Computer Algebra
Independent Integration Tests

Summer 2024

6-Hyperbolic-functions/6.6-Hyperbolic-cosecant/318-6.6.3

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**175**]. This is test number [318].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (175)	0.00 (0)
Rubi	99.43 (174)	0.57 (1)
Fricas	96.57 (169)	3.43 (6)
Maple	77.71 (136)	22.29 (39)
Maxima	62.29 (109)	37.71 (66)
Giac	61.14 (107)	38.86 (68)
Mupad	52.00 (91)	48.00 (84)
Reduce	44.57 (78)	55.43 (97)
Sympy	1.71 (3)	98.29 (172)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

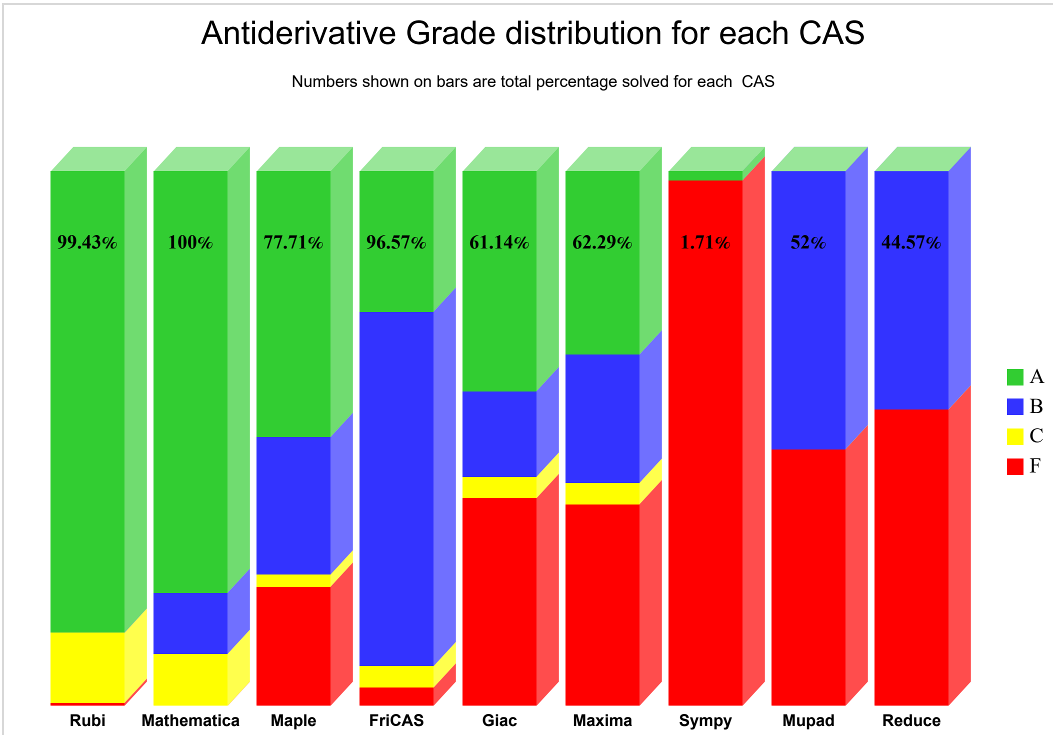
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

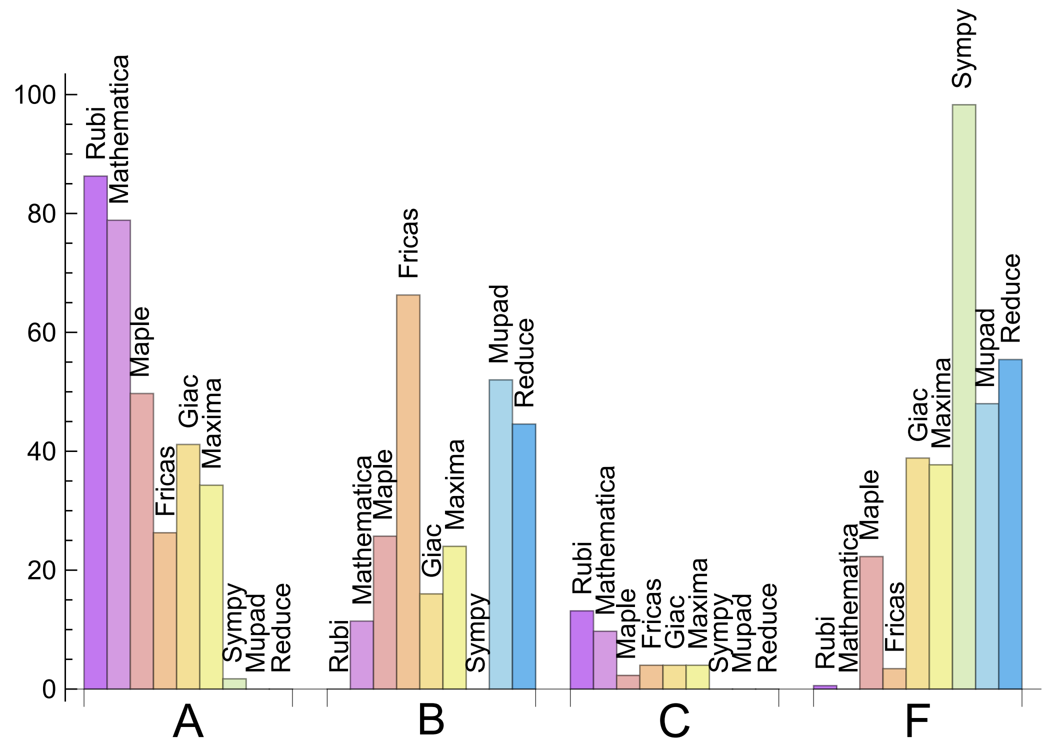
System	% A grade	% B grade	% C grade	% F grade
Rubi	86.286	0.000	13.143	0.571
Mathematica	78.857	11.429	9.714	0.000
Maple	49.714	25.714	2.286	22.286
Giac	41.143	16.000	4.000	38.857
Maxima	34.286	24.000	4.000	37.714
Fricas	26.286	66.286	4.000	3.429
Sympy	1.714	0.000	0.000	98.286
Mupad	0.000	52.000	0.000	48.000
Reduce	0.000	44.571	0.000	55.429

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Rubi	1	100.00	0.00	0.00
Fricas	6	100.00	0.00	0.00
Maple	39	100.00	0.00	0.00
Maxima	66	96.97	0.00	3.03
Giac	68	60.29	35.29	4.41
Mupad	84	0.00	100.00	0.00
Reduce	97	100.00	0.00	0.00
Sympy	172	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.09
Fricas	0.10
Giac	0.12
Reduce	0.23
Sympy	0.31
Rubi	0.37
Mathematica	0.39
Mupad	2.51
Maple	4.71

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	27.00	1.60	22.00	1.42
Rubi	70.99	1.10	68.00	1.05
Mathematica	74.43	1.25	62.00	1.00
Giac	92.58	1.66	62.00	1.46
Maxima	105.02	1.76	75.00	1.55
Maple	105.18	1.78	88.00	1.44
Mupad	151.23	2.36	81.00	2.16
Reduce	240.64	2.93	97.00	2.18
Fricas	415.37	5.06	154.00	3.19

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

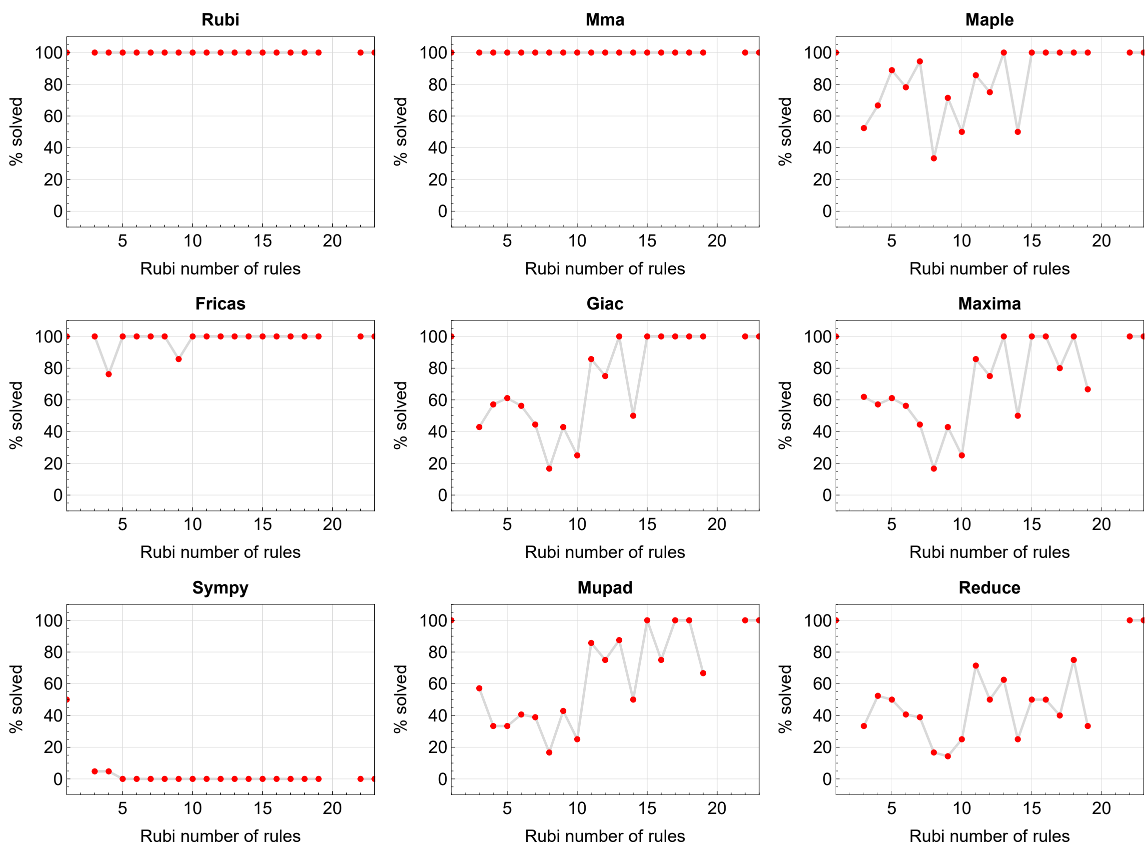


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

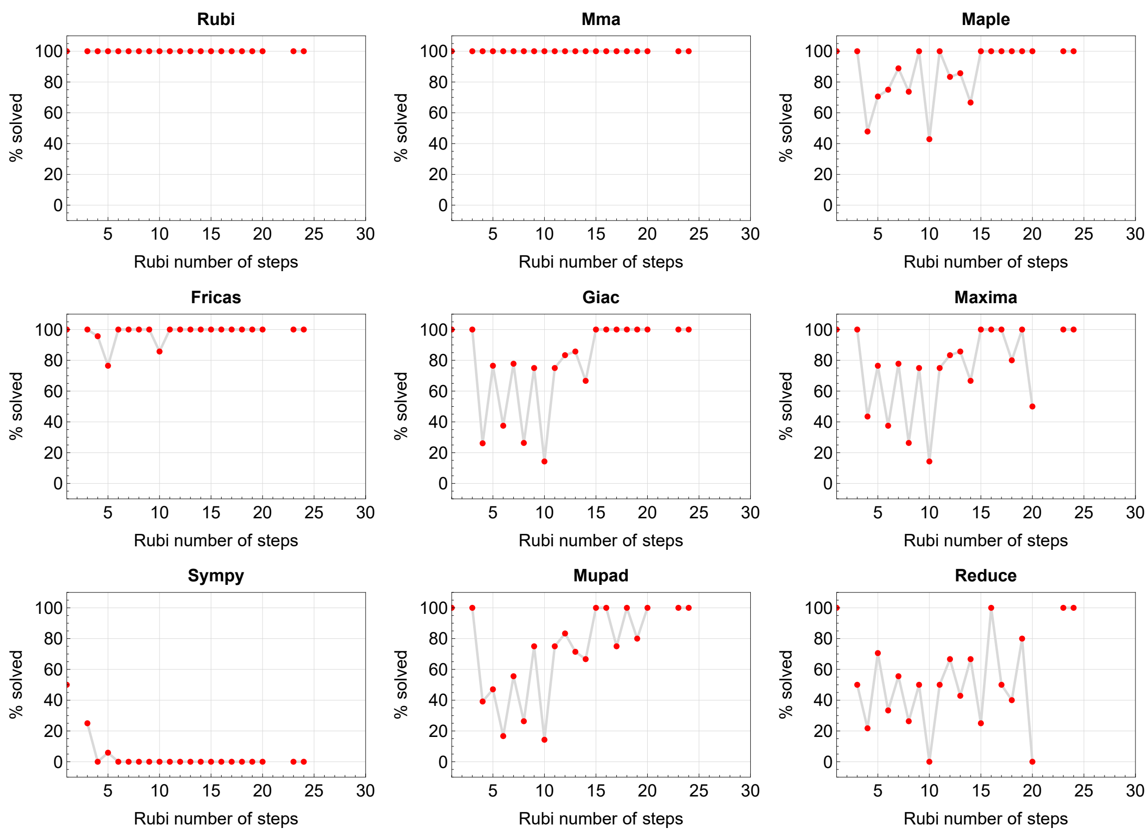


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

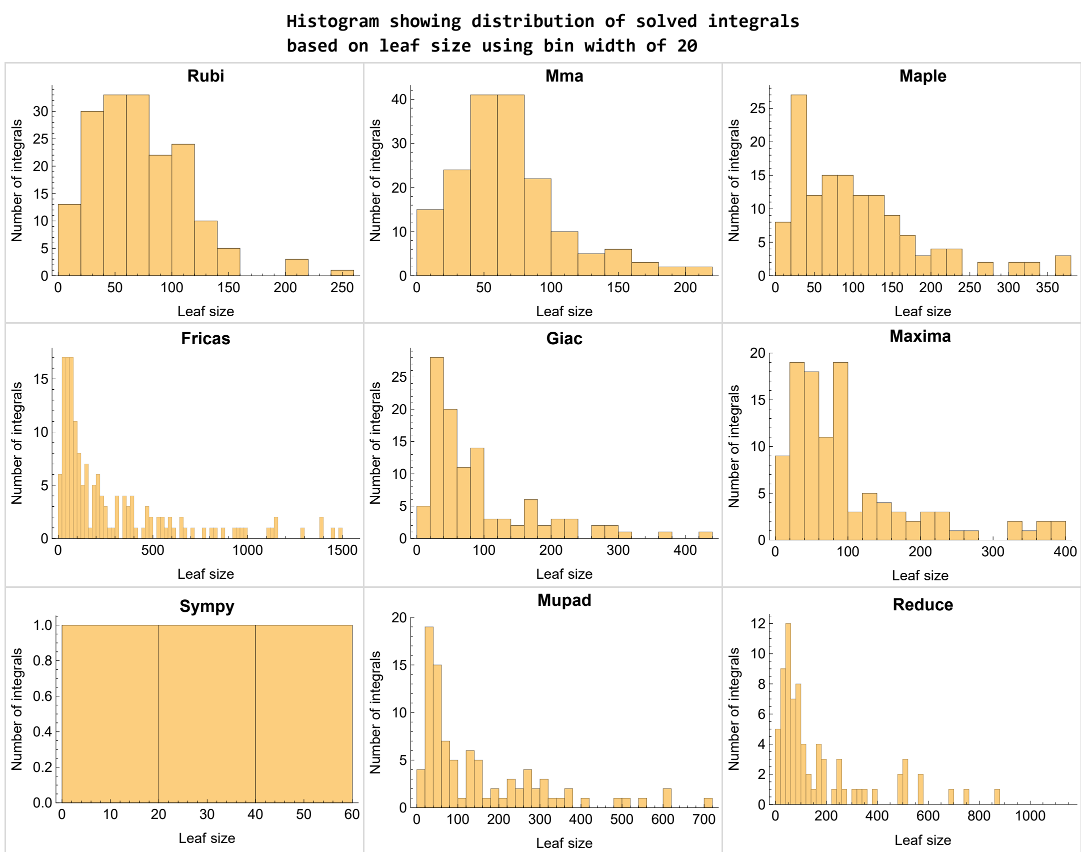


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

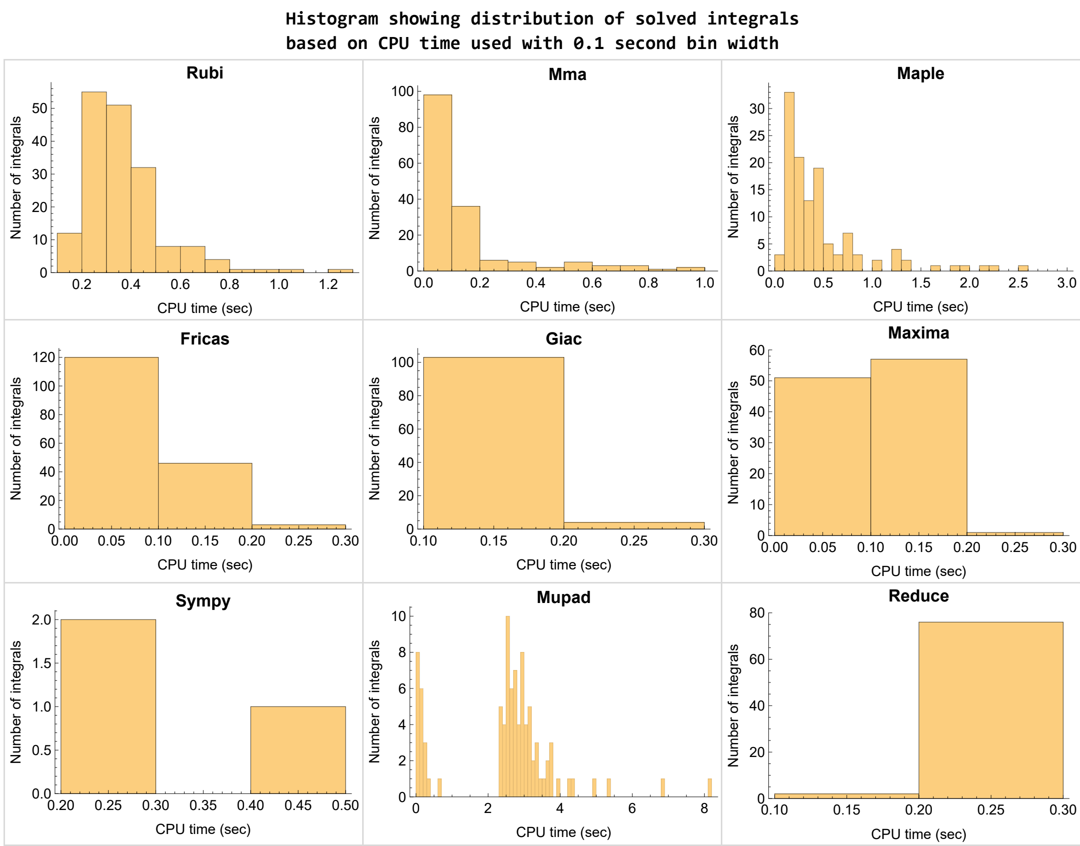


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

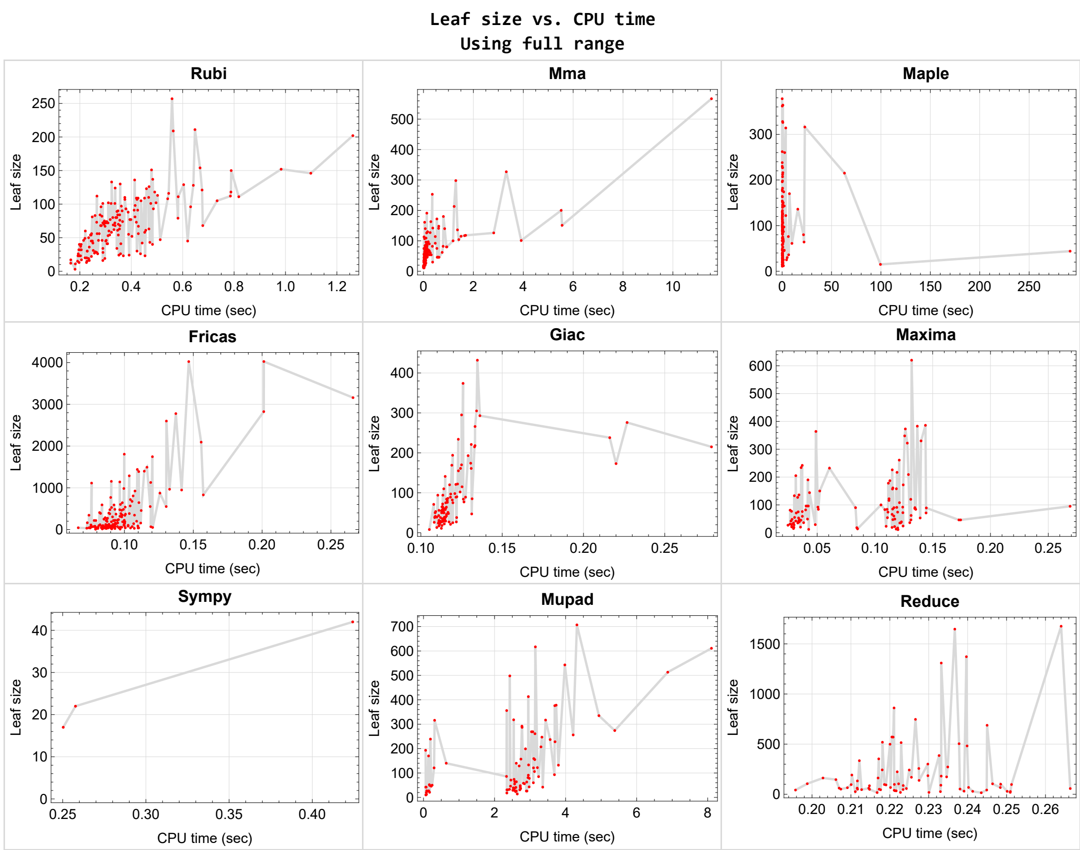


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{74, 75, 76, 130, 131, 132, 134, 135, 136, 137, 139, 142, 143, 144, 146, 147, 148, 149, 150, 151, 154, 155\}$

Mathematica $\{51, 164\}$

Maple $\{125, 126, 127, 128\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

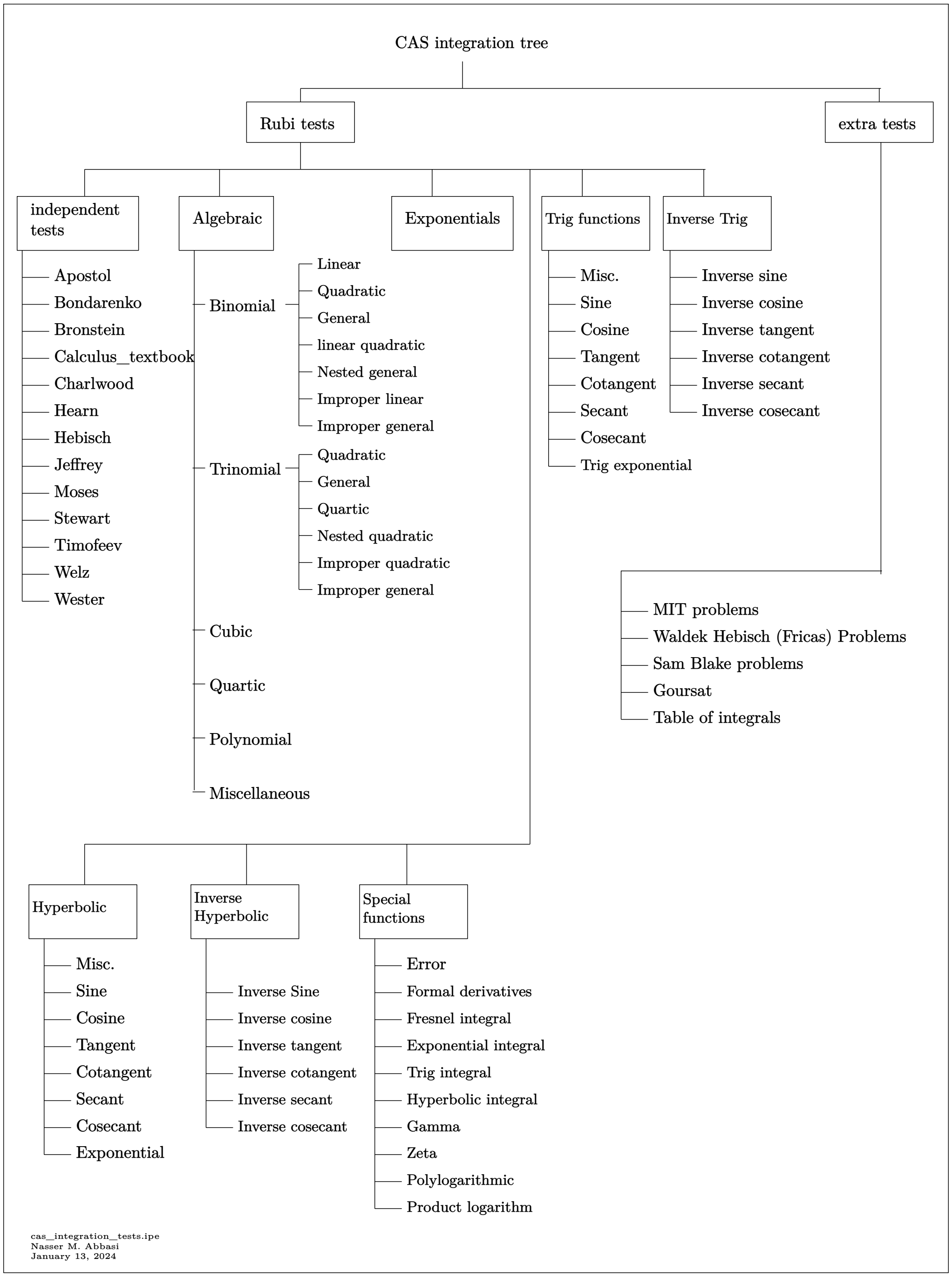
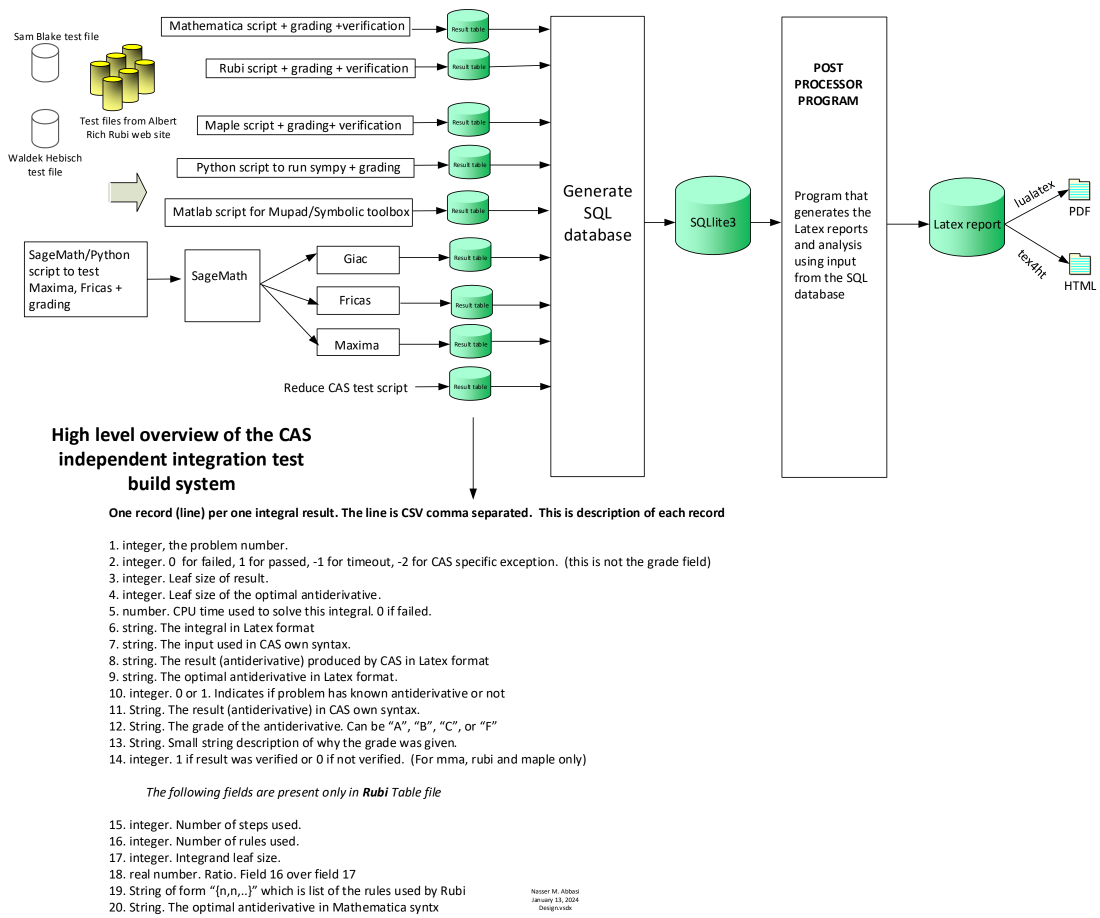


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 80, 81, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 96, 97, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 117, 118, 119, 120, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 170, 171, 172, 173, 174, 175 }

B grade { }

C grade { 3, 4, 5, 6, 42, 43, 44, 77, 78, 79, 82, 83, 93, 95, 98, 100, 116, 121, 123, 160, 167, 168, 169 }

F normal fail { 114 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 92, 93, 94, 95, 96, 97, 98, 100, 102, 104, 105, 106, 107, 108, 109, 111, 114, 116, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 135, 137, 138, 139, 141, 143, 145, 147, 149, 152, 153, 156, 157, 158, 160, 163, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

B grade { 3, 5, 23, 24, 55, 67, 68, 69, 70, 71, 72, 89, 91, 103, 110, 112, 159, 161, 162, 164 }

C grade { 99, 101, 113, 115, 117, 132, 134, 136, 140, 142, 144, 146, 148, 150, 151, 154, 155 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 11, 42, 43, 44, 45, 49, 50, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 79, 80, 81, 82, 83, 85, 87, 88, 93, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 111, 113, 114, 115, 116, 117, 118, 119, 121, 123, 129, 130, 131, 132, 133, 134, 135, 140, 141, 143, 144, 145, 146, 147, 148, 150, 151, 160, 165, 166, 167, 168, 169, 170, 172, 174 }

B grade { 8, 9, 10, 12, 17, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 46, 47, 48, 76, 77, 78, 84, 86, 89, 90, 91, 92, 94, 101, 110, 112, 120, 122, 124, 136, 138, 142, 152, 171, 173, 175 }

C grade { 125, 126, 127, 128 }

F normal fail { 13, 14, 15, 16, 18, 19, 20, 21, 36, 37, 38, 39, 40, 41, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 137, 139, 149, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 8, 9, 15, 16, 38, 49, 50, 62, 63, 66, 84, 97, 107, 108, 117, 118, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 151, 152, 153, 154, 155, 161, 162, 171, 172 }

B grade { 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 17, 18, 19, 20, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 145, 160, 163, 164, 165, 166, 167, 168, 169, 170, 173, 174, 175 }

C grade { 22, 23, 24, 25, 26, 27, 28 }

F normal fail { 21, 150, 156, 157, 158, 159 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 29, 30, 31, 32, 33, 34, 35, 45, 46, 47, 48, 49, 50, 62, 63, 64, 65, 66, 67, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 84, 87, 93, 95, 97, 98, 99, 105, 106, 107, 114, 115, 116, 117, 118, 119, 123, 126, 127, 128, 129, 130, 131, 133, 145, 162, 165, 166 }

B grade { 3, 4, 5, 6, 42, 43, 44, 68, 69, 70, 76, 83, 85, 86, 88, 89, 92, 94, 96, 100, 101, 102, 103, 104, 108, 109, 110, 111, 112, 113, 120, 121, 122, 124, 125, 141, 153, 160, 161, 167, 168, 169 }

C grade { 22, 23, 24, 25, 26, 27, 28 }

F normal fail { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 36, 37, 38, 39, 40, 41, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 132, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 163, 164, 170, 171, 172, 173, 174, 175 }

F(-1) timedout fail { }

F(-2) exception fail { 90, 91 }

Giac

A grade { 2, 4, 6, 29, 30, 31, 33, 34, 35, 42, 43, 44, 45, 46, 47, 48, 49, 50, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 87, 93, 94, 95, 96, 97, 98, 100, 102, 103, 104, 105, 106, 107, 108, 114, 116, 117, 118, 119, 123, 125, 126, 127, 128, 129, 130, 131, 161, 162, 166, 168 }

B grade { 1, 3, 5, 32, 68, 85, 86, 88, 89, 90, 91, 92, 99, 101, 109, 110, 111, 112, 113, 115, 120, 121, 122, 124, 160, 165, 167, 169 }

C grade { 22, 23, 24, 25, 26, 27, 28 }

F normal fail { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 36, 37, 38, 39, 40, 41, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 132, 133, 135, 156, 157, 158, 159, 163, 164 }

F(-1) timedout fail { 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 170, 171, 172, 173, 174, 175 }

F(-2) exception fail { 134, 136, 148 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 25, 32, 42, 43, 44, 45, 49, 50, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 133, 141, 145, 153, 160, 161, 162, 165, 166, 167, 168, 169 }

C grade { }

F normal fail { }

F(-1) timedout fail { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 76, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 163, 164, 170, 171, 172, 173, 174, 175 }

F(-2) exception fail { }

Sympy

A grade { 1, 73, 165 }

B grade { }

C grade { }

F normal fail { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

F(-1) timedout fail { }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 42, 43, 44, 45, 46, 47, 48, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 160, 161, 162, 165, 166, 167, 168, 169 }

C grade { }

F normal fail { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 36, 37, 38, 39, 40, 41, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 84, 85, 86, 87, 88, 89, 90, 91, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 163, 164, 170, 171, 172, 173, 174, 175 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	15	14	38	17	27	27	27
N.S.	1	1.00	1.00	1.25	1.17	3.17	1.42	2.25	2.25	2.25
time (sec)	N/A	0.164	0.003	0.083	0.085	0.075	0.250	0.113	0.251	0.091

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	18	43	0	18	29	18
N.S.	1	1.00	1.00	1.09	1.64	3.91	0.00	1.64	2.64	1.64
time (sec)	N/A	0.182	0.022	0.259	0.084	0.066	0.000	0.113	0.241	2.346

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	42	75	27	84	387	0	84	162	86
N.S.	1	1.24	2.21	0.79	2.47	11.38	0.00	2.47	4.76	2.53
time (sec)	N/A	0.251	0.018	0.257	0.051	0.083	0.000	0.116	0.217	2.336

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	32	35	23	90	164	0	31	57	31
N.S.	1	1.23	1.35	0.88	3.46	6.31	0.00	1.19	2.19	1.19
time (sec)	N/A	0.201	0.013	0.281	0.083	0.073	0.000	0.111	0.266	2.370

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	70	113	41	133	1114	0	110	301	193
N.S.	1	1.27	2.05	0.75	2.42	20.25	0.00	2.00	5.47	3.51
time (sec)	N/A	0.350	0.022	0.312	0.030	0.076	0.000	0.114	0.230	0.060

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	46	56	33	205	344	0	42	93	42
N.S.	1	1.10	1.33	0.79	4.88	8.19	0.00	1.00	2.21	1.00
time (sec)	N/A	0.214	0.020	0.337	0.032	0.074	0.000	0.116	0.219	0.076

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	61	101	0	194	0	0	18	0
N.S.	1	1.00	0.76	1.26	0.00	2.42	0.00	0.00	0.22	0.00
time (sec)	N/A	0.344	0.093	0.225	0.000	0.087	0.000	0.000	0.232	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	57	154	0	96	0	0	16	0
N.S.	1	1.00	0.75	2.03	0.00	1.26	0.00	0.00	0.21	0.00
time (sec)	N/A	0.355	0.169	0.162	0.000	0.081	0.000	0.000	0.246	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	87	0	24	0	0	9	0
N.S.	1	1.00	0.89	1.61	0.00	0.44	0.00	0.00	0.17	0.00
time (sec)	N/A	0.244	0.153	0.182	0.000	0.093	0.000	0.000	0.241	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	108	0	150	0	0	18	0
N.S.	1	1.00	0.93	2.00	0.00	2.78	0.00	0.00	0.33	0.00
time (sec)	N/A	0.241	0.034	0.231	0.000	0.100	0.000	0.000	0.203	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	63	100	0	223	0	0	18	0
N.S.	1	1.00	0.79	1.25	0.00	2.79	0.00	0.00	0.22	0.00
time (sec)	N/A	0.328	0.053	0.180	0.000	0.093	0.000	0.000	0.253	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	67	164	0	370	0	0	18	0
N.S.	1	1.00	0.84	2.05	0.00	4.62	0.00	0.00	0.22	0.00
time (sec)	N/A	0.340	0.096	0.165	0.000	0.101	0.000	0.000	0.216	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	118	79	0	0	484	0	0	24	0
N.S.	1	1.02	0.68	0.00	0.00	4.17	0.00	0.00	0.21	0.00
time (sec)	N/A	0.491	0.131	0.000	0.000	0.093	0.000	0.000	0.217	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	66	0	0	217	0	0	24	0
N.S.	1	1.00	0.75	0.00	0.00	2.47	0.00	0.00	0.27	0.00
time (sec)	N/A	0.388	0.087	0.000	0.000	0.079	0.000	0.000	0.235	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	60	0	0	107	0	0	20	0
N.S.	1	1.00	0.71	0.00	0.00	1.27	0.00	0.00	0.24	0.00
time (sec)	N/A	0.355	0.055	0.000	0.000	0.087	0.000	0.000	0.233	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	0	0	27	0	0	12	0
N.S.	1	1.00	0.96	0.00	0.00	0.48	0.00	0.00	0.21	0.00
time (sec)	N/A	0.267	0.037	0.000	0.000	0.091	0.000	0.000	0.214	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	52	227	0	154	0	0	24	0
N.S.	1	1.00	0.93	4.05	0.00	2.75	0.00	0.00	0.43	0.00
time (sec)	N/A	0.275	0.045	0.347	0.000	0.100	0.000	0.000	0.210	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	73	0	0	232	0	0	24	0
N.S.	1	1.00	0.81	0.00	0.00	2.58	0.00	0.00	0.27	0.00
time (sec)	N/A	0.364	0.075	0.000	0.000	0.095	0.000	0.000	0.259	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	68	0	0	378	0	0	24	0
N.S.	1	1.00	0.76	0.00	0.00	4.20	0.00	0.00	0.27	0.00
time (sec)	N/A	0.348	0.106	0.000	0.000	0.089	0.000	0.000	0.229	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	126	80	0	0	483	0	0	24	0
N.S.	1	1.07	0.68	0.00	0.00	4.09	0.00	0.00	0.20	0.00
time (sec)	N/A	0.469	0.132	0.000	0.000	0.099	0.000	0.000	0.211	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	90	0	0	0	0	0	14	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.304	0.123	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	C	C	F	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	49	51	114	74	117	0	72	181	0
N.S.	1	1.22	1.28	2.85	1.85	2.92	0.00	1.80	4.52	0.00
time (sec)	N/A	0.220	0.155	0.223	0.120	0.082	0.000	0.113	0.233	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	C	C	F	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	26	51	99	49	67	0	57	98	0
N.S.	1	1.08	2.12	4.12	2.04	2.79	0.00	2.38	4.08	0.00
time (sec)	N/A	0.194	0.041	0.119	0.117	0.077	0.000	0.113	0.221	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	C	C	F	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	17	67	19	15	0	27	17	0
N.S.	1	1.00	5.67	22.33	6.33	5.00	0.00	9.00	5.67	0.00
time (sec)	N/A	0.181	0.005	0.113	0.115	0.084	0.000	0.117	0.217	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	C	C	F	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	15	13	58	11	14	0	25	15	31
N.S.	1	1.15	1.00	4.46	0.85	1.08	0.00	1.92	1.15	2.38
time (sec)	N/A	0.191	0.003	0.118	0.120	0.079	0.000	0.120	0.243	2.349

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	C	C	F	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	37	27	118	23	26	0	38	31	0
N.S.	1	1.12	0.82	3.58	0.70	0.79	0.00	1.15	0.94	0.00
time (sec)	N/A	0.203	0.036	0.110	0.110	0.081	0.000	0.114	0.239	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	C	C	F	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	60	33	178	35	38	0	50	47	0
N.S.	1	1.22	0.67	3.63	0.71	0.78	0.00	1.02	0.96	0.00
time (sec)	N/A	0.221	0.046	0.118	0.122	0.088	0.000	0.114	0.223	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	C	C	F	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	83	39	238	47	50	0	62	61	0
N.S.	1	1.28	0.60	3.66	0.72	0.77	0.00	0.95	0.94	0.00
time (sec)	N/A	0.259	0.054	0.128	0.116	0.081	0.000	0.114	0.207	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	82	51	123	92	1128	0	75	185	0
N.S.	1	1.26	0.78	1.89	1.42	17.35	0.00	1.15	2.85	0.00
time (sec)	N/A	0.286	0.129	0.237	0.124	0.119	0.000	0.115	0.230	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	56	39	103	60	340	0	58	100	0
N.S.	1	1.22	0.85	2.24	1.30	7.39	0.00	1.26	2.17	0.00
time (sec)	N/A	0.245	0.082	0.121	0.116	0.100	0.000	0.115	0.248	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	30	17	67	24	97	0	29	18	0
N.S.	1	1.15	0.65	2.58	0.92	3.73	0.00	1.12	0.69	0.00
time (sec)	N/A	0.229	0.006	0.128	0.124	0.084	0.000	0.115	0.223	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	17	13	58	17	83	0	24	19	33
N.S.	1	1.31	1.00	4.46	1.31	6.38	0.00	1.85	1.46	2.54
time (sec)	N/A	0.206	0.025	0.115	0.120	0.092	0.000	0.112	0.251	2.422

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	50	27	130	35	285	0	41	35	0
N.S.	1	1.39	0.75	3.61	0.97	7.92	0.00	1.14	0.97	0.00
time (sec)	N/A	0.234	0.045	0.123	0.122	0.086	0.000	0.109	0.221	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	81	36	196	53	590	0	53	51	0
N.S.	1	1.47	0.65	3.56	0.96	10.73	0.00	0.96	0.93	0.00
time (sec)	N/A	0.248	0.049	0.136	0.139	0.092	0.000	0.109	0.207	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	112	42	262	71	984	0	65	65	0
N.S.	1	1.51	0.57	3.54	0.96	13.30	0.00	0.88	0.88	0.00
time (sec)	N/A	0.266	0.068	0.121	0.144	0.100	0.000	0.113	0.209	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	150	68	0	0	1389	0	0	16	0
N.S.	1	1.11	0.50	0.00	0.00	10.29	0.00	0.00	0.12	0.00
time (sec)	N/A	0.789	0.140	0.000	0.000	0.110	0.000	0.000	0.249	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	108	56	0	0	395	0	0	14	0
N.S.	1	1.33	0.69	0.00	0.00	4.88	0.00	0.00	0.17	0.00
time (sec)	N/A	0.542	0.099	0.000	0.000	0.089	0.000	0.000	0.240	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	77	42	0	0	60	0	0	11	0
N.S.	1	1.38	0.75	0.00	0.00	1.07	0.00	0.00	0.20	0.00
time (sec)	N/A	0.394	0.036	0.000	0.000	0.075	0.000	0.000	0.217	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	81	43	0	0	127	0	0	16	0
N.S.	1	1.31	0.69	0.00	0.00	2.05	0.00	0.00	0.26	0.00
time (sec)	N/A	0.414	0.070	0.000	0.000	0.089	0.000	0.000	0.235	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	110	57	0	0	407	0	0	16	0
N.S.	1	1.24	0.64	0.00	0.00	4.57	0.00	0.00	0.18	0.00
time (sec)	N/A	0.468	0.078	0.000	0.000	0.082	0.000	0.000	0.219	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	154	71	0	0	718	0	0	16	0
N.S.	1	1.14	0.53	0.00	0.00	5.32	0.00	0.00	0.12	0.00
time (sec)	N/A	0.668	0.103	0.000	0.000	0.104	0.000	0.000	0.223	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	84	59	72	620	2825	0	51	146	498
N.S.	1	0.51	0.36	0.44	3.78	17.23	0.00	0.31	0.89	3.04
time (sec)	N/A	0.278	0.057	0.800	0.132	0.201	0.000	0.113	0.206	2.429

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	68	47	60	322	1493	0	39	104	356
N.S.	1	0.58	0.40	0.51	2.73	12.65	0.00	0.33	0.88	3.02
time (sec)	N/A	0.281	0.047	0.762	0.128	0.116	0.000	0.109	0.246	2.341

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	46	33	46	120	529	0	27	60	46
N.S.	1	0.74	0.53	0.74	1.94	8.53	0.00	0.44	0.97	0.74
time (sec)	N/A	0.272	0.036	0.115	0.131	0.091	0.000	0.111	0.215	2.449

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	29	13	81	0	13	18	71
N.S.	1	1.00	1.00	1.81	0.81	5.06	0.00	0.81	1.12	4.44
time (sec)	N/A	0.237	0.026	0.135	0.118	0.079	0.000	0.110	0.230	2.460

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	30	24	89	22	253	0	26	29	0
N.S.	1	0.83	0.67	2.47	0.61	7.03	0.00	0.72	0.81	0.00
time (sec)	N/A	0.229	0.043	0.122	0.117	0.085	0.000	0.111	0.250	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	63	38	230	46	1141	0	50	57	0
N.S.	1	0.73	0.44	2.67	0.53	13.27	0.00	0.58	0.66	0.00
time (sec)	N/A	0.354	0.055	0.122	0.131	0.097	0.000	0.114	0.224	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	93	55	362	72	2600	0	76	87	0
N.S.	1	0.70	0.42	2.74	0.55	19.70	0.00	0.58	0.66	0.00
time (sec)	N/A	0.486	0.107	0.144	0.124	0.130	0.000	0.116	0.223	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	54	27	35	32	0	29	18	26
N.S.	1	1.00	1.69	0.84	1.09	1.00	0.00	0.91	0.56	0.81
time (sec)	N/A	0.196	0.311	0.287	0.030	0.086	0.000	0.114	0.212	2.912

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	54	27	35	32	0	29	19	26
N.S.	1	1.00	1.69	0.84	1.09	1.00	0.00	0.91	0.59	0.81
time (sec)	N/A	0.206	0.248	0.310	0.033	0.090	0.000	0.112	0.265	2.896

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	111	136	0	0	566	0	0	67	0
N.S.	1	1.04	1.27	0.00	0.00	5.29	0.00	0.00	0.63	0.00
time (sec)	N/A	0.583	1.363	0.000	0.000	0.104	0.000	0.000	0.246	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	100	0	0	461	0	0	40	0
N.S.	1	1.00	1.39	0.00	0.00	6.40	0.00	0.00	0.56	0.00
time (sec)	N/A	0.309	1.186	0.000	0.000	0.098	0.000	0.000	0.249	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	80	0	0	383	0	0	16	0
N.S.	1	1.00	2.00	0.00	0.00	9.58	0.00	0.00	0.40	0.00
time (sec)	N/A	0.205	0.921	0.000	0.000	0.104	0.000	0.000	0.265	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	118	0	0	551	0	0	68	0
N.S.	1	1.00	1.30	0.00	0.00	6.05	0.00	0.00	0.75	0.00
time (sec)	N/A	0.425	1.689	0.000	0.000	0.130	0.000	0.000	0.239	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	129	327	0	0	873	0	0	104	0
N.S.	1	1.05	2.66	0.00	0.00	7.10	0.00	0.00	0.85	0.00
time (sec)	N/A	0.604	3.320	0.000	0.000	0.126	0.000	0.000	0.241	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	80	0	0	383	0	0	17	0
N.S.	1	1.00	2.00	0.00	0.00	9.58	0.00	0.00	0.42	0.00
time (sec)	N/A	0.198	0.937	0.000	0.000	0.097	0.000	0.000	0.215	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	117	0	0	551	0	0	69	0
N.S.	1	1.00	1.29	0.00	0.00	6.05	0.00	0.00	0.76	0.00
time (sec)	N/A	0.411	1.648	0.000	0.000	0.119	0.000	0.000	0.243	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	46	0	0	212	0	0	12	0
N.S.	1	1.00	2.00	0.00	0.00	9.22	0.00	0.00	0.52	0.00
time (sec)	N/A	0.193	0.624	0.000	0.000	0.092	0.000	0.000	0.232	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	46	0	0	212	0	0	13	0
N.S.	1	1.00	2.00	0.00	0.00	9.22	0.00	0.00	0.57	0.00
time (sec)	N/A	0.196	0.583	0.000	0.000	0.087	0.000	0.000	0.232	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	46	0	0	218	0	0	12	0
N.S.	1	1.00	2.00	0.00	0.00	9.48	0.00	0.00	0.52	0.00
time (sec)	N/A	0.203	0.579	0.000	0.000	0.095	0.000	0.000	0.249	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	46	0	0	218	0	0	13	0
N.S.	1	1.00	2.00	0.00	0.00	9.48	0.00	0.00	0.57	0.00
time (sec)	N/A	0.200	0.550	0.000	0.000	0.092	0.000	0.000	0.212	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	63	63	65	71	79	0	62	13	64
N.S.	1	1.09	1.09	1.12	1.22	1.36	0.00	1.07	0.22	1.10
time (sec)	N/A	0.469	0.152	1.053	0.032	0.083	0.000	0.115	0.203	2.615

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	52	56	53	59	67	0	50	13	52
N.S.	1	1.13	1.22	1.15	1.28	1.46	0.00	1.09	0.28	1.13
time (sec)	N/A	0.402	0.152	0.737	0.037	0.119	0.000	0.109	0.213	0.165

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	42	46	39	47	55	0	38	13	38
N.S.	1	1.17	1.28	1.08	1.31	1.53	0.00	1.06	0.36	1.06
time (sec)	N/A	0.442	0.147	0.520	0.035	0.089	0.000	0.112	0.263	2.568

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	24	35	25	31	39	0	24	11	24
N.S.	1	1.20	1.75	1.25	1.55	1.95	0.00	1.20	0.55	1.20
time (sec)	N/A	0.392	0.090	0.359	0.027	0.110	0.000	0.111	0.218	2.593

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	13	27	11	12	8	0	8	13	10
N.S.	1	0.93	1.93	0.79	0.86	0.57	0.00	0.57	0.93	0.71
time (sec)	N/A	0.226	0.095	0.165	0.043	0.079	0.000	0.105	0.215	0.066

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	26	46	19	27	31	0	22	13	26
N.S.	1	1.53	2.71	1.12	1.59	1.82	0.00	1.29	0.76	1.53
time (sec)	N/A	0.358	0.116	0.278	0.025	0.085	0.000	0.112	0.227	2.612

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	70	35	53	77	0	46	13	60
N.S.	1	1.00	2.69	1.35	2.04	2.96	0.00	1.77	0.50	2.31
time (sec)	N/A	0.436	0.199	0.447	0.034	0.090	0.000	0.116	0.217	3.093

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	47	90	53	77	120	0	50	13	63
N.S.	1	1.27	2.43	1.43	2.08	3.24	0.00	1.35	0.35	1.70
time (sec)	N/A	0.513	0.232	0.592	0.028	0.092	0.000	0.113	0.224	2.906

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	116	567	92	234	1440	0	169	571	239
N.S.	1	1.06	5.20	0.84	2.15	13.21	0.00	1.55	5.24	2.19
time (sec)	N/A	0.545	11.547	0.717	0.036	0.109	0.000	0.119	0.221	0.196

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	77	151	66	136	769	0	122	386	170
N.S.	1	1.03	2.01	0.88	1.81	10.25	0.00	1.63	5.15	2.27
time (sec)	N/A	0.318	5.558	0.470	0.034	0.090	0.000	0.123	0.233	0.142

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	75	37	44	222	0	59	133	74
N.S.	1	1.00	2.21	1.09	1.29	6.53	0.00	1.74	3.91	2.18
time (sec)	N/A	0.344	0.537	0.347	0.035	0.100	0.000	0.114	0.212	2.549

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	20	19	44	22	32	34	42
N.S.	1	1.00	1.00	1.18	1.12	2.59	1.29	1.88	2.00	2.47
time (sec)	N/A	0.165	0.001	0.109	0.030	0.083	0.258	0.113	0.222	0.063

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	53	64	82	85	186	0	84	67	121
N.S.	1	0.98	1.19	1.52	1.57	3.44	0.00	1.56	1.24	2.24
time (sec)	N/A	0.310	0.195	0.208	0.115	0.099	0.000	0.126	0.248	0.298

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	121	142	176	187	645	0	161	570	269
N.S.	1	1.20	1.41	1.74	1.85	6.39	0.00	1.59	5.64	2.66
time (sec)	N/A	0.675	0.373	0.289	0.113	0.111	0.000	0.124	0.221	3.040

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	202	213	328	373	2094	0	293	1647	0
N.S.	1	1.24	1.31	2.01	2.29	12.85	0.00	1.80	10.10	0.00
time (sec)	N/A	1.262	1.229	0.452	0.126	0.156	0.000	0.136	0.237	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	146	104	198	157	807	0	155	243	199
N.S.	1	1.36	0.97	1.85	1.47	7.54	0.00	1.45	2.27	1.86
time (sec)	N/A	1.099	1.415	0.425	0.115	0.106	0.000	0.123	0.218	2.871

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	105	82	152	116	456	0	115	173	157
N.S.	1	1.31	1.02	1.90	1.45	5.70	0.00	1.44	2.16	1.96
time (sec)	N/A	0.734	0.787	0.273	0.112	0.112	0.000	0.125	0.235	2.742

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	77	61	92	84	238	0	86	105	129
N.S.	1	1.35	1.07	1.61	1.47	4.18	0.00	1.51	1.84	2.26
time (sec)	N/A	0.398	0.272	0.207	0.111	0.096	0.000	0.117	0.199	2.692

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	45	45	35	54	111	0	56	42	49
N.S.	1	1.22	1.22	0.95	1.46	3.00	0.00	1.51	1.14	1.32
time (sec)	N/A	0.301	0.021	0.112	0.111	0.076	0.000	0.111	0.196	0.175

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	58	67	49	83	156	0	82	90	287
N.S.	1	1.16	1.34	0.98	1.66	3.12	0.00	1.64	1.80	5.74
time (sec)	N/A	0.452	0.235	0.170	0.109	0.112	0.000	0.119	0.233	2.775

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	79	92	73	100	345	0	98	242	292
N.S.	1	1.34	1.56	1.24	1.69	5.85	0.00	1.66	4.10	4.95
time (sec)	N/A	0.582	0.472	0.199	0.105	0.107	0.000	0.121	0.225	2.771

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	111	148	108	158	947	0	141	519	617
N.S.	1	1.34	1.78	1.30	1.90	11.41	0.00	1.70	6.25	7.43
time (sec)	N/A	0.818	0.613	0.267	0.119	0.142	0.000	0.115	0.218	3.149

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	43	32	116	42	43	0	38	13	41
N.S.	1	1.13	0.84	3.05	1.11	1.13	0.00	1.00	0.34	1.08
time (sec)	N/A	0.472	0.053	0.089	0.032	0.101	0.000	0.108	0.212	2.600

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	23	19	15	39	36	0	35	13	39
N.S.	1	1.21	1.00	0.79	2.05	1.89	0.00	1.84	0.68	2.05
time (sec)	N/A	0.369	0.008	99.528	0.035	0.087	0.000	0.113	0.236	2.528

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	24	20	30	30	31	0	26	13	29
N.S.	1	1.20	1.00	1.50	1.50	1.55	0.00	1.30	0.65	1.45
time (sec)	N/A	0.352	0.043	5.132	0.028	0.084	0.000	0.108	0.219	2.516

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	25	21	28	0	21	11	12
N.S.	1	1.00	1.00	1.56	1.31	1.75	0.00	1.31	0.69	0.75
time (sec)	N/A	0.289	0.008	1.240	0.028	0.087	0.000	0.118	0.220	0.084

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	30	20	30	41	52	0	51	11	46
N.S.	1	1.07	0.71	1.07	1.46	1.86	0.00	1.82	0.39	1.64
time (sec)	N/A	0.422	0.022	0.556	0.034	0.080	0.000	0.117	0.219	0.217

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	23	64	31	81	32	0	27	13	31
N.S.	1	1.21	3.37	1.63	4.26	1.68	0.00	1.42	0.68	1.63
time (sec)	N/A	0.453	0.065	0.509	0.027	0.073	0.000	0.122	0.248	2.712

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	45	32	61	0	142	0	94	13	122
N.S.	1	1.12	0.80	1.52	0.00	3.55	0.00	2.35	0.32	3.05
time (sec)	N/A	0.619	0.045	9.920	0.000	0.094	0.000	0.110	0.214	3.206

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	40	96	44	0	68	0	55	13	207
N.S.	1	1.38	3.31	1.52	0.00	2.34	0.00	1.90	0.45	7.14
time (sec)	N/A	0.483	0.100	291.459	0.000	0.076	0.000	0.113	0.208	3.306

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	101	97	378	242	1398	0	194	335	228
N.S.	1	0.99	0.95	3.71	2.37	13.71	0.00	1.90	3.28	2.24
time (sec)	N/A	0.476	0.192	0.080	0.037	0.114	0.000	0.120	0.212	3.702

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	152	180	215	217	924	0	221	257	247
N.S.	1	1.22	1.44	1.72	1.74	7.39	0.00	1.77	2.06	1.98
time (sec)	N/A	0.983	0.799	63.115	0.119	0.108	0.000	0.131	0.227	3.333

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	56	56	136	127	476	0	97	169	121
N.S.	1	0.98	0.98	2.39	2.23	8.35	0.00	1.70	2.96	2.12
time (sec)	N/A	0.427	0.027	15.882	0.033	0.089	0.000	0.118	0.226	3.096

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	96	80	122	122	304	0	121	104	159
N.S.	1	1.25	1.04	1.58	1.58	3.95	0.00	1.57	1.35	2.06
time (sec)	N/A	0.631	0.098	3.731	0.112	0.098	0.000	0.122	0.222	3.104

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	19	19	31	48	80	0	39	52	20
N.S.	1	0.95	0.95	1.55	2.40	4.00	0.00	1.95	2.60	1.00
time (sec)	N/A	0.297	0.009	0.645	0.028	0.103	0.000	0.115	0.238	0.080

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	54	56	72	66	57	0	89	49	93
N.S.	1	0.84	0.88	1.12	1.03	0.89	0.00	1.39	0.77	1.45
time (sec)	N/A	0.350	0.049	0.267	0.115	0.085	0.000	0.122	0.215	3.686

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	73	67	81	91	256	0	85	162	133
N.S.	1	1.22	1.12	1.35	1.52	4.27	0.00	1.42	2.70	2.22
time (sec)	N/A	0.420	0.171	0.467	0.119	0.087	0.000	0.131	0.203	2.958

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	123	127	143	161	675	0	218	353	256
N.S.	1	1.37	1.41	1.59	1.79	7.50	0.00	2.42	3.92	2.84
time (sec)	N/A	0.459	0.266	2.200	0.116	0.106	0.000	0.133	0.217	4.213

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	128	114	170	226	1155	0	174	482	269
N.S.	1	1.23	1.10	1.63	2.17	11.11	0.00	1.67	4.63	2.59
time (sec)	N/A	0.642	0.472	7.418	0.115	0.091	0.000	0.131	0.240	3.075

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	209	163	316	348	2778	0	374	861	513
N.S.	1	1.39	1.09	2.11	2.32	18.52	0.00	2.49	5.74	3.42
time (sec)	N/A	0.563	0.295	22.982	0.126	0.137	0.000	0.126	0.221	6.875

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	101	75	97	144	304	0	120	13	274
N.S.	1	0.93	0.69	0.89	1.32	2.79	0.00	1.10	0.12	2.51
time (sec)	N/A	0.289	0.139	1.855	0.043	0.091	0.000	0.118	0.210	5.384

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	61	126	68	96	124	0	58	13	237
N.S.	1	1.17	2.42	1.31	1.85	2.38	0.00	1.12	0.25	4.56
time (sec)	N/A	0.456	0.126	1.224	0.042	0.092	0.000	0.115	0.243	3.565

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	73	61	71	96	186	0	94	13	140
N.S.	1	0.95	0.79	0.92	1.25	2.42	0.00	1.22	0.17	1.82
time (sec)	N/A	0.265	0.086	0.819	0.040	0.091	0.000	0.114	0.216	0.640

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	41	71	35	42	50	0	34	13	85
N.S.	1	1.14	1.97	0.97	1.17	1.39	0.00	0.94	0.36	2.36
time (sec)	N/A	0.359	0.103	0.550	0.027	0.120	0.000	0.115	0.233	3.239

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	39	35	45	71	0	53	11	50
N.S.	1	1.00	0.87	0.78	1.00	1.58	0.00	1.18	0.24	1.11
time (sec)	N/A	0.244	0.028	0.388	0.036	0.106	0.000	0.114	0.224	0.244

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	15	11	0	11	11	14
N.S.	1	1.00	1.00	1.15	1.15	0.85	0.00	0.85	0.85	1.08
time (sec)	N/A	0.197	0.007	0.427	0.030	0.079	0.000	0.113	0.231	2.640

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	15	22	18	20	16	0	17	13	21
N.S.	1	1.36	2.00	1.64	1.82	1.45	0.00	1.55	1.18	1.91
time (sec)	N/A	0.228	0.073	0.812	0.036	0.091	0.000	0.111	0.195	0.171

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	20	12	12	36	40	0	39	13	27
N.S.	1	1.67	1.00	1.00	3.00	3.33	0.00	3.25	1.08	2.25
time (sec)	N/A	0.227	0.009	1.337	0.038	0.110	0.000	0.115	0.201	0.147

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	34	76	46	55	86	0	43	13	56
N.S.	1	1.26	2.81	1.70	2.04	3.19	0.00	1.59	0.48	2.07
time (sec)	N/A	0.289	0.050	2.198	0.030	0.093	0.000	0.111	0.217	2.841

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	36	30	25	75	100	0	65	13	81
N.S.	1	1.20	1.00	0.83	2.50	3.33	0.00	2.17	0.43	2.70
time (sec)	N/A	0.237	0.012	4.462	0.034	0.091	0.000	0.113	0.215	2.942

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	55	140	76	96	157	0	71	13	106
N.S.	1	1.28	3.26	1.77	2.23	3.65	0.00	1.65	0.30	2.47
time (sec)	N/A	0.389	0.072	6.309	0.042	0.093	0.000	0.108	0.222	3.123

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	257	253	364	383	4025	0	432	1309	611
N.S.	1	1.32	1.30	1.88	1.97	20.75	0.00	2.23	6.75	3.15
time (sec)	N/A	0.559	0.354	0.776	0.137	0.201	0.000	0.135	0.233	8.106

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	B	F	A	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	0	141	207	261	1746	0	215	688	707
N.S.	1	0.00	0.77	1.13	1.43	9.54	0.00	1.17	3.76	3.86
time (sec)	N/A	0.000	0.604	0.431	0.121	0.120	0.000	0.133	0.245	4.318

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	151	191	182	172	965	0	234	515	335
N.S.	1	1.34	1.69	1.61	1.52	8.54	0.00	2.07	4.56	2.96
time (sec)	N/A	0.479	0.134	0.436	0.125	0.133	0.000	0.123	0.223	4.938

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	112	82	95	108	349	0	102	224	376
N.S.	1	1.12	0.82	0.95	1.08	3.49	0.00	1.02	2.24	3.76
time (sec)	N/A	0.786	0.356	0.236	0.121	0.098	0.000	0.124	0.222	3.695

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	84	63	97	74	75	0	89	68	132
N.S.	1	1.38	1.03	1.59	1.21	1.23	0.00	1.46	1.11	2.16
time (sec)	N/A	0.347	0.043	0.244	0.111	0.087	0.000	0.118	0.240	3.798

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	22	11	21	28	27	0	22	26	25
N.S.	1	1.16	0.58	1.11	1.47	1.42	0.00	1.16	1.37	1.32
time (sec)	N/A	0.245	0.007	0.233	0.030	0.087	0.000	0.114	0.233	0.095

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	68	75	84	90	141	0	89	61	316
N.S.	1	1.19	1.32	1.47	1.58	2.47	0.00	1.56	1.07	5.54
time (sec)	N/A	0.678	0.099	0.431	0.116	0.107	0.000	0.127	0.212	0.313

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	41	37	75	82	199	0	80	192	261
N.S.	1	1.28	1.16	2.34	2.56	6.22	0.00	2.50	6.00	8.16
time (sec)	N/A	0.308	0.036	0.638	0.040	0.109	0.000	0.116	0.210	3.138

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	118	172	150	178	831	0	161	504	378
N.S.	1	1.34	1.95	1.70	2.02	9.44	0.00	1.83	5.73	4.30
time (sec)	N/A	0.788	0.537	1.085	0.112	0.157	0.000	0.131	0.238	3.734

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	75	83	174	190	1288	0	170	747	155
N.S.	1	1.07	1.19	2.49	2.71	18.40	0.00	2.43	10.67	2.21
time (sec)	N/A	0.296	0.093	1.605	0.042	0.104	0.000	0.125	0.227	3.132

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	211	298	260	330	3160	0	305	1372	543
N.S.	1	1.15	1.63	1.42	1.80	17.27	0.00	1.67	7.50	2.97
time (sec)	N/A	0.648	1.296	2.582	0.140	0.266	0.000	0.134	0.240	3.978

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	124	130	314	364	4024	0	295	1676	317
N.S.	1	1.04	1.09	2.64	3.06	33.82	0.00	2.48	14.08	2.66
time (sec)	N/A	0.337	0.165	3.711	0.049	0.147	0.000	0.125	0.264	3.436

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	199	113	84	87	386	592	0	90	138	413
N.S.	1	0.57	0.42	0.44	1.94	2.97	0.00	0.45	0.69	2.08
time (sec)	N/A	0.502	0.060	1.352	0.144	0.084	0.000	0.122	0.228	2.952

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	147	93	72	65	209	315	0	77	96	91
N.S.	1	0.63	0.49	0.44	1.42	2.14	0.00	0.52	0.65	0.62
time (sec)	N/A	0.379	0.046	1.280	0.129	0.097	0.000	0.127	0.210	2.951

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	40	84	121	0	64	50	78
N.S.	1	1.00	0.97	0.69	1.45	2.09	0.00	1.10	0.86	1.34
time (sec)	N/A	0.323	0.033	0.333	0.135	0.086	0.000	0.116	0.212	2.980

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	29	39	42	0	48	46	0
N.S.	1	1.00	0.96	0.63	0.85	0.91	0.00	1.04	1.00	0.00
time (sec)	N/A	0.303	0.027	0.327	0.123	0.074	0.000	0.117	0.213	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	61	48	106	36	66	0	75	26	0
N.S.	1	0.82	0.65	1.43	0.49	0.89	0.00	1.01	0.35	0.00
time (sec)	N/A	0.347	0.038	0.736	0.130	0.088	0.000	0.120	0.211	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	74	76	216	62	126	0	193	67	0
N.S.	1	0.46	0.47	1.33	0.38	0.78	0.00	1.19	0.41	0.00
time (sec)	N/A	0.366	0.046	0.713	0.131	0.093	0.000	0.129	0.220	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	102	109	326	90	218	0	266	97	0
N.S.	1	0.41	0.44	1.30	0.36	0.87	0.00	1.06	0.39	0.00
time (sec)	N/A	0.496	0.102	0.719	0.135	0.078	0.000	0.133	0.219	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	101	80	125	0	72	0	0	23	0
N.S.	1	1.25	0.99	1.54	0.00	0.89	0.00	0.00	0.28	0.00
time (sec)	N/A	0.336	0.126	0.401	0.000	0.095	0.000	0.000	0.233	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	28	44	39	46	48	0	0	23	42
N.S.	1	0.93	1.47	1.30	1.53	1.60	0.00	0.00	0.77	1.40
time (sec)	N/A	0.290	0.035	0.115	0.173	0.086	0.000	0.000	0.216	3.356

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	107	60	127	0	94	0	0	23	0
N.S.	1	0.90	0.50	1.07	0.00	0.79	0.00	0.00	0.19	0.00
time (sec)	N/A	0.422	0.085	0.420	0.000	0.100	0.000	0.000	0.212	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	73	74	97	0	92	0	0	23	0
N.S.	1	1.06	1.07	1.41	0.00	1.33	0.00	0.00	0.33	0.00
time (sec)	N/A	0.294	0.098	0.140	0.000	0.088	0.000	0.000	0.248	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	72	57	109	0	63	0	0	21	0
N.S.	1	1.20	0.95	1.82	0.00	1.05	0.00	0.00	0.35	0.00
time (sec)	N/A	0.289	0.108	0.341	0.000	0.092	0.000	0.000	0.239	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	68	75	0	0	86	0	0	20	0
N.S.	1	1.13	1.25	0.00	0.00	1.43	0.00	0.00	0.33	0.00
time (sec)	N/A	0.284	0.079	0.000	0.000	0.110	0.000	0.000	0.223	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	90	0	11	0	0	14	0
N.S.	1	1.00	0.93	1.96	0.00	0.24	0.00	0.00	0.30	0.00
time (sec)	N/A	0.287	0.081	0.224	0.000	0.097	0.000	0.000	0.207	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	54	0	0	43	0	0	14	0
N.S.	1	1.00	1.32	0.00	0.00	1.05	0.00	0.00	0.34	0.00
time (sec)	N/A	0.306	0.083	0.000	0.000	0.101	0.000	0.000	0.254	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	54	58	126	0	72	0	0	14	0
N.S.	1	0.73	0.78	1.70	0.00	0.97	0.00	0.00	0.19	0.00
time (sec)	N/A	0.316	0.070	0.427	0.000	0.085	0.000	0.000	0.216	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	29	33	38	89	37	0	0	14	58
N.S.	1	1.16	1.32	1.52	3.56	1.48	0.00	0.00	0.56	2.32
time (sec)	N/A	0.260	0.026	0.131	0.144	0.104	0.000	0.000	0.225	2.813

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	70	60	112	0	57	0	0	14	0
N.S.	1	1.09	0.94	1.75	0.00	0.89	0.00	0.00	0.22	0.00
time (sec)	N/A	0.293	0.072	0.351	0.000	0.097	0.000	0.000	0.263	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	133	95	121	0	110	0	0	23	0
N.S.	1	1.04	0.74	0.95	0.00	0.86	0.00	0.00	0.18	0.00
time (sec)	N/A	0.324	0.148	0.142	0.000	0.100	0.000	0.000	0.240	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	130	80	133	0	80	0	0	23	0
N.S.	1	1.10	0.68	1.13	0.00	0.68	0.00	0.00	0.19	0.00
time (sec)	N/A	0.357	0.124	0.336	0.000	0.089	0.000	0.000	0.211	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	28	44	47	46	56	0	0	23	42
N.S.	1	0.93	1.47	1.57	1.53	1.87	0.00	0.00	0.77	1.40
time (sec)	N/A	0.265	0.038	0.111	0.174	0.098	0.000	0.000	0.212	2.725

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	136	63	140	0	103	0	0	23	0
N.S.	1	0.84	0.39	0.86	0.00	0.64	0.00	0.00	0.14	0.00
time (sec)	N/A	0.413	0.093	0.421	0.000	0.088	0.000	0.000	0.249	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	102	87	113	0	102	0	0	23	0
N.S.	1	1.06	0.91	1.18	0.00	1.06	0.00	0.00	0.24	0.00
time (sec)	N/A	0.311	0.131	0.145	0.000	0.103	0.000	0.000	0.249	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	101	65	124	0	71	0	0	23	0
N.S.	1	1.17	0.76	1.44	0.00	0.83	0.00	0.00	0.27	0.00
time (sec)	N/A	0.321	0.094	0.356	0.000	0.091	0.000	0.000	0.222	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	89	88	0	0	94	0	0	23	0
N.S.	1	0.98	0.97	0.00	0.00	1.03	0.00	0.00	0.25	0.00
time (sec)	N/A	0.344	0.116	0.000	0.000	0.091	0.000	0.000	0.227	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	110	60	152	0	0	0	0	21	0
N.S.	1	0.85	0.46	1.17	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.375	0.120	0.413	0.000	0.000	0.000	0.000	0.223	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	63	130	0	106	0	0	20	0
N.S.	1	1.00	0.66	1.35	0.00	1.10	0.00	0.00	0.21	0.00
time (sec)	N/A	0.282	0.077	0.122	0.000	0.089	0.000	0.000	0.203	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	163	0	60	0	0	21	0
N.S.	1	1.00	0.81	2.43	0.00	0.90	0.00	0.00	0.31	0.00
time (sec)	N/A	0.428	0.095	0.260	0.000	0.085	0.000	0.000	0.220	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	31	33	0	87	29	0	0	21	29
N.S.	1	1.15	1.22	0.00	3.22	1.07	0.00	0.00	0.78	1.07
time (sec)	N/A	0.252	0.030	0.000	0.134	0.107	0.000	0.000	0.252	2.554

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	72	66	0	0	43	0	0	21	0
N.S.	1	1.04	0.96	0.00	0.00	0.62	0.00	0.00	0.30	0.00
time (sec)	N/A	0.297	0.088	0.000	0.000	0.096	0.000	0.000	0.213	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	67	53	0	0	78	0	0	21	0
N.S.	1	0.97	0.77	0.00	0.00	1.13	0.00	0.00	0.30	0.00
time (sec)	N/A	0.298	0.077	0.000	0.000	0.088	0.000	0.000	0.198	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	62	62	0	0	0	0	0	13	0
N.S.	1	0.89	0.89	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.333	0.749	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	68	126	0	0	0	0	0	60	0
N.S.	1	0.97	1.80	0.00	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.356	2.816	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	69	101	0	0	0	0	0	78	0
N.S.	1	0.99	1.44	0.00	0.00	0.00	0.00	0.00	1.11	0.00
time (sec)	N/A	0.330	3.918	0.000	0.000	0.000	0.000	0.000	0.232	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	68	200	0	0	0	0	0	0	0
N.S.	1	0.97	2.86	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.337	5.520	0.000	0.000	0.000	0.000	0.000	0.268	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	137	30	80	95	187	0	215	97	65
N.S.	1	3.26	0.71	1.90	2.26	4.45	0.00	5.12	2.31	1.55
time (sec)	N/A	0.484	0.366	21.656	0.269	0.100	0.000	0.279	0.251	2.567

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	B	A	F	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	27	62	0	76	48	0	38	42	48
N.S.	1	1.04	2.38	0.00	2.92	1.85	0.00	1.46	1.62	1.85
time (sec)	N/A	0.257	0.139	0.000	0.037	0.079	0.000	0.122	0.245	2.724

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	A	A	F	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	65	0	49	49	0	39	39	36
N.S.	1	1.00	2.50	0.00	1.88	1.88	0.00	1.50	1.50	1.38
time (sec)	N/A	0.269	0.073	0.000	0.042	0.086	0.000	0.110	0.221	2.734

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	97	115	0	0	475	0	0	30	0
N.S.	1	1.08	1.28	0.00	0.00	5.28	0.00	0.00	0.33	0.00
time (sec)	N/A	0.361	1.506	0.000	0.000	0.097	0.000	0.000	0.245	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	107	140	0	0	539	0	0	35	0
N.S.	1	1.62	2.12	0.00	0.00	8.17	0.00	0.00	0.53	0.00
time (sec)	N/A	0.375	0.849	0.000	0.000	0.101	0.000	0.000	0.246	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	23	22	65	42	173	50	43
N.S.	1	1.00	1.00	1.15	1.10	3.25	2.10	8.65	2.50	2.15
time (sec)	N/A	0.220	0.039	0.243	0.028	0.093	0.424	0.220	0.217	2.974

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	29	71	0	28	45	25
N.S.	1	1.00	1.00	1.05	1.53	3.74	0.00	1.47	2.37	1.32
time (sec)	N/A	0.234	0.060	0.632	0.048	0.078	0.000	0.113	0.218	2.588

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	60	107	45	150	643	0	238	272	140
N.S.	1	1.09	1.95	0.82	2.73	11.69	0.00	4.33	4.95	2.55
time (sec)	N/A	0.313	0.045	1.952	0.052	0.098	0.000	0.216	0.235	2.612

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	45	56	36	92	272	0	47	85	55
N.S.	1	1.07	1.33	0.86	2.19	6.48	0.00	1.12	2.02	1.31
time (sec)	N/A	0.249	0.039	6.621	0.051	0.088	0.000	0.131	0.215	2.565

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	98	161	64	232	1806	0	276	499	318
N.S.	1	1.10	1.81	0.72	2.61	20.29	0.00	3.10	5.61	3.57
time (sec)	N/A	0.415	0.046	22.184	0.061	0.100	0.000	0.227	0.220	2.538

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	84	144	0	318	0	0	31	0
N.S.	1	1.00	0.77	1.32	0.00	2.92	0.00	0.00	0.28	0.00
time (sec)	N/A	0.420	0.146	1.265	0.000	0.097	0.000	0.000	0.232	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	80	212	0	159	0	0	29	0
N.S.	1	1.00	0.76	2.02	0.00	1.51	0.00	0.00	0.28	0.00
time (sec)	N/A	0.438	0.079	0.470	0.000	0.095	0.000	0.000	0.236	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	72	66	120	0	39	0	0	18	0
N.S.	1	1.03	0.94	1.71	0.00	0.56	0.00	0.00	0.26	0.00
time (sec)	N/A	0.302	0.062	0.422	0.000	0.085	0.000	0.000	0.212	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	72	68	146	0	248	0	0	31	0
N.S.	1	1.03	0.97	2.09	0.00	3.54	0.00	0.00	0.44	0.00
time (sec)	N/A	0.336	0.057	0.439	0.000	0.085	0.000	0.000	0.219	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	86	143	0	370	0	0	31	0
N.S.	1	1.00	0.79	1.31	0.00	3.39	0.00	0.00	0.28	0.00
time (sec)	N/A	0.452	0.086	0.449	0.000	0.094	0.000	0.000	0.234	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	95	227	0	602	0	0	31	0
N.S.	1	1.00	0.87	2.08	0.00	5.52	0.00	0.00	0.28	0.00
time (sec)	N/A	0.426	0.129	0.458	0.000	0.097	0.000	0.000	0.223	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [48] had the largest ratio of [1.8999999999999991]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	6	0.500
2	A	5	4	1.00	8	0.500
3	C	7	7	1.24	8	0.875
4	C	4	3	1.23	8	0.375
5	C	11	11	1.27	8	1.375
6	C	5	4	1.10	8	0.500
7	A	6	6	1.00	10	0.600
8	A	6	6	1.00	10	0.600
9	A	4	4	1.00	10	0.400
10	A	4	4	1.00	10	0.400
11	A	6	6	1.00	10	0.600
12	A	6	6	1.00	10	0.600
13	A	8	8	1.02	12	0.667
14	A	6	6	1.00	12	0.500
15	A	6	6	1.00	12	0.500
16	A	4	4	1.00	12	0.333
17	A	4	4	1.00	12	0.333
18	A	6	6	1.00	12	0.500
19	A	6	6	1.00	12	0.500
20	A	8	8	1.07	12	0.667
21	A	4	4	1.00	10	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	6	5	1.22	10	0.500
23	A	5	4	1.08	10	0.400
24	A	4	3	1.00	10	0.300
25	A	4	3	1.15	10	0.300
26	A	5	4	1.12	10	0.400
27	A	6	5	1.22	10	0.500
28	A	7	6	1.28	10	0.600
29	A	7	6	1.26	10	0.600
30	A	6	5	1.22	10	0.500
31	A	5	4	1.15	10	0.400
32	A	4	3	1.31	10	0.300
33	A	5	4	1.39	10	0.400
34	A	6	5	1.47	10	0.500
35	A	7	6	1.51	10	0.600
36	A	14	14	1.11	10	1.400
37	A	10	10	1.33	10	1.000
38	A	8	8	1.38	10	0.800
39	A	8	8	1.31	10	0.800
40	A	10	10	1.24	10	1.000
41	A	14	14	1.14	10	1.400
42	C	8	7	0.51	10	0.700
43	C	8	7	0.58	10	0.700
44	C	8	7	0.74	10	0.700
45	A	8	7	1.00	10	0.700
46	A	7	7	0.83	10	0.700
47	A	13	13	0.73	10	1.300
48	A	19	19	0.70	10	1.900
49	A	3	3	1.00	15	0.200
50	A	3	3	1.00	15	0.200
51	A	12	11	1.04	17	0.647
52	A	7	6	1.00	17	0.353
53	A	4	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
54	A	10	9	1.00	17	0.529
55	A	13	12	1.05	17	0.706
56	A	4	3	1.00	17	0.176
57	A	10	9	1.00	17	0.529
58	A	4	3	1.00	12	0.250
59	A	4	3	1.00	12	0.250
60	A	4	3	1.00	12	0.250
61	A	4	3	1.00	12	0.250
62	A	17	16	1.09	13	1.231
63	A	18	17	1.13	13	1.308
64	A	15	15	1.17	13	1.154
65	A	13	13	1.20	11	1.182
66	A	4	4	0.93	11	0.364
67	A	9	9	1.53	13	0.692
68	A	13	13	1.00	13	1.000
69	A	20	19	1.27	13	1.462
70	A	5	5	1.06	12	0.417
71	A	3	3	1.03	12	0.250
72	A	11	10	1.00	12	0.833
73	A	1	1	1.00	10	0.100
74	A	7	6	0.98	12	0.500
75	A	14	13	1.20	12	1.083
76	A	17	16	1.24	12	1.333
77	C	24	23	1.36	13	1.769
78	C	18	17	1.31	13	1.308
79	C	12	11	1.35	11	1.000
80	A	8	7	1.22	11	0.636
81	A	13	12	1.16	13	0.923
82	C	17	16	1.34	13	1.231
83	C	19	18	1.34	13	1.385
84	A	19	18	1.13	13	1.385
85	A	15	14	1.21	13	1.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	15	15	1.20	13	1.154
87	A	10	9	1.00	11	0.818
88	A	18	17	1.07	11	1.545
89	A	17	16	1.21	13	1.231
90	A	20	19	1.12	13	1.462
91	A	18	17	1.38	13	1.308
92	A	12	11	0.99	13	0.846
93	C	18	17	1.22	13	1.308
94	A	12	11	0.98	13	0.846
95	C	15	14	1.25	13	1.077
96	A	12	11	0.95	11	1.000
97	A	14	13	0.84	11	1.182
98	C	14	13	1.22	13	1.000
99	A	14	13	1.37	13	1.000
100	C	16	15	1.23	13	1.154
101	A	16	15	1.39	13	1.154
102	A	7	6	0.93	13	0.462
103	A	13	13	1.17	13	1.000
104	A	7	6	0.95	13	0.462
105	A	11	11	1.14	13	0.846
106	A	7	6	1.00	11	0.545
107	A	6	5	1.00	11	0.455
108	A	5	5	1.36	13	0.385
109	A	7	6	1.67	13	0.462
110	A	7	7	1.26	13	0.538
111	A	7	6	1.20	13	0.462
112	A	12	12	1.28	13	0.923
113	A	7	6	1.32	13	0.462
114	F	0	0	N/A	0.000	N/A
115	A	6	5	1.34	13	0.385
116	C	23	22	1.12	13	1.692
117	A	7	6	1.38	11	0.545

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	7	6	1.16	11	0.545
119	A	19	18	1.19	13	1.385
120	A	7	6	1.28	13	0.462
121	C	19	18	1.34	13	1.385
122	A	6	5	1.07	13	0.385
123	C	9	9	1.15	13	0.692
124	A	7	6	1.04	13	0.462
125	A	7	6	0.57	25	0.240
126	A	7	6	0.63	25	0.240
127	A	5	4	1.00	25	0.160
128	A	5	4	1.00	25	0.160
129	A	6	5	0.82	25	0.200
130	A	7	6	0.46	25	0.240
131	A	7	6	0.41	25	0.240
132	A	7	6	1.25	15	0.400
133	A	4	3	0.93	15	0.200
134	A	10	9	0.90	15	0.600
135	A	7	6	1.06	15	0.400
136	A	6	5	1.20	13	0.385
137	A	7	6	1.13	11	0.545
138	A	6	5	1.00	15	0.333
139	A	6	5	1.00	15	0.333
140	A	8	7	0.73	15	0.467
141	A	4	3	1.16	15	0.200
142	A	6	5	1.09	15	0.333
143	A	9	8	1.04	15	0.533
144	A	8	7	1.10	15	0.467
145	A	4	3	0.93	15	0.200
146	A	11	10	0.84	15	0.667
147	A	8	7	1.06	15	0.467
148	A	7	6	1.17	15	0.400
149	A	8	7	0.98	15	0.467

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	10	9	0.85	13	0.692
151	A	8	7	1.00	11	0.636
152	A	8	7	1.00	15	0.467
153	A	4	3	1.15	15	0.200
154	A	6	5	1.04	15	0.333
155	A	7	6	0.97	15	0.400
156	A	5	4	0.89	11	0.364
157	A	5	4	0.97	13	0.308
158	A	5	4	0.99	13	0.308
159	A	5	4	0.97	13	0.308
160	C	1	1	3.26	45	0.022
161	A	4	3	1.04	15	0.200
162	A	5	4	1.00	15	0.267
163	A	4	3	1.08	20	0.150
164	A	4	3	1.62	21	0.143
165	A	5	4	1.00	15	0.267
166	A	6	5	1.00	17	0.294
167	C	9	8	1.09	17	0.471
168	C	5	4	1.07	17	0.235
169	C	13	12	1.10	17	0.706
170	A	8	7	1.00	19	0.368
171	A	8	7	1.00	19	0.368
172	A	6	5	1.03	19	0.263
173	A	6	5	1.03	19	0.263
174	A	8	7	1.00	19	0.368
175	A	8	7	1.00	19	0.368

CHAPTER 3

Listing of integrals

3.1 $\int \mathrm{csch}(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 12

$$\int \mathrm{csch}(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}$$

output
```
-arctanh(cosh(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \mathrm{csch}(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}$$

input
```
Integrate[Csch[a + b*x],x]
```

output
```
-(ArcTanh[Cosh[a + b*x]]/b)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\csc(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$i\int \csc(ia+ibx)dx$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}$$

input
```
Int[Csch[a + b*x],x]
```

output
```
-(ArcTanh[Cosh[a + b*x]]/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	15
default	$\frac{\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	15
parallelrisch	$\frac{\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	15
risch	$-\frac{\ln(1+e^{bx+a})}{b}+\frac{\ln(e^{bx+a}-1)}{b}$	29

input
```
int(csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*ln(tanh(1/2*b*x+1/2*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(12) = 24$.

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 3.17

$$\int \operatorname{csch}(a+bx)\,dx = -\frac{\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)-\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-1\right)}{b}$$

input
```
integrate(csch(b*x+a),x, algorithm="fricas")
```

output
```
-(log(cosh(b*x + a) + sinh(b*x + a) + 1) - log(cosh(b*x + a) + sinh(b*x +
a) - 1))/b
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \operatorname{csch}(a+bx)\,dx = \begin{cases} \frac{\log\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} & \text{for } b \neq 0 \\ x \operatorname{csch}(a) & \text{otherwise} \end{cases}$$

input
```
integrate(csch(b*x+a),x)
```

output
```
Piecewise((log(tanh(a/2 + b*x/2))/b, Ne(b, 0)), (x*csch(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \operatorname{csch}(a+bx)\,dx = \frac{\log\left(\tanh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}{b}$$

input
```
integrate(csch(b*x+a),x, algorithm="maxima")
```

output
```
log(tanh(1/2*b*x + 1/2*a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(12) = 24.

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int \operatorname{csch}(a+bx)\,dx = -\frac{\log\left(e^{(bx+a)}+1\right)-\log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

input
```
integrate(csch(b*x+a),x, algorithm="giac")
```

output
```
-(log(e^(b*x + a) + 1) - log(abs(e^(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int \operatorname{csch}(a+bx)\,dx = -\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

input
```
int(1/sinh(a + b*x),x)
```

output
```
-(2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int \operatorname{csch}(a+bx)\,dx = \frac{\log(e^{bx+a}-1)-\log(e^{bx+a}+1)}{b}$$

input
```
int(csch(b*x+a),x)
```

output
```
(log(e**(a + b*x) - 1) - log(e**(a + b*x) + 1))/b
```

3.2 $\int \operatorname{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 11

$$\int \operatorname{csch}^2(a+bx)\,dx=-\frac{\coth(a+bx)}{b}$$

output
```
-coth(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}^2(a+bx)\,dx=-\frac{\coth(a+bx)}{b}$$

input
```
Integrate[Csch[a + b*x]^2,x]
```

output
```
-(Coth[a + b*x]/b)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 25, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\csc(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int \csc(ia+ibx)^2dx$$

$$\Big\downarrow \text{4254}$$

$$-\frac{i\int 1d(-i\coth(a+bx))}{b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\coth(a+bx)}{b}$$

input
```
Int[Csch[a + b*x]^2,x]
```

output
```
-(Coth[a + b*x]/b)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\frac{\coth(bx+a)}{b}$	12
default	$-\frac{\coth(bx+a)}{b}$	12
risch	$-\frac{2}{b(e^{2bx+2a}-1)}$	19
parallelrisch	$\frac{-\coth\left(\frac{bx}{2}+\frac{a}{2}\right)-\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b}$	29

input
```
int(csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-coth(b*x+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(11) = 22$.

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.91

$$\int \operatorname{csch}^2(a+bx)\,dx = -\frac{2}{b\cosh(bx+a)^2 + 2\,b\cosh(bx+a)\sinh(bx+a) + b\sinh(bx+a)^2 - b}$$

input
```
integrate(csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
-2/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^
2 - b)
```

Sympy [F]

$$\int \operatorname{csch}^2(a+bx)\,dx = \int \operatorname{csch}^2(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**2,x)
```

output
```
Integral(csch(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int \operatorname{csch}^2(a+bx)\,dx = \frac{2}{b(e^{(-2\,bx-2\,a)} - 1)}$$

input
```
integrate(csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
2/(b*(e^(-2*b*x - 2*a) - 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int \operatorname{csch}^2(a+bx)\,dx = -\frac{2}{b(e^{(2\,bx+2\,a)}-1)}$$

input
```
integrate(csch(b*x+a)^2,x, algorithm="giac")
```

output
```
-2/(b*(e^(2*b*x + 2*a) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int \operatorname{csch}^2(a+bx)\,dx = -\frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(1/sinh(a + b*x)^2,x)
```

output
```
-2/(b*(exp(2*a + 2*b*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.64

$$\int \operatorname{csch}^2(a+bx)\,dx = -\frac{2e^{2bx+2a}}{b\,(e^{2bx+2a}-1)}$$

input
```
int(csch(b*x+a)^2,x)
```

output
```
( - 2*e**(2*a + 2*b*x))/(b*(e**(2*a + 2*b*x) - 1))
```

3.3 $\int \operatorname{csch}^3(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 34

$$\int \operatorname{csch}^3(a+bx)\,dx=\frac{\operatorname{arctanh}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{2b}$$

output
```
1/2*arctanh(cosh(b*x+a))/b-1/2*coth(b*x+a)*csch(b*x+a)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 75 vs. 2(34) = 68.

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int \operatorname{csch}^3(a+bx)\,dx=-\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b}+\frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}-\frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}-\frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}$$

input
```
Integrate[Csch[a + b*x]^3,x]
```

output
```
-1/8*Csch[(a + b*x)/2]^2/b + Log[Cosh[(a + b*x)/2]]/(2*b) - Log[Sinh[(a +
b*x)/2]]/(2*b) - Sech[(a + b*x)/2]^2/(8*b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {3042, 26, 4255, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \csc(ia+ibx)^3dx$$

$$\Big\downarrow \text{4255}$$

$$-i\left(\frac{1}{2}\int -i\operatorname{csch}(a+bx)dx-\frac{i\coth(a+bx)\operatorname{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{1}{2}i\int \operatorname{csch}(a+bx)dx-\frac{i\coth(a+bx)\operatorname{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{1}{2}i\int i\csc(ia+ibx)dx-\frac{i\coth(a+bx)\operatorname{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{1}{2}\int \csc(ia+ibx)dx-\frac{i\coth(a+bx)\operatorname{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{4257}$$

$$-i\left(\frac{i\operatorname{arctanh}(\cosh(a+bx))}{2b}-\frac{i\coth(a+bx)\operatorname{csch}(a+bx)}{2b}\right)$$

input
```
Int[Csch[a + b*x]^3,x]
```

output
```
(-I)*(((I/2)*ArcTanh[Cosh[a + b*x]])/b - ((I/2)*Coth[a + b*x]*Csch[a + b*x
])/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{-\frac{\operatorname{csch}(bx+a)\coth(bx+a)}{2}+\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	27
default	$\frac{-\frac{\operatorname{csch}(bx+a)\coth(bx+a)}{2}+\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	27
parallelrisch	$\frac{\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-\coth\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-4\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}$	43
risch	$-\frac{e^{bx+a}\left(1+e^{2bx+2a}\right)}{b\left(e^{2bx+2a}-1\right)^{2}}-\frac{\ln\left(e^{bx+a}-1\right)}{2b}+\frac{\ln\left(1+e^{bx+a}\right)}{2b}$	65

input
```
int(csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2*csch(b*x+a)*coth(b*x+a)+arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(30) = 60.

Time = 0.08 (sec) , antiderivative size = 387, normalized size of antiderivative = 11.38

$$\int \operatorname{csch}^3(a+bx)\,dx = -\frac{2\,\cosh\left(bx+a\right)^{3}+6\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+2\,\sinh\left(bx+a\right)^{3}-\left(\cosh\left(bx+a\right)^{4}+4\,\cosh\left(bx+\right.\right.}{}$$

input
```
integrate(csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*cosh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x + a)^2 + 2*sinh(b*x + a
)^3 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4
 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh
(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b
*x + a) + 1) + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b
*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2
+ 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a)
 + sinh(b*x + a) - 1) + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 2*cosh(b
*x + a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b
*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a
)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{csch}^3(a+bx)\,dx = \int \operatorname{csch}^3(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**3,x)
```

output
```
Integral(csch(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(30) = 60$.

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.47

$$\int \operatorname{csch}^3(a+bx)\,dx = \frac{\log\left(e^{(-bx-a)}+1\right)}{2\,b} - \frac{\log\left(e^{(-bx-a)}-1\right)}{2\,b} + \frac{e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{b(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1)}$$

input
```
integrate(csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*log(e^(-b*x - a) + 1)/b - 1/2*log(e^(-b*x - a) - 1)/b + (e^(-b*x - a)
+ e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(30) = 60$.

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.47

$$\int \operatorname{csch}^3(a+bx)\,dx = -\frac{\frac{4\left(e^{(bx+a)}+e^{(-bx-a)}\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4} - \log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) + \log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{4\,b}$$

input
```
integrate(csch(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/4*(4*(e^(b*x + a) + e^(-b*x - a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4)
- log(e^(b*x + a) + e^(-b*x - a) + 2) + log(e^(b*x + a) + e^(-b*x - a) - 2
))/b
```

Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.53

$$\int \operatorname{csch}^3(a+bx)\,dx = \frac{\operatorname{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(1/sinh(a + b*x)^3,x)
```

output
```
atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b)/(-b^2)^(1/2) - (2*exp(a + b*x))/(b*
(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a +
2*b*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 4.76

$$\int \operatorname{csch}^3(a+bx)\,dx$$
$$= \frac{-e^{4bx+4a}\log\left(e^{bx+a}-1\right)+e^{4bx+4a}\log\left(e^{bx+a}+1\right)-2e^{3bx+3a}+2e^{2bx+2a}\log\left(e^{bx+a}-1\right)-2e^{2bx+2a}\log\left(e^{bx+a}\right.}{2b\left(e^{4bx+4a}-2e^{2bx+2a}+1\right)}$$

input
```
int(csch(b*x+a)^3,x)
```

output
```
( - e**(4*a + 4*b*x)*log(e**(a + b*x) - 1) + e**(4*a + 4*b*x)*log(e**(a +
b*x) + 1) - 2*e**(3*a + 3*b*x) + 2*e**(2*a + 2*b*x)*log(e**(a + b*x) - 1)
- 2*e**(2*a + 2*b*x)*log(e**(a + b*x) + 1) - 2*e**(a + b*x) - log(e**(a +
b*x) - 1) + log(e**(a + b*x) + 1))/(2*b*(e**(4*a + 4*b*x) - 2*e**(2*a + 2*
b*x) + 1))
```

3.4 $\int \operatorname{csch}^4(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 26

$$\int \operatorname{csch}^4(a+bx)\,dx=\frac{\coth(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}$$

output
```
coth(b*x+a)/b-1/3*coth(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35

$$\int \operatorname{csch}^4(a+bx)\,dx=\frac{2\coth(a+bx)}{3b}-\frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{3b}$$

input
```
Integrate[Csch[a + b*x]^4,x]
```

output
```
(2*Coth[a + b*x])/(3*b) - (Coth[a + b*x]*Csch[a + b*x]^2)/(3*b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc(ia+ibx)^4dx$$
$$\Big\downarrow \text{4254}$$
$$\frac{i\int \left(1-\coth^2(a+bx)\right)d(-i\coth(a+bx))}{b}$$
$$\Big\downarrow \text{2009}$$
$$\frac{i\left(\frac{1}{3}i\coth^3(a+bx)-i\coth(a+bx)\right)}{b}$$

input
```
Int[Csch[a + b*x]^4,x]
```

output
```
(I*((-I)*Coth[a + b*x] + (I/3)*Coth[a + b*x]^3))/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}(bx+a)^{2}}{3}\right)\coth(bx+a)}{b}$	23
default	$\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}(bx+a)^{2}}{3}\right)\coth(bx+a)}{b}$	23
risch	$-\frac{4\left(3\,\mathrm{e}^{2bx+2a}-1\right)}{3b\left(\mathrm{e}^{2bx+2a}-1\right)^{3}}$	32
parallelrisch	$\frac{-\coth\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}-\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}+9\coth\left(\frac{bx}{2}+\frac{a}{2}\right)+9\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)}{24b}$	55

input
```
int(csch(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(2/3-1/3*csch(b*x+a)^2)*coth(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(24) = 48.

Time = 0.07 (sec) , antiderivative size = 164, normalized size of antiderivative = 6.31

$$\int \operatorname{csch}^4(a+bx)\,dx =$$

$$-\frac{}{3\left(b\cosh\left(bx+a\right)^{5}+5\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{4}+b\sinh\left(bx+a\right)^{5}-3\,b\cosh\left(bx+a\right)^{3}+\left(10\,b\cos\right.\right.}$$

input
```
integrate(csch(b*x+a)^4,x, algorithm="fricas")
```

output
```
-8/3*(cosh(b*x + a) + 2*sinh(b*x + a))/(b*cosh(b*x + a)^5 + 5*b*cosh(b*x +
 a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5 - 3*b*cosh(b*x + a)^3 + (10*b*cosh
(b*x + a)^2 - 3*b)*sinh(b*x + a)^3 + (10*b*cosh(b*x + a)^3 - 9*b*cosh(b*x
+ a))*sinh(b*x + a)^2 + 2*b*cosh(b*x + a) + (5*b*cosh(b*x + a)^4 - 9*b*cos
h(b*x + a)^2 + 4*b)*sinh(b*x + a))
```

Sympy [F]

$$\int \operatorname{csch}^4(a+bx)\,dx = \int \operatorname{csch}^4{(a+bx)}\,dx$$

input
```
integrate(csch(b*x+a)**4,x)
```

output
```
Integral(csch(a + b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(24) = 48.

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.46

$$\int \operatorname{csch}^4(a+bx)\,dx = \frac{4\,e^{(-2\,bx-2\,a)}}{b(3\,e^{(-2\,bx-2\,a)} - 3\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)} - 1)} - \frac{4}{3\,b(3\,e^{(-2\,bx-2\,a)} - 3\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)} - 1)}$$

input
```
integrate(csch(b*x+a)^4,x, algorithm="maxima")
```

output
```
4*e^(-2*b*x - 2*a)/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + e^(-6*b*x
 - 6*a) - 1)) - 4/3/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + e^(-6*b*
x - 6*a) - 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \operatorname{csch}^4(a+bx)\,dx = -\frac{4\left(3\,e^{(2\,bx+2\,a)}-1\right)}{3\,b\left(e^{(2\,bx+2\,a)}-1\right)^3}$$

input
```
integrate(csch(b*x+a)^4,x, algorithm="giac")
```

output
```
-4/3*(3*e^(2*b*x + 2*a) - 1)/(b*(e^(2*b*x + 2*a) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \operatorname{csch}^4(a+bx)\,dx = -\frac{4\left(3\,\mathrm{e}^{2\,a+2\,b\,x}-1\right)}{3\,b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)^3}$$

input
```
int(1/sinh(a + b*x)^4,x)
```

output
```
-(4*(3*exp(2*a + 2*b*x) - 1))/(3*b*(exp(2*a + 2*b*x) - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.19

$$\int \operatorname{csch}^4(a+bx)\,dx = \frac{-4e^{2bx+2a}+\frac{4}{3}}{b\left(e^{6bx+6a}-3e^{4bx+4a}+3e^{2bx+2a}-1\right)}$$

input
```
int(csch(b*x+a)^4,x)
```

output
```
(4*( - 3*e**(2*a + 2*b*x) + 1))/(3*b*(e**(6*a + 6*b*x) - 3*e**(4*a + 4*b*x
) + 3*e**(2*a + 2*b*x) - 1))
```

3.5 $\int \operatorname{csch}^5(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 55

$$\int \operatorname{csch}^5(a+bx)\,dx = -\frac{3\operatorname{arctanh}(\cosh(a+bx))}{8b} + \frac{3\coth(a+bx)\operatorname{csch}(a+bx)}{8b} - \frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}$$

output
```
-3/8*arctanh(cosh(b*x+a))/b+3/8*coth(b*x+a)*csch(b*x+a)/b-1/4*coth(b*x+a)*
csch(b*x+a)^3/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 113 vs. 2(55) = 110.

Time = 0.02 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.05

$$\int \operatorname{csch}^5(a+bx)\,dx = \frac{3\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b} - \frac{\operatorname{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b} - \frac{3\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{8b} + \frac{3\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{8b} + \frac{3\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b} + \frac{\operatorname{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b}$$

input
```
Integrate[Csch[a + b*x]^5,x]
```

output
```
(3*Csch[(a + b*x)/2]^2)/(32*b) - Csch[(a + b*x)/2]^4/(64*b) - (3*Log[Cosh[
(a + b*x)/2]])/(8*b) + (3*Log[Sinh[(a + b*x)/2]])/(8*b) + (3*Sech[(a + b*x
)/2]^2)/(32*b) + Sech[(a + b*x)/2]^4/(64*b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.375$, Rules used = {3042, 26, 4255, 26, 3042, 26, 4255, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\csc(ia+ibx)^5dx$$

$$\Big\downarrow \text{26}$$

$$i\int \csc(ia+ibx)^5dx$$

$$\Big\downarrow \text{4255}$$

$$i\left(\frac{3}{4}\int i\operatorname{csch}^3(a+bx)dx+\frac{i\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{3}{4}i\int \operatorname{csch}^3(a+bx)dx+\frac{i\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{3}{4}i\int -i\csc(ia+ibx)^3dx+\frac{i\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{3}{4}\int \csc(ia+ibx)^3dx+\frac{i\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}\right)$$

$$\Big\downarrow \text{4255}$$

$$i\left(\frac{3}{4}\left(\frac{1}{2}\int -i\text{csch}(a+bx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)+\frac{i\coth(a+bx)\text{csch}^3(a+bx)}{4b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{3}{4}\left(-\frac{1}{2}i\int \text{csch}(a+bx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)+\frac{i\coth(a+bx)\text{csch}^3(a+bx)}{4b}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{3}{4}\left(-\frac{1}{2}i\int i\csc(ia+ibx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)+\frac{i\coth(a+bx)\text{csch}^3(a+bx)}{4b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{3}{4}\left(\frac{1}{2}\int \csc(ia+ibx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)+\frac{i\coth(a+bx)\text{csch}^3(a+bx)}{4b}\right)$$

$$\Big\downarrow \text{4257}$$

$$i\left(\frac{3}{4}\left(\frac{i\text{arctanh}(\cosh(a+bx))}{2b}-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)+\frac{i\coth(a+bx)\text{csch}^3(a+bx)}{4b}\right)$$

input
```
Int[Csch[a + b*x]^5,x]
```

output
```
I*(((I/4)*Coth[a + b*x]*Csch[a + b*x]^3)/b + (3*(((I/2)*ArcTanh[Cosh[a + b
*x]])/b - ((I/2)*Coth[a + b*x]*Csch[a + b*x])/b))/4)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\left(-\frac{\operatorname{csch}(bx+a)^{3}}{4}+\frac{3\operatorname{csch}(bx+a)}{8}\right)\coth(bx+a)-\frac{3\operatorname{arctanh}\left(e^{bx+a}\right)}{4}}{b}$	41
default	$\frac{\left(-\frac{\operatorname{csch}(bx+a)^{3}}{4}+\frac{3\operatorname{csch}(bx+a)}{8}\right)\coth(bx+a)-\frac{3\operatorname{arctanh}\left(e^{bx+a}\right)}{4}}{b}$	41
parallelrisch	$\frac{\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}-\coth\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}-8\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+8\coth\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+24\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{64b}$	69
risch	$\frac{e^{bx+a}\left(3\,e^{6bx+6a}-11\,e^{4bx+4a}-11\,e^{2bx+2a}+3\right)}{4b\left(e^{2bx+2a}-1\right)^{4}}+\frac{3\ln\left(e^{bx+a}-1\right)}{8b}-\frac{3\ln\left(1+e^{bx+a}\right)}{8b}$	89

input
```
int(csch(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*((-1/4*csch(b*x+a)^3+3/8*csch(b*x+a))*coth(b*x+a)-3/4*arctanh(exp(b*x+
a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1114 vs. 2(49) = 98.

Time = 0.08 (sec) , antiderivative size = 1114, normalized size of antiderivative = 20.25

$$\int \operatorname{csch}^5(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/8*(6*cosh(b*x + a)^7 + 42*cosh(b*x + a)*sinh(b*x + a)^6 + 6*sinh(b*x + a
)^7 + 2*(63*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^5 - 22*cosh(b*x + a)^5 + 1
0*(21*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b*x + a)^4 + 2*(105*cosh(b*
x + a)^4 - 110*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^3 - 22*cosh(b*x + a)^3
+ 2*(63*cosh(b*x + a)^5 - 110*cosh(b*x + a)^3 - 33*cosh(b*x + a))*sinh(b*x
 + a)^2 - 3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x
+ a)^8 + 4*(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8
*(7*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x +
a)^4 - 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*
cosh(b*x + a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 +
4*(7*cosh(b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*
x + a)^2 - 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*
cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + si
nh(b*x + a) + 1) + 3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 +
sinh(b*x + a)^8 + 4*(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x +
 a)^6 + 8*(7*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*co
sh(b*x + a)^4 - 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^
4 + 8*(7*cosh(b*x + a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x
+ a)^3 + 4*(7*cosh(b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1
)*sinh(b*x + a)^2 - 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x...
```

Sympy [F]

$$\int \operatorname{csch}^5(a+bx)\,dx=\int \operatorname{csch}^5{(a+bx)}\,dx$$

input
```
integrate(csch(b*x+a)**5,x)
```

output
```
Integral(csch(a + b*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(49) = 98$.

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.42

$$\int \operatorname{csch}^5(a+bx)\,dx = -\frac{3\log\left(e^{(-bx-a)}+1\right)}{8b} + \frac{3\log\left(e^{(-bx-a)}-1\right)}{8b} - \frac{3e^{(-bx-a)} - 11e^{(-3bx-3a)} - 11e^{(-5bx-5a)} + 3e^{(-7bx-7a)}}{4b(4e^{(-2bx-2a)} - 6e^{(-4bx-4a)} + 4e^{(-6bx-6a)} - e^{(-8bx-8a)} - 1)}$$

input
```
integrate(csch(b*x+a)^5,x, algorithm="maxima")
```

output
```
-3/8*log(e^(-b*x - a) + 1)/b + 3/8*log(e^(-b*x - a) - 1)/b - 1/4*(3*e^(-b*
x - a) - 11*e^(-3*b*x - 3*a) - 11*e^(-5*b*x - 5*a) + 3*e^(-7*b*x - 7*a))/(
b*(4*e^(-2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) - e^(-8*b*
x - 8*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(49) = 98$.

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.00

$$\int \operatorname{csch}^5(a+bx)\,dx = \frac{\frac{4\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3-20e^{(bx+a)}-20e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4\right)^2} - 3\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) + 3\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{16b}$$

input
```
integrate(csch(b*x+a)^5,x, algorithm="giac")
```

output
```
1/16*(4*(3*(e^(b*x + a) + e^(-b*x - a))^3 - 20*e^(b*x + a) - 20*e^(-b*x -
a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4)^2 - 3*log(e^(b*x + a) + e^(-b*x -
 a) + 2) + 3*log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 193, normalized size of antiderivative = 3.51

$$\int \operatorname{csch}^5(a+bx)\,dx = \frac{3\,\mathrm{e}^{a+b\,x}}{4\,b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)} - \frac{\mathrm{e}^{a+b\,x}}{2\,b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}-3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}-1\right)} - \frac{4\,\mathrm{e}^{3\,a+3\,b\,x}}{b\,\left(6\,\mathrm{e}^{4\,a+4\,b\,x}-4\,\mathrm{e}^{2\,a+2\,b\,x}-4\,\mathrm{e}^{6\,a+6\,b\,x}+\mathrm{e}^{8\,a+8\,b\,x}+1\right)} - \frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{4\,\sqrt{-b^2}}$$

input
```
int(1/sinh(a + b*x)^5,x)
```

output
```
(3*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) - 1)) - exp(a + b*x)/(2*b*(exp(4*a
 + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (2*exp(a + b*x))/(b*(3*exp(2*a + 2*
b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (4*exp(3*a + 3*b*x))/
(b*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*exp(6*a + 6*b*x) + exp(8*a
 + 8*b*x) + 1)) - (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(4*(-b^2)^(1/
2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 301, normalized size of antiderivative = 5.47

$$\int \operatorname{csch}^5(a+bx)\,dx = \frac{3e^{8bx+8a}\log\left(e^{bx+a}-1\right) - 3e^{8bx+8a}\log\left(e^{bx+a}+1\right) + 6e^{7bx+7a} - 12e^{6bx+6a}\log\left(e^{bx+a}-1\right) + 12e^{6bx+6a}\log\left(e^{b\cdots\right.}{\cdots}$$

input
```
int(csch(b*x+a)^5,x)
```

output
```
(3*e**(8*a + 8*b*x)*log(e**(a + b*x) - 1) - 3*e**(8*a + 8*b*x)*log(e**(a +
 b*x) + 1) + 6*e**(7*a + 7*b*x) - 12*e**(6*a + 6*b*x)*log(e**(a + b*x) - 1
) + 12*e**(6*a + 6*b*x)*log(e**(a + b*x) + 1) - 22*e**(5*a + 5*b*x) + 18*e
**(4*a + 4*b*x)*log(e**(a + b*x) - 1) - 18*e**(4*a + 4*b*x)*log(e**(a + b*
x) + 1) - 22*e**(3*a + 3*b*x) - 12*e**(2*a + 2*b*x)*log(e**(a + b*x) - 1)
+ 12*e**(2*a + 2*b*x)*log(e**(a + b*x) + 1) + 6*e**(a + b*x) + 3*log(e**(a
 + b*x) - 1) - 3*log(e**(a + b*x) + 1))/(8*b*(e**(8*a + 8*b*x) - 4*e**(6*a
 + 6*b*x) + 6*e**(4*a + 4*b*x) - 4*e**(2*a + 2*b*x) + 1))
```

3.6 $\int \operatorname{csch}^6(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 42

$$\int \operatorname{csch}^6(a+bx)\,dx=-\frac{\coth(a+bx)}{b}+\frac{2\coth^3(a+bx)}{3b}-\frac{\coth^5(a+bx)}{5b}$$

output
```
-coth(b*x+a)/b+2/3*coth(b*x+a)^3/b-1/5*coth(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.33

$$\int \operatorname{csch}^6(a+bx)\,dx=-\frac{8\coth(a+bx)}{15b}+\frac{4\coth(a+bx)\operatorname{csch}^2(a+bx)}{15b}-\frac{\coth(a+bx)\operatorname{csch}^4(a+bx)}{5b}$$

input
```
Integrate[Csch[a + b*x]^6,x]
```

output
```
(-8*Coth[a + b*x])/(15*b) + (4*Coth[a + b*x]*Csch[a + b*x]^2)/(15*b) - (Co
th[a + b*x]*Csch[a + b*x]^4)/(5*b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 25, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^6(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\csc(ia+ibx)^6dx$$

$$\Big\downarrow \text{25}$$

$$-\int \csc(ia+ibx)^6dx$$

$$\Big\downarrow \text{4254}$$

$$-\frac{i\int \left(\coth^4(a+bx)-2\coth^2(a+bx)+1\right)d(-i\coth(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-\frac{1}{5}i\coth^5(a+bx)+\frac{2}{3}i\coth^3(a+bx)-i\coth(a+bx)\right)}{b}$$

input
```
Int[Csch[a + b*x]^6,x]
```

output
```
((-I)*((-I)*Coth[a + b*x] + ((2*I)/3)*Coth[a + b*x]^3 - (I/5)*Coth[a + b*x
]^5))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\left(-\frac{8}{15}-\frac{\operatorname{csch}(bx+a)^{4}}{5}+\frac{4\operatorname{csch}(bx+a)^{2}}{15}\right)\coth(bx+a)}{b}$
default	$\frac{\left(-\frac{8}{15}-\frac{\operatorname{csch}(bx+a)^{4}}{5}+\frac{4\operatorname{csch}(bx+a)^{2}}{15}\right)\coth(bx+a)}{b}$
risch	$-\frac{16\left(10\,{\mathrm e}^{4bx+4a}-5\,{\mathrm e}^{2bx+2a}+1\right)}{15b\left({\mathrm e}^{2bx+2a}-1\right)^{5}}$
parallelrisch	$\frac{-3\coth\left(\frac{bx}{2}+\frac{a}{2}\right)^{5}-3\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{5}+25\coth\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}+25\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}-150\coth\left(\frac{bx}{2}+\frac{a}{2}\right)-150\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)}{480b}$

input
```
int(csch(b*x+a)^6,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-8/15-1/5*csch(b*x+a)^4+4/15*csch(b*x+a)^2)*coth(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(38) = 76$.

