x], x], x, Cot[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[
NonfreeFactors[Cot[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u],
x]]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{-\frac{\operatorname{Si}(-1+\coth(bx+a))\cos(1)}{2}-\frac{\operatorname{Ci}(-1+\coth(bx+a))\sin(1)}{2}+\frac{\operatorname{Si}(1+\coth(bx+a))\cos(1)}{2}-\frac{\operatorname{Ci}(1+\coth(bx+a))\sin(1)}{2}}{b}$
default	$\frac{-\frac{\operatorname{Si}(-1+\coth(bx+a))\cos(1)}{2}-\frac{\operatorname{Ci}(-1+\coth(bx+a))\sin(1)}{2}+\frac{\operatorname{Si}(1+\coth(bx+a))\cos(1)}{2}-\frac{\operatorname{Ci}(1+\coth(bx+a))\sin(1)}{2}}{b}$
risch	$\frac{i e^{-i} \operatorname{expIntegral}_1\left(-\frac{2i e^{-a}}{e^{2bx+a}-e^{-a}}-2i\right)}{4b}-\frac{\pi \operatorname{csgn}\left(\frac{e^{-a}}{-e^{2bx+a}+e^{-a}}\right)e^{i}}{4b}-\frac{\operatorname{Si}\left(\frac{2 e^{-a}}{e^{2bx+a}-e^{-a}}\right)e^{i}}{2b}-\frac{i e^{i} \operatorname{expIntegral}_1\left(\right.}{4b}$

input
```
int(sin(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2*Si(-1+coth(b*x+a))*cos(1)-1/2*Ci(-1+coth(b*x+a))*sin(1)+1/2*Si(1
+coth(b*x+a))*cos(1)-1/2*Ci(1+coth(b*x+a))*sin(1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.81

$$\int \sin(\coth(a+bx))\,dx$$
$$= \frac{\left(i\cos(1)^2 - 2\cos(1)\sin(1) - i\sin(1)^2 - i\right)\operatorname{Ci}\left(\frac{\cosh(bx+a)+\sinh(bx+a)}{\sinh(bx+a)}\right) + \left(i\cos(1)^2 - 2\cos(1)\sin(1) - \right.}{}$$

input
```
integrate(sin(coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*((I*cos(1)^2 - 2*cos(1)*sin(1) - I*sin(1)^2 - I)*cos_integral((cosh(b*
x + a) + sinh(b*x + a))/sinh(b*x + a)) + (I*cos(1)^2 - 2*cos(1)*sin(1) - I
*sin(1)^2 - I)*cos_integral(2/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x
+ a) + sinh(b*x + a)^2 - 1)) + (cos(1)^2 + 2*I*cos(1)*sin(1) - sin(1)^2 +
1)*sin_integral((cosh(b*x + a) + sinh(b*x + a))/sinh(b*x + a)) - (cos(1)^2
 + 2*I*cos(1)*sin(1) - sin(1)^2 + 1)*sin_integral(2/(cosh(b*x + a)^2 + 2*c
osh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)))/(b*cos(1) + I*b*sin(1)
)
```

Sympy [F]

$$\int \sin(\coth(a+bx))\,dx = \int \sin\left(\coth\left(a+bx\right)\right)dx$$

input
```
integrate(sin(coth(b*x+a)),x)
```

output
```
Integral(sin(coth(a + b*x)), x)
```

Maxima [F]

$$\int \sin(\coth(a+bx))\,dx = \int \sin\left(\coth\left(bx+a\right)\right)\,dx$$

input
```
integrate(sin(coth(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(sin(coth(b*x + a)), x)
```

Giac [F]

$$\int \sin(\coth(a+bx))\,dx = \int \sin\left(\coth\left(bx+a\right)\right)\,dx$$

input
```
integrate(sin(coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(sin(coth(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin(\coth(a+bx))\,dx = \int \sin\left(\coth(a+b\,x)\right)\,dx$$

input
```
int(sin(coth(a + b*x)),x)
```

output
```
int(sin(coth(a + b*x)), x)
```

Reduce [F]

$$\int \sin(\coth(a+bx))\,dx = \int \sin\left(\coth\left(bx+a\right)\right)dx$$

input
```
int(sin(coth(b*x+a)),x)
```

output
```
int(sin(coth(a + b*x)),x)
```

3.225 $\int \csc(\coth(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 7

$$\int \csc(\coth(a+bx))\,dx = \frac{1}{2}\text{Int}\left(-\frac{\csc(\coth(a+bx))\text{csch}^2(a+bx)}{1-\coth(a+bx)},x\right) - \frac{1}{2}\text{Int}\left(\frac{\csc(\coth(a+bx))\text{csch}^2(a+bx)}{1+\coth(a+bx)},x\right)$$

