Computer Algebra
Independent Integration Tests

Summer 2024

6-Hyperbolic-functions/6.5-Hyperbolic-secant/314-6.5.3

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**201**]. This is test number [314].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.50 (200)	0.50 (1)
Mathematica	95.52 (192)	4.48 (9)
Fricas	91.04 (183)	8.96 (18)
Maple	69.65 (140)	30.35 (61)
Giac	55.22 (111)	44.78 (90)
Reduce	53.23 (107)	46.77 (94)
Mupad	46.77 (94)	53.23 (107)
Maxima	44.78 (90)	55.22 (111)
Sympy	6.97 (14)	93.03 (187)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

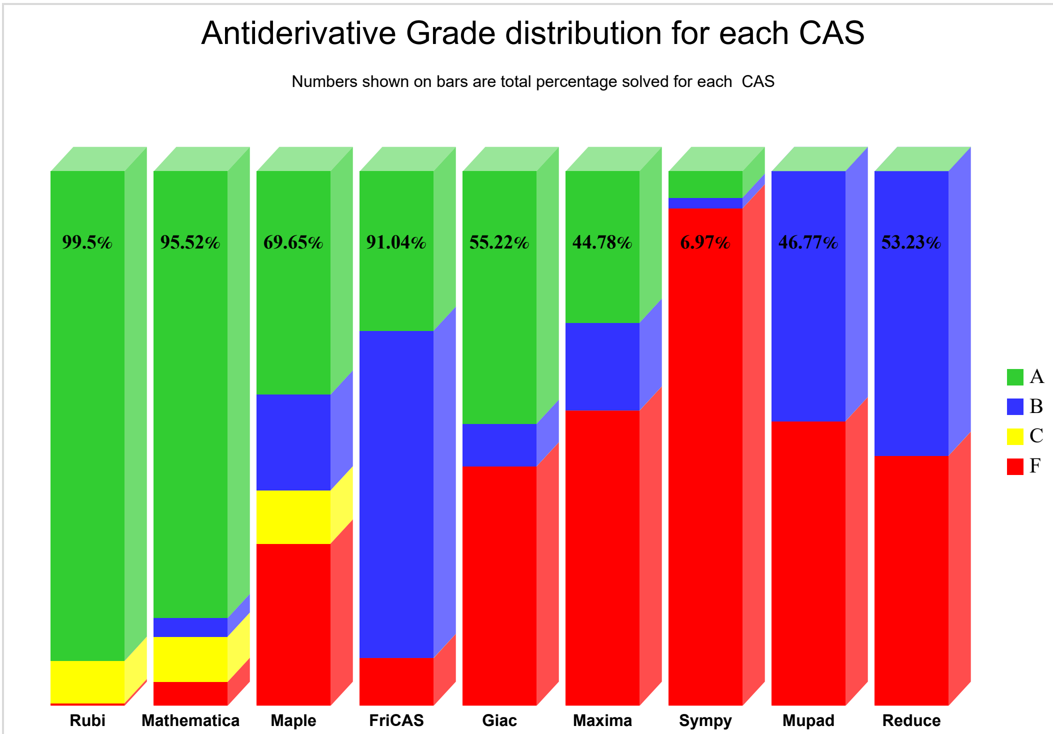
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

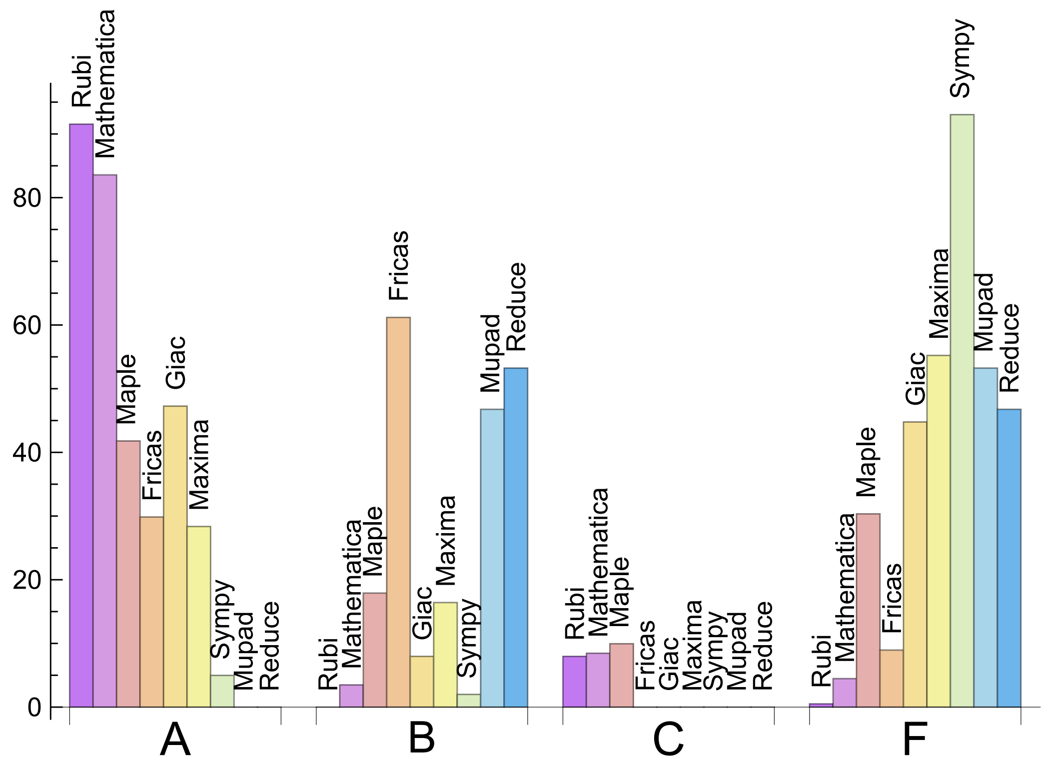
System	% A grade	% B grade	% C grade	% F grade
Rubi	91.542	0.000	7.960	0.498
Mathematica	83.582	3.483	8.458	4.478
Giac	47.264	7.960	0.000	44.776
Maple	41.791	17.910	9.950	30.348
Fricas	29.851	61.194	0.000	8.955
Maxima	28.358	16.418	0.000	55.224
Sympy	4.975	1.990	0.000	93.035
Mupad	0.000	46.766	0.000	53.234
Reduce	0.000	53.234	0.000	46.766

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	1	100.00	0.00	0.00
Mathematica	9	100.00	0.00	0.00
Fricas	18	88.89	11.11	0.00
Maple	61	100.00	0.00	0.00
Giac	90	63.33	27.78	8.89
Reduce	94	100.00	0.00	0.00
Mupad	107	0.00	100.00	0.00
Maxima	111	81.98	0.00	18.02
Sympy	187	95.72	4.28	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.08
Giac	0.12
Fricas	0.23
Mathematica	0.38
Reduce	0.47
Rubi	0.49
Mupad	2.30
Maple	8.03
Sympy	8.17

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	70.51	1.01	58.50	0.97
Giac	72.92	1.28	52.00	1.18
Sympy	78.71	1.43	42.50	1.19
Maxima	88.78	1.71	62.00	1.56
Rubi	95.09	1.07	69.00	1.00
Maple	105.48	1.71	82.50	1.31
Mupad	154.05	2.38	75.50	2.14
Reduce	163.70	2.30	83.00	1.78
Fricas	1253.59	10.84	231.00	4.99

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

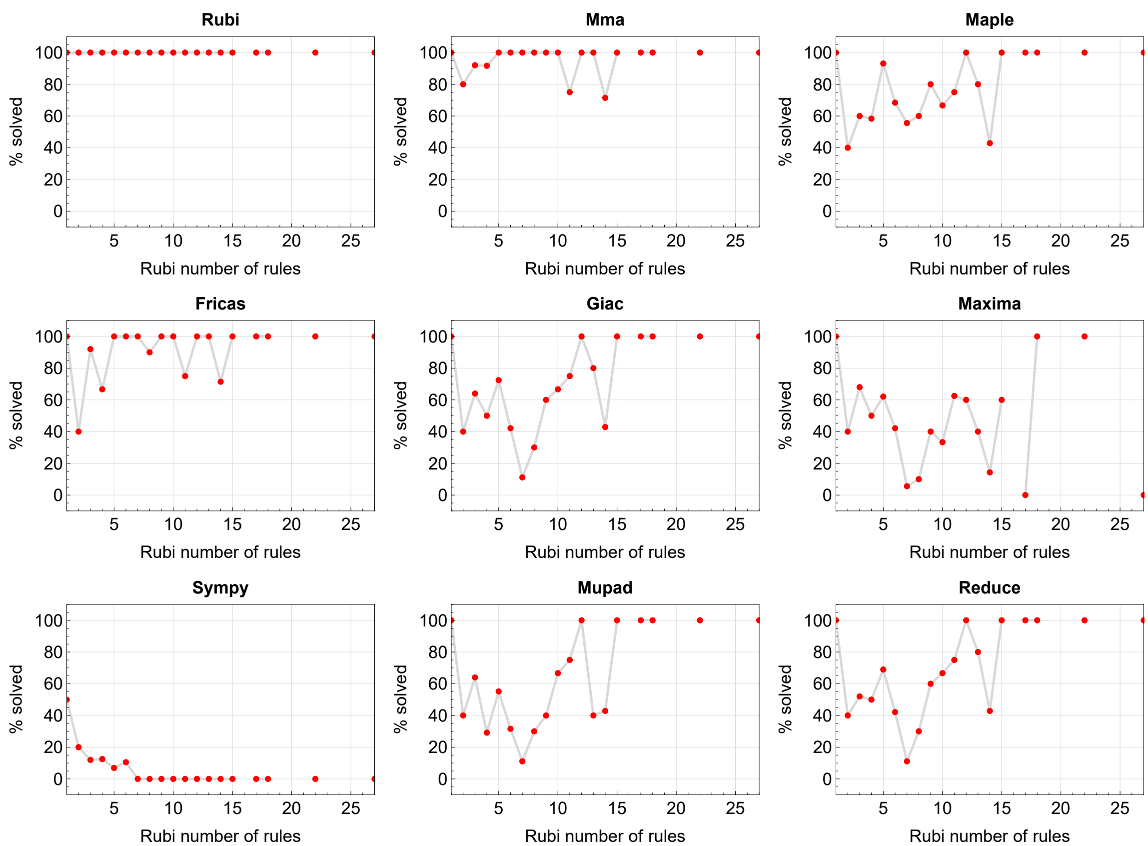


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

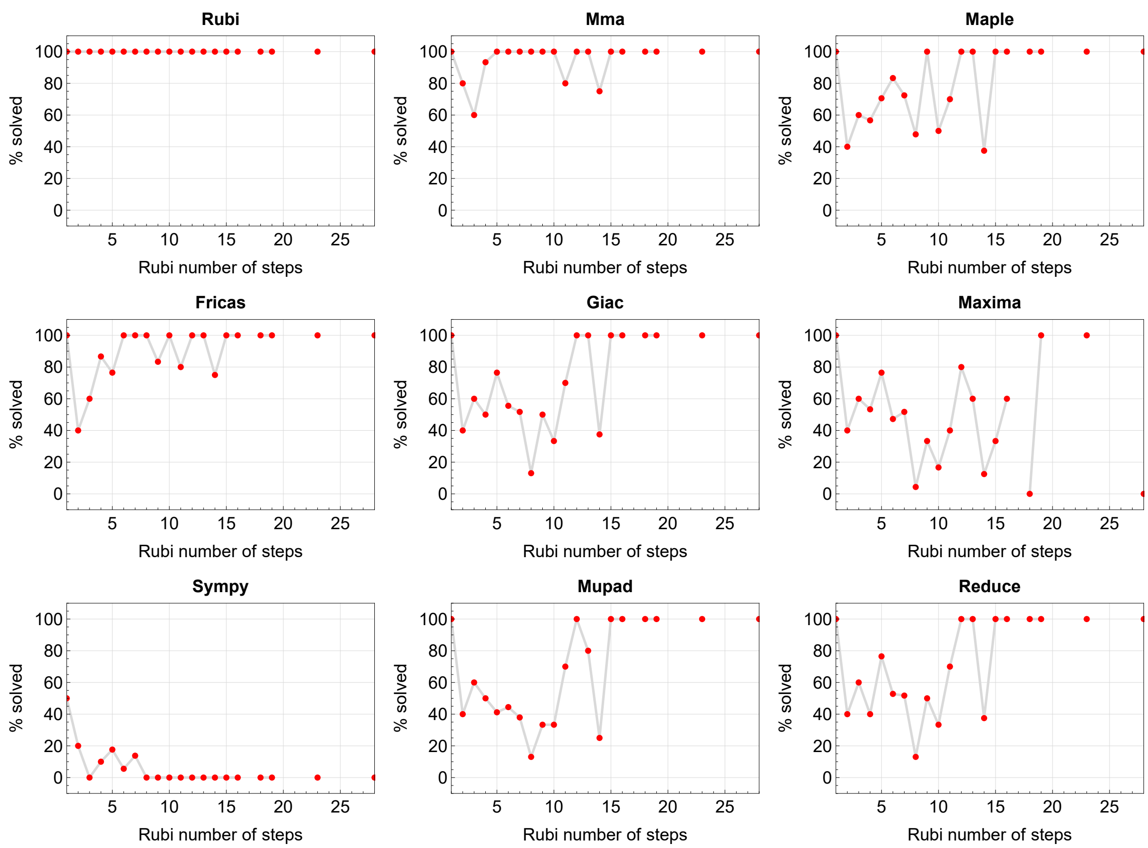


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

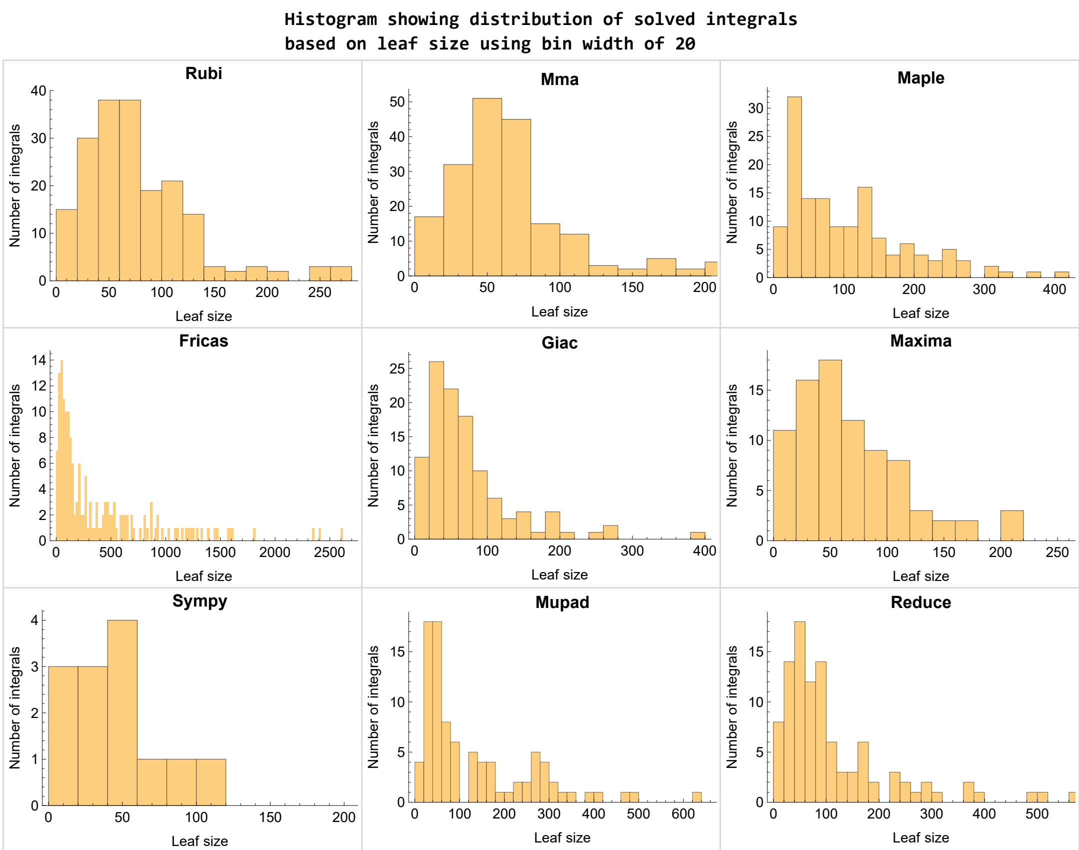


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

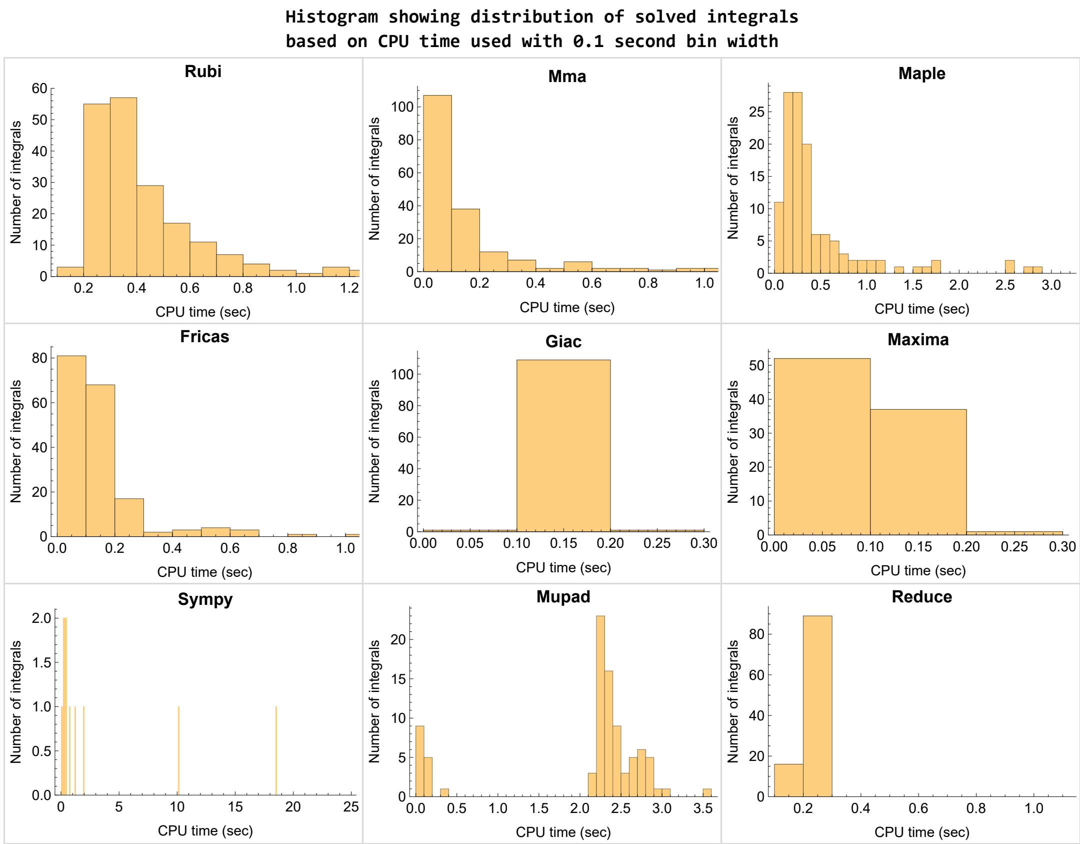


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

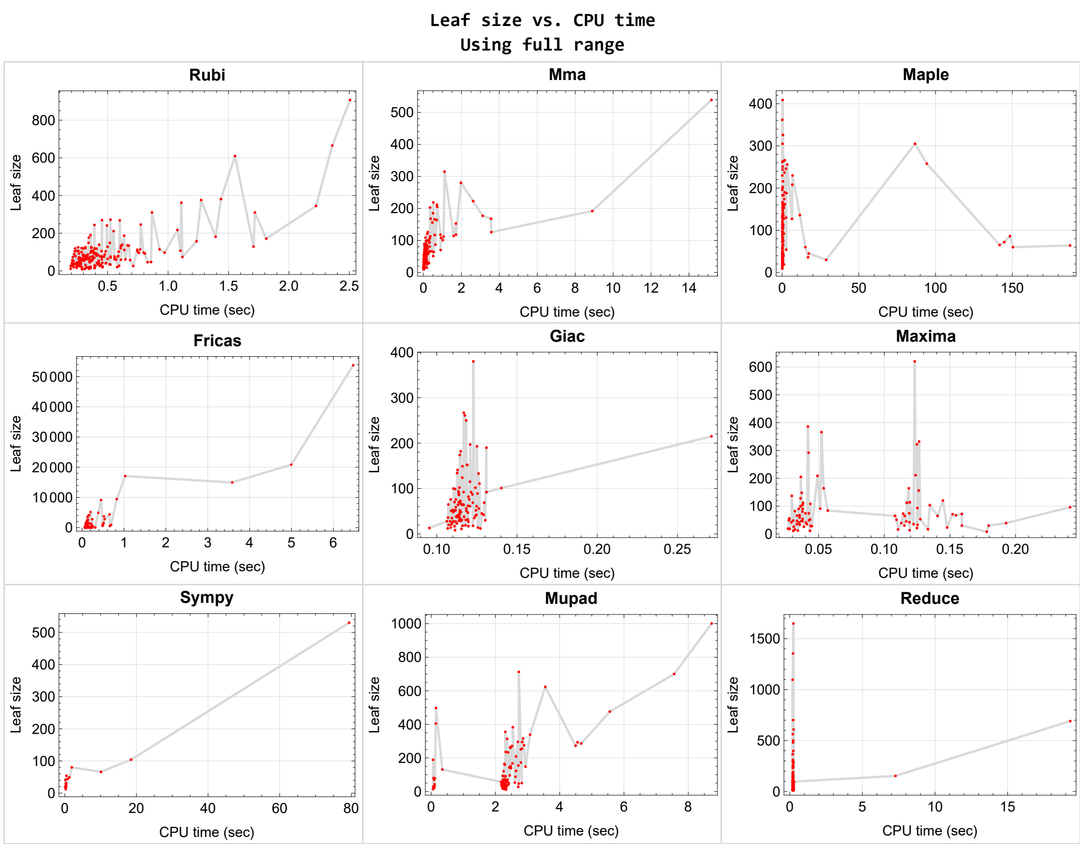


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {125, 126, 129, 133, 134, 137, 142, 143, 146, 156, 157, 158, 160, 161, 162, 163, 165, 166, 168, 169, 170, 172, 173, 174, 175, 176, 177, 180, 181}

Mathematica {190}

Maple {151, 152, 153}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```python
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

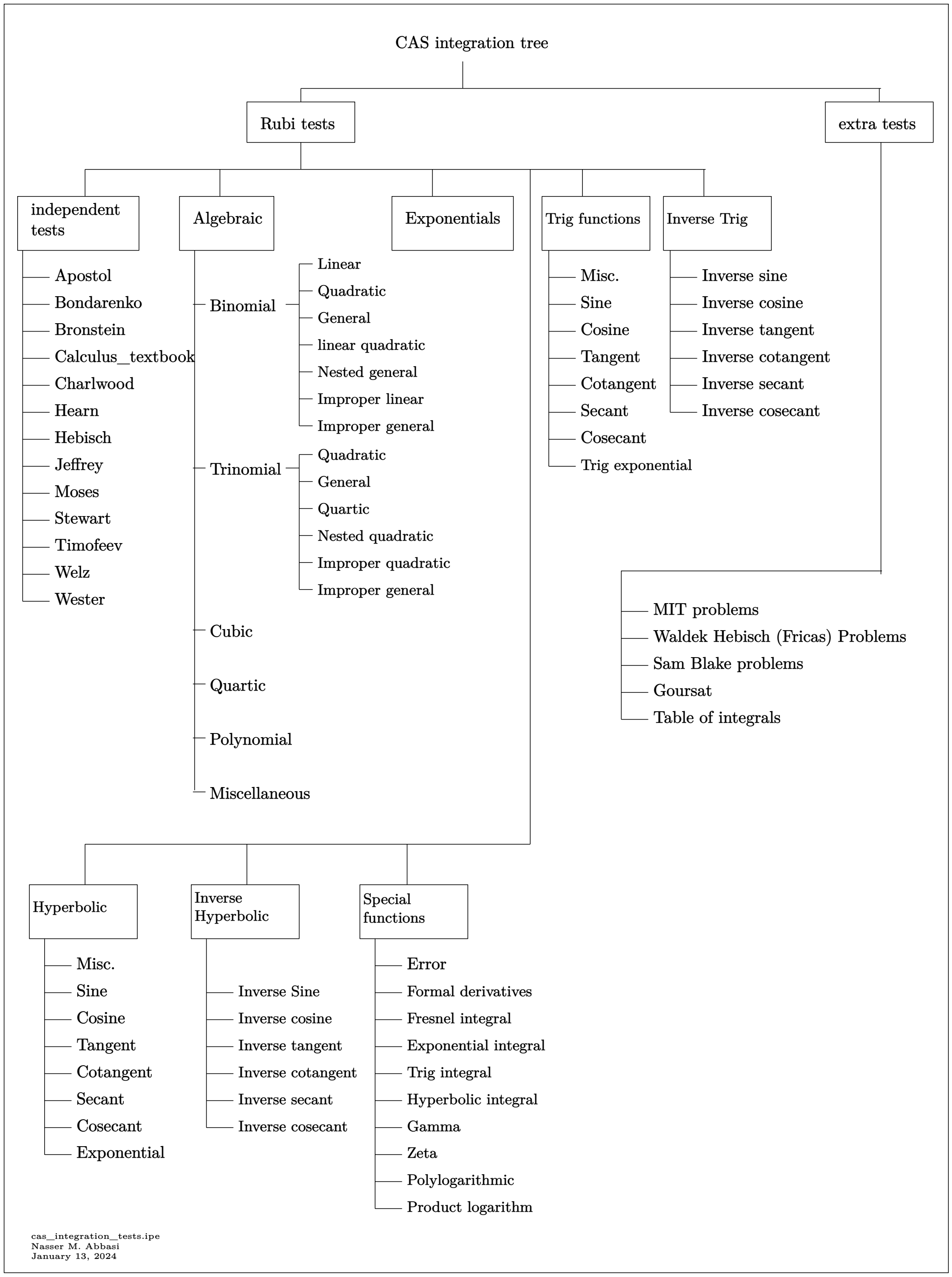
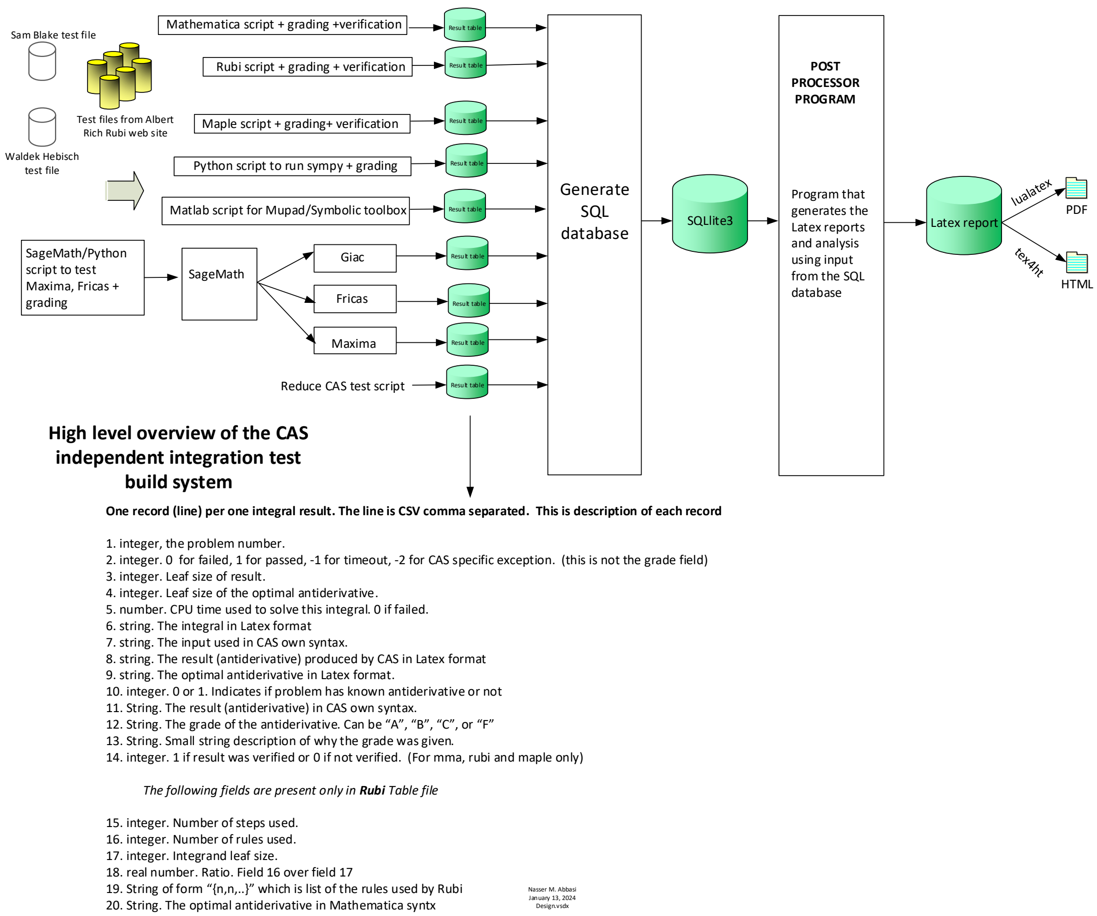


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 48, 49, 50, 51, 52, 54, 55, 57, 60, 61, 62, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 195, 196, 197, 198, 199, 200, 201 }

B grade { }

C grade { 4, 6, 7, 8, 45, 46, 47, 53, 56, 58, 59, 68, 69, 123, 186, 194 }

F normal fail { 145 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 133, 134, 135, 136, 137, 140, 151, 152, 153, 154, 155, 156, 157, 159, 161, 163, 164, 165, 167, 169, 171, 173, 175, 178, 179, 182, 183, 184, 186, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201 }

B grade { 27, 85, 86, 132, 185, 187, 188 }

C grade { 142, 143, 144, 145, 146, 158, 160, 162, 166, 168, 170, 172, 174, 176, 177, 180, 181 }

F normal fail { 130, 131, 138, 139, 141, 147, 148, 149, 150 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 45, 46, 47, 52, 54, 55, 56, 57, 58, 59, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 87, 88, 89, 90, 91, 92, 93, 98, 99, 100, 101, 102, 103, 104, 108, 109, 110, 111, 112, 114, 116, 118, 119, 120, 121, 122, 123, 124, 127, 135, 144, 154, 155, 156, 157, 159, 161, 167, 169, 171, 173, 177, 186, 191, 192, 193, 194, 195, 197 }

B grade { 9, 11, 12, 13, 14, 19, 28, 29, 30, 31, 35, 36, 37, 38, 48, 49, 50, 51, 53, 60, 61, 95, 96, 97, 105, 106, 113, 115, 117, 164, 178, 196, 198, 199, 200, 201 }

C grade { 24, 25, 26, 27, 32, 33, 34, 107, 151, 152, 153, 158, 160, 162, 166, 168, 170, 172, 174, 176 }

F normal fail { 15, 16, 17, 18, 20, 21, 22, 23, 39, 40, 41, 42, 43, 44, 78, 79, 80, 81, 82, 83, 84, 85, 86, 94, 125, 126, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 163, 165, 175, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 10, 11, 17, 18, 27, 28, 29, 30, 31, 34, 41, 52, 53, 54, 64, 70, 71, 72, 73, 76, 77, 90, 91, 99, 100, 107, 108, 110, 118, 119, 120, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 177, 178, 179, 180, 181, 191, 197, 198 }

B grade { 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 19, 20, 21, 22, 24, 25, 26, 32, 33, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 95, 96, 97, 98, 101, 102, 103, 104, 105, 106, 109, 111, 112, 113, 114, 115, 116, 117, 121, 122, 123, 124, 125, 126, 127, 128, 129, 133, 134, 135, 136, 137, 142, 143, 144, 145, 146, 151, 152, 153, 163, 171, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 199, 200, 201 }

C grade { }

F normal fail { 23, 94, 130, 131, 138, 139, 140, 141, 147, 148, 149, 176, 182, 183, 184, 185 }

F(-1) timedout fail { 132, 150 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 48, 49, 50, 51, 52, 54, 56, 64, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 88, 89, 90, 107, 108, 109, 110, 111, 117, 119, 120, 122, 152, 153, 154, 155, 156, 157, 159, 171, 179, 191, 192 }

B grade { 3, 4, 5, 6, 7, 8, 24, 25, 45, 46, 47, 53, 55, 57, 58, 59, 61, 63, 87, 103, 104, 105, 106, 112, 113, 115, 124, 151, 167, 186, 187, 188, 194 }

C grade { }

F normal fail { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 39, 40, 41, 42, 43, 44, 78, 79, 80, 81, 82, 83, 84, 85, 86, 94, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 158, 160, 161, 162, 163, 164, 165, 166, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 189, 190, 193, 195, 196, 197, 198, 199, 200, 201 }

F(-1) timedout fail { }

F(-2) exception fail { 60, 62, 65, 67, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 102, 114, 116, 118, 121, 123 }

Giac

A grade { 1, 2, 4, 6, 7, 8, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 87, 88, 89, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 114, 116, 118, 119, 120, 121, 122, 123, 151, 152, 153, 154, 155, 156, 157, 187, 188, 191, 192, 194, 195 }

B grade { 3, 5, 26, 58, 59, 66, 83, 85, 86, 106, 113, 115, 117, 124, 186, 193 }

C grade { }

F normal fail { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 39, 40, 41, 42, 43, 44, 94, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 158, 159, 161, 182, 183, 184, 185, 189, 190 }

F(-1) timedout fail { 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 196, 197, 198, 199, 200, 201 }

F(-2) exception fail { 78, 79, 80, 81, 82, 84, 160, 162 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 28, 35, 45, 46, 47, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 87, 88, 89, 90, 91, 92, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 127, 135, 144, 151, 152, 153, 159, 167, 171, 179, 186, 187, 188, 191, 192, 193, 194, 195 }

C grade { }

F normal fail { }

F(-1) timedout fail { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 49, 50, 51, 78, 79, 80, 81, 82, 83, 84, 85, 86, 93, 94, 125, 126, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 189, 190, 196, 197, 198, 199, 200, 201 }

F(-2) exception fail { }

Sympy

A grade { 28, 29, 30, 31, 35, 36, 37, 38, 90, 191 }

B grade { 1, 108, 119, 157 }

C grade { }

F normal fail { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 32, 33, 34, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 197, 198, 199, 200 }

F(-1) timedout fail { 15, 24, 45, 151, 152, 153, 196, 201 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 87, 88, 89, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 151, 152, 153, 154, 155, 156, 157, 186, 187, 188, 191, 192, 193, 194, 195 }

C grade { }

F normal fail { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 39, 40, 41, 42, 43, 44, 78, 79, 80, 81, 82, 83, 84, 85, 86, 94, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 189, 190, 196, 197, 198, 199, 200, 201 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	12	12	11	19	19	12	13	23
N.S.	1	1.00	1.09	1.09	1.00	1.73	1.73	1.09	1.18	2.09
time (sec)	N/A	0.287	0.003	0.150	0.043	0.072	0.351	0.127	0.192	0.073

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	18	41	0	18	29	18
N.S.	1	1.00	1.00	1.10	1.80	4.10	0.00	1.80	2.90	1.80
time (sec)	N/A	0.310	0.005	0.135	0.028	0.064	0.000	0.116	0.218	2.224

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	27	65	267	0	76	97	81
N.S.	1	1.00	1.00	0.79	1.91	7.85	0.00	2.24	2.85	2.38
time (sec)	N/A	0.420	0.019	0.341	0.108	0.078	0.000	0.120	0.296	0.075

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	32	26	23	90	164	0	31	57	31
N.S.	1	1.23	1.00	0.88	3.46	6.31	0.00	1.19	2.19	1.19
time (sec)	N/A	0.317	0.006	0.217	0.038	0.065	0.000	0.118	0.206	2.199

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	60	55	41	112	812	0	102	191	189
N.S.	1	1.09	1.00	0.75	2.04	14.76	0.00	1.85	3.47	3.44
time (sec)	N/A	0.571	0.012	0.598	0.117	0.085	0.000	0.124	0.225	0.060

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	46	41	33	205	344	0	42	93	42
N.S.	1	1.12	1.00	0.80	5.00	8.39	0.00	1.02	2.27	1.02
time (sec)	N/A	0.355	0.009	0.289	0.036	0.072	0.000	0.129	0.263	2.186

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	27	19	17	49	116	0	18	36	30
N.S.	1	1.42	1.00	0.89	2.58	6.11	0.00	0.95	1.89	1.58
time (sec)	N/A	0.195	0.005	0.200	0.027	0.071	0.000	0.123	0.230	0.071

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	40	35	27	137	280	0	30	65	30
N.S.	1	1.14	1.00	0.77	3.91	8.00	0.00	0.86	1.86	0.86
time (sec)	N/A	0.236	0.005	0.356	0.030	0.097	0.000	0.130	0.223	2.274

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	51	190	0	190	0	0	18	0
N.S.	1	1.00	0.77	2.88	0.00	2.88	0.00	0.00	0.27	0.00
time (sec)	N/A	0.340	0.062	1.602	0.000	0.084	0.000	0.000	0.242	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	49	103	0	96	0	0	16	0
N.S.	1	1.00	0.79	1.66	0.00	1.55	0.00	0.00	0.26	0.00
time (sec)	N/A	0.329	0.039	0.386	0.000	0.082	0.000	0.000	0.240	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	135	0	24	0	0	9	0
N.S.	1	1.00	1.00	3.38	0.00	0.60	0.00	0.00	0.22	0.00
time (sec)	N/A	0.243	0.027	0.281	0.000	0.092	0.000	0.000	0.214	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	135	0	150	0	0	18	0
N.S.	1	1.00	1.00	3.38	0.00	3.75	0.00	0.00	0.45	0.00
time (sec)	N/A	0.250	0.031	0.826	0.000	0.082	0.000	0.000	0.214	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	53	174	0	223	0	0	18	0
N.S.	1	1.00	0.80	2.64	0.00	3.38	0.00	0.00	0.27	0.00
time (sec)	N/A	0.331	0.039	1.347	0.000	0.096	0.000	0.000	0.255	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	59	188	0	370	0	0	18	0
N.S.	1	1.00	0.89	2.85	0.00	5.61	0.00	0.00	0.27	0.00
time (sec)	N/A	0.323	0.057	2.836	0.000	0.097	0.000	0.000	0.243	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	104	68	0	0	478	0	0	24	0
N.S.	1	1.02	0.67	0.00	0.00	4.69	0.00	0.00	0.24	0.00
time (sec)	N/A	0.484	0.164	0.000	0.000	0.091	0.000	0.000	0.217	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	56	0	0	215	0	0	24	0
N.S.	1	1.00	0.76	0.00	0.00	2.91	0.00	0.00	0.32	0.00
time (sec)	N/A	0.358	0.064	0.000	0.000	0.083	0.000	0.000	0.251	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	52	0	0	107	0	0	20	0
N.S.	1	1.00	0.74	0.00	0.00	1.53	0.00	0.00	0.29	0.00
time (sec)	N/A	0.390	0.041	0.000	0.000	0.086	0.000	0.000	0.243	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	27	0	0	12	0
N.S.	1	1.00	1.00	0.00	0.00	0.64	0.00	0.00	0.29	0.00
time (sec)	N/A	0.428	0.026	0.000	0.000	0.082	0.000	0.000	0.207	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	244	0	154	0	0	24	0
N.S.	1	1.00	1.00	5.81	0.00	3.67	0.00	0.00	0.57	0.00
time (sec)	N/A	0.427	0.032	0.279	0.000	0.098	0.000	0.000	0.216	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	63	0	0	231	0	0	24	0
N.S.	1	1.00	0.83	0.00	0.00	3.04	0.00	0.00	0.32	0.00
time (sec)	N/A	0.584	0.059	0.000	0.000	0.089	0.000	0.000	0.257	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	64	0	0	379	0	0	24	0
N.S.	1	1.00	0.84	0.00	0.00	4.99	0.00	0.00	0.32	0.00
time (sec)	N/A	0.557	0.071	0.000	0.000	0.084	0.000	0.000	0.221	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	112	70	0	0	483	0	0	24	0
N.S.	1	1.08	0.67	0.00	0.00	4.64	0.00	0.00	0.23	0.00
time (sec)	N/A	0.745	0.109	0.000	0.000	0.093	0.000	0.000	0.205	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	60	0	0	0	0	0	14	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.324	0.050	0.000	0.000	0.000	0.000	0.000	0.245	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	104	81	230	156	1604	0	124	279	0
N.S.	1	1.16	0.90	2.56	1.73	17.82	0.00	1.38	3.10	0.00
time (sec)	N/A	0.228	0.064	6.793	0.126	0.085	0.000	0.116	0.252	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	73	65	208	112	812	0	102	191	0
N.S.	1	1.12	1.00	3.20	1.72	12.49	0.00	1.57	2.94	0.00
time (sec)	N/A	0.212	0.044	6.654	0.117	0.087	0.000	0.114	0.232	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	42	46	183	65	267	0	76	97	0
N.S.	1	1.05	1.15	4.58	1.62	6.68	0.00	1.90	2.42	0.00
time (sec)	N/A	0.205	0.032	0.270	0.141	0.086	0.000	0.112	0.209	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	30	130	11	19	0	12	13	0
N.S.	1	1.00	2.73	11.82	1.00	1.73	0.00	1.09	1.18	0.00
time (sec)	N/A	0.194	0.014	0.245	0.031	0.086	0.000	0.107	0.262	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	26	22	97	26	10	29	23	26	53
N.S.	1	1.18	1.00	4.41	1.18	0.45	1.32	1.05	1.18	2.41
time (sec)	N/A	0.197	0.039	0.252	0.031	0.089	0.293	0.113	0.189	2.605

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	57	44	201	54	32	54	48	53	0
N.S.	1	1.12	0.86	3.94	1.06	0.63	1.06	0.94	1.04	0.00
time (sec)	N/A	0.208	0.049	0.249	0.028	0.085	0.450	0.114	0.203	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	88	47	305	82	66	80	70	79	0
N.S.	1	1.16	0.62	4.01	1.08	0.87	1.05	0.92	1.04	0.00
time (sec)	N/A	0.223	0.059	0.255	0.032	0.084	1.959	0.114	0.205	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	119	57	409	100	108	104	92	103	0
N.S.	1	1.18	0.56	4.05	0.99	1.07	1.03	0.91	1.02	0.00
time (sec)	N/A	0.231	0.095	0.295	0.036	0.078	18.507	0.117	0.219	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	78	38	127	72	1082	0	65	115	0
N.S.	1	1.20	0.58	1.95	1.11	16.65	0.00	1.00	1.77	0.00
time (sec)	N/A	0.234	0.026	6.440	0.159	0.099	0.000	0.107	0.235	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	53	24	106	39	310	0	48	56	0
N.S.	1	1.15	0.52	2.30	0.85	6.74	0.00	1.04	1.22	0.00
time (sec)	N/A	0.218	0.024	0.096	0.193	0.093	0.000	0.115	0.193	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	28	17	72	8	145	0	8	8	0
N.S.	1	1.12	0.68	2.88	0.32	5.80	0.00	0.32	0.32	0.00
time (sec)	N/A	0.212	0.006	0.092	0.178	0.104	0.000	0.111	0.236	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	16	13	58	17	79	12	14	19	33
N.S.	1	1.23	1.00	4.46	1.31	6.08	0.92	1.08	1.46	2.54
time (sec)	N/A	0.320	0.021	0.089	0.133	0.080	0.279	0.109	0.195	0.108

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	47	25	130	35	277	31	29	35	0
N.S.	1	1.31	0.69	3.61	0.97	7.69	0.86	0.81	0.97	0.00
time (sec)	N/A	0.349	0.013	0.103	0.122	0.093	0.407	0.114	0.207	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	77	33	196	53	580	49	41	51	0
N.S.	1	1.40	0.60	3.56	0.96	10.55	0.89	0.75	0.93	0.00
time (sec)	N/A	0.377	0.023	0.087	0.127	0.098	1.298	0.113	0.247	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	107	39	262	71	970	66	53	65	0
N.S.	1	1.45	0.53	3.54	0.96	13.11	0.89	0.72	0.88	0.00
time (sec)	N/A	0.380	0.027	0.093	0.152	0.127	10.112	0.110	0.250	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	107	63	0	0	1382	0	0	16	0
N.S.	1	0.88	0.52	0.00	0.00	11.42	0.00	0.00	0.13	0.00
time (sec)	N/A	1.102	0.084	0.000	0.000	0.116	0.000	0.000	0.223	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	75	47	0	0	391	0	0	14	0
N.S.	1	1.09	0.68	0.00	0.00	5.67	0.00	0.00	0.20	0.00
time (sec)	N/A	0.470	0.040	0.000	0.000	0.105	0.000	0.000	0.216	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	53	36	0	0	60	0	0	11	0
N.S.	1	1.15	0.78	0.00	0.00	1.30	0.00	0.00	0.24	0.00
time (sec)	N/A	0.369	0.027	0.000	0.000	0.101	0.000	0.000	0.254	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	57	38	0	0	126	0	0	16	0
N.S.	1	1.19	0.79	0.00	0.00	2.62	0.00	0.00	0.33	0.00
time (sec)	N/A	0.367	0.043	0.000	0.000	0.091	0.000	0.000	0.223	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	47	0	0	407	0	0	16	0
N.S.	1	1.00	0.61	0.00	0.00	5.29	0.00	0.00	0.21	0.00
time (sec)	N/A	0.454	0.081	0.000	0.000	0.104	0.000	0.000	0.206	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	111	63	0	0	718	0	0	16	0
N.S.	1	0.92	0.52	0.00	0.00	5.93	0.00	0.00	0.13	0.00
time (sec)	N/A	0.639	0.079	0.000	0.000	0.116	0.000	0.000	0.261	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	84	54	72	620	2804	0	51	146	498
N.S.	1	0.52	0.33	0.44	3.80	17.20	0.00	0.31	0.90	3.06
time (sec)	N/A	0.296	0.150	144.807	0.123	0.241	0.000	0.111	0.198	0.151

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	68	42	60	322	1475	0	39	104	356
N.S.	1	0.58	0.36	0.51	2.75	12.61	0.00	0.33	0.89	3.04
time (sec)	N/A	0.273	0.082	150.441	0.125	0.128	0.000	0.112	0.203	2.310

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	46	30	46	120	516	0	27	60	46
N.S.	1	0.75	0.49	0.75	1.97	8.46	0.00	0.44	0.98	0.75
time (sec)	N/A	0.256	0.043	0.121	0.145	0.100	0.000	0.112	0.222	2.269

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	29	13	81	0	13	18	71
N.S.	1	1.00	1.00	1.93	0.87	5.40	0.00	0.87	1.20	4.73
time (sec)	N/A	0.360	0.006	0.101	0.119	0.079	0.000	0.095	0.218	2.235

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	29	23	89	30	253	0	28	29	0
N.S.	1	0.81	0.64	2.47	0.83	7.03	0.00	0.78	0.81	0.00
time (sec)	N/A	0.392	0.032	0.104	0.115	0.087	0.000	0.106	0.212	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	62	38	230	65	1141	0	52	57	0
N.S.	1	0.72	0.44	2.67	0.76	13.27	0.00	0.60	0.66	0.00
time (sec)	N/A	0.557	0.045	0.115	0.116	0.112	0.000	0.111	0.212	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	92	55	362	103	2600	0	76	87	0
N.S.	1	0.70	0.42	2.74	0.78	19.70	0.00	0.58	0.66	0.00
time (sec)	N/A	0.809	0.072	0.110	0.135	0.143	0.000	0.109	0.231	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	46	28	60	54	36	0	42	55	59
N.S.	1	1.05	0.64	1.36	1.23	0.82	0.00	0.95	1.25	1.34
time (sec)	N/A	0.831	0.172	15.178	0.038	0.074	0.000	0.114	0.198	2.335

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	31	23	54	46	30	0	37	45	53
N.S.	1	1.35	1.00	2.35	2.00	1.30	0.00	1.61	1.96	2.30
time (sec)	N/A	0.444	0.035	2.734	0.037	0.074	0.000	0.114	0.230	2.224

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	26	16	42	42	14	0	28	39	41
N.S.	1	0.96	0.59	1.56	1.56	0.52	0.00	1.04	1.44	1.52
time (sec)	N/A	0.399	0.030	0.818	0.029	0.095	0.000	0.108	0.272	2.234

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	19	16	23	35	50	0	32	34	15
N.S.	1	1.12	0.94	1.35	2.06	2.94	0.00	1.88	2.00	0.88
time (sec)	N/A	0.345	0.014	0.184	0.033	0.091	0.000	0.114	0.217	2.263

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	42	44	20	48	103	0	52	87	51
N.S.	1	1.27	1.33	0.61	1.45	3.12	0.00	1.58	2.64	1.55
time (sec)	N/A	0.487	0.035	0.221	0.031	0.096	0.000	0.112	0.213	2.249

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	90	71	0	31	40	91
N.S.	1	1.00	1.09	1.00	3.91	3.09	0.00	1.35	1.74	3.96
time (sec)	N/A	0.458	0.189	0.259	0.035	0.074	0.000	0.116	0.235	2.269

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	59	59	38	99	630	0	90	238	121
N.S.	1	1.28	1.28	0.83	2.15	13.70	0.00	1.96	5.17	2.63
time (sec)	N/A	0.610	0.119	0.358	0.036	0.086	0.000	0.114	0.229	2.255

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	40	39	39	292	219	0	59	82	236
N.S.	1	1.18	1.15	1.15	8.59	6.44	0.00	1.74	2.41	6.94
time (sec)	N/A	0.485	0.148	0.793	0.042	0.070	0.000	0.112	0.210	2.321

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	157	219	305	0	1812	0	197	257	275
N.S.	1	1.19	1.66	2.31	0.00	13.73	0.00	1.49	1.95	2.08
time (sec)	N/A	1.236	0.523	86.602	0.000	0.123	0.000	0.121	0.208	2.891

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	66	136	128	490	0	87	163	123
N.S.	1	1.00	1.08	2.23	2.10	8.03	0.00	1.43	2.67	2.02
time (sec)	N/A	0.648	0.144	11.555	0.036	0.092	0.000	0.122	0.232	2.643

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	97	76	130	0	536	0	100	104	173
N.S.	1	1.18	0.93	1.59	0.00	6.54	0.00	1.22	1.27	2.11
time (sec)	N/A	0.971	0.123	2.513	0.000	0.098	0.000	0.110	0.193	2.792

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	19	19	31	46	78	0	34	48	20
N.S.	1	0.95	0.95	1.55	2.30	3.90	0.00	1.70	2.40	1.00
time (sec)	N/A	0.458	0.008	0.465	0.038	0.101	0.000	0.114	0.194	0.085

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	59	50	48	59	58	0	65	66	148
N.S.	1	1.11	0.94	0.91	1.11	1.09	0.00	1.23	1.25	2.79
time (sec)	N/A	0.376	0.078	0.243	0.036	0.086	0.000	0.116	0.225	2.936

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	77	75	77	0	452	0	64	165	151
N.S.	1	1.17	1.14	1.17	0.00	6.85	0.00	0.97	2.50	2.29
time (sec)	N/A	0.436	0.218	0.308	0.000	0.114	0.000	0.113	0.242	2.469

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	128	93	82	148	828	0	174	501	255
N.S.	1	1.51	1.09	0.96	1.74	9.74	0.00	2.05	5.89	3.00
time (sec)	N/A	0.499	0.311	0.424	0.037	0.102	0.000	0.114	0.236	2.823

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	135	156	127	0	2340	0	149	481	295
N.S.	1	1.22	1.41	1.14	0.00	21.08	0.00	1.34	4.33	2.66
time (sec)	N/A	0.664	0.467	0.993	0.000	0.122	0.000	0.116	0.225	2.832

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	76	63	39	80	139	0	86	91	88
N.S.	1	1.13	0.94	0.58	1.19	2.07	0.00	1.28	1.36	1.31
time (sec)	N/A	0.522	0.275	0.347	0.038	0.076	0.000	0.114	0.183	2.625

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	61	53	31	66	100	0	70	75	70
N.S.	1	1.13	0.98	0.57	1.22	1.85	0.00	1.30	1.39	1.30
time (sec)	N/A	0.451	0.044	0.234	0.034	0.080	0.000	0.111	0.243	2.378

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	42	45	24	56	70	0	51	61	52
N.S.	1	1.02	1.10	0.59	1.37	1.71	0.00	1.24	1.49	1.27
time (sec)	N/A	0.416	0.029	0.174	0.028	0.086	0.000	0.109	0.209	2.210

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	27	32	15	41	47	0	35	45	34
N.S.	1	1.04	1.23	0.58	1.58	1.81	0.00	1.35	1.73	1.31
time (sec)	N/A	0.595	0.173	0.124	0.035	0.087	0.000	0.109	0.226	2.246

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	10	9	12	14	0	11	15	11
N.S.	1	1.00	0.91	0.82	1.09	1.27	0.00	1.00	1.36	1.00
time (sec)	N/A	0.346	0.008	0.081	0.034	0.072	0.000	0.115	0.262	2.327

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	15	19	23	29	0	20	30	31
N.S.	1	1.00	0.75	0.95	1.15	1.45	0.00	1.00	1.50	1.55
time (sec)	N/A	0.579	0.035	0.186	0.148	0.073	0.000	0.107	0.210	2.250

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	30	33	45	127	0	36	66	58
N.S.	1	1.00	1.15	1.27	1.73	4.88	0.00	1.38	2.54	2.23
time (sec)	N/A	0.712	0.063	0.209	0.117	0.076	0.000	0.107	0.197	2.370

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	47	41	46	73	325	0	48	122	73
N.S.	1	1.04	0.91	1.02	1.62	7.22	0.00	1.07	2.71	1.62
time (sec)	N/A	0.859	0.080	0.350	0.114	0.080	0.000	0.110	0.211	2.301

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	58	23	33	48	0	29	41	24
N.S.	1	1.00	2.00	0.79	1.14	1.66	0.00	1.00	1.41	0.83
time (sec)	N/A	0.206	0.241	0.137	0.036	0.072	0.000	0.115	0.226	2.331

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	59	23	35	50	0	29	42	24
N.S.	1	1.00	1.97	0.77	1.17	1.67	0.00	0.97	1.40	0.80
time (sec)	N/A	0.215	0.244	0.166	0.041	0.096	0.000	0.115	0.246	2.263

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	102	99	0	0	924	0	0	58	0
N.S.	1	1.04	1.01	0.00	0.00	9.43	0.00	0.00	0.59	0.00
time (sec)	N/A	0.592	0.290	0.000	0.000	0.105	0.000	0.000	0.222	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	75	0	0	697	0	0	34	0
N.S.	1	1.00	1.14	0.00	0.00	10.56	0.00	0.00	0.52	0.00
time (sec)	N/A	0.312	0.130	0.000	0.000	0.125	0.000	0.000	0.274	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	60	0	0	637	0	0	14	0
N.S.	1	1.00	1.62	0.00	0.00	17.22	0.00	0.00	0.38	0.00
time (sec)	N/A	0.209	0.082	0.000	0.000	0.092	0.000	0.000	0.215	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	118	0	0	868	0	0	28	0
N.S.	1	1.00	1.39	0.00	0.00	10.21	0.00	0.00	0.33	0.00
time (sec)	N/A	0.421	0.897	0.000	0.000	0.112	0.000	0.000	0.208	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	120	177	0	0	1190	0	0	38	0
N.S.	1	1.05	1.55	0.00	0.00	10.44	0.00	0.00	0.33	0.00
time (sec)	N/A	0.608	3.121	0.000	0.000	0.128	0.000	0.000	0.240	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	70	0	0	642	0	101	16	0
N.S.	1	1.00	1.84	0.00	0.00	16.89	0.00	2.66	0.42	0.00
time (sec)	N/A	0.225	0.905	0.000	0.000	0.098	0.000	0.140	0.219	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	118	0	0	871	0	0	31	0
N.S.	1	1.00	1.36	0.00	0.00	10.01	0.00	0.00	0.36	0.00
time (sec)	N/A	0.476	1.714	0.000	0.000	0.108	0.000	0.000	0.226	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	39	0	0	457	0	52	10	0
N.S.	1	1.00	2.05	0.00	0.00	24.05	0.00	2.74	0.53	0.00
time (sec)	N/A	0.329	0.029	0.000	0.000	0.081	0.000	0.124	0.200	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	51	0	0	462	0	69	12	0
N.S.	1	1.00	2.43	0.00	0.00	22.00	0.00	3.29	0.57	0.00
time (sec)	N/A	0.286	0.513	0.000	0.000	0.100	0.000	0.122	0.257	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	114	86	92	211	1028	0	141	367	233
N.S.	1	1.07	0.80	0.86	1.97	9.61	0.00	1.32	3.43	2.18
time (sec)	N/A	0.770	0.175	1.137	0.124	0.090	0.000	0.113	0.203	2.394

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	74	61	66	114	521	0	92	237	165
N.S.	1	1.01	0.84	0.90	1.56	7.14	0.00	1.26	3.25	2.26
time (sec)	N/A	0.452	0.099	0.657	0.119	0.096	0.000	0.121	0.211	2.278

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	34	41	157	0	43	90	70
N.S.	1	1.00	0.97	1.03	1.24	4.76	0.00	1.30	2.73	2.12
time (sec)	N/A	0.523	0.045	0.329	0.040	0.085	0.000	0.114	0.219	0.091

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	17	17	16	26	24	17	20	38
N.S.	1	1.00	1.06	1.06	1.00	1.62	1.50	1.06	1.25	2.38
time (sec)	N/A	0.244	0.003	0.102	0.032	0.083	0.373	0.111	0.226	2.222

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	60	83	0	270	0	56	70	131
N.S.	1	1.00	1.02	1.41	0.00	4.58	0.00	0.95	1.19	2.22
time (sec)	N/A	0.303	0.123	0.170	0.000	0.130	0.000	0.118	0.240	0.349

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	134	203	164	0	1207	0	134	563	296
N.S.	1	1.23	1.86	1.50	0.00	11.07	0.00	1.23	5.17	2.72
time (sec)	N/A	0.676	0.348	0.256	0.000	0.109	0.000	0.113	0.208	2.698

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	217	205	251	0	4125	0	261	1649	0
N.S.	1	1.25	1.18	1.45	0.00	23.84	0.00	1.51	9.53	0.00
time (sec)	N/A	1.078	0.722	0.383	0.000	0.171	0.000	0.118	0.251	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	168	0	0	0	0	0	26	0
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.259	3.561	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	171	126	264	0	2402	0	182	311	251
N.S.	1	1.17	0.86	1.81	0.00	16.45	0.00	1.25	2.13	1.72
time (sec)	N/A	1.812	0.226	0.675	0.000	0.115	0.000	0.115	0.211	2.788

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	129	99	203	0	1562	0	133	248	209
N.S.	1	1.15	0.88	1.81	0.00	13.95	0.00	1.19	2.21	1.87
time (sec)	N/A	1.707	0.143	0.386	0.000	0.113	0.000	0.126	0.249	2.643

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	96	78	153	0	860	0	92	175	167
N.S.	1	1.13	0.92	1.80	0.00	10.12	0.00	1.08	2.06	1.96
time (sec)	N/A	0.759	0.103	0.257	0.000	0.100	0.000	0.131	0.197	2.529

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	65	57	94	0	430	0	62	110	139
N.S.	1	1.05	0.92	1.52	0.00	6.94	0.00	1.00	1.77	2.24
time (sec)	N/A	0.377	0.135	0.192	0.000	0.101	0.000	0.119	0.226	2.475

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	41	36	0	165	0	32	44	43
N.S.	1	1.00	0.98	0.86	0.00	3.93	0.00	0.76	1.05	1.02
time (sec)	N/A	0.278	0.024	0.083	0.000	0.095	0.000	0.119	0.229	0.107

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	51	0	219	0	45	68	286
N.S.	1	1.00	1.00	0.94	0.00	4.06	0.00	0.83	1.26	5.30
time (sec)	N/A	0.425	0.070	0.229	0.000	0.107	0.000	0.128	0.262	4.670

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	67	63	73	0	504	0	61	177	294
N.S.	1	1.05	0.98	1.14	0.00	7.88	0.00	0.95	2.77	4.59
time (sec)	N/A	0.560	0.117	0.348	0.000	0.105	0.000	0.125	0.203	4.556

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	97	82	109	0	1444	0	89	365	476
N.S.	1	1.11	0.94	1.25	0.00	16.60	0.00	1.02	4.20	5.47
time (sec)	N/A	0.797	0.183	0.545	0.000	0.162	0.000	0.125	0.198	5.559

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	57	60	75	93	686	0	69	169	143
N.S.	1	1.19	1.25	1.56	1.94	14.29	0.00	1.44	3.52	2.98
time (sec)	N/A	0.686	0.254	0.682	0.125	0.086	0.000	0.130	0.225	2.431

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	26	38	25	74	437	0	61	138	96
N.S.	1	0.72	1.06	0.69	2.06	12.14	0.00	1.69	3.83	2.67
time (sec)	N/A	0.436	0.053	0.454	0.119	0.083	0.000	0.121	0.207	2.444

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	35	41	59	51	210	0	42	83	67
N.S.	1	1.13	1.32	1.90	1.65	6.77	0.00	1.35	2.68	2.16
time (sec)	N/A	0.516	0.150	0.318	0.109	0.088	0.000	0.125	0.204	2.293

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	15	10	34	33	85	0	35	52	33
N.S.	1	1.07	0.71	2.43	2.36	6.07	0.00	2.50	3.71	2.36
time (sec)	N/A	0.390	0.026	0.217	0.125	0.084	0.000	0.123	0.237	2.207

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	15	15	31	16	14	0	14	12	25
N.S.	1	1.07	1.07	2.21	1.14	1.00	0.00	1.00	0.86	1.79
time (sec)	N/A	0.394	0.167	0.168	0.111	0.086	0.000	0.126	0.209	0.077

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	12	17	18	16	19	17	16	14
N.S.	1	1.00	1.33	1.89	2.00	1.78	2.11	1.89	1.78	1.56
time (sec)	N/A	0.289	0.002	0.107	0.027	0.079	0.093	0.120	0.196	0.056

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	36	44	38	52	136	0	56	106	65
N.S.	1	0.90	1.10	0.95	1.30	3.40	0.00	1.40	2.65	1.62
time (sec)	N/A	0.262	0.036	0.154	0.044	0.094	0.000	0.126	0.197	2.244

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	33	36	47	46	0	40	65	94
N.S.	1	1.11	0.87	0.95	1.24	1.21	0.00	1.05	1.71	2.47
time (sec)	N/A	0.411	0.207	0.151	0.034	0.082	0.000	0.124	0.237	2.224

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	57	66	56	108	773	0	94	294	160
N.S.	1	0.84	0.97	0.82	1.59	11.37	0.00	1.38	4.32	2.35
time (sec)	N/A	0.283	0.110	0.210	0.044	0.110	0.000	0.113	0.203	2.292

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	64	69	63	105	151	0	64	153	264
N.S.	1	1.16	1.25	1.15	1.91	2.75	0.00	1.16	2.78	4.80
time (sec)	N/A	0.501	0.181	0.244	0.038	0.097	0.000	0.117	7.277	2.457

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	126	121	246	332	4077	0	267	1355	316
N.S.	1	1.04	1.00	2.03	2.74	33.69	0.00	2.21	11.20	2.61
time (sec)	N/A	0.366	0.202	2.530	0.127	0.154	0.000	0.117	0.223	2.860

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	185	266	0	4914	0	250	1097	1001
N.S.	1	1.00	0.99	1.42	0.00	26.28	0.00	1.34	5.87	5.35
time (sec)	N/A	0.637	0.504	1.746	0.000	0.386	0.000	0.118	0.192	8.728

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	77	76	158	164	1280	0	152	609	155
N.S.	1	1.07	1.06	2.19	2.28	17.78	0.00	2.11	8.46	2.15
time (sec)	N/A	0.300	0.133	1.004	0.119	0.116	0.000	0.119	0.261	2.712

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	114	113	152	0	1254	0	111	398	700
N.S.	1	1.21	1.20	1.62	0.00	13.34	0.00	1.18	4.23	7.45
time (sec)	N/A	0.930	0.354	0.720	0.000	0.214	0.000	0.127	0.242	7.563

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	43	38	75	67	200	0	73	162	260
N.S.	1	1.23	1.09	2.14	1.91	5.71	0.00	2.09	4.63	7.43
time (sec)	N/A	0.456	0.075	0.383	0.155	0.111	0.000	0.115	0.210	2.467

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	74	62	84	0	193	0	52	51	273
N.S.	1	1.19	1.00	1.35	0.00	3.11	0.00	0.84	0.82	4.40
time (sec)	N/A	1.119	0.087	0.301	0.000	0.106	0.000	0.115	0.215	4.495

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	22	11	21	26	27	41	19	24	23
N.S.	1	1.16	0.58	1.11	1.37	1.42	2.16	1.00	1.26	1.21
time (sec)	N/A	0.360	0.003	0.165	0.039	0.086	0.157	0.118	0.179	0.097

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	81	63	78	67	81	0	67	87	271
N.S.	1	1.23	0.95	1.18	1.02	1.23	0.00	1.02	1.32	4.11
time (sec)	N/A	0.409	0.119	0.292	0.036	0.111	0.000	0.115	0.209	2.518

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	104	81	104	0	646	0	82	230	383
N.S.	1	0.91	0.71	0.91	0.00	5.67	0.00	0.72	2.02	3.36
time (sec)	N/A	0.747	0.328	0.411	0.000	0.101	0.000	0.120	0.212	2.540

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	139	107	113	164	1222	0	193	701	339
N.S.	1	1.23	0.95	1.00	1.45	10.81	0.00	1.71	6.20	3.00
time (sec)	N/A	0.430	0.334	0.658	0.054	0.146	0.000	0.125	0.232	3.078

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	181	166	153	0	3530	0	190	691	713
N.S.	1	0.87	0.80	0.74	0.00	17.05	0.00	0.92	3.34	3.44
time (sec)	N/A	1.393	0.587	1.075	0.000	0.128	0.000	0.131	19.313	2.728

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	211	164	162	366	5181	0	380	15	623
N.S.	1	1.19	0.92	0.91	2.06	29.11	0.00	2.13	0.08	3.50
time (sec)	N/A	0.550	0.692	1.739	0.052	0.201	0.000	0.123	200.021	3.552

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	169	132	153	0	0	4363	0	0	173	0
N.S.	1	0.78	0.91	0.00	0.00	25.82	0.00	0.00	1.02	0.00
time (sec)	N/A	0.609	1.709	0.000	0.000	0.654	0.000	0.000	0.272	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	81	87	0	0	1589	0	0	113	0
N.S.	1	0.81	0.87	0.00	0.00	15.89	0.00	0.00	1.13	0.00
time (sec)	N/A	0.317	0.629	0.000	0.000	0.513	0.000	0.000	0.274	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	50	50	43	0	605	0	0	53	47
N.S.	1	0.98	0.98	0.84	0.00	11.86	0.00	0.00	1.04	0.92
time (sec)	N/A	0.274	0.155	0.141	0.000	0.488	0.000	0.000	0.223	2.724

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	118	100	0	0	9170	0	0	20	0
N.S.	1	1.11	0.94	0.00	0.00	86.51	0.00	0.00	0.19	0.00
time (sec)	N/A	0.409	0.216	0.000	0.000	0.452	0.000	0.000	0.236	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	268	223	0	0	17083	0	0	22	0
N.S.	1	1.75	1.46	0.00	0.00	111.65	0.00	0.00	0.14	0.00
time (sec)	N/A	0.601	2.619	0.000	0.000	1.027	0.000	0.000	0.269	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	344	345	0	0	0	0	0	0	22	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	2.224	0.000	0.000	0.000	0.000	0.000	0.000	0.217	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	0	0	0	0	0	0	13	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.256	0.000	0.000	0.000	0.000	0.000	0.000	0.241	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	539	0	0	0	0	0	22	0
N.S.	1	1.00	2.19	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.774	15.184	0.000	0.000	0.000	0.000	0.000	0.258	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	123	112	0	0	2813	0	0	34	0
N.S.	1	0.83	0.76	0.00	0.00	19.01	0.00	0.00	0.23	0.00
time (sec)	N/A	0.366	1.062	0.000	0.000	0.540	0.000	0.000	0.229	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	71	66	0	0	925	0	0	34	0
N.S.	1	0.90	0.84	0.00	0.00	11.71	0.00	0.00	0.43	0.00
time (sec)	N/A	0.526	0.438	0.000	0.000	0.693	0.000	0.000	0.212	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	0	558	0	0	32	27
N.S.	1	1.00	1.00	0.84	0.00	18.00	0.00	0.00	1.03	0.87
time (sec)	N/A	0.252	0.119	0.155	0.000	0.663	0.000	0.000	0.259	2.709

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	118	100	0	0	9458	0	0	32	0
N.S.	1	1.11	0.94	0.00	0.00	89.23	0.00	0.00	0.30	0.00
time (sec)	N/A	0.395	0.282	0.000	0.000	0.827	0.000	0.000	0.258	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	272	280	0	0	20851	0	0	34	0
N.S.	1	1.55	1.60	0.00	0.00	119.15	0.00	0.00	0.19	0.00
time (sec)	N/A	0.523	1.976	0.000	0.000	4.994	0.000	0.000	0.281	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	610	610	0	0	0	0	0	0	34	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.554	0.000	0.000	0.000	0.000	0.000	0.000	0.245	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	0	0	0	0	0	0	34	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.718	0.000	0.000	0.000	0.000	0.000	0.000	0.234	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	168	0	0	0	0	0	26	0
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.246	0.378	0.000	0.000	0.000	0.000	0.000	0.255	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	0	0	0	0	0	0	34	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.110	0.000	0.000	0.000	0.000	0.000	0.000	0.315	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	119	117	0	0	3745	0	0	50	0
N.S.	1	0.80	0.79	0.00	0.00	25.30	0.00	0.00	0.34	0.00
time (sec)	N/A	0.393	0.589	0.000	0.000	0.520	0.000	0.000	0.265	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	79	66	0	0	1107	0	0	50	0
N.S.	1	0.90	0.75	0.00	0.00	12.58	0.00	0.00	0.57	0.00
time (sec)	N/A	0.588	0.323	0.000	0.000	0.494	0.000	0.000	0.244	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	53	43	46	0	917	0	0	48	50
N.S.	1	0.98	0.80	0.85	0.00	16.98	0.00	0.00	0.89	0.93
time (sec)	N/A	0.269	0.155	0.128	0.000	0.502	0.000	0.000	0.249	2.820

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	B	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	0	212	0	0	14962	0	0	48	0
N.S.	1	0.00	1.49	0.00	0.00	105.37	0.00	0.00	0.34	0.00
time (sec)	N/A	0.000	0.704	0.000	0.000	3.585	0.000	0.000	0.291	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	217	310	315	0	0	53763	0	0	50	0
N.S.	1	1.43	1.45	0.00	0.00	247.76	0.00	0.00	0.23	0.00
time (sec)	N/A	0.867	1.106	0.000	0.000	6.474	0.000	0.000	0.342	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	907	907	0	0	0	0	0	0	50	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	2.505	0.000	0.000	0.000	0.000	0.000	0.000	0.274	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	344	381	0	0	0	0	0	0	50	0
N.S.	1	1.11	0.00	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	1.438	0.000	0.000	0.000	0.000	0.000	0.000	0.257	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	347	376	0	0	0	0	0	0	42	0
N.S.	1	1.08	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.273	0.000	0.000	0.000	0.000	0.000	0.000	0.260	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	665	665	0	0	0	0	0	0	50	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	2.359	0.000	0.000	0.000	0.000	0.000	0.000	0.739	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	191	105	84	86	386	589	0	64	138	405
N.S.	1	0.55	0.44	0.45	2.02	3.08	0.00	0.34	0.72	2.12
time (sec)	N/A	0.451	0.064	148.546	0.042	0.081	0.000	0.116	0.246	0.144

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	141	87	72	65	209	315	0	51	96	91
N.S.	1	0.62	0.51	0.46	1.48	2.23	0.00	0.36	0.68	0.65
time (sec)	N/A	0.336	0.051	141.811	0.049	0.185	0.000	0.118	0.269	2.316

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	44	38	84	120	0	38	50	78
N.S.	1	1.00	0.79	0.68	1.50	2.14	0.00	0.68	0.89	1.39
time (sec)	N/A	0.297	0.045	0.256	0.057	0.180	0.000	0.112	0.234	0.119

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	66	21	42	0	21	22	0
N.S.	1	1.00	0.95	1.50	0.48	0.95	0.00	0.48	0.50	0.00
time (sec)	N/A	0.276	0.030	0.530	0.125	0.158	0.000	0.109	0.221	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	59	48	106	29	66	0	25	26	0
N.S.	1	0.80	0.65	1.43	0.39	0.89	0.00	0.34	0.35	0.00
time (sec)	N/A	0.311	0.037	0.564	0.044	0.173	0.000	0.110	0.263	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	74	81	216	74	126	0	71	67	0
N.S.	1	0.46	0.50	1.33	0.46	0.78	0.00	0.44	0.41	0.00
time (sec)	N/A	0.334	0.073	0.572	0.042	0.194	0.000	0.115	0.268	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	102	109	326	112	218	530	99	97	0
N.S.	1	0.41	0.44	1.30	0.45	0.87	2.12	0.40	0.39	0.00
time (sec)	N/A	0.366	0.105	0.573	0.038	0.206	79.386	0.111	0.267	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	149	77	130	0	81	0	0	23	0
N.S.	1	1.38	0.71	1.20	0.00	0.75	0.00	0.00	0.21	0.00
time (sec)	N/A	0.518	0.130	0.359	0.000	0.203	0.000	0.000	0.218	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	27	44	39	30	48	0	0	23	42
N.S.	1	0.96	1.57	1.39	1.07	1.71	0.00	0.00	0.82	1.50
time (sec)	N/A	0.245	0.035	0.092	0.179	0.163	0.000	0.000	0.270	2.405

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	241	65	134	0	120	0	0	23	0
N.S.	1	1.19	0.32	0.66	0.00	0.59	0.00	0.00	0.11	0.00
time (sec)	N/A	0.494	0.089	0.322	0.000	0.113	0.000	0.000	0.248	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	72	77	97	0	90	0	0	23	0
N.S.	1	1.07	1.15	1.45	0.00	1.34	0.00	0.00	0.34	0.00
time (sec)	N/A	0.264	0.104	0.099	0.000	0.184	0.000	0.000	0.229	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	121	58	114	0	71	0	0	21	0
N.S.	1	1.39	0.67	1.31	0.00	0.82	0.00	0.00	0.24	0.00
time (sec)	N/A	0.284	0.108	0.269	0.000	0.225	0.000	0.000	0.249	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	64	75	0	0	100	0	0	20	0
N.S.	1	1.08	1.27	0.00	0.00	1.69	0.00	0.00	0.34	0.00
time (sec)	N/A	0.259	0.088	0.000	0.000	0.202	0.000	0.000	0.248	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	167	0	27	0	0	14	0
N.S.	1	1.00	1.00	4.64	0.00	0.75	0.00	0.00	0.39	0.00
time (sec)	N/A	0.278	0.069	0.325	0.000	0.101	0.000	0.000	0.227	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	55	0	0	57	0	0	14	0
N.S.	1	1.00	1.38	0.00	0.00	1.42	0.00	0.00	0.35	0.00
time (sec)	N/A	0.261	0.094	0.000	0.000	0.161	0.000	0.000	0.218	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	190	59	134	0	94	0	0	14	0
N.S.	1	1.39	0.43	0.98	0.00	0.69	0.00	0.00	0.10	0.00
time (sec)	N/A	0.359	0.090	0.339	0.000	0.113	0.000	0.000	0.277	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	28	33	38	42	37	0	0	14	58
N.S.	1	1.22	1.43	1.65	1.83	1.61	0.00	0.00	0.61	2.52
time (sec)	N/A	0.244	0.029	0.112	0.119	0.176	0.000	0.000	0.234	2.170

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	119	65	117	0	67	0	0	14	0
N.S.	1	1.49	0.81	1.46	0.00	0.84	0.00	0.00	0.18	0.00
time (sec)	N/A	0.299	0.078	0.293	0.000	0.212	0.000	0.000	0.223	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	126	98	121	0	109	0	0	23	0
N.S.	1	1.03	0.80	0.99	0.00	0.89	0.00	0.00	0.19	0.00
time (sec)	N/A	0.316	0.140	0.128	0.000	0.232	0.000	0.000	0.252	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	177	77	138	0	89	0	0	23	0
N.S.	1	1.26	0.55	0.98	0.00	0.63	0.00	0.00	0.16	0.00
time (sec)	N/A	0.345	0.131	0.269	0.000	0.112	0.000	0.000	0.237	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	27	44	47	30	56	0	0	23	42
N.S.	1	0.96	1.57	1.68	1.07	2.00	0.00	0.00	0.82	1.50
time (sec)	N/A	0.255	0.035	0.103	0.159	0.194	0.000	0.000	0.223	2.220

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	269	65	147	0	129	0	0	23	0
N.S.	1	1.07	0.26	0.59	0.00	0.51	0.00	0.00	0.09	0.00
time (sec)	N/A	0.453	0.094	0.332	0.000	0.111	0.000	0.000	0.227	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	100	90	113	0	101	0	0	23	0
N.S.	1	1.09	0.98	1.23	0.00	1.10	0.00	0.00	0.25	0.00
time (sec)	N/A	0.284	0.124	0.098	0.000	0.246	0.000	0.000	0.289	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	149	61	129	0	79	0	0	23	0
N.S.	1	1.34	0.55	1.16	0.00	0.71	0.00	0.00	0.21	0.00
time (sec)	N/A	0.342	0.085	0.279	0.000	0.314	0.000	0.000	0.233	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	87	88	0	0	109	0	0	23	0
N.S.	1	0.99	1.00	0.00	0.00	1.24	0.00	0.00	0.26	0.00
time (sec)	N/A	0.298	0.123	0.000	0.000	0.205	0.000	0.000	0.216	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	244	65	159	0	0	0	0	21	0
N.S.	1	1.14	0.30	0.74	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.391	0.128	0.337	0.000	0.000	0.000	0.000	0.230	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	64	131	0	106	0	0	20	0
N.S.	1	1.00	0.70	1.42	0.00	1.15	0.00	0.00	0.22	0.00
time (sec)	N/A	0.263	0.081	0.098	0.000	0.104	0.000	0.000	0.254	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	45	127	0	82	0	0	21	0
N.S.	1	1.00	0.80	2.27	0.00	1.46	0.00	0.00	0.38	0.00
time (sec)	N/A	0.352	0.099	0.467	0.000	0.177	0.000	0.000	0.233	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	30	32	0	39	28	0	0	21	28
N.S.	1	1.20	1.28	0.00	1.56	1.12	0.00	0.00	0.84	1.12
time (sec)	N/A	0.243	0.027	0.000	0.113	0.096	0.000	0.000	0.221	2.248

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	121	65	0	0	56	0	0	21	0
N.S.	1	1.32	0.71	0.00	0.00	0.61	0.00	0.00	0.23	0.00
time (sec)	N/A	0.317	0.089	0.000	0.000	0.102	0.000	0.000	0.312	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	64	51	0	0	93	0	0	21	0
N.S.	1	0.97	0.77	0.00	0.00	1.41	0.00	0.00	0.32	0.00
time (sec)	N/A	0.289	0.085	0.000	0.000	0.213	0.000	0.000	0.231	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	63	64	0	0	0	0	0	13	0
N.S.	1	0.89	0.90	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.347	0.538	0.000	0.000	0.000	0.000	0.000	0.236	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	69	126	0	0	0	0	0	60	0
N.S.	1	0.97	1.77	0.00	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.326	3.576	0.000	0.000	0.000	0.000	0.000	0.306	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	70	101	0	0	0	0	0	78	0
N.S.	1	0.99	1.42	0.00	0.00	0.00	0.00	0.00	1.10	0.00
time (sec)	N/A	0.326	1.015	0.000	0.000	0.000	0.000	0.000	0.229	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	69	192	0	0	0	0	0	0	0
N.S.	1	0.97	2.70	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.330	8.900	0.000	0.000	0.000	0.000	0.000	0.243	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	139	29	30	96	189	0	215	95	66
N.S.	1	3.48	0.72	0.75	2.40	4.72	0.00	5.38	2.38	1.65
time (sec)	N/A	0.621	0.286	28.712	0.242	0.110	0.000	0.271	0.251	2.310

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	B	B	F	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	26	62	0	74	48	0	38	42	49
N.S.	1	1.04	2.48	0.00	2.96	1.92	0.00	1.52	1.68	1.96
time (sec)	N/A	0.253	0.138	0.000	0.040	0.134	0.000	0.117	0.273	2.412

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	B	B	F	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	64	0	49	49	0	37	39	36
N.S.	1	1.00	2.56	0.00	1.96	1.96	0.00	1.48	1.56	1.44
time (sec)	N/A	0.267	0.071	0.000	0.043	0.178	0.000	0.113	0.242	2.367

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	96	114	0	0	474	0	0	30	0
N.S.	1	1.08	1.28	0.00	0.00	5.33	0.00	0.00	0.34	0.00
time (sec)	N/A	0.359	1.589	0.000	0.000	0.249	0.000	0.000	0.260	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	106	108	0	0	538	0	0	31	0
N.S.	1	1.63	1.66	0.00	0.00	8.28	0.00	0.00	0.48	0.00
time (sec)	N/A	0.364	0.928	0.000	0.000	0.239	0.000	0.000	0.277	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	20	20	19	34	44	27	21	41
N.S.	1	1.00	1.05	1.05	1.00	1.79	2.32	1.42	1.11	2.16
time (sec)	N/A	0.219	0.038	0.723	0.027	0.183	0.785	0.120	0.249	2.309

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	28	70	0	28	45	24
N.S.	1	1.00	1.00	1.06	1.56	3.89	0.00	1.56	2.50	1.33
time (sec)	N/A	0.243	0.059	1.198	0.045	0.172	0.000	0.110	0.226	2.329

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	53	55	45	0	452	0	115	151	139
N.S.	1	0.96	1.00	0.82	0.00	8.22	0.00	2.09	2.75	2.53
time (sec)	N/A	0.302	0.041	17.402	0.000	0.088	0.000	0.121	0.227	2.487

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	45	42	36	91	272	0	47	85	55
N.S.	1	1.07	1.00	0.86	2.17	6.48	0.00	1.12	2.02	1.31
time (sec)	N/A	0.248	0.041	16.915	0.051	0.178	0.000	0.119	0.270	2.325

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	64	0	1326	0	152	290	314
N.S.	1	1.00	1.00	0.72	0.00	14.90	0.00	1.71	3.26	3.53
time (sec)	N/A	0.392	0.046	187.708	0.000	0.209	0.000	0.119	0.252	2.364

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	95	74	258	0	315	0	0	31	0
N.S.	1	0.98	0.76	2.66	0.00	3.25	0.00	0.00	0.32	0.00
time (sec)	N/A	0.399	0.116	94.312	0.000	0.126	0.000	0.000	0.243	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	91	72	141	0	159	0	0	29	0
N.S.	1	0.98	0.77	1.52	0.00	1.71	0.00	0.00	0.31	0.00
time (sec)	N/A	0.407	0.073	0.618	0.000	0.168	0.000	0.000	0.250	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	183	0	39	0	0	18	0
N.S.	1	1.00	1.00	3.16	0.00	0.67	0.00	0.00	0.31	0.00
time (sec)	N/A	0.290	0.046	0.475	0.000	0.107	0.000	0.000	0.224	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	183	0	248	0	0	31	0
N.S.	1	1.00	1.00	3.16	0.00	4.28	0.00	0.00	0.53	0.00
time (sec)	N/A	0.294	0.053	0.965	0.000	0.241	0.000	0.000	0.218	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	95	76	237	0	370	0	0	31	0
N.S.	1	0.98	0.78	2.44	0.00	3.81	0.00	0.00	0.32	0.00
time (sec)	N/A	0.389	0.081	1.559	0.000	0.116	0.000	0.000	0.258	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	95	87	256	0	602	0	0	31	0
N.S.	1	0.98	0.90	2.64	0.00	6.21	0.00	0.00	0.32	0.00
time (sec)	N/A	0.385	0.097	3.300	0.000	0.247	0.000	0.000	0.248	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [123] had the largest ratio of [2.07691999999999988]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	6	0.333
2	A	4	3	1.00	8	0.375
3	A	4	4	1.00	8	0.500
4	C	4	3	1.23	8	0.375
5	A	6	6	1.09	8	0.750
6	C	4	3	1.12	8	0.375
7	C	4	3	1.42	6	0.500
8	C	4	3	1.14	6	0.500
9	A	6	6	1.00	10	0.600
10	A	6	6	1.00	10	0.600
11	A	4	4	1.00	10	0.400
12	A	4	4	1.00	10	0.400
13	A	6	6	1.00	10	0.600
14	A	6	6	1.00	10	0.600
15	A	8	8	1.02	12	0.667
16	A	6	6	1.00	12	0.500
17	A	6	6	1.00	12	0.500
18	A	4	4	1.00	12	0.333
19	A	4	4	1.00	12	0.333
20	A	6	6	1.00	12	0.500
21	A	6	6	1.00	12	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	8	8	1.08	12	0.667
23	A	4	4	1.00	10	0.400
24	A	7	6	1.16	12	0.500
25	A	6	5	1.12	12	0.417
26	A	5	4	1.05	12	0.333
27	A	4	3	1.00	12	0.250
28	A	4	3	1.18	12	0.250
29	A	5	4	1.12	12	0.333
30	A	6	5	1.16	12	0.417
31	A	7	6	1.18	12	0.500
32	A	7	6	1.20	10	0.600
33	A	6	5	1.15	10	0.500
34	A	5	4	1.12	10	0.400
35	A	4	3	1.23	10	0.300
36	A	5	4	1.31	10	0.400
37	A	6	5	1.40	10	0.500
38	A	7	6	1.45	10	0.600
39	A	14	14	0.88	10	1.400
40	A	10	10	1.09	10	1.000
41	A	8	8	1.15	10	0.800
42	A	8	8	1.19	10	0.800
43	A	10	10	1.00	10	1.000
44	A	14	14	0.92	10	1.400
45	C	6	5	0.52	10	0.500
46	C	6	5	0.58	10	0.500
47	C	6	5	0.75	10	0.500
48	A	6	5	1.00	10	0.500
49	A	5	5	0.81	10	0.500
50	A	9	9	0.72	10	0.900
51	A	13	13	0.70	10	1.300
52	A	16	15	1.05	13	1.154
53	C	16	15	1.35	13	1.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	11	11	0.96	13	0.846
55	A	12	11	1.12	11	1.000
56	C	19	18	1.27	11	1.636
57	A	13	12	1.00	13	0.923
58	C	23	22	1.28	13	1.692
59	C	16	15	1.18	13	1.154
60	A	15	14	1.19	13	1.077
61	A	12	11	1.00	13	0.846
62	A	14	13	1.18	13	1.000
63	A	12	11	0.95	11	1.000
64	A	14	13	1.11	11	1.182
65	A	11	10	1.17	13	0.769
66	A	15	14	1.51	13	1.077
67	A	16	15	1.22	13	1.154
68	C	13	12	1.13	13	0.923
69	C	11	10	1.13	13	0.769
70	A	9	9	1.02	13	0.692
71	A	8	8	1.04	11	0.727
72	A	2	2	1.00	11	0.182
73	A	5	5	1.00	13	0.385
74	A	7	7	1.00	13	0.538
75	A	11	10	1.04	13	0.769
76	A	3	3	1.00	12	0.250
77	A	3	3	1.00	13	0.231
78	A	10	9	1.04	14	0.643
79	A	7	6	1.00	14	0.429
80	A	4	3	1.00	14	0.214
81	A	8	7	1.00	14	0.500
82	A	11	10	1.05	14	0.714
83	A	4	3	1.00	15	0.200
84	A	8	7	1.00	15	0.467
85	A	4	3	1.00	10	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	4	3	1.00	10	0.300
87	A	5	5	1.07	12	0.417
88	A	3	3	1.01	12	0.250
89	A	7	6	1.00	12	0.500
90	A	1	1	1.00	10	0.100
91	A	6	5	1.00	12	0.417
92	A	11	10	1.23	12	0.833
93	A	14	13	1.25	12	1.083
94	A	2	2	1.00	14	0.143
95	A	18	17	1.17	13	1.308
96	A	16	15	1.15	13	1.154
97	A	13	12	1.13	13	0.923
98	A	10	9	1.05	11	0.818
99	A	6	5	1.00	11	0.455
100	A	9	8	1.00	13	0.615
101	A	11	10	1.05	13	0.769
102	A	13	12	1.11	13	0.923
103	A	11	11	1.19	13	0.846
104	A	7	6	0.72	13	0.462
105	A	6	6	1.13	13	0.462
106	A	7	6	1.07	13	0.462
107	A	4	4	1.07	13	0.308
108	A	5	4	1.00	11	0.364
109	A	7	6	0.90	11	0.545
110	A	10	10	1.11	13	0.769
111	A	7	6	0.84	13	0.462
112	A	12	12	1.16	13	0.923
113	A	6	5	1.04	13	0.385
114	A	8	8	1.00	13	0.615
115	A	6	5	1.07	13	0.385
116	A	12	11	1.21	13	0.846
117	A	6	5	1.23	13	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	15	14	1.19	13	1.077
119	A	7	6	1.16	11	0.545
120	A	6	5	1.23	11	0.455
121	A	18	17	0.91	13	1.308
122	A	6	5	1.23	13	0.385
123	C	28	27	0.87	13	2.077
124	A	6	5	1.19	13	0.385
125	A	8	7	0.78	23	0.304
126	A	7	6	0.81	23	0.261
127	A	7	6	0.98	21	0.286
128	A	7	6	1.11	21	0.286
129	A	14	13	1.75	23	0.565
130	A	14	14	1.00	23	0.609
131	A	2	2	1.00	14	0.143
132	A	4	4	1.00	23	0.174
133	A	8	7	0.83	23	0.304
134	A	7	6	0.90	23	0.261
135	A	6	5	1.00	21	0.238
136	A	7	6	1.11	21	0.286
137	A	8	7	1.55	23	0.304
138	A	3	3	1.00	23	0.130
139	A	11	11	1.00	23	0.478
140	A	2	2	1.00	14	0.143
141	A	4	4	1.00	23	0.174
142	A	8	7	0.80	23	0.304
143	A	7	6	0.90	23	0.261
144	A	7	6	0.98	21	0.286
145	F	0	0	N/A	0.000	N/A
146	A	8	7	1.43	23	0.304
147	A	3	3	1.00	23	0.130
148	A	14	14	1.11	23	0.609
149	A	11	11	1.08	14	0.786

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	4	4	1.00	23	0.174
151	A	7	6	0.55	25	0.240
152	A	7	6	0.62	25	0.240
153	A	5	4	1.00	25	0.160
154	A	5	4	1.00	25	0.160
155	A	6	5	0.80	25	0.200
156	A	7	6	0.46	25	0.240
157	A	7	6	0.41	25	0.240
158	A	7	6	1.38	15	0.400
159	A	4	3	0.96	15	0.200
160	A	9	8	1.19	15	0.533
161	A	7	6	1.07	15	0.400
162	A	6	5	1.39	13	0.385
163	A	7	6	1.08	11	0.545
164	A	6	5	1.00	15	0.333
165	A	6	5	1.00	15	0.333
166	A	7	6	1.39	15	0.400
167	A	4	3	1.22	15	0.200
168	A	6	5	1.49	15	0.333
169	A	9	8	1.03	15	0.533
170	A	8	7	1.26	15	0.467
171	A	4	3	0.96	15	0.200
172	A	10	9	1.07	15	0.600
173	A	8	7	1.09	15	0.467
174	A	7	6	1.34	15	0.400
175	A	8	7	0.99	15	0.467
176	A	9	8	1.14	13	0.615
177	A	8	7	1.00	11	0.636
178	A	8	7	1.00	15	0.467
179	A	4	3	1.20	15	0.200
180	A	6	5	1.32	15	0.333
181	A	7	6	0.97	15	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	5	4	0.89	11	0.364
183	A	5	4	0.97	13	0.308
184	A	5	4	0.99	13	0.308
185	A	5	4	0.97	13	0.308
186	C	1	1	3.48	44	0.023
187	A	4	3	1.04	15	0.200
188	A	5	4	1.00	15	0.267
189	A	4	3	1.08	20	0.150
190	A	4	3	1.63	21	0.143
191	A	4	3	1.00	15	0.200
192	A	5	4	1.00	17	0.235
193	A	6	5	0.96	17	0.294
194	C	5	4	1.07	17	0.235
195	A	8	7	1.00	17	0.412
196	A	8	7	0.98	19	0.368
197	A	8	7	0.98	19	0.368
198	A	6	5	1.00	19	0.263
199	A	6	5	1.00	19	0.263
200	A	8	7	0.98	19	0.368
201	A	8	7	0.98	19	0.368

CHAPTER 3

Listing of integrals

3.1 $\int \operatorname{sech}(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \operatorname{sech}(a+bx)\,dx = \frac{\arctan(\sinh(a+bx))}{b}$$

output
```
arctan(sinh(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \operatorname{sech}(a+bx)\,dx = -\frac{\cot^{-1}(\sinh(a+bx))}{b}$$

input
```
Integrate[Sech[a + b*x],x]
```

output
```
-(ArcCot[Sinh[a + b*x]]/b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \csc\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \ 4257$$

$$\frac{\arctan(\sinh(a+bx))}{b}$$

input
```
Int[Sech[a + b*x],x]
```

output
```
ArcTan[Sinh[a + b*x]]/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\arctan(\sinh(bx+a))}{b}$	12
default	$\frac{\arctan(\sinh(bx+a))}{b}$	12
risch	$\frac{i\ln(e^{bx+a}+i)}{b}-\frac{i\ln(e^{bx+a}-i)}{b}$	34
parallelrisch	$-\frac{i\left(\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-i\right)-\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+i\right)\right)}{b}$	36

input
```
int(sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
arctan(sinh(b*x+a))/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \operatorname{sech}(a+bx)\,dx=\frac{2\arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b}$$

input
```
integrate(sech(b*x+a),x, algorithm="fricas")
```

output
```
2*arctan(cosh(b*x + a) + sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \operatorname{sech}(a+bx)\,dx=\begin{cases}\frac{2\operatorname{atan}\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} & \text{for } b\neq 0\\ x\operatorname{sech}\left(a\right) & \text{otherwise}\end{cases}$$

input
```
integrate(sech(b*x+a),x)
```

output
```
Piecewise((2*atan(tanh(a/2 + b*x/2))/b, Ne(b, 0)), (x*sech(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}(a+bx)\,dx = \frac{\arctan(\sinh(bx+a))}{b}$$

input
```
integrate(sech(b*x+a),x, algorithm="maxima")
```

output
```
arctan(sinh(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \operatorname{sech}(a+bx)\,dx = \frac{2\arctan\left(e^{(bx+a)}\right)}{b}$$

input
```
integrate(sech(b*x+a),x, algorithm="giac")
```

output
```
2*arctan(e^(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \operatorname{sech}(a+bx)\,dx=\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}$$

input
```
int(1/cosh(a + b*x),x)
```

output
```
(2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \operatorname{sech}(a+bx)\,dx=\frac{2atan\left(e^{bx+a}\right)}{b}$$

input
```
int(sech(b*x+a),x)
```

output
```
(2*atan(e**(a + b*x)))/b
```

3.2 $\int \operatorname{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 10

$$\int \operatorname{sech}^2(a+bx)\,dx=\frac{\tanh(a+bx)}{b}$$

output
```
tanh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(a+bx)\,dx=\frac{\tanh(a+bx)}{b}$$

input
```
Integrate[Sech[a + b*x]^2,x]
```

output
```
Tanh[a + b*x]/b
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{4254}$$

$$\frac{i\int 1d(-i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\tanh(a+bx)}{b}$$

input
```
Int[Sech[a + b*x]^2,x]
```

output
```
Tanh[a + b*x]/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\tanh(bx+a)}{b}$	11
default	$\frac{\tanh(bx+a)}{b}$	11
risch	$-\frac{2}{b(1+e^{2bx+2a})}$	19
parallelrisch	$\frac{\sinh(bx+a)}{\cosh(bx+a)b}$	19

input
```
int(sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
tanh(b*x+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(10) = 20.

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 4.10

$$\int \operatorname{sech}^2(a+bx)\,dx = -\frac{2}{b\cosh(bx+a)^2+2\,b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2+b}$$

input
```
integrate(sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
-2/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^
2 + b)
```

Sympy [F]

$$\int \operatorname{sech}^2(a+bx)\,dx = \int \operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)**2,x)
```

output
```
Integral(sech(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \operatorname{sech}^2(a+bx)\,dx = \frac{2}{b(e^{(-2\,bx-2\,a)}+1)}$$

input
```
integrate(sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
2/(b*(e^(-2*b*x - 2*a) + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \operatorname{sech}^2(a+bx)\,dx = -\frac{2}{b(e^{(2\,bx+2\,a)}+1)}$$

input
```
integrate(sech(b*x+a)^2,x, algorithm="giac")
```

output
```
-2/(b*(e^(2*b*x + 2*a) + 1))
```

Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \mathrm{sech}^2(a+bx)\,dx = -\frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/cosh(a + b*x)^2,x)
```

output
```
-2/(b*(exp(2*a + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.90

$$\int \mathrm{sech}^2(a+bx)\,dx = \frac{2e^{2bx+2a}}{b\left(e^{2bx+2a}+1\right)}$$

input
```
int(sech(b*x+a)^2,x)
```

output
```
(2*e**(2*a + 2*b*x))/(b*(e**(2*a + 2*b*x) + 1))
```

3.3 $\int \operatorname{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 34

$$\int \operatorname{sech}^3(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{2b}+\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}$$

output
```
1/2*arctan(sinh(b*x+a))/b+1/2*sech(b*x+a)*tanh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^3(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{2b}+\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}$$

input
```
Integrate[Sech[a + b*x]^3,x]
```

output
```
ArcTan[Sinh[a + b*x]]/(2*b) + (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{2}\int \operatorname{sech}(a+bx)dx+\frac{\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}+\frac{1}{2}\int \csc\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4257}$$

$$\frac{\arctan(\sinh(a+bx))}{2b}+\frac{\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

input
```
Int[Sech[a + b*x]^3,x]
```

output
```
ArcTan[Sinh[a + b*x]]/(2*b) + (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\frac{\operatorname{sech}(bx+a)\tanh(bx+a)}{2}+\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	27
default	$\frac{\frac{\operatorname{sech}(bx+a)\tanh(bx+a)}{2}+\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	27
risch	$\frac{\mathrm{e}^{bx+a}\left(\mathrm{e}^{2bx+2a}-1\right)}{b\left(1+\mathrm{e}^{2bx+2a}\right)^{2}}+\frac{i\ln\left(\mathrm{e}^{bx+a}+i\right)}{2b}-\frac{i\ln\left(\mathrm{e}^{bx+a}-i\right)}{2b}$	68
parallelrisch	$\frac{i(-1-\cosh(2bx+2a))\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-i\right)+i(1+\cosh(2bx+2a))\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+i\right)+2\sinh(bx+a)}{2b(1+\cosh(2bx+2a))}$	84

input
```
int(sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*sech(b*x+a)*tanh(b*x+a)+arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(30) = 60$.

Time = 0.08 (sec) , antiderivative size = 267, normalized size of antiderivative = 7.85

$$\int \operatorname{sech}^3(a+bx)\,dx = \frac{\cosh(bx+a)^3 + 3\cosh(bx+a)\sinh(bx+a)^2 + \sinh(bx+a)^3 + \left(\cosh(bx+a)^4 + 4\cosh(bx+a)\sinh\right.}{b\cosh(bx+a)^4 + 4\,b\cosh(}$$

input
```
integrate(sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (co
sh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*c
osh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)
^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a
)) + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))/(b*cosh(b*x +
a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*
x + a)^2 + 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a
)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{sech}^3(a+bx)\,dx = \int \operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)**3,x)
```

output
```
Integral(sech(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(30) = 60$.

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.91

$$\int \operatorname{sech}^3(a+bx)\,dx = -\frac{\arctan\left(e^{(-bx-a)}\right)}{b} + \frac{e^{(-bx-a)} - e^{(-3\,bx-3\,a)}}{b(2\,e^{(-2\,bx-2\,a)} + e^{(-4\,bx-4\,a)} + 1)}$$

input
```
integrate(sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
-arctan(e^(-b*x - a))/b + (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*
x - 2*a) + e^(-4*b*x - 4*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(30) = 60$.

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.24

$$\int \operatorname{sech}^3(a+bx)\,dx = \frac{\pi + \frac{4\left(e^{(bx+a)} - e^{(-bx-a)}\right)}{\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2 + 4} + 2\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)} - 1\right)e^{(-bx-a)}\right)}{4\,b}$$

input
```
integrate(sech(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*(pi + 4*(e^(b*x + a) - e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 +
 4) + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.38

$$\int \operatorname{sech}^3(a+bx)\,dx = \frac{\operatorname{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{2\,e^{a+b\,x}}{b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} + \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/cosh(a + b*x)^3,x)
```

output
```
atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b)/(b^2)^(1/2) - (2*exp(a + b*x))/(b*(2
*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) + exp(a + b*x)/(b*(exp(2*a + 2*
b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.85

$$\int \operatorname{sech}^3(a+bx)\,dx = \frac{e^{4bx+4a}atan\left(e^{bx+a}\right)+2e^{2bx+2a}atan\left(e^{bx+a}\right)+atan\left(e^{bx+a}\right)+e^{3bx+3a}-e^{bx+a}}{b\left(e^{4bx+4a}+2e^{2bx+2a}+1\right)}$$

input
```
int(sech(b*x+a)^3,x)
```

output
```
(e**(4*a + 4*b*x)*atan(e**(a + b*x)) + 2*e**(2*a + 2*b*x)*atan(e**(a + b*x
)) + atan(e**(a + b*x)) + e**(3*a + 3*b*x) - e**(a + b*x))/(b*(e**(4*a + 4
*b*x) + 2*e**(2*a + 2*b*x) + 1))
```

3.4 $\int \operatorname{sech}^4(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 26

$$\int \operatorname{sech}^4(a+bx)\,dx=\frac{\tanh(a+bx)}{b}-\frac{\tanh^3(a+bx)}{3b}$$

output
```
tanh(b*x+a)/b-1/3*tanh(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^4(a+bx)\,dx=\frac{\tanh(a+bx)}{b}-\frac{\tanh^3(a+bx)}{3b}$$

input
```
Integrate[Sech[a + b*x]^4,x]
```

output
```
Tanh[a + b*x]/b - Tanh[a + b*x]^3/(3*b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(ia+ibx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{4254}$$

$$\frac{i\int \left(1-\tanh^2(a+bx)\right)d(-i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(\frac{1}{3}i\tanh^3(a+bx)-i\tanh(a+bx)\right)}{b}$$

input
```
Int[Sech[a + b*x]^4,x]
```

output
```
(I*((-I)*Tanh[a + b*x] + (I/3)*Tanh[a + b*x]^3))/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(bx+a)^2}{3}\right)\tanh(bx+a)}{b}$	23
default	$\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(bx+a)^2}{3}\right)\tanh(bx+a)}{b}$	23
risch	$-\frac{4(3\,\mathrm{e}^{2bx+2a}+1)}{3b(1+\mathrm{e}^{2bx+2a})^3}$	32
parallelrisch	$\frac{6\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^5+4\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^3+6\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)}{3b\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^3}$	59

input
```
int(sech(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(2/3+1/3*sech(b*x+a)^2)*tanh(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(24) = 48.

Time = 0.07 (sec) , antiderivative size = 164, normalized size of antiderivative = 6.31

$$\int \operatorname{sech}^4(a+bx)\,dx =$$

$$-\frac{}{3\left(b\cosh\left(bx+a\right)^5+5\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4+b\sinh\left(bx+a\right)^5+3\,b\cosh\left(bx+a\right)^3+\left(10\,b\cos\right.\right.}$$

input
```
integrate(sech(b*x+a)^4,x, algorithm="fricas")
```

output
```
-8/3*(2*cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^5 + 5*b*cosh(b*x +
 a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + (10*b*cosh
(b*x + a)^2 + 3*b)*sinh(b*x + a)^3 + (10*b*cosh(b*x + a)^3 + 9*b*cosh(b*x
+ a))*sinh(b*x + a)^2 + 4*b*cosh(b*x + a) + (5*b*cosh(b*x + a)^4 + 9*b*cos
h(b*x + a)^2 + 2*b)*sinh(b*x + a))
```

Sympy [F]

$$\int \operatorname{sech}^4(a + bx) \, dx = \int \operatorname{sech}^4 (a + bx) \, dx$$

input
```
integrate(sech(b*x+a)**4,x)
```

output
```
Integral(sech(a + b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(24) = 48.

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.46

$$\int \operatorname{sech}^4(a + bx) \, dx = \frac{4 e^{(-2 bx - 2 a)}}{b(3 e^{(-2 bx - 2 a)} + 3 e^{(-4 bx - 4 a)} + e^{(-6 bx - 6 a)} + 1)} + \frac{4}{3 b(3 e^{(-2 bx - 2 a)} + 3 e^{(-4 bx - 4 a)} + e^{(-6 bx - 6 a)} + 1)}$$

input
```
integrate(sech(b*x+a)^4,x, algorithm="maxima")
```

output
```
4*e^(-2*b*x - 2*a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x
 - 6*a) + 1)) + 4/3/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*
x - 6*a) + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \operatorname{sech}^4(a+bx)\,dx = -\frac{4\left(3\,e^{(2\,bx+2\,a)}+1\right)}{3\,b\left(e^{(2\,bx+2\,a)}+1\right)^3}$$

input
```
integrate(sech(b*x+a)^4,x, algorithm="giac")
```

output
```
-4/3*(3*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \operatorname{sech}^4(a+bx)\,dx = -\frac{4\left(3\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}{3\,b\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)^3}$$

input
```
int(1/cosh(a + b*x)^4,x)
```

output
```
-(4*(3*exp(2*a + 2*b*x) + 1))/(3*b*(exp(2*a + 2*b*x) + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.19

$$\int \operatorname{sech}^4(a+bx)\,dx = \frac{-4e^{2bx+2a}-\frac{4}{3}}{b\left(e^{6bx+6a}+3e^{4bx+4a}+3e^{2bx+2a}+1\right)}$$

input
```
int(sech(b*x+a)^4,x)
```

output
```
(4*( - 3*e**(2*a + 2*b*x) - 1))/(3*b*(e**(6*a + 6*b*x) + 3*e**(4*a + 4*b*x
) + 3*e**(2*a + 2*b*x) + 1))
```

3.5 $\int \operatorname{sech}^5(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 55

$$\int \operatorname{sech}^5(a+bx)\,dx=\frac{3\arctan(\sinh(a+bx))}{8b}+\frac{3\operatorname{sech}(a+bx)\tanh(a+bx)}{8b}+\frac{\operatorname{sech}^3(a+bx)\tanh(a+bx)}{4b}$$

output
```
3/8*arctan(sinh(b*x+a))/b+3/8*sech(b*x+a)*tanh(b*x+a)/b+1/4*sech(b*x+a)^3*
tanh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^5(a+bx)\,dx=\frac{3\arctan(\sinh(a+bx))}{8b}+\frac{3\operatorname{sech}(a+bx)\tanh(a+bx)}{8b}+\frac{\operatorname{sech}^3(a+bx)\tanh(a+bx)}{4b}$$

input
```
Integrate[Sech[a + b*x]^5,x]
```

output
```
(3*ArcTan[Sinh[a + b*x]])/(8*b) + (3*Sech[a + b*x]*Tanh[a + b*x])/(8*b) +
(Sech[a + b*x]^3*Tanh[a + b*x])/(4*b)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}^5(a+bx) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(ia+ibx+\frac{\pi}{2}\right)^5 dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{3}{4}\int \text{sech}^3(a+bx)dx+\frac{\tanh(a+bx)\text{sech}^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(a+bx)\text{sech}^3(a+bx)}{4b}+\frac{3}{4}\int \csc\left(ia+ibx+\frac{\pi}{2}\right)^3 dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \text{sech}(a+bx)dx+\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}\right)+\frac{\tanh(a+bx)\text{sech}^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(a+bx)\text{sech}^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}+\frac{1}{2}\int \csc\left(ia+ibx+\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \text{4257}$$

$$\frac{3}{4}\left(\frac{\arctan(\sinh(a+bx))}{2b}+\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}\right)+\frac{\tanh(a+bx)\text{sech}^3(a+bx)}{4b}$$

input
```
Int[Sech[a + b*x]^5,x]
```

output
```
(Sech[a + b*x]^3*Tanh[a + b*x])/(4*b) + (3*(ArcTan[Sinh[a + b*x]]/(2*b) +
(Sech[a + b*x]*Tanh[a + b*x])/(2*b)))/4
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)+\frac{3\arctan\left(e^{bx+a}\right)}{4}}{b}$
default	$\frac{\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)+\frac{3\arctan\left(e^{bx+a}\right)}{4}}{b}$
risch	$\frac{e^{bx+a}\left(3\,e^{6bx+6a}+11\,e^{4bx+4a}-11\,e^{2bx+2a}-3\right)}{4b\left(1+e^{2bx+2a}\right)^4}+\frac{3i\ln\left(e^{bx+a}+i\right)}{8b}-\frac{3i\ln\left(e^{bx+a}-i\right)}{8b}$
parallelrisch	$\frac{3i(-\cosh(4bx+4a)-4\cosh(2bx+2a)-3)\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-i\right)+3i(3+\cosh(4bx+4a)+4\cosh(2bx+2a))\ln\left(\tanh\left(\frac{bx}{2}+\right.\right.}{8b(3+\cosh(4bx+4a)+4\cosh(2bx+2a))}$

input
```
int(sech(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*((1/4*sech(b*x+a)^3+3/8*sech(b*x+a))*tanh(b*x+a)+3/4*arctan(exp(b*x+a)
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 812 vs. $2(49) = 98$.

Time = 0.08 (sec) , antiderivative size = 812, normalized size of antiderivative = 14.76

$$\int \operatorname{sech}^5(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sech(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/4*(3*cosh(b*x + a)^7 + 21*cosh(b*x + a)*sinh(b*x + a)^6 + 3*sinh(b*x + a
)^7 + (63*cosh(b*x + a)^2 + 11)*sinh(b*x + a)^5 + 11*cosh(b*x + a)^5 + 5*(
21*cosh(b*x + a)^3 + 11*cosh(b*x + a))*sinh(b*x + a)^4 + (105*cosh(b*x + a
)^4 + 110*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^3 - 11*cosh(b*x + a)^3 + (63
*cosh(b*x + a)^5 + 110*cosh(b*x + a)^3 - 33*cosh(b*x + a))*sinh(b*x + a)^2
 + 3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8
+ 4*(7*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cos
h(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 +
30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*
x + a)^5 + 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*co
sh(b*x + a)^6 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^
2 + 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*
x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b
*x + a)) + (21*cosh(b*x + a)^6 + 55*cosh(b*x + a)^4 - 33*cosh(b*x + a)^2 -
 3)*sinh(b*x + a) - 3*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a
)*sinh(b*x + a)^7 + b*sinh(b*x + a)^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(
b*x + a)^2 + b)*sinh(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x +
a))*sinh(b*x + a)^5 + 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b
*cosh(b*x + a)^2 + 3*b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*co
sh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 4*b*cosh(b*x + a)^...
```

Sympy [F]

$$\int \operatorname{sech}^5(a+bx)\,dx=\int \operatorname{sech}^5(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)**5,x)
```

output
```
Integral(sech(a + b*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(49) = 98.

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.04

$$\int \operatorname{sech}^5(a+bx)\,dx=-\frac{3\arctan\left(e^{(-bx-a)}\right)}{4b}+\frac{3e^{(-bx-a)}+11e^{(-3bx-3a)}-11e^{(-5bx-5a)}-3e^{(-7bx-7a)}}{4b(4e^{(-2bx-2a)}+6e^{(-4bx-4a)}+4e^{(-6bx-6a)}+e^{(-8bx-8a)}+1)}$$

input
```
integrate(sech(b*x+a)^5,x, algorithm="maxima")
```

output
```
-3/4*arctan(e^(-b*x - a))/b + 1/4*(3*e^(-b*x - a) + 11*e^(-3*b*x - 3*a) -
11*e^(-5*b*x - 5*a) - 3*e^(-7*b*x - 7*a))/(b*(4*e^(-2*b*x - 2*a) + 6*e^(-4
*b*x - 4*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(49) = 98.

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.85

$$\int \operatorname{sech}^5(a+bx)\,dx=\frac{3\pi+\frac{4\left(3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+20e^{(bx+a)}-20e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^2}+6\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)}-1\right)e^{(-bx-a)}\right)}{16b}$$

input
```
integrate(sech(b*x+a)^5,x, algorithm="giac")
```

output
```
1/16*(3*pi + 4*(3*(e^(b*x + a) - e^(-b*x - a))^3 + 20*e^(b*x + a) - 20*e^(
-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4)^2 + 6*arctan(1/2*(e^(2*b*x
 + 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 189, normalized size of antiderivative = 3.44

$$\int \operatorname{sech}^5(a+bx)\,dx = \frac{3\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{4\,\sqrt{b^2}}+\frac{e^{a+b\,x}}{2\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}-\frac{2\,e^{a+b\,x}}{b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)}-\frac{4\,e^{3\,a+3\,b\,x}}{b\,\left(4\,e^{2\,a+2\,b\,x}+6\,e^{4\,a+4\,b\,x}+4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)}+\frac{3\,e^{a+b\,x}}{4\,b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/cosh(a + b*x)^5,x)
```

output
```
(3*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(4*(b^2)^(1/2)) + exp(a + b*x)/(
2*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (2*exp(a + b*x))/(b*(3*
exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) - (4*exp(3*
a + 3*b*x))/(b*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6*a + 6*b*
x) + exp(8*a + 8*b*x) + 1)) + (3*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) + 1)
)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 191, normalized size of antiderivative = 3.47

$$\int \operatorname{sech}^5(a+bx)\,dx$$
$$= \frac{3e^{8bx+8a}atan\left(e^{bx+a}\right)+12e^{6bx+6a}atan\left(e^{bx+a}\right)+18e^{4bx+4a}atan\left(e^{bx+a}\right)+12e^{2bx+2a}atan\left(e^{bx+a}\right)+3atan\left(e^{b}\right.}{4b\left(e^{8bx+8a}+4e^{6bx+6a}+6e^{4bx+4a}+4e^{2bx+2a}+1\right)}$$

input
```
int(sech(b*x+a)^5,x)
```

output
```
(3*e**(8*a + 8*b*x)*atan(e**(a + b*x)) + 12*e**(6*a + 6*b*x)*atan(e**(a +
b*x)) + 18*e**(4*a + 4*b*x)*atan(e**(a + b*x)) + 12*e**(2*a + 2*b*x)*atan(
e**(a + b*x)) + 3*atan(e**(a + b*x)) + 3*e**(7*a + 7*b*x) + 11*e**(5*a + 5
*b*x) - 11*e**(3*a + 3*b*x) - 3*e**(a + b*x))/(4*b*(e**(8*a + 8*b*x) + 4*e
**(6*a + 6*b*x) + 6*e**(4*a + 4*b*x) + 4*e**(2*a + 2*b*x) + 1))
```

3.6 $\int \operatorname{sech}^6(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \operatorname{sech}^6(a+bx)\,dx=\frac{\tanh(a+bx)}{b}-\frac{2\tanh^3(a+bx)}{3b}+\frac{\tanh^5(a+bx)}{5b}$$

output
```
tanh(b*x+a)/b-2/3*tanh(b*x+a)^3/b+1/5*tanh(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^6(a+bx)\,dx=\frac{\tanh(a+bx)}{b}-\frac{2\tanh^3(a+bx)}{3b}+\frac{\tanh^5(a+bx)}{5b}$$

input
```
Integrate[Sech[a + b*x]^6,x]
```

output
```
Tanh[a + b*x]/b - (2*Tanh[a + b*x]^3)/(3*b) + Tanh[a + b*x]^5/(5*b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^6(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(ia+ibx+\frac{\pi}{2}\right)^6 dx$$

$$\Big\downarrow \text{ 4254}$$

$$\frac{i\int \left(\tanh^4(a+bx)-2\tanh^2(a+bx)+1\right)d(-i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{i\left(-\frac{1}{5}i\tanh^5(a+bx)+\frac{2}{3}i\tanh^3(a+bx)-i\tanh(a+bx)\right)}{b}$$

input
```
Int[Sech[a + b*x]^6,x]
```

output
```
(I*((-I)*Tanh[a + b*x] + ((2*I)/3)*Tanh[a + b*x]^3 - (I/5)*Tanh[a + b*x]^5
))/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(bx+a)^4}{5}+\frac{4\operatorname{sech}(bx+a)^2}{15}\right)\tanh(bx+a)}{b}$	33
default	$\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(bx+a)^4}{5}+\frac{4\operatorname{sech}(bx+a)^2}{15}\right)\tanh(bx+a)}{b}$	33
risch	$-\frac{16\left(10\,e^{4bx+4a}+5\,e^{2bx+2a}+1\right)}{15b\left(1+e^{2bx+2a}\right)^5}$	43
parallelrisch	$\frac{\frac{8\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^7}{3}+\frac{8\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{3}+\frac{116\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^5}{15}+2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^9}{b\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^5}$	85

input
```
int(sech(b*x+a)^6,x,method=_RETURNVERBOSE)
```

output
```
1/b*(8/15+1/5*sech(b*x+a)^4+4/15*sech(b*x+a)^2)*tanh(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(37) = 74$.

Time = 0.07 (sec) , antiderivative size = 344, normalized size of antiderivative = 8.39

$$\int \operatorname{sech}^6(a+bx)\,dx =$$

$$-\frac{}{15\left(b\cosh\left(bx+a\right)^8+8\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^7+b\sinh\left(bx+a\right)^8+5\,b\cosh\left(bx+a\right)^6+\left(28\,b\,\mathrm{co}\right.\right.}$$

input
```
integrate(sech(b*x+a)^6,x, algorithm="fricas")
```

output
```
-16/15*(11*cosh(b*x + a)^2 + 18*cosh(b*x + a)*sinh(b*x + a) + 11*sinh(b*x
+ a)^2 + 5)/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sin
h(b*x + a)^8 + 5*b*cosh(b*x + a)^6 + (28*b*cosh(b*x + a)^2 + 5*b)*sinh(b*x
 + a)^6 + 2*(28*b*cosh(b*x + a)^3 + 15*b*cosh(b*x + a))*sinh(b*x + a)^5 +
10*b*cosh(b*x + a)^4 + 5*(14*b*cosh(b*x + a)^4 + 15*b*cosh(b*x + a)^2 + 2*
b)*sinh(b*x + a)^4 + 4*(14*b*cosh(b*x + a)^5 + 25*b*cosh(b*x + a)^3 + 10*b
*cosh(b*x + a))*sinh(b*x + a)^3 + 11*b*cosh(b*x + a)^2 + (28*b*cosh(b*x +
a)^6 + 75*b*cosh(b*x + a)^4 + 60*b*cosh(b*x + a)^2 + 11*b)*sinh(b*x + a)^2
 + 2*(4*b*cosh(b*x + a)^7 + 15*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3 +
9*b*cosh(b*x + a))*sinh(b*x + a) + 5*b)
```

Sympy [F]

$$\int \operatorname{sech}^6(a+bx)\,dx = \int \operatorname{sech}^6\left(a+bx\right)dx$$

input
```
integrate(sech(b*x+a)**6,x)
```

output
```
Integral(sech(a + b*x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(37) = 74$.

Time = 0.04 (sec) , antiderivative size = 205, normalized size of antiderivative = 5.00

$$\int \operatorname{sech}^6(a+bx)\,dx$$
$$= \frac{16\,e^{(-2\,bx-2\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)} + \frac{32\,e^{(-4\,bx-4\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)} + \frac{16}{15\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)}$$

input
```
integrate(sech(b*x+a)^6,x, algorithm="maxima")
```

output
```
16/3*e^(-2*b*x - 2*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^
(-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) + 32/3*e^(-
4*b*x - 4*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x -
 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) + 16/15/(b*(5*e^(-2*
b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a
) + e^(-10*b*x - 10*a) + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \operatorname{sech}^6(a+bx)\,dx = -\frac{16\left(10\,e^{(4\,bx+4\,a)}+5\,e^{(2\,bx+2\,a)}+1\right)}{15\,b\left(e^{(2\,bx+2\,a)}+1\right)^5}$$

input
```
integrate(sech(b*x+a)^6,x, algorithm="giac")
```

output
```
-16/15*(10*e^(4*b*x + 4*a) + 5*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) +
1)^5)
```

Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \operatorname{sech}^6(a+bx)\,dx = -\frac{16\left(5\,e^{2\,a+2\,b\,x}+10\,e^{4\,a+4\,b\,x}+1\right)}{15\,b\left(e^{2\,a+2\,b\,x}+1\right)^5}$$

input
```
int(1/cosh(a + b*x)^6,x)
```

output
```
-(16*(5*exp(2*a + 2*b*x) + 10*exp(4*a + 4*b*x) + 1))/(15*b*(exp(2*a + 2*b*
x) + 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.27

$$\int \operatorname{sech}^6(a+bx)\,dx = \frac{-\frac{32e^{4bx+4a}}{3}-\frac{16e^{2bx+2a}}{3}-\frac{16}{15}}{b\left(e^{10bx+10a}+5e^{8bx+8a}+10e^{6bx+6a}+10e^{4bx+4a}+5e^{2bx+2a}+1\right)}$$

input
```
int(sech(b*x+a)^6,x)
```

output
```
(16*( - 10*e**(4*a + 4*b*x) - 5*e**(2*a + 2*b*x) - 1))/(15*b*(e**(10*a + 1
0*b*x) + 5*e**(8*a + 8*b*x) + 10*e**(6*a + 6*b*x) + 10*e**(4*a + 4*b*x) +
5*e**(2*a + 2*b*x) + 1))
```

3.7 $\int \operatorname{sech}^4(7x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 19

$$\int \operatorname{sech}^4(7x)\,dx = \frac{1}{7}\tanh(7x) - \frac{1}{21}\tanh^3(7x)$$

output
```
1/7*tanh(7*x)-1/21*tanh(7*x)^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^4(7x)\,dx = \frac{1}{7}\tanh(7x) - \frac{1}{21}\tanh^3(7x)$$

input
```
Integrate[Sech[7*x]^4,x]
```

output
```
Tanh[7*x]/7 - Tanh[7*x]^3/21
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^4(7x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(\frac{\pi}{2}+7ix\right)^4 dx$$

$$\Big\downarrow \; 4254$$

$$\frac{1}{7}i\int \left(1-\tanh^2(7x)\right)d(-i\tanh(7x))$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{7}i\left(\frac{1}{3}i\tanh^3(7x)-i\tanh(7x)\right)$$

input
```
Int[Sech[7*x]^4,x]
```

output
```
(I/7)*((-I)*Tanh[7*x] + (I/3)*Tanh[7*x]^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(7x)^2}{3}\right)\tanh(7x)}{7}$	17
default	$\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(7x)^2}{3}\right)\tanh(7x)}{7}$	17
risch	$-\frac{4(3\,e^{14x}+1)}{21(e^{14x}+1)^3}$	19
parallelrisch	$\frac{6\tanh\left(\frac{7x}{2}\right)^5+4\tanh\left(\frac{7x}{2}\right)^3+6\tanh\left(\frac{7x}{2}\right)}{21\left(1+\tanh\left(\frac{7x}{2}\right)^2\right)^3}$	36

input
```
int(sech(7*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/7*(2/3+1/3*sech(7*x)^2)*tanh(7*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(15) = 30.

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 6.11

$$\int \operatorname{sech}^4(7x)\,dx =$$

$$-\frac{8\,(2\,\cosh\,(}{21\,\left(\cosh\,(7\,x)^5+5\,\cosh\,(7\,x)\sinh\,(7\,x)^4+\sinh\,(7\,x)^5+\left(10\,\cosh\,(7\,x)^2+3\right)\sinh\,(7\,x)^3+3\,\cosh\,(7\,x)}$$

input
```
integrate(sech(7*x)^4,x, algorithm="fricas")
```

output
```
-8/21*(2*cosh(7*x) + sinh(7*x))/(cosh(7*x)^5 + 5*cosh(7*x)*sinh(7*x)^4 + s
inh(7*x)^5 + (10*cosh(7*x)^2 + 3)*sinh(7*x)^3 + 3*cosh(7*x)^3 + (10*cosh(7
*x)^3 + 9*cosh(7*x))*sinh(7*x)^2 + (5*cosh(7*x)^4 + 9*cosh(7*x)^2 + 2)*sin
h(7*x) + 4*cosh(7*x))
```

Sympy [F]

$$\int \operatorname{sech}^4(7x)\,dx = \int \operatorname{sech}^4(7x)\,dx$$

input
```
integrate(sech(7*x)**4,x)
```

output
```
Integral(sech(7*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(15) = 30$.

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.58

$$\int \operatorname{sech}^4(7x)\,dx = \frac{4\,e^{(-14\,x)}}{7\,(3\,e^{(-14\,x)} + 3\,e^{(-28\,x)} + e^{(-42\,x)} + 1)} + \frac{4}{21\,(3\,e^{(-14\,x)} + 3\,e^{(-28\,x)} + e^{(-42\,x)} + 1)}$$

input
```
integrate(sech(7*x)^4,x, algorithm="maxima")
```

output
```
4/7*e^(-14*x)/(3*e^(-14*x) + 3*e^(-28*x) + e^(-42*x) + 1) + 4/21/(3*e^(-14
*x) + 3*e^(-28*x) + e^(-42*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \operatorname{sech}^4(7x)\,dx = -\frac{4\,\left(3\,e^{(14\,x)}+1\right)}{21\,\left(e^{(14\,x)}+1\right)^3}$$

input
```
integrate(sech(7*x)^4,x, algorithm="giac")
```

output
```
-4/21*(3*e^(14*x) + 1)/(e^(14*x) + 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \operatorname{sech}^4(7x)\,dx = -\frac{2\,\left(3\,\mathrm{e}^{14\,x}-3\,\mathrm{e}^{28\,x}-\mathrm{e}^{42\,x}+1\right)}{21\,\left(\mathrm{e}^{14\,x}+1\right)^3}$$

input
```
int(1/cosh(7*x)^4,x)
```

output
```
-(2*(3*exp(14*x) - 3*exp(28*x) - exp(42*x) + 1))/(21*(exp(14*x) + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \operatorname{sech}^4(7x)\,dx = \frac{-12e^{14x}-4}{21e^{42x}+63e^{28x}+63e^{14x}+21}$$

input
```
int(sech(7*x)^4,x)
```

output
```
(4*( - 3*e**(14*x) - 1))/(21*(e**(42*x) + 3*e**(28*x) + 3*e**(14*x) + 1))
```

3.8 $\int \operatorname{sech}^6(\pi x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 35

$$\int \operatorname{sech}^6(\pi x)\,dx = \frac{\tanh(\pi x)}{\pi} - \frac{2\tanh^3(\pi x)}{3\pi} + \frac{\tanh^5(\pi x)}{5\pi}$$

output
```
tanh(Pi*x)/Pi-2/3*tanh(Pi*x)^3/Pi+1/5*tanh(Pi*x)^5/Pi
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^6(\pi x)\,dx = \frac{\tanh(\pi x)}{\pi} - \frac{2\tanh^3(\pi x)}{3\pi} + \frac{\tanh^5(\pi x)}{5\pi}$$

input
```
Integrate[Sech[Pi*x]^6,x]
```

output
```
Tanh[Pi*x]/Pi - (2*Tanh[Pi*x]^3)/(3*Pi) + Tanh[Pi*x]^5/(5*Pi)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^6(\pi x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(\frac{\pi}{2} + i\pi x\right)^6 dx$$

$$\Big\downarrow \text{4254}$$

$$\frac{i \int \left(\tanh^4(\pi x) - 2\tanh^2(\pi x) + 1\right) d(-i\tanh(\pi x))}{\pi}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-\frac{1}{5} i \tanh^5(\pi x) + \frac{2}{3} i \tanh^3(\pi x) - i\tanh(\pi x)\right)}{\pi}$$

input
```
Int[Sech[Pi*x]^6,x]
```

output
```
(I*((-I)*Tanh[Pi*x] + ((2*I)/3)*Tanh[Pi*x]^3 - (I/5)*Tanh[Pi*x]^5))/Pi
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(\pi x)^{4}}{5}+\frac{4\operatorname{sech}(\pi x)^{2}}{15}\right)\tanh(\pi x)}{\pi}$	27
default	$\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(\pi x)^{4}}{5}+\frac{4\operatorname{sech}(\pi x)^{2}}{15}\right)\tanh(\pi x)}{\pi}$	27
risch	$-\frac{16\left(10\,{\mathrm e}^{4\pi x}+5\,{\mathrm e}^{2\pi x}+1\right)}{15\pi\left({\mathrm e}^{2\pi x}+1\right)^{5}}$	31
parallelrisch	$\frac{2\tanh\left(\frac{\pi x}{2}\right)+\frac{8\tanh\left(\frac{\pi x}{2}\right)^{7}}{3}+2\tanh\left(\frac{\pi x}{2}\right)^{9}+\frac{8\tanh\left(\frac{\pi x}{2}\right)^{3}}{3}+\frac{116\tanh\left(\frac{\pi x}{2}\right)^{5}}{15}}{\pi\left(1+\tanh\left(\frac{\pi x}{2}\right)^{2}\right)^{5}}$	61

input
```
int(sech(Pi*x)^6,x,method=_RETURNVERBOSE)
```

output
```
1/Pi*(8/15+1/5*sech(Pi*x)^4+4/15*sech(Pi*x)^2)*tanh(Pi*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(31) = 62$.

Time = 0.10 (sec) , antiderivative size = 280, normalized size of antiderivative = 8.00

$$\int \operatorname{sech}^6(\pi x)\,dx =$$

$$-\frac{}{15\left(5\pi+\pi\cosh(\pi x)^{8}+8\pi\cosh(\pi x)\sinh(\pi x)^{7}+\pi\sinh(\pi x)^{8}+5\pi\cosh(\pi x)^{6}+\left(5\pi+28\pi\cosh(\right.\right.}$$

input
```
integrate(sech(pi*x)^6,x, algorithm="fricas")
```

output
```
-16/15*(11*cosh(pi*x)^2 + 18*cosh(pi*x)*sinh(pi*x) + 11*sinh(pi*x)^2 + 5)/
(5*pi + pi*cosh(pi*x)^8 + 8*pi*cosh(pi*x)*sinh(pi*x)^7 + pi*sinh(pi*x)^8 +
 5*pi*cosh(pi*x)^6 + (5*pi + 28*pi*cosh(pi*x)^2)*sinh(pi*x)^6 + 2*(28*pi*c
osh(pi*x)^3 + 15*pi*cosh(pi*x))*sinh(pi*x)^5 + 10*pi*cosh(pi*x)^4 + 5*(2*p
i + 14*pi*cosh(pi*x)^4 + 15*pi*cosh(pi*x)^2)*sinh(pi*x)^4 + 4*(14*pi*cosh(
pi*x)^5 + 25*pi*cosh(pi*x)^3 + 10*pi*cosh(pi*x))*sinh(pi*x)^3 + 11*pi*cosh
(pi*x)^2 + (11*pi + 28*pi*cosh(pi*x)^6 + 75*pi*cosh(pi*x)^4 + 60*pi*cosh(p
i*x)^2)*sinh(pi*x)^2 + 2*(4*pi*cosh(pi*x)^7 + 15*pi*cosh(pi*x)^5 + 20*pi*c
osh(pi*x)^3 + 9*pi*cosh(pi*x))*sinh(pi*x))
```

Sympy [F]

$$\int \operatorname{sech}^6(\pi x)\,dx = \int \operatorname{sech}^6(\pi x)\,dx$$

input
```
integrate(sech(pi*x)**6,x)
```

output
```
Integral(sech(pi*x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(31) = 62$.

Time = 0.03 (sec) , antiderivative size = 137, normalized size of antiderivative = 3.91

$$\int \operatorname{sech}^6(\pi x)\,dx = \frac{16\,e^{(-2\,\pi x)}}{3\,\pi(5\,e^{(-2\,\pi x)} + 10\,e^{(-4\,\pi x)} + 10\,e^{(-6\,\pi x)} + 5\,e^{(-8\,\pi x)} + e^{(-10\,\pi x)} + 1)} + \frac{32\,e^{(-4\,\pi x)}}{3\,\pi(5\,e^{(-2\,\pi x)} + 10\,e^{(-4\,\pi x)} + 10\,e^{(-6\,\pi x)} + 5\,e^{(-8\,\pi x)} + e^{(-10\,\pi x)} + 1)} + \frac{16}{15\,\pi(5\,e^{(-2\,\pi x)} + 10\,e^{(-4\,\pi x)} + 10\,e^{(-6\,\pi x)} + 5\,e^{(-8\,\pi x)} + e^{(-10\,\pi x)} + 1)}$$

input
```
integrate(sech(pi*x)^6,x, algorithm="maxima")
```

output
```
16/3*e^(-2*pi*x)/(pi*(5*e^(-2*pi*x) + 10*e^(-4*pi*x) + 10*e^(-6*pi*x) + 5*
e^(-8*pi*x) + e^(-10*pi*x) + 1)) + 32/3*e^(-4*pi*x)/(pi*(5*e^(-2*pi*x) + 1
0*e^(-4*pi*x) + 10*e^(-6*pi*x) + 5*e^(-8*pi*x) + e^(-10*pi*x) + 1)) + 16/1
5/(pi*(5*e^(-2*pi*x) + 10*e^(-4*pi*x) + 10*e^(-6*pi*x) + 5*e^(-8*pi*x) + e
^(-10*pi*x) + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \operatorname{sech}^6(\pi x)\,dx = -\frac{16\left(10\,e^{(4\,\pi x)} + 5\,e^{(2\,\pi x)} + 1\right)}{15\,\pi\left(e^{(2\,\pi x)} + 1\right)^5}$$

input
```
integrate(sech(pi*x)^6,x, algorithm="giac")
```

output
```
-16/15*(10*e^(4*pi*x) + 5*e^(2*pi*x) + 1)/(pi*(e^(2*pi*x) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \operatorname{sech}^6(\pi x)\,dx = -\frac{16\left(5\,\mathrm{e}^{2\,\Pi\,x} + 10\,\mathrm{e}^{4\,\Pi\,x} + 1\right)}{15\,\Pi\,\left(\mathrm{e}^{2\,\Pi\,x} + 1\right)^5}$$

input
```
int(1/cosh(Pi*x)^6,x)
```

output
```
-(16*(5*exp(2*Pi*x) + 10*exp(4*Pi*x) + 1))/(15*Pi*(exp(2*Pi*x) + 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.86

$$\int \operatorname{sech}^6(\pi x)\,dx = \frac{-\frac{32e^{4\pi x}}{3}-\frac{16e^{2\pi x}}{3}-\frac{16}{15}}{\pi\left(e^{10\pi x}+5e^{8\pi x}+10e^{6\pi x}+10e^{4\pi x}+5e^{2\pi x}+1\right)}$$

input
```
int(sech(Pi*x)^6,x)
```

output
```
(16*( - 10*e**(4*pi*x) - 5*e**(2*pi*x) - 1))/(15*pi*(e**(10*pi*x) + 5*e**(
8*pi*x) + 10*e**(6*pi*x) + 10*e**(4*pi*x) + 5*e**(2*pi*x) + 1))
```

3.9 $\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 66

$$\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx = -\frac{2i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\sqrt{\operatorname{sech}(a+bx)}}{3b} + \frac{2\operatorname{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{3b}$$

output
```
-2/3*I*cosh(b*x+a)^(1/2)*InverseJacobiAM(1/2*I*(b*x+a),2^(1/2))*sech(b*x+a
)^(1/2)/b+2/3*sech(b*x+a)^(3/2)*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.77

$$\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx = \frac{2\operatorname{sech}^{\frac{3}{2}}(a+bx)\left(-i\cosh^{\frac{3}{2}}(a+bx)\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)+\sinh(a+bx)\right)}{3b}$$

input
```
Integrate[Sech[a + b*x]^(5/2),x]
```

output
```
(2*Sech[a + b*x]^(3/2)*((-I)*Cosh[a + b*x]^(3/2)*EllipticF[(I/2)*(a + b*x)
, 2] + Sinh[a + b*x]))/(3*b)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(ia+ibx+\frac{\pi}{2}\right)^{5/2}dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{3}\int \sqrt{\operatorname{sech}(a+bx)}dx+\frac{2\sinh(a+bx)\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(a+bx)\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}+\frac{1}{3}\int \sqrt{\csc\left(ia+ibx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{3}\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\int \frac{1}{\sqrt{\cosh(a+bx)}}dx+\frac{2\sinh(a+bx)\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(a+bx)\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}+\frac{1}{3}\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\int \frac{1}{\sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sinh(a+bx)\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b}$$

input
```
Int[Sech[a + b*x]^(5/2),x]
```

output
```
(((-2*I)/3)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a
+ b*x]])/b + (2*Sech[a + b*x]^(3/2)*Sinh[a + b*x])/(3*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(54) = 108$.

Time = 1.60 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.88

method	result
default	$\frac{\sqrt{\left(2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\left(\frac{\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}}{3\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-\frac{1}{2}\right)^{2}}+\frac{2\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\sqrt{-2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1}\ \mathrm{E}}{3\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sinh\left(\right.}}\right.}{\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\,b}$

input
```
int(sech(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(1/3*cosh(1/2*b*
x+1/2*a)*(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/(cosh(1/2*b
*x+1/2*a)^2-1/2)^2+2/3*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2
*a)^2+1)^(1/2)/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)*Ellip
ticF(cosh(1/2*b*x+1/2*a),2^(1/2)))/sinh(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2
*a)^2-1)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(53) = 106$.

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.88

$$\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx = \frac{2\left(\sqrt{2}\left(\cosh(bx+a)^2 + 2\cosh(bx+a)\sinh(bx+a) + \sinh(bx+a)^2 - 1\right)\sqrt{\frac{\cosh(bx+a)+\sinh(b}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx}}}{3\left(b\cosh(\right.}$$

input
```
integrate(sech(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(2)*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +
 a)^2 - 1)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(
b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + (sqrt(2)*cosh(b*x + a)^2
+ 2*sqrt(2)*cosh(b*x + a)*sinh(b*x + a) + sqrt(2)*sinh(b*x + a)^2 + sqrt(2
))*weierstrassPInverse(-4, 0, cosh(b*x + a) + sinh(b*x + a)))/(b*cosh(b*x
+ a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

Sympy [F]

$$\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx = \int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)**(5/2),x)
```

output
```
Integral(sech(a + b*x)**(5/2), x)
```

Maxima [F]

$$\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx = \int \operatorname{sech}(bx+a)^{\frac{5}{2}}\,dx$$

input
```
integrate(sech(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sech(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx = \int \operatorname{sech}(bx+a)^{\frac{5}{2}}\,dx$$

input
```
integrate(sech(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx=\int \left(\frac{1}{\cosh\left(a+b\,x\right)}\right)^{5/2}\,dx$$

input
```
int((1/cosh(a + b*x))^(5/2),x)
```

output
```
int((1/cosh(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx=\int \sqrt{\operatorname{sech}\left(bx+a\right)}\,\operatorname{sech}(bx+a)^{2}\,dx$$

input
```
int(sech(b*x+a)^(5/2),x)
```

output
```
int(sqrt(sech(a + b*x))*sech(a + b*x)**2,x)
```

3.10 $\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 62

$$\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx=\frac{2i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{b}+\frac{2\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)}{b}$$

output
```
2*I*cosh(b*x+a)^(1/2)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))*sech(b*x+a)
^(1/2)/b+2*sech(b*x+a)^(1/2)*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

$$\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx=\frac{2\sqrt{\operatorname{sech}(a+bx)}\left(i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)+\sinh(a+bx)\right)}{b}$$

input
```
Integrate[Sech[a + b*x]^(3/2),x]
```

output
```
(2*Sqrt[Sech[a + b*x]]*(I*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2
] + Sinh[a + b*x]))/b
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(ia+ibx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{2\sinh(a+bx)\sqrt{\operatorname{sech}(a+bx)}}{b}-\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(a+bx)\sqrt{\operatorname{sech}(a+bx)}}{b}-\int \frac{1}{\sqrt{\csc\left(ia+ibx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4258}$$

$$\frac{2\sinh(a+bx)\sqrt{\operatorname{sech}(a+bx)}}{b}-\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\int \sqrt{\cosh(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(a+bx)\sqrt{\operatorname{sech}(a+bx)}}{b}-\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\int \sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\sinh(a+bx)\sqrt{\operatorname{sech}(a+bx)}}{b}+\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

input
```
Int[Sech[a + b*x]^(3/2),x]
```

output
```
((2*I)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x
]])/b + (2*Sqrt[Sech[a + b*x]]*Sinh[a + b*x])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.66

method	result	size
default	$\frac{4\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)+2\sqrt{-2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\,\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\operatorname{EllipticE}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)}{\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\,b}$	103

input
```
int(sech(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(2*sinh(1/2*b*x+1/2*a)^2*cosh(1/2*b*x+1/2*a)+(-2*sinh(1/2*b*x+1/2*a)^2-1
)^(1/2)*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cosh(1/2*b*x+1/2*a),2^(1/
2)))/sinh(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.55

$$\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx = \frac{2\left(\sqrt{2}\sqrt{\frac{\cosh(bx+a)+\sinh(bx+a)}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2+1}}(\cosh(bx+a)+\sinh(bx+a))+\sqrt{2}\text{weierstrassZeta}\right.}{b}$$

input
```
integrate(sech(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2*(sqrt(2)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(
b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1))*(cosh(b*x + a) + sinh(b*x +
 a)) + sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(b*x
+ a) + sinh(b*x + a))))/b
```

Sympy [F]

$$\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx = \int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)**(3/2),x)
```

output
```
Integral(sech(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx = \int \operatorname{sech}(bx+a)^{\frac{3}{2}}\,dx$$

input
```
integrate(sech(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sech(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx = \int \operatorname{sech}(bx+a)^{\frac{3}{2}}\,dx$$

input
```
integrate(sech(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx = \int \left(\frac{1}{\cosh(a+b\,x)}\right)^{3/2}\,dx$$

input
```
int((1/cosh(a + b*x))^(3/2),x)
```

output
```
int((1/cosh(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx = \int \sqrt{\operatorname{sech}(bx+a)}\,\operatorname{sech}(bx+a)\,dx$$

input
```
int(sech(b*x+a)^(3/2),x)
```

output
```
int(sqrt(sech(a + b*x))*sech(a + b*x),x)
```

3.11 $\int \sqrt{\text{sech}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 40

$$\int \sqrt{\text{sech}(a+bx)}\,dx=-\frac{2i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\sqrt{\text{sech}(a+bx)}}{b}$$

output
```
-2*I*cosh(b*x+a)^(1/2)*InverseJacobiAM(1/2*I*(b*x+a),2^(1/2))*sech(b*x+a)^
(1/2)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \sqrt{\text{sech}(a+bx)}\,dx=-\frac{2i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\sqrt{\text{sech}(a+bx)}}{b}$$

input
```
Integrate[Sqrt[Sech[a + b*x]],x]
```

output
```
((-2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*
x]])/b
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\text{sech}(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(ia+ibx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\int \frac{1}{\sqrt{\cosh(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\int \frac{1}{\sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b}$$

input
```
Int[Sqrt[Sech[a + b*x]],x]
```

output
```
((-2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*
x]])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(34) = 68$.

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.38

method	result	size
default	$\frac{2\sqrt{\left(2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^2}\,\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^2}\,\sqrt{-2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1}\ \mathrm{EllipticF}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)}{\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^4+\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^2}\,\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1}\,b}$	135

input
```
int(sech(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-sinh(1/2*b*x
+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)/(2*sinh(1/2*b*x+1/2*a)
^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))/sin
h(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int \sqrt{\operatorname{sech}(a+bx)}\,dx = \frac{2\sqrt{2}\text{weierstrassPInverse}(-4,0,\cosh(bx+a)+\sinh(bx+a))}{b}$$

input
```
integrate(sech(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(2)*weierstrassPInverse(-4, 0, cosh(b*x + a) + sinh(b*x + a))/b
```

Sympy [F]

$$\int \sqrt{\operatorname{sech}(a+bx)}\,dx = \int \sqrt{\operatorname{sech}(a+bx)}\,dx$$

input
```
integrate(sech(b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(sech(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{\operatorname{sech}(a+bx)}\,dx = \int \sqrt{\operatorname{sech}(bx+a)}\,dx$$

input
```
integrate(sech(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sech(b*x + a)), x)
```

Giac [F]

$$\int \sqrt{\operatorname{sech}(a+bx)}\,dx = \int \sqrt{\operatorname{sech}(bx+a)}\,dx$$

input
```
integrate(sech(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sech(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\operatorname{sech}(a+bx)}\,dx = \int \sqrt{\frac{1}{\cosh(a+b\,x)}}\,dx$$

input
```
int((1/cosh(a + b*x))^(1/2),x)
```

output
```
int((1/cosh(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\operatorname{sech}(a+bx)}\,dx = \int \sqrt{\operatorname{sech}(bx+a)}dx$$

input
```
int(sech(b*x+a)^(1/2),x)
```

output
```
int(sqrt(sech(a + b*x)),x)
```

3.12 $\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 40

$$\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}} \, dx = -\frac{2i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{b}$$

output
```
-2*I*cosh(b*x+a)^(1/2)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))*sech(b*x+a
)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}} \, dx = -\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}}$$

input
```
Integrate[1/Sqrt[Sech[a + b*x]],x]
```

output
```
((-2*I)*EllipticE[(I/2)*(a + b*x), 2])/(b*Sqrt[Cosh[a + b*x]]*Sqrt[Sech[a
+ b*x]])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\text{sech}(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(ia+ibx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\int \sqrt{\cosh(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\int \sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

input
```
Int[1/Sqrt[Sech[a + b*x]],x]
```

output
```
((-2*I)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*
x]])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(38) = 76$.

Time = 0.83 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.38

method	result
default	$-\frac{2\sqrt{\left(2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sqrt{-2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1}\,\operatorname{EllipticE}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)}{\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\,b}$
risch	$\frac{\sqrt{2}}{b\sqrt{\frac{e^{bx+a}}{1+e^{2bx+2a}}}}+\frac{\left(-\frac{2\left(1+e^{2bx+2a}\right)}{\sqrt{e^{bx+a}\left(1+e^{2bx+2a}\right)}}+\frac{i\sqrt{-i\left(e^{bx+a}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{bx+a}-i\right)}\,\sqrt{i e^{bx+a}}\left(-2i\,\operatorname{EllipticE}\left(\sqrt{-i\left(e^{bx+a}+i\right)},\frac{\sqrt{2}}{2}\right)+i\,\text{Ellipti}\right.}{\sqrt{e^{3bx+3a}+e^{bx+a}}}\right.}{b\sqrt{\frac{e^{bx+a}}{1+e^{2bx+2a}}}\left(1+e^{2bx+2a}\right)}$

input
```
int(1/sech(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-sinh(1/2*b*
x+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cosh(1/2*b*
x+1/2*a),2^(1/2))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/si
nh(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(36) = 72.

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 3.75

$$\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx = -\frac{\sqrt{2}\left(\cosh\left(bx+a\right)^2 + 2\cosh\left(bx+a\right)\sinh\left(bx+a\right) + \sinh\left(bx+a\right)^2 + 1\right)\sqrt{\frac{\cosh(bx+a)+\sinh(bx}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx}}}{}$$

input
```
integrate(1/sech(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(2)*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)
^2 + 1)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x
 + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + 2*(sqrt(2)*cosh(b*x + a) + s
qrt(2)*sinh(b*x + a))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
sh(b*x + a) + sinh(b*x + a))))/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

Sympy [F]

$$\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx = \int \frac{1}{\sqrt{\operatorname{sech}\left(a+bx\right)}}\,dx$$

input
```
integrate(1/sech(b*x+a)**(1/2),x)
```

output
```
Integral(1/sqrt(sech(a + b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx = \int \frac{1}{\sqrt{\operatorname{sech}\left(bx+a\right)}}\,dx$$

input
```
integrate(1/sech(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(sech(b*x + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx = \int \frac{1}{\sqrt{\operatorname{sech}\left(bx+a\right)}}\,dx$$

input
```
integrate(1/sech(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(sech(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\cosh(a+b\,x)}}}\,dx$$

input
```
int(1/(1/cosh(a + b*x))^(1/2),x)
```

output
```
int(1/(1/cosh(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx = \int \frac{\sqrt{\operatorname{sech}(bx+a)}}{\operatorname{sech}(bx+a)}dx$$

input
```
int(1/sech(b*x+a)^(1/2),x)
```

output
```
int(sqrt(sech(a + b*x))/sech(a + b*x),x)
```

3.13 $\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 66

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\sqrt{\operatorname{sech}(a+bx)}}{3b} + \frac{2\sinh(a+bx)}{3b\sqrt{\operatorname{sech}(a+bx)}}$$

output
```
-2/3*I*cosh(b*x+a)^(1/2)*InverseJacobiAM(1/2*I*(b*x+a),2^(1/2))*sech(b*x+a
)^(1/2)/b+2/3*sinh(b*x+a)/b/sech(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \frac{\sqrt{\operatorname{sech}(a+bx)}\left(-2i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)+\sinh(2(a+bx))\right)}{3b}$$

input
```
Integrate[Sech[a + b*x]^(-3/2),x]
```

output
```
(Sqrt[Sech[a + b*x]]*((-2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x)
, 2] + Sinh[2*(a + b*x)]))/(3*b)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{sech}^{\frac{3}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(ia+ibx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\frac{1}{3}\int \sqrt{\text{sech}(a+bx)}dx+\frac{2\sinh(a+bx)}{3b\sqrt{\text{sech}(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(a+bx)}{3b\sqrt{\text{sech}(a+bx)}}+\frac{1}{3}\int \sqrt{\csc\left(ia+ibx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{3}\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\int \frac{1}{\sqrt{\cosh(a+bx)}}dx+\frac{2\sinh(a+bx)}{3b\sqrt{\text{sech}(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(a+bx)}{3b\sqrt{\text{sech}(a+bx)}}+\frac{1}{3}\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\int \frac{1}{\sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sinh(a+bx)}{3b\sqrt{\text{sech}(a+bx)}}-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b}$$

input
```
Int[Sech[a + b*x]^(-3/2),x]
```

output
```
(((-2*I)/3)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a
+ b*x]])/b + (2*Sinh[a + b*x])/(3*b*Sqrt[Sech[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(54) = 108$.

Time = 1.35 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.64

method	result
default	$\frac{2\sqrt{\left(2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\left(4\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{5}-6\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}+\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\sqrt{-2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1}\ \mathrm{Ellipti}}{3\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\, b}$

input
```
int(1/sech(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(4*cosh(1/2*
b*x+1/2*a)^5-6*cosh(1/2*b*x+1/2*a)^3+(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*co
sh(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))+2*cosh
(1/2*b*x+1/2*a))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/sin
h(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(53) = 106$.

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.38

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \frac{\sqrt{2}\left(\cosh(bx+a)^4 + 4\cosh(bx+a)^3\sinh(bx+a) + 6\cosh(bx+a)^2\sinh(bx+a)^2 + 4\cosh(bx+a)\,\mathrm{s}\right.}{}$$

input
```
integrate(1/sech(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(2)*(cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b
*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a
)^4 - 1)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*
x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + 4*(sqrt(2)*cosh(b*x + a)^2
+ 2*sqrt(2)*cosh(b*x + a)*sinh(b*x + a) + sqrt(2)*sinh(b*x + a)^2)*weierst
rassPInverse(-4, 0, cosh(b*x + a) + sinh(b*x + a)))/(b*cosh(b*x + a)^2 + 2
*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F]

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(1/sech(b*x+a)**(3/2),x)
```

output
```
Integral(sech(a + b*x)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\operatorname{sech}(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sech(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sech(b*x + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\operatorname{sech}(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sech(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\left(\frac{1}{\cosh(a+b\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(1/cosh(a + b*x))^(3/2),x)
```

output
```
int(1/(1/cosh(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\operatorname{sech}(bx+a)}}{\operatorname{sech}(bx+a)^2}\,dx$$

input
```
int(1/sech(b*x+a)^(3/2),x)
```

output
```
int(sqrt(sech(a + b*x))/sech(a + b*x)**2,x)
```

3.14 $\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)} dx$

Optimal result

Integrand size = 10, antiderivative size = 66

$$\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)} dx = -\frac{6i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{5b} + \frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)}$$

output
```
-6/5*I*cosh(b*x+a)^(1/2)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))*sech(b*x
+a)^(1/2)/b+2/5*sinh(b*x+a)/b/sech(b*x+a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)} dx = \frac{\sqrt{\operatorname{sech}(a+bx)}\left(-12i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)+\sinh(a+bx)+\sinh(3(a+bx))\right)}{10b}$$

input
```
Integrate[Sech[a + b*x]^(-5/2),x]
```

output
```
(Sqrt[Sech[a + b*x]]*((-12*I)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x
), 2] + Sinh[a + b*x] + Sinh[3*(a + b*x)]))/(10*b)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(ia+ibx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\frac{3}{5}\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}dx+\frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)}+\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(ia+ibx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4258}$$

$$\frac{3}{5}\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\int \sqrt{\cosh(a+bx)}dx+\frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)}+\frac{3}{5}\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\int \sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)}-\frac{6i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)|2\right)}{5b}$$

input
```
Int[Sech[a + b*x]^(-5/2),x]
```

output
```
(((-6*I)/5)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a
+ b*x]])/b + (2*Sinh[a + b*x])/(5*b*Sech[a + b*x]^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(58) = 116$.

Time = 2.84 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.85

method	result
default	$\frac{2\sqrt{\left(2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\left(8\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{7}-16\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{5}+10\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}-3\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\sqrt{-2\cos}\right.}{5\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-}}$

input
```
int(1/sech(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(8*cosh(1/2*
b*x+1/2*a)^7-16*cosh(1/2*b*x+1/2*a)^5+10*cosh(1/2*b*x+1/2*a)^3-3*(-sinh(1/
2*b*x+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cosh(1/
2*b*x+1/2*a),2^(1/2))-2*cosh(1/2*b*x+1/2*a))/(2*sinh(1/2*b*x+1/2*a)^4+sinh
(1/2*b*x+1/2*a)^2)^(1/2)/sinh(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(
1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(56) = 112$.

Time = 0.10 (sec) , antiderivative size = 370, normalized size of antiderivative = 5.61

$$\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx$$
$$= \frac{\sqrt{2}\left(\cosh\left(bx+a\right)^6 + 6\cosh\left(bx+a\right)\sinh\left(bx+a\right)^5 + \sinh\left(bx+a\right)^6 + \left(15\cosh\left(bx+a\right)^2 - 11\right)\sinh\left(bx\right.}{}$$

input
```
integrate(1/sech(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
1/20*(sqrt(2)*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*
x + a)^6 + (15*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^4 - 11*cosh(b*x + a)^4
+ 4*(5*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x
+ a)^4 - 66*cosh(b*x + a)^2 - 13)*sinh(b*x + a)^2 - 13*cosh(b*x + a)^2 + 2
*(3*cosh(b*x + a)^5 - 22*cosh(b*x + a)^3 - 13*cosh(b*x + a))*sinh(b*x + a)
 - 1)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x +
 a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) - 24*(sqrt(2)*cosh(b*x + a)^3 +
3*sqrt(2)*cosh(b*x + a)^2*sinh(b*x + a) + 3*sqrt(2)*cosh(b*x + a)*sinh(b*x
 + a)^2 + sqrt(2)*sinh(b*x + a)^3)*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cosh(b*x + a) + sinh(b*x + a))))/(b*cosh(b*x + a)^3 + 3*b*cosh(
b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x + a)*sinh(b*x + a)^2 + b*sinh(b*x
+ a)^3)
```

Sympy [F]

$$\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx$$

input
```
integrate(1/sech(b*x+a)**(5/2),x)
```

output
```
Integral(sech(a + b*x)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\operatorname{sech}(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sech(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sech(b*x + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\operatorname{sech}(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sech(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\left(\frac{1}{\cosh(a+b\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(1/cosh(a + b*x))^(5/2),x)
```

output
```
int(1/(1/cosh(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\operatorname{sech}(bx+a)}}{\operatorname{sech}(bx+a)^3}dx$$

input
```
int(1/sech(b*x+a)^(5/2),x)
```

output
```
int(sqrt(sech(a + b*x))/sech(a + b*x)**3,x)
```

3.15 $\int (b\operatorname{sech}(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 102

$$\int (b\operatorname{sech}(c+dx))^{7/2}\,dx = \frac{6ib^4E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5d\sqrt{\cosh(c+dx)}\sqrt{b\operatorname{sech}(c+dx)}} + \frac{6b^3\sqrt{b\operatorname{sech}(c+dx)}\sinh(c+dx)}{5d} + \frac{2b(b\operatorname{sech}(c+dx))^{5/2}\sinh(c+dx)}{5d}$$

output
```
6/5*I*b^4*EllipticE(I*sinh(1/2*d*x+1/2*c),2^(1/2))/d/cosh(d*x+c)^(1/2)/(b*
sech(d*x+c))^(1/2)+6/5*b^3*(b*sech(d*x+c))^(1/2)*sinh(d*x+c)/d+2/5*b*(b*se
ch(d*x+c))^(5/2)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

$$\int (b\operatorname{sech}(c+dx))^{7/2}\,dx = \frac{b^2(b\operatorname{sech}(c+dx))^{3/2}\left(6i\cosh^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}i(c+dx)\middle|2\right)+3\sinh(2(c+dx))+2\tanh(c+dx\right.}{5d}$$

input
```
Integrate[(b*Sech[c + d*x])^(7/2),x]
```

output
```
(b^2*(b*Sech[c + d*x])^(3/2)*((6*I)*Cosh[c + d*x]^(3/2)*EllipticE[(I/2)*(c
 + d*x), 2] + 3*Sinh[2*(c + d*x)] + 2*Tanh[c + d*x]))/(5*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\text{sech}(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^{7/2}dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{3}{5}b^2\int (b\text{sech}(c+dx))^{3/2}dx+\frac{2b\sinh(c+dx)(b\text{sech}(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sinh(c+dx)(b\text{sech}(c+dx))^{5/2}}{5d}+\frac{3}{5}b^2\int \left(b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{3}{5}b^2\left(\frac{2b\sinh(c+dx)\sqrt{b\text{sech}(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}dx\right)+\frac{2b\sinh(c+dx)(b\text{sech}(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sinh(c+dx)(b\text{sech}(c+dx))^{5/2}}{5d}+\frac{3}{5}b^2\left(\frac{2b\sinh(c+dx)\sqrt{b\text{sech}(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$\frac{3}{5}b^2\left(\frac{2b\sinh(c+dx)\sqrt{b\text{sech}(c+dx)}}{d}-\frac{b^2\int\sqrt{\cosh(c+dx)}dx}{\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}\right)+\frac{2b\sinh(c+dx)(b\text{sech}(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sinh(c+dx)(b\text{sech}(c+dx))^{5/2}}{5d}+\frac{3}{5}b^2\left(\frac{2b\sinh(c+dx)\sqrt{b\text{sech}(c+dx)}}{d}-\frac{b^2\int\sqrt{\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$\frac{2b\sinh(c+dx)(b\text{sech}(c+dx))^{5/2}}{5d}+\frac{3}{5}b^2\left(\frac{2b\sinh(c+dx)\sqrt{b\text{sech}(c+dx)}}{d}+\frac{2ib^2E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}\right)$$

input

```
Int[(b*Sech[c + d*x])^(7/2),x]
```

output

```
(2*b*(b*Sech[c + d*x])^(5/2)*Sinh[c + d*x])/(5*d) + (3*b^2*(((2*I)*b^2*Ell
ipticE[(I/2)*(c + d*x), 2])/(d*Sqrt[Cosh[c + d*x]]*Sqrt[b*Sech[c + d*x]])
+ (2*b*Sqrt[b*Sech[c + d*x]]*Sinh[c + d*x])/d))/5
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [F]

$$\int (b\operatorname{sech}(dx+c))^{\frac{7}{2}}\,dx$$

input
```
int((b*sech(d*x+c))^(7/2),x)
```

output
```
int((b*sech(d*x+c))^(7/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(88) = 176.

Time = 0.09 (sec) , antiderivative size = 478, normalized size of antiderivative = 4.69

$$\int (b\operatorname{sech}(c+dx))^{7/2}\,dx = \frac{2\left(3\sqrt{2}\left(b^3\cosh(dx+c)^4+4\,b^3\cosh(dx+c)\sinh(dx+c)^3+b^3\sinh(dx+c)^4+2\,b^3\cosh(\right.\right.}{}$$

input
```
integrate((b*sech(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
2/5*(3*sqrt(2)*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3
+ b^3*sinh(d*x + c)^4 + 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x +
c)^2 + b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*
sinh(d*x + c))*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
osh(d*x + c) + sinh(d*x + c))) + sqrt(2)*(3*b^3*cosh(d*x + c)^5 + 15*b^3*c
osh(d*x + c)*sinh(d*x + c)^4 + 3*b^3*sinh(d*x + c)^5 + 8*b^3*cosh(d*x + c)
^3 + b^3*cosh(d*x + c) + 2*(15*b^3*cosh(d*x + c)^2 + 4*b^3)*sinh(d*x + c)^
3 + 6*(5*b^3*cosh(d*x + c)^3 + 4*b^3*cosh(d*x + c))*sinh(d*x + c)^2 + (15*
b^3*cosh(d*x + c)^4 + 24*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c))*sqrt((b
*cosh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(
d*x + c) + sinh(d*x + c)^2 + 1)))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*s
inh(d*x + c)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x
 + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sin
h(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int (b\text{sech}(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((b*sech(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\text{sech}(c+dx))^{7/2}\,dx = \int (b\,\text{sech}\,(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((b*sech(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sech(d*x + c))^(7/2), x)
```

Giac [F]

$$\int (b\text{sech}(c+dx))^{7/2}\,dx = \int (b\,\text{sech}\,(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((b*sech(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sech(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\text{sech}(c+dx))^{7/2}\,dx = \int \left(\frac{b}{\cosh\,(c+d\,x)}\right)^{7/2}\,dx$$

input
```
int((b/cosh(c + d*x))^(7/2),x)
```

output
```
int((b/cosh(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int (b\text{sech}(c+dx))^{7/2}\,dx = \sqrt{b}\left(\int \sqrt{\text{sech}\,(dx+c)}\,\text{sech}(dx+c)^3\,dx\right)b^3$$

input
```
int((b*sech(d*x+c))^(7/2),x)
```

output
```
sqrt(b)*int(sqrt(sech(c + d*x))*sech(c + d*x)**3,x)*b**3
```

3.16 $\int (b\operatorname{sech}(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 74

$$\int (b\operatorname{sech}(c+dx))^{5/2}\,dx = -\frac{2ib^2\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\operatorname{sech}(c+dx)}}{3d} + \frac{2b(b\operatorname{sech}(c+dx))^{3/2}\sinh(c+dx)}{3d}$$

output

```
-2/3*I*b^2*cosh(d*x+c)^(1/2)*InverseJacobiAM(1/2*I*(d*x+c),2^(1/2))*(b*sec
h(d*x+c))^(1/2)/d+2/3*b*(b*sech(d*x+c))^(3/2)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

$$\int (b\operatorname{sech}(c+dx))^{5/2}\,dx = \frac{2b^2\sqrt{b\operatorname{sech}(c+dx)}\left(-i\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)+\tanh(c+dx)\right)}{3d}$$

input

```
Integrate[(b*Sech[c + d*x])^(5/2),x]
```

output
```
(2*b^2*Sqrt[b*Sech[c + d*x]]*((-I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c
+ d*x), 2] + Tanh[c + d*x]))/(3*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\text{sech}(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{3}b^2\int \sqrt{b\text{sech}(c+dx)}dx+\frac{2b\sinh(c+dx)(b\text{sech}(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sinh(c+dx)(b\text{sech}(c+dx))^{3/2}}{3d}+\frac{1}{3}b^2\int \sqrt{b\csc\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{3}b^2\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}\int \frac{1}{\sqrt{\cosh(c+dx)}}dx+\frac{2b\sinh(c+dx)(b\text{sech}(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sinh(c+dx)(b\text{sech}(c+dx))^{3/2}}{3d}+\frac{1}{3}b^2\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}\int \frac{1}{\sqrt{\sin\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\frac{2b\sinh(c+dx)(b\text{sech}(c+dx))^{3/2}}{3d} - \frac{2ib^2\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\text{sech}(c+dx)}}{3d}$$

input
```
Int[(b*Sech[c + d*x])^(5/2),x]
```

output
```
(((-2*I)/3)*b^2*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*S
ech[c + d*x]])/d + (2*b*(b*Sech[c + d*x])^(3/2)*Sinh[c + d*x])/(3*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [F]

$$\int (b \operatorname{sech}(dx+c))^{\frac{5}{2}} dx$$

input
```
int((b*sech(d*x+c))^(5/2),x)
```

output
```
int((b*sech(d*x+c))^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(61) = 122$.

Time = 0.08 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.91

$$\int (b\text{sech}(c+dx))^{5/2} dx = \frac{2\left(\sqrt{2}\left(b^2\cosh(dx+c)^2 + 2b^2\cosh(dx+c)\sinh(dx+c) + b^2\sinh(dx+c)^2 + b^2\right)\sqrt{b}\text{weiers}\right.}{}$$

input
```
integrate((b*sech(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(2)*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^
2*sinh(d*x + c)^2 + b^2)*sqrt(b)*weierstrassPInverse(-4, 0, cosh(d*x + c)
+ sinh(d*x + c)) + sqrt(2)*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh
(d*x + c) + b^2*sinh(d*x + c)^2 - b^2)*sqrt((b*cosh(d*x + c) + b*sinh(d*x
+ c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 +
 1)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x +
c)^2 + d)
```

Sympy [F]

$$\int (b\text{sech}(c+dx))^{5/2}\,dx = \int (b\,\text{sech}\,(c+dx))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*sech(d*x+c))**(5/2),x)
```

output
```
Integral((b*sech(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int (b\text{sech}(c+dx))^{5/2}\,dx = \int (b\,\text{sech}\,(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*sech(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sech(d*x + c))^(5/2), x)
```

Giac [F]

$$\int (b\text{sech}(c+dx))^{5/2}\,dx = \int (b\,\text{sech}\,(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*sech(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sech(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\text{sech}(c+dx))^{5/2}\,dx = \int \left(\frac{b}{\cosh\left(c+d\,x\right)}\right)^{5/2}\,dx$$

input
```
int((b/cosh(c + d*x))^(5/2),x)
```

output
```
int((b/cosh(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int (b\text{sech}(c+dx))^{5/2}\,dx = \sqrt{b}\left(\int \sqrt{\text{sech}\left(dx+c\right)}\,\text{sech}(dx+c)^2\,dx\right)b^2$$

input
```
int((b*sech(d*x+c))^(5/2),x)
```

output
```
sqrt(b)*int(sqrt(sech(c + d*x))*sech(c + d*x)**2,x)*b**2
```

3.17 $\int (b\text{sech}(c + dx))^{3/2} dx$

Optimal result

Integrand size = 12, antiderivative size = 70

$$\int (b\text{sech}(c+dx))^{3/2}\,dx = \frac{2ib^2E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}} + \frac{2b\sqrt{b\text{sech}(c+dx)}\sinh(c+dx)}{d}$$

output
```
2*I*b^2*EllipticE(I*sinh(1/2*d*x+1/2*c),2^(1/2))/d/cosh(d*x+c)^(1/2)/(b*se
ch(d*x+c))^(1/2)+2*b*(b*sech(d*x+c))^(1/2)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.74

$$\int (b\text{sech}(c+dx))^{3/2}\,dx = \frac{2b\sqrt{b\text{sech}(c+dx)}\left(i\sqrt{\cosh(c+dx)}E\left(\frac{1}{2}i(c+dx)\middle|2\right)+\sinh(c+dx)\right)}{d}$$

input
```
Integrate[(b*Sech[c + d*x])^(3/2),x]
```

output
```
(2*b*Sqrt[b*Sech[c + d*x]]*(I*Sqrt[Cosh[c + d*x]]*EllipticE[(I/2)*(c + d*x
), 2] + Sinh[c + d*x]))/d
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\text{sech}(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{2b\sinh(c+dx)\sqrt{b\text{sech}(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sinh(c+dx)\sqrt{b\text{sech}(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4258}$$

$$\frac{2b\sinh(c+dx)\sqrt{b\text{sech}(c+dx)}}{d}-\frac{b^2\int \sqrt{\cosh(c+dx)}dx}{\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sinh(c+dx)\sqrt{b\text{sech}(c+dx)}}{d}-\frac{b^2\int \sqrt{\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2b\sinh(c+dx)\sqrt{b\text{sech}(c+dx)}}{d}+\frac{2ib^2E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}$$

input
```
Int[(b*Sech[c + d*x])^(3/2),x]
```

output
```
((2*I)*b^2*EllipticE[(I/2)*(c + d*x), 2])/(d*Sqrt[Cosh[c + d*x]]*Sqrt[b*Se
ch[c + d*x]]) + (2*b*Sqrt[b*Sech[c + d*x]]*Sinh[c + d*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [F]

$$\int (b \operatorname{sech}(dx + c))^{\frac{3}{2}} dx$$

input
```
int((b*sech(d*x+c))^(3/2),x)
```

output
```
int((b*sech(d*x+c))^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.53

$$\int (b\mathrm{sech}(c+dx))^{3/2}\,dx = \frac{2\left(\sqrt{2}b^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cosh(dx+c)+\sinh(dx+c)))+\right.}{d}$$

input
```
integrate((b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2*(sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(
d*x + c) + sinh(d*x + c))) + sqrt(2)*(b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((b*cosh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 + 1)))/d
```

Sympy [F]

$$\int (b\mathrm{sech}(c+dx))^{3/2}\,dx = \int (b\,\mathrm{sech}\,(c+dx))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sech(d*x+c))**(3/2),x)
```

output
```
Integral((b*sech(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int (b\mathrm{sech}(c+dx))^{3/2}\,dx = \int (b\,\mathrm{sech}\,(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sech(d*x + c))^(3/2), x)
```

Giac [F]

$$\int (b\text{sech}(c+dx))^{3/2}\,dx = \int (b\,\text{sech}\,(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sech(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\text{sech}(c+dx))^{3/2}\,dx = \int \left(\frac{b}{\cosh(c+d\,x)}\right)^{3/2}\,dx$$

input
```
int((b/cosh(c + d*x))^(3/2),x)
```

output
```
int((b/cosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int (b\text{sech}(c+dx))^{3/2}\,dx = \sqrt{b}\left(\int \sqrt{\text{sech}\,(dx+c)}\,\text{sech}(dx+c)\,dx\right)b$$

input
```
int((b*sech(d*x+c))^(3/2),x)
```

output
```
sqrt(b)*int(sqrt(sech(c + d*x))*sech(c + d*x),x)*b
```

3.18 $\int \sqrt{b\text{sech}(c + dx)}\, dx$

Optimal result

Integrand size = 12, antiderivative size = 42

$$\int \sqrt{b\text{sech}(c + dx)}\, dx = -\frac{2i\sqrt{\cosh(c + dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c + dx), 2\right)\sqrt{b\text{sech}(c + dx)}}{d}$$

output
```
-2*I*cosh(d*x+c)^(1/2)*InverseJacobiAM(1/2*I*(d*x+c),2^(1/2))*(b*sech(d*x+
c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \sqrt{b\text{sech}(c + dx)}\, dx = -\frac{2i\sqrt{\cosh(c + dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c + dx), 2\right)\sqrt{b\text{sech}(c + dx)}}{d}$$

input
```
Integrate[Sqrt[b*Sech[c + d*x]],x]
```

output
```
((-2*I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Sech[c +
d*x]])/d
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\text{sech}(c+dx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{b\csc\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 4258$$

$$\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}\int \frac{1}{\sqrt{\cosh(c+dx)}}dx$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}\int \frac{1}{\sqrt{\sin\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \ 3120$$

$$-\frac{2i\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\text{sech}(c+dx)}}{d}$$

input
```
Int[Sqrt[b*Sech[c + d*x]],x]
```

output
```
((-2*I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Sech[c + 
d*x]])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [F]

$$\int \sqrt{b\ \mathrm{sech}\left(dx+c\right)}dx$$

input
```
int((b*sech(d*x+c))^(1/2),x)
```

output
```
int((b*sech(d*x+c))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

$$\int \sqrt{b\mathrm{sech}(c+dx)}\,dx = \frac{2\sqrt{2}\sqrt{b}\mathrm{weierstrassPInverse}(-4,0,\cosh\left(dx+c\right)+\sinh\left(dx+c\right))}{d}$$

input
```
integrate((b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cosh(d*x + c) + sinh(d*x + c)
)/d
```

Sympy [F]

$$\int \sqrt{b\text{sech}(c+dx)}\,dx = \int \sqrt{b\,\text{sech}\,(c+dx)}\,dx$$

input
```
integrate((b*sech(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{b\text{sech}(c+dx)}\,dx = \int \sqrt{b\,\text{sech}\,(dx+c)}\,dx$$

input
```
integrate((b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sech(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{b\text{sech}(c+dx)}\,dx = \int \sqrt{b\,\text{sech}\,(dx+c)}\,dx$$

input
```
integrate((b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sech(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\text{sech}(c+dx)}\,dx = \int \sqrt{\frac{b}{\cosh(c+d\,x)}}\,dx$$

input
```
int((b/cosh(c + d*x))^(1/2),x)
```

output
```
int((b/cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{b\text{sech}(c+dx)}\,dx = \sqrt{b}\left(\int \sqrt{\text{sech}\,(dx+c)}dx\right)$$

input
```
int((b*sech(d*x+c))^(1/2),x)
```

output
```
sqrt(b)*int(sqrt(sech(c + d*x)),x)
```

3.19 $\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 42

$$\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx=-\frac{2iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}$$

output
```
-2*I*EllipticE(I*sinh(1/2*d*x+1/2*c),2^(1/2))/d/cosh(d*x+c)^(1/2)/(b*sech(
d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx=-\frac{2iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}$$

input
```
Integrate[1/Sqrt[b*Sech[c + d*x]],x]
```

output
```
((-2*I)*EllipticE[(I/2)*(c + d*x), 2])/(d*Sqrt[Cosh[c + d*x]]*Sqrt[b*Sech[
c + d*x]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4258}$$

$$\frac{\int \sqrt{\cosh(c+dx)}dx}{\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}$$

input
```
Int[1/Sqrt[b*Sech[c + d*x]],x]
```

output
```
((-2*I)*EllipticE[(I/2)*(c + d*x), 2])/(d*Sqrt[Cosh[c + d*x]]*Sqrt[b*Sech[
c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(40) = 80$.

Time = 0.28 (sec) , antiderivative size = 244, normalized size of antiderivative = 5.81

method	result
risch	$\frac{\sqrt{2}}{d\sqrt{\frac{b\,e^{dx+c}}{e^{2dx+2c}+1}}}+\frac{\left(-\frac{2\left(b\,e^{2dx+2c}+b\right)}{b\sqrt{e^{dx+c}\left(b\,e^{2dx+2c}+b\right)}}+\frac{i\sqrt{-i\left(e^{dx+c}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{dx+c}-i\right)}\,\sqrt{i\,e^{dx+c}}\,\left(-2i\,\mathrm{EllipticE}\left(\sqrt{-i\left(e^{dx+c}+i\right)},\frac{\sqrt{2}}{2}\right)+i\,\mathrm{Ellip}\cdots\right.}{\sqrt{b\,e^{3dx+3c}+b\,e^{dx+c}}}\cdots\right.}{d\sqrt{\frac{b\,e^{dx+c}}{e^{2dx+2c}+1}}\left(e^{2dx+2c}+1\right)\cdots}$

input
```
int(1/(b*sech(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*2^(1/2)/(b*exp(d*x+c)/(exp(d*x+c)^2+1))^(1/2)+1/d*(-2*(b*exp(d*x+c)^2+
b)/b/(exp(d*x+c)*(b*exp(d*x+c)^2+b))^(1/2)+I*(-I*(exp(d*x+c)+I))^(1/2)*2^(
1/2)*(I*(exp(d*x+c)-I))^(1/2)*(I*exp(d*x+c))^(1/2)/(b*exp(d*x+c)^3+b*exp(d
*x+c))^(1/2)*(-2*I*EllipticE((-I*(exp(d*x+c)+I))^(1/2),1/2*2^(1/2))+I*Elli
pticF((-I*(exp(d*x+c)+I))^(1/2),1/2*2^(1/2))))*2^(1/2)/(b*exp(d*x+c)/(exp(
d*x+c)^2+1))^(1/2)*(b*exp(d*x+c)*(exp(d*x+c)^2+1))^(1/2)/(exp(d*x+c)^2+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(38) = 76$.

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 3.67

$$\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx =$$

$$-\frac{2\sqrt{2}\sqrt{b}(\cosh(dx+c)+\sinh(dx+c))\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cosh(dx+c$$

input
```
integrate(1/(b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-(2*sqrt(2)*sqrt(b)*(cosh(d*x + c) + sinh(d*x + c))*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cosh(d*x + c) + sinh(d*x + c))) + sqrt(2)*(cos
h(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt((
b*cosh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh
(d*x + c) + sinh(d*x + c)^2 + 1)))/(b*d*cosh(d*x + c) + b*d*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{sech}\,(c+dx)}}\,dx$$

input
```
integrate(1/(b*sech(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{sech}\,(dx+c)}}\,dx$$

input
```
integrate(1/(b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*sech(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{sech}\,(dx+c)}}\,dx$$

input
```
integrate(1/(b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*sech(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx = \int \frac{1}{\sqrt{\frac{b}{\cosh(c+d\,x)}}}\,dx$$

input
```
int(1/(b/cosh(c + d*x))^(1/2),x)
```

output
```
int(1/(b/cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{b\operatorname{sech}(c+dx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\operatorname{sech}(dx+c)}}{\operatorname{sech}(dx+c)}dx\right)}{b}$$

input
```
int(1/(b*sech(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*int(sqrt(sech(c + d*x))/sech(c + d*x),x))/b
```

3.20 $\int \frac{1}{(b\operatorname{sech}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 76

$$\int \frac{1}{(b\operatorname{sech}(c+dx))^{3/2}}\,dx = -\frac{2i\sqrt{\cosh(c+dx)}\,\operatorname{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\operatorname{sech}(c+dx)}}{3b^2d}+\frac{2\sinh(c+dx)}{3bd\sqrt{b\operatorname{sech}(c+dx)}}$$

output
```
-2/3*I*cosh(d*x+c)^(1/2)*InverseJacobiAM(1/2*I*(d*x+c),2^(1/2))*(b*sech(d*
x+c))^(1/2)/b^2/d+2/3*sinh(d*x+c)/b/d/(b*sech(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

$$\int \frac{1}{(b\operatorname{sech}(c+dx))^{3/2}}\,dx = \frac{\operatorname{sech}^2(c+dx)\left(-2i\sqrt{\cosh(c+dx)}\,\operatorname{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)+\sinh(2(c+dx))\right)}{3d(b\operatorname{sech}(c+dx))^{3/2}}$$

input
```
Integrate[(b*Sech[c + d*x])^(-3/2),x]
```

output
```
(Sech[c + d*x]^2*((-2*I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]
 + Sinh[2*(c + d*x)]))/(3*d*(b*Sech[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\text{sech}(c+dx))^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 4256$$

$$\frac{\int \sqrt{b\text{sech}(c+dx)}dx}{3b^2}+\frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}}+\frac{\int \sqrt{b\csc\left(ic+idx+\frac{\pi}{2}\right)}dx}{3b^2}$$

$$\Big\downarrow \; 4258$$

$$\frac{\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}\int \frac{1}{\sqrt{\cosh(c+dx)}}dx}{3b^2}+\frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}}+\frac{\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}\int \frac{1}{\sqrt{\sin\left(ic+idx+\frac{\pi}{2}\right)}}dx}{3b^2}$$

$$\Big\downarrow \; 3120$$

$$\frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}}-\frac{2i\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\text{sech}(c+dx)}}{3b^2d}$$

input
```
Int[(b*Sech[c + d*x])^(-3/2),x]
```

output
```
(((-2*I)/3)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Sech[
c + d*x]])/(b^2*d) + (2*Sinh[c + d*x])/(3*b*d*Sqrt[b*Sech[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [F]

$$\int \frac{1}{(b\,\mathrm{sech}\,(dx+c))^{\frac{3}{2}}}dx$$

input
```
int(1/(b*sech(d*x+c))^(3/2),x)
```

output
```
int(1/(b*sech(d*x+c))^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(63) = 126$.

Time = 0.09 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.04

$$\int \frac{1}{(b\mathrm{sech}(c+dx))^{3/2}}\,dx = \frac{4\sqrt{2}\left(\cosh{(dx+c)}^2+2\cosh{(dx+c)}\sinh{(dx+c)}+\sinh{(dx+c)}^2\right)\sqrt{b}\mathrm{weiers}}{}$$

input
```
integrate(1/(b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/6*(4*sqrt(2)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
 + c)^2)*sqrt(b)*weierstrassPInverse(-4, 0, cosh(d*x + c) + sinh(d*x + c))
 + sqrt(2)*(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x
 + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^
4 - 1)*sqrt((b*cosh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)))/(b^2*d*cosh(d*x + c)^2 + 2
*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{1}{(b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\operatorname{sech}(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*sech(d*x+c))**(3/2),x)
```

output
```
Integral((b*sech(c + d*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\operatorname{sech}(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sech(d*x + c))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\operatorname{sech}(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sech(d*x + c))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(b/cosh(c + d*x))^(3/2),x)
```

output
```
int(1/(b/cosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(b\text{sech}(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\text{sech}(dx+c)}}{\text{sech}(dx+c)^2}dx\right)}{b^2}$$

input
```
int(1/(b*sech(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*int(sqrt(sech(c + d*x))/sech(c + d*x)**2,x))/b**2
```

3.21 $\int \frac{1}{(b\,\text{sech}(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 76

$$\int \frac{1}{(b\,\text{sech}(c+dx))^{5/2}}\,dx = -\frac{6iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5b^2d\sqrt{\cosh(c+dx)}\sqrt{b\,\text{sech}(c+dx)}}+\frac{2\sinh(c+dx)}{5bd(b\,\text{sech}(c+dx))^{3/2}}$$

output
```
-6/5*I*EllipticE(I*sinh(1/2*d*x+1/2*c),2^(1/2))/b^2/d/cosh(d*x+c)^(1/2)/(b
*sech(d*x+c))^(1/2)+2/5*sinh(d*x+c)/b/d/(b*sech(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \frac{1}{(b\,\text{sech}(c+dx))^{5/2}}\,dx = \frac{\sqrt{b\,\text{sech}(c+dx)}\left(-12i\sqrt{\cosh(c+dx)}E\left(\frac{1}{2}i(c+dx)\middle|2\right)+\sinh(c+dx)+\sinh(3\right.}{10b^3d}$$

input
```
Integrate[(b*Sech[c + d*x])^(-5/2),x]
```

output
```
(Sqrt[b*Sech[c + d*x]]*((-12*I)*Sqrt[Cosh[c + d*x]]*EllipticE[(I/2)*(c + d
*x), 2] + Sinh[c + d*x] + Sinh[3*(c + d*x)]))/(10*b^3*d)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\text{sech}(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\frac{3\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}dx}{5b^2}+\frac{2\sinh(c+dx)}{5bd(b\text{sech}(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(c+dx)}{5bd(b\text{sech}(c+dx))^{3/2}}+\frac{3\int \frac{1}{\sqrt{b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx}{5b^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{3\int \sqrt{\cosh(c+dx)}dx}{5b^2\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}+\frac{2\sinh(c+dx)}{5bd(b\text{sech}(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(c+dx)}{5bd(b\text{sech}(c+dx))^{3/2}}+\frac{3\int \sqrt{\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{5b^2\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\sinh(c+dx)}{5bd(b\text{sech}(c+dx))^{3/2}}-\frac{6iE\left(\frac{1}{2}i(c+dx)|2\right)}{5b^2d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}$$

input
```
Int[(b*Sech[c + d*x])^(-5/2),x]
```

output
```
(((-6*I)/5)*EllipticE[(I/2)*(c + d*x), 2])/(b^2*d*Sqrt[Cosh[c + d*x]]*Sqrt
[b*Sech[c + d*x]]) + (2*Sinh[c + d*x])/(5*b*d*(b*Sech[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [F]

$$\int \frac{1}{(b\,\mathrm{sech}\,(dx+c))^{\frac{5}{2}}}dx$$

input
```
int(1/(b*sech(d*x+c))^(5/2),x)
```

output
```
int(1/(b*sech(d*x+c))^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(66) = 132$.

Time = 0.08 (sec) , antiderivative size = 379, normalized size of antiderivative = 4.99

$$\int \frac{1}{(b\mathrm{sech}(c+dx))^{5/2}}\,dx = -\frac{24\sqrt{2}\left(\cosh\left(dx+c\right)^3+3\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)+3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+\sinh\left(dx+c\right)^3\right)}{}$$

input
```
integrate(1/(b*sech(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/20*(24*sqrt(2)*(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*c
osh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3)*sqrt(b)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cosh(d*x + c) + sinh(d*x + c))) - sqrt(2)*
(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6 + (15
*cosh(d*x + c)^2 - 11)*sinh(d*x + c)^4 - 11*cosh(d*x + c)^4 + 4*(5*cosh(d*
x + c)^3 - 11*cosh(d*x + c))*sinh(d*x + c)^3 + (15*cosh(d*x + c)^4 - 66*co
sh(d*x + c)^2 - 13)*sinh(d*x + c)^2 - 13*cosh(d*x + c)^2 + 2*(3*cosh(d*x +
 c)^5 - 22*cosh(d*x + c)^3 - 13*cosh(d*x + c))*sinh(d*x + c) - 1)*sqrt((b*
cosh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2 + 1)))/(b^3*d*cosh(d*x + c)^3 + 3*b^3*d*cosh(d*x
 + c)^2*sinh(d*x + c) + 3*b^3*d*cosh(d*x + c)*sinh(d*x + c)^2 + b^3*d*sinh
(d*x + c)^3)
```

Sympy [F]

$$\int \frac{1}{(b\mathrm{sech}(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(b\operatorname{sech}\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*sech(d*x+c))**(5/2),x)
```

output
```
Integral((b*sech(c + d*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\text{sech}(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\,\text{sech}\,(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*sech(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sech(d*x + c))^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(b\text{sech}(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\,\text{sech}\,(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*sech(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sech(d*x + c))^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\text{sech}(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(\frac{b}{\cosh(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(b/cosh(c + d*x))^(5/2),x)
```

output
```
int(1/(b/cosh(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(b\text{sech}(c+dx))^{5/2}}\,dx=\frac{\sqrt{b}\left(\int \frac{\sqrt{\text{sech}(dx+c)}}{\text{sech}(dx+c)^3}dx\right)}{b^3}$$

input
```
int(1/(b*sech(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*int(sqrt(sech(c + d*x))/sech(c + d*x)**3,x))/b**3
```

3.22 $\int \frac{1}{(b\text{sech}(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 104

$$\int \frac{1}{(b\text{sech}(c+dx))^{7/2}}\,dx = -\frac{10i\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\text{sech}(c+dx)}}{21b^4d} + \frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}} + \frac{10\sinh(c+dx)}{21b^3d\sqrt{b\text{sech}(c+dx)}}$$

output
```
-10/21*I*cosh(d*x+c)^(1/2)*InverseJacobiAM(1/2*I*(d*x+c),2^(1/2))*(b*sech(
d*x+c))^(1/2)/b^4/d+2/7*sinh(d*x+c)/b/d/(b*sech(d*x+c))^(5/2)+10/21*sinh(d
*x+c)/b^3/d/(b*sech(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.67

$$\int \frac{1}{(b\text{sech}(c+dx))^{7/2}}\,dx = \frac{\sqrt{b\text{sech}(c+dx)}\left(-40i\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)+26\sinh(2(c+\right.}{84b^4d}$$

input
```
Integrate[(b*Sech[c + d*x])^(-7/2),x]
```

output
```
(Sqrt[b*Sech[c + d*x]]*((-40*I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d
*x), 2] + 26*Sinh[2*(c + d*x)] + 3*Sinh[4*(c + d*x)]))/(84*b^4*d)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\text{sech}(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\frac{5\int \frac{1}{(b\text{sech}(c+dx))^{3/2}}dx}{7b^2}+\frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}}+\frac{5\int \frac{1}{\left(b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7b^2}$$

$$\Big\downarrow \text{4256}$$

$$\frac{5\left(\frac{\int \sqrt{b\text{sech}(c+dx)}dx}{3b^2}+\frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}}\right)}{7b^2}+\frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}}+\frac{5\left(\frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}}+\frac{\int \sqrt{b\csc\left(ic+idx+\frac{\pi}{2}\right)}dx}{3b^2}\right)}{7b^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{5\left(\frac{\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}\int\frac{1}{\sqrt{\cosh(c+dx)}}dx}{3b^2}+\frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}}\right)}{7b^2}+\frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}}+\frac{5\left(\frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}}+\frac{\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}\int\frac{1}{\sqrt{\sin\left(ic+idx+\frac{\pi}{2}\right)}}dx}{3b^2}\right)}{7b^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}}+\frac{5\left(\frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}}-\frac{2i\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\text{sech}(c+dx)}}{3b^2d}\right)}{7b^2}$$

input
```
Int[(b*Sech[c + d*x])^(-7/2),x]
```

output
```
(2*Sinh[c + d*x])/(7*b*d*(b*Sech[c + d*x])^(5/2)) + (5*((((-2*I)/3)*Sqrt[C
osh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Sech[c + d*x]])/(b^2*d)
 + (2*Sinh[c + d*x])/(3*b*d*Sqrt[b*Sech[c + d*x]])))/(7*b^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
 + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [F]

$$\int \frac{1}{\left(b\,\mathrm{sech}\left(dx+c\right)\right)^{\frac{7}{2}}}dx$$

input
```
int(1/(b*sech(d*x+c))^(7/2),x)
```

output
```
int(1/(b*sech(d*x+c))^(7/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(87) = 174$.

Time = 0.09 (sec) , antiderivative size = 483, normalized size of antiderivative = 4.64

$$\int \frac{1}{(b\mathrm{sech}(c+dx))^{7/2}}\,dx = \frac{80\sqrt{2}\left(\cosh\left(dx+c\right)^{4}+4\,\cosh\left(dx+c\right)^{3}\sinh\left(dx+c\right)+6\,\cosh\left(dx+c\right)^{2}\sinh\left(\right.\right.}{}$$

input
```
integrate(1/(b*sech(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/168*(80*sqrt(2)*(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*c
osh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*
x + c)^4)*sqrt(b)*weierstrassPInverse(-4, 0, cosh(d*x + c) + sinh(d*x + c)
) + sqrt(2)*(3*cosh(d*x + c)^8 + 24*cosh(d*x + c)*sinh(d*x + c)^7 + 3*sinh
(d*x + c)^8 + 2*(42*cosh(d*x + c)^2 + 13)*sinh(d*x + c)^6 + 26*cosh(d*x +
c)^6 + 12*(14*cosh(d*x + c)^3 + 13*cosh(d*x + c))*sinh(d*x + c)^5 + 30*(7*
cosh(d*x + c)^4 + 13*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(21*cosh(d*x + c
)^5 + 65*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 2*(42*cosh(d*x + c)^6 + 195*co
sh(d*x + c)^4 - 13)*sinh(d*x + c)^2 - 26*cosh(d*x + c)^2 + 4*(6*cosh(d*x +
 c)^7 + 39*cosh(d*x + c)^5 - 13*cosh(d*x + c))*sinh(d*x + c) - 3)*sqrt((b*
cosh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2 + 1)))/(b^4*d*cosh(d*x + c)^4 + 4*b^4*d*cosh(d*x
 + c)^3*sinh(d*x + c) + 6*b^4*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^4*d*
cosh(d*x + c)*sinh(d*x + c)^3 + b^4*d*sinh(d*x + c)^4)
```

Sympy [F]

$$\int \frac{1}{(b\text{sech}(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(b\operatorname{sech}\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(b*sech(d*x+c))**(7/2),x)
```

output
```
Integral((b*sech(c + d*x))**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\text{sech}(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(b\operatorname{sech}\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(b*sech(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sech(d*x + c))^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(b\text{sech}(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\,\text{sech}\,(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(b*sech(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sech(d*x + c))^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\text{sech}(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(\frac{b}{\cosh(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int(1/(b/cosh(c + d*x))^(7/2),x)
```

output
```
int(1/(b/cosh(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{(b\text{sech}(c+dx))^{7/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\text{sech}(dx+c)}}{\text{sech}(dx+c)^4}dx\right)}{b^4}$$

input
```
int(1/(b*sech(d*x+c))^(7/2),x)
```

output
```
(sqrt(b)*int(sqrt(sech(c + d*x))/sech(c + d*x)**4,x))/b**4
```

3.23 $\int (b\text{sech}(c+dx))^n\,dx$

Optimal result

Integrand size = 10, antiderivative size = 75

$$\int (b\text{sech}(c+dx))^n\,dx = -\frac{b\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cosh^2(c+dx)\right)(b\text{sech}(c+dx))^{-1+n}\sinh(c+dx)}{d(1-n)\sqrt{-\sinh^2(c+dx)}}$$

output
```
-b*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cosh(d*x+c)^2)*(b*sech(d*x+c))^(
-1+n)*sinh(d*x+c)/d/(1-n)/(-sinh(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int (b\text{sech}(c+dx))^n\,dx = -\frac{\coth(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\text{sech}^2(c+dx)\right)(b\text{sech}(c+dx))^n\sqrt{\tanh^2(c+dx)}}{dn}$$

input
```
Integrate[(b*Sech[c + d*x])^n,x]
```

output
```
-((Coth[c + d*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sech[c + d*x]^2]*(
b*Sech[c + d*x])^n*Sqrt[Tanh[c + d*x]^2])/(d*n))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\text{sech}(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4259}$$

$$\left(\frac{\cosh(c+dx)}{b}\right)^n (b\text{sech}(c+dx))^n \int \left(\frac{\cosh(c+dx)}{b}\right)^{-n} dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{\cosh(c+dx)}{b}\right)^n (b\text{sech}(c+dx))^n \int \left(\frac{\sin\left(ic+idx+\frac{\pi}{2}\right)}{b}\right)^{-n} dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{b\sinh(c+dx)(b\text{sech}(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cosh^2(c+dx)\right)}{d(1-n)\sqrt{-\sinh^2(c+dx)}}$$

input
```
Int[(b*Sech[c + d*x])^n,x]
```

output
```
-((b*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cosh[c + d*x]^2]*(b*Sech
[c + d*x])^(-1 + n)*Sinh[c + d*x])/(d*(1 - n)*Sqrt[-Sinh[c + d*x]^2]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int (b\,\mathrm{sech}\,(dx+c))^n\,dx$$

input
```
int((b*sech(d*x+c))^n,x)
```

output
```
int((b*sech(d*x+c))^n,x)
```

Fricas [F]

$$\int (b\mathrm{sech}(c+dx))^n\,dx=\int (b\,\mathrm{sech}\,(dx+c))^n\;dx$$

input
```
integrate((b*sech(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*sech(d*x + c))^n, x)
```

Sympy [F]

$$\int (b\text{sech}(c+dx))^n\,dx = \int (b\,\text{sech}\,(c+dx))^n\,dx$$

input
```
integrate((b*sech(d*x+c))**n,x)
```

output
```
Integral((b*sech(c + d*x))**n, x)
```

Maxima [F]

$$\int (b\text{sech}(c+dx))^n\,dx = \int (b\,\text{sech}\,(dx+c))^n\,dx$$

input
```
integrate((b*sech(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*sech(d*x + c))^n, x)
```

Giac [F]

$$\int (b\text{sech}(c+dx))^n\,dx = \int (b\,\text{sech}\,(dx+c))^n\,dx$$

input
```
integrate((b*sech(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*sech(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\text{sech}(c+dx))^n\,dx = \int \left(\frac{b}{\cosh(c+d\,x)}\right)^n\,dx$$

input
```
int((b/cosh(c + d*x))^n,x)
```

output
```
int((b/cosh(c + d*x))^n, x)
```

Reduce [F]

$$\int (b\text{sech}(c+dx))^n\,dx = b^n\left(\int \text{sech}(dx+c)^n\,dx\right)$$

input
```
int((b*sech(d*x+c))^n,x)
```

output
```
b**n*int(sech(c + d*x)**n,x)
```

3.24 $\int \text{sech}^2(a+bx)^{7/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int \text{sech}^2(a+bx)^{7/2}\,dx = \frac{5\arcsin(\tanh(a+bx))}{16b} + \frac{5\sqrt{\text{sech}^2(a+bx)}\tanh(a+bx)}{16b} + \frac{5\text{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{24b} + \frac{\text{sech}^2(a+bx)^{5/2}\tanh(a+bx)}{6b}$$

output
```
5/16*arcsin(tanh(b*x+a))/b+5/16*(sech(b*x+a)^2)^(1/2)*tanh(b*x+a)/b+5/24*(
sech(b*x+a)^2)^(3/2)*tanh(b*x+a)/b+1/6*(sech(b*x+a)^2)^(5/2)*tanh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int \text{sech}^2(a+bx)^{7/2}\,dx = \frac{\text{sech}(a+bx)\left(15\arctan(\sinh(a+bx)) + 15\text{sech}(a+bx)\tanh(a+bx) + 10\text{sech}^3(a+bx)\tanh\right.}{48b\sqrt{\text{sech}^2(a+bx)}}$$

input
```
Integrate[(Sech[a + b*x]^2)^(7/2),x]
```

output
```
(Sech[a + b*x]*(15*ArcTan[Sinh[a + b*x]] + 15*Sech[a + b*x]*Tanh[a + b*x]
+ 10*Sech[a + b*x]^3*Tanh[a + b*x] + 8*Sech[a + b*x]^5*Tanh[a + b*x]))/(48
*b*Sqrt[Sech[a + b*x]^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4610, 211, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^2(a+bx)^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\sec(ia+ibx)^2\right)^{7/2}dx$$

$$\Big\downarrow \text{4610}$$

$$\frac{\int \left(1-\tanh^2(a+bx)\right)^{5/2}d\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{5}{6}\int \left(1-\tanh^2(a+bx)\right)^{3/2}d\tanh(a+bx)+\frac{1}{6}\tanh(a+bx)\left(1-\tanh^2(a+bx)\right)^{5/2}}{b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{5}{6}\left(\frac{3}{4}\int \sqrt{1-\tanh^2(a+bx)}d\tanh(a+bx)+\frac{1}{4}\tanh(a+bx)\left(1-\tanh^2(a+bx)\right)^{3/2}\right)+\frac{1}{6}\tanh(a+bx)\left(1-\tan\right.}{b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-\tanh^2(a+bx)}}d\tanh(a+bx)+\frac{1}{2}\sqrt{1-\tanh^2(a+bx)}\tanh(a+bx)\right)+\frac{1}{4}\tanh(a+bx)\left(1-\tanh^2(a\right.\right.}{b}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\arcsin(\tanh(a+bx))+\frac{1}{2}\tanh(a+bx)\sqrt{1-\tanh^2(a+bx)}\right)+\frac{1}{4}\tanh(a+bx)\left(1-\tanh^2(a+bx)\right)^{3/2}\right)}{b}$$

input
```
Int[(Sech[a + b*x]^2)^(7/2),x]
```

output
```
((Tanh[a + b*x]*(1 - Tanh[a + b*x]^2)^(5/2))/6 + (5*((Tanh[a + b*x]*(1 - T
anh[a + b*x]^2)^(3/2))/4 + (3*(ArcSin[Tanh[a + b*x]]/2 + (Tanh[a + b*x]*Sq
rt[1 - Tanh[a + b*x]^2])/2))/4))/6)/b
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.79 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.56

method	result
risch	$\frac{\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^{2}}}\,\left(15\,e^{10bx+10a}+85\,e^{8bx+8a}+198\,e^{6bx+6a}-198\,e^{4bx+4a}-85\,e^{2bx+2a}-15\right)}{24\left(1+e^{2bx+2a}\right)^{5}b}+\frac{5i\ln\left(e^{bx}+ie^{-a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^{2}}}\,(1\cdots}{16b}$

input
```
int((sech(b*x+a)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/(1+exp(2*b*x+2*a))^5*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*(1
5*exp(10*b*x+10*a)+85*exp(8*b*x+8*a)+198*exp(6*b*x+6*a)-198*exp(4*b*x+4*a)
-85*exp(2*b*x+2*a)-15)/b+5/16*I*ln(exp(b*x)+I*exp(-a))/b*(1/(1+exp(2*b*x+2
*a))^2*exp(2*b*x+2*a))^(1/2)*(1+exp(2*b*x+2*a))*exp(-b*x-a)-5/16*I*ln(exp(
b*x)-I*exp(-a))/b*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*(1+exp(2*b
*x+2*a))*exp(-b*x-a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1604 vs. 2(76) = 152.