Time = 0.07 (sec) , antiderivative size = 344, normalized size of antiderivative = 8.19

$$\int \operatorname{csch}^6(a+bx)\,dx =$$

$$-\frac{}{15\left(b\cosh\left(bx+a\right)^{8}+8\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{7}+b\sinh\left(bx+a\right)^{8}-5\,b\cosh\left(bx+a\right)^{6}+\left(28\,b\,\mathrm{co}\right.\right.}$$

input
```
integrate(csch(b*x+a)^6,x, algorithm="fricas")
```

output
```
-16/15*(11*cosh(b*x + a)^2 + 18*cosh(b*x + a)*sinh(b*x + a) + 11*sinh(b*x
+ a)^2 - 5)/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sin
h(b*x + a)^8 - 5*b*cosh(b*x + a)^6 + (28*b*cosh(b*x + a)^2 - 5*b)*sinh(b*x
 + a)^6 + 2*(28*b*cosh(b*x + a)^3 - 15*b*cosh(b*x + a))*sinh(b*x + a)^5 +
10*b*cosh(b*x + a)^4 + 5*(14*b*cosh(b*x + a)^4 - 15*b*cosh(b*x + a)^2 + 2*
b)*sinh(b*x + a)^4 + 4*(14*b*cosh(b*x + a)^5 - 25*b*cosh(b*x + a)^3 + 10*b
*cosh(b*x + a))*sinh(b*x + a)^3 - 11*b*cosh(b*x + a)^2 + (28*b*cosh(b*x +
a)^6 - 75*b*cosh(b*x + a)^4 + 60*b*cosh(b*x + a)^2 - 11*b)*sinh(b*x + a)^2
 + 2*(4*b*cosh(b*x + a)^7 - 15*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3 -
9*b*cosh(b*x + a))*sinh(b*x + a) + 5*b)
```

Sympy [F]

$$\int \operatorname{csch}^6(a+bx)\,dx = \int \operatorname{csch}^6\left(a+bx\right)dx$$

input
```
integrate(csch(b*x+a)**6,x)
```

output
```
Integral(csch(a + b*x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(38) = 76$.

Time = 0.03 (sec) , antiderivative size = 205, normalized size of antiderivative = 4.88

$$\int \operatorname{csch}^6(a+bx)\,dx = -\frac{16\,e^{(-2\,bx-2\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)} - 10\,e^{(-4\,bx-4\,a)} + 10\,e^{(-6\,bx-6\,a)} - 5\,e^{(-8\,bx-8\,a)} + e^{(-10\,bx-10\,a)} - 1)} + \frac{32\,e^{(-4\,bx-4\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)} - 10\,e^{(-4\,bx-4\,a)} + 10\,e^{(-6\,bx-6\,a)} - 5\,e^{(-8\,bx-8\,a)} + e^{(-10\,bx-10\,a)} - 1)} + \frac{16}{15\,b(5\,e^{(-2\,bx-2\,a)} - 10\,e^{(-4\,bx-4\,a)} + 10\,e^{(-6\,bx-6\,a)} - 5\,e^{(-8\,bx-8\,a)} + e^{(-10\,bx-10\,a)} - 1)}$$

input
```
integrate(csch(b*x+a)^6,x, algorithm="maxima")
```

output
```
-16/3*e^(-2*b*x - 2*a)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e
^(-6*b*x - 6*a) - 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1)) + 32/3*e^(
-4*b*x - 4*a)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x
- 6*a) - 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1)) + 16/15/(b*(5*e^(-2
*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) - 5*e^(-8*b*x - 8*
a) + e^(-10*b*x - 10*a) - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}^6(a+bx)\,dx = -\frac{16\left(10\,e^{(4\,bx+4\,a)} - 5\,e^{(2\,bx+2\,a)} + 1\right)}{15\,b(e^{(2\,bx+2\,a)} - 1)^5}$$

input
```
integrate(csch(b*x+a)^6,x, algorithm="giac")
```

output
```
-16/15*(10*e^(4*b*x + 4*a) - 5*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) -
1)^5)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}^6(a+bx)\,dx = -\frac{16\,\left(10\,\mathrm{e}^{4\,a+4\,b\,x}-5\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}{15\,b\,{\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}^5}$$

input
```
int(1/sinh(a + b*x)^6,x)
```

output
```
-(16*(10*exp(4*a + 4*b*x) - 5*exp(2*a + 2*b*x) + 1))/(15*b*(exp(2*a + 2*b*
x) - 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.21

$$\int \operatorname{csch}^6(a+bx)\,dx = \frac{-\frac{32e^{4bx+4a}}{3}+\frac{16e^{2bx+2a}}{3}-\frac{16}{15}}{b\left(e^{10bx+10a}-5e^{8bx+8a}+10e^{6bx+6a}-10e^{4bx+4a}+5e^{2bx+2a}-1\right)}$$

input
```
int(csch(b*x+a)^6,x)
```

output
```
(16*( - 10*e**(4*a + 4*b*x) + 5*e**(2*a + 2*b*x) - 1))/(15*b*(e**(10*a + 1
0*b*x) - 5*e**(8*a + 8*b*x) + 10*e**(6*a + 6*b*x) - 10*e**(4*a + 4*b*x) +
5*e**(2*a + 2*b*x) - 1))
```

3.7 $\int \text{csch}^{\frac{5}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 80

$$\int \text{csch}^{\frac{5}{2}}(a+bx)\,dx = -\frac{2\cosh(a+bx)\text{csch}^{\frac{3}{2}}(a+bx)}{3b} + \frac{2i\sqrt{\text{csch}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{3b}$$

output
```
-2/3*cosh(b*x+a)*csch(b*x+a)^(3/2)/b+2/3*I*csch(b*x+a)^(1/2)*InverseJacobi
AM(1/2*I*a-1/4*Pi+1/2*I*b*x,2^(1/2))*(I*sinh(b*x+a))^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int \text{csch}^{\frac{5}{2}}(a+bx)\,dx = -\frac{2\sqrt{\text{csch}(a+bx)}\left(\coth(a+bx)+i\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)\sqrt{i\sinh(a+bx)}\right)}{3b}$$

input
```
Integrate[Csch[a + b*x]^(5/2),x]
```

output
```
(-2*Sqrt[Csch[a + b*x]]*(Coth[a + b*x] + I*EllipticF[((-2*I)*a + Pi - (2*I
)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]))/(3*b)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (i\csc(ia+ibx))^{5/2}dx$$

$$\Big\downarrow \text{ 4255}$$

$$-\frac{1}{3}\int \sqrt{\operatorname{csch}(a+bx)}dx-\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}-\frac{1}{3}\int \sqrt{i\csc(ia+ibx)}dx$$

$$\Big\downarrow \text{ 4258}$$

$$-\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}-\frac{1}{3}\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\int \frac{1}{\sqrt{i\sinh(a+bx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}-\frac{1}{3}\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\int \frac{1}{\sqrt{\sin(ia+ibx)}}dx$$

$$\Big\downarrow \text{ 3120}$$

$$-\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2i\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{3b}$$

input
```
Int[Csch[a + b*x]^(5/2),x]
```

output
```
(-2*Cosh[a + b*x]*Csch[a + b*x]^(3/2))/(3*b) + (((2*I)/3)*Sqrt[Csch[a + b*
x]]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

method	result	size
default	$-\frac{i\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}\,\text{EllipticF}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)\sinh(bx+a)+2\cosh(bx+a)^{2}}{3\sinh(bx+a)^{\frac{3}{2}}\cosh(bx+a)b}$	101

input
```
int(csch(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/sinh(b*x+a)^(3/2)*(I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a)
)^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2
))*sinh(b*x+a)+2*cosh(b*x+a)^2)/cosh(b*x+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(58) = 116.

Time = 0.09 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.42

$$\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx =$$

$$-\frac{2\left(\sqrt{2}\left(\cosh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}+1\right)\sqrt{\frac{\cosh(bx+a)+\sin}{\cosh(bx+a)^{2}+2\,\cosh(bx+a)\sinh}}\right.}{3\left(b\cosh\right.}$$

input
```
integrate(csch(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(2)*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x
+ a)^2 + 1)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh
(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + (sqrt(2)*cosh(b*x + a)^2
 + 2*sqrt(2)*cosh(b*x + a)*sinh(b*x + a) + sqrt(2)*sinh(b*x + a)^2 - sqrt(
2))*weierstrassPInverse(4, 0, cosh(b*x + a) + sinh(b*x + a)))/(b*cosh(b*x
+ a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F]

$$\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx=\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**(5/2),x)
```

output
```
Integral(csch(a + b*x)**(5/2), x)
```

Maxima [F]

$$\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx=\int \operatorname{csch}(bx+a)^{\frac{5}{2}}\,dx$$

input
```
integrate(csch(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(csch(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx=\int \operatorname{csch}(bx+a)^{\frac{5}{2}}\,dx$$

input
```
integrate(csch(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx = \int \left(\frac{1}{\sinh\left(a+b\,x\right)}\right)^{5/2} dx$$

input
```
int((1/sinh(a + b*x))^(5/2),x)
```

output
```
int((1/sinh(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx = \int \sqrt{\operatorname{csch}\left(bx+a\right)}\,\operatorname{csch}(bx+a)^2\,dx$$

input
```
int(csch(b*x+a)^(5/2),x)
```

output
```
int(sqrt(csch(a + b*x))*csch(a + b*x)**2,x)
```

3.8 $\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 76

$$\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx=-\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b}-\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

output
```
-2*cosh(b*x+a)*csch(b*x+a)^(1/2)/b+2*I*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*
b*x),2^(1/2))/b/csch(b*x+a)^(1/2)/(I*sinh(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx =-\frac{2\sqrt{\operatorname{csch}(a+bx)}\left(\cosh(a+bx)-E\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|2\right)\sqrt{i\sinh(a+bx)}\right)}{b}$$

input
```
Integrate[Csch[a + b*x]^(3/2),x]
```

output
```
(-2*Sqrt[Csch[a + b*x]]*(Cosh[a + b*x] - EllipticE[((-2*I)*a + Pi - (2*I)*
b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]))/b
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (i\csc(ia+ibx))^{3/2}dx$$

$$\Big\downarrow \text{4255}$$

$$\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}dx-\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b}+\int \frac{1}{\sqrt{i\csc(ia+ibx)}}dx$$

$$\Big\downarrow \text{4258}$$

$$-\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b}+\frac{\int \sqrt{i\sinh(a+bx)}dx}{\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b}+\frac{\int \sqrt{\sin(ia+ibx)}dx}{\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b}-\frac{2iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

input
```
Int[Csch[a + b*x]^(3/2),x]
```

output
```
(-2*Cosh[a + b*x]*Sqrt[Csch[a + b*x]])/b - ((2*I)*EllipticE[(I*a - Pi/2 +
I*b*x)/2, 2])/(b*Sqrt[Csch[a + b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(63) = 126$.

Time = 0.16 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.03

method	result
default	$\frac{2\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}\,\operatorname{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)-\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+}}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$

input
```
int(csch(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(2*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))
^(1/2)*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))-(1-I*sinh(b*x+a))^(1
/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1-I*s
inh(b*x+a))^(1/2),1/2*2^(1/2))-2*cosh(b*x+a)^2)/cosh(b*x+a)/sinh(b*x+a)^(1
/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.26

$$\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx = -\frac{2\left(\sqrt{2}\sqrt{\frac{\cosh(bx+a)+\sinh(bx+a)}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-1}}(\cosh(bx+a)+\sinh(bx+a))+\sqrt{2}\text{weierstrassZe}$$
$$b$$

input
```
integrate(csch(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(2)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh
(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1))*(cosh(b*x + a) + sinh(b*x
+ a)) + sqrt(2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(b*x +
 a) + sinh(b*x + a))))/b
```

Sympy [F]

$$\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx = \int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**(3/2),x)
```

output
```
Integral(csch(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx = \int \operatorname{csch}(bx+a)^{\frac{3}{2}}\,dx$$

input
```
integrate(csch(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(csch(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx = \int \operatorname{csch}(bx+a)^{\frac{3}{2}}\,dx$$

input
```
integrate(csch(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx = \int \left(\frac{1}{\sinh(a+b\,x)}\right)^{3/2}\,dx$$

input
```
int((1/sinh(a + b*x))^(3/2),x)
```

output
```
int((1/sinh(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx = \int \sqrt{\operatorname{csch}\left(bx+a\right)}\operatorname{csch}(bx+a)\,dx$$

input
```
int(csch(b*x+a)^(3/2),x)
```

output
```
int(sqrt(csch(a + b*x))*csch(a + b*x),x)
```

3.9 $\int \sqrt{\operatorname{csch}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \sqrt{\operatorname{csch}(a+bx)}\,dx = -\frac{2i\sqrt{\operatorname{csch}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{b}$$

output
```
-2*I*csch(b*x+a)^(1/2)*InverseJacobiAM(1/2*I*a-1/4*Pi+1/2*I*b*x,2^(1/2))*(
I*sinh(b*x+a))^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \sqrt{\operatorname{csch}(a+bx)}\,dx = \frac{2\operatorname{csch}^{\frac{3}{2}}(a+bx)\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)(i\sinh(a+bx))^{3/2}}{b}$$

input
```
Integrate[Sqrt[Csch[a + b*x]],x]
```

output
```
(2*Csch[a + b*x]^(3/2)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*(I*Sinh
[a + b*x])^(3/2))/b
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\operatorname{csch}(a+bx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{i\csc(ia+ibx)}dx$$

$$\Big\downarrow \text{ 4258}$$

$$\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\int \frac{1}{\sqrt{i\sinh(a+bx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\int \frac{1}{\sqrt{\sin(ia+ibx)}}dx$$

$$\Big\downarrow \text{ 3120}$$

$$-\frac{2i\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{b}$$

input
```
Int[Sqrt[Csch[a + b*x]],x]
```

output
```
((-2*I)*Sqrt[Csch[a + b*x]]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Si
nh[a + b*x]])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(42) = 84.

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.61

method	result	size
default	$\frac{i\sqrt{-i(\sinh(bx+a)+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(bx+a)+i)}\,\sqrt{i\sinh(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{-i(\sinh(bx+a)+i)},\frac{\sqrt{2}}{2}\right)}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$	87

input
```
int(csch(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*(-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*x+a)+I))^(1/2)*(I*sinh(b
*x+a))^(1/2)*EllipticF((-I*(sinh(b*x+a)+I))^(1/2),1/2*2^(1/2))/cosh(b*x+a)
/sinh(b*x+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.44

$$\int \sqrt{\operatorname{csch}(a+bx)}\,dx = \frac{2\sqrt{2}\text{weierstrassPInverse}(4,0,\cosh{(bx+a)}+\sinh{(bx+a)})}{b}$$

input
```
integrate(csch(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(2)*weierstrassPInverse(4, 0, cosh(b*x + a) + sinh(b*x + a))/b
```

Sympy [F]

$$\int \sqrt{\operatorname{csch}(a+bx)}\,dx = \int \sqrt{\operatorname{csch}{(a+bx)}}\,dx$$

input
```
integrate(csch(b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(csch(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{\operatorname{csch}(a+bx)}\,dx = \int \sqrt{\operatorname{csch}{(bx+a)}}\,dx$$

input
```
integrate(csch(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(csch(b*x + a)), x)
```

Giac [F]

$$\int \sqrt{\operatorname{csch}(a+bx)}\,dx = \int \sqrt{\operatorname{csch}\left(bx+a\right)}\,dx$$

input
```
integrate(csch(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(csch(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\operatorname{csch}(a+bx)}\,dx = \int \sqrt{\frac{1}{\sinh\left(a+b\,x\right)}}\,dx$$

input
```
int((1/sinh(a + b*x))^(1/2),x)
```

output
```
int((1/sinh(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\operatorname{csch}(a+bx)}\,dx = \int \sqrt{\operatorname{csch}\left(bx+a\right)}dx$$

input
```
int(csch(b*x+a)^(1/2),x)
```

output
```
int(sqrt(csch(a + b*x)),x)
```

3.10 $\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = -\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

output
```
2*I*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))/b/csch(b*x+a)^(1/2)/(
I*sinh(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = \frac{2\sqrt{\operatorname{csch}(a+bx)}E\left(\frac{1}{2}\left(\frac{\pi}{2}-i(a+bx)\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{b}$$

input
```
Integrate[1/Sqrt[Csch[a + b*x]],x]
```

output
```
(2*Sqrt[Csch[a + b*x]]*EllipticE[(Pi/2 - I*(a + b*x))/2, 2]*Sqrt[I*Sinh[a
+ b*x]])/b
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{i\csc(ia+ibx)}}dx$$

$$\Big\downarrow \text{4258}$$

$$\frac{\int \sqrt{i\sinh(a+bx)}dx}{\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\sin(ia+ibx)}dx}{\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

input
```
Int[1/Sqrt[Csch[a + b*x]],x]
```

output
```
((-2*I)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2])/(b*Sqrt[Csch[a + b*x]]*Sqrt[
I*Sinh[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(43) = 86$.

Time = 0.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.00

method	result
default	$\frac{\sqrt{-i(\sinh(bx+a)+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(bx+a)+i)}\,\sqrt{i\sinh(bx+a)}\,\left(2\,\text{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)-\text{EllipticF}\left(\sqrt{1-i\sinh(bx+a}\right.\right.}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$
risch	$\frac{\sqrt{2}}{b\sqrt{\frac{e^{bx+a}}{e^{2bx+2a}-1}}}-\frac{\left(\frac{2\,e^{2bx+2a}-2}{\sqrt{e^{bx+a}\left(e^{2bx+2a}-1\right)}}-\frac{\sqrt{1+e^{bx+a}}\,\sqrt{-2\,e^{bx+a}+2}\,\sqrt{-e^{bx+a}}\,\left(-2\,\text{EllipticE}\left(\sqrt{1+e^{bx+a}},\frac{\sqrt{2}}{2}\right)+\text{EllipticF}\left(\sqrt{1+e^{bx+a}},\right.\right.}{\sqrt{e^{3bx+3a}-e^{bx+a}}}\right.}{b\sqrt{\frac{e^{bx+a}}{e^{2bx+2a}-1}}\left(e^{2bx+2a}-1\right)}$

input
```
int(1/csch(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*x+a)+I))^(1/2)*(I*sinh(b*x
+a))^(1/2)*(2*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))-EllipticF((1-
I*sinh(b*x+a))^(1/2),1/2*2^(1/2)))/cosh(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(39) = 78$.

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.78

$$\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx =$$
$$-\frac{\sqrt{2}\left(\cosh\left(bx+a\right)^2+2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^2-1\right)\sqrt{\frac{\cosh(bx+a)+\sinh(b}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx}}}{}$$

input
```
integrate(1/csch(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(2)*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)
^2 - 1)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x
 + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + 2*(sqrt(2)*cosh(b*x + a) + s
qrt(2)*sinh(b*x + a))*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh
(b*x + a) + sinh(b*x + a))))/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

Sympy [F]

$$\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = \int \frac{1}{\sqrt{\operatorname{csch}\left(a+bx\right)}}\,dx$$

input
```
integrate(1/csch(b*x+a)**(1/2),x)
```

output
```
Integral(1/sqrt(csch(a + b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = \int \frac{1}{\sqrt{\operatorname{csch}(bx+a)}}\,dx$$

input
```
integrate(1/csch(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(csch(b*x + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = \int \frac{1}{\sqrt{\operatorname{csch}(bx+a)}}\,dx$$

input
```
integrate(1/csch(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(csch(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\sinh(a+b\,x)}}}\,dx$$

input
```
int(1/(1/sinh(a + b*x))^(1/2),x)
```

output
```
int(1/(1/sinh(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\mathrm{csch}(a+bx)}}\,dx = \int \frac{\sqrt{\mathrm{csch}\,(bx+a)}}{\mathrm{csch}\,(bx+a)}dx$$

input
```
int(1/csch(b*x+a)^(1/2),x)
```

output
```
int(sqrt(csch(a + b*x))/csch(a + b*x),x)
```

3.11 $\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 80

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}} + \frac{2i\sqrt{\operatorname{csch}(a+bx)}\,\operatorname{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{3b}$$

output
```
2/3*cosh(b*x+a)/b/csch(b*x+a)^(1/2)+2/3*I*csch(b*x+a)^(1/2)*InverseJacobiA
M(1/2*I*a-1/4*Pi+1/2*I*b*x,2^(1/2))*(I*sinh(b*x+a))^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \frac{\sqrt{\operatorname{csch}(a+bx)}\left(-2i\,\operatorname{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)\sqrt{i\sinh(a+bx)}+\sinh(2(a+bx))\right)}{3b}$$

input
```
Integrate[Csch[a + b*x]^(-3/2),x]
```

output
```
(Sqrt[Csch[a + b*x]]*((-2*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*S
qrt[I*Sinh[a + b*x]] + Sinh[2*(a + b*x)]))/(3*b)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(i\csc(ia+ibx))^{3/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}}-\frac{1}{3}\int \sqrt{\operatorname{csch}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}}-\frac{1}{3}\int \sqrt{i\csc(ia+ibx)}dx$$

$$\Big\downarrow \text{4258}$$

$$\frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}}-\frac{1}{3}\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\int \frac{1}{\sqrt{i\sinh(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}}-\frac{1}{3}\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\int \frac{1}{\sqrt{\sin(ia+ibx)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}}+\frac{2i\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{3b}$$

input
```
Int[Csch[a + b*x]^(-3/2),x]
```

output
```
(2*Cosh[a + b*x])/(3*b*Sqrt[Csch[a + b*x]]) + (((2*I)/3)*Sqrt[Csch[a + b*x
]]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

method	result	size
default	$\frac{-\frac{i\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)}{3}+\frac{2\cosh(bx+a)^{2}\sinh(bx+a)}{3}}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$	100

input
```
int(1/csch(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/3*I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*
x+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/3*cosh(b*x+a)
^2*sinh(b*x+a))/cosh(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(58) = 116.

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.79

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \frac{\sqrt{2}\left(\cosh(bx+a)^4 + 4\cosh(bx+a)^3\sinh(bx+a) + 6\cosh(bx+a)^2\sinh(bx+a)^2 + 4\cosh(bx+a)\,\mathrm{s}\right.}{}$$

input
```
integrate(1/csch(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(2)*(cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b
*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a
)^4 - 1)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*
x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) - 4*(sqrt(2)*cosh(b*x + a)^2
+ 2*sqrt(2)*cosh(b*x + a)*sinh(b*x + a) + sqrt(2)*sinh(b*x + a)^2)*weierst
rassPInverse(4, 0, cosh(b*x + a) + sinh(b*x + a)))/(b*cosh(b*x + a)^2 + 2*
b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F]

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(1/csch(b*x+a)**(3/2),x)
```

output
```
Integral(csch(a + b*x)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\operatorname{csch}\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/csch(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(csch(b*x + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\operatorname{csch}\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/csch(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{{\left(\frac{1}{\sinh(a+b\,x)}\right)}^{3/2}}\,dx$$

input
```
int(1/(1/sinh(a + b*x))^(3/2),x)
```

output
```
int(1/(1/sinh(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\operatorname{csch}(bx+a)}}{\operatorname{csch}(bx+a)^2}dx$$

input
```
int(1/csch(b*x+a)^(3/2),x)
```

output
```
int(sqrt(csch(a + b*x))/csch(a + b*x)**2,x)
```

3.12 $\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 80

$$\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx=\frac{2\cosh(a+bx)}{5b\operatorname{csch}^{\frac{3}{2}}(a+bx)}+\frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{5b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

output
```
2/5*cosh(b*x+a)/b/csch(b*x+a)^(3/2)-6/5*I*EllipticE(cos(1/2*I*a+1/4*Pi+1/2
*I*b*x),2^(1/2))/b/csch(b*x+a)^(1/2)/(I*sinh(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

$$\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx =\frac{2\left(\cosh(a+bx)-3\operatorname{csch}^2(a+bx)E\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|2\right)\sqrt{i\sinh(a+bx)}\right)}{5b\operatorname{csch}^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[Csch[a + b*x]^(-5/2),x]
```

output
```
(2*(Cosh[a + b*x] - 3*Csch[a + b*x]^2*EllipticE[((-2*I)*a + Pi - (2*I)*b*x
)/4, 2]*Sqrt[I*Sinh[a + b*x]]))/(5*b*Csch[a + b*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(i\csc(ia+ibx))^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\frac{2\cosh(a+bx)}{5b\operatorname{csch}^{\frac{3}{2}}(a+bx)}-\frac{3}{5}\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\cosh(a+bx)}{5b\operatorname{csch}^{\frac{3}{2}}(a+bx)}-\frac{3}{5}\int \frac{1}{\sqrt{i\csc(ia+ibx)}}dx$$

$$\Big\downarrow \text{4258}$$

$$\frac{2\cosh(a+bx)}{5b\operatorname{csch}^{\frac{3}{2}}(a+bx)}-\frac{3\int \sqrt{i\sinh(a+bx)}dx}{5\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\cosh(a+bx)}{5b\operatorname{csch}^{\frac{3}{2}}(a+bx)}-\frac{3\int \sqrt{\sin(ia+ibx)}dx}{5\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\cosh(a+bx)}{5b\operatorname{csch}^{\frac{3}{2}}(a+bx)}+\frac{6iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)|2\right)}{5b\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

input
```
Int[Csch[a + b*x]^(-5/2),x]
```

output
```
(2*Cosh[a + b*x])/(5*b*Csch[a + b*x]^(3/2)) + (((6*I)/5)*EllipticE[(I*a -
Pi/2 + I*b*x)/2, 2])/(b*Sqrt[Csch[a + b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(63) = 126$.

Time = 0.16 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.05

method	result
default	$\frac{-\frac{6\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}\ \text{EllipticE}\left(\sqrt{1-i\sinh(bx+a)}, \frac{\sqrt{2}}{2}\right)}{5}+\frac{3\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx}{5}}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$

input
```
int(1/csch(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-6/5*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+
a))^(1/2)*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+3/5*(1-I*sinh(b*x
+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF
((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/5*cosh(b*x+a)^4-2/5*cosh(b*x+a)^2)
/cosh(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(59) = 118$.

Time = 0.10 (sec) , antiderivative size = 370, normalized size of antiderivative = 4.62

$$\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \frac{\sqrt{2}\left(\cosh(bx+a)^6+6\cosh(bx+a)\sinh(bx+a)^5+\sinh(bx+a)^6+\left(15\cosh(bx+a)^2+11\right)\sinh(bx\right.}{}$$

input
```
integrate(1/csch(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
1/20*(sqrt(2)*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*
x + a)^6 + (15*cosh(b*x + a)^2 + 11)*sinh(b*x + a)^4 + 11*cosh(b*x + a)^4
+ 4*(5*cosh(b*x + a)^3 + 11*cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x
+ a)^4 + 66*cosh(b*x + a)^2 - 13)*sinh(b*x + a)^2 - 13*cosh(b*x + a)^2 + 2
*(3*cosh(b*x + a)^5 + 22*cosh(b*x + a)^3 - 13*cosh(b*x + a))*sinh(b*x + a)
 + 1)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x +
 a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + 24*(sqrt(2)*cosh(b*x + a)^3 +
3*sqrt(2)*cosh(b*x + a)^2*sinh(b*x + a) + 3*sqrt(2)*cosh(b*x + a)*sinh(b*x
 + a)^2 + sqrt(2)*sinh(b*x + a)^3)*weierstrassZeta(4, 0, weierstrassPInver
se(4, 0, cosh(b*x + a) + sinh(b*x + a))))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*
x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x + a)*sinh(b*x + a)^2 + b*sinh(b*x +
a)^3)
```

Sympy [F]

$$\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx=\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx$$

input
```
integrate(1/csch(b*x+a)**(5/2),x)
```

output
```
Integral(csch(a + b*x)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx=\int \frac{1}{\operatorname{csch}\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/csch(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(csch(b*x + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx=\int \frac{1}{\operatorname{csch}\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/csch(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\left(\frac{1}{\sinh(a+b\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(1/sinh(a + b*x))^(5/2),x)
```

output
```
int(1/(1/sinh(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\operatorname{csch}(bx+a)}}{\operatorname{csch}(bx+a)^3}dx$$

input
```
int(1/csch(b*x+a)^(5/2),x)
```

output
```
int(sqrt(csch(a + b*x))/csch(a + b*x)**3,x)
```

3.13 $\int (b\operatorname{csch}(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 116

$$\int (b\operatorname{csch}(c+dx))^{7/2}\,dx = \frac{6b^3\cosh(c+dx)\sqrt{b\operatorname{csch}(c+dx)}}{5d} - \frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{5/2}}{5d} + \frac{6ib^4E\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{5d\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}$$

output
```
6/5*b^3*cosh(d*x+c)*(b*csch(d*x+c))^(1/2)/d-2/5*b*cosh(d*x+c)*(b*csch(d*x+
c))^(5/2)/d-6/5*I*b^4*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d/(
b*csch(d*x+c))^(1/2)/(I*sinh(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int (b\operatorname{csch}(c+dx))^{7/2}\,dx = -\frac{2b^3\sqrt{b\operatorname{csch}(c+dx)}\left(-3\cosh(c+dx)+\coth(c+dx)\operatorname{csch}(c+dx)+3E\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)\sqrt{i\sinh\right.}{5d}$$

input
```
Integrate[(b*Csch[c + d*x])^(7/2),x]
```

output
```
(-2*b^3*Sqrt[b*Csch[c + d*x]]*(-3*Cosh[c + d*x] + Coth[c + d*x]*Csch[c + d
*x] + 3*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]])
)/(5*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\text{csch}(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (ib\csc(ic+idx))^{7/2}dx$$

$$\Big\downarrow \; 4255$$

$$-\frac{3}{5}b^2\int (b\text{csch}(c+dx))^{3/2}dx-\frac{2b\cosh(c+dx)(b\text{csch}(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2b\cosh(c+dx)(b\text{csch}(c+dx))^{5/2}}{5d}-\frac{3}{5}b^2\int (ib\csc(ic+idx))^{3/2}dx$$

$$\Big\downarrow \; 4255$$

$$-\frac{3}{5}b^2\left(b^2\int \frac{1}{\sqrt{b\text{csch}(c+dx)}}dx-\frac{2b\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{d}\right)-\frac{2b\cosh(c+dx)(b\text{csch}(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2b\cosh(c+dx)(b\text{csch}(c+dx))^{5/2}}{5d}-\frac{3}{5}b^2\left(-\frac{2b\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{d}+b^2\int \frac{1}{\sqrt{ib\csc(ic+idx)}}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$-\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{5/2}}{5d}-\frac{3}{5}b^2\left(-\frac{2b\cosh(c+dx)\sqrt{b\operatorname{csch}(c+dx)}}{d}+\frac{b^2\int\sqrt{i\sinh(c+dx)}dx}{\sqrt{i\sinh(c+dx)}\sqrt{b\operatorname{csch}(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{5/2}}{5d}-\frac{3}{5}b^2\left(-\frac{2b\cosh(c+dx)\sqrt{b\operatorname{csch}(c+dx)}}{d}+\frac{b^2\int\sqrt{\sin(ic+idx)}dx}{\sqrt{i\sinh(c+dx)}\sqrt{b\operatorname{csch}(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{5/2}}{5d}-\frac{3}{5}b^2\left(-\frac{2b\cosh(c+dx)\sqrt{b\operatorname{csch}(c+dx)}}{d}-\frac{2ib^2E\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)|2\right)}{d\sqrt{i\sinh(c+dx)}\sqrt{b\operatorname{csch}(c+dx)}}\right)$$

input
```
Int[(b*Csch[c + d*x])^(7/2),x]
```

output
```
(-2*b*Cosh[c + d*x]*(b*Csch[c + d*x])^(5/2))/(5*d) - (3*b^2*((-2*b*Cosh[c
+ d*x]*Sqrt[b*Csch[c + d*x]])/d - ((2*I)*b^2*EllipticE[(I*c - Pi/2 + I*d*x
)/2, 2])/(d*Sqrt[b*Csch[c + d*x]]*Sqrt[I*Sinh[c + d*x]])))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [F]

$$\int (\mathrm{csch}\,(dx+c)\,b)^{\frac{7}{2}}\,dx$$

input
```
int((csch(d*x+c)*b)^(7/2),x)
```

output
```
int((csch(d*x+c)*b)^(7/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. 2(91) = 182.

Time = 0.09 (sec) , antiderivative size = 484, normalized size of antiderivative = 4.17

$$\int (b\mathrm{csch}(c+dx))^{7/2}\,dx = \frac{2\left(3\sqrt{2}\left(b^3\cosh\left(dx+c\right)^4+4\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+b^3\sinh\left(dx+c\right)^4-2\,b^3\cosh\left(\right.\right.\right.}{}$$

input
```
integrate((b*csch(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
2/5*(3*sqrt(2)*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3
+ b^3*sinh(d*x + c)^4 - 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x +
c)^2 - b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*
sinh(d*x + c))*sqrt(b)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos
h(d*x + c) + sinh(d*x + c))) + sqrt(2)*(3*b^3*cosh(d*x + c)^5 + 15*b^3*cos
h(d*x + c)*sinh(d*x + c)^4 + 3*b^3*sinh(d*x + c)^5 - 8*b^3*cosh(d*x + c)^3
 + b^3*cosh(d*x + c) + 2*(15*b^3*cosh(d*x + c)^2 - 4*b^3)*sinh(d*x + c)^3
+ 6*(5*b^3*cosh(d*x + c)^3 - 4*b^3*cosh(d*x + c))*sinh(d*x + c)^2 + (15*b^
3*cosh(d*x + c)^4 - 24*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c))*sqrt((b*c
osh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*
x + c) + sinh(d*x + c)^2 - 1)))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sin
h(d*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x +
 c)^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(
d*x + c) + d)
```

Sympy [F]

$$\int (b\text{csch}(c+dx))^{7/2}\,dx = \int (b\operatorname{csch}(c+dx))^{\frac{7}{2}}\,dx$$

input
```
integrate((b*csch(d*x+c))**(7/2),x)
```

output
```
Integral((b*csch(c + d*x))**(7/2), x)
```

Maxima [F]

$$\int (b\text{csch}(c+dx))^{7/2}\,dx = \int (b\operatorname{csch}(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((b*csch(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*csch(d*x + c))^(7/2), x)
```

Giac [F]

$$\int (b\text{csch}(c+dx))^{7/2}\,dx=\int (b\,\text{csch}\,(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((b*csch(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*csch(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\text{csch}(c+dx))^{7/2}\,dx=\int \left(\frac{b}{\sinh\left(c+d\,x\right)}\right)^{7/2}\,dx$$

input
```
int((b/sinh(c + d*x))^(7/2),x)
```

output
```
int((b/sinh(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int (b\text{csch}(c+dx))^{7/2}\,dx=\sqrt{b}\left(\int \sqrt{\text{csch}\,(dx+c)}\,\text{csch}(dx+c)^3\,dx\right)b^3$$

input
```
int((b*csch(d*x+c))^(7/2),x)
```

output
```
sqrt(b)*int(sqrt(csch(c + d*x))*csch(c + d*x)**3,x)*b**3
```

3.14 $\int (b\operatorname{csch}(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 88

$$\int (b\operatorname{csch}(c+dx))^{5/2}\,dx = -\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{3/2}}{3d} + \frac{2ib^2\sqrt{b\operatorname{csch}(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{3d}$$

output
```
-2/3*b*cosh(d*x+c)*(b*csch(d*x+c))^(3/2)/d+2/3*I*b^2*(b*csch(d*x+c))^(1/2)
*InverseJacobiAM(1/2*I*c-1/4*Pi+1/2*I*d*x,2^(1/2))*(I*sinh(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\int (b\operatorname{csch}(c+dx))^{5/2}\,dx = -\frac{2b^2\sqrt{b\operatorname{csch}(c+dx)}\left(\coth(c+dx)+i\,\text{EllipticF}\left(\frac{1}{4}(-2ic+\pi-2idx),2\right)\sqrt{i\sinh(c+dx)}\right)}{3d}$$

input
```
Integrate[(b*Csch[c + d*x])^(5/2),x]
```

output
```
(-2*b^2*Sqrt[b*Csch[c + d*x]]*(Coth[c + d*x] + I*EllipticF[((-2*I)*c + Pi
- (2*I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/(3*d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\operatorname{csch}(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (ib\csc(ic+idx))^{5/2}dx$$

$$\Big\downarrow \; 4255$$

$$-\frac{1}{3}b^2\int \sqrt{b\operatorname{csch}(c+dx)}dx-\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{3/2}}{3d}-\frac{1}{3}b^2\int \sqrt{ib\csc(ic+idx)}dx$$

$$\Big\downarrow \; 4258$$

$$-\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{3/2}}{3d}-\frac{1}{3}b^2\sqrt{i\sinh(c+dx)}\sqrt{b\operatorname{csch}(c+dx)}\int \frac{1}{\sqrt{i\sinh(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$-\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{3/2}}{3d}-\frac{1}{3}b^2\sqrt{i\sinh(c+dx)}\sqrt{b\operatorname{csch}(c+dx)}\int \frac{1}{\sqrt{\sin(ic+idx)}}dx$$

$$\Big\downarrow \; 3120$$

$$-\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{3/2}}{3d}+\frac{2ib^2\sqrt{i\sinh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)\sqrt{b\operatorname{csch}(c+dx)}}{3d}$$

input
```
Int[(b*Csch[c + d*x])^(5/2),x]
```

output
```
(-2*b*Cosh[c + d*x]*(b*Csch[c + d*x])^(3/2))/(3*d) + (((2*I)/3)*b^2*Sqrt[b
*Csch[c + d*x]]*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]]
)/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [F]

$$\int (\operatorname{csch}(dx+c)\,b)^{\frac{5}{2}}\,dx$$

input
```
int((csch(d*x+c)*b)^(5/2),x)
```

output
```
int((csch(d*x+c)*b)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(66) = 132.

Time = 0.08 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.47

$$\int (b\operatorname{csch}(c+dx))^{5/2}\,dx =$$

$$-\frac{2\left(\sqrt{2}\left(b^2\cosh(dx+c)^2+2\,b^2\cosh(dx+c)\sinh(dx+c)+b^2\sinh(dx+c)^2-b^2\right)\sqrt{b}\text{weierstrassPInvers}\right.}{3\left(d\,\text{co}\right.}$$

input
```
integrate((b*csch(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(2)*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b
^2*sinh(d*x + c)^2 - b^2)*sqrt(b)*weierstrassPInverse(4, 0, cosh(d*x + c)
+ sinh(d*x + c)) + sqrt(2)*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh
(d*x + c) + b^2*sinh(d*x + c)^2 + b^2)*sqrt((b*cosh(d*x + c) + b*sinh(d*x
+ c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 -
 1)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x +
c)^2 - d)
```

Sympy [F]

$$\int (b\text{csch}(c+dx))^{5/2}\,dx = \int (b\operatorname{csch}(c+dx))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*csch(d*x+c))**(5/2),x)
```

output
```
Integral((b*csch(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int (b\text{csch}(c+dx))^{5/2}\,dx = \int (b\operatorname{csch}(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*csch(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*csch(d*x + c))^(5/2), x)
```

Giac [F]

$$\int (b\text{csch}(c+dx))^{5/2}\,dx = \int (b\operatorname{csch}(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*csch(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*csch(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\text{csch}(c+dx))^{5/2}\,dx=\int \left(\frac{b}{\sinh\left(c+d\,x\right)}\right)^{5/2}\,dx$$

input
```
int((b/sinh(c + d*x))^(5/2),x)
```

output
```
int((b/sinh(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int (b\text{csch}(c+dx))^{5/2}\,dx=\sqrt{b}\left(\int \sqrt{\text{csch}\left(dx+c\right)}\,\text{csch}(dx+c)^2\,dx\right)b^2$$

input
```
int((b*csch(d*x+c))^(5/2),x)
```

output
```
sqrt(b)*int(sqrt(csch(c + d*x))*csch(c + d*x)**2,x)*b**2
```

3.15 $\int (b\text{csch}(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 84

$$\int (b\text{csch}(c+dx))^{3/2}\,dx = -\frac{2b\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{d} - \frac{2ib^2E\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d\sqrt{b\text{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}$$

output
```
-2*b*cosh(d*x+c)*(b*csch(d*x+c))^(1/2)/d+2*I*b^2*EllipticE(cos(1/2*I*c+1/4
*Pi+1/2*I*d*x),2^(1/2))/d/(b*csch(d*x+c))^(1/2)/(I*sinh(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int (b\text{csch}(c+dx))^{3/2}\,dx = -\frac{2b\sqrt{b\text{csch}(c+dx)}\left(\cosh(c+dx)-E\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)\sqrt{i\sinh(c+dx)}\right)}{d}$$

input
```
Integrate[(b*Csch[c + d*x])^(3/2),x]
```

output
```
(-2*b*Sqrt[b*Csch[c + d*x]]*(Cosh[c + d*x] - EllipticE[((-2*I)*c + Pi - (2
*I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/d
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\text{csch}(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (ib\csc(ic+idx))^{3/2}dx$$

$$\Big\downarrow \text{4255}$$

$$b^2\int \frac{1}{\sqrt{b\text{csch}(c+dx)}}dx-\frac{2b\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{d}+b^2\int \frac{1}{\sqrt{ib\csc(ic+idx)}}dx$$

$$\Big\downarrow \text{4258}$$

$$-\frac{2b\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{d}+\frac{b^2\int \sqrt{i\sinh(c+dx)}dx}{\sqrt{i\sinh(c+dx)}\sqrt{b\text{csch}(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{d}+\frac{b^2\int \sqrt{\sin(ic+idx)}dx}{\sqrt{i\sinh(c+dx)}\sqrt{b\text{csch}(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2b\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{d}-\frac{2ib^2E\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{i\sinh(c+dx)}\sqrt{b\text{csch}(c+dx)}}$$

input
```
Int[(b*Csch[c + d*x])^(3/2),x]
```

output
```
(-2*b*Cosh[c + d*x]*Sqrt[b*Csch[c + d*x]])/d - ((2*I)*b^2*EllipticE[(I*c -
 Pi/2 + I*d*x)/2, 2])/(d*Sqrt[b*Csch[c + d*x]]*Sqrt[I*Sinh[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [F]

$$\int (\operatorname{csch}(dx+c)\,b)^{\frac{3}{2}}\,dx$$

input
```
int((csch(d*x+c)*b)^(3/2),x)
```

output
```
int((csch(d*x+c)*b)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.27

$$\int (b\text{csch}(c+dx))^{3/2}\,dx =$$

$$-\frac{2\left(\sqrt{2}b^{\frac{3}{2}}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cosh(dx+c)+\sinh(dx+c)))+\sqrt{2}(b\cosh(dx\right.}{d}$$

input
```
integrate((b*csch(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(2)*b^(3/2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(d
*x + c) + sinh(d*x + c))) + sqrt(2)*(b*cosh(d*x + c) + b*sinh(d*x + c))*sq
rt((b*cosh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*
sinh(d*x + c) + sinh(d*x + c)^2 - 1)))/d
```

Sympy [F]

$$\int (b\text{csch}(c+dx))^{3/2}\,dx = \int (b\,\text{csch}\,(c+dx))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*csch(d*x+c))**(3/2),x)
```

output
```
Integral((b*csch(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int (b\text{csch}(c+dx))^{3/2}\,dx = \int (b\,\text{csch}\,(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*csch(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*csch(d*x + c))^(3/2), x)
```

Giac [F]

$$\int (b\text{csch}(c+dx))^{3/2}\,dx = \int \left(b\,\text{csch}\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*csch(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*csch(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\text{csch}(c+dx))^{3/2}\,dx = \int \left(\frac{b}{\sinh\left(c+d\,x\right)}\right)^{3/2}\,dx$$

input
```
int((b/sinh(c + d*x))^(3/2),x)
```

output
```
int((b/sinh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int (b\text{csch}(c+dx))^{3/2}\,dx = \sqrt{b}\left(\int \sqrt{\text{csch}\left(dx+c\right)}\,\text{csch}(dx+c)\,dx\right)b$$

input
```
int((b*csch(d*x+c))^(3/2),x)
```

output
```
sqrt(b)*int(sqrt(csch(c + d*x))*csch(c + d*x),x)*b
```

3.16 $\int \sqrt{b\operatorname{csch}(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \sqrt{b\operatorname{csch}(c+dx)}\,dx = -\frac{2i\sqrt{b\operatorname{csch}(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{d}$$

output
```
-2*I*(b*csch(d*x+c))^(1/2)*InverseJacobiAM(1/2*I*c-1/4*Pi+1/2*I*d*x,2^(1/2
))*(I*sinh(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \sqrt{b\operatorname{csch}(c+dx)}\,dx = \frac{2i\sqrt{b\operatorname{csch}(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(\frac{\pi}{2}-i(c+dx)\right),2\right)\sqrt{i\sinh(c+dx)}}{d}$$

input
```
Integrate[Sqrt[b*Csch[c + d*x]],x]
```

output
```
((2*I)*Sqrt[b*Csch[c + d*x]]*EllipticF[(Pi/2 - I*(c + d*x))/2, 2]*Sqrt[I*S
inh[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\text{csch}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{ib\csc(ic+idx)}dx$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{i\sinh(c+dx)}\sqrt{b\text{csch}(c+dx)}\int \frac{1}{\sqrt{i\sinh(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{i\sinh(c+dx)}\sqrt{b\text{csch}(c+dx)}\int \frac{1}{\sqrt{\sin(ic+idx)}}dx$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2i\sqrt{i\sinh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)\sqrt{b\text{csch}(c+dx)}}{d}$$

input
```
Int[Sqrt[b*Csch[c + d*x]],x]
```

output
```
((-2*I)*Sqrt[b*Csch[c + d*x]]*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*
Sinh[c + d*x]])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [F]

$$\int \sqrt{\operatorname{csch}\left(dx+c\right)b}dx$$

input
```
int((csch(d*x+c)*b)^(1/2),x)
```

output
```
int((csch(d*x+c)*b)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.48

$$\int \sqrt{b\operatorname{csch}(c+dx)}\,dx = \frac{2\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(4,0,\cosh\left(dx+c\right)+\sinh\left(dx+c\right))}{d}$$

input
```
integrate((b*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(2)*sqrt(b)*weierstrassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c))
/d
```

Sympy [F]

$$\int \sqrt{b\text{csch}(c+dx)}\,dx = \int \sqrt{b\,\text{csch}\,(c+dx)}\,dx$$

input
```
integrate((b*csch(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*csch(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{b\text{csch}(c+dx)}\,dx = \int \sqrt{b\,\text{csch}\,(dx+c)}\,dx$$

input
```
integrate((b*csch(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*csch(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{b\text{csch}(c+dx)}\,dx = \int \sqrt{b\,\text{csch}\,(dx+c)}\,dx$$

input
```
integrate((b*csch(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*csch(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\text{csch}(c+dx)}\,dx = \int \sqrt{\frac{b}{\sinh\left(c+d\,x\right)}}\,dx$$

input
```
int((b/sinh(c + d*x))^(1/2),x)
```

output
```
int((b/sinh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{b\text{csch}(c+dx)}\,dx = \sqrt{b}\left(\int \sqrt{\text{csch}\left(dx+c\right)}dx\right)$$

input
```
int((b*csch(d*x+c))^(1/2),x)
```

output
```
sqrt(b)*int(sqrt(csch(c + d*x)),x)
```

3.17 $\int \frac{1}{\sqrt{b\operatorname{csch}(c+dx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{\sqrt{b\operatorname{csch}(c+dx)}}\,dx=-\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}$$

output
```
2*I*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d/(b*csch(d*x+c))^(1/
2)/(I*sinh(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{b\operatorname{csch}(c+dx)}}\,dx=\frac{2iE\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)}{d\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}$$

input
```
Integrate[1/Sqrt[b*Csch[c + d*x]],x]
```

output
```
((2*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2])/(d*Sqrt[b*Csch[c + d*x
]]*Sqrt[I*Sinh[c + d*x]])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\text{csch}(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{ib\csc(ic+idx)}}dx$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{\int \sqrt{i\sinh(c+dx)}dx}{\sqrt{i\sinh(c+dx)}\sqrt{b\text{csch}(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sqrt{\sin(ic+idx)}dx}{\sqrt{i\sinh(c+dx)}\sqrt{b\text{csch}(c+dx)}}$$

$$\Big\downarrow \text{ 3119}$$

$$-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{i\sinh(c+dx)}\sqrt{b\text{csch}(c+dx)}}$$

input
```
Int[1/Sqrt[b*Csch[c + d*x]],x]
```

output
```
((-2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/(d*Sqrt[b*Csch[c + d*x]]*Sqr
t[I*Sinh[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(45) = 90$.

Time $= 0.35$ (sec) , antiderivative size $= 227$, normalized size of antiderivative $= 4.05$

method	result
risch	$\frac{\sqrt{2}}{d\sqrt{\frac{e^{dx+c}b}{e^{2dx+2c}-1}}}-\frac{\left(\frac{2b\,e^{2dx+2c}-2b}{b\sqrt{e^{dx+c}\left(b\,e^{2dx+2c}-b\right)}}-\frac{\sqrt{e^{dx+c}+1}\,\sqrt{-2\,e^{dx+c}+2}\,\sqrt{-e^{dx+c}}\left(-2\,\text{EllipticE}\left(\sqrt{e^{dx+c}+1},\frac{\sqrt{2}}{2}\right)+\text{EllipticF}\left(\sqrt{e^{dx+c}+1}\ldots\right)\right)}{\sqrt{b\,e^{3dx+3c}-b\,e^{dx+c}}}\right)\ldots}{d\sqrt{\frac{e^{dx+c}b}{e^{2dx+2c}-1}}\left(e^{2dx+2c}-1\right)}$

input
```
int(1/(csch(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*2^(1/2)/(exp(d*x+c)*b/(exp(d*x+c)^2-1))^(1/2)-1/d*(2*(b*exp(d*x+c)^2-b
)/b/(exp(d*x+c)*(b*exp(d*x+c)^2-b))^(1/2)-(exp(d*x+c)+1)^(1/2)*(-2*exp(d*x
+c)+2)^(1/2)*(-exp(d*x+c))^(1/2)/(b*exp(d*x+c)^3-b*exp(d*x+c))^(1/2)*(-2*E
llipticE((exp(d*x+c)+1)^(1/2),1/2*2^(1/2))+EllipticF((exp(d*x+c)+1)^(1/2),
1/2*2^(1/2))))*2^(1/2)/(exp(d*x+c)*b/(exp(d*x+c)^2-1))^(1/2)*(exp(d*x+c)*b
*(exp(d*x+c)^2-1))^(1/2)/(exp(d*x+c)^2-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(41) = 82$.

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.75

$$\int \frac{1}{\sqrt{b\text{csch}(c+dx)}}\,dx = -\frac{2\sqrt{2}\sqrt{b}(\cosh(dx+c)+\sinh(dx+c))\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cosh(dx+c)+}{b}$$

input
```
integrate(1/(b*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-(2*sqrt(2)*sqrt(b)*(cosh(d*x + c) + sinh(d*x + c))*weierstrassZeta(4, 0,
weierstrassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c))) + sqrt(2)*(cosh(
d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt((b*
cosh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2 - 1)))/(b*d*cosh(d*x + c) + b*d*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\text{csch}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{csch}\,(c+dx)}}\,dx$$

input
```
integrate(1/(b*csch(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(b*csch(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\text{csch}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{csch}\,(dx+c)}}\,dx$$

input
```
integrate(1/(b*csch(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*csch(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{b\text{csch}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{csch}\,(dx+c)}}\,dx$$

input
```
integrate(1/(b*csch(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*csch(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\text{csch}(c+dx)}}\,dx = \int \frac{1}{\sqrt{\frac{b}{\sinh(c+d\,x)}}}\,dx$$

input
```
int(1/(b/sinh(c + d*x))^(1/2),x)
```

output
```
int(1/(b/sinh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{b\text{csch}(c+dx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\text{csch}(dx+c)}}{\text{csch}(dx+c)}dx\right)}{b}$$

input
```
int(1/(b*csch(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*int(sqrt(csch(c + d*x))/csch(c + d*x),x))/b
```

3.18 $\int \frac{1}{(b\operatorname{csch}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int \frac{1}{(b\operatorname{csch}(c+dx))^{3/2}}\,dx = \frac{2\cosh(c+dx)}{3bd\sqrt{b\operatorname{csch}(c+dx)}} + \frac{2i\sqrt{b\operatorname{csch}(c+dx)}\,\operatorname{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{3b^2d}$$

output
```
2/3*cosh(d*x+c)/b/d/(b*csch(d*x+c))^(1/2)+2/3*I*(b*csch(d*x+c))^(1/2)*Inve
rseJacobiAM(1/2*I*c-1/4*Pi+1/2*I*d*x,2^(1/2))*(I*sinh(d*x+c))^(1/2)/b^2/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.81

$$\int \frac{1}{(b\operatorname{csch}(c+dx))^{3/2}}\,dx = \frac{\operatorname{csch}^2(c+dx)\left(-2i\,\operatorname{EllipticF}\left(\frac{1}{4}(-2ic+\pi-2idx),2\right)\sqrt{i\sinh(c+dx)}+\sinh(2\right.}{3d(b\operatorname{csch}(c+dx))^{3/2}}$$

input
```
Integrate[(b*Csch[c + d*x])^(-3/2),x]
```

output
```
(Csch[c + d*x]^2*((-2*I)*EllipticF[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[
I*Sinh[c + d*x]] + Sinh[2*(c + d*x)]))/(3*d*(b*Csch[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\text{csch}(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(ib\csc(ic+idx))^{3/2}}dx$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{2\cosh(c+dx)}{3bd\sqrt{b\text{csch}(c+dx)}}-\frac{\int \sqrt{b\text{csch}(c+dx)}dx}{3b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\cosh(c+dx)}{3bd\sqrt{b\text{csch}(c+dx)}}-\frac{\int \sqrt{ib\csc(ic+idx)}dx}{3b^2}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{2\cosh(c+dx)}{3bd\sqrt{b\text{csch}(c+dx)}}-\frac{\sqrt{i\sinh(c+dx)}\sqrt{b\text{csch}(c+dx)}\int \frac{1}{\sqrt{i\sinh(c+dx)}}dx}{3b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\cosh(c+dx)}{3bd\sqrt{b\text{csch}(c+dx)}}-\frac{\sqrt{i\sinh(c+dx)}\sqrt{b\text{csch}(c+dx)}\int \frac{1}{\sqrt{\sin(ic+idx)}}dx}{3b^2}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2\cosh(c+dx)}{3bd\sqrt{b\text{csch}(c+dx)}}+\frac{2i\sqrt{i\sinh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)\sqrt{b\text{csch}(c+dx)}}{3b^2d}$$

input
```
Int[(b*Csch[c + d*x])^(-3/2),x]
```

output
```
(2*Cosh[c + d*x])/(3*b*d*Sqrt[b*Csch[c + d*x]]) + (((2*I)/3)*Sqrt[b*Csch[c
 + d*x]]*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(b^2*
d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [F]

$$\int \frac{1}{\left(\operatorname{csch}\left(dx+c\right) b\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(csch(d*x+c)*b)^(3/2),x)
```

output
```
int(1/(csch(d*x+c)*b)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(68) = 136$.

Time = 0.10 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.58

$$\int \frac{1}{(b\text{csch}(c+dx))^{3/2}}\,dx =$$
$$-\frac{4\sqrt{2}\left(\cosh\left(dx+c\right)^2+2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^2\right)\sqrt{b}\text{weierstrassPInverse}(4,0,\cosh\left(\right.}{}$$

input
```
integrate(1/(b*csch(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(4*sqrt(2)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*
x + c)^2)*sqrt(b)*weierstrassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c))
 - sqrt(2)*(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x
 + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^
4 - 1)*sqrt((b*cosh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)))/(b^2*d*cosh(d*x + c)^2 + 2
*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{1}{(b\text{csch}(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(b\,\text{csch}\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*csch(d*x+c))**(3/2),x)
```

output
```
Integral((b*csch(c + d*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\text{csch}(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\,\text{csch}\,(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*csch(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*csch(d*x + c))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(b\text{csch}(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\,\text{csch}\,(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*csch(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*csch(d*x + c))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\text{csch}(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(\frac{b}{\sinh(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(b/sinh(c + d*x))^(3/2),x)
```

output
```
int(1/(b/sinh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(b\operatorname{csch}(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\operatorname{csch}(dx+c)}}{\operatorname{csch}(dx+c)^2}dx\right)}{b^2}$$

input
```
int(1/(b*csch(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*int(sqrt(csch(c + d*x))/csch(c + d*x)**2,x))/b**2
```

3.19 $\int \frac{1}{(b\,\mathrm{csch}(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int \frac{1}{(b\mathrm{csch}(c+dx))^{5/2}}\,dx=\frac{2\cosh(c+dx)}{5bd(b\mathrm{csch}(c+dx))^{3/2}}+\frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{5b^2d\sqrt{b\mathrm{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}$$

output
```
2/5*cosh(d*x+c)/b/d/(b*csch(d*x+c))^(3/2)-6/5*I*EllipticE(cos(1/2*I*c+1/4*
Pi+1/2*I*d*x),2^(1/2))/b^2/d/(b*csch(d*x+c))^(1/2)/(I*sinh(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \frac{1}{(b\mathrm{csch}(c+dx))^{5/2}}\,dx=\frac{-\frac{6iE\left(\frac{1}{4}(-2ic+\pi-2idx)|2\right)}{\sqrt{i\sinh(c+dx)}}+\sinh(2(c+dx))}{5b^2d\sqrt{b\mathrm{csch}(c+dx)}}$$

input
```
Integrate[(b*Csch[c + d*x])^(-5/2),x]
```

output
```
(((-6*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2])/Sqrt[I*Sinh[c + d*x]
] + Sinh[2*(c + d*x)])/(5*b^2*d*Sqrt[b*Csch[c + d*x]])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\text{csch}(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(ib\csc(ic+idx))^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\frac{2\cosh(c+dx)}{5bd(b\text{csch}(c+dx))^{3/2}}-\frac{3\int \frac{1}{\sqrt{b\text{csch}(c+dx)}}dx}{5b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\cosh(c+dx)}{5bd(b\text{csch}(c+dx))^{3/2}}-\frac{3\int \frac{1}{\sqrt{ib\csc(ic+idx)}}dx}{5b^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2\cosh(c+dx)}{5bd(b\text{csch}(c+dx))^{3/2}}-\frac{3\int \sqrt{i\sinh(c+dx)}dx}{5b^2\sqrt{i\sinh(c+dx)}\sqrt{b\text{csch}(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\cosh(c+dx)}{5bd(b\text{csch}(c+dx))^{3/2}}-\frac{3\int \sqrt{\sin(ic+idx)}dx}{5b^2\sqrt{i\sinh(c+dx)}\sqrt{b\text{csch}(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\cosh(c+dx)}{5bd(b\text{csch}(c+dx))^{3/2}}+\frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5b^2d\sqrt{i\sinh(c+dx)}\sqrt{b\text{csch}(c+dx)}}$$

input
```
Int[(b*Csch[c + d*x])^(-5/2),x]
```

output
```
(2*Cosh[c + d*x])/(5*b*d*(b*Csch[c + d*x])^(3/2)) + (((6*I)/5)*EllipticE[(
I*c - Pi/2 + I*d*x)/2, 2])/(b^2*d*Sqrt[b*Csch[c + d*x]]*Sqrt[I*Sinh[c + d*
x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [F]

$$\int \frac{1}{(\operatorname{csch}(dx+c)\,b)^{\frac{5}{2}}}dx$$

input
```
int(1/(csch(d*x+c)*b)^(5/2),x)
```

output
```
int(1/(csch(d*x+c)*b)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(69) = 138$.

Time = 0.09 (sec) , antiderivative size = 378, normalized size of antiderivative = 4.20

$$\int \frac{1}{(b\text{csch}(c+dx))^{5/2}}\,dx = \frac{24\sqrt{2}\left(\cosh\left(dx+c\right)^3 + 3\cosh\left(dx+c\right)^2\sinh\left(dx+c\right) + 3\cosh\left(dx+c\right)\sinh\left(d\right.\right.}{}$$

input
```
integrate(1/(b*csch(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/20*(24*sqrt(2)*(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*co
sh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3)*sqrt(b)*weierstrassZeta(4,
0, weierstrassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c))) + sqrt(2)*(co
sh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6 + (15*co
sh(d*x + c)^2 + 11)*sinh(d*x + c)^4 + 11*cosh(d*x + c)^4 + 4*(5*cosh(d*x +
 c)^3 + 11*cosh(d*x + c))*sinh(d*x + c)^3 + (15*cosh(d*x + c)^4 + 66*cosh(
d*x + c)^2 - 13)*sinh(d*x + c)^2 - 13*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c)
^5 + 22*cosh(d*x + c)^3 - 13*cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt((b*cos
h(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
+ c) + sinh(d*x + c)^2 - 1)))/(b^3*d*cosh(d*x + c)^3 + 3*b^3*d*cosh(d*x +
c)^2*sinh(d*x + c) + 3*b^3*d*cosh(d*x + c)*sinh(d*x + c)^2 + b^3*d*sinh(d*
x + c)^3)
```

Sympy [F]

$$\int \frac{1}{(b\text{csch}(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(b\operatorname{csch}\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*csch(d*x+c))**(5/2),x)
```

output
```
Integral((b*csch(c + d*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\text{csch}(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\,\text{csch}\,(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*csch(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*csch(d*x + c))^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(b\text{csch}(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\,\text{csch}\,(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*csch(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*csch(d*x + c))^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\text{csch}(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(\frac{b}{\sinh(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(b/sinh(c + d*x))^(5/2),x)
```

output
```
int(1/(b/sinh(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(b\text{csch}(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\text{csch}(dx+c)}}{\text{csch}(dx+c)^3}dx\right)}{b^3}$$

input
```
int(1/(b*csch(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*int(sqrt(csch(c + d*x))/csch(c + d*x)**3,x))/b**3
```

3.20 $\int \frac{1}{(b\text{csch}(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 118

$$\int \frac{1}{(b\text{csch}(c+dx))^{7/2}}\,dx = \frac{2\cosh(c+dx)}{7bd(b\text{csch}(c+dx))^{5/2}} - \frac{10\cosh(c+dx)}{21b^3d\sqrt{b\text{csch}(c+dx)}} - \frac{10i\sqrt{b\text{csch}(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{21b^4d}$$

output
```
2/7*cosh(d*x+c)/b/d/(b*csch(d*x+c))^(5/2)-10/21*cosh(d*x+c)/b^3/d/(b*csch(
d*x+c))^(1/2)-10/21*I*(b*csch(d*x+c))^(1/2)*InverseJacobiAM(1/2*I*c-1/4*Pi
+1/2*I*d*x,2^(1/2))*(I*sinh(d*x+c))^(1/2)/b^4/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.68

$$\int \frac{1}{(b\text{csch}(c+dx))^{7/2}}\,dx = \frac{\sqrt{b\text{csch}(c+dx)}\left(40i\,\text{EllipticF}\left(\frac{1}{4}(-2ic+\pi-2idx),2\right)\sqrt{i\sinh(c+dx)}-26\,\text{si}\right.}{84b^4d}$$

input
```
Integrate[(b*Csch[c + d*x])^(-7/2),x]
```

output
```
(Sqrt[b*Csch[c + d*x]]*((40*I)*EllipticF[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]
*Sqrt[I*Sinh[c + d*x]] - 26*Sinh[2*(c + d*x)] + 3*Sinh[4*(c + d*x)]))/(84*
b^4*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\text{csch}(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(ib\csc(ic+idx))^{7/2}}dx$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{2\cosh(c+dx)}{7bd(b\text{csch}(c+dx))^{5/2}}-\frac{5\int \frac{1}{(b\text{csch}(c+dx))^{3/2}}dx}{7b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\cosh(c+dx)}{7bd(b\text{csch}(c+dx))^{5/2}}-\frac{5\int \frac{1}{(ib\csc(ic+idx))^{3/2}}dx}{7b^2}$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{2\cosh(c+dx)}{7bd(b\text{csch}(c+dx))^{5/2}}-\frac{5\left(\frac{2\cosh(c+dx)}{3bd\sqrt{b\text{csch}(c+dx)}}-\frac{\int \sqrt{b\text{csch}(c+dx)}dx}{3b^2}\right)}{7b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\cosh(c+dx)}{7bd(b\text{csch}(c+dx))^{5/2}}-\frac{5\left(\frac{2\cosh(c+dx)}{3bd\sqrt{b\text{csch}(c+dx)}}-\frac{\int \sqrt{ib\csc(ic+idx)}dx}{3b^2}\right)}{7b^2}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{2\cosh(c+dx)}{7bd(b\operatorname{csch}(c+dx))^{5/2}}-\frac{5\left(\frac{2\cosh(c+dx)}{3bd\sqrt{b\operatorname{csch}(c+dx)}}-\frac{\sqrt{i\sinh(c+dx)}\sqrt{b\operatorname{csch}(c+dx)}\int\frac{1}{\sqrt{i\sinh(c+dx)}}dx}{3b^2}\right)}{7b^2}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2\cosh(c+dx)}{7bd(b\operatorname{csch}(c+dx))^{5/2}}-\frac{5\left(\frac{2\cosh(c+dx)}{3bd\sqrt{b\operatorname{csch}(c+dx)}}-\frac{\sqrt{i\sinh(c+dx)}\sqrt{b\operatorname{csch}(c+dx)}\int\frac{1}{\sqrt{\sin(ic+idx)}}dx}{3b^2}\right)}{7b^2}$$

$$\Big\downarrow \textbf{3120}$$

$$\frac{2\cosh(c+dx)}{7bd(b\operatorname{csch}(c+dx))^{5/2}}-\frac{5\left(\frac{2\cosh(c+dx)}{3bd\sqrt{b\operatorname{csch}(c+dx)}}+\frac{2i\sqrt{i\sinh(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)\sqrt{b\operatorname{csch}(c+dx)}}{3b^2d}\right)}{7b^2}$$

input
```
Int[(b*Csch[c + d*x])^(-7/2),x]
```

output
```
(2*Cosh[c + d*x])/(7*b*d*(b*Csch[c + d*x])^(5/2)) - (5*((2*Cosh[c + d*x])/
(3*b*d*Sqrt[b*Csch[c + d*x]]) + (((2*I)/3)*Sqrt[b*Csch[c + d*x]]*EllipticF
[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(b^2*d)))/(7*b^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [F]

$$\int \frac{1}{\left(\operatorname{csch}\left(dx+c\right)b\right)^{\frac{7}{2}}}dx$$

input
```
int(1/(csch(d*x+c)*b)^(7/2),x)
```

output
```
int(1/(csch(d*x+c)*b)^(7/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(92) = 184$.

Time = 0.10 (sec) , antiderivative size = 483, normalized size of antiderivative = 4.09

$$\int \frac{1}{(b\operatorname{csch}(c+dx))^{7/2}}\,dx = \frac{80\sqrt{2}\left(\cosh\left(dx+c\right)^4+4\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)+6\cosh\left(dx+c\right)^2\sinh\left(\right.\right.}{}$$

input
```
integrate(1/(b*csch(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/168*(80*sqrt(2)*(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*c
osh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*
x + c)^4)*sqrt(b)*weierstrassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c))
 + sqrt(2)*(3*cosh(d*x + c)^8 + 24*cosh(d*x + c)*sinh(d*x + c)^7 + 3*sinh(
d*x + c)^8 + 2*(42*cosh(d*x + c)^2 - 13)*sinh(d*x + c)^6 - 26*cosh(d*x + c
)^6 + 12*(14*cosh(d*x + c)^3 - 13*cosh(d*x + c))*sinh(d*x + c)^5 + 30*(7*c
osh(d*x + c)^4 - 13*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(21*cosh(d*x + c)
^5 - 65*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 2*(42*cosh(d*x + c)^6 - 195*cos
h(d*x + c)^4 + 13)*sinh(d*x + c)^2 + 26*cosh(d*x + c)^2 + 4*(6*cosh(d*x +
c)^7 - 39*cosh(d*x + c)^5 + 13*cosh(d*x + c))*sinh(d*x + c) - 3)*sqrt((b*c
osh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*
x + c) + sinh(d*x + c)^2 - 1)))/(b^4*d*cosh(d*x + c)^4 + 4*b^4*d*cosh(d*x
+ c)^3*sinh(d*x + c) + 6*b^4*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^4*d*c
osh(d*x + c)*sinh(d*x + c)^3 + b^4*d*sinh(d*x + c)^4)
```

Sympy [F]

$$\int \frac{1}{(b\text{csch}(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\operatorname{csch}(c+dx))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(b*csch(d*x+c))**(7/2),x)
```

output
```
Integral((b*csch(c + d*x))**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\text{csch}(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\operatorname{csch}(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(b*csch(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*csch(d*x + c))^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(b\text{csch}(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(b\operatorname{csch}\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(b*csch(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*csch(d*x + c))^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\text{csch}(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(\frac{b}{\sinh(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int(1/(b/sinh(c + d*x))^(7/2),x)
```

output
```
int(1/(b/sinh(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{(b\text{csch}(c+dx))^{7/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\operatorname{csch}(dx+c)}}{\operatorname{csch}(dx+c)^4}dx\right)}{b^4}$$

input
```
int(1/(b*csch(d*x+c))^(7/2),x)
```

output
```
(sqrt(b)*int(sqrt(csch(c + d*x))/csch(c + d*x)**4,x))/b**4
```

3.21 $\int (b\,\text{csch}(c+dx))^n\,dx$

Optimal result

Integrand size = 10, antiderivative size = 74

$$\int (b\,\text{csch}(c+dx))^n\,dx = \frac{b\cosh(c+dx)(b\,\text{csch}(c+dx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},-\sinh^2(c+dx)\right)}{d(1-n)\sqrt{\cosh^2(c+dx)}}$$

output
```
b*cosh(d*x+c)*(b*csch(d*x+c))^(-1+n)*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n
],-sinh(d*x+c)^2)/d/(1-n)/(cosh(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.22

$$\int (b\,\text{csch}(c+dx))^n\,dx = -\frac{2^n\left(\frac{e^{c+dx}}{-1+e^{2(c+dx)}}\right)^n\left(-1+e^{2(c+dx)}\right)\text{csch}^{-n}(c+dx)(b\,\text{csch}(c+dx))^n\,\text{Hypergeometric2F1}\left(1,1-\frac{n}{2},1+\frac{n}{2}\right.}{dn}$$

input
```
Integrate[(b*Csch[c + d*x])^n,x]
```

output
```
-((2^n*(E^(c + d*x)/(-1 + E^(2*(c + d*x))))^n*(-1 + E^(2*(c + d*x)))*(b*Cs
ch[c + d*x])^n*Hypergeometric2F1[1, 1 - n/2, 1 + n/2, E^(2*(c + d*x))])/(d
*n*Csch[c + d*x]^n))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\text{csch}(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (ib\csc(ic+idx))^n dx$$

$$\Big\downarrow \text{4259}$$

$$\left(\frac{\sinh(c+dx)}{b}\right)^n(b\text{csch}(c+dx))^n\int\left(\frac{\sinh(c+dx)}{b}\right)^{-n}dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{\sinh(c+dx)}{b}\right)^n(b\text{csch}(c+dx))^n\int\left(-\frac{i\sin(ic+idx)}{b}\right)^{-n}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{b\cosh(c+dx)(b\text{csch}(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},-\sinh^2(c+dx)\right)}{d(1-n)\sqrt{\cosh^2(c+dx)}}$$

input
```
Int[(b*Csch[c + d*x])^n,x]
```

output
```
(b*Cosh[c + d*x]*(b*Csch[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (1 - n)
/2, (3 - n)/2, -Sinh[c + d*x]^2])/(d*(1 - n)*Sqrt[Cosh[c + d*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Maple [F]

$$\int (\operatorname{csch}(dx+c)\, b)^n\, dx$$

input
```
int((csch(d*x+c)*b)^n,x)
```

output
```
int((csch(d*x+c)*b)^n,x)
```

Fricas [F]

$$\int (b\operatorname{csch}(c+dx))^n\, dx = \int (b \operatorname{csch}(dx+c))^n\, dx$$

input
```
integrate((b*csch(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*csch(d*x + c))^n, x)
```

Sympy [F]

$$\int (b\operatorname{csch}(c+dx))^n\,dx = \int (b\operatorname{csch}(c+dx))^n\,dx$$

input
```
integrate((b*csch(d*x+c))**n,x)
```

output
```
Integral((b*csch(c + d*x))**n, x)
```

Maxima [F]

$$\int (b\operatorname{csch}(c+dx))^n\,dx = \int (b\operatorname{csch}(dx+c))^n\,dx$$

input
```
integrate((b*csch(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*csch(d*x + c))^n, x)
```

Giac [F]

$$\int (b\operatorname{csch}(c+dx))^n\,dx = \int (b\operatorname{csch}(dx+c))^n\,dx$$

input
```
integrate((b*csch(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*csch(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\operatorname{csch}(c+dx))^n\,dx = \int \left(\frac{b}{\sinh(c+d\,x)}\right)^n\,dx$$

input
```
int((b/sinh(c + d*x))^n,x)
```

output
```
int((b/sinh(c + d*x))^n, x)
```

Reduce [F]

$$\int (b\operatorname{csch}(c+dx))^n\,dx = b^n\left(\int \operatorname{csch}(dx+c)^n\,dx\right)$$

input
```
int((b*csch(d*x+c))^n,x)
```

output
```
b**n*int(csch(c + d*x)**n,x)
```

3.22 $\int \left(-\operatorname{csch}^2(x)\right)^{5/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 40

$$\int \left(-\operatorname{csch}^2(x)\right)^{5/2}\,dx = \frac{3}{8}\arcsin(\coth(x)) + \frac{3}{8}\coth(x)\sqrt{-\operatorname{csch}^2(x)} + \frac{1}{4}\coth(x)\left(-\operatorname{csch}^2(x)\right)^{3/2}$$

output
```
3/8*arcsin(coth(x))+3/8*coth(x)*(-csch(x)^2)^(1/2)+1/4*coth(x)*(-csch(x)^2
)^(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \left(-\operatorname{csch}^2(x)\right)^{5/2}\,dx = \frac{1}{64}\left(-\operatorname{csch}^2(x)\right)^{5/2}\sinh(x)\left(-22\cosh(x)+6\left(\cosh(3x)+4\left(-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\right.\right.\right.$$

input
```
Integrate[(-Csch[x]^2)^(5/2),x]
```

output
```
((-Csch[x]^2)^(5/2)*Sinh[x]*(-22*Cosh[x] + 6*(Cosh[3*x] + 4*(-Log[Cosh[x/2
]] + Log[Sinh[x/2]])*Sinh[x]^4)))/64
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4610, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-\text{csch}^2(x)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\sec \left(\frac{\pi }{2}+i x\right)^2\right)^{5/2} dx$$

$$\Big\downarrow \text{4610}$$

$$\int \left(1-\coth^2(x)\right)^{3/2}d\coth (x)$$

$$\Big\downarrow \text{211}$$

$$\frac{3}{4} \int \sqrt{1-\coth^2(x)}d\coth (x)+\frac{1}{4} \coth (x) \left(1-\coth^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{3}{4} \left(\frac{1}{2} \int \frac{1}{\sqrt{1-\coth^2(x)}}d\coth (x)+\frac{1}{2} \sqrt{1-\coth^2(x)} \coth (x)\right)+\frac{1}{4} \coth (x) \left(1-\coth^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{223}$$

$$\frac{3}{4} \left(\frac{1}{2} \arcsin (\coth (x))+\frac{1}{2} \coth (x) \sqrt{1-\coth^2(x)}\right)+\frac{1}{4} \coth (x) \left(1-\coth^2(x)\right)^{3/2}$$

input
```
Int[(-Csch[x]^2)^(5/2),x]
```

output
```
(Coth[x]*(1 - Coth[x]^2)^(3/2))/4 + (3*(ArcSin[Coth[x]]/2 + (Coth[x]*Sqrt[
1 - Coth[x]^2])/2))/4
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(30) = 60$.

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.85

method	result	s
risch	$\frac{\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}\,\left(3\,e^{6x}-11\,e^{4x}-11\,e^{2x}+3\right)}{4\left(e^{2x}-1\right)^{3}}-\frac{3\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}\,e^{-x}\left(e^{2x}-1\right)\ln\left(1+e^{x}\right)}{8}+\frac{3\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}\,e^{-x}\left(e^{2x}-1\right)\ln\left(e^{x}-1\right)}{8}$	1

input
```
int((-csch(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/(exp(2*x)-1)^3*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*(3*exp(6*x)-11*exp(4*x
)-11*exp(2*x)+3)-3/8*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*exp(-x)*(exp(2*x)-1)
*ln(1+exp(x))+3/8*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*exp(-x)*(exp(2*x)-1)*ln
(exp(x)-1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.92

$$\int \left(-\operatorname{csch}^2(x)\right)^{5/2} dx =$$

$$-\frac{3 \left(i e^{(8x)} - 4i e^{(6x)} + 6i e^{(4x)} - 4i e^{(2x)} + i\right) \log\left(e^x + 1\right) + 3 \left(-i e^{(8x)} + 4i e^{(6x)} - 6i e^{(4x)} + 4i e^{(2x)} - i\right) \mathrm{l}}{8 \left(e^{(8x)} - 4 e^{(6x)} + 6 e^{(4x)} - 4 e^{(2x)} + 1\right)}$$

input
```
integrate((-csch(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/8*(3*(I*e^(8*x) - 4*I*e^(6*x) + 6*I*e^(4*x) - 4*I*e^(2*x) + I)*log(e^x
+ 1) + 3*(-I*e^(8*x) + 4*I*e^(6*x) - 6*I*e^(4*x) + 4*I*e^(2*x) - I)*log(e^
x - 1) - 6*I*e^(7*x) + 22*I*e^(5*x) + 22*I*e^(3*x) - 6*I*e^x)/(e^(8*x) - 4
*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)
```

Sympy [F]

$$\int \left(-\operatorname{csch}^2(x)\right)^{5/2} dx = \int \left(-\operatorname{csch}^2(x)\right)^{\frac{5}{2}} dx$$

input
```
integrate((-csch(x)**2)**(5/2),x)
```

output
```
Integral((-csch(x)**2)**(5/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.85

$$\int \left(-\operatorname{csch}^2(x)\right)^{5/2} dx = \frac{3i\,e^{(-x)} - 11i\,e^{(-3x)} - 11i\,e^{(-5x)} + 3i\,e^{(-7x)}}{4\left(4\,e^{(-2x)} - 6\,e^{(-4x)} + 4\,e^{(-6x)} - e^{(-8x)} - 1\right)} + \frac{3}{8}i\,\log\left(e^{(-x)} + 1\right) - \frac{3}{8}i\,\log\left(e^{(-x)} - 1\right)$$

input
```
integrate((-csch(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/4*(3*I*e^(-x) - 11*I*e^(-3*x) - 11*I*e^(-5*x) + 3*I*e^(-7*x))/(4*e^(-2*x
) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 3/8*I*log(e^(-x) + 1) - 3/8*
I*log(e^(-x) - 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.80

$$\int \left(-\operatorname{csch}^2(x)\right)^{5/2} dx = -\frac{1}{16}\left(\frac{4i\left(3\left(e^{(-x)} + e^{x}\right)^3 - 20\,e^{(-x)} - 20\,e^{x}\right)}{\left(\left(e^{(-x)} + e^{x}\right)^2 - 4\right)^2} - 3i\,\log\left(e^{(-x)} + e^{x} + 2\right) + 3i\,\log\left(e^{(-x)} + e^{x} - 2\right)\right)\operatorname{sgn}\left(-e^{(3x)} + e^{x}\right)$$

input
```
integrate((-csch(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/16*(4*I*(3*(e^(-x) + e^x)^3 - 20*e^(-x) - 20*e^x)/((e^(-x) + e^x)^2 - 4
)^2 - 3*I*log(e^(-x) + e^x + 2) + 3*I*log(e^(-x) + e^x - 2))*sgn(-e^(3*x)
+ e^x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(-\operatorname{csch}^2(x)\right)^{5/2} \, dx = \int \left(-\frac{1}{\sinh\left(x\right)^2}\right)^{5/2} \,d x$$

input
```
int((-1/sinh(x)^2)^(5/2),x)
```

output
```
int((-1/sinh(x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 181, normalized size of antiderivative = 4.52

$$\int \left(-\operatorname{csch}^2(x)\right)^{5/2} \, dx = \frac{i(3e^{8x}\log(e^x-1)-3e^{8x}\log(e^x+1)+6e^{7x}-12e^{6x}\log(e^x-1)+12e^{6x}\log(e^x+1)}{}$$

input
```
int((-csch(x)^2)^(5/2),x)
```

output
```
(i*(3*e**(8*x)*log(e**x - 1) - 3*e**(8*x)*log(e**x + 1) + 6*e**(7*x) - 12*
e**(6*x)*log(e**x - 1) + 12*e**(6*x)*log(e**x + 1) - 22*e**(5*x) + 18*e**(
4*x)*log(e**x - 1) - 18*e**(4*x)*log(e**x + 1) - 22*e**(3*x) - 12*e**(2*x)
*log(e**x - 1) + 12*e**(2*x)*log(e**x + 1) + 6*e**x + 3*log(e**x - 1) - 3*
log(e**x + 1)))/(8*(e**(8*x) - 4*e**(6*x) + 6*e**(4*x) - 4*e**(2*x) + 1))
```

3.23 $\int \left(-\operatorname{csch}^2(x)\right)^{3/2} dx$

Optimal result

Integrand size = 10, antiderivative size = 24

$$\int \left(-\operatorname{csch}^2(x)\right)^{3/2} dx = \frac{1}{2}\arcsin(\coth(x)) + \frac{1}{2}\coth(x)\sqrt{-\operatorname{csch}^2(x)}$$

output
```
1/2*arcsin(coth(x))+1/2*coth(x)*(-csch(x)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(24) = 48.

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \left(-\operatorname{csch}^2(x)\right)^{3/2} dx = \frac{1}{8}\sqrt{-\operatorname{csch}^2(x)}\left(\operatorname{csch}^2\left(\frac{x}{2}\right) - 4\log\left(\cosh\left(\frac{x}{2}\right)\right) + 4\log\left(\sinh\left(\frac{x}{2}\right)\right) + \operatorname{sech}^2\left(\frac{x}{2}\right)\right)\sinh(x)$$

input
```
Integrate[(-Csch[x]^2)^(3/2),x]
```

output
```
(Sqrt[-Csch[x]^2]*(Csch[x/2]^2 - 4*Log[Cosh[x/2]] + 4*Log[Sinh[x/2]] + Sec
h[x/2]^2)*Sinh[x])/8
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4610, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-\operatorname{csch}^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\sec\left(\frac{\pi}{2}+ix\right)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{4610}$$

$$\int \sqrt{1-\coth^2(x)}d\coth(x)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-\coth^2(x)}}d\coth(x)+\frac{1}{2}\sqrt{1-\coth^2(x)}\coth(x)$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2}\arcsin(\coth(x))+\frac{1}{2}\coth(x)\sqrt{1-\coth^2(x)}$$

input
```
Int[(-Csch[x]^2)^(3/2),x]
```

output
```
ArcSin[Coth[x]]/2 + (Coth[x]*Sqrt[1 - Coth[x]^2])/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(18) = 36$.

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 4.12

method	result	size
risch	$\frac{\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}\,\left(e^{2x}+1\right)}{e^{2x}-1}+\frac{\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}\,e^{-x}\left(e^{2x}-1\right)\ln\left(e^{x}-1\right)}{2}-\frac{\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}\,e^{-x}\left(e^{2x}-1\right)\ln\left(1+e^{x}\right)}{2}$	99

input
```
int((-csch(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(exp(2*x)-1)*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*(exp(2*x)+1)+1/2*(-exp(2*x
)/(exp(2*x)-1)^2)^(1/2)*exp(-x)*(exp(2*x)-1)*ln(exp(x)-1)-1/2*(-exp(2*x)/(
exp(2*x)-1)^2)^(1/2)*exp(-x)*(exp(2*x)-1)*ln(1+exp(x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.79

$$\int \left(-\operatorname{csch}^2(x)\right)^{3/2} dx = \frac{\left(-i e^{(4x)} + 2i e^{(2x)} - i\right) \log\left(e^x + 1\right) + \left(i e^{(4x)} - 2i e^{(2x)} + i\right) \log\left(e^x - 1\right) + 2i e^{(3x)}}{2\left(e^{(4x)} - 2e^{(2x)} + 1\right)}$$

input
```
integrate((-csch(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((-I*e^(4*x) + 2*I*e^(2*x) - I)*log(e^x + 1) + (I*e^(4*x) - 2*I*e^(2*x
) + I)*log(e^x - 1) + 2*I*e^(3*x) + 2*I*e^x)/(e^(4*x) - 2*e^(2*x) + 1)
```

Sympy [F]

$$\int \left(-\operatorname{csch}^2(x)\right)^{3/2} dx = \int \left(-\operatorname{csch}^2(x)\right)^{\frac{3}{2}} dx$$

input
```
integrate((-csch(x)**2)**(3/2),x)
```

output
```
Integral((-csch(x)**2)**(3/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \left(-\operatorname{csch}^2(x)\right)^{3/2} dx = \frac{i e^{(-x)} + i e^{(-3x)}}{2e^{(-2x)} - e^{(-4x)} - 1} + \frac{1}{2} i \log\left(e^{(-x)} + 1\right) - \frac{1}{2} i \log\left(e^{(-x)} - 1\right)$$

input
```
integrate((-csch(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
(I*e^(-x) + I*e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 1/2*I*log(e^(-x) + 1
) - 1/2*I*log(e^(-x) - 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.38

$$\int \left(-\operatorname{csch}^2(x)\right)^{3/2} dx = -\frac{1}{4}\left(\frac{4i\left(e^{(-x)}+e^{x}\right)}{\left(e^{(-x)}+e^{x}\right)^2-4} - i\log\left(e^{(-x)}+e^{x}+2\right) + i\log\left(e^{(-x)}+e^{x}-2\right)\right)\operatorname{sgn}\left(-e^{(3x)}+e^{x}\right)$$

input
```
integrate((-csch(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/4*(4*I*(e^(-x) + e^x)/((e^(-x) + e^x)^2 - 4) - I*log(e^(-x) + e^x + 2)
+ I*log(e^(-x) + e^x - 2))*sgn(-e^(3*x) + e^x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(-\operatorname{csch}^2(x)\right)^{3/2} dx = \int \left(-\frac{1}{\sinh(x)^2}\right)^{3/2} dx$$

input
```
int((-1/sinh(x)^2)^(3/2),x)
```

output
```
int((-1/sinh(x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 4.08

$$\int \left(-\operatorname{csch}^2(x)\right)^{3/2} dx = \frac{i(e^{4x}\log(e^x-1) - e^{4x}\log(e^x+1) + 2e^{3x} - 2e^{2x}\log(e^x-1) + 2e^{2x}\log(e^x+1) + 2e}{2e^{4x}-4e^{2x}+2}$$

input
```
int((-csch(x)^2)^(3/2),x)
```

output
```
(i*(e**(4*x)*log(e**x - 1) - e**(4*x)*log(e**x + 1) + 2*e**(3*x) - 2*e**(2
*x)*log(e**x - 1) + 2*e**(2*x)*log(e**x + 1) + 2*e**x + log(e**x - 1) - lo
g(e**x + 1)))/(2*(e**(4*x) - 2*e**(2*x) + 1))
```

3.24 $\int \sqrt{-\operatorname{csch}^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 3

$$\int \sqrt{-\operatorname{csch}^2(x)}\,dx = \arcsin(\coth(x))$$

output
```
arcsin(coth(x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 17 vs. 2(3) = 6.

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 5.67

$$\int \sqrt{-\operatorname{csch}^2(x)}\,dx = -\operatorname{arctanh}(\cosh(x))\sqrt{-\operatorname{csch}^2(x)}\sinh(x)$$

input
```
Integrate[Sqrt[-Csch[x]^2],x]
```

output
```
-(ArcTanh[Cosh[x]]*Sqrt[-Csch[x]^2]*Sinh[x])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4610, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-\text{csch}^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sec\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{4610}$$

$$\int \frac{1}{\sqrt{1-\coth^2(x)}}d\coth(x)$$

$$\Big\downarrow \text{223}$$

$$\arcsin(\coth(x))$$

input
```
Int[Sqrt[-Csch[x]^2],x]
```

output
```
ArcSin[Coth[x]]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(3) = 6$.

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 22.33

method	result	size
risch	$-\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}\, e^{-x}\left(e^{2x}-1\right)\ln\left(1+e^{x}\right)+\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}\, e^{-x}\left(e^{2x}-1\right)\ln\left(e^{x}-1\right)$	67

input
```
int((-csch(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*exp(-x)*(exp(2*x)-1)*ln(1+exp(x))+(-exp(
2*x)/(exp(2*x)-1)^2)^(1/2)*exp(-x)*(exp(2*x)-1)*ln(exp(x)-1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 5.00

$$\int \sqrt{-\mathrm{csch}^2(x)}\, dx = -i \log\left(e^{x}+1\right) + i \log\left(e^{x}-1\right)$$

input
```
integrate((-csch(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-I*log(e^x + 1) + I*log(e^x - 1)
```

Sympy [F]

$$\int \sqrt{-\operatorname{csch}^2(x)}\,dx = \int \sqrt{-\operatorname{csch}^2(x)}\,dx$$

input
```
integrate((-csch(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(-csch(x)**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 6.33

$$\int \sqrt{-\operatorname{csch}^2(x)}\,dx = i\log\left(e^{(-x)}+1\right) - i\log\left(e^{(-x)}-1\right)$$

input
```
integrate((-csch(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
I*log(e^(-x) + 1) - I*log(e^(-x) - 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 9.00

$$\int \sqrt{-\operatorname{csch}^2(x)}\,dx = (i\log\left(e^x+1\right) - i\log\left(|e^x-1|\right))\operatorname{sgn}\left(-e^{(3x)}+e^x\right)$$

input
```
integrate((-csch(x)^2)^(1/2),x, algorithm="giac")
```

output
```
(I*log(e^x + 1) - I*log(abs(e^x - 1)))*sgn(-e^(3*x) + e^x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{-\operatorname{csch}^2(x)}\,dx = \int \sqrt{-\frac{1}{\sinh(x)^2}}\,dx$$

input
```
int((-1/sinh(x)^2)^(1/2),x)
```

output
```
int((-1/sinh(x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 5.67

$$\int \sqrt{-\operatorname{csch}^2(x)}\,dx = i(\log(e^x - 1) - \log(e^x + 1))$$

input
```
int((-csch(x)^2)^(1/2),x)
```

output
```
i*(log(e**x - 1) - log(e**x + 1))
```

3.25 $\int \frac{1}{\sqrt{-\operatorname{csch}^2(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 13

$$\int \frac{1}{\sqrt{-\operatorname{csch}^2(x)}}\,dx = \frac{\coth(x)}{\sqrt{-\operatorname{csch}^2(x)}}$$

output
```
coth(x)/(-csch(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-\operatorname{csch}^2(x)}}\,dx = \frac{\coth(x)}{\sqrt{-\operatorname{csch}^2(x)}}$$

input
```
Integrate[1/Sqrt[-Csch[x]^2],x]
```

output
```
Coth[x]/Sqrt[-Csch[x]^2]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4610, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-\text{csch}^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sec\left(\frac{\pi}{2}+ix\right)^2}}dx$$

$$\Big\downarrow \text{4610}$$

$$\int \frac{1}{\left(1-\coth^2(x)\right)^{3/2}}d\coth(x)$$

$$\Big\downarrow \text{208}$$

$$\frac{\coth(x)}{\sqrt{1-\coth^2(x)}}$$

input
```
Int[1/Sqrt[-Csch[x]^2],x]
```

output
```
Coth[x]/Sqrt[1 - Coth[x]^2]
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(11) = 22.

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 4.46

method	result	size
risch	$\frac{e^{2x}}{2\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}\,(e^{2x}-1)}+\frac{1}{2(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}}$	58

input
```
int(1/(-csch(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)/(exp(2*x)-1)*exp(2*x)+1/2/(exp(2*x)-1
)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{-\operatorname{csch}^2(x)}}\,dx=\frac{1}{2}\left(-i\,e^{(2\,x)}-i\right)e^{(-x)}$$

input
```
integrate(1/(-csch(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(-I*e^(2*x) - I)*e^(-x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-\operatorname{csch}^2(x)}}\,dx = \int \frac{1}{\sqrt{-\operatorname{csch}^2(x)}}\,dx$$

input
```
integrate(1/(-csch(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(-csch(x)**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{-\operatorname{csch}^2(x)}}\,dx = \frac{1}{2}i\,e^{(-x)} + \frac{1}{2}i\,e^{x}$$

input
```
integrate(1/(-csch(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*I*e^(-x) + 1/2*I*e^x
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{1}{\sqrt{-\operatorname{csch}^2(x)}}\,dx = -\frac{-i\,e^{(-x)} - i\,e^{x}}{2\,\operatorname{sgn}\left(-e^{(3\,x)} + e^{x}\right)}$$

input
```
integrate(1/(-csch(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(-I*e^(-x) - I*e^x)/sgn(-e^(3*x) + e^x)
```

Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.38

$$\int \frac{1}{\sqrt{-\mathrm{csch}^2(x)}}\,dx = -\mathrm{e}^{-2\,x}\,\sqrt{-\frac{1}{{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)}^2}}\,\left(\frac{\mathrm{e}^{4\,x}}{4}-\frac{1}{4}\right)$$

input
```
int(1/(-1/sinh(x)^2)^(1/2),x)
```

output
```
-exp(-2*x)*(-1/(exp(-x)/2 - exp(x)/2)^2)^(1/2)*(exp(4*x)/4 - 1/4)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{-\mathrm{csch}^2(x)}}\,dx = -\frac{i(e^{2x}+1)}{2e^x}$$

input
```
int(1/(-csch(x)^2)^(1/2),x)
```

output
```
( - i*(e**(2*x) + 1))/(2*e**x)
```

3.26 $\int \frac{1}{\left(-\text{csch}^2(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 33

$$\int \frac{1}{\left(-\text{csch}^2(x)\right)^{3/2}}\,dx=\frac{\coth(x)}{3\left(-\text{csch}^2(x)\right)^{3/2}}+\frac{2\coth(x)}{3\sqrt{-\text{csch}^2(x)}}$$

output
```
1/3*coth(x)/(-csch(x)^2)^(3/2)+2/3*coth(x)/(-csch(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(-\text{csch}^2(x)\right)^{3/2}}\,dx=\frac{9\coth(x)-\cosh(3x)\text{csch}(x)}{12\sqrt{-\text{csch}^2(x)}}$$

input
```
Integrate[(-Csch[x]^2)^(-3/2),x]
```

output
```
(9*Coth[x] - Cosh[3*x]*Csch[x])/(12*Sqrt[-Csch[x]^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4610, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(-\text{csch}^2(x)\right)^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\left(\sec\left(\frac{\pi}{2}+ix\right)^2\right)^{3/2}}dx$$
$$\Big\downarrow \text{4610}$$
$$\int \frac{1}{\left(1-\coth^2(x)\right)^{5/2}}d\coth(x)$$
$$\Big\downarrow \text{209}$$
$$\frac{2}{3}\int \frac{1}{\left(1-\coth^2(x)\right)^{3/2}}d\coth(x)+\frac{\coth(x)}{3\left(1-\coth^2(x)\right)^{3/2}}$$
$$\Big\downarrow \text{208}$$
$$\frac{2\coth(x)}{3\sqrt{1-\coth^2(x)}}+\frac{\coth(x)}{3\left(1-\coth^2(x)\right)^{3/2}}$$

input
```
Int[(-Csch[x]^2)^(-3/2),x]
```

output
```
Coth[x]/(3*(1 - Coth[x]^2)^(3/2)) + (2*Coth[x])/(3*Sqrt[1 - Coth[x]^2])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(25) = 50$.

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 3.58

method	result	size
risch	$-\frac{e^{4x}}{24(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}}+\frac{3\,e^{2x}}{8\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}\,(e^{2x}-1)}+\frac{3}{8(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}}-\frac{e^{-2x}}{24(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}}$	118

input
```
int(1/(-csch(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*exp(4*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+3/8/(-exp(2*x
)/(exp(2*x)-1)^2)^(1/2)/(exp(2*x)-1)*exp(2*x)+3/8/(exp(2*x)-1)/(-exp(2*x)/
(exp(2*x)-1)^2)^(1/2)-1/24*exp(-2*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^
2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{3/2}} \, dx = \frac{1}{24} \left(i\, e^{(6\,x)} - 9i\, e^{(4\,x)} - 9i\, e^{(2\,x)} + i\right) e^{(-3\,x)}$$

input
```
integrate(1/(-csch(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/24*(I*e^(6*x) - 9*I*e^(4*x) - 9*I*e^(2*x) + I)*e^(-3*x)
```

Sympy [F]

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{3/2}} \, dx = \int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/(-csch(x)**2)**(3/2),x)
```

output
```
Integral((-csch(x)**2)**(-3/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{3/2}} \, dx = -\frac{1}{24} i\, e^{(3\,x)} + \frac{3}{8} i\, e^{(-x)} - \frac{1}{24} i\, e^{(-3\,x)} + \frac{3}{8} i\, e^{x}$$

input
```
integrate(1/(-csch(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/24*I*e^(3*x) + 3/8*I*e^(-x) - 1/24*I*e^(-3*x) + 3/8*I*e^x
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{3/2}}\,dx = \frac{\left(9i\,e^{(2\,x)} - i\right)e^{(-3\,x)} - i\,e^{(3\,x)} + 9i\,e^{x}}{24\,\operatorname{sgn}\left(-e^{(3\,x)} + e^{x}\right)}$$

input
```
integrate(1/(-csch(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/24*((9*I*e^(2*x) - I)*e^(-3*x) - I*e^(3*x) + 9*I*e^x)/sgn(-e^(3*x) + e^x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(-\frac{1}{\sinh(x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(-1/sinh(x)^2)^(3/2),x)
```

output
```
int(1/(-1/sinh(x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{3/2}}\,dx = \frac{i(e^{6x} - 9e^{4x} - 9e^{2x} + 1)}{24e^{3x}}$$

input
```
int(1/(-csch(x)^2)^(3/2),x)
```

output
```
(i*(e**(6*x) - 9*e**(4*x) - 9*e**(2*x) + 1))/(24*e**(3*x))
```

3.27 $\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 49

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{5/2}}\,dx=\frac{\coth(x)}{5\left(-\operatorname{csch}^2(x)\right)^{5/2}}+\frac{4\coth(x)}{15\left(-\operatorname{csch}^2(x)\right)^{3/2}}+\frac{8\coth(x)}{15\sqrt{-\operatorname{csch}^2(x)}}$$

output
```
1/5*coth(x)/(-csch(x)^2)^(5/2)+4/15*coth(x)/(-csch(x)^2)^(3/2)+8/15*coth(x
)/(-csch(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{5/2}}\,dx=\frac{(150\cosh(x)-25\cosh(3x)+3\cosh(5x))\operatorname{csch}(x)}{240\sqrt{-\operatorname{csch}^2(x)}}$$

input
```
Integrate[(-Csch[x]^2)^(-5/2),x]
```

output
```
((150*Cosh[x] - 25*Cosh[3*x] + 3*Cosh[5*x])*Csch[x])/(240*Sqrt[-Csch[x]^2]
)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4610, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\sec\left(\frac{\pi}{2}+ix\right)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4610}$$

$$\int \frac{1}{\left(1-\coth^2(x)\right)^{7/2}}d\coth(x)$$

$$\Big\downarrow \text{209}$$

$$\frac{4}{5}\int \frac{1}{\left(1-\coth^2(x)\right)^{5/2}}d\coth(x)+\frac{\coth(x)}{5\left(1-\coth^2(x)\right)^{5/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{\left(1-\coth^2(x)\right)^{3/2}}d\coth(x)+\frac{\coth(x)}{3\left(1-\coth^2(x)\right)^{3/2}}\right)+\frac{\coth(x)}{5\left(1-\coth^2(x)\right)^{5/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\coth(x)}{5\left(1-\coth^2(x)\right)^{5/2}}+\frac{4}{5}\left(\frac{2\coth(x)}{3\sqrt{1-\coth^2(x)}}+\frac{\coth(x)}{3\left(1-\coth^2(x)\right)^{3/2}}\right)$$

input
```
Int[(-Csch[x]^2)^(-5/2),x]
```

output
```
Coth[x]/(5*(1 - Coth[x]^2)^(5/2)) + (4*(Coth[x]/(3*(1 - Coth[x]^2)^(3/2))
+ (2*Coth[x])/(3*Sqrt[1 - Coth[x]^2])))/5
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(37) = 74.

Time = 0.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 3.63

method	result
risch	$\frac{e^{6x}}{160(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}}-\frac{5\,e^{4x}}{96(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}}+\frac{5\,e^{2x}}{16\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}\,(e^{2x}-1)}+\frac{5}{16(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}}-\frac{5}{96(e^{2x}-1)}$

input
```
int(1/(-csch(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/160*exp(6*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)-5/96*exp(4*x)
/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+5/16/(-exp(2*x)/(exp(2*x)-1
)^2)^(1/2)/(exp(2*x)-1)*exp(2*x)+5/16/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)
^2)^(1/2)-5/96*exp(-2*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+1/1
60*exp(-4*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{5/2}}\,dx = \frac{1}{480}\left(-3i\,e^{(10\,x)} + 25i\,e^{(8\,x)} - 150i\,e^{(6\,x)} - 150i\,e^{(4\,x)} + 25i\,e^{(2\,x)} - 3i\right)e^{(-5\,x)}$$

input
```
integrate(1/(-csch(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/480*(-3*I*e^(10*x) + 25*I*e^(8*x) - 150*I*e^(6*x) - 150*I*e^(4*x) + 25*I
*e^(2*x) - 3*I)*e^(-5*x)
```

Sympy [F]

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(-\operatorname{csch}^2\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-csch(x)**2)**(5/2),x)
```

output
```
Integral((-csch(x)**2)**(-5/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{5/2}}\,dx = \frac{1}{160}i\,e^{(5\,x)} - \frac{5}{96}i\,e^{(3\,x)} + \frac{5}{16}i\,e^{(-x)} - \frac{5}{96}i\,e^{(-3\,x)} + \frac{1}{160}i\,e^{(-5\,x)} + \frac{5}{16}i\,e^{x}$$

input
```
integrate(1/(-csch(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/160*I*e^(5*x) - 5/96*I*e^(3*x) + 5/16*I*e^(-x) - 5/96*I*e^(-3*x) + 1/160
*I*e^(-5*x) + 5/16*I*e^x
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(-\text{csch}^2(x)\right)^{5/2}}\,dx = -\frac{\left(-150i\,e^{(4\,x)}+25i\,e^{(2\,x)}-3i\right)e^{(-5\,x)}-3i\,e^{(5\,x)}+25i\,e^{(3\,x)}-150i\,e^{x}}{480\,\text{sgn}\left(-e^{(3\,x)}+e^{x}\right)}$$

input
```
integrate(1/(-csch(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/480*((-150*I*e^(4*x) + 25*I*e^(2*x) - 3*I)*e^(-5*x) - 3*I*e^(5*x) + 25*
I*e^(3*x) - 150*I*e^x)/sgn(-e^(3*x) + e^x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(-\text{csch}^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(-\frac{1}{\sinh(x)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(-1/sinh(x)^2)^(5/2),x)
```

output
```
int(1/(-1/sinh(x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{5/2}}\,dx = \frac{i(-3e^{10x}+25e^{8x}-150e^{6x}-150e^{4x}+25e^{2x}-3)}{480e^{5x}}$$

input
```
int(1/(-csch(x)^2)^(5/2),x)
```

output
```
(i*( - 3*e**(10*x) + 25*e**(8*x) - 150*e**(6*x) - 150*e**(4*x) + 25*e**(2*
x) - 3))/(480*e**(5*x))
```

3.28 $\int \frac{1}{\left(-\mathrm{csch}^2(x)\right)^{7/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 65

$$\int \frac{1}{\left(-\mathrm{csch}^2(x)\right)^{7/2}}\,dx = \frac{\coth(x)}{7\left(-\mathrm{csch}^2(x)\right)^{7/2}} + \frac{6\coth(x)}{35\left(-\mathrm{csch}^2(x)\right)^{5/2}} + \frac{8\coth(x)}{35\left(-\mathrm{csch}^2(x)\right)^{3/2}} + \frac{16\coth(x)}{35\sqrt{-\mathrm{csch}^2(x)}}$$

output
```
1/7*coth(x)/(-csch(x)^2)^(7/2)+6/35*coth(x)/(-csch(x)^2)^(5/2)+8/35*coth(x
)/(-csch(x)^2)^(3/2)+16/35*coth(x)/(-csch(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.60

$$\int \frac{1}{\left(-\mathrm{csch}^2(x)\right)^{7/2}}\,dx = \frac{(1225\cosh(x) - 245\cosh(3x) + 49\cosh(5x) - 5\cosh(7x))\mathrm{csch}(x)}{2240\sqrt{-\mathrm{csch}^2(x)}}$$

input
```
Integrate[(-Csch[x]^2)^(-7/2),x]
```

output
```
((1225*Cosh[x] - 245*Cosh[3*x] + 49*Cosh[5*x] - 5*Cosh[7*x])*Csch[x])/(224
0*Sqrt[-Csch[x]^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4610, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(-\text{csch}^2(x)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\sec\left(\frac{\pi}{2}+ix\right)^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{4610}$$

$$\int \frac{1}{\left(1-\coth^2(x)\right)^{9/2}}d\coth(x)$$

$$\Big\downarrow \text{209}$$

$$\frac{6}{7}\int \frac{1}{\left(1-\coth^2(x)\right)^{7/2}}d\coth(x)+\frac{\coth(x)}{7\left(1-\coth^2(x)\right)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{6}{7}\left(\frac{4}{5}\int \frac{1}{\left(1-\coth^2(x)\right)^{5/2}}d\coth(x)+\frac{\coth(x)}{5\left(1-\coth^2(x)\right)^{5/2}}\right)+\frac{\coth(x)}{7\left(1-\coth^2(x)\right)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{\left(1-\coth^2(x)\right)^{3/2}}d\coth(x)+\frac{\coth(x)}{3\left(1-\coth^2(x)\right)^{3/2}}\right)+\frac{\coth(x)}{5\left(1-\coth^2(x)\right)^{5/2}}\right)+\frac{\coth(x)}{7\left(1-\coth^2(x)\right)^{7/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\coth(x)}{7\left(1-\coth^2(x)\right)^{7/2}}+\frac{6}{7}\left(\frac{\coth(x)}{5\left(1-\coth^2(x)\right)^{5/2}}+\frac{4}{5}\left(\frac{2\coth(x)}{3\sqrt{1-\coth^2(x)}}+\frac{\coth(x)}{3\left(1-\coth^2(x)\right)^{3/2}}\right)\right)$$

input
```
Int[(-Csch[x]^2)^(-7/2),x]
```

output
```
Coth[x]/(7*(1 - Coth[x]^2)^(7/2)) + (6*(Coth[x]/(5*(1 - Coth[x]^2)^(5/2))
+ (4*(Coth[x]/(3*(1 - Coth[x]^2)^(3/2)) + (2*Coth[x])/(3*Sqrt[1 - Coth[x]^
2])))/5))/7
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(49) = 98.

Time = 0.13 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.66

method	result
risch	$-\frac{e^{8x}}{896(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}}+\frac{7\,e^{6x}}{640(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}}-\frac{7\,e^{4x}}{128(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}}+\frac{35\,e^{2x}}{128\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}\,(e^{2x}-1)}+\frac{}{128(}$

input
```
int(1/(-csch(x)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/896*exp(8*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+7/640*exp(6*
x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)-7/128*exp(4*x)/(exp(2*x)-
1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+35/128/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2
)/(exp(2*x)-1)*exp(2*x)+35/128/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/
2)-7/128*exp(-2*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+7/640*exp
(-4*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)-1/896*exp(-6*x)/(exp(
2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{1}{(-\operatorname{csch}^2(x))^{7/2}}\,dx = \frac{1}{4480}\left(5i\,e^{(14\,x)}-49i\,e^{(12\,x)}+245i\,e^{(10\,x)}-1225i\,e^{(8\,x)}-1225i\,e^{(6\,x)}+245i\,e^{(4\,x)}-\right.$$

input
```
integrate(1/(-csch(x)^2)^(7/2),x, algorithm="fricas")
```

output
```
1/4480*(5*I*e^(14*x) - 49*I*e^(12*x) + 245*I*e^(10*x) - 1225*I*e^(8*x) - 1
225*I*e^(6*x) + 245*I*e^(4*x) - 49*I*e^(2*x) + 5*I)*e^(-7*x)
```

Sympy [F]

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{7/2}}\,dx = \int \frac{1}{\left(-\operatorname{csch}^2\left(x\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(-csch(x)**2)**(7/2),x)
```

output
```
Integral((-csch(x)**2)**(-7/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{7/2}}\,dx = -\frac{1}{896}i\,e^{(7\,x)} + \frac{7}{640}i\,e^{(5\,x)} - \frac{7}{128}i\,e^{(3\,x)} + \frac{35}{128}i\,e^{(-x)} - \frac{7}{128}i\,e^{(-3\,x)} + \frac{7}{640}i\,e^{(-5\,x)} - \frac{1}{896}i\,e^{(-7\,x)} + \frac{35}{128}i\,e^{x}$$

input
```
integrate(1/(-csch(x)^2)^(7/2),x, algorithm="maxima")
```

output
```
-1/896*I*e^(7*x) + 7/640*I*e^(5*x) - 7/128*I*e^(3*x) + 35/128*I*e^(-x) - 7
/128*I*e^(-3*x) + 7/640*I*e^(-5*x) - 1/896*I*e^(-7*x) + 35/128*I*e^x
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{7/2}}\,dx = -\frac{\left(-1225i\,e^{(6\,x)} + 245i\,e^{(4\,x)} - 49i\,e^{(2\,x)} + 5i\right)e^{(-7\,x)} + 5i\,e^{(7\,x)} - 49i\,e^{(5\,x)} + 245i\,e^{(3\,x)} - 1225i\,e^{x}}{4480\,\operatorname{sgn}\left(-e^{(3\,x)} + e^{x}\right)}$$

input
```
integrate(1/(-csch(x)^2)^(7/2),x, algorithm="giac")
```

output
```
-1/4480*((-1225*I*e^(6*x) + 245*I*e^(4*x) - 49*I*e^(2*x) + 5*I)*e^(-7*x) +
 5*I*e^(7*x) - 49*I*e^(5*x) + 245*I*e^(3*x) - 1225*I*e^x)/sgn(-e^(3*x) + e
^x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{7/2}}\,dx = \int \frac{1}{\left(-\frac{1}{\sinh(x)^2}\right)^{7/2}}\,dx$$

input
```
int(1/(-1/sinh(x)^2)^(7/2),x)
```

output
```
int(1/(-1/sinh(x)^2)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{7/2}}\,dx = \frac{i(5e^{14x} - 49e^{12x} + 245e^{10x} - 1225e^{8x} - 1225e^{6x} + 245e^{4x} - 49e^{2x} + 5)}{4480e^{7x}}$$

input
```
int(1/(-csch(x)^2)^(7/2),x)
```

output
```
(i*(5*e**(14*x) - 49*e**(12*x) + 245*e**(10*x) - 1225*e**(8*x) - 1225*e**(
6*x) + 245*e**(4*x) - 49*e**(2*x) + 5))/(4480*e**(7*x))
```

3.29 $\int \left(a\operatorname{csch}^2(x)\right)^{5/2} dx$

Optimal result

Integrand size = 10, antiderivative size = 65

$$\int \left(a\operatorname{csch}^2(x)\right)^{5/2} dx = -\frac{3}{8}a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}\right) + \frac{3}{8}a^2\coth(x)\sqrt{a\operatorname{csch}^2(x)} - \frac{1}{4}a\coth(x)\left(a\operatorname{csch}^2(x)\right)^{3/2}$$

output
```
-3/8*a^(5/2)*arctanh(a^(1/2)*coth(x)/(a*csch(x)^2)^(1/2))+3/8*a^2*coth(x)*
(a*csch(x)^2)^(1/2)-1/4*a*coth(x)*(a*csch(x)^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \left(a\operatorname{csch}^2(x)\right)^{5/2} dx = \frac{1}{64}\left(a\operatorname{csch}^2(x)\right)^{5/2}\sinh(x)\left(-22\cosh(x)+6\left(\cosh(3x)+4\left(-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\right.\right.\right.\right.$$

input
```
Integrate[(a*Csch[x]^2)^(5/2),x]
```

output
```
((a*Csch[x]^2)^(5/2)*Sinh[x]*(-22*Cosh[x] + 6*(Cosh[3*x] + 4*(-Log[Cosh[x/
2]] + Log[Sinh[x/2]])*Sinh[x]^4)))/64
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4610, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\text{csch}^2(x)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-a \sec\left(\frac{\pi}{2}+ix\right)^2\right)^{5/2} dx$$

$$\Big\downarrow \text{4610}$$

$$-a\int \left(a\coth^2(x)-a\right)^{3/2} d\coth(x)$$

$$\Big\downarrow \text{211}$$

$$-a\left(\frac{1}{4}\coth(x)\left(a\coth^2(x)-a\right)^{3/2}-\frac{3}{4}a\int \sqrt{a\coth^2(x)-a}d\coth(x)\right)$$

$$\Big\downarrow \text{211}$$

$$-a\left(\frac{1}{4}\coth(x)\left(a\coth^2(x)-a\right)^{3/2}-\frac{3}{4}a\left(\frac{1}{2}\coth(x)\sqrt{a\coth^2(x)-a}-\frac{1}{2}a\int \frac{1}{\sqrt{a\coth^2(x)-a}}d\coth(x)\right)\right)$$

$$\Big\downarrow \text{224}$$

$$-a\left(\frac{1}{4}\coth(x)\left(a\coth^2(x)-a\right)^{3/2}-\frac{3}{4}a\left(\frac{1}{2}\coth(x)\sqrt{a\coth^2(x)-a}-\frac{1}{2}a\int \frac{1}{1-\frac{a\coth^2(x)}{a\coth^2(x)-a}}d\frac{\coth(x)}{\sqrt{a\coth^2(x)-a}}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$-a\left(\frac{1}{4}\coth(x)\left(a\coth^2(x)-a\right)^{3/2}-\frac{3}{4}a\left(\frac{1}{2}\coth(x)\sqrt{a\coth^2(x)-a}-\frac{1}{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\coth^2(x)-a}}\right)\right)\right)$$

input
```
Int[(a*Csch[x]^2)^(5/2),x]
```

output
```
-(a*((Coth[x]*(-a + a*Coth[x]^2)^(3/2))/4 - (3*a*(-1/2*(Sqrt[a]*ArcTanh[(S
qrt[a]*Coth[x])/Sqrt[-a + a*Coth[x]^2]]) + (Coth[x]*Sqrt[-a + a*Coth[x]^2]
)/2))/4))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(49) = 98.

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.89

method	result
risch	$\frac{a^{2} \sqrt{\frac{a\,{\mathrm e}^{2 x}}{\left({\mathrm e}^{2 x}-1\right)^{2}}}\, \left(3\,{\mathrm e}^{6 x}-11\,{\mathrm e}^{4 x}-11\,{\mathrm e}^{2 x}+3\right)}{4 \left({\mathrm e}^{2 x}-1\right)^{3}}+\frac{3 a^{2} {\mathrm e}^{-x} \left({\mathrm e}^{2 x}-1\right) \sqrt{\frac{a\,{\mathrm e}^{2 x}}{\left({\mathrm e}^{2 x}-1\right)^{2}}}\, \ln\left({\mathrm e}^{x}-1\right)}{8}-\frac{3 a^{2} {\mathrm e}^{-x} \left({\mathrm e}^{2 x}-1\right) \sqrt{\frac{a\,{\mathrm e}^{2 x}}{\left({\mathrm e}^{2 x}-1\right)^{2}}}\, \ln\left(1+{\mathrm e}^{x}\right)}{8}$

input

```
int((a*csch(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/4*a^2/(exp(2*x)-1)^3*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*(3*exp(6*x)-11*ex
p(4*x)-11*exp(2*x)+3)+3/8*a^2*exp(-x)*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1
)^2)^(1/2)*ln(exp(x)-1)-3/8*a^2*exp(-x)*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)
-1)^2)^(1/2)*ln(1+exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1128 vs. 2(49) = 98.