output
```
1/2*Defer(Int)(-csc(coth(b*x+a))*csch(b*x+a)^2/(1-coth(b*x+a)),x)-1/2*Defe
r(Int)(csc(coth(b*x+a))*csch(b*x+a)^2/(1+coth(b*x+a)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \csc(\coth(a+bx))\,dx = \int \csc(\coth(a+bx))\,dx$$

input
```
Integrate[Csc[Coth[a + b*x]],x]
```

output
```
Integrate[Csc[Coth[a + b*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(\coth(a+bx))\,dx$$

$$\Big\downarrow \ 4852$$

$$\frac{\int \frac{\csc(\coth(a+bx))}{1-\coth^2(a+bx)}d\coth(a+bx)}{b}$$

$$\Big\downarrow \ 7276$$

$$\frac{\int \left(\frac{\csc(\coth(a+bx))}{2(\coth(a+bx)+1)}-\frac{\csc(\coth(a+bx))}{2(\coth(a+bx)-1)}\right)d\coth(a+bx)}{b}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{1}{2}\int \frac{\csc(\coth(a+bx))}{\coth(a+bx)+1}d\coth(a+bx)-\frac{1}{2}\int \frac{\csc(\coth(a+bx))}{\coth(a+bx)-1}d\coth(a+bx)}{b}$$

input
```
Int[Csc[Coth[a + b*x]],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \csc(\coth(bx+a))\,dx$$

input
```
int(csc(coth(b*x+a)),x)
```

output
```
int(csc(coth(b*x+a)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \csc(\coth(a+bx))\,dx = \int \csc(\coth(bx+a))\,dx$$

input
```
integrate(csc(coth(b*x+a)),x, algorithm="fricas")
```

output
```
integral(csc(coth(b*x + a)), x)
```

Sympy [N/A]

Not integrable

Time = 9.87 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int \csc(\coth(a+bx))\,dx = \int \csc(\coth(a+bx))\,dx$$

input
```
integrate(csc(coth(b*x+a)),x)
```

output
```
Integral(csc(coth(a + b*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \csc(\coth(a+bx))\,dx = \int \csc\left(\coth\left(bx+a\right)\right)\,dx$$

input
```
integrate(csc(coth(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(csc(coth(b*x + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \csc(\coth(a+bx))\,dx = \int \csc\left(\coth\left(bx+a\right)\right)\,dx$$

input
```
integrate(csc(coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(csc(coth(b*x + a)), x)
```

Mupad [N/A]

Not integrable

Time = 3.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

$$\int \csc(\coth(a+bx))\,dx = \int \frac{1}{\sin\left(\coth\left(a+b\,x\right)\right)}\,dx$$

input
```
int(1/sin(coth(a + b*x)),x)
```

output
```
int(1/sin(coth(a + b*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \csc(\coth(a+bx))\,dx = \int \csc\left(\coth\left(bx+a\right)\right)dx$$

input
```
int(csc(coth(b*x+a)),x)
```

output
```
int(csc(coth(a + b*x)),x)
```

3.226 $\int \cos^3(\coth(a+bx))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 157

$$\int \cos^3(\coth(a+bx))\,dx = -\frac{\cos(3)\,\text{CosIntegral}(3-3\coth(a+bx))}{8b} - \frac{3\cos(1)\,\text{CosIntegral}(1-\coth(a+bx))}{8b} + \frac{3\cos(1)\,\text{CosIntegral}(1+\coth(a+bx))}{8b} + \frac{\cos(3)\,\text{CosIntegral}(3+3\coth(a+bx))}{8b} - \frac{\sin(3)\text{Si}(3-3\coth(a+bx))}{8b} - \frac{3\sin(1)\text{Si}(1-\coth(a+bx))}{8b} + \frac{3\sin(1)\text{Si}(1+\coth(a+bx))}{8b} + \frac{\sin(3)\text{Si}(3+3\coth(a+bx))}{8b}$$

output
```
-1/8*cos(3)*Ci(3-3*coth(b*x+a))/b-3/8*cos(1)*Ci(1-coth(b*x+a))/b+3/8*cos(1
)*Ci(1+coth(b*x+a))/b+1/8*cos(3)*Ci(3+3*coth(b*x+a))/b+1/8*sin(3)*Si(-3+3*
coth(b*x+a))/b+3/8*sin(1)*Si(-1+coth(b*x+a))/b+3/8*sin(1)*Si(1+coth(b*x+a)
)/b+1/8*sin(3)*Si(3+3*coth(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