Time = 0.09 (sec) , antiderivative size = 1604, normalized size of antiderivative = 17.82

$$\int \operatorname{sech}^2(a+bx)^{7/2}\,dx = \text{Too large to display}$$

input
```
integrate((sech(b*x+a)^2)^(7/2),x, algorithm="fricas")
```

output
```
1/24*(15*cosh(b*x + a)^11 + 165*cosh(b*x + a)*sinh(b*x + a)^10 + 15*sinh(b
*x + a)^11 + 5*(165*cosh(b*x + a)^2 + 17)*sinh(b*x + a)^9 + 85*cosh(b*x +
a)^9 + 45*(55*cosh(b*x + a)^3 + 17*cosh(b*x + a))*sinh(b*x + a)^8 + 18*(27
5*cosh(b*x + a)^4 + 170*cosh(b*x + a)^2 + 11)*sinh(b*x + a)^7 + 198*cosh(b
*x + a)^7 + 42*(165*cosh(b*x + a)^5 + 170*cosh(b*x + a)^3 + 33*cosh(b*x +
a))*sinh(b*x + a)^6 + 18*(385*cosh(b*x + a)^6 + 595*cosh(b*x + a)^4 + 231*
cosh(b*x + a)^2 - 11)*sinh(b*x + a)^5 - 198*cosh(b*x + a)^5 + 90*(55*cosh(
b*x + a)^7 + 119*cosh(b*x + a)^5 + 77*cosh(b*x + a)^3 - 11*cosh(b*x + a))*
sinh(b*x + a)^4 + 5*(495*cosh(b*x + a)^8 + 1428*cosh(b*x + a)^6 + 1386*cos
h(b*x + a)^4 - 396*cosh(b*x + a)^2 - 17)*sinh(b*x + a)^3 - 85*cosh(b*x + a
)^3 + 3*(275*cosh(b*x + a)^9 + 1020*cosh(b*x + a)^7 + 1386*cosh(b*x + a)^5
 - 660*cosh(b*x + a)^3 - 85*cosh(b*x + a))*sinh(b*x + a)^2 + 15*(cosh(b*x
+ a)^12 + 12*cosh(b*x + a)*sinh(b*x + a)^11 + sinh(b*x + a)^12 + 6*(11*cos
h(b*x + a)^2 + 1)*sinh(b*x + a)^10 + 6*cosh(b*x + a)^10 + 20*(11*cosh(b*x
+ a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^9 + 15*(33*cosh(b*x + a)^4 + 18*co
sh(b*x + a)^2 + 1)*sinh(b*x + a)^8 + 15*cosh(b*x + a)^8 + 24*(33*cosh(b*x
+ a)^5 + 30*cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a)^7 + 4*(231*co
sh(b*x + a)^6 + 315*cosh(b*x + a)^4 + 105*cosh(b*x + a)^2 + 5)*sinh(b*x +
a)^6 + 20*cosh(b*x + a)^6 + 24*(33*cosh(b*x + a)^7 + 63*cosh(b*x + a)^5 +
35*cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a)^5 + 15*(33*cosh(b*x...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{sech}^2(a+bx)^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((sech(b*x+a)**2)**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(76) = 152$.

Time = 0.13 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.73

$$\int \operatorname{sech}^2(a+bx)^{7/2}\,dx = -\frac{5\arctan\left(e^{(-bx-a)}\right)}{8\,b} + \frac{15\,e^{(-bx-a)} + 85\,e^{(-3\,bx-3\,a)} + 198\,e^{(-5\,bx-5\,a)} - 198\,e^{(-7\,bx-7\,a)} - 85\,e^{(-9\,bx-9\,a)} - 15\,e^{(-11\,bx-11\,a)}}{24\,b(6\,e^{(-2\,bx-2\,a)} + 15\,e^{(-4\,bx-4\,a)} + 20\,e^{(-6\,bx-6\,a)} + 15\,e^{(-8\,bx-8\,a)} + 6\,e^{(-10\,bx-10\,a)} + e^{(-12\,bx-12\,a)} + 1)}$$

input
```
integrate((sech(b*x+a)^2)^(7/2),x, algorithm="maxima")
```

output
```
-5/8*arctan(e^(-b*x - a))/b + 1/24*(15*e^(-b*x - a) + 85*e^(-3*b*x - 3*a)
+ 198*e^(-5*b*x - 5*a) - 198*e^(-7*b*x - 7*a) - 85*e^(-9*b*x - 9*a) - 15*e
^(-11*b*x - 11*a))/(b*(6*e^(-2*b*x - 2*a) + 15*e^(-4*b*x - 4*a) + 20*e^(-6
*b*x - 6*a) + 15*e^(-8*b*x - 8*a) + 6*e^(-10*b*x - 10*a) + e^(-12*b*x - 12
*a) + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.38

$$\int \operatorname{sech}^2(a+bx)^{7/2}\,dx = \frac{15\,\pi + \frac{4\left(15\left(e^{(bx+a)}-e^{(-bx-a)}\right)^5+160\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+528\,e^{(bx+a)}-528\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^3} + 30\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2}\right.\right.}{96\,b}$$

input
```
integrate((sech(b*x+a)^2)^(7/2),x, algorithm="giac")
```

output
```
1/96*(15*pi + 4*(15*(e^(b*x + a) - e^(-b*x - a))^5 + 160*(e^(b*x + a) - e^
(-b*x - a))^3 + 528*e^(b*x + a) - 528*e^(-b*x - a))/((e^(b*x + a) - e^(-b*
x - a))^2 + 4)^3 + 30*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [F(-1)]

Timed out.

$$\int \mathrm{sech}^2(a+bx)^{7/2}\,dx=\int \left(\frac{1}{\cosh{(a+b\,x)}^2}\right)^{7/2}\,dx$$

input
```
int((1/cosh(a + b*x)^2)^(7/2),x)
```

output
```
int((1/cosh(a + b*x)^2)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 279, normalized size of antiderivative = 3.10

$$\int \mathrm{sech}^2(a+bx)^{7/2}\,dx=\frac{15e^{12bx+12a}atan\left(e^{bx+a}\right)+90e^{10bx+10a}atan\left(e^{bx+a}\right)+225e^{8bx+8a}atan\left(e^{bx+a}\right)+300e^{6bx+6a}atan(}{24b\left(e^{12bx+1}\right.}$$

input
```
int((sech(b*x+a)^2)^(7/2),x)
```

output
```
(15*e**(12*a + 12*b*x)*atan(e**(a + b*x)) + 90*e**(10*a + 10*b*x)*atan(e**
(a + b*x)) + 225*e**(8*a + 8*b*x)*atan(e**(a + b*x)) + 300*e**(6*a + 6*b*x
)*atan(e**(a + b*x)) + 225*e**(4*a + 4*b*x)*atan(e**(a + b*x)) + 90*e**(2*
a + 2*b*x)*atan(e**(a + b*x)) + 15*atan(e**(a + b*x)) + 15*e**(11*a + 11*b
*x) + 85*e**(9*a + 9*b*x) + 198*e**(7*a + 7*b*x) - 198*e**(5*a + 5*b*x) -
85*e**(3*a + 3*b*x) - 15*e**(a + b*x))/(24*b*(e**(12*a + 12*b*x) + 6*e**(1
0*a + 10*b*x) + 15*e**(8*a + 8*b*x) + 20*e**(6*a + 6*b*x) + 15*e**(4*a + 4
*b*x) + 6*e**(2*a + 2*b*x) + 1))
```

3.25 $\int \operatorname{sech}^2(a+bx)^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 65

$$\int \operatorname{sech}^2(a+bx)^{5/2}\,dx=\frac{3\arcsin(\tanh(a+bx))}{8b}+\frac{3\sqrt{\operatorname{sech}^2(a+bx)}\tanh(a+bx)}{8b}+\frac{\operatorname{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{4b}$$

output
```
3/8*arcsin(tanh(b*x+a))/b+3/8*(sech(b*x+a)^2)^(1/2)*tanh(b*x+a)/b+1/4*(sec
h(b*x+a)^2)^(3/2)*tanh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(a+bx)^{5/2}\,dx=\frac{\operatorname{sech}(a+bx)\left(3\arctan(\sinh(a+bx))+3\operatorname{sech}(a+bx)\tanh(a+bx)+2\operatorname{sech}^3(a+bx)\tanh(a+\right.}{8b\sqrt{\operatorname{sech}^2(a+bx)}}$$

input
```
Integrate[(Sech[a + b*x]^2)^(5/2),x]
```

output
```
(Sech[a + b*x]*(3*ArcTan[Sinh[a + b*x]] + 3*Sech[a + b*x]*Tanh[a + b*x] +
2*Sech[a + b*x]^3*Tanh[a + b*x]))/(8*b*Sqrt[Sech[a + b*x]^2])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4610, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}^2(a+bx)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\sec(ia+ibx)^2\right)^{5/2}dx$$

$$\Big\downarrow \text{4610}$$

$$\frac{\int \left(1-\tanh^2(a+bx)\right)^{3/2}d\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{3}{4}\int \sqrt{1-\tanh^2(a+bx)}d\tanh(a+bx)+\frac{1}{4}\tanh(a+bx)\left(1-\tanh^2(a+bx)\right)^{3/2}}{b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-\tanh^2(a+bx)}}d\tanh(a+bx)+\frac{1}{2}\sqrt{1-\tanh^2(a+bx)}\tanh(a+bx)\right)+\frac{1}{4}\tanh(a+bx)\left(1-\tanh^2(a+b\right.}{b}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\arcsin(\tanh(a+bx))+\frac{1}{2}\tanh(a+bx)\sqrt{1-\tanh^2(a+bx)}\right)+\frac{1}{4}\tanh(a+bx)\left(1-\tanh^2(a+bx)\right)^{3/2}}{b}$$

input
```
Int[(Sech[a + b*x]^2)^(5/2),x]
```

output
```
((Tanh[a + b*x]*(1 - Tanh[a + b*x]^2)^(3/2))/4 + (3*(ArcSin[Tanh[a + b*x]]
/2 + (Tanh[a + b*x]*Sqrt[1 - Tanh[a + b*x]^2])/2))/4)/b
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.65 (sec) , antiderivative size = 208, normalized size of antiderivative = 3.20

method	result
risch	$\frac{\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^{2}}}\,\left(3\,e^{6bx+6a}+11\,e^{4bx+4a}-11\,e^{2bx+2a}-3\right)}{4\left(1+e^{2bx+2a}\right)^{3} b}+\frac{3 i \ln\left(e^{bx}+i e^{-a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^{2}}}\,\left(1+e^{2bx+2a}\right) e^{-bx-a}}{8 b}-\frac{3 i \ln\left(e^{bx}\right.}{}$

input
```
int((sech(b*x+a)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/(1+exp(2*b*x+2*a))^3*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*(3*
exp(6*b*x+6*a)+11*exp(4*b*x+4*a)-11*exp(2*b*x+2*a)-3)/b+3/8*I*ln(exp(b*x)+
I*exp(-a))/b*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*(1+exp(2*b*x+2*
a))*exp(-b*x-a)-3/8*I*ln(exp(b*x)-I*exp(-a))/b*(1/(1+exp(2*b*x+2*a))^2*exp
(2*b*x+2*a))^(1/2)*(1+exp(2*b*x+2*a))*exp(-b*x-a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 812 vs. $2(55) = 110$.

Time = 0.09 (sec) , antiderivative size = 812, normalized size of antiderivative = 12.49

$$\int \operatorname{sech}^2(a+bx)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((sech(b*x+a)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/4*(3*cosh(b*x + a)^7 + 21*cosh(b*x + a)*sinh(b*x + a)^6 + 3*sinh(b*x + a
)^7 + (63*cosh(b*x + a)^2 + 11)*sinh(b*x + a)^5 + 11*cosh(b*x + a)^5 + 5*(
21*cosh(b*x + a)^3 + 11*cosh(b*x + a))*sinh(b*x + a)^4 + (105*cosh(b*x + a
)^4 + 110*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^3 - 11*cosh(b*x + a)^3 + (63
*cosh(b*x + a)^5 + 110*cosh(b*x + a)^3 - 33*cosh(b*x + a))*sinh(b*x + a)^2
 + 3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8
+ 4*(7*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cos
h(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 +
30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*
x + a)^5 + 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*co
sh(b*x + a)^6 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^
2 + 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*
x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b
*x + a)) + (21*cosh(b*x + a)^6 + 55*cosh(b*x + a)^4 - 33*cosh(b*x + a)^2 -
 3)*sinh(b*x + a) - 3*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a
)*sinh(b*x + a)^7 + b*sinh(b*x + a)^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(
b*x + a)^2 + b)*sinh(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x +
a))*sinh(b*x + a)^5 + 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b
*cosh(b*x + a)^2 + 3*b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*co
sh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 4*b*cosh(b*x + a)^...
```

Sympy [F]

$$\int \operatorname{sech}^2(a+bx)^{5/2}\,dx = \int \left(\operatorname{sech}^2(a+bx)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((sech(b*x+a)**2)**(5/2),x)
```

output
```
Integral((sech(a + b*x)**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(55) = 110.

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.72

$$\int \operatorname{sech}^2(a+bx)^{5/2}\,dx = -\frac{3\arctan\left(e^{(-bx-a)}\right)}{4\,b} + \frac{3\,e^{(-bx-a)} + 11\,e^{(-3\,bx-3\,a)} - 11\,e^{(-5\,bx-5\,a)} - 3\,e^{(-7\,bx-7\,a)}}{4\,b(4\,e^{(-2\,bx-2\,a)} + 6\,e^{(-4\,bx-4\,a)} + 4\,e^{(-6\,bx-6\,a)} + e^{(-8\,bx-8\,a)} + 1)}$$

input
```
integrate((sech(b*x+a)^2)^(5/2),x, algorithm="maxima")
```

output
```
-3/4*arctan(e^(-b*x - a))/b + 1/4*(3*e^(-b*x - a) + 11*e^(-3*b*x - 3*a) -
11*e^(-5*b*x - 5*a) - 3*e^(-7*b*x - 7*a))/(b*(4*e^(-2*b*x - 2*a) + 6*e^(-4
*b*x - 4*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.57

$$\int \operatorname{sech}^2(a+bx)^{5/2}\,dx = \frac{3\,\pi + \frac{4\left(3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+20\,e^{(bx+a)}-20\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^2} + 6\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{16\,b}$$

input
```
integrate((sech(b*x+a)^2)^(5/2),x, algorithm="giac")
```

output
```
1/16*(3*pi + 4*(3*(e^(b*x + a) - e^(-b*x - a))^3 + 20*e^(b*x + a) - 20*e^(
-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4)^2 + 6*arctan(1/2*(e^(2*b*x
 + 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^2(a+bx)^{5/2}\,dx = \int \left(\frac{1}{\cosh(a+b\,x)^2}\right)^{5/2}\,dx$$

input
```
int((1/cosh(a + b*x)^2)^(5/2),x)
```

output
```
int((1/cosh(a + b*x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.94

$$\int \operatorname{sech}^2(a+bx)^{5/2}\,dx = \frac{3e^{8bx+8a}atan\left(e^{bx+a}\right)+12e^{6bx+6a}atan\left(e^{bx+a}\right)+18e^{4bx+4a}atan\left(e^{bx+a}\right)+12e^{2bx+2a}atan\left(e^{bx+a}\right)}{4b\left(e^{8bx+8a}+4e^{6bx+6a}+6e^{4bx+4a}+4e^{2bx\cdots}\right.}$$

input
```
int((sech(b*x+a)^2)^(5/2),x)
```

output
```
(3*e**(8*a + 8*b*x)*atan(e**(a + b*x)) + 12*e**(6*a + 6*b*x)*atan(e**(a +
b*x)) + 18*e**(4*a + 4*b*x)*atan(e**(a + b*x)) + 12*e**(2*a + 2*b*x)*atan(
e**(a + b*x)) + 3*atan(e**(a + b*x)) + 3*e**(7*a + 7*b*x) + 11*e**(5*a + 5
*b*x) - 11*e**(3*a + 3*b*x) - 3*e**(a + b*x))/(4*b*(e**(8*a + 8*b*x) + 4*e
**(6*a + 6*b*x) + 6*e**(4*a + 4*b*x) + 4*e**(2*a + 2*b*x) + 1))
```

3.26 $\int \operatorname{sech}^2(a+bx)^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \operatorname{sech}^2(a+bx)^{3/2}\,dx=\frac{\arcsin(\tanh(a+bx))}{2b}+\frac{\sqrt{\operatorname{sech}^2(a+bx)}\tanh(a+bx)}{2b}$$

output
```
1/2*arcsin(tanh(b*x+a))/b+1/2*(sech(b*x+a)^2)^(1/2)*tanh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \operatorname{sech}^2(a+bx)^{3/2}\,dx=\frac{\operatorname{sech}(a+bx)(\arctan(\sinh(a+bx))+\operatorname{sech}(a+bx)\tanh(a+bx))}{2b\sqrt{\operatorname{sech}^2(a+bx)}}$$

input
```
Integrate[(Sech[a + b*x]^2)^(3/2),x]
```

output
```
(Sech[a + b*x]*(ArcTan[Sinh[a + b*x]] + Sech[a + b*x]*Tanh[a + b*x]))/(2*b
*Sqrt[Sech[a + b*x]^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4610, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^2(a+bx)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\sec(ia+ibx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4610}$$

$$\frac{\int \sqrt{1-\tanh^2(a+bx)}d\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{1}{2}\int \frac{1}{\sqrt{1-\tanh^2(a+bx)}}d\tanh(a+bx)+\frac{1}{2}\sqrt{1-\tanh^2(a+bx)}\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{1}{2}\arcsin(\tanh(a+bx))+\frac{1}{2}\tanh(a+bx)\sqrt{1-\tanh^2(a+bx)}}{b}$$

input
```
Int[(Sech[a + b*x]^2)^(3/2),x]
```

output
```
(ArcSin[Tanh[a + b*x]]/2 + (Tanh[a + b*x]*Sqrt[1 - Tanh[a + b*x]^2])/2)/b
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 4.58

method	result
risch	$\frac{\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}\,\left(e^{2bx+2a}-1\right)}{\left(1+e^{2bx+2a}\right)b}+\frac{i\ln\left(e^{bx}+ie^{-a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}\,\left(1+e^{2bx+2a}\right)e^{-bx-a}}{2b}-\frac{i\ln\left(e^{bx}-ie^{-a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}\,(1+e}{2b}$

input
```
int((sech(b*x+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(1+exp(2*b*x+2*a))*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*(exp(2*
b*x+2*a)-1)/b+1/2*I*ln(exp(b*x)+I*exp(-a))/b*(1/(1+exp(2*b*x+2*a))^2*exp(2
*b*x+2*a))^(1/2)*(1+exp(2*b*x+2*a))*exp(-b*x-a)-1/2*I*ln(exp(b*x)-I*exp(-a
))/b*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*(1+exp(2*b*x+2*a))*exp(
-b*x-a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(34) = 68$.

Time = 0.09 (sec) , antiderivative size = 267, normalized size of antiderivative = 6.68

$$\int \operatorname{sech}^2(a+bx)^{3/2}\,dx = \frac{\cosh(bx+a)^3 + 3\cosh(bx+a)\sinh(bx+a)^2 + \sinh(bx+a)^3 + \left(\cosh(bx+a)^4 + 4\cosh(b\cdots\right.}{b\cosh(bx+a)^4\cdots}$$

input
```
integrate((sech(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

output
```
(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (co
sh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*c
osh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)
^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a
)) + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))/(b*cosh(b*x +
a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*
x + a)^2 + 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a
)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{sech}^2(a+bx)^{3/2}\,dx = \int \left(\operatorname{sech}^2(a+bx)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((sech(b*x+a)**2)**(3/2),x)
```

output
```
Integral((sech(a + b*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.62

$$\int \operatorname{sech}^2(a+bx)^{3/2}\,dx = -\frac{\arctan\left(e^{(-bx-a)}\right)}{b} + \frac{e^{(-bx-a)} - e^{(-3\,bx-3\,a)}}{b(2\,e^{(-2\,bx-2\,a)} + e^{(-4\,bx-4\,a)} + 1)}$$

input
```
integrate((sech(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

output
```
-arctan(e^(-b*x - a))/b + (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*
x - 2*a) + e^(-4*b*x - 4*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(34) = 68.

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.90

$$\int \operatorname{sech}^2(a+bx)^{3/2}\,dx = \frac{\pi + \frac{4\left(e^{(bx+a)} - e^{(-bx-a)}\right)}{\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2 + 4} + 2\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)} - 1\right)e^{(-bx-a)}\right)}{4\,b}$$

input
```
integrate((sech(b*x+a)^2)^(3/2),x, algorithm="giac")
```

output
```
1/4*(pi + 4*(e^(b*x + a) - e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 +
 4) + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^2(a+bx)^{3/2}\,dx = \int \left(\frac{1}{\cosh\left(a+b\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int((1/cosh(a + b*x)^2)^(3/2),x)
```

output
```
int((1/cosh(a + b*x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.42

$$\int \operatorname{sech}^2(a + bx)^{3/2} \, dx = \frac{e^{4bx+4a} atan\left(e^{bx+a}\right) + 2e^{2bx+2a} atan\left(e^{bx+a}\right) + atan\left(e^{bx+a}\right) + e^{3bx+3a} - e^{bx+a}}{b\left(e^{4bx+4a} + 2e^{2bx+2a} + 1\right)}$$

input
```
int((sech(b*x+a)^2)^(3/2),x)
```

output
```
(e**(4*a + 4*b*x)*atan(e**(a + b*x)) + 2*e**(2*a + 2*b*x)*atan(e**(a + b*x
)) + atan(e**(a + b*x)) + e**(3*a + 3*b*x) - e**(a + b*x))/(b*(e**(4*a + 4
*b*x) + 2*e**(2*a + 2*b*x) + 1))
```

3.27 $\int \sqrt{\operatorname{sech}^2(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 11

$$\int \sqrt{\operatorname{sech}^2(a+bx)}\,dx = \frac{\arcsin(\tanh(a+bx))}{b}$$

output
```
arcsin(tanh(b*x+a))/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 30 vs. 2(11) = 22.

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.73

$$\int \sqrt{\operatorname{sech}^2(a+bx)}\,dx = -\frac{\cot^{-1}(\sinh(a+bx))\cosh(a+bx)\sqrt{\operatorname{sech}^2(a+bx)}}{b}$$

input
```
Integrate[Sqrt[Sech[a + b*x]^2],x]
```

output
```
-((ArcCot[Sinh[a + b*x]]*Cosh[a + b*x]*Sqrt[Sech[a + b*x]^2])/b)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4610, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\operatorname{sech}^2(a+bx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{\sec(ia+ibx)^2}dx$$

$$\Big\downarrow \text{ 4610}$$

$$\frac{\int \frac{1}{\sqrt{1-\tanh^2(a+bx)}}d\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{\arcsin(\tanh(a+bx))}{b}$$

input
```
Int[Sqrt[Sech[a + b*x]^2],x]
```

output
```
ArcSin[Tanh[a + b*x]]/b
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 130, normalized size of antiderivative = 11.82

method	result	size
risch	$\frac{i\ln\left(e^{bx}+ie^{-a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^{2}}}\,\left(1+e^{2bx+2a}\right)e^{-bx-a}}{b}-\frac{i\ln\left(e^{bx}-ie^{-a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^{2}}}\,\left(1+e^{2bx+2a}\right)e^{-bx-a}}{b}$	130

input
```
int((sech(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*ln(exp(b*x)+I*exp(-a))/b*(1+exp(2*b*x+2*a))*(1/(1+exp(2*b*x+2*a))^2*exp(
2*b*x+2*a))^(1/2)*exp(-b*x-a)-I*ln(exp(b*x)-I*exp(-a))/b*(1+exp(2*b*x+2*a)
)*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*exp(-b*x-a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \sqrt{\operatorname{sech}^2(a+bx)}\,dx=\frac{2\arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b}$$

input
```
integrate((sech(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
2*arctan(cosh(b*x + a) + sinh(b*x + a))/b
```

Sympy [F]

$$\int \sqrt{\operatorname{sech}^2(a+bx)}\,dx = \int \sqrt{\operatorname{sech}^2(a+bx)}\,dx$$

input
```
integrate((sech(b*x+a)**2)**(1/2),x)
```

output
```
Integral(sqrt(sech(a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sqrt{\operatorname{sech}^2(a+bx)}\,dx = \frac{\arctan(\sinh(bx+a))}{b}$$

input
```
integrate((sech(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
arctan(sinh(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \sqrt{\operatorname{sech}^2(a+bx)}\,dx = \frac{2\arctan\left(e^{(bx+a)}\right)}{b}$$

input
```
integrate((sech(b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(e^(b*x + a))/b
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\operatorname{sech}^2(a+bx)}\,dx = \int \sqrt{\frac{1}{\cosh{(a+b\,x)}^2}}\,dx$$

input
```
int((1/cosh(a + b*x)^2)^(1/2),x)
```

output
```
int((1/cosh(a + b*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \sqrt{\operatorname{sech}^2(a+bx)}\,dx = \frac{2atan\left(e^{bx+a}\right)}{b}$$

input
```
int((sech(b*x+a)^2)^(1/2),x)
```

output
```
(2*atan(e**(a + b*x)))/b
```

3.28 $\int \frac{1}{\sqrt{\operatorname{sech}^2(a+bx)}} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \frac{1}{\sqrt{\operatorname{sech}^2(a+bx)}} \, dx = \frac{\tanh(a+bx)}{b\sqrt{\operatorname{sech}^2(a+bx)}}$$

output
```
tanh(b*x+a)/b/(sech(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\operatorname{sech}^2(a+bx)}} \, dx = \frac{\tanh(a+bx)}{b\sqrt{\operatorname{sech}^2(a+bx)}}$$

input
```
Integrate[1/Sqrt[Sech[a + b*x]^2],x]
```

output
```
Tanh[a + b*x]/(b*Sqrt[Sech[a + b*x]^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4610, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\operatorname{sech}^2(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sec(ia+ibx)^2}}dx$$

$$\Big\downarrow \text{4610}$$

$$\frac{\int \frac{1}{(1-\tanh^2(a+bx))^{3/2}}d\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{208}$$

$$\frac{\tanh(a+bx)}{b\sqrt{1-\tanh^2(a+bx)}}$$

input
```
Int[1/Sqrt[Sech[a + b*x]^2],x]
```

output
```
Tanh[a + b*x]/(b*Sqrt[1 - Tanh[a + b*x]^2])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(20) = 40$.

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 4.41

method	result	size
risch	$\frac{e^{2bx+2a}}{2b(1+e^{2bx+2a})\sqrt{\frac{e^{2bx+2a}}{(1+e^{2bx+2a})^2}}}-\frac{1}{2b(1+e^{2bx+2a})\sqrt{\frac{e^{2bx+2a}}{(1+e^{2bx+2a})^2}}}$	97

input
```
int(1/(sech(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*exp
(2*b*x+2*a)-1/2/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a
))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{\operatorname{sech}^2(a+bx)}}\,dx=\frac{\sinh{(bx+a)}}{b}$$

input
```
integrate(1/(sech(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
sinh(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{1}{\sqrt{\operatorname{sech}^2(a+bx)}}\,dx = \begin{cases} \frac{\tanh{(a+bx)}}{b\sqrt{\operatorname{sech}^2{(a+bx)}}} & \text{for } b\neq 0 \\ \frac{x}{\sqrt{\operatorname{sech}^2{(a)}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(sech(b*x+a)**2)**(1/2),x)
```

output
```
Piecewise((tanh(a + b*x)/(b*sqrt(sech(a + b*x)**2)), Ne(b, 0)), (x/sqrt(se
ch(a)**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{\operatorname{sech}^2(a+bx)}}\,dx = \frac{e^{(bx+a)}}{2\,b} - \frac{e^{(-bx-a)}}{2\,b}$$

input
```
integrate(1/(sech(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*e^(b*x + a)/b - 1/2*e^(-b*x - a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{\operatorname{sech}^2(a+bx)}}\,dx = \frac{e^{(bx+a)} - e^{(-bx-a)}}{2\,b}$$

input
```
integrate(1/(sech(b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*(e^(b*x + a) - e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int \frac{1}{\sqrt{\operatorname{sech}^2(a+bx)}}\,dx = \frac{e^{-2\,a-2\,b\,x}\,\left(e^{4\,a+4\,b\,x}-1\right)\,\sqrt{\frac{4\,e^{2\,a+2\,b\,x}}{{\left(e^{2\,a+2\,b\,x}+1\right)}^2}}}{4\,b}$$

input
```
int(1/(1/cosh(a + b*x)^2)^(1/2),x)
```

output
```
(exp(- 2*a - 2*b*x)*(exp(4*a + 4*b*x) - 1)*((4*exp(2*a + 2*b*x))/(exp(2*a
+ 2*b*x) + 1)^2)^(1/2))/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{\operatorname{sech}^2(a+bx)}}\,dx = \frac{e^{2bx+2a}-1}{2e^{bx+a}b}$$

input
```
int(1/(sech(b*x+a)^2)^(1/2),x)
```

output
```
(e**(2*a + 2*b*x) - 1)/(2*e**(a + b*x)*b)
```

3.29 $\int \frac{1}{\text{sech}^2(a+bx)^{3/2}} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 51

$$\int \frac{1}{\text{sech}^2(a+bx)^{3/2}} \, dx = \frac{\tanh(a+bx)}{3b\text{sech}^2(a+bx)^{3/2}} + \frac{2\tanh(a+bx)}{3b\sqrt{\text{sech}^2(a+bx)}}$$

output
```
1/3*tanh(b*x+a)/b/(sech(b*x+a)^2)^(3/2)+2/3*tanh(b*x+a)/b/(sech(b*x+a)^2)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int \frac{1}{\text{sech}^2(a+bx)^{3/2}} \, dx = \frac{3\text{sech}^2(a+bx)\tanh(a+bx) + \tanh^3(a+bx)}{3b\text{sech}^2(a+bx)^{3/2}}$$

input
```
Integrate[(Sech[a + b*x]^2)^(-3/2),x]
```

output
```
(3*Sech[a + b*x]^2*Tanh[a + b*x] + Tanh[a + b*x]^3)/(3*b*(Sech[a + b*x]^2)
^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4610, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(\sec(ia+ibx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 4610$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(a+bx)\right)^{5/2}}d\tanh(a+bx)}{b}$$

$$\Big\downarrow \; 209$$

$$\frac{\frac{2}{3}\int \frac{1}{\left(1-\tanh^2(a+bx)\right)^{3/2}}d\tanh(a+bx)+\frac{\tanh(a+bx)}{3\left(1-\tanh^2(a+bx)\right)^{3/2}}}{b}$$

$$\Big\downarrow \; 208$$

$$\frac{\frac{2\tanh(a+bx)}{3\sqrt{1-\tanh^2(a+bx)}}+\frac{\tanh(a+bx)}{3\left(1-\tanh^2(a+bx)\right)^{3/2}}}{b}$$

input
```
Int[(Sech[a + b*x]^2)^(-3/2),x]
```

output
```
(Tanh[a + b*x]/(3*(1 - Tanh[a + b*x]^2)^(3/2)) + (2*Tanh[a + b*x])/(3*Sqrt
[1 - Tanh[a + b*x]^2]))/b
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(43) = 86$.

Time = 0.25 (sec) , antiderivative size = 201, normalized size of antiderivative = 3.94

method	result
risch	$\frac{e^{4bx+4a}}{24b(1+e^{2bx+2a})\sqrt{\frac{e^{2bx+2a}}{(1+e^{2bx+2a})^2}}}+\frac{3\,e^{2bx+2a}}{8b(1+e^{2bx+2a})\sqrt{\frac{e^{2bx+2a}}{(1+e^{2bx+2a})^2}}}-\frac{3}{8b(1+e^{2bx+2a})\sqrt{\frac{e^{2bx+2a}}{(1+e^{2bx+2a})^2}}}-\frac{e^{-2bx}}{24b(1+e^{2bx+2a})}$

input
```
int(1/(sech(b*x+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*ex
p(4*b*x+4*a)+3/8/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*
a))^(1/2)*exp(2*b*x+2*a)-3/8/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*
exp(2*b*x+2*a))^(1/2)-1/24/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*ex
p(2*b*x+2*a))^(1/2)*exp(-2*b*x-2*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{3/2}}\,dx = \frac{\sinh(bx+a)^3 + 3\left(\cosh(bx+a)^2+3\right)\sinh(bx+a)}{12\,b}$$

input
```
integrate(1/(sech(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/12*(sinh(b*x + a)^3 + 3*(cosh(b*x + a)^2 + 3)*sinh(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{3/2}}\,dx = \begin{cases} -\frac{2\tanh^3(a+bx)}{3b(\operatorname{sech}^2(a+bx))^{\frac{3}{2}}} + \frac{\tanh(a+bx)}{b(\operatorname{sech}^2(a+bx))^{\frac{3}{2}}} & \text{for } b \neq 0 \\ \frac{x}{(\operatorname{sech}^2(a))^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(sech(b*x+a)**2)**(3/2),x)
```

output
```
Piecewise((-2*tanh(a + b*x)**3/(3*b*(sech(a + b*x)**2)**(3/2)) + tanh(a +
b*x)/(b*(sech(a + b*x)**2)**(3/2)), Ne(b, 0)), (x/(sech(a)**2)**(3/2), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{3/2}}\,dx = \frac{e^{(3\,bx+3\,a)}}{24\,b} + \frac{3\,e^{(bx+a)}}{8\,b} - \frac{3\,e^{(-bx-a)}}{8\,b} - \frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

input
```
integrate(1/(sech(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/24*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b - 3/8*e^(-b*x - a)/b - 1/24*e^(
-3*b*x - 3*a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{3/2}}\,dx = -\frac{\left(9\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}-e^{(3\,bx+3\,a)}-9\,e^{(bx+a)}}{24\,b}$$

input
```
integrate(1/(sech(b*x+a)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/24*((9*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) - e^(3*b*x + 3*a) - 9*e^(b
*x + a))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{3/2}}\,dx = \int \frac{1}{{\left(\frac{1}{{\cosh\left(a+b\,x\right)}^2}\right)}^{3/2}}\,dx$$

input
```
int(1/(1/cosh(a + b*x)^2)^(3/2),x)
```

output
```
int(1/(1/cosh(a + b*x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{3/2}}\,dx = \frac{e^{6bx+6a}+9e^{4bx+4a}-9e^{2bx+2a}-1}{24e^{3bx+3a}b}$$

input
```
int(1/(sech(b*x+a)^2)^(3/2),x)
```

output
```
(e**(6*a + 6*b*x) + 9*e**(4*a + 4*b*x) - 9*e**(2*a + 2*b*x) - 1)/(24*e**(3
*a + 3*b*x)*b)
```

3.30 $\int \frac{1}{\operatorname{sech}^2(a+bx)^{5/2}} dx$

Optimal result

Integrand size = 12, antiderivative size = 76

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{5/2}} dx = \frac{\tanh(a+bx)}{5b\operatorname{sech}^2(a+bx)^{5/2}} + \frac{4\tanh(a+bx)}{15b\operatorname{sech}^2(a+bx)^{3/2}} + \frac{8\tanh(a+bx)}{15b\sqrt{\operatorname{sech}^2(a+bx)}}$$

output
```
1/5*tanh(b*x+a)/b/(sech(b*x+a)^2)^(5/2)+4/15*tanh(b*x+a)/b/(sech(b*x+a)^2)
^(3/2)+8/15*tanh(b*x+a)/b/(sech(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.62

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{5/2}} dx = \frac{(15+10\sinh^2(a+bx)+3\sinh^4(a+bx))\tanh(a+bx)}{15b\sqrt{\operatorname{sech}^2(a+bx)}}$$

input
```
Integrate[(Sech[a + b*x]^2)^(-5/2),x]
```

output
```
((15 + 10*Sinh[a + b*x]^2 + 3*Sinh[a + b*x]^4)*Tanh[a + b*x])/(15*b*Sqrt[S
ech[a + b*x]^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 4610, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{sech}^2(a+bx)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\sec(ia+ibx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4610}$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(a+bx)\right)^{7/2}}d\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{209}$$

$$\frac{\frac{4}{5}\int \frac{1}{\left(1-\tanh^2(a+bx)\right)^{5/2}}d\tanh(a+bx)+\frac{\tanh(a+bx)}{5\left(1-\tanh^2(a+bx)\right)^{5/2}}}{b}$$

$$\Big\downarrow \text{209}$$

$$\frac{\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{\left(1-\tanh^2(a+bx)\right)^{3/2}}d\tanh(a+bx)+\frac{\tanh(a+bx)}{3\left(1-\tanh^2(a+bx)\right)^{3/2}}\right)+\frac{\tanh(a+bx)}{5\left(1-\tanh^2(a+bx)\right)^{5/2}}}{b}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{\tanh(a+bx)}{5\left(1-\tanh^2(a+bx)\right)^{5/2}}+\frac{4}{5}\left(\frac{2\tanh(a+bx)}{3\sqrt{1-\tanh^2(a+bx)}}+\frac{\tanh(a+bx)}{3\left(1-\tanh^2(a+bx)\right)^{3/2}}\right)}{b}$$

input
```
Int[(Sech[a + b*x]^2)^(-5/2),x]
```

output
```
(Tanh[a + b*x]/(5*(1 - Tanh[a + b*x]^2)^(5/2)) + (4*(Tanh[a + b*x]/(3*(1 -
 Tanh[a + b*x]^2)^(3/2)) + (2*Tanh[a + b*x])/(3*Sqrt[1 - Tanh[a + b*x]^2])
))/5)/b
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(64) = 128$.

Time = 0.26 (sec) , antiderivative size = 305, normalized size of antiderivative = 4.01

method	result
risch	$\frac{e^{6bx+6a}}{160b\left(1+e^{2bx+2a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}}+\frac{5\,e^{4bx+4a}}{96b\left(1+e^{2bx+2a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}}+\frac{5\,e^{2bx+2a}}{16b\left(1+e^{2bx+2a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}}-\frac{}{16b(1+e^{2bx+2a}}$

input
```
int(1/(sech(b*x+a)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/160/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*e
xp(6*b*x+6*a)+5/96/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+
2*a))^(1/2)*exp(4*b*x+4*a)+5/16/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))
^2*exp(2*b*x+2*a))^(1/2)*exp(2*b*x+2*a)-5/16/b/(1+exp(2*b*x+2*a))/(1/(1+ex
p(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)-5/96/b/(1+exp(2*b*x+2*a))/(1/(1+exp(
2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*exp(-2*b*x-2*a)-1/160/b/(1+exp(2*b*x+2
*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*exp(-4*b*x-4*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{5/2}}\,dx = \frac{3\sinh(bx+a)^5 + 5\left(6\cosh(bx+a)^2+5\right)\sinh(bx+a)^3 + 15\left(\cosh(bx+a)^4 + \right.}{240\,b}$$

input
```
integrate(1/(sech(b*x+a)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/240*(3*sinh(b*x + a)^5 + 5*(6*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^3 + 15*
(cosh(b*x + a)^4 + 5*cosh(b*x + a)^2 + 10)*sinh(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{5/2}}\,dx = \begin{cases} \frac{8\tanh^5(a+bx)}{15b\left(\operatorname{sech}^2(a+bx)\right)^{\frac{5}{2}}} - \frac{4\tanh^3(a+bx)}{3b\left(\operatorname{sech}^2(a+bx)\right)^{\frac{5}{2}}} + \frac{\tanh(a+bx)}{b\left(\operatorname{sech}^2(a+bx)\right)^{\frac{5}{2}}} & \text{for } b \neq 0 \\ \frac{x}{\left(\operatorname{sech}^2(a)\right)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(sech(b*x+a)**2)**(5/2),x)
```

output
```
Piecewise((8*tanh(a + b*x)**5/(15*b*(sech(a + b*x)**2)**(5/2)) - 4*tanh(a
+ b*x)**3/(3*b*(sech(a + b*x)**2)**(5/2)) + tanh(a + b*x)/(b*(sech(a + b*x
)**2)**(5/2)), Ne(b, 0)), (x/(sech(a)**2)**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{5/2}}\,dx = \frac{e^{(5\,bx+5\,a)}}{160\,b} + \frac{5\,e^{(3\,bx+3\,a)}}{96\,b} + \frac{5\,e^{(bx+a)}}{16\,b} - \frac{5\,e^{(-bx-a)}}{16\,b} - \frac{5\,e^{(-3\,bx-3\,a)}}{96\,b} - \frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

input
```
integrate(1/(sech(b*x+a)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/160*e^(5*b*x + 5*a)/b + 5/96*e^(3*b*x + 3*a)/b + 5/16*e^(b*x + a)/b - 5/
16*e^(-b*x - a)/b - 5/96*e^(-3*b*x - 3*a)/b - 1/160*e^(-5*b*x - 5*a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{5/2}}\,dx = -\frac{\left(150\,e^{(4\,bx+4\,a)} + 25\,e^{(2\,bx+2\,a)} + 3\right)e^{(-5\,bx-5\,a)} - 3\,e^{(5\,bx+5\,a)} - 25\,e^{(3\,bx+3\,a)} - 150\,e^{(bx+a)}}{480\,b}$$

input
```
integrate(1/(sech(b*x+a)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/480*((150*e^(4*b*x + 4*a) + 25*e^(2*b*x + 2*a) + 3)*e^(-5*b*x - 5*a) -
3*e^(5*b*x + 5*a) - 25*e^(3*b*x + 3*a) - 150*e^(b*x + a))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{5/2}}\,dx = \int \frac{1}{\left(\frac{1}{\cosh(a+b\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(1/cosh(a + b*x)^2)^(5/2),x)
```

output
```
int(1/(1/cosh(a + b*x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{5/2}}\,dx = \frac{3e^{10bx+10a} + 25e^{8bx+8a} + 150e^{6bx+6a} - 150e^{4bx+4a} - 25e^{2bx+2a} - 3}{480e^{5bx+5a}b}$$

input
```
int(1/(sech(b*x+a)^2)^(5/2),x)
```

output
```
(3*e**(10*a + 10*b*x) + 25*e**(8*a + 8*b*x) + 150*e**(6*a + 6*b*x) - 150*e
**(4*a + 4*b*x) - 25*e**(2*a + 2*b*x) - 3)/(480*e**(5*a + 5*b*x)*b)
```

3.31 $\int \frac{1}{\operatorname{sech}^2(a+bx)^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 101

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{7/2}}\,dx=\frac{\tanh(a+bx)}{7b\operatorname{sech}^2(a+bx)^{7/2}}+\frac{6\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{5/2}}+\frac{8\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{3/2}}+\frac{16\tanh(a+bx)}{35b\sqrt{\operatorname{sech}^2(a+bx)}}$$

output
```
1/7*tanh(b*x+a)/b/(sech(b*x+a)^2)^(7/2)+6/35*tanh(b*x+a)/b/(sech(b*x+a)^2)
^(5/2)+8/35*tanh(b*x+a)/b/(sech(b*x+a)^2)^(3/2)+16/35*tanh(b*x+a)/b/(sech(
b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.56

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{7/2}}\,dx=\frac{(35+35\sinh^2(a+bx)+21\sinh^4(a+bx)+5\sinh^6(a+bx))\tanh(a+bx)}{35b\sqrt{\operatorname{sech}^2(a+bx)}}$$

input
```
Integrate[(Sech[a + b*x]^2)^(-7/2),x]
```

output
```
((35 + 35*Sinh[a + b*x]^2 + 21*Sinh[a + b*x]^4 + 5*Sinh[a + b*x]^6)*Tanh[a
 + b*x])/(35*b*Sqrt[Sech[a + b*x]^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4610, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\text{sech}^2(a+bx)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\sec(ia+ibx)^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{4610}$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(a+bx)\right)^{9/2}}d\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{209}$$

$$\frac{\frac{6}{7}\int \frac{1}{\left(1-\tanh^2(a+bx)\right)^{7/2}}d\tanh(a+bx)+\frac{\tanh(a+bx)}{7\left(1-\tanh^2(a+bx)\right)^{7/2}}}{b}$$

$$\Big\downarrow \text{209}$$

$$\frac{\frac{6}{7}\left(\frac{4}{5}\int \frac{1}{\left(1-\tanh^2(a+bx)\right)^{5/2}}d\tanh(a+bx)+\frac{\tanh(a+bx)}{5\left(1-\tanh^2(a+bx)\right)^{5/2}}\right)+\frac{\tanh(a+bx)}{7\left(1-\tanh^2(a+bx)\right)^{7/2}}}{b}$$

$$\Big\downarrow \text{209}$$

$$\frac{\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{\left(1-\tanh^2(a+bx)\right)^{3/2}}d\tanh(a+bx)+\frac{\tanh(a+bx)}{3\left(1-\tanh^2(a+bx)\right)^{3/2}}\right)+\frac{\tanh(a+bx)}{5\left(1-\tanh^2(a+bx)\right)^{5/2}}\right)+\frac{\tanh(a+bx)}{7\left(1-\tanh^2(a+bx)\right)^{7/2}}}{b}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{\tanh(a+bx)}{7(1-\tanh^2(a+bx))^{7/2}}+\frac{6}{7}\left(\frac{\tanh(a+bx)}{5(1-\tanh^2(a+bx))^{5/2}}+\frac{4}{5}\left(\frac{2\tanh(a+bx)}{3\sqrt{1-\tanh^2(a+bx)}}+\frac{\tanh(a+bx)}{3(1-\tanh^2(a+bx))^{3/2}}\right)\right)}{b}$$

input
```
Int[(Sech[a + b*x]^2)^(-7/2),x]
```

output
```
(Tanh[a + b*x]/(7*(1 - Tanh[a + b*x]^2)^(7/2)) + (6*(Tanh[a + b*x]/(5*(1 -
 Tanh[a + b*x]^2)^(5/2)) + (4*(Tanh[a + b*x]/(3*(1 - Tanh[a + b*x]^2)^(3/2
)) + (2*Tanh[a + b*x])/(3*Sqrt[1 - Tanh[a + b*x]^2])))/5))/7)/b
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(85) = 170$.

Time = 0.30 (sec) , antiderivative size = 409, normalized size of antiderivative = 4.05

method	result
risch	$\frac{e^{8bx+8a}}{896b(1+e^{2bx+2a})\sqrt{\frac{e^{2bx+2a}}{(1+e^{2bx+2a})^2}}} + \frac{7\,e^{6bx+6a}}{640b(1+e^{2bx+2a})\sqrt{\frac{e^{2bx+2a}}{(1+e^{2bx+2a})^2}}} + \frac{7\,e^{4bx+4a}}{128b(1+e^{2bx+2a})\sqrt{\frac{e^{2bx+2a}}{(1+e^{2bx+2a})^2}}} + \frac{3}{128b(1+e^{2b}}$

input
```
int(1/(sech(b*x+a)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/896/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*e
xp(8*b*x+8*a)+7/640/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x
+2*a))^(1/2)*exp(6*b*x+6*a)+7/128/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a
))^2*exp(2*b*x+2*a))^(1/2)*exp(4*b*x+4*a)+35/128/b/(1+exp(2*b*x+2*a))/(1/(
1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*exp(2*b*x+2*a)-35/128/b/(1+exp(2
*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)-7/128/b/(1+exp(2*
b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*exp(-2*b*x-2*a)-7/
640/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*exp
(-4*b*x-4*a)-1/896/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+
2*a))^(1/2)*exp(-6*b*x-6*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.07

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{7/2}}\,dx = \frac{5\,\sinh(bx+a)^7 + 7\left(15\,\cosh(bx+a)^2+7\right)\sinh(bx+a)^5 + 35\left(5\,\cosh(bx+a)^4\right.}{}$$

input
```
integrate(1/(sech(b*x+a)^2)^(7/2),x, algorithm="fricas")
```

output
```
1/2240*(5*sinh(b*x + a)^7 + 7*(15*cosh(b*x + a)^2 + 7)*sinh(b*x + a)^5 + 3
5*(5*cosh(b*x + a)^4 + 14*cosh(b*x + a)^2 + 7)*sinh(b*x + a)^3 + 35*(cosh(
b*x + a)^6 + 7*cosh(b*x + a)^4 + 21*cosh(b*x + a)^2 + 35)*sinh(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 18.51 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{7/2}}\,dx = \begin{cases} -\frac{16\tanh^7(a+bx)}{35b(\operatorname{sech}^2(a+bx))^{\frac{7}{2}}} + \frac{8\tanh^5(a+bx)}{5b(\operatorname{sech}^2(a+bx))^{\frac{7}{2}}} - \frac{2\tanh^3(a+bx)}{b(\operatorname{sech}^2(a+bx))^{\frac{7}{2}}} + \frac{\tanh(a+bx)}{b(\operatorname{sech}^2(a+bx))^{\frac{7}{2}}} & \text{for } b \neq 0 \\ \frac{x}{(\operatorname{sech}^2(a))^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(sech(b*x+a)**2)**(7/2),x)
```

output
```
Piecewise((-16*tanh(a + b*x)**7/(35*b*(sech(a + b*x)**2)**(7/2)) + 8*tanh(
a + b*x)**5/(5*b*(sech(a + b*x)**2)**(7/2)) - 2*tanh(a + b*x)**3/(b*(sech(
a + b*x)**2)**(7/2)) + tanh(a + b*x)/(b*(sech(a + b*x)**2)**(7/2)), Ne(b,
0)), (x/(sech(a)**2)**(7/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{7/2}}\,dx = \frac{\left(49\,e^{(-2\,bx-2\,a)} + 245\,e^{(-4\,bx-4\,a)} + 1225\,e^{(-6\,bx-6\,a)} + 5\right)e^{(7\,bx+7\,a)}}{4480\,b} - \frac{1225\,e^{(-bx-a)} + 245\,e^{(-3\,bx-3\,a)} + 49\,e^{(-5\,bx-5\,a)} + 5\,e^{(-7\,bx-7\,a)}}{4480\,b}$$

input
```
integrate(1/(sech(b*x+a)^2)^(7/2),x, algorithm="maxima")
```

output
```
1/4480*(49*e^(-2*b*x - 2*a) + 245*e^(-4*b*x - 4*a) + 1225*e^(-6*b*x - 6*a)
 + 5)*e^(7*b*x + 7*a)/b - 1/4480*(1225*e^(-b*x - a) + 245*e^(-3*b*x - 3*a)
 + 49*e^(-5*b*x - 5*a) + 5*e^(-7*b*x - 7*a))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{7/2}}\,dx = -\frac{\left(1225\,e^{(6\,bx+6\,a)}+245\,e^{(4\,bx+4\,a)}+49\,e^{(2\,bx+2\,a)}+5\right)e^{(-7\,bx-7\,a)}-5\,e^{(7\,bx+7\,a)}-49\,e^{(5\,bx+5\,a)}-245\,e^{(3\,bx+3\,a)}}{4480\,b}$$

input
```
integrate(1/(sech(b*x+a)^2)^(7/2),x, algorithm="giac")
```

output
```
-1/4480*((1225*e^(6*b*x + 6*a) + 245*e^(4*b*x + 4*a) + 49*e^(2*b*x + 2*a)
+ 5)*e^(-7*b*x - 7*a) - 5*e^(7*b*x + 7*a) - 49*e^(5*b*x + 5*a) - 245*e^(3*
b*x + 3*a) - 1225*e^(b*x + a))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{7/2}}\,dx = \int \frac{1}{\left(\frac{1}{\cosh(a+b\,x)^2}\right)^{7/2}}\,dx$$

input
```
int(1/(1/cosh(a + b*x)^2)^(7/2),x)
```

output
```
int(1/(1/cosh(a + b*x)^2)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{7/2}}\,dx = \frac{5e^{14bx+14a}+49e^{12bx+12a}+245e^{10bx+10a}+1225e^{8bx+8a}-1225e^{6bx+6a}-245e^{4bx+4a}}{4480e^{7bx+7a}b}$$

input
```
int(1/(sech(b*x+a)^2)^(7/2),x)
```

output

```
(5*e**(14*a + 14*b*x) + 49*e**(12*a + 12*b*x) + 245*e**(10*a + 10*b*x) + 1
225*e**(8*a + 8*b*x) - 1225*e**(6*a + 6*b*x) - 245*e**(4*a + 4*b*x) - 49*e
**(2*a + 2*b*x) - 5)/(4480*e**(7*a + 7*b*x)*b)
```

3.32 $\int \left(a\text{sech}^2(x)\right)^{5/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 65

$$\int \left(a\text{sech}^2(x)\right)^{5/2}\,dx = \frac{3}{8}a^{5/2}\arctan\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right) + \frac{3}{8}a^2\sqrt{a\text{sech}^2(x)}\tanh(x) + \frac{1}{4}a\left(a\text{sech}^2(x)\right)^{3/2}\tanh(x)$$

output
```
3/8*a^(5/2)*arctan(a^(1/2)*tanh(x)/(a*sech(x)^2)^(1/2))+3/8*a^2*(a*sech(x)
^2)^(1/2)*tanh(x)+1/4*a*(a*sech(x)^2)^(3/2)*tanh(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

$$\int \left(a\text{sech}^2(x)\right)^{5/2}\,dx = \frac{1}{8}\cosh(x)\left(a\text{sech}^2(x)\right)^{5/2}\left(3\arctan(\sinh(x))\cosh^4(x)+2\sinh(x)+3\cosh^2(x)\sinh(x\right.$$

input
```
Integrate[(a*Sech[x]^2)^(5/2),x]
```

output
```
(Cosh[x]*(a*Sech[x]^2)^(5/2)*(3*ArcTan[Sinh[x]]*Cosh[x]^4 + 2*Sinh[x] + 3*
Cosh[x]^2*Sinh[x]))/8
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4610, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\text{sech}^2(x)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sec(ix)^2\right)^{5/2} dx$$

$$\Big\downarrow \text{4610}$$

$$a\int \left(a-a\tanh^2(x)\right)^{3/2} d\tanh(x)$$

$$\Big\downarrow \text{211}$$

$$a\left(\frac{3}{4}a\int \sqrt{a-a\tanh^2(x)}d\tanh(x)+\frac{1}{4}\tanh(x)\left(a-a\tanh^2(x)\right)^{3/2}\right)$$

$$\Big\downarrow \text{211}$$

$$a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{a-a\tanh^2(x)}}d\tanh(x)+\frac{1}{2}\tanh(x)\sqrt{a-a\tanh^2(x)}\right)+\frac{1}{4}\tanh(x)\left(a-a\tanh^2(x)\right)^{3/2}\right)$$

$$\Big\downarrow \text{224}$$

$$a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\frac{a\tanh^2(x)}{a-a\tanh^2(x)}+1}d\frac{\tanh(x)}{\sqrt{a-a\tanh^2(x)}}+\frac{1}{2}\tanh(x)\sqrt{a-a\tanh^2(x)}\right)+\frac{1}{4}\tanh(x)\left(a-a\tanh^2(x)\right)^{3/2}\right)$$

$$\Big\downarrow \text{216}$$

$$a\left(\frac{3}{4}a\left(\frac{1}{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-a\tanh^2(x)}}\right)+\frac{1}{2}\tanh(x)\sqrt{a-a\tanh^2(x)}\right)+\frac{1}{4}\tanh(x)\left(a-a\tanh^2(x)\right)^{3/2}\right)$$

input
```
Int[(a*Sech[x]^2)^(5/2),x]
```

output
```
a*((Tanh[x]*(a - a*Tanh[x]^2)^(3/2))/4 + (3*a*((Sqrt[a]*ArcTan[(Sqrt[a]*Ta
nh[x])/Sqrt[a - a*Tanh[x]^2]])/2 + (Tanh[x]*Sqrt[a - a*Tanh[x]^2])/2))/4)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.44 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.95

method	result
risch	$\frac{a^{2} \sqrt{\frac{{\mathrm e}^{2 x} a}{\left({\mathrm e}^{2 x}+1\right)^{2}}}\, \left(3 \,{\mathrm e}^{6 x}+11 \,{\mathrm e}^{4 x}-11 \,{\mathrm e}^{2 x}-3\right)}{4 \left({\mathrm e}^{2 x}+1\right)^{3}}+\frac{3 i a^{2} {\mathrm e}^{-x} \left({\mathrm e}^{2 x}+1\right) \sqrt{\frac{{\mathrm e}^{2 x} a}{\left({\mathrm e}^{2 x}+1\right)^{2}}}\, \ln \left({\mathrm e}^{x}+i\right)}{8}-\frac{3 i a^{2} {\mathrm e}^{-x} \left({\mathrm e}^{2 x}+1\right) \sqrt{\frac{{\mathrm e}^{2 x} a}{\left({\mathrm e}^{2 x}+1\right)^{2}}}\, \ln \left({\mathrm e}^{x}-i\right)}{8}$

input
```
int((sech(x)^2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2/(exp(2*x)+1)^3*(exp(2*x)*a/(exp(2*x)+1)^2)^(1/2)*(3*exp(6*x)+11*ex
p(4*x)-11*exp(2*x)-3)+3/8*I*a^2*exp(-x)*(exp(2*x)+1)*(exp(2*x)*a/(exp(2*x)
+1)^2)^(1/2)*ln(exp(x)+I)-3/8*I*a^2*exp(-x)*(exp(2*x)+1)*(exp(2*x)*a/(exp(
2*x)+1)^2)^(1/2)*ln(exp(x)-I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1082 vs. 2(49) = 98.

Time = 0.10 (sec) , antiderivative size = 1082, normalized size of antiderivative = 16.65

$$\int \left(a \operatorname{sech}^2(x)\right)^{5/2}\, dx = \text{Too large to display}$$

input
```
integrate((a*sech(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/4*(3*a^2*cosh(x)^7 + 3*(a^2*e^(2*x) + a^2)*sinh(x)^7 + 11*a^2*cosh(x)^5
+ 21*(a^2*cosh(x)*e^(2*x) + a^2*cosh(x))*sinh(x)^6 + (63*a^2*cosh(x)^2 + 1
1*a^2 + (63*a^2*cosh(x)^2 + 11*a^2)*e^(2*x))*sinh(x)^5 - 11*a^2*cosh(x)^3
+ 5*(21*a^2*cosh(x)^3 + 11*a^2*cosh(x) + (21*a^2*cosh(x)^3 + 11*a^2*cosh(x
))*e^(2*x))*sinh(x)^4 + (105*a^2*cosh(x)^4 + 110*a^2*cosh(x)^2 - 11*a^2 +
(105*a^2*cosh(x)^4 + 110*a^2*cosh(x)^2 - 11*a^2)*e^(2*x))*sinh(x)^3 - 3*a^
2*cosh(x) + (63*a^2*cosh(x)^5 + 110*a^2*cosh(x)^3 - 33*a^2*cosh(x) + (63*a
^2*cosh(x)^5 + 110*a^2*cosh(x)^3 - 33*a^2*cosh(x))*e^(2*x))*sinh(x)^2 + 3*
(a^2*cosh(x)^8 + (a^2*e^(2*x) + a^2)*sinh(x)^8 + 4*a^2*cosh(x)^6 + 8*(a^2*
cosh(x)*e^(2*x) + a^2*cosh(x))*sinh(x)^7 + 4*(7*a^2*cosh(x)^2 + a^2 + (7*a
^2*cosh(x)^2 + a^2)*e^(2*x))*sinh(x)^6 + 6*a^2*cosh(x)^4 + 8*(7*a^2*cosh(x
)^3 + 3*a^2*cosh(x) + (7*a^2*cosh(x)^3 + 3*a^2*cosh(x))*e^(2*x))*sinh(x)^5
 + 2*(35*a^2*cosh(x)^4 + 30*a^2*cosh(x)^2 + 3*a^2 + (35*a^2*cosh(x)^4 + 30
*a^2*cosh(x)^2 + 3*a^2)*e^(2*x))*sinh(x)^4 + 4*a^2*cosh(x)^2 + 8*(7*a^2*co
sh(x)^5 + 10*a^2*cosh(x)^3 + 3*a^2*cosh(x) + (7*a^2*cosh(x)^5 + 10*a^2*cos
h(x)^3 + 3*a^2*cosh(x))*e^(2*x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 + 15*a^2*c
osh(x)^4 + 9*a^2*cosh(x)^2 + a^2 + (7*a^2*cosh(x)^6 + 15*a^2*cosh(x)^4 + 9
*a^2*cosh(x)^2 + a^2)*e^(2*x))*sinh(x)^2 + a^2 + (a^2*cosh(x)^8 + 4*a^2*co
sh(x)^6 + 6*a^2*cosh(x)^4 + 4*a^2*cosh(x)^2 + a^2)*e^(2*x) + 8*(a^2*cosh(x
)^7 + 3*a^2*cosh(x)^5 + 3*a^2*cosh(x)^3 + a^2*cosh(x) + (a^2*cosh(x)^7 ...
```

Sympy [F]

$$\int \left(a\mathrm{sech}^2(x)\right)^{5/2}\,dx = \int \left(a\,\mathrm{sech}^2\left(x\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*sech(x)**2)**(5/2),x)
```

output
```
Integral((a*sech(x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int \left(a\text{sech}^2(x)\right)^{5/2} dx = \frac{3}{4}a^{\frac{5}{2}}\arctan\left(e^x\right) + \frac{3a^{\frac{5}{2}}e^{(7x)} + 11a^{\frac{5}{2}}e^{(5x)} - 11a^{\frac{5}{2}}e^{(3x)} - 3a^{\frac{5}{2}}e^x}{4\left(e^{(8x)} + 4e^{(6x)} + 6e^{(4x)} + 4e^{(2x)} + 1\right)}$$

input
```
integrate((a*sech(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
3/4*a^(5/2)*arctan(e^x) + 1/4*(3*a^(5/2)*e^(7*x) + 11*a^(5/2)*e^(5*x) - 11
*a^(5/2)*e^(3*x) - 3*a^(5/2)*e^x)/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(
2*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \left(a\text{sech}^2(x)\right)^{5/2} dx = \frac{1}{16}\left(3\pi - \frac{4\left(3\left(e^{(-x)} - e^x\right)^3 + 20e^{(-x)} - 20e^x\right)}{\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)^2} + 6\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)\right.$$

input
```
integrate((a*sech(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/16*(3*pi - 4*(3*(e^(-x) - e^x)^3 + 20*e^(-x) - 20*e^x)/((e^(-x) - e^x)^2
 + 4)^2 + 6*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*a^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\text{sech}^2(x)\right)^{5/2} dx = \int \left(\frac{a}{\cosh(x)^2}\right)^{5/2} dx$$

input
```
int((a/cosh(x)^2)^(5/2),x)
```

output
```
int((a/cosh(x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.77

$$\int \left(a\text{sech}^2(x)\right)^{5/2} dx = \frac{\sqrt{a}\, a^2(3e^{8x}atan(e^x)+12e^{6x}atan(e^x)+18e^{4x}atan(e^x)+12e^{2x}atan(e^x)+3atan(e^x)}{4e^{8x}+16e^{6x}+24e^{4x}+16e^{2x}+4}$$

input
```
int((a*sech(x)^2)^(5/2),x)
```

output
```
(sqrt(a)*a**2*(3*e**(8*x)*atan(e**x) + 12*e**(6*x)*atan(e**x) + 18*e**(4*x
)*atan(e**x) + 12*e**(2*x)*atan(e**x) + 3*atan(e**x) + 3*e**(7*x) + 11*e**
(5*x) - 11*e**(3*x) - 3*e**x))/(4*(e**(8*x) + 4*e**(6*x) + 6*e**(4*x) + 4*
e**(2*x) + 1))
```

3.33 $\int \left(a\,\text{sech}^2(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \left(a\,\text{sech}^2(x)\right)^{3/2}\,dx=\frac{1}{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\,\text{sech}^2(x)}}\right)+\frac{1}{2}a\sqrt{a\,\text{sech}^2(x)}\tanh(x)$$

output
```
1/2*a^(3/2)*arctan(a^(1/2)*tanh(x)/(a*sech(x)^2)^(1/2))+1/2*a*(a*sech(x)^2
)^(1/2)*tanh(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int \left(a\,\text{sech}^2(x)\right)^{3/2}\,dx=\frac{1}{2}a\sqrt{a\,\text{sech}^2(x)}(\arctan(\sinh(x))\cosh(x)+\tanh(x))$$

input
```
Integrate[(a*Sech[x]^2)^(3/2),x]
```

output
```
(a*Sqrt[a*Sech[x]^2]*(ArcTan[Sinh[x]]*Cosh[x] + Tanh[x]))/2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4610, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\text{sech}^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sec(ix)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{4610}$$

$$a\int \sqrt{a-a\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{211}$$

$$a\left(\frac{1}{2}a\int \frac{1}{\sqrt{a-a\tanh^2(x)}}d\tanh(x)+\frac{1}{2}\tanh(x)\sqrt{a-a\tanh^2(x)}\right)$$

$$\Big\downarrow \text{224}$$

$$a\left(\frac{1}{2}a\int \frac{1}{\frac{a\tanh^2(x)}{a-a\tanh^2(x)}+1}d\frac{\tanh(x)}{\sqrt{a-a\tanh^2(x)}}+\frac{1}{2}\tanh(x)\sqrt{a-a\tanh^2(x)}\right)$$

$$\Big\downarrow \text{216}$$

$$a\left(\frac{1}{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-a\tanh^2(x)}}\right)+\frac{1}{2}\tanh(x)\sqrt{a-a\tanh^2(x)}\right)$$

input
```
Int[(a*Sech[x]^2)^(3/2),x]
```

output
```
a*((Sqrt[a]*ArcTan[(Sqrt[a]*Tanh[x])/Sqrt[a - a*Tanh[x]^2]])/2 + (Tanh[x]*
Sqrt[a - a*Tanh[x]^2])/2)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.30

method	result	size
risch	$\frac{a\sqrt{\frac{e^{2x}a}{\left(e^{2x}+1\right)^{2}}}\,\left(e^{2x}-1\right)}{e^{2x}+1}+\frac{i a\,e^{-x}\left(e^{2x}+1\right)\sqrt{\frac{e^{2x}a}{\left(e^{2x}+1\right)^{2}}}\,\ln\left(e^{x}+i\right)}{2}-\frac{i a\,e^{-x}\left(e^{2x}+1\right)\sqrt{\frac{e^{2x}a}{\left(e^{2x}+1\right)^{2}}}\,\ln\left(e^{x}-i\right)}{2}$	106

input
```
int((sech(x)^2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a/(exp(2*x)+1)*(exp(2*x)*a/(exp(2*x)+1)^2)^(1/2)*(exp(2*x)-1)+1/2*I*a*exp(
-x)*(exp(2*x)+1)*(exp(2*x)*a/(exp(2*x)+1)^2)^(1/2)*ln(exp(x)+I)-1/2*I*a*ex
p(-x)*(exp(2*x)+1)*(exp(2*x)*a/(exp(2*x)+1)^2)^(1/2)*ln(exp(x)-I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(34) = 68$.

Time = 0.09 (sec) , antiderivative size = 310, normalized size of antiderivative = 6.74

$$\int \left(a\mathrm{sech}^2(x)\right)^{3/2}\,dx = \frac{\left(a\cosh\left(x\right)\right)^3 + \left(ae^{(2\,x)} + a\right)\sinh\left(x\right)^3 + 3\left(a\cosh\left(x\right)e^{(2\,x)} + a\cosh\left(x\right)\right)\sinh\left(x\right)^2 +}{}$$

input
```
integrate((a*sech(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
(a*cosh(x)^3 + (a*e^(2*x) + a)*sinh(x)^3 + 3*(a*cosh(x)*e^(2*x) + a*cosh(x
))*sinh(x)^2 + (a*cosh(x)^4 + (a*e^(2*x) + a)*sinh(x)^4 + 4*(a*cosh(x)*e^(
2*x) + a*cosh(x))*sinh(x)^3 + 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 + (3*a*cosh
(x)^2 + a)*e^(2*x) + a)*sinh(x)^2 + (a*cosh(x)^4 + 2*a*cosh(x)^2 + a)*e^(2
*x) + 4*(a*cosh(x)^3 + a*cosh(x) + (a*cosh(x)^3 + a*cosh(x))*e^(2*x))*sinh
(x) + a)*arctan(cosh(x) + sinh(x)) - a*cosh(x) + (a*cosh(x)^3 - a*cosh(x))
*e^(2*x) + (3*a*cosh(x)^2 + (3*a*cosh(x)^2 - a)*e^(2*x) - a)*sinh(x))*sqrt
(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x/(4*cosh(x)*e^x*sinh(x)^3 + e^x*sinh(x)^4
 + 2*(3*cosh(x)^2 + 1)*e^x*sinh(x)^2 + 4*(cosh(x)^3 + cosh(x))*e^x*sinh(x)
 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^x)
```

Sympy [F]

$$\int \left(a\mathrm{sech}^2(x)\right)^{3/2}\,dx = \int \left(a\operatorname{sech}^2\left(x\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sech(x)**2)**(3/2),x)
```

output
```
Integral((a*sech(x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \left(a\text{sech}^2(x)\right)^{3/2}\,dx = a^{\frac{3}{2}}\arctan\left(e^x\right) + \frac{a^{\frac{3}{2}}e^{(3x)} - a^{\frac{3}{2}}e^x}{e^{(4x)} + 2e^{(2x)} + 1}$$

input
```
integrate((a*sech(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
a^(3/2)*arctan(e^x) + (a^(3/2)*e^(3*x) - a^(3/2)*e^x)/(e^(4*x) + 2*e^(2*x)
 + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \left(a\text{sech}^2(x)\right)^{3/2}\,dx = \frac{1}{4}\left(\pi - \frac{4\left(e^{(-x)} - e^x\right)}{\left(e^{(-x)} - e^x\right)^2 + 4} + 2\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)\right)a^{\frac{3}{2}}$$

input
```
integrate((a*sech(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/4*(pi - 4*(e^(-x) - e^x)/((e^(-x) - e^x)^2 + 4) + 2*arctan(1/2*(e^(2*x)
- 1)*e^(-x)))*a^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\text{sech}^2(x)\right)^{3/2}\,dx = \int \left(\frac{a}{\cosh(x)^2}\right)^{3/2}\,dx$$

input
```
int((a/cosh(x)^2)^(3/2),x)
```

output
```
int((a/cosh(x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \left(a\text{sech}^2(x)\right)^{3/2} dx = \frac{\sqrt{a}\, a(e^{4x} atan(e^x) + 2e^{2x} atan(e^x) + atan(e^x) + e^{3x} - e^x)}{e^{4x} + 2e^{2x} + 1}$$

input
```
int((a*sech(x)^2)^(3/2),x)
```

output
```
(sqrt(a)*a*(e**(4*x)*atan(e**x) + 2*e**(2*x)*atan(e**x) + atan(e**x) + e**
(3*x) - e**x))/(e**(4*x) + 2*e**(2*x) + 1)
```

3.34 $\int \sqrt{a\text{sech}^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 25

$$\int \sqrt{a\,\text{sech}^2(x)}\,dx=\sqrt{a}\arctan\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\,\text{sech}^2(x)}}\right)$$

output
```
a^(1/2)*arctan(a^(1/2)*tanh(x)/(a*sech(x)^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \sqrt{a\,\text{sech}^2(x)}\,dx=-\cot^{-1}(\sinh(x))\cosh(x)\sqrt{a\,\text{sech}^2(x)}$$

input
```
Integrate[Sqrt[a*Sech[x]^2],x]
```

output
```
-(ArcCot[Sinh[x]]*Cosh[x]*Sqrt[a*Sech[x]^2])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4610, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\text{sech}^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\sec(ix)^2}dx$$

$$\Big\downarrow \text{4610}$$

$$a\int \frac{1}{\sqrt{a-a\tanh^2(x)}}d\tanh(x)$$

$$\Big\downarrow \text{224}$$

$$a\int \frac{1}{\frac{a\tanh^2(x)}{a-a\tanh^2(x)}+1}d\frac{\tanh(x)}{\sqrt{a-a\tanh^2(x)}}$$

$$\Big\downarrow \text{216}$$

$$\sqrt{a}\arctan\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-a\tanh^2(x)}}\right)$$

input
```
Int[Sqrt[a*Sech[x]^2],x]
```

output
```
Sqrt[a]*ArcTan[(Sqrt[a]*Tanh[x])/Sqrt[a - a*Tanh[x]^2]]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.88

method	result	size
risch	$i\sqrt{\frac{e^{2x}a}{\left(e^{2x}+1\right)^{2}}}\, e^{-x}\left(e^{2x}+1\right)\ln\left(e^{x}+i\right)-i\sqrt{\frac{e^{2x}a}{\left(e^{2x}+1\right)^{2}}}\, e^{-x}\left(e^{2x}+1\right)\ln\left(e^{x}-i\right)$	72

input
```
int((sech(x)^2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*(exp(2*x)*a/(exp(2*x)+1)^2)^(1/2)*exp(-x)*(exp(2*x)+1)*ln(exp(x)+I)-I*(e
xp(2*x)*a/(exp(2*x)+1)^2)^(1/2)*exp(-x)*(exp(2*x)+1)*ln(exp(x)-I)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 5.80

$$\int \sqrt{a\text{sech}^2(x)}\,dx = \left[\sqrt{-a}\log\left(\frac{2\,a\cosh(x)\,e^{x}\sinh(x) + ae^{x}\sinh(x)^2 + 2\left(\cosh(x)\,e^{(2\,x)} + \left(e^{(2\,x)} + 1\right)\sinh(x) + \cosh(x)\right)\cdots}{2\,\cosh(x)\,e^{x}\sinh(x) + e^{x}\sinh(x)^2 + \left(\cosh(x)^2 + \cdots\right.}\right.\right.$$
$$\left.+\sinh(x))\right]$$

input
```
integrate((a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[sqrt(-a)*log((2*a*cosh(x)*e^x*sinh(x) + a*e^x*sinh(x)^2 + 2*(cosh(x)*e^(2
*x) + (e^(2*x) + 1)*sinh(x) + cosh(x))*sqrt(-a)*sqrt(a/(e^(4*x) + 2*e^(2*x
) + 1))*e^x + (a*cosh(x)^2 - a)*e^x)/(2*cosh(x)*e^x*sinh(x) + e^x*sinh(x)^
2 + (cosh(x)^2 + 1)*e^x)), 2*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) +
1)*arctan(cosh(x) + sinh(x))]
```

Sympy [F]

$$\int \sqrt{a\text{sech}^2(x)}\,dx = \int \sqrt{a\,\text{sech}^2(x)}\,dx$$

input
```
integrate((a*sech(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a*sech(x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.32

$$\int \sqrt{a\text{sech}^2(x)}\,dx = 2\sqrt{a}\arctan\left(e^x\right)$$

input
```
integrate((a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(a)*arctan(e^x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.32

$$\int \sqrt{a\text{sech}^2(x)}\,dx = 2\sqrt{a}\arctan\left(e^x\right)$$

input
```
integrate((a*sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(a)*arctan(e^x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\text{sech}^2(x)}\,dx = \int \sqrt{\frac{a}{\cosh\left(x\right)^2}}\,dx$$

input
```
int((a/cosh(x)^2)^(1/2),x)
```

output
```
int((a/cosh(x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.32

$$\int \sqrt{a\text{sech}^2(x)}\,dx = 2\sqrt{a}\,atan(e^x)$$

input
```
int((a*sech(x)^2)^(1/2),x)
```

output
```
2*sqrt(a)*atan(e**x)
```

3.35 $\int \frac{1}{\sqrt{a\textbf{sech}^2(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 13

$$\int \frac{1}{\sqrt{a\operatorname{sech}^2(x)}}\,dx = \frac{\tanh(x)}{\sqrt{a\operatorname{sech}^2(x)}}$$

output
```
tanh(x)/(a*sech(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a\operatorname{sech}^2(x)}}\,dx = \frac{\tanh(x)}{\sqrt{a\operatorname{sech}^2(x)}}$$

input
```
Integrate[1/Sqrt[a*Sech[x]^2],x]
```

output
```
Tanh[x]/Sqrt[a*Sech[x]^2]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4610, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\text{sech}^2(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{a\sec(ix)^2}}dx$$

$$\Big\downarrow \text{ 4610}$$

$$a\int \frac{1}{(a-a\tanh^2(x))^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{ 208}$$

$$\frac{\tanh(x)}{\sqrt{a-a\tanh^2(x)}}$$

input
```
Int[1/Sqrt[a*Sech[x]^2],x]
```

output
```
Tanh[x]/Sqrt[a - a*Tanh[x]^2]
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(11) = 22.

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 4.46

method	result	size
risch	$\frac{e^{2x}}{2\sqrt{\frac{e^{2x}a}{\left(e^{2x}+1\right)^{2}}}\,(e^{2x}+1)}-\frac{1}{2(e^{2x}+1)\sqrt{\frac{e^{2x}a}{\left(e^{2x}+1\right)^{2}}}}$	58

input
```
int(1/(sech(x)^2*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(exp(2*x)*a/(exp(2*x)+1)^2)^(1/2)/(exp(2*x)+1)*exp(2*x)-1/2/(exp(2*x)+
1)/(exp(2*x)*a/(exp(2*x)+1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(11) = 22.

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 6.08

$$\int \frac{1}{\sqrt{a\mathrm{sech}^2(x)}}\,dx = \frac{\left(\left(e^{(2\,x)}+1\right)\sinh\left(x\right)^2+\cosh\left(x\right)^2+\left(\cosh\left(x\right)^2-1\right)e^{(2\,x)}+2\left(\cosh\left(x\right)e^{(2\,x)}+\cosh\left(x\right)\right)\sinh\left(x\right)-1\right)\sqrt{\frac{\cdot}{e}}}{2\left(a\cosh\left(x\right)e^{x}+ae^{x}\sinh\left(x\right)\right)}$$

input
```
integrate(1/(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 - 1)*e^(2*x) + 2*(co
sh(x)*e^(2*x) + cosh(x))*sinh(x) - 1)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^
x/(a*cosh(x)*e^x + a*e^x*sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{a\text{sech}^2(x)}}\,dx = \frac{\tanh(x)}{\sqrt{a\,\text{sech}^2(x)}}$$

input
```
integrate(1/(a*sech(x)**2)**(1/2),x)
```

output
```
tanh(x)/sqrt(a*sech(x)**2)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{a\text{sech}^2(x)}}\,dx = -\frac{e^{(-x)}}{2\sqrt{a}} + \frac{e^x}{2\sqrt{a}}$$

input
```
integrate(1/(a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*e^(-x)/sqrt(a) + 1/2*e^x/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{a\text{sech}^2(x)}}\,dx = -\frac{e^{(-x)} - e^x}{2\sqrt{a}}$$

input
```
integrate(1/(a*sech(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(e^(-x) - e^x)/sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.54

$$\int \frac{1}{\sqrt{a\text{sech}^2(x)}}\,dx = -\frac{\left(\frac{\mathrm{e}^{-2\,x}}{2}-\frac{\mathrm{e}^{2\,x}}{2}\right)\,\sqrt{\frac{1}{{\left(\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}\right)}^2}}}{2\,\sqrt{a}}$$

input
```
int(1/(a/cosh(x)^2)^(1/2),x)
```

output
```
-((exp(-2*x)/2 - exp(2*x)/2)*(1/(exp(-x)/2 + exp(x)/2)^2)^(1/2))/(2*a^(1/2
))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{1}{\sqrt{a\text{sech}^2(x)}}\,dx = \frac{\sqrt{a}\,(e^{2x}-1)}{2e^x a}$$

input
```
int(1/(a*sech(x)^2)^(1/2),x)
```

output
```
(sqrt(a)*(e**(2*x) - 1))/(2*e**x*a)
```

3.36 $\int \frac{1}{\left(a\text{sech}^2(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \frac{1}{\left(a\,\text{sech}^2(x)\right)^{3/2}}\,dx=\frac{\tanh(x)}{3\left(a\,\text{sech}^2(x)\right)^{3/2}}+\frac{2\tanh(x)}{3a\sqrt{a\,\text{sech}^2(x)}}$$

output
```
1/3*tanh(x)/(a*sech(x)^2)^(3/2)+2/3*tanh(x)/a/(a*sech(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \frac{1}{\left(a\,\text{sech}^2(x)\right)^{3/2}}\,dx=\frac{\left(3+\sinh^2(x)\right)\tanh(x)}{3a\sqrt{a\,\text{sech}^2(x)}}$$

input
```
Integrate[(a*Sech[x]^2)^(-3/2),x]
```

output
```
((3 + Sinh[x]^2)*Tanh[x])/(3*a*Sqrt[a*Sech[x]^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4610, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sec(ix)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4610}$$

$$a\int \frac{1}{\left(a-a\tanh^2(x)\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow \text{209}$$

$$a\left(\frac{2\int \frac{1}{\left(a-a\tanh^2(x)\right)^{3/2}}d\tanh(x)}{3a}+\frac{\tanh(x)}{3a\left(a-a\tanh^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$a\left(\frac{2\tanh(x)}{3a^2\sqrt{a-a\tanh^2(x)}}+\frac{\tanh(x)}{3a\left(a-a\tanh^2(x)\right)^{3/2}}\right)$$

input
```
Int[(a*Sech[x]^2)^(-3/2),x]
```

output
```
a*(Tanh[x]/(3*a*(a - a*Tanh[x]^2)^(3/2)) + (2*Tanh[x])/(3*a^2*Sqrt[a - a*T
anh[x]^2]))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(28) = 56.

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 3.61

method	result	size
risch	$\frac{\mathrm{e}^{4x}}{24a\left(\mathrm{e}^{2x}+1\right)\sqrt{\frac{\mathrm{e}^{2x}a}{\left(\mathrm{e}^{2x}+1\right)^{2}}}}+\frac{3\,\mathrm{e}^{2x}}{8a\left(\mathrm{e}^{2x}+1\right)\sqrt{\frac{\mathrm{e}^{2x}a}{\left(\mathrm{e}^{2x}+1\right)^{2}}}}-\frac{3}{8\sqrt{\frac{\mathrm{e}^{2x}a}{\left(\mathrm{e}^{2x}+1\right)^{2}}}\,a\left(\mathrm{e}^{2x}+1\right)}-\frac{\mathrm{e}^{-2x}}{24a\left(\mathrm{e}^{2x}+1\right)\sqrt{\frac{\mathrm{e}^{2x}a}{\left(\mathrm{e}^{2x}+1\right)^{2}}}}$	130

input
```
int(1/(sech(x)^2*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/a*exp(4*x)/(exp(2*x)+1)/(exp(2*x)*a/(exp(2*x)+1)^2)^(1/2)+3/8/a*exp(2
*x)/(exp(2*x)+1)/(exp(2*x)*a/(exp(2*x)+1)^2)^(1/2)-3/8/(exp(2*x)*a/(exp(2*
x)+1)^2)^(1/2)/a/(exp(2*x)+1)-1/24/a*exp(-2*x)/(exp(2*x)+1)/(exp(2*x)*a/(e
xp(2*x)+1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(28) = 56$.

Time = 0.09 (sec) , antiderivative size = 277, normalized size of antiderivative = 7.69

$$\int \frac{1}{\left(a\mathrm{sech}^2(x)\right)^{3/2}}\,dx = \frac{\left(\left(e^{(2\,x)}+1\right)\sinh\left(x\right)^6+\cosh\left(x\right)^6+6\left(\cosh\left(x\right)e^{(2\,x)}+\cosh\left(x\right)\right)\sinh\left(x\right)^5+3\left(5\,\mathrm{co}\right.}{}$$

input
```
integrate(1/(a*sech(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/24*((e^(2*x) + 1)*sinh(x)^6 + cosh(x)^6 + 6*(cosh(x)*e^(2*x) + cosh(x))*
sinh(x)^5 + 3*(5*cosh(x)^2 + (5*cosh(x)^2 + 3)*e^(2*x) + 3)*sinh(x)^4 + 9*
cosh(x)^4 + 4*(5*cosh(x)^3 + (5*cosh(x)^3 + 9*cosh(x))*e^(2*x) + 9*cosh(x)
)*sinh(x)^3 + 3*(5*cosh(x)^4 + 18*cosh(x)^2 + (5*cosh(x)^4 + 18*cosh(x)^2
- 3)*e^(2*x) - 3)*sinh(x)^2 - 9*cosh(x)^2 + (cosh(x)^6 + 9*cosh(x)^4 - 9*c
osh(x)^2 - 1)*e^(2*x) + 6*(cosh(x)^5 + 6*cosh(x)^3 + (cosh(x)^5 + 6*cosh(x
)^3 - 3*cosh(x))*e^(2*x) - 3*cosh(x))*sinh(x) - 1)*sqrt(a/(e^(4*x) + 2*e^(
2*x) + 1))*e^x/(a^2*cosh(x)^3*e^x + 3*a^2*cosh(x)^2*e^x*sinh(x) + 3*a^2*co
sh(x)*e^x*sinh(x)^2 + a^2*e^x*sinh(x)^3)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a\mathrm{sech}^2(x)\right)^{3/2}}\,dx = -\frac{2\tanh^3\left(x\right)}{3\left(a\operatorname{sech}^2\left(x\right)\right)^{\frac{3}{2}}}+\frac{\tanh\left(x\right)}{\left(a\operatorname{sech}^2\left(x\right)\right)^{\frac{3}{2}}}$$

input
```
integrate(1/(a*sech(x)**2)**(3/2),x)
```

output
```
-2*tanh(x)**3/(3*(a*sech(x)**2)**(3/2)) + tanh(x)/(a*sech(x)**2)**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{3/2}}\,dx = \frac{e^{(3x)}}{24\,a^{\frac{3}{2}}} - \frac{3\,e^{(-x)}}{8\,a^{\frac{3}{2}}} - \frac{e^{(-3x)}}{24\,a^{\frac{3}{2}}} + \frac{3\,e^{x}}{8\,a^{\frac{3}{2}}}$$

input
```
integrate(1/(a*sech(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/24*e^(3*x)/a^(3/2) - 3/8*e^(-x)/a^(3/2) - 1/24*e^(-3*x)/a^(3/2) + 3/8*e^
x/a^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{3/2}}\,dx = -\frac{\left(9\,e^{(2x)}+1\right)e^{(-3x)} - e^{(3x)} - 9\,e^{x}}{24\,a^{\frac{3}{2}}}$$

input
```
integrate(1/(a*sech(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/24*((9*e^(2*x) + 1)*e^(-3*x) - e^(3*x) - 9*e^x)/a^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{a}{\cosh(x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(a/cosh(x)^2)^(3/2),x)
```

output
```
int(1/(a/cosh(x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a\operatorname{sech}^2(x)\right)^{3/2}}\,dx = \frac{\sqrt{a}\,(e^{6x}+9e^{4x}-9e^{2x}-1)}{24e^{3x}a^2}$$

input
```
int(1/(a*sech(x)^2)^(3/2),x)
```

output
```
(sqrt(a)*(e**(6*x) + 9*e**(4*x) - 9*e**(2*x) - 1))/(24*e**(3*x)*a**2)
```

3.37 $\int \frac{1}{\left(a\text{sech}^2(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 55

$$\int \frac{1}{\left(a\,\text{sech}^2(x)\right)^{5/2}}\,dx=\frac{\tanh(x)}{5\left(a\,\text{sech}^2(x)\right)^{5/2}}+\frac{4\tanh(x)}{15a\left(a\,\text{sech}^2(x)\right)^{3/2}}+\frac{8\tanh(x)}{15a^2\sqrt{a\,\text{sech}^2(x)}}$$

output
```
1/5*tanh(x)/(a*sech(x)^2)^(5/2)+4/15*tanh(x)/a/(a*sech(x)^2)^(3/2)+8/15*ta
nh(x)/a^2/(a*sech(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.60

$$\int \frac{1}{\left(a\,\text{sech}^2(x)\right)^{5/2}}\,dx=\frac{\left(15+10\sinh^2(x)+3\sinh^4(x)\right)\tanh(x)}{15a^2\sqrt{a\,\text{sech}^2(x)}}$$

input
```
Integrate[(a*Sech[x]^2)^(-5/2),x]
```

output
```
((15 + 10*Sinh[x]^2 + 3*Sinh[x]^4)*Tanh[x])/(15*a^2*Sqrt[a*Sech[x]^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4610, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a\sec(ix)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4610}$$

$$a\int \frac{1}{\left(a-a\tanh^2(x)\right)^{7/2}}d\tanh(x)$$

$$\Big\downarrow \text{ 209}$$

$$a\left(\frac{4\int \frac{1}{\left(a-a\tanh^2(x)\right)^{5/2}}d\tanh(x)}{5a}+\frac{\tanh(x)}{5a\left(a-a\tanh^2(x)\right)^{5/2}}\right)$$

$$\Big\downarrow \text{ 209}$$

$$a\left(\frac{4\left(\frac{2\int \frac{1}{\left(a-a\tanh^2(x)\right)^{3/2}}d\tanh(x)}{3a}+\frac{\tanh(x)}{3a\left(a-a\tanh^2(x)\right)^{3/2}}\right)}{5a}+\frac{\tanh(x)}{5a\left(a-a\tanh^2(x)\right)^{5/2}}\right)$$

$$\Big\downarrow \text{ 208}$$

$$a\left(\frac{4\left(\frac{2\tanh(x)}{3a^2\sqrt{a-a\tanh^2(x)}}+\frac{\tanh(x)}{3a\left(a-a\tanh^2(x)\right)^{3/2}}\right)}{5a}+\frac{\tanh(x)}{5a\left(a-a\tanh^2(x)\right)^{5/2}}\right)$$

input
```
Int[(a*Sech[x]^2)^(-5/2),x]
```

output
```
a*(Tanh[x]/(5*a*(a - a*Tanh[x]^2)^(5/2)) + (4*(Tanh[x]/(3*a*(a - a*Tanh[x]
^2)^(3/2)) + (2*Tanh[x])/(3*a^2*Sqrt[a - a*Tanh[x]^2])))/(5*a))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(43) = 86$.

Time = 0.09 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.56

method	result
risch	$\frac{e^{6x}}{160a^2(e^{2x}+1)\sqrt{\frac{e^{2x}a}{(e^{2x}+1)^2}}}+\frac{5\,e^{4x}}{96a^2(e^{2x}+1)\sqrt{\frac{e^{2x}a}{(e^{2x}+1)^2}}}+\frac{5\,e^{2x}}{16a^2(e^{2x}+1)\sqrt{\frac{e^{2x}a}{(e^{2x}+1)^2}}}-\frac{5}{16\sqrt{\frac{e^{2x}a}{(e^{2x}+1)^2}}\,(e^{2x}+1)a^2}-\frac{}{96a^2(e^{2x}}$

input
```
int(1/(sech(x)^2*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/160/a^2*exp(6*x)/(exp(2*x)+1)/(exp(2*x)*a/(exp(2*x)+1)^2)^(1/2)+5/96/a^2
*exp(4*x)/(exp(2*x)+1)/(exp(2*x)*a/(exp(2*x)+1)^2)^(1/2)+5/16/a^2*exp(2*x)
/(exp(2*x)+1)/(exp(2*x)*a/(exp(2*x)+1)^2)^(1/2)-5/16/(exp(2*x)*a/(exp(2*x)
+1)^2)^(1/2)/(exp(2*x)+1)/a^2-5/96/a^2*exp(-2*x)/(exp(2*x)+1)/(exp(2*x)*a/
(exp(2*x)+1)^2)^(1/2)-1/160/a^2*exp(-4*x)/(exp(2*x)+1)/(exp(2*x)*a/(exp(2*
x)+1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(43) = 86$.

Time = 0.10 (sec) , antiderivative size = 580, normalized size of antiderivative = 10.55

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*sech(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/480*(3*(e^(2*x) + 1)*sinh(x)^10 + 3*cosh(x)^10 + 30*(cosh(x)*e^(2*x) + c
osh(x))*sinh(x)^9 + 5*(27*cosh(x)^2 + (27*cosh(x)^2 + 5)*e^(2*x) + 5)*sinh
(x)^8 + 25*cosh(x)^8 + 40*(9*cosh(x)^3 + (9*cosh(x)^3 + 5*cosh(x))*e^(2*x)
 + 5*cosh(x))*sinh(x)^7 + 10*(63*cosh(x)^4 + 70*cosh(x)^2 + (63*cosh(x)^4
+ 70*cosh(x)^2 + 15)*e^(2*x) + 15)*sinh(x)^6 + 150*cosh(x)^6 + 4*(189*cosh
(x)^5 + 350*cosh(x)^3 + (189*cosh(x)^5 + 350*cosh(x)^3 + 225*cosh(x))*e^(2
*x) + 225*cosh(x))*sinh(x)^5 + 10*(63*cosh(x)^6 + 175*cosh(x)^4 + 225*cosh
(x)^2 + (63*cosh(x)^6 + 175*cosh(x)^4 + 225*cosh(x)^2 - 15)*e^(2*x) - 15)*
sinh(x)^4 - 150*cosh(x)^4 + 40*(9*cosh(x)^7 + 35*cosh(x)^5 + 75*cosh(x)^3
+ (9*cosh(x)^7 + 35*cosh(x)^5 + 75*cosh(x)^3 - 15*cosh(x))*e^(2*x) - 15*co
sh(x))*sinh(x)^3 + 5*(27*cosh(x)^8 + 140*cosh(x)^6 + 450*cosh(x)^4 - 180*c
osh(x)^2 + (27*cosh(x)^8 + 140*cosh(x)^6 + 450*cosh(x)^4 - 180*cosh(x)^2 -
 5)*e^(2*x) - 5)*sinh(x)^2 - 25*cosh(x)^2 + (3*cosh(x)^10 + 25*cosh(x)^8 +
 150*cosh(x)^6 - 150*cosh(x)^4 - 25*cosh(x)^2 - 3)*e^(2*x) + 10*(3*cosh(x)
^9 + 20*cosh(x)^7 + 90*cosh(x)^5 - 60*cosh(x)^3 + (3*cosh(x)^9 + 20*cosh(x
)^7 + 90*cosh(x)^5 - 60*cosh(x)^3 - 5*cosh(x))*e^(2*x) - 5*cosh(x))*sinh(x
) - 3)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x/(a^3*cosh(x)^5*e^x + 5*a^3*co
sh(x)^4*e^x*sinh(x) + 10*a^3*cosh(x)^3*e^x*sinh(x)^2 + 10*a^3*cosh(x)^2*e^
x*sinh(x)^3 + 5*a^3*cosh(x)*e^x*sinh(x)^4 + a^3*e^x*sinh(x)^5)
```

Sympy [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a\mathrm{sech}^2(x)\right)^{5/2}}\,dx = \frac{8\tanh^5(x)}{15\left(a\,\mathrm{sech}^2(x)\right)^{\frac{5}{2}}} - \frac{4\tanh^3(x)}{3\left(a\,\mathrm{sech}^2(x)\right)^{\frac{5}{2}}} + \frac{\tanh(x)}{\left(a\,\mathrm{sech}^2(x)\right)^{\frac{5}{2}}}$$

input
```
integrate(1/(a*sech(x)**2)**(5/2),x)
```

output
```
8*tanh(x)**5/(15*(a*sech(x)**2)**(5/2)) - 4*tanh(x)**3/(3*(a*sech(x)**2)**
(5/2)) + tanh(x)/(a*sech(x)**2)**(5/2)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a\mathrm{sech}^2(x)\right)^{5/2}}\,dx = \frac{e^{(5x)}}{160\,a^{\frac{5}{2}}} + \frac{5\,e^{(3x)}}{96\,a^{\frac{5}{2}}} - \frac{5\,e^{(-x)}}{16\,a^{\frac{5}{2}}} - \frac{5\,e^{(-3x)}}{96\,a^{\frac{5}{2}}} - \frac{e^{(-5x)}}{160\,a^{\frac{5}{2}}} + \frac{5\,e^{x}}{16\,a^{\frac{5}{2}}}$$

input
```
integrate(1/(a*sech(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/160*e^(5*x)/a^(5/2) + 5/96*e^(3*x)/a^(5/2) - 5/16*e^(-x)/a^(5/2) - 5/96*
e^(-3*x)/a^(5/2) - 1/160*e^(-5*x)/a^(5/2) + 5/16*e^x/a^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(a\mathrm{sech}^2(x)\right)^{5/2}}\,dx = -\frac{\left(150\,e^{(4x)} + 25\,e^{(2x)} + 3\right)e^{(-5x)} - 3\,e^{(5x)} - 25\,e^{(3x)} - 150\,e^{x}}{480\,a^{\frac{5}{2}}}$$

input
```
integrate(1/(a*sech(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/480*((150*e^(4*x) + 25*e^(2*x) + 3)*e^(-5*x) - 3*e^(5*x) - 25*e^(3*x) -
 150*e^x)/a^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{5/2}} dx = \int \frac{1}{\left(\frac{a}{\cosh(x)^2}\right)^{5/2}} dx$$

input
```
int(1/(a/cosh(x)^2)^(5/2),x)
```

output
```
int(1/(a/cosh(x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{5/2}} dx = \frac{\sqrt{a}\,(3e^{10x} + 25e^{8x} + 150e^{6x} - 150e^{4x} - 25e^{2x} - 3)}{480e^{5x}a^3}$$

input
```
int(1/(a*sech(x)^2)^(5/2),x)
```

output
```
(sqrt(a)*(3*e**(10*x) + 25*e**(8*x) + 150*e**(6*x) - 150*e**(4*x) - 25*e**
(2*x) - 3))/(480*e**(5*x)*a**3)
```

3.38 $\int \frac{1}{\left(a\,\mathbf{sech}^2(x)\right)^{7/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 74

$$\int \frac{1}{\left(a\,\text{sech}^2(x)\right)^{7/2}}\,dx=\frac{\tanh(x)}{7\left(a\,\text{sech}^2(x)\right)^{7/2}}+\frac{6\tanh(x)}{35a\left(a\,\text{sech}^2(x)\right)^{5/2}}+\frac{8\tanh(x)}{35a^2\left(a\,\text{sech}^2(x)\right)^{3/2}}+\frac{16\tanh(x)}{35a^3\sqrt{a\,\text{sech}^2(x)}}$$

output
```
1/7*tanh(x)/(a*sech(x)^2)^(7/2)+6/35*tanh(x)/a/(a*sech(x)^2)^(5/2)+8/35*ta
nh(x)/a^2/(a*sech(x)^2)^(3/2)+16/35*tanh(x)/a^3/(a*sech(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.53

$$\int \frac{1}{\left(a\,\text{sech}^2(x)\right)^{7/2}}\,dx=\frac{\left(35+35\sinh^2(x)+21\sinh^4(x)+5\sinh^6(x)\right)\tanh(x)}{35a^3\sqrt{a\,\text{sech}^2(x)}}$$

input
```
Integrate[(a*Sech[x]^2)^(-7/2),x]
```

output
```
((35 + 35*Sinh[x]^2 + 21*Sinh[x]^4 + 5*Sinh[x]^6)*Tanh[x])/(35*a^3*Sqrt[a*
Sech[x]^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.45, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4610, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sec(ix)^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{4610}$$

$$a\int \frac{1}{\left(a-a\tanh^2(x)\right)^{9/2}}d\tanh(x)$$

$$\Big\downarrow \text{209}$$

$$a\left(\frac{6\int \frac{1}{\left(a-a\tanh^2(x)\right)^{7/2}}d\tanh(x)}{7a}+\frac{\tanh(x)}{7a\left(a-a\tanh^2(x)\right)^{7/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$a\left(\frac{6\left(\frac{4\int \frac{1}{\left(a-a\tanh^2(x)\right)^{5/2}}d\tanh(x)}{5a}+\frac{\tanh(x)}{5a\left(a-a\tanh^2(x)\right)^{5/2}}\right)}{7a}+\frac{\tanh(x)}{7a\left(a-a\tanh^2(x)\right)^{7/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$a\left(\frac{6\left(\frac{4\left(\frac{2\int \frac{1}{\left(a-a\tanh^2(x)\right)^{3/2}}d\tanh(x)}{3a}+\frac{\tanh(x)}{3a\left(a-a\tanh^2(x)\right)^{3/2}}\right)}{5a}+\frac{\tanh(x)}{5a\left(a-a\tanh^2(x)\right)^{5/2}}\right)}{7a}+\frac{\tanh(x)}{7a\left(a-a\tanh^2(x)\right)^{7/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$a\left(\frac{6\left(\frac{4\left(\frac{2\tanh(x)}{3a^2\sqrt{a-a\tanh^2(x)}}+\frac{\tanh(x)}{3a\left(a-a\tanh^2(x)\right)^{3/2}}\right)}{5a}+\frac{\tanh(x)}{5a\left(a-a\tanh^2(x)\right)^{5/2}}\right)}{7a}+\frac{\tanh(x)}{7a\left(a-a\tanh^2(x)\right)^{7/2}}\right)$$

input
```
Int[(a*Sech[x]^2)^(-7/2),x]
```

output
```
a*(Tanh[x]/(7*a*(a - a*Tanh[x]^2)^(7/2)) + (6*(Tanh[x]/(5*a*(a - a*Tanh[x]
^2)^(5/2)) + (4*(Tanh[x]/(3*a*(a - a*Tanh[x]^2)^(3/2)) + (2*Tanh[x])/(3*a^
2*Sqrt[a - a*Tanh[x]^2])))/(5*a)))/(7*a))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(58) = 116$.

Time = 0.09 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.54

method	result
risch	$\frac{e^{8x}}{896a^3(e^{2x}+1)\sqrt{\frac{e^{2x}a}{(e^{2x}+1)^2}}}+\frac{7\,e^{6x}}{640a^3(e^{2x}+1)\sqrt{\frac{e^{2x}a}{(e^{2x}+1)^2}}}+\frac{7\,e^{4x}}{128a^3(e^{2x}+1)\sqrt{\frac{e^{2x}a}{(e^{2x}+1)^2}}}+\frac{35\,e^{2x}}{128a^3(e^{2x}+1)\sqrt{\frac{e^{2x}a}{(e^{2x}+1)^2}}}-\frac{}{128\sqrt{}}$

input
```
int(1/(sech(x)^2*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/896/a^3*exp(8*x)/(exp(2*x)+1)/(exp(2*x)*a/(exp(2*x)+1)^2)^(1/2)+7/640/a^
3*exp(6*x)/(exp(2*x)+1)/(exp(2*x)*a/(exp(2*x)+1)^2)^(1/2)+7/128/a^3*exp(4*
x)/(exp(2*x)+1)/(exp(2*x)*a/(exp(2*x)+1)^2)^(1/2)+35/128/a^3*exp(2*x)/(exp
(2*x)+1)/(exp(2*x)*a/(exp(2*x)+1)^2)^(1/2)-35/128/(exp(2*x)*a/(exp(2*x)+1)
^2)^(1/2)/(exp(2*x)+1)/a^3-7/128/a^3*exp(-2*x)/(exp(2*x)+1)/(exp(2*x)*a/(e
xp(2*x)+1)^2)^(1/2)-7/640/a^3*exp(-4*x)/(exp(2*x)+1)/(exp(2*x)*a/(exp(2*x)
+1)^2)^(1/2)-1/896/a^3*exp(-6*x)/(exp(2*x)+1)/(exp(2*x)*a/(exp(2*x)+1)^2)^
(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 970 vs. $2(58) = 116$.

Time = 0.13 (sec) , antiderivative size = 970, normalized size of antiderivative = 13.11

$$\int \frac{1}{\left(a\operatorname{sech}^2(x)\right)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*sech(x)^2)^(7/2),x, algorithm="fricas")
```

output
```
1/4480*(5*(e^(2*x) + 1)*sinh(x)^14 + 5*cosh(x)^14 + 70*(cosh(x)*e^(2*x) +
cosh(x))*sinh(x)^13 + 7*(65*cosh(x)^2 + (65*cosh(x)^2 + 7)*e^(2*x) + 7)*si
nh(x)^12 + 49*cosh(x)^12 + 28*(65*cosh(x)^3 + (65*cosh(x)^3 + 21*cosh(x))*
e^(2*x) + 21*cosh(x))*sinh(x)^11 + 7*(715*cosh(x)^4 + 462*cosh(x)^2 + (715
*cosh(x)^4 + 462*cosh(x)^2 + 35)*e^(2*x) + 35)*sinh(x)^10 + 245*cosh(x)^10
 + 70*(143*cosh(x)^5 + 154*cosh(x)^3 + (143*cosh(x)^5 + 154*cosh(x)^3 + 35
*cosh(x))*e^(2*x) + 35*cosh(x))*sinh(x)^9 + 35*(429*cosh(x)^6 + 693*cosh(x
)^4 + 315*cosh(x)^2 + (429*cosh(x)^6 + 693*cosh(x)^4 + 315*cosh(x)^2 + 35)
*e^(2*x) + 35)*sinh(x)^8 + 1225*cosh(x)^8 + 8*(2145*cosh(x)^7 + 4851*cosh(
x)^5 + 3675*cosh(x)^3 + (2145*cosh(x)^7 + 4851*cosh(x)^5 + 3675*cosh(x)^3
+ 1225*cosh(x))*e^(2*x) + 1225*cosh(x))*sinh(x)^7 + 7*(2145*cosh(x)^8 + 64
68*cosh(x)^6 + 7350*cosh(x)^4 + 4900*cosh(x)^2 + (2145*cosh(x)^8 + 6468*co
sh(x)^6 + 7350*cosh(x)^4 + 4900*cosh(x)^2 - 175)*e^(2*x) - 175)*sinh(x)^6
- 1225*cosh(x)^6 + 14*(715*cosh(x)^9 + 2772*cosh(x)^7 + 4410*cosh(x)^5 + 4
900*cosh(x)^3 + (715*cosh(x)^9 + 2772*cosh(x)^7 + 4410*cosh(x)^5 + 4900*co
sh(x)^3 - 525*cosh(x))*e^(2*x) - 525*cosh(x))*sinh(x)^5 + 35*(143*cosh(x)^
10 + 693*cosh(x)^8 + 1470*cosh(x)^6 + 2450*cosh(x)^4 - 525*cosh(x)^2 + (14
3*cosh(x)^10 + 693*cosh(x)^8 + 1470*cosh(x)^6 + 2450*cosh(x)^4 - 525*cosh(
x)^2 - 7)*e^(2*x) - 7)*sinh(x)^4 - 245*cosh(x)^4 + 140*(13*cosh(x)^11 + 77
*cosh(x)^9 + 210*cosh(x)^7 + 490*cosh(x)^5 - 175*cosh(x)^3 + (13*cosh(x...
```

Sympy [A] (verification not implemented)

Time = 10.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a\operatorname{sech}^2(x)\right)^{7/2}}\,dx = -\frac{16\tanh^7(x)}{35\left(a\operatorname{sech}^2(x)\right)^{\frac{7}{2}}} + \frac{8\tanh^5(x)}{5\left(a\operatorname{sech}^2(x)\right)^{\frac{7}{2}}} - \frac{2\tanh^3(x)}{\left(a\operatorname{sech}^2(x)\right)^{\frac{7}{2}}} + \frac{\tanh(x)}{\left(a\operatorname{sech}^2(x)\right)^{\frac{7}{2}}}$$

input
```
integrate(1/(a*sech(x)**2)**(7/2),x)
```

output
```
-16*tanh(x)**7/(35*(a*sech(x)**2)**(7/2)) + 8*tanh(x)**5/(5*(a*sech(x)**2)
**(7/2)) - 2*tanh(x)**3/(a*sech(x)**2)**(7/2) + tanh(x)/(a*sech(x)**2)**(7
/2)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a\operatorname{sech}^2(x)\right)^{7/2}}\,dx = \frac{e^{(7x)}}{896a^{\frac{7}{2}}} + \frac{7e^{(5x)}}{640a^{\frac{7}{2}}} + \frac{7e^{(3x)}}{128a^{\frac{7}{2}}} - \frac{35e^{(-x)}}{128a^{\frac{7}{2}}} - \frac{7e^{(-3x)}}{128a^{\frac{7}{2}}} - \frac{7e^{(-5x)}}{640a^{\frac{7}{2}}} - \frac{e^{(-7x)}}{896a^{\frac{7}{2}}} + \frac{35e^{x}}{128a^{\frac{7}{2}}}$$

input
```
integrate(1/(a*sech(x)^2)^(7/2),x, algorithm="maxima")
```

output
```
1/896*e^(7*x)/a^(7/2) + 7/640*e^(5*x)/a^(7/2) + 7/128*e^(3*x)/a^(7/2) - 35
/128*e^(-x)/a^(7/2) - 7/128*e^(-3*x)/a^(7/2) - 7/640*e^(-5*x)/a^(7/2) - 1/
896*e^(-7*x)/a^(7/2) + 35/128*e^x/a^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.72

$$\int \frac{1}{\left(a\,\mathrm{sech}^2(x)\right)^{7/2}}\,dx = -\frac{\left(1225\,e^{(6\,x)} + 245\,e^{(4\,x)} + 49\,e^{(2\,x)} + 5\right)e^{(-7\,x)} - 5\,e^{(7\,x)} - 49\,e^{(5\,x)} - 245\,e^{(3\,x)} - 1225\,e^{x}}{4480\,a^{\frac{7}{2}}}$$

input
```
integrate(1/(a*sech(x)^2)^(7/2),x, algorithm="giac")
```

output
```
-1/4480*((1225*e^(6*x) + 245*e^(4*x) + 49*e^(2*x) + 5)*e^(-7*x) - 5*e^(7*x
) - 49*e^(5*x) - 245*e^(3*x) - 1225*e^x)/a^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\,\mathrm{sech}^2(x)\right)^{7/2}}\,dx = \int \frac{1}{\left(\frac{a}{\cosh(x)^2}\right)^{7/2}}\,dx$$

input
```
int(1/(a/cosh(x)^2)^(7/2),x)
```

output
```
int(1/(a/cosh(x)^2)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a\,\mathrm{sech}^2(x)\right)^{7/2}}\,dx = \frac{\sqrt{a}\,(5e^{14x} + 49e^{12x} + 245e^{10x} + 1225e^{8x} - 1225e^{6x} - 245e^{4x} - 49e^{2x} - 5)}{4480e^{7x}a^4}$$

input
```
int(1/(a*sech(x)^2)^(7/2),x)
```

output
```
(sqrt(a)*(5*e**(14*x) + 49*e**(12*x) + 245*e**(10*x) + 1225*e**(8*x) - 122
5*e**(6*x) - 245*e**(4*x) - 49*e**(2*x) - 5))/(4480*e**(7*x)*a**4)
```

3.39 $\int \left(a\text{sech}^3(x)\right)^{5/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 121

$$\begin{aligned}\int \left(a\text{sech}^3(x)\right)^{5/2}\,dx &= \frac{154}{195}ia^2\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\text{sech}^3(x)}\\&+\frac{154}{195}a^2\cosh(x)\sqrt{a\text{sech}^3(x)}\sinh(x)+\frac{154}{585}a^2\sqrt{a\text{sech}^3(x)}\tanh(x)\\&+\frac{22}{117}a^2\text{sech}^2(x)\sqrt{a\text{sech}^3(x)}\tanh(x)+\frac{2}{13}a^2\text{sech}^4(x)\sqrt{a\text{sech}^3(x)}\tanh(x)\end{aligned}$$

output

```
154/195*I*a^2*cosh(x)^(3/2)*EllipticE(I*sinh(1/2*x),2^(1/2))*(a*sech(x)^3)
^(1/2)+154/195*a^2*cosh(x)*(a*sech(x)^3)^(1/2)*sinh(x)+154/585*a^2*(a*sech
(x)^3)^(1/2)*tanh(x)+22/117*a^2*sech(x)^2*(a*sech(x)^3)^(1/2)*tanh(x)+2/13
*a^2*sech(x)^4*(a*sech(x)^3)^(1/2)*tanh(x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

$$\int \left(a\text{sech}^3(x)\right)^{5/2}\,dx=\frac{2}{585}a\text{sech}(x)\left(a\text{sech}^3(x)\right)^{3/2}\left(231i\cosh^{\frac{11}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)+55\cosh(x)\sinh(x)+77\cos\right.$$

input

```
Integrate[(a*Sech[x]^3)^(5/2),x]
```

output
```
(2*a*Sech[x]*(a*Sech[x]^3)^(3/2)*((231*I)*Cosh[x]^(11/2)*EllipticE[(I/2)*x
, 2] + 55*Cosh[x]*Sinh[x] + 77*Cosh[x]^3*Sinh[x] + 231*Cosh[x]^5*Sinh[x] +
 45*Tanh[x]))/585
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.88, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.400$, Rules used = {3042, 4611, 3042, 4255, 3042, 4255, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\text{sech}^3(x)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a \sec(ix)^3\right)^{5/2} dx$$

$$\Big\downarrow \text{4611}$$

$$\frac{a^2\sqrt{a\text{sech}^3(x)}\int \text{sech}^{\frac{15}{2}}(x)dx}{\text{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\text{sech}^3(x)}\int \csc\left(ix+\frac{\pi}{2}\right)^{15/2} dx}{\text{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{a^2\sqrt{a\text{sech}^3(x)}\left(\frac{11}{13}\int \text{sech}^{\frac{11}{2}}(x)dx+\frac{2}{13}\sinh(x)\text{sech}^{\frac{13}{2}}(x)\right)}{\text{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\text{sech}^3(x)}\left(\frac{2}{13}\sinh(x)\text{sech}^{\frac{13}{2}}(x)+\frac{11}{13}\int \csc\left(ix+\frac{\pi}{2}\right)^{11/2} dx\right)}{\text{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \ 4255$$

$$\frac{a^2\sqrt{a\text{sech}^3(x)}\left(\frac{11}{13}\left(\frac{7}{9}\int \text{sech}^{\frac{7}{2}}(x)dx+\frac{2}{9}\sinh(x)\text{sech}^{\frac{9}{2}}(x)\right)+\frac{2}{13}\sinh(x)\text{sech}^{\frac{13}{2}}(x)\right)}{\text{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \ 3042$$

$$\frac{a^2\sqrt{a\text{sech}^3(x)}\left(\frac{2}{13}\sinh(x)\text{sech}^{\frac{13}{2}}(x)+\frac{11}{13}\left(\frac{2}{9}\sinh(x)\text{sech}^{\frac{9}{2}}(x)+\frac{7}{9}\int \csc\left(ix+\frac{\pi}{2}\right)^{7/2}dx\right)\right)}{\text{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \ 4255$$

$$\frac{a^2\sqrt{a\text{sech}^3(x)}\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\int \text{sech}^{\frac{3}{2}}(x)dx+\frac{2}{5}\sinh(x)\text{sech}^{\frac{5}{2}}(x)\right)+\frac{2}{9}\sinh(x)\text{sech}^{\frac{9}{2}}(x)\right)+\frac{2}{13}\sinh(x)\text{sech}^{\frac{13}{2}}(x)\right)}{\text{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \ 3042$$

$$\frac{a^2\sqrt{a\text{sech}^3(x)}\left(\frac{2}{13}\sinh(x)\text{sech}^{\frac{13}{2}}(x)+\frac{11}{13}\left(\frac{2}{9}\sinh(x)\text{sech}^{\frac{9}{2}}(x)+\frac{7}{9}\left(\frac{2}{5}\sinh(x)\text{sech}^{\frac{5}{2}}(x)+\frac{3}{5}\int \csc\left(ix+\frac{\pi}{2}\right)^{3/2}dx\right)\right)\right)}{\text{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \ 4255$$

$$\frac{a^2\sqrt{a\text{sech}^3(x)}\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\left(2\sinh(x)\sqrt{\text{sech}(x)}-\int \frac{1}{\sqrt{\text{sech}(x)}}dx\right)+\frac{2}{5}\sinh(x)\text{sech}^{\frac{5}{2}}(x)\right)+\frac{2}{9}\sinh(x)\text{sech}^{\frac{9}{2}}(x)\right)+\right.}{\text{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \ 3042$$

$$\frac{a^2\sqrt{a\text{sech}^3(x)}\left(\frac{2}{13}\sinh(x)\text{sech}^{\frac{13}{2}}(x)+\frac{11}{13}\left(\frac{2}{9}\sinh(x)\text{sech}^{\frac{9}{2}}(x)+\frac{7}{9}\left(\frac{2}{5}\sinh(x)\text{sech}^{\frac{5}{2}}(x)+\frac{3}{5}\left(2\sinh(x)\sqrt{\text{sech}(x)}-\right.\right.\right.\right.}{\text{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \ 4258$$

$$\frac{a^2\sqrt{a\text{sech}^3(x)}\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\left(2\sinh(x)\sqrt{\text{sech}(x)}-\sqrt{\cosh(x)}\sqrt{\text{sech}(x)}\int \sqrt{\cosh(x)}dx\right)+\frac{2}{5}\sinh(x)\text{sech}^{\frac{5}{2}}(x)\right)+\frac{2}{9}\text{si}\right.\right.}{\text{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \ 3042$$

$$\frac{a^2\sqrt{a\text{sech}^3(x)}\left(\frac{2}{13}\sinh(x)\text{sech}^{\frac{13}{2}}(x)+\frac{11}{13}\left(\frac{2}{9}\sinh(x)\text{sech}^{\frac{9}{2}}(x)+\frac{7}{9}\left(\frac{2}{5}\sinh(x)\text{sech}^{\frac{5}{2}}(x)+\frac{3}{5}\left(2\sinh(x)\sqrt{\text{sech}(x)}-\right.\right.\right.\right.}{\text{sech}^{\frac{3}{2}}(x)}$$

$\Big\downarrow$ 3119

$$\frac{a^2\sqrt{a\text{sech}^3(x)}\left(\frac{2}{13}\sinh(x)\text{sech}^{\frac{13}{2}}(x)+\frac{11}{13}\left(\frac{2}{9}\sinh(x)\text{sech}^{\frac{9}{2}}(x)+\frac{7}{9}\left(\frac{2}{5}\sinh(x)\text{sech}^{\frac{5}{2}}(x)+\frac{3}{5}\left(2\sinh(x)\sqrt{\text{sech}(x)}+2\right.\right.\right.\right.}{\text{sech}^{\frac{3}{2}}(x)}$$

input
```
Int[(a*Sech[x]^3)^(5/2),x]
```

output
```
(a^2*Sqrt[a*Sech[x]^3]*((2*Sech[x]^(13/2)*Sinh[x])/13 + (11*((2*Sech[x]^(9
/2)*Sinh[x])/9 + (7*((2*Sech[x]^(5/2)*Sinh[x])/5 + (3*((2*I)*Sqrt[Cosh[x]]
*EllipticE[(I/2)*x, 2]*Sqrt[Sech[x]] + 2*Sqrt[Sech[x]]*Sinh[x]))/5))/9))/1
3))/Sech[x]^(3/2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [F]

$$\int \left(a\,\mathrm{sech}\left(x\right)^{3}\right)^{\frac{5}{2}}dx$$

input
```
int((a*sech(x)^3)^(5/2),x)
```

output
```
int((a*sech(x)^3)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1382 vs. $2(98) = 196$.

Time = 0.12 (sec) , antiderivative size = 1382, normalized size of antiderivative = 11.42

$$\int \left(a\mathrm{sech}^3(x)\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a*sech(x)^3)^(5/2),x, algorithm="fricas")
```

output
```
2/585*(231*sqrt(2)*(a^2*cosh(x)^12 + 12*a^2*cosh(x)*sinh(x)^11 + a^2*sinh(
x)^12 + 6*a^2*cosh(x)^10 + 6*(11*a^2*cosh(x)^2 + a^2)*sinh(x)^10 + 15*a^2*
cosh(x)^8 + 20*(11*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^9 + 15*(33*a^2*c
osh(x)^4 + 18*a^2*cosh(x)^2 + a^2)*sinh(x)^8 + 20*a^2*cosh(x)^6 + 24*(33*a
^2*cosh(x)^5 + 30*a^2*cosh(x)^3 + 5*a^2*cosh(x))*sinh(x)^7 + 4*(231*a^2*co
sh(x)^6 + 315*a^2*cosh(x)^4 + 105*a^2*cosh(x)^2 + 5*a^2)*sinh(x)^6 + 15*a^
2*cosh(x)^4 + 24*(33*a^2*cosh(x)^7 + 63*a^2*cosh(x)^5 + 35*a^2*cosh(x)^3 +
 5*a^2*cosh(x))*sinh(x)^5 + 15*(33*a^2*cosh(x)^8 + 84*a^2*cosh(x)^6 + 70*a
^2*cosh(x)^4 + 20*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 6*a^2*cosh(x)^2 + 20*(1
1*a^2*cosh(x)^9 + 36*a^2*cosh(x)^7 + 42*a^2*cosh(x)^5 + 20*a^2*cosh(x)^3 +
 3*a^2*cosh(x))*sinh(x)^3 + 6*(11*a^2*cosh(x)^10 + 45*a^2*cosh(x)^8 + 70*a
^2*cosh(x)^6 + 50*a^2*cosh(x)^4 + 15*a^2*cosh(x)^2 + a^2)*sinh(x)^2 + a^2
+ 12*(a^2*cosh(x)^11 + 5*a^2*cosh(x)^9 + 10*a^2*cosh(x)^7 + 10*a^2*cosh(x)
^5 + 5*a^2*cosh(x)^3 + a^2*cosh(x))*sinh(x))*sqrt(a)*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cosh(x) + sinh(x))) + sqrt(2)*(231*a^2*cosh(x
)^13 + 3003*a^2*cosh(x)*sinh(x)^12 + 231*a^2*sinh(x)^13 + 1540*a^2*cosh(x)
^11 + 154*(117*a^2*cosh(x)^2 + 10*a^2)*sinh(x)^11 + 4367*a^2*cosh(x)^9 + 1
694*(39*a^2*cosh(x)^3 + 10*a^2*cosh(x))*sinh(x)^10 + 11*(15015*a^2*cosh(x)
^4 + 7700*a^2*cosh(x)^2 + 397*a^2)*sinh(x)^9 + 6808*a^2*cosh(x)^7 + 33*(90
09*a^2*cosh(x)^5 + 7700*a^2*cosh(x)^3 + 1191*a^2*cosh(x))*sinh(x)^8 + 4...
```

Sympy [F]

$$\int \left(a\text{sech}^3(x)\right)^{5/2} dx = \int \left(a \operatorname{sech}^{3}{\left(x \right)}\right)^{\frac{5}{2}}\, dx$$

input
```
integrate((a*sech(x)**3)**(5/2),x)
```

output
```
Integral((a*sech(x)**3)**(5/2), x)
```

Maxima [F]

$$\int \left(a\text{sech}^3(x)\right)^{5/2} dx = \int \left(a\,\text{sech}\,(x)^3\right)^{\frac{5}{2}} dx$$

input
```
integrate((a*sech(x)^3)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sech(x)^3)^(5/2), x)
```

Giac [F]

$$\int \left(a\text{sech}^3(x)\right)^{5/2} dx = \int \left(a\,\text{sech}\,(x)^3\right)^{\frac{5}{2}} dx$$

input
```
integrate((a*sech(x)^3)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sech(x)^3)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\text{sech}^3(x)\right)^{5/2} dx = \int \left(\frac{a}{\cosh\,(x)^3}\right)^{5/2} dx$$

input
```
int((a/cosh(x)^3)^(5/2),x)
```

output
```
int((a/cosh(x)^3)^(5/2), x)
```

Reduce [F]

$$\int \left(a\,\mathrm{sech}^3(x)\right)^{5/2} dx = \sqrt{a}\left(\int \sqrt{\mathrm{sech}\,(x)}\,\mathrm{sech}(x)^7\,dx\right)a^2$$

input
```
int((a*sech(x)^3)^(5/2),x)
```

output
```
sqrt(a)*int(sqrt(sech(x))*sech(x)**7,x)*a**2
```

3.40 $\int \left(a\operatorname{sech}^3(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 69

$$\int \left(a\operatorname{sech}^3(x)\right)^{3/2}\,dx=-\frac{10}{21}ia\cosh^{\frac{3}{2}}(x)\operatorname{EllipticF}\left(\frac{ix}{2},2\right)\sqrt{a\operatorname{sech}^3(x)}+\frac{10}{21}a\sqrt{a\operatorname{sech}^3(x)}\sinh(x)+\frac{2}{7}a\operatorname{sech}(x)\sqrt{a\operatorname{sech}^3(x)}\tanh(x)$$

output
```
-10/21*I*a*cosh(x)^(3/2)*InverseJacobiAM(1/2*I*x,2^(1/2))*(a*sech(x)^3)^(1
/2)+10/21*a*(a*sech(x)^3)^(1/2)*sinh(x)+2/7*a*sech(x)*(a*sech(x)^3)^(1/2)*
tanh(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.68

$$\int \left(a\operatorname{sech}^3(x)\right)^{3/2}\,dx=\frac{2}{21}a\operatorname{sech}(x)\sqrt{a\operatorname{sech}^3(x)}\left(-5i\cosh^{\frac{5}{2}}(x)\operatorname{EllipticF}\left(\frac{ix}{2},2\right)+5\cosh(x)\sinh(x)+3\tanh(x)\right)$$

input
```
Integrate[(a*Sech[x]^3)^(3/2),x]
```

output
```
(2*a*Sech[x]*Sqrt[a*Sech[x]^3]*((-5*I)*Cosh[x]^(5/2)*EllipticF[(I/2)*x, 2]
 + 5*Cosh[x]*Sinh[x] + 3*Tanh[x]))/21
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4611, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\text{sech}^3(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sec(ix)^3\right)^{3/2}dx$$

$$\Big\downarrow \text{4611}$$

$$\frac{a\sqrt{a\text{sech}^3(x)}\int \text{sech}^{\frac{9}{2}}(x)dx}{\text{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{a\text{sech}^3(x)}\int \csc\left(ix+\frac{\pi}{2}\right)^{9/2}dx}{\text{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{a\sqrt{a\text{sech}^3(x)}\left(\frac{5}{7}\int \text{sech}^{\frac{5}{2}}(x)dx+\frac{2}{7}\sinh(x)\text{sech}^{\frac{7}{2}}(x)\right)}{\text{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{a\text{sech}^3(x)}\left(\frac{2}{7}\sinh(x)\text{sech}^{\frac{7}{2}}(x)+\frac{5}{7}\int \csc\left(ix+\frac{\pi}{2}\right)^{5/2}dx\right)}{\text{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{a\sqrt{a\text{sech}^3(x)}\left(\frac{5}{7}\left(\frac{1}{3}\int\sqrt{\text{sech}(x)}dx+\frac{2}{3}\sinh(x)\text{sech}^{\frac{3}{2}}(x)\right)+\frac{2}{7}\sinh(x)\text{sech}^{\frac{7}{2}}(x)\right)}{\text{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{a\text{sech}^3(x)}\left(\frac{2}{7}\sinh(x)\text{sech}^{\frac{7}{2}}(x)+\frac{5}{7}\left(\frac{2}{3}\sinh(x)\text{sech}^{\frac{3}{2}}(x)+\frac{1}{3}\int\sqrt{\csc\left(ix+\frac{\pi}{2}\right)}dx\right)\right)}{\text{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a\sqrt{a\text{sech}^3(x)}\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cosh(x)}\sqrt{\text{sech}(x)}\int\frac{1}{\sqrt{\cosh(x)}}dx+\frac{2}{3}\sinh(x)\text{sech}^{\frac{3}{2}}(x)\right)+\frac{2}{7}\sinh(x)\text{sech}^{\frac{7}{2}}(x)\right)}{\text{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{a\text{sech}^3(x)}\left(\frac{2}{7}\sinh(x)\text{sech}^{\frac{7}{2}}(x)+\frac{5}{7}\left(\frac{2}{3}\sinh(x)\text{sech}^{\frac{3}{2}}(x)+\frac{1}{3}\sqrt{\cosh(x)}\sqrt{\text{sech}(x)}\int\frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}}dx\right)\right)}{\text{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{a\sqrt{a\text{sech}^3(x)}\left(\frac{2}{7}\sinh(x)\text{sech}^{\frac{7}{2}}(x)+\frac{5}{7}\left(\frac{2}{3}\sinh(x)\text{sech}^{\frac{3}{2}}(x)-\frac{2}{3}i\sqrt{\cosh(x)}\sqrt{\text{sech}(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)\right)\right)}{\text{sech}^{\frac{3}{2}}(x)}$$

input

```
Int[(a*Sech[x]^3)^(3/2),x]
```

output

```
(a*Sqrt[a*Sech[x]^3]*((2*Sech[x]^(7/2)*Sinh[x])/7 + (5*(((-2*I)/3)*Sqrt[Co
sh[x]]*EllipticF[(I/2)*x, 2]*Sqrt[Sech[x]] + (2*Sech[x]^(3/2)*Sinh[x])/3))
/7))/Sech[x]^(3/2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [F]

$$\int \left(a\,\mathrm{sech}\left(x\right)^{3}\right)^{\frac{3}{2}}dx$$

input
```
int((a*sech(x)^3)^(3/2),x)
```

output
```
int((a*sech(x)^3)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(51) = 102$.

Time = 0.10 (sec) , antiderivative size = 391, normalized size of antiderivative = 5.67

$$\int \left(a\text{sech}^3(x)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((a*sech(x)^3)^(3/2),x, algorithm="fricas")
```

output
```
2/21*(5*sqrt(2)*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 + 3*a*c
osh(x)^4 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x
))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*sinh(
x)^2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(a)*we
ierstrassPInverse(-4, 0, cosh(x) + sinh(x)) + sqrt(2)*(5*a*cosh(x)^6 + 30*
a*cosh(x)*sinh(x)^5 + 5*a*sinh(x)^6 + 17*a*cosh(x)^4 + (75*a*cosh(x)^2 + 1
7*a)*sinh(x)^4 + 4*(25*a*cosh(x)^3 + 17*a*cosh(x))*sinh(x)^3 - 17*a*cosh(x
)^2 + (75*a*cosh(x)^4 + 102*a*cosh(x)^2 - 17*a)*sinh(x)^2 + 2*(15*a*cosh(x
)^5 + 34*a*cosh(x)^3 - 17*a*cosh(x))*sinh(x) - 5*a)*sqrt((a*cosh(x) + a*si
nh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)))/(cosh(x)^6 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3*cosh(x)^4
+ 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1
)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + cosh(x))*sinh(x)
+ 1)
```

Sympy [F]

$$\int \left(a\text{sech}^3(x)\right)^{3/2} dx = \int \left(a \operatorname{sech}^3(x)\right)^{\frac{3}{2}} dx$$

input
```
integrate((a*sech(x)**3)**(3/2),x)
```

output
```
Integral((a*sech(x)**3)**(3/2), x)
```

Maxima [F]

$$\int \left(a\text{sech}^3(x)\right)^{3/2} \, dx = \int \left(a\,\text{sech}\,(x)^3\right)^{\frac{3}{2}} \, dx$$

input
```
integrate((a*sech(x)^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sech(x)^3)^(3/2), x)
```

Giac [F]

$$\int \left(a\text{sech}^3(x)\right)^{3/2} \, dx = \int \left(a\,\text{sech}\,(x)^3\right)^{\frac{3}{2}} \, dx$$

input
```
integrate((a*sech(x)^3)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sech(x)^3)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\text{sech}^3(x)\right)^{3/2} \, dx = \int \left(\frac{a}{\cosh\,(x)^3}\right)^{3/2} \, dx$$

input
```
int((a/cosh(x)^3)^(3/2),x)
```

output
```
int((a/cosh(x)^3)^(3/2), x)
```

Reduce [F]

$$\int \left(a\operatorname{sech}^3(x)\right)^{3/2} dx = \sqrt{a}\left(\int \sqrt{\operatorname{sech}(x)}\,\operatorname{sech}(x)^4\, dx\right) a$$

input
```
int((a*sech(x)^3)^(3/2),x)
```

output
```
sqrt(a)*int(sqrt(sech(x))*sech(x)**4,x)*a
```

3.41 $\int \sqrt{a\,\mathrm{sech}^3(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \sqrt{a\,\mathrm{sech}^3(x)}\,dx = 2i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\,\mathrm{sech}^3(x)}+2\cosh(x)\sqrt{a\,\mathrm{sech}^3(x)}\sinh(x)$$

output
```
2*I*cosh(x)^(3/2)*EllipticE(I*sinh(1/2*x),2^(1/2))*(a*sech(x)^3)^(1/2)+2*c
osh(x)*(a*sech(x)^3)^(1/2)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sqrt{a\,\mathrm{sech}^3(x)}\,dx = 2\cosh(x)\sqrt{a\,\mathrm{sech}^3(x)}\left(i\sqrt{\cosh(x)}E\left(\frac{ix}{2}\middle|2\right)+\sinh(x)\right)$$

input
```
Integrate[Sqrt[a*Sech[x]^3],x]
```

output
```
2*Cosh[x]*Sqrt[a*Sech[x]^3]*(I*Sqrt[Cosh[x]]*EllipticE[(I/2)*x, 2] + Sinh[
x])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 4611, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\operatorname{sech}^3(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\sec(ix)^3}dx$$

$$\Big\downarrow \text{4611}$$

$$\frac{\sqrt{a\operatorname{sech}^3(x)}\int \operatorname{sech}^{\frac{3}{2}}(x)dx}{\operatorname{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a\operatorname{sech}^3(x)}\int \csc\left(ix+\frac{\pi}{2}\right)^{3/2}dx}{\operatorname{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{a\operatorname{sech}^3(x)}\left(2\sinh(x)\sqrt{\operatorname{sech}(x)}-\int \frac{1}{\sqrt{\operatorname{sech}(x)}}dx\right)}{\operatorname{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a\operatorname{sech}^3(x)}\left(2\sinh(x)\sqrt{\operatorname{sech}(x)}-\int \frac{1}{\sqrt{\csc\left(ix+\frac{\pi}{2}\right)}}dx\right)}{\operatorname{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\sqrt{a\operatorname{sech}^3(x)}\left(2\sinh(x)\sqrt{\operatorname{sech}(x)}-\sqrt{\cosh(x)}\sqrt{\operatorname{sech}(x)}\int \sqrt{\cosh(x)}dx\right)}{\operatorname{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a\text{sech}^3(x)}\left(2\sinh(x)\sqrt{\text{sech}(x)}-\sqrt{\cosh(x)}\sqrt{\text{sech}(x)}\int\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}dx\right)}{\text{sech}^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sqrt{a\text{sech}^3(x)}\left(2\sinh(x)\sqrt{\text{sech}(x)}+2i\sqrt{\cosh(x)}\sqrt{\text{sech}(x)}E\left(\frac{ix}{2}\middle|2\right)\right)}{\text{sech}^{\frac{3}{2}}(x)}$$

input
```
Int[Sqrt[a*Sech[x]^3],x]
```

output
```
(Sqrt[a*Sech[x]^3]*((2*I)*Sqrt[Cosh[x]]*EllipticE[(I/2)*x, 2]*Sqrt[Sech[x]
] + 2*Sqrt[Sech[x]]*Sinh[x]))/Sech[x]^(3/2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [F]

$$\int \sqrt{a\,\mathrm{sech}\,(x)^{3}}dx$$

input
```
int((a*sech(x)^3)^(1/2),x)
```

output
```
int((a*sech(x)^3)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \sqrt{a\mathrm{sech}^{3}(x)}\,dx = 2\sqrt{2}\sqrt{\frac{a\cosh(x)+a\sinh(x)}{\cosh(x)^{2}+2\cosh(x)\sinh(x)+\sinh(x)^{2}+1}}(\cosh(x)+\sinh(x)) + 2\sqrt{2}\sqrt{a}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cosh(x)+\sinh(x)))$$

input
```
integrate((a*sech(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(2)*sqrt((a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + si
nh(x)^2 + 1))*(cosh(x) + sinh(x)) + 2*sqrt(2)*sqrt(a)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cosh(x) + sinh(x)))
```

Sympy [F]

$$\int \sqrt{a\text{sech}^3(x)}\,dx = \int \sqrt{a\,\text{sech}^3\,(x)}\,dx$$

input
```
integrate((a*sech(x)**3)**(1/2),x)
```

output
```
Integral(sqrt(a*sech(x)**3), x)
```

Maxima [F]

$$\int \sqrt{a\text{sech}^3(x)}\,dx = \int \sqrt{a\,\text{sech}\,(x)^3}\,dx$$

input
```
integrate((a*sech(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sech(x)^3), x)
```

Giac [F]

$$\int \sqrt{a\text{sech}^3(x)}\,dx = \int \sqrt{a\,\text{sech}\,(x)^3}\,dx$$

input
```
integrate((a*sech(x)^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sech(x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\text{sech}^3(x)}\,dx = \int \sqrt{\frac{a}{\cosh{(x)}^3}}\,dx$$

input
```
int((a/cosh(x)^3)^(1/2),x)
```

output
```
int((a/cosh(x)^3)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a\text{sech}^3(x)}\,dx = \sqrt{a}\left(\int \sqrt{\text{sech}\,(x)}\,\text{sech}(x)\,dx\right)$$

input
```
int((a*sech(x)^3)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sech(x))*sech(x),x)
```

3.42 $\int \frac{1}{\sqrt{a\text{sech}^3(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 48

$$\int \frac{1}{\sqrt{a\text{sech}^3(x)}}\,dx=-\frac{2i\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{3\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}}+\frac{2\tanh(x)}{3\sqrt{a\text{sech}^3(x)}}$$

output
```
-2/3*I*InverseJacobiAM(1/2*I*x,2^(1/2))/cosh(x)^(3/2)/(a*sech(x)^3)^(1/2)+
2/3*tanh(x)/(a*sech(x)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{a\text{sech}^3(x)}}\,dx=\frac{-\frac{2i\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{\cosh^{\frac{3}{2}}(x)}+2\tanh(x)}{3\sqrt{a\text{sech}^3(x)}}$$

input
```
Integrate[1/Sqrt[a*Sech[x]^3],x]
```

output
```
(((-2*I)*EllipticF[(I/2)*x, 2])/Cosh[x]^(3/2) + 2*Tanh[x])/(3*Sqrt[a*Sech[
x]^3])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 4611, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\text{sech}^3(x)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt{a\sec(ix)^3}}dx$$

$$\Big\downarrow \; 4611$$

$$\frac{\text{sech}^{\frac{3}{2}}(x)\int \frac{1}{\text{sech}^{\frac{3}{2}}(x)}dx}{\sqrt{a\text{sech}^3(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\text{sech}^{\frac{3}{2}}(x)\int \frac{1}{\csc\left(ix+\frac{\pi}{2}\right)^{3/2}}dx}{\sqrt{a\text{sech}^3(x)}}$$

$$\Big\downarrow \; 4256$$

$$\frac{\text{sech}^{\frac{3}{2}}(x)\left(\frac{1}{3}\int \sqrt{\text{sech}(x)}dx+\frac{2\sinh(x)}{3\sqrt{\text{sech}(x)}}\right)}{\sqrt{a\text{sech}^3(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\text{sech}^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{3\sqrt{\text{sech}(x)}}+\frac{1}{3}\int \sqrt{\csc\left(ix+\frac{\pi}{2}\right)}dx\right)}{\sqrt{a\text{sech}^3(x)}}$$

$$\Big\downarrow \; 4258$$

$$\frac{\operatorname{sech}^{\frac{3}{2}}(x)\left(\frac{1}{3}\sqrt{\cosh(x)}\sqrt{\operatorname{sech}(x)}\int\frac{1}{\sqrt{\cosh(x)}}dx+\frac{2\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}\right)}{\sqrt{a\operatorname{sech}^3(x)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\operatorname{sech}^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}+\frac{1}{3}\sqrt{\cosh(x)}\sqrt{\operatorname{sech}(x)}\int\frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}}dx\right)}{\sqrt{a\operatorname{sech}^3(x)}}$$

$$\Big\downarrow \ 3120$$

$$\frac{\operatorname{sech}^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}-\frac{2}{3}i\sqrt{\cosh(x)}\sqrt{\operatorname{sech}(x)}\operatorname{EllipticF}\left(\frac{ix}{2},2\right)\right)}{\sqrt{a\operatorname{sech}^3(x)}}$$

input
```
Int[1/Sqrt[a*Sech[x]^3],x]
```

output
```
(Sech[x]^(3/2)*(((-2*I)/3)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2]*Sqrt[Sech[x
]] + (2*Sinh[x])/(3*Sqrt[Sech[x]])))/Sqrt[a*Sech[x]^3]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\sqrt{a\,\mathrm{sech}\,(x)^{3}}}dx$$

input
```
int(1/(a*sech(x)^3)^(1/2),x)
```

output
```
int(1/(a*sech(x)^3)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(34) = 68.

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.62

$$\int \frac{1}{\sqrt{a\mathrm{sech}^3(x)}}\,dx = \frac{4\sqrt{2}\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2\right)\sqrt{a}\mathrm{weierstrassPInverse}(-4, 0, \cosh(x) + \sinh(x)) + \sqrt{}}{6\left(a\cosh(x)\right.}$$

input
```
integrate(1/(a*sech(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(4*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(a)*weierst
rassPInverse(-4, 0, cosh(x) + sinh(x)) + sqrt(2)*(cosh(x)^4 + 4*cosh(x)^3*
sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)*sqr
t((a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1))
)/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\text{sech}^3(x)}}\,dx = \int \frac{1}{\sqrt{a\,\text{sech}^3\left(x\right)}}\,dx$$

input
```
integrate(1/(a*sech(x)**3)**(1/2),x)
```

output
```
Integral(1/sqrt(a*sech(x)**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a\text{sech}^3(x)}}\,dx = \int \frac{1}{\sqrt{a\,\text{sech}\left(x\right)^3}}\,dx$$

input
```
integrate(1/(a*sech(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*sech(x)^3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a\operatorname{sech}^3(x)}}\,dx = \int \frac{1}{\sqrt{a\operatorname{sech}(x)^3}}\,dx$$

input
```
integrate(1/(a*sech(x)^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a*sech(x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\operatorname{sech}^3(x)}}\,dx = \int \frac{1}{\sqrt{\frac{a}{\cosh(x)^3}}}\,dx$$

input
```
int(1/(a/cosh(x)^3)^(1/2),x)
```

output
```
int(1/(a/cosh(x)^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a\operatorname{sech}^3(x)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\operatorname{sech}(x)}}{\operatorname{sech}(x)^2}dx\right)}{a}$$

input
```
int(1/(a*sech(x)^3)^(1/2),x)
```

output
```
(sqrt(a)*int(sqrt(sech(x))/sech(x)**2,x))/a
```

3.43 $\int \frac{1}{\left(a\operatorname{sech}^3(x)\right)^{3/2}} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 77

$$\int \frac{1}{\left(a\operatorname{sech}^3(x)\right)^{3/2}} \, dx = -\frac{14iE\left(\frac{ix}{2}\middle|2\right)}{15a\cosh^{\frac{3}{2}}(x)\sqrt{a\operatorname{sech}^3(x)}}+\frac{14\sinh(x)}{45a\sqrt{a\operatorname{sech}^3(x)}}+\frac{2\cosh^2(x)\sinh(x)}{9a\sqrt{a\operatorname{sech}^3(x)}}$$

output
```
-14/15*I*EllipticE(I*sinh(1/2*x),2^(1/2))/a/cosh(x)^(3/2)/(a*sech(x)^3)^(1
/2)+14/45*sinh(x)/a/(a*sech(x)^3)^(1/2)+2/9*cosh(x)^2*sinh(x)/a/(a*sech(x)
^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61

$$\int \frac{1}{\left(a\operatorname{sech}^3(x)\right)^{3/2}} \, dx = \frac{-\frac{84iE\left(\frac{ix}{2}|2\right)}{\cosh^{\frac{3}{2}}(x)}+33\sinh(x)+5\sinh(3x)}{90a\sqrt{a\operatorname{sech}^3(x)}}$$

input
```
Integrate[(a*Sech[x]^3)^(-3/2),x]
```

output
```
(((-84*I)*EllipticE[(I/2)*x, 2])/Cosh[x]^(3/2) + 33*Sinh[x] + 5*Sinh[3*x])
/(90*a*Sqrt[a*Sech[x]^3])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4611, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sec(ix)^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{4611}$$

$$\frac{\text{sech}^{\frac{3}{2}}(x)\int \frac{1}{\text{sech}^{\frac{9}{2}}(x)}dx}{a\sqrt{a\text{sech}^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{sech}^{\frac{3}{2}}(x)\int \frac{1}{\csc\left(ix+\frac{\pi}{2}\right)^{9/2}}dx}{a\sqrt{a\text{sech}^3(x)}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{\text{sech}^{\frac{3}{2}}(x)\left(\frac{7}{9}\int \frac{1}{\text{sech}^{\frac{5}{2}}(x)}dx+\frac{2\sinh(x)}{9\text{sech}^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\text{sech}^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{sech}^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{9\text{sech}^{\frac{7}{2}}(x)}+\frac{7}{9}\int \frac{1}{\csc\left(ix+\frac{\pi}{2}\right)^{5/2}}dx\right)}{a\sqrt{a\text{sech}^3(x)}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{\operatorname{sech}^{\frac{3}{2}}(x)\left(\frac{7}{9}\left(\frac{3}{5}\int\frac{1}{\sqrt{\operatorname{sech}(x)}}dx+\frac{2\sinh(x)}{5\operatorname{sech}^{\frac{3}{2}}(x)}\right)+\frac{2\sinh(x)}{9\operatorname{sech}^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\operatorname{sech}^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\operatorname{sech}^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{9\operatorname{sech}^{\frac{7}{2}}(x)}+\frac{7}{9}\left(\frac{2\sinh(x)}{5\operatorname{sech}^{\frac{3}{2}}(x)}+\frac{3}{5}\int\frac{1}{\sqrt{\csc\left(ix+\frac{\pi}{2}\right)}}dx\right)\right)}{a\sqrt{a\operatorname{sech}^3(x)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\operatorname{sech}^{\frac{3}{2}}(x)\left(\frac{7}{9}\left(\frac{3}{5}\sqrt{\cosh(x)}\sqrt{\operatorname{sech}(x)}\int\sqrt{\cosh(x)}dx+\frac{2\sinh(x)}{5\operatorname{sech}^{\frac{3}{2}}(x)}\right)+\frac{2\sinh(x)}{9\operatorname{sech}^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\operatorname{sech}^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\operatorname{sech}^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{9\operatorname{sech}^{\frac{7}{2}}(x)}+\frac{7}{9}\left(\frac{2\sinh(x)}{5\operatorname{sech}^{\frac{3}{2}}(x)}+\frac{3}{5}\sqrt{\cosh(x)}\sqrt{\operatorname{sech}(x)}\int\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}dx\right)\right)}{a\sqrt{a\operatorname{sech}^3(x)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\operatorname{sech}^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{9\operatorname{sech}^{\frac{7}{2}}(x)}+\frac{7}{9}\left(\frac{2\sinh(x)}{5\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{6}{5}i\sqrt{\cosh(x)}\sqrt{\operatorname{sech}(x)}E\left(\frac{ix}{2}\middle|2\right)\right)\right)}{a\sqrt{a\operatorname{sech}^3(x)}}$$

input
```
Int[(a*Sech[x]^3)^(-3/2),x]
```

output
```
(Sech[x]^(3/2)*((2*Sinh[x])/(9*Sech[x]^(7/2)) + (7*(((-6*I)/5)*Sqrt[Cosh[x
]]*EllipticE[(I/2)*x, 2]*Sqrt[Sech[x]] + (2*Sinh[x])/(5*Sech[x]^(3/2))))/9
))/(a*Sqrt[a*Sech[x]^3])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\left(a \operatorname{sech}\left(x\right)^{3}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(a*sech(x)^3)^(3/2),x)
```

output
```
int(1/(a*sech(x)^3)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(62) = 124$.

Time = 0.10 (sec) , antiderivative size = 407, normalized size of antiderivative = 5.29

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*sech(x)^3)^(3/2),x, algorithm="fricas")
```

output
```
-1/720*(672*sqrt(2)*(cosh(x)^5 + 5*cosh(x)^4*sinh(x) + 10*cosh(x)^3*sinh(x
)^2 + 10*cosh(x)^2*sinh(x)^3 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5)*sqrt(a)*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(x) + sinh(x))) - sqrt
(2)*(5*cosh(x)^10 + 50*cosh(x)*sinh(x)^9 + 5*sinh(x)^10 + (225*cosh(x)^2 +
 43)*sinh(x)^8 + 43*cosh(x)^8 + 8*(75*cosh(x)^3 + 43*cosh(x))*sinh(x)^7 +
2*(525*cosh(x)^4 + 602*cosh(x)^2 - 149)*sinh(x)^6 - 298*cosh(x)^6 + 4*(315
*cosh(x)^5 + 602*cosh(x)^3 - 447*cosh(x))*sinh(x)^5 + 2*(525*cosh(x)^6 + 1
505*cosh(x)^4 - 2235*cosh(x)^2 - 187)*sinh(x)^4 - 374*cosh(x)^4 + 8*(75*co
sh(x)^7 + 301*cosh(x)^5 - 745*cosh(x)^3 - 187*cosh(x))*sinh(x)^3 + (225*co
sh(x)^8 + 1204*cosh(x)^6 - 4470*cosh(x)^4 - 2244*cosh(x)^2 - 43)*sinh(x)^2
 - 43*cosh(x)^2 + 2*(25*cosh(x)^9 + 172*cosh(x)^7 - 894*cosh(x)^5 - 748*co
sh(x)^3 - 43*cosh(x))*sinh(x) - 5)*sqrt((a*cosh(x) + a*sinh(x))/(cosh(x)^2
 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)))/(a^2*cosh(x)^5 + 5*a^2*cosh(x)^4*s
inh(x) + 10*a^2*cosh(x)^3*sinh(x)^2 + 10*a^2*cosh(x)^2*sinh(x)^3 + 5*a^2*c
osh(x)*sinh(x)^4 + a^2*sinh(x)^5)
```

Sympy [F]

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\operatorname{sech}^3{(x)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sech(x)**3)**(3/2),x)
```

output
```
Integral((a*sech(x)**3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\,\text{sech}\,(x)^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sech(x)^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sech(x)^3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\,\text{sech}\,(x)^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sech(x)^3)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sech(x)^3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{a}{\cosh(x)^3}\right)^{3/2}}\,dx$$

input
```
int(1/(a/cosh(x)^3)^(3/2),x)
```

output
```
int(1/(a/cosh(x)^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a\operatorname{sech}^3(x)\right)^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\operatorname{sech}(x)}}{\operatorname{sech}(x)^5}dx\right)}{a^2}$$

input
```
int(1/(a*sech(x)^3)^(3/2),x)
```

output
```
(sqrt(a)*int(sqrt(sech(x))/sech(x)**5,x))/a**2
```

3.44 $\int \frac{1}{\left(a\operatorname{sech}^3(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 121

$$\int \frac{1}{\left(a\operatorname{sech}^3(x)\right)^{5/2}}\,dx = -\frac{26i\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{77a^2\cosh^{\frac{3}{2}}(x)\sqrt{a\operatorname{sech}^3(x)}}+\frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{26\cosh^3(x)\sinh(x)}{165a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{26\tanh(x)}{77a^2\sqrt{a\operatorname{sech}^3(x)}}$$

output
```
-26/77*I*InverseJacobiAM(1/2*I*x,2^(1/2))/a^2/cosh(x)^(3/2)/(a*sech(x)^3)^
(1/2)+78/385*cosh(x)*sinh(x)/a^2/(a*sech(x)^3)^(1/2)+26/165*cosh(x)^3*sinh
(x)/a^2/(a*sech(x)^3)^(1/2)+2/15*cosh(x)^5*sinh(x)/a^2/(a*sech(x)^3)^(1/2)
+26/77*tanh(x)/a^2/(a*sech(x)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

$$\int \frac{1}{\left(a\operatorname{sech}^3(x)\right)^{5/2}}\,dx = \frac{\cosh(x)\sqrt{a\operatorname{sech}^3(x)}\left(-24960i\sqrt{\cosh(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)+19122\sinh(2x)+4406\,\text{s}\right.}{73920a^3}$$

input
```
Integrate[(a*Sech[x]^3)^(-5/2),x]
```

output
```
(Cosh[x]*Sqrt[a*Sech[x]^3]*((-24960*I)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2]
 + 19122*Sinh[2*x] + 4406*Sinh[4*x] + 826*Sinh[6*x] + 77*Sinh[8*x]))/(7392
0*a^3)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.400$, Rules used = {3042, 4611, 3042, 4256, 3042, 4256, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sec(ix)^3\right)^{5/2}}dx$$

$$\Big\downarrow \text{4611}$$

$$\frac{\text{sech}^{\frac{3}{2}}(x)\int \frac{1}{\text{sech}^{\frac{15}{2}}(x)}dx}{a^2\sqrt{a\text{sech}^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{sech}^{\frac{3}{2}}(x)\int \frac{1}{\csc\left(ix+\frac{\pi}{2}\right)^{15/2}}dx}{a^2\sqrt{a\text{sech}^3(x)}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{\text{sech}^{\frac{3}{2}}(x)\left(\frac{13}{15}\int \frac{1}{\text{sech}^{\frac{11}{2}}(x)}dx+\frac{2\sinh(x)}{15\text{sech}^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\text{sech}^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{sech}^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{15\text{sech}^{\frac{13}{2}}(x)}+\frac{13}{15}\int\frac{1}{\csc\left(ix+\frac{\pi}{2}\right)^{11/2}}dx\right)}{a^2\sqrt{a\text{sech}^3(x)}}$$

$\downarrow$ 4256

$$\frac{\text{sech}^{\frac{3}{2}}(x)\left(\frac{13}{15}\left(\frac{9}{11}\int\frac{1}{\text{sech}^{\frac{7}{2}}(x)}dx+\frac{2\sinh(x)}{11\text{sech}^{\frac{9}{2}}(x)}\right)+\frac{2\sinh(x)}{15\text{sech}^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\text{sech}^3(x)}}$$

$\downarrow$ 3042

$$\frac{\text{sech}^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{15\text{sech}^{\frac{13}{2}}(x)}+\frac{13}{15}\left(\frac{2\sinh(x)}{11\text{sech}^{\frac{9}{2}}(x)}+\frac{9}{11}\int\frac{1}{\csc\left(ix+\frac{\pi}{2}\right)^{7/2}}dx\right)\right)}{a^2\sqrt{a\text{sech}^3(x)}}$$

$\downarrow$ 4256

$$\frac{\text{sech}^{\frac{3}{2}}(x)\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\int\frac{1}{\text{sech}^{\frac{3}{2}}(x)}dx+\frac{2\sinh(x)}{7\text{sech}^{\frac{5}{2}}(x)}\right)+\frac{2\sinh(x)}{11\text{sech}^{\frac{9}{2}}(x)}\right)+\frac{2\sinh(x)}{15\text{sech}^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\text{sech}^3(x)}}$$

$\downarrow$ 3042

$$\frac{\text{sech}^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{15\text{sech}^{\frac{13}{2}}(x)}+\frac{13}{15}\left(\frac{2\sinh(x)}{11\text{sech}^{\frac{9}{2}}(x)}+\frac{9}{11}\left(\frac{2\sinh(x)}{7\text{sech}^{\frac{5}{2}}(x)}+\frac{5}{7}\int\frac{1}{\csc\left(ix+\frac{\pi}{2}\right)^{3/2}}dx\right)\right)\right)}{a^2\sqrt{a\text{sech}^3(x)}}$$

$\downarrow$ 4256

$$\frac{\text{sech}^{\frac{3}{2}}(x)\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\left(\frac{1}{3}\int\sqrt{\text{sech}(x)}dx+\frac{2\sinh(x)}{3\sqrt{\text{sech}(x)}}\right)+\frac{2\sinh(x)}{7\text{sech}^{\frac{5}{2}}(x)}\right)+\frac{2\sinh(x)}{11\text{sech}^{\frac{9}{2}}(x)}\right)+\frac{2\sinh(x)}{15\text{sech}^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\text{sech}^3(x)}}$$

$\downarrow$ 3042

$$\frac{\text{sech}^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{15\text{sech}^{\frac{13}{2}}(x)}+\frac{13}{15}\left(\frac{2\sinh(x)}{11\text{sech}^{\frac{9}{2}}(x)}+\frac{9}{11}\left(\frac{2\sinh(x)}{7\text{sech}^{\frac{5}{2}}(x)}+\frac{5}{7}\left(\frac{2\sinh(x)}{3\sqrt{\text{sech}(x)}}+\frac{1}{3}\int\sqrt{\csc\left(ix+\frac{\pi}{2}\right)}dx\right)\right)\right)\right)}{a^2\sqrt{a\text{sech}^3(x)}}$$

$\downarrow$ 4258

$$\frac{\text{sech}^{\frac{3}{2}}(x)\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cosh(x)}\sqrt{\text{sech}(x)}\int\frac{1}{\sqrt{\cosh(x)}}dx+\frac{2\sinh(x)}{3\sqrt{\text{sech}(x)}}\right)+\frac{2\sinh(x)}{7\text{sech}^{\frac{5}{2}}(x)}\right)+\frac{2\sinh(x)}{11\text{sech}^{\frac{9}{2}}(x)}\right)+\frac{2\sinh(x)}{15\text{sech}^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\text{sech}^3(x)}}$$

$\downarrow$ 3042

$$\frac{\text{sech}^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{15\text{sech}^{\frac{13}{2}}(x)}+\frac{13}{15}\left(\frac{2\sinh(x)}{11\text{sech}^{\frac{9}{2}}(x)}+\frac{9}{11}\left(\frac{2\sinh(x)}{7\text{sech}^{\frac{5}{2}}(x)}+\frac{5}{7}\left(\frac{2\sinh(x)}{3\sqrt{\text{sech}(x)}}+\frac{1}{3}\sqrt{\cosh(x)}\sqrt{\text{sech}(x)}\int\frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}}dx\right)\right)\right)\right)}{a^2\sqrt{a\text{sech}^3(x)}}$$

$\downarrow$ 3120

$$\frac{\text{sech}^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{15\text{sech}^{\frac{13}{2}}(x)}+\frac{13}{15}\left(\frac{2\sinh(x)}{11\text{sech}^{\frac{9}{2}}(x)}+\frac{9}{11}\left(\frac{2\sinh(x)}{7\text{sech}^{\frac{5}{2}}(x)}+\frac{5}{7}\left(\frac{2\sinh(x)}{3\sqrt{\text{sech}(x)}}-\frac{2}{3}i\sqrt{\cosh(x)}\sqrt{\text{sech}(x)}\text{EllipticF}\left(\frac{ix}{2},2\right)\right)\right)\right)\right)}{a^2\sqrt{a\text{sech}^3(x)}}$$

input
```
Int[(a*Sech[x]^3)^(-5/2),x]
```

output
```
(Sech[x]^(3/2)*((2*Sinh[x])/(15*Sech[x]^(13/2)) + (13*((2*Sinh[x])/(11*Sec
h[x]^(9/2)) + (9*((2*Sinh[x])/(7*Sech[x]^(5/2)) + (5*(((-2*I)/3)*Sqrt[Cosh
[x]]*EllipticF[(I/2)*x, 2]*Sqrt[Sech[x]] + (2*Sinh[x])/(3*Sqrt[Sech[x]])))
/7))/11))/15))/(a^2*Sqrt[a*Sech[x]^3])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\left(a\,\mathrm{sech}\left(x\right)^{3}\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(a*sech(x)^3)^(5/2),x)
```

output
```
int(1/(a*sech(x)^3)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 718 vs. $2(95) = 190$.

Time = 0.12 (sec) , antiderivative size = 718, normalized size of antiderivative = 5.93

$$\int \frac{1}{\left(a\mathrm{sech}^3(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*sech(x)^3)^(5/2),x, algorithm="fricas")
```

output
```
1/147840*(49920*sqrt(2)*(cosh(x)^8 + 8*cosh(x)^7*sinh(x) + 28*cosh(x)^6*si
nh(x)^2 + 56*cosh(x)^5*sinh(x)^3 + 70*cosh(x)^4*sinh(x)^4 + 56*cosh(x)^3*s
inh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8)*sqrt(
a)*weierstrassPInverse(-4, 0, cosh(x) + sinh(x)) + sqrt(2)*(77*cosh(x)^16
+ 1232*cosh(x)*sinh(x)^15 + 77*sinh(x)^16 + 14*(660*cosh(x)^2 + 59)*sinh(x
)^14 + 826*cosh(x)^14 + 196*(220*cosh(x)^3 + 59*cosh(x))*sinh(x)^13 + 2*(7
0070*cosh(x)^4 + 37583*cosh(x)^2 + 2203)*sinh(x)^12 + 4406*cosh(x)^12 + 8*
(42042*cosh(x)^5 + 37583*cosh(x)^3 + 6609*cosh(x))*sinh(x)^11 + 2*(308308*
cosh(x)^6 + 413413*cosh(x)^4 + 145398*cosh(x)^2 + 9561)*sinh(x)^10 + 19122
*cosh(x)^10 + 4*(220220*cosh(x)^7 + 413413*cosh(x)^5 + 242330*cosh(x)^3 +
47805*cosh(x))*sinh(x)^9 + 6*(165165*cosh(x)^8 + 413413*cosh(x)^6 + 363495
*cosh(x)^4 + 143415*cosh(x)^2)*sinh(x)^8 + 16*(55055*cosh(x)^9 + 177177*co
sh(x)^7 + 218097*cosh(x)^5 + 143415*cosh(x)^3)*sinh(x)^7 + 2*(308308*cosh(
x)^10 + 1240239*cosh(x)^8 + 2035572*cosh(x)^6 + 2007810*cosh(x)^4 - 9561)*
sinh(x)^6 - 19122*cosh(x)^6 + 4*(84084*cosh(x)^11 + 413413*cosh(x)^9 + 872
388*cosh(x)^7 + 1204686*cosh(x)^5 - 28683*cosh(x))*sinh(x)^5 + 2*(70070*co
sh(x)^12 + 413413*cosh(x)^10 + 1090485*cosh(x)^8 + 2007810*cosh(x)^6 - 143
415*cosh(x)^2 - 2203)*sinh(x)^4 - 4406*cosh(x)^4 + 8*(5390*cosh(x)^13 + 37
583*cosh(x)^11 + 121165*cosh(x)^9 + 286830*cosh(x)^7 - 47805*cosh(x)^3 - 2
203*cosh(x))*sinh(x)^3 + 2*(4620*cosh(x)^14 + 37583*cosh(x)^12 + 145398...
```

Sympy [F]

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\,\text{sech}^3\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*sech(x)**3)**(5/2),x)
```

output
```
Integral((a*sech(x)**3)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\,\text{sech}\,(x)^3\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*sech(x)^3)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sech(x)^3)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\,\text{sech}\,(x)^3\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*sech(x)^3)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sech(x)^3)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(\frac{a}{\cosh(x)^3}\right)^{5/2}}\,dx$$

input
```
int(1/(a/cosh(x)^3)^(5/2),x)
```

output
```
int(1/(a/cosh(x)^3)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a\operatorname{sech}^3(x)\right)^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\operatorname{sech}(x)}}{\operatorname{sech}(x)^8}dx\right)}{a^3}$$

input
```
int(1/(a*sech(x)^3)^(5/2),x)
```

output
```
(sqrt(a)*int(sqrt(sech(x))/sech(x)**8,x))/a**3
```

3.45 $\int \left(a\,\text{sech}^4(x)\right)^{7/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 163

$$\begin{aligned}\int \left(a\,\text{sech}^4(x)\right)^{7/2}\,dx &= a^3\cosh(x)\sqrt{a\,\text{sech}^4(x)}\sinh(x)\\ &\quad - 2a^3\sqrt{a\,\text{sech}^4(x)}\sinh^2(x)\tanh(x) + 3a^3\sqrt{a\,\text{sech}^4(x)}\sinh^2(x)\tanh^3(x)\\ &\quad - \frac{20}{7}a^3\sqrt{a\,\text{sech}^4(x)}\sinh^2(x)\tanh^5(x) + \frac{5}{3}a^3\sqrt{a\,\text{sech}^4(x)}\sinh^2(x)\tanh^7(x)\\ &\quad - \frac{6}{11}a^3\sqrt{a\,\text{sech}^4(x)}\sinh^2(x)\tanh^9(x) + \frac{1}{13}a^3\sqrt{a\,\text{sech}^4(x)}\sinh^2(x)\tanh^{11}(x)\end{aligned}$$

output

```
a^3*cosh(x)*(a*sech(x)^4)^(1/2)*sinh(x)-2*a^3*(a*sech(x)^4)^(1/2)*sinh(x)^
2*tanh(x)+3*a^3*(a*sech(x)^4)^(1/2)*sinh(x)^2*tanh(x)^3-20/7*a^3*(a*sech(x
)^4)^(1/2)*sinh(x)^2*tanh(x)^5+5/3*a^3*(a*sech(x)^4)^(1/2)*sinh(x)^2*tanh(
x)^7-6/11*a^3*(a*sech(x)^4)^(1/2)*sinh(x)^2*tanh(x)^9+1/13*a^3*(a*sech(x)^
4)^(1/2)*sinh(x)^2*tanh(x)^11
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.33

$$\int \left(a\text{sech}^4(x)\right)^{7/2}\,dx = \frac{\cosh(x)(2048+2380\cosh(2x)+1093\cosh(4x)+378\cosh(6x)+92\cosh(8x)+14$$

input
```
Integrate[(a*Sech[x]^4)^(7/2),x]
```

output
```
(Cosh[x]*(2048 + 2380*Cosh[2*x] + 1093*Cosh[4*x] + 378*Cosh[6*x] + 92*Cosh
[8*x] + 14*Cosh[10*x] + Cosh[12*x])*(a*Sech[x]^4)^(7/2)*Sinh[x])/6006
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.52, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4611, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\text{sech}^4(x)\right)^{7/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a\sec(ix)^4\right)^{7/2}dx$$

$$\Big\downarrow \text{ 4611}$$

$$a^3\cosh^2(x)\sqrt{a\text{sech}^4(x)}\int \text{sech}^{14}(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$a^3\cosh^2(x)\sqrt{a\text{sech}^4(x)}\int \csc\left(ix+\frac{\pi}{2}\right)^{14}dx$$

$$\Big\downarrow \text{ 4254}$$

$$ia^3\cosh^2(x)\sqrt{a\text{sech}^4(x)}\int\left(\tanh^{12}(x)-6\tanh^{10}(x)+15\tanh^8(x)-20\tanh^6(x)+15\tanh^4(x)-6\tanh^2(x)+\right.$$

$\big\downarrow$ 2009

$$ia^3\cosh^2(x)\left(-\frac{1}{13}i\tanh^{13}(x)+\frac{6}{11}i\tanh^{11}(x)-\frac{5}{3}i\tanh^9(x)+\frac{20}{7}i\tanh^7(x)-3i\tanh^5(x)+2i\tanh^3(x)-i\tan\right.$$

input
```
Int[(a*Sech[x]^4)^(7/2),x]
```

output
```
I*a^3*Cosh[x]^2*Sqrt[a*Sech[x]^4]*((-I)*Tanh[x] + (2*I)*Tanh[x]^3 - (3*I)*
Tanh[x]^5 + ((20*I)/7)*Tanh[x]^7 - ((5*I)/3)*Tanh[x]^9 + ((6*I)/11)*Tanh[x
]^11 - (I/13)*Tanh[x]^13)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 144.81 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.44

method	result	size
risch	$-\frac{2048 a^{3} {\mathrm e}^{-2 x} \sqrt{\frac{a \,{\mathrm e}^{4 x}}{\left({\mathrm e}^{2 x}+1\right)^{4}}}\, \left(1716 \,{\mathrm e}^{12 x}+1287 \,{\mathrm e}^{10 x}+715 \,{\mathrm e}^{8 x}+286 \,{\mathrm e}^{6 x}+78 \,{\mathrm e}^{4 x}+13 \,{\mathrm e}^{2 x}+1\right)}{3003 \left({\mathrm e}^{2 x}+1\right)^{11}}$	72

input
```
int((a*sech(x)^4)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2048/3003*a^3*exp(-2*x)/(exp(2*x)+1)^11*(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)
*(1716*exp(12*x)+1287*exp(10*x)+715*exp(8*x)+286*exp(6*x)+78*exp(4*x)+13*e
xp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2804 vs. 2(141) = 282.

Time = 0.24 (sec) , antiderivative size = 2804, normalized size of antiderivative = 17.20

$$\int \left(a\text{sech}^4(x)\right)^{7/2}\,dx=\text{Too large to display}$$

input
```
integrate((a*sech(x)^4)^(7/2),x, algorithm="fricas")
```

output
```
-2048/3003*(1716*a^3*cosh(x)^12 + 1287*a^3*cosh(x)^10 + 1716*(a^3*e^(4*x)
+ 2*a^3*e^(2*x) + a^3)*sinh(x)^12 + 20592*(a^3*cosh(x)*e^(4*x) + 2*a^3*cos
h(x)*e^(2*x) + a^3*cosh(x))*sinh(x)^11 + 715*a^3*cosh(x)^8 + 1287*(88*a^3*
cosh(x)^2 + a^3 + (88*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(88*a^3*cosh(x)^2 +
 a^3)*e^(2*x))*sinh(x)^10 + 4290*(88*a^3*cosh(x)^3 + 3*a^3*cosh(x) + (88*a
^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(4*x) + 2*(88*a^3*cosh(x)^3 + 3*a^3*cosh(x
))*e^(2*x))*sinh(x)^9 + 286*a^3*cosh(x)^6 + 715*(1188*a^3*cosh(x)^4 + 81*a
^3*cosh(x)^2 + a^3 + (1188*a^3*cosh(x)^4 + 81*a^3*cosh(x)^2 + a^3)*e^(4*x)
 + 2*(1188*a^3*cosh(x)^4 + 81*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^8 + 11
44*(1188*a^3*cosh(x)^5 + 135*a^3*cosh(x)^3 + 5*a^3*cosh(x) + (1188*a^3*cos
h(x)^5 + 135*a^3*cosh(x)^3 + 5*a^3*cosh(x))*e^(4*x) + 2*(1188*a^3*cosh(x)^
5 + 135*a^3*cosh(x)^3 + 5*a^3*cosh(x))*e^(2*x))*sinh(x)^7 + 78*a^3*cosh(x)
^4 + 286*(5544*a^3*cosh(x)^6 + 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 + a^3
+ (5544*a^3*cosh(x)^6 + 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 + a^3)*e^(4*x
) + 2*(5544*a^3*cosh(x)^6 + 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 + a^3)*e^
(2*x))*sinh(x)^6 + 572*(2376*a^3*cosh(x)^7 + 567*a^3*cosh(x)^5 + 70*a^3*co
sh(x)^3 + 3*a^3*cosh(x) + (2376*a^3*cosh(x)^7 + 567*a^3*cosh(x)^5 + 70*a^3
*cosh(x)^3 + 3*a^3*cosh(x))*e^(4*x) + 2*(2376*a^3*cosh(x)^7 + 567*a^3*cosh
(x)^5 + 70*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(2*x))*sinh(x)^5 + 13*a^3*cosh
(x)^2 + 26*(32670*a^3*cosh(x)^8 + 10395*a^3*cosh(x)^6 + 1925*a^3*cosh(x...
```

Sympy [F(-1)]

Timed out.

$$\int \left(a\text{sech}^4(x)\right)^{7/2} dx = \text{Timed out}$$

input
```
integrate((a*sech(x)**4)**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(141) = 282$.

Time = 0.12 (sec) , antiderivative size = 620, normalized size of antiderivative = 3.80

$$\int \left(a\text{sech}^4(x)\right)^{7/2} dx = \text{Too large to display}$$

input
```
integrate((a*sech(x)^4)^(7/2),x, algorithm="maxima")
```

output
```
2048/231*a^(7/2)*e^(-2*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*
e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*
x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*
x) + 1) + 4096/77*a^(7/2)*e^(-4*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*
x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 128
7*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x)
 + e^(-26*x) + 1) + 4096/21*a^(7/2)*e^(-6*x)/(13*e^(-2*x) + 78*e^(-4*x) + 2
86*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14
*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e
^(-24*x) + e^(-26*x) + 1) + 10240/21*a^(7/2)*e^(-8*x)/(13*e^(-2*x) + 78*e^
(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1
716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22
*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 6144/7*a^(7/2)*e^(-10*x)/(13*e^(-2*x
) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-
12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) +
78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 8192/7*a^(7/2)*e^(-12*x)/(1
3*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) +
1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(
-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 2048/3003*a^(7/2)/
(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*...
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.31

$$\int \left(a\text{sech}^4(x)\right)^{7/2}\,dx = -\frac{2048\,a^{\frac{7}{2}}\left(1716\,e^{(12\,x)}+1287\,e^{(10\,x)}+715\,e^{(8\,x)}+286\,e^{(6\,x)}+78\,e^{(4\,x)}+13\,e^{(2\,x)}+1\right)}{3003\,\left(e^{(2\,x)}+1\right)^{13}}$$

input
```
integrate((a*sech(x)^4)^(7/2),x, algorithm="giac")
```

output
```
-2048/3003*a^(7/2)*(1716*e^(12*x) + 1287*e^(10*x) + 715*e^(8*x) + 286*e^(6
*x) + 78*e^(4*x) + 13*e^(2*x) + 1)/(e^(2*x) + 1)^13
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.06

$$\int \left(a\text{sech}^4(x)\right)^{7/2}\,dx = \text{Too large to display}$$

input
```
int((a/cosh(x)^4)^(7/2),x)
```

output
```
(1536*a^3*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*exp(4*x) + 4*
exp(6*x) + exp(8*x) + 1))/((exp(2*x) + 1)^8*(exp(2*x) + 2*exp(4*x) + exp(6
*x))) - (2048*a^3*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*exp(4
*x) + 4*exp(6*x) + exp(8*x) + 1))/(7*(exp(2*x) + 1)^7*(exp(2*x) + 2*exp(4*
x) + exp(6*x))) - (10240*a^3*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x
) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/(3*(exp(2*x) + 1)^9*(exp(2*x)
 + 2*exp(4*x) + exp(6*x))) + (4096*a^3*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*
(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/((exp(2*x) + 1)^10*
(exp(2*x) + 2*exp(4*x) + exp(6*x))) - (30720*a^3*(a/(exp(-x)/2 + exp(x)/2)
^4)^(1/2)*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/(11*(exp(
2*x) + 1)^11*(exp(2*x) + 2*exp(4*x) + exp(6*x))) + (1024*a^3*(a/(exp(-x)/2
 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1
))/((exp(2*x) + 1)^12*(exp(2*x) + 2*exp(4*x) + exp(6*x))) - (2048*a^3*(a/(
exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp
(8*x) + 1))/(13*(exp(2*x) + 1)^13*(exp(2*x) + 2*exp(4*x) + exp(6*x)))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.90

$$\int \left(a\operatorname{sech}^4(x)\right)^{7/2}\,dx = \frac{2048\sqrt{a}\,a^3(-1716e^{12x}-128}{3003e^{26x}+39039e^{24x}+234234e^{22x}+858858e^{20x}+2147145e^{18x}+3864861e^{16x}+5}$$

input
```
int((a*sech(x)^4)^(7/2),x)
```

output
```
(2048*sqrt(a)*a**3*( - 1716*e**(12*x) - 1287*e**(10*x) - 715*e**(8*x) - 28
6*e**(6*x) - 78*e**(4*x) - 13*e**(2*x) - 1))/(3003*(e**(26*x) + 13*e**(24*
x) + 78*e**(22*x) + 286*e**(20*x) + 715*e**(18*x) + 1287*e**(16*x) + 1716*
e**(14*x) + 1716*e**(12*x) + 1287*e**(10*x) + 715*e**(8*x) + 286*e**(6*x)
+ 78*e**(4*x) + 13*e**(2*x) + 1))
```

3.46 $\int \left(a\,\mathrm{sech}^4(x)\right)^{5/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 117

$$\int \left(a\,\mathrm{sech}^4(x)\right)^{5/2}\,dx = a^2\cosh(x)\sqrt{a\,\mathrm{sech}^4(x)}\sinh(x) - \frac{4}{3}a^2\sqrt{a\,\mathrm{sech}^4(x)}\sinh^2(x)\tanh(x) + \frac{6}{5}a^2\sqrt{a\,\mathrm{sech}^4(x)}\sinh^2(x)\tanh^3(x) - \frac{4}{7}a^2\sqrt{a\,\mathrm{sech}^4(x)}\sinh^2(x)\tanh^5(x) + \frac{1}{9}a^2\sqrt{a\,\mathrm{sech}^4(x)}\sinh^2(x)\tanh^7(x)$$

output
```
a^2*cosh(x)*(a*sech(x)^4)^(1/2)*sinh(x)-4/3*a^2*(a*sech(x)^4)^(1/2)*sinh(x
)^2*tanh(x)+6/5*a^2*(a*sech(x)^4)^(1/2)*sinh(x)^2*tanh(x)^3-4/7*a^2*(a*sec
h(x)^4)^(1/2)*sinh(x)^2*tanh(x)^5+1/9*a^2*(a*sech(x)^4)^(1/2)*sinh(x)^2*ta
nh(x)^7
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.36

$$\int \left(a\,\mathrm{sech}^4(x)\right)^{5/2}\,dx = \frac{1}{315}\cosh(x)(128 + 130\cosh(2x) + 46\cosh(4x) + 10\cosh(6x) + \cosh(8x))\left(a\,\mathrm{sech}^4(x)\right)^{5/2}\sinh(x)$$

input
```
Integrate[(a*Sech[x]^4)^(5/2),x]
```

output
```
(Cosh[x]*(128 + 130*Cosh[2*x] + 46*Cosh[4*x] + 10*Cosh[6*x] + Cosh[8*x])*(
a*Sech[x]^4)^(5/2)*Sinh[x])/315
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.58, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4611, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\text{sech}^4(x)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sec(ix)^4\right)^{5/2} dx$$

$$\Big\downarrow \text{4611}$$

$$a^2\cosh^2(x)\sqrt{a\text{sech}^4(x)}\int \text{sech}^{10}(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a^2\cosh^2(x)\sqrt{a\text{sech}^4(x)}\int \csc\left(ix+\frac{\pi}{2}\right)^{10} dx$$

$$\Big\downarrow \text{4254}$$

$$ia^2\cosh^2(x)\sqrt{a\text{sech}^4(x)}\int \left(\tanh^8(x)-4\tanh^6(x)+6\tanh^4(x)-4\tanh^2(x)+1\right)d(-i\tanh(x))$$

$$\Big\downarrow \text{2009}$$

$$ia^2\cosh^2(x)\left(-\frac{1}{9}i\tanh^9(x)+\frac{4}{7}i\tanh^7(x)-\frac{6}{5}i\tanh^5(x)+\frac{4}{3}i\tanh^3(x)-i\tanh(x)\right)\sqrt{a\text{sech}^4(x)}$$

input
```
Int[(a*Sech[x]^4)^(5/2),x]
```

output
```
I*a^2*Cosh[x]^2*Sqrt[a*Sech[x]^4]*((-I)*Tanh[x] + ((4*I)/3)*Tanh[x]^3 - ((
6*I)/5)*Tanh[x]^5 + ((4*I)/7)*Tanh[x]^7 - (I/9)*Tanh[x]^9)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 150.44 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.51

method	result	size
risch	$-\frac{256 a^{2} {\mathrm e}^{-2 x} \sqrt{\frac{a \,{\mathrm e}^{4 x}}{\left({\mathrm e}^{2 x}+1\right)^{4}}}\, \left(126 \,{\mathrm e}^{8 x}+84 \,{\mathrm e}^{6 x}+36 \,{\mathrm e}^{4 x}+9 \,{\mathrm e}^{2 x}+1\right)}{315 \left({\mathrm e}^{2 x}+1\right)^{7}}$	60

input
```
int((a*sech(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-256/315*a^2*exp(-2*x)/(exp(2*x)+1)^7*(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*(1
26*exp(8*x)+84*exp(6*x)+36*exp(4*x)+9*exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1475 vs. $2(99) = 198$.

Time = 0.13 (sec) , antiderivative size = 1475, normalized size of antiderivative = 12.61

$$\int \left(a\text{sech}^4(x)\right)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((a*sech(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
-256/315*(126*a^2*cosh(x)^8 + 126*(a^2*e^(4*x) + 2*a^2*e^(2*x) + a^2)*sinh
(x)^8 + 84*a^2*cosh(x)^6 + 1008*(a^2*cosh(x)*e^(4*x) + 2*a^2*cosh(x)*e^(2*
x) + a^2*cosh(x))*sinh(x)^7 + 84*(42*a^2*cosh(x)^2 + a^2 + (42*a^2*cosh(x)
^2 + a^2)*e^(4*x) + 2*(42*a^2*cosh(x)^2 + a^2)*e^(2*x))*sinh(x)^6 + 36*a^2
*cosh(x)^4 + 504*(14*a^2*cosh(x)^3 + a^2*cosh(x) + (14*a^2*cosh(x)^3 + a^2
*cosh(x))*e^(4*x) + 2*(14*a^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh(x)^5
+ 36*(245*a^2*cosh(x)^4 + 35*a^2*cosh(x)^2 + a^2 + (245*a^2*cosh(x)^4 + 35
*a^2*cosh(x)^2 + a^2)*e^(4*x) + 2*(245*a^2*cosh(x)^4 + 35*a^2*cosh(x)^2 +
a^2)*e^(2*x))*sinh(x)^4 + 9*a^2*cosh(x)^2 + 48*(147*a^2*cosh(x)^5 + 35*a^2
*cosh(x)^3 + 3*a^2*cosh(x) + (147*a^2*cosh(x)^5 + 35*a^2*cosh(x)^3 + 3*a^2
*cosh(x))*e^(4*x) + 2*(147*a^2*cosh(x)^5 + 35*a^2*cosh(x)^3 + 3*a^2*cosh(x
))*e^(2*x))*sinh(x)^3 + 9*(392*a^2*cosh(x)^6 + 140*a^2*cosh(x)^4 + 24*a^2*
cosh(x)^2 + a^2 + (392*a^2*cosh(x)^6 + 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^
2 + a^2)*e^(4*x) + 2*(392*a^2*cosh(x)^6 + 140*a^2*cosh(x)^4 + 24*a^2*cosh(
x)^2 + a^2)*e^(2*x))*sinh(x)^2 + a^2 + (126*a^2*cosh(x)^8 + 84*a^2*cosh(x)
^6 + 36*a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 + a^2)*e^(4*x) + 2*(126*a^2*cosh(x
)^8 + 84*a^2*cosh(x)^6 + 36*a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 + a^2)*e^(2*x)
 + 18*(56*a^2*cosh(x)^7 + 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 + a^2*cosh(x)
 + (56*a^2*cosh(x)^7 + 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 + a^2*cosh(x))*e
^(4*x) + 2*(56*a^2*cosh(x)^7 + 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 + a^2...
```

Sympy [F]

$$\int \left(a\text{sech}^4(x)\right)^{5/2} dx = \int \left(a \operatorname{sech}^4(x)\right)^{\frac{5}{2}} dx$$

input
```
integrate((a*sech(x)**4)**(5/2),x)
```

output
```
Integral((a*sech(x)**4)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(99) = 198.

Time = 0.12 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.75

$$\int \left(a\text{sech}^4(x)\right)^{5/2} dx = \frac{256 a^{\frac{5}{2}} e^{(-2x)}}{35 \left(9 e^{(-2x)} + 36 e^{(-4x)} + 84 e^{(-6x)} + 126 e^{(-8x)} + 126 e^{(-10x)} + 84 e^{(-12x)} + 36 e^{(-\cdots} \right.} + \frac{1024 a^{\frac{5}{2}} e^{(-4x)}}{35 \left(9 e^{(-2x)} + 36 e^{(-4x)} + 84 e^{(-6x)} + 126 e^{(-8x)} + 126 e^{(-10x)} + 84 e^{(-12x)} + 36 e^{(-14x)} + 9 e^{(-16x)} + e^{(-18\cdots} \right.} + \frac{1024 a^{\frac{5}{2}} e^{(-6x)}}{15 \left(9 e^{(-2x)} + 36 e^{(-4x)} + 84 e^{(-6x)} + 126 e^{(-8x)} + 126 e^{(-10x)} + 84 e^{(-12x)} + 36 e^{(-14x)} + 9 e^{(-16x)} + e^{(-18\cdots} \right.} + \frac{512 a^{\frac{5}{2}} e^{(-8x)}}{5 \left(9 e^{(-2x)} + 36 e^{(-4x)} + 84 e^{(-6x)} + 126 e^{(-8x)} + 126 e^{(-10x)} + 84 e^{(-12x)} + 36 e^{(-14x)} + 9 e^{(-16x)} + e^{(-18\cdots} \right.} + \frac{256 a^{\frac{5}{2}}}{315 \left(9 e^{(-2x)} + 36 e^{(-4x)} + 84 e^{(-6x)} + 126 e^{(-8x)} + 126 e^{(-10x)} + 84 e^{(-12x)} + 36 e^{(-14x)} + 9 e^{(-16x)} + e^{(-\cdots} \right.}$$

input
```
integrate((a*sech(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
256/35*a^(5/2)*e^(-2*x)/(9*e^(-2*x) + 36*e^(-4*x) + 84*e^(-6*x) + 126*e^(-
8*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^(-14*x) + 9*e^(-16*x) + e^(-18*
x) + 1) + 1024/35*a^(5/2)*e^(-4*x)/(9*e^(-2*x) + 36*e^(-4*x) + 84*e^(-6*x)
 + 126*e^(-8*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^(-14*x) + 9*e^(-16*x
) + e^(-18*x) + 1) + 1024/15*a^(5/2)*e^(-6*x)/(9*e^(-2*x) + 36*e^(-4*x) +
84*e^(-6*x) + 126*e^(-8*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^(-14*x) +
 9*e^(-16*x) + e^(-18*x) + 1) + 512/5*a^(5/2)*e^(-8*x)/(9*e^(-2*x) + 36*e^
(-4*x) + 84*e^(-6*x) + 126*e^(-8*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^
(-14*x) + 9*e^(-16*x) + e^(-18*x) + 1) + 256/315*a^(5/2)/(9*e^(-2*x) + 36*
e^(-4*x) + 84*e^(-6*x) + 126*e^(-8*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*
e^(-14*x) + 9*e^(-16*x) + e^(-18*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.33

$$\int \left(a\text{sech}^4(x)\right)^{5/2}\,dx = -\frac{256\,a^{\frac{5}{2}}\left(126\,e^{(8\,x)}+84\,e^{(6\,x)}+36\,e^{(4\,x)}+9\,e^{(2\,x)}+1\right)}{315\,\left(e^{(2\,x)}+1\right)^{9}}$$

input
```
integrate((a*sech(x)^4)^(5/2),x, algorithm="giac")
```

output
```
-256/315*a^(5/2)*(126*e^(8*x) + 84*e^(6*x) + 36*e^(4*x) + 9*e^(2*x) + 1)/(
e^(2*x) + 1)^9
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 356, normalized size of antiderivative = 3.04

$$\int \left(a\mathrm{sech}^4(x)\right)^{5/2} dx = \frac{256\,a^2\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}+\frac{{\mathrm{e}}^{x}}{2}\right)}^4}}\,\left(4\,{\mathrm{e}}^{2\,x}+6\,{\mathrm{e}}^{4\,x}+4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{3\,{\left({\mathrm{e}}^{2\,x}+1\right)}^6\,\left({\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)} - \frac{128\,a^2\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}+\frac{{\mathrm{e}}^{x}}{2}\right)}^4}}\,\left(4\,{\mathrm{e}}^{2\,x}+6\,{\mathrm{e}}^{4\,x}+4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{5\,{\left({\mathrm{e}}^{2\,x}+1\right)}^5\,\left({\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)} - \frac{768\,a^2\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}+\frac{{\mathrm{e}}^{x}}{2}\right)}^4}}\,\left(4\,{\mathrm{e}}^{2\,x}+6\,{\mathrm{e}}^{4\,x}+4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{7\,{\left({\mathrm{e}}^{2\,x}+1\right)}^7\,\left({\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)} + \frac{64\,a^2\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}+\frac{{\mathrm{e}}^{x}}{2}\right)}^4}}\,\left(4\,{\mathrm{e}}^{2\,x}+6\,{\mathrm{e}}^{4\,x}+4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{{\left({\mathrm{e}}^{2\,x}+1\right)}^8\,\left({\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)} - \frac{128\,a^2\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}+\frac{{\mathrm{e}}^{x}}{2}\right)}^4}}\,\left(4\,{\mathrm{e}}^{2\,x}+6\,{\mathrm{e}}^{4\,x}+4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{9\,{\left({\mathrm{e}}^{2\,x}+1\right)}^9\,\left({\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)}$$

input
```
int((a/cosh(x)^4)^(5/2),x)
```

output
```
(256*a^2*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*exp(4*x) + 4*e
xp(6*x) + exp(8*x) + 1))/(3*(exp(2*x) + 1)^6*(exp(2*x) + 2*exp(4*x) + exp(
6*x))) - (128*a^2*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*exp(4
*x) + 4*exp(6*x) + exp(8*x) + 1))/(5*(exp(2*x) + 1)^5*(exp(2*x) + 2*exp(4*
x) + exp(6*x))) - (768*a^2*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x)
+ 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/(7*(exp(2*x) + 1)^7*(exp(2*x) +
 2*exp(4*x) + exp(6*x))) + (64*a^2*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*e
xp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/((exp(2*x) + 1)^8*(exp(
2*x) + 2*exp(4*x) + exp(6*x))) - (128*a^2*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/
2)*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/(9*(exp(2*x) + 1
)^9*(exp(2*x) + 2*exp(4*x) + exp(6*x)))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

$$\int \left(a\text{sech}^4(x)\right)^{5/2} dx = \frac{256\sqrt{a}\,a^2(-126e^{8x} - 84e^{6x} - 36e^{4x} - 9e^{2x} - 1)}{315e^{18x} + 2835e^{16x} + 11340e^{14x} + 26460e^{12x} + 39690e^{10x} + 39690e^{8x} + 26460e^{6x} + \cdots}$$

input

```
int((a*sech(x)^4)^(5/2),x)
```

output

```
(256*sqrt(a)*a**2*( - 126*e**(8*x) - 84*e**(6*x) - 36*e**(4*x) - 9*e**(2*x
) - 1))/(315*(e**(18*x) + 9*e**(16*x) + 36*e**(14*x) + 84*e**(12*x) + 126*
e**(10*x) + 126*e**(8*x) + 84*e**(6*x) + 36*e**(4*x) + 9*e**(2*x) + 1))
```

3.47 $\int \left(a\operatorname{sech}^4(x)\right)^{3/2} dx$

Optimal result

Integrand size = 10, antiderivative size = 61

$$\int \left(a\operatorname{sech}^4(x)\right)^{3/2} dx = a\cosh(x)\sqrt{a\operatorname{sech}^4(x)}\sinh(x) - \frac{2}{3}a\sqrt{a\operatorname{sech}^4(x)}\sinh^2(x)\tanh(x) + \frac{1}{5}a\sqrt{a\operatorname{sech}^4(x)}\sinh^2(x)\tanh^3(x)$$

output
```
a*cosh(x)*(a*sech(x)^4)^(1/2)*sinh(x)-2/3*a*(a*sech(x)^4)^(1/2)*sinh(x)^2*
tanh(x)+1/5*a*(a*sech(x)^4)^(1/2)*sinh(x)^2*tanh(x)^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.49

$$\int \left(a\operatorname{sech}^4(x)\right)^{3/2} dx = \frac{1}{15}\cosh(x)(8 + 6\cosh(2x) + \cosh(4x))\left(a\operatorname{sech}^4(x)\right)^{3/2}\sinh(x)$$

input
```
Integrate[(a*Sech[x]^4)^(3/2),x]
```

output
```
(Cosh[x]*(8 + 6*Cosh[2*x] + Cosh[4*x])*(a*Sech[x]^4)^(3/2)*Sinh[x])/15
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4611, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\text{sech}^4(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a \sec(ix)^4\right)^{3/2} dx$$

$$\Big\downarrow \text{4611}$$

$$a \cosh^2(x)\sqrt{a\text{sech}^4(x)} \int \text{sech}^6(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a \cosh^2(x)\sqrt{a\text{sech}^4(x)} \int \csc\left(ix+\frac{\pi}{2}\right)^6 dx$$

$$\Big\downarrow \text{4254}$$

$$ia \cosh^2(x)\sqrt{a\text{sech}^4(x)} \int \left(\tanh^4(x)-2\tanh^2(x)+1\right) d(-i\tanh(x))$$

$$\Big\downarrow \text{2009}$$

$$ia \cosh^2(x)\left(-\frac{1}{5}i\tanh^5(x)+\frac{2}{3}i\tanh^3(x)-i\tanh(x)\right)\sqrt{a\text{sech}^4(x)}$$

input
```
Int[(a*Sech[x]^4)^(3/2),x]
```

output
```
I*a*Cosh[x]^2*Sqrt[a*Sech[x]^4]*((-I)*Tanh[x] + ((2*I)/3)*Tanh[x]^3 - (I/5
)*Tanh[x]^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{16 a\,{\mathrm e}^{-2 x} \sqrt{\frac{a\,{\mathrm e}^{4 x}}{\left({\mathrm e}^{2 x}+1\right)^{4}}}\, \left(10\,{\mathrm e}^{4 x}+5\,{\mathrm e}^{2 x}+1\right)}{15 \left({\mathrm e}^{2 x}+1\right)^{3}}$	46

input
```
int((a*sech(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-16/15*a*exp(-2*x)/(exp(2*x)+1)^3*(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*(10*ex
p(4*x)+5*exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(51) = 102$.

Time = 0.10 (sec) , antiderivative size = 516, normalized size of antiderivative = 8.46

$$\int \left(a\text{sech}^4(x)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((a*sech(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
-16/15*(10*a*cosh(x)^4 + 10*(a*e^(4*x) + 2*a*e^(2*x) + a)*sinh(x)^4 + 40*(
a*cosh(x)*e^(4*x) + 2*a*cosh(x)*e^(2*x) + a*cosh(x))*sinh(x)^3 + 5*a*cosh(
x)^2 + 5*(12*a*cosh(x)^2 + (12*a*cosh(x)^2 + a)*e^(4*x) + 2*(12*a*cosh(x)^
2 + a)*e^(2*x) + a)*sinh(x)^2 + (10*a*cosh(x)^4 + 5*a*cosh(x)^2 + a)*e^(4*
x) + 2*(10*a*cosh(x)^4 + 5*a*cosh(x)^2 + a)*e^(2*x) + 10*(4*a*cosh(x)^3 +
a*cosh(x) + (4*a*cosh(x)^3 + a*cosh(x))*e^(4*x) + 2*(4*a*cosh(x)^3 + a*cos
h(x))*e^(2*x))*sinh(x) + a)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^
(2*x) + 1))*e^(2*x)/(10*cosh(x)*e^(2*x)*sinh(x)^9 + e^(2*x)*sinh(x)^10 + 5
*(9*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^8 + 40*(3*cosh(x)^3 + cosh(x))*e^(2*x)*
sinh(x)^7 + 10*(21*cosh(x)^4 + 14*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^6 + 4*(63
*cosh(x)^5 + 70*cosh(x)^3 + 15*cosh(x))*e^(2*x)*sinh(x)^5 + 10*(21*cosh(x)
^6 + 35*cosh(x)^4 + 15*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^4 + 40*(3*cosh(x)^7
+ 7*cosh(x)^5 + 5*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^3 + 5*(9*cosh(x)^8
+ 28*cosh(x)^6 + 30*cosh(x)^4 + 12*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 10*(
cosh(x)^9 + 4*cosh(x)^7 + 6*cosh(x)^5 + 4*cosh(x)^3 + cosh(x))*e^(2*x)*sin
h(x) + (cosh(x)^10 + 5*cosh(x)^8 + 10*cosh(x)^6 + 10*cosh(x)^4 + 5*cosh(x)
^2 + 1)*e^(2*x))
```

Sympy [F]

$$\int \left(a\text{sech}^4(x)\right)^{3/2} dx = \int \left(a\,\text{sech}^4\left(x\right)\right)^{\frac{3}{2}} dx$$

input
```
integrate((a*sech(x)**4)**(3/2),x)
```

output
```
Integral((a*sech(x)**4)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(51) = 102$.

Time = 0.14 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.97

$$\int \left(a\operatorname{sech}^4(x)\right)^{3/2} dx = \frac{16 a^{\frac{3}{2}} e^{(-2x)}}{3 \left(5 e^{(-2x)} + 10 e^{(-4x)} + 10 e^{(-6x)} + 5 e^{(-8x)} + e^{(-10x)} + 1\right)} + \frac{32 a^{\frac{3}{2}} e^{(-4x)}}{3 \left(5 e^{(-2x)} + 10 e^{(-4x)} + 10 e^{(-6x)} + 5 e^{(-8x)} + e^{(-10x)} + 1\right)} + \frac{16 a^{\frac{3}{2}}}{15 \left(5 e^{(-2x)} + 10 e^{(-4x)} + 10 e^{(-6x)} + 5 e^{(-8x)} + e^{(-10x)} + 1\right)}$$

input
```
integrate((a*sech(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
16/3*a^(3/2)*e^(-2*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(-8*x)
 + e^(-10*x) + 1) + 32/3*a^(3/2)*e^(-4*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e
^(-6*x) + 5*e^(-8*x) + e^(-10*x) + 1) + 16/15*a^(3/2)/(5*e^(-2*x) + 10*e^(
-4*x) + 10*e^(-6*x) + 5*e^(-8*x) + e^(-10*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.44

$$\int \left(a\operatorname{sech}^4(x)\right)^{3/2} dx = -\frac{16 a^{\frac{3}{2}} \left(10 e^{(4x)} + 5 e^{(2x)} + 1\right)}{15 \left(e^{(2x)} + 1\right)^5}$$

input
```
integrate((a*sech(x)^4)^(3/2),x, algorithm="giac")
```

output
```
-16/15*a^(3/2)*(10*e^(4*x) + 5*e^(2*x) + 1)/(e^(2*x) + 1)^5
```

Mupad [B] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \left(a\,\mathrm{sech}^4(x)\right)^{3/2}\,dx=-\frac{4\,a\,\mathrm{e}^{-2\,x}\,\sqrt{\frac{a}{{\left(\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}\right)}^4}}\,\left(5\,\mathrm{e}^{2\,x}+10\,\mathrm{e}^{4\,x}+1\right)}{15\,{\left(\mathrm{e}^{2\,x}+1\right)}^3}$$

input
```
int((a/cosh(x)^4)^(3/2),x)
```

output
```
-(4*a*exp(-2*x)*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(5*exp(2*x) + 10*exp(4*
x) + 1))/(15*(exp(2*x) + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \left(a\,\mathrm{sech}^4(x)\right)^{3/2}\,dx=\frac{16\sqrt{a}\,a(-10e^{4x}-5e^{2x}-1)}{15e^{10x}+75e^{8x}+150e^{6x}+150e^{4x}+75e^{2x}+15}$$

input
```
int((a*sech(x)^4)^(3/2),x)
```

output
```
(16*sqrt(a)*a*( - 10*e**(4*x) - 5*e**(2*x) - 1))/(15*(e**(10*x) + 5*e**(8*
x) + 10*e**(6*x) + 10*e**(4*x) + 5*e**(2*x) + 1))
```

3.48 $\int \sqrt{a\text{sech}^4(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 15

$$\int \sqrt{a\text{sech}^4(x)}\,dx = \cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x)$$

output
```
cosh(x)*(a*sech(x)^4)^(1/2)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sqrt{a\text{sech}^4(x)}\,dx = \cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x)$$

input
```
Integrate[Sqrt[a*Sech[x]^4],x]
```

output
```
Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4611, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\text{sech}^4(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{a\sec(ix)^4}dx$$

$$\Big\downarrow \text{ 4611}$$

$$\cosh^2(x)\sqrt{a\text{sech}^4(x)}\int \text{sech}^2(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\cosh^2(x)\sqrt{a\text{sech}^4(x)}\int \csc\left(ix+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{ 4254}$$

$$i\cosh^2(x)\sqrt{a\text{sech}^4(x)}\int 1d(-i\tanh(x))$$

$$\Big\downarrow \text{ 24}$$

$$\sinh(x)\cosh(x)\sqrt{a\text{sech}^4(x)}$$

input
```
Int[Sqrt[a*Sech[x]^4],x]
```

output
```
Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(13) = 26$.