Time = 0.12 (sec) , antiderivative size = 1128, normalized size of antiderivative = 17.35

$$\int \left(a\text{csch}^2(x)\right)^{5/2}\,dx = \text{Too large to display}$$

input

```
integrate((a*csch(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/8*(6*a^2*cosh(x)^7 - 6*(a^2*e^(2*x) - a^2)*sinh(x)^7 - 22*a^2*cosh(x)^5
 - 42*(a^2*cosh(x)*e^(2*x) - a^2*cosh(x))*sinh(x)^6 + 2*(63*a^2*cosh(x)^2
- 11*a^2 - (63*a^2*cosh(x)^2 - 11*a^2)*e^(2*x))*sinh(x)^5 - 22*a^2*cosh(x)
^3 + 10*(21*a^2*cosh(x)^3 - 11*a^2*cosh(x) - (21*a^2*cosh(x)^3 - 11*a^2*co
sh(x))*e^(2*x))*sinh(x)^4 + 2*(105*a^2*cosh(x)^4 - 110*a^2*cosh(x)^2 - 11*
a^2 - (105*a^2*cosh(x)^4 - 110*a^2*cosh(x)^2 - 11*a^2)*e^(2*x))*sinh(x)^3
+ 6*a^2*cosh(x) + 2*(63*a^2*cosh(x)^5 - 110*a^2*cosh(x)^3 - 33*a^2*cosh(x)
 - (63*a^2*cosh(x)^5 - 110*a^2*cosh(x)^3 - 33*a^2*cosh(x))*e^(2*x))*sinh(x
)^2 - 2*(3*a^2*cosh(x)^7 - 11*a^2*cosh(x)^5 - 11*a^2*cosh(x)^3 + 3*a^2*cos
h(x))*e^(2*x) + 3*(a^2*cosh(x)^8 - (a^2*e^(2*x) - a^2)*sinh(x)^8 - 4*a^2*c
osh(x)^6 - 8*(a^2*cosh(x)*e^(2*x) - a^2*cosh(x))*sinh(x)^7 + 4*(7*a^2*cosh
(x)^2 - a^2 - (7*a^2*cosh(x)^2 - a^2)*e^(2*x))*sinh(x)^6 + 6*a^2*cosh(x)^4
 + 8*(7*a^2*cosh(x)^3 - 3*a^2*cosh(x) - (7*a^2*cosh(x)^3 - 3*a^2*cosh(x))*
e^(2*x))*sinh(x)^5 + 2*(35*a^2*cosh(x)^4 - 30*a^2*cosh(x)^2 + 3*a^2 - (35*
a^2*cosh(x)^4 - 30*a^2*cosh(x)^2 + 3*a^2)*e^(2*x))*sinh(x)^4 - 4*a^2*cosh(
x)^2 + 8*(7*a^2*cosh(x)^5 - 10*a^2*cosh(x)^3 + 3*a^2*cosh(x) - (7*a^2*cosh
(x)^5 - 10*a^2*cosh(x)^3 + 3*a^2*cosh(x))*e^(2*x))*sinh(x)^3 + 4*(7*a^2*co
sh(x)^6 - 15*a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 - a^2 - (7*a^2*cosh(x)^6 - 15
*a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 - a^2)*e^(2*x))*sinh(x)^2 + a^2 - (a^2*co
sh(x)^8 - 4*a^2*cosh(x)^6 + 6*a^2*cosh(x)^4 - 4*a^2*cosh(x)^2 + a^2)*e^...
```

Sympy [F]

$$\int \left(a\text{csch}^2(x)\right)^{5/2} dx = \int \left(a \operatorname{csch}^2(x)\right)^{\frac{5}{2}} dx$$

input
```
integrate((a*csch(x)**2)**(5/2),x)
```

output
```
Integral((a*csch(x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.42

$$\int \left(a\text{csch}^2(x)\right)^{5/2} dx = \frac{3}{8} a^{\frac{5}{2}} \log\left(e^{(-x)} + 1\right) - \frac{3}{8} a^{\frac{5}{2}} \log\left(e^{(-x)} - 1\right) + \frac{3 a^{\frac{5}{2}} e^{(-x)} - 11 a^{\frac{5}{2}} e^{(-3x)} - 11 a^{\frac{5}{2}} e^{(-5x)} + 3 a^{\frac{5}{2}} e^{(-7x)}}{4 \left(4 e^{(-2x)} - 6 e^{(-4x)} + 4 e^{(-6x)} - e^{(-8x)} - 1\right)}$$

input
```
integrate((a*csch(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
3/8*a^(5/2)*log(e^(-x) + 1) - 3/8*a^(5/2)*log(e^(-x) - 1) + 1/4*(3*a^(5/2)
*e^(-x) - 11*a^(5/2)*e^(-3*x) - 11*a^(5/2)*e^(-5*x) + 3*a^(5/2)*e^(-7*x))/
(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15

$$\int \left(a\text{csch}^2(x)\right)^{5/2} dx = \frac{1}{16} a^{\frac{5}{2}} \left(\frac{4 \left(3 \left(e^{(-x)} + e^{x}\right)^3 - 20 e^{(-x)} - 20 e^{x}\right)}{\left(\left(e^{(-x)} + e^{x}\right)^2 - 4\right)^2} - 3 \log\left(e^{(-x)} + e^{x} + 2\right) + 3 \log\left(e^{(-x)} + e^{x} - 2\right)\right) \text{sgn}\left(e^{(3x)} - e^{x}\right)$$

input
```
integrate((a*csch(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/16*a^(5/2)*(4*(3*(e^(-x) + e^x)^3 - 20*e^(-x) - 20*e^x)/((e^(-x) + e^x)^
2 - 4)^2 - 3*log(e^(-x) + e^x + 2) + 3*log(e^(-x) + e^x - 2))*sgn(e^(3*x)
- e^x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\text{csch}^2(x)\right)^{5/2} dx = \int \left(\frac{a}{\sinh(x)^2}\right)^{5/2} dx$$

input
```
int((a/sinh(x)^2)^(5/2),x)
```

output
```
int((a/sinh(x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.85

$$\int \left(a\text{csch}^2(x)\right)^{5/2} dx = \frac{\sqrt{a}\, a^2(3e^{8x}\log(e^x-1) - 3e^{8x}\log(e^x+1) + 6e^{7x} - 12e^{6x}\log(e^x-1) + 12e^{6x}\log(e^x$$

input
```
int((a*csch(x)^2)^(5/2),x)
```

output
```
(sqrt(a)*a**2*(3*e**(8*x)*log(e**x - 1) - 3*e**(8*x)*log(e**x + 1) + 6*e**
(7*x) - 12*e**(6*x)*log(e**x - 1) + 12*e**(6*x)*log(e**x + 1) - 22*e**(5*x
) + 18*e**(4*x)*log(e**x - 1) - 18*e**(4*x)*log(e**x + 1) - 22*e**(3*x) -
12*e**(2*x)*log(e**x - 1) + 12*e**(2*x)*log(e**x + 1) + 6*e**x + 3*log(e**
x - 1) - 3*log(e**x + 1)))/(8*(e**(8*x) - 4*e**(6*x) + 6*e**(4*x) - 4*e**(
2*x) + 1))
```

3.30 $\int \left(a\operatorname{csch}^2(x)\right)^{3/2} dx$

Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \left(a\operatorname{csch}^2(x)\right)^{3/2} dx = \frac{1}{2}a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}\right) - \frac{1}{2}a\coth(x)\sqrt{a\operatorname{csch}^2(x)}$$

output
```
1/2*a^(3/2)*arctanh(a^(1/2)*coth(x)/(a*csch(x)^2)^(1/2))-1/2*a*coth(x)*(a*
csch(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \left(a\operatorname{csch}^2(x)\right)^{3/2} dx = -\frac{1}{2}a\sqrt{a\operatorname{csch}^2(x)}\left(\coth(x)\operatorname{csch}(x) - \log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right)\right)\sinh(x)$$

input
```
Integrate[(a*Csch[x]^2)^(3/2),x]
```

output
```
-1/2*(a*Sqrt[a*Csch[x]^2]*(Coth[x]*Csch[x] - Log[Cosh[x/2]] + Log[Sinh[x/2
]])*Sinh[x])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4610, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\text{csch}^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-a\sec\left(\frac{\pi}{2}+ix\right)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{4610}$$

$$-a\int \sqrt{a\coth^2(x)-a}d\coth(x)$$

$$\Big\downarrow \text{211}$$

$$-a\left(\frac{1}{2}\coth(x)\sqrt{a\coth^2(x)-a}-\frac{1}{2}a\int \frac{1}{\sqrt{a\coth^2(x)-a}}d\coth(x)\right)$$

$$\Big\downarrow \text{224}$$

$$-a\left(\frac{1}{2}\coth(x)\sqrt{a\coth^2(x)-a}-\frac{1}{2}a\int \frac{1}{1-\frac{a\coth^2(x)}{a\coth^2(x)-a}}d\frac{\coth(x)}{\sqrt{a\coth^2(x)-a}}\right)$$

$$\Big\downarrow \text{219}$$

$$-a\left(\frac{1}{2}\coth(x)\sqrt{a\coth^2(x)-a}-\frac{1}{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\coth^2(x)-a}}\right)\right)$$

input
```
Int[(a*Csch[x]^2)^(3/2),x]
```

output
```
-(a*(-1/2*(Sqrt[a]*ArcTanh[(Sqrt[a]*Coth[x])/Sqrt[-a + a*Coth[x]^2]]) + (C
oth[x]*Sqrt[-a + a*Coth[x]^2])/2))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(34) = 68$.

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.24

method	result	size
risch	$-\frac{a\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^{2}}}\,\left(e^{2x}+1\right)}{e^{2x}-1}+\frac{a\,e^{-x}\left(e^{2x}-1\right)\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^{2}}}\,\ln\left(1+e^{x}\right)}{2}-\frac{a\,e^{-x}\left(e^{2x}-1\right)\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^{2}}}\,\ln\left(e^{x}-1\right)}{2}$	103

input
```
int((a*csch(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-a/(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*(exp(2*x)+1)+1/2*a*exp(-
x)*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(1+exp(x))-1/2*a*exp(-
x)*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(34) = 68$.

Time = 0.10 (sec) , antiderivative size = 340, normalized size of antiderivative = 7.39

$$\int \left(a\text{csch}^2(x)\right)^{3/2} dx = \frac{\left(2\,a\cosh\left(x\right)^3 - 2\left(ae^{(2\,x)} - a\right)\sinh\left(x\right)^3 - 6\left(a\cosh\left(x\right)e^{(2\,x)} - a\cosh\left(x\right)\right)\sinh\left(x\right)^2}{}$$

input
```
integrate((a*csch(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(2*a*cosh(x)^3 - 2*(a*e^(2*x) - a)*sinh(x)^3 - 6*(a*cosh(x)*e^(2*x) -
a*cosh(x))*sinh(x)^2 + 2*a*cosh(x) - 2*(a*cosh(x)^3 + a*cosh(x))*e^(2*x) -
 (a*cosh(x)^4 - (a*e^(2*x) - a)*sinh(x)^4 - 4*(a*cosh(x)*e^(2*x) - a*cosh(
x))*sinh(x)^3 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - (3*a*cosh(x)^2 - a)*e^(
2*x) - a)*sinh(x)^2 - (a*cosh(x)^4 - 2*a*cosh(x)^2 + a)*e^(2*x) + 4*(a*cos
h(x)^3 - a*cosh(x) - (a*cosh(x)^3 - a*cosh(x))*e^(2*x))*sinh(x) + a)*log((
cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) + 2*(3*a*cosh(x)^2 - (3*a*
cosh(x)^2 + a)*e^(2*x) + a)*sinh(x))*sqrt(a/(e^(4*x) - 2*e^(2*x) + 1))*e^x
/(4*cosh(x)*e^x*sinh(x)^3 + e^x*sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*e^x*sinh(x
)^2 + 4*(cosh(x)^3 - cosh(x))*e^x*sinh(x) + (cosh(x)^4 - 2*cosh(x)^2 + 1)*
e^x)
```

Sympy [F]

$$\int \left(a\text{csch}^2(x)\right)^{3/2} dx = \int \left(a\operatorname{csch}^2\left(x\right)\right)^{\frac{3}{2}}\, dx$$

input
```
integrate((a*csch(x)**2)**(3/2),x)
```

output
```
Integral((a*csch(x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \left(a\text{csch}^2(x)\right)^{3/2}\,dx = -\frac{1}{2}\,a^{\frac{3}{2}}\log\left(e^{(-x)}+1\right)+\frac{1}{2}\,a^{\frac{3}{2}}\log\left(e^{(-x)}-1\right)-\frac{a^{\frac{3}{2}}e^{(-x)}+a^{\frac{3}{2}}e^{(-3\,x)}}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}$$

input
```
integrate((a*csch(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*a^(3/2)*log(e^(-x) + 1) + 1/2*a^(3/2)*log(e^(-x) - 1) - (a^(3/2)*e^(-
x) + a^(3/2)*e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \left(a\text{csch}^2(x)\right)^{3/2}\,dx = -\frac{1}{4}\,a^{\frac{3}{2}}\left(\frac{4\left(e^{(-x)}+e^{x}\right)}{\left(e^{(-x)}+e^{x}\right)^{2}-4}-\log\left(e^{(-x)}+e^{x}+2\right)+\log\left(e^{(-x)}+e^{x}-2\right)\right)\text{sgn}\left(e^{(3\,x)}-e^{x}\right)$$

input
```
integrate((a*csch(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/4*a^(3/2)*(4*(e^(-x) + e^x)/((e^(-x) + e^x)^2 - 4) - log(e^(-x) + e^x +
 2) + log(e^(-x) + e^x - 2))*sgn(e^(3*x) - e^x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\text{csch}^2(x)\right)^{3/2} dx = \int \left(\frac{a}{\sinh(x)^2}\right)^{3/2} dx$$

input
```
int((a/sinh(x)^2)^(3/2),x)
```

output
```
int((a/sinh(x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.17

$$\int \left(a\text{csch}^2(x)\right)^{3/2} dx = \frac{\sqrt{a}\, a(-e^{4x}\log(e^x-1)+e^{4x}\log(e^x+1)-2e^{3x}+2e^{2x}\log(e^x-1)-2e^{2x}\log(e^x+1)}{2e^{4x}-4e^{2x}+2}$$

input
```
int((a*csch(x)^2)^(3/2),x)
```

output
```
(sqrt(a)*a*( - e**(4*x)*log(e**x - 1) + e**(4*x)*log(e**x + 1) - 2*e**(3*x
) + 2*e**(2*x)*log(e**x - 1) - 2*e**(2*x)*log(e**x + 1) - 2*e**x - log(e**
x - 1) + log(e**x + 1)))/(2*(e**(4*x) - 2*e**(2*x) + 1))
```

3.31 $\int \sqrt{a\operatorname{csch}^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 26

$$\int \sqrt{a\operatorname{csch}^2(x)}\,dx = -\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}\right)$$

output
```
-a^(1/2)*arctanh(a^(1/2)*coth(x)/(a*csch(x)^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \sqrt{a\operatorname{csch}^2(x)}\,dx = -\operatorname{arctanh}(\cosh(x))\sqrt{a\operatorname{csch}^2(x)}\sinh(x)$$

input
```
Integrate[Sqrt[a*Csch[x]^2],x]
```

output
```
-(ArcTanh[Cosh[x]]*Sqrt[a*Csch[x]^2]*Sinh[x])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4610, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\text{csch}^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{-a\sec\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{ 4610}$$

$$-a\int \frac{1}{\sqrt{a\coth^2(x)-a}}d\coth(x)$$

$$\Big\downarrow \text{ 224}$$

$$-a\int \frac{1}{1-\frac{a\coth^2(x)}{a\coth^2(x)-a}}d\frac{\coth(x)}{\sqrt{a\coth^2(x)-a}}$$

$$\Big\downarrow \text{ 219}$$

$$-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\coth^2(x)-a}}\right)$$

input
```
Int[Sqrt[a*Csch[x]^2],x]
```

output
```
-(Sqrt[a]*ArcTanh[(Sqrt[a]*Coth[x])/Sqrt[-a + a*Coth[x]^2]])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(20) = 40$.

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.58

method	result	size
risch	$\mathrm{e}^{-x}(\mathrm{e}^{2x}-1)\sqrt{\frac{a\,\mathrm{e}^{2x}}{(\mathrm{e}^{2x}-1)^2}}\,\ln(\mathrm{e}^{x}-1)-\mathrm{e}^{-x}(\mathrm{e}^{2x}-1)\sqrt{\frac{a\,\mathrm{e}^{2x}}{(\mathrm{e}^{2x}-1)^2}}\,\ln(1+\mathrm{e}^{x})$	67

input
```
int((a*csch(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
exp(-x)*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(exp(x)-1)-exp(-x
)*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(1+exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(20) = 40$.

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.73

$$\int \sqrt{a\text{csch}^2(x)}\,dx$$
$$= \left[\sqrt{\frac{a}{e^{(4x)} - 2e^{(2x)} + 1}}\left(e^{(2x)} - 1\right)\log\left(\frac{\cosh(x) + \sinh(x) - 1}{\cosh(x) + \sinh(x) + 1}\right), 2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a}{e^{(4x)}-2e^{(2x)}+1}}\left(e^{(2x)} - 1\right)e^x}{a\cosh(x)e^x + ae^x\sinh(x)}\right)\right]$$

input
```
integrate((a*csch(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[sqrt(a/(e^(4*x) - 2*e^(2*x) + 1))*(e^(2*x) - 1)*log((cosh(x) + sinh(x) -
1)/(cosh(x) + sinh(x) + 1)), 2*sqrt(-a)*arctan(sqrt(-a)*sqrt(a/(e^(4*x) -
2*e^(2*x) + 1))*(e^(2*x) - 1)*e^x/(a*cosh(x)*e^x + a*e^x*sinh(x)))]
```

Sympy [F]

$$\int \sqrt{a\text{csch}^2(x)}\,dx = \int \sqrt{a\,\text{csch}^2\,(x)}\,dx$$

input
```
integrate((a*csch(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a*csch(x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \sqrt{a\operatorname{csch}^2(x)}\,dx = \sqrt{a}\log\left(e^{(-x)}+1\right) - \sqrt{a}\log\left(e^{(-x)}-1\right)$$

input
```
integrate((a*csch(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a)*log(e^(-x) + 1) - sqrt(a)*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \sqrt{a\operatorname{csch}^2(x)}\,dx = -\sqrt{a}(\log\left(e^x+1\right) - \log\left(|e^x-1|\right))\operatorname{sgn}\left(e^{(3x)}-e^x\right)$$

input
```
integrate((a*csch(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(a)*(log(e^x + 1) - log(abs(e^x - 1)))*sgn(e^(3*x) - e^x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\operatorname{csch}^2(x)}\,dx = \int \sqrt{\frac{a}{\sinh(x)^2}}\,dx$$

input
```
int((a/sinh(x)^2)^(1/2),x)
```

output
```
int((a/sinh(x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \sqrt{a\text{csch}^2(x)}\,dx = \sqrt{a}\,(\log(e^x-1)-\log(e^x+1))$$

input
```
int((a*csch(x)^2)^(1/2),x)
```

output
```
sqrt(a)*(log(e**x - 1) - log(e**x + 1))
```

3.32 $\int \frac{1}{\sqrt{a\mathbf{csch}^2(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 13

$$\int \frac{1}{\sqrt{a\text{csch}^2(x)}}\,dx=\frac{\coth(x)}{\sqrt{a\text{csch}^2(x)}}$$

output
```
coth(x)/(a*csch(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a\text{csch}^2(x)}}\,dx=\frac{\coth(x)}{\sqrt{a\text{csch}^2(x)}}$$

input
```
Integrate[1/Sqrt[a*Csch[x]^2],x]
```

output
```
Coth[x]/Sqrt[a*Csch[x]^2]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4610, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\text{csch}^2(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{-a\sec\left(\frac{\pi}{2}+ix\right)^2}}dx$$

$$\Big\downarrow \text{ 4610}$$

$$-a\int \frac{1}{\left(a\coth^2(x)-a\right)^{3/2}}d\coth(x)$$

$$\Big\downarrow \text{ 208}$$

$$\frac{\coth(x)}{\sqrt{a\coth^2(x)-a}}$$

input
```
Int[1/Sqrt[a*Csch[x]^2],x]
```

output
```
Coth[x]/Sqrt[-a + a*Coth[x]^2]
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(11) = 22$.

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 4.46

method	result	size
risch	$\frac{e^{2x}}{2\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^{2}}}\,\left(e^{2x}-1\right)}+\frac{1}{2\left(e^{2x}-1\right)\sqrt{\frac{a\,e^{2x}}{\left(e^{2x}-1\right)^{2}}}}$	58

input
```
int(1/(a*csch(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)/(exp(2*x)-1)*exp(2*x)+1/2/(exp(2*x)-
1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(11) = 22$.

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 6.38

$$\int \frac{1}{\sqrt{a\text{csch}^2(x)}}\,dx = \frac{\left(\left(e^{(2x)}-1\right)\sinh{(x)}^2-\cosh{(x)}^2+\left(\cosh{(x)}^2+1\right)e^{(2x)}+2\left(\cosh{(x)}\,e^{(2x)}-\cosh{(x)}\right)\sinh{(x)}-1\right)\sqrt{\cdots}}{2\left(a\cosh{(x)}\,e^{x}+ae^{x}\sinh{(x)}\right)}$$

input
```
integrate(1/(a*csch(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*((e^(2*x) - 1)*sinh(x)^2 - cosh(x)^2 + (cosh(x)^2 + 1)*e^(2*x) + 2*(co
sh(x)*e^(2*x) - cosh(x))*sinh(x) - 1)*sqrt(a/(e^(4*x) - 2*e^(2*x) + 1))*e^
x/(a*cosh(x)*e^x + a*e^x*sinh(x))
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\text{csch}^2(x)}}\,dx = \int \frac{1}{\sqrt{a\,\text{csch}^2\left(x\right)}}\,dx$$

input
```
integrate(1/(a*csch(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a*csch(x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{a\text{csch}^2(x)}}\,dx = -\frac{e^{(-x)}}{2\sqrt{a}} - \frac{e^x}{2\sqrt{a}}$$

input
```
integrate(1/(a*csch(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*e^(-x)/sqrt(a) - 1/2*e^x/sqrt(a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(11) = 22.

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int \frac{1}{\sqrt{a\text{csch}^2(x)}}\,dx = \frac{e^{(-x)} + e^x}{2\sqrt{a}\text{sgn}\left(e^{(3x)} - e^x\right)}$$

input
```
integrate(1/(a*csch(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*(e^(-x) + e^x)/(sqrt(a)*sgn(e^(3*x) - e^x))
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.54

$$\int \frac{1}{\sqrt{a\mathrm{csch}^2(x)}}\,dx = -\frac{\left(\frac{\mathrm{e}^{-2\,x}}{2}-\frac{\mathrm{e}^{2\,x}}{2}\right)\,\sqrt{\frac{1}{{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)}^2}}}{2\,\sqrt{a}}$$

input
```
int(1/(a/sinh(x)^2)^(1/2),x)
```

output
```
-((exp(-2*x)/2 - exp(2*x)/2)*(1/(exp(-x)/2 - exp(x)/2)^2)^(1/2))/(2*a^(1/2
))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{1}{\sqrt{a\mathrm{csch}^2(x)}}\,dx = \frac{\sqrt{a}\,(e^{2x}+1)}{2e^{x}a}$$

input
```
int(1/(a*csch(x)^2)^(1/2),x)
```

output
```
(sqrt(a)*(e**(2*x) + 1))/(2*e**x*a)
```

3.33 $\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{3/2}}\,dx = \frac{\coth(x)}{3\left(a\operatorname{csch}^2(x)\right)^{3/2}} - \frac{2\coth(x)}{3a\sqrt{a\operatorname{csch}^2(x)}}$$

output
```
1/3*coth(x)/(a*csch(x)^2)^(3/2)-2/3*coth(x)/a/(a*csch(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{3/2}}\,dx = \frac{(-9\cosh(x)+\cosh(3x))\operatorname{csch}^3(x)}{12\left(a\operatorname{csch}^2(x)\right)^{3/2}}$$

input
```
Integrate[(a*Csch[x]^2)^(-3/2),x]
```

output
```
((-9*Cosh[x] + Cosh[3*x])*Csch[x]^3)/(12*(a*Csch[x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.39, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4610, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\text{csch}^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(-a\sec\left(\frac{\pi}{2}+ix\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4610}$$

$$-a\int \frac{1}{\left(a\coth^2(x)-a\right)^{5/2}}d\coth(x)$$

$$\Big\downarrow \text{209}$$

$$-a\left(-\frac{2\int \frac{1}{\left(a\coth^2(x)-a\right)^{3/2}}d\coth(x)}{3a}-\frac{\coth(x)}{3a\left(a\coth^2(x)-a\right)^{3/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$-a\left(\frac{2\coth(x)}{3a^2\sqrt{a\coth^2(x)-a}}-\frac{\coth(x)}{3a\left(a\coth^2(x)-a\right)^{3/2}}\right)$$

input
```
Int[(a*Csch[x]^2)^(-3/2),x]
```

output
```
-(a*(-1/3*Coth[x]/(a*(-a + a*Coth[x]^2)^(3/2)) + (2*Coth[x])/(3*a^2*Sqrt[-
a + a*Coth[x]^2])))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(28) = 56$.

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 3.61

method	result	size
risch	$\frac{e^{4x}}{24a(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}}-\frac{3\,e^{2x}}{8a(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}}-\frac{3}{8\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}\,(e^{2x}-1)a}+\frac{e^{-2x}}{24a(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}}$	130

input
```
int(1/(a*csch(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/a*exp(4*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-3/8/a*exp(2
*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-3/8/(a*exp(2*x)/(exp(2*
x)-1)^2)^(1/2)/(exp(2*x)-1)/a+1/24/a*exp(-2*x)/(exp(2*x)-1)/(a*exp(2*x)/(e
xp(2*x)-1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(28) = 56$.

Time = 0.09 (sec) , antiderivative size = 285, normalized size of antiderivative = 7.92

$$\int \frac{1}{\left(a\text{csch}^2(x)\right)^{3/2}}\,dx = \frac{\left(\left(e^{(2x)} - 1\right)\sinh\left(x\right)^6 - \cosh\left(x\right)^6 + 6\left(\cosh\left(x\right)e^{(2x)} - \cosh\left(x\right)\right)\sinh\left(x\right)^5 - 3\left(5\text{ co}\right.\right.}{}$$

input
```
integrate(1/(a*csch(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/24*((e^(2*x) - 1)*sinh(x)^6 - cosh(x)^6 + 6*(cosh(x)*e^(2*x) - cosh(x))*
sinh(x)^5 - 3*(5*cosh(x)^2 - (5*cosh(x)^2 - 3)*e^(2*x) - 3)*sinh(x)^4 + 9*
cosh(x)^4 - 4*(5*cosh(x)^3 - (5*cosh(x)^3 - 9*cosh(x))*e^(2*x) - 9*cosh(x)
)*sinh(x)^3 - 3*(5*cosh(x)^4 - 18*cosh(x)^2 - (5*cosh(x)^4 - 18*cosh(x)^2
- 3)*e^(2*x) - 3)*sinh(x)^2 + 9*cosh(x)^2 + (cosh(x)^6 - 9*cosh(x)^4 - 9*c
osh(x)^2 + 1)*e^(2*x) - 6*(cosh(x)^5 - 6*cosh(x)^3 - (cosh(x)^5 - 6*cosh(x
)^3 - 3*cosh(x))*e^(2*x) - 3*cosh(x))*sinh(x) - 1)*sqrt(a/(e^(4*x) - 2*e^(
2*x) + 1))*e^x/(a^2*cosh(x)^3*e^x + 3*a^2*cosh(x)^2*e^x*sinh(x) + 3*a^2*co
sh(x)*e^x*sinh(x)^2 + a^2*e^x*sinh(x)^3)
```

Sympy [F]

$$\int \frac{1}{\left(a\text{csch}^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\operatorname{csch}^2\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*csch(x)**2)**(3/2),x)
```

output
```
Integral((a*csch(x)**2)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(\text{acsch}^2(x)\right)^{3/2}}\,dx = -\frac{e^{(3x)}}{24\,a^{\frac{3}{2}}} + \frac{3\,e^{(-x)}}{8\,a^{\frac{3}{2}}} - \frac{e^{(-3x)}}{24\,a^{\frac{3}{2}}} + \frac{3\,e^{x}}{8\,a^{\frac{3}{2}}}$$

input
```
integrate(1/(a*csch(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/24*e^(3*x)/a^(3/2) + 3/8*e^(-x)/a^(3/2) - 1/24*e^(-3*x)/a^(3/2) + 3/8*e
^x/a^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(\text{acsch}^2(x)\right)^{3/2}}\,dx = -\frac{\left(9\,e^{(2x)} - 1\right)e^{(-3x)} - e^{(3x)} + 9\,e^{x}}{24\,a^{\frac{3}{2}}\,\text{sgn}\left(e^{(3x)} - e^{x}\right)}$$

input
```
integrate(1/(a*csch(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/24*((9*e^(2*x) - 1)*e^(-3*x) - e^(3*x) + 9*e^x)/(a^(3/2)*sgn(e^(3*x) -
e^x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(\text{acsch}^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{a}{\sinh(x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(a/sinh(x)^2)^(3/2),x)
```

output
```
int(1/(a/sinh(x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{3/2}}\,dx = \frac{\sqrt{a}\,(e^{6x} - 9e^{4x} - 9e^{2x} + 1)}{24e^{3x}a^2}$$

input
```
int(1/(a*csch(x)^2)^(3/2),x)
```

output
```
(sqrt(a)*(e**(6*x) - 9*e**(4*x) - 9*e**(2*x) + 1))/(24*e**(3*x)*a**2)
```

3.34 $\int \frac{1}{\left(a\text{csch}^2(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 55

$$\int \frac{1}{\left(a\text{csch}^2(x)\right)^{5/2}}\,dx=\frac{\coth(x)}{5\left(a\text{csch}^2(x)\right)^{5/2}}-\frac{4\coth(x)}{15a\left(a\text{csch}^2(x)\right)^{3/2}}+\frac{8\coth(x)}{15a^2\sqrt{a\text{csch}^2(x)}}$$

output
```
1/5*coth(x)/(a*csch(x)^2)^(5/2)-4/15*coth(x)/a/(a*csch(x)^2)^(3/2)+8/15*co
th(x)/a^2/(a*csch(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{1}{\left(a\text{csch}^2(x)\right)^{5/2}}\,dx=\frac{(150\cosh(x)-25\cosh(3x)+3\cosh(5x))\sqrt{a\text{csch}^2(x)}\sinh(x)}{240a^3}$$

input
```
Integrate[(a*Csch[x]^2)^(-5/2),x]
```

output
```
((150*Cosh[x] - 25*Cosh[3*x] + 3*Cosh[5*x])*Sqrt[a*Csch[x]^2]*Sinh[x])/(24
0*a^3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.47, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4610, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\text{csch}^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(-a\sec\left(\frac{\pi}{2}+ix\right)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4610}$$

$$-a\int \frac{1}{\left(a\coth^2(x)-a\right)^{7/2}}d\coth(x)$$

$$\Big\downarrow \text{209}$$

$$-a\left(-\frac{4\int \frac{1}{\left(a\coth^2(x)-a\right)^{5/2}}d\coth(x)}{5a}-\frac{\coth(x)}{5a\left(a\coth^2(x)-a\right)^{5/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$-a\left(-\frac{4\left(-\frac{2\int \frac{1}{\left(a\coth^2(x)-a\right)^{3/2}}d\coth(x)}{3a}-\frac{\coth(x)}{3a\left(a\coth^2(x)-a\right)^{3/2}}\right)}{5a}-\frac{\coth(x)}{5a\left(a\coth^2(x)-a\right)^{5/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$-a\left(-\frac{4\left(\frac{2\coth(x)}{3a^2\sqrt{a\coth^2(x)-a}}-\frac{\coth(x)}{3a\left(a\coth^2(x)-a\right)^{3/2}}\right)}{5a}-\frac{\coth(x)}{5a\left(a\coth^2(x)-a\right)^{5/2}}\right)$$

input
```
Int[(a*Csch[x]^2)^(-5/2),x]
```

output
```
-(a*(-1/5*Coth[x]/(a*(-a + a*Coth[x]^2)^(5/2)) - (4*(-1/3*Coth[x]/(a*(-a +
 a*Coth[x]^2)^(3/2)) + (2*Coth[x])/(3*a^2*Sqrt[-a + a*Coth[x]^2])))/(5*a))
)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(43) = 86$.

Time = 0.14 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.56

method	result
risch	$\frac{e^{6x}}{160a^2(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}} - \frac{5\,e^{4x}}{96a^2(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}} + \frac{5\,e^{2x}}{16a^2(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}} + \frac{5}{16\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}\,(e^{2x}-1)a^2} - \frac{}{96a^2(e^{2x}}$

input
```
int(1/(a*csch(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/160/a^2*exp(6*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-5/96/a^2
*exp(4*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)+5/16/a^2*exp(2*x)
/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)+5/16/(a*exp(2*x)/(exp(2*x)
-1)^2)^(1/2)/(exp(2*x)-1)/a^2-5/96/a^2*exp(-2*x)/(exp(2*x)-1)/(a*exp(2*x)/
(exp(2*x)-1)^2)^(1/2)+1/160/a^2*exp(-4*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*
x)-1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(43) = 86$.

Time = 0.09 (sec) , antiderivative size = 590, normalized size of antiderivative = 10.73

$$\int \frac{1}{\left(a\text{csch}^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*csch(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/480*(3*(e^(2*x) - 1)*sinh(x)^10 - 3*cosh(x)^10 + 30*(cosh(x)*e^(2*x) - c
osh(x))*sinh(x)^9 - 5*(27*cosh(x)^2 - (27*cosh(x)^2 - 5)*e^(2*x) - 5)*sinh
(x)^8 + 25*cosh(x)^8 - 40*(9*cosh(x)^3 - (9*cosh(x)^3 - 5*cosh(x))*e^(2*x)
 - 5*cosh(x))*sinh(x)^7 - 10*(63*cosh(x)^4 - 70*cosh(x)^2 - (63*cosh(x)^4
- 70*cosh(x)^2 + 15)*e^(2*x) + 15)*sinh(x)^6 - 150*cosh(x)^6 - 4*(189*cosh
(x)^5 - 350*cosh(x)^3 - (189*cosh(x)^5 - 350*cosh(x)^3 + 225*cosh(x))*e^(2
*x) + 225*cosh(x))*sinh(x)^5 - 10*(63*cosh(x)^6 - 175*cosh(x)^4 + 225*cosh
(x)^2 - (63*cosh(x)^6 - 175*cosh(x)^4 + 225*cosh(x)^2 + 15)*e^(2*x) + 15)*
sinh(x)^4 - 150*cosh(x)^4 - 40*(9*cosh(x)^7 - 35*cosh(x)^5 + 75*cosh(x)^3
- (9*cosh(x)^7 - 35*cosh(x)^5 + 75*cosh(x)^3 + 15*cosh(x))*e^(2*x) + 15*co
sh(x))*sinh(x)^3 - 5*(27*cosh(x)^8 - 140*cosh(x)^6 + 450*cosh(x)^4 + 180*c
osh(x)^2 - (27*cosh(x)^8 - 140*cosh(x)^6 + 450*cosh(x)^4 + 180*cosh(x)^2 -
 5)*e^(2*x) - 5)*sinh(x)^2 + 25*cosh(x)^2 + (3*cosh(x)^10 - 25*cosh(x)^8 +
 150*cosh(x)^6 + 150*cosh(x)^4 - 25*cosh(x)^2 + 3)*e^(2*x) - 10*(3*cosh(x)
^9 - 20*cosh(x)^7 + 90*cosh(x)^5 + 60*cosh(x)^3 - (3*cosh(x)^9 - 20*cosh(x
)^7 + 90*cosh(x)^5 + 60*cosh(x)^3 - 5*cosh(x))*e^(2*x) - 5*cosh(x))*sinh(x
) - 3)*sqrt(a/(e^(4*x) - 2*e^(2*x) + 1))*e^x/(a^3*cosh(x)^5*e^x + 5*a^3*co
sh(x)^4*e^x*sinh(x) + 10*a^3*cosh(x)^3*e^x*sinh(x)^2 + 10*a^3*cosh(x)^2*e^
x*sinh(x)^3 + 5*a^3*cosh(x)*e^x*sinh(x)^4 + a^3*e^x*sinh(x)^5)
```

Sympy [F]

$$\int \frac{1}{\left(a\mathrm{csch}^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\,\mathrm{csch}^2\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*csch(x)**2)**(5/2),x)
```

output
```
Integral((a*csch(x)**2)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a\mathrm{csch}^2(x)\right)^{5/2}}\,dx = -\frac{e^{(5\,x)}}{160\,a^{\frac{5}{2}}} + \frac{5\,e^{(3\,x)}}{96\,a^{\frac{5}{2}}} - \frac{5\,e^{(-x)}}{16\,a^{\frac{5}{2}}} + \frac{5\,e^{(-3\,x)}}{96\,a^{\frac{5}{2}}} - \frac{e^{(-5\,x)}}{160\,a^{\frac{5}{2}}} - \frac{5\,e^{x}}{16\,a^{\frac{5}{2}}}$$

input
```
integrate(1/(a*csch(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/160*e^(5*x)/a^(5/2) + 5/96*e^(3*x)/a^(5/2) - 5/16*e^(-x)/a^(5/2) + 5/96
*e^(-3*x)/a^(5/2) - 1/160*e^(-5*x)/a^(5/2) - 5/16*e^x/a^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a\mathrm{csch}^2(x)\right)^{5/2}}\,dx = \frac{\left(150\,e^{(4\,x)} - 25\,e^{(2\,x)} + 3\right)e^{(-5\,x)} + 3\,e^{(5\,x)} - 25\,e^{(3\,x)} + 150\,e^{x}}{480\,a^{\frac{5}{2}}\mathrm{sgn}\left(e^{(3\,x)} - e^{x}\right)}$$

input
```
integrate(1/(a*csch(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/480*((150*e^(4*x) - 25*e^(2*x) + 3)*e^(-5*x) + 3*e^(5*x) - 25*e^(3*x) +
150*e^x)/(a^(5/2)*sgn(e^(3*x) - e^x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\text{csch}^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(\frac{a}{\sinh(x)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(a/sinh(x)^2)^(5/2),x)
```

output
```
int(1/(a/sinh(x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a\text{csch}^2(x)\right)^{5/2}}\,dx = \frac{\sqrt{a}\,(3e^{10x}-25e^{8x}+150e^{6x}+150e^{4x}-25e^{2x}+3)}{480e^{5x}a^3}$$

input
```
int(1/(a*csch(x)^2)^(5/2),x)
```

output
```
(sqrt(a)*(3*e**(10*x) - 25*e**(8*x) + 150*e**(6*x) + 150*e**(4*x) - 25*e**
(2*x) + 3))/(480*e**(5*x)*a**3)
```

3.35 $\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{7/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 74

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{7/2}}\,dx = \frac{\coth(x)}{7\left(a\operatorname{csch}^2(x)\right)^{7/2}} - \frac{6\coth(x)}{35a\left(a\operatorname{csch}^2(x)\right)^{5/2}} + \frac{8\coth(x)}{35a^2\left(a\operatorname{csch}^2(x)\right)^{3/2}} - \frac{16\coth(x)}{35a^3\sqrt{a\operatorname{csch}^2(x)}}$$

output
```
1/7*coth(x)/(a*csch(x)^2)^(7/2)-6/35*coth(x)/a/(a*csch(x)^2)^(5/2)+8/35*co
th(x)/a^2/(a*csch(x)^2)^(3/2)-16/35*coth(x)/a^3/(a*csch(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.57

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{7/2}}\,dx = \frac{(-1225\cosh(x)+245\cosh(3x)-49\cosh(5x)+5\cosh(7x))\sqrt{a\operatorname{csch}^2(x)}\sinh(x)}{2240a^4}$$

input
```
Integrate[(a*Csch[x]^2)^(-7/2),x]
```

output
```
((-1225*Cosh[x] + 245*Cosh[3*x] - 49*Cosh[5*x] + 5*Cosh[7*x])*Sqrt[a*Csch[
x]^2]*Sinh[x])/(2240*a^4)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.51, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4610, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\text{csch}^2(x)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(-a\sec\left(\frac{\pi}{2}+ix\right)^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{4610}$$

$$-a\int \frac{1}{\left(a\coth^2(x)-a\right)^{9/2}}d\coth(x)$$

$$\Big\downarrow \text{209}$$

$$-a\left(-\frac{6\int \frac{1}{\left(a\coth^2(x)-a\right)^{7/2}}d\coth(x)}{7a}-\frac{\coth(x)}{7a\left(a\coth^2(x)-a\right)^{7/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$-a\left(-\frac{6\left(-\frac{4\int \frac{1}{\left(a\coth^2(x)-a\right)^{5/2}}d\coth(x)}{5a}-\frac{\coth(x)}{5a\left(a\coth^2(x)-a\right)^{5/2}}\right)}{7a}-\frac{\coth(x)}{7a\left(a\coth^2(x)-a\right)^{7/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$-a\left(-\frac{6\left(-\frac{4\left(-\frac{2\int \frac{1}{\left(a\coth^2(x)-a\right)^{3/2}}d\coth(x)}{3a}-\frac{\coth(x)}{3a\left(a\coth^2(x)-a\right)^{3/2}}\right)}{5a}-\frac{\coth(x)}{5a\left(a\coth^2(x)-a\right)^{5/2}}\right)}{7a}-\frac{\coth(x)}{7a\left(a\coth^2(x)-a\right)^{7/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$-a\left(-\frac{6\left(-\frac{4\left(\frac{2\coth(x)}{3a^2\sqrt{a\coth^2(x)-a}}-\frac{\coth(x)}{3a\left(a\coth^2(x)-a\right)^{3/2}}\right)}{5a}-\frac{\coth(x)}{5a\left(a\coth^2(x)-a\right)^{5/2}}\right)}{7a}-\frac{\coth(x)}{7a\left(a\coth^2(x)-a\right)^{7/2}}\right)$$

input
```
Int[(a*Csch[x]^2)^(-7/2),x]
```

output
```
-(a*(-1/7*Coth[x]/(a*(-a + a*Coth[x]^2)^(7/2)) - (6*(-1/5*Coth[x]/(a*(-a +
 a*Coth[x]^2)^(5/2)) - (4*(-1/3*Coth[x]/(a*(-a + a*Coth[x]^2)^(3/2)) + (2*
Coth[x])/(3*a^2*Sqrt[-a + a*Coth[x]^2])))/(5*a)))/(7*a)))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(58) = 116$.

Time = 0.12 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.54

method	result
risch	$\frac{e^{8x}}{896a^3(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}}-\frac{7\,e^{6x}}{640a^3(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}}+\frac{7\,e^{4x}}{128a^3(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}}-\frac{35\,e^{2x}}{128a^3(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}}-\frac{}{128}$

input
```
int(1/(a*csch(x)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/896/a^3*exp(8*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-7/640/a^
3*exp(6*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)+7/128/a^3*exp(4*
x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-35/128/a^3*exp(2*x)/(exp
(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-35/128/(a*exp(2*x)/(exp(2*x)-1)
^2)^(1/2)/(exp(2*x)-1)/a^3+7/128/a^3*exp(-2*x)/(exp(2*x)-1)/(a*exp(2*x)/(e
xp(2*x)-1)^2)^(1/2)-7/640/a^3*exp(-4*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)
-1)^2)^(1/2)+1/896/a^3*exp(-6*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^
(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 984 vs. $2(58) = 116$.

Time = 0.10 (sec) , antiderivative size = 984, normalized size of antiderivative = 13.30

$$\int \frac{1}{\left(a\text{csch}^2(x)\right)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*csch(x)^2)^(7/2),x, algorithm="fricas")
```

output
```
1/4480*(5*(e^(2*x) - 1)*sinh(x)^14 - 5*cosh(x)^14 + 70*(cosh(x)*e^(2*x) -
cosh(x))*sinh(x)^13 - 7*(65*cosh(x)^2 - (65*cosh(x)^2 - 7)*e^(2*x) - 7)*si
nh(x)^12 + 49*cosh(x)^12 - 28*(65*cosh(x)^3 - (65*cosh(x)^3 - 21*cosh(x))*
e^(2*x) - 21*cosh(x))*sinh(x)^11 - 7*(715*cosh(x)^4 - 462*cosh(x)^2 - (715
*cosh(x)^4 - 462*cosh(x)^2 + 35)*e^(2*x) + 35)*sinh(x)^10 - 245*cosh(x)^10
 - 70*(143*cosh(x)^5 - 154*cosh(x)^3 - (143*cosh(x)^5 - 154*cosh(x)^3 + 35
*cosh(x))*e^(2*x) + 35*cosh(x))*sinh(x)^9 - 35*(429*cosh(x)^6 - 693*cosh(x
)^4 + 315*cosh(x)^2 - (429*cosh(x)^6 - 693*cosh(x)^4 + 315*cosh(x)^2 - 35)
*e^(2*x) - 35)*sinh(x)^8 + 1225*cosh(x)^8 - 8*(2145*cosh(x)^7 - 4851*cosh(
x)^5 + 3675*cosh(x)^3 - (2145*cosh(x)^7 - 4851*cosh(x)^5 + 3675*cosh(x)^3
- 1225*cosh(x))*e^(2*x) - 1225*cosh(x))*sinh(x)^7 - 7*(2145*cosh(x)^8 - 64
68*cosh(x)^6 + 7350*cosh(x)^4 - 4900*cosh(x)^2 - (2145*cosh(x)^8 - 6468*co
sh(x)^6 + 7350*cosh(x)^4 - 4900*cosh(x)^2 - 175)*e^(2*x) - 175)*sinh(x)^6
+ 1225*cosh(x)^6 - 14*(715*cosh(x)^9 - 2772*cosh(x)^7 + 4410*cosh(x)^5 - 4
900*cosh(x)^3 - (715*cosh(x)^9 - 2772*cosh(x)^7 + 4410*cosh(x)^5 - 4900*co
sh(x)^3 - 525*cosh(x))*e^(2*x) - 525*cosh(x))*sinh(x)^5 - 35*(143*cosh(x)^
10 - 693*cosh(x)^8 + 1470*cosh(x)^6 - 2450*cosh(x)^4 - 525*cosh(x)^2 - (14
3*cosh(x)^10 - 693*cosh(x)^8 + 1470*cosh(x)^6 - 2450*cosh(x)^4 - 525*cosh(
x)^2 + 7)*e^(2*x) + 7)*sinh(x)^4 - 245*cosh(x)^4 - 140*(13*cosh(x)^11 - 77
*cosh(x)^9 + 210*cosh(x)^7 - 490*cosh(x)^5 - 175*cosh(x)^3 - (13*cosh(x...
```

Sympy [F]

$$\int \frac{1}{\left(a\text{csch}^2(x)\right)^{7/2}}\,dx = \int \frac{1}{\left(a\,\text{csch}^2\left(x\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a*csch(x)**2)**(7/2),x)
```

output
```
Integral((a*csch(x)**2)**(-7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a\text{csch}^2(x)\right)^{7/2}}\,dx = -\frac{e^{(7\,x)}}{896\,a^{\frac{7}{2}}} + \frac{7\,e^{(5\,x)}}{640\,a^{\frac{7}{2}}} - \frac{7\,e^{(3\,x)}}{128\,a^{\frac{7}{2}}} + \frac{35\,e^{(-x)}}{128\,a^{\frac{7}{2}}} - \frac{7\,e^{(-3\,x)}}{128\,a^{\frac{7}{2}}} + \frac{7\,e^{(-5\,x)}}{640\,a^{\frac{7}{2}}} - \frac{e^{(-7\,x)}}{896\,a^{\frac{7}{2}}} + \frac{35\,e^{x}}{128\,a^{\frac{7}{2}}}$$

input
```
integrate(1/(a*csch(x)^2)^(7/2),x, algorithm="maxima")
```

output
```
-1/896*e^(7*x)/a^(7/2) + 7/640*e^(5*x)/a^(7/2) - 7/128*e^(3*x)/a^(7/2) + 3
5/128*e^(-x)/a^(7/2) - 7/128*e^(-3*x)/a^(7/2) + 7/640*e^(-5*x)/a^(7/2) - 1
/896*e^(-7*x)/a^(7/2) + 35/128*e^x/a^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a\text{csch}^2(x)\right)^{7/2}}\,dx = -\frac{\left(1225\,e^{(6\,x)} - 245\,e^{(4\,x)} + 49\,e^{(2\,x)} - 5\right)e^{(-7\,x)} - 5\,e^{(7\,x)} + 49\,e^{(5\,x)} - 245\,e^{(3\,x)} + 1225\,e^{x}}{4480\,a^{\frac{7}{2}}\text{sgn}\left(e^{(3\,x)} - e^{x}\right)}$$

input
```
integrate(1/(a*csch(x)^2)^(7/2),x, algorithm="giac")
```

output
```
-1/4480*((1225*e^(6*x) - 245*e^(4*x) + 49*e^(2*x) - 5)*e^(-7*x) - 5*e^(7*x
) + 49*e^(5*x) - 245*e^(3*x) + 1225*e^x)/(a^(7/2)*sgn(e^(3*x) - e^x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\text{csch}^2(x)\right)^{7/2}}\,dx = \int \frac{1}{\left(\frac{a}{\sinh(x)^2}\right)^{7/2}}\,dx$$

input
```
int(1/(a/sinh(x)^2)^(7/2),x)
```

output
```
int(1/(a/sinh(x)^2)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a\text{csch}^2(x)\right)^{7/2}}\,dx = \frac{\sqrt{a}\,(5e^{14x} - 49e^{12x} + 245e^{10x} - 1225e^{8x} - 1225e^{6x} + 245e^{4x} - 49e^{2x} + 5)}{4480e^{7x}a^4}$$

input
```
int(1/(a*csch(x)^2)^(7/2),x)
```

output
```
(sqrt(a)*(5*e**(14*x) - 49*e**(12*x) + 245*e**(10*x) - 1225*e**(8*x) - 122
5*e**(6*x) + 245*e**(4*x) - 49*e**(2*x) + 5))/(4480*e**(7*x)*a**4)
```

3.36 $\int \left(a\operatorname{csch}^3(x)\right)^{5/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 135

$$
\begin{aligned}
\int \left(a\operatorname{csch}^3(x)\right)^{5/2}\,dx = & -\frac{154}{585}a^2\coth(x)\sqrt{a\operatorname{csch}^3(x)} \\
& + \frac{22}{117}a^2\coth(x)\operatorname{csch}^2(x)\sqrt{a\operatorname{csch}^3(x)} - \frac{2}{13}a^2\coth(x)\operatorname{csch}^4(x)\sqrt{a\operatorname{csch}^3(x)} \\
& + \frac{154}{195}a^2\cosh(x)\sqrt{a\operatorname{csch}^3(x)}\sinh(x) - \frac{154ia^2\sqrt{a\operatorname{csch}^3(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sinh^2(x)}{195\sqrt{i\sinh(x)}}
\end{aligned}
$$

output
```
-154/585*a^2*coth(x)*(a*csch(x)^3)^(1/2)+22/117*a^2*coth(x)*csch(x)^2*(a*c
sch(x)^3)^(1/2)-2/13*a^2*coth(x)*csch(x)^4*(a*csch(x)^3)^(1/2)+154/195*a^2
*cosh(x)*(a*csch(x)^3)^(1/2)*sinh(x)-154/195*I*a^2*(a*csch(x)^3)^(1/2)*Ell
ipticE(cos(1/4*Pi+1/2*I*x),2^(1/2))*sinh(x)^2/(I*sinh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.50

$$\int \left(a\text{csch}^3(x)\right)^{5/2} dx = -\frac{2}{585}a^2\sqrt{a\text{csch}^3(x)}\left(-231\cosh(x)+\coth(x)\text{csch}(x)\left(77-55\text{csch}^2(x)+45\text{csch}^4(x)\right)+231E\left(\frac{1}{4}(\pi-2ix)\middle|2\right)\sqrt{i\sinh(x)}\right)\sinh(x)$$

input
```
Integrate[(a*Csch[x]^3)^(5/2),x]
```

output
```
(-2*a^2*Sqrt[a*Csch[x]^3]*(-231*Cosh[x] + Coth[x]*Csch[x]*(77 - 55*Csch[x]
^2 + 45*Csch[x]^4) + 231*EllipticE[(Pi - (2*I)*x)/4, 2]*Sqrt[I*Sinh[x]])*S
inh[x])/585
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.400, Rules used = {3042, 4611, 3042, 4255, 3042, 4255, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\text{csch}^3(x)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(ia\sec\left(\frac{\pi}{2}+ix\right)^3\right)^{5/2} dx$$

$$\Big\downarrow \text{4611}$$

$$-\frac{a^2\sqrt{a\text{csch}^3(x)}\int(i\text{csch}(x))^{15/2}dx}{(i\text{csch}(x))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a^2\sqrt{a\text{csch}^3(x)}\int(-\csc(ix))^{15/2}dx}{(i\text{csch}(x))^{3/2}}$$

$$\Big\downarrow \ 4255$$

$$-\frac{a^2\sqrt{a\text{csch}^3(x)}\left(\frac{11}{13}\int(i\text{csch}(x))^{11/2}dx-\frac{2}{13}i\cosh(x)(i\text{csch}(x))^{13/2}\right)}{(i\text{csch}(x))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a^2\sqrt{a\text{csch}^3(x)}\left(\frac{11}{13}\int(-\csc(ix))^{11/2}dx-\frac{2}{13}i\cosh(x)(i\text{csch}(x))^{13/2}\right)}{(i\text{csch}(x))^{3/2}}$$

$$\Big\downarrow \ 4255$$

$$-\frac{a^2\sqrt{a\text{csch}^3(x)}\left(\frac{11}{13}\left(\frac{7}{9}\int(i\text{csch}(x))^{7/2}dx-\frac{2}{9}i\cosh(x)(i\text{csch}(x))^{9/2}\right)-\frac{2}{13}i\cosh(x)(i\text{csch}(x))^{13/2}\right)}{(i\text{csch}(x))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a^2\sqrt{a\text{csch}^3(x)}\left(\frac{11}{13}\left(\frac{7}{9}\int(-\csc(ix))^{7/2}dx-\frac{2}{9}i\cosh(x)(i\text{csch}(x))^{9/2}\right)-\frac{2}{13}i\cosh(x)(i\text{csch}(x))^{13/2}\right)}{(i\text{csch}(x))^{3/2}}$$

$$\Big\downarrow \ 4255$$

$$-\frac{a^2\sqrt{a\text{csch}^3(x)}\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\int(i\text{csch}(x))^{3/2}dx-\frac{2}{5}i\cosh(x)(i\text{csch}(x))^{5/2}\right)-\frac{2}{9}i\cosh(x)(i\text{csch}(x))^{9/2}\right)-\frac{2}{13}i\cosh(x)(i\right.}{(i\text{csch}(x))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a^2\sqrt{a\text{csch}^3(x)}\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\int(-\csc(ix))^{3/2}dx-\frac{2}{5}i\cosh(x)(i\text{csch}(x))^{5/2}\right)-\frac{2}{9}i\cosh(x)(i\text{csch}(x))^{9/2}\right)-\frac{2}{13}i\cosh(x)(\right.}{(i\text{csch}(x))^{3/2}}$$

$$\Big\downarrow \ 4255$$

$$-\frac{a^2\sqrt{a\text{csch}^3(x)}\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\left(-\int\frac{1}{\sqrt{i\text{csch}(x)}}dx-2i\cosh(x)\sqrt{i\text{csch}(x)}\right)-\frac{2}{5}i\cosh(x)(i\text{csch}(x))^{5/2}\right)-\frac{2}{9}i\cosh(x)\right.\right.}{(i\text{csch}(x))^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{a^2\sqrt{a\text{csch}^3(x)}\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\left(-\int\frac{1}{\sqrt{-\csc(ix)}}dx-2i\cosh(x)\sqrt{i\text{csch}(x)}\right)-\frac{2}{5}i\cosh(x)(i\text{csch}(x))^{5/2}\right)-\frac{2}{9}i\cosh(x)(i\text{cs}\right.\right.}{(i\text{csch}(x))^{3/2}}$$

$\Big\downarrow$ 4258

$$-\frac{a^2\sqrt{a\text{csch}^3(x)}\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\left(-\frac{\int\sqrt{i\sinh(x)}dx}{\sqrt{i\sinh(x)}\sqrt{i\text{csch}(x)}}-2i\cosh(x)\sqrt{i\text{csch}(x)}\right)-\frac{2}{5}i\cosh(x)(i\text{csch}(x))^{5/2}\right)-\frac{2}{9}i\cosh\right.\right.}{(i\text{csch}(x))^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{a^2\sqrt{a\text{csch}^3(x)}\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\left(-\frac{\int\sqrt{\sin(ix)}dx}{\sqrt{i\sinh(x)}\sqrt{i\text{csch}(x)}}-2i\cosh(x)\sqrt{i\text{csch}(x)}\right)-\frac{2}{5}i\cosh(x)(i\text{csch}(x))^{5/2}\right)-\frac{2}{9}i\cosh\right.\right.}{(i\text{csch}(x))^{3/2}}$$

$\Big\downarrow$ 3119

$$-\frac{a^2\sqrt{a\text{csch}^3(x)}\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\left(-2i\cosh(x)\sqrt{i\text{csch}(x)}-\frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}|2\right)}{\sqrt{i\sinh(x)}\sqrt{i\text{csch}(x)}}\right)-\frac{2}{5}i\cosh(x)(i\text{csch}(x))^{5/2}\right)-\frac{2}{9}i\cosh\right.\right.}{(i\text{csch}(x))^{3/2}}$$

input
```
Int[(a*Csch[x]^3)^(5/2),x]
```

output
```
-((a^2*Sqrt[a*Csch[x]^3]*(((-2*I)/13)*Cosh[x]*(I*Csch[x])^(13/2) + (11*(((
-2*I)/9)*Cosh[x]*(I*Csch[x])^(9/2) + (7*(((-2*I)/5)*Cosh[x]*(I*Csch[x])^(5
/2) + (3*((-2*I)*Cosh[x]*Sqrt[I*Csch[x]] - ((2*I)*EllipticE[Pi/4 - (I/2)*x
, 2])/(Sqrt[I*Csch[x]]*Sqrt[I*Sinh[x]])))/5))/9))/13))/(I*Csch[x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [F]

$$\int \left(a\,\mathrm{csch}\left(x\right)^{3}\right)^{\frac{5}{2}}dx$$

input
```
int((a*csch(x)^3)^(5/2),x)
```

output
```
int((a*csch(x)^3)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1389 vs. $2(106) = 212$.

Time = 0.11 (sec) , antiderivative size = 1389, normalized size of antiderivative = 10.29

$$\int \left(a\text{csch}^3(x)\right)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((a*csch(x)^3)^(5/2),x, algorithm="fricas")
```

output
```
2/585*(231*sqrt(2)*(a^2*cosh(x)^12 + 12*a^2*cosh(x)*sinh(x)^11 + a^2*sinh(
x)^12 - 6*a^2*cosh(x)^10 + 6*(11*a^2*cosh(x)^2 - a^2)*sinh(x)^10 + 15*a^2*
cosh(x)^8 + 20*(11*a^2*cosh(x)^3 - 3*a^2*cosh(x))*sinh(x)^9 + 15*(33*a^2*c
osh(x)^4 - 18*a^2*cosh(x)^2 + a^2)*sinh(x)^8 - 20*a^2*cosh(x)^6 + 24*(33*a
^2*cosh(x)^5 - 30*a^2*cosh(x)^3 + 5*a^2*cosh(x))*sinh(x)^7 + 4*(231*a^2*co
sh(x)^6 - 315*a^2*cosh(x)^4 + 105*a^2*cosh(x)^2 - 5*a^2)*sinh(x)^6 + 15*a^
2*cosh(x)^4 + 24*(33*a^2*cosh(x)^7 - 63*a^2*cosh(x)^5 + 35*a^2*cosh(x)^3 -
 5*a^2*cosh(x))*sinh(x)^5 + 15*(33*a^2*cosh(x)^8 - 84*a^2*cosh(x)^6 + 70*a
^2*cosh(x)^4 - 20*a^2*cosh(x)^2 + a^2)*sinh(x)^4 - 6*a^2*cosh(x)^2 + 20*(1
1*a^2*cosh(x)^9 - 36*a^2*cosh(x)^7 + 42*a^2*cosh(x)^5 - 20*a^2*cosh(x)^3 +
 3*a^2*cosh(x))*sinh(x)^3 + 6*(11*a^2*cosh(x)^10 - 45*a^2*cosh(x)^8 + 70*a
^2*cosh(x)^6 - 50*a^2*cosh(x)^4 + 15*a^2*cosh(x)^2 - a^2)*sinh(x)^2 + a^2
+ 12*(a^2*cosh(x)^11 - 5*a^2*cosh(x)^9 + 10*a^2*cosh(x)^7 - 10*a^2*cosh(x)
^5 + 5*a^2*cosh(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(a)*weierstrassZeta(4, 0,
 weierstrassPInverse(4, 0, cosh(x) + sinh(x))) + sqrt(2)*(231*a^2*cosh(x)^
13 + 3003*a^2*cosh(x)*sinh(x)^12 + 231*a^2*sinh(x)^13 - 1540*a^2*cosh(x)^1
1 + 154*(117*a^2*cosh(x)^2 - 10*a^2)*sinh(x)^11 + 4367*a^2*cosh(x)^9 + 169
4*(39*a^2*cosh(x)^3 - 10*a^2*cosh(x))*sinh(x)^10 + 11*(15015*a^2*cosh(x)^4
 - 7700*a^2*cosh(x)^2 + 397*a^2)*sinh(x)^9 - 6808*a^2*cosh(x)^7 + 33*(9009
*a^2*cosh(x)^5 - 7700*a^2*cosh(x)^3 + 1191*a^2*cosh(x))*sinh(x)^8 + 4*(...
```

Sympy [F]

$$\int \left(a\mathrm{csch}^3(x)\right)^{5/2} dx = \int \left(a \operatorname{csch}^3(x)\right)^{\frac{5}{2}} dx$$

input
```
integrate((a*csch(x)**3)**(5/2),x)
```

output
```
Integral((a*csch(x)**3)**(5/2), x)
```

Maxima [F]

$$\int \left(a\mathrm{csch}^3(x)\right)^{5/2} dx = \int \left(a \operatorname{csch}(x)^3\right)^{\frac{5}{2}} dx$$

input
```
integrate((a*csch(x)^3)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*csch(x)^3)^(5/2), x)
```

Giac [F]

$$\int \left(a\mathrm{csch}^3(x)\right)^{5/2} dx = \int \left(a \operatorname{csch}(x)^3\right)^{\frac{5}{2}} dx$$

input
```
integrate((a*csch(x)^3)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*csch(x)^3)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\text{csch}^3(x)\right)^{5/2} dx = \int \left(\frac{a}{\sinh(x)^3}\right)^{5/2} dx$$

input
```
int((a/sinh(x)^3)^(5/2),x)
```

output
```
int((a/sinh(x)^3)^(5/2), x)
```

Reduce [F]

$$\int \left(a\text{csch}^3(x)\right)^{5/2} dx = \sqrt{a}\left(\int \sqrt{\text{csch}(x)}\,\text{csch}(x)^7\,dx\right)a^2$$

input
```
int((a*csch(x)^3)^(5/2),x)
```

output
```
sqrt(a)*int(sqrt(csch(x))*csch(x)**7,x)*a**2
```

3.37 $\int \left(a\text{csch}^3(x)\right)^{3/2} dx$

Optimal result

Integrand size = 10, antiderivative size = 81

$$\int \left(a\text{csch}^3(x)\right)^{3/2} dx = \frac{10}{21}a\cosh(x)\sqrt{a\text{csch}^3(x)} - \frac{2}{7}a\coth(x)\text{csch}(x)\sqrt{a\text{csch}^3(x)} + \frac{10}{21}ia\sqrt{a\text{csch}^3(x)}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{i\sinh(x)}\sinh(x)$$

output
```
10/21*a*cosh(x)*(a*csch(x)^3)^(1/2)-2/7*a*coth(x)*csch(x)*(a*csch(x)^3)^(1
/2)-10/21*I*a*(a*csch(x)^3)^(1/2)*InverseJacobiAM(-1/4*Pi+1/2*I*x,2^(1/2))
*(I*sinh(x))^(1/2)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.69

$$\int \left(a\text{csch}^3(x)\right)^{3/2} dx = -\frac{2}{21}a\sqrt{a\text{csch}^3(x)}\left(\coth(x)\left(-5+3\text{csch}^2(x)\right) - 5i\,\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),2\right)\sqrt{i\sinh(x)}\right)\sinh(x)$$

input
```
Integrate[(a*Csch[x]^3)^(3/2),x]
```

output
```
(-2*a*Sqrt[a*Csch[x]^3]*(Coth[x]*(-5 + 3*Csch[x]^2) - (5*I)*EllipticF[(Pi
- (2*I)*x)/4, 2]*Sqrt[I*Sinh[x]])*Sinh[x])/21
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.33, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4611, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a \text{csch}^3(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(i a \sec \left(\frac{\pi}{2}+i x\right)^3\right)^{3/2} dx$$

$$\Big\downarrow \text{4611}$$

$$\frac{i a \sqrt{a \text{csch}^3(x)} \int (i \text{csch}(x))^{9/2}dx}{(i \text{csch}(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i a \sqrt{a \text{csch}^3(x)} \int (-\csc(i x))^{9/2}dx}{(i \text{csch}(x))^{3/2}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{i a \sqrt{a \text{csch}^3(x)} \left(\frac{5}{7} \int (i \text{csch}(x))^{5/2}dx-\frac{2}{7} i \cosh(x) (i \text{csch}(x))^{7/2}\right)}{(i \text{csch}(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i a \sqrt{a \text{csch}^3(x)} \left(\frac{5}{7} \int (-\csc(i x))^{5/2}dx-\frac{2}{7} i \cosh(x) (i \text{csch}(x))^{7/2}\right)}{(i \text{csch}(x))^{3/2}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{ia\sqrt{a\text{csch}^3(x)}\left(\frac{5}{7}\left(\frac{1}{3}\int\sqrt{i\text{csch}(x)}dx-\frac{2}{3}i\cosh(x)(i\text{csch}(x))^{3/2}\right)-\frac{2}{7}i\cosh(x)(i\text{csch}(x))^{7/2}\right)}{(i\text{csch}(x))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{ia\sqrt{a\text{csch}^3(x)}\left(\frac{5}{7}\left(\frac{1}{3}\int\sqrt{-\csc(ix)}dx-\frac{2}{3}i\cosh(x)(i\text{csch}(x))^{3/2}\right)-\frac{2}{7}i\cosh(x)(i\text{csch}(x))^{7/2}\right)}{(i\text{csch}(x))^{3/2}}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{ia\sqrt{a\text{csch}^3(x)}\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{i\sinh(x)}\sqrt{i\text{csch}(x)}\int\frac{1}{\sqrt{i\sinh(x)}}dx-\frac{2}{3}i\cosh(x)(i\text{csch}(x))^{3/2}\right)-\frac{2}{7}i\cosh(x)(i\text{csch}(x))^{7/2}\right)}{(i\text{csch}(x))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{ia\sqrt{a\text{csch}^3(x)}\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{i\sinh(x)}\sqrt{i\text{csch}(x)}\int\frac{1}{\sqrt{\sin(ix)}}dx-\frac{2}{3}i\cosh(x)(i\text{csch}(x))^{3/2}\right)-\frac{2}{7}i\cosh(x)(i\text{csch}(x))^{7/2}\right)}{(i\text{csch}(x))^{3/2}}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{ia\sqrt{a\text{csch}^3(x)}\left(\frac{5}{7}\left(\frac{2}{3}i\sqrt{i\sinh(x)}\sqrt{i\text{csch}(x)}\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)-\frac{2}{3}i\cosh(x)(i\text{csch}(x))^{3/2}\right)-\frac{2}{7}i\cosh(x)(i\text{csch}(x\right.}{(i\text{csch}(x))^{3/2}}$$

input
```
Int[(a*Csch[x]^3)^(3/2),x]
```

output
```
(I*a*Sqrt[a*Csch[x]^3]*(((-2*I)/7)*Cosh[x]*(I*Csch[x])^(7/2) + (5*(((-2*I)
/3)*Cosh[x]*(I*Csch[x])^(3/2) + ((2*I)/3)*Sqrt[I*Csch[x]]*EllipticF[Pi/4 -
 (I/2)*x, 2]*Sqrt[I*Sinh[x]]))/7))/(I*Csch[x])^(3/2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [F]

$$\int \left(a\,\mathrm{csch}\left(x\right)^{3}\right)^{\frac{3}{2}}dx$$

input
```
int((a*csch(x)^3)^(3/2),x)
```

output
```
int((a*csch(x)^3)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(59) = 118$.

Time = 0.09 (sec) , antiderivative size = 395, normalized size of antiderivative = 4.88

$$\int \left(a\mathrm{csch}^3(x)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((a*csch(x)^3)^(3/2),x, algorithm="fricas")
```

output
```
2/21*(5*sqrt(2)*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 - 3*a*c
osh(x)^4 + 3*(5*a*cosh(x)^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 3*a*cosh(x
))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^2 + a)*sinh(
x)^2 + 6*(a*cosh(x)^5 - 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) - a)*sqrt(a)*we
ierstrassPInverse(4, 0, cosh(x) + sinh(x)) + sqrt(2)*(5*a*cosh(x)^6 + 30*a
*cosh(x)*sinh(x)^5 + 5*a*sinh(x)^6 - 17*a*cosh(x)^4 + (75*a*cosh(x)^2 - 17
*a)*sinh(x)^4 + 4*(25*a*cosh(x)^3 - 17*a*cosh(x))*sinh(x)^3 - 17*a*cosh(x)
^2 + (75*a*cosh(x)^4 - 102*a*cosh(x)^2 - 17*a)*sinh(x)^2 + 2*(15*a*cosh(x)
^5 - 34*a*cosh(x)^3 - 17*a*cosh(x))*sinh(x) + 5*a)*sqrt((a*cosh(x) + a*sin
h(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)))/(cosh(x)^6 + 6*cos
h(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 +
 4*(5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 - 6*cosh(x)^2 + 1)
*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 - 2*cosh(x)^3 + cosh(x))*sinh(x) -
 1)
```

Sympy [F]

$$\int \left(a\mathrm{csch}^3(x)\right)^{3/2} dx = \int \left(a\operatorname{csch}^3(x)\right)^{\frac{3}{2}} dx$$

input
```
integrate((a*csch(x)**3)**(3/2),x)
```

output
```
Integral((a*csch(x)**3)**(3/2), x)
```

Maxima [F]

$$\int \left(a\text{csch}^3(x)\right)^{3/2}\,dx = \int \left(a\,\text{csch}\left(x\right)^3\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*csch(x)^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*csch(x)^3)^(3/2), x)
```

Giac [F]

$$\int \left(a\text{csch}^3(x)\right)^{3/2}\,dx = \int \left(a\,\text{csch}\left(x\right)^3\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*csch(x)^3)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*csch(x)^3)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\text{csch}^3(x)\right)^{3/2}\,dx = \int \left(\frac{a}{\sinh\left(x\right)^3}\right)^{3/2}\,dx$$

input
```
int((a/sinh(x)^3)^(3/2),x)
```

output
```
int((a/sinh(x)^3)^(3/2), x)
```

Reduce [F]

$$\int \left(a\mathrm{csch}^3(x)\right)^{3/2} \, dx = \sqrt{a}\left(\int \sqrt{\mathrm{csch}\,(x)}\,\mathrm{csch}(x)^4\,dx\right)a$$

input
```
int((a*csch(x)^3)^(3/2),x)
```

output
```
sqrt(a)*int(sqrt(csch(x))*csch(x)**4,x)*a
```

3.38 $\int \sqrt{a\operatorname{csch}^3(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 56

$$\int \sqrt{a\operatorname{csch}^3(x)}\,dx = -2i\sqrt{a\operatorname{csch}^3(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)(i\sinh(x))^{3/2} - 2\cosh(x)\sqrt{a\operatorname{csch}^3(x)}\sinh(x)$$

output
```
-2*I*(a*csch(x)^3)^(1/2)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2))*(I*sinh(x)
)^(3/2)-2*cosh(x)*(a*csch(x)^3)^(1/2)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.75

$$\int \sqrt{a\operatorname{csch}^3(x)}\,dx = -2\sqrt{a\operatorname{csch}^3(x)}\left(\cosh(x)-E\left(\frac{1}{4}(\pi-2ix)\middle|2\right)\sqrt{i\sinh(x)}\right)\sinh(x)$$

input
```
Integrate[Sqrt[a*Csch[x]^3],x]
```

output
```
-2*Sqrt[a*Csch[x]^3]*(Cosh[x] - EllipticE[(Pi - (2*I)*x)/4, 2]*Sqrt[I*Sinh
[x]])*Sinh[x]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.38, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 4611, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\text{csch}^3(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{ia\sec\left(\frac{\pi}{2}+ix\right)^3}dx$$

$$\Big\downarrow \text{4611}$$

$$\frac{\sqrt{a\text{csch}^3(x)}\int (i\text{csch}(x))^{3/2}dx}{(i\text{csch}(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a\text{csch}^3(x)}\int (-\csc(ix))^{3/2}dx}{(i\text{csch}(x))^{3/2}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{a\text{csch}^3(x)}\left(-\int \frac{1}{\sqrt{i\text{csch}(x)}}dx-2i\cosh(x)\sqrt{i\text{csch}(x)}\right)}{(i\text{csch}(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a\text{csch}^3(x)}\left(-\int \frac{1}{\sqrt{-\csc(ix)}}dx-2i\cosh(x)\sqrt{i\text{csch}(x)}\right)}{(i\text{csch}(x))^{3/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\sqrt{a\text{csch}^3(x)}\left(-\frac{\int \sqrt{i\sinh(x)}dx}{\sqrt{i\sinh(x)}\sqrt{i\text{csch}(x)}}-2i\cosh(x)\sqrt{i\text{csch}(x)}\right)}{(i\text{csch}(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a\operatorname{csch}^3(x)}\left(-\frac{\int \sqrt{\sin(ix)}dx}{\sqrt{i\sinh(x)}\sqrt{i\operatorname{csch}(x)}}-2i\cosh(x)\sqrt{i\operatorname{csch}(x)}\right)}{(i\operatorname{csch}(x))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sqrt{a\operatorname{csch}^3(x)}\left(-2i\cosh(x)\sqrt{i\operatorname{csch}(x)}-\frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}|2\right)}{\sqrt{i\sinh(x)}\sqrt{i\operatorname{csch}(x)}}\right)}{(i\operatorname{csch}(x))^{3/2}}$$

input
```
Int[Sqrt[a*Csch[x]^3],x]
```

output
```
(Sqrt[a*Csch[x]^3]*((-2*I)*Cosh[x]*Sqrt[I*Csch[x]] - ((2*I)*EllipticE[Pi/4
 - (I/2)*x, 2])/(Sqrt[I*Csch[x]]*Sqrt[I*Sinh[x]])))/(I*Csch[x])^(3/2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [F]

$$\int \sqrt{a\,\mathrm{csch}\,(x)^{3}}dx$$

input
```
int((a*csch(x)^3)^(1/2),x)
```

output
```
int((a*csch(x)^3)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \sqrt{a\mathrm{csch}^3(x)}\,dx &= -2\sqrt{2}\sqrt{\frac{a\cosh(x)+a\sinh(x)}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1}}(\cosh(x)+\sinh(x)) \\ &\quad -2\sqrt{2}\sqrt{a}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cosh(x)+\sinh(x)))\end{aligned}$$

input
```
integrate((a*csch(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(2)*sqrt((a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2 - 1))*(cosh(x) + sinh(x)) - 2*sqrt(2)*sqrt(a)*weierstrassZeta(4,
0, weierstrassPInverse(4, 0, cosh(x) + sinh(x)))
```

Sympy [F]

$$\int \sqrt{a\text{csch}^3(x)}\,dx = \int \sqrt{a\,\text{csch}^3\,(x)}\,dx$$

input
```
integrate((a*csch(x)**3)**(1/2),x)
```

output
```
Integral(sqrt(a*csch(x)**3), x)
```

Maxima [F]

$$\int \sqrt{a\text{csch}^3(x)}\,dx = \int \sqrt{a\,\text{csch}\,(x)^3}\,dx$$

input
```
integrate((a*csch(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*csch(x)^3), x)
```

Giac [F]

$$\int \sqrt{a\text{csch}^3(x)}\,dx = \int \sqrt{a\,\text{csch}\,(x)^3}\,dx$$

input
```
integrate((a*csch(x)^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*csch(x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\text{csch}^3(x)}\,dx = \int \sqrt{\frac{a}{\sinh(x)^3}}\,dx$$

input
```
int((a/sinh(x)^3)^(1/2),x)
```

output
```
int((a/sinh(x)^3)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a\text{csch}^3(x)}\,dx = \sqrt{a}\left(\int \sqrt{\text{csch}(x)}\,\text{csch}(x)\,dx\right)$$

input
```
int((a*csch(x)^3)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(csch(x))*csch(x),x)
```

3.39 $\int \frac{1}{\sqrt{a\text{csch}^3(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 62

$$\int \frac{1}{\sqrt{a\text{csch}^3(x)}}\,dx = \frac{2\coth(x)}{3\sqrt{a\text{csch}^3(x)}} - \frac{2i\text{csch}^2(x)\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{i\sinh(x)}}{3\sqrt{a\text{csch}^3(x)}}$$

output
```
2/3*coth(x)/(a*csch(x)^3)^(1/2)+2/3*I*csch(x)^2*InverseJacobiAM(-1/4*Pi+1/
2*I*x,2^(1/2))*(I*sinh(x))^(1/2)/(a*csch(x)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{a\text{csch}^3(x)}}\,dx = \frac{2\left(\coth(x)+\frac{\text{csch}(x)\,\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),2\right)}{\sqrt{i\sinh(x)}}\right)}{3\sqrt{a\text{csch}^3(x)}}$$

input
```
Integrate[1/Sqrt[a*Csch[x]^3],x]
```

output
```
(2*(Coth[x] + (Csch[x]*EllipticF[(Pi - (2*I)*x)/4, 2])/Sqrt[I*Sinh[x]]))/(
3*Sqrt[a*Csch[x]^3])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 4611, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\text{csch}^3(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{ia\sec\left(\frac{\pi}{2}+ix\right)^3}}dx$$

$$\Big\downarrow \text{ 4611}$$

$$\frac{(i\text{csch}(x))^{3/2}\int \frac{1}{(i\text{csch}(x))^{3/2}}dx}{\sqrt{a\text{csch}^3(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(i\text{csch}(x))^{3/2}\int \frac{1}{(-\csc(ix))^{3/2}}dx}{\sqrt{a\text{csch}^3(x)}}$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{(i\text{csch}(x))^{3/2}\left(\frac{1}{3}\int \sqrt{i\text{csch}(x)}dx-\frac{2i\cosh(x)}{3\sqrt{i\text{csch}(x)}}\right)}{\sqrt{a\text{csch}^3(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(i\text{csch}(x))^{3/2}\left(\frac{1}{3}\int \sqrt{-\csc(ix)}dx-\frac{2i\cosh(x)}{3\sqrt{i\text{csch}(x)}}\right)}{\sqrt{a\text{csch}^3(x)}}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{(i\operatorname{csch}(x))^{3/2}\left(\frac{1}{3}\sqrt{i\sinh(x)}\sqrt{i\operatorname{csch}(x)}\int\frac{1}{\sqrt{i\sinh(x)}}dx-\frac{2i\cosh(x)}{3\sqrt{i\operatorname{csch}(x)}}\right)}{\sqrt{a\operatorname{csch}^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(i\operatorname{csch}(x))^{3/2}\left(\frac{1}{3}\sqrt{i\sinh(x)}\sqrt{i\operatorname{csch}(x)}\int\frac{1}{\sqrt{\sin(ix)}}dx-\frac{2i\cosh(x)}{3\sqrt{i\operatorname{csch}(x)}}\right)}{\sqrt{a\operatorname{csch}^3(x)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{(i\operatorname{csch}(x))^{3/2}\left(\frac{2}{3}i\sqrt{i\sinh(x)}\sqrt{i\operatorname{csch}(x)}\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)-\frac{2i\cosh(x)}{3\sqrt{i\operatorname{csch}(x)}}\right)}{\sqrt{a\operatorname{csch}^3(x)}}$$

input
```
Int[1/Sqrt[a*Csch[x]^3],x]
```

output
```
((I*Csch[x])^(3/2)*((((-2*I)/3)*Cosh[x])/Sqrt[I*Csch[x]] + ((2*I)/3)*Sqrt[
I*Csch[x]]*EllipticF[Pi/4 - (I/2)*x, 2]*Sqrt[I*Sinh[x]]))/Sqrt[a*Csch[x]^3
]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\sqrt{a\operatorname{csch}\left(x\right)^{3}}}dx$$

input
```
int(1/(a*csch(x)^3)^(1/2),x)
```

output
```
int(1/(a*csch(x)^3)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(44) = 88.

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sqrt{a\operatorname{csch}^3(x)}}\,dx = -\frac{4\sqrt{2}\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}\right)\sqrt{a}\text{weierstrassPInverse}(4,0,\cosh\left(x\right)+\sinh\left(x\right))-\sqrt{}}{6\left(a\cosh\left(x\right.\right.}$$

input
```
integrate(1/(a*csch(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(4*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(a)*weiers
trassPInverse(4, 0, cosh(x) + sinh(x)) - sqrt(2)*(cosh(x)^4 + 4*cosh(x)^3*
sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)*sqr
t((a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1))
)/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\text{csch}^3(x)}}\,dx = \int \frac{1}{\sqrt{a\operatorname{csch}^{3}{\left(x \right)}}}\,dx$$

input
```
integrate(1/(a*csch(x)**3)**(1/2),x)
```

output
```
Integral(1/sqrt(a*csch(x)**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a\text{csch}^3(x)}}\,dx = \int \frac{1}{\sqrt{a\operatorname{csch}\left(x\right)^{3}}}\,dx$$

input
```
integrate(1/(a*csch(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*csch(x)^3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a\text{csch}^3(x)}}\,dx = \int \frac{1}{\sqrt{a\,\text{csch}\,(x)^3}}\,dx$$

input
```
integrate(1/(a*csch(x)^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a*csch(x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\text{csch}^3(x)}}\,dx = \int \frac{1}{\sqrt{\frac{a}{\sinh(x)^3}}}\,dx$$

input
```
int(1/(a/sinh(x)^3)^(1/2),x)
```

output
```
int(1/(a/sinh(x)^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a\text{csch}^3(x)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\text{csch}(x)}}{\text{csch}(x)^2}dx\right)}{a}$$

input
```
int(1/(a*csch(x)^3)^(1/2),x)
```

output
```
(sqrt(a)*int(sqrt(csch(x))/csch(x)**2,x))/a
```

3.40 $\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 89

$$\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{3/2}}\,dx = -\frac{14\cosh(x)}{45a\sqrt{a\operatorname{csch}^3(x)}} + \frac{14i\operatorname{csch}(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{15a\sqrt{a\operatorname{csch}^3(x)}\sqrt{i\sinh(x)}} + \frac{2\cosh(x)\sinh^2(x)}{9a\sqrt{a\operatorname{csch}^3(x)}}$$

output
```
-14/45*cosh(x)/a/(a*csch(x)^3)^(1/2)+14/15*I*csch(x)*EllipticE(cos(1/4*Pi+
1/2*I*x),2^(1/2))/a/(a*csch(x)^3)^(1/2)/(I*sinh(x))^(1/2)+2/9*cosh(x)*sinh
(x)^2/a/(a*csch(x)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.64

$$\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{3/2}}\,dx = \frac{-33\cosh(x)+5\cosh(3x)+84\operatorname{csch}^2(x)E\left(\frac{1}{4}(\pi-2ix)\middle|2\right)\sqrt{i\sinh(x)}}{90a\sqrt{a\operatorname{csch}^3(x)}}$$

input
```
Integrate[(a*Csch[x]^3)^(-3/2),x]
```

output
```
(-33*Cosh[x] + 5*Cosh[3*x] + 84*Csch[x]^2*EllipticE[(Pi - (2*I)*x)/4, 2]*S
qrt[I*Sinh[x]])/(90*a*Sqrt[a*Csch[x]^3])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4611, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\text{csch}^3(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(ia\sec\left(\frac{\pi}{2}+ix\right)^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{4611}$$

$$-\frac{i(i\text{csch}(x))^{3/2}\int \frac{1}{(i\text{csch}(x))^{9/2}}dx}{a\sqrt{a\text{csch}^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i(i\text{csch}(x))^{3/2}\int \frac{1}{(-\csc(ix))^{9/2}}dx}{a\sqrt{a\text{csch}^3(x)}}$$

$$\Big\downarrow \text{4256}$$

$$-\frac{i(i\text{csch}(x))^{3/2}\left(\frac{7}{9}\int \frac{1}{(i\text{csch}(x))^{5/2}}dx-\frac{2i\cosh(x)}{9(i\text{csch}(x))^{7/2}}\right)}{a\sqrt{a\text{csch}^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i(i\text{csch}(x))^{3/2}\left(\frac{7}{9}\int \frac{1}{(-\csc(ix))^{5/2}}dx-\frac{2i\cosh(x)}{9(i\text{csch}(x))^{7/2}}\right)}{a\sqrt{a\text{csch}^3(x)}}$$

$$\Big\downarrow \; 4256$$

$$-\frac{i(i\text{csch}(x))^{3/2}\left(\frac{7}{9}\left(\frac{3}{5}\int\frac{1}{\sqrt{i\text{csch}(x)}}dx-\frac{2i\cosh(x)}{5(i\text{csch}(x))^{3/2}}\right)-\frac{2i\cosh(x)}{9(i\text{csch}(x))^{7/2}}\right)}{a\sqrt{a\text{csch}^3(x)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{i(i\text{csch}(x))^{3/2}\left(\frac{7}{9}\left(\frac{3}{5}\int\frac{1}{\sqrt{-\csc(ix)}}dx-\frac{2i\cosh(x)}{5(i\text{csch}(x))^{3/2}}\right)-\frac{2i\cosh(x)}{9(i\text{csch}(x))^{7/2}}\right)}{a\sqrt{a\text{csch}^3(x)}}$$

$$\Big\downarrow \; 4258$$

$$-\frac{i(i\text{csch}(x))^{3/2}\left(\frac{7}{9}\left(\frac{3\int\sqrt{i\sinh(x)}dx}{5\sqrt{i\sinh(x)}\sqrt{i\text{csch}(x)}}-\frac{2i\cosh(x)}{5(i\text{csch}(x))^{3/2}}\right)-\frac{2i\cosh(x)}{9(i\text{csch}(x))^{7/2}}\right)}{a\sqrt{a\text{csch}^3(x)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{i(i\text{csch}(x))^{3/2}\left(\frac{7}{9}\left(\frac{3\int\sqrt{\sin(ix)}dx}{5\sqrt{i\sinh(x)}\sqrt{i\text{csch}(x)}}-\frac{2i\cosh(x)}{5(i\text{csch}(x))^{3/2}}\right)-\frac{2i\cosh(x)}{9(i\text{csch}(x))^{7/2}}\right)}{a\sqrt{a\text{csch}^3(x)}}$$

$$\Big\downarrow \; 3119$$

$$-\frac{i(i\text{csch}(x))^{3/2}\left(\frac{7}{9}\left(\frac{6iE\left(\frac{\pi}{4}-\frac{ix}{2}|2\right)}{5\sqrt{i\sinh(x)}\sqrt{i\text{csch}(x)}}-\frac{2i\cosh(x)}{5(i\text{csch}(x))^{3/2}}\right)-\frac{2i\cosh(x)}{9(i\text{csch}(x))^{7/2}}\right)}{a\sqrt{a\text{csch}^3(x)}}$$

input
```
Int[(a*Csch[x]^3)^(-3/2),x]
```

output
```
((-I)*(I*Csch[x])^(3/2)*((((-2*I)/9)*Cosh[x])/(I*Csch[x])^(7/2) + (7*((((-
2*I)/5)*Cosh[x])/(I*Csch[x])^(3/2) + (((6*I)/5)*EllipticE[Pi/4 - (I/2)*x,
2])/(Sqrt[I*Csch[x]]*Sqrt[I*Sinh[x]])))/9))/(a*Sqrt[a*Csch[x]^3])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\left(a\,\mathrm{csch}\left(x\right)^{3}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(a*csch(x)^3)^(3/2),x)
```

output
```
int(1/(a*csch(x)^3)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(68) = 136$.

Time = 0.08 (sec) , antiderivative size = 407, normalized size of antiderivative = 4.57

$$\int \frac{1}{\left(a\text{csch}^3(x)\right)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/(a*csch(x)^3)^(3/2),x, algorithm="fricas")
```

output
```
-1/720*(672*sqrt(2)*(cosh(x)^5 + 5*cosh(x)^4*sinh(x) + 10*cosh(x)^3*sinh(x
)^2 + 10*cosh(x)^2*sinh(x)^3 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5)*sqrt(a)*we
ierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(x) + sinh(x))) - sqrt(2
)*(5*cosh(x)^10 + 50*cosh(x)*sinh(x)^9 + 5*sinh(x)^10 + (225*cosh(x)^2 - 4
3)*sinh(x)^8 - 43*cosh(x)^8 + 8*(75*cosh(x)^3 - 43*cosh(x))*sinh(x)^7 + 2*
(525*cosh(x)^4 - 602*cosh(x)^2 - 149)*sinh(x)^6 - 298*cosh(x)^6 + 4*(315*c
osh(x)^5 - 602*cosh(x)^3 - 447*cosh(x))*sinh(x)^5 + 2*(525*cosh(x)^6 - 150
5*cosh(x)^4 - 2235*cosh(x)^2 + 187)*sinh(x)^4 + 374*cosh(x)^4 + 8*(75*cosh
(x)^7 - 301*cosh(x)^5 - 745*cosh(x)^3 + 187*cosh(x))*sinh(x)^3 + (225*cosh
(x)^8 - 1204*cosh(x)^6 - 4470*cosh(x)^4 + 2244*cosh(x)^2 - 43)*sinh(x)^2 -
 43*cosh(x)^2 + 2*(25*cosh(x)^9 - 172*cosh(x)^7 - 894*cosh(x)^5 + 748*cosh
(x)^3 - 43*cosh(x))*sinh(x) + 5)*sqrt((a*cosh(x) + a*sinh(x))/(cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)))/(a^2*cosh(x)^5 + 5*a^2*cosh(x)^4*sin
h(x) + 10*a^2*cosh(x)^3*sinh(x)^2 + 10*a^2*cosh(x)^2*sinh(x)^3 + 5*a^2*cos
h(x)*sinh(x)^4 + a^2*sinh(x)^5)
```

Sympy [F]

$$\int \frac{1}{\left(a\text{csch}^3(x)\right)^{3/2}} \, dx = \int \frac{1}{\left(a \operatorname{csch}^{3}(x)\right)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/(a*csch(x)**3)**(3/2),x)
```

output
```
Integral((a*csch(x)**3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a\text{csch}^3(x))^{3/2}}\,dx = \int \frac{1}{(a\,\text{csch}\,(x)^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*csch(x)^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*csch(x)^3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a\text{csch}^3(x))^{3/2}}\,dx = \int \frac{1}{(a\,\text{csch}\,(x)^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*csch(x)^3)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*csch(x)^3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\text{csch}^3(x))^{3/2}}\,dx = \int \frac{1}{\left(\frac{a}{\sinh(x)^3}\right)^{3/2}}\,dx$$

input
```
int(1/(a/sinh(x)^3)^(3/2),x)
```

output
```
int(1/(a/sinh(x)^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\operatorname{csch}(x)}}{\operatorname{csch}(x)^5}dx\right)}{a^2}$$

input
```
int(1/(a*csch(x)^3)^(3/2),x)
```

output
```
(sqrt(a)*int(sqrt(csch(x))/csch(x)**5,x))/a**2
```

3.41 $\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 135

$$\begin{aligned}\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{5/2}}\,dx =& -\frac{26\coth(x)}{77a^2\sqrt{a\operatorname{csch}^3(x)}} \\ &+ \frac{26i\operatorname{csch}^2(x)\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{i\sinh(x)}}{77a^2\sqrt{a\operatorname{csch}^3(x)}} \\ &+ \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{26\cosh(x)\sinh^3(x)}{165a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{2\cosh(x)\sinh^5(x)}{15a^2\sqrt{a\operatorname{csch}^3(x)}}\end{aligned}$$

output
```
-26/77*coth(x)/a^2/(a*csch(x)^3)^(1/2)-26/77*I*csch(x)^2*InverseJacobiAM(-
1/4*Pi+1/2*I*x,2^(1/2))*(I*sinh(x))^(1/2)/a^2/(a*csch(x)^3)^(1/2)+78/385*c
osh(x)*sinh(x)/a^2/(a*csch(x)^3)^(1/2)-26/165*cosh(x)*sinh(x)^3/a^2/(a*csc
h(x)^3)^(1/2)+2/15*cosh(x)*sinh(x)^5/a^2/(a*csch(x)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.53

$$\int \frac{1}{\left(a\text{csch}^3(x)\right)^{5/2}}\,dx = \frac{\sqrt{a\text{csch}^3(x)}\sinh(x)\left(24960i\,\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),2\right)\sqrt{i\sinh(x)}-19122\sinh(2x)\right.}{73920a^3}$$

input
```
Integrate[(a*Csch[x]^3)^(-5/2),x]
```

output
```
(Sqrt[a*Csch[x]^3]*Sinh[x]*((24960*I)*EllipticF[(Pi - (2*I)*x)/4, 2]*Sqrt[
I*Sinh[x]] - 19122*Sinh[2*x] + 4406*Sinh[4*x] - 826*Sinh[6*x] + 77*Sinh[8*
x]))/(73920*a^3)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.400$, Rules used = {3042, 4611, 3042, 4256, 3042, 4256, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\text{csch}^3(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(ia\sec\left(\frac{\pi}{2}+ix\right)^3\right)^{5/2}}dx$$

$$\Big\downarrow \text{4611}$$

$$-\frac{(i\text{csch}(x))^{3/2}\int \frac{1}{(i\text{csch}(x))^{15/2}}dx}{a^2\sqrt{a\text{csch}^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(i\text{csch}(x))^{3/2}\int \frac{1}{(-\csc(ix))^{15/2}}dx}{a^2\sqrt{a\text{csch}^3(x)}}$$

$$\Big\downarrow \; 4256$$

$$-\frac{(i\text{csch}(x))^{3/2}\left(\frac{13}{15}\int \frac{1}{(i\text{csch}(x))^{11/2}}dx-\frac{2i\cosh(x)}{15(i\text{csch}(x))^{13/2}}\right)}{a^2\sqrt{a\text{csch}^3(x)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{(i\text{csch}(x))^{3/2}\left(\frac{13}{15}\int \frac{1}{(-\csc(ix))^{11/2}}dx-\frac{2i\cosh(x)}{15(i\text{csch}(x))^{13/2}}\right)}{a^2\sqrt{a\text{csch}^3(x)}}$$

$$\Big\downarrow \; 4256$$

$$-\frac{(i\text{csch}(x))^{3/2}\left(\frac{13}{15}\left(\frac{9}{11}\int \frac{1}{(i\text{csch}(x))^{7/2}}dx-\frac{2i\cosh(x)}{11(i\text{csch}(x))^{9/2}}\right)-\frac{2i\cosh(x)}{15(i\text{csch}(x))^{13/2}}\right)}{a^2\sqrt{a\text{csch}^3(x)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{(i\text{csch}(x))^{3/2}\left(\frac{13}{15}\left(\frac{9}{11}\int \frac{1}{(-\csc(ix))^{7/2}}dx-\frac{2i\cosh(x)}{11(i\text{csch}(x))^{9/2}}\right)-\frac{2i\cosh(x)}{15(i\text{csch}(x))^{13/2}}\right)}{a^2\sqrt{a\text{csch}^3(x)}}$$

$$\Big\downarrow \; 4256$$

$$-\frac{(i\text{csch}(x))^{3/2}\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\int \frac{1}{(i\text{csch}(x))^{3/2}}dx-\frac{2i\cosh(x)}{7(i\text{csch}(x))^{5/2}}\right)-\frac{2i\cosh(x)}{11(i\text{csch}(x))^{9/2}}\right)-\frac{2i\cosh(x)}{15(i\text{csch}(x))^{13/2}}\right)}{a^2\sqrt{a\text{csch}^3(x)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{(i\text{csch}(x))^{3/2}\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\int \frac{1}{(-\csc(ix))^{3/2}}dx-\frac{2i\cosh(x)}{7(i\text{csch}(x))^{5/2}}\right)-\frac{2i\cosh(x)}{11(i\text{csch}(x))^{9/2}}\right)-\frac{2i\cosh(x)}{15(i\text{csch}(x))^{13/2}}\right)}{a^2\sqrt{a\text{csch}^3(x)}}$$

$$\Big\downarrow \; 4256$$

$$-\frac{(i\text{csch}(x))^{3/2}\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\left(\frac{1}{3}\int \sqrt{i\text{csch}(x)}dx-\frac{2i\cosh(x)}{3\sqrt{i\text{csch}(x)}}\right)-\frac{2i\cosh(x)}{7(i\text{csch}(x))^{5/2}}\right)-\frac{2i\cosh(x)}{11(i\text{csch}(x))^{9/2}}\right)-\frac{2i\cosh(x)}{15(i\text{csch}(x))^{13}}\right.}{a^2\sqrt{a\text{csch}^3(x)}}$$

$\downarrow$ 3042

$$-\frac{(i\text{csch}(x))^{3/2}\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\left(\frac{1}{3}\int\sqrt{-\csc(ix)}dx-\frac{2i\cosh(x)}{3\sqrt{i\text{csch}(x)}}\right)-\frac{2i\cosh(x)}{7(i\text{csch}(x))^{5/2}}\right)-\frac{2i\cosh(x)}{11(i\text{csch}(x))^{9/2}}\right)-\frac{2i\cosh(x)}{15(i\text{csch}(x))^{1}}\right.}{a^2\sqrt{a\text{csch}^3(x)}}$$

$\downarrow$ 4258

$$-\frac{(i\text{csch}(x))^{3/2}\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{i\sinh(x)}\sqrt{i\text{csch}(x)}\int\frac{1}{\sqrt{i\sinh(x)}}dx-\frac{2i\cosh(x)}{3\sqrt{i\text{csch}(x)}}\right)-\frac{2i\cosh(x)}{7(i\text{csch}(x))^{5/2}}\right)-\frac{2i\cosh(x)}{11(i\text{csch}(x))}\right.\right.}{a^2\sqrt{a\text{csch}^3(x)}}$$

$\downarrow$ 3042

$$-\frac{(i\text{csch}(x))^{3/2}\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{i\sinh(x)}\sqrt{i\text{csch}(x)}\int\frac{1}{\sqrt{\sin(ix)}}dx-\frac{2i\cosh(x)}{3\sqrt{i\text{csch}(x)}}\right)-\frac{2i\cosh(x)}{7(i\text{csch}(x))^{5/2}}\right)-\frac{2i\cosh(x)}{11(i\text{csch}(x))^{9/}}\right.\right.}{a^2\sqrt{a\text{csch}^3(x)}}$$

$\downarrow$ 3120

$$-\frac{(i\text{csch}(x))^{3/2}\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\left(\frac{2}{3}i\sqrt{i\sinh(x)}\sqrt{i\text{csch}(x)}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)-\frac{2i\cosh(x)}{3\sqrt{i\text{csch}(x)}}\right)-\frac{2i\cosh(x)}{7(i\text{csch}(x))^{5/2}}\right)-\frac{}{11(}\right.\right.}{a^2\sqrt{a\text{csch}^3(x)}}$$

input
```
Int[(a*Csch[x]^3)^(-5/2),x]
```

output
```
-(((I*Csch[x])^(3/2)*((((-2*I)/15)*Cosh[x])/(I*Csch[x])^(13/2) + (13*((((-
2*I)/11)*Cosh[x])/(I*Csch[x])^(9/2) + (9*((((-2*I)/7)*Cosh[x])/(I*Csch[x])
^(5/2) + (5*((((-2*I)/3)*Cosh[x])/Sqrt[I*Csch[x]] + ((2*I)/3)*Sqrt[I*Csch[
x]]*EllipticF[Pi/4 - (I/2)*x, 2]*Sqrt[I*Sinh[x]]))/7))/11))/15))/(a^2*Sqrt
[a*Csch[x]^3]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\left(a\,\mathrm{csch}\left(x\right)^{3}\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(a*csch(x)^3)^(5/2),x)
```

output
```
int(1/(a*csch(x)^3)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 718 vs. $2(105) = 210$.

Time = 0.10 (sec) , antiderivative size = 718, normalized size of antiderivative = 5.32

$$\int \frac{1}{\left(a\text{csch}^3(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate(1/(a*csch(x)^3)^(5/2),x, algorithm="fricas")
```

output

```
1/147840*(49920*sqrt(2)*(cosh(x)^8 + 8*cosh(x)^7*sinh(x) + 28*cosh(x)^6*si
nh(x)^2 + 56*cosh(x)^5*sinh(x)^3 + 70*cosh(x)^4*sinh(x)^4 + 56*cosh(x)^3*s
inh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8)*sqrt(
a)*weierstrassPInverse(4, 0, cosh(x) + sinh(x)) + sqrt(2)*(77*cosh(x)^16 +
 1232*cosh(x)*sinh(x)^15 + 77*sinh(x)^16 + 14*(660*cosh(x)^2 - 59)*sinh(x)
^14 - 826*cosh(x)^14 + 196*(220*cosh(x)^3 - 59*cosh(x))*sinh(x)^13 + 2*(70
070*cosh(x)^4 - 37583*cosh(x)^2 + 2203)*sinh(x)^12 + 4406*cosh(x)^12 + 8*(
42042*cosh(x)^5 - 37583*cosh(x)^3 + 6609*cosh(x))*sinh(x)^11 + 2*(308308*c
osh(x)^6 - 413413*cosh(x)^4 + 145398*cosh(x)^2 - 9561)*sinh(x)^10 - 19122*
cosh(x)^10 + 4*(220220*cosh(x)^7 - 413413*cosh(x)^5 + 242330*cosh(x)^3 - 4
7805*cosh(x))*sinh(x)^9 + 6*(165165*cosh(x)^8 - 413413*cosh(x)^6 + 363495*
cosh(x)^4 - 143415*cosh(x)^2)*sinh(x)^8 + 16*(55055*cosh(x)^9 - 177177*cos
h(x)^7 + 218097*cosh(x)^5 - 143415*cosh(x)^3)*sinh(x)^7 + 2*(308308*cosh(x
)^10 - 1240239*cosh(x)^8 + 2035572*cosh(x)^6 - 2007810*cosh(x)^4 + 9561)*s
inh(x)^6 + 19122*cosh(x)^6 + 4*(84084*cosh(x)^11 - 413413*cosh(x)^9 + 8723
88*cosh(x)^7 - 1204686*cosh(x)^5 + 28683*cosh(x))*sinh(x)^5 + 2*(70070*cos
h(x)^12 - 413413*cosh(x)^10 + 1090485*cosh(x)^8 - 2007810*cosh(x)^6 + 1434
15*cosh(x)^2 - 2203)*sinh(x)^4 - 4406*cosh(x)^4 + 8*(5390*cosh(x)^13 - 375
83*cosh(x)^11 + 121165*cosh(x)^9 - 286830*cosh(x)^7 + 47805*cosh(x)^3 - 22
03*cosh(x))*sinh(x)^3 + 2*(4620*cosh(x)^14 - 37583*cosh(x)^12 + 145398*...
```

Sympy [F]

$$\int \frac{1}{\left(a\text{csch}^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\operatorname{csch}^{3}{\left(x \right)}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*csch(x)**3)**(5/2),x)
```

output
```
Integral((a*csch(x)**3)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a\text{csch}^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\operatorname{csch}\left(x\right)^{3}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*csch(x)^3)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*csch(x)^3)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a\text{csch}^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\operatorname{csch}\left(x\right)^{3}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*csch(x)^3)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*csch(x)^3)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(\frac{a}{\sinh(x)^3}\right)^{5/2}}\,dx$$

input
```
int(1/(a/sinh(x)^3)^(5/2),x)
```

output
```
int(1/(a/sinh(x)^3)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\operatorname{csch}(x)}}{\operatorname{csch}(x)^8}dx\right)}{a^3}$$

input
```
int(1/(a*csch(x)^3)^(5/2),x)
```

output
```
(sqrt(a)*int(sqrt(csch(x))/csch(x)**8,x))/a**3
```

3.42 $\int \left(a\operatorname{csch}^4(x)\right)^{7/2} dx$

Optimal result

Integrand size = 10, antiderivative size = 164

$$\begin{aligned}\int \left(a\operatorname{csch}^4(x)\right)^{7/2} dx &= 2a^3\cosh^2(x)\coth(x)\sqrt{a\operatorname{csch}^4(x)} \\ &- 3a^3\cosh^2(x)\coth^3(x)\sqrt{a\operatorname{csch}^4(x)} + \frac{20}{7}a^3\cosh^2(x)\coth^5(x)\sqrt{a\operatorname{csch}^4(x)} \\ &- \frac{5}{3}a^3\cosh^2(x)\coth^7(x)\sqrt{a\operatorname{csch}^4(x)} + \frac{6}{11}a^3\cosh^2(x)\coth^9(x)\sqrt{a\operatorname{csch}^4(x)} \\ &- \frac{1}{13}a^3\cosh^2(x)\coth^{11}(x)\sqrt{a\operatorname{csch}^4(x)} - a^3\cosh(x)\sqrt{a\operatorname{csch}^4(x)}\sinh(x)\end{aligned}$$

output
```
2*a^3*cosh(x)^2*coth(x)*(a*csch(x)^4)^(1/2)-3*a^3*cosh(x)^2*coth(x)^3*(a*c
sch(x)^4)^(1/2)+20/7*a^3*cosh(x)^2*coth(x)^5*(a*csch(x)^4)^(1/2)-5/3*a^3*c
osh(x)^2*coth(x)^7*(a*csch(x)^4)^(1/2)+6/11*a^3*cosh(x)^2*coth(x)^9*(a*csc
h(x)^4)^(1/2)-1/13*a^3*cosh(x)^2*coth(x)^11*(a*csch(x)^4)^(1/2)-a^3*cosh(x
)*(a*csch(x)^4)^(1/2)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.36

$$\int \left(a \text{csch}^4(x)\right)^{7/2} \, dx =$$
$$-\frac{a^3 \cosh(x) \sqrt{a \text{csch}^4(x)} \left(1024 - 512 \text{csch}^2(x) + 384 \text{csch}^4(x) - 320 \text{csch}^6(x) + 280 \text{csch}^8(x) - 252 \text{csch}^{10}(x)\right.}{3003}$$

input
```
Integrate[(a*Csch[x]^4)^(7/2),x]
```

output
```
-1/3003*(a^3*Cosh[x]*Sqrt[a*Csch[x]^4]*(1024 - 512*Csch[x]^2 + 384*Csch[x]
^4 - 320*Csch[x]^6 + 280*Csch[x]^8 - 252*Csch[x]^10 + 231*Csch[x]^12)*Sinh
[x])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.51, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3042, 4611, 25, 3042, 25, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a \text{csch}^4(x)\right)^{7/2} \, dx$$
$$\Big\downarrow \; 3042$$
$$\int \left(a \sec\left(\frac{\pi}{2} + ix\right)^4\right)^{7/2} dx$$
$$\Big\downarrow \; 4611$$
$$-a^3 \sinh^2(x) \sqrt{a \text{csch}^4(x)} \int -\text{csch}^{14}(x) dx$$
$$\Big\downarrow \; 25$$

$$a^3 \sinh^2(x)\sqrt{a\text{csch}^4(x)}\int \text{csch}^{14}(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a^3 \sinh^2(x)\sqrt{a\text{csch}^4(x)}\int -\csc(ix)^{14}dx$$

$$\Big\downarrow \text{25}$$

$$-a^3 \sinh^2(x)\sqrt{a\text{csch}^4(x)}\int \csc(ix)^{14}dx$$

$$\Big\downarrow \text{4254}$$

$$-ia^3 \sinh^2(x)\sqrt{a\text{csch}^4(x)}\int \left(\coth^{12}(x)-6\coth^{10}(x)+15\coth^8(x)-20\coth^6(x)+15\coth^4(x)-6\coth^2(x)+\right.$$

$$\Big\downarrow \text{2009}$$

$$-ia^3 \sinh^2(x)\left(-\frac{1}{13}i\coth^{13}(x)+\frac{6}{11}i\coth^{11}(x)-\frac{5}{3}i\coth^9(x)+\frac{20}{7}i\coth^7(x)-3i\coth^5(x)+2i\coth^3(x)-i\,\text{co}\right.$$

input
```
Int[(a*Csch[x]^4)^(7/2),x]
```

output
```
(-I)*a^3*((-I)*Coth[x] + (2*I)*Coth[x]^3 - (3*I)*Coth[x]^5 + ((20*I)/7)*Co
th[x]^7 - ((5*I)/3)*Coth[x]^9 + ((6*I)/11)*Coth[x]^11 - (I/13)*Coth[x]^13)
*Sqrt[a*Csch[x]^4]*Sinh[x]^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.44

method	result	size
risch	$-\frac{2048 a^{3} {\mathrm e}^{-2 x} \sqrt{\frac{a \,{\mathrm e}^{4 x}}{\left({\mathrm e}^{2 x}-1\right)^{4}}}\, \left(1716 \,{\mathrm e}^{12 x}-1287 \,{\mathrm e}^{10 x}+715 \,{\mathrm e}^{8 x}-286 \,{\mathrm e}^{6 x}+78 \,{\mathrm e}^{4 x}-13 \,{\mathrm e}^{2 x}+1\right)}{3003 \left({\mathrm e}^{2 x}-1\right)^{11}}$	72

input
```
int((a*csch(x)^4)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2048/3003*a^3*exp(-2*x)/(exp(2*x)-1)^11*(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)
*(1716*exp(12*x)-1287*exp(10*x)+715*exp(8*x)-286*exp(6*x)+78*exp(4*x)-13*e
xp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2825 vs. 2(142) = 284.

Time = 0.20 (sec) , antiderivative size = 2825, normalized size of antiderivative = 17.23

$$\int \left(a\text{csch}^4(x)\right)^{7/2} dx = \text{Too large to display}$$

input
```
integrate((a*csch(x)^4)^(7/2),x, algorithm="fricas")
```

output
```
-2048/3003*(1716*a^3*cosh(x)^12 - 1287*a^3*cosh(x)^10 + 1716*(a^3*e^(4*x)
- 2*a^3*e^(2*x) + a^3)*sinh(x)^12 + 20592*(a^3*cosh(x)*e^(4*x) - 2*a^3*cos
h(x)*e^(2*x) + a^3*cosh(x))*sinh(x)^11 + 715*a^3*cosh(x)^8 + 1287*(88*a^3*
cosh(x)^2 - a^3 + (88*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2*(88*a^3*cosh(x)^2 -
 a^3)*e^(2*x))*sinh(x)^10 + 4290*(88*a^3*cosh(x)^3 - 3*a^3*cosh(x) + (88*a
^3*cosh(x)^3 - 3*a^3*cosh(x))*e^(4*x) - 2*(88*a^3*cosh(x)^3 - 3*a^3*cosh(x
))*e^(2*x))*sinh(x)^9 - 286*a^3*cosh(x)^6 + 715*(1188*a^3*cosh(x)^4 - 81*a
^3*cosh(x)^2 + a^3 + (1188*a^3*cosh(x)^4 - 81*a^3*cosh(x)^2 + a^3)*e^(4*x)
 - 2*(1188*a^3*cosh(x)^4 - 81*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^8 + 11
44*(1188*a^3*cosh(x)^5 - 135*a^3*cosh(x)^3 + 5*a^3*cosh(x) + (1188*a^3*cos
h(x)^5 - 135*a^3*cosh(x)^3 + 5*a^3*cosh(x))*e^(4*x) - 2*(1188*a^3*cosh(x)^
5 - 135*a^3*cosh(x)^3 + 5*a^3*cosh(x))*e^(2*x))*sinh(x)^7 + 78*a^3*cosh(x)
^4 + 286*(5544*a^3*cosh(x)^6 - 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 - a^3
+ (5544*a^3*cosh(x)^6 - 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 - a^3)*e^(4*x
) - 2*(5544*a^3*cosh(x)^6 - 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 - a^3)*e^
(2*x))*sinh(x)^6 + 572*(2376*a^3*cosh(x)^7 - 567*a^3*cosh(x)^5 + 70*a^3*co
sh(x)^3 - 3*a^3*cosh(x) + (2376*a^3*cosh(x)^7 - 567*a^3*cosh(x)^5 + 70*a^3
*cosh(x)^3 - 3*a^3*cosh(x))*e^(4*x) - 2*(2376*a^3*cosh(x)^7 - 567*a^3*cosh
(x)^5 + 70*a^3*cosh(x)^3 - 3*a^3*cosh(x))*e^(2*x))*sinh(x)^5 - 13*a^3*cosh
(x)^2 + 26*(32670*a^3*cosh(x)^8 - 10395*a^3*cosh(x)^6 + 1925*a^3*cosh(x...
```

Sympy [F]

$$\int \left(a\text{csch}^4(x)\right)^{7/2} \, dx = \int \left(a \operatorname{csch}^{4}{\left(x \right)}\right)^{\frac{7}{2}}\, dx$$

input
```
integrate((a*csch(x)**4)**(7/2),x)
```

output
```
Integral((a*csch(x)**4)**(7/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(142) = 284$.