$$\int \cos^3(\coth(a+bx))\,dx$$
$$= \frac{-2\cos(3)\,\text{CosIntegral}(3-3\coth(a+bx))-6\cos(1)\,\text{CosIntegral}(1-\coth(a+bx))+6\cos(1)\,\text{CosInteg}}{}$$

input
```
Integrate[Cos[Coth[a + b*x]]^3,x]
```

output
```
(-2*Cos[3]*CosIntegral[3 - 3*Coth[a + b*x]] - 6*Cos[1]*CosIntegral[1 - Cot
h[a + b*x]] + 6*Cos[1]*CosIntegral[1 + Coth[a + b*x]] + 2*Cos[3]*CosIntegr
al[3 + 3*Coth[a + b*x]] - 2*Sin[3]*SinIntegral[3 - 3*Coth[a + b*x]] - 6*Si
n[1]*SinIntegral[1 - Coth[a + b*x]] + 6*Sin[1]*SinIntegral[1 + Coth[a + b*
x]] + 2*Sin[3]*SinIntegral[3 + 3*Coth[a + b*x]])/(16*b)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4852, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(\coth(a+bx))\,dx$$
$$\Big\downarrow \; 4852$$
$$\frac{\int \frac{\cos^3(\coth(a+bx))}{1-\coth^2(a+bx)}d\coth(a+bx)}{b}$$
$$\Big\downarrow \; 7276$$
$$\frac{\int \left(\frac{\cos^3(\coth(a+bx))}{2(\coth(a+bx)+1)}-\frac{\cos^3(\coth(a+bx))}{2(\coth(a+bx)-1)}\right)d\coth(a+bx)}{b}$$
$$\Big\downarrow \; 2009$$

$$-\frac{1}{8}\cos(3)\,\text{CosIntegral}(3-3\coth(a+bx))-\frac{3}{8}\cos(1)\,\text{CosIntegral}(1-\coth(a+bx))+\frac{3}{8}\cos(1)\,\text{CosIntegral}(\coth$$

input
```
Int[Cos[Coth[a + b*x]]^3,x]
```

output
```
(-1/8*(Cos[3]*CosIntegral[3 - 3*Coth[a + b*x]]) - (3*Cos[1]*CosIntegral[1
- Coth[a + b*x]])/8 + (3*Cos[1]*CosIntegral[1 + Coth[a + b*x]])/8 + (Cos[3
]*CosIntegral[3 + 3*Coth[a + b*x]])/8 - (Sin[3]*SinIntegral[3 - 3*Coth[a +
 b*x]])/8 - (3*Sin[1]*SinIntegral[1 - Coth[a + b*x]])/8 + (3*Sin[1]*SinInt
egral[1 + Coth[a + b*x]])/8 + (Sin[3]*SinIntegral[3 + 3*Coth[a + b*x]])/8)
/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4852
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cot[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*
x^2), Cot[v]/d, u, x], x], x, Cot[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[
NonfreeFactors[Cot[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u],
x]]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{\frac{\mathrm{Si}(-3+3\coth(bx+a))\sin(3)}{8}-\frac{\mathrm{Ci}(-3+3\coth(bx+a))\cos(3)}{8}+\frac{\mathrm{Si}(3+3\coth(bx+a))\sin(3)}{8}+\frac{\mathrm{Ci}(3+3\coth(bx+a))\cos(3)}{8}+\frac{3\ \mathrm{Si}(-1+\mathrm{co}}{}}{b}$
default	$\frac{\frac{\mathrm{Si}(-3+3\coth(bx+a))\sin(3)}{8}-\frac{\mathrm{Ci}(-3+3\coth(bx+a))\cos(3)}{8}+\frac{\mathrm{Si}(3+3\coth(bx+a))\sin(3)}{8}+\frac{\mathrm{Ci}(3+3\coth(bx+a))\cos(3)}{8}+\frac{3\ \mathrm{Si}(-1+\mathrm{co}}{}}{b}$
risch	$-\frac{e^{3i}\,\mathrm{expIntegral}_1\left(\frac{6ie^{-a}}{e^{2bx+a}-e^{-a}}+6i\right)}{16b}+\frac{e^{-3i}\,\mathrm{expIntegral}_1\left(\frac{6ie^{-a}}{e^{2bx+a}-e^{-a}}\right)}{16b}-\frac{e^{-3i}\,\mathrm{expIntegral}_1\left(-\frac{6ie^{-a}}{e^{2bx+a}-e^{-a}}-6\right.}{16b}$