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

method	result	size
risch	$-2\sqrt{\frac{a\,{\mathrm e}^{4x}}{\left({\mathrm e}^{2x}+1\right)^{4}}}\,{\mathrm e}^{-2x}\left({\mathrm e}^{2x}+1\right)$	29

input
```
int((a*sech(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*exp(-2*x)*(exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(13) = 26$.

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.40

$$\int \sqrt{a\text{sech}^4(x)}\,dx = -\frac{2\sqrt{\frac{a}{e^{(8x)}+4e^{(6x)}+6e^{(4x)}+4e^{(2x)}+1}}\left(e^{(4x)}+2e^{(2x)}+1\right)e^{(2x)}}{2\cosh(x)e^{(2x)}\sinh(x)+e^{(2x)}\sinh(x)^2+\left(\cosh(x)^2+1\right)e^{(2x)}}$$

input
```
integrate((a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*(e^(4*x) + 2*
e^(2*x) + 1)*e^(2*x)/(2*cosh(x)*e^(2*x)*sinh(x) + e^(2*x)*sinh(x)^2 + (cos
h(x)^2 + 1)*e^(2*x))
```

Sympy [F]

$$\int \sqrt{a\text{sech}^4(x)}\,dx = \int \sqrt{a\operatorname{sech}^4(x)}\,dx$$

input
```
integrate((a*sech(x)**4)**(1/2),x)
```

output
```
Integral(sqrt(a*sech(x)**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sqrt{a\text{sech}^4(x)}\,dx = \frac{2\sqrt{a}}{e^{(-2x)}+1}$$

input
```
integrate((a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(a)/(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sqrt{a\text{sech}^4(x)}\,dx = -\frac{2\sqrt{a}}{e^{(2x)}+1}$$

input
```
integrate((a*sech(x)^4)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(a)/(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 4.73

$$\int \sqrt{a\text{sech}^4(x)}\,dx = -\frac{\sqrt{a}\,\sqrt{\frac{1}{\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^4}}\,\left(2\,e^{2\,x}+3\,e^{4\,x}+2\,e^{6\,x}+\frac{e^{8\,x}}{2}+\frac{1}{2}\right)}{\left(e^{2\,x}+1\right)\,\left(e^{2\,x}+2\,e^{4\,x}+e^{6\,x}\right)}$$

input
```
int((a/cosh(x)^4)^(1/2),x)
```

output
```
-(a^(1/2)*(1/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(2*exp(2*x) + 3*exp(4*x) + 2*
exp(6*x) + exp(8*x)/2 + 1/2))/((exp(2*x) + 1)*(exp(2*x) + 2*exp(4*x) + exp
(6*x)))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \sqrt{a\text{sech}^4(x)}\,dx = \frac{2e^{2x}\sqrt{a}}{e^{2x}+1}$$

input
```
int((a*sech(x)^4)^(1/2),x)
```

output
```
(2*e**(2*x)*sqrt(a))/(e**(2*x) + 1)
```

3.49 $\int \frac{1}{\sqrt{a\text{sech}^4(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \frac{1}{\sqrt{a\text{sech}^4(x)}}\,dx = \frac{x\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}} + \frac{\tanh(x)}{2\sqrt{a\text{sech}^4(x)}}$$

output
```
1/2*x*sech(x)^2/(a*sech(x)^4)^(1/2)+1/2*tanh(x)/(a*sech(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{a\text{sech}^4(x)}}\,dx = \frac{x\text{sech}^2(x) + \tanh(x)}{2\sqrt{a\text{sech}^4(x)}}$$

input
```
Integrate[1/Sqrt[a*Sech[x]^4],x]
```

output
```
(x*Sech[x]^2 + Tanh[x])/(2*Sqrt[a*Sech[x]^4])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4611, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\text{sech}^4(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{a\sec(ix)^4}}dx$$

$$\Big\downarrow \text{ 4611}$$

$$\frac{\text{sech}^2(x)\int \cosh^2(x)dx}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\text{sech}^2(x)\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\text{sech}^2(x)\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\text{sech}^2(x)\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)}{\sqrt{a\text{sech}^4(x)}}$$

input
```
Int[1/Sqrt[a*Sech[x]^4],x]
```

output
```
(Sech[x]^2*(x/2 + (Cosh[x]*Sinh[x])/2))/Sqrt[a*Sech[x]^4]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(28) = 56$.

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.47

method	result	size
risch	$\frac{e^{2x} x}{2\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}+1\right)^{4}}}\,\left(e^{2x}+1\right)^{2}}+\frac{e^{4x}}{8\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}+1\right)^{4}}}\,\left(e^{2x}+1\right)^{2}}-\frac{1}{8\left(e^{2x}+1\right)^{2}\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}+1\right)^{4}}}}$	89

input
```
int(1/(a*sech(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp(2*x)*x+1/8/(a*exp
(4*x)/(exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2*exp(4*x)-1/8/(exp(2*x)+1)^2/(a*
exp(4*x)/(exp(2*x)+1)^4)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(28) = 56$.

Time = 0.09 (sec) , antiderivative size = 253, normalized size of antiderivative = 7.03

$$\int \frac{1}{\sqrt{a\text{sech}^4(x)}}\,dx = \frac{\left(\left(e^{(4x)}+2e^{(2x)}+1\right)\sinh(x)^4+\cosh(x)^4+4\left(\cosh(x)e^{(4x)}+2\cosh(x)e^{(2x)}+\cosh(x)\right)\sinh(x)^3+4\right.}{}$$

input
```
integrate(1/(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
1/8*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x)
+ 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 4*x*cosh(x)^2 + 2*(3*cosh(x)^2
+ (3*cosh(x)^2 + 2*x)*e^(4*x) + 2*(3*cosh(x)^2 + 2*x)*e^(2*x) + 2*x)*sinh(
x)^2 + (cosh(x)^4 + 4*x*cosh(x)^2 - 1)*e^(4*x) + 2*(cosh(x)^4 + 4*x*cosh(x
)^2 - 1)*e^(2*x) + 4*(cosh(x)^3 + 2*x*cosh(x) + (cosh(x)^3 + 2*x*cosh(x))*
e^(4*x) + 2*(cosh(x)^3 + 2*x*cosh(x))*e^(2*x))*sinh(x) - 1)*sqrt(a/(e^(8*x
) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)/(a*cosh(x)^2*e^(2*x) +
 2*a*cosh(x)*e^(2*x)*sinh(x) + a*e^(2*x)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\text{sech}^4(x)}}\,dx = \int \frac{1}{\sqrt{a\,\text{sech}^4(x)}}\,dx$$

input
```
integrate(1/(a*sech(x)**4)**(1/2),x)
```

output
```
Integral(1/sqrt(a*sech(x)**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{a\text{sech}^4(x)}}\,dx = -\frac{\left(\sqrt{a}e^{(-4\,x)} - \sqrt{a}\right)e^{(2\,x)}}{8\,a} + \frac{x}{2\,\sqrt{a}}$$

input
```
integrate(1/(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*(sqrt(a)*e^(-4*x) - sqrt(a))*e^(2*x)/a + 1/2*x/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{a\text{sech}^4(x)}}\,dx = -\frac{\left(2\,e^{(2\,x)}+1\right)e^{(-2\,x)} - 4\,x - e^{(2\,x)}}{8\,\sqrt{a}}$$

input
```
integrate(1/(a*sech(x)^4)^(1/2),x, algorithm="giac")
```

output
```
-1/8*((2*e^(2*x) + 1)*e^(-2*x) - 4*x - e^(2*x))/sqrt(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\text{sech}^4(x)}}\,dx = \int \frac{1}{\sqrt{\frac{a}{\cosh(x)^4}}}\,dx$$

input
```
int(1/(a/cosh(x)^4)^(1/2),x)
```

output
```
int(1/(a/cosh(x)^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{a\mathrm{sech}^4(x)}}\,dx = \frac{\sqrt{a}\,(e^{4x} + 4e^{2x}x - 1)}{8e^{2x}a}$$

input
```
int(1/(a*sech(x)^4)^(1/2),x)
```

output
```
(sqrt(a)*(e**(4*x) + 4*e**(2*x)*x - 1))/(8*e**(2*x)*a)
```

3.50 $\int \frac{1}{\left(a\mathbf{sech}^4(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 86

$$\int \frac{1}{\left(a\operatorname{sech}^4(x)\right)^{3/2}}\,dx = \frac{5x\operatorname{sech}^2(x)}{16a\sqrt{a\operatorname{sech}^4(x)}} + \frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\operatorname{sech}^4(x)}} + \frac{\cosh^3(x)\sinh(x)}{6a\sqrt{a\operatorname{sech}^4(x)}} + \frac{5\tanh(x)}{16a\sqrt{a\operatorname{sech}^4(x)}}$$

output

```
5/16*x*sech(x)^2/a/(a*sech(x)^4)^(1/2)+5/24*cosh(x)*sinh(x)/a/(a*sech(x)^4
)^(1/2)+1/6*cosh(x)^3*sinh(x)/a/(a*sech(x)^4)^(1/2)+5/16*tanh(x)/a/(a*sech
(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.44

$$\int \frac{1}{\left(a\operatorname{sech}^4(x)\right)^{3/2}}\,dx = \frac{\operatorname{sech}^6(x)(60x + 45\sinh(2x) + 9\sinh(4x) + \sinh(6x))}{192\left(a\operatorname{sech}^4(x)\right)^{3/2}}$$

input

```
Integrate[(a*Sech[x]^4)^(-3/2),x]
```

output
```
(Sech[x]^6*(60*x + 45*Sinh[2*x] + 9*Sinh[4*x] + Sinh[6*x]))/(192*(a*Sech[x
]^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {3042, 4611, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\text{sech}^4(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sec(ix)^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{4611}$$

$$\frac{\text{sech}^2(x)\int \cosh^6(x)dx}{a\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{sech}^2(x)\int \sin\left(ix+\frac{\pi}{2}\right)^6dx}{a\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\text{sech}^2(x)\left(\frac{5}{6}\int \cosh^4(x)dx+\frac{1}{6}\sinh(x)\cosh^5(x)\right)}{a\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{sech}^2(x)\left(\frac{1}{6}\sinh(x)\cosh^5(x)+\frac{5}{6}\int \sin\left(ix+\frac{\pi}{2}\right)^4dx\right)}{a\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\operatorname{sech}^2(x)\left(\frac{5}{6}\left(\frac{3}{4}\int \cosh^2(x)dx+\frac{1}{4}\sinh(x)\cosh^3(x)\right)+\frac{1}{6}\sinh(x)\cosh^5(x)\right)}{a\sqrt{a\operatorname{sech}^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\operatorname{sech}^2(x)\left(\frac{1}{6}\sinh(x)\cosh^5(x)+\frac{5}{6}\left(\frac{1}{4}\sinh(x)\cosh^3(x)+\frac{3}{4}\int \sin\left(ix+\frac{\pi}{2}\right)^2dx\right)\right)}{a\sqrt{a\operatorname{sech}^4(x)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\operatorname{sech}^2(x)\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)+\frac{1}{4}\sinh(x)\cosh^3(x)\right)+\frac{1}{6}\sinh(x)\cosh^5(x)\right)}{a\sqrt{a\operatorname{sech}^4(x)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\operatorname{sech}^2(x)\left(\frac{1}{6}\sinh(x)\cosh^5(x)+\frac{5}{6}\left(\frac{1}{4}\sinh(x)\cosh^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)\right)\right)}{a\sqrt{a\operatorname{sech}^4(x)}}$$

input
```
Int[(a*Sech[x]^4)^(-3/2),x]
```

output
```
(Sech[x]^2*((Cosh[x]^5*Sinh[x])/6 + (5*((Cosh[x]^3*Sinh[x])/4 + (3*(x/2 +
(Cosh[x]*Sinh[x])/2))/4))/6))/(a*Sqrt[a*Sech[x]^4])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(70) = 140$.

Time = 0.12 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.67

method	result
risch	$\frac{5\,\mathrm{e}^{2x} x}{16a(\mathrm{e}^{2x}+1)^2\sqrt{\frac{a\,\mathrm{e}^{4x}}{(\mathrm{e}^{2x}+1)^4}}}+\frac{\mathrm{e}^{8x}}{384a(\mathrm{e}^{2x}+1)^2\sqrt{\frac{a\,\mathrm{e}^{4x}}{(\mathrm{e}^{2x}+1)^4}}}+\frac{3\,\mathrm{e}^{6x}}{128a(\mathrm{e}^{2x}+1)^2\sqrt{\frac{a\,\mathrm{e}^{4x}}{(\mathrm{e}^{2x}+1)^4}}}+\frac{15\,\mathrm{e}^{4x}}{128a(\mathrm{e}^{2x}+1)^2\sqrt{\frac{a\,\mathrm{e}^{4x}}{(\mathrm{e}^{2x}+1)^4}}}-\frac{}{128\sqrt{\cdots}}$

input
```
int(1/(a*sech(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
5/16/a*exp(2*x)/(exp(2*x)+1)^2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*x+1/384/a
*exp(8*x)/(exp(2*x)+1)^2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)+3/128/a*exp(6*x
)/(exp(2*x)+1)^2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)+15/128/a*exp(4*x)/(exp(
2*x)+1)^2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)-15/128/(a*exp(4*x)/(exp(2*x)+1
)^4)^(1/2)/(exp(2*x)+1)^2/a-3/128/a*exp(-2*x)/(exp(2*x)+1)^2/(a*exp(4*x)/(
exp(2*x)+1)^4)^(1/2)-1/384/a*exp(-4*x)/(exp(2*x)+1)^2/(a*exp(4*x)/(exp(2*x
)+1)^4)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1141 vs. $2(70) = 140$.

Time = 0.11 (sec) , antiderivative size = 1141, normalized size of antiderivative = 13.27

$$\int \frac{1}{\left(a\mathrm{sech}^4(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*sech(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
1/384*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^12 + cosh(x)^12 + 12*(cosh(x)*e^(
4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^11 + 3*(22*cosh(x)^2 + (22*cos
h(x)^2 + 3)*e^(4*x) + 2*(22*cosh(x)^2 + 3)*e^(2*x) + 3)*sinh(x)^10 + 9*cos
h(x)^10 + 10*(22*cosh(x)^3 + (22*cosh(x)^3 + 9*cosh(x))*e^(4*x) + 2*(22*co
sh(x)^3 + 9*cosh(x))*e^(2*x) + 9*cosh(x))*sinh(x)^9 + 45*(11*cosh(x)^4 + 9
*cosh(x)^2 + (11*cosh(x)^4 + 9*cosh(x)^2 + 1)*e^(4*x) + 2*(11*cosh(x)^4 +
9*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^8 + 45*cosh(x)^8 + 72*(11*cosh(x)^5
+ 15*cosh(x)^3 + (11*cosh(x)^5 + 15*cosh(x)^3 + 5*cosh(x))*e^(4*x) + 2*(11
*cosh(x)^5 + 15*cosh(x)^3 + 5*cosh(x))*e^(2*x) + 5*cosh(x))*sinh(x)^7 + 12
0*x*cosh(x)^6 + 6*(154*cosh(x)^6 + 315*cosh(x)^4 + 210*cosh(x)^2 + (154*co
sh(x)^6 + 315*cosh(x)^4 + 210*cosh(x)^2 + 20*x)*e^(4*x) + 2*(154*cosh(x)^6
 + 315*cosh(x)^4 + 210*cosh(x)^2 + 20*x)*e^(2*x) + 20*x)*sinh(x)^6 + 36*(2
2*cosh(x)^7 + 63*cosh(x)^5 + 70*cosh(x)^3 + 20*x*cosh(x) + (22*cosh(x)^7 +
 63*cosh(x)^5 + 70*cosh(x)^3 + 20*x*cosh(x))*e^(4*x) + 2*(22*cosh(x)^7 + 6
3*cosh(x)^5 + 70*cosh(x)^3 + 20*x*cosh(x))*e^(2*x))*sinh(x)^5 + 45*(11*cos
h(x)^8 + 42*cosh(x)^6 + 70*cosh(x)^4 + 40*x*cosh(x)^2 + (11*cosh(x)^8 + 42
*cosh(x)^6 + 70*cosh(x)^4 + 40*x*cosh(x)^2 - 1)*e^(4*x) + 2*(11*cosh(x)^8
+ 42*cosh(x)^6 + 70*cosh(x)^4 + 40*x*cosh(x)^2 - 1)*e^(2*x) - 1)*sinh(x)^4
 - 45*cosh(x)^4 + 20*(11*cosh(x)^9 + 54*cosh(x)^7 + 126*cosh(x)^5 + 120*x*
cosh(x)^3 + (11*cosh(x)^9 + 54*cosh(x)^7 + 126*cosh(x)^5 + 120*x*cosh(x...
```

Sympy [F]

$$\int \frac{1}{\left(a\text{sech}^4(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\operatorname{sech}^{4}(x)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sech(x)**4)**(3/2),x)
```

output
```
Integral((a*sech(x)**4)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(a\text{sech}^4(x)\right)^{3/2}}\,dx = \frac{\left(9\sqrt{a}e^{(-2x)} + 45\sqrt{a}e^{(-4x)} - 45\sqrt{a}e^{(-8x)} - 9\sqrt{a}e^{(-10x)} - \sqrt{a}e^{(-12x)} + \sqrt{a}\right)e^{(6x)}}{384\,a^2} + \frac{5\,x}{16\,a^{\frac{3}{2}}}$$

input
```
integrate(1/(a*sech(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
1/384*(9*sqrt(a)*e^(-2*x) + 45*sqrt(a)*e^(-4*x) - 45*sqrt(a)*e^(-8*x) - 9*
sqrt(a)*e^(-10*x) - sqrt(a)*e^(-12*x) + sqrt(a))*e^(6*x)/a^2 + 5/16*x/a^(3
/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

$$\int \frac{1}{\left(a\text{sech}^4(x)\right)^{3/2}}\,dx = -\frac{\left(110\,e^{(6x)} + 45\,e^{(4x)} + 9\,e^{(2x)} + 1\right)e^{(-6x)} - 120\,x - e^{(6x)} - 9\,e^{(4x)} - 45\,e^{(2x)}}{384\,a^{\frac{3}{2}}}$$

input
```
integrate(1/(a*sech(x)^4)^(3/2),x, algorithm="giac")
```

output
```
-1/384*((110*e^(6*x) + 45*e^(4*x) + 9*e^(2*x) + 1)*e^(-6*x) - 120*x - e^(6
*x) - 9*e^(4*x) - 45*e^(2*x))/a^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\text{sech}^4(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{a}{\cosh(x)^4}\right)^{3/2}}\,dx$$

input
```
int(1/(a/cosh(x)^4)^(3/2),x)
```

output
```
int(1/(a/cosh(x)^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.66

$$\int \frac{1}{\left(a\text{sech}^4(x)\right)^{3/2}}\,dx = \frac{\sqrt{a}\,(e^{12x} + 9e^{10x} + 45e^{8x} + 120e^{6x}x - 45e^{4x} - 9e^{2x} - 1)}{384e^{6x}a^2}$$

input
```
int(1/(a*sech(x)^4)^(3/2),x)
```

output
```
(sqrt(a)*(e**(12*x) + 9*e**(10*x) + 45*e**(8*x) + 120*e**(6*x)*x - 45*e**(
4*x) - 9*e**(2*x) - 1))/(384*e**(6*x)*a**2)
```

3.51 $\int \frac{1}{\left(a\mathbf{sech}^4(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 132

$$\int \frac{1}{\left(a\mathrm{sech}^4(x)\right)^{5/2}}\,dx=\frac{63x\mathrm{sech}^2(x)}{256a^2\sqrt{a\mathrm{sech}^4(x)}}+\frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\mathrm{sech}^4(x)}}+\frac{21\cosh^3(x)\sinh(x)}{160a^2\sqrt{a\mathrm{sech}^4(x)}}+\frac{9\cosh^5(x)\sinh(x)}{80a^2\sqrt{a\mathrm{sech}^4(x)}}+\frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\mathrm{sech}^4(x)}}+\frac{63\tanh(x)}{256a^2\sqrt{a\mathrm{sech}^4(x)}}$$

output
```
63/256*x*sech(x)^2/a^2/(a*sech(x)^4)^(1/2)+21/128*cosh(x)*sinh(x)/a^2/(a*s
ech(x)^4)^(1/2)+21/160*cosh(x)^3*sinh(x)/a^2/(a*sech(x)^4)^(1/2)+9/80*cosh
(x)^5*sinh(x)/a^2/(a*sech(x)^4)^(1/2)+1/10*cosh(x)^7*sinh(x)/a^2/(a*sech(x
)^4)^(1/2)+63/256*tanh(x)/a^2/(a*sech(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.42

$$\int \frac{1}{\left(a\mathrm{sech}^4(x)\right)^{5/2}}\,dx=\frac{\cosh^2(x)\sqrt{a\mathrm{sech}^4(x)}(2520x+2100\sinh(2x)+600\sinh(4x)+150\sinh(6x)+25\,\mathrm{si}}{10240a^3}$$

input
```
Integrate[(a*Sech[x]^4)^(-5/2),x]
```

output
```
(Cosh[x]^2*Sqrt[a*Sech[x]^4]*(2520*x + 2100*Sinh[2*x] + 600*Sinh[4*x] + 15
0*Sinh[6*x] + 25*Sinh[8*x] + 2*Sinh[10*x]))/(10240*a^3)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.70, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {3042, 4611, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\text{sech}^4(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sec(ix)^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{4611}$$

$$\frac{\text{sech}^2(x)\int \cosh^{10}(x)dx}{a^2\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{sech}^2(x)\int \sin\left(ix+\frac{\pi}{2}\right)^{10}dx}{a^2\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\text{sech}^2(x)\left(\frac{9}{10}\int \cosh^8(x)dx+\frac{1}{10}\sinh(x)\cosh^9(x)\right)}{a^2\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{sech}^2(x)\left(\frac{1}{10}\sinh(x)\cosh^9(x)+\frac{9}{10}\int \sin\left(ix+\frac{\pi}{2}\right)^8dx\right)}{a^2\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\text{sech}^2(x)\left(\frac{9}{10}\left(\frac{7}{8}\int\cosh^6(x)dx+\frac{1}{8}\sinh(x)\cosh^7(x)\right)+\frac{1}{10}\sinh(x)\cosh^9(x)\right)}{a^2\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\text{sech}^2(x)\left(\frac{1}{10}\sinh(x)\cosh^9(x)+\frac{9}{10}\left(\frac{1}{8}\sinh(x)\cosh^7(x)+\frac{7}{8}\int\sin\left(ix+\frac{\pi}{2}\right)^6dx\right)\right)}{a^2\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\text{sech}^2(x)\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\int\cosh^4(x)dx+\frac{1}{6}\sinh(x)\cosh^5(x)\right)+\frac{1}{8}\sinh(x)\cosh^7(x)\right)+\frac{1}{10}\sinh(x)\cosh^9(x)\right)}{a^2\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\text{sech}^2(x)\left(\frac{1}{10}\sinh(x)\cosh^9(x)+\frac{9}{10}\left(\frac{1}{8}\sinh(x)\cosh^7(x)+\frac{7}{8}\left(\frac{1}{6}\sinh(x)\cosh^5(x)+\frac{5}{6}\int\sin\left(ix+\frac{\pi}{2}\right)^4dx\right)\right)\right)}{a^2\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\text{sech}^2(x)\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int\cosh^2(x)dx+\frac{1}{4}\sinh(x)\cosh^3(x)\right)+\frac{1}{6}\sinh(x)\cosh^5(x)\right)+\frac{1}{8}\sinh(x)\cosh^7(x)\right)+\frac{1}{10}\sinh(x\right.}{a^2\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\text{sech}^2(x)\left(\frac{1}{10}\sinh(x)\cosh^9(x)+\frac{9}{10}\left(\frac{1}{8}\sinh(x)\cosh^7(x)+\frac{7}{8}\left(\frac{1}{6}\sinh(x)\cosh^5(x)+\frac{5}{6}\left(\frac{1}{4}\sinh(x)\cosh^3(x)+\frac{3}{4}\int\sin\right.\right.\right.\right.}{a^2\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\text{sech}^2(x)\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)+\frac{1}{4}\sinh(x)\cosh^3(x)\right)+\frac{1}{6}\sinh(x)\cosh^5(x)\right)+\frac{1}{8}\sinh(x)\cosh^7\right.\right.\right.}{a^2\sqrt{a\text{sech}^4(x)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\text{sech}^2(x)\left(\frac{1}{10}\sinh(x)\cosh^9(x)+\frac{9}{10}\left(\frac{1}{8}\sinh(x)\cosh^7(x)+\frac{7}{8}\left(\frac{1}{6}\sinh(x)\cosh^5(x)+\frac{5}{6}\left(\frac{1}{4}\sinh(x)\cosh^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\right.\right.\right.\right.\right.}{a^2\sqrt{a\text{sech}^4(x)}}$$

input
```
Int[(a*Sech[x]^4)^(-5/2),x]
```

output
```
(Sech[x]^2*((Cosh[x]^9*Sinh[x])/10 + (9*((Cosh[x]^7*Sinh[x])/8 + (7*((Cosh
[x]^5*Sinh[x])/6 + (5*((Cosh[x]^3*Sinh[x])/4 + (3*(x/2 + (Cosh[x]*Sinh[x])
/2))/4))/6))/8))/10))/(a^2*Sqrt[a*Sech[x]^4])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(108) = 216$.

Time = 0.11 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.74

method	result
risch	$\frac{63\,\mathrm{e}^{2x} x}{256 a^{2} \left(\mathrm{e}^{2x}+1\right)^{2} \sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(\mathrm{e}^{2x}+1\right)^{4}}}}+\frac{\mathrm{e}^{12x}}{10240 a^{2} \left(\mathrm{e}^{2x}+1\right)^{2} \sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(\mathrm{e}^{2x}+1\right)^{4}}}}+\frac{5\,\mathrm{e}^{10x}}{4096 a^{2} \left(\mathrm{e}^{2x}+1\right)^{2} \sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(\mathrm{e}^{2x}+1\right)^{4}}}}+\frac{15\,\mathrm{e}^{8x}}{2048 a^{2} \left(\mathrm{e}^{2x}+1\right)^{2} \sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(\mathrm{e}^{2x}+1\right)^{4}}}}$

input
```
int(1/(a*sech(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
63/256/a^2*exp(2*x)/(exp(2*x)+1)^2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)*x+1/1
0240/a^2*exp(12*x)/(exp(2*x)+1)^2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)+5/4096
/a^2*exp(10*x)/(exp(2*x)+1)^2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)+15/2048/a^
2*exp(8*x)/(exp(2*x)+1)^2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)+15/512/a^2*exp
(6*x)/(exp(2*x)+1)^2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)+105/1024/a^2*exp(4*
x)/(exp(2*x)+1)^2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)-105/1024/(a*exp(4*x)/(
exp(2*x)+1)^4)^(1/2)/(exp(2*x)+1)^2/a^2-15/512/a^2*exp(-2*x)/(exp(2*x)+1)^
2/(a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)-15/2048/a^2*exp(-4*x)/(exp(2*x)+1)^2/(
a*exp(4*x)/(exp(2*x)+1)^4)^(1/2)-5/4096/a^2*exp(-6*x)/(exp(2*x)+1)^2/(a*ex
p(4*x)/(exp(2*x)+1)^4)^(1/2)-1/10240/a^2*exp(-8*x)/(exp(2*x)+1)^2/(a*exp(4
*x)/(exp(2*x)+1)^4)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2600 vs. $2(108) = 216$.

Time = 0.14 (sec) , antiderivative size = 2600, normalized size of antiderivative = 19.70

$$\int \frac{1}{\left(a\text{sech}^4(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*sech(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
1/20480*(2*(e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^20 + 2*cosh(x)^20 + 40*(cosh(
x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^19 + 5*(76*cosh(x)^2 + (
76*cosh(x)^2 + 5)*e^(4*x) + 2*(76*cosh(x)^2 + 5)*e^(2*x) + 5)*sinh(x)^18 +
 25*cosh(x)^18 + 30*(76*cosh(x)^3 + (76*cosh(x)^3 + 15*cosh(x))*e^(4*x) +
2*(76*cosh(x)^3 + 15*cosh(x))*e^(2*x) + 15*cosh(x))*sinh(x)^17 + 15*(646*c
osh(x)^4 + 255*cosh(x)^2 + (646*cosh(x)^4 + 255*cosh(x)^2 + 10)*e^(4*x) +
2*(646*cosh(x)^4 + 255*cosh(x)^2 + 10)*e^(2*x) + 10)*sinh(x)^16 + 150*cosh
(x)^16 + 48*(646*cosh(x)^5 + 425*cosh(x)^3 + (646*cosh(x)^5 + 425*cosh(x)^
3 + 50*cosh(x))*e^(4*x) + 2*(646*cosh(x)^5 + 425*cosh(x)^3 + 50*cosh(x))*e
^(2*x) + 50*cosh(x))*sinh(x)^15 + 60*(1292*cosh(x)^6 + 1275*cosh(x)^4 + 30
0*cosh(x)^2 + (1292*cosh(x)^6 + 1275*cosh(x)^4 + 300*cosh(x)^2 + 10)*e^(4*
x) + 2*(1292*cosh(x)^6 + 1275*cosh(x)^4 + 300*cosh(x)^2 + 10)*e^(2*x) + 10
)*sinh(x)^14 + 600*cosh(x)^14 + 120*(1292*cosh(x)^7 + 1785*cosh(x)^5 + 700
*cosh(x)^3 + (1292*cosh(x)^7 + 1785*cosh(x)^5 + 700*cosh(x)^3 + 70*cosh(x)
)*e^(4*x) + 2*(1292*cosh(x)^7 + 1785*cosh(x)^5 + 700*cosh(x)^3 + 70*cosh(x
))*e^(2*x) + 70*cosh(x))*sinh(x)^13 + 60*(4199*cosh(x)^8 + 7735*cosh(x)^6
+ 4550*cosh(x)^4 + 910*cosh(x)^2 + (4199*cosh(x)^8 + 7735*cosh(x)^6 + 4550
*cosh(x)^4 + 910*cosh(x)^2 + 35)*e^(4*x) + 2*(4199*cosh(x)^8 + 7735*cosh(x
)^6 + 4550*cosh(x)^4 + 910*cosh(x)^2 + 35)*e^(2*x) + 35)*sinh(x)^12 + 2100
*cosh(x)^12 + 80*(4199*cosh(x)^9 + 9945*cosh(x)^7 + 8190*cosh(x)^5 + 27...
```

Sympy [F]

$$\int \frac{1}{\left(a\mathrm{sech}^4(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\,\mathrm{sech}^4\,(x)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*sech(x)**4)**(5/2),x)
```

output
```
Integral((a*sech(x)**4)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a\operatorname{sech}^4(x)\right)^{5/2}}\,dx = \frac{\left(25\,\sqrt{a}e^{(-2\,x)} + 150\,\sqrt{a}e^{(-4\,x)} + 600\,\sqrt{a}e^{(-6\,x)} + 2100\,\sqrt{a}e^{(-8\,x)} - 2100\,\sqrt{a}e^{(-12\,x)} - \cdots\right.}{20480\,a \cdots} + \frac{63\,x}{256\,a^{\frac{5}{2}}}$$

input
```
integrate(1/(a*sech(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
1/20480*(25*sqrt(a)*e^(-2*x) + 150*sqrt(a)*e^(-4*x) + 600*sqrt(a)*e^(-6*x)
 + 2100*sqrt(a)*e^(-8*x) - 2100*sqrt(a)*e^(-12*x) - 600*sqrt(a)*e^(-14*x)
- 150*sqrt(a)*e^(-16*x) - 25*sqrt(a)*e^(-18*x) - 2*sqrt(a)*e^(-20*x) + 2*s
qrt(a))*e^(10*x)/a^3 + 63/256*x/a^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.58

$$\int \frac{1}{\left(a\operatorname{sech}^4(x)\right)^{5/2}}\,dx = -\frac{\left(5754\,e^{(10\,x)} + 2100\,e^{(8\,x)} + 600\,e^{(6\,x)} + 150\,e^{(4\,x)} + 25\,e^{(2\,x)} + 2\right)e^{(-10\,x)} - 5040\,x - 2\,e^{(10\,x)} - 25\,e^{(8\,x)} - 15\cdots}{20480\,a^{\frac{5}{2}}}$$

input
```
integrate(1/(a*sech(x)^4)^(5/2),x, algorithm="giac")
```

output
```
-1/20480*((5754*e^(10*x) + 2100*e^(8*x) + 600*e^(6*x) + 150*e^(4*x) + 25*e
^(2*x) + 2)*e^(-10*x) - 5040*x - 2*e^(10*x) - 25*e^(8*x) - 150*e^(6*x) - 6
00*e^(4*x) - 2100*e^(2*x))/a^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\text{sech}^4(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(\frac{a}{\cosh(x)^4}\right)^{5/2}}\,dx$$

input
```
int(1/(a/cosh(x)^4)^(5/2),x)
```

output
```
int(1/(a/cosh(x)^4)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.66

$$\int \frac{1}{\left(a\text{sech}^4(x)\right)^{5/2}}\,dx = \frac{\sqrt{a}\,(2e^{20x} + 25e^{18x} + 150e^{16x} + 600e^{14x} + 2100e^{12x} + 5040e^{10x}x - 2100e^{8x} - 600e^{6\cdots}}{20480e^{10x}a^3}$$

input
```
int(1/(a*sech(x)^4)^(5/2),x)
```

output
```
(sqrt(a)*(2*e**(20*x) + 25*e**(18*x) + 150*e**(16*x) + 600*e**(14*x) + 210
0*e**(12*x) + 5040*e**(10*x)*x - 2100*e**(8*x) - 600*e**(6*x) - 150*e**(4*
x) - 25*e**(2*x) - 2))/(20480*e**(10*x)*a**3)
```

3.52 $\int \frac{\sinh^4(x)}{a+a\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int \frac{\sinh^4(x)}{a+a\text{sech}(x)}\,dx = -\frac{x}{8a}-\frac{\cosh(x)\sinh(x)}{8a}+\frac{\cosh^3(x)\sinh(x)}{4a}-\frac{\sinh^3(x)}{3a}$$

output
```
-1/8*x/a-1/8*cosh(x)*sinh(x)/a+1/4*cosh(x)^3*sinh(x)/a-1/3*sinh(x)^3/a
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int \frac{\sinh^4(x)}{a+a\text{sech}(x)}\,dx = \frac{24\sinh(x)-8\sinh(3x)+3(-4x+\sinh(4x))}{96a}$$

input
```
Integrate[Sinh[x]^4/(a + a*Sech[x]),x]
```

output
```
(24*Sinh[x] - 8*Sinh[3*x] + 3*(-4*x + Sinh[4*x]))/(96*a)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.154, Rules used = {3042, 4360, 25, 25, 3042, 3318, 25, 3042, 25, 3044, 15, 3048, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(x)}{a\text{sech}(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(-\frac{\pi}{2}+ix\right)^4}{a-a\csc\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{4360}$$

$$\int -\frac{\sinh^4(x)\cosh(x)}{a(-\cosh(x))-a}dx$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{\cosh(x)\sinh^4(x)}{\cosh(x)a+a}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\sinh^4(x)\cosh(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)\cos\left(\frac{\pi}{2}+ix\right)^4}{a+a\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3318}$$

$$\frac{\int -\cosh(x)\sinh^2(x)dx}{a}-\frac{\int -\cosh^2(x)\sinh^2(x)dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \cosh^2(x)\sinh^2(x)dx}{a}-\frac{\int \cosh(x)\sinh^2(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\cos(ix)^2\sin(ix)^2dx}{a}-\frac{\int -\cos(ix)\sin(ix)^2dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \cos(ix)\sin(ix)^2dx}{a}-\frac{\int \cos(ix)^2\sin(ix)^2dx}{a}$$

$$\Big\downarrow \text{3044}$$

$$-\frac{i\int -\sinh^2(x)d(i\sinh(x))}{a}-\frac{\int \cos(ix)^2\sin(ix)^2dx}{a}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\sinh^3(x)}{3a}-\frac{\int \cos(ix)^2\sin(ix)^2dx}{a}$$

$$\Big\downarrow \text{3048}$$

$$-\frac{\frac{1}{4}\int \cosh^2(x)dx-\frac{1}{4}\sinh(x)\cosh^3(x)}{a}-\frac{\sinh^3(x)}{3a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh^3(x)}{3a}-\frac{-\frac{1}{4}\sinh(x)\cosh^3(x)+\frac{1}{4}\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{\frac{1}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\frac{1}{4}\sinh(x)\cosh^3(x)}{a}-\frac{\sinh^3(x)}{3a}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\sinh^3(x)}{3a}-\frac{\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-\frac{1}{4}\sinh(x)\cosh^3(x)}{a}$$

input
```
Int[Sinh[x]^4/(a + a*Sech[x]),x]
```

output
```
-1/3*Sinh[x]^3/a - (-1/4*(Cosh[x]^3*Sinh[x]) + (x/2 + (Cosh[x]*Sinh[x])/2)
/4)/a
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 15.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36

method	result
risch	$-\frac{x}{8a}+\frac{e^{4x}}{64a}-\frac{e^{3x}}{24a}+\frac{e^{x}}{8a}-\frac{e^{-x}}{8a}+\frac{e^{-3x}}{24a}-\frac{e^{-4x}}{64a}$
default	$\frac{-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)+1\right)^{4}}+\frac{5}{6\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{7}{8\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{16}{128\tanh\left(\frac{x}{2}\right)+128}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}+\frac{5}{6\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}+\frac{[illegible]}{8(t[illegible]}}{a}$

input
```
int(sinh(x)^4/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/8*x/a+1/64/a*exp(4*x)-1/24/a*exp(3*x)+1/8/a*exp(x)-1/8/a*exp(-x)+1/24/a
*exp(-3*x)-1/64/a*exp(-4*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{\sinh^4(x)}{a+a\text{sech}(x)}\,dx = \frac{(3\cosh(x)-2)\sinh(x)^3+3\left(\cosh(x)^3-2\cosh(x)^2+2\right)\sinh(x)-3x}{24a}$$

input
```
integrate(sinh(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
1/24*((3*cosh(x) - 2)*sinh(x)^3 + 3*(cosh(x)^3 - 2*cosh(x)^2 + 2)*sinh(x)
- 3*x)/a
```

Sympy [F]

$$\int \frac{\sinh^4(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\sinh^4(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

input
```
integrate(sinh(x)**4/(a+a*sech(x)),x)
```

output
```
Integral(sinh(x)**4/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23

$$\int \frac{\sinh^4(x)}{a + a\operatorname{sech}(x)}\,dx = -\frac{\left(8\,e^{(-x)} - 24\,e^{(-3\,x)} - 3\right)e^{(4\,x)}}{192\,a} - \frac{x}{8\,a} - \frac{24\,e^{(-x)} - 8\,e^{(-3\,x)} + 3\,e^{(-4\,x)}}{192\,a}$$

input
```
integrate(sinh(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
-1/192*(8*e^(-x) - 24*e^(-3*x) - 3)*e^(4*x)/a - 1/8*x/a - 1/192*(24*e^(-x)
 - 8*e^(-3*x) + 3*e^(-4*x))/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{\sinh^4(x)}{a + a\operatorname{sech}(x)}\,dx = -\frac{\left(24\,e^{(3\,x)} - 8\,e^{x} + 3\right)e^{(-4\,x)} + 24\,x - 3\,e^{(4\,x)} + 8\,e^{(3\,x)} - 24\,e^{x}}{192\,a}$$

input
```
integrate(sinh(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

output
```
-1/192*((24*e^(3*x) - 8*e^x + 3)*e^(-4*x) + 24*x - 3*e^(4*x) + 8*e^(3*x) -
 24*e^x)/a
```

Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.34

$$\int \frac{\sinh^4(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{e^{-3x}}{24a} - \frac{e^{-x}}{8a} - \frac{e^{3x}}{24a} - \frac{e^{-4x}}{64a} + \frac{e^{4x}}{64a} - \frac{x}{8a} + \frac{e^{x}}{8a}$$

input
```
int(sinh(x)^4/(a + a/cosh(x)),x)
```

output
```
exp(-3*x)/(24*a) - exp(-x)/(8*a) - exp(3*x)/(24*a) - exp(-4*x)/(64*a) + ex
p(4*x)/(64*a) - x/(8*a) + exp(x)/(8*a)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

$$\int \frac{\sinh^4(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{3e^{8x} - 8e^{7x} + 24e^{5x} - 24e^{4x}x - 24e^{3x} + 8e^{x} - 3}{192e^{4x}a}$$

input
```
int(sinh(x)^4/(a+a*sech(x)),x)
```

output
```
(3*e**(8*x) - 8*e**(7*x) + 24*e**(5*x) - 24*e**(4*x)*x - 24*e**(3*x) + 8*e
**x - 3)/(192*e**(4*x)*a)
```

3.53 $\int \frac{\sinh^3(x)}{a+a\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{\sinh^3(x)}{a+a\text{sech}(x)}\,dx=\frac{\cosh^3(x)}{3a}-\frac{\sinh^2(x)}{2a}$$

output
```
1/3*cosh(x)^3/a-1/2*sinh(x)^2/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(x)}{a+a\text{sech}(x)}\,dx=\frac{-7+3\cosh(x)-3\cosh(2x)+\cosh(3x)}{12a}$$

input
```
Integrate[Sinh[x]^3/(a + a*Sech[x]),x]
```

output
```
(-7 + 3*Cosh[x] - 3*Cosh[2*x] + Cosh[3*x])/(12*a)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.154$, Rules used = {3042, 26, 4360, 26, 25, 3042, 26, 3314, 26, 3042, 26, 3044, 15, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{a\text{sech}(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cos\left(-\frac{\pi}{2}+ix\right)^3}{a-a\csc\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^3}{a-a\csc\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4360}$$

$$i\int \frac{i\cosh(x)\sinh^3(x)}{-\cosh(x)a-a}dx$$

$$\Big\downarrow \text{26}$$

$$-\int -\frac{\cosh(x)\sinh^3(x)}{\cosh(x)a+a}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\sinh^3(x)\cosh(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin\left(\frac{\pi}{2}+ix\right)\cos\left(\frac{\pi}{2}+ix\right)^3}{a+a\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos\left(ix+\frac{\pi}{2}\right)^3\sin\left(ix+\frac{\pi}{2}\right)}{\sin\left(ix+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{3314}$$

$$-i\left(\frac{\int -i\cosh(x)\sinh(x)dx}{a}-\frac{\int -i\cosh^2(x)\sinh(x)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\int \cosh^2(x)\sinh(x)dx}{a}-\frac{i\int \cosh(x)\sinh(x)dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\int -i\cos(ix)^2\sin(ix)dx}{a}-\frac{i\int -i\cos(ix)\sin(ix)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{\int \cos(ix)^2\sin(ix)dx}{a}-\frac{\int \cos(ix)\sin(ix)dx}{a}\right)$$

$$\Big\downarrow \text{3044}$$

$$-i\left(\frac{i\int i\sinh(x)d(i\sinh(x))}{a}+\frac{\int \cos(ix)^2\sin(ix)dx}{a}\right)$$

$$\Big\downarrow \text{15}$$

$$-i\left(\frac{\int \cos(ix)^2\sin(ix)dx}{a}-\frac{i\sinh^2(x)}{2a}\right)$$

$$\Big\downarrow \text{3045}$$

$$-i\left(\frac{i\int \cosh^2(x)d\cosh(x)}{a}-\frac{i\sinh^2(x)}{2a}\right)$$

$$\Big\downarrow \text{15}$$

$$-i\left(\frac{i\cosh^3(x)}{3a}-\frac{i\sinh^2(x)}{2a}\right)$$

input
```
Int[Sinh[x]^3/(a + a*Sech[x]),x]
```

output
```
(-I)*(((I/3)*Cosh[x]^3)/a - ((I/2)*Sinh[x]^2)/a)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3314
```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[Cos[e + f
*x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[1/(b*d)   Int[Cos[e + f*x]^(p
 - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &
& IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p
+ 1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(19) = 38.

Time = 2.73 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.35

method	result	size
risch	$\frac{e^{3x}}{24a}-\frac{e^{2x}}{8a}+\frac{e^{x}}{8a}+\frac{e^{-x}}{8a}-\frac{e^{-2x}}{8a}+\frac{e^{-3x}}{24a}$	54
default	$\frac{\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{8}{8\tanh\left(\frac{x}{2}\right)+8}-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}}{a}$	67

input
```
int(sinh(x)^3/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
1/24/a*exp(3*x)-1/8/a*exp(2*x)+1/8/a*exp(x)+1/8/a*exp(-x)-1/8/a*exp(-2*x)+
1/24/a*exp(-3*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \frac{\sinh^3(x)}{a+a\text{sech}(x)}\,dx=\frac{\cosh(x)^3+3(\cosh(x)-1)\sinh(x)^2-3\cosh(x)^2+3\cosh(x)}{12a}$$

input
```
integrate(sinh(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
1/12*(cosh(x)^3 + 3*(cosh(x) - 1)*sinh(x)^2 - 3*cosh(x)^2 + 3*cosh(x))/a
```

Sympy [F]

$$\int \frac{\sinh^3(x)}{a+a\text{sech}(x)}\,dx=\frac{\int\frac{\sinh^3(x)}{\text{sech}(x)+1}\,dx}{a}$$

input
```
integrate(sinh(x)**3/(a+a*sech(x)),x)
```

output
```
Integral(sinh(x)**3/(sech(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00

$$\int \frac{\sinh^3(x)}{a + a\text{sech}(x)}\, dx = -\frac{\left(3\,e^{(-x)} - 3\,e^{(-2\,x)} - 1\right)e^{(3\,x)}}{24\,a} + \frac{3\,e^{(-x)} - 3\,e^{(-2\,x)} + e^{(-3\,x)}}{24\,a}$$

input
```
integrate(sinh(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
-1/24*(3*e^(-x) - 3*e^(-2*x) - 1)*e^(3*x)/a + 1/24*(3*e^(-x) - 3*e^(-2*x)
+ e^(-3*x))/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \frac{\sinh^3(x)}{a + a\text{sech}(x)}\, dx = \frac{\left(3\,e^{(2\,x)} - 3\,e^{x} + 1\right)e^{(-3\,x)} + e^{(3\,x)} - 3\,e^{(2\,x)} + 3\,e^{x}}{24\,a}$$

input
```
integrate(sinh(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

output
```
1/24*((3*e^(2*x) - 3*e^x + 1)*e^(-3*x) + e^(3*x) - 3*e^(2*x) + 3*e^x)/a
```

Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{\sinh^3(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{e^{-x}}{8\,a} - \frac{e^{-2\,x}}{8\,a} - \frac{e^{2\,x}}{8\,a} + \frac{e^{-3\,x}}{24\,a} + \frac{e^{3\,x}}{24\,a} + \frac{e^{x}}{8\,a}$$

input
```
int(sinh(x)^3/(a + a/cosh(x)),x)
```

output
```
exp(-x)/(8*a) - exp(-2*x)/(8*a) - exp(2*x)/(8*a) + exp(-3*x)/(24*a) + exp(
3*x)/(24*a) + exp(x)/(8*a)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.96

$$\int \frac{\sinh^3(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{e^{6x} - 3e^{5x} + 3e^{4x} + 3e^{2x} - 3e^{x} + 1}{24e^{3x}a}$$

input
```
int(sinh(x)^3/(a+a*sech(x)),x)
```

output
```
(e**(6*x) - 3*e**(5*x) + 3*e**(4*x) + 3*e**(2*x) - 3*e**x + 1)/(24*e**(3*x
)*a)
```

3.54 $\int \frac{\sinh^2(x)}{a+a\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{\sinh^2(x)}{a+a\text{sech}(x)}\,dx=\frac{x}{2a}-\frac{\sinh(x)}{a}+\frac{\cosh(x)\sinh(x)}{2a}$$

output
```
1/2*x/a-sinh(x)/a+1/2*cosh(x)*sinh(x)/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \frac{\sinh^2(x)}{a+a\text{sech}(x)}\,dx=\frac{x+(-2+\cosh(x))\sinh(x)}{2a}$$

input
```
Integrate[Sinh[x]^2/(a + a*Sech[x]),x]
```

output
```
(x + (-2 + Cosh[x])*Sinh[x])/(2*a)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.846, Rules used = {3042, 25, 4360, 3042, 25, 25, 3318, 3042, 3115, 24, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{a\text{sech}(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^2}{a-a\csc\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{a-a\csc\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4360}$$

$$-\int \frac{\cosh(x)\sinh^2(x)}{-\cosh(x)a-a}dx$$

$$\Big\downarrow \text{3042}$$

$$-\int -\frac{\cos\left(ix+\frac{\pi}{2}\right)^2\sin\left(ix+\frac{\pi}{2}\right)}{-\sin\left(ix+\frac{\pi}{2}\right)a-a}dx$$

$$\Big\downarrow \text{25}$$

$$\int -\frac{\sin\left(\frac{\pi}{2}+ix\right)\cos\left(\frac{\pi}{2}+ix\right)^2}{a+a\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(ix+\frac{\pi}{2}\right)^2\sin\left(ix+\frac{\pi}{2}\right)}{\sin\left(ix+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{3318}$$

$$\frac{\int \cosh^2(x)dx}{a}-\frac{\int \cosh(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a}-\frac{\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)}{a}-\frac{\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)}{a}-\frac{\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{3117}$$

$$\frac{\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)}{a}-\frac{\sinh(x)}{a}$$

input
```
Int[Sinh[x]^2/(a + a*Sech[x]),x]
```

output
```
-(Sinh[x]/a) + (x/2 + (Cosh[x]*Sinh[x])/2)/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

method	result	size
risch	$\frac{x}{2a}+\frac{{\mathrm e}^{2x}}{8a}-\frac{{\mathrm e}^{x}}{2a}+\frac{{\mathrm e}^{-x}}{2a}-\frac{{\mathrm e}^{-2x}}{8a}$	42
default	$\frac{-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}}{a}$	65

input
```
int(sinh(x)^2/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a+1/8/a*exp(2*x)-1/2/a*exp(x)+1/2/a*exp(-x)-1/8/a*exp(-2*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int \frac{\sinh^2(x)}{a + a\text{sech}(x)}\,dx = \frac{(\cosh(x) - 2)\sinh(x) + x}{2\,a}$$

input
```
integrate(sinh(x)^2/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
1/2*((cosh(x) - 2)*sinh(x) + x)/a
```

Sympy [F]

$$\int \frac{\sinh^2(x)}{a + a\text{sech}(x)}\,dx = \frac{\int \frac{\sinh^2(x)}{\text{sech}(x)+1}\,dx}{a}$$

input
```
integrate(sinh(x)**2/(a+a*sech(x)),x)
```

output
```
Integral(sinh(x)**2/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \frac{\sinh^2(x)}{a + a\text{sech}(x)}\,dx = -\frac{(4\,e^{(-x)} - 1)e^{(2\,x)}}{8\,a} + \frac{x}{2\,a} + \frac{4\,e^{(-x)} - e^{(-2\,x)}}{8\,a}$$

input
```
integrate(sinh(x)^2/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
-1/8*(4*e^(-x) - 1)*e^(2*x)/a + 1/2*x/a + 1/8*(4*e^(-x) - e^(-2*x))/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{\sinh^2(x)}{a+a\text{sech}(x)}\,dx = \frac{(4\,e^x-1)e^{(-2\,x)}+4\,x+e^{(2\,x)}-4\,e^x}{8\,a}$$

input
```
integrate(sinh(x)^2/(a+a*sech(x)),x, algorithm="giac")
```

output
```
1/8*((4*e^x - 1)*e^(-2*x) + 4*x + e^(2*x) - 4*e^x)/a
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{\sinh^2(x)}{a+a\text{sech}(x)}\,dx = \frac{e^{-x}}{2\,a}-\frac{e^{-2\,x}}{8\,a}+\frac{e^{2\,x}}{8\,a}+\frac{x}{2\,a}-\frac{e^{x}}{2\,a}$$

input
```
int(sinh(x)^2/(a + a/cosh(x)),x)
```

output
```
exp(-x)/(2*a) - exp(-2*x)/(8*a) + exp(2*x)/(8*a) + x/(2*a) - exp(x)/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{\sinh^2(x)}{a+a\text{sech}(x)}\,dx = \frac{e^{4x}-4e^{3x}+4e^{2x}x+4e^x-1}{8e^{2x}a}$$

input
```
int(sinh(x)^2/(a+a*sech(x)),x)
```

output
```
(e**(4*x) - 4*e**(3*x) + 4*e**(2*x)*x + 4*e**x - 1)/(8*e**(2*x)*a)
```

3.55 $\int \frac{\sinh(x)}{a+a\text{sech}(x)} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{\sinh(x)}{a+a\text{sech}(x)} \, dx = \frac{\cosh(x)}{a} - \frac{\log(1+\cosh(x))}{a}$$

output
```
cosh(x)/a-ln(1+cosh(x))/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\sinh(x)}{a+a\text{sech}(x)} \, dx = \frac{\cosh(x) - 2\log\left(\cosh\left(\frac{x}{2}\right)\right)}{a}$$

input
```
Integrate[Sinh[x]/(a + a*Sech[x]),x]
```

output
```
(Cosh[x] - 2*Log[Cosh[x/2]])/a
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 26, 4360, 26, 25, 3042, 26, 3312, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{a\text{sech}(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cos\left(-\frac{\pi}{2}+ix\right)}{a-a\csc\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)}{a-a\csc\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4360}$$

$$-i\int -\frac{i\cosh(x)\sinh(x)}{-\cosh(x)a-a}dx$$

$$\Big\downarrow \text{26}$$

$$-\int -\frac{\cosh(x)\sinh(x)}{\cosh(x)a+a}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\sinh(x)\cosh(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin\left(\frac{\pi}{2}+ix\right)\cos\left(\frac{\pi}{2}+ix\right)}{a+a\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos\left(ix+\frac{\pi}{2}\right)\sin\left(ix+\frac{\pi}{2}\right)}{\sin\left(ix+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{3312}$$

$$\frac{\int \frac{\cosh(x)}{\cosh(x)a+a}d(a\cosh(x))}{a}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{a\cosh(x)}{\cosh(x)a+a}d(a\cosh(x))}{a^2}$$

$$\Big\downarrow \; 49$$

$$\frac{\int \left(1-\frac{a}{\cosh(x)a+a}\right)d(a\cosh(x))}{a^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{a\cosh(x)-a\log(a\cosh(x)+a)}{a^2}$$

input
```
Int[Sinh[x]/(a + a*Sech[x]),x]
```

output
```
(a*Cosh[x] - a*Log[a + a*Cosh[x]])/a^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

method	result	size
derivativedivides	$-\frac{\ln(1+\operatorname{sech}(x))-\frac{1}{\operatorname{sech}(x)}-\ln(\operatorname{sech}(x))}{a}$	23
default	$-\frac{\ln(1+\operatorname{sech}(x))-\frac{1}{\operatorname{sech}(x)}-\ln(\operatorname{sech}(x))}{a}$	23
risch	$\frac{x}{a}+\frac{e^{x}}{2a}+\frac{e^{-x}}{2a}-\frac{2\ln(1+e^{x})}{a}$	33

input
```
int(sinh(x)/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/a*(ln(1+sech(x))-1/sech(x)-ln(sech(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(17) = 34$.

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.94

$$\int \frac{\sinh(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{2\,x\cosh\left(x\right) + \cosh\left(x\right)^{2} - 4\,(\cosh\left(x\right) + \sinh\left(x\right))\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) + 2\,(x + \cosh\left(x\right))\sinh\left(x\right) + \sinh\left(x\right)^{2} + 1}{2\,(a\cosh\left(x\right) + a\sinh\left(x\right))}$$

input
```
integrate(sinh(x)/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
1/2*(2*x*cosh(x) + cosh(x)^2 - 4*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x)
 + 1) + 2*(x + cosh(x))*sinh(x) + sinh(x)^2 + 1)/(a*cosh(x) + a*sinh(x))
```

Sympy [F]

$$\int \frac{\sinh(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\sinh\left(x\right)}{\operatorname{sech}\left(x\right)+1}\,dx}{a}$$

input
```
integrate(sinh(x)/(a+a*sech(x)),x)
```

output
```
Integral(sinh(x)/(sech(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \frac{\sinh(x)}{a + a\operatorname{sech}(x)}\,dx = -\frac{x}{a} + \frac{e^{(-x)}}{2\,a} + \frac{e^{x}}{2\,a} - \frac{2\,\log\left(e^{(-x)} + 1\right)}{a}$$

input
```
integrate(sinh(x)/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
-x/a + 1/2*e^(-x)/a + 1/2*e^x/a - 2*log(e^(-x) + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.88

$$\int \frac{\sinh(x)}{a + a\text{sech}(x)}\,dx = \frac{x}{a} + \frac{e^{(-x)}}{2\,a} + \frac{e^{x}}{2\,a} - \frac{2\,\log\left(e^{x} + 1\right)}{a}$$

input
```
integrate(sinh(x)/(a+a*sech(x)),x, algorithm="giac")
```

output
```
x/a + 1/2*e^(-x)/a + 1/2*e^x/a - 2*log(e^x + 1)/a
```

Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\sinh(x)}{a + a\text{sech}(x)}\,dx = -\frac{\ln\left(\cosh(x) + 1\right) - \cosh(x)}{a}$$

input
```
int(sinh(x)/(a + a/cosh(x)),x)
```

output
```
-(log(cosh(x) + 1) - cosh(x))/a
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.00

$$\int \frac{\sinh(x)}{a + a\text{sech}(x)}\,dx = \frac{e^{2x} - 4e^{x}\log(e^{x} + 1) + 2e^{x}x + 1}{2e^{x}a}$$

input
```
int(sinh(x)/(a+a*sech(x)),x)
```

output
```
(e**(2*x) - 4*e**x*log(e**x + 1) + 2*e**x*x + 1)/(2*e**x*a)
```

3.56 $\int \frac{\operatorname{csch}(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \frac{\operatorname{csch}(x)}{a+a\operatorname{sech}(x)}\,dx=-\frac{\operatorname{arctanh}(\cosh(x))}{2a}-\frac{\coth(x)\operatorname{csch}(x)}{2a}+\frac{\operatorname{csch}^2(x)}{2a}$$

output
```
-1/2*arctanh(cosh(x))/a-1/2*coth(x)*csch(x)/a+1/2*csch(x)^2/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{\operatorname{csch}(x)}{a+a\operatorname{sech}(x)}\,dx=-\frac{\left(1+2\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)\right)\operatorname{sech}(x)}{2a(1+\operatorname{sech}(x))}$$

input
```
Integrate[Csch[x]/(a + a*Sech[x]),x]
```

output
```
-1/2*((1 + 2*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]]))*Sech[x])/(a*(1
 + Sech[x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.636$, Rules used = {3042, 26, 4359, 26, 25, 3042, 26, 3185, 26, 3042, 26, 3086, 15, 3091, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(x)}{a\operatorname{sech}(x)+a}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i}{\cos\left(-\frac{\pi}{2}+ix\right)\left(a-a\csc\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{1}{\cos\left(ix-\frac{\pi}{2}\right)\left(a-a\csc\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \; 4359$$

$$i\int \frac{i\coth(x)}{-\cosh(x)a-a}dx$$

$$\Big\downarrow \; 26$$

$$-\int -\frac{\coth(x)}{\cosh(x)a+a}dx$$

$$\Big\downarrow \; 25$$

$$\int \frac{\coth(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i\tan\left(-\frac{\pi}{2}+ix\right)}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{\tan\left(ix-\frac{\pi}{2}\right)}{a-a\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3185}$$

$$-i\left(\frac{\int i\coth^2(x)\text{csch}(x)dx}{a}+\frac{\int -i\coth(x)\text{csch}^2(x)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\int \coth^2(x)\text{csch}(x)dx}{a}-\frac{i\int \coth(x)\text{csch}^2(x)dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\int -i\sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}-\frac{i\int i\sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)dx}{a}+\frac{\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}\right)$$

$$\Big\downarrow \text{3086}$$

$$-i\left(\frac{\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}-\frac{i\int -i\text{csch}(x)d(-i\text{csch}(x))}{a}\right)$$

$$\Big\downarrow \text{15}$$

$$-i\left(\frac{\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}+\frac{i\text{csch}^2(x)}{2a}\right)$$

$$\Big\downarrow \text{3091}$$

$$-i\left(\frac{-\frac{1}{2}\int -i\text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)}{a}+\frac{i\text{csch}^2(x)}{2a}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{\frac{1}{2}i\int \text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)}{a}+\frac{i\text{csch}^2(x)}{2a}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{\frac{1}{2}i\int i\csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)}{a}+\frac{i\text{csch}^2(x)}{2a}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{-\frac{1}{2}\int \csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)}{a}+\frac{i\text{csch}^2(x)}{2a}\right)$$

$$\Big\downarrow \text{4257}$$

$$-i\left(\frac{-\frac{1}{2}i\text{arctanh}(\cosh(x))-\frac{1}{2}i\coth(x)\text{csch}(x)}{a}+\frac{i\text{csch}^2(x)}{2a}\right)$$

input
```
Int[Csch[x]/(a + a*Sech[x]),x]
```

output
```
(-I)*(((I/2)*Csch[x]^2)/a + ((-1/2*I)*ArcTanh[Cosh[x]] - (I/2)*Coth[x]*Csc
h[x])/a)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4359
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_.), x_Symbol] :> Int[Cot[e + f*x]^p*(b + a*Sin[e + f*x])^m, x] /; FreeQ[{a
, b, e, f, p}, x] && IntegerQ[m] && EqQ[m, p]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^{2}}{2}+\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2a}$	20
risch	$-\frac{e^{x}}{\left(1+e^{x}\right)^{2} a}-\frac{\ln\left(1+e^{x}\right)}{2a}+\frac{\ln\left(e^{x}-1\right)}{2a}$	35

input
```
int(csch(x)/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(1/2*tanh(1/2*x)^2+ln(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(27) = 54.

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.12

$$\int \frac{\operatorname{csch}(x)}{a + a\operatorname{sech}(x)}\,dx = -\frac{\left(\cosh\left(x\right)^{2} + 2\left(\cosh\left(x\right) + 1\right)\sinh\left(x\right) + \sinh\left(x\right)^{2} + 2\cosh\left(x\right) + 1\right)\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) - (\mathrm{co}}{2\left(a\cosh\left(x\right)^{2} + a\sinh\left(x\right)^{2} +\right.}$$

input
```
integrate(csch(x)/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
-1/2*((cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*lo
g(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^
2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*cosh(x) + 2*sinh(x))/(a*
cosh(x)^2 + a*sinh(x)^2 + 2*a*cosh(x) + 2*(a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\operatorname{csch}\left(x\right)}{\operatorname{sech}\left(x\right)+1}\,dx}{a}$$

input
```
integrate(csch(x)/(a+a*sech(x)),x)
```

output
```
Integral(csch(x)/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

$$\int \frac{\operatorname{csch}(x)}{a + a\operatorname{sech}(x)}\,dx = -\frac{e^{(-x)}}{2\,ae^{(-x)} + ae^{(-2\,x)} + a} - \frac{\log\left(e^{(-x)} + 1\right)}{2\,a} + \frac{\log\left(e^{(-x)} - 1\right)}{2\,a}$$

input
```
integrate(csch(x)/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
-e^(-x)/(2*a*e^(-x) + a*e^(-2*x) + a) - 1/2*log(e^(-x) + 1)/a + 1/2*log(e^
(-x) - 1)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.58

$$\int \frac{\operatorname{csch}(x)}{a + a\operatorname{sech}(x)}\,dx = -\frac{\log\left(e^{(-x)} + e^{x} + 2\right)}{4\,a} + \frac{\log\left(e^{(-x)} + e^{x} - 2\right)}{4\,a} + \frac{e^{(-x)} + e^{x} - 2}{4\,a(e^{(-x)} + e^{x} + 2)}$$

input
```
integrate(csch(x)/(a+a*sech(x)),x, algorithm="giac")
```

output
```
-1/4*log(e^(-x) + e^x + 2)/a + 1/4*log(e^(-x) + e^x - 2)/a + 1/4*(e^(-x) +
 e^x - 2)/(a*(e^(-x) + e^x + 2))
```

Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \frac{\operatorname{csch}(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{1}{a\,\left(\mathrm{e}^{2\,x} + 2\,\mathrm{e}^{x} + 1\right)} - \frac{1}{a\,\left(\mathrm{e}^{x} + 1\right)} - \frac{\operatorname{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{-a^2}}{a}\right)}{\sqrt{-a^2}}$$

input
```
int(1/(sinh(x)*(a + a/cosh(x))),x)
```

output
```
1/(a*(exp(2*x) + 2*exp(x) + 1)) - 1/(a*(exp(x) + 1)) - atan((exp(x)*(-a^2)
^(1/2))/a)/(-a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.64

$$\int \frac{\operatorname{csch}(x)}{a + a\operatorname{sech}(x)}\, dx$$
$$= \frac{e^{2x}\log(e^x - 1) - e^{2x}\log(e^x + 1) + e^{2x} + 2e^x\log(e^x - 1) - 2e^x\log(e^x + 1) + \log(e^x - 1) - \log(e^x + 1) + 1}{2a\left(e^{2x} + 2e^x + 1\right)}$$

input
```
int(csch(x)/(a+a*sech(x)),x)
```

output
```
(e**(2*x)*log(e**x - 1) - e**(2*x)*log(e**x + 1) + e**(2*x) + 2*e**x*log(e
**x - 1) - 2*e**x*log(e**x + 1) + log(e**x - 1) - log(e**x + 1) + 1)/(2*a*
(e**(2*x) + 2*e**x + 1))
```

3.57 $\int \frac{\operatorname{csch}^2(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{\operatorname{csch}^2(x)}{a+a\operatorname{sech}(x)}\,dx=-\frac{\coth^3(x)}{3a}+\frac{\operatorname{csch}^3(x)}{3a}$$

output
```
-1/3*coth(x)^3/a+1/3*csch(x)^3/a
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{csch}^2(x)}{a+a\operatorname{sech}(x)}\,dx=-\frac{(3+2\cosh(x)+\cosh(2x))\operatorname{csch}(x)}{6a(1+\cosh(x))}$$

input
```
Integrate[Csch[x]^2/(a + a*Sech[x]),x]
```

output
```
-1/6*((3 + 2*Cosh[x] + Cosh[2*x])*Csch[x])/(a*(1 + Cosh[x]))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923, Rules used = {3042, 25, 4360, 3042, 3318, 25, 3042, 25, 3086, 15, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(x)}{a\text{sech}(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\cos\left(-\frac{\pi}{2}+ix\right)^2\left(a-a\csc\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\cos\left(ix-\frac{\pi}{2}\right)^2\left(a-a\csc\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4360}$$

$$-\int \frac{\coth(x)\text{csch}(x)}{-\cosh(x)a-a}dx$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\sin\left(ix-\frac{\pi}{2}\right)}{\cos\left(ix-\frac{\pi}{2}\right)^2\left(a\sin\left(ix-\frac{\pi}{2}\right)-a\right)}dx$$

$$\Big\downarrow \text{3318}$$

$$\frac{\int \coth^2(x)\text{csch}^2(x)dx}{a}+\frac{\int -\coth(x)\text{csch}^3(x)dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \coth^2(x)\text{csch}^2(x)dx}{a}-\frac{\int \coth(x)\text{csch}^3(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}-\frac{\int -\sec\left(ix-\frac{\pi}{2}\right)^3\tan\left(ix-\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \sec \left(ix-\frac{\pi}{2}\right)^3 \tan \left(ix-\frac{\pi}{2}\right) dx}{a}+\frac{\int \sec \left(ix-\frac{\pi}{2}\right)^2 \tan \left(ix-\frac{\pi}{2}\right)^2 dx}{a}$$

$$\Big\downarrow \text{ 3086}$$

$$\frac{\int \sec \left(ix-\frac{\pi}{2}\right)^2 \tan \left(ix-\frac{\pi}{2}\right)^2 dx}{a}-\frac{i\int -\text{csch}^2(x)d(-i\text{csch}(x))}{a}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\text{csch}^3(x)}{3a}+\frac{\int \sec \left(ix-\frac{\pi}{2}\right)^2 \tan \left(ix-\frac{\pi}{2}\right)^2 dx}{a}$$

$$\Big\downarrow \text{ 3087}$$

$$\frac{\text{csch}^3(x)}{3a}-\frac{i\int -\coth^2(x)d(i\coth(x))}{a}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\text{csch}^3(x)}{3a}-\frac{\coth^3(x)}{3a}$$

input
```
Int[Csch[x]^2/(a + a*Sech[x]),x]
```

output
```
-1/3*Coth[x]^3/a + Csch[x]^3/(3*a)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{-\frac{\tanh\left(\frac{x}{2}\right)^{3}}{3}-\frac{1}{\tanh\left(\frac{x}{2}\right)}}{4a}$	23
risch	$-\frac{2\left(3\,\mathrm{e}^{2x}+2\,\mathrm{e}^{x}+1\right)}{3\left(1+\mathrm{e}^{x}\right)^{3}a\left(\mathrm{e}^{x}-1\right)}$	30

input
```
int(csch(x)^2/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*(-1/3*tanh(1/2*x)^3-1/tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(19) = 38$.

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.09

$$\int \frac{\operatorname{csch}^2(x)}{a + a\operatorname{sech}(x)}\,dx = -\frac{4\,(2\,\cosh(x) + \sinh(x) + 1)}{3\,\left(a\cosh(x)^3 + a\sinh(x)^3 + 2\,a\cosh(x)^2 + (3\,a\cosh(x) + 2\,a)\sinh(x)^2 - a\cosh(x) + (3\,a\cosh(x)^2\right.}$$

input
```
integrate(csch(x)^2/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
-4/3*(2*cosh(x) + sinh(x) + 1)/(a*cosh(x)^3 + a*sinh(x)^3 + 2*a*cosh(x)^2
+ (3*a*cosh(x) + 2*a)*sinh(x)^2 - a*cosh(x) + (3*a*cosh(x)^2 + 4*a*cosh(x)
 + a)*sinh(x) - 2*a)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\operatorname{csch}^2(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

input
```
integrate(csch(x)**2/(a+a*sech(x)),x)
```

output
```
Integral(csch(x)**2/(sech(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(19) = 38$.

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.91

$$\int \frac{\operatorname{csch}^2(x)}{a + a\operatorname{sech}(x)}\,dx = -\frac{4\,e^{(-x)}}{3\left(2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a\right)} - \frac{2\,e^{(-2\,x)}}{2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a} - \frac{2}{3\left(2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a\right)}$$

input
```
integrate(csch(x)^2/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
-4/3*e^(-x)/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 2*e^(-2*x)/(2*a
*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 2/3/(2*a*e^(-x) - 2*a*e^(-3*x)
- a*e^(-4*x) + a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{\operatorname{csch}^2(x)}{a + a\operatorname{sech}(x)}\,dx = -\frac{1}{2\,a(e^x - 1)} + \frac{3\,e^{(2\,x)} + 1}{6\,a(e^x + 1)^3}$$

input
```
integrate(csch(x)^2/(a+a*sech(x)),x, algorithm="giac")
```

output
```
-1/2/(a*(e^x - 1)) + 1/6*(3*e^(2*x) + 1)/(a*(e^x + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.96

$$\int \frac{\operatorname{csch}^2(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\frac{e^{2x}}{6a}+\frac{1}{6a}-\frac{e^x}{3a}}{3e^{2x}+e^{3x}+3e^x+1}-\frac{\frac{1}{6a}-\frac{e^x}{6a}}{e^{2x}+2e^x+1}-\frac{1}{2a(e^x-1)}+\frac{1}{6a(e^x+1)}$$

input
```
int(1/(sinh(x)^2*(a + a/cosh(x))),x)
```

output
```
(exp(2*x)/(6*a) + 1/(6*a) - exp(x)/(3*a))/(3*exp(2*x) + exp(3*x) + 3*exp(x
) + 1) - (1/(6*a) - exp(x)/(6*a))/(exp(2*x) + 2*exp(x) + 1) - 1/(2*a*(exp(
x) - 1)) + 1/(6*a*(exp(x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{\operatorname{csch}^2(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{-2e^{2x}-\frac{4e^x}{3}-\frac{2}{3}}{a(e^{4x}+2e^{3x}-2e^x-1)}$$

input
```
int(csch(x)^2/(a+a*sech(x)),x)
```

output
```
(2*( - 3*e**(2*x) - 2*e**x - 1))/(3*a*(e**(4*x) + 2*e**(3*x) - 2*e**x - 1)
)
```

3.58 $\int \frac{\operatorname{csch}^3(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{\operatorname{csch}^3(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{\operatorname{arctanh}(\cosh(x))}{8a}-\frac{\coth(x)\operatorname{csch}(x)}{8a}-\frac{\coth(x)\operatorname{csch}^3(x)}{4a}+\frac{\operatorname{csch}^4(x)}{4a}$$

output
```
1/8*arctanh(cosh(x))/a-1/8*coth(x)*csch(x)/a-1/4*coth(x)*csch(x)^3/a+1/4*c
sch(x)^4/a
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

$$\int \frac{\operatorname{csch}^3(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{\cosh^2\left(\frac{x}{2}\right)\left(-2\operatorname{csch}^2\left(\frac{x}{2}\right)+4\log\left(\cosh\left(\frac{x}{2}\right)\right)-4\log\left(\sinh\left(\frac{x}{2}\right)\right)+\operatorname{sech}^4\left(\frac{x}{2}\right)\right)\operatorname{sech}(x)}{16(a+a\operatorname{sech}(x))}$$

input
```
Integrate[Csch[x]^3/(a + a*Sech[x]),x]
```

output
```
(Cosh[x/2]^2*(-2*Csch[x/2]^2 + 4*Log[Cosh[x/2]] - 4*Log[Sinh[x/2]] + Sech[
x/2]^4)*Sech[x])/(16*(a + a*Sech[x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 1.692$, Rules used = {3042, 26, 4360, 26, 25, 3042, 26, 3314, 26, 3042, 26, 3086, 15, 3091, 26, 3042, 26, 4255, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^3(x)}{a\text{sech}(x)+a}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int -\frac{i}{\cos\left(-\frac{\pi}{2}+ix\right)^3\left(a-a\csc\left(-\frac{\pi}{2}+ix\right)\right)}dx$$
$$\Big\downarrow \text{ 26}$$
$$-i\int \frac{1}{\cos\left(ix-\frac{\pi}{2}\right)^3\left(a-a\csc\left(ix-\frac{\pi}{2}\right)\right)}dx$$
$$\Big\downarrow \text{ 4360}$$
$$-i\int -\frac{i\coth(x)\text{csch}^2(x)}{-\cosh(x)a-a}dx$$
$$\Big\downarrow \text{ 26}$$
$$-\int -\frac{\coth(x)\text{csch}^2(x)}{\cosh(x)a+a}dx$$
$$\Big\downarrow \text{ 25}$$
$$\int \frac{\coth(x)\text{csch}^2(x)}{a\cosh(x)+a}dx$$
$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\sin\left(-\frac{\pi}{2}+ix\right)}{\cos\left(-\frac{\pi}{2}+ix\right)^3\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\sin\left(ix-\frac{\pi}{2}\right)}{\cos\left(ix-\frac{\pi}{2}\right)^3\left(a-a\sin\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 3314}$$

$$i\left(\frac{\int -i\coth^2(x)\text{csch}^3(x)dx}{a}+\frac{\int i\coth(x)\text{csch}^4(x)dx}{a}\right)$$

$$\Big\downarrow \text{ 26}$$

$$i\left(\frac{i\int \coth(x)\text{csch}^4(x)dx}{a}-\frac{i\int \coth^2(x)\text{csch}^3(x)dx}{a}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$i\left(\frac{i\int -i\sec\left(ix-\frac{\pi}{2}\right)^4\tan\left(ix-\frac{\pi}{2}\right)dx}{a}-\frac{i\int i\sec\left(ix-\frac{\pi}{2}\right)^3\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}\right)$$

$$\Big\downarrow \text{ 26}$$

$$i\left(\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^4\tan\left(ix-\frac{\pi}{2}\right)dx}{a}+\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^3\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}\right)$$

$$\Big\downarrow \text{ 3086}$$

$$i\left(\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^3\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}-\frac{i\int i\text{csch}^3(x)d(-i\text{csch}(x))}{a}\right)$$

$$\Big\downarrow \text{ 15}$$

$$i\left(\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^3\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}-\frac{i\text{csch}^4(x)}{4a}\right)$$

$$\Big\downarrow \text{ 3091}$$

$$i\left(\frac{\frac{1}{4}i\coth(x)\text{csch}^3(x)-\frac{1}{4}\int i\text{csch}^3(x)dx}{a}-\frac{i\text{csch}^4(x)}{4a}\right)$$

$$\Big\downarrow \text{ 26}$$

$$i\left(\frac{\frac{1}{4}i\coth(x)\text{csch}^3(x)-\frac{1}{4}i\int \text{csch}^3(x)dx}{a}-\frac{i\text{csch}^4(x)}{4a}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{\frac{1}{4}i\coth(x)\text{csch}^3(x)-\frac{1}{4}i\int -i\csc(ix)^3dx}{a}-\frac{i\text{csch}^4(x)}{4a}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{\frac{1}{4}i\coth(x)\text{csch}^3(x)-\frac{1}{4}\int \csc(ix)^3dx}{a}-\frac{i\text{csch}^4(x)}{4a}\right)$$

$$\Big\downarrow \text{4255}$$

$$i\left(\frac{\frac{1}{4}\left(\frac{1}{2}i\coth(x)\text{csch}(x)-\frac{1}{2}\int -i\text{csch}(x)dx\right)+\frac{1}{4}i\coth(x)\text{csch}^3(x)}{a}-\frac{i\text{csch}^4(x)}{4a}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{\frac{1}{4}\left(\frac{1}{2}i\int \text{csch}(x)dx+\frac{1}{2}i\coth(x)\text{csch}(x)\right)+\frac{1}{4}i\coth(x)\text{csch}^3(x)}{a}-\frac{i\text{csch}^4(x)}{4a}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{\frac{1}{4}\left(\frac{1}{2}i\int i\csc(ix)dx+\frac{1}{2}i\coth(x)\text{csch}(x)\right)+\frac{1}{4}i\coth(x)\text{csch}^3(x)}{a}-\frac{i\text{csch}^4(x)}{4a}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{\frac{1}{4}\left(\frac{1}{2}i\coth(x)\text{csch}(x)-\frac{1}{2}\int \csc(ix)dx\right)+\frac{1}{4}i\coth(x)\text{csch}^3(x)}{a}-\frac{i\text{csch}^4(x)}{4a}\right)$$

$$\Big\downarrow \text{4257}$$

$$i\left(\frac{\frac{1}{4}\left(\frac{1}{2}i\coth(x)\text{csch}(x)-\frac{1}{2}i\text{arctanh}(\cosh(x))\right)+\frac{1}{4}i\coth(x)\text{csch}^3(x)}{a}-\frac{i\text{csch}^4(x)}{4a}\right)$$

input
```
Int[Csch[x]^3/(a + a*Sech[x]),x]
```

output
```
I*(((-1/4*I)*Csch[x]^4)/a + ((I/4)*Coth[x]*Csch[x]^3 + ((-1/2*I)*ArcTanh[C
osh[x]] + (I/2)*Coth[x]*Csch[x])/4)/a)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3314
```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[Cos[e + f
*x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[1/(b*d)   Int[Cos[e + f*x]^(p
 - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &
& IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p
+ 1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^{4}}{4}-\frac{\tanh\left(\frac{x}{2}\right)^{2}}{2}-\frac{1}{2\tanh\left(\frac{x}{2}\right)^{2}}-\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{8a}$	38
risch	$-\frac{{\rm e}^{x}\left({\rm e}^{4x}+2\,{\rm e}^{3x}+10\,{\rm e}^{2x}+2\,{\rm e}^{x}+1\right)}{4\left(1+{\rm e}^{x}\right)^{4}a\left({\rm e}^{x}-1\right)^{2}}-\frac{\ln\left({\rm e}^{x}-1\right)}{8a}+\frac{\ln\left(1+{\rm e}^{x}\right)}{8a}$	63

input
```
int(csch(x)^3/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
1/8/a*(1/4*tanh(1/2*x)^4-1/2*tanh(1/2*x)^2-1/2/tanh(1/2*x)^2-ln(tanh(1/2*x
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(38) = 76$.

Time = 0.09 (sec) , antiderivative size = 630, normalized size of antiderivative = 13.70

$$\int \frac{\operatorname{csch}^3(x)}{a + a\operatorname{sech}(x)} \, dx = \text{Too large to display}$$

input

```
integrate(csch(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

output

```
-1/8*(2*cosh(x)^5 + 2*(5*cosh(x) + 2)*sinh(x)^4 + 2*sinh(x)^5 + 4*cosh(x)^
4 + 4*(5*cosh(x)^2 + 4*cosh(x) + 5)*sinh(x)^3 + 20*cosh(x)^3 + 4*(5*cosh(x
)^3 + 6*cosh(x)^2 + 15*cosh(x) + 1)*sinh(x)^2 + 4*cosh(x)^2 - (cosh(x)^6 +
 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^2 + 1
0*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 - cosh
(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4 + 20*cosh(x)^3 - 6*cosh(x
)^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh(x)^4
 - 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*log(c
osh(x) + sinh(x) + 1) + (cosh(x)^6 + 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)
^6 + 2*cosh(x)^5 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 +
 4*(5*cosh(x)^3 + 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15
*cosh(x)^4 + 20*cosh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh
(x)^2 + 2*(3*cosh(x)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x)
 + 1)*sinh(x) + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*(5*cosh(x)^4
 + 8*cosh(x)^3 + 30*cosh(x)^2 + 4*cosh(x) + 1)*sinh(x) + 2*cosh(x))/(a*cos
h(x)^6 + a*sinh(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(x)^5 - a*c
osh(x)^4 + (15*a*cosh(x)^2 + 10*a*cosh(x) - a)*sinh(x)^4 - 4*a*cosh(x)^3 +
 4*(5*a*cosh(x)^3 + 5*a*cosh(x)^2 - a*cosh(x) - a)*sinh(x)^3 - a*cosh(x)^2
 + (15*a*cosh(x)^4 + 20*a*cosh(x)^3 - 6*a*cosh(x)^2 - 12*a*cosh(x) - a)*si
nh(x)^2 + 2*a*cosh(x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x)^4 - 2*a*cosh(x)^...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\operatorname{csch}^3(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

input
```
integrate(csch(x)**3/(a+a*sech(x)),x)
```

output
```
Integral(csch(x)**3/(sech(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(38) = 76$.

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.15

$$\int \frac{\operatorname{csch}^3(x)}{a+a\operatorname{sech}(x)}\,dx = -\frac{e^{(-x)}+2\,e^{(-2\,x)}+10\,e^{(-3\,x)}+2\,e^{(-4\,x)}+e^{(-5\,x)}}{4\,(2\,ae^{(-x)}-ae^{(-2\,x)}-4\,ae^{(-3\,x)}-ae^{(-4\,x)}+2\,ae^{(-5\,x)}+ae^{(-6\,x)}+a)} + \frac{\log\left(e^{(-x)}+1\right)}{8\,a} - \frac{\log\left(e^{(-x)}-1\right)}{8\,a}$$

input
```
integrate(csch(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
-1/4*(e^(-x) + 2*e^(-2*x) + 10*e^(-3*x) + 2*e^(-4*x) + e^(-5*x))/(2*a*e^(-
x) - a*e^(-2*x) - 4*a*e^(-3*x) - a*e^(-4*x) + 2*a*e^(-5*x) + a*e^(-6*x) +
a) + 1/8*log(e^(-x) + 1)/a - 1/8*log(e^(-x) - 1)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(38) = 76.

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.96

$$\int \frac{\operatorname{csch}^3(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\log\left(e^{(-x)}+e^x+2\right)}{16\,a} - \frac{\log\left(e^{(-x)}+e^x-2\right)}{16\,a} + \frac{e^{(-x)}+e^x-6}{16\,a(e^{(-x)}+e^x-2)} - \frac{3\left(e^{(-x)}+e^x\right)^2+12\,e^{(-x)}+12\,e^x-4}{32\,a(e^{(-x)}+e^x+2)^2}$$

input
```
integrate(csch(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

output
```
1/16*log(e^(-x) + e^x + 2)/a - 1/16*log(e^(-x) + e^x - 2)/a + 1/16*(e^(-x)
 + e^x - 6)/(a*(e^(-x) + e^x - 2)) - 1/32*(3*(e^(-x) + e^x)^2 + 12*e^(-x)
+ 12*e^x - 4)/(a*(e^(-x) + e^x + 2)^2)
```

Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.63

$$\int \frac{\operatorname{csch}^3(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{1}{2\,a\,\left(\mathrm{e}^{2\,x}+2\,\mathrm{e}^{x}+1\right)} - \frac{1}{4\,a\,\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{x}+1\right)} + \frac{1}{2\,a\,\left(6\,\mathrm{e}^{2\,x}+4\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}+4\,\mathrm{e}^{x}+1\right)} - \frac{1}{4\,a\,\left(\mathrm{e}^{x}-1\right)} + \frac{\operatorname{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{-a^2}}{a}\right)}{4\,\sqrt{-a^2}} - \frac{1}{a\,\left(3\,\mathrm{e}^{2\,x}+\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}+1\right)}$$

input
```
int(1/(sinh(x)^3*(a + a/cosh(x))),x)
```

output
```
1/(2*a*(exp(2*x) + 2*exp(x) + 1)) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)) + 1/
(2*a*(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp(x) + 1)) - 1/(4*a*(exp(x)
 - 1)) + atan((exp(x)*(-a^2)^(1/2))/a)/(4*(-a^2)^(1/2)) - 1/(a*(3*exp(2*x)
 + exp(3*x) + 3*exp(x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 238, normalized size of antiderivative = 5.17

$$\int \frac{\operatorname{csch}^3(x)}{a + a\operatorname{sech}(x)}\,dx$$
$$= \frac{-e^{6x}\log(e^x-1) + e^{6x}\log(e^x+1) + e^{6x} - 2e^{5x}\log(e^x-1) + 2e^{5x}\log(e^x+1) + e^{4x}\log(e^x-1) - e^{4x}\log(e}{}$$

input
```
int(csch(x)^3/(a+a*sech(x)),x)
```

output
```
( - e**(6*x)*log(e**x - 1) + e**(6*x)*log(e**x + 1) + e**(6*x) - 2*e**(5*x
)*log(e**x - 1) + 2*e**(5*x)*log(e**x + 1) + e**(4*x)*log(e**x - 1) - e**(
4*x)*log(e**x + 1) - 5*e**(4*x) + 4*e**(3*x)*log(e**x - 1) - 4*e**(3*x)*lo
g(e**x + 1) - 24*e**(3*x) + e**(2*x)*log(e**x - 1) - e**(2*x)*log(e**x + 1
) - 5*e**(2*x) - 2*e**x*log(e**x - 1) + 2*e**x*log(e**x + 1) - log(e**x -
1) + log(e**x + 1) + 1)/(8*a*(e**(6*x) + 2*e**(5*x) - e**(4*x) - 4*e**(3*x
) - e**(2*x) + 2*e**x + 1))
```

3.59 $\int \frac{\operatorname{csch}^4(x)}{a+a\operatorname{sech}(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{\operatorname{csch}^4(x)}{a+a\operatorname{sech}(x)} \, dx = \frac{\coth^3(x)}{3a} - \frac{\coth^5(x)}{5a} + \frac{\operatorname{csch}^5(x)}{5a}$$

output
```
1/3*coth(x)^3/a-1/5*coth(x)^5/a+1/5*csch(x)^5/a
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int \frac{\operatorname{csch}^4(x)}{a+a\operatorname{sech}(x)} \, dx = \frac{(-15-6\cosh(x)-2\cosh(2x)+2\cosh(3x)+\cosh(4x))\operatorname{csch}^3(x)}{60a(1+\cosh(x))}$$

input
```
Integrate[Csch[x]^4/(a + a*Sech[x]),x]
```

output
```
((-15 - 6*Cosh[x] - 2*Cosh[2*x] + 2*Cosh[3*x] + Cosh[4*x])*Csch[x]^3)/(60*
a*(1 + Cosh[x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.154$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3318, 25, 3042, 25, 3086, 15, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^4(x)}{a\text{sech}(x)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\cos\left(-\frac{\pi}{2}+ix\right)^4\left(a-a\csc\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{ 4360}$$

$$\int -\frac{\coth(x)\text{csch}^3(x)}{a(-\cosh(x))-a}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int -\frac{\coth(x)\text{csch}^3(x)}{\cosh(x)a+a}dx$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{\coth(x)\text{csch}^3(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\sin\left(-\frac{\pi}{2}+ix\right)}{\cos\left(-\frac{\pi}{2}+ix\right)^4\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\sin\left(ix-\frac{\pi}{2}\right)}{\cos\left(ix-\frac{\pi}{2}\right)^4\left(a-a\sin\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 3318}$$

$$-\frac{\int -\coth^2(x)\text{csch}^4(x)dx}{a}-\frac{\int \coth(x)\text{csch}^5(x)dx}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \coth^2(x)\text{csch}^4(x)dx}{a}-\frac{\int \coth(x)\text{csch}^5(x)dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int -\sec\left(ix-\frac{\pi}{2}\right)^4\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}-\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^5\tan\left(ix-\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^5\tan\left(ix-\frac{\pi}{2}\right)dx}{a}-\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^4\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}$$

$$\Big\downarrow \; 3086$$

$$\frac{i\int \text{csch}^4(x)d(-i\text{csch}(x))}{a}-\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^4\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}$$

$$\Big\downarrow \; 15$$

$$\frac{\text{csch}^5(x)}{5a}-\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^4\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}$$

$$\Big\downarrow \; 3087$$

$$\frac{\text{csch}^5(x)}{5a}+\frac{i\int -\coth^2(x)\left(1-\coth^2(x)\right)d(i\coth(x))}{a}$$

$$\Big\downarrow \; 244$$

$$\frac{\text{csch}^5(x)}{5a}+\frac{i\int \left(\coth^4(x)-\coth^2(x)\right)d(i\coth(x))}{a}$$

$$\Big\downarrow \; 2009$$

$$\frac{\text{csch}^5(x)}{5a}+\frac{i\left(\frac{1}{5}i\coth^5(x)-\frac{1}{3}i\coth^3(x)\right)}{a}$$

input
```
Int[Csch[x]^4/(a + a*Sech[x]),x]
```

output
```
(I*((-1/3*I)*Coth[x]^3 + (I/5)*Coth[x]^5))/a + Csch[x]^5/(5*a)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{-\frac{\tanh\left(\frac{x}{2}\right)^5}{5}+\frac{2\tanh\left(\frac{x}{2}\right)^3}{3}+\frac{2}{\tanh\left(\frac{x}{2}\right)}-\frac{1}{3\tanh\left(\frac{x}{2}\right)^3}}{16a}$	39
risch	$-\frac{4\left(15\,\mathrm{e}^{4x}+6\,\mathrm{e}^{3x}+2\,\mathrm{e}^{2x}-2\,\mathrm{e}^{x}-1\right)}{15\left(\mathrm{e}^{x}-1\right)^3 a\left(1+\mathrm{e}^{x}\right)^5}$	42

input
```
int(csch(x)^4/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
1/16/a*(-1/5*tanh(1/2*x)^5+2/3*tanh(1/2*x)^3+2/tanh(1/2*x)-1/3/tanh(1/2*x)
^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(28) = 56$.

Time = 0.07 (sec) , antiderivative size = 219, normalized size of antiderivative = 6.44

$$\int \frac{\operatorname{csch}^4(x)}{a+a\operatorname{sech}(x)}\,dx =$$

$$-\frac{}{15\left(a\cosh(x)^6+a\sinh(x)^6+2\,a\cosh(x)^5+2\left(3\,a\cosh(x)+a\right)\sinh(x)^5-2\,a\cosh(x)^4+\left(15\,a\cosh\right.\right.}$$

input
```
integrate(csch(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
-8/15*(7*cosh(x)^2 + 4*(4*cosh(x) + 1)*sinh(x) + 7*sinh(x)^2 + 2*cosh(x) +
 1)/(a*cosh(x)^6 + a*sinh(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(
x)^5 - 2*a*cosh(x)^4 + (15*a*cosh(x)^2 + 10*a*cosh(x) - 2*a)*sinh(x)^4 - 6
*a*cosh(x)^3 + 2*(10*a*cosh(x)^3 + 10*a*cosh(x)^2 - 4*a*cosh(x) - 3*a)*sin
h(x)^3 - a*cosh(x)^2 + (15*a*cosh(x)^4 + 20*a*cosh(x)^3 - 12*a*cosh(x)^2 -
 18*a*cosh(x) - a)*sinh(x)^2 + 4*a*cosh(x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x
)^4 - 4*a*cosh(x)^3 - 9*a*cosh(x)^2 + a*cosh(x) + 4*a)*sinh(x) + 2*a)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\operatorname{csch}^4(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

input
```
integrate(csch(x)**4/(a+a*sech(x)),x)
```

output
```
Integral(csch(x)**4/(sech(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(28) = 56$.

Time = 0.04 (sec) , antiderivative size = 292, normalized size of antiderivative = 8.59

$$\begin{aligned}
&\int \frac{\operatorname{csch}^4(x)}{a + a\operatorname{sech}(x)}\,dx \\
&= \frac{8\,e^{(-x)}}{15\,(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a)} \\
&\quad - \frac{8\,e^{(-2\,x)}}{15\,(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a)} \\
&\quad - \frac{8\,e^{(-3\,x)}}{5\,(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a)} \\
&\quad - \frac{4\,e^{(-4\,x)}}{2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a} \\
&\quad + \frac{4}{15\,(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a)}
\end{aligned}$$

input
```
integrate(csch(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
8/15*e^(-x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a
*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 8/15*e^(-2*x)/(2*a*e^(-x) - 2
*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) -
a*e^(-8*x) + a) - 8/5*e^(-3*x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) +
 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 4*e^(-4*x)
/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) -
 2*a*e^(-7*x) - a*e^(-8*x) + a) + 4/15/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^
(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(28) = 56$.

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.74

$$\int \frac{\operatorname{csch}^4(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{3\,e^{(2\,x)} - 12\,e^{x} + 5}{24\,a(e^{x} - 1)^3} - \frac{15\,e^{(4\,x)} + 60\,e^{(3\,x)} + 10\,e^{(2\,x)} + 20\,e^{x} + 7}{120\,a(e^{x} + 1)^5}$$

input
```
integrate(csch(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

output
```
1/24*(3*e^(2*x) - 12*e^x + 5)/(a*(e^x - 1)^3) - 1/120*(15*e^(4*x) + 60*e^(
3*x) + 10*e^(2*x) + 20*e^x + 7)/(a*(e^x + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 236, normalized size of antiderivative = 6.94

$$\begin{aligned}\int \frac{\operatorname{csch}^4(x)}{a + a\operatorname{sech}(x)}\,dx &= \frac{1}{6\,a\,\left(3\,e^{2\,x} - e^{3\,x} - 3\,e^{x} + 1\right)} \\ &\quad - \frac{\frac{3\,e^{2\,x}}{40\,a} + \frac{e^{3\,x}}{40\,a} + \frac{1}{40\,a} - \frac{e^{x}}{8\,a}}{6\,e^{2\,x} + 4\,e^{3\,x} + e^{4\,x} + 4\,e^{x} + 1} - \frac{\frac{e^{2\,x}}{40\,a} - \frac{1}{24\,a} + \frac{e^{x}}{20\,a}}{3\,e^{2\,x} + e^{3\,x} + 3\,e^{x} + 1} \\ &\quad - \frac{\frac{e^{3\,x}}{10\,a} - \frac{e^{2\,x}}{4\,a} + \frac{e^{4\,x}}{40\,a} + \frac{1}{40\,a} + \frac{e^{x}}{10\,a}}{10\,e^{2\,x} + 10\,e^{3\,x} + 5\,e^{4\,x} + e^{5\,x} + 5\,e^{x} + 1} \\ &\quad - \frac{1}{4\,a\,\left(e^{2\,x} - 2\,e^{x} + 1\right)} + \frac{1}{8\,a\,\left(e^{x} - 1\right)} - \frac{1}{20\,a\,\left(e^{x} + 1\right)}\end{aligned}$$

input
```
int(1/(sinh(x)^4*(a + a/cosh(x))),x)
```

output
```
1/(6*a*(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1)) - ((3*exp(2*x))/(40*a) + ex
p(3*x)/(40*a) + 1/(40*a) - exp(x)/(8*a))/(6*exp(2*x) + 4*exp(3*x) + exp(4*
x) + 4*exp(x) + 1) - (exp(2*x)/(40*a) - 1/(24*a) + exp(x)/(20*a))/(3*exp(2
*x) + exp(3*x) + 3*exp(x) + 1) - (exp(3*x)/(10*a) - exp(2*x)/(4*a) + exp(4
*x)/(40*a) + 1/(40*a) + exp(x)/(10*a))/(10*exp(2*x) + 10*exp(3*x) + 5*exp(
4*x) + exp(5*x) + 5*exp(x) + 1) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)) + 1/(8
*a*(exp(x) - 1)) - 1/(20*a*(exp(x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.41

$$\int \frac{\operatorname{csch}^4(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{-4e^{4x}-\frac{8e^{3x}}{5}-\frac{8e^{2x}}{15}+\frac{8e^{x}}{15}+\frac{4}{15}}{a\left(e^{8x}+2e^{7x}-2e^{6x}-6e^{5x}+6e^{3x}+2e^{2x}-2e^{x}-1\right)}$$

input
```
int(csch(x)^4/(a+a*sech(x)),x)
```

output
```
(4*( - 15*e**(4*x) - 6*e**(3*x) - 2*e**(2*x) + 2*e**x + 1))/(15*a*(e**(8*x
) + 2*e**(7*x) - 2*e**(6*x) - 6*e**(5*x) + 6*e**(3*x) + 2*e**(2*x) - 2*e**
x - 1))
```

3.60 $\int \frac{\sinh^4(x)}{a+b\,\mathrm{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 132

$$\int \frac{\sinh^4(x)}{a+b\,\mathrm{sech}(x)}\,dx = \frac{(3a^4-12a^2b^2+8b^4)\,x}{8a^5} - \frac{2(a-b)^{3/2}b(a+b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5} + \frac{(8b(a^2-b^2)-a(3a^2-4b^2)\cosh(x))\sinh(x)}{8a^4} - \frac{(4b-3a\cosh(x))\sinh^3(x)}{12a^2}$$

output
```
1/8*(3*a^4-12*a^2*b^2+8*b^4)*x/a^5-2*(a-b)^(3/2)*b*(a+b)^(3/2)*arctan((a-b
)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a^5+1/8*(8*b*(a^2-b^2)-a*(3*a^2-4*b^2)*co
sh(x))*sinh(x)/a^4-1/12*(4*b-3*a*cosh(x))*sinh(x)^3/a^2
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.66

$$\int \frac{\sinh^4(x)}{a+b\operatorname{sech}(x)}\,dx$$

$$= \frac{36a^4x - 144a^2b^2x + 96b^4x + \frac{192a^4b\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{384a^2b^3\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{192b^5\arctan\left(\frac{(-a+b)\tan}{\sqrt{a^2-}}\right.}{\sqrt{a^2-b^2}}}{96a^5}$$

input
```
Integrate[Sinh[x]^4/(a + b*Sech[x]),x]
```

output
```
(36*a^4*x - 144*a^2*b^2*x + 96*b^4*x + (192*a^4*b*ArcTan[((-a + b)*Tanh[x/
2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (384*a^2*b^3*ArcTan[((-a + b)*Tanh
[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (192*b^5*ArcTan[((-a + b)*Tanh[
x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 24*a*b*(5*a^2 - 4*b^2)*Sinh[x] -
 24*a^2*(a^2 - b^2)*Sinh[2*x] - 8*a^3*b*Sinh[3*x] + 3*a^4*Sinh[4*x])/(96*a
^5)
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.19, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.077$, Rules used = {3042, 4360, 25, 25, 3042, 3344, 3042, 25, 3344, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(x)}{a+b\operatorname{sech}(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(-\frac{\pi}{2}+ix\right)^4}{a-b\csc\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{4360}$$

$$\int -\frac{\sinh^4(x)\cosh(x)}{-a\cosh(x)-b}dx$$

$$\Big\downarrow \; 25$$

$$-\int -\frac{\cosh(x)\sinh^4(x)}{b+a\cosh(x)}dx$$

$$\Big\downarrow \; 25$$

$$\int \frac{\sinh^4(x)\cosh(x)}{a\cosh(x)+b}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)\cos\left(\frac{\pi}{2}+ix\right)^4}{b+a\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \; 3344$$

$$\frac{\int \frac{\left(ab-\left(3a^2-4b^2\right)\cosh(x)\right)\sinh^2(x)}{b+a\cosh(x)}dx}{4a^2}-\frac{\sinh^3(x)(4b-3a\cosh(x))}{12a^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\sinh^3(x)(4b-3a\cosh(x))}{12a^2}+\frac{\int -\frac{\cos\left(ix+\frac{\pi}{2}\right)^2\left(ab+\left(4b^2-3a^2\right)\sin\left(ix+\frac{\pi}{2}\right)\right)}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx}{4a^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\sinh^3(x)(4b-3a\cosh(x))}{12a^2}-\frac{\int \frac{\cos\left(ix+\frac{\pi}{2}\right)^2\left(ab-\left(3a^2-4b^2\right)\sin\left(ix+\frac{\pi}{2}\right)\right)}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx}{4a^2}$$

$$\Big\downarrow \; 3344$$

$$-\frac{\frac{\int \frac{ab\left(5a^2-4b^2\right)-\left(3a^4-12b^2a^2+8b^4\right)\cosh(x)}{b+a\cosh(x)}dx}{2a^2}-\frac{\sinh(x)\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)\cosh(x)\right)}{2a^2}}{4a^2}-\frac{\sinh^3(x)(4b-3a\cosh(x))}{12a^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\sinh^3(x)(4b-3a\cosh(x))}{12a^2}-\frac{-\frac{\sinh(x)\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)\cosh(x)\right)}{2a^2}+\frac{\int \frac{ab\left(5a^2-4b^2\right)+\left(-3a^4+12b^2a^2-8b^4\right)\sin\left(ix+\frac{\pi}{2}\right)}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx}{2a^2}}{4a^2}$$

$$\Big\downarrow \text{3214}$$

$$-\frac{\frac{\frac{8b(a^2-b^2)^2\int \frac{1}{b+a\cosh(x)}dx}{a}-\frac{x(3a^4-12a^2b^2+8b^4)}{a}}{2a^2}-\frac{\sinh(x)(8b(a^2-b^2)-a(3a^2-4b^2)\cosh(x))}{2a^2}}{4a^2}-\frac{\sinh^3(x)(4b-3a\cosh(x))}{12a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh^3(x)(4b-3a\cosh(x))}{12a^2}-\frac{-\frac{\sinh(x)(8b(a^2-b^2)-a(3a^2-4b^2)\cosh(x))}{2a^2}+\frac{-\frac{x(3a^4-12a^2b^2+8b^4)}{a}+\frac{8b(a^2-b^2)^2\int \frac{1}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx}{a}}{2a^2}}{4a^2}$$

$$\Big\downarrow \text{3138}$$

$$-\frac{\frac{\frac{16b(a^2-b^2)^2\int \frac{1}{(a-b)\tanh^2\left(\frac{x}{2}\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a}-\frac{x(3a^4-12a^2b^2+8b^4)}{a}}{2a^2}-\frac{\sinh(x)(8b(a^2-b^2)-a(3a^2-4b^2)\cosh(x))}{2a^2}}{4a^2}-\frac{\sinh^3(x)(4b-3a\cosh(x))}{12a^2}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\sinh^3(x)(4b-3a\cosh(x))}{12a^2}-\frac{\frac{\frac{16b(a^2-b^2)^2\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}-\frac{x(3a^4-12a^2b^2+8b^4)}{a}}{2a^2}-\frac{\sinh(x)(8b(a^2-b^2)-a(3a^2-4b^2)\cosh(x))}{2a^2}}{4a^2}$$

input
```
Int[Sinh[x]^4/(a + b*Sech[x]),x]
```

output
```
-1/12*((4*b - 3*a*Cosh[x])*Sinh[x]^3)/a^2 - ((-(((3*a^4 - 12*a^2*b^2 + 8*b
^4)*x)/a) + (16*b*(a^2 - b^2)^2*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]
])/(a*Sqrt[a - b]*Sqrt[a + b]))/(2*a^2) - ((8*b*(a^2 - b^2) - a*(3*a^2 - 4
*b^2)*Cosh[x])*Sinh[x])/(2*a^2))/(4*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(116) = 232.

Time = 86.60 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.31

method	result
risch	$\frac{3x}{8a} - \frac{3x\,b^2}{2a^3} + \frac{x\,b^4}{a^5} + \frac{e^{4x}}{64a} - \frac{b\,e^{3x}}{24a^2} - \frac{e^{2x}}{8a} + \frac{e^{2x}b^2}{8a^3} + \frac{5b\,e^{x}}{8a^2} - \frac{b^3e^{x}}{2a^4} - \frac{5b\,e^{-x}}{8a^2} + \frac{b^3e^{-x}}{2a^4} + \frac{e^{-2x}}{8a} - \frac{e^{-2x}b^2}{8a^3} + \frac{b\,e^{-3x}}{24a^2}$
default	$-\frac{1}{4a\left(\tanh\left(\frac{x}{2}\right)+1\right)^4} - \frac{-3a-2b}{6a^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^3} - \frac{-a^2+4ab+4b^2}{8a^3\left(\tanh\left(\frac{x}{2}\right)+1\right)^2} + \frac{\left(3a^4-12a^2b^2+8b^4\right)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8a^5} - \frac{3a^3+8a^2b-4a\,b^2-}{8a^4\left(\tanh\left(\frac{x}{2}\right)+1\right)}$

input
```
int(sinh(x)^4/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
3/8*x/a-3/2*x/a^3*b^2+x/a^5*b^4+1/64/a*exp(x)^4-1/24*b/a^2*exp(x)^3-1/8/a*
exp(x)^2+1/8/a^3*exp(x)^2*b^2+5/8*b/a^2*exp(x)-1/2*b^3/a^4*exp(x)-5/8*b/a^
2/exp(x)+1/2*b^3/a^4/exp(x)+1/8/a/exp(x)^2-1/8/a^3/exp(x)^2*b^2+1/24*b/a^2
/exp(x)^3-1/64/a/exp(x)^4+(-a^2+b^2)^(1/2)*b/a^3*ln(exp(x)-((-a^2+b^2)^(1/
2)-b)/a)-(-a^2+b^2)^(1/2)*b^3/a^5*ln(exp(x)-((-a^2+b^2)^(1/2)-b)/a)-(-a^2+
b^2)^(1/2)*b/a^3*ln(exp(x)+(b+(-a^2+b^2)^(1/2))/a)+(-a^2+b^2)^(1/2)*b^3/a^
5*ln(exp(x)+(b+(-a^2+b^2)^(1/2))/a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 866 vs. 2(115) = 230.

Time = 0.12 (sec) , antiderivative size = 1812, normalized size of antiderivative = 13.73

$$\int \frac{\sinh^4(x)}{a + b\,\mathrm{sech}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
[1/192*(3*a^4*cosh(x)^8 + 3*a^4*sinh(x)^8 - 8*a^3*b*cosh(x)^7 + 8*(3*a^4*c
osh(x) - a^3*b)*sinh(x)^7 - 24*(a^4 - a^2*b^2)*cosh(x)^6 + 4*(21*a^4*cosh(
x)^2 - 14*a^3*b*cosh(x) - 6*a^4 + 6*a^2*b^2)*sinh(x)^6 + 24*(3*a^4 - 12*a^
2*b^2 + 8*b^4)*x*cosh(x)^4 + 24*(5*a^3*b - 4*a*b^3)*cosh(x)^5 + 24*(7*a^4*
cosh(x)^3 - 7*a^3*b*cosh(x)^2 + 5*a^3*b - 4*a*b^3 - 6*(a^4 - a^2*b^2)*cosh
(x))*sinh(x)^5 + 8*a^3*b*cosh(x) + 2*(105*a^4*cosh(x)^4 - 140*a^3*b*cosh(x
)^3 - 180*(a^4 - a^2*b^2)*cosh(x)^2 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x +
60*(5*a^3*b - 4*a*b^3)*cosh(x))*sinh(x)^4 - 3*a^4 - 24*(5*a^3*b - 4*a*b^3)
*cosh(x)^3 + 8*(21*a^4*cosh(x)^5 - 35*a^3*b*cosh(x)^4 - 15*a^3*b + 12*a*b^
3 - 60*(a^4 - a^2*b^2)*cosh(x)^3 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x*cosh(
x) + 30*(5*a^3*b - 4*a*b^3)*cosh(x)^2)*sinh(x)^3 + 24*(a^4 - a^2*b^2)*cosh
(x)^2 + 12*(7*a^4*cosh(x)^6 - 14*a^3*b*cosh(x)^5 - 30*(a^4 - a^2*b^2)*cosh
(x)^4 + 2*a^4 - 2*a^2*b^2 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x*cosh(x)^2 +
20*(5*a^3*b - 4*a*b^3)*cosh(x)^3 - 6*(5*a^3*b - 4*a*b^3)*cosh(x))*sinh(x)^
2 - 192*((a^2*b - b^3)*cosh(x)^4 + 4*(a^2*b - b^3)*cosh(x)^3*sinh(x) + 6*(
a^2*b - b^3)*cosh(x)^2*sinh(x)^2 + 4*(a^2*b - b^3)*cosh(x)*sinh(x)^3 + (a^
2*b - b^3)*sinh(x)^4)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2
+ 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^
2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cos
h(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) + 8*(3*a^4*cosh(x)^7 - 7*a^3*b*c...
```

Sympy [F]

$$\int \frac{\sinh^4(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\sinh^4(x)}{a + b\operatorname{sech}(x)}\,dx$$

input
```
integrate(sinh(x)**4/(a+b*sech(x)),x)
```

output
```
Integral(sinh(x)**4/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^4(x)}{a + b\text{sech}(x)} dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.49

$$\begin{aligned}\int \frac{\sinh^4(x)}{a + b\text{sech}(x)} dx &= \frac{3 a^3 e^{(4 x)} - 8 a^2 b e^{(3 x)} - 24 a^3 e^{(2 x)} + 24 a b^2 e^{(2 x)} + 120 a^2 b e^x - 96 b^3 e^x}{192 a^4} \\ &+ \frac{(3 a^4 - 12 a^2 b^2 + 8 b^4) x}{8 a^5} \\ &+ \frac{(8 a^3 b e^x - 3 a^4 - 24 (5 a^3 b - 4 a b^3) e^{(3 x)} + 24 (a^4 - a^2 b^2) e^{(2 x)}) e^{(-4 x)}}{192 a^5} \\ &- \frac{2 (a^4 b - 2 a^2 b^3 + b^5) \arctan\left(\frac{a e^x + b}{\sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2} a^5}\end{aligned}$$

input
```
integrate(sinh(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

output
```
1/192*(3*a^3*e^(4*x) - 8*a^2*b*e^(3*x) - 24*a^3*e^(2*x) + 24*a*b^2*e^(2*x)
 + 120*a^2*b*e^x - 96*b^3*e^x)/a^4 + 1/8*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x/a^
5 + 1/192*(8*a^3*b*e^x - 3*a^4 - 24*(5*a^3*b - 4*a*b^3)*e^(3*x) + 24*(a^4
- a^2*b^2)*e^(2*x))*e^(-4*x)/a^5 - 2*(a^4*b - 2*a^2*b^3 + b^5)*arctan((a*e
^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^5)
```

Mupad [B] (verification not implemented)

Time = 2.89 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.08

$$\int \frac{\sinh^4(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{e^{4x}}{64a} - \frac{e^{-4x}}{64a} + \frac{x\,(3a^4-12a^2b^2+8b^4)}{8a^5} - \frac{e^{-x}\,(5a^2b-4b^3)}{8a^4} + \frac{e^{-2x}\,(a^2-b^2)}{8a^3} - \frac{e^{2x}\,(a^2-b^2)}{8a^3} + \frac{b\,e^{-3x}}{24a^2} - \frac{b\,e^{3x}}{24a^2} + \frac{e^{x}\,(5a^2b-4b^3)}{8a^4} + \frac{b\ln\left(\frac{2e^x\,(a^4b-2a^2b^3+b^5)}{a^6} - \frac{2b\,(a+b)^{3/2}\,(a+b\,e^x)\,(b-a)^{3/2}}{a^6}\right)(a+b)^{3/2}\,(b-a)^{3/2}}{a^5} - \frac{b\ln\left(\frac{2e^x\,(a^4b-2a^2b^3+b^5)}{a^6} + \frac{2b\,(a+b)^{3/2}\,(a+b\,e^x)\,(b-a)^{3/2}}{a^6}\right)(a+b)^{3/2}\,(b-a)^{3/2}}{a^5}$$

input
```
int(sinh(x)^4/(a + b/cosh(x)),x)
```

output
```
exp(4*x)/(64*a) - exp(-4*x)/(64*a) + (x*(3*a^4 + 8*b^4 - 12*a^2*b^2))/(8*a
^5) - (exp(-x)*(5*a^2*b - 4*b^3))/(8*a^4) + (exp(-2*x)*(a^2 - b^2))/(8*a^3
) - (exp(2*x)*(a^2 - b^2))/(8*a^3) + (b*exp(-3*x))/(24*a^2) - (b*exp(3*x))
/(24*a^2) + (exp(x)*(5*a^2*b - 4*b^3))/(8*a^4) + (b*log((2*exp(x)*(a^4*b +
 b^5 - 2*a^2*b^3))/a^6 - (2*b*(a + b)^(3/2)*(a + b*exp(x))*(b - a)^(3/2))/
a^6)*(a + b)^(3/2)*(b - a)^(3/2))/a^5 - (b*log((2*exp(x)*(a^4*b + b^5 - 2*
a^2*b^3))/a^6 + (2*b*(a + b)^(3/2)*(a + b*exp(x))*(b - a)^(3/2))/a^6)*(a +
 b)^(3/2)*(b - a)^(3/2))/a^5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.95

$$\int \frac{\sinh^4(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{-384e^{4x}\sqrt{a^2-b^2}\,atan\left(\frac{e^x a+b}{\sqrt{a^2-b^2}}\right)a^2b + 384e^{4x}\sqrt{a^2-b^2}\,atan\left(\frac{e^x a+b}{\sqrt{a^2-b^2}}\right)b^3 + 3e^{8x}a^4 - 8e^{7x}a^3b - 24e^{6x}a^4 + \cdots}{}$$

input
```
int(sinh(x)^4/(a+b*sech(x)),x)
```

output
```
( - 384*e**(4*x)*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2))*a*
*2*b + 384*e**(4*x)*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2))
*b**3 + 3*e**(8*x)*a**4 - 8*e**(7*x)*a**3*b - 24*e**(6*x)*a**4 + 24*e**(6*
x)*a**2*b**2 + 120*e**(5*x)*a**3*b - 96*e**(5*x)*a*b**3 + 72*e**(4*x)*a**4
*x - 288*e**(4*x)*a**2*b**2*x + 192*e**(4*x)*b**4*x - 120*e**(3*x)*a**3*b
+ 96*e**(3*x)*a*b**3 + 24*e**(2*x)*a**4 - 24*e**(2*x)*a**2*b**2 + 8*e**x*a
**3*b - 3*a**4)/(192*e**(4*x)*a**5)
```

3.61 $\int \frac{\sinh^3(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{\sinh^3(x)}{a+b\text{sech}(x)}\,dx=-\frac{(a^2-b^2)\cosh(x)}{a^3}-\frac{b\cosh^2(x)}{2a^2}+\frac{\cosh^3(x)}{3a}+\frac{b(a^2-b^2)\log(b+a\cosh(x))}{a^4}$$

output
```
-(a^2-b^2)*cosh(x)/a^3-1/2*b*cosh(x)^2/a^2+1/3*cosh(x)^3/a+b*(a^2-b^2)*ln(
b+a*cosh(x))/a^4
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^3(x)}{a+b\text{sech}(x)}\,dx=\frac{(-9a^3+12ab^2)\cosh(x)-3a^2b\cosh(2x)+a^3\cosh(3x)+12a^2b\log(b+a\cosh(x))-12b^3\log(b+a\cosh($$
$$12a^4$$

input
```
Integrate[Sinh[x]^3/(a + b*Sech[x]),x]
```

output
```
((-9*a^3 + 12*a*b^2)*Cosh[x] - 3*a^2*b*Cosh[2*x] + a^3*Cosh[3*x] + 12*a^2*
b*Log[b + a*Cosh[x]] - 12*b^3*Log[b + a*Cosh[x]])/(12*a^4)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {3042, 26, 4360, 26, 25, 3042, 26, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{a+b\text{sech}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cos\left(-\frac{\pi}{2}+ix\right)^3}{a-b\csc\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^3}{a-b\csc\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4360}$$

$$i\int \frac{i\cosh(x)\sinh^3(x)}{-b-a\cosh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$-\int -\frac{\cosh(x)\sinh^3(x)}{b+a\cosh(x)}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\sinh^3(x)\cosh(x)}{a\cosh(x)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin\left(\frac{\pi}{2}+ix\right)\cos\left(\frac{\pi}{2}+ix\right)^3}{b+a\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{\cos\left(ix+\frac{\pi}{2}\right)^3\sin\left(ix+\frac{\pi}{2}\right)}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3316$$

$$-\frac{\int \frac{\cosh(x)(a^2-a^2\cosh^2(x))}{b+a\cosh(x)}d(a\cosh(x))}{a^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{a\cosh(x)(a^2-a^2\cosh^2(x))}{b+a\cosh(x)}d(a\cosh(x))}{a^4}$$

$$\Big\downarrow \; 522$$

$$-\frac{\int \left(-\cosh^2(x)a^2+\left(1-\frac{b^2}{a^2}\right)a^2+b\cosh(x)a+\frac{b^3-a^2b}{b+a\cosh(x)}\right)d(a\cosh(x))}{a^4}$$

$$\Big\downarrow \; 2009$$

$$-\frac{-\frac{1}{3}a^3\cosh^3(x)+a(a^2-b^2)\cosh(x)-b(a^2-b^2)\log(a\cosh(x)+b)+\frac{1}{2}a^2b\cosh^2(x)}{a^4}$$

input
```
Int[Sinh[x]^3/(a + b*Sech[x]),x]
```

output
```
-((a*(a^2 - b^2)*Cosh[x] + (a^2*b*Cosh[x]^2)/2 - (a^3*Cosh[x]^3)/3 - b*(a^
2 - b^2)*Log[b + a*Cosh[x]])/a^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(57) = 114$.

Time = 11.56 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.23

method	result
risch	$-\frac{xb}{a^2}+\frac{x\,b^3}{a^4}+\frac{e^{3x}}{24a}-\frac{b\,e^{2x}}{8a^2}-\frac{3\,e^{x}}{8a}+\frac{e^{x}b^2}{2a^3}-\frac{3\,e^{-x}}{8a}+\frac{e^{-x}b^2}{2a^3}-\frac{b\,e^{-2x}}{8a^2}+\frac{e^{-3x}}{24a}+\frac{b\ln\left(e^{2x}+\frac{2b\,e^{x}}{a}+1\right)}{a^2}-\frac{b^3\ln\left(e^{2x}\right.}{a}$
default	$-\frac{a+b}{2a^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{1}{3a\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}-\frac{b\left(a^2-b^2\right)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a^4}-\frac{a^2-ab-2b^2}{2a^3\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{b\left(a^3-a^2b-a\,b^2+b^3\right)\ln\left(a\tan\right.}{a^4(a}$

input
```
int(sinh(x)^3/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
-x*b/a^2+x/a^4*b^3+1/24/a*exp(3*x)-1/8*b/a^2*exp(2*x)-3/8/a*exp(x)+1/2/a^3
*exp(x)*b^2-3/8/a*exp(-x)+1/2/a^3*exp(-x)*b^2-1/8*b/a^2*exp(-2*x)+1/24/a*e
xp(-3*x)+b/a^2*ln(exp(2*x)+2*b/a*exp(x)+1)-b^3/a^4*ln(exp(2*x)+2*b/a*exp(x
)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(57) = 114$.

Time = 0.09 (sec) , antiderivative size = 490, normalized size of antiderivative = 8.03

$$\int \frac{\sinh^3(x)}{a + b\text{sech}(x)}\, dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
1/24*(a^3*cosh(x)^6 + a^3*sinh(x)^6 - 3*a^2*b*cosh(x)^5 + 3*(2*a^3*cosh(x)
 - a^2*b)*sinh(x)^5 - 24*(a^2*b - b^3)*x*cosh(x)^3 - 3*(3*a^3 - 4*a*b^2)*c
osh(x)^4 + 3*(5*a^3*cosh(x)^2 - 5*a^2*b*cosh(x) - 3*a^3 + 4*a*b^2)*sinh(x)
^4 - 3*a^2*b*cosh(x) + 2*(10*a^3*cosh(x)^3 - 15*a^2*b*cosh(x)^2 - 12*(a^2*
b - b^3)*x - 6*(3*a^3 - 4*a*b^2)*cosh(x))*sinh(x)^3 + a^3 - 3*(3*a^3 - 4*a
*b^2)*cosh(x)^2 + 3*(5*a^3*cosh(x)^4 - 10*a^2*b*cosh(x)^3 - 3*a^3 + 4*a*b^
2 - 24*(a^2*b - b^3)*x*cosh(x) - 6*(3*a^3 - 4*a*b^2)*cosh(x)^2)*sinh(x)^2
+ 24*((a^2*b - b^3)*cosh(x)^3 + 3*(a^2*b - b^3)*cosh(x)^2*sinh(x) + 3*(a^2
*b - b^3)*cosh(x)*sinh(x)^2 + (a^2*b - b^3)*sinh(x)^3)*log(2*(a*cosh(x) +
b)/(cosh(x) - sinh(x))) + 3*(2*a^3*cosh(x)^5 - 5*a^2*b*cosh(x)^4 - 24*(a^2
*b - b^3)*x*cosh(x)^2 - 4*(3*a^3 - 4*a*b^2)*cosh(x)^3 - a^2*b - 2*(3*a^3 -
 4*a*b^2)*cosh(x))*sinh(x))/(a^4*cosh(x)^3 + 3*a^4*cosh(x)^2*sinh(x) + 3*a
^4*cosh(x)*sinh(x)^2 + a^4*sinh(x)^3)
```

Sympy [F]

$$\int \frac{\sinh^3(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\sinh^3(x)}{a + b\,\text{sech}\,(x)}\,dx$$

input
```
integrate(sinh(x)**3/(a+b*sech(x)),x)
```

output
```
Integral(sinh(x)**3/(a + b*sech(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(57) = 114.

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.10

$$\begin{aligned}\int \frac{\sinh^3(x)}{a + b\text{sech}(x)}\,dx = &-\frac{\left(3\,abe^{(-x)} - a^2 + 3\,(3\,a^2 - 4\,b^2)e^{(-2\,x)}\right)e^{(3\,x)}}{24\,a^3} \\ &- \frac{3\,abe^{(-2\,x)} - a^2e^{(-3\,x)} + 3\,(3\,a^2 - 4\,b^2)e^{(-x)}}{24\,a^3} \\ &+ \frac{(a^2b - b^3)x}{a^4} + \frac{(a^2b - b^3)\log\left(2\,be^{(-x)} + ae^{(-2\,x)} + a\right)}{a^4}\end{aligned}$$

input
```
integrate(sinh(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
-1/24*(3*a*b*e^(-x) - a^2 + 3*(3*a^2 - 4*b^2)*e^(-2*x))*e^(3*x)/a^3 - 1/24
*(3*a*b*e^(-2*x) - a^2*e^(-3*x) + 3*(3*a^2 - 4*b^2)*e^(-x))/a^3 + (a^2*b -
 b^3)*x/a^4 + (a^2*b - b^3)*log(2*b*e^(-x) + a*e^(-2*x) + a)/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.43

$$\int \frac{\sinh^3(x)}{a + b\text{sech}(x)}\,dx = \frac{a^2\left(e^{(-x)} + e^x\right)^3 - 3\,ab\left(e^{(-x)} + e^x\right)^2 - 12\,a^2\left(e^{(-x)} + e^x\right) + 12\,b^2\left(e^{(-x)} + e^x\right)}{24\,a^3} + \frac{(a^2b - b^3)\log\left(\left|a\left(e^{(-x)} + e^x\right) + 2\,b\right|\right)}{a^4}$$

input
```
integrate(sinh(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

output
```
1/24*(a^2*(e^(-x) + e^x)^3 - 3*a*b*(e^(-x) + e^x)^2 - 12*a^2*(e^(-x) + e^x
) + 12*b^2*(e^(-x) + e^x))/a^3 + (a^2*b - b^3)*log(abs(a*(e^(-x) + e^x) +
2*b))/a^4
```

Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.02

$$\int \frac{\sinh^3(x)}{a + b\text{sech}(x)}\,dx = \frac{e^{-3\,x}}{24\,a} + \frac{e^{3\,x}}{24\,a} - \frac{x\,(a^2\,b - b^3)}{a^4} - \frac{e^x\,(3\,a^2 - 4\,b^2)}{8\,a^3} - \frac{b\,e^{-2\,x}}{8\,a^2} - \frac{b\,e^{2\,x}}{8\,a^2} + \frac{\ln\left(a + 2\,b\,e^x + a\,e^{2\,x}\right)\,(a^2\,b - b^3)}{a^4} - \frac{e^{-x}\,(3\,a^2 - 4\,b^2)}{8\,a^3}$$

input
```
int(sinh(x)^3/(a + b/cosh(x)),x)
```

output
```
exp(-3*x)/(24*a) + exp(3*x)/(24*a) - (x*(a^2*b - b^3))/a^4 - (exp(x)*(3*a^
2 - 4*b^2))/(8*a^3) - (b*exp(-2*x))/(8*a^2) - (b*exp(2*x))/(8*a^2) + (log(
a + 2*b*exp(x) + a*exp(2*x))*(a^2*b - b^3))/a^4 - (exp(-x)*(3*a^2 - 4*b^2)
)/(8*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.67

$$\int \frac{\sinh^3(x)}{a+b\text{sech}(x)}\,dx = \frac{e^{6x}a^3 - 3e^{5x}a^2b - 9e^{4x}a^3 + 12e^{4x}a\,b^2 + 24e^{3x}\log(e^{2x}a + 2e^{x}b + a)\,a^2b - 24e^{3x}\log(e^{2x}a + 2e^{x}b + a)\,b^3 - }{24e^{3x}a^4}$$

input
```
int(sinh(x)^3/(a+b*sech(x)),x)
```

output
```
(e**(6*x)*a**3 - 3*e**(5*x)*a**2*b - 9*e**(4*x)*a**3 + 12*e**(4*x)*a*b**2
+ 24*e**(3*x)*log(e**(2*x)*a + 2*e**x*b + a)*a**2*b - 24*e**(3*x)*log(e**(
2*x)*a + 2*e**x*b + a)*b**3 - 24*e**(3*x)*a**2*b*x + 24*e**(3*x)*b**3*x -
9*e**(2*x)*a**3 + 12*e**(2*x)*a*b**2 - 3*e**x*a**2*b + a**3)/(24*e**(3*x)*
a**4)
```

3.62 $\int \frac{\sinh^2(x)}{a+b\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 82

$$\int \frac{\sinh^2(x)}{a+b\text{sech}(x)}\,dx=-\frac{(a^2-2b^2)\,x}{2a^3}+\frac{2\sqrt{a-b}b\sqrt{a+b}\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3}-\frac{(2b-a\cosh(x))\sinh(x)}{2a^2}$$

output
```
-1/2*(a^2-2*b^2)*x/a^3+2*(a-b)^(1/2)*b*(a+b)^(1/2)*arctan((a-b)^(1/2)*tanh
(1/2*x)/(a+b)^(1/2))/a^3-1/2*(2*b-a*cosh(x))*sinh(x)/a^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int \frac{\sinh^2(x)}{a+b\text{sech}(x)}\,dx=\frac{-2a^2x+4b^2x-8b\sqrt{a^2-b^2}\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)-4ab\sinh(x)+a^2\sinh(2x)}{4a^3}$$

input
```
Integrate[Sinh[x]^2/(a + b*Sech[x]),x]
```

output
```
(-2*a^2*x + 4*b^2*x - 8*b*Sqrt[a^2 - b^2]*ArcTan[((-a + b)*Tanh[x/2])/Sqrt
[a^2 - b^2]] - 4*a*b*Sinh[x] + a^2*Sinh[2*x])/(4*a^3)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 25, 4360, 3042, 25, 25, 3344, 25, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{a+b\text{sech}(x)} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^2}{a-b\csc\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{a-b\csc\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4360}$$

$$-\int \frac{\cosh(x)\sinh^2(x)}{-b-a\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$-\int -\frac{\cos\left(ix+\frac{\pi}{2}\right)^2\sin\left(ix+\frac{\pi}{2}\right)}{-b-a\sin\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{25}$$

$$\int -\frac{\sin\left(\frac{\pi}{2}+ix\right)\cos\left(\frac{\pi}{2}+ix\right)^2}{b+a\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(ix+\frac{\pi}{2}\right)^2\sin\left(ix+\frac{\pi}{2}\right)}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3344}$$

$$-\frac{\int -\frac{ab-(a^2-2b^2)\cosh(x)}{b+a\cosh(x)}dx}{2a^2}-\frac{\sinh(x)(2b-a\cosh(x))}{2a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{ab-(a^2-2b^2)\cosh(x)}{b+a\cosh(x)}dx}{2a^2}-\frac{\sinh(x)(2b-a\cosh(x))}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)(2b-a\cosh(x))}{2a^2}+\frac{\int \frac{ab+(2b^2-a^2)\sin\left(ix+\frac{\pi}{2}\right)}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx}{2a^2}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{2b(a^2-b^2)\int \frac{1}{b+a\cosh(x)}dx}{a}-\frac{x(a^2-2b^2)}{a}}{2a^2}-\frac{\sinh(x)(2b-a\cosh(x))}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)(2b-a\cosh(x))}{2a^2}+\frac{-\frac{x(a^2-2b^2)}{a}+\frac{2b(a^2-b^2)\int \frac{1}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx}{a}}{2a^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{4b(a^2-b^2)\int \frac{1}{(a-b)\tanh^2\left(\frac{x}{2}\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a}-\frac{x(a^2-2b^2)}{a}}{2a^2}-\frac{\sinh(x)(2b-a\cosh(x))}{2a^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{4b(a^2-b^2)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}-\frac{x(a^2-2b^2)}{a}}{2a^2}-\frac{\sinh(x)(2b-a\cosh(x))}{2a^2}$$

input
```
Int[Sinh[x]^2/(a + b*Sech[x]),x]
```

output
```
(-(((a^2 - 2*b^2)*x)/a) + (4*b*(a^2 - b^2)*ArcTan[(Sqrt[a - b]*Tanh[x/2])/
Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]))/(2*a^2) - ((2*b - a*Cosh[x])*Si
nh[x])/(2*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.59

method	result
risch	$-\frac{x}{2a}+\frac{x\,b^{2}}{a^{3}}+\frac{\mathrm{e}^{2x}}{8a}-\frac{b\,\mathrm{e}^{x}}{2a^{2}}+\frac{b\,\mathrm{e}^{-x}}{2a^{2}}-\frac{\mathrm{e}^{-2x}}{8a}+\frac{\sqrt{-a^{2}+b^{2}}\,b\ln\left(\mathrm{e}^{x}+\frac{b+\sqrt{-a^{2}+b^{2}}}{a}\right)}{a^{3}}-\frac{\sqrt{-a^{2}+b^{2}}\,b\ln\left(\mathrm{e}^{x}-\frac{\sqrt{-a^{2}+b^{2}}-b}{a}\right)}{a^{3}}$
default	$\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{-a-2b}{2a^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\left(a^{2}-2b^{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a^{3}}-\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{-a-2b}{2a^{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{(-a^{2}+2b^{2}}{}$

input
```
int(sinh(x)^2/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/a+x/a^3*b^2+1/8/a*exp(x)^2-1/2*b/a^2*exp(x)+1/2*b/a^2/exp(x)-1/8/a/
exp(x)^2+(-a^2+b^2)^(1/2)*b/a^3*ln(exp(x)+(b+(-a^2+b^2)^(1/2))/a)-(-a^2+b^
2)^(1/2)*b/a^3*ln(exp(x)-((-a^2+b^2)^(1/2)-b)/a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(67) = 134$.

Time = 0.10 (sec) , antiderivative size = 536, normalized size of antiderivative = 6.54

$$\int \frac{\sinh^2(x)}{a+b\operatorname{sech}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
[1/8*(a^2*cosh(x)^4 + a^2*sinh(x)^4 - 4*a*b*cosh(x)^3 - 4*(a^2 - 2*b^2)*x*
cosh(x)^2 + 4*(a^2*cosh(x) - a*b)*sinh(x)^3 + 4*a*b*cosh(x) + 2*(3*a^2*cos
h(x)^2 - 6*a*b*cosh(x) - 2*(a^2 - 2*b^2)*x)*sinh(x)^2 + 8*(b*cosh(x)^2 + 2
*b*cosh(x)*sinh(x) + b*sinh(x)^2)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^
2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x)
+ 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)
^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) - a^2 + 4*(a^2*cosh(x)^
3 - 3*a*b*cosh(x)^2 - 2*(a^2 - 2*b^2)*x*cosh(x) + a*b)*sinh(x))/(a^3*cosh(
x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2), 1/8*(a^2*cosh(x)^4 + a^2*si
nh(x)^4 - 4*a*b*cosh(x)^3 - 4*(a^2 - 2*b^2)*x*cosh(x)^2 + 4*(a^2*cosh(x) -
 a*b)*sinh(x)^3 + 4*a*b*cosh(x) + 2*(3*a^2*cosh(x)^2 - 6*a*b*cosh(x) - 2*(
a^2 - 2*b^2)*x)*sinh(x)^2 - 16*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh
(x)^2)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)
) - a^2 + 4*(a^2*cosh(x)^3 - 3*a*b*cosh(x)^2 - 2*(a^2 - 2*b^2)*x*cosh(x) +
 a*b)*sinh(x))/(a^3*cosh(x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2)]
```

Sympy [F]

$$\int \frac{\sinh^2(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\sinh^2(x)}{a + b\,\text{sech}(x)}\,dx$$

input
```
integrate(sinh(x)**2/(a+b*sech(x)),x)
```

output
```
Integral(sinh(x)**2/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^2(x)}{a + b\text{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.22

$$\int \frac{\sinh^2(x)}{a+b\text{sech}(x)}\,dx = \frac{ae^{(2x)}-4\,be^x}{8\,a^2} - \frac{(a^2-2\,b^2)x}{2\,a^3} + \frac{(4\,abe^x-a^2)e^{(-2\,x)}}{8\,a^3} + \frac{2\,(a^2b-b^3)\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}a^3}$$

input
```
integrate(sinh(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

output
```
1/8*(a*e^(2*x) - 4*b*e^x)/a^2 - 1/2*(a^2 - 2*b^2)*x/a^3 + 1/8*(4*a*b*e^x -
 a^2)*e^(-2*x)/a^3 + 2*(a^2*b - b^3)*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(
sqrt(a^2 - b^2)*a^3)
```

Mupad [B] (verification not implemented)

Time = 2.79 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.11

$$\int \frac{\sinh^2(x)}{a+b\text{sech}(x)}\,dx = \frac{e^{2x}}{8\,a} - \frac{e^{-2x}}{8\,a} - \frac{b\,e^x}{2\,a^2} + \frac{b\,e^{-x}}{2\,a^2} - \frac{x\,(a^2-2\,b^2)}{2\,a^3} + \frac{b\,\ln\left(-\frac{2\,b\,e^x\,(a^2-b^2)}{a^4} - \frac{2\,b\,\sqrt{a+b}\,(a+b\,e^x)\,\sqrt{b-a}}{a^4}\right)\sqrt{a+b}\,\sqrt{b-a}}{a^3} - \frac{b\,\ln\left(\frac{2\,b\,\sqrt{a+b}\,(a+b\,e^x)\,\sqrt{b-a}}{a^4} - \frac{2\,b\,e^x\,(a^2-b^2)}{a^4}\right)\sqrt{a+b}\,\sqrt{b-a}}{a^3}$$

input
```
int(sinh(x)^2/(a + b/cosh(x)),x)
```

output
```
exp(2*x)/(8*a) - exp(-2*x)/(8*a) - (b*exp(x))/(2*a^2) + (b*exp(-x))/(2*a^2
) - (x*(a^2 - 2*b^2))/(2*a^3) + (b*log(- (2*b*exp(x)*(a^2 - b^2))/a^4 - (2
*b*(a + b)^(1/2)*(a + b*exp(x))*(b - a)^(1/2))/a^4)*(a + b)^(1/2)*(b - a)^
(1/2))/a^3 - (b*log((2*b*(a + b)^(1/2)*(a + b*exp(x))*(b - a)^(1/2))/a^4 -
 (2*b*exp(x)*(a^2 - b^2))/a^4)*(a + b)^(1/2)*(b - a)^(1/2))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.27

$$\int \frac{\sinh^2(x)}{a + b\text{sech}(x)} \, dx = \frac{16e^{2x}\sqrt{a^2 - b^2}\, atan\left(\frac{e^x a+b}{\sqrt{a^2-b^2}}\right) b + e^{4x}a^2 - 4e^{3x}ab - 4e^{2x}a^2x + 8e^{2x}b^2x + 4e^x ab - a^2}{8e^{2x}a^3}$$

input
```
int(sinh(x)^2/(a+b*sech(x)),x)
```

output
```
(16*e**(2*x)*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2))*b + e*
*(4*x)*a**2 - 4*e**(3*x)*a*b - 4*e**(2*x)*a**2*x + 8*e**(2*x)*b**2*x + 4*e
**x*a*b - a**2)/(8*e**(2*x)*a**3)
```

3.63 $\int \frac{\sinh(x)}{a+b\text{sech}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{\sinh(x)}{a+b\text{sech}(x)}\,dx = \frac{\cosh(x)}{a} - \frac{b\log(b+a\cosh(x))}{a^2}$$

output
```
cosh(x)/a-b*ln(b+a*cosh(x))/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\sinh(x)}{a+b\text{sech}(x)}\,dx = \frac{a\cosh(x)-b\log(b+a\cosh(x))}{a^2}$$

input
```
Integrate[Sinh[x]/(a + b*Sech[x]),x]
```

output
```
(a*Cosh[x] - b*Log[b + a*Cosh[x]])/a^2
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 26, 4360, 26, 25, 3042, 26, 3312, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{a+b\text{sech}(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\cos\left(-\frac{\pi}{2}+ix\right)}{a-b\csc\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)}{a-b\csc\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4360}$$

$$-i\int -\frac{i\cosh(x)\sinh(x)}{-b-a\cosh(x)}dx$$

$$\Big\downarrow \text{ 26}$$

$$-\int -\frac{\cosh(x)\sinh(x)}{b+a\cosh(x)}dx$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{\sinh(x)\cosh(x)}{a\cosh(x)+b}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\sin\left(\frac{\pi}{2}+ix\right)\cos\left(\frac{\pi}{2}+ix\right)}{b+a\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\cos\left(ix+\frac{\pi}{2}\right)\sin\left(ix+\frac{\pi}{2}\right)}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3312}$$

$$\frac{\int \frac{\cosh(x)}{b+a\cosh(x)}d(a\cosh(x))}{a}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{a\cosh(x)}{b+a\cosh(x)}d(a\cosh(x))}{a^2}$$
$$\Big\downarrow \text{49}$$
$$\frac{\int \left(1-\frac{b}{b+a\cosh(x)}\right)d(a\cosh(x))}{a^2}$$
$$\Big\downarrow \text{2009}$$
$$\frac{a\cosh(x)-b\log(a\cosh(x)+b)}{a^2}$$

input
```
Int[Sinh[x]/(a + b*Sech[x]),x]
```

output
```
(a*Cosh[x] - b*Log[b + a*Cosh[x]])/a^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

method	result	size
derivativedivides	$\frac{1}{a\,\operatorname{sech}(x)}+\frac{b\ln(\operatorname{sech}(x))}{a^2}-\frac{b\ln(a+b\,\operatorname{sech}(x))}{a^2}$	31
default	$\frac{1}{a\,\operatorname{sech}(x)}+\frac{b\ln(\operatorname{sech}(x))}{a^2}-\frac{b\ln(a+b\,\operatorname{sech}(x))}{a^2}$	31
risch	$\frac{xb}{a^2}+\frac{e^x}{2a}+\frac{e^{-x}}{2a}-\frac{b\ln\left(e^{2x}+\frac{2b\,e^x}{a}+1\right)}{a^2}$	45

input
```
int(sinh(x)/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a/sech(x)+b/a^2*ln(sech(x))-b/a^2*ln(a+b*sech(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(20) = 40$.

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.90

$$\int \frac{\sinh(x)}{a + b\text{sech}(x)}\, dx = \frac{2\, bx\cosh\left(x\right) + a\cosh\left(x\right)^{2} + a\sinh\left(x\right)^{2} - 2\left(b\cosh\left(x\right) + b\sinh\left(x\right)\right)\log\left(\frac{2\left(a\cosh(x)+b\right)}{\cosh(x)-\sinh(x)}\right) + 2\left(bx + a\cosh\right.}{2\left(a^{2}\cosh\left(x\right) + a^{2}\sinh\left(x\right)\right)}$$

input
```
integrate(sinh(x)/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
1/2*(2*b*x*cosh(x) + a*cosh(x)^2 + a*sinh(x)^2 - 2*(b*cosh(x) + b*sinh(x))
*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) + 2*(b*x + a*cosh(x))*sinh(x)
+ a)/(a^2*cosh(x) + a^2*sinh(x))
```

Sympy [F]

$$\int \frac{\sinh(x)}{a + b\text{sech}(x)}\, dx = \int \frac{\sinh\left(x\right)}{a + b\,\text{sech}\left(x\right)}\, dx$$

input
```
integrate(sinh(x)/(a+b*sech(x)),x)
```

output
```
Integral(sinh(x)/(a + b*sech(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(20) = 40$.

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int \frac{\sinh(x)}{a + b\text{sech}(x)}\, dx = -\frac{bx}{a^{2}} + \frac{e^{(-x)}}{2\,a} + \frac{e^{x}}{2\,a} - \frac{b\log\left(2\,be^{(-x)} + ae^{(-2\,x)} + a\right)}{a^{2}}$$

input
```
integrate(sinh(x)/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
-b*x/a^2 + 1/2*e^(-x)/a + 1/2*e^x/a - b*log(2*b*e^(-x) + a*e^(-2*x) + a)/a
^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{\sinh(x)}{a + b\text{sech}(x)} \, dx = \frac{e^{(-x)} + e^{x}}{2\,a} - \frac{b \log\left(\left|a\left(e^{(-x)} + e^{x}\right) + 2\,b\right|\right)}{a^{2}}$$

input
```
integrate(sinh(x)/(a+b*sech(x)),x, algorithm="giac")
```

output
```
1/2*(e^(-x) + e^x)/a - b*log(abs(a*(e^(-x) + e^x) + 2*b))/a^2
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{a + b\text{sech}(x)} \, dx = \frac{\cosh(x)}{a} - \frac{b\,\ln\left(b + a\cosh(x)\right)}{a^{2}}$$

input
```
int(sinh(x)/(a + b/cosh(x)),x)
```

output
```
cosh(x)/a - (b*log(b + a*cosh(x)))/a^2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.40

$$\int \frac{\sinh(x)}{a + b\mathrm{sech}(x)}\,dx = \frac{e^{2x}a - 2e^x\log(e^{2x}a + 2e^xb + a)\,b + 2e^xbx + a}{2e^xa^2}$$

input
```
int(sinh(x)/(a+b*sech(x)),x)
```

output
```
(e**(2*x)*a - 2*e**x*log(e**(2*x)*a + 2*e**x*b + a)*b + 2*e**x*b*x + a)/(2
*e**x*a**2)
```

3.64 $\int \frac{\text{csch}(x)}{a+b\text{sech}(x)} dx$

Optimal result

Integrand size = 11, antiderivative size = 53

$$\int \frac{\text{csch}(x)}{a+b\text{sech}(x)}\,dx=\frac{\log(1-\cosh(x))}{2(a+b)}-\frac{\log(1+\cosh(x))}{2(a-b)}+\frac{b\log(b+a\cosh(x))}{a^2-b^2}$$

output
```
ln(1-cosh(x))/(2*a+2*b)-ln(1+cosh(x))/(2*a-2*b)+b*ln(b+a*cosh(x))/(a^2-b^2
)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{\text{csch}(x)}{a+b\text{sech}(x)}\,dx=\frac{\log\left(\cosh\left(\frac{x}{2}\right)\right)}{-a+b}-\frac{b\log(b+a\cosh(x))}{-a^2+b^2}+\frac{\log\left(\sinh\left(\frac{x}{2}\right)\right)}{a+b}$$

input
```
Integrate[Csch[x]/(a + b*Sech[x]),x]
```

output
```
Log[Cosh[x/2]]/(-a + b) - (b*Log[b + a*Cosh[x]])/(-a^2 + b^2) + Log[Sinh[x
/2]]/(a + b)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.182, Rules used = {3042, 26, 4359, 26, 25, 3042, 26, 3200, 587, 16, 452, 219, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}(x)}{a+b\text{sech}(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i}{\cos\left(-\frac{\pi}{2}+ix\right)\left(a-b\csc\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{1}{\cos\left(ix-\frac{\pi}{2}\right)\left(a-b\csc\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 4359}$$

$$i\int \frac{i\coth(x)}{-b-a\cosh(x)}dx$$

$$\Big\downarrow \text{ 26}$$

$$-\int -\frac{\coth(x)}{b+a\cosh(x)}dx$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{\coth(x)}{a\cosh(x)+b}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\tan\left(-\frac{\pi}{2}+ix\right)}{b-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\tan\left(ix-\frac{\pi}{2}\right)}{b-a\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3200}$$

$$-\int \frac{a\cosh(x)}{(b+a\cosh(x))\left(a^2-a^2\cosh^2(x)\right)}d(a\cosh(x))$$

$\downarrow$ 587

$$\frac{b\int \frac{1}{b+a\cosh(x)}d(a\cosh(x))}{a^2-b^2}-\frac{\int \frac{a^2-ab\cosh(x)}{a^2-a^2\cosh^2(x)}d(a\cosh(x))}{a^2-b^2}$$

$\downarrow$ 16

$$\frac{b\log(a\cosh(x)+b)}{a^2-b^2}-\frac{\int \frac{a^2-ab\cosh(x)}{a^2-a^2\cosh^2(x)}d(a\cosh(x))}{a^2-b^2}$$

$\downarrow$ 452

$$\frac{b\log(a\cosh(x)+b)}{a^2-b^2}-\frac{a^2\int \frac{1}{a^2-a^2\cosh^2(x)}d(a\cosh(x))-b\int \frac{a\cosh(x)}{a^2-a^2\cosh^2(x)}d(a\cosh(x))}{a^2-b^2}$$

$\downarrow$ 219

$$\frac{b\log(a\cosh(x)+b)}{a^2-b^2}-\frac{a\text{arctanh}(\cosh(x))-b\int \frac{a\cosh(x)}{a^2-a^2\cosh^2(x)}d(a\cosh(x))}{a^2-b^2}$$

$\downarrow$ 240

$$\frac{b\log(a\cosh(x)+b)}{a^2-b^2}-\frac{\frac{1}{2}b\log\left(a^2-a^2\cosh^2(x)\right)+a\text{arctanh}(\cosh(x))}{a^2-b^2}$$

input
```
Int[Csch[x]/(a + b*Sech[x]),x]
```

output
```
(b*Log[b + a*Cosh[x]])/(a^2 - b^2) - (a*ArcTanh[Cosh[x]] + (b*Log[a^2 - a^
2*Cosh[x]^2])/2)/(a^2 - b^2)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 587
```
Int[(x_.)/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[(-
c)*(d/(b*c^2 + a*d^2))   Int[1/(c + d*x), x], x] + Simp[1/(b*c^2 + a*d^2)
 Int[(a*d + b*c*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
^2 + a*d^2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

rule 4359
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_.), x_Symbol] :> Int[Cot[e + f*x]^p*(b + a*Sin[e + f*x])^m, x] /; FreeQ[{a
, b, e, f, p}, x] && IntegerQ[m] && EqQ[m, p]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{b\ln\left(a\tanh\left(\frac{x}{2}\right)^{2}-b\tanh\left(\frac{x}{2}\right)^{2}+a+b\right)}{(a+b)(a-b)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a+b}$	48
risch	$-\frac{x}{a+b}+\frac{x}{a-b}-\frac{2bx}{a^{2}-b^{2}}+\frac{\ln\left(\mathrm{e}^{x}-1\right)}{a+b}-\frac{\ln\left(1+\mathrm{e}^{x}\right)}{a-b}+\frac{b\ln\left(\mathrm{e}^{2x}+\frac{2b\,\mathrm{e}^{x}}{a}+1\right)}{a^{2}-b^{2}}$	87

input
```
int(csch(x)/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
b/(a+b)/(a-b)*ln(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2+a+b)+1/(a+b)*ln(tanh(1/2*
x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{b\log\left(\frac{2\,(a\cosh(x)+b)}{\cosh(x)-\sinh(x)}\right)-(a+b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+(a-b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)}{a^2-b^2}$$

input
```
integrate(csch(x)/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
(b*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) - (a + b)*log(cosh(x) + sinh
(x) + 1) + (a - b)*log(cosh(x) + sinh(x) - 1))/(a^2 - b^2)
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{a + b\operatorname{sech}(x)}\,dx = \int \frac{\operatorname{csch}(x)}{a + b\operatorname{sech}(x)}\,dx$$

input
```
integrate(csch(x)/(a+b*sech(x)),x)
```

output
```
Integral(csch(x)/(a + b*sech(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(x)}{a + b\operatorname{sech}(x)}\,dx = \frac{b\log\left(2\,be^{(-x)} + ae^{(-2\,x)} + a\right)}{a^2 - b^2} - \frac{\log\left(e^{(-x)} + 1\right)}{a - b} + \frac{\log\left(e^{(-x)} - 1\right)}{a + b}$$

input
```
integrate(csch(x)/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
b*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^2 - b^2) - log(e^(-x) + 1)/(a - b) +
 log(e^(-x) - 1)/(a + b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.23

$$\int \frac{\operatorname{csch}(x)}{a + b\operatorname{sech}(x)}\,dx = \frac{ab\log\left(\left|a\left(e^{(-x)} + e^x\right) + 2\,b\right|\right)}{a^3 - ab^2} - \frac{\log\left(e^{(-x)} + e^x + 2\right)}{2\,(a - b)} + \frac{\log\left(e^{(-x)} + e^x - 2\right)}{2\,(a + b)}$$

input
```
integrate(csch(x)/(a+b*sech(x)),x, algorithm="giac")
```

output
```
a*b*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^3 - a*b^2) - 1/2*log(e^(-x) + e^x
+ 2)/(a - b) + 1/2*log(e^(-x) + e^x - 2)/(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.94 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.79

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{\ln\left(128\,a\,b-32\,a^2-128\,b^2+32\,a^2\,\mathrm{e}^x+128\,b^2\,\mathrm{e}^x-128\,a\,b\,\mathrm{e}^x\right)}{a+b} - \frac{\ln\left(-128\,a\,b-32\,a^2-128\,b^2-32\,a^2\,\mathrm{e}^x-128\,b^2\,\mathrm{e}^x-128\,a\,b\,\mathrm{e}^x\right)}{a-b} + \frac{b\,\ln\left(16\,a\,b^2-4\,a^3\,\mathrm{e}^{2\,x}-4\,a^3+32\,b^3\,\mathrm{e}^x-8\,a^2\,b\,\mathrm{e}^x+16\,a\,b^2\,\mathrm{e}^{2\,x}\right)}{a^2-b^2}$$

input
```
int(1/(sinh(x)*(a + b/cosh(x))),x)
```

output
```
log(128*a*b - 32*a^2 - 128*b^2 + 32*a^2*exp(x) + 128*b^2*exp(x) - 128*a*b*
exp(x))/(a + b) - log(- 128*a*b - 32*a^2 - 128*b^2 - 32*a^2*exp(x) - 128*b
^2*exp(x) - 128*a*b*exp(x))/(a - b) + (b*log(16*a*b^2 - 4*a^3*exp(2*x) - 4
*a^3 + 32*b^3*exp(x) - 8*a^2*b*exp(x) + 16*a*b^2*exp(2*x)))/(a^2 - b^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{\log(e^x-1)\,a-\log(e^x-1)\,b-\log(e^x+1)\,a-\log(e^x+1)\,b+\log(e^{2x}a+2e^xb+a)\,b}{a^2-b^2}$$

input
```
int(csch(x)/(a+b*sech(x)),x)
```

output
```
(log(e**x - 1)*a - log(e**x - 1)*b - log(e**x + 1)*a - log(e**x + 1)*b + l
og(e**(2*x)*a + 2*e**x*b + a)*b)/(a**2 - b**2)
```

3.65 $\int \frac{\mathrm{csch}^2(x)}{a+b\,\mathrm{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 66

$$\int \frac{\mathrm{csch}^2(x)}{a+b\,\mathrm{sech}(x)}\,dx=\frac{2ab\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{(b-a\cosh(x))\mathrm{csch}(x)}{a^2-b^2}$$

output
```
2*a*b*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^(3/2)+
(b-a*cosh(x))*csch(x)/(a^2-b^2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int \frac{\mathrm{csch}^2(x)}{a+b\,\mathrm{sech}(x)}\,dx=\frac{1}{2}\left(-\frac{4ab\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{\coth\left(\frac{x}{2}\right)}{a+b}+\frac{\tanh\left(\frac{x}{2}\right)}{-a+b}\right)$$

input
```
Integrate[Csch[x]^2/(a + b*Sech[x]),x]
```

output
```
((-4*a*b*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) -
 Coth[x/2]/(a + b) + Tanh[x/2]/(-a + b))/2
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {3042, 25, 4360, 3042, 3345, 25, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(x)}{a+b\text{sech}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\cos\left(-\frac{\pi}{2}+ix\right)^2\left(a-b\csc\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\cos\left(ix-\frac{\pi}{2}\right)^2\left(a-b\csc\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4360}$$

$$-\int \frac{\coth(x)\text{csch}(x)}{-b-a\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\sin\left(ix-\frac{\pi}{2}\right)}{\cos\left(ix-\frac{\pi}{2}\right)^2\left(a\sin\left(ix-\frac{\pi}{2}\right)-b\right)}dx$$

$$\Big\downarrow \text{3345}$$

$$\frac{\text{csch}(x)(b-a\cosh(x))}{a^2-b^2}-\frac{\int -\frac{ab}{b+a\cosh(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{ab}{b+a\cosh(x)}dx}{a^2-b^2}+\frac{\text{csch}(x)(b-a\cosh(x))}{a^2-b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{ab\int \frac{1}{b+a\cosh(x)}dx}{a^2-b^2}+\frac{\text{csch}(x)(b-a\cosh(x))}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{csch}(x)(b-a\cosh(x))}{a^2-b^2}+\frac{ab\int\frac{1}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{2ab\int\frac{1}{(a-b)\tanh^2\left(\frac{x}{2}\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2}+\frac{\text{csch}(x)(b-a\cosh(x))}{a^2-b^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{2ab\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\,(a^2-b^2)}+\frac{\text{csch}(x)(b-a\cosh(x))}{a^2-b^2}$$

input
```
Int[Csch[x]^2/(a + b*Sech[x]),x]
```

output
```
(2*a*b*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a +
b]*(a^2 - b^2)) + ((b - a*Cosh[x])*Csch[x])/(a^2 - b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2(a-b)}-\frac{1}{2(a+b)\tanh\left(\frac{x}{2}\right)}+\frac{2ab\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}$	77
risch	$-\frac{2(-b\,e^{x}+a)}{(e^{2x}-1)(a^2-b^2)}-\frac{ba\ln\left(e^{x}+\frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}+\frac{ba\ln\left(e^{x}+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}$	165

input
```
int(csch(x)^2/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*tanh(1/2*x)/(a-b)-1/2/(a+b)/tanh(1/2*x)+2*a/(a-b)/(a+b)*b/((a-b)*(a+b
))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a-b)*(a+b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(58) = 116.

Time = 0.11 (sec) , antiderivative size = 452, normalized size of antiderivative = 6.85

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{sech}(x)}\,dx$$
$$= \left[\frac{2\,a^3 - 2\,ab^2 - \left(ab\cosh(x)^2 + 2\,ab\cosh(x)\sinh(x) + ab\sinh(x)^2 - ab\right)\sqrt{-a^2+b^2}\log\left(\frac{a^2\cosh(x)^2+a^2\sinh}{}\right.}{a^4 - 2\,a^2b^2 + b^4 - (a^4 - 2\,a^2b^2 + b^4)\cosh(x)^2 - 2} \right.$$

input
```
integrate(csch(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
[(2*a^3 - 2*a*b^2 - (a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2
 - a*b)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x
) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*co
sh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*co
sh(x) + b)*sinh(x) + a)) - 2*(a^2*b - b^3)*cosh(x) - 2*(a^2*b - b^3)*sinh(
x))/(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 -
2*a^2*b^2 + b^4)*cosh(x)*sinh(x) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2), 2*(
a^3 - a*b^2 + (a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 - a*b
)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) - (
a^2*b - b^3)*cosh(x) - (a^2*b - b^3)*sinh(x))/(a^4 - 2*a^2*b^2 + b^4 - (a^
4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)
 - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2)]
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{sech}(x)}\,dx = \int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{sech}(x)}\,dx$$

input
```
integrate(csch(x)**2/(a+b*sech(x)),x)
```

output
```
Integral(csch(x)**2/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^2(x)}{a + b\operatorname{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csch(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{csch}^2(x)}{a + b\operatorname{sech}(x)}\,dx = \frac{2\,ab\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}} + \frac{2\,(be^x-a)}{(a^2-b^2)(e^{(2\,x)}-1)}$$

input
```
integrate(csch(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

output
```
2*a*b*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) + 2*(b*e^x - a
)/((a^2 - b^2)*(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.29

$$\int \frac{\operatorname{csch}^2(x)}{a + b\operatorname{sech}(x)}\,dx = \frac{a\,b\,\ln\left(-\frac{2\,b\,e^x}{a^2-b^2}-\frac{2\,b\,(a+b\,e^x)}{(a+b)^{3/2}\,(b-a)^{3/2}}\right)}{(a+b)^{3/2}\,(b-a)^{3/2}} - \frac{\frac{2\,a}{a^2-b^2}-\frac{2\,b\,e^x}{a^2-b^2}}{e^{2\,x}-1} - \frac{a\,b\,\ln\left(\frac{2\,b\,(a+b\,e^x)}{(a+b)^{3/2}\,(b-a)^{3/2}}-\frac{2\,b\,e^x}{a^2-b^2}\right)}{(a+b)^{3/2}\,(b-a)^{3/2}}$$

input
```
int(1/(sinh(x)^2*(a + b/cosh(x))),x)
```

output
```
(a*b*log(- (2*b*exp(x))/(a^2 - b^2) - (2*b*(a + b*exp(x)))/((a + b)^(3/2)*
(b - a)^(3/2))))/((a + b)^(3/2)*(b - a)^(3/2)) - ((2*a)/(a^2 - b^2) - (2*b
*exp(x))/(a^2 - b^2))/(exp(2*x) - 1) - (a*b*log((2*b*(a + b*exp(x)))/((a +
 b)^(3/2)*(b - a)^(3/2)) - (2*b*exp(x))/(a^2 - b^2)))/((a + b)^(3/2)*(b -
a)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.50

$$\int \frac{\operatorname{csch}^2(x)}{a + b\operatorname{sech}(x)}\,dx$$
$$= \frac{2e^{2x}\sqrt{a^2-b^2}\,atan\left(\frac{e^x a+b}{\sqrt{a^2-b^2}}\right)ab - 2\sqrt{a^2-b^2}\,atan\left(\frac{e^x a+b}{\sqrt{a^2-b^2}}\right)ab - 2e^{2x}a^3 + 2e^{2x}a\,b^2 + 2e^x a^2 b - 2e^x b^3}{e^{2x}a^4 - 2e^{2x}a^2b^2 + e^{2x}b^4 - a^4 + 2a^2b^2 - b^4}$$

input
```
int(csch(x)^2/(a+b*sech(x)),x)
```

output
```
(2*(e**(2*x)*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2))*a*b -
sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2))*a*b - e**(2*x)*a**3
 + e**(2*x)*a*b**2 + e**x*a**2*b - e**x*b**3))/(e**(2*x)*a**4 - 2*e**(2*x)
*a**2*b**2 + e**(2*x)*b**4 - a**4 + 2*a**2*b**2 - b**4)
```

3.66 $\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 85

$$\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{(b-a\cosh(x))\operatorname{csch}^2(x)}{2(a^2-b^2)} - \frac{a\log(1-\cosh(x))}{4(a+b)^2} + \frac{a\log(1+\cosh(x))}{4(a-b)^2} - \frac{a^2b\log(b+a\cosh(x))}{(a^2-b^2)^2}$$

output
```
(b-a*cosh(x))*csch(x)^2/(2*a^2-2*b^2)-1/4*a*ln(1-cosh(x))/(a+b)^2+1/4*a*ln
(1+cosh(x))/(a-b)^2-a^2*b*ln(b+a*cosh(x))/(a^2-b^2)^2
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{1}{8}\left(-\frac{\operatorname{csch}^2\left(\frac{x}{2}\right)}{a+b} + \frac{4a\left((a+b)^2\log\left(\cosh\left(\frac{x}{2}\right)\right)-2ab\log(b+a\cosh(x))-(a-b)^2\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)}{(a-b)^2(a+b)^2} - \frac{\operatorname{sech}^2\left(\frac{x}{2}\right)}{a-b}\right)$$

input
```
Integrate[Csch[x]^3/(a + b*Sech[x]),x]
```

output
```
(-(Csch[x/2]^2/(a + b)) + (4*a*((a + b)^2*Log[Cosh[x/2]] - 2*a*b*Log[b + a
*Cosh[x]] - (a - b)^2*Log[Sinh[x/2]]))/((a - b)^2*(a + b)^2) - Sech[x/2]^2
/(a - b))/8
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.51, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.077$, Rules used = {3042, 26, 4360, 26, 25, 3042, 26, 3316, 25, 27, 593, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{sech}(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\cos\left(-\frac{\pi}{2}+ix\right)^3\left(a-b\csc\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\cos \left(ix-\frac{\pi}{2}\right)^3 \left(a-b \csc \left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \ 4360$$

$$-i\int -\frac{i \coth(x)\text{csch}^2(x)}{-b-a \cosh(x)}dx$$

$$\Big\downarrow \ 26$$

$$-\int -\frac{\coth(x)\text{csch}^2(x)}{b+a \cosh(x)}dx$$

$$\Big\downarrow \ 25$$

$$\int \frac{\coth(x)\text{csch}^2(x)}{a \cosh(x)+b}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{i \sin \left(-\frac{\pi}{2}+ix\right)}{\cos \left(-\frac{\pi}{2}+ix\right)^3 \left(b-a \sin \left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \ 26$$

$$i\int \frac{\sin \left(ix-\frac{\pi}{2}\right)}{\cos \left(ix-\frac{\pi}{2}\right)^3 \left(b-a \sin \left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \ 3316$$

$$-a^3\int -\frac{\cosh(x)}{(b+a \cosh(x)) \left(a^2-a^2 \cosh^2(x)\right)^2}d(a \cosh(x))$$

$$\Big\downarrow \ 25$$

$$a^3\int \frac{\cosh(x)}{(b+a \cosh(x)) \left(a^2-a^2 \cosh^2(x)\right)^2}d(a \cosh(x))$$

$$\Big\downarrow \ 27$$

$$a^2\int \frac{a \cosh(x)}{(b+a \cosh(x)) \left(a^2-a^2 \cosh^2(x)\right)^2}d(a \cosh(x))$$

$$\Big\downarrow \ 593$$

$$a^2 \left(\frac{\int -\frac{b-a \cosh(x)}{(b+a \cosh(x)) \left(a^2-a^2 \cosh^2(x)\right)}d(a \cosh(x))}{2 \left(a^2-b^2\right)}-\frac{b-a \cosh(x)}{2 \left(a^2-b^2\right) \left(a^2-a^2 \cosh^2(x)\right)}\right)$$

$$\Big\downarrow \ 25$$

$$a^2\left(-\frac{\int \frac{b-a\cosh(x)}{(b+a\cosh(x))\left(a^2-a^2\cosh^2(x)\right)}d(a\cosh(x))}{2\left(a^2-b^2\right)}-\frac{b-a\cosh(x)}{2\left(a^2-b^2\right)\left(a^2-a^2\cosh^2(x)\right)}\right)$$

$$\Big\downarrow \; 657$$

$$a^2\left(-\frac{\int \left(\frac{-a-b}{2a(a-b)(\cosh(x)a+a)}+\frac{b-a}{2a(a+b)(a-a\cosh(x))}+\frac{2b}{(a-b)(a+b)(b+a\cosh(x))}\right)d(a\cosh(x))}{2\left(a^2-b^2\right)}-\frac{b-a\cosh(x)}{2\left(a^2-b^2\right)\left(a^2-a^2\cos\right.}\right.$$

$$\Big\downarrow \; 2009$$

$$a^2\left(-\frac{b-a\cosh(x)}{2\left(a^2-b^2\right)\left(a^2-a^2\cosh^2(x)\right)}-\frac{\frac{2b\log(a\cosh(x)+b)}{a^2-b^2}+\frac{(a-b)\log(a-a\cosh(x))}{2a(a+b)}-\frac{(a+b)\log(a\cosh(x)+a)}{2a(a-b)}}{2\left(a^2-b^2\right)}\right)$$

input
```
Int[Csch[x]^3/(a + b*Sech[x]),x]
```

output
```
a^2*(-1/2*(b - a*Cosh[x])/((a^2 - b^2)*(a^2 - a^2*Cosh[x]^2)) - (((a - b)*
Log[a - a*Cosh[x]])/(2*a*(a + b)) - ((a + b)*Log[a + a*Cosh[x]])/(2*a*(a -
 b)) + (2*b*Log[b + a*Cosh[x]])/(a^2 - b^2))/(2*(a^2 - b^2)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

method	result
default	$\frac{\tanh\left(\frac{x}{2}\right)^2}{8a-8b}-\frac{a^2 b\ln\left(a\tanh\left(\frac{x}{2}\right)^2-b\tanh\left(\frac{x}{2}\right)^2+a+b\right)}{(a+b)^2(a-b)^2}-\frac{1}{8(a+b)\tanh\left(\frac{x}{2}\right)^2}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2(a+b)^2}$
risch	$\frac{xa}{2a^2+4ab+2b^2}-\frac{xa}{2(a^2-2ab+b^2)}+\frac{2a^2bx}{a^4-2a^2b^2+b^4}-\frac{e^x\left(e^{2x}a-2b\,e^x+a\right)}{\left(e^{2x}-1\right)^2(a^2-b^2)}-\frac{a\ln(e^x-1)}{2(a^2+2ab+b^2)}+\frac{a\ln(1+e^x)}{2a^2-4ab+2b^2}-\frac{a^2 b\ln\left(e^{2x}-\right.}{a^4-2a^2}$

input
```
int(csch(x)^3/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
1/8*tanh(1/2*x)^2/(a-b)-a^2*b/(a+b)^2/(a-b)^2*ln(a*tanh(1/2*x)^2-b*tanh(1/
2*x)^2+a+b)-1/8/(a+b)/tanh(1/2*x)^2-1/2/(a+b)^2*a*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(80) = 160$.

Time = 0.10 (sec) , antiderivative size = 828, normalized size of antiderivative = 9.74

$$\int \frac{\operatorname{csch}^3(x)}{a + b\operatorname{sech}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
-1/2*(2*(a^3 - a*b^2)*cosh(x)^3 + 2*(a^3 - a*b^2)*sinh(x)^3 - 4*(a^2*b - b
^3)*cosh(x)^2 - 2*(2*a^2*b - 2*b^3 - 3*(a^3 - a*b^2)*cosh(x))*sinh(x)^2 +
2*(a^3 - a*b^2)*cosh(x) + 2*(a^2*b*cosh(x)^4 + 4*a^2*b*cosh(x)*sinh(x)^3 +
 a^2*b*sinh(x)^4 - 2*a^2*b*cosh(x)^2 + a^2*b + 2*(3*a^2*b*cosh(x)^2 - a^2*
b)*sinh(x)^2 + 4*(a^2*b*cosh(x)^3 - a^2*b*cosh(x))*sinh(x))*log(2*(a*cosh(
x) + b)/(cosh(x) - sinh(x))) - ((a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3
 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^4
+ a^3 + 2*a^2*b + a*b^2 - 2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2 - 2*(a^3 + 2
*a^2*b + a*b^2 - 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3
+ 2*a^2*b + a*b^2)*cosh(x)^3 - (a^3 + 2*a^2*b + a*b^2)*cosh(x))*sinh(x))*l
og(cosh(x) + sinh(x) + 1) + ((a^3 - 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 -
2*a^2*b + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 - 2*a^2*b + a*b^2)*sinh(x)^4 + a
^3 - 2*a^2*b + a*b^2 - 2*(a^3 - 2*a^2*b + a*b^2)*cosh(x)^2 - 2*(a^3 - 2*a^
2*b + a*b^2 - 3*(a^3 - 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 - 2
*a^2*b + a*b^2)*cosh(x)^3 - (a^3 - 2*a^2*b + a*b^2)*cosh(x))*sinh(x))*log(
cosh(x) + sinh(x) - 1) + 2*(a^3 - a*b^2 + 3*(a^3 - a*b^2)*cosh(x)^2 - 4*(a
^2*b - b^3)*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4
- 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^4 +
 a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*
a^2*b^2 + b^4 - 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{a + b\operatorname{sech}(x)}\,dx = \int \frac{\operatorname{csch}^3(x)}{a + b\operatorname{sech}(x)}\,dx$$

input
```
integrate(csch(x)**3/(a+b*sech(x)),x)
```

output
```
Integral(csch(x)**3/(a + b*sech(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.74

$$\int \frac{\operatorname{csch}^3(x)}{a + b\operatorname{sech}(x)}\,dx = -\frac{a^2 b \log\left(2\,be^{(-x)} + ae^{(-2\,x)} + a\right)}{a^4 - 2\,a^2b^2 + b^4} + \frac{a \log\left(e^{(-x)} + 1\right)}{2\,(a^2 - 2\,ab + b^2)} - \frac{a \log\left(e^{(-x)} - 1\right)}{2\,(a^2 + 2\,ab + b^2)} - \frac{ae^{(-x)} - 2\,be^{(-2\,x)} + ae^{(-3\,x)}}{a^2 - b^2 - 2\,(a^2 - b^2)e^{(-2\,x)} + (a^2 - b^2)e^{(-4\,x)}}$$

input
```
integrate(csch(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
-a^2*b*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*a*lo
g(e^(-x) + 1)/(a^2 - 2*a*b + b^2) - 1/2*a*log(e^(-x) - 1)/(a^2 + 2*a*b + b
^2) - (a*e^(-x) - 2*b*e^(-2*x) + a*e^(-3*x))/(a^2 - b^2 - 2*(a^2 - b^2)*e^
(-2*x) + (a^2 - b^2)*e^(-4*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(80) = 160$.

Time = 0.11 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.05

$$\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{sech}(x)}\,dx = -\frac{a^3b\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{a^5-2\,a^3b^2+ab^4}+\frac{a\log\left(e^{(-x)}+e^x+2\right)}{4\,(a^2-2\,ab+b^2)}-\frac{a\log\left(e^{(-x)}+e^x-2\right)}{4\,(a^2+2\,ab+b^2)} - \frac{a^2b\left(e^{(-x)}+e^x\right)^2+2\,a^3\left(e^{(-x)}+e^x\right)-2\,ab^2\left(e^{(-x)}+e^x\right)-8\,a^2b+4\,b^3}{2\,(a^4-2\,a^2b^2+b^4)\left(\left(e^{(-x)}+e^x\right)^2-4\right)}$$

input
```
integrate(csch(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

output
```
-a^3*b*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^5 - 2*a^3*b^2 + a*b^4) + 1/4*a*
log(e^(-x) + e^x + 2)/(a^2 - 2*a*b + b^2) - 1/4*a*log(e^(-x) + e^x - 2)/(a
^2 + 2*a*b + b^2) - 1/2*(a^2*b*(e^(-x) + e^x)^2 + 2*a^3*(e^(-x) + e^x) - 2
*a*b^2*(e^(-x) + e^x) - 8*a^2*b + 4*b^3)/((a^4 - 2*a^2*b^2 + b^4)*((e^(-x)
 + e^x)^2 - 4))
```

Mupad [B] (verification not implemented)

Time = 2.82 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.00

$$\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{\frac{2\,(a^2\,b-b^3)}{(a^2-b^2)^2}+\frac{e^x\,(a\,b^2-a^3)}{(a^2-b^2)^2}}{e^{2\,x}-1}+\frac{\frac{2\,b}{a^2-b^2}-\frac{2\,a\,e^x}{a^2-b^2}}{e^{4\,x}-2\,e^{2\,x}+1}-\frac{a\,\ln\left(e^x-1\right)}{2\,a^2+4\,a\,b+2\,b^2}+\frac{a\,\ln\left(e^x+1\right)}{2\,a^2-4\,a\,b+2\,b^2} - \frac{a^2\,b\,\ln\left(a^6\,e^{2\,x}+a^6+a^2\,b^4-14\,a^4\,b^2+a^2\,b^4\,e^{2\,x}-14\,a^4\,b^2\,e^{2\,x}+2\,a\,b^5\,e^x+2\,a^5\,b\,e^x-28\,a^3\,b^3\,e^x\right)}{a^4-2\,a^2\,b^2+b^4}$$

input
```
int(1/(sinh(x)^3*(a + b/cosh(x))),x)
```

output
```
((2*(a^2*b - b^3))/(a^2 - b^2)^2 + (exp(x)*(a*b^2 - a^3))/(a^2 - b^2)^2)/(
exp(2*x) - 1) + ((2*b)/(a^2 - b^2) - (2*a*exp(x))/(a^2 - b^2))/(exp(4*x) -
 2*exp(2*x) + 1) - (a*log(exp(x) - 1))/(4*a*b + 2*a^2 + 2*b^2) + (a*log(ex
p(x) + 1))/(2*a^2 - 4*a*b + 2*b^2) - (a^2*b*log(a^6*exp(2*x) + a^6 + a^2*b
^4 - 14*a^4*b^2 + a^2*b^4*exp(2*x) - 14*a^4*b^2*exp(2*x) + 2*a*b^5*exp(x)
+ 2*a^5*b*exp(x) - 28*a^3*b^3*exp(x)))/(a^4 + b^4 - 2*a^2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 501, normalized size of antiderivative = 5.89

$$\int \frac{\operatorname{csch}^3(x)}{a + b\operatorname{sech}(x)}\,dx = \frac{-2b^3 - \log(e^x-1)\,a^3 + \log(e^x+1)\,a^3 - 2e^{3x}a^3 - 2e^x a^3 + 2e^{3x}a\,b^2 + 2e^{2x}\log(e^x-1)\,a^3 - 2e^{2x}\log(e^x +}{}$$

input
```
int(csch(x)^3/(a+b*sech(x)),x)
```

output
```
( - e**(4*x)*log(e**x - 1)*a**3 + 2*e**(4*x)*log(e**x - 1)*a**2*b - e**(4*
x)*log(e**x - 1)*a*b**2 + e**(4*x)*log(e**x + 1)*a**3 + 2*e**(4*x)*log(e**
x + 1)*a**2*b + e**(4*x)*log(e**x + 1)*a*b**2 - 2*e**(4*x)*log(e**(2*x)*a
+ 2*e**x*b + a)*a**2*b + 2*e**(4*x)*a**2*b - 2*e**(4*x)*b**3 - 2*e**(3*x)*
a**3 + 2*e**(3*x)*a*b**2 + 2*e**(2*x)*log(e**x - 1)*a**3 - 4*e**(2*x)*log(
e**x - 1)*a**2*b + 2*e**(2*x)*log(e**x - 1)*a*b**2 - 2*e**(2*x)*log(e**x +
 1)*a**3 - 4*e**(2*x)*log(e**x + 1)*a**2*b - 2*e**(2*x)*log(e**x + 1)*a*b*
*2 + 4*e**(2*x)*log(e**(2*x)*a + 2*e**x*b + a)*a**2*b - 2*e**x*a**3 + 2*e*
*x*a*b**2 - log(e**x - 1)*a**3 + 2*log(e**x - 1)*a**2*b - log(e**x - 1)*a*
b**2 + log(e**x + 1)*a**3 + 2*log(e**x + 1)*a**2*b + log(e**x + 1)*a*b**2
- 2*log(e**(2*x)*a + 2*e**x*b + a)*a**2*b + 2*a**2*b - 2*b**3)/(2*(e**(4*x
)*a**4 - 2*e**(4*x)*a**2*b**2 + e**(4*x)*b**4 - 2*e**(2*x)*a**4 + 4*e**(2*
x)*a**2*b**2 - 2*e**(2*x)*b**4 + a**4 - 2*a**2*b**2 + b**4))
```

3.67 $\int \frac{\mathrm{csch}^4(x)}{a+b\,\mathrm{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 111

$$\int \frac{\mathrm{csch}^4(x)}{a+b\,\mathrm{sech}(x)}\,dx=-\frac{2a^3b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}-\frac{(3a^2b-a(2a^2+b^2)\cosh(x))\,\mathrm{csch}(x)}{3\left(a^2-b^2\right)^2}+\frac{(b-a\cosh(x))\mathrm{csch}^3(x)}{3\left(a^2-b^2\right)}$$

output
```
-2*a^3*b*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(5/2)/(a+b)^(5/
2)-1/3*(3*a^2*b-a*(2*a^2+b^2)*cosh(x))*csch(x)/(a^2-b^2)^2+(b-a*cosh(x))*c
sch(x)^3/(3*a^2-3*b^2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.41

$$\int \frac{\mathrm{csch}^4(x)}{a+b\,\mathrm{sech}(x)}\,dx = \frac{(b+a\cosh(x))\mathrm{sech}(x)\left(\frac{48a^3b\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{2(4a+b)\coth\left(\frac{x}{2}\right)}{(a+b)^2}+\frac{8\mathrm{csch}^3(x)\sinh^4\left(\frac{x}{2}\right)}{a-b}-\frac{\mathrm{csch}^4\left(\frac{x}{2}\right)\sinh(x)}{2(a+b)}+\right.}{24(a+b\,\mathrm{sech}(x))}$$

input
```
Integrate[Csch[x]^4/(a + b*Sech[x]),x]
```

output
```
((b + a*Cosh[x])*Sech[x]*((48*a^3*b*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 -
 b^2]])/(a^2 - b^2)^(5/2) + (2*(4*a + b)*Coth[x/2])/(a + b)^2 + (8*Csch[x]
^3*Sinh[x/2]^4)/(a - b) - (Csch[x/2]^4*Sinh[x])/(2*(a + b)) + (8*a*Tanh[x/
2])/(a - b)^2 - (2*b*Tanh[x/2])/(a - b)^2))/(24*(a + b*Sech[x]))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.22, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.154$, Rules used = {3042, 4360, 25, 25, 3042, 25, 3345, 25, 3042, 25, 3345, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^4(x)}{a+b\text{sech}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos\left(-\frac{\pi}{2}+ix\right)^4\left(a-b\csc\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{4360}$$

$$\int -\frac{\coth(x)\text{csch}^3(x)}{-a\cosh(x)-b}dx$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{\coth(x)\text{csch}^3(x)}{b+a\cosh(x)}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\coth(x)\text{csch}^3(x)}{a\cosh(x)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin\left(-\frac{\pi}{2}+ix\right)}{\cos\left(-\frac{\pi}{2}+ix\right)^4\left(b-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin\left(ix-\frac{\pi}{2}\right)}{\cos\left(ix-\frac{\pi}{2}\right)^4\left(b-a\sin\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3345}$$

$$\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}-\frac{\int -\frac{\left(ab-2a^2\cosh(x)\right)\operatorname{csch}^2(x)}{b+a\cosh(x)}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(ab-2a^2\cosh(x)\right)\operatorname{csch}^2(x)}{b+a\cosh(x)}dx}{3\left(a^2-b^2\right)}+\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}+\frac{\int -\frac{2\sin\left(ix-\frac{\pi}{2}\right)a^2+ba}{\cos\left(ix-\frac{\pi}{2}\right)^2\left(b-a\sin\left(ix-\frac{\pi}{2}\right)\right)}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}-\frac{\int \frac{2\sin\left(ix-\frac{\pi}{2}\right)a^2+ba}{\cos\left(ix-\frac{\pi}{2}\right)^2\left(b-a\sin\left(ix-\frac{\pi}{2}\right)\right)}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3345}$$

$$\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}-\frac{\frac{\int \frac{3a^3b}{b+a\cosh(x)}dx}{a^2-b^2}+\frac{\operatorname{csch}(x)\left(3a^2b-a\left(2a^2+b^2\right)\cosh(x)\right)}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}-\frac{\frac{3a^3b\int \frac{1}{b+a\cosh(x)}dx}{a^2-b^2}+\frac{\operatorname{csch}(x)\left(3a^2b-a\left(2a^2+b^2\right)\cosh(x)\right)}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}-\frac{\frac{\operatorname{csch}(x)\left(3a^2b-a\left(2a^2+b^2\right)\cosh(x)\right)}{a^2-b^2}+\frac{3a^3b\int \frac{1}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}-\frac{\frac{6a^3b\int \frac{1}{(a-b)\tanh^2\left(\frac{x}{2}\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2}+\frac{\operatorname{csch}(x)\left(3a^2b-a\left(2a^2+b^2\right)\cosh(x)\right)}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3(a^2-b^2)}-\frac{\frac{\operatorname{csch}(x)(3a^2b-a(2a^2+b^2)\cosh(x))}{a^2-b^2}+\frac{6a^3b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}}{3(a^2-b^2)}$$

input
```
Int[Csch[x]^4/(a + b*Sech[x]),x]
```

output
```
((b - a*Cosh[x])*Csch[x]^3)/(3*(a^2 - b^2)) - ((6*a^3*b*ArcTan[(Sqrt[a - b
]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)) + ((3*a^2
*b - a*(2*a^2 + b^2)*Cosh[x])*Csch[x])/(a^2 - b^2))/(3*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.14

method	result
default	$-\frac{2a^3 b \arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)^2(a+b)^2\sqrt{(a-b)(a+b)}}-\frac{\frac{a\tanh\left(\frac{x}{2}\right)^3}{3}-\frac{b\tanh\left(\frac{x}{2}\right)^3}{3}-3a\tanh\left(\frac{x}{2}\right)+\tanh\left(\frac{x}{2}\right)b}{8(a-b)^2}-\frac{1}{24(a+b)\tanh\left(\frac{x}{2}\right)^3}-\frac{-3a-b}{8(a+b)^2\tanh\left(\frac{x}{2}\right)}$
risch	$-\frac{2\left(3a^2 b\,e^{5x}-3a b^2 e^{4x}-10a^2 b\,e^{3x}+4b^3 e^{3x}+6a^3 e^{2x}+3b\,e^x a^2-2a^3-a b^2\right)}{3\left(e^{2x}-1\right)^3\left(a^2-b^2\right)^2}-\frac{b a^3\ln\left(e^x+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,(a+b)^2(a-b)^2}+\frac{b a^3\ln\left(e^x+\frac{b\sqrt{}}{}\right.}{\sqrt{-a^2+b^2}\,(}$

input
```
int(csch(x)^4/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/(a-b)^2/(a+b)^2*a^3*b/((a-b)*(a+b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a-
b)*(a+b))^(1/2))-1/8/(a-b)^2*(1/3*a*tanh(1/2*x)^3-1/3*b*tanh(1/2*x)^3-3*a*
tanh(1/2*x)+tanh(1/2*x)*b)-1/24/(a+b)/tanh(1/2*x)^3-1/8/(a+b)^2*(-3*a-b)/t
anh(1/2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1129 vs. $2(98) = 196$.