Time = 0.13 (sec) , antiderivative size = 620, normalized size of antiderivative = 3.78

$$\int \left(a\text{csch}^4(x)\right)^{7/2} dx = \text{Too large to display}$$

input
```
integrate((a*csch(x)^4)^(7/2),x, algorithm="maxima")
```

output
```
-2048/231*a^(7/2)*e^(-2*x)/(13*e^(-2*x) - 78*e^(-4*x) + 286*e^(-6*x) - 715
*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) + 1716*e^(-14*x) - 1287*e^(-16
*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22*x) - 13*e^(-24*x) + e^(-26
*x) - 1) + 4096/77*a^(7/2)*e^(-4*x)/(13*e^(-2*x) - 78*e^(-4*x) + 286*e^(-6
*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) + 1716*e^(-14*x) - 12
87*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22*x) - 13*e^(-24*x)
 + e^(-26*x) - 1) - 4096/21*a^(7/2)*e^(-6*x)/(13*e^(-2*x) - 78*e^(-4*x) +
286*e^(-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) + 1716*e^(-1
4*x) - 1287*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22*x) - 13*
e^(-24*x) + e^(-26*x) - 1) + 10240/21*a^(7/2)*e^(-8*x)/(13*e^(-2*x) - 78*e
^(-4*x) + 286*e^(-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) +
1716*e^(-14*x) - 1287*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-2
2*x) - 13*e^(-24*x) + e^(-26*x) - 1) - 6144/7*a^(7/2)*e^(-10*x)/(13*e^(-2*
x) - 78*e^(-4*x) + 286*e^(-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(
-12*x) + 1716*e^(-14*x) - 1287*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) +
 78*e^(-22*x) - 13*e^(-24*x) + e^(-26*x) - 1) + 8192/7*a^(7/2)*e^(-12*x)/(
13*e^(-2*x) - 78*e^(-4*x) + 286*e^(-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) -
 1716*e^(-12*x) + 1716*e^(-14*x) - 1287*e^(-16*x) + 715*e^(-18*x) - 286*e^
(-20*x) + 78*e^(-22*x) - 13*e^(-24*x) + e^(-26*x) - 1) + 2048/3003*a^(7/2)
/(13*e^(-2*x) - 78*e^(-4*x) + 286*e^(-6*x) - 715*e^(-8*x) + 1287*e^(-10...
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.31

$$\int \left(a\operatorname{csch}^4(x)\right)^{7/2} dx = -\frac{2048 a^{\frac{7}{2}} \left(1716 e^{(12 x)} - 1287 e^{(10 x)} + 715 e^{(8 x)} - 286 e^{(6 x)} + 78 e^{(4 x)} - 13 e^{(2 x)} + 1\right)}{3003 \left(e^{(2 x)} - 1\right)^{13}}$$

input
```
integrate((a*csch(x)^4)^(7/2),x, algorithm="giac")
```

output
```
-2048/3003*a^(7/2)*(1716*e^(12*x) - 1287*e^(10*x) + 715*e^(8*x) - 286*e^(6
*x) + 78*e^(4*x) - 13*e^(2*x) + 1)/(e^(2*x) - 1)^13
```

Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.04

$$\int \left(a\operatorname{csch}^4(x)\right)^{7/2} dx = \text{Too large to display}$$

input
```
int((a/sinh(x)^4)^(7/2),x)
```

output
```
- (2048*a^3*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4*exp(2*x) -
4*exp(6*x) + exp(8*x) + 1))/(7*(exp(2*x) - 1)^7*(exp(2*x) - 2*exp(4*x) + e
xp(6*x))) - (1536*a^3*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4*e
xp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/((exp(2*x) - 1)^8*(exp(2*x) - 2*exp(
4*x) + exp(6*x))) - (10240*a^3*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4
*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/(3*(exp(2*x) - 1)^9*(exp(2*
x) - 2*exp(4*x) + exp(6*x))) - (4096*a^3*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2
)*(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/((exp(2*x) - 1)^1
0*(exp(2*x) - 2*exp(4*x) + exp(6*x))) - (30720*a^3*(a/(exp(-x)/2 - exp(x)/
2)^4)^(1/2)*(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/(11*(ex
p(2*x) - 1)^11*(exp(2*x) - 2*exp(4*x) + exp(6*x))) - (1024*a^3*(a/(exp(-x)
/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) +
 1))/((exp(2*x) - 1)^12*(exp(2*x) - 2*exp(4*x) + exp(6*x))) - (2048*a^3*(a
/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + e
xp(8*x) + 1))/(13*(exp(2*x) - 1)^13*(exp(2*x) - 2*exp(4*x) + exp(6*x)))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.89

$$\int \left(a\mathrm{csch}^4(x)\right)^{7/2} dx = \frac{2048\sqrt{a}\,a^3(-1716e^{12x}+12\ldots}{3003e^{26x}-39039e^{24x}+234234e^{22x}-858858e^{20x}+2147145e^{18x}-3864861e^{16x}+\ldots}$$

input
```
int((a*csch(x)^4)^(7/2),x)
```

output
```
(2048*sqrt(a)*a**3*( - 1716*e**(12*x) + 1287*e**(10*x) - 715*e**(8*x) + 28
6*e**(6*x) - 78*e**(4*x) + 13*e**(2*x) - 1))/(3003*(e**(26*x) - 13*e**(24*
x) + 78*e**(22*x) - 286*e**(20*x) + 715*e**(18*x) - 1287*e**(16*x) + 1716*
e**(14*x) - 1716*e**(12*x) + 1287*e**(10*x) - 715*e**(8*x) + 286*e**(6*x)
- 78*e**(4*x) + 13*e**(2*x) - 1))
```

3.43 $\int \left(a\operatorname{csch}^4(x)\right)^{5/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 118

$$\int \left(a\operatorname{csch}^4(x)\right)^{5/2}\,dx=\frac{4}{3}a^2\cosh^2(x)\coth(x)\sqrt{a\operatorname{csch}^4(x)}-\frac{6}{5}a^2\cosh^2(x)\coth^3(x)\sqrt{a\operatorname{csch}^4(x)}+\frac{4}{7}a^2\cosh^2(x)\coth^5(x)\sqrt{a\operatorname{csch}^4(x)}-\frac{1}{9}a^2\cosh^2(x)\coth^7(x)\sqrt{a\operatorname{csch}^4(x)}-a^2\cosh(x)\sqrt{a\operatorname{csch}^4(x)}\sinh(x)$$

output
```
4/3*a^2*cosh(x)^2*coth(x)*(a*csch(x)^4)^(1/2)-6/5*a^2*cosh(x)^2*coth(x)^3*
(a*csch(x)^4)^(1/2)+4/7*a^2*cosh(x)^2*coth(x)^5*(a*csch(x)^4)^(1/2)-1/9*a^
2*cosh(x)^2*coth(x)^7*(a*csch(x)^4)^(1/2)-a^2*cosh(x)*(a*csch(x)^4)^(1/2)*
sinh(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.40

$$\int \left(a\operatorname{csch}^4(x)\right)^{5/2}\,dx=-\frac{1}{315}a^2\cosh(x)\sqrt{a\operatorname{csch}^4(x)}\left(128-64\operatorname{csch}^2(x)+48\operatorname{csch}^4(x)-40\operatorname{csch}^6(x)+35\operatorname{csch}^8(x)\right)\sinh(x)$$

input
```
Integrate[(a*Csch[x]^4)^(5/2),x]
```

output
```
-1/315*(a^2*Cosh[x]*Sqrt[a*Csch[x]^4]*(128 - 64*Csch[x]^2 + 48*Csch[x]^4 -
 40*Csch[x]^6 + 35*Csch[x]^8)*Sinh[x])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.58, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3042, 4611, 25, 3042, 25, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\text{csch}^4(x)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sec\left(\frac{\pi}{2}+ix\right)^4\right)^{5/2} dx$$

$$\Big\downarrow \text{4611}$$

$$-a^2\sinh^2(x)\sqrt{a\text{csch}^4(x)}\int -\text{csch}^{10}(x)dx$$

$$\Big\downarrow \text{25}$$

$$a^2\sinh^2(x)\sqrt{a\text{csch}^4(x)}\int \text{csch}^{10}(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a^2\sinh^2(x)\sqrt{a\text{csch}^4(x)}\int -\csc(ix)^{10}dx$$

$$\Big\downarrow \text{25}$$

$$-a^2\sinh^2(x)\sqrt{a\text{csch}^4(x)}\int \csc(ix)^{10}dx$$

$$\Big\downarrow \text{4254}$$

$$-ia^2\sinh^2(x)\sqrt{a\text{csch}^4(x)}\int\left(\coth^8(x)-4\coth^6(x)+6\coth^4(x)-4\coth^2(x)+1\right)d(-i\coth(x))$$

$$\Big\downarrow \text{2009}$$

$$-ia^2\sinh^2(x)\left(-\frac{1}{9}i\coth^9(x)+\frac{4}{7}i\coth^7(x)-\frac{6}{5}i\coth^5(x)+\frac{4}{3}i\coth^3(x)-i\coth(x)\right)\sqrt{a\text{csch}^4(x)}$$

input
```
Int[(a*Csch[x]^4)^(5/2),x]
```

output
```
(-I)*a^2*((-I)*Coth[x] + ((4*I)/3)*Coth[x]^3 - ((6*I)/5)*Coth[x]^5 + ((4*I
)/7)*Coth[x]^7 - (I/9)*Coth[x]^9)*Sqrt[a*Csch[x]^4]*Sinh[x]^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.51

method	result	size
risch	$-\frac{256a^{2}\mathrm{e}^{-2x}\sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(\mathrm{e}^{2x}-1\right)^{4}}}\,\left(126\,\mathrm{e}^{8x}-84\,\mathrm{e}^{6x}+36\,\mathrm{e}^{4x}-9\,\mathrm{e}^{2x}+1\right)}{315\left(\mathrm{e}^{2x}-1\right)^{7}}$	60

input
```
int((a*csch(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-256/315*a^2*exp(-2*x)/(exp(2*x)-1)^7*(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*(1
26*exp(8*x)-84*exp(6*x)+36*exp(4*x)-9*exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1493 vs. $2(100) = 200$.

Time = 0.12 (sec) , antiderivative size = 1493, normalized size of antiderivative = 12.65

$$\int \left(a\mathrm{csch}^4(x)\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a*csch(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
-256/315*(126*a^2*cosh(x)^8 + 126*(a^2*e^(4*x) - 2*a^2*e^(2*x) + a^2)*sinh
(x)^8 - 84*a^2*cosh(x)^6 + 1008*(a^2*cosh(x)*e^(4*x) - 2*a^2*cosh(x)*e^(2*
x) + a^2*cosh(x))*sinh(x)^7 + 84*(42*a^2*cosh(x)^2 - a^2 + (42*a^2*cosh(x)
^2 - a^2)*e^(4*x) - 2*(42*a^2*cosh(x)^2 - a^2)*e^(2*x))*sinh(x)^6 + 36*a^2
*cosh(x)^4 + 504*(14*a^2*cosh(x)^3 - a^2*cosh(x) + (14*a^2*cosh(x)^3 - a^2
*cosh(x))*e^(4*x) - 2*(14*a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x))*sinh(x)^5
 + 36*(245*a^2*cosh(x)^4 - 35*a^2*cosh(x)^2 + a^2 + (245*a^2*cosh(x)^4 - 35
*a^2*cosh(x)^2 + a^2)*e^(4*x) - 2*(245*a^2*cosh(x)^4 - 35*a^2*cosh(x)^2 +
a^2)*e^(2*x))*sinh(x)^4 - 9*a^2*cosh(x)^2 + 48*(147*a^2*cosh(x)^5 - 35*a^2
*cosh(x)^3 + 3*a^2*cosh(x) + (147*a^2*cosh(x)^5 - 35*a^2*cosh(x)^3 + 3*a^2
*cosh(x))*e^(4*x) - 2*(147*a^2*cosh(x)^5 - 35*a^2*cosh(x)^3 + 3*a^2*cosh(x
))*e^(2*x))*sinh(x)^3 + 9*(392*a^2*cosh(x)^6 - 140*a^2*cosh(x)^4 + 24*a^2*
cosh(x)^2 - a^2 + (392*a^2*cosh(x)^6 - 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^
2 - a^2)*e^(4*x) - 2*(392*a^2*cosh(x)^6 - 140*a^2*cosh(x)^4 + 24*a^2*cosh(
x)^2 - a^2)*e^(2*x))*sinh(x)^2 + a^2 + (126*a^2*cosh(x)^8 - 84*a^2*cosh(x)
^6 + 36*a^2*cosh(x)^4 - 9*a^2*cosh(x)^2 + a^2)*e^(4*x) - 2*(126*a^2*cosh(x
)^8 - 84*a^2*cosh(x)^6 + 36*a^2*cosh(x)^4 - 9*a^2*cosh(x)^2 + a^2)*e^(2*x)
 + 18*(56*a^2*cosh(x)^7 - 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 - a^2*cosh(x)
 + (56*a^2*cosh(x)^7 - 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 - a^2*cosh(x))*e
^(4*x) - 2*(56*a^2*cosh(x)^7 - 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 - a^2...
```

Sympy [F]

$$\int \left(a\text{csch}^4(x)\right)^{5/2}\,dx = \int \left(a\operatorname{csch}^4\left(x\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*csch(x)**4)**(5/2),x)
```

output
```
Integral((a*csch(x)**4)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(100) = 200$.

Time = 0.13 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.73

$$\int \left(a\operatorname{csch}^4(x)\right)^{5/2}\,dx =$$

$$-\frac{256\,a^{\frac{5}{2}}e^{(-2\,x)}}{35\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-1\cdots}}$$
$$+\frac{1024\,a^{\frac{5}{2}}e^{(-4\,x)}}{35\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-1\cdots}}$$
$$-\frac{1024\,a^{\frac{5}{2}}e^{(-6\,x)}}{15\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-1\cdots}}$$
$$+\frac{512\,a^{\frac{5}{2}}e^{(-8\,x)}}{5\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-18\cdots}}$$
$$+\frac{256\,a^{\frac{5}{2}}}{315\,(9\,e^{(-2\,x)} - 36\,e^{(-4\,x)} + 84\,e^{(-6\,x)} - 126\,e^{(-8\,x)} + 126\,e^{(-10\,x)} - 84\,e^{(-12\,x)} + 36\,e^{(-14\,x)} - 9\,e^{(-16\,x)} + e^{(-\cdots}}$$

input
```
integrate((a*csch(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
-256/35*a^(5/2)*e^(-2*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(
-8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18
*x) - 1) + 1024/35*a^(5/2)*e^(-4*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x
) - 126*e^(-8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*
x) + e^(-18*x) - 1) - 1024/15*a^(5/2)*e^(-6*x)/(9*e^(-2*x) - 36*e^(-4*x) +
 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x)
- 9*e^(-16*x) + e^(-18*x) - 1) + 512/5*a^(5/2)*e^(-8*x)/(9*e^(-2*x) - 36*e
^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e
^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 256/315*a^(5/2)/(9*e^(-2*x) - 36
*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36
*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.33

$$\int \left(a\text{csch}^4(x)\right)^{5/2}\,dx = -\frac{256\,a^{\frac{5}{2}}\left(126\,e^{(8\,x)} - 84\,e^{(6\,x)} + 36\,e^{(4\,x)} - 9\,e^{(2\,x)} + 1\right)}{315\left(e^{(2\,x)} - 1\right)^9}$$

input
```
integrate((a*csch(x)^4)^(5/2),x, algorithm="giac")
```

output
```
-256/315*a^(5/2)*(126*e^(8*x) - 84*e^(6*x) + 36*e^(4*x) - 9*e^(2*x) + 1)/(
e^(2*x) - 1)^9
```

Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 356, normalized size of antiderivative = 3.02

$$\begin{aligned}\int \left(a\text{csch}^4(x)\right)^{5/2}\,dx = &-\frac{128\,a^2\sqrt{\frac{a}{\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^4}}\left(6\,e^{4\,x} - 4\,e^{2\,x} - 4\,e^{6\,x} + e^{8\,x} + 1\right)}{5\left(e^{2\,x}-1\right)^5\left(e^{2\,x} - 2\,e^{4\,x} + e^{6\,x}\right)}\\ &- \frac{256\,a^2\sqrt{\frac{a}{\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^4}}\left(6\,e^{4\,x} - 4\,e^{2\,x} - 4\,e^{6\,x} + e^{8\,x} + 1\right)}{3\left(e^{2\,x}-1\right)^6\left(e^{2\,x} - 2\,e^{4\,x} + e^{6\,x}\right)}\\ &- \frac{768\,a^2\sqrt{\frac{a}{\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^4}}\left(6\,e^{4\,x} - 4\,e^{2\,x} - 4\,e^{6\,x} + e^{8\,x} + 1\right)}{7\left(e^{2\,x}-1\right)^7\left(e^{2\,x} - 2\,e^{4\,x} + e^{6\,x}\right)}\\ &- \frac{64\,a^2\sqrt{\frac{a}{\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^4}}\left(6\,e^{4\,x} - 4\,e^{2\,x} - 4\,e^{6\,x} + e^{8\,x} + 1\right)}{\left(e^{2\,x}-1\right)^8\left(e^{2\,x} - 2\,e^{4\,x} + e^{6\,x}\right)}\\ &- \frac{128\,a^2\sqrt{\frac{a}{\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^4}}\left(6\,e^{4\,x} - 4\,e^{2\,x} - 4\,e^{6\,x} + e^{8\,x} + 1\right)}{9\left(e^{2\,x}-1\right)^9\left(e^{2\,x} - 2\,e^{4\,x} + e^{6\,x}\right)}\end{aligned}$$

input
```
int((a/sinh(x)^4)^(5/2),x)
```

output
```
- (128*a^2*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4*exp(2*x) - 4
*exp(6*x) + exp(8*x) + 1))/(5*(exp(2*x) - 1)^5*(exp(2*x) - 2*exp(4*x) + ex
p(6*x))) - (256*a^2*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4*exp
(2*x) - 4*exp(6*x) + exp(8*x) + 1))/(3*(exp(2*x) - 1)^6*(exp(2*x) - 2*exp(
4*x) + exp(6*x))) - (768*a^2*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x
) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/(7*(exp(2*x) - 1)^7*(exp(2*x)
 - 2*exp(4*x) + exp(6*x))) - (64*a^2*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6
*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/((exp(2*x) - 1)^8*(ex
p(2*x) - 2*exp(4*x) + exp(6*x))) - (128*a^2*(a/(exp(-x)/2 - exp(x)/2)^4)^(
1/2)*(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/(9*(exp(2*x) -
 1)^9*(exp(2*x) - 2*exp(4*x) + exp(6*x)))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.88

$$\int \left(a\text{csch}^4(x)\right)^{5/2} dx = \frac{256\sqrt{a}\,a^2(-126e^{8x}+84e^{6x}-36e^{4x}+9e^{2x}-1)}{315e^{18x}-2835e^{16x}+11340e^{14x}-26460e^{12x}+39690e^{10x}-39690e^{8x}+26460e^{6x}-}$$

input
```
int((a*csch(x)^4)^(5/2),x)
```

output
```
(256*sqrt(a)*a**2*( - 126*e**(8*x) + 84*e**(6*x) - 36*e**(4*x) + 9*e**(2*x
) - 1))/(315*(e**(18*x) - 9*e**(16*x) + 36*e**(14*x) - 84*e**(12*x) + 126*
e**(10*x) - 126*e**(8*x) + 84*e**(6*x) - 36*e**(4*x) + 9*e**(2*x) - 1))
```

3.44 $\int (a\mathrm{csch}^4(x))^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 62

$$\int \left(a\,\mathrm{csch}^4(x)\right)^{3/2}\,dx = \frac{2}{3}a\cosh^2(x)\coth(x)\sqrt{a\,\mathrm{csch}^4(x)} - \frac{1}{5}a\cosh^2(x)\coth^3(x)\sqrt{a\,\mathrm{csch}^4(x)} - a\cosh(x)\sqrt{a\,\mathrm{csch}^4(x)}\sinh(x)$$

output
```
2/3*a*cosh(x)^2*coth(x)*(a*csch(x)^4)^(1/2)-1/5*a*cosh(x)^2*coth(x)^3*(a*c
sch(x)^4)^(1/2)-a*cosh(x)*(a*csch(x)^4)^(1/2)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.53

$$\int \left(a\,\mathrm{csch}^4(x)\right)^{3/2}\,dx = -\frac{1}{15}a\cosh(x)\sqrt{a\,\mathrm{csch}^4(x)}\left(8-4\mathrm{csch}^2(x)+3\mathrm{csch}^4(x)\right)\sinh(x)$$

input
```
Integrate[(a*Csch[x]^4)^(3/2),x]
```

output
```
-1/15*(a*Cosh[x]*Sqrt[a*Csch[x]^4]*(8 - 4*Csch[x]^2 + 3*Csch[x]^4)*Sinh[x]
)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3042, 4611, 25, 3042, 25, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\text{csch}^4(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a \sec\left(\frac{\pi}{2}+ix\right)^4\right)^{3/2} dx$$

$$\Big\downarrow \text{4611}$$

$$-a\sinh^2(x)\sqrt{a\text{csch}^4(x)}\int -\text{csch}^6(x)dx$$

$$\Big\downarrow \text{25}$$

$$a\sinh^2(x)\sqrt{a\text{csch}^4(x)}\int \text{csch}^6(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a\sinh^2(x)\sqrt{a\text{csch}^4(x)}\int -\csc(ix)^6dx$$

$$\Big\downarrow \text{25}$$

$$-a\sinh^2(x)\sqrt{a\text{csch}^4(x)}\int \csc(ix)^6dx$$

$$\Big\downarrow \text{4254}$$

$$-ia\sinh^2(x)\sqrt{a\text{csch}^4(x)}\int \left(\coth^4(x)-2\coth^2(x)+1\right)d(-i\coth(x))$$

$$\Big\downarrow \text{2009}$$

$$-ia\sinh^2(x)\left(-\frac{1}{5}i\coth^5(x)+\frac{2}{3}i\coth^3(x)-i\coth(x)\right)\sqrt{a\text{csch}^4(x)}$$

input
```
Int[(a*Csch[x]^4)^(3/2),x]
```

output
```
(-I)*a*((-I)*Coth[x] + ((2*I)/3)*Coth[x]^3 - (I/5)*Coth[x]^5)*Sqrt[a*Csch[
x]^4]*Sinh[x]^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{16 a \,{\mathrm e}^{-2 x} \sqrt{\frac{a \,{\mathrm e}^{4 x}}{\left({\mathrm e}^{2 x}-1\right)^{4}}}\, \left(10 \,{\mathrm e}^{4 x}-5 \,{\mathrm e}^{2 x}+1\right)}{15 \left({\mathrm e}^{2 x}-1\right)^{3}}$	46

input
```
int((a*csch(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-16/15*a*exp(-2*x)/(exp(2*x)-1)^3*(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*(10*ex
p(4*x)-5*exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(52) = 104$.

Time = 0.09 (sec) , antiderivative size = 529, normalized size of antiderivative = 8.53

$$\int \left(a\text{csch}^4(x)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((a*csch(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
-16/15*(10*a*cosh(x)^4 + 10*(a*e^(4*x) - 2*a*e^(2*x) + a)*sinh(x)^4 + 40*(
a*cosh(x)*e^(4*x) - 2*a*cosh(x)*e^(2*x) + a*cosh(x))*sinh(x)^3 - 5*a*cosh(
x)^2 + 5*(12*a*cosh(x)^2 + (12*a*cosh(x)^2 - a)*e^(4*x) - 2*(12*a*cosh(x)^
2 - a)*e^(2*x) - a)*sinh(x)^2 + (10*a*cosh(x)^4 - 5*a*cosh(x)^2 + a)*e^(4*
x) - 2*(10*a*cosh(x)^4 - 5*a*cosh(x)^2 + a)*e^(2*x) + 10*(4*a*cosh(x)^3 -
a*cosh(x) + (4*a*cosh(x)^3 - a*cosh(x))*e^(4*x) - 2*(4*a*cosh(x)^3 - a*cos
h(x))*e^(2*x))*sinh(x) + a)*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^
(2*x) + 1))*e^(2*x)/(10*cosh(x)*e^(2*x)*sinh(x)^9 + e^(2*x)*sinh(x)^10 + 5
*(9*cosh(x)^2 - 1)*e^(2*x)*sinh(x)^8 + 40*(3*cosh(x)^3 - cosh(x))*e^(2*x)*
sinh(x)^7 + 10*(21*cosh(x)^4 - 14*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^6 + 4*(63
*cosh(x)^5 - 70*cosh(x)^3 + 15*cosh(x))*e^(2*x)*sinh(x)^5 + 10*(21*cosh(x)
^6 - 35*cosh(x)^4 + 15*cosh(x)^2 - 1)*e^(2*x)*sinh(x)^4 + 40*(3*cosh(x)^7
- 7*cosh(x)^5 + 5*cosh(x)^3 - cosh(x))*e^(2*x)*sinh(x)^3 + 5*(9*cosh(x)^8
- 28*cosh(x)^6 + 30*cosh(x)^4 - 12*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 10*(
cosh(x)^9 - 4*cosh(x)^7 + 6*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*e^(2*x)*sin
h(x) + (cosh(x)^10 - 5*cosh(x)^8 + 10*cosh(x)^6 - 10*cosh(x)^4 + 5*cosh(x)
^2 - 1)*e^(2*x))
```

Sympy [F]

$$\int \left(a\text{csch}^4(x)\right)^{3/2} dx = \int \left(a\operatorname{csch}^4(x)\right)^{\frac{3}{2}} dx$$

input
```
integrate((a*csch(x)**4)**(3/2),x)
```

output
```
Integral((a*csch(x)**4)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(52) = 104.

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.94

$$\int \left(a\text{csch}^4(x)\right)^{3/2} dx = -\frac{16\,a^{\frac{3}{2}}e^{(-2\,x)}}{3\,(5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1)} + \frac{32\,a^{\frac{3}{2}}e^{(-4\,x)}}{3\,(5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1)} + \frac{16\,a^{\frac{3}{2}}}{15\,(5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1)}$$

input
```
integrate((a*csch(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
-16/3*a^(3/2)*e^(-2*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x
) + e^(-10*x) - 1) + 32/3*a^(3/2)*e^(-4*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*
e^(-6*x) - 5*e^(-8*x) + e^(-10*x) - 1) + 16/15*a^(3/2)/(5*e^(-2*x) - 10*e^
(-4*x) + 10*e^(-6*x) - 5*e^(-8*x) + e^(-10*x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.44

$$\int \left(a\text{csch}^4(x)\right)^{3/2}\,dx = -\frac{16\,a^{\frac{3}{2}}\left(10\,e^{(4\,x)}-5\,e^{(2\,x)}+1\right)}{15\left(e^{(2\,x)}-1\right)^5}$$

input
```
integrate((a*csch(x)^4)^(3/2),x, algorithm="giac")
```

output
```
-16/15*a^(3/2)*(10*e^(4*x) - 5*e^(2*x) + 1)/(e^(2*x) - 1)^5
```

Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\int \left(a\text{csch}^4(x)\right)^{3/2}\,dx = -\frac{4\,a\,\mathrm{e}^{-2\,x}\,\sqrt{\frac{a}{{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)}^4}}\,\left(10\,\mathrm{e}^{4\,x}-5\,\mathrm{e}^{2\,x}+1\right)}{15\,{\left(\mathrm{e}^{2\,x}-1\right)}^3}$$

input
```
int((a/sinh(x)^4)^(3/2),x)
```

output
```
-(4*a*exp(-2*x)*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(10*exp(4*x) - 5*exp(2*
x) + 1))/(15*(exp(2*x) - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \left(a\text{csch}^4(x)\right)^{3/2}\,dx = \frac{16\sqrt{a}\,a(-10e^{4x}+5e^{2x}-1)}{15e^{10x}-75e^{8x}+150e^{6x}-150e^{4x}+75e^{2x}-15}$$

input
```
int((a*csch(x)^4)^(3/2),x)
```

output
```
(16*sqrt(a)*a*( - 10*e**(4*x) + 5*e**(2*x) - 1))/(15*(e**(10*x) - 5*e**(8*
x) + 10*e**(6*x) - 10*e**(4*x) + 5*e**(2*x) - 1))
```

3.45 $\int \sqrt{a \text{csch}^4(x)}\, dx$

Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \sqrt{a\text{csch}^4(x)}\, dx = -\cosh(x)\sqrt{a\text{csch}^4(x)}\sinh(x)$$

output
```
-cosh(x)*(a*csch(x)^4)^(1/2)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{a\text{csch}^4(x)}\, dx = -\cosh(x)\sqrt{a\text{csch}^4(x)}\sinh(x)$$

input
```
Integrate[Sqrt[a*Csch[x]^4],x]
```

output
```
-(Cosh[x]*Sqrt[a*Csch[x]^4]*Sinh[x])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3042, 4611, 25, 3042, 25, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a \text{csch}^4(x)} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a \sec \left(\frac{\pi}{2}+i x\right)^4} dx$$

$$\Big\downarrow \text{4611}$$

$$\sinh^2(x) \left(-\sqrt{a \text{csch}^4(x)}\right) \int -\text{csch}^2(x)dx$$

$$\Big\downarrow \text{25}$$

$$\sinh^2(x)\sqrt{a \text{csch}^4(x)} \int \text{csch}^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\sinh^2(x)\sqrt{a \text{csch}^4(x)} \int -\csc(ix)^2dx$$

$$\Big\downarrow \text{25}$$

$$\sinh^2(x) \left(-\sqrt{a \text{csch}^4(x)}\right) \int \csc(ix)^2dx$$

$$\Big\downarrow \text{4254}$$

$$-i\sinh^2(x)\sqrt{a \text{csch}^4(x)} \int 1d(-i\coth(x))$$

$$\Big\downarrow \text{24}$$

$$\sinh(x)(-\cosh(x))\sqrt{a \text{csch}^4(x)}$$

input
```
Int[Sqrt[a*Csch[x]^4],x]
```

output
```
-(Cosh[x]*Sqrt[a*Csch[x]^4]*Sinh[x])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

method	result	size
risch	$-2\sqrt{\frac{a\,\mathrm{e}^{4x}}{(\mathrm{e}^{2x}-1)^4}}\,\mathrm{e}^{-2x}(\mathrm{e}^{2x}-1)$	29

input
```
int((a*csch(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*exp(-2*x)*(exp(2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \sqrt{a\text{csch}^4(x)}\,dx = -\frac{2\sqrt{\frac{a}{e^{(8x)}-4e^{(6x)}+6e^{(4x)}-4e^{(2x)}+1}}\left(e^{(4x)}-2e^{(2x)}+1\right)e^{(2x)}}{2\cosh(x)e^{(2x)}\sinh(x)+e^{(2x)}\sinh(x)^2+\left(\cosh(x)^2-1\right)e^{(2x)}}$$

input
```
integrate((a*csch(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1))*(e^(4*x) - 2*
e^(2*x) + 1)*e^(2*x)/(2*cosh(x)*e^(2*x)*sinh(x) + e^(2*x)*sinh(x)^2 + (cos
h(x)^2 - 1)*e^(2*x))
```

Sympy [F]

$$\int \sqrt{a\text{csch}^4(x)}\,dx = \int \sqrt{a\,\text{csch}^4(x)}\,dx$$

input
```
integrate((a*csch(x)**4)**(1/2),x)
```

output
```
Integral(sqrt(a*csch(x)**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \sqrt{a\text{csch}^4(x)}\,dx = \frac{2\sqrt{a}}{e^{(-2x)}-1}$$

input
```
integrate((a*csch(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(a)/(e^(-2*x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \sqrt{a\operatorname{csch}^4(x)}\,dx = -\frac{2\sqrt{a}}{e^{(2x)}-1}$$

input
```
integrate((a*csch(x)^4)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(a)/(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 71, normalized size of antiderivative = 4.44

$$\int \sqrt{a\operatorname{csch}^4(x)}\,dx = -\frac{\sqrt{a}\,\sqrt{\frac{1}{\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^4}}\,\left(3\,e^{4\,x}-2\,e^{2\,x}-2\,e^{6\,x}+\frac{e^{8\,x}}{2}+\frac{1}{2}\right)}{\left(e^{2\,x}-1\right)\,\left(e^{2\,x}-2\,e^{4\,x}+e^{6\,x}\right)}$$

input
```
int((a/sinh(x)^4)^(1/2),x)
```

output
```
-(a^(1/2)*(1/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(3*exp(4*x) - 2*exp(2*x) - 2*
exp(6*x) + exp(8*x)/2 + 1/2))/((exp(2*x) - 1)*(exp(2*x) - 2*exp(4*x) + exp
(6*x)))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \sqrt{a\operatorname{csch}^4(x)}\,dx = -\frac{2e^{2x}\sqrt{a}}{e^{2x}-1}$$

input
```
int((a*csch(x)^4)^(1/2),x)
```

output
```
( - 2*e**(2*x)*sqrt(a))/(e**(2*x) - 1)
```

3.46 $\int \frac{1}{\sqrt{a\operatorname{csch}^4(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \frac{1}{\sqrt{a\operatorname{csch}^4(x)}}\,dx=\frac{\coth(x)}{2\sqrt{a\operatorname{csch}^4(x)}}-\frac{x\operatorname{csch}^2(x)}{2\sqrt{a\operatorname{csch}^4(x)}}$$

output
```
1/2*coth(x)/(a*csch(x)^4)^(1/2)-1/2*x*csch(x)^2/(a*csch(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{a\operatorname{csch}^4(x)}}\,dx=\frac{\coth(x)-x\operatorname{csch}^2(x)}{2\sqrt{a\operatorname{csch}^4(x)}}$$

input
```
Integrate[1/Sqrt[a*Csch[x]^4],x]
```

output
```
(Coth[x] - x*Csch[x]^2)/(2*Sqrt[a*Csch[x]^4])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3042, 4611, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\text{csch}^4(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\sec\left(\frac{\pi}{2}+ix\right)^4}}dx$$

$$\Big\downarrow \text{4611}$$

$$-\frac{\text{csch}^2(x)\int -\sinh^2(x)dx}{\sqrt{a\text{csch}^4(x)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\text{csch}^2(x)\int \sinh^2(x)dx}{\sqrt{a\text{csch}^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{csch}^2(x)\int -\sin(ix)^2dx}{\sqrt{a\text{csch}^4(x)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\text{csch}^2(x)\int \sin(ix)^2dx}{\sqrt{a\text{csch}^4(x)}}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{\text{csch}^2(x)\left(\frac{\int 1dx}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{\sqrt{a\text{csch}^4(x)}}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\operatorname{csch}^2(x)\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)}{\sqrt{a\operatorname{csch}^4(x)}}$$

input
```
Int[1/Sqrt[a*Csch[x]^4],x]
```

output
```
-((Csch[x]^2*(x/2 - (Cosh[x]*Sinh[x])/2))/Sqrt[a*Csch[x]^4])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(28) = 56.

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.47

method	result	size
risch	$-\frac{e^{2x}x}{2\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^{4}}}\left(e^{2x}-1\right)^{2}}+\frac{e^{4x}}{8\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^{4}}}\left(e^{2x}-1\right)^{2}}-\frac{1}{8\left(e^{2x}-1\right)^{2}\sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^{4}}}}$	89

input
```
int(1/(a*csch(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)/(exp(2*x)-1)^2*exp(2*x)*x+1/8/(a*ex
p(4*x)/(exp(2*x)-1)^4)^(1/2)/(exp(2*x)-1)^2*exp(4*x)-1/8/(exp(2*x)-1)^2/(a
*exp(4*x)/(exp(2*x)-1)^4)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(28) = 56.

Time = 0.09 (sec) , antiderivative size = 253, normalized size of antiderivative = 7.03

$$\int \frac{1}{\sqrt{a\text{csch}^4(x)}}\,dx$$
$$= \frac{\left(\left(e^{(4x)}-2\,e^{(2x)}+1\right)\sinh(x)^4+\cosh(x)^4+4\left(\cosh(x)\,e^{(4x)}-2\,\cosh(x)\,e^{(2x)}+\cosh(x)\right)\sinh(x)^3-\cdots}{\cdots}$$

input
```
integrate(1/(a*csch(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
1/8*((e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x)
- 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 - 4*x*cosh(x)^2 + 2*(3*cosh(x)^2
+ (3*cosh(x)^2 - 2*x)*e^(4*x) - 2*(3*cosh(x)^2 - 2*x)*e^(2*x) - 2*x)*sinh(
x)^2 + (cosh(x)^4 - 4*x*cosh(x)^2 - 1)*e^(4*x) - 2*(cosh(x)^4 - 4*x*cosh(x
)^2 - 1)*e^(2*x) + 4*(cosh(x)^3 - 2*x*cosh(x) + (cosh(x)^3 - 2*x*cosh(x))*
e^(4*x) - 2*(cosh(x)^3 - 2*x*cosh(x))*e^(2*x))*sinh(x) - 1)*sqrt(a/(e^(8*x
) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1))*e^(2*x)/(a*cosh(x)^2*e^(2*x) +
 2*a*cosh(x)*e^(2*x)*sinh(x) + a*e^(2*x)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\text{csch}^4(x)}}\,dx = \int \frac{1}{\sqrt{a\operatorname{csch}^4(x)}}\,dx$$

input
```
integrate(1/(a*csch(x)**4)**(1/2),x)
```

output
```
Integral(1/sqrt(a*csch(x)**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \frac{1}{\sqrt{a\text{csch}^4(x)}}\,dx = -\frac{\left(e^{(-4x)} - 1\right)e^{(2x)}}{8\sqrt{a}} - \frac{x}{2\sqrt{a}}$$

input
```
integrate(1/(a*csch(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*(e^(-4*x) - 1)*e^(2*x)/sqrt(a) - 1/2*x/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt{a\text{csch}^4(x)}}\,dx = \frac{\left(2\,e^{(2\,x)}-1\right)e^{(-2\,x)}-4\,x+e^{(2\,x)}}{8\,\sqrt{a}}$$

input
```
integrate(1/(a*csch(x)^4)^(1/2),x, algorithm="giac")
```

output
```
1/8*((2*e^(2*x) - 1)*e^(-2*x) - 4*x + e^(2*x))/sqrt(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\text{csch}^4(x)}}\,dx = \int \frac{1}{\sqrt{\frac{a}{\sinh(x)^4}}}\,dx$$

input
```
int(1/(a/sinh(x)^4)^(1/2),x)
```

output
```
int(1/(a/sinh(x)^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{a\text{csch}^4(x)}}\,dx = \frac{\sqrt{a}\,(e^{4x}-4e^{2x}x-1)}{8e^{2x}a}$$

input
```
int(1/(a*csch(x)^4)^(1/2),x)
```

output
```
(sqrt(a)*(e**(4*x) - 4*e**(2*x)*x - 1))/(8*e**(2*x)*a)
```

3.47 $\int \frac{1}{\left(a\mathbf{csch}^4(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 86

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{3/2}}\,dx = \frac{5\coth(x)}{16a\sqrt{a\operatorname{csch}^4(x)}} - \frac{5x\operatorname{csch}^2(x)}{16a\sqrt{a\operatorname{csch}^4(x)}} - \frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\operatorname{csch}^4(x)}} + \frac{\cosh(x)\sinh^3(x)}{6a\sqrt{a\operatorname{csch}^4(x)}}$$

output
```
5/16*coth(x)/a/(a*csch(x)^4)^(1/2)-5/16*x*csch(x)^2/a/(a*csch(x)^4)^(1/2)-
5/24*cosh(x)*sinh(x)/a/(a*csch(x)^4)^(1/2)+1/6*cosh(x)*sinh(x)^3/a/(a*csch
(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.44

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{3/2}}\,dx = \frac{\operatorname{csch}^6(x)(-60x + 45\sinh(2x) - 9\sinh(4x) + \sinh(6x))}{192\left(a\operatorname{csch}^4(x)\right)^{3/2}}$$

input
```
Integrate[(a*Csch[x]^4)^(-3/2),x]
```

output
```
(Csch[x]^6*(-60*x + 45*Sinh[2*x] - 9*Sinh[4*x] + Sinh[6*x]))/(192*(a*Csch[
x]^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.73, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {3042, 4611, 25, 3042, 25, 3115, 3042, 3115, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sec\left(\frac{\pi}{2}+ix\right)^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{4611}$$

$$-\frac{\operatorname{csch}^2(x)\int -\sinh^6(x)dx}{a\sqrt{a\operatorname{csch}^4(x)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\operatorname{csch}^2(x)\int \sinh^6(x)dx}{a\sqrt{a\operatorname{csch}^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\operatorname{csch}^2(x)\int -\sin(ix)^6dx}{a\sqrt{a\operatorname{csch}^4(x)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\operatorname{csch}^2(x)\int \sin(ix)^6dx}{a\sqrt{a\operatorname{csch}^4(x)}}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{\operatorname{csch}^2(x)\left(\frac{5}{6}\int \sinh^4(x)dx-\frac{1}{6}\sinh^5(x)\cosh(x)\right)}{a\sqrt{a\operatorname{csch}^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\operatorname{csch}^2(x)\left(-\frac{1}{6}\sinh^5(x)\cosh(x)+\frac{5}{6}\int \sin(ix)^4dx\right)}{a\sqrt{a\operatorname{csch}^4(x)}}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{\operatorname{csch}^2(x)\left(\frac{5}{6}\left(\frac{3}{4}\int -\sinh^2(x)dx+\frac{1}{4}\sinh^3(x)\cosh(x)\right)-\frac{1}{6}\sinh^5(x)\cosh(x)\right)}{a\sqrt{a\operatorname{csch}^4(x)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\operatorname{csch}^2(x)\left(\frac{5}{6}\left(\frac{1}{4}\sinh^3(x)\cosh(x)-\frac{3}{4}\int \sinh^2(x)dx\right)-\frac{1}{6}\sinh^5(x)\cosh(x)\right)}{a\sqrt{a\operatorname{csch}^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\operatorname{csch}^2(x)\left(-\frac{1}{6}\sinh^5(x)\cosh(x)+\frac{5}{6}\left(\frac{1}{4}\sinh^3(x)\cosh(x)-\frac{3}{4}\int -\sin(ix)^2dx\right)\right)}{a\sqrt{a\operatorname{csch}^4(x)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\operatorname{csch}^2(x)\left(-\frac{1}{6}\sinh^5(x)\cosh(x)+\frac{5}{6}\left(\frac{1}{4}\sinh^3(x)\cosh(x)+\frac{3}{4}\int \sin(ix)^2dx\right)\right)}{a\sqrt{a\operatorname{csch}^4(x)}}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{\operatorname{csch}^2(x)\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)+\frac{1}{4}\sinh^3(x)\cosh(x)\right)-\frac{1}{6}\sinh^5(x)\cosh(x)\right)}{a\sqrt{a\operatorname{csch}^4(x)}}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\operatorname{csch}^2(x)\left(\frac{5}{6}\left(\frac{1}{4}\sinh^3(x)\cosh(x)+\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)\right)-\frac{1}{6}\sinh^5(x)\cosh(x)\right)}{a\sqrt{a\operatorname{csch}^4(x)}}$$

input
```
Int[(a*Csch[x]^4)^(-3/2),x]
```

output
```
-((Csch[x]^2*(-1/6*(Cosh[x]*Sinh[x]^5) + (5*((Cosh[x]*Sinh[x]^3)/4 + (3*(x
/2 - (Cosh[x]*Sinh[x])/2))/4))/6))/(a*Sqrt[a*Csch[x]^4]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(70) = 140$.

Time = 0.12 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.67

method	result
risch	$-\frac{5\,e^{2x} x}{16 a \left(e^{2x}-1\right)^{2} \sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^{4}}}}+\frac{e^{8x}}{384 a \left(e^{2x}-1\right)^{2} \sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^{4}}}}-\frac{3\,e^{6x}}{128 a \left(e^{2x}-1\right)^{2} \sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^{4}}}}+\frac{15\,e^{4x}}{128 a \left(e^{2x}-1\right)^{2} \sqrt{\frac{a\,e^{4x}}{\left(e^{2x}-1\right)^{4}}}}-$ 128

input
```
int(1/(a*csch(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-5/16/a*exp(2*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*x+1/384/
a*exp(8*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-3/128/a*exp(6*
x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)+15/128/a*exp(4*x)/(exp
(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-15/128/(a*exp(4*x)/(exp(2*x)-
1)^4)^(1/2)/(exp(2*x)-1)^2/a+3/128/a*exp(-2*x)/(exp(2*x)-1)^2/(a*exp(4*x)/
(exp(2*x)-1)^4)^(1/2)-1/384/a*exp(-4*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*
x)-1)^4)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1141 vs. $2(70) = 140$.

Time = 0.10 (sec) , antiderivative size = 1141, normalized size of antiderivative = 13.27

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*csch(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
1/384*((e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^12 + cosh(x)^12 + 12*(cosh(x)*e^(
4*x) - 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^11 + 3*(22*cosh(x)^2 + (22*cos
h(x)^2 - 3)*e^(4*x) - 2*(22*cosh(x)^2 - 3)*e^(2*x) - 3)*sinh(x)^10 - 9*cos
h(x)^10 + 10*(22*cosh(x)^3 + (22*cosh(x)^3 - 9*cosh(x))*e^(4*x) - 2*(22*co
sh(x)^3 - 9*cosh(x))*e^(2*x) - 9*cosh(x))*sinh(x)^9 + 45*(11*cosh(x)^4 - 9
*cosh(x)^2 + (11*cosh(x)^4 - 9*cosh(x)^2 + 1)*e^(4*x) - 2*(11*cosh(x)^4 -
9*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^8 + 45*cosh(x)^8 + 72*(11*cosh(x)^5
- 15*cosh(x)^3 + (11*cosh(x)^5 - 15*cosh(x)^3 + 5*cosh(x))*e^(4*x) - 2*(11
*cosh(x)^5 - 15*cosh(x)^3 + 5*cosh(x))*e^(2*x) + 5*cosh(x))*sinh(x)^7 - 12
0*x*cosh(x)^6 + 6*(154*cosh(x)^6 - 315*cosh(x)^4 + 210*cosh(x)^2 + (154*co
sh(x)^6 - 315*cosh(x)^4 + 210*cosh(x)^2 - 20*x)*e^(4*x) - 2*(154*cosh(x)^6
 - 315*cosh(x)^4 + 210*cosh(x)^2 - 20*x)*e^(2*x) - 20*x)*sinh(x)^6 + 36*(2
2*cosh(x)^7 - 63*cosh(x)^5 + 70*cosh(x)^3 - 20*x*cosh(x) + (22*cosh(x)^7 -
 63*cosh(x)^5 + 70*cosh(x)^3 - 20*x*cosh(x))*e^(4*x) - 2*(22*cosh(x)^7 - 6
3*cosh(x)^5 + 70*cosh(x)^3 - 20*x*cosh(x))*e^(2*x))*sinh(x)^5 + 45*(11*cos
h(x)^8 - 42*cosh(x)^6 + 70*cosh(x)^4 - 40*x*cosh(x)^2 + (11*cosh(x)^8 - 42
*cosh(x)^6 + 70*cosh(x)^4 - 40*x*cosh(x)^2 - 1)*e^(4*x) - 2*(11*cosh(x)^8
- 42*cosh(x)^6 + 70*cosh(x)^4 - 40*x*cosh(x)^2 - 1)*e^(2*x) - 1)*sinh(x)^4
 - 45*cosh(x)^4 + 20*(11*cosh(x)^9 - 54*cosh(x)^7 + 126*cosh(x)^5 - 120*x*
cosh(x)^3 + (11*cosh(x)^9 - 54*cosh(x)^7 + 126*cosh(x)^5 - 120*x*cosh(x...
```

Sympy [F]

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{3/2}}\, dx = \int \frac{1}{\left(a \operatorname{csch}^{4}{\left(x \right)}\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate(1/(a*csch(x)**4)**(3/2),x)
```

output
```
Integral((a*csch(x)**4)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.53

$$\int \frac{1}{\left(\operatorname{acsch}^4(x)\right)^{3/2}}\,dx = -\frac{\left(9\,e^{(-2\,x)} - 45\,e^{(-4\,x)} + 45\,e^{(-8\,x)} - 9\,e^{(-10\,x)} + e^{(-12\,x)} - 1\right)e^{(6\,x)}}{384\,a^{\frac{3}{2}}} - \frac{5\,x}{16\,a^{\frac{3}{2}}}$$

input
```
integrate(1/(a*csch(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
-1/384*(9*e^(-2*x) - 45*e^(-4*x) + 45*e^(-8*x) - 9*e^(-10*x) + e^(-12*x) -
 1)*e^(6*x)/a^(3/2) - 5/16*x/a^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.58

$$\int \frac{1}{\left(\operatorname{acsch}^4(x)\right)^{3/2}}\,dx = \frac{\left(110\,e^{(6\,x)} - 45\,e^{(4\,x)} + 9\,e^{(2\,x)} - 1\right)e^{(-6\,x)} - 120\,x + e^{(6\,x)} - 9\,e^{(4\,x)} + 45\,e^{(2\,x)}}{384\,a^{\frac{3}{2}}}$$

input
```
integrate(1/(a*csch(x)^4)^(3/2),x, algorithm="giac")
```

output
```
1/384*((110*e^(6*x) - 45*e^(4*x) + 9*e^(2*x) - 1)*e^(-6*x) - 120*x + e^(6*
x) - 9*e^(4*x) + 45*e^(2*x))/a^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(\operatorname{acsch}^4(x)\right)^{3/2}}\,dx = \int \frac{1}{{\left(\frac{a}{{\sinh\left(x\right)}^4}\right)}^{3/2}}\,dx$$

input
```
int(1/(a/sinh(x)^4)^(3/2),x)
```

output
```
int(1/(a/sinh(x)^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.66

$$\int \frac{1}{\left(a\text{csch}^4(x)\right)^{3/2}}\,dx = \frac{\sqrt{a}\,(e^{12x} - 9e^{10x} + 45e^{8x} - 120e^{6x}x - 45e^{4x} + 9e^{2x} - 1)}{384e^{6x}a^2}$$

input
```
int(1/(a*csch(x)^4)^(3/2),x)
```

output
```
(sqrt(a)*(e**(12*x) - 9*e**(10*x) + 45*e**(8*x) - 120*e**(6*x)*x - 45*e**(
4*x) + 9*e**(2*x) - 1))/(384*e**(6*x)*a**2)
```

3.48 $\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 132

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{5/2}}\,dx = \frac{63\coth(x)}{256a^2\sqrt{a\operatorname{csch}^4(x)}} - \frac{63x\operatorname{csch}^2(x)}{256a^2\sqrt{a\operatorname{csch}^4(x)}} - \frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\operatorname{csch}^4(x)}} + \frac{21\cosh(x)\sinh^3(x)}{160a^2\sqrt{a\operatorname{csch}^4(x)}} - \frac{9\cosh(x)\sinh^5(x)}{80a^2\sqrt{a\operatorname{csch}^4(x)}} + \frac{\cosh(x)\sinh^7(x)}{10a^2\sqrt{a\operatorname{csch}^4(x)}}$$

output
```
63/256*coth(x)/a^2/(a*csch(x)^4)^(1/2)-63/256*x*csch(x)^2/a^2/(a*csch(x)^4
)^(1/2)-21/128*cosh(x)*sinh(x)/a^2/(a*csch(x)^4)^(1/2)+21/160*cosh(x)*sinh
(x)^3/a^2/(a*csch(x)^4)^(1/2)-9/80*cosh(x)*sinh(x)^5/a^2/(a*csch(x)^4)^(1/
2)+1/10*cosh(x)*sinh(x)^7/a^2/(a*csch(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.42

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{5/2}}\,dx = \frac{\sqrt{a\operatorname{csch}^4(x)}\sinh^2(x)(-2520x + 2100\sinh(2x) - 600\sinh(4x) + 150\sinh(6x) - 25}{10240a^3}$$

input
```
Integrate[(a*Csch[x]^4)^(-5/2),x]
```

output
```
(Sqrt[a*Csch[x]^4]*Sinh[x]^2*(-2520*x + 2100*Sinh[2*x] - 600*Sinh[4*x] + 1
50*Sinh[6*x] - 25*Sinh[8*x] + 2*Sinh[10*x]))/(10240*a^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.70, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 1.900$, Rules used = {3042, 4611, 25, 3042, 25, 3115, 3042, 3115, 25, 3042, 25, 3115, 3042, 3115, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sec\left(\frac{\pi}{2}+ix\right)^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{4611}$$

$$-\frac{\operatorname{csch}^2(x)\int -\sinh^{10}(x)dx}{a^2\sqrt{a\operatorname{csch}^4(x)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\operatorname{csch}^2(x)\int \sinh^{10}(x)dx}{a^2\sqrt{a\operatorname{csch}^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\operatorname{csch}^2(x)\int -\sin(ix)^{10}dx}{a^2\sqrt{a\operatorname{csch}^4(x)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\operatorname{csch}^2(x)\int \sin(ix)^{10}dx}{a^2\sqrt{a\operatorname{csch}^4(x)}}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{\operatorname{csch}^2(x)\left(\frac{9}{10}\int \sinh^8(x)dx-\frac{1}{10}\sinh^9(x)\cosh(x)\right)}{a^2\sqrt{a\operatorname{csch}^4(x)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\operatorname{csch}^2(x)\left(-\frac{1}{10}\sinh^9(x)\cosh(x)+\frac{9}{10}\int \sin(ix)^8dx\right)}{a^2\sqrt{a\operatorname{csch}^4(x)}}$$

$$\Big\downarrow \; 3115$$

$$-\frac{\operatorname{csch}^2(x)\left(\frac{9}{10}\left(\frac{7}{8}\int -\sinh^6(x)dx+\frac{1}{8}\sinh^7(x)\cosh(x)\right)-\frac{1}{10}\sinh^9(x)\cosh(x)\right)}{a^2\sqrt{a\operatorname{csch}^4(x)}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\operatorname{csch}^2(x)\left(\frac{9}{10}\left(\frac{1}{8}\sinh^7(x)\cosh(x)-\frac{7}{8}\int \sinh^6(x)dx\right)-\frac{1}{10}\sinh^9(x)\cosh(x)\right)}{a^2\sqrt{a\operatorname{csch}^4(x)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\operatorname{csch}^2(x)\left(-\frac{1}{10}\sinh^9(x)\cosh(x)+\frac{9}{10}\left(\frac{1}{8}\sinh^7(x)\cosh(x)-\frac{7}{8}\int -\sin(ix)^6dx\right)\right)}{a^2\sqrt{a\operatorname{csch}^4(x)}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\operatorname{csch}^2(x)\left(-\frac{1}{10}\sinh^9(x)\cosh(x)+\frac{9}{10}\left(\frac{1}{8}\sinh^7(x)\cosh(x)+\frac{7}{8}\int \sin(ix)^6dx\right)\right)}{a^2\sqrt{a\operatorname{csch}^4(x)}}$$

$$\Big\downarrow \; 3115$$

$$-\frac{\operatorname{csch}^2(x)\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\int \sinh^4(x)dx-\frac{1}{6}\sinh^5(x)\cosh(x)\right)+\frac{1}{8}\sinh^7(x)\cosh(x)\right)-\frac{1}{10}\sinh^9(x)\cosh(x)\right)}{a^2\sqrt{a\operatorname{csch}^4(x)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\operatorname{csch}^2(x)\left(-\frac{1}{10}\sinh^9(x)\cosh(x)+\frac{9}{10}\left(\frac{1}{8}\sinh^7(x)\cosh(x)+\frac{7}{8}\left(-\frac{1}{6}\sinh^5(x)\cosh(x)+\frac{5}{6}\int \sin(ix)^4dx\right)\right)\right)}{a^2\sqrt{a\operatorname{csch}^4(x)}}$$

$$\Big\downarrow \; 3115$$

$$-\frac{\operatorname{csch}^2(x)\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int -\sinh^2(x)dx+\frac{1}{4}\sinh^3(x)\cosh(x)\right)-\frac{1}{6}\sinh^5(x)\cosh(x)\right)+\frac{1}{8}\sinh^7(x)\cosh(x)\right)-\frac{1}{10}\sin\right.}{a^2\sqrt{a\operatorname{csch}^4(x)}}$$

$\Big\downarrow$ 25

$$-\frac{\text{csch}^2(x)\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{1}{4}\sinh^3(x)\cosh(x)-\frac{3}{4}\int\sinh^2(x)dx\right)-\frac{1}{6}\sinh^5(x)\cosh(x)\right)+\frac{1}{8}\sinh^7(x)\cosh(x)\right)-\frac{1}{10}\sinh^9\right.}{a^2\sqrt{a\text{csch}^4(x)}}$$

$\Big\downarrow$ 3042

$$-\frac{\text{csch}^2(x)\left(-\frac{1}{10}\sinh^9(x)\cosh(x)+\frac{9}{10}\left(\frac{1}{8}\sinh^7(x)\cosh(x)+\frac{7}{8}\left(-\frac{1}{6}\sinh^5(x)\cosh(x)+\frac{5}{6}\left(\frac{1}{4}\sinh^3(x)\cosh(x)-\frac{3}{4}\right.\right.\right.\right.}{a^2\sqrt{a\text{csch}^4(x)}}$$

$\Big\downarrow$ 25

$$-\frac{\text{csch}^2(x)\left(-\frac{1}{10}\sinh^9(x)\cosh(x)+\frac{9}{10}\left(\frac{1}{8}\sinh^7(x)\cosh(x)+\frac{7}{8}\left(-\frac{1}{6}\sinh^5(x)\cosh(x)+\frac{5}{6}\left(\frac{1}{4}\sinh^3(x)\cosh(x)+\frac{3}{4}\right.\right.\right.\right.}{a^2\sqrt{a\text{csch}^4(x)}}$$

$\Big\downarrow$ 3115

$$-\frac{\text{csch}^2(x)\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)+\frac{1}{4}\sinh^3(x)\cosh(x)\right)-\frac{1}{6}\sinh^5(x)\cosh(x)\right)+\frac{1}{8}\sinh^7(x)\cos\right.\right.}{a^2\sqrt{a\text{csch}^4(x)}}$$

$\Big\downarrow$ 24

$$-\frac{\text{csch}^2(x)\left(\frac{9}{10}\left(\frac{1}{8}\sinh^7(x)\cosh(x)+\frac{7}{8}\left(\frac{5}{6}\left(\frac{1}{4}\sinh^3(x)\cosh(x)+\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)\right)-\frac{1}{6}\sinh^5(x)\cosh(x)\right)\right)\right.}{a^2\sqrt{a\text{csch}^4(x)}}$$

input
```
Int[(a*Csch[x]^4)^(-5/2),x]
```

output
```
-((Csch[x]^2*(-1/10*(Cosh[x]*Sinh[x]^9) + (9*((Cosh[x]*Sinh[x]^7)/8 + (7*(
-1/6*(Cosh[x]*Sinh[x]^5) + (5*((Cosh[x]*Sinh[x]^3)/4 + (3*(x/2 - (Cosh[x]*
Sinh[x])/2))/4))/6))/8))/10))/(a^2*Sqrt[a*Csch[x]^4]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(108) = 216$.

Time = 0.14 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.74

method	result
risch	$-\frac{63\,\mathrm{e}^{2x} x}{256 a^{2} \left(\mathrm{e}^{2x}-1\right)^{2} \sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(\mathrm{e}^{2x}-1\right)^{4}}}}+\frac{\mathrm{e}^{12x}}{10240 a^{2} \left(\mathrm{e}^{2x}-1\right)^{2} \sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(\mathrm{e}^{2x}-1\right)^{4}}}}-\frac{5\,\mathrm{e}^{10x}}{4096 a^{2} \left(\mathrm{e}^{2x}-1\right)^{2} \sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(\mathrm{e}^{2x}-1\right)^{4}}}}+\frac{15\,\mathrm{e}^{8x}}{2048 a^{2} \left(\mathrm{e}^{2x}-1\right)^{2} \sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(\mathrm{e}^{2x}-1\right)}}}$

input
```
int(1/(a*csch(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-63/256/a^2*exp(2*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*x+1/
10240/a^2*exp(12*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-5/409
6/a^2*exp(10*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)+15/2048/a
^2*exp(8*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-15/512/a^2*ex
p(6*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)+105/1024/a^2*exp(4
*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-105/1024/(a*exp(4*x)/
(exp(2*x)-1)^4)^(1/2)/(exp(2*x)-1)^2/a^2+15/512/a^2*exp(-2*x)/(exp(2*x)-1)
^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-15/2048/a^2*exp(-4*x)/(exp(2*x)-1)^2/
(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)+5/4096/a^2*exp(-6*x)/(exp(2*x)-1)^2/(a*e
xp(4*x)/(exp(2*x)-1)^4)^(1/2)-1/10240/a^2*exp(-8*x)/(exp(2*x)-1)^2/(a*exp(
4*x)/(exp(2*x)-1)^4)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2600 vs. $2(108) = 216$.

Time = 0.13 (sec) , antiderivative size = 2600, normalized size of antiderivative = 19.70

$$\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*csch(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
1/20480*(2*(e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^20 + 2*cosh(x)^20 + 40*(cosh(
x)*e^(4*x) - 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^19 + 5*(76*cosh(x)^2 + (
76*cosh(x)^2 - 5)*e^(4*x) - 2*(76*cosh(x)^2 - 5)*e^(2*x) - 5)*sinh(x)^18 -
 25*cosh(x)^18 + 30*(76*cosh(x)^3 + (76*cosh(x)^3 - 15*cosh(x))*e^(4*x) -
2*(76*cosh(x)^3 - 15*cosh(x))*e^(2*x) - 15*cosh(x))*sinh(x)^17 + 15*(646*c
osh(x)^4 - 255*cosh(x)^2 + (646*cosh(x)^4 - 255*cosh(x)^2 + 10)*e^(4*x) -
2*(646*cosh(x)^4 - 255*cosh(x)^2 + 10)*e^(2*x) + 10)*sinh(x)^16 + 150*cosh
(x)^16 + 48*(646*cosh(x)^5 - 425*cosh(x)^3 + (646*cosh(x)^5 - 425*cosh(x)^
3 + 50*cosh(x))*e^(4*x) - 2*(646*cosh(x)^5 - 425*cosh(x)^3 + 50*cosh(x))*e
^(2*x) + 50*cosh(x))*sinh(x)^15 + 60*(1292*cosh(x)^6 - 1275*cosh(x)^4 + 30
0*cosh(x)^2 + (1292*cosh(x)^6 - 1275*cosh(x)^4 + 300*cosh(x)^2 - 10)*e^(4*
x) - 2*(1292*cosh(x)^6 - 1275*cosh(x)^4 + 300*cosh(x)^2 - 10)*e^(2*x) - 10
)*sinh(x)^14 - 600*cosh(x)^14 + 120*(1292*cosh(x)^7 - 1785*cosh(x)^5 + 700
*cosh(x)^3 + (1292*cosh(x)^7 - 1785*cosh(x)^5 + 700*cosh(x)^3 - 70*cosh(x)
)*e^(4*x) - 2*(1292*cosh(x)^7 - 1785*cosh(x)^5 + 700*cosh(x)^3 - 70*cosh(x
))*e^(2*x) - 70*cosh(x))*sinh(x)^13 + 60*(4199*cosh(x)^8 - 7735*cosh(x)^6
+ 4550*cosh(x)^4 - 910*cosh(x)^2 + (4199*cosh(x)^8 - 7735*cosh(x)^6 + 4550
*cosh(x)^4 - 910*cosh(x)^2 + 35)*e^(4*x) - 2*(4199*cosh(x)^8 - 7735*cosh(x
)^6 + 4550*cosh(x)^4 - 910*cosh(x)^2 + 35)*e^(2*x) + 35)*sinh(x)^12 + 2100
*cosh(x)^12 + 80*(4199*cosh(x)^9 - 9945*cosh(x)^7 + 8190*cosh(x)^5 - 27...
```

Sympy [F]

$$\int \frac{1}{\left(a\text{csch}^4(x)\right)^{5/2}}\,dx=\int \frac{1}{\left(a\operatorname{csch}^{4}{\left(x\right)}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*csch(x)**4)**(5/2),x)
```

output
```
Integral((a*csch(x)**4)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.55

$$\int \frac{1}{\left(a\text{csch}^4(x)\right)^{5/2}}\,dx = -\frac{\left(25\,e^{(-2\,x)} - 150\,e^{(-4\,x)} + 600\,e^{(-6\,x)} - 2100\,e^{(-8\,x)} + 2100\,e^{(-12\,x)} - 600\,e^{(-14\,x)} + 150\,e^{(-16\,x)} - 25\,e^{(-18\,x)}\right.}{20480\,a^{\frac{5}{2}}} - \frac{63\,x}{256\,a^{\frac{5}{2}}}$$

input
```
integrate(1/(a*csch(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
-1/20480*(25*e^(-2*x) - 150*e^(-4*x) + 600*e^(-6*x) - 2100*e^(-8*x) + 2100
*e^(-12*x) - 600*e^(-14*x) + 150*e^(-16*x) - 25*e^(-18*x) + 2*e^(-20*x) -
2)*e^(10*x)/a^(5/2) - 63/256*x/a^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.58

$$\int \frac{1}{\left(a\text{csch}^4(x)\right)^{5/2}}\,dx = \frac{\left(5754\,e^{(10\,x)} - 2100\,e^{(8\,x)} + 600\,e^{(6\,x)} - 150\,e^{(4\,x)} + 25\,e^{(2\,x)} - 2\right)e^{(-10\,x)} - 5040\,x + 2}{20480\,a^{\frac{5}{2}}}$$

input
```
integrate(1/(a*csch(x)^4)^(5/2),x, algorithm="giac")
```

output
```
1/20480*((5754*e^(10*x) - 2100*e^(8*x) + 600*e^(6*x) - 150*e^(4*x) + 25*e^
(2*x) - 2)*e^(-10*x) - 5040*x + 2*e^(10*x) - 25*e^(8*x) + 150*e^(6*x) - 60
0*e^(4*x) + 2100*e^(2*x))/a^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\text{csch}^4(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(\frac{a}{\sinh(x)^4}\right)^{5/2}}\,dx$$

input
```
int(1/(a/sinh(x)^4)^(5/2),x)
```

output
```
int(1/(a/sinh(x)^4)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.66

$$\int \frac{1}{\left(a\text{csch}^4(x)\right)^{5/2}}\,dx = \frac{\sqrt{a}\,(2e^{20x} - 25e^{18x} + 150e^{16x} - 600e^{14x} + 2100e^{12x} - 5040e^{10x}x - 2100e^{8x} + 600e^{6}}{20480e^{10x}a^3}$$

input
```
int(1/(a*csch(x)^4)^(5/2),x)
```

output
```
(sqrt(a)*(2*e**(20*x) - 25*e**(18*x) + 150*e**(16*x) - 600*e**(14*x) + 210
0*e**(12*x) - 5040*e**(10*x)*x - 2100*e**(8*x) + 600*e**(6*x) - 150*e**(4*
x) + 25*e**(2*x) - 2))/(20480*e**(10*x)*a**3)
```

3.49 $\int \frac{1}{a+ia\text{csch}(a+bx)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{a+ia\text{csch}(a+bx)}\,dx=\frac{x}{a}-\frac{\coth(a+bx)}{b(a+ia\text{csch}(a+bx))}$$

output
```
x/a-coth(b*x+a)/b/(a+I*a*csch(b*x+a))
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int \frac{1}{a+ia\text{csch}(a+bx)}\,dx=\frac{1}{b}+\frac{x}{a}-\frac{2\sinh\left(\frac{1}{2}(a+bx)\right)}{ab\left(\cosh\left(\frac{1}{2}(a+bx)\right)-i\sinh\left(\frac{1}{2}(a+bx)\right)\right)}$$

input
```
Integrate[(a + I*a*Csch[a + b*x])^(-1),x]
```

output
```
b^(-1) + x/a - (2*Sinh[(a + b*x)/2])/(a*b*(Cosh[(a + b*x)/2] - I*Sinh[(a + 
 b*x)/2]))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4264, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+ia\text{csch}(a+bx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{a-a\csc(ia+ibx)}dx$$
$$\Big\downarrow \text{4264}$$
$$-\frac{\int -adx}{a^2}-\frac{\coth(a+bx)}{b(a+ia\text{csch}(a+bx))}$$
$$\Big\downarrow \text{24}$$
$$\frac{x}{a}-\frac{\coth(a+bx)}{b(a+ia\text{csch}(a+bx))}$$

input
```
Int[(a + I*a*Csch[a + b*x])^(-1),x]
```

output
```
x/a - Coth[a + b*x]/(b*(a + I*a*Csch[a + b*x]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4264
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Simp[1/(a^2*(2*n + 1))
   Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Int
egerQ[2*n]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{x}{a}+\frac{2i}{ba\left(\mathrm{e}^{bx+a}+i\right)}$	27
parallelrisch	$\frac{ibx+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)(bx-2i)}{ba\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+i\right)}$	44
derivativedivides	$\frac{-\frac{2}{\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+i}-\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{ab}$	54
default	$\frac{-\frac{2}{\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+i}-\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{ab}$	54

input
```
int(1/(a+I*a*csch(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
x/a+2*I/b/a/(exp(b*x+a)+I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+i a\operatorname{csch}(a+bx)}\,dx=\frac{bxe^{(bx+a)}+i\,bx+2i}{abe^{(bx+a)}+i\,ab}$$

input
```
integrate(1/(a+I*a*csch(b*x+a)),x, algorithm="fricas")
```

output
```
(b*x*e^(b*x + a) + I*b*x + 2*I)/(a*b*e^(b*x + a) + I*a*b)
```

Sympy [F]

$$\int \frac{1}{a + i a \operatorname{csch}(a + bx)} \, dx = -\frac{i \int \frac{1}{\operatorname{csch}(a+bx)-i} \, dx}{a}$$

input
```
integrate(1/(a+I*a*csch(b*x+a)),x)
```

output
```
-I*Integral(1/(csch(a + b*x) - I), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{1}{a + i a \operatorname{csch}(a + bx)} \, dx = \frac{bx + a}{ab} + \frac{2i}{(ae^{(-bx-a)} - i\, a)b}$$

input
```
integrate(1/(a+I*a*csch(b*x+a)),x, algorithm="maxima")
```

output
```
(b*x + a)/(a*b) + 2*I/((a*e^(-b*x - a) - I*a)*b)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{1}{a + i a \operatorname{csch}(a + bx)} \, dx = \frac{\frac{bx+a}{a} + \frac{2i}{a\left(e^{(bx+a)}+i\right)}}{b}$$

input
```
integrate(1/(a+I*a*csch(b*x+a)),x, algorithm="giac")
```

output
```
((b*x + a)/a + 2*I/(a*(e^(b*x + a) + I)))/b
```

Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{a + ia\operatorname{csch}(a + bx)}\,dx = \frac{x}{a} + \frac{2i}{a\,b\,\left(e^{a+b\,x} + 1i\right)}$$

input
```
int(1/(a + (a*1i)/sinh(a + b*x)),x)
```

output
```
x/a + 2i/(a*b*(exp(a + b*x) + 1i))
```

Reduce [F]

$$\int \frac{1}{a + ia\operatorname{csch}(a + bx)}\,dx = \frac{\int \frac{1}{\operatorname{csch}(bx+a)i+1}dx}{a}$$

input
```
int(1/(a+I*a*csch(b*x+a)),x)
```

output
```
int(1/(csch(a + b*x)*i + 1),x)/a
```

3.50 $\int \frac{1}{a-ia\text{csch}(a+bx)} dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{a-ia\text{csch}(a+bx)} dx = \frac{x}{a} - \frac{\coth(a+bx)}{b(a-ia\text{csch}(a+bx))}$$

output
```
x/a-coth(b*x+a)/b/(a-I*a*csch(b*x+a))
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int \frac{1}{a-ia\text{csch}(a+bx)} dx = \frac{1}{b} + \frac{x}{a} - \frac{2\sinh\left(\frac{1}{2}(a+bx)\right)}{ab\left(\cosh\left(\frac{1}{2}(a+bx)\right)+i\sinh\left(\frac{1}{2}(a+bx)\right)\right)}$$

input
```
Integrate[(a - I*a*Csch[a + b*x])^(-1),x]
```

output
```
b^(-1) + x/a - (2*Sinh[(a + b*x)/2])/(a*b*(Cosh[(a + b*x)/2] + I*Sinh[(a +
 b*x)/2]))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4264, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-iacsch(a+bx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{a+a\csc(ia+ibx)}dx$$
$$\Big\downarrow \text{4264}$$
$$-\frac{\int -adx}{a^2}-\frac{\coth(a+bx)}{b(a-iacsch(a+bx))}$$
$$\Big\downarrow \text{24}$$
$$\frac{x}{a}-\frac{\coth(a+bx)}{b(a-iacsch(a+bx))}$$

input
```
Int[(a - I*a*Csch[a + b*x])^(-1),x]
```

output
```
x/a - Coth[a + b*x]/(b*(a - I*a*Csch[a + b*x]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4264
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Simp[1/(a^2*(2*n + 1))
   Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Int
egerQ[2*n]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{x}{a}-\frac{2i}{ba\left(\mathrm{e}^{bx+a}-i\right)}$	27
parallelrisch	$\frac{\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)(-bx-2i)+ibx}{ba\left(-\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+i\right)}$	47
derivativedivides	$\frac{-\frac{2}{\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-i}-\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{ab}$	54
default	$\frac{-\frac{2}{\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-i}-\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{ab}$	54

input
```
int(1/(a-I*a*csch(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
x/a-2*I/b/a/(exp(b*x+a)-I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{a-iacsch(a+bx)}\,dx=\frac{bxe^{(bx+a)}-i\,bx-2i}{abe^{(bx+a)}-i\,ab}$$

input
```
integrate(1/(a-I*a*csch(b*x+a)),x, algorithm="fricas")
```

output
```
(b*x*e^(b*x + a) - I*b*x - 2*I)/(a*b*e^(b*x + a) - I*a*b)
```

Sympy [F]

$$\int \frac{1}{a - i a\operatorname{csch}(a + bx)}\, dx = \frac{i \int \frac{1}{\operatorname{csch}(a+bx)+i}\, dx}{a}$$

input
```
integrate(1/(a-I*a*csch(b*x+a)),x)
```

output
```
I*Integral(1/(csch(a + b*x) + I), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{1}{a - i a\operatorname{csch}(a + bx)}\, dx = \frac{bx + a}{ab} - \frac{2i}{(ae^{(-bx-a)} + i\,a)b}$$

input
```
integrate(1/(a-I*a*csch(b*x+a)),x, algorithm="maxima")
```

output
```
(b*x + a)/(a*b) - 2*I/((a*e^(-b*x - a) + I*a)*b)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{1}{a - i a\operatorname{csch}(a + bx)}\, dx = \frac{\frac{bx+a}{a} - \frac{2i}{a(e^{(bx+a)}-i)}}{b}$$

input
```
integrate(1/(a-I*a*csch(b*x+a)),x, algorithm="giac")
```

output
```
((b*x + a)/a - 2*I/(a*(e^(b*x + a) - I)))/b
```

Mupad [B] (verification not implemented)

Time = 2.90 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{a - ia\text{csch}(a + bx)} \, dx = \frac{x}{a} - \frac{2\mathrm{i}}{a\,b\,\left(\mathrm{e}^{a+b\,x} - \mathrm{i}\right)}$$

input
```
int(1/(a - (a*1i)/sinh(a + b*x)),x)
```

output
```
x/a - 2i/(a*b*(exp(a + b*x) - 1i))
```

Reduce [F]

$$\int \frac{1}{a - ia\text{csch}(a + bx)} \, dx = -\frac{\int \frac{1}{\text{csch}(bx+a)i-1}dx}{a}$$

input
```
int(1/(a-I*a*csch(b*x+a)),x)
```

output
```
( - int(1/(csch(a + b*x)*i - 1),x))/a
```

3.51 $\int (a+ia\text{csch}(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 107

$$\int (a+ia\text{csch}(c+dx))^{5/2}\,dx=\frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}+\frac{14a^3\coth(c+dx)}{3d\sqrt{a+ia\text{csch}(c+dx)}}+\frac{2a^2\coth(c+dx)\sqrt{a+ia\text{csch}(c+dx)}}{3d}$$

output
```
2*a^(5/2)*arctanh(a^(1/2)*coth(d*x+c)/(a+I*a*csch(d*x+c))^(1/2))/d+14/3*a^
3*coth(d*x+c)/d/(a+I*a*csch(d*x+c))^(1/2)+2/3*a^2*coth(d*x+c)*(a+I*a*csch(
d*x+c))^(1/2)/d
```

Mathematica [A] (warning: unable to verify)

Time = 1.36 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.27

$$\int (a+ia\text{csch}(c+dx))^{5/2}\,dx=\frac{2a^2\sqrt{a+ia\text{csch}(c+dx)}\left(-7i+\coth(c+dx)+\frac{3(-1)^{3/4}\text{arctanh}\left((-1)^{3/4}\sqrt{i+\text{csch}(c+dx)}\right)\coth(c+\cdots}{(-i+\text{csch}(c+dx))\sqrt{i+\text{csch}(c+dx)}}\cdots\right.}{3d}$$

input
```
Integrate[(a + I*a*Csch[c + d*x])^(5/2),x]
```

output
```
(2*a^2*Sqrt[a + I*a*Csch[c + d*x]]*(-7*I + Coth[c + d*x] + (3*(-1)^(3/4)*A
rcTanh[(-1)^(3/4)*Sqrt[I + Csch[c + d*x]]]*Coth[c + d*x])/((-I + Csch[c +
d*x])*Sqrt[I + Csch[c + d*x]]) + (14*Sinh[(c + d*x)/2])/(Cosh[(c + d*x)/2]
 - I*Sinh[(c + d*x)/2])))/(3*d)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {3042, 4262, 27, 3042, 4403, 26, 3042, 26, 4261, 216, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\text{csch}(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a-a\csc(ic+idx))^{5/2}dx$$

$$\Big\downarrow \text{4262}$$

$$\frac{2}{3}a\int \frac{1}{2}\sqrt{i\text{csch}(c+dx)a+a}(7i\text{csch}(c+dx)a+3a)dx+\frac{2a^2\coth(c+dx)\sqrt{a+ia\text{csch}(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}a\int \sqrt{i\text{csch}(c+dx)a+a}(7i\text{csch}(c+dx)a+3a)dx+\frac{2a^2\coth(c+dx)\sqrt{a+ia\text{csch}(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}a\int (3a-7a\csc(ic+idx))\sqrt{a-a\csc(ic+idx)}dx+\frac{2a^2\coth(c+dx)\sqrt{a+ia\text{csch}(c+dx)}}{3d}$$

$$\Big\downarrow \text{4403}$$

$$\frac{1}{3}a\left(3a\int \sqrt{i\text{csch}(c+dx)a+a}dx-7a\int -i\text{csch}(c+dx)\sqrt{i\text{csch}(c+dx)a+a}dx\right)+\frac{2a^2\coth(c+dx)\sqrt{a+ia\text{csch}(c+dx)}}{3d}$$

$\Big\downarrow$ 26

$$\frac{1}{3}a\left(3a\int \sqrt{i\text{csch}(c+dx)a+a}dx+7ia\int \text{csch}(c+dx)\sqrt{i\text{csch}(c+dx)a+a}dx\right)+\frac{2a^2\coth(c+dx)\sqrt{a+ia\text{csch}(c+dx)}}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}a\left(3a\int \sqrt{a-a\csc(ic+idx)}dx+7ia\int i\csc(ic+idx)\sqrt{a-a\csc(ic+idx)}dx\right)+\frac{2a^2\coth(c+dx)\sqrt{a+ia\text{csch}(c+dx)}}{3d}$$

$\Big\downarrow$ 26

$$\frac{1}{3}a\left(3a\int \sqrt{a-a\csc(ic+idx)}dx-7a\int \csc(ic+idx)\sqrt{a-a\csc(ic+idx)}dx\right)+\frac{2a^2\coth(c+dx)\sqrt{a+ia\text{csch}(c+dx)}}{3d}$$

$\Big\downarrow$ 4261

$$\frac{1}{3}a\left(-\frac{6ia^2\int \frac{1}{a-\frac{a^2\coth^2(c+dx)}{i\text{csch}(c+dx)a+a}}d\frac{ia\coth(c+dx)}{\sqrt{i\text{csch}(c+dx)a+a}}}{d}-7a\int \csc(ic+idx)\sqrt{a-a\csc(ic+idx)}dx\right)+\frac{2a^2\coth(c+dx)\sqrt{a+ia\text{csch}(c+dx)}}{3d}$$

$\Big\downarrow$ 216

$$\frac{1}{3}a\left(\frac{6a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}-7a\int \csc(ic+idx)\sqrt{a-a\csc(ic+idx)}dx\right)+\frac{2a^2\coth(c+dx)\sqrt{a+ia\text{csch}(c+dx)}}{3d}$$

$\Big\downarrow$ 4279

$$\frac{2a^2\coth(c+dx)\sqrt{a+ia\text{csch}(c+dx)}}{3d}+\frac{1}{3}a\left(\frac{6a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}+\frac{14a^2\coth(c+dx)}{d\sqrt{a+ia\text{csch}(c+dx)}}\right)$$

input
```
Int[(a + I*a*Csch[c + d*x])^(5/2),x]
```

output
```
(2*a^2*Coth[c + d*x]*Sqrt[a + I*a*Csch[c + d*x]])/(3*d) + (a*((6*a^(3/2)*A
rcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/d + (14*a^2*C
oth[c + d*x])/(d*Sqrt[a + I*a*Csch[c + d*x]])))/3
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4262
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[a/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Inte
gerQ[2*n]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4403
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Simp[c   Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Si
mp[d   Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (a+ia\ \text{csch}\,(dx+c))^{\frac{5}{2}}\,dx$$

input
```
int((a+I*a*csch(d*x+c))^(5/2),x)
```

output
```
int((a+I*a*csch(d*x+c))^(5/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(87) = 174$.

Time = 0.10 (sec) , antiderivative size = 566, normalized size of antiderivative = 5.29

$$\int (a+ia\text{csch}(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*csch(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/6*(3*sqrt(a^5/d^2)*(d*e^(2*d*x + 2*c) - d)*log(2*(a^3*e^(d*x + c) + I*a^
3 + sqrt(a^5/d^2)*(d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) - 1)))*e
^(-d*x - c)/d) - 3*sqrt(a^5/d^2)*(d*e^(2*d*x + 2*c) - d)*log(2*(a^3*e^(d*x
 + c) + I*a^3 - sqrt(a^5/d^2)*(d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2
*c) - 1)))*e^(-d*x - c)/d) + 3*sqrt(a^5/d^2)*(d*e^(2*d*x + 2*c) - d)*log(2
*(sqrt(a^5/d^2)*(a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d) + (a^3*e
^(3*d*x + 3*c) - 2*I*a^3*e^(2*d*x + 2*c) - a^3*e^(d*x + c) + 2*I*a^3)*sqrt
(a/(e^(2*d*x + 2*c) - 1)))*e^(-2*d*x - 2*c)/d) - 3*sqrt(a^5/d^2)*(d*e^(2*d
*x + 2*c) - d)*log(-2*(sqrt(a^5/d^2)*(a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x +
 c) - 2*a*d) - (a^3*e^(3*d*x + 3*c) - 2*I*a^3*e^(2*d*x + 2*c) - a^3*e^(d*x
 + c) + 2*I*a^3)*sqrt(a/(e^(2*d*x + 2*c) - 1)))*e^(-2*d*x - 2*c)/d) + 8*(4
*a^2*e^(3*d*x + 3*c) - 3*I*a^2*e^(2*d*x + 2*c) - 3*a^2*e^(d*x + c) + 4*I*a
^2)*sqrt(a/(e^(2*d*x + 2*c) - 1)))/(d*e^(2*d*x + 2*c) - d)
```

Sympy [F]

$$\int (a+ia\text{csch}(c+dx))^{5/2}\,dx=\int (ia\operatorname{csch}{(c+dx)}+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+I*a*csch(d*x+c))**(5/2),x)
```

output
```
Integral((I*a*csch(c + d*x) + a)**(5/2), x)
```

Maxima [F]

$$\int (a+ia\text{csch}(c+dx))^{5/2}\,dx=\int (i\,a\operatorname{csch}(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+I*a*csch(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*csch(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int (a+iacsch(c+dx))^{5/2}\,dx=\int (i\,a\operatorname{csch}\left(dx+c\right)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+I*a*csch(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*csch(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+iacsch(c+dx))^{5/2}\,dx=\int \left(a+\frac{a\,1\mathrm{i}}{\sinh\left(c+d\,x\right)}\right)^{5/2}\,dx$$

input
```
int((a + (a*1i)/sinh(c + d*x))^(5/2),x)
```

output
```
int((a + (a*1i)/sinh(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int (a+iacsch(c+dx))^{5/2}\,dx=\sqrt{a}\,a^2\left(\int \sqrt{\operatorname{csch}\left(dx+c\right)i+1}dx+2\left(\int \sqrt{\operatorname{csch}\left(dx+c\right)i+1}\operatorname{csch}(dx+c)\,dx\right)i-\left(\int \sqrt{\operatorname{csch}\left(dx+c\right)i+1}\operatorname{csch}(dx+c)^2\,dx\right)\right)$$

input
```
int((a+I*a*csch(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(csch(c + d*x)*i + 1),x) + 2*int(sqrt(csch(c + d*x)*
i + 1)*csch(c + d*x),x)*i - int(sqrt(csch(c + d*x)*i + 1)*csch(c + d*x)**2
,x))
```

3.52 $\int (a+ia\operatorname{csch}(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 72

$$\int (a+ia\operatorname{csch}(c+dx))^{3/2}\,dx=\frac{2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{d}+\frac{2a^2\coth(c+dx)}{d\sqrt{a+ia\operatorname{csch}(c+dx)}}$$

output
```
2*a^(3/2)*arctanh(a^(1/2)*coth(d*x+c)/(a+I*a*csch(d*x+c))^(1/2))/d+2*a^2*c
oth(d*x+c)/d/(a+I*a*csch(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.39

$$\int (a+ia\operatorname{csch}(c+dx))^{3/2}\,dx=$$
$$-\frac{2ia\coth(c+dx)\sqrt{a+ia\operatorname{csch}(c+dx)}\left(-\sqrt[4]{-1}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{i+\operatorname{csch}(c+dx)}\right)+\sqrt{i+\operatorname{csch}(c+dx}\right.}{d(-i+\operatorname{csch}(c+dx))\sqrt{i+\operatorname{csch}(c+dx)}}$$

input
```
Integrate[(a + I*a*Csch[c + d*x])^(3/2),x]
```

output
```
((-2*I)*a*Coth[c + d*x]*Sqrt[a + I*a*Csch[c + d*x]]*(-((-1)^(1/4)*ArcTanh[
(-1)^(3/4)*Sqrt[I + Csch[c + d*x]]]) + Sqrt[I + Csch[c + d*x]]))/(d*(-I +
Csch[c + d*x])*Sqrt[I + Csch[c + d*x]])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 4262, 27, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + ia\text{csch}(c + dx))^{3/2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a - a\csc(ic + idx))^{3/2} dx$$

$$\Big\downarrow \text{4262}$$

$$2a \int \frac{1}{2}\sqrt{i\text{csch}(c + dx)a + a}dx + \frac{2a^2\coth(c + dx)}{d\sqrt{a + ia\text{csch}(c + dx)}}$$

$$\Big\downarrow \text{27}$$

$$a \int \sqrt{i\text{csch}(c + dx)a + a}dx + \frac{2a^2\coth(c + dx)}{d\sqrt{a + ia\text{csch}(c + dx)}}$$

$$\Big\downarrow \text{3042}$$

$$a \int \sqrt{a - a\csc(ic + idx)}dx + \frac{2a^2\coth(c + dx)}{d\sqrt{a + ia\text{csch}(c + dx)}}$$

$$\Big\downarrow \text{4261}$$

$$\frac{2a^2\coth(c + dx)}{d\sqrt{a + ia\text{csch}(c + dx)}} - \frac{2ia^2 \int \frac{1}{a - \frac{a^2\coth^2(c+dx)}{i\text{csch}(c+dx)a+a}} d\frac{ia\coth(c+dx)}{\sqrt{i\text{csch}(c+dx)a+a}}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}+\frac{2a^2\coth(c+dx)}{d\sqrt{a+ia\text{csch}(c+dx)}}$$

input
```
Int[(a + I*a*Csch[c + d*x])^(3/2),x]
```

output
```
(2*a^(3/2)*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/d
 + (2*a^2*Coth[c + d*x])/(d*Sqrt[a + I*a*Csch[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4262
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[a/(n - 1)
    Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Inte
gerQ[2*n]
```

Maple [F]

$$\int (a + ia \operatorname{csch}(dx + c))^{\frac{3}{2}} dx$$

input
```
int((a+I*a*csch(d*x+c))^(3/2),x)
```

output
```
int((a+I*a*csch(d*x+c))^(3/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(60) = 120$.

Time = 0.10 (sec) , antiderivative size = 461, normalized size of antiderivative = 6.40

$$\int (a + ia\text{csch}(c + dx))^{3/2}\, dx = \frac{\sqrt{\frac{a^3}{d^2}}\, d \log\left(\frac{2\left(a^2 e^{(dx+c)} + i\, a^2 + \left(de^{(2\,dx+2\,c)} - d\right)\sqrt{\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\right)e^{(-dx-c)}}{d}\right) - \sqrt{\frac{a^3}{d^2}}\, d \log\left(\frac{2\left(a^2 e^{(dx+c)} + i\, a\right.}{}\right.}{}$$

input
```
integrate((a+I*a*csch(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(a^3/d^2)*d*log(2*(a^2*e^(d*x + c) + I*a^2 + (d*e^(2*d*x + 2*c) -
 d)*sqrt(a^3/d^2)*sqrt(a/(e^(2*d*x + 2*c) - 1)))*e^(-d*x - c)/d) - sqrt(a^
3/d^2)*d*log(2*(a^2*e^(d*x + c) + I*a^2 - (d*e^(2*d*x + 2*c) - d)*sqrt(a^3
/d^2)*sqrt(a/(e^(2*d*x + 2*c) - 1)))*e^(-d*x - c)/d) + sqrt(a^3/d^2)*d*log
(2*((a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d)*sqrt(a^3/d^2) + (a^2
*e^(3*d*x + 3*c) - 2*I*a^2*e^(2*d*x + 2*c) - a^2*e^(d*x + c) + 2*I*a^2)*sq
rt(a/(e^(2*d*x + 2*c) - 1)))*e^(-2*d*x - 2*c)/d) - sqrt(a^3/d^2)*d*log(-2*
((a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d)*sqrt(a^3/d^2) - (a^2*e^
(3*d*x + 3*c) - 2*I*a^2*e^(2*d*x + 2*c) - a^2*e^(d*x + c) + 2*I*a^2)*sqrt(
a/(e^(2*d*x + 2*c) - 1)))*e^(-2*d*x - 2*c)/d) + 4*(a*e^(d*x + c) - I*a)*sq
rt(a/(e^(2*d*x + 2*c) - 1)))/d
```

Sympy [F]

$$\int (a+iacsch(c+dx))^{3/2}\,dx=\int \left(ia\,\text{csch}\,(c+dx)+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+I*a*csch(d*x+c))**(3/2),x)
```

output
```
Integral((I*a*csch(c + d*x) + a)**(3/2), x)
```

Maxima [F]

$$\int (a+iacsch(c+dx))^{3/2}\,dx=\int \left(i\,a\,\text{csch}\,(dx+c)+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+I*a*csch(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*csch(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int (a+iacsch(c+dx))^{3/2}\,dx=\int \left(i\,a\,\text{csch}\,(dx+c)+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+I*a*csch(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*csch(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\operatorname{csch}(c+dx))^{3/2}\,dx=\int \left(a+\frac{a\,1\mathrm{i}}{\sinh (c+d\,x)}\right)^{3/2}\,dx$$

input
```
int((a + (a*1i)/sinh(c + d*x))^(3/2),x)
```

output
```
int((a + (a*1i)/sinh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int (a+ia\operatorname{csch}(c+dx))^{3/2}\,dx=\sqrt{a}\,a\left(\int \sqrt{\operatorname{csch}(dx+c)\,i+1}dx+\left(\int \sqrt{\operatorname{csch}(dx+c)\,i+1}\operatorname{csch}(dx+c)\,dx\right)i\right)$$

input
```
int((a+I*a*csch(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(csch(c + d*x)*i + 1),x) + int(sqrt(csch(c + d*x)*i + 1
)*csch(c + d*x),x)*i)
```

3.53 $\int \sqrt{a+ia\text{csch}(c+dx)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \sqrt{a+ia\text{csch}(c+dx)}\,dx = \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}$$

output
```
2*a^(1/2)*arctanh(a^(1/2)*coth(d*x+c)/(a+I*a*csch(d*x+c))^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.00

$$\int \sqrt{a+ia\text{csch}(c+dx)}\,dx = \frac{2(-1)^{3/4}\text{arctanh}\left((-1)^{3/4}\sqrt{i+\text{csch}(c+dx)}\right)\coth(c+dx)\sqrt{a+ia\text{csch}(c+dx)}}{d(-i+\text{csch}(c+dx))\sqrt{i+\text{csch}(c+dx)}}$$

input
```
Integrate[Sqrt[a + I*a*Csch[c + d*x]],x]
```

output
```
(2*(-1)^(3/4)*ArcTanh[(-1)^(3/4)*Sqrt[I + Csch[c + d*x]]]*Coth[c + d*x]*Sq
rt[a + I*a*Csch[c + d*x]])/(d*(-I + Csch[c + d*x])*Sqrt[I + Csch[c + d*x]]
)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\text{csch}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a-a\csc(ic+idx)}dx$$

$$\Big\downarrow \text{4261}$$

$$-\frac{2ia\int \frac{1}{a-\frac{a^2\coth^2(c+dx)}{i\text{csch}(c+dx)a+a}}d\frac{ia\coth(c+dx)}{\sqrt{i\text{csch}(c+dx)a+a}}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}$$

input
```
Int[Sqrt[a + I*a*Csch[c + d*x]],x]
```

output
```
(2*Sqrt[a]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sqrt{a + ia \,\mathrm{csch}\,(dx + c)}dx$$

input
```
int((a+I*a*csch(d*x+c))^(1/2),x)
```

output
```
int((a+I*a*csch(d*x+c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(32) = 64$.

Time = 0.10 (sec) , antiderivative size = 383, normalized size of antiderivative = 9.58

$$\int \sqrt{a + ia\text{csch}(c + dx)}\, dx$$
$$= \frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(\frac{2\left(\left(de^{(2\,dx+2\,c)} - d\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\sqrt{\frac{a}{d^2}} + ae^{(dx+c)} + i\,a\right)e^{(-dx-c)}}{d}\right)$$
$$- \frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(-\frac{2\left(\left(de^{(2\,dx+2\,c)} - d\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\sqrt{\frac{a}{d^2}} - ae^{(dx+c)} - i\,a\right)e^{(-dx-c)}}{d}\right)$$
$$+ \frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(\frac{2\left(\left(ae^{(3\,dx+3\,c)} - 2i\,ae^{(2\,dx+2\,c)} - ae^{(dx+c)} + 2i\,a\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}} + \left(ade^{(2\,dx+2\,c)} - i\,ade^{(dx+c)}\right.\right.}{d}\right.$$
$$- \frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(\frac{2\left(\left(ae^{(3\,dx+3\,c)} - 2i\,ae^{(2\,dx+2\,c)} - ae^{(dx+c)} + 2i\,a\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}} - \left(ade^{(2\,dx+2\,c)} - i\,ade^{(dx+c)}\right.\right.}{d}\right.$$

input
```
integrate((a+I*a*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(a/d^2)*log(2*((d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) - 1
))*sqrt(a/d^2) + a*e^(d*x + c) + I*a)*e^(-d*x - c)/d) - 1/2*sqrt(a/d^2)*lo
g(-2*((d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) - 1))*sqrt(a/d^2) -
a*e^(d*x + c) - I*a)*e^(-d*x - c)/d) + 1/2*sqrt(a/d^2)*log(2*((a*e^(3*d*x
+ 3*c) - 2*I*a*e^(2*d*x + 2*c) - a*e^(d*x + c) + 2*I*a)*sqrt(a/(e^(2*d*x +
 2*c) - 1)) + (a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d)*sqrt(a/d^2
))*e^(-2*d*x - 2*c)/d) - 1/2*sqrt(a/d^2)*log(2*((a*e^(3*d*x + 3*c) - 2*I*a
*e^(2*d*x + 2*c) - a*e^(d*x + c) + 2*I*a)*sqrt(a/(e^(2*d*x + 2*c) - 1)) -
(a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d)*sqrt(a/d^2))*e^(-2*d*x -
 2*c)/d)
```

Sympy [F]

$$\int \sqrt{a + ia\text{csch}(c + dx)}\,dx = \int \sqrt{ia\,\text{csch}\,(c + dx) + a}\,dx$$

input
```
integrate((a+I*a*csch(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*csch(c + d*x) + a), x)
```

Maxima [F]

$$\int \sqrt{a + ia\text{csch}(c + dx)}\,dx = \int \sqrt{i\,a\,\text{csch}\,(dx + c) + a}\,dx$$

input
```
integrate((a+I*a*csch(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(I*a*csch(d*x + c) + a), x)
```

Giac [F]

$$\int \sqrt{a + ia\text{csch}(c + dx)}\,dx = \int \sqrt{i\,a\,\text{csch}\,(dx + c) + a}\,dx$$

input
```
integrate((a+I*a*csch(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*csch(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+ia\text{csch}(c+dx)}\,dx=\int \sqrt{a+\frac{a\,1\text{i}}{\sinh\left(c+d\,x\right)}}\,dx$$

input
```
int((a + (a*1i)/sinh(c + d*x))^(1/2),x)
```

output
```
int((a + (a*1i)/sinh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+ia\text{csch}(c+dx)}\,dx=\sqrt{a}\left(\int \sqrt{\text{csch}\left(dx+c\right)i+1}dx\right)$$

input
```
int((a+I*a*csch(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(csch(c + d*x)*i + 1),x)
```

3.54 $\int \frac{1}{\sqrt{a+ia\text{csch}(c+dx)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 91

$$\int \frac{1}{\sqrt{a+ia\text{csch}(c+dx)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a+ia\text{csch}(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
2*arctanh(a^(1/2)*coth(d*x+c)/(a+I*a*csch(d*x+c))^(1/2))/a^(1/2)/d-2^(1/2)
*arctanh(1/2*a^(1/2)*coth(d*x+c)*2^(1/2)/(a+I*a*csch(d*x+c))^(1/2))/a^(1/2
)/d
```

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{a+ia\text{csch}(c+dx)}}\,dx = \frac{\sqrt{a}\left(2\arctan\left(\frac{\sqrt{ia(i+\text{csch}(c+dx))}}{\sqrt{a}}\right)-\sqrt{2}\arctan\left(\frac{\sqrt{ia(i+\text{csch}(c+dx))}}{\sqrt{2}\sqrt{a}}\right)\right)\coth(c+dx)}{d\sqrt{ia(i+\text{csch}(c+dx))}\sqrt{a+ia\text{csch}(c+dx)}}$$

input
```
Integrate[1/Sqrt[a + I*a*Csch[c + d*x]],x]
```

output
```
(Sqrt[a]*(2*ArcTan[Sqrt[I*a*(I + Csch[c + d*x])]/Sqrt[a]] - Sqrt[2]*ArcTan
[Sqrt[I*a*(I + Csch[c + d*x])]/(Sqrt[2]*Sqrt[a])])*Coth[c + d*x])/(d*Sqrt[
I*a*(I + Csch[c + d*x])]*Sqrt[a + I*a*Csch[c + d*x]])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 4263, 26, 3042, 26, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+ia\text{csch}(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a-a\csc(ic+idx)}}dx$$

$$\Big\downarrow \text{4263}$$

$$\int -\frac{i\text{csch}(c+dx)}{\sqrt{i\text{csch}(c+dx)a+a}}dx+\frac{\int \sqrt{i\text{csch}(c+dx)a+a}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \sqrt{i\text{csch}(c+dx)a+a}dx}{a}-i\int \frac{\text{csch}(c+dx)}{\sqrt{i\text{csch}(c+dx)a+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sqrt{a-a\csc (ic+idx)}dx}{a}-i\int \frac{i\csc (ic+idx)}{\sqrt{a-a\csc (ic+idx)}}dx$$

$$\Big\downarrow \text{ 26}$$

$$\int \frac{\csc (ic+idx)}{\sqrt{a-a\csc (ic+idx)}}dx+\frac{\int \sqrt{a-a\csc (ic+idx)}dx}{a}$$

$$\Big\downarrow \text{ 4261}$$

$$\int \frac{\csc (ic+idx)}{\sqrt{a-a\csc (ic+idx)}}dx-\frac{2i\int \frac{1}{a-\frac{a^2\coth ^2(c+dx)}{i\text{csch}(c+dx)a+a}}d\frac{ia\coth (c+dx)}{\sqrt{i\text{csch}(c+dx)a+a}}}{d}$$

$$\Big\downarrow \text{ 216}$$

$$\int \frac{\csc (ic+idx)}{\sqrt{a-a\csc (ic+idx)}}dx+\frac{2\text{arctanh}\left(\frac{\sqrt{a}\coth (c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{\sqrt{a}d}$$

$$\Big\downarrow \text{ 4282}$$

$$\frac{2i\int \frac{1}{2a-\frac{a^2\coth ^2(c+dx)}{i\text{csch}(c+dx)a+a}}d\frac{ia\coth (c+dx)}{\sqrt{i\text{csch}(c+dx)a+a}}}{d}+\frac{2\text{arctanh}\left(\frac{\sqrt{a}\coth (c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{\sqrt{a}d}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a}\coth (c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\coth (c+dx)}{\sqrt{2}\sqrt{a+ia\text{csch}(c+dx)}}\right)}{\sqrt{a}d}$$

input
```
Int[1/Sqrt[a + I*a*Csch[c + d*x]],x]
```

output
```
(2*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/(Sqrt[a]*
d) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Csch[c
 + d*x]])])/(Sqrt[a]*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4263
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[1/a   I
nt[Sqrt[a + b*Csc[c + d*x]], x], x] - Simp[b/a   Int[Csc[c + d*x]/Sqrt[a +
b*Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{1}{\sqrt{a + ia \operatorname{csch}(dx + c)}} dx$$

input
```
int(1/(a+I*a*csch(d*x+c))^(1/2),x)
```

output
```
int(1/(a+I*a*csch(d*x+c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 551 vs. 2(72) = 144.

Time = 0.13 (sec) , antiderivative size = 551, normalized size of antiderivative = 6.05

$$\int \frac{1}{\sqrt{a + ia\text{csch}(c + dx)}}\, dx = \text{Too large to display}$$

input
```
integrate(1/(a+I*a*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*sqrt(1/(a*d^2))*log(2*(sqrt(2)*(a*d*e^(2*d*x + 2*c) - a*d)*sq
rt(a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) + a*e^(d*x + c) - I*a)*e^(-d*x
 - c)) + 1/2*sqrt(2)*sqrt(1/(a*d^2))*log(-2*(sqrt(2)*(a*d*e^(2*d*x + 2*c)
- a*d)*sqrt(a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) - a*e^(d*x + c) + I*a
)*e^(-d*x - c)) + 1/2*sqrt(1/(a*d^2))*log(2*((d*e^(2*d*x + 2*c) - d)*sqrt(
a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) + e^(d*x + c) + I)*e^(-d*x - c)/d
) - 1/2*sqrt(1/(a*d^2))*log(-2*((d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x +
 2*c) - 1))*sqrt(1/(a*d^2)) - e^(d*x + c) - I)*e^(-d*x - c)/d) + 1/2*sqrt(
1/(a*d^2))*log(2*((a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d)*sqrt(1
/(a*d^2)) + sqrt(a/(e^(2*d*x + 2*c) - 1))*(e^(3*d*x + 3*c) - 2*I*e^(2*d*x
+ 2*c) - e^(d*x + c) + 2*I))*e^(-2*d*x - 2*c)/d) - 1/2*sqrt(1/(a*d^2))*log
(-2*((a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d)*sqrt(1/(a*d^2)) - s
qrt(a/(e^(2*d*x + 2*c) - 1))*(e^(3*d*x + 3*c) - 2*I*e^(2*d*x + 2*c) - e^(d
*x + c) + 2*I))*e^(-2*d*x - 2*c)/d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a + ia\text{csch}(c + dx)}}\, dx = \int \frac{1}{\sqrt{ia \operatorname{csch}(c + dx) + a}}\, dx$$

input
```
integrate(1/(a+I*a*csch(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(I*a*csch(c + d*x) + a), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a + ia\text{csch}(c + dx)}}\,dx = \int \frac{1}{\sqrt{i\,a\,\text{csch}\,(dx + c) + a}}\,dx$$

input
```
integrate(1/(a+I*a*csch(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(I*a*csch(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a + ia\text{csch}(c + dx)}}\,dx = \int \frac{1}{\sqrt{i\,a\,\text{csch}\,(dx + c) + a}}\,dx$$

input
```
integrate(1/(a+I*a*csch(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(I*a*csch(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + ia\text{csch}(c + dx)}}\,dx = \int \frac{1}{\sqrt{a + \frac{a\,1i}{\sinh(c+d\,x)}}}\,dx$$

input
```
int(1/(a + (a*1i)/sinh(c + d*x))^(1/2),x)
```

output
```
int(1/(a + (a*1i)/sinh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+ia\text{csch}(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\text{csch}(dx+c)i+1}}{\text{csch}(dx+c)^2+1}dx-\left(\int \frac{\sqrt{\text{csch}(dx+c)i+1}\,\text{csch}(dx+c)}{\text{csch}(dx+c)^2+1}dx\right)i\right)}{a}$$

input
```
int(1/(a+I*a*csch(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(int(sqrt(csch(c + d*x)*i + 1)/(csch(c + d*x)**2 + 1),x) - int((s
qrt(csch(c + d*x)*i + 1)*csch(c + d*x))/(csch(c + d*x)**2 + 1),x)*i))/a
```

3.55 $\int \frac{1}{(a+ia\operatorname{csch}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 123

$$\int \frac{1}{(a+ia\operatorname{csch}(c+dx))^{3/2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{a^{3/2}d} - \frac{5\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\coth(c+dx)}{2d(a+ia\operatorname{csch}(c+dx))^{3/2}}$$

output
```
2*arctanh(a^(1/2)*coth(d*x+c)/(a+I*a*csch(d*x+c))^(1/2))/a^(3/2)/d-5/4*arc
tanh(1/2*a^(1/2)*coth(d*x+c)*2^(1/2)/(a+I*a*csch(d*x+c))^(1/2))*2^(1/2)/a^
(3/2)/d-1/2*coth(d*x+c)/d/(a+I*a*csch(d*x+c))^(3/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 327 vs. 2(123) = 246.

Time = 3.32 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.66

$$\int \frac{1}{(a+ia\operatorname{csch}(c+dx))^{3/2}}\,dx = \frac{\left(-2\sqrt{a} - 8\arctan\left(\frac{\sqrt{ia(i+\operatorname{csch}(c+dx))}}{\sqrt{a}}\right)\sqrt{ia(i+\operatorname{csch}(c+dx))} + 5\sqrt{2}\operatorname{arc}\right.}{}$$

input
```
Integrate[(a + I*a*Csch[c + d*x])^(-3/2),x]
```

output
```
((-2*Sqrt[a] - 8*ArcTan[Sqrt[I*a*(I + Csch[c + d*x])]/Sqrt[a]]*Sqrt[I*a*(I
 + Csch[c + d*x])] + 5*Sqrt[2]*ArcTan[Sqrt[I*a*(I + Csch[c + d*x])]/(Sqrt[
2]*Sqrt[a])]*Sqrt[I*a*(I + Csch[c + d*x])] + I*Csch[c + d*x]*(2*Sqrt[a] -
8*ArcTan[Sqrt[I*a*(I + Csch[c + d*x])]/Sqrt[a]]*Sqrt[I*a*(I + Csch[c + d*x
])] + 5*Sqrt[2]*ArcTan[Sqrt[I*a*(I + Csch[c + d*x])]/(Sqrt[2]*Sqrt[a])]*Sq
rt[I*a*(I + Csch[c + d*x])]))*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]))/(
4*a^(3/2)*d*(I + Csch[c + d*x])*Sqrt[a + I*a*Csch[c + d*x]]*(Cosh[(c + d*x
)/2] - I*Sinh[(c + d*x)/2]))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {3042, 4264, 27, 3042, 4408, 26, 3042, 26, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\text{csch}(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a-a\csc(ic+idx))^{3/2}}dx$$

$$\Big\downarrow \text{4264}$$

$$-\frac{\int -\frac{4a-ia\text{csch}(c+dx)}{2\sqrt{i\text{csch}(c+dx)a+a}}dx}{2a^2}-\frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4a-ia\text{csch}(c+dx)}{\sqrt{i\text{csch}(c+dx)a+a}}dx}{4a^2}-\frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc(ic+idx)a+4a}{\sqrt{a-a\csc(ic+idx)}}dx}{4a^2}-\frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4408}$$

$$\frac{5a\int -\frac{i\text{csch}(c+dx)}{\sqrt{i\text{csch}(c+dx)a+a}}dx+4\int \sqrt{i\text{csch}(c+dx)a+a}dx}{4a^2}-\frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx))^{3/2}}$$

$$\Big\downarrow \text{26}$$

$$\frac{4\int \sqrt{i\text{csch}(c+dx)a+a}dx-5ia\int \frac{\text{csch}(c+dx)}{\sqrt{i\text{csch}(c+dx)a+a}}dx}{4a^2}-\frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \sqrt{a-a\csc(ic+idx)}dx-5ia\int \frac{i\csc(ic+idx)}{\sqrt{a-a\csc(ic+idx)}}dx}{4a^2}-\frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx))^{3/2}}$$

$$\Big\downarrow \text{26}$$

$$\frac{5a\int \frac{\csc(ic+idx)}{\sqrt{a-a\csc(ic+idx)}}dx+4\int \sqrt{a-a\csc(ic+idx)}dx}{4a^2}-\frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4261}$$

$$\frac{5a\int \frac{\csc(ic+idx)}{\sqrt{a-a\csc(ic+idx)}}dx-\frac{8ia\int \frac{1}{a-\frac{a^2\coth^2(c+dx)}{i\text{csch}(c+dx)a+a}}d\frac{ia\coth(c+dx)}{\sqrt{i\text{csch}(c+dx)a+a}}}{d}}{4a^2}-\frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx))^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{5a\int \frac{\csc(ic+idx)}{\sqrt{a-a\csc(ic+idx)}}dx+\frac{8\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}}{4a^2}-\frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4282}$$

$$\frac{\frac{10ia\int \frac{1}{2a-\frac{a^2\coth^2(c+dx)}{i\text{csch}(c+dx)a+a}}d\frac{ia\coth(c+dx)}{\sqrt{i\text{csch}(c+dx)a+a}}}{d}+\frac{8\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}}{4a^2}-\frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx))^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{8\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{d}-\frac{5\sqrt{2}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{d}}{4a^2}-\frac{\coth(c+dx)}{2d(a+ia\operatorname{csch}(c+dx))^{3/2}}$$

input
```
Int[(a + I*a*Csch[c + d*x])^(-3/2),x]
```

output
```
((8*Sqrt[a]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/
d - (5*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/(Sqrt[2]*Sqrt[a + I
*a*Csch[c + d*x]])])/d)/(4*a^2) - Coth[c + d*x]/(2*d*(a + I*a*Csch[c + d*x
])^(3/2))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4264
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Simp[1/(a^2*(2*n + 1))
   Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Int
egerQ[2*n]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{1}{(a + ia\ \mathrm{csch}\,(dx + c))^{\frac{3}{2}}} dx$$

input
```
int(1/(a+I*a*csch(d*x+c))^(3/2),x)
```

output
```
int(1/(a+I*a*csch(d*x+c))^(3/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 873 vs. $2(96) = 192$.