input
```
int(cos(coth(b*x+a))^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*Si(-3+3*coth(b*x+a))*sin(3)-1/8*Ci(-3+3*coth(b*x+a))*cos(3)+1/8*S
i(3+3*coth(b*x+a))*sin(3)+1/8*Ci(3+3*coth(b*x+a))*cos(3)+3/8*Si(-1+coth(b*
x+a))*sin(1)-3/8*Ci(-1+coth(b*x+a))*cos(1)+3/8*Si(1+coth(b*x+a))*sin(1)+3/
8*Ci(1+coth(b*x+a))*cos(1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 698, normalized size of antiderivative = 4.45

$$\int \cos^3(\coth(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(coth(b*x+a))^3,x, algorithm="fricas")
```

output
```
1/16*((cos(3)^2*cos(1) - (cos(1) + I*sin(1))*sin(3)^2 + 2*I*(cos(3)*cos(1)
 + I*cos(3)*sin(1))*sin(3) + I*(cos(3)^2 + 1)*sin(1) + cos(1))*cos_integra
l(3*(cosh(b*x + a) + sinh(b*x + a))/sinh(b*x + a)) - 3*(-2*I*cos(3)*cos(1)
*sin(1) + cos(3)*sin(1)^2 - (cos(1)^2 + 1)*cos(3) - I*(cos(1)^2 + 2*I*cos(
1)*sin(1) - sin(1)^2 + 1)*sin(3))*cos_integral((cosh(b*x + a) + sinh(b*x +
 a))/sinh(b*x + a)) - (cos(3)^2*cos(1) - (cos(1) + I*sin(1))*sin(3)^2 + 2*
I*(cos(3)*cos(1) + I*cos(3)*sin(1))*sin(3) + I*(cos(3)^2 + 1)*sin(1) + cos
(1))*cos_integral(6/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sin
h(b*x + a)^2 - 1)) - 3*(2*I*cos(3)*cos(1)*sin(1) - cos(3)*sin(1)^2 + (cos(
1)^2 + 1)*cos(3) + I*(cos(1)^2 + 2*I*cos(1)*sin(1) - sin(1)^2 + 1)*sin(3))
*cos_integral(2/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*
x + a)^2 - 1)) + (-I*cos(3)^2*cos(1) - (-I*cos(1) + sin(1))*sin(3)^2 - 2*I
*(I*cos(3)*cos(1) - cos(3)*sin(1))*sin(3) + I*(-I*cos(3)^2 + I)*sin(1) + I
*cos(1))*sin_integral(3*(cosh(b*x + a) + sinh(b*x + a))/sinh(b*x + a)) + 3
*(2*cos(3)*cos(1)*sin(1) + I*cos(3)*sin(1)^2 - (I*cos(1)^2 - I)*cos(3) - I
*(I*cos(1)^2 - 2*cos(1)*sin(1) - I*sin(1)^2 - I)*sin(3))*sin_integral((cos
h(b*x + a) + sinh(b*x + a))/sinh(b*x + a)) + (-I*cos(3)^2*cos(1) - (-I*cos
(1) + sin(1))*sin(3)^2 - 2*I*(I*cos(3)*cos(1) - cos(3)*sin(1))*sin(3) + I*
(-I*cos(3)^2 + I)*sin(1) + I*cos(1))*sin_integral(6/(cosh(b*x + a)^2 + 2*c
osh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + 3*(2*cos(3)*cos(1)...
```

Sympy [F]

$$\int \cos^3(\coth(a+bx))\,dx = \int \cos^3(\coth(a+bx))\,dx$$

input
```
integrate(cos(coth(b*x+a))**3,x)
```

output
```
Integral(cos(coth(a + b*x))**3, x)
```

Maxima [F]

$$\int \cos^3(\coth(a+bx))\,dx = \int \cos\left(\coth\left(bx+a\right)\right)^3\,dx$$

input
```
integrate(cos(coth(b*x+a))^3,x, algorithm="maxima")
```

output
```
integrate(cos(coth(b*x + a))^3, x)
```

Giac [F]

$$\int \cos^3(\coth(a+bx))\,dx = \int \cos\left(\coth\left(bx+a\right)\right)^3\,dx$$