Time = 0.12 (sec) , antiderivative size = 2340, normalized size of antiderivative = 21.08

$$\int \frac{\operatorname{csch}^4(x)}{a + b\operatorname{sech}(x)}\, dx = \text{Too large to display}$$

input

```
integrate(csch(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

output

```
[-1/3*(6*(a^4*b - a^2*b^3)*cosh(x)^5 + 6*(a^4*b - a^2*b^3)*sinh(x)^5 - 4*a
^5 + 2*a^3*b^2 + 2*a*b^4 - 6*(a^3*b^2 - a*b^4)*cosh(x)^4 - 6*(a^3*b^2 - a*
b^4 - 5*(a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4 - 4*(5*a^4*b - 7*a^2*b^3 + 2*
b^5)*cosh(x)^3 - 4*(5*a^4*b - 7*a^2*b^3 + 2*b^5 - 15*(a^4*b - a^2*b^3)*cos
h(x)^2 + 6*(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 + 12*(a^5 - a^3*b^2)*cosh(
x)^2 + 12*(a^5 - a^3*b^2 + 5*(a^4*b - a^2*b^3)*cosh(x)^3 - 3*(a^3*b^2 - a*
b^4)*cosh(x)^2 - (5*a^4*b - 7*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x)^2 + 3*(a^3
*b*cosh(x)^6 + 6*a^3*b*cosh(x)*sinh(x)^5 + a^3*b*sinh(x)^6 - 3*a^3*b*cosh(
x)^4 + 3*a^3*b*cosh(x)^2 + 3*(5*a^3*b*cosh(x)^2 - a^3*b)*sinh(x)^4 - a^3*b
 + 4*(5*a^3*b*cosh(x)^3 - 3*a^3*b*cosh(x))*sinh(x)^3 + 3*(5*a^3*b*cosh(x)^
4 - 6*a^3*b*cosh(x)^2 + a^3*b)*sinh(x)^2 + 6*(a^3*b*cosh(x)^5 - 2*a^3*b*co
sh(x)^3 + a^3*b*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^
2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x)
+ 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)
^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) + 6*(a^4*b - a^2*b^3)*c
osh(x) + 6*(a^4*b - a^2*b^3 + 5*(a^4*b - a^2*b^3)*cosh(x)^4 - 4*(a^3*b^2 -
 a*b^4)*cosh(x)^3 - 2*(5*a^4*b - 7*a^2*b^3 + 2*b^5)*cosh(x)^2 + 4*(a^5 - a
^3*b^2)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 +
 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6)*sinh(x)^6 - a^6 + 3*a^4*b^2 - 3*a^2*b^4 + b^6 - 3*...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{sech}(x)}\,dx=\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{sech}(x)}\,dx$$

input
```
integrate(csch(x)**4/(a+b*sech(x)),x)
```

output
```
Integral(csch(x)**4/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{sech}(x)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(csch(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.34

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{sech}(x)}\,dx=-\frac{2\,a^3b\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}} - \frac{2\left(3\,a^2be^{(5\,x)}-3\,ab^2e^{(4\,x)}-10\,a^2be^{(3\,x)}+4\,b^3e^{(3\,x)}+6\,a^3e^{(2\,x)}+3\,a^2be^x-2\,a^3-ab^2\right)}{3\,(a^4-2\,a^2b^2+b^4)(e^{(2\,x)}-1)^3}$$

input
```
integrate(csch(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

output
```
-2*a^3*b*arctan((a*e^x + b)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*sqrt
(a^2 - b^2)) - 2/3*(3*a^2*b*e^(5*x) - 3*a*b^2*e^(4*x) - 10*a^2*b*e^(3*x) +
 4*b^3*e^(3*x) + 6*a^3*e^(2*x) + 3*a^2*b*e^x - 2*a^3 - a*b^2)/((a^4 - 2*a^
2*b^2 + b^4)*(e^(2*x) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.66

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{\frac{4\,(a\,b^2-a^3)}{(a^2-b^2)^2}+\frac{8\,e^x\,(a^2\,b-b^3)}{3\,(a^2-b^2)^2}}{e^{4\,x}-2\,e^{2\,x}+1}-\frac{\frac{8\,a}{3\,(a^2-b^2)}-\frac{8\,b\,e^x}{3\,(a^2-b^2)}}{3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1}+\frac{\frac{2\,a\,b^2}{(a^2-b^2)^2}-\frac{2\,a^2\,b\,e^x}{(a^2-b^2)^2}}{e^{2\,x}-1}+\frac{a^3\,b\,\ln\left(\frac{2\,a^2\,b\,e^x}{(a^2-b^2)^2}-\frac{2\,a^2\,b\,(a+b\,e^x)}{(a+b)^{5/2}\,(b-a)^{5/2}}\right)}{(a+b)^{5/2}\,(b-a)^{5/2}}-\frac{a^3\,b\,\ln\left(\frac{2\,a^2\,b\,e^x}{(a^2-b^2)^2}+\frac{2\,a^2\,b\,(a+b\,e^x)}{(a+b)^{5/2}\,(b-a)^{5/2}}\right)}{(a+b)^{5/2}\,(b-a)^{5/2}}$$

input
```
int(1/(sinh(x)^4*(a + b/cosh(x))),x)
```

output
```
((4*(a*b^2 - a^3))/(a^2 - b^2)^2 + (8*exp(x)*(a^2*b - b^3))/(3*(a^2 - b^2)
^2))/(exp(4*x) - 2*exp(2*x) + 1) - ((8*a)/(3*(a^2 - b^2)) - (8*b*exp(x))/(
3*(a^2 - b^2)))/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) + ((2*a*b^2)/(a^2
 - b^2)^2 - (2*a^2*b*exp(x))/(a^2 - b^2)^2)/(exp(2*x) - 1) + (a^3*b*log((2
*a^2*b*exp(x))/(a^2 - b^2)^2 - (2*a^2*b*(a + b*exp(x)))/((a + b)^(5/2)*(b
- a)^(5/2))))/((a + b)^(5/2)*(b - a)^(5/2)) - (a^3*b*log((2*a^2*b*exp(x))/
(a^2 - b^2)^2 + (2*a^2*b*(a + b*exp(x)))/((a + b)^(5/2)*(b - a)^(5/2))))/(
(a + b)^(5/2)*(b - a)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 481, normalized size of antiderivative = 4.33

$$\int \frac{\operatorname{csch}^4(x)}{a + b\operatorname{sech}(x)}\,dx$$
$$= \frac{-6e^{6x}\sqrt{a^2-b^2}\,atan\left(\frac{e^x a+b}{\sqrt{a^2-b^2}}\right)a^3b + 18e^{4x}\sqrt{a^2-b^2}\,atan\left(\frac{e^x a+b}{\sqrt{a^2-b^2}}\right)a^3b - 18e^{2x}\sqrt{a^2-b^2}\,atan\left(\frac{e^x a+b}{\sqrt{a^2-b^2}}\right)a^3b}{3e^{6x}a^6 - 9e^{6x}a^4b^2 + 9e^{6x}a^2b^4 - 3e^{6x}b^6 -}$$

input
```
int(csch(x)^4/(a+b*sech(x)),x)
```

output
```
(2*( - 3*e**(6*x)*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2))*a
**3*b + 9*e**(4*x)*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2))*
a**3*b - 9*e**(2*x)*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2))
*a**3*b + 3*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2))*a**3*b
+ e**(6*x)*a**3*b**2 - e**(6*x)*a*b**4 - 3*e**(5*x)*a**4*b + 3*e**(5*x)*a*
*2*b**3 + 10*e**(3*x)*a**4*b - 14*e**(3*x)*a**2*b**3 + 4*e**(3*x)*b**5 - 6
*e**(2*x)*a**5 + 9*e**(2*x)*a**3*b**2 - 3*e**(2*x)*a*b**4 - 3*e**x*a**4*b
+ 3*e**x*a**2*b**3 + 2*a**5 - 2*a**3*b**2))/(3*(e**(6*x)*a**6 - 3*e**(6*x)
*a**4*b**2 + 3*e**(6*x)*a**2*b**4 - e**(6*x)*b**6 - 3*e**(4*x)*a**6 + 9*e*
*(4*x)*a**4*b**2 - 9*e**(4*x)*a**2*b**4 + 3*e**(4*x)*b**6 + 3*e**(2*x)*a**
6 - 9*e**(2*x)*a**4*b**2 + 9*e**(2*x)*a**2*b**4 - 3*e**(2*x)*b**6 - a**6 +
 3*a**4*b**2 - 3*a**2*b**4 + b**6))
```

3.68 $\int \frac{\cosh^4(x)}{a+a\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{\cosh^4(x)}{a+a\text{sech}(x)}\,dx = \frac{15x}{8a}-\frac{4\sinh(x)}{a}+\frac{15\cosh(x)\sinh(x)}{8a}+\frac{5\cosh^3(x)\sinh(x)}{4a}-\frac{\cosh^3(x)\sinh(x)}{a+a\text{sech}(x)}-\frac{4\sinh^3(x)}{3a}$$

output
```
15/8*x/a-4*sinh(x)/a+15/8*cosh(x)*sinh(x)/a+5/4*cosh(x)^3*sinh(x)/a-cosh(x
)^3*sinh(x)/(a+a*sech(x))-4/3*sinh(x)^3/a
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \frac{\cosh^4(x)}{a+a\text{sech}(x)}\,dx = \frac{\text{sech}\left(\frac{x}{2}\right)\left(360x\cosh\left(\frac{x}{2}\right)-360\sinh\left(\frac{x}{2}\right)-120\sinh\left(\frac{3x}{2}\right)+40\sinh\left(\frac{5x}{2}\right)-5\sinh\left(\frac{7x}{2}\right)+3\sinh\left(\frac{9x}{2}\right)\right)}{192a}$$

input
```
Integrate[Cosh[x]^4/(a + a*Sech[x]),x]
```

output
```
(Sech[x/2]*(360*x*Cosh[x/2] - 360*Sinh[x/2] - 120*Sinh[(3*x)/2] + 40*Sinh[
(5*x)/2] - 5*Sinh[(7*x)/2] + 3*Sinh[(9*x)/2]))/(192*a)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$, Rules used = {3042, 4306, 25, 3042, 4274, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(x)}{a\text{sech}(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(\frac{\pi}{2}+ix\right)^4\left(a+a\csc\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{4306}$$

$$-\frac{\int -\cosh^4(x)(5a-4a\text{sech}(x))dx}{a^2}-\frac{\sinh(x)\cosh^3(x)}{a\text{sech}(x)+a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \cosh^4(x)(5a-4a\text{sech}(x))dx}{a^2}-\frac{\sinh(x)\cosh^3(x)}{a\text{sech}(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)\cosh^3(x)}{a\text{sech}(x)+a}+\frac{\int \frac{5a-4a\csc\left(ix+\frac{\pi}{2}\right)}{\csc\left(ix+\frac{\pi}{2}\right)^4}dx}{a^2}$$

$$\Big\downarrow \text{4274}$$

$$\frac{5a\int \cosh^4(x)dx-4a\int \cosh^3(x)dx}{a^2}-\frac{\sinh(x)\cosh^3(x)}{a\text{sech}(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)\cosh^3(x)}{a\text{sech}(x)+a}+\frac{5a\int \sin\left(ix+\frac{\pi}{2}\right)^4dx-4a\int \sin\left(ix+\frac{\pi}{2}\right)^3dx}{a^2}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{\sinh(x)\cosh^3(x)}{a\text{sech}(x)+a}+\frac{5a\int \sin\left(ix+\frac{\pi}{2}\right)^4dx-4ia\int \left(\sinh^2(x)+1\right)d(-i\sinh(x))}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sinh(x)\cosh^3(x)}{a\text{sech}(x)+a}+\frac{5a\int \sin\left(ix+\frac{\pi}{2}\right)^4dx-4ia\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{\sinh(x)\cosh^3(x)}{a\text{sech}(x)+a}+\frac{5a\left(\frac{3}{4}\int \cosh^2(x)dx+\frac{1}{4}\sinh(x)\cosh^3(x)\right)-4ia\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)\cosh^3(x)}{a\text{sech}(x)+a}+\frac{5a\left(\frac{1}{4}\sinh(x)\cosh^3(x)+\frac{3}{4}\int \sin\left(ix+\frac{\pi}{2}\right)^2dx\right)-4ia\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{\sinh(x)\cosh^3(x)}{a\text{sech}(x)+a}+\frac{5a\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)+\frac{1}{4}\sinh(x)\cosh^3(x)\right)-4ia\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\sinh(x)\cosh^3(x)}{a\text{sech}(x)+a}+\frac{5a\left(\frac{1}{4}\sinh(x)\cosh^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)\right)-4ia\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2}$$

input

```
Int[Cosh[x]^4/(a + a*Sech[x]),x]
```

output

```
-((Cosh[x]^3*Sinh[x])/(a + a*Sech[x])) + ((-4*I)*a*((-I)*Sinh[x] - (I/3)*S
inh[x]^3) + 5*a*((Cosh[x]^3*Sinh[x])/4 + (3*(x/2 + (Cosh[x]*Sinh[x])/2))/4
))/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

method	result
parallelrisch	$\frac{180x+\tanh\left(\frac{x}{2}\right)\left(-221+3\cosh(4x)-2\cosh(3x)+38\cosh(2x)-82\cosh(x)\right)}{96a}$
risch	$\frac{3\,\mathrm{e}^{5x}-5\,\mathrm{e}^{4x}+40\,\mathrm{e}^{3x}-120\,\mathrm{e}^{2x}+552+120\,\mathrm{e}^{-x}-40\,\mathrm{e}^{-2x}+5\,\mathrm{e}^{-3x}+360x\,\mathrm{e}^{x}-168\,\mathrm{e}^{x}-3\,\mathrm{e}^{-4x}+360x}{192(1+\mathrm{e}^{x})a}$
default	$\frac{-\tanh\left(\frac{x}{2}\right)-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)+1\right)^{4}}+\frac{5}{6\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{15}{8\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{25}{8\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{15\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}+\frac{\ }{6(\mathrm{t}}}{a}$

input
```
int(cosh(x)^4/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
1/96*(180*x+tanh(1/2*x)*(-221+3*cosh(4*x)-2*cosh(3*x)+38*cosh(2*x)-82*cosh
(x)))/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(59) = 118.

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.07

$$\int \frac{\cosh^4(x)}{a+a\operatorname{sech}(x)}\,dx$$
$$= \frac{3\cosh(x)^5+(15\cosh(x)-8)\sinh(x)^4+3\sinh(x)^5-8\cosh(x)^4+\left(30\cosh(x)^2-8\cosh(x)+35\right)}{}$$

input
```
integrate(cosh(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
1/192*(3*cosh(x)^5 + (15*cosh(x) - 8)*sinh(x)^4 + 3*sinh(x)^5 - 8*cosh(x)^
4 + (30*cosh(x)^2 - 8*cosh(x) + 35)*sinh(x)^3 + 45*cosh(x)^3 + (30*cosh(x)
^3 - 48*cosh(x)^2 + 135*cosh(x) - 160)*sinh(x)^2 + 24*(15*x - 2)*cosh(x) -
 160*cosh(x)^2 + (15*cosh(x)^4 - 8*cosh(x)^3 + 105*cosh(x)^2 + 360*x - 160
*cosh(x) - 288)*sinh(x) + 360*x + 552)/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\cosh^4(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\cosh^4(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

input
```
integrate(cosh(x)**4/(a+a*sech(x)),x)
```

output
```
Integral(cosh(x)**4/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int \frac{\cosh^4(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{15\,x}{8\,a} + \frac{168\,e^{(-x)} - 48\,e^{(-2\,x)} + 8\,e^{(-3\,x)} - 3\,e^{(-4\,x)}}{192\,a} - \frac{5\,e^{(-x)} - 40\,e^{(-2\,x)} + 120\,e^{(-3\,x)} + 552\,e^{(-4\,x)} - 3}{192\,(ae^{(-4\,x)} + ae^{(-5\,x)})}$$

input
```
integrate(cosh(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
15/8*x/a + 1/192*(168*e^(-x) - 48*e^(-2*x) + 8*e^(-3*x) - 3*e^(-4*x))/a -
1/192*(5*e^(-x) - 40*e^(-2*x) + 120*e^(-3*x) + 552*e^(-4*x) - 3)/(a*e^(-4*
x) + a*e^(-5*x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.28

$$\int \frac{\cosh^4(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{15\,x}{8\,a} + \frac{\left(552\,e^{(4\,x)} + 120\,e^{(3\,x)} - 40\,e^{(2\,x)} + 5\,e^{x} - 3\right)e^{(-4\,x)}}{192\,a(e^{x} + 1)} + \frac{3\,a^3e^{(4\,x)} - 8\,a^3e^{(3\,x)} + 48\,a^3e^{(2\,x)} - 168\,a^3e^{x}}{192\,a^4}$$

input
```
integrate(cosh(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

output
```
15/8*x/a + 1/192*(552*e^(4*x) + 120*e^(3*x) - 40*e^(2*x) + 5*e^x - 3)*e^(-
4*x)/(a*(e^x + 1)) + 1/192*(3*a^3*e^(4*x) - 8*a^3*e^(3*x) + 48*a^3*e^(2*x)
 - 168*a^3*e^x)/a^4
```

Mupad [B] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \frac{\cosh^4(x)}{a+a\text{sech}(x)}\,dx = \frac{7\,\mathrm{e}^{-x}}{8\,a}-\frac{\mathrm{e}^{-2\,x}}{4\,a}+\frac{\mathrm{e}^{2\,x}}{4\,a}+\frac{\mathrm{e}^{-3\,x}}{24\,a}-\frac{\mathrm{e}^{3\,x}}{24\,a}-\frac{\mathrm{e}^{-4\,x}}{64\,a}+\frac{\mathrm{e}^{4\,x}}{64\,a}+\frac{15\,x}{8\,a}+\frac{2}{a\,\left(\mathrm{e}^x+1\right)}-\frac{7\,\mathrm{e}^x}{8\,a}$$

input
```
int(cosh(x)^4/(a + a/cosh(x)),x)
```

output
```
(7*exp(-x))/(8*a) - exp(-2*x)/(4*a) + exp(2*x)/(4*a) + exp(-3*x)/(24*a) -
exp(3*x)/(24*a) - exp(-4*x)/(64*a) + exp(4*x)/(64*a) + (15*x)/(8*a) + 2/(a
*(exp(x) + 1)) - (7*exp(x))/(8*a)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.36

$$\int \frac{\cosh^4(x)}{a+a\text{sech}(x)}\,dx = \frac{3e^{9x}-5e^{8x}+40e^{7x}-120e^{6x}+360e^{5x}x-720e^{5x}+360e^{4x}x+120e^{3x}-40e^{2x}+5e^{x}-3}{192e^{4x}a\left(e^{x}+1\right)}$$

input
```
int(cosh(x)^4/(a+a*sech(x)),x)
```

output
```
(3*e**(9*x) - 5*e**(8*x) + 40*e**(7*x) - 120*e**(6*x) + 360*e**(5*x)*x - 7
20*e**(5*x) + 360*e**(4*x)*x + 120*e**(3*x) - 40*e**(2*x) + 5*e**x - 3)/(1
92*e**(4*x)*a*(e**x + 1))
```

3.69 $\int \frac{\cosh^3(x)}{a+a\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{\cosh^3(x)}{a+a\text{sech}(x)}\,dx=-\frac{3x}{2a}+\frac{4\sinh(x)}{a}-\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh^2(x)\sinh(x)}{a+a\text{sech}(x)}+\frac{4\sinh^3(x)}{3a}$$

output
```
-3/2*x/a+4*sinh(x)/a-3/2*cosh(x)*sinh(x)/a-cosh(x)^2*sinh(x)/(a+a*sech(x))
+4/3*sinh(x)^3/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{\cosh^3(x)}{a+a\text{sech}(x)}\,dx = \frac{\text{sech}\left(\frac{x}{2}\right)\left(-36x\cosh\left(\frac{x}{2}\right)+45\sinh\left(\frac{x}{2}\right)+18\sinh\left(\frac{3x}{2}\right)-2\sinh\left(\frac{5x}{2}\right)+\sinh\left(\frac{7x}{2}\right)\right)}{24a}$$

input
```
Integrate[Cosh[x]^3/(a + a*Sech[x]),x]
```

output
```
(Sech[x/2]*(-36*x*Cosh[x/2] + 45*Sinh[x/2] + 18*Sinh[(3*x)/2] - 2*Sinh[(5*
x)/2] + Sinh[(7*x)/2]))/(24*a)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {3042, 4306, 25, 3042, 4274, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{a\text{sech}(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(\frac{\pi}{2}+ix\right)^3\left(a+a\csc\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{4306}$$

$$-\frac{\int -\cosh^3(x)(4a-3a\text{sech}(x))dx}{a^2}-\frac{\sinh(x)\cosh^2(x)}{a\text{sech}(x)+a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \cosh^3(x)(4a-3a\text{sech}(x))dx}{a^2}-\frac{\sinh(x)\cosh^2(x)}{a\text{sech}(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)\cosh^2(x)}{a\text{sech}(x)+a}+\frac{\int \frac{4a-3a\csc\left(ix+\frac{\pi}{2}\right)}{\csc\left(ix+\frac{\pi}{2}\right)^3}dx}{a^2}$$

$$\Big\downarrow \text{4274}$$

$$\frac{4a\int \cosh^3(x)dx-3a\int \cosh^2(x)dx}{a^2}-\frac{\sinh(x)\cosh^2(x)}{a\text{sech}(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)\cosh^2(x)}{a\text{sech}(x)+a}+\frac{4a\int \sin\left(ix+\frac{\pi}{2}\right)^3dx-3a\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a^2}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{\sinh(x)\cosh^2(x)}{a\text{sech}(x)+a}+\frac{4ia\int \left(\sinh^2(x)+1\right)d(-i\sinh(x))-3a\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sinh(x)\cosh^2(x)}{a\text{sech}(x)+a}+\frac{4ia\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)-3a\int\sin\left(ix+\frac{\pi}{2}\right)^2dx}{a^2}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{\sinh(x)\cosh^2(x)}{a\text{sech}(x)+a}+\frac{-3a\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)+4ia\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\sinh(x)\cosh^2(x)}{a\text{sech}(x)+a}+\frac{-3a\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)+4ia\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2}$$

input
```
Int[Cosh[x]^3/(a + a*Sech[x]),x]
```

output
```
-((Cosh[x]^2*Sinh[x])/(a + a*Sech[x])) + (-3*a*(x/2 + (Cosh[x]*Sinh[x])/2)
 + (4*I)*a*((-I)*Sinh[x] - (I/3)*Sinh[x]^3))/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.57

method	result
parallelrisch	$\frac{-18x+\tanh\left(\frac{x}{2}\right)\left(31+\cosh(3x)-\cosh(2x)+17\cosh(x)\right)}{12a}$
risch	$\frac{e^{4x}-2\,e^{3x}+18\,e^{2x}-69-18\,e^{-x}+2\,e^{-2x}-36x\,e^{x}+21\,e^{x}-e^{-3x}-36x}{24(1+e^{x})a}$
default	$\frac{\tanh\left(\frac{x}{2}\right)-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{5}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{5}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)}}{a}$

input
```
int(cosh(x)^3/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
1/12*(-18*x+tanh(1/2*x)*(31+cosh(3*x)-cosh(2*x)+17*cosh(x)))/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(48) = 96$.

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.85

$$\int \frac{\cosh^3(x)}{a + a\text{sech}(x)}\,dx$$
$$= \frac{\cosh(x)^4 + (4\cosh(x) - 1)\sinh(x)^3 + \sinh(x)^4 - 3\cosh(x)^3 + (6\cosh(x)^2 - 9\cosh(x) + 20)\sinh(x)^2 \ldots}{24\,(a\cos \ldots}$$

input
```
integrate(cosh(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
1/24*(cosh(x)^4 + (4*cosh(x) - 1)*sinh(x)^3 + sinh(x)^4 - 3*cosh(x)^3 + (6
*cosh(x)^2 - 9*cosh(x) + 20)*sinh(x)^2 - 3*(12*x - 1)*cosh(x) + 20*cosh(x)
^2 + (4*cosh(x)^3 - 3*cosh(x)^2 - 36*x + 32*cosh(x) + 39)*sinh(x) - 36*x -
 69)/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\cosh^3(x)}{a + a\text{sech}(x)}\,dx = \frac{\int \frac{\cosh^3(x)}{\text{sech}(x)+1}\,dx}{a}$$

input
```
integrate(cosh(x)**3/(a+a*sech(x)),x)
```

output
```
Integral(cosh(x)**3/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^3(x)}{a + a\operatorname{sech}(x)}\,dx = -\frac{3\,x}{2\,a} - \frac{21\,e^{(-x)} - 3\,e^{(-2\,x)} + e^{(-3\,x)}}{24\,a} - \frac{2\,e^{(-x)} - 18\,e^{(-2\,x)} - 69\,e^{(-3\,x)} - 1}{24\left(ae^{(-3\,x)} + ae^{(-4\,x)}\right)}$$

input
```
integrate(cosh(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
-3/2*x/a - 1/24*(21*e^(-x) - 3*e^(-2*x) + e^(-3*x))/a - 1/24*(2*e^(-x) - 1
8*e^(-2*x) - 69*e^(-3*x) - 1)/(a*e^(-3*x) + a*e^(-4*x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{\cosh^3(x)}{a + a\operatorname{sech}(x)}\,dx = -\frac{3\,x}{2\,a} - \frac{\left(69\,e^{(3\,x)} + 18\,e^{(2\,x)} - 2\,e^{x} + 1\right)e^{(-3\,x)}}{24\,a(e^{x} + 1)} + \frac{a^2e^{(3\,x)} - 3\,a^2e^{(2\,x)} + 21\,a^2e^{x}}{24\,a^3}$$

input
```
integrate(cosh(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

output
```
-3/2*x/a - 1/24*(69*e^(3*x) + 18*e^(2*x) - 2*e^x + 1)*e^(-3*x)/(a*(e^x + 1
)) + 1/24*(a^2*e^(3*x) - 3*a^2*e^(2*x) + 21*a^2*e^x)/a^3
```

Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{\cosh^3(x)}{a + a\text{sech}(x)}\,dx = \frac{e^{-2x}}{8a} - \frac{7e^{-x}}{8a} - \frac{e^{2x}}{8a} - \frac{e^{-3x}}{24a} + \frac{e^{3x}}{24a} - \frac{3x}{2a} - \frac{2}{a(e^x+1)} + \frac{7e^x}{8a}$$

input
```
int(cosh(x)^3/(a + a/cosh(x)),x)
```

output
```
exp(-2*x)/(8*a) - (7*exp(-x))/(8*a) - exp(2*x)/(8*a) - exp(-3*x)/(24*a) +
exp(3*x)/(24*a) - (3*x)/(2*a) - 2/(a*(exp(x) + 1)) + (7*exp(x))/(8*a)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.39

$$\int \frac{\cosh^3(x)}{a + a\text{sech}(x)}\,dx = \frac{e^{7x} - 2e^{6x} + 18e^{5x} - 36e^{4x}x + 90e^{4x} - 36e^{3x}x - 18e^{2x} + 2e^x - 1}{24e^{3x}a(e^x+1)}$$

input
```
int(cosh(x)^3/(a+a*sech(x)),x)
```

output
```
(e**(7*x) - 2*e**(6*x) + 18*e**(5*x) - 36*e**(4*x)*x + 90*e**(4*x) - 36*e*
*(3*x)*x - 18*e**(2*x) + 2*e**x - 1)/(24*e**(3*x)*a*(e**x + 1))
```

3.70 $\int \frac{\cosh^2(x)}{a+a\mathbf{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \frac{\cosh^2(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{3x}{2a}-\frac{2\sinh(x)}{a}+\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh(x)\sinh(x)}{a+a\operatorname{sech}(x)}$$

output
```
3/2*x/a-2*sinh(x)/a+3/2*cosh(x)*sinh(x)/a-cosh(x)*sinh(x)/(a+a*sech(x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int \frac{\cosh^2(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{\operatorname{sech}\left(\frac{x}{2}\right)\left(12x\cosh\left(\frac{x}{2}\right)-12\sinh\left(\frac{x}{2}\right)-3\sinh\left(\frac{3x}{2}\right)+\sinh\left(\frac{5x}{2}\right)\right)}{8a}$$

input
```
Integrate[Cosh[x]^2/(a + a*Sech[x]),x]
```

output
```
(Sech[x/2]*(12*x*Cosh[x/2] - 12*Sinh[x/2] - 3*Sinh[(3*x)/2] + Sinh[(5*x)/2
]))/(8*a)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {3042, 4306, 25, 3042, 4274, 3042, 3115, 24, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{a\text{sech}(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(\frac{\pi}{2}+ix\right)^2\left(a+a\csc\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{4306}$$

$$-\frac{\int -\cosh^2(x)(3a-2a\text{sech}(x))dx}{a^2}-\frac{\sinh(x)\cosh(x)}{a\text{sech}(x)+a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \cosh^2(x)(3a-2a\text{sech}(x))dx}{a^2}-\frac{\sinh(x)\cosh(x)}{a\text{sech}(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)\cosh(x)}{a\text{sech}(x)+a}+\frac{\int \frac{3a-2a\csc\left(ix+\frac{\pi}{2}\right)}{\csc\left(ix+\frac{\pi}{2}\right)^2}dx}{a^2}$$

$$\Big\downarrow \text{4274}$$

$$\frac{3a\int \cosh^2(x)dx-2a\int \cosh(x)dx}{a^2}-\frac{\sinh(x)\cosh(x)}{a\text{sech}(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)\cosh(x)}{a\text{sech}(x)+a}+\frac{3a\int \sin\left(ix+\frac{\pi}{2}\right)^2dx-2a\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a^2}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{\sinh(x)\cosh(x)}{a\text{sech}(x)+a}+\frac{3a\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-2a\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\sinh(x)\cosh(x)}{a\text{sech}(x)+a}+\frac{3a\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-2a\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a^2}$$

$$\Big\downarrow \text{3117}$$

$$\frac{3a\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-2a\sinh(x)}{a^2}-\frac{\sinh(x)\cosh(x)}{a\text{sech}(x)+a}$$

input
```
Int[Cosh[x]^2/(a + a*Sech[x]),x]
```

output
```
-((Cosh[x]*Sinh[x])/(a + a*Sech[x])) + (-2*a*Sinh[x] + 3*a*(x/2 + (Cosh[x]
*Sinh[x])/2))/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.59

method	result	siz
parallelrisch	$\frac{6x+\sinh(2x)-4\sinh(x)-4\tanh\left(\frac{x}{2}\right)}{4a}$	24
risch	$\frac{e^{3x}-3\,e^{2x}+20+3\,e^{-x}+12x\,e^{x}-4\,e^{x}-e^{-2x}+12x}{8(1+e^{x})a}$	48
default	$\frac{-\tanh\left(\frac{x}{2}\right)+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}}{a}$	70

input
```
int(cosh(x)^2/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*(6*x+sinh(2*x)-4*sinh(x)-4*tanh(1/2*x))/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.71

$$\int \frac{\cosh^2(x)}{a+a\text{sech}(x)}\,dx$$
$$= \frac{\cosh(x)^3+(3\cosh(x)-4)\sinh(x)^2+\sinh(x)^3+(12\,x-1)\cosh(x)-4\cosh(x)^2+\left(3\cosh(x)^2+1\right.}{8\,(a\cosh(x)+a\sinh(x)+a)}$$

input
```
integrate(cosh(x)^2/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
1/8*(cosh(x)^3 + (3*cosh(x) - 4)*sinh(x)^2 + sinh(x)^3 + (12*x - 1)*cosh(x
) - 4*cosh(x)^2 + (3*cosh(x)^2 + 12*x - 4*cosh(x) - 7)*sinh(x) + 12*x + 20
)/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\cosh^2(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\cosh^2(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

input
```
integrate(cosh(x)**2/(a+a*sech(x)),x)
```

output
```
Integral(cosh(x)**2/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.37

$$\int \frac{\cosh^2(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{3\,x}{2\,a} + \frac{4\,e^{(-x)} - e^{(-2\,x)}}{8\,a} - \frac{3\,e^{(-x)} + 20\,e^{(-2\,x)} - 1}{8\,\left(ae^{(-2\,x)} + ae^{(-3\,x)}\right)}$$

input
```
integrate(cosh(x)^2/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
3/2*x/a + 1/8*(4*e^(-x) - e^(-2*x))/a - 1/8*(3*e^(-x) + 20*e^(-2*x) - 1)/(
a*e^(-2*x) + a*e^(-3*x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \frac{\cosh^2(x)}{a+a\text{sech}(x)}\,dx = \frac{3\,x}{2\,a} + \frac{\left(20\,e^{(2\,x)}+3\,e^{x}-1\right)e^{(-2\,x)}}{8\,a(e^{x}+1)} + \frac{ae^{(2\,x)}-4\,ae^{x}}{8\,a^2}$$

input
```
integrate(cosh(x)^2/(a+a*sech(x)),x, algorithm="giac")
```

output
```
3/2*x/a + 1/8*(20*e^(2*x) + 3*e^x - 1)*e^(-2*x)/(a*(e^x + 1)) + 1/8*(a*e^(
2*x) - 4*a*e^x)/a^2
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27

$$\int \frac{\cosh^2(x)}{a+a\text{sech}(x)}\,dx = \frac{e^{-x}}{2\,a} - \frac{e^{-2\,x}}{8\,a} + \frac{e^{2\,x}}{8\,a} + \frac{3\,x}{2\,a} + \frac{2}{a\,\left(e^{x}+1\right)} - \frac{e^{x}}{2\,a}$$

input
```
int(cosh(x)^2/(a + a/cosh(x)),x)
```

output
```
exp(-x)/(2*a) - exp(-2*x)/(8*a) + exp(2*x)/(8*a) + (3*x)/(2*a) + 2/(a*(exp
(x) + 1)) - exp(x)/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.49

$$\int \frac{\cosh^2(x)}{a+a\text{sech}(x)}\,dx = \frac{e^{5x}-3e^{4x}+12e^{3x}x-24e^{3x}+12e^{2x}x+3e^{x}-1}{8e^{2x}a\left(e^{x}+1\right)}$$

input
```
int(cosh(x)^2/(a+a*sech(x)),x)
```

output
```
(e**(5*x) - 3*e**(4*x) + 12*e**(3*x)*x - 24*e**(3*x) + 12*e**(2*x)*x + 3*e
**x - 1)/(8*e**(2*x)*a*(e**x + 1))
```

3.71 $\int \frac{\cosh(x)}{a+a\,\mathrm{sech}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 26

$$\int \frac{\cosh(x)}{a+a\,\mathrm{sech}(x)}\,dx=-\frac{x}{a}+\frac{2\sinh(x)}{a}-\frac{\sinh(x)}{a+a\,\mathrm{sech}(x)}$$

output
```
-x/a+2*sinh(x)/a-sinh(x)/(a+a*sech(x))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \frac{\cosh(x)}{a+a\,\mathrm{sech}(x)}\,dx=\frac{-2x+\mathrm{sech}\left(\frac{x}{2}\right)\sinh\left(\frac{3x}{2}\right)+3\tanh\left(\frac{x}{2}\right)}{2a}$$

input
```
Integrate[Cosh[x]/(a + a*Sech[x]),x]
```

output
```
(-2*x + Sech[x/2]*Sinh[(3*x)/2] + 3*Tanh[x/2])/(2*a)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {3042, 4306, 25, 3042, 4274, 24, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{a\text{sech}(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(\frac{\pi}{2}+ix\right)\left(a+a\csc\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{4306}$$

$$-\frac{\int -\cosh(x)(2a-a\text{sech}(x))dx}{a^2}-\frac{\sinh(x)}{a\text{sech}(x)+a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \cosh(x)(2a-a\text{sech}(x))dx}{a^2}-\frac{\sinh(x)}{a\text{sech}(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)}{a\text{sech}(x)+a}+\frac{\int \frac{2a-a\csc\left(ix+\frac{\pi}{2}\right)}{\csc\left(ix+\frac{\pi}{2}\right)}dx}{a^2}$$

$$\Big\downarrow \text{4274}$$

$$\frac{2a\int \cosh(x)dx-a\int 1dx}{a^2}-\frac{\sinh(x)}{a\text{sech}(x)+a}$$

$$\Big\downarrow \text{24}$$

$$\frac{2a\int \cosh(x)dx-ax}{a^2}-\frac{\sinh(x)}{a\text{sech}(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)}{a\text{sech}(x)+a}+\frac{-ax+2a\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a^2}$$

$$\Big\downarrow \text{3117}$$

$$\frac{2a\sinh(x) - ax}{a^2} - \frac{\sinh(x)}{a\text{sech}(x) + a}$$

input
```
Int[Cosh[x]/(a + a*Sech[x]),x]
```

output
```
-(Sinh[x]/(a + a*Sech[x])) + (-(a*x) + 2*a*Sinh[x])/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.58

method	result	size
parallelrisch	$\frac{\sinh(x)-x+\tanh\left(\frac{x}{2}\right)}{a}$	15
risch	$-\frac{x}{a}+\frac{e^{x}}{2a}-\frac{e^{-x}}{2a}-\frac{2}{(1+e^{x})a}$	35
default	$\frac{\tanh\left(\frac{x}{2}\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)+1}-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$	46

input
```
int(cosh(x)/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
(sinh(x)-x+tanh(1/2*x))/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.81

$$\int \frac{\cosh(x)}{a+a\operatorname{sech}(x)}\,dx$$
$$= -\frac{2\,x\cosh\left(x\right)-\cosh\left(x\right)^{2}+2\left(x-\cosh\left(x\right)-1\right)\sinh\left(x\right)-\sinh\left(x\right)^{2}+2\,x+5}{2\left(a\cosh\left(x\right)+a\sinh\left(x\right)+a\right)}$$

input
```
integrate(cosh(x)/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
-1/2*(2*x*cosh(x) - cosh(x)^2 + 2*(x - cosh(x) - 1)*sinh(x) - sinh(x)^2 +
2*x + 5)/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\cosh(x)}{a + a\text{sech}(x)}\,dx = \frac{\int \frac{\cosh(x)}{\text{sech}(x)+1}\,dx}{a}$$

input
```
integrate(cosh(x)/(a+a*sech(x)),x)
```

output
```
Integral(cosh(x)/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{\cosh(x)}{a + a\text{sech}(x)}\,dx = -\frac{x}{a} + \frac{5\,e^{(-x)} + 1}{2\,(ae^{(-x)} + ae^{(-2\,x)})} - \frac{e^{(-x)}}{2\,a}$$

input
```
integrate(cosh(x)/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
-x/a + 1/2*(5*e^(-x) + 1)/(a*e^(-x) + a*e^(-2*x)) - 1/2*e^(-x)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35

$$\int \frac{\cosh(x)}{a + a\text{sech}(x)}\,dx = -\frac{x}{a} - \frac{(5\,e^{x} + 1)e^{(-x)}}{2\,a(e^{x} + 1)} + \frac{e^{x}}{2\,a}$$

input
```
integrate(cosh(x)/(a+a*sech(x)),x, algorithm="giac")
```

output
```
-x/a - 1/2*(5*e^x + 1)*e^(-x)/(a*(e^x + 1)) + 1/2*e^x/a
```

Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{\cosh(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{e^x}{2\,a} - \frac{x}{a} - \frac{2}{a\,(e^x+1)} - \frac{e^{-x}}{2\,a}$$

input
```
int(cosh(x)/(a + a/cosh(x)),x)
```

output
```
exp(x)/(2*a) - x/a - 2/(a*(exp(x) + 1)) - exp(-x)/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.73

$$\int \frac{\cosh(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{e^{3x}-2e^{2x}x+6e^{2x}-2e^{x}x-1}{2e^{x}a\,(e^{x}+1)}$$

input
```
int(cosh(x)/(a+a*sech(x)),x)
```

output
```
(e**(3*x) - 2*e**(2*x)*x + 6*e**(2*x) - 2*e**x*x - 1)/(2*e**x*a*(e**x + 1)
)
```

3.72 $\int \frac{\operatorname{sech}(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{\operatorname{sech}(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{\tanh(x)}{a+a\operatorname{sech}(x)}$$

output
```
tanh(x)/(a+a*sech(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{sech}(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{\tanh\left(\frac{x}{2}\right)}{a}$$

input
```
Integrate[Sech[x]/(a + a*Sech[x]),x]
```

output
```
Tanh[x/2]/a
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(x)}{a\text{sech}(x) + a} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(\frac{\pi}{2} + ix\right)}{a + a\csc\left(\frac{\pi}{2} + ix\right)}dx$$

$$\Big\downarrow \text{4281}$$

$$\frac{\tanh(x)}{a\text{sech}(x) + a}$$

input
```
Int[Sech[x]/(a + a*Sech[x]),x]
```

output
```
Tanh[x]/(a + a*Sech[x])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)}{a}$	9
parallelrisch	$\frac{\tanh\left(\frac{x}{2}\right)}{a}$	9
risch	$-\frac{2}{(1+e^x)a}$	12

input
```
int(sech(x)/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a*tanh(1/2*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{\text{sech}(x)}{a+a\text{sech}(x)}\,dx = -\frac{2}{a\cosh(x)+a\sinh(x)+a}$$

input
```
integrate(sech(x)/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
-2/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\text{sech}(x)}{a+a\text{sech}(x)}\,dx = \frac{\int \frac{\text{sech}(x)}{\text{sech}(x)+1}\,dx}{a}$$

input
```
integrate(sech(x)/(a+a*sech(x)),x)
```

output
```
Integral(sech(x)/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{sech}(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{2}{ae^{(-x)} + a}$$

input
```
integrate(sech(x)/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
2/(a*e^(-x) + a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(x)}{a + a\operatorname{sech}(x)}\,dx = -\frac{2}{a(e^x + 1)}$$

input
```
integrate(sech(x)/(a+a*sech(x)),x, algorithm="giac")
```

output
```
-2/(a*(e^x + 1))
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(x)}{a + a\operatorname{sech}(x)}\,dx = -\frac{2}{a\,\left(\mathrm{e}^x + 1\right)}$$

input
```
int(1/(cosh(x)*(a + a/cosh(x))),x)
```

output
```
-2/(a*(exp(x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{\operatorname{sech}(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{2e^x}{a\left(e^x + 1\right)}$$

input
```
int(sech(x)/(a+a*sech(x)),x)
```

output
```
(2*e**x)/(a*(e**x + 1))
```

3.73 $\int \frac{\text{sech}^2(x)}{a+a\text{sech}(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{\text{sech}^2(x)}{a+a\text{sech}(x)} \, dx = \frac{\arctan(\sinh(x))}{a} - \frac{\tanh(x)}{a+a\text{sech}(x)}$$

output
```
arctan(sinh(x))/a-tanh(x)/(a+a*sech(x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{\text{sech}^2(x)}{a+a\text{sech}(x)} \, dx = -\frac{\cot^{-1}(\sinh(x)) + \tanh\left(\frac{x}{2}\right)}{a}$$

input
```
Integrate[Sech[x]^2/(a + a*Sech[x]),x]
```

output
```
-((ArcCot[Sinh[x]] + Tanh[x/2])/a)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 4276, 3042, 4257, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(x)}{a\text{sech}(x)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(\frac{\pi}{2}+ix\right)^2}{a+a\csc\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 4276}$$

$$\frac{\int \text{sech}(x)dx}{a}-\int \frac{\text{sech}(x)}{\text{sech}(x)a+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}-\int \frac{\csc\left(ix+\frac{\pi}{2}\right)}{\csc\left(ix+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\arctan(\sinh(x))}{a}-\int \frac{\csc\left(ix+\frac{\pi}{2}\right)}{\csc\left(ix+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{ 4281}$$

$$\frac{\arctan(\sinh(x))}{a}-\frac{\tanh(x)}{a\text{sech}(x)+a}$$

input
```
Int[Sech[x]^2/(a + a*Sech[x]),x]
```

output
```
ArcTan[Sinh[x]]/a - Tanh[x]/(a + a*Sech[x])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4276
```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[1/b   Int[Csc[e + f*x], x], x] - Simp[a/b   Int[Csc[e + f*x]/(
a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{-\tanh\left(\frac{x}{2}\right)+2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	19
parallelrisch	$-\frac{-i\left(-\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)\right)+\tanh\left(\frac{x}{2}\right)}{a}$	33
risch	$\frac{2}{(1+e^{x})a}+\frac{i\ln(e^{x}+i)}{a}-\frac{i\ln(e^{x}-i)}{a}$	37

input
```
int(sech(x)^2/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-tanh(1/2*x)+2*arctan(tanh(1/2*x)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{\operatorname{sech}^2(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{2\left((\cosh(x)+\sinh(x)+1)\arctan(\cosh(x)+\sinh(x))+1\right)}{a\cosh(x)+a\sinh(x)+a}$$

input
```
integrate(sech(x)^2/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
2*((cosh(x) + sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 1)/(a*cosh(x) + a*s
inh(x) + a)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\operatorname{sech}^2(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

input
```
integrate(sech(x)**2/(a+a*sech(x)),x)
```

output
```
Integral(sech(x)**2/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\operatorname{sech}^2(x)}{a+a\operatorname{sech}(x)}\,dx = -\frac{2\arctan\left(e^{(-x)}\right)}{a} - \frac{2}{ae^{(-x)}+a}$$

input
```
integrate(sech(x)^2/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
-2*arctan(e^(-x))/a - 2/(a*e^(-x) + a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{2\arctan(e^x)}{a}+\frac{2}{a(e^x+1)}$$

input
```
integrate(sech(x)^2/(a+a*sech(x)),x, algorithm="giac")
```

output
```
2*arctan(e^x)/a + 2/(a*(e^x + 1))
```

Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{\operatorname{sech}^2(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{2}{a\,(e^x+1)}+\frac{2\,\operatorname{atan}\left(\frac{e^x\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

input
```
int(1/(cosh(x)^2*(a + a/cosh(x))),x)
```

output
```
2/(a*(exp(x) + 1)) + (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int \frac{\operatorname{sech}^2(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{2e^x\,atan(e^x)+2atan(e^x)-2e^x}{a\,(e^x+1)}$$

input
```
int(sech(x)^2/(a+a*sech(x)),x)
```

output
```
(2*(e**x*atan(e**x) + atan(e**x) - e**x))/(a*(e**x + 1))
```

3.74 $\int \frac{\operatorname{sech}^3(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{\operatorname{sech}^3(x)}{a+a\operatorname{sech}(x)}\,dx=-\frac{\arctan(\sinh(x))}{a}+\frac{\tanh(x)}{a}+\frac{\tanh(x)}{a+a\operatorname{sech}(x)}$$

output
```
-arctan(sinh(x))/a+tanh(x)/a+tanh(x)/(a+a*sech(x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\operatorname{sech}^3(x)}{a+a\operatorname{sech}(x)}\,dx=-\frac{\arctan(\sinh(x))+\arctan(\sinh(x))\operatorname{sech}(x)-2\tanh(x)-\operatorname{sech}(x)\tanh(x)}{a+a\operatorname{sech}(x)}$$

input
```
Integrate[Sech[x]^3/(a + a*Sech[x]),x]
```

output
```
-((ArcTan[Sinh[x]] + ArcTan[Sinh[x]]*Sech[x] - 2*Tanh[x] - Sech[x]*Tanh[x]
)/(a + a*Sech[x]))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3042, 4277, 3042, 4276, 3042, 4257, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^3(x)}{a\text{sech}(x)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(\frac{\pi}{2}+ix\right)^3}{a+a\csc\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 4277}$$

$$\frac{\tanh(x)}{a}-\int \frac{\text{sech}^2(x)}{\text{sech}(x)a+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tanh(x)}{a}-\int \frac{\csc\left(ix+\frac{\pi}{2}\right)^2}{\csc\left(ix+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{ 4276}$$

$$-\frac{\int \text{sech}(x)dx}{a}+\int \frac{\text{sech}(x)}{\text{sech}(x)a+a}dx+\frac{\tanh(x)}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}+\int \frac{\csc\left(ix+\frac{\pi}{2}\right)}{\csc\left(ix+\frac{\pi}{2}\right)a+a}dx+\frac{\tanh(x)}{a}$$

$$\Big\downarrow \text{ 4257}$$

$$\int \frac{\csc\left(ix+\frac{\pi}{2}\right)}{\csc\left(ix+\frac{\pi}{2}\right)a+a}dx-\frac{\arctan(\sinh(x))}{a}+\frac{\tanh(x)}{a}$$

$$\Big\downarrow \text{ 4281}$$

$$-\frac{\arctan(\sinh(x))}{a}+\frac{\tanh(x)}{a}+\frac{\tanh(x)}{a\text{sech}(x)+a}$$

input
```
Int[Sech[x]^3/(a + a*Sech[x]),x]
```

output
```
-(ArcTan[Sinh[x]]/a) + Tanh[x]/a + Tanh[x]/(a + a*Sech[x])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4276
```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[1/b   Int[Csc[e + f*x], x], x] - Simp[a/b   Int[Csc[e + f*x]/(
a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4277
```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[-Cot[e + f*x]/(b*f), x] - Simp[a/b   Int[Csc[e + f*x]^2/(a + b
*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)+\frac{2\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^{2}+1}-2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	33
parallelrisch	$\frac{-i\cosh(x)\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)+i\cosh(x)\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+2\tanh\left(\frac{x}{2}\right)\cosh(x)+\tanh\left(\frac{x}{2}\right)}{\cosh(x)a}$	48
risch	$-\frac{2(e^{2x}+e^{x}+2)}{a(e^{2x}+1)(1+e^{x})}+\frac{i\ln(e^{x}-i)}{a}-\frac{i\ln(e^{x}+i)}{a}$	53

input
```
int(sech(x)^3/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a*(tanh(1/2*x)+2*tanh(1/2*x)/(tanh(1/2*x)^2+1)-2*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(26) = 52.

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 4.88

$$\int \frac{\operatorname{sech}^3(x)}{a+a\operatorname{sech}(x)}\,dx = -\frac{2\left(\left(\cosh(x)^3+(3\cosh(x)+1)\sinh(x)^2+\sinh(x)^3+\cosh(x)^2+\left(3\cosh(x)^2+2\cosh(x)+1\right)\sin\right.\right.}{a\cosh(x)^3+a\sinh(x)^3+a\cosh(x)^2+(3a\cosh(x)+a}$$

input
```
integrate(sech(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
-2*((cosh(x)^3 + (3*cosh(x) + 1)*sinh(x)^2 + sinh(x)^3 + cosh(x)^2 + (3*co
sh(x)^2 + 2*cosh(x) + 1)*sinh(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x))
+ cosh(x)^2 + (2*cosh(x) + 1)*sinh(x) + sinh(x)^2 + cosh(x) + 2)/(a*cosh(x
)^3 + a*sinh(x)^3 + a*cosh(x)^2 + (3*a*cosh(x) + a)*sinh(x)^2 + a*cosh(x)
+ (3*a*cosh(x)^2 + 2*a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\operatorname{sech}^3(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

input
```
integrate(sech(x)**3/(a+a*sech(x)),x)
```

output
```
Integral(sech(x)**3/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.73

$$\int \frac{\operatorname{sech}^3(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{2\left(e^{(-x)} + e^{(-2x)} + 2\right)}{ae^{(-x)} + ae^{(-2x)} + ae^{(-3x)} + a} + \frac{2\arctan\left(e^{(-x)}\right)}{a}$$

input
```
integrate(sech(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
2*(e^(-x) + e^(-2*x) + 2)/(a*e^(-x) + a*e^(-2*x) + a*e^(-3*x) + a) + 2*arc
tan(e^(-x))/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \frac{\operatorname{sech}^3(x)}{a + a\operatorname{sech}(x)}\,dx = -\frac{2\arctan\left(e^{x}\right)}{a} - \frac{2\left(e^{(2x)} + e^{x} + 2\right)}{a(e^{(3x)} + e^{(2x)} + e^{x} + 1)}$$

input
```
integrate(sech(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

output
```
-2*arctan(e^x)/a - 2*(e^(2*x) + e^x + 2)/(a*(e^(3*x) + e^(2*x) + e^x + 1))
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.23

$$\int \frac{\operatorname{sech}^3(x)}{a + a\operatorname{sech}(x)}\, dx = -\frac{\frac{2\,e^{2\,x}}{a} + \frac{4}{a} + \frac{2\,e^{x}}{a}}{e^{2\,x} + e^{3\,x} + e^{x} + 1} - \frac{2\,\operatorname{atan}\left(\frac{e^{x}\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

input
```
int(1/(cosh(x)^3*(a + a/cosh(x))),x)
```

output
```
- ((2*exp(2*x))/a + 4/a + (2*exp(x))/a)/(exp(2*x) + exp(3*x) + exp(x) + 1)
 - (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.54

$$\int \frac{\operatorname{sech}^3(x)}{a + a\operatorname{sech}(x)}\, dx = \frac{-2e^{3x}\,atan(e^{x}) - 2e^{2x}\,atan(e^{x}) - 2e^{x}\,atan(e^{x}) - 2atan(e^{x}) + 2e^{3x} - 2}{a\left(e^{3x} + e^{2x} + e^{x} + 1\right)}$$

input
```
int(sech(x)^3/(a+a*sech(x)),x)
```

output
```
(2*( - e**(3*x)*atan(e**x) - e**(2*x)*atan(e**x) - e**x*atan(e**x) - atan(
e**x) + e**(3*x) - 1))/(a*(e**(3*x) + e**(2*x) + e**x + 1))
```

3.75 $\int \frac{\text{sech}^4(x)}{a+a\text{sech}(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 45

$$\int \frac{\text{sech}^4(x)}{a+a\text{sech}(x)} \, dx = \frac{3\arctan(\sinh(x))}{2a} - \frac{2\tanh(x)}{a} + \frac{3\text{sech}(x)\tanh(x)}{2a} - \frac{\text{sech}^2(x)\tanh(x)}{a+a\text{sech}(x)}$$

output
```
3/2*arctan(sinh(x))/a-2*tanh(x)/a+3/2*sech(x)*tanh(x)/a-sech(x)^2*tanh(x)/
(a+a*sech(x))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{\text{sech}^4(x)}{a+a\text{sech}(x)} \, dx = \frac{3\arctan(\sinh(x)) + 2\text{sech}^3(x)\tanh\left(\frac{x}{2}\right) - 6\tanh(x) + 3\text{sech}(x)\tanh(x) + 2\tanh^3(x)}{2a}$$

input
```
Integrate[Sech[x]^4/(a + a*Sech[x]),x]
```

output
```
(3*ArcTan[Sinh[x]] + 2*Sech[x]^3*Tanh[x/2] - 6*Tanh[x] + 3*Sech[x]*Tanh[x]
 + 2*Tanh[x]^3)/(2*a)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {3042, 4305, 3042, 4274, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^4(x)}{a\text{sech}(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(\frac{\pi}{2}+ix\right)^4}{a+a\csc\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{4305}$$

$$-\frac{\int \text{sech}^2(x)(2a-3a\text{sech}(x))dx}{a^2}-\frac{\tanh(x)\text{sech}^2(x)}{a\text{sech}(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(x)\text{sech}^2(x)}{a\text{sech}(x)+a}-\frac{\int \csc\left(ix+\frac{\pi}{2}\right)^2\left(2a-3a\csc\left(ix+\frac{\pi}{2}\right)\right)dx}{a^2}$$

$$\Big\downarrow \text{4274}$$

$$-\frac{2a\int \text{sech}^2(x)dx-3a\int \text{sech}^3(x)dx}{a^2}-\frac{\tanh(x)\text{sech}^2(x)}{a\text{sech}(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(x)\text{sech}^2(x)}{a\text{sech}(x)+a}-\frac{2a\int \csc\left(ix+\frac{\pi}{2}\right)^2dx-3a\int \csc\left(ix+\frac{\pi}{2}\right)^3dx}{a^2}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\tanh(x)\text{sech}^2(x)}{a\text{sech}(x)+a}-\frac{2ia\int 1d(-i\tanh(x))-3a\int \csc\left(ix+\frac{\pi}{2}\right)^3dx}{a^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\tanh(x)\text{sech}^2(x)}{a\text{sech}(x)+a}-\frac{2a\tanh(x)-3a\int\csc\left(ix+\frac{\pi}{2}\right)^3dx}{a^2}$$

$$\Big\downarrow \text{4255}$$

$$-\frac{2a\tanh(x)-3a\left(\frac{\int\text{sech}(x)dx}{2}+\frac{1}{2}\tanh(x)\text{sech}(x)\right)}{a^2}-\frac{\tanh(x)\text{sech}^2(x)}{a\text{sech}(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(x)\text{sech}^2(x)}{a\text{sech}(x)+a}-\frac{2a\tanh(x)-3a\left(\frac{1}{2}\tanh(x)\text{sech}(x)+\frac{1}{2}\int\csc\left(ix+\frac{\pi}{2}\right)dx\right)}{a^2}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{2a\tanh(x)-3a\left(\frac{1}{2}\arctan(\sinh(x))+\frac{1}{2}\tanh(x)\text{sech}(x)\right)}{a^2}-\frac{\tanh(x)\text{sech}^2(x)}{a\text{sech}(x)+a}$$

input
```
Int[Sech[x]^4/(a + a*Sech[x]),x]
```

output
```
-((Sech[x]^2*Tanh[x])/(a + a*Sech[x])) - (2*a*Tanh[x] - 3*a*(ArcTan[Sinh[x
]]/2 + (Sech[x]*Tanh[x])/2))/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4305
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Simp[d^2/(a*b)   Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
 - 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ
[a^2 - b^2, 0] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{-\tanh\left(\frac{x}{2}\right)+\frac{-3\tanh\left(\frac{x}{2}\right)^{3}-\tanh\left(\frac{x}{2}\right)}{\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{2}}+3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	46
risch	$\frac{3\,\mathrm{e}^{4x}+3\,\mathrm{e}^{3x}+5\,\mathrm{e}^{2x}+\mathrm{e}^{x}+4}{(1+\mathrm{e}^{x})a(\mathrm{e}^{2x}+1)^{2}}+\frac{3i\ln(\mathrm{e}^{x}+i)}{2a}-\frac{3i\ln(\mathrm{e}^{x}-i)}{2a}$	66
parallelrisch	$\frac{3i(-1-\cosh(2x))\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+3i(1+\cosh(2x))\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)-2\tanh\left(\frac{x}{2}\right)(\cosh(x)+2\cosh(2x)+1)}{2a(1+\cosh(2x))}$	67

input
```
int(sech(x)^4/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-tanh(1/2*x)+2*(-3/2*tanh(1/2*x)^3-1/2*tanh(1/2*x))/(tanh(1/2*x)^2+1)
^2+3*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(41) = 82$.

Time = 0.08 (sec) , antiderivative size = 325, normalized size of antiderivative = 7.22

$$\int \frac{\operatorname{sech}^4(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{3\cosh(x)^4 + 3(4\cosh(x)+1)\sinh(x)^3 + 3\sinh(x)^4 + 3\cosh(x)^3 + (18\cosh(x)^2 + 9\cosh(x) + 5)\,s}{}$$

input
```
integrate(sech(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
(3*cosh(x)^4 + 3*(4*cosh(x) + 1)*sinh(x)^3 + 3*sinh(x)^4 + 3*cosh(x)^3 + (
18*cosh(x)^2 + 9*cosh(x) + 5)*sinh(x)^2 + 3*(cosh(x)^5 + (5*cosh(x) + 1)*s
inh(x)^4 + sinh(x)^5 + cosh(x)^4 + 2*(5*cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)
^3 + 2*cosh(x)^3 + 2*(5*cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x)^2
 + 2*cosh(x)^2 + (5*cosh(x)^4 + 4*cosh(x)^3 + 6*cosh(x)^2 + 4*cosh(x) + 1)
*sinh(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x)) + 5*cosh(x)^2 + (12*cosh
(x)^3 + 9*cosh(x)^2 + 10*cosh(x) + 1)*sinh(x) + cosh(x) + 4)/(a*cosh(x)^5
+ a*sinh(x)^5 + a*cosh(x)^4 + (5*a*cosh(x) + a)*sinh(x)^4 + 2*a*cosh(x)^3
+ 2*(5*a*cosh(x)^2 + 2*a*cosh(x) + a)*sinh(x)^3 + 2*a*cosh(x)^2 + 2*(5*a*c
osh(x)^3 + 3*a*cosh(x)^2 + 3*a*cosh(x) + a)*sinh(x)^2 + a*cosh(x) + (5*a*c
osh(x)^4 + 4*a*cosh(x)^3 + 6*a*cosh(x)^2 + 4*a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\operatorname{sech}^4(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

input
```
integrate(sech(x)**4/(a+a*sech(x)),x)
```

output
```
Integral(sech(x)**4/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.62

$$\int \frac{\operatorname{sech}^4(x)}{a+a\operatorname{sech}(x)}\,dx = -\frac{e^{(-x)}+5\,e^{(-2\,x)}+3\,e^{(-3\,x)}+3\,e^{(-4\,x)}+4}{ae^{(-x)}+2\,ae^{(-2\,x)}+2\,ae^{(-3\,x)}+ae^{(-4\,x)}+ae^{(-5\,x)}+a} - \frac{3\,\arctan\left(e^{(-x)}\right)}{a}$$

input
```
integrate(sech(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
-(e^(-x) + 5*e^(-2*x) + 3*e^(-3*x) + 3*e^(-4*x) + 4)/(a*e^(-x) + 2*a*e^(-2
*x) + 2*a*e^(-3*x) + a*e^(-4*x) + a*e^(-5*x) + a) - 3*arctan(e^(-x))/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{sech}^4(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{3\,\arctan\left(e^{x}\right)}{a} + \frac{e^{(3\,x)}+2\,e^{(2\,x)}-e^{x}+2}{a(e^{(2\,x)}+1)^2} + \frac{2}{a(e^{x}+1)}$$

input
```
integrate(sech(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

output
```
3*arctan(e^x)/a + (e^(3*x) + 2*e^(2*x) - e^x + 2)/(a*(e^(2*x) + 1)^2) + 2/
(a*(e^x + 1))
```

Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.62

$$\int \frac{\operatorname{sech}^4(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{2}{a\,\left(e^{x}+1\right)} + \frac{\frac{2}{a}+\frac{e^{x}}{a}}{e^{2\,x}+1} + \frac{3\,\mathrm{atan}\left(\frac{e^{x}\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}} - \frac{2\,e^{x}}{a\,\left(2\,e^{2\,x}+e^{4\,x}+1\right)}$$

input
```
int(1/(cosh(x)^4*(a + a/cosh(x))),x)
```

output
```
2/(a*(exp(x) + 1)) + (2/a + exp(x)/a)/(exp(2*x) + 1) + (3*atan((exp(x)*(a^
2)^(1/2))/a))/(a^2)^(1/2) - (2*exp(x))/(a*(2*exp(2*x) + exp(4*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.71

$$\int \frac{\operatorname{sech}^4(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{3e^{5x}atan(e^x)+3e^{4x}atan(e^x)+6e^{3x}atan(e^x)+6e^{2x}atan(e^x)+3e^{x}atan(e^x)+3atan(e^x)-3e^{5x}-3e^{3x}-}{a\left(e^{5x}+e^{4x}+2e^{3x}+2e^{2x}+e^{x}+1\right)}$$

input
```
int(sech(x)^4/(a+a*sech(x)),x)
```

output
```
(3*e**(5*x)*atan(e**x) + 3*e**(4*x)*atan(e**x) + 6*e**(3*x)*atan(e**x) + 6
*e**(2*x)*atan(e**x) + 3*e**x*atan(e**x) + 3*atan(e**x) - 3*e**(5*x) - 3*e
**(3*x) - e**(2*x) - 2*e**x + 1)/(a*(e**(5*x) + e**(4*x) + 2*e**(3*x) + 2*
e**(2*x) + e**x + 1))
```

3.76 $\int \frac{1}{a+a\text{sech}(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 29

$$\int \frac{1}{a+a\text{sech}(c+dx)}\,dx=\frac{x}{a}-\frac{\tanh(c+dx)}{d(a+a\text{sech}(c+dx))}$$

output
```
x/a-tanh(d*x+c)/d/(a+a*sech(d*x+c))
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.00

$$\int \frac{1}{a+a\text{sech}(c+dx)}\,dx = \frac{\text{sech}\left(\frac{c}{2}\right)\text{sech}\left(\frac{1}{2}(c+dx)\right)\left(dx\cosh\left(\frac{dx}{2}\right)+dx\cosh\left(c+\frac{dx}{2}\right)-2\sinh\left(\frac{dx}{2}\right)\right)}{2ad}$$

input
```
Integrate[(a + a*Sech[c + d*x])^(-1),x]
```

output
```
(Sech[c/2]*Sech[(c + d*x)/2]*(d*x*Cosh[(d*x)/2] + d*x*Cosh[c + (d*x)/2] -
2*Sinh[(d*x)/2]))/(2*a*d)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4264, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\text{sech}(c+dx)+a}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{a+a\csc\left(ic+idx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{4264}$$
$$-\frac{\int -adx}{a^2}-\frac{\tanh(c+dx)}{d(a\text{sech}(c+dx)+a)}$$
$$\Big\downarrow \text{24}$$
$$\frac{x}{a}-\frac{\tanh(c+dx)}{d(a\text{sech}(c+dx)+a)}$$

input
```
Int[(a + a*Sech[c + d*x])^(-1),x]
```

output
```
x/a - Tanh[c + d*x]/(d*(a + a*Sech[c + d*x]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4264
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Simp[1/(a^2*(2*n + 1))
   Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Int
egerQ[2*n]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

method	result	size
parallelrisch	$\frac{dx-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$	23
risch	$\frac{x}{a}+\frac{2}{da\left(\mathrm{e}^{dx+c}+1\right)}$	25
derivativedivides	$\frac{-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	46
default	$\frac{-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	46

input
```
int(1/(a+a*sech(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(d*x-tanh(1/2*d*x+1/2*c))/a/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.66

$$\int \frac{1}{a+a\text{sech}(c+dx)}\,dx=\frac{dx\cosh\left(dx+c\right)+dx\sinh\left(dx+c\right)+dx+2}{ad\cosh\left(dx+c\right)+ad\sinh\left(dx+c\right)+ad}$$

input
```
integrate(1/(a+a*sech(d*x+c)),x, algorithm="fricas")
```

output
```
(d*x*cosh(d*x + c) + d*x*sinh(d*x + c) + d*x + 2)/(a*d*cosh(d*x + c) + a*d
*sinh(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{1}{a+a\operatorname{sech}(c+dx)}\,dx = \frac{\int \frac{1}{\operatorname{sech}(c+dx)+1}\,dx}{a}$$

input
```
integrate(1/(a+a*sech(d*x+c)),x)
```

output
```
Integral(1/(sech(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+a\operatorname{sech}(c+dx)}\,dx = \frac{dx+c}{ad} - \frac{2}{(ae^{(-dx-c)}+a)d}$$

input
```
integrate(1/(a+a*sech(d*x+c)),x, algorithm="maxima")
```

output
```
(d*x + c)/(a*d) - 2/((a*e^(-d*x - c) + a)*d)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+a\operatorname{sech}(c+dx)}\,dx = \frac{\frac{dx+c}{a} + \frac{2}{a\left(e^{(dx+c)}+1\right)}}{d}$$

input
```
integrate(1/(a+a*sech(d*x+c)),x, algorithm="giac")
```

output
```
((d*x + c)/a + 2/(a*(e^(d*x + c) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{1}{a+a\operatorname{sech}(c+dx)}\,dx=\frac{x}{a}+\frac{2}{a\,d\,\left(\mathrm{e}^{c+d\,x}+1\right)}$$

input
```
int(1/(a + a/cosh(c + d*x)),x)
```

output
```
x/a + 2/(a*d*(exp(c + d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int \frac{1}{a+a\operatorname{sech}(c+dx)}\,dx=\frac{e^{dx+c}dx-2e^{dx+c}+dx}{ad\,(e^{dx+c}+1)}$$

input
```
int(1/(a+a*sech(d*x+c)),x)
```

output
```
(e**(c + d*x)*d*x - 2*e**(c + d*x) + d*x)/(a*d*(e**(c + d*x) + 1))
```

3.77 $\int \frac{1}{a-a\text{sech}(c+dx)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{1}{a-a\text{sech}(c+dx)}\,dx=\frac{x}{a}-\frac{\tanh(c+dx)}{d(a-a\text{sech}(c+dx))}$$

output
```
x/a-tanh(d*x+c)/d/(a-a*sech(d*x+c))
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.97

$$\int \frac{1}{a-a\text{sech}(c+dx)}\,dx =\frac{\text{csch}\left(\frac{c}{2}\right)\text{csch}\left(\frac{1}{2}(c+dx)\right)\left(-dx\cosh\left(\frac{dx}{2}\right)+dx\cosh\left(c+\frac{dx}{2}\right)+2\sinh\left(\frac{dx}{2}\right)\right)}{2ad}$$

input
```
Integrate[(a - a*Sech[c + d*x])^(-1),x]
```

output
```
(Csch[c/2]*Csch[(c + d*x)/2]*(-(d*x*Cosh[(d*x)/2]) + d*x*Cosh[c + (d*x)/2]
 + 2*Sinh[(d*x)/2]))/(2*a*d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4264, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-a\text{sech}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{a-a\csc\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4264}$$

$$-\frac{\int -adx}{a^2}-\frac{\tanh(c+dx)}{d(a-a\text{sech}(c+dx))}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{x}{a}-\frac{\tanh(c+dx)}{d(a-a\text{sech}(c+dx))}$$

input
```
Int[(a - a*Sech[c + d*x])^(-1),x]
```

output
```
x/a - Tanh[c + d*x]/(d*(a - a*Sech[c + d*x]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4264
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Simp[1/(a^2*(2*n + 1))
   Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Int
egerQ[2*n]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result	size
parallelrisch	$\frac{dx-\coth\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}$	23
risch	$\frac{x}{a}-\frac{2}{da\left(\mathrm{e}^{dx+c}-1\right)}$	25
derivativedivides	$\frac{-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{1}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	48
default	$\frac{-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{1}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	48

input
```
int(1/(a-a*sech(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(d*x-coth(1/2*d*x+1/2*c))/d/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.67

$$\int \frac{1}{a-a\text{sech}(c+dx)}\,dx=\frac{dx\cosh\left(dx+c\right)+dx\sinh\left(dx+c\right)-dx-2}{ad\cosh\left(dx+c\right)+ad\sinh\left(dx+c\right)-ad}$$

input
```
integrate(1/(a-a*sech(d*x+c)),x, algorithm="fricas")
```

output
```
(d*x*cosh(d*x + c) + d*x*sinh(d*x + c) - d*x - 2)/(a*d*cosh(d*x + c) + a*d
*sinh(d*x + c) - a*d)
```

Sympy [F]

$$\int \frac{1}{a - a\operatorname{sech}(c + dx)} dx = -\frac{\int \frac{1}{\operatorname{sech}(c+dx)-1} dx}{a}$$

input
```
integrate(1/(a-a*sech(d*x+c)),x)
```

output
```
-Integral(1/(sech(c + d*x) - 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int \frac{1}{a - a\operatorname{sech}(c + dx)} dx = \frac{dx + c}{ad} + \frac{2}{(ae^{(-dx-c)} - a)d}$$

input
```
integrate(1/(a-a*sech(d*x+c)),x, algorithm="maxima")
```

output
```
(d*x + c)/(a*d) + 2/((a*e^(-d*x - c) - a)*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{1}{a - a\operatorname{sech}(c + dx)} dx = \frac{\frac{dx+c}{a} - \frac{2}{a(e^{(dx+c)}-1)}}{d}$$

input
```
integrate(1/(a-a*sech(d*x+c)),x, algorithm="giac")
```

output
```
((d*x + c)/a - 2/(a*(e^(d*x + c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{1}{a - a\text{sech}(c + dx)} \, dx = \frac{x}{a} - \frac{2}{a\,d\,\left(e^{c+d\,x} - 1\right)}$$

input
```
int(1/(a - a/cosh(c + d*x)),x)
```

output
```
x/a - 2/(a*d*(exp(c + d*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \frac{1}{a - a\text{sech}(c + dx)} \, dx = \frac{e^{dx+c}dx - 2e^{dx+c} - dx}{ad\,(e^{dx+c} - 1)}$$

input
```
int(1/(a-a*sech(d*x+c)),x)
```

output
```
(e**(c + d*x)*d*x - 2*e**(c + d*x) - d*x)/(a*d*(e**(c + d*x) - 1))
```

3.78 $\int (a+a\text{sech}(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 98

$$\int (a+a\text{sech}(c+dx))^{5/2}\,dx = \frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{d} + \frac{14a^3\tanh(c+dx)}{3d\sqrt{a+a\text{sech}(c+dx)}} + \frac{2a^2\sqrt{a+a\text{sech}(c+dx)}\tanh(c+dx)}{3d}$$

output
```
2*a^(5/2)*arctanh(a^(1/2)*tanh(d*x+c)/(a+a*sech(d*x+c))^(1/2))/d+14/3*a^3*
tanh(d*x+c)/d/(a+a*sech(d*x+c))^(1/2)+2/3*a^2*(a+a*sech(d*x+c))^(1/2)*tanh
(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int (a+a\text{sech}(c+dx))^{5/2}\,dx = \frac{a^2\text{sech}\left(\frac{1}{2}(c+dx)\right)\text{sech}(c+dx)\sqrt{a(1+\text{sech}(c+dx))}\left(3\sqrt{2}\text{arcsinh}\left(\sqrt{2}\sinh\left(\frac{1}{2}(c+dx)\right)\right)\text{co}\right.}{3d}$$

input
```
Integrate[(a + a*Sech[c + d*x])^(5/2),x]
```

output
```
(a^2*Sech[(c + d*x)/2]*Sech[c + d*x]*Sqrt[a*(1 + Sech[c + d*x])]*(3*Sqrt[2
]*ArcSinh[Sqrt[2]*Sinh[(c + d*x)/2]]*Cosh[c + d*x]^(3/2) - 6*Sinh[(c + d*x
)/2] + 8*Sinh[(3*(c + d*x))/2]))/(3*d)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3042, 4262, 27, 3042, 4403, 3042, 4261, 216, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\text{sech}(c+dx)+a)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+a\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4262}$$

$$\frac{2}{3}a\int \frac{1}{2}\sqrt{\text{sech}(c+dx)a+a}(7\text{sech}(c+dx)a+3a)dx+\frac{2a^2\tanh(c+dx)\sqrt{a\text{sech}(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}a\int \sqrt{\text{sech}(c+dx)a+a}(7\text{sech}(c+dx)a+3a)dx+\frac{2a^2\tanh(c+dx)\sqrt{a\text{sech}(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a^2\tanh(c+dx)\sqrt{a\text{sech}(c+dx)+a}}{3d}+\frac{1}{3}a\int \sqrt{\csc\left(ic+idx+\frac{\pi}{2}\right)a+a}\left(7\csc\left(ic+idx+\frac{\pi}{2}\right)a+3a\right)dx$$

$$\Big\downarrow \text{4403}$$

$$\frac{1}{3}a\left(3a\int \sqrt{\text{sech}(c+dx)a+a}dx+7a\int \text{sech}(c+dx)\sqrt{\text{sech}(c+dx)a+a}dx\right)+\frac{2a^2\tanh(c+dx)\sqrt{a\text{sech}(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a^2\tanh(c+dx)\sqrt{a\text{sech}(c+dx)+a}}{3d}+\frac{1}{3}a\left(3a\int\sqrt{\csc\left(ic+idx+\frac{\pi}{2}\right)a+a}dx+7a\int\csc\left(ic+idx+\frac{\pi}{2}\right)\sqrt{\csc\left(ic+idx+\frac{\pi}{2}\right)a+a}dx\right)$$

$$\Big\downarrow \text{4261}$$

$$\frac{2a^2\tanh(c+dx)\sqrt{a\text{sech}(c+dx)+a}}{3d}+\frac{1}{3}a\left(\frac{6ia^2\int\frac{1}{a-\frac{a^2\tanh^2(c+dx)}{\text{sech}(c+dx)a+a}}d\left(-\frac{ia\tanh(c+dx)}{\sqrt{\text{sech}(c+dx)a+a}}\right)}{d}+7a\int\csc\left(ic+idx+\frac{\pi}{2}\right)\sqrt{\csc\left(ic+idx+\frac{\pi}{2}\right)a+a}dx\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{2a^2\tanh(c+dx)\sqrt{a\text{sech}(c+dx)+a}}{3d}+\frac{1}{3}a\left(\frac{6a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\text{sech}(c+dx)+a}}\right)}{d}+7a\int\csc\left(ic+idx+\frac{\pi}{2}\right)\sqrt{\csc\left(ic+idx+\frac{\pi}{2}\right)a+a}dx\right)$$

$$\Big\downarrow \text{4279}$$

$$\frac{2a^2\tanh(c+dx)\sqrt{a\text{sech}(c+dx)+a}}{3d}+\frac{1}{3}a\left(\frac{6a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\text{sech}(c+dx)+a}}\right)}{d}+\frac{14a^2\tanh(c+dx)}{d\sqrt{a\text{sech}(c+dx)+a}}\right)$$

input
```
Int[(a + a*Sech[c + d*x])^(5/2),x]
```

output
```
(2*a^2*Sqrt[a + a*Sech[c + d*x]]*Tanh[c + d*x])/(3*d) + (a*((6*a^(3/2)*Arc
Tanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/d + (14*a^2*Tanh[
c + d*x])/(d*Sqrt[a + a*Sech[c + d*x]])))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4262
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[a/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Inte
gerQ[2*n]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4403
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Simp[c   Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Si
mp[d   Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (a + a \operatorname{sech}(dx + c))^{\frac{5}{2}} dx$$

input
```
int((a+a*sech(d*x+c))^(5/2),x)
```

output
```
int((a+a*sech(d*x+c))^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. 2(84) = 168.

Time = 0.11 (sec) , antiderivative size = 924, normalized size of antiderivative = 9.43

$$\int (a + a\text{sech}(c + dx))^{5/2} \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sech(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/6*(3*(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh
(d*x + c)^2 + a^2)*sqrt(a)*log(-(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 - 3
*a*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^3 + 5*a*cosh(
d*x + c)^2 + (6*a*cosh(d*x + c)^2 - 9*a*cosh(d*x + c) + 5*a)*sinh(d*x + c)
^2 + (cosh(d*x + c)^5 + (5*cosh(d*x + c) - 3)*sinh(d*x + c)^4 + sinh(d*x +
 c)^5 - 3*cosh(d*x + c)^4 + (10*cosh(d*x + c)^2 - 12*cosh(d*x + c) + 5)*si
nh(d*x + c)^3 + 5*cosh(d*x + c)^3 + (10*cosh(d*x + c)^3 - 18*cosh(d*x + c)
^2 + 15*cosh(d*x + c) - 7)*sinh(d*x + c)^2 - 7*cosh(d*x + c)^2 + (5*cosh(d
*x + c)^4 - 12*cosh(d*x + c)^3 + 15*cosh(d*x + c)^2 - 14*cosh(d*x + c) + 4
)*sinh(d*x + c) + 4*cosh(d*x + c) - 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 4*a*cosh(d*x + c) +
 (4*a*cosh(d*x + c)^3 - 9*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) - 4*a)*si
nh(d*x + c) + 4*a)/(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*
cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3)) + 3*(a^2*cosh(d*x + c)^2
 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(d*x + c)^2 + a^2)*sqrt(a)*
log((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + (cosh(d*x + c)^3 + (3*cosh(d*
x + c) + 1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + cosh(d*x + c)^2 + (3*cosh(
d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) + 1)*sqrt(
a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 + 1)) + a*cosh(d*x + c) + (2*a*cosh(d*x + c) + a)*sinh(d*x + c) + a)...
```

Sympy [F]

$$\int (a + a\text{sech}(c + dx))^{5/2} \, dx = \int (a \operatorname{sech}(c + dx) + a)^{\frac{5}{2}} \, dx$$

input
```
integrate((a+a*sech(d*x+c))**(5/2),x)
```

output
```
Integral((a*sech(c + d*x) + a)**(5/2), x)
```

Maxima [F]

$$\int (a + a\text{sech}(c + dx))^{5/2} \, dx = \int (a\,\text{sech}\,(dx + c) + a)^{\frac{5}{2}} \, dx$$

input
```
integrate((a+a*sech(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sech(d*x + c) + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int (a + a\text{sech}(c + dx))^{5/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sech(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\text{sech}(c + dx))^{5/2} \, dx = \int \left(a + \frac{a}{\cosh\left(c + d\,x\right)}\right)^{5/2} \, dx$$

input
```
int((a + a/cosh(c + d*x))^(5/2),x)
```

output
```
int((a + a/cosh(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int (a + a\operatorname{sech}(c + dx))^{5/2}\,dx = \sqrt{a}\,a^2\left(\int \sqrt{\operatorname{sech}(dx + c) + 1}dx + \int \sqrt{\operatorname{sech}(dx + c) + 1}\operatorname{sech}(dx + c)^2\,dx + 2\left(\int \sqrt{\operatorname{sech}(dx + c) + 1}\operatorname{sech}(dx + c)\,dx\right)\right)$$

input
```
int((a+a*sech(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sech(c + d*x) + 1),x) + int(sqrt(sech(c + d*x) + 1)
*sech(c + d*x)**2,x) + 2*int(sqrt(sech(c + d*x) + 1)*sech(c + d*x),x))
```

3.79 $\int (a+a\text{sech}(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 66

$$\int (a+a\text{sech}(c+dx))^{3/2}\,dx=\frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{d}+\frac{2a^2\tanh(c+dx)}{d\sqrt{a+a\text{sech}(c+dx)}}$$

output
```
2*a^(3/2)*arctanh(a^(1/2)*tanh(d*x+c)/(a+a*sech(d*x+c))^(1/2))/d+2*a^2*tan
h(d*x+c)/d/(a+a*sech(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int (a+a\text{sech}(c+dx))^{3/2}\,dx=\frac{a\text{sech}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\text{sech}(c+dx))}\left(\sqrt{2}\text{arcsinh}\left(\sqrt{2}\sinh\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cosh(c+dx)}+\right.}{d}$$

input
```
Integrate[(a + a*Sech[c + d*x])^(3/2),x]
```

output
```
(a*Sech[(c + d*x)/2]*Sqrt[a*(1 + Sech[c + d*x])]*(Sqrt[2]*ArcSinh[Sqrt[2]*
Sinh[(c + d*x)/2]]*Sqrt[Cosh[c + d*x]] + 2*Sinh[(c + d*x)/2]))/d
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4262, 27, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\text{sech}(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+a\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4262}$$

$$2a\int \frac{1}{2}\sqrt{\text{sech}(c+dx)a+a}dx+\frac{2a^2\tanh(c+dx)}{d\sqrt{a\text{sech}(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$a\int \sqrt{\text{sech}(c+dx)a+a}dx+\frac{2a^2\tanh(c+dx)}{d\sqrt{a\text{sech}(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a^2\tanh(c+dx)}{d\sqrt{a\text{sech}(c+dx)+a}}+a\int \sqrt{\csc\left(ic+idx+\frac{\pi}{2}\right)a+adx}$$

$$\Big\downarrow \text{4261}$$

$$\frac{2a^2\tanh(c+dx)}{d\sqrt{a\text{sech}(c+dx)+a}}+\frac{2ia^2\int \frac{1}{a-\frac{a^2\tanh^2(c+dx)}{\text{sech}(c+dx)a+a}}d\left(-\frac{ia\tanh(c+dx)}{\sqrt{\text{sech}(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\operatorname{sech}(c+dx)+a}}\right)}{d}+\frac{2a^2\tanh(c+dx)}{d\sqrt{a\operatorname{sech}(c+dx)+a}}$$

input
```
Int[(a + a*Sech[c + d*x])^(3/2),x]
```

output
```
(2*a^(3/2)*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/d +
 (2*a^2*Tanh[c + d*x])/(d*Sqrt[a + a*Sech[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4262
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[a/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Inte
gerQ[2*n]
```

Maple [F]

$$\int (a + a\ \mathrm{sech}\,(dx + c))^{\frac{3}{2}}\,dx$$

input
```
int((a+a*sech(d*x+c))^(3/2),x)
```

output
```
int((a+a*sech(d*x+c))^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. 2(58) = 116.

Time = 0.12 (sec) , antiderivative size = 697, normalized size of antiderivative = 10.56

$$\int (a + a\mathrm{sech}(c + dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/2*(a^(3/2)*log(-(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 - 3*a*cosh(d*x +
c)^3 + (4*a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (
6*a*cosh(d*x + c)^2 - 9*a*cosh(d*x + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x
 + c)^5 + (5*cosh(d*x + c) - 3)*sinh(d*x + c)^4 + sinh(d*x + c)^5 - 3*cosh
(d*x + c)^4 + (10*cosh(d*x + c)^2 - 12*cosh(d*x + c) + 5)*sinh(d*x + c)^3
+ 5*cosh(d*x + c)^3 + (10*cosh(d*x + c)^3 - 18*cosh(d*x + c)^2 + 15*cosh(d
*x + c) - 7)*sinh(d*x + c)^2 - 7*cosh(d*x + c)^2 + (5*cosh(d*x + c)^4 - 12
*cosh(d*x + c)^3 + 15*cosh(d*x + c)^2 - 14*cosh(d*x + c) + 4)*sinh(d*x + c
) + 4*cosh(d*x + c) - 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 4*a*cosh(d*x + c) + (4*a*cosh(d*x
 + c)^3 - 9*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) - 4*a)*sinh(d*x + c) +
4*a)/(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*cosh(d*x + c)*
sinh(d*x + c)^2 + sinh(d*x + c)^3)) + a^(3/2)*log((a*cosh(d*x + c)^2 + a*s
inh(d*x + c)^2 + (cosh(d*x + c)^3 + (3*cosh(d*x + c) + 1)*sinh(d*x + c)^2
+ sinh(d*x + c)^3 + cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 + 2*cosh(d*x + c)
 + 1)*sinh(d*x + c) + cosh(d*x + c) + 1)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 +
 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + a*cosh(d*x + c) +
 (2*a*cosh(d*x + c) + a)*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(d*x + c)
)) + 4*(a*cosh(d*x + c) + a*sinh(d*x + c) - a)*sqrt(a/(cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)))/d
```

Sympy [F]

$$\int (a + a\text{sech}(c + dx))^{3/2}\,dx = \int (a\,\text{sech}\,(c + dx) + a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sech(d*x+c))**(3/2),x)
```

output
```
Integral((a*sech(c + d*x) + a)**(3/2), x)
```

Maxima [F]

$$\int (a + a\text{sech}(c + dx))^{3/2} \, dx = \int (a \operatorname{sech}(dx + c) + a)^{\frac{3}{2}} \, dx$$

input
```
integrate((a+a*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sech(d*x + c) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int (a + a\text{sech}(c + dx))^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sech(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\text{sech}(c + dx))^{3/2} \, dx = \int \left(a + \frac{a}{\cosh(c + d\,x)}\right)^{3/2} \, dx$$

input
```
int((a + a/cosh(c + d*x))^(3/2),x)
```

output
```
int((a + a/cosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int (a + a\text{sech}(c + dx))^{3/2} \, dx = \sqrt{a}\, a\left(\int \sqrt{\text{sech}\,(dx + c) + 1}dx + \int \sqrt{\text{sech}\,(dx + c) + 1}\,\text{sech}(dx + c)\,dx\right)$$

input
```
int((a+a*sech(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sech(c + d*x) + 1),x) + int(sqrt(sech(c + d*x) + 1)*se
ch(c + d*x),x))
```

3.80 $\int \sqrt{a+a\text{sech}(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 37

$$\int \sqrt{a+a\text{sech}(c+dx)}\,dx=\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{d}$$

output
```
2*a^(1/2)*arctanh(a^(1/2)*tanh(d*x+c)/(a+a*sech(d*x+c))^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \sqrt{a+a\text{sech}(c+dx)}\,dx=\frac{\sqrt{2}\,\text{arcsinh}\left(\sqrt{2}\sinh\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cosh(c+dx)}\,\text{sech}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\text{sech}(c+dx))}}{d}$$

input
```
Integrate[Sqrt[a + a*Sech[c + d*x]],x]
```

output
```
(Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[(c + d*x)/2]]*Sqrt[Cosh[c + d*x]]*Sech[(c + 
d*x)/2]*Sqrt[a*(1 + Sech[c + d*x])])/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\text{sech}(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+a\csc\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4261}$$

$$\frac{2ia\int \frac{1}{a-\frac{a^2\tanh^2(c+dx)}{\text{sech}(c+dx)a+a}}d\left(-\frac{ia\tanh(c+dx)}{\sqrt{\text{sech}(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\text{sech}(c+dx)+a}}\right)}{d}$$

input
```
Int[Sqrt[a + a*Sech[c + d*x]],x]
```

output
```
(2*Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sqrt{a + a \operatorname{sech}(dx + c)}dx$$

input
```
int((a+a*sech(d*x+c))^(1/2),x)
```

output
```
int((a+a*sech(d*x+c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(31) = 62$.

Time = 0.09 (sec) , antiderivative size = 637, normalized size of antiderivative = 17.22

$$\int \sqrt{a + a\operatorname{sech}(c + dx)}\, dx = \text{Too large to display}$$

input
```
integrate((a+a*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(a)*log(-(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 - 3*a*cosh(d*x +
c)^3 + (4*a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (
6*a*cosh(d*x + c)^2 - 9*a*cosh(d*x + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x
 + c)^5 + (5*cosh(d*x + c) - 3)*sinh(d*x + c)^4 + sinh(d*x + c)^5 - 3*cosh
(d*x + c)^4 + (10*cosh(d*x + c)^2 - 12*cosh(d*x + c) + 5)*sinh(d*x + c)^3
+ 5*cosh(d*x + c)^3 + (10*cosh(d*x + c)^3 - 18*cosh(d*x + c)^2 + 15*cosh(d
*x + c) - 7)*sinh(d*x + c)^2 - 7*cosh(d*x + c)^2 + (5*cosh(d*x + c)^4 - 12
*cosh(d*x + c)^3 + 15*cosh(d*x + c)^2 - 14*cosh(d*x + c) + 4)*sinh(d*x + c
) + 4*cosh(d*x + c) - 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 4*a*cosh(d*x + c) + (4*a*cosh(d*x
 + c)^3 - 9*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) - 4*a)*sinh(d*x + c) +
4*a)/(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*cosh(d*x + c)*
sinh(d*x + c)^2 + sinh(d*x + c)^3)) + sqrt(a)*log((a*cosh(d*x + c)^2 + a*s
inh(d*x + c)^2 + (cosh(d*x + c)^3 + (3*cosh(d*x + c) + 1)*sinh(d*x + c)^2
+ sinh(d*x + c)^3 + cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 + 2*cosh(d*x + c)
 + 1)*sinh(d*x + c) + cosh(d*x + c) + 1)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 +
 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + a*cosh(d*x + c) +
 (2*a*cosh(d*x + c) + a)*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(d*x + c)
)))/d
```

Sympy [F]

$$\int \sqrt{a + a\text{sech}(c + dx)}\, dx = \int \sqrt{a\,\text{sech}\,(c + dx) + a}\, dx$$

input
```
integrate((a+a*sech(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*sech(c + d*x) + a), x)
```

Maxima [F]

$$\int \sqrt{a + a\text{sech}(c + dx)}\, dx = \int \sqrt{a\, \text{sech}\,(dx + c) + a}\, dx$$

input
```
integrate((a+a*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sech(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a + a\text{sech}(c + dx)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sech(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + a\text{sech}(c + dx)}\, dx = \int \sqrt{a + \frac{a}{\cosh\,(c + d\,x)}}\, dx$$

input
```
int((a + a/cosh(c + d*x))^(1/2),x)
```

output
```
int((a + a/cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+a\operatorname{sech}(c+dx)}\,dx = \sqrt{a}\left(\int \sqrt{\operatorname{sech}\left(dx+c\right)+1}dx\right)$$

input
```
int((a+a*sech(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sech(c + d*x) + 1),x)
```

3.81 $\int \frac{1}{\sqrt{a+a\text{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 85

$$\int \frac{1}{\sqrt{a+a\text{sech}(c+dx)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a+a\text{sech}(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
2*arctanh(a^(1/2)*tanh(d*x+c)/(a+a*sech(d*x+c))^(1/2))/a^(1/2)/d-2^(1/2)*a
rctanh(1/2*a^(1/2)*tanh(d*x+c)*2^(1/2)/(a+a*sech(d*x+c))^(1/2))/a^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39

$$\int \frac{1}{\sqrt{a+a\text{sech}(c+dx)}}\,dx = \frac{(1+e^{c+dx})\left(\sqrt{2}\text{arcsinh}(e^{c+dx}) - 2\text{arctanh}\left(\frac{-1+e^{c+dx}}{\sqrt{2}\sqrt{1+e^{2(c+dx)}}}\right) - \sqrt{2}\text{arctanh}\left(\sqrt{1+e^{2(c+dx)}}\right)\right)}{\sqrt{2}d\sqrt{1+e^{2(c+dx)}}\sqrt{a(1+\text{sech}(c+dx))}}$$

input
```
Integrate[1/Sqrt[a + a*Sech[c + d*x]],x]
```

output
```
((1 + E^(c + d*x))*(Sqrt[2]*ArcSinh[E^(c + d*x)] - 2*ArcTanh[(-1 + E^(c +
d*x))/(Sqrt[2]*Sqrt[1 + E^(2*(c + d*x))])] - Sqrt[2]*ArcTanh[Sqrt[1 + E^(2
*(c + d*x))]]))/(Sqrt[2]*d*Sqrt[1 + E^(2*(c + d*x))]*Sqrt[a*(1 + Sech[c +
d*x])])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4263, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\text{sech}(c+dx)+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{a+a\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4263}$$

$$\frac{\int \sqrt{\text{sech}(c+dx)a+a}dx}{a}-\int \frac{\text{sech}(c+dx)}{\sqrt{\text{sech}(c+dx)a+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sqrt{\csc\left(ic+idx+\frac{\pi}{2}\right)a+a}dx}{a}-\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(ic+idx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{ 4261}$$

$$\frac{2i\int \frac{1}{a-\frac{a^2\tanh^2(c+dx)}{\text{sech}(c+dx)a+a}}d\left(-\frac{ia\tanh(c+dx)}{\sqrt{\text{sech}(c+dx)a+a}}\right)}{d}-\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(ic+idx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{216}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\text{sech}(c+dx)+a}}\right)}{\sqrt{a}d}-\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(ic+idx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4282}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\text{sech}(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2i\int \frac{1}{2a-\frac{a^2\tanh^2(c+dx)}{\text{sech}(c+dx)a+a}}d\left(-\frac{ia\tanh(c+dx)}{\sqrt{\text{sech}(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\text{sech}(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a\text{sech}(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[1/Sqrt[a + a*Sech[c + d*x]],x]
```

output
```
(2*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/(Sqrt[a]*d)
 - (Sqrt[2]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sech[c + d
*x]])])/(Sqrt[a]*d)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4263
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[1/a   I
nt[Sqrt[a + b*Csc[c + d*x]], x], x] - Simp[b/a   Int[Csc[c + d*x]/Sqrt[a +
b*Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{1}{\sqrt{a + a\ \mathrm{sech}\left(dx + c\right)}}dx$$

input
```
int(1/(a+a*sech(d*x+c))^(1/2),x)
```

output
```
int(1/(a+a*sech(d*x+c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 868 vs. $2(70) = 140$.

Time = 0.11 (sec) , antiderivative size = 868, normalized size of antiderivative = 10.21

$$\int \frac{1}{\sqrt{a + a\mathrm{sech}(c + dx)}}\, dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*sqrt(a)*log(-(3*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c) - 1)*sin
h(d*x + c) + 3*sinh(d*x + c)^2 - 2*sqrt(2)*(cosh(d*x + c)^3 + (3*cosh(d*x
+ c) - 1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 - cosh(d*x + c)^2 + (3*cosh(d*
x + c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) - 1)*sqrt(a/
(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1))/s
qrt(a) - 2*cosh(d*x + c) + 3)/(cosh(d*x + c)^2 + 2*(cosh(d*x + c) + 1)*sin
h(d*x + c) + sinh(d*x + c)^2 + 2*cosh(d*x + c) + 1)) + sqrt(a)*log(-(a*cos
h(d*x + c)^4 + a*sinh(d*x + c)^4 - 3*a*cosh(d*x + c)^3 + (4*a*cosh(d*x + c
) - 3*a)*sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 - 9*
a*cosh(d*x + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x + c)^5 + (5*cosh(d*x +
c) - 3)*sinh(d*x + c)^4 + sinh(d*x + c)^5 - 3*cosh(d*x + c)^4 + (10*cosh(d
*x + c)^2 - 12*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 5*cosh(d*x + c)^3 + (1
0*cosh(d*x + c)^3 - 18*cosh(d*x + c)^2 + 15*cosh(d*x + c) - 7)*sinh(d*x +
c)^2 - 7*cosh(d*x + c)^2 + (5*cosh(d*x + c)^4 - 12*cosh(d*x + c)^3 + 15*co
sh(d*x + c)^2 - 14*cosh(d*x + c) + 4)*sinh(d*x + c) + 4*cosh(d*x + c) - 4)
*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*
x + c)^2 + 1)) - 4*a*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 - 9*a*cosh(d*x +
 c)^2 + 10*a*cosh(d*x + c) - 4*a)*sinh(d*x + c) + 4*a)/(cosh(d*x + c)^3 +
3*cosh(d*x + c)^2*sinh(d*x + c) + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d
*x + c)^3)) + sqrt(a)*log((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + (cos...
```

Sympy [F]

$$\int \frac{1}{\sqrt{a + a\text{sech}(c + dx)}}\,dx = \int \frac{1}{\sqrt{a\,\text{sech}\,(c + dx) + a}}\,dx$$

input
```
integrate(1/(a+a*sech(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a*sech(c + d*x) + a), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a + a\text{sech}(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\,\text{sech}\,(dx+c)+a}}\,dx$$

input
```
integrate(1/(a+a*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*sech(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a + a\text{sech}(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sech(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + a\text{sech}(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cosh(c+d\,x)}}}\,dx$$

input
```
int(1/(a + a/cosh(c + d*x))^(1/2),x)
```

output
```
int(1/(a + a/cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a + a\operatorname{sech}(c + dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\operatorname{sech}(dx+c)+1}}{\operatorname{sech}(dx+c)+1}dx\right)}{a}$$

input
```
int(1/(a+a*sech(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int(sqrt(sech(c + d*x) + 1)/(sech(c + d*x) + 1),x))/a
```

3.82 $\int \frac{1}{(a+a\text{sech}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 114

$$\int \frac{1}{(a+a\text{sech}(c+dx))^{3/2}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{a^{3/2}d}-\frac{5\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a+a\text{sech}(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\tanh(c+dx)}{2d(a+a\text{sech}(c+dx))^{3/2}}$$

output
```
2*arctanh(a^(1/2)*tanh(d*x+c)/(a+a*sech(d*x+c))^(1/2))/a^(3/2)/d-5/4*arcta
nh(1/2*a^(1/2)*tanh(d*x+c)*2^(1/2)/(a+a*sech(d*x+c))^(1/2))*2^(1/2)/a^(3/2
)/d-1/2*tanh(d*x+c)/d/(a+a*sech(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 3.12 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.55

$$\int \frac{1}{(a+a\text{sech}(c+dx))^{3/2}}\,dx=\frac{\cosh^2\left(\frac{1}{2}(c+dx)\right)\text{sech}(c+dx)\left(4\left(1+e^{c+dx}\right)\text{arcsinh}\left(e^{c+dx}\right)+5\sqrt{2}\left(1+e^{c\cdots}\right.\right.}{2d\sqrt{\cdots}}$$

input
```
Integrate[(a + a*Sech[c + d*x])^(-3/2),x]
```

output
```
(Cosh[(c + d*x)/2]^2*Sech[c + d*x]*(4*(1 + E^(c + d*x))*ArcSinh[E^(c + d*x
)] + 5*Sqrt[2]*(1 + E^(c + d*x))*ArcTanh[(1 - E^(c + d*x))/(Sqrt[2]*Sqrt[1
 + E^(2*(c + d*x))])] - 4*(1 + E^(c + d*x))*ArcTanh[Sqrt[1 + E^(2*(c + d*x
))]] - 2*Sqrt[1 + E^(2*(c + d*x))]*Tanh[(c + d*x)/2]))/(2*d*Sqrt[1 + E^(2*
(c + d*x))]*(a*(1 + Sech[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4264, 27, 3042, 4408, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\text{sech}(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+a\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4264}$$

$$-\frac{\int -\frac{4a-a\text{sech}(c+dx)}{2\sqrt{\text{sech}(c+dx)a+a}}dx}{2a^2}-\frac{\tanh(c+dx)}{2d(a\text{sech}(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4a-a\text{sech}(c+dx)}{\sqrt{\text{sech}(c+dx)a+a}}dx}{4a^2}-\frac{\tanh(c+dx)}{2d(a\text{sech}(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(c+dx)}{2d(a\text{sech}(c+dx)+a)^{3/2}}+\frac{\int \frac{4a-a\csc\left(ic+idx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(ic+idx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}$$

$$\Big\downarrow \text{4408}$$

$$\frac{4\int \sqrt{\text{sech}(c+dx)a+a}dx-5a\int \frac{\text{sech}(c+dx)}{\sqrt{\text{sech}(c+dx)a+a}}dx}{4a^2}-\frac{\tanh(c+dx)}{2d(a\text{sech}(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\tanh(c+dx)}{2d(a\text{sech}(c+dx)+a)^{3/2}}+\frac{4\int \sqrt{\csc\left(ic+idx+\frac{\pi}{2}\right)a+a}dx-5a\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(ic+idx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}$$

$$\Big\downarrow \ 4261$$

$$-\frac{\tanh(c+dx)}{2d(a\text{sech}(c+dx)+a)^{3/2}}+\frac{\frac{8ia\int \frac{1}{a-\frac{a^2\tanh^2(c+dx)}{\text{sech}(c+dx)a+a}}d\left(-\frac{ia\tanh(c+dx)}{\sqrt{\text{sech}(c+dx)a+a}}\right)}{d}-5a\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(ic+idx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}$$

$$\Big\downarrow \ 216$$

$$-\frac{\tanh(c+dx)}{2d(a\text{sech}(c+dx)+a)^{3/2}}+\frac{\frac{8\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\text{sech}(c+dx)+a}}\right)}{d}-5a\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(ic+idx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}$$

$$\Big\downarrow \ 4282$$

$$-\frac{\tanh(c+dx)}{2d(a\text{sech}(c+dx)+a)^{3/2}}+\frac{\frac{8\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\text{sech}(c+dx)+a}}\right)}{d}-\frac{10ia\int \frac{1}{2a-\frac{a^2\tanh^2(c+dx)}{\text{sech}(c+dx)a+a}}d\left(-\frac{ia\tanh(c+dx)}{\sqrt{\text{sech}(c+dx)a+a}}\right)}{d}}{4a^2}$$

$$\Big\downarrow \ 216$$

$$\frac{\frac{8\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\text{sech}(c+dx)+a}}\right)}{d}-\frac{5\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a\text{sech}(c+dx)+a}}\right)}{d}}{4a^2}-\frac{\tanh(c+dx)}{2d(a\text{sech}(c+dx)+a)^{3/2}}$$

input
```
Int[(a + a*Sech[c + d*x])^(-3/2),x]
```

output
```
((8*Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/d
- (5*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ech[c + d*x]])])/d)/(4*a^2) - Tanh[c + d*x]/(2*d*(a + a*Sech[c + d*x])^(3/
2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4264
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Simp[1/(a^2*(2*n + 1))
   Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Int
egerQ[2*n]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{1}{(a + a\ \mathrm{sech}\,(dx + c))^{\frac{3}{2}}}dx$$

input
```
int(1/(a+a*sech(d*x+c))^(3/2),x)
```

output
```
int(1/(a+a*sech(d*x+c))^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1190 vs. 2(93) = 186.

Time = 0.13 (sec) , antiderivative size = 1190, normalized size of antiderivative = 10.44

$$\int \frac{1}{(a + a\mathrm{sech}(c + dx))^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/8*(5*sqrt(2)*(cosh(d*x + c)^2 + 2*(cosh(d*x + c) + 1)*sinh(d*x + c) + si
nh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sqrt(a)*log(-(3*a*cosh(d*x + c)^2 + 3
*a*sinh(d*x + c)^2 - 2*sqrt(2)*(cosh(d*x + c)^3 + (3*cosh(d*x + c) - 1)*si
nh(d*x + c)^2 + sinh(d*x + c)^3 - cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 - 2
*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) - 1)*sqrt(a)*sqrt(a/(cos
h(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 2*a
*cosh(d*x + c) + 2*(3*a*cosh(d*x + c) - a)*sinh(d*x + c) + 3*a)/(cosh(d*x
+ c)^2 + 2*(cosh(d*x + c) + 1)*sinh(d*x + c) + sinh(d*x + c)^2 + 2*cosh(d*
x + c) + 1)) + 4*(cosh(d*x + c)^2 + 2*(cosh(d*x + c) + 1)*sinh(d*x + c) +
sinh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sqrt(a)*log(-(a*cosh(d*x + c)^4 + a
*sinh(d*x + c)^4 - 3*a*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) - 3*a)*sinh(d*
x + c)^3 + 5*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 - 9*a*cosh(d*x + c)
+ 5*a)*sinh(d*x + c)^2 + (cosh(d*x + c)^5 + (5*cosh(d*x + c) - 3)*sinh(d*x
 + c)^4 + sinh(d*x + c)^5 - 3*cosh(d*x + c)^4 + (10*cosh(d*x + c)^2 - 12*c
osh(d*x + c) + 5)*sinh(d*x + c)^3 + 5*cosh(d*x + c)^3 + (10*cosh(d*x + c)^
3 - 18*cosh(d*x + c)^2 + 15*cosh(d*x + c) - 7)*sinh(d*x + c)^2 - 7*cosh(d*
x + c)^2 + (5*cosh(d*x + c)^4 - 12*cosh(d*x + c)^3 + 15*cosh(d*x + c)^2 -
14*cosh(d*x + c) + 4)*sinh(d*x + c) + 4*cosh(d*x + c) - 4)*sqrt(a)*sqrt(a/
(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) -
 4*a*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 - 9*a*cosh(d*x + c)^2 + 10*a*...
```

Sympy [F]

$$\int \frac{1}{(a+a\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\operatorname{sech}(c+dx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*sech(d*x+c))**(3/2),x)
```

output
```
Integral((a*sech(c + d*x) + a)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a + a\operatorname{sech}(c + dx))^{3/2}}\,dx = \int \frac{1}{(a\operatorname{sech}(dx + c) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sech(d*x + c) + a)^(-3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a + a\operatorname{sech}(c + dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sech(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a\operatorname{sech}(c + dx))^{3/2}}\,dx = \int \frac{1}{\left(a + \frac{a}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(a + a/cosh(c + d*x))^(3/2),x)
```

output
```
int(1/(a + a/cosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+a\operatorname{sech}(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\operatorname{sech}(dx+c)+1}}{\operatorname{sech}(dx+c)^2+2\operatorname{sech}(dx+c)+1}dx\right)}{a^2}$$

input
```
int(1/(a+a*sech(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int(sqrt(sech(c + d*x) + 1)/(sech(c + d*x)**2 + 2*sech(c + d*x) +
 1),x))/a**2
```

3.83 $\int \sqrt{a - a\text{sech}(c + dx)}\, dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \sqrt{a - a\text{sech}(c + dx)}\, dx = \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-a\text{sech}(c+dx)}}\right)}{d}$$

output
```
2*a^(1/2)*arctanh(a^(1/2)*tanh(d*x+c)/(a-a*sech(d*x+c))^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

$$\int \sqrt{a - a\text{sech}(c + dx)}\, dx = \frac{\sqrt{1+e^{2(c+dx)}}\left(\text{arcsinh}\left(e^{c+dx}\right) + \text{arctanh}\left(\sqrt{1+e^{2(c+dx)}}\right)\right)\sqrt{a - a\text{sech}(c + dx)}}{d\left(-1+e^{c+dx}\right)}$$

input
```
Integrate[Sqrt[a - a*Sech[c + d*x]],x]
```

output
```
(Sqrt[1 + E^(2*(c + d*x))]*(ArcSinh[E^(c + d*x)] + ArcTanh[Sqrt[1 + E^(2*(
c + d*x))]])*Sqrt[a - a*Sech[c + d*x]])/(d*(-1 + E^(c + d*x)))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-a\operatorname{sech}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a-a\csc\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4261}$$

$$-\frac{2ia\int \frac{1}{a-\frac{a^2\tanh^2(c+dx)}{a-a\operatorname{sech}(c+dx)}}d\frac{ia\tanh(c+dx)}{\sqrt{a-a\operatorname{sech}(c+dx)}}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-a\operatorname{sech}(c+dx)}}\right)}{d}$$

input
```
Int[Sqrt[a - a*Sech[c + d*x]],x]
```

output
```
(2*Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a - a*Sech[c + d*x]]])/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sqrt{a - a\,\text{sech}\left(dx + c\right)}dx$$

input
```
int((a-a*sech(d*x+c))^(1/2),x)
```

output
```
int((a-a*sech(d*x+c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(32) = 64$.

Time = 0.10 (sec) , antiderivative size = 642, normalized size of antiderivative = 16.89

$$\int \sqrt{a - a\text{sech}(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a-a*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(a)*log((a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + 3*a*cosh(d*x + c
)^3 + (4*a*cosh(d*x + c) + 3*a)*sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (6
*a*cosh(d*x + c)^2 + 9*a*cosh(d*x + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x
+ c)^5 + (5*cosh(d*x + c) + 3)*sinh(d*x + c)^4 + sinh(d*x + c)^5 + 3*cosh(
d*x + c)^4 + (10*cosh(d*x + c)^2 + 12*cosh(d*x + c) + 5)*sinh(d*x + c)^3 +
 5*cosh(d*x + c)^3 + (10*cosh(d*x + c)^3 + 18*cosh(d*x + c)^2 + 15*cosh(d*
x + c) + 7)*sinh(d*x + c)^2 + 7*cosh(d*x + c)^2 + (5*cosh(d*x + c)^4 + 12*
cosh(d*x + c)^3 + 15*cosh(d*x + c)^2 + 14*cosh(d*x + c) + 4)*sinh(d*x + c)
 + 4*cosh(d*x + c) + 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*
sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + 4*a*cosh(d*x + c) + (4*a*cosh(d*x
+ c)^3 + 9*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) + 4*a)*sinh(d*x + c) + 4
*a)/(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*cosh(d*x + c)*s
inh(d*x + c)^2 + sinh(d*x + c)^3)) + sqrt(a)*log(-(a*cosh(d*x + c)^2 + a*s
inh(d*x + c)^2 + (cosh(d*x + c)^3 + (3*cosh(d*x + c) - 1)*sinh(d*x + c)^2
+ sinh(d*x + c)^3 - cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 - 2*cosh(d*x + c)
 + 1)*sinh(d*x + c) + cosh(d*x + c) - 1)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 +
 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - a*cosh(d*x + c) +
 (2*a*cosh(d*x + c) - a)*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(d*x + c)
)))/d
```

Sympy [F]

$$\int \sqrt{a - a\text{sech}(c + dx)}\,dx = \int \sqrt{-a\,\text{sech}\,(c + dx) + a}\,dx$$

input
```
integrate((a-a*sech(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(-a*sech(c + d*x) + a), x)
```

Maxima [F]

$$\int \sqrt{a-a\text{sech}(c+dx)}\,dx=\int \sqrt{-a\,\text{sech}\,(dx+c)+a}\,dx$$

input
```
integrate((a-a*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a*sech(d*x + c) + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(32)=64$.

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.66

$$\int \sqrt{a-a\text{sech}(c+dx)}\,dx = -\frac{\frac{2\,a\arctan\left(-\frac{\sqrt{a}e^{(dx+c)}-\sqrt{ae^{(2\,dx+2\,c)}+a}}{\sqrt{-a}}\right)\text{sgn}\left(e^{(dx+c)}-1\right)}{\sqrt{-a}}+\sqrt{a}\log\left(\left|-\sqrt{a}e^{(dx+c)}+\sqrt{ae^{(2\,dx+2\,c)}+a}\right|\right)\text{sgn}\left(e^{(dx+c)}-1\right)}{d}$$

input
```
integrate((a-a*sech(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-(2*a*arctan(-(sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x + 2*c) + a))/sqrt(-a)
)*sgn(e^(d*x + c) - 1)/sqrt(-a) + sqrt(a)*log(abs(-sqrt(a)*e^(d*x + c) + s
qrt(a*e^(2*d*x + 2*c) + a)))*sgn(e^(d*x + c) - 1))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a-a\text{sech}(c+dx)}\,dx=\int \sqrt{a-\frac{a}{\cosh\left(c+d\,x\right)}}\,dx$$

input
```
int((a - a/cosh(c + d*x))^(1/2),x)
```

output
```
int((a - a/cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a-a\text{sech}(c+dx)}\,dx=\sqrt{a}\left(\int \sqrt{-\text{sech}\left(dx+c\right)+1}dx\right)$$

input
```
int((a-a*sech(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt( - sech(c + d*x) + 1),x)
```

3.84 $\int \frac{1}{\sqrt{a-a\,\text{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 87

$$\int \frac{1}{\sqrt{a-a\,\text{sech}(c+dx)}}\,dx = \frac{2\,\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-a\,\text{sech}(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a-a\,\text{sech}(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
2*arctanh(a^(1/2)*tanh(d*x+c)/(a-a*sech(d*x+c))^(1/2))/a^(1/2)/d-2^(1/2)*a
rctanh(1/2*a^(1/2)*tanh(d*x+c)*2^(1/2)/(a-a*sech(d*x+c))^(1/2))/a^(1/2)/d
```

Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{a-a\,\text{sech}(c+dx)}}\,dx = \frac{\left(-1+e^{c+dx}\right)\left(\sqrt{2}\,\text{arcsinh}\left(e^{c+dx}\right) - 2\,\text{arctanh}\left(\frac{1+e^{c+dx}}{\sqrt{2}\sqrt{1+e^{2(c+dx)}}}\right) + \sqrt{2}\,\text{arctanh}\left(\sqrt{1+e^{2(c+dx)}}\right)\right)}{\sqrt{2}d\sqrt{1+e^{2(c+dx)}}\sqrt{a-a\,\text{sech}(c+dx)}}$$

input
```
Integrate[1/Sqrt[a - a*Sech[c + d*x]],x]
```

output
```
((-1 + E^(c + d*x))*(Sqrt[2]*ArcSinh[E^(c + d*x)] - 2*ArcTanh[(1 + E^(c +
d*x))/(Sqrt[2]*Sqrt[1 + E^(2*(c + d*x))])] + Sqrt[2]*ArcTanh[Sqrt[1 + E^(2
*(c + d*x))]]))/(Sqrt[2]*d*Sqrt[1 + E^(2*(c + d*x))]*Sqrt[a - a*Sech[c + d
*x]])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 4263, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-a\text{sech}(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{a-a\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4263}$$

$$\int \frac{\text{sech}(c+dx)}{\sqrt{a-a\text{sech}(c+dx)}}dx+\frac{\int \sqrt{a-a\text{sech}(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)}{\sqrt{a-a\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx+\frac{\int \sqrt{a-a\csc\left(ic+idx+\frac{\pi}{2}\right)}dx}{a}$$

$$\Big\downarrow \text{ 4261}$$

$$\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)}{\sqrt{a-a\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx-\frac{2i\int \frac{1}{a-\frac{a^2\tanh^2(c+dx)}{a-a\text{sech}(c+dx)}}d\frac{ia\tanh(c+dx)}{\sqrt{a-a\text{sech}(c+dx)}}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-a\text{sech}(c+dx)}}\right)}{\sqrt{a}d}+\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)}{\sqrt{a-a\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4282}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-a\text{sech}(c+dx)}}\right)}{\sqrt{a}d}+\frac{2i\int \frac{1}{2a-\frac{a^2\tanh^2(c+dx)}{a-a\text{sech}(c+dx)}}d\frac{ia\tanh(c+dx)}{\sqrt{a-a\text{sech}(c+dx)}}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-a\text{sech}(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a-a\text{sech}(c+dx)}}\right)}{\sqrt{a}d}$$

input
```
Int[1/Sqrt[a - a*Sech[c + d*x]],x]
```

output
```
(2*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a - a*Sech[c + d*x]]])/(Sqrt[a]*d)
 - (Sqrt[2]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sech[c + d
*x]])])/(Sqrt[a]*d)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4263
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[1/a   I
nt[Sqrt[a + b*Csc[c + d*x]], x], x] - Simp[b/a   Int[Csc[c + d*x]/Sqrt[a +
b*Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{1}{\sqrt{a - a\,\mathrm{sech}\,(dx + c)}}dx$$

input
```
int(1/(a-a*sech(d*x+c))^(1/2),x)
```

output
```
int(1/(a-a*sech(d*x+c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 871 vs. $2(72) = 144$.

Time = 0.11 (sec) , antiderivative size = 871, normalized size of antiderivative = 10.01

$$\int \frac{1}{\sqrt{a - a\mathrm{sech}(c + dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-a*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*sqrt(a)*log(-(3*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c) + 1)*sin
h(d*x + c) + 3*sinh(d*x + c)^2 - 2*sqrt(2)*(cosh(d*x + c)^3 + (3*cosh(d*x
+ c) + 1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + cosh(d*x + c)^2 + (3*cosh(d*
x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) + 1)*sqrt(a/
(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1))/s
qrt(a) + 2*cosh(d*x + c) + 3)/(cosh(d*x + c)^2 + 2*(cosh(d*x + c) - 1)*sin
h(d*x + c) + sinh(d*x + c)^2 - 2*cosh(d*x + c) + 1)) + sqrt(a)*log((a*cosh
(d*x + c)^4 + a*sinh(d*x + c)^4 + 3*a*cosh(d*x + c)^3 + (4*a*cosh(d*x + c)
 + 3*a)*sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 9*a
*cosh(d*x + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x + c)^5 + (5*cosh(d*x + c
) + 3)*sinh(d*x + c)^4 + sinh(d*x + c)^5 + 3*cosh(d*x + c)^4 + (10*cosh(d*
x + c)^2 + 12*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 5*cosh(d*x + c)^3 + (10
*cosh(d*x + c)^3 + 18*cosh(d*x + c)^2 + 15*cosh(d*x + c) + 7)*sinh(d*x + c
)^2 + 7*cosh(d*x + c)^2 + (5*cosh(d*x + c)^4 + 12*cosh(d*x + c)^3 + 15*cos
h(d*x + c)^2 + 14*cosh(d*x + c) + 4)*sinh(d*x + c) + 4*cosh(d*x + c) + 4)*
sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
 + c)^2 + 1)) + 4*a*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 9*a*cosh(d*x +
c)^2 + 10*a*cosh(d*x + c) + 4*a)*sinh(d*x + c) + 4*a)/(cosh(d*x + c)^3 + 3
*cosh(d*x + c)^2*sinh(d*x + c) + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*
x + c)^3)) + sqrt(a)*log(-(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + (cos...
```

Sympy [F]

$$\int \frac{1}{\sqrt{a - a\mathrm{sech}(c + dx)}}\,dx = \int \frac{1}{\sqrt{-a\,\mathrm{sech}\,(c + dx) + a}}\,dx$$

input
```
integrate(1/(a-a*sech(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(-a*sech(c + d*x) + a), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a - a\text{sech}(c + dx)}}\,dx = \int \frac{1}{\sqrt{-a\,\text{sech}\,(dx + c) + a}}\,dx$$

input
```
integrate(1/(a-a*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-a*sech(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a - a\text{sech}(c + dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-a*sech(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a - a\text{sech}(c + dx)}}\,dx = \int \frac{1}{\sqrt{a - \frac{a}{\cosh(c+d\,x)}}}\,dx$$

input
```
int(1/(a - a/cosh(c + d*x))^(1/2),x)
```

output
```
int(1/(a - a/cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a - a\text{sech}(c + dx)}}\,dx = -\frac{\sqrt{a}\left(\int \frac{\sqrt{-\text{sech}(dx+c)+1}}{\text{sech}(dx+c)-1}dx\right)}{a}$$

input
```
int(1/(a-a*sech(d*x+c))^(1/2),x)
```

output
```
( - sqrt(a)*int(sqrt( - sech(c + d*x) + 1)/(sech(c + d*x) - 1),x))/a
```

3.85 $\int \sqrt{3+3\mathrm{sech}(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 19

$$\int \sqrt{3+3\mathrm{sech}(x)}\,dx = 2\sqrt{3}\mathrm{arctanh}\left(\frac{\tanh(x)}{\sqrt{1+\mathrm{sech}(x)}}\right)$$

output
```
2*3^(1/2)*arctanh(tanh(x)/(1+sech(x))^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(19) = 38.

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \sqrt{3+3\mathrm{sech}(x)}\,dx = \sqrt{6}\mathrm{arcsinh}\left(\sqrt{2}\sinh\left(\frac{x}{2}\right)\right)\sqrt{\cosh(x)}\mathrm{sech}\left(\frac{x}{2}\right)\sqrt{1+\mathrm{sech}(x)}$$

input
```
Integrate[Sqrt[3 + 3*Sech[x]],x]
```

output
```
Sqrt[6]*ArcSinh[Sqrt[2]*Sinh[x/2]]*Sqrt[Cosh[x]]*Sech[x/2]*Sqrt[1 + Sech[x
]]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3\text{sech}(x)+3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{3+3\csc\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{4261}$$

$$6i\int \frac{1}{3-\frac{3\tanh^2(x)}{\text{sech}(x)+1}}d\left(-\frac{i\sqrt{3}\tanh(x)}{\sqrt{\text{sech}(x)+1}}\right)$$

$$\Big\downarrow \text{216}$$

$$2\sqrt{3}\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{\text{sech}(x)+1}}\right)$$

input
```
Int[Sqrt[3 + 3*Sech[x]],x]
```

output
```
2*Sqrt[3]*ArcTanh[Tanh[x]/Sqrt[1 + Sech[x]]]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sqrt{3 + 3\,\mathrm{sech}\,(x)}dx$$

input
```
int((3+3*sech(x))^(1/2),x)
```

output
```
int((3+3*sech(x))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(15) = 30$.

Time = 0.08 (sec) , antiderivative size = 457, normalized size of antiderivative = 24.05

$$\int \sqrt{3 + 3\mathrm{sech}(x)}\,dx = \text{Too large to display}$$

input
```
integrate((3+3*sech(x))^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(3)*log(-(3*cosh(x)^4 + 3*(4*cosh(x) - 3)*sinh(x)^3 + 3*sinh(x)^4
- 9*cosh(x)^3 + 3*(6*cosh(x)^2 - 9*cosh(x) + 5)*sinh(x)^2 + 15*cosh(x)^2 +
 3*(4*cosh(x)^3 - 9*cosh(x)^2 + 10*cosh(x) - 4)*sinh(x) + sqrt(3)*(sqrt(3)
*cosh(x)^5 + sqrt(3)*sinh(x)^5 + (5*sqrt(3)*cosh(x) - 3*sqrt(3))*sinh(x)^4
 - 3*sqrt(3)*cosh(x)^4 + (10*sqrt(3)*cosh(x)^2 - 12*sqrt(3)*cosh(x) + 5*sq
rt(3))*sinh(x)^3 + 5*sqrt(3)*cosh(x)^3 + (10*sqrt(3)*cosh(x)^3 - 18*sqrt(3
)*cosh(x)^2 + 15*sqrt(3)*cosh(x) - 7*sqrt(3))*sinh(x)^2 - 7*sqrt(3)*cosh(x
)^2 + (5*sqrt(3)*cosh(x)^4 - 12*sqrt(3)*cosh(x)^3 + 15*sqrt(3)*cosh(x)^2 -
 14*sqrt(3)*cosh(x) + 4*sqrt(3))*sinh(x) + 4*sqrt(3)*cosh(x) - 4*sqrt(3))/
sqrt(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1) - 12*cosh(x) + 12)/(co
sh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)) + 1/2*sq
rt(3)*log((3*cosh(x)^2 + 3*(2*cosh(x) + 1)*sinh(x) + 3*sinh(x)^2 + sqrt(3)
*(sqrt(3)*cosh(x)^3 + sqrt(3)*sinh(x)^3 + (3*sqrt(3)*cosh(x) + sqrt(3))*si
nh(x)^2 + sqrt(3)*cosh(x)^2 + (3*sqrt(3)*cosh(x)^2 + 2*sqrt(3)*cosh(x) + s
qrt(3))*sinh(x) + sqrt(3)*cosh(x) + sqrt(3))/sqrt(cosh(x)^2 + 2*cosh(x)*si
nh(x) + sinh(x)^2 + 1) + 3*cosh(x) + 3)/(cosh(x) + sinh(x)))
```

Sympy [F]

$$\int \sqrt{3+3\text{sech}(x)}\,dx = \sqrt{3}\int \sqrt{\operatorname{sech}(x)+1}\,dx$$

input
```
integrate((3+3*sech(x))**(1/2),x)
```

output
```
sqrt(3)*Integral(sqrt(sech(x) + 1), x)
```

Maxima [F]

$$\int \sqrt{3+3\text{sech}(x)}\,dx = \int \sqrt{3\,\operatorname{sech}(x)+3}\,dx$$

input
```
integrate((3+3*sech(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(3*sech(x) + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(15) = 30$.

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.74

$$\int \sqrt{3+3\text{sech}(x)}\,dx = -\sqrt{3}\left(\log\left(\sqrt{e^{(2x)}+1}-e^x+1\right)+\log\left(\sqrt{e^{(2x)}+1}-e^x\right)-\log\left(-\sqrt{e^{(2x)}+1}+e^x+1\right)\right)$$

input
```
integrate((3+3*sech(x))^(1/2),x, algorithm="giac")
```

output
```
-sqrt(3)*(log(sqrt(e^(2*x) + 1) - e^x + 1) + log(sqrt(e^(2*x) + 1) - e^x)
- log(-sqrt(e^(2*x) + 1) + e^x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{3+3\text{sech}(x)}\,dx = \int \sqrt{\frac{3}{\cosh(x)}+3}\,dx$$

input
```
int((3/cosh(x) + 3)^(1/2),x)
```

output
```
int((3/cosh(x) + 3)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{3+3\text{sech}(x)}\,dx = \sqrt{3}\left(\int \sqrt{\text{sech}(x)+1}dx\right)$$

input
```
int((3+3*sech(x))^(1/2),x)
```

output
```
sqrt(3)*int(sqrt(sech(x) + 1),x)
```

3.86 $\int \sqrt{3-3\text{sech}(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 21

$$\int \sqrt{3-3\text{sech}(x)}\,dx=2\sqrt{3}\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{1-\text{sech}(x)}}\right)$$

output
```
2*3^(1/2)*arctanh(tanh(x)/(1-sech(x))^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(21) = 42.

Time = 0.51 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.43

$$\int \sqrt{3-3\text{sech}(x)}\,dx=\frac{\sqrt{3}\sqrt{1+e^{2x}}\left(\text{arcsinh}(e^x)+\text{arctanh}\left(\sqrt{1+e^{2x}}\right)\right)\sqrt{1-\text{sech}(x)}}{-1+e^x}$$

input
```
Integrate[Sqrt[3 - 3*Sech[x]],x]
```

output
```
(Sqrt[3]*Sqrt[1 + E^(2*x)]*(ArcSinh[E^x] + ArcTanh[Sqrt[1 + E^(2*x)]])*Sqr
t[1 - Sech[x]])/(-1 + E^x)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3-3\text{sech}(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{3-3\csc\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 4261}$$

$$-6i\int \frac{1}{3-\frac{3\tanh^2(x)}{1-\text{sech}(x)}}d\frac{i\sqrt{3}\tanh(x)}{\sqrt{1-\text{sech}(x)}}$$

$$\Big\downarrow \text{ 216}$$

$$2\sqrt{3}\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{1-\text{sech}(x)}}\right)$$

input
```
Int[Sqrt[3 - 3*Sech[x]],x]
```

output
```
2*Sqrt[3]*ArcTanh[Tanh[x]/Sqrt[1 - Sech[x]]]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sqrt{3 - 3\,\mathrm{sech}\,(x)}dx$$

input
```
int((3-3*sech(x))^(1/2),x)
```

output
```
int((3-3*sech(x))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(17) = 34$.

Time = 0.10 (sec) , antiderivative size = 462, normalized size of antiderivative = 22.00

$$\int \sqrt{3 - 3\mathrm{sech}(x)}\,dx = \text{Too large to display}$$

input
```
integrate((3-3*sech(x))^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(3)*log((3*cosh(x)^4 + 3*(4*cosh(x) + 3)*sinh(x)^3 + 3*sinh(x)^4 +
 9*cosh(x)^3 + 3*(6*cosh(x)^2 + 9*cosh(x) + 5)*sinh(x)^2 + 15*cosh(x)^2 +
3*(4*cosh(x)^3 + 9*cosh(x)^2 + 10*cosh(x) + 4)*sinh(x) + sqrt(3)*(sqrt(3)*
cosh(x)^5 + sqrt(3)*sinh(x)^5 + (5*sqrt(3)*cosh(x) + 3*sqrt(3))*sinh(x)^4
+ 3*sqrt(3)*cosh(x)^4 + (10*sqrt(3)*cosh(x)^2 + 12*sqrt(3)*cosh(x) + 5*sqr
t(3))*sinh(x)^3 + 5*sqrt(3)*cosh(x)^3 + (10*sqrt(3)*cosh(x)^3 + 18*sqrt(3)
*cosh(x)^2 + 15*sqrt(3)*cosh(x) + 7*sqrt(3))*sinh(x)^2 + 7*sqrt(3)*cosh(x)
^2 + (5*sqrt(3)*cosh(x)^4 + 12*sqrt(3)*cosh(x)^3 + 15*sqrt(3)*cosh(x)^2 +
14*sqrt(3)*cosh(x) + 4*sqrt(3))*sinh(x) + 4*sqrt(3)*cosh(x) + 4*sqrt(3))/s
qrt(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1) + 12*cosh(x) + 12)/(cos
h(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)) + 1/2*sqr
t(3)*log(-(3*cosh(x)^2 + 3*(2*cosh(x) - 1)*sinh(x) + 3*sinh(x)^2 + sqrt(3)
*(sqrt(3)*cosh(x)^3 + sqrt(3)*sinh(x)^3 + (3*sqrt(3)*cosh(x) - sqrt(3))*si
nh(x)^2 - sqrt(3)*cosh(x)^2 + (3*sqrt(3)*cosh(x)^2 - 2*sqrt(3)*cosh(x) + s
qrt(3))*sinh(x) + sqrt(3)*cosh(x) - sqrt(3))/sqrt(cosh(x)^2 + 2*cosh(x)*si
nh(x) + sinh(x)^2 + 1) - 3*cosh(x) + 3)/(cosh(x) + sinh(x)))
```

Sympy [F]

$$\int \sqrt{3 - 3\text{sech}(x)}\, dx = \sqrt{3} \int \sqrt{1 - \text{sech}\,(x)}\, dx$$

input
```
integrate((3-3*sech(x))**(1/2),x)
```

output
```
sqrt(3)*Integral(sqrt(1 - sech(x)), x)
```

Maxima [F]

$$\int \sqrt{3 - 3\text{sech}(x)}\, dx = \int \sqrt{-3\,\text{sech}\,(x) + 3}\, dx$$

input
```
integrate((3-3*sech(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-3*sech(x) + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.29

$$\int \sqrt{3-3\text{sech}(x)}\,dx$$
$$=\sqrt{3}\Big(\log\left(\sqrt{e^{(2x)}+1}-e^{x}+1\right)\text{sgn}(e^{x}-1)-\log\left(\sqrt{e^{(2x)}+1}-e^{x}\right)\text{sgn}(e^{x}-1)-\log\left(-\sqrt{e^{(2x)}+1}+\right.$$

input
```
integrate((3-3*sech(x))^(1/2),x, algorithm="giac")
```

output
```
sqrt(3)*(log(sqrt(e^(2*x) + 1) - e^x + 1)*sgn(e^x - 1) - log(sqrt(e^(2*x)
+ 1) - e^x)*sgn(e^x - 1) - log(-sqrt(e^(2*x) + 1) + e^x + 1)*sgn(e^x - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{3-3\text{sech}(x)}\,dx = \int \sqrt{3-\frac{3}{\cosh(x)}}\,dx$$

input
```
int((3 - 3/cosh(x))^(1/2),x)
```

output
```
int((3 - 3/cosh(x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{3-3\text{sech}(x)}\,dx = \sqrt{3}\left(\int \sqrt{-\text{sech}(x)+1}dx\right)$$

input
```
int((3-3*sech(x))^(1/2),x)
```

output
```
sqrt(3)*int(sqrt( - sech(x) + 1),x)
```

3.87 $\int (a + b\text{sech}(c + dx))^4 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 107

$$\int (a + b\text{sech}(c + dx))^4 \, dx = a^4x + \frac{2ab(2a^2 + b^2)\arctan(\sinh(c + dx))}{d} + \frac{b^2(17a^2 + 2b^2)\tanh(c + dx)}{3d} + \frac{4ab^3\text{sech}(c + dx)\tanh(c + dx)}{3d} + \frac{b^2(a + b\text{sech}(c + dx))^2\tanh(c + dx)}{3d}$$

output
```
a^4*x+2*a*b*(2*a^2+b^2)*arctan(sinh(d*x+c))/d+1/3*b^2*(17*a^2+2*b^2)*tanh(
d*x+c)/d+4/3*a*b^3*sech(d*x+c)*tanh(d*x+c)/d+1/3*b^2*(a+b*sech(d*x+c))^2*t
anh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int (a+b\text{sech}(c+dx))^4\,dx$$
$$=\frac{3a(a^3dx-4a^2b\cot^{-1}(\sinh(c+dx))+2b^3\arctan(\sinh(c+dx)))+3b^2(6a^2+b^2+2ab\text{sech}(c+dx))\tanh}{3d}$$

input
```
Integrate[(a + b*Sech[c + d*x])^4,x]
```

output
```
(3*a*(a^3*d*x - 4*a^2*b*ArcCot[Sinh[c + d*x]] + 2*b^3*ArcTan[Sinh[c + d*x]
]) + 3*b^2*(6*a^2 + b^2 + 2*a*b*Sech[c + d*x])*Tanh[c + d*x] - b^4*Tanh[c
+ d*x]^3)/(3*d)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 4269, 3042, 4536, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{sech}(c+dx))^4\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \left(a+b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^4dx$$
$$\Big\downarrow \text{4269}$$
$$\frac{1}{3}\int (a+b\text{sech}(c+dx))\left(3a^3+8b^2\text{sech}^2(c+dx)a+b\left(9a^2+2b^2\right)\text{sech}(c+dx)\right)dx+\frac{b^2\tanh(c+dx)(a+b\text{sech}(c+dx))^2}{3d}$$
$$\Big\downarrow \text{3042}$$

$$\frac{b^2\tanh(c+dx)(a+b\text{sech}(c+dx))^2}{3d}+$$
$$\frac{1}{3}\int\left(a+b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)\left(3a^3+8b^2\csc\left(ic+idx+\frac{\pi}{2}\right)^2a+b(9a^2+2b^2)\csc\left(ic+idx+\frac{\pi}{2}\right)\right)dx$$

$\big\downarrow$ 4536

$$\frac{1}{3}\left(\frac{1}{2}\int\left(6a^4+12b(2a^2+b^2)\,\text{sech}(c+dx)a+2b^2(17a^2+2b^2)\,\text{sech}^2(c+dx)\right)dx+\frac{4ab^3\tanh(c+dx)\text{sech}(c+d\right.$$
$$\frac{b^2\tanh(c+dx)(a+b\text{sech}(c+dx))^2}{3d}$$

$\big\downarrow$ 2009

$$\frac{1}{3}\left(\frac{1}{2}\left(6a^4x+\frac{12ab(2a^2+b^2)\arctan(\sinh(c+dx))}{d}+\frac{2b^2(17a^2+2b^2)\tanh(c+dx)}{d}\right)+\frac{4ab^3\tanh(c+dx)\text{sech}(\right.$$
$$\frac{b^2\tanh(c+dx)(a+b\text{sech}(c+dx))^2}{3d}$$

input
```
Int[(a + b*Sech[c + d*x])^4,x]
```

output
```
(b^2*(a + b*Sech[c + d*x])^2*Tanh[c + d*x])/(3*d) + ((4*a*b^3*Sech[c + d*x
]*Tanh[c + d*x])/d + (6*a^4*x + (12*a*b*(2*a^2 + b^2)*ArcTan[Sinh[c + d*x]
])/d + (2*b^2*(17*a^2 + 2*b^2)*Tanh[c + d*x])/d)/2)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4269
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[1/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*
a^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4536
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Simp[1/2   Int[Simp[2*A*a + (2*B*a + b*(2*
A + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, e, f, A, B, C}, x]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{a^4(dx+c)+8a^3b\arctan\left(e^{dx+c}\right)+6a^2b^2\tanh(dx+c)+4a\,b^3\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b^4\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c}{3}\right.}{d}$
default	$\frac{a^4(dx+c)+8a^3b\arctan\left(e^{dx+c}\right)+6a^2b^2\tanh(dx+c)+4a\,b^3\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b^4\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c}{3}\right.}{d}$
parts	$a^4x+\frac{b^4\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{d}+\frac{4a\,b^3\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)}{d}+\frac{6a^2b^2\tanh(dx+c)}{d}$ -
risch	$a^4x-\frac{4b^2\left(-3ab\,e^{5dx+5c}+9a^2e^{4dx+4c}+18a^2e^{2dx+2c}+3b^2e^{2dx+2c}+3ab\,e^{dx+c}+9a^2+b^2\right)}{3d\left(e^{2dx+2c}+1\right)^3}+\frac{4ia^3b\ln\left(e^{dx+c}+i\right)}{d}+$
parallelrisch	$\frac{-36i\left(\frac{\cosh(3dx+3c)}{3}+\cosh(dx+c)\right)b\left(a^2+\frac{b^2}{2}\right)a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)+36i\left(\frac{\cosh(3dx+3c)}{3}+\cosh(dx+c)\right)b\left(a^2+\frac{b^2}{2}\right)a\ln}{3d(\mathrm{co}}$

input
```
int((a+b*sech(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*(d*x+c)+8*a^3*b*arctan(exp(d*x+c))+6*a^2*b^2*tanh(d*x+c)+4*a*b^3*
(1/2*sech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+b^4*(2/3+1/3*sech(d*x+c)^
2)*tanh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(101) = 202$.

Time = 0.09 (sec) , antiderivative size = 1028, normalized size of antiderivative = 9.61

$$\int (a + b\text{sech}(c + dx))^4 \, dx = \text{Too large to display}$$

input
```
integrate((a+b*sech(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/3*(3*a^4*d*x*cosh(d*x + c)^6 + 3*a^4*d*x*sinh(d*x + c)^6 + 12*a*b^3*cosh
(d*x + c)^5 + 3*a^4*d*x + 6*(3*a^4*d*x*cosh(d*x + c) + 2*a*b^3)*sinh(d*x +
 c)^5 - 12*a*b^3*cosh(d*x + c) + 9*(a^4*d*x - 4*a^2*b^2)*cosh(d*x + c)^4 +
 3*(15*a^4*d*x*cosh(d*x + c)^2 + 3*a^4*d*x + 20*a*b^3*cosh(d*x + c) - 12*a
^2*b^2)*sinh(d*x + c)^4 - 36*a^2*b^2 - 4*b^4 + 12*(5*a^4*d*x*cosh(d*x + c)
^3 + 10*a*b^3*cosh(d*x + c)^2 + 3*(a^4*d*x - 4*a^2*b^2)*cosh(d*x + c))*sin
h(d*x + c)^3 + 3*(3*a^4*d*x - 24*a^2*b^2 - 4*b^4)*cosh(d*x + c)^2 + 3*(15*
a^4*d*x*cosh(d*x + c)^4 + 40*a*b^3*cosh(d*x + c)^3 + 3*a^4*d*x - 24*a^2*b^
2 - 4*b^4 + 18*(a^4*d*x - 4*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 12
*((2*a^3*b + a*b^3)*cosh(d*x + c)^6 + 6*(2*a^3*b + a*b^3)*cosh(d*x + c)*si
nh(d*x + c)^5 + (2*a^3*b + a*b^3)*sinh(d*x + c)^6 + 3*(2*a^3*b + a*b^3)*co
sh(d*x + c)^4 + 3*(2*a^3*b + a*b^3 + 5*(2*a^3*b + a*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^4 + 2*a^3*b + a*b^3 + 4*(5*(2*a^3*b + a*b^3)*cosh(d*x + c)^3
 + 3*(2*a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(2*a^3*b + a*b^3
)*cosh(d*x + c)^2 + 3*(5*(2*a^3*b + a*b^3)*cosh(d*x + c)^4 + 2*a^3*b + a*b
^3 + 6*(2*a^3*b + a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 6*((2*a^3*b +
a*b^3)*cosh(d*x + c)^5 + 2*(2*a^3*b + a*b^3)*cosh(d*x + c)^3 + (2*a^3*b +
a*b^3)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c))
 + 6*(3*a^4*d*x*cosh(d*x + c)^5 + 10*a*b^3*cosh(d*x + c)^4 - 2*a*b^3 + 6*(
a^4*d*x - 4*a^2*b^2)*cosh(d*x + c)^3 + (3*a^4*d*x - 24*a^2*b^2 - 4*b^4)...
```

Sympy [F]

$$\int (a + b\text{sech}(c + dx))^4 \, dx = \int (a + b \operatorname{sech}(c + dx))^4 \, dx$$

input
```
integrate((a+b*sech(d*x+c))**4,x)
```

output
```
Integral((a + b*sech(c + d*x))**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(101) = 202$.

Time = 0.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.97

$$\begin{aligned}
&\int (a + b\text{sech}(c + dx))^4 \, dx \\
&= a^4 x - 4\, ab^3 \left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)} \right) \\
&\quad + \frac{4}{3} b^4 \left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} + 1)} + \frac{1}{d(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} + 1)} \right. \\
&\quad + \frac{4\,a^3 b \arctan\left(\sinh\left(dx + c\right)\right)}{d} + \frac{12\,a^2 b^2}{d(e^{(-2\,dx-2\,c)} + 1)}
\end{aligned}$$

input
```
integrate((a+b*sech(d*x+c))^4,x, algorithm="maxima")
```

output
```
a^4*x - 4*a*b^3*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c)
)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 4/3*b^4*(3*e^(-2*d*x
- 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)
) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))
) + 4*a^3*b*arctan(sinh(d*x + c))/d + 12*a^2*b^2/(d*(e^(-2*d*x - 2*c) + 1)
)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.32

$$\int (a+b\text{sech}(c+dx))^4\,dx$$
$$=\frac{3\,(dx+c)a^4+12\,(2\,a^3b+ab^3)\arctan\left(e^{(dx+c)}\right)+\frac{4\,\left(3\,ab^3e^{(5\,dx+5\,c)}-9\,a^2b^2e^{(4\,dx+4\,c)}-18\,a^2b^2e^{(2\,dx+2\,c)}-3\,b^4e^{(2\,dx+2\,c)}-3}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

input
```
integrate((a+b*sech(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*a^4 + 12*(2*a^3*b + a*b^3)*arctan(e^(d*x + c)) + 4*(3*a*b
^3*e^(5*d*x + 5*c) - 9*a^2*b^2*e^(4*d*x + 4*c) - 18*a^2*b^2*e^(2*d*x + 2*c
) - 3*b^4*e^(2*d*x + 2*c) - 3*a*b^3*e^(d*x + c) - 9*a^2*b^2 - b^4)/(e^(2*d
*x + 2*c) + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.18

$$\int (a+b\text{sech}(c+dx))^4\,dx=a^4\,x-\frac{\frac{12\,a^2\,b^2}{d}-\frac{4\,a\,b^3\,e^{c+d\,x}}{d}}{e^{2\,c+2\,d\,x}+1}-\frac{\frac{4\,b^4}{d}+\frac{8\,a\,b^3\,e^{c+d\,x}}{d}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1}$$
$$+\frac{8\,b^4}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}$$
$$+\frac{4\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(a\,b^3\,\sqrt{d^2}+2\,a^3\,b\,\sqrt{d^2}\right)}{d\,\sqrt{4\,a^6\,b^2+4\,a^4\,b^4+a^2\,b^6}}\right)\,\sqrt{4\,a^6\,b^2+4\,a^4\,b^4+a^2\,b^6}}{\sqrt{d^2}}$$

input
```
int((a + b/cosh(c + d*x))^4,x)
```

output
```
a^4*x - ((12*a^2*b^2)/d - (4*a*b^3*exp(c + d*x))/d)/(exp(2*c + 2*d*x) + 1)
 - ((4*b^4)/d + (8*a*b^3*exp(c + d*x))/d)/(2*exp(2*c + 2*d*x) + exp(4*c +
4*d*x) + 1) + (8*b^4)/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(
6*c + 6*d*x) + 1)) + (4*atan((exp(d*x)*exp(c)*(a*b^3*(d^2)^(1/2) + 2*a^3*b
*(d^2)^(1/2)))/(d*(a^2*b^6 + 4*a^4*b^4 + 4*a^6*b^2)^(1/2)))*(a^2*b^6 + 4*a
^4*b^4 + 4*a^6*b^2)^(1/2))/(d^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 367, normalized size of antiderivative = 3.43

$$\int (a + b\operatorname{sech}(c + dx))^4 \, dx$$
$$= \frac{24e^{6dx+6c} atan\left(e^{dx+c}\right) a^3b + 12e^{6dx+6c} atan\left(e^{dx+c}\right) a\, b^3 + 72e^{4dx+4c} atan\left(e^{dx+c}\right) a^3b + 36e^{4dx+4c} atan\left(e^{dx+c}\right.}{}$$

input
```
int((a+b*sech(d*x+c))^4,x)
```

output
```
(24*e**(6*c + 6*d*x)*atan(e**(c + d*x))*a**3*b + 12*e**(6*c + 6*d*x)*atan(
e**(c + d*x))*a*b**3 + 72*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**3*b + 36*
e**(4*c + 4*d*x)*atan(e**(c + d*x))*a*b**3 + 72*e**(2*c + 2*d*x)*atan(e**(
c + d*x))*a**3*b + 36*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a*b**3 + 24*atan
(e**(c + d*x))*a**3*b + 12*atan(e**(c + d*x))*a*b**3 + 3*e**(6*c + 6*d*x)*
a**4*d*x + 12*e**(6*c + 6*d*x)*a**2*b**2 + 12*e**(5*c + 5*d*x)*a*b**3 + 9*
e**(4*c + 4*d*x)*a**4*d*x + 9*e**(2*c + 2*d*x)*a**4*d*x - 36*e**(2*c + 2*d
*x)*a**2*b**2 - 12*e**(2*c + 2*d*x)*b**4 - 12*e**(c + d*x)*a*b**3 + 3*a**4
*d*x - 24*a**2*b**2 - 4*b**4)/(3*d*(e**(6*c + 6*d*x) + 3*e**(4*c + 4*d*x)
+ 3*e**(2*c + 2*d*x) + 1))
```

3.88 $\int (a+b\text{sech}(c+dx))^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 73

$$\int (a+b\text{sech}(c+dx))^3\,dx = a^3x + \frac{b(6a^2+b^2)\arctan(\sinh(c+dx))}{2d} + \frac{5ab^2\tanh(c+dx)}{2d} + \frac{b^2(a+b\text{sech}(c+dx))\tanh(c+dx)}{2d}$$

output
```
a^3*x+1/2*b*(6*a^2+b^2)*arctan(sinh(d*x+c))/d+5/2*a*b^2*tanh(d*x+c)/d+1/2*
b^2*(a+b*sech(d*x+c))*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int (a+b\text{sech}(c+dx))^3\,dx = \frac{2a^3dx - 6a^2b\cot^{-1}(\sinh(c+dx)) + b^3\arctan(\sinh(c+dx)) + b^2(6a+b\text{sech}(c+dx))\tanh(c+dx)}{2d}$$

input
```
Integrate[(a + b*Sech[c + d*x])^3,x]
```

output
```
(2*a^3*d*x - 6*a^2*b*ArcCot[Sinh[c + d*x]] + b^3*ArcTan[Sinh[c + d*x]] + b
^2*(6*a + b*Sech[c + d*x])*Tanh[c + d*x])/(2*d)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4269, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{sech}(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4269}$$

$$\frac{1}{2}\int \left(2a^3+5b^2\text{sech}^2(c+dx)a+b\left(6a^2+b^2\right)\text{sech}(c+dx)\right)dx+\frac{b^2\tanh(c+dx)(a+b\text{sech}(c+dx))}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(2a^3x+\frac{b\left(6a^2+b^2\right)\arctan(\sinh(c+dx))}{d}+\frac{5ab^2\tanh(c+dx)}{d}\right)+\frac{b^2\tanh(c+dx)(a+b\text{sech}(c+dx))}{2d}$$

input
```
Int[(a + b*Sech[c + d*x])^3,x]
```

output
```
(b^2*(a + b*Sech[c + d*x])*Tanh[c + d*x])/(2*d) + (2*a^3*x + (b*(6*a^2 + b
^2)*ArcTan[Sinh[c + d*x]])/d + (5*a*b^2*Tanh[c + d*x])/d)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4269
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[1/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*
a^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{a^3(dx+c)+6a^2b\arctan(e^{dx+c})+3\tanh(dx+c)a\,b^2+b^3\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(e^{dx+c})\right)}{d}$
default	$\frac{a^3(dx+c)+6a^2b\arctan(e^{dx+c})+3\tanh(dx+c)a\,b^2+b^3\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(e^{dx+c})\right)}{d}$
parts	$a^3x+\frac{b^3\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(e^{dx+c})\right)}{d}+\frac{3a\,b^2\tanh(dx+c)}{d}+\frac{3a^2b\arctan(\sinh(dx+c))}{d}$
parallelrisch	$\frac{-3i\left(a^2+\frac{b^2}{6}\right)b(\cosh(2dx+2c)+1)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)+3i\left(a^2+\frac{b^2}{6}\right)b(\cosh(2dx+2c)+1)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)+a^3dx}{d(\cosh(2dx+2c)+1)}$
risch	$a^3x-\frac{b^2\left(-b\,e^{3dx+3c}+6a\,e^{2dx+2c}+b\,e^{dx+c}+6a\right)}{d\left(e^{2dx+2c}+1\right)^2}+\frac{3ib\ln(e^{dx+c}+i)a^2}{d}+\frac{ib^3\ln(e^{dx+c}+i)}{2d}-\frac{3ib\ln(e^{dx+c}-i)a^2}{d}$

input
```
int((a+b*sech(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(d*x+c)+6*a^2*b*arctan(exp(d*x+c))+3*tanh(d*x+c)*a*b^2+b^3*(1/2*s
ech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(67) = 134$.

Time = 0.10 (sec) , antiderivative size = 521, normalized size of antiderivative = 7.14

$$\int (a + b\text{sech}(c + dx))^3 \, dx$$
$$= \frac{a^3 dx \cosh{(dx + c)}^4 + a^3 dx \sinh{(dx + c)}^4 + b^3 \cosh{(dx + c)}^3 + a^3 dx - b^3 \cosh{(dx + c)} + (4\, a^3 dx \cosh{(}}{}$$

input
```
integrate((a+b*sech(d*x+c))^3,x, algorithm="fricas")
```

output
```
(a^3*d*x*cosh(d*x + c)^4 + a^3*d*x*sinh(d*x + c)^4 + b^3*cosh(d*x + c)^3 +
 a^3*d*x - b^3*cosh(d*x + c) + (4*a^3*d*x*cosh(d*x + c) + b^3)*sinh(d*x +
c)^3 - 6*a*b^2 + 2*(a^3*d*x - 3*a*b^2)*cosh(d*x + c)^2 + (6*a^3*d*x*cosh(d
*x + c)^2 + 2*a^3*d*x + 3*b^3*cosh(d*x + c) - 6*a*b^2)*sinh(d*x + c)^2 + (
(6*a^2*b + b^3)*cosh(d*x + c)^4 + 4*(6*a^2*b + b^3)*cosh(d*x + c)*sinh(d*x
 + c)^3 + (6*a^2*b + b^3)*sinh(d*x + c)^4 + 6*a^2*b + b^3 + 2*(6*a^2*b + b
^3)*cosh(d*x + c)^2 + 2*(6*a^2*b + b^3 + 3*(6*a^2*b + b^3)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 4*((6*a^2*b + b^3)*cosh(d*x + c)^3 + (6*a^2*b + b^3)*c
osh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + (4*a^
3*d*x*cosh(d*x + c)^3 + 3*b^3*cosh(d*x + c)^2 - b^3 + 4*(a^3*d*x - 3*a*b^2
)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sin
h(d*x + c)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x +
 c)^2 + d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(
d*x + c) + d)
```

Sympy [F]

$$\int (a + b\text{sech}(c + dx))^3 \, dx = \int (a + b \operatorname{sech}{(c + dx)})^3 \, dx$$

input
```
integrate((a+b*sech(d*x+c))**3,x)
```

output
```
Integral((a + b*sech(c + d*x))**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.56

$$\int (a + b\text{sech}(c + dx))^3 \, dx = a^3 x - b^3 \left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)} \right) + \frac{3\,a^2 b \arctan\left(\sinh\left(dx + c\right)\right)}{d} + \frac{6\,ab^2}{d(e^{(-2\,dx-2\,c)} + 1)}$$

input
```
integrate((a+b*sech(d*x+c))^3,x, algorithm="maxima")
```

output
```
a^3*x - b^3*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d
*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 3*a^2*b*arctan(sinh(d*x +
 c))/d + 6*a*b^2/(d*(e^(-2*d*x - 2*c) + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

$$\int (a + b\text{sech}(c + dx))^3 \, dx = \frac{(dx + c)a^3 + (6\,a^2 b + b^3) \arctan\left(e^{(dx+c)}\right) + \frac{b^3 e^{(3\,dx+3\,c)} - 6\,ab^2 e^{(2\,dx+2\,c)} - b^3 e^{(dx+c)} - 6\,ab^2}{\left(e^{(2\,dx+2\,c)}+1\right)^2}}{d}$$

input
```
integrate((a+b*sech(d*x+c))^3,x, algorithm="giac")
```

output
```
((d*x + c)*a^3 + (6*a^2*b + b^3)*arctan(e^(d*x + c)) + (b^3*e^(3*d*x + 3*c
) - 6*a*b^2*e^(2*d*x + 2*c) - b^3*e^(d*x + c) - 6*a*b^2)/(e^(2*d*x + 2*c)
+ 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.26

$$\int (a+b\operatorname{sech}(c+dx))^3\,dx = a^3\,x-\frac{\frac{6\,a\,b^2}{d}-\frac{b^3\,\mathrm{e}^{c+d\,x}}{d}}{\mathrm{e}^{2\,c+2\,d\,x}+1}+\frac{\operatorname{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(b^3\,\sqrt{d^2}+6\,a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{36\,a^4\,b^2+12\,a^2\,b^4+b^6}}\right)\,\sqrt{36\,a^4\,b^2+12\,a^2\,b^4+b^6}}{\sqrt{d^2}}-\frac{2\,b^3\,\mathrm{e}^{c+d\,x}}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

input
```
int((a + b/cosh(c + d*x))^3,x)
```

output
```
a^3*x - ((6*a*b^2)/d - (b^3*exp(c + d*x))/d)/(exp(2*c + 2*d*x) + 1) + (ata
n((exp(d*x)*exp(c)*(b^3*(d^2)^(1/2) + 6*a^2*b*(d^2)^(1/2)))/(d*(b^6 + 12*a
^2*b^4 + 36*a^4*b^2)^(1/2)))*(b^6 + 12*a^2*b^4 + 36*a^4*b^2)^(1/2))/(d^2)^
(1/2) - (2*b^3*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1
))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.25

$$\int (a+b\operatorname{sech}(c+dx))^3\,dx = \frac{6e^{4dx+4c}atan\left(e^{dx+c}\right)a^2b+e^{4dx+4c}atan\left(e^{dx+c}\right)b^3+12e^{2dx+2c}atan\left(e^{dx+c}\right)a^2b+2e^{2dx+2c}atan\left(e^{dx+c}\right)b^3+ \cdots}{d\left(e^{4dx+\cdots}\right.}$$

input
```
int((a+b*sech(d*x+c))^3,x)
```

output
```
(6*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**2*b + e**(4*c + 4*d*x)*atan(e**(
c + d*x))*b**3 + 12*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**2*b + 2*e**(2*c
 + 2*d*x)*atan(e**(c + d*x))*b**3 + 6*atan(e**(c + d*x))*a**2*b + atan(e**
(c + d*x))*b**3 + e**(4*c + 4*d*x)*a**3*d*x + 3*e**(4*c + 4*d*x)*a*b**2 +
e**(3*c + 3*d*x)*b**3 + 2*e**(2*c + 2*d*x)*a**3*d*x - e**(c + d*x)*b**3 +
a**3*d*x - 3*a*b**2)/(d*(e**(4*c + 4*d*x) + 2*e**(2*c + 2*d*x) + 1))
```

3.89 $\int (a+b\text{sech}(c+dx))^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int (a+b\text{sech}(c+dx))^2\,dx = a^2x+\frac{2ab\arctan(\sinh(c+dx))}{d}+\frac{b^2\tanh(c+dx)}{d}$$

output
```
a^2*x+2*a*b*arctan(sinh(d*x+c))/d+b^2*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int (a+b\text{sech}(c+dx))^2\,dx = \frac{a(adx-2b\cot^{-1}(\sinh(c+dx)))+b^2\tanh(c+dx)}{d}$$

input
```
Integrate[(a + b*Sech[c + d*x])^2,x]
```

output
```
(a*(a*d*x - 2*b*ArcCot[Sinh[c + d*x]]) + b^2*Tanh[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4260, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{sech}(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4260}$$

$$2ab\int \text{sech}(c+dx)dx+b^2\int \text{sech}^2(c+dx)dx+a^2x$$

$$\Big\downarrow \text{3042}$$

$$2ab\int \csc\left(ic+idx+\frac{\pi}{2}\right)dx+b^2\int \csc\left(ic+idx+\frac{\pi}{2}\right)^2dx+a^2x$$

$$\Big\downarrow \text{4254}$$

$$2ab\int \csc\left(ic+idx+\frac{\pi}{2}\right)dx+\frac{ib^2\int 1d(-i\tanh(c+dx))}{d}+a^2x$$

$$\Big\downarrow \text{24}$$

$$2ab\int \csc\left(ic+idx+\frac{\pi}{2}\right)dx+a^2x+\frac{b^2\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$a^2x+\frac{2ab\arctan(\sinh(c+dx))}{d}+\frac{b^2\tanh(c+dx)}{d}$$

input
```
Int[(a + b*Sech[c + d*x])^2,x]
```

output
```
a^2*x + (2*a*b*ArcTan[Sinh[c + d*x]])/d + (b^2*Tanh[c + d*x])/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4260
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Simp[2*a*b   Int[Csc[c + d*x], x], x] + Simp[b^2   Int[Csc[c + d*x]^2, x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

method	result
parts	$a^2x+\frac{2ab\arctan(\sinh(dx+c))}{d}+\frac{b^2\tanh(dx+c)}{d}$
derivativedivides	$\frac{a^2(dx+c)+4ab\arctan(\mathrm{e}^{dx+c})+b^2\tanh(dx+c)}{d}$
default	$\frac{a^2(dx+c)+4ab\arctan(\mathrm{e}^{dx+c})+b^2\tanh(dx+c)}{d}$
risch	$a^2x-\frac{2b^2}{d(\mathrm{e}^{2dx+2c}+1)}+\frac{2iba\ln(\mathrm{e}^{dx+c}+i)}{d}-\frac{2iba\ln(\mathrm{e}^{dx+c}-i)}{d}$
parallelrisch	$\frac{-2i\cosh(dx+c)ba\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)+2i\cosh(dx+c)ba\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)+a^2dx\cosh(dx+c)+b^2\sinh(dx+c)}{\cosh(dx+c)d}$

input
```
int((a+b*sech(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+2*a*b*arctan(sinh(d*x+c))/d+b^2*tanh(d*x+c)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(33) = 66$.

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 4.76

$$\int (a + b\text{sech}(c + dx))^2 \, dx$$
$$= \frac{a^2 dx \cosh{(dx + c)}^2 + 2\,a^2 dx \cosh{(dx + c)} \sinh{(dx + c)} + a^2 dx \sinh{(dx + c)}^2 + a^2 dx - 2\,b^2 + 4\,(ab \cosh}{d \cosh{(dx + c)}^2 + 2\,d \cosh{(dx +}}$$

input
```
integrate((a+b*sech(d*x+c))^2,x, algorithm="fricas")
```

output
```
(a^2*d*x*cosh(d*x + c)^2 + 2*a^2*d*x*cosh(d*x + c)*sinh(d*x + c) + a^2*d*x
*sinh(d*x + c)^2 + a^2*d*x - 2*b^2 + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d
*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + a*b)*arctan(cosh(d*x + c) +
sinh(d*x + c)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*s
inh(d*x + c)^2 + d)
```

Sympy [F]

$$\int (a + b\text{sech}(c + dx))^2 \, dx = \int (a + b \operatorname{sech}{(c + dx)})^2 \, dx$$

input
```
integrate((a+b*sech(d*x+c))**2,x)
```

output
```
Integral((a + b*sech(c + d*x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int (a+b\text{sech}(c+dx))^2\,dx = a^2x + \frac{2\,ab\arctan\left(\sinh\left(dx+c\right)\right)}{d} + \frac{2\,b^2}{d(e^{(-2\,dx-2\,c)}+1)}$$

input
```
integrate((a+b*sech(d*x+c))^2,x, algorithm="maxima")
```

output
```
a^2*x + 2*a*b*arctan(sinh(d*x + c))/d + 2*b^2/(d*(e^(-2*d*x - 2*c) + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

$$\int (a+b\text{sech}(c+dx))^2\,dx = \frac{(dx+c)a^2 + 4\,ab\arctan\left(e^{(dx+c)}\right) - \frac{2\,b^2}{e^{(2\,dx+2\,c)}+1}}{d}$$

input
```
integrate((a+b*sech(d*x+c))^2,x, algorithm="giac")
```

output
```
((d*x + c)*a^2 + 4*a*b*arctan(e^(d*x + c)) - 2*b^2/(e^(2*d*x + 2*c) + 1))/
d
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.12

$$\int (a+b\text{sech}(c+dx))^2\,dx = a^2\,x - \frac{2\,b^2}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)} + \frac{4\,\mathrm{atan}\left(\frac{a\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{d^2}}$$

input
```
int((a + b/cosh(c + d*x))^2,x)
```

output
```
a^2*x - (2*b^2)/(d*(exp(2*c + 2*d*x) + 1)) + (4*atan((a*b*exp(d*x)*exp(c)*
(d^2)^(1/2))/(d*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(d^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.73

$$\int (a + b\text{sech}(c + dx))^2 \, dx = \frac{4e^{2dx+2c}atan(e^{dx+c})\,ab + 4atan(e^{dx+c})\,ab + e^{2dx+2c}a^2dx + 2e^{2dx+2c}b^2 + a^2dx}{d\,(e^{2dx+2c} + 1)}$$

input
```
int((a+b*sech(d*x+c))^2,x)
```

output
```
(4*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a*b + 4*atan(e**(c + d*x))*a*b + e*
*(2*c + 2*d*x)*a**2*d*x + 2*e**(2*c + 2*d*x)*b**2 + a**2*d*x)/(d*(e**(2*c
+ 2*d*x) + 1))
```

3.90 $\int (a + b\text{sech}(c + dx))\, dx$

Optimal result

Integrand size = 10, antiderivative size = 16

$$\int (a + b\text{sech}(c + dx))\, dx = ax + \frac{b \arctan(\sinh(c + dx))}{d}$$

output
```
a*x+b*arctan(sinh(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (a + b\text{sech}(c + dx))\, dx = ax - \frac{b \cot^{-1}(\sinh(c + dx))}{d}$$

input
```
Integrate[a + b*Sech[c + d*x],x]
```

output
```
a*x - (b*ArcCot[Sinh[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\text{sech}(c + dx)) \, dx$$

$$\Big\downarrow \; 2009$$

$$ax + \frac{b \arctan(\sinh(c + dx))}{d}$$

input
```
Int[a + b*Sech[c + d*x],x]
```

output
```
a*x + (b*ArcTan[Sinh[c + d*x]])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
default	$xa + \frac{b \arctan(\sinh(dx+c))}{d}$	17
parts	$xa + \frac{b \arctan(\sinh(dx+c))}{d}$	17
derivativedivides	$\frac{(dx+c)a + b \arctan(\sinh(dx+c))}{d}$	22
risch	$xa + \frac{ib \ln(e^{dx+c}+i)}{d} - \frac{ib \ln(e^{dx+c}-i)}{d}$	39
parallelrisch	$\frac{ib\left(\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)\right)}{d} + xa$	41

input
```
int(a+b*sech(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
x*a+b*arctan(sinh(d*x+c))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int (a + b\text{sech}(c + dx))\, dx = \frac{adx + 2\, b \arctan \left(\cosh \left(dx + c\right) + \sinh \left(dx + c\right)\right)}{d}$$

input
```
integrate(a+b*sech(d*x+c),x, algorithm="fricas")
```

output
```
(a*d*x + 2*b*arctan(cosh(d*x + c) + sinh(d*x + c)))/d
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int (a + b\text{sech}(c + dx))\, dx = ax + b \left(\begin{cases} \frac{2\,\text{atan}\left(\tanh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} & \text{for}\ d \neq 0 \\ x\,\text{sech}\left(c\right) & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*sech(d*x+c),x)
```

output
```
a*x + b*Piecewise((2*atan(tanh(c/2 + d*x/2))/d, Ne(d, 0)), (x*sech(c), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a + b\text{sech}(c + dx))\,dx = ax + \frac{b\arctan(\sinh(dx + c))}{d}$$

input
```
integrate(a+b*sech(d*x+c),x, algorithm="maxima")
```

output
```
a*x + b*arctan(sinh(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (a + b\text{sech}(c + dx))\,dx = ax + \frac{2\,b\arctan\left(e^{(dx+c)}\right)}{d}$$

input
```
integrate(a+b*sech(d*x+c),x, algorithm="giac")
```

output
```
a*x + 2*b*arctan(e^(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.38

$$\int (a + b\text{sech}(c + dx))\,dx = a\,x + \frac{2\,\text{atan}\left(\frac{b\,e^{d\,x}\,e^{c}\,\sqrt{d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{d^2}}$$

input
```
int(a + b/cosh(c + d*x),x)
```

output
```
a*x + (2*atan((b*exp(d*x)*exp(c)*(d^2)^(1/2))/(d*(b^2)^(1/2)))*(b^2)^(1/2)
)/(d^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (a + b\text{sech}(c + dx))\, dx = \frac{2atan\left(e^{dx+c}\right) b + adx}{d}$$

input
```
int(a+b*sech(d*x+c),x)
```

output
```
(2*atan(e**(c + d*x))*b + a*d*x)/d
```

3.91 $\int \frac{1}{a+b\operatorname{sech}(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int \frac{1}{a+b\operatorname{sech}(c+dx)}\,dx=\frac{x}{a}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d}$$

output
```
x/a-2*b*arctan((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/(a-b)^(1/2)/
(a+b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{1}{a+b\operatorname{sech}(c+dx)}\,dx=\frac{\frac{c}{d}+x+\frac{2b\arctan\left(\frac{(-a+b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}}{a}$$

input
```
Integrate[(a + b*Sech[c + d*x])^(-1),x]
```

output
```
(c/d + x + (2*b*ArcTan[((-a + b)*Tanh[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(Sqr
t[a^2 - b^2]*d))/a
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4270, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\text{sech}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4270}$$

$$\frac{x}{a}-\frac{\int \frac{1}{\frac{a\cosh(c+dx)}{b}+1}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x}{a}-\frac{\int \frac{1}{\frac{a\sin\left(ic+idx+\frac{\pi}{2}\right)}{b}+1}dx}{a}$$

$$\Big\downarrow \text{3138}$$

$$\frac{x}{a}+\frac{2i\int \frac{1}{\frac{a+b}{b}-\left(1-\frac{a}{b}\right)\tanh^2\left(\frac{1}{2}(c+dx)\right)}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

$$\Big\downarrow \text{221}$$

$$\frac{x}{a}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[(a + b*Sech[c + d*x])^(-1),x]
```

output
```
x/a - (2*b*ArcTan[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a
- b]*Sqrt[a + b]*d)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4270
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Simp[1/a   Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.41

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}-\frac{2b\arctan\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a\sqrt{(a-b)(a+b)}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}}{d}$	83
default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}-\frac{2b\arctan\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a\sqrt{(a-b)(a+b)}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}}{d}$	83
risch	$\frac{x}{a}-\frac{b\ln\left(e^{dx+c}+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,da}+\frac{b\ln\left(e^{dx+c}+\frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,da}$	136

input
```
int(1/(a+b*sech(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a*ln(tanh(1/2*d*x+1/2*c)-1)-2*b/a/((a-b)*(a+b))^(1/2)*arctan((a-b)
*tanh(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))+1/a*ln(tanh(1/2*d*x+1/2*c)+1))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 270, normalized size of antiderivative = 4.58

$$\int \frac{1}{a+b\mathrm{sech}(c+dx)}\,dx$$
$$=\left[\frac{(a^2-b^2)dx-\sqrt{-a^2+b^2}b\log\left(\frac{a^2\cosh(dx+c)^2+a^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)-a^2+2\,b^2+2\,(a^2\cosh(dx+c)+ab)\sinh(dx+c)}{a\cosh(dx+c)^2+a\sinh(dx+c)^2+2\,b\cosh(dx+c)+2\,(a\cosh(dx+c)+}\right.}{(a^3-ab^2)d}\right.$$

input
```
integrate(1/(a+b*sech(d*x+c)),x, algorithm="fricas")
```

output
```
[((a^2 - b^2)*d*x - sqrt(-a^2 + b^2)*b*log((a^2*cosh(d*x + c)^2 + a^2*sinh
(d*x + c)^2 + 2*a*b*cosh(d*x + c) - a^2 + 2*b^2 + 2*(a^2*cosh(d*x + c) + a
*b)*sinh(d*x + c) + 2*sqrt(-a^2 + b^2)*(a*cosh(d*x + c) + a*sinh(d*x + c)
+ b))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(a*co
sh(d*x + c) + b)*sinh(d*x + c) + a)))/((a^3 - a*b^2)*d), ((a^2 - b^2)*d*x
+ 2*sqrt(a^2 - b^2)*b*arctan(-(a*cosh(d*x + c) + a*sinh(d*x + c) + b)/sqrt
(a^2 - b^2)))/((a^3 - a*b^2)*d)]
```

Sympy [F]

$$\int \frac{1}{a+b\mathrm{sech}(c+dx)}\,dx=\int \frac{1}{a+b\,\mathrm{sech}\,(c+dx)}\,dx$$

input
```
integrate(1/(a+b*sech(d*x+c)),x)
```

output
```
Integral(1/(a + b*sech(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a + b\text{sech}(c + dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*sech(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{1}{a + b\text{sech}(c + dx)}\,dx = -\frac{\frac{2\,b\arctan\left(\frac{ae^{(dx+c)}+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}a} - \frac{dx+c}{a}}{d}$$

input
```
integrate(1/(a+b*sech(d*x+c)),x, algorithm="giac")
```

output
```
-(2*b*arctan((a*e^(d*x + c) + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a) - (d
*x + c)/a)/d
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.22

$$\int \frac{1}{a + b\text{sech}(c + dx)}\,dx = \frac{x}{a} + \frac{b\,\ln\left(\frac{2\,b\,e^{c+d\,x}}{a^2} - \frac{2\,b\,\left(a+b\,e^{c+d\,x}\right)}{a^2\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a\,d\,\sqrt{a+b}\,\sqrt{b-a}} - \frac{b\,\ln\left(\frac{2\,b\,e^{c+d\,x}}{a^2} + \frac{2\,b\,\left(a+b\,e^{c+d\,x}\right)}{a^2\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a\,d\,\sqrt{a+b}\,\sqrt{b-a}}$$

input
```
int(1/(a + b/cosh(c + d*x)),x)
```

output
```
x/a + (b*log((2*b*exp(c + d*x))/a^2 - (2*b*(a + b*exp(c + d*x)))/(a^2*(a +
 b)^(1/2)*(b - a)^(1/2))))/(a*d*(a + b)^(1/2)*(b - a)^(1/2)) - (b*log((2*b
*exp(c + d*x))/a^2 + (2*b*(a + b*exp(c + d*x)))/(a^2*(a + b)^(1/2)*(b - a)
^(1/2))))/(a*d*(a + b)^(1/2)*(b - a)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19

$$\int \frac{1}{a + b\operatorname{sech}(c + dx)}\, dx = \frac{-2\sqrt{a^2 - b^2}\, atan\left(\frac{e^{dx+c}a+b}{\sqrt{a^2-b^2}}\right) b + a^2dx - b^2dx}{ad\,(a^2 - b^2)}$$

input
```
int(1/(a+b*sech(d*x+c)),x)
```

output
```
( - 2*sqrt(a**2 - b**2)*atan((e**(c + d*x)*a + b)/sqrt(a**2 - b**2))*b + a
**2*d*x - b**2*d*x)/(a*d*(a**2 - b**2))
```

3.92 $\int \frac{1}{(a+b\,\mathbf{sech}(c+dx))^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 109

$$\int \frac{1}{(a+b\,\mathrm{sech}(c+dx))^2}\,dx = \frac{x}{a^2} - \frac{2b(2a^2-b^2)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{b^2\tanh(c+dx)}{a\,(a^2-b^2)\,d(a+b\,\mathrm{sech}(c+dx))}$$

output
```
x/a^2-2*b*(2*a^2-b^2)*arctan((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/
a^2/(a-b)^(3/2)/(a+b)^(3/2)/d+b^2*tanh(d*x+c)/a/(a^2-b^2)/d/(a+b*sech(d*x+
c))
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.86

$$\int \frac{1}{(a+b\,\mathrm{sech}(c+dx))^2}\,dx = \frac{a\left(\left(a^2-b^2\right)^{3/2}(c+dx) + (4a^2b-2b^3)\arctan\left(\frac{(-a+b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\right)\cosh(c+dx) + b\left(\left(a^2-b^2\right)^{3/2}(c+\right.}{a^2(a-b)(a+b)\sqrt{a^2-b^2}\,d(b+a\cosh}$$

input
```
Integrate[(a + b*Sech[c + d*x])^(-2),x]
```

output
```
(a*((a^2 - b^2)^(3/2)*(c + d*x) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh[
(c + d*x)/2])/Sqrt[a^2 - b^2]])*Cosh[c + d*x] + b*((a^2 - b^2)^(3/2)*(c +
d*x) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh[(c + d*x)/2])/Sqrt[a^2 - b^
2]] + a*b*Sqrt[a^2 - b^2]*Sinh[c + d*x]))/(a^2*(a - b)*(a + b)*Sqrt[a^2 -
b^2]*d*(b + a*Cosh[c + d*x]))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 4272, 25, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{sech}(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4272}$$

$$\frac{b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)(a+b\text{sech}(c+dx))}-\frac{\int -\frac{a^2-b\text{sech}(c+dx)a-b^2}{a+b\text{sech}(c+dx)}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a^2-b\text{sech}(c+dx)a-b^2}{a+b\text{sech}(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)(a+b\text{sech}(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)(a+b\text{sech}(c+dx))}+\frac{\int \frac{a^2-b\csc\left(ic+idx+\frac{\pi}{2}\right)a-b^2}{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4407}$$

$$\frac{\frac{x\left(a^2-b^2\right)}{a}-\frac{b\left(2a^2-b^2\right)\int \frac{\text{sech}(c+dx)}{a+b\text{sech}(c+dx)}dx}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)(a+b\text{sech}(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\tanh(c+dx)}{ad\,(a^2-b^2)\,(a+b\text{sech}(c+dx))}+\frac{\frac{x(a^2-b^2)}{a}-\frac{b(2a^2-b^2)\int\frac{\csc\left(ic+idx+\frac{\pi}{2}\right)}{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}dx}{a}}{a\,(a^2-b^2)}$$

$$\Big\downarrow \text{4318}$$

$$\frac{\frac{x(a^2-b^2)}{a}-\frac{(2a^2-b^2)\int\frac{1}{\frac{a\cosh(c+dx)}{b}+1}dx}{a}}{a\,(a^2-b^2)}+\frac{b^2\tanh(c+dx)}{ad\,(a^2-b^2)\,(a+b\text{sech}(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\tanh(c+dx)}{ad\,(a^2-b^2)\,(a+b\text{sech}(c+dx))}+\frac{\frac{x(a^2-b^2)}{a}-\frac{(2a^2-b^2)\int\frac{1}{\frac{a\sin\left(ic+idx+\frac{\pi}{2}\right)}{b}+1}dx}{a}}{a\,(a^2-b^2)}$$

$$\Big\downarrow \text{3138}$$

$$\frac{b^2\tanh(c+dx)}{ad\,(a^2-b^2)\,(a+b\text{sech}(c+dx))}+\frac{\frac{x(a^2-b^2)}{a}+\frac{2i(2a^2-b^2)\int\frac{1}{\frac{a+b}{b}-\left(1-\frac{a}{b}\right)\tanh^2\left(\frac{1}{2}(c+dx)\right)}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ad}}{a\,(a^2-b^2)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{x(a^2-b^2)}{a}-\frac{2b(2a^2-b^2)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a\,(a^2-b^2)}+\frac{b^2\tanh(c+dx)}{ad\,(a^2-b^2)\,(a+b\text{sech}(c+dx))}$$

input

```
Int[(a + b*Sech[c + d*x])^(-2),x]
```

output

```
(((a^2 - b^2)*x)/a - (2*b*(2*a^2 - b^2)*ArcTan[(Sqrt[a - b]*Tanh[(c + d*x)
/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d))/(a*(a^2 - b^2)) + (b^2*T
anh[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sech[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.50

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2}-\frac{2b\left(-\frac{ab\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2-b^2)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}+\frac{(2a^2-b^2)\arctan\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{a^2}-}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2}-\frac{2b\left(-\frac{ab\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2-b^2)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}+\frac{(2a^2-b^2)\arctan\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{a^2}-}{d}$
risch	$\frac{x}{a^2}-\frac{2b^2(b\,e^{dx+c}+a)}{d\,a^2(a^2-b^2)(a\,e^{2dx+2c}+2b\,e^{dx+c}+a)}-\frac{2b\ln\left(e^{dx+c}+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d}+\frac{b^3\ln\left(e^{dx+c}+\frac{b\sqrt{-a^2+b^2}+a^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d\,a}$

input
```
int(1/(a+b*sech(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-2*b/a^2*(-a*b/(a^2-b^2)*tanh(1/2*d*x+
1/2*c)/(tanh(1/2*d*x+1/2*c)^2*a-tanh(1/2*d*x+1/2*c)^2*b+a+b)+(2*a^2-b^2)/(
a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tanh(1/2*d*x+1/2*c)/((a-b)*(a+
b))^(1/2)))-1/a^2*ln(tanh(1/2*d*x+1/2*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(100) = 200$.

Time = 0.11 (sec) , antiderivative size = 1207, normalized size of antiderivative = 11.07

$$\int \frac{1}{(a+b\operatorname{sech}(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sech(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-(2*a^3*b^2 - 2*a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c)^2 - (
a^5 - 2*a^3*b^2 + a*b^4)*d*x*sinh(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d
*x + (2*a^3*b - a*b^3 + (2*a^3*b - a*b^3)*cosh(d*x + c)^2 + (2*a^3*b - a*b
^3)*sinh(d*x + c)^2 + 2*(2*a^2*b^2 - b^4)*cosh(d*x + c) + 2*(2*a^2*b^2 - b
^4 + (2*a^3*b - a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + b^2)*log(
(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) - a^2 + 2
*b^2 + 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(-a^2 + b^2)*(a*c
osh(d*x + c) + a*sinh(d*x + c) + b))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^
2 + 2*b*cosh(d*x + c) + 2*(a*cosh(d*x + c) + b)*sinh(d*x + c) + a)) + 2*(a
^2*b^3 - b^5 - (a^4*b - 2*a^2*b^3 + b^5)*d*x)*cosh(d*x + c) + 2*(a^2*b^3 -
 b^5 - (a^5 - 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c) - (a^4*b - 2*a^2*b^3 +
b^5)*d*x)*sinh(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^2 +
(a^7 - 2*a^5*b^2 + a^3*b^4)*d*sinh(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2
*b^5)*d*cosh(d*x + c) + (a^7 - 2*a^5*b^2 + a^3*b^4)*d + 2*((a^7 - 2*a^5*b^
2 + a^3*b^4)*d*cosh(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)*sinh(d*x +
 c)), -(2*a^3*b^2 - 2*a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c)^
2 - (a^5 - 2*a^3*b^2 + a*b^4)*d*x*sinh(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b
^4)*d*x - 2*(2*a^3*b - a*b^3 + (2*a^3*b - a*b^3)*cosh(d*x + c)^2 + (2*a^3*
b - a*b^3)*sinh(d*x + c)^2 + 2*(2*a^2*b^2 - b^4)*cosh(d*x + c) + 2*(2*a^2*
b^2 - b^4 + (2*a^3*b - a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - ...
```

Sympy [F]

$$\int \frac{1}{(a+b\text{sech}(c+dx))^2}\,dx=\int \frac{1}{\left(a+b\operatorname{sech}\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(1/(a+b*sech(d*x+c))**2,x)
```

output
```
Integral((a + b*sech(c + d*x))**(-2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\operatorname{sech}(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*sech(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.23

$$\int \frac{1}{(a+b\operatorname{sech}(c+dx))^2}\,dx = -\frac{\frac{2\,(2\,a^2b-b^3)\arctan\left(\frac{ae^{(dx+c)}+b}{\sqrt{a^2-b^2}}\right)}{(a^4-a^2b^2)\sqrt{a^2-b^2}}+\frac{2\,(b^3e^{(dx+c)}+ab^2)}{(a^4-a^2b^2)\left(ae^{(2\,dx+2\,c)}+2\,be^{(dx+c)}+a\right)}-\frac{dx+c}{a^2}}{d}$$

input
```
integrate(1/(a+b*sech(d*x+c))^2,x, algorithm="giac")
```

output
```
-(2*(2*a^2*b - b^3)*arctan((a*e^(d*x + c) + b)/sqrt(a^2 - b^2))/((a^4 - a^
2*b^2)*sqrt(a^2 - b^2)) + 2*(b^3*e^(d*x + c) + a*b^2)/((a^4 - a^2*b^2)*(a*
e^(2*d*x + 2*c) + 2*b*e^(d*x + c) + a)) - (d*x + c)/a^2)/d
```

Mupad [B] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.72

$$\int \frac{1}{(a+b\text{sech}(c+dx))^2}\,dx = \frac{\frac{2\,b^2}{d\,(a\,b^2-a^3)}+\frac{2\,b^3\,e^{c+d\,x}}{a\,d\,(a\,b^2-a^3)}}{a+2\,b\,e^{c+d\,x}+a\,e^{2\,c+2\,d\,x}}+\frac{x}{a^2} + \frac{b\,\ln\left(\frac{2\,e^{c+d\,x}\,(2\,a^2\,b-b^3)}{a^3\,(a^2-b^2)}-\frac{2\,b\,(2\,a^2-b^2)\,(a+b\,e^{c+d\,x})}{a^3\,(a+b)^{3/2}\,(b-a)^{3/2}}\right)\,(2\,a^2-b^2)}{a^2\,d\,(a+b)^{3/2}\,(b-a)^{3/2}} - \frac{b\,\ln\left(\frac{2\,e^{c+d\,x}\,(2\,a^2\,b-b^3)}{a^3\,(a^2-b^2)}+\frac{2\,b\,(2\,a^2-b^2)\,(a+b\,e^{c+d\,x})}{a^3\,(a+b)^{3/2}\,(b-a)^{3/2}}\right)\,(2\,a^2-b^2)}{a^2\,d\,(a+b)^{3/2}\,(b-a)^{3/2}}$$

input
```
int(1/(a + b/cosh(c + d*x))^2,x)
```

output
```
((2*b^2)/(d*(a*b^2 - a^3)) + (2*b^3*exp(c + d*x))/(a*d*(a*b^2 - a^3)))/(a
+ 2*b*exp(c + d*x) + a*exp(2*c + 2*d*x)) + x/a^2 + (b*log((2*exp(c + d*x)*
(2*a^2*b - b^3))/(a^3*(a^2 - b^2)) - (2*b*(2*a^2 - b^2)*(a + b*exp(c + d*x
)))/(a^3*(a + b)^(3/2)*(b - a)^(3/2)))*(2*a^2 - b^2))/(a^2*d*(a + b)^(3/2)
*(b - a)^(3/2)) - (b*log((2*exp(c + d*x)*(2*a^2*b - b^3))/(a^3*(a^2 - b^2)
) + (2*b*(2*a^2 - b^2)*(a + b*exp(c + d*x)))/(a^3*(a + b)^(3/2)*(b - a)^(3
/2)))*(2*a^2 - b^2))/(a^2*d*(a + b)^(3/2)*(b - a)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 563, normalized size of antiderivative = 5.17

$$\int \frac{1}{(a+b\text{sech}(c+dx))^2}\,dx = \frac{-4e^{2dx+2c}\sqrt{a^2-b^2}\,atan\left(\frac{e^{dx+c}a+b}{\sqrt{a^2-b^2}}\right)a^3b+2e^{2dx+2c}\sqrt{a^2-b^2}\,atan\left(\frac{e^{dx+c}a+b}{\sqrt{a^2-b^2}}\right)a\,b^3-8e^{dx+c}\sqrt{a^2-b^2}\,atan\left(\right.}{}$$

input
```
int(1/(a+b*sech(d*x+c))^2,x)
```

output
```
( - 4*e**(2*c + 2*d*x)*sqrt(a**2 - b**2)*atan((e**(c + d*x)*a + b)/sqrt(a*
*2 - b**2))*a**3*b + 2*e**(2*c + 2*d*x)*sqrt(a**2 - b**2)*atan((e**(c + d*
x)*a + b)/sqrt(a**2 - b**2))*a*b**3 - 8*e**(c + d*x)*sqrt(a**2 - b**2)*ata
n((e**(c + d*x)*a + b)/sqrt(a**2 - b**2))*a**2*b**2 + 4*e**(c + d*x)*sqrt(
a**2 - b**2)*atan((e**(c + d*x)*a + b)/sqrt(a**2 - b**2))*b**4 - 4*sqrt(a*
*2 - b**2)*atan((e**(c + d*x)*a + b)/sqrt(a**2 - b**2))*a**3*b + 2*sqrt(a*
*2 - b**2)*atan((e**(c + d*x)*a + b)/sqrt(a**2 - b**2))*a*b**3 + e**(2*c +
 2*d*x)*a**5*d*x - 2*e**(2*c + 2*d*x)*a**3*b**2*d*x + e**(2*c + 2*d*x)*a**
3*b**2 + e**(2*c + 2*d*x)*a*b**4*d*x - e**(2*c + 2*d*x)*a*b**4 + 2*e**(c +
 d*x)*a**4*b*d*x - 4*e**(c + d*x)*a**2*b**3*d*x + 2*e**(c + d*x)*b**5*d*x
+ a**5*d*x - 2*a**3*b**2*d*x - a**3*b**2 + a*b**4*d*x + a*b**4)/(a**2*d*(e
**(2*c + 2*d*x)*a**5 - 2*e**(2*c + 2*d*x)*a**3*b**2 + e**(2*c + 2*d*x)*a*b
**4 + 2*e**(c + d*x)*a**4*b - 4*e**(c + d*x)*a**2*b**3 + 2*e**(c + d*x)*b*
*5 + a**5 - 2*a**3*b**2 + a*b**4))
```

3.93 $\int \frac{1}{(a+b\,\mathrm{sech}(c+dx))^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 173

$$\int \frac{1}{(a+b\,\mathrm{sech}(c+dx))^3}\,dx = \frac{x}{a^3}-\frac{b(6a^4-5a^2b^2+2b^4)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} + \frac{b^2\tanh(c+dx)}{2a\,(a^2-b^2)\,d(a+b\,\mathrm{sech}(c+dx))^2} + \frac{b^2(5a^2-2b^2)\tanh(c+dx)}{2a^2\,(a^2-b^2)^2\,d(a+b\,\mathrm{sech}(c+dx))}$$

output
```
x/a^3-b*(6*a^4-5*a^2*b^2+2*b^4)*arctan((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+
b)^(1/2))/a^3/(a-b)^(5/2)/(a+b)^(5/2)/d+1/2*b^2*tanh(d*x+c)/a/(a^2-b^2)/d/
(a+b*sech(d*x+c))^2+1/2*b^2*(5*a^2-2*b^2)*tanh(d*x+c)/a^2/(a^2-b^2)^2/d/(a
+b*sech(d*x+c))
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.18

$$\int \frac{1}{(a+b\operatorname{sech}(c+dx))^3}\,dx$$
$$= \frac{(b+a\cosh(c+dx))\operatorname{sech}^3(c+dx)\left(2(c+dx)(b+a\cosh(c+dx))^2+\frac{2b(6a^4-5a^2b^2+2b^4)\arctan\left(\frac{(-a+b)\tanh\left(\frac{1}{2}(c-\right.}{\sqrt{a^2-b^2}}\right.}{(a^2-b^2)^{5/2}}\right.}{2a^3d(a+b\operatorname{sech}(c+dx))^3}$$

input
```
Integrate[(a + b*Sech[c + d*x])^(-3),x]
```

output
```
((b + a*Cosh[c + d*x])*Sech[c + d*x]^3*(2*(c + d*x)*(b + a*Cosh[c + d*x])^
2 + (2*b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[((-a + b)*Tanh[(c + d*x)/2])/S
qrt[a^2 - b^2]]*(b + a*Cosh[c + d*x])^2)/(a^2 - b^2)^(5/2) + (a*b^3*Sinh[c
 + d*x])/((-a + b)*(a + b)) + (3*a*b^2*(2*a^2 - b^2)*(b + a*Cosh[c + d*x])
*Sinh[c + d*x])/((a - b)^2*(a + b)^2)))/(2*a^3*d*(a + b*Sech[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.25, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {3042, 4272, 25, 3042, 4548, 25, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\operatorname{sech}(c+dx))^3}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\left(a+b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \text{ 4272}$$

$$\frac{b^2\tanh(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\text{sech}(c+dx)\right)^2}-\frac{\int -\frac{b^2\text{sech}^2(c+dx)-2ab\text{sech}(c+dx)+2\left(a^2-b^2\right)}{(a+b\text{sech}(c+dx))^2}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{b^2\text{sech}^2(c+dx)-2ab\text{sech}(c+dx)+2\left(a^2-b^2\right)}{(a+b\text{sech}(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\tanh(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\text{sech}(c+dx)\right)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b^2\tanh(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\text{sech}(c+dx)\right)^2}+\frac{\int \frac{b^2\csc\left(ic+idx+\frac{\pi}{2}\right)^2-2ab\csc\left(ic+idx+\frac{\pi}{2}\right)+2\left(a^2-b^2\right)}{\left(a+b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^2}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 4548}$$

$$\frac{\frac{b^2\left(5a^2-2b^2\right)\tanh(c+dx)}{ad(a^2-b^2)(a+b\text{sech}(c+dx))}-\frac{\int -\frac{2\left(a^2-b^2\right)^2-ab\left(4a^2-b^2\right)\text{sech}(c+dx)}{a+b\text{sech}(c+dx)}dx}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}+\frac{b^2\tanh(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\text{sech}(c+dx)\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{2\left(a^2-b^2\right)^2-ab\left(4a^2-b^2\right)\text{sech}(c+dx)}{a+b\text{sech}(c+dx)}dx}{a(a^2-b^2)}+\frac{b^2\left(5a^2-2b^2\right)\tanh(c+dx)}{ad(a^2-b^2)(a+b\text{sech}(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\tanh(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\text{sech}(c+dx)\right)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b^2\tanh(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\text{sech}(c+dx)\right)^2}+\frac{\frac{b^2\left(5a^2-2b^2\right)\tanh(c+dx)}{ad(a^2-b^2)(a+b\text{sech}(c+dx))}+\frac{\int \frac{2\left(a^2-b^2\right)^2-ab\left(4a^2-b^2\right)\csc\left(ic+idx+\frac{\pi}{2}\right)}{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}dx}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 4407}$$

$$\frac{\frac{\frac{2x\left(a^2-b^2\right)^2}{a}-\frac{b\left(6a^4-5a^2b^2+2b^4\right)\int \frac{\text{sech}(c+dx)}{a+b\text{sech}(c+dx)}dx}{a}}{a(a^2-b^2)}+\frac{b^2\left(5a^2-2b^2\right)\tanh(c+dx)}{ad(a^2-b^2)(a+b\text{sech}(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\tanh(c+dx)}{2ad\left(a^2-b^2\right)\left(a+b\text{sech}(c+dx)\right)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b^2\tanh(c+dx)}{2ad\left(a^2-b^2\right)(a+b\operatorname{sech}(c+dx))^2}+\frac{\frac{b^2(5a^2-2b^2)\tanh(c+dx)}{ad(a^2-b^2)(a+b\operatorname{sech}(c+dx))}+\frac{\frac{2x\left(a^2-b^2\right)^2}{a}-\frac{b\left(6a^4-5a^2b^2+2b^4\right)\int\frac{\csc\left(ic+idx+\frac{\pi}{2}\right)}{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}dx}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

$\big\downarrow$ 4318

$$\frac{\frac{\frac{2x\left(a^2-b^2\right)^2}{a}-\frac{\left(6a^4-5a^2b^2+2b^4\right)\int\frac{1}{\frac{a\cosh(c+dx)}{b}+1}dx}{a}}{a(a^2-b^2)}+\frac{b^2(5a^2-2b^2)\tanh(c+dx)}{ad(a^2-b^2)(a+b\operatorname{sech}(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\tanh(c+dx)}{2ad\left(a^2-b^2\right)(a+b\operatorname{sech}(c+dx))^2}$$

$\big\downarrow$ 3042

$$\frac{b^2\tanh(c+dx)}{2ad\left(a^2-b^2\right)(a+b\operatorname{sech}(c+dx))^2}+\frac{\frac{b^2(5a^2-2b^2)\tanh(c+dx)}{ad(a^2-b^2)(a+b\operatorname{sech}(c+dx))}+\frac{\frac{2x\left(a^2-b^2\right)^2}{a}-\frac{\left(6a^4-5a^2b^2+2b^4\right)\int\frac{1}{\frac{a\sin\left(ic+idx+\frac{\pi}{2}\right)}{b}+1}dx}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

$\big\downarrow$ 3138

$$\frac{b^2\tanh(c+dx)}{2ad\left(a^2-b^2\right)(a+b\operatorname{sech}(c+dx))^2}+\frac{\frac{b^2(5a^2-2b^2)\tanh(c+dx)}{ad(a^2-b^2)(a+b\operatorname{sech}(c+dx))}+\frac{\frac{2x\left(a^2-b^2\right)^2}{a}+\frac{2i\left(6a^4-5a^2b^2+2b^4\right)\int\frac{1}{\frac{a+b}{b}-\left(1-\frac{a}{b}\right)\tanh^2\left(\frac{1}{2}(c+dx)\right)}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ad}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

$\big\downarrow$ 221

$$\frac{b^2\tanh(c+dx)}{2ad\left(a^2-b^2\right)(a+b\operatorname{sech}(c+dx))^2}+\frac{\frac{b^2(5a^2-2b^2)\tanh(c+dx)}{ad(a^2-b^2)(a+b\operatorname{sech}(c+dx))}+\frac{\frac{2x\left(a^2-b^2\right)^2}{a}-\frac{2b\left(6a^4-5a^2b^2+2b^4\right)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

input

```
Int[(a + b*Sech[c + d*x])^(-3),x]
```

output
```
(b^2*Tanh[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sech[c + d*x])^2) + (((2*(a^
2 - b^2)^2*x)/a - (2*b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(Sqrt[a - b]*Tan
h[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d))/(a*(a^2 - b^2
)) + (b^2*(5*a^2 - 2*b^2)*Tanh[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sech[c +
d*x])))/(2*a*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.45

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3}-\frac{2b\left(\frac{-\frac{(6a^2+ab-2b^2)ab\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)(a^2+2ab+b^2)}-\frac{(6a^2-ab-2b^2)ab\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)(a^2-2ab+b^2)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}+\frac{(6\cdots}{\cdots}\right)}{a^3}}{d}$
default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3}-\frac{2b\left(\frac{-\frac{(6a^2+ab-2b^2)ab\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)(a^2+2ab+b^2)}-\frac{(6a^2-ab-2b^2)ab\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)(a^2-2ab+b^2)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}+\frac{(6\cdots}{\cdots}\right)}{a^3}}{d}$
risch	$\frac{x}{a^3}-\frac{b^2\left(7a^3b\,e^{3dx+3c}-4a\,b^3e^{3dx+3c}+6a^4e^{2dx+2c}+9a^2b^2e^{2dx+2c}-6b^4e^{2dx+2c}+17a^3b\,e^{dx+c}-8b^3e^{dx+c}a+6a^4-3a^2\cdots\right.}{a^3(a^2-b^2)^2d\left(a\,e^{2dx+2c}+2b\,e^{dx+c}+a\right)^2}$

input
```
int(1/(a+b*sech(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-2*b/
a^3*((-1/2*(6*a^2+a*b-2*b^2)*a*b/(a-b)/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)
^3-1/2*(6*a^2-a*b-2*b^2)*a*b/(a+b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c))/(t
anh(1/2*d*x+1/2*c)^2*a-tanh(1/2*d*x+1/2*c)^2*b+a+b)^2+1/2*(6*a^4-5*a^2*b^2
+2*b^4)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tanh(1/2*d*x+
1/2*c)/((a-b)*(a+b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2000 vs. $2(160) = 320$.