Time = 0.13 (sec) , antiderivative size = 873, normalized size of antiderivative = 7.10

$$\int \frac{1}{(a + ia\mathrm{csch}(c + dx))^{3/2}} dx = \text{Too large to display}$$

input
```
integrate(1/(a+I*a*csch(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(5*sqrt(1/2)*(a^2*d*e^(2*d*x + 2*c) + 2*I*a^2*d*e^(d*x + c) - a^2*d)*
sqrt(1/(a^3*d^2))*log(2*(2*sqrt(1/2)*(a^2*d*e^(2*d*x + 2*c) - a^2*d)*sqrt(
a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a^3*d^2)) + a*e^(d*x + c) - I*a)*e^(-d*x
- c)) - 5*sqrt(1/2)*(a^2*d*e^(2*d*x + 2*c) + 2*I*a^2*d*e^(d*x + c) - a^2*d
)*sqrt(1/(a^3*d^2))*log(-2*(2*sqrt(1/2)*(a^2*d*e^(2*d*x + 2*c) - a^2*d)*sq
rt(a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a^3*d^2)) - a*e^(d*x + c) + I*a)*e^(-d
*x - c)) - 2*(a^2*d*e^(2*d*x + 2*c) + 2*I*a^2*d*e^(d*x + c) - a^2*d)*sqrt(
1/(a^3*d^2))*log(2*((a*d*e^(2*d*x + 2*c) - a*d)*sqrt(a/(e^(2*d*x + 2*c) -
1))*sqrt(1/(a^3*d^2)) + e^(d*x + c) + I)*e^(-d*x - c)/(a*d)) + 2*(a^2*d*e^
(2*d*x + 2*c) + 2*I*a^2*d*e^(d*x + c) - a^2*d)*sqrt(1/(a^3*d^2))*log(-2*((
a*d*e^(2*d*x + 2*c) - a*d)*sqrt(a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a^3*d^2))
 - e^(d*x + c) - I)*e^(-d*x - c)/(a*d)) - 2*(a^2*d*e^(2*d*x + 2*c) + 2*I*a
^2*d*e^(d*x + c) - a^2*d)*sqrt(1/(a^3*d^2))*log(2*((a^2*d*e^(2*d*x + 2*c)
- I*a^2*d*e^(d*x + c) - 2*a^2*d)*sqrt(1/(a^3*d^2)) + sqrt(a/(e^(2*d*x + 2*
c) - 1))*(e^(3*d*x + 3*c) - 2*I*e^(2*d*x + 2*c) - e^(d*x + c) + 2*I))*e^(-
2*d*x - 2*c)/(a*d)) + 2*(a^2*d*e^(2*d*x + 2*c) + 2*I*a^2*d*e^(d*x + c) - a
^2*d)*sqrt(1/(a^3*d^2))*log(-2*((a^2*d*e^(2*d*x + 2*c) - I*a^2*d*e^(d*x +
c) - 2*a^2*d)*sqrt(1/(a^3*d^2)) - sqrt(a/(e^(2*d*x + 2*c) - 1))*(e^(3*d*x
+ 3*c) - 2*I*e^(2*d*x + 2*c) - e^(d*x + c) + 2*I))*e^(-2*d*x - 2*c)/(a*d))
 + 2*sqrt(a/(e^(2*d*x + 2*c) - 1))*(e^(3*d*x + 3*c) - I*e^(2*d*x + 2*c)...
```

Sympy [F]

$$\int \frac{1}{(a+ia\operatorname{csch}(c+dx))^{3/2}}\,dx = \int \frac{1}{(ia\operatorname{csch}(c+dx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+I*a*csch(d*x+c))**(3/2),x)
```

output
```
Integral((I*a*csch(c + d*x) + a)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a + i a \operatorname{csch}(c + dx))^{3/2}} \, dx = \int \frac{1}{(i\, a \operatorname{csch}(dx + c) + a)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/(a+I*a*csch(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*csch(d*x + c) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a + i a \operatorname{csch}(c + dx))^{3/2}} \, dx = \int \frac{1}{(i\, a \operatorname{csch}(dx + c) + a)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/(a+I*a*csch(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*csch(d*x + c) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + i a \operatorname{csch}(c + dx))^{3/2}} \, dx = \int \frac{1}{\left(a + \frac{a\,1\mathrm{i}}{\sinh(c+d\,x)}\right)^{3/2}} \, dx$$

input
```
int(1/(a + (a*1i)/sinh(c + d*x))^(3/2),x)
```

output
```
int(1/(a + (a*1i)/sinh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+ia\operatorname{csch}(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\operatorname{csch}(dx+c)i+1}}{\operatorname{csch}(dx+c)^3 i+\operatorname{csch}(dx+c)^2+\operatorname{csch}(dx+c)i+1}dx - \left(\int \frac{\sqrt{\operatorname{csch}(dx+c)i+1}\operatorname{csch}(dx}{\operatorname{csch}(dx+c)^3 i+\operatorname{csch}(dx+c)^2+\operatorname{csch}}\right.\right.}{a^2}$$

input
```
int(1/(a+I*a*csch(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*(int(sqrt(csch(c + d*x)*i + 1)/(csch(c + d*x)**3*i + csch(c + d*x
)**2 + csch(c + d*x)*i + 1),x) - int((sqrt(csch(c + d*x)*i + 1)*csch(c + d
*x))/(csch(c + d*x)**3*i + csch(c + d*x)**2 + csch(c + d*x)*i + 1),x)*i))/
a**2
```

3.56 $\int \sqrt{a-ia\text{csch}(c+dx)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \sqrt{a-ia\text{csch}(c+dx)}\,dx=\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{d}$$

output
```
2*a^(1/2)*arctanh(a^(1/2)*coth(d*x+c)/(a-I*a*csch(d*x+c))^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.00

$$\int \sqrt{a-ia\text{csch}(c+dx)}\,dx$$
$$=-\frac{2(-1)^{3/4}\arctan\left((-1)^{3/4}\sqrt{-i+\text{csch}(c+dx)}\right)\coth(c+dx)\sqrt{a-ia\text{csch}(c+dx)}}{d\sqrt{-i+\text{csch}(c+dx)}(i+\text{csch}(c+dx))}$$

input
```
Integrate[Sqrt[a - I*a*Csch[c + d*x]],x]
```

output
```
(-2*(-1)^(3/4)*ArcTan[(-1)^(3/4)*Sqrt[-I + Csch[c + d*x]]]*Coth[c + d*x]*S
qrt[a - I*a*Csch[c + d*x]])/(d*Sqrt[-I + Csch[c + d*x]]*(I + Csch[c + d*x]
))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a - ia\text{csch}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a + a\csc(ic+idx)}dx$$

$$\Big\downarrow \text{4261}$$

$$\frac{2ia\int \frac{1}{a-\frac{a^2\coth^2(c+dx)}{a-ia\text{csch}(c+dx)}}d\left(-\frac{ia\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{d}$$

input
```
Int[Sqrt[a - I*a*Csch[c + d*x]],x]
```

output
```
(2*Sqrt[a]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a - I*a*Csch[c + d*x]]])/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sqrt{a - ia \operatorname{csch}(dx + c)}dx$$

input
```
int((a-I*a*csch(d*x+c))^(1/2),x)
```

output
```
int((a-I*a*csch(d*x+c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(32) = 64$.

Time = 0.10 (sec) , antiderivative size = 383, normalized size of antiderivative = 9.58

$$\int \sqrt{a - ia\operatorname{csch}(c + dx)}\, dx$$

$$= \frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(\frac{2\left(\left(de^{(2\,dx+2\,c)} - d\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\sqrt{\frac{a}{d^2}} + ae^{(dx+c)} - i\,a\right)e^{(-dx-c)}}{d}\right)$$

$$- \frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(-\frac{2\left(\left(de^{(2\,dx+2\,c)} - d\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\sqrt{\frac{a}{d^2}} - ae^{(dx+c)} + i\,a\right)e^{(-dx-c)}}{d}\right)$$

$$+ \frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(\frac{2\left(\left(ae^{(3\,dx+3\,c)} + 2i\,ae^{(2\,dx+2\,c)} - ae^{(dx+c)} - 2i\,a\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}} + \left(ade^{(2\,dx+2\,c)} + i\,ade^{(dx+c)}\right.\right.}{d}\right.$$

$$- \frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(\frac{2\left(\left(ae^{(3\,dx+3\,c)} + 2i\,ae^{(2\,dx+2\,c)} - ae^{(dx+c)} - 2i\,a\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}} - \left(ade^{(2\,dx+2\,c)} + i\,ade^{(dx+c)}\right.\right.}{d}\right.$$

input
```
integrate((a-I*a*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(a/d^2)*log(2*((d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) - 1
))*sqrt(a/d^2) + a*e^(d*x + c) - I*a)*e^(-d*x - c)/d) - 1/2*sqrt(a/d^2)*lo
g(-2*((d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) - 1))*sqrt(a/d^2) -
a*e^(d*x + c) + I*a)*e^(-d*x - c)/d) + 1/2*sqrt(a/d^2)*log(2*((a*e^(3*d*x
+ 3*c) + 2*I*a*e^(2*d*x + 2*c) - a*e^(d*x + c) - 2*I*a)*sqrt(a/(e^(2*d*x +
 2*c) - 1)) + (a*d*e^(2*d*x + 2*c) + I*a*d*e^(d*x + c) - 2*a*d)*sqrt(a/d^2
))*e^(-2*d*x - 2*c)/d) - 1/2*sqrt(a/d^2)*log(2*((a*e^(3*d*x + 3*c) + 2*I*a
*e^(2*d*x + 2*c) - a*e^(d*x + c) - 2*I*a)*sqrt(a/(e^(2*d*x + 2*c) - 1)) -
(a*d*e^(2*d*x + 2*c) + I*a*d*e^(d*x + c) - 2*a*d)*sqrt(a/d^2))*e^(-2*d*x -
 2*c)/d)
```

Sympy [F]

$$\int \sqrt{a - ia\text{csch}(c + dx)}\, dx = \int \sqrt{-ia \operatorname{csch}(c + dx) + a}\, dx$$

input
```
integrate((a-I*a*csch(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(-I*a*csch(c + d*x) + a), x)
```

Maxima [F]

$$\int \sqrt{a - ia\text{csch}(c + dx)}\, dx = \int \sqrt{-i\, a \operatorname{csch}(dx + c) + a}\, dx$$

input
```
integrate((a-I*a*csch(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-I*a*csch(d*x + c) + a), x)
```

Giac [F]

$$\int \sqrt{a - ia\text{csch}(c + dx)}\, dx = \int \sqrt{-i\, a \operatorname{csch}(dx + c) + a}\, dx$$

input
```
integrate((a-I*a*csch(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-I*a*csch(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a - ia\text{csch}(c + dx)}\, dx = \int \sqrt{a - \frac{a\, 1\text{i}}{\sinh{(c + d\, x)}}}\, dx$$

input
```
int((a - (a*1i)/sinh(c + d*x))^(1/2),x)
```

output
```
int((a - (a*1i)/sinh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a - ia\text{csch}(c + dx)}\, dx = \sqrt{a} \left(\int \sqrt{-\text{csch}\,(dx + c)\, i + 1}dx \right)$$

input
```
int((a-I*a*csch(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt( - csch(c + d*x)*i + 1),x)
```

3.57 $\int \frac{1}{\sqrt{a-ia\,\mathrm{csch}(c+dx)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 91

$$\int \frac{1}{\sqrt{a-iacsch(c+dx)}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a-ia\text{csch}(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
2*arctanh(a^(1/2)*coth(d*x+c)/(a-I*a*csch(d*x+c))^(1/2))/a^(1/2)/d-2^(1/2)
*arctanh(1/2*a^(1/2)*coth(d*x+c)*2^(1/2)/(a-I*a*csch(d*x+c))^(1/2))/a^(1/2
)/d
```

Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.29

$$\int \frac{1}{\sqrt{a-ia\text{csch}(c+dx)}}\,dx = \frac{\sqrt{a}\left(2\arctan\left(\frac{\sqrt{-ia(-i+\text{csch}(c+dx))}}{\sqrt{a}}\right)-\sqrt{2}\arctan\left(\frac{\sqrt{-ia(-i+\text{csch}(c+dx))}}{\sqrt{2}\sqrt{a}}\right)\right)\coth(c+dx)}{d\sqrt{a(-1-i\text{csch}(c+dx))}\sqrt{a-ia\text{csch}(c+dx)}}$$

input
```
Integrate[1/Sqrt[a - I*a*Csch[c + d*x]],x]
```

output
```
(Sqrt[a]*(2*ArcTan[Sqrt[(-I)*a*(-I + Csch[c + d*x])]/Sqrt[a]] - Sqrt[2]*Ar
cTan[Sqrt[(-I)*a*(-I + Csch[c + d*x])]/(Sqrt[2]*Sqrt[a])])*Coth[c + d*x])/
(d*Sqrt[a*(-1 - I*Csch[c + d*x])]*Sqrt[a - I*a*Csch[c + d*x]])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {3042, 4263, 26, 3042, 26, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-ia\text{csch}(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt{a+a\csc(ic+idx)}}dx$$

$$\Big\downarrow \; 4263$$

$$\frac{\int \sqrt{a-ia\text{csch}(c+dx)}dx}{a}-\int -\frac{i\text{csch}(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\operatorname{csch}(c+dx)}{\sqrt{a-ia\operatorname{csch}(c+dx)}}dx+\frac{\int \sqrt{a-ia\operatorname{csch}(c+dx)}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$i\int \frac{i\csc(ic+idx)}{\sqrt{\csc(ic+idx)a+a}}dx+\frac{\int \sqrt{\csc(ic+idx)a+a}dx}{a}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\int \sqrt{\csc(ic+idx)a+a}dx}{a}-\int \frac{\csc(ic+idx)}{\sqrt{\csc(ic+idx)a+a}}dx$$

$$\Big\downarrow \text{ 4261}$$

$$\frac{2i\int \frac{1}{a-\frac{a^2\coth^2(c+dx)}{a-ia\operatorname{csch}(c+dx)}}d\left(-\frac{ia\coth(c+dx)}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{d}-\int \frac{\csc(ic+idx)}{\sqrt{\csc(ic+idx)a+a}}dx$$

$$\Big\downarrow \text{ 216}$$

$$\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}d}-\int \frac{\csc(ic+idx)}{\sqrt{\csc(ic+idx)a+a}}dx$$

$$\Big\downarrow \text{ 4282}$$

$$\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}d}-\frac{2i\int \frac{1}{2a-\frac{a^2\coth^2(c+dx)}{a-ia\operatorname{csch}(c+dx)}}d\left(-\frac{ia\coth(c+dx)}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{d}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a-ia\operatorname{csch}(c+dx)}}\right)}{\sqrt{a}d}$$

input
```
Int[1/Sqrt[a - I*a*Csch[c + d*x]],x]
```

output
```
(2*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a - I*a*Csch[c + d*x]]])/(Sqrt[a]*
d) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/(Sqrt[2]*Sqrt[a - I*a*Csch[c
 + d*x]])])/(Sqrt[a]*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4263
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[1/a   I
nt[Sqrt[a + b*Csc[c + d*x]], x], x] - Simp[b/a   Int[Csc[c + d*x]/Sqrt[a +
b*Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{1}{\sqrt{a - ia \operatorname{csch}(dx + c)}} dx$$

input
```
int(1/(a-I*a*csch(d*x+c))^(1/2),x)
```

output
```
int(1/(a-I*a*csch(d*x+c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(72) = 144$.

Time = 0.12 (sec) , antiderivative size = 551, normalized size of antiderivative = 6.05

$$\int \frac{1}{\sqrt{a - ia\text{csch}(c + dx)}} \, dx = \text{Too large to display}$$

input
```
integrate(1/(a-I*a*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*sqrt(1/(a*d^2))*log(2*(sqrt(2)*(a*d*e^(2*d*x + 2*c) - a*d)*sq
rt(a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) + a*e^(d*x + c) + I*a)*e^(-d*x
 - c)) + 1/2*sqrt(2)*sqrt(1/(a*d^2))*log(-2*(sqrt(2)*(a*d*e^(2*d*x + 2*c)
- a*d)*sqrt(a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) - a*e^(d*x + c) - I*a
)*e^(-d*x - c)) + 1/2*sqrt(1/(a*d^2))*log(2*((d*e^(2*d*x + 2*c) - d)*sqrt(
a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) + e^(d*x + c) - I)*e^(-d*x - c)/d
) - 1/2*sqrt(1/(a*d^2))*log(-2*((d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x +
 2*c) - 1))*sqrt(1/(a*d^2)) - e^(d*x + c) + I)*e^(-d*x - c)/d) + 1/2*sqrt(
1/(a*d^2))*log(2*((a*d*e^(2*d*x + 2*c) + I*a*d*e^(d*x + c) - 2*a*d)*sqrt(1
/(a*d^2)) + sqrt(a/(e^(2*d*x + 2*c) - 1))*(e^(3*d*x + 3*c) + 2*I*e^(2*d*x
+ 2*c) - e^(d*x + c) - 2*I))*e^(-2*d*x - 2*c)/d) - 1/2*sqrt(1/(a*d^2))*log
(-2*((a*d*e^(2*d*x + 2*c) + I*a*d*e^(d*x + c) - 2*a*d)*sqrt(1/(a*d^2)) - s
qrt(a/(e^(2*d*x + 2*c) - 1))*(e^(3*d*x + 3*c) + 2*I*e^(2*d*x + 2*c) - e^(d
*x + c) - 2*I))*e^(-2*d*x - 2*c)/d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a - ia\text{csch}(c + dx)}} \, dx = \int \frac{1}{\sqrt{-ia \operatorname{csch}(c + dx) + a}} \, dx$$

input
```
integrate(1/(a-I*a*csch(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(-I*a*csch(c + d*x) + a), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a - ia\text{csch}(c + dx)}}\,dx = \int \frac{1}{\sqrt{-i\,a\,\text{csch}\,(dx + c) + a}}\,dx$$

input
```
integrate(1/(a-I*a*csch(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-I*a*csch(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a - ia\text{csch}(c + dx)}}\,dx = \int \frac{1}{\sqrt{-i\,a\,\text{csch}\,(dx + c) + a}}\,dx$$

input
```
integrate(1/(a-I*a*csch(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-I*a*csch(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a - ia\text{csch}(c + dx)}}\,dx = \int \frac{1}{\sqrt{a - \frac{a\,1\text{i}}{\sinh(c+d\,x)}}}\,dx$$

input
```
int(1/(a - (a*1i)/sinh(c + d*x))^(1/2),x)
```

output
```
int(1/(a - (a*1i)/sinh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a - ia\text{csch}(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{-\text{csch}(dx+c)i+1}}{\text{csch}(dx+c)^2+1}dx + \left(\int \frac{\sqrt{-\text{csch}(dx+c)i+1}\,\text{csch}(dx+c)}{\text{csch}(dx+c)^2+1}dx\right)i\right)}{a}$$

input
```
int(1/(a-I*a*csch(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(int(sqrt( - csch(c + d*x)*i + 1)/(csch(c + d*x)**2 + 1),x) + int
((sqrt( - csch(c + d*x)*i + 1)*csch(c + d*x))/(csch(c + d*x)**2 + 1),x)*i)
)/a
```

3.58 $\int \sqrt{3+3i\text{csch}(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 23

$$\int \sqrt{3+3i\text{csch}(x)}\,dx = 2\sqrt{3}\text{arctanh}\left(\frac{\coth(x)}{\sqrt{1+i\text{csch}(x)}}\right)$$

output
```
2*3^(1/2)*arctanh(coth(x)/(1+I*csch(x))^(1/2))
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00

$$\int \sqrt{3+3i\text{csch}(x)}\,dx = \frac{2\sqrt{3}\arctan\left(\sqrt{-1+i\text{csch}(x)}\right)\coth(x)}{\sqrt{-1+i\text{csch}(x)}\sqrt{1+i\text{csch}(x)}}$$

input
```
Integrate[Sqrt[3 + (3*I)*Csch[x]],x]
```

output
```
(2*Sqrt[3]*ArcTan[Sqrt[-1 + I*Csch[x]]]*Coth[x])/(Sqrt[-1 + I*Csch[x]]*Sqr
t[1 + I*Csch[x]])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3+3i\text{csch}(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{3-3\csc(ix)}dx$$

$$\Big\downarrow \text{4261}$$

$$-6i\int \frac{1}{3-\frac{3\coth^2(x)}{i\text{csch}(x)+1}}d\frac{i\sqrt{3}\coth(x)}{\sqrt{i\text{csch}(x)+1}}$$

$$\Big\downarrow \text{216}$$

$$2\sqrt{3}\text{arctanh}\left(\frac{\coth(x)}{\sqrt{1+i\text{csch}(x)}}\right)$$

input
```
Int[Sqrt[3 + (3*I)*Csch[x]],x]
```

output
```
2*Sqrt[3]*ArcTanh[Coth[x]/Sqrt[1 + I*Csch[x]]]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sqrt{3+3i\,\mathrm{csch}\,(x)}dx$$

input
```
int((3+3*I*csch(x))^(1/2),x)
```

output
```
int((3+3*I*csch(x))^(1/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(17) = 34.

Time = 0.09 (sec) , antiderivative size = 212, normalized size of antiderivative = 9.22

$$\begin{aligned}\int \sqrt{3+3i\mathrm{csch}(x)}\,dx &= \frac{1}{2}\sqrt{3}\log\left(2\left(\frac{\sqrt{3}\left(\sqrt{3}e^{(2x)}-\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}+3e^{x}+3i\right)e^{(-x)}\right)\\ &-\frac{1}{2}\sqrt{3}\log\left(-2\left(\frac{\sqrt{3}\left(\sqrt{3}e^{(2x)}-\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}-3e^{x}-3i\right)e^{(-x)}\right)\\ &+\frac{1}{2}\sqrt{3}\log\left(6\left(\sqrt{3}e^{(2x)}-i\sqrt{3}e^{x}+\frac{\sqrt{3}\left(e^{(3x)}-2i\,e^{(2x)}-e^{x}+2i\right)}{\sqrt{e^{(2x)}-1}}-2\sqrt{3}\right)e^{(-2x)}\right)\\ &-\frac{1}{2}\sqrt{3}\log\left(-6\left(\sqrt{3}e^{(2x)}-i\sqrt{3}e^{x}-\frac{\sqrt{3}\left(e^{(3x)}-2i\,e^{(2x)}-e^{x}+2i\right)}{\sqrt{e^{(2x)}-1}}-2\sqrt{3}\right)e^{(-2x)}\right)\end{aligned}$$

input
```
integrate((3+3*I*csch(x))^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(3)*log(2*(sqrt(3)*(sqrt(3)*e^(2*x) - sqrt(3))/sqrt(e^(2*x) - 1) +
 3*e^x + 3*I)*e^(-x)) - 1/2*sqrt(3)*log(-2*(sqrt(3)*(sqrt(3)*e^(2*x) - sqr
t(3))/sqrt(e^(2*x) - 1) - 3*e^x - 3*I)*e^(-x)) + 1/2*sqrt(3)*log(6*(sqrt(3
)*e^(2*x) - I*sqrt(3)*e^x + sqrt(3)*(e^(3*x) - 2*I*e^(2*x) - e^x + 2*I)/sq
rt(e^(2*x) - 1) - 2*sqrt(3))*e^(-2*x)) - 1/2*sqrt(3)*log(-6*(sqrt(3)*e^(2*
x) - I*sqrt(3)*e^x - sqrt(3)*(e^(3*x) - 2*I*e^(2*x) - e^x + 2*I)/sqrt(e^(2
*x) - 1) - 2*sqrt(3))*e^(-2*x))
```

Sympy [F]

$$\int \sqrt{3+3i\text{csch}(x)}\,dx = \sqrt{3}\int \sqrt{i\,\text{csch}\,(x)+1}\,dx$$

input
```
integrate((3+3*I*csch(x))**(1/2),x)
```

output
```
sqrt(3)*Integral(sqrt(I*csch(x) + 1), x)
```

Maxima [F]

$$\int \sqrt{3+3i\text{csch}(x)}\,dx = \int \sqrt{3i\,\text{csch}\,(x)+3}\,dx$$

input
```
integrate((3+3*I*csch(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(3*I*csch(x) + 3), x)
```

Giac [F]

$$\int \sqrt{3+3i\text{csch}(x)}\,dx = \int \sqrt{3i\,\text{csch}\,(x)+3}\,dx$$

input
```
integrate((3+3*I*csch(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(3*I*csch(x) + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{3+3i\text{csch}(x)}\,dx = \int \sqrt{3+\frac{3\text{i}}{\sinh\,(x)}}\,dx$$

input
```
int((3i/sinh(x) + 3)^(1/2),x)
```

output
```
int((3i/sinh(x) + 3)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{3+3i\text{csch}(x)}\,dx = \sqrt{3}\left(\int \sqrt{\text{csch}\,(x)\,i+1}dx\right)$$

input
```
int((3+3*I*csch(x))^(1/2),x)
```

output
```
sqrt(3)*int(sqrt(csch(x)*i + 1),x)
```

3.59 $\int \sqrt{3-3i\text{csch}(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 23

$$\int \sqrt{3-3i\text{csch}(x)}\,dx = 2\sqrt{3}\text{arctanh}\left(\frac{\coth(x)}{\sqrt{1-i\text{csch}(x)}}\right)$$

output
```
2*3^(1/2)*arctanh(coth(x)/(1-I*csch(x))^(1/2))
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00

$$\int \sqrt{3-3i\text{csch}(x)}\,dx = \frac{2\sqrt{3}\arctan\left(\sqrt{-1-i\text{csch}(x)}\right)\coth(x)}{\sqrt{-1-i\text{csch}(x)}\sqrt{1-i\text{csch}(x)}}$$

input
```
Integrate[Sqrt[3 - (3*I)*Csch[x]],x]
```

output
```
(2*Sqrt[3]*ArcTan[Sqrt[-1 - I*Csch[x]]]*Coth[x])/(Sqrt[-1 - I*Csch[x]]*Sqr
t[1 - I*Csch[x]])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3-3i\text{csch}(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{3+3\csc(ix)}dx$$

$$\Big\downarrow \text{ 4261}$$

$$6i\int \frac{1}{3-\frac{3\coth^2(x)}{1-i\text{csch}(x)}}d\left(-\frac{i\sqrt{3}\coth(x)}{\sqrt{1-i\text{csch}(x)}}\right)$$

$$\Big\downarrow \text{ 216}$$

$$2\sqrt{3}\text{arctanh}\left(\frac{\coth(x)}{\sqrt{1-i\text{csch}(x)}}\right)$$

input
```
Int[Sqrt[3 - (3*I)*Csch[x]],x]
```

output
```
2*Sqrt[3]*ArcTanh[Coth[x]/Sqrt[1 - I*Csch[x]]]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sqrt{3-3i\,\text{csch}\,(x)}dx$$

input
```
int((3-3*I*csch(x))^(1/2),x)
```

output
```
int((3-3*I*csch(x))^(1/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(17) = 34$.

Time = 0.09 (sec) , antiderivative size = 212, normalized size of antiderivative = 9.22

$$\begin{aligned}\int \sqrt{3-3i\text{csch}(x)}\,dx = &\frac{1}{2}\sqrt{3}\log\left(2\left(\frac{\sqrt{3}\left(\sqrt{3}e^{(2\,x)}-\sqrt{3}\right)}{\sqrt{e^{(2\,x)}-1}}+3\,e^{x}-3i\right)e^{(-x)}\right)\\&-\frac{1}{2}\sqrt{3}\log\left(-2\left(\frac{\sqrt{3}\left(\sqrt{3}e^{(2\,x)}-\sqrt{3}\right)}{\sqrt{e^{(2\,x)}-1}}-3\,e^{x}+3i\right)e^{(-x)}\right)\\&+\frac{1}{2}\sqrt{3}\log\left(6\left(\sqrt{3}e^{(2\,x)}+i\sqrt{3}e^{x}+\frac{\sqrt{3}\left(e^{(3\,x)}+2i\,e^{(2\,x)}-e^{x}-2i\right)}{\sqrt{e^{(2\,x)}-1}}-2\sqrt{3}\right)e^{(-2\,x)}\right)\\&-\frac{1}{2}\sqrt{3}\log\left(-6\left(\sqrt{3}e^{(2\,x)}+i\sqrt{3}e^{x}-\frac{\sqrt{3}\left(e^{(3\,x)}+2i\,e^{(2\,x)}-e^{x}-2i\right)}{\sqrt{e^{(2\,x)}-1}}-2\sqrt{3}\right)e^{(-2\,x)}\right)\end{aligned}$$

input
```
integrate((3-3*I*csch(x))^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(3)*log(2*(sqrt(3)*(sqrt(3)*e^(2*x) - sqrt(3))/sqrt(e^(2*x) - 1) +
 3*e^x - 3*I)*e^(-x)) - 1/2*sqrt(3)*log(-2*(sqrt(3)*(sqrt(3)*e^(2*x) - sqr
t(3))/sqrt(e^(2*x) - 1) - 3*e^x + 3*I)*e^(-x)) + 1/2*sqrt(3)*log(6*(sqrt(3
)*e^(2*x) + I*sqrt(3)*e^x + sqrt(3)*(e^(3*x) + 2*I*e^(2*x) - e^x - 2*I)/sq
rt(e^(2*x) - 1) - 2*sqrt(3))*e^(-2*x)) - 1/2*sqrt(3)*log(-6*(sqrt(3)*e^(2*
x) + I*sqrt(3)*e^x - sqrt(3)*(e^(3*x) + 2*I*e^(2*x) - e^x - 2*I)/sqrt(e^(2
*x) - 1) - 2*sqrt(3))*e^(-2*x))
```

Sympy [F]

$$\int \sqrt{3-3i\text{csch}(x)}\,dx=\sqrt{3}\int \sqrt{-i\operatorname{csch}(x)+1}\,dx$$

input
```
integrate((3-3*I*csch(x))**(1/2),x)
```

output
```
sqrt(3)*Integral(sqrt(-I*csch(x) + 1), x)
```

Maxima [F]

$$\int \sqrt{3-3i\text{csch}(x)}\,dx=\int \sqrt{-3i\operatorname{csch}(x)+3}\,dx$$

input
```
integrate((3-3*I*csch(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-3*I*csch(x) + 3), x)
```

Giac [F]

$$\int \sqrt{3-3i\text{csch}(x)}\,dx = \int \sqrt{-3i\,\text{csch}\,(x)+3}\,dx$$

input
```
integrate((3-3*I*csch(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-3*I*csch(x) + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{3-3i\text{csch}(x)}\,dx = \int \sqrt{3-\frac{3\text{i}}{\sinh\,(x)}}\,dx$$

input
```
int((3 - 3i/sinh(x))^(1/2),x)
```

output
```
int((3 - 3i/sinh(x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{3-3i\text{csch}(x)}\,dx = \sqrt{3}\left(\int \sqrt{-\text{csch}\,(x)\,i+1}dx\right)$$

input
```
int((3-3*I*csch(x))^(1/2),x)
```

output
```
sqrt(3)*int(sqrt( - csch(x)*i + 1),x)
```

3.60 $\int \sqrt{-3+3i\text{csch}(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 23

$$\int \sqrt{-3+3i\text{csch}(x)}\,dx=-2\sqrt{3}\arctan\left(\frac{\coth(x)}{\sqrt{-1+i\text{csch}(x)}}\right)$$

output
```
-2*3^(1/2)*arctan(coth(x)/(-1+I*csch(x))^(1/2))
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00

$$\int \sqrt{-3+3i\text{csch}(x)}\,dx=-\frac{2\sqrt{3}\text{arctanh}\left(\sqrt{1+i\text{csch}(x)}\right)\coth(x)}{\sqrt{-1+i\text{csch}(x)}\sqrt{1+i\text{csch}(x)}}$$

input
```
Integrate[Sqrt[-3 + (3*I)*Csch[x]],x]
```

output
```
(-2*Sqrt[3]*ArcTanh[Sqrt[1 + I*Csch[x]]]*Coth[x])/(Sqrt[-1 + I*Csch[x]]*Sq
rt[1 + I*Csch[x]])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4261, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-3+3i\text{csch}(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-3-3\csc(ix)}dx$$

$$\Big\downarrow \text{4261}$$

$$-6i\int \frac{1}{-\frac{3\coth^2(x)}{i\text{csch}(x)-1}-3}d\frac{i\sqrt{3}\coth(x)}{\sqrt{i\text{csch}(x)-1}}$$

$$\Big\downarrow \text{220}$$

$$-2\sqrt{3}\arctan\left(\frac{\coth(x)}{\sqrt{-1+i\text{csch}(x)}}\right)$$

input
```
Int[Sqrt[-3 + (3*I)*Csch[x]],x]
```

output
```
-2*Sqrt[3]*ArcTan[Coth[x]/Sqrt[-1 + I*Csch[x]]]
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sqrt{-3+3i\,\mathrm{csch}\,(x)}dx$$

input
```
int((-3+3*I*csch(x))^(1/2),x)
```

output
```
int((-3+3*I*csch(x))^(1/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(17) = 34.

Time = 0.09 (sec) , antiderivative size = 218, normalized size of antiderivative = 9.48

$$\int \sqrt{-3+3i\mathrm{csch}(x)}\,dx = -\frac{1}{2}i\,\sqrt{3}\log\left(-2\left(\frac{\sqrt{3}\big(i\,\sqrt{3}e^{(2\,x)}-i\,\sqrt{3}\big)}{\sqrt{e^{(2\,x)}-1}}-3i\,e^{x}-3\right)e^{(-x)}\right) + \frac{1}{2}i\,\sqrt{3}\log\left(-2\left(\frac{\sqrt{3}\big(-i\,\sqrt{3}e^{(2\,x)}+i\,\sqrt{3}\big)}{\sqrt{e^{(2\,x)}-1}}-3i\,e^{x}-3\right)e^{(-x)}\right) - \frac{1}{2}i\,\sqrt{3}\log\left(-6\left(i\,\sqrt{3}e^{(2\,x)}-\sqrt{3}e^{x}+\frac{\sqrt{3}\big(-i\,e^{(3\,x)}+2\,e^{(2\,x)}+i\,e^{x}-2\big)}{\sqrt{e^{(2\,x)}-1}}-2i\,\sqrt{3}\right)e^{(-2\,x)}\right) + \frac{1}{2}i\,\sqrt{3}\log\left(-6\left(-i\,\sqrt{3}e^{(2\,x)}+\sqrt{3}e^{x}+\frac{\sqrt{3}\big(-i\,e^{(3\,x)}+2\,e^{(2\,x)}+i\,e^{x}-2\big)}{\sqrt{e^{(2\,x)}-1}}+2i\,\sqrt{3}\right)e^{(-2\,x)}\right)$$

input
```
integrate((-3+3*I*csch(x))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*I*sqrt(3)*log(-2*(sqrt(3)*(I*sqrt(3)*e^(2*x) - I*sqrt(3))/sqrt(e^(2*x
) - 1) - 3*I*e^x - 3)*e^(-x)) + 1/2*I*sqrt(3)*log(-2*(sqrt(3)*(-I*sqrt(3)*
e^(2*x) + I*sqrt(3))/sqrt(e^(2*x) - 1) - 3*I*e^x - 3)*e^(-x)) - 1/2*I*sqrt
(3)*log(-6*(I*sqrt(3)*e^(2*x) - sqrt(3)*e^x + sqrt(3)*(-I*e^(3*x) + 2*e^(2
*x) + I*e^x - 2)/sqrt(e^(2*x) - 1) - 2*I*sqrt(3))*e^(-2*x)) + 1/2*I*sqrt(3
)*log(-6*(-I*sqrt(3)*e^(2*x) + sqrt(3)*e^x + sqrt(3)*(-I*e^(3*x) + 2*e^(2*
x) + I*e^x - 2)/sqrt(e^(2*x) - 1) + 2*I*sqrt(3))*e^(-2*x))
```

Sympy [F]

$$\int \sqrt{-3+3i\text{csch}(x)}\,dx = \sqrt{3}\int \sqrt{i\,\text{csch}\,(x)-1}\,dx$$

input
```
integrate((-3+3*I*csch(x))**(1/2),x)
```

output
```
sqrt(3)*Integral(sqrt(I*csch(x) - 1), x)
```

Maxima [F]

$$\int \sqrt{-3+3i\text{csch}(x)}\,dx = \int \sqrt{3i\ \text{csch}\,(x)-3}\,dx$$

input
```
integrate((-3+3*I*csch(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(3*I*csch(x) - 3), x)
```

Giac [F]

$$\int \sqrt{-3+3i\text{csch}(x)}\,dx = \int \sqrt{3i\,\text{csch}\,(x) - 3}\,dx$$

input
```
integrate((-3+3*I*csch(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(3*I*csch(x) - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{-3+3i\text{csch}(x)}\,dx = \int \sqrt{-3+\frac{3\text{i}}{\sinh\,(x)}}\,dx$$

input
```
int((3i/sinh(x) - 3)^(1/2),x)
```

output
```
int((3i/sinh(x) - 3)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{-3+3i\text{csch}(x)}\,dx = \sqrt{3}\left(\int \sqrt{\text{csch}\,(x)\,i-1}dx\right)$$

input
```
int((-3+3*I*csch(x))^(1/2),x)
```

output
```
sqrt(3)*int(sqrt(csch(x)*i - 1),x)
```

3.61 $\int \sqrt{-3-3i\text{csch}(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 23

$$\int \sqrt{-3-3i\text{csch}(x)}\,dx = -2\sqrt{3}\arctan\left(\frac{\coth(x)}{\sqrt{-1-i\text{csch}(x)}}\right)$$

output
```
-2*3^(1/2)*arctan(coth(x)/(-1-I*csch(x))^(1/2))
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00

$$\int \sqrt{-3-3i\text{csch}(x)}\,dx = -\frac{2\sqrt{3}\text{arctanh}\left(\sqrt{1-i\text{csch}(x)}\right)\coth(x)}{\sqrt{-1-i\text{csch}(x)}\sqrt{1-i\text{csch}(x)}}$$

input
```
Integrate[Sqrt[-3 - (3*I)*Csch[x]],x]
```

output
```
(-2*Sqrt[3]*ArcTanh[Sqrt[1 - I*Csch[x]]]*Coth[x])/(Sqrt[-1 - I*Csch[x]]*Sq
rt[1 - I*Csch[x]])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4261, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-3-3i\text{csch}(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{-3+3\csc(ix)}dx$$
$$\Big\downarrow \text{4261}$$
$$6i\int \frac{1}{-\frac{3\coth^2(x)}{-i\text{csch}(x)-1}-3}d\left(-\frac{i\sqrt{3}\coth(x)}{\sqrt{-i\text{csch}(x)-1}}\right)$$
$$\Big\downarrow \text{220}$$
$$-2\sqrt{3}\arctan\left(\frac{\coth(x)}{\sqrt{-1-i\text{csch}(x)}}\right)$$

input
```
Int[Sqrt[-3 - (3*I)*Csch[x]],x]
```

output
```
-2*Sqrt[3]*ArcTan[Coth[x]/Sqrt[-1 - I*Csch[x]]]
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sqrt{-3-3i\,\mathrm{csch}\,(x)}dx$$

input
```
int((-3-3*I*csch(x))^(1/2),x)
```

output
```
int((-3-3*I*csch(x))^(1/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(17) = 34$.

Time = 0.09 (sec) , antiderivative size = 218, normalized size of antiderivative = 9.48

$$\begin{aligned}\int \sqrt{-3-3i\mathrm{csch}(x)}\,dx &= -\frac{1}{2}i\sqrt{3}\log\left(-2\left(\frac{\sqrt{3}(i\sqrt{3}e^{(2x)}-i\sqrt{3})}{\sqrt{e^{(2x)}-1}}-3i\,e^{x}+3\right)e^{(-x)}\right)\\&+\frac{1}{2}i\sqrt{3}\log\left(-2\left(\frac{\sqrt{3}(-i\sqrt{3}e^{(2x)}+i\sqrt{3})}{\sqrt{e^{(2x)}-1}}-3i\,e^{x}+3\right)e^{(-x)}\right)\\&-\frac{1}{2}i\sqrt{3}\log\left(-6\left(i\sqrt{3}e^{(2x)}+\sqrt{3}e^{x}+\frac{\sqrt{3}(-i\,e^{(3x)}-2\,e^{(2x)}+i\,e^{x}+2)}{\sqrt{e^{(2x)}-1}}-2i\sqrt{3}\right)e^{(-2x)}\right)\\&+\frac{1}{2}i\sqrt{3}\log\left(-6\left(-i\sqrt{3}e^{(2x)}-\sqrt{3}e^{x}+\frac{\sqrt{3}(-i\,e^{(3x)}-2\,e^{(2x)}+i\,e^{x}+2)}{\sqrt{e^{(2x)}-1}}+2i\sqrt{3}\right)e^{(-2x)}\right)\end{aligned}$$

input
```
integrate((-3-3*I*csch(x))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*I*sqrt(3)*log(-2*(sqrt(3)*(I*sqrt(3)*e^(2*x) - I*sqrt(3))/sqrt(e^(2*x
) - 1) - 3*I*e^x + 3)*e^(-x)) + 1/2*I*sqrt(3)*log(-2*(sqrt(3)*(-I*sqrt(3)*
e^(2*x) + I*sqrt(3))/sqrt(e^(2*x) - 1) - 3*I*e^x + 3)*e^(-x)) - 1/2*I*sqrt
(3)*log(-6*(I*sqrt(3)*e^(2*x) + sqrt(3)*e^x + sqrt(3)*(-I*e^(3*x) - 2*e^(2
*x) + I*e^x + 2)/sqrt(e^(2*x) - 1) - 2*I*sqrt(3))*e^(-2*x)) + 1/2*I*sqrt(3
)*log(-6*(-I*sqrt(3)*e^(2*x) - sqrt(3)*e^x + sqrt(3)*(-I*e^(3*x) - 2*e^(2*
x) + I*e^x + 2)/sqrt(e^(2*x) - 1) + 2*I*sqrt(3))*e^(-2*x))
```

Sympy [F]

$$\int \sqrt{-3-3i\text{csch}(x)}\,dx=\sqrt{3}\int \sqrt{-i\,\text{csch}\,(x)-1}\,dx$$

input
```
integrate((-3-3*I*csch(x))**(1/2),x)
```

output
```
sqrt(3)*Integral(sqrt(-I*csch(x) - 1), x)
```

Maxima [F]

$$\int \sqrt{-3-3i\text{csch}(x)}\,dx=\int \sqrt{-3i\ \text{csch}\,(x)-3}\,dx$$

input
```
integrate((-3-3*I*csch(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-3*I*csch(x) - 3), x)
```

Giac [F]

$$\int \sqrt{-3-3i\operatorname{csch}(x)}\,dx = \int \sqrt{-3i\,\operatorname{csch}(x)-3}\,dx$$

input
```
integrate((-3-3*I*csch(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-3*I*csch(x) - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{-3-3i\operatorname{csch}(x)}\,dx = \int \sqrt{-3-\frac{3i}{\sinh(x)}}\,dx$$

input
```
int((- 3i/sinh(x) - 3)^(1/2),x)
```

output
```
int((- 3i/sinh(x) - 3)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{-3-3i\operatorname{csch}(x)}\,dx = \sqrt{3}\left(\int \sqrt{\operatorname{csch}(x)\,i+1}dx\right)i$$

input
```
int((-3-3*I*csch(x))^(1/2),x)
```

output
```
sqrt(3)*int(sqrt(csch(x)*i + 1),x)*i
```

3.62 $\int \frac{\sinh^4(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 58

$$\int \frac{\sinh^4(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{15ix}{8} - 4\cosh(x) + \frac{4\cosh^3(x)}{3} + \frac{15}{8}i\cosh(x)\sinh(x) - \frac{5}{4}i\cosh(x)\sinh^3(x) - \frac{\cosh(x)\sinh^3(x)}{i+\operatorname{csch}(x)}$$

output
```
-15/8*I*x-4*cosh(x)+4/3*cosh(x)^3+15/8*I*cosh(x)*sinh(x)-5/4*I*cosh(x)*sin
h(x)^3-cosh(x)*sinh(x)^3/(I+csch(x))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

$$\int \frac{\sinh^4(x)}{i+\operatorname{csch}(x)}\,dx = \frac{1}{96}\left(-180ix - 168\cosh(x) + 8\cosh(3x) + \frac{192\sinh\left(\frac{x}{2}\right)}{-i\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)} + 48i\sinh(2x) - 3i\sinh(4x)\right)$$

input
```
Integrate[Sinh[x]^4/(I + Csch[x]),x]
```

output
```
((-180*I)*x - 168*Cosh[x] + 8*Cosh[3*x] + (192*Sinh[x/2])/((-I)*Cosh[x/2]
+ Sinh[x/2]) + (48*I)*Sinh[2*x] - (3*I)*Sinh[4*x])/96
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.231$, Rules used = {3042, 4306, 25, 3042, 4274, 26, 3042, 26, 3113, 2009, 3115, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(x)}{\operatorname{csch}(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(i\csc(ix)+i)\csc(ix)^4}dx$$

$$\Big\downarrow \text{4306}$$

$$\int -\left((5i-4\operatorname{csch}(x))\sinh^4(x)\right)dx-\frac{\sinh^3(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

$$\Big\downarrow \text{25}$$

$$-\int (5i-4\operatorname{csch}(x))\sinh^4(x)dx-\frac{\sinh^3(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{5i-4i\csc(ix)}{\csc(ix)^4}dx-\frac{\sinh^3(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

$$\Big\downarrow \text{4274}$$

$$-5i\int \sinh^4(x)dx+4i\int -i\sinh^3(x)dx-\frac{\sinh^3(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

$$\Big\downarrow \text{26}$$

$$-5i\int \sinh^4(x)dx+4\int \sinh^3(x)dx-\frac{\sinh^3(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

$$\Big\downarrow \text{3042}$$

$$4\int i\sin(ix)^3dx-5i\int\sin(ix)^4dx-\frac{\sinh^3(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

$$\Big\downarrow \text{26}$$

$$4i\int\sin(ix)^3dx-5i\int\sin(ix)^4dx-\frac{\sinh^3(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

$$\Big\downarrow \text{3113}$$

$$-5i\int\sin(ix)^4dx-4\int\left(1-\cosh^2(x)\right)d\cosh(x)-\frac{\sinh^3(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

$$\Big\downarrow \text{2009}$$

$$-5i\int\sin(ix)^4dx-4\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)-\frac{\sinh^3(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

$$\Big\downarrow \text{3115}$$

$$-5i\left(\frac{3}{4}\int-\sinh^2(x)dx+\frac{1}{4}\sinh^3(x)\cosh(x)\right)-4\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)-\frac{\sinh^3(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

$$\Big\downarrow \text{25}$$

$$-5i\left(\frac{1}{4}\sinh^3(x)\cosh(x)-\frac{3}{4}\int\sinh^2(x)dx\right)-4\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)-\frac{\sinh^3(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

$$\Big\downarrow \text{3042}$$

$$-5i\left(\frac{1}{4}\sinh^3(x)\cosh(x)-\frac{3}{4}\int-\sin(ix)^2dx\right)-4\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)-\frac{\sinh^3(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

$$\Big\downarrow \text{25}$$

$$-5i\left(\frac{1}{4}\sinh^3(x)\cosh(x)+\frac{3}{4}\int\sin(ix)^2dx\right)-4\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)-\frac{\sinh^3(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

$$\Big\downarrow \text{3115}$$

$$-5i\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)+\frac{1}{4}\sinh^3(x)\cosh(x)\right)-4\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)-\frac{\sinh^3(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

$$\Big\downarrow \text{24}$$

$$-4\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)-5i\left(\frac{1}{4}\sinh^3(x)\cosh(x)+\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)\right)-\frac{\sinh^3(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

input
```
Int[Sinh[x]^4/(I + Csch[x]),x]
```

output
```
-4*(Cosh[x] - Cosh[x]^3/3) - (Cosh[x]*Sinh[x]^3)/(I + Csch[x]) - (5*I)*((C
osh[x]*Sinh[x]^3)/4 + (3*(x/2 - (Cosh[x]*Sinh[x])/2))/4)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{15ix}{8}-\frac{i\mathrm{e}^{4x}}{64}+\frac{\mathrm{e}^{3x}}{24}+\frac{i\mathrm{e}^{2x}}{4}-\frac{7\,\mathrm{e}^{x}}{8}-\frac{7\,\mathrm{e}^{-x}}{8}-\frac{i\mathrm{e}^{-2x}}{4}+\frac{\mathrm{e}^{-3x}}{24}+\frac{i\mathrm{e}^{-4x}}{64}-\frac{2}{\mathrm{e}^{x}-i}$
default	$-\frac{i}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}+\frac{15i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8}+\frac{\frac{3}{2}+\frac{7i}{8}}{\tanh\left(\frac{x}{2}\right)-1}+\frac{-\frac{1}{2}+\frac{5i}{8}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{-\frac{1}{3}-\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}+\frac{2i}{\tanh\left(\frac{x}{2}\right)-i}-\frac{15i\,\mathrm{l}}{}$
parallelrisch	$\frac{\left(-360i\sinh\left(\frac{x}{2}\right)-360\cosh\left(\frac{x}{2}\right)\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\left(360i\sinh\left(\frac{x}{2}\right)+360\cosh\left(\frac{x}{2}\right)\right)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-432i\cosh\left(\frac{x}{2}\right)-120i\cosh\left(\frac{3x}{2}\right.}{192i\cosh\left(\frac{x}{2}\right)-192\,\mathrm{s}}$

input
```
int(sinh(x)^4/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-15/8*I*x-1/64*I*exp(x)^4+1/24*exp(x)^3+1/4*I*exp(x)^2-7/8*exp(x)-7/8/exp(
x)-1/4*I/exp(x)^2+1/24/exp(x)^3+1/64*I/exp(x)^4-2/(exp(x)-I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.36

$$\int \frac{\sinh^4(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{24\,(15i\,x-7i)e^{(5\,x)}+24\,(15\,x+23)e^{(4\,x)}+3i\,e^{(9\,x)}-5\,e^{(8\,x)}-40i\,e^{(7\,x)}+120\,e^{(6\,x)}-120i\,e^{(3\,x)}+40\,e}{192\,\left(e^{(5\,x)}-i\,e^{(4\,x)}\right)}$$

input
```
integrate(sinh(x)^4/(I+csch(x)),x, algorithm="fricas")
```

output
```
-1/192*(24*(15*I*x - 7*I)*e^(5*x) + 24*(15*x + 23)*e^(4*x) + 3*I*e^(9*x) -
 5*e^(8*x) - 40*I*e^(7*x) + 120*e^(6*x) - 120*I*e^(3*x) + 40*e^(2*x) + 5*I
*e^x - 3)/(e^(5*x) - I*e^(4*x))
```

Sympy [F]

$$\int \frac{\sinh^4(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\sinh^4(x)}{\operatorname{csch}(x)+i}\,dx$$

input
```
integrate(sinh(x)**4/(I+csch(x)),x)
```

output
```
Integral(sinh(x)**4/(csch(x) + I), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22

$$\int \frac{\sinh^4(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{15}{8}i\,x - \frac{-5i\,e^{(-x)}+40\,e^{(-2\,x)}+120i\,e^{(-3\,x)}+552\,e^{(-4\,x)}-3}{16\,\left(12i\,e^{(-4\,x)}+12\,e^{(-5\,x)}\right)} - \frac{7}{8}\,e^{(-x)} - \frac{1}{4}i\,e^{(-2\,x)} + \frac{1}{24}\,e^{(-3\,x)} + \frac{1}{64}i\,e^{(-4\,x)}$$

input
```
integrate(sinh(x)^4/(I+csch(x)),x, algorithm="maxima")
```

output
```
-15/8*I*x - 1/16*(-5*I*e^(-x) + 40*e^(-2*x) + 120*I*e^(-3*x) + 552*e^(-4*x
) - 3)/(12*I*e^(-4*x) + 12*e^(-5*x)) - 7/8*e^(-x) - 1/4*I*e^(-2*x) + 1/24*
e^(-3*x) + 1/64*I*e^(-4*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{\sinh^4(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{15}{8}i\,x - \frac{\left(552\,e^{(4\,x)} - 120i\,e^{(3\,x)} + 40\,e^{(2\,x)} + 5i\,e^{x} - 3\right)e^{(-4\,x)}}{192\,(e^{x} - i)} - \frac{1}{64}i\,e^{(4\,x)} + \frac{1}{24}\,e^{(3\,x)} + \frac{1}{4}i\,e^{(2\,x)} - \frac{7}{8}\,e^{x}$$

input
```
integrate(sinh(x)^4/(I+csch(x)),x, algorithm="giac")
```

output
```
-15/8*I*x - 1/192*(552*e^(4*x) - 120*I*e^(3*x) + 40*e^(2*x) + 5*I*e^x - 3)
*e^(-4*x)/(e^x - I) - 1/64*I*e^(4*x) + 1/24*e^(3*x) + 1/4*I*e^(2*x) - 7/8*
e^x
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

$$\int \frac{\sinh^4(x)}{i+\operatorname{csch}(x)}\,dx = \frac{e^{-3\,x}}{24} - \frac{7\,e^{-x}}{8} - \frac{e^{-2\,x}\,1i}{4} + \frac{e^{2\,x}\,1i}{4} - \frac{x\,15i}{8} + \frac{e^{3\,x}}{24} + \frac{e^{-4\,x}\,1i}{64} - \frac{e^{4\,x}\,1i}{64} - \frac{7\,e^{x}}{8} - \frac{2}{e^{x} - i}$$

input
```
int(sinh(x)^4/(1/sinh(x) + 1i),x)
```

output
```
(exp(2*x)*1i)/4 - (7*exp(-x))/8 - (exp(-2*x)*1i)/4 - (x*15i)/8 + exp(-3*x)
/24 + exp(3*x)/24 + (exp(-4*x)*1i)/64 - (exp(4*x)*1i)/64 - (7*exp(x))/8 -
2/(exp(x) - 1i)
```

Reduce [F]

$$\int \frac{\sinh^4(x)}{i + \operatorname{csch}(x)}\,dx = \int \frac{\sinh(x)^4}{\operatorname{csch}(x) + i}dx$$

input
```
int(sinh(x)^4/(I+csch(x)),x)
```

output
```
int(sinh(x)**4/(csch(x) + i),x)
```

3.63 $\int \frac{\sinh^3(x)}{i+\operatorname{csch}(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{\sinh^3(x)}{i+\operatorname{csch}(x)} \, dx = -\frac{3x}{2}+4i\cosh(x)-\frac{4}{3}i\cosh^3(x)+\frac{3}{2}\cosh(x)\sinh(x)-\frac{\cosh(x)\sinh^2(x)}{i+\operatorname{csch}(x)}$$

output
```
-3/2*x+4*I*cosh(x)-4/3*I*cosh(x)^3+3/2*cosh(x)*sinh(x)-cosh(x)*sinh(x)^2/(
I+csch(x))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \frac{\sinh^3(x)}{i+\operatorname{csch}(x)} \, dx = \frac{1}{12}\left(21i\cosh(x)-i\cosh(3x)+3\left(-6x+\frac{8\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)}+\sinh(2x)\right)\right)$$

input
```
Integrate[Sinh[x]^3/(I + Csch[x]),x]
```

output
```
((21*I)*Cosh[x] - I*Cosh[3*x] + 3*(-6*x + (8*Sinh[x/2])/(Cosh[x/2] + I*Sin
h[x/2]) + Sinh[2*x]))/12
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.308, Rules used = {3042, 26, 26, 4306, 26, 3042, 26, 4274, 25, 26, 3042, 25, 26, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{\operatorname{csch}(x)+i}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i}{(i\csc(ix)+i)\csc(ix)^3}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int -\frac{i}{\csc(ix)^3(\csc(ix)+1)}dx$$

$$\Big\downarrow \text{ 26}$$

$$\int \frac{1}{\csc(ix)^3(1+\csc(ix))}dx$$

$$\Big\downarrow \text{ 4306}$$

$$\frac{i\sinh^2(x)\cosh(x)}{1-i\operatorname{csch}(x)}-\int i(3i\operatorname{csch}(x)+4)\sinh^3(x)dx$$

$$\Big\downarrow \text{ 26}$$

$$\frac{i\sinh^2(x)\cosh(x)}{1-i\operatorname{csch}(x)}-i\int (3i\operatorname{csch}(x)+4)\sinh^3(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{i\sinh^2(x)\cosh(x)}{1-i\operatorname{csch}(x)}-i\int \frac{i(4-3\csc(ix))}{\csc(ix)^3}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{4-3\csc(ix)}{\csc(ix)^3}dx+\frac{i\sinh^2(x)\cosh(x)}{1-i\text{csch}(x)}$$

$$\Big\downarrow \text{4274}$$

$$4\int -i\sinh^3(x)dx-3\int -\sinh^2(x)dx+\frac{i\sinh^2(x)\cosh(x)}{1-i\text{csch}(x)}$$

$$\Big\downarrow \text{25}$$

$$4\int -i\sinh^3(x)dx+3\int \sinh^2(x)dx+\frac{i\sinh^2(x)\cosh(x)}{1-i\text{csch}(x)}$$

$$\Big\downarrow \text{26}$$

$$-4i\int \sinh^3(x)dx+3\int \sinh^2(x)dx+\frac{i\sinh^2(x)\cosh(x)}{1-i\text{csch}(x)}$$

$$\Big\downarrow \text{3042}$$

$$3\int -\sin(ix)^2dx-4i\int i\sin(ix)^3dx+\frac{i\sinh^2(x)\cosh(x)}{1-i\text{csch}(x)}$$

$$\Big\downarrow \text{25}$$

$$-3\int \sin(ix)^2dx-4i\int i\sin(ix)^3dx+\frac{i\sinh^2(x)\cosh(x)}{1-i\text{csch}(x)}$$

$$\Big\downarrow \text{26}$$

$$-3\int \sin(ix)^2dx+4\int \sin(ix)^3dx+\frac{i\sinh^2(x)\cosh(x)}{1-i\text{csch}(x)}$$

$$\Big\downarrow \text{3113}$$

$$-3\int \sin(ix)^2dx+4i\int \left(1-\cosh^2(x)\right)d\cosh(x)+\frac{i\sinh^2(x)\cosh(x)}{1-i\text{csch}(x)}$$

$$\Big\downarrow \text{2009}$$

$$-3\int \sin(ix)^2dx+4i\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)+\frac{i\sinh^2(x)\cosh(x)}{1-i\text{csch}(x)}$$

$$\Big\downarrow \text{3115}$$

$$-3\left(\frac{\int 1dx}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)+4i\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)+\frac{i\sinh^2(x)\cosh(x)}{1-i\text{csch}(x)}$$

$$\Big\downarrow \text{24}$$

$$4i\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)-3\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)+\frac{i\sinh^2(x)\cosh(x)}{1-i\text{csch}(x)}$$

input
```
Int[Sinh[x]^3/(I + Csch[x]),x]
```

output
```
(4*I)*(Cosh[x] - Cosh[x]^3/3) + (I*Cosh[x]*Sinh[x]^2)/(1 - I*Csch[x]) - 3*
(x/2 - (Cosh[x]*Sinh[x])/2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{3x}{2}-\frac{i\mathrm{e}^{3x}}{24}+\frac{\mathrm{e}^{2x}}{8}+\frac{7i\mathrm{e}^{x}}{8}+\frac{7i\mathrm{e}^{-x}}{8}-\frac{\mathrm{e}^{-2x}}{8}-\frac{i\mathrm{e}^{-3x}}{24}+\frac{2i}{\mathrm{e}^{x}-i}$
default	$-\frac{i}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{\frac{1}{2}+\frac{3i}{2}}{\tanh\left(\frac{x}{2}\right)+1}+\frac{-\frac{1}{2}+\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}+\frac{2}{\tanh\left(\frac{x}{2}\right)-i}+\frac{i}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}+\frac{}{(\tanh}$
parallelrisch	$\frac{\left(36i\sinh\left(\frac{x}{2}\right)+36\cosh\left(\frac{x}{2}\right)\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\left(-36i\sinh\left(\frac{x}{2}\right)-36\cosh\left(\frac{x}{2}\right)\right)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+85i\cosh\left(\frac{x}{2}\right)+18i\cosh\left(\frac{3x}{2}\right)+2i\,c}{24\cosh\left(\frac{x}{2}\right)+24i\sinh\left(\frac{x}{2}\right)}$

input
```
int(sinh(x)^3/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-3/2*x-1/24*I*exp(x)^3+1/8*exp(x)^2+7/8*I*exp(x)+7/8*I/exp(x)-1/8/exp(x)^2
-1/24*I/exp(x)^3+2*I/(exp(x)-I)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.46

$$\int \frac{\sinh^3(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{3\,(12\,x-7)e^{(4\,x)}+3\,(-12i\,x-23i)e^{(3\,x)}+i\,e^{(7\,x)}-2\,e^{(6\,x)}-18i\,e^{(5\,x)}-18\,e^{(2\,x)}-2i\,e^{x}+1}{24\,\left(e^{(4\,x)}-i\,e^{(3\,x)}\right)}$$

input
```
integrate(sinh(x)^3/(I+csch(x)),x, algorithm="fricas")
```

output
```
-1/24*(3*(12*x - 7)*e^(4*x) + 3*(-12*I*x - 23*I)*e^(3*x) + I*e^(7*x) - 2*e
^(6*x) - 18*I*e^(5*x) - 18*e^(2*x) - 2*I*e^x + 1)/(e^(4*x) - I*e^(3*x))
```

Sympy [F]

$$\int \frac{\sinh^3(x)}{i + \operatorname{csch}(x)}\,dx = \int \frac{\sinh^3{(x)}}{\operatorname{csch}{(x)} + i}\,dx$$

input
```
integrate(sinh(x)**3/(I+csch(x)),x)
```

output
```
Integral(sinh(x)**3/(csch(x) + I), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

$$\int \frac{\sinh^3(x)}{i + \operatorname{csch}(x)}\,dx = -\frac{3}{2}\,x + \frac{2i\,e^{(-x)} - 18\,e^{(-2\,x)} + 69i\,e^{(-3\,x)} + 1}{8\,(3i\,e^{(-3\,x)} + 3\,e^{(-4\,x)})} + \frac{7}{8}i\,e^{(-x)} - \frac{1}{8}\,e^{(-2\,x)} - \frac{1}{24}i\,e^{(-3\,x)}$$

input
```
integrate(sinh(x)^3/(I+csch(x)),x, algorithm="maxima")
```

output
```
-3/2*x + 1/8*(2*I*e^(-x) - 18*e^(-2*x) + 69*I*e^(-3*x) + 1)/(3*I*e^(-3*x)
+ 3*e^(-4*x)) + 7/8*I*e^(-x) - 1/8*e^(-2*x) - 1/24*I*e^(-3*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \frac{\sinh^3(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{3}{2}\,x - \frac{\left(-69i\,e^{(3\,x)} - 18\,e^{(2\,x)} - 2i\,e^{x} + 1\right)e^{(-3\,x)}}{24\,(e^{x}-i)} - \frac{1}{24}i\,e^{(3\,x)} + \frac{1}{8}\,e^{(2\,x)} + \frac{7}{8}i\,e^{x}$$

input
```
integrate(sinh(x)^3/(I+csch(x)),x, algorithm="giac")
```

output
```
-3/2*x - 1/24*(-69*I*e^(3*x) - 18*e^(2*x) - 2*I*e^x + 1)*e^(-3*x)/(e^x - I
) - 1/24*I*e^(3*x) + 1/8*e^(2*x) + 7/8*I*e^x
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{\sinh^3(x)}{i+\operatorname{csch}(x)}\,dx = \frac{e^{2\,x}}{8} + \frac{e^{-x}\,7i}{8} - \frac{e^{-2\,x}}{8} - \frac{3\,x}{2} - \frac{e^{-3\,x}\,1i}{24} - \frac{e^{3\,x}\,1i}{24} + \frac{e^{x}\,7i}{8} + \frac{2i}{e^{x}-i}$$

input
```
int(sinh(x)^3/(1/sinh(x) + 1i),x)
```

output
```
(exp(-x)*7i)/8 - (3*x)/2 - exp(-2*x)/8 + exp(2*x)/8 - (exp(-3*x)*1i)/24 -
(exp(3*x)*1i)/24 + (exp(x)*7i)/8 + 2i/(exp(x) - 1i)
```

Reduce [F]

$$\int \frac{\sinh^3(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\sinh(x)^3}{\operatorname{csch}(x)+i}dx$$

input
```
int(sinh(x)^3/(I+csch(x)),x)
```

output
```
int(sinh(x)**3/(csch(x) + i),x)
```

3.64 $\int \frac{\sinh^2(x)}{i+\operatorname{csch}(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{\sinh^2(x)}{i+\operatorname{csch}(x)} \, dx = \frac{3ix}{2} + 2\cosh(x) - \frac{3}{2} i \cosh(x)\sinh(x) - \frac{\cosh(x)\sinh(x)}{i+\operatorname{csch}(x)}$$

output
```
3/2*I*x+2*cosh(x)-3/2*I*cosh(x)*sinh(x)-cosh(x)*sinh(x)/(I+csch(x))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \frac{\sinh^2(x)}{i+\operatorname{csch}(x)} \, dx = \cosh(x) + \frac{1}{4} i \left(6x - \frac{8\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)} - \sinh(2x)\right)$$

input
```
Integrate[Sinh[x]^2/(I + Csch[x]),x]
```

output
```
Cosh[x] + (I/4)*(6*x - (8*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x/2]) - Sinh[2*x]
)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.154, Rules used = {3042, 25, 26, 4306, 3042, 25, 4274, 25, 26, 3042, 25, 26, 3115, 24, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{\text{csch}(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{(i\csc(ix)+i)\csc(ix)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{i}{\csc(ix)^2(\csc(ix)+1)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\csc(ix)^2(\csc(ix)+1)}dx$$

$$\Big\downarrow \text{4306}$$

$$i\left(\frac{\sinh(x)\cosh(x)}{1-i\text{csch}(x)}-\int (2i\text{csch}(x)+3)\sinh^2(x)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{\sinh(x)\cosh(x)}{1-i\text{csch}(x)}-\int -\frac{3-2\csc(ix)}{\csc(ix)^2}dx\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(\int \frac{3-2\csc(ix)}{\csc(ix)^2}dx+\frac{\sinh(x)\cosh(x)}{1-i\text{csch}(x)}\right)$$

$$\Big\downarrow \text{4274}$$

$$i\left(3\int -\sinh^2(x)dx-2\int i\sinh(x)dx+\frac{\sinh(x)\cosh(x)}{1-i\text{csch}(x)}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-3\int \sinh^2(x)dx-2\int i\sinh(x)dx+\frac{\sinh(x)\cosh(x)}{1-i\text{csch}(x)}\right)$$

$$\Big\downarrow \text{ 26}$$

$$i\left(-3\int \sinh^2(x)dx-2i\int \sinh(x)dx+\frac{\sinh(x)\cosh(x)}{1-i\text{csch}(x)}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$i\left(-2i\int -i\sin(ix)dx-3\int -\sin(ix)^2dx+\frac{\sinh(x)\cosh(x)}{1-i\text{csch}(x)}\right)$$

$$\Big\downarrow \text{ 25}$$

$$i\left(-2i\int -i\sin(ix)dx+3\int \sin(ix)^2dx+\frac{\sinh(x)\cosh(x)}{1-i\text{csch}(x)}\right)$$

$$\Big\downarrow \text{ 26}$$

$$i\left(-2\int \sin(ix)dx+3\int \sin(ix)^2dx+\frac{\sinh(x)\cosh(x)}{1-i\text{csch}(x)}\right)$$

$$\Big\downarrow \text{ 3115}$$

$$i\left(-2\int \sin(ix)dx+3\left(\frac{\int 1dx}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)+\frac{\sinh(x)\cosh(x)}{1-i\text{csch}(x)}\right)$$

$$\Big\downarrow \text{ 24}$$

$$i\left(-2\int \sin(ix)dx+3\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)+\frac{\sinh(x)\cosh(x)}{1-i\text{csch}(x)}\right)$$

$$\Big\downarrow \text{ 3118}$$

$$i\left(-2i\cosh(x)+3\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)+\frac{\sinh(x)\cosh(x)}{1-i\text{csch}(x)}\right)$$

input
```
Int[Sinh[x]^2/(I + Csch[x]),x]
```

output
```
I*((-2*I)*Cosh[x] + (Cosh[x]*Sinh[x])/(1 - I*Csch[x]) + 3*(x/2 - (Cosh[x]*
Sinh[x])/2))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

method	result
risch	$\frac{3ix}{2}-\frac{i\mathrm{e}^{2x}}{8}+\frac{\mathrm{e}^{x}}{2}+\frac{\mathrm{e}^{-x}}{2}+\frac{i\mathrm{e}^{-2x}}{8}+\frac{2}{\mathrm{e}^{x}-i}$
default	$-\frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{-1-\frac{i}{2}}{\tanh\left(\frac{x}{2}\right)-1}-\frac{2i}{\tanh\left(\frac{x}{2}\right)-i}+\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}+$
parallelrisch	$\frac{\left(12i\sinh\left(\frac{x}{2}\right)+12\cosh\left(\frac{x}{2}\right)\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\left(-12i\sinh\left(\frac{x}{2}\right)-12\cosh\left(\frac{x}{2}\right)\right)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+8i\cosh\left(\frac{x}{2}\right)+3i\cosh\left(\frac{3x}{2}\right)+i\cosh}{8i\cosh\left(\frac{x}{2}\right)-8\sinh\left(\frac{x}{2}\right)}$

input
```
int(sinh(x)^2/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
3/2*I*x-1/8*I*exp(x)^2+1/2*exp(x)+1/2/exp(x)+1/8*I/exp(x)^2+2/(exp(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(26) = 52.

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.53

$$\int \frac{\sinh^2(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{4\,(-3i\,x+i)e^{(3\,x)}-4\,(3\,x+5)e^{(2\,x)}+i\,e^{(5\,x)}-3\,e^{(4\,x)}+3i\,e^{x}-1}{8\,\left(e^{(3\,x)}-i\,e^{(2\,x)}\right)}$$

input
```
integrate(sinh(x)^2/(I+csch(x)),x, algorithm="fricas")
```

output
```
-1/8*(4*(-3*I*x + I)*e^(3*x) - 4*(3*x + 5)*e^(2*x) + I*e^(5*x) - 3*e^(4*x)
 + 3*I*e^x - 1)/(e^(3*x) - I*e^(2*x))
```

Sympy [F]

$$\int \frac{\sinh^2(x)}{i + \operatorname{csch}(x)}\,dx = \int \frac{\sinh^2(x)}{\operatorname{csch}(x) + i}\,dx$$

input
```
integrate(sinh(x)**2/(I+csch(x)),x)
```

output
```
Integral(sinh(x)**2/(csch(x) + I), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31

$$\int \frac{\sinh^2(x)}{i + \operatorname{csch}(x)}\,dx = \frac{3}{2}i\,x + \frac{3i\,e^{(-x)} + 20\,e^{(-2\,x)} + 1}{4\,\left(2i\,e^{(-2\,x)} + 2\,e^{(-3\,x)}\right)} + \frac{1}{2}\,e^{(-x)} + \frac{1}{8}i\,e^{(-2\,x)}$$

input
```
integrate(sinh(x)^2/(I+csch(x)),x, algorithm="maxima")
```

output
```
3/2*I*x + 1/4*(3*I*e^(-x) + 20*e^(-2*x) + 1)/(2*I*e^(-2*x) + 2*e^(-3*x)) +
 1/2*e^(-x) + 1/8*I*e^(-2*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\sinh^2(x)}{i + \operatorname{csch}(x)}\,dx = \frac{3}{2}i\,x + \frac{\left(20\,e^{(2\,x)} - 3i\,e^{x} + 1\right)e^{(-2\,x)}}{8\,(e^{x} - i)} - \frac{1}{8}i\,e^{(2\,x)} + \frac{1}{2}\,e^{x}$$

input
```
integrate(sinh(x)^2/(I+csch(x)),x, algorithm="giac")
```

output
```
3/2*I*x + 1/8*(20*e^(2*x) - 3*I*e^x + 1)*e^(-2*x)/(e^x - I) - 1/8*I*e^(2*x
) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\sinh^2(x)}{i+\operatorname{csch}(x)}\,dx = \frac{x\,3\mathrm{i}}{2}+\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{-2\,x}\,1\mathrm{i}}{8}-\frac{\mathrm{e}^{2\,x}\,1\mathrm{i}}{8}+\frac{\mathrm{e}^{x}}{2}+\frac{2}{\mathrm{e}^{x}-\mathrm{i}}$$

input
```
int(sinh(x)^2/(1/sinh(x) + 1i),x)
```

output
```
(x*3i)/2 + exp(-x)/2 + (exp(-2*x)*1i)/8 - (exp(2*x)*1i)/8 + exp(x)/2 + 2/(
exp(x) - 1i)
```

Reduce [F]

$$\int \frac{\sinh^2(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\sinh(x)^2}{\operatorname{csch}(x)+i}dx$$

input
```
int(sinh(x)^2/(I+csch(x)),x)
```

output
```
int(sinh(x)**2/(csch(x) + i),x)
```

3.65 $\int \frac{\sinh(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{\sinh(x)}{i+\operatorname{csch}(x)}\,dx = x-2i\cosh(x)-\frac{\cosh(x)}{i+\operatorname{csch}(x)}$$

output
```
x-2*I*cosh(x)-cosh(x)/(I+csch(x))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{\sinh(x)}{i+\operatorname{csch}(x)}\,dx = x-i\cosh(x)-\frac{2\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)}$$

input
```
Integrate[Sinh[x]/(I + Csch[x]),x]
```

output
```
x - I*Cosh[x] - (2*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x/2])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.182, Rules used = {3042, 26, 26, 4306, 26, 3042, 26, 4274, 24, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{\text{csch}(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{(i\csc(ix)+i)\csc(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int -\frac{i}{\csc(ix)(\csc(ix)+1)}dx$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{1}{\csc(ix)(\csc(ix)+1)}dx$$

$$\Big\downarrow \text{4306}$$

$$\int -i(i\text{csch}(x)+2)\sinh(x)dx+\frac{i\cosh(x)}{1-i\text{csch}(x)}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\cosh(x)}{1-i\text{csch}(x)}-i\int (i\text{csch}(x)+2)\sinh(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\cosh(x)}{1-i\text{csch}(x)}-i\int -\frac{i(2-\csc(ix))}{\csc(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$\frac{i\cosh(x)}{1-i\text{csch}(x)}-\int \frac{2-\csc(ix)}{\csc(ix)}dx$$

$$\Big\downarrow \text{4274}$$

$$\int 1dx-2\int i\sinh(x)dx+\frac{i\cosh(x)}{1-i\text{csch}(x)}$$

$$\Big\downarrow \text{ 24}$$

$$-2\int i\sinh(x)dx+x+\frac{i\cosh(x)}{1-i\text{csch}(x)}$$

$$\Big\downarrow \text{ 26}$$

$$-2i\int \sinh(x)dx+x+\frac{i\cosh(x)}{1-i\text{csch}(x)}$$

$$\Big\downarrow \text{ 3042}$$

$$-2i\int -i\sin(ix)dx+x+\frac{i\cosh(x)}{1-i\text{csch}(x)}$$

$$\Big\downarrow \text{ 26}$$

$$-2\int \sin(ix)dx+x+\frac{i\cosh(x)}{1-i\text{csch}(x)}$$

$$\Big\downarrow \text{ 3118}$$

$$x-2i\cosh(x)+\frac{i\cosh(x)}{1-i\text{csch}(x)}$$

input
```
Int[Sinh[x]/(I + Csch[x]),x]
```

output
```
x - (2*I)*Cosh[x] + (I*Cosh[x])/(1 - I*Csch[x])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

method	result	size
risch	$x-\frac{i\,\mathrm{e}^{x}}{2}-\frac{i\,\mathrm{e}^{-x}}{2}-\frac{2 i}{\mathrm{e}^{x}-i}$	25
default	$\frac{i}{\tanh\left(\frac{x}{2}\right)-1}-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{i}{\tanh\left(\frac{x}{2}\right)+1}+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{2}{\tanh\left(\frac{x}{2}\right)-i}$	51
parallelrisch	$\frac{(2 i x+7)\cosh\left(\frac{x}{2}\right)+(i-2 x)\sinh\left(\frac{x}{2}\right)+i\sinh\left(\frac{3 x}{2}\right)+\cosh\left(\frac{3 x}{2}\right)}{2 i\cosh\left(\frac{x}{2}\right)-2\sinh\left(\frac{x}{2}\right)}$	52

input
```
int(sinh(x)/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
x-1/2*I*exp(x)-1/2*I/exp(x)-2*I/(exp(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(16) = 32.

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \frac{\sinh(x)}{i+\operatorname{csch}(x)}\,dx = \frac{(2\,x-1)e^{(2\,x)}+(-2i\,x-5i)e^{x}-i\,e^{(3\,x)}-1}{2\,(e^{(2\,x)}-i\,e^{x})}$$

input
```
integrate(sinh(x)/(I+csch(x)),x, algorithm="fricas")
```

output
```
1/2*((2*x - 1)*e^(2*x) + (-2*I*x - 5*I)*e^x - I*e^(3*x) - 1)/(e^(2*x) - I*
e^x)
```

Sympy [F]

$$\int \frac{\sinh(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\sinh(x)}{\operatorname{csch}(x)+i}\,dx$$

input
```
integrate(sinh(x)/(I+csch(x)),x)
```

output
```
Integral(sinh(x)/(csch(x) + I), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{\sinh(x)}{i+\operatorname{csch}(x)}\,dx = x-\frac{5i\,e^{(-x)}-1}{2\,(i\,e^{(-x)}+e^{(-2\,x)})}-\frac{1}{2}i\,e^{(-x)}$$

input
```
integrate(sinh(x)/(I+csch(x)),x, algorithm="maxima")
```

output
```
x - 1/2*(5*I*e^(-x) - 1)/(I*e^(-x) + e^(-2*x)) - 1/2*I*e^(-x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\sinh(x)}{i+\operatorname{csch}(x)}\,dx = x + \frac{(-5i\,e^{x}-1)e^{(-x)}}{2\,(e^{x}-i)} - \frac{1}{2}i\,e^{x}$$

input
```
integrate(sinh(x)/(I+csch(x)),x, algorithm="giac")
```

output
```
x + 1/2*(-5*I*e^x - 1)*e^(-x)/(e^x - I) - 1/2*I*e^x
```

Mupad [B] (verification not implemented)

Time = 2.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\sinh(x)}{i+\operatorname{csch}(x)}\,dx = x - \frac{e^{-x}\,1\mathrm{i}}{2} - \frac{e^{x}\,1\mathrm{i}}{2} - \frac{2\mathrm{i}}{e^{x}-\mathrm{i}}$$

input
```
int(sinh(x)/(1/sinh(x) + 1i),x)
```

output
```
x - (exp(-x)*1i)/2 - (exp(x)*1i)/2 - 2i/(exp(x) - 1i)
```

Reduce [F]

$$\int \frac{\sinh(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\sinh(x)}{\operatorname{csch}(x)+i}dx$$

input
```
int(sinh(x)/(I+csch(x)),x)
```

output
```
int(sinh(x)/(csch(x) + i),x)
```

3.66 $\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx = \frac{i\coth(x)}{i+\operatorname{csch}(x)}$$

output
```
I*coth(x)/(I+csch(x))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx = \frac{2\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)}$$

input
```
Integrate[Csch[x]/(I + Csch[x]),x]
```

output
```
(2*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x/2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 26, 26, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}(x)}{\text{csch}(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\csc(ix)}{i\csc(ix)+i}dx$$

$$\Big\downarrow \text{26}$$

$$i\int -\frac{i\csc(ix)}{\csc(ix)+1}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\csc(ix)}{1+\csc(ix)}dx$$

$$\Big\downarrow \text{4281}$$

$$\frac{\coth(x)}{1-i\text{csch}(x)}$$

input
```
Int[Csch[x]/(I + Csch[x]),x]
```

output
```
Coth[x]/(1 - I*Csch[x])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{2i}{{\mathrm e}^{x}-i}$	11
default	$\frac{2}{\tanh\left(\frac{x}{2}\right)-i}$	12
parallelrisch	$-\frac{2}{-\tanh\left(\frac{x}{2}\right)+i}$	14

input
```
int(csch(x)/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
2*I/(exp(x)-I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx = \frac{2i}{e^x-i}$$

input
```
integrate(csch(x)/(I+csch(x)),x, algorithm="fricas")
```

output
```
2*I/(e^x - I)
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{csch}(x)}{\operatorname{csch}(x)+i}\,dx$$

input
```
integrate(csch(x)/(I+csch(x)),x)
```

output
```
Integral(csch(x)/(csch(x) + I), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{2}{i\,e^{(-x)}-1}$$

input
```
integrate(csch(x)/(I+csch(x)),x, algorithm="maxima")
```

output
```
-2/(I*e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx = \frac{2i}{e^x-i}$$

input
```
integrate(csch(x)/(I+csch(x)),x, algorithm="giac")
```

output
```
2*I/(e^x - I)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\mathrm{csch}(x)}{i + \mathrm{csch}(x)}\, dx = \frac{2\mathrm{i}}{\mathrm{e}^x - \mathrm{i}}$$

input
```
int(1/(sinh(x)*(1/sinh(x) + 1i)),x)
```

output
```
2i/(exp(x) - 1i)
```

Reduce [F]

$$\int \frac{\mathrm{csch}(x)}{i + \mathrm{csch}(x)}\, dx = -\left(\int \frac{1}{\mathrm{csch}\,(x) + i} dx\right) i + x$$

input
```
int(csch(x)/(I+csch(x)),x)
```

output
```
- int(1/(csch(x) + i),x)*i + x
```

3.67 $\int \frac{\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 17

$$\int \frac{\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)} \, dx = -\operatorname{arctanh}(\cosh(x)) + \frac{\coth(x)}{i+\operatorname{csch}(x)}$$

output
```
-arctanh(cosh(x))+coth(x)/(I+csch(x))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 46 vs. 2(17) = 34.

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.71

$$\int \frac{\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)} \, dx = -\log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right) - \frac{2i\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)}$$

input
```
Integrate[Csch[x]^2/(I + Csch[x]),x]
```

output
```
-Log[Cosh[x/2]] + Log[Sinh[x/2]] - ((2*I)*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x
/2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {3042, 25, 26, 4276, 26, 3042, 26, 4257, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(x)}{\operatorname{csch}(x)+i}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\csc(ix)^2}{i\csc(ix)+i}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int -\frac{i\csc(ix)^2}{\csc(ix)+1}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\csc(ix)^2}{\csc(ix)+1}dx$$

$$\Big\downarrow \text{ 4276}$$

$$i\left(\int -i\operatorname{csch}(x)dx-\int -\frac{i\operatorname{csch}(x)}{1-i\operatorname{csch}(x)}dx\right)$$

$$\Big\downarrow \text{ 26}$$

$$i\left(i\int \frac{\operatorname{csch}(x)}{1-i\operatorname{csch}(x)}dx-i\int \operatorname{csch}(x)dx\right)$$

$$\Big\downarrow \text{ 3042}$$

$$i\left(i\int \frac{i\csc(ix)}{\csc(ix)+1}dx-i\int i\csc(ix)dx\right)$$

$$\Big\downarrow \text{ 26}$$

$$i\left(\int \csc(ix)dx-\int \frac{\csc(ix)}{\csc(ix)+1}dx\right)$$

$$\Big\downarrow \text{ 4257}$$

$$i\left(i\text{arctanh}(\cosh(x))-\int\frac{\csc(ix)}{\csc(ix)+1}dx\right)$$
$$\Big\downarrow \text{4281}$$
$$i\left(i\text{arctanh}(\cosh(x))-\frac{\coth(x)}{1-i\text{csch}(x)}\right)$$

input
```
Int[Csch[x]^2/(I + Csch[x]),x]
```

output
```
I*(I*ArcTanh[Cosh[x]] - Coth[x]/(1 - I*Csch[x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4276
```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[1/b   Int[Csc[e + f*x], x], x] - Simp[a/b   Int[Csc[e + f*x]/(
a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

method	result	size
default	$-\frac{2i}{\tanh\left(\frac{x}{2}\right)-i}+\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	19
risch	$\frac{2}{\mathrm{e}^{x}-i}-\ln\left(1+\mathrm{e}^{x}\right)+\ln\left(\mathrm{e}^{x}-1\right)$	23
parallelrisch	$\frac{2i+\ln\left(\tanh\left(\frac{x}{2}\right)\right)\left(-\tanh\left(\frac{x}{2}\right)+i\right)}{-\tanh\left(\frac{x}{2}\right)+i}$	31

input
```
int(csch(x)^2/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*I/(tanh(1/2*x)-I)+ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(15) = 30.

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

$$\int \frac{\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{(e^x-i)\log(e^x+1)-(e^x-i)\log(e^x-1)-2}{e^x-i}$$

input
```
integrate(csch(x)^2/(I+csch(x)),x, algorithm="fricas")
```

output
```
-((e^x - I)*log(e^x + 1) - (e^x - I)*log(e^x - 1) - 2)/(e^x - I)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{i + \operatorname{csch}(x)}\,dx = \int \frac{\operatorname{csch}^2(x)}{\operatorname{csch}(x) + i}\,dx$$

input
```
integrate(csch(x)**2/(I+csch(x)),x)
```

output
```
Integral(csch(x)**2/(csch(x) + I), x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{\operatorname{csch}^2(x)}{i + \operatorname{csch}(x)}\,dx = \frac{2}{e^{(-x)} + i} - \log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right)$$

input
```
integrate(csch(x)^2/(I+csch(x)),x, algorithm="maxima")
```

output
```
2/(e^(-x) + I) - log(e^(-x) + 1) + log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \frac{\operatorname{csch}^2(x)}{i + \operatorname{csch}(x)}\,dx = \frac{2}{e^x - i} - \log\left(e^x + 1\right) + \log\left(|e^x - 1|\right)$$

input
```
integrate(csch(x)^2/(I+csch(x)),x, algorithm="giac")
```

output
```
2/(e^x - I) - log(e^x + 1) + log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{\operatorname{csch}^2(x)}{i + \operatorname{csch}(x)} \, dx = \ln(2 - 2\,e^x) - \ln(-2\,e^x - 2) + \frac{2}{e^x - i}$$

input
```
int(1/(sinh(x)^2*(1/sinh(x) + 1i)),x)
```

output
```
log(2 - 2*exp(x)) - log(- 2*exp(x) - 2) + 2/(exp(x) - 1i)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^2(x)}{i + \operatorname{csch}(x)} \, dx = \int \frac{\operatorname{csch}(x)^2}{\operatorname{csch}(x) + i} dx$$

input
```
int(csch(x)^2/(I+csch(x)),x)
```

output
```
int(csch(x)**2/(csch(x) + i),x)
```

3.68 $\int \frac{\operatorname{csch}^3(x)}{i+\operatorname{csch}(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{\operatorname{csch}^3(x)}{i+\operatorname{csch}(x)} dx = i\operatorname{arctanh}(\cosh(x)) - \coth(x) - \frac{i\coth(x)}{i+\operatorname{csch}(x)}$$

output
```
I*arctanh(cosh(x))-coth(x)-I*coth(x)/(I+csch(x))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 70 vs. 2(26) = 52.

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.69

$$\int \frac{\operatorname{csch}^3(x)}{i+\operatorname{csch}(x)} dx = -\frac{1}{2}\coth\left(\frac{x}{2}\right) + i\log\left(\cosh\left(\frac{x}{2}\right)\right) - i\log\left(\sinh\left(\frac{x}{2}\right)\right) - \frac{2\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)} - \frac{1}{2}\tanh\left(\frac{x}{2}\right)$$

input
```
Integrate[Csch[x]^3/(I + Csch[x]),x]
```

output
```
-1/2*Coth[x/2] + I*Log[Cosh[x/2]] - I*Log[Sinh[x/2]] - (2*Sinh[x/2])/(Cosh
[x/2] + I*Sinh[x/2]) - Tanh[x/2]/2
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 26, 26, 4277, 25, 3042, 25, 4276, 26, 3042, 26, 4257, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(x)}{\operatorname{csch}(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\csc(ix)^3}{i\csc(ix)+i}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int -\frac{i\csc(ix)^3}{\csc(ix)+1}dx$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{\csc(ix)^3}{\csc(ix)+1}dx$$

$$\Big\downarrow \text{4277}$$

$$-\coth(x)+\int -\frac{\operatorname{csch}^2(x)}{1-i\operatorname{csch}(x)}dx$$

$$\Big\downarrow \text{25}$$

$$-\coth(x)-\int \frac{\operatorname{csch}^2(x)}{1-i\operatorname{csch}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$-\coth(x)-\int -\frac{\csc(ix)^2}{\csc(ix)+1}dx$$

$$\Big\downarrow \text{25}$$

$$-\coth(x)+\int \frac{\csc(ix)^2}{\csc(ix)+1}dx$$

$$\Big\downarrow \; 4276$$

$$\int -i\text{csch}(x)dx-\int -\frac{i\text{csch}(x)}{1-i\text{csch}(x)}dx-\coth(x)$$

$$\Big\downarrow \; 26$$

$$-i\int \text{csch}(x)dx+i\int \frac{\text{csch}(x)}{1-i\text{csch}(x)}dx-\coth(x)$$

$$\Big\downarrow \; 3042$$

$$-i\int i\csc(ix)dx+i\int \frac{i\csc(ix)}{\csc(ix)+1}dx-\coth(x)$$

$$\Big\downarrow \; 26$$

$$\int \csc(ix)dx-\int \frac{\csc(ix)}{\csc(ix)+1}dx-\coth(x)$$

$$\Big\downarrow \; 4257$$

$$-\int \frac{\csc(ix)}{\csc(ix)+1}dx+i\text{arctanh}(\cosh(x))-\coth(x)$$

$$\Big\downarrow \; 4281$$

$$i\text{arctanh}(\cosh(x))-\coth(x)-\frac{\coth(x)}{1-i\text{csch}(x)}$$

input
```
Int[Csch[x]^3/(I + Csch[x]),x]
```

output
```
I*ArcTanh[Cosh[x]] - Coth[x] - Coth[x]/(1 - I*Csch[x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4276
```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[1/b   Int[Csc[e + f*x], x], x] - Simp[a/b   Int[Csc[e + f*x]/(
a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4277
```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[-Cot[e + f*x]/(b*f), x] - Simp[a/b   Int[Csc[e + f*x]^2/(a + b
*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2}-\frac{1}{2\tanh\left(\frac{x}{2}\right)}-i\ln\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{2}{\tanh\left(\frac{x}{2}\right)-i}$	35
risch	$-\frac{2i\left(\mathrm{e}^{2x}-2-i\,\mathrm{e}^{x}\right)}{\left(\mathrm{e}^{2x}-1\right)\left(\mathrm{e}^{x}-i\right)}+i\ln\left(1+\mathrm{e}^{x}\right)-i\ln\left(\mathrm{e}^{x}-1\right)$	47
parallelrisch	$\frac{2i\ln\left(\tanh\left(\frac{x}{2}\right)\right)\tanh\left(\frac{x}{2}\right)-i\coth\left(\frac{x}{2}\right)+\tanh\left(\frac{x}{2}\right)^{2}+2\ln\left(\tanh\left(\frac{x}{2}\right)\right)+6}{-2\tanh\left(\frac{x}{2}\right)+2i}$	47

input
```
int(csch(x)^3/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*tanh(1/2*x)-1/2/tanh(1/2*x)-I*ln(tanh(1/2*x))-2/(tanh(1/2*x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(20) = 40$.

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.96

$$\int \frac{\operatorname{csch}^3(x)}{i + \operatorname{csch}(x)}\,dx = \frac{\left(i\,e^{(3\,x)} + e^{(2\,x)} - i\,e^{x} - 1\right)\log\left(e^{x} + 1\right) + \left(-i\,e^{(3\,x)} - e^{(2\,x)} + i\,e^{x} + 1\right)\log\left(e^{x} - 1\right) - 2i\,e^{(2\,x)} - 2\,e^{x} + 4i}{e^{(3\,x)} - i\,e^{(2\,x)} - e^{x} + i}$$

input
```
integrate(csch(x)^3/(I+csch(x)),x, algorithm="fricas")
```

output
```
((I*e^(3*x) + e^(2*x) - I*e^x - 1)*log(e^x + 1) + (-I*e^(3*x) - e^(2*x) +
I*e^x + 1)*log(e^x - 1) - 2*I*e^(2*x) - 2*e^x + 4*I)/(e^(3*x) - I*e^(2*x)
- e^x + I)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{i + \operatorname{csch}(x)}\,dx = \int \frac{\operatorname{csch}^3(x)}{\operatorname{csch}(x) + i}\,dx$$

input
```
integrate(csch(x)**3/(I+csch(x)),x)
```

output
```
Integral(csch(x)**3/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(20) = 40.

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.04

$$\int \frac{\operatorname{csch}^3(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{2\left(e^{(-x)} - i\,e^{(-2\,x)} + 2i\right)}{e^{(-x)} - i\,e^{(-2\,x)} - e^{(-3\,x)} + i} + i\,\log\left(e^{(-x)} + 1\right) - i\,\log\left(e^{(-x)} - 1\right)$$

input
```
integrate(csch(x)^3/(I+csch(x)),x, algorithm="maxima")
```

output
```
-2*(e^(-x) - I*e^(-2*x) + 2*I)/(e^(-x) - I*e^(-2*x) - e^(-3*x) + I) + I*lo
g(e^(-x) + 1) - I*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(20) = 40.

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

$$\int \frac{\operatorname{csch}^3(x)}{i+\operatorname{csch}(x)}\,dx = \frac{2\left(e^{(2\,x)} - i\,e^{x} - 2\right)}{i\,e^{(3\,x)} + e^{(2\,x)} - i\,e^{x} - 1} + i\,\log\left(e^{x} + 1\right) - i\,\log\left(|e^{x} - 1|\right)$$

input
```
integrate(csch(x)^3/(I+csch(x)),x, algorithm="giac")
```

output
```
2*(e^(2*x) - I*e^x - 2)/(I*e^(3*x) + e^(2*x) - I*e^x - 1) + I*log(e^x + 1)
 - I*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 3.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.31

$$\int \frac{\operatorname{csch}^3(x)}{i+\operatorname{csch}(x)}\,dx = -\ln\left(e^x\,2i-2i\right)\,1i+\ln\left(e^x\,2i+2i\right)\,1i+\frac{e^{2x}\,2i+2\,e^x-4i}{e^{2x}\,1i-e^{3x}+e^x-i}$$

input
```
int(1/(sinh(x)^3*(1/sinh(x) + 1i)),x)
```

output
```
log(exp(x)*2i + 2i)*1i - log(exp(x)*2i - 2i)*1i + (exp(2*x)*2i + 2*exp(x)
- 4i)/(exp(2*x)*1i - exp(3*x) + exp(x) - 1i)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^3(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{csch}(x)^3}{\operatorname{csch}(x)+i}dx$$

input
```
int(csch(x)^3/(I+csch(x)),x)
```

output
```
int(csch(x)**3/(csch(x) + i),x)
```

3.69 $\int \frac{\operatorname{csch}^4(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{\operatorname{csch}^4(x)}{i+\operatorname{csch}(x)}\,dx=\frac{3}{2}\operatorname{arctanh}(\cosh(x))+2i\coth(x)-\frac{3}{2}\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)}$$

output
```
3/2*arctanh(cosh(x))+2*I*coth(x)-3/2*coth(x)*csch(x)+coth(x)*csch(x)^2/(I+
csch(x))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 90 vs. 2(37) = 74.

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.43

$$\int \frac{\operatorname{csch}^4(x)}{i+\operatorname{csch}(x)}\,dx=\frac{1}{8}\left(4i\coth\left(\frac{x}{2}\right)-\operatorname{csch}^2\left(\frac{x}{2}\right)+12\log\left(\cosh\left(\frac{x}{2}\right)\right)-12\log\left(\sinh\left(\frac{x}{2}\right)\right)-\operatorname{sech}^2\left(\frac{x}{2}\right)+\frac{16\sinh\left(\frac{x}{2}\right)}{-i\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)}+4i\tanh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Csch[x]^4/(I + Csch[x]),x]
```

output
```
((4*I)*Coth[x/2] - Csch[x/2]^2 + 12*Log[Cosh[x/2]] - 12*Log[Sinh[x/2]] - S
ech[x/2]^2 + (16*Sinh[x/2])/((-I)*Cosh[x/2] + Sinh[x/2]) + (4*I)*Tanh[x/2]
)/8
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 1.462$, Rules used = {3042, 4305, 25, 3042, 25, 26, 4274, 25, 26, 3042, 25, 26, 4254, 24, 4255, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^4(x)}{\text{csch}(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(ix)^4}{i\csc(ix)+i}dx$$

$$\Big\downarrow \text{4305}$$

$$\int -\left((2i-3\text{csch}(x))\text{csch}^2(x)\right)dx+\frac{\coth(x)\text{csch}^2(x)}{\text{csch}(x)+i}$$

$$\Big\downarrow \text{25}$$

$$\frac{\coth(x)\text{csch}^2(x)}{\text{csch}(x)+i}-\int (2i-3\text{csch}(x))\text{csch}^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\coth(x)\text{csch}^2(x)}{\text{csch}(x)+i}-\int -\left((2i-3i\csc(ix))\csc(ix)^2\right)dx$$

$$\Big\downarrow \text{25}$$

$$\int i(2-3\csc(ix))\csc(ix)^2dx+\frac{\coth(x)\text{csch}^2(x)}{\text{csch}(x)+i}$$

$$\Big\downarrow \text{26}$$

$$i\int (2-3\csc (ix))\csc (ix)^2dx+\frac{\coth (x)\text{csch}^2(x)}{\text{csch}(x)+i}$$

$$\Big\downarrow \text{4274}$$

$$i\left(2\int -\text{csch}^2(x)dx-3\int i\text{csch}^3(x)dx\right)+\frac{\coth (x)\text{csch}^2(x)}{\text{csch}(x)+i}$$

$$\Big\downarrow \text{25}$$

$$i\left(-2\int \text{csch}^2(x)dx-3\int i\text{csch}^3(x)dx\right)+\frac{\coth (x)\text{csch}^2(x)}{\text{csch}(x)+i}$$

$$\Big\downarrow \text{26}$$

$$i\left(-2\int \text{csch}^2(x)dx-3i\int \text{csch}^3(x)dx\right)+\frac{\coth (x)\text{csch}^2(x)}{\text{csch}(x)+i}$$

$$\Big\downarrow \text{3042}$$

$$i\left(-2\int -\csc (ix)^2dx-3i\int -i\csc (ix)^3dx\right)+\frac{\coth (x)\text{csch}^2(x)}{\text{csch}(x)+i}$$

$$\Big\downarrow \text{25}$$

$$i\left(2\int \csc (ix)^2dx-3i\int -i\csc (ix)^3dx\right)+\frac{\coth (x)\text{csch}^2(x)}{\text{csch}(x)+i}$$

$$\Big\downarrow \text{26}$$

$$i\left(2\int \csc (ix)^2dx-3\int \csc (ix)^3dx\right)+\frac{\coth (x)\text{csch}^2(x)}{\text{csch}(x)+i}$$

$$\Big\downarrow \text{4254}$$

$$i\left(2i\int 1d(-i\coth (x))-3\int \csc (ix)^3dx\right)+\frac{\coth (x)\text{csch}^2(x)}{\text{csch}(x)+i}$$

$$\Big\downarrow \text{24}$$

$$i\left(2\coth (x)-3\int \csc (ix)^3dx\right)+\frac{\coth (x)\text{csch}^2(x)}{\text{csch}(x)+i}$$

$$\Big\downarrow \text{4255}$$

$$i\left(2\coth (x)-3\left(\frac{1}{2}\int -i\text{csch}(x)dx-\frac{1}{2}i\coth (x)\text{csch}(x)\right)\right)+\frac{\coth (x)\text{csch}^2(x)}{\text{csch}(x)+i}$$

$$\Big\downarrow \text{26}$$

$$i \left(2 \coth(x) - 3 \left(-\frac{1}{2} i \int \operatorname{csch}(x) dx - \frac{1}{2} i \coth(x) \operatorname{csch}(x)\right)\right) + \frac{\coth(x) \operatorname{csch}^2(x)}{\operatorname{csch}(x) + i}$$

$$\Big\downarrow \text{ 3042}$$

$$i \left(2 \coth(x) - 3 \left(-\frac{1}{2} i \int i \csc(ix) dx - \frac{1}{2} i \coth(x) \operatorname{csch}(x)\right)\right) + \frac{\coth(x) \operatorname{csch}^2(x)}{\operatorname{csch}(x) + i}$$

$$\Big\downarrow \text{ 26}$$

$$i \left(2 \coth(x) - 3 \left(\frac{1}{2} \int \csc(ix) dx - \frac{1}{2} i \coth(x) \operatorname{csch}(x)\right)\right) + \frac{\coth(x) \operatorname{csch}^2(x)}{\operatorname{csch}(x) + i}$$

$$\Big\downarrow \text{ 4257}$$

$$i \left(2 \coth(x) - 3 \left(\frac{1}{2} i \operatorname{arctanh}(\cosh(x)) - \frac{1}{2} i \coth(x) \operatorname{csch}(x)\right)\right) + \frac{\coth(x) \operatorname{csch}^2(x)}{\operatorname{csch}(x) + i}$$

input
```
Int[Csch[x]^4/(I + Csch[x]),x]
```

output
```
(Coth[x]*Csch[x]^2)/(I + Csch[x]) + I*(2*Coth[x] - 3*((I/2)*ArcTanh[Cosh[x
]] - (I/2)*Coth[x]*Csch[x]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4305
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Simp[d^2/(a*b)   Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
 - 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ
[a^2 - b^2, 0] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

method	result	size
default	$\frac{i\tanh\left(\frac{x}{2}\right)}{2}+\frac{\tanh\left(\frac{x}{2}\right)^{2}}{8}-\frac{1}{8\tanh\left(\frac{x}{2}\right)^{2}}+\frac{i}{2\tanh\left(\frac{x}{2}\right)}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2}+\frac{2i}{\tanh\left(\frac{x}{2}\right)-i}$	53
risch	$-\frac{-5\,\mathrm{e}^{2x}-3i\mathrm{e}^{3x}+3\,\mathrm{e}^{4x}+4+i\mathrm{e}^{x}}{\left(\mathrm{e}^{2x}-1\right)^{2}\left(\mathrm{e}^{x}-i\right)}+\frac{3\ln\left(1+\mathrm{e}^{x}\right)}{2}-\frac{3\ln\left(\mathrm{e}^{x}-1\right)}{2}$	59
parallelrisch	$\frac{\left(-12i+12\tanh\left(\frac{x}{2}\right)\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right)-i\coth\left(\frac{x}{2}\right)^{2}-3i\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)^{3}-24i-3\coth\left(\frac{x}{2}\right)}{-8\tanh\left(\frac{x}{2}\right)+8i}$	63

input
```
int(csch(x)^4/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*tanh(1/2*x)+1/8*tanh(1/2*x)^2-1/8/tanh(1/2*x)^2+1/2*I/tanh(1/2*x)-3/
2*ln(tanh(1/2*x))+2*I/(tanh(1/2*x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(29) = 58$.

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.24

$$\int \frac{\operatorname{csch}^4(x)}{i+\operatorname{csch}(x)}\,dx = \frac{3\left(e^{(5x)} - i\,e^{(4x)} - 2\,e^{(3x)} + 2i\,e^{(2x)} + e^x - i\right)\log\left(e^x+1\right) - 3\left(e^{(5x)} - i\,e^{(4x)} - 2\,e^{(3x)} + 2i\,e^{(2x)} + e^x - i}{2\left(e^{(5x)} - i\,e^{(4x)} - 2\,e^{(3x)} + 2i\,e^{(2x)} + e^x - i\right)}$$

input
```
integrate(csch(x)^4/(I+csch(x)),x, algorithm="fricas")
```

output
```
1/2*(3*(e^(5*x) - I*e^(4*x) - 2*e^(3*x) + 2*I*e^(2*x) + e^x - I)*log(e^x +
 1) - 3*(e^(5*x) - I*e^(4*x) - 2*e^(3*x) + 2*I*e^(2*x) + e^x - I)*log(e^x
- 1) - 6*e^(4*x) + 6*I*e^(3*x) + 10*e^(2*x) - 2*I*e^x - 8)/(e^(5*x) - I*e^
(4*x) - 2*e^(3*x) + 2*I*e^(2*x) + e^x - I)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{csch}^4(x)}{\operatorname{csch}(x)+i}\,dx$$

input
```
integrate(csch(x)**4/(I+csch(x)),x)
```

output
```
Integral(csch(x)**4/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(29) = 58.

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.08

$$\int \frac{\operatorname{csch}^4(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{-i\,e^{(-x)} - 5\,e^{(-2\,x)} + 3i\,e^{(-3\,x)} + 3\,e^{(-4\,x)} + 4}{e^{(-x)} - 2i\,e^{(-2\,x)} - 2\,e^{(-3\,x)} + i\,e^{(-4\,x)} + e^{(-5\,x)} + i} + \frac{3}{2}\,\log\left(e^{(-x)} + 1\right) - \frac{3}{2}\,\log\left(e^{(-x)} - 1\right)$$

input
```
integrate(csch(x)^4/(I+csch(x)),x, algorithm="maxima")
```

output
```
-(-I*e^(-x) - 5*e^(-2*x) + 3*I*e^(-3*x) + 3*e^(-4*x) + 4)/(e^(-x) - 2*I*e^
(-2*x) - 2*e^(-3*x) + I*e^(-4*x) + e^(-5*x) + I) + 3/2*log(e^(-x) + 1) - 3
/2*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int \frac{\operatorname{csch}^4(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{e^{(3\,x)} - 2i\,e^{(2\,x)} + e^{x} + 2i}{\left(e^{(2\,x)} - 1\right)^2} - \frac{2i}{i\,e^{x} + 1} + \frac{3}{2}\,\log\left(e^{x} + 1\right) - \frac{3}{2}\,\log\left(|e^{x} - 1|\right)$$

input
```
integrate(csch(x)^4/(I+csch(x)),x, algorithm="giac")
```

output
```
-(e^(3*x) - 2*I*e^(2*x) + e^x + 2*I)/(e^(2*x) - 1)^2 - 2*I/(I*e^x + 1) + 3
/2*log(e^x + 1) - 3/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.70

$$\int \frac{\operatorname{csch}^4(x)}{i+\operatorname{csch}(x)}\,dx = \frac{3\,\ln\left(3\,e^x+3\right)}{2}-\frac{3\,\ln\left(3\,e^x-3\right)}{2}-\frac{e^x}{e^{2\,x}-1}-\frac{2\,e^x}{\left(e^{2\,x}-1\right)^2}-\frac{2}{e^x-\mathrm{i}}+\frac{2\mathrm{i}}{e^{2\,x}-1}$$

input
```
int(1/(sinh(x)^4*(1/sinh(x) + 1i)),x)
```

output
```
(3*log(3*exp(x) + 3))/2 - (3*log(3*exp(x) - 3))/2 - exp(x)/(exp(2*x) - 1)
- (2*exp(x))/(exp(2*x) - 1)^2 - 2/(exp(x) - 1i) + 2i/(exp(2*x) - 1)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^4(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{csch}(x)^4}{\operatorname{csch}(x)+i}dx$$

input
```
int(csch(x)^4/(I+csch(x)),x)
```

output
```
int(csch(x)**4/(csch(x) + i),x)
```

3.70 $\int (a+b\text{csch}(c+dx))^4\,dx$

Optimal result

Integrand size = 12, antiderivative size = 109

$$\int (a+b\text{csch}(c+dx))^4\,dx = a^4x-\frac{2ab(2a^2-b^2)\,\text{arctanh}(\cosh(c+dx))}{d} - \frac{b^2(17a^2-2b^2)\coth(c+dx)}{3d} - \frac{4ab^3\coth(c+dx)\text{csch}(c+dx)}{3d} - \frac{b^2\coth(c+dx)(a+b\text{csch}(c+dx))^2}{3d}$$

output
```
a^4*x-2*a*b*(2*a^2-b^2)*arctanh(cosh(d*x+c))/d-1/3*b^2*(17*a^2-2*b^2)*coth
(d*x+c)/d-4/3*a*b^3*coth(d*x+c)*csch(d*x+c)/d-1/3*b^2*coth(d*x+c)*(a+b*csc
h(d*x+c))^2/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 567 vs. $2(109) = 218$.

Time = 11.55 (sec) , antiderivative size = 567, normalized size of antiderivative = 5.20

$$
\begin{aligned}
\int (a+b\operatorname{csch}(c+dx))^4\,dx &= \frac{a^4(c+dx)(a+b\operatorname{csch}(c+dx))^4\sinh^4(c+dx)}{d(b+a\sinh(c+dx))^4} \\
&+ \frac{\left(-9a^2b^2\cosh\left(\frac{1}{2}(c+dx)\right)+b^4\cosh\left(\frac{1}{2}(c+dx)\right)\right)\operatorname{csch}\left(\frac{1}{2}(c+dx)\right)(a+b\operatorname{csch}(c+dx))^4\sinh^4(c+dx)}{3d(b+a\sinh(c+dx))^4} \\
&- \frac{ab^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)(a+b\operatorname{csch}(c+dx))^4\sinh^4(c+dx)}{2d(b+a\sinh(c+dx))^4} \\
&- \frac{b^4\coth\left(\frac{1}{2}(c+dx)\right)\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)(a+b\operatorname{csch}(c+dx))^4\sinh^4(c+dx)}{24d(b+a\sinh(c+dx))^4} \\
&+ \frac{2(-2a^3b+ab^3)(a+b\operatorname{csch}(c+dx))^4\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)\sinh^4(c+dx)}{d(b+a\sinh(c+dx))^4} \\
&- \frac{2(-2a^3b+ab^3)(a+b\operatorname{csch}(c+dx))^4\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\sinh^4(c+dx)}{d(b+a\sinh(c+dx))^4} \\
&- \frac{ab^3(a+b\operatorname{csch}(c+dx))^4\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)\sinh^4(c+dx)}{2d(b+a\sinh(c+dx))^4} \\
&+ \frac{(a+b\operatorname{csch}(c+dx))^4\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(-9a^2b^2\sinh\left(\frac{1}{2}(c+dx)\right)+b^4\sinh\left(\frac{1}{2}(c+dx)\right)\right)\sinh^4(c+dx)}{3d(b+a\sinh(c+dx))^4} \\
&+ \frac{b^4(a+b\operatorname{csch}(c+dx))^4\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)\sinh^4(c+dx)\tanh\left(\frac{1}{2}(c+dx)\right)}{24d(b+a\sinh(c+dx))^4}
\end{aligned}
$$

input
```
Integrate[(a + b*Csch[c + d*x])^4,x]
```

output
```
(a^4*(c + d*x)*(a + b*Csch[c + d*x])^4*Sinh[c + d*x]^4)/(d*(b + a*Sinh[c +
 d*x])^4) + ((-9*a^2*b^2*Cosh[(c + d*x)/2] + b^4*Cosh[(c + d*x)/2])*Csch[(
c + d*x)/2]*(a + b*Csch[c + d*x])^4*Sinh[c + d*x]^4)/(3*d*(b + a*Sinh[c +
d*x])^4) - (a*b^3*Csch[(c + d*x)/2]^2*(a + b*Csch[c + d*x])^4*Sinh[c + d*x
]^4)/(2*d*(b + a*Sinh[c + d*x])^4) - (b^4*Coth[(c + d*x)/2]*Csch[(c + d*x)
/2]^2*(a + b*Csch[c + d*x])^4*Sinh[c + d*x]^4)/(24*d*(b + a*Sinh[c + d*x])
^4) + (2*(-2*a^3*b + a*b^3)*(a + b*Csch[c + d*x])^4*Log[Cosh[(c + d*x)/2]]
*Sinh[c + d*x]^4)/(d*(b + a*Sinh[c + d*x])^4) - (2*(-2*a^3*b + a*b^3)*(a +
 b*Csch[c + d*x])^4*Log[Sinh[(c + d*x)/2]]*Sinh[c + d*x]^4)/(d*(b + a*Sinh
[c + d*x])^4) - (a*b^3*(a + b*Csch[c + d*x])^4*Sech[(c + d*x)/2]^2*Sinh[c
+ d*x]^4)/(2*d*(b + a*Sinh[c + d*x])^4) + ((a + b*Csch[c + d*x])^4*Sech[(c
 + d*x)/2]*(-9*a^2*b^2*Sinh[(c + d*x)/2] + b^4*Sinh[(c + d*x)/2])*Sinh[c +
 d*x]^4)/(3*d*(b + a*Sinh[c + d*x])^4) + (b^4*(a + b*Csch[c + d*x])^4*Sech
[(c + d*x)/2]^2*Sinh[c + d*x]^4*Tanh[(c + d*x)/2])/(24*d*(b + a*Sinh[c + d
*x])^4)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4269, 3042, 4536, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{csch}(c+dx))^4\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+ib\csc(ic+idx))^4dx$$

$$\Big\downarrow \text{ 4269}$$

$$\frac{1}{3}\int (a+b\text{csch}(c+dx))\left(3a^3+8b^2\text{csch}^2(c+dx)a+b\left(9a^2-2b^2\right)\text{csch}(c+dx)\right)dx-\frac{b^2\coth(c+dx)(a+b\text{csch}(c+dx))^2}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b^2\coth(c+dx)(a+b\text{csch}(c+dx))^2}{3d}+\frac{1}{3}\int(a+ib\csc(ic+idx))\left(3a^3-8b^2\csc(ic+idx)^2a+ib\left(9a^2-2b^2\right)\csc(ic+idx)\right)dx$$

$\big\downarrow$ 4536

$$\frac{1}{3}\left(\frac{1}{2}\int\left(6a^4+12b\left(2a^2-b^2\right)\text{csch}(c+dx)a+2b^2\left(17a^2-2b^2\right)\text{csch}^2(c+dx)\right)dx-\frac{4ab^3\coth(c+dx)\text{csch}(c+dx)}{d}\right)-\frac{b^2\coth(c+dx)(a+b\text{csch}(c+dx))^2}{3d}$$

$\big\downarrow$ 2009

$$\frac{1}{3}\left(\frac{1}{2}\left(6a^4x-\frac{12ab\left(2a^2-b^2\right)\text{arctanh}(\cosh(c+dx))}{d}-\frac{2b^2\left(17a^2-2b^2\right)\coth(c+dx)}{d}\right)-\frac{4ab^3\coth(c+dx)\text{csch}(c+dx)}{d}\right)-\frac{b^2\coth(c+dx)(a+b\text{csch}(c+dx))^2}{3d}$$

input

```
Int[(a + b*Csch[c + d*x])^4,x]
```

output

```
-1/3*(b^2*Coth[c + d*x]*(a + b*Csch[c + d*x])^2)/d + ((6*a^4*x - (12*a*b*(
2*a^2 - b^2)*ArcTanh[Cosh[c + d*x]])/d - (2*b^2*(17*a^2 - 2*b^2)*Coth[c +
d*x])/d)/2 - (4*a*b^3*Coth[c + d*x]*Csch[c + d*x])/d)/3
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4269

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[1/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*
a^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4536
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Simp[1/2   Int[Simp[2*A*a + (2*B*a + b*(2*
A + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, e, f, A, B, C}, x]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{a^4(dx+c)-8a^3b\,\text{arctanh}(e^{dx+c})-6a^2b^2\coth(dx+c)+4ab^3\left(-\frac{\text{csch}(dx+c)\coth(dx+c)}{2}+\text{arctanh}(e^{dx+c})\right)+b^4\left(\frac{2}{3}-\frac{\text{csch}(\ldots}{}\right)}{d}$
default	$\frac{a^4(dx+c)-8a^3b\,\text{arctanh}(e^{dx+c})-6a^2b^2\coth(dx+c)+4ab^3\left(-\frac{\text{csch}(dx+c)\coth(dx+c)}{2}+\text{arctanh}(e^{dx+c})\right)+b^4\left(\frac{2}{3}-\frac{\text{csch}(\ldots}{}\right)}{d}$
parts	$a^4x+\frac{b^4\left(\frac{2}{3}-\frac{\text{csch}(dx+c)^2}{3}\right)\coth(dx+c)}{d}+\frac{4ab^3\left(-\frac{\text{csch}(dx+c)\coth(dx+c)}{2}+\text{arctanh}(e^{dx+c})\right)}{d}-\frac{6a^2b^2\coth(dx+c\ldots}{d}$
parallelrisch	$\frac{48(2a^3b-ab^3)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^3b^4-12\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^2ab^3+9(-8a^2b^2+b^4)\coth\left(\frac{dx}{2}+\frac{c}{2}\right)-\tanh\left(\frac{dx}{2}\ldots\right.}{24d}$
risch	$a^4x-\frac{4b^2(3ab\,e^{5dx+5c}+9a^2e^{4dx+4c}-18a^2e^{2dx+2c}+3b^2e^{2dx+2c}-3ab\,e^{dx+c}+9a^2-b^2)}{3d(e^{2dx+2c}-1)^3}-\frac{4a^3b\ln(e^{dx+c}+1)}{d}+\ldots$

input
```
int((a+csch(d*x+c)*b)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*(d*x+c)-8*a^3*b*arctanh(exp(d*x+c))-6*a^2*b^2*coth(d*x+c)+4*a*b^3
*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b^4*(2/3-1/3*csch(d*x+
c)^2)*coth(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1440 vs. 2(103) = 206.

Time = 0.11 (sec) , antiderivative size = 1440, normalized size of antiderivative = 13.21

$$\int (a+b\,\text{csch}(c+dx))^4\,dx = \text{Too large to display}$$

input
```
integrate((a+b*csch(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/3*(3*a^4*d*x*cosh(d*x + c)^6 + 3*a^4*d*x*sinh(d*x + c)^6 - 12*a*b^3*cosh
(d*x + c)^5 - 3*a^4*d*x + 6*(3*a^4*d*x*cosh(d*x + c) - 2*a*b^3)*sinh(d*x +
 c)^5 + 12*a*b^3*cosh(d*x + c) - 9*(a^4*d*x + 4*a^2*b^2)*cosh(d*x + c)^4 +
 3*(15*a^4*d*x*cosh(d*x + c)^2 - 3*a^4*d*x - 20*a*b^3*cosh(d*x + c) - 12*a
^2*b^2)*sinh(d*x + c)^4 - 36*a^2*b^2 + 4*b^4 + 12*(5*a^4*d*x*cosh(d*x + c)
^3 - 10*a*b^3*cosh(d*x + c)^2 - 3*(a^4*d*x + 4*a^2*b^2)*cosh(d*x + c))*sin
h(d*x + c)^3 + 3*(3*a^4*d*x + 24*a^2*b^2 - 4*b^4)*cosh(d*x + c)^2 + 3*(15*
a^4*d*x*cosh(d*x + c)^4 - 40*a*b^3*cosh(d*x + c)^3 + 3*a^4*d*x + 24*a^2*b^
2 - 4*b^4 - 18*(a^4*d*x + 4*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 6*
((2*a^3*b - a*b^3)*cosh(d*x + c)^6 + 6*(2*a^3*b - a*b^3)*cosh(d*x + c)*sin
h(d*x + c)^5 + (2*a^3*b - a*b^3)*sinh(d*x + c)^6 - 3*(2*a^3*b - a*b^3)*cos
h(d*x + c)^4 - 3*(2*a^3*b - a*b^3 - 5*(2*a^3*b - a*b^3)*cosh(d*x + c)^2)*s
inh(d*x + c)^4 - 2*a^3*b + a*b^3 + 4*(5*(2*a^3*b - a*b^3)*cosh(d*x + c)^3
- 3*(2*a^3*b - a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(2*a^3*b - a*b^3)
*cosh(d*x + c)^2 + 3*(5*(2*a^3*b - a*b^3)*cosh(d*x + c)^4 + 2*a^3*b - a*b^
3 - 6*(2*a^3*b - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 6*((2*a^3*b - a
*b^3)*cosh(d*x + c)^5 - 2*(2*a^3*b - a*b^3)*cosh(d*x + c)^3 + (2*a^3*b - a
*b^3)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1)
 + 6*((2*a^3*b - a*b^3)*cosh(d*x + c)^6 + 6*(2*a^3*b - a*b^3)*cosh(d*x + c
)*sinh(d*x + c)^5 + (2*a^3*b - a*b^3)*sinh(d*x + c)^6 - 3*(2*a^3*b - a*...
```

Sympy [F]

$$\int (a + b\text{csch}(c + dx))^4 \, dx = \int (a + b \operatorname{csch}(c + dx))^4 \, dx$$

input
```
integrate((a+b*csch(d*x+c))**4,x)
```

output
```
Integral((a + b*csch(c + d*x))**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(103) = 206$.

Time = 0.04 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.15

$$\int (a + b\operatorname{csch}(c + dx))^4 \, dx = a^4x + 2\,ab^3\left(\frac{\log\left(e^{(-dx-c)} + 1\right)}{d} - \frac{\log\left(e^{(-dx-c)} - 1\right)}{d} + \frac{2\left(e^{(-dx-c)} + e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} - 1)}\right) + \frac{4}{3}\,b^4\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)} - 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} - 1)} - \frac{1}{d(3\,e^{(-2\,dx-2\,c)} - 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} - 1)}\right) + \frac{4\,a^3b\log\left(\tanh\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}{d} + \frac{12\,a^2b^2}{d(e^{(-2\,dx-2\,c)} - 1)}$$

input
```
integrate((a+b*csch(d*x+c))^4,x, algorithm="maxima")
```

output
```
a^4*x + 2*a*b^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^
(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) -
 1))) + 4/3*b^4*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x -
 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x -
4*c) + e^(-6*d*x - 6*c) - 1))) + 4*a^3*b*log(tanh(1/2*d*x + 1/2*c))/d + 12
*a^2*b^2/(d*(e^(-2*d*x - 2*c) - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.55

$$\int (a + b\operatorname{csch}(c + dx))^4 \, dx = \frac{3\,(dx + c)a^4 - 6\,(2\,a^3b - ab^3)\log\left(e^{(dx+c)} + 1\right) + 6\,(2\,a^3b - ab^3)\log\left(\left|e^{(dx+c)} - 1\right|\right) - \frac{4\,(3\,ab^3e^{(5\,dx+5\,c)} + 9\,a^2b^2 \cdots}{\cdots}}{3\,d}$$

input
```
integrate((a+b*csch(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*a^4 - 6*(2*a^3*b - a*b^3)*log(e^(d*x + c) + 1) + 6*(2*a^3
*b - a*b^3)*log(abs(e^(d*x + c) - 1)) - 4*(3*a*b^3*e^(5*d*x + 5*c) + 9*a^2
*b^2*e^(4*d*x + 4*c) - 18*a^2*b^2*e^(2*d*x + 2*c) + 3*b^4*e^(2*d*x + 2*c)
- 3*a*b^3*e^(d*x + c) + 9*a^2*b^2 - b^4)/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.19

$$\int (a+b\operatorname{csch}(c+dx))^4\,dx$$
$$= a^4\,x - \frac{\frac{12\,a^2\,b^2}{d}+\frac{4\,a\,b^3\,\mathrm{e}^{c+d\,x}}{d}}{\mathrm{e}^{2\,c+2\,d\,x}-1} - \frac{\frac{4\,b^4}{d}+\frac{8\,a\,b^3\,\mathrm{e}^{c+d\,x}}{d}}{\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1} - \frac{8\,b^4}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)} + \frac{4\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(a\,b^3\,\sqrt{-d^2}-2\,a^3\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{4\,a^6\,b^2-4\,a^4\,b^4+a^2\,b^6}}\right)\,\sqrt{4\,a^6\,b^2-4\,a^4\,b^4+a^2\,b^6}}{\sqrt{-d^2}}$$

input
```
int((a + b/sinh(c + d*x))^4,x)
```

output
```
a^4*x - ((12*a^2*b^2)/d + (4*a*b^3*exp(c + d*x))/d)/(exp(2*c + 2*d*x) - 1)
 - ((4*b^4)/d + (8*a*b^3*exp(c + d*x))/d)/(exp(4*c + 4*d*x) - 2*exp(2*c +
2*d*x) + 1) - (8*b^4)/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(
6*c + 6*d*x) - 1)) + (4*atan((exp(d*x)*exp(c)*(a*b^3*(-d^2)^(1/2) - 2*a^3*
b*(-d^2)^(1/2)))/(d*(a^2*b^6 - 4*a^4*b^4 + 4*a^6*b^2)^(1/2)))*(a^2*b^6 - 4
*a^4*b^4 + 4*a^6*b^2)^(1/2))/(-d^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 571, normalized size of antiderivative = 5.24

$$\int (a+b\operatorname{csch}(c+dx))^4\,dx$$
$$= \frac{-3a^4dx+4b^4-12\log\left(e^{dx+c}-1\right)a^3b+12\log\left(e^{dx+c}+1\right)a^3b-36e^{4dx+4c}\log\left(e^{dx+c}-1\right)a^3b+36e^{4dx+4c}}{}$$

input
```
int((a+b*csch(d*x+c))^4,x)
```

output
```
(12*e**(6*c + 6*d*x)*log(e**(c + d*x) - 1)*a**3*b - 6*e**(6*c + 6*d*x)*log
(e**(c + d*x) - 1)*a*b**3 - 12*e**(6*c + 6*d*x)*log(e**(c + d*x) + 1)*a**3
*b + 6*e**(6*c + 6*d*x)*log(e**(c + d*x) + 1)*a*b**3 + 3*e**(6*c + 6*d*x)*
a**4*d*x - 12*e**(6*c + 6*d*x)*a**2*b**2 - 12*e**(5*c + 5*d*x)*a*b**3 - 36
*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**3*b + 18*e**(4*c + 4*d*x)*log(e
**(c + d*x) - 1)*a*b**3 + 36*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**3*b
 - 18*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a*b**3 - 9*e**(4*c + 4*d*x)*a
**4*d*x + 36*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**3*b - 18*e**(2*c +
2*d*x)*log(e**(c + d*x) - 1)*a*b**3 - 36*e**(2*c + 2*d*x)*log(e**(c + d*x)
 + 1)*a**3*b + 18*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a*b**3 + 9*e**(2*
c + 2*d*x)*a**4*d*x + 36*e**(2*c + 2*d*x)*a**2*b**2 - 12*e**(2*c + 2*d*x)*
b**4 + 12*e**(c + d*x)*a*b**3 - 12*log(e**(c + d*x) - 1)*a**3*b + 6*log(e*
*(c + d*x) - 1)*a*b**3 + 12*log(e**(c + d*x) + 1)*a**3*b - 6*log(e**(c + d
*x) + 1)*a*b**3 - 3*a**4*d*x - 24*a**2*b**2 + 4*b**4)/(3*d*(e**(6*c + 6*d*
x) - 3*e**(4*c + 4*d*x) + 3*e**(2*c + 2*d*x) - 1))
```

3.71 $\int (a+b\text{csch}(c+dx))^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 75

$$\int (a+b\text{csch}(c+dx))^3\,dx = a^3x-\frac{b(6a^2-b^2)\,\text{arctanh}(\cosh(c+dx))}{2d}-\frac{5ab^2\coth(c+dx)}{2d}-\frac{b^2\coth(c+dx)(a+b\text{csch}(c+dx))}{2d}$$

output
```
a^3*x-1/2*b*(6*a^2-b^2)*arctanh(cosh(d*x+c))/d-5/2*a*b^2*coth(d*x+c)/d-1/2
*b^2*coth(d*x+c)*(a+b*csch(d*x+c))/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 151 vs. 2(75) = 150.

Time = 5.56 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.01

$$\int (a+b\text{csch}(c+dx))^3\,dx = -\frac{-8a^3c-8a^3dx+12ab^2\coth\left(\frac{1}{2}(c+dx)\right)+b^3\text{csch}^2\left(\frac{1}{2}(c+dx)\right)+24a^2b\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)-4b^3\text{l}}{}$$

input
```
Integrate[(a + b*Csch[c + d*x])^3,x]
```

output
```
-1/8*(-8*a^3*c - 8*a^3*d*x + 12*a*b^2*Coth[(c + d*x)/2] + b^3*Csch[(c + d*
x)/2]^2 + 24*a^2*b*Log[Cosh[(c + d*x)/2]] - 4*b^3*Log[Cosh[(c + d*x)/2]] -
 24*a^2*b*Log[Sinh[(c + d*x)/2]] + 4*b^3*Log[Sinh[(c + d*x)/2]] + b^3*Sech
[(c + d*x)/2]^2 + 12*a*b^2*Tanh[(c + d*x)/2])/d
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4269, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{csch}(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ib\csc(ic+idx))^3dx$$

$$\Big\downarrow \text{4269}$$

$$\frac{1}{2}\int \left(2a^3+5b^2\text{csch}^2(c+dx)a+b\left(6a^2-b^2\right)\text{csch}(c+dx)\right)dx-\frac{b^2\coth(c+dx)(a+b\text{csch}(c+dx))}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(2a^3x-\frac{b\left(6a^2-b^2\right)\text{arctanh}(\cosh(c+dx))}{d}-\frac{5ab^2\coth(c+dx)}{d}\right)-\frac{b^2\coth(c+dx)(a+b\text{csch}(c+dx))}{2d}$$

input
```
Int[(a + b*Csch[c + d*x])^3,x]
```

output
```
(2*a^3*x - (b*(6*a^2 - b^2)*ArcTanh[Cosh[c + d*x]])/d - (5*a*b^2*Coth[c +
d*x])/d)/2 - (b^2*Coth[c + d*x]*(a + b*Csch[c + d*x]))/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4269
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[1/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*
a^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{a^{3}(dx+c)-6a^{2}b\,\operatorname{arctanh}\left(e^{dx+c}\right)-3a\,b^{2}\coth(dx+c)+b^{3}\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)}{d}$
default	$\frac{a^{3}(dx+c)-6a^{2}b\,\operatorname{arctanh}\left(e^{dx+c}\right)-3a\,b^{2}\coth(dx+c)+b^{3}\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)}{d}$
parts	$a^{3}x+\frac{b^{3}\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)}{d}-\frac{3a\,b^{2}\coth(dx+c)}{d}+\frac{3a^{2}b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$
parallelrisch	$\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b^{3}-\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b^{3}+8a^{3}dx+24\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^{2}b-4\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^{3}-12\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a\,b^{2}}{8d}$
risch	$a^{3}x-\frac{b^{2}\left(b\,e^{3dx+3c}+6a\,e^{2dx+2c}+b\,e^{dx+c}-6a\right)}{d\left(e^{2dx+2c}-1\right)^{2}}-\frac{3b\ln\left(e^{dx+c}+1\right)a^{2}}{d}+\frac{b^{3}\ln\left(e^{dx+c}+1\right)}{2d}+\frac{3b\ln\left(e^{dx+c}-1\right)a^{2}}{d}-$

input
```
int((a+csch(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(d*x+c)-6*a^2*b*arctanh(exp(d*x+c))-3*a*b^2*coth(d*x+c)+b^3*(-1/2
*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 769 vs. $2(69) = 138$.