input
```
integrate(cos(coth(b*x+a))^3,x, algorithm="giac")
```

output
```
integrate(cos(coth(b*x + a))^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(\coth(a+bx))\,dx = \int \cos\left(\coth(a+b\,x)\right)^3\,dx$$

input
```
int(cos(coth(a + b*x))^3,x)
```

output
```
int(cos(coth(a + b*x))^3, x)
```

Reduce [F]

$$\int \cos^3(\coth(a+bx))\,dx = \int \cos\left(\coth\left(bx+a\right)\right)^3 dx$$

input
```
int(cos(coth(b*x+a))^3,x)
```

output
```
int(cos(coth(a + b*x))**3,x)
```

3.227 $\int \cos^2(\coth(a+bx))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 115

$$\int \cos^2(\coth(a+bx))\,dx = -\frac{\cos(2)\,\text{CosIntegral}(2-2\coth(a+bx))}{4b} + \frac{\cos(2)\,\text{CosIntegral}(2+2\coth(a+bx))}{4b} - \frac{\log(1-\coth(a+bx))}{4b} + \frac{\log(1+\coth(a+bx))}{4b} - \frac{\sin(2)\text{Si}(2-2\coth(a+bx))}{4b} + \frac{\sin(2)\text{Si}(2+2\coth(a+bx))}{4b}$$

output
```
-1/4*cos(2)*Ci(2-2*coth(b*x+a))/b+1/4*cos(2)*Ci(2+2*coth(b*x+a))/b-1/4*ln(
1-coth(b*x+a))/b+1/4*ln(1+coth(b*x+a))/b+1/4*sin(2)*Si(-2+2*coth(b*x+a))/b
+1/4*sin(2)*Si(2+2*coth(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$\int \cos^2(\coth(a+bx))\,dx$$
$$= \frac{-\cos(2)\,\text{CosIntegral}(2-2\coth(a+bx))+\cos(2)\,\text{CosIntegral}(2(1+\coth(a+bx)))-\log(1-\coth(a+}{4b}$$

input
```
Integrate[Cos[Coth[a + b*x]]^2,x]
```

output
```
(-(Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]]) + Cos[2]*CosIntegral[2*(1 + Co
th[a + b*x])] - Log[1 - Coth[a + b*x]] + Log[1 + Coth[a + b*x]] - Sin[2]*S
inIntegral[2 - 2*Coth[a + b*x]] + Sin[2]*SinIntegral[2*(1 + Coth[a + b*x])
])/(4*b)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4852, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(\coth(a+bx))\,dx$$

$$\Big\downarrow \text{4852}$$

$$\frac{\int \frac{\cos^2(\coth(a+bx))}{1-\coth^2(a+bx)}d\coth(a+bx)}{b}$$

$$\Big\downarrow \text{7276}$$

$$\frac{\int \left(\frac{\cos^2(\coth(a+bx))}{2(\coth(a+bx)+1)}-\frac{\cos^2(\coth(a+bx))}{2(\coth(a+bx)-1)}\right)d\coth(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{4}\cos(2)\,\text{CosIntegral}(2-2\coth(a+bx))+\frac{1}{4}\cos(2)\,\text{CosIntegral}(2\coth(a+bx)+2)-\frac{1}{4}\sin(2)\text{Si}(2-2\coth(a}{b}$$

input
```
Int[Cos[Coth[a + b*x]]^2,x]
```

output
```
(-1/4*(Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]]) + (Cos[2]*CosIntegral[2 +
2*Coth[a + b*x]])/4 - Log[1 - Coth[a + b*x]]/4 + Log[1 + Coth[a + b*x]]/4
- (Sin[2]*SinIntegral[2 - 2*Coth[a + b*x]])/4 + (Sin[2]*SinIntegral[2 + 2*
Coth[a + b*x]])/4)/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4852
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cot[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*
x^2), Cot[v]/d, u, x], x], x, Cot[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[
NonfreeFactors[Cot[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u],
x]]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{\frac{\mathrm{Si}(-2+2\coth(bx+a))\sin(2)}{4}-\frac{\mathrm{Ci}(-2+2\coth(bx+a))\cos(2)}{4}+\frac{\mathrm{Si}(2+2\coth(bx+a))\sin(2)}{4}+\frac{\mathrm{Ci}(2+2\coth(bx+a))\cos(2)}{4}-\frac{\ln(-1+\coth}{4}}{b}$
default	$\frac{\frac{\mathrm{Si}(-2+2\coth(bx+a))\sin(2)}{4}-\frac{\mathrm{Ci}(-2+2\coth(bx+a))\cos(2)}{4}+\frac{\mathrm{Si}(2+2\coth(bx+a))\sin(2)}{4}+\frac{\mathrm{Ci}(2+2\coth(bx+a))\cos(2)}{4}-\frac{\ln(-1+\coth}{4}}{b}$
risch	$-\frac{e^{2i}\,\mathrm{expIntegral}_1\left(\frac{4i e^{-a}}{e^{2bx+a}-e^{-a}}+4i\right)}{8b}+\frac{e^{-2i}\,\mathrm{expIntegral}_1\left(\frac{4i e^{-a}}{e^{2bx+a}-e^{-a}}\right)}{8b}-\frac{e^{-2i}\,\mathrm{expIntegral}_1\left(-\frac{4i e^{-a}}{e^{2bx+a}-e^{-a}}-4\right.}{8b}$