Time = 0.17 (sec) , antiderivative size = 4125, normalized size of antiderivative = 23.84

$$\int \frac{1}{(a+b\text{sech}(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sech(d*x+c))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(a+b\text{sech}(c+dx))^3}\,dx = \int \frac{1}{(a+b\,\text{sech}\,(c+dx))^3}\,dx$$

input
```
integrate(1/(a+b*sech(d*x+c))**3,x)
```

output
```
Integral((a + b*sech(c + d*x))**(-3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\operatorname{sech}(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*sech(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.51

$$\int \frac{1}{(a+b\operatorname{sech}(c+dx))^3}\,dx = -\frac{\frac{(6a^4b-5a^2b^3+2b^5)\arctan\left(\frac{ae^{(dx+c)}+b}{\sqrt{a^2-b^2}}\right)}{(a^7-2a^5b^2+a^3b^4)\sqrt{a^2-b^2}} + \frac{7a^3b^3e^{(3dx+3c)}-4ab^5e^{(3dx+3c)}+6a^4b^2e^{(2dx+2c)}+9a^2b^4e^{(2dx+2c)}-6b^6e^{(2dx+2c)}+17a}{(a^7-2a^5b^2+a^3b^4)\left(ae^{(2dx+2c)}+2be^{(dx+c)}+a\right)^2}}{d}$$

input
```
integrate(1/(a+b*sech(d*x+c))^3,x, algorithm="giac")
```

output
```
-((6*a^4*b - 5*a^2*b^3 + 2*b^5)*arctan((a*e^(d*x + c) + b)/sqrt(a^2 - b^2)
)/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(a^2 - b^2)) + (7*a^3*b^3*e^(3*d*x + 3*
c) - 4*a*b^5*e^(3*d*x + 3*c) + 6*a^4*b^2*e^(2*d*x + 2*c) + 9*a^2*b^4*e^(2*
d*x + 2*c) - 6*b^6*e^(2*d*x + 2*c) + 17*a^3*b^3*e^(d*x + c) - 8*a*b^5*e^(d
*x + c) + 6*a^4*b^2 - 3*a^2*b^4)/((a^7 - 2*a^5*b^2 + a^3*b^4)*(a*e^(2*d*x
+ 2*c) + 2*b*e^(d*x + c) + a)^2) - (d*x + c)/a^3)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\operatorname{sech}(c+dx))^3}\,dx = \int \frac{1}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^3}\,dx$$

input
```
int(1/(a + b/cosh(c + d*x))^3,x)
```

output
```
int(1/(a + b/cosh(c + d*x))^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1649, normalized size of antiderivative = 9.53

$$\int \frac{1}{(a+b\operatorname{sech}(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*sech(d*x+c))^3,x)
```

output
```
( - 24*e**(4*c + 4*d*x)*sqrt(a**2 - b**2)*atan((e**(c + d*x)*a + b)/sqrt(a
**2 - b**2))*a**6*b + 20*e**(4*c + 4*d*x)*sqrt(a**2 - b**2)*atan((e**(c +
d*x)*a + b)/sqrt(a**2 - b**2))*a**4*b**3 - 8*e**(4*c + 4*d*x)*sqrt(a**2 -
b**2)*atan((e**(c + d*x)*a + b)/sqrt(a**2 - b**2))*a**2*b**5 - 96*e**(3*c
+ 3*d*x)*sqrt(a**2 - b**2)*atan((e**(c + d*x)*a + b)/sqrt(a**2 - b**2))*a*
*5*b**2 + 80*e**(3*c + 3*d*x)*sqrt(a**2 - b**2)*atan((e**(c + d*x)*a + b)/
sqrt(a**2 - b**2))*a**3*b**4 - 32*e**(3*c + 3*d*x)*sqrt(a**2 - b**2)*atan(
(e**(c + d*x)*a + b)/sqrt(a**2 - b**2))*a*b**6 - 48*e**(2*c + 2*d*x)*sqrt(
a**2 - b**2)*atan((e**(c + d*x)*a + b)/sqrt(a**2 - b**2))*a**6*b - 56*e**(
2*c + 2*d*x)*sqrt(a**2 - b**2)*atan((e**(c + d*x)*a + b)/sqrt(a**2 - b**2)
)*a**4*b**3 + 64*e**(2*c + 2*d*x)*sqrt(a**2 - b**2)*atan((e**(c + d*x)*a +
 b)/sqrt(a**2 - b**2))*a**2*b**5 - 32*e**(2*c + 2*d*x)*sqrt(a**2 - b**2)*a
tan((e**(c + d*x)*a + b)/sqrt(a**2 - b**2))*b**7 - 96*e**(c + d*x)*sqrt(a*
*2 - b**2)*atan((e**(c + d*x)*a + b)/sqrt(a**2 - b**2))*a**5*b**2 + 80*e**
(c + d*x)*sqrt(a**2 - b**2)*atan((e**(c + d*x)*a + b)/sqrt(a**2 - b**2))*a
**3*b**4 - 32*e**(c + d*x)*sqrt(a**2 - b**2)*atan((e**(c + d*x)*a + b)/sqr
t(a**2 - b**2))*a*b**6 - 24*sqrt(a**2 - b**2)*atan((e**(c + d*x)*a + b)/sq
rt(a**2 - b**2))*a**6*b + 20*sqrt(a**2 - b**2)*atan((e**(c + d*x)*a + b)/s
qrt(a**2 - b**2))*a**4*b**3 - 8*sqrt(a**2 - b**2)*atan((e**(c + d*x)*a + b
)/sqrt(a**2 - b**2))*a**2*b**5 + 4*e**(4*c + 4*d*x)*a**8*d*x - 12*e**(4...
```

3.94 $\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 106

$$\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{ad}$$

output
```
2*(a+b)^(1/2)*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),(
a+b)/a,((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*
x+c))/(a-b))^(1/2)/a/d
```

Mathematica [A] (verified)

Time = 3.56 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \frac{2b\sqrt{b+a\cosh(c+dx)}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a+b}\sqrt{a\cosh(c+dx)}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1+\text{sech}(c+dx))}{-a+b}}\tanh\left(\frac{1}{2}(c+dx\right.}{\sqrt{a}\sqrt{a+b}d\sqrt{a\cosh(c+dx)}\sqrt{-\frac{b(-1+\text{sech}(c+dx))}{a+b}}\sqrt{a+b\text{sech}(c+dx)}}$$

input
```
Integrate[1/Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
(2*b*Sqrt[b + a*Cosh[c + d*x]]*EllipticPi[(a + b)/a, ArcSin[(Sqrt[a]*Sqrt[
b + a*Cosh[c + d*x]])/(Sqrt[a + b]*Sqrt[a*Cosh[c + d*x]])], (a + b)/(a - b
)]*Sqrt[(b*(1 + Sech[c + d*x]))/(-a + b)]*Tanh[(c + d*x)/2])/(Sqrt[a]*Sqrt
[a + b]*d*Sqrt[a*Cosh[c + d*x]]*Sqrt[-((b*(-1 + Sech[c + d*x]))/(a + b))]*
Sqrt[a + b*Sech[c + d*x]])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4271}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4271}$$

$$\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}$$

input
```
Int[1/Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
(2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a +
 b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{1}{\sqrt{a + b\ \mathrm{sech}\,(dx + c)}} dx$$

input
```
int(1/(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int(1/(a+b*sech(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a + b\mathrm{sech}(c + dx)}}\, dx = \int \frac{1}{\sqrt{b\,\mathrm{sech}\,(dx + c) + a}}\, dx$$

input
```
integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(1/sqrt(b*sech(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a + b\text{sech}(c + dx)}}\, dx = \int \frac{1}{\sqrt{a + b\,\text{sech}\,(c + dx)}}\, dx$$

input
```
integrate(1/(a+b*sech(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a + b\text{sech}(c + dx)}}\, dx = \int \frac{1}{\sqrt{b\,\text{sech}\,(dx + c) + a}}\, dx$$

input
```
integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*sech(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a + b\text{sech}(c + dx)}}\, dx = \int \frac{1}{\sqrt{b\,\text{sech}\,(dx + c) + a}}\, dx$$

input
```
integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + b\text{sech}(c + dx)}}\,dx = \int \frac{1}{\sqrt{a + \frac{b}{\cosh(c+d\,x)}}}\,dx$$

input
```
int(1/(a + b/cosh(c + d*x))^(1/2),x)
```

output
```
int(1/(a + b/cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a + b\text{sech}(c + dx)}}\,dx = \int \frac{\sqrt{\text{sech}\,(dx + c)\,b + a}}{\text{sech}\,(dx + c)\,b + a}\,dx$$

input
```
int(1/(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sech(c + d*x)*b + a)/(sech(c + d*x)*b + a),x)
```

3.95 $\int \frac{\cosh^4(x)}{a+b\,\mathrm{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 146

$$\int \frac{\cosh^4(x)}{a+b\,\mathrm{sech}(x)}\,dx = \frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5} - \frac{2b^5\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5\sqrt{a-b}\sqrt{a+b}} - \frac{b(2a^2+3b^2)\sinh(x)}{3a^4} + \frac{(3a^2+4b^2)\cosh(x)\sinh(x)}{8a^3} - \frac{b\cosh^2(x)\sinh(x)}{3a^2} + \frac{\cosh^3(x)\sinh(x)}{4a}$$

output
```
1/8*(3*a^4+4*a^2*b^2+8*b^4)*x/a^5-2*b^5*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+
b)^(1/2))/a^5/(a-b)^(1/2)/(a+b)^(1/2)-1/3*b*(2*a^2+3*b^2)*sinh(x)/a^4+1/8*
(3*a^2+4*b^2)*cosh(x)*sinh(x)/a^3-1/3*b*cosh(x)^2*sinh(x)/a^2+1/4*cosh(x)^
3*sinh(x)/a
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

$$\int \frac{\cosh^4(x)}{a+b\text{sech}(x)}\,dx = \frac{12(3a^4+4a^2b^2+8b^4)\,x+\frac{192b^5\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-24ab(3a^2+4b^2)\sinh(x)+24a^2(a^2+b^2)\sinh(2x)}{96a^5}$$

input
```
Integrate[Cosh[x]^4/(a + b*Sech[x]),x]
```

output
```
(12*(3*a^4 + 4*a^2*b^2 + 8*b^4)*x + (192*b^5*ArcTan[((-a + b)*Tanh[x/2])/S
qrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - 24*a*b*(3*a^2 + 4*b^2)*Sinh[x] + 24*a^2
*(a^2 + b^2)*Sinh[2*x] - 8*a^3*b*Sinh[3*x] + 3*a^4*Sinh[4*x])/(96*a^5)
```

Rubi [A] (verified)

Time = 1.81 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.17, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.308, Rules used = {3042, 4340, 25, 3042, 4592, 3042, 4592, 3042, 4592, 27, 3042, 4407, 3042, 4318, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(x)}{a+b\text{sech}(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\csc\left(\frac{\pi}{2}+ix\right)^4\left(a+b\csc\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \; 4340$$

$$\frac{\int -\frac{\cosh^3(x)\left(-3b\text{sech}^2(x)-3a\text{sech}(x)+4b\right)}{a+b\text{sech}(x)}dx}{4a}+\frac{\sinh(x)\cosh^3(x)}{4a}$$

$$\Big\downarrow \; 25$$

$$\frac{\sinh(x)\cosh^3(x)}{4a}-\frac{\int \frac{\cosh^3(x)\left(-3b\,\text{sech}^2(x)-3a\,\text{sech}(x)+4b\right)}{a+b\,\text{sech}(x)}dx}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh(x)\cosh^3(x)}{4a}-\frac{\int \frac{-3b\csc\left(ix+\frac{\pi}{2}\right)^2-3a\csc\left(ix+\frac{\pi}{2}\right)+4b}{\csc\left(ix+\frac{\pi}{2}\right)^3\left(a+b\csc\left(ix+\frac{\pi}{2}\right)\right)}dx}{4a}$$

$$\Big\downarrow \; 4592$$

$$\frac{\sinh(x)\cosh^3(x)}{4a}-\frac{\frac{4b\sinh(x)\cosh^2(x)}{3a}-\frac{\int \frac{\cosh^2(x)\left(-8b^2\text{sech}^2(x)+ab\,\text{sech}(x)+3\left(3a^2+4b^2\right)\right)}{a+b\,\text{sech}(x)}dx}{3a}}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh(x)\cosh^3(x)}{4a}-\frac{\frac{4b\sinh(x)\cosh^2(x)}{3a}-\frac{\int \frac{-8b^2\csc\left(ix+\frac{\pi}{2}\right)^2+ab\csc\left(ix+\frac{\pi}{2}\right)+3\left(3a^2+4b^2\right)}{\csc\left(ix+\frac{\pi}{2}\right)^2\left(a+b\csc\left(ix+\frac{\pi}{2}\right)\right)}dx}{3a}}{4a}$$

$$\Big\downarrow \; 4592$$

$$\frac{\sinh(x)\cosh^3(x)}{4a}-\frac{\frac{4b\sinh(x)\cosh^2(x)}{3a}-\frac{\frac{3\left(3a^2+4b^2\right)\sinh(x)\cosh(x)}{2a}-\frac{\int \frac{\cosh(x)\left(-3b\left(3a^2+4b^2\right)\text{sech}^2(x)-a\left(9a^2-4b^2\right)\text{sech}(x)+8b\left(2a^2+3b^2\right)\right)}{a+b\,\text{sech}(x)}dx}{2a}}{3a}}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh(x)\cosh^3(x)}{4a}-\frac{\frac{4b\sinh(x)\cosh^2(x)}{3a}-\frac{\frac{3\left(3a^2+4b^2\right)\sinh(x)\cosh(x)}{2a}-\frac{\int \frac{-3b\left(3a^2+4b^2\right)\csc\left(ix+\frac{\pi}{2}\right)^2-a\left(9a^2-4b^2\right)\csc\left(ix+\frac{\pi}{2}\right)+8b\left(2a^2+3b^2\right)}{\csc\left(ix+\frac{\pi}{2}\right)\left(a+b\csc\left(ix+\frac{\pi}{2}\right)\right)}dx}{2a}}{3a}}{4a}$$

$$\Big\downarrow \; 4592$$

$$\frac{\sinh(x)\cosh^3(x)}{4a}-\frac{\frac{4b\sinh(x)\cosh^2(x)}{3a}-\frac{\frac{3\left(3a^2+4b^2\right)\sinh(x)\cosh(x)}{2a}-\frac{\frac{8b\left(2a^2+3b^2\right)\sinh(x)}{a}-\frac{\int \frac{3\left(3a^4+4b^2a^2+b\left(3a^2+4b^2\right)\text{sech}(x)a+8b^4\right)}{a+b\,\text{sech}(x)}dx}{a}}{2a}}{3a}}{4a}$$

$\Big\downarrow$ 27

$$\frac{\sinh(x)\cosh^3(x)}{4a}-\frac{\frac{4b\sinh(x)\cosh^2(x)}{3a}-\frac{\frac{3\left(3a^2+4b^2\right)\sinh(x)\cosh(x)}{2a}-\frac{\frac{8b\left(2a^2+3b^2\right)\sinh(x)}{a}-\frac{3\int\frac{3a^4+4b^2a^2+b\left(3a^2+4b^2\right)\text{sech}(x)a+8b^4}{a+b\,\text{sech}(x)}dx}{a}}{2a}}{3a}}{4a}$$

$\Big\downarrow$ 3042

$$\frac{\sinh(x)\cosh^3(x)}{4a}-\frac{\frac{4b\sinh(x)\cosh^2(x)}{3a}-\frac{\frac{3\left(3a^2+4b^2\right)\sinh(x)\cosh(x)}{2a}-\frac{\frac{8b\left(2a^2+3b^2\right)\sinh(x)}{a}-\frac{3\int\frac{3a^4+4b^2a^2+b\left(3a^2+4b^2\right)\csc\left(ix+\frac{\pi}{2}\right)a+8b^4}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx}{a}}{2a}}{3a}}{4a}$$

$\Big\downarrow$ 4407

$$\frac{\sinh(x)\cosh^3(x)}{4a}-\frac{\frac{4b\sinh(x)\cosh^2(x)}{3a}-\frac{\frac{3\left(3a^2+4b^2\right)\sinh(x)\cosh(x)}{2a}-\frac{\frac{8b\left(2a^2+3b^2\right)\sinh(x)}{a}-\frac{3\left(\frac{x\left(3a^4+4a^2b^2+8b^4\right)}{a}-\frac{8b^5\int\frac{\text{sech}(x)}{a+b\,\text{sech}(x)}dx}{a}\right)}{a}}{2a}}{3a}}{4a}$$

$\Big\downarrow$ 3042

$$\frac{\sinh(x)\cosh^3(x)}{4a}-\frac{\frac{4b\sinh(x)\cosh^2(x)}{3a}-\frac{\frac{3\left(3a^2+4b^2\right)\sinh(x)\cosh(x)}{2a}-\frac{\frac{8b\left(2a^2+3b^2\right)\sinh(x)}{a}-\frac{3\left(\frac{x\left(3a^4+4a^2b^2+8b^4\right)}{a}-\frac{8b^5\int\frac{\csc\left(ix+\frac{\pi}{2}\right)}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx}{a}\right)}{a}}{2a}}{3a}}{4a}$$

$\Big\downarrow$ 4318

$$\frac{\sinh(x)\cosh^3(x)}{4a}-\frac{\frac{4b\sinh(x)\cosh^2(x)}{3a}-\frac{\frac{3\left(3a^2+4b^2\right)\sinh(x)\cosh(x)}{2a}-\frac{\frac{8b\left(2a^2+3b^2\right)\sinh(x)}{a}-\frac{3\left(\frac{x\left(3a^4+4a^2b^2+8b^4\right)}{a}-\frac{8b^4\int\frac{1}{\frac{a\cosh(x)}{b}+1}dx}{a}\right)}{a}}{2a}}{3a}}{4a}$$

$\Big\downarrow$ 3042

$$\frac{\sinh(x)\cosh^3(x)}{4a}-\frac{\frac{4b\sinh(x)\cosh^2(x)}{3a}-\frac{\frac{3(3a^2+4b^2)\sinh(x)\cosh(x)}{2a}-\frac{\frac{8b(2a^2+3b^2)\sinh(x)}{a}-\frac{3\left(\frac{x(3a^4+4a^2b^2+8b^4)}{a}-\frac{8b^4\int\frac{1}{\frac{a\sin\left(ix+\frac{\pi}{2}\right)}{b}+1}dx}{a}\right)}{a}}{2a}}{3a}}{4a}$$

$\big\downarrow$ 3138

$$\frac{\sinh(x)\cosh^3(x)}{4a}-\frac{\frac{4b\sinh(x)\cosh^2(x)}{3a}-\frac{\frac{3(3a^2+4b^2)\sinh(x)\cosh(x)}{2a}-\frac{\frac{8b(2a^2+3b^2)\sinh(x)}{a}-\frac{3\left(\frac{x(3a^4+4a^2b^2+8b^4)}{a}-\frac{16b^4\int\frac{1}{\frac{a+b}{b}-\left(1-\frac{a}{b}\right)\tanh^2\left(\frac{x}{2}\right)}d\tanh\left(\frac{x}{2}\right)}{a}\right)}{a}}{2a}}{3a}}{4a}$$

$\big\downarrow$ 218

$$\frac{\sinh(x)\cosh^3(x)}{4a}-\frac{\frac{4b\sinh(x)\cosh^2(x)}{3a}-\frac{\frac{3(3a^2+4b^2)\sinh(x)\cosh(x)}{2a}-\frac{\frac{8b(2a^2+3b^2)\sinh(x)}{a}-\frac{3\left(\frac{x(3a^4+4a^2b^2+8b^4)}{a}-\frac{16b^5\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}\right)}{a}}{2a}}{3a}}{4a}$$

input
```
Int[Cosh[x]^4/(a + b*Sech[x]),x]
```

output
```
(Cosh[x]^3*Sinh[x])/(4*a) - ((4*b*Cosh[x]^2*Sinh[x])/(3*a) - ((3*(3*a^2 +
4*b^2)*Cosh[x]*Sinh[x])/(2*a) - ((-3*(((3*a^4 + 4*a^2*b^2 + 8*b^4)*x)/a -
(16*b^5*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a
 + b])))/a + (8*b*(2*a^2 + 3*b^2)*Sinh[x])/a)/(2*a))/(3*a))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4340
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Sim
p[1/(a*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n
- a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(126) = 252$.

Time = 0.68 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.81

method	result
risch	$\frac{3x}{8a}+\frac{x\,b^2}{2a^3}+\frac{x\,b^4}{a^5}+\frac{e^{4x}}{64a}-\frac{b\,e^{3x}}{24a^2}+\frac{e^{2x}}{8a}+\frac{e^{2x}b^2}{8a^3}-\frac{3b\,e^{x}}{8a^2}-\frac{b^3e^{x}}{2a^4}+\frac{3b\,e^{-x}}{8a^2}+\frac{b^3e^{-x}}{2a^4}-\frac{e^{-2x}}{8a}-\frac{e^{-2x}b^2}{8a^3}+\frac{b\,e^{-3x}}{24a^2}$
default	$-\frac{1}{4a\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}-\frac{-3a-2b}{6a^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}-\frac{7a^2+4ab+4b^2}{8a^3\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{\left(3a^4+4a^2b^2+8b^4\right)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8a^5}-\frac{-5a^3-8a^2b-4a\,b^2}{8a^4\left(\tanh\left(\frac{x}{2}\right)+1\right.}$

input
```
int(cosh(x)^4/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
3/8*x/a+1/2*x/a^3*b^2+x/a^5*b^4+1/64/a*exp(x)^4-1/24*b/a^2*exp(x)^3+1/8/a*
exp(x)^2+1/8/a^3*exp(x)^2*b^2-3/8*b/a^2*exp(x)-1/2*b^3/a^4*exp(x)+3/8*b/a^
2/exp(x)+1/2*b^3/a^4/exp(x)-1/8/a/exp(x)^2-1/8/a^3/exp(x)^2*b^2+1/24*b/a^2
/exp(x)^3-1/64/a/exp(x)^4-1/(-a^2+b^2)^(1/2)*b^5/a^5*ln(exp(x)+(b*(-a^2+b^
2)^(1/2)+a^2-b^2)/(-a^2+b^2)^(1/2)/a)+1/(-a^2+b^2)^(1/2)*b^5/a^5*ln(exp(x)
+(b*(-a^2+b^2)^(1/2)-a^2+b^2)/(-a^2+b^2)^(1/2)/a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1161 vs. $2(126) = 252$.

Time = 0.11 (sec) , antiderivative size = 2402, normalized size of antiderivative = 16.45

$$\int \frac{\cosh^4(x)}{a + b\operatorname{sech}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
[1/192*(3*(a^6 - a^4*b^2)*cosh(x)^8 + 3*(a^6 - a^4*b^2)*sinh(x)^8 - 8*(a^5
*b - a^3*b^3)*cosh(x)^7 - 8*(a^5*b - a^3*b^3 - 3*(a^6 - a^4*b^2)*cosh(x))*
sinh(x)^7 + 24*(a^6 - a^2*b^4)*cosh(x)^6 + 4*(6*a^6 - 6*a^2*b^4 + 21*(a^6
- a^4*b^2)*cosh(x)^2 - 14*(a^5*b - a^3*b^3)*cosh(x))*sinh(x)^6 - 3*a^6 + 3
*a^4*b^2 + 24*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x)^4 - 24*(3*a^
5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^5 - 24*(3*a^5*b + a^3*b^3 - 4*a*b^5 - 7*(
a^6 - a^4*b^2)*cosh(x)^3 + 7*(a^5*b - a^3*b^3)*cosh(x)^2 - 6*(a^6 - a^2*b^
4)*cosh(x))*sinh(x)^5 + 2*(105*(a^6 - a^4*b^2)*cosh(x)^4 - 140*(a^5*b - a^
3*b^3)*cosh(x)^3 + 180*(a^6 - a^2*b^4)*cosh(x)^2 + 12*(3*a^6 + a^4*b^2 + 4
*a^2*b^4 - 8*b^6)*x - 60*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x))*sinh(x)^4
+ 24*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^3 + 8*(9*a^5*b + 3*a^3*b^3 - 12
*a*b^5 + 21*(a^6 - a^4*b^2)*cosh(x)^5 - 35*(a^5*b - a^3*b^3)*cosh(x)^4 + 6
0*(a^6 - a^2*b^4)*cosh(x)^3 + 12*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*c
osh(x) - 30*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^2)*sinh(x)^3 - 24*(a^6 -
 a^2*b^4)*cosh(x)^2 + 12*(7*(a^6 - a^4*b^2)*cosh(x)^6 - 2*a^6 + 2*a^2*b^4
- 14*(a^5*b - a^3*b^3)*cosh(x)^5 + 30*(a^6 - a^2*b^4)*cosh(x)^4 + 12*(3*a^
6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x)^2 - 20*(3*a^5*b + a^3*b^3 - 4*a
*b^5)*cosh(x)^3 + 6*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x))*sinh(x)^2 - 192
*(b^5*cosh(x)^4 + 4*b^5*cosh(x)^3*sinh(x) + 6*b^5*cosh(x)^2*sinh(x)^2 + 4*
b^5*cosh(x)*sinh(x)^3 + b^5*sinh(x)^4)*sqrt(-a^2 + b^2)*log((a^2*cosh(x...
```

Sympy [F]

$$\int \frac{\cosh^4(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\cosh^4{(x)}}{a + b\,\text{sech}\,(x)}\,dx$$

input
```
integrate(cosh(x)**4/(a+b*sech(x)),x)
```

output
```
Integral(cosh(x)**4/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^4(x)}{a + b\text{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.25

$$\int \frac{\cosh^4(x)}{a+b\mathrm{sech}(x)}\,dx = -\frac{2\,b^5\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}a^5} + \frac{3\,a^3e^{(4\,x)}-8\,a^2be^{(3\,x)}+24\,a^3e^{(2\,x)}+24\,ab^2e^{(2\,x)}-72\,a^2be^x-96\,b^3e^x}{192\,a^4} + \frac{(3\,a^4+4\,a^2b^2+8\,b^4)x}{8\,a^5} + \frac{\left(8\,a^3be^x-3\,a^4+24\,(3\,a^3b+4\,ab^3)e^{(3\,x)}-24\,(a^4+a^2b^2)e^{(2\,x)}\right)e^{(-4\,x)}}{192\,a^5}$$

input
```
integrate(cosh(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

output
```
-2*b^5*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^5) + 1/192*(
3*a^3*e^(4*x) - 8*a^2*b*e^(3*x) + 24*a^3*e^(2*x) + 24*a*b^2*e^(2*x) - 72*a
^2*b*e^x - 96*b^3*e^x)/a^4 + 1/8*(3*a^4 + 4*a^2*b^2 + 8*b^4)*x/a^5 + 1/192
*(8*a^3*b*e^x - 3*a^4 + 24*(3*a^3*b + 4*a*b^3)*e^(3*x) - 24*(a^4 + a^2*b^2
)*e^(2*x))*e^(-4*x)/a^5
```

Mupad [B] (verification not implemented)

Time = 2.79 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.72

$$\int \frac{\cosh^4(x)}{a+b\mathrm{sech}(x)}\,dx = \frac{e^{4\,x}}{64\,a} - \frac{e^{-4\,x}}{64\,a} + \frac{x\,(3\,a^4+4\,a^2\,b^2+8\,b^4)}{8\,a^5} - \frac{e^{-2\,x}\,(a^2+b^2)}{8\,a^3} + \frac{e^{2\,x}\,(a^2+b^2)}{8\,a^3} + \frac{e^{-x}\,(3\,a^2\,b+4\,b^3)}{8\,a^4} + \frac{b\,e^{-3\,x}}{24\,a^2} - \frac{b\,e^{3\,x}}{24\,a^2} - \frac{e^x\,(3\,a^2\,b+4\,b^3)}{8\,a^4} + \frac{b^5\,\ln\left(\frac{2\,b^5\,e^x}{a^6}-\frac{2\,b^5\,(a+b\,e^x)}{a^6\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^5\,\sqrt{a+b}\,\sqrt{b-a}} - \frac{b^5\,\ln\left(\frac{2\,b^5\,e^x}{a^6}+\frac{2\,b^5\,(a+b\,e^x)}{a^6\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^5\,\sqrt{a+b}\,\sqrt{b-a}}$$

input
```
int(cosh(x)^4/(a + b/cosh(x)),x)
```

output
```
exp(4*x)/(64*a) - exp(-4*x)/(64*a) + (x*(3*a^4 + 8*b^4 + 4*a^2*b^2))/(8*a^
5) - (exp(-2*x)*(a^2 + b^2))/(8*a^3) + (exp(2*x)*(a^2 + b^2))/(8*a^3) + (e
xp(-x)*(3*a^2*b + 4*b^3))/(8*a^4) + (b*exp(-3*x))/(24*a^2) - (b*exp(3*x))/
(24*a^2) - (exp(x)*(3*a^2*b + 4*b^3))/(8*a^4) + (b^5*log((2*b^5*exp(x))/a^
6 - (2*b^5*(a + b*exp(x)))/(a^6*(a + b)^(1/2)*(b - a)^(1/2))))/(a^5*(a + b
)^(1/2)*(b - a)^(1/2)) - (b^5*log((2*b^5*exp(x))/a^6 + (2*b^5*(a + b*exp(x
)))/(a^6*(a + b)^(1/2)*(b - a)^(1/2))))/(a^5*(a + b)^(1/2)*(b - a)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.13

$$\int \frac{\cosh^4(x)}{a + b\mathrm{sech}(x)}\,dx = \frac{-384e^{4x}\sqrt{a^2-b^2}\,atan\left(\frac{e^x a+b}{\sqrt{a^2-b^2}}\right) b^5 + 3e^{8x}a^6 - 3e^{8x}a^4b^2 - 8e^{7x}a^5b + 8e^{7x}a^3b^3 + 24e^{6x}a^6 - 24e^{6x}a^2b^4 - 7}{}$$

input
```
int(cosh(x)^4/(a+b*sech(x)),x)
```

output
```
( - 384*e**(4*x)*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2))*b*
*5 + 3*e**(8*x)*a**6 - 3*e**(8*x)*a**4*b**2 - 8*e**(7*x)*a**5*b + 8*e**(7*
x)*a**3*b**3 + 24*e**(6*x)*a**6 - 24*e**(6*x)*a**2*b**4 - 72*e**(5*x)*a**5
*b - 24*e**(5*x)*a**3*b**3 + 96*e**(5*x)*a*b**5 + 72*e**(4*x)*a**6*x + 24*
e**(4*x)*a**4*b**2*x + 96*e**(4*x)*a**2*b**4*x - 192*e**(4*x)*b**6*x + 72*
e**(3*x)*a**5*b + 24*e**(3*x)*a**3*b**3 - 96*e**(3*x)*a*b**5 - 24*e**(2*x)
*a**6 + 24*e**(2*x)*a**2*b**4 + 8*e**x*a**5*b - 8*e**x*a**3*b**3 - 3*a**6
+ 3*a**4*b**2)/(192*e**(4*x)*a**5*(a**2 - b**2))
```

3.96 $\int \frac{\cosh^3(x)}{a+b\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 112

$$\int \frac{\cosh^3(x)}{a+b\text{sech}(x)}\,dx = -\frac{b(a^2+2b^2)\,x}{2a^4}+\frac{2b^4\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}}+\frac{(2a^2+3b^2)\sinh(x)}{3a^3}-\frac{b\cosh(x)\sinh(x)}{2a^2}+\frac{\cosh^2(x)\sinh(x)}{3a}$$

output
```
-1/2*b*(a^2+2*b^2)*x/a^4+2*b^4*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))
/a^4/(a-b)^(1/2)/(a+b)^(1/2)+1/3*(2*a^2+3*b^2)*sinh(x)/a^3-1/2*b*cosh(x)*s
inh(x)/a^2+1/3*cosh(x)^2*sinh(x)/a
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^3(x)}{a+b\text{sech}(x)}\,dx = \frac{-6b(a^2+2b^2)\,x-\frac{24b^4\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+3a(3a^2+4b^2)\sinh(x)-3a^2b\sinh(2x)+a^3\sinh(3x)}{12a^4}$$

input
```
Integrate[Cosh[x]^3/(a + b*Sech[x]),x]
```

output
```
(-6*b*(a^2 + 2*b^2)*x - (24*b^4*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2
]])/Sqrt[a^2 - b^2] + 3*a*(3*a^2 + 4*b^2)*Sinh[x] - 3*a^2*b*Sinh[2*x] + a^
3*Sinh[3*x])/(12*a^4)
```

Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.15, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.154$, Rules used = {3042, 4340, 25, 3042, 4592, 3042, 4592, 27, 3042, 4407, 3042, 4318, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{a+b\text{sech}(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\csc\left(\frac{\pi}{2}+ix\right)^3\left(a+b\csc\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{ 4340}$$

$$\frac{\int -\frac{\cosh^2(x)\left(-2b\text{sech}^2(x)-2a\text{sech}(x)+3b\right)}{a+b\text{sech}(x)}dx}{3a}+\frac{\sinh(x)\cosh^2(x)}{3a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sinh(x)\cosh^2(x)}{3a}-\frac{\int \frac{\cosh^2(x)\left(-2b\text{sech}^2(x)-2a\text{sech}(x)+3b\right)}{a+b\text{sech}(x)}dx}{3a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sinh(x)\cosh^2(x)}{3a}-\frac{\int \frac{-2b\csc\left(ix+\frac{\pi}{2}\right)^2-2a\csc\left(ix+\frac{\pi}{2}\right)+3b}{\csc\left(ix+\frac{\pi}{2}\right)^2\left(a+b\csc\left(ix+\frac{\pi}{2}\right)\right)}dx}{3a}$$

$$\Big\downarrow \text{ 4592}$$

$$\frac{\sinh(x)\cosh^2(x)}{3a}-\frac{\frac{3b\sinh(x)\cosh(x)}{2a}-\frac{\int \frac{\cosh(x)\left(-3b^2\text{sech}^2(x)+ab\,\text{sech}(x)+2\left(2a^2+3b^2\right)\right)}{a+b\,\text{sech}(x)}dx}{2a}}{3a}$$

$\Big\downarrow$ 3042

$$\frac{\sinh(x)\cosh^2(x)}{3a}-\frac{\frac{3b\sinh(x)\cosh(x)}{2a}-\frac{\int \frac{-3b^2\csc\left(ix+\frac{\pi}{2}\right)^2+ab\csc\left(ix+\frac{\pi}{2}\right)+2\left(2a^2+3b^2\right)}{\csc\left(ix+\frac{\pi}{2}\right)\left(a+b\csc\left(ix+\frac{\pi}{2}\right)\right)}dx}{2a}}{3a}$$

$\Big\downarrow$ 4592

$$\frac{\sinh(x)\cosh^2(x)}{3a}-\frac{\frac{3b\sinh(x)\cosh(x)}{2a}-\frac{\frac{2\left(2a^2+3b^2\right)\sinh(x)}{a}-\frac{\int \frac{3\left(a\,\text{sech}(x)b^2+\left(a^2+2b^2\right)b\right)}{a+b\,\text{sech}(x)}dx}{a}}{2a}}{3a}$$

$\Big\downarrow$ 27

$$\frac{\sinh(x)\cosh^2(x)}{3a}-\frac{\frac{3b\sinh(x)\cosh(x)}{2a}-\frac{\frac{2\left(2a^2+3b^2\right)\sinh(x)}{a}-\frac{3\int \frac{a\,\text{sech}(x)b^2+\left(a^2+2b^2\right)b}{a+b\,\text{sech}(x)}dx}{a}}{2a}}{3a}$$

$\Big\downarrow$ 3042

$$\frac{\sinh(x)\cosh^2(x)}{3a}-\frac{\frac{3b\sinh(x)\cosh(x)}{2a}-\frac{\frac{2\left(2a^2+3b^2\right)\sinh(x)}{a}-\frac{3\int \frac{a\csc\left(ix+\frac{\pi}{2}\right)b^2+\left(a^2+2b^2\right)b}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx}{a}}{2a}}{3a}$$

$\Big\downarrow$ 4407

$$\frac{\sinh(x)\cosh^2(x)}{3a}-\frac{\frac{3b\sinh(x)\cosh(x)}{2a}-\frac{\frac{2\left(2a^2+3b^2\right)\sinh(x)}{a}-\frac{3\left(\frac{bx\left(a^2+2b^2\right)}{a}-\frac{2b^4\int \frac{\text{sech}(x)}{a+b\,\text{sech}(x)}dx}{a}\right)}{a}}{2a}}{3a}$$

$\Big\downarrow$ 3042

$$\frac{\sinh(x)\cosh^2(x)}{3a}-\frac{\frac{3b\sinh(x)\cosh(x)}{2a}-\frac{\frac{2\left(2a^2+3b^2\right)\sinh(x)}{a}-\frac{3\left(\frac{bx\left(a^2+2b^2\right)}{a}-\frac{2b^4\int \frac{\csc\left(ix+\frac{\pi}{2}\right)}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx}{a}\right)}{a}}{2a}}{3a}$$

$\Big\downarrow$ 4318

$$\frac{\sinh(x)\cosh^2(x)}{3a}-\frac{\frac{3b\sinh(x)\cosh(x)}{2a}-\frac{\frac{2(2a^2+3b^2)\sinh(x)}{a}-\frac{3\left(\frac{bx(a^2+2b^2)}{a}-\frac{2b^3\int\frac{1}{\frac{a\cosh(x)}{b}+1}dx}{a}\right)}{a}}{2a}}{3a}$$

$\Big\downarrow$ 3042

$$\frac{\sinh(x)\cosh^2(x)}{3a}-\frac{\frac{3b\sinh(x)\cosh(x)}{2a}-\frac{\frac{2(2a^2+3b^2)\sinh(x)}{a}-\frac{3\left(\frac{bx(a^2+2b^2)}{a}-\frac{2b^3\int\frac{1}{\frac{a\sin\left(ix+\frac{\pi}{2}\right)}{b}+1}dx}{a}\right)}{a}}{2a}}{3a}$$

$\Big\downarrow$ 3138

$$\frac{\sinh(x)\cosh^2(x)}{3a}-\frac{\frac{3b\sinh(x)\cosh(x)}{2a}-\frac{\frac{2(2a^2+3b^2)\sinh(x)}{a}-\frac{3\left(\frac{bx(a^2+2b^2)}{a}-\frac{4b^3\int\frac{1}{\frac{a+b}{b}-\left(1-\frac{a}{b}\right)\tanh^2\left(\frac{x}{2}\right)}d\tanh\left(\frac{x}{2}\right)}{a}\right)}{a}}{2a}}{3a}$$

$\Big\downarrow$ 218

$$\frac{\sinh(x)\cosh^2(x)}{3a}-\frac{\frac{3b\sinh(x)\cosh(x)}{2a}-\frac{\frac{2(2a^2+3b^2)\sinh(x)}{a}-\frac{3\left(\frac{bx(a^2+2b^2)}{a}-\frac{4b^4\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}\right)}{a}}{2a}}{3a}$$

input
```
Int[Cosh[x]^3/(a + b*Sech[x]),x]
```

output
```
(Cosh[x]^2*Sinh[x])/(3*a) - ((3*b*Cosh[x]*Sinh[x])/(2*a) - ((-3*((b*(a^2 +
 2*b^2)*x)/a - (4*b^4*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*Sqrt
[a - b]*Sqrt[a + b])))/a + (2*(2*a^2 + 3*b^2)*Sinh[x])/a)/(2*a))/(3*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4340
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Sim
p[1/(a*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n
- a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(94) = 188.

Time = 0.39 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.81

method	result
default	$\frac{2b^4 \arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^4\sqrt{(a-b)(a+b)}} - \frac{1}{3a\left(\tanh\left(\frac{x}{2}\right)+1\right)^3} - \frac{-a-b}{2a^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2} - \frac{2a^2+ab+2b^2}{2a^3\left(\tanh\left(\frac{x}{2}\right)+1\right)} - \frac{b\left(a^2+2b^2\right)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2a^4}$ -
risch	$-\frac{xb}{2a^2} - \frac{x\,b^3}{a^4} + \frac{e^{3x}}{24a} - \frac{b\,e^{2x}}{8a^2} + \frac{3\,e^{x}}{8a} + \frac{e^{x}b^2}{2a^3} - \frac{3\,e^{-x}}{8a} - \frac{e^{-x}b^2}{2a^3} + \frac{b\,e^{-2x}}{8a^2} - \frac{e^{-3x}}{24a} - \frac{b^4\ln\left(e^{x}+\frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,a^4}$ +

input
```
int(cosh(x)^3/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
2*b^4/a^4/((a-b)*(a+b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a-b)*(a+b))^(1/2)
)-1/3/a/(tanh(1/2*x)+1)^3-1/2*(-a-b)/a^2/(tanh(1/2*x)+1)^2-1/2*(2*a^2+a*b+
2*b^2)/a^3/(tanh(1/2*x)+1)-1/2*b*(a^2+2*b^2)/a^4*ln(tanh(1/2*x)+1)-1/3/a/(
tanh(1/2*x)-1)^3-1/2*(a+b)/a^2/(tanh(1/2*x)-1)^2-1/2*(2*a^2+a*b+2*b^2)/a^3
/(tanh(1/2*x)-1)+1/2*b*(a^2+2*b^2)/a^4*ln(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(94) = 188$.

Time = 0.11 (sec) , antiderivative size = 1562, normalized size of antiderivative = 13.95

$$\int \frac{\cosh^3(x)}{a + b\text{sech}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
[1/24*((a^5 - a^3*b^2)*cosh(x)^6 + (a^5 - a^3*b^2)*sinh(x)^6 - 3*(a^4*b -
a^2*b^3)*cosh(x)^5 - 3*(a^4*b - a^2*b^3 - 2*(a^5 - a^3*b^2)*cosh(x))*sinh(
x)^5 - a^5 + a^3*b^2 - 12*(a^4*b + a^2*b^3 - 2*b^5)*x*cosh(x)^3 + 3*(3*a^5
 + a^3*b^2 - 4*a*b^4)*cosh(x)^4 + 3*(3*a^5 + a^3*b^2 - 4*a*b^4 + 5*(a^5 -
a^3*b^2)*cosh(x)^2 - 5*(a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4 + 2*(10*(a^5 -
 a^3*b^2)*cosh(x)^3 - 15*(a^4*b - a^2*b^3)*cosh(x)^2 - 6*(a^4*b + a^2*b^3
- 2*b^5)*x + 6*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x)^3 - 3*(3*a^5 +
 a^3*b^2 - 4*a*b^4)*cosh(x)^2 - 3*(3*a^5 + a^3*b^2 - 4*a*b^4 - 5*(a^5 - a^
3*b^2)*cosh(x)^4 + 10*(a^4*b - a^2*b^3)*cosh(x)^3 + 12*(a^4*b + a^2*b^3 -
2*b^5)*x*cosh(x) - 6*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^2)*sinh(x)^2 - 24
*(b^4*cosh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*
sinh(x)^3)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cos
h(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(-a^2 + b^2)*(a
*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a
*cosh(x) + b)*sinh(x) + a)) + 3*(a^4*b - a^2*b^3)*cosh(x) + 3*(2*(a^5 - a^
3*b^2)*cosh(x)^5 + a^4*b - a^2*b^3 - 5*(a^4*b - a^2*b^3)*cosh(x)^4 - 12*(a
^4*b + a^2*b^3 - 2*b^5)*x*cosh(x)^2 + 4*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x
)^3 - 2*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x))/((a^6 - a^4*b^2)*cos
h(x)^3 + 3*(a^6 - a^4*b^2)*cosh(x)^2*sinh(x) + 3*(a^6 - a^4*b^2)*cosh(x)*s
inh(x)^2 + (a^6 - a^4*b^2)*sinh(x)^3), 1/24*((a^5 - a^3*b^2)*cosh(x)^6 ...
```

Sympy [F]

$$\int \frac{\cosh^3(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\cosh^3{(x)}}{a + b\,\text{sech}\,(x)}\,dx$$

input
```
integrate(cosh(x)**3/(a+b*sech(x)),x)
```

output
```
Integral(cosh(x)**3/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(x)}{a + b\text{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.19

$$\int \frac{\cosh^3(x)}{a + b\text{sech}(x)}\,dx = \frac{2\,b^4\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}a^4} + \frac{a^2e^{(3\,x)} - 3\,abe^{(2\,x)} + 9\,a^2e^x + 12\,b^2e^x}{24\,a^3} - \frac{(a^2b + 2\,b^3)x}{2\,a^4} + \frac{\left(3\,a^2be^x - a^3 - 3\,(3\,a^3 + 4\,ab^2)e^{(2\,x)}\right)e^{(-3\,x)}}{24\,a^4}$$

input
```
integrate(cosh(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

output
```
2*b^4*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^4) + 1/24*(a^
2*e^(3*x) - 3*a*b*e^(2*x) + 9*a^2*e^x + 12*b^2*e^x)/a^3 - 1/2*(a^2*b + 2*b
^3)*x/a^4 + 1/24*(3*a^2*b*e^x - a^3 - 3*(3*a^3 + 4*a*b^2)*e^(2*x))*e^(-3*x
)/a^4
```

Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.87

$$\int \frac{\cosh^3(x)}{a+b\text{sech}(x)}\,dx = \frac{e^{3x}}{24a} - \frac{e^{-3x}}{24a} - \frac{x(a^2b+2b^3)}{2a^4} + \frac{e^x(3a^2+4b^2)}{8a^3} + \frac{b\,e^{-2x}}{8a^2} - \frac{b\,e^{2x}}{8a^2} - \frac{e^{-x}(3a^2+4b^2)}{8a^3} + \frac{b^4\ln\left(-\frac{2b^4e^x}{a^5} - \frac{2b^4(a+b\,e^x)}{a^5\sqrt{a+b}\sqrt{b-a}}\right)}{a^4\sqrt{a+b}\sqrt{b-a}} - \frac{b^4\ln\left(\frac{2b^4(a+b\,e^x)}{a^5\sqrt{a+b}\sqrt{b-a}} - \frac{2b^4e^x}{a^5}\right)}{a^4\sqrt{a+b}\sqrt{b-a}}$$

input
```
int(cosh(x)^3/(a + b/cosh(x)),x)
```

output
```
exp(3*x)/(24*a) - exp(-3*x)/(24*a) - (x*(a^2*b + 2*b^3))/(2*a^4) + (exp(x)
*(3*a^2 + 4*b^2))/(8*a^3) + (b*exp(-2*x))/(8*a^2) - (b*exp(2*x))/(8*a^2) -
 (exp(-x)*(3*a^2 + 4*b^2))/(8*a^3) + (b^4*log(- (2*b^4*exp(x))/a^5 - (2*b^
4*(a + b*exp(x)))/(a^5*(a + b)^(1/2)*(b - a)^(1/2))))/(a^4*(a + b)^(1/2)*(
b - a)^(1/2)) - (b^4*log((2*b^4*(a + b*exp(x)))/(a^5*(a + b)^(1/2)*(b - a)
^(1/2)) - (2*b^4*exp(x))/a^5))/(a^4*(a + b)^(1/2)*(b - a)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.21

$$\int \frac{\cosh^3(x)}{a+b\text{sech}(x)}\,dx = \frac{48e^{3x}\sqrt{a^2-b^2}\,atan\left(\frac{e^xa+b}{\sqrt{a^2-b^2}}\right)b^4 + e^{6x}a^5 - e^{6x}a^3b^2 - 3e^{5x}a^4b + 3e^{5x}a^2b^3 + 9e^{4x}a^5 + 3e^{4x}a^3b^2 - 12e^{4x}a\,b^4}{24e^{3x}a^4(}$$

input
```
int(cosh(x)^3/(a+b*sech(x)),x)
```

output

```
(48*e**(3*x)*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2))*b**4 +
 e**(6*x)*a**5 - e**(6*x)*a**3*b**2 - 3*e**(5*x)*a**4*b + 3*e**(5*x)*a**2*
b**3 + 9*e**(4*x)*a**5 + 3*e**(4*x)*a**3*b**2 - 12*e**(4*x)*a*b**4 - 12*e*
*(3*x)*a**4*b*x - 12*e**(3*x)*a**2*b**3*x + 24*e**(3*x)*b**5*x - 9*e**(2*x
)*a**5 - 3*e**(2*x)*a**3*b**2 + 12*e**(2*x)*a*b**4 + 3*e**x*a**4*b - 3*e**
x*a**2*b**3 - a**5 + a**3*b**2)/(24*e**(3*x)*a**4*(a**2 - b**2))
```

3.97 $\int \frac{\cosh^2(x)}{a+b\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 85

$$\int \frac{\cosh^2(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{(a^2+2b^2)\,x}{2a^3} - \frac{2b^3\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}} - \frac{b\sinh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a}$$

output
```
1/2*(a^2+2*b^2)*x/a^3-2*b^3*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a^
3/(a-b)^(1/2)/(a+b)^(1/2)-b*sinh(x)/a^2+1/2*cosh(x)*sinh(x)/a
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{\cosh^2(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{2a^2x+4b^2x+\frac{8b^3\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-4ab\sinh(x)+a^2\sinh(2x)}{4a^3}$$

input
```
Integrate[Cosh[x]^2/(a + b*Sech[x]),x]
```

output
```
(2*a^2*x + 4*b^2*x + (8*b^3*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/
Sqrt[a^2 - b^2] - 4*a*b*Sinh[x] + a^2*Sinh[2*x])/(4*a^3)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$, Rules used = {3042, 4340, 25, 3042, 4592, 3042, 4407, 3042, 4318, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{a+b\text{sech}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(\frac{\pi}{2}+ix\right)^2\left(a+b\csc\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{4340}$$

$$\frac{\int -\frac{\cosh(x)\left(-b\text{sech}^2(x)-a\text{sech}(x)+2b\right)}{a+b\text{sech}(x)}dx}{2a}+\frac{\sinh(x)\cosh(x)}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh(x)\cosh(x)}{2a}-\frac{\int \frac{\cosh(x)\left(-b\text{sech}^2(x)-a\text{sech}(x)+2b\right)}{a+b\text{sech}(x)}dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)\cosh(x)}{2a}-\frac{\int \frac{-b\csc\left(ix+\frac{\pi}{2}\right)^2-a\csc\left(ix+\frac{\pi}{2}\right)+2b}{\csc\left(ix+\frac{\pi}{2}\right)\left(a+b\csc\left(ix+\frac{\pi}{2}\right)\right)}dx}{2a}$$

$$\Big\downarrow \text{4592}$$

$$\frac{\sinh(x)\cosh(x)}{2a}-\frac{\frac{2b\sinh(x)}{a}-\frac{\int \frac{a^2+b\text{sech}(x)a+2b^2}{a+b\text{sech}(x)}dx}{a}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)\cosh(x)}{2a}-\frac{\frac{2b\sinh(x)}{a}-\frac{\int \frac{a^2+b\csc\left(ix+\frac{\pi}{2}\right)a+2b^2}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx}{a}}{2a}$$

$\Big\downarrow$ 4407

$$\frac{\sinh(x)\cosh(x)}{2a}-\frac{\frac{2b\sinh(x)}{a}-\frac{\frac{x(a^2+2b^2)}{a}-\frac{2b^3\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}dx}{a}}{a}}{2a}$$

$\Big\downarrow$ 3042

$$\frac{\sinh(x)\cosh(x)}{2a}-\frac{\frac{2b\sinh(x)}{a}-\frac{\frac{x(a^2+2b^2)}{a}-\frac{2b^3\int \frac{\csc\left(ix+\frac{\pi}{2}\right)}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx}{a}}{a}}{2a}$$

$\Big\downarrow$ 4318

$$\frac{\sinh(x)\cosh(x)}{2a}-\frac{\frac{2b\sinh(x)}{a}-\frac{\frac{x(a^2+2b^2)}{a}-\frac{2b^2\int \frac{1}{\frac{a\cosh(x)}{b}+1}dx}{a}}{a}}{2a}$$

$\Big\downarrow$ 3042

$$\frac{\sinh(x)\cosh(x)}{2a}-\frac{\frac{2b\sinh(x)}{a}-\frac{\frac{x(a^2+2b^2)}{a}-\frac{2b^2\int \frac{1}{\frac{a\sin\left(ix+\frac{\pi}{2}\right)}{b}+1}dx}{a}}{a}}{2a}$$

$\Big\downarrow$ 3138

$$\frac{\sinh(x)\cosh(x)}{2a}-\frac{\frac{2b\sinh(x)}{a}-\frac{\frac{x(a^2+2b^2)}{a}-\frac{4b^2\int \frac{1}{\frac{a+b}{b}-\left(1-\frac{a}{b}\right)\tanh^2\left(\frac{x}{2}\right)}d\tanh\left(\frac{x}{2}\right)}{a}}{a}}{2a}$$

$\Big\downarrow$ 218

$$\frac{\sinh(x)\cosh(x)}{2a}-\frac{\frac{2b\sinh(x)}{a}-\frac{\frac{x(a^2+2b^2)}{a}-\frac{4b^3\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}}{a}}{2a}$$

input
```
Int[Cosh[x]^2/(a + b*Sech[x]),x]
```

output
```
(Cosh[x]*Sinh[x])/(2*a) - (-((((a^2 + 2*b^2)*x)/a - (4*b^3*ArcTan[(Sqrt[a
- b]*Tanh[x/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]))/a) + (2*b*Sinh[
x])/a)/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4340
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Sim
p[1/(a*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n
- a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(71) = 142$.

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.80

method	result
default	$-\frac{2b^3\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^3\sqrt{(a-b)(a+b)}}+\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{-a-2b}{2a^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\left(-a^2-2b^2\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a^3}-\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}$
risch	$\frac{x}{2a}+\frac{x\,b^2}{a^3}+\frac{e^{2x}}{8a}-\frac{b\,e^{x}}{2a^2}+\frac{b\,e^{-x}}{2a^2}-\frac{e^{-2x}}{8a}-\frac{b^3\ln\left(e^{x}+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,a^3}+\frac{b^3\ln\left(e^{x}+\frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,a^3}$

input
```
int(cosh(x)^2/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*b^3/a^3/((a-b)*(a+b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a-b)*(a+b))^(1/2
))+1/2/a/(tanh(1/2*x)-1)^2-1/2*(-a-2*b)/a^2/(tanh(1/2*x)-1)+1/2/a^3*(-a^2-
2*b^2)*ln(tanh(1/2*x)-1)-1/2/a/(tanh(1/2*x)+1)^2-1/2*(-a-2*b)/a^2/(tanh(1/
2*x)+1)+1/2*(a^2+2*b^2)/a^3*ln(tanh(1/2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(71) = 142$.

Time = 0.10 (sec) , antiderivative size = 860, normalized size of antiderivative = 10.12

$$\int \frac{\cosh^2(x)}{a + b\text{sech}(x)} \, dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
[1/8*((a^4 - a^2*b^2)*cosh(x)^4 + (a^4 - a^2*b^2)*sinh(x)^4 - a^4 + a^2*b^
2 + 4*(a^4 + a^2*b^2 - 2*b^4)*x*cosh(x)^2 - 4*(a^3*b - a*b^3)*cosh(x)^3 -
4*(a^3*b - a*b^3 - (a^4 - a^2*b^2)*cosh(x))*sinh(x)^3 + 2*(3*(a^4 - a^2*b^
2)*cosh(x)^2 + 2*(a^4 + a^2*b^2 - 2*b^4)*x - 6*(a^3*b - a*b^3)*cosh(x))*si
nh(x)^2 - 8*(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2)*sqrt(-
a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^
2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh
(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sin
h(x) + a)) + 4*(a^3*b - a*b^3)*cosh(x) + 4*(a^3*b - a*b^3 + (a^4 - a^2*b^2
)*cosh(x)^3 + 2*(a^4 + a^2*b^2 - 2*b^4)*x*cosh(x) - 3*(a^3*b - a*b^3)*cosh
(x)^2)*sinh(x))/((a^5 - a^3*b^2)*cosh(x)^2 + 2*(a^5 - a^3*b^2)*cosh(x)*sin
h(x) + (a^5 - a^3*b^2)*sinh(x)^2), 1/8*((a^4 - a^2*b^2)*cosh(x)^4 + (a^4 -
 a^2*b^2)*sinh(x)^4 - a^4 + a^2*b^2 + 4*(a^4 + a^2*b^2 - 2*b^4)*x*cosh(x)^
2 - 4*(a^3*b - a*b^3)*cosh(x)^3 - 4*(a^3*b - a*b^3 - (a^4 - a^2*b^2)*cosh(
x))*sinh(x)^3 + 2*(3*(a^4 - a^2*b^2)*cosh(x)^2 + 2*(a^4 + a^2*b^2 - 2*b^4)
*x - 6*(a^3*b - a*b^3)*cosh(x))*sinh(x)^2 + 16*(b^3*cosh(x)^2 + 2*b^3*cosh
(x)*sinh(x) + b^3*sinh(x)^2)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x
) + b)/sqrt(a^2 - b^2)) + 4*(a^3*b - a*b^3)*cosh(x) + 4*(a^3*b - a*b^3 + (
a^4 - a^2*b^2)*cosh(x)^3 + 2*(a^4 + a^2*b^2 - 2*b^4)*x*cosh(x) - 3*(a^3*b
- a*b^3)*cosh(x)^2)*sinh(x))/((a^5 - a^3*b^2)*cosh(x)^2 + 2*(a^5 - a^3*...
```

Sympy [F]

$$\int \frac{\cosh^2(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\cosh^2(x)}{a + b\,\text{sech}(x)}\,dx$$

input
```
integrate(cosh(x)**2/(a+b*sech(x)),x)
```

output
```
Integral(cosh(x)**2/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^2(x)}{a + b\text{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08

$$\int \frac{\cosh^2(x)}{a + b\text{sech}(x)}\,dx = -\frac{2\,b^3\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}a^3} + \frac{ae^{(2\,x)} - 4\,be^x}{8\,a^2} + \frac{(a^2+2\,b^2)x}{2\,a^3} + \frac{(4\,abe^x - a^2)e^{(-2\,x)}}{8\,a^3}$$

input
```
integrate(cosh(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

output
```
-2*b^3*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^3) + 1/8*(a*
e^(2*x) - 4*b*e^x)/a^2 + 1/2*(a^2 + 2*b^2)*x/a^3 + 1/8*(4*a*b*e^x - a^2)*e
^(-2*x)/a^3
```

Mupad [B] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.96

$$\int \frac{\cosh^2(x)}{a+b\text{sech}(x)}\,dx = \frac{e^{2x}}{8a} - \frac{e^{-2x}}{8a} - \frac{b\,e^{x}}{2a^2} + \frac{b\,e^{-x}}{2a^2} + \frac{x\,(a^2+2b^2)}{2a^3} + \frac{b^3\ln\left(\frac{2b^3e^x}{a^4} - \frac{2b^3(a+b\,e^x)}{a^4\sqrt{a+b}\sqrt{b-a}}\right)}{a^3\sqrt{a+b}\sqrt{b-a}} - \frac{b^3\ln\left(\frac{2b^3e^x}{a^4} + \frac{2b^3(a+b\,e^x)}{a^4\sqrt{a+b}\sqrt{b-a}}\right)}{a^3\sqrt{a+b}\sqrt{b-a}}$$

input
```
int(cosh(x)^2/(a + b/cosh(x)),x)
```

output
```
exp(2*x)/(8*a) - exp(-2*x)/(8*a) - (b*exp(x))/(2*a^2) + (b*exp(-x))/(2*a^2
) + (x*(a^2 + 2*b^2))/(2*a^3) + (b^3*log((2*b^3*exp(x))/a^4 - (2*b^3*(a +
b*exp(x)))/(a^4*(a + b)^(1/2)*(b - a)^(1/2))))/(a^3*(a + b)^(1/2)*(b - a)^
(1/2)) - (b^3*log((2*b^3*exp(x))/a^4 + (2*b^3*(a + b*exp(x)))/(a^4*(a + b)
^(1/2)*(b - a)^(1/2))))/(a^3*(a + b)^(1/2)*(b - a)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.06

$$\int \frac{\cosh^2(x)}{a+b\text{sech}(x)}\,dx = \frac{-16e^{2x}\sqrt{a^2-b^2}\,atan\left(\frac{e^x a+b}{\sqrt{a^2-b^2}}\right)b^3 + e^{4x}a^4 - e^{4x}a^2b^2 - 4e^{3x}a^3b + 4e^{3x}a\,b^3 + 4e^{2x}a^4x + 4e^{2x}a^2b^2x - 8e^{2x}b}{8e^{2x}a^3\,(a^2-b^2)}$$

input
```
int(cosh(x)^2/(a+b*sech(x)),x)
```

output
```
( - 16*e**(2*x)*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2))*b**
3 + e**(4*x)*a**4 - e**(4*x)*a**2*b**2 - 4*e**(3*x)*a**3*b + 4*e**(3*x)*a*
b**3 + 4*e**(2*x)*a**4*x + 4*e**(2*x)*a**2*b**2*x - 8*e**(2*x)*b**4*x + 4*
e**x*a**3*b - 4*e**x*a*b**3 - a**4 + a**2*b**2)/(8*e**(2*x)*a**3*(a**2 - b
**2))
```

3.98 $\int \frac{\cosh(x)}{a+b\text{sech}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 62

$$\int \frac{\cosh(x)}{a+b\text{sech}(x)}\,dx=-\frac{bx}{a^2}+\frac{2b^2\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}}+\frac{\sinh(x)}{a}$$

output
```
-b*x/a^2+2*b^2*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a^2/(a-b)^(1/2)
/(a+b)^(1/2)+sinh(x)/a
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \frac{\cosh(x)}{a+b\text{sech}(x)}\,dx=\frac{b\left(-x-\frac{2b\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)+a\sinh(x)}{a^2}$$

input
```
Integrate[Cosh[x]/(a + b*Sech[x]),x]
```

output
```
(b*(-x - (2*b*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2
]) + a*Sinh[x])/a^2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {3042, 4340, 25, 27, 3042, 4270, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{a+b\text{sech}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(\frac{\pi}{2}+ix\right)\left(a+b\csc\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{4340}$$

$$\frac{\int -\frac{b}{a+b\text{sech}(x)}dx}{a}+\frac{\sinh(x)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh(x)}{a}-\frac{\int \frac{b}{a+b\text{sech}(x)}dx}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sinh(x)}{a}-\frac{b\int \frac{1}{a+b\text{sech}(x)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)}{a}-\frac{b\int \frac{1}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx}{a}$$

$$\Big\downarrow \text{4270}$$

$$\frac{\sinh(x)}{a}-\frac{b\left(\frac{x}{a}-\frac{\int \frac{1}{\frac{a\cosh(x)}{b}+1}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)}{a}-\frac{b\left(\frac{x}{a}-\frac{\int \frac{1}{\frac{a\sin\left(ix+\frac{\pi}{2}\right)}{b}+1}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\sinh(x)}{a}-\frac{b\left(\frac{x}{a}-\frac{2\int \frac{1}{\frac{a+b}{b}-\left(1-\frac{a}{b}\right)\tanh^2\left(\frac{x}{2}\right)}d\tanh\left(\frac{x}{2}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sinh(x)}{a}-\frac{b\left(\frac{x}{a}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}\right)}{a}$$

input
```
Int[Cosh[x]/(a + b*Sech[x]),x]
```

output
```
-((b*(x/a - (2*b*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*Sqrt[a -
b]*Sqrt[a + b])))/a) + Sinh[x]/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4270
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Simp[1/a   Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[a^2 - b^2, 0]
```

rule 4340
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Sim
p[1/(a*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n
- a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.52

method	result	size
default	$-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a^{2}}+\frac{2b^{2}\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^{2}\sqrt{(a-b)(a+b)}}-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a^{2}}$	94
risch	$-\frac{xb}{a^{2}}+\frac{\mathrm{e}^{x}}{2a}-\frac{\mathrm{e}^{-x}}{2a}-\frac{b^{2}\ln\left(\mathrm{e}^{x}+\frac{b\sqrt{-a^{2}+b^{2}}-a^{2}+b^{2}}{\sqrt{-a^{2}+b^{2}}\,a}\right)}{\sqrt{-a^{2}+b^{2}}\,a^{2}}+\frac{b^{2}\ln\left(\mathrm{e}^{x}+\frac{b\sqrt{-a^{2}+b^{2}}+a^{2}-b^{2}}{\sqrt{-a^{2}+b^{2}}\,a}\right)}{\sqrt{-a^{2}+b^{2}}\,a^{2}}$	144

input
```
int(cosh(x)/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/a/(tanh(1/2*x)+1)-b/a^2*ln(tanh(1/2*x)+1)+2*b^2/a^2/((a-b)*(a+b))^(1/2)
*arctan((a-b)*tanh(1/2*x)/((a-b)*(a+b))^(1/2))-1/a/(tanh(1/2*x)-1)+b/a^2*l
n(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(52) = 104.

Time = 0.10 (sec) , antiderivative size = 430, normalized size of antiderivative = 6.94

$$\int \frac{\cosh(x)}{a + b\text{sech}(x)} \, dx$$
$$= \left[-\frac{a^3 - ab^2 + 2\,(a^2b - b^3)x\cosh(x) - (a^3 - ab^2)\cosh(x)^2 - (a^3 - ab^2)\sinh(x)^2 + 2\,(b^2\cosh(x) + b^2\sinh(x)\right.$$
$$-\frac{a^3 - ab^2 + 2\,(a^2b - b^3)x\cosh(x) - (a^3 - ab^2)\cosh(x)^2 - (a^3 - ab^2)\sinh(x)^2 + 4\,(b^2\cosh(x) + b^2\sin}{2\,((a^4 - a^2b^2)\cosh(x) + (a}$$

input
```
integrate(cosh(x)/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
[-1/2*(a^3 - a*b^2 + 2*(a^2*b - b^3)*x*cosh(x) - (a^3 - a*b^2)*cosh(x)^2 -
 (a^3 - a*b^2)*sinh(x)^2 + 2*(b^2*cosh(x) + b^2*sinh(x))*sqrt(-a^2 + b^2)*
log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*
cosh(x) + a*b)*sinh(x) - 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(
a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a))
+ 2*((a^2*b - b^3)*x - (a^3 - a*b^2)*cosh(x))*sinh(x))/((a^4 - a^2*b^2)*co
sh(x) + (a^4 - a^2*b^2)*sinh(x)), -1/2*(a^3 - a*b^2 + 2*(a^2*b - b^3)*x*co
sh(x) - (a^3 - a*b^2)*cosh(x)^2 - (a^3 - a*b^2)*sinh(x)^2 + 4*(b^2*cosh(x)
 + b^2*sinh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a
^2 - b^2)) + 2*((a^2*b - b^3)*x - (a^3 - a*b^2)*cosh(x))*sinh(x))/((a^4 -
a^2*b^2)*cosh(x) + (a^4 - a^2*b^2)*sinh(x))]
```

Sympy [F]

$$\int \frac{\cosh(x)}{a + b\text{sech}(x)} \, dx = \int \frac{\cosh(x)}{a + b\,\text{sech}(x)} \, dx$$

input
```
integrate(cosh(x)/(a+b*sech(x)),x)
```

output
```
Integral(cosh(x)/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh(x)}{a + b\text{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{a + b\text{sech}(x)}\,dx = \frac{2\,b^2\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}a^2} - \frac{bx}{a^2} - \frac{e^{(-x)}}{2\,a} + \frac{e^x}{2\,a}$$

input
```
integrate(cosh(x)/(a+b*sech(x)),x, algorithm="giac")
```

output
```
2*b^2*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^2) - b*x/a^2
- 1/2*e^(-x)/a + 1/2*e^x/a
```

Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.24

$$\int \frac{\cosh(x)}{a+b\mathrm{sech}(x)}\,dx = \frac{\mathrm{e}^x}{2\,a}-\frac{\mathrm{e}^{-x}}{2\,a}-\frac{b\,x}{a^2}+\frac{b^2\,\ln\left(-\frac{2\,b^2\,\mathrm{e}^x}{a^3}-\frac{2\,b^2\,(a+b\,\mathrm{e}^x)}{a^3\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^2\,\sqrt{a+b}\,\sqrt{b-a}} - \frac{b^2\,\ln\left(\frac{2\,b^2\,(a+b\,\mathrm{e}^x)}{a^3\,\sqrt{a+b}\,\sqrt{b-a}}-\frac{2\,b^2\,\mathrm{e}^x}{a^3}\right)}{a^2\,\sqrt{a+b}\,\sqrt{b-a}}$$

input

```
int(cosh(x)/(a + b/cosh(x)),x)
```

output

```
exp(x)/(2*a) - exp(-x)/(2*a) - (b*x)/a^2 + (b^2*log(- (2*b^2*exp(x))/a^3 -
 (2*b^2*(a + b*exp(x)))/(a^3*(a + b)^(1/2)*(b - a)^(1/2))))/(a^2*(a + b)^(
1/2)*(b - a)^(1/2)) - (b^2*log((2*b^2*(a + b*exp(x)))/(a^3*(a + b)^(1/2)*(
b - a)^(1/2)) - (2*b^2*exp(x))/a^3))/(a^2*(a + b)^(1/2)*(b - a)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.77

$$\int \frac{\cosh(x)}{a+b\mathrm{sech}(x)}\,dx = \frac{4e^x\sqrt{a^2-b^2}\,atan\left(\frac{e^xa+b}{\sqrt{a^2-b^2}}\right)b^2+e^{2x}a^3-e^{2x}a\,b^2-2e^xa^2bx+2e^xb^3x-a^3+a\,b^2}{2e^xa^2\,(a^2-b^2)}$$

input

```
int(cosh(x)/(a+b*sech(x)),x)
```

output

```
(4*e**x*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2))*b**2 + e**(
2*x)*a**3 - e**(2*x)*a*b**2 - 2*e**x*a**2*b*x + 2*e**x*b**3*x - a**3 + a*b
**2)/(2*e**x*a**2*(a**2 - b**2))
```

3.99 $\int \frac{\text{sech}(x)}{a+b\text{sech}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 42

$$\int \frac{\text{sech}(x)}{a+b\text{sech}(x)}\,dx=\frac{2\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

output
```
2*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{\text{sech}(x)}{a+b\text{sech}(x)}\,dx=-\frac{2\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

input
```
Integrate[Sech[x]/(a + b*Sech[x]),x]
```

output
```
(-2*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 4318, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(x)}{a+b\text{sech}(x)} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc \left(\frac{\pi }{2}+i x\right)}{a+b \csc \left(\frac{\pi }{2}+i x\right)}dx$$
$$\Big\downarrow \text{4318}$$
$$\frac{\int \frac{1}{\frac{a \cosh (x)}{b}+1}dx}{b}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \frac{1}{\frac{a \sin \left(i x+\frac{\pi }{2}\right)}{b}+1}dx}{b}$$
$$\Big\downarrow \text{3138}$$
$$\frac{2 \int \frac{1}{\frac{a+b}{b}-\left(1-\frac{a}{b}\right) \tanh ^2\left(\frac{x}{2}\right)}d\tanh \left(\frac{x}{2}\right)}{b}$$
$$\Big\downarrow \text{218}$$
$$\frac{2 \arctan \left(\frac{\sqrt{a-b} \tanh \left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b} \sqrt{a+b}}$$

input
```
Int[Sech[x]/(a + b*Sech[x]),x]
```

output
```
(2*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{2\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}$	36
risch	$-\frac{\ln\left(e^{x}+\frac{b\sqrt{-a^{2}+b^{2}}-a^{2}+b^{2}}{\sqrt{-a^{2}+b^{2}}\,a}\right)}{\sqrt{-a^{2}+b^{2}}}+\frac{\ln\left(e^{x}+\frac{b\sqrt{-a^{2}+b^{2}}+a^{2}-b^{2}}{\sqrt{-a^{2}+b^{2}}\,a}\right)}{\sqrt{-a^{2}+b^{2}}}$	109

input
```
int(sech(x)/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
2/((a-b)*(a+b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a-b)*(a+b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.93

$$\int \frac{\mathrm{sech}(x)}{a + b\mathrm{sech}(x)}\,dx = \left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)-a^2+2\,b^2+2\,(a^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{-a^2+b^2}(a\cosh(x)+a\sinh(x)+b)}{a\cosh(x)^2+a\sinh(x)^2+2\,b\cosh(x)+2\,(a\cosh(x)+b)\sinh(x)+a}\right)}{a^2-b^2}, -\frac{2\,\arctan\left(-\frac{a\cosh(x)+a\sinh(x)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right]$$

input
```
integrate(sech(x)/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
[-sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^
2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(-a^2 + b^2)*(a*cosh(x)
+ a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x)
+ b)*sinh(x) + a))/(a^2 - b^2), -2*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqr
t(a^2 - b^2))/sqrt(a^2 - b^2)]
```

Sympy [F]

$$\int \frac{\mathrm{sech}(x)}{a + b\mathrm{sech}(x)}\,dx = \int \frac{\mathrm{sech}\,(x)}{a + b\,\mathrm{sech}\,(x)}\,dx$$

input
```
integrate(sech(x)/(a+b*sech(x)),x)
```

output
```
Integral(sech(x)/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sech(x)/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{2\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

input
```
integrate(sech(x)/(a+b*sech(x)),x, algorithm="giac")
```

output
```
2*arctan((a*e^x + b)/sqrt(a^2 - b^2))/sqrt(a^2 - b^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{a^2-b^2}}+\frac{a\,\mathrm{e}^x}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

input
```
int(1/(cosh(x)*(a + b/cosh(x))),x)
```

output
```
(2*atan(b/(a^2 - b^2)^(1/2) + (a*exp(x))/(a^2 - b^2)^(1/2)))/(a^2 - b^2)^(
1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \frac{\operatorname{sech}(x)}{a + b\operatorname{sech}(x)}\,dx = \frac{2\sqrt{a^2 - b^2}\,atan\left(\frac{e^x a + b}{\sqrt{a^2 - b^2}}\right)}{a^2 - b^2}$$

input
```
int(sech(x)/(a+b*sech(x)),x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2)))/(a**2 - b**2)
```

3.100 $\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx=\frac{\arctan(\sinh(x))}{b}-\frac{2a\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}}$$

output
```
arctan(sinh(x))/b-2*a*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(1
/2)/b/(a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx=\frac{2\left(\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{a\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{b}$$

input
```
Integrate[Sech[x]^2/(a + b*Sech[x]),x]
```

output
```
(2*(ArcTan[Tanh[x/2]] + (a*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/S
qrt[a^2 - b^2]))/b
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {3042, 4276, 3042, 4257, 4318, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(x)}{a+b\text{sech}(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(\frac{\pi}{2}+ix\right)^2}{a+b\csc\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 4276}$$

$$\frac{\int \text{sech}(x)dx}{b}-\frac{a\int \frac{\text{sech}(x)}{a+b\text{sech}(x)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{b}-\frac{a\int \frac{\csc\left(ix+\frac{\pi}{2}\right)}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\arctan(\sinh(x))}{b}-\frac{a\int \frac{\csc\left(ix+\frac{\pi}{2}\right)}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{ 4318}$$

$$\frac{\arctan(\sinh(x))}{b}-\frac{a\int \frac{1}{\frac{a\cosh(x)}{b}+1}dx}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\arctan(\sinh(x))}{b}-\frac{a\int \frac{1}{\frac{a\sin\left(ix+\frac{\pi}{2}\right)}{b}+1}dx}{b^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\arctan(\sinh(x))}{b}-\frac{2a\int \frac{1}{\frac{a+b}{b}-\left(1-\frac{a}{b}\right)\tanh^2\left(\frac{x}{2}\right)}d\tanh\left(\frac{x}{2}\right)}{b^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan(\sinh(x))}{b}-\frac{2a\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[Sech[x]^2/(a + b*Sech[x]),x]
```

output
```
ArcTan[Sinh[x]]/b - (2*a*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqr
t[a - b]*b*Sqrt[a + b])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4276
```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[1/b   Int[Csc[e + f*x], x], x] - Simp[a/b   Int[Csc[e + f*x]/(
a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{2a\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b\sqrt{(a-b)(a+b)}}+\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{b}$	51
risch	$\frac{i\ln(\mathrm{e}^{x}+i)}{b}-\frac{i\ln(\mathrm{e}^{x}-i)}{b}-\frac{a\ln\left(\mathrm{e}^{x}+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,b}+\frac{a\ln\left(\mathrm{e}^{x}+\frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,b}$	141

input
```
int(sech(x)^2/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*a/b/((a-b)*(a+b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a-b)*(a+b))^(1/2))+2
/b*arctan(tanh(1/2*x))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 219, normalized size of antiderivative = 4.06

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx$$
$$=\left[-\frac{\sqrt{-a^2+b^2}a\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)-a^2+2\,b^2+2\,(a^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{-a^2+b^2}(a\cosh(x)+a\sinh(x)+b}{a\cosh(x)^2+a\sinh(x)^2+2\,b\cosh(x)+2\,(a\cosh(x)+b)\sinh(x)+a}\right.}{a^2b-b^3}\right.$$

input
```
integrate(sech(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
[-(sqrt(-a^2 + b^2)*a*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) -
 a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(
x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(
x) + b)*sinh(x) + a)) - 2*(a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^2*b -
b^3), 2*(sqrt(a^2 - b^2)*a*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 -
b^2)) + (a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^2*b - b^3)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx = \int \frac{\operatorname{sech}^2{(x)}}{a+b\operatorname{sech}{(x)}}\,dx$$

input
```
integrate(sech(x)**2/(a+b*sech(x)),x)
```

output
```
Integral(sech(x)**2/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sech(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx = -\frac{2\,a\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}b} + \frac{2\,\arctan\left(e^x\right)}{b}$$

input
```
integrate(sech(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

output
```
-2*a*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*b) + 2*arctan(e^
x)/b
```

Mupad [B] (verification not implemented)

Time = 4.67 (sec) , antiderivative size = 286, normalized size of antiderivative = 5.30

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx$$
$$= \frac{a\,\ln\left(64\,a\,b^4-64\,a^3\,b^2+128\,b^5\,\mathrm{e}^x-64\,a\,b^3\,\sqrt{b^2-a^2}+32\,a^3\,b\,\sqrt{b^2-a^2}+32\,a^4\,b\,\mathrm{e}^x-128\,b^4\,\mathrm{e}^x\,\sqrt{b^2-a^2}\right.}{b\,\sqrt{b^2-a^2}}$$
$$-\frac{\ln\left(\mathrm{e}^x-\mathrm{i}\right)\,1\mathrm{i}-\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,1\mathrm{i}}{b}$$
$$-\frac{a\,\ln\left(64\,a\,b^4-64\,a^3\,b^2+128\,b^5\,\mathrm{e}^x+64\,a\,b^3\,\sqrt{b^2-a^2}-32\,a^3\,b\,\sqrt{b^2-a^2}+32\,a^4\,b\,\mathrm{e}^x+128\,b^4\,\mathrm{e}^x\,\sqrt{b^2-\right.}{b\,\sqrt{b^2-a^2}}$$

input
```
int(1/(cosh(x)^2*(a + b/cosh(x))),x)
```

output
```
(a*log(64*a*b^4 - 64*a^3*b^2 + 128*b^5*exp(x) - 64*a*b^3*(b^2 - a^2)^(1/2)
 + 32*a^3*b*(b^2 - a^2)^(1/2) + 32*a^4*b*exp(x) - 128*b^4*exp(x)*(b^2 - a^
2)^(1/2) - 160*a^2*b^3*exp(x) + 96*a^2*b^2*exp(x)*(b^2 - a^2)^(1/2)))/(b*(
b^2 - a^2)^(1/2)) - (log(exp(x) - 1i)*1i - log(exp(x) + 1i)*1i)/b - (a*log
(64*a*b^4 - 64*a^3*b^2 + 128*b^5*exp(x) + 64*a*b^3*(b^2 - a^2)^(1/2) - 32*
a^3*b*(b^2 - a^2)^(1/2) + 32*a^4*b*exp(x) + 128*b^4*exp(x)*(b^2 - a^2)^(1/
2) - 160*a^2*b^3*exp(x) - 96*a^2*b^2*exp(x)*(b^2 - a^2)^(1/2)))/(b*(b^2 -
a^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{2atan(e^x)\,a^2-2atan(e^x)\,b^2-2\sqrt{a^2-b^2}\,atan\left(\frac{e^xa+b}{\sqrt{a^2-b^2}}\right)a}{b\,(a^2-b^2)}$$

input
```
int(sech(x)^2/(a+b*sech(x)),x)
```

output
```
(2*(atan(e**x)*a**2 - atan(e**x)*b**2 - sqrt(a**2 - b**2)*atan((e**x*a + b
)/sqrt(a**2 - b**2))*a))/(b*(a**2 - b**2))
```

3.101 $\int \frac{\text{sech}^3(x)}{a+b\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{\text{sech}^3(x)}{a+b\text{sech}(x)}\,dx=-\frac{a\arctan(\sinh(x))}{b^2}+\frac{2a^2\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+b}}+\frac{\tanh(x)}{b}$$

output
```
-a*arctan(sinh(x))/b^2+2*a^2*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(
a-b)^(1/2)/b^2/(a+b)^(1/2)+tanh(x)/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{\text{sech}^3(x)}{a+b\text{sech}(x)}\,dx=\frac{-2a\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{2a^2\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+b\tanh(x)}{b^2}$$

input
```
Integrate[Sech[x]^3/(a + b*Sech[x]),x]
```

output
```
(-2*a*ArcTan[Tanh[x/2]] - (2*a^2*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^
2]])/Sqrt[a^2 - b^2] + b*Tanh[x])/b^2
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {3042, 4277, 3042, 4276, 3042, 4257, 4318, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^3(x)}{a+b\text{sech}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(\frac{\pi}{2}+ix\right)^3}{a+b\csc\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{4277}$$

$$\frac{\tanh(x)}{b}-\frac{a\int \frac{\text{sech}^2(x)}{a+b\text{sech}(x)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)}{b}-\frac{a\int \frac{\csc\left(ix+\frac{\pi}{2}\right)^2}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{4276}$$

$$\frac{\tanh(x)}{b}-\frac{a\left(\frac{\int \text{sech}(x)dx}{b}-\frac{a\int \frac{\text{sech}(x)}{a+b\text{sech}(x)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)}{b}-\frac{a\left(\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{b}-\frac{a\int \frac{\csc\left(ix+\frac{\pi}{2}\right)}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\tanh(x)}{b}-\frac{a\left(\frac{\arctan(\sinh(x))}{b}-\frac{a\int\frac{\csc\left(ix+\frac{\pi}{2}\right)}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx}{b}\right)}{b}$$

$\Big\downarrow$ 4318

$$\frac{\tanh(x)}{b}-\frac{a\left(\frac{\arctan(\sinh(x))}{b}-\frac{a\int\frac{1}{\frac{a\cosh(x)}{b}+1}dx}{b^2}\right)}{b}$$

$\Big\downarrow$ 3042

$$\frac{\tanh(x)}{b}-\frac{a\left(\frac{\arctan(\sinh(x))}{b}-\frac{a\int\frac{1}{\frac{a\sin\left(ix+\frac{\pi}{2}\right)}{b}+1}dx}{b^2}\right)}{b}$$

$\Big\downarrow$ 3138

$$\frac{\tanh(x)}{b}-\frac{a\left(\frac{\arctan(\sinh(x))}{b}-\frac{2a\int\frac{1}{\frac{a+b}{b}-\left(1-\frac{a}{b}\right)\tanh^2\left(\frac{x}{2}\right)}d\tanh\left(\frac{x}{2}\right)}{b^2}\right)}{b}$$

$\Big\downarrow$ 218

$$\frac{\tanh(x)}{b}-\frac{a\left(\frac{\arctan(\sinh(x))}{b}-\frac{2a\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}\right)}{b}$$

input
```
Int[Sech[x]^3/(a + b*Sech[x]),x]
```

output
```
-((a*(ArcTan[Sinh[x]]/b - (2*a*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]]
)/(Sqrt[a - b]*b*Sqrt[a + b])))/b) + Tanh[x]/b
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4276
```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[1/b   Int[Csc[e + f*x], x], x] - Simp[a/b   Int[Csc[e + f*x]/(
a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4277
```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[-Cot[e + f*x]/(b*f), x] - Simp[a/b   Int[Csc[e + f*x]^2/(a + b
*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{2\left(-\frac{b\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^{2}+1}+a\arctan\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{b^{2}}+\frac{2a^{2}\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^{2}\sqrt{(a-b)(a+b)}}$	73
risch	$-\frac{2}{b(e^{2x}+1)}-\frac{a^{2}\ln\left(e^{x}+\frac{b\sqrt{-a^{2}+b^{2}}-a^{2}+b^{2}}{\sqrt{-a^{2}+b^{2}}\,a}\right)}{\sqrt{-a^{2}+b^{2}}\,b^{2}}+\frac{a^{2}\ln\left(e^{x}+\frac{b\sqrt{-a^{2}+b^{2}}+a^{2}-b^{2}}{\sqrt{-a^{2}+b^{2}}\,a}\right)}{\sqrt{-a^{2}+b^{2}}\,b^{2}}+\frac{ia\ln(e^{x}-i)}{b^{2}}-\frac{ia\ln(e^{x}+i)}{b^{2}}$	160

input
```
int(sech(x)^3/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/b^2*(-b*tanh(1/2*x)/(tanh(1/2*x)^2+1)+a*arctan(tanh(1/2*x)))+2/b^2*a^2/
((a-b)*(a+b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a-b)*(a+b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(54) = 108.

Time = 0.11 (sec) , antiderivative size = 504, normalized size of antiderivative = 7.88

$$\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{sech}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
[-(2*a^2*b - 2*b^3 + (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^
2 + a^2)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(
x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(-a^2 + b^2)*(a*c
osh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*c
osh(x) + b)*sinh(x) + a)) + 2*(a^3 - a*b^2 + (a^3 - a*b^2)*cosh(x)^2 + 2*(
a^3 - a*b^2)*cosh(x)*sinh(x) + (a^3 - a*b^2)*sinh(x)^2)*arctan(cosh(x) + s
inh(x)))/(a^2*b^2 - b^4 + (a^2*b^2 - b^4)*cosh(x)^2 + 2*(a^2*b^2 - b^4)*co
sh(x)*sinh(x) + (a^2*b^2 - b^4)*sinh(x)^2), -2*(a^2*b - b^3 + (a^2*cosh(x)
^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2)*sqrt(a^2 - b^2)*arctan(-
(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) + (a^3 - a*b^2 + (a^3 - a*b^2
)*cosh(x)^2 + 2*(a^3 - a*b^2)*cosh(x)*sinh(x) + (a^3 - a*b^2)*sinh(x)^2)*a
rctan(cosh(x) + sinh(x)))/(a^2*b^2 - b^4 + (a^2*b^2 - b^4)*cosh(x)^2 + 2*(
a^2*b^2 - b^4)*cosh(x)*sinh(x) + (a^2*b^2 - b^4)*sinh(x)^2)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{a + b\operatorname{sech}(x)}\, dx = \int \frac{\operatorname{sech}^3(x)}{a + b\operatorname{sech}(x)}\, dx$$

input
```
integrate(sech(x)**3/(a+b*sech(x)),x)
```

output
```
Integral(sech(x)**3/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^3(x)}{a + b\operatorname{sech}(x)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(sech(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{2\,a^2\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}b^2} - \frac{2\,a\arctan\left(e^x\right)}{b^2} - \frac{2}{b(e^{(2\,x)}+1)}$$

input
```
integrate(sech(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

output
```
2*a^2*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*b^2) - 2*a*arct
an(e^x)/b^2 - 2/(b*(e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 4.56 (sec) , antiderivative size = 294, normalized size of antiderivative = 4.59

$$\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{sech}(x)}\,dx$$
$$= \frac{a^2\,\ln\left(64\,a^3\,b-64\,a\,b^3+32\,a^3\,\sqrt{b^2-a^2}-32\,a^4\,e^x-128\,b^4\,e^x-64\,a\,b^2\,\sqrt{b^2-a^2}-128\,b^3\,e^x\,\sqrt{b^2-a^2}+\right.}{b^2\,\sqrt{b^2-a^2}}$$
$$+\frac{a\,\left(\ln\left(32\,e^x-32\mathrm{i}\right)\,1\mathrm{i}-\ln\left(32\,e^x+32\mathrm{i}\right)\,1\mathrm{i}\right)}{b^2}-\frac{2}{b+b\,e^{2\,x}}$$
$$-\frac{a^2\,\ln\left(64\,a\,b^3-64\,a^3\,b+32\,a^3\,\sqrt{b^2-a^2}+32\,a^4\,e^x+128\,b^4\,e^x-64\,a\,b^2\,\sqrt{b^2-a^2}-128\,b^3\,e^x\,\sqrt{b^2-a^2}\right.}{b^2\,\sqrt{b^2-a^2}}$$

input
```
int(1/(cosh(x)^3*(a + b/cosh(x))),x)
```

output
```
(a*(log(32*exp(x) - 32i)*1i - log(32*exp(x) + 32i)*1i))/b^2 - 2/(b + b*exp
(2*x)) + (a^2*log(64*a^3*b - 64*a*b^3 + 32*a^3*(b^2 - a^2)^(1/2) - 32*a^4*
exp(x) - 128*b^4*exp(x) - 64*a*b^2*(b^2 - a^2)^(1/2) - 128*b^3*exp(x)*(b^2
 - a^2)^(1/2) + 160*a^2*b^2*exp(x) + 96*a^2*b*exp(x)*(b^2 - a^2)^(1/2)))/(
b^2*(b^2 - a^2)^(1/2)) - (a^2*log(64*a*b^3 - 64*a^3*b + 32*a^3*(b^2 - a^2)
^(1/2) + 32*a^4*exp(x) + 128*b^4*exp(x) - 64*a*b^2*(b^2 - a^2)^(1/2) - 128
*b^3*exp(x)*(b^2 - a^2)^(1/2) - 160*a^2*b^2*exp(x) + 96*a^2*b*exp(x)*(b^2
- a^2)^(1/2)))/(b^2*(b^2 - a^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.77

$$\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{-2e^{2x}atan(e^x)\,a^3+2e^{2x}atan(e^x)\,a\,b^2-2atan(e^x)\,a^3+2atan(e^x)\,a\,b^2+2e^{2x}\sqrt{a^2-b^2}\,atan\left(\frac{e^x a+b}{\sqrt{a^2-b^2}}\right)a^2 \cdots}{b^2\left(e^{2x}a^2-e^{2x}b^2+a^2-b^2\right)}$$

input
```
int(sech(x)^3/(a+b*sech(x)),x)
```

output
```
(2*( - e**(2*x)*atan(e**x)*a**3 + e**(2*x)*atan(e**x)*a*b**2 - atan(e**x)*
a**3 + atan(e**x)*a*b**2 + e**(2*x)*sqrt(a**2 - b**2)*atan((e**x*a + b)/sq
rt(a**2 - b**2))*a**2 + sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b*
*2))*a**2 + e**(2*x)*a**2*b - e**(2*x)*b**3))/(b**2*(e**(2*x)*a**2 - e**(2
*x)*b**2 + a**2 - b**2))
```

3.102 $\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 87

$$\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{sech}(x)}\,dx=\frac{(2a^2+b^2)\arctan(\sinh(x))}{2b^3}-\frac{2a^3\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^3\sqrt{a+b}}-\frac{a\tanh(x)}{b^2}+\frac{\operatorname{sech}(x)\tanh(x)}{2b}$$

output
```
1/2*(2*a^2+b^2)*arctan(sinh(x))/b^3-2*a^3*arctan((a-b)^(1/2)*tanh(1/2*x)/(
a+b)^(1/2))/(a-b)^(1/2)/b^3/(a+b)^(1/2)-a*tanh(x)/b^2+1/2*sech(x)*tanh(x)/
b
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{sech}(x)}\,dx$$
$$=\frac{2(2a^2+b^2)\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{4a^3\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+b(-2a+b\operatorname{sech}(x))\tanh(x)}{2b^3}$$

input
```
Integrate[Sech[x]^4/(a + b*Sech[x]),x]
```

output
```
(2*(2*a^2 + b^2)*ArcTan[Tanh[x/2]] + (4*a^3*ArcTan[((-a + b)*Tanh[x/2])/Sq
rt[a^2 - b^2]])/Sqrt[a^2 - b^2] + b*(-2*a + b*Sech[x])*Tanh[x])/(2*b^3)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$, Rules used = {3042, 4338, 3042, 4570, 3042, 4486, 3042, 4257, 4318, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^4(x)}{a+b\text{sech}(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(\frac{\pi}{2}+ix\right)^4}{a+b\csc\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 4338}$$

$$\frac{\int \frac{\text{sech}(x)\left(-2a\text{sech}^2(x)+b\text{sech}(x)+a\right)}{a+b\text{sech}(x)}dx}{2b}+\frac{\tanh(x)\text{sech}(x)}{2b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tanh(x)\text{sech}(x)}{2b}+\frac{\int \frac{\csc\left(ix+\frac{\pi}{2}\right)\left(-2a\csc\left(ix+\frac{\pi}{2}\right)^2+b\csc\left(ix+\frac{\pi}{2}\right)+a\right)}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx}{2b}$$

$$\Big\downarrow \text{ 4570}$$

$$\frac{\frac{\int \frac{\text{sech}(x)\left(ab+\left(2a^2+b^2\right)\text{sech}(x)\right)}{a+b\text{sech}(x)}dx}{b}-\frac{2a\tanh(x)}{b}}{2b}+\frac{\tanh(x)\text{sech}(x)}{2b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tanh(x)\text{sech}(x)}{2b}+\frac{-\frac{2a\tanh(x)}{b}+\frac{\int \frac{\csc\left(ix+\frac{\pi}{2}\right)\left(ab+\left(2a^2+b^2\right)\csc\left(ix+\frac{\pi}{2}\right)\right)}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx}{b}}{2b}$$

$$\Big\downarrow \text{ 4486}$$

$$\frac{\frac{\frac{\left(2a^2+b^2\right)\int \text{sech}(x)dx}{b}-\frac{2a^3\int \frac{\text{sech}(x)}{a+b\text{sech}(x)}dx}{b}}{b}-\frac{2a\tanh(x)}{b}}{2b}+\frac{\tanh(x)\text{sech}(x)}{2b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tanh(x)\text{sech}(x)}{2b}+\frac{-\frac{2a\tanh(x)}{b}+\frac{\frac{\left(2a^2+b^2\right)\int \csc\left(ix+\frac{\pi}{2}\right)dx}{b}-\frac{2a^3\int \frac{\csc\left(ix+\frac{\pi}{2}\right)}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx}{b}}{b}}{2b}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\tanh(x)\text{sech}(x)}{2b}+\frac{-\frac{2a\tanh(x)}{b}+\frac{\frac{\left(2a^2+b^2\right)\arctan(\sinh(x))}{b}-\frac{2a^3\int \frac{\csc\left(ix+\frac{\pi}{2}\right)}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx}{b}}{b}}{2b}$$

$$\Big\downarrow \text{ 4318}$$

$$\frac{\frac{\frac{\left(2a^2+b^2\right)\arctan(\sinh(x))}{b}-\frac{2a^3\int \frac{1}{\frac{a\cosh(x)}{b}+1}dx}{b^2}}{b}-\frac{2a\tanh(x)}{b}}{2b}+\frac{\tanh(x)\text{sech}(x)}{2b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tanh(x)\text{sech}(x)}{2b}+\frac{-\frac{2a\tanh(x)}{b}+\frac{\frac{\left(2a^2+b^2\right)\arctan(\sinh(x))}{b}-\frac{2a^3\int \frac{1}{\frac{a\sin\left(ix+\frac{\pi}{2}\right)}{b}+1}dx}{b^2}}{b}}{2b}$$

$$\Big\downarrow \text{ 3138}$$

$$\frac{\frac{\frac{\left(2a^2+b^2\right)\arctan(\sinh(x))}{b}-\frac{4a^3\int \frac{1}{\frac{a+b}{b}-\left(1-\frac{a}{b}\right)\tanh^2\left(\frac{x}{2}\right)}d\tanh\left(\frac{x}{2}\right)}{b^2}}{b}-\frac{2a\tanh(x)}{b}}{2b}+\frac{\tanh(x)\text{sech}(x)}{2b}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\frac{\left(2a^2+b^2\right)\arctan(\sinh(x))}{b}-\frac{4a^3\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}}{b}-\frac{2a\tanh(x)}{b}}{2b}+\frac{\tanh(x)\text{sech}(x)}{2b}$$

input
```
Int[Sech[x]^4/(a + b*Sech[x]),x]
```

output
```
(Sech[x]*Tanh[x])/(2*b) + ((((2*a^2 + b^2)*ArcTan[Sinh[x]])/b - (4*a^3*Arc
Tan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]))/b -
 (2*a*Tanh[x])/b)/(2*b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4338
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 3)/(b*f*
(n - 2))), x] + Simp[d^3/(b*(n - 2))   Int[(d*Csc[e + f*x])^(n - 3)*(Simp[a
*(n - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x]/(a + b*Csc
[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && Gt
Q[n, 3]
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.25

method	result
default	$-\frac{2a^3 \arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^3\sqrt{(a-b)(a+b)}}+\frac{\frac{2\left(\left(-ab-\frac{1}{2}b^2\right)\tanh\left(\frac{x}{2}\right)^3+\left(-ab+\frac{1}{2}b^2\right)\tanh\left(\frac{x}{2}\right)\right)}{\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^2}+\left(2a^2+b^2\right)\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{b^3}$
risch	$\frac{b\,e^{3x}+2\,e^{2x}a-b\,e^{x}+2a}{\left(e^{2x}+1\right)^2 b^2}-\frac{a^3\ln\left(e^x+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,b^3}+\frac{a^3\ln\left(e^x+\frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,b^3}+\frac{i\ln(e^x+i)a^2}{b^3}+\frac{i\ln(e^x+i)}{2b}-$

input
```
int(sech(x)^4/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/b^3*a^3/((a-b)*(a+b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a-b)*(a+b))^(1/2
))+2/b^3*(((-a*b-1/2*b^2)*tanh(1/2*x)^3+(-a*b+1/2*b^2)*tanh(1/2*x))/(tanh(
1/2*x)^2+1)^2+1/2*(2*a^2+b^2)*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(73) = 146$.

Time = 0.16 (sec) , antiderivative size = 1444, normalized size of antiderivative = 16.60

$$\int \frac{\operatorname{sech}^4(x)}{a + b\operatorname{sech}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
[(2*a^3*b - 2*a*b^3 + (a^2*b^2 - b^4)*cosh(x)^3 + (a^2*b^2 - b^4)*sinh(x)^
3 + 2*(a^3*b - a*b^3)*cosh(x)^2 + (2*a^3*b - 2*a*b^3 + 3*(a^2*b^2 - b^4)*c
osh(x))*sinh(x)^2 - (a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)
^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*
cosh(x)^3 + a^3*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^
2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x)
+ 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)
^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) + ((2*a^4 - a^2*b^2 - b
^4)*cosh(x)^4 + 4*(2*a^4 - a^2*b^2 - b^4)*cosh(x)*sinh(x)^3 + (2*a^4 - a^2
*b^2 - b^4)*sinh(x)^4 + 2*a^4 - a^2*b^2 - b^4 + 2*(2*a^4 - a^2*b^2 - b^4)*
cosh(x)^2 + 2*(2*a^4 - a^2*b^2 - b^4 + 3*(2*a^4 - a^2*b^2 - b^4)*cosh(x)^2
)*sinh(x)^2 + 4*((2*a^4 - a^2*b^2 - b^4)*cosh(x)^3 + (2*a^4 - a^2*b^2 - b^
4)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2*b^2 - b^4)*cosh(x) -
 (a^2*b^2 - b^4 - 3*(a^2*b^2 - b^4)*cosh(x)^2 - 4*(a^3*b - a*b^3)*cosh(x))
*sinh(x))/(a^2*b^3 - b^5 + (a^2*b^3 - b^5)*cosh(x)^4 + 4*(a^2*b^3 - b^5)*c
osh(x)*sinh(x)^3 + (a^2*b^3 - b^5)*sinh(x)^4 + 2*(a^2*b^3 - b^5)*cosh(x)^2
 + 2*(a^2*b^3 - b^5 + 3*(a^2*b^3 - b^5)*cosh(x)^2)*sinh(x)^2 + 4*((a^2*b^3
 - b^5)*cosh(x)^3 + (a^2*b^3 - b^5)*cosh(x))*sinh(x)), (2*a^3*b - 2*a*b^3
+ (a^2*b^2 - b^4)*cosh(x)^3 + (a^2*b^2 - b^4)*sinh(x)^3 + 2*(a^3*b - a*b^3
)*cosh(x)^2 + (2*a^3*b - 2*a*b^3 + 3*(a^2*b^2 - b^4)*cosh(x))*sinh(x)^2...
```

Sympy [F]

$$\int \frac{\mathrm{sech}^4(x)}{a + b\mathrm{sech}(x)}\,dx = \int \frac{\mathrm{sech}^4(x)}{a + b\,\mathrm{sech}(x)}\,dx$$

input
```
integrate(sech(x)**4/(a+b*sech(x)),x)
```

output
```
Integral(sech(x)**4/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\mathrm{sech}^4(x)}{a + b\mathrm{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sech(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \frac{\mathrm{sech}^4(x)}{a + b\mathrm{sech}(x)}\,dx = -\frac{2\,a^3 \arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}b^3} + \frac{(2\,a^2+b^2)\arctan\left(e^x\right)}{b^3} + \frac{be^{(3\,x)} + 2\,ae^{(2\,x)} - be^x + 2\,a}{b^2\left(e^{(2\,x)}+1\right)^2}$$

input
```
integrate(sech(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

output
```
-2*a^3*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*b^3) + (2*a^2
+ b^2)*arctan(e^x)/b^3 + (b*e^(3*x) + 2*a*e^(2*x) - b*e^x + 2*a)/(b^2*(e^(
2*x) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 5.56 (sec) , antiderivative size = 476, normalized size of antiderivative = 5.47

$$\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{e^x}{b+b\,e^{2x}} - \frac{2\,e^x}{b+2\,b\,e^{2x}+b\,e^{4x}} + \frac{2\,a}{b^2\,e^{2x}+b^2} - \frac{\ln\left(1+e^x\,1i\right)\,1i-\ln\left(e^x+1i\right)\,1i}{2\,b} - \frac{a^2\,\left(\ln\left(1+e^x\,1i\right)\,1i-\ln\left(e^x+1i\right)\,1i\right)}{b^3} - \frac{a^3\,\ln\left(16\,a\,b^5-48\,a^5\,b-24\,a^5\,\sqrt{b^2-a^2}+32\,a^3\,b^3+24\,a^6\,e^x+32\,b^6\,e^x+16\,a\,b^4\,\sqrt{b^2-a^2}+40\,a^3\,b^2\,\sqrt{\cdots}\right.}{b^3\,\sqrt{b^2\cdots}} + \frac{a^3\,\ln\left(16\,a\,b^5-48\,a^5\,b+24\,a^5\,\sqrt{b^2-a^2}+32\,a^3\,b^3+24\,a^6\,e^x+32\,b^6\,e^x-16\,a\,b^4\,\sqrt{b^2-a^2}-40\,a^3\,b^2\,\sqrt{\cdots}\right.}{b^3\,\sqrt{b^2\cdots}}$$

input
```
int(1/(cosh(x)^4*(a + b/cosh(x))),x)
```

output
```
exp(x)/(b + b*exp(2*x)) - (2*exp(x))/(b + 2*b*exp(2*x) + b*exp(4*x)) + (2*
a)/(b^2*exp(2*x) + b^2) - (log(exp(x)*1i + 1)*1i - log(exp(x) + 1i)*1i)/(2
*b) - (a^2*(log(exp(x)*1i + 1)*1i - log(exp(x) + 1i)*1i))/b^3 - (a^3*log(1
6*a*b^5 - 48*a^5*b - 24*a^5*(b^2 - a^2)^(1/2) + 32*a^3*b^3 + 24*a^6*exp(x)
 + 32*b^6*exp(x) + 16*a*b^4*(b^2 - a^2)^(1/2) + 40*a^3*b^2*(b^2 - a^2)^(1/
2) + 32*b^5*exp(x)*(b^2 - a^2)^(1/2) + 56*a^2*b^4*exp(x) - 112*a^4*b^2*exp
(x) + 72*a^2*b^3*exp(x)*(b^2 - a^2)^(1/2) - 72*a^4*b*exp(x)*(b^2 - a^2)^(1
/2)))/(b^3*(b^2 - a^2)^(1/2)) + (a^3*log(16*a*b^5 - 48*a^5*b + 24*a^5*(b^2
 - a^2)^(1/2) + 32*a^3*b^3 + 24*a^6*exp(x) + 32*b^6*exp(x) - 16*a*b^4*(b^2
 - a^2)^(1/2) - 40*a^3*b^2*(b^2 - a^2)^(1/2) - 32*b^5*exp(x)*(b^2 - a^2)^(
1/2) + 56*a^2*b^4*exp(x) - 112*a^4*b^2*exp(x) - 72*a^2*b^3*exp(x)*(b^2 - a
^2)^(1/2) + 72*a^4*b*exp(x)*(b^2 - a^2)^(1/2)))/(b^3*(b^2 - a^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 365, normalized size of antiderivative = 4.20

$$\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{sech}(x)}\,dx$$
$$= \frac{2e^{4x}atan(e^x)\,a^4 - e^{4x}atan(e^x)\,a^2b^2 - e^{4x}atan(e^x)\,b^4 + 4e^{2x}atan(e^x)\,a^4 - 2e^{2x}atan(e^x)\,a^2b^2 - 2e^{2x}atan(e}{}$$

input
```
int(sech(x)^4/(a+b*sech(x)),x)
```

output
```
(2*e**(4*x)*atan(e**x)*a**4 - e**(4*x)*atan(e**x)*a**2*b**2 - e**(4*x)*ata
n(e**x)*b**4 + 4*e**(2*x)*atan(e**x)*a**4 - 2*e**(2*x)*atan(e**x)*a**2*b**
2 - 2*e**(2*x)*atan(e**x)*b**4 + 2*atan(e**x)*a**4 - atan(e**x)*a**2*b**2
- atan(e**x)*b**4 - 2*e**(4*x)*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a*
*2 - b**2))*a**3 - 4*e**(2*x)*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**
2 - b**2))*a**3 - 2*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2))
*a**3 - e**(4*x)*a**3*b + e**(4*x)*a*b**3 + e**(3*x)*a**2*b**2 - e**(3*x)*
b**4 - e**x*a**2*b**2 + e**x*b**4 + a**3*b - a*b**3)/(b**3*(e**(4*x)*a**2
- e**(4*x)*b**2 + 2*e**(2*x)*a**2 - 2*e**(2*x)*b**2 + a**2 - b**2))
```

3.103 $\int \frac{\tanh^6(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 48

$$\int \frac{\tanh^6(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{x}{a}-\frac{3\arctan(\sinh(x))}{8a}-\frac{(8-3\operatorname{sech}(x))\tanh(x)}{8a}-\frac{(4-3\operatorname{sech}(x))\tanh^3(x)}{12a}$$

output
```
x/a-3/8*arctan(sinh(x))/a-1/8*(8-3*sech(x))*tanh(x)/a-1/12*(4-3*sech(x))*t
anh(x)^3/a
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int \frac{\tanh^6(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{\cosh^2\left(\frac{x}{2}\right)\operatorname{sech}(x)\left(6\left(4x-3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)\right)+\left(-32+15\operatorname{sech}(x)+8\operatorname{sech}^2(x)-6\operatorname{sech}^3(x)\right)\tanh(x)\right)}{12a(1+\operatorname{sech}(x))}$$

input
```
Integrate[Tanh[x]^6/(a + a*Sech[x]),x]
```

output
```
(Cosh[x/2]^2*Sech[x]*(6*(4*x - 3*ArcTan[Tanh[x/2]]) + (-32 + 15*Sech[x] +
8*Sech[x]^2 - 6*Sech[x]^3)*Tanh[x]))/(12*a*(1 + Sech[x]))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {3042, 25, 4376, 25, 3042, 4369, 25, 3042, 25, 4369, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^6(x)}{a\text{sech}(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cot\left(\frac{\pi}{2}+ix\right)^6}{a+a\csc\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cot\left(ix+\frac{\pi}{2}\right)^6}{\csc\left(ix+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4376}$$

$$-\frac{\int -\left((a-a\text{sech}(x))\tanh^4(x)\right)dx}{a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int (a-a\text{sech}(x))\tanh^4(x)dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \cot\left(ix+\frac{\pi}{2}\right)^4\left(a-a\csc\left(ix+\frac{\pi}{2}\right)\right)dx}{a^2}$$

$$\Big\downarrow \text{4369}$$

$$\frac{-\frac{1}{4}\int -\left((4a-3a\text{sech}(x))\tanh^2(x)\right)dx-\frac{1}{12}\tanh^3(x)(4a-3a\text{sech}(x))}{a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{4}\int (4a-3a\text{sech}(x))\tanh^2(x)dx-\frac{1}{12}\tanh^3(x)(4a-3a\text{sech}(x))}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{1}{12}\tanh^3(x)(4a-3a\text{sech}(x))+\frac{1}{4}\int -\cot\left(ix+\frac{\pi}{2}\right)^2\left(4a-3a\csc\left(ix+\frac{\pi}{2}\right)\right)dx}{a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{1}{12}\tanh^3(x)(4a-3a\text{sech}(x))-\frac{1}{4}\int \cot\left(ix+\frac{\pi}{2}\right)^2\left(4a-3a\csc\left(ix+\frac{\pi}{2}\right)\right)dx}{a^2}$$

$$\Big\downarrow \text{4369}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int (8a-3a\text{sech}(x))dx-\frac{1}{2}\tanh(x)(8a-3a\text{sech}(x))\right)-\frac{1}{12}\tanh^3(x)(4a-3a\text{sech}(x))}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}(8ax-3a\arctan(\sinh(x)))-\frac{1}{2}\tanh(x)(8a-3a\text{sech}(x))\right)-\frac{1}{12}\tanh^3(x)(4a-3a\text{sech}(x))}{a^2}$$

input
```
Int[Tanh[x]^6/(a + a*Sech[x]),x]
```

output
```
(-1/12*((4*a - 3*a*Sech[x])*Tanh[x]^3) + ((8*a*x - 3*a*ArcTan[Sinh[x]])/2
- ((8*a - 3*a*Sech[x])*Tanh[x])/2)/4)/a^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.56

method	result	size
default	$\frac{-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{2\left(-\frac{11\tanh\left(\frac{x}{2}\right)^{7}}{8}-\frac{137\tanh\left(\frac{x}{2}\right)^{5}}{24}-\frac{71\tanh\left(\frac{x}{2}\right)^{3}}{24}-\frac{5\tanh\left(\frac{x}{2}\right)}{8}\right)}{\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{4}}-\frac{3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{4}+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$	75
risch	$\frac{x}{a}+\frac{15\,{\rm e}^{7x}+48\,{\rm e}^{6x}-9\,{\rm e}^{5x}+96\,{\rm e}^{4x}+9\,{\rm e}^{3x}+80\,{\rm e}^{2x}-15\,{\rm e}^{x}+32}{12\left({\rm e}^{2x}+1\right)^{4}a}+\frac{3i\ln\left({\rm e}^{x}-i\right)}{8a}-\frac{3i\ln\left({\rm e}^{x}+i\right)}{8a}$	86

input
```
int(tanh(x)^6/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
64/a*(-1/64*ln(tanh(1/2*x)-1)+1/32*(-11/8*tanh(1/2*x)^7-137/24*tanh(1/2*x)
^5-71/24*tanh(1/2*x)^3-5/8*tanh(1/2*x))/(tanh(1/2*x)^2+1)^4-3/256*arctan(t
anh(1/2*x))+1/64*ln(tanh(1/2*x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(42) = 84$.

Time = 0.09 (sec) , antiderivative size = 686, normalized size of antiderivative = 14.29

$$\int \frac{\tanh^6(x)}{a + a\text{sech}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^6/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
1/12*(12*x*cosh(x)^8 + 12*x*sinh(x)^8 + 3*(32*x*cosh(x) + 5)*sinh(x)^7 + 4
8*(x + 1)*cosh(x)^6 + 15*cosh(x)^7 + 3*(112*x*cosh(x)^2 + 16*x + 35*cosh(x
) + 16)*sinh(x)^6 + 3*(224*x*cosh(x)^3 + 96*(x + 1)*cosh(x) + 105*cosh(x)^
2 - 3)*sinh(x)^5 + 24*(3*x + 4)*cosh(x)^4 - 9*cosh(x)^5 + 3*(280*x*cosh(x)
^4 + 240*(x + 1)*cosh(x)^2 + 175*cosh(x)^3 + 24*x - 15*cosh(x) + 32)*sinh(
x)^4 + 3*(224*x*cosh(x)^5 + 320*(x + 1)*cosh(x)^3 + 175*cosh(x)^4 + 32*(3*
x + 4)*cosh(x) - 30*cosh(x)^2 + 3)*sinh(x)^3 + 16*(3*x + 5)*cosh(x)^2 + 9*
cosh(x)^3 + (336*x*cosh(x)^6 + 720*(x + 1)*cosh(x)^4 + 315*cosh(x)^5 + 144
*(3*x + 4)*cosh(x)^2 - 90*cosh(x)^3 + 48*x + 27*cosh(x) + 80)*sinh(x)^2 -
9*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(
x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)
^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(
x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2
+ 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 +
cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + (96*x*cosh(x)^7 + 288*(x
 + 1)*cosh(x)^5 + 105*cosh(x)^6 + 96*(3*x + 4)*cosh(x)^3 - 45*cosh(x)^4 +
32*(3*x + 5)*cosh(x) + 27*cosh(x)^2 - 15)*sinh(x) + 12*x - 15*cosh(x) + 32
)/(a*cosh(x)^8 + 8*a*cosh(x)*sinh(x)^7 + a*sinh(x)^8 + 4*a*cosh(x)^6 + 4*(
7*a*cosh(x)^2 + a)*sinh(x)^6 + 8*(7*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^5 +
 6*a*cosh(x)^4 + 2*(35*a*cosh(x)^4 + 30*a*cosh(x)^2 + 3*a)*sinh(x)^4 + ...
```

Sympy [F]

$$\int \frac{\tanh^6(x)}{a+a\,\mathrm{sech}(x)}\,dx=\frac{\int \frac{\tanh^6(x)}{\mathrm{sech}(x)+1}\,dx}{a}$$

input
```
integrate(tanh(x)**6/(a+a*sech(x)),x)
```

output
```
Integral(tanh(x)**6/(sech(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(42) = 84.

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.94

$$\begin{aligned}\int \frac{\tanh^6(x)}{a+a\,\mathrm{sech}(x)}\,dx &= \frac{x}{a}\\ &\quad+\frac{15\,e^{(-x)}-80\,e^{(-2\,x)}-9\,e^{(-3\,x)}-96\,e^{(-4\,x)}+9\,e^{(-5\,x)}-48\,e^{(-6\,x)}-15\,e^{(-7\,x)}-32}{12\,\left(4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a\right)}\\ &\quad+\frac{3\,\arctan\left(e^{(-x)}\right)}{4\,a}\end{aligned}$$

input
```
integrate(tanh(x)^6/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
x/a + 1/12*(15*e^(-x) - 80*e^(-2*x) - 9*e^(-3*x) - 96*e^(-4*x) + 9*e^(-5*x
) - 48*e^(-6*x) - 15*e^(-7*x) - 32)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(
-6*x) + a*e^(-8*x) + a) + 3/4*arctan(e^(-x))/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\int \frac{\tanh^6(x)}{a + a\text{sech}(x)}\,dx = \frac{x}{a} - \frac{3\arctan\left(e^x\right)}{4a} + \frac{15\,e^{(7\,x)} + 48\,e^{(6\,x)} - 9\,e^{(5\,x)} + 96\,e^{(4\,x)} + 9\,e^{(3\,x)} + 80\,e^{(2\,x)} - 15\,e^x + 32}{12\,a(e^{(2\,x)} + 1)^4}$$

input
```
integrate(tanh(x)^6/(a+a*sech(x)),x, algorithm="giac")
```

output
```
x/a - 3/4*arctan(e^x)/a + 1/12*(15*e^(7*x) + 48*e^(6*x) - 9*e^(5*x) + 96*e
^(4*x) + 9*e^(3*x) + 80*e^(2*x) - 15*e^x + 32)/(a*(e^(2*x) + 1)^4)
```

Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.98

$$\int \frac{\tanh^6(x)}{a + a\text{sech}(x)}\,dx = \frac{\frac{8}{3\,a} + \frac{6\,e^x}{a}}{3\,e^{2\,x} + 3\,e^{4\,x} + e^{6\,x} + 1} - \frac{\frac{4}{a} + \frac{9\,e^x}{2\,a}}{2\,e^{2\,x} + e^{4\,x} + 1} + \frac{x}{a} + \frac{\frac{4}{a} + \frac{5\,e^x}{4\,a}}{e^{2\,x} + 1} - \frac{3\,\text{atan}\left(\frac{e^x\,\sqrt{a^2}}{a}\right)}{4\,\sqrt{a^2}} - \frac{4\,e^x}{a\,\left(4\,e^{2\,x} + 6\,e^{4\,x} + 4\,e^{6\,x} + e^{8\,x} + 1\right)}$$

input
```
int(tanh(x)^6/(a + a/cosh(x)),x)
```

output
```
(8/(3*a) + (6*exp(x))/a)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (4/a +
 (9*exp(x))/(2*a))/(2*exp(2*x) + exp(4*x) + 1) + x/a + (4/a + (5*exp(x))/(
4*a))/(exp(2*x) + 1) - (3*atan((exp(x)*(a^2)^(1/2))/a))/(4*(a^2)^(1/2)) -
(4*exp(x))/(a*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 169, normalized size of antiderivative = 3.52

$$\int \frac{\tanh^6(x)}{a + a\text{sech}(x)}\,dx = \frac{-9e^{8x}atan(e^x) - 36e^{6x}atan(e^x) - 54e^{4x}atan(e^x) - 36e^{2x}atan(e^x) - 9atan(e^x) + 12e^{8x}x - 12e^{8x} + 15e^{7}}{12a\left(e^{8x} + 4e^{6x} + 6e^{4x} + 4e^{2x}\right.}$$

input
```
int(tanh(x)^6/(a+a*sech(x)),x)
```

output
```
( - 9*e**(8*x)*atan(e**x) - 36*e**(6*x)*atan(e**x) - 54*e**(4*x)*atan(e**x
) - 36*e**(2*x)*atan(e**x) - 9*atan(e**x) + 12*e**(8*x)*x - 12*e**(8*x) +
15*e**(7*x) + 48*e**(6*x)*x - 9*e**(5*x) + 72*e**(4*x)*x + 24*e**(4*x) + 9
*e**(3*x) + 48*e**(2*x)*x + 32*e**(2*x) - 15*e**x + 12*x + 20)/(12*a*(e**(
8*x) + 4*e**(6*x) + 6*e**(4*x) + 4*e**(2*x) + 1))
```

3.104 $\int \frac{\tanh^5(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{\tanh^5(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\log(\cosh(x))}{a}+\frac{\operatorname{sech}(x)}{a}+\frac{\operatorname{sech}^2(x)}{2a}-\frac{\operatorname{sech}^3(x)}{3a}$$

output
```
ln(cosh(x))/a+sech(x)/a+1/2*sech(x)^2/a-1/3*sech(x)^3/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\tanh^5(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{(2+6\cosh(2x)+3\cosh(3x)\log(\cosh(x))+\cosh(x)(6+9\log(\cosh(x))))\operatorname{sech}^3(x)}{12a}$$

input
```
Integrate[Tanh[x]^5/(a + a*Sech[x]),x]
```

output
```
((2 + 6*Cosh[2*x] + 3*Cosh[3*x]*Log[Cosh[x]] + Cosh[x]*(6 + 9*Log[Cosh[x]]
))*Sech[x]^3)/(12*a)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 26, 4367, 27, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(x)}{a\text{sech}(x)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\cot\left(\frac{\pi}{2}+ix\right)^5}{a+a\csc\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\cot\left(ix+\frac{\pi}{2}\right)^5}{\csc\left(ix+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{ 4367}$$

$$\frac{\int a^3(1-\cosh(x))^2(\cosh(x)+1)\text{sech}^4(x)d\cosh(x)}{a^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int (1-\cosh(x))^2(\cosh(x)+1)\text{sech}^4(x)d\cosh(x)}{a}$$

$$\Big\downarrow \text{ 84}$$

$$\frac{\int \left(\text{sech}^4(x)-\text{sech}^3(x)-\text{sech}^2(x)+\text{sech}(x)\right)d\cosh(x)}{a}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{1}{3}\text{sech}^3(x)+\frac{\text{sech}^2(x)}{2}+\text{sech}(x)+\log(\cosh(x))}{a}$$

input
```
Int[Tanh[x]^5/(a + a*Sech[x]),x]
```

output
```
(Log[Cosh[x]] + Sech[x] + Sech[x]^2/2 - Sech[x]^3/3)/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{\operatorname{sech}(x)-\frac{\operatorname{sech}(x)^{3}}{3}+\frac{\operatorname{sech}(x)^{2}}{2}-\ln(\operatorname{sech}(x))}{a}$	25
default	$\frac{\operatorname{sech}(x)-\frac{\operatorname{sech}(x)^{3}}{3}+\frac{\operatorname{sech}(x)^{2}}{2}-\ln(\operatorname{sech}(x))}{a}$	25
risch	$-\frac{x}{a}+\frac{2\,{\rm e}^{x}\left(3\,{\rm e}^{4x}+3\,{\rm e}^{3x}+2\,{\rm e}^{2x}+3\,{\rm e}^{x}+3\right)}{3\left({\rm e}^{2x}+1\right)^{3}a}+\frac{\ln\left({\rm e}^{2x}+1\right)}{a}$	58

input
```
int(tanh(x)^5/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a*(sech(x)-1/3*sech(x)^3+1/2*sech(x)^2-ln(sech(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(32) = 64$.

Time = 0.08 (sec) , antiderivative size = 437, normalized size of antiderivative = 12.14

$$\int \frac{\tanh^5(x)}{a + a\text{sech}(x)} \, dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^5/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
-1/3*(3*x*cosh(x)^6 + 3*x*sinh(x)^6 + 6*(3*x*cosh(x) - 1)*sinh(x)^5 + 3*(3
*x - 2)*cosh(x)^4 - 6*cosh(x)^5 + 3*(15*x*cosh(x)^2 + 3*x - 10*cosh(x) - 2
)*sinh(x)^4 + 4*(15*x*cosh(x)^3 + 3*(3*x - 2)*cosh(x) - 15*cosh(x)^2 - 1)*
sinh(x)^3 + 3*(3*x - 2)*cosh(x)^2 - 4*cosh(x)^3 + 3*(15*x*cosh(x)^4 + 6*(3
*x - 2)*cosh(x)^2 - 20*cosh(x)^3 + 3*x - 4*cosh(x) - 2)*sinh(x)^2 - 3*(cos
h(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4 +
 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6*
cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + cosh
(x))*sinh(x) + 1)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 6*(3*x*cosh(x)^5 +
2*(3*x - 2)*cosh(x)^3 - 5*cosh(x)^4 + (3*x - 2)*cosh(x) - 2*cosh(x)^2 - 1)
*sinh(x) + 3*x - 6*cosh(x))/(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(
x)^6 + 3*a*cosh(x)^4 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5*a*cosh(x)^3
+ 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 + 6*a*cosh(x)^
2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a
)
```

Sympy [F]

$$\int \frac{\tanh^5(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\tanh^5(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

input
```
integrate(tanh(x)**5/(a+a*sech(x)),x)
```

output
```
Integral(tanh(x)**5/(sech(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(32) = 64$.

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.06

$$\int \frac{\tanh^5(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{x}{a} + \frac{2\left(3\,e^{(-x)} + 3\,e^{(-2\,x)} + 2\,e^{(-3\,x)} + 3\,e^{(-4\,x)} + 3\,e^{(-5\,x)}\right)}{3\,(3\,ae^{(-2\,x)} + 3\,ae^{(-4\,x)} + ae^{(-6\,x)} + a)} + \frac{\log\left(e^{(-2\,x)} + 1\right)}{a}$$

input
```
integrate(tanh(x)^5/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
x/a + 2/3*(3*e^(-x) + 3*e^(-2*x) + 2*e^(-3*x) + 3*e^(-4*x) + 3*e^(-5*x))/(
3*a*e^(-2*x) + 3*a*e^(-4*x) + a*e^(-6*x) + a) + log(e^(-2*x) + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.69

$$\int \frac{\tanh^5(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{\log\left(e^{(-x)} + e^{x}\right)}{a} - \frac{11\left(e^{(-x)} + e^{x}\right)^3 - 12\left(e^{(-x)} + e^{x}\right)^2 - 12\,e^{(-x)} - 12\,e^{x} + 16}{6\,a(e^{(-x)} + e^{x})^3}$$

input
```
integrate(tanh(x)^5/(a+a*sech(x)),x, algorithm="giac")
```

output
```
log(e^(-x) + e^x)/a - 1/6*(11*(e^(-x) + e^x)^3 - 12*(e^(-x) + e^x)^2 - 12*
e^(-x) - 12*e^x + 16)/(a*(e^(-x) + e^x)^3)
```

Mupad [B] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.67

$$\int \frac{\tanh^5(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\ln\left(e^{2x}+1\right)}{a} - \frac{\frac{2}{a}+\frac{8\,e^x}{3\,a}}{2\,e^{2x}+e^{4x}+1} - \frac{x}{a} + \frac{\frac{2}{a}+\frac{2\,e^x}{a}}{e^{2x}+1} + \frac{8\,e^x}{3\,a\,\left(3\,e^{2x}+3\,e^{4x}+e^{6x}+1\right)}$$

input
```
int(tanh(x)^5/(a + a/cosh(x)),x)
```

output
```
log(exp(2*x) + 1)/a - (2/a + (8*exp(x))/(3*a))/(2*exp(2*x) + exp(4*x) + 1)
 - x/a + (2/a + (2*exp(x))/a)/(exp(2*x) + 1) + (8*exp(x))/(3*a*(3*exp(2*x)
 + 3*exp(4*x) + exp(6*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.83

$$\int \frac{\tanh^5(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{3e^{6x}\log(e^{2x}+1)-3e^{6x}x-2e^{6x}+6e^{5x}+9e^{4x}\log(e^{2x}+1)-9e^{4x}x+4e^{3x}+9e^{2x}\log(e^{2x}+1)-9e^{2x}x+}{3a\left(e^{6x}+3e^{4x}+3e^{2x}+1\right)}$$

input
```
int(tanh(x)^5/(a+a*sech(x)),x)
```

output
```
(3*e**(6*x)*log(e**(2*x) + 1) - 3*e**(6*x)*x - 2*e**(6*x) + 6*e**(5*x) + 9
*e**(4*x)*log(e**(2*x) + 1) - 9*e**(4*x)*x + 4*e**(3*x) + 9*e**(2*x)*log(e
**(2*x) + 1) - 9*e**(2*x)*x + 6*e**x + 3*log(e**(2*x) + 1) - 3*x - 2)/(3*a
*(e**(6*x) + 3*e**(4*x) + 3*e**(2*x) + 1))
```

3.105 $\int \frac{\tanh^4(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\tanh^4(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{x}{a}-\frac{\arctan(\sinh(x))}{2a}-\frac{(2-\operatorname{sech}(x))\tanh(x)}{2a}$$

output
```
x/a-1/2*arctan(sinh(x))/a-1/2*(2-sech(x))*tanh(x)/a
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\tanh^4(x)}{a+a\operatorname{sech}(x)}\,dx =\frac{\cosh^2\left(\frac{x}{2}\right)\operatorname{sech}(x)\left(2\left(x-\arctan\left(\tanh\left(\frac{x}{2}\right)\right)\right)+(-2+\operatorname{sech}(x))\tanh(x)\right)}{a(1+\operatorname{sech}(x))}$$

input
```
Integrate[Tanh[x]^4/(a + a*Sech[x]),x]
```

output
```
(Cosh[x/2]^2*Sech[x]*(2*(x - ArcTan[Tanh[x/2]]) + (-2 + Sech[x])*Tanh[x]))
/(a*(1 + Sech[x]))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 4376, 3042, 25, 4369, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(x)}{a\text{sech}(x)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cot\left(\frac{\pi}{2}+ix\right)^4}{a+a\csc\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 4376}$$

$$\frac{\int (a-a\text{sech}(x))\tanh^2(x)dx}{a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int -\cot\left(ix+\frac{\pi}{2}\right)^2\left(a-a\csc\left(ix+\frac{\pi}{2}\right)\right)dx}{a^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \cot\left(ix+\frac{\pi}{2}\right)^2\left(a-a\csc\left(ix+\frac{\pi}{2}\right)\right)dx}{a^2}$$

$$\Big\downarrow \text{ 4369}$$

$$-\frac{\frac{1}{2}\tanh(x)(2a-a\text{sech}(x))-\frac{1}{2}\int (2a-a\text{sech}(x))dx}{a^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{1}{2}(a\arctan(\sinh(x))-2ax)+\frac{1}{2}\tanh(x)(2a-a\text{sech}(x))}{a^2}$$

input
```
Int[Tanh[x]^4/(a + a*Sech[x]),x]
```

output
```
-(((-2*a*x + a*ArcTan[Sinh[x]])/2 + ((2*a - a*Sech[x])*Tanh[x])/2)/a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(27) = 54$.

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

method	result	size
default	$\frac{\frac{2\left(-\frac{3\tanh\left(\frac{x}{2}\right)^{3}}{2}-\frac{\tanh\left(\frac{x}{2}\right)}{2}\right)}{\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{2}}-\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$	59
risch	$\frac{x}{a}+\frac{e^{3x}+2\,e^{2x}-e^{x}+2}{\left(e^{2x}+1\right)^{2}a}+\frac{i\ln\left(e^{x}-i\right)}{2a}-\frac{i\ln\left(e^{x}+i\right)}{2a}$	59

input
```
int(tanh(x)^4/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
16/a*(1/8*(-3/2*tanh(1/2*x)^3-1/2*tanh(1/2*x))/(tanh(1/2*x)^2+1)^2-1/16*ar
ctan(tanh(1/2*x))-1/16*ln(tanh(1/2*x)-1)+1/16*ln(tanh(1/2*x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(25) = 50$.

Time = 0.09 (sec) , antiderivative size = 210, normalized size of antiderivative = 6.77

$$\int \frac{\tanh^4(x)}{a + a\text{sech}(x)}\,dx = \frac{x\cosh(x)^4 + x\sinh(x)^4 + (4x\cosh(x) + 1)\sinh(x)^3 + 2(x+1)\cosh(x)^2 + \cosh(x)^3 + (6x\cosh(x)^2}{}$$

input
```
integrate(tanh(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
(x*cosh(x)^4 + x*sinh(x)^4 + (4*x*cosh(x) + 1)*sinh(x)^3 + 2*(x + 1)*cosh(
x)^2 + cosh(x)^3 + (6*x*cosh(x)^2 + 2*x + 3*cosh(x) + 2)*sinh(x)^2 - (cosh
(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 +
2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x
)) + (4*x*cosh(x)^3 + 4*(x + 1)*cosh(x) + 3*cosh(x)^2 - 1)*sinh(x) + x - c
osh(x) + 2)/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(
x)^2 + 2*(3*a*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(
x) + a)
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{a + a\text{sech}(x)}\,dx = \frac{\int \frac{\tanh^4(x)}{\text{sech}(x)+1}\,dx}{a}$$

input
```
integrate(tanh(x)**4/(a+a*sech(x)),x)
```

output
```
Integral(tanh(x)**4/(sech(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(25) = 50.

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \frac{\tanh^4(x)}{a + a\text{sech}(x)}\, dx = \frac{x}{a} + \frac{e^{(-x)} - 2\, e^{(-2\, x)} - e^{(-3\, x)} - 2}{2\, ae^{(-2\, x)} + ae^{(-4\, x)} + a} + \frac{\arctan\left(e^{(-x)}\right)}{a}$$

input
```
integrate(tanh(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
x/a + (e^(-x) - 2*e^(-2*x) - e^(-3*x) - 2)/(2*a*e^(-2*x) + a*e^(-4*x) + a)
 + arctan(e^(-x))/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{\tanh^4(x)}{a + a\text{sech}(x)}\, dx = \frac{x}{a} - \frac{\arctan\left(e^{x}\right)}{a} + \frac{e^{(3\, x)} + 2\, e^{(2\, x)} - e^{x} + 2}{a\left(e^{(2\, x)} + 1\right)^{2}}$$

input
```
integrate(tanh(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

output
```
x/a - arctan(e^x)/a + (e^(3*x) + 2*e^(2*x) - e^x + 2)/(a*(e^(2*x) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.16

$$\int \frac{\tanh^4(x)}{a + a\text{sech}(x)}\, dx = \frac{x}{a} + \frac{\frac{2}{a} + \frac{e^{x}}{a}}{e^{2\,x} + 1} - \frac{\operatorname{atan}\left(\frac{e^{x}\sqrt{a^2}}{a}\right)}{\sqrt{a^2}} - \frac{2\, e^{x}}{a\,\left(2\, e^{2\,x} + e^{4\,x} + 1\right)}$$

input
```
int(tanh(x)^4/(a + a/cosh(x)),x)
```

output
```
x/a + (2/a + exp(x)/a)/(exp(2*x) + 1) - atan((exp(x)*(a^2)^(1/2))/a)/(a^2)
^(1/2) - (2*exp(x))/(a*(2*exp(2*x) + exp(4*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.68

$$\int \frac{\tanh^4(x)}{a + a\text{sech}(x)} \, dx = \frac{-e^{4x}atan(e^x) - 2e^{2x}atan(e^x) - atan(e^x) + e^{4x}x - e^{4x} + e^{3x} + 2e^{2x}x - e^x + x + 1}{a\,(e^{4x} + 2e^{2x} + 1)}$$

input
```
int(tanh(x)^4/(a+a*sech(x)),x)
```

output
```
( - e**(4*x)*atan(e**x) - 2*e**(2*x)*atan(e**x) - atan(e**x) + e**(4*x)*x
- e**(4*x) + e**(3*x) + 2*e**(2*x)*x - e**x + x + 1)/(a*(e**(4*x) + 2*e**(
2*x) + 1))
```

3.106 $\int \frac{\tanh^3(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{\tanh^3(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{\log(\cosh(x))}{a}+\frac{\operatorname{sech}(x)}{a}$$

output
```
ln(cosh(x))/a+sech(x)/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\tanh^3(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{\log(\cosh(x))+\operatorname{sech}(x)}{a}$$

input
```
Integrate[Tanh[x]^3/(a + a*Sech[x]),x]
```

output
```
(Log[Cosh[x]] + Sech[x])/a
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 26, 4367, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{a\text{sech}(x)+a}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int -\frac{i\cot\left(\frac{\pi}{2}+ix\right)^3}{a+a\csc\left(\frac{\pi}{2}+ix\right)}dx$$
$$\Big\downarrow \text{ 26}$$
$$-i\int \frac{\cot\left(ix+\frac{\pi}{2}\right)^3}{\csc\left(ix+\frac{\pi}{2}\right)a+a}dx$$
$$\Big\downarrow \text{ 4367}$$
$$-\frac{\int a(1-\cosh(x))\text{sech}^2(x)d\cosh(x)}{a^2}$$
$$\Big\downarrow \text{ 27}$$
$$-\frac{\int (1-\cosh(x))\text{sech}^2(x)d\cosh(x)}{a}$$
$$\Big\downarrow \text{ 49}$$
$$-\frac{\int \left(\text{sech}^2(x)-\text{sech}(x)\right)d\cosh(x)}{a}$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{-\text{sech}(x)-\log(\cosh(x))}{a}$$

input
```
Int[Tanh[x]^3/(a + a*Sech[x]),x]
```

output
```
-((-Log[Cosh[x]] - Sech[x])/a)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.43

method	result	size
risch	$-\frac{x}{a}+\frac{2\,{\mathrm e}^{x}}{a\left({\mathrm e}^{2 x}+1\right)}+\frac{\ln\left({\mathrm e}^{2 x}+1\right)}{a}$	34
default	$\frac{-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{8}{4\tanh\left(\frac{x}{2}\right)^{2}+4}+\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$	48

input
```
int(tanh(x)^3/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
-x/a+2/a*exp(x)/(exp(2*x)+1)+1/a*ln(exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(14) = 28$.

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 6.07

$$\int \frac{\tanh^3(x)}{a + a\text{sech}(x)}\, dx = -\frac{x \cosh(x)^2 + x \sinh(x)^2 - \left(\cosh(x)^2 + 2 \cosh(x)\sinh(x) + \sinh(x)^2 + 1\right) \log\left(\frac{2 \cosh(x)}{\cosh(x) - \sinh(x)}\right) + 2 (x}{a \cosh(x)^2 + 2\, a \cosh(x)\sinh(x) + a \sinh(x)^2 + a}$$

input
```
integrate(tanh(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
-(x*cosh(x)^2 + x*sinh(x)^2 - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 +
 1)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*(x*cosh(x) - 1)*sinh(x) + x - 2
*cosh(x))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{a + a\text{sech}(x)}\, dx = \frac{\int \frac{\tanh^3(x)}{\text{sech}(x)+1}\, dx}{a}$$

input
```
integrate(tanh(x)**3/(a+a*sech(x)),x)
```

output
```
Integral(tanh(x)**3/(sech(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.36

$$\int \frac{\tanh^3(x)}{a + a\text{sech}(x)}\,dx = \frac{x}{a} + \frac{2\,e^{(-x)}}{ae^{(-2\,x)} + a} + \frac{\log\left(e^{(-2\,x)} + 1\right)}{a}$$

input
```
integrate(tanh(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
x/a + 2*e^(-x)/(a*e^(-2*x) + a) + log(e^(-2*x) + 1)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(14) = 28$.

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.50

$$\int \frac{\tanh^3(x)}{a + a\text{sech}(x)}\,dx = \frac{\log\left(e^{(-x)} + e^x\right)}{a} - \frac{e^{(-x)} + e^x - 2}{a(e^{(-x)} + e^x)}$$

input
```
integrate(tanh(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

output
```
log(e^(-x) + e^x)/a - (e^(-x) + e^x - 2)/(a*(e^(-x) + e^x))
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.36

$$\int \frac{\tanh^3(x)}{a + a\text{sech}(x)}\,dx = \frac{\ln\left(e^{2\,x} + 1\right)}{a} - \frac{x}{a} + \frac{2\,e^x}{a\,\left(e^{2\,x} + 1\right)}$$

input
```
int(tanh(x)^3/(a + a/cosh(x)),x)
```

output
```
log(exp(2*x) + 1)/a - x/a + (2*exp(x))/(a*(exp(2*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 3.71

$$\int \frac{\tanh^3(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{e^{2x}\log(e^{2x}+1)-e^{2x}x+2e^{x}+\log(e^{2x}+1)-x}{a\,(e^{2x}+1)}$$

input
```
int(tanh(x)^3/(a+a*sech(x)),x)
```

output
```
(e**(2*x)*log(e**(2*x) + 1) - e**(2*x)*x + 2*e**x + log(e**(2*x) + 1) - x)
/(a*(e**(2*x) + 1))
```

3.107 $\int \frac{\tanh^2(x)}{a+a\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{\tanh^2(x)}{a+a\text{sech}(x)}\,dx=\frac{x}{a}-\frac{\arctan(\sinh(x))}{a}$$

output
```
x/a-arctan(sinh(x))/a
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{\tanh^2(x)}{a+a\text{sech}(x)}\,dx=\frac{x-2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

input
```
Integrate[Tanh[x]^2/(a + a*Sech[x]),x]
```

output
```
(x - 2*ArcTan[Tanh[x/2]])/a
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 25, 4376, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)}{a\text{sech}(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cot\left(\frac{\pi}{2}+ix\right)^2}{a+a\csc\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cot\left(ix+\frac{\pi}{2}\right)^2}{\csc\left(ix+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4376}$$

$$-\frac{\int (a\text{sech}(x)-a)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\arctan(\sinh(x))-ax}{a^2}$$

input
```
Int[Tanh[x]^2/(a + a*Sech[x]),x]
```

output
```
-((-(a*x) + a*ArcTan[Sinh[x]])/a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

method	result	size
risch	$\frac{x}{a}+\frac{i \ln \left({\mathrm e}^{x}-i\right)}{a}-\frac{i \ln \left({\mathrm e}^{x}+i\right)}{a}$	31
default	$\frac{-\ln \left(\tanh \left(\frac{x}{2}\right)-1\right)+\ln \left(\tanh \left(\frac{x}{2}\right)+1\right)-2 \arctan \left(\tanh \left(\frac{x}{2}\right)\right)}{a}$	32

input
```
int(tanh(x)^2/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
x/a+I/a*ln(exp(x)-I)-I/a*ln(exp(x)+I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^2(x)}{a + a\text{sech}(x)}\,dx = \frac{x - 2\arctan\left(\cosh\left(x\right) + \sinh\left(x\right)\right)}{a}$$

input
```
integrate(tanh(x)^2/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
(x - 2*arctan(cosh(x) + sinh(x)))/a
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{a + a\text{sech}(x)}\,dx = \frac{\int \frac{\tanh^2(x)}{\text{sech}(x)+1}\,dx}{a}$$

input
```
integrate(tanh(x)**2/(a+a*sech(x)),x)
```

output
```
Integral(tanh(x)**2/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tanh^2(x)}{a + a\text{sech}(x)}\,dx = \frac{x}{a} + \frac{2\arctan\left(e^{(-x)}\right)}{a}$$

input
```
integrate(tanh(x)^2/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
x/a + 2*arctan(e^(-x))/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^2(x)}{a + a\text{sech}(x)}\,dx = \frac{x}{a} - \frac{2\,\arctan\left(e^x\right)}{a}$$

input
```
integrate(tanh(x)^2/(a+a*sech(x)),x, algorithm="giac")
```

output
```
x/a - 2*arctan(e^x)/a
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{\tanh^2(x)}{a + a\text{sech}(x)}\,dx = \frac{x}{a} - \frac{2\,\mathrm{atan}\left(\frac{e^x\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

input
```
int(tanh(x)^2/(a + a/cosh(x)),x)
```

output
```
x/a - (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\tanh^2(x)}{a + a\text{sech}(x)}\,dx = \frac{-2atan(e^x) + x}{a}$$

input
```
int(tanh(x)^2/(a+a*sech(x)),x)
```

output
```
( - 2*atan(e**x) + x)/a
```

3.108 $\int \frac{\tanh(x)}{a+a\,\mathrm{sech}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 9

$$\int \frac{\tanh(x)}{a+a\,\mathrm{sech}(x)}\,dx = \frac{\log(1+\cosh(x))}{a}$$

output
```
ln(1+cosh(x))/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \frac{\tanh(x)}{a+a\,\mathrm{sech}(x)}\,dx = \frac{2\log\left(\cosh\left(\frac{x}{2}\right)\right)}{a}$$

input
```
Integrate[Tanh[x]/(a + a*Sech[x]),x]
```

output
```
(2*Log[Cosh[x/2]])/a
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 26, 4367, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{a\text{sech}(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cot\left(\frac{\pi}{2}+ix\right)}{a+a\csc\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cot\left(ix+\frac{\pi}{2}\right)}{\csc\left(ix+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4367}$$

$$\int \frac{1}{a\cosh(x)+a}d\cosh(x)$$

$$\Big\downarrow \text{16}$$

$$\frac{\log(\cosh(x)+1)}{a}$$

input
```
Int[Tanh[x]/(a + a*Sech[x]),x]
```

output
```
Log[1 + Cosh[x]]/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.89

method	result	size
derivativedivides	$-\frac{-\ln(1+\operatorname{sech}(x))+\ln(\operatorname{sech}(x))}{a}$	17
default	$-\frac{-\ln(1+\operatorname{sech}(x))+\ln(\operatorname{sech}(x))}{a}$	17
risch	$-\frac{x}{a}+\frac{2\ln(1+e^{x})}{a}$	18

input
```
int(tanh(x)/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/a*(-ln(1+sech(x))+ln(sech(x)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.78

$$\int \frac{\tanh(x)}{a + a\text{sech}(x)} \, dx = -\frac{x - 2 \log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right)}{a}$$

input
```
integrate(tanh(x)/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
-(x - 2*log(cosh(x) + sinh(x) + 1))/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(7) = 14$.

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.11

$$\int \frac{\tanh(x)}{a + a\text{sech}(x)} \, dx = \frac{x}{a} - \frac{\log\left(\tanh\left(x\right) + 1\right)}{a} + \frac{\log\left(\text{sech}\left(x\right) + 1\right)}{a}$$

input
```
integrate(tanh(x)/(a+a*sech(x)),x)
```

output
```
x/a - log(tanh(x) + 1)/a + log(sech(x) + 1)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.00

$$\int \frac{\tanh(x)}{a + a\text{sech}(x)} \, dx = \frac{x}{a} + \frac{2 \log\left(e^{(-x)} + 1\right)}{a}$$

input
```
integrate(tanh(x)/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
x/a + 2*log(e^(-x) + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.89

$$\int \frac{\tanh(x)}{a + a\text{sech}(x)}\,dx = -\frac{x}{a} + \frac{2\log\left(e^x + 1\right)}{a}$$

input
```
integrate(tanh(x)/(a+a*sech(x)),x, algorithm="giac")
```

output
```
-x/a + 2*log(e^x + 1)/a
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.56

$$\int \frac{\tanh(x)}{a + a\text{sech}(x)}\,dx = -\frac{x - 2\ln\left(\mathrm{e}^x + 1\right)}{a}$$

input
```
int(tanh(x)/(a + a/cosh(x)),x)
```

output
```
-(x - 2*log(exp(x) + 1))/a
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.78

$$\int \frac{\tanh(x)}{a + a\text{sech}(x)}\,dx = \frac{2\log(e^x + 1) - x}{a}$$

input
```
int(tanh(x)/(a+a*sech(x)),x)
```

output
```
(2*log(e**x + 1) - x)/a
```

3.109 $\int \frac{\coth(x)}{a+a\,\mathrm{sech}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 40

$$\int \frac{\coth(x)}{a+a\,\mathrm{sech}(x)}\,dx=\frac{1}{2a(1+\cosh(x))}+\frac{\log(1-\cosh(x))}{4a}+\frac{3\log(1+\cosh(x))}{4a}$$

output
```
1/2/a/(1+cosh(x))+1/4*ln(1-cosh(x))/a+3/4*ln(1+cosh(x))/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \frac{\coth(x)}{a+a\,\mathrm{sech}(x)}\,dx=\frac{\left(1+2\cosh^2\left(\frac{x}{2}\right)\left(3\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)\right)\mathrm{sech}(x)}{2a(1+\mathrm{sech}(x))}$$

input
```
Integrate[Coth[x]/(a + a*Sech[x]),x]
```

output
```
((1 + 2*Cosh[x/2]^2*(3*Log[Cosh[x/2]] + Log[Sinh[x/2]]))*Sech[x])/(2*a*(1
+ Sech[x]))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 26, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{a\text{sech}(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\cot\left(\frac{\pi}{2}+ix\right)\left(a+a\csc\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\cot\left(ix+\frac{\pi}{2}\right)\left(\csc\left(ix+\frac{\pi}{2}\right)a+a\right)}dx$$

$$\Big\downarrow \text{4367}$$

$$-a^2\int \frac{\cosh^2(x)}{a^3(1-\cosh(x))(\cosh(x)+1)^2}d\cosh(x)$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\cosh^2(x)}{(1-\cosh(x))(\cosh(x)+1)^2}d\cosh(x)}{a}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\int \left(-\frac{3}{4(\cosh(x)+1)}+\frac{1}{2(\cosh(x)+1)^2}-\frac{1}{4(\cosh(x)-1)}\right)d\cosh(x)}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{2(\cosh(x)+1)}-\frac{1}{4}\log(1-\cosh(x))-\frac{3}{4}\log(\cosh(x)+1)}{a}$$

input
```
Int[Coth[x]/(a + a*Sech[x]),x]
```

output
```
-((-1/2*1/(1 + Cosh[x]) - Log[1 - Cosh[x]]/4 - (3*Log[1 + Cosh[x]])/4)/a)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{-\frac{\tanh\left(\frac{x}{2}\right)^{2}}{2}-2\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)\right)-2\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2a}$	38
risch	$-\frac{x}{a}+\frac{e^{x}}{\left(1+e^{x}\right)^{2}a}+\frac{3\ln\left(1+e^{x}\right)}{2a}+\frac{\ln\left(e^{x}-1\right)}{2a}$	40

input
```
int(coth(x)/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(-1/2*tanh(1/2*x)^2-2*ln(tanh(1/2*x)-1)+ln(tanh(1/2*x))-2*ln(tanh(1/
2*x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(34) = 68$.

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.40

$$\int \frac{\coth(x)}{a+a\text{sech}(x)}\,dx = -\frac{2\,x\cosh\left(x\right)^{2} + 2\,x\sinh\left(x\right)^{2} + 2\left(2\,x-1\right)\cosh\left(x\right) - 3\left(\cosh\left(x\right)^{2} + 2\left(\cosh\left(x\right)+1\right)\sinh\left(x\right) + \sinh\left(x\right)\right.}{}$$

input
```
integrate(coth(x)/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
-1/2*(2*x*cosh(x)^2 + 2*x*sinh(x)^2 + 2*(2*x - 1)*cosh(x) - 3*(cosh(x)^2 +
 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x
) + 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)
*log(cosh(x) + sinh(x) - 1) + 2*(2*x*cosh(x) + 2*x - 1)*sinh(x) + 2*x)/(a*
cosh(x)^2 + a*sinh(x)^2 + 2*a*cosh(x) + 2*(a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\coth(x)}{a+a\text{sech}(x)}\,dx = \frac{\int \frac{\coth\left(x\right)}{\operatorname{sech}\left(x\right)+1}\,dx}{a}$$

input
```
integrate(coth(x)/(a+a*sech(x)),x)
```

output
```
Integral(coth(x)/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30

$$\int \frac{\coth(x)}{a + a\mathrm{sech}(x)}\, dx = \frac{x}{a} + \frac{e^{(-x)}}{2\, ae^{(-x)} + ae^{(-2\, x)} + a} + \frac{3\, \log\left(e^{(-x)} + 1\right)}{2\, a} + \frac{\log\left(e^{(-x)} - 1\right)}{2\, a}$$

input
```
integrate(coth(x)/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
x/a + e^(-x)/(2*a*e^(-x) + a*e^(-2*x) + a) + 3/2*log(e^(-x) + 1)/a + 1/2*l
og(e^(-x) - 1)/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int \frac{\coth(x)}{a + a\mathrm{sech}(x)}\, dx = \frac{3\, \log\left(e^{(-x)} + e^{x} + 2\right)}{4\, a} + \frac{\log\left(e^{(-x)} + e^{x} - 2\right)}{4\, a} - \frac{3\, e^{(-x)} + 3\, e^{x} + 2}{4\, a(e^{(-x)} + e^{x} + 2)}$$

input
```
integrate(coth(x)/(a+a*sech(x)),x, algorithm="giac")
```

output
```
3/4*log(e^(-x) + e^x + 2)/a + 1/4*log(e^(-x) + e^x - 2)/a - 1/4*(3*e^(-x)
+ 3*e^x + 2)/(a*(e^(-x) + e^x + 2))
```

Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.62

$$\int \frac{\coth(x)}{a + a\mathrm{sech}(x)}\, dx = \frac{\ln\left(\mathrm{e}^{2\, x} - 1\right)}{a} - \frac{x}{a} - \frac{1}{a + 2\, a\, \mathrm{e}^{x} + a\, \mathrm{e}^{2\, x}} + \frac{\mathrm{atan}\left(\frac{\mathrm{e}^{x}\, \sqrt{-a^2}}{a}\right)}{\sqrt{-a^2}} + \frac{1}{a + a\, \mathrm{e}^{x}}$$

input
```
int(coth(x)/(a + a/cosh(x)),x)
```

output
```
log(exp(2*x) - 1)/a - x/a - 1/(a + 2*a*exp(x) + a*exp(2*x)) + atan((exp(x)
*(-a^2)^(1/2))/a)/(-a^2)^(1/2) + 1/(a + a*exp(x))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.65

$$\int \frac{\coth(x)}{a + a\text{sech}(x)}\,dx$$
$$= \frac{e^{2x}\log(e^x-1)+3e^{2x}\log(e^x+1)-2e^{2x}x-e^{2x}+2e^x\log(e^x-1)+6e^x\log(e^x+1)-4e^x x+\log(e^x-1)}{2a\,(e^{2x}+2e^x+1)}$$

input
```
int(coth(x)/(a+a*sech(x)),x)
```

output
```
(e**(2*x)*log(e**x - 1) + 3*e**(2*x)*log(e**x + 1) - 2*e**(2*x)*x - e**(2*
x) + 2*e**x*log(e**x - 1) + 6*e**x*log(e**x + 1) - 4*e**x*x + log(e**x - 1
) + 3*log(e**x + 1) - 2*x - 1)/(2*a*(e**(2*x) + 2*e**x + 1))
```

3.110 $\int \frac{\coth^2(x)}{a+a\text{sech}(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{\coth^2(x)}{a+a\text{sech}(x)} \, dx = \frac{x}{a} - \frac{\coth(x)(3-2\text{sech}(x))}{3a} - \frac{\coth^3(x)(1-\text{sech}(x))}{3a}$$

output
```
x/a-1/3*coth(x)*(3-2*sech(x))/a-1/3*coth(x)^3*(1-sech(x))/a
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{\coth^2(x)}{a+a\text{sech}(x)} \, dx = \frac{6x-4\coth(x)-2\text{csch}(x)+6x\text{sech}(x)-4\tanh(x)}{6a+6a\text{sech}(x)}$$

input
```
Integrate[Coth[x]^2/(a + a*Sech[x]),x]
```

output
```
(6*x - 4*Coth[x] - 2*Csch[x] + 6*x*Sech[x] - 4*Tanh[x])/(6*a + 6*a*Sech[x]
)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {3042, 25, 4376, 25, 3042, 4370, 3042, 25, 4370, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{a\text{sech}(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\cot\left(\frac{\pi}{2}+ix\right)^2\left(a+a\csc\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\cot\left(ix+\frac{\pi}{2}\right)^2\left(\csc\left(ix+\frac{\pi}{2}\right)a+a\right)}dx$$

$$\Big\downarrow \text{4376}$$

$$-\frac{\int -\coth^4(x)(a-a\text{sech}(x))dx}{a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \coth^4(x)(a-a\text{sech}(x))dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a-a\csc\left(ix+\frac{\pi}{2}\right)}{\cot\left(ix+\frac{\pi}{2}\right)^4}dx}{a^2}$$

$$\Big\downarrow \text{4370}$$

$$\frac{\frac{1}{3}\int \coth^2(x)(3a-2a\text{sech}(x))dx-\frac{1}{3}\coth^3(x)(a-a\text{sech}(x))}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{1}{3}\coth^3(x)(a-a\text{sech}(x))+\frac{1}{3}\int -\frac{3a-2a\csc\left(ix+\frac{\pi}{2}\right)}{\cot\left(ix+\frac{\pi}{2}\right)^2}dx}{a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{1}{3}\coth^3(x)(a-a\text{sech}(x))-\frac{1}{3}\int \frac{3a-2a\csc\left(ix+\frac{\pi}{2}\right)}{\cot\left(ix+\frac{\pi}{2}\right)^2}dx}{a^2}$$

$$\Big\downarrow \text{4370}$$

$$\frac{\frac{1}{3}\left(-\int -3adx-\coth(x)(3a-2a\text{sech}(x))\right)-\frac{1}{3}\coth^3(x)(a-a\text{sech}(x))}{a^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{1}{3}(3ax-\coth(x)(3a-2a\text{sech}(x)))-\frac{1}{3}\coth^3(x)(a-a\text{sech}(x))}{a^2}$$

input
```
Int[Coth[x]^2/(a + a*Sech[x]),x]
```

output
```
(-1/3*(Coth[x]^3*(a - a*Sech[x])) + (3*a*x - Coth[x]*(3*a - 2*a*Sech[x]))/
3)/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{x}{a}+\frac{2\,{\mathrm e}^{3 x}-\frac{10\,{\mathrm e}^{x}}{3}-\frac{8}{3}}{a \left(1+{\mathrm e}^{x}\right)^{3} \left({\mathrm e}^{x}-1\right)}$	36
default	$\frac{-\frac{\tanh\left(\frac{x}{2}\right)^{3}}{3}-4\tanh\left(\frac{x}{2}\right)+4\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-4\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)}}{4a}$	47

input
```
int(coth(x)^2/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
x/a+2/3*(3*exp(3*x)-5*exp(x)-4)/a/(1+exp(x))^3/(exp(x)-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \frac{\coth^2(x)}{a+a\operatorname{sech}(x)}\,dx = -\frac{2\,\cosh\left(x\right)^{2}-\left(\left(3\,x+4\right)\cosh\left(x\right)+3\,x+4\right)\sinh\left(x\right)+2\,\sinh\left(x\right)^{2}+\cosh\left(x\right)}{3\,\left(a\cosh\left(x\right)+a\right)\sinh\left(x\right)}$$

input
```
integrate(coth(x)^2/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
-1/3*(2*cosh(x)^2 - ((3*x + 4)*cosh(x) + 3*x + 4)*sinh(x) + 2*sinh(x)^2 +
cosh(x))/((a*cosh(x) + a)*sinh(x))
```

Sympy [F]

$$\int \frac{\coth^2(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\coth^2(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

input
```
integrate(coth(x)**2/(a+a*sech(x)),x)
```

output
```
Integral(coth(x)**2/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{\coth^2(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{x}{a} - \frac{2\left(5\,e^{(-x)} - 3\,e^{(-3\,x)} + 4\right)}{3\left(2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a\right)}$$

input
```
integrate(coth(x)^2/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
x/a - 2/3*(5*e^(-x) - 3*e^(-3*x) + 4)/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4
*x) + a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \frac{\coth^2(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{x}{a} - \frac{1}{2\,a(e^x - 1)} + \frac{15\,e^{(2\,x)} + 24\,e^x + 13}{6\,a(e^x + 1)^3}$$

input
```
integrate(coth(x)^2/(a+a*sech(x)),x, algorithm="giac")
```

output
```
x/a - 1/2/(a*(e^x - 1)) + 1/6*(15*e^(2*x) + 24*e^x + 13)/(a*(e^x + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.47

$$\int \frac{\coth^2(x)}{a + a\operatorname{sech}(x)}\, dx = \frac{\frac{5\,\mathrm{e}^{2\,x}}{6\,a} + \frac{5}{6\,a} + \frac{\mathrm{e}^{x}}{a}}{3\,\mathrm{e}^{2\,x} + \mathrm{e}^{3\,x} + 3\,\mathrm{e}^{x} + 1} + \frac{\frac{1}{2\,a} + \frac{5\,\mathrm{e}^{x}}{6\,a}}{\mathrm{e}^{2\,x} + 2\,\mathrm{e}^{x} + 1} + \frac{x}{a} - \frac{1}{2\,a\,\left(\mathrm{e}^{x} - 1\right)} + \frac{5}{6\,a\,\left(\mathrm{e}^{x} + 1\right)}$$

input
```
int(coth(x)^2/(a + a/cosh(x)),x)
```

output
```
((5*exp(2*x))/(6*a) + 5/(6*a) + exp(x)/a)/(3*exp(2*x) + exp(3*x) + 3*exp(x
) + 1) + (1/(2*a) + (5*exp(x))/(6*a))/(exp(2*x) + 2*exp(x) + 1) + x/a - 1/
(2*a*(exp(x) - 1)) + 5/(6*a*(exp(x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \frac{\coth^2(x)}{a + a\operatorname{sech}(x)}\, dx = \frac{3e^{4x}x - 3e^{4x} + 6e^{3x}x - 6e^{x}x - 4e^{x} - 3x - 5}{3a\left(e^{4x} + 2e^{3x} - 2e^{x} - 1\right)}$$

input
```
int(coth(x)^2/(a+a*sech(x)),x)
```

output
```
(3*e**(4*x)*x - 3*e**(4*x) + 6*e**(3*x)*x - 6*e**x*x - 4*e**x - 3*x - 5)/(
3*a*(e**(4*x) + 2*e**(3*x) - 2*e**x - 1))
```

3.111 $\int \frac{\coth^3(x)}{a+a\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \frac{\coth^3(x)}{a+a\text{sech}(x)}\,dx=\frac{1}{8a(1-\cosh(x))}-\frac{1}{8a(1+\cosh(x))^2}+\frac{3}{4a(1+\cosh(x))}+\frac{5\log(1-\cosh(x))}{16a}+\frac{11\log(1+\cosh(x))}{16a}$$

output
```
1/8/a/(1-cosh(x))-1/8/a/(1+cosh(x))^2+3/4/a/(1+cosh(x))+5/16*ln(1-cosh(x))
/a+11/16*ln(1+cosh(x))/a
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{\coth^3(x)}{a+a\text{sech}(x)}\,dx=\frac{\left(12-2\coth^2\left(\frac{x}{2}\right)+4\cosh^2\left(\frac{x}{2}\right)\left(11\log\left(\cosh\left(\frac{x}{2}\right)\right)+5\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)-\text{sech}^2\left(\frac{x}{2}\right)\right)\text{sech}(x)}{16a(1+\text{sech}(x))}$$

input
```
Integrate[Coth[x]^3/(a + a*Sech[x]),x]
```

output
```
((12 - 2*Coth[x/2]^2 + 4*Cosh[x/2]^2*(11*Log[Cosh[x/2]] + 5*Log[Sinh[x/2]]
) - Sech[x/2]^2)*Sech[x])/(16*a*(1 + Sech[x]))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 26, 4367, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(x)}{a\text{sech}(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\cot\left(\frac{\pi}{2}+ix\right)^3\left(a+a\csc\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\cot\left(ix+\frac{\pi}{2}\right)^3\left(\csc\left(ix+\frac{\pi}{2}\right)a+a\right)}dx$$

$$\Big\downarrow \text{4367}$$

$$a^4\int \frac{\cosh^4(x)}{a^5(1-\cosh(x))^2(\cosh(x)+1)^3}d\cosh(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cosh^4(x)}{(1-\cosh(x))^2(\cosh(x)+1)^3}d\cosh(x)}{a}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\frac{11}{16(\cosh(x)+1)}-\frac{3}{4(\cosh(x)+1)^2}+\frac{1}{4(\cosh(x)+1)^3}+\frac{5}{16(\cosh(x)-1)}+\frac{1}{8(\cosh(x)-1)^2}\right)d\cosh(x)}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{8(1-\cosh(x))}+\frac{3}{4(\cosh(x)+1)}-\frac{1}{8(\cosh(x)+1)^2}+\frac{5}{16}\log(1-\cosh(x))+\frac{11}{16}\log(\cosh(x)+1)}{a}$$

input
```
Int[Coth[x]^3/(a + a*Sech[x]),x]
```

output
```
(1/(8*(1 - Cosh[x])) - 1/(8*(1 + Cosh[x])^2) + 3/(4*(1 + Cosh[x])) + (5*Lo
g[1 - Cosh[x]])/16 + (11*Log[1 + Cosh[x]])/16)/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{-\frac{\tanh\left(\frac{x}{2}\right)^{4}}{4}-\frac{5\tanh\left(\frac{x}{2}\right)^{2}}{2}-\frac{1}{2\tanh\left(\frac{x}{2}\right)^{2}}+5\ln\left(\tanh\left(\frac{x}{2}\right)\right)-8\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-8\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8a}$	56
risch	$-\frac{x}{a}+\frac{\mathrm{e}^{x}\left(5\,\mathrm{e}^{4x}-6\,\mathrm{e}^{3x}-14\,\mathrm{e}^{2x}-6\,\mathrm{e}^{x}+5\right)}{4a\left(\mathrm{e}^{x}-1\right)^{2}\left(1+\mathrm{e}^{x}\right)^{4}}+\frac{11\ln\left(1+\mathrm{e}^{x}\right)}{8a}+\frac{5\ln\left(\mathrm{e}^{x}-1\right)}{8a}$	71

input
```
int(coth(x)^3/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
1/8/a*(-1/4*tanh(1/2*x)^4-5/2*tanh(1/2*x)^2-1/2/tanh(1/2*x)^2+5*ln(tanh(1/
2*x))-8*ln(tanh(1/2*x)-1)-8*ln(tanh(1/2*x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 773 vs. 2(56) = 112.

Time = 0.11 (sec) , antiderivative size = 773, normalized size of antiderivative = 11.37

$$\int \frac{\coth^3(x)}{a+a\operatorname{sech}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
-1/8*(8*x*cosh(x)^6 + 8*x*sinh(x)^6 + 2*(8*x - 5)*cosh(x)^5 + 2*(24*x*cosh
(x) + 8*x - 5)*sinh(x)^5 - 4*(2*x - 3)*cosh(x)^4 + 2*(60*x*cosh(x)^2 + 5*(
8*x - 5)*cosh(x) - 4*x + 6)*sinh(x)^4 - 4*(8*x - 7)*cosh(x)^3 + 4*(40*x*co
sh(x)^3 + 5*(8*x - 5)*cosh(x)^2 - 4*(2*x - 3)*cosh(x) - 8*x + 7)*sinh(x)^3
 - 4*(2*x - 3)*cosh(x)^2 + 4*(30*x*cosh(x)^4 + 5*(8*x - 5)*cosh(x)^3 - 6*(
2*x - 3)*cosh(x)^2 - 3*(8*x - 7)*cosh(x) - 2*x + 3)*sinh(x)^2 + 2*(8*x - 5
)*cosh(x) - 11*(cosh(x)^6 + 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*co
sh(x)^5 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cos
h(x)^3 + 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^
4 + 20*cosh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2
*(3*cosh(x)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sin
h(x) + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) + 1) - 5*(cosh(x)^6 + 2*(3*cos
h(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^2 + 10*cosh(x)
 - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 - cosh(x) - 1)*
sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4 + 20*cosh(x)^3 - 6*cosh(x)^2 - 12*
cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh(x)^4 - 2*cosh
(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*log(cosh(x) +
sinh(x) - 1) + 2*(24*x*cosh(x)^5 + 5*(8*x - 5)*cosh(x)^4 - 8*(2*x - 3)*cos
h(x)^3 - 6*(8*x - 7)*cosh(x)^2 - 4*(2*x - 3)*cosh(x) + 8*x - 5)*sinh(x) +
8*x)/(a*cosh(x)^6 + a*sinh(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*s...
```

Sympy [F]

$$\int \frac{\coth^3(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{\int \frac{\coth^3(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

input
```
integrate(coth(x)**3/(a+a*sech(x)),x)
```

output
```
Integral(coth(x)**3/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.59

$$\int \frac{\coth^3(x)}{a + a\text{sech}(x)}\,dx = \frac{x}{a} + \frac{5\,e^{(-x)} - 6\,e^{(-2\,x)} - 14\,e^{(-3\,x)} - 6\,e^{(-4\,x)} + 5\,e^{(-5\,x)}}{4\,(2\,ae^{(-x)} - ae^{(-2\,x)} - 4\,ae^{(-3\,x)} - ae^{(-4\,x)} + 2\,ae^{(-5\,x)} + ae^{(-6\,x)} + a)} + \frac{11\,\log\left(e^{(-x)} + 1\right)}{8\,a} + \frac{5\,\log\left(e^{(-x)} - 1\right)}{8\,a}$$

input
```
integrate(coth(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
x/a + 1/4*(5*e^(-x) - 6*e^(-2*x) - 14*e^(-3*x) - 6*e^(-4*x) + 5*e^(-5*x))/
(2*a*e^(-x) - a*e^(-2*x) - 4*a*e^(-3*x) - a*e^(-4*x) + 2*a*e^(-5*x) + a*e^
(-6*x) + a) + 11/8*log(e^(-x) + 1)/a + 5/8*log(e^(-x) - 1)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.38

$$\int \frac{\coth^3(x)}{a + a\text{sech}(x)}\,dx = \frac{11\,\log\left(e^{(-x)} + e^{x} + 2\right)}{16\,a} + \frac{5\,\log\left(e^{(-x)} + e^{x} - 2\right)}{16\,a} - \frac{5\,e^{(-x)} + 5\,e^{x} - 6}{16\,a(e^{(-x)} + e^{x} - 2)} - \frac{33\,\left(e^{(-x)} + e^{x}\right)^2 + 84\,e^{(-x)} + 84\,e^{x} + 52}{32\,a(e^{(-x)} + e^{x} + 2)^2}$$

input
```
integrate(coth(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

output
```
11/16*log(e^(-x) + e^x + 2)/a + 5/16*log(e^(-x) + e^x - 2)/a - 1/16*(5*e^(
-x) + 5*e^x - 6)/(a*(e^(-x) + e^x - 2)) - 1/32*(33*(e^(-x) + e^x)^2 + 84*e
^(-x) + 84*e^x + 52)/(a*(e^(-x) + e^x + 2)^2)
```

Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.35

$$\int \frac{\coth^3(x)}{a + a\text{sech}(x)} \, dx = \frac{\ln(9\,e^{2\,x} - 9)}{a} - \frac{x}{a} - \frac{1}{2\,(a + 4\,a\,e^{x} + 6\,a\,e^{2\,x} + 4\,a\,e^{3\,x} + a\,e^{4\,x})} + \frac{1}{a + 3\,a\,e^{x} + 3\,a\,e^{2\,x} + a\,e^{3\,x}} - \frac{1}{4\,(a - 2\,a\,e^{x} + a\,e^{2\,x})} - \frac{2}{a + 2\,a\,e^{x} + a\,e^{2\,x}} + \frac{3\,\text{atan}\left(\frac{e^{x}\,\sqrt{-a^2}}{a}\right)}{4\,\sqrt{-a^2}} + \frac{3}{2\,(a + a\,e^{x})} + \frac{1}{4\,(a - a\,e^{x})}$$

input
```
int(coth(x)^3/(a + a/cosh(x)),x)
```

output
```
log(9*exp(2*x) - 9)/a - x/a - 1/(2*(a + 4*a*exp(x) + 6*a*exp(2*x) + 4*a*ex
p(3*x) + a*exp(4*x))) + 1/(a + 3*a*exp(x) + 3*a*exp(2*x) + a*exp(3*x)) - 1
/(4*(a - 2*a*exp(x) + a*exp(2*x))) - 2/(a + 2*a*exp(x) + a*exp(2*x)) + (3*
atan((exp(x)*(-a^2)^(1/2))/a))/(4*(-a^2)^(1/2)) + 3/(2*(a + a*exp(x))) + 1
/(4*(a - a*exp(x)))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 294, normalized size of antiderivative = 4.32

$$\int \frac{\coth^3(x)}{a + a\text{sech}(x)} \, dx = \frac{-5 - 8x - 7e^{4x} + 10e^{5x}\log(e^{x} - 1) + 22e^{5x}\log(e^{x} + 1) - 20e^{3x}\log(e^{x} - 1) - 44e^{3x}\log(e^{x} + 1) - 16e^{5x}x}{}$$

input
```
int(coth(x)^3/(a+a*sech(x)),x)
```

output
```
(5*e**(6*x)*log(e**x - 1) + 11*e**(6*x)*log(e**x + 1) - 8*e**(6*x)*x - 5*e
**(6*x) + 10*e**(5*x)*log(e**x - 1) + 22*e**(5*x)*log(e**x + 1) - 16*e**(5
*x)*x - 5*e**(4*x)*log(e**x - 1) - 11*e**(4*x)*log(e**x + 1) + 8*e**(4*x)*
x - 7*e**(4*x) - 20*e**(3*x)*log(e**x - 1) - 44*e**(3*x)*log(e**x + 1) + 3
2*e**(3*x)*x - 8*e**(3*x) - 5*e**(2*x)*log(e**x - 1) - 11*e**(2*x)*log(e**
x + 1) + 8*e**(2*x)*x - 7*e**(2*x) + 10*e**x*log(e**x - 1) + 22*e**x*log(e
**x + 1) - 16*e**x*x + 5*log(e**x - 1) + 11*log(e**x + 1) - 8*x - 5)/(8*a*
(e**(6*x) + 2*e**(5*x) - e**(4*x) - 4*e**(3*x) - e**(2*x) + 2*e**x + 1))
```

3.112 $\int \frac{\coth^4(x)}{a+a\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 55

$$\int \frac{\coth^4(x)}{a+a\text{sech}(x)}\,dx=\frac{x}{a}-\frac{\coth(x)(15-8\text{sech}(x))}{15a}-\frac{\coth^3(x)(5-4\text{sech}(x))}{15a}-\frac{\coth^5(x)(1-\text{sech}(x))}{5a}$$

output
```
x/a-1/15*coth(x)*(15-8*sech(x))/a-1/15*coth(x)^3*(5-4*sech(x))/a-1/5*coth(
x)^5*(1-sech(x))/a
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \frac{\coth^4(x)}{a+a\text{sech}(x)}\,dx = \frac{\text{csch}^3(x)\text{sech}(x)(-25+8\cosh(x)+16\cosh(2x)-16\cosh(3x)-23\cosh(4x)-90x\sinh(x)-30x\sinh(}{120a(1+\text{sech}(x))}$$

input
```
Integrate[Coth[x]^4/(a + a*Sech[x]),x]
```

output
```
(Csch[x]^3*Sech[x]*(-25 + 8*Cosh[x] + 16*Cosh[2*x] - 16*Cosh[3*x] - 23*Cos
h[4*x] - 90*x*Sinh[x] - 30*x*Sinh[2*x] + 30*x*Sinh[3*x] + 15*x*Sinh[4*x]))
/(120*a*(1 + Sech[x]))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$, Rules used = {3042, 4376, 3042, 25, 4370, 25, 3042, 4370, 3042, 25, 4370, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(x)}{a\text{sech}(x)+a}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\cot\left(\frac{\pi}{2}+ix\right)^4\left(a+a\csc\left(\frac{\pi}{2}+ix\right)\right)}dx$$
$$\Big\downarrow \text{ 4376}$$
$$\frac{\int \coth^6(x)(a-a\text{sech}(x))dx}{a^2}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int -\frac{a-a\csc\left(ix+\frac{\pi}{2}\right)}{\cot\left(ix+\frac{\pi}{2}\right)^6}dx}{a^2}$$
$$\Big\downarrow \text{ 25}$$
$$-\frac{\int \frac{a-a\csc\left(ix+\frac{\pi}{2}\right)}{\cot\left(ix+\frac{\pi}{2}\right)^6}dx}{a^2}$$
$$\Big\downarrow \text{ 4370}$$
$$-\frac{\frac{1}{5}\int -\coth^4(x)(5a-4a\text{sech}(x))dx+\frac{1}{5}\coth^5(x)(a-a\text{sech}(x))}{a^2}$$
$$\Big\downarrow \text{ 25}$$
$$-\frac{\frac{1}{5}\coth^5(x)(a-a\text{sech}(x))-\frac{1}{5}\int \coth^4(x)(5a-4a\text{sech}(x))dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{1}{5}\coth^5(x)(a-a\text{sech}(x))-\frac{1}{5}\int \frac{5a-4a\csc\left(ix+\frac{\pi}{2}\right)}{\cot\left(ix+\frac{\pi}{2}\right)^4}dx}{a^2}$$

$$\Big\downarrow \text{4370}$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\coth^3(x)(5a-4a\text{sech}(x))-\frac{1}{3}\int \coth^2(x)(15a-8a\text{sech}(x))dx\right)+\frac{1}{5}\coth^5(x)(a-a\text{sech}(x))}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{1}{5}\coth^5(x)(a-a\text{sech}(x))+\frac{1}{5}\left(\frac{1}{3}\coth^3(x)(5a-4a\text{sech}(x))-\frac{1}{3}\int -\frac{15a-8a\csc\left(ix+\frac{\pi}{2}\right)}{\cot\left(ix+\frac{\pi}{2}\right)^2}dx\right)}{a^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{5}\coth^5(x)(a-a\text{sech}(x))+\frac{1}{5}\left(\frac{1}{3}\coth^3(x)(5a-4a\text{sech}(x))+\frac{1}{3}\int \frac{15a-8a\csc\left(ix+\frac{\pi}{2}\right)}{\cot\left(ix+\frac{\pi}{2}\right)^2}dx\right)}{a^2}$$

$$\Big\downarrow \text{4370}$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(\int -15adx+\coth(x)(15a-8a\text{sech}(x))\right)+\frac{1}{3}\coth^3(x)(5a-4a\text{sech}(x))\right)+\frac{1}{5}\coth^5(x)(a-a\text{sech}(x))}{a^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\frac{1}{5}\coth^5(x)(a-a\text{sech}(x))+\frac{1}{5}\left(\frac{1}{3}\coth^3(x)(5a-4a\text{sech}(x))+\frac{1}{3}(\coth(x)(15a-8a\text{sech}(x))-15ax)\right)}{a^2}$$

input
```
Int[Coth[x]^4/(a + a*Sech[x]),x]
```

output
```
-(((Coth[x]^5*(a - a*Sech[x]))/5 + ((Coth[x]^3*(5*a - 4*a*Sech[x]))/3 + (-
15*a*x + Coth[x]*(15*a - 8*a*Sech[x]))/3)/5)/a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{-\frac{\tanh\left(\frac{x}{2}\right)^{5}}{5}-2\tanh\left(\frac{x}{2}\right)^{3}-16\tanh\left(\frac{x}{2}\right)-\frac{1}{3\tanh\left(\frac{x}{2}\right)^{3}}-\frac{6}{\tanh\left(\frac{x}{2}\right)}+16\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-16\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{16a}$	63
risch	$\frac{x}{a}+\frac{2\,\mathrm{e}^{7x}-2\,\mathrm{e}^{6x}-\frac{26\,\mathrm{e}^{5x}}{3}-\frac{10\,\mathrm{e}^{4x}}{3}+\frac{146\,\mathrm{e}^{3x}}{15}+\frac{62\,\mathrm{e}^{2x}}{15}-\frac{62\,\mathrm{e}^{x}}{15}-\frac{46}{15}}{a\left(1+\mathrm{e}^{x}\right)^{5}\left(\mathrm{e}^{x}-1\right)^{3}}$	66

input
```
int(coth(x)^4/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
1/16/a*(-1/5*tanh(1/2*x)^5-2*tanh(1/2*x)^3-16*tanh(1/2*x)-1/3/tanh(1/2*x)^
3-6/tanh(1/2*x)+16*ln(tanh(1/2*x)+1)-16*ln(tanh(1/2*x)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(47) = 94.

Time = 0.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.75

$$\int \frac{\coth^4(x)}{a+a\operatorname{sech}(x)}\,dx = -\frac{23\,\cosh\left(x\right)^{4} - 2\,\left(2\,\left(15\,x + 23\right)\cosh\left(x\right) + 15\,x + 23\right)\sinh\left(x\right)^{3} + 23\,\sinh\left(x\right)^{4} + 16\,\cosh\left(x\right)^{3} + 2\,\left(69\,\mathrm{c}\right.}{30\,\left(\left(2\,a\cosh\right.\right.}$$

input
```
integrate(coth(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

output
```
-1/30*(23*cosh(x)^4 - 2*(2*(15*x + 23)*cosh(x) + 15*x + 23)*sinh(x)^3 + 23
*sinh(x)^4 + 16*cosh(x)^3 + 2*(69*cosh(x)^2 + 24*cosh(x) - 8)*sinh(x)^2 -
16*cosh(x)^2 - 2*(2*(15*x + 23)*cosh(x)^3 + 3*(15*x + 23)*cosh(x)^2 - 2*(1
5*x + 23)*cosh(x) - 45*x - 69)*sinh(x) - 8*cosh(x) + 25)/((2*a*cosh(x) + a
)*sinh(x)^3 + (2*a*cosh(x)^3 + 3*a*cosh(x)^2 - 2*a*cosh(x) - 3*a)*sinh(x))
```

Sympy [F]

$$\int \frac{\coth^4(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\coth^4(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

input
```
integrate(coth(x)**4/(a+a*sech(x)),x)
```

output
```
Integral(coth(x)**4/(sech(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(47) = 94.

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.91

$$\int \frac{\coth^4(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{x}{a} - \frac{2\left(31\,e^{(-x)} - 31\,e^{(-2\,x)} - 73\,e^{(-3\,x)} + 25\,e^{(-4\,x)} + 65\,e^{(-5\,x)} + 15\,e^{(-6\,x)} - 15\,e^{(-7\,x)} + 23\right)}{15\left(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a\right)}$$

input
```
integrate(coth(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

output
```
x/a - 2/15*(31*e^(-x) - 31*e^(-2*x) - 73*e^(-3*x) + 25*e^(-4*x) + 65*e^(-5
*x) + 15*e^(-6*x) - 15*e^(-7*x) + 23)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(
-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

$$\int \frac{\coth^4(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{x}{a} - \frac{21\,e^{(2\,x)} - 36\,e^{x} + 19}{24\,a(e^{x} - 1)^{3}} + \frac{115\,e^{(4\,x)} + 380\,e^{(3\,x)} + 530\,e^{(2\,x)} + 340\,e^{x} + 91}{40\,a(e^{x} + 1)^{5}}$$

input
```
integrate(coth(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

output
```
x/a - 1/24*(21*e^(2*x) - 36*e^x + 19)/(a*(e^x - 1)^3) + 1/40*(115*e^(4*x)
+ 380*e^(3*x) + 530*e^(2*x) + 340*e^x + 91)/(a*(e^x + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 264, normalized size of antiderivative = 4.80

$$\int \frac{\coth^4(x)}{a + a\mathrm{sech}(x)}\,dx = \frac{\frac{9\,\mathrm{e}^{2\,x}}{4\,a}+\frac{3\,\mathrm{e}^{3\,x}}{2\,a}+\frac{23\,\mathrm{e}^{4\,x}}{40\,a}+\frac{23}{40\,a}+\frac{3\,\mathrm{e}^{x}}{2\,a}}{10\,\mathrm{e}^{2\,x}+10\,\mathrm{e}^{3\,x}+5\,\mathrm{e}^{4\,x}+\mathrm{e}^{5\,x}+5\,\mathrm{e}^{x}+1} + \frac{\frac{9\,\mathrm{e}^{2\,x}}{8\,a}+\frac{23\,\mathrm{e}^{3\,x}}{40\,a}+\frac{3}{8\,a}+\frac{9\,\mathrm{e}^{x}}{8\,a}}{6\,\mathrm{e}^{2\,x}+4\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}+4\,\mathrm{e}^{x}+1} + \frac{\frac{23\,\mathrm{e}^{2\,x}}{40\,a}+\frac{3}{8\,a}+\frac{3\,\mathrm{e}^{x}}{4\,a}}{3\,\mathrm{e}^{2\,x}+\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}+1} + \frac{\frac{3}{8\,a}+\frac{23\,\mathrm{e}^{x}}{40\,a}}{\mathrm{e}^{2\,x}+2\,\mathrm{e}^{x}+1} + \frac{1}{6\,a\,\left(3\,\mathrm{e}^{2\,x}-\mathrm{e}^{3\,x}-3\,\mathrm{e}^{x}+1\right)} - \frac{1}{4\,a\,\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{x}+1\right)} + \frac{x}{a} - \frac{7}{8\,a\,\left(\mathrm{e}^{x}-1\right)} + \frac{23}{40\,a\,\left(\mathrm{e}^{x}+1\right)}$$

input
```
int(coth(x)^4/(a + a/cosh(x)),x)
```

output
```
((9*exp(2*x))/(4*a) + (3*exp(3*x))/(2*a) + (23*exp(4*x))/(40*a) + 23/(40*a
) + (3*exp(x))/(2*a))/(10*exp(2*x) + 10*exp(3*x) + 5*exp(4*x) + exp(5*x) +
 5*exp(x) + 1) + ((9*exp(2*x))/(8*a) + (23*exp(3*x))/(40*a) + 3/(8*a) + (9
*exp(x))/(8*a))/(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp(x) + 1) + ((23
*exp(2*x))/(40*a) + 3/(8*a) + (3*exp(x))/(4*a))/(3*exp(2*x) + exp(3*x) + 3
*exp(x) + 1) + (3/(8*a) + (23*exp(x))/(40*a))/(exp(2*x) + 2*exp(x) + 1) +
1/(6*a*(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1)) - 1/(4*a*(exp(2*x) - 2*exp(
x) + 1)) + x/a - 7/(8*a*(exp(x) - 1)) + 23/(40*a*(exp(x) + 1))
```

Reduce [B] (verification not implemented)

Time = 7.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.78

$$\int \frac{\coth^4(x)}{a + a\mathrm{sech}(x)}\,dx = \frac{15e^{8x}x - 15e^{8x} + 30e^{7x}x - 30e^{6x}x - 90e^{5x}x - 40e^{5x} - 50e^{4x} + 90e^{3x}x + 56e^{3x} + 30e^{2x}x + 32e^{2x} - 30e^{x}}{15a\left(e^{8x} + 2e^{7x} - 2e^{6x} - 6e^{5x} + 6e^{3x} + 2e^{2x} - 2e^{x} - 1\right)}$$

input
```
int(coth(x)^4/(a+a*sech(x)),x)
```

output
```
(15*e**(8*x)*x - 15*e**(8*x) + 30*e**(7*x)*x - 30*e**(6*x)*x - 90*e**(5*x)
*x - 40*e**(5*x) - 50*e**(4*x) + 90*e**(3*x)*x + 56*e**(3*x) + 30*e**(2*x)
*x + 32*e**(2*x) - 30*e***x*x - 32*e***x - 15*x - 31)/(15*a*(e**(8*x) + 2*e*
*(7*x) - 2*e**(6*x) - 6*e**(5*x) + 6*e**(3*x) + 2*e**(2*x) - 2*e***x - 1))
```

3.113 $\int \frac{\tanh^7(x)}{a+b\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 121

$$\int \frac{\tanh^7(x)}{a+b\text{sech}(x)}\,dx=\frac{\log(\cosh(x))}{a}-\frac{(a^2-b^2)^3\log(a+b\text{sech}(x))}{ab^6}+\frac{(a^4-3a^2b^2+3b^4)\,\text{sech}(x)}{b^5}-\frac{a(a^2-3b^2)\,\text{sech}^2(x)}{2b^4}+\frac{(a^2-3b^2)\,\text{sech}^3(x)}{3b^3}-\frac{a\text{sech}^4(x)}{4b^2}+\frac{\text{sech}^5(x)}{5b}$$

output
```
ln(cosh(x))/a-(a^2-b^2)^3*ln(a+b*sech(x))/a/b^6+(a^4-3*a^2*b^2+3*b^4)*sech
(x)/b^5-1/2*a*(a^2-3*b^2)*sech(x)^2/b^4+1/3*(a^2-3*b^2)*sech(x)^3/b^3-1/4*
a*sech(x)^4/b^2+1/5*sech(x)^5/b
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^7(x)}{a+b\text{sech}(x)}\,dx=\frac{\log(\cosh(x))}{a}-\frac{(a^2-b^2)^3\log(a+b\text{sech}(x))}{ab^6}+\frac{(a^4-3a^2b^2+3b^4)\,\text{sech}(x)}{b^5}-\frac{a(a^2-3b^2)\,\text{sech}^2(x)}{2b^4}+\frac{(a^2-3b^2)\,\text{sech}^3(x)}{3b^3}-\frac{a\text{sech}^4(x)}{4b^2}+\frac{\text{sech}^5(x)}{5b}$$

input
```
Integrate[Tanh[x]^7/(a + b*Sech[x]),x]
```

output
```
Log[Cosh[x]]/a - ((a^2 - b^2)^3*Log[a + b*Sech[x]])/(a*b^6) + ((a^4 - 3*a^
2*b^2 + 3*b^4)*Sech[x])/b^5 - (a*(a^2 - 3*b^2)*Sech[x]^2)/(2*b^4) + ((a^2
- 3*b^2)*Sech[x]^3)/(3*b^3) - (a*Sech[x]^4)/(4*b^2) + Sech[x]^5/(5*b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 26, 4373, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^7(x)}{a+b\text{sech}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cot\left(\frac{\pi}{2}+ix\right)^7}{a+b\csc\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cot\left(ix+\frac{\pi}{2}\right)^7}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4373}$$

$$-\frac{\int \frac{\cosh(x)\left(b^2-b^2\text{sech}^2(x)\right)^3}{b(a+b\text{sech}(x))}d(b\text{sech}(x))}{b^6}$$

$$\Big\downarrow \text{522}$$

$$-\frac{\int \left(\frac{\cosh(x)b^5}{a}-\text{sech}^4(x)b^4+a\text{sech}^3(x)b^3-\left(a^2-3b^2\right)\text{sech}^2(x)b^2+a\left(a^2-3b^2\right)\text{sech}(x)b-a^4\left(\frac{3\left(b^2-a^2\right)b^2}{a^4}+1\right)\right.}{b^6}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{2}ab^2(a^2-3b^2)\operatorname{sech}^2(x)+\frac{(a^2-b^2)^3\log(a+b\operatorname{sech}(x))}{a}-\frac{1}{3}b^3(a^2-3b^2)\operatorname{sech}^3(x)-b(a^4-3a^2b^2+3b^4)\operatorname{sech}(x)+\frac{b^6}{\cdots}}{b^6}$$

input
```
Int[Tanh[x]^7/(a + b*Sech[x]),x]
```

output
```
-(((b^6*Log[b*Sech[x]])/a + ((a^2 - b^2)^3*Log[a + b*Sech[x]])/a - b*(a^4
- 3*a^2*b^2 + 3*b^4)*Sech[x] + (a*b^2*(a^2 - 3*b^2)*Sech[x]^2)/2 - (b^3*(a
^2 - 3*b^2)*Sech[x]^3)/3 + (a*b^4*Sech[x]^4)/4 - (b^5*Sech[x]^5)/5)/b^6)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(113) = 226$.

Time = 2.53 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.03

method	result
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{(a-b)^3\left(a^3+3a^2b+3a\,b^2+b^3\right)\ln\left(a\tanh\left(\frac{x}{2}\right)^2-b\tanh\left(\frac{x}{2}\right)^2+a+b\right)}{a\,b^6}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}+\frac{-\frac{4b^4(a+4b)}{\left(\tanh\left(\frac{x}{2}\right)^2+1\right)}}{}$
risch	$-\frac{x}{a}+\frac{2\,\mathrm{e}^{x}\left(15a^4\mathrm{e}^{8x}-45a^2b^2\mathrm{e}^{8x}+45b^4\mathrm{e}^{8x}-15a^3b\,\mathrm{e}^{7x}+45a\,b^3\mathrm{e}^{7x}+60a^4\mathrm{e}^{6x}-160a^2b^2\mathrm{e}^{6x}+120b^4\mathrm{e}^{6x}-45a^3b\,\mathrm{e}^{5x}+105a\,b^3\mathrm{e}^{5x}+90\,\mathrm{e}\right.}{15}$

input
```
int(tanh(x)^7/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/a*ln(tanh(1/2*x)-1)-(a-b)^3*(a^3+3*a^2*b+3*a*b^2+b^3)/a/b^6*ln(a*tanh(1
/2*x)^2-b*tanh(1/2*x)^2+a+b)-1/a*ln(tanh(1/2*x)+1)+1/b^6*(-4*b^4*(a+4*b)/(
tanh(1/2*x)^2+1)^4+a*(a^4-3*a^2*b^2+3*b^4)*ln(tanh(1/2*x)^2+1)+32/5*b^5/(t
anh(1/2*x)^2+1)^5-2*b^2*(a^3+2*a^2*b-2*b^3)/(tanh(1/2*x)^2+1)^2+2*b*(a^4+a
^3*b-2*a^2*b^2-2*a*b^3+b^4)/(tanh(1/2*x)^2+1)+8/3*b^3*(a^2+3*a*b+3*b^2)/(t
anh(1/2*x)^2+1)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4077 vs. $2(113) = 226$.

Time = 0.15 (sec) , antiderivative size = 4077, normalized size of antiderivative = 33.69

$$\int \frac{\tanh^7(x)}{a+b\mathrm{sech}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^7/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh^7(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\tanh^7(x)}{a + b\,\text{sech}\,(x)}\,dx$$

input
```
integrate(tanh(x)**7/(a+b*sech(x)),x)
```

output
```
Integral(tanh(x)**7/(a + b*sech(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(113) = 226$.

Time = 0.13 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.74

$$\int \frac{\tanh^7(x)}{a + b\text{sech}(x)}\,dx = \frac{2\,(15\,(a^4 - 3\,a^2b^2 + 3\,b^4)e^{(-x)} - 15\,(a^3b - 3\,ab^3)e^{(-2\,x)} + 20\,(3\,a^4 - 8\,a^2b^2 + 6\,b^4)e^{(-3\,x)} - 15\,(3\,a^3b - 7\,a}{15\,(5\,b^5} + \frac{x}{a} + \frac{(a^5 - 3\,a^3b^2 + 3\,ab^4)\log\left(e^{(-2\,x)} + 1\right)}{b^6} - \frac{(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6)\log\left(2\,be^{(-x)} + ae^{(-2\,x)} + a\right)}{ab^6}$$

input
```
integrate(tanh(x)^7/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
2/15*(15*(a^4 - 3*a^2*b^2 + 3*b^4)*e^(-x) - 15*(a^3*b - 3*a*b^3)*e^(-2*x)
+ 20*(3*a^4 - 8*a^2*b^2 + 6*b^4)*e^(-3*x) - 15*(3*a^3*b - 7*a*b^3)*e^(-4*x
) + 2*(45*a^4 - 115*a^2*b^2 + 99*b^4)*e^(-5*x) - 15*(3*a^3*b - 7*a*b^3)*e^
(-6*x) + 20*(3*a^4 - 8*a^2*b^2 + 6*b^4)*e^(-7*x) - 15*(a^3*b - 3*a*b^3)*e^
(-8*x) + 15*(a^4 - 3*a^2*b^2 + 3*b^4)*e^(-9*x))/(5*b^5*e^(-2*x) + 10*b^5*e
^(-4*x) + 10*b^5*e^(-6*x) + 5*b^5*e^(-8*x) + b^5*e^(-10*x) + b^5) + x/a +
(a^5 - 3*a^3*b^2 + 3*a*b^4)*log(e^(-2*x) + 1)/b^6 - (a^6 - 3*a^4*b^2 + 3*a
^2*b^4 - b^6)*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a*b^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(113) = 226$.