Time = 0.09 (sec) , antiderivative size = 769, normalized size of antiderivative = 10.25

$$\int (a + b\text{csch}(c + dx))^3 \, dx = \text{Too large to display}$$

input
```
integrate((a+b*csch(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(2*a^3*d*x*cosh(d*x + c)^4 + 2*a^3*d*x*sinh(d*x + c)^4 - 2*b^3*cosh(d*
x + c)^3 + 2*a^3*d*x - 2*b^3*cosh(d*x + c) + 2*(4*a^3*d*x*cosh(d*x + c) -
b^3)*sinh(d*x + c)^3 + 12*a*b^2 - 4*(a^3*d*x + 3*a*b^2)*cosh(d*x + c)^2 +
2*(6*a^3*d*x*cosh(d*x + c)^2 - 2*a^3*d*x - 3*b^3*cosh(d*x + c) - 6*a*b^2)*
sinh(d*x + c)^2 - ((6*a^2*b - b^3)*cosh(d*x + c)^4 + 4*(6*a^2*b - b^3)*cos
h(d*x + c)*sinh(d*x + c)^3 + (6*a^2*b - b^3)*sinh(d*x + c)^4 + 6*a^2*b - b
^3 - 2*(6*a^2*b - b^3)*cosh(d*x + c)^2 - 2*(6*a^2*b - b^3 - 3*(6*a^2*b - b
^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((6*a^2*b - b^3)*cosh(d*x + c)^3
- (6*a^2*b - b^3)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d
*x + c) + 1) + ((6*a^2*b - b^3)*cosh(d*x + c)^4 + 4*(6*a^2*b - b^3)*cosh(d
*x + c)*sinh(d*x + c)^3 + (6*a^2*b - b^3)*sinh(d*x + c)^4 + 6*a^2*b - b^3
- 2*(6*a^2*b - b^3)*cosh(d*x + c)^2 - 2*(6*a^2*b - b^3 - 3*(6*a^2*b - b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((6*a^2*b - b^3)*cosh(d*x + c)^3 - (
6*a^2*b - b^3)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x
+ c) - 1) + 2*(4*a^3*d*x*cosh(d*x + c)^3 - 3*b^3*cosh(d*x + c)^2 - b^3 - 4
*(a^3*d*x + 3*a*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*
d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2
+ 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*c
osh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]

$$\int (a + b\text{csch}(c + dx))^3 \, dx = \int (a + b \operatorname{csch}(c + dx))^3 \, dx$$

input
```
integrate((a+b*csch(d*x+c))**3,x)
```

output
```
Integral((a + b*csch(c + d*x))**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.81

$$\int (a+b\text{csch}(c+dx))^3\,dx$$
$$= a^3x + \frac{1}{2}b^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right) + \frac{3\,a^2b\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d} + \frac{6\,ab^2}{d(e^{(-2\,dx-2\,c)}-1)}$$

input
```
integrate((a+b*csch(d*x+c))^3,x, algorithm="maxima")
```

output
```
a^3*x + 1/2*b^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^
(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) -
 1))) + 3*a^2*b*log(tanh(1/2*d*x + 1/2*c))/d + 6*a*b^2/(d*(e^(-2*d*x - 2*c
) - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.63

$$\int (a+b\text{csch}(c+dx))^3\,dx$$
$$= \frac{2\,(dx+c)a^3 - (6\,a^2b-b^3)\log\left(e^{(dx+c)}+1\right) + (6\,a^2b-b^3)\log\left(\left|e^{(dx+c)}-1\right|\right) - \frac{2\left(b^3e^{(3\,dx+3\,c)}+6\,ab^2e^{(2\,dx+2\,c)}+\right.}{\left(e^{(2\,dx+2\,c)}-1\right)^2}}{2\,d}$$

input
```
integrate((a+b*csch(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(2*(d*x + c)*a^3 - (6*a^2*b - b^3)*log(e^(d*x + c) + 1) + (6*a^2*b - b
^3)*log(abs(e^(d*x + c) - 1)) - 2*(b^3*e^(3*d*x + 3*c) + 6*a*b^2*e^(2*d*x
+ 2*c) + b^3*e^(d*x + c) - 6*a*b^2)/(e^(2*d*x + 2*c) - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.27

$$\int (a+b\operatorname{csch}(c+dx))^3\,dx = a^3\,x - \frac{\frac{6\,a\,b^2}{d}+\frac{b^3\,e^{c+d\,x}}{d}}{e^{2\,c+2\,d\,x}-1} + \frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(b^3\,\sqrt{-d^2}-6\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{36\,a^4\,b^2-12\,a^2\,b^4+b^6}}\right)\,\sqrt{36\,a^4\,b^2-12\,a^2\,b^4+b^6}}{\sqrt{-d^2}} - \frac{2\,b^3\,e^{c+d\,x}}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b/sinh(c + d*x))^3,x)
```

output
```
a^3*x - ((6*a*b^2)/d + (b^3*exp(c + d*x))/d)/(exp(2*c + 2*d*x) - 1) + (ata
n((exp(d*x)*exp(c)*(b^3*(-d^2)^(1/2) - 6*a^2*b*(-d^2)^(1/2)))/(d*(b^6 - 12
*a^2*b^4 + 36*a^4*b^2)^(1/2)))*(b^6 - 12*a^2*b^4 + 36*a^4*b^2)^(1/2))/(-d^
2)^(1/2) - (2*b^3*exp(c + d*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x)
+ 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 386, normalized size of antiderivative = 5.15

$$\int (a+b\operatorname{csch}(c+dx))^3\,dx$$
$$= \frac{6e^{4dx+4c}\log\left(e^{dx+c}-1\right)a^2b - e^{4dx+4c}\log\left(e^{dx+c}-1\right)b^3 - 6e^{4dx+4c}\log\left(e^{dx+c}+1\right)a^2b + e^{4dx+4c}\log\left(e^{dx+c}+\right.}{}$$

input
```
int((a+b*csch(d*x+c))^3,x)
```

output
```
(6*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**2*b - e**(4*c + 4*d*x)*log(e*
*(c + d*x) - 1)*b**3 - 6*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**2*b + e
**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*b**3 + 2*e**(4*c + 4*d*x)*a**3*d*x -
 6*e**(4*c + 4*d*x)*a*b**2 - 2*e**(3*c + 3*d*x)*b**3 - 12*e**(2*c + 2*d*x)
*log(e**(c + d*x) - 1)*a**2*b + 2*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*b
**3 + 12*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a**2*b - 2*e**(2*c + 2*d*x
)*log(e**(c + d*x) + 1)*b**3 - 4*e**(2*c + 2*d*x)*a**3*d*x - 2*e**(c + d*x
)*b**3 + 6*log(e**(c + d*x) - 1)*a**2*b - log(e**(c + d*x) - 1)*b**3 - 6*l
og(e**(c + d*x) + 1)*a**2*b + log(e**(c + d*x) + 1)*b**3 + 2*a**3*d*x + 6*
a*b**2)/(2*d*(e**(4*c + 4*d*x) - 2*e**(2*c + 2*d*x) + 1))
```

3.72 $\int (a + b\text{csch}(c + dx))^2 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 34

$$\int (a + b\text{csch}(c + dx))^2 \, dx = a^2x - \frac{2ab\text{arctanh}(\cosh(c + dx))}{d} - \frac{b^2\coth(c + dx)}{d}$$

output
```
a^2*x-2*a*b*arctanh(cosh(d*x+c))/d-b^2*coth(d*x+c)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 75 vs. 2(34) = 68.

Time = 0.54 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int (a + b\text{csch}(c + dx))^2 \, dx = -\frac{b^2\coth\left(\frac{1}{2}(c + dx)\right) - 2a\left(ac + adx - 2b\log\left(\cosh\left(\frac{1}{2}(c + dx)\right)\right) + 2b\log\left(\sinh\left(\frac{1}{2}(c + dx)\right)\right)\right) + b^2\tanh}{2d}$$

input
```
Integrate[(a + b*Csch[c + d*x])^2,x]
```

output
```
-1/2*(b^2*Coth[(c + d*x)/2] - 2*a*(a*c + a*d*x - 2*b*Log[Cosh[(c + d*x)/2]
] + 2*b*Log[Sinh[(c + d*x)/2]]) + b^2*Tanh[(c + d*x)/2])/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 4260, 25, 26, 3042, 25, 26, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{csch}(c+dx))^2 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ib\csc(ic+idx))^2dx$$

$$\Big\downarrow \text{4260}$$

$$2iab\int -i\text{csch}(c+dx)dx-b^2\int -\text{csch}^2(c+dx)dx+a^2x$$

$$\Big\downarrow \text{25}$$

$$2iab\int -i\text{csch}(c+dx)dx+b^2\int \text{csch}^2(c+dx)dx+a^2x$$

$$\Big\downarrow \text{26}$$

$$2ab\int \text{csch}(c+dx)dx+b^2\int \text{csch}^2(c+dx)dx+a^2x$$

$$\Big\downarrow \text{3042}$$

$$2ab\int i\csc(ic+idx)dx+b^2\int -\csc(ic+idx)^2dx+a^2x$$

$$\Big\downarrow \text{25}$$

$$2ab\int i\csc(ic+idx)dx-b^2\int \csc(ic+idx)^2dx+a^2x$$

$$\Big\downarrow \text{26}$$

$$2iab\int \csc(ic+idx)dx-b^2\int \csc(ic+idx)^2dx+a^2x$$

$$\Big\downarrow \text{4254}$$

$$2iab\int \csc(ic+idx)dx-\frac{ib^2\int 1d(-i\coth(c+dx))}{d}+a^2x$$

$$\Big\downarrow \text{24}$$

$$2iab\int \csc(ic+idx)dx+a^2x-\frac{b^2\coth(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$a^2x-\frac{2ab\text{arctanh}(\cosh(c+dx))}{d}-\frac{b^2\coth(c+dx)}{d}$$

input
```
Int[(a + b*Csch[c + d*x])^2,x]
```

output
```
a^2*x - (2*a*b*ArcTanh[Cosh[c + d*x]])/d - (b^2*Coth[c + d*x])/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4260
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Simp[2*a*b   Int[Csc[c + d*x], x], x] + Simp[b^2   Int[Csc[c + d*x]^2, x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{a^{2}(dx+c)-4ab\,\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)-b^{2}\coth(dx+c)}{d}$	37
default	$\frac{a^{2}(dx+c)-4ab\,\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)-b^{2}\coth(dx+c)}{d}$	37
parts	$a^{2}x-\frac{b^{2}\coth(dx+c)}{d}+\frac{2ab\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$	38
parallelrisch	$\frac{2a^{2}dx+4\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab-b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\coth\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}$	52
risch	$a^{2}x-\frac{2b^{2}}{d\left(\mathrm{e}^{2dx+2c}-1\right)}-\frac{2ab\ln\left(\mathrm{e}^{dx+c}+1\right)}{d}+\frac{2ab\ln\left(\mathrm{e}^{dx+c}-1\right)}{d}$	60

input
```
int((a+csch(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(d*x+c)-4*a*b*arctanh(exp(d*x+c))-b^2*coth(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(34) = 68$.

Time = 0.10 (sec) , antiderivative size = 222, normalized size of antiderivative = 6.53

$$\int (a+b\operatorname{csch}(c+dx))^2\,dx$$
$$=\frac{a^2dx\cosh{(dx+c)}^2+2\,a^2dx\cosh{(dx+c)}\sinh{(dx+c)}+a^2dx\sinh{(dx+c)}^2-a^2dx-2\,b^2-2\,(ab\cosh$$

input
```
integrate((a+b*csch(d*x+c))^2,x, algorithm="fricas")
```

output
```
(a^2*d*x*cosh(d*x + c)^2 + 2*a^2*d*x*cosh(d*x + c)*sinh(d*x + c) + a^2*d*x
*sinh(d*x + c)^2 - a^2*d*x - 2*b^2 - 2*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d
*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 - a*b)*log(cosh(d*x + c) + sin
h(d*x + c) + 1) + 2*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x +
c) + a*b*sinh(d*x + c)^2 - a*b)*log(cosh(d*x + c) + sinh(d*x + c) - 1))/(d
*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 - d
)
```

Sympy [F]

$$\int (a+b\mathrm{csch}(c+dx))^2\,dx=\int \left(a+b\operatorname{csch}\left(c+dx\right)\right)^2\,dx$$

input
```
integrate((a+b*csch(d*x+c))**2,x)
```

output
```
Integral((a + b*csch(c + d*x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

$$\int (a+b\mathrm{csch}(c+dx))^2\,dx=a^2x+\frac{2\,ab\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}+\frac{2\,b^2}{d(e^{(-2\,dx-2\,c)}-1)}$$

input
```
integrate((a+b*csch(d*x+c))^2,x, algorithm="maxima")
```

output
```
a^2*x + 2*a*b*log(tanh(1/2*d*x + 1/2*c))/d + 2*b^2/(d*(e^(-2*d*x - 2*c) -
1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.74

$$\int (a + b\operatorname{csch}(c + dx))^2 \, dx = \frac{(dx + c)a^2 - 2\,ab\log\left(e^{(dx+c)} + 1\right) + 2\,ab\log\left(\left|e^{(dx+c)} - 1\right|\right) - \frac{2\,b^2}{e^{(2\,dx+2\,c)}-1}}{d}$$

input
```
integrate((a+b*csch(d*x+c))^2,x, algorithm="giac")
```

output
```
((d*x + c)*a^2 - 2*a*b*log(e^(d*x + c) + 1) + 2*a*b*log(abs(e^(d*x + c) -
1)) - 2*b^2/(e^(2*d*x + 2*c) - 1))/d
```

Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.18

$$\int (a + b\operatorname{csch}(c + dx))^2 \, dx = a^2\,x - \frac{2\,b^2}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x} - 1\right)} - \frac{4\,\mathrm{atan}\left(\frac{a\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{-d^2}}$$

input
```
int((a + b/sinh(c + d*x))^2,x)
```

output
```
a^2*x - (2*b^2)/(d*(exp(2*c + 2*d*x) - 1)) - (4*atan((a*b*exp(d*x)*exp(c)*
(-d^2)^(1/2))/(d*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(-d^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.91

$$\int (a + b\operatorname{csch}(c + dx))^2 \, dx = \frac{2e^{2dx+2c}\log\left(e^{dx+c} - 1\right)ab - 2e^{2dx+2c}\log\left(e^{dx+c} + 1\right)ab + e^{2dx+2c}a^2dx - 2e^{2dx+2c}b^2 - 2\log\left(e^{dx+c} - 1\right)ab -}{d\left(e^{2dx+2c} - 1\right)}$$

input
```
int((a+b*csch(d*x+c))^2,x)
```

output
```
(2*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a*b - 2*e**(2*c + 2*d*x)*log(e**
(c + d*x) + 1)*a*b + e**(2*c + 2*d*x)*a**2*d*x - 2*e**(2*c + 2*d*x)*b**2 -
 2*log(e**(c + d*x) - 1)*a*b + 2*log(e**(c + d*x) + 1)*a*b - a**2*d*x)/(d*
(e**(2*c + 2*d*x) - 1))
```

3.73 $\int (a + b\,\mathrm{csch}(c + dx))\,dx$

Optimal result

Integrand size = 10, antiderivative size = 17

$$\int (a + b\,\mathrm{csch}(c + dx))\,dx = ax - \frac{b\,\mathrm{arctanh}(\cosh(c + dx))}{d}$$

output
```
a*x-b*arctanh(cosh(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int (a + b\,\mathrm{csch}(c + dx))\,dx = ax - \frac{b\,\mathrm{arctanh}(\cosh(c + dx))}{d}$$

input
```
Integrate[a + b*Csch[c + d*x],x]
```

output
```
a*x - (b*ArcTanh[Cosh[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{csch}(c+dx))\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax-\frac{b\text{arctanh}(\cosh(c+dx))}{d}$$

input
```
Int[a + b*Csch[c + d*x],x]
```

output
```
a*x - (b*ArcTanh[Cosh[c + d*x]])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

method	result	size
default	$xa+\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$	20
parallelrisch	$xa+\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$	20
parts	$xa+\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$	20
derivativedivides	$\frac{(dx+c)a+b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$	25
risch	$xa+\frac{b\ln\left(e^{dx+c}-1\right)}{d}-\frac{b\ln\left(e^{dx+c}+1\right)}{d}$	34

input
```
int(a+csch(d*x+c)*b,x,method=_RETURNVERBOSE)
```

output
```
x*a+b/d*ln(tanh(1/2*d*x+1/2*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(17) = 34.

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.59

$$\int (a+b\operatorname{csch}(c+dx))\,dx = \frac{adx-b\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+1\right)+b\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)-1\right)}{d}$$

input
```
integrate(a+b*csch(d*x+c),x, algorithm="fricas")
```

output
```
(a*d*x - b*log(cosh(d*x + c) + sinh(d*x + c) + 1) + b*log(cosh(d*x + c) +
sinh(d*x + c) - 1))/d
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int (a+b\operatorname{csch}(c+dx))\,dx = ax+b\left(\begin{cases}\frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} & \text{for } d\neq 0\\ x\operatorname{csch}(c) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a+b*csch(d*x+c),x)
```

output
```
a*x + b*Piecewise((log(tanh(c/2 + d*x/2))/d, Ne(d, 0)), (x*csch(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int (a+b\operatorname{csch}(c+dx))\,dx = ax+\frac{b\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

input
```
integrate(a+b*csch(d*x+c),x, algorithm="maxima")
```

output
```
a*x + b*log(tanh(1/2*d*x + 1/2*c))/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.88

$$\int (a+b\operatorname{csch}(c+dx))\,dx = ax-\frac{b\left(\log\left(e^{(dx+c)}+1\right)-\log\left(\left|e^{(dx+c)}-1\right|\right)\right)}{d}$$

input
```
integrate(a+b*csch(d*x+c),x, algorithm="giac")
```

output
```
a*x - b*(log(e^(d*x + c) + 1) - log(abs(e^(d*x + c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.47

$$\int (a + b\text{csch}(c + dx))\, dx = a\,x - \frac{2\,\text{atan}\left(\frac{b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{-d^2}}$$

input
```
int(a + b/sinh(c + d*x),x)
```

output
```
a*x - (2*atan((b*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(b^2)^(1/2)))*(b^2)^(1/2
))/(-d^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.00

$$\int (a + b\text{csch}(c + dx))\, dx = \frac{\log\left(e^{dx+c} - 1\right) b - \log\left(e^{dx+c} + 1\right) b + adx}{d}$$

input
```
int(a+b*csch(d*x+c),x)
```

output
```
(log(e**(c + d*x) - 1)*b - log(e**(c + d*x) + 1)*b + a*d*x)/d
```

3.74 $\int \frac{1}{a+b\,\mathrm{csch}(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 54

$$\int \frac{1}{a+b\,\mathrm{csch}(c+dx)}\,dx=\frac{x}{a}+\frac{2b\,\mathrm{arctanh}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d}$$

output
```
x/a+2*b*arctanh((a-b*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/a/(a^2+b^2)^(1/
2)/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

$$\int \frac{1}{a+b\,\mathrm{csch}(c+dx)}\,dx=\frac{\frac{c}{d}+x-\frac{2b\arctan\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,d}}{a}$$

input
```
Integrate[(a + b*Csch[c + d*x])^(-1),x]
```

output
```
(c/d + x - (2*b*ArcTan[(a - b*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(Sqrt[
-a^2 - b^2]*d))/a
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4270, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\text{csch}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+ib\csc(ic+idx)}dx$$

$$\Big\downarrow \text{4270}$$

$$\frac{x}{a}-\frac{\int \frac{1}{\frac{a\sinh(c+dx)}{b}+1}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x}{a}-\frac{\int \frac{1}{1-\frac{ia\sin(ic+idx)}{b}}dx}{a}$$

$$\Big\downarrow \text{3139}$$

$$\frac{x}{a}+\frac{2i\int \frac{1}{-\tanh^2\left(\frac{1}{2}(c+dx)\right)+\frac{2a\tanh\left(\frac{1}{2}(c+dx)\right)}{b}+1}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

$$\Big\downarrow \text{1083}$$

$$\frac{x}{a}-\frac{4i\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(\frac{a^2}{b^2}+1\right)}d\left(2i\tanh\left(\frac{1}{2}(c+dx)\right)-\frac{2ia}{b}\right)}{ad}$$

$$\Big\downarrow \text{217}$$

$$\frac{x}{a}-\frac{2b\text{arctanh}\left(\frac{b\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}$$

input
```
Int[(a + b*Csch[c + d*x])^(-1),x]
```

output
```
x/a - (2*b*ArcTanh[(b*Tanh[(c + d*x)/2])/(2*Sqrt[a^2 + b^2])])/(a*Sqrt[a^2
 + b^2]*d)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 4270
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Simp[1/a   Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.52

method	result	size
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}+\frac{2b\,\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}}{d}$	82
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}+\frac{2b\,\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}}{d}$	82
risch	$\frac{x}{a}+\frac{b\ln\left(\mathrm{e}^{dx+c}+\frac{b\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,da}-\frac{b\ln\left(\mathrm{e}^{dx+c}+\frac{b\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,da}$	124

input
```
int(1/(a+csch(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a*ln(tanh(1/2*d*x+1/2*c)+1)-1/a*ln(tanh(1/2*d*x+1/2*c)-1)+2/a*b/(a^
2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*d*x+1/2*c)+2*a)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(51) = 102.

Time = 0.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.44

$$\int \frac{1}{a+b\operatorname{csch}(c+dx)}\,dx$$
$$=\frac{(a^2+b^2)dx+\sqrt{a^2+b^2}b\log\left(\frac{a^2\cosh(dx+c)^2+a^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+a^2+2\,b^2+2\,(a^2\cosh(dx+c)+ab)\sinh(dx+c)+2\,\sqrt{}}{a\cosh(dx+c)^2+a\sinh(dx+c)^2+2\,b\cosh(dx+c)+2\,(a\cosh(dx+c)+b)\sinh}\right.}{(a^3+ab^2)d}$$

input
```
integrate(1/(a+b*csch(d*x+c)),x, algorithm="fricas")
```

output
```
((a^2 + b^2)*d*x + sqrt(a^2 + b^2)*b*log((a^2*cosh(d*x + c)^2 + a^2*sinh(d
*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*b^2 + 2*(a^2*cosh(d*x + c) + a*b
)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(a*cosh(d*x + c) + a*sinh(d*x + c) + b
))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(a*cosh(
d*x + c) + b)*sinh(d*x + c) - a))/((a^3 + a*b^2)*d)
```

Sympy [F]

$$\int \frac{1}{a + b\text{csch}(c + dx)}\,dx = \int \frac{1}{a + b\,\text{csch}\,(c + dx)}\,dx$$

input
```
integrate(1/(a+b*csch(d*x+c)),x)
```

output
```
Integral(1/(a + b*csch(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.57

$$\int \frac{1}{a + b\text{csch}(c + dx)}\,dx = -\frac{b\log\left(\frac{ae^{(-dx-c)}-b-\sqrt{a^2+b^2}}{ae^{(-dx-c)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}ad} + \frac{dx+c}{ad}$$

input
```
integrate(1/(a+b*csch(d*x+c)),x, algorithm="maxima")
```

output
```
-b*log((a*e^(-d*x - c) - b - sqrt(a^2 + b^2))/(a*e^(-d*x - c) - b + sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a*d) + (d*x + c)/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.56

$$\int \frac{1}{a + b\text{csch}(c + dx)}\,dx = -\frac{\frac{b\log\left(\frac{\left|2\,ae^{(dx+c)}+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^{(dx+c)}+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a} - \frac{dx+c}{a}}{d}$$

input
```
integrate(1/(a+b*csch(d*x+c)),x, algorithm="giac")
```

output
```
-(b*log(abs(2*a*e^(d*x + c) + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^(d*x + c)
 + 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) - (d*x + c)/a)/d
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.24

$$\int \frac{1}{a+b\operatorname{csch}(c+dx)}\,dx = \frac{x}{a} - \frac{b\ln\left(\frac{2\,b\,e^{c+d\,x}}{a^2} - \frac{2\,b\,(a-b\,e^{c+d\,x})}{a^2\,\sqrt{a^2+b^2}}\right)}{a\,d\,\sqrt{a^2+b^2}} + \frac{b\ln\left(\frac{2\,b\,e^{c+d\,x}}{a^2} + \frac{2\,b\,(a-b\,e^{c+d\,x})}{a^2\,\sqrt{a^2+b^2}}\right)}{a\,d\,\sqrt{a^2+b^2}}$$

input
```
int(1/(a + b/sinh(c + d*x)),x)
```

output
```
x/a - (b*log((2*b*exp(c + d*x))/a^2 - (2*b*(a - b*exp(c + d*x)))/(a^2*(a^2
 + b^2)^(1/2))))/(a*d*(a^2 + b^2)^(1/2)) + (b*log((2*b*exp(c + d*x))/a^2 +
 (2*b*(a - b*exp(c + d*x)))/(a^2*(a^2 + b^2)^(1/2))))/(a*d*(a^2 + b^2)^(1/
2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24

$$\int \frac{1}{a+b\operatorname{csch}(c+dx)}\,dx = \frac{-2\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}ai+bi}{\sqrt{a^2+b^2}}\right)bi + a^2dx + b^2dx}{ad\,(a^2+b^2)}$$

input
```
int(1/(a+b*csch(d*x+c)),x)
```

output
```
( - 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*a*i + b*i)/sqrt(a**2 + b**2))*b
*i + a**2*d*x + b**2*d*x)/(a*d*(a**2 + b**2))
```

3.75 $\int \frac{1}{(a+b\mathbf{csch}(c+dx))^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 101

$$\int \frac{1}{(a+b\operatorname{csch}(c+dx))^2}\,dx = \frac{x}{a^2}+\frac{2b(2a^2+b^2)\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d} - \frac{b^2\coth(c+dx)}{a\left(a^2+b^2\right)d(a+b\operatorname{csch}(c+dx))}$$

output
```
x/a^2+2*b*(2*a^2+b^2)*arctanh((a-b*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/a
^2/(a^2+b^2)^(3/2)/d-b^2*coth(d*x+c)/a/(a^2+b^2)/d/(a+b*csch(d*x+c))
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.41

$$\int \frac{1}{(a+b\operatorname{csch}(c+dx))^2}\,dx = \frac{\operatorname{csch}(c+dx)\left(-\frac{ab^2\coth(c+dx)}{a^2+b^2}+(c+dx)(a+b\operatorname{csch}(c+dx))+\frac{2b(2a^2+b^2)\arctan\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)(a+b\operatorname{csch}(c+}{\left(-a^2-b^2\right)^{3/2}}\right.}{a^2d(a+b\operatorname{csch}(c+dx))^2}$$

input
```
Integrate[(a + b*Csch[c + d*x])^(-2),x]
```

output
```
(Csch[c + d*x]*(-((a*b^2*Coth[c + d*x])/(a^2 + b^2)) + (c + d*x)*(a + b*Cs
ch[c + d*x]) + (2*b*(2*a^2 + b^2)*ArcTan[(a - b*Tanh[(c + d*x)/2])/Sqrt[-a
^2 - b^2]]*(a + b*Csch[c + d*x]))/(-a^2 - b^2)^(3/2))*(b + a*Sinh[c + d*x]
))/(a^2*d*(a + b*Csch[c + d*x])^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.20, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 4272, 25, 3042, 4407, 26, 3042, 26, 4318, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{csch}(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ib\csc(ic+idx))^2}dx$$

$$\Big\downarrow \text{4272}$$

$$-\frac{\int -\frac{a^2-b\text{csch}(c+dx)a+b^2}{a+b\text{csch}(c+dx)}dx}{a(a^2+b^2)}-\frac{b^2\coth(c+dx)}{ad(a^2+b^2)(a+b\text{csch}(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a^2-b\text{csch}(c+dx)a+b^2}{a+b\text{csch}(c+dx)}dx}{a(a^2+b^2)}-\frac{b^2\coth(c+dx)}{ad(a^2+b^2)(a+b\text{csch}(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b^2\coth(c+dx)}{ad(a^2+b^2)(a+b\text{csch}(c+dx))}+\frac{\int \frac{a^2-ib\csc(ic+idx)a+b^2}{a+ib\csc(ic+idx)}dx}{a(a^2+b^2)}$$

$$\Big\downarrow \text{4407}$$

$$-\frac{b^2\coth(c+dx)}{ad\left(a^2+b^2\right)(a+b\text{csch}(c+dx))}+\frac{\frac{x\left(a^2+b^2\right)}{a}-\frac{ib\left(2a^2+b^2\right)\int -\frac{i\text{csch}(c+dx)}{a+b\text{csch}(c+dx)}dx}{a}}{a\left(a^2+b^2\right)}$$

$\big\downarrow$ 26

$$\frac{\frac{x\left(a^2+b^2\right)}{a}-\frac{b\left(2a^2+b^2\right)\int \frac{\text{csch}(c+dx)}{a+b\text{csch}(c+dx)}dx}{a}}{a\left(a^2+b^2\right)}-\frac{b^2\coth(c+dx)}{ad\left(a^2+b^2\right)(a+b\text{csch}(c+dx))}$$

$\big\downarrow$ 3042

$$-\frac{b^2\coth(c+dx)}{ad\left(a^2+b^2\right)(a+b\text{csch}(c+dx))}+\frac{\frac{x\left(a^2+b^2\right)}{a}-\frac{b\left(2a^2+b^2\right)\int \frac{i\csc(ic+idx)}{a+ib\csc(ic+idx)}dx}{a}}{a\left(a^2+b^2\right)}$$

$\big\downarrow$ 26

$$-\frac{b^2\coth(c+dx)}{ad\left(a^2+b^2\right)(a+b\text{csch}(c+dx))}+\frac{\frac{x\left(a^2+b^2\right)}{a}-\frac{ib\left(2a^2+b^2\right)\int \frac{\csc(ic+idx)}{a+ib\csc(ic+idx)}dx}{a}}{a\left(a^2+b^2\right)}$$

$\big\downarrow$ 4318

$$\frac{\frac{x\left(a^2+b^2\right)}{a}-\frac{\left(2a^2+b^2\right)\int \frac{1}{\frac{a\sinh(c+dx)}{b}+1}dx}{a}}{a\left(a^2+b^2\right)}-\frac{b^2\coth(c+dx)}{ad\left(a^2+b^2\right)(a+b\text{csch}(c+dx))}$$

$\big\downarrow$ 3042

$$-\frac{b^2\coth(c+dx)}{ad\left(a^2+b^2\right)(a+b\text{csch}(c+dx))}+\frac{\frac{x\left(a^2+b^2\right)}{a}-\frac{\left(2a^2+b^2\right)\int \frac{1}{1-\frac{ia\sin(ic+idx)}{b}}dx}{a}}{a\left(a^2+b^2\right)}$$

$\big\downarrow$ 3139

$$-\frac{b^2\coth(c+dx)}{ad\left(a^2+b^2\right)(a+b\text{csch}(c+dx))}+\frac{\frac{x\left(a^2+b^2\right)}{a}+\frac{2i\left(2a^2+b^2\right)\int \frac{1}{-\tanh^2\left(\frac{1}{2}(c+dx)\right)+\frac{2a\tanh\left(\frac{1}{2}(c+dx)\right)}{b}+1}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ad}}{a\left(a^2+b^2\right)}$$

$\big\downarrow$ 1083

$$-\frac{b^2\coth(c+dx)}{ad\left(a^2+b^2\right)(a+b\text{csch}(c+dx))}+\frac{\frac{x\left(a^2+b^2\right)}{a}-\frac{4i\left(2a^2+b^2\right)\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(\frac{a^2}{b^2}+1\right)}d\left(2i\tanh\left(\frac{1}{2}(c+dx)\right)-\frac{2ia}{b}\right)}{ad}}{a\left(a^2+b^2\right)}$$

$\big\downarrow$ 217

$$\frac{\frac{x(a^2+b^2)}{a}-\frac{2b(2a^2+b^2)\text{arctanh}\left(\frac{b\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}}{a\left(a^2+b^2\right)}-\frac{b^2\coth(c+dx)}{ad\left(a^2+b^2\right)\left(a+b\text{csch}(c+dx)\right)}$$

input
```
Int[(a + b*Csch[c + d*x])^(-2),x]
```

output
```
(((a^2 + b^2)*x)/a - (2*b*(2*a^2 + b^2)*ArcTanh[(b*Tanh[(c + d*x)/2])/(2*S
qrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d))/(a*(a^2 + b^2)) - (b^2*Coth[c + d
*x])/(a*(a^2 + b^2)*d*(a + b*Csch[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.74

method	result
derivativedivides	$\frac{-\frac{2b\left(\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a^2}{2a^2+2b^2}+\frac{ab}{2a^2+2b^2}}{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b}{2}+a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{b}{2}}-\frac{2\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{\left(2a^2+2b^2\right)\sqrt{a^2+b^2}}\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}+\frac{\ln\left(\tan\ldots\right.}{}}{d}$
default	$\frac{-\frac{2b\left(\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a^2}{2a^2+2b^2}+\frac{ab}{2a^2+2b^2}}{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b}{2}+a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{b}{2}}-\frac{2\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{\left(2a^2+2b^2\right)\sqrt{a^2+b^2}}\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}+\frac{\ln\left(\tan\ldots\right.}{}}{d}$
risch	$\frac{x}{a^2}-\frac{2b^2\left(-b\,\mathrm{e}^{dx+c}+a\right)}{d\,a^2\left(a^2+b^2\right)\left(a\,\mathrm{e}^{2dx+2c}+2b\,\mathrm{e}^{dx+c}-a\right)}+\frac{2b\ln\left(\mathrm{e}^{dx+c}+\frac{\left(a^2+b^2\right)^{\frac{3}{2}}b+a^4+2a^2b^2+b^4}{a\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}d}+\frac{b^3\ln\left(\mathrm{e}^{dx+c}+\frac{\left(a^2+b^{\ldots}\right.}{}\right.}{\left(a^2+b^{\ldots}\right.}$

input
```
int(1/(a+csch(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*b/a^2*((1/2*a^2/(a^2+b^2)*tanh(1/2*d*x+1/2*c)+1/2*b*a/(a^2+b^2))/(
-1/2*tanh(1/2*d*x+1/2*c)^2*b+a*tanh(1/2*d*x+1/2*c)+1/2*b)-2*(2*a^2+b^2)/(2
*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*d*x+1/2*c)+2*a)/(a^
2+b^2)^(1/2)))-1/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+1/a^2*ln(tanh(1/2*d*x+1/2*c
)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 645 vs. $2(98) = 196$.

Time = 0.11 (sec) , antiderivative size = 645, normalized size of antiderivative = 6.39

$$\int \frac{1}{(a+b\text{csch}(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*csch(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(2*a^3*b^2 + 2*a*b^4 - (a^5 + 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c)^2 - (a
^5 + 2*a^3*b^2 + a*b^4)*d*x*sinh(d*x + c)^2 + (a^5 + 2*a^3*b^2 + a*b^4)*d*
x + (2*a^3*b + a*b^3 - (2*a^3*b + a*b^3)*cosh(d*x + c)^2 - (2*a^3*b + a*b^
3)*sinh(d*x + c)^2 - 2*(2*a^2*b^2 + b^4)*cosh(d*x + c) - 2*(2*a^2*b^2 + b^
4 + (2*a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)*log((a
^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*b
^2 + 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(a*cosh
(d*x + c) + a*sinh(d*x + c) + b))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 +
 2*b*cosh(d*x + c) + 2*(a*cosh(d*x + c) + b)*sinh(d*x + c) - a)) - 2*(a^2*
b^3 + b^5 + (a^4*b + 2*a^2*b^3 + b^5)*d*x)*cosh(d*x + c) - 2*(a^2*b^3 + b^
5 + (a^5 + 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5
)*d*x)*sinh(d*x + c))/((a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^2 + (a^
7 + 2*a^5*b^2 + a^3*b^4)*d*sinh(d*x + c)^2 + 2*(a^6*b + 2*a^4*b^3 + a^2*b^
5)*d*cosh(d*x + c) - (a^7 + 2*a^5*b^2 + a^3*b^4)*d + 2*((a^7 + 2*a^5*b^2 +
 a^3*b^4)*d*cosh(d*x + c) + (a^6*b + 2*a^4*b^3 + a^2*b^5)*d)*sinh(d*x + c)
)
```

Sympy [F]

$$\int \frac{1}{(a+b\mathrm{csch}(c+dx))^2}\,dx = \int \frac{1}{(a+b\operatorname{csch}(c+dx))^2}\,dx$$

input
```
integrate(1/(a+b*csch(d*x+c))**2,x)
```

output
```
Integral((a + b*csch(c + d*x))**(-2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.85

$$\int \frac{1}{(a+b\mathrm{csch}(c+dx))^2}\,dx = -\frac{(2a^2b+b^3)\log\left(\frac{ae^{(-dx-c)}-b-\sqrt{a^2+b^2}}{ae^{(-dx-c)}-b+\sqrt{a^2+b^2}}\right)}{(a^4+a^2b^2)\sqrt{a^2+b^2}d} - \frac{2\left(b^3e^{(-dx-c)}+ab^2\right)}{(a^5+a^3b^2+2(a^4b+a^2b^3)e^{(-dx-c)}-(a^5+a^3b^2)e^{(-2dx-2c)})d} + \frac{dx+c}{a^2d}$$

input
```
integrate(1/(a+b*csch(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(2*a^2*b + b^3)*log((a*e^(-d*x - c) - b - sqrt(a^2 + b^2))/(a*e^(-d*x - c
) - b + sqrt(a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)*d) - 2*(b^3*e^(
-d*x - c) + a*b^2)/((a^5 + a^3*b^2 + 2*(a^4*b + a^2*b^3)*e^(-d*x - c) - (a
^5 + a^3*b^2)*e^(-2*d*x - 2*c))*d) + (d*x + c)/(a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.59

$$\int \frac{1}{(a + b\text{csch}(c + dx))^2} dx$$
$$= -\frac{\frac{(2a^2b+b^3)\log\left(\frac{|2ae^{(dx+c)}+2b-2\sqrt{a^2+b^2}|}{|2ae^{(dx+c)}+2b+2\sqrt{a^2+b^2}|}\right)}{(a^4+a^2b^2)\sqrt{a^2+b^2}} - \frac{2(b^3e^{(dx+c)}-ab^2)}{(a^4+a^2b^2)(ae^{(2dx+2c)}+2be^{(dx+c)}-a)} - \frac{dx+c}{a^2}}{d}$$

input
```
integrate(1/(a+b*csch(d*x+c))^2,x, algorithm="giac")
```

output
```
-((2*a^2*b + b^3)*log(abs(2*a*e^(d*x + c) + 2*b - 2*sqrt(a^2 + b^2))/abs(2
*a*e^(d*x + c) + 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2
)) - 2*(b^3*e^(d*x + c) - a*b^2)/((a^4 + a^2*b^2)*(a*e^(2*d*x + 2*c) + 2*b
*e^(d*x + c) - a)) - (d*x + c)/a^2)/d
```

Mupad [B] (verification not implemented)

Time = 3.04 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.66

$$\int \frac{1}{(a + b\text{csch}(c + dx))^2} dx = \frac{x}{a^2} - \frac{\frac{2b^2}{d(a^3+ab^2)} - \frac{2b^3 e^{c+dx}}{ad(a^3+ab^2)}}{2b e^{c+dx} - a + a e^{2c+2dx}}$$
$$- \frac{b \ln\left(\frac{2e^{c+dx}(2a^2b+b^3)}{a^3(a^2+b^2)} - \frac{2b(2a^2+b^2)(a-be^{c+dx})}{a^3(a^2+b^2)^{3/2}}\right)(2a^2+b^2)}{a^2 d (a^2+b^2)^{3/2}}$$
$$+ \frac{b \ln\left(\frac{2e^{c+dx}(2a^2b+b^3)}{a^3(a^2+b^2)} + \frac{2b(2a^2+b^2)(a-be^{c+dx})}{a^3(a^2+b^2)^{3/2}}\right)(2a^2+b^2)}{a^2 d (a^2+b^2)^{3/2}}$$

input
```
int(1/(a + b/sinh(c + d*x))^2,x)
```

output
```
x/a^2 - ((2*b^2)/(d*(a*b^2 + a^3)) - (2*b^3*exp(c + d*x))/(a*d*(a*b^2 + a^
3)))/(2*b*exp(c + d*x) - a + a*exp(2*c + 2*d*x)) - (b*log((2*exp(c + d*x)*
(2*a^2*b + b^3))/(a^3*(a^2 + b^2)) - (2*b*(2*a^2 + b^2)*(a - b*exp(c + d*x
)))/(a^3*(a^2 + b^2)^(3/2)))*(2*a^2 + b^2))/(a^2*d*(a^2 + b^2)^(3/2)) + (b
*log((2*exp(c + d*x)*(2*a^2*b + b^3))/(a^3*(a^2 + b^2)) + (2*b*(2*a^2 + b^
2)*(a - b*exp(c + d*x)))/(a^3*(a^2 + b^2)^(3/2)))*(2*a^2 + b^2))/(a^2*d*(a
^2 + b^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 570, normalized size of antiderivative = 5.64

$$\int \frac{1}{(a + b\text{csch}(c + dx))^2}\, dx$$
$$= \frac{-4e^{2dx+2c}\sqrt{a^2+b^2}\, atan\left(\frac{e^{dx+c}ai+bi}{\sqrt{a^2+b^2}}\right) a^3bi - 2e^{2dx+2c}\sqrt{a^2+b^2}\, atan\left(\frac{e^{dx+c}ai+bi}{\sqrt{a^2+b^2}}\right) a\,b^3i - 8e^{dx+c}\sqrt{a^2+b^2}\, ata}{}$$

input
```
int(1/(a+b*csch(d*x+c))^2,x)
```

output
```
( - 4*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*a*i + b*i)/sqr
t(a**2 + b**2))*a**3*b*i - 2*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(
c + d*x)*a*i + b*i)/sqrt(a**2 + b**2))*a*b**3*i - 8*e**(c + d*x)*sqrt(a**2
 + b**2)*atan((e**(c + d*x)*a*i + b*i)/sqrt(a**2 + b**2))*a**2*b**2*i - 4*
e**(c + d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*a*i + b*i)/sqrt(a**2 + b
**2))*b**4*i + 4*sqrt(a**2 + b**2)*atan((e**(c + d*x)*a*i + b*i)/sqrt(a**2
 + b**2))*a**3*b*i + 2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*a*i + b*i)/sqr
t(a**2 + b**2))*a*b**3*i + e**(2*c + 2*d*x)*a**5*d*x + 2*e**(2*c + 2*d*x)*
a**3*b**2*d*x - e**(2*c + 2*d*x)*a**3*b**2 + e**(2*c + 2*d*x)*a*b**4*d*x -
 e**(2*c + 2*d*x)*a*b**4 + 2*e**(c + d*x)*a**4*b*d*x + 4*e**(c + d*x)*a**2
*b**3*d*x + 2*e**(c + d*x)*b**5*d*x - a**5*d*x - 2*a**3*b**2*d*x - a**3*b*
*2 - a*b**4*d*x - a*b**4)/(a**2*d*(e**(2*c + 2*d*x)*a**5 + 2*e**(2*c + 2*d
*x)*a**3*b**2 + e**(2*c + 2*d*x)*a*b**4 + 2*e**(c + d*x)*a**4*b + 4*e**(c
+ d*x)*a**2*b**3 + 2*e**(c + d*x)*b**5 - a**5 - 2*a**3*b**2 - a*b**4))
```

3.76 $\int \frac{1}{(a+b\operatorname{csch}(c+dx))^3} dx$

Optimal result

Integrand size = 12, antiderivative size = 163

$$\int \frac{1}{(a+b\operatorname{csch}(c+dx))^3} dx = \frac{x}{a^3} + \frac{b(6a^4+5a^2b^2+2b^4)\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3(a^2+b^2)^{5/2}d} - \frac{b^2\coth(c+dx)}{2a(a^2+b^2)d(a+b\operatorname{csch}(c+dx))^2} - \frac{b^2(5a^2+2b^2)\coth(c+dx)}{2a^2(a^2+b^2)^2d(a+b\operatorname{csch}(c+dx))}$$

output
```
x/a^3+b*(6*a^4+5*a^2*b^2+2*b^4)*arctanh((a-b*tanh(1/2*d*x+1/2*c))/(a^2+b^2
)^(1/2))/a^3/(a^2+b^2)^(5/2)/d-1/2*b^2*coth(d*x+c)/a/(a^2+b^2)/d/(a+b*csch
(d*x+c))^2-1/2*b^2*(5*a^2+2*b^2)*coth(d*x+c)/a^2/(a^2+b^2)^2/d/(a+b*csch(d
*x+c))
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.31

$$\int \frac{1}{(a+b\text{csch}(c+dx))^3}\,dx$$

$$= \frac{\text{csch}^2(c+dx)(b+a\sinh(c+dx))\left(\frac{ab^3\coth(c+dx)}{a^2+b^2} - \frac{3ab^2(2a^2+b^2)\coth(c+dx)(b+a\sinh(c+dx))}{(a^2+b^2)^2} + 2(c+dx)\text{csch}(c+\right.}{2a^3d(a+b\text{csch}(c+}$$

input
```
Integrate[(a + b*Csch[c + d*x])^(-3),x]
```

output
```
(Csch[c + d*x]^2*(b + a*Sinh[c + d*x])*((a*b^3*Coth[c + d*x])/(a^2 + b^2)
- (3*a*b^2*(2*a^2 + b^2)*Coth[c + d*x]*(b + a*Sinh[c + d*x]))/(a^2 + b^2)^
2 + 2*(c + d*x)*Csch[c + d*x]*(b + a*Sinh[c + d*x])^2 - (2*b*(6*a^4 + 5*a^
2*b^2 + 2*b^4)*ArcTan[(a - b*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]]*Csch[c +
 d*x]*(b + a*Sinh[c + d*x])^2)/(-a^2 - b^2)^(5/2)))/(2*a^3*d*(a + b*Csch[c
 + d*x])^3)
```

Rubi [A] (warning: unable to verify)

Time = 1.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.24, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {3042, 4272, 25, 3042, 4548, 25, 3042, 4407, 26, 3042, 26, 4318, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\text{csch}(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ib\csc(ic+idx))^3}dx$$

$$\Big\downarrow \text{4272}$$

$$-\frac{\int -\frac{b^2\text{csch}^2(c+dx)-2ab\text{csch}(c+dx)+2(a^2+b^2)}{(a+b\text{csch}(c+dx))^2}dx}{2a(a^2+b^2)}-\frac{b^2\coth(c+dx)}{2ad(a^2+b^2)(a+b\text{csch}(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b^2\text{csch}^2(c+dx)-2ab\text{csch}(c+dx)+2(a^2+b^2)}{(a+b\text{csch}(c+dx))^2}dx}{2a(a^2+b^2)}-\frac{b^2\coth(c+dx)}{2ad(a^2+b^2)(a+b\text{csch}(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b^2\coth(c+dx)}{2ad(a^2+b^2)(a+b\text{csch}(c+dx))^2}+\frac{\int \frac{-b^2\csc(ic+idx)^2-2iab\csc(ic+idx)+2(a^2+b^2)}{(a+ib\csc(ic+idx))^2}dx}{2a(a^2+b^2)}$$

$$\Big\downarrow \text{4548}$$

$$\frac{-\frac{\int -\frac{2(a^2+b^2)^2-ab(4a^2+b^2)\text{csch}(c+dx)}{a+b\text{csch}(c+dx)}dx}{a(a^2+b^2)}-\frac{b^2(5a^2+2b^2)\coth(c+dx)}{ad(a^2+b^2)(a+b\text{csch}(c+dx))}}{2a(a^2+b^2)}-\frac{b^2\coth(c+dx)}{2ad(a^2+b^2)(a+b\text{csch}(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{2(a^2+b^2)^2-ab(4a^2+b^2)\text{csch}(c+dx)}{a+b\text{csch}(c+dx)}dx}{a(a^2+b^2)}-\frac{b^2(5a^2+2b^2)\coth(c+dx)}{ad(a^2+b^2)(a+b\text{csch}(c+dx))}}{2a(a^2+b^2)}-\frac{b^2\coth(c+dx)}{2ad(a^2+b^2)(a+b\text{csch}(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b^2\coth(c+dx)}{2ad(a^2+b^2)(a+b\text{csch}(c+dx))^2}+\frac{-\frac{b^2(5a^2+2b^2)\coth(c+dx)}{ad(a^2+b^2)(a+b\text{csch}(c+dx))}+\frac{\int \frac{2(a^2+b^2)^2-iab(4a^2+b^2)\csc(ic+idx)}{a+ib\csc(ic+idx)}dx}{a(a^2+b^2)}}{2a(a^2+b^2)}$$

$$\Big\downarrow \text{4407}$$

$$-\frac{b^2\coth(c+dx)}{2ad(a^2+b^2)(a+b\text{csch}(c+dx))^2}+\frac{-\frac{b^2(5a^2+2b^2)\coth(c+dx)}{ad(a^2+b^2)(a+b\text{csch}(c+dx))}+\frac{\frac{2x(a^2+b^2)^2}{a}-\frac{ib(6a^4+5a^2b^2+2b^4)\int -\frac{i\text{csch}(c+dx)}{a+b\text{csch}(c+dx)}dx}{a}}{a(a^2+b^2)}}{2a(a^2+b^2)}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{\frac{2x(a^2+b^2)^2}{a}-\frac{b(6a^4+5a^2b^2+2b^4)\int\frac{\operatorname{csch}(c+dx)}{a+b\operatorname{csch}(c+dx)}dx}{a}}{a(a^2+b^2)}-\frac{b^2(5a^2+2b^2)\coth(c+dx)}{ad(a^2+b^2)(a+b\operatorname{csch}(c+dx))}}{2a(a^2+b^2)}-\frac{b^2\coth(c+dx)}{2ad(a^2+b^2)(a+b\operatorname{csch}(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b^2\coth(c+dx)}{2ad(a^2+b^2)(a+b\operatorname{csch}(c+dx))^2}+\frac{-\frac{b^2(5a^2+2b^2)\coth(c+dx)}{ad(a^2+b^2)(a+b\operatorname{csch}(c+dx))}+\frac{\frac{2x(a^2+b^2)^2}{a}-\frac{b(6a^4+5a^2b^2+2b^4)\int\frac{i\csc(ic+idx)}{a+ib\csc(ic+idx)}dx}{a}}{a(a^2+b^2)}}{2a(a^2+b^2)}$$

$$\Big\downarrow \; 26$$

$$-\frac{b^2\coth(c+dx)}{2ad(a^2+b^2)(a+b\operatorname{csch}(c+dx))^2}+\frac{-\frac{b^2(5a^2+2b^2)\coth(c+dx)}{ad(a^2+b^2)(a+b\operatorname{csch}(c+dx))}+\frac{\frac{2x(a^2+b^2)^2}{a}-\frac{ib(6a^4+5a^2b^2+2b^4)\int\frac{\csc(ic+idx)}{a+ib\csc(ic+idx)}dx}{a}}{a(a^2+b^2)}}{2a(a^2+b^2)}$$

$$\Big\downarrow \; 4318$$

$$\frac{\frac{\frac{2x(a^2+b^2)^2}{a}-\frac{(6a^4+5a^2b^2+2b^4)\int\frac{1}{\frac{a\sinh(c+dx)}{b}+1}dx}{a}}{a(a^2+b^2)}-\frac{b^2(5a^2+2b^2)\coth(c+dx)}{ad(a^2+b^2)(a+b\operatorname{csch}(c+dx))}}{2a(a^2+b^2)}-\frac{b^2\coth(c+dx)}{2ad(a^2+b^2)(a+b\operatorname{csch}(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b^2\coth(c+dx)}{2ad(a^2+b^2)(a+b\operatorname{csch}(c+dx))^2}+\frac{-\frac{b^2(5a^2+2b^2)\coth(c+dx)}{ad(a^2+b^2)(a+b\operatorname{csch}(c+dx))}+\frac{\frac{2x(a^2+b^2)^2}{a}-\frac{(6a^4+5a^2b^2+2b^4)\int\frac{1}{1-\frac{ia\sin(ic+idx)}{b}}dx}{a}}{a(a^2+b^2)}}{2a(a^2+b^2)}$$

$$\Big\downarrow \; 3139$$

$$-\frac{b^2\coth(c+dx)}{2ad(a^2+b^2)(a+b\operatorname{csch}(c+dx))^2}+\frac{-\frac{b^2(5a^2+2b^2)\coth(c+dx)}{ad(a^2+b^2)(a+b\operatorname{csch}(c+dx))}+\frac{\frac{2x(a^2+b^2)^2}{a}+\frac{2i(6a^4+5a^2b^2+2b^4)\int\frac{1}{-\tanh^2\left(\frac{1}{2}(c+dx)\right)+\frac{2a\tanh\left(\frac{1}{2}(c+dx)\right)}{b}+1}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ad}}{a(a^2+b^2)}}{2a(a^2+b^2)}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{b^2\coth(c+dx)}{2ad\left(a^2+b^2\right)(a+b\text{csch}(c+dx))^2}+\frac{-\frac{b^2\left(5a^2+2b^2\right)\coth(c+dx)}{ad(a^2+b^2)(a+b\text{csch}(c+dx))}+\frac{\frac{2x\left(a^2+b^2\right)^2}{a}-\frac{4i\left(6a^4+5a^2b^2+2b^4\right)\int\frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(\frac{a^2}{b^2}+1\right)}d\left(2i\tanh\left(\frac{1}{2}(c+dx)\right)-\frac{2ia}{b}\right)}{ad}}{a(a^2+b^2)}}{2a\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\frac{2x\left(a^2+b^2\right)^2}{a}-\frac{2b\left(6a^4+5a^2b^2+2b^4\right)\text{arctanh}\left(\frac{b\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}}{a(a^2+b^2)}-\frac{b^2\left(5a^2+2b^2\right)\coth(c+dx)}{ad(a^2+b^2)(a+b\text{csch}(c+dx))}}{2a\left(a^2+b^2\right)}-\frac{b^2\coth(c+dx)}{2ad\left(a^2+b^2\right)(a+b\text{csch}(c+dx))^2}$$

input
```
Int[(a + b*Csch[c + d*x])^(-3),x]
```

output
```
-1/2*(b^2*Coth[c + d*x])/(a*(a^2 + b^2)*d*(a + b*Csch[c + d*x])^2) + (((2*
(a^2 + b^2)^2*x)/a - (2*b*(6*a^4 + 5*a^2*b^2 + 2*b^4)*ArcTanh[(b*Tanh[(c +
 d*x)/2])/(2*Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d))/(a*(a^2 + b^2)) - (
b^2*(5*a^2 + 2*b^2)*Coth[c + d*x])/(a*(a^2 + b^2)*d*(a + b*Csch[c + d*x]))
)/(2*a*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(154) = 308$.

Time = 0.45 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.01

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3}-\frac{2b\left(\frac{-\frac{b a^2\left(4a^2+b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2\left(a^4+2a^2b^2+b^4\right)}+\frac{4a\left(10a^4-a^2b^2-2b^4\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a^4+16a^2b^2+8b^4}+\frac{4}{}}{\left(-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+2a\tanh\left(\frac{dx}{2}\right.\right.}\right.}{}}{d}$
default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3}-\frac{2b\left(\frac{-\frac{b a^2\left(4a^2+b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2\left(a^4+2a^2b^2+b^4\right)}+\frac{4a\left(10a^4-a^2b^2-2b^4\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a^4+16a^2b^2+8b^4}+\frac{4}{}}{\left(-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+2a\tanh\left(\frac{dx}{2}\right.\right.}\right.}{}}{d}$
risch	$\frac{x}{a^3}-\frac{b^2\left(-7a^3b\,e^{3dx+3c}-4a\,b^3e^{3dx+3c}+6a^4e^{2dx+2c}-9\,e^{2dx+2c}a^2b^2-6b^4e^{2dx+2c}+17a^3b\,e^{dx+c}+8b^3e^{dx+c}a-6a^4-3\right.}{a^3\left(a^2+b^2\right)^2d\left(a\,e^{2dx+2c}+2b\,e^{dx+c}-a\right)^2}$

input
```
int(1/(a+csch(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-2/a^
3*b*(4*(-1/8*b*a^2*(4*a^2+b^2)/(a^4+2*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^3+1
/8*a*(10*a^4-a^2*b^2-2*b^4)/(a^4+2*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^2+1/8*
a^2*b*(16*a^2+7*b^2)/(a^4+2*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)+1/8*a*b^2*(5*
a^2+2*b^2)/(a^4+2*a^2*b^2+b^4))/(-tanh(1/2*d*x+1/2*c)^2*b+2*a*tanh(1/2*d*x
+1/2*c)+b)^2-2*(6*a^4+5*a^2*b^2+2*b^4)/(4*a^4+8*a^2*b^2+4*b^4)/(a^2+b^2)^(
1/2)*arctanh(1/2*(-2*b*tanh(1/2*d*x+1/2*c)+2*a)/(a^2+b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2094 vs. $2(156) = 312$.

Time = 0.16 (sec) , antiderivative size = 2094, normalized size of antiderivative = 12.85

$$\int \frac{1}{(a + b\text{csch}(c + dx))^3} dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*csch(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(12*a^6*b^2 + 18*a^4*b^4 + 6*a^2*b^6 + 2*(a^8 + 3*a^6*b^2 + 3*a^4*b^4
+ a^2*b^6)*d*x*cosh(d*x + c)^4 + 2*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)
*d*x*sinh(d*x + c)^4 + 2*(7*a^5*b^3 + 11*a^3*b^5 + 4*a*b^7 + 4*(a^7*b + 3*
a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*x)*cosh(d*x + c)^3 + 2*(7*a^5*b^3 + 11*a^3*
b^5 + 4*a*b^7 + 4*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d*x*cosh(d*x + c
) + 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*x)*sinh(d*x + c)^3 + 2*(a^
8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d*x - 2*(6*a^6*b^2 - 3*a^4*b^4 - 15*a
^2*b^6 - 6*b^8 + 2*(a^8 + a^6*b^2 - 3*a^4*b^4 - 5*a^2*b^6 - 2*b^8)*d*x)*co
sh(d*x + c)^2 - 2*(6*a^6*b^2 - 3*a^4*b^4 - 15*a^2*b^6 - 6*b^8 - 6*(a^8 + 3
*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d*x*cosh(d*x + c)^2 + 2*(a^8 + a^6*b^2 - 3
*a^4*b^4 - 5*a^2*b^6 - 2*b^8)*d*x - 3*(7*a^5*b^3 + 11*a^3*b^5 + 4*a*b^7 +
4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*x)*cosh(d*x + c))*sinh(d*x + c
)^2 + (6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5 + (6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5)
*cosh(d*x + c)^4 + (6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*sinh(d*x + c)^4 + 4*(
6*a^5*b^2 + 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c)^3 + 4*(6*a^5*b^2 + 5*a^3*b^
4 + 2*a*b^6 + (6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c))*sinh(d*x +
c)^3 - 2*(6*a^6*b - 7*a^4*b^3 - 8*a^2*b^5 - 4*b^7)*cosh(d*x + c)^2 - 2*(6*
a^6*b - 7*a^4*b^3 - 8*a^2*b^5 - 4*b^7 - 3*(6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5
)*cosh(d*x + c)^2 - 6*(6*a^5*b^2 + 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c))*sin
h(d*x + c)^2 - 4*(6*a^5*b^2 + 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c) - 4*(6...
```

Sympy [F]

$$\int \frac{1}{(a+b\mathrm{csch}(c+dx))^3}\,dx = \int \frac{1}{\left(a+b\,\mathrm{csch}\,(c+dx)\right)^3}\,dx$$

input
```
integrate(1/(a+b*csch(d*x+c))**3,x)
```

output
```
Integral((a + b*csch(c + d*x))**(-3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(156) = 312.

Time = 0.13 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.29

$$\int \frac{1}{(a+b\mathrm{csch}(c+dx))^3}\,dx = -\frac{(6\,a^4b+5\,a^2b^3+2\,b^5)\log\left(\frac{ae^{(-dx-c)}-b-\sqrt{a^2+b^2}}{ae^{(-dx-c)}-b+\sqrt{a^2+b^2}}\right)}{2\,(a^7+2\,a^5b^2+a^3b^4)\sqrt{a^2+b^2}d} - \frac{6\,a^4b^2+3\,a^2b^4+(17\,a^3b^3+8\,ab^5)e^{(-dx-c)}-3\,(2\,a^4b^2-3\,a^2b^4-2\,b^6)e^{(-2\,dx-}}{(a^9+2\,a^7b^2+a^5b^4+4\,(a^8b+2\,a^6b^3+a^4b^5)e^{(-dx-c)}-2\,(a^9-3\,a^5b^4-2\,a^3b^6)e^{(-2\,dx-2\,c)}-4\,(a^8b+2}\, + \frac{dx+c}{a^3d}$$

input
```
integrate(1/(a+b*csch(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*(6*a^4*b + 5*a^2*b^3 + 2*b^5)*log((a*e^(-d*x - c) - b - sqrt(a^2 + b^
2))/(a*e^(-d*x - c) - b + sqrt(a^2 + b^2)))/((a^7 + 2*a^5*b^2 + a^3*b^4)*s
qrt(a^2 + b^2)*d) - (6*a^4*b^2 + 3*a^2*b^4 + (17*a^3*b^3 + 8*a*b^5)*e^(-d*
x - c) - 3*(2*a^4*b^2 - 3*a^2*b^4 - 2*b^6)*e^(-2*d*x - 2*c) - (7*a^3*b^3 +
 4*a*b^5)*e^(-3*d*x - 3*c))/((a^9 + 2*a^7*b^2 + a^5*b^4 + 4*(a^8*b + 2*a^6
*b^3 + a^4*b^5)*e^(-d*x - c) - 2*(a^9 - 3*a^5*b^4 - 2*a^3*b^6)*e^(-2*d*x -
 2*c) - 4*(a^8*b + 2*a^6*b^3 + a^4*b^5)*e^(-3*d*x - 3*c) + (a^9 + 2*a^7*b^
2 + a^5*b^4)*e^(-4*d*x - 4*c))*d) + (d*x + c)/(a^3*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.80

$$\int \frac{1}{(a+b\text{csch}(c+dx))^3}\,dx =$$

$$-\frac{\frac{(6\,a^4b+5\,a^2b^3+2\,b^5)\log\left(\frac{\left|2\,ae^{(dx+c)}+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^{(dx+c)}+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^7+2\,a^5b^2+a^3b^4)\sqrt{a^2+b^2}} - \frac{2\,(7\,a^3b^3e^{(3\,dx+3\,c)}+4\,ab^5e^{(3\,dx+3\,c)}-6\,a^4b^2e^{(2\,dx+2\,c)}+9\,a^2b^4e^{(2\,dx+2\,c)}+6\,b^6e^{(}}{(a^7+2\,a^5b^2+a^3b^4)\left(ae^{(2\,dx+2\,c)}+2\,b\right.}}{2\,d}$$

input
```
integrate(1/(a+b*csch(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2*((6*a^4*b + 5*a^2*b^3 + 2*b^5)*log(abs(2*a*e^(d*x + c) + 2*b - 2*sqrt
(a^2 + b^2))/abs(2*a*e^(d*x + c) + 2*b + 2*sqrt(a^2 + b^2)))/((a^7 + 2*a^5
*b^2 + a^3*b^4)*sqrt(a^2 + b^2)) - 2*(7*a^3*b^3*e^(3*d*x + 3*c) + 4*a*b^5*
e^(3*d*x + 3*c) - 6*a^4*b^2*e^(2*d*x + 2*c) + 9*a^2*b^4*e^(2*d*x + 2*c) +
6*b^6*e^(2*d*x + 2*c) - 17*a^3*b^3*e^(d*x + c) - 8*a*b^5*e^(d*x + c) + 6*a
^4*b^2 + 3*a^2*b^4)/((a^7 + 2*a^5*b^2 + a^3*b^4)*(a*e^(2*d*x + 2*c) + 2*b*
e^(d*x + c) - a)^2) - 2*(d*x + c)/a^3)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\text{csch}(c+dx))^3}\,dx = \int \frac{1}{\left(a+\frac{b}{\sinh(c+d\,x)}\right)^3}\,dx$$

input
```
int(1/(a + b/sinh(c + d*x))^3,x)
```

output
```
int(1/(a + b/sinh(c + d*x))^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1647, normalized size of antiderivative = 10.10

$$\int \frac{1}{(a+b\operatorname{csch}(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*csch(d*x+c))^3,x)
```

output
```
( - 24*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*a*i + b*i)/sq
rt(a**2 + b**2))*a**6*b*i - 20*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e*
*(c + d*x)*a*i + b*i)/sqrt(a**2 + b**2))*a**4*b**3*i - 8*e**(4*c + 4*d*x)*
sqrt(a**2 + b**2)*atan((e**(c + d*x)*a*i + b*i)/sqrt(a**2 + b**2))*a**2*b*
*5*i - 96*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*a*i + b*i)
/sqrt(a**2 + b**2))*a**5*b**2*i - 80*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*at
an((e**(c + d*x)*a*i + b*i)/sqrt(a**2 + b**2))*a**3*b**4*i - 32*e**(3*c +
3*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*a*i + b*i)/sqrt(a**2 + b**2))*
a*b**6*i + 48*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*a*i +
b*i)/sqrt(a**2 + b**2))*a**6*b*i - 56*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*a
tan((e**(c + d*x)*a*i + b*i)/sqrt(a**2 + b**2))*a**4*b**3*i - 64*e**(2*c +
 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*a*i + b*i)/sqrt(a**2 + b**2))
*a**2*b**5*i - 32*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*a*
i + b*i)/sqrt(a**2 + b**2))*b**7*i + 96*e**(c + d*x)*sqrt(a**2 + b**2)*ata
n((e**(c + d*x)*a*i + b*i)/sqrt(a**2 + b**2))*a**5*b**2*i + 80*e**(c + d*x
)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*a*i + b*i)/sqrt(a**2 + b**2))*a**3*
b**4*i + 32*e**(c + d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*a*i + b*i)/s
qrt(a**2 + b**2))*a*b**6*i - 24*sqrt(a**2 + b**2)*atan((e**(c + d*x)*a*i +
 b*i)/sqrt(a**2 + b**2))*a**6*b*i - 20*sqrt(a**2 + b**2)*atan((e**(c + d*x
)*a*i + b*i)/sqrt(a**2 + b**2))*a**4*b**3*i - 8*sqrt(a**2 + b**2)*atan(...
```

3.77 $\int \frac{\sinh^3(x)}{a+b\text{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 107

$$\int \frac{\sinh^3(x)}{a+b\text{csch}(x)}\,dx = \frac{b(a^2-2b^2)\,x}{2a^4} - \frac{2b^4\text{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}} - \frac{(2a^2-3b^2)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a}$$

output
```
1/2*b*(a^2-2*b^2)*x/a^4-2*b^4*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/a
^4/(a^2+b^2)^(1/2)-1/3*(2*a^2-3*b^2)*cosh(x)/a^3-1/2*b*cosh(x)*sinh(x)/a^2
+1/3*cosh(x)*sinh(x)^2/a
```

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.97

$$\int \frac{\sinh^3(x)}{a+b\text{csch}(x)}\,dx = \frac{(-9a^3+12ab^2)\cosh(x)+a^3\cosh(3x)+3b\left(2a^2x-4b^2x+\frac{8b^3\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-a^2\sinh(2x)\right)}{12a^4}$$

input
```
Integrate[Sinh[x]^3/(a + b*Csch[x]),x]
```

output
```
((-9*a^3 + 12*a*b^2)*Cosh[x] + a^3*Cosh[3*x] + 3*b*(2*a^2*x - 4*b^2*x + (8
*b^3*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - a^2*Si
nh[2*x]))/(12*a^4)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.36, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.769, Rules used = {3042, 26, 4340, 26, 3042, 25, 4592, 26, 26, 3042, 26, 4592, 27, 3042, 4407, 26, 3042, 26, 4318, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{a+b\text{csch}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\csc(ix)^3(a+ib\csc(ix))}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\csc(ix)^3(a+ib\csc(ix))}dx$$

$$\Big\downarrow \text{4340}$$

$$i\left(\frac{\int \frac{i\left(2b\text{csch}^2(x)+2a\text{csch}(x)+3b\right)\sinh^2(x)}{a+b\text{csch}(x)}dx}{3a}-\frac{i\sinh^2(x)\cosh(x)}{3a}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\int \frac{\left(2b\text{csch}^2(x)+2a\text{csch}(x)+3b\right)\sinh^2(x)}{a+b\text{csch}(x)}dx}{3a}-\frac{i\sinh^2(x)\cosh(x)}{3a}\right)$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{i\int -\frac{-2b\csc(ix)^2+2ia\csc(ix)+3b}{\csc(ix)^2(a+ib\csc(ix))}dx}{3a}-\frac{i\sinh^2(x)\cosh(x)}{3a}\right)$$

$$\Big\downarrow \; 25$$

$$i\left(-\frac{i\int \frac{-2b\csc(ix)^2+2ia\csc(ix)+3b}{\csc(ix)^2(a+ib\csc(ix))}dx}{3a}-\frac{i\sinh^2(x)\cosh(x)}{3a}\right)$$

$$\Big\downarrow \; 4592$$

$$i\left(-\frac{i\left(-\frac{3b\sinh(x)\cosh(x)}{2a}-\frac{\int -\frac{i\left(-3ib^2\text{csch}^2(x)+iab\text{csch}(x)+2\left(2ia^2-3ib^2\right)\right)\sinh(x)}{a+b\text{csch}(x)}dx}{2a}\right)}{3a}-\frac{i\sinh^2(x)\cosh(x)}{3a}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(-\frac{i\left(-\frac{3b\sinh(x)\cosh(x)}{2a}+\frac{i\int \frac{i\left(-3b^2\text{csch}^2(x)+ab\text{csch}(x)+2\left(2a^2-3b^2\right)\right)\sinh(x)}{a+b\text{csch}(x)}dx}{2a}\right)}{3a}-\frac{i\sinh^2(x)\cosh(x)}{3a}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(-\frac{i\left(-\frac{\int \frac{\left(-3b^2\text{csch}^2(x)+ab\text{csch}(x)+2\left(2a^2-3b^2\right)\right)\sinh(x)}{a+b\text{csch}(x)}dx}{2a}-\frac{3b\sinh(x)\cosh(x)}{2a}\right)}{3a}-\frac{i\sinh^2(x)\cosh(x)}{3a}\right)$$

$$\Big\downarrow \; 3042$$

$$i\left(-\frac{i\left(-\frac{3b\sinh(x)\cosh(x)}{2a}-\frac{\int -\frac{i\left(3b^2\csc(ix)^2+iab\csc(ix)+2\left(2a^2-3b^2\right)\right)}{\csc(ix)(a+ib\csc(ix))}dx}{2a}\right)}{3a}-\frac{i\sinh^2(x)\cosh(x)}{3a}\right)$$

$\downarrow$ 26

$$i\left(-\frac{i\left(-\frac{3b\sinh(x)\cosh(x)}{2a}+\frac{i\int \frac{3b^2\csc(ix)^2+iab\csc(ix)+2\left(2a^2-3b^2\right)}{\csc(ix)(a+ib\csc(ix))}dx}{2a}\right)}{3a}-\frac{i\sinh^2(x)\cosh(x)}{3a}\right)$$

$\downarrow$ 4592

$$i\left(-\frac{i\left(-\frac{3b\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{2i\left(2a^2-3b^2\right)\cosh(x)}{a}-\frac{\int \frac{3i\left(a\text{csch}(x)b^2+\left(a^2-2b^2\right)b\right)}{a+b\text{csch}(x)}dx}{a}\right)}{2a}\right)}{3a}-\frac{i\sinh^2(x)\cosh(x)}{3a}\right)$$

$\downarrow$ 27

$$i\left(-\frac{i\left(-\frac{3b\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{2i\left(2a^2-3b^2\right)\cosh(x)}{a}-\frac{3i\int\frac{a\operatorname{csch}(x)b^2+\left(a^2-2b^2\right)b}{a+b\operatorname{csch}(x)}dx}{a}\right)}{2a}\right)}{3a}-\frac{i\sinh^2(x)\cosh(x)}{3a}\right)$$

$\downarrow$ 3042

$$i\left(-\frac{i\left(-\frac{3b\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{2i\left(2a^2-3b^2\right)\cosh(x)}{a}-\frac{3i\int\frac{ia\csc(ix)b^2+\left(a^2-2b^2\right)b}{a+ib\csc(ix)}dx}{a}\right)}{2a}\right)}{3a}-\frac{i\sinh^2(x)\cosh(x)}{3a}\right)$$

$\downarrow$ 4407

$$i\left(-\frac{i\left(-\frac{3b\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{2i(2a^2-3b^2)\cosh(x)}{a}-\frac{3i\left(\frac{bx(a^2-2b^2)}{a}+\frac{2ib^4\int-\frac{i\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}dx}{a}\right)}{a}\right)}{2a}\right)}{3a}-\frac{i\sinh^2(x)\cosh(x)}{3a}\right)$$

$$\Big\downarrow\ 26$$

$$i\left(-\frac{i\left(-\frac{3b\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{2i\left(2a^2-3b^2\right)\cosh(x)}{a}-\frac{3i\left(\frac{2b^4\int\frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}dx}{a}+\frac{bx\left(a^2-2b^2\right)}{a}\right)}{a}\right)}{2a}\right)}{3a}-\frac{i\sinh^2(x)\cosh(x)}{3a}\right)$$

$\downarrow$ 3042

$$i\left(-\frac{i\left(-\frac{3b\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{2i\left(2a^2-3b^2\right)\cosh(x)}{a}-\frac{3i\left(\frac{bx\left(a^2-2b^2\right)}{a}+\frac{2b^4\int\frac{i\csc(ix)}{a+ib\csc(ix)}dx}{a}\right)}{a}\right)}{2a}\right)}{3a}-\frac{i\sinh^2(x)\cosh(x)}{3a}\right)$$

$\downarrow$ 26

$$i\left(-\frac{i\left(-\frac{3b\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{2i\left(2a^2-3b^2\right)\cosh(x)}{a}-\frac{3i\left(\frac{bx\left(a^2-2b^2\right)}{a}+\frac{2ib^4\int\frac{\csc(ix)}{a+ib\csc(ix)}dx}{a}\right)}{a}\right)}{2a}\right)}{3a}-\frac{i\sinh^2(x)\cosh(x)}{3a}\right)$$

$$\Big\downarrow \; 4318$$

$$i\left(-\frac{i\left(-\frac{3b\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{2i\left(2a^2-3b^2\right)\cosh(x)}{a}-\frac{3i\left(\frac{2b^3\int\frac{1}{\frac{a\sinh(x)}{b}+1}dx}{a}+\frac{bx\left(a^2-2b^2\right)}{a}\right)}{a}\right)}{2a}\right)}{3a}-\frac{i\sinh^2(x)\cosh(x)}{3a}\right)$$

$$\Big\downarrow \; 3042$$

$$i\left(-\frac{i\left(-\frac{3b\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{2i\left(2a^2-3b^2\right)\cosh(x)}{a}-\frac{3i\left(\frac{bx\left(a^2-2b^2\right)}{a}+\frac{2b^3\int\frac{1}{1-\frac{ia\sin(ix)}{b}}dx}{a}\right)}{a}\right)}{2a}\right)}{3a}-\frac{i\sinh^2(x)\cosh(x)}{3a}\right)$$

$\downarrow$ 3139

$$i\left(-\frac{i\left(-\frac{3b\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{2i(2a^2-3b^2)\cosh(x)}{a}-\frac{3i\left(\frac{4b^3\int\frac{1}{-\tanh^2\left(\frac{x}{2}\right)+\frac{2a\tanh\left(\frac{x}{2}\right)}{b}+1}d\tanh\left(\frac{x}{2}\right)}{a}+\frac{bx(a^2-2b^2)}{a}\right)}{a}\right)}{2a}\right)}{3a}-\frac{i\sinh^2(x)\cosh(x)}{3a}\right.$$

$\downarrow$ 1083

$$i\left(-\frac{i\left(-\frac{3b\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{2i\left(2a^2-3b^2\right)\cosh(x)}{a}-\frac{3i\left(\frac{bx\left(a^2-2b^2\right)}{a}-\frac{8b^3\int\frac{1}{4\left(\frac{a^2}{b^2}+1\right)-\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)^2}d\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{a}\right)}{a}\right)}{2a}\right)}{3a}-\frac{i\sin\ldots}{}\right.$$

$\downarrow$ 219

$$i\left(-\frac{i\left(-\frac{3b\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{2i(2a^2-3b^2)\cosh(x)}{a}-\frac{3i\left(\frac{bx(a^2-2b^2)}{a}-\frac{4b^4\operatorname{arctanh}\left(\frac{b\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\right)}{a}\right)}{2a}\right)}{3a}-\frac{i\sinh^2(x)\cosh(x}{3a}\right.$$

input
```
Int[Sinh[x]^3/(a + b*Csch[x]),x]
```

output
```
I*(((-1/3*I)*Cosh[x]*Sinh[x]^2)/a - ((I/3)*(((I/2)*(((-3*I)*((b*(a^2 - 2*b
^2)*x)/a - (4*b^4*ArcTanh[(b*((2*a)/b - 2*Tanh[x/2]))/(2*Sqrt[a^2 + b^2])]
)/(a*Sqrt[a^2 + b^2])))/a + ((2*I)*(2*a^2 - 3*b^2)*Cosh[x])/a))/a - (3*b*C
osh[x]*Sinh[x])/(2*a)))/a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4340
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Sim
p[1/(a*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n
- a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(93) = 186$.

Time = 0.42 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.85

method	result
default	$-\frac{2b^4 \operatorname{arctanh}\left(\frac{-2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}}+\frac{1}{3a\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}-\frac{a-b}{2a^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{a^2+ab-2b^2}{2a^3\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{b\left(a^2-2b^2\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right.}{2a^4}$
risch	$\frac{xb}{2a^2}-\frac{x\,b^3}{a^4}+\frac{e^{3x}}{24a}-\frac{b\,e^{2x}}{8a^2}-\frac{3\,e^{x}}{8a}+\frac{e^{x}b^2}{2a^3}-\frac{3\,e^{-x}}{8a}+\frac{e^{-x}b^2}{2a^3}+\frac{b\,e^{-2x}}{8a^2}+\frac{e^{-3x}}{24a}+\frac{b^4\ln\left(e^{x}+\frac{b\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,a^4}-$

input
```
int(sinh(x)^3/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*b^4/a^4/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*tanh(1/2*x)*b+2*a)/(a^2+b^2)^(1
/2))+1/3/a/(tanh(1/2*x)+1)^3-1/2*(a-b)/a^2/(tanh(1/2*x)+1)^2-1/2*(a^2+a*b-
2*b^2)/a^3/(tanh(1/2*x)+1)+1/2*b*(a^2-2*b^2)/a^4*ln(tanh(1/2*x)+1)-1/3/a/(
tanh(1/2*x)-1)^3-1/2*(a+b)/a^2/(tanh(1/2*x)-1)^2-1/2*(-a^2+a*b+2*b^2)/a^3/
(tanh(1/2*x)-1)-1/2*b*(a^2-2*b^2)/a^4*ln(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 807 vs. $2(95) = 190$.

Time = 0.11 (sec) , antiderivative size = 807, normalized size of antiderivative = 7.54

$$\int \frac{\sinh^3(x)}{a + b\text{csch}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
1/24*((a^5 + a^3*b^2)*cosh(x)^6 + (a^5 + a^3*b^2)*sinh(x)^6 - 3*(a^4*b + a
^2*b^3)*cosh(x)^5 - 3*(a^4*b + a^2*b^3 - 2*(a^5 + a^3*b^2)*cosh(x))*sinh(x
)^5 + a^5 + a^3*b^2 + 12*(a^4*b - a^2*b^3 - 2*b^5)*x*cosh(x)^3 - 3*(3*a^5
- a^3*b^2 - 4*a*b^4)*cosh(x)^4 - 3*(3*a^5 - a^3*b^2 - 4*a*b^4 - 5*(a^5 + a
^3*b^2)*cosh(x)^2 + 5*(a^4*b + a^2*b^3)*cosh(x))*sinh(x)^4 + 2*(10*(a^5 +
a^3*b^2)*cosh(x)^3 - 15*(a^4*b + a^2*b^3)*cosh(x)^2 + 6*(a^4*b - a^2*b^3 -
 2*b^5)*x - 6*(3*a^5 - a^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x)^3 - 3*(3*a^5 -
a^3*b^2 - 4*a*b^4)*cosh(x)^2 - 3*(3*a^5 - a^3*b^2 - 4*a*b^4 - 5*(a^5 + a^3
*b^2)*cosh(x)^4 + 10*(a^4*b + a^2*b^3)*cosh(x)^3 - 12*(a^4*b - a^2*b^3 - 2
*b^5)*x*cosh(x) + 6*(3*a^5 - a^3*b^2 - 4*a*b^4)*cosh(x)^2)*sinh(x)^2 + 24*
(b^4*cosh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*s
inh(x)^3)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(
x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(a*co
sh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*co
sh(x) + b)*sinh(x) - a)) + 3*(a^4*b + a^2*b^3)*cosh(x) + 3*(2*(a^5 + a^3*b
^2)*cosh(x)^5 + a^4*b + a^2*b^3 - 5*(a^4*b + a^2*b^3)*cosh(x)^4 + 12*(a^4*
b - a^2*b^3 - 2*b^5)*x*cosh(x)^2 - 4*(3*a^5 - a^3*b^2 - 4*a*b^4)*cosh(x)^3
 - 2*(3*a^5 - a^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x))/((a^6 + a^4*b^2)*cosh(x
)^3 + 3*(a^6 + a^4*b^2)*cosh(x)^2*sinh(x) + 3*(a^6 + a^4*b^2)*cosh(x)*sinh
(x)^2 + (a^6 + a^4*b^2)*sinh(x)^3)
```

Sympy [F]

$$\int \frac{\sinh^3(x)}{a + b\text{csch}(x)}\,dx = \int \frac{\sinh^3{(x)}}{a + b\,\text{csch}\,(x)}\,dx$$

input
```
integrate(sinh(x)**3/(a+b*csch(x)),x)
```

output
```
Integral(sinh(x)**3/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.47

$$\int \frac{\sinh^3(x)}{a + b\text{csch}(x)}\,dx = \frac{b^4 \log\left(\frac{ae^{(-x)} - b - \sqrt{a^2+b^2}}{ae^{(-x)} - b + \sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^4} - \frac{\left(3\,abe^{(-x)} - a^2 + 3\,(3\,a^2 - 4\,b^2)e^{(-2\,x)}\right)e^{(3\,x)}}{24\,a^3} + \frac{3\,abe^{(-2\,x)} + a^2e^{(-3\,x)} - 3\,(3\,a^2 - 4\,b^2)e^{(-x)}}{24\,a^3} + \frac{(a^2b - 2\,b^3)x}{2\,a^4}$$

input
```
integrate(sinh(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
b^4*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))
/(sqrt(a^2 + b^2)*a^4) - 1/24*(3*a*b*e^(-x) - a^2 + 3*(3*a^2 - 4*b^2)*e^(-
2*x))*e^(3*x)/a^3 + 1/24*(3*a*b*e^(-2*x) + a^2*e^(-3*x) - 3*(3*a^2 - 4*b^2
)*e^(-x))/a^3 + 1/2*(a^2*b - 2*b^3)*x/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.45

$$\int \frac{\sinh^3(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{b^4\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^4} + \frac{a^2e^{(3\,x)}-3\,abe^{(2\,x)}-9\,a^2e^x+12\,b^2e^x}{24\,a^3} + \frac{(a^2b-2\,b^3)x}{2\,a^4} + \frac{\left(3\,a^2be^x+a^3-3\,(3\,a^3-4\,ab^2)e^{(2\,x)}\right)e^{(-3\,x)}}{24\,a^4}$$

input
```
integrate(sinh(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

output
```
b^4*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*a^4) + 1/24*(a^2*e^(3*x) - 3*a*b*e^(2*x) - 9
*a^2*e^x + 12*b^2*e^x)/a^3 + 1/2*(a^2*b - 2*b^3)*x/a^4 + 1/24*(3*a^2*b*e^x
 + a^3 - 3*(3*a^3 - 4*a*b^2)*e^(2*x))*e^(-3*x)/a^4
```

Mupad [B] (verification not implemented)

Time = 2.87 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.86

$$\int \frac{\sinh^3(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\mathrm{e}^{-3\,x}}{24\,a} + \frac{\mathrm{e}^{3\,x}}{24\,a} + \frac{x\,(a^2\,b-2\,b^3)}{2\,a^4} - \frac{\mathrm{e}^{x}\,(3\,a^2-4\,b^2)}{8\,a^3} + \frac{b\,\mathrm{e}^{-2\,x}}{8\,a^2} - \frac{b\,\mathrm{e}^{2\,x}}{8\,a^2} - \frac{\mathrm{e}^{-x}\,(3\,a^2-4\,b^2)}{8\,a^3} - \frac{b^4\,\ln\left(-\frac{2\,b^4\,\mathrm{e}^{x}}{a^5}-\frac{2\,b^4\,(a-b\,\mathrm{e}^{x})}{a^5\,\sqrt{a^2+b^2}}\right)}{a^4\,\sqrt{a^2+b^2}} + \frac{b^4\,\ln\left(\frac{2\,b^4\,(a-b\,\mathrm{e}^{x})}{a^5\,\sqrt{a^2+b^2}}-\frac{2\,b^4\,\mathrm{e}^{x}}{a^5}\right)}{a^4\,\sqrt{a^2+b^2}}$$

input
```
int(sinh(x)^3/(a + b/sinh(x)),x)
```

output
```
exp(-3*x)/(24*a) + exp(3*x)/(24*a) + (x*(a^2*b - 2*b^3))/(2*a^4) - (exp(x)
*(3*a^2 - 4*b^2))/(8*a^3) + (b*exp(-2*x))/(8*a^2) - (b*exp(2*x))/(8*a^2) -
 (exp(-x)*(3*a^2 - 4*b^2))/(8*a^3) - (b^4*log(- (2*b^4*exp(x))/a^5 - (2*b^
4*(a - b*exp(x)))/(a^5*(a^2 + b^2)^(1/2))))/(a^4*(a^2 + b^2)^(1/2)) + (b^4
*log((2*b^4*(a - b*exp(x)))/(a^5*(a^2 + b^2)^(1/2)) - (2*b^4*exp(x))/a^5))
/(a^4*(a^2 + b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.27

$$\int \frac{\sinh^3(x)}{a+b\,\text{csch}(x)}\,dx = \frac{48e^{3x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x ai+bi}{\sqrt{a^2+b^2}}\right)b^4 i + e^{6x}a^5 + e^{6x}a^3b^2 - 3e^{5x}a^4b - 3e^{5x}a^2b^3 - 9e^{4x}a^5 + 3e^{4x}a^3b^2 + 12e^{4x}a\,b}{24e^{3x}a^4\,(}$$

input
```
int(sinh(x)^3/(a+b*csch(x)),x)
```

output
```
(48*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2))*b*
*4*i + e**(6*x)*a**5 + e**(6*x)*a**3*b**2 - 3*e**(5*x)*a**4*b - 3*e**(5*x)
*a**2*b**3 - 9*e**(4*x)*a**5 + 3*e**(4*x)*a**3*b**2 + 12*e**(4*x)*a*b**4 +
 12*e**(3*x)*a**4*b*x - 12*e**(3*x)*a**2*b**3*x - 24*e**(3*x)*b**5*x - 9*e
**(2*x)*a**5 + 3*e**(2*x)*a**3*b**2 + 12*e**(2*x)*a*b**4 + 3*e**x*a**4*b +
 3*e**x*a**2*b**3 + a**5 + a**3*b**2)/(24*e**(3*x)*a**4*(a**2 + b**2))
```

3.78 $\int \frac{\sinh^2(x)}{a+b\text{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \frac{\sinh^2(x)}{a+b\text{csch}(x)}\,dx = -\frac{(a^2-2b^2)\,x}{2a^3}+\frac{2b^3\text{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}} - \frac{b\cosh(x)}{a^2}+\frac{\cosh(x)\sinh(x)}{2a}$$

output
```
-1/2*(a^2-2*b^2)*x/a^3+2*b^3*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/a^
3/(a^2+b^2)^(1/2)-b*cosh(x)/a^2+1/2*cosh(x)*sinh(x)/a
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \frac{\sinh^2(x)}{a+b\text{csch}(x)}\,dx = \frac{-2a^2x+4b^2x-\frac{8b^3\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-4ab\cosh(x)+a^2\sinh(2x)}{4a^3}$$

input
```
Integrate[Sinh[x]^2/(a + b*Csch[x]),x]
```

output
```
(-2*a^2*x + 4*b^2*x - (8*b^3*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/S
qrt[-a^2 - b^2] - 4*a*b*Cosh[x] + a^2*Sinh[2*x])/(4*a^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.31, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.308$, Rules used = {3042, 25, 4340, 3042, 26, 4592, 26, 3042, 4407, 26, 3042, 26, 4318, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{a+b\text{csch}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\csc(ix)^2(a+ib\csc(ix))}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\csc(ix)^2(a+ib\csc(ix))}dx$$

$$\Big\downarrow \text{4340}$$

$$\frac{\sinh(x)\cosh(x)}{2a}-\frac{\int \frac{\left(b\text{csch}^2(x)+a\text{csch}(x)+2b\right)\sinh(x)}{a+b\text{csch}(x)}dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)\cosh(x)}{2a}-\frac{\int -\frac{i(-b\csc(ix)^2+ia\csc(ix)+2b)}{\csc(ix)(a+ib\csc(ix))}dx}{2a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\sinh(x)\cosh(x)}{2a}+\frac{i\int \frac{-b\csc(ix)^2+ia\csc(ix)+2b}{\csc(ix)(a+ib\csc(ix))}dx}{2a}$$

$$\Big\downarrow \text{4592}$$

$$\frac{\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{2ib\cosh(x)}{a}-\frac{\int -\frac{i\left(a^2+b\text{csch}(x)a-2b^2\right)}{a+b\text{csch}(x)}dx}{a}\right)}{2a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{i\int \frac{a^2+b\text{csch}(x)a-2b^2}{a+b\text{csch}(x)}dx}{a}+\frac{2ib\cosh(x)}{a}\right)}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{i\int \frac{a^2+ib\csc(ix)a-2b^2}{a+ib\csc(ix)}dx}{a}+\frac{2ib\cosh(x)}{a}\right)}{2a}$$

$$\Big\downarrow \text{4407}$$

$$\frac{\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{i\left(\frac{x\left(a^2-2b^2\right)}{a}+\frac{2ib^3\int -\frac{i\text{csch}(x)}{a+b\text{csch}(x)}dx}{a}\right)}{a}+\frac{2ib\cosh(x)}{a}\right)}{2a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{i\left(\frac{2b^3\int \frac{\text{csch}(x)}{a+b\text{csch}(x)}dx}{a}+\frac{x\left(a^2-2b^2\right)}{a}\right)}{a}+\frac{2ib\cosh(x)}{a}\right)}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{i\left(\frac{x\left(a^2-2b^2\right)}{a}+\frac{2b^3\int \frac{i\csc(ix)}{a+ib\csc(ix)}dx}{a}\right)}{a}+\frac{2ib\cosh(x)}{a}\right)}{2a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{i\left(\frac{x(a^2-2b^2)}{a}+\frac{2ib^3\int\frac{\csc(ix)}{a+ib\csc(ix)}dx}{a}\right)}{a}+\frac{2ib\cosh(x)}{a}\right)}{2a}$$

$\big\downarrow$ 4318

$$\frac{\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{i\left(\frac{2b^2\int\frac{1}{\frac{a\sinh(x)}{b}+1}dx}{a}+\frac{x(a^2-2b^2)}{a}\right)}{a}+\frac{2ib\cosh(x)}{a}\right)}{2a}$$

$\big\downarrow$ 3042

$$\frac{\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{i\left(\frac{x(a^2-2b^2)}{a}+\frac{2b^2\int\frac{1}{1-\frac{ia\sin(ix)}{b}}dx}{a}\right)}{a}+\frac{2ib\cosh(x)}{a}\right)}{2a}$$

$\big\downarrow$ 3139

$$\frac{\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{i\left(\frac{4b^2\int\frac{1}{-\tanh^2\left(\frac{x}{2}\right)+\frac{2a\tanh\left(\frac{x}{2}\right)}{b}+1}d\tanh\left(\frac{x}{2}\right)}{a}+\frac{x(a^2-2b^2)}{a}\right)}{a}+\frac{2ib\cosh(x)}{a}\right)}{2a}$$

$\big\downarrow$ 1083

$$\frac{\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{i\left(\frac{x(a^2-2b^2)}{a}-\frac{8b^2\int\frac{1}{4\left(\frac{a^2}{b^2}+1\right)-\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)^2}d\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{a}\right)}{a}+\frac{2ib\cosh(x)}{a}\right)}{2a}$$

$\big\downarrow$ 219

$$\frac{\sinh(x)\cosh(x)}{2a}+\frac{i\left(\frac{i\left(\frac{x(a^2-2b^2)}{a}-\frac{4b^3\operatorname{arctanh}\left(\frac{b\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\right)}{a}+\frac{2ib\cosh(x)}{a}\right)}{2a}$$

input
```
Int[Sinh[x]^2/(a + b*Csch[x]),x]
```

output
```
((I/2)*((I*(((a^2 - 2*b^2)*x)/a - (4*b^3*ArcTanh[(b*((2*a)/b - 2*Tanh[x/2]
))/(2*Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2])))/a + ((2*I)*b*Cosh[x])/a))/a
 + (Cosh[x]*Sinh[x])/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4340
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Sim
p[1/(a*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n
- a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(70) = 140$.

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.90

method	result
default	$\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{-a-2b}{2a^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\left(a^{2}-2b^{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a^{3}}-\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{-a+2b}{2a^{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{\left(-a^{2}+2b^{2}\right.}{}$
risch	$-\frac{x}{2a}+\frac{x\,b^{2}}{a^{3}}+\frac{\mathrm{e}^{2x}}{8a}-\frac{b\,\mathrm{e}^{x}}{2a^{2}}-\frac{b\,\mathrm{e}^{-x}}{2a^{2}}-\frac{\mathrm{e}^{-2x}}{8a}+\frac{b^{3}\ln\left(\mathrm{e}^{x}+\frac{b\sqrt{a^{2}+b^{2}}+a^{2}+b^{2}}{\sqrt{a^{2}+b^{2}}\,a}\right)}{\sqrt{a^{2}+b^{2}}\,a^{3}}-\frac{b^{3}\ln\left(\mathrm{e}^{x}+\frac{b\sqrt{a^{2}+b^{2}}-a^{2}-b^{2}}{\sqrt{a^{2}+b^{2}}\,a}\right)}{\sqrt{a^{2}+b^{2}}\,a^{3}}$

input
```
int(sinh(x)^2/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2/a/(tanh(1/2*x)-1)^2-1/2*(-a-2*b)/a^2/(tanh(1/2*x)-1)+1/2*(a^2-2*b^2)/a
^3*ln(tanh(1/2*x)-1)-1/2/a/(tanh(1/2*x)+1)^2-1/2*(-a+2*b)/a^2/(tanh(1/2*x)
+1)+1/2/a^3*(-a^2+2*b^2)*ln(tanh(1/2*x)+1)+2*b^3/a^3/(a^2+b^2)^(1/2)*arcta
nh(1/2*(-2*tanh(1/2*x)*b+2*a)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(72) = 144$.

Time = 0.11 (sec) , antiderivative size = 456, normalized size of antiderivative = 5.70

$$\int \frac{\sinh^2(x)}{a+b\text{csch}(x)}\,dx$$
$$=\frac{(a^4+a^2b^2)\cosh(x)^4+(a^4+a^2b^2)\sinh(x)^4-a^4-a^2b^2-4\,(a^4-a^2b^2-2\,b^4)x\cosh(x)^2-4\,(a^3b+ab^3}{}$$

input
```
integrate(sinh(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
1/8*((a^4 + a^2*b^2)*cosh(x)^4 + (a^4 + a^2*b^2)*sinh(x)^4 - a^4 - a^2*b^2
 - 4*(a^4 - a^2*b^2 - 2*b^4)*x*cosh(x)^2 - 4*(a^3*b + a*b^3)*cosh(x)^3 - 4
*(a^3*b + a*b^3 - (a^4 + a^2*b^2)*cosh(x))*sinh(x)^3 + 2*(3*(a^4 + a^2*b^2
)*cosh(x)^2 - 2*(a^4 - a^2*b^2 - 2*b^4)*x - 6*(a^3*b + a*b^3)*cosh(x))*sin
h(x)^2 + 8*(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2)*sqrt(a^
2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2
+ 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x)
 + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x
) - a)) - 4*(a^3*b + a*b^3)*cosh(x) - 4*(a^3*b + a*b^3 - (a^4 + a^2*b^2)*c
osh(x)^3 + 2*(a^4 - a^2*b^2 - 2*b^4)*x*cosh(x) + 3*(a^3*b + a*b^3)*cosh(x)
^2)*sinh(x))/((a^5 + a^3*b^2)*cosh(x)^2 + 2*(a^5 + a^3*b^2)*cosh(x)*sinh(x
) + (a^5 + a^3*b^2)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{\sinh^2(x)}{a + b\text{csch}(x)}\,dx = \int \frac{\sinh^2(x)}{a + b\,\text{csch}(x)}\,dx$$

input
```
integrate(sinh(x)**2/(a+b*csch(x)),x)
```

output
```
Integral(sinh(x)**2/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.45

$$\int \frac{\sinh^2(x)}{a + b\text{csch}(x)}\,dx = -\frac{b^3 \log\left(\frac{ae^{(-x)} - b - \sqrt{a^2+b^2}}{ae^{(-x)} - b + \sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^3} - \frac{\left(4\,be^{(-x)} - a\right)e^{(2\,x)}}{8\,a^2} - \frac{4\,be^{(-x)} + ae^{(-2\,x)}}{8\,a^2} - \frac{(a^2 - 2\,b^2)x}{2\,a^3}$$

input
```
integrate(sinh(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
-b^3*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2))
)/(sqrt(a^2 + b^2)*a^3) - 1/8*(4*b*e^(-x) - a)*e^(2*x)/a^2 - 1/8*(4*b*e^(-
x) + a*e^(-2*x))/a^2 - 1/2*(a^2 - 2*b^2)*x/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.44

$$\int \frac{\sinh^2(x)}{a+b\text{csch}(x)}\,dx = -\frac{b^3\log\left(\frac{|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}|}{|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}|}\right)}{\sqrt{a^2+b^2}a^3}+\frac{ae^{(2\,x)}-4\,be^x}{8\,a^2} - \frac{(a^2-2\,b^2)x}{2\,a^3}-\frac{(4\,abe^x+a^2)e^{(-2\,x)}}{8\,a^3}$$

input
```
integrate(sinh(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

output
```
-b^3*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt
(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3) + 1/8*(a*e^(2*x) - 4*b*e^x)/a^2 - 1/2*
(a^2 - 2*b^2)*x/a^3 - 1/8*(4*a*b*e^x + a^2)*e^(-2*x)/a^3
```

Mupad [B] (verification not implemented)

Time = 2.74 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.96

$$\int \frac{\sinh^2(x)}{a+b\text{csch}(x)}\,dx = \frac{e^{2\,x}}{8\,a}-\frac{e^{-2\,x}}{8\,a}-\frac{b\,e^x}{2\,a^2}-\frac{b\,e^{-x}}{2\,a^2}-\frac{x\,(a^2-2\,b^2)}{2\,a^3} - \frac{b^3\,\ln\left(\frac{2\,b^3\,e^x}{a^4}-\frac{2\,b^3\,(a-b\,e^x)}{a^4\,\sqrt{a^2+b^2}}\right)}{a^3\,\sqrt{a^2+b^2}}+\frac{b^3\,\ln\left(\frac{2\,b^3\,e^x}{a^4}+\frac{2\,b^3\,(a-b\,e^x)}{a^4\,\sqrt{a^2+b^2}}\right)}{a^3\,\sqrt{a^2+b^2}}$$

input
```
int(sinh(x)^2/(a + b/sinh(x)),x)
```

output
```
exp(2*x)/(8*a) - exp(-2*x)/(8*a) - (b*exp(x))/(2*a^2) - (b*exp(-x))/(2*a^2
) - (x*(a^2 - 2*b^2))/(2*a^3) - (b^3*log((2*b^3*exp(x))/a^4 - (2*b^3*(a -
b*exp(x)))/(a^4*(a^2 + b^2)^(1/2))))/(a^3*(a^2 + b^2)^(1/2)) + (b^3*log((2
*b^3*exp(x))/a^4 + (2*b^3*(a - b*exp(x)))/(a^4*(a^2 + b^2)^(1/2))))/(a^3*(
a^2 + b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.16

$$\int \frac{\sinh^2(x)}{a + b\operatorname{csch}(x)}\,dx$$
$$= \frac{-16e^{2x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x ai+bi}{\sqrt{a^2+b^2}}\right)b^3i + e^{4x}a^4 + e^{4x}a^2b^2 - 4e^{3x}a^3b - 4e^{3x}a\,b^3 - 4e^{2x}a^4x + 4e^{2x}a^2b^2x + 8e^{2x}}{8e^{2x}a^3\left(a^2+b^2\right)}$$

input
```
int(sinh(x)^2/(a+b*csch(x)),x)
```

output
```
( - 16*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2))
*b**3*i + e**(4*x)*a**4 + e**(4*x)*a**2*b**2 - 4*e**(3*x)*a**3*b - 4*e**(3
*x)*a*b**3 - 4*e**(2*x)*a**4*x + 4*e**(2*x)*a**2*b**2*x + 8*e**(2*x)*b**4*
x - 4*e**x*a**3*b - 4*e**x*a*b**3 - a**4 - a**2*b**2)/(8*e**(2*x)*a**3*(a*
*2 + b**2))
```

3.79 $\int \frac{\sinh(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 57

$$\int \frac{\sinh(x)}{a+b\operatorname{csch}(x)}\,dx=-\frac{bx}{a^2}-\frac{2b^2\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{\cosh(x)}{a}$$

output
```
-b*x/a^2-2*b^2*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/a^2/(a^2+b^2)^(1
/2)+cosh(x)/a
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{\sinh(x)}{a+b\operatorname{csch}(x)}\,dx=\frac{b\left(-x+\frac{2b\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}\right)+a\cosh(x)}{a^2}$$

input
```
Integrate[Sinh[x]/(a + b*Csch[x]),x]
```

output
```
(b*(-x + (2*b*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
) + a*Cosh[x])/a^2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.35, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 26, 4340, 26, 27, 3042, 4270, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{a+b\text{csch}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\csc(ix)(a+ib\csc(ix))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\csc(ix)(a+ib\csc(ix))}dx$$

$$\Big\downarrow \text{4340}$$

$$-i\left(\frac{\int -\frac{ib}{a+b\text{csch}(x)}dx}{a}+\frac{i\cosh(x)}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\cosh(x)}{a}-\frac{i\int \frac{b}{a+b\text{csch}(x)}dx}{a}\right)$$

$$\Big\downarrow \text{27}$$

$$-i\left(\frac{i\cosh(x)}{a}-\frac{ib\int \frac{1}{a+b\text{csch}(x)}dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\cosh(x)}{a}-\frac{ib\int \frac{1}{a+ib\csc(ix)}dx}{a}\right)$$

$\Big\downarrow$ 4270

$$-i\left(\frac{i\cosh(x)}{a}-\frac{ib\left(\frac{x}{a}-\frac{\int \frac{1}{\frac{a\sinh(x)}{b}+1}dx}{a}\right)}{a}\right)$$

$\Big\downarrow$ 3042

$$-i\left(\frac{i\cosh(x)}{a}-\frac{ib\left(\frac{x}{a}-\frac{\int \frac{1}{1-\frac{ia\sin(ix)}{b}}dx}{a}\right)}{a}\right)$$

$\Big\downarrow$ 3139

$$-i\left(\frac{i\cosh(x)}{a}-\frac{ib\left(\frac{x}{a}-\frac{2\int \frac{1}{-\tanh^2\left(\frac{x}{2}\right)+\frac{2a\tanh\left(\frac{x}{2}\right)}{b}+1}d\tanh\left(\frac{x}{2}\right)}{a}\right)}{a}\right)$$

$\Big\downarrow$ 1083

$$-i\left(\frac{i\cosh(x)}{a}-\frac{ib\left(\frac{4\int \frac{1}{4\left(\frac{a^2}{b^2}+1\right)-\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)^2}d\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{a}+\frac{x}{a}\right)}{a}\right)$$

$\Big\downarrow$ 219

$$-i\left(\frac{i\cosh(x)}{a}-\frac{ib\left(\frac{2b\text{arctanh}\left(\frac{b\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{x}{a}\right)}{a}\right)$$

input
```
Int[Sinh[x]/(a + b*Csch[x]),x]
```

output
```
(-I)*(((-I)*b*(x/a + (2*b*ArcTanh[(b*((2*a)/b - 2*Tanh[x/2]))/(2*Sqrt[a^2
+ b^2])])/(a*Sqrt[a^2 + b^2])))/a + (I*Cosh[x])/a)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 4270
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Simp[1/a   Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[a^2 - b^2, 0]
```

rule 4340
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Sim
p[1/(a*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n
- a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.61

method	result	size
default	$-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a^{2}}+\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a^{2}}-\frac{2b^{2}\operatorname{arctanh}\left(\frac{-2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{a^{2}+b^{2}}}\right)}{a^{2}\sqrt{a^{2}+b^{2}}}$	92
risch	$-\frac{xb}{a^{2}}+\frac{e^{x}}{2a}+\frac{e^{-x}}{2a}+\frac{b^{2}\ln\left(e^{x}+\frac{b\sqrt{a^{2}+b^{2}}-a^{2}-b^{2}}{\sqrt{a^{2}+b^{2}}\,a}\right)}{\sqrt{a^{2}+b^{2}}\,a^{2}}-\frac{b^{2}\ln\left(e^{x}+\frac{b\sqrt{a^{2}+b^{2}}+a^{2}+b^{2}}{\sqrt{a^{2}+b^{2}}\,a}\right)}{\sqrt{a^{2}+b^{2}}\,a^{2}}$	132

input
```
int(sinh(x)/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/a/(tanh(1/2*x)-1)+b/a^2*ln(tanh(1/2*x)-1)+1/a/(tanh(1/2*x)+1)-b/a^2*ln(
tanh(1/2*x)+1)-2*b^2/a^2/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*tanh(1/2*x)*b+2*a
)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(53) = 106$.