input
```
int(cos(coth(b*x+a))^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4*Si(-2+2*coth(b*x+a))*sin(2)-1/4*Ci(-2+2*coth(b*x+a))*cos(2)+1/4*S
i(2+2*coth(b*x+a))*sin(2)+1/4*Ci(2+2*coth(b*x+a))*cos(2)-1/4*ln(-1+coth(b*
x+a))+1/4*ln(1+coth(b*x+a)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.00

$$\int \cos^2(\coth(a+bx))\,dx$$

$$= \frac{4\,bx\cos(2)+4i\,bx\sin(2)+\left(\cos(2)^2+2i\,\cos(2)\sin(2)-\sin(2)^2+1\right)\mathrm{Ci}\left(\frac{2(\cosh(bx+a)+\sinh(bx+a))}{\sinh(bx+a)}\right)-(c}{}$$

input
```
integrate(cos(coth(b*x+a))^2,x, algorithm="fricas")
```

output
```
1/8*(4*b*x*cos(2) + 4*I*b*x*sin(2) + (cos(2)^2 + 2*I*cos(2)*sin(2) - sin(2
)^2 + 1)*cos_integral(2*(cosh(b*x + a) + sinh(b*x + a))/sinh(b*x + a)) - (
cos(2)^2 + 2*I*cos(2)*sin(2) - sin(2)^2 + 1)*cos_integral(4/(cosh(b*x + a)
^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + (-I*cos(2)^2
+ 2*cos(2)*sin(2) + I*sin(2)^2 + I)*sin_integral(2*(cosh(b*x + a) + sinh(b
*x + a))/sinh(b*x + a)) + (-I*cos(2)^2 + 2*cos(2)*sin(2) + I*sin(2)^2 + I)
*sin_integral(4/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*
x + a)^2 - 1)))/(b*cos(2) + I*b*sin(2))
```

Sympy [F]

$$\int \cos^2(\coth(a+bx))\,dx = \int \cos^2\left(\coth\left(a+bx\right)\right)\,dx$$

input
```
integrate(cos(coth(b*x+a))**2,x)
```

output
```
Integral(cos(coth(a + b*x))**2, x)
```

Maxima [F]

$$\int \cos^2(\coth(a+bx))\,dx = \int \cos\left(\coth\left(bx+a\right)\right)^2\,dx$$

input
```
integrate(cos(coth(b*x+a))^2,x, algorithm="maxima")
```

output
```
1/2*x + 1/2*integrate(cos(2*(e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1)),
x)
```

Giac [F]

$$\int \cos^2(\coth(a+bx))\,dx = \int \cos\left(\coth\left(bx+a\right)\right)^2\,dx$$

input
```
integrate(cos(coth(b*x+a))^2,x, algorithm="giac")
```

output
```
integrate(cos(coth(b*x + a))^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(\coth(a+bx))\,dx = \int \cos\left(\coth(a+b\,x)\right)^2\,dx$$

input
```
int(cos(coth(a + b*x))^2,x)
```

output
```
int(cos(coth(a + b*x))^2, x)
```

Reduce [F]

$$\int \cos^2(\coth(a+bx))\,dx = \int \cos\left(\coth\left(bx+a\right)\right)^2\,dx$$

input
```
int(cos(coth(b*x+a))^2,x)
```

output
```
int(cos(coth(a + b*x))**2,x)
```

3.228 $\int \cos(\coth(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 77

$$\int \cos(\coth(a+bx))\,dx = -\frac{\cos(1)\,\text{CosIntegral}(1-\coth(a+bx))}{2b} + \frac{\cos(1)\,\text{CosIntegral}(1+\coth(a+bx))}{2b} - \frac{\sin(1)\text{Si}(1-\coth(a+bx))}{2b} + \frac{\sin(1)\text{Si}(1+\coth(a+bx))}{2b}$$

output
```
-1/2*cos(1)*Ci(1-coth(b*x+a))/b+1/2*cos(1)*Ci(1+coth(b*x+a))/b+1/2*sin(1)*
Si(-1+coth(b*x+a))/b+1/2*sin(1)*Si(1+coth(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int \cos(\coth(a+bx))\,dx = -\frac{\cos(1)\,\text{CosIntegral}(1-\coth(a+bx)) - \cos(1)\,\text{CosIntegral}(1+\coth(a+bx)) + \sin(1)\text{Si}(1-\coth(a -}{2b}$$

input
```
Integrate[Cos[Coth[a + b*x]],x]
```

output
```
-1/2*(Cos[1]*CosIntegral[1 - Coth[a + b*x]] - Cos[1]*CosIntegral[1 + Coth[
a + b*x]] + Sin[1]*SinIntegral[1 - Coth[a + b*x]] - Sin[1]*SinIntegral[1 +
 Coth[a + b*x]])/b
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4852, 3815, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(\coth(a+bx))\,dx$$