Time = 0.12 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.21

$$\int \frac{\tanh^7(x)}{a + b\text{sech}(x)}\,dx = \frac{(a^5 - 3a^3b^2 + 3ab^4)\log\left(e^{(-x)} + e^x\right)}{b^6} - \frac{(a^6 - 3a^4b^2 + 3a^2b^4 - b^6)\log\left(\left|a\left(e^{(-x)} + e^x\right) + 2b\right|\right)}{ab^6} - \frac{137a^5\left(e^{(-x)} + e^x\right)^5 - 411a^3b^2\left(e^{(-x)} + e^x\right)^5 + 411ab^4\left(e^{(-x)} + e^x\right)^5 - 120a^4b\left(e^{(-x)} + e^x\right)^4 + 360a^2b^3\left(\cdots\right.}{\cdots}$$

input
```
integrate(tanh(x)^7/(a+b*sech(x)),x, algorithm="giac")
```

output
```
(a^5 - 3*a^3*b^2 + 3*a*b^4)*log(e^(-x) + e^x)/b^6 - (a^6 - 3*a^4*b^2 + 3*a
^2*b^4 - b^6)*log(abs(a*(e^(-x) + e^x) + 2*b))/(a*b^6) - 1/60*(137*a^5*(e^
(-x) + e^x)^5 - 411*a^3*b^2*(e^(-x) + e^x)^5 + 411*a*b^4*(e^(-x) + e^x)^5
- 120*a^4*b*(e^(-x) + e^x)^4 + 360*a^2*b^3*(e^(-x) + e^x)^4 - 360*b^5*(e^(
-x) + e^x)^4 + 120*a^3*b^2*(e^(-x) + e^x)^3 - 360*a*b^4*(e^(-x) + e^x)^3 -
 160*a^2*b^3*(e^(-x) + e^x)^2 + 480*b^5*(e^(-x) + e^x)^2 + 240*a*b^4*(e^(-
x) + e^x) - 384*b^5)/(b^6*(e^(-x) + e^x)^5)
```

Mupad [B] (verification not implemented)

Time = 2.86 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.61

$$\int \frac{\tanh^7(x)}{a + b\text{sech}(x)}\,dx = \frac{\frac{8a}{b^2} - \frac{8e^x(5a^2 - 27b^2)}{15b^3}}{3e^{2x} + 3e^{4x} + e^{6x} + 1} - \frac{\frac{4a}{b^2} + \frac{64e^x}{5b}}{4e^{2x} + 6e^{4x} + 4e^{6x} + e^{8x} + 1} + \frac{\frac{8e^x(a^2 - 3b^2)}{3b^3} + \frac{2(a^4 - 5a^2b^2)}{ab^4}}{2e^{2x} + e^{4x} + 1} - \frac{x}{a} + \frac{\frac{2e^x(a^4 - 3a^2b^2 + 3b^4)}{b^5} - \frac{2(a^4 - 3a^2b^2)}{ab^4}}{e^{2x} + 1} + \frac{32e^x}{5b(5e^{2x} + 10e^{4x} + 10e^{6x} + 5e^{8x} + e^{10x} + 1)} + \frac{\ln(e^{2x} + 1)(a^5 - 3a^3b^2 + 3ab^4)}{b^6} - \frac{\ln(a + 2be^x + ae^{2x})(a^6 - 3a^4b^2 + 3a^2b^4 - b^6)}{ab^6}$$

input
```
int(tanh(x)^7/(a + b/cosh(x)),x)
```

output
```
((8*a)/b^2 - (8*exp(x)*(5*a^2 - 27*b^2))/(15*b^3))/(3*exp(2*x) + 3*exp(4*x
) + exp(6*x) + 1) - ((4*a)/b^2 + (64*exp(x))/(5*b))/(4*exp(2*x) + 6*exp(4*
x) + 4*exp(6*x) + exp(8*x) + 1) + ((8*exp(x)*(a^2 - 3*b^2))/(3*b^3) + (2*(
a^4 - 5*a^2*b^2))/(a*b^4))/(2*exp(2*x) + exp(4*x) + 1) - x/a + ((2*exp(x)*
(a^4 + 3*b^4 - 3*a^2*b^2))/b^5 - (2*(a^4 - 3*a^2*b^2))/(a*b^4))/(exp(2*x)
+ 1) + (32*exp(x))/(5*b*(5*exp(2*x) + 10*exp(4*x) + 10*exp(6*x) + 5*exp(8*
x) + exp(10*x) + 1)) + (log(exp(2*x) + 1)*(3*a*b^4 + a^5 - 3*a^3*b^2))/b^6
 - (log(a + 2*b*exp(x) + a*exp(2*x))*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/
(a*b^6)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1355, normalized size of antiderivative = 11.20

$$\int \frac{\tanh^7(x)}{a + b\mathrm{sech}(x)}\,dx = \text{Too large to display}$$

input
```
int(tanh(x)^7/(a+b*sech(x)),x)
```

output

```
(15*e**(10*x)*log(e**(2*x) + 1)*a**6 - 45*e**(10*x)*log(e**(2*x) + 1)*a**4
*b**2 + 45*e**(10*x)*log(e**(2*x) + 1)*a**2*b**4 - 15*e**(10*x)*log(e**(2*
x)*a + 2*e**x*b + a)*a**6 + 45*e**(10*x)*log(e**(2*x)*a + 2*e**x*b + a)*a*
*4*b**2 - 45*e**(10*x)*log(e**(2*x)*a + 2*e**x*b + a)*a**2*b**4 + 15*e**(1
0*x)*log(e**(2*x)*a + 2*e**x*b + a)*b**6 + 6*e**(10*x)*a**4*b**2 - 18*e**(
10*x)*a**2*b**4 - 15*e**(10*x)*b**6*x + 30*e**(9*x)*a**5*b - 90*e**(9*x)*a
**3*b**3 + 90*e**(9*x)*a*b**5 + 75*e**(8*x)*log(e**(2*x) + 1)*a**6 - 225*e
**(8*x)*log(e**(2*x) + 1)*a**4*b**2 + 225*e**(8*x)*log(e**(2*x) + 1)*a**2*
b**4 - 75*e**(8*x)*log(e**(2*x)*a + 2*e**x*b + a)*a**6 + 225*e**(8*x)*log(
e**(2*x)*a + 2*e**x*b + a)*a**4*b**2 - 225*e**(8*x)*log(e**(2*x)*a + 2*e**
x*b + a)*a**2*b**4 + 75*e**(8*x)*log(e**(2*x)*a + 2*e**x*b + a)*b**6 - 75*
e**(8*x)*b**6*x + 120*e**(7*x)*a**5*b - 320*e**(7*x)*a**3*b**3 + 240*e**(7
*x)*a*b**5 + 150*e**(6*x)*log(e**(2*x) + 1)*a**6 - 450*e**(6*x)*log(e**(2*
x) + 1)*a**4*b**2 + 450*e**(6*x)*log(e**(2*x) + 1)*a**2*b**4 - 150*e**(6*x
)*log(e**(2*x)*a + 2*e**x*b + a)*a**6 + 450*e**(6*x)*log(e**(2*x)*a + 2*e*
*x*b + a)*a**4*b**2 - 450*e**(6*x)*log(e**(2*x)*a + 2*e**x*b + a)*a**2*b**
4 + 150*e**(6*x)*log(e**(2*x)*a + 2*e**x*b + a)*b**6 - 30*e**(6*x)*a**4*b*
*2 + 30*e**(6*x)*a**2*b**4 - 150*e**(6*x)*b**6*x + 180*e**(5*x)*a**5*b - 4
60*e**(5*x)*a**3*b**3 + 396*e**(5*x)*a*b**5 + 150*e**(4*x)*log(e**(2*x) +
1)*a**6 - 450*e**(4*x)*log(e**(2*x) + 1)*a**4*b**2 + 450*e**(4*x)*log(e...
```

3.114 $\int \frac{\tanh^6(x)}{a+b\,\mathrm{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 187

$$\begin{aligned}\int \frac{\tanh^6(x)}{a+b\,\mathrm{sech}(x)}\,dx &= \frac{x}{a}-\frac{3\arctan(\sinh(x))}{8b}-\frac{(a^2-3b^2)\arctan(\sinh(x))}{2b^3}\\&\quad-\frac{(a^4-3a^2b^2+3b^4)\arctan(\sinh(x))}{b^5}\\&\quad+\frac{2(a-b)^{5/2}(a+b)^{5/2}\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab^5}\\&\quad+\frac{a\tanh(x)}{b^2}+\frac{a(a^2-3b^2)\tanh(x)}{b^4}-\frac{3\,\mathrm{sech}(x)\tanh(x)}{8b}\\&\quad-\frac{(a^2-3b^2)\,\mathrm{sech}(x)\tanh(x)}{2b^3}-\frac{\mathrm{sech}^3(x)\tanh(x)}{4b}-\frac{a\tanh^3(x)}{3b^2}\end{aligned}$$

output
```
x/a-3/8*arctan(sinh(x))/b-1/2*(a^2-3*b^2)*arctan(sinh(x))/b^3-(a^4-3*a^2*b
^2+3*b^4)*arctan(sinh(x))/b^5+2*(a-b)^(5/2)*(a+b)^(5/2)*arctan((a-b)^(1/2)
*tanh(1/2*x)/(a+b)^(1/2))/a/b^5+a*tanh(x)/b^2+a*(a^2-3*b^2)*tanh(x)/b^4-3/
8*sech(x)*tanh(x)/b-1/2*(a^2-3*b^2)*sech(x)*tanh(x)/b^3-1/4*sech(x)^3*tanh
(x)/b-1/3*a*tanh(x)^3/b^2
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.99

$$\int \frac{\tanh^6(x)}{a+b\text{sech}(x)}\,dx$$

$$= \frac{-12(8a^4-20a^2b^2+15b^4)\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{48\left(b^5\sqrt{a^2-b^2}x-2(a^2-b^2)^3\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)\right)}{a\sqrt{a^2-b^2}}+b(-12a^2b+$$

input

```
Integrate[Tanh[x]^6/(a + b*Sech[x]),x]
```

output

```
(-12*(8*a^4 - 20*a^2*b^2 + 15*b^4)*ArcTan[Tanh[x/2]] + (48*(b^5*Sqrt[a^2 -
 b^2]*x - 2*(a^2 - b^2)^3*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]]))/(
a*Sqrt[a^2 - b^2]) + b*(-12*a^2*b + 15*b^3 + 4*a*(9*a^2 - 17*b^2)*Cosh[x]
+ 3*b*(-4*a^2 + 9*b^2)*Cosh[2*x] + 12*a^3*Cosh[3*x] - 28*a*b^2*Cosh[3*x])*
Sech[x]^3*Tanh[x])/(48*b^5)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {3042, 25, 4386, 25, 3042, 25, 3376, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^6(x)}{a+b\text{sech}(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cot\left(\frac{\pi}{2}+ix\right)^6}{a+b\csc\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cot\left(ix+\frac{\pi}{2}\right)^6}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4386$$

$$-\int -\frac{\sinh(x)\tanh^5(x)}{b+a\cosh(x)}dx$$

$$\Big\downarrow \; 25$$

$$\int \frac{\sinh(x)\tanh^5(x)}{a\cosh(x)+b}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\cos\left(\frac{\pi}{2}+ix\right)^6}{\sin\left(\frac{\pi}{2}+ix\right)^5\left(b+a\sin\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\cos\left(ix+\frac{\pi}{2}\right)^6}{\sin\left(ix+\frac{\pi}{2}\right)^5\left(b+a\sin\left(ix+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \; 3376$$

$$-\int \left(\frac{\text{sech}^5(x)}{b}-\frac{a\text{sech}^4(x)}{b^2}+\frac{(a^2-3b^2)\,\text{sech}^3(x)}{b^3}+\frac{(3ab^2-a^3)\,\text{sech}^2(x)}{b^4}+\frac{(a^4-3b^2a^2+3b^4)\,\text{sech}(x)}{b^5}-\frac{1}{a}-\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{(a^2-3b^2)\arctan(\sinh(x))}{2b^3}+\frac{a(a^2-3b^2)\tanh(x)}{b^4}-\frac{(a^2-3b^2)\tanh(x)\text{sech}(x)}{2b^3}-\frac{(a^4-3a^2b^2+3b^4)\arctan(\sinh(x))}{b^5}+\frac{2(a-b)^{5/2}(a+b)^{5/2}\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab^5}-\frac{a\tanh^3(x)}{3b^2}+\frac{a\tanh(x)}{b^2}+\frac{x}{a}-\frac{3\arctan(\sinh(x))}{8b}-\frac{\tanh(x)\text{sech}^3(x)}{4b}-\frac{3\tanh(x)\text{sech}(x)}{8b}$$

input
```
Int[Tanh[x]^6/(a + b*Sech[x]),x]
```

output
```
x/a - (3*ArcTan[Sinh[x]])/(8*b) - ((a^2 - 3*b^2)*ArcTan[Sinh[x]])/(2*b^3)
- ((a^4 - 3*a^2*b^2 + 3*b^4)*ArcTan[Sinh[x]])/b^5 + (2*(a - b)^(5/2)*(a +
b)^(5/2)*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*b^5) + (a*Tanh[x]
)/b^2 + (a*(a^2 - 3*b^2)*Tanh[x])/b^4 - (3*Sech[x]*Tanh[x])/(8*b) - ((a^2
- 3*b^2)*Sech[x]*Tanh[x])/(2*b^3) - (Sech[x]^3*Tanh[x])/(4*b) - (a*Tanh[x]
^3)/(3*b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3376
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

rule 4386
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.42

method	result
default	$-\frac{2\left(\frac{\left(-a^{3}b-\frac{1}{2}a^{2}b^{2}+2ab^{3}+\frac{7}{8}b^{4}\right)\tanh\left(\frac{x}{2}\right)^{7}+\left(-3a^{3}b-\frac{1}{2}a^{2}b^{2}+\frac{15}{8}b^{4}+\frac{22}{3}ab^{3}\right)\tanh\left(\frac{x}{2}\right)^{5}+\left(\frac{1}{2}a^{2}b^{2}-\frac{15}{8}b^{4}-3a^{3}b+\frac{22}{3}ab^{3}\right)\tanh\left(\frac{x}{2}\right)^{3}+\left(-a^{3}b\cdots\right.}{\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{4}}\cdots\right.}{b^{5}}$
risch	$\frac{x}{a}-\frac{12a^{2}b\,e^{7x}-27b^{3}e^{7x}+24a^{3}e^{6x}-72a\,b^{2}e^{6x}+12a^{2}b\,e^{5x}-3\,e^{5x}b^{3}+72a^{3}e^{4x}-168a\,b^{2}e^{4x}-12a^{2}b\,e^{3x}+3b^{3}e^{3x}+72a^{3}e^{2x}-152a\,b^{2}e^{2\cdots}}{12b^{4}\left(e^{2x}+1\right)^{4}}$

input
```
int(tanh(x)^6/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/b^5*(((-a^3*b-1/2*a^2*b^2+2*a*b^3+7/8*b^4)*tanh(1/2*x)^7+(-3*a^3*b-1/2*
a^2*b^2+15/8*b^4+22/3*a*b^3)*tanh(1/2*x)^5+(1/2*a^2*b^2-15/8*b^4-3*a^3*b+2
2/3*a*b^3)*tanh(1/2*x)^3+(-a^3*b+2*a*b^3+1/2*a^2*b^2-7/8*b^4)*tanh(1/2*x))
/(tanh(1/2*x)^2+1)^4+1/8*(8*a^4-20*a^2*b^2+15*b^4)*arctan(tanh(1/2*x)))-1/
a*ln(tanh(1/2*x)-1)+1/a*ln(tanh(1/2*x)+1)+2/a*(a-b)^3*(a^3+3*a^2*b+3*a*b^2
+b^3)/b^5/((a-b)*(a+b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a-b)*(a+b))^(1/2)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2417 vs. $2(165) = 330$.

Time = 0.39 (sec) , antiderivative size = 4914, normalized size of antiderivative = 26.28

$$\int \frac{\tanh^6(x)}{a + b\text{sech}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^6/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh^6(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\tanh^6(x)}{a + b\,\text{sech}\,(x)}\,dx$$

input
```
integrate(tanh(x)**6/(a+b*sech(x)),x)
```

output
```
Integral(tanh(x)**6/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tanh^6(x)}{a + b\text{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(tanh(x)^6/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.34

$$\int \frac{\tanh^6(x)}{a + b\text{sech}(x)}\,dx$$
$$= \frac{x}{a} - \frac{(8\,a^4 - 20\,a^2b^2 + 15\,b^4)\arctan\left(e^x\right)}{4\,b^5} + \frac{2\,(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6)\arctan\left(\frac{ae^x + b}{\sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2}ab^5}$$
$$- \frac{12\,a^2be^{(7\,x)} - 27\,b^3e^{(7\,x)} + 24\,a^3e^{(6\,x)} - 72\,ab^2e^{(6\,x)} + 12\,a^2be^{(5\,x)} - 3\,b^3e^{(5\,x)} + 72\,a^3e^{(4\,x)} - 168\,ab^2e^{(4\,x)}}{12\,b^4(e^{(2\,x)} + 1}$$

input
```
integrate(tanh(x)^6/(a+b*sech(x)),x, algorithm="giac")
```

output
```
x/a - 1/4*(8*a^4 - 20*a^2*b^2 + 15*b^4)*arctan(e^x)/b^5 + 2*(a^6 - 3*a^4*b
^2 + 3*a^2*b^4 - b^6)*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)
*a*b^5) - 1/12*(12*a^2*b*e^(7*x) - 27*b^3*e^(7*x) + 24*a^3*e^(6*x) - 72*a*
b^2*e^(6*x) + 12*a^2*b*e^(5*x) - 3*b^3*e^(5*x) + 72*a^3*e^(4*x) - 168*a*b^
2*e^(4*x) - 12*a^2*b*e^(3*x) + 3*b^3*e^(3*x) + 72*a^3*e^(2*x) - 152*a*b^2*
e^(2*x) - 12*a^2*b*e^x + 27*b^3*e^x + 24*a^3 - 56*a*b^2)/(b^4*(e^(2*x) + 1
)^4)
```

Mupad [B] (verification not implemented)

Time = 8.73 (sec) , antiderivative size = 1001, normalized size of antiderivative = 5.35

$$\int \frac{\tanh^6(x)}{a + b\text{sech}(x)} \, dx = \text{Too large to display}$$

input
```
int(tanh(x)^6/(a + b/cosh(x)),x)
```

output
```
((8*a)/(3*b^2) + (6*exp(x))/b)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) -
((exp(x)*(4*a^2 - 9*b^2))/(4*b^3) + (2*(a^4 - 3*a^2*b^2))/(a*b^4))/(exp(2*
x) + 1) - ((4*a)/b^2 - (exp(x)*(4*a^2 - 13*b^2))/(2*b^3))/(2*exp(2*x) + ex
p(4*x) + 1) + x/a + (log(exp(x) - 1i)*(a^4*8i + b^4*15i - a^2*b^2*20i))/(8
*b^5) - (log(exp(x) + 1i)*(a^4*8i + b^4*15i - a^2*b^2*20i))/(8*b^5) - (4*e
xp(x))/(b*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1)) + (log(((
-(a + b)^5*(a - b)^5)^(1/2)*((128*a^12 + 64*b^12 - 834*a^2*b^10 + 2385*a^4
*b^8 - 3160*a^6*b^6 + 2240*a^8*b^4 - 832*a^10*b^2 - 900*a*b^11*exp(x) + 19
2*a^11*b*exp(x) + 3075*a^3*b^9*exp(x) - 4360*a^5*b^7*exp(x) + 3200*a^7*b^5
*exp(x) - 1216*a^9*b^3*exp(x))/(2*a^6*b^8) - ((-(a + b)^5*(a - b)^5)^(1/2)
*((4*(a^2 - b^2)*(16*a*b^4 + 16*a^5 - 32*a^3*b^2 + 32*b^5*exp(x) + 28*a^4*
b*exp(x) - 57*a^2*b^3*exp(x)))/(a^6*b^2) + (32*(-(a + b)^5*(a - b)^5)^(1/2
)*(3*a*b^2 - 2*a^3 + 4*b^3*exp(x) - 3*a^2*b*exp(x)))/(a^6*b^3)))/(a*b^5)))
/(a*b^5) - ((a^2 - b^2)^3*(8*a^4 + 15*b^4 - 20*a^2*b^2)*(30*a*b^4 + 16*a^5
 - 40*a^3*b^2 + 52*b^5*exp(x) + 28*a^4*b*exp(x) - 71*a^2*b^3*exp(x)))/(2*a
^6*b^12))*(-(a + b)^5*(a - b)^5)^(1/2))/(a*b^5) - (log(- ((-(a + b)^5*(a -
 b)^5)^(1/2)*((128*a^12 + 64*b^12 - 834*a^2*b^10 + 2385*a^4*b^8 - 3160*a^6
*b^6 + 2240*a^8*b^4 - 832*a^10*b^2 - 900*a*b^11*exp(x) + 192*a^11*b*exp(x)
 + 3075*a^3*b^9*exp(x) - 4360*a^5*b^7*exp(x) + 3200*a^7*b^5*exp(x) - 1216*
a^9*b^3*exp(x))/(2*a^6*b^8) + ((-(a + b)^5*(a - b)^5)^(1/2)*((4*(a^2 - ...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1097, normalized size of antiderivative = 5.87

$$\int \frac{\tanh^6(x)}{a + b\text{sech}(x)} \, dx = \text{Too large to display}$$

input
```
int(tanh(x)^6/(a+b*sech(x)),x)
```

output

```
( - 24*e**(8*x)*atan(e**x)*a**5 + 60*e**(8*x)*atan(e**x)*a**3*b**2 - 45*e*
*(8*x)*atan(e**x)*a*b**4 - 96*e**(6*x)*atan(e**x)*a**5 + 240*e**(6*x)*atan
(e**x)*a**3*b**2 - 180*e**(6*x)*atan(e**x)*a*b**4 - 144*e**(4*x)*atan(e**x
)*a**5 + 360*e**(4*x)*atan(e**x)*a**3*b**2 - 270*e**(4*x)*atan(e**x)*a*b**
4 - 96*e**(2*x)*atan(e**x)*a**5 + 240*e**(2*x)*atan(e**x)*a**3*b**2 - 180*
e**(2*x)*atan(e**x)*a*b**4 - 24*atan(e**x)*a**5 + 60*atan(e**x)*a**3*b**2
- 45*atan(e**x)*a*b**4 + 24*e**(8*x)*sqrt(a**2 - b**2)*atan((e**x*a + b)/s
qrt(a**2 - b**2))*a**4 - 48*e**(8*x)*sqrt(a**2 - b**2)*atan((e**x*a + b)/s
qrt(a**2 - b**2))*a**2*b**2 + 24*e**(8*x)*sqrt(a**2 - b**2)*atan((e**x*a +
 b)/sqrt(a**2 - b**2))*b**4 + 96*e**(6*x)*sqrt(a**2 - b**2)*atan((e**x*a +
 b)/sqrt(a**2 - b**2))*a**4 - 192*e**(6*x)*sqrt(a**2 - b**2)*atan((e**x*a
+ b)/sqrt(a**2 - b**2))*a**2*b**2 + 96*e**(6*x)*sqrt(a**2 - b**2)*atan((e*
*x*a + b)/sqrt(a**2 - b**2))*b**4 + 144*e**(4*x)*sqrt(a**2 - b**2)*atan((e
**x*a + b)/sqrt(a**2 - b**2))*a**4 - 288*e**(4*x)*sqrt(a**2 - b**2)*atan((
e**x*a + b)/sqrt(a**2 - b**2))*a**2*b**2 + 144*e**(4*x)*sqrt(a**2 - b**2)*
atan((e**x*a + b)/sqrt(a**2 - b**2))*b**4 + 96*e**(2*x)*sqrt(a**2 - b**2)*
atan((e**x*a + b)/sqrt(a**2 - b**2))*a**4 - 192*e**(2*x)*sqrt(a**2 - b**2)
*atan((e**x*a + b)/sqrt(a**2 - b**2))*a**2*b**2 + 96*e**(2*x)*sqrt(a**2 -
b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2))*b**4 + 24*sqrt(a**2 - b**2)*ata
n((e**x*a + b)/sqrt(a**2 - b**2))*a**4 - 48*sqrt(a**2 - b**2)*atan((e**...
```

3.115 $\int \frac{\tanh^5(x)}{a+b\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int \frac{\tanh^5(x)}{a+b\text{sech}(x)}\,dx = \frac{\log(\cosh(x))}{a} + \frac{(a^2-b^2)^2\log(a+b\text{sech}(x))}{ab^4} - \frac{(a^2-2b^2)\,\text{sech}(x)}{b^3} + \frac{a\text{sech}^2(x)}{2b^2} - \frac{\text{sech}^3(x)}{3b}$$

output
```
ln(cosh(x))/a+(a^2-b^2)^2*ln(a+b*sech(x))/a/b^4-(a^2-2*b^2)*sech(x)/b^3+1/
2*a*sech(x)^2/b^2-1/3*sech(x)^3/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int \frac{\tanh^5(x)}{a+b\text{sech}(x)}\,dx = -\frac{-\frac{b^4\log(\cosh(x))}{a} - \frac{(a^2-b^2)^2\log(a+b\text{sech}(x))}{a} + b(a^2-2b^2)\,\text{sech}(x) - \frac{1}{2}ab^2\text{sech}^2(x) + \frac{1}{3}b^3\text{sech}^3(x)}{b^4}$$

input
```
Integrate[Tanh[x]^5/(a + b*Sech[x]),x]
```

output
```
-((-((b^4*Log[Cosh[x]])/a) - ((a^2 - b^2)^2*Log[a + b*Sech[x]])/a + b*(a^2
 - 2*b^2)*Sech[x] - (a*b^2*Sech[x]^2)/2 + (b^3*Sech[x]^3)/3)/b^4)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 26, 4373, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(x)}{a+b\text{sech}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cot\left(\frac{\pi}{2}+ix\right)^5}{a+b\csc\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cot\left(ix+\frac{\pi}{2}\right)^5}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4373}$$

$$-\frac{\int \frac{\cosh(x)\left(b^2-b^2\text{sech}^2(x)\right)^2}{b(a+b\text{sech}(x))}d(b\text{sech}(x))}{b^4}$$

$$\Big\downarrow \text{522}$$

$$-\frac{\int \left(\frac{\cosh(x)b^3}{a}+\text{sech}^2(x)b^2-a\text{sech}(x)b+a^2\left(1-\frac{2b^2}{a^2}\right)-\frac{\left(a^2-b^2\right)^2}{a(a+b\text{sech}(x))}\right)d(b\text{sech}(x))}{b^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(a^2-2b^2\right)\text{sech}(x)-\frac{\left(a^2-b^2\right)^2\log(a+b\text{sech}(x))}{a}+\frac{b^4\log(b\text{sech}(x))}{a}-\frac{1}{2}ab^2\text{sech}^2(x)+\frac{1}{3}b^3\text{sech}^3(x)}{b^4}$$

input
```
Int[Tanh[x]^5/(a + b*Sech[x]),x]
```

output
```
-(((b^4*Log[b*Sech[x]])/a - ((a^2 - b^2)^2*Log[a + b*Sech[x]])/a + b*(a^2
- 2*b^2)*Sech[x] - (a*b^2*Sech[x]^2)/2 + (b^3*Sech[x]^3)/3)/b^4)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(68) = 136$.

Time = 1.00 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.19

method	result
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{\frac{8b^{3}}{3\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{3}}+a\left(a^{2}-2b^{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)-\frac{2b^{2}(a+2b)}{\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{2}}+\frac{2b\left(a^{2}+ab-b^{2}\right)}{\tanh\left(\frac{x}{2}\right)^{2}+1}}{b^{4}}+$
risch	$-\frac{x}{a}-\frac{2\,\mathrm{e}^{x}\left(3\,\mathrm{e}^{4x}a^{2}-6b^{2}\mathrm{e}^{4x}-3ab\,\mathrm{e}^{3x}+6\,\mathrm{e}^{2x}a^{2}-8b^{2}\mathrm{e}^{2x}-3a\,\mathrm{e}^{x}b+3a^{2}-6b^{2}\right)}{3b^{3}\left(\mathrm{e}^{2x}+1\right)^{3}}-\frac{a^{3}\ln\left(\mathrm{e}^{2x}+1\right)}{b^{4}}+\frac{2a\ln\left(\mathrm{e}^{2x}+1\right)}{b^{2}}+\frac{a^{3}\ln\left(\mathrm{e}^{2x}+\right.}{b^{4}}$

input
```
int(tanh(x)^5/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/a*ln(tanh(1/2*x)-1)-1/a*ln(tanh(1/2*x)+1)-1/b^4*(8/3*b^3/(tanh(1/2*x)^2
+1)^3+a*(a^2-2*b^2)*ln(tanh(1/2*x)^2+1)-2*b^2*(a+2*b)/(tanh(1/2*x)^2+1)^2+
2*b*(a^2+a*b-b^2)/(tanh(1/2*x)^2+1))+(a-b)^2*(a^2+2*a*b+b^2)/b^4/a*ln(a*ta
nh(1/2*x)^2-b*tanh(1/2*x)^2+a+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1280 vs. $2(68) = 136$.

Time = 0.12 (sec) , antiderivative size = 1280, normalized size of antiderivative = 17.78

$$\int \frac{\tanh^5(x)}{a + b\text{sech}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^5/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
-1/3*(3*b^4*x*cosh(x)^6 + 3*b^4*x*sinh(x)^6 + 6*(a^3*b - 2*a*b^3)*cosh(x)^
5 + 6*(3*b^4*x*cosh(x) + a^3*b - 2*a*b^3)*sinh(x)^5 + 3*b^4*x + 3*(3*b^4*x
 - 2*a^2*b^2)*cosh(x)^4 + 3*(15*b^4*x*cosh(x)^2 + 3*b^4*x - 2*a^2*b^2 + 10
*(a^3*b - 2*a*b^3)*cosh(x))*sinh(x)^4 + 4*(3*a^3*b - 4*a*b^3)*cosh(x)^3 +
4*(15*b^4*x*cosh(x)^3 + 3*a^3*b - 4*a*b^3 + 15*(a^3*b - 2*a*b^3)*cosh(x)^2
 + 3*(3*b^4*x - 2*a^2*b^2)*cosh(x))*sinh(x)^3 + 3*(3*b^4*x - 2*a^2*b^2)*co
sh(x)^2 + 3*(15*b^4*x*cosh(x)^4 + 3*b^4*x - 2*a^2*b^2 + 20*(a^3*b - 2*a*b^
3)*cosh(x)^3 + 6*(3*b^4*x - 2*a^2*b^2)*cosh(x)^2 + 4*(3*a^3*b - 4*a*b^3)*c
osh(x))*sinh(x)^2 + 6*(a^3*b - 2*a*b^3)*cosh(x) - 3*((a^4 - 2*a^2*b^2 + b^
4)*cosh(x)^6 + 6*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^5 + (a^4 - 2*a^2*
b^2 + b^4)*sinh(x)^6 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 3*(a^4 - 2*a^
2*b^2 + b^4 + 5*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 + a^4 - 2*a^2
*b^2 + b^4 + 4*(5*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2 +
 b^4)*cosh(x))*sinh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 + 3*(5*(a^4
 - 2*a^2*b^2 + b^4)*cosh(x)^4 + a^4 - 2*a^2*b^2 + b^4 + 6*(a^4 - 2*a^2*b^2
 + b^4)*cosh(x)^2)*sinh(x)^2 + 6*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 2*(a
^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)
)*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) + 3*((a^4 - 2*a^2*b^2)*cosh(x
)^6 + 6*(a^4 - 2*a^2*b^2)*cosh(x)*sinh(x)^5 + (a^4 - 2*a^2*b^2)*sinh(x)^6
+ 3*(a^4 - 2*a^2*b^2)*cosh(x)^4 + 3*(a^4 - 2*a^2*b^2 + 5*(a^4 - 2*a^2*b...
```

Sympy [F]

$$\int \frac{\tanh^5(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\tanh^5(x)}{a + b\,\text{sech}(x)}\,dx$$

input
```
integrate(tanh(x)**5/(a+b*sech(x)),x)
```

output
```
Integral(tanh(x)**5/(a + b*sech(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(68) = 136$.

Time = 0.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.28

$$\int \frac{\tanh^5(x)}{a + b\mathrm{sech}(x)}\,dx = \frac{2\left(3\,abe^{(-2\,x)} + 3\,abe^{(-4\,x)} - 3\,(a^2 - 2\,b^2)e^{(-x)} - 2\,(3\,a^2 - 4\,b^2)e^{(-3\,x)} - 3\,(a^2 - 2\,b^2)e^{(-5\,x)}\right)}{3\left(3\,b^3e^{(-2\,x)} + 3\,b^3e^{(-4\,x)} + b^3e^{(-6\,x)} + b^3\right)} + \frac{x}{a} - \frac{(a^3 - 2\,ab^2)\log\left(e^{(-2\,x)} + 1\right)}{b^4} + \frac{(a^4 - 2\,a^2b^2 + b^4)\log\left(2\,be^{(-x)} + ae^{(-2\,x)} + a\right)}{ab^4}$$

input
```
integrate(tanh(x)^5/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
2/3*(3*a*b*e^(-2*x) + 3*a*b*e^(-4*x) - 3*(a^2 - 2*b^2)*e^(-x) - 2*(3*a^2 -
 4*b^2)*e^(-3*x) - 3*(a^2 - 2*b^2)*e^(-5*x))/(3*b^3*e^(-2*x) + 3*b^3*e^(-4
*x) + b^3*e^(-6*x) + b^3) + x/a - (a^3 - 2*a*b^2)*log(e^(-2*x) + 1)/b^4 +
(a^4 - 2*a^2*b^2 + b^4)*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(68) = 136$.

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.11

$$\int \frac{\tanh^5(x)}{a + b\mathrm{sech}(x)}\,dx = -\frac{(a^3 - 2\,ab^2)\log\left(e^{(-x)} + e^x\right)}{b^4} + \frac{(a^4 - 2\,a^2b^2 + b^4)\log\left(\left|a\left(e^{(-x)} + e^x\right) + 2\,b\right|\right)}{ab^4} + \frac{11\,a^3\left(e^{(-x)} + e^x\right)^3 - 22\,ab^2\left(e^{(-x)} + e^x\right)^3 - 12\,a^2b\left(e^{(-x)} + e^x\right)^2 + 24\,b^3\left(e^{(-x)} + e^x\right)^2 + 12\,ab^2\left(e^{(-x)} + e\right.}{6\,b^4\left(e^{(-x)} + e^x\right)^3}$$

input
```
integrate(tanh(x)^5/(a+b*sech(x)),x, algorithm="giac")
```

output
```
-(a^3 - 2*a*b^2)*log(e^(-x) + e^x)/b^4 + (a^4 - 2*a^2*b^2 + b^4)*log(abs(a
*(e^(-x) + e^x) + 2*b))/(a*b^4) + 1/6*(11*a^3*(e^(-x) + e^x)^3 - 22*a*b^2*
(e^(-x) + e^x)^3 - 12*a^2*b*(e^(-x) + e^x)^2 + 24*b^3*(e^(-x) + e^x)^2 + 1
2*a*b^2*(e^(-x) + e^x) - 16*b^3)/(b^4*(e^(-x) + e^x)^3)
```

Mupad [B] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.15

$$\int \frac{\tanh^5(x)}{a+b\text{sech}(x)}\,dx = \frac{\frac{2\,a}{b^2}-\frac{2\,e^x\,\left(a^2-2\,b^2\right)}{b^3}}{e^{2\,x}+1}-\frac{x}{a}-\frac{\frac{2\,a}{b^2}+\frac{8\,e^x}{3\,b}}{2\,e^{2\,x}+e^{4\,x}+1} + \frac{\ln\left(e^{2\,x}+1\right)\,\left(2\,a\,b^2-a^3\right)}{b^4}+\frac{8\,e^x}{3\,b\,\left(3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1\right)} + \frac{\ln\left(a+2\,b\,e^x+a\,e^{2\,x}\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}{a\,b^4}$$

input
```
int(tanh(x)^5/(a + b/cosh(x)),x)
```

output
```
((2*a)/b^2 - (2*exp(x)*(a^2 - 2*b^2))/b^3)/(exp(2*x) + 1) - x/a - ((2*a)/b
^2 + (8*exp(x))/(3*b))/(2*exp(2*x) + exp(4*x) + 1) + (log(exp(2*x) + 1)*(2
*a*b^2 - a^3))/b^4 + (8*exp(x))/(3*b*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) +
 1)) + (log(a + 2*b*exp(x) + a*exp(2*x))*(a^4 + b^4 - 2*a^2*b^2))/(a*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 609, normalized size of antiderivative = 8.46

$$\int \frac{\tanh^5(x)}{a+b\text{sech}(x)}\,dx = \frac{6e^{6x}\log(e^{2x}+1)\,a^2b^2-6e^{6x}\log(e^{2x}a+2e^xb+a)\,a^2b^2+18e^{4x}\log(e^{2x}+1)\,a^2b^2-18e^{4x}\log(e^{2x}a+2e^xb+}{}$$

input
```
int(tanh(x)^5/(a+b*sech(x)),x)
```

output
```
( - 3*e**(6*x)*log(e**(2*x) + 1)*a**4 + 6*e**(6*x)*log(e**(2*x) + 1)*a**2*
b**2 + 3*e**(6*x)*log(e**(2*x)*a + 2*e**x*b + a)*a**4 - 6*e**(6*x)*log(e**
(2*x)*a + 2*e**x*b + a)*a**2*b**2 + 3*e**(6*x)*log(e**(2*x)*a + 2*e**x*b +
 a)*b**4 - 2*e**(6*x)*a**2*b**2 - 3*e**(6*x)*b**4*x - 6*e**(5*x)*a**3*b +
12*e**(5*x)*a*b**3 - 9*e**(4*x)*log(e**(2*x) + 1)*a**4 + 18*e**(4*x)*log(e
**(2*x) + 1)*a**2*b**2 + 9*e**(4*x)*log(e**(2*x)*a + 2*e**x*b + a)*a**4 -
18*e**(4*x)*log(e**(2*x)*a + 2*e**x*b + a)*a**2*b**2 + 9*e**(4*x)*log(e**(
2*x)*a + 2*e**x*b + a)*b**4 - 9*e**(4*x)*b**4*x - 12*e**(3*x)*a**3*b + 16*
e**(3*x)*a*b**3 - 9*e**(2*x)*log(e**(2*x) + 1)*a**4 + 18*e**(2*x)*log(e**(
2*x) + 1)*a**2*b**2 + 9*e**(2*x)*log(e**(2*x)*a + 2*e**x*b + a)*a**4 - 18*
e**(2*x)*log(e**(2*x)*a + 2*e**x*b + a)*a**2*b**2 + 9*e**(2*x)*log(e**(2*x
)*a + 2*e**x*b + a)*b**4 - 9*e**(2*x)*b**4*x - 6*e**x*a**3*b + 12*e**x*a*b
**3 - 3*log(e**(2*x) + 1)*a**4 + 6*log(e**(2*x) + 1)*a**2*b**2 + 3*log(e**
(2*x)*a + 2*e**x*b + a)*a**4 - 6*log(e**(2*x)*a + 2*e**x*b + a)*a**2*b**2
+ 3*log(e**(2*x)*a + 2*e**x*b + a)*b**4 - 2*a**2*b**2 - 3*b**4*x)/(3*a*b**
4*(e**(6*x) + 3*e**(4*x) + 3*e**(2*x) + 1))
```

3.116 $\int \frac{\tanh^4(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 94

$$\int \frac{\tanh^4(x)}{a+b\text{sech}(x)}\,dx = \frac{x}{a}+\frac{(2a^2-3b^2)\arctan(\sinh(x))}{2b^3} - \frac{2(a-b)^{3/2}(a+b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab^3} - \frac{a\tanh(x)}{b^2}+\frac{\text{sech}(x)\tanh(x)}{2b}$$

output
```
x/a+1/2*(2*a^2-3*b^2)*arctan(sinh(x))/b^3-2*(a-b)^(3/2)*(a+b)^(3/2)*arctan
((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a/b^3-a*tanh(x)/b^2+1/2*sech(x)*tanh
(x)/b
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.20

$$\int \frac{\tanh^4(x)}{a+b\text{sech}(x)}\,dx = \frac{(b+a\cosh(x))\text{sech}^2(x)\left(2\left(b^3x+a(2a^2-3b^2)\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+2(a^2-b^2)^{3/2}\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)\right.\right.}{2ab^3(a+b\text{sech}(x))}$$

input
```
Integrate[Tanh[x]^4/(a + b*Sech[x]),x]
```

output
```
((b + a*Cosh[x])*Sech[x]^2*(2*(b^3*x + a*(2*a^2 - 3*b^2)*ArcTan[Tanh[x/2]]
 + 2*(a^2 - b^2)^(3/2)*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])*Cosh[
x] + a*b*(-2*a*Sinh[x] + b*Tanh[x])))/(2*a*b^3*(a + b*Sech[x]))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {3042, 4386, 3042, 3372, 25, 3042, 3536, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(x)}{a+b\text{sech}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot\left(\frac{\pi}{2}+ix\right)^4}{a+b\csc\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{4386}$$

$$\int \frac{\sinh(x)\tanh^3(x)}{a\cosh(x)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(\frac{\pi}{2}+ix\right)^4}{\sin\left(\frac{\pi}{2}+ix\right)^3\left(b+a\sin\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3372}$$

$$-\frac{\int -\frac{\left(2a^2+b\cosh(x)a-3b^2+2b^2\cosh^2(x)\right)\text{sech}(x)}{b+a\cosh(x)}dx}{2b^2}-\frac{a\tanh(x)}{b^2}+\frac{\tanh(x)\text{sech}(x)}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(2a^2+b\cosh(x)a-3b^2+2b^2\cosh^2(x)\right)\text{sech}(x)}{b+a\cosh(x)}dx}{2b^2}-\frac{a\tanh(x)}{b^2}+\frac{\tanh(x)\text{sech}(x)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2a^2+b\sin\left(ix+\frac{\pi}{2}\right)a-3b^2+2b^2\sin\left(ix+\frac{\pi}{2}\right)^2}{\sin\left(ix+\frac{\pi}{2}\right)\left(b+a\sin\left(ix+\frac{\pi}{2}\right)\right)}dx}{2b^2}-\frac{a\tanh(x)}{b^2}+\frac{\tanh(x)\text{sech}(x)}{2b}$$

$$\Big\downarrow \text{3536}$$

$$\frac{-\frac{2(a^2-b^2)^2\int\frac{1}{b+a\cosh(x)}dx}{ab}+\frac{(2a^2-3b^2)\int\text{sech}(x)dx}{b}+\frac{2b^2x}{a}}{2b^2}-\frac{a\tanh(x)}{b^2}+\frac{\tanh(x)\text{sech}(x)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{2(a^2-b^2)^2\int\frac{1}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx}{ab}+\frac{(2a^2-3b^2)\int\csc\left(ix+\frac{\pi}{2}\right)dx}{b}+\frac{2b^2x}{a}}{2b^2}-\frac{a\tanh(x)}{b^2}+\frac{\tanh(x)\text{sech}(x)}{2b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{(2a^2-3b^2)\int\csc\left(ix+\frac{\pi}{2}\right)dx}{b}-\frac{4(a^2-b^2)^2\int\frac{1}{(a-b)\tanh^2\left(\frac{x}{2}\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{ab}+\frac{2b^2x}{a}}{2b^2}-\frac{a\tanh(x)}{b^2}+\frac{\tanh(x)\text{sech}(x)}{2b}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(2a^2-3b^2)\int\csc\left(ix+\frac{\pi}{2}\right)dx}{b}-\frac{4(a^2-b^2)^2\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab\sqrt{a-b}\sqrt{a+b}}+\frac{2b^2x}{a}}{2b^2}-\frac{a\tanh(x)}{b^2}+\frac{\tanh(x)\text{sech}(x)}{2b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{(2a^2-3b^2)\arctan(\sinh(x))}{b}-\frac{4(a^2-b^2)^2\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab\sqrt{a-b}\sqrt{a+b}}+\frac{2b^2x}{a}}{2b^2}-\frac{a\tanh(x)}{b^2}+\frac{\tanh(x)\text{sech}(x)}{2b}$$

input
```
Int[Tanh[x]^4/(a + b*Sech[x]),x]
```

output
```
((2*b^2*x)/a + ((2*a^2 - 3*b^2)*ArcTan[Sinh[x]])/b - (4*(a^2 - b^2)^2*ArcT
an[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*b*Sqrt[a + b]))/(2
*b^2) - (a*Tanh[x])/b^2 + (Sech[x]*Tanh[x])/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3372
```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Si
mp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x]
)^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x] - Simp[1/(a^2*d^2*(n + 1)*(n + 2
))   Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])^(n + 2)*Simp[a^2*n*(n + 2)
 - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f*x] - (a^2*(n + 1)*(n + 2)
- b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x], x], x]) /; FreeQ[{a, b,
d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
 &&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

rule 3536
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[C*(x/(b*d)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d))   Int[1/(a + b*Sin[e + f*x]), x], x] - Simp[(c^2*C - B*c*d
 + A*d^2)/(d*(b*c - a*d))   Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a
, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4386
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.62

method	result
default	$-\frac{2(a-b)^2(a^2+2ab+b^2)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a\,b^3\sqrt{(a-b)(a+b)}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}+\frac{\frac{2\left(\left(-ab-\frac{1}{2}b^2\right)\tanh\left(\frac{x}{2}\right)^3+\left(-ab+\frac{1}{2}b^2\right)\tanh\left(\frac{x}{2}\right)\right)}{\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^2}+(2a^2-}{b^3}$
risch	$\frac{x}{a}+\frac{b\,e^{3x}+2\,e^{2x}a-b\,e^{x}+2a}{(e^{2x}+1)^2b^2}+\frac{i\ln(e^x+i)a^2}{b^3}-\frac{3i\ln(e^x+i)}{2b}-\frac{i\ln(e^x-i)a^2}{b^3}+\frac{3i\ln(e^x-i)}{2b}+\frac{\sqrt{-a^2+b^2}\,a\ln\left(e^x-\frac{\sqrt{-a^2+b^2}-}{a}\right.}{b^3}$

input
```
int(tanh(x)^4/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/a*(a-b)^2*(a^2+2*a*b+b^2)/b^3/((a-b)*(a+b))^(1/2)*arctan((a-b)*tanh(1/2
*x)/((a-b)*(a+b))^(1/2))+1/a*ln(tanh(1/2*x)+1)+2/b^3*(((-a*b-1/2*b^2)*tanh
(1/2*x)^3+(-a*b+1/2*b^2)*tanh(1/2*x))/(tanh(1/2*x)^2+1)^2+1/2*(2*a^2-3*b^2
)*arctan(tanh(1/2*x)))-1/a*ln(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(80) = 160$.

Time = 0.21 (sec) , antiderivative size = 1254, normalized size of antiderivative = 13.34

$$\int \frac{\tanh^4(x)}{a+b\operatorname{sech}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
[(b^3*x*cosh(x)^4 + b^3*x*sinh(x)^4 + a*b^2*cosh(x)^3 + b^3*x - a*b^2*cosh
(x) + (4*b^3*x*cosh(x) + a*b^2)*sinh(x)^3 + 2*a^2*b + 2*(b^3*x + a^2*b)*co
sh(x)^2 + (6*b^3*x*cosh(x)^2 + 2*b^3*x + 3*a*b^2*cosh(x) + 2*a^2*b)*sinh(x
)^2 - ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^2 - b^
2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)^2 + a^2
- b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 + (a^2 - b^2)*cosh
(x))*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*
cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)
*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2
*(a*cosh(x) + b)*sinh(x) + a)) + ((2*a^3 - 3*a*b^2)*cosh(x)^4 + 4*(2*a^3 -
 3*a*b^2)*cosh(x)*sinh(x)^3 + (2*a^3 - 3*a*b^2)*sinh(x)^4 + 2*a^3 - 3*a*b^
2 + 2*(2*a^3 - 3*a*b^2)*cosh(x)^2 + 2*(2*a^3 - 3*a*b^2 + 3*(2*a^3 - 3*a*b^
2)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 - 3*a*b^2)*cosh(x)^3 + (2*a^3 - 3*a*b^
2)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) + (4*b^3*x*cosh(x)^3 + 3*a*
b^2*cosh(x)^2 - a*b^2 + 4*(b^3*x + a^2*b)*cosh(x))*sinh(x))/(a*b^3*cosh(x)
^4 + 4*a*b^3*cosh(x)*sinh(x)^3 + a*b^3*sinh(x)^4 + 2*a*b^3*cosh(x)^2 + a*b
^3 + 2*(3*a*b^3*cosh(x)^2 + a*b^3)*sinh(x)^2 + 4*(a*b^3*cosh(x)^3 + a*b^3*
cosh(x))*sinh(x)), (b^3*x*cosh(x)^4 + b^3*x*sinh(x)^4 + a*b^2*cosh(x)^3 +
b^3*x - a*b^2*cosh(x) + (4*b^3*x*cosh(x) + a*b^2)*sinh(x)^3 + 2*a^2*b + 2*
(b^3*x + a^2*b)*cosh(x)^2 + (6*b^3*x*cosh(x)^2 + 2*b^3*x + 3*a*b^2*cosh...
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\tanh^4{(x)}}{a + b\operatorname{sech}{(x)}}\,dx$$

input
```
integrate(tanh(x)**4/(a+b*sech(x)),x)
```

output
```
Integral(tanh(x)**4/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tanh^4(x)}{a + b\text{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(tanh(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.18

$$\int \frac{\tanh^4(x)}{a + b\text{sech}(x)}\,dx = \frac{x}{a} + \frac{(2\,a^2 - 3\,b^2)\arctan(e^x)}{b^3} - \frac{2\,(a^4 - 2\,a^2b^2 + b^4)\arctan\left(\frac{ae^x + b}{\sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2}ab^3} + \frac{be^{(3\,x)} + 2\,ae^{(2\,x)} - be^x + 2\,a}{b^2\left(e^{(2\,x)} + 1\right)^2}$$

input
```
integrate(tanh(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

output
```
x/a + (2*a^2 - 3*b^2)*arctan(e^x)/b^3 - 2*(a^4 - 2*a^2*b^2 + b^4)*arctan((
a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a*b^3) + (b*e^(3*x) + 2*a*e^(
2*x) - b*e^x + 2*a)/(b^2*(e^(2*x) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 7.56 (sec) , antiderivative size = 700, normalized size of antiderivative = 7.45

$$\int \frac{\tanh^4(x)}{a + b\text{sech}(x)}\,dx = \text{Too large to display}$$

input
```
int(tanh(x)^4/(a + b/cosh(x)),x)
```

output
```
((2*a)/b^2 + exp(x)/b)/(exp(2*x) + 1) + x/a - (log(exp(x) - 1i)*(a^2*2i -
b^2*3i))/(2*b^3) + (log(exp(x) + 1i)*(a^2*2i - b^2*3i))/(2*b^3) - (2*exp(x
))/(b*(2*exp(2*x) + exp(4*x) + 1)) + (log((((64*a^8 + 32*b^8 - 272*a^2*b^6
 + 456*a^4*b^4 - 288*a^6*b^2 - 288*a*b^7*exp(x) + 96*a^7*b*exp(x) + 600*a^
3*b^5*exp(x) - 416*a^5*b^3*exp(x))/(a^6*b^4) - (((16*(a^2 - b^2)*(4*a*b^2
- 4*a^3 + 8*b^3*exp(x) - 7*a^2*b*exp(x)))/a^6 + (32*(-(a + b)^3*(a - b)^3)
^(1/2)*(3*a*b^2 - 2*a^3 + 4*b^3*exp(x) - 3*a^2*b*exp(x)))/(a^6*b))*(-(a +
b)^3*(a - b)^3)^(1/2))/(a*b^3))*(-(a + b)^3*(a - b)^3)^(1/2))/(a*b^3) - (8
*(a^2 - b^2)^2*(2*a^2 - 3*b^2)*(6*a*b^2 - 4*a^3 + 10*b^3*exp(x) - 7*a^2*b*
exp(x)))/(a^6*b^6))*(-(a + b)^3*(a - b)^3)^(1/2))/(a*b^3) - (log(- (((64*a
^8 + 32*b^8 - 272*a^2*b^6 + 456*a^4*b^4 - 288*a^6*b^2 - 288*a*b^7*exp(x) +
 96*a^7*b*exp(x) + 600*a^3*b^5*exp(x) - 416*a^5*b^3*exp(x))/(a^6*b^4) + ((
(16*(a^2 - b^2)*(4*a*b^2 - 4*a^3 + 8*b^3*exp(x) - 7*a^2*b*exp(x)))/a^6 - (
32*(-(a + b)^3*(a - b)^3)^(1/2)*(3*a*b^2 - 2*a^3 + 4*b^3*exp(x) - 3*a^2*b*
exp(x)))/(a^6*b))*(-(a + b)^3*(a - b)^3)^(1/2))/(a*b^3))*(-(a + b)^3*(a -
b)^3)^(1/2))/(a*b^3) - (8*(a^2 - b^2)^2*(2*a^2 - 3*b^2)*(6*a*b^2 - 4*a^3 +
 10*b^3*exp(x) - 7*a^2*b*exp(x)))/(a^6*b^6))*(-(a + b)^3*(a - b)^3)^(1/2))
/(a*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 398, normalized size of antiderivative = 4.23

$$\int \frac{\tanh^4(x)}{a + b\text{sech}(x)}\,dx = \frac{2e^{4x}atan(e^x)\,a^3 - 3e^{4x}atan(e^x)\,a\,b^2 + 4e^{2x}atan(e^x)\,a^3 - 6e^{2x}atan(e^x)\,a\,b^2 + 2atan(e^x)\,a^3 - 3atan(e^x)\,a}{}$$

input
```
int(tanh(x)^4/(a+b*sech(x)),x)
```

output

```
(2*e**(4*x)*atan(e**x)*a**3 - 3*e**(4*x)*atan(e**x)*a*b**2 + 4*e**(2*x)*at
an(e**x)*a**3 - 6*e**(2*x)*atan(e**x)*a*b**2 + 2*atan(e**x)*a**3 - 3*atan(
e**x)*a*b**2 - 2*e**(4*x)*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 -
b**2))*a**2 + 2*e**(4*x)*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b
**2))*b**2 - 4*e**(2*x)*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b*
*2))*a**2 + 4*e**(2*x)*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**
2))*b**2 - 2*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2))*a**2 +
 2*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2))*b**2 - e**(4*x)*
a**2*b + e**(4*x)*b**3*x + e**(3*x)*a*b**2 + 2*e**(2*x)*b**3*x - e**x*a*b*
*2 + a**2*b + b**3*x)/(a*b**3*(e**(4*x) + 2*e**(2*x) + 1))
```

3.117 $\int \frac{\tanh^3(x)}{a+b\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{\tanh^3(x)}{a+b\operatorname{sech}(x)}\,dx=\frac{\log(\cosh(x))}{a}+\frac{\left(1-\frac{a^2}{b^2}\right)\log(a+b\operatorname{sech}(x))}{a}+\frac{\operatorname{sech}(x)}{b}$$

output
```
ln(cosh(x))/a+(1-a^2/b^2)*ln(a+b*sech(x))/a+sech(x)/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{\tanh^3(x)}{a+b\operatorname{sech}(x)}\,dx=\frac{\log(\cosh(x))}{a}+\frac{\log(a+b\operatorname{sech}(x))}{a}-\frac{a\log(a+b\operatorname{sech}(x))}{b^2}+\frac{\operatorname{sech}(x)}{b}$$

input
```
Integrate[Tanh[x]^3/(a + b*Sech[x]),x]
```

output
```
Log[Cosh[x]]/a + Log[a + b*Sech[x]]/a - (a*Log[a + b*Sech[x]])/b^2 + Sech[
x]/b
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 26, 4373, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{a+b\text{sech}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cot\left(\frac{\pi}{2}+ix\right)^3}{a+b\csc\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cot\left(ix+\frac{\pi}{2}\right)^3}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4373}$$

$$-\frac{\int \frac{\cosh(x)\left(b^2-b^2\text{sech}^2(x)\right)}{b(a+b\text{sech}(x))}d(b\text{sech}(x))}{b^2}$$

$$\Big\downarrow \text{522}$$

$$-\frac{\int \left(\frac{a^2-b^2}{a(a+b\text{sech}(x))}+\frac{b\cosh(x)}{a}-1\right)d(b\text{sech}(x))}{b^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{(a^2-b^2)\log(a+b\text{sech}(x))}{a}+\frac{b^2\log(b\text{sech}(x))}{a}-b\text{sech}(x)}{b^2}$$

input
```
Int[Tanh[x]^3/(a + b*Sech[x]),x]
```

output
```
-(((b^2*Log[b*Sech[x]])/a + ((a^2 - b^2)*Log[a + b*Sech[x]])/a - b*Sech[x]
)/b^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(35) = 70$.

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.14

method	result
risch	$-\frac{x}{a}+\frac{2\,\mathrm{e}^{x}}{b\left(\mathrm{e}^{2x}+1\right)}+\frac{a\ln\left(\mathrm{e}^{2x}+1\right)}{b^{2}}-\frac{a\ln\left(\mathrm{e}^{2x}+\frac{2b\,\mathrm{e}^{x}}{a}+1\right)}{b^{2}}+\frac{\ln\left(\mathrm{e}^{2x}+\frac{2b\,\mathrm{e}^{x}}{a}+1\right)}{a}$
default	$\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)+\frac{2b}{\tanh\left(\frac{x}{2}\right)^{2}+1}}{b^{2}}-\frac{(a-b)(a+b)\ln\left(a\tanh\left(\frac{x}{2}\right)^{2}-b\tanh\left(\frac{x}{2}\right)^{2}+a+b\right)}{a\,b^{2}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$

input
```
int(tanh(x)^3/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
-x/a+2/b*exp(x)/(exp(2*x)+1)+a/b^2*ln(exp(2*x)+1)-a/b^2*ln(exp(2*x)+2*b/a*
exp(x)+1)+1/a*ln(exp(2*x)+2*b/a*exp(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(35) = 70$.

Time = 0.11 (sec) , antiderivative size = 200, normalized size of antiderivative = 5.71

$$\int \frac{\tanh^3(x)}{a + b\text{sech}(x)}\,dx = -\frac{b^2x\cosh{(x)}^2 + b^2x\sinh{(x)}^2 + b^2x - 2\,ab\cosh{(x)} + \left((a^2 - b^2)\cosh{(x)}^2 + 2\,(a^2 - b^2)\cosh{(x)}\sinh{(x}\right.}{}$$

input
```
integrate(tanh(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
-(b^2*x*cosh(x)^2 + b^2*x*sinh(x)^2 + b^2*x - 2*a*b*cosh(x) + ((a^2 - b^2)
*cosh(x)^2 + 2*(a^2 - b^2)*cosh(x)*sinh(x) + (a^2 - b^2)*sinh(x)^2 + a^2 -
 b^2)*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) - (a^2*cosh(x)^2 + 2*a^2*
cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2)*log(2*cosh(x)/(cosh(x) - sinh(x)))
+ 2*(b^2*x*cosh(x) - a*b)*sinh(x))/(a*b^2*cosh(x)^2 + 2*a*b^2*cosh(x)*sinh
(x) + a*b^2*sinh(x)^2 + a*b^2)
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\tanh^3{(x)}}{a + b\,\text{sech}\,(x)}\,dx$$

input
```
integrate(tanh(x)**3/(a+b*sech(x)),x)
```

output
```
Integral(tanh(x)**3/(a + b*sech(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.91

$$\int \frac{\tanh^3(x)}{a + b\text{sech}(x)}\, dx = \frac{x}{a} + \frac{2\, e^{(-x)}}{be^{(-2\, x)} + b} + \frac{a \log\left(e^{(-2\, x)} + 1\right)}{b^2} - \frac{(a^2 - b^2) \log\left(2\, be^{(-x)} + ae^{(-2\, x)} + a\right)}{ab^2}$$

input
```
integrate(tanh(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
x/a + 2*e^(-x)/(b*e^(-2*x) + b) + a*log(e^(-2*x) + 1)/b^2 - (a^2 - b^2)*lo
g(2*b*e^(-x) + a*e^(-2*x) + a)/(a*b^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(35) = 70.

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.09

$$\int \frac{\tanh^3(x)}{a + b\text{sech}(x)}\, dx = \frac{a \log\left(e^{(-x)} + e^x\right)}{b^2} - \frac{(a^2 - b^2) \log\left(\left|a\left(e^{(-x)} + e^x\right) + 2\, b\right|\right)}{ab^2} - \frac{a\left(e^{(-x)} + e^x\right) - 2\, b}{b^2\left(e^{(-x)} + e^x\right)}$$

input
```
integrate(tanh(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

output
```
a*log(e^(-x) + e^x)/b^2 - (a^2 - b^2)*log(abs(a*(e^(-x) + e^x) + 2*b))/(a*
b^2) - (a*(e^(-x) + e^x) - 2*b)/(b^2*(e^(-x) + e^x))
```

Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 260, normalized size of antiderivative = 7.43

$$\int \frac{\tanh^3(x)}{a+b\text{sech}(x)}\,dx = \frac{2\,\mathrm{e}^{x}}{b+b\,\mathrm{e}^{2\,x}}-\frac{x}{a} + \frac{\ln\left(16\,a^5\,\mathrm{e}^{2\,x}+4\,a\,b^4+16\,a^5-16\,a^3\,b^2+8\,b^5\,\mathrm{e}^{x}-16\,a^3\,b^2\,\mathrm{e}^{2\,x}+32\,a^4\,b\,\mathrm{e}^{x}+4\,a\,b^4\,\mathrm{e}^{2\,x}-32\,a^2\,b^3\,\mathrm{e}^{x}\right)}{a} - \frac{a\,\ln\left(16\,a^5\,\mathrm{e}^{2\,x}+4\,a\,b^4+16\,a^5-16\,a^3\,b^2+8\,b^5\,\mathrm{e}^{x}-16\,a^3\,b^2\,\mathrm{e}^{2\,x}+32\,a^4\,b\,\mathrm{e}^{x}+4\,a\,b^4\,\mathrm{e}^{2\,x}-32\,a^2\,b^3\,\mathrm{e}^{x}\right)}{b^2} + \frac{a\,\ln\left(16\,a^6\,\mathrm{e}^{2\,x}-4\,b^6\,\mathrm{e}^{2\,x}+16\,a^6-4\,b^6+20\,a^2\,b^4-32\,a^4\,b^2+20\,a^2\,b^4\,\mathrm{e}^{2\,x}-32\,a^4\,b^2\,\mathrm{e}^{2\,x}\right)}{b^2}$$

input
```
int(tanh(x)^3/(a + b/cosh(x)),x)
```

output
```
(2*exp(x))/(b + b*exp(2*x)) - x/a + log(16*a^5*exp(2*x) + 4*a*b^4 + 16*a^5
 - 16*a^3*b^2 + 8*b^5*exp(x) - 16*a^3*b^2*exp(2*x) + 32*a^4*b*exp(x) + 4*a
*b^4*exp(2*x) - 32*a^2*b^3*exp(x))/a - (a*log(16*a^5*exp(2*x) + 4*a*b^4 +
16*a^5 - 16*a^3*b^2 + 8*b^5*exp(x) - 16*a^3*b^2*exp(2*x) + 32*a^4*b*exp(x)
 + 4*a*b^4*exp(2*x) - 32*a^2*b^3*exp(x)))/b^2 + (a*log(16*a^6*exp(2*x) - 4
*b^6*exp(2*x) + 16*a^6 - 4*b^6 + 20*a^2*b^4 - 32*a^4*b^2 + 20*a^2*b^4*exp(
2*x) - 32*a^4*b^2*exp(2*x)))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 162, normalized size of antiderivative = 4.63

$$\int \frac{\tanh^3(x)}{a+b\text{sech}(x)}\,dx = \frac{e^{2x}\log(e^{2x}+1)\,a^2-e^{2x}\log(e^{2x}a+2e^{x}b+a)\,a^2+e^{2x}\log(e^{2x}a+2e^{x}b+a)\,b^2-e^{2x}b^2x+2e^{x}ab+\log(e^{2x}}{a\,b^2\,(e^{2x}+1)}$$

input
```
int(tanh(x)^3/(a+b*sech(x)),x)
```

output
```
(e**(2*x)*log(e**(2*x) + 1)*a**2 - e**(2*x)*log(e**(2*x)*a + 2*e**x*b + a)
*a**2 + e**(2*x)*log(e**(2*x)*a + 2*e**x*b + a)*b**2 - e**(2*x)*b**2*x + 2
*e**x*a*b + log(e**(2*x) + 1)*a**2 - log(e**(2*x)*a + 2*e**x*b + a)*a**2 +
 log(e**(2*x)*a + 2*e**x*b + a)*b**2 - b**2*x)/(a*b**2*(e**(2*x) + 1))
```

3.118 $\int \frac{\tanh^2(x)}{a+b\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 62

$$\int \frac{\tanh^2(x)}{a+b\text{sech}(x)}\,dx=\frac{x}{a}-\frac{\arctan(\sinh(x))}{b}+\frac{2\sqrt{a-b}\sqrt{a+b}\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab}$$

output
```
x/a-arctan(sinh(x))/b+2*(a-b)^(1/2)*(a+b)^(1/2)*arctan((a-b)^(1/2)*tanh(1/
2*x)/(a+b)^(1/2))/a/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^2(x)}{a+b\text{sech}(x)}\,dx=\frac{bx-2a\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-2\sqrt{a^2-b^2}\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{ab}$$

input
```
Integrate[Tanh[x]^2/(a + b*Sech[x]),x]
```

output
```
(b*x - 2*a*ArcTan[Tanh[x/2]] - 2*Sqrt[a^2 - b^2]*ArcTan[((-a + b)*Tanh[x/2
])/Sqrt[a^2 - b^2]])/(a*b)
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.077, Rules used = {3042, 25, 4382, 3042, 4539, 25, 3042, 4257, 4407, 3042, 4318, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)}{a+b\text{sech}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cot\left(\frac{\pi}{2}+ix\right)^2}{a+b\csc\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cot\left(ix+\frac{\pi}{2}\right)^2}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4382}$$

$$-\int \frac{\text{sech}^2(x)-1}{a+b\text{sech}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\csc\left(ix+\frac{\pi}{2}\right)^2-1}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4539}$$

$$-\frac{\int -\frac{b+a\text{sech}(x)}{a+b\text{sech}(x)}dx}{b}-\frac{\int \text{sech}(x)dx}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b+a\text{sech}(x)}{a+b\text{sech}(x)}dx}{b}-\frac{\int \text{sech}(x)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b+a\csc\left(ix+\frac{\pi}{2}\right)}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx}{b}-\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \ 4257$$

$$-\frac{\arctan(\sinh(x))}{b}+\frac{\int \frac{b+a\csc\left(ix+\frac{\pi}{2}\right)}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \ 4407$$

$$\frac{\frac{(a^2-b^2)\int \frac{\text{sech}(x)}{a+b\,\text{sech}(x)}dx}{a}+\frac{bx}{a}}{b}-\frac{\arctan(\sinh(x))}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\arctan(\sinh(x))}{b}+\frac{\frac{bx}{a}+\frac{(a^2-b^2)\int \frac{\csc\left(ix+\frac{\pi}{2}\right)}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx}{a}}{b}$$

$$\Big\downarrow \ 4318$$

$$\frac{\frac{(a^2-b^2)\int \frac{1}{\frac{a\cosh(x)}{b}+1}dx}{ab}+\frac{bx}{a}}{b}-\frac{\arctan(\sinh(x))}{b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\arctan(\sinh(x))}{b}+\frac{\frac{bx}{a}+\frac{(a^2-b^2)\int \frac{1}{\frac{a\sin\left(ix+\frac{\pi}{2}\right)}{b}+1}dx}{ab}}{b}$$

$$\Big\downarrow \ 3138$$

$$\frac{\frac{2(a^2-b^2)\int \frac{1}{\frac{a+b}{b}-\left(1-\frac{a}{b}\right)\tanh^2\left(\frac{x}{2}\right)}d\tanh\left(\frac{x}{2}\right)}{ab}+\frac{bx}{a}}{b}-\frac{\arctan(\sinh(x))}{b}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{2(a^2-b^2)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}+\frac{bx}{a}}{b}-\frac{\arctan(\sinh(x))}{b}$$

input
```
Int[Tanh[x]^2/(a + b*Sech[x]),x]
```

output
```
-(ArcTan[Sinh[x]]/b) + ((b*x)/a + (2*(a^2 - b^2)*ArcTan[(Sqrt[a - b]*Tanh[
x/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4382
```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4539
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)), x_Symbol] :> Simp[C/b   Int[Csc[e + f*x], x], x] + Simp[1/b   In
t[(A*b - a*C*Csc[e + f*x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e,
f, A, C}, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.35

method	result	size
default	$\frac{2(a-b)(a+b)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{ab\sqrt{(a-b)(a+b)}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{b}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$	84
risch	$\frac{x}{a}+\frac{\sqrt{-a^2+b^2}\,\ln\left(e^x+\frac{b+\sqrt{-a^2+b^2}}{a}\right)}{ba}-\frac{\sqrt{-a^2+b^2}\,\ln\left(e^x-\frac{\sqrt{-a^2+b^2}-b}{a}\right)}{ba}+\frac{i\ln(e^x-i)}{b}-\frac{i\ln(e^x+i)}{b}$	113

input
```
int(tanh(x)^2/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
2/a*(a-b)*(a+b)/b/((a-b)*(a+b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a-b)*(a+b
))^(1/2))-1/a*ln(tanh(1/2*x)-1)-2/b*arctan(tanh(1/2*x))+1/a*ln(tanh(1/2*x)
+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 3.11

$$\int \frac{\tanh^2(x)}{a+b\text{sech}(x)}\,dx$$
$$=\left[\frac{bx-2\,a\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+\sqrt{-a^2+b^2}\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)-a^2+2\,b^2+2\,(a^2\cosh(x)+}{a\cosh(x)^2+a\sinh(x)^2+2\,b\cosh(x)+2}\right.}{ab}\right.$$

input
```
integrate(tanh(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
[(b*x - 2*a*arctan(cosh(x) + sinh(x)) + sqrt(-a^2 + b^2)*log((a^2*cosh(x)^
2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*si
nh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*s
inh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)))/(a*b), (b*x - 2*
a*arctan(cosh(x) + sinh(x)) - 2*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sin
h(x) + b)/sqrt(a^2 - b^2)))/(a*b)]
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{a+b\text{sech}(x)}\,dx=\int \frac{\tanh^2{(x)}}{a+b\operatorname{sech}{(x)}}\,dx$$

input
```
integrate(tanh(x)**2/(a+b*sech(x)),x)
```

output
```
Integral(tanh(x)**2/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tanh^2(x)}{a+b\text{sech}(x)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(tanh(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{\tanh^2(x)}{a+b\text{sech}(x)}\,dx=\frac{x}{a}-\frac{2\,\arctan\left(e^x\right)}{b}+\frac{2\,\sqrt{a^2-b^2}\,\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{ab}$$

input
```
integrate(tanh(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

output
```
x/a - 2*arctan(e^x)/b + 2*sqrt(a^2 - b^2)*arctan((a*e^x + b)/sqrt(a^2 - b^
2))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 4.50 (sec) , antiderivative size = 273, normalized size of antiderivative = 4.40

$$\int \frac{\tanh^2(x)}{a+b\text{sech}(x)}\,dx=\frac{\ln\left(\mathrm{e}^x-\mathrm{i}\right)\,1\mathrm{i}-\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,1\mathrm{i}}{b}+\frac{\ln\left(2\,a\,b^3-2\,a^3\,b+a^3\,\sqrt{b^2-a^2}+a^4\,\mathrm{e}^x+4\,b^4\,\mathrm{e}^x-2\,a\,b^2\,\sqrt{b^2-a^2}-4\,b^3\,\mathrm{e}^x\,\sqrt{b^2-a^2}-5\,a^2\,b^2\,\mathrm{e}^x+3\,a\right.}{}$$

input
```
int(tanh(x)^2/(a + b/cosh(x)),x)
```

output
```
(log(exp(x) - 1i)*1i - log(exp(x) + 1i)*1i)/b + (log(2*a*b^3 - 2*a^3*b + a
^3*(b^2 - a^2)^(1/2) + a^4*exp(x) + 4*b^4*exp(x) - 2*a*b^2*(b^2 - a^2)^(1/
2) - 4*b^3*exp(x)*(b^2 - a^2)^(1/2) - 5*a^2*b^2*exp(x) + 3*a^2*b*exp(x)*(b
^2 - a^2)^(1/2))*(b^2 - a^2)^(1/2) - log(2*a*b^3 - 2*a^3*b - a^3*(b^2 - a^
2)^(1/2) + a^4*exp(x) + 4*b^4*exp(x) + 2*a*b^2*(b^2 - a^2)^(1/2) + 4*b^3*e
xp(x)*(b^2 - a^2)^(1/2) - 5*a^2*b^2*exp(x) - 3*a^2*b*exp(x)*(b^2 - a^2)^(1
/2))*(b^2 - a^2)^(1/2) + b*x)/(a*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \frac{\tanh^2(x)}{a + b\text{sech}(x)}\,dx = \frac{-2atan(e^x)\,a + 2\sqrt{a^2-b^2}\,atan\left(\frac{e^x a+b}{\sqrt{a^2-b^2}}\right) + bx}{ab}$$

input
```
int(tanh(x)^2/(a+b*sech(x)),x)
```

output
```
( - 2*atan(e**x)*a + 2*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**
2)) + b*x)/(a*b)
```

3.119 $\int \frac{\tanh(x)}{a+b\,\mathrm{sech}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{\tanh(x)}{a+b\,\mathrm{sech}(x)}\,dx=\frac{\log(\cosh(x))}{a}+\frac{\log(a+b\,\mathrm{sech}(x))}{a}$$

output
```
ln(cosh(x))/a+ln(a+b*sech(x))/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{\tanh(x)}{a+b\,\mathrm{sech}(x)}\,dx=\frac{\log(b+a\cosh(x))}{a}$$

input
```
Integrate[Tanh[x]/(a + b*Sech[x]),x]
```

output
```
Log[b + a*Cosh[x]]/a
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 26, 4373, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{a+b\text{sech}(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\cot\left(\frac{\pi}{2}+ix\right)}{a+b\csc\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\cot\left(ix+\frac{\pi}{2}\right)}{a+b\csc\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4373}$$

$$-\int \frac{\cosh(x)}{b(a+b\text{sech}(x))}d(b\text{sech}(x))$$

$$\Big\downarrow \text{ 47}$$

$$\frac{\int \frac{1}{a+b\text{sech}(x)}d(b\text{sech}(x))}{a}-\frac{\int \frac{\cosh(x)}{b}d(b\text{sech}(x))}{a}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{\int \frac{1}{a+b\text{sech}(x)}d(b\text{sech}(x))}{a}-\frac{\log(b\text{sech}(x))}{a}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\log(a+b\text{sech}(x))}{a}-\frac{\log(b\text{sech}(x))}{a}$$

input
```
Int[Tanh[x]/(a + b*Sech[x]),x]
```

output
```
-(Log[b*Sech[x]]/a) + Log[a + b*Sech[x]]/a
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$-\frac{\ln(\operatorname{sech}(x))}{a}+\frac{\ln(a+b\operatorname{sech}(x))}{a}$	21
default	$-\frac{\ln(\operatorname{sech}(x))}{a}+\frac{\ln(a+b\operatorname{sech}(x))}{a}$	21
risch	$-\frac{x}{a}+\frac{\ln\left(e^{2x}+\frac{2b\,e^{x}}{a}+1\right)}{a}$	27

input
```
int(tanh(x)/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/a*ln(sech(x))+ln(a+b*sech(x))/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{\tanh(x)}{a+b\text{sech}(x)}\,dx = -\frac{x-\log\left(\frac{2\,(a\cosh(x)+b)}{\cosh(x)-\sinh(x)}\right)}{a}$$

input
```
integrate(tanh(x)/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
-(x - log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))))/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(15) = 30.

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

$$\int \frac{\tanh(x)}{a+b\text{sech}(x)}\,dx = \begin{cases} \frac{\tilde{\infty}}{\operatorname{sech}(x)} & \text{for } a=0 \wedge b=0 \\ \frac{1}{b\operatorname{sech}(x)} & \text{for } a=0 \\ \frac{x-\log(\tanh(x)+1)}{a} & \text{for } b=0 \\ \frac{x}{a}+\frac{\log\left(\frac{a}{b}+\operatorname{sech}(x)\right)}{a}-\frac{\log(\tanh(x)+1)}{a} & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(x)/(a+b*sech(x)),x)
```

output
```
Piecewise((zoo/sech(x), Eq(a, 0) & Eq(b, 0)), (1/(b*sech(x)), Eq(a, 0)), (
(x - log(tanh(x) + 1))/a, Eq(b, 0)), (x/a + log(a/b + sech(x))/a - log(tan
h(x) + 1)/a, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{\tanh(x)}{a + b\text{sech}(x)}\, dx = \frac{x}{a} + \frac{\log\left(2\,be^{(-x)} + ae^{(-2\,x)} + a\right)}{a}$$

input
```
integrate(tanh(x)/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
x/a + log(2*b*e^(-x) + a*e^(-2*x) + a)/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(x)}{a + b\text{sech}(x)}\, dx = \frac{\log\left(\left|a\left(e^{(-x)} + e^{x}\right) + 2\,b\right|\right)}{a}$$

input
```
integrate(tanh(x)/(a+b*sech(x)),x, algorithm="giac")
```

output
```
log(abs(a*(e^(-x) + e^x) + 2*b))/a
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{\tanh(x)}{a + b\text{sech}(x)}\, dx = -\frac{x - \ln\left(a + 2\,b\,\mathrm{e}^{x} + a\,\mathrm{e}^{2\,x}\right)}{a}$$

input
```
int(tanh(x)/(a + b/cosh(x)),x)
```

output
```
-(x - log(a + 2*b*exp(x) + a*exp(2*x)))/a
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{\tanh(x)}{a + b\text{sech}(x)}\,dx = \frac{\log(e^{2x}a + 2e^{x}b + a) - x}{a}$$

input
```
int(tanh(x)/(a+b*sech(x)),x)
```

output
```
(log(e**(2*x)*a + 2*e**x*b + a) - x)/a
```

3.120 $\int \frac{\coth(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 66

$$\int \frac{\coth(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{\log(\cosh(x))}{a} + \frac{\log(1-\operatorname{sech}(x))}{2(a+b)} + \frac{\log(1+\operatorname{sech}(x))}{2(a-b)} - \frac{b^2\log(a+b\operatorname{sech}(x))}{a\,(a^2-b^2)}$$

output
```
ln(cosh(x))/a+ln(1-sech(x))/(2*a+2*b)+ln(1+sech(x))/(2*a-2*b)-b^2*ln(a+b*s
ech(x))/a/(a^2-b^2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int \frac{\coth(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{1}{2}\left(\frac{2\log(\cosh(x))}{a} + \frac{\log(1-\operatorname{sech}(x))}{a+b} + \frac{\log(1+\operatorname{sech}(x))}{a-b} - \frac{2b^2\log(a+b\operatorname{sech}(x))}{a^3-ab^2}\right)$$

input
```
Integrate[Coth[x]/(a + b*Sech[x]),x]
```

output
```
((2*Log[Cosh[x]])/a + Log[1 - Sech[x]]/(a + b) + Log[1 + Sech[x]]/(a - b)
- (2*b^2*Log[a + b*Sech[x]])/(a^3 - a*b^2))/2
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 26, 4373, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{a+b\text{sech}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\cot\left(\frac{\pi}{2}+ix\right)\left(a+b\csc\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\cot\left(ix+\frac{\pi}{2}\right)\left(a+b\csc\left(ix+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4373}$$

$$-b^2\int \frac{\cosh(x)}{b(a+b\text{sech}(x))\left(b^2-b^2\text{sech}^2(x)\right)}d(b\text{sech}(x))$$

$$\Big\downarrow \text{615}$$

$$-b^2\int \left(\frac{\cosh(x)}{ab^3}+\frac{1}{2b^2(a+b)(b-b\text{sech}(x))}+\frac{1}{a(a-b)(a+b)(a+b\text{sech}(x))}-\frac{1}{2(a-b)b^2(\text{sech}(x)b+b)}\right)d(b\text{se}$$

$$\Big\downarrow \text{2009}$$

$$-b^2\left(\frac{\log(a+b\text{sech}(x))}{a\left(a^2-b^2\right)}+\frac{\log(b\text{sech}(x))}{ab^2}-\frac{\log(b-b\text{sech}(x))}{2b^2(a+b)}-\frac{\log(b\text{sech}(x)+b)}{2b^2(a-b)}\right)$$

input
```
Int[Coth[x]/(a + b*Sech[x]),x]
```

output
```
-(b^2*(Log[b*Sech[x]]/(a*b^2) - Log[b - b*Sech[x]]/(2*b^2*(a + b)) + Log[a
 + b*Sech[x]]/(a*(a^2 - b^2)) - Log[b + b*Sech[x]]/(2*(a - b)*b^2)))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a+b}-\frac{b^{2} \ln\left(a \tanh\left(\frac{x}{2}\right)^{2}-b \tanh\left(\frac{x}{2}\right)^{2}+a+b\right)}{a \left(a+b\right) \left(a-b\right)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$	78
risch	$\frac{x}{a}-\frac{x}{a+b}-\frac{x}{a-b}+\frac{2 x \,b^{2}}{a \left(a^{2}-b^{2}\right)}+\frac{\ln\left({\mathrm e}^{x}-1\right)}{a+b}+\frac{\ln\left(1+{\mathrm e}^{x}\right)}{a-b}-\frac{b^{2} \ln\left({\mathrm e}^{2 x}+\frac{2 b \,{\mathrm e}^{x}}{a}+1\right)}{a \left(a^{2}-b^{2}\right)}$	103

input
```
int(coth(x)/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
1/(a+b)*ln(tanh(1/2*x))-b^2/a/(a+b)/(a-b)*ln(a*tanh(1/2*x)^2-b*tanh(1/2*x)
^2+a+b)-1/a*ln(tanh(1/2*x)-1)-1/a*ln(tanh(1/2*x)+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

$$\int \frac{\coth(x)}{a+b\text{sech}(x)}\,dx = -\frac{b^2\log\left(\frac{2\,(a\cosh(x)+b)}{\cosh(x)-\sinh(x)}\right)+(a^2-b^2)x-(a^2+ab)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-(a^2-ab)\log\left(\cosh\left(x\right)\right.}{a^3-ab^2}$$

input
```
integrate(coth(x)/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
-(b^2*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) + (a^2 - b^2)*x - (a^2 +
a*b)*log(cosh(x) + sinh(x) + 1) - (a^2 - a*b)*log(cosh(x) + sinh(x) - 1))/
(a^3 - a*b^2)
```

Sympy [F]

$$\int \frac{\coth(x)}{a+b\text{sech}(x)}\,dx = \int \frac{\coth\left(x\right)}{a+b\,\text{sech}\left(x\right)}\,dx$$

input
```
integrate(coth(x)/(a+b*sech(x)),x)
```

output
```
Integral(coth(x)/(a + b*sech(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{\coth(x)}{a + b\text{sech}(x)}\, dx = -\frac{b^2 \log\left(2\, be^{(-x)} + ae^{(-2\, x)} + a\right)}{a^3 - ab^2} + \frac{x}{a} + \frac{\log\left(e^{(-x)} + 1\right)}{a - b} + \frac{\log\left(e^{(-x)} - 1\right)}{a + b}$$

input
```
integrate(coth(x)/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
-b^2*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^3 - a*b^2) + x/a + log(e^(-x) + 1
)/(a - b) + log(e^(-x) - 1)/(a + b)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{\coth(x)}{a + b\text{sech}(x)}\, dx = -\frac{b^2 \log\left(\left|a\left(e^{(-x)} + e^x\right) + 2\, b\right|\right)}{a^3 - ab^2} + \frac{\log\left(e^{(-x)} + e^x + 2\right)}{2\,(a - b)} + \frac{\log\left(e^{(-x)} + e^x - 2\right)}{2\,(a + b)}$$

input
```
integrate(coth(x)/(a+b*sech(x)),x, algorithm="giac")
```

output
```
-b^2*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^3 - a*b^2) + 1/2*log(e^(-x) + e^x
 + 2)/(a - b) + 1/2*log(e^(-x) + e^x - 2)/(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 271, normalized size of antiderivative = 4.11

$$\int \frac{\coth(x)}{a+b\text{sech}(x)}\,dx$$
$$= \frac{\ln\left(64\,a\,b^4+32\,a^4\,b+32\,b^5+96\,a^2\,b^3+64\,a^3\,b^2+32\,b^5\,\mathrm{e}^x+64\,a\,b^4\,\mathrm{e}^x+32\,a^4\,b\,\mathrm{e}^x+96\,a^2\,b^3\,\mathrm{e}^x+64\,a^3\,b^2\right.}{a-b}$$
$$-\frac{x}{a}$$
$$+\frac{\ln\left(64\,a\,b^4-32\,a^4\,b-32\,b^5-96\,a^2\,b^3+64\,a^3\,b^2+32\,b^5\,\mathrm{e}^x-64\,a\,b^4\,\mathrm{e}^x+32\,a^4\,b\,\mathrm{e}^x+96\,a^2\,b^3\,\mathrm{e}^x-64\,a^3\right.}{a+b}$$
$$+\frac{b^2\,\ln\left(4\,a^5\,\mathrm{e}^{2\,x}+4\,a\,b^4+4\,a^5+4\,a^3\,b^2+8\,b^5\,\mathrm{e}^x+4\,a^3\,b^2\,\mathrm{e}^{2\,x}+8\,a^4\,b\,\mathrm{e}^x+4\,a\,b^4\,\mathrm{e}^{2\,x}+8\,a^2\,b^3\,\mathrm{e}^x\right)}{a\,b^2-a^3}$$

input
```
int(coth(x)/(a + b/cosh(x)),x)
```

output
```
log(64*a*b^4 + 32*a^4*b + 32*b^5 + 96*a^2*b^3 + 64*a^3*b^2 + 32*b^5*exp(x)
 + 64*a*b^4*exp(x) + 32*a^4*b*exp(x) + 96*a^2*b^3*exp(x) + 64*a^3*b^2*exp(
x))/(a - b) - x/a + log(64*a*b^4 - 32*a^4*b - 32*b^5 - 96*a^2*b^3 + 64*a^3
*b^2 + 32*b^5*exp(x) - 64*a*b^4*exp(x) + 32*a^4*b*exp(x) + 96*a^2*b^3*exp(
x) - 64*a^3*b^2*exp(x))/(a + b) + (b^2*log(4*a^5*exp(2*x) + 4*a*b^4 + 4*a^
5 + 4*a^3*b^2 + 8*b^5*exp(x) + 4*a^3*b^2*exp(2*x) + 8*a^4*b*exp(x) + 4*a*b
^4*exp(2*x) + 8*a^2*b^3*exp(x)))/(a*b^2 - a^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.32

$$\int \frac{\coth(x)}{a+b\text{sech}(x)}\,dx$$
$$= \frac{\log(e^x-1)\,a^2-\log(e^x-1)\,ab+\log(e^x+1)\,a^2+\log(e^x+1)\,ab-\log(e^{2x}a+2e^xb+a)\,b^2-a^2x+b^2x}{a\,(a^2-b^2)}$$

input
```
int(coth(x)/(a+b*sech(x)),x)
```

output
```
(log(e**x - 1)*a**2 - log(e**x - 1)*a*b + log(e**x + 1)*a**2 + log(e**x +
1)*a*b - log(e**(2*x)*a + 2*e**x*b + a)*b**2 - a**2*x + b**2*x)/(a*(a**2 -
 b**2))
```

3.121 $\int \frac{\coth^2(x)}{a+b\,\mathrm{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 114

$$\int \frac{\coth^2(x)}{a+b\,\mathrm{sech}(x)}\,dx = \frac{ax}{a^2-b^2} - \frac{b^2x}{a\left(a^2-b^2\right)} + \frac{2b^3\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}} - \frac{a\coth(x)}{a^2-b^2} + \frac{b\,\mathrm{csch}(x)}{a^2-b^2}$$

output
```
a*x/(a^2-b^2)-b^2*x/a/(a^2-b^2)+2*b^3*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)
^(1/2))/a/(a-b)^(3/2)/(a+b)^(3/2)-a*coth(x)/(a^2-b^2)+b*csch(x)/(a^2-b^2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.71

$$\int \frac{\coth^2(x)}{a+b\,\mathrm{sech}(x)}\,dx = \frac{a^2x - b^2x + \frac{2b^3\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - a^2\coth(x) + ab\,\mathrm{csch}(x)}{a^3-ab^2}$$

input
```
Integrate[Coth[x]^2/(a + b*Sech[x]),x]
```

output
```
(a^2*x - b^2*x + (2*b^3*ArcTan[((a - b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sqrt[
a^2 - b^2] - a^2*Coth[x] + a*b*Csch[x])/(a^3 - a*b^2)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.91, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.308, Rules used = {3042, 25, 4386, 25, 3042, 3381, 25, 3042, 25, 3086, 24, 3214, 3042, 3138, 218, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{a+b\text{sech}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\cot\left(\frac{\pi}{2}+ix\right)^2\left(a+b\csc\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\cot\left(ix+\frac{\pi}{2}\right)^2\left(a+b\csc\left(ix+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4386}$$

$$-\int -\frac{\cosh(x)\coth^2(x)}{b+a\cosh(x)}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\cosh(x)\coth^2(x)}{a\cosh(x)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(-\frac{\pi}{2}+ix\right)^3}{\cos\left(-\frac{\pi}{2}+ix\right)^2\left(b-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3381}$$

$$\frac{b^2\int -\frac{\cosh(x)}{b+a\cosh(x)}dx}{a^2-b^2}-\frac{a\int -\coth^2(x)dx}{a^2-b^2}-\frac{b\int \coth(x)\text{csch}(x)dx}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b^2\int \frac{\cosh(x)}{b+a\cosh(x)}dx}{a^2-b^2}+\frac{a\int \coth^2(x)dx}{a^2-b^2}-\frac{b\int \coth(x)\text{csch}(x)dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b^2\int \frac{\sin\left(ix+\frac{\pi}{2}\right)}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{a\int -\tan\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)dx}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b^2\int \frac{\sin\left(ix+\frac{\pi}{2}\right)}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)dx}{a^2-b^2}$$

$$\Big\downarrow \text{3086}$$

$$-\frac{b^2\int \frac{\sin\left(ix+\frac{\pi}{2}\right)}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}+\frac{ib\int 1d(-i\text{csch}(x))}{a^2-b^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{b^2\int \frac{\sin\left(ix+\frac{\pi}{2}\right)}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}+\frac{b\text{csch}(x)}{a^2-b^2}$$

$$\Big\downarrow \text{3214}$$

$$-\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b^2\left(\frac{x}{a}-\frac{b\int \frac{1}{b+a\cosh(x)}dx}{a}\right)}{a^2-b^2}+\frac{b\text{csch}(x)}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b^2\left(\frac{x}{a}-\frac{b\int \frac{1}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx}{a}\right)}{a^2-b^2}-\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}+\frac{b\text{csch}(x)}{a^2-b^2}$$

$$\Big\downarrow \text{3138}$$

$$-\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b^2\left(\frac{x}{a}-\frac{2b\int \frac{1}{(a-b)\tanh^2\left(\frac{x}{2}\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a}\right)}{a^2-b^2}+\frac{b\text{csch}(x)}{a^2-b^2}$$

$$\Big\downarrow \text{218}$$

$$-\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b^2\left(\frac{x}{a}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}\right)}{a^2-b^2}+\frac{b\operatorname{csch}(x)}{a^2-b^2}$$

$$\Big\downarrow \text{3954}$$

$$-\frac{a(\coth(x)-\int 1dx)}{a^2-b^2}-\frac{b^2\left(\frac{x}{a}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}\right)}{a^2-b^2}+\frac{b\operatorname{csch}(x)}{a^2-b^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{b^2\left(\frac{x}{a}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}\right)}{a^2-b^2}-\frac{a(\coth(x)-x)}{a^2-b^2}+\frac{b\operatorname{csch}(x)}{a^2-b^2}$$

input
```
Int[Coth[x]^2/(a + b*Sech[x]),x]
```

output
```
-((b^2*(x/a - (2*b*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*Sqrt[a
- b]*Sqrt[a + b])))/(a^2 - b^2)) - (a*(-x + Coth[x]))/(a^2 - b^2) + (b*Csc
h[x])/(a^2 - b^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3381
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a*(d^2/(a^2
- b^2))   Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Simp[
b*(d/(a^2 - b^2))   Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x]
 - Simp[a^2*(d^2/(g^2*(a^2 - b^2)))   Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[
e + f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1
]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4386
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2(a-b)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{1}{2(a+b)\tanh\left(\frac{x}{2}\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}+\frac{2b^{3}\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a(a-b)(a+b)\sqrt{(a-b)(a+b)}}$	104
risch	$\frac{x}{a}-\frac{2(-b\,\mathrm{e}^{x}+a)}{(\mathrm{e}^{2x}-1)(a^{2}-b^{2})}-\frac{b^{3}\ln\left(\mathrm{e}^{x}+\frac{b\sqrt{-a^{2}+b^{2}}-a^{2}+b^{2}}{\sqrt{-a^{2}+b^{2}}\,a}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)a}+\frac{b^{3}\ln\left(\mathrm{e}^{x}+\frac{b\sqrt{-a^{2}+b^{2}}+a^{2}-b^{2}}{\sqrt{-a^{2}+b^{2}}\,a}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)(a-b)a}$	178

input
```
int(coth(x)^2/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*tanh(1/2*x)/(a-b)-1/a*ln(tanh(1/2*x)-1)-1/2/(a+b)/tanh(1/2*x)+1/a*ln(
tanh(1/2*x)+1)+2/a/(a-b)/(a+b)*b^3/((a-b)*(a+b))^(1/2)*arctan((a-b)*tanh(1
/2*x)/((a-b)*(a+b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(104) = 208.

Time = 0.10 (sec) , antiderivative size = 646, normalized size of antiderivative = 5.67

$$\int \frac{\coth^2(x)}{a+b\operatorname{sech}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
[(2*a^4 - 2*a^2*b^2 - (a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)^2 - (a^4 - 2*a^2*b
^2 + b^4)*x*sinh(x)^2 - (b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(
x)^2 - b^3)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*co
sh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(
a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(
a*cosh(x) + b)*sinh(x) + a)) + (a^4 - 2*a^2*b^2 + b^4)*x - 2*(a^3*b - a*b^
3)*cosh(x) - 2*(a^3*b - a*b^3 + (a^4 - 2*a^2*b^2 + b^4)*x*cosh(x))*sinh(x)
)/(a^5 - 2*a^3*b^2 + a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*cosh(x)^2 - 2*(a^5
- 2*a^3*b^2 + a*b^4)*cosh(x)*sinh(x) - (a^5 - 2*a^3*b^2 + a*b^4)*sinh(x)^2
), (2*a^4 - 2*a^2*b^2 - (a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)^2 - (a^4 - 2*a^2
*b^2 + b^4)*x*sinh(x)^2 + 2*(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*s
inh(x)^2 - b^3)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a
^2 - b^2)) + (a^4 - 2*a^2*b^2 + b^4)*x - 2*(a^3*b - a*b^3)*cosh(x) - 2*(a^
3*b - a*b^3 + (a^4 - 2*a^2*b^2 + b^4)*x*cosh(x))*sinh(x))/(a^5 - 2*a^3*b^2
 + a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*cosh(x)^2 - 2*(a^5 - 2*a^3*b^2 + a*b^
4)*cosh(x)*sinh(x) - (a^5 - 2*a^3*b^2 + a*b^4)*sinh(x)^2)]
```

Sympy [F]

$$\int \frac{\coth^2(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\coth^2(x)}{a + b\,\text{sech}(x)}\,dx$$

input
```
integrate(coth(x)**2/(a+b*sech(x)),x)
```

output
```
Integral(coth(x)**2/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\coth^2(x)}{a + b\text{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(coth(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.72

$$\int \frac{\coth^2(x)}{a+b\text{sech}(x)}\,dx = \frac{2\,b^3\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{(a^3-ab^2)\sqrt{a^2-b^2}} + \frac{x}{a} + \frac{2\,(be^x-a)}{(a^2-b^2)(e^{(2\,x)}-1)}$$

input
```
integrate(coth(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

output
```
2*b^3*arctan((a*e^x + b)/sqrt(a^2 - b^2))/((a^3 - a*b^2)*sqrt(a^2 - b^2))
+ x/a + 2*(b*e^x - a)/((a^2 - b^2)*(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.36

$$\int \frac{\coth^2(x)}{a+b\text{sech}(x)}\,dx = \frac{x}{a} - \frac{\frac{2\,a}{a^2-b^2}-\frac{2\,b\,e^x}{a^2-b^2}}{e^{2\,x}-1} - \frac{2\,\text{atan}\left(\left(e^x\left(\frac{2\,b^3}{a^3\,(a\,b^2-a^3)\,(a^2-b^2)\,\sqrt{b^6}} - \frac{2\,\left(a\,b^3\,\sqrt{b^6}-a^3\,b\,\sqrt{b^6}\right)}{a^2\,b^2\,(a\,b^2-a^3)\,\sqrt{a^2\,(a^2-b^2)^3}\,\sqrt{a^8-3\,a^6\,b^2+3\,a^4\,b^4-a^2\,b^6}}\right) + \frac{2\,\left(a^4\right.}{a^2\,b^2\,(a\,b^2-a^3)\,\sqrt{a^2\,(a^2}}\right.\right.}{\sqrt{a^8-3\,a^6\,b^2+3\,a^4\,b^4}}$$

input
```
int(coth(x)^2/(a + b/cosh(x)),x)
```

output
```
x/a - ((2*a)/(a^2 - b^2) - (2*b*exp(x))/(a^2 - b^2))/(exp(2*x) - 1) - (2*a
tan((exp(x)*((2*b^3)/(a^3*(a*b^2 - a^3)*(a^2 - b^2)*(b^6)^(1/2)) - (2*(a*b
^3*(b^6)^(1/2) - a^3*b*(b^6)^(1/2)))/(a^2*b^2*(a*b^2 - a^3)*(a^2*(a^2 - b^
2)^3)^(1/2)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)^(1/2))) + (2*(a^4*(b^6
)^(1/2) - a^2*b^2*(b^6)^(1/2)))/(a^2*b^2*(a*b^2 - a^3)*(a^2*(a^2 - b^2)^3)
^(1/2)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)^(1/2)))*((a^4*(a^8 - a^2*b^
6 + 3*a^4*b^4 - 3*a^6*b^2)^(1/2))/2 - (a^2*b^2*(a^8 - a^2*b^6 + 3*a^4*b^4
- 3*a^6*b^2)^(1/2))/2))*(b^6)^(1/2))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^
2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.02

$$\int \frac{\coth^2(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{2e^{2x}\sqrt{a^2-b^2}\,atan\left(\frac{e^x a+b}{\sqrt{a^2-b^2}}\right)b^3 - 2\sqrt{a^2-b^2}\,atan\left(\frac{e^x a+b}{\sqrt{a^2-b^2}}\right)b^3 + e^{2x}a^4x - 2e^{2x}a^4 - 2e^{2x}a^2b^2x + 2e^{2x}a^2b^2}{a\left(e^{2x}a^4 - 2e^{2x}a^2b^2 + e^{2x}b^4 - a^4 + 2a^2b^2 - b^4\right)}$$

input
```
int(coth(x)^2/(a+b*sech(x)),x)
```

output
```
(2*e**(2*x)*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2))*b**3 -
2*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2))*b**3 + e**(2*x)*a
**4*x - 2*e**(2*x)*a**4 - 2*e**(2*x)*a**2*b**2*x + 2*e**(2*x)*a**2*b**2 +
e**(2*x)*b**4*x + 2*e**x*a**3*b - 2*e**x*a*b**3 - a**4*x + 2*a**2*b**2*x -
 b**4*x)/(a*(e**(2*x)*a**4 - 2*e**(2*x)*a**2*b**2 + e**(2*x)*b**4 - a**4 +
 2*a**2*b**2 - b**4))
```

3.122 $\int \frac{\coth^3(x)}{a+b\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 113

$$\int \frac{\coth^3(x)}{a+b\text{sech}(x)}\,dx = \frac{\log(\cosh(x))}{a} + \frac{(2a+3b)\log(1-\text{sech}(x))}{4(a+b)^2} + \frac{(2a-3b)\log(1+\text{sech}(x))}{4(a-b)^2} + \frac{b^4\log(a+b\text{sech}(x))}{a\left(a^2-b^2\right)^2} - \frac{1}{4(a+b)(1-\text{sech}(x))} - \frac{1}{4(a-b)(1+\text{sech}(x))}$$

output
```
ln(cosh(x))/a+1/4*(2*a+3*b)*ln(1-sech(x))/(a+b)^2+1/4*(2*a-3*b)*ln(1+sech(
x))/(a-b)^2+b^4*ln(a+b*sech(x))/a/(a^2-b^2)^2-1/4/(a+b)/(1-sech(x))-1/4/(a
-b)/(1+sech(x))
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\int \frac{\coth^3(x)}{a+b\text{sech}(x)}\,dx = \frac{1}{4}\left(\frac{4\log(\cosh(x))}{a} + \frac{(2a+3b)\log(1-\text{sech}(x))}{(a+b)^2} + \frac{(2a-3b)\log(1+\text{sech}(x))}{(a-b)^2} + \frac{4b^4\log(a+b\text{sech}(x))}{a(a-b)^2(a+b)^2} + \frac{1}{(a+b)(-1+\text{sech}(x))} - \frac{1}{(a-b)(1+\text{sech}(x))}\right)$$

input
```
Integrate[Coth[x]^3/(a + b*Sech[x]),x]
```

output
```
((4*Log[Cosh[x]])/a + ((2*a + 3*b)*Log[1 - Sech[x]])/(a + b)^2 + ((2*a - 3
*b)*Log[1 + Sech[x]])/(a - b)^2 + (4*b^4*Log[a + b*Sech[x]])/(a*(a - b)^2*
(a + b)^2) + 1/((a + b)*(-1 + Sech[x])) - 1/((a - b)*(1 + Sech[x])))/4
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 26, 4373, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(x)}{a+b\text{sech}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\cot\left(\frac{\pi}{2}+ix\right)^3\left(a+b\csc\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\cot\left(ix+\frac{\pi}{2}\right)^3\left(a+b\csc\left(ix+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4373}$$

$$-b^4\int \frac{\cosh(x)}{b(a+b\text{sech}(x))\left(b^2-b^2\text{sech}^2(x)\right)^2}d(b\text{sech}(x))$$

$$\Big\downarrow \text{615}$$

$$-b^4\int \left(\frac{3b-2a}{4(a-b)^2b^4(\text{sech}(x)b+b)}+\frac{\cosh(x)}{ab^5}+\frac{2a+3b}{4b^4(a+b)^2(b-b\text{sech}(x))}-\frac{1}{a(a-b)^2(a+b)^2(a+b\text{sech}(x))}+\right.$$

$$\Big\downarrow \text{2009}$$

$$-b^4\left(-\frac{\log(a+b\text{sech}(x))}{a\left(a^2-b^2\right)^2}+\frac{\log(b\text{sech}(x))}{ab^4}-\frac{(2a+3b)\log(b-b\text{sech}(x))}{4b^4(a+b)^2}-\frac{(2a-3b)\log(b\text{sech}(x)+b)}{4b^4(a-b)^2}+\frac{}{4b^3(a\right.$$

input
```
Int[Coth[x]^3/(a + b*Sech[x]),x]
```

output
```
-(b^4*(Log[b*Sech[x]]/(a*b^4) - ((2*a + 3*b)*Log[b - b*Sech[x]])/(4*b^4*(a
 + b)^2) - Log[a + b*Sech[x]]/(a*(a^2 - b^2)^2) - ((2*a - 3*b)*Log[b + b*S
ech[x]])/(4*(a - b)^2*b^4) + 1/(4*b^3*(a + b)*(b - b*Sech[x])) + 1/(4*(a -
 b)*b^3*(b + b*Sech[x]))))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00

method	result
default	$-\frac{\tanh\left(\frac{x}{2}\right)^{2}}{8(a-b)}+\frac{b^{4}\ln\left(a\tanh\left(\frac{x}{2}\right)^{2}-b\tanh\left(\frac{x}{2}\right)^{2}+a+b\right)}{(a-b)^{2}(a+b)^{2}a}-\frac{1}{8(a+b)\tanh\left(\frac{x}{2}\right)^{2}}+\frac{(4a+6b)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4(a+b)^{2}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{\ln}{}$
risch	$\frac{x}{a}-\frac{xa}{a^{2}+2ab+b^{2}}-\frac{3xb}{2(a^{2}+2ab+b^{2})}-\frac{xa}{a^{2}-2ab+b^{2}}+\frac{3xb}{2(a^{2}-2ab+b^{2})}-\frac{2x\,b^{4}}{(a^{4}-2a^{2}b^{2}+b^{4})a}-\frac{e^{x}\left(-e^{2x}b+2\,e^{x}a-b\right)}{\left(e^{2x}-1\right)^{2}(a^{2}-b^{2})}+\frac{a\ln(}{a^{2}+2}$

input
```
int(coth(x)^3/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/8*tanh(1/2*x)^2/(a-b)+1/(a-b)^2*b^4/(a+b)^2/a*ln(a*tanh(1/2*x)^2-b*tanh
(1/2*x)^2+a+b)-1/8/(a+b)/tanh(1/2*x)^2+1/4/(a+b)^2*(4*a+6*b)*ln(tanh(1/2*x
))-1/a*ln(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1222 vs. $2(103) = 206$.

Time = 0.15 (sec) , antiderivative size = 1222, normalized size of antiderivative = 10.81

$$\int \frac{\coth^3(x)}{a+b\text{sech}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
-1/2*(2*(a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)^4 + 2*(a^4 - 2*a^2*b^2 + b^4)*x*
sinh(x)^4 - 2*(a^3*b - a*b^3)*cosh(x)^3 - 2*(a^3*b - a*b^3 - 4*(a^4 - 2*a^
2*b^2 + b^4)*x*cosh(x))*sinh(x)^3 + 4*(a^4 - a^2*b^2 - (a^4 - 2*a^2*b^2 +
b^4)*x)*cosh(x)^2 + 2*(2*a^4 - 2*a^2*b^2 + 6*(a^4 - 2*a^2*b^2 + b^4)*x*cos
h(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*x - 3*(a^3*b - a*b^3)*cosh(x))*sinh(x)^
2 + 2*(a^4 - 2*a^2*b^2 + b^4)*x - 2*(a^3*b - a*b^3)*cosh(x) - 2*(b^4*cosh(
x)^4 + 4*b^4*cosh(x)*sinh(x)^3 + b^4*sinh(x)^4 - 2*b^4*cosh(x)^2 + b^4 + 2
*(3*b^4*cosh(x)^2 - b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 - b^4*cosh(x))*sinh(
x))*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) - ((2*a^4 + a^3*b - 4*a^2*b
^2 - 3*a*b^3)*cosh(x)^4 + 4*(2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cosh(x)*
sinh(x)^3 + (2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*sinh(x)^4 + 2*a^4 + a^3*
b - 4*a^2*b^2 - 3*a*b^3 - 2*(2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cosh(x)^
2 - 2*(2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3 - 3*(2*a^4 + a^3*b - 4*a^2*b^2
- 3*a*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)
*cosh(x)^3 - (2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cosh(x))*sinh(x))*log(c
osh(x) + sinh(x) + 1) - ((2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(x)^4 +
 4*(2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(x)*sinh(x)^3 + (2*a^4 - a^3*
b - 4*a^2*b^2 + 3*a*b^3)*sinh(x)^4 + 2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3 -
 2*(2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(x)^2 - 2*(2*a^4 - a^3*b - 4*
a^2*b^2 + 3*a*b^3 - 3*(2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(x)^2)*...
```

Sympy [F]

$$\int \frac{\coth^3(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\coth^3(x)}{a + b\,\text{sech}(x)}\,dx$$

input
```
integrate(coth(x)**3/(a+b*sech(x)),x)
```

output
```
Integral(coth(x)**3/(a + b*sech(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.45

$$\int \frac{\coth^3(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{b^4\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^5-2\,a^3b^2+ab^4} + \frac{(2\,a-3\,b)\log\left(e^{(-x)}+1\right)}{2\,(a^2-2\,ab+b^2)} + \frac{(2\,a+3\,b)\log\left(e^{(-x)}-1\right)}{2\,(a^2+2\,ab+b^2)} + \frac{be^{(-x)}-2\,ae^{(-2\,x)}+be^{(-3\,x)}}{a^2-b^2-2\,(a^2-b^2)e^{(-2\,x)}+(a^2-b^2)e^{(-4\,x)}} + \frac{x}{a}$$

input
```
integrate(coth(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
b^4*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^5 - 2*a^3*b^2 + a*b^4) + 1/2*(2*a
- 3*b)*log(e^(-x) + 1)/(a^2 - 2*a*b + b^2) + 1/2*(2*a + 3*b)*log(e^(-x) -
1)/(a^2 + 2*a*b + b^2) + (b*e^(-x) - 2*a*e^(-2*x) + b*e^(-3*x))/(a^2 - b^2
 - 2*(a^2 - b^2)*e^(-2*x) + (a^2 - b^2)*e^(-4*x)) + x/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.71

$$\int \frac{\coth^3(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{b^4\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{a^5-2\,a^3b^2+ab^4} + \frac{(2\,a-3\,b)\log\left(e^{(-x)}+e^x+2\right)}{4\,(a^2-2\,ab+b^2)} + \frac{(2\,a+3\,b)\log\left(e^{(-x)}+e^x-2\right)}{4\,(a^2+2\,ab+b^2)} - \frac{a^3\left(e^{(-x)}+e^x\right)^2-2\,ab^2\left(e^{(-x)}+e^x\right)^2-2\,a^2b\left(e^{(-x)}+e^x\right)+2\,b^3\left(e^{(-x)}+e^x\right)+4\,ab^2}{2\,(a^4-2\,a^2b^2+b^4)\left(\left(e^{(-x)}+e^x\right)^2-4\right)}$$

input
```
integrate(coth(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

output
```
b^4*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^5 - 2*a^3*b^2 + a*b^4) + 1/4*(2*a
- 3*b)*log(e^(-x) + e^x + 2)/(a^2 - 2*a*b + b^2) + 1/4*(2*a + 3*b)*log(e^(
-x) + e^x - 2)/(a^2 + 2*a*b + b^2) - 1/2*(a^3*(e^(-x) + e^x)^2 - 2*a*b^2*(
e^(-x) + e^x)^2 - 2*a^2*b*(e^(-x) + e^x) + 2*b^3*(e^(-x) + e^x) + 4*a*b^2)
/((a^4 - 2*a^2*b^2 + b^4)*((e^(-x) + e^x)^2 - 4))
```

Mupad [B] (verification not implemented)

Time = 3.08 (sec) , antiderivative size = 339, normalized size of antiderivative = 3.00

$$\int \frac{\coth^3(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{\ln\left(e^x-1\right)\,\left(2\,a+3\,b\right)}{2\,a^2+4\,a\,b+2\,b^2}-\frac{x}{a}-\frac{\frac{2\,a}{a^2-b^2}-\frac{2\,b\,e^x}{a^2-b^2}}{e^{4\,x}-2\,e^{2\,x}+1} - \frac{\frac{2\,\left(a^4-a^2\,b^2\right)}{a\,{\left(a^2-b^2\right)}^2}-\frac{e^x\,\left(a^2\,b-b^3\right)}{{\left(a^2-b^2\right)}^2}}{e^{2\,x}-1}+\frac{\ln\left(e^x+1\right)\,\left(2\,a-3\,b\right)}{2\,a^2-4\,a\,b+2\,b^2} + \frac{b^4\,\ln\left(4\,a^9\,e^{2\,x}+4\,a\,b^8+4\,a^9+7\,a^3\,b^6+14\,a^5\,b^4-17\,a^7\,b^2+8\,b^9\,e^x+7\,a^3\,b^6\,e^{2\,x}+14\,a^5\,b^4\,e^{2\,x}-17\,a^7}{a^5-2\,a^3\,b^2+a\,b^4}$$

input
```
int(coth(x)^3/(a + b/cosh(x)),x)
```

output
```
(log(exp(x) - 1)*(2*a + 3*b))/(4*a*b + 2*a^2 + 2*b^2) - x/a - ((2*a)/(a^2
- b^2) - (2*b*exp(x))/(a^2 - b^2))/(exp(4*x) - 2*exp(2*x) + 1) - ((2*(a^4
- a^2*b^2))/(a*(a^2 - b^2)^2) - (exp(x)*(a^2*b - b^3))/(a^2 - b^2)^2)/(exp
(2*x) - 1) + (log(exp(x) + 1)*(2*a - 3*b))/(2*a^2 - 4*a*b + 2*b^2) + (b^4*
log(4*a^9*exp(2*x) + 4*a*b^8 + 4*a^9 + 7*a^3*b^6 + 14*a^5*b^4 - 17*a^7*b^2
 + 8*b^9*exp(x) + 7*a^3*b^6*exp(2*x) + 14*a^5*b^4*exp(2*x) - 17*a^7*b^2*ex
p(2*x) + 8*a^8*b*exp(x) + 4*a*b^8*exp(2*x) + 14*a^2*b^7*exp(x) + 28*a^4*b^
5*exp(x) - 34*a^6*b^3*exp(x)))/(a*b^4 + a^5 - 2*a^3*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 701, normalized size of antiderivative = 6.20

$$\int \frac{\coth^3(x)}{a+b\operatorname{sech}(x)}\,dx$$
$$= \frac{3\log(e^x-1)\,a\,b^3-3\log(e^x+1)\,a\,b^3-2e^{4x}a^4-2e^{4x}a^4x-2e^{4x}b^4x+2e^{3x}a^3b-2e^{3x}a\,b^3+2e^x a^3b+2e^{4}}{}$$

input
```
int(coth(x)^3/(a+b*sech(x)),x)
```

output
```
(2*e**(4*x)*log(e**x - 1)*a**4 - e**(4*x)*log(e**x - 1)*a**3*b - 4*e**(4*x
)*log(e**x - 1)*a**2*b**2 + 3*e**(4*x)*log(e**x - 1)*a*b**3 + 2*e**(4*x)*l
og(e**x + 1)*a**4 + e**(4*x)*log(e**x + 1)*a**3*b - 4*e**(4*x)*log(e**x +
1)*a**2*b**2 - 3*e**(4*x)*log(e**x + 1)*a*b**3 + 2*e**(4*x)*log(e**(2*x)*a
 + 2*e**x*b + a)*b**4 - 2*e**(4*x)*a**4*x - 2*e**(4*x)*a**4 + 4*e**(4*x)*a
**2*b**2*x + 2*e**(4*x)*a**2*b**2 - 2*e**(4*x)*b**4*x + 2*e**(3*x)*a**3*b
- 2*e**(3*x)*a*b**3 - 4*e**(2*x)*log(e**x - 1)*a**4 + 2*e**(2*x)*log(e**x
- 1)*a**3*b + 8*e**(2*x)*log(e**x - 1)*a**2*b**2 - 6*e**(2*x)*log(e**x - 1
)*a*b**3 - 4*e**(2*x)*log(e**x + 1)*a**4 - 2*e**(2*x)*log(e**x + 1)*a**3*b
 + 8*e**(2*x)*log(e**x + 1)*a**2*b**2 + 6*e**(2*x)*log(e**x + 1)*a*b**3 -
4*e**(2*x)*log(e**(2*x)*a + 2*e**x*b + a)*b**4 + 4*e**(2*x)*a**4*x - 8*e**
(2*x)*a**2*b**2*x + 4*e**(2*x)*b**4*x + 2*e**x*a**3*b - 2*e**x*a*b**3 + 2*
log(e**x - 1)*a**4 - log(e**x - 1)*a**3*b - 4*log(e**x - 1)*a**2*b**2 + 3*
log(e**x - 1)*a*b**3 + 2*log(e**x + 1)*a**4 + log(e**x + 1)*a**3*b - 4*log
(e**x + 1)*a**2*b**2 - 3*log(e**x + 1)*a*b**3 + 2*log(e**(2*x)*a + 2*e**x*
b + a)*b**4 - 2*a**4*x - 2*a**4 + 4*a**2*b**2*x + 2*a**2*b**2 - 2*b**4*x)/
(2*a*(e**(4*x)*a**4 - 2*e**(4*x)*a**2*b**2 + e**(4*x)*b**4 - 2*e**(2*x)*a*
*4 + 4*e**(2*x)*a**2*b**2 - 2*e**(2*x)*b**4 + a**4 - 2*a**2*b**2 + b**4))
```

3.123 $\int \frac{\coth^4(x)}{a+b\,\mathrm{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 207

$$\int \frac{\coth^4(x)}{a+b\,\mathrm{sech}(x)}\,dx = -\frac{ab^2x}{(a^2-b^2)^2}+\frac{b^4x}{a(a^2-b^2)^2}+\frac{ax}{a^2-b^2} - \frac{2b^5\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}}+\frac{ab^2\coth(x)}{(a^2-b^2)^2}-\frac{a\coth(x)}{a^2-b^2} - \frac{a\coth^3(x)}{3(a^2-b^2)}-\frac{b^3\mathrm{csch}(x)}{(a^2-b^2)^2}+\frac{b\,\mathrm{csch}(x)}{a^2-b^2}+\frac{b\,\mathrm{csch}^3(x)}{3(a^2-b^2)}$$

output
```
-a*b^2*x/(a^2-b^2)^2+b^4*x/a/(a^2-b^2)^2+a*x/(a^2-b^2)-2*b^5*arctan((a-b)^
(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a/(a-b)^(5/2)/(a+b)^(5/2)+a*b^2*coth(x)/(a^
2-b^2)^2-a*coth(x)/(a^2-b^2)-a*coth(x)^3/(3*a^2-3*b^2)-b^3*csch(x)/(a^2-b^
2)^2+b*csch(x)/(a^2-b^2)+b*csch(x)^3/(3*a^2-3*b^2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.80

$$\int \frac{\coth^4(x)}{a+b\operatorname{sech}(x)}\,dx$$
$$= \frac{(b+a\cosh(x))\operatorname{sech}(x)\left(\frac{24x}{a}+\frac{48b^5\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a(a^2-b^2)^{5/2}}-\frac{2(8a+11b)\coth\left(\frac{x}{2}\right)}{(a+b)^2}+\frac{8\operatorname{csch}^3(x)\sinh^4\left(\frac{x}{2}\right)}{a-b}-\frac{\operatorname{csch}^4\left(\frac{x}{2}\right)\sin}{2(a+b)}\right.}{24(a+b\operatorname{sech}(x))}$$

input
```
Integrate[Coth[x]^4/(a + b*Sech[x]),x]
```

output
```
((b + a*Cosh[x])*Sech[x]*((24*x)/a + (48*b^5*ArcTan[((-a + b)*Tanh[x/2])/S
qrt[a^2 - b^2]])/(a*(a^2 - b^2)^(5/2)) - (2*(8*a + 11*b)*Coth[x/2])/(a + b
)^2 + (8*Csch[x]^3*Sinh[x/2]^4)/(a - b) - (Csch[x/2]^4*Sinh[x])/(2*(a + b)
) - (16*a*Tanh[x/2])/(a - b)^2 + (22*b*Tanh[x/2])/(a - b)^2))/(24*(a + b*S
ech[x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.39 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.87, number of steps used = 28, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.077, Rules used = {3042, 4386, 3042, 25, 3381, 25, 3042, 25, 3086, 2009, 3381, 25, 3042, 25, 3086, 24, 3214, 3042, 3138, 218, 3954, 24, 25, 3042, 25, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(x)}{a+b\operatorname{sech}(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cot\left(\frac{\pi}{2}+ix\right)^4\left(a+b\csc\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{4386}$$

$$\int \frac{\cosh(x)\coth^4(x)}{a\cosh(x)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin\left(-\frac{\pi}{2}+ix\right)^5}{\cos\left(-\frac{\pi}{2}+ix\right)^4\left(b-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin\left(ix-\frac{\pi}{2}\right)^5}{\cos\left(ix-\frac{\pi}{2}\right)^4\left(b-a\sin\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3381}$$

$$\frac{a\int \coth^4(x)dx}{a^2-b^2}-\frac{b^2\int \frac{\cosh(x)\coth^2(x)}{b+a\cosh(x)}dx}{a^2-b^2}+\frac{b\int -\coth^3(x)\text{csch}(x)dx}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{a\int \coth^4(x)dx}{a^2-b^2}-\frac{b^2\int \frac{\cosh(x)\coth^2(x)}{b+a\cosh(x)}dx}{a^2-b^2}-\frac{b\int \coth^3(x)\text{csch}(x)dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^4dx}{a^2-b^2}-\frac{b^2\int \frac{\sin\left(ix-\frac{\pi}{2}\right)^3}{\cos\left(ix-\frac{\pi}{2}\right)^2\left(b-a\sin\left(ix-\frac{\pi}{2}\right)\right)}dx}{a^2-b^2}-\frac{b\int -\sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^3dx}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^4dx}{a^2-b^2}-\frac{b^2\int \frac{\sin\left(ix-\frac{\pi}{2}\right)^3}{\cos\left(ix-\frac{\pi}{2}\right)^2\left(b-a\sin\left(ix-\frac{\pi}{2}\right)\right)}dx}{a^2-b^2}+\frac{b\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^3dx}{a^2-b^2}$$

$$\Big\downarrow \text{3086}$$

$$\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^4dx}{a^2-b^2}-\frac{ib\int \left(-\text{csch}^2(x)-1\right)d(-i\text{csch}(x))}{a^2-b^2}-\frac{b^2\int \frac{\sin\left(ix-\frac{\pi}{2}\right)^3}{\cos\left(ix-\frac{\pi}{2}\right)^2\left(b-a\sin\left(ix-\frac{\pi}{2}\right)\right)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^4dx}{a^2-b^2}-\frac{b^2\int \frac{\sin\left(ix-\frac{\pi}{2}\right)^3}{\cos\left(ix-\frac{\pi}{2}\right)^2\left(b-a\sin\left(ix-\frac{\pi}{2}\right)\right)}dx}{a^2-b^2}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3381}$$

$$\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^4dx}{a^2-b^2}-\frac{b^2\left(\frac{b^2\int -\frac{\cosh(x)}{b+a\cosh(x)}dx}{a^2-b^2}-\frac{a\int -\coth^2(x)dx}{a^2-b^2}-\frac{b\int \coth(x)\text{csch}(x)dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^4dx}{a^2-b^2}-\frac{b^2\left(-\frac{b^2\int \frac{\cosh(x)}{b+a\cosh(x)}dx}{a^2-b^2}+\frac{a\int \coth^2(x)dx}{a^2-b^2}-\frac{b\int \coth(x)\text{csch}(x)dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^4dx}{a^2-b^2}-\frac{b^2\left(-\frac{b^2\int \frac{\sin\left(ix+\frac{\pi}{2}\right)}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{a\int -\tan\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^4dx}{a^2-b^2}-\frac{b^2\left(-\frac{b^2\int \frac{\sin\left(ix+\frac{\pi}{2}\right)}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 3086}$$

$$\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^4dx}{a^2-b^2}-\frac{b^2\left(-\frac{b^2\int \frac{\sin\left(ix+\frac{\pi}{2}\right)}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}+\frac{ib\int 1d(-i\text{csch}(x))}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^4dx}{a^2-b^2}-\frac{b^2\left(-\frac{b^2\int \frac{\sin\left(ix+\frac{\pi}{2}\right)}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}+\frac{b\text{csch}(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$\big\downarrow$ 3214

$$\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^4dx}{a^2-b^2}-\frac{b^2\left(-\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b^2\left(\frac{x}{a}-\frac{b\int \frac{1}{b+a\cosh(x)}dx}{a}\right)}{a^2-b^2}+\frac{b\text{csch}(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$\big\downarrow$ 3042

$$\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^4dx}{a^2-b^2}-\frac{b^2\left(-\frac{b^2\left(\frac{x}{a}-\frac{b\int \frac{1}{b+a\sin\left(ix+\frac{\pi}{2}\right)}dx}{a}\right)}{a^2-b^2}-\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}+\frac{b\text{csch}(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$\big\downarrow$ 3138

$$\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^4dx}{a^2-b^2}-\frac{b^2\left(-\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b^2\left(\frac{x}{a}-\frac{2b\int \frac{1}{(a-b)\tanh^2\left(\frac{x}{2}\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a}\right)}{a^2-b^2}+\frac{b\text{csch}(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$\big\downarrow$ 218

$$-\frac{b^2\left(-\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b^2\left(\frac{x}{a}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}\right)}{a^2-b^2}+\frac{b\operatorname{csch}(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\int \tan\left(ix+\frac{\pi}{2}\right)^4dx}{a^2-b^2}-\frac{ib\left(\frac{1}{3}i\operatorname{csch}^3(x)+i\operatorname{csch}(x)\right)}{a^2-b^2}$$

$\downarrow$ 3954

$$-\frac{b^2\left(-\frac{a\left(\coth(x)-\int 1dx\right)}{a^2-b^2}-\frac{b^2\left(\frac{x}{a}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}\right)}{a^2-b^2}+\frac{b\operatorname{csch}(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(-\int -\coth^2(x)dx-\frac{1}{3}\coth^3(x)\right)}{a^2-b^2}-\frac{ib\left(\frac{1}{3}i\operatorname{csch}^3(x)+i\operatorname{csch}(x)\right)}{a^2-b^2}$$

$\downarrow$ 24

$$\frac{a\left(-\int -\coth^2(x)dx-\frac{1}{3}\coth^3(x)\right)}{a^2-b^2}-\frac{b^2\left(-\frac{b^2\left(\frac{x}{a}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}\right)}{a^2-b^2}-\frac{a(\coth(x)-x)}{a^2-b^2}+\frac{b\operatorname{csch}(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(\frac{1}{3}i\operatorname{csch}^3(x)+i\operatorname{csch}(x)\right)}{a^2-b^2}$$

$\downarrow$ 25

$$\frac{a\left(\int \coth^2(x)dx-\frac{\coth^3(x)}{3}\right)}{a^2-b^2}-\frac{b^2\left(-\frac{b^2\left(\frac{x}{a}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}\right)}{a^2-b^2}-\frac{a(\coth(x)-x)}{a^2-b^2}+\frac{b\operatorname{csch}(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(\frac{1}{3}i\operatorname{csch}^3(x)+i\operatorname{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(-\frac{\coth^3(x)}{3}+\int -\tan\left(ix+\frac{\pi}{2}\right)^2dx\right)}{a^2-b^2}-\frac{b^2\left(-\frac{b^2\left(\frac{x}{a}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}\right)}{a^2-b^2}-\frac{a(\coth(x)-x)}{a^2-b^2}+\frac{b\text{csch}(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{a\left(-\frac{1}{3}\coth^3(x)-\int \tan\left(ix+\frac{\pi}{2}\right)^2dx\right)}{a^2-b^2}-\frac{b^2\left(-\frac{b^2\left(\frac{x}{a}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}\right)}{a^2-b^2}-\frac{a(\coth(x)-x)}{a^2-b^2}+\frac{b\text{csch}(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3954}$$

$$\frac{a\left(\int 1dx-\frac{1}{3}\coth^3(x)-\coth(x)\right)}{a^2-b^2}-\frac{b^2\left(-\frac{b^2\left(\frac{x}{a}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}\right)}{a^2-b^2}-\frac{a(\coth(x)-x)}{a^2-b^2}+\frac{b\text{csch}(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{b^2\left(-\frac{b^2\left(\frac{x}{a}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}\right)}{a^2-b^2}-\frac{a(\coth(x)-x)}{a^2-b^2}+\frac{b\text{csch}(x)}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\left(x-\frac{1}{3}\coth^3(x)-\coth(x)\right)}{a^2-b^2}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

input
```
Int[Coth[x]^4/(a + b*Sech[x]),x]
```

output
```
(a*(x - Coth[x] - Coth[x]^3/3))/(a^2 - b^2) - (b^2*(-((b^2*(x/a - (2*b*Arc
Tan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b])))/(a
^2 - b^2)) - (a*(-x + Coth[x]))/(a^2 - b^2) + (b*Csch[x])/(a^2 - b^2)))/(a
^2 - b^2) - (I*b*(I*Csch[x] + (I/3)*Csch[x]^3))/(a^2 - b^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3381
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a*(d^2/(a^2
- b^2))   Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Simp[
b*(d/(a^2 - b^2))   Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x]
 - Simp[a^2*(d^2/(g^2*(a^2 - b^2)))   Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[
e + f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1
]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4386
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.74

method	result
default	$-\frac{\frac{a\tanh\left(\frac{x}{2}\right)^{3}}{3}-\frac{b\tanh\left(\frac{x}{2}\right)^{3}}{3}+5a\tanh\left(\frac{x}{2}\right)-7\tanh\left(\frac{x}{2}\right)b}{8(a-b)^{2}}-\frac{1}{24(a+b)\tanh\left(\frac{x}{2}\right)^{3}}-\frac{5a+7b}{8(a+b)^{2}\tanh\left(\frac{x}{2}\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{\ln(\tan}{}$
risch	$\frac{x}{a}-\frac{2\left(-3a^{2}b\,e^{5x}+6\,e^{5x}b^{3}+6a^{3}e^{4x}-9a\,b^{2}e^{4x}+2a^{2}b\,e^{3x}-8b^{3}e^{3x}-6a^{3}e^{2x}+12a\,b^{2}e^{2x}-3b\,e^{x}a^{2}+6b^{3}e^{x}+4a^{3}-7a\,b^{2}\right)}{3(a^{2}-b^{2})^{2}(e^{2x}-1)^{3}}-\frac{b^{5}\ln\left(e^{x}+\right.}{\sqrt{-a^{2}+}}$

input
```
int(coth(x)^4/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/8/(a-b)^2*(1/3*a*tanh(1/2*x)^3-1/3*b*tanh(1/2*x)^3+5*a*tanh(1/2*x)-7*ta
nh(1/2*x)*b)-1/24/(a+b)/tanh(1/2*x)^3-1/8*(5*a+7*b)/(a+b)^2/tanh(1/2*x)+1/
a*ln(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)-2/a/(a-b)^2/(a+b)^2*b^5/((a-b)*(
a+b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a-b)*(a+b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1725 vs. 2(193) = 386.

Time = 0.13 (sec) , antiderivative size = 3530, normalized size of antiderivative = 17.05

$$\int \frac{\coth^4(x)}{a+b\operatorname{sech}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth^4(x)}{a+b\operatorname{sech}(x)}\,dx = \int \frac{\coth^4{(x)}}{a+b\operatorname{sech}{(x)}}\,dx$$

input
```
integrate(coth(x)**4/(a+b*sech(x)),x)
```

output
```
Integral(coth(x)**4/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\coth^4(x)}{a+b\operatorname{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(coth(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.92

$$\int \frac{\coth^4(x)}{a+b\operatorname{sech}(x)}\,dx = -\frac{2\,b^5\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{(a^5-2\,a^3b^2+ab^4)\sqrt{a^2-b^2}}+\frac{x}{a} + \frac{2\left(3\,a^2be^{(5\,x)}-6\,b^3e^{(5\,x)}-6\,a^3e^{(4\,x)}+9\,ab^2e^{(4\,x)}-2\,a^2be^{(3\,x)}+8\,b^3e^{(3\,x)}+6\,a^3e^{(2\,x)}-12\,ab^2e^{(2\,x)}+3\,a^2\cdots\right.}{3\,(a^4-2\,a^2b^2+b^4)(e^{(2\,x)}-1)^3}$$

input
```
integrate(coth(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

output
```
-2*b^5*arctan((a*e^x + b)/sqrt(a^2 - b^2))/((a^5 - 2*a^3*b^2 + a*b^4)*sqrt
(a^2 - b^2)) + x/a + 2/3*(3*a^2*b*e^(5*x) - 6*b^3*e^(5*x) - 6*a^3*e^(4*x)
+ 9*a*b^2*e^(4*x) - 2*a^2*b*e^(3*x) + 8*b^3*e^(3*x) + 6*a^3*e^(2*x) - 12*a
*b^2*e^(2*x) + 3*a^2*b*e^x - 6*b^3*e^x - 4*a^3 + 7*a*b^2)/((a^4 - 2*a^2*b^
2 + b^4)*(e^(2*x) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 713, normalized size of antiderivative = 3.44

$$\int \frac{\coth^4(x)}{a+b\mathrm{sech}(x)}\,dx$$
$$= \frac{x}{a} - \frac{\frac{8a}{3(a^2-b^2)} - \frac{8b\,e^x}{3(a^2-b^2)}}{3e^{2x}-3e^{4x}+e^{6x}-1} - \frac{\frac{2(2a^4-3a^2b^2)}{a(a^2-b^2)^2} - \frac{2e^x(a^2b-2b^3)}{(a^2-b^2)^2}}{e^{2x}-1} - \frac{\frac{4(a^4-a^2b^2)}{a(a^2-b^2)^2} - \frac{8e^x(a^2b-b^3)}{3(a^2-b^2)^2}}{e^{4x}-2e^{2x}+1}$$
$$- \frac{2\,\mathrm{atan}\left(\left(\left(e^x\left(\frac{2b^5}{a^3(a^2-b^2)^2\sqrt{b^{10}}(a^5-2a^3b^2+ab^4)} + \frac{2\left(a\,b^5\sqrt{b^{10}}-2a^3b^3\sqrt{b^{10}}+a^5\,b\sqrt{b^{10}}\right)}{a^2b^4\sqrt{a^2(a^2-b^2)^5}\,(a^5-2a^3b^2+ab^4)\sqrt{a^{12}-5a^{10}b^2+10a^8b^4-10a^6b^6+5a^4}}\right.\right.\right.\right.}{}$$

input
```
int(coth(x)^4/(a + b/cosh(x)),x)
```

output
```
x/a - ((8*a)/(3*(a^2 - b^2)) - (8*b*exp(x))/(3*(a^2 - b^2)))/(3*exp(2*x) -
 3*exp(4*x) + exp(6*x) - 1) - ((2*(2*a^4 - 3*a^2*b^2))/(a*(a^2 - b^2)^2) -
 (2*exp(x)*(a^2*b - 2*b^3))/(a^2 - b^2)^2)/(exp(2*x) - 1) - ((4*(a^4 - a^2
*b^2))/(a*(a^2 - b^2)^2) - (8*exp(x)*(a^2*b - b^3))/(3*(a^2 - b^2)^2))/(ex
p(4*x) - 2*exp(2*x) + 1) - (2*atan((exp(x)*((2*b^5)/(a^3*(a^2 - b^2)^2*(b^
10)^(1/2)*(a*b^4 + a^5 - 2*a^3*b^2)) + (2*(a*b^5*(b^10)^(1/2) - 2*a^3*b^3*
(b^10)^(1/2) + a^5*b*(b^10)^(1/2)))/(a^2*b^4*(a^2*(a^2 - b^2)^5)^(1/2)*(a*
b^4 + a^5 - 2*a^3*b^2)*(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*
b^4 - 5*a^10*b^2)^(1/2))) + (2*(a^6*(b^10)^(1/2) + a^2*b^4*(b^10)^(1/2) -
2*a^4*b^2*(b^10)^(1/2)))/(a^2*b^4*(a^2*(a^2 - b^2)^5)^(1/2)*(a*b^4 + a^5 -
 2*a^3*b^2)*(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^1
0*b^2)^(1/2)))*((a^6*(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^
4 - 5*a^10*b^2)^(1/2))/2 + (a^2*b^4*(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*
b^6 + 10*a^8*b^4 - 5*a^10*b^2)^(1/2))/2 - a^4*b^2*(a^12 - a^2*b^10 + 5*a^4
*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2)^(1/2)))*(b^10)^(1/2))/(a^12 -
 a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 19.31 (sec) , antiderivative size = 691, normalized size of antiderivative = 3.34

$$\int \frac{\coth^4(x)}{a+b\operatorname{sech}(x)}\,dx$$
$$= \frac{-4e^{6x}a^6 - 9a^2b^4x + 6\sqrt{a^2-b^2}\,atan\left(\frac{e^x a+b}{\sqrt{a^2-b^2}}\right)b^5 + 3e^{6x}a^6x + 9e^{2x}a^6x + 9a^4b^2x - 3a^6x + 3b^6x + 6e^{5x}a^5b}{}$$

input
```
int(coth(x)^4/(a+b*sech(x)),x)
```

output
```
( - 6*e**(6*x)*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2))*b**5
 + 18*e**(4*x)*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2))*b**5
 - 18*e**(2*x)*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2))*b**5
 + 6*sqrt(a**2 - b**2)*atan((e**x*a + b)/sqrt(a**2 - b**2))*b**5 + 3*e**(6
*x)*a**6*x - 4*e**(6*x)*a**6 - 9*e**(6*x)*a**4*b**2*x + 10*e**(6*x)*a**4*b
**2 + 9*e**(6*x)*a**2*b**4*x - 6*e**(6*x)*a**2*b**4 - 3*e**(6*x)*b**6*x +
6*e**(5*x)*a**5*b - 18*e**(5*x)*a**3*b**3 + 12*e**(5*x)*a*b**5 - 9*e**(4*x
)*a**6*x + 27*e**(4*x)*a**4*b**2*x - 27*e**(4*x)*a**2*b**4*x + 9*e**(4*x)*
b**6*x - 4*e**(3*x)*a**5*b + 20*e**(3*x)*a**3*b**3 - 16*e**(3*x)*a*b**5 +
9*e**(2*x)*a**6*x - 27*e**(2*x)*a**4*b**2*x - 6*e**(2*x)*a**4*b**2 + 27*e*
*(2*x)*a**2*b**4*x + 6*e**(2*x)*a**2*b**4 - 9*e**(2*x)*b**6*x + 6*e**x*a**
5*b - 18*e**x*a**3*b**3 + 12*e**x*a*b**5 - 3*a**6*x - 4*a**6 + 9*a**4*b**2
*x + 12*a**4*b**2 - 9*a**2*b**4*x - 8*a**2*b**4 + 3*b**6*x)/(3*a*(e**(6*x)
*a**6 - 3*e**(6*x)*a**4*b**2 + 3*e**(6*x)*a**2*b**4 - e**(6*x)*b**6 - 3*e*
*(4*x)*a**6 + 9*e**(4*x)*a**4*b**2 - 9*e**(4*x)*a**2*b**4 + 3*e**(4*x)*b**
6 + 3*e**(2*x)*a**6 - 9*e**(2*x)*a**4*b**2 + 9*e**(2*x)*a**2*b**4 - 3*e**(
2*x)*b**6 - a**6 + 3*a**4*b**2 - 3*a**2*b**4 + b**6))
```

3.124 $\int \frac{\coth^5(x)}{a+b\,\mathrm{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 178

$$\int \frac{\coth^5(x)}{a+b\,\mathrm{sech}(x)}\,dx = \frac{\log(\cosh(x))}{a} + \frac{(8a^2+21ab+15b^2)\log(1-\mathrm{sech}(x))}{16(a+b)^3} + \frac{(8a^2-21ab+15b^2)\log(1+\mathrm{sech}(x))}{16(a-b)^3} - \frac{b^6\log(a+b\,\mathrm{sech}(x))}{a\,(a^2-b^2)^3} - \frac{1}{16(a+b)(1-\mathrm{sech}(x))^2} - \frac{5a+7b}{16(a+b)^2(1-\mathrm{sech}(x))} - \frac{1}{16(a-b)(1+\mathrm{sech}(x))^2} - \frac{5a-7b}{16(a-b)^2(1+\mathrm{sech}(x))}$$

output
```
ln(cosh(x))/a+1/16*(8*a^2+21*a*b+15*b^2)*ln(1-sech(x))/(a+b)^3+1/16*(8*a^2
-21*a*b+15*b^2)*ln(1+sech(x))/(a-b)^3-b^6*ln(a+b*sech(x))/a/(a^2-b^2)^3-1/
16/(a+b)/(1-sech(x))^2-1/16*(5*a+7*b)/(a+b)^2/(1-sech(x))-1/16/(a-b)/(1+se
ch(x))^2-1/16*(5*a-7*b)/(a-b)^2/(1+sech(x))
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.92

$$\int \frac{\coth^5(x)}{a+b\text{sech}(x)}\,dx=\frac{1}{16}\left(\frac{16\log(\cosh(x))}{a}+\frac{(8a^2+21ab+15b^2)\log(1-\text{sech}(x))}{(a+b)^3}+\frac{(8a^2-21ab+15b^2)\log(1+\text{sech}(x))}{(a-b)^3}-\frac{16b^6\log(a+b\text{sech}(x))}{a(a-b)^3(a+b)^3}-\frac{1}{(a+b)(-1+\text{sech}(x))^2}+\frac{5a+7b}{(a+b)^2(-1+\text{sech}(x))}-\frac{1}{(a-b)(1+\text{sech}(x))^2}+\frac{-5a+7b}{(a-b)^2(1+\text{sech}(x))}\right)$$

input
```
Integrate[Coth[x]^5/(a + b*Sech[x]),x]
```

output
```
((16*Log[Cosh[x]])/a + ((8*a^2 + 21*a*b + 15*b^2)*Log[1 - Sech[x]])/(a + b
)^3 + ((8*a^2 - 21*a*b + 15*b^2)*Log[1 + Sech[x]])/(a - b)^3 - (16*b^6*Log
[a + b*Sech[x]])/(a*(a - b)^3*(a + b)^3) - 1/((a + b)*(-1 + Sech[x])^2) +
(5*a + 7*b)/((a + b)^2*(-1 + Sech[x])) - 1/((a - b)*(1 + Sech[x])^2) + (-5
*a + 7*b)/((a - b)^2*(1 + Sech[x])))/16
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 26, 4373, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^5(x)}{a+b\text{sech}(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\cot\left(\frac{\pi}{2}+ix\right)^5\left(a+b\csc\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\cot\left(ix+\frac{\pi}{2}\right)^5\left(a+b\csc\left(ix+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4373}$$

$$-b^6\int \frac{\cosh(x)}{b(a+b\text{sech}(x))\left(b^2-b^2\text{sech}^2(x)\right)^3}d(b\text{sech}(x))$$

$$\Big\downarrow \text{615}$$

$$-b^6\int \left(\frac{7b-5a}{16(a-b)^2b^5(\text{sech}(x)b+b)^2}+\frac{\cosh(x)}{ab^7}+\frac{8a^2+21ba+15b^2}{16b^6(a+b)^3(b-b\text{sech}(x))}+\frac{1}{a(a-b)^3(a+b)^3(a+b\text{sech}(x))}\right.$$

$$\Big\downarrow \text{2009}$$

$$-b^6\left(\frac{\log(a+b\text{sech}(x))}{a\left(a^2-b^2\right)^3}-\frac{\left(8a^2+21ab+15b^2\right)\log(b-b\text{sech}(x))}{16b^6(a+b)^3}-\frac{\left(8a^2-21ab+15b^2\right)\log(b\text{sech}(x)+b)}{16b^6(a-b)^3}+\log\right.$$

input
```
Int[Coth[x]^5/(a + b*Sech[x]),x]
```

output
```
-(b^6*(Log[b*Sech[x]]/(a*b^6) - ((8*a^2 + 21*a*b + 15*b^2)*Log[b - b*Sech[
x]])/(16*b^6*(a + b)^3) + Log[a + b*Sech[x]]/(a*(a^2 - b^2)^3) - ((8*a^2 -
 21*a*b + 15*b^2)*Log[b + b*Sech[x]])/(16*(a - b)^3*b^6) + 1/(16*b^4*(a +
b)*(b - b*Sech[x])^2) + (5*a + 7*b)/(16*b^5*(a + b)^2*(b - b*Sech[x])) + 1
/(16*(a - b)*b^4*(b + b*Sech[x])^2) + (5*a - 7*b)/(16*(a - b)^2*b^5*(b + b
*Sech[x]))))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.91

method	result
default	$-\frac{\left(a\tanh\left(\frac{x}{2}\right)^{2}-b\tanh\left(\frac{x}{2}\right)^{2}+6a-8b\right)^{2}}{64(a-b)^{3}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{b^{6}\ln\left(a\tanh\left(\frac{x}{2}\right)^{2}-b\tanh\left(\frac{x}{2}\right)^{2}+a+b\right)}{(a-b)^{3}(a+b)^{3}a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{}{64}$
risch	$\frac{x}{a}-\frac{x\,a^{2}}{a^{3}+3a^{2}b+3a\,b^{2}+b^{3}}-\frac{21xab}{8(a^{3}+3a^{2}b+3a\,b^{2}+b^{3})}-\frac{15x\,b^{2}}{8(a^{3}+3a^{2}b+3a\,b^{2}+b^{3})}-\frac{x\,a^{2}}{a^{3}-3a^{2}b+3a\,b^{2}-b^{3}}+\frac{21xab}{8(a^{3}-3a^{2}b+3a\,b^{2}-b}$

input
```
int(coth(x)^5/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/64*(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2+6*a-8*b)^2/(a-b)^3-1/a*ln(tanh(1/2*
x)-1)-1/(a-b)^3*b^6/(a+b)^3/a*ln(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2+a+b)-1/a*
ln(tanh(1/2*x)+1)-1/64/(a+b)/tanh(1/2*x)^4-1/32*(6*a+8*b)/(a+b)^2/tanh(1/2
*x)^2+1/16/(a+b)^3*(16*a^2+42*a*b+30*b^2)*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5181 vs. $2(162) = 324$.

Time = 0.20 (sec) , antiderivative size = 5181, normalized size of antiderivative = 29.11

$$\int \frac{\coth^5(x)}{a+b\text{sech}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^5/(a+b*sech(x)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth^5(x)}{a+b\text{sech}(x)}\,dx = \int \frac{\coth^5(x)}{a+b\,\text{sech}(x)}\,dx$$

input
```
integrate(coth(x)**5/(a+b*sech(x)),x)
```

output
```
Integral(coth(x)**5/(a + b*sech(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(162) = 324$.

Time = 0.05 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.06

$$\int \frac{\coth^5(x)}{a+b\text{sech}(x)}\,dx = -\frac{b^6\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^7-3\,a^5b^2+3\,a^3b^4-ab^6} + \frac{(8\,a^2-21\,ab+15\,b^2)\log\left(e^{(-x)}+1\right)}{8\,(a^3-3\,a^2b+3\,ab^2-b^3)} + \frac{(8\,a^2+21\,ab+15\,b^2)\log\left(e^{(-x)}-1\right)}{8\,(a^3+3\,a^2b+3\,ab^2+b^3)} + \frac{(5\,a^2b-9\,b^3)e^{(-x)}-8\,(2\,a^3-3\,ab^2)e^{(-2\,x)}+(3\,a^2b+b^3)e^{(-3\,x)}+16\,(a^3-2\,ab^2)e^{(-4\,x)}+(3\,a^2b+b^3)e^{\cdots}}{4\,(a^4-2\,a^2b^2+b^4-4\,(a^4-2\,a^2b^2+b^4)e^{(-2\,x)}+6\,(a^4-2\,a^2b^2+b^4)e^{(-4\,x)}-4\,(a^4-2\,a^2b^2\cdots} + \frac{x}{a}$$

input
```
integrate(coth(x)^5/(a+b*sech(x)),x, algorithm="maxima")
```

output
```
-b^6*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6
) + 1/8*(8*a^2 - 21*a*b + 15*b^2)*log(e^(-x) + 1)/(a^3 - 3*a^2*b + 3*a*b^2
 - b^3) + 1/8*(8*a^2 + 21*a*b + 15*b^2)*log(e^(-x) - 1)/(a^3 + 3*a^2*b + 3
*a*b^2 + b^3) + 1/4*((5*a^2*b - 9*b^3)*e^(-x) - 8*(2*a^3 - 3*a*b^2)*e^(-2*
x) + (3*a^2*b + b^3)*e^(-3*x) + 16*(a^3 - 2*a*b^2)*e^(-4*x) + (3*a^2*b + b
^3)*e^(-5*x) - 8*(2*a^3 - 3*a*b^2)*e^(-6*x) + (5*a^2*b - 9*b^3)*e^(-7*x))/
(a^4 - 2*a^2*b^2 + b^4 - 4*(a^4 - 2*a^2*b^2 + b^4)*e^(-2*x) + 6*(a^4 - 2*a
^2*b^2 + b^4)*e^(-4*x) - 4*(a^4 - 2*a^2*b^2 + b^4)*e^(-6*x) + (a^4 - 2*a^2
*b^2 + b^4)*e^(-8*x)) + x/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(162) = 324$.

Time = 0.12 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.13

$$\int \frac{\coth^5(x)}{a+b\operatorname{sech}(x)}\,dx$$
$$= -\frac{b^6\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{a^7-3\,a^5b^2+3\,a^3b^4-ab^6}+\frac{(8\,a^2-21\,ab+15\,b^2)\log\left(e^{(-x)}+e^x+2\right)}{16\,(a^3-3\,a^2b+3\,ab^2-b^3)}$$
$$+\frac{(8\,a^2+21\,ab+15\,b^2)\log\left(e^{(-x)}+e^x-2\right)}{16\,(a^3+3\,a^2b+3\,ab^2+b^3)}$$
$$-\frac{3\,a^5\left(e^{(-x)}+e^x\right)^4-9\,a^3b^2\left(e^{(-x)}+e^x\right)^4+9\,ab^4\left(e^{(-x)}+e^x\right)^4-5\,a^4b\left(e^{(-x)}+e^x\right)^3+14\,a^2b^3\left(e^{(-x)}+e^x\right)}{}$$

input
```
integrate(coth(x)^5/(a+b*sech(x)),x, algorithm="giac")
```

output
```
-b^6*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6
) + 1/16*(8*a^2 - 21*a*b + 15*b^2)*log(e^(-x) + e^x + 2)/(a^3 - 3*a^2*b +
3*a*b^2 - b^3) + 1/16*(8*a^2 + 21*a*b + 15*b^2)*log(e^(-x) + e^x - 2)/(a^3
 + 3*a^2*b + 3*a*b^2 + b^3) - 1/4*(3*a^5*(e^(-x) + e^x)^4 - 9*a^3*b^2*(e^(
-x) + e^x)^4 + 9*a*b^4*(e^(-x) + e^x)^4 - 5*a^4*b*(e^(-x) + e^x)^3 + 14*a^
2*b^3*(e^(-x) + e^x)^3 - 9*b^5*(e^(-x) + e^x)^3 - 8*a^5*(e^(-x) + e^x)^2 +
 32*a^3*b^2*(e^(-x) + e^x)^2 - 48*a*b^4*(e^(-x) + e^x)^2 + 12*a^4*b*(e^(-x
) + e^x) - 40*a^2*b^3*(e^(-x) + e^x) + 28*b^5*(e^(-x) + e^x) - 16*a^3*b^2
+ 64*a*b^4)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*((e^(-x) + e^x)^2 - 4)^2)
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 623, normalized size of antiderivative = 3.50

$$\int \frac{\coth^5(x)}{a+b\mathrm{sech}(x)}\,dx = \frac{\ln(e^x-1)\,(8a^2+21ab+15b^2)}{8a^3+24a^2b+24ab^2+8b^3} - \frac{\frac{2(4a^4-5a^2b^2)}{a(a^2-b^2)^2} - \frac{e^x(9a^2b-13b^3)}{2(a^2-b^2)^2}}{e^{4x}-2e^{2x}+1}$$
$$- \frac{\frac{2(2a^6-5a^4b^2+3a^2b^4)}{a(a^2-b^2)^3} - \frac{e^x(5a^4b-14a^2b^3+9b^5)}{4(a^2-b^2)^3}}{e^{2x}-1} - \frac{\frac{8(a^4-a^2b^2)}{a(a^2-b^2)^2} - \frac{6e^x(a^2b-b^3)}{(a^2-b^2)^2}}{3e^{2x}-3e^{4x}+e^{6x}-1}$$
$$- \frac{x}{a} - \frac{\frac{4a}{a^2-b^2} - \frac{4be^x}{a^2-b^2}}{6e^{4x}-4e^{2x}-4e^{6x}+e^{8x}+1} + \frac{\ln(e^x+1)\,(8a^2-21ab+15b^2)}{8a^3-24a^2b+24ab^2-8b^3}$$
$$+ \frac{b^6\ln(64a^{13}e^{2x}+64ab^{12}+64a^{13}+159a^3b^{10}+492a^5b^8-1214a^7b^6+1020a^9b^4-393a^{11}b^2+128}{}$$

input
```
int(coth(x)^5/(a + b/cosh(x)),x)
```

output
```
(log(exp(x) - 1)*(21*a*b + 8*a^2 + 15*b^2))/(24*a*b^2 + 24*a^2*b + 8*a^3 +
 8*b^3) - ((2*(4*a^4 - 5*a^2*b^2))/(a*(a^2 - b^2)^2) - (exp(x)*(9*a^2*b -
13*b^3))/(2*(a^2 - b^2)^2))/(exp(4*x) - 2*exp(2*x) + 1) - ((2*(2*a^6 + 3*a
^2*b^4 - 5*a^4*b^2))/(a*(a^2 - b^2)^3) - (exp(x)*(5*a^4*b + 9*b^5 - 14*a^2
*b^3))/(4*(a^2 - b^2)^3))/(exp(2*x) - 1) - ((8*(a^4 - a^2*b^2))/(a*(a^2 -
b^2)^2) - (6*exp(x)*(a^2*b - b^3))/(a^2 - b^2)^2)/(3*exp(2*x) - 3*exp(4*x)
 + exp(6*x) - 1) - x/a - ((4*a)/(a^2 - b^2) - (4*b*exp(x))/(a^2 - b^2))/(6
*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1) + (log(exp(x) + 1)*(8*
a^2 - 21*a*b + 15*b^2))/(24*a*b^2 - 24*a^2*b + 8*a^3 - 8*b^3) + (b^6*log(6
4*a^13*exp(2*x) + 64*a*b^12 + 64*a^13 + 159*a^3*b^10 + 492*a^5*b^8 - 1214*
a^7*b^6 + 1020*a^9*b^4 - 393*a^11*b^2 + 128*b^13*exp(x) + 159*a^3*b^10*exp
(2*x) + 492*a^5*b^8*exp(2*x) - 1214*a^7*b^6*exp(2*x) + 1020*a^9*b^4*exp(2*
x) - 393*a^11*b^2*exp(2*x) + 128*a^12*b*exp(x) + 64*a*b^12*exp(2*x) + 318*
a^2*b^11*exp(x) + 984*a^4*b^9*exp(x) - 2428*a^6*b^7*exp(x) + 2040*a^8*b^5*
exp(x) - 786*a^10*b^3*exp(x)))/(a*b^6 - a^7 - 3*a^3*b^4 + 3*a^5*b^2)
```

Reduce [F]

$$\int \frac{\coth^5(x)}{a + b\text{sech}(x)}\, dx = \int \frac{\coth(x)^5}{a + b\,\text{sech}(x)} dx$$

input
```
int(coth(x)^5/(a+b*sech(x)),x)
```

output
```
int(coth(x)^5/(a+b*sech(x)),x)
```

3.125 $\int \sqrt{a+b\operatorname{sech}(c+dx)} \tanh^5(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 169

$$\int \sqrt{a+b\operatorname{sech}(c+dx)} \tanh^5(c+dx)\,dx = \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{a+b\operatorname{sech}(c+dx)}}{d} + \frac{2a(a^2-2b^2)\,(a+b\operatorname{sech}(c+dx))^{3/2}}{3b^4d} - \frac{2(3a^2-2b^2)\,(a+b\operatorname{sech}(c+dx))^{5/2}}{5b^4d} + \frac{6a(a+b\operatorname{sech}(c+dx))^{7/2}}{7b^4d} - \frac{2(a+b\operatorname{sech}(c+dx))^{9/2}}{9b^4d}$$

output
```
2*a^(1/2)*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/d-2*(a+b*sech(d*x+c))^(
1/2)/d+2/3*a*(a^2-2*b^2)*(a+b*sech(d*x+c))^(3/2)/b^4/d-2/5*(3*a^2-2*b^2)*(
a+b*sech(d*x+c))^(5/2)/b^4/d+6/7*a*(a+b*sech(d*x+c))^(7/2)/b^4/d-2/9*(a+b*
sech(d*x+c))^(9/2)/b^4/d
```

Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.91

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^5(c+dx)\,dx =$$

$$-\frac{-2\sqrt{a}b^4\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)+2b^4\sqrt{a+b\text{sech}(c+dx)}-\frac{2}{3}a(a^2-2b^2)\,(a+b\text{sech}(c+dx))^{3/2}+}{b^4d}$$

input
```
Integrate[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^5,x]
```

output
```
-((-2*Sqrt[a]*b^4*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]] + 2*b^4*Sqrt[
a + b*Sech[c + d*x]] - (2*a*(a^2 - 2*b^2)*(a + b*Sech[c + d*x])^(3/2))/3 +
 (2*(3*a^2 - 2*b^2)*(a + b*Sech[c + d*x])^(5/2))/5 - (6*a*(a + b*Sech[c +
d*x])^(7/2))/7 + (2*(a + b*Sech[c + d*x])^(9/2))/9)/(b^4*d))
```

Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4373, 517, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^5(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int i\cot\left(ic+idx+\frac{\pi}{2}\right)^5\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \cot\left(\frac{1}{2}(2ic+\pi)+idx\right)^5\sqrt{a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)}dx$$

$$\Big\downarrow \text{ 4373}$$

$$-\frac{\int \frac{\cosh(c+dx)\sqrt{a+b\text{sech}(c+dx)}\left(b^2-b^2\text{sech}^2(c+dx)\right)^2}{b}d(b\text{sech}(c+dx))}{b^4d}$$

$$\Big\downarrow \text{ 517}$$

$$-\frac{2\int -\frac{b^2\text{sech}^2(c+dx)\left(b^4\text{sech}^4(c+dx)-2ab^2\text{sech}^2(c+dx)+a^2-b^2\right)^2}{a-b^2\text{sech}^2(c+dx)}d\sqrt{a+b\text{sech}(c+dx)}}{b^4d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\int \frac{b^2\text{sech}^2(c+dx)\left(b^4\text{sech}^4(c+dx)-2ab^2\text{sech}^2(c+dx)+a^2-b^2\right)^2}{a-b^2\text{sech}^2(c+dx)}d\sqrt{a+b\text{sech}(c+dx)}}{b^4d}$$

$$\Big\downarrow \text{ 1584}$$

$$\frac{2\int \left(-b^8\text{sech}^8(c+dx)+3ab^6\text{sech}^6(c+dx)-b^4\left(3a^2-2b^2\right)\text{sech}^4(c+dx)+ab^2\left(a^2-2b^2\right)\text{sech}^2(c+dx)-b^4+\right.}{b^4d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2\left(\frac{1}{5}b^5\left(3a^2-2b^2\right)\text{sech}^5(c+dx)-\frac{1}{3}ab^3\left(a^2-2b^2\right)\text{sech}^3(c+dx)-\sqrt{a}b^4\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)-\frac{3}{7}ab^7\text{sec}\right.}{b^4d}$$

input
```
Int[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^5,x]
```

output
```
(-2*(-(Sqrt[a]*b^4*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]]) + b^5*Sech[
c + d*x] - (a*b^3*(a^2 - 2*b^2)*Sech[c + d*x]^3)/3 + (b^5*(3*a^2 - 2*b^2)*
Sech[c + d*x]^5)/5 - (3*a*b^7*Sech[c + d*x]^7)/7 + (b^9*Sech[c + d*x]^9)/9
))/(b^4*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 517
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> Simp[2*(e^m/d^(m + 2*p + 1))   Subst[Int[x^(2*n + 1)*(-c + x^
2)^m*(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4)^p, x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && ILtQ[m, 0] && IntegerQ[n + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sqrt{a+b\,\mathrm{sech}\,(dx+c)}\,\tanh\,(dx+c)^5\,dx$$

input
```
int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x)
```

output
```
int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2052 vs. $2(145) = 290$.

Time = 0.65 (sec) , antiderivative size = 4363, normalized size of antiderivative = 25.82

$$\int \sqrt{a+b\mathrm{sech}(c+dx)}\tanh^5(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \sqrt{a+b\mathrm{sech}(c+dx)}\tanh^5(c+dx)\,dx = \int \sqrt{a+b\,\mathrm{sech}\,(c+dx)}\tanh^5\,(c+dx)\,dx$$

input
```
integrate((a+b*sech(d*x+c))**(1/2)*tanh(d*x+c)**5,x)
```

output
```
Integral(sqrt(a + b*sech(c + d*x))*tanh(c + d*x)**5, x)
```

Maxima [F]

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^5(c+dx)\,dx=\int \sqrt{b\,\text{sech}\left(dx+c\right)+a}\tanh\left(dx+c\right)^5\,dx$$

input
```
integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^5, x)
```

Giac [F]

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^5(c+dx)\,dx=\int \sqrt{b\,\text{sech}\left(dx+c\right)+a}\tanh\left(dx+c\right)^5\,dx$$

input
```
integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^5(c+dx)\,dx=\int \tanh(c+d\,x)^5\,\sqrt{a+\frac{b}{\cosh\left(c+d\,x\right)}}\,dx$$

input
```
int(tanh(c + d*x)^5*(a + b/cosh(c + d*x))^(1/2),x)
```

output
```
int(tanh(c + d*x)^5*(a + b/cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^5(c+dx)\,dx$$
$$=\frac{-10\sqrt{\text{sech}\,(dx+c)\,b+a}\,\tanh\,(dx+c)^4-16\sqrt{\text{sech}\,(dx+c)\,b+a}\,\tanh\,(dx+c)^2-64\sqrt{\text{sech}\,(dx+c)\,b}}{}$$

input

```
int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x)
```

output

```
( - 10*sqrt(sech(c + d*x)*b + a)*tanh(c + d*x)**4 - 16*sqrt(sech(c + d*x)*
b + a)*tanh(c + d*x)**2 - 64*sqrt(sech(c + d*x)*b + a) + 5*int((sqrt(sech(
c + d*x)*b + a)*tanh(c + d*x)**5)/(sech(c + d*x)*b + a),x)*a*d + 8*int((sq
rt(sech(c + d*x)*b + a)*tanh(c + d*x)**3)/(sech(c + d*x)*b + a),x)*a*d + 3
2*int((sqrt(sech(c + d*x)*b + a)*tanh(c + d*x))/(sech(c + d*x)*b + a),x)*a
*d)/(45*d)
```

3.126 $\int \sqrt{a+b\text{sech}(c+dx)}\tanh^3(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^3(c+dx)\,dx = \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{a+b\text{sech}(c+dx)}}{d} - \frac{2a(a+b\text{sech}(c+dx))^{3/2}}{3b^2d} + \frac{2(a+b\text{sech}(c+dx))^{5/2}}{5b^2d}$$

output
```
2*a^(1/2)*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/d-2*(a+b*sech(d*x+c))^(
1/2)/d-2/3*a*(a+b*sech(d*x+c))^(3/2)/b^2/d+2/5*(a+b*sech(d*x+c))^(5/2)/b^2
/d
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.87

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^3(c+dx)\,dx$$
$$=\frac{2\left(15\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)+\frac{\sqrt{a+b\text{sech}(c+dx)}\left(-2a^2-15b^2+ab\text{sech}(c+dx)+3b^2\text{sech}^2(c+dx)\right)}{b^2}\right)}{15d}$$

input
```
Integrate[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^3,x]
```

output
```
(2*(15*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]] + (Sqrt[a + b*Se
ch[c + d*x]]*(-2*a^2 - 15*b^2 + a*b*Sech[c + d*x] + 3*b^2*Sech[c + d*x]^2)
)/b^2))/(15*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 26, 4373, 517, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i\cot\left(ic+idx+\frac{\pi}{2}\right)^3\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \cot\left(\frac{1}{2}(2ic+\pi)+idx\right)^3\sqrt{a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)}dx$$

$$\Big\downarrow \text{ 4373}$$

$$-\frac{\int \frac{\cosh(c+dx)\sqrt{a+b\text{sech}(c+dx)}\left(b^2-b^2\text{sech}^2(c+dx)\right)}{b}d(b\text{sech}(c+dx))}{b^2d}$$

$\Big\downarrow$ 517

$$-\frac{2\int \frac{b^2\text{sech}^2(c+dx)\left(b^4\text{sech}^4(c+dx)-2ab^2\text{sech}^2(c+dx)+a^2-b^2\right)}{a-b^2\text{sech}^2(c+dx)}d\sqrt{a+b\text{sech}(c+dx)}}{b^2d}$$

$\Big\downarrow$ 1584

$$-\frac{2\int \left(-b^4\text{sech}^4(c+dx)+ab^2\text{sech}^2(c+dx)+b^2-\frac{ab^2}{a-b^2\text{sech}^2(c+dx)}\right)d\sqrt{a+b\text{sech}(c+dx)}}{b^2d}$$

$\Big\downarrow$ 2009

$$-\frac{2\left(-\sqrt{a}b^2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)+\frac{1}{3}ab^3\text{sech}^3(c+dx)-\frac{1}{5}b^5\text{sech}^5(c+dx)+b^3\text{sech}(c+dx)\right)}{b^2d}$$

input
```
Int[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^3,x]
```

output
```
(-2*(-(Sqrt[a]*b^2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]]) + b^3*Sech[
c + d*x] + (a*b^3*Sech[c + d*x]^3)/3 - (b^5*Sech[c + d*x]^5)/5))/(b^2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 517
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> Simp[2*(e^m/d^(m + 2*p + 1))   Subst[Int[x^(2*n + 1)*(-c + x^
2)^m*(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4)^p, x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && ILtQ[m, 0] && IntegerQ[n + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sqrt{a + b\,\mathrm{sech}\,(dx + c)}\,\tanh\,(dx + c)^3\,dx$$

input
```
int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x)
```

output
```
int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(84) = 168$.

Time = 0.51 (sec) , antiderivative size = 1589, normalized size of antiderivative = 15.89

$$\int \sqrt{a + b\mathrm{sech}(c + dx)}\tanh^3(c + dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x, algorithm="fricas")
```

output
```
[1/30*(15*(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2
*sinh(d*x + c)^4 + 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + b^2)
*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(
d*x + c))*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*
a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*
b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2
+ 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(
d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) +
b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh
(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3
+ 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*
sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a
*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(
cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 4*(2
*a*b*cosh(d*x + c)^3 - (2*a^2 + 15*b^2)*cosh(d*x + c)^4 - (2*a^2 + 15*b^2)
*sinh(d*x + c)^4 + 2*(a*b - 2*(2*a^2 + 15*b^2)*cosh(d*x + c))*sinh(d*x + c
)^3 + 2*a*b*cosh(d*x + c) - 2*(2*a^2 + 9*b^2)*cosh(d*x + c)^2 + 2*(3*a*b*c
osh(d*x + c) - 3*(2*a^2 + 15*b^2)*cosh(d*x + c)^2 - 2*a^2 - 9*b^2)*sinh(d*
x + c)^2 - 2*a^2 - 15*b^2 + 2*(3*a*b*cosh(d*x + c)^2 - 2*(2*a^2 + 15*b^2)*
cosh(d*x + c)^3 + a*b - 2*(2*a^2 + 9*b^2)*cosh(d*x + c))*sinh(d*x + c))...
```

Sympy [F]

$$\int \sqrt{a + b\operatorname{sech}(c + dx)}\tanh^3(c + dx)\,dx = \int \sqrt{a + b\operatorname{sech}(c + dx)}\tanh^3(c + dx)\,dx$$

input
```
integrate((a+b*sech(d*x+c))**(1/2)*tanh(d*x+c)**3,x)
```

output
```
Integral(sqrt(a + b*sech(c + d*x))*tanh(c + d*x)**3, x)
```

Maxima [F]

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^3(c+dx)\,dx=\int \sqrt{b\,\text{sech}\,(dx+c)+a}\tanh\,(dx+c)^3\;dx$$

input
```
integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^3, x)
```

Giac [F]

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^3(c+dx)\,dx=\int \sqrt{b\,\text{sech}\,(dx+c)+a}\tanh\,(dx+c)^3\;dx$$

input
```
integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^3(c+dx)\,dx=\int \tanh(c+d\,x)^3\,\sqrt{a+\frac{b}{\cosh\,(c+d\,x)}}\,dx$$

input
```
int(tanh(c + d*x)^3*(a + b/cosh(c + d*x))^(1/2),x)
```

output
```
int(tanh(c + d*x)^3*(a + b/cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^3(c+dx)\,dx$$
$$= \frac{-2\sqrt{\text{sech}\,(dx+c)\,b+a}\,\tanh\,(dx+c)^2-8\sqrt{\text{sech}\,(dx+c)\,b+a}+\left(\int \frac{\sqrt{\text{sech}(dx+c)b+a}\,\tanh(dx+c)^3}{\text{sech}(dx+c)b+a}dx\right)ad+}{5d}$$

input
```
int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x)
```

output
```
( - 2*sqrt(sech(c + d*x)*b + a)*tanh(c + d*x)**2 - 8*sqrt(sech(c + d*x)*b
+ a) + int((sqrt(sech(c + d*x)*b + a)*tanh(c + d*x)**3)/(sech(c + d*x)*b +
 a),x)*a*d + 4*int((sqrt(sech(c + d*x)*b + a)*tanh(c + d*x))/(sech(c + d*x
)*b + a),x)*a*d)/(5*d)
```

3.127 $\int \sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)\,dx = \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{a+b\text{sech}(c+dx)}}{d}$$

output
```
2*a^(1/2)*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/d-2*(a+b*sech(d*x+c))^(
1/2)/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)\,dx = -\frac{-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)+2\sqrt{a+b\text{sech}(c+dx)}}{d}$$

input
```
Integrate[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x],x]
```

output
```
-((-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]] + 2*Sqrt[a + b*Se
ch[c + d*x]])/d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 26, 4373, 60, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\cot\left(ic+idx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \cot\left(\frac{1}{2}(2ic+\pi)+idx\right)\sqrt{a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)}dx$$

$$\Big\downarrow \text{4373}$$

$$-\frac{\int \frac{\cosh(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{b}d(b\text{sech}(c+dx))}{d}$$

$$\Big\downarrow \text{60}$$

$$-\frac{a\int \frac{\cosh(c+dx)}{b\sqrt{a+b\text{sech}(c+dx)}}d(b\text{sech}(c+dx))+2\sqrt{a+b\text{sech}(c+dx)}}{d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2a\int \frac{1}{b^2\text{sech}^2(c+dx)-a}d\sqrt{a+b\text{sech}(c+dx)}+2\sqrt{a+b\text{sech}(c+dx)}}{d}$$

$$\Big\downarrow \text{220}$$

$$-\frac{2\sqrt{a+b\text{sech}(c+dx)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d}$$

input
```
Int[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x],x]
```

output
```
-((-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]] + 2*Sqrt[a + b*Se
ch[c + d*x]])/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{2\sqrt{a+b\,\operatorname{sech}(dx+c)}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\operatorname{sech}(dx+c)}}{\sqrt{a}}\right)}{d}$	43
default	$-\frac{2\sqrt{a+b\,\operatorname{sech}(dx+c)}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\operatorname{sech}(dx+c)}}{\sqrt{a}}\right)}{d}$	43

input
```
int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(2*(a+b*sech(d*x+c))^(1/2)-2*a^(1/2)*arctanh((a+b*sech(d*x+c))^(1/2)/
a^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(43) = 86$.

Time = 0.49 (sec) , antiderivative size = 605, normalized size of antiderivative = 11.86

$$\int \sqrt{a+b\operatorname{sech}(c+dx)}\tanh(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*
cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*co
sh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12
*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x
+ c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*s
inh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x
 + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*
b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt
((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*c
osh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh
(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 4*sqrt((
a*cosh(d*x + c) + b)/cosh(d*x + c)))/d, -(sqrt(-a)*arctan((a*cosh(d*x + c)
^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*
x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(a^2*cosh(d
*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*(a^2*cosh(
d*x + c) + a*b)*sinh(d*x + c))) + 2*sqrt((a*cosh(d*x + c) + b)/cosh(d*x +
c)))/d]
```

Sympy [F]

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)\,dx=\int \sqrt{a+b\operatorname{sech}\left(c+dx\right)}\tanh\left(c+dx\right)dx$$

input
```
integrate((a+b*sech(d*x+c))**(1/2)*tanh(d*x+c),x)
```

output
```
Integral(sqrt(a + b*sech(c + d*x))*tanh(c + d*x), x)
```

Maxima [F]

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)\,dx=\int \sqrt{b\operatorname{sech}\left(dx+c\right)+a}\tanh\left(dx+c\right)\,dx$$

input
```
integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c), x)
```

Giac [F]

$$\int \sqrt{a+b\mathrm{sech}(c+dx)}\tanh(c+dx)\,dx=\int \sqrt{b\,\mathrm{sech}\left(dx+c\right)+a}\tanh\left(dx+c\right)\,dx$$

input
```
integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \sqrt{a+b\mathrm{sech}(c+dx)}\tanh(c+dx)\,dx=\frac{2\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}{d}$$

input
```
int(tanh(c + d*x)*(a + b/cosh(c + d*x))^(1/2),x)
```

output
```
(2*a^(1/2)*atanh((a + b/cosh(c + d*x))^(1/2)/a^(1/2)))/d - (2*(a + b/cosh(
c + d*x))^(1/2))/d
```

Reduce [F]

$$\int \sqrt{a+b\mathrm{sech}(c+dx)}\tanh(c+dx)\,dx$$
$$=\frac{-2\sqrt{\mathrm{sech}\left(dx+c\right)b+a}+\left(\int \frac{\sqrt{\mathrm{sech}(dx+c)b+a}\,\tanh(dx+c)}{\mathrm{sech}(dx+c)b+a}dx\right)ad}{d}$$

input
```
int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c),x)
```

output
```
( - 2*sqrt(sech(c + d*x)*b + a) + int((sqrt(sech(c + d*x)*b + a)*tanh(c +
d*x))/(sech(c + d*x)*b + a),x)*a*d)/d
```

3.128 $\int \coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{\sqrt{a-b}\,\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d} - \frac{\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

output
```
2*a^(1/2)*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/d-(a-b)^(1/2)*arctanh((
a+b*sech(d*x+c))^(1/2)/(a-b)^(1/2))/d-(a+b)^(1/2)*arctanh((a+b*sech(d*x+c)
)^(1/2)/(a+b)^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.94

$$\int \coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx =$$

$$-\frac{-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)+\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)+\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

input
```
Integrate[Coth[c + d*x]*Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
-((-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]] + Sqrt[a - b]*Arc
Tanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]] + Sqrt[a + b]*ArcTanh[Sqrt[a +
 b*Sech[c + d*x]]/Sqrt[a + b]])/d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 26, 4373, 561, 1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}{\cot\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sqrt{a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)}}{\cot\left(\frac{1}{2}(2ic+\pi)+idx\right)}dx$$

$$\Big\downarrow \text{4373}$$

$$-\frac{b^2\int \frac{\cosh(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{b\left(b^2-b^2\text{sech}^2(c+dx)\right)}d(b\text{sech}(c+dx))}{d}$$

$$\Big\downarrow \; 561$$

$$-\frac{2b^2\int \frac{b^2\text{sech}^2(c+dx)}{\left(a-b^2\text{sech}^2(c+dx)\right)\left(b^4\text{sech}^4(c+dx)-2ab^2\text{sech}^2(c+dx)+a^2-b^2\right)}d\sqrt{a+b\text{sech}(c+dx)}}{d}$$

$$\Big\downarrow \; 1610$$

$$-\frac{2b^2\int \left(-\frac{a}{b^2\left(a-b^2\text{sech}^2(c+dx)\right)}+\frac{a+b}{2b^2\left(-b^2\text{sech}^2(c+dx)+a+b\right)}+\frac{b-a}{2b^2\left(b^2\text{sech}^2(c+dx)-a+b\right)}\right)d\sqrt{a+b\text{sech}(c+dx)}}{d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{2b^2\left(-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{b^2}+\frac{\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{2b^2}+\frac{\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{2b^2}\right)}{d}$$

input
```
Int[Coth[c + d*x]*Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
(-2*b^2*(-((Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/b^2) + (Sq
rt[a - b]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]])/(2*b^2) + (Sqrt[
a + b]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]])/(2*b^2)))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \coth \left(dx+c\right) \sqrt{a+b\,\mathrm{sech}\left(dx+c\right)}dx$$

input
```
int(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(88) = 176$.

Time = 0.45 (sec) , antiderivative size = 9170, normalized size of antiderivative = 86.51

$$\int \coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = \int \sqrt{a+b\,\text{sech}\,(c+dx)}\coth\,(c+dx)\,dx$$

input
```
integrate(coth(d*x+c)*(a+b*sech(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sech(c + d*x))*coth(c + d*x), x)
```

Maxima [F]

$$\int \coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = \int \sqrt{b\,\text{sech}\,(dx+c)+a}\coth\,(dx+c)\,dx$$

input
```
integrate(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c), x)
```

Giac [F]

$$\int \coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx=\int \sqrt{b\,\text{sech}\,(dx+c)+a}\,\coth\,(dx+c)\;dx$$

input
```
integrate(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx=\int \coth(c+d\,x)\,\sqrt{a+\frac{b}{\cosh\,(c+d\,x)}}\,dx$$

input
```
int(coth(c + d*x)*(a + b/cosh(c + d*x))^(1/2),x)
```

output
```
int(coth(c + d*x)*(a + b/cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx=\int \sqrt{\text{sech}\,(dx+c)\,b+a}\;\coth\,(dx+c)\,dx$$

input
```
int(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sech(c + d*x)*b + a)*coth(c + d*x),x)
```

3.129 $\int \coth^3(c+dx)\sqrt{a+b\operatorname{sech}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 153

$$\int \coth^3(c+dx)\sqrt{a+b\operatorname{sech}(c+dx)}\,dx = \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{(4a-3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4\sqrt{a-b}d} - \frac{(4a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)}{4\sqrt{a+b}d} - \frac{\coth^2(c+dx)\sqrt{a+b\operatorname{sech}(c+dx)}}{2d}$$

output
```
2*a^(1/2)*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/d-1/4*(4*a-3*b)*arctanh
((a+b*sech(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)/d-1/4*(4*a+3*b)*arctanh(
(a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)/d-1/2*coth(d*x+c)^2*(a+b*
sech(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 2.62 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.46

$$\int \coth^3(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx$$
$$= \frac{\frac{b\arctan\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} + 8\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right) - 4\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right) + \frac{b\text{arctan}}{}}{4d}$$

input
```
Integrate[Coth[c + d*x]^3*Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
((b*ArcTan[Sqrt[a + b*Sech[c + d*x]]/Sqrt[-a + b]])/Sqrt[-a + b] + 8*Sqrt[
a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]] - 4*Sqrt[a - b]*ArcTanh[Sqrt
[a + b*Sech[c + d*x]]/Sqrt[a - b]] + (b*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/
Sqrt[a + b]])/Sqrt[a + b] - 4*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Sech[c + d*x]
]/Sqrt[a + b]] + Sqrt[a + b*Sech[c + d*x]]/(-1 + Sech[c + d*x]) - Sqrt[a +
 b*Sech[c + d*x]]/(1 + Sech[c + d*x]))/(4*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.75, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 26, 4373, 561, 25, 1652, 25, 1484, 1492, 27, 1406, 220, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}{\cot\left(ic+idx+\frac{\pi}{2}\right)^3}dx$$
$$\Big\downarrow \text{26}$$

$$i\int \frac{\sqrt{a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)}}{\cot\left(\frac{1}{2}(2ic+\pi)+idx\right)^3}dx$$

$$\Big\downarrow \text{4373}$$

$$-\frac{b^4\int \frac{\cosh(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{b\left(b^2-b^2\text{sech}^2(c+dx)\right)^2}d(b\text{sech}(c+dx))}{d}$$

$$\Big\downarrow \text{561}$$

$$-\frac{2b^4\int -\frac{b^2\text{sech}^2(c+dx)}{\left(a-b^2\text{sech}^2(c+dx)\right)\left(b^4\text{sech}^4(c+dx)-2ab^2\text{sech}^2(c+dx)+a^2-b^2\right)^2}d\sqrt{a+b\text{sech}(c+dx)}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{2b^4\int \frac{b^2\text{sech}^2(c+dx)}{\left(a-b^2\text{sech}^2(c+dx)\right)\left(b^4\text{sech}^4(c+dx)-2ab^2\text{sech}^2(c+dx)+a^2-b^2\right)^2}d\sqrt{a+b\text{sech}(c+dx)}}{d}$$

$$\Big\downarrow \text{1652}$$

$$-\frac{2b^4\left(\frac{\int -\frac{a^2-b^2\text{sech}^2(c+dx)a-b^2}{\left(b^4\text{sech}^4(c+dx)-2ab^2\text{sech}^2(c+dx)+a^2-b^2\right)^2}d\sqrt{a+b\text{sech}(c+dx)}}{b^2}+\frac{a\int \frac{1}{\left(a-b^2\text{sech}^2(c+dx)\right)\left(b^4\text{sech}^4(c+dx)-2ab^2\text{sech}^2(c+dx)\right.}}{b^2}\right.}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2b^4\left(\frac{a\int \frac{1}{\left(a-b^2\text{sech}^2(c+dx)\right)\left(b^4\text{sech}^4(c+dx)-2ab^2\text{sech}^2(c+dx)+a^2-b^2\right)}d\sqrt{a+b\text{sech}(c+dx)}}{b^2}-\frac{\int \frac{a^2-b^2\text{sech}^2(c+dx)a-b^2}{\left(b^4\text{sech}^4(c+dx)-2ab^2\text{sech}^2(c+dx)+a^2\right.}}{b^2}\right.}{d}$$

$$\Big\downarrow \text{1484}$$

$$-\frac{2b^4\left(\frac{a\int \left(\frac{1}{2b^2\left(-b^2\text{sech}^2(c+dx)+a+b\right)}-\frac{1}{2b^2\left(b^2\text{sech}^2(c+dx)-a+b\right)}-\frac{1}{b^2\left(a-b^2\text{sech}^2(c+dx)\right)}\right)d\sqrt{a+b\text{sech}(c+dx)}}{b^2}-\frac{\int \frac{a^2-b^2\text{se}}{\left(b^4\text{sech}^4(c+dx)-\right.}}{}\right.}{d}$$

$$\Big\downarrow \text{1492}$$

$$-\frac{2b^4\left(\frac{a\int\left(\frac{1}{2b^2\left(-b^2\text{sech}^2(c+dx)+a+b\right)}-\frac{1}{2b^2\left(b^2\text{sech}^2(c+dx)-a+b\right)}-\frac{1}{b^2\left(a-b^2\text{sech}^2(c+dx)\right)}\right)d\sqrt{a+b\text{sech}(c+dx)}}{b^2}-\frac{\sqrt{a+bS\cdots}}{4\left(a^2-2ab^2\text{sech}^2(c+\cdots\right.}\cdots\right.}{d}$$

$\downarrow$ 27

$$-\frac{2b^4\left(\frac{a\int\left(\frac{1}{2b^2\left(-b^2\text{sech}^2(c+dx)+a+b\right)}-\frac{1}{2b^2\left(b^2\text{sech}^2(c+dx)-a+b\right)}-\frac{1}{b^2\left(a-b^2\text{sech}^2(c+dx)\right)}\right)d\sqrt{a+b\text{sech}(c+dx)}}{b^2}-\frac{\frac{3}{4}\int\frac{\cdots}{b^4\text{sech}^4(c+dx)-\cdots}\cdots}{\cdots}\right.}{d}$$

$\downarrow$ 1406

$$-\frac{2b^4\left(\frac{a\int\left(\frac{1}{2b^2\left(-b^2\text{sech}^2(c+dx)+a+b\right)}-\frac{1}{2b^2\left(b^2\text{sech}^2(c+dx)-a+b\right)}-\frac{1}{b^2\left(a-b^2\text{sech}^2(c+dx)\right)}\right)d\sqrt{a+b\text{sech}(c+dx)}}{b^2}-\frac{\frac{3}{4}\left(\frac{\int\frac{1}{b^2\text{sech}^2(c+dx)\cdots}\cdots}{\cdots}\right.}{\cdots}\right.}{d}$$

$\downarrow$ 220

$$-\frac{2b^4\left(\frac{a\int\left(\frac{1}{2b^2\left(-b^2\text{sech}^2(c+dx)+a+b\right)}-\frac{1}{2b^2\left(b^2\text{sech}^2(c+dx)-a+b\right)}-\frac{1}{b^2\left(a-b^2\text{sech}^2(c+dx)\right)}\right)d\sqrt{a+b\text{sech}(c+dx)}}{b^2}-\frac{\sqrt{a+bS\cdots}}{4\left(a^2-2ab^2\text{sech}^2(c+\cdots\right.}\cdots\right.}{d}$$

$\downarrow$ 2009

$$-\frac{2b^4\left(\frac{a\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}b^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a-b}}\right)}{2b^2\sqrt{a-b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)}{2b^2\sqrt{a+b}}\right)}{b^2}-\frac{\frac{\sqrt{a+b\operatorname{sec}}}{4\left(a^2-2ab^2\operatorname{sech}^2(c+dx\right.}}{}\right.}{d}$$

input
```
Int[Coth[c + d*x]^3*Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
(-2*b^4*((a*(-(ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]]/(Sqrt[a]*b^2)) +
 ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]]/(2*Sqrt[a - b]*b^2) + ArcT
anh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]]/(2*b^2*Sqrt[a + b])))/b^2 - ((3
*(ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]]/(2*Sqrt[a - b]*b) - ArcTa
nh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]]/(2*b*Sqrt[a + b])))/4 + Sqrt[a +
 b*Sech[c + d*x]]/(4*(a^2 - b^2 - 2*a*b^2*Sech[c + d*x]^2 + b^4*Sech[c + d
*x]^4)))/b^2))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1652
```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_) +
 (e_.)*(x_)^2), x_Symbol] :> Simp[f^2/(c*d^2 - b*d*e + a*e^2)   Int[(f*x)^(
m - 2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^p, x], x] - Simp[d*e*(f^2/(c*d^2
 - b*d*e + a*e^2))   Int[(f*x)^(m - 2)*((a + b*x^2 + c*x^4)^(p + 1)/(d + e*
x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \coth\left(dx+c\right)^{3} \sqrt{a+b\,\mathrm{sech}\left(dx+c\right)}dx$$

input
```
int(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1538 vs. $2(127) = 254$.

Time = 1.03 (sec) , antiderivative size = 17083, normalized size of antiderivative = 111.65

$$\int \coth^3(c+dx)\sqrt{a+b\mathrm{sech}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \coth^3(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx=\int \sqrt{a+b\,\text{sech}\,(c+dx)}\coth^3{(c+dx)}\,dx$$

input
```
integrate(coth(d*x+c)**3*(a+b*sech(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sech(c + d*x))*coth(c + d*x)**3, x)
```

Maxima [F]

$$\int \coth^3(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx=\int \sqrt{b\,\text{sech}\,(dx+c)+a}\coth{(dx+c)}^3\,dx$$

input
```
integrate(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^3, x)
```

Giac [F]

$$\int \coth^3(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx=\int \sqrt{b\,\text{sech}\,(dx+c)+a}\coth{(dx+c)}^3\,dx$$

input
```
integrate(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^3(c+dx)\sqrt{a+b\operatorname{sech}(c+dx)}\,dx=\int \coth(c+d\,x)^3\,\sqrt{a+\frac{b}{\cosh\left(c+d\,x\right)}}\,dx$$

input
```
int(coth(c + d*x)^3*(a + b/cosh(c + d*x))^(1/2),x)
```

output
```
int(coth(c + d*x)^3*(a + b/cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \coth^3(c+dx)\sqrt{a+b\operatorname{sech}(c+dx)}\,dx=\int \sqrt{\operatorname{sech}\left(dx+c\right)b+a}\,\coth\left(dx+c\right)^3dx$$

input
```
int(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sech(c + d*x)*b + a)*coth(c + d*x)**3,x)
```

3.130 $\int \sqrt{a+b\,\text{sech}(c+dx)}\tanh^2(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 344

$$\int \sqrt{a+b\,\text{sech}(c+dx)}\tanh^2(c+dx)\,dx = -\frac{2a(a-b)\sqrt{a+b}\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{3b^2d}$$
$$-\frac{2\sqrt{a+b}(a+2b)\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{3bd}$$
$$+\frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{d}$$
$$-\frac{2\sqrt{a+b\,\text{sech}(c+dx)}\tanh(c+dx)}{3d}$$

output
```
-2/3*a*(a-b)*(a+b)^(1/2)*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+
b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(
d*x+c))/(a-b))^(1/2)/b^2/d-2/3*(a+b)^(1/2)*(a+2*b)*coth(d*x+c)*EllipticF((
a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))
/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b/d+2*(a+b)^(1/2)*coth(d*x+
c)*EllipticPi((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1
/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/d-2/
3*(a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)/d
```

Mathematica [F]

$$\int \sqrt{a+b\operatorname{sech}(c+dx)}\tanh^2(c+dx)\,dx=\int \sqrt{a+b\operatorname{sech}(c+dx)}\tanh^2(c+dx)\,dx$$

input
```
Integrate[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^2,x]
```

output
```
Integrate[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^2, x]
```

Rubi [A] (verified)

Time = 2.22 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3042, 25, 4382, 3042, 4545, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(c+dx)\sqrt{a+b\operatorname{sech}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\cot\left(ic+idx+\frac{\pi}{2}\right)^2\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \cot\left(\frac{1}{2}(2ic+\pi)+idx\right)^2 \sqrt{a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)}dx$$

$$\Big\downarrow \text{4382}$$

$$-\int \sqrt{a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)}\left(\csc^2\left(\frac{1}{2}(2ic+\pi)+idx\right)-1\right)dx$$

$$\Big\downarrow \text{3042}$$

$$-\int \sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}\left(\csc\left(ic+idx+\frac{\pi}{2}\right)^2-1\right)dx$$

$$\Big\downarrow \text{4545}$$

$$-\frac{2}{3}\int -\frac{-a\text{sech}^2(c+dx)+2b\text{sech}(c+dx)+3a}{2\sqrt{a+b\text{sech}(c+dx)}}dx-\frac{2\tanh(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{-a\text{sech}^2(c+dx)+2b\text{sech}(c+dx)+3a}{\sqrt{a+b\text{sech}(c+dx)}}dx-\frac{2\tanh(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\tanh(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{3d}+\frac{1}{3}\int \frac{-a\csc\left(ic+idx+\frac{\pi}{2}\right)^2+2b\csc\left(ic+idx+\frac{\pi}{2}\right)+3a}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4546}$$

$$\frac{1}{3}\left(\int \frac{3a+(a+2b)\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}dx-a\int \frac{\text{sech}(c+dx)(\text{sech}(c+dx)+1)}{\sqrt{a+b\text{sech}(c+dx)}}dx\right)-\frac{2\tanh(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\tanh(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{3d}+\frac{1}{3}\left(\int \frac{3a+(a+2b)\csc\left(ic+idx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx-a\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)\left(\csc\left(ic+idx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{4409}$$

$$-\frac{2\tanh(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{3d}+$$
$$\frac{1}{3}\left(-a\int\frac{\csc\left(ic+idx+\frac{\pi}{2}\right)\left(\csc\left(ic+idx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx+3a\int\frac{1}{\sqrt{a+b\text{sech}(c+dx)}}dx+(a+2b)\int\frac{\text{sech}(c}{\sqrt{a+bsec}}\right.$$

$\downarrow$ 3042

$$-\frac{2\tanh(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{3d}+$$
$$\frac{1}{3}\left(3a\int\frac{1}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx+(a+2b)\int\frac{\csc\left(ic+idx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx-a\int\frac{\csc\left(ic+idx+\frac{\pi}{2}\right)(\csc}{\sqrt{a+b\csc(ic}}\right.$$

$\downarrow$ 4271

$$-\frac{2\tanh(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{3d}+$$
$$\frac{1}{3}\left((a+2b)\int\frac{\csc\left(ic+idx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx-a\int\frac{\csc\left(ic+idx+\frac{\pi}{2}\right)\left(\csc\left(ic+idx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx+\frac{6\sqrt{a+b}\cot}{}\right.$$

$\downarrow$ 4319

$$-\frac{2\tanh(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{3d}+$$
$$\frac{1}{3}\left(-a\int\frac{\csc\left(ic+idx+\frac{\pi}{2}\right)\left(\csc\left(ic+idx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}(a+2b)\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(}{}}}{}\right.$$

$\downarrow$ 4492

$$\frac{1}{3}\left(-\frac{2a(a-b)\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2}{}\right.$$
$$\frac{2\tanh(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{3d}$$

input
```
Int[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^2,x]
```

output
```
((-2*a*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a +
 b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*(a
 + 2*b)*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sech[c + d*x]))/(a - b))])/(b*d) + (6*Sqrt[a + b]*Coth[c + d*x]*EllipticP
i[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a
 - b))])/d)/3 - (2*Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x])/(3*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4382
```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4545
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^
m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Csc[e + f*x])^(m - 1)*Simp
[a*A*(m + 1) + (A*b*(m + 1) + b*C*m)*Csc[e + f*x] + a*C*m*Csc[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m,
 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sqrt{a+b\,\mathrm{sech}\,(dx+c)}\,\tanh\,(dx+c)^2\,dx$$

input
```
int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x)
```

output
```
int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x)
```

Fricas [F]

$$\int \sqrt{a+b\mathrm{sech}(c+dx)}\tanh^2(c+dx)\,dx = \int \sqrt{b\,\mathrm{sech}\,(dx+c)+a}\tanh\,(dx+c)^2\;dx$$

input
```
integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^2, x)
```

Sympy [F]

$$\int \sqrt{a+b\mathrm{sech}(c+dx)}\tanh^2(c+dx)\,dx = \int \sqrt{a+b\,\mathrm{sech}\,(c+dx)}\tanh^2\,(c+dx)\,dx$$

input
```
integrate((a+b*sech(d*x+c))**(1/2)*tanh(d*x+c)**2,x)
```

output
```
Integral(sqrt(a + b*sech(c + d*x))*tanh(c + d*x)**2, x)
```

Maxima [F]

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^2(c+dx)\,dx=\int \sqrt{b\,\text{sech}\,(dx+c)+a}\tanh\,(dx+c)^2\,dx$$

input
```
integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^2, x)
```

Giac [F]

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^2(c+dx)\,dx=\int \sqrt{b\,\text{sech}\,(dx+c)+a}\tanh\,(dx+c)^2\,dx$$

input
```
integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^2(c+dx)\,dx=\int \tanh(c+d\,x)^2\,\sqrt{a+\frac{b}{\cosh\,(c+d\,x)}}\,dx$$

input
```
int(tanh(c + d*x)^2*(a + b/cosh(c + d*x))^(1/2),x)
```

output
```
int(tanh(c + d*x)^2*(a + b/cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^2(c+dx)\,dx=\int \sqrt{\text{sech}\,(dx+c)\,b+a}\,\tanh\,(dx+c)^2\,dx$$

input
```
int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x)
```

output
```
int(sqrt(sech(c + d*x)*b + a)*tanh(c + d*x)**2,x)
```

3.131 $\int \sqrt{a+b\,\text{sech}(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 125

$$\int \sqrt{a+b\,\text{sech}(c+dx)}\,dx$$
$$= \frac{2\coth(c+dx)\,\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\,\text{sech}(c+dx)}}\right),\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\text{sech}(c+dx))}{a+b\,\text{sech}(c+dx)}}\sqrt{\frac{b(1+\text{sech}(c+dx))}{a+b\,\text{sech}(c+dx)}}(a+}{\sqrt{a+b}\,d}$$

output
```
2*coth(d*x+c)*EllipticPi((a+b)^(1/2)/(a+b*sech(d*x+c))^(1/2),a/(a+b),((a-b
)/(a+b))^(1/2))*(-b*(1-sech(d*x+c))/(a+b*sech(d*x+c)))^(1/2)*(b*(1+sech(d*
x+c))/(a+b*sech(d*x+c)))^(1/2)*(a+b*sech(d*x+c))/(a+b)^(1/2)/d
```

Mathematica [F]

$$\int \sqrt{a+b\,\text{sech}(c+dx)}\,dx = \int \sqrt{a+b\,\text{sech}(c+dx)}\,dx$$

input
```
Integrate[Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
Integrate[Sqrt[a + b*Sech[c + d*x]], x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4267}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\text{sech}(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4267}$$

$$\frac{2\coth(c+dx)\sqrt{-\frac{b(1-\text{sech}(c+dx))}{a+b\text{sech}(c+dx)}}\sqrt{\frac{b(\text{sech}(c+dx)+1)}{a+b\text{sech}(c+dx)}}(a+b\text{sech}(c+dx))\,\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\text{sech}(c+dx)}}\right)\right.}{d\sqrt{a+b}}$$

input
```
Int[Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
(2*Coth[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sech[
c + d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sech[c + d*x]))/(a + b*Sech[c
+ d*x]))]*Sqrt[(b*(1 + Sech[c + d*x]))/(a + b*Sech[c + d*x])]*(a + b*Sech[
c + d*x]))/(Sqrt[a + b]*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4267
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sqrt{a + b\ \mathrm{sech}\left(dx + c\right)}dx$$

input
```
int((a+b*sech(d*x+c))^(1/2),x)
```

output
```
int((a+b*sech(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \sqrt{a + b\mathrm{sech}(c + dx)}\, dx = \int \sqrt{b\,\mathrm{sech}\left(dx + c\right) + a}\, dx$$

input
```
integrate((a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sech(d*x + c) + a), x)
```

Sympy [F]

$$\int \sqrt{a + b\mathrm{sech}(c + dx)}\, dx = \int \sqrt{a + b\,\mathrm{sech}\left(c + dx\right)}\, dx$$

input
```
integrate((a+b*sech(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{a + b\text{sech}(c + dx)}\, dx = \int \sqrt{b \operatorname{sech}(dx + c) + a}\, dx$$

input
```
integrate((a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sech(d*x + c) + a), x)
```

Giac [F]

$$\int \sqrt{a + b\text{sech}(c + dx)}\, dx = \int \sqrt{b \operatorname{sech}(dx + c) + a}\, dx$$

input
```
integrate((a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + b\text{sech}(c + dx)}\, dx = \int \sqrt{a + \frac{b}{\cosh(c + d\, x)}}\, dx$$

input
```
int((a + b/cosh(c + d*x))^(1/2),x)
```

output
```
int((a + b/cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a + b\text{sech}(c + dx)}\, dx = \int \sqrt{\text{sech}\,(dx + c)\, b + a}dx$$

input
```
int((a+b*sech(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sech(c + d*x)*b + a),x)
```

3.132 $\int \coth^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 246

$$\int \coth^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx$$
$$= \frac{\sqrt{a+b}\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{d}$$
$$-\frac{\coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{d}$$
$$+\frac{2\coth(c+dx)\,\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\text{sech}(c+dx)}}\right),\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\text{sech}(c+dx))}{a+b\text{sech}(c+dx)}}\sqrt{\frac{b(1+\text{sech}(c+dx))}{a+b\text{sech}(c+dx)}}(a \cdots}{\sqrt{a+b}d}$$

output
```
(a+b)^(1/2)*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+
b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b)
)^(1/2)/d-coth(d*x+c)*(a+b*sech(d*x+c))^(1/2)/d+2*coth(d*x+c)*EllipticPi((
a+b)^(1/2)/(a+b*sech(d*x+c))^(1/2),a/(a+b),((a-b)/(a+b))^(1/2))*(-b*(1-sec
h(d*x+c))/(a+b*sech(d*x+c)))^(1/2)*(b*(1+sech(d*x+c))/(a+b*sech(d*x+c)))^(
1/2)*(a+b*sech(d*x+c))/(a+b)^(1/2)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 539 vs. $2(246) = 492$.

Time = 15.18 (sec) , antiderivative size = 539, normalized size of antiderivative = 2.19

$$\int \coth^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = -\frac{\coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{d}$$

$$+\frac{\sqrt{a+b\text{sech}(c+dx)}\left(\frac{2\sqrt{b}(a-a\cosh(c+dx))^{3/2}\sqrt{\frac{(a+b)(a+a\cosh(c+dx))}{(a-b)(a-a\cosh(c+dx))}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a}\sqrt{b+a\cosh(c+dx)}}{\sqrt{b}\sqrt{a-a\cosh(c+dx)}}\right),-\frac{2b}{a-b}\right)\sinh(c}{a^{3/2}\sqrt{-1+\cosh(c+dx)}\sqrt{1+\cosh(c+dx)}\sqrt{-\frac{a(a+b)\cosh(c+dx)}{b(a-a\cosh(c+dx))}}\left(-\frac{a-a\cosh(c+dx)}{a}\right)^{3/2}\sqrt{\frac{a+a\cosh(c+dx)}{a}}\sqrt{\text{sec}}}\right.}{2d\sqrt{b+a}}$$

input
```
Integrate[Coth[c + d*x]^2*Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
-((Coth[c + d*x]*Sqrt[a + b*Sech[c + d*x]])/d) + (Sqrt[a + b*Sech[c + d*x]
]*((2*Sqrt[b]*(a - a*Cosh[c + d*x])^(3/2)*Sqrt[((a + b)*(a + a*Cosh[c + d*
x]))/((a - b)*(a - a*Cosh[c + d*x]))]*EllipticF[ArcSin[(Sqrt[a]*Sqrt[b + a
*Cosh[c + d*x]])/(Sqrt[b]*Sqrt[a - a*Cosh[c + d*x]])], (-2*b)/(a - b)]*Sin
h[c + d*x])/(a^(3/2)*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[1 + Cosh[c + d*x]]*Sqrt
[-((a*(a + b)*Cosh[c + d*x])/(b*(a - a*Cosh[c + d*x])))]*(-((a - a*Cosh[c
+ d*x])/a))^(3/2)*Sqrt[(a + a*Cosh[c + d*x])/a]*Sqrt[Sech[c + d*x]]) - (4*
b*(a - a*Cosh[c + d*x])*EllipticPi[(a + b)/a, ArcSin[(Sqrt[a]*Sqrt[b + a*C
osh[c + d*x]])/(Sqrt[a + b]*Sqrt[a*Cosh[c + d*x]])], (a + b)/(a - b)]*Sqrt
[-((b*(a + a*Cosh[c + d*x])*Sech[c + d*x])/(a*(a - b)))]*Sinh[c + d*x])/(S
qrt[a]*Sqrt[a + b]*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[a*Cosh[c + d*x]]*Sqrt[1 +
 Cosh[c + d*x]]*Sqrt[-((a - a*Cosh[c + d*x])/a)]*Sqrt[(a + a*Cosh[c + d*x]
)/a]*Sqrt[Sech[c + d*x]]*Sqrt[-((b*(a - a*Cosh[c + d*x])*Sech[c + d*x])/(a
*(a + b)))])))/(2*d*Sqrt[b + a*Cosh[c + d*x]]*Sqrt[Sech[c + d*x]])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 25, 4384, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}{\cot\left(ic+idx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sqrt{a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)}}{\cot\left(\frac{1}{2}(2ic+\pi)+idx\right)^2}dx$$

$$\Big\downarrow \text{4384}$$

$$-\int \left(-\sqrt{a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)}\text{csch}^2(c+dx)-\sqrt{a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}+\frac{2\coth(c+dx)\sqrt{-\frac{b(1-\text{sech}(c+dx))}{a+b\text{sech}(c+dx)}}\sqrt{\frac{b(\text{sech}(c+dx)+1)}{a+b\text{sech}(c+dx)}}(a+b\text{sech}(c+dx))\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\text{sech}(c+dx)}}\right.\right.}{d\sqrt{a+b}}$$
$$\frac{\coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{d}$$

input
```
Int[Coth[c + d*x]^2*Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
(Sqrt[a + b]*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b
*(1 + Sech[c + d*x]))/(a - b))])/d - (Coth[c + d*x]*Sqrt[a + b*Sech[c + d*
x]])/d + (2*Coth[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a
+ b*Sech[c + d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sech[c + d*x]))/(a +
b*Sech[c + d*x]))]*Sqrt[(b*(1 + Sech[c + d*x]))/(a + b*Sech[c + d*x])]*(a
+ b*Sech[c + d*x]))/(Sqrt[a + b]*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4384
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &&
ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Maple [F]

$$\int \coth (dx+c)^2 \sqrt{a+b\,\mathrm{sech}\,(dx+c)}dx$$

input
```
int(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \coth^2(c+dx)\sqrt{a+b\mathrm{sech}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \coth^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx=\int \sqrt{a+b\,\text{sech}\,(c+dx)}\coth^2{(c+dx)}\,dx$$

input
```
integrate(coth(d*x+c)**2*(a+b*sech(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sech(c + d*x))*coth(c + d*x)**2, x)
```

Maxima [F]

$$\int \coth^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx=\int \sqrt{b\,\text{sech}\,(dx+c)+a}\coth{(dx+c)}^2\,dx$$

input
```
integrate(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^2, x)
```

Giac [F]

$$\int \coth^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx=\int \sqrt{b\,\text{sech}\,(dx+c)+a}\coth{(dx+c)}^2\,dx$$

input
```
integrate(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx=\int \coth(c+d\,x)^2\,\sqrt{a+\frac{b}{\cosh\left(c+d\,x\right)}}\,dx$$

input
```
int(coth(c + d*x)^2*(a + b/cosh(c + d*x))^(1/2),x)
```

output
```
int(coth(c + d*x)^2*(a + b/cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \coth^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx=\int \sqrt{\text{sech}\left(dx+c\right)b+a}\;\coth\left(dx+c\right)^2 dx$$

input
```
int(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sech(c + d*x)*b + a)*coth(c + d*x)**2,x)
```

3.133 $\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{2a(a^2-2b^2)\sqrt{a+b\operatorname{sech}(c+dx)}}{b^4d} - \frac{2(3a^2-2b^2)(a+b\operatorname{sech}(c+dx))^{3/2}}{3b^4d} + \frac{6a(a+b\operatorname{sech}(c+dx))^{5/2}}{5b^4d} - \frac{2(a+b\operatorname{sech}(c+dx))^{7/2}}{7b^4d}$$

output
```
2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(1/2)/d+2*a*(a^2-2*b^2)*(a+b*
sech(d*x+c))^(1/2)/b^4/d-2/3*(3*a^2-2*b^2)*(a+b*sech(d*x+c))^(3/2)/b^4/d+6
/5*a*(a+b*sech(d*x+c))^(5/2)/b^4/d-2/7*(a+b*sech(d*x+c))^(7/2)/b^4/d
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.76

$$\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \frac{2\left(\frac{105\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{a+b\text{sech}(c+dx)}\left(48a^3-140ab^2+(-24a^2b+70b^3)\text{sech}(c+dx)+18ab^2\text{sech}^2(c+dx)-15b^3\text{sec}\right.}{b^4}\right.}{105d}$$

input
```
Integrate[Tanh[c + d*x]^5/Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
(2*((105*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/Sqrt[a] + (Sqrt[a + b
*Sech[c + d*x]]*(48*a^3 - 140*a*b^2 + (-24*a^2*b + 70*b^3)*Sech[c + d*x] +
 18*a*b^2*Sech[c + d*x]^2 - 15*b^3*Sech[c + d*x]^3))/b^4))/(105*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.83, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4373, 517, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i\cot\left(ic+idx+\frac{\pi}{2}\right)^5}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\cot\left(\frac{1}{2}(2ic+\pi)+idx\right)^5}{\sqrt{a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)}}dx$$

$$\Big\downarrow \; 4373$$

$$-\frac{\int \frac{\cosh(c+dx)\left(b^2-b^2\text{sech}^2(c+dx)\right)^2}{b\sqrt{a+b\text{sech}(c+dx)}}d(b\text{sech}(c+dx))}{b^4d}$$

$$\Big\downarrow \; 517$$

$$-\frac{2\int -\frac{\left(b^4\text{sech}^4(c+dx)-2ab^2\text{sech}^2(c+dx)+a^2-b^2\right)^2}{a-b^2\text{sech}^2(c+dx)}d\sqrt{a+b\text{sech}(c+dx)}}{b^4d}$$

$$\Big\downarrow \; 25$$

$$\frac{2\int \frac{\left(b^4\text{sech}^4(c+dx)-2ab^2\text{sech}^2(c+dx)+a^2-b^2\right)^2}{a-b^2\text{sech}^2(c+dx)}d\sqrt{a+b\text{sech}(c+dx)}}{b^4d}$$

$$\Big\downarrow \; 1467$$

$$\frac{2\int \left(-b^6\text{sech}^6(c+dx)+3ab^4\text{sech}^4(c+dx)-b^2\left(3a^2-2b^2\right)\text{sech}^2(c+dx)+a^3-2ab^2+\frac{b^4}{a-b^2\text{sech}^2(c+dx)}\right)d\sqrt{a}}{b^4d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{2\left(-a\left(a^2-2b^2\right)\sqrt{a+b\text{sech}(c+dx)}+\frac{1}{3}b^3\left(3a^2-2b^2\right)\text{sech}^3(c+dx)-\frac{b^4\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{3}{5}ab^5\text{sech}\right.}{b^4d}$$

input
```
Int[Tanh[c + d*x]^5/Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
(-2*(-((b^4*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/Sqrt[a]) + (b^3*(3
*a^2 - 2*b^2)*Sech[c + d*x]^3)/3 - (3*a*b^5*Sech[c + d*x]^5)/5 + (b^7*Sech
[c + d*x]^7)/7 - a*(a^2 - 2*b^2)*Sqrt[a + b*Sech[c + d*x]]))/(b^4*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 517
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> Simp[2*(e^m/d^(m + 2*p + 1))   Subst[Int[x^(2*n + 1)*(-c + x^
2)^m*(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4)^p, x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && ILtQ[m, 0] && IntegerQ[n + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\tanh (dx+c)^5}{\sqrt{a+b\,\mathrm{sech}\,(dx+c)}}dx$$

input
```
int(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1277 vs. $2(128) = 256$.