Time = 0.10 (sec) , antiderivative size = 238, normalized size of antiderivative = 4.18

$$\int \frac{\sinh(x)}{a + b\text{csch}(x)}\, dx$$
$$= \frac{a^3 + ab^2 - 2\left(a^2b + b^3\right)x\cosh\left(x\right) + \left(a^3 + ab^2\right)\cosh\left(x\right)^2 + \left(a^3 + ab^2\right)\sinh\left(x\right)^2 + 2\left(b^2\cosh\left(x\right) + b^2\sinh\left(\right.\right.}{}$$

input
```
integrate(sinh(x)/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
1/2*(a^3 + a*b^2 - 2*(a^2*b + b^3)*x*cosh(x) + (a^3 + a*b^2)*cosh(x)^2 + (
a^3 + a*b^2)*sinh(x)^2 + 2*(b^2*cosh(x) + b^2*sinh(x))*sqrt(a^2 + b^2)*log
((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cos
h(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*co
sh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) - 2*
((a^2*b + b^3)*x - (a^3 + a*b^2)*cosh(x))*sinh(x))/((a^4 + a^2*b^2)*cosh(x
) + (a^4 + a^2*b^2)*sinh(x))
```

Sympy [F]

$$\int \frac{\sinh(x)}{a + b\text{csch}(x)}\, dx = \int \frac{\sinh\left(x\right)}{a + b\operatorname{csch}\left(x\right)}\, dx$$

input
```
integrate(sinh(x)/(a+b*csch(x)),x)
```

output
```
Integral(sinh(x)/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.47

$$\int \frac{\sinh(x)}{a+b\text{csch}(x)}\,dx = \frac{b^2\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^2} - \frac{bx}{a^2} + \frac{e^{(-x)}}{2\,a} + \frac{e^x}{2\,a}$$

input
```
integrate(sinh(x)/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
b^2*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))
/(sqrt(a^2 + b^2)*a^2) - b*x/a^2 + 1/2*e^(-x)/a + 1/2*e^x/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.51

$$\int \frac{\sinh(x)}{a+b\text{csch}(x)}\,dx = \frac{b^2\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^2} - \frac{bx}{a^2} + \frac{e^{(-x)}}{2\,a} + \frac{e^x}{2\,a}$$

input
```
integrate(sinh(x)/(a+b*csch(x)),x, algorithm="giac")
```

output
```
b^2*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*a^2) - b*x/a^2 + 1/2*e^(-x)/a + 1/2*e^x/a
```

Mupad [B] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.26

$$\int \frac{\sinh(x)}{a+b\text{csch}(x)}\,dx = \frac{\mathrm{e}^{-x}}{2\,a} + \frac{\mathrm{e}^x}{2\,a} - \frac{b\,x}{a^2} - \frac{b^2\,\ln\left(-\frac{2\,b^2\,\mathrm{e}^x}{a^3} - \frac{2\,b^2\,(a-b\,\mathrm{e}^x)}{a^3\,\sqrt{a^2+b^2}}\right)}{a^2\,\sqrt{a^2+b^2}} + \frac{b^2\,\ln\left(\frac{2\,b^2\,(a-b\,\mathrm{e}^x)}{a^3\,\sqrt{a^2+b^2}} - \frac{2\,b^2\,\mathrm{e}^x}{a^3}\right)}{a^2\,\sqrt{a^2+b^2}}$$

input
```
int(sinh(x)/(a + b/sinh(x)),x)
```

output
```
exp(-x)/(2*a) + exp(x)/(2*a) - (b*x)/a^2 - (b^2*log(- (2*b^2*exp(x))/a^3 -
 (2*b^2*(a - b*exp(x)))/(a^3*(a^2 + b^2)^(1/2))))/(a^2*(a^2 + b^2)^(1/2))
+ (b^2*log((2*b^2*(a - b*exp(x)))/(a^3*(a^2 + b^2)^(1/2)) - (2*b^2*exp(x))
/a^3))/(a^2*(a^2 + b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.84

$$\int \frac{\sinh(x)}{a + b\text{csch}(x)}\,dx = \frac{4e^x\sqrt{a^2+b^2}\,atan\left(\frac{e^x ai+bi}{\sqrt{a^2+b^2}}\right)b^2i + e^{2x}a^3 + e^{2x}a\,b^2 - 2e^x a^2bx - 2e^x b^3x + a^3 + a\,b^2}{2e^x a^2\left(a^2+b^2\right)}$$

input
```
int(sinh(x)/(a+b*csch(x)),x)
```

output
```
(4*e**x*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2))*b**2*i
+ e**(2*x)*a**3 + e**(2*x)*a*b**2 - 2*e**x*a**2*b*x - 2*e**x*b**3*x + a**3
 + a*b**2)/(2*e**x*a**2*(a**2 + b**2))
```

3.80 $\int \frac{\text{csch}(x)}{a+b\text{csch}(x)} dx$

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{\text{csch}(x)}{a+b\text{csch}(x)} dx = -\frac{2\text{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

output
```
-2*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{\text{csch}(x)}{a+b\text{csch}(x)} dx = \frac{2\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}$$

input
```
Integrate[Csch[x]/(a + b*Csch[x]),x]
```

output
```
(2*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 26, 4318, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}(x)}{a+b\text{csch}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\csc(ix)}{a+ib\csc(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\csc(ix)}{a+ib\csc(ix)}dx$$

$$\Big\downarrow \text{4318}$$

$$\frac{\int \frac{1}{\frac{a\sinh(x)}{b}+1}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{1-\frac{ia\sin(ix)}{b}}dx}{b}$$

$$\Big\downarrow \text{3139}$$

$$\frac{2\int \frac{1}{-\tanh^2\left(\frac{x}{2}\right)+\frac{2a\tanh\left(\frac{x}{2}\right)}{b}+1}d\tanh\left(\frac{x}{2}\right)}{b}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{4\int \frac{1}{4\left(\frac{a^2}{b^2}+1\right)-\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)^2}d\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{b\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

input
```
Int[Csch[x]/(a + b*Csch[x]),x]
```

output
```
(-2*ArcTanh[(b*((2*a)/b - 2*Tanh[x/2]))/(2*Sqrt[a^2 + b^2])])/Sqrt[a^2 + b
^2]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{-2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$	35
risch	$\frac{\ln\left(\mathrm{e}^x+\frac{b\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}}-\frac{\ln\left(\mathrm{e}^x+\frac{b\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}}$	97

input
```
int(csch(x)/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*tanh(1/2*x)*b+2*a)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(33) = 66.

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.00

$$\int \frac{\mathrm{csch}(x)}{a+b\mathrm{csch}(x)}\,dx = \frac{\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)+a^2+2\,b^2+2\,(a^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2+b^2}(a\cosh(x)+a\sinh(x)+b)}{a\cosh(x)^2+a\sinh(x)^2+2\,b\cosh(x)+2\,(a\cosh(x)+b)\sinh(x)-a}\right)}{\sqrt{a^2+b^2}}$$

input
```
integrate(csch(x)/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*
cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a
*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a))/s
qrt(a^2 + b^2)
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{a + b\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{csch}(x)}{a + b\operatorname{csch}(x)}\,dx$$

input
```
integrate(csch(x)/(a+b*csch(x)),x)
```

output
```
Integral(csch(x)/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int \frac{\operatorname{csch}(x)}{a + b\operatorname{csch}(x)}\,dx = \frac{\log\left(\frac{ae^{(-x)} - b - \sqrt{a^2+b^2}}{ae^{(-x)} - b + \sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

input
```
integrate(csch(x)/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/sqr
t(a^2 + b^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.51

$$\int \frac{\operatorname{csch}(x)}{a + b\operatorname{csch}(x)}\,dx = \frac{\log\left(\frac{\left|2\,ae^x + 2\,b - 2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x + 2\,b + 2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}$$

input
```
integrate(csch(x)/(a+b*csch(x)),x, algorithm="giac")
```

output
```
log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^2
+ b^2)))/sqrt(a^2 + b^2)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx=\frac{2\operatorname{atan}\left(\frac{b}{\sqrt{-a^2-b^2}}+\frac{a\,e^x}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}$$

input
```
int(1/(sinh(x)*(a + b/sinh(x))),x)
```

output
```
(2*atan(b/(- a^2 - b^2)^(1/2) + (a*exp(x))/(- a^2 - b^2)^(1/2)))/(- a^2 -
b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx=\frac{2\sqrt{a^2+b^2}\,atan\left(\frac{e^x ai+bi}{\sqrt{a^2+b^2}}\right)i}{a^2+b^2}$$

input
```
int(csch(x)/(a+b*csch(x)),x)
```

output
```
(2*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2))*i)/(a**2 + b
**2)
```

3.81 $\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 50

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{\operatorname{arctanh}(\cosh(x))}{b} + \frac{2a\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$$

output
```
-arctanh(cosh(x))/b+2*a*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/b/(a^2+
b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.34

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{-\frac{2a\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - \log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right)}{b}$$

input
```
Integrate[Csch[x]^2/(a + b*Csch[x]),x]
```

output
```
((-2*a*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - Log[
Cosh[x/2]] + Log[Sinh[x/2]])/b
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.16, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923, Rules used = {3042, 25, 4276, 26, 3042, 26, 4257, 4318, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(x)}{a+b\text{csch}(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\csc(ix)^2}{a+ib\csc(ix)}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\csc(ix)^2}{a+ib\csc(ix)}dx$$

$$\Big\downarrow \text{ 4276}$$

$$\frac{i\int -i\text{csch}(x)dx}{b}-\frac{ia\int -\frac{i\text{csch}(x)}{a+b\text{csch}(x)}dx}{b}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\int \text{csch}(x)dx}{b}-\frac{a\int \frac{\text{csch}(x)}{a+b\text{csch}(x)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int i\csc(ix)dx}{b}-\frac{a\int \frac{i\csc(ix)}{a+ib\csc(ix)}dx}{b}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{i\int \csc(ix)dx}{b}-\frac{ia\int \frac{\csc(ix)}{a+ib\csc(ix)}dx}{b}$$

$$\Big\downarrow \text{ 4257}$$

$$-\frac{\text{arctanh}(\cosh(x))}{b}-\frac{ia\int \frac{\csc(ix)}{a+ib\csc(ix)}dx}{b}$$

$$\Big\downarrow \text{ 4318}$$

$$-\frac{a\int \frac{1}{\frac{a\sinh(x)}{b}+1}dx}{b^2}-\frac{\operatorname{arctanh}(\cosh(x))}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\operatorname{arctanh}(\cosh(x))}{b}-\frac{a\int \frac{1}{1-\frac{ia\sin(ix)}{b}}dx}{b^2}$$

$$\Big\downarrow \; 3139$$

$$-\frac{2a\int \frac{1}{-\tanh^2\left(\frac{x}{2}\right)+\frac{2a\tanh\left(\frac{x}{2}\right)}{b}+1}d\tanh\left(\frac{x}{2}\right)}{b^2}-\frac{\operatorname{arctanh}(\cosh(x))}{b}$$

$$\Big\downarrow \; 1083$$

$$\frac{4a\int \frac{1}{4\left(\frac{a^2}{b^2}+1\right)-\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)^2}d\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{b^2}-\frac{\operatorname{arctanh}(\cosh(x))}{b}$$

$$\Big\downarrow \; 219$$

$$\frac{2a\operatorname{arctanh}\left(\frac{b\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{\operatorname{arctanh}(\cosh(x))}{b}$$

input
```
Int[Csch[x]^2/(a + b*Csch[x]),x]
```

output
```
-(ArcTanh[Cosh[x]]/b) + (2*a*ArcTanh[(b*((2*a)/b - 2*Tanh[x/2]))/(2*Sqrt[a
^2 + b^2])])/(b*Sqrt[a^2 + b^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4276
```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[1/b   Int[Csc[e + f*x], x], x] - Simp[a/b   Int[Csc[e + f*x]/(
a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{2a \operatorname{arctanh}\left(\frac{-2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b}$	49
risch	$\frac{a\ln\left(\mathrm{e}^x+\frac{b\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,b}-\frac{a\ln\left(\mathrm{e}^x+\frac{b\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,b}-\frac{\ln(1+\mathrm{e}^x)}{b}+\frac{\ln(\mathrm{e}^x-1)}{b}$	124

input
```
int(csch(x)^2/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
2*a/b/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*tanh(1/2*x)*b+2*a)/(a^2+b^2)^(1/2))+
1/b*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(46) = 92.

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.12

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx$$
$$= \frac{\sqrt{a^2+b^2}\,a\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)+a^2+2\,b^2+2\,(a^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^2+b^2}(a\cosh(x)+a\sinh(x)+b)}{a\cosh(x)^2+a\sinh(x)^2+2\,b\cosh(x)+2\,(a\cosh(x)+b)\sinh(x)-a}\right)-(}{a^2b+b^3}$$

input
```
integrate(csch(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
(sqrt(a^2 + b^2)*a*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^
2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) +
 a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) +
 b)*sinh(x) - a)) - (a^2 + b^2)*log(cosh(x) + sinh(x) + 1) + (a^2 + b^2)*l
og(cosh(x) + sinh(x) - 1))/(a^2*b + b^3)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{a + b\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{csch}^2(x)}{a + b\operatorname{csch}(x)}\,dx$$

input
```
integrate(csch(x)**2/(a+b*csch(x)),x)
```

output
```
Integral(csch(x)**2/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.66

$$\int \frac{\operatorname{csch}^2(x)}{a + b\operatorname{csch}(x)}\,dx = -\frac{a\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b} - \frac{\log\left(e^{(-x)}+1\right)}{b} + \frac{\log\left(e^{(-x)}-1\right)}{b}$$

input
```
integrate(csch(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
-a*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*b) - log(e^(-x) + 1)/b + log(e^(-x) - 1)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.64

$$\int \frac{\operatorname{csch}^2(x)}{a + b\operatorname{csch}(x)}\,dx = -\frac{a\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b} - \frac{\log\left(e^x+1\right)}{b} + \frac{\log\left(\left|e^x-1\right|\right)}{b}$$

input
```
integrate(csch(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

output
```
-a*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*b) - log(e^x + 1)/b + log(abs(e^x - 1))/b
```

Mupad [B] (verification not implemented)

Time = 2.77 (sec) , antiderivative size = 287, normalized size of antiderivative = 5.74

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\ln\left(32\,b-32\,b\,e^x\right)}{b} - \frac{\ln\left(32\,b+32\,b\,e^x\right)}{b} + \frac{a\,\ln\left(128\,b^5\,e^x-64\,a^3\,b^2-64\,a\,b^4-128\,b^4\,e^x\,\sqrt{a^2+b^2}+32\,a^4\,b\,e^x+160\,a^2\,b^3\,e^x+64\,a\,b^3\,\sqrt{a^2+b^2}+\right.}{a^2\,b+b^3} - \frac{a\,\ln\left(64\,a\,b^4+64\,a^3\,b^2-128\,b^5\,e^x-128\,b^4\,e^x\,\sqrt{a^2+b^2}-32\,a^4\,b\,e^x-160\,a^2\,b^3\,e^x+64\,a\,b^3\,\sqrt{a^2+b^2}+\right.}{a^2\,b+b^3}$$

input
```
int(1/(sinh(x)^2*(a + b/sinh(x))),x)
```

output
```
log(32*b - 32*b*exp(x))/b - log(32*b + 32*b*exp(x))/b + (a*log(128*b^5*exp
(x) - 64*a^3*b^2 - 64*a*b^4 - 128*b^4*exp(x)*(a^2 + b^2)^(1/2) + 32*a^4*b*
exp(x) + 160*a^2*b^3*exp(x) + 64*a*b^3*(a^2 + b^2)^(1/2) + 32*a^3*b*(a^2 +
 b^2)^(1/2) - 96*a^2*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^2
*b + b^3) - (a*log(64*a*b^4 + 64*a^3*b^2 - 128*b^5*exp(x) - 128*b^4*exp(x)
*(a^2 + b^2)^(1/2) - 32*a^4*b*exp(x) - 160*a^2*b^3*exp(x) + 64*a*b^3*(a^2
+ b^2)^(1/2) + 32*a^3*b*(a^2 + b^2)^(1/2) - 96*a^2*b^2*exp(x)*(a^2 + b^2)^
(1/2))*(a^2 + b^2)^(1/2))/(a^2*b + b^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.80

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{-2\sqrt{a^2+b^2}\,atan\left(\frac{e^x ai+bi}{\sqrt{a^2+b^2}}\right)ai+\log(e^x-1)\,a^2+\log(e^x-1)\,b^2-\log(e^x+1)\,a^2-\log(e^x+1)\,b^2}{b\,(a^2+b^2)}$$

input
```
int(csch(x)^2/(a+b*csch(x)),x)
```

output
```
( - 2*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2))*a*i + log
(e**x - 1)*a**2 + log(e**x - 1)*b**2 - log(e**x + 1)*a**2 - log(e**x + 1)*
b**2)/(b*(a**2 + b**2))
```

3.82 $\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{csch}(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 59

$$\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{csch}(x)} dx = \frac{a\operatorname{arctanh}(\cosh(x))}{b^2} - \frac{2a^2\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} - \frac{\coth(x)}{b}$$

output
```
a*arctanh(cosh(x))/b^2-2*a^2*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/b^
2/(a^2+b^2)^(1/2)-coth(x)/b
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.56

$$\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{csch}(x)} dx = \frac{\operatorname{csch}\left(\frac{x}{2}\right)\operatorname{sech}\left(\frac{x}{2}\right)\left(-b\cosh(x)+a\left(\frac{2a\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)\sinh(x)\right)}{2b^2}$$

input
```
Integrate[Csch[x]^3/(a + b*Csch[x]),x]
```

output
```
(Csch[x/2]*Sech[x/2]*(-(b*Cosh[x]) + a*((2*a*ArcTan[(a - b*Tanh[x/2])/Sqrt
[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + Log[Cosh[x/2]] - Log[Sinh[x/2]])*Sinh[x]
))/(2*b^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.34, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.231$, Rules used = {3042, 26, 4277, 25, 3042, 25, 4276, 26, 3042, 26, 4257, 4318, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^3(x)}{a+b\text{csch}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\csc(ix)^3}{a+ib\csc(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\csc(ix)^3}{a+ib\csc(ix)}dx$$

$$\Big\downarrow \text{4277}$$

$$-i\left(\frac{ia\int -\frac{\text{csch}^2(x)}{a+b\text{csch}(x)}dx}{b}-\frac{i\coth(x)}{b}\right)$$

$$\Big\downarrow \text{25}$$

$$-i\left(-\frac{ia\int \frac{\text{csch}^2(x)}{a+b\text{csch}(x)}dx}{b}-\frac{i\coth(x)}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{ia\int -\frac{\csc(ix)^2}{a+ib\csc(ix)}dx}{b}-\frac{i\coth(x)}{b}\right)$$

$\Big\downarrow$ 25

$$-i\left(\frac{ia\int \frac{\csc(ix)^2}{a+ib\csc(ix)}dx}{b}-\frac{i\coth(x)}{b}\right)$$

$\Big\downarrow$ 4276

$$-i\left(\frac{ia\left(\frac{ia\int -\frac{i\text{csch}(x)}{a+b\text{csch}(x)}dx}{b}-\frac{i\int -i\text{csch}(x)dx}{b}\right)}{b}-\frac{i\coth(x)}{b}\right)$$

$\Big\downarrow$ 26

$$-i\left(\frac{ia\left(\frac{a\int \frac{\text{csch}(x)}{a+b\text{csch}(x)}dx}{b}-\frac{\int \text{csch}(x)dx}{b}\right)}{b}-\frac{i\coth(x)}{b}\right)$$

$\Big\downarrow$ 3042

$$-i\left(\frac{ia\left(\frac{a\int \frac{i\csc(ix)}{a+ib\csc(ix)}dx}{b}-\frac{\int i\csc(ix)dx}{b}\right)}{b}-\frac{i\coth(x)}{b}\right)$$

$\Big\downarrow$ 26

$$-i\left(\frac{ia\left(\frac{ia\int \frac{\csc(ix)}{a+ib\csc(ix)}dx}{b}-\frac{i\int \csc(ix)dx}{b}\right)}{b}-\frac{i\coth(x)}{b}\right)$$

$\Big\downarrow$ 4257

$$-i\left(\frac{ia\left(\frac{\text{arctanh}(\cosh(x))}{b}+\frac{ia\int \frac{\csc(ix)}{a+ib\csc(ix)}dx}{b}\right)}{b}-\frac{i\coth(x)}{b}\right)$$

$\Big\downarrow$ 4318

$$-i\left(\frac{ia\left(\frac{a\int \frac{1}{\frac{a\sinh(x)}{b}+1}dx}{b^2}+\frac{\text{arctanh}(\cosh(x))}{b}\right)}{b}-\frac{i\coth(x)}{b}\right)$$

$\Big\downarrow$ 3042

$$-i\left(\frac{ia\left(\frac{\text{arctanh}(\cosh(x))}{b}+\frac{a\int \frac{1}{1-\frac{ia\sin(ix)}{b}}dx}{b^2}\right)}{b}-\frac{i\coth(x)}{b}\right)$$

$\Big\downarrow$ 3139

$$-i\left(\frac{ia\left(\frac{2a\int \frac{1}{-\tanh^2\left(\frac{x}{2}\right)+\frac{2a\tanh\left(\frac{x}{2}\right)}{b}+1}d\tanh\left(\frac{x}{2}\right)}{b^2}+\frac{\text{arctanh}(\cosh(x))}{b}\right)}{b}-\frac{i\coth(x)}{b}\right)$$

$\Big\downarrow$ 1083

$$-i\left(\frac{ia\left(\frac{\text{arctanh}(\cosh(x))}{b}-\frac{4a\int \frac{1}{4\left(\frac{a^2}{b^2}+1\right)-\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)^2}d\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{b^2}\right)}{b}-\frac{i\coth(x)}{b}\right)$$

$\Big\downarrow$ 219

$$-i\left(\frac{ia\left(\frac{\text{arctanh}(\cosh(x))}{b}-\frac{2a\text{arctanh}\left(\frac{b\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}\right)}{b}-\frac{i\coth(x)}{b}\right)$$

input
```
Int[Csch[x]^3/(a + b*Csch[x]),x]
```

output
```
(-I)*((I*a*(ArcTanh[Cosh[x]]/b - (2*a*ArcTanh[(b*((2*a)/b - 2*Tanh[x/2]))/
(2*Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2])))/b - (I*Coth[x])/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4276
```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[1/b   Int[Csc[e + f*x], x], x] - Simp[a/b   Int[Csc[e + f*x]/(
a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4277
```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[-Cot[e + f*x]/(b*f), x] - Simp[a/b   Int[Csc[e + f*x]^2/(a + b
*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2b}-\frac{1}{2b\tanh\left(\frac{x}{2}\right)}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b^{2}}-\frac{2a^{2}\operatorname{arctanh}\left(\frac{-2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{a^{2}+b^{2}}}\right)}{b^{2}\sqrt{a^{2}+b^{2}}}$	73
risch	$-\frac{2}{b\left(e^{2x}-1\right)}+\frac{a^{2}\ln\left(e^{x}+\frac{b\sqrt{a^{2}+b^{2}}-a^{2}-b^{2}}{\sqrt{a^{2}+b^{2}}\,a}\right)}{\sqrt{a^{2}+b^{2}}\,b^{2}}-\frac{a^{2}\ln\left(e^{x}+\frac{b\sqrt{a^{2}+b^{2}}+a^{2}+b^{2}}{\sqrt{a^{2}+b^{2}}\,a}\right)}{\sqrt{a^{2}+b^{2}}\,b^{2}}+\frac{a\ln\left(1+e^{x}\right)}{b^{2}}-\frac{a\ln\left(e^{x}-1\right)}{b^{2}}$	143

input
```
int(csch(x)^3/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*tanh(1/2*x)/b-1/2/b/tanh(1/2*x)-1/b^2*a*ln(tanh(1/2*x))-2/b^2*a^2/(a^
2+b^2)^(1/2)*arctanh(1/2*(-2*tanh(1/2*x)*b+2*a)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(55) = 110$.

Time = 0.11 (sec) , antiderivative size = 345, normalized size of antiderivative = 5.85

$$\int \frac{\operatorname{csch}^3(x)}{a + b\operatorname{csch}(x)}\,dx = \frac{2\,a^2b + 2\,b^3 - \left(a^2\cosh\left(x\right)^2 + 2\,a^2\cosh\left(x\right)\sinh\left(x\right) + a^2\sinh\left(x\right)^2 - a^2\right)\sqrt{a^2 + b^2}\log\left(\frac{a^2\cosh(x)^2 + a^2\sinh(x)^2\cdots}{\cdots}\right)\cdots}{\cdots}$$

input
```
integrate(csch(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
(2*a^2*b + 2*b^3 - (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2
- a^2)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x)
+ a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(a*cosh(
x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(
x) + b)*sinh(x) - a)) + (a^3 + a*b^2 - (a^3 + a*b^2)*cosh(x)^2 - 2*(a^3 +
a*b^2)*cosh(x)*sinh(x) - (a^3 + a*b^2)*sinh(x)^2)*log(cosh(x) + sinh(x) +
1) - (a^3 + a*b^2 - (a^3 + a*b^2)*cosh(x)^2 - 2*(a^3 + a*b^2)*cosh(x)*sinh
(x) - (a^3 + a*b^2)*sinh(x)^2)*log(cosh(x) + sinh(x) - 1))/(a^2*b^2 + b^4
- (a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^2*b^2 + b^4)*cosh(x)*sinh(x) - (a^2*b^2
 + b^4)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{a + b\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{csch}^3\left(x\right)}{a + b\operatorname{csch}\left(x\right)}\,dx$$

input
```
integrate(csch(x)**3/(a+b*csch(x)),x)
```

output
```
Integral(csch(x)**3/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

$$\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{a^2\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b^2} + \frac{a\log\left(e^{(-x)}+1\right)}{b^2} - \frac{a\log\left(e^{(-x)}-1\right)}{b^2} + \frac{2}{be^{(-2x)}-b}$$

input
```
integrate(csch(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
a^2*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))
/(sqrt(a^2 + b^2)*b^2) + a*log(e^(-x) + 1)/b^2 - a*log(e^(-x) - 1)/b^2 + 2
/(b*e^(-2*x) - b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.66

$$\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{a^2\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^2} + \frac{a\log\left(e^x+1\right)}{b^2} - \frac{a\log\left(\left|e^x-1\right|\right)}{b^2} - \frac{2}{b\left(e^{(2x)}-1\right)}$$

input
```
integrate(csch(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

output
```
a^2*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) + a*log(e^x + 1)/b^2 - a*log(abs(e^x -
1))/b^2 - 2/(b*(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.77 (sec) , antiderivative size = 292, normalized size of antiderivative = 4.95

$$\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{2}{b-b\,\mathrm{e}^{2\,x}} - \frac{a\,\ln\left(32\,\mathrm{e}^{x}-32\right)}{b^2} + \frac{a\,\ln\left(32\,\mathrm{e}^{x}+32\right)}{b^2}$$
$$+\frac{a^2\,\ln\left(32\,a^4\,\mathrm{e}^{x}-64\,a\,b^3-64\,a^3\,b-32\,a^3\,\sqrt{a^2+b^2}+128\,b^4\,\mathrm{e}^{x}+128\,b^3\,\mathrm{e}^{x}\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,\mathrm{e}^{x}-64\right.}{a^2\,b^2+b^4}$$
$$-\frac{a^2\,\ln\left(32\,a^3\,\sqrt{a^2+b^2}-64\,a\,b^3-64\,a^3\,b+32\,a^4\,\mathrm{e}^{x}+128\,b^4\,\mathrm{e}^{x}-128\,b^3\,\mathrm{e}^{x}\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,\mathrm{e}^{x}+64\right.}{a^2\,b^2+b^4}$$

input
```
int(1/(sinh(x)^3*(a + b/sinh(x))),x)
```

output
```
2/(b - b*exp(2*x)) - (a*log(32*exp(x) - 32))/b^2 + (a*log(32*exp(x) + 32))
/b^2 + (a^2*log(32*a^4*exp(x) - 64*a*b^3 - 64*a^3*b - 32*a^3*(a^2 + b^2)^(
1/2) + 128*b^4*exp(x) + 128*b^3*exp(x)*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp
(x) - 64*a*b^2*(a^2 + b^2)^(1/2) + 96*a^2*b*exp(x)*(a^2 + b^2)^(1/2))*(a^2
 + b^2)^(1/2))/(b^4 + a^2*b^2) - (a^2*log(32*a^3*(a^2 + b^2)^(1/2) - 64*a*
b^3 - 64*a^3*b + 32*a^4*exp(x) + 128*b^4*exp(x) - 128*b^3*exp(x)*(a^2 + b^
2)^(1/2) + 160*a^2*b^2*exp(x) + 64*a*b^2*(a^2 + b^2)^(1/2) - 96*a^2*b*exp(
x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(b^4 + a^2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 242, normalized size of antiderivative = 4.10

$$\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{csch}(x)}\,dx$$
$$=\frac{2e^{2x}\sqrt{a^2+b^2}\,atan\left(\frac{e^{x}ai+bi}{\sqrt{a^2+b^2}}\right)a^2i-2\sqrt{a^2+b^2}\,atan\left(\frac{e^{x}ai+bi}{\sqrt{a^2+b^2}}\right)a^2i-e^{2x}\log(e^{x}-1)\,a^3-e^{2x}\log(e^{x}-1)\,a\,b^2}{b}$$

input
```
int(csch(x)^3/(a+b*csch(x)),x)
```

output

```
(2*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2))*a**
2*i - 2*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2))*a**2*i
- e**(2*x)*log(e**x - 1)*a**3 - e**(2*x)*log(e**x - 1)*a*b**2 + e**(2*x)*l
og(e**x + 1)*a**3 + e**(2*x)*log(e**x + 1)*a*b**2 - 2*e**(2*x)*a**2*b - 2*
e**(2*x)*b**3 + log(e**x - 1)*a**3 + log(e**x - 1)*a*b**2 - log(e**x + 1)*
a**3 - log(e**x + 1)*a*b**2)/(b**2*(e**(2*x)*a**2 + e**(2*x)*b**2 - a**2 -
 b**2))
```

3.83 $\int \frac{\text{csch}^4(x)}{a+b\text{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 83

$$\int \frac{\text{csch}^4(x)}{a+b\text{csch}(x)}\,dx = -\frac{(2a^2-b^2)\,\text{arctanh}(\cosh(x))}{2b^3} + \frac{2a^3\text{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\text{csch}(x)}{2b}$$

output
```
-1/2*(2*a^2-b^2)*arctanh(cosh(x))/b^3+2*a^3*arctanh((a-b*tanh(1/2*x))/(a^2
+b^2)^(1/2))/b^3/(a^2+b^2)^(1/2)+a*coth(x)/b^2-1/2*coth(x)*csch(x)/b
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.78

$$\int \frac{\text{csch}^4(x)}{a+b\text{csch}(x)}\,dx = -\frac{\frac{16a^3\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 4ab\coth\left(\frac{x}{2}\right) + b^2\text{csch}^2\left(\frac{x}{2}\right) + 8a^2\log\left(\cosh\left(\frac{x}{2}\right)\right) - 4b^2\log\left(\cosh\left(\frac{x}{2}\right)\right) - 8a^2\log}{8b^3}$$

input
```
Integrate[Csch[x]^4/(a + b*Csch[x]),x]
```

output
```
-1/8*((16*a^3*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
 - 4*a*b*Coth[x/2] + b^2*Csch[x/2]^2 + 8*a^2*Log[Cosh[x/2]] - 4*b^2*Log[Co
sh[x/2]] - 8*a^2*Log[Sinh[x/2]] + 4*b^2*Log[Sinh[x/2]] + b^2*Sech[x/2]^2 -
 4*a*b*Tanh[x/2])/b^3
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.34, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.385$, Rules used = {3042, 4338, 26, 3042, 26, 4570, 3042, 26, 4486, 26, 3042, 26, 4257, 4318, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^4(x)}{a+b\text{csch}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(ix)^4}{a+ib\csc(ix)}dx$$

$$\Big\downarrow \text{4338}$$

$$-\frac{\coth(x)\text{csch}(x)}{2b}-\frac{i\int -\frac{i\text{csch}(x)\left(2a\text{csch}^2(x)+b\text{csch}(x)+a\right)}{a+b\text{csch}(x)}dx}{2b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\int \frac{\text{csch}(x)\left(2a\text{csch}^2(x)+b\text{csch}(x)+a\right)}{a+b\text{csch}(x)}dx}{2b}-\frac{\coth(x)\text{csch}(x)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\coth(x)\text{csch}(x)}{2b}-\frac{\int \frac{i\csc(ix)\left(-2a\csc(ix)^2+ib\csc(ix)+a\right)}{a+ib\csc(ix)}dx}{2b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\coth(x)\text{csch}(x)}{2b}-\frac{i\int \frac{\csc(ix)\left(-2a\csc(ix)^2+ib\csc(ix)+a\right)}{a+ib\csc(ix)}dx}{2b}$$

$\downarrow$ 4570

$$-\frac{\coth(x)\text{csch}(x)}{2b}-\frac{i\left(\frac{2ia\coth(x)}{b}-\frac{i\int \frac{\text{csch}(x)\left(ab-\left(2a^2-b^2\right)\text{csch}(x)\right)}{a+b\text{csch}(x)}dx}{b}\right)}{2b}$$

$\downarrow$ 3042

$$-\frac{\coth(x)\text{csch}(x)}{2b}-\frac{i\left(\frac{2ia\coth(x)}{b}-\frac{i\int \frac{i\csc(ix)\left(ab-i\left(2a^2-b^2\right)\csc(ix)\right)}{a+ib\csc(ix)}dx}{b}\right)}{2b}$$

$\downarrow$ 26

$$-\frac{\coth(x)\text{csch}(x)}{2b}-\frac{i\left(\frac{\int \frac{\csc(ix)\left(ab-i\left(2a^2-b^2\right)\csc(ix)\right)}{a+ib\csc(ix)}dx}{b}+\frac{2ia\coth(x)}{b}\right)}{2b}$$

$\downarrow$ 4486

$$-\frac{\coth(x)\text{csch}(x)}{2b}-\frac{i\left(\frac{\frac{2a^3\int -\frac{i\text{csch}(x)}{a+b\text{csch}(x)}dx}{b}-\frac{\left(2a^2-b^2\right)\int -i\text{csch}(x)dx}{b}}{b}+\frac{2ia\coth(x)}{b}\right)}{2b}$$

$\downarrow$ 26

$$-\frac{\coth(x)\text{csch}(x)}{2b}-\frac{i\left(\frac{\frac{i\left(2a^2-b^2\right)\int \text{csch}(x)dx}{b}-\frac{2ia^3\int \frac{\text{csch}(x)}{a+b\text{csch}(x)}dx}{b}}{b}+\frac{2ia\coth(x)}{b}\right)}{2b}$$

$\downarrow$ 3042

$$-\frac{\coth(x)\text{csch}(x)}{2b}-\frac{i\left(\frac{\frac{i\left(2a^2-b^2\right)\int i\csc(ix)dx}{b}-\frac{2ia^3\int \frac{i\csc(ix)}{a+ib\csc(ix)}dx}{b}}{b}+\frac{2ia\coth(x)}{b}\right)}{2b}$$

$\downarrow$ 26

$$-\frac{\coth(x)\text{csch}(x)}{2b}-\frac{i\left(\frac{\frac{2a^3\int\frac{\csc(ix)}{a+ib\csc(ix)}dx}{b}-\frac{\left(2a^2-b^2\right)\int\csc(ix)dx}{b}}{b}+\frac{2ia\coth(x)}{b}\right)}{2b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\coth(x)\text{csch}(x)}{2b}-\frac{i\left(\frac{\frac{2a^3\int\frac{\csc(ix)}{a+ib\csc(ix)}dx}{b}-\frac{i\left(2a^2-b^2\right)\text{arctanh}(\cosh(x))}{b}}{b}+\frac{2ia\coth(x)}{b}\right)}{2b}$$

$$\Big\downarrow \text{4318}$$

$$-\frac{\coth(x)\text{csch}(x)}{2b}-\frac{i\left(\frac{-\frac{2ia^3\int\frac{1}{\frac{a\sinh(x)}{b}+1}dx}{b^2}-\frac{i\left(2a^2-b^2\right)\text{arctanh}(\cosh(x))}{b}}{b}+\frac{2ia\coth(x)}{b}\right)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\coth(x)\text{csch}(x)}{2b}-\frac{i\left(\frac{-\frac{2ia^3\int\frac{1}{1-\frac{ia\sin(ix)}{b}}dx}{b^2}-\frac{i\left(2a^2-b^2\right)\text{arctanh}(\cosh(x))}{b}}{b}+\frac{2ia\coth(x)}{b}\right)}{2b}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{\coth(x)\text{csch}(x)}{2b}-\frac{i\left(\frac{-\frac{4ia^3\int\frac{1}{-\tanh^2\left(\frac{x}{2}\right)+\frac{2a\tanh\left(\frac{x}{2}\right)}{b}+1}d\tanh\left(\frac{x}{2}\right)}{b^2}-\frac{i\left(2a^2-b^2\right)\text{arctanh}(\cosh(x))}{b}}{b}+\frac{2ia\coth(x)}{b}\right)}{2b}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{\coth(x)\text{csch}(x)}{2b}-\frac{i\left(\frac{\frac{8ia^3\int\frac{1}{4\left(\frac{a^2}{b^2}+1\right)-\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)^2}d\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{b^2}-\frac{i\left(2a^2-b^2\right)\text{arctanh}(\cosh(x))}{b}}{b}+\frac{2ia\coth(x)}{b}\right)}{2b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\coth(x)\text{csch}(x)}{2b}-\frac{i\left(\frac{\frac{4ia^3\text{arctanh}\left(\frac{b\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{i(2a^2-b^2)\text{arctanh}(\cosh(x))}{b}}{b}+\frac{2ia\coth(x)}{b}\right)}{2b}$$

input
```
Int[Csch[x]^4/(a + b*Csch[x]),x]
```

output
```
((-1/2*I)*((((-I)*(2*a^2 - b^2)*ArcTanh[Cosh[x]])/b + ((4*I)*a^3*ArcTanh[(
b*((2*a)/b - 2*Tanh[x/2]))/(2*Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]))/b +
((2*I)*a*Coth[x])/b))/b - (Coth[x]*Csch[x])/(2*b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4338
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 3)/(b*f*
(n - 2))), x] + Simp[d^3/(b*(n - 2))   Int[(d*Csc[e + f*x])^(n - 3)*(Simp[a
*(n - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x]/(a + b*Csc
[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && Gt
Q[n, 3]
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.30

method	result
default	$\frac{\frac{b\tanh\left(\frac{x}{2}\right)^{2}}{2}+2a\tanh\left(\frac{x}{2}\right)}{4b^{2}}+\frac{2a^{3}\operatorname{arctanh}\left(\frac{-2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{a^{2}+b^{2}}}\right)}{b^{3}\sqrt{a^{2}+b^{2}}}-\frac{1}{8b\tanh\left(\frac{x}{2}\right)^{2}}+\frac{\left(4a^{2}-2b^{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4b^{3}}+\frac{a}{2b^{2}\tanh\left(\frac{x}{2}\right)}$
risch	$\frac{-b\,e^{3x}+2\,e^{2x}a-b\,e^{x}-2a}{\left(e^{2x}-1\right)^{2}b^{2}}+\frac{\ln(e^{x}-1)a^{2}}{b^{3}}-\frac{\ln(e^{x}-1)}{2b}-\frac{\ln(1+e^{x})a^{2}}{b^{3}}+\frac{\ln(1+e^{x})}{2b}+\frac{a^{3}\ln\left(e^{x}+\frac{b\sqrt{a^{2}+b^{2}}+a^{2}+b^{2}}{\sqrt{a^{2}+b^{2}}\,a}\right)}{\sqrt{a^{2}+b^{2}}\,b^{3}}-\frac{a^{3}\ln\left(\right.}{}$

input
```
int(csch(x)^4/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
1/4/b^2*(1/2*b*tanh(1/2*x)^2+2*a*tanh(1/2*x))+2/b^3*a^3/(a^2+b^2)^(1/2)*ar
ctanh(1/2*(-2*tanh(1/2*x)*b+2*a)/(a^2+b^2)^(1/2))-1/8/b/tanh(1/2*x)^2+1/4/
b^3*(4*a^2-2*b^2)*ln(tanh(1/2*x))+1/2/b^2*a/tanh(1/2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 947 vs. 2(75) = 150.

Time = 0.14 (sec) , antiderivative size = 947, normalized size of antiderivative = 11.41

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{csch}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
-1/2*(4*a^3*b + 4*a*b^3 + 2*(a^2*b^2 + b^4)*cosh(x)^3 + 2*(a^2*b^2 + b^4)*
sinh(x)^3 - 4*(a^3*b + a*b^3)*cosh(x)^2 - 2*(2*a^3*b + 2*a*b^3 - 3*(a^2*b^
2 + b^4)*cosh(x))*sinh(x)^2 - 2*(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 +
 a^3*sinh(x)^4 - 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 - a^3)*sinh(x)
^2 + 4*(a^3*cosh(x)^3 - a^3*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cos
h(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*
b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 +
 a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) + 2*(a^2*b^2
+ b^4)*cosh(x) + ((2*a^4 + a^2*b^2 - b^4)*cosh(x)^4 + 4*(2*a^4 + a^2*b^2 -
 b^4)*cosh(x)*sinh(x)^3 + (2*a^4 + a^2*b^2 - b^4)*sinh(x)^4 + 2*a^4 + a^2*
b^2 - b^4 - 2*(2*a^4 + a^2*b^2 - b^4)*cosh(x)^2 - 2*(2*a^4 + a^2*b^2 - b^4
 - 3*(2*a^4 + a^2*b^2 - b^4)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^4 + a^2*b^2 -
b^4)*cosh(x)^3 - (2*a^4 + a^2*b^2 - b^4)*cosh(x))*sinh(x))*log(cosh(x) + s
inh(x) + 1) - ((2*a^4 + a^2*b^2 - b^4)*cosh(x)^4 + 4*(2*a^4 + a^2*b^2 - b^
4)*cosh(x)*sinh(x)^3 + (2*a^4 + a^2*b^2 - b^4)*sinh(x)^4 + 2*a^4 + a^2*b^2
 - b^4 - 2*(2*a^4 + a^2*b^2 - b^4)*cosh(x)^2 - 2*(2*a^4 + a^2*b^2 - b^4 -
3*(2*a^4 + a^2*b^2 - b^4)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^4 + a^2*b^2 - b^4
)*cosh(x)^3 - (2*a^4 + a^2*b^2 - b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh
(x) - 1) + 2*(a^2*b^2 + b^4 + 3*(a^2*b^2 + b^4)*cosh(x)^2 - 4*(a^3*b + a*b
^3)*cosh(x))*sinh(x))/(a^2*b^3 + b^5 + (a^2*b^3 + b^5)*cosh(x)^4 + 4*(a...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{csch}(x)}\,dx=\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{csch}(x)}\,dx$$

input
```
integrate(csch(x)**4/(a+b*csch(x)),x)
```

output
```
Integral(csch(x)**4/(a + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(75) = 150$.

Time = 0.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.90

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{a^3\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b^3} + \frac{be^{(-x)}+2\,ae^{(-2\,x)}+be^{(-3\,x)}-2\,a}{2\,b^2e^{(-2\,x)}-b^2e^{(-4\,x)}-b^2} - \frac{(2\,a^2-b^2)\log\left(e^{(-x)}+1\right)}{2\,b^3} + \frac{(2\,a^2-b^2)\log\left(e^{(-x)}-1\right)}{2\,b^3}$$

input
```
integrate(csch(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
-a^3*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2))
)/(sqrt(a^2 + b^2)*b^3) + (b*e^(-x) + 2*a*e^(-2*x) + b*e^(-3*x) - 2*a)/(2*
b^2*e^(-2*x) - b^2*e^(-4*x) - b^2) - 1/2*(2*a^2 - b^2)*log(e^(-x) + 1)/b^3
 + 1/2*(2*a^2 - b^2)*log(e^(-x) - 1)/b^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.70

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{a^3\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^3} - \frac{(2\,a^2-b^2)\log\left(e^x+1\right)}{2\,b^3} + \frac{(2\,a^2-b^2)\log\left(|e^x-1|\right)}{2\,b^3} - \frac{be^{(3\,x)}-2\,ae^{(2\,x)}+be^x+2\,a}{b^2\left(e^{(2\,x)}-1\right)^2}$$

input
```
integrate(csch(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

output
```
-a^3*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt
(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3) - 1/2*(2*a^2 - b^2)*log(e^x + 1)/b^3 +
 1/2*(2*a^2 - b^2)*log(abs(e^x - 1))/b^3 - (b*e^(3*x) - 2*a*e^(2*x) + b*e^
x + 2*a)/(b^2*(e^(2*x) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 3.15 (sec) , antiderivative size = 617, normalized size of antiderivative = 7.43

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{e^x}{b-b\,e^{2x}} - \frac{2\,e^x}{b-2\,b\,e^{2x}+b\,e^{4x}}$$
$$-\frac{\ln\left(24\,a^4+4\,b^4-20\,a^2\,b^2-24\,a^4\,e^x-4\,b^4\,e^x+20\,a^2\,b^2\,e^x\right)}{2\,b}$$
$$+\frac{\ln\left(24\,a^4+4\,b^4-20\,a^2\,b^2+24\,a^4\,e^x+4\,b^4\,e^x-20\,a^2\,b^2\,e^x\right)}{2\,b}+\frac{2\,a}{b^2\,e^{2x}-b^2}$$
$$+\frac{a^2\,\ln\left(24\,a^4+4\,b^4-20\,a^2\,b^2-24\,a^4\,e^x-4\,b^4\,e^x+20\,a^2\,b^2\,e^x\right)}{b^3}$$
$$-\frac{a^2\,\ln\left(24\,a^4+4\,b^4-20\,a^2\,b^2+24\,a^4\,e^x+4\,b^4\,e^x-20\,a^2\,b^2\,e^x\right)}{b^3}$$
$$-\frac{a^3\,\ln\left(16\,a\,b^5-24\,a^5\,\sqrt{a^2+b^2}-48\,a^5\,b-32\,a^3\,b^3+24\,a^6\,e^x-32\,b^6\,e^x-40\,a^3\,b^2\,\sqrt{a^2+b^2}-32\,b^5\,e^x\right.}{a}$$
$$+\frac{a^3\,\ln\left(24\,a^5\,\sqrt{a^2+b^2}+16\,a\,b^5-48\,a^5\,b-32\,a^3\,b^3+24\,a^6\,e^x-32\,b^6\,e^x+40\,a^3\,b^2\,\sqrt{a^2+b^2}+32\,b^5\,e^x\right.}{a}$$

input
```
int(1/(sinh(x)^4*(a + b/sinh(x))),x)
```

output
```
exp(x)/(b - b*exp(2*x)) - (2*exp(x))/(b - 2*b*exp(2*x) + b*exp(4*x)) - log
(24*a^4 + 4*b^4 - 20*a^2*b^2 - 24*a^4*exp(x) - 4*b^4*exp(x) + 20*a^2*b^2*e
xp(x))/(2*b) + log(24*a^4 + 4*b^4 - 20*a^2*b^2 + 24*a^4*exp(x) + 4*b^4*exp
(x) - 20*a^2*b^2*exp(x))/(2*b) + (2*a)/(b^2*exp(2*x) - b^2) + (a^2*log(24*
a^4 + 4*b^4 - 20*a^2*b^2 - 24*a^4*exp(x) - 4*b^4*exp(x) + 20*a^2*b^2*exp(x
)))/b^3 - (a^2*log(24*a^4 + 4*b^4 - 20*a^2*b^2 + 24*a^4*exp(x) + 4*b^4*exp
(x) - 20*a^2*b^2*exp(x)))/b^3 - (a^3*log(16*a*b^5 - 24*a^5*(a^2 + b^2)^(1/
2) - 48*a^5*b - 32*a^3*b^3 + 24*a^6*exp(x) - 32*b^6*exp(x) - 40*a^3*b^2*(a
^2 + b^2)^(1/2) - 32*b^5*exp(x)*(a^2 + b^2)^(1/2) + 56*a^2*b^4*exp(x) + 11
2*a^4*b^2*exp(x) + 16*a*b^4*(a^2 + b^2)^(1/2) + 72*a^4*b*exp(x)*(a^2 + b^2
)^(1/2) + 72*a^2*b^3*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(b^5 + a
^2*b^3) + (a^3*log(24*a^5*(a^2 + b^2)^(1/2) + 16*a*b^5 - 48*a^5*b - 32*a^3
*b^3 + 24*a^6*exp(x) - 32*b^6*exp(x) + 40*a^3*b^2*(a^2 + b^2)^(1/2) + 32*b
^5*exp(x)*(a^2 + b^2)^(1/2) + 56*a^2*b^4*exp(x) + 112*a^4*b^2*exp(x) - 16*
a*b^4*(a^2 + b^2)^(1/2) - 72*a^4*b*exp(x)*(a^2 + b^2)^(1/2) - 72*a^2*b^3*e
xp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(b^5 + a^2*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 519, normalized size of antiderivative = 6.25

$$\int \frac{\operatorname{csch}^4(x)}{a + b\operatorname{csch}(x)}\,dx$$
$$= \frac{-e^{4x}\log(e^x - 1)\,b^4 + e^{4x}\log(e^x - 1)\,a^2b^2 - e^{4x}\log(e^x + 1)\,a^2b^2 + e^{4x}\log(e^x + 1)\,b^4 + \log(e^x - 1)\,a^2b^2 - \mathrm{l}}{}$$

input
```
int(csch(x)^4/(a+b*csch(x)),x)
```

output
```
( - 4*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2))*
a**3*i + 8*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b*
*2))*a**3*i - 4*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2))
*a**3*i + 2*e**(4*x)*log(e**x - 1)*a**4 + e**(4*x)*log(e**x - 1)*a**2*b**2
 - e**(4*x)*log(e**x - 1)*b**4 - 2*e**(4*x)*log(e**x + 1)*a**4 - e**(4*x)*
log(e**x + 1)*a**2*b**2 + e**(4*x)*log(e**x + 1)*b**4 + 2*e**(4*x)*a**3*b
+ 2*e**(4*x)*a*b**3 - 2*e**(3*x)*a**2*b**2 - 2*e**(3*x)*b**4 - 4*e**(2*x)*
log(e**x - 1)*a**4 - 2*e**(2*x)*log(e**x - 1)*a**2*b**2 + 2*e**(2*x)*log(e
**x - 1)*b**4 + 4*e**(2*x)*log(e**x + 1)*a**4 + 2*e**(2*x)*log(e**x + 1)*a
**2*b**2 - 2*e**(2*x)*log(e**x + 1)*b**4 - 2*e**x*a**2*b**2 - 2*e**x*b**4
+ 2*log(e**x - 1)*a**4 + log(e**x - 1)*a**2*b**2 - log(e**x - 1)*b**4 - 2*
log(e**x + 1)*a**4 - log(e**x + 1)*a**2*b**2 + log(e**x + 1)*b**4 - 2*a**3
*b - 2*a*b**3)/(2*b**3*(e**(4*x)*a**2 + e**(4*x)*b**2 - 2*e**(2*x)*a**2 -
2*e**(2*x)*b**2 + a**2 + b**2))
```

3.84 $\int \frac{\cosh^4(x)}{i+\operatorname{csch}(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{\cosh^4(x)}{i+\operatorname{csch}(x)} \, dx = \frac{ix}{8} + \frac{\cosh^3(x)}{3} + \frac{1}{8} i \cosh(x) \sinh(x) - \frac{1}{4} i \cosh^3(x) \sinh(x)$$

output
```
1/8*I*x+1/3*cosh(x)^3+1/8*I*cosh(x)*sinh(x)-1/4*I*cosh(x)^3*sinh(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{\cosh^4(x)}{i+\operatorname{csch}(x)} \, dx = \frac{ix}{8} + \frac{\cosh(x)}{4} + \frac{1}{12} \cosh(3x) - \frac{1}{32} i \sinh(4x)$$

input
```
Integrate[Cosh[x]^4/(I + Csch[x]),x]
```

output
```
(I/8)*x + Cosh[x]/4 + Cosh[3*x]/12 - (I/32)*Sinh[4*x]
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.385, Rules used = {3042, 4360, 26, 3042, 26, 26, 3318, 25, 26, 3042, 25, 26, 3045, 15, 3048, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(x)}{\operatorname{csch}(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^4}{i\csc(ix)+i}dx$$

$$\Big\downarrow \text{4360}$$

$$\int \frac{i\sinh(x)\cosh^4(x)}{-\sinh(x)+i}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cosh^4(x)\sinh(x)}{i-\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$i\int -\frac{i\cos(ix)^4\sin(ix)}{i\sin(ix)+i}dx$$

$$\Big\downarrow \text{26}$$

$$\int -\frac{i\sin(ix)\cos(ix)^4}{1+\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ix)^4\sin(ix)}{\sin(ix)+1}dx$$

$$\Big\downarrow \text{3318}$$

$$-i\left(-\int -\cosh^2(x)\sinh^2(x)dx+\int i\cosh^2(x)\sinh(x)dx\right)$$

$$\Big\downarrow \text{25}$$

$$-i\left(\int \cosh^2(x)\sinh^2(x)dx+\int i\cosh^2(x)\sinh(x)dx\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\int \cosh^2(x)\sinh^2(x)dx+i\int \cosh^2(x)\sinh(x)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(i\int -i\cos(ix)^2\sin(ix)dx+\int -\cos(ix)^2\sin(ix)^2dx\right)$$

$$\Big\downarrow \text{25}$$

$$-i\left(i\int -i\cos(ix)^2\sin(ix)dx-\int \cos(ix)^2\sin(ix)^2dx\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\int \cos(ix)^2\sin(ix)dx-\int \cos(ix)^2\sin(ix)^2dx\right)$$

$$\Big\downarrow \text{3045}$$

$$-i\left(i\int \cosh^2(x)d\cosh(x)-\int \cos(ix)^2\sin(ix)^2dx\right)$$

$$\Big\downarrow \text{15}$$

$$-i\left(\frac{1}{3}i\cosh^3(x)-\int \cos(ix)^2\sin(ix)^2dx\right)$$

$$\Big\downarrow \text{3048}$$

$$-i\left(-\frac{1}{4}\int \cosh^2(x)dx+\frac{1}{3}i\cosh^3(x)+\frac{1}{4}\sinh(x)\cosh^3(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{1}{4}\int \sin\left(ix+\frac{\pi}{2}\right)^2dx+\frac{1}{3}i\cosh^3(x)+\frac{1}{4}\sinh(x)\cosh^3(x)\right)$$

$$\Big\downarrow \text{3115}$$

$$-i\left(\frac{1}{4}\left(-\frac{\int 1dx}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)+\frac{1}{3}i\cosh^3(x)+\frac{1}{4}\sinh(x)\cosh^3(x)\right)$$

$$\Big\downarrow \text{24}$$

$$-i\left(\frac{1}{3}i\cosh^3(x)+\frac{1}{4}\sinh(x)\cosh^3(x)+\frac{1}{4}\left(-\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)\right)$$

input
```
Int[Cosh[x]^4/(I + Csch[x]),x]
```

output
```
(-I)*((I/3)*Cosh[x]^3 + (Cosh[x]^3*Sinh[x])/4 + (-1/2*x - (Cosh[x]*Sinh[x]
)/2)/4)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(27) = 54$.

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.05

$$\frac{i}{4\left(\tanh\left(\frac{x}{2}\right)+1\right)^{4}}+\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8}+\frac{\frac{1}{3}-\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{\frac{1}{2}-\frac{i}{8}}{\tanh\left(\frac{x}{2}\right)+1}+\frac{-\frac{1}{2}+\frac{3i}{8}}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)\right.}{8}$$

input
```
int(cosh(x)^4/(I+csch(x)),x)
```

output
```
1/4*I/(tanh(1/2*x)+1)^4+1/8*I*ln(tanh(1/2*x)+1)+(1/3-1/2*I)/(tanh(1/2*x)+1
)^3+(1/2-1/8*I)/(tanh(1/2*x)+1)+(-1/2+3/8*I)/(tanh(1/2*x)+1)^2-1/8*I*ln(ta
nh(1/2*x)-1)-1/4*I/(tanh(1/2*x)-1)^4-(1/3+1/2*I)/(tanh(1/2*x)-1)^3-(1/2+3/
8*I)/(tanh(1/2*x)-1)^2-(1/2+1/8*I)/(tanh(1/2*x)-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \frac{\cosh^4(x)}{i+\operatorname{csch}(x)}\,dx$$
$$=\frac{1}{192}\left(24i\,xe^{(4\,x)}-3i\,e^{(8\,x)}+8\,e^{(7\,x)}+24\,e^{(5\,x)}+24\,e^{(3\,x)}+8\,e^{x}+3i\right)e^{(-4\,x)}$$

input
```
integrate(cosh(x)^4/(I+csch(x)),x, algorithm="fricas")
```

output
```
1/192*(24*I*x*e^(4*x) - 3*I*e^(8*x) + 8*e^(7*x) + 24*e^(5*x) + 24*e^(3*x)
+ 8*e^x + 3*I)*e^(-4*x)
```

Sympy [F]

$$\int \frac{\cosh^4(x)}{i+\operatorname{csch}(x)}\,dx=\int \frac{\cosh^4{(x)}}{\operatorname{csch}{(x)}+i}\,dx$$

input
```
integrate(cosh(x)**4/(I+csch(x)),x)
```

output
```
Integral(cosh(x)**4/(csch(x) + I), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{\cosh^4(x)}{i+\operatorname{csch}(x)}\,dx=\frac{1}{192}\left(8\,e^{(-x)}+24\,e^{(-3\,x)}-3i\right)e^{(4\,x)}+\frac{1}{8}i\,x+\frac{1}{8}\,e^{(-x)}+\frac{1}{24}\,e^{(-3\,x)}+\frac{1}{64}i\,e^{(-4\,x)}$$

input
```
integrate(cosh(x)^4/(I+csch(x)),x, algorithm="maxima")
```

output
```
1/192*(8*e^(-x) + 24*e^(-3*x) - 3*I)*e^(4*x) + 1/8*I*x + 1/8*e^(-x) + 1/24
*e^(-3*x) + 1/64*I*e^(-4*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^4(x)}{i+\text{csch}(x)}\,dx = \frac{1}{192}\left(24\,e^{(3\,x)} + 8\,e^{x} + 3i\right)e^{(-4\,x)} + \frac{1}{8}i\,x - \frac{1}{64}i\,e^{(4\,x)} + \frac{1}{24}\,e^{(3\,x)} + \frac{1}{8}\,e^{x}$$

input
```
integrate(cosh(x)^4/(I+csch(x)),x, algorithm="giac")
```

output
```
1/192*(24*e^(3*x) + 8*e^x + 3*I)*e^(-4*x) + 1/8*I*x - 1/64*I*e^(4*x) + 1/2
4*e^(3*x) + 1/8*e^x
```

Mupad [B] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \frac{\cosh^4(x)}{i+\text{csch}(x)}\,dx = \frac{e^{-x}}{8} + \frac{e^{-3\,x}}{24} + \frac{e^{3\,x}}{24} + \frac{e^{x}}{8} + \frac{x\,1i}{8} + \frac{e^{-4\,x}\,1i}{64} - \frac{e^{4\,x}\,1i}{64}$$

input
```
int(cosh(x)^4/(1/sinh(x) + 1i),x)
```

output
```
(x*1i)/8 + exp(-x)/8 + exp(-3*x)/24 + exp(3*x)/24 + (exp(-4*x)*1i)/64 - (e
xp(4*x)*1i)/64 + exp(x)/8
```

Reduce [F]

$$\int \frac{\cosh^4(x)}{i+\operatorname{csch}(x)}\,dx=\int \frac{\cosh\left(x\right)^{4}}{\operatorname{csch}\left(x\right)+i}dx$$

input
```
int(cosh(x)^4/(I+csch(x)),x)
```

output
```
int(cosh(x)**4/(csch(x) + i),x)
```

3.85 $\int \frac{\cosh^3(x)}{i+\text{csch}(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{\cosh^3(x)}{i+\text{csch}(x)} \, dx = \frac{\sinh^2(x)}{2} - \frac{1}{3} i \sinh^3(x)$$

output
```
1/2*sinh(x)^2-1/3*I*sinh(x)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^3(x)}{i+\text{csch}(x)} \, dx = \frac{\sinh^2(x)}{2} - \frac{1}{3} i \sinh^3(x)$$

input
```
Integrate[Cosh[x]^3/(I + Csch[x]),x]
```

output
```
Sinh[x]^2/2 - (I/3)*Sinh[x]^3
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.077$, Rules used = {3042, 4360, 26, 3042, 26, 26, 3314, 25, 26, 3042, 25, 26, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{\operatorname{csch}(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^3}{i\csc(ix)+i}dx$$

$$\Big\downarrow \text{4360}$$

$$\int \frac{i\sinh(x)\cosh^3(x)}{-\sinh(x)+i}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cosh^3(x)\sinh(x)}{i-\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$i\int -\frac{i\cos(ix)^3\sin(ix)}{i\sin(ix)+i}dx$$

$$\Big\downarrow \text{26}$$

$$\int -\frac{i\sin(ix)\cos(ix)^3}{1+\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ix)^3\sin(ix)}{\sin(ix)+1}dx$$

$$\Big\downarrow \text{3314}$$

$$-i\left(-\int -\cosh(x)\sinh^2(x)dx+\int i\cosh(x)\sinh(x)dx\right)$$

$$\Big\downarrow \text{25}$$

$$-i\left(\int \cosh(x)\sinh^2(x)dx+\int i\cosh(x)\sinh(x)dx\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(\int \cosh(x)\sinh^2(x)dx+i\int \cosh(x)\sinh(x)dx\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(i\int -i\cos(ix)\sin(ix)dx+\int -\cos(ix)\sin(ix)^2dx\right)$$

$$\Big\downarrow \text{ 25}$$

$$-i\left(i\int -i\cos(ix)\sin(ix)dx-\int \cos(ix)\sin(ix)^2dx\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(\int \cos(ix)\sin(ix)dx-\int \cos(ix)\sin(ix)^2dx\right)$$

$$\Big\downarrow \text{ 3044}$$

$$-i\left(i\int -\sinh^2(x)d(i\sinh(x))-i\int i\sinh(x)d(i\sinh(x))\right)$$

$$\Big\downarrow \text{ 15}$$

$$-i\left(\frac{\sinh^3(x)}{3}+\frac{1}{2}i\sinh^2(x)\right)$$

input
```
Int[Cosh[x]^3/(I + Csch[x]),x]
```

output
```
(-I)*((I/2)*Sinh[x]^2 + Sinh[x]^3/3)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3314
```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[Cos[e + f
*x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[1/(b*d)   Int[Cos[e + f*x]^(p
 - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &
& IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p
+ 1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 99.53 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-\frac{i}{3\operatorname{csch}(x)^{3}}+\frac{1}{2\operatorname{csch}(x)^{2}}$	15
default	$-\frac{i}{3\operatorname{csch}(x)^{3}}+\frac{1}{2\operatorname{csch}(x)^{2}}$	15
risch	$-\frac{i\mathrm{e}^{3x}}{24}+\frac{\mathrm{e}^{2x}}{8}+\frac{i\mathrm{e}^{x}}{8}-\frac{i\mathrm{e}^{-x}}{8}+\frac{\mathrm{e}^{-2x}}{8}+\frac{i\mathrm{e}^{-3x}}{24}$	40

input
```
int(cosh(x)^3/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/csch(x)^3+1/2/csch(x)^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(13) = 26.

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{\cosh^3(x)}{i+\operatorname{csch}(x)}\,dx = \frac{1}{24}\left(-i\,e^{(6\,x)}+3\,e^{(5\,x)}+3i\,e^{(4\,x)}-3i\,e^{(2\,x)}+3\,e^{x}+i\right)e^{(-3\,x)}$$

input
```
integrate(cosh(x)^3/(I+csch(x)),x, algorithm="fricas")
```

output
```
1/24*(-I*e^(6*x) + 3*e^(5*x) + 3*I*e^(4*x) - 3*I*e^(2*x) + 3*e^x + I)*e^(-
3*x)
```

Sympy [F]

$$\int \frac{\cosh^3(x)}{i + \operatorname{csch}(x)}\,dx = \int \frac{\cosh^3(x)}{\operatorname{csch}(x) + i}\,dx$$

input
```
integrate(cosh(x)**3/(I+csch(x)),x)
```

output
```
Integral(cosh(x)**3/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(13) = 26.

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{\cosh^3(x)}{i + \operatorname{csch}(x)}\,dx = \frac{1}{24}\left(3\,e^{(-x)} + 3i\,e^{(-2\,x)} - i\right)e^{(3\,x)} - \frac{1}{8}i\,e^{(-x)} + \frac{1}{8}\,e^{(-2\,x)} + \frac{1}{24}i\,e^{(-3\,x)}$$

input
```
integrate(cosh(x)^3/(I+csch(x)),x, algorithm="maxima")
```

output
```
1/24*(3*e^(-x) + 3*I*e^(-2*x) - I)*e^(3*x) - 1/8*I*e^(-x) + 1/8*e^(-2*x) +
 1/24*I*e^(-3*x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(13) = 26.

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{\cosh^3(x)}{i + \operatorname{csch}(x)}\,dx = -\frac{1}{24}\left(3i\,e^{(2\,x)} - 3\,e^{x} - i\right)e^{(-3\,x)} - \frac{1}{24}i\,e^{(3\,x)} + \frac{1}{8}\,e^{(2\,x)} + \frac{1}{8}i\,e^{x}$$

input
```
integrate(cosh(x)^3/(I+csch(x)),x, algorithm="giac")
```

output
```
-1/24*(3*I*e^(2*x) - 3*e^x - I)*e^(-3*x) - 1/24*I*e^(3*x) + 1/8*e^(2*x) +
1/8*I*e^x
```

Mupad [B] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{\cosh^3(x)}{i+\operatorname{csch}(x)}\,dx = \frac{e^{-2x}}{8} - \frac{e^{-x}\,1i}{8} + \frac{e^{2x}}{8} + \frac{e^{-3x}\,1i}{24} - \frac{e^{3x}\,1i}{24} + \frac{e^{x}\,1i}{8}$$

input
```
int(cosh(x)^3/(1/sinh(x) + 1i),x)
```

output
```
exp(-2*x)/8 - (exp(-x)*1i)/8 + exp(2*x)/8 + (exp(-3*x)*1i)/24 - (exp(3*x)*
1i)/24 + (exp(x)*1i)/8
```

Reduce [F]

$$\int \frac{\cosh^3(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\cosh(x)^3}{\operatorname{csch}(x)+i}dx$$

input
```
int(cosh(x)^3/(I+csch(x)),x)
```

output
```
int(cosh(x)**3/(csch(x) + i),x)
```

3.86 $\int \frac{\cosh^2(x)}{i+\operatorname{csch}(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{\cosh^2(x)}{i+\operatorname{csch}(x)} dx = \frac{ix}{2} + \cosh(x) - \frac{1}{2} i \cosh(x) \sinh(x)$$

output
```
1/2*I*x+cosh(x)-1/2*I*cosh(x)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^2(x)}{i+\operatorname{csch}(x)} dx = \frac{ix}{2} + \cosh(x) - \frac{1}{4} i \sinh(2x)$$

input
```
Integrate[Cosh[x]^2/(I + Csch[x]),x]
```

output
```
(I/2)*x + Cosh[x] - (I/4)*Sinh[2*x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.154, Rules used = {3042, 4360, 26, 3042, 26, 26, 3318, 25, 26, 3042, 25, 26, 3115, 24, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{\operatorname{csch}(x)+i} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^2}{i\csc(ix)+i}dx$$

$$\Big\downarrow \text{4360}$$

$$\int \frac{i\sinh(x)\cosh^2(x)}{-\sinh(x)+i}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cosh^2(x)\sinh(x)}{i-\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$i\int -\frac{i\cos(ix)^2\sin(ix)}{i\sin(ix)+i}dx$$

$$\Big\downarrow \text{26}$$

$$\int -\frac{i\sin(ix)\cos(ix)^2}{1+\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ix)^2\sin(ix)}{\sin(ix)+1}dx$$

$$\Big\downarrow \text{3318}$$

$$-i\left(-\int -\sinh^2(x)dx+\int i\sinh(x)dx\right)$$

$$\Big\downarrow \text{25}$$

$$-i\left(\int \sinh^2(x)dx+\int i\sinh(x)dx\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(\int \sinh^2(x)dx+i\int \sinh(x)dx\right)$$

$$\Big\downarrow \; 3042$$

$$-i\left(i\int -i\sin(ix)dx+\int -\sin(ix)^2dx\right)$$

$$\Big\downarrow \; 25$$

$$-i\left(i\int -i\sin(ix)dx-\int \sin(ix)^2dx\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(\int \sin(ix)dx-\int \sin(ix)^2dx\right)$$

$$\Big\downarrow \; 3115$$

$$-i\left(-\frac{\int 1dx}{2}+\int \sin(ix)dx+\frac{1}{2}\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \; 24$$

$$-i\left(\int \sin(ix)dx-\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \; 3118$$

$$-i\left(-\frac{x}{2}+i\cosh(x)+\frac{1}{2}\sinh(x)\cosh(x)\right)$$

input
```
Int[Cosh[x]^2/(I + Csch[x]),x]
```

output
```
(-I)*(-1/2*x + I*Cosh[x] + (Cosh[x]*Sinh[x])/2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(14) = 28.

Time = 5.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

method	result	size
risch	$\frac{ix}{2}-\frac{i\,\mathrm{e}^{2x}}{8}+\frac{\mathrm{e}^{x}}{2}+\frac{\mathrm{e}^{-x}}{2}+\frac{i\,\mathrm{e}^{-2x}}{8}$	30
default	$-\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{-1-\frac{i}{2}}{\tanh\left(\frac{x}{2}\right)-1}+\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}+\frac{1-\frac{i}{2}}{\tanh\left(\frac{x}{2}\right)+1}$	68

input
```
int(cosh(x)^2/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*x-1/8*I*exp(x)^2+1/2*exp(x)+1/2/exp(x)+1/8*I/exp(x)^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{\cosh^2(x)}{i+\operatorname{csch}(x)}\,dx = \frac{1}{8}\,\left(4i\,xe^{(2\,x)}-i\,e^{(4\,x)}+4\,e^{(3\,x)}+4\,e^{x}+i\right)e^{(-2\,x)}$$

input
```
integrate(cosh(x)^2/(I+csch(x)),x, algorithm="fricas")
```

output
```
1/8*(4*I*x*e^(2*x) - I*e^(4*x) + 4*e^(3*x) + 4*e^x + I)*e^(-2*x)
```

Sympy [F]

$$\int \frac{\cosh^2(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\cosh^2{(x)}}{\operatorname{csch}{(x)}+i}\,dx$$

input
```
integrate(cosh(x)**2/(I+csch(x)),x)
```

output
```
Integral(cosh(x)**2/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(12) = 24$.

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int \frac{\cosh^2(x)}{i+\operatorname{csch}(x)}\,dx = \frac{1}{8}\left(4\,e^{(-x)}-i\right)e^{(2\,x)}+\frac{1}{2}i\,x+\frac{1}{2}\,e^{(-x)}+\frac{1}{8}i\,e^{(-2\,x)}$$

input
```
integrate(cosh(x)^2/(I+csch(x)),x, algorithm="maxima")
```

output
```
1/8*(4*e^(-x) - I)*e^(2*x) + 1/2*I*x + 1/2*e^(-x) + 1/8*I*e^(-2*x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{\cosh^2(x)}{i+\operatorname{csch}(x)}\,dx = \frac{1}{8}\left(4\,e^{x}+i\right)e^{(-2\,x)}+\frac{1}{2}i\,x-\frac{1}{8}i\,e^{(2\,x)}+\frac{1}{2}\,e^{x}$$

input
```
integrate(cosh(x)^2/(I+csch(x)),x, algorithm="giac")
```

output
```
1/8*(4*e^x + I)*e^(-2*x) + 1/2*I*x - 1/8*I*e^(2*x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{\cosh^2(x)}{i+\operatorname{csch}(x)}\,dx=\frac{e^{-x}}{2}+\frac{e^{x}}{2}+\frac{x\,1i}{2}+\frac{e^{-2\,x}\,1i}{8}-\frac{e^{2\,x}\,1i}{8}$$

input
```
int(cosh(x)^2/(1/sinh(x) + 1i),x)
```

output
```
(x*1i)/2 + exp(-x)/2 + (exp(-2*x)*1i)/8 - (exp(2*x)*1i)/8 + exp(x)/2
```

Reduce [F]

$$\int \frac{\cosh^2(x)}{i+\operatorname{csch}(x)}\,dx=\int \frac{\cosh(x)^2}{\operatorname{csch}(x)+i}dx$$

input
```
int(cosh(x)^2/(I+csch(x)),x)
```

output
```
int(cosh(x)**2/(csch(x) + i),x)
```

3.87 $\int \frac{\cosh(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{\cosh(x)}{i+\operatorname{csch}(x)}\,dx=\log(i-\sinh(x))-i\sinh(x)$$

output
```
ln(I-sinh(x))-I*sinh(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{i+\operatorname{csch}(x)}\,dx=\log(i-\sinh(x))-i\sinh(x)$$

input
```
Integrate[Cosh[x]/(I + Csch[x]),x]
```

output
```
Log[I - Sinh[x]] - I*Sinh[x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {3042, 4360, 26, 3042, 26, 26, 3312, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{\operatorname{csch}(x)+i}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(ix)}{i\csc(ix)+i}dx$$

$$\Big\downarrow \; 4360$$

$$\int \frac{i\sinh(x)\cosh(x)}{-\sinh(x)+i}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\cosh(x)\sinh(x)}{i-\sinh(x)}dx$$

$$\Big\downarrow \; 3042$$

$$i\int -\frac{i\cos(ix)\sin(ix)}{i\sin(ix)+i}dx$$

$$\Big\downarrow \; 26$$

$$\int -\frac{i\sin(ix)\cos(ix)}{1+\sin(ix)}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{\cos(ix)\sin(ix)}{\sin(ix)+1}dx$$

$$\Big\downarrow \; 3312$$

$$-\int \frac{i\sinh(x)}{i\sinh(x)+1}d(i\sinh(x))$$

$$\Big\downarrow \; 49$$

$$-\int \left(1+\frac{1}{-i\sinh(x)-1}\right)d(i\sinh(x))$$

$$\Big\downarrow \text{2009}$$

$$\log(1+i\sinh(x))-i\sinh(x)$$

input
```
Int[Cosh[x]/(I + Csch[x]),x]
```

output
```
Log[1 + I*Sinh[x]] - I*Sinh[x]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

method	result	size
risch	$-x-\frac{i e^{x}}{2}+\frac{i e^{-x}}{2}+2\ln\left(e^{x}-i\right)$	25
derivativedivides	$\frac{\ln\left(1+\operatorname{csch}(x)^{2}\right)}{2}-i\arctan\left(\operatorname{csch}\left(x\right)\right)-\frac{i}{\operatorname{csch}(x)}-\ln\left(\operatorname{csch}\left(x\right)\right)$	29
default	$\frac{\ln\left(1+\operatorname{csch}(x)^{2}\right)}{2}-i\arctan\left(\operatorname{csch}\left(x\right)\right)-\frac{i}{\operatorname{csch}(x)}-\ln\left(\operatorname{csch}\left(x\right)\right)$	29

input
```
int(cosh(x)/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-x-1/2*I*exp(x)+1/2*I/exp(x)+2*ln(exp(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(12) = 24.

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int \frac{\cosh(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{1}{2}\left(2\,xe^{x}-4\,e^{x}\log\left(e^{x}-i\right)+i\,e^{(2\,x)}-i\right)e^{(-x)}$$

input
```
integrate(cosh(x)/(I+csch(x)),x, algorithm="fricas")
```

output
```
-1/2*(2*x*e^x - 4*e^x*log(e^x - I) + I*e^(2*x) - I)*e^(-x)
```

Sympy [F]

$$\int \frac{\cosh(x)}{i+\operatorname{csch}(x)}\,dx=\int \frac{\cosh(x)}{\operatorname{csch}(x)+i}\,dx$$

input
```
integrate(cosh(x)/(I+csch(x)),x)
```

output
```
Integral(cosh(x)/(csch(x) + I), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \frac{\cosh(x)}{i+\operatorname{csch}(x)}\,dx=x+\frac{1}{2}i\,e^{(-x)}-\frac{1}{2}i\,e^{x}+2\,\log\left(e^{(-x)}+i\right)$$

input
```
integrate(cosh(x)/(I+csch(x)),x, algorithm="maxima")
```

output
```
x + 1/2*I*e^(-x) - 1/2*I*e^x + 2*log(e^(-x) + I)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \frac{\cosh(x)}{i+\operatorname{csch}(x)}\,dx=-x+\frac{1}{2}i\,e^{(-x)}-\frac{1}{2}i\,e^{x}+2\,\log\left(e^{x}-i\right)$$

input
```
integrate(cosh(x)/(I+csch(x)),x, algorithm="giac")
```

output
```
-x + 1/2*I*e^(-x) - 1/2*I*e^x + 2*log(e^x - I)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\cosh(x)}{i + \operatorname{csch}(x)}\, dx = \ln\left(\sinh(x) - \mathrm{i}\right) - \sinh(x)\ 1\mathrm{i}$$

input
```
int(cosh(x)/(1/sinh(x) + 1i),x)
```

output
```
log(sinh(x) - 1i) - sinh(x)*1i
```

Reduce [F]

$$\int \frac{\cosh(x)}{i + \operatorname{csch}(x)}\, dx = \int \frac{\cosh\left(x\right)}{\operatorname{csch}\left(x\right) + i} dx$$

input
```
int(cosh(x)/(I+csch(x)),x)
```

output
```
int(cosh(x)/(csch(x) + i),x)
```

3.88 $\int \frac{\text{sech}(x)}{i+\text{csch}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 28

$$\int \frac{\text{sech}(x)}{i+\text{csch}(x)}\,dx = -\frac{1}{2}i\arctan(\sinh(x)) - \frac{\text{sech}^2(x)}{2} + \frac{1}{2}i\text{sech}(x)\tanh(x)$$

output
```
-1/2*I*arctan(sinh(x))-1/2*sech(x)^2+1/2*I*sech(x)*tanh(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{\text{sech}(x)}{i+\text{csch}(x)}\,dx = -\frac{1}{2}i\left(\arctan(\sinh(x)) + \frac{1}{i-\sinh(x)}\right)$$

input
```
Integrate[Sech[x]/(I + Csch[x]),x]
```

output
```
(-1/2*I)*(ArcTan[Sinh[x]] + (I - Sinh[x])^(-1))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.545, Rules used = {3042, 4359, 26, 3042, 26, 26, 3185, 25, 26, 3042, 25, 26, 3086, 15, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(x)}{\text{csch}(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ix)(i\csc(ix)+i)}dx$$

$$\Big\downarrow \text{4359}$$

$$\int \frac{i\tanh(x)}{-\sinh(x)+i}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\tanh(x)}{i-\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$i\int -\frac{i\tan(ix)}{i\sin(ix)+i}dx$$

$$\Big\downarrow \text{26}$$

$$\int -\frac{i\tan(ix)}{1+\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ix)}{\sin(ix)+1}dx$$

$$\Big\downarrow \text{3185}$$

$$-i\left(-\int -\text{sech}(x)\tanh^2(x)dx+\int i\text{sech}^2(x)\tanh(x)dx\right)$$

$$\Big\downarrow \text{25}$$

$$-i\left(\int \text{sech}(x)\tanh^2(x)dx+\int i\text{sech}^2(x)\tanh(x)dx\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(\int \text{sech}(x)\tanh^2(x)dx+i\int \text{sech}^2(x)\tanh(x)dx\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(i\int -i\sec(ix)^2\tan(ix)dx+\int -\sec(ix)\tan(ix)^2dx\right)$$

$$\Big\downarrow \text{ 25}$$

$$-i\left(i\int -i\sec(ix)^2\tan(ix)dx-\int \sec(ix)\tan(ix)^2dx\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(\int \sec(ix)^2\tan(ix)dx-\int \sec(ix)\tan(ix)^2dx\right)$$

$$\Big\downarrow \text{ 3086}$$

$$-i\left(-i\int \text{sech}(x)d\text{sech}(x)-\int \sec(ix)\tan(ix)^2dx\right)$$

$$\Big\downarrow \text{ 15}$$

$$-i\left(-\int \sec(ix)\tan(ix)^2dx-\frac{1}{2}i\text{sech}^2(x)\right)$$

$$\Big\downarrow \text{ 3091}$$

$$-i\left(\frac{\int \text{sech}(x)dx}{2}-\frac{1}{2}i\text{sech}^2(x)-\frac{1}{2}\tanh(x)\text{sech}(x)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(\frac{1}{2}\int \csc\left(ix+\frac{\pi}{2}\right)dx-\frac{1}{2}i\text{sech}^2(x)-\frac{1}{2}\tanh(x)\text{sech}(x)\right)$$

$$\Big\downarrow \text{ 4257}$$

$$-i\left(\frac{1}{2}\arctan(\sinh(x))-\frac{1}{2}i\text{sech}^2(x)-\frac{1}{2}\tanh(x)\text{sech}(x)\right)$$

input

```
Int[Sech[x]/(I + Csch[x]),x]
```

output
```
(-I)*(ArcTan[Sinh[x]]/2 - (I/2)*Sech[x]^2 - (Sech[x]*Tanh[x])/2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4359
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_.), x_Symbol] :> Int[Cot[e + f*x]^p*(b + a*Sin[e + f*x])^m, x] /; FreeQ[{a
, b, e, f, p}, x] && IntegerQ[m] && EqQ[m, p]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

method	result	size
risch	$\frac{i\,\mathrm{e}^{x}}{\left(\mathrm{e}^{x}-i\right)^{2}}+\frac{\ln\left(\mathrm{e}^{x}+i\right)}{2}-\frac{\ln\left(\mathrm{e}^{x}-i\right)}{2}$	30
default	$-\frac{i}{\tanh\left(\frac{x}{2}\right)-i}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)}{2}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{2}$	43

input
```
int(sech(x)/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
I*exp(x)/(exp(x)-I)^2+1/2*ln(exp(x)+I)-1/2*ln(exp(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(18) = 36.

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \frac{\operatorname{sech}(x)}{i+\operatorname{csch}(x)}\,dx = \frac{\left(e^{(2x)}-2i\,e^{x}-1\right)\log\left(e^{x}+i\right)-\left(e^{(2x)}-2i\,e^{x}-1\right)\log\left(e^{x}-i\right)+2i\,e^{x}}{2\left(e^{(2x)}-2i\,e^{x}-1\right)}$$

input
```
integrate(sech(x)/(I+csch(x)),x, algorithm="fricas")
```

output
```
1/2*((e^(2*x) - 2*I*e^x - 1)*log(e^x + I) - (e^(2*x) - 2*I*e^x - 1)*log(e^
x - I) + 2*I*e^x)/(e^(2*x) - 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{i+\operatorname{csch}(x)}\,dx=\int \frac{\operatorname{sech}\left(x\right)}{\operatorname{csch}\left(x\right)+i}\,dx$$

input
```
integrate(sech(x)/(I+csch(x)),x)
```

output
```
Integral(sech(x)/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(18) = 36$.

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{\operatorname{sech}(x)}{i+\operatorname{csch}(x)}\,dx=-\frac{2i\,e^{(-x)}}{4i\,e^{(-x)}+2\,e^{(-2\,x)}-2}-\frac{1}{2}\,\log\left(e^{(-x)}+i\right)+\frac{1}{2}\,\log\left(e^{(-x)}-i\right)$$

input
```
integrate(sech(x)/(I+csch(x)),x, algorithm="maxima")
```

output
```
-2*I*e^(-x)/(4*I*e^(-x) + 2*e^(-2*x) - 2) - 1/2*log(e^(-x) + I) + 1/2*log(
e^(-x) - I)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(18) = 36.

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.82

$$\int \frac{\operatorname{sech}(x)}{i+\operatorname{csch}(x)}\,dx = \frac{e^{(-x)}-e^{x}-2i}{4\left(e^{(-x)}-e^{x}+2i\right)}+\frac{1}{4}\log\left(-e^{(-x)}+e^{x}+2i\right)-\frac{1}{4}\log\left(-e^{(-x)}+e^{x}-2i\right)$$

input
```
integrate(sech(x)/(I+csch(x)),x, algorithm="giac")
```

output
```
1/4*(e^(-x) - e^x - 2*I)/(e^(-x) - e^x + 2*I) + 1/4*log(-e^(-x) + e^x + 2*
I) - 1/4*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \frac{\operatorname{sech}(x)}{i+\operatorname{csch}(x)}\,dx = \frac{\ln\left(-1+\mathrm{e}^{x}\,1\mathrm{i}\right)}{2}-\frac{\ln\left(1+\mathrm{e}^{x}\,1\mathrm{i}\right)}{2}+\frac{1}{1-\mathrm{e}^{2\,x}+\mathrm{e}^{x}\,2\mathrm{i}}+\frac{1\mathrm{i}}{\mathrm{e}^{x}-\mathrm{i}}$$

input
```
int(1/(cosh(x)*(1/sinh(x) + 1i)),x)
```

output
```
log(exp(x)*1i - 1)/2 - log(exp(x)*1i + 1)/2 + 1/(exp(x)*2i - exp(2*x) + 1)
 + 1i/(exp(x) - 1i)
```

Reduce [F]

$$\int \frac{\operatorname{sech}(x)}{i + \operatorname{csch}(x)}\, dx = \int \frac{\operatorname{sech}(x)}{\operatorname{csch}(x) + i} dx$$

input
```
int(sech(x)/(I+csch(x)),x)
```

output
```
int(sech(x)/(csch(x) + i),x)
```

3.89 $\int \frac{\operatorname{sech}^2(x)}{i+\operatorname{csch}(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{\operatorname{sech}^2(x)}{i+\operatorname{csch}(x)} \, dx = -\frac{1}{3}\operatorname{sech}^3(x) - \frac{1}{3} i \tanh^3(x)$$

output
```
-1/3*sech(x)^3-1/3*I*tanh(x)^3
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 64 vs. 2(19) = 38.

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.37

$$\int \frac{\operatorname{sech}^2(x)}{i+\operatorname{csch}(x)} \, dx = \frac{-3+\cosh(x)+\cosh(2x)-2i\sinh(x)+i\cosh(x)\sinh(x)}{6\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3}$$

input
```
Integrate[Sech[x]^2/(I + Csch[x]),x]
```

output
```
(-3 + Cosh[x] + Cosh[2*x] - (2*I)*Sinh[x] + I*Cosh[x]*Sinh[x])/(6*(Cosh[x/
2] - I*Sinh[x/2])*(Cosh[x/2] + I*Sinh[x/2])^3)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.231, Rules used = {3042, 4360, 26, 3042, 26, 26, 3318, 25, 26, 3042, 25, 26, 3086, 15, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(x)}{\text{csch}(x)+i}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\cos(ix)^2(i\csc(ix)+i)}dx$$

$$\Big\downarrow \text{ 4360}$$

$$\int \frac{i\tanh(x)\text{sech}(x)}{-\sinh(x)+i}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\text{sech}(x)\tanh(x)}{i-\sinh(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$i\int -\frac{i\sin(ix)}{\cos(ix)^2(i\sin(ix)+i)}dx$$

$$\Big\downarrow \text{ 26}$$

$$\int -\frac{i\sin(ix)}{(1+\sin(ix))\cos(ix)^2}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\sin(ix)}{\cos(ix)^2(\sin(ix)+1)}dx$$

$$\Big\downarrow \text{ 3318}$$

$$-i\left(-\int -\text{sech}^2(x)\tanh^2(x)dx+\int i\text{sech}^3(x)\tanh(x)dx\right)$$

$$\Big\downarrow \text{ 25}$$

$$-i\left(\int \text{sech}^2(x)\tanh^2(x)dx+\int i\text{sech}^3(x)\tanh(x)dx\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\int \text{sech}^2(x)\tanh^2(x)dx+i\int \text{sech}^3(x)\tanh(x)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(i\int -i\sec(ix)^3\tan(ix)dx+\int -\sec(ix)^2\tan(ix)^2dx\right)$$

$$\Big\downarrow \text{25}$$

$$-i\left(i\int -i\sec(ix)^3\tan(ix)dx-\int \sec(ix)^2\tan(ix)^2dx\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\int \sec(ix)^3\tan(ix)dx-\int \sec(ix)^2\tan(ix)^2dx\right)$$

$$\Big\downarrow \text{3086}$$

$$-i\left(-i\int \text{sech}^2(x)d\text{sech}(x)-\int \sec(ix)^2\tan(ix)^2dx\right)$$

$$\Big\downarrow \text{15}$$

$$-i\left(-\int \sec(ix)^2\tan(ix)^2dx-\frac{1}{3}i\text{sech}^3(x)\right)$$

$$\Big\downarrow \text{3087}$$

$$-i\left(i\int -\tanh^2(x)d(i\tanh(x))-\frac{1}{3}i\text{sech}^3(x)\right)$$

$$\Big\downarrow \text{15}$$

$$-i\left(\frac{\tanh^3(x)}{3}-\frac{1}{3}i\text{sech}^3(x)\right)$$

input
```
Int[Sech[x]^2/(I + Csch[x]),x]
```

output
```
(-I)*((-1/3*I)*Sech[x]^3 + Tanh[x]^3/3)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(14) = 28.

Time = 0.51 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

method	result	size
risch	$\frac{2i(-2i\,\mathrm{e}^{x}+3\,\mathrm{e}^{2x}-1)}{3(\mathrm{e}^{x}+i)(\mathrm{e}^{x}-i)^{3}}$	31
default	$-\frac{i}{2(\tanh(\frac{x}{2})+i)}+\frac{i}{2\tanh(\frac{x}{2})-2i}-\frac{2i}{3(\tanh(\frac{x}{2})-i)^{3}}-\frac{1}{(\tanh(\frac{x}{2})-i)^{2}}$	49

input
```
int(sech(x)^2/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*(-2*I*exp(x)+3*exp(2*x)-1)/(exp(x)+I)/(exp(x)-I)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(13) = 26.

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int \frac{\operatorname{sech}^2(x)}{i+\operatorname{csch}(x)}\,dx=-\frac{2\left(-3i\,e^{(2\,x)}-2\,e^{x}+i\right)}{3\left(e^{(4\,x)}-2i\,e^{(3\,x)}-2i\,e^{x}-1\right)}$$

input
```
integrate(sech(x)^2/(I+csch(x)),x, algorithm="fricas")
```

output
```
-2/3*(-3*I*e^(2*x) - 2*e^x + I)/(e^(4*x) - 2*I*e^(3*x) - 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{sech}^2{(x)}}{\operatorname{csch}{(x)}+i}\,dx$$

input
```
integrate(sech(x)**2/(I+csch(x)),x)
```

output
```
Integral(sech(x)**2/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(13) = 26$.

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 4.26

$$\int \frac{\operatorname{sech}^2(x)}{i+\operatorname{csch}(x)}\,dx = \frac{8\,e^{(-x)}}{12i\,e^{(-x)}+12i\,e^{(-3\,x)}+6\,e^{(-4\,x)}-6} - \frac{12i\,e^{(-2\,x)}}{12i\,e^{(-x)}+12i\,e^{(-3\,x)}+6\,e^{(-4\,x)}-6} + \frac{4i}{12i\,e^{(-x)}+12i\,e^{(-3\,x)}+6\,e^{(-4\,x)}-6}$$

input
```
integrate(sech(x)^2/(I+csch(x)),x, algorithm="maxima")
```

output
```
8*e^(-x)/(12*I*e^(-x) + 12*I*e^(-3*x) + 6*e^(-4*x) - 6) - 12*I*e^(-2*x)/(1
2*I*e^(-x) + 12*I*e^(-3*x) + 6*e^(-4*x) - 6) + 4*I/(12*I*e^(-x) + 12*I*e^(
-3*x) + 6*e^(-4*x) - 6)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{\operatorname{sech}^2(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{i}{2\,(i\,e^x-1)}+\frac{3\,e^{(2\,x)}-1}{6\,(e^x-i)^3}$$

input
```
integrate(sech(x)^2/(I+csch(x)),x, algorithm="giac")
```

output
```
-1/2*I/(I*e^x - 1) + 1/6*(3*e^(2*x) - 1)/(e^x - I)^3
```

Mupad [B] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \frac{\operatorname{sech}^2(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{\frac{2}{3}-2\,\mathrm{e}^{2\,x}+\frac{\mathrm{e}^x\,4\mathrm{i}}{3}}{(\mathrm{e}^x+1\mathrm{i})\,\left(1+\mathrm{e}^x\,1\mathrm{i}\right)^3}$$

input
```
int(1/(cosh(x)^2*(1/sinh(x) + 1i)),x)
```

output
```
-((exp(x)*4i)/3 - 2*exp(2*x) + 2/3)/((exp(x) + 1i)*(exp(x)*1i + 1)^3)
```

Reduce [F]

$$\int \frac{\operatorname{sech}^2(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{sech}(x)^2}{\operatorname{csch}(x)+i}dx$$

input
```
int(sech(x)^2/(I+csch(x)),x)
```

output
```
int(sech(x)**2/(csch(x) + i),x)
```

3.90 $\int \frac{\operatorname{sech}^3(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{\operatorname{sech}^3(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{1}{8}i\arctan(\sinh(x)) - \frac{\operatorname{sech}^4(x)}{4} - \frac{1}{8}i\operatorname{sech}(x)\tanh(x) + \frac{1}{4}i\operatorname{sech}^3(x)\tanh(x)$$

output
```
-1/8*I*arctan(sinh(x))-1/4*sech(x)^4-1/8*I*sech(x)*tanh(x)+1/4*I*sech(x)^3
*tanh(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{sech}^3(x)}{i+\operatorname{csch}(x)}\,dx = \frac{1}{8}\left(-i\arctan(\sinh(x)) + \frac{1}{(-i+\sinh(x))^2} - \frac{i}{i+\sinh(x)}\right)$$

input
```
Integrate[Sech[x]^3/(I + Csch[x]),x]
```

output
```
((-I)*ArcTan[Sinh[x]] + (-I + Sinh[x])^(-2) - I/(I + Sinh[x]))/8
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.462, Rules used = {3042, 4360, 26, 3042, 26, 26, 3314, 25, 26, 3042, 25, 26, 3086, 15, 3091, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^3(x)}{\text{csch}(x)+i}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\cos(ix)^3(i\csc(ix)+i)}dx$$

$$\Big\downarrow \text{ 4360}$$

$$\int \frac{i\tanh(x)\text{sech}^2(x)}{-\sinh(x)+i}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\text{sech}^2(x)\tanh(x)}{i-\sinh(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$i\int -\frac{i\sin(ix)}{\cos(ix)^3(i\sin(ix)+i)}dx$$

$$\Big\downarrow \text{ 26}$$

$$\int -\frac{i\sin(ix)}{(1+\sin(ix))\cos(ix)^3}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\sin(ix)}{\cos(ix)^3(\sin(ix)+1)}dx$$

$$\Big\downarrow \text{ 3314}$$

$$-i\left(-\int -\text{sech}^3(x)\tanh^2(x)dx+\int i\text{sech}^4(x)\tanh(x)dx\right)$$

$$\Big\downarrow \text{ 25}$$

$$-i\left(\int \text{sech}^3(x)\tanh^2(x)dx+\int i\text{sech}^4(x)\tanh(x)dx\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\int \text{sech}^3(x)\tanh^2(x)dx+i\int \text{sech}^4(x)\tanh(x)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(i\int -i\sec(ix)^4\tan(ix)dx+\int -\sec(ix)^3\tan(ix)^2dx\right)$$

$$\Big\downarrow \text{25}$$

$$-i\left(i\int -i\sec(ix)^4\tan(ix)dx-\int \sec(ix)^3\tan(ix)^2dx\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\int \sec(ix)^4\tan(ix)dx-\int \sec(ix)^3\tan(ix)^2dx\right)$$

$$\Big\downarrow \text{3086}$$

$$-i\left(-i\int \text{sech}^3(x)d\text{sech}(x)-\int \sec(ix)^3\tan(ix)^2dx\right)$$

$$\Big\downarrow \text{15}$$

$$-i\left(-\int \sec(ix)^3\tan(ix)^2dx-\frac{1}{4}i\text{sech}^4(x)\right)$$

$$\Big\downarrow \text{3091}$$

$$-i\left(\frac{1}{4}\int \text{sech}^3(x)dx-\frac{1}{4}i\text{sech}^4(x)-\frac{1}{4}\tanh(x)\text{sech}^3(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{1}{4}\int \csc\left(ix+\frac{\pi}{2}\right)^3dx-\frac{1}{4}i\text{sech}^4(x)-\frac{1}{4}\tanh(x)\text{sech}^3(x)\right)$$

$$\Big\downarrow \text{4255}$$

$$-i\left(\frac{1}{4}\left(\frac{\int \text{sech}(x)dx}{2}+\frac{1}{2}\tanh(x)\text{sech}(x)\right)-\frac{1}{4}i\text{sech}^4(x)-\frac{1}{4}\tanh(x)\text{sech}^3(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{1}{4}\left(\frac{1}{2}\tanh(x)\text{sech}(x)+\frac{1}{2}\int \csc\left(ix+\frac{\pi}{2}\right)dx\right)-\frac{1}{4}i\text{sech}^4(x)-\frac{1}{4}\tanh(x)\text{sech}^3(x)\right)$$

$$\Big\downarrow \text{4257}$$

$$-i\left(\frac{1}{4}\left(\frac{1}{2}\arctan(\sinh(x))+\frac{1}{2}\tanh(x)\text{sech}(x)\right)-\frac{1}{4}i\text{sech}^4(x)-\frac{1}{4}\tanh(x)\text{sech}^3(x)\right)$$

input
```
Int[Sech[x]^3/(I + Csch[x]),x]
```

output
```
(-I)*((-1/4*I)*Sech[x]^4 - (Sech[x]^3*Tanh[x])/4 + (ArcTan[Sinh[x]]/2 + (S
ech[x]*Tanh[x])/2)/4)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3314
```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[Cos[e + f
*x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[1/(b*d)   Int[Cos[e + f*x]^(p
 - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &
& IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p
+ 1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(29) = 58.

Time = 9.92 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.52

method	result
risch	$-\frac{i e^{x}\left(-2 i e^{3 x}+e^{4 x}+2 i e^{x}-10 e^{2 x}+1\right)}{4\left(e^{x}-i\right)^{4}\left(e^{x}+i\right)^{2}}+\frac{\ln \left(e^{x}+i\right)}{8}-\frac{\ln \left(e^{x}-i\right)}{8}$
default	$\frac{i}{4 \tanh \left(\frac{x}{2}\right)+4 i}+\frac{1}{4\left(\tanh \left(\frac{x}{2}\right)+i\right)^{2}}+\frac{\ln \left(\tanh \left(\frac{x}{2}\right)+i\right)}{8}+\frac{i}{\left(\tanh \left(\frac{x}{2}\right)-i\right)^{3}}-\frac{i}{2\left(\tanh \left(\frac{x}{2}\right)-i\right)}-\frac{1}{2\left(\tanh \left(\frac{x}{2}\right)-i\right)^{4}}+\frac{1}{\left(\tanh \left(\frac{x}{2}\right)-\right.}$

input
```
int(sech(x)^3/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*exp(x)*(-2*I*exp(x)^3+exp(x)^4+2*I*exp(x)-10*exp(x)^2+1)/(exp(x)-I)
^4/(exp(x)+I)^2+1/8*ln(exp(x)+I)-1/8*ln(exp(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(26) = 52.

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.55

$$\int \frac{\operatorname{sech}^3(x)}{i+\operatorname{csch}(x)}\,dx = \frac{\left(e^{(6x)}-2i\,e^{(5x)}+e^{(4x)}-4i\,e^{(3x)}-e^{(2x)}-2i\,e^{x}-1\right)\log\left(e^{x}+i\right)-\left(e^{(6x)}-2i\,e^{(5x)}+e^{(4x)}-4i\,e^{(3x)}-\cdots\right.}{8\left(e^{(6x)}-2i\,e^{(5x)}+e^{(4x)}-4i\,e^{(3x)}-e^{(2x}\cdots\right.}$$

input
```
integrate(sech(x)^3/(I+csch(x)),x, algorithm="fricas")
```

output
```
1/8*((e^(6*x) - 2*I*e^(5*x) + e^(4*x) - 4*I*e^(3*x) - e^(2*x) - 2*I*e^x -
1)*log(e^x + I) - (e^(6*x) - 2*I*e^(5*x) + e^(4*x) - 4*I*e^(3*x) - e^(2*x)
 - 2*I*e^x - 1)*log(e^x - I) - 2*I*e^(5*x) - 4*e^(4*x) + 20*I*e^(3*x) + 4*
e^(2*x) - 2*I*e^x)/(e^(6*x) - 2*I*e^(5*x) + e^(4*x) - 4*I*e^(3*x) - e^(2*x
) - 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{i+\operatorname{csch}(x)}\,dx=\int \frac{\operatorname{sech}^3(x)}{\operatorname{csch}(x)+i}\,dx$$

input
```
integrate(sech(x)**3/(I+csch(x)),x)
```

output
```
Integral(sech(x)**3/(csch(x) + I), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^3(x)}{i+\operatorname{csch}(x)}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(sech(x)^3/(I+csch(x)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(26) = 52$.

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.35

$$\int \frac{\operatorname{sech}^3(x)}{i+\operatorname{csch}(x)}\,dx=-\frac{-i\,e^{(-x)}+i\,e^{x}-6}{16\,\left(-i\,e^{(-x)}+i\,e^{x}-2\right)}+\frac{3\,\left(e^{(-x)}-e^{x}\right)^{2}+12i\,e^{(-x)}-12i\,e^{x}+4}{32\,\left(e^{(-x)}-e^{x}+2i\right)^{2}}+\frac{1}{16}\log\left(-e^{(-x)}+e^{x}+2i\right)-\frac{1}{16}\log\left(-e^{(-x)}+e^{x}-2i\right)$$

input
```
integrate(sech(x)^3/(I+csch(x)),x, algorithm="giac")
```

output
```
-1/16*(-I*e^(-x) + I*e^x - 6)/(-I*e^(-x) + I*e^x - 2) + 1/32*(3*(e^(-x) -
e^x)^2 + 12*I*e^(-x) - 12*I*e^x + 4)/(e^(-x) - e^x + 2*I)^2 + 1/16*log(-e^
(-x) + e^x + 2*I) - 1/16*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 3.21 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.05

$$\int \frac{\operatorname{sech}^3(x)}{i+\operatorname{csch}(x)}\,dx = \frac{\ln\left(-\frac{1}{4}+\frac{e^x\,1i}{4}\right)}{8} - \frac{\ln\left(\frac{1}{4}+\frac{e^x\,1i}{4}\right)}{8} - \frac{1i}{e^{2x}\,3i-e^{3x}+3\,e^x-i} - \frac{1}{4\,\left(e^{2x}-1+e^x\,2i\right)} - \frac{1}{2\,\left(e^{4x}-6\,e^{2x}+1-e^{3x}\,4i+e^x\,4i\right)} - \frac{1}{2\,\left(1-e^{2x}+e^x\,2i\right)} - \frac{1i}{4\,\left(e^x+1i\right)}$$

input
```
int(1/(cosh(x)^3*(1/sinh(x) + 1i)),x)
```

output
```
log((exp(x)*1i)/4 - 1/4)/8 - log((exp(x)*1i)/4 + 1/4)/8 - 1i/(exp(2*x)*3i
- exp(3*x) + 3*exp(x) - 1i) - 1/(4*(exp(2*x) + exp(x)*2i - 1)) - 1/(2*(exp
(4*x) - exp(3*x)*4i - 6*exp(2*x) + exp(x)*4i + 1)) - 1/(2*(exp(x)*2i - exp
(2*x) + 1)) - 1i/(4*(exp(x) + 1i))
```

Reduce [F]

$$\int \frac{\operatorname{sech}^3(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{sech}(x)^3}{\operatorname{csch}(x)+i}dx$$

input
```
int(sech(x)^3/(I+csch(x)),x)
```

output
```
int(sech(x)**3/(csch(x) + i),x)
```

3.91 $\int \frac{\operatorname{sech}^4(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \frac{\operatorname{sech}^4(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{1}{5}\operatorname{sech}^5(x)-\frac{1}{3}i\tanh^3(x)+\frac{1}{5}i\tanh^5(x)$$

output
```
-1/5*sech(x)^5-1/3*I*tanh(x)^3+1/5*I*tanh(x)^5
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 96 vs. 2(29) = 58.

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 3.31

$$\int \frac{\operatorname{sech}^4(x)}{i+\operatorname{csch}(x)}\,dx = \frac{-240+54\cosh(x)+32\cosh(2x)+18\cosh(3x)+16\cosh(4x)-96i\sinh(x)+18i\sinh(2x)-32i\sinh(3\ldots}{960\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^3\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^5}$$

input
```
Integrate[Sech[x]^4/(I + Csch[x]),x]
```

output
```
(-240 + 54*Cosh[x] + 32*Cosh[2*x] + 18*Cosh[3*x] + 16*Cosh[4*x] - (96*I)*S
inh[x] + (18*I)*Sinh[2*x] - (32*I)*Sinh[3*x] + (9*I)*Sinh[4*x])/(960*(Cosh
[x/2] - I*Sinh[x/2])^3*(Cosh[x/2] + I*Sinh[x/2])^5)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.308$, Rules used = {3042, 4360, 26, 3042, 26, 26, 3318, 25, 26, 3042, 25, 26, 3086, 15, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^4(x)}{\text{csch}(x)+i} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ix)^4(i\csc(ix)+i)}dx$$

$$\Big\downarrow \text{4360}$$

$$\int \frac{i\tanh(x)\text{sech}^3(x)}{-\sinh(x)+i}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\text{sech}^3(x)\tanh(x)}{i-\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$i\int -\frac{i\sin(ix)}{\cos(ix)^4(i\sin(ix)+i)}dx$$

$$\Big\downarrow \text{26}$$

$$\int -\frac{i\sin(ix)}{(1+\sin(ix))\cos(ix)^4}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ix)}{\cos(ix)^4(\sin(ix)+1)}dx$$

$$\Big\downarrow \text{3318}$$

$$-i\left(-\int -\text{sech}^4(x)\tanh^2(x)dx+\int i\text{sech}^5(x)\tanh(x)dx\right)$$

$$\Big\downarrow \text{25}$$

$$-i\left(\int \text{sech}^4(x)\tanh^2(x)dx+\int i\text{sech}^5(x)\tanh(x)dx\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\int \text{sech}^4(x)\tanh^2(x)dx+i\int \text{sech}^5(x)\tanh(x)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(i\int -i\sec(ix)^5\tan(ix)dx+\int -\sec(ix)^4\tan(ix)^2dx\right)$$

$$\Big\downarrow \text{25}$$

$$-i\left(i\int -i\sec(ix)^5\tan(ix)dx-\int \sec(ix)^4\tan(ix)^2dx\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\int \sec(ix)^5\tan(ix)dx-\int \sec(ix)^4\tan(ix)^2dx\right)$$

$$\Big\downarrow \text{3086}$$

$$-i\left(-i\int \text{sech}^4(x)d\text{sech}(x)-\int \sec(ix)^4\tan(ix)^2dx\right)$$

$$\Big\downarrow \text{15}$$

$$-i\left(-\int \sec(ix)^4\tan(ix)^2dx-\frac{1}{5}i\text{sech}^5(x)\right)$$

$$\Big\downarrow \text{3087}$$

$$-i\left(i\int -\tanh^2(x)\left(1-\tanh^2(x)\right)d(i\tanh(x))-\frac{1}{5}i\text{sech}^5(x)\right)$$

$$\Big\downarrow \text{244}$$

$$-i\left(i\int \left(\tanh^4(x)-\tanh^2(x)\right)d(i\tanh(x))-\frac{1}{5}i\text{sech}^5(x)\right)$$

$$\Big\downarrow \text{2009}$$

$$-i\left(i\left(\frac{1}{5}i\tanh^5(x)-\frac{1}{3}i\tanh^3(x)\right)-\frac{1}{5}i\text{sech}^5(x)\right)$$

input
```
Int[Sech[x]^4/(I + Csch[x]),x]
```

output
```
(-I)*((-1/5*I)*Sech[x]^5 + I*((-1/3*I)*Tanh[x]^3 + (I/5)*Tanh[x]^5))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(21) = 42$.

Time = 291.46 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.52

method	result
risch	$\frac{4i\left(-6i\,\mathrm{e}^{3x}+15\,\mathrm{e}^{4x}-2i\,\mathrm{e}^{x}-2\,\mathrm{e}^{2x}-1\right)}{15\left(\mathrm{e}^{x}+i\right)^{3}\left(\mathrm{e}^{x}-i\right)^{5}}$
default	$\frac{i}{6\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{3i}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)}-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{4i}{3\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}}+\frac{3i}{8\left(\tanh\left(\frac{x}{2}\right)-i\right)}+\frac{2i}{5\left(\tanh\left(\frac{x}{2}\right)-i\right)^{5}}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right.\right.}$

input
```
int(sech(x)^4/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
4/15*I*(-6*I*exp(x)^3+15*exp(x)^4-2*I*exp(x)-2*exp(x)^2-1)/(exp(x)+I)^3/(e
xp(x)-I)^5
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(19) = 38$.

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.34

$$\int \frac{\operatorname{sech}^4(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{4\left(-15i\,e^{(4\,x)}-6\,e^{(3\,x)}+2i\,e^{(2\,x)}-2\,e^{x}+i\right)}{15\left(e^{(8\,x)}-2i\,e^{(7\,x)}+2\,e^{(6\,x)}-6i\,e^{(5\,x)}-6i\,e^{(3\,x)}-2\,e^{(2\,x)}-2i\,e^{x}-1\right)}$$

input
```
integrate(sech(x)^4/(I+csch(x)),x, algorithm="fricas")
```

output
```
-4/15*(-15*I*e^(4*x) - 6*e^(3*x) + 2*I*e^(2*x) - 2*e^x + I)/(e^(8*x) - 2*I
*e^(7*x) + 2*e^(6*x) - 6*I*e^(5*x) - 6*I*e^(3*x) - 2*e^(2*x) - 2*I*e^x - 1
)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{sech}^4(x)}{\operatorname{csch}(x)+i}\,dx$$

input
```
integrate(sech(x)**4/(I+csch(x)),x)
```

output
```
Integral(sech(x)**4/(csch(x) + I), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^4(x)}{i+\operatorname{csch}(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sech(x)^4/(I+csch(x)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(19) = 38$.

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.90

$$\int \frac{\operatorname{sech}^4(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{-3i\,e^{(2\,x)}+12\,e^x+5i}{24\,(i\,e^x-1)^3}+\frac{15\,e^{(4\,x)}-60i\,e^{(3\,x)}-10\,e^{(2\,x)}+20i\,e^x+7}{120\,(e^x-i)^5}$$

input
```
integrate(sech(x)^4/(I+csch(x)),x, algorithm="giac")
```

output
```
-1/24*(-3*I*e^(2*x) + 12*e^x + 5*I)/(I*e^x - 1)^3 + 1/120*(15*e^(4*x) - 60
*I*e^(3*x) - 10*e^(2*x) + 20*I*e^x + 7)/(e^x - I)^5
```

Mupad [B] (verification not implemented)

Time = 3.31 (sec) , antiderivative size = 207, normalized size of antiderivative = 7.14

$$\int \frac{\operatorname{sech}^4(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{1\mathrm{i}}{4\,\left(\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}\right)}+\frac{1}{20\,\left(\mathrm{e}^{x}-\mathrm{i}\right)} -\frac{1}{8\,\left(\mathrm{e}^{x}+1\mathrm{i}\right)}+\frac{\frac{\mathrm{e}^{3\,x}}{40}-\frac{\mathrm{e}^{2\,x}\,3\mathrm{i}}{40}+\frac{\mathrm{e}^{x}}{8}+\frac{1}{40}\mathrm{i}}{\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1-\mathrm{e}^{3\,x}\,4\mathrm{i}+\mathrm{e}^{x}\,4\mathrm{i}} -\frac{\frac{\mathrm{e}^{2\,x}}{40}+\frac{1}{24}-\frac{\mathrm{e}^{x}\,1\mathrm{i}}{20}}{\mathrm{e}^{2\,x}\,3\mathrm{i}-\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}-\mathrm{i}}-\frac{1}{6\,\left(\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^{x}-\mathrm{i}\right)} +\frac{\frac{\mathrm{e}^{2\,x}}{4}+\frac{\mathrm{e}^{4\,x}}{40}+\frac{1}{40}-\frac{\mathrm{e}^{3\,x}\,1\mathrm{i}}{10}+\frac{\mathrm{e}^{x}\,1\mathrm{i}}{10}}{\mathrm{e}^{2\,x}\,10\mathrm{i}-10\,\mathrm{e}^{3\,x}-\mathrm{e}^{4\,x}\,5\mathrm{i}+\mathrm{e}^{5\,x}+5\,\mathrm{e}^{x}-\mathrm{i}}$$

input
```
int(1/(cosh(x)^4*(1/sinh(x) + 1i)),x)
```

output
```
1/(20*(exp(x) - 1i)) - 1i/(4*(exp(2*x) + exp(x)*2i - 1)) - 1/(8*(exp(x) +
1i)) + (exp(3*x)/40 - (exp(2*x)*3i)/40 + exp(x)/8 + 1i/40)/(exp(4*x) - exp
(3*x)*4i - 6*exp(2*x) + exp(x)*4i + 1) - (exp(2*x)/40 - (exp(x)*1i)/20 + 1
/24)/(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) - 1/(6*(exp(2*x)*3i + exp(3*
x) - 3*exp(x) - 1i)) + (exp(2*x)/4 - (exp(3*x)*1i)/10 + exp(4*x)/40 + (exp
(x)*1i)/10 + 1/40)/(exp(2*x)*10i - 10*exp(3*x) - exp(4*x)*5i + exp(5*x) +
5*exp(x) - 1i)
```

Reduce [F]

$$\int \frac{\operatorname{sech}^4(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{sech}(x)^4}{\operatorname{csch}(x)+i}dx$$

input
```
int(sech(x)^4/(I+csch(x)),x)
```

output
```
int(sech(x)**4/(csch(x) + i),x)
```

3.92 $\int \frac{\cosh^5(x)}{a+b\text{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 102

$$\int \frac{\cosh^5(x)}{a+b\text{csch}(x)}\,dx = -\frac{b(a^2+b^2)^2\log(b+a\sinh(x))}{a^6} + \frac{(a^2+b^2)^2\sinh(x)}{a^5} - \frac{b(2a^2+b^2)\sinh^2(x)}{2a^4} + \frac{(2a^2+b^2)\sinh^3(x)}{3a^3} - \frac{b\sinh^4(x)}{4a^2} + \frac{\sinh^5(x)}{5a}$$

output
```
-b*(a^2+b^2)^2*ln(b+a*sinh(x))/a^6+(a^2+b^2)^2*sinh(x)/a^5-1/2*b*(2*a^2+b^
2)*sinh(x)^2/a^4+1/3*(2*a^2+b^2)*sinh(x)^3/a^3-1/4*b*sinh(x)^4/a^2+1/5*sin
h(x)^5/a
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.95

$$\int \frac{\cosh^5(x)}{a+b\text{csch}(x)}\,dx = \frac{-60b(a^2+b^2)^2\log(b+a\sinh(x)) + 60a(a^2+b^2)^2\sinh(x) - 30a^2b(2a^2+b^2)\sinh^2(x) + 20a^3(2a^2+b^2)\,\mathrm{s}}{60a^6}$$

input
```
Integrate[Cosh[x]^5/(a + b*Csch[x]),x]
```

output
```
(-60*b*(a^2 + b^2)^2*Log[b + a*Sinh[x]] + 60*a*(a^2 + b^2)^2*Sinh[x] - 30*
a^2*b*(2*a^2 + b^2)*Sinh[x]^2 + 20*a^3*(2*a^2 + b^2)*Sinh[x]^3 - 15*a^4*b*
Sinh[x]^4 + 12*a^5*Sinh[x]^5)/(60*a^6)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {3042, 4360, 26, 26, 3042, 26, 3316, 26, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^5(x)}{a+b\text{csch}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^5}{a+ib\csc(ix)}dx$$

$$\Big\downarrow \text{4360}$$

$$\int \frac{i\sinh(x)\cosh^5(x)}{ia\sinh(x)+ib}dx$$

$$\Big\downarrow \text{26}$$

$$i\int -\frac{i\cosh^5(x)\sinh(x)}{b+a\sinh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\sinh(x)\cosh^5(x)}{a\sinh(x)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)\cos(ix)^5}{b-ia\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ix)^5\sin(ix)}{b-ia\sin(ix)}dx$$

$\big\downarrow$ 3316

$$-\frac{i\int \frac{i\sinh(x)\left(\sinh^2(x)a^2+a^2\right)^2}{b+a\sinh(x)}d(a\sinh(x))}{a^5}$$

$\big\downarrow$ 26

$$\frac{\int \frac{\sinh(x)\left(\sinh^2(x)a^2+a^2\right)^2}{b+a\sinh(x)}d(a\sinh(x))}{a^5}$$

$\big\downarrow$ 27

$$\frac{\int \frac{a\sinh(x)\left(\sinh^2(x)a^2+a^2\right)^2}{b+a\sinh(x)}d(a\sinh(x))}{a^6}$$

$\big\downarrow$ 522

$$\frac{\int \left(a^4\sinh^4(x)-a^3b\sinh^3(x)+a^2\left(2a^2+b^2\right)\sinh^2(x)-ab\left(2a^2+b^2\right)\sinh(x)+\left(a^2+b^2\right)^2-\frac{b\left(a^2+b^2\right)^2}{b+a\sinh(x)}\right)d(a\sin}{a^6}$$

$\big\downarrow$ 2009

$$\frac{\frac{1}{5}a^5\sinh^5(x)-\frac{1}{4}a^4b\sinh^4(x)-\frac{1}{2}a^2b\left(2a^2+b^2\right)\sinh^2(x)+a\left(a^2+b^2\right)^2\sinh(x)-b\left(a^2+b^2\right)^2\log(a\sinh(x)+b)+}{a^6}$$

input
```
Int[Cosh[x]^5/(a + b*Csch[x]),x]
```

output
```
(-(b*(a^2 + b^2)^2*Log[b + a*Sinh[x]]) + a*(a^2 + b^2)^2*Sinh[x] - (a^2*b*
(2*a^2 + b^2)*Sinh[x]^2)/2 + (a^3*(2*a^2 + b^2)*Sinh[x]^3)/3 - (a^4*b*Sinh
[x]^4)/4 + (a^5*Sinh[x]^5)/5)/a^6
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. 2(94) = 188.