$$\Big\downarrow \text{4852}$$

$$\frac{\int \frac{\cos(\coth(a+bx))}{1-\coth^2(a+bx)}d\coth(a+bx)}{b}$$

$$\Big\downarrow \text{3815}$$

$$\frac{\int \left(\frac{\cos(\coth(a+bx))}{2(1-\coth(a+bx))}+\frac{\cos(\coth(a+bx))}{2(\coth(a+bx)+1)}\right)d\coth(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}\cos(1)\,\text{CosIntegral}(1-\coth(a+bx))+\frac{1}{2}\cos(1)\,\text{CosIntegral}(\coth(a+bx)+1)-\frac{1}{2}\sin(1)\text{Si}(1-\coth(a+bx}{b}$$

input
```
Int[Cos[Coth[a + b*x]],x]
```

output
```
(-1/2*(Cos[1]*CosIntegral[1 - Coth[a + b*x]]) + (Cos[1]*CosIntegral[1 + Co
th[a + b*x]])/2 - (Sin[1]*SinIntegral[1 - Coth[a + b*x]])/2 + (Sin[1]*SinI
ntegral[1 + Coth[a + b*x]])/2)/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3815
```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 4852
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cot[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*
x^2), Cot[v]/d, u, x], x], x, Cot[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[
NonfreeFactors[Cot[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u],
x]]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{\frac{\operatorname{Si}(-1+\coth(bx+a))\sin(1)}{2}-\frac{\operatorname{Ci}(-1+\coth(bx+a))\cos(1)}{2}+\frac{\operatorname{Si}(1+\coth(bx+a))\sin(1)}{2}+\frac{\operatorname{Ci}(1+\coth(bx+a))\cos(1)}{2}}{b}$
default	$\frac{\frac{\operatorname{Si}(-1+\coth(bx+a))\sin(1)}{2}-\frac{\operatorname{Ci}(-1+\coth(bx+a))\cos(1)}{2}+\frac{\operatorname{Si}(1+\coth(bx+a))\sin(1)}{2}+\frac{\operatorname{Ci}(1+\coth(bx+a))\cos(1)}{2}}{b}$
risch	$-\frac{e^{i}\operatorname{expIntegral}_1\left(\frac{2ie^{-a}}{e^{2bx+a}-e^{-a}}+2i\right)}{4b}+\frac{e^{-i}\operatorname{expIntegral}_1\left(\frac{2ie^{-a}}{e^{2bx+a}-e^{-a}}\right)}{4b}-\frac{e^{-i}\operatorname{expIntegral}_1\left(-\frac{2ie^{-a}}{e^{2bx+a}-e^{-a}}-2i\right)}{4b}$

input
```
int(cos(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*Si(-1+coth(b*x+a))*sin(1)-1/2*Ci(-1+coth(b*x+a))*cos(1)+1/2*Si(1+
coth(b*x+a))*sin(1)+1/2*Ci(1+coth(b*x+a))*cos(1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.81

$$\int \cos(\coth(a+bx))\,dx$$
$$= \frac{\left(\cos(1)^2 + 2i\cos(1)\sin(1) - \sin(1)^2 + 1\right)\operatorname{Ci}\left(\frac{\cosh(bx+a)+\sinh(bx+a)}{\sinh(bx+a)}\right) - \left(\cos(1)^2 + 2i\cos(1)\sin(1) - \sin\right.}{}$$

input
```
integrate(cos(coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*((cos(1)^2 + 2*I*cos(1)*sin(1) - sin(1)^2 + 1)*cos_integral((cosh(b*x
+ a) + sinh(b*x + a))/sinh(b*x + a)) - (cos(1)^2 + 2*I*cos(1)*sin(1) - sin
(1)^2 + 1)*cos_integral(2/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 - 1)) + (-I*cos(1)^2 + 2*cos(1)*sin(1) + I*sin(1)^2 + I
)*sin_integral((cosh(b*x + a) + sinh(b*x + a))/sinh(b*x + a)) + (-I*cos(1)
^2 + 2*cos(1)*sin(1) + I*sin(1)^2 + I)*sin_integral(2/(cosh(b*x + a)^2 + 2
*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)))/(b*cos(1) + I*b*sin(
1))
```

Sympy [F]

$$\int \cos(\coth(a+bx))\,dx = \int \cos\left(\coth\left(a+bx\right)\right)dx$$

input
```
integrate(cos(coth(b*x+a)),x)
```

output
```
Integral(cos(coth(a + b*x)), x)
```

Maxima [F]

$$\int \cos(\coth(a+bx))\,dx=\int \cos\left(\coth\left(bx+a\right)\right)\,dx$$

input
```
integrate(cos(coth(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(cos(coth(b*x + a)), x)
```

Giac [F]

$$\int \cos(\coth(a+bx))\,dx=\int \cos\left(\coth\left(bx+a\right)\right)\,dx$$

input
```
integrate(cos(coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(cos(coth(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(\coth(a+bx))\,dx=\int \cos\left(\coth(a+b\,x)\right)\,dx$$