Time = 0.54 (sec) , antiderivative size = 2813, normalized size of antiderivative = 19.01

$$\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\mathrm{sech}(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/210*(105*(b^4*cosh(d*x + c)^6 + 6*b^4*cosh(d*x + c)*sinh(d*x + c)^5 + b
^4*sinh(d*x + c)^6 + 3*b^4*cosh(d*x + c)^4 + 3*b^4*cosh(d*x + c)^2 + 3*(5*
b^4*cosh(d*x + c)^2 + b^4)*sinh(d*x + c)^4 + b^4 + 4*(5*b^4*cosh(d*x + c)^
3 + 3*b^4*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^4*cosh(d*x + c)^4 + 6*b^
4*cosh(d*x + c)^2 + b^4)*sinh(d*x + c)^2 + 6*(b^4*cosh(d*x + c)^5 + 2*b^4*
cosh(d*x + c)^3 + b^4*cosh(d*x + c))*sinh(d*x + c))*sqrt(a)*log(-(2*a^2*co
sh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*c
osh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*
cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 +
 b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 +
 b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*
x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c)^2
 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh
(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh(
d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*
a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c
)*sinh(d*x + c) + sinh(d*x + c)^2)) + 16*((12*a^4 - 35*a^2*b^2)*cosh(d*x +
 c)^6 + (12*a^4 - 35*a^2*b^2)*sinh(d*x + c)^6 - (12*a^3*b - 35*a*b^3)*cosh
(d*x + c)^5 - (12*a^3*b - 35*a*b^3 - 6*(12*a^4 - 35*a^2*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^5 + 3*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)^4 + (36*a^4 -...
```

Sympy [F]

$$\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx=\int \frac{\tanh^5{(c+dx)}}{\sqrt{a+b\operatorname{sech}{(c+dx)}}}\,dx$$

input
```
integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c))**(1/2),x)
```

output
```
Integral(tanh(c + d*x)**5/sqrt(a + b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx=\int \frac{\tanh\left(dx+c\right)^5}{\sqrt{b\,\text{sech}\left(dx+c\right)+a}}\,dx$$

input
```
integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(d*x + c)^5/sqrt(b*sech(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx=\int \frac{\tanh\left(dx+c\right)^5}{\sqrt{b\,\text{sech}\left(dx+c\right)+a}}\,dx$$

input
```
integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(tanh(d*x + c)^5/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx=\int \frac{\tanh(c+d\,x)^5}{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}\,dx$$

input
```
int(tanh(c + d*x)^5/(a + b/cosh(c + d*x))^(1/2),x)
```

output
```
int(tanh(c + d*x)^5/(a + b/cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\sqrt{\text{sech}\,(dx+c)\,b+a}\,\tanh\,(dx+c)^5}{\text{sech}\,(dx+c)\,b+a}dx$$

input
```
int(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sech(c + d*x)*b + a)*tanh(c + d*x)**5)/(sech(c + d*x)*b + a),x)
```

3.134 $\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{2a\sqrt{a+b\text{sech}(c+dx)}}{b^2d} + \frac{2(a+b\text{sech}(c+dx))^{3/2}}{3b^2d}$$

output
```
2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(1/2)/d-2*a*(a+b*sech(d*x+c))
^(1/2)/b^2/d+2/3*(a+b*sech(d*x+c))^(3/2)/b^2/d
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \frac{2\left(\frac{3\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{(-2a+b\text{sech}(c+dx))\sqrt{a+b\text{sech}(c+dx)}}{b^2}\right)}{3d}$$

input
```
Integrate[Tanh[c + d*x]^3/Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
(2*((3*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/Sqrt[a] + ((-2*a + b*Se
ch[c + d*x])*Sqrt[a + b*Sech[c + d*x]])/b^2))/(3*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 26, 4373, 517, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cot\left(ic+idx+\frac{\pi}{2}\right)^3}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cot\left(\frac{1}{2}(2ic+\pi)+idx\right)^3}{\sqrt{a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)}}dx$$

$$\Big\downarrow \text{4373}$$

$$-\frac{\int \frac{\cosh(c+dx)\left(b^2-b^2\text{sech}^2(c+dx)\right)}{b\sqrt{a+b\text{sech}(c+dx)}}d(b\text{sech}(c+dx))}{b^2d}$$

$$\Big\downarrow \text{517}$$

$$-\frac{2\int \frac{b^4\text{sech}^4(c+dx)-2ab^2\text{sech}^2(c+dx)+a^2-b^2}{a-b^2\text{sech}^2(c+dx)}d\sqrt{a+b\text{sech}(c+dx)}}{b^2d}$$

$$\Big\downarrow \text{1467}$$

$$-\frac{2\int \left(-\text{sech}^2(c+dx)b^2-\frac{b^2}{a-b^2\text{sech}^2(c+dx)}+a\right)d\sqrt{a+b\text{sech}(c+dx)}}{b^2d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(-\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}}+a\sqrt{a+b\text{sech}(c+dx)}-\frac{1}{3}b^3\text{sech}^3(c+dx)\right)}{b^2d}$$

input
```
Int[Tanh[c + d*x]^3/Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
(-2*(-((b^2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/Sqrt[a]) - (b^3*Se
ch[c + d*x]^3)/3 + a*Sqrt[a + b*Sech[c + d*x]]))/(b^2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 517
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> Simp[2*(e^m/d^(m + 2*p + 1))   Subst[Int[x^(2*n + 1)*(-c + x^
2)^m*(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4)^p, x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && ILtQ[m, 0] && IntegerQ[n + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\tanh\left(dx+c\right)^{3}}{\sqrt{a+b\,\mathrm{sech}\left(dx+c\right)}}dx$$

input
```
int(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(67) = 134.

Time = 0.69 (sec) , antiderivative size = 925, normalized size of antiderivative = 11.71

$$\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\mathrm{sech}(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sin
h(d*x + c)^2 + b^2)*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x +
 c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c
)^3 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d
*x + c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 +
2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d
*x + c) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2
+ 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d
*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a
)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x +
c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d
*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^
2)) - 8*(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 - a*b*cosh(d*x + c) + a
^2 + (2*a^2*cosh(d*x + c) - a*b)*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)
/cosh(d*x + c)))/(a*b^2*d*cosh(d*x + c)^2 + 2*a*b^2*d*cosh(d*x + c)*sinh(d
*x + c) + a*b^2*d*sinh(d*x + c)^2 + a*b^2*d), -1/3*(3*(b^2*cosh(d*x + c)^2
 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b^2)*sqrt(-a)
*arctan((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*co
sh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/co
sh(d*x + c))/(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*...
```

Sympy [F]

$$\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh^3(c+dx)}{\sqrt{a+b\,\text{sech}\,(c+dx)}}\,dx$$

input
```
integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c))**(1/2),x)
```

output
```
Integral(tanh(c + d*x)**3/sqrt(a + b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx=\int \frac{\tanh\left(dx+c\right)^3}{\sqrt{b\,\text{sech}\left(dx+c\right)+a}}\,dx$$

input
```
integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(d*x + c)^3/sqrt(b*sech(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx=\int \frac{\tanh\left(dx+c\right)^3}{\sqrt{b\,\text{sech}\left(dx+c\right)+a}}\,dx$$

input
```
integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(tanh(d*x + c)^3/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx=\int \frac{\tanh(c+d\,x)^3}{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}\,dx$$

input
```
int(tanh(c + d*x)^3/(a + b/cosh(c + d*x))^(1/2),x)
```

output
```
int(tanh(c + d*x)^3/(a + b/cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(dx+c\right)b+a}\,\tanh\left(dx+c\right)^{3}}{\operatorname{sech}\left(dx+c\right)b+a}dx$$

input
```
int(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sech(c + d*x)*b + a)*tanh(c + d*x)**3)/(sech(c + d*x)*b + a),x)
```

3.135 $\int \frac{\tanh(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \frac{\tanh(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

output
```
2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
Integrate[Tanh[c + d*x]/Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
(2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 26, 4373, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cot\left(ic+idx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cot\left(\frac{1}{2}(2ic+\pi)+idx\right)}{\sqrt{a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)}}dx$$

$$\Big\downarrow \text{4373}$$

$$-\frac{\int \frac{\cosh(c+dx)}{b\sqrt{a+b\text{sech}(c+dx)}}d(b\text{sech}(c+dx))}{d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2\int \frac{1}{b^2\text{sech}^2(c+dx)-a}d\sqrt{a+b\text{sech}(c+dx)}}{d}$$

$$\Big\downarrow \text{220}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
Int[Tanh[c + d*x]/Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
(2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\operatorname{sech}(dx+c)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$	26
default	$\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\operatorname{sech}(dx+c)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$	26

input
```
int(tanh(d*x+c)/(a+b*sech(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(25) = 50$.

Time = 0.66 (sec) , antiderivative size = 558, normalized size of antiderivative = 18.00

$$\int \frac{\tanh(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x
+ c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x +
c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh
(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 +
a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x +
 c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2
*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*
x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(
d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x +
 c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)
^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2))/(sqrt(a)*d), -sqrt(
-a)*arctan((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a
*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)
/cosh(d*x + c))/(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*
x + c) + a^2 + 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)))/(a*d)]
```

Sympy [F]

$$\int \frac{\tanh(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh(c+dx)}{\sqrt{a+b\,\text{sech}(c+dx)}}\,dx$$

input
```
integrate(tanh(d*x+c)/(a+b*sech(d*x+c))**(1/2),x)
```

output
```
Integral(tanh(c + d*x)/sqrt(a + b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\tanh(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx=\int \frac{\tanh(dx+c)}{\sqrt{b\,\text{sech}\,(dx+c)+a}}\,dx$$

input
```
integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(d*x + c)/sqrt(b*sech(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\tanh(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx=\int \frac{\tanh(dx+c)}{\sqrt{b\,\text{sech}\,(dx+c)+a}}\,dx$$

input
```
integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(tanh(d*x + c)/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{\tanh(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx=\frac{2\,\text{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

input
```
int(tanh(c + d*x)/(a + b/cosh(c + d*x))^(1/2),x)
```

output
```
(2*atanh((a + b/cosh(c + d*x))^(1/2)/a^(1/2)))/(a^(1/2)*d)
```

Reduce [F]

$$\int \frac{\tanh(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\sqrt{\text{sech}\,(dx+c)\,b+a}\,\tanh\,(dx+c)}{\text{sech}\,(dx+c)\,b+a}dx$$

input
```
int(tanh(d*x+c)/(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sech(c + d*x)*b + a)*tanh(c + d*x))/(sech(c + d*x)*b + a),x)
```

3.136 $\int \frac{\coth(c+dx)}{\sqrt{a+b\,\text{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\coth(c+dx)}{\sqrt{a+b\,\text{sech}(c+dx)}}\,dx=\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b\,\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}-\frac{\text{arctanh}\left(\frac{\sqrt{a+b\,\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d}-\frac{\text{arctanh}\left(\frac{\sqrt{a+b\,\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d}$$

output
```
2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(1/2)/d-arctanh((a+b*sech(d*x
+c))^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)/d-arctanh((a+b*sech(d*x+c))^(1/2)/(a+b
)^(1/2))/(a+b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.94

$$\int \frac{\coth(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = -\frac{-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+\frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{d}$$

input
```
Integrate[Coth[c + d*x]/Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
-(((-2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/Sqrt[a] + ArcTanh[Sqrt[
a + b*Sech[c + d*x]]/Sqrt[a - b]]/Sqrt[a - b] + ArcTanh[Sqrt[a + b*Sech[c
+ d*x]]/Sqrt[a + b]]/Sqrt[a + b])/d)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 26, 4373, 561, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\cot\left(ic+idx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\cot\left(\frac{1}{2}(2ic+\pi)+idx\right)\sqrt{a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)}}dx$$

$$\Big\downarrow \text{4373}$$

$$-\frac{b^2\int \frac{\cosh(c+dx)}{b\sqrt{a+b\text{sech}(c+dx)}\left(b^2-b^2\text{sech}^2(c+dx)\right)}d(b\text{sech}(c+dx))}{d}$$

$$\Big\downarrow \text{561}$$

$$-\frac{2b^2\int \frac{1}{\left(a-b^2\text{sech}^2(c+dx)\right)\left(b^4\text{sech}^4(c+dx)-2ab^2\text{sech}^2(c+dx)+a^2-b^2\right)}d\sqrt{a+b\text{sech}(c+dx)}}{d}$$

$$\Big\downarrow \text{1484}$$

$$-\frac{2b^2\int \left(\frac{1}{2b^2\left(-b^2\text{sech}^2(c+dx)+a+b\right)}-\frac{1}{2b^2\left(b^2\text{sech}^2(c+dx)-a+b\right)}-\frac{1}{b^2\left(a-b^2\text{sech}^2(c+dx)\right)}\right)d\sqrt{a+b\text{sech}(c+dx)}}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2b^2\left(-\frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}b^2}+\frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{2b^2\sqrt{a-b}}+\frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{2b^2\sqrt{a+b}}\right)}{d}$$

input
```
Int[Coth[c + d*x]/Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
(-2*b^2*(-(ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]]/(Sqrt[a]*b^2)) + Arc
Tanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]]/(2*Sqrt[a - b]*b^2) + ArcTanh[
Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]]/(2*b^2*Sqrt[a + b])))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\coth (dx + c)}{\sqrt{a + b \operatorname{sech}(dx + c)}}dx$$

input
```
int(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(88) = 176$.

Time = 0.83 (sec) , antiderivative size = 9458, normalized size of antiderivative = 89.23

$$\int \frac{\coth(c + dx)}{\sqrt{a + b\text{sech}(c + dx)}}\, dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\coth(c+dx)}{\sqrt{a+b\,\text{sech}\,(c+dx)}}\,dx$$

input
```
integrate(coth(d*x+c)/(a+b*sech(d*x+c))**(1/2),x)
```

output
```
Integral(coth(c + d*x)/sqrt(a + b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\coth(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\coth(dx+c)}{\sqrt{b\,\text{sech}\,(dx+c)+a}}\,dx$$

input
```
integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(coth(d*x + c)/sqrt(b*sech(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\coth(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\coth(dx+c)}{\sqrt{b\,\text{sech}\,(dx+c)+a}}\,dx$$

input
```
integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(coth(d*x + c)/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx=\int \frac{\coth(c+d\,x)}{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}\,dx$$

input
```
int(coth(c + d*x)/(a + b/cosh(c + d*x))^(1/2),x)
```

output
```
int(coth(c + d*x)/(a + b/cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\coth(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx=\int \frac{\sqrt{\text{sech}\,(dx+c)\,b+a}\,\coth\,(dx+c)}{\text{sech}\,(dx+c)\,b+a}dx$$

input
```
int(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sech(c + d*x)*b + a)*coth(c + d*x))/(sech(c + d*x)*b + a),x)
```

3.137 $\int \frac{\coth^3(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 175

$$\int \frac{\coth^3(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx = \frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{(4a-5b)\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{3/2}d} - \frac{(4a+5b)\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)}{4(a+b)^{3/2}d} - \frac{\coth^2(c+dx)(a-b\,\mathrm{sech}(c+dx))\sqrt{a+b\,\mathrm{sech}(c+dx)}}{2\left(a^2-b^2\right)d}$$

output
```
2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(1/2)/d-1/4*(4*a-5*b)*arctanh
((a+b*sech(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/d-1/4*(4*a+5*b)*arctanh(
(a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/d-1/2*coth(d*x+c)^2*(a-b*
sech(d*x+c))*(a+b*sech(d*x+c))^(1/2)/(a^2-b^2)/d
```

Mathematica [A] (verified)

Time = 1.98 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.60

$$\int \frac{\coth^3(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx =$$

$$-\frac{-\frac{b^2\arctan\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{-a+b}}\right)}{(-a+b)^{3/2}}-\frac{8b\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{4b\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{S}}}{}\right)}{(a+b)^{3/}}}{}$$

input
```
Integrate[Coth[c + d*x]^3/Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
-1/4*(-((b^2*ArcTan[Sqrt[a + b*Sech[c + d*x]]/Sqrt[-a + b]])/(-a + b)^(3/2
)) - (8*b*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/Sqrt[a] + (4*b*ArcTa
nh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]])/Sqrt[a - b] + (b^2*ArcTanh[Sqrt
[a + b*Sech[c + d*x]]/Sqrt[a + b]])/(a + b)^(3/2) - (4*a*ArcTanh[Sqrt[a +
b*Sech[c + d*x]]/Sqrt[a + b]])/Sqrt[a + b] + 4*Sqrt[a + b]*ArcTanh[Sqrt[a
+ b*Sech[c + d*x]]/Sqrt[a + b]] - (b*Sqrt[a + b*Sech[c + d*x]])/((a + b)*(
-1 + Sech[c + d*x])) + (b*Sqrt[a + b*Sech[c + d*x]])/((a - b)*(1 + Sech[c
+ d*x])))/(b*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.55, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4373, 561, 25, 1567, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\cot\left(ic+idx+\frac{\pi}{2}\right)^3\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{1}{\cot\left(\frac{1}{2}(2ic+\pi)+idx\right)^3\sqrt{a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)}}dx$$

$$\Big\downarrow \; 4373$$

$$-\frac{b^4\int \frac{\cosh(c+dx)}{b\sqrt{a+b\text{sech}(c+dx)}\left(b^2-b^2\text{sech}^2(c+dx)\right)^2}d(b\text{sech}(c+dx))}{d}$$

$$\Big\downarrow \; 561$$

$$-\frac{2b^4\int -\frac{1}{\left(a-b^2\text{sech}^2(c+dx)\right)\left(b^4\text{sech}^4(c+dx)-2ab^2\text{sech}^2(c+dx)+a^2-b^2\right)^2}d\sqrt{a+b\text{sech}(c+dx)}}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{2b^4\int \frac{1}{\left(a-b^2\text{sech}^2(c+dx)\right)\left(b^4\text{sech}^4(c+dx)-2ab^2\text{sech}^2(c+dx)+a^2-b^2\right)^2}d\sqrt{a+b\text{sech}(c+dx)}}{d}$$

$$\Big\downarrow \; 1567$$

$$\frac{2b^4\int \left(-\frac{1}{2b^4\left(-b^2\text{sech}^2(c+dx)+a+b\right)}+\frac{1}{2b^4\left(b^2\text{sech}^2(c+dx)-a+b\right)}-\frac{1}{4b^3\left(-b^2\text{sech}^2(c+dx)+a+b\right)^2}+\frac{1}{4b^3\left(b^2\text{sech}^2(c+dx)-a+b\right)^2}\right.}{d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{2b^4\left(-\frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}b^4}+\frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{2b^4\sqrt{a-b}}+\frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{2b^4\sqrt{a+b}}-\frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(}{\sqrt{a-b}}\right.}{8b^3(a-b)^{3/2}}\right.}{d}$$

input
```
Int[Coth[c + d*x]^3/Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
(-2*b^4*(-(ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]]/(Sqrt[a]*b^4)) + Arc
Tanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]]/(2*Sqrt[a - b]*b^4) - ArcTanh[
Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]]/(8*(a - b)^(3/2)*b^3) + ArcTanh[Sqr
t[a + b*Sech[c + d*x]]/Sqrt[a + b]]/(8*b^3*(a + b)^(3/2)) + ArcTanh[Sqrt[a
 + b*Sech[c + d*x]]/Sqrt[a + b]]/(2*b^4*Sqrt[a + b]) - Sech[c + d*x]/(8*(a
 - b)*b^2*(a - b - b^2*Sech[c + d*x]^2)) + Sech[c + d*x]/(8*b^2*(a + b)*(a
 + b - b^2*Sech[c + d*x]^2))))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 1567
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\coth\left(dx+c\right)^{3}}{\sqrt{a+b\,\mathrm{sech}\left(dx+c\right)}}dx$$

input
```
int(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2009 vs. $2(150) = 300$.

Time = 4.99 (sec) , antiderivative size = 20851, normalized size of antiderivative = 119.15

$$\int \frac{\coth^3(c+dx)}{\sqrt{a+b\mathrm{sech}(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx=\int \frac{\coth^3{(c+dx)}}{\sqrt{a+b\,\text{sech}\,(c+dx)}}\,dx$$

input
```
integrate(coth(d*x+c)**3/(a+b*sech(d*x+c))**(1/2),x)
```

output
```
Integral(coth(c + d*x)**3/sqrt(a + b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\coth^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx=\int \frac{\coth{(dx+c)}^3}{\sqrt{b\,\text{sech}\,(dx+c)+a}}\,dx$$

input
```
integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(coth(d*x + c)^3/sqrt(b*sech(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\coth^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx=\int \frac{\coth{(dx+c)}^3}{\sqrt{b\,\text{sech}\,(dx+c)+a}}\,dx$$

input
```
integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(coth(d*x + c)^3/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx=\int \frac{{\mathrm{coth}\left(c+d\,x\right)}^3}{\sqrt{a+\frac{b}{\mathrm{cosh}\left(c+d\,x\right)}}}\,dx$$

input
```
int(coth(c + d*x)^3/(a + b/cosh(c + d*x))^(1/2),x)
```

output
```
int(coth(c + d*x)^3/(a + b/cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\coth^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx=\int \frac{\sqrt{\mathrm{sech}\left(d x +c \right) b +a}\, \mathrm{coth}\left(d x +c \right)^{3}}{\mathrm{sech}\left(d x +c \right) b +a}d x$$

input
```
int(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sech(c + d*x)*b + a)*coth(c + d*x)**3)/(sech(c + d*x)*b + a),x)
```

3.138 $\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 610

$$\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx = -\frac{4(a-b)\sqrt{a+b}\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{b^2d}$$
$$+\frac{2(a-b)\sqrt{a+b}(8a^2+9b^2)\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{se}}{}}}{15b^4d}$$
$$-\frac{4\sqrt{a+b}\coth(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{bd}$$
$$+\frac{2\sqrt{a+b}(8a^2-2ab+9b^2)\coth(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{}}{15b^3d}$$
$$+\frac{2\sqrt{a+b}\coth(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+}{a-b}}}{ad}$$
$$-\frac{8a\sqrt{a+b\operatorname{sech}(c+dx)}\tanh(c+dx)}{15b^2d}$$
$$+\frac{2\operatorname{sech}(c+dx)\sqrt{a+b\operatorname{sech}(c+dx)}\tanh(c+dx)}{5bd}$$

output
```
-4*(a-b)*(a+b)^(1/2)*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(
1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+
c))/(a-b))^(1/2)/b^2/d+2/15*(a-b)*(a+b)^(1/2)*(8*a^2+9*b^2)*coth(d*x+c)*El
lipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec
h(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b^4/d-4*(a+b)^(1/2
)*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^
(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b/
d+2/15*(a+b)^(1/2)*(8*a^2-2*a*b+9*b^2)*coth(d*x+c)*EllipticF((a+b*sech(d*x
+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2
)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b^3/d+2*(a+b)^(1/2)*coth(d*x+c)*Ellipti
cPi((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(b*(1
-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/d-8/15*a*(a+
b*sech(d*x+c))^(1/2)*tanh(d*x+c)/b^2/d+2/5*sech(d*x+c)*(a+b*sech(d*x+c))^(
1/2)*tanh(d*x+c)/b/d
```

Mathematica [F]

$$\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh^4(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$

input
```
Integrate[Tanh[c + d*x]^4/Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
Integrate[Tanh[c + d*x]^4/Sqrt[a + b*Sech[c + d*x]], x]
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4383, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}dx$$

$\Big\downarrow$ 3042

$$\int \frac{\cot\left(ic+idx+\frac{\pi}{2}\right)^4}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$\Big\downarrow$ 4383

$$\int \left(\frac{\text{sech}^4(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}-\frac{2\text{sech}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}+\frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\right)dx$$

$\Big\downarrow$ 2009

$$\frac{2(a-b)\sqrt{a+b}(8a^2+9b^2)\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^4d}$$
$$\frac{2\sqrt{a+b}(8a^2-2ab+9b^2)\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)\right.}{15b^3d}$$
$$\frac{4(a-b)\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-$$
$$\frac{4\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}+$$
$$\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}-$$
$$\frac{8a\tanh(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{15b^2d}+\frac{2\tanh(c+dx)\text{sech}(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{5bd}$$

input
```
Int[Tanh[c + d*x]^4/Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
(-4*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c +
 d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b^2*d) + (2*(a - b)*Sqrt[a +
b]*(8*a^2 + 9*b^2)*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sech[c + d*x]))/(a - b))])/(15*b^4*d) - (4*Sqrt[a + b]*Coth[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a
 - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x])
)/(a - b))])/(b*d) + (2*Sqrt[a + b]*(8*a^2 - 2*a*b + 9*b^2)*Coth[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a -
b))])/(15*b^3*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcS
in[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d) -
(8*a*Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x])/(15*b^2*d) + (2*Sech[c + d*x
]*Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x])/(5*b*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4383
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Csc[c + d
*x]^2)^(m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && I
GtQ[m/2, 0] && IntegerQ[n - 1/2]
```

Maple [F]

$$\int \frac{\tanh(dx+c)^4}{\sqrt{a+b\,\mathrm{sech}(dx+c)}}dx$$

input
```
int(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\mathrm{sech}(c+dx)}}\,dx = \int \frac{\tanh(dx+c)^4}{\sqrt{b\,\mathrm{sech}(dx+c)+a}}\,dx$$

input
```
integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(tanh(d*x + c)^4/sqrt(b*sech(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\mathrm{sech}(c+dx)}}\,dx = \int \frac{\tanh^4(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx$$

input
```
integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c))**(1/2),x)
```

output
```
Integral(tanh(c + d*x)**4/sqrt(a + b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh(dx+c)^4}{\sqrt{b\,\text{sech}(dx+c)+a}}\,dx$$

input
```
integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(d*x + c)^4/sqrt(b*sech(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh(dx+c)^4}{\sqrt{b\,\text{sech}(dx+c)+a}}\,dx$$

input
```
integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(tanh(d*x + c)^4/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh(c+d\,x)^4}{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}\,dx$$

input
```
int(tanh(c + d*x)^4/(a + b/cosh(c + d*x))^(1/2),x)
```

output
```
int(tanh(c + d*x)^4/(a + b/cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\sqrt{\text{sech}\,(dx+c)\,b+a}\,\tanh\,(dx+c)^4}{\text{sech}\,(dx+c)\,b+a}dx$$

input
```
int(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sech(c + d*x)*b + a)*tanh(c + d*x)**4)/(sech(c + d*x)*b + a),x)
```

3.139 $\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 310

$$\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx = -\frac{2(a-b)\sqrt{a+b}\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}(c+dx))}{a-b}}}{b^2d} - \frac{2\sqrt{a+b}\coth(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}(c+dx))}{a-b}}}{bd} + \frac{2\sqrt{a+b}\coth(c+dx)\,\mathrm{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}(c+dx))}{a-b}}}{ad}$$

output
```
-2*(a-b)*(a+b)^(1/2)*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(
1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+
c))/(a-b))^(1/2)/b^2/d-2*(a+b)^(1/2)*coth(d*x+c)*EllipticF((a+b*sech(d*x+c
))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*
(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b/d+2*(a+b)^(1/2)*coth(d*x+c)*EllipticPi(
(a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(b*(1-sec
h(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/d
```

Mathematica [F]

$$\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx=\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$

input
```
Integrate[Tanh[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
Integrate[Tanh[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]], x]
```

Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3042, 25, 4382, 3042, 4547, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\cot\left(ic+idx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\cot\left(\frac{1}{2}(2ic+\pi)+idx\right)^2}{\sqrt{a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)}}dx$$

$$\Big\downarrow \text{ 4382}$$

$$-\int \frac{\csc^2\left(\frac{1}{2}(2ic+\pi)+idx\right)-1}{\sqrt{a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$-\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)^2-1}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4547}$$

$$-\int \frac{-\text{sech}(c+dx)-1}{\sqrt{a+b\text{sech}(c+dx)}}dx-\int \frac{\text{sech}(c+dx)(\text{sech}(c+dx)+1)}{\sqrt{a+b\text{sech}(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$-\int \frac{-\csc\left(ic+idx+\frac{\pi}{2}\right)-1}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx-\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)\left(\csc\left(ic+idx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4409}$$

$$-\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)\left(\csc\left(ic+idx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx+\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}dx+\int \frac{\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx+\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx-\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)\left(\csc\left(ic+idx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4271}$$

$$\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx-\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)\left(\csc\left(ic+idx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}$$

$$\Big\downarrow \text{ 4319}$$

$$-\int \frac{\csc \left(ic+idx+\frac{\pi}{2}\right) \left(\csc \left(ic+idx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc \left(ic+idx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}+\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}$$

$$\Big\downarrow \text{4492}$$

$$-\frac{2(a-b)\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}+\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}$$

input
```
Int[Tanh[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
(-2*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c +
 d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*Coth[
c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/
(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x
]))/(a - b))])/(b*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a,
ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4382
```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4547
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Simp[C   Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f
*x]]), x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\tanh (dx+c)^2}{\sqrt{a+b\,\mathrm{sech}\,(dx+c)}}dx$$

input
```
int(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\mathrm{sech}(c+dx)}}\,dx = \int \frac{\tanh (dx+c)^2}{\sqrt{b\,\mathrm{sech}\,(dx+c)+a}}\,dx$$

input
```
integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(tanh(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\mathrm{sech}(c+dx)}}\,dx = \int \frac{\tanh^2 (c+dx)}{\sqrt{a+b\,\mathrm{sech}\,(c+dx)}}\,dx$$

input
```
integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c))**(1/2),x)
```

output
```
Integral(tanh(c + d*x)**2/sqrt(a + b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx=\int \frac{\tanh\left(dx+c\right)^2}{\sqrt{b\,\text{sech}\left(dx+c\right)+a}}\,dx$$

input
```
integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx=\int \frac{\tanh\left(dx+c\right)^2}{\sqrt{b\,\text{sech}\left(dx+c\right)+a}}\,dx$$

input
```
integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(tanh(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx=\int \frac{\tanh(c+d\,x)^2}{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}\,dx$$

input
```
int(tanh(c + d*x)^2/(a + b/cosh(c + d*x))^(1/2),x)
```

output
```
int(tanh(c + d*x)^2/(a + b/cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\sqrt{\text{sech}\,(dx+c)\,b+a}\,\tanh\,(dx+c)^2}{\text{sech}\,(dx+c)\,b+a}dx$$

input
```
int(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sech(c + d*x)*b + a)*tanh(c + d*x)**2)/(sech(c + d*x)*b + a),x)
```

3.140 $\int \frac{1}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 106

$$\int \frac{1}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx = \frac{2\sqrt{a+b}\coth(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx}{a-b}}}{ad}$$

output

```
2*(a+b)^(1/2)*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),(
a+b)/a,((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*
x+c))/(a-b))^(1/2)/a/d
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx = \frac{2b\sqrt{b+a\cosh(c+dx)}\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a+b}\sqrt{a\cosh(c+dx)}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1+\operatorname{sech}(c+dx))}{-a+b}}\tanh\left(\frac{1}{2}(c+dx\right.}{\sqrt{a}\sqrt{a+b}d\sqrt{a\cosh(c+dx)}\sqrt{-\frac{b(-1+\operatorname{sech}(c+dx))}{a+b}}\sqrt{a+b\operatorname{sech}(c+dx)}}$$

input
```
Integrate[1/Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
(2*b*Sqrt[b + a*Cosh[c + d*x]]*EllipticPi[(a + b)/a, ArcSin[(Sqrt[a]*Sqrt[
b + a*Cosh[c + d*x]])/(Sqrt[a + b]*Sqrt[a*Cosh[c + d*x]])], (a + b)/(a - b
)]*Sqrt[(b*(1 + Sech[c + d*x]))/(-a + b)]*Tanh[(c + d*x)/2])/(Sqrt[a]*Sqrt
[a + b]*d*Sqrt[a*Cosh[c + d*x]]*Sqrt[-((b*(-1 + Sech[c + d*x]))/(a + b))]*
Sqrt[a + b*Sech[c + d*x]])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4271}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4271}$$

$$\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}$$

input
```
Int[1/Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
(2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a +
 b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{1}{\sqrt{a + b \operatorname{sech}(dx + c)}} dx$$

input
```
int(1/(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int(1/(a+b*sech(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a + b\text{sech}(c + dx)}} dx = \int \frac{1}{\sqrt{b \operatorname{sech}(dx + c) + a}} dx$$

input
```
integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(1/sqrt(b*sech(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a + b\text{sech}(c + dx)}}\, dx = \int \frac{1}{\sqrt{a + b\,\text{sech}\,(c + dx)}}\, dx$$

input
```
integrate(1/(a+b*sech(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a + b\text{sech}(c + dx)}}\, dx = \int \frac{1}{\sqrt{b\,\text{sech}\,(dx + c) + a}}\, dx$$

input
```
integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*sech(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a + b\text{sech}(c + dx)}}\, dx = \int \frac{1}{\sqrt{b\,\text{sech}\,(dx + c) + a}}\, dx$$

input
```
integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + b\text{sech}(c + dx)}}\,dx = \int \frac{1}{\sqrt{a + \frac{b}{\cosh(c+d\,x)}}}\,dx$$

input
```
int(1/(a + b/cosh(c + d*x))^(1/2),x)
```

output
```
int(1/(a + b/cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a + b\text{sech}(c + dx)}}\,dx = \int \frac{\sqrt{\text{sech}\,(dx + c)\,b + a}}{\text{sech}\,(dx + c)\,b + a}dx$$

input
```
int(1/(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int(sqrt(sech(c + d*x)*b + a)/(sech(c + d*x)*b + a),x)
```

3.141 $\int \frac{\coth^2(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 362

$$\begin{aligned}
\int \frac{\coth^2(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx
&= \frac{\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{\sqrt{a+b}d} \\
&\quad -\frac{\coth(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{\sqrt{a+b}d} \\
&\quad +\frac{2\sqrt{a+b}\coth(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{ad} \\
&\quad -\frac{\coth(c+dx)}{d\sqrt{a+b\operatorname{sech}(c+dx)}}-\frac{b^2\tanh(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\operatorname{sech}(c+dx)}}
\end{aligned}$$

output
```
coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1
/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/(a+b
)^(1/2)/d-coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)
/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^
(1/2)/(a+b)^(1/2)/d+2*(a+b)^(1/2)*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))
^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^
(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/d-coth(d*x+c)/d/(a+b*sech(d*x+c))
^(1/2)-b^2*tanh(d*x+c)/(a^2-b^2)/d/(a+b*sech(d*x+c))^(1/2)
```

Mathematica [F]

$$\int \frac{\coth^2(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx=\int \frac{\coth^2(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx$$

input
```
Integrate[Coth[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
Integrate[Coth[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]], x]
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}=0.174$, Rules used = {3042, 25, 4384, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\cot\left(ic+idx+\frac{\pi}{2}\right)^2\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\cot\left(\frac{1}{2}(2ic+\pi)+idx\right)^2\sqrt{a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)}}dx$$

$\Big\downarrow$ 4384

$$-\int \left(-\frac{\text{csch}^2(c+dx)}{\sqrt{a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)}}-\frac{1}{\sqrt{a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)}}\right)dx$$

$\Big\downarrow$ 2009

$$-\frac{b^2\tanh(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}-\frac{\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d\sqrt{a+b}}+\frac{\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d\sqrt{a+b}}+\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}-\frac{\coth(c+dx)}{d\sqrt{a+b\text{sech}(c+dx)}}$$

input
```
Int[Coth[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]],x]
```

output
```
(Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c
 + d*x]))/(a - b))])/(Sqrt[a + b]*d) - (Coth[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(Sqrt[a + b]*d
) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*S
ech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/
(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d) - Coth[c + d*x]/(
d*Sqrt[a + b*Sech[c + d*x]]) - (b^2*Tanh[c + d*x])/((a^2 - b^2)*d*Sqrt[a +
 b*Sech[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4384
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &&
ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Maple [F]

$$\int \frac{\coth\left(dx+c\right)^{2}}{\sqrt{a+b\,\mathrm{sech}\left(dx+c\right)}}dx$$

input
```
int(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{\coth^2(c+dx)}{\sqrt{a+b\mathrm{sech}(c+dx)}}\,dx=\int \frac{\coth\left(dx+c\right)^{2}}{\sqrt{b\,\mathrm{sech}\left(dx+c\right)+a}}\,dx$$

input
```
integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(coth(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{\coth^2(c+dx)}{\sqrt{a+b\mathrm{sech}(c+dx)}}\,dx = \int \frac{\coth^2(c+dx)}{\sqrt{a+b\,\mathrm{sech}\,(c+dx)}}\,dx$$

input
```
integrate(coth(d*x+c)**2/(a+b*sech(d*x+c))**(1/2),x)
```

output
```
Integral(coth(c + d*x)**2/sqrt(a + b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\coth^2(c+dx)}{\sqrt{a+b\mathrm{sech}(c+dx)}}\,dx = \int \frac{\coth\,(dx+c)^2}{\sqrt{b\,\mathrm{sech}\,(dx+c)+a}}\,dx$$

input
```
integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(coth(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\coth^2(c+dx)}{\sqrt{a+b\mathrm{sech}(c+dx)}}\,dx = \int \frac{\coth\,(dx+c)^2}{\sqrt{b\,\mathrm{sech}\,(dx+c)+a}}\,dx$$

input
```
integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(coth(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\coth(c+d\,x)^2}{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}\,dx$$

input
```
int(coth(c + d*x)^2/(a + b/cosh(c + d*x))^(1/2),x)
```

output
```
int(coth(c + d*x)^2/(a + b/cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\coth^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\sqrt{\text{sech}\,(dx+c)\,b+a}\,\coth\,(dx+c)^2}{\text{sech}\,(dx+c)\,b+a}dx$$

input
```
int(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x)
```

output
```
int((sqrt(sech(c + d*x)*b + a)*coth(c + d*x)**2)/(sech(c + d*x)*b + a),x)
```

3.142 $\int \frac{\tanh^5(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{\tanh^5(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{2(a^2-b^2)^2}{ab^4d\sqrt{a+b\text{sech}(c+dx)}} - \frac{2(3a^2-2b^2)\sqrt{a+b\text{sech}(c+dx)}}{b^4d} + \frac{2a(a+b\text{sech}(c+dx))^{3/2}}{b^4d} - \frac{2(a+b\text{sech}(c+dx))^{5/2}}{5b^4d}$$

output
```
2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-2*(a^2-b^2)^2/a/b^4/d
/(a+b*sech(d*x+c))^(1/2)-2*(3*a^2-2*b^2)*(a+b*sech(d*x+c))^(1/2)/b^4/d+2*a
*(a+b*sech(d*x+c))^(3/2)/b^4/d-2/5*(a+b*sech(d*x+c))^(5/2)/b^4/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.59 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.79

$$\int \frac{\tanh^5(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx =$$
$$-\frac{2\left(5b^4\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{b\text{sech}(c+dx)}{a}\right)+a\left(4a(4a^2-5b^2)+2b(4a^2-5b^2)\,\text{sech}(c+dx)-\right.\right.}{5ab^4d\sqrt{a+b\text{sech}(c+dx)}}$$

input
```
Integrate[Tanh[c + d*x]^5/(a + b*Sech[c + d*x])^(3/2),x]
```

output
```
(-2*(5*b^4*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sech[c + d*x])/a] + a*(4
*a*(4*a^2 - 5*b^2) + 2*b*(4*a^2 - 5*b^2)*Sech[c + d*x] - 2*a*b^2*Sech[c +
d*x]^2 + b^3*Sech[c + d*x]^3)))/(5*a*b^4*d*Sqrt[a + b*Sech[c + d*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.80, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4373, 517, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\cot\left(ic+idx+\frac{\pi}{2}\right)^5}{\left(a+b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{\cot\left(\frac{1}{2}(2ic+\pi)+idx\right)^5}{\left(a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 4373$$

$$-\frac{\int \frac{\cosh(c+dx)\left(b^2-b^2\text{sech}^2(c+dx)\right)^2}{b(a+b\text{sech}(c+dx))^{3/2}}d(b\text{sech}(c+dx))}{b^4d}$$

$$\Big\downarrow \; 517$$

$$-\frac{2\int -\frac{\cosh^2(c+dx)\left(b^4\text{sech}^4(c+dx)-2ab^2\text{sech}^2(c+dx)+a^2-b^2\right)^2}{b^2\left(a-b^2\text{sech}^2(c+dx)\right)}d\sqrt{a+b\text{sech}(c+dx)}}{b^4d}$$

$$\Big\downarrow \; 25$$

$$\frac{2\int \frac{\cosh^2(c+dx)\left(b^4\text{sech}^4(c+dx)-2ab^2\text{sech}^2(c+dx)+a^2-b^2\right)^2}{b^2\left(a-b^2\text{sech}^2(c+dx)\right)}d\sqrt{a+b\text{sech}(c+dx)}}{b^4d}$$

$$\Big\downarrow \; 1584$$

$$\frac{2\int \left(-\text{sech}^4(c+dx)b^4+\frac{b^4}{a\left(a-b^2\text{sech}^2(c+dx)\right)}+3a\text{sech}^2(c+dx)b^2-3a^2\left(1-\frac{2b^2}{3a^2}\right)+\frac{\left(a^2-b^2\right)^2\cosh^2(c+dx)}{ab^2}\right)d\sqrt{a+}}{b^4d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{2\left(-\frac{b^4\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\left(a^2-b^2\right)^2\cosh(c+dx)}{ab}+\left(3a^2-2b^2\right)\sqrt{a+b\text{sech}(c+dx)}-ab^3\text{sech}^3(c+dx)+\right.}{b^4d}$$

input
```
Int[Tanh[c + d*x]^5/(a + b*Sech[c + d*x])^(3/2),x]
```

output
```
(-2*(-((b^4*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/a^(3/2)) + ((a^2 -
 b^2)^2*Cosh[c + d*x])/(a*b) - a*b^3*Sech[c + d*x]^3 + (b^5*Sech[c + d*x]^
5)/5 + (3*a^2 - 2*b^2)*Sqrt[a + b*Sech[c + d*x]]))/(b^4*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 517
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> Simp[2*(e^m/d^(m + 2*p + 1))   Subst[Int[x^(2*n + 1)*(-c + x^
2)^m*(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4)^p, x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && ILtQ[m, 0] && IntegerQ[n + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\tanh(dx+c)^5}{(a+b\,\mathrm{sech}(dx+c))^{\frac{3}{2}}}dx$$

input
```
int(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x)
```

output
```
int(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1743 vs. $2(132) = 264$.

Time = 0.52 (sec) , antiderivative size = 3745, normalized size of antiderivative = 25.30

$$\int \frac{\tanh^5(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh^5(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh^5(c+dx)}{(a+b\,\mathrm{sech}(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c))**(3/2),x)
```

output
```
Integral(tanh(c + d*x)**5/(a + b*sech(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh^5(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh\left(dx+c\right)^5}{\left(b\,\text{sech}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(d*x + c)^5/(b*sech(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\tanh^5(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh\left(dx+c\right)^5}{\left(b\,\text{sech}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(tanh(d*x + c)^5/(b*sech(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^5(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh(c+d\,x)^5}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(tanh(c + d*x)^5/(a + b/cosh(c + d*x))^(3/2),x)
```

output
```
int(tanh(c + d*x)^5/(a + b/cosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\tanh^5(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\text{sech}\,(dx+c)\,b+a}\,\tanh\,(dx+c)^5}{\text{sech}\,(dx+c)^2\,b^2+2\,\text{sech}\,(dx+c)\,ab+a^2}dx$$

input
```
int(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sech(c + d*x)*b + a)*tanh(c + d*x)**5)/(sech(c + d*x)**2*b**2 +
2*sech(c + d*x)*a*b + a**2),x)
```

3.143 $\int \frac{\tanh^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int \frac{\tanh^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2(a^2-b^2)}{ab^2d\sqrt{a+b\text{sech}(c+dx)}} + \frac{2\sqrt{a+b\text{sech}(c+dx)}}{b^2d}$$

output
```
2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+2*(a^2-b^2)/a/b^2/d/(
a+b*sech(d*x+c))^(1/2)+2*(a+b*sech(d*x+c))^(1/2)/b^2/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\int \frac{\tanh^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \frac{2\left(-b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{b\text{sech}(c+dx)}{a}\right)+a(2a+b\text{sech}(c+\right.}{ab^2d\sqrt{a+b\text{sech}(c+dx)}}$$

input
```
Integrate[Tanh[c + d*x]^3/(a + b*Sech[c + d*x])^(3/2),x]
```

output
```
(2*(-(b^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sech[c + d*x])/a]) + a*(2
*a + b*Sech[c + d*x])))/(a*b^2*d*Sqrt[a + b*Sech[c + d*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 26, 4373, 517, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cot\left(ic+idx+\frac{\pi}{2}\right)^3}{\left(a+b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cot\left(\frac{1}{2}(2ic+\pi)+idx\right)^3}{\left(a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4373}$$

$$-\frac{\int \frac{\cosh(c+dx)\left(b^2-b^2\text{sech}^2(c+dx)\right)}{b(a+b\text{sech}(c+dx))^{3/2}}d(b\text{sech}(c+dx))}{b^2d}$$

$$\Big\downarrow \text{517}$$

$$-\frac{2\int \frac{\cosh^2(c+dx)\left(b^4\text{sech}^4(c+dx)-2ab^2\text{sech}^2(c+dx)+a^2-b^2\right)}{b^2\left(a-b^2\text{sech}^2(c+dx)\right)}d\sqrt{a+b\text{sech}(c+dx)}}{b^2d}$$

$$\Big\downarrow \text{1584}$$

$$-\frac{2\int \left(-\frac{b^2}{a\left(a-b^2\text{sech}^2(c+dx)\right)}-1+\frac{(a^2-b^2)\cosh^2(c+dx)}{ab^2}\right)d\sqrt{a+b\text{sech}(c+dx)}}{b^2d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(-\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{(a^2-b^2)\cosh(c+dx)}{ab}-\sqrt{a+b\text{sech}(c+dx)}\right)}{b^2d}$$

input
```
Int[Tanh[c + d*x]^3/(a + b*Sech[c + d*x])^(3/2),x]
```

output
```
(-2*(-((b^2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/a^(3/2)) - ((a^2 -
 b^2)*Cosh[c + d*x])/(a*b) - Sqrt[a + b*Sech[c + d*x]]))/(b^2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 517
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> Simp[2*(e^m/d^(m + 2*p + 1))   Subst[Int[x^(2*n + 1)*(-c + x^
2)^m*(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4)^p, x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && ILtQ[m, 0] && IntegerQ[n + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\tanh \left(dx+c\right)^{3}}{\left(a+b\,\mathrm{sech}\left(dx+c\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x)
```

output
```
int(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(78) = 156$.

Time = 0.49 (sec) , antiderivative size = 1107, normalized size of antiderivative = 12.58

$$\int \frac{\tanh^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((a*b^2*cosh(d*x + c)^2 + a*b^2*sinh(d*x + c)^2 + 2*b^3*cosh(d*x + c)
 + a*b^2 + 2*(a*b^2*cosh(d*x + c) + b^3)*sinh(d*x + c))*sqrt(a)*log(-(2*a^
2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a
^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2 + b
^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4*a
^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)
^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cos
h(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x +
c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*
cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/c
osh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b +
 (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x
 + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 4*(2*a^2*b*cosh(d*x + c) + 2*a^3
 - a*b^2 + (2*a^3 - a*b^2)*cosh(d*x + c)^2 + (2*a^3 - a*b^2)*sinh(d*x + c)
^2 + 2*(a^2*b + (2*a^3 - a*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh
(d*x + c) + b)/cosh(d*x + c)))/(a^3*b^2*d*cosh(d*x + c)^2 + a^3*b^2*d*sinh
(d*x + c)^2 + 2*a^2*b^3*d*cosh(d*x + c) + a^3*b^2*d + 2*(a^3*b^2*d*cosh(d*
x + c) + a^2*b^3*d)*sinh(d*x + c)), -((a*b^2*cosh(d*x + c)^2 + a*b^2*sinh(
d*x + c)^2 + 2*b^3*cosh(d*x + c) + a*b^2 + 2*(a*b^2*cosh(d*x + c) + b^3)*s
inh(d*x + c))*sqrt(-a)*arctan((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + ...
```

Sympy [F]

$$\int \frac{\tanh^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh^3{(c+dx)}}{(a+b\operatorname{sech}{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c))**(3/2),x)
```

output
```
Integral(tanh(c + d*x)**3/(a + b*sech(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh\left(dx+c\right)^3}{\left(b\,\text{sech}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(d*x + c)^3/(b*sech(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\tanh^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh\left(dx+c\right)^3}{\left(b\,\text{sech}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(tanh(d*x + c)^3/(b*sech(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh(c+d\,x)^3}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(tanh(c + d*x)^3/(a + b/cosh(c + d*x))^(3/2),x)
```

output
```
int(tanh(c + d*x)^3/(a + b/cosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\tanh^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\text{sech}\,(dx+c)\,b+a}\,\tanh\,(dx+c)^3}{\text{sech}\,(dx+c)^2\,b^2+2\,\text{sech}\,(dx+c)\,ab+a^2}dx$$

input
```
int(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sech(c + d*x)*b + a)*tanh(c + d*x)**3)/(sech(c + d*x)**2*b**2 +
2*sech(c + d*x)*a*b + a**2),x)
```

3.144 $\int \frac{\tanh(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{\tanh(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{2}{ad\sqrt{a+b\text{sech}(c+dx)}}$$

output
```
2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-2/a/d/(a+b*sech(d*x+c
))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{\tanh(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx=-\frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{b\text{sech}(c+dx)}{a}\right)}{ad\sqrt{a+b\text{sech}(c+dx)}}$$

input
```
Integrate[Tanh[c + d*x]/(a + b*Sech[c + d*x])^(3/2),x]
```

output
```
(-2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sech[c + d*x])/a])/(a*d*Sqrt[a
+ b*Sech[c + d*x]])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 26, 4373, 61, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cot\left(ic+idx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cot\left(\frac{1}{2}(2ic+\pi)+idx\right)}{\left(a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4373}$$

$$-\frac{\int \frac{\cosh(c+dx)}{b(a+b\text{sech}(c+dx))^{3/2}}d(b\text{sech}(c+dx))}{d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{\frac{\int \frac{\cosh(c+dx)}{b\sqrt{a+b\text{sech}(c+dx)}}d(b\text{sech}(c+dx))}{a}+\frac{2}{a\sqrt{a+b\text{sech}(c+dx)}}}{d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{2\int \frac{1}{b^2\text{sech}^2(c+dx)-a}d\sqrt{a+b\text{sech}(c+dx)}}{a}+\frac{2}{a\sqrt{a+b\text{sech}(c+dx)}}}{d}$$

$$\Big\downarrow \text{220}$$

$$-\frac{-\frac{2}{a\sqrt{a+b\operatorname{sech}(c+dx)}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}}}{d}$$

input
```
Int[Tanh[c + d*x]/(a + b*Sech[c + d*x])^(3/2),x]
```

output
```
-(((-2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[a +
 b*Sech[c + d*x]]))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{\frac{2}{a\sqrt{a+b\,\operatorname{sech}\left(dx+c\right)}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\operatorname{sech}\left(dx+c\right)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{d}$	46
default	$-\frac{\frac{2}{a\sqrt{a+b\,\operatorname{sech}\left(dx+c\right)}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\operatorname{sech}\left(dx+c\right)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{d}$	46

input
```
int(tanh(d*x+c)/(a+b*sech(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(2/a/(a+b*sech(d*x+c))^(1/2)-2/a^(3/2)*arctanh((a+b*sech(d*x+c))^(1/2
)/a^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(46) = 92.

Time = 0.50 (sec) , antiderivative size = 917, normalized size of antiderivative = 16.98

$$\int \frac{\tanh(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(a*co
sh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 +
2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a
*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2
+ (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x +
 c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)
^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*
cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x +
c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*s
inh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4
*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(
d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
+ sinh(d*x + c)^2)) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x +
c) + a*sinh(d*x + c)^2 + a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a^
3*d*cosh(d*x + c)^2 + a^3*d*sinh(d*x + c)^2 + 2*a^2*b*d*cosh(d*x + c) + a^
3*d + 2*(a^3*d*cosh(d*x + c) + a^2*b*d)*sinh(d*x + c)), -((a*cosh(d*x + c)
^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(a*cosh(d*x + c) + b)*sinh(
d*x + c) + a)*sqrt(-a)*arctan((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*c
osh(d*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(-a)*sqrt((a
*cosh(d*x + c) + b)/cosh(d*x + c))/(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x ...
```

Sympy [F]

$$\int \frac{\tanh(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh{(c+dx)}}{(a+b\operatorname{sech}{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(d*x+c)/(a+b*sech(d*x+c))**(3/2),x)
```

output
```
Integral(tanh(c + d*x)/(a + b*sech(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh(dx+c)}{(b\operatorname{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(d*x + c)/(b*sech(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\tanh(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh(dx+c)}{(b\operatorname{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(tanh(d*x + c)/(b*sech(d*x + c) + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 2.82 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{\tanh(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx = \frac{2\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(c+dx)}}}{\sqrt{a}}\right)}{a^{3/2}\,d} - \frac{2}{a\,d\,\sqrt{a+\frac{b}{\cosh(c+dx)}}}$$

input
```
int(tanh(c + d*x)/(a + b/cosh(c + d*x))^(3/2),x)
```

output
```
(2*atanh((a + b/cosh(c + d*x))^(1/2)/a^(1/2)))/(a^(3/2)*d) - 2/(a*d*(a + b
/cosh(c + d*x))^(1/2))
```

Reduce [F]

$$\int \frac{\tanh(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\text{sech}\,(dx+c)\,b+a}\,\tanh\,(dx+c)}{\text{sech}\,(dx+c)^2\,b^2+2\,\text{sech}\,(dx+c)\,ab+a^2}dx$$

input
```
int(tanh(d*x+c)/(a+b*sech(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sech(c + d*x)*b + a)*tanh(c + d*x))/(sech(c + d*x)**2*b**2 + 2*s
ech(c + d*x)*a*b + a**2),x)
```

3.145 $\int \frac{\coth(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 142

$$\int \frac{\coth(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d} + \frac{2b^2}{a\left(a^2-b^2\right)d\sqrt{a+b\text{sech}(c+dx)}}$$

output
```
2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-arctanh((a+b*sech(d*x
+c))^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/d-arctanh((a+b*sech(d*x+c))^(1/2)/(a+b
)^(1/2))/(a+b)^(3/2)/d+2*b^2/a/(a^2-b^2)/d/(a+b*sech(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.70 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.49

$$\int \frac{\coth(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx = -\frac{-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\operatorname{sech}(c+dx)}{a-b}\right)}{(a-b)\sqrt{a+b\operatorname{sech}(c+dx)}} + \frac{a\,\text{Hyperge}\cdots}{\cdots}}{bd}$$

input
```
Integrate[Coth[c + d*x]/(a + b*Sech[c + d*x])^(3/2),x]
```

output
```
-((-(ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]]/Sqrt[a - b]) + ArcTanh
[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]]/Sqrt[a + b] - (a*Hypergeometric2F1
[-1/2, 1, 1/2, (a + b*Sech[c + d*x])/(a - b)])/((a - b)*Sqrt[a + b*Sech[c
+ d*x]]) + (a*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sech[c + d*x])/(a + b
)])/((a + b)*Sqrt[a + b*Sech[c + d*x]]) + (2*b*Hypergeometric2F1[-1/2, 1,
1/2, 1 + (b*Sech[c + d*x])/a])/(a*Sqrt[a + b*Sech[c + d*x]]))/(b*d))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

Failed to integrate

input
```
Int[Coth[c + d*x]/(a + b*Sech[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \frac{\coth (dx+c)}{(a+b\,\mathrm{sech}\,(dx+c))^{\frac{3}{2}}}dx$$

input
```
int(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x)
```

output
```
int(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1273 vs. $2(122) = 244$.

Time = 3.58 (sec) , antiderivative size = 14962, normalized size of antiderivative = 105.37

$$\int \frac{\coth(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth (c+dx)}{(a+b\,\mathrm{sech}\,(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(d*x+c)/(a+b*sech(d*x+c))**(3/2),x)
```

output
```
Integral(coth(c + d*x)/(a + b*sech(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\coth(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth\left(dx+c\right)}{\left(b\,\text{sech}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(coth(d*x + c)/(b*sech(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\coth(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth\left(dx+c\right)}{\left(b\,\text{sech}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(coth(d*x + c)/(b*sech(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth(c+d\,x)}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(coth(c + d*x)/(a + b/cosh(c + d*x))^(3/2),x)
```

output
```
int(coth(c + d*x)/(a + b/cosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\coth(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\text{sech}\,(dx+c)\,b+a}\,\coth\,(dx+c)}{\text{sech}\,(dx+c)^2\,b^2+2\,\text{sech}\,(dx+c)\,ab+a^2}dx$$

input
```
int(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sech(c + d*x)*b + a)*coth(c + d*x))/(sech(c + d*x)**2*b**2 + 2*s
ech(c + d*x)*a*b + a**2),x)
```

3.146 $\int \frac{\coth^3(c+dx)}{(a+b\,\text{sech}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 217

$$\int \frac{\coth^3(c+dx)}{(a+b\,\text{sech}(c+dx))^{3/2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{(4a-7b)\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{5/2}d} - \frac{(4a+7b)\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{4(a+b)^{5/2}d} - \frac{2b^4}{a\left(a^2-b^2\right)^2 d\sqrt{a+b\text{sech}(c+dx)}} - \frac{\coth^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}\left(a^2+b^2-2ab\text{sech}(c+dx)\right)}{2\left(a^2-b^2\right)^2 d}$$

output
```
2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-1/4*(4*a-7*b)*arctanh
((a+b*sech(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/d-1/4*(4*a+7*b)*arctanh(
(a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/d-2*b^4/a/(a^2-b^2)^2/d/(
a+b*sech(d*x+c))^(1/2)-1/2*coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2)*(a^2+b^2-
2*a*b*sech(d*x+c))/(a^2-b^2)^2/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.11 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.45

$$\int \frac{\coth^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx =$$

$$-\frac{-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{2a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\text{sech}(c+dx)}{a-b}\right)}{(a-b)\sqrt{a+b\text{sech}(c+dx)}}+\frac{2a\,\text{Hyp}}{}}{}$$

input
```
Integrate[Coth[c + d*x]^3/(a + b*Sech[c + d*x])^(3/2),x]
```

output
```
-1/2*((-2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]])/Sqrt[a - b] + (2
*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]])/Sqrt[a + b] - (2*a*Hyperg
eometric2F1[-1/2, 1, 1/2, (a + b*Sech[c + d*x])/(a - b)])/((a - b)*Sqrt[a
+ b*Sech[c + d*x]]) + (2*a*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sech[c +
 d*x])/(a + b)])/((a + b)*Sqrt[a + b*Sech[c + d*x]]) + (4*b*Hypergeometric
2F1[-1/2, 1, 1/2, 1 + (b*Sech[c + d*x])/a])/(a*Sqrt[a + b*Sech[c + d*x]])
+ (b^2*Hypergeometric2F1[-1/2, 2, 1/2, (a + b*Sech[c + d*x])/(a - b)])/((a
 - b)^2*Sqrt[a + b*Sech[c + d*x]]) - (b^2*Hypergeometric2F1[-1/2, 2, 1/2,
(a + b*Sech[c + d*x])/(a + b)])/((a + b)^2*Sqrt[a + b*Sech[c + d*x]]))/(b*
d)
```

Rubi [A] (warning: unable to verify)

Time = 0.87 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.43, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4373, 561, 25, 1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\cot\left(ic+idx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$\Big\downarrow$ 26

$$i\int \frac{1}{\cot\left(\frac{1}{2}(2ic+\pi)+idx\right)^3\left(a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)\right)^{3/2}}dx$$

$\Big\downarrow$ 4373

$$-\frac{b^4\int \frac{\cosh(c+dx)}{b(a+b\text{sech}(c+dx))^{3/2}\left(b^2-b^2\text{sech}^2(c+dx)\right)^2}d(b\text{sech}(c+dx))}{d}$$

$\Big\downarrow$ 561

$$-\frac{2b^4\int -\frac{\cosh^2(c+dx)}{b^2\left(a-b^2\text{sech}^2(c+dx)\right)\left(b^4\text{sech}^4(c+dx)-2ab^2\text{sech}^2(c+dx)+a^2-b^2\right)^2}d\sqrt{a+b\text{sech}(c+dx)}}{d}$$

$\Big\downarrow$ 25

$$\frac{2b^4\int \frac{\cosh^2(c+dx)}{b^2\left(a-b^2\text{sech}^2(c+dx)\right)\left(b^4\text{sech}^4(c+dx)-2ab^2\text{sech}^2(c+dx)+a^2-b^2\right)^2}d\sqrt{a+b\text{sech}(c+dx)}}{d}$$

$\Big\downarrow$ 1674

$$\frac{2b^4\int \left(\frac{\cosh^2(c+dx)}{a(a-b)^2b^2(a+b)^2}+\frac{1}{ab^4\left(a-b^2\text{sech}^2(c+dx)\right)}+\frac{3b-2a}{4(a-b)^2b^4\left(-b^2\text{sech}^2(c+dx)+a-b\right)}+\frac{-2a-3b}{4b^4(a+b)^2\left(-b^2\text{sech}^2(c+dx)+a+b\right)}+\right.}{d}$$

$\Big\downarrow$ 2009

$$-\frac{2b^4\left(-\frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}b^4}+\frac{\cosh(c+dx)}{ab(a^2-b^2)^2}+\frac{(2a-3b)\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4b^4(a-b)^{5/2}}+\frac{(2a+3b)\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+d}}{\sqrt{a+b}}\right.}{4b^4(a+b)^{5/2}}\right.}{}$$

input
```
Int[Coth[c + d*x]^3/(a + b*Sech[c + d*x])^(3/2),x]
```

output
```
(-2*b^4*(-(ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]]/(a^(3/2)*b^4)) + ((2
*a - 3*b)*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(5/2)
*b^4) - ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]]/(8*(a - b)^(5/2)*b^
3) + ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]]/(8*b^3*(a + b)^(5/2))
+ ((2*a + 3*b)*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]])/(4*b^4*(a +
 b)^(5/2)) + Cosh[c + d*x]/(a*b*(a^2 - b^2)^2) - Sech[c + d*x]/(8*(a - b)^
2*b^2*(a - b - b^2*Sech[c + d*x]^2)) + Sech[c + d*x]/(8*b^2*(a + b)^2*(a +
 b - b^2*Sech[c + d*x]^2))))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\coth \left(dx+c\right)^{3}}{\left(a+b\,\mathrm{sech}\left(dx+c\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x)
```

output
```
int(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6123 vs. $2(193) = 386$.

Time = 6.47 (sec) , antiderivative size = 53763, normalized size of antiderivative = 247.76

$$\int \frac{\coth^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth^3(c+dx)}{(a+b\operatorname{sech}(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(d*x+c)**3/(a+b*sech(d*x+c))**(3/2),x)
```

output
```
Integral(coth(c + d*x)**3/(a + b*sech(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\coth^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth(dx+c)^3}{(b\operatorname{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(coth(d*x + c)^3/(b*sech(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\coth^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth(dx+c)^3}{(b\operatorname{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(coth(d*x + c)^3/(b*sech(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx=\int \frac{\coth(c+d\,x)^3}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(coth(c + d*x)^3/(a + b/cosh(c + d*x))^(3/2),x)
```

output
```
int(coth(c + d*x)^3/(a + b/cosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\coth^3(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{\operatorname{sech}(dx+c)\,b+a}\,\coth(dx+c)^3}{\operatorname{sech}(dx+c)^2\,b^2+2\operatorname{sech}(dx+c)\,ab+a^2}dx$$

input
```
int(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sech(c + d*x)*b + a)*coth(c + d*x)**3)/(sech(c + d*x)**2*b**2 +
2*sech(c + d*x)*a*b + a**2),x)
```

3.147 $\int \frac{\tanh^4(c+dx)}{(a+b\,\mathrm{sech}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 907

$$\int \frac{\tanh^4(c+dx)}{(a+b\,\mathrm{sech}(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

output
```
-2*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))
^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a
/(a+b)^(1/2)/d+4*a*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/
2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c)
)/(a-b))^(1/2)/b^2/(a+b)^(1/2)/d-2/3*a*(8*a^2-5*b^2)*coth(d*x+c)*EllipticE
((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c
))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b^4/(a+b)^(1/2)/d+2*coth(
d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*
(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/(a+b)^(
1/2)/d+4*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/
(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(
1/2)/b/(a+b)^(1/2)/d-2/3*(2*a+b)*(4*a+b)*coth(d*x+c)*EllipticF((a+b*sech(d
*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1
/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b^3/(a+b)^(1/2)/d+2*(a+b)^(1/2)*coth(
d*x+c)*EllipticPi((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b)
)^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/
a^2/d-4*a*tanh(d*x+c)/(a^2-b^2)/d/(a+b*sech(d*x+c))^(1/2)+2*b^2*tanh(d*x+c
)/a/(a^2-b^2)/d/(a+b*sech(d*x+c))^(1/2)-2*a^2*sech(d*x+c)*tanh(d*x+c)/b/(a
^2-b^2)/d/(a+b*sech(d*x+c))^(1/2)+2/3*(4*a^2-b^2)*(a+b*sech(d*x+c))^(1/2)*
tanh(d*x+c)/b^2/(a^2-b^2)/d
```

Mathematica [F]

$$\int \frac{\tanh^4(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx=\int \frac{\tanh^4(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$$

input
```
Integrate[Tanh[c + d*x]^4/(a + b*Sech[c + d*x])^(3/2),x]
```

output
```
Integrate[Tanh[c + d*x]^4/(a + b*Sech[c + d*x])^(3/2), x]
```

Rubi [A] (verified)

Time = 2.51 (sec) , antiderivative size = 907, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 4383, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cot\left(ic+idx+\frac{\pi}{2}\right)^4}{\left(a+b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4383}$$

$$\int \left(\frac{\text{sech}^4(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}-\frac{2\text{sech}^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}+\frac{1}{(a+b\text{sech}(c+dx))^{3/2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2\text{sech}(c+dx)\tanh(c+dx)a^2}{b\left(a^2-b^2\right)d\sqrt{a+b\text{sech}(c+dx)}}-\frac{4\tanh(c+dx)a}{\left(a^2-b^2\right)d\sqrt{a+b\text{sech}(c+dx)}}-$$

$$\frac{2(8a^2-5b^2)\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}a}{3b^4\sqrt{a+b}d}+$$

$$\frac{4\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}a}{b^2\sqrt{a+b}d}+$$

$$\frac{2\left(4a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)}{3b^2\left(a^2-b^2\right)d}-$$

$$\frac{2(2a+b)(4a+b)\coth(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}}{3b^3\sqrt{a+b}d}$$

$$\frac{4\coth(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}}{b\sqrt{a+b}d}+$$

$$\frac{2b^2\tanh(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\text{sech}(c+dx)}a}-$$

$$\frac{2\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}}{\sqrt{a+b}da}+$$

$$\frac{2\coth(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}}{\sqrt{a+b}da}+$$

$$\frac{2\sqrt{a+b}\coth(c+dx)\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}}{da^2}$$

input
```
Int[Tanh[c + d*x]^4/(a + b*Sech[c + d*x])^(3/2),x]
```

output
```
(-2*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
h[c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) + (4*a*Coth[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b^2*
Sqrt[a + b]*d) - (2*a*(8*a^2 - 5*b^2)*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c +
d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a +
b]*d) + (2*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a
 + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(
1 + Sech[c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) + (4*Coth[c + d*x]*Ellipt
icF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/
(b*Sqrt[a + b]*d) - (2*(2*a + b)*(4*a + b)*Coth[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(3*b^3*Sqrt
[a + b]*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqr
t[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a^2*d) - (4*a
*Tanh[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sech[c + d*x]]) + (2*b^2*Tanh...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4383
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Csc[c + d
*x]^2)^(m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && I
GtQ[m/2, 0] && IntegerQ[n - 1/2]
```

Maple [F]

$$\int \frac{\tanh(dx+c)^4}{(a+b\,\mathrm{sech}(dx+c))^{\frac{3}{2}}}dx$$

input
```
int(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x)
```

output
```
int(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x)
```

Fricas [F]

$$\int \frac{\tanh^4(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh(dx+c)^4}{(b\,\mathrm{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^4/(b^2*sech(d*x + c)^2 +
2*a*b*sech(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\tanh^4(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh^4(c+dx)}{(a+b\,\mathrm{sech}(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c))**(3/2),x)
```

output
```
Integral(tanh(c + d*x)**4/(a + b*sech(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh^4(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh\left(dx+c\right)^4}{\left(b\,\text{sech}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(d*x + c)^4/(b*sech(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\tanh^4(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh\left(dx+c\right)^4}{\left(b\,\text{sech}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(tanh(d*x + c)^4/(b*sech(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^4(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh(c+d\,x)^4}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(tanh(c + d*x)^4/(a + b/cosh(c + d*x))^(3/2),x)
```

output
```
int(tanh(c + d*x)^4/(a + b/cosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\tanh^4(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\text{sech}\,(dx+c)\,b+a}\,\tanh\,(dx+c)^4}{\text{sech}\,(dx+c)^2\,b^2+2\,\text{sech}\,(dx+c)\,ab+a^2}dx$$

input
```
int(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sech(c + d*x)*b + a)*tanh(c + d*x)**4)/(sech(c + d*x)**2*b**2 +
2*sech(c + d*x)*a*b + a**2),x)
```

3.148 $\int \frac{\tanh^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 344

$$\int \frac{\tanh^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \frac{2(a-b)\sqrt{a+b}\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(}{a+b}}}{ab^2d}$$
$$+\frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{abd}$$
$$+\frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx)}{a-b}}}{a^2d}$$
$$-\frac{2\tanh(c+dx)}{ad\sqrt{a+b\text{sech}(c+dx)}}$$

output
```
2*(a-b)*(a+b)^(1/2)*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1
/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c
))/(a-b))^(1/2)/a/b^2/d+2*(a+b)^(1/2)*coth(d*x+c)*EllipticF((a+b*sech(d*x+
c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)
*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/b/d+2*(a+b)^(1/2)*coth(d*x+c)*Elliptic
Pi((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(b*(1-
sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a^2/d-2*tanh(d*
x+c)/a/d/(a+b*sech(d*x+c))^(1/2)
```

Mathematica [F]

$$\int \frac{\tanh^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$$

input
```
Integrate[Tanh[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2),x]
```

output
```
Integrate[Tanh[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2), x]
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 25, 4382, 3042, 4549, 27, 3042, 4547, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{\cot\left(ic+idx+\frac{\pi}{2}\right)^2}{\left(a+b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\cot\left(\frac{1}{2}(2ic+\pi)+idx\right)^2}{\left(a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \ 4382$$

$$-\int \frac{\csc^2\left(\frac{1}{2}(2ic+\pi)+idx\right)-1}{\left(a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$-\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)^2-1}{\left(a+b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$\Big\downarrow$ 4549

$$\frac{2\int \frac{a^2-b^2+(a^2-b^2)\text{sech}^2(c+dx)}{2\sqrt{a+b\text{sech}(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2\tanh(c+dx)}{ad\sqrt{a+b\text{sech}(c+dx)}}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{a^2-b^2+(a^2-b^2)\text{sech}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2\tanh(c+dx)}{ad\sqrt{a+b\text{sech}(c+dx)}}$$

$\Big\downarrow$ 3042

$$-\frac{2\tanh(c+dx)}{ad\sqrt{a+b\text{sech}(c+dx)}}+\frac{\int \frac{a^2-b^2+(a^2-b^2)\csc\left(ic+idx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}$$

$\Big\downarrow$ 4547

$$\frac{\left(a^2-b^2\right)\int \frac{\text{sech}(c+dx)(\text{sech}(c+dx)+1)}{\sqrt{a+b\text{sech}(c+dx)}}dx+\int \frac{a^2-b^2-(a^2-b^2)\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2\tanh(c+dx)}{ad\sqrt{a+b\text{sech}(c+dx)}}$$

$\Big\downarrow$ 3042

$$-\frac{2\tanh(c+dx)}{ad\sqrt{a+b\text{sech}(c+dx)}}+\frac{\left(a^2-b^2\right)\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)\left(\csc\left(ic+idx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx+\int \frac{a^2-b^2+(b^2-a^2)\csc\left(ic+idx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}$$

$\Big\downarrow$ 4409

$$-\frac{2\tanh(c+dx)}{ad\sqrt{a+b\text{sech}(c+dx)}}+\frac{\left(a^2-b^2\right)\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)\left(\csc\left(ic+idx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx+\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}dx-\left(a^2-b^2\right)\int \frac{\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}dx}{a\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$-\frac{2\tanh(c+dx)}{ad\sqrt{a+b\text{sech}(c+dx)}}+\frac{(a^2-b^2)\int\frac{1}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx-(a^2-b^2)\int\frac{\csc\left(ic+idx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx+(a^2-b^2)\int\frac{\csc\left(ic+idx+\frac{\pi}{2}\right)\left(\csc\left(ic+idx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}d}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4271}$$

$$-\frac{2\tanh(c+dx)}{ad\sqrt{a+b\text{sech}(c+dx)}}+\frac{-(a^2-b^2)\int\frac{\csc\left(ic+idx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx+(a^2-b^2)\int\frac{\csc\left(ic+idx+\frac{\pi}{2}\right)\left(\csc\left(ic+idx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}(a^2-b^2)\coth(c+dx)\sqrt{\frac{b(1-\text{S}}{}}}{}}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4319}$$

$$-\frac{2\tanh(c+dx)}{ad\sqrt{a+b\text{sech}(c+dx)}}+\frac{(a^2-b^2)\int\frac{\csc\left(ic+idx+\frac{\pi}{2}\right)\left(\csc\left(ic+idx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}(a^2-b^2)\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticF}\left(\text{arc}\right.}{bd}}{a}$$

$$\Big\downarrow \text{4492}$$

$$\frac{\frac{2\sqrt{a+b}(a^2-b^2)\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}+\frac{2(a-b)\sqrt{a+b}(a^2-b^2)\text{co}}{}}{}$$

$$\frac{2\tanh(c+dx)}{ad\sqrt{a+b\text{sech}(c+dx)}}$$

input
```
Int[Tanh[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2),x]
```

output
```
((2*(a - b)*Sqrt[a + b]*(a^2 - b^2)*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b^2*d) + (2*Sqrt[
a + b]*(a^2 - b^2)*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b*d) + (2*Sqrt[a + b]*(a^2 - b^2)*
Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sech[c + d*x]))/(a - b))])/(a*d))/(a*(a^2 - b^2)) - (2*Tanh[c + d*x])
/(a*d*Sqrt[a + b*Sech[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4382
```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4547
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Simp[C   Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f
*x]]), x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4549
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 + a^2*C)*Cot[e + f*x]*((a + b*Csc[
e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 -
 b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*
(A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m
] && LtQ[m, -1]
```

Maple [F]

$$\int \frac{\tanh (dx + c)^2}{(a + b \operatorname{sech}(dx + c))^{\frac{3}{2}}} dx$$

input
```
int(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x)
```

output
```
int(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x)
```

Fricas [F]

$$\int \frac{\tanh^2(c + dx)}{(a + b\operatorname{sech}(c + dx))^{3/2}} dx = \int \frac{\tanh (dx + c)^2}{(b \operatorname{sech}(dx + c) + a)^{\frac{3}{2}}} dx$$

input
```
integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^2/(b^2*sech(d*x + c)^2 +
2*a*b*sech(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\tanh^2(c + dx)}{(a + b\operatorname{sech}(c + dx))^{3/2}} dx = \int \frac{\tanh^2(c + dx)}{(a + b \operatorname{sech}(c + dx))^{\frac{3}{2}}} dx$$

input
```
integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c))**(3/2),x)
```

output
```
Integral(tanh(c + d*x)**2/(a + b*sech(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx=\int \frac{\tanh\left(dx+c\right)^2}{\left(b\,\text{sech}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(d*x + c)^2/(b*sech(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\tanh^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx=\int \frac{\tanh\left(dx+c\right)^2}{\left(b\,\text{sech}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(tanh(d*x + c)^2/(b*sech(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx=\int \frac{\tanh(c+d\,x)^2}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(tanh(c + d*x)^2/(a + b/cosh(c + d*x))^(3/2),x)
```

output
```
int(tanh(c + d*x)^2/(a + b/cosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\tanh^2(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\mathrm{sech}\,(dx+c)\,b+a}\,\tanh\,(dx+c)^2}{\mathrm{sech}\,(dx+c)^2\,b^2+2\,\mathrm{sech}\,(dx+c)\,ab+a^2}dx$$

input
```
int(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sech(c + d*x)*b + a)*tanh(c + d*x)**2)/(sech(c + d*x)**2*b**2 +
2*sech(c + d*x)*a*b + a**2),x)
```

3.149 $\int \frac{1}{(a+b\,\mathrm{sech}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 347

$$\int \frac{1}{(a+b\,\mathrm{sech}(c+dx))^{3/2}}\,dx = -\frac{2\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$
$$+\frac{2\coth(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$
$$+\frac{2\sqrt{a+b}\coth(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}(c+dx))}{a-b}}}{a^2d}$$
$$+\frac{2b^2\tanh(c+dx)}{a(a^2-b^2)\,d\sqrt{a+b\,\mathrm{sech}(c+dx)}}$$

output
```
-2*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))
^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a
/(a+b)^(1/2)/d+2*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2)
,((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/
(a-b))^(1/2)/a/(a+b)^(1/2)/d+2*(a+b)^(1/2)*coth(d*x+c)*EllipticPi((a+b*sec
h(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c)
)/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a^2/d+2*b^2*tanh(d*x+c)/a/
(a^2-b^2)/d/(a+b*sech(d*x+c))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx=\int \frac{1}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx$$

input
```
Integrate[(a + b*Sech[c + d*x])^(-3/2),x]
```

output
```
Integrate[(a + b*Sech[c + d*x])^(-3/2), x]
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {3042, 4272, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4272}$$

$$\frac{2b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}-\frac{2\int -\frac{a^2-b\text{sech}(c+dx)a-b^2-b^2\text{sech}^2(c+dx)}{2\sqrt{a+b\text{sech}(c+dx)}}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{a^2-b\text{sech}(c+dx)a-b^2-b^2\text{sech}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{2b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}+\frac{\int \frac{a^2-b\csc\left(ic+idx+\frac{\pi}{2}\right)a-b^2-b^2\csc\left(ic+idx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 4546$$

$$\frac{\int \frac{a^2-b^2+\left(b^2-ab\right)\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}dx-b^2\int \frac{\text{sech}(c+dx)(\text{sech}(c+dx)+1)}{\sqrt{a+b\text{sech}(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{2b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}+\frac{\int \frac{a^2-b^2+\left(b^2-ab\right)\csc\left(ic+idx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx-b^2\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)\left(\csc\left(ic+idx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 4409$$

$$\frac{2b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}+\frac{\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}dx+b^2\left(-\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)\left(\csc\left(ic+idx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx\right)-b(a-b)\int \frac{\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{2b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}+\frac{\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx+b^2\left(-\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)\left(\csc\left(ic+idx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx\right)-b(a-b)\int \frac{\csc\left(ic+idx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4271}$$

$$\frac{2b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}+\frac{b^2\left(-\int\frac{\csc\left(ic+idx+\frac{\pi}{2}\right)\left(\csc\left(ic+idx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx\right)-b(a-b)\int\frac{\csc\left(ic+idx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}\left(a^2-b^2\right)\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c}{a+b}}}{}}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4319}$$

$$\frac{2b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}+\frac{b^2\left(-\int\frac{\csc\left(ic+idx+\frac{\pi}{2}\right)\left(\csc\left(ic+idx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(ic+idx+\frac{\pi}{2}\right)}}dx\right)+\frac{2\sqrt{a+b}\left(a^2-b^2\right)\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\right.}{ad}}{a\,(}$$

$$\Big\downarrow \text{4492}$$

$$\frac{\frac{2\sqrt{a+b}\left(a^2-b^2\right)\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}+\frac{2(a-b)\sqrt{a+b}\coth(}{}}{}$$

$$\frac{2b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}$$

input

```
Int[(a + b*Sech[c + d*x])^(-3/2),x]
```

output

```
((-2*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/d + (2*(a - b)*Sqrt[a + b]*Co
th[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c +
d*x]))/(a - b))])/d + (2*Sqrt[a + b]*(a^2 - b^2)*Coth[c + d*x]*EllipticPi[
(a + b)/a, ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a -
 b))])/(a*d))/(a*(a^2 - b^2)) + (2*b^2*Tanh[c + d*x])/(a*(a^2 - b^2)*d*Sqr
t[a + b*Sech[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{1}{(a + b\,\mathrm{sech}\,(dx + c))^{\frac{3}{2}}} dx$$

input
```
int(1/(a+b*sech(d*x+c))^(3/2),x)
```

output
```
int(1/(a+b*sech(d*x+c))^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{(a + b\mathrm{sech}(c + dx))^{3/2}}\, dx = \int \frac{1}{(b\,\mathrm{sech}\,(dx + c) + a)^{\frac{3}{2}}}\, dx$$

input
```
integrate(1/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sech(d*x + c) + a)/(b^2*sech(d*x + c)^2 + 2*a*b*sech(d*x +
 c) + a^2), x)
```

Sympy [F]

$$\int \frac{1}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\,\text{sech}\,(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sech(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sech(c + d*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\,\text{sech}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sech(d*x + c) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\,\text{sech}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sech(d*x + c) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(a + b/cosh(c + d*x))^(3/2),x)
```

output
```
int(1/(a + b/cosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{\operatorname{sech}(dx+c)\,b+a}}{\operatorname{sech}(dx+c)^2\,b^2+2\operatorname{sech}(dx+c)\,ab+a^2}dx$$

input
```
int(1/(a+b*sech(d*x+c))^(3/2),x)
```

output
```
int(sqrt(sech(c + d*x)*b + a)/(sech(c + d*x)**2*b**2 + 2*sech(c + d*x)*a*b
 + a**2),x)
```

3.150 $\int \frac{\coth^2(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 665

$$
\begin{aligned}
\int \frac{\coth^2(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx &= \frac{4a\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1-}{}}}{(a-b)(a+b)^{3/2}d} \\
&- \frac{2\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d} \\
&- \frac{(3a-b)\coth(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{(a-b)(a+b)^{3/2}d} \\
&+ \frac{2\coth(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d} \\
&+ \frac{2\sqrt{a+b}\coth(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx)}{a-b}}}{a^2d} \\
&- \frac{\coth(c+dx)}{d(a+b\operatorname{sech}(c+dx))^{3/2}} - \frac{b^2\tanh(c+dx)}{(a^2-b^2)d(a+b\operatorname{sech}(c+dx))^{3/2}} \\
&- \frac{4ab^2\tanh(c+dx)}{(a^2-b^2)^2d\sqrt{a+b\operatorname{sech}(c+dx)}} + \frac{2b^2\tanh(c+dx)}{a(a^2-b^2)d\sqrt{a+b\operatorname{sech}(c+dx)}}
\end{aligned}
$$

output
```
4*a*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b)
)^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/
(a-b)/(a+b)^(3/2)/d-2*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^
(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x
+c))/(a-b))^(1/2)/a/(a+b)^(1/2)/d-(3*a-b)*coth(d*x+c)*EllipticF((a+b*sech(
d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(
1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/(a-b)/(a+b)^(3/2)/d+2*coth(d*x+c)*El
lipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec
h(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/(a+b)^(1/2)/d+2*
(a+b)^(1/2)*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),(a+
b)/a,((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+
c))/(a-b))^(1/2)/a^2/d-coth(d*x+c)/d/(a+b*sech(d*x+c))^(3/2)-b^2*tanh(d*x+
c)/(a^2-b^2)/d/(a+b*sech(d*x+c))^(3/2)-4*a*b^2*tanh(d*x+c)/(a^2-b^2)^2/d/(
a+b*sech(d*x+c))^(1/2)+2*b^2*tanh(d*x+c)/a/(a^2-b^2)/d/(a+b*sech(d*x+c))^(
1/2)
```

Mathematica [F]

$$\int \frac{\coth^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx=\int \frac{\coth^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$$

input
```
Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2),x]
```

output
```
Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2), x]
```

Rubi [A] (verified)

Time = 2.36 (sec) , antiderivative size = 665, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 25, 4384, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\cot\left(ic+idx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\cot\left(\frac{1}{2}(2ic+\pi)+idx\right)^2\left(a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4384}$$

$$-\int \left(-\frac{\text{csch}^2(c+dx)}{\left(a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)\right)^{3/2}}-\frac{1}{\left(a+b\csc\left(\frac{1}{2}(2ic+\pi)+idx\right)\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{a^2d}+\frac{2b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}-\frac{4ab^2\tanh(c+dx)}{d\left(a^2-b^2\right)^2\sqrt{a+b\text{sech}(c+dx)}}-\frac{b^2\tanh(c+dx)}{d\left(a^2-b^2\right)(a+b\text{sech}(c+dx))^{3/2}}+\frac{2\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}-\frac{(3a-b)\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d(a-b)(a+b)^{3/2}}-\frac{2\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}+\frac{4a\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d(a-b)(a+b)^{3/2}}-\frac{\coth(c+dx)}{d(a+b\text{sech}(c+dx))^{3/2}}$$

input
```
Int[Coth[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2),x]
```

output
```
(4*a*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Se
ch[c + d*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*d) - (2*Coth[c + d*x]*Elli
pticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt
[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))]
)/(a*Sqrt[a + b]*d) - ((3*a - b)*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b
*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/((a - b)*(a + b)^(3/2
)*d) + (2*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sech[c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) + (2*Sqrt[a + b]*Coth[c +
d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech
[c + d*x]))/(a - b))])/(a^2*d) - Coth[c + d*x]/(d*(a + b*Sech[c + d*x])^(3
/2)) - (b^2*Tanh[c + d*x])/((a^2 - b^2)*d*(a + b*Sech[c + d*x])^(3/2)) - (
4*a*b^2*Tanh[c + d*x])/((a^2 - b^2)^2*d*Sqrt[a + b*Sech[c + d*x]]) + (2*b^
2*Tanh[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sech[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4384
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &&
ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Maple [F]

$$\int \frac{\coth(dx+c)^2}{(a+b\,\mathrm{sech}(dx+c))^{\frac{3}{2}}}dx$$

input
```
int(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x)
```

output
```
int(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\coth^2(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\coth^2(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth^2(c+dx)}{(a+b\,\mathrm{sech}(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(d*x+c)**2/(a+b*sech(d*x+c))**(3/2),x)
```

output
```
Integral(coth(c + d*x)**2/(a + b*sech(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\coth^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth(dx+c)^2}{(b\operatorname{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(coth(d*x + c)^2/(b*sech(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\coth^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth(dx+c)^2}{(b\operatorname{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(coth(d*x + c)^2/(b*sech(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth(c+d\,x)^2}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(coth(c + d*x)^2/(a + b/cosh(c + d*x))^(3/2),x)
```

output
```
int(coth(c + d*x)^2/(a + b/cosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\coth^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\text{sech}\,(dx+c)\,b+a}\,\coth\,(dx+c)^2}{\text{sech}\,(dx+c)^2\,b^2+2\,\text{sech}\,(dx+c)\,ab+a^2}dx$$

input
```
int(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x)
```

output
```
int((sqrt(sech(c + d*x)*b + a)*coth(c + d*x)**2)/(sech(c + d*x)**2*b**2 +
2*sech(c + d*x)*a*b + a**2),x)
```

3.151 $\int e^{c(a+bx)}\mathbf{sech}^2(ac+bcx)^{7/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 191

$$\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{7/2}\,dx=\frac{32\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{3bc\left(1+e^{2c(a+bx)}\right)^6}-\frac{192\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{5bc\left(1+e^{2c(a+bx)}\right)^5}+\frac{48\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^4}-\frac{64\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{3bc\left(1+e^{2c(a+bx)}\right)^3}$$

output
```
32/3*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^6-
192/5*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^5
+48*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^4-6
4/3*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.44

$$\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{7/2}\,dx = -\frac{16\left(1+6e^{2c(a+bx)}+15e^{4c(a+bx)}+20e^{6c(a+bx)}\right)\cosh(c(a+bx))\sqrt{\text{sech}^2(c(a+bx))}}{15bc\left(1+e^{2c(a+bx)}\right)^6}$$

input
```
Integrate[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(7/2),x]
```

output
```
(-16*(1 + 6*E^(2*c*(a + b*x)) + 15*E^(4*c*(a + b*x)) + 20*E^(6*c*(a + b*x)
))*Cosh[c*(a + b*x)]*Sqrt[Sech[c*(a + b*x)]^2])/(15*b*c*(1 + E^(2*c*(a + b
*x)))^6)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.55, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {7271, 2720, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{7/2}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}\int e^{c(a+bx)}\text{sech}^7(ac+bxc)dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}\int \frac{128e^{7c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^7}de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{128\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\int \frac{e^{7c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^7}de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \; 243$$

$$\frac{64\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\int \frac{e^{3c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^7}de^{2c(a+bx)}}{bc}$$

$$\Big\downarrow \; 53$$

$$\frac{64\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\int \left(\frac{1}{\left(1+e^{2c(a+bx)}\right)^4}-\frac{3}{\left(1+e^{2c(a+bx)}\right)^5}+\frac{3}{\left(1+e^{2c(a+bx)}\right)^6}-\frac{1}{\left(1+e^{2c(a+bx)}\right)^7}\right)de^{2c(a+bx)}}{bc}$$

$$\Big\downarrow \; 2009$$

$$\frac{64\left(-\frac{1}{3\left(e^{2c(a+bx)}+1\right)^3}+\frac{3}{4\left(e^{2c(a+bx)}+1\right)^4}-\frac{3}{5\left(e^{2c(a+bx)}+1\right)^5}+\frac{1}{6\left(e^{2c(a+bx)}+1\right)^6}\right)\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc}$$

input
```
Int[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(7/2),x]
```

output
```
(64*(1/(6*(1 + E^(2*c*(a + b*x)))^6) - 3/(5*(1 + E^(2*c*(a + b*x)))^5) + 3
/(4*(1 + E^(2*c*(a + b*x)))^4) - 1/(3*(1 + E^(2*c*(a + b*x)))^3))*Cosh[a*c
 + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(b*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 148.55 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.45

method	result	siz
default	$\frac{\operatorname{csgn}(\operatorname{sech}(c(bx+a)))\left(\frac{\tanh(c(bx+a))^6}{6}+\frac{\tanh(c(bx+a))^5}{5}-\frac{\tanh(c(bx+a))^4}{2}-\frac{2\tanh(c(bx+a))^3}{3}+\frac{\tanh(c(bx+a))^2}{2}+\tanh(c(bx+a))\right)}{bc}$	86
risch	$-\frac{16\left(20\,e^{6c(bx+a)}+15\,e^{4c(bx+a)}+6\,e^{2c(bx+a)}+1\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)^2}}\,e^{-c(bx+a)}}{15bc\left(1+e^{2c(bx+a)}\right)^5}$	91

input
```
int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(sech(c*(b*x+a)))/b/c*(1/6*tanh(c*(b*x+a))^6+1/5*tanh(c*(b*x+a))^5-1/2
*tanh(c*(b*x+a))^4-2/3*tanh(c*(b*x+a))^3+1/2*tanh(c*(b*x+a))^2+tanh(c*(b*x
+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(173) = 346$.

Time = 0.08 (sec) , antiderivative size = 589, normalized size of antiderivative = 3.08

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{7/2}\,dx =$$

$$-\frac{}{15\left(bc\cosh\left(bcx+ac\right)^9+9\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^8+bc\sinh\left(bcx+ac\right)^9+6\,bc\cosh\left(bcx+ac\right)\right.}$$

input
```
integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(7/2),x, algorithm="fricas")
```

output
```
-16/15*(21*cosh(b*c*x + a*c)^3 + 63*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2
+ 19*sinh(b*c*x + a*c)^3 + 3*(19*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c
) + 21*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^9 + 9*b*c*cosh(b*c*x + a*
c)*sinh(b*c*x + a*c)^8 + b*c*sinh(b*c*x + a*c)^9 + 6*b*c*cosh(b*c*x + a*c)
^7 + 6*(6*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^7 + 15*b*c*cosh
(b*c*x + a*c)^5 + 42*(2*b*c*cosh(b*c*x + a*c)^3 + b*c*cosh(b*c*x + a*c))*s
inh(b*c*x + a*c)^6 + 3*(42*b*c*cosh(b*c*x + a*c)^4 + 42*b*c*cosh(b*c*x + a
*c)^2 + 5*b*c)*sinh(b*c*x + a*c)^5 + 21*b*c*cosh(b*c*x + a*c)^3 + 3*(42*b*
c*cosh(b*c*x + a*c)^5 + 70*b*c*cosh(b*c*x + a*c)^3 + 25*b*c*cosh(b*c*x + a
*c))*sinh(b*c*x + a*c)^4 + (84*b*c*cosh(b*c*x + a*c)^6 + 210*b*c*cosh(b*c*
x + a*c)^4 + 150*b*c*cosh(b*c*x + a*c)^2 + 19*b*c)*sinh(b*c*x + a*c)^3 + 2
1*b*c*cosh(b*c*x + a*c) + 3*(12*b*c*cosh(b*c*x + a*c)^7 + 42*b*c*cosh(b*c*
x + a*c)^5 + 50*b*c*cosh(b*c*x + a*c)^3 + 21*b*c*cosh(b*c*x + a*c))*sinh(b
*c*x + a*c)^2 + 3*(3*b*c*cosh(b*c*x + a*c)^8 + 14*b*c*cosh(b*c*x + a*c)^6
+ 25*b*c*cosh(b*c*x + a*c)^4 + 19*b*c*cosh(b*c*x + a*c)^2 + 3*b*c)*sinh(b*
c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)**2)**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(173) = 346.

Time = 0.04 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.02

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{7/2}\,dx = -\frac{64\,e^{(6\,bcx+6\,ac)}}{3\,bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)} - \frac{16\,e^{(4\,bcx+4\,ac)}}{bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)} - \frac{32\,e^{(2\,bcx+2\,ac)}}{5\,bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)} - \frac{16}{15\,bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)}$$

input
```
integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(7/2),x, algorithm="maxima")
```

output
```
-64/3*e^(6*b*c*x + 6*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10*b*c*x + 10
*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x +
4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1)) - 16*e^(4*b*c*x + 4*a*c)/(b*c*(e^(12*
b*c*x + 12*a*c) + 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^
(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1)) -
 32/5*e^(2*b*c*x + 2*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10*b*c*x + 10
*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x +
4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1)) - 16/15/(b*c*(e^(12*b*c*x + 12*a*c) +
 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c)
 + 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.34

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{7/2}\,dx = -\frac{16\left(20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1\right)}{15\,bc\left(e^{(2\,bcx+2\,ac)}+1\right)^6}$$

input
```
integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(7/2),x, algorithm="giac")
```

output
```
-16/15*(20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2
*a*c) + 1)/(b*c*(e^(2*b*c*x + 2*a*c) + 1)^6)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.12

$$\begin{aligned}\int e^{c(a+bx)}\operatorname{sech}^2(ac&+bcx)^{7/2}\,dx = \frac{24\sqrt{\frac{1}{\left(\frac{e^{a\,c+b\,c\,x}}{2}+\frac{e^{-a\,c-b\,c\,x}}{2}\right)^2}}\left(2\,e^{2\,a\,c+2\,b\,c\,x}+e^{4\,a\,c+4\,b\,c\,x}+1\right)}{b\,c\left(e^{a\,c+b\,c\,x}+e^{3\,a\,c+3\,b\,c\,x}\right)\left(e^{2\,a\,c+2\,b\,c\,x}+1\right)^4}\\&-\frac{32\sqrt{\frac{1}{\left(\frac{e^{a\,c+b\,c\,x}}{2}+\frac{e^{-a\,c-b\,c\,x}}{2}\right)^2}}\left(2\,e^{2\,a\,c+2\,b\,c\,x}+e^{4\,a\,c+4\,b\,c\,x}+1\right)}{3\,b\,c\left(e^{a\,c+b\,c\,x}+e^{3\,a\,c+3\,b\,c\,x}\right)\left(e^{2\,a\,c+2\,b\,c\,x}+1\right)^3}\\&-\frac{96\sqrt{\frac{1}{\left(\frac{e^{a\,c+b\,c\,x}}{2}+\frac{e^{-a\,c-b\,c\,x}}{2}\right)^2}}\left(2\,e^{2\,a\,c+2\,b\,c\,x}+e^{4\,a\,c+4\,b\,c\,x}+1\right)}{5\,b\,c\left(e^{a\,c+b\,c\,x}+e^{3\,a\,c+3\,b\,c\,x}\right)\left(e^{2\,a\,c+2\,b\,c\,x}+1\right)^5}\\&+\frac{16\sqrt{\frac{1}{\left(\frac{e^{a\,c+b\,c\,x}}{2}+\frac{e^{-a\,c-b\,c\,x}}{2}\right)^2}}\left(2\,e^{2\,a\,c+2\,b\,c\,x}+e^{4\,a\,c+4\,b\,c\,x}+1\right)}{3\,b\,c\left(e^{a\,c+b\,c\,x}+e^{3\,a\,c+3\,b\,c\,x}\right)\left(e^{2\,a\,c+2\,b\,c\,x}+1\right)^6}\end{aligned}$$

input
```
int(exp(c*(a + b*x))*(1/cosh(a*c + b*c*x)^2)^(7/2),x)
```

output
```
(24*(1/(exp(a*c + b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1/2)*(2*exp(2*a*c +
 2*b*c*x) + exp(4*a*c + 4*b*c*x) + 1))/(b*c*(exp(a*c + b*c*x) + exp(3*a*c
+ 3*b*c*x))*(exp(2*a*c + 2*b*c*x) + 1)^4) - (32*(1/(exp(a*c + b*c*x)/2 + e
xp(- a*c - b*c*x)/2)^2)^(1/2)*(2*exp(2*a*c + 2*b*c*x) + exp(4*a*c + 4*b*c*
x) + 1))/(3*b*c*(exp(a*c + b*c*x) + exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b
*c*x) + 1)^3) - (96*(1/(exp(a*c + b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1/2
)*(2*exp(2*a*c + 2*b*c*x) + exp(4*a*c + 4*b*c*x) + 1))/(5*b*c*(exp(a*c + b
*c*x) + exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x) + 1)^5) + (16*(1/(exp(
a*c + b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1/2)*(2*exp(2*a*c + 2*b*c*x) +
exp(4*a*c + 4*b*c*x) + 1))/(3*b*c*(exp(a*c + b*c*x) + exp(3*a*c + 3*b*c*x)
)*(exp(2*a*c + 2*b*c*x) + 1)^6)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.72

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{7/2}\,dx = \frac{-\frac{64e^{6bcx+6ac}}{3}-16e^{4bcx+4ac}-\frac{32e^{2bcx+2ac}}{5}-\frac{16}{15}}{bc\left(e^{12bcx+12ac}+6e^{10bcx+10ac}+15e^{8bcx+8ac}+20e^{6bcx+6ac}+15e^{4bcx+4ac}+6e^{2bcx+2ac}+1\right)}$$

input
```
int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(7/2),x)
```

output
```
(16*( - 20*e**(6*a*c + 6*b*c*x) - 15*e**(4*a*c + 4*b*c*x) - 6*e**(2*a*c +
2*b*c*x) - 1))/(15*b*c*(e**(12*a*c + 12*b*c*x) + 6*e**(10*a*c + 10*b*c*x)
+ 15*e**(8*a*c + 8*b*c*x) + 20*e**(6*a*c + 6*b*c*x) + 15*e**(4*a*c + 4*b*c
*x) + 6*e**(2*a*c + 2*b*c*x) + 1))
```

3.152 $\int e^{c(a+bx)}\mathbf{sech}^2(ac+bcx)^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 141

$$\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{5/2}\,dx=-\frac{4\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^4}+\frac{32\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{3bc\left(1+e^{2c(a+bx)}\right)^3}-\frac{8\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^2}$$

output
```
-4*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^4+32
/3*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^3-8*
cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.51

$$\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{5/2}\,dx=-\frac{4\left(1+4e^{2c(a+bx)}+6e^{4c(a+bx)}\right)\cosh(c(a+bx))\sqrt{\text{sech}^2(c(a+bx))}}{3bc\left(1+e^{2c(a+bx)}\right)^4}$$

input
```
Integrate[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(5/2),x]
```

output
```
(-4*(1 + 4*E^(2*c*(a + b*x)) + 6*E^(4*c*(a + b*x)))*Cosh[c*(a + b*x)]*Sqrt
[Sech[c*(a + b*x)]^2])/(3*b*c*(1 + E^(2*c*(a + b*x)))^4)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.62, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {7271, 2720, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{5/2}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}\int e^{c(a+bx)}\text{sech}^5(ac+bxc)dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}\int \frac{32e^{5c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^5}de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{32\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}\int \frac{e^{5c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^5}de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{243}$$

$$\frac{16\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}\int \frac{e^{2c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^5}de^{2c(a+bx)}}{bc}$$

$$\Big\downarrow \text{53}$$

$$\frac{16\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}\int \left(\frac{1}{\left(1+e^{2c(a+bx)}\right)^3}-\frac{2}{\left(1+e^{2c(a+bx)}\right)^4}+\frac{1}{\left(1+e^{2c(a+bx)}\right)^5}\right)de^{2c(a+bx)}}{bc}$$

$$\Big\downarrow \text{2009}$$

$$\frac{16\left(-\frac{1}{2(e^{2c(a+bx)}+1)^2}+\frac{2}{3(e^{2c(a+bx)}+1)^3}-\frac{1}{4(e^{2c(a+bx)}+1)^4}\right)\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc}$$

input
```
Int[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(5/2),x]
```

output
```
(16*(-1/4*1/(1 + E^(2*c*(a + b*x)))^4 + 2/(3*(1 + E^(2*c*(a + b*x)))^3) -
1/(2*(1 + E^(2*c*(a + b*x)))^2))*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^
2])/(b*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 141.81 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.46

method	result	size
default	$-\frac{\operatorname{csgn}\left(\operatorname{sech}\left(c \left(b x +a \right)\right)\right) \left(\frac{\tanh \left(c \left(b x +a \right)\right)^{4}}{4}+\frac{\tanh \left(c \left(b x +a \right)\right)^{3}}{3}-\frac{\tanh \left(c \left(b x +a \right)\right)^{2}}{2}-\tanh \left(c \left(b x +a \right)\right)\right)}{c b}$	65
risch	$-\frac{4 \left(6 \,{\mathrm e}^{4 c \left(b x +a \right)}+4 \,{\mathrm e}^{2 c \left(b x +a \right)}+1\right) \sqrt{\frac{{\mathrm e}^{2 c \left(b x +a \right)}}{\left(1+{\mathrm e}^{2 c \left(b x +a \right)}\right)^{2}}}\, {\mathrm e}^{-c \left(b x +a \right)}}{3 b c \left(1+{\mathrm e}^{2 c \left(b x +a \right)}\right)^{3}}$	80

input
```
int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-csgn(sech(c*(b*x+a)))/c/b*(1/4*tanh(c*(b*x+a))^4+1/3*tanh(c*(b*x+a))^3-1/
2*tanh(c*(b*x+a))^2-tanh(c*(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(130) = 260.

Time = 0.19 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.23

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{5/2}\,dx =$$

$$-\frac{}{3\left(bc\cosh\left(bcx+ac\right)^{6}+6\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{5}+bc\sinh\left(bcx+ac\right)^{6}+4\,bc\cosh\left(bcx+ac\right)^{4}\right.}$$

input
```
integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

output
```
-4/3*(7*cosh(b*c*x + a*c)^2 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 7*s
inh(b*c*x + a*c)^2 + 4)/(b*c*cosh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)
*sinh(b*c*x + a*c)^5 + b*c*sinh(b*c*x + a*c)^6 + 4*b*c*cosh(b*c*x + a*c)^4
 + (15*b*c*cosh(b*c*x + a*c)^2 + 4*b*c)*sinh(b*c*x + a*c)^4 + 7*b*c*cosh(b
*c*x + a*c)^2 + 4*(5*b*c*cosh(b*c*x + a*c)^3 + 4*b*c*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c)^3 + (15*b*c*cosh(b*c*x + a*c)^4 + 24*b*c*cosh(b*c*x + a*c)
^2 + 7*b*c)*sinh(b*c*x + a*c)^2 + 4*b*c + 2*(3*b*c*cosh(b*c*x + a*c)^5 + 8
*b*c*cosh(b*c*x + a*c)^3 + 5*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.48

$$\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{5/2}\,dx = -\frac{8\,e^{(4\,bcx+4\,ac)}}{bc(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1)} - \frac{16\,e^{(2\,bcx+2\,ac)}}{3\,bc(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1)} - \frac{4}{3\,bc(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1)}$$

input
```
integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

output
```
-8*e^(4*b*c*x + 4*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) +
 6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1)) - 16/3*e^(2*b*c*x + 2
*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4
*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1)) - 4/3/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^
(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.36

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{5/2}\,dx = -\frac{4\left(6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}{3\,bc\left(e^{(2\,bcx+2\,ac)}+1\right)^4}$$

input
```
integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

output
```
-4/3*(6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x
+ 2*a*c) + 1)^4)
```

Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.65

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{5/2}\,dx = -\frac{2\,e^{-a\,c-b\,c\,x}\,\sqrt{\frac{1}{{\left(\frac{e^{a\,c+b\,c\,x}}{2}+\frac{e^{-a\,c-b\,c\,x}}{2}\right)}^2}}\,\left(4\,e^{2\,a\,c+2\,b\,c\,x}+6\,e^{4\,a\,c+4\,b\,c\,x}+1\right)}{3\,b\,c\,{\left(e^{2\,a\,c+2\,b\,c\,x}+1\right)}^3}$$

input
```
int(exp(c*(a + b*x))*(1/cosh(a*c + b*c*x)^2)^(5/2),x)
```

output
```
-(2*exp(- a*c - b*c*x)*(1/(exp(a*c + b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(
1/2)*(4*exp(2*a*c + 2*b*c*x) + 6*exp(4*a*c + 4*b*c*x) + 1))/(3*b*c*(exp(2*
a*c + 2*b*c*x) + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.68

$$\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{5/2}\,dx = \frac{-8e^{4bcx+4ac}-\frac{16e^{2bcx+2ac}}{3}-\frac{4}{3}}{bc\left(e^{8bcx+8ac}+4e^{6bcx+6ac}+6e^{4bcx+4ac}+4e^{2bcx+2ac}+1\right)}$$

input
```
int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(5/2),x)
```

output
```
(4*( - 6*e**(4*a*c + 4*b*c*x) - 4*e**(2*a*c + 2*b*c*x) - 1))/(3*b*c*(e**(8
*a*c + 8*b*c*x) + 4*e**(6*a*c + 6*b*c*x) + 6*e**(4*a*c + 4*b*c*x) + 4*e**(
2*a*c + 2*b*c*x) + 1))
```

3.153 $\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 56

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{3/2}\,dx = \frac{2e^{4c(a+bx)}\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^2}$$

output
```
2*exp(4*c*(b*x+a))*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*
c*(b*x+a)))^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{3/2}\,dx = \frac{e^{3c(a+bx)}\sqrt{\operatorname{sech}^2(c(a+bx))}}{bc+bce^{2c(a+bx)}}$$

input
```
Integrate[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(3/2),x]
```

output
```
(E^(3*c*(a + b*x))*Sqrt[Sech[c*(a + b*x)]^2])/(b*c + b*c*E^(2*c*(a + b*x))
)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {7271, 2720, 27, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{3/2}\,dx$$

$$\Big\downarrow \; 7271$$

$$\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}\int e^{c(a+bx)}\text{sech}^3(ac+bxc)dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}\int \frac{8e^{3c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^3}de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \; 27$$

$$\frac{8\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}\int \frac{e^{3c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^3}de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \; 242$$

$$\frac{2e^{4c(a+bx)}\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^2}$$

input
```
Int[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(3/2),x]
```

output
```
(2*E^(4*c*(a + b*x))*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(b*c*(1
+ E^(2*c*(a + b*x)))^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{\operatorname{csgn}(\operatorname{sech}(c(bx+a)))\left(\frac{\tanh(c(bx+a))^2}{2}+\tanh(c(bx+a))\right)}{cb}$	38
risch	$-\frac{2\left(2\,\mathrm{e}^{2c(bx+a)}+1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2}}\,\mathrm{e}^{-c(bx+a)}}{bc\left(1+\mathrm{e}^{2c(bx+a)}\right)}$	69

input
```
int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(sech(c*(b*x+a)))/c/b*(1/2*tanh(c*(b*x+a))^2+tanh(c*(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(52) = 104.

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.14

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{3/2}\,dx =$$

$$-\frac{2\,(3\,\cosh\,(bcx+ac)+\sinh\,(bcx+ac))}{bc\cosh\,(bcx+ac)^3+3\,bc\cosh\,(bcx+ac)\sinh\,(bcx+ac)^2+bc\sinh\,(bcx+ac)^3+3\,bc\cosh\,(bcx+ac)+ (}$$

input
```
integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

output
```
-2*(3*cosh(b*c*x + a*c) + sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^3 + 3*
b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 + b*c*sinh(b*c*x + a*c)^3 + 3*b*
c*cosh(b*c*x + a*c) + (3*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.50

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{3/2}\,dx = -\frac{4\,e^{(2\,bcx+2\,ac)}}{bc(e^{(4\,bcx+4\,ac)}+2\,e^{(2\,bcx+2\,ac)}+1)} - \frac{2}{bc(e^{(4\,bcx+4\,ac)}+2\,e^{(2\,bcx+2\,ac)}+1)}$$

input
```
integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

output
```
-4*e^(2*b*c*x + 2*a*c)/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c) +
 1)) - 2/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.68

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{3/2}\,dx = -\frac{2\left(2\,e^{(2\,bcx+2\,ac)}+1\right)}{bc(e^{(2\,bcx+2\,ac)}+1)^2}$$

input
```
integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

output
```
-2*(2*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x + 2*a*c) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.39

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{3/2}\,dx = -\frac{\mathrm{e}^{-a\,c-b\,c\,x}\,\left(2\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)\,\sqrt{\frac{1}{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}+\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}}{b\,c\,\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)}$$

input
```
int(exp(c*(a + b*x))*(1/cosh(a*c + b*c*x)^2)^(3/2),x)
```

output
```
-(exp(- a*c - b*c*x)*(2*exp(2*a*c + 2*b*c*x) + 1)*(1/(exp(a*c + b*c*x)/2 +
 exp(- a*c - b*c*x)/2)^2)^(1/2))/(b*c*(exp(2*a*c + 2*b*c*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{3/2}\,dx=\frac{2e^{4bcx+4ac}}{bc\,(e^{4bcx+4ac}+2e^{2bcx+2ac}+1)}$$

input
```
int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(3/2),x)
```

output
```
(2*e**(4*a*c + 4*b*c*x))/(b*c*(e**(4*a*c + 4*b*c*x) + 2*e**(2*a*c + 2*b*c*
x) + 1))
```

3.154 $\int e^{c(a+bx)}\sqrt{\mathrm{sech}^2(ac+bcx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 44

$$\int e^{c(a+bx)}\sqrt{\mathrm{sech}^2(ac+bcx)}\,dx=\frac{\cosh(ac+bcx)\log\left(1+e^{2c(a+bx)}\right)\sqrt{\mathrm{sech}^2(ac+bcx)}}{bc}$$

output
```
cosh(b*c*x+a*c)*ln(1+exp(2*c*(b*x+a)))*(sech(b*c*x+a*c)^2)^(1/2)/b/c
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int e^{c(a+bx)}\sqrt{\mathrm{sech}^2(ac+bcx)}\,dx=\frac{\cosh(c(a+bx))\log\left(1+e^{2c(a+bx)}\right)\sqrt{\mathrm{sech}^2(c(a+bx))}}{bc}$$

input
```
Integrate[E^(c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2],x]
```

output
```
(Cosh[c*(a + b*x)]*Log[1 + E^(2*c*(a + b*x))]*Sqrt[Sech[c*(a + b*x)]^2])/(
b*c)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {7271, 2720, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\sqrt{\text{sech}^2(ac+bcx)}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}\int e^{c(a+bx)}\text{sech}(ac+bxc)dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}\int \frac{2e^{c(a+bx)}}{1+e^{2c(a+bx)}}de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}\int \frac{e^{c(a+bx)}}{1+e^{2c(a+bx)}}de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{240}$$

$$\frac{\log\left(e^{2c(a+bx)}+1\right)\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{bc}$$

input
```
Int[E^(c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2],x]
```

output
```
(Cosh[a*c + b*c*x]*Log[1 + E^(2*c*(a + b*x))]*Sqrt[Sech[a*c + b*c*x]^2])/(
b*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.50

method	result	size
risch	$\frac{\ln\left(\mathrm{e}^{2bxc}+\mathrm{e}^{-2ac}\right)\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}}}\,\mathrm{e}^{-c(bx+a)}}{bc}$	66

input
```
int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(exp(2*b*x*c)+exp(-2*a*c))/b/c*(1+exp(2*c*(b*x+a)))*(1/(1+exp(2*c*(b*x+a
)))^2*exp(2*c*(b*x+a)))^(1/2)*exp(-c*(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int e^{c(a+bx)}\sqrt{\operatorname{sech}^2(ac+bcx)}\,dx = \frac{\log\left(\frac{2\cosh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

output
```
log(2*cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c)))/(b*c)
```

Sympy [F]

$$\int e^{c(a+bx)}\sqrt{\operatorname{sech}^2(ac+bcx)}\,dx = e^{ac}\int\sqrt{\operatorname{sech}^2(ac+bcx)}e^{bcx}\,dx$$

input
```
integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)**2)**(1/2),x)
```

output
```
exp(a*c)*Integral(sqrt(sech(a*c + b*c*x)**2)*exp(b*c*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int e^{c(a+bx)}\sqrt{\operatorname{sech}^2(ac+bcx)}\,dx = \frac{\log\left(e^{(2bcx+2ac)}+1\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

output
```
log(e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int e^{c(a+bx)}\sqrt{\operatorname{sech}^2(ac+bcx)}\,dx = \frac{\log\left(e^{(2\,bcx+2\,ac)}+1\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

output
```
log(e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\sqrt{\operatorname{sech}^2(ac+bcx)}\,dx = \int e^{c\,(a+b\,x)}\,\sqrt{\frac{1}{{\cosh\left(a\,c+b\,c\,x\right)}^2}}\,dx$$

input
```
int(exp(c*(a + b*x))*(1/cosh(a*c + b*c*x)^2)^(1/2),x)
```

output
```
int(exp(c*(a + b*x))*(1/cosh(a*c + b*c*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

$$\int e^{c(a+bx)}\sqrt{\operatorname{sech}^2(ac+bcx)}\,dx = \frac{\log\left(e^{2bcx+2ac}+1\right)}{bc}$$

input
```
int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(1/2),x)
```

output
```
log(e**(2*a*c + 2*b*c*x) + 1)/(b*c)
```

3.155 $\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\,dx=\frac{e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{4bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{x\operatorname{sech}(ac+bcx)}{2\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

output
```
1/4*exp(2*c*(b*x+a))*sech(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+1/2*x*s
ech(b*c*x+a*c)/(sech(b*c*x+a*c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\,dx=\frac{\left(e^{2c(a+bx)}+2bcx\right)\operatorname{sech}(c(a+bx))}{4bc\sqrt{\operatorname{sech}^2(c(a+bx))}}$$

input
```
Integrate[E^(c*(a + b*x))/Sqrt[Sech[a*c + b*c*x]^2],x]
```

output
```
((E^(2*c*(a + b*x)) + 2*b*c*x)*Sech[c*(a + b*x)])/(4*b*c*Sqrt[Sech[c*(a +
b*x)]^2])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7271, 2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{sech}^2(ac+bcx)}}dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\operatorname{sech}(ac+bcx)\int e^{c(a+bx)}\cosh(ac+bxc)dx}{\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\operatorname{sech}(ac+bcx)\int \frac{1}{2}e^{-c(a+bx)}\left(1+e^{2c(a+bx)}\right)de^{c(a+bx)}}{bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\operatorname{sech}(ac+bcx)\int e^{-c(a+bx)}\left(1+e^{2c(a+bx)}\right)de^{c(a+bx)}}{2bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$\Big\downarrow \text{244}$$

$$\frac{\operatorname{sech}(ac+bcx)\int \left(e^{-c(a+bx)}+e^{c(a+bx)}\right)de^{c(a+bx)}}{2bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{1}{2}e^{2c(a+bx)}+\log\left(e^{c(a+bx)}\right)\right)\operatorname{sech}(ac+bcx)}{2bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

input
```
Int[E^(c*(a + b*x))/Sqrt[Sech[a*c + b*c*x]^2],x]
```

output
```
((E^(2*c*(a + b*x))/2 + Log[E^(c*(a + b*x))])*Sech[a*c + b*c*x])/(2*b*c*Sq
rt[Sech[a*c + b*c*x]^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.43

method	result	size
risch	$\frac{x\,\mathrm{e}^{c(bx+a)}}{2\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}}}}+\frac{\mathrm{e}^{3c(bx+a)}}{4bc\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}}}}$	106

input
```
int(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/
2)*exp(c*(b*x+a))+1/4/b/c/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*e
xp(2*c*(b*x+a)))^(1/2)*exp(3*c*(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\,dx = \frac{(2\,bcx+1)\cosh\left(bcx+ac\right)-(2\,bcx-1)\sinh\left(bcx+ac\right)}{4\left(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right)\right)}$$

input
```
integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*((2*b*c*x + 1)*cosh(b*c*x + a*c) - (2*b*c*x - 1)*sinh(b*c*x + a*c))/(b
*c*cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [F]

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\,dx = e^{ac}\int \frac{e^{bcx}}{\sqrt{\operatorname{sech}^2\left(ac+bcx\right)}}\,dx$$

input
```
integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)**2)**(1/2),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)/sqrt(sech(a*c + b*c*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.39

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\,dx = \frac{1}{2}\,x + \frac{a}{2\,b} + \frac{e^{(2\,bcx+2\,ac)}}{4\,bc}$$

input
```
integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*x + 1/2*a/b + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.34

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\,dx = \frac{2\,bcx + e^{(2\,bcx+2\,ac)}}{4\,bc}$$

input
```
integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*(2*b*c*x + e^(2*b*c*x + 2*a*c))/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\,dx = \int \frac{\mathrm{e}^{c\,(a+b\,x)}}{\sqrt{\frac{1}{\cosh(a\,c+b\,c\,x)^2}}}\,dx$$

input
```
int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(1/2),x)
```

output
```
int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.35

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\,dx=\frac{e^{2bcx+2ac}+2bcx}{4bc}$$

input
```
int(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2),x)
```

output
```
(e**(2*a*c + 2*b*c*x) + 2*b*c*x)/(4*b*c)
```

3.156 $\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 162

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{3/2}}\,dx = -\frac{e^{-2c(a+bx)}\operatorname{sech}(ac+bcx)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{3e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{e^{4c(a+bx)}\operatorname{sech}(ac+bcx)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{3x\operatorname{sech}(ac+bcx)}{8\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

output
```
-1/16*sech(b*c*x+a*c)/b/c/exp(2*c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2)+3/16*
exp(2*c*(b*x+a))*sech(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+1/32*exp(4*
c*(b*x+a))*sech(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+3/8*x*sech(b*c*x+
a*c)/(sech(b*c*x+a*c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.50

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{3/2}}\,dx = \frac{\left(-\frac{1}{16}e^{-2c(a+bx)} + \frac{3}{16}e^{2c(a+bx)} + \frac{1}{32}e^{4c(a+bx)} + \frac{3bcx}{8}\right)\operatorname{sech}^3(c(a+bx))}{bc\operatorname{sech}^2(c(a+bx))^{3/2}}$$

input
```
Integrate[E^(c*(a + b*x))/(Sech[a*c + b*c*x]^2)^(3/2),x]
```

output
```
((-1/16*1/E^(2*c*(a + b*x)) + (3*E^(2*c*(a + b*x)))/16 + E^(4*c*(a + b*x))
/32 + (3*b*c*x)/8)*Sech[c*(a + b*x)]^3)/(b*c*(Sech[c*(a + b*x)]^2)^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.46, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {7271, 2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{3/2}}dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\operatorname{sech}(ac+bcx)\int e^{c(a+bx)}\cosh^3(ac+bxc)dx}{\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\operatorname{sech}(ac+bcx)\int \frac{1}{8}e^{-3c(a+bx)}\left(1+e^{2c(a+bx)}\right)^3de^{c(a+bx)}}{bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\operatorname{sech}(ac+bcx)\int e^{-3c(a+bx)}\left(1+e^{2c(a+bx)}\right)^3de^{c(a+bx)}}{8bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\operatorname{sech}(ac+bcx)\int e^{-2c(a+bx)}\left(1+e^{2c(a+bx)}\right)^3de^{2c(a+bx)}}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\operatorname{sech}(ac+bcx)\int \left(3+e^{-2c(a+bx)}+3e^{-c(a+bx)}+e^{2c(a+bx)}\right)de^{2c(a+bx)}}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-e^{-c(a+bx)}+\frac{7}{2}e^{2c(a+bx)}+3\log\left(e^{2c(a+bx)}\right)\right)\operatorname{sech}(ac+bcx)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

input
```
Int[E^(c*(a + b*x))/(Sech[a*c + b*c*x]^2)^(3/2),x]
```

output
```
((-E^(-(c*(a + b*x))) + (7*E^(2*c*(a + b*x)))/2 + 3*Log[E^(2*c*(a + b*x))]
)*Sech[a*c + b*c*x])/(16*b*c*Sqrt[Sech[a*c + b*c*x]^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.33

method	result
risch	$\frac{3x\,\mathrm{e}^{c(bx+a)}}{8\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}}}}+\frac{\mathrm{e}^{5c(bx+a)}}{32bc\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}}}}+\frac{3\,\mathrm{e}^{3c(bx+a)}}{16bc\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}}}}-\frac{}{16bc(1}$

input
```
int(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/8*x/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/
2)*exp(c*(b*x+a))+1/32/b/c/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*
exp(2*c*(b*x+a)))^(1/2)*exp(5*c*(b*x+a))+3/16/b/c/(1+exp(2*c*(b*x+a)))/(1/
(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)*exp(3*c*(b*x+a))-1/16/b/c/(
1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)*exp(
-c*(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.78

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{3/2}}\,dx = -\frac{\cosh(bcx+ac)^3+3\cosh(bcx+ac)\sinh(bcx+ac)^2-3\sinh(bcx+ac)^3-6\,(2\,bcx+1)\cosh(bcx+ac}{32\,(bc\cosh(bcx+ac)-bc\sinh(bcx+ac))}$$

input
```
integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/32*(cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 - 3*s
inh(b*c*x + a*c)^3 - 6*(2*b*c*x + 1)*cosh(b*c*x + a*c) + 3*(4*b*c*x - 3*co
sh(b*c*x + a*c)^2 - 2)*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c) - b*c*sin
h(b*c*x + a*c))
```

Sympy [F]

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{3/2}}\,dx = e^{ac}\int \frac{e^{bcx}}{\left(\operatorname{sech}^2\left(ac+bcx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)**2)**(3/2),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)/(sech(a*c + b*c*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.46

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{3/2}}\,dx = \frac{3\,(bcx+ac)}{8\,bc} + \frac{e^{(4\,bcx+4\,ac)}}{32\,bc} + \frac{3\,e^{(2\,bcx+2\,ac)}}{16\,bc} - \frac{e^{(-2\,bcx-2\,ac)}}{16\,bc}$$

input
```
integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

output
```
3/8*(b*c*x + a*c)/(b*c) + 1/32*e^(4*b*c*x + 4*a*c)/(b*c) + 3/16*e^(2*b*c*x
 + 2*a*c)/(b*c) - 1/16*e^(-2*b*c*x - 2*a*c)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.44

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{3/2}}\,dx = \frac{\left(12\,bcxe^{(2\,ac)} - 2\left(3\,e^{(2\,bcx+2\,ac)}+1\right)e^{(-2\,bcx)} + e^{(4\,bcx+6\,ac)} + 6\,e^{(2\,bcx+4\,ac)}\right)e^{(-2\,ac)}}{32\,bc}$$

input
```
integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

output
```
1/32*(12*b*c*x*e^(2*a*c) - 2*(3*e^(2*b*c*x + 2*a*c) + 1)*e^(-2*b*c*x) + e^
(4*b*c*x + 6*a*c) + 6*e^(2*b*c*x + 4*a*c))*e^(-2*a*c)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{3/2}}\,dx = \int \frac{e^{c\,(a+b\,x)}}{\left(\frac{1}{\cosh(a\,c+b\,c\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(3/2),x)
```

output
```
int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.41

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{3/2}}\,dx = \frac{e^{6bcx+6ac} + 6e^{4bcx+4ac} + 12e^{2bcx+2ac}bcx - 2}{32e^{2bcx+2ac}bc}$$

input
```
int(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(3/2),x)
```

output
```
(e**(6*a*c + 6*b*c*x) + 6*e**(4*a*c + 4*b*c*x) + 12*e**(2*a*c + 2*b*c*x)*b
*c*x - 2)/(32*e**(2*a*c + 2*b*c*x)*b*c)
```

3.157 $\int \frac{e^{c(a+bx)}}{\mathrm{sech}^2(ac+bcx)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 250

$$\int \frac{e^{c(a+bx)}}{\mathrm{sech}^2(ac+bcx)^{5/2}}\,dx = -\frac{e^{-4c(a+bx)}\mathrm{sech}(ac+bcx)}{128bc\sqrt{\mathrm{sech}^2(ac+bcx)}} - \frac{5e^{-2c(a+bx)}\mathrm{sech}(ac+bcx)}{64bc\sqrt{\mathrm{sech}^2(ac+bcx)}} + \frac{5e^{2c(a+bx)}\mathrm{sech}(ac+bcx)}{32bc\sqrt{\mathrm{sech}^2(ac+bcx)}} + \frac{5e^{4c(a+bx)}\mathrm{sech}(ac+bcx)}{128bc\sqrt{\mathrm{sech}^2(ac+bcx)}} + \frac{e^{6c(a+bx)}\mathrm{sech}(ac+bcx)}{192bc\sqrt{\mathrm{sech}^2(ac+bcx)}} + \frac{5x\,\mathrm{sech}(ac+bcx)}{16\sqrt{\mathrm{sech}^2(ac+bcx)}}$$

output
```
-1/128*sech(b*c*x+a*c)/b/c/exp(4*c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2)-5/64
*sech(b*c*x+a*c)/b/c/exp(2*c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2)+5/32*exp(2
*c*(b*x+a))*sech(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+5/128*exp(4*c*(b
*x+a))*sech(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+1/192*exp(6*c*(b*x+a)
)*sech(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+5/16*x*sech(b*c*x+a*c)/(se
ch(b*c*x+a*c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.44

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{5/2}}\,dx = \frac{\left(-\frac{1}{128}e^{-4c(a+bx)}-\frac{5}{64}e^{-2c(a+bx)}+\frac{5}{32}e^{2c(a+bx)}+\frac{5}{128}e^{4c(a+bx)}+\frac{1}{192}e^{6c(a+bx)}+\frac{5bcx}{16}\right)}{bc\operatorname{sech}^2(c(a+bx))^{5/2}}$$

input
```
Integrate[E^(c*(a + b*x))/(Sech[a*c + b*c*x]^2)^(5/2),x]
```

output
```
((-1/128*1/E^(4*c*(a + b*x)) - 5/(64*E^(2*c*(a + b*x))) + (5*E^(2*c*(a + b
*x)))/32 + (5*E^(4*c*(a + b*x)))/128 + E^(6*c*(a + b*x))/192 + (5*b*c*x)/1
6)*Sech[c*(a + b*x)]^5)/(b*c*(Sech[c*(a + b*x)]^2)^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.41, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {7271, 2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{5/2}}dx$$

$$\Big\downarrow \text{ 7271}$$

$$\frac{\operatorname{sech}(ac+bcx)\int e^{c(a+bx)}\cosh^5(ac+bxc)dx}{\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\operatorname{sech}(ac+bcx)\int \frac{1}{32}e^{-5c(a+bx)}\left(1+e^{2c(a+bx)}\right)^5de^{c(a+bx)}}{bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\operatorname{sech}(ac+bcx)\int e^{-5c(a+bx)}\left(1+e^{2c(a+bx)}\right)^5de^{c(a+bx)}}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\operatorname{sech}(ac+bcx)\int e^{-3c(a+bx)}\left(1+e^{2c(a+bx)}\right)^5de^{2c(a+bx)}}{64bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\operatorname{sech}(ac+bcx)\int \left(10+e^{-3c(a+bx)}+5e^{-2c(a+bx)}+10e^{-c(a+bx)}+6e^{2c(a+bx)}\right)de^{2c(a+bx)}}{64bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{1}{2}e^{-2c(a+bx)}-5e^{-c(a+bx)}+\frac{25}{2}e^{2c(a+bx)}+\frac{1}{3}e^{3c(a+bx)}+10\log\left(e^{2c(a+bx)}\right)\right)\operatorname{sech}(ac+bcx)}{64bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

input
```
Int[E^(c*(a + b*x))/(Sech[a*c + b*c*x]^2)^(5/2),x]
```

output
```
((-1/2*1/E^(2*c*(a + b*x)) - 5/E^(c*(a + b*x)) + (25*E^(2*c*(a + b*x)))/2
+ E^(3*c*(a + b*x))/3 + 10*Log[E^(2*c*(a + b*x))])*Sech[a*c + b*c*x])/(64*
b*c*Sqrt[Sech[a*c + b*c*x]^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.30

method	result
risch	$\frac{5x\,\mathrm{e}^{c(bx+a)}}{16\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}}}}+\frac{\mathrm{e}^{7c(bx+a)}}{192bc\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}}}}+\frac{5\,\mathrm{e}^{5c(bx+a)}}{128bc\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}}}}+\frac{}{32b}$

input
```
int(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
5/16*x/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1
/2)*exp(c*(b*x+a))+1/192/b/c/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^
2*exp(2*c*(b*x+a)))^(1/2)*exp(7*c*(b*x+a))+5/128/b/c/(1+exp(2*c*(b*x+a)))/
(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)*exp(5*c*(b*x+a))+5/32/b/
c/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)*e
xp(3*c*(b*x+a))-5/64/b/c/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*ex
p(2*c*(b*x+a)))^(1/2)*exp(-c*(b*x+a))-1/128/b/c/(1+exp(2*c*(b*x+a)))/(1/(1
+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)*exp(-3*c*(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.87

$$\int \frac{e^{c(a+bx)}}{\text{sech}^2(ac+bcx)^{5/2}}\,dx = -\frac{\cosh(bcx+ac)^5+5\,\cosh(bcx+ac)\sinh(bcx+ac)^4-5\,\sinh(bcx+ac)^5-5\left(10\,\cosh(bcx+ac)^2+9\right)}{}$$

input
```
integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/384*(cosh(b*c*x + a*c)^5 + 5*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 - 5*
sinh(b*c*x + a*c)^5 - 5*(10*cosh(b*c*x + a*c)^2 + 9)*sinh(b*c*x + a*c)^3 +
 15*cosh(b*c*x + a*c)^3 + 5*(2*cosh(b*c*x + a*c)^3 + 9*cosh(b*c*x + a*c))*
sinh(b*c*x + a*c)^2 - 60*(2*b*c*x + 1)*cosh(b*c*x + a*c) - 5*(5*cosh(b*c*x
 + a*c)^4 - 24*b*c*x + 27*cosh(b*c*x + a*c)^2 + 12)*sinh(b*c*x + a*c))/(b*
c*cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(238) = 476.

Time = 79.39 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.12

$$\int \frac{e^{c(a+bx)}}{\text{sech}^2(ac+bcx)^{5/2}}\,dx = \begin{cases} x \\ \frac{xe^{ac}}{\left(\text{sech}^2(ac)\right)^{\frac{5}{2}}} \\ x \\ -\frac{5xe^{ac}e^{bcx}\tanh^5(ac+bcx)}{16\left(\text{sech}^2(ac+bcx)\right)^{\frac{5}{2}}}+\frac{5xe^{ac}e^{bcx}\tanh^4(ac+bcx)}{16\left(\text{sech}^2(ac+bcx)\right)^{\frac{5}{2}}}+\frac{5xe^{ac}e^{bcx}\tanh^3(ac+bcx)}{8\left(\text{sech}^2(ac+bcx)\right)^{\frac{5}{2}}}-\frac{5xe^{ac}e^{bcx}\,\text{t}}{8(\text{sech}^2} \end{cases}$$

input
```
integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)**2)**(5/2),x)
```

output
```
Piecewise((x, Eq(b, 0) & Eq(c, 0)), (x*exp(a*c)/(sech(a*c)**2)**(5/2), Eq(
b, 0)), (x, Eq(c, 0)), (-5*x*exp(a*c)*exp(b*c*x)*tanh(a*c + b*c*x)**5/(16*
(sech(a*c + b*c*x)**2)**(5/2)) + 5*x*exp(a*c)*exp(b*c*x)*tanh(a*c + b*c*x)
**4/(16*(sech(a*c + b*c*x)**2)**(5/2)) + 5*x*exp(a*c)*exp(b*c*x)*tanh(a*c
+ b*c*x)**3/(8*(sech(a*c + b*c*x)**2)**(5/2)) - 5*x*exp(a*c)*exp(b*c*x)*ta
nh(a*c + b*c*x)**2/(8*(sech(a*c + b*c*x)**2)**(5/2)) - 5*x*exp(a*c)*exp(b*
c*x)*tanh(a*c + b*c*x)/(16*(sech(a*c + b*c*x)**2)**(5/2)) + 5*x*exp(a*c)*e
xp(b*c*x)/(16*(sech(a*c + b*c*x)**2)**(5/2)) + 8*exp(a*c)*exp(b*c*x)*tanh(
a*c + b*c*x)**5/(15*b*c*(sech(a*c + b*c*x)**2)**(5/2)) - 53*exp(a*c)*exp(b
*c*x)*tanh(a*c + b*c*x)**4/(240*b*c*(sech(a*c + b*c*x)**2)**(5/2)) - 331*e
xp(a*c)*exp(b*c*x)*tanh(a*c + b*c*x)**3/(240*b*c*(sech(a*c + b*c*x)**2)**(
5/2)) + 131*exp(a*c)*exp(b*c*x)*tanh(a*c + b*c*x)**2/(240*b*c*(sech(a*c +
b*c*x)**2)**(5/2)) + 253*exp(a*c)*exp(b*c*x)*tanh(a*c + b*c*x)/(240*b*c*(s
ech(a*c + b*c*x)**2)**(5/2)) - 11*exp(a*c)*exp(b*c*x)/(30*b*c*(sech(a*c +
b*c*x)**2)**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.45

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{5/2}}\,dx = \frac{5\,(bcx+ac)}{16\,bc} + \frac{e^{(6\,bcx+6\,ac)}}{192\,bc} + \frac{5\,e^{(4\,bcx+4\,ac)}}{128\,bc} + \frac{5\,e^{(2\,bcx+2\,ac)}}{32\,bc} - \frac{5\,e^{(-2\,bcx-2\,ac)}}{64\,bc} - \frac{e^{(-4\,bcx-4\,ac)}}{128\,bc}$$

input
```
integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

output
```
5/16*(b*c*x + a*c)/(b*c) + 1/192*e^(6*b*c*x + 6*a*c)/(b*c) + 5/128*e^(4*b*
c*x + 4*a*c)/(b*c) + 5/32*e^(2*b*c*x + 2*a*c)/(b*c) - 5/64*e^(-2*b*c*x - 2
*a*c)/(b*c) - 1/128*e^(-4*b*c*x - 4*a*c)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.40

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{5/2}}\,dx = \frac{\left(120\,bcxe^{(4\,ac)} - 3\left(30\,e^{(4\,bcx+4\,ac)} + 10\,e^{(2\,bcx+2\,ac)} + 1\right)e^{(-4\,bcx)} + 2\,e^{(6\,bcx+10\,ac)} + \right.}{384\,bc}$$

input
```
integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

output
```
1/384*(120*b*c*x*e^(4*a*c) - 3*(30*e^(4*b*c*x + 4*a*c) + 10*e^(2*b*c*x + 2
*a*c) + 1)*e^(-4*b*c*x) + 2*e^(6*b*c*x + 10*a*c) + 15*e^(4*b*c*x + 8*a*c)
+ 60*e^(2*b*c*x + 6*a*c))*e^(-4*a*c)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{5/2}}\,dx = \int \frac{e^{c\,(a+b\,x)}}{\left(\frac{1}{\cosh(a\,c+b\,c\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(5/2),x)
```

output
```
int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.39

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{5/2}}\,dx = \frac{2e^{10bcx+10ac} + 15e^{8bcx+8ac} + 60e^{6bcx+6ac} + 120e^{4bcx+4ac}bcx - 30e^{2bcx+2ac} - 3}{384e^{4bcx+4ac}bc}$$

input
```
int(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(5/2),x)
```

output
```
(2*e**(10*a*c + 10*b*c*x) + 15*e**(8*a*c + 8*b*c*x) + 60*e**(6*a*c + 6*b*c
*x) + 120*e**(4*a*c + 4*b*c*x)*b*c*x - 30*e**(2*a*c + 2*b*c*x) - 3)/(384*e
**(4*a*c + 4*b*c*x)*b*c)
```

3.158 $\int \frac{x^5}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 108

$$\int \frac{x^5}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{2x^2}{21c^4\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{x^6}{7\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\operatorname{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{21c^5\left(c^4+\frac{1}{x^4}\right)x\sqrt{\operatorname{sech}(2\log(cx))}}$$

output
```
2/21*x^2/c^4/sech(2*ln(c*x))^(1/2)+1/7*x^6/sech(2*ln(c*x))^(1/2)+1/21*((c^
4+1/x^4)/(c^2+1/x^2)^2)^(1/2)*(c^2+1/x^2)*InverseJacobiAM(2*arccot(c*x),1/
2*2^(1/2))/c^5/(c^4+1/x^4)/x/sech(2*ln(c*x))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.71

$$\int \frac{x^5}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{\sqrt{1+c^4x^4}\sqrt{\frac{c^2x^2}{2+2c^4x^4}}\left(\left(1+c^4x^4\right)^{3/2}-\operatorname{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-c^4x^4\right)\right)}{7c^6}$$

input
```
Integrate[x^5/Sqrt[Sech[2*Log[c*x]]],x]
```

output
```
(Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*((1 + c^4*x^4)^(3/2) -
Hypergeometric2F1[-1/2, 1/4, 5/4, -(c^4*x^4)]))/(7*c^6)
```

Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.38, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6085, 6083, 858, 809, 847, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{\operatorname{sech}(2\log(cx))}}dx$$

$$\Big\downarrow \text{6085}$$

$$\frac{\int \frac{c^5x^5}{\sqrt{\operatorname{sech}(2\log(cx))}}d(cx)}{c^6}$$

$$\Big\downarrow \text{6083}$$

$$\frac{\int c^6\sqrt{1+\frac{1}{c^4x^4}}x^6d(cx)}{c^7x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{\int \frac{\sqrt{c^4x^4+1}}{c^8x^8}d\frac{1}{cx}}{c^7x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$\Big\downarrow \text{809}$$

$$-\frac{\frac{2}{7}\int \frac{1}{c^4x^4\sqrt{c^4x^4+1}}d\frac{1}{cx}-\frac{\sqrt{c^4x^4+1}}{7c^7x^7}}{c^7x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{\frac{2}{7}\left(-\frac{1}{3}\int \frac{1}{\sqrt{c^4x^4+1}}d\frac{1}{cx}-\frac{\sqrt{c^4x^4+1}}{3c^3x^3}\right)-\frac{\sqrt{c^4x^4+1}}{7c^7x^7}}{c^7x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$\Big\downarrow \text{761}$$

$$-\frac{\frac{2}{7}\left(-\frac{(c^2x^2+1)\sqrt{\frac{c^4x^4+1}{(c^2x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{1}{cx}\right),\frac{1}{2}\right)}{6\sqrt{c^4x^4+1}}-\frac{\sqrt{c^4x^4+1}}{3c^3x^3}\right)-\frac{\sqrt{c^4x^4+1}}{7c^7x^7}}{c^7x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}$$

input
```
Int[x^5/Sqrt[Sech[2*Log[c*x]]],x]
```

output
```
-((-1/7*Sqrt[1 + c^4*x^4]/(c^7*x^7) + (2*(-1/3*Sqrt[1 + c^4*x^4]/(c^3*x^3)
 - ((1 + c^2*x^2)*Sqrt[(1 + c^4*x^4)/(1 + c^2*x^2)^2]*EllipticF[2*ArcTan[1
/(c*x)], 1/2])/(6*Sqrt[1 + c^4*x^4])))/7)/(c^7*Sqrt[1 + 1/(c^4*x^4)]*x*Sqr
t[Sech[2*Log[c*x]]]))
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6083
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6085
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.20

method	result	size
risch	$\frac{x^{2}\left(3 c^{4} x^{4}+2\right) \sqrt{2}}{42 c^{4} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}-\frac{\sqrt{-i c^{2} x^{2}+1}\, \sqrt{i c^{2} x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{i c^{2}}, i\right) \sqrt{2}\, x}{21 c^{4} \sqrt{i c^{2}}\, \left(c^{4} x^{4}+1\right) \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}$	130

input
```
int(x^5/sech(2*ln(x*c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/42*x^2*(3*c^4*x^4+2)/c^4*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)-1/21/c^4/(I
*c^2)^(1/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)*EllipticF(
x*(I*c^2)^(1/2),I)*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = -\frac{4\sqrt{\frac{1}{2}}\sqrt{c^4}c\left(-\frac{1}{c^4}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{1}{c^4}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)-\sqrt{2}(3c^8x^8+5c^4x^4+2)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{42c^6}$$

input
```
integrate(x^5/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

output
```
-1/42*(4*sqrt(1/2)*sqrt(c^4)*c*(-1/c^4)^(3/4)*elliptic_f(arcsin((-1/c^4)^(
1/4)/x), -1) - sqrt(2)*(3*c^8*x^8 + 5*c^4*x^4 + 2)*sqrt(c^2*x^2/(c^4*x^4 +
 1)))/c^6
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^5}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(x**5/sech(2*ln(c*x))**(1/2),x)
```

output
```
Integral(x**5/sqrt(sech(2*log(c*x))), x)
```

Maxima [F]

$$\int \frac{x^5}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^5}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(x^5/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^5/sqrt(sech(2*log(c*x))), x)
```

Giac [F]

$$\int \frac{x^5}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^5}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(x^5/sech(2*log(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^5/sqrt(sech(2*log(c*x))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^5}{\sqrt{\frac{1}{\cosh(2\,\ln(c\,x))}}}\,dx$$

input
```
int(x^5/(1/cosh(2*log(c*x)))^(1/2),x)
```

output
```
int(x^5/(1/cosh(2*log(c*x)))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^5}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}\,x^5}{\operatorname{sech}\left(2\log\left(cx\right)\right)}dx$$

input
```
int(x^5/sech(2*log(c*x))^(1/2),x)
```

output
```
int((sqrt(sech(2*log(c*x)))*x**5)/sech(2*log(c*x)),x)
```

3.159 $\int \frac{x^4}{\sqrt{\text{sech}(2\log(cx))}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{x^4}{\sqrt{\text{sech}(2\log(cx))}}\,dx = \frac{\left(c^4+\frac{1}{x^4}\right)x^5}{6c^4\sqrt{\text{sech}(2\log(cx))}}$$

output
```
1/6*(c^4+1/x^4)*x^5/c^4/sech(2*ln(c*x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.57

$$\int \frac{x^4}{\sqrt{\text{sech}(2\log(cx))}}\,dx = \frac{(1+c^4x^4)^2\sqrt{\frac{c^2x^2}{2+2c^4x^4}}}{6c^6x}$$

input
```
Integrate[x^4/Sqrt[Sech[2*Log[c*x]]],x]
```

output
```
((1 + c^4*x^4)^2*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)])/(6*c^6*x)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6085, 6083, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$$

$$\Big\downarrow \text{ 6085}$$

$$\frac{\int \frac{c^4x^4}{\sqrt{\operatorname{sech}(2\log(cx))}}d(cx)}{c^5}$$

$$\Big\downarrow \text{ 6083}$$

$$\frac{\int c^5\sqrt{1+\frac{1}{c^4x^4}}x^5d(cx)}{c^6x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{x^5\left(\frac{1}{c^4x^4}+1\right)}{6\sqrt{\operatorname{sech}(2\log(cx))}}$$

input
```
Int[x^4/Sqrt[Sech[2*Log[c*x]]],x]
```

output
```
((1 + 1/(c^4*x^4))*x^5)/(6*Sqrt[Sech[2*Log[c*x]]])
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 6083
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6085
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

method	result	size
risch	$\frac{\sqrt{2}\, x \left(c^{4} x^{4}+1\right)}{12 \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}\, c^{4}}$	39

input
```
int(x^4/sech(2*ln(x*c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)*(c^4*x^4+1)/c^4
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \frac{x^4}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{\sqrt{2}(c^8x^8 + 2\,c^4x^4 + 1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{12\,c^6x}$$

input
```
integrate(x^4/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(2)*(c^8*x^8 + 2*c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1))/(c^6*x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^4}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(x**4/sech(2*ln(c*x))**(1/2),x)
```

output
```
Integral(x**4/sqrt(sech(2*log(c*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^4}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{(\sqrt{2}c^4x^4 + \sqrt{2})\sqrt{c^4x^4 + 1}}{12\,c^5}$$

input
```
integrate(x^4/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
```

output
```
1/12*(sqrt(2)*c^4*x^4 + sqrt(2))*sqrt(c^4*x^4 + 1)/c^5
```

Giac [F]

$$\int \frac{x^4}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^4}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$$

input
```
integrate(x^4/sech(2*log(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(sech(2*log(c*x))), x)
```

Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.50

$$\int \frac{x^4}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{\left(c^4\,x^4+1\right)^2\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4+1}}}{12\,c^6\,x}$$

input
```
int(x^4/(1/cosh(2*log(c*x)))^(1/2),x)
```

output
```
((c^4*x^4 + 1)^2*((2*c^2*x^2)/(c^4*x^4 + 1))^(1/2))/(12*c^6*x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{\sqrt{\operatorname{sech}(2\log(cx))}\,x^4}{\operatorname{sech}(2\log(cx))}dx$$

input
```
int(x^4/sech(2*log(c*x))^(1/2),x)
```

output
```
int((sqrt(sech(2*log(c*x)))*x**4)/sech(2*log(c*x)),x)
```

3.160 $\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 203

$$\begin{aligned}\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx &= \frac{2}{5c^4\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{2}{5c^4\left(c^2+\frac{1}{x^2}\right)x^2\sqrt{\operatorname{sech}(2\log(cx))}}\\ &\quad + \frac{x^4}{5\sqrt{\operatorname{sech}(2\log(cx))}}\\ &\quad + \frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)E\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{5c^3\left(c^4+\frac{1}{x^4}\right)x\sqrt{\operatorname{sech}(2\log(cx))}}\\ &\quad - \frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\operatorname{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{5c^3\left(c^4+\frac{1}{x^4}\right)x\sqrt{\operatorname{sech}(2\log(cx))}}\end{aligned}$$

output
```
2/5/c^4/sech(2*ln(c*x))^(1/2)-2/5/c^4/(c^2+1/x^2)/x^2/sech(2*ln(c*x))^(1/2
)+1/5*x^4/sech(2*ln(c*x))^(1/2)+2/5*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)*(c^2
+1/x^2)*EllipticE(sin(2*arccot(c*x)),1/2*2^(1/2))/c^3/(c^4+1/x^4)/x/sech(2
*ln(c*x))^(1/2)-1/5*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)*(c^2+1/x^2)*InverseJ
acobiAM(2*arccot(c*x),1/2*2^(1/2))/c^3/(c^4+1/x^4)/x/sech(2*ln(c*x))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.32

$$\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{\left(\frac{c^2x^2}{1+c^4x^4}\right)^{3/2}(1+c^4x^4)^{3/2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-c^4x^4\right)}{3\sqrt{2}c^4}$$

input
```
Integrate[x^3/Sqrt[Sech[2*Log[c*x]]],x]
```

output
```
(((c^2*x^2)/(1 + c^4*x^4))^(3/2)*(1 + c^4*x^4)^(3/2)*Hypergeometric2F1[-1/
2, 3/4, 7/4, -(c^4*x^4)])/(3*Sqrt[2]*c^4)
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {6085, 6083, 858, 809, 847, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$$

$$\Big\downarrow \text{6085}$$

$$\frac{\int \frac{c^3x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}d(cx)}{c^4}$$

$$\Big\downarrow \text{6083}$$

$$\frac{\int c^4\sqrt{1+\frac{1}{c^4x^4}}x^4d(cx)}{c^5x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$\Big\downarrow \text{ 858}$$

$$-\frac{\int \frac{\sqrt{c^4x^4+1}}{c^6x^6}d\frac{1}{cx}}{c^5x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$\Big\downarrow \text{ 809}$$

$$-\frac{\frac{2}{5}\int \frac{1}{c^2x^2\sqrt{c^4x^4+1}}d\frac{1}{cx}-\frac{\sqrt{c^4x^4+1}}{5c^5x^5}}{c^5x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{\frac{2}{5}\left(\int \frac{c^2x^2}{\sqrt{c^4x^4+1}}d\frac{1}{cx}-\frac{\sqrt{c^4x^4+1}}{cx}\right)-\frac{\sqrt{c^4x^4+1}}{5c^5x^5}}{c^5x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$\Big\downarrow \text{ 834}$$

$$-\frac{\frac{2}{5}\left(\int \frac{1}{\sqrt{c^4x^4+1}}d\frac{1}{cx}-\int \frac{1-c^2x^2}{\sqrt{c^4x^4+1}}d\frac{1}{cx}-\frac{\sqrt{c^4x^4+1}}{cx}\right)-\frac{\sqrt{c^4x^4+1}}{5c^5x^5}}{c^5x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$\Big\downarrow \text{ 761}$$

$$-\frac{\frac{2}{5}\left(-\int \frac{1-c^2x^2}{\sqrt{c^4x^4+1}}d\frac{1}{cx}+\frac{(c^2x^2+1)\sqrt{\frac{c^4x^4+1}{(c^2x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{1}{cx}\right),\frac{1}{2}\right)}{2\sqrt{c^4x^4+1}}-\frac{\sqrt{c^4x^4+1}}{cx}\right)-\frac{\sqrt{c^4x^4+1}}{5c^5x^5}}{c^5x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$\Big\downarrow \text{ 1510}$$

$$-\frac{\frac{2}{5}\left(\frac{(c^2x^2+1)\sqrt{\frac{c^4x^4+1}{(c^2x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{1}{cx}\right),\frac{1}{2}\right)}{2\sqrt{c^4x^4+1}}-\frac{(c^2x^2+1)\sqrt{\frac{c^4x^4+1}{(c^2x^2+1)^2}}E\left(2\arctan\left(\frac{1}{cx}\right)|\frac{1}{2}\right)}{\sqrt{c^4x^4+1}}-\frac{\sqrt{c^4x^4+1}}{cx}+\frac{\sqrt{c^4x^4+1}}{cx(c^2x^2+1)}\right)-\frac{\sqrt{c}}{5}}{c^5x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}$$

input
```
Int[x^3/Sqrt[Sech[2*Log[c*x]]],x]
```

output
```
-((-1/5*Sqrt[1 + c^4*x^4]/(c^5*x^5) + (2*(-(Sqrt[1 + c^4*x^4]/(c*x)) + Sqr
t[1 + c^4*x^4]/(c*x*(1 + c^2*x^2)) - ((1 + c^2*x^2)*Sqrt[(1 + c^4*x^4)/(1
+ c^2*x^2)^2]*EllipticE[2*ArcTan[1/(c*x)], 1/2])/Sqrt[1 + c^4*x^4] + ((1 +
 c^2*x^2)*Sqrt[(1 + c^4*x^4)/(1 + c^2*x^2)^2]*EllipticF[2*ArcTan[1/(c*x)],
 1/2])/(2*Sqrt[1 + c^4*x^4])))/5)/(c^5*Sqrt[1 + 1/(c^4*x^4)]*x*Sqrt[Sech[2
*Log[c*x]]]))
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 6083
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6085
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{\sqrt{2}\,x^{4}}{10\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}+\frac{i\sqrt{-ic^{2}x^{2}+1}\,\sqrt{ic^{2}x^{2}+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{ic^{2}},i\right)-\operatorname{EllipticE}\left(x\sqrt{ic^{2}},i\right)\right)\sqrt{2}\,x}{5\sqrt{ic^{2}}\,\left(c^{4}x^{4}+1\right)c^{2}\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}$	134

input
```
int(x^3/sech(2*ln(x*c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*2^(1/2)*x^4/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/5*I/(I*c^2)^(1/2)*(1-I*c^2*
x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)/c^2*(EllipticF(x*(I*c^2)^(1/2),
I)-EllipticE(x*(I*c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.59

$$\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{4\sqrt{\frac{1}{2}}\sqrt{c^4}cx^2\left(-\frac{1}{c^4}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{1}{c^4}\right)^{\frac{1}{4}}}{x}\right)|-1\right) - 4\sqrt{\frac{1}{2}}\sqrt{c^4}cx^2\left(-\frac{1}{c^4}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{1}{c^4}\right)^{\frac{1}{4}}}{x}\right)|-1\right) + \sqrt{2}(c}{10\,c^6x^2}$$

input
```
integrate(x^3/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

output
```
1/10*(4*sqrt(1/2)*sqrt(c^4)*c*x^2*(-1/c^4)^(3/4)*elliptic_e(arcsin((-1/c^4
)^(1/4)/x), -1) - 4*sqrt(1/2)*sqrt(c^4)*c*x^2*(-1/c^4)^(3/4)*elliptic_f(ar
csin((-1/c^4)^(1/4)/x), -1) + sqrt(2)*(c^8*x^8 + 3*c^4*x^4 + 2)*sqrt(c^2*x
^2/(c^4*x^4 + 1)))/(c^6*x^2)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^3}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(x**3/sech(2*ln(c*x))**(1/2),x)
```

output
```
Integral(x**3/sqrt(sech(2*log(c*x))), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^3}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(x^3/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(sech(2*log(c*x))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/sech(2*log(c*x))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly exception caught Unable to
 convert to real %%{poly1[1.000000000000000000000000000000,0.000000000000
000000000
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^3}{\sqrt{\frac{1}{\cosh(2\,\ln(c\,x))}}}\,dx$$

input
```
int(x^3/(1/cosh(2*log(c*x)))^(1/2),x)
```

output
```
int(x^3/(1/cosh(2*log(c*x)))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}\,x^3}{\operatorname{sech}\left(2\log\left(cx\right)\right)}dx$$

input

```
int(x^3/sech(2*log(c*x))^(1/2),x)
```

output

```
int((sqrt(sech(2*log(c*x)))*x**3)/sech(2*log(c*x)),x)
```

3.161 $\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx=\frac{x^3}{4\sqrt{\operatorname{sech}(2\log(cx))}}+\frac{\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^4x^4}}\right)}{4c^4\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

output
```
1/4*x^3/sech(2*ln(c*x))^(1/2)+1/4*arctanh((1+1/c^4/x^4)^(1/2))/c^4/(1+1/c^
4/x^4)^(1/2)/x/sech(2*ln(c*x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

$$\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx=\frac{x\left(c^2x^2\sqrt{1+c^4x^4}+\operatorname{arcsinh}(c^2x^2)\right)}{4\sqrt{2}c^2\sqrt{\frac{c^2x^2}{1+c^4x^4}}\sqrt{1+c^4x^4}}$$

input
```
Integrate[x^2/Sqrt[Sech[2*Log[c*x]]],x]
```

output
```
(x*(c^2*x^2*Sqrt[1 + c^4*x^4] + ArcSinh[c^2*x^2]))/(4*Sqrt[2]*c^2*Sqrt[(c^
2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6085, 6083, 798, 51, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}dx$$

$$\Big\downarrow \text{6085}$$

$$\frac{\int \frac{c^2x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}d(cx)}{c^3}$$

$$\Big\downarrow \text{6083}$$

$$\frac{\int c^3\sqrt{1+\frac{1}{c^4x^4}}x^3d(cx)}{c^4x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$\Big\downarrow \text{798}$$

$$-\frac{\int \frac{\sqrt{1+\frac{1}{c^4x^4}}}{c^2x^2}d\frac{1}{c^4x^4}}{4c^4x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$\Big\downarrow \text{51}$$

$$-\frac{\frac{1}{2}\int \frac{1}{c\sqrt{1+\frac{1}{c^4x^4}}x}d\frac{1}{c^4x^4}-\frac{\sqrt{\frac{1}{c^4x^4}+1}}{cx}}{4c^4x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\int \frac{1}{c^2x^2-1}d\sqrt{1+\frac{1}{c^4x^4}}-\frac{\sqrt{\frac{1}{c^4x^4}+1}}{cx}}{4c^4x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$\Big\downarrow \text{220}$$

$$-\frac{-\operatorname{arctanh}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)-\frac{\sqrt{\frac{1}{c^4x^4}+1}}{cx}}{4c^4x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}$$

input
```
Int[x^2/Sqrt[Sech[2*Log[c*x]]],x]
```

output
```
-1/4*(-(Sqrt[1 + 1/(c^4*x^4)]/(c*x)) - ArcTanh[Sqrt[1 + 1/(c^4*x^4)]])/(c^
4*Sqrt[1 + 1/(c^4*x^4)]*x*Sqrt[Sech[2*Log[c*x]]])
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6083
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6085
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.45

method	result	size
risch	$\frac{\sqrt{2}\,x^{3}}{8\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}+\frac{\ln\left(\frac{c^{4}x^{2}}{\sqrt{c^{4}}}+\sqrt{c^{4}x^{4}+1}\right)\sqrt{2}\,x}{8\sqrt{c^{4}}\,\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}\,\sqrt{c^{4}x^{4}+1}}$	97

input
```
int(x^2/sech(2*ln(x*c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*2^(1/2)*x^3/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/8*ln(c^4*x^2/(c^4)^(1/2)+(c^
4*x^4+1)^(1/2))/(c^4)^(1/2)*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)/(c^4*x^4
+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.34

$$\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{2\sqrt{2}(c^5x^5+cx)\sqrt{\frac{c^2x^2}{c^4x^4+1}}+\sqrt{2}\log\left(-2\,c^4x^4-2\,(c^5x^5+cx)\sqrt{\frac{c^2x^2}{c^4x^4+1}}-1\right)}{16\,c^3}$$

input
```
integrate(x^2/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

output
```
1/16*(2*sqrt(2)*(c^5*x^5 + c*x)*sqrt(c^2*x^2/(c^4*x^4 + 1)) + sqrt(2)*log(
-2*c^4*x^4 - 2*(c^5*x^5 + c*x)*sqrt(c^2*x^2/(c^4*x^4 + 1)) - 1))/c^3
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^2}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(x**2/sech(2*ln(c*x))**(1/2),x)
```

output
```
Integral(x**2/sqrt(sech(2*log(c*x))), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^2}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(x^2/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(sech(2*log(c*x))), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^2}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(x^2/sech(2*log(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(sech(2*log(c*x))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^2}{\sqrt{\frac{1}{\cosh(2\ln(c\,x))}}}\,dx$$

input
```
int(x^2/(1/cosh(2*log(c*x)))^(1/2),x)
```

output
```
int(x^2/(1/cosh(2*log(c*x)))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}\,x^2}{\operatorname{sech}\left(2\log\left(cx\right)\right)}dx$$

input
```
int(x^2/sech(2*log(c*x))^(1/2),x)
```

output
```
int((sqrt(sech(2*log(c*x)))*x**2)/sech(2*log(c*x)),x)
```

3.162 $\int \frac{x}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 87

$$\int \frac{x}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{x^2}{3\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\operatorname{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{3c\left(c^4+\frac{1}{x^4}\right)x\sqrt{\operatorname{sech}(2\log(cx))}}$$

output
```
1/3*x^2/sech(2*ln(c*x))^(1/2)-1/3*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)*(c^2+1
/x^2)*InverseJacobiAM(2*arccot(c*x),1/2*2^(1/2))/c/(c^4+1/x^4)/x/sech(2*ln
(c*x))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{\sqrt{1+c^4x^4}\sqrt{\frac{c^2x^2}{2+2c^4x^4}}\operatorname{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-c^4x^4\right)}{c^2}$$

input
```
Integrate[x/Sqrt[Sech[2*Log[c*x]]],x]
```

output
```
(Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*Hypergeometric2F1[-1/2,
 1/4, 5/4, -(c^4*x^4)])/c^2
```

Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.39, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6085, 6083, 858, 809, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\text{sech}(2\log(cx))}}dx$$

$$\Big\downarrow \text{ 6085}$$

$$\frac{\int \frac{cx}{\sqrt{\text{sech}(2\log(cx))}}d(cx)}{c^2}$$

$$\Big\downarrow \text{ 6083}$$

$$\frac{\int c^2\sqrt{1+\frac{1}{c^4x^4}}x^2d(cx)}{c^3x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\text{sech}(2\log(cx))}}$$

$$\Big\downarrow \text{ 858}$$

$$-\frac{\int \frac{\sqrt{c^4x^4+1}}{c^4x^4}d\frac{1}{cx}}{c^3x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\text{sech}(2\log(cx))}}$$

$$\Big\downarrow \text{ 809}$$

$$-\frac{\frac{2}{3}\int \frac{1}{\sqrt{c^4x^4+1}}d\frac{1}{cx}-\frac{\sqrt{c^4x^4+1}}{3c^3x^3}}{c^3x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\text{sech}(2\log(cx))}}$$

$$\Big\downarrow \text{ 761}$$

$$-\frac{\frac{(c^2x^2+1)\sqrt{\frac{c^4x^4+1}{(c^2x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{1}{cx}\right),\frac{1}{2}\right)}{3\sqrt{c^4x^4+1}}-\frac{\sqrt{c^4x^4+1}}{3c^3x^3}}{c^3x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}$$

input
```
Int[x/Sqrt[Sech[2*Log[c*x]]],x]
```

output
```
-((-1/3*Sqrt[1 + c^4*x^4]/(c^3*x^3) + ((1 + c^2*x^2)*Sqrt[(1 + c^4*x^4)/(1
 + c^2*x^2)^2]*EllipticF[2*ArcTan[1/(c*x)], 1/2])/(3*Sqrt[1 + c^4*x^4]))/(
c^3*Sqrt[1 + 1/(c^4*x^4)]*x*Sqrt[Sech[2*Log[c*x]]]))
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6083
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6085
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.31

method	result	size
risch	$\frac{\sqrt{2}\, x^{2}}{6 \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}+\frac{\sqrt{-i c^{2} x^{2}+1}\, \sqrt{i c^{2} x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{i c^{2}}, i\right) \sqrt{2}\, x}{3 \sqrt{i c^{2}}\, \left(c^{4} x^{4}+1\right) \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}$	114

input
```
int(x/sech(2*ln(x*c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/2)*x^2/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/3/(I*c^2)^(1/2)*(1-I*c^2*x^2
)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)*EllipticF(x*(I*c^2)^(1/2),I)*2^(1/
2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int \frac{x}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{4\sqrt{\frac{1}{2}}\sqrt{c^{4}}c\left(-\frac{1}{c^{4}}\right)^{\frac{3}{4}} F(\arcsin\left(\frac{\left(-\frac{1}{c^{4}}\right)^{\frac{1}{4}}}{x}\right) \mid -1) + \sqrt{2}(c^{4}x^{4}+1)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}{6\,c^{2}}$$

input
```
integrate(x/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

output
```
1/6*(4*sqrt(1/2)*sqrt(c^4)*c*(-1/c^4)^(3/4)*elliptic_f(arcsin((-1/c^4)^(1/
4)/x), -1) + sqrt(2)*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1)))/c^2
```

Sympy [F]

$$\int \frac{x}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(x/sech(2*ln(c*x))**(1/2),x)
```

output
```
Integral(x/sqrt(sech(2*log(c*x))), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(x/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(sech(2*log(c*x))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/sech(2*log(c*x))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly exception caught Unable to
 convert to real %%{poly1[1.00000000000000000000000000000,0.000000000000
000000000
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x}{\sqrt{\frac{1}{\cosh(2\ln(c\,x))}}}\,dx$$

input
```
int(x/(1/cosh(2*log(c*x)))^(1/2),x)
```

output
```
int(x/(1/cosh(2*log(c*x)))^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{\sqrt{\operatorname{sech}(2\log(cx))}\,x}{\operatorname{sech}(2\log(cx))}dx$$

input
```
int(x/sech(2*log(c*x))^(1/2),x)
```

output
```
int((sqrt(sech(2*log(c*x)))*x)/sech(2*log(c*x)),x)
```

3.163 $\int \frac{1}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 59

$$\int \frac{1}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx=\frac{x}{2\sqrt{\operatorname{sech}(2\log(cx))}}-\frac{\operatorname{csch}^{-1}(c^2x^2)}{2c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

output
```
1/2*x/sech(2*ln(c*x))^(1/2)-1/2*arccsch(c^2*x^2)/c^2/(1+1/c^4/x^4)^(1/2)/x
/sech(2*ln(c*x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int \frac{1}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx=\frac{x\left(\sqrt{1+c^4x^4}-\operatorname{arctanh}\left(\sqrt{1+c^4x^4}\right)\right)}{2\sqrt{2}\sqrt{\frac{c^2x^2}{1+c^4x^4}}\sqrt{1+c^4x^4}}$$

input
```
Integrate[1/Sqrt[Sech[2*Log[c*x]]],x]
```

output
```
(x*(Sqrt[1 + c^4*x^4] - ArcTanh[Sqrt[1 + c^4*x^4]]))/(2*Sqrt[2]*Sqrt[(c^2*
x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {6079, 6077, 858, 807, 247, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\text{sech}(2\log(cx))}}dx$$
$$\Big\downarrow \text{6079}$$
$$\frac{\int \frac{1}{\sqrt{\text{sech}(2\log(cx))}}d(cx)}{c}$$
$$\Big\downarrow \text{6077}$$
$$\frac{\int c\sqrt{1+\frac{1}{c^4x^4}}xd(cx)}{c^2x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\text{sech}(2\log(cx))}}$$
$$\Big\downarrow \text{858}$$
$$-\frac{\int \frac{\sqrt{c^4x^4+1}}{c^3x^3}d\frac{1}{cx}}{c^2x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\text{sech}(2\log(cx))}}$$
$$\Big\downarrow \text{807}$$
$$-\frac{\int \frac{\sqrt{c^2x^2+1}}{c^2x^2}d(c^2x^2)}{2c^2x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\text{sech}(2\log(cx))}}$$
$$\Big\downarrow \text{247}$$
$$-\frac{\int \frac{1}{\sqrt{c^2x^2+1}}d(c^2x^2)-c^2x^2\sqrt{c^2x^2+1}}{2c^2x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\text{sech}(2\log(cx))}}$$
$$\Big\downarrow \text{222}$$
$$-\frac{\text{arcsinh}(c^2x^2)-c^2x^2\sqrt{c^2x^2+1}}{2c^2x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\text{sech}(2\log(cx))}}$$

input
```
Int[1/Sqrt[Sech[2*Log[c*x]]],x]
```

output
```
-1/2*(-(c^2*x^2*Sqrt[1 + c^2*x^2]) + ArcSinh[c^2*x^2])/(c^2*Sqrt[1 + 1/(c^
4*x^4)]*x*Sqrt[Sech[2*Log[c*x]]])
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6077
```
Int[Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Simp[Sech[d*(a
+ b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*d*p))   Int[1/(x^(b
*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; FreeQ[{a, b, d, p}, x] &&
  !IntegerQ[p]
```

rule 6079
```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

Maple [F]

$$\int \frac{1}{\sqrt{\operatorname{sech}\left(2\ln\left(xc\right)\right)}}dx$$

input
```
int(1/sech(2*ln(x*c))^(1/2),x)
```

output
```
int(1/sech(2*ln(x*c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(49) = 98.

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

$$\int \frac{1}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{\sqrt{2}cx\log\left(\frac{c^5x^5+2\,cx-2\,(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{cx^5}\right)+2\,\sqrt{2}(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{8\,c^2x}$$

input
```
integrate(1/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

output
```
1/8*(sqrt(2)*c*x*log((c^5*x^5 + 2*c*x - 2*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*
x^4 + 1)))/(c*x^5)) + 2*sqrt(2)*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1)))
/(c^2*x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{1}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(1/sech(2*ln(c*x))**(1/2),x)
```

output
```
Integral(1/sqrt(sech(2*log(c*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{1}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(1/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(sech(2*log(c*x))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \text{Timed out}$$

input
```
integrate(1/sech(2*log(c*x))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\cosh(2\,\ln(c\,x))}}}\,dx$$

input
```
int(1/(1/cosh(2*log(c*x)))^(1/2),x)
```

output
```
int(1/(1/cosh(2*log(c*x)))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{\operatorname{sech}(2\log(cx))}dx$$

input
```
int(1/sech(2*log(c*x))^(1/2),x)
```

output
```
int(sqrt(sech(2*log(c*x)))/sech(2*log(c*x)),x)
```

3.164 $\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x}\,dx = -i\sqrt{\cosh(2\log(cx))}\,\operatorname{EllipticF}(i\log(cx),2)\sqrt{\operatorname{sech}(2\log(cx))}$$

output
```
-I*cosh(2*ln(c*x))^(1/2)*InverseJacobiAM(I*ln(c*x),2^(1/2))*sech(2*ln(c*x)
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x}\,dx = -i\sqrt{\cosh(2\log(cx))}\,\operatorname{EllipticF}(i\log(cx),2)\sqrt{\operatorname{sech}(2\log(cx))}$$

input
```
Integrate[Sqrt[Sech[2*Log[c*x]]]/x,x]
```

output
```
(-I)*Sqrt[Cosh[2*Log[c*x]]]*EllipticF[I*Log[c*x], 2]*Sqrt[Sech[2*Log[c*x]]
]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3039, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x}\,dx$$

$$\Big\downarrow \; 3039$$

$$\int \sqrt{\operatorname{sech}(2\log(cx))}d\log(cx)$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{\csc\left(\frac{\pi}{2}+2i\log(cx)\right)}d\log(cx)$$

$$\Big\downarrow \; 4258$$

$$\sqrt{\operatorname{sech}(2\log(cx))}\sqrt{\cosh(2\log(cx))}\int \frac{1}{\sqrt{\cosh(2\log(cx))}}d\log(cx)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\operatorname{sech}(2\log(cx))}\sqrt{\cosh(2\log(cx))}\int \frac{1}{\sqrt{\sin\left(2i\log(cx)+\frac{\pi}{2}\right)}}d\log(cx)$$

$$\Big\downarrow \; 3120$$

$$-i\sqrt{\operatorname{sech}(2\log(cx))}\sqrt{\cosh(2\log(cx))}\operatorname{EllipticF}(i\log(cx),2)$$

input
```
Int[Sqrt[Sech[2*Log[c*x]]]/x,x]
```

output
```
(-I)*Sqrt[Cosh[2*Log[c*x]]]*EllipticF[I*Log[c*x], 2]*Sqrt[Sech[2*Log[c*x]]
]
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(32) = 64$.

Time = 0.32 (sec) , antiderivative size = 167, normalized size of antiderivative = 4.64

method	result	size
derivativedivides	$\frac{\sqrt{\left(2\left(\frac{xc}{2}+\frac{1}{2xc}\right)^{2}-1\right)\left(\frac{xc}{2}-\frac{1}{2xc}\right)^{2}}\,\sqrt{-\left(\frac{xc}{2}-\frac{1}{2xc}\right)^{2}}\,\sqrt{-2\left(\frac{xc}{2}+\frac{1}{2xc}\right)^{2}+1}\;\mathrm{EllipticF}\left(\frac{xc}{2}+\frac{1}{2xc},\sqrt{2}\right)}{\sqrt{2\left(\frac{xc}{2}-\frac{1}{2xc}\right)^{4}+\left(\frac{xc}{2}-\frac{1}{2xc}\right)^{2}}\,\left(\frac{xc}{2}-\frac{1}{2xc}\right)\sqrt{2\left(\frac{xc}{2}+\frac{1}{2xc}\right)^{2}-1}}$	167
default	$\frac{\sqrt{\left(2\left(\frac{xc}{2}+\frac{1}{2xc}\right)^{2}-1\right)\left(\frac{xc}{2}-\frac{1}{2xc}\right)^{2}}\,\sqrt{-\left(\frac{xc}{2}-\frac{1}{2xc}\right)^{2}}\,\sqrt{-2\left(\frac{xc}{2}+\frac{1}{2xc}\right)^{2}+1}\;\mathrm{EllipticF}\left(\frac{xc}{2}+\frac{1}{2xc},\sqrt{2}\right)}{\sqrt{2\left(\frac{xc}{2}-\frac{1}{2xc}\right)^{4}+\left(\frac{xc}{2}-\frac{1}{2xc}\right)^{2}}\,\left(\frac{xc}{2}-\frac{1}{2xc}\right)\sqrt{2\left(\frac{xc}{2}+\frac{1}{2xc}\right)^{2}-1}}$	167

input
```
int(sech(2*ln(x*c))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
((2*(1/2*x*c+1/2/x/c)^2-1)*(1/2*x*c-1/2/x/c)^2)^(1/2)*(-(1/2*x*c-1/2/x/c)^
2)^(1/2)*(-2*(1/2*x*c+1/2/x/c)^2+1)^(1/2)/(2*(1/2*x*c-1/2/x/c)^4+(1/2*x*c-
1/2/x/c)^2)^(1/2)*EllipticF(1/2*x*c+1/2/x/c,2^(1/2))/(1/2*x*c-1/2/x/c)/(2*
(1/2*x*c+1/2/x/c)^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x}\,dx = -\frac{\sqrt{2}(-c^4)^{\frac{3}{4}}\,F\left(\arcsin\left((-c^4)^{\frac{1}{4}}\,x\right)\,|-1\right)}{c^3}$$

input
```
integrate(sech(2*log(c*x))^(1/2)/x,x, algorithm="fricas")
```

output
```
-sqrt(2)*(-c^4)^(3/4)*elliptic_f(arcsin((-c^4)^(1/4)*x), -1)/c^3
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x}\,dx$$

input
```
integrate(sech(2*ln(c*x))**(1/2)/x,x)
```

output
```
Integral(sqrt(sech(2*log(c*x)))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}{x}\,dx$$

input
```
integrate(sech(2*log(c*x))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(sech(2*log(c*x)))/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x}\,dx = \text{Timed out}$$

input
```
integrate(sech(2*log(c*x))^(1/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x}\,dx = \int \frac{\sqrt{\frac{1}{\cosh(2\,\ln(c\,x))}}}{x}\,dx$$

input
```
int((1/cosh(2*log(c*x)))^(1/2)/x,x)
```

output
```
int((1/cosh(2*log(c*x)))^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x}dx$$

input
```
int(sech(2*log(c*x))^(1/2)/x,x)
```

output
```
int(sqrt(sech(2*log(c*x)))/x,x)
```

3.165 $\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^2}\,dx = -\frac{1}{2}c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\operatorname{csch}^{-1}\left(c^2x^2\right)\sqrt{\operatorname{sech}(2\log(cx))}$$

output
```
-1/2*c^2*(1+1/c^4/x^4)^(1/2)*x*arccsch(c^2*x^2)*sech(2*ln(c*x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^2}\,dx = -\frac{\sqrt{1+c^4x^4}\sqrt{\frac{c^2x^2}{2+2c^4x^4}}\operatorname{arctanh}\left(\sqrt{1+c^4x^4}\right)}{x}$$

input
```
Integrate[Sqrt[Sech[2*Log[c*x]]]/x^2,x]
```

output
```
-((Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*ArcTanh[Sqrt[1 + c^4*
x^4]])/x)
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6085, 6083, 858, 807, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\text{sech}(2\log(cx))}}{x^2}dx$$

$$\Big\downarrow \text{6085}$$

$$c\int \frac{\sqrt{\text{sech}(2\log(cx))}}{c^2x^2}d(cx)$$

$$\Big\downarrow \text{6083}$$

$$c^2x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\text{sech}(2\log(cx))}\int \frac{1}{c^3\sqrt{1+\frac{1}{c^4x^4}}x^3}d(cx)$$

$$\Big\downarrow \text{858}$$

$$-c^2x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\text{sech}(2\log(cx))}\int \frac{1}{cx\sqrt{c^4x^4+1}}d\frac{1}{cx}$$

$$\Big\downarrow \text{807}$$

$$-\frac{1}{2}c^2x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\text{sech}(2\log(cx))}\int \frac{1}{\sqrt{c^2x^2+1}}d(c^2x^2)$$

$$\Big\downarrow \text{222}$$

$$-\frac{1}{2}c^2x\sqrt{\frac{1}{c^4x^4}+1}\text{arcsinh}\left(c^2x^2\right)\sqrt{\text{sech}(2\log(cx))}$$

input
```
Int[Sqrt[Sech[2*Log[c*x]]]/x^2,x]
```

output
```
-1/2*(c^2*Sqrt[1 + 1/(c^4*x^4)]*x*ArcSinh[c^2*x^2]*Sqrt[Sech[2*Log[c*x]]])
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6083
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6085
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\ln(xc))}}{x^2}dx$$

input
```
int(sech(2*ln(x*c))^(1/2)/x^2,x)
```

output
```
int(sech(2*ln(x*c))^(1/2)/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^2}\,dx = \frac{1}{4}\sqrt{2}c\log\left(\frac{c^5x^5+2\,cx-2\,(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{cx^5}\right)$$

input
```
integrate(sech(2*log(c*x))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*c*log((c^5*x^5 + 2*c*x - 2*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4
 + 1)))/(c*x^5))
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^2}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x^2}\,dx$$

input
```
integrate(sech(2*ln(c*x))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(sech(2*log(c*x)))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^2}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}{x^2}\,dx$$

input
```
integrate(sech(2*log(c*x))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(sech(2*log(c*x)))/x^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate(sech(2*log(c*x))^(1/2)/x^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^2}\,dx = \int \frac{\sqrt{\frac{1}{\cosh(2\ln(c\,x))}}}{x^2}\,dx$$

input
```
int((1/cosh(2*log(c*x)))^(1/2)/x^2,x)
```

output
```
int((1/cosh(2*log(c*x)))^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^2}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x^2}dx$$

input
```
int(sech(2*log(c*x))^(1/2)/x^2,x)
```

output
```
int(sqrt(sech(2*log(c*x)))/x**2,x)
```

3.166 $\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 137

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^3}\,dx = -\frac{\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}{c^2+\frac{1}{x^2}} + c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)xE\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)\sqrt{\operatorname{sech}(2\log(cx))} - \frac{1}{2}c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)x\,\text{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)\sqrt{\operatorname{sech}(2\log(cx))}$$

output
```
-(c^4+1/x^4)*sech(2*ln(c*x))^(1/2)/(c^2+1/x^2)+c*((c^4+1/x^4)/(c^2+1/x^2)^
2)^(1/2)*(c^2+1/x^2)*x*EllipticE(sin(2*arccot(c*x)),1/2*2^(1/2))*sech(2*ln
(c*x))^(1/2)-1/2*c*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)*(c^2+1/x^2)*x*Inverse
JacobiAM(2*arccot(c*x),1/2*2^(1/2))*sech(2*ln(c*x))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^3}\,dx = -\frac{c^2 \operatorname{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-c^4x^4\right)}{\sqrt{1+c^4x^4}\sqrt{\frac{c^2x^2}{2+2c^4x^4}}}$$

input
```
Integrate[Sqrt[Sech[2*Log[c*x]]]/x^3,x]
```

output
```
-((c^2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c^4*x^4)])/(Sqrt[1 + c^4*x^4]*S
qrt[(c^2*x^2)/(2 + 2*c^4*x^4)]))
```

Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.39, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6085, 6083, 858, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^3}dx$$

$$\Big\downarrow \text{6085}$$

$$c^2\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{c^3x^3}d(cx)$$

$$\Big\downarrow \text{6083}$$

$$c^3x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}\int \frac{1}{c^4\sqrt{1+\frac{1}{c^4x^4}}x^4}d(cx)$$

$$\Big\downarrow \text{858}$$

$$-c^3x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}\int \frac{c^2x^2}{\sqrt{c^4x^4+1}}d\frac{1}{cx}$$

$$\Big\downarrow \text{834}$$

$$-c^3x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\text{sech}(2\log(cx))}\left(\int \frac{1}{\sqrt{c^4x^4+1}}d\frac{1}{cx}-\int \frac{1-c^2x^2}{\sqrt{c^4x^4+1}}d\frac{1}{cx}\right)$$

$$\Big\downarrow \text{761}$$

$$-c^3x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\text{sech}(2\log(cx))}\left(\frac{(c^2x^2+1)\sqrt{\frac{c^4x^4+1}{(c^2x^2+1)^2}}\text{EllipticF}\left(2\arctan\left(\frac{1}{cx}\right),\frac{1}{2}\right)}{2\sqrt{c^4x^4+1}}-\int \frac{1-c^2x^2}{\sqrt{c^4x^4+1}}d\frac{1}{cx}\right)$$

$$\Big\downarrow \text{1510}$$

$$-c^3x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\text{sech}(2\log(cx))}\left(\frac{(c^2x^2+1)\sqrt{\frac{c^4x^4+1}{(c^2x^2+1)^2}}\text{EllipticF}\left(2\arctan\left(\frac{1}{cx}\right),\frac{1}{2}\right)}{2\sqrt{c^4x^4+1}}-\frac{(c^2x^2+1)\sqrt{\frac{c^4x^4+1}{(c^2x^2+1)^2}}}{\sqrt{c^4x^4}}\right.$$

input
```
Int[Sqrt[Sech[2*Log[c*x]]]/x^3,x]
```

output
```
-(c^3*Sqrt[1 + 1/(c^4*x^4)]*x*(Sqrt[1 + c^4*x^4]/(c*x*(1 + c^2*x^2)) - ((1
 + c^2*x^2)*Sqrt[(1 + c^4*x^4)/(1 + c^2*x^2)^2]*EllipticE[2*ArcTan[1/(c*x)
], 1/2])/Sqrt[1 + c^4*x^4] + ((1 + c^2*x^2)*Sqrt[(1 + c^4*x^4)/(1 + c^2*x^
2)^2]*EllipticF[2*ArcTan[1/(c*x)], 1/2])/(2*Sqrt[1 + c^4*x^4]))*Sqrt[Sech[
2*Log[c*x]]])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 6083
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6085
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.98

method	result	size
risch	$-\frac{\left(c^{4} x^{4}+1\right) \sqrt{2}\, \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}{x^{2}}+\frac{i c^{2} \sqrt{-i c^{2} x^{2}+1}\, \sqrt{i c^{2} x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \sqrt{i c^{2}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{i c^{2}}, i\right)\right) \sqrt{2}\, \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}{\sqrt{i c^{2}}\, x}$	134

input
```
int(sech(2*ln(x*c))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-(c^4*x^4+1)/x^2*2^(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)+I*c^2/(I*c^2)^(1/2)*(
1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)*(EllipticF(x*(I*c^2)^(1/2),I)-Ellip
ticE(x*(I*c^2)^(1/2),I))*2^(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^3}\,dx = -\frac{\sqrt{2}(-c^4)^{\frac{3}{4}}\,cx^2E\left(\arcsin\left((-c^4)^{\frac{1}{4}}\,x\right)\,|-1\right)-\sqrt{2}(-c^4)^{\frac{3}{4}}\,cx^2F\left(\arcsin\left((-c^4)^{\frac{1}{4}}\,x\right)\,|-1\right)+\sqrt{2}(c^4x^4+1)}{x^2}$$

input
```
integrate(sech(2*log(c*x))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
-(sqrt(2)*(-c^4)^(3/4)*c*x^2*elliptic_e(arcsin((-c^4)^(1/4)*x), -1) - sqrt
(2)*(-c^4)^(3/4)*c*x^2*elliptic_f(arcsin((-c^4)^(1/4)*x), -1) + sqrt(2)*(c
^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1)))/x^2
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^3}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x^3}\,dx$$

input
```
integrate(sech(2*ln(c*x))**(1/2)/x**3,x)
```

output
```
Integral(sqrt(sech(2*log(c*x)))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^3}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}{x^3}\,dx$$

input
```
integrate(sech(2*log(c*x))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(sech(2*log(c*x)))/x^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(sech(2*log(c*x))^(1/2)/x^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^3}\,dx = \int \frac{\sqrt{\frac{1}{\cosh(2\,\ln(c\,x))}}}{x^3}\,dx$$

input
```
int((1/cosh(2*log(c*x)))^(1/2)/x^3,x)
```

output
```
int((1/cosh(2*log(c*x)))^(1/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^3}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x^3}dx$$

input
```
int(sech(2*log(c*x))^(1/2)/x^3,x)
```

output
```
int(sqrt(sech(2*log(c*x)))/x**3,x)
```

3.167 $\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^4}\,dx = -\frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x\sqrt{\operatorname{sech}(2\log(cx))}$$

output
```
-1/2*(c^4+1/x^4)*x*sech(2*ln(c*x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^4}\,dx = -\frac{c^2}{2x\sqrt{\frac{c^2x^2}{2+2c^4x^4}}}$$

input
```
Integrate[Sqrt[Sech[2*Log[c*x]]]/x^4,x]
```

output
```
-1/2*c^2/(x*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6085, 6083, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\text{sech}(2\log(cx))}}{x^4}\,dx$$
$$\Big\downarrow \text{ 6085}$$
$$c^3\int \frac{\sqrt{\text{sech}(2\log(cx))}}{c^4x^4}d(cx)$$
$$\Big\downarrow \text{ 6083}$$
$$c^4x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\text{sech}(2\log(cx))}\int \frac{1}{c^5\sqrt{1+\frac{1}{c^4x^4}}x^5}d(cx)$$
$$\Big\downarrow \text{ 793}$$
$$-\frac{1}{2}c^4x\left(\frac{1}{c^4x^4}+1\right)\sqrt{\text{sech}(2\log(cx))}$$

input
```
Int[Sqrt[Sech[2*Log[c*x]]]/x^4,x]
```

output
```
-1/2*(c^4*(1 + 1/(c^4*x^4))*x*Sqrt[Sech[2*Log[c*x]]])
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 6083
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6085
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

method	result	size
risch	$-\frac{\left(c^{4} x^{4}+1\right) \sqrt{2}\, \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}{2 x^{3}}$	38

input
```
int(sech(2*ln(x*c))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c^4*x^4+1)/x^3*2^(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^4}\,dx = -\frac{\sqrt{2}(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{2\,x^3}$$

input
```
integrate(sech(2*log(c*x))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1))/x^3
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^4}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x^4}\,dx$$

input
```
integrate(sech(2*ln(c*x))**(1/2)/x**4,x)
```

output
```
Integral(sqrt(sech(2*log(c*x)))/x**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^4}\,dx = -\frac{1}{2}\,c^3\left(\frac{\sqrt{2}}{\sqrt{\frac{1}{c^4x^4}+1}} + \frac{\sqrt{2}}{c^4x^4\sqrt{\frac{1}{c^4x^4}+1}}\right)$$

input
```
integrate(sech(2*log(c*x))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
-1/2*c^3*(sqrt(2)/sqrt(1/(c^4*x^4) + 1) + sqrt(2)/(c^4*x^4*sqrt(1/(c^4*x^4
) + 1)))
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(sech(2*log(c*x))^(1/2)/x^4,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{\sqrt{\mathrm{sech}(2\log(cx))}}{x^4}\,dx = -\frac{\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4+1}}}{2\,x^3} - \frac{c^4\,x\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4+1}}}{2}$$

input
```
int((1/cosh(2*log(c*x)))^(1/2)/x^4,x)
```

output
```
- ((2*c^2*x^2)/(c^4*x^4 + 1))^(1/2)/(2*x^3) - (c^4*x*((2*c^2*x^2)/(c^4*x^4
 + 1))^(1/2))/2
```

Reduce [F]

$$\int \frac{\sqrt{\mathrm{sech}(2\log(cx))}}{x^4}\,dx = \int \frac{\sqrt{\mathrm{sech}\,(2\log\,(cx))}}{x^4}dx$$

input
```
int(sech(2*log(c*x))^(1/2)/x^4,x)
```

output
```
int(sqrt(sech(2*log(c*x)))/x**4,x)
```

3.168 $\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx = -\frac{1}{3}\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}+\frac{1}{6}c^3\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)x\,\operatorname{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)\sqrt{\operatorname{sech}(2\log(cx))}$$

output
```
-1/3*(c^4+1/x^4)*sech(2*ln(c*x))^(1/2)+1/6*c^3*((c^4+1/x^4)/(c^2+1/x^2)^2)
^(1/2)*(c^2+1/x^2)*x*InverseJacobiAM(2*arccot(c*x),1/2*2^(1/2))*sech(2*ln(
c*x))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx = -\frac{\sqrt{2}\sqrt{\frac{c^2x^2}{1+c^4x^4}}\sqrt{1+c^4x^4}\,\operatorname{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-c^4x^4\right)}{3x^4}$$

input
```
Integrate[Sqrt[Sech[2*Log[c*x]]]/x^5,x]
```

output
```
-1/3*(Sqrt[2]*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4]*Hypergeometr
ic2F1[-3/4, 1/2, 1/4, -(c^4*x^4)])/x^4
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.49, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6085, 6083, 858, 843, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}dx$$

$$\Big\downarrow \text{6085}$$

$$c^4\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{c^5x^5}d(cx)$$

$$\Big\downarrow \text{6083}$$

$$c^5x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}\int \frac{1}{c^6\sqrt{1+\frac{1}{c^4x^4}}x^6}d(cx)$$

$$\Big\downarrow \text{858}$$

$$-c^5x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}\int \frac{c^4x^4}{\sqrt{c^4x^4+1}}d\frac{1}{cx}$$

$$\Big\downarrow \text{843}$$

$$-c^5x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}\left(\frac{\sqrt{c^4x^4+1}}{3cx}-\frac{1}{3}\int \frac{1}{\sqrt{c^4x^4+1}}d\frac{1}{cx}\right)$$

$$\Big\downarrow \text{761}$$

$$-c^5x\sqrt{\frac{1}{c^4x^4}+1}\left(\frac{\sqrt{c^4x^4+1}}{3cx}-\frac{(c^2x^2+1)\sqrt{\frac{c^4x^4+1}{(c^2x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{1}{cx}\right),\frac{1}{2}\right)}{6\sqrt{c^4x^4+1}}\right)\sqrt{\operatorname{sech}(2\log(cx))}$$

input
```
Int[Sqrt[Sech[2*Log[c*x]]]/x^5,x]
```

output
```
-(c^5*Sqrt[1 + 1/(c^4*x^4)]*x*(Sqrt[1 + c^4*x^4]/(3*c*x) - ((1 + c^2*x^2)*
Sqrt[(1 + c^4*x^4)/(1 + c^2*x^2)^2]*EllipticF[2*ArcTan[1/(c*x)], 1/2])/(6*
Sqrt[1 + c^4*x^4]))*Sqrt[Sech[2*Log[c*x]]])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6083
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6085
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.46

method	result	size
risch	$-\frac{(c^4x^4+1)\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{3x^4}-\frac{c^4\sqrt{-ic^2x^2+1}\,\sqrt{ic^2x^2+1}\,\text{EllipticF}\left(x\sqrt{ic^2},i\right)\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{3\sqrt{ic^2}\,x}$	117

input
```
int(sech(2*ln(x*c))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(c^4*x^4+1)/x^4*2^(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)-1/3*c^4/(I*c^2)^(
1/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)*EllipticF(x*(I*c^2)^(1/2),I)*
2^(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx = \frac{\sqrt{2}(-c^4)^{\frac{3}{4}}\,cx^4F(\arcsin\left(\left(-c^4\right)^{\frac{1}{4}}x\right)\,|-1)-\sqrt{2}(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{3\,x^4}$$

input
```
integrate(sech(2*log(c*x))^(1/2)/x^5,x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*(-c^4)^(3/4)*c*x^4*elliptic_f(arcsin((-c^4)^(1/4)*x), -1) - s
qrt(2)*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1)))/x^4
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx = \int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx$$

input
```
integrate(sech(2*ln(c*x))**(1/2)/x**5,x)
```

output
```
Integral(sqrt(sech(2*log(c*x)))/x**5, x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx = \int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx$$

input
```
integrate(sech(2*log(c*x))^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(sech(2*log(c*x)))/x^5, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate(sech(2*log(c*x))^(1/2)/x^5,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx = \int \frac{\sqrt{\frac{1}{\cosh(2\,\ln(c\,x))}}}{x^5}\,dx$$

input
```
int((1/cosh(2*log(c*x)))^(1/2)/x^5,x)
```

output
```
int((1/cosh(2*log(c*x)))^(1/2)/x^5, x)
```

Reduce [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x^5}dx$$

input
```
int(sech(2*log(c*x))^(1/2)/x^5,x)
```

output
```
int(sqrt(sech(2*log(c*x)))/x**5,x)
```

3.169 $\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 122

$$\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{x}{32c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^9}{12\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^4x^4}}\right)}{32c^{12}\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

output
```
1/32*x/c^4/(c^4+1/x^4)/sech(2*ln(c*x))^(3/2)+1/16*x^5/(c^4+1/x^4)/sech(2*l
n(c*x))^(3/2)+1/12*x^9/sech(2*ln(c*x))^(3/2)-1/32*arctanh((1+1/c^4/x^4)^(1
/2))/c^12/(1+1/c^4/x^4)^(3/2)/x^3/sech(2*ln(c*x))^(3/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{c^3x^3\sqrt{1+c^4x^4}(3+14c^4x^4+8c^8x^8)-3cx\operatorname{arcsinh}(c^2x^2)}{192\sqrt{2}c^9\sqrt{\frac{c^2x^2}{1+c^4x^4}}\sqrt{1+c^4x^4}}$$

input
```
Integrate[x^8/Sech[2*Log[c*x]]^(3/2),x]
```

output
```
(c^3*x^3*Sqrt[1 + c^4*x^4]*(3 + 14*c^4*x^4 + 8*c^8*x^8) - 3*c*x*ArcSinh[c^
2*x^2])/(192*Sqrt[2]*c^9*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {6085, 6083, 798, 51, 51, 52, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}dx$$

$$\Big\downarrow \text{6085}$$

$$\frac{\int \frac{c^8x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}d(cx)}{c^9}$$

$$\Big\downarrow \text{6083}$$

$$\frac{\int c^{11}\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^{11}d(cx)}{c^{12}x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{798}$$

$$-\frac{\int \frac{\left(1+\frac{1}{c^4x^4}\right)^{3/2}}{c^4x^4}d\frac{1}{c^4x^4}}{4c^{12}x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{51}$$

$$-\frac{\frac{1}{2}\int \frac{\sqrt{1+\frac{1}{c^4x^4}}}{c^3x^3}d\frac{1}{c^4x^4}-\frac{\left(\frac{1}{c^4x^4}+1\right)^{3/2}}{3c^3x^3}}{4c^{12}x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{51}$$

$$-\frac{\frac{1}{2}\left(\frac{1}{4}\int \frac{1}{c^2\sqrt{1+\frac{1}{c^4x^4}}x^2}d\frac{1}{c^4x^4}-\frac{\sqrt{\frac{1}{c^4x^4}+1}}{2c^2x^2}\right)-\frac{\left(\frac{1}{c^4x^4}+1\right)^{3/2}}{3c^3x^3}}{4c^{12}x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{52}$$

$$-\frac{\frac{1}{2}\left(\frac{1}{4}\left(-\frac{1}{2}\int \frac{1}{c\sqrt{1+\frac{1}{c^4x^4}}x}d\frac{1}{c^4x^4}-\frac{\sqrt{\frac{1}{c^4x^4}+1}}{cx}\right)-\frac{\sqrt{\frac{1}{c^4x^4}+1}}{2c^2x^2}\right)-\frac{\left(\frac{1}{c^4x^4}+1\right)^{3/2}}{3c^3x^3}}{4c^{12}x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{1}{2}\left(\frac{1}{4}\left(-\int \frac{1}{c^2x^2-1}d\sqrt{1+\frac{1}{c^4x^4}}-\frac{\sqrt{\frac{1}{c^4x^4}+1}}{cx}\right)-\frac{\sqrt{\frac{1}{c^4x^4}+1}}{2c^2x^2}\right)-\frac{\left(\frac{1}{c^4x^4}+1\right)^{3/2}}{3c^3x^3}}{4c^{12}x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{220}$$

$$-\frac{\frac{1}{2}\left(\frac{1}{4}\left(\operatorname{arctanh}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)-\frac{\sqrt{\frac{1}{c^4x^4}+1}}{cx}\right)-\frac{\sqrt{\frac{1}{c^4x^4}+1}}{2c^2x^2}\right)-\frac{\left(\frac{1}{c^4x^4}+1\right)^{3/2}}{3c^3x^3}}{4c^{12}x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

input
```
Int[x^8/Sech[2*Log[c*x]]^(3/2),x]
```

output
```
-1/4*(-1/3*(1 + 1/(c^4*x^4))^(3/2)/(c^3*x^3) + (-1/2*Sqrt[1 + 1/(c^4*x^4)]
/(c^2*x^2) + (-(Sqrt[1 + 1/(c^4*x^4)]/(c*x)) + ArcTanh[Sqrt[1 + 1/(c^4*x^4
)]])/4)/2)/(c^12*(1 + 1/(c^4*x^4))^(3/2)*x^3*Sech[2*Log[c*x]]^(3/2))
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6083
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6085
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.99

method	result	size
risch	$\frac{x^{3}\left(8 c^{8} x^{8}+14 c^{4} x^{4}+3\right) \sqrt{2}}{384 c^{6} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}-\frac{\ln\left(\frac{c^{4} x^{2}}{\sqrt{c^{4}}}+\sqrt{c^{4} x^{4}+1}\right) \sqrt{2}\, x}{128 c^{6} \sqrt{c^{4}}\, \sqrt{c^{4} x^{4}+1}\, \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}$	121

input
```
int(x^8/sech(2*ln(x*c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/384*x^3*(8*c^8*x^8+14*c^4*x^4+3)/c^6*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)
-1/128/c^6*ln(c^4*x^2/(c^4)^(1/2)+(c^4*x^4+1)^(1/2))/(c^4)^(1/2)*2^(1/2)*x
/(c^4*x^4+1)^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.89

$$\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{2\sqrt{2}(8\,c^{13}x^{13} + 22\,c^9x^9 + 17\,c^5x^5 + 3\,cx)\sqrt{\frac{c^2x^2}{c^4x^4+1}} + 3\sqrt{2}\log\left(-2\,c^4x^4 + 2\,(c^5x^5 + cx)\sqrt{\frac{c^2x^2}{c^4x^4+1}} - 1\right)}{768\,c^9}$$

input
```
integrate(x^8/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

output
```
1/768*(2*sqrt(2)*(8*c^13*x^13 + 22*c^9*x^9 + 17*c^5*x^5 + 3*c*x)*sqrt(c^2*
x^2/(c^4*x^4 + 1)) + 3*sqrt(2)*log(-2*c^4*x^4 + 2*(c^5*x^5 + c*x)*sqrt(c^2
*x^2/(c^4*x^4 + 1)) - 1))/c^9
```

Sympy [F]

$$\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$$

input
```
integrate(x**8/sech(2*ln(c*x))**(3/2),x)
```

output
```
Integral(x**8/sech(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^8}{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^8/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^8/sech(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

input
```
integrate(x^8/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^8}{{\left(\frac{1}{\cosh\left(2\,\ln\left(c\,x\right)\right)}\right)}^{3/2}}\,dx$$

input
```
int(x^8/(1/cosh(2*log(c*x)))^(3/2),x)
```

output
```
int(x^8/(1/cosh(2*log(c*x)))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}\,x^8}{\operatorname{sech}\left(2\log\left(cx\right)\right)^2}dx$$

input
```
int(x^8/sech(2*log(c*x))^(3/2),x)
```

output
```
int((sqrt(sech(2*log(c*x)))*x**8)/sech(2*log(c*x))**2,x)
```

3.170 $\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 141

$$\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{4}{77c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{6x^4}{77\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\operatorname{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{77c^5\left(c^4+\frac{1}{x^4}\right)^2x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

output
```
4/77/c^4/(c^4+1/x^4)/sech(2*ln(c*x))^(3/2)+6/77*x^4/(c^4+1/x^4)/sech(2*ln(
c*x))^(3/2)+1/11*x^8/sech(2*ln(c*x))^(3/2)+2/77*((c^4+1/x^4)/(c^2+1/x^2)^2
)^(1/2)*(c^2+1/x^2)*InverseJacobiAM(2*arccot(c*x),1/2*2^(1/2))/c^5/(c^4+1/
x^4)^2/x^3/sech(2*ln(c*x))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.55

$$\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\sqrt{1+c^4x^4}\sqrt{\frac{c^2x^2}{2+2c^4x^4}}\left((1+c^4x^4)^{5/2} - \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-c^4x^4\right)\right)}{22c^8}$$

input
```
Integrate[x^7/Sech[2*Log[c*x]]^(3/2),x]
```

output
```
(Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*((1 + c^4*x^4)^(5/2) -
Hypergeometric2F1[-3/2, 1/4, 5/4, -(c^4*x^4)]))/(22*c^8)
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {6085, 6083, 858, 809, 809, 847, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$$

$$\Big\downarrow \ 6085$$

$$\frac{\int \frac{c^7x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}d(cx)}{c^8}$$

$$\Big\downarrow \ 6083$$

$$\frac{\int c^{10}\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^{10}d(cx)}{c^{11}x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \ 858$$

$$-\frac{\int \frac{\left(c^4x^4+1\right)^{3/2}}{c^{12}x^{12}}d\frac{1}{cx}}{c^{11}x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{ 809}$$

$$-\frac{\frac{6}{11}\int \frac{\sqrt{c^4x^4+1}}{c^8x^8}d\frac{1}{cx}-\frac{\left(c^4x^4+1\right)^{3/2}}{11c^{11}x^{11}}}{c^{11}x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{ 809}$$

$$-\frac{\frac{6}{11}\left(\frac{2}{7}\int \frac{1}{c^4x^4\sqrt{c^4x^4+1}}d\frac{1}{cx}-\frac{\sqrt{c^4x^4+1}}{7c^7x^7}\right)-\frac{\left(c^4x^4+1\right)^{3/2}}{11c^{11}x^{11}}}{c^{11}x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{\frac{6}{11}\left(\frac{2}{7}\left(-\frac{1}{3}\int \frac{1}{\sqrt{c^4x^4+1}}d\frac{1}{cx}-\frac{\sqrt{c^4x^4+1}}{3c^3x^3}\right)-\frac{\sqrt{c^4x^4+1}}{7c^7x^7}\right)-\frac{\left(c^4x^4+1\right)^{3/2}}{11c^{11}x^{11}}}{c^{11}x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{ 761}$$

$$-\frac{\frac{6}{11}\left(\frac{2}{7}\left(-\frac{\left(c^2x^2+1\right)\sqrt{\frac{c^4x^4+1}{\left(c^2x^2+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{1}{cx}\right),\frac{1}{2}\right)}{6\sqrt{c^4x^4+1}}-\frac{\sqrt{c^4x^4+1}}{3c^3x^3}\right)-\frac{\sqrt{c^4x^4+1}}{7c^7x^7}\right)-\frac{\left(c^4x^4+1\right)^{3/2}}{11c^{11}x^{11}}}{c^{11}x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

input
```
Int[x^7/Sech[2*Log[c*x]]^(3/2),x]
```

output
```
-((-1/11*(1 + c^4*x^4)^(3/2)/(c^11*x^11) + (6*(-1/7*Sqrt[1 + c^4*x^4]/(c^7
*x^7) + (2*(-1/3*Sqrt[1 + c^4*x^4]/(c^3*x^3) - ((1 + c^2*x^2)*Sqrt[(1 + c^
4*x^4)/(1 + c^2*x^2)^2]*EllipticF[2*ArcTan[1/(c*x)], 1/2])/(6*Sqrt[1 + c^4
*x^4])))/7))/11)/(c^11*(1 + 1/(c^4*x^4))^(3/2)*x^3*Sech[2*Log[c*x]]^(3/2))
)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6083
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6085
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.98

method	result	size
risch	$\frac{x^{2}\left(7 c^{8} x^{8}+13 c^{4} x^{4}+4\right) \sqrt{2}}{308 c^{6} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}-\frac{\sqrt{-i c^{2} x^{2}+1}\, \sqrt{i c^{2} x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{i c^{2}}, i\right) \sqrt{2}\, x}{77 c^{6} \sqrt{i c^{2}}\, \left(c^{4} x^{4}+1\right) \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}$	138

input
```
int(x^7/sech(2*ln(x*c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/308*x^2*(7*c^8*x^8+13*c^4*x^4+4)/c^6*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)
-1/77/c^6/(I*c^2)^(1/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1
)*EllipticF(x*(I*c^2)^(1/2),I)*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.63

$$\int \frac{x^{7}}{\operatorname{sech}^{\frac{3}{2}}(2 \log(cx))}\, dx = -\frac{8 \sqrt{\frac{1}{2}}\sqrt{c^{4}}\, c\left(-\frac{1}{c^{4}}\right)^{\frac{3}{4}} F(\arcsin\left(\frac{\left(-\frac{1}{c^{4}}\right)^{\frac{1}{4}}}{x}\right) \mid -1) - \sqrt{2}(7\, c^{12} x^{12} + 20\, c^{8} x^{8} + 17\, c^{4} x^{4} + 4)\sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}{308\, c^{8}}$$

input
```
integrate(x^7/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

output
```
-1/308*(8*sqrt(1/2)*sqrt(c^4)*c*(-1/c^4)^(3/4)*elliptic_f(arcsin((-1/c^4)^
(1/4)/x), -1) - sqrt(2)*(7*c^12*x^12 + 20*c^8*x^8 + 17*c^4*x^4 + 4)*sqrt(c
^2*x^2/(c^4*x^4 + 1)))/c^8
```

Sympy [F]

$$\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$$

input
```
integrate(x**7/sech(2*ln(c*x))**(3/2),x)
```

output
```
Integral(x**7/sech(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^7}{\operatorname{sech}(2\log(cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^7/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^7/sech(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

input
```
integrate(x^7/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^7}{\left(\frac{1}{\cosh(2\ln(c\,x))}\right)^{3/2}}\,dx$$

input
```
int(x^7/(1/cosh(2*log(c*x)))^(3/2),x)
```

output
```
int(x^7/(1/cosh(2*log(c*x)))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{\sqrt{\operatorname{sech}(2\log(cx))}\,x^7}{\operatorname{sech}(2\log(cx))^2}dx$$

input
```
int(x^7/sech(2*log(c*x))^(3/2),x)
```

output
```
int((sqrt(sech(2*log(c*x)))*x**7)/sech(2*log(c*x))**2,x)
```

3.171 $\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\left(c^4+\frac{1}{x^4}\right)x^7}{10c^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

output
```
1/10*(c^4+1/x^4)*x^7/c^4/sech(2*ln(c*x))^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.57

$$\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{(1+c^4x^4)^3\sqrt{\frac{c^2x^2}{2+2c^4x^4}}}{20c^8x}$$

input
```
Integrate[x^6/Sech[2*Log[c*x]]^(3/2),x]
```

output
```
((1 + c^4*x^4)^3*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)])/(20*c^8*x)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6085, 6083, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}dx$$

$$\Big\downarrow \text{ 6085}$$

$$\frac{\int \frac{c^6x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}d(cx)}{c^7}$$

$$\Big\downarrow \text{ 6083}$$

$$\frac{\int c^9\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^9d(cx)}{c^{10}x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{x^7\left(\frac{1}{c^4x^4}+1\right)}{10\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

input
```
Int[x^6/Sech[2*Log[c*x]]^(3/2),x]
```

output
```
((1 + 1/(c^4*x^4))*x^7)/(10*Sech[2*Log[c*x]]^(3/2))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 6083
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6085
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

method	result	size
risch	$\frac{\sqrt{2}\, x \left(c^{8} x^{8}+2 c^{4} x^{4}+1\right)}{40 c^{6} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}$	47

input
```
int(x^6/sech(2*ln(x*c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/40*2^(1/2)/c^6*x/(c^2*x^2/(c^4*x^4+1))^(1/2)*(c^8*x^8+2*c^4*x^4+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(24) = 48.