Time = 0.08 (sec) , antiderivative size = 378, normalized size of antiderivative = 3.71

$$-\frac{1}{5a\left(\tanh\left(\frac{x}{2}\right)+1\right)^{5}}-\frac{-2a+b}{4a^{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{4}}-\frac{11a^{2}-6ab+4b^{2}}{12a^{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{-7a^{3}+9a^{2}b-4a\,b^{2}+4b^{3}}{8a^{4}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{8a^{4}-7a}{8}$$

input
```
int(cosh(x)^5/(a+b*csch(x)),x)
```

output
```
-1/5/a/(tanh(1/2*x)+1)^5-1/4*(-2*a+b)/a^2/(tanh(1/2*x)+1)^4-1/12*(11*a^2-6
*a*b+4*b^2)/a^3/(tanh(1/2*x)+1)^3-1/8*(-7*a^3+9*a^2*b-4*a*b^2+4*b^3)/a^4/(
tanh(1/2*x)+1)^2-1/8*(8*a^4-7*a^3*b+16*a^2*b^2-4*a*b^3+8*b^4)/a^5/(tanh(1/
2*x)+1)+b*(a^4+2*a^2*b^2+b^4)/a^6*ln(tanh(1/2*x)+1)-1/5/a/(tanh(1/2*x)-1)^
5-1/4*(2*a+b)/a^2/(tanh(1/2*x)-1)^4-1/12*(11*a^2+6*a*b+4*b^2)/a^3/(tanh(1/
2*x)-1)^3-1/8*(7*a^3+9*a^2*b+4*a*b^2+4*b^3)/a^4/(tanh(1/2*x)-1)^2-1/8*(8*a
^4+7*a^3*b+16*a^2*b^2+4*a*b^3+8*b^4)/a^5/(tanh(1/2*x)-1)+b*(a^4+2*a^2*b^2+
b^4)/a^6*ln(tanh(1/2*x)-1)-2*b/a^6*(1/2*a^4+a^2*b^2+1/2*b^4)*ln(b*tanh(1/2
*x)^2-2*a*tanh(1/2*x)-b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1398 vs. 2(94) = 188.

Time = 0.11 (sec) , antiderivative size = 1398, normalized size of antiderivative = 13.71

$$\int \frac{\cosh^5(x)}{a+b\operatorname{csch}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^5/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
1/960*(6*a^5*cosh(x)^10 + 6*a^5*sinh(x)^10 - 15*a^4*b*cosh(x)^9 + 15*(4*a^
5*cosh(x) - a^4*b)*sinh(x)^9 + 10*(5*a^5 + 4*a^3*b^2)*cosh(x)^8 + 5*(54*a^
5*cosh(x)^2 - 27*a^4*b*cosh(x) + 10*a^5 + 8*a^3*b^2)*sinh(x)^8 - 60*(3*a^4
*b + 2*a^2*b^3)*cosh(x)^7 + 20*(36*a^5*cosh(x)^3 - 27*a^4*b*cosh(x)^2 - 9*
a^4*b - 6*a^2*b^3 + 4*(5*a^5 + 4*a^3*b^2)*cosh(x))*sinh(x)^7 + 960*(a^4*b
+ 2*a^2*b^3 + b^5)*x*cosh(x)^5 + 60*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)
^6 + 20*(63*a^5*cosh(x)^4 - 63*a^4*b*cosh(x)^3 + 15*a^5 + 42*a^3*b^2 + 24*
a*b^4 + 14*(5*a^5 + 4*a^3*b^2)*cosh(x)^2 - 21*(3*a^4*b + 2*a^2*b^3)*cosh(x
))*sinh(x)^6 - 15*a^4*b*cosh(x) + 2*(756*a^5*cosh(x)^5 - 945*a^4*b*cosh(x)
^4 + 280*(5*a^5 + 4*a^3*b^2)*cosh(x)^3 - 630*(3*a^4*b + 2*a^2*b^3)*cosh(x)
^2 + 480*(a^4*b + 2*a^2*b^3 + b^5)*x + 180*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*
cosh(x))*sinh(x)^5 - 6*a^5 - 60*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^4 +
 10*(126*a^5*cosh(x)^6 - 189*a^4*b*cosh(x)^5 - 30*a^5 - 84*a^3*b^2 - 48*a*
b^4 + 70*(5*a^5 + 4*a^3*b^2)*cosh(x)^4 - 210*(3*a^4*b + 2*a^2*b^3)*cosh(x)
^3 + 480*(a^4*b + 2*a^2*b^3 + b^5)*x*cosh(x) + 90*(5*a^5 + 14*a^3*b^2 + 8*
a*b^4)*cosh(x)^2)*sinh(x)^4 - 60*(3*a^4*b + 2*a^2*b^3)*cosh(x)^3 + 20*(36*
a^5*cosh(x)^7 - 63*a^4*b*cosh(x)^6 + 28*(5*a^5 + 4*a^3*b^2)*cosh(x)^5 - 9*
a^4*b - 6*a^2*b^3 - 105*(3*a^4*b + 2*a^2*b^3)*cosh(x)^4 + 480*(a^4*b + 2*a
^2*b^3 + b^5)*x*cosh(x)^2 + 60*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^3 -
12*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x))*sinh(x)^3 - 10*(5*a^5 + 4*a^...
```

Sympy [F]

$$\int \frac{\cosh^5(x)}{a + b\text{csch}(x)}\,dx = \int \frac{\cosh^5{(x)}}{a + b\,\text{csch}\,(x)}\,dx$$

input
```
integrate(cosh(x)**5/(a+b*csch(x)),x)
```

output
```
Integral(cosh(x)**5/(a + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(94) = 188$.

Time = 0.04 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.37

$$\int \frac{\cosh^5(x)}{a + b\text{csch}(x)}\,dx = -\frac{\left(15\,a^3be^{(-x)} - 6\,a^4 - 10\,(5\,a^4 + 4\,a^2b^2)e^{(-2\,x)} + 60\,(3\,a^3b + 2\,ab^3)e^{(-3\,x)} - 60\,(5\,a^4 + 14\,a^2b^2 + 8\,b^4)e^{(-}}{960\,a^5} - \frac{15\,a^3be^{(-4\,x)} + 6\,a^4e^{(-5\,x)} + 60\,(5\,a^4 + 14\,a^2b^2 + 8\,b^4)e^{(-x)} + 60\,(3\,a^3b + 2\,ab^3)e^{(-2\,x)} + 10\,(5\,a^4 + 4\,a^2b^2}{960\,a^5} - \frac{(a^4b + 2\,a^2b^3 + b^5)x}{a^6} - \frac{(a^4b + 2\,a^2b^3 + b^5)\log\left(-2\,be^{(-x)} + ae^{(-2\,x)} - a\right)}{a^6}$$

input
```
integrate(cosh(x)^5/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
-1/960*(15*a^3*b*e^(-x) - 6*a^4 - 10*(5*a^4 + 4*a^2*b^2)*e^(-2*x) + 60*(3*
a^3*b + 2*a*b^3)*e^(-3*x) - 60*(5*a^4 + 14*a^2*b^2 + 8*b^4)*e^(-4*x))*e^(5
*x)/a^5 - 1/960*(15*a^3*b*e^(-4*x) + 6*a^4*e^(-5*x) + 60*(5*a^4 + 14*a^2*b
^2 + 8*b^4)*e^(-x) + 60*(3*a^3*b + 2*a*b^3)*e^(-2*x) + 10*(5*a^4 + 4*a^2*b
^2)*e^(-3*x))/a^5 - (a^4*b + 2*a^2*b^3 + b^5)*x/a^6 - (a^4*b + 2*a^2*b^3 +
 b^5)*log(-2*b*e^(-x) + a*e^(-2*x) - a)/a^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(94) = 188$.

Time = 0.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.90

$$\int \frac{\cosh^5(x)}{a + b\text{csch}(x)}\,dx = -\frac{6\,a^4\left(e^{(-x)} - e^x\right)^5 + 15\,a^3b\left(e^{(-x)} - e^x\right)^4 + 80\,a^4\left(e^{(-x)} - e^x\right)^3 + 40\,a^2b^2\left(e^{(-x)} - e^x\right)^3 + 240\,a^3b\left(e^{(-x)} -\right.}{960\,a^5} - \frac{(a^4b + 2\,a^2b^3 + b^5)\log\left(\left|-a\left(e^{(-x)} - e^x\right) + 2\,b\right|\right)}{a^6}$$

input
```
integrate(cosh(x)^5/(a+b*csch(x)),x, algorithm="giac")
```

output
```
-1/960*(6*a^4*(e^(-x) - e^x)^5 + 15*a^3*b*(e^(-x) - e^x)^4 + 80*a^4*(e^(-x
) - e^x)^3 + 40*a^2*b^2*(e^(-x) - e^x)^3 + 240*a^3*b*(e^(-x) - e^x)^2 + 12
0*a*b^3*(e^(-x) - e^x)^2 + 480*a^4*(e^(-x) - e^x) + 960*a^2*b^2*(e^(-x) -
e^x) + 480*b^4*(e^(-x) - e^x))/a^5 - (a^4*b + 2*a^2*b^3 + b^5)*log(abs(-a*
(e^(-x) - e^x) + 2*b))/a^6
```

Mupad [B] (verification not implemented)

Time = 3.70 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.24

$$\int \frac{\cosh^5(x)}{a+b\text{csch}(x)}\,dx = \frac{e^{5x}}{160\,a}-\frac{e^{-5x}}{160\,a}-\frac{e^{-2x}\,(3\,a^2\,b+2\,b^3)}{16\,a^4}-\frac{e^{2x}\,(3\,a^2\,b+2\,b^3)}{16\,a^4}+\frac{e^{x}\,(5\,a^4+14\,a^2\,b^2+8\,b^4)}{16\,a^5}-\frac{b\,e^{-4x}}{64\,a^2}-\frac{b\,e^{4x}}{64\,a^2}-\frac{\ln\left(2\,b\,e^{x}-a+a\,e^{2x}\right)\,(a^4\,b+2\,a^2\,b^3+b^5)}{a^6}-\frac{e^{-x}\,(5\,a^4+14\,a^2\,b^2+8\,b^4)}{16\,a^5}-\frac{e^{-3x}\,(5\,a^2+4\,b^2)}{96\,a^3}+\frac{e^{3x}\,(5\,a^2+4\,b^2)}{96\,a^3}+\frac{b\,x\,(a^2+b^2)^2}{a^6}$$

input
```
int(cosh(x)^5/(a + b/sinh(x)),x)
```

output
```
exp(5*x)/(160*a) - exp(-5*x)/(160*a) - (exp(-2*x)*(3*a^2*b + 2*b^3))/(16*a
^4) - (exp(2*x)*(3*a^2*b + 2*b^3))/(16*a^4) + (exp(x)*(5*a^4 + 8*b^4 + 14*
a^2*b^2))/(16*a^5) - (b*exp(-4*x))/(64*a^2) - (b*exp(4*x))/(64*a^2) - (log
(2*b*exp(x) - a + a*exp(2*x))*(a^4*b + b^5 + 2*a^2*b^3))/a^6 - (exp(-x)*(5
*a^4 + 8*b^4 + 14*a^2*b^2))/(16*a^5) - (exp(-3*x)*(5*a^2 + 4*b^2))/(96*a^3
) + (exp(3*x)*(5*a^2 + 4*b^2))/(96*a^3) + (b*x*(a^2 + b^2)^2)/a^6
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 335, normalized size of antiderivative = 3.28

$$\int \frac{\cosh^5(x)}{a+b\text{csch}(x)}\,dx$$
$$= \frac{6e^{10x}a^5 - 15e^{9x}a^4b + 50e^{8x}a^5 + 40e^{8x}a^3b^2 - 180e^{7x}a^4b - 120e^{7x}a^2b^3 + 300e^{6x}a^5 + 840e^{6x}a^3b^2 + 480e^{6x}a}{}$$

input
```
int(cosh(x)^5/(a+b*csch(x)),x)
```

output
```
(6*e**(10*x)*a**5 - 15*e**(9*x)*a**4*b + 50*e**(8*x)*a**5 + 40*e**(8*x)*a*
*3*b**2 - 180*e**(7*x)*a**4*b - 120*e**(7*x)*a**2*b**3 + 300*e**(6*x)*a**5
 + 840*e**(6*x)*a**3*b**2 + 480*e**(6*x)*a*b**4 - 960*e**(5*x)*log(e**(2*x
)*a + 2*e**x*b - a)*a**4*b - 1920*e**(5*x)*log(e**(2*x)*a + 2*e**x*b - a)*
a**2*b**3 - 960*e**(5*x)*log(e**(2*x)*a + 2*e**x*b - a)*b**5 + 960*e**(5*x
)*a**4*b*x + 1920*e**(5*x)*a**2*b**3*x + 960*e**(5*x)*b**5*x - 300*e**(4*x
)*a**5 - 840*e**(4*x)*a**3*b**2 - 480*e**(4*x)*a*b**4 - 180*e**(3*x)*a**4*
b - 120*e**(3*x)*a**2*b**3 - 50*e**(2*x)*a**5 - 40*e**(2*x)*a**3*b**2 - 15
*e**x*a**4*b - 6*a**5)/(960*e**(5*x)*a**6)
```

3.93 $\int \frac{\cosh^4(x)}{a+b\,\mathbf{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 125

$$\int \frac{\cosh^4(x)}{a+b\text{csch}(x)}\,dx = \frac{(3a^4+12a^2b^2+8b^4)\,x}{8a^5} + \frac{2b(a^2+b^2)^{3/2}\,\text{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5} - \frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2} - \frac{\cosh(x)\left(8b(a^2+b^2)-a(3a^2+4b^2)\sinh(x)\right)}{8a^4}$$

output
```
1/8*(3*a^4+12*a^2*b^2+8*b^4)*x/a^5+2*b*(a^2+b^2)^(3/2)*arctanh((a-b*tanh(1
/2*x))/(a^2+b^2)^(1/2))/a^5-1/12*cosh(x)^3*(4*b-3*a*sinh(x))/a^2-1/8*cosh(
x)*(8*b*(a^2+b^2)-a*(3*a^2+4*b^2)*sinh(x))/a^4
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.44

$$\int \frac{\cosh^4(x)}{a+b\text{csch}(x)}\,dx = \frac{-24ab(5a^2+4b^2)\cosh(x) - 8a^3b\cosh(3x) + 3\left(12a^4x+48a^2b^2x+32b^4x+64a^2b\sqrt{-a^2-b^2}\,\arctan\left(\frac{a-\cdots}{\sqrt{\cdots}}\right.\right.}{96a^5}$$

input
```
Integrate[Cosh[x]^4/(a + b*Csch[x]),x]
```

output
```
(-24*a*b*(5*a^2 + 4*b^2)*Cosh[x] - 8*a^3*b*Cosh[3*x] + 3*(12*a^4*x + 48*a^
2*b^2*x + 32*b^4*x + 64*a^2*b*Sqrt[-a^2 - b^2]*ArcTan[(a - b*Tanh[x/2])/Sq
rt[-a^2 - b^2]] + 64*b^3*Sqrt[-a^2 - b^2]*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a
^2 - b^2]] + 8*a^2*(a^2 + b^2)*Sinh[2*x] + a^4*Sinh[4*x]))/(96*a^5)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.22, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.308$, Rules used = {3042, 4360, 26, 26, 3042, 26, 3344, 26, 3042, 3344, 25, 3042, 3214, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(x)}{a+b\text{csch}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^4}{a+ib\csc(ix)}dx$$

$$\Big\downarrow \text{4360}$$

$$\int \frac{i\sinh(x)\cosh^4(x)}{ia\sinh(x)+ib}dx$$

$$\Big\downarrow \text{26}$$

$$i\int -\frac{i\cosh^4(x)\sinh(x)}{b+a\sinh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\sinh(x)\cosh^4(x)}{a\sinh(x)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)\cos(ix)^4}{b-ia\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ix)^4\sin(ix)}{b-ia\sin(ix)}dx$$

$$\Big\downarrow \text{3344}$$

$$-i\left(-\frac{\int \frac{i\cosh^2(x)\left(ab-\left(3a^2+4b^2\right)\sinh(x)\right)}{b+a\sinh(x)}dx}{4a^2}-\frac{i\cosh^3(x)(4b-3a\sinh(x))}{12a^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{i\int \frac{\cosh^2(x)\left(ab-\left(3a^2+4b^2\right)\sinh(x)\right)}{b+a\sinh(x)}dx}{4a^2}-\frac{i\cosh^3(x)(4b-3a\sinh(x))}{12a^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{i\int \frac{\cos(ix)^2\left(ab+i\left(3a^2+4b^2\right)\sin(ix)\right)}{b-ia\sin(ix)}dx}{4a^2}-\frac{i\cosh^3(x)(4b-3a\sinh(x))}{12a^2}\right)$$

$$\Big\downarrow \text{3344}$$

$$-i\left(-\frac{i\left(\frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+4b^2\right)\sinh(x)\right)}{2a^2}-\frac{\int -\frac{ab\left(5a^2+4b^2\right)-\left(3a^4+12b^2a^2+8b^4\right)\sinh(x)}{b+a\sinh(x)}dx}{2a^2}\right)}{4a^2}-\frac{i\cosh^3(x)(4b-3a\sinh(x))}{12a^2}\right)$$

$$\Big\downarrow \text{25}$$

$$-i\left(-\frac{i\left(\frac{\int \frac{ab\left(5a^2+4b^2\right)-\left(3a^4+12b^2a^2+8b^4\right)\sinh(x)}{b+a\sinh(x)}dx}{2a^2}+\frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+4b^2\right)\sinh(x)\right)}{2a^2}\right)}{4a^2}-\frac{i\cosh^3(x)(4b-3a\sinh(x))}{12a^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{i\left(\frac{\cosh(x)\left(8b(a^2+b^2)-a(3a^2+4b^2)\sinh(x)\right)}{2a^2}+\frac{\int\frac{ab\left(5a^2+4b^2\right)+i\left(3a^4+12b^2a^2+8b^4\right)\sin(ix)}{b-ia\sin(ix)}dx}{2a^2}\right)}{4a^2}-\frac{i\cosh^3(x)(4b-3a\sinh(x)}{12a^2}\right.$$

$$\Big\downarrow \ 3214$$

$$-i\left(-\frac{i\left(\frac{\frac{8b\left(a^2+b^2\right)^2\int\frac{1}{b+a\sinh(x)}dx}{a}-\frac{x\left(3a^4+12a^2b^2+8b^4\right)}{a}}{2a^2}+\frac{\cosh(x)\left(8b(a^2+b^2)-a(3a^2+4b^2)\sinh(x)\right)}{2a^2}\right)}{4a^2}-\frac{i\cosh^3(x)(4b-3a\sinh}{12a^2}\right.$$

$$\Big\downarrow \ 3042$$

$$-i\left(-\frac{i\left(\frac{\cosh(x)\left(8b(a^2+b^2)-a(3a^2+4b^2)\sinh(x)\right)}{2a^2}+\frac{-\frac{x\left(3a^4+12a^2b^2+8b^4\right)}{a}+\frac{8b\left(a^2+b^2\right)^2\int\frac{1}{b-ia\sin(ix)}dx}{a}}{2a^2}\right)}{4a^2}-\frac{i\cosh^3(x)(4b-3a\sin}{12a^2}\right.$$

$$\Big\downarrow \ 3139$$

$$-i\left(-\frac{i\left(\frac{\frac{16b\left(a^2+b^2\right)^2\int\frac{1}{-b\tanh^2\left(\frac{x}{2}\right)+2a\tanh\left(\frac{x}{2}\right)+b}d\tanh\left(\frac{x}{2}\right)}{a}-\frac{x\left(3a^4+12a^2b^2+8b^4\right)}{a}}{2a^2}+\frac{\cosh(x)\left(8b(a^2+b^2)-a(3a^2+4b^2)\sinh(x)\right)}{2a^2}\right)}{4a^2}-\frac{i\,\mathrm{co}}{}\right.$$

$$\Big\downarrow \ 1083$$

$$-i\left(-\frac{i\left(\frac{-\frac{32b\left(a^2+b^2\right)^2\int\frac{1}{4\left(a^2+b^2\right)-\left(2a-2b\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2a-2b\tanh\left(\frac{x}{2}\right)\right)}{a}-\frac{x\left(3a^4+12a^2b^2+8b^4\right)}{a}}{2a^2}+\frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+4b^2\right)\sinh(x)\right)}{2a^2}\right.}{4a^2}\right.$$

$$\Big\downarrow \text{ 219}$$

$$-i\left(-\frac{i\cosh^3(x)(4b-3a\sinh(x))}{12a^2}-\frac{i\left(\frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+4b^2\right)\sinh(x)\right)}{2a^2}+\frac{-\frac{16b\left(a^2+b^2\right)^{3/2}\text{arctanh}\left(\frac{2a-2b\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a}-}{2a^2}\right.}{4a^2}\right.$$

input
```
Int[Cosh[x]^4/(a + b*Csch[x]),x]
```

output
```
(-I)*(((-1/12*I)*Cosh[x]^3*(4*b - 3*a*Sinh[x]))/a^2 - ((I/4)*((-(((3*a^4 +
 12*a^2*b^2 + 8*b^4)*x)/a) - (16*b*(a^2 + b^2)^(3/2)*ArcTanh[(2*a - 2*b*Ta
nh[x/2])/(2*Sqrt[a^2 + b^2])])/a)/(2*a^2) + (Cosh[x]*(8*b*(a^2 + b^2) - a*
(3*a^2 + 4*b^2)*Sinh[x]))/(2*a^2)))/a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 63.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.72

method	result
risch	$\frac{3x}{8a}+\frac{3x\,b^{2}}{2a^{3}}+\frac{x\,b^{4}}{a^{5}}+\frac{\mathrm{e}^{4x}}{64a}-\frac{b\,\mathrm{e}^{3x}}{24a^{2}}+\frac{\mathrm{e}^{2x}}{8a}+\frac{\mathrm{e}^{2x}b^{2}}{8a^{3}}-\frac{5b\,\mathrm{e}^{x}}{8a^{2}}-\frac{b^{3}\mathrm{e}^{x}}{2a^{4}}-\frac{5b\,\mathrm{e}^{-x}}{8a^{2}}-\frac{b^{3}\mathrm{e}^{-x}}{2a^{4}}-\frac{\mathrm{e}^{-2x}}{8a}-\frac{\mathrm{e}^{-2x}b^{2}}{8a^{3}}-\frac{b\,\mathrm{e}^{-3x}}{24a^{2}}$
default	$\frac{1}{4a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}-\frac{-3a-2b}{6a^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{-7a^{2}-4ab-4b^{2}}{8a^{3}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{\left(-3a^{4}-12a^{2}b^{2}-8b^{4}\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8a^{5}}-\frac{-5a^{3}-12a^{2}b-4ab}{8a^{4}\left(\tanh\left(\frac{x}{2}\right)-\right.}$

input
```
int(cosh(x)^4/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
3/8*x/a+3/2*x/a^3*b^2+x/a^5*b^4+1/64/a*exp(x)^4-1/24*b/a^2*exp(x)^3+1/8/a*
exp(x)^2+1/8/a^3*exp(x)^2*b^2-5/8*b/a^2*exp(x)-1/2*b^3/a^4*exp(x)-5/8*b/a^
2/exp(x)-1/2*b^3/a^4/exp(x)-1/8/a/exp(x)^2-1/8/a^3/exp(x)^2*b^2-1/24*b/a^2
/exp(x)^3-1/64/a/exp(x)^4+(a^2+b^2)^(3/2)*b/a^5*ln(exp(x)+(b+(a^2+b^2)^(1/
2))/a)-(a^2+b^2)^(3/2)*b/a^5*ln(exp(x)-(-b+(a^2+b^2)^(1/2))/a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. 2(114) = 228.

Time = 0.11 (sec) , antiderivative size = 924, normalized size of antiderivative = 7.39

$$\int \frac{\cosh^4(x)}{a+b\operatorname{csch}(x)}\,dx=\text{Too large to display}$$

input
```
integrate(cosh(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
1/192*(3*a^4*cosh(x)^8 + 3*a^4*sinh(x)^8 - 8*a^3*b*cosh(x)^7 + 8*(3*a^4*co
sh(x) - a^3*b)*sinh(x)^7 + 24*(a^4 + a^2*b^2)*cosh(x)^6 + 4*(21*a^4*cosh(x
)^2 - 14*a^3*b*cosh(x) + 6*a^4 + 6*a^2*b^2)*sinh(x)^6 + 24*(3*a^4 + 12*a^2
*b^2 + 8*b^4)*x*cosh(x)^4 - 24*(5*a^3*b + 4*a*b^3)*cosh(x)^5 + 24*(7*a^4*c
osh(x)^3 - 7*a^3*b*cosh(x)^2 - 5*a^3*b - 4*a*b^3 + 6*(a^4 + a^2*b^2)*cosh(
x))*sinh(x)^5 - 8*a^3*b*cosh(x) + 2*(105*a^4*cosh(x)^4 - 140*a^3*b*cosh(x)
^3 + 180*(a^4 + a^2*b^2)*cosh(x)^2 + 12*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x - 6
0*(5*a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^4 - 3*a^4 - 24*(5*a^3*b + 4*a*b^3)*
cosh(x)^3 + 8*(21*a^4*cosh(x)^5 - 35*a^3*b*cosh(x)^4 - 15*a^3*b - 12*a*b^3
 + 60*(a^4 + a^2*b^2)*cosh(x)^3 + 12*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x*cosh(x
) - 30*(5*a^3*b + 4*a*b^3)*cosh(x)^2)*sinh(x)^3 - 24*(a^4 + a^2*b^2)*cosh(
x)^2 + 12*(7*a^4*cosh(x)^6 - 14*a^3*b*cosh(x)^5 + 30*(a^4 + a^2*b^2)*cosh(
x)^4 - 2*a^4 - 2*a^2*b^2 + 12*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x*cosh(x)^2 - 2
0*(5*a^3*b + 4*a*b^3)*cosh(x)^3 - 6*(5*a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^2
 + 192*((a^2*b + b^3)*cosh(x)^4 + 4*(a^2*b + b^3)*cosh(x)^3*sinh(x) + 6*(a
^2*b + b^3)*cosh(x)^2*sinh(x)^2 + 4*(a^2*b + b^3)*cosh(x)*sinh(x)^3 + (a^2
*b + b^3)*sinh(x)^4)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 +
2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 +
 b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x
) + 2*(a*cosh(x) + b)*sinh(x) - a)) + 8*(3*a^4*cosh(x)^7 - 7*a^3*b*cosh...
```

Sympy [F]

$$\int \frac{\cosh^4(x)}{a + b\text{csch}(x)}\,dx = \int \frac{\cosh^4{(x)}}{a + b\operatorname{csch}{(x)}}\,dx$$

input
```
integrate(cosh(x)**4/(a+b*csch(x)),x)
```

output
```
Integral(cosh(x)**4/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.74

$$\int \frac{\cosh^4(x)}{a + b\text{csch}(x)}\, dx = -\frac{\left(8\,a^2be^{(-x)} - 3\,a^3 - 24\,(a^3 + ab^2)e^{(-2\,x)} + 24\,(5\,a^2b + 4\,b^3)e^{(-3\,x)}\right)e^{(4\,x)}}{192\,a^4} - \frac{8\,a^2be^{(-3\,x)} + 3\,a^3e^{(-4\,x)} + 24\,(5\,a^2b + 4\,b^3)e^{(-x)} + 24\,(a^3 + ab^2)e^{(-2\,x)}}{192\,a^4} + \frac{(3\,a^4 + 12\,a^2b^2 + 8\,b^4)x}{8\,a^5} - \frac{(a^4b + 2\,a^2b^3 + b^5)\log\left(\frac{ae^{(-x)} - b - \sqrt{a^2+b^2}}{ae^{(-x)} - b + \sqrt{a^2+b^2}}\right)}{\sqrt{a^2 + b^2}a^5}$$

input
```
integrate(cosh(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
-1/192*(8*a^2*b*e^(-x) - 3*a^3 - 24*(a^3 + a*b^2)*e^(-2*x) + 24*(5*a^2*b +
 4*b^3)*e^(-3*x))*e^(4*x)/a^4 - 1/192*(8*a^2*b*e^(-3*x) + 3*a^3*e^(-4*x) +
 24*(5*a^2*b + 4*b^3)*e^(-x) + 24*(a^3 + a*b^2)*e^(-2*x))/a^4 + 1/8*(3*a^4
 + 12*a^2*b^2 + 8*b^4)*x/a^5 - (a^4*b + 2*a^2*b^3 + b^5)*log((a*e^(-x) - b
 - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^5
)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.77

$$\int \frac{\cosh^4(x)}{a + b\text{csch}(x)}\, dx = \frac{3\,a^3e^{(4\,x)} - 8\,a^2be^{(3\,x)} + 24\,a^3e^{(2\,x)} + 24\,ab^2e^{(2\,x)} - 120\,a^2be^x - 96\,b^3e^x}{192\,a^4} + \frac{(3\,a^4 + 12\,a^2b^2 + 8\,b^4)x}{8\,a^5} - \frac{\left(8\,a^3be^x + 3\,a^4 + 24\,(5\,a^3b + 4\,ab^3)e^{(3\,x)} + 24\,(a^4 + a^2b^2)e^{(2\,x)}\right)e^{(-4\,x)}}{192\,a^5} - \frac{(a^4b + 2\,a^2b^3 + b^5)\log\left(\frac{\left|2\,ae^x + 2\,b - 2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x + 2\,b + 2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2 + b^2}a^5}$$

input
```
integrate(cosh(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

output
```
1/192*(3*a^3*e^(4*x) - 8*a^2*b*e^(3*x) + 24*a^3*e^(2*x) + 24*a*b^2*e^(2*x)
 - 120*a^2*b*e^x - 96*b^3*e^x)/a^4 + 1/8*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x/a^
5 - 1/192*(8*a^3*b*e^x + 3*a^4 + 24*(5*a^3*b + 4*a*b^3)*e^(3*x) + 24*(a^4
+ a^2*b^2)*e^(2*x))*e^(-4*x)/a^5 - (a^4*b + 2*a^2*b^3 + b^5)*log(abs(2*a*e
^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^2 + b^2)))/(sqr
t(a^2 + b^2)*a^5)
```

Mupad [B] (verification not implemented)

Time = 3.33 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.98

$$\int \frac{\cosh^4(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{e^{4x}}{64a}-\frac{e^{-4x}}{64a}+\frac{x\,(3a^4+12a^2b^2+8b^4)}{8a^5} - \frac{e^{-2x}\,(a^2+b^2)}{8a^3}+\frac{e^{2x}\,(a^2+b^2)}{8a^3}-\frac{e^{-x}\,(5a^2b+4b^3)}{8a^4} - \frac{b\,e^{-3x}}{24a^2}-\frac{b\,e^{3x}}{24a^2}-\frac{e^{x}\,(5a^2b+4b^3)}{8a^4} - \frac{b\ln\left(\frac{2b\,e^{x}\,(a^2+b^2)^2}{a^6}-\frac{2b\,(a-b\,e^{x})\,(a^2+b^2)^{3/2}}{a^6}\right)(a^2+b^2)^{3/2}}{a^5} + \frac{b\ln\left(\frac{2b\,(a-b\,e^{x})\,(a^2+b^2)^{3/2}}{a^6}+\frac{2b\,e^{x}\,(a^2+b^2)^2}{a^6}\right)(a^2+b^2)^{3/2}}{a^5}$$

input
```
int(cosh(x)^4/(a + b/sinh(x)),x)
```

output
```
exp(4*x)/(64*a) - exp(-4*x)/(64*a) + (x*(3*a^4 + 8*b^4 + 12*a^2*b^2))/(8*a
^5) - (exp(-2*x)*(a^2 + b^2))/(8*a^3) + (exp(2*x)*(a^2 + b^2))/(8*a^3) - (
exp(-x)*(5*a^2*b + 4*b^3))/(8*a^4) - (b*exp(-3*x))/(24*a^2) - (b*exp(3*x))
/(24*a^2) - (exp(x)*(5*a^2*b + 4*b^3))/(8*a^4) - (b*log((2*b*exp(x)*(a^2 +
 b^2)^2)/a^6 - (2*b*(a - b*exp(x))*(a^2 + b^2)^(3/2))/a^6)*(a^2 + b^2)^(3/
2))/a^5 + (b*log((2*b*(a - b*exp(x))*(a^2 + b^2)^(3/2))/a^6 + (2*b*exp(x)*
(a^2 + b^2)^2)/a^6)*(a^2 + b^2)^(3/2))/a^5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.06

$$\int \frac{\cosh^4(x)}{a+b\operatorname{csch}(x)}\,dx$$
$$= \frac{-384e^{4x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x ai+bi}{\sqrt{a^2+b^2}}\right)a^2bi-384e^{4x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x ai+bi}{\sqrt{a^2+b^2}}\right)b^3i+3e^{8x}a^4-8e^{7x}a^3b+24e^{6x}a^4}{}$$

input
```
int(cosh(x)^4/(a+b*csch(x)),x)
```

output
```
( - 384*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2)
)*a**2*b*i - 384*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**
2 + b**2))*b**3*i + 3*e**(8*x)*a**4 - 8*e**(7*x)*a**3*b + 24*e**(6*x)*a**4
 + 24*e**(6*x)*a**2*b**2 - 120*e**(5*x)*a**3*b - 96*e**(5*x)*a*b**3 + 72*e
**(4*x)*a**4*x + 288*e**(4*x)*a**2*b**2*x + 192*e**(4*x)*b**4*x - 120*e**(
3*x)*a**3*b - 96*e**(3*x)*a*b**3 - 24*e**(2*x)*a**4 - 24*e**(2*x)*a**2*b**
2 - 8*e**x*a**3*b - 3*a**4)/(192*e**(4*x)*a**5)
```

3.94 $\int \frac{\cosh^3(x)}{a+b\text{csch}(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 57

$$\int \frac{\cosh^3(x)}{a+b\text{csch}(x)} dx = -\frac{b(a^2+b^2)\log(b+a\sinh(x))}{a^4} + \frac{(a^2+b^2)\sinh(x)}{a^3} - \frac{b\sinh^2(x)}{2a^2} + \frac{\sinh^3(x)}{3a}$$

output
```
-b*(a^2+b^2)*ln(b+a*sinh(x))/a^4+(a^2+b^2)*sinh(x)/a^3-1/2*b*sinh(x)^2/a^2
+1/3*sinh(x)^3/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \frac{\cosh^3(x)}{a+b\text{csch}(x)} dx = \frac{-6b(a^2+b^2)\log(b+a\sinh(x)) + 6a(a^2+b^2)\sinh(x) - 3a^2b\sinh^2(x) + 2a^3\sinh^3(x)}{6a^4}$$

input
```
Integrate[Cosh[x]^3/(a + b*Csch[x]),x]
```

output
```
(-6*b*(a^2 + b^2)*Log[b + a*Sinh[x]] + 6*a*(a^2 + b^2)*Sinh[x] - 3*a^2*b*S
inh[x]^2 + 2*a^3*Sinh[x]^3)/(6*a^4)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {3042, 4360, 26, 26, 3042, 26, 3316, 26, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{a + b\text{csch}(x)} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^3}{a + ib\csc(ix)}dx$$

$$\Big\downarrow \text{4360}$$

$$\int \frac{i\sinh(x)\cosh^3(x)}{ia\sinh(x) + ib}dx$$

$$\Big\downarrow \text{26}$$

$$i\int -\frac{i\cosh^3(x)\sinh(x)}{b + a\sinh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\sinh(x)\cosh^3(x)}{a\sinh(x) + b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)\cos(ix)^3}{b - ia\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ix)^3\sin(ix)}{b - ia\sin(ix)}dx$$

$$\Big\downarrow \text{3316}$$

$$\frac{i\int -\frac{i\sinh(x)(\sinh^2(x)a^2+a^2)}{b+a\sinh(x)}d(a\sinh(x))}{a^3}$$

$\Big\downarrow$ 26

$$\frac{\int \frac{\sinh(x)(\sinh^2(x)a^2+a^2)}{b+a\sinh(x)}d(a\sinh(x))}{a^3}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{a\sinh(x)(\sinh^2(x)a^2+a^2)}{b+a\sinh(x)}d(a\sinh(x))}{a^4}$$

$\Big\downarrow$ 522

$$\frac{\int \left(\sinh^2(x)a^2+\left(\frac{b^2}{a^2}+1\right)a^2-b\sinh(x)a-\frac{b(a^2+b^2)}{b+a\sinh(x)}\right)d(a\sinh(x))}{a^4}$$

$\Big\downarrow$ 2009

$$\frac{\frac{1}{3}a^3\sinh^3(x)+a(a^2+b^2)\sinh(x)-b(a^2+b^2)\log(a\sinh(x)+b)-\frac{1}{2}a^2b\sinh^2(x)}{a^4}$$

input
```
Int[Cosh[x]^3/(a + b*Csch[x]),x]
```

output
```
(-(b*(a^2 + b^2)*Log[b + a*Sinh[x]]) + a*(a^2 + b^2)*Sinh[x] - (a^2*b*Sinh
[x]^2)/2 + (a^3*Sinh[x]^3)/3)/a^4
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(53) = 106$.

Time = 15.88 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.39

method	result
risch	$\frac{xb}{a^2}+\frac{x\,b^3}{a^4}+\frac{e^{3x}}{24a}-\frac{b\,e^{2x}}{8a^2}+\frac{3\,e^{x}}{8a}+\frac{e^{x}b^2}{2a^3}-\frac{3\,e^{-x}}{8a}-\frac{e^{-x}b^2}{2a^3}-\frac{b\,e^{-2x}}{8a^2}-\frac{e^{-3x}}{24a}-\frac{b\ln\left(e^{2x}+\frac{2b\,e^{x}}{a}-1\right)}{a^2}-\frac{b^3\ln\left(e^{2x}+2\ldots\right.}{a^4}$
default	$-\frac{1}{3a\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}-\frac{-a+b}{2a^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{2a^2-ab+2b^2}{2a^3\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{\left(a^2+b^2\right)b\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a^4}-\frac{1}{3a\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{\ldots}{2a^2(\tan\ldots}$

input
```
int(cosh(x)^3/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
x*b/a^2+x/a^4*b^3+1/24/a*exp(3*x)-1/8*b/a^2*exp(2*x)+3/8/a*exp(x)+1/2/a^3*
exp(x)*b^2-3/8/a*exp(-x)-1/2/a^3*exp(-x)*b^2-1/8*b/a^2*exp(-2*x)-1/24/a*ex
p(-3*x)-1/a^2*b*ln(exp(2*x)+2*b/a*exp(x)-1)-1/a^4*b^3*ln(exp(2*x)+2*b/a*ex
p(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(53) = 106$.

Time = 0.09 (sec) , antiderivative size = 476, normalized size of antiderivative = 8.35

$$\int \frac{\cosh^3(x)}{a + b\text{csch}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
1/24*(a^3*cosh(x)^6 + a^3*sinh(x)^6 - 3*a^2*b*cosh(x)^5 + 3*(2*a^3*cosh(x)
 - a^2*b)*sinh(x)^5 + 24*(a^2*b + b^3)*x*cosh(x)^3 + 3*(3*a^3 + 4*a*b^2)*c
osh(x)^4 + 3*(5*a^3*cosh(x)^2 - 5*a^2*b*cosh(x) + 3*a^3 + 4*a*b^2)*sinh(x)
^4 - 3*a^2*b*cosh(x) + 2*(10*a^3*cosh(x)^3 - 15*a^2*b*cosh(x)^2 + 12*(a^2*
b + b^3)*x + 6*(3*a^3 + 4*a*b^2)*cosh(x))*sinh(x)^3 - a^3 - 3*(3*a^3 + 4*a
*b^2)*cosh(x)^2 + 3*(5*a^3*cosh(x)^4 - 10*a^2*b*cosh(x)^3 - 3*a^3 - 4*a*b^
2 + 24*(a^2*b + b^3)*x*cosh(x) + 6*(3*a^3 + 4*a*b^2)*cosh(x)^2)*sinh(x)^2
- 24*((a^2*b + b^3)*cosh(x)^3 + 3*(a^2*b + b^3)*cosh(x)^2*sinh(x) + 3*(a^2
*b + b^3)*cosh(x)*sinh(x)^2 + (a^2*b + b^3)*sinh(x)^3)*log(2*(a*sinh(x) +
b)/(cosh(x) - sinh(x))) + 3*(2*a^3*cosh(x)^5 - 5*a^2*b*cosh(x)^4 + 24*(a^2
*b + b^3)*x*cosh(x)^2 + 4*(3*a^3 + 4*a*b^2)*cosh(x)^3 - a^2*b - 2*(3*a^3 +
 4*a*b^2)*cosh(x))*sinh(x))/(a^4*cosh(x)^3 + 3*a^4*cosh(x)^2*sinh(x) + 3*a
^4*cosh(x)*sinh(x)^2 + a^4*sinh(x)^3)
```

Sympy [F]

$$\int \frac{\cosh^3(x)}{a + b\text{csch}(x)}\,dx = \int \frac{\cosh^3(x)}{a + b\,\text{csch}\,(x)}\,dx$$

input
```
integrate(cosh(x)**3/(a+b*csch(x)),x)
```

output
```
Integral(cosh(x)**3/(a + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(53) = 106$.

Time = 0.03 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.23

$$\int \frac{\cosh^3(x)}{a + b\text{csch}(x)}\,dx = -\frac{\left(3\,abe^{(-x)} - a^2 - 3\,(3\,a^2 + 4\,b^2)e^{(-2\,x)}\right)e^{(3\,x)}}{24\,a^3} - \frac{3\,abe^{(-2\,x)} + a^2e^{(-3\,x)} + 3\,(3\,a^2 + 4\,b^2)e^{(-x)}}{24\,a^3} - \frac{(a^2b + b^3)x}{a^4} - \frac{(a^2b + b^3)\log\left(-2\,be^{(-x)} + ae^{(-2\,x)} - a\right)}{a^4}$$

input
```
integrate(cosh(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
-1/24*(3*a*b*e^(-x) - a^2 - 3*(3*a^2 + 4*b^2)*e^(-2*x))*e^(3*x)/a^3 - 1/24
*(3*a*b*e^(-2*x) + a^2*e^(-3*x) + 3*(3*a^2 + 4*b^2)*e^(-x))/a^3 - (a^2*b +
 b^3)*x/a^4 - (a^2*b + b^3)*log(-2*b*e^(-x) + a*e^(-2*x) - a)/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.70

$$\int \frac{\cosh^3(x)}{a + b\text{csch}(x)}\,dx = -\frac{a^2\left(e^{(-x)} - e^x\right)^3 + 3\,ab\left(e^{(-x)} - e^x\right)^2 + 12\,a^2\left(e^{(-x)} - e^x\right) + 12\,b^2\left(e^{(-x)} - e^x\right)}{24\,a^3} - \frac{(a^2b + b^3)\log\left(\left|-a\left(e^{(-x)} - e^x\right) + 2\,b\right|\right)}{a^4}$$

input
```
integrate(cosh(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

output
```
-1/24*(a^2*(e^(-x) - e^x)^3 + 3*a*b*(e^(-x) - e^x)^2 + 12*a^2*(e^(-x) - e^
x) + 12*b^2*(e^(-x) - e^x))/a^3 - (a^2*b + b^3)*log(abs(-a*(e^(-x) - e^x)
+ 2*b))/a^4
```

Mupad [B] (verification not implemented)

Time = 3.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.12

$$\int \frac{\cosh^3(x)}{a+b\operatorname{csch}(x)}\,dx=\frac{e^{3x}}{24\,a}-\frac{e^{-3x}}{24\,a}+\frac{x\,(a^2\,b+b^3)}{a^4}+\frac{e^x\,(3\,a^2+4\,b^2)}{8\,a^3}-\frac{b\,e^{-2x}}{8\,a^2}-\frac{b\,e^{2x}}{8\,a^2}-\frac{\ln\left(2\,b\,e^x-a+a\,e^{2x}\right)\,(a^2\,b+b^3)}{a^4}-\frac{e^{-x}\,(3\,a^2+4\,b^2)}{8\,a^3}$$

input
```
int(cosh(x)^3/(a + b/sinh(x)),x)
```

output
```
exp(3*x)/(24*a) - exp(-3*x)/(24*a) + (x*(a^2*b + b^3))/a^4 + (exp(x)*(3*a^
2 + 4*b^2))/(8*a^3) - (b*exp(-2*x))/(8*a^2) - (b*exp(2*x))/(8*a^2) - (log(
2*b*exp(x) - a + a*exp(2*x))*(a^2*b + b^3))/a^4 - (exp(-x)*(3*a^2 + 4*b^2)
)/(8*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.96

$$\int \frac{\cosh^3(x)}{a+b\operatorname{csch}(x)}\,dx=\frac{e^{6x}a^3-3e^{5x}a^2b+9e^{4x}a^3+12e^{4x}a\,b^2-24e^{3x}\log(e^{2x}a+2e^xb-a)\,a^2b-24e^{3x}\log(e^{2x}a+2e^xb-a)\,b^3+2}{24e^{3x}a^4}$$

input
```
int(cosh(x)^3/(a+b*csch(x)),x)
```

output
```
(e**(6*x)*a**3 - 3*e**(5*x)*a**2*b + 9*e**(4*x)*a**3 + 12*e**(4*x)*a*b**2
- 24*e**(3*x)*log(e**(2*x)*a + 2*e**x*b - a)*a**2*b - 24*e**(3*x)*log(e**(
2*x)*a + 2*e**x*b - a)*b**3 + 24*e**(3*x)*a**2*b*x + 24*e**(3*x)*b**3*x -
9*e**(2*x)*a**3 - 12*e**(2*x)*a*b**2 - 3*e**x*a**2*b - a**3)/(24*e**(3*x)*
a**4)
```

3.95 $\int \frac{\cosh^2(x)}{a+b\,\mathrm{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 77

$$\int \frac{\cosh^2(x)}{a+b\,\mathrm{csch}(x)}\,dx = \frac{(a^2+2b^2)\,x}{2a^3} + \frac{2b\sqrt{a^2+b^2}\,\mathrm{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3} - \frac{\cosh(x)(2b-a\sinh(x))}{2a^2}$$

output
```
1/2*(a^2+2*b^2)*x/a^3+2*b*(a^2+b^2)^(1/2)*arctanh((a-b*tanh(1/2*x))/(a^2+b
^2)^(1/2))/a^3-1/2*cosh(x)*(2*b-a*sinh(x))/a^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04

$$\int \frac{\cosh^2(x)}{a+b\,\mathrm{csch}(x)}\,dx = \frac{2a^2x+4b^2x+8b\sqrt{-a^2-b^2}\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)-4ab\cosh(x)+a^2\sinh(2x)}{4a^3}$$

input
```
Integrate[Cosh[x]^2/(a + b*Csch[x]),x]
```

output
```
(2*a^2*x + 4*b^2*x + 8*b*Sqrt[-a^2 - b^2]*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a
^2 - b^2]] - 4*a*b*Cosh[x] + a^2*Sinh[2*x])/(4*a^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.25, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.077$, Rules used = {3042, 4360, 26, 26, 3042, 26, 3344, 26, 3042, 3214, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{a+b\text{csch}(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(ix)^2}{a+ib\csc(ix)}dx$$

$$\Big\downarrow \text{ 4360}$$

$$\int \frac{i\sinh(x)\cosh^2(x)}{ia\sinh(x)+ib}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int -\frac{i\cosh^2(x)\sinh(x)}{b+a\sinh(x)}dx$$

$$\Big\downarrow \text{ 26}$$

$$\int \frac{\sinh(x)\cosh^2(x)}{a\sinh(x)+b}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\sin(ix)\cos(ix)^2}{b-ia\sin(ix)}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\cos(ix)^2\sin(ix)}{b-ia\sin(ix)}dx$$

$$\Big\downarrow \ 3344$$

$$-i\left(-\frac{\int \frac{i\left(ab-\left(a^2+2b^2\right)\sinh(x)\right)}{b+a\sinh(x)}dx}{2a^2}-\frac{i\cosh(x)(2b-a\sinh(x))}{2a^2}\right)$$

$$\Big\downarrow \ 26$$

$$-i\left(-\frac{i\int \frac{ab-\left(a^2+2b^2\right)\sinh(x)}{b+a\sinh(x)}dx}{2a^2}-\frac{i\cosh(x)(2b-a\sinh(x))}{2a^2}\right)$$

$$\Big\downarrow \ 3042$$

$$-i\left(-\frac{i\int \frac{ab+i\left(a^2+2b^2\right)\sin(ix)}{b-ia\sin(ix)}dx}{2a^2}-\frac{i\cosh(x)(2b-a\sinh(x))}{2a^2}\right)$$

$$\Big\downarrow \ 3214$$

$$-i\left(-\frac{i\left(\frac{2b\left(a^2+b^2\right)\int \frac{1}{b+a\sinh(x)}dx}{a}-\frac{x\left(a^2+2b^2\right)}{a}\right)}{2a^2}-\frac{i\cosh(x)(2b-a\sinh(x))}{2a^2}\right)$$

$$\Big\downarrow \ 3042$$

$$-i\left(-\frac{i\left(-\frac{x\left(a^2+2b^2\right)}{a}+\frac{2b\left(a^2+b^2\right)\int \frac{1}{b-ia\sin(ix)}dx}{a}\right)}{2a^2}-\frac{i\cosh(x)(2b-a\sinh(x))}{2a^2}\right)$$

$$\Big\downarrow \ 3139$$

$$-i\left(-\frac{i\left(\frac{4b\left(a^2+b^2\right)\int \frac{1}{-b\tanh^2\left(\frac{x}{2}\right)+2a\tanh\left(\frac{x}{2}\right)+b}d\tanh\left(\frac{x}{2}\right)}{a}-\frac{x\left(a^2+2b^2\right)}{a}\right)}{2a^2}-\frac{i\cosh(x)(2b-a\sinh(x))}{2a^2}\right)$$

$$\Big\downarrow \ 1083$$

$$-i\left(-\frac{i\left(-\frac{8b\left(a^2+b^2\right)\int \frac{1}{4\left(a^2+b^2\right)-\left(2a-2b\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2a-2b\tanh\left(\frac{x}{2}\right)\right)}{a}-\frac{x\left(a^2+2b^2\right)}{a}\right)}{2a^2}-\frac{i\cosh(x)(2b-a\sinh(x))}{2a^2}\right)$$

$$\downarrow 219$$

$$-i\left(-\frac{i\left(-\frac{4b\sqrt{a^2+b^2}\text{arctanh}\left(\frac{2a-2b\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a}-\frac{x(a^2+2b^2)}{a}\right)}{2a^2}-\frac{i\cosh(x)(2b-a\sinh(x))}{2a^2}\right)$$

input
```
Int[Cosh[x]^2/(a + b*Csch[x]),x]
```

output
```
(-I)*(((-1/2*I)*(-(((a^2 + 2*b^2)*x)/a) - (4*b*Sqrt[a^2 + b^2]*ArcTanh[(2*
a - 2*b*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/a))/a^2 - ((I/2)*Cosh[x]*(2*b - a
*Sinh[x]))/a^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 3.73 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.58

method	result
risch	$\frac{x}{2a}+\frac{x\,b^{2}}{a^{3}}+\frac{{\mathrm e}^{2x}}{8a}-\frac{b\,{\mathrm e}^{x}}{2a^{2}}-\frac{b\,{\mathrm e}^{-x}}{2a^{2}}-\frac{{\mathrm e}^{-2x}}{8a}+\frac{\sqrt{a^{2}+b^{2}}\,b\ln\left({\mathrm e}^{x}+\frac{b+\sqrt{a^{2}+b^{2}}}{a}\right)}{a^{3}}-\frac{\sqrt{a^{2}+b^{2}}\,b\ln\left({\mathrm e}^{x}-\frac{-b+\sqrt{a^{2}+b^{2}}}{a}\right)}{a^{3}}$
default	$\frac{2b\sqrt{a^{2}+b^{2}}\,\operatorname{arctanh}\left(\frac{-2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{a^{2}+b^{2}}}\right)}{a^{3}}-\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{-a+2b}{2a^{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{\left(a^{2}+2b^{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2a^{3}}+\frac{1}{2a(\tanh(\frac{x}{2}}$

input
```
int(cosh(x)^2/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a+x/a^3*b^2+1/8/a*exp(x)^2-1/2*b/a^2*exp(x)-1/2*b/a^2/exp(x)-1/8/a/e
xp(x)^2+(a^2+b^2)^(1/2)*b/a^3*ln(exp(x)+(b+(a^2+b^2)^(1/2))/a)-(a^2+b^2)^(
1/2)*b/a^3*ln(exp(x)-(-b+(a^2+b^2)^(1/2))/a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(68) = 136$.

Time = 0.10 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.95

$$\int \frac{\cosh^2(x)}{a+b\text{csch}(x)}\,dx = \frac{a^2\cosh(x)^4 + a^2\sinh(x)^4 - 4\,ab\cosh(x)^3 + 4\,(a^2+2\,b^2)x\cosh(x)^2 + 4\,(a^2\cosh(x) - ab)\sinh(x)^3 - 4}{}$$

input
```
integrate(cosh(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
1/8*(a^2*cosh(x)^4 + a^2*sinh(x)^4 - 4*a*b*cosh(x)^3 + 4*(a^2 + 2*b^2)*x*c
osh(x)^2 + 4*(a^2*cosh(x) - a*b)*sinh(x)^3 - 4*a*b*cosh(x) + 2*(3*a^2*cosh
(x)^2 - 6*a*b*cosh(x) + 2*(a^2 + 2*b^2)*x)*sinh(x)^2 + 8*(b*cosh(x)^2 + 2*
b*cosh(x)*sinh(x) + b*sinh(x)^2)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*
sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) +
2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2
+ 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) - a^2 + 4*(a^2*cosh(x)^3 -
 3*a*b*cosh(x)^2 + 2*(a^2 + 2*b^2)*x*cosh(x) - a*b)*sinh(x))/(a^3*cosh(x)^
2 + 2*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2)
```

Sympy [F]

$$\int \frac{\cosh^2(x)}{a+b\text{csch}(x)}\,dx = \int \frac{\cosh^2(x)}{a+b\operatorname{csch}(x)}\,dx$$

input
```
integrate(cosh(x)**2/(a+b*csch(x)),x)
```

output
```
Integral(cosh(x)**2/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.58

$$\int \frac{\cosh^2(x)}{a+b\text{csch}(x)}\,dx = -\frac{(4\,be^{(-x)}-a)e^{(2\,x)}}{8\,a^2}-\frac{4\,be^{(-x)}+ae^{(-2\,x)}}{8\,a^2}+\frac{(a^2+2\,b^2)x}{2\,a^3}-\frac{(a^2b+b^3)\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^3}$$

input
```
integrate(cosh(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
-1/8*(4*b*e^(-x) - a)*e^(2*x)/a^2 - 1/8*(4*b*e^(-x) + a*e^(-2*x))/a^2 + 1/
2*(a^2 + 2*b^2)*x/a^3 - (a^2*b + b^3)*log((a*e^(-x) - b - sqrt(a^2 + b^2))
/(a*e^(-x) - b + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.57

$$\int \frac{\cosh^2(x)}{a+b\text{csch}(x)}\,dx = \frac{ae^{(2\,x)}-4\,be^x}{8\,a^2}+\frac{(a^2+2\,b^2)x}{2\,a^3}-\frac{(4\,abe^x+a^2)e^{(-2\,x)}}{8\,a^3}-\frac{(a^2b+b^3)\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^3}$$

input
```
integrate(cosh(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

output
```
1/8*(a*e^(2*x) - 4*b*e^x)/a^2 + 1/2*(a^2 + 2*b^2)*x/a^3 - 1/8*(4*a*b*e^x +
 a^2)*e^(-2*x)/a^3 - (a^2*b + b^3)*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^
2))/abs(2*a*e^x + 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3)
```

Mupad [B] (verification not implemented)

Time = 3.10 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.06

$$\int \frac{\cosh^2(x)}{a+b\text{csch}(x)}\,dx = \frac{e^{2x}}{8a} - \frac{e^{-2x}}{8a} - \frac{b\,e^{x}}{2a^2} - \frac{b\,e^{-x}}{2a^2} + \frac{x\,(a^2+2b^2)}{2a^3} - \frac{b\,\ln\left(\frac{2\,b\,e^{x}\,(a^2+b^2)}{a^4} - \frac{2\,b\,(a-b\,e^{x})\,\sqrt{a^2+b^2}}{a^4}\right)\sqrt{a^2+b^2}}{a^3} + \frac{b\,\ln\left(\frac{2\,b\,(a-b\,e^{x})\,\sqrt{a^2+b^2}}{a^4} + \frac{2\,b\,e^{x}\,(a^2+b^2)}{a^4}\right)\sqrt{a^2+b^2}}{a^3}$$

input
```
int(cosh(x)^2/(a + b/sinh(x)),x)
```

output
```
exp(2*x)/(8*a) - exp(-2*x)/(8*a) - (b*exp(x))/(2*a^2) - (b*exp(-x))/(2*a^2
) + (x*(a^2 + 2*b^2))/(2*a^3) - (b*log((2*b*exp(x)*(a^2 + b^2))/a^4 - (2*b
*(a - b*exp(x))*(a^2 + b^2)^(1/2))/a^4)*(a^2 + b^2)^(1/2))/a^3 + (b*log((2
*b*(a - b*exp(x))*(a^2 + b^2)^(1/2))/a^4 + (2*b*exp(x)*(a^2 + b^2))/a^4)*(
a^2 + b^2)^(1/2))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.35

$$\int \frac{\cosh^2(x)}{a+b\text{csch}(x)}\,dx = \frac{-16e^{2x}\sqrt{a^2+b^2}\,atan\left(\frac{e^{x}ai+bi}{\sqrt{a^2+b^2}}\right)bi + e^{4x}a^2 - 4e^{3x}ab + 4e^{2x}a^2x + 8e^{2x}b^2x - 4e^{x}ab - a^2}{8e^{2x}a^3}$$

input
```
int(cosh(x)^2/(a+b*csch(x)),x)
```

output
```
( - 16*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2))
*b*i + e**(4*x)*a**2 - 4*e**(3*x)*a*b + 4*e**(2*x)*a**2*x + 8*e**(2*x)*b**
2*x - 4*e**x*a*b - a**2)/(8*e**(2*x)*a**3)
```

3.96 $\int \frac{\cosh(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{\cosh(x)}{a+b\text{csch}(x)}\,dx = -\frac{b\log(b+a\sinh(x))}{a^2}+\frac{\sinh(x)}{a}$$

output
```
-b*ln(b+a*sinh(x))/a^2+sinh(x)/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\cosh(x)}{a+b\text{csch}(x)}\,dx = \frac{-b\log(b+a\sinh(x))+a\sinh(x)}{a^2}$$

input
```
Integrate[Cosh[x]/(a + b*Csch[x]),x]
```

output
```
(-(b*Log[b + a*Sinh[x]]) + a*Sinh[x])/a^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4360, 26, 26, 3042, 26, 3312, 26, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{a+b\text{csch}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)}{a+ib\csc(ix)}dx$$

$$\Big\downarrow \text{4360}$$

$$\int \frac{i\sinh(x)\cosh(x)}{ia\sinh(x)+ib}dx$$

$$\Big\downarrow \text{26}$$

$$i\int -\frac{i\cosh(x)\sinh(x)}{b+a\sinh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\sinh(x)\cosh(x)}{a\sinh(x)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)\cos(ix)}{b-ia\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ix)\sin(ix)}{b-ia\sin(ix)}dx$$

$$\Big\downarrow \text{3312}$$

$$-\frac{i\int \frac{i\sinh(x)}{b+a\sinh(x)}d(a\sinh(x))}{a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\sinh(x)}{b+a\sinh(x)}d(a\sinh(x))}{a}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{a\sinh(x)}{b+a\sinh(x)}d(a\sinh(x))}{a^2}$$

$$\Big\downarrow \; 49$$

$$\frac{\int \left(1-\frac{b}{b+a\sinh(x)}\right)d(a\sinh(x))}{a^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{a\sinh(x)-b\log(a\sinh(x)+b)}{a^2}$$

input
```
Int[Cosh[x]/(a + b*Csch[x]),x]
```

output
```
(-(b*Log[b + a*Sinh[x]]) + a*Sinh[x])/a^2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

method	result	size
derivativedivides	$-\frac{b \ln(a+b\ \mathrm{csch}(x))}{a^2}+\frac{1}{a\ \mathrm{csch}(x)}+\frac{b \ln(\mathrm{csch}(x))}{a^2}$	31
default	$-\frac{b \ln(a+b\ \mathrm{csch}(x))}{a^2}+\frac{1}{a\ \mathrm{csch}(x)}+\frac{b \ln(\mathrm{csch}(x))}{a^2}$	31
risch	$\frac{xb}{a^2}+\frac{\mathrm{e}^{x}}{2a}-\frac{\mathrm{e}^{-x}}{2a}-\frac{b \ln\left(\mathrm{e}^{2x}+\frac{2b\,\mathrm{e}^{x}}{a}-1\right)}{a^2}$	45

input
```
int(cosh(x)/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-b/a^2*ln(a+b*csch(x))+1/a/csch(x)+b/a^2*ln(csch(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(20) = 40$.

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 4.00

$$\int \frac{\cosh(x)}{a + b\text{csch}(x)}\,dx = \frac{2\,bx\cosh\left(x\right) + a\cosh\left(x\right)^2 + a\sinh\left(x\right)^2 - 2\left(b\cosh\left(x\right) + b\sinh\left(x\right)\right)\log\left(\frac{2\,(a\sinh(x)+b)}{\cosh(x)-\sinh(x)}\right) + 2\left(bx + a\cosh\right.}{2\left(a^2\cosh\left(x\right) + a^2\sinh\left(x\right)\right)}$$

input
```
integrate(cosh(x)/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
1/2*(2*b*x*cosh(x) + a*cosh(x)^2 + a*sinh(x)^2 - 2*(b*cosh(x) + b*sinh(x))
*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + 2*(b*x + a*cosh(x))*sinh(x)
- a)/(a^2*cosh(x) + a^2*sinh(x))
```

Sympy [F]

$$\int \frac{\cosh(x)}{a + b\text{csch}(x)}\,dx = \int \frac{\cosh\left(x\right)}{a + b\,\text{csch}\left(x\right)}\,dx$$

input
```
integrate(cosh(x)/(a+b*csch(x)),x)
```

output
```
Integral(cosh(x)/(a + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(20) = 40$.

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.40

$$\int \frac{\cosh(x)}{a + b\text{csch}(x)}\,dx = -\frac{bx}{a^2} - \frac{e^{(-x)}}{2\,a} + \frac{e^x}{2\,a} - \frac{b\log\left(-2\,be^{(-x)} + ae^{(-2\,x)} - a\right)}{a^2}$$

input
```
integrate(cosh(x)/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
-b*x/a^2 - 1/2*e^(-x)/a + 1/2*e^x/a - b*log(-2*b*e^(-x) + a*e^(-2*x) - a)/
a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \frac{\cosh(x)}{a + b\text{csch}(x)}\,dx = -\frac{e^{(-x)} - e^{x}}{2\,a} - \frac{b\log\left(\left|-a\left(e^{(-x)} - e^{x}\right) + 2\,b\right|\right)}{a^2}$$

input
```
integrate(cosh(x)/(a+b*csch(x)),x, algorithm="giac")
```

output
```
-1/2*(e^(-x) - e^x)/a - b*log(abs(-a*(e^(-x) - e^x) + 2*b))/a^2
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{a + b\text{csch}(x)}\,dx = \frac{\sinh(x)}{a} - \frac{b\,\ln\left(b + a\sinh(x)\right)}{a^2}$$

input
```
int(cosh(x)/(a + b/sinh(x)),x)
```

output
```
sinh(x)/a - (b*log(b + a*sinh(x)))/a^2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \frac{\cosh(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{e^{2x}a-2e^{x}\log\left(e^{2x}a+2e^{x}b-a\right)b+2e^{x}bx-a}{2e^{x}a^{2}}$$

input
```
int(cosh(x)/(a+b*csch(x)),x)
```

output
```
(e**(2*x)*a - 2*e**x*log(e**(2*x)*a + 2*e**x*b - a)*b + 2*e**x*b*x - a)/(2
*e**x*a**2)
```

3.97 $\int \frac{\text{sech}(x)}{a+b\text{csch}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 64

$$\int \frac{\text{sech}(x)}{a+b\text{csch}(x)}\,dx=\frac{\log(i-\sinh(x))}{2(ia+b)}-\frac{\log(i+\sinh(x))}{2(ia-b)}-\frac{b\log(b+a\sinh(x))}{a^2+b^2}$$

output
```
ln(I-sinh(x))/(2*I*a+2*b)-ln(I+sinh(x))/(2*I*a-2*b)-b*ln(b+a*sinh(x))/(a^2
+b^2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{\text{sech}(x)}{a+b\text{csch}(x)}\,dx =\frac{(-ia+b)\log(i-\sinh(x))+(ia+b)\log(i+\sinh(x))-2b\log(b+a\sinh(x))}{2(a^2+b^2)}$$

input
```
Integrate[Sech[x]/(a + b*Csch[x]),x]
```

output
```
(((-I)*a + b)*Log[I - Sinh[x]] + (I*a + b)*Log[I + Sinh[x]] - 2*b*Log[b +
a*Sinh[x]])/(2*(a^2 + b^2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.182, Rules used = {3042, 4359, 26, 26, 3042, 26, 3200, 25, 587, 16, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(x)}{a+b\text{csch}(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\cos(ix)(a+ib\csc(ix))}dx$$

$$\Big\downarrow \text{ 4359}$$

$$\int \frac{i\tanh(x)}{ia\sinh(x)+ib}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int -\frac{i\tanh(x)}{b+a\sinh(x)}dx$$

$$\Big\downarrow \text{ 26}$$

$$\int \frac{\tanh(x)}{a\sinh(x)+b}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\tan(ix)}{b-ia\sin(ix)}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\tan(ix)}{b-ia\sin(ix)}dx$$

$$\Big\downarrow \text{ 3200}$$

$$-\int -\frac{a\sinh(x)}{(b+a\sinh(x))\left(\sinh^2(x)a^2+a^2\right)}d(a\sinh(x))$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{a\sinh(x)}{(a^2\sinh^2(x)+a^2)(a\sinh(x)+b)}d(a\sinh(x))$$

$\Big\downarrow$ 587

$$\frac{\int \frac{a^2+b\sinh(x)a}{\sinh^2(x)a^2+a^2}d(a\sinh(x))}{a^2+b^2}-\frac{b\int \frac{1}{b+a\sinh(x)}d(a\sinh(x))}{a^2+b^2}$$

$\Big\downarrow$ 16

$$\frac{\int \frac{a^2+b\sinh(x)a}{\sinh^2(x)a^2+a^2}d(a\sinh(x))}{a^2+b^2}-\frac{b\log(a\sinh(x)+b)}{a^2+b^2}$$

$\Big\downarrow$ 452

$$\frac{b\int \frac{a\sinh(x)}{\sinh^2(x)a^2+a^2}d(a\sinh(x))+a^2\int \frac{1}{\sinh^2(x)a^2+a^2}d(a\sinh(x))}{a^2+b^2}-\frac{b\log(a\sinh(x)+b)}{a^2+b^2}$$

$\Big\downarrow$ 216

$$\frac{b\int \frac{a\sinh(x)}{\sinh^2(x)a^2+a^2}d(a\sinh(x))+a\arctan(\sinh(x))}{a^2+b^2}-\frac{b\log(a\sinh(x)+b)}{a^2+b^2}$$

$\Big\downarrow$ 240

$$\frac{\frac{1}{2}b\log\left(a^2\sinh^2(x)+a^2\right)+a\arctan(\sinh(x))}{a^2+b^2}-\frac{b\log(a\sinh(x)+b)}{a^2+b^2}$$

input
```
Int[Sech[x]/(a + b*Csch[x]),x]
```

output
```
-((b*Log[b + a*Sinh[x]])/(a^2 + b^2)) + (a*ArcTan[Sinh[x]] + (b*Log[a^2 +
a^2*Sinh[x]^2])/2)/(a^2 + b^2)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 587
```
Int[(x_.)/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[(-
c)*(d/(b*c^2 + a*d^2))   Int[1/(c + d*x), x], x] + Simp[1/(b*c^2 + a*d^2)
 Int[(a*d + b*c*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
^2 + a*d^2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

rule 4359
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_.), x_Symbol] :> Int[Cot[e + f*x]^p*(b + a*Sin[e + f*x])^m, x] /; FreeQ[{a
, b, e, f, p}, x] && IntegerQ[m] && EqQ[m, p]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{2b\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)+4a\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{2a^{2}+2b^{2}}-\frac{2b\ln\left(-b\tanh\left(\frac{x}{2}\right)^{2}+2a\tanh\left(\frac{x}{2}\right)+b\right)}{2a^{2}+2b^{2}}$	72
risch	$\frac{i\ln(\mathrm{e}^{x}+i)a}{a^{2}+b^{2}}+\frac{\ln(\mathrm{e}^{x}+i)b}{a^{2}+b^{2}}-\frac{i\ln(\mathrm{e}^{x}-i)a}{a^{2}+b^{2}}+\frac{\ln(\mathrm{e}^{x}-i)b}{a^{2}+b^{2}}-\frac{b\ln\left(\mathrm{e}^{2x}+\frac{2b\,\mathrm{e}^{x}}{a}-1\right)}{a^{2}+b^{2}}$	101

input
```
int(sech(x)/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
4/(2*a^2+2*b^2)*(1/2*b*ln(tanh(1/2*x)^2+1)+a*arctan(tanh(1/2*x)))-2*b/(2*a
^2+2*b^2)*ln(-b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{sech}(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{2\,a\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)-b\log\left(\frac{2\,(a\sinh(x)+b)}{\cosh(x)-\sinh(x)}\right)+b\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^{2}+b^{2}}$$

input
```
integrate(sech(x)/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
(2*a*arctan(cosh(x) + sinh(x)) - b*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x
))) + b*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^2 + b^2)
```

Sympy [F]

$$\int \frac{\mathrm{sech}(x)}{a+b\mathrm{csch}(x)}\,dx=\int \frac{\mathrm{sech}\,(x)}{a+b\,\mathrm{csch}\,(x)}\,dx$$

input
```
integrate(sech(x)/(a+b*csch(x)),x)
```

output
```
Integral(sech(x)/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int \frac{\mathrm{sech}(x)}{a+b\mathrm{csch}(x)}\,dx=-\frac{2\,a\arctan\left(e^{(-x)}\right)}{a^2+b^2}-\frac{b\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{a^2+b^2}+\frac{b\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2}$$

input
```
integrate(sech(x)/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
-2*a*arctan(e^(-x))/(a^2 + b^2) - b*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^2
 + b^2) + b*log(e^(-2*x) + 1)/(a^2 + b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.39

$$\int \frac{\mathrm{sech}(x)}{a+b\mathrm{csch}(x)}\,dx=-\frac{ab\log\left(\left|-a\left(e^{(-x)}-e^x\right)+2\,b\right|\right)}{a^3+ab^2}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)a}{2\left(a^2+b^2\right)}+\frac{b\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\left(a^2+b^2\right)}$$

input
```
integrate(sech(x)/(a+b*csch(x)),x, algorithm="giac")
```

output
```
-a*b*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^3 + a*b^2) + 1/2*(pi + 2*arctan(
1/2*(e^(2*x) - 1)*e^(-x)))*a/(a^2 + b^2) + 1/2*b*log((e^(-x) - e^x)^2 + 4)
/(a^2 + b^2)
```

Mupad [B] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.45

$$\int \frac{\text{sech}(x)}{a + b\text{csch}(x)}\,dx = \frac{\ln\left(1 + e^x\,1i\right)}{b + a\,1i} - \frac{b\,\ln\left(a^3\,e^{2\,x} - 4\,a\,b^2 - a^3 + 8\,b^3\,e^x + 2\,a^2\,b\,e^x + 4\,a\,b^2\,e^{2\,x}\right)}{a^2 + b^2} + \frac{\ln\left(e^x + 1i\right)\,1i}{a + b\,1i}$$

input
```
int(1/(cosh(x)*(a + b/sinh(x))),x)
```

output
```
log(exp(x)*1i + 1)/(a*1i + b) + (log(exp(x) + 1i)*1i)/(a + b*1i) - (b*log(
a^3*exp(2*x) - 4*a*b^2 - a^3 + 8*b^3*exp(x) + 2*a^2*b*exp(x) + 4*a*b^2*exp
(2*x)))/(a^2 + b^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.77

$$\int \frac{\text{sech}(x)}{a + b\text{csch}(x)}\,dx = \frac{2atan(e^x)\,a + \log(e^{2x} + 1)\,b - \log(e^{2x}a + 2e^x b - a)\,b}{a^2 + b^2}$$

input
```
int(sech(x)/(a+b*csch(x)),x)
```

output
```
(2*atan(e**x)*a + log(e**(2*x) + 1)*b - log(e**(2*x)*a + 2*e**x*b - a)*b)/
(a**2 + b**2)
```

3.98 $\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{csch}(x)}\,dx=\frac{2ab\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{\operatorname{sech}(x)(b-a\sinh(x))}{a^2+b^2}$$

output
```
2*a*b*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)-sech(x)*(
b-a*sinh(x))/(a^2+b^2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{csch}(x)}\,dx=\frac{-b\operatorname{sech}(x)+a\left(-\frac{2b\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\tanh(x)\right)}{a^2+b^2}$$

input
```
Integrate[Sech[x]^2/(a + b*Csch[x]),x]
```

output
```
(-(b*Sech[x]) + a*((-2*b*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[
-a^2 - b^2] + Tanh[x]))/(a^2 + b^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4360, 26, 26, 3042, 26, 3345, 26, 27, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{csch}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ix)^2(a+ib\csc(ix))}dx$$

$$\Big\downarrow \text{4360}$$

$$\int \frac{i\tanh(x)\operatorname{sech}(x)}{ia\sinh(x)+ib}dx$$

$$\Big\downarrow \text{26}$$

$$i\int -\frac{i\operatorname{sech}(x)\tanh(x)}{b+a\sinh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh(x)\operatorname{sech}(x)}{a\sinh(x)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)}{\cos(ix)^2(b-ia\sin(ix))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ix)}{\cos(ix)^2(b-ia\sin(ix))}dx$$

$$\Big\downarrow \text{3345}$$

$$-i\left(-\frac{\int \frac{iab}{b+a\sinh(x)}dx}{a^2+b^2}-\frac{i\text{sech}(x)(b-a\sinh(x))}{a^2+b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{i\int \frac{ab}{b+a\sinh(x)}dx}{a^2+b^2}-\frac{i\text{sech}(x)(b-a\sinh(x))}{a^2+b^2}\right)$$

$$\Big\downarrow \text{27}$$

$$-i\left(-\frac{iab\int \frac{1}{b+a\sinh(x)}dx}{a^2+b^2}-\frac{i\text{sech}(x)(b-a\sinh(x))}{a^2+b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{iab\int \frac{1}{b-ia\sin(ix)}dx}{a^2+b^2}-\frac{i\text{sech}(x)(b-a\sinh(x))}{a^2+b^2}\right)$$

$$\Big\downarrow \text{3139}$$

$$-i\left(-\frac{2iab\int \frac{1}{-b\tanh^2\left(\frac{x}{2}\right)+2a\tanh\left(\frac{x}{2}\right)+b}d\tanh\left(\frac{x}{2}\right)}{a^2+b^2}-\frac{i\text{sech}(x)(b-a\sinh(x))}{a^2+b^2}\right)$$

$$\Big\downarrow \text{1083}$$

$$-i\left(\frac{4iab\int \frac{1}{4(a^2+b^2)-\left(2a-2b\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2a-2b\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}-\frac{i\text{sech}(x)(b-a\sinh(x))}{a^2+b^2}\right)$$

$$\Big\downarrow \text{219}$$

$$-i\left(\frac{2iab\text{arctanh}\left(\frac{2a-2b\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{i\text{sech}(x)(b-a\sinh(x))}{a^2+b^2}\right)$$

input
```
Int[Sech[x]^2/(a + b*Csch[x]),x]
```

output
```
(-I)*(((2*I)*a*b*ArcTanh[(2*a - 2*b*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/(a^2
+ b^2)^(3/2) - (I*Sech[x]*(b - a*Sinh[x]))/(a^2 + b^2))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.35

method	result	size
default	$\frac{4ab\,\operatorname{arctanh}\left(\frac{-2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{a^2+b^2}}\right)}{(2a^2+2b^2)\sqrt{a^2+b^2}}-\frac{2\left(-a\tanh\left(\frac{x}{2}\right)+b\right)}{(a^2+b^2)\left(\tanh\left(\frac{x}{2}\right)^2+1\right)}$	81
risch	$-\frac{2(b\,e^x+a)}{(e^{2x}+1)(a^2+b^2)}+\frac{ba\ln\left(e^x+\frac{(a^2+b^2)^{\frac{3}{2}}b+a^4+2a^2b^2+b^4}{a(a^2+b^2)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{ba\ln\left(e^x+\frac{(a^2+b^2)^{\frac{3}{2}}b-a^4-2a^2b^2-b^4}{a(a^2+b^2)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}}$	142

input
```
int(sech(x)^2/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
4*a*b/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*tanh(1/2*x)*b+2*a)/(a^
2+b^2)^(1/2))-2/(a^2+b^2)*(-a*tanh(1/2*x)+b)/(tanh(1/2*x)^2+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(56) = 112$.

Time = 0.09 (sec) , antiderivative size = 256, normalized size of antiderivative = 4.27

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{csch}(x)}\,dx =$$

$$-\frac{2\,a^3+2\,ab^2-\left(ab\cosh\left(x\right)^2+2\,ab\cosh\left(x\right)\sinh\left(x\right)+ab\sinh\left(x\right)^2+ab\right)\sqrt{a^2+b^2}\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(\cdots}{\cdots}\right.}{a^4+2\,a^2b^2+b^4+\left(a^4+2\,a^2b^2+b^4\right)\cosh\left(x\right)^2+2\,(\cdots}$$

input
```
integrate(sech(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
-(2*a^3 + 2*a*b^2 - (a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2
 + a*b)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x)
 + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh
(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh
(x) + b)*sinh(x) - a)) + 2*(a^2*b + b^3)*cosh(x) + 2*(a^2*b + b^3)*sinh(x)
)/(a^4 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^4 + 2*
a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{csch}(x)}\,dx$$

input
```
integrate(sech(x)**2/(a+b*csch(x)),x)
```

output
```
Integral(sech(x)**2/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.52

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{ab\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{2\left(be^{(-x)}-a\right)}{a^2+b^2+(a^2+b^2)e^{(-2x)}}$$

input
```
integrate(sech(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
-a*b*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2))
)/(a^2 + b^2)^(3/2) - 2*(b*e^(-x) - a)/(a^2 + b^2 + (a^2 + b^2)*e^(-2*x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.42

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{csch}(x)}\,dx=-\frac{ab\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\,(be^x+a)}{(a^2+b^2)(e^{(2\,x)}+1)}$$

input
```
integrate(sech(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

output
```
-a*b*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt
(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b*e^x + a)/((a^2 + b^2)*(e^(2*x) + 1)
)
```

Mupad [B] (verification not implemented)

Time = 2.96 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.22

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{csch}(x)}\,dx=\frac{a\,b\,\ln\left(\frac{2\,b\,(a-b\,e^x)}{(a^2+b^2)^{3/2}}+\frac{2\,b\,e^x}{a^2+b^2}\right)}{(a^2+b^2)^{3/2}}-\frac{a\,b\,\ln\left(\frac{2\,b\,e^x}{a^2+b^2}-\frac{2\,b\,(a-b\,e^x)}{(a^2+b^2)^{3/2}}\right)}{(a^2+b^2)^{3/2}}-\frac{\frac{2\,a}{a^2+b^2}+\frac{2\,b\,e^x}{a^2+b^2}}{e^{2\,x}+1}$$

input
```
int(1/(cosh(x)^2*(a + b/sinh(x))),x)
```

output
```
(a*b*log((2*b*(a - b*exp(x)))/(a^2 + b^2)^(3/2) + (2*b*exp(x))/(a^2 + b^2)
))/(a^2 + b^2)^(3/2) - (a*b*log((2*b*exp(x))/(a^2 + b^2) - (2*b*(a - b*exp
(x)))/(a^2 + b^2)^(3/2)))/(a^2 + b^2)^(3/2) - ((2*a)/(a^2 + b^2) + (2*b*ex
p(x))/(a^2 + b^2))/(exp(2*x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.70

$$\int \frac{\operatorname{sech}^2(x)}{a + b\operatorname{csch}(x)}\,dx = \frac{-2e^{2x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x ai+bi}{\sqrt{a^2+b^2}}\right)abi - 2\sqrt{a^2+b^2}\,atan\left(\frac{e^x ai+bi}{\sqrt{a^2+b^2}}\right)abi + 2e^{2x}a^3 + 2e^{2x}a\,b^2 - 2e^x a^2 b - 2e^x b^3}{e^{2x}a^4 + 2e^{2x}a^2b^2 + e^{2x}b^4 + a^4 + 2a^2b^2 + b^4}$$

input
```
int(sech(x)^2/(a+b*csch(x)),x)
```

output
```
(2*( - e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2))
*a*b*i - sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2))*a*b*i
+ e**(2*x)*a**3 + e**(2*x)*a*b**2 - e**x*a**2*b - e**x*b**3))/(e**(2*x)*a*
*4 + 2*e**(2*x)*a**2*b**2 + e**(2*x)*b**4 + a**4 + 2*a**2*b**2 + b**4)
```

3.99 $\int \frac{\mathbf{sech}^3(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 90

$$\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{a(a^2-b^2)\arctan(\sinh(x))}{2\left(a^2+b^2\right)^2} + \frac{a^2b\log(\cosh(x))}{\left(a^2+b^2\right)^2} - \frac{a^2b\log(b+a\sinh(x))}{\left(a^2+b^2\right)^2} - \frac{\operatorname{sech}^2(x)(b-a\sinh(x))}{2\left(a^2+b^2\right)}$$

output
```
1/2*a*(a^2-b^2)*arctan(sinh(x))/(a^2+b^2)^2+a^2*b*ln(cosh(x))/(a^2+b^2)^2-
a^2*b*ln(b+a*sinh(x))/(a^2+b^2)^2-sech(x)^2*(b-a*sinh(x))/(2*a^2+2*b^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.41

$$\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{i\operatorname{csch}(x)(b+a\sinh(x))\left(\frac{a\log(i-\sinh(x))}{(a-ib)^2} - \frac{a\log(i+\sinh(x))}{(a+ib)^2} - \frac{4ia^2b\log(b+a\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{1}{(ia+b)(i-\sinh(x))} + \frac{i}{(a+ib)(i+\sinh(x))}\right)}{4(a+b\operatorname{csch}(x))}$$

input
```
Integrate[Sech[x]^3/(a + b*Csch[x]),x]
```

output
```
((-1/4*I)*Csch[x]*(b + a*Sinh[x])*((a*Log[I - Sinh[x]])/(a - I*b)^2 - (a*L
og[I + Sinh[x]])/(a + I*b)^2 - ((4*I)*a^2*b*Log[b + a*Sinh[x]])/(a^2 + b^2
)^2 + 1/((I*a + b)*(I - Sinh[x])) + I/((a + I*b)*(I + Sinh[x]))))/(a + b*C
sch[x])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.37, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4360, 26, 26, 3042, 26, 3316, 26, 27, 593, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^3(x)}{a+b\text{csch}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ix)^3(a+ib\csc(ix))}dx$$

$$\Big\downarrow \text{4360}$$

$$\int \frac{i\tanh(x)\text{sech}^2(x)}{ia\sinh(x)+ib}dx$$

$$\Big\downarrow \text{26}$$

$$i\int -\frac{i\text{sech}^2(x)\tanh(x)}{b+a\sinh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh(x)\text{sech}^2(x)}{a\sinh(x)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)}{\cos(ix)^3(b-ia\sin(ix))}dx$$

$$\Big\downarrow \textbf{26}$$

$$-i\int \frac{\sin(ix)}{\cos(ix)^3(b-ia\sin(ix))}dx$$

$$\Big\downarrow \textbf{3316}$$

$$-ia^3\int \frac{i\sinh(x)}{(b+a\sinh(x))\left(\sinh^2(x)a^2+a^2\right)^2}d(a\sinh(x))$$

$$\Big\downarrow \textbf{26}$$

$$a^3\int \frac{\sinh(x)}{(b+a\sinh(x))\left(\sinh^2(x)a^2+a^2\right)^2}d(a\sinh(x))$$

$$\Big\downarrow \textbf{27}$$

$$a^2\int \frac{a\sinh(x)}{(b+a\sinh(x))\left(\sinh^2(x)a^2+a^2\right)^2}d(a\sinh(x))$$

$$\Big\downarrow \textbf{593}$$

$$a^2\left(\frac{\int -\frac{b-a\sinh(x)}{(b+a\sinh(x))(\sinh^2(x)a^2+a^2)}d(a\sinh(x))}{2\left(a^2+b^2\right)}-\frac{b-a\sinh(x)}{2\left(a^2+b^2\right)\left(a^2\sinh^2(x)+a^2\right)}\right)$$

$$\Big\downarrow \textbf{25}$$

$$a^2\left(-\frac{\int \frac{b-a\sinh(x)}{(b+a\sinh(x))(\sinh^2(x)a^2+a^2)}d(a\sinh(x))}{2\left(a^2+b^2\right)}-\frac{b-a\sinh(x)}{2\left(a^2+b^2\right)\left(a^2\sinh^2(x)+a^2\right)}\right)$$

$$\Big\downarrow \textbf{657}$$

$$a^2\left(-\frac{\int \left(\frac{2b}{(a^2+b^2)(b+a\sinh(x))}+\frac{-a^2-2b\sinh(x)a+b^2}{(a^2+b^2)(\sinh^2(x)a^2+a^2)}\right)d(a\sinh(x))}{2\left(a^2+b^2\right)}-\frac{b-a\sinh(x)}{2\left(a^2+b^2\right)\left(a^2\sinh^2(x)+a^2\right)}\right)$$

$$\Big\downarrow \textbf{2009}$$

$$a^2\left(-\frac{-\frac{(a^2-b^2)\arctan(\sinh(x))}{a(a^2+b^2)}-\frac{b\log(a^2\sinh^2(x)+a^2)}{a^2+b^2}+\frac{2b\log(a\sinh(x)+b)}{a^2+b^2}}{2\left(a^2+b^2\right)}-\frac{b-a\sinh(x)}{2\left(a^2+b^2\right)\left(a^2\sinh^2(x)+a^2\right)}\right)$$

input

```
Int[Sech[x]^3/(a + b*Csch[x]),x]
```

output
```
a^2*(-1/2*(-(((a^2 - b^2)*ArcTan[Sinh[x]])/(a*(a^2 + b^2))) + (2*b*Log[b +
 a*Sinh[x]])/(a^2 + b^2) - (b*Log[a^2 + a^2*Sinh[x]^2])/(a^2 + b^2))/(a^2
+ b^2) - (b - a*Sinh[x])/(2*(a^2 + b^2)*(a^2 + a^2*Sinh[x]^2)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.59

method	result
default	$-\frac{a^{2} b \ln\left(-b \tanh\left(\frac{x}{2}\right)^{2}+2 a \tanh\left(\frac{x}{2}\right)+b\right)}{\left(a^{2}+b^{2}\right)^{2}}+\frac{\frac{2\left(\left(-\frac{1}{2} a^{3}-\frac{1}{2} a b^{2}\right) \tanh\left(\frac{x}{2}\right)^{3}+\left(a^{2} b+b^{3}\right) \tanh\left(\frac{x}{2}\right)^{2}+\left(\frac{1}{2} a^{3}+\frac{1}{2} a b^{2}\right) \tanh\left(\frac{x}{2}\right)\right)}{\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{2}}+a\left(a b \ln\left(\mathrm{ta}\right.\right.}{\left(a^{2}+b^{2}\right)^{2}}$
risch	$\frac{e^{x}\left(e^{2x} a-2 b\, e^{x}-a\right)}{\left(e^{2x}+1\right)^{2}\left(a^{2}+b^{2}\right)}-\frac{i a^{3} \ln\left(e^{x}-i\right)}{2\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}+\frac{i a \ln\left(e^{x}-i\right) b^{2}}{2 a^{4}+4 a^{2} b^{2}+2 b^{4}}+\frac{a^{2} \ln\left(e^{x}-i\right) b}{a^{4}+2 a^{2} b^{2}+b^{4}}+\frac{i a^{3} \ln\left(e^{x}+i\right)}{2 a^{4}+4 a^{2} b^{2}+2 b^{4}}-\frac{i a \ln\left(e^{x}+i\right) b^{2}}{2\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}+\frac{a}{a^{4}}$

input
```
int(sech(x)^3/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-a^2*b/(a^2+b^2)^2*ln(-b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b)+2/(a^2+b^2)^2*((
(-1/2*a^3-1/2*a*b^2)*tanh(1/2*x)^3+(a^2*b+b^3)*tanh(1/2*x)^2+(1/2*a^3+1/2*
a*b^2)*tanh(1/2*x))/(tanh(1/2*x)^2+1)^2+1/2*a*(a*b*ln(tanh(1/2*x)^2+1)+(a^
2-b^2)*arctan(tanh(1/2*x))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 675 vs. $2(87) = 174$.

Time = 0.11 (sec) , antiderivative size = 675, normalized size of antiderivative = 7.50

$$\int \frac{\operatorname{sech}^3(x)}{a + b\operatorname{csch}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
((a^3 + a*b^2)*cosh(x)^3 + (a^3 + a*b^2)*sinh(x)^3 - 2*(a^2*b + b^3)*cosh(
x)^2 - (2*a^2*b + 2*b^3 - 3*(a^3 + a*b^2)*cosh(x))*sinh(x)^2 + ((a^3 - a*b
^2)*cosh(x)^4 + 4*(a^3 - a*b^2)*cosh(x)*sinh(x)^3 + (a^3 - a*b^2)*sinh(x)^
4 + a^3 - a*b^2 + 2*(a^3 - a*b^2)*cosh(x)^2 + 2*(a^3 - a*b^2 + 3*(a^3 - a*
b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 - a*b^2)*cosh(x)^3 + (a^3 - a*b^2)*cos
h(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^3 + a*b^2)*cosh(x) - (a^2*b*
cosh(x)^4 + 4*a^2*b*cosh(x)*sinh(x)^3 + a^2*b*sinh(x)^4 + 2*a^2*b*cosh(x)^
2 + a^2*b + 2*(3*a^2*b*cosh(x)^2 + a^2*b)*sinh(x)^2 + 4*(a^2*b*cosh(x)^3 +
 a^2*b*cosh(x))*sinh(x))*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + (a^2
*b*cosh(x)^4 + 4*a^2*b*cosh(x)*sinh(x)^3 + a^2*b*sinh(x)^4 + 2*a^2*b*cosh(
x)^2 + a^2*b + 2*(3*a^2*b*cosh(x)^2 + a^2*b)*sinh(x)^2 + 4*(a^2*b*cosh(x)^
3 + a^2*b*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) - (a^3 + a*
b^2 - 3*(a^3 + a*b^2)*cosh(x)^2 + 4*(a^2*b + b^3)*cosh(x))*sinh(x))/((a^4
+ 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^3
 + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*
a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 +
 b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 +
 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{a + b\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{sech}^3(x)}{a + b\operatorname{csch}(x)}\,dx$$

input
```
integrate(sech(x)**3/(a+b*csch(x)),x)
```

output
```
Integral(sech(x)**3/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.79

$$\int \frac{\operatorname{sech}^3(x)}{a + b\operatorname{csch}(x)}\,dx = -\frac{a^2 b \log\left(-2\,be^{(-x)} + ae^{(-2\,x)} - a\right)}{a^4 + 2\,a^2b^2 + b^4} + \frac{a^2 b \log\left(e^{(-2\,x)} + 1\right)}{a^4 + 2\,a^2b^2 + b^4} - \frac{(a^3 - ab^2)\arctan\left(e^{(-x)}\right)}{a^4 + 2\,a^2b^2 + b^4} + \frac{ae^{(-x)} - 2\,be^{(-2\,x)} - ae^{(-3\,x)}}{a^2 + b^2 + 2\,(a^2 + b^2)e^{(-2\,x)} + (a^2 + b^2)e^{(-4\,x)}}$$

input
```
integrate(sech(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
-a^2*b*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^4 + 2*a^2*b^2 + b^4) + a^2*b*l
og(e^(-2*x) + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^3 - a*b^2)*arctan(e^(-x))/(a
^4 + 2*a^2*b^2 + b^4) + (a*e^(-x) - 2*b*e^(-2*x) - a*e^(-3*x))/(a^2 + b^2
+ 2*(a^2 + b^2)*e^(-2*x) + (a^2 + b^2)*e^(-4*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(87) = 174.

Time = 0.13 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.42

$$\int \frac{\operatorname{sech}^3(x)}{a + b\operatorname{csch}(x)}\,dx = -\frac{a^3 b \log\left(\left|-a\left(e^{(-x)} - e^x\right) + 2\,b\right|\right)}{a^5 + 2\,a^3b^2 + ab^4} + \frac{a^2 b \log\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}{2\,(a^4 + 2\,a^2b^2 + b^4)} + \frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)(a^3 - ab^2)}{4\,(a^4 + 2\,a^2b^2 + b^4)} - \frac{a^2b\left(e^{(-x)} - e^x\right)^2 + 2\,a^3\left(e^{(-x)} - e^x\right) + 2\,ab^2\left(e^{(-x)} - e^x\right) + 8\,a^2b + 4\,b^3}{2\,(a^4 + 2\,a^2b^2 + b^4)\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}$$

input
```
integrate(sech(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

output
```
-a^3*b*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^5 + 2*a^3*b^2 + a*b^4) + 1/2*a
^2*b*log((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) + 1/4*(pi + 2*arcta
n(1/2*(e^(2*x) - 1)*e^(-x)))*(a^3 - a*b^2)/(a^4 + 2*a^2*b^2 + b^4) - 1/2*(
a^2*b*(e^(-x) - e^x)^2 + 2*a^3*(e^(-x) - e^x) + 2*a*b^2*(e^(-x) - e^x) + 8
*a^2*b + 4*b^3)/((a^4 + 2*a^2*b^2 + b^4)*((e^(-x) - e^x)^2 + 4))
```

Mupad [B] (verification not implemented)

Time = 4.21 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.84

$$\int \frac{\mathrm{sech}^3(x)}{a+b\mathrm{csch}(x)}\,dx = \frac{\frac{2\,b}{a^2+b^2}-\frac{2\,a\,\mathrm{e}^x}{a^2+b^2}}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1}-\frac{\frac{2\,\left(a^2\,b+b^3\right)}{{\left(a^2+b^2\right)}^2}-\frac{\mathrm{e}^x\,\left(a^3+a\,b^2\right)}{{\left(a^2+b^2\right)}^2}}{\mathrm{e}^{2\,x}+1}+\frac{a\,\ln\left(\mathrm{e}^x+1\mathrm{i}\right)}{2\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)} - \frac{a^2\,b\,\ln\left(a^6\,\mathrm{e}^{2\,x}-a^6-a^2\,b^4-14\,a^4\,b^2+a^2\,b^4\,\mathrm{e}^{2\,x}+14\,a^4\,b^2\,\mathrm{e}^{2\,x}+2\,a\,b^5\,\mathrm{e}^x+2\,a^5\,b\,\mathrm{e}^x+28\,a^3\,b^3\,\mathrm{e}^x\right)}{a^4+2\,a^2\,b^2+b^4} + \frac{a\,\ln\left(1+\mathrm{e}^x\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)}$$

input
```
int(1/(cosh(x)^3*(a + b/sinh(x))),x)
```

output
```
((2*b)/(a^2 + b^2) - (2*a*exp(x))/(a^2 + b^2))/(2*exp(2*x) + exp(4*x) + 1)
 - ((2*(a^2*b + b^3))/(a^2 + b^2)^2 - (exp(x)*(a*b^2 + a^3))/(a^2 + b^2)^2
)/(exp(2*x) + 1) + (a*log(exp(x)*1i + 1)*1i)/(2*(a*b*2i - a^2 + b^2)) + (a
*log(exp(x) + 1i))/(2*(2*a*b - a^2*1i + b^2*1i)) - (a^2*b*log(a^6*exp(2*x)
 - a^6 - a^2*b^4 - 14*a^4*b^2 + a^2*b^4*exp(2*x) + 14*a^4*b^2*exp(2*x) + 2
*a*b^5*exp(x) + 2*a^5*b*exp(x) + 28*a^3*b^3*exp(x)))/(a^4 + b^4 + 2*a^2*b^
2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 353, normalized size of antiderivative = 3.92

$$\int \frac{\operatorname{sech}^3(x)}{a + b\operatorname{csch}(x)}\, dx$$
$$= \frac{e^{4x} atan(e^x)\, a^3 - e^{4x} atan(e^x)\, a\, b^2 + 2e^{2x} atan(e^x)\, a^3 - 2e^{2x} atan(e^x)\, a\, b^2 + atan(e^x)\, a^3 - atan(e^x)\, a\, b^2 +}{}$$

input
```
int(sech(x)^3/(a+b*csch(x)),x)
```

output
```
(e**(4*x)*atan(e**x)*a**3 - e**(4*x)*atan(e**x)*a*b**2 + 2*e**(2*x)*atan(e
**x)*a**3 - 2*e**(2*x)*atan(e**x)*a*b**2 + atan(e**x)*a**3 - atan(e**x)*a*
b**2 + e**(4*x)*log(e**(2*x) + 1)*a**2*b - e**(4*x)*log(e**(2*x)*a + 2*e**
x*b - a)*a**2*b + e**(4*x)*a**2*b + e**(4*x)*b**3 + e**(3*x)*a**3 + e**(3*
x)*a*b**2 + 2*e**(2*x)*log(e**(2*x) + 1)*a**2*b - 2*e**(2*x)*log(e**(2*x)*
a + 2*e**x*b - a)*a**2*b - e**x*a**3 - e**x*a*b**2 + log(e**(2*x) + 1)*a**
2*b - log(e**(2*x)*a + 2*e**x*b - a)*a**2*b + a**2*b + b**3)/(e**(4*x)*a**
4 + 2*e**(4*x)*a**2*b**2 + e**(4*x)*b**4 + 2*e**(2*x)*a**4 + 4*e**(2*x)*a*
*2*b**2 + 2*e**(2*x)*b**4 + a**4 + 2*a**2*b**2 + b**4)
```

3.100 $\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 104

$$\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{2a^3b\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\,(a^2+b^2)} - \frac{\operatorname{sech}(x)\,(3a^2b-a(2a^2-b^2)\sinh(x))}{3\,(a^2+b^2)^2}$$

output

```
2*a^3*b*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)-sech(x)
^3*(b-a*sinh(x))/(3*a^2+3*b^2)-1/3*sech(x)*(3*a^2*b-a*(2*a^2-b^2)*sinh(x))
/(a^2+b^2)^2
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{\frac{6a^3b\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + 3a^2b\operatorname{sech}(x) + b(a^2+b^2)\operatorname{sech}^3(x) + (-2a^3+ab^2)\tanh(x) - a(a^2+b^2)\operatorname{sech}^2(}{3\,(a^2+b^2)^2}$$

input
```
Integrate[Sech[x]^4/(a + b*Csch[x]),x]
```

output
```
-1/3*((6*a^3*b*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2
] + 3*a^2*b*Sech[x] + b*(a^2 + b^2)*Sech[x]^3 + (-2*a^3 + a*b^2)*Tanh[x] -
 a*(a^2 + b^2)*Sech[x]^2*Tanh[x])/(a^2 + b^2)^2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.23, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.154$, Rules used = {3042, 4360, 26, 26, 3042, 26, 3345, 26, 3042, 3345, 27, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^4(x)}{a+b\text{csch}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ix)^4(a+ib\csc(ix))}dx$$

$$\Big\downarrow \text{4360}$$

$$\int \frac{i\tanh(x)\text{sech}^3(x)}{ia\sinh(x)+ib}dx$$

$$\Big\downarrow \text{26}$$

$$i\int -\frac{i\text{sech}^3(x)\tanh(x)}{b+a\sinh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh(x)\text{sech}^3(x)}{a\sinh(x)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)}{\cos(ix)^4(b-ia\sin(ix))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ix)}{\cos(ix)^4(b-ia\sin(ix))}dx$$

$$\Big\downarrow \text{3345}$$

$$-i\left(-\frac{\int \frac{i\text{sech}^2(x)\left(ab-2a^2\sinh(x)\right)}{b+a\sinh(x)}dx}{3\left(a^2+b^2\right)}-\frac{i\text{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{i\int \frac{\text{sech}^2(x)\left(ab-2a^2\sinh(x)\right)}{b+a\sinh(x)}dx}{3\left(a^2+b^2\right)}-\frac{i\text{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{i\int \frac{2i\sin(ix)a^2+ba}{\cos(ix)^2(b-ia\sin(ix))}dx}{3\left(a^2+b^2\right)}-\frac{i\text{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)}\right)$$

$$\Big\downarrow \text{3345}$$

$$-i\left(-\frac{i\left(\frac{\text{sech}(x)\left(3a^2b-a\left(2a^2-b^2\right)\sinh(x)\right)}{a^2+b^2}-\frac{\int -\frac{3a^3b}{b+a\sinh(x)}dx}{a^2+b^2}\right)}{3\left(a^2+b^2\right)}-\frac{i\text{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$-i\left(-\frac{i\left(\frac{3a^3b\int \frac{1}{b+a\sinh(x)}dx}{a^2+b^2}+\frac{\text{sech}(x)\left(3a^2b-a\left(2a^2-b^2\right)\sinh(x)\right)}{a^2+b^2}\right)}{3\left(a^2+b^2\right)}-\frac{i\text{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{i\left(\frac{\text{sech}(x)\left(3a^2b-a\left(2a^2-b^2\right)\sinh(x)\right)}{a^2+b^2}+\frac{3a^3b\int \frac{1}{b-ia\sin(ix)}dx}{a^2+b^2}\right)}{3\left(a^2+b^2\right)}-\frac{i\text{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)}\right)$$

$$\Big\downarrow \text{3139}$$

$$-i\left(-\frac{i\left(\frac{6a^3b\int \frac{1}{-b\tanh^2\left(\frac{x}{2}\right)+2a\tanh\left(\frac{x}{2}\right)+b}d\tanh\left(\frac{x}{2}\right)}{a^2+b^2}+\frac{\text{sech}(x)(3a^2b-a(2a^2-b^2)\sinh(x))}{a^2+b^2}\right)}{3\left(a^2+b^2\right)}-\frac{i\text{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)}\right)$$

$\downarrow$ 1083

$$-i\left(-\frac{i\left(\frac{\text{sech}(x)(3a^2b-a(2a^2-b^2)\sinh(x))}{a^2+b^2}-\frac{12a^3b\int \frac{1}{4\left(a^2+b^2\right)-\left(2a-2b\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2a-2b\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\right)}{3\left(a^2+b^2\right)}-\frac{i\text{sech}^3(x)(b-a\sinh(x$$

$\downarrow$ 219

$$-i\left(-\frac{i\text{sech}^3(x)(b-a\sinh(x))}{3\left(a^2+b^2\right)}-\frac{i\left(\frac{\text{sech}(x)(3a^2b-a(2a^2-b^2)\sinh(x))}{a^2+b^2}-\frac{6a^3b\,\text{arctanh}\left(\frac{2a-2b\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}\right)}{3\left(a^2+b^2\right)}\right)$$

input
```
Int[Sech[x]^4/(a + b*Csch[x]),x]
```

output
```
(-I)*(((-1/3*I)*Sech[x]^3*(b - a*Sinh[x]))/(a^2 + b^2) - ((I/3)*((-6*a^3*b
*ArcTanh[(2*a - 2*b*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/(a^2 + b^2)^(3/2) + (
Sech[x]*(3*a^2*b - a*(2*a^2 - b^2)*Sinh[x]))/(a^2 + b^2)))/(a^2 + b^2))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 7.42 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.63

method	result
default	$-\frac{2\left(-\tanh\left(\frac{x}{2}\right)^{5} a^{3}+\left(2a^{2}b+b^{3}\right)\tanh\left(\frac{x}{2}\right)^{4}+\left(-\frac{2}{3}a^{3}+\frac{4}{3}a\,b^{2}\right)\tanh\left(\frac{x}{2}\right)^{3}+2a^{2}b\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)a^{3}+\frac{4a^{2}b}{3}+\frac{b^{3}}{3}\right)}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{3}}+\frac{4a^{3}b\,\text{arctanh}}{(2a^{4}+4a^{2}}$
risch	$-\frac{2\left(3a^{2}b\,e^{5x}-3a\,b^{2}e^{4x}+10a^{2}b\,e^{3x}+4b^{3}e^{3x}+6a^{3}e^{2x}+3b\,e^{x}a^{2}+2a^{3}-a\,b^{2}\right)}{3\left(a^{2}+b^{2}\right)^{2}\left(e^{2x}+1\right)^{3}}+\frac{b\,a^{3}\ln\left(e^{x}+\frac{\left(a^{2}+b^{2}\right)^{\frac{5}{2}}b+a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}}{a\left(a^{2}+b^{2}\right)^{\frac{5}{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{5}{2}}}-$

input
```
int(sech(x)^4/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/(a^4+2*a^2*b^2+b^4)*(-tanh(1/2*x)^5*a^3+(2*a^2*b+b^3)*tanh(1/2*x)^4+(-2
/3*a^3+4/3*a*b^2)*tanh(1/2*x)^3+2*a^2*b*tanh(1/2*x)^2-tanh(1/2*x)*a^3+4/3*
a^2*b+1/3*b^3)/(tanh(1/2*x)^2+1)^3+4*a^3*b/(2*a^4+4*a^2*b^2+2*b^4)/(a^2+b^
2)^(1/2)*arctanh(1/2*(-2*tanh(1/2*x)*b+2*a)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1155 vs. 2(97) = 194.

Time = 0.09 (sec) , antiderivative size = 1155, normalized size of antiderivative = 11.11

$$\int \frac{\text{sech}^4(x)}{a+b\text{csch}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
-1/3*(6*(a^4*b + a^2*b^3)*cosh(x)^5 + 6*(a^4*b + a^2*b^3)*sinh(x)^5 + 4*a^
5 + 2*a^3*b^2 - 2*a*b^4 - 6*(a^3*b^2 + a*b^4)*cosh(x)^4 - 6*(a^3*b^2 + a*b
^4 - 5*(a^4*b + a^2*b^3)*cosh(x))*sinh(x)^4 + 4*(5*a^4*b + 7*a^2*b^3 + 2*b
^5)*cosh(x)^3 + 4*(5*a^4*b + 7*a^2*b^3 + 2*b^5 + 15*(a^4*b + a^2*b^3)*cosh
(x)^2 - 6*(a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 + 12*(a^5 + a^3*b^2)*cosh(x
)^2 + 12*(a^5 + a^3*b^2 + 5*(a^4*b + a^2*b^3)*cosh(x)^3 - 3*(a^3*b^2 + a*b
^4)*cosh(x)^2 + (5*a^4*b + 7*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x)^2 - 3*(a^3*
b*cosh(x)^6 + 6*a^3*b*cosh(x)*sinh(x)^5 + a^3*b*sinh(x)^6 + 3*a^3*b*cosh(x
)^4 + 3*a^3*b*cosh(x)^2 + 3*(5*a^3*b*cosh(x)^2 + a^3*b)*sinh(x)^4 + a^3*b
+ 4*(5*a^3*b*cosh(x)^3 + 3*a^3*b*cosh(x))*sinh(x)^3 + 3*(5*a^3*b*cosh(x)^4
 + 6*a^3*b*cosh(x)^2 + a^3*b)*sinh(x)^2 + 6*(a^3*b*cosh(x)^5 + 2*a^3*b*cos
h(x)^3 + a^3*b*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*
sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) +
2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2
+ 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) + 6*(a^4*b + a^2*b^3)*cosh
(x) + 6*(a^4*b + a^2*b^3 + 5*(a^4*b + a^2*b^3)*cosh(x)^4 - 4*(a^3*b^2 + a*
b^4)*cosh(x)^3 + 2*(5*a^4*b + 7*a^2*b^3 + 2*b^5)*cosh(x)^2 + 4*(a^5 + a^3*
b^2)*cosh(x))*sinh(x))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 6*
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)*sinh(x)^5 + (a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6)*sinh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 3*(a^...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{csch}(x)}\,dx=\int \frac{\operatorname{sech}^4{(x)}}{a+b\operatorname{csch}{(x)}}\,dx$$

input
```
integrate(sech(x)**4/(a+b*csch(x)),x)
```

output
```
Integral(sech(x)**4/(a + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(97) = 194$.

Time = 0.11 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.17

$$\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{a^3 b \log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(3\,a^2be^{(-x)}-6\,a^3e^{(-2\,x)}+3\,ab^2e^{(-4\,x)}+3\,a^2be^{(-5\,x)}-2\,a^3+ab^2+2\,(5\,a^2b+2\,b^3)e^{(-3\,x)}\right)}{3\,(a^4+2\,a^2b^2+b^4+3\,(a^4+2\,a^2b^2+b^4)e^{(-2\,x)}+3\,(a^4+2\,a^2b^2+b^4)e^{(-4\,x)}+(a^4+2\,a^2b^2+b^4)e^{(-6\,x)})}$$

input
```
integrate(sech(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
-a^3*b*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2
)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2/3*(3*a^2*b*e^(-x) - 6*a^
3*e^(-2*x) + 3*a*b^2*e^(-4*x) + 3*a^2*b*e^(-5*x) - 2*a^3 + a*b^2 + 2*(5*a^
2*b + 2*b^3)*e^(-3*x))/(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*
e^(-2*x) + 3*(a^4 + 2*a^2*b^2 + b^4)*e^(-4*x) + (a^4 + 2*a^2*b^2 + b^4)*e^
(-6*x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.67

$$\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{a^3 b \log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(3\,a^2be^{(5\,x)}-3\,ab^2e^{(4\,x)}+10\,a^2be^{(3\,x)}+4\,b^3e^{(3\,x)}+6\,a^3e^{(2\,x)}+3\,a^2be^x+2\,a^3-ab^2\right)}{3\,(a^4+2\,a^2b^2+b^4)(e^{(2\,x)}+1)^3}$$

input
```
integrate(sech(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

output
```
-a^3*b*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sq
rt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2/3*(3*a^2*b*e
^(5*x) - 3*a*b^2*e^(4*x) + 10*a^2*b*e^(3*x) + 4*b^3*e^(3*x) + 6*a^3*e^(2*x
) + 3*a^2*b*e^x + 2*a^3 - a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(e^(2*x) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 3.08 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.59

$$\int \frac{\mathrm{sech}^4(x)}{a + b\mathrm{csch}(x)}\,dx = \frac{\frac{2\,a\,b^2}{(a^2+b^2)^2} - \frac{2\,a^2\,b\,\mathrm{e}^x}{(a^2+b^2)^2}}{\mathrm{e}^{2\,x}+1} - \frac{\frac{4\,(a^3+a\,b^2)}{(a^2+b^2)^2} + \frac{8\,\mathrm{e}^x\,(a^2\,b+b^3)}{3\,(a^2+b^2)^2}}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1} + \frac{\frac{8\,a}{3\,(a^2+b^2)} + \frac{8\,b\,\mathrm{e}^x}{3\,(a^2+b^2)}}{3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1} - \frac{a^3\,b\,\ln\left(\frac{2\,a^2\,b\,\mathrm{e}^x}{(a^2+b^2)^2} - \frac{2\,a^2\,b\,(a-b\,\mathrm{e}^x)}{(a^2+b^2)^{5/2}}\right)}{(a^2+b^2)^{5/2}} + \frac{a^3\,b\,\ln\left(\frac{2\,a^2\,b\,(a-b\,\mathrm{e}^x)}{(a^2+b^2)^{5/2}} + \frac{2\,a^2\,b\,\mathrm{e}^x}{(a^2+b^2)^2}\right)}{(a^2+b^2)^{5/2}}$$

input
```
int(1/(cosh(x)^4*(a + b/sinh(x))),x)
```

output
```
((2*a*b^2)/(a^2 + b^2)^2 - (2*a^2*b*exp(x))/(a^2 + b^2)^2)/(exp(2*x) + 1)
- ((4*(a*b^2 + a^3))/(a^2 + b^2)^2 + (8*exp(x)*(a^2*b + b^3))/(3*(a^2 + b^
2)^2))/(2*exp(2*x) + exp(4*x) + 1) + ((8*a)/(3*(a^2 + b^2)) + (8*b*exp(x))
/(3*(a^2 + b^2)))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (a^3*b*log((2
*a^2*b*exp(x))/(a^2 + b^2)^2 - (2*a^2*b*(a - b*exp(x)))/(a^2 + b^2)^(5/2))
)/(a^2 + b^2)^(5/2) + (a^3*b*log((2*a^2*b*(a - b*exp(x)))/(a^2 + b^2)^(5/2
) + (2*a^2*b*exp(x))/(a^2 + b^2)^2))/(a^2 + b^2)^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 482, normalized size of antiderivative = 4.63

$$\int \frac{\mathrm{sech}^4(x)}{a + b\mathrm{csch}(x)}\,dx = \frac{-6e^{6x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x ai+bi}{\sqrt{a^2+b^2}}\right)a^3bi - 18e^{4x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x ai+bi}{\sqrt{a^2+b^2}}\right)a^3bi - 18e^{2x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x ai+bi}{\sqrt{a^2+b^2}}\right)a}{3e^{6x}a^6 + 9e^{6x}a^4b^2 + 9e^{6x}a^2b^4 + 3e^{6x}b^6}$$

input
```
int(sech(x)^4/(a+b*csch(x)),x)
```

output
```
(2*( - 3*e**(6*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2
))*a**3*b*i - 9*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2
 + b**2))*a**3*b*i - 9*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sq
rt(a**2 + b**2))*a**3*b*i - 3*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt
(a**2 + b**2))*a**3*b*i - e**(6*x)*a**3*b**2 - e**(6*x)*a*b**4 - 3*e**(5*x
)*a**4*b - 3*e**(5*x)*a**2*b**3 - 10*e**(3*x)*a**4*b - 14*e**(3*x)*a**2*b*
*3 - 4*e**(3*x)*b**5 - 6*e**(2*x)*a**5 - 9*e**(2*x)*a**3*b**2 - 3*e**(2*x)
*a*b**4 - 3*e**x*a**4*b - 3*e**x*a**2*b**3 - 2*a**5 - 2*a**3*b**2))/(3*(e*
*(6*x)*a**6 + 3*e**(6*x)*a**4*b**2 + 3*e**(6*x)*a**2*b**4 + e**(6*x)*b**6
+ 3*e**(4*x)*a**6 + 9*e**(4*x)*a**4*b**2 + 9*e**(4*x)*a**2*b**4 + 3*e**(4*
x)*b**6 + 3*e**(2*x)*a**6 + 9*e**(2*x)*a**4*b**2 + 9*e**(2*x)*a**2*b**4 +
3*e**(2*x)*b**6 + a**6 + 3*a**4*b**2 + 3*a**2*b**4 + b**6))
```

3.101 $\int \frac{\text{sech}^5(x)}{a+b\text{csch}(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 150

$$\int \frac{\text{sech}^5(x)}{a+b\text{csch}(x)} \, dx = \frac{a(3a^4-6a^2b^2-b^4)\arctan(\sinh(x))}{8\,(a^2+b^2)^3} + \frac{a^4b\log(\cosh(x))}{(a^2+b^2)^3} - \frac{a^4b\log(b+a\sinh(x))}{(a^2+b^2)^3} - \frac{a^2b\text{sech}^2(x)}{2\,(a^2+b^2)^2} - \frac{\text{sech}^4(x)(b-a\sinh(x))}{4\,(a^2+b^2)} + \frac{a(3a^2-b^2)\,\text{sech}(x)\tanh(x)}{8\,(a^2+b^2)^2}$$

output
```
1/8*a*(3*a^4-6*a^2*b^2-b^4)*arctan(sinh(x))/(a^2+b^2)^3+a^4*b*ln(cosh(x))/
(a^2+b^2)^3-a^4*b*ln(b+a*sinh(x))/(a^2+b^2)^3-1/2*a^2*b*sech(x)^2/(a^2+b^2
)^2-sech(x)^4*(b-a*sinh(x))/(4*a^2+4*b^2)+1/8*a*(3*a^2-b^2)*sech(x)*tanh(x
)/(a^2+b^2)^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.09

$$\int \frac{\text{sech}^5(x)}{a + b\text{csch}(x)}\,dx$$
$$= \frac{a(ia-b)^3(3a-ib)\log(i-\sinh(x)) - a(3a+ib)(ia+b)^3\log(i+\sinh(x)) - 16a^4b\log(b+a\sinh(x)) -}{}$$

input
```
Integrate[Sech[x]^5/(a + b*Csch[x]),x]
```

output
```
(a*(I*a - b)^3*(3*a - I*b)*Log[I - Sinh[x]] - a*(3*a + I*b)*(I*a + b)^3*Lo
g[I + Sinh[x]] - 16*a^4*b*Log[b + a*Sinh[x]] - 8*a^2*b*(a^2 + b^2)*Sech[x]
^2 - 4*b*(a^2 + b^2)^2*Sech[x]^4 + 2*a*(3*a^4 + 2*a^2*b^2 - b^4)*Sech[x]*T
anh[x] + 4*a*(a^2 + b^2)^2*Sech[x]^3*Tanh[x])/(16*(a^2 + b^2)^3)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.39, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.154$, Rules used = {3042, 4360, 26, 26, 3042, 26, 3316, 26, 27, 593, 25, 686, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^5(x)}{a + b\text{csch}(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\cos(ix)^5(a + ib\csc(ix))}dx$$

$$\Big\downarrow \; 4360$$

$$\int \frac{i\tanh(x)\text{sech}^4(x)}{ia\sinh(x) + ib}dx$$

$$\Big\downarrow \text{26}$$

$$i\int -\frac{i\text{sech}^4(x)\tanh(x)}{b+a\sinh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh(x)\text{sech}^4(x)}{a\sinh(x)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)}{\cos(ix)^5(b-ia\sin(ix))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ix)}{\cos(ix)^5(b-ia\sin(ix))}dx$$

$$\Big\downarrow \text{3316}$$

$$ia^5\int -\frac{i\sinh(x)}{(b+a\sinh(x))\left(\sinh^2(x)a^2+a^2\right)^3}d(a\sinh(x))$$

$$\Big\downarrow \text{26}$$

$$a^5\int \frac{\sinh(x)}{(b+a\sinh(x))\left(\sinh^2(x)a^2+a^2\right)^3}d(a\sinh(x))$$

$$\Big\downarrow \text{27}$$

$$a^4\int \frac{a\sinh(x)}{(b+a\sinh(x))\left(\sinh^2(x)a^2+a^2\right)^3}d(a\sinh(x))$$

$$\Big\downarrow \text{593}$$

$$a^4\left(\frac{\int -\frac{b-3a\sinh(x)}{(b+a\sinh(x))\left(\sinh^2(x)a^2+a^2\right)^2}d(a\sinh(x))}{4\left(a^2+b^2\right)}-\frac{b-a\sinh(x)}{4\left(a^2+b^2\right)\left(a^2\sinh^2(x)+a^2\right)^2}\right)$$

$$\Big\