input
```
int(cos(coth(a + b*x)),x)
```

output
```
int(cos(coth(a + b*x)), x)
```

Reduce [F]

$$\int \cos(\coth(a+bx))\,dx = \int \cos\left(\coth\left(bx+a\right)\right)dx$$

input
```
int(cos(coth(b*x+a)),x)
```

output
```
int(cos(coth(a + b*x)),x)
```

3.229 $\int \sec(\coth(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 7

$$\int \sec(\coth(a+bx))\,dx = \frac{1}{2}\text{Int}\left(-\frac{\operatorname{csch}^2(a+bx)\sec(\coth(a+bx))}{1-\coth(a+bx)},x\right) - \frac{1}{2}\text{Int}\left(\frac{\operatorname{csch}^2(a+bx)\sec(\coth(a+bx))}{1+\coth(a+bx)},x\right)$$

output
```
1/2*Defer(Int)(-csch(b*x+a)^2*sec(coth(b*x+a))/(1-coth(b*x+a)),x)-1/2*Defe
r(Int)(csch(b*x+a)^2*sec(coth(b*x+a))/(1+coth(b*x+a)),x)
```

Mathematica [N/A]

Not integrable

Time = 4.57 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \sec(\coth(a+bx))\,dx = \int \sec(\coth(a+bx))\,dx$$

input
```
Integrate[Sec[Coth[a + b*x]],x]
```

output
```
Integrate[Sec[Coth[a + b*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(\coth(a+bx))\,dx$$

$$\Big\downarrow \text{4852}$$

$$\frac{\int \frac{\sec(\coth(a+bx))}{1-\coth^2(a+bx)}d\coth(a+bx)}{b}$$

$$\Big\downarrow \text{7276}$$

$$\frac{\int \left(\frac{\sec(\coth(a+bx))}{2(\coth(a+bx)+1)}-\frac{\sec(\coth(a+bx))}{2(\coth(a+bx)-1)}\right)d\coth(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\int \frac{\sec(\coth(a+bx))}{\coth(a+bx)+1}d\coth(a+bx)-\frac{1}{2}\int \frac{\sec(\coth(a+bx))}{\coth(a+bx)-1}d\coth(a+bx)}{b}$$

input
```
Int[Sec[Coth[a + b*x]],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \sec(\coth(bx+a))\,dx$$

input
```
int(sec(coth(b*x+a)),x)
```

output
```
int(sec(coth(b*x+a)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \sec(\coth(a+bx))\,dx = \int \sec(\coth(bx+a))\;dx$$

input
```
integrate(sec(coth(b*x+a)),x, algorithm="fricas")
```

output
```
integral(sec(coth(b*x + a)), x)
```

Sympy [N/A]

Not integrable

Time = 3.70 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int \sec(\coth(a+bx))\,dx = \int \sec(\coth(a+bx))\,dx$$

input
```
integrate(sec(coth(b*x+a)),x)
```

output
```
Integral(sec(coth(a + b*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \sec(\coth(a+bx))\,dx = \int \sec\left(\coth\left(bx+a\right)\right)\,dx$$

input
```
integrate(sec(coth(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(sec(coth(b*x + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \sec(\coth(a+bx))\,dx = \int \sec\left(\coth\left(bx+a\right)\right)\,dx$$

input
```
integrate(sec(coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(sec(coth(b*x + a)), x)
```

Mupad [N/A]

Not integrable

Time = 3.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

$$\int \sec(\coth(a+bx))\,dx = \int \frac{1}{\cos\left(\coth\left(a+b\,x\right)\right)}\,dx$$

input
```
int(1/cos(coth(a + b*x)),x)
```

output
```
int(1/cos(coth(a + b*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \sec(\coth(a+bx))\,dx = \int \sec\left(\coth\left(bx+a\right)\right)dx$$

input
```
int(sec(coth(b*x+a)),x)
```

output
```
int(sec(coth(a + b*x)),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                      convert(ExpnType_result,string)," vs. order ",
                      convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file