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

$$\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\sqrt{2}(c^{12}x^{12} + 3\,c^8x^8 + 3\,c^4x^4 + 1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{40\,c^8x}$$

input
```
integrate(x^6/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

output
```
1/40*sqrt(2)*(c^12*x^12 + 3*c^8*x^8 + 3*c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4
 + 1))/(c^8*x)
```

Sympy [F]

$$\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

input
```
integrate(x**6/sech(2*ln(c*x))**(3/2),x)
```

output
```
Integral(x**6/sech(2*log(c*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\left(\sqrt{2}c^4x^4 + \sqrt{2}\right)\left(c^4x^4 + 1\right)^{\frac{3}{2}}}{40\,c^7}$$

input
```
integrate(x^6/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

output
```
1/40*(sqrt(2)*c^4*x^4 + sqrt(2))*(c^4*x^4 + 1)^(3/2)/c^7
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

input
```
integrate(x^6/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.50

$$\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{(c^4x^4+1)^3\sqrt{\frac{2c^2x^2}{c^4x^4+1}}}{40\,c^8\,x}$$

input
```
int(x^6/(1/cosh(2*log(c*x)))^(3/2),x)
```

output
```
((c^4*x^4 + 1)^3*((2*c^2*x^2)/(c^4*x^4 + 1))^(1/2))/(40*c^8*x)
```

Reduce [F]

$$\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{\sqrt{\operatorname{sech}(2\log(cx))}\,x^6}{\operatorname{sech}(2\log(cx))^2}dx$$

input
```
int(x^6/sech(2*log(c*x))^(3/2),x)
```

output
```
int((sqrt(sech(2*log(c*x)))*x**6)/sech(2*log(c*x))**2,x)
```

3.172 $\int \frac{x^5}{\text{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 251

$$\begin{aligned}\int \frac{x^5}{\text{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = & -\frac{4}{15c^4\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)x^4\text{sech}^{\frac{3}{2}}(2\log(cx))} \\ & + \frac{4}{15c^4\left(c^4+\frac{1}{x^4}\right)x^2\text{sech}^{\frac{3}{2}}(2\log(cx))} \\ & + \frac{2x^2}{15\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\text{sech}^{\frac{3}{2}}(2\log(cx))} \\ & + \frac{4\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)E\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{15c^3\left(c^4+\frac{1}{x^4}\right)^2x^3\text{sech}^{\frac{3}{2}}(2\log(cx))} \\ & - \frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{15c^3\left(c^4+\frac{1}{x^4}\right)^2x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}\end{aligned}$$

output
```
-4/15/c^4/(c^4+1/x^4)/(c^2+1/x^2)/x^4/sech(2*ln(c*x))^(3/2)+4/15/c^4/(c^4+
1/x^4)/x^2/sech(2*ln(c*x))^(3/2)+2/15*x^2/(c^4+1/x^4)/sech(2*ln(c*x))^(3/2
)+1/9*x^6/sech(2*ln(c*x))^(3/2)+4/15*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)*(c^
2+1/x^2)*EllipticE(sin(2*arccot(c*x)),1/2*2^(1/2))/c^3/(c^4+1/x^4)^2/x^3/s
ech(2*ln(c*x))^(3/2)-2/15*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)*(c^2+1/x^2)*In
verseJacobiAM(2*arccot(c*x),1/2*2^(1/2))/c^3/(c^4+1/x^4)^2/x^3/sech(2*ln(c
*x))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.26

$$\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\left(\frac{c^2x^2}{1+c^4x^4}\right)^{3/2}(1+c^4x^4)^{3/2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-c^4x^4\right)}{6\sqrt{2}c^6}$$

input
```
Integrate[x^5/Sech[2*Log[c*x]]^(3/2),x]
```

output
```
(((c^2*x^2)/(1 + c^4*x^4))^(3/2)*(1 + c^4*x^4)^(3/2)*Hypergeometric2F1[-3/
2, 3/4, 7/4, -(c^4*x^4)])/(6*Sqrt[2]*c^6)
```

Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6085, 6083, 858, 809, 809, 847, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$$

$$\Big\downarrow \text{ 6085}$$

$$\frac{\int \frac{c^5x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}d(cx)}{c^6}$$

$\Big\downarrow$ 6083

$$\frac{\int c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^8d(cx)}{c^9x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$\Big\downarrow$ 858

$$-\frac{\int \frac{\left(c^4x^4+1\right)^{3/2}}{c^{10}x^{10}}d\frac{1}{cx}}{c^9x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$\Big\downarrow$ 809

$$-\frac{\frac{2}{3}\int \frac{\sqrt{c^4x^4+1}}{c^6x^6}d\frac{1}{cx}-\frac{\left(c^4x^4+1\right)^{3/2}}{9c^9x^9}}{c^9x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$\Big\downarrow$ 809

$$-\frac{\frac{2}{3}\left(\frac{2}{5}\int \frac{1}{c^2x^2\sqrt{c^4x^4+1}}d\frac{1}{cx}-\frac{\sqrt{c^4x^4+1}}{5c^5x^5}\right)-\frac{\left(c^4x^4+1\right)^{3/2}}{9c^9x^9}}{c^9x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$\Big\downarrow$ 847

$$-\frac{\frac{2}{3}\left(\frac{2}{5}\left(\int \frac{c^2x^2}{\sqrt{c^4x^4+1}}d\frac{1}{cx}-\frac{\sqrt{c^4x^4+1}}{cx}\right)-\frac{\sqrt{c^4x^4+1}}{5c^5x^5}\right)-\frac{\left(c^4x^4+1\right)^{3/2}}{9c^9x^9}}{c^9x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$\Big\downarrow$ 834

$$-\frac{\frac{2}{3}\left(\frac{2}{5}\left(\int \frac{1}{\sqrt{c^4x^4+1}}d\frac{1}{cx}-\int \frac{1-c^2x^2}{\sqrt{c^4x^4+1}}d\frac{1}{cx}-\frac{\sqrt{c^4x^4+1}}{cx}\right)-\frac{\sqrt{c^4x^4+1}}{5c^5x^5}\right)-\frac{\left(c^4x^4+1\right)^{3/2}}{9c^9x^9}}{c^9x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$\Big\downarrow$ 761

$$-\frac{\frac{2}{3}\left(\frac{2}{5}\left(-\int \frac{1-c^2x^2}{\sqrt{c^4x^4+1}}d\frac{1}{cx}+\frac{\left(c^2x^2+1\right)\sqrt{\frac{c^4x^4+1}{\left(c^2x^2+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{1}{cx}\right),\frac{1}{2}\right)}{2\sqrt{c^4x^4+1}}-\frac{\sqrt{c^4x^4+1}}{cx}\right)-\frac{\sqrt{c^4x^4+1}}{5c^5x^5}\right)-\frac{\left(c^4x^4+1\right)^{3/2}}{9c^9x^9}}{c^9x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$\Big\downarrow$ 1510

$$-\frac{\frac{2}{3}\left(\frac{2}{5}\left(\frac{(c^2x^2+1)\sqrt{\frac{c^4x^4+1}{(c^2x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{1}{cx}\right),\frac{1}{2}\right)}{2\sqrt{c^4x^4+1}}-\frac{(c^2x^2+1)\sqrt{\frac{c^4x^4+1}{(c^2x^2+1)^2}}E\left(2\arctan\left(\frac{1}{cx}\right)|\frac{1}{2}\right)}{\sqrt{c^4x^4+1}}-\frac{\sqrt{c^4x^4+1}}{cx}+\frac{\sqrt{c^4x^4+1}}{cx(c^2x^2+1)}\right)-\right.}{c^9x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

input
```
Int[x^5/Sech[2*Log[c*x]]^(3/2),x]
```

output
```
-((-1/9*(1 + c^4*x^4)^(3/2)/(c^9*x^9) + (2*(-1/5*Sqrt[1 + c^4*x^4]/(c^5*x^
5) + (2*(-(Sqrt[1 + c^4*x^4]/(c*x)) + Sqrt[1 + c^4*x^4]/(c*x*(1 + c^2*x^2)
) - ((1 + c^2*x^2)*Sqrt[(1 + c^4*x^4)/(1 + c^2*x^2)^2]*EllipticE[2*ArcTan[
1/(c*x)], 1/2])/Sqrt[1 + c^4*x^4] + ((1 + c^2*x^2)*Sqrt[(1 + c^4*x^4)/(1 +
 c^2*x^2)^2]*EllipticF[2*ArcTan[1/(c*x)], 1/2])/(2*Sqrt[1 + c^4*x^4])))/5)
)/3)/(c^9*(1 + 1/(c^4*x^4))^(3/2)*x^3*Sech[2*Log[c*x]]^(3/2)))
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 6083
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6085
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.59

method	result	size
risch	$\frac{x^{4}\left(5 c^{4} x^{4}+11\right) \sqrt{2}}{180 c^{2} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}+\frac{i \sqrt{-i c^{2} x^{2}+1}\, \sqrt{i c^{2} x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \sqrt{i c^{2}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{i c^{2}}, i\right)\right) \sqrt{2}\, x}{15 \sqrt{i c^{2}}\, \left(c^{4} x^{4}+1\right) c^{4} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}$	147

input
```
int(x^5/sech(2*ln(x*c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/180*x^4*(5*c^4*x^4+11)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/15*I/(I
*c^2)^(1/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)/c^4*(Ellip
ticF(x*(I*c^2)^(1/2),I)-EllipticE(x*(I*c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(
c^4*x^4+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.51

$$\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{24\sqrt{\frac{1}{2}}\sqrt{c^4}cx^2\left(-\frac{1}{c^4}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{1}{c^4}\right)^{\frac{1}{4}}}{x}\right)\mid -1\right) - 24\sqrt{\frac{1}{2}}\sqrt{c^4}cx^2\left(-\frac{1}{c^4}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{1}{c^4}\right)^{\frac{1}{4}}}{x}\right)\mid -1\right) + \sqrt{2}}{180\,c^8x^2}$$

input
```
integrate(x^5/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

output
```
1/180*(24*sqrt(1/2)*sqrt(c^4)*c*x^2*(-1/c^4)^(3/4)*elliptic_e(arcsin((-1/c
^4)^(1/4)/x), -1) - 24*sqrt(1/2)*sqrt(c^4)*c*x^2*(-1/c^4)^(3/4)*elliptic_f
(arcsin((-1/c^4)^(1/4)/x), -1) + sqrt(2)*(5*c^12*x^12 + 16*c^8*x^8 + 23*c^
4*x^4 + 12)*sqrt(c^2*x^2/(c^4*x^4 + 1)))/(c^8*x^2)
```

Sympy [F]

$$\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$$

input
```
integrate(x**5/sech(2*ln(c*x))**(3/2),x)
```

output
```
Integral(x**5/sech(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^5}{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^5/sech(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

input
```
integrate(x^5/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^5}{{\left(\frac{1}{\cosh\left(2\,\ln\left(c\,x\right)\right)}\right)}^{3/2}}\,dx$$

input
```
int(x^5/(1/cosh(2*log(c*x)))^(3/2),x)
```

output
```
int(x^5/(1/cosh(2*log(c*x)))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{\sqrt{\operatorname{sech}(2\log(cx))}\,x^5}{\operatorname{sech}(2\log(cx))^2}dx$$

input
```
int(x^5/sech(2*log(c*x))^(3/2),x)
```

output
```
int((sqrt(sech(2*log(c*x)))*x**5)/sech(2*log(c*x))**2,x)
```

3.173 $\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{3x}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^5}{8\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{3\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^4x^4}}\right)}{16c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

output
```
3/16*x/(c^4+1/x^4)/sech(2*ln(c*x))^(3/2)+1/8*x^5/sech(2*ln(c*x))^(3/2)+3/1
6*arctanh((1+1/c^4/x^4)^(1/2))/c^8/(1+1/c^4/x^4)^(3/2)/x^3/sech(2*ln(c*x))
^(3/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

$$\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{c^3x^3\sqrt{1+c^4x^4}(5+2c^4x^4)+3cx\operatorname{arcsinh}(c^2x^2)}{32\sqrt{2}c^5\sqrt{\frac{c^2x^2}{1+c^4x^4}}\sqrt{1+c^4x^4}}$$

input
```
Integrate[x^4/Sech[2*Log[c*x]]^(3/2),x]
```

output
```
(c^3*x^3*Sqrt[1 + c^4*x^4]*(5 + 2*c^4*x^4) + 3*c*x*ArcSinh[c^2*x^2])/(32*S
qrt[2]*c^5*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {6085, 6083, 798, 51, 51, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}dx$$

$$\Big\downarrow \text{6085}$$

$$\frac{\int \frac{c^4x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}d(cx)}{c^5}$$

$$\Big\downarrow \text{6083}$$

$$\frac{\int c^7\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^7d(cx)}{c^8x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{798}$$

$$-\frac{\int \frac{\left(1+\frac{1}{c^4x^4}\right)^{3/2}}{c^3x^3}d\frac{1}{c^4x^4}}{4c^8x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{51}$$

$$-\frac{\frac{3}{4}\int \frac{\sqrt{1+\frac{1}{c^4x^4}}}{c^2x^2}d\frac{1}{c^4x^4}-\frac{\left(\frac{1}{c^4x^4}+1\right)^{3/2}}{2c^2x^2}}{4c^8x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{51}$$

$$-\frac{\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{c\sqrt{1+\frac{1}{c^4x^4}}x}d\frac{1}{c^4x^4}-\frac{\sqrt{\frac{1}{c^4x^4}+1}}{cx}\right)-\frac{\left(\frac{1}{c^4x^4}+1\right)^{3/2}}{2c^2x^2}}{4c^8x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{3}{4}\left(\int \frac{1}{c^2x^2-1}d\sqrt{1+\frac{1}{c^4x^4}}-\frac{\sqrt{\frac{1}{c^4x^4}+1}}{cx}\right)-\frac{\left(\frac{1}{c^4x^4}+1\right)^{3/2}}{2c^2x^2}}{4c^8x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{220}$$

$$-\frac{\frac{3}{4}\left(-\operatorname{arctanh}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)-\frac{\sqrt{\frac{1}{c^4x^4}+1}}{cx}\right)-\frac{\left(\frac{1}{c^4x^4}+1\right)^{3/2}}{2c^2x^2}}{4c^8x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

input
```
Int[x^4/Sech[2*Log[c*x]]^(3/2),x]
```

output
```
-1/4*(-1/2*(1 + 1/(c^4*x^4))^(3/2)/(c^2*x^2) + (3*(-(Sqrt[1 + 1/(c^4*x^4)]
/(c*x)) - ArcTanh[Sqrt[1 + 1/(c^4*x^4)]]))/4)/(c^8*(1 + 1/(c^4*x^4))^(3/2)
*x^3*Sech[2*Log[c*x]]^(3/2))
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6083
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6085
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.23

method	result	size
risch	$\frac{x^{3}\left(2 c^{4} x^{4}+5\right) \sqrt{2}}{64 c^{2} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}+\frac{3 \ln \left(\frac{c^{4} x^{2}}{\sqrt{c^{4}}}+\sqrt{c^{4} x^{4}+1}\right) \sqrt{2}\, x}{64 \sqrt{c^{4}}\, c^{2} \sqrt{c^{4} x^{4}+1}\, \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}$	113

input
```
int(x^4/sech(2*ln(x*c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/64*x^3*(2*c^4*x^4+5)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(1/2)+3/64*ln(c^4
*x^2/(c^4)^(1/2)+(c^4*x^4+1)^(1/2))/(c^4)^(1/2)*2^(1/2)/c^2*x/(c^4*x^4+1)^
(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.10

$$\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$$
$$= \frac{2\sqrt{2}(2\,c^9x^9 + 7\,c^5x^5 + 5\,cx)\sqrt{\frac{c^2x^2}{c^4x^4+1}} + 3\sqrt{2}\log\left(-2\,c^4x^4 - 2\,(c^5x^5 + cx)\sqrt{\frac{c^2x^2}{c^4x^4+1}} - 1\right)}{128\,c^5}$$

input
```
integrate(x^4/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

output
```
1/128*(2*sqrt(2)*(2*c^9*x^9 + 7*c^5*x^5 + 5*c*x)*sqrt(c^2*x^2/(c^4*x^4 + 1
)) + 3*sqrt(2)*log(-2*c^4*x^4 - 2*(c^5*x^5 + c*x)*sqrt(c^2*x^2/(c^4*x^4 +
1)) - 1))/c^5
```

Sympy [F]

$$\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$$

input
```
integrate(x**4/sech(2*ln(c*x))**(3/2),x)
```

output
```
Integral(x**4/sech(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^4}{\operatorname{sech}(2\,\log(cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sech(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

input
```
integrate(x^4/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^4}{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

input
```
int(x^4/(1/cosh(2*log(c*x)))^(3/2),x)
```

output
```
int(x^4/(1/cosh(2*log(c*x)))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{\sqrt{\operatorname{sech}(2\log(cx))}\,x^4}{\operatorname{sech}(2\log(cx))^2}dx$$

input
```
int(x^4/sech(2*log(c*x))^(3/2),x)
```

output
```
int((sqrt(sech(2*log(c*x)))*x**4)/sech(2*log(c*x))**2,x)
```

3.174 $\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 111

$$\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{2}{7\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^4}{7\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{7c\left(c^4+\frac{1}{x^4}\right)^2x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

output
```
2/7/(c^4+1/x^4)/sech(2*ln(c*x))^(3/2)+1/7*x^4/sech(2*ln(c*x))^(3/2)-2/7*((
c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)*(c^2+1/x^2)*InverseJacobiAM(2*arccot(c*x),
1/2*2^(1/2))/c/(c^4+1/x^4)^2/x^3/sech(2*ln(c*x))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.55

$$\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\sqrt{1+c^4x^4}\sqrt{\frac{c^2x^2}{2+2c^4x^4}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-c^4x^4\right)}{2c^4}$$

input
```
Integrate[x^3/Sech[2*Log[c*x]]^(3/2),x]
```

output
```
(Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*Hypergeometric2F1[-3/2,
 1/4, 5/4, -(c^4*x^4)])/(2*c^4)
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.34, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6085, 6083, 858, 809, 809, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\text{sech}^{\frac{3}{2}}(2\log(cx))}dx$$

$$\Big\downarrow \text{ 6085}$$

$$\frac{\int \frac{c^3x^3}{\text{sech}^{\frac{3}{2}}(2\log(cx))}d(cx)}{c^4}$$

$$\Big\downarrow \text{ 6083}$$

$$\frac{\int c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^6d(cx)}{c^7x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{ 858}$$

$$-\frac{\int \frac{\left(c^4x^4+1\right)^{3/2}}{c^8x^8}d\frac{1}{cx}}{c^7x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{ 809}$$

$$-\frac{\frac{6}{7}\int \frac{\sqrt{c^4x^4+1}}{c^4x^4}d\frac{1}{cx}-\frac{\left(c^4x^4+1\right)^{3/2}}{7c^7x^7}}{c^7x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{ 809}$$

$$-\frac{\frac{6}{7}\left(\frac{2}{3}\int \frac{1}{\sqrt{c^4x^4+1}}d\frac{1}{cx}-\frac{\sqrt{c^4x^4+1}}{3c^3x^3}\right)-\frac{\left(c^4x^4+1\right)^{3/2}}{7c^7x^7}}{c^7x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{761}$$

$$-\frac{\frac{6}{7}\left(\frac{\left(c^2x^2+1\right)\sqrt{\frac{c^4x^4+1}{\left(c^2x^2+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{1}{cx}\right),\frac{1}{2}\right)}{3\sqrt{c^4x^4+1}}-\frac{\sqrt{c^4x^4+1}}{3c^3x^3}\right)-\frac{\left(c^4x^4+1\right)^{3/2}}{7c^7x^7}}{c^7x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

input
```
Int[x^3/Sech[2*Log[c*x]]^(3/2),x]
```

output
```
-((-1/7*(1 + c^4*x^4)^(3/2)/(c^7*x^7) + (6*(-1/3*Sqrt[1 + c^4*x^4]/(c^3*x^
3) + ((1 + c^2*x^2)*Sqrt[(1 + c^4*x^4)/(1 + c^2*x^2)^2]*EllipticF[2*ArcTan
[1/(c*x)], 1/2])/(3*Sqrt[1 + c^4*x^4])))/7)/(c^7*(1 + 1/(c^4*x^4))^(3/2)*x
^3*Sech[2*Log[c*x]]^(3/2)))
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6083
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6085
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.16

method	result	size
risch	$\frac{x^{2}\left(c^{4} x^{4}+3\right) \sqrt{2}}{28 c^{2} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}+\frac{\sqrt{-i c^{2} x^{2}+1}\, \sqrt{i c^{2} x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{i c^{2}}, i\right) \sqrt{2}\, x}{7 \sqrt{i c^{2}}\, \left(c^{4} x^{4}+1\right) c^{2} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}$	129

input
```
int(x^3/sech(2*ln(x*c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/28*x^2*(c^4*x^4+3)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/7/(I*c^2)^(
1/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)*EllipticF(x*(I*c^
2)^(1/2),I)*2^(1/2)/c^2*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.71

$$\int \frac{x^{3}}{\operatorname{sech}^{\frac{3}{2}}(2 \log(cx))}\, dx = \frac{8 \sqrt{\frac{1}{2}} \sqrt{c^{4}} c \left(-\frac{1}{c^{4}}\right)^{\frac{3}{4}} F(\arcsin\left(\frac{\left(-\frac{1}{c^{4}}\right)^{\frac{1}{4}}}{x}\right) \mid -1) + \sqrt{2} (c^{8} x^{8} + 4 c^{4} x^{4} + 3) \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}{28 c^{4}}$$

input
```
integrate(x^3/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

output
```
1/28*(8*sqrt(1/2)*sqrt(c^4)*c*(-1/c^4)^(3/4)*elliptic_f(arcsin((-1/c^4)^(1
/4)/x), -1) + sqrt(2)*(c^8*x^8 + 4*c^4*x^4 + 3)*sqrt(c^2*x^2/(c^4*x^4 + 1)
))/c^4
```

Sympy [F]

$$\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

input
```
integrate(x**3/sech(2*ln(c*x))**(3/2),x)
```

output
```
Integral(x**3/sech(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^3}{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sech(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

input
```
integrate(x^3/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^3}{\left(\frac{1}{\cosh(2\ln(c\,x))}\right)^{3/2}}\,dx$$

input
```
int(x^3/(1/cosh(2*log(c*x)))^(3/2),x)
```

output
```
int(x^3/(1/cosh(2*log(c*x)))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{\sqrt{\operatorname{sech}(2\log(cx))}\,x^3}{\operatorname{sech}(2\log(cx))^2}dx$$

input
```
int(x^3/sech(2*log(c*x))^(3/2),x)
```

output
```
int((sqrt(sech(2*log(c*x)))*x**3)/sech(2*log(c*x))**2,x)
```

3.175 $\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{1}{2\left(c^4+\frac{1}{x^4}\right)x\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^3}{6\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\operatorname{csch}^{-1}(c^2x^2)}{2c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

output
```
1/2/(c^4+1/x^4)/x/sech(2*ln(c*x))^(3/2)+1/6*x^3/sech(2*ln(c*x))^(3/2)-1/2*
arccsch(c^2*x^2)/c^6/(1+1/c^4/x^4)^(3/2)/x^3/sech(2*ln(c*x))^(3/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{x\left(\sqrt{1+c^4x^4}(4+c^4x^4)-3\operatorname{arctanh}\left(\sqrt{1+c^4x^4}\right)\right)}{12\sqrt{2}c^2\sqrt{\frac{c^2x^2}{1+c^4x^4}}\sqrt{1+c^4x^4}}$$

input
```
Integrate[x^2/Sech[2*Log[c*x]]^(3/2),x]
```

output
```
(x*(Sqrt[1 + c^4*x^4]*(4 + c^4*x^4) - 3*ArcTanh[Sqrt[1 + c^4*x^4]]))/(12*S
qrt[2]*c^2*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {6085, 6083, 858, 807, 247, 247, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\text{sech}^{\frac{3}{2}}(2\log(cx))}dx$$

$$\Big\downarrow \text{6085}$$

$$\frac{\int \frac{c^2x^2}{\text{sech}^{\frac{3}{2}}(2\log(cx))}d(cx)}{c^3}$$

$$\Big\downarrow \text{6083}$$

$$\frac{\int c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^5d(cx)}{c^6x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{858}$$

$$-\frac{\int \frac{\left(c^4x^4+1\right)^{3/2}}{c^7x^7}d\frac{1}{cx}}{c^6x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{807}$$

$$-\frac{\int \frac{\left(c^2x^2+1\right)^{3/2}}{c^4x^4}d\left(c^2x^2\right)}{2c^6x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{247}$$

$$-\frac{\int \frac{\sqrt{c^2x^2+1}}{c^2x^2}d\left(c^2x^2\right)-\frac{\left(c^2x^2+1\right)^{3/2}}{3c^3x^3}}{2c^6x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{247}$$

$$-\frac{\int \frac{1}{\sqrt{c^2x^2+1}}d(c^2x^2) - c^2x^2\sqrt{c^2x^2+1} - \frac{(c^2x^2+1)^{3/2}}{3c^3x^3}}{2c^6x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \; 222$$

$$-\frac{\operatorname{arcsinh}(c^2x^2) - c^2x^2\sqrt{c^2x^2+1} - \frac{(c^2x^2+1)^{3/2}}{3c^3x^3}}{2c^6x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

input
```
Int[x^2/Sech[2*Log[c*x]]^(3/2),x]
```

output
```
-1/2*(-(c^2*x^2*Sqrt[1 + c^2*x^2]) - (1 + c^2*x^2)^(3/2)/(3*c^3*x^3) + Arc
Sinh[c^2*x^2])/(c^6*(1 + 1/(c^4*x^4))^(3/2)*x^3*Sech[2*Log[c*x]]^(3/2))
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6083
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6085
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{x^2}{\operatorname{sech}\left(2\ln\left(xc\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2/sech(2*ln(x*c))^(3/2),x)
```

output
```
int(x^2/sech(2*ln(x*c))^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.24

$$\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{3\sqrt{2}cx\log\left(\frac{c^5x^5+2\,cx-2\,(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{cx^5}\right)+2\sqrt{2}(c^8x^8+5\,c^4x^4+4)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{48\,c^4x}$$

input
```
integrate(x^2/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

output
```
1/48*(3*sqrt(2)*c*x*log((c^5*x^5 + 2*c*x - 2*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c
^4*x^4 + 1)))/(c*x^5)) + 2*sqrt(2)*(c^8*x^8 + 5*c^4*x^4 + 4)*sqrt(c^2*x^2/
(c^4*x^4 + 1)))/(c^4*x)
```

Sympy [F]

$$\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$$

input
```
integrate(x**2/sech(2*ln(c*x))**(3/2),x)
```

output
```
Integral(x**2/sech(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^2}{\operatorname{sech}(2\log(cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sech(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

input
```
integrate(x^2/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^2}{\left(\frac{1}{\cosh(2\ln(c\,x))}\right)^{3/2}}\,dx$$

input
```
int(x^2/(1/cosh(2*log(c*x)))^(3/2),x)
```

output
```
int(x^2/(1/cosh(2*log(c*x)))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{\sqrt{\operatorname{sech}(2\log(cx))}\,x^2}{\operatorname{sech}(2\log(cx))^2}dx$$

input
```
int(x^2/sech(2*log(c*x))^(3/2),x)
```

output
```
int((sqrt(sech(2*log(c*x)))*x**2)/sech(2*log(c*x))**2,x)
```

3.176 $\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 214

$$\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{12}{5\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)x^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{6}{5\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{12c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)E\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{5\left(c^4+\frac{1}{x^4}\right)^2x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{6c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\operatorname{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{5\left(c^4+\frac{1}{x^4}\right)^2x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

output
```
-12/5/(c^4+1/x^4)/(c^2+1/x^2)/x^4/sech(2*ln(c*x))^(3/2)+6/5/(c^4+1/x^4)/x^
2/sech(2*ln(c*x))^(3/2)+1/5*x^2/sech(2*ln(c*x))^(3/2)+12/5*c*((c^4+1/x^4)/
(c^2+1/x^2)^2)^(1/2)*(c^2+1/x^2)*EllipticE(sin(2*arccot(c*x)),1/2*2^(1/2))
/(c^4+1/x^4)^2/x^3/sech(2*ln(c*x))^(3/2)-6/5*c*((c^4+1/x^4)/(c^2+1/x^2)^2)
^(1/2)*(c^2+1/x^2)*InverseJacobiAM(2*arccot(c*x),1/2*2^(1/2))/(c^4+1/x^4)^
2/x^3/sech(2*ln(c*x))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.30

$$\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},\frac{3}{4},-c^4x^4\right)}{2\sqrt{2}c^2\sqrt{\frac{c^2x^2}{1+c^4x^4}}\sqrt{1+c^4x^4}}$$

input
```
Integrate[x/Sech[2*Log[c*x]]^(3/2),x]
```

output
```
-1/2*Hypergeometric2F1[-3/2, -1/4, 3/4, -(c^4*x^4)]/(Sqrt[2]*c^2*Sqrt[(c^2
*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {6085, 6083, 858, 809, 809, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}dx$$

$$\Big\downarrow \text{6085}$$

$$\frac{\int \frac{cx}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}d(cx)}{c^2}$$

$$\Big\downarrow \text{6083}$$

$$\frac{\int c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^4d(cx)}{c^5x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{858}$$

$$-\frac{\int \frac{\left(c^4x^4+1\right)^{3/2}}{c^6x^6}d\frac{1}{cx}}{c^5x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$\Big\downarrow$ 809

$$-\frac{\frac{6}{5}\int \frac{\sqrt{c^4x^4+1}}{c^2x^2}d\frac{1}{cx}-\frac{\left(c^4x^4+1\right)^{3/2}}{5c^5x^5}}{c^5x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$\Big\downarrow$ 809

$$-\frac{\frac{6}{5}\left(2\int \frac{c^2x^2}{\sqrt{c^4x^4+1}}d\frac{1}{cx}-\frac{\sqrt{c^4x^4+1}}{cx}\right)-\frac{\left(c^4x^4+1\right)^{3/2}}{5c^5x^5}}{c^5x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$\Big\downarrow$ 834

$$-\frac{\frac{6}{5}\left(2\left(\int \frac{1}{\sqrt{c^4x^4+1}}d\frac{1}{cx}-\int \frac{1-c^2x^2}{\sqrt{c^4x^4+1}}d\frac{1}{cx}\right)-\frac{\sqrt{c^4x^4+1}}{cx}\right)-\frac{\left(c^4x^4+1\right)^{3/2}}{5c^5x^5}}{c^5x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$\Big\downarrow$ 761

$$-\frac{\frac{6}{5}\left(2\left(\frac{\left(c^2x^2+1\right)\sqrt{\frac{c^4x^4+1}{\left(c^2x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{1}{cx}\right),\frac{1}{2}\right)}{2\sqrt{c^4x^4+1}}-\int \frac{1-c^2x^2}{\sqrt{c^4x^4+1}}d\frac{1}{cx}\right)-\frac{\sqrt{c^4x^4+1}}{cx}\right)-\frac{\left(c^4x^4+1\right)^{3/2}}{5c^5x^5}}{c^5x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$\Big\downarrow$ 1510

$$-\frac{\frac{6}{5}\left(2\left(\frac{\left(c^2x^2+1\right)\sqrt{\frac{c^4x^4+1}{\left(c^2x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{1}{cx}\right),\frac{1}{2}\right)}{2\sqrt{c^4x^4+1}}-\frac{\left(c^2x^2+1\right)\sqrt{\frac{c^4x^4+1}{\left(c^2x^2+1\right)^2}}E\left(2\arctan\left(\frac{1}{cx}\right)|\frac{1}{2}\right)}{\sqrt{c^4x^4+1}}+\frac{\sqrt{c^4x^4+1}}{cx(c^2x^2+1)}\right)-\frac{\sqrt{c^4x^4+1}}{cx}\right)}{c^5x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

input

```
Int[x/Sech[2*Log[c*x]]^(3/2),x]
```

output
```
-((-1/5*(1 + c^4*x^4)^(3/2)/(c^5*x^5) + (6*(-(Sqrt[1 + c^4*x^4]/(c*x)) + 2
*(Sqrt[1 + c^4*x^4]/(c*x*(1 + c^2*x^2)) - ((1 + c^2*x^2)*Sqrt[(1 + c^4*x^4
)/(1 + c^2*x^2)^2]*EllipticE[2*ArcTan[1/(c*x)], 1/2])/Sqrt[1 + c^4*x^4] +
((1 + c^2*x^2)*Sqrt[(1 + c^4*x^4)/(1 + c^2*x^2)^2]*EllipticF[2*ArcTan[1/(c
*x)], 1/2])/(2*Sqrt[1 + c^4*x^4]))))/5)/(c^5*(1 + 1/(c^4*x^4))^(3/2)*x^3*S
ech[2*Log[c*x]]^(3/2)))
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 6083
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6085
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{(c^8x^8-4c^4x^4-5)\sqrt{2}}{20(c^4x^4+1)c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}}+\frac{3i\sqrt{-ic^2x^2+1}\,\sqrt{ic^2x^2+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{ic^2},i\right)-\operatorname{EllipticE}\left(x\sqrt{ic^2},i\right)\right)\sqrt{2}\,x}{5\sqrt{ic^2}\,(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$	159

input
```
int(x/sech(2*ln(x*c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*(c^8*x^8-4*c^4*x^4-5)/(c^4*x^4+1)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(
1/2)+3/5*I/(I*c^2)^(1/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+
1)*(EllipticF(x*(I*c^2)^(1/2),I)-EllipticE(x*(I*c^2)^(1/2),I))*2^(1/2)*x/(
c^2*x^2/(c^4*x^4+1))^(1/2)
```

Fricas [F]

$$\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx=\int \frac{x}{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

output
```
integral(x/sech(2*log(c*x))^(3/2), x)
```

Sympy [F]

$$\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x}{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

input
```
integrate(x/sech(2*ln(c*x))**(3/2),x)
```

output
```
Integral(x/sech(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x}{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/sech(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

input
```
integrate(x/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x}{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

input
```
int(x/(1/cosh(2*log(c*x)))^(3/2),x)
```

output
```
int(x/(1/cosh(2*log(c*x)))^(3/2), x)
```

Reduce [F]

$$\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{\sqrt{\operatorname{sech}(2\log(cx))}\,x}{\operatorname{sech}(2\log(cx))^{2}}dx$$

input
```
int(x/sech(2*log(c*x))^(3/2),x)
```

output
```
int((sqrt(sech(2*log(c*x)))*x)/sech(2*log(c*x))**2,x)
```

3.177 $\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Contents

Optimal result

Integrand size = 11, antiderivative size = 92

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{3}{4\left(c^4+\frac{1}{x^4}\right)x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x}{4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{3\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^4x^4}}\right)}{4c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

output
```
-3/4/(c^4+1/x^4)/x^3/sech(2*ln(c*x))^(3/2)+1/4*x/sech(2*ln(c*x))^(3/2)+3/4
*arctanh((1+1/c^4/x^4)^(1/2))/c^4/(1+1/c^4/x^4)^(3/2)/x^3/sech(2*ln(c*x))^
(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{\sqrt{1+c^4x^4}\sqrt{\frac{c^2x^2}{2+2c^4x^4}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{2},\frac{1}{2},-c^4x^4\right)}{4c^4x^3}$$

input
```
Integrate[Sech[2*Log[c*x]]^(-3/2),x]
```

output
```
-1/4*(Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*Hypergeometric2F1[
-3/2, -1/2, 1/2, -(c^4*x^4)])/(c^4*x^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {6079, 6077, 798, 51, 60, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}dx$$

$$\Big\downarrow \text{6079}$$

$$\frac{\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}d(cx)}{c}$$

$$\Big\downarrow \text{6077}$$

$$\frac{\int c^3\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3d(cx)}{c^4x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{798}$$

$$-\frac{\int \frac{\left(1+\frac{1}{c^4x^4}\right)^{3/2}}{c^2x^2}d\frac{1}{c^4x^4}}{4c^4x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{51}$$

$$-\frac{\frac{3}{2}\int \frac{\sqrt{1+\frac{1}{c^4x^4}}}{cx}d\frac{1}{c^4x^4}-\frac{\left(\frac{1}{c^4x^4}+1\right)^{3/2}}{cx}}{4c^4x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{60}$$

$$-\frac{\frac{3}{2}\left(\int \frac{1}{c\sqrt{1+\frac{1}{c^4x^4}}x}d\frac{1}{c^4x^4}+2\sqrt{\frac{1}{c^4x^4}+1}\right)-\frac{\left(\frac{1}{c^4x^4}+1\right)^{3/2}}{cx}}{4c^4x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{\frac{3}{2}\left(2\int \frac{1}{c^2x^2-1}d\sqrt{1+\frac{1}{c^4x^4}}+2\sqrt{\frac{1}{c^4x^4}+1}\right)-\frac{\left(\frac{1}{c^4x^4}+1\right)^{3/2}}{cx}}{4c^4x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{ 220}$$

$$-\frac{\frac{3}{2}\left(2\sqrt{\frac{1}{c^4x^4}+1}-2\operatorname{arctanh}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)\right)-\frac{\left(\frac{1}{c^4x^4}+1\right)^{3/2}}{cx}}{4c^4x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

input
```
Int[Sech[2*Log[c*x]]^(-3/2),x]
```

output
```
-1/4*(-((1 + 1/(c^4*x^4))^(3/2)/(c*x)) + (3*(2*Sqrt[1 + 1/(c^4*x^4)] - 2*A
rcTanh[Sqrt[1 + 1/(c^4*x^4)]]))/2)/(c^4*(1 + 1/(c^4*x^4))^(3/2)*x^3*Sech[2
*Log[c*x]]^(3/2))
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6077
```
Int[Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Simp[Sech[d*(a
+ b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*d*p))   Int[1/(x^(b
*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; FreeQ[{a, b, d, p}, x] &&
  !IntegerQ[p]
```

rule 6079
```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.42

method	result	size
risch	$\frac{\left(c^{8} x^{8}-c^{4} x^{4}-2\right) \sqrt{2}}{16 x \left(c^{4} x^{4}+1\right) c^{2} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}+\frac{3 c^{2} \ln \left(\frac{c^{4} x^{2}}{\sqrt{c^{4}}}+\sqrt{c^{4} x^{4}+1}\right) \sqrt{2}\, x}{16 \sqrt{c^{4}}\, \sqrt{c^{4} x^{4}+1}\, \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}$	131

input
```
int(1/sech(2*ln(x*c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*(c^8*x^8-c^4*x^4-2)/x/(c^4*x^4+1)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(
1/2)+3/16*c^2*ln(c^4*x^2/(c^4)^(1/2)+(c^4*x^4+1)^(1/2))/(c^4)^(1/2)*2^(1/2
)*x/(c^4*x^4+1)^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.15

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{3\sqrt{2}c^3x^3\log\left(-2\,c^4x^4 - 2\,(c^5x^5 + cx)\sqrt{\frac{c^2x^2}{c^4x^4+1}} - 1\right) + 2\sqrt{2}(c^8x^8 - c^4x^4 - 2)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{32\,c^4x^3}$$

input
```
integrate(1/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

output
```
1/32*(3*sqrt(2)*c^3*x^3*log(-2*c^4*x^4 - 2*(c^5*x^5 + c*x)*sqrt(c^2*x^2/(c
^4*x^4 + 1)) - 1) + 2*sqrt(2)*(c^8*x^8 - c^4*x^4 - 2)*sqrt(c^2*x^2/(c^4*x^
4 + 1)))/(c^4*x^3)
```

Sympy [F]

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$$

input
```
integrate(1/sech(2*ln(c*x))**(3/2),x)
```

output
```
Integral(sech(2*log(c*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{1}{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sech(2*log(c*x))^(-3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

input
```
integrate(1/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{1}{{\left(\frac{1}{\cosh\left(2\,\ln\left(c\,x\right)\right)}\right)}^{3/2}}\,dx$$

input
```
int(1/(1/cosh(2*log(c*x)))^(3/2),x)
```

output
```
int(1/(1/cosh(2*log(c*x)))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{\operatorname{sech}\left(2\log\left(cx\right)\right)^{2}}dx$$

input
```
int(1/sech(2*log(c*x))^(3/2),x)
```

output
```
int(sqrt(sech(2*log(c*x)))/sech(2*log(c*x))**2,x)
```

3.178 $\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = i\sqrt{\cosh(2\log(cx))}E(i\log(cx)|2)\sqrt{\operatorname{sech}(2\log(cx))} + \sqrt{\operatorname{sech}(2\log(cx))}\sinh(2\log(cx))$$

output
```
I*cosh(2*ln(c*x))^(1/2)*EllipticE(I*(1/2*c*x-1/2/c/x),2^(1/2))*sech(2*ln(c
*x))^(1/2)+sech(2*ln(c*x))^(1/2)*sinh(2*ln(c*x))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \frac{\frac{iE(i\log(cx)|2)}{\sqrt{\cosh(2\log(cx))}} + \tanh(2\log(cx))}{\sqrt{\operatorname{sech}(2\log(cx))}}$$

input
```
Integrate[Sech[2*Log[c*x]]^(3/2)/x,x]
```

output
```
((I*EllipticE[I*Log[c*x], 2])/Sqrt[Cosh[2*Log[c*x]]] + Tanh[2*Log[c*x]])/S
qrt[Sech[2*Log[c*x]]]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3039, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\int \operatorname{sech}^{\frac{3}{2}}(2\log(cx))d\log(cx)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(\frac{\pi}{2}+2i\log(cx)\right)^{3/2}d\log(cx)$$

$$\Big\downarrow \text{4255}$$

$$\sinh(2\log(cx))\sqrt{\operatorname{sech}(2\log(cx))}-\int \frac{1}{\sqrt{\operatorname{sech}(2\log(cx))}}d\log(cx)$$

$$\Big\downarrow \text{3042}$$

$$\sinh(2\log(cx))\sqrt{\operatorname{sech}(2\log(cx))}-\int \frac{1}{\sqrt{\csc\left(2i\log(cx)+\frac{\pi}{2}\right)}}d\log(cx)$$

$$\Big\downarrow \text{4258}$$

$$\sinh(2\log(cx))\sqrt{\operatorname{sech}(2\log(cx))}-\sqrt{\operatorname{sech}(2\log(cx))}\sqrt{\cosh(2\log(cx))}\int \sqrt{\cosh(2\log(cx))}d\log(cx)$$

$$\Big\downarrow \text{3042}$$

$$\sinh(2\log(cx))\sqrt{\operatorname{sech}(2\log(cx))}-\sqrt{\operatorname{sech}(2\log(cx))}\sqrt{\cosh(2\log(cx))}\int \sqrt{\sin\left(2i\log(cx)+\frac{\pi}{2}\right)}d\log(cx)$$

$$\Big\downarrow \text{3119}$$

$$\sinh(2\log(cx))\sqrt{\operatorname{sech}(2\log(cx))}+i\sqrt{\operatorname{sech}(2\log(cx))}\sqrt{\cosh(2\log(cx))}E(i\log(cx)|2)$$

input
```
Int[Sech[2*Log[c*x]]^(3/2)/x,x]
```

output
```
I*Sqrt[Cosh[2*Log[c*x]]]*EllipticE[I*Log[c*x], 2]*Sqrt[Sech[2*Log[c*x]]] +
 Sqrt[Sech[2*Log[c*x]]]*Sinh[2*Log[c*x]]
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(59) = 118$.

Time = 0.47 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.27

method	result	size
derivativedivides	$\frac{2\left(\frac{xc}{2}+\frac{1}{2xc}\right)\left(\frac{xc}{2}-\frac{1}{2xc}\right)^{2}+\sqrt{-2\left(\frac{xc}{2}-\frac{1}{2xc}\right)^{2}-1}\,\sqrt{-\left(\frac{xc}{2}-\frac{1}{2xc}\right)^{2}}\,\operatorname{EllipticE}\left(\frac{xc}{2}+\frac{1}{2xc},\sqrt{2}\right)}{\left(\frac{xc}{2}-\frac{1}{2xc}\right)\sqrt{2\left(\frac{xc}{2}+\frac{1}{2xc}\right)^{2}-1}}$	127
default	$\frac{2\left(\frac{xc}{2}+\frac{1}{2xc}\right)\left(\frac{xc}{2}-\frac{1}{2xc}\right)^{2}+\sqrt{-2\left(\frac{xc}{2}-\frac{1}{2xc}\right)^{2}-1}\,\sqrt{-\left(\frac{xc}{2}-\frac{1}{2xc}\right)^{2}}\,\operatorname{EllipticE}\left(\frac{xc}{2}+\frac{1}{2xc},\sqrt{2}\right)}{\left(\frac{xc}{2}-\frac{1}{2xc}\right)\sqrt{2\left(\frac{xc}{2}+\frac{1}{2xc}\right)^{2}-1}}$	127

input
```
int(sech(2*ln(x*c))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(2*(1/2*x*c+1/2/x/c)*(1/2*x*c-1/2/x/c)^2+(-2*(1/2*x*c-1/2/x/c)^2-1)^(1/2)*
(-(1/2*x*c-1/2/x/c)^2)^(1/2)*EllipticE(1/2*x*c+1/2/x/c,2^(1/2)))/(1/2*x*c-
1/2/x/c)/(2*(1/2*x*c+1/2/x/c)^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.46

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \frac{\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}c^3x^2+\sqrt{2}(-c^4)^{\frac{3}{4}}\,E\left(\arcsin\left((-c^4)^{\frac{1}{4}}\,x\right)\,|-1\right)-\sqrt{2}(-c^4)^{\frac{3}{4}}\,F\left(\arcsin\left((-c^4)^{\frac{1}{4}}\,x\right)\,|-1\right)}{c}$$

input
```
integrate(sech(2*log(c*x))^(3/2)/x,x, algorithm="fricas")
```

output
```
(sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 + 1))*c^3*x^2 + sqrt(2)*(-c^4)^(3/4)*ellipt
ic_e(arcsin((-c^4)^(1/4)*x), -1) - sqrt(2)*(-c^4)^(3/4)*elliptic_f(arcsin(
(-c^4)^(1/4)*x), -1))/c
```

Sympy [F]

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx$$

input
```
integrate(sech(2*ln(c*x))**(3/2)/x,x)
```

output
```
Integral(sech(2*log(c*x))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \int \frac{\operatorname{sech}(2\log(cx))^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(sech(2*log(c*x))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(sech(2*log(c*x))^(3/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \text{Timed out}$$

input
```
integrate(sech(2*log(c*x))^(3/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \int \frac{\left(\frac{1}{\cosh(2\ln(c\,x))}\right)^{3/2}}{x}\,dx$$

input
```
int((1/cosh(2*log(c*x)))^(3/2)/x,x)
```

output
```
int((1/cosh(2*log(c*x)))^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}\operatorname{sech}(2\log(cx))}{x}dx$$

input
```
int(sech(2*log(c*x))^(3/2)/x,x)
```

output
```
int((sqrt(sech(2*log(c*x)))*sech(2*log(c*x)))/x,x)
```

3.179 $\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^2} dx = \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))$$

output
```
1/2*(c^4+1/x^4)*x^3*sech(2*ln(c*x))^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^2} dx = \sqrt{2}c^2x\sqrt{\frac{c^2x^2}{1+c^4x^4}}$$

input
```
Integrate[Sech[2*Log[c*x]]^(3/2)/x^2,x]
```

output
```
Sqrt[2]*c^2*x*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6085, 6083, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}dx$$

$$\Big\downarrow \text{ 6085}$$

$$c\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{c^2x^2}d(cx)$$

$$\Big\downarrow \text{ 6083}$$

$$c^4x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\int \frac{1}{c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^5}d(cx)$$

$$\Big\downarrow \text{ 793}$$

$$\frac{1}{2}c^4x^3\left(\frac{1}{c^4x^4}+1\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))$$

input
```
Int[Sech[2*Log[c*x]]^(3/2)/x^2,x]
```

output
```
(c^4*(1 + 1/(c^4*x^4))*x^3*Sech[2*Log[c*x]]^(3/2))/2
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 6083
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6085
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\operatorname{sech}\left(2\ln\left(xc\right)\right)^{\frac{3}{2}}}{x^{2}}dx$$

input
```
int(sech(2*ln(x*c))^(3/2)/x^2,x)
```

output
```
int(sech(2*ln(x*c))^(3/2)/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = \sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}c^2x$$

input
```
integrate(sech(2*log(c*x))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 + 1))*c^2*x
```

Sympy [F]

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = \int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx$$

input
```
integrate(sech(2*ln(c*x))**(3/2)/x**2,x)
```

output
```
Integral(sech(2*log(c*x))**(3/2)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = c\left(\frac{\sqrt{2}}{\left(\frac{1}{c^4x^4}+1\right)^{\frac{3}{2}}} + \frac{\sqrt{2}}{c^4x^4\left(\frac{1}{c^4x^4}+1\right)^{\frac{3}{2}}}\right)$$

input
```
integrate(sech(2*log(c*x))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
c*(sqrt(2)/(1/(c^4*x^4) + 1)^(3/2) + sqrt(2)/(c^4*x^4*(1/(c^4*x^4) + 1)^(3
/2)))
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = \text{Timed out}$$

input
```
integrate(sech(2*log(c*x))^(3/2)/x^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = c^2\,x\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4+1}}$$

input
```
int((1/cosh(2*log(c*x)))^(3/2)/x^2,x)
```

output
```
c^2*x*((2*c^2*x^2)/(c^4*x^4 + 1))^(1/2)
```

Reduce [F]

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}\,\operatorname{sech}(2\log(cx))}{x^2}dx$$

input
```
int(sech(2*log(c*x))^(3/2)/x^2,x)
```

output
```
int((sqrt(sech(2*log(c*x)))*sech(2*log(c*x)))/x**2,x)
```

3.180 $\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx)) - \frac{\left(c^4+\frac{1}{x^4}\right)\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)x^3\operatorname{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{4c}$$

output
```
1/2*(c^4+1/x^4)*x^2*sech(2*ln(c*x))^(3/2)-1/4*(c^4+1/x^4)*((c^4+1/x^4)/(c^
2+1/x^2)^2)^(1/2)*(c^2+1/x^2)*x^3*InverseJacobiAM(2*arccot(c*x),1/2*2^(1/2
))*sech(2*ln(c*x))^(3/2)/c
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \sqrt{2}c^2\sqrt{\frac{c^2x^2}{1+c^4x^4}}\left(1 + \sqrt{1+c^4x^4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-c^4x^4\right)\right)$$

input
```
Integrate[Sech[2*Log[c*x]]^(3/2)/x^3,x]
```

output
```
Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*(1 + Sqrt[1 + c^4*x^4]*Hypergeom
etric2F1[1/4, 1/2, 5/4, -(c^4*x^4)])
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6085, 6083, 858, 817, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx$$

$$\Big\downarrow \text{ 6085}$$

$$c^2\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{c^3x^3}d(cx)$$

$$\Big\downarrow \text{ 6083}$$

$$c^5x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\int \frac{1}{c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^6}d(cx)$$

$$\Big\downarrow \text{ 858}$$

$$-c^5x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\int\frac{c^4x^4}{(c^4x^4+1)^{3/2}}d\frac{1}{cx}$$

$$\Big\downarrow \text{817}$$

$$-c^5x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\left(\frac{1}{2}\int\frac{1}{\sqrt{c^4x^4+1}}d\frac{1}{cx}-\frac{1}{2cx\sqrt{c^4x^4+1}}\right)$$

$$\Big\downarrow \text{761}$$

$$-c^5x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\left(\frac{(c^2x^2+1)\sqrt{\frac{c^4x^4+1}{(c^2x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{1}{cx}\right),\frac{1}{2}\right)}{4\sqrt{c^4x^4+1}}-\frac{1}{2cx\sqrt{c^4x^4+1}}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))$$

input
```
Int[Sech[2*Log[c*x]]^(3/2)/x^3,x]
```

output
```
-(c^5*(1 + 1/(c^4*x^4))^(3/2)*x^3*(-1/2*1/(c*x*Sqrt[1 + c^4*x^4]) + ((1 +
c^2*x^2)*Sqrt[(1 + c^4*x^4)/(1 + c^2*x^2)^2]*EllipticF[2*ArcTan[1/(c*x)],
1/2])/(4*Sqrt[1 + c^4*x^4]))*Sech[2*Log[c*x]]^(3/2))
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6083
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6085
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\operatorname{sech}\left(2\ln\left(xc\right)\right)^{\frac{3}{2}}}{x^{3}}dx$$

input
```
int(sech(2*ln(x*c))^(3/2)/x^3,x)
```

output
```
int(sech(2*ln(x*c))^(3/2)/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.61

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \frac{\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}c^3 - \sqrt{2}(-c^4)^{\frac{3}{4}}\,F(\arcsin\left((-c^4)^{\frac{1}{4}}\,x\right)\,|-1)}{c}$$

input
```
integrate(sech(2*log(c*x))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
(sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 + 1))*c^3 - sqrt(2)*(-c^4)^(3/4)*elliptic_f
(arcsin((-c^4)^(1/4)*x), -1))/c
```

Sympy [F]

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log{(cx)})}{x^3}\,dx$$

input
```
integrate(sech(2*ln(c*x))**(3/2)/x**3,x)
```

output
```
Integral(sech(2*log(c*x))**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \int \frac{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate(sech(2*log(c*x))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sech(2*log(c*x))^(3/2)/x^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(sech(2*log(c*x))^(3/2)/x^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \int \frac{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}{x^3}\,dx$$

input
```
int((1/cosh(2*log(c*x)))^(3/2)/x^3,x)
```

output
```
int((1/cosh(2*log(c*x)))^(3/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}\operatorname{sech}(2\log(cx))}{x^3}dx$$

input
```
int(sech(2*log(c*x))^(3/2)/x^3,x)
```

output
```
int((sqrt(sech(2*log(c*x)))*sech(2*log(c*x)))/x**3,x)
```

3.181 $\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 66

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x\operatorname{sech}^{\frac{3}{2}}(2\log(cx)) - \frac{1}{2}c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{-1}(c^2x^2)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))$$

output
```
1/2*(c^4+1/x^4)*x*sech(2*ln(c*x))^(3/2)-1/2*c^6*(1+1/c^4/x^4)^(3/2)*x^3*ar
ccsch(c^2*x^2)*sech(2*ln(c*x))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.77

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \frac{\sqrt{2}c^2\sqrt{\frac{c^2x^2}{1+c^4x^4}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+c^4x^4\right)}{x}$$

input
```
Integrate[Sech[2*Log[c*x]]^(3/2)/x^4,x]
```

output
```
(Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Hypergeometric2F1[-1/2, 1, 1/2,
 1 + c^4*x^4])/x
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6085, 6083, 858, 807, 252, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}dx$$

$$\Big\downarrow \text{6085}$$

$$c^3\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{c^4x^4}d(cx)$$

$$\Big\downarrow \text{6083}$$

$$c^6x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\int \frac{1}{c^7\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^7}d(cx)$$

$$\Big\downarrow \text{858}$$

$$-c^6x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\int \frac{c^5x^5}{(c^4x^4+1)^{3/2}}d\frac{1}{cx}$$

$$\Big\downarrow \text{807}$$

$$-\frac{1}{2}c^6x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\int \frac{c^2x^2}{(c^2x^2+1)^{3/2}}d(c^2x^2)$$

$$\Big\downarrow \text{252}$$

$$-\frac{1}{2}c^6x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\left(\int \frac{1}{\sqrt{c^2x^2+1}}d(c^2x^2)-\frac{c^2x^2}{\sqrt{c^2x^2+1}}\right)$$

$$\Big\downarrow \text{222}$$

$$-\frac{1}{2}c^6x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\left(\operatorname{arcsinh}(c^2x^2)-\frac{c^2x^2}{\sqrt{c^2x^2+1}}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))$$

input
```
Int[Sech[2*Log[c*x]]^(3/2)/x^4,x]
```

output
```
-1/2*(c^6*(1 + 1/(c^4*x^4))^(3/2)*x^3*(-((c^2*x^2)/Sqrt[1 + c^2*x^2]) + Ar
cSinh[c^2*x^2])*Sech[2*Log[c*x]]^(3/2))
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6083
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6085
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\operatorname{sech}(2\ln(xc))^{\frac{3}{2}}}{x^4}dx$$

input
```
int(sech(2*ln(x*c))^(3/2)/x^4,x)
```

output
```
int(sech(2*ln(x*c))^(3/2)/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.41

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \frac{\sqrt{2}c^3x\log\left(\frac{c^5x^5+2\,cx-2\,(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{cx^5}\right)+2\,\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}c^2}{2\,x}$$

input
```
integrate(sech(2*log(c*x))^(3/2)/x^4,x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*c^3*x*log((c^5*x^5 + 2*c*x - 2*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^
4*x^4 + 1)))/(c*x^5)) + 2*sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 + 1))*c^2)/x
```

Sympy [F]

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \int \frac{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}{x^4}\,dx$$

input
```
integrate(sech(2*ln(c*x))**(3/2)/x**4,x)
```

output
```
Integral(sech(2*log(c*x))**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \int \frac{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate(sech(2*log(c*x))^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sech(2*log(c*x))^(3/2)/x^4, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(sech(2*log(c*x))^(3/2)/x^4,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \int \frac{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}{x^4}\,dx$$

input
```
int((1/cosh(2*log(c*x)))^(3/2)/x^4,x)
```

output
```
int((1/cosh(2*log(c*x)))^(3/2)/x^4, x)
```

Reduce [F]

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \int \frac{\sqrt{\operatorname{sech}(2\log(cx))}\,\operatorname{sech}(2\log(cx))}{x^4}dx$$

input
```
int(sech(2*log(c*x))^(3/2)/x^4,x)
```

output
```
int((sqrt(sech(2*log(c*x)))*sech(2*log(c*x)))/x**4,x)
```

3.182 $\int \operatorname{sech}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 71

$$\int \operatorname{sech}(a+b\log(cx^n))\,dx = \frac{2e^{-a}x(cx^n)^{-b}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1-\frac{1}{bn}\right),\frac{1}{2}\left(3-\frac{1}{bn}\right),-e^{-2a}(cx^n)^{-2b}\right)}{1-bn}$$

output
```
2*x*hypergeom([1, 1/2-1/2/b/n],[3/2-1/2/b/n],-1/exp(2*a)/((c*x^n)^(2*b)))/
exp(a)/(-b*n+1)/((c*x^n)^b)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

$$\int \operatorname{sech}(a+b\log(cx^n))\,dx = \frac{2e^{a}x(cx^n)^{b}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{1}{bn}\right),\frac{1}{2}\left(3+\frac{1}{bn}\right),-e^{2a}(cx^n)^{2b}\right)}{1+bn}$$

input
```
Integrate[Sech[a + b*Log[c*x^n]],x]
```

output
```
(2*E^a*x*(c*x^n)^b*Hypergeometric2F1[1, (1 + 1/(b*n))/2, (3 + 1/(b*n))/2,
-(E^(2*a)*(c*x^n)^(2*b))])/(1 + b*n)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6079, 6081, 795, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{6079}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\operatorname{sech}(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{6081}$$

$$\frac{2e^{-a}x(cx^n)^{-1/n}\int \frac{(cx^n)^{-b+\frac{1}{n}-1}}{e^{-2a}(cx^n)^{-2b}+1}d(cx^n)}{n}$$

$$\Big\downarrow \text{795}$$

$$\frac{2e^{-a}x(cx^n)^{-1/n}\int \frac{(cx^n)^{b+\frac{1}{n}-1}}{(cx^n)^{2b}+e^{-2a}}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{2e^a x(cx^n)^b\,\text{Hypergeometric2F1}\left(1,\frac{b+\frac{1}{n}}{2b},\frac{1}{2}\left(3+\frac{1}{bn}\right),-e^{2a}(cx^n)^{2b}\right)}{bn+1}$$

input
```
Int[Sech[a + b*Log[c*x^n]],x]
```

output
```
(2*E^a*x*(c*x^n)^b*Hypergeometric2F1[1, (b + n^(-1))/(2*b), (3 + 1/(b*n))/
2, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + b*n)
```

Defintions of rubi rules used

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6079
```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6081
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[2^p/E^(a*d*p)   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b
*d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [F]

$$\int \operatorname{sech}\left(a+b\ln\left(c\,x^{n}\right)\right)dx$$

input
```
int(sech(a+b*ln(c*x^n)),x)
```

output
```
int(sech(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int \operatorname{sech}(a + b\log(cx^n)) \, dx = \int \operatorname{sech}(b\log(cx^n) + a) \, dx$$

input
```
integrate(sech(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
integral(sech(b*log(c*x^n) + a), x)
```

Sympy [F]

$$\int \operatorname{sech}(a + b\log(cx^n)) \, dx = \int \operatorname{sech}(a + b\log(cx^n)) \, dx$$

input
```
integrate(sech(a+b*ln(c*x**n)),x)
```

output
```
Integral(sech(a + b*log(c*x**n)), x)
```

Maxima [F]

$$\int \operatorname{sech}(a + b\log(cx^n)) \, dx = \int \operatorname{sech}(b\log(cx^n) + a) \, dx$$

input
```
integrate(sech(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
integrate(sech(b*log(c*x^n) + a), x)
```

Giac [F]

$$\int \operatorname{sech}(a+b\log(cx^n))\,dx=\int \operatorname{sech}(b\log(cx^n)+a)\,dx$$

input
```
integrate(sech(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
integrate(sech(b*log(c*x^n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}(a+b\log(cx^n))\,dx=\int \frac{1}{\cosh(a+b\,\ln(c\,x^n))}\,dx$$

input
```
int(1/cosh(a + b*log(c*x^n)),x)
```

output
```
int(1/cosh(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int \operatorname{sech}(a+b\log(cx^n))\,dx=\int \operatorname{sech}(\log(x^nc)\,b+a)\,dx$$

input
```
int(sech(a+b*log(c*x^n)),x)
```

output
```
int(sech(log(x**n*c)*b + a),x)
```

3.183 $\int \operatorname{sech}^2(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 71

$$\int \operatorname{sech}^2(a+b\log(cx^n))\,dx = \frac{4e^{-2a}x(cx^n)^{-2b}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(2-\frac{1}{bn}\right),\frac{1}{2}\left(4-\frac{1}{bn}\right),-e^{-2a}(cx^n)^{-2b}\right)}{1-2bn}$$

output
```
4*x*hypergeom([2, 1-1/2/b/n],[2-1/2/b/n],-1/exp(2*a)/((c*x^n)^(2*b)))/exp(
2*a)/(-2*b*n+1)/((c*x^n)^(2*b))
```

Mathematica [A] (verified)

Time = 3.58 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.77

$$\int \operatorname{sech}^2(a+b\log(cx^n))\,dx = \frac{x\left(-\frac{e^{2a}(cx^n)^{2b}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{2bn},2+\frac{1}{2bn},-e^{2a}(cx^n)^{2b}\right)}{1+2bn}+\text{Hypergeometric2F1}\left(1,\frac{1}{2bn},1+\frac{1}{2bn},-e^{2a}(cx^n)^{2b}\right.\right.}{bn}$$

input
```
Integrate[Sech[a + b*Log[c*x^n]]^2,x]
```

output
```
(x*(-((E^(2*a)*(c*x^n)^(2*b)*Hypergeometric2F1[1, 1 + 1/(2*b*n), 2 + 1/(2*
b*n), -(E^(2*a)*(c*x^n)^(2*b))])/(1 + 2*b*n)) + Hypergeometric2F1[1, 1/(2*
b*n), 1 + 1/(2*b*n), -(E^(2*a)*(c*x^n)^(2*b))] + Tanh[a + b*Log[c*x^n]]))/
(b*n)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6079, 6081, 795, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^2(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{ 6079}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\operatorname{sech}^2(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{ 6081}$$

$$\frac{4e^{-2a}x(cx^n)^{-1/n}\int \frac{(cx^n)^{-2b+\frac{1}{n}-1}}{\left(e^{-2a}(cx^n)^{-2b}+1\right)^2}d(cx^n)}{n}$$

$$\Big\downarrow \text{ 795}$$

$$\frac{4e^{-2a}x(cx^n)^{-1/n}\int \frac{(cx^n)^{2b+\frac{1}{n}-1}}{\left((cx^n)^{2b}+e^{-2a}\right)^2}d(cx^n)}{n}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{4e^{2a}x(cx^n)^{2b}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(2+\frac{1}{bn}\right),\frac{1}{2}\left(4+\frac{1}{bn}\right),-e^{2a}(cx^n)^{2b}\right)}{2bn+1}$$

input
```
Int[Sech[a + b*Log[c*x^n]]^2,x]
```

output
```
(4*E^(2*a)*x*(c*x^n)^(2*b)*Hypergeometric2F1[2, (2 + 1/(b*n))/2, (4 + 1/(b
*n))/2, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + 2*b*n)
```

Defintions of rubi rules used

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6079
```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6081
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[2^p/E^(a*d*p)   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b
*d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [F]

$$\int \mathrm{sech}\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}dx$$

input
```
int(sech(a+b*ln(c*x^n))^2,x)
```

output
```
int(sech(a+b*ln(c*x^n))^2,x)
```

Fricas [F]

$$\int \operatorname{sech}^2(a+b\log(cx^n))\,dx=\int \operatorname{sech}(b\log(cx^n)+a)^2\,dx$$

input
```
integrate(sech(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
integral(sech(b*log(c*x^n) + a)^2, x)
```

Sympy [F]

$$\int \operatorname{sech}^2(a+b\log(cx^n))\,dx=\int \operatorname{sech}^2(a+b\log(cx^n))\,dx$$

input
```
integrate(sech(a+b*ln(c*x**n))**2,x)
```

output
```
Integral(sech(a + b*log(c*x**n))**2, x)
```

Maxima [F]

$$\int \operatorname{sech}^2(a+b\log(cx^n))\,dx=\int \operatorname{sech}(b\log(cx^n)+a)^2\,dx$$

input
```
integrate(sech(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
-2*x/(b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) + 4*integrate(1/2/(b*c^(2*
b)*n*e^(2*b*log(x^n) + 2*a) + b*n), x)
```

Giac [F]

$$\int \operatorname{sech}^2(a+b\log(cx^n))\,dx = \int \operatorname{sech}(b\log(cx^n)+a)^2\,dx$$

input
```
integrate(sech(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
integrate(sech(b*log(c*x^n) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^2(a+b\log(cx^n))\,dx = \int \frac{1}{\cosh(a+b\ln(c\,x^n))^2}\,dx$$

input
```
int(1/cosh(a + b*log(c*x^n))^2,x)
```

output
```
int(1/cosh(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int \operatorname{sech}^2(a+b\log(cx^n))\,dx = 4e^{2a}c^{2b}\left(\int \frac{x^{2bn}}{x^{4bn}e^{4a}c^{4b}+2x^{2bn}e^{2a}c^{2b}+1}dx\right)$$

input
```
int(sech(a+b*log(c*x^n))^2,x)
```

output
```
4*e**(2*a)*c**(2*b)*int(x**(2*b*n)/(x**(4*b*n)*e**(4*a)*c**(4*b) + 2*x**(2
*b*n)*e**(2*a)*c**(2*b) + 1),x)
```

3.184 $\int \mathrm{sech}^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 71

$$\int \mathrm{sech}^3(a+b\log(cx^n))\,dx = \frac{8e^{-3a}x(cx^n)^{-3b}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3-\frac{1}{bn}\right),\frac{1}{2}\left(5-\frac{1}{bn}\right),-e^{-2a}(cx^n)^{-2b}\right)}{1-3bn}$$

output
```
8*x*hypergeom([3, 3/2-1/2/b/n],[5/2-1/2/b/n],-1/exp(2*a)/((c*x^n)^(2*b)))/
exp(3*a)/(-3*b*n+1)/((c*x^n)^(3*b))
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.42

$$\int \mathrm{sech}^3(a+b\log(cx^n))\,dx = \frac{x\left(2e^a(-1+bn)(cx^n)^b\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{1}{bn}\right),\frac{1}{2}\left(3+\frac{1}{bn}\right),-e^{2a}(cx^n)^{2b}\right)+\mathrm{sech}(a+b\log(cx\right.}{2b^2n^2}$$

input
```
Integrate[Sech[a + b*Log[c*x^n]]^3,x]
```

output
```
(x*(2*E^a*(-1 + b*n)*(c*x^n)^b*Hypergeometric2F1[1, (1 + 1/(b*n))/2, (3 +
1/(b*n))/2, -(E^(2*a)*(c*x^n)^(2*b))] + Sech[a + b*Log[c*x^n]]*(1 + b*n*Ta
nh[a + b*Log[c*x^n]])))/(2*b^2*n^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6079, 6081, 795, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^3(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{6079}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\operatorname{sech}^3(a+b\log(cx^n))d(cx^n)}{n}$$

$$\Big\downarrow \text{6081}$$

$$\frac{8e^{-3a}x(cx^n)^{-1/n}\int \frac{(cx^n)^{-3b+\frac{1}{n}-1}}{\left(e^{-2a}(cx^n)^{-2b}+1\right)^3}d(cx^n)}{n}$$

$$\Big\downarrow \text{795}$$

$$\frac{8e^{-3a}x(cx^n)^{-1/n}\int \frac{(cx^n)^{3b+\frac{1}{n}-1}}{\left((cx^n)^{2b}+e^{-2a}\right)^3}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{8e^{3a}x(cx^n)^{3b}\,\text{Hypergeometric2F1}\left(3,\frac{3b+\frac{1}{n}}{2b},\frac{1}{2}\left(5+\frac{1}{bn}\right),-e^{2a}(cx^n)^{2b}\right)}{3bn+1}$$

input
```
Int[Sech[a + b*Log[c*x^n]]^3,x]
```

output
```
(8*E^(3*a)*x*(c*x^n)^(3*b)*Hypergeometric2F1[3, (3*b + n^(-1))/(2*b), (5 +
 1/(b*n))/2, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + 3*b*n)
```

Defintions of rubi rules used

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6079
```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6081
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[2^p/E^(a*d*p)   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b
*d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [F]

$$\int \operatorname{sech}\left(a+b\ln\left(c\,x^{n}\right)\right)^{3}dx$$

input
```
int(sech(a+b*ln(c*x^n))^3,x)
```

output
```
int(sech(a+b*ln(c*x^n))^3,x)
```

Fricas [F]

$$\int \operatorname{sech}^3(a+b\log(cx^n))\,dx=\int \operatorname{sech}(b\log(cx^n)+a)^3\,dx$$

input
```
integrate(sech(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
integral(sech(b*log(c*x^n) + a)^3, x)
```

Sympy [F]

$$\int \operatorname{sech}^3(a+b\log(cx^n))\,dx=\int \operatorname{sech}^3(a+b\log(cx^n))\,dx$$

input
```
integrate(sech(a+b*ln(c*x**n))**3,x)
```

output
```
Integral(sech(a + b*log(c*x**n))**3, x)
```

Maxima [F]

$$\int \operatorname{sech}^3(a+b\log(cx^n))\,dx=\int \operatorname{sech}(b\log(cx^n)+a)^3\,dx$$

input
```
integrate(sech(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
8*(b^2*c^b*n^2 - c^b)*integrate(1/8*e^(b*log(x^n) + a)/(b^2*c^(2*b)*n^2*e^
(2*b*log(x^n) + 2*a) + b^2*n^2), x) + ((b*c^(3*b)*n + c^(3*b))*x*e^(3*b*lo
g(x^n) + 3*a) - (b*c^b*n - c^b)*x*e^(b*log(x^n) + a))/(b^2*c^(4*b)*n^2*e^(
4*b*log(x^n) + 4*a) + 2*b^2*c^(2*b)*n^2*e^(2*b*log(x^n) + 2*a) + b^2*n^2)
```

Giac [F]

$$\int \operatorname{sech}^3(a+b\log(cx^n))\,dx = \int \operatorname{sech}(b\log(cx^n)+a)^3\,dx$$

input
```
integrate(sech(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output
```
integrate(sech(b*log(c*x^n) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^3(a+b\log(cx^n))\,dx = \int \frac{1}{\cosh(a+b\,\ln(c\,x^n))^3}\,dx$$

input
```
int(1/cosh(a + b*log(c*x^n))^3,x)
```

output
```
int(1/cosh(a + b*log(c*x^n))^3, x)
```

Reduce [F]

$$\int \operatorname{sech}^3(a+b\log(cx^n))\,dx = 8e^{3a}c^{3b}\left(\int \frac{x^{3bn}}{x^{6bn}e^{6a}c^{6b}+3x^{4bn}e^{4a}c^{4b}+3x^{2bn}e^{2a}c^{2b}+1}dx\right)$$

input
```
int(sech(a+b*log(c*x^n))^3,x)
```

output
```
8*e**(3*a)*c**(3*b)*int(x**(3*b*n)/(x**(6*b*n)*e**(6*a)*c**(6*b) + 3*x**(4
*b*n)*e**(4*a)*c**(4*b) + 3*x**(2*b*n)*e**(2*a)*c**(2*b) + 1),x)
```

3.185 $\int \text{sech}^4(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 71

$$\int \text{sech}^4(a+b\log(cx^n))\,dx = \frac{16e^{-4a}x(cx^n)^{-4b}\,\text{Hypergeometric2F1}\left(4,\frac{1}{2}\left(4-\frac{1}{bn}\right),\frac{1}{2}\left(6-\frac{1}{bn}\right),-e^{-2a}(cx^n)^{-2b}\right)}{1-4bn}$$

output
```
16*x*hypergeom([4, 2-1/2/b/n],[3-1/2/b/n],-1/exp(2*a)/((c*x^n)^(2*b)))/exp
(4*a)/(-4*b*n+1)/((c*x^n)^(4*b))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 192 vs. 2(71) = 142.

Time = 8.90 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.70

$$\int \text{sech}^4(a+b\log(cx^n))\,dx = \frac{x\left(-2e^{2a}(-1+2bn)(cx^n)^{2b}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{2bn},2+\frac{1}{2bn},-e^{2a}(cx^n)^{2b}\right)+(-2+8b^2n^2)\,\text{Hype}\ldots}{\ldots}$$

input
```
Integrate[Sech[a + b*Log[c*x^n]]^4,x]
```

output
```
(x*(-2*E^(2*a)*(-1 + 2*b*n)*(c*x^n)^(2*b)*Hypergeometric2F1[1, 1 + 1/(2*b*
n), 2 + 1/(2*b*n), -(E^(2*a)*(c*x^n)^(2*b))] + (-2 + 8*b^2*n^2)*Hypergeome
tric2F1[1, 1/(2*b*n), 1 + 1/(2*b*n), -(E^(2*a)*(c*x^n)^(2*b))] + Sech[a +
b*Log[c*x^n]]^2*(2*b*n + (-1 + 8*b^2*n^2 + (-1 + 4*b^2*n^2)*Cosh[2*(a + b*
Log[c*x^n])])*Tanh[a + b*Log[c*x^n]])))/(12*b^3*n^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6079, 6081, 795, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^4(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{6079}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\operatorname{sech}^4(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{6081}$$

$$\frac{16e^{-4a}x(cx^n)^{-1/n}\int \frac{(cx^n)^{-4b+\frac{1}{n}-1}}{\left(e^{-2a}(cx^n)^{-2b}+1\right)^4}d(cx^n)}{n}$$

$$\Big\downarrow \text{795}$$

$$\frac{16e^{-4a}x(cx^n)^{-1/n}\int \frac{(cx^n)^{4b+\frac{1}{n}-1}}{\left((cx^n)^{2b}+e^{-2a}\right)^4}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{16e^{4a}x(cx^n)^{4b}\,\text{Hypergeometric2F1}\left(4,\frac{1}{2}\left(4+\frac{1}{bn}\right),\frac{1}{2}\left(6+\frac{1}{bn}\right),-e^{2a}(cx^n)^{2b}\right)}{4bn+1}$$

input
```
Int[Sech[a + b*Log[c*x^n]]^4,x]
```

output
```
(16*E^(4*a)*x*(c*x^n)^(4*b)*Hypergeometric2F1[4, (4 + 1/(b*n))/2, (6 + 1/(
b*n))/2, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + 4*b*n)
```

Defintions of rubi rules used

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6079
```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6081
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[2^p/E^(a*d*p)   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b
*d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [F]

$$\int \operatorname{sech}\left(a+b\ln\left(c\,x^{n}\right)\right)^{4}dx$$

input
```
int(sech(a+b*ln(c*x^n))^4,x)
```

output
```
int(sech(a+b*ln(c*x^n))^4,x)
```

Fricas [F]

$$\int \operatorname{sech}^4(a+b\log(cx^n))\,dx = \int \operatorname{sech}\left(b\log\left(cx^n\right)+a\right)^4\,dx$$

input
```
integrate(sech(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

output
```
integral(sech(b*log(c*x^n) + a)^4, x)
```

Sympy [F]

$$\int \operatorname{sech}^4(a+b\log(cx^n))\,dx = \int \operatorname{sech}^4\left(a+b\log\left(cx^n\right)\right)dx$$

input
```
integrate(sech(a+b*ln(c*x**n))**4,x)
```

output
```
Integral(sech(a + b*log(c*x**n))**4, x)
```

Maxima [F]

$$\int \operatorname{sech}^4(a+b\log(cx^n))\,dx = \int \operatorname{sech}\left(b\log\left(cx^n\right)+a\right)^4\,dx$$

input
```
integrate(sech(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

output
```
16*(4*b^2*n^2 - 1)*integrate(1/48/(b^3*c^(2*b)*n^3*e^(2*b*log(x^n) + 2*a)
+ b^3*n^3), x) + 1/3*((2*b*c^(4*b)*n + c^(4*b))*x*e^(4*b*log(x^n) + 4*a) -
 2*(6*b^2*c^(2*b)*n^2 - b*c^(2*b)*n - c^(2*b))*x*e^(2*b*log(x^n) + 2*a) -
(4*b^2*n^2 - 1)*x)/(b^3*c^(6*b)*n^3*e^(6*b*log(x^n) + 6*a) + 3*b^3*c^(4*b)
*n^3*e^(4*b*log(x^n) + 4*a) + 3*b^3*c^(2*b)*n^3*e^(2*b*log(x^n) + 2*a) + b
^3*n^3)
```

Giac [F]

$$\int \operatorname{sech}^4(a+b\log(cx^n))\,dx = \int \operatorname{sech}(b\log(cx^n)+a)^4\,dx$$

input
```
integrate(sech(a+b*log(c*x^n))^4,x, algorithm="giac")
```

output
```
integrate(sech(b*log(c*x^n) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^4(a+b\log(cx^n))\,dx = \int \frac{1}{\cosh(a+b\ln(c\,x^n))^4}\,dx$$

input
```
int(1/cosh(a + b*log(c*x^n))^4,x)
```

output
```
int(1/cosh(a + b*log(c*x^n))^4, x)
```

Reduce [F]

$$\int \operatorname{sech}^4(a+b\log(cx^n))\,dx = \text{too large to display}$$

input
```
int(sech(a+b*log(c*x^n))^4,x)
```

output

```
(16*e**(2*a)*c**(2*b)*(8*x**(6*b*n)*e**(6*a)*c**(6*b)*int(x**(2*b*n)/(4*x*
*(8*b*n)*e**(8*a)*c**(8*b)*b*n - x**(8*b*n)*e**(8*a)*c**(8*b) + 16*x**(6*b
*n)*e**(6*a)*c**(6*b)*b*n - 4*x**(6*b*n)*e**(6*a)*c**(6*b) + 24*x**(4*b*n)
*e**(4*a)*c**(4*b)*b*n - 6*x**(4*b*n)*e**(4*a)*c**(4*b) + 16*x**(2*b*n)*e*
*(2*a)*c**(2*b)*b*n - 4*x**(2*b*n)*e**(2*a)*c**(2*b) + 4*b*n - 1),x)*b**2*
n**2 + 2*x**(6*b*n)*e**(6*a)*c**(6*b)*int(x**(2*b*n)/(4*x**(8*b*n)*e**(8*a
)*c**(8*b)*b*n - x**(8*b*n)*e**(8*a)*c**(8*b) + 16*x**(6*b*n)*e**(6*a)*c**
(6*b)*b*n - 4*x**(6*b*n)*e**(6*a)*c**(6*b) + 24*x**(4*b*n)*e**(4*a)*c**(4*
b)*b*n - 6*x**(4*b*n)*e**(4*a)*c**(4*b) + 16*x**(2*b*n)*e**(2*a)*c**(2*b)*
b*n - 4*x**(2*b*n)*e**(2*a)*c**(2*b) + 4*b*n - 1),x)*b*n - x**(6*b*n)*e**(
6*a)*c**(6*b)*int(x**(2*b*n)/(4*x**(8*b*n)*e**(8*a)*c**(8*b)*b*n - x**(8*b
*n)*e**(8*a)*c**(8*b) + 16*x**(6*b*n)*e**(6*a)*c**(6*b)*b*n - 4*x**(6*b*n)
*e**(6*a)*c**(6*b) + 24*x**(4*b*n)*e**(4*a)*c**(4*b)*b*n - 6*x**(4*b*n)*e*
*(4*a)*c**(4*b) + 16*x**(2*b*n)*e**(2*a)*c**(2*b)*b*n - 4*x**(2*b*n)*e**(2
*a)*c**(2*b) + 4*b*n - 1),x) + 24*x**(4*b*n)*e**(4*a)*c**(4*b)*int(x**(2*b
*n)/(4*x**(8*b*n)*e**(8*a)*c**(8*b)*b*n - x**(8*b*n)*e**(8*a)*c**(8*b) + 1
6*x**(6*b*n)*e**(6*a)*c**(6*b)*b*n - 4*x**(6*b*n)*e**(6*a)*c**(6*b) + 24*x
**(4*b*n)*e**(4*a)*c**(4*b)*b*n - 6*x**(4*b*n)*e**(4*a)*c**(4*b) + 16*x**(
2*b*n)*e**(2*a)*c**(2*b)*b*n - 4*x**(2*b*n)*e**(2*a)*c**(2*b) + 4*b*n - 1)
,x)*b**2*n**2 + 6*x**(4*b*n)*e**(4*a)*c**(4*b)*int(x**(2*b*n)/(4*x**(8*...
```

3.186 $\int ((1-b^2n^2)\,\mathbf{sech}(a+b\log(cx^n))+2b^2n^2\mathbf{sech}^3(a$

Optimal result

Integrand size = 44, antiderivative size = 40

$$\int \left(\left(1-b^2n^2\right)\operatorname{sech}(a+b\log(cx^n))+2b^2n^2\operatorname{sech}^3(a+b\log(cx^n))\right)\,dx = x\operatorname{sech}(a+b\log(cx^n))+bnx\operatorname{sech}(a+b\log(cx^n))\tanh(a+b\log(cx^n))$$

output
```
x*sech(a+b*ln(c*x^n))+b*n*x*sech(a+b*ln(c*x^n))*tanh(a+b*ln(c*x^n))
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \left(\left(1-b^2n^2\right)\operatorname{sech}(a+b\log(cx^n))+2b^2n^2\operatorname{sech}^3(a+b\log(cx^n))\right)\,dx = x\operatorname{sech}(a+b\log(cx^n))\,(1+bn\tanh(a+b\log(cx^n)))$$

input
```
Integrate[(1 - b^2*n^2)*Sech[a + b*Log[c*x^n]] + 2*b^2*n^2*Sech[a + b*Log[
c*x^n]]^3,x]
```

output
```
x*Sech[a + b*Log[c*x^n]]*(1 + b*n*Tanh[a + b*Log[c*x^n]])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.62 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.48, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2b^2n^2\text{sech}^3(a+b\log(cx^n))+\left(1-b^2n^2\right)\text{sech}(a+b\log(cx^n))\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{16e^{3a}b^2n^2x(cx^n)^{3b}\text{Hypergeometric2F1}\left(3,\frac{3b+\frac{1}{n}}{2b},\frac{1}{2}\left(5+\frac{1}{bn}\right),-e^{2a}(cx^n)^{2b}\right)}{3bn+1}+2e^ax(1-bn)\left(cx^n\right)^b\text{Hypergeometric2F1}\left(1,\frac{b+\frac{1}{n}}{2b},\frac{1}{2}\left(3+\frac{1}{bn}\right),-e^{2a}(cx^n)^{2b}\right)$$

input
```
Int[(1 - b^2*n^2)*Sech[a + b*Log[c*x^n]] + 2*b^2*n^2*Sech[a + b*Log[c*x^n]
]^3,x]
```

output
```
2*E^a*(1 - b*n)*x*(c*x^n)^b*Hypergeometric2F1[1, (b + n^(-1))/(2*b), (3 +
1/(b*n))/2, -(E^(2*a)*(c*x^n)^(2*b))] + (16*b^2*E^(3*a)*n^2*x*(c*x^n)^(3*b
)*Hypergeometric2F1[3, (3*b + n^(-1))/(2*b), (5 + 1/(b*n))/2, -(E^(2*a)*(c
*x^n)^(2*b))])/(1 + 3*b*n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 28.71 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

method	result
parallelrisch	$\operatorname{sech}\left(a+b\ln\left(c\,x^{n}\right)\right)\left(1+\tanh\left(a+b\ln\left(c\,x^{n}\right)\right)bn\right)x$
risch	$\frac{2c^{b}(x^{n})^{b}x\left(nb(x^{n})^{2b}c^{2b}\mathrm{e}^{3a}\mathrm{e}^{-\frac{3ib\pi\,\mathrm{csgn}(ic\,x^{n})^{3}}{2}}\mathrm{e}^{\frac{3ib\pi\,\mathrm{csgn}(ic\,x^{n})^{2}\mathrm{csgn}(ic)}{2}}\mathrm{e}^{\frac{3ib\pi\,\mathrm{csgn}(ix^{n})\mathrm{csgn}(ic\,x^{n})^{2}}{2}}\mathrm{e}^{-\frac{3ib\pi\,\mathrm{csgn}(ix^{n})\mathrm{csgn}(ic\,x}{2}}\right.}{}$

input
```
int((-b^2*n^2+1)*sech(a+b*ln(c*x^n))+2*b^2*n^2*sech(a+b*ln(c*x^n))^3,x,met
hod=_RETURNVERBOSE)
```

output
```
sech(a+b*ln(c*x^n))*(1+tanh(a+b*ln(c*x^n))*b*n)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(40) = 80$.

Time = 0.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.72

$$\int \left(\left(1-b^{2}n^{2}\right)\operatorname{sech}(a+b\log\left(cx^{n}\right))+2b^{2}n^{2}\operatorname{sech}^{3}(a+b\log\left(cx^{n}\right))\right)\,dx$$
$$=\frac{2\left((bn+1)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\,(bn+1)x\cosh\left(bn\log\right.\right.}{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+3\,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+\mathrm{s}}$$

input
```
integrate((-b^2*n^2+1)*sech(a+b*log(c*x^n))+2*b^2*n^2*sech(a+b*log(c*x^n))
^3,x, algorithm="fricas")
```

output
```
2*((b*n + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(b*n + 1)*x*cosh(b*n*
log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + (b*n + 1)*x*sinh(
b*n*log(x) + b*log(c) + a)^2 - (b*n - 1)*x)/(cosh(b*n*log(x) + b*log(c) +
a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2
 + sinh(b*n*log(x) + b*log(c) + a)^3 + (3*cosh(b*n*log(x) + b*log(c) + a)^
2 + 1)*sinh(b*n*log(x) + b*log(c) + a) + 3*cosh(b*n*log(x) + b*log(c) + a)
)
```

Sympy [F]

$$\int \left(\left(1-b^2n^2\right)\operatorname{sech}(a+b\log\left(cx^n\right))+2b^2n^2\operatorname{sech}^3(a+b\log\left(cx^n\right))\right)\,dx$$
$$=\int \left(2b^2n^2\operatorname{sech}^2\left(a+b\log\left(cx^n\right)\right)-b^2n^2+1\right)\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)dx$$

input
```
integrate((-b**2*n**2+1)*sech(a+b*ln(c*x**n))+2*b**2*n**2*sech(a+b*ln(c*x*
*n))**3,x)
```

output
```
Integral((2*b**2*n**2*sech(a + b*log(c*x**n))**2 - b**2*n**2 + 1)*sech(a +
 b*log(c*x**n)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(40) = 80$.

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.40

$$\int \left(\left(1-b^2n^2\right)\operatorname{sech}(a+b\log\left(cx^n\right))+2b^2n^2\operatorname{sech}^3(a+b\log\left(cx^n\right))\right)\,dx$$
$$=\frac{2\left(\left(bc^{3\,b}n+c^{3\,b}\right)xe^{\left(3\,b\log\left(x^n\right)+3\,a\right)}-\left(bc^bn-c^b\right)xe^{\left(b\log\left(x^n\right)+a\right)}\right)}{c^{4\,b}e^{\left(4\,b\log\left(x^n\right)+4\,a\right)}+2\,c^{2\,b}e^{\left(2\,b\log\left(x^n\right)+2\,a\right)}+1}$$

input
```
integrate((-b^2*n^2+1)*sech(a+b*log(c*x^n))+2*b^2*n^2*sech(a+b*log(c*x^n))
^3,x, algorithm="maxima")
```

output
```
2*((b*c^(3*b)*n + c^(3*b))*x*e^(3*b*log(x^n) + 3*a) - (b*c^b*n - c^b)*x*e^
(b*log(x^n) + a))/(c^(4*b)*e^(4*b*log(x^n) + 4*a) + 2*c^(2*b)*e^(2*b*log(x
^n) + 2*a) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(40) = 80$.

Time = 0.27 (sec) , antiderivative size = 215, normalized size of antiderivative = 5.38

$$\int \left(\left(1 - b^2 n^2\right) \operatorname{sech}(a + b \log(cx^n)) + 2b^2 n^2 \operatorname{sech}^3(a + b \log(cx^n)) \right) dx = \frac{2\, bc^{3\, b} n x x^{3\, bn} e^{(3\, a)}}{c^{4\, b} x^{4\, bn} e^{(4\, a)} + 2\, c^{2\, b} x^{2\, bn} e^{(2\, a)} + 1} - \frac{2\, bc^{b} n x x^{bn} e^{a}}{c^{4\, b} x^{4\, bn} e^{(4\, a)} + 2\, c^{2\, b} x^{2\, bn} e^{(2\, a)} + 1} + \frac{2\, c^{3\, b} x x^{3\, bn} e^{(3\, a)}}{c^{4\, b} x^{4\, bn} e^{(4\, a)} + 2\, c^{2\, b} x^{2\, bn} e^{(2\, a)} + 1} + \frac{2\, c^{b} x x^{bn} e^{a}}{c^{4\, b} x^{4\, bn} e^{(4\, a)} + 2\, c^{2\, b} x^{2\, bn} e^{(2\, a)} + 1}$$

input
```
integrate((-b^2*n^2+1)*sech(a+b*log(c*x^n))+2*b^2*n^2*sech(a+b*log(c*x^n))
^3,x, algorithm="giac")
```

output
```
2*b*c^(3*b)*n*x*x^(3*b*n)*e^(3*a)/(c^(4*b)*x^(4*b*n)*e^(4*a) + 2*c^(2*b)*x
^(2*b*n)*e^(2*a) + 1) - 2*b*c^b*n*x*x^(b*n)*e^a/(c^(4*b)*x^(4*b*n)*e^(4*a)
 + 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1) + 2*c^(3*b)*x*x^(3*b*n)*e^(3*a)/(c^(4*
b)*x^(4*b*n)*e^(4*a) + 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1) + 2*c^b*x*x^(b*n)*
e^a/(c^(4*b)*x^(4*b*n)*e^(4*a) + 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int \left(\left(1 - b^2 n^2\right) \operatorname{sech}(a + b \log(cx^n)) + 2b^2 n^2 \operatorname{sech}^3(a + b \log(cx^n)) \right) dx = \frac{2\, x\, e^{a} \left(c\, x^n\right)^b \left(e^{2\, a} \left(c\, x^n\right)^{2\, b} - b\, n + b\, n\, e^{2\, a} \left(c\, x^n\right)^{2\, b} + 1\right)}{\left(e^{2\, a} \left(c\, x^n\right)^{2\, b} + 1\right)^2}$$

input
```
int((2*b^2*n^2)/cosh(a + b*log(c*x^n))^3 - (b^2*n^2 - 1)/cosh(a + b*log(c*
x^n)),x)
```

output
```
(2*x*exp(a)*(c*x^n)^b*(exp(2*a)*(c*x^n)^(2*b) - b*n + b*n*exp(2*a)*(c*x^n)
^(2*b) + 1))/(exp(2*a)*(c*x^n)^(2*b) + 1)^2
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.38

$$\int \left((1-b^2n^2)\,\text{sech}(a+b\log(cx^n))+2b^2n^2\text{sech}^3(a+b\log(cx^n))\right)\,dx = \frac{2x^{bn}e^{a}c^{b}x\left(x^{2bn}e^{2a}c^{2b}bn+x^{2bn}e^{2a}c^{2b}-bn+1\right)}{x^{4bn}e^{4a}c^{4b}+2x^{2bn}e^{2a}c^{2b}+1}$$

input
```
int((-b^2*n^2+1)*sech(a+b*log(c*x^n))+2*b^2*n^2*sech(a+b*log(c*x^n))^3,x)
```

output
```
(2*x**(b*n)*e**a*c**b*x*(x**(2*b*n)*e**(2*a)*c**(2*b)*b*n + x**(2*b*n)*e**
(2*a)*c**(2*b) - b*n + 1))/(x**(4*b*n)*e**(4*a)*c**(4*b) + 2*x**(2*b*n)*e*
*(2*a)*c**(2*b) + 1)
```

3.187 $\int \operatorname{sech}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \operatorname{sech}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx=\frac{2c^6e^{-a}}{\left(c^4+\frac{e^{-2a}}{x^2}\right)^2}$$

output
```
2*c^6/exp(a)/(c^4+1/exp(2*a)/x^2)^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 62 vs. 2(25) = 50.

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.48

$$\int \operatorname{sech}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx = -\frac{2(\cosh(a)-\sinh(a))\left(2c^4x^2+\cosh^2(a)-2\cosh(a)\sinh(a)+\sinh^2(a)\right)}{c^2\left((1+c^4x^2)\cosh(a)+(-1+c^4x^2)\sinh(a)\right)^2}$$

input
```
Integrate[Sech[a + 2*Log[c*Sqrt[x]]]^3,x]
```

output
```
(-2*(Cosh[a] - Sinh[a])*(2*c^4*x^2 + Cosh[a]^2 - 2*Cosh[a]*Sinh[a] + Sinh[
a]^2))/(c^2*((1 + c^4*x^2)*Cosh[a] + (-1 + c^4*x^2)*Sinh[a])^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6079, 6081, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx$$

$$\Big\downarrow \text{6079}$$

$$\frac{2\int c\sqrt{x}\operatorname{sech}^3\left(a+2\log\left(c\sqrt{x}\right)\right)d\left(c\sqrt{x}\right)}{c^2}$$

$$\Big\downarrow \text{6081}$$

$$\frac{16e^{-3a}\int \frac{1}{c^5\left(1+\frac{e^{-2a}}{c^4x^2}\right)^3x^{5/2}}d\left(c\sqrt{x}\right)}{c^2}$$

$$\Big\downarrow \text{793}$$

$$\frac{2e^{-a}}{c^2\left(\frac{e^{-2a}}{c^4x^2}+1\right)^2}$$

input
```
Int[Sech[a + 2*Log[c*Sqrt[x]]]^3,x]
```

output
```
2/(c^2*E^a*(1 + 1/(c^4*E^(2*a)*x^2))^2)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 6079
```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6081
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[2^p/E^(a*d*p)   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b
*d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [F]

$$\int \operatorname{sech}\left(a+2\ln\left(c\sqrt{x}\right)\right)^{3} dx$$

input
```
int(sech(a+2*ln(c*x^(1/2)))^3,x)
```

output
```
int(sech(a+2*ln(c*x^(1/2)))^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(23) = 46$.

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.92

$$\int \operatorname{sech}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx = -\frac{2\left(2\,c^4x^2e^{(2\,a)}+1\right)}{c^{10}x^4e^{(5\,a)}+2\,c^6x^2e^{(3\,a)}+c^2e^a}$$

input
```
integrate(sech(a+2*log(c*x^(1/2)))^3,x, algorithm="fricas")
```

output
```
-2*(2*c^4*x^2*e^(2*a) + 1)/(c^10*x^4*e^(5*a) + 2*c^6*x^2*e^(3*a) + c^2*e^a
)
```

Sympy [F]

$$\int \operatorname{sech}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx=\int \operatorname{sech}^3\left(a+2\log\left(c\sqrt{x}\right)\right)dx$$

input
```
integrate(sech(a+2*ln(c*x**(1/2)))**3,x)
```

output
```
Integral(sech(a + 2*log(c*sqrt(x)))**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(23) = 46$.

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.96

$$\int \operatorname{sech}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx=-\frac{2\left(\frac{2\,c^4x^2e^{(2\,a)}}{c^8x^4e^{(5\,a)}+2\,c^4x^2e^{(3\,a)}+e^a}+\frac{1}{c^8x^4e^{(5\,a)}+2\,c^4x^2e^{(3\,a)}+e^a}\right)}{c^2}$$

input
```
integrate(sech(a+2*log(c*x^(1/2)))^3,x, algorithm="maxima")
```

output
```
-2*(2*c^4*x^2*e^(2*a)/(c^8*x^4*e^(5*a) + 2*c^4*x^2*e^(3*a) + e^a) + 1/(c^8
*x^4*e^(5*a) + 2*c^4*x^2*e^(3*a) + e^a))/c^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int \operatorname{sech}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx=-\frac{2\left(2\,c^4x^2e^{(2\,a)}+1\right)e^{(-a)}}{\left(c^4x^2e^{(2\,a)}+1\right)^2c^2}$$

input
```
integrate(sech(a+2*log(c*x^(1/2)))^3,x, algorithm="giac")
```

output
```
-2*(2*c^4*x^2*e^(2*a) + 1)*e^(-a)/((c^4*x^2*e^(2*a) + 1)^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \operatorname{sech}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx=-\frac{\frac{2\,\mathrm{e}^{-a}}{c^2}+4\,c^2\,x^2\,\mathrm{e}^{a}}{\mathrm{e}^{4\,a}\,c^8\,x^4+2\,\mathrm{e}^{2\,a}\,c^4\,x^2+1}$$

input
```
int(1/cosh(a + 2*log(c*x^(1/2)))^3,x)
```

output
```
-((2*exp(-a))/c^2 + 4*c^2*x^2*exp(a))/(2*c^4*x^2*exp(2*a) + c^8*x^4*exp(4*
a) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int \operatorname{sech}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx=\frac{2e^{3a}c^6x^4}{e^{4a}c^8x^4+2e^{2a}c^4x^2+1}$$

input
```
int(sech(a+2*log(c*x^(1/2)))^3,x)
```

output
```
(2*e**(3*a)*c**6*x**4)/(e**(4*a)*c**8*x**4 + 2*e**(2*a)*c**4*x**2 + 1)
```

3.188 $\int \operatorname{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \operatorname{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx=\frac{2c^2e^{-3a}}{\left(e^{-2a}+\frac{c^4}{x^2}\right)^2}$$

output
```
2*c^2/exp(3*a)/(exp(-2*a)+c^4/x^2)^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 64 vs. 2(25) = 50.

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.56

$$\int \operatorname{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx = -\frac{2c^6((c^4+2x^2)\cosh(a)+(c^4-2x^2)\sinh(a))\,(\cosh(2a)+\sinh(2a))}{((c^4+x^2)\cosh(a)+(c^4-x^2)\sinh(a))^2}$$

input
```
Integrate[Sech[a + 2*Log[c/Sqrt[x]]]^3,x]
```

output
```
(-2*c^6*((c^4 + 2*x^2)*Cosh[a] + (c^4 - 2*x^2)*Sinh[a])*(Cosh[2*a] + Sinh[
2*a]))/((c^4 + x^2)*Cosh[a] + (c^4 - x^2)*Sinh[a])^2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6079, 6081, 795, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)dx$$

$$\Big\downarrow \text{ 6079}$$

$$-2c^2\int \frac{x^{3/2}\operatorname{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)}{c^3}d\frac{c}{\sqrt{x}}$$

$$\Big\downarrow \text{ 6081}$$

$$-16e^{-3a}c^2\int \frac{x^{9/2}}{c^9\left(\frac{e^{-2a}x^2}{c^4}+1\right)^3}d\frac{c}{\sqrt{x}}$$

$$\Big\downarrow \text{ 795}$$

$$-16e^{-3a}c^2\int \frac{c^3}{\left(\frac{c^4}{x^2}+e^{-2a}\right)^3x^{3/2}}d\frac{c}{\sqrt{x}}$$

$$\Big\downarrow \text{ 793}$$

$$\frac{2e^{-3a}c^2}{\left(e^{-2a}+\frac{c^4}{x^2}\right)^2}$$

input
```
Int[Sech[a + 2*Log[c/Sqrt[x]]]^3,x]
```

output
```
(2*c^2)/(E^(3*a)*(E^(-2*a) + c^4/x^2)^2)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 6079
```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6081
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[2^p/E^(a*d*p)   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b
*d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [F]

$$\int \operatorname{sech}\left(a+2\ln\left(\frac{c}{\sqrt{x}}\right)\right)^3 dx$$

input
```
int(sech(a+2*ln(c/x^(1/2)))^3,x)
```

output
```
int(sech(a+2*ln(c/x^(1/2)))^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(23) = 46$.

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \operatorname{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx = -\frac{2\left(c^{10}e^{(5\,a)}+2\,c^6x^2e^{(3\,a)}\right)}{c^8e^{(4\,a)}+2\,c^4x^2e^{(2\,a)}+x^4}$$

input
```
integrate(sech(a+2*log(c/x^(1/2)))^3,x, algorithm="fricas")
```

output
```
-2*(c^10*e^(5*a) + 2*c^6*x^2*e^(3*a))/(c^8*e^(4*a) + 2*c^4*x^2*e^(2*a) + x
^4)
```

Sympy [F]

$$\int \operatorname{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx = \int \operatorname{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx$$

input
```
integrate(sech(a+2*ln(c/x**(1/2)))**3,x)
```

output
```
Integral(sech(a + 2*log(c/sqrt(x)))**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(23) = 46$.

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \operatorname{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx = -\frac{2\left(c^{10}e^{(5\,a)}+2\,c^6x^2e^{(3\,a)}\right)}{c^8e^{(4\,a)}+2\,c^4x^2e^{(2\,a)}+x^4}$$

input
```
integrate(sech(a+2*log(c/x^(1/2)))^3,x, algorithm="maxima")
```

output
```
-2*(c^10*e^(5*a) + 2*c^6*x^2*e^(3*a))/(c^8*e^(4*a) + 2*c^4*x^2*e^(2*a) + x
^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \operatorname{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx=-\frac{2\left(c^{10}e^{(5\,a)}+2\,c^6x^2e^{(3\,a)}\right)}{\left(c^4e^{(2\,a)}+x^2\right)^2}$$

input
```
integrate(sech(a+2*log(c/x^(1/2)))^3,x, algorithm="giac")
```

output
```
-2*(c^10*e^(5*a) + 2*c^6*x^2*e^(3*a))/(c^4*e^(2*a) + x^2)^2
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \operatorname{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx=\frac{2\,c^2\,x^4\,\mathrm{e}^a}{\mathrm{e}^{4\,a}\,c^8+2\,\mathrm{e}^{2\,a}\,c^4\,x^2+x^4}$$

input
```
int(1/cosh(a + 2*log(c/x^(1/2)))^3,x)
```

output
```
(2*c^2*x^4*exp(a))/(c^8*exp(4*a) + x^4 + 2*c^4*x^2*exp(2*a))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int \operatorname{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx=\frac{2e^{a}c^{2}x^{4}}{e^{4a}c^{8}+2e^{2a}c^{4}x^{2}+x^{4}}$$

input
```
int(sech(a+2*log(c/x^(1/2)))^3,x)
```

output
```
(2*e**a*c**2*x**4)/(e**(4*a)*c**8 + 2*e**(2*a)*c**4*x**2 + x**4)
```

3.189 $\int \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 89

$$\int \operatorname{sech}^p\left(a+\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx = \frac{e^{2a}(2-p)x\left(cx^n\right)^{-\frac{2}{n(2-p)}}\left(1+e^{-2a}\left(cx^n\right)^{\frac{2}{n(2-p)}}\right)\operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

output
```
1/2*exp(2*a)*(2-p)*x*(1+(c*x^n)^(2/n/(2-p))/exp(2*a))*sech(a-ln(c*x^n)/n/(
2-p))^p/(1-p)/((c*x^n)^(2/n/(2-p)))
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.28

$$\int \operatorname{sech}^p\left(a+\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx = -\frac{2^{-1+p}(-2+p)x\left(\frac{e^a\left(cx^n\right)^{\frac{1}{2n-np}}}{e^{2a}+\left(cx^n\right)^{-\frac{2}{n(-2+p)}}}\right)^p\left(-1+e^{2a}\left(cx^n\right)^{\frac{2}{n(-2+p)}}\left(-1+\left(1+e^{-2a}\left(cx^n\right)^{-\frac{2}{n(-2+p)}}\right)^p\right)\right)}{-1+p}$$

input
```
Integrate[Sech[a + Log[c*x^n]/(n*(-2 + p))]^p,x]
```

output
```
-((2^(-1 + p)*(-2 + p)*x*((E^a*(c*x^n)^(2*n - n*p)^(-1))/(E^(2*a) + (c*x^n
)^(-2/(n*(-2 + p)))))^p*(-1 + E^(2*a)*(c*x^n)^(2/(n*(-2 + p)))*(-1 + (1 +
1/(E^(2*a)*(c*x^n)^(2/(n*(-2 + p)))))^p)))/(-1 + p))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6079, 6083, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(p-2)}\right)dx$$
$$\Big\downarrow \text{6079}$$
$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)d(cx^n)}{n}$$
$$\Big\downarrow \text{6083}$$
$$\frac{x(cx^n)^{-\frac{p}{n(2-p)}-\frac{1}{n}}\left(e^{-2a}(cx^n)^{\frac{2}{n(2-p)}}+1\right)^p\operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)\int (cx^n)^{\frac{p}{2n-np}+\frac{1}{n}-1}\left(e^{-2a}(cx^n)^{\frac{2}{n(2-p)}}+1\right)^{-p}d(cx^n)}{n}$$
$$\Big\downarrow \text{793}$$
$$\frac{e^{2a}(2-p)x(cx^n)^{-\frac{p}{n(2-p)}-\frac{1}{n}}\left(e^{-2a}(cx^n)^{\frac{2}{n(2-p)}}+1\right)\operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

input
```
Int[Sech[a + Log[c*x^n]/(n*(-2 + p))]^p,x]
```

output
```
(E^(2*a)*(2 - p)*x*(c*x^n)^(-n^(-1) - p/(n*(2 - p)))*(1 + (c*x^n)^(2/(n*(2
 - p)))/E^(2*a))*Sech[a - Log[c*x^n]/(n*(2 - p))]^p)/(2*(1 - p))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 6079
```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6083
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \operatorname{sech}\left(a+\frac{\ln\left(c\,x^{n}\right)}{n\left(-2+p\right)}\right)^{p}dx$$

input
```
int(sech(a+ln(c*x^n)/n/(-2+p))^p,x)
```

output
```
int(sech(a+ln(c*x^n)/n/(-2+p))^p,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(76) = 152$.

Time = 0.25 (sec) , antiderivative size = 474, normalized size of antiderivative = 5.33

$$\int \operatorname{sech}^{p}\left(a+\frac{\log\left(cx^{n}\right)}{n(-2+p)}\right)\,dx = \frac{(p-2)x\cosh\left(p\log\left(\frac{2\left(\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)+\sinh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)\right)}{\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)^{2}+2\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)\sinh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)+\sinh\left(an\right.}\right.\right.}{}$$

input
```
integrate(sech(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
```

output
```
((p - 2)*x*cosh(p*log(2*(cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2
*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)))/(cosh((a*n*p
 - 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 + 2*cosh((a*n*p - 2*a*n + n*l
og(x) + log(c))/(n*p - 2*n))*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p
 - 2*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 + 1)))*
cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)) + (p - 2)*x*cosh((a*
n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n))*sinh(p*log(2*(cosh((a*n*p -
2*a*n + n*log(x) + log(c))/(n*p - 2*n)) + sinh((a*n*p - 2*a*n + n*log(x) +
 log(c))/(n*p - 2*n)))/(cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*
n))^2 + 2*cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n))*sinh((a*n*
p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)) + sinh((a*n*p - 2*a*n + n*log(
x) + log(c))/(n*p - 2*n))^2 + 1))))/((p - 1)*cosh((a*n*p - 2*a*n + n*log(x
) + log(c))/(n*p - 2*n)) - (p - 1)*sinh((a*n*p - 2*a*n + n*log(x) + log(c)
)/(n*p - 2*n)))
```

Sympy [F]

$$\int \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\int \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n\,(p-2)}\right)dx$$

input
```
integrate(sech(a+ln(c*x**n)/n/(-2+p))**p,x)
```

output
```
Integral(sech(a + log(c*x**n)/(n*(p - 2)))**p, x)
```

Maxima [F]

$$\int \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\int \operatorname{sech}\left(a+\frac{\log(cx^n)}{n(p-2)}\right)^p\,dx$$

input
```
integrate(sech(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
```

output
```
integrate(sech(a + log(c*x^n)/(n*(p - 2)))^p, x)
```

Giac [F]

$$\int \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\int \operatorname{sech}\left(a+\frac{\log(cx^n)}{n(p-2)}\right)^p\,dx$$

input
```
integrate(sech(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
```

output
```
integrate(sech(a + log(c*x^n)/(n*(p - 2)))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\int \left(\frac{1}{\cosh\left(a+\frac{\ln(c\,x^n)}{n\,(p-2)}\right)}\right)^p\,dx$$

input
```
int((1/cosh(a + log(c*x^n)/(n*(p - 2))))^p,x)
```

output
```
int((1/cosh(a + log(c*x^n)/(n*(p - 2))))^p, x)
```

Reduce [F]

$$\int \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\int \operatorname{sech}\left(\frac{\log(x^n c)+anp-2an}{np-2n}\right)^p dx$$

input
```
int(sech(a+log(c*x^n)/n/(-2+p))^p,x)
```

output
```
int(sech((log(x**n*c) + a*n*p - 2*a*n)/(n*p - 2*n))**p,x)
```

3.190 $\int \operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \operatorname{sech}^p\left(a-\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=\frac{(2-p)x\left(1+e^{-2a}\left(cx^n\right)^{-\frac{2}{n(2-p)}}\right)\operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

output
```
(2-p)*x*(1+1/exp(2*a)/((c*x^n)^(2/n/(2-p))))*sech(a+ln(c*x^n)/n/(2-p))^p/(
2-2*p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.93 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.66

$$\int \operatorname{sech}^p\left(a-\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=\frac{2^{-1+p}e^{-a}(-2+p)x\left(cx^n\right)^{\frac{1}{n(-2+p)}}\left(\frac{e^{\frac{a(2+p)}{-2+p}}\left(cx^n\right)^{\frac{1}{n(-2+p)}}}{e^{\frac{2ap}{-2+p}}+e^{\frac{4a}{-2+p}}\left(cx^n\right)^{\frac{2}{n(-2+p)}}}\right)^{-1+p}}{-1+p}$$

input
```
Integrate[Sech[a - Log[c*x^n]/(n*(-2 + p))]^p,x]
```

output
```
(2^(-1 + p)*(-2 + p)*x*(c*x^n)^(1/(n*(-2 + p)))*((E^((a*(2 + p))/(-2 + p))
*(c*x^n)^(1/(n*(-2 + p))))/(E^((2*a*p)/(-2 + p)) + E^((4*a)/(-2 + p))*(c*x
^n)^(2/(n*(-2 + p)))))^(-1 + p))/(E^a*(-1 + p))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.63, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6079, 6083, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^p\left(a-\frac{\log\left(cx^n\right)}{n(p-2)}\right)dx$$

$$\Big\downarrow \text{ 6079}$$

$$\frac{x\left(cx^n\right)^{-1/n}\int \left(cx^n\right)^{\frac{1}{n}-1}\operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)d\left(cx^n\right)}{n}$$

$$\Big\downarrow \text{ 6083}$$

$$\frac{x\left(cx^n\right)^{\frac{p}{n(2-p)}-\frac{1}{n}}\left(e^{-2a}\left(cx^n\right)^{-\frac{2}{n(2-p)}}+1\right)^p\operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)\int \left(cx^n\right)^{\frac{1-\frac{p}{2-p}}{n}-1}\left(e^{-2a}\left(cx^n\right)^{-\frac{2}{n(2-p)}}+1\right)^{-p}d\left(cx^n\right)}{n}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{(2-p)x\left(cx^n\right)^{\frac{2(1-p)}{n(2-p)}+\frac{p}{n(2-p)}-\frac{1}{n}}\left(e^{-2a}\left(cx^n\right)^{-\frac{2}{n(2-p)}}+1\right)\operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

input
```
Int[Sech[a - Log[c*x^n]/(n*(-2 + p))]^p,x]
```

output
```
((2 - p)*x*(c*x^n)^(-n^(-1) + (2*(1 - p))/(n*(2 - p)) + p/(n*(2 - p)))*(1
+ 1/(E^(2*a)*(c*x^n)^(2/(n*(2 - p)))))*Sech[a + Log[c*x^n]/(n*(2 - p))]^p)
/(2*(1 - p))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 6079
```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6083
```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \operatorname{sech}\left(a-\frac{\ln\left(c\,x^{n}\right)}{n\left(-2+p\right)}\right)^{p}dx$$

input
```
int(sech(a-ln(c*x^n)/n/(-2+p))^p,x)
```

output
```
int(sech(a-ln(c*x^n)/n/(-2+p))^p,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 538 vs. 2(55) = 110.

Time = 0.24 (sec) , antiderivative size = 538, normalized size of antiderivative = 8.28

$$\int \operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\,dx = \frac{(p-2)x\cosh\left(p\log\left(\frac{2\left(\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)+\sinh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)\right)}{\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)^{2}+2\,\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)\sinh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)+\sin}\right.\right.}{}$$

input
```
integrate(sech(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
```

output
```
((p - 2)*x*cosh(p*log(2*(cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p -
2*n)) + sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)))/(cosh(-(a*
n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 + 2*cosh(-(a*n*p - 2*a*n -
 n*log(x) - log(c))/(n*p - 2*n))*sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))
/(n*p - 2*n)) + sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 +
 1)))*cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + (p - 2)*x*c
osh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))*sinh(p*log(2*(cosh(-
(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + sinh(-(a*n*p - 2*a*n -
n*log(x) - log(c))/(n*p - 2*n)))/(cosh(-(a*n*p - 2*a*n - n*log(x) - log(c)
)/(n*p - 2*n))^2 + 2*cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)
)*sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + sinh(-(a*n*p -
2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 + 1))))/((p - 1)*cosh(-(a*n*p -
2*a*n - n*log(x) - log(c))/(n*p - 2*n)) - (p - 1)*sinh(-(a*n*p - 2*a*n - n
*log(x) - log(c))/(n*p - 2*n)))
```

Sympy [F]

$$\int \operatorname{sech}^p\left(a-\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=\int \operatorname{sech}^p\left(a-\frac{\log\left(cx^n\right)}{n\left(p-2\right)}\right)dx$$

input
```
integrate(sech(a-ln(c*x**n)/n/(-2+p))**p,x)
```

output
```
Integral(sech(a - log(c*x**n)/(n*(p - 2)))**p, x)
```

Maxima [F]

$$\int \operatorname{sech}^p\left(a-\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=\int \operatorname{sech}\left(a-\frac{\log\left(cx^n\right)}{n(p-2)}\right)^p\,dx$$

input
```
integrate(sech(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
```

output
```
integrate(sech(-a + log(c*x^n)/(n*(p - 2)))^p, x)
```

Giac [F]

$$\int \operatorname{sech}^p\left(a-\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=\int \operatorname{sech}\left(a-\frac{\log\left(cx^n\right)}{n(p-2)}\right)^p\,dx$$

input
```
integrate(sech(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
```

output
```
integrate(sech(a - log(c*x^n)/(n*(p - 2)))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^p\left(a-\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=\int \left(\frac{1}{\cosh\left(a-\frac{\ln(c\,x^n)}{n\,(p-2)}\right)}\right)^p\,dx$$

input
```
int((1/cosh(a - log(c*x^n)/(n*(p - 2))))^p,x)
```

output
```
int((1/cosh(a - log(c*x^n)/(n*(p - 2))))^p, x)
```

Reduce [F]

$$\int \operatorname{sech}^p\left(a-\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=\int \operatorname{sech}\left(\frac{\log(x^nc)-anp+2an}{np-2n}\right)^p dx$$

input
```
int(sech(a-log(c*x^n)/n/(-2+p))^p,x)
```

output
```
int(sech((log(x**n*c) - a*n*p + 2*a*n)/(n*p - 2*n))**p,x)
```

3.191 $\int \frac{\operatorname{sech}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{\operatorname{sech}(a+b\log(cx^n))}{x}\,dx = \frac{\arctan(\sinh(a+b\log(cx^n)))}{bn}$$

output
```
arctan(sinh(a+b*ln(c*x^n)))/b/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\operatorname{sech}(a+b\log(cx^n))}{x}\,dx = -\frac{\cot^{-1}(\sinh(a+b\log(cx^n)))}{bn}$$

input
```
Integrate[Sech[a + b*Log[c*x^n]]/x,x]
```

output
```
-(ArcCot[Sinh[a + b*Log[c*x^n]]]/(b*n))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3039, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{ 3039}$$

$$\frac{\int \text{sech}(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \csc\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\arctan(\sinh(a+b\log(cx^n)))}{bn}$$

input
```
Int[Sech[a + b*Log[c*x^n]]/x,x]
```

output
```
ArcTan[Sinh[a + b*Log[c*x^n]]]/(b*n)
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\arctan(\sinh(a+b\ln(c\,x^n)))}{bn}$
default	$\frac{\arctan(\sinh(a+b\ln(c\,x^n)))}{bn}$
parallelrisch	$-\frac{i\left(\ln\left(\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)-i\right)-\ln\left(\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)+i\right)\right)}{bn}$
risch	$\frac{i\ln\left(c^b(x^n)^b \mathrm{e}^a \mathrm{e}^{-\frac{ib\pi\,\mathrm{csgn}(ic\,x^n)^3}{2}} \mathrm{e}^{\frac{ib\pi\,\mathrm{csgn}(ic\,x^n)^2\,\mathrm{csgn}(ic)}{2}} \mathrm{e}^{\frac{ib\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}{2}} \mathrm{e}^{-\frac{ib\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)}{2}}\right.}{bn}$

input
```
int(sech(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
```

output
```
arctan(sinh(a+b*ln(c*x^n)))/b/n
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{\operatorname{sech}(a+b\log(cx^n))}{x}\,dx = \frac{2\arctan\left(\cosh\left(bn\log(x)+b\log(c)+a\right)+\sinh\left(bn\log(x)+b\log(c)+a\right)\right)}{bn}$$

input
```
integrate(sech(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

output
```
2*arctan(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)
)/(b*n)
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.32

$$\int \frac{\operatorname{sech}(a+b\log(cx^n))}{x}\,dx = -\begin{cases} -\log(x)\operatorname{sech}(a) & \text{for } b=0 \\ -\log(x)\operatorname{sech}(a+b\log(c)) & \text{for } n=0 \\ -\frac{2\operatorname{atan}\left(\tanh\left(\frac{a}{2}+\frac{b\log(cx^n)}{2}\right)\right)}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(sech(a+b*ln(c*x**n))/x,x)
```

output
```
-Piecewise((-log(x)*sech(a), Eq(b, 0)), (-log(x)*sech(a + b*log(c)), Eq(n,
 0)), (-2*atan(tanh(a/2 + b*log(c*x**n)/2))/(b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(a+b\log(cx^n))}{x}\,dx = \frac{\arctan(\sinh(b\log(cx^n)+a))}{bn}$$

input
```
integrate(sech(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

output
```
arctan(sinh(b*log(c*x^n) + a))/(b*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{\operatorname{sech}(a+b\log(cx^n))}{x}\,dx = \frac{2\arctan\left(\frac{c^{2b}x^{bn}e^a}{c^b}\right)}{bn}$$

input
```
integrate(sech(a+b*log(c*x^n))/x,x, algorithm="giac")
```

output
```
2*arctan(c^(2*b)*x^(b*n)*e^a/c^b)/(b*n)
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

$$\int \frac{\operatorname{sech}(a+b\log(cx^n))}{x}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{-a}\,\sqrt{b^2\,n^2}}{b\,n\,{\left(c\,x^n\right)}^b}\right)}{\sqrt{b^2\,n^2}}$$

input
```
int(1/(x*cosh(a + b*log(c*x^n))),x)
```

output
```
-(2*atan((exp(-a)*(b^2*n^2)^(1/2))/(b*n*(c*x^n)^b)))/(b^2*n^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}(a+b\log(cx^n))}{x}\,dx = \frac{2atan(x^{bn}e^ac^b)}{bn}$$

input
```
int(sech(a+b*log(c*x^n))/x,x)
```

output
```
(2*atan(x**(b*n)*e**a*c**b))/(b*n)
```

3.192 $\int \frac{\text{sech}^2(a+b\log(cx^n))}{x} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 18

$$\int \frac{\text{sech}^2(a+b\log(cx^n))}{x} \, dx = \frac{\tanh(a+b\log(cx^n))}{bn}$$

output
```
tanh(a+b*ln(c*x^n))/b/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}^2(a+b\log(cx^n))}{x} \, dx = \frac{\tanh(a+b\log(cx^n))}{bn}$$

input
```
Integrate[Sech[a + b*Log[c*x^n]]^2/x,x]
```

output
```
Tanh[a + b*Log[c*x^n]]/(b*n)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3039, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{ 3039}$$

$$\frac{\int \text{sech}^2(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \csc\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^2d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 4254}$$

$$\frac{i\int 1d(-i\tanh(a+b\log(cx^n)))}{bn}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\tanh(a+b\log(cx^n))}{bn}$$

input
```
Int[Sech[a + b*Log[c*x^n]]^2/x,x]
```

output
```
Tanh[a + b*Log[c*x^n]]/(b*n)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\tanh(a+b\ln(c\,x^n))}{bn}$
default	$\frac{\tanh(a+b\ln(c\,x^n))}{bn}$
parallelrisch	$\frac{\sinh(a+b\ln(c\,x^n))}{\cosh(a+b\ln(c\,x^n))bn}$
risch	$-\frac{2}{bn\left((x^n)^{2b}c^{2b}\mathrm{e}^{2a}\mathrm{e}^{ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}\mathrm{e}^{-ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)}\mathrm{e}^{-ib\pi\,\mathrm{csgn}(ic\,x^n)^3}\mathrm{e}^{ib\pi\,\mathrm{csgn}(ic\,x^n)^2\,\mathrm{csgn}(ic}\right.}$

input
```
int(sech(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
```

output
```
tanh(a+b*ln(c*x^n))/b/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(18) = 36$.

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.89

$$\int \frac{\operatorname{sech}^2(a + b\log(cx^n))}{x}\,dx =$$

$$-\frac{2}{bn\cosh(bn\log(x) + b\log(c) + a)^2 + 2\,bn\cosh(bn\log(x) + b\log(c) + a)\sinh(bn\log(x) + b\log(c) +}$$

input
```
integrate(sech(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

output
```
-2/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*b*n*cosh(b*n*log(x) + b*log(
c) + a)*sinh(b*n*log(x) + b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) +
 a)^2 + b*n)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(a + b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{sech}^2(a + b\log(cx^n))}{x}\,dx$$

input
```
integrate(sech(a+b*ln(c*x**n))**2/x,x)
```

output
```
Integral(sech(a + b*log(c*x**n))**2/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \frac{\operatorname{sech}^2(a + b\log(cx^n))}{x}\,dx = -\frac{2}{bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)} + bn}$$

input
```
integrate(sech(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

output
```
-2/(b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \frac{\operatorname{sech}^2(a+b\log(cx^n))}{x}\,dx = -\frac{2}{(c^{2\,b}x^{2\,bn}e^{(2\,a)}+1)bn}$$

input
```
integrate(sech(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

output
```
-2/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1)*b*n)
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{\operatorname{sech}^2(a+b\log(cx^n))}{x}\,dx = -\frac{2}{b\,n+b\,n\,\mathrm{e}^{2\,a}\,{(c\,x^n)}^{2\,b}}$$

input
```
int(1/(x*cosh(a + b*log(c*x^n))^2),x)
```

output
```
-2/(b*n + b*n*exp(2*a)*(c*x^n)^(2*b))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.50

$$\int \frac{\operatorname{sech}^2(a+b\log(cx^n))}{x}\,dx = \frac{2x^{2bn}e^{2a}c^{2b}}{bn\,(x^{2bn}e^{2a}c^{2b}+1)}$$

input
```
int(sech(a+b*log(c*x^n))^2/x,x)
```

output
```
(2*x**(2*b*n)*e**(2*a)*c**(2*b))/(b*n*(x**(2*b*n)*e**(2*a)*c**(2*b) + 1))
```

3.193 $\int \frac{\operatorname{sech}^3(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \frac{\operatorname{sech}^3(a+b\log(cx^n))}{x}\,dx = \frac{\arctan(\sinh(a+b\log(cx^n)))}{2bn} + \frac{\operatorname{sech}(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{2bn}$$

output
```
1/2*arctan(sinh(a+b*ln(c*x^n)))/b/n+1/2*sech(a+b*ln(c*x^n))*tanh(a+b*ln(c*
x^n))/b/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^3(a+b\log(cx^n))}{x}\,dx = \frac{\arctan(\sinh(a+b\log(cx^n)))}{2bn} + \frac{\operatorname{sech}(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{2bn}$$

input
```
Integrate[Sech[a + b*Log[c*x^n]]^3/x,x]
```

output
```
ArcTan[Sinh[a + b*Log[c*x^n]]]/(2*b*n) + (Sech[a + b*Log[c*x^n]]*Tanh[a +
b*Log[c*x^n]])/(2*b*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3039, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^3(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \text{sech}^3(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^3d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{1}{2}\int \text{sech}(a+b\log(cx^n))d\log(cx^n)+\frac{\tanh(a+b\log(cx^n))\text{sech}(a+b\log(cx^n))}{2b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\tanh(a+b\log(cx^n))\text{sech}(a+b\log(cx^n))}{2b}+\frac{1}{2}\int \csc\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{\arctan(\sinh(a+b\log(cx^n)))}{2b}+\frac{\tanh(a+b\log(cx^n))\text{sech}(a+b\log(cx^n))}{2b}}{n}$$

input
```
Int[Sech[a + b*Log[c*x^n]]^3/x,x]
```

output
```
(ArcTan[Sinh[a + b*Log[c*x^n]]]/(2*b) + (Sech[a + b*Log[c*x^n]]*Tanh[a + b
*Log[c*x^n]])/(2*b))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 17.40 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{\frac{\operatorname{sech}(a+b\ln(c\,x^n))\tanh(a+b\ln(c\,x^n))}{2}+\arctan\left(\mathrm{e}^{a+b\ln(c\,x^n)}\right)}{nb}$
default	$\frac{\frac{\operatorname{sech}(a+b\ln(c\,x^n))\tanh(a+b\ln(c\,x^n))}{2}+\arctan\left(\mathrm{e}^{a+b\ln(c\,x^n)}\right)}{nb}$
parallelrisch	$\frac{i(-1-\cosh(2b\ln(c\,x^n)+2a))\ln\left(\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)-i\right)+i(\cosh(2b\ln(c\,x^n)+2a)+1)\ln\left(\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)+i\right)+}{2bn\left(\cosh\left(4b\ln\left(\sqrt{c\,x^n}\right)+2a\right)+1\right)}$
risch	$\frac{c^b(x^n)^b\left(\left((x^n)^{2b}c^{2b}\mathrm{e}^{3a}\mathrm{e}^{-\frac{3ib\pi\operatorname{csgn}(ic\,x^n)^3}{2}}\mathrm{e}^{\frac{3ib\pi\operatorname{csgn}(ic\,x^n)^2\operatorname{csgn}(ic)}{2}}\mathrm{e}^{\frac{3ib\pi\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)^2}{2}}\mathrm{e}^{-\frac{3ib\pi\operatorname{csgn}(ix^n)\operatorname{csgn}(i}{2}}\right.\right.}{bn\left((x^n)^{2b}c^{2b}\mathrm{e}^{2a}\mathrm{e}^{ib\pi\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)^2}\mathrm{e}^{-ib\pi\operatorname{csgn}(ix^n)\operatorname{cs}}\right.}$

input
```
int(sech(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(1/2*sech(a+b*ln(c*x^n))*tanh(a+b*ln(c*x^n))+arctan(exp(a+b*ln(c*x^n
))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(51) = 102$.

Time = 0.09 (sec) , antiderivative size = 452, normalized size of antiderivative = 8.22

$$\int \frac{\operatorname{sech}^3(a + b\log(cx^n))}{x}\,dx = \text{Too large to display}$$

input
```
integrate(sech(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

output
```
(cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a)*sin
h(b*n*log(x) + b*log(c) + a)^2 + sinh(b*n*log(x) + b*log(c) + a)^3 + (cosh
(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n
*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cosh(
b*n*log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) + a)^2 + 2*co
sh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 + c
osh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*arcta
n(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)) + (3*
cosh(b*n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a) - c
osh(b*n*log(x) + b*log(c) + a))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 4
*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + b
*n*sinh(b*n*log(x) + b*log(c) + a)^4 + 2*b*n*cosh(b*n*log(x) + b*log(c) +
a)^2 + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + b*n)*sinh(b*n*log(x) +
 b*log(c) + a)^2 + b*n + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + b*n*co
sh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(a+b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{sech}^3(a+b\log(cx^n))}{x}\,dx$$

input
```
integrate(sech(a+b*ln(c*x**n))**3/x,x)
```

output
```
Integral(sech(a + b*log(c*x**n))**3/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^3(a+b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{sech}(b\log(cx^n)+a)^3}{x}\,dx$$

input
```
integrate(sech(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

output
```
8*c^b*integrate(1/8*e^(b*log(x^n) + a)/(c^(2*b)*x*e^(2*b*log(x^n) + 2*a) +
 x), x) + (c^(3*b)*e^(3*b*log(x^n) + 3*a) - c^b*e^(b*log(x^n) + a))/(b*c^(
4*b)*n*e^(4*b*log(x^n) + 4*a) + 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(51) = 102$.

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.09

$$\int \frac{\operatorname{sech}^3(a+b\log(cx^n))}{x}\,dx = c^{3b}\left(\frac{\arctan\left(\frac{c^{2b}x^{bn}e^a}{c^b}\right)e^{(-3a)}}{bc^{2b}c^bn} + \frac{\left(c^{2b}x^{3bn}e^{(2a)} - x^{bn}\right)e^{(-2a)}}{\left(c^{2b}x^{2bn}e^{(2a)}+1\right)^2bc^{2b}n}\right)e^{(3a)}$$

input
```
integrate(sech(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

output
```
c^(3*b)*(arctan(c^(2*b)*x^(b*n)*e^a/c^b)*e^(-3*a)/(b*c^(2*b)*c^b*n) + (c^(
2*b)*x^(3*b*n)*e^(2*a) - x^(b*n))*e^(-2*a)/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1
)^2*b*c^(2*b)*n))*e^(3*a)
```

Mupad [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.53

$$\int \frac{\operatorname{sech}^3(a+b\log(cx^n))}{x}\,dx = \frac{2\,\mathrm{e}^{-a}}{(c\,x^n)^b\,\left(b\,n+\frac{2\,b\,n\,\mathrm{e}^{-2\,a}}{(c\,x^n)^{2\,b}}+\frac{b\,n\,\mathrm{e}^{-4\,a}}{(c\,x^n)^{4\,b}}\right)} - \frac{\mathrm{e}^{-a}}{(c\,x^n)^b\,\left(b\,n+\frac{b\,n\,\mathrm{e}^{-2\,a}}{(c\,x^n)^{2\,b}}\right)} - \frac{\mathrm{atan}\left(\frac{\mathrm{e}^{-a}\,\sqrt{b^2\,n^2}}{b\,n\,(c\,x^n)^b}\right)}{\sqrt{b^2\,n^2}}$$

input
```
int(1/(x*cosh(a + b*log(c*x^n))^3),x)
```

output
```
(2*exp(-a))/((c*x^n)^b*(b*n + (2*b*n*exp(-2*a))/(c*x^n)^(2*b) + (b*n*exp(-
4*a))/(c*x^n)^(4*b))) - exp(-a)/((c*x^n)^b*(b*n + (b*n*exp(-2*a))/(c*x^n)^
(2*b))) - atan((exp(-a)*(b^2*n^2)^(1/2))/(b*n*(c*x^n)^b))/(b^2*n^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.75

$$\int \frac{\operatorname{sech}^3(a+b\log(cx^n))}{x}\,dx = \frac{x^{4bn}e^{4a}c^{4b}\,atan\left(x^{bn}e^{a}c^{b}\right)+2x^{2bn}e^{2a}c^{2b}\,atan\left(x^{bn}e^{a}c^{b}\right)+atan\left(x^{bn}e^{a}c^{b}\right)+x^{3bn}e^{3a}c^{3b}-x^{bn}e^{a}c^{b}}{bn\left(x^{4bn}e^{4a}c^{4b}+2x^{2bn}e^{2a}c^{2b}+1\right)}$$

input
```
int(sech(a+b*log(c*x^n))^3/x,x)
```

output

```
(x**(4*b*n)*e**(4*a)*c**(4*b)*atan(x**(b*n)*e**a*c**b) + 2*x**(2*b*n)*e**(
2*a)*c**(2*b)*atan(x**(b*n)*e**a*c**b) + atan(x**(b*n)*e**a*c**b) + x**(3*
b*n)*e**(3*a)*c**(3*b) - x**(b*n)*e**a*c**b)/(b*n*(x**(4*b*n)*e**(4*a)*c**
(4*b) + 2*x**(2*b*n)*e**(2*a)*c**(2*b) + 1))
```

3.194 $\int \frac{\operatorname{sech}^4(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{\operatorname{sech}^4(a+b\log(cx^n))}{x}\,dx = \frac{\tanh(a+b\log(cx^n))}{bn} - \frac{\tanh^3(a+b\log(cx^n))}{3bn}$$

output
```
tanh(a+b*ln(c*x^n))/b/n-1/3*tanh(a+b*ln(c*x^n))^3/b/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^4(a+b\log(cx^n))}{x}\,dx = \frac{\tanh(a+b\log(cx^n))}{bn} - \frac{\tanh^3(a+b\log(cx^n))}{3bn}$$

input
```
Integrate[Sech[a + b*Log[c*x^n]]^4/x,x]
```

output
```
Tanh[a + b*Log[c*x^n]]/(b*n) - Tanh[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3039, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^4(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \operatorname{sech}^4(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^4d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4254}$$

$$\frac{i\int \left(1-\tanh^2(a+b\log(cx^n))\right)d(-i\tanh(a+b\log(cx^n)))}{bn}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(\frac{1}{3}i\tanh^3(a+b\log(cx^n))-i\tanh(a+b\log(cx^n))\right)}{bn}$$

input
```
Int[Sech[a + b*Log[c*x^n]]^4/x,x]
```

output
```
(I*((-I)*Tanh[a + b*Log[c*x^n]] + (I/3)*Tanh[a + b*Log[c*x^n]]^3))/(b*n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 16.92 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(a+b\ln(c\,x^{n}))^{2}}{3}\right)\tanh(a+b\ln(c\,x^{n}))}{nb}$
default	$\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(a+b\ln(c\,x^{n}))^{2}}{3}\right)\tanh(a+b\ln(c\,x^{n}))}{nb}$
parallelrisch	$\frac{6\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)^{5}+4\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)^{3}+6\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)}{3bn\left(1+\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)^{2}\right)^{3}}$
risch	$-\frac{4\left(3(x^{n})^{2b}c^{2b}\mathrm{e}^{2a}\mathrm{e}^{ib\pi\,\operatorname{csgn}(ix^{n})\operatorname{csgn}(ic\,x^{n})^{2}}\mathrm{e}^{-ib\pi\,\operatorname{csgn}(ix^{n})\operatorname{csgn}(ic\,x^{n})\operatorname{csgn}(ic)}\mathrm{e}^{-ib\pi\operatorname{csgn}(ic\,x^{n})^{3}}\mathrm{e}^{ib\pi\operatorname{csgn}(ic\,x^{n})^{2}\operatorname{csgn}(\cdots}{3bn\left((x^{n})^{2b}c^{2b}\mathrm{e}^{2a}\mathrm{e}^{ib\pi\,\operatorname{csgn}(ix^{n})\operatorname{csgn}(ic\,x^{n})^{2}}\mathrm{e}^{-ib\pi\,\operatorname{csgn}(ix^{n})\operatorname{csgn}(ic\,x^{n})\operatorname{csgn}(ic)}\mathrm{e}^{-ib\pi\operatorname{csgn}(ic\,x^{n})^{3}}\mathrm{e}^{ib\pi\operatorname{csgn}(ic\,x^{n})^{2}\operatorname{csgn}(\cdots}$

input
```
int(sech(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(2/3+1/3*sech(a+b*ln(c*x^n))^2)*tanh(a+b*ln(c*x^n))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(40) = 80$.

Time = 0.18 (sec) , antiderivative size = 272, normalized size of antiderivative = 6.48

$$\int \frac{\operatorname{sech}^4(a + b\log(cx^n))}{x}\,dx =$$

$$-\frac{}{3\left(bn\cosh\left(bn\log\left(x\right) + b\log\left(c\right) + a\right)^5 + 5\,bn\cosh\left(bn\log\left(x\right) + b\log\left(c\right) + a\right)\sinh\left(bn\log\left(x\right) + b\log\left(c\right)\right.\right.}$$

input
```
integrate(sech(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

output
```
-8/3*(2*cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))
/(b*n*cosh(b*n*log(x) + b*log(c) + a)^5 + 5*b*n*cosh(b*n*log(x) + b*log(c)
 + a)*sinh(b*n*log(x) + b*log(c) + a)^4 + b*n*sinh(b*n*log(x) + b*log(c) +
 a)^5 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + (10*b*n*cosh(b*n*log(x)
+ b*log(c) + a)^2 + 3*b*n)*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*b*n*cosh(
b*n*log(x) + b*log(c) + a) + (10*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 9
*b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^2 +
(5*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*b*n*cosh(b*n*log(x) + b*log(c
) + a)^2 + 2*b*n)*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(a + b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{sech}^4\left(a + b\log\left(cx^n\right)\right)}{x}\,dx$$

input
```
integrate(sech(a+b*ln(c*x**n))**4/x,x)
```

output
```
Integral(sech(a + b*log(c*x**n))**4/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(40) = 80$.

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.17

$$\int \frac{\operatorname{sech}^4(a+b\log(cx^n))}{x}\,dx = -\frac{4\left(3\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)}+1\right)}{3\,\left(bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)}+3\,bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}+3\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn\right)}$$

input
```
integrate(sech(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

output
```
-4/3*(3*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)/(b*c^(6*b)*n*e^(6*b*log(x^n) +
 6*a) + 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^
n) + 2*a) + b*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^4(a+b\log(cx^n))}{x}\,dx = -\frac{4\left(3\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1\right)}{3\,\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}+1\right)^3bn}$$

input
```
integrate(sech(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

output
```
-4/3*(3*c^(2*b)*x^(2*b*n)*e^(2*a) + 1)/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1)^3*
b*n)
```

Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31

$$\int \frac{\operatorname{sech}^4(a+b\log(cx^n))}{x}\,dx = \frac{4\,e^{4\,a}\,\left(c\,x^n\right)^{4\,b}\,\left(e^{2\,a}\,\left(c\,x^n\right)^{2\,b}+3\right)}{3\,b\,n\,\left(e^{2\,a}\,\left(c\,x^n\right)^{2\,b}+1\right)^3}$$

input
```
int(1/(x*cosh(a + b*log(c*x^n))^4),x)
```

output
```
(4*exp(4*a)*(c*x^n)^(4*b)*(exp(2*a)*(c*x^n)^(2*b) + 3))/(3*b*n*(exp(2*a)*(
c*x^n)^(2*b) + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.02

$$\int \frac{\operatorname{sech}^4(a+b\log(cx^n))}{x}\,dx = \frac{-4x^{2bn}e^{2a}c^{2b}-\frac{4}{3}}{bn\left(x^{6bn}e^{6a}c^{6b}+3x^{4bn}e^{4a}c^{4b}+3x^{2bn}e^{2a}c^{2b}+1\right)}$$

input
```
int(sech(a+b*log(c*x^n))^4/x,x)
```

output
```
(4*( - 3*x**(2*b*n)*e**(2*a)*c**(2*b) - 1))/(3*b*n*(x**(6*b*n)*e**(6*a)*c*
*(6*b) + 3*x**(4*b*n)*e**(4*a)*c**(4*b) + 3*x**(2*b*n)*e**(2*a)*c**(2*b) +
 1))
```

3.195 $\int \frac{\text{sech}^5(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 89

$$\int \frac{\text{sech}^5(a+b\log(cx^n))}{x}\,dx = \frac{3\arctan(\sinh(a+b\log(cx^n)))}{8bn} + \frac{3\text{sech}(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{8bn} + \frac{\text{sech}^3(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{4bn}$$

output
```
3/8*arctan(sinh(a+b*ln(c*x^n)))/b/n+3/8*sech(a+b*ln(c*x^n))*tanh(a+b*ln(c*
x^n))/b/n+1/4*sech(a+b*ln(c*x^n))^3*tanh(a+b*ln(c*x^n))/b/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}^5(a+b\log(cx^n))}{x}\,dx = \frac{3\arctan(\sinh(a+b\log(cx^n)))}{8bn} + \frac{3\text{sech}(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{8bn} + \frac{\text{sech}^3(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{4bn}$$

input
```
Integrate[Sech[a + b*Log[c*x^n]]^5/x,x]
```

output
```
(3*ArcTan[Sinh[a + b*Log[c*x^n]]])/(8*b*n) + (3*Sech[a + b*Log[c*x^n]]*Tan
h[a + b*Log[c*x^n]])/(8*b*n) + (Sech[a + b*Log[c*x^n]]^3*Tanh[a + b*Log[c*
x^n]])/(4*b*n)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3039, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^5(a+b\log(cx^n))}{x}dx$$
$$\Big\downarrow \text{3039}$$
$$\frac{\int \text{sech}^5(a+b\log(cx^n))d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \csc\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^5d\log(cx^n)}{n}$$
$$\Big\downarrow \text{4255}$$
$$\frac{\frac{3}{4}\int \text{sech}^3(a+b\log(cx^n))d\log(cx^n)+\frac{\tanh(a+b\log(cx^n))\text{sech}^3(a+b\log(cx^n))}{4b}}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\frac{\tanh(a+b\log(cx^n))\text{sech}^3(a+b\log(cx^n))}{4b}+\frac{3}{4}\int \csc\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^3d\log(cx^n)}{n}$$
$$\Big\downarrow \text{4255}$$
$$\frac{\frac{3}{4}\left(\frac{1}{2}\int \text{sech}(a+b\log(cx^n))d\log(cx^n)+\frac{\tanh(a+b\log(cx^n))\text{sech}(a+b\log(cx^n))}{2b}\right)+\frac{\tanh(a+b\log(cx^n))\text{sech}^3(a+b\log(cx^n))}{4b}}{n}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\tanh(a+b\log(cx^n))\text{sech}^3(a+b\log(cx^n))}{4b}+\frac{3}{4}\left(\frac{\tanh(a+b\log(cx^n))\text{sech}(a+b\log(cx^n))}{2b}+\frac{1}{2}\int\csc\left(ia+ib\log\left(cx^n\right)+\frac{\pi}{2}\right)d\log\left(cx^n\right)\right)}{n}$$

$\Big\downarrow$ 4257

$$\frac{\frac{3}{4}\left(\frac{\arctan(\sinh(a+b\log(cx^n)))}{2b}+\frac{\tanh(a+b\log(cx^n))\text{sech}(a+b\log(cx^n))}{2b}\right)+\frac{\tanh(a+b\log(cx^n))\text{sech}^3(a+b\log(cx^n))}{4b}}{n}$$

input
```
Int[Sech[a + b*Log[c*x^n]]^5/x,x]
```

output
```
((Sech[a + b*Log[c*x^n]]^3*Tanh[a + b*Log[c*x^n]])/(4*b) + (3*(ArcTan[Sinh
[a + b*Log[c*x^n]]]/(2*b) + (Sech[a + b*Log[c*x^n]]*Tanh[a + b*Log[c*x^n]]
)/(2*b)))/4)/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 187.71 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{\left(\frac{\operatorname{sech}(a+b\ln(c\,x^n))^3}{4}+\frac{3\operatorname{sech}(a+b\ln(c\,x^n))}{8}\right)\tanh(a+b\ln(c\,x^n))+\frac{3\arctan\left(\mathrm{e}^{a+b\ln(c\,x^n)}\right)}{4}}{nb}$
default	$\frac{\left(\frac{\operatorname{sech}(a+b\ln(c\,x^n))^3}{4}+\frac{3\operatorname{sech}(a+b\ln(c\,x^n))}{8}\right)\tanh(a+b\ln(c\,x^n))+\frac{3\arctan\left(\mathrm{e}^{a+b\ln(c\,x^n)}\right)}{4}}{nb}$
parallelrisch	$\frac{3i(-\cosh(4b\ln(c\,x^n)+4a)-4\cosh(2b\ln(c\,x^n)+2a)-3)\ln\left(\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)-i\right)+3i(\cosh(4b\ln(c\,x^n)+4a)+4\cosh}{8bn(\cosh(4b\ln(c\,x^n)+4a)+4\cosh(2b}$
risch	Expression too large to display

input
```
int(sech(a+b*ln(c*x^n))^5/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*((1/4*sech(a+b*ln(c*x^n))^3+3/8*sech(a+b*ln(c*x^n)))*tanh(a+b*ln(c*x
^n))+3/4*arctan(exp(a+b*ln(c*x^n))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1326 vs. 2(83) = 166.

Time = 0.21 (sec) , antiderivative size = 1326, normalized size of antiderivative = 14.90

$$\int \frac{\operatorname{sech}^5(a+b\log(cx^n))}{x}\,dx = \text{Too large to display}$$

input
```
integrate(sech(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

output
```
1/4*(3*cosh(b*n*log(x) + b*log(c) + a)^7 + 21*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a)^6 + 3*sinh(b*n*log(x) + b*log(c) + a)^
7 + (63*cosh(b*n*log(x) + b*log(c) + a)^2 + 11)*sinh(b*n*log(x) + b*log(c)
 + a)^5 + 11*cosh(b*n*log(x) + b*log(c) + a)^5 + 5*(21*cosh(b*n*log(x) + b
*log(c) + a)^3 + 11*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*l
og(c) + a)^4 + (105*cosh(b*n*log(x) + b*log(c) + a)^4 + 110*cosh(b*n*log(x
) + b*log(c) + a)^2 - 11)*sinh(b*n*log(x) + b*log(c) + a)^3 - 11*cosh(b*n*
log(x) + b*log(c) + a)^3 + (63*cosh(b*n*log(x) + b*log(c) + a)^5 + 110*cos
h(b*n*log(x) + b*log(c) + a)^3 - 33*cosh(b*n*log(x) + b*log(c) + a))*sinh(
b*n*log(x) + b*log(c) + a)^2 + 3*(cosh(b*n*log(x) + b*log(c) + a)^8 + 8*co
sh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^7 + sinh(b*n
*log(x) + b*log(c) + a)^8 + 4*(7*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*si
nh(b*n*log(x) + b*log(c) + a)^6 + 4*cosh(b*n*log(x) + b*log(c) + a)^6 + 8*
(7*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a))*
sinh(b*n*log(x) + b*log(c) + a)^5 + 2*(35*cosh(b*n*log(x) + b*log(c) + a)^
4 + 30*cosh(b*n*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x) + b*log(c) +
 a)^4 + 6*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*cosh(b*n*log(x) + b*log
(c) + a)^5 + 10*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*
log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*cosh(b*n*log(x) + b*
log(c) + a)^6 + 15*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*cosh(b*n*log(x...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^5(a + b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{sech}^5\left(a + b\log\left(cx^n\right)\right)}{x}\,dx$$

input
```
integrate(sech(a+b*ln(c*x**n))**5/x,x)
```

output
```
Integral(sech(a + b*log(c*x**n))**5/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^5(a+b\log(cx^n))}{x}\,dx=\int \frac{\operatorname{sech}(b\log(cx^n)+a)^5}{x}\,dx$$

input
```
integrate(sech(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

output
```
96*c^b*integrate(1/128*e^(b*log(x^n) + a)/(c^(2*b)*x*e^(2*b*log(x^n) + 2*a
) + x), x) + 1/4*(3*c^(7*b)*e^(7*b*log(x^n) + 7*a) + 11*c^(5*b)*e^(5*b*log
(x^n) + 5*a) - 11*c^(3*b)*e^(3*b*log(x^n) + 3*a) - 3*c^b*e^(b*log(x^n) + a
))/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) + 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6
*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 4*b*c^(2*b)*n*e^(2*b*log(x^n)
 + 2*a) + b*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.71

$$\int \frac{\operatorname{sech}^5(a+b\log(cx^n))}{x}\,dx$$
$$=\frac{1}{4}c^{5\,b}\left(\frac{3\arctan\left(\frac{c^{2\,b}x^{bn}e^{a}}{c^{b}}\right)e^{(-5\,a)}}{bc^{4\,b}c^{b}n}+\frac{\left(3\,c^{6\,b}x^{7\,bn}e^{(6\,a)}+11\,c^{4\,b}x^{5\,bn}e^{(4\,a)}-11\,c^{2\,b}x^{3\,bn}e^{(2\,a)}-3\,x^{bn}\right)e^{(-4\,a)}}{\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}+1\right)^{4}bc^{4\,b}n}\right)e$$

input
```
integrate(sech(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

output
```
1/4*c^(5*b)*(3*arctan(c^(2*b)*x^(b*n)*e^a/c^b)*e^(-5*a)/(b*c^(4*b)*c^b*n)
+ (3*c^(6*b)*x^(7*b*n)*e^(6*a) + 11*c^(4*b)*x^(5*b*n)*e^(4*a) - 11*c^(2*b)
*x^(3*b*n)*e^(2*a) - 3*x^(b*n))*e^(-4*a)/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1)^
4*b*c^(4*b)*n))*e^(5*a)
```

Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.53

$$\int \frac{\operatorname{sech}^5(a+b\log(cx^n))}{x}\,dx = \frac{2\,e^{-a}}{(c\,x^n)^b\,\left(b\,n+\frac{3\,b\,n\,e^{-2\,a}}{(c\,x^n)^{2\,b}}+\frac{3\,b\,n\,e^{-4\,a}}{(c\,x^n)^{4\,b}}+\frac{b\,n\,e^{-6\,a}}{(c\,x^n)^{6\,b}}\right)} - \frac{3\,\mathrm{atan}\left(\frac{e^{-a}\,\sqrt{b^2\,n^2}}{b\,n\,(c\,x^n)^b}\right)}{4\,\sqrt{b^2\,n^2}} - \frac{3\,e^{-a}}{4\,(c\,x^n)^b\,\left(b\,n+\frac{b\,n\,e^{-2\,a}}{(c\,x^n)^{2\,b}}\right)} + \frac{4\,e^{-3\,a}}{(c\,x^n)^{3\,b}\,\left(b\,n+\frac{4\,b\,n\,e^{-2\,a}}{(c\,x^n)^{2\,b}}+\frac{6\,b\,n\,e^{-4\,a}}{(c\,x^n)^{4\,b}}+\frac{4\,b\,n\,e^{-6\,a}}{(c\,x^n)^{6\,b}}+\frac{b\,n\,e^{-8\,a}}{(c\,x^n)^{8\,b}}\right)} - \frac{e^{-a}}{2\,(c\,x^n)^b\,\left(b\,n+\frac{2\,b\,n\,e^{-2\,a}}{(c\,x^n)^{2\,b}}+\frac{b\,n\,e^{-4\,a}}{(c\,x^n)^{4\,b}}\right)}$$

input
```
int(1/(x*cosh(a + b*log(c*x^n))^5),x)
```

output
```
(2*exp(-a))/((c*x^n)^b*(b*n + (3*b*n*exp(-2*a))/(c*x^n)^(2*b) + (3*b*n*exp
(-4*a))/(c*x^n)^(4*b) + (b*n*exp(-6*a))/(c*x^n)^(6*b))) - (3*atan((exp(-a)
*(b^2*n^2)^(1/2))/(b*n*(c*x^n)^b)))/(4*(b^2*n^2)^(1/2)) - (3*exp(-a))/(4*(
c*x^n)^b*(b*n + (b*n*exp(-2*a))/(c*x^n)^(2*b))) + (4*exp(-3*a))/((c*x^n)^(
3*b)*(b*n + (4*b*n*exp(-2*a))/(c*x^n)^(2*b) + (6*b*n*exp(-4*a))/(c*x^n)^(4
*b) + (4*b*n*exp(-6*a))/(c*x^n)^(6*b) + (b*n*exp(-8*a))/(c*x^n)^(8*b))) -
exp(-a)/(2*(c*x^n)^b*(b*n + (2*b*n*exp(-2*a))/(c*x^n)^(2*b) + (b*n*exp(-4*
a))/(c*x^n)^(4*b)))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 290, normalized size of antiderivative = 3.26

$$\int \frac{\operatorname{sech}^5(a+b\log(cx^n))}{x}\,dx = \frac{3x^{8bn}e^{8a}c^{8b}atan\left(x^{bn}e^{a}c^{b}\right)+12x^{6bn}e^{6a}c^{6b}atan\left(x^{bn}e^{a}c^{b}\right)+18x^{4bn}e^{4a}c^{4b}atan\left(x^{bn}e^{a}c^{b}\right)+12x^{2bn}e^{2a}c^{2b}atan\left(\right.}{4bn\left(x^{8bn}e^{8a}c^{8b}+4x^{6bn}e^{6a}c^{6b}+6x^{4bn}e^{4a}c^{4\right.}}$$

input
```
int(sech(a+b*log(c*x^n))^5/x,x)
```

output

```
(3*x**(8*b*n)*e**(8*a)*c**(8*b)*atan(x**(b*n)*e**a*c**b) + 12*x**(6*b*n)*e
**(6*a)*c**(6*b)*atan(x**(b*n)*e**a*c**b) + 18*x**(4*b*n)*e**(4*a)*c**(4*b
)*atan(x**(b*n)*e**a*c**b) + 12*x**(2*b*n)*e**(2*a)*c**(2*b)*atan(x**(b*n)
*e**a*c**b) + 3*atan(x**(b*n)*e**a*c**b) + 3*x**(7*b*n)*e**(7*a)*c**(7*b)
+ 11*x**(5*b*n)*e**(5*a)*c**(5*b) - 11*x**(3*b*n)*e**(3*a)*c**(3*b) - 3*x*
*(b*n)*e**a*c**b)/(4*b*n*(x**(8*b*n)*e**(8*a)*c**(8*b) + 4*x**(6*b*n)*e**(
6*a)*c**(6*b) + 6*x**(4*b*n)*e**(4*a)*c**(4*b) + 4*x**(2*b*n)*e**(2*a)*c**
(2*b) + 1))
```

3.196 $\int \frac{\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \frac{\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2i\sqrt{\cosh(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{3bn} + \frac{2\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{3bn}$$

output
```
-2/3*I*cosh(a+b*ln(c*x^n))^(1/2)*InverseJacobiAM(1/2*I*(a+b*ln(c*x^n)),2^(
1/2))*sech(a+b*ln(c*x^n))^(1/2)/b/n+2/3*sech(a+b*ln(c*x^n))^(3/2)*sinh(a+b
*ln(c*x^n))/b/n
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int \frac{\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))\left(-i\cosh^{\frac{3}{2}}(a+b\log(cx^n))\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)+\sinh(a+b\log(ca

input
```
Integrate[Sech[a + b*Log[c*x^n]]^(5/2)/x,x]
```

output
```
(2*Sech[a + b*Log[c*x^n]]^(3/2)*((-I)*Cosh[a + b*Log[c*x^n]]^(3/2)*Ellipti
cF[(I/2)*(a + b*Log[c*x^n]), 2] + Sinh[a + b*Log[c*x^n]]))/(3*b*n)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3039, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^{\frac{5}{2}}(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \text{sech}^{\frac{5}{2}}(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^{5/2}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{1}{3}\int \sqrt{\text{sech}(a+b\log(cx^n))}d\log(cx^n)+\frac{2\sinh(a+b\log(cx^n))\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}{3b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}{3b}+\frac{1}{3}\int \sqrt{\csc\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{1}{3}\sqrt{\text{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}\int \frac{1}{\sqrt{\cosh(a+b\log(cx^n))}}d\log(cx^n)+\frac{2\sinh(a+b\log(cx^n))\text{sech}^{\frac{3}{2}}(a+b\log(cx}{3b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}{3b}+\frac{1}{3}\sqrt{\text{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\int\frac{1}{\sqrt{\sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}}d\log}{n}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}{3b}-\frac{2i\sqrt{\text{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)}{3b}}{n}$$

input
```
Int[Sech[a + b*Log[c*x^n]]^(5/2)/x,x]
```

output
```
((((-2*I)/3)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n
]), 2]*Sqrt[Sech[a + b*Log[c*x^n]]])/b + (2*Sech[a + b*Log[c*x^n]]^(3/2)*S
inh[a + b*Log[c*x^n]])/(3*b))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(85) = 170$.

Time = 94.31 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.66

method	result
derivativedivides	$\frac{\sqrt{\left(2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\left(\frac{\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right.}}{3\left(\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-\frac{1}{2}\right)^2}\right.}{n\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{2\cosh\left(\frac{a}{2}+\frac{b\ln\right.}}}$
default	$\frac{\sqrt{\left(2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\left(\frac{\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right.}}{3\left(\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-\frac{1}{2}\right)^2}\right.}{n\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{2\cosh\left(\frac{a}{2}+\frac{b\ln\right.}}}$

input
```
int(sech(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1
/2)*(1/3*cosh(1/2*a+1/2*b*ln(c*x^n))*(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh
(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(cosh(1/2*a+1/2*b*ln(c*x^n))^2-1/2)^2+2/3
*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^2+
1)^(1/2)/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2)))/sinh(1/2*a+1/2*b*ln(c
*x^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(84) = 168$.

Time = 0.13 (sec) , antiderivative size = 315, normalized size of antiderivative = 3.25

$$\int \frac{\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$$
$$= \frac{2\left(\sqrt{2}(\cosh(bn\log(x)+b\log(c)+a)^2+2\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+\right.}{}$$

input
```
integrate(sech(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

output
```
2/3*(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a
)^2 - 1)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c
) + a))/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c)
+ a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 +
 1)) + (sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*log
(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log
(x) + b*log(c) + a)^2 + sqrt(2))*weierstrassPInverse(-4, 0, cosh(b*n*log(x
) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x)
 + b*log(c) + a)^2 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x)
 + b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(sech(a+b*ln(c*x**n))**(5/2)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{sech}(b\log(cx^n)+a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate(sech(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

output
```
integrate(sech(b*log(c*x^n) + a)^(5/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(sech(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\left(\frac{1}{\cosh(a+b\,\ln(c\,x^n))}\right)^{5/2}}{x}\,dx$$

input
```
int((1/cosh(a + b*log(c*x^n)))^(5/2)/x,x)
```

output
```
int((1/cosh(a + b*log(c*x^n)))^(5/2)/x, x)
```

Reduce [F]

$$\int \frac{\operatorname{sech}^{\frac{5}{2}}(a + b \log(cx^n))}{x} \, dx = \int \frac{\sqrt{\operatorname{sech}(\log(x^n c) b + a)} \operatorname{sech}(\log(x^n c) b + a)^2}{x} dx$$

input
```
int(sech(a+b*log(c*x^n))^(5/2)/x,x)
```

output
```
int((sqrt(sech(log(x**n*c)*b + a))*sech(log(x**n*c)*b + a)**2)/x,x)
```

3.197 $\int \frac{\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 93

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2i\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{bn} + \frac{2\sqrt{\operatorname{sech}(a+b\log(cx^n))}\sinh(a+b\log(cx^n))}{bn}$$

output
```
2*I*cosh(a+b*ln(c*x^n))^(1/2)*EllipticE(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1
/2))*sech(a+b*ln(c*x^n))^(1/2)/b/n+2*sech(a+b*ln(c*x^n))^(1/2)*sinh(a+b*ln
(c*x^n))/b/n
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.77

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2\sqrt{\operatorname{sech}(a+b\log(cx^n))}\left(i\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)+\sinh(a+b\log(cx^n))\right)}{bn}$$

input
```
Integrate[Sech[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
(2*Sqrt[Sech[a + b*Log[c*x^n]]]*(I*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[
(I/2)*(a + b*Log[c*x^n]), 2] + Sinh[a + b*Log[c*x^n]]))/(b*n)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3039, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{ 3039}$$

$$\frac{\int \operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \csc\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^{3/2}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{b}-\int \frac{1}{\sqrt{\operatorname{sech}(a+b\log(cx^n))}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{b}-\int \frac{1}{\sqrt{\csc\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))\sqrt{\text{sech}(a+b\log(cx^n))}}{b}-\sqrt{\text{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\int\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}d\,\text{l}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))\sqrt{\text{sech}(a+b\log(cx^n))}}{b}-\sqrt{\text{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\int\sqrt{\sin\left(ia+ib\log\left(cx^n\right)+\right.}}{n}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))\sqrt{\text{sech}(a+b\log(cx^n))}}{b}+\frac{2i\sqrt{\text{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))|2\right)}{b}}{n}$$

input
```
Int[Sech[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
(((2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2
]*Sqrt[Sech[a + b*Log[c*x^n]]])/b + (2*Sqrt[Sech[a + b*Log[c*x^n]]]*Sinh[a
 + b*Log[c*x^n]])/b)/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.52

method	result
derivativedivides	$\frac{4\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+2\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\ \operatorname{EllipticE}\left(\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right),\sqrt{2}\right)\sqrt{-2\sinh}}{n\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1}\,b}$
default	$\frac{4\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+2\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\ \operatorname{EllipticE}\left(\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right),\sqrt{2}\right)\sqrt{-2\sinh}}{n\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1}\,b}$

input
```
int(sech(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/n*(2*cosh(1/2*a+1/2*b*ln(c*x^n))*sinh(1/2*a+1/2*b*ln(c*x^n))^2+(-sinh(1/
2*a+1/2*b*ln(c*x^n))^2)^(1/2)*EllipticE(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2
))*(-2*sinh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2))/sinh(1/2*a+1/2*b*ln(c*x^n))
/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.71

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \frac{2\left(\sqrt{2}\sqrt{\frac{\cosh(bn\log(x)+b\log(c)+a)+\sinh(bn\log(x)+b\log(c)+a)}{\cosh(bn\log(x)+b\log(c)+a)^2+2\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)+\sinh(bn\log(x)+b\log(c)+a)^2+1}}(\cosh(\right.}{}$$

input
```
integrate(sech(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

output
```
2*(sqrt(2)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log
(c) + a))/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c
) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2
 + 1))*(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))
 + sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(b*n*log(
x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{sech}^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx$$

input
```
integrate(sech(a+b*ln(c*x**n))**(3/2)/x,x)
```

output
```
Integral(sech(a + b*log(c*x**n))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{sech}(b\log(cx^n) + a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(sech(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(sech(b*log(c*x^n) + a)^(3/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(sech(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \int \frac{\left(\frac{1}{\cosh(a+b\,\ln(c\,x^n))}\right)^{3/2}}{x}\,dx$$

input
```
int((1/cosh(a + b*log(c*x^n)))^(3/2)/x,x)
```

output
```
int((1/cosh(a + b*log(c*x^n)))^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \int \frac{\sqrt{\operatorname{sech}(\log(x^n c)\,b + a)}\,\operatorname{sech}(\log(x^n c)\,b + a)}{x}dx$$

input
```
int(sech(a+b*log(c*x^n))^(3/2)/x,x)
```

output
```
int((sqrt(sech(log(x**n*c)*b + a))*sech(log(x**n*c)*b + a))/x,x)
```

3.198 $\int \frac{\sqrt{\text{sech}(a+b\log(cx^n))}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{\sqrt{\text{sech}(a+b\log(cx^n))}}{x}\,dx = -\frac{2i\sqrt{\cosh(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)\sqrt{\text{sech}(a+b\log(cx^n))}}{bn}$$

output
```
-2*I*cosh(a+b*ln(c*x^n))^(1/2)*InverseJacobiAM(1/2*I*(a+b*ln(c*x^n)),2^(1/
2))*sech(a+b*ln(c*x^n))^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\text{sech}(a+b\log(cx^n))}}{x}\,dx = -\frac{2i\sqrt{\cosh(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)\sqrt{\text{sech}(a+b\log(cx^n))}}{bn}$$

input
```
Integrate[Sqrt[Sech[a + b*Log[c*x^n]]]/x,x]
```

output
```
((-2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2
]*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3039, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \sqrt{\operatorname{sech}(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\csc\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}\int \frac{1}{\sqrt{\cosh(a+b\log(cx^n))}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}\int \frac{1}{\sqrt{\sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2i\sqrt{\operatorname{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}\operatorname{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)}{bn}$$

input
```
Int[Sqrt[Sech[a + b*Log[c*x^n]]]/x,x]
```

output
```
((-2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2
]*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n)
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(52) = 104$.

Time = 0.48 (sec) , antiderivative size = 183, normalized size of antiderivative = 3.16

method	result
derivativedivides	$\frac{2\sqrt{\left(2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{-2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+1}\ \text{Elliptic}}{n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}}$
default	$\frac{2\sqrt{\left(2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{-2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+1}\ \text{Elliptic}}{n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}}$

input
```
int(sech(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1
/2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))
^2+1)^(1/2)/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2
)^(1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/sinh(1/2*a+1/2*b*ln
(c*x^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{x}\,dx = \frac{2\sqrt{2}\text{weierstrassPInverse}(-4,0,\cosh(bn\log(x)+b\log(c)+a)+\sinh(bn\log(x)+b\log(c)+a))}{bn}$$

input
```
integrate(sech(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

output
```
2*sqrt(2)*weierstrassPInverse(-4, 0, cosh(b*n*log(x) + b*log(c) + a) + sin
h(b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{x}\,dx = \int \frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{x}\,dx$$

input
```
integrate(sech(a+b*ln(c*x**n))**(1/2)/x,x)
```

output
```
Integral(sqrt(sech(a + b*log(c*x**n)))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{x}\,dx = \int \frac{\sqrt{\operatorname{sech}(b\log(cx^n)+a)}}{x}\,dx$$

input
```
integrate(sech(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(sech(b*log(c*x^n) + a))/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{x}\,dx = \text{Timed out}$$

input
```
integrate(sech(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{x}\,dx = \int \frac{\sqrt{\frac{1}{\cosh(a+b\,\ln(c\,x^n))}}}{x}\,dx$$

input
```
int((1/cosh(a + b*log(c*x^n)))^(1/2)/x,x)
```

output
```
int((1/cosh(a + b*log(c*x^n)))^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(\log\left(x^nc\right)b+a\right)}}{x}dx$$

input
```
int(sech(a+b*log(c*x^n))^(1/2)/x,x)
```

output
```
int(sqrt(sech(log(x**n*c)*b + a))/x,x)
```

3.199 $\int \frac{1}{x\sqrt{\operatorname{sech}(a+b\log(cx^n))}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{1}{x\sqrt{\operatorname{sech}(a+b\log(cx^n))}}\,dx = -\frac{2i\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{bn}$$

output
```
-2*I*cosh(a+b*ln(c*x^n))^(1/2)*EllipticE(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(
1/2))*sech(a+b*ln(c*x^n))^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\operatorname{sech}(a+b\log(cx^n))}}\,dx = -\frac{2iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn\sqrt{\cosh(a+b\log(cx^n))}\sqrt{\operatorname{sech}(a+b\log(cx^n))}}$$

input
```
Integrate[1/(x*Sqrt[Sech[a + b*Log[c*x^n]]]),x]
```

output
```
((-2*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n*Sqrt[Cosh[a + b*Log[c
*x^n]]]*Sqrt[Sech[a + b*Log[c*x^n]]])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3039, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\operatorname{sech}(a+b\log(cx^n))}}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\sqrt{\operatorname{sech}(a+b\log(cx^n))}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{\csc\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}\int\sqrt{\cosh(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}\int\sqrt{\sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2i\sqrt{\operatorname{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn}$$

input
```
Int[1/(x*Sqrt[Sech[a + b*Log[c*x^n]]]),x]
```

output
```
((-2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2
]*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n)
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(56) = 112$.

Time = 0.96 (sec) , antiderivative size = 183, normalized size of antiderivative = 3.16

method	result
derivativedivides	$-\frac{2\sqrt{\left(2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{-2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+1}\ \mathrm{Ellip}}{n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\ \sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)}}$
default	$-\frac{2\sqrt{\left(2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{-2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+1}\ \mathrm{Ellip}}{n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\ \sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)}}$

input
```
int(1/x/sech(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n)
)^2+1)^(1/2)*EllipticE(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/(2*sinh(1/2*a+
1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sinh(1/2*a+1/2*b*l
n(c*x^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(54) = 108$.

Time = 0.24 (sec) , antiderivative size = 248, normalized size of antiderivative = 4.28

$$\int \frac{1}{x\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}\,dx =$$

$$-\frac{\sqrt{2}\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+2\,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+\right.\right.}{}$$

input
```
integrate(1/x/sech(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c
) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2
 + 1)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) +
 a))/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a
)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)
) + 2*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x) +
 b*log(c) + a))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(b*n
*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))))/(b*n*cosh(b*n
*log(x) + b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]

$$\int \frac{1}{x\sqrt{\operatorname{sech}(a+b\log(cx^n))}}\,dx = \int \frac{1}{x\sqrt{\operatorname{sech}(a+b\log(cx^n))}}\,dx$$

input
```
integrate(1/x/sech(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(sech(a + b*log(c*x**n)))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{\operatorname{sech}(a+b\log(cx^n))}}\,dx = \int \frac{1}{x\sqrt{\operatorname{sech}(b\log(cx^n)+a)}}\,dx$$

input
```
integrate(1/x/sech(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sqrt(sech(b*log(c*x^n) + a))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\operatorname{sech}(a+b\log(cx^n))}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/sech(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}\,dx=\int \frac{1}{x\,\sqrt{\frac{1}{\cosh(a+b\,\ln(c\,x^n))}}}\,dx$$

input
```
int(1/(x*(1/cosh(a + b*log(c*x^n)))^(1/2)),x)
```

output
```
int(1/(x*(1/cosh(a + b*log(c*x^n)))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}\,dx=\int \frac{\sqrt{\operatorname{sech}\left(\log\left(x^n c\right)b+a\right)}}{\operatorname{sech}\left(\log\left(x^n c\right)b+a\right)x}dx$$

input
```
int(1/x/sech(a+b*log(c*x^n))^(1/2),x)
```

output
```
int(sqrt(sech(log(x**n*c)*b + a))/(sech(log(x**n*c)*b + a)*x),x)
```

3.200 $\int \frac{1}{x\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \frac{1}{x\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = -\frac{2i\sqrt{\cosh(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)\sqrt{\text{sech}(a+b\log(cx^n))}}{3bn} + \frac{2\sinh(a+b\log(cx^n))}{3bn\sqrt{\text{sech}(a+b\log(cx^n))}}$$

output
```
-2/3*I*cosh(a+b*ln(c*x^n))^(1/2)*InverseJacobiAM(1/2*I*(a+b*ln(c*x^n)),2^(
1/2))*sech(a+b*ln(c*x^n))^(1/2)/b/n+2/3*sinh(a+b*ln(c*x^n))/b/n/sech(a+b*l
n(c*x^n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}\left(-2i\sqrt{\cosh(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)+\sinh(2(a+b\log$$
$$3bn$$

input
```
Integrate[1/(x*Sech[a + b*Log[c*x^n]]^(3/2)),x]
```

output
```
(Sqrt[Sech[a + b*Log[c*x^n]]]*((-2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*Ellipti
cF[(I/2)*(a + b*Log[c*x^n]), 2] + Sinh[2*(a + b*Log[c*x^n])]))/(3*b*n)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3039, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}dx$$
$$\Big\downarrow \text{ 3039}$$
$$\frac{\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}d\log(cx^n)}{n}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int \frac{1}{\csc\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^{3/2}}d\log(cx^n)}{n}$$
$$\Big\downarrow \text{ 4256}$$

$$\frac{\frac{1}{3}\int \sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}d\log\left(cx^n\right)+\frac{2\sinh(a+b\log(cx^n))}{3b\sqrt{\operatorname{sech}(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))}{3b\sqrt{\operatorname{sech}(a+b\log(cx^n))}}+\frac{1}{3}\int \sqrt{\csc\left(ia+ib\log\left(cx^n\right)+\frac{\pi}{2}\right)}d\log\left(cx^n\right)}{n}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{\frac{1}{3}\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\int \frac{1}{\sqrt{\cosh(a+b\log(cx^n))}}d\log\left(cx^n\right)+\frac{2\sinh(a+b\log(cx^n))}{3b\sqrt{\operatorname{sech}(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))}{3b\sqrt{\operatorname{sech}(a+b\log(cx^n))}}+\frac{1}{3}\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\int \frac{1}{\sqrt{\sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}}d\log\left(cx^n\right)}{n}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))}{3b\sqrt{\operatorname{sech}(a+b\log(cx^n))}}-\frac{2i\sqrt{\operatorname{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}\operatorname{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)}{3b}}{n}$$

input
```
Int[1/(x*Sech[a + b*Log[c*x^n]]^(3/2)),x]
```

output
```
((((-2*I)/3)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n
]), 2]*Sqrt[Sech[a + b*Log[c*x^n]]])/b + (2*Sinh[a + b*Log[c*x^n]])/(3*b*S
qrt[Sech[a + b*Log[c*x^n]]]))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(85) = 170$.

Time = 1.56 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.44

method	result
derivativedivides	$\frac{2\sqrt{\left(2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\left(4\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^5-6\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^3+\sqrt{-\sinh\left(\frac{a}{2}+\right.}\right.}{3n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\sinh\left(\right.}$
default	$\frac{2\sqrt{\left(2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\left(4\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^5-6\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^3+\sqrt{-\sinh\left(\frac{a}{2}+\right.}\right.}{3n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\sinh\left(\right.}$

input
```
int(1/x/sech(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^
(1/2)*(4*cosh(1/2*a+1/2*b*ln(c*x^n))^5-6*cosh(1/2*a+1/2*b*ln(c*x^n))^3+(-s
inh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^2+1)^(
1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))+2*cosh(1/2*a+1/2*b*ln(
c*x^n)))/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)/sinh(1/2*a+1/2*b*ln(c*x^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)
/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(84) = 168$.

Time = 0.12 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.81

$$\int \frac{1}{x\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\sqrt{2}\left(\cosh(bn\log(x)+b\log(c)+a)^4+4\cosh(bn\log(x)+b\log(c)+a)^3\sinh(bn\log(x)+b\log(c)+a\right)}{}$$

input
```
integrate(1/x/sech(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*lo
g(c) + a)^3*sinh(b*n*log(x) + b*log(c) + a) + 6*cosh(b*n*log(x) + b*log(c)
 + a)^2*sinh(b*n*log(x) + b*log(c) + a)^2 + 4*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4
- 1)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) +
a))/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1))
 + 4*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*log(x
) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x
) + b*log(c) + a)^2)*weierstrassPInverse(-4, 0, cosh(b*n*log(x) + b*log(c)
 + a) + sinh(b*n*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(c)
+ a)^2 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c)
+ a) + b*n*sinh(b*n*log(x) + b*log(c) + a)^2)
```

Sympy [F]

$$\int \frac{1}{x\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\,\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$

input
```
integrate(1/x/sech(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Integral(1/(x*sech(a + b*log(c*x**n))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\,\text{sech}\,(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/sech(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sech(b*log(c*x^n) + a)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/x/sech(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\left(\frac{1}{\cosh(a+b\ln(c\,x^n))}\right)^{3/2}}\,dx$$

input
```
int(1/(x*(1/cosh(a + b*log(c*x^n)))^(3/2)),x)
```

output
```
int(1/(x*(1/cosh(a + b*log(c*x^n)))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{\sqrt{\text{sech}(\log(x^n c)\,b+a)}}{\text{sech}(\log(x^n c)\,b+a)^2\,x}\,dx$$

input
```
int(1/x/sech(a+b*log(c*x^n))^(3/2),x)
```

output
```
int(sqrt(sech(log(x**n*c)*b + a))/(sech(log(x**n*c)*b + a)**2*x),x)
```

3.201 $\int \frac{1}{x\text{sech}^{\frac{5}{2}}(a+b\log(cx^n))} dx$

Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \frac{1}{x\text{sech}^{\frac{5}{2}}(a+b\log(cx^n))} dx = -\frac{6i\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)\sqrt{\text{sech}(a+b\log(cx^n))}}{5bn} + \frac{2\sinh(a+b\log(cx^n))}{5bn\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}$$

output
```
-6/5*I*cosh(a+b*ln(c*x^n))^(1/2)*EllipticE(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2
^(1/2))*sech(a+b*ln(c*x^n))^(1/2)/b/n+2/5*sinh(a+b*ln(c*x^n))/b/n/sech(a+b
*ln(c*x^n))^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}\left(-12i\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)+\sinh(a+b\log(cx^n))+\right.}{10bn}$$

input
```
Integrate[1/(x*Sech[a + b*Log[c*x^n]]^(5/2)),x]
```

output
```
(Sqrt[Sech[a + b*Log[c*x^n]]]*((-12*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*Ellipt
icE[(I/2)*(a + b*Log[c*x^n]), 2] + Sinh[a + b*Log[c*x^n]] + Sinh[3*(a + b*
Log[c*x^n])]))/(10*b*n)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3039, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\csc\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^{5/2}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4256}$$

$$\frac{\frac{3}{5}\int \frac{1}{\sqrt{\text{sech}(a+b\log(cx^n))}}d\log(cx^n)+\frac{2\sinh(a+b\log(cx^n))}{5b\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))}{5b\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}+\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{3}{5}\sqrt{\text{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}\int \sqrt{\cosh(a+b\log(cx^n))}d\log(cx^n)+\frac{2\sinh(a+b\log(cx^n))}{5b\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))}{5b\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}+\frac{3}{5}\sqrt{\text{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}\int \sqrt{\sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))}{5b\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}-\frac{6i\sqrt{\text{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))|2\right)}{5b}}{n}$$

input
```
Int[1/(x*Sech[a + b*Log[c*x^n]]^(5/2)),x]
```

output
```
((((-6*I)/5)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[(I/2)*(a + b*Log[c*x^n
]), 2]*Sqrt[Sech[a + b*Log[c*x^n]]])/b + (2*Sinh[a + b*Log[c*x^n]])/(5*b*S
ech[a + b*Log[c*x^n]]^(3/2)))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(89) = 178$.

Time = 3.30 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.64

method	result
derivativedivides	$\frac{2\sqrt{\left(2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\left(8\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^7-16\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^5+10\cosh\left(\frac{a}{2}+\right.\right.}{5n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(\right.}}}$
default	$\frac{2\sqrt{\left(2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\left(8\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^7-16\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^5+10\cosh\left(\frac{a}{2}+\right.\right.}{5n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(\right.}}}$

input
```
int(1/x/sech(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^
(1/2)*(8*cosh(1/2*a+1/2*b*ln(c*x^n))^7-16*cosh(1/2*a+1/2*b*ln(c*x^n))^5+10
*cosh(1/2*a+1/2*b*ln(c*x^n))^3-3*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-
2*cosh(1/2*a+1/2*b*ln(c*x^n))^2+1)^(1/2)*EllipticE(cosh(1/2*a+1/2*b*ln(c*x
^n)),2^(1/2))-2*cosh(1/2*a+1/2*b*ln(c*x^n)))/(2*sinh(1/2*a+1/2*b*ln(c*x^n)
)^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sinh(1/2*a+1/2*b*ln(c*x^n))/(2*co
sh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(87) = 174$.

Time = 0.25 (sec) , antiderivative size = 602, normalized size of antiderivative = 6.21

$$\int \frac{1}{x\text{sech}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/sech(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

output
```
1/20*(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^6 + 6*cosh(b*n*log(x) + b*l
og(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^5 + sinh(b*n*log(x) + b*log(c)
+ a)^6 + (15*cosh(b*n*log(x) + b*log(c) + a)^2 - 11)*sinh(b*n*log(x) + b*l
og(c) + a)^4 - 11*cosh(b*n*log(x) + b*log(c) + a)^4 + 4*(5*cosh(b*n*log(x)
 + b*log(c) + a)^3 - 11*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) +
 b*log(c) + a)^3 + (15*cosh(b*n*log(x) + b*log(c) + a)^4 - 66*cosh(b*n*log
(x) + b*log(c) + a)^2 - 13)*sinh(b*n*log(x) + b*log(c) + a)^2 - 13*cosh(b*
n*log(x) + b*log(c) + a)^2 + 2*(3*cosh(b*n*log(x) + b*log(c) + a)^5 - 22*c
osh(b*n*log(x) + b*log(c) + a)^3 - 13*cosh(b*n*log(x) + b*log(c) + a))*sin
h(b*n*log(x) + b*log(c) + a) - 1)*sqrt((cosh(b*n*log(x) + b*log(c) + a) +
sinh(b*n*log(x) + b*log(c) + a))/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*co
sh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*l
og(x) + b*log(c) + a)^2 + 1)) - 24*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a
)^3 + 3*sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2*sinh(b*n*log(x) + b*log(
c) + a) + 3*sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*lo
g(c) + a)^2 + sqrt(2)*sinh(b*n*log(x) + b*log(c) + a)^3)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cosh(b*n*log(x) + b*log(c) + a) + sinh(b*
n*log(x) + b*log(c) + a))))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*b*n
*cosh(b*n*log(x) + b*log(c) + a)^2*sinh(b*n*log(x) + b*log(c) + a) + 3*b*n
*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 + b*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\operatorname{sech}^{\frac{5}{2}}\left(a+b\log\left(cx^{n}\right)\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x/sech(a+b*ln(c*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x\text{sech}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\,\text{sech}\,(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/sech(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sech(b*log(c*x^n) + a)^(5/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\text{sech}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/x/sech(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\text{sech}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\,\left(\frac{1}{\cosh(a+b\,\ln(c\,x^n))}\right)^{5/2}}\,dx$$

input
```
int(1/(x*(1/cosh(a + b*log(c*x^n)))^(5/2)),x)
```

output
```
int(1/(x*(1/cosh(a + b*log(c*x^n)))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{\sqrt{\operatorname{sech}(\log(x^n c)\,b+a)}}{\operatorname{sech}(\log(x^n c)\,b+a)^3 x}dx$$

input
```
int(1/x/sech(a+b*log(c*x^n))^(5/2),x)
```

output
```
int(sqrt(sech(log(x**n*c)*b + a))/(sech(log(x**n*c)*b + a)**3*x),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(4,m1)    #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)    #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)    #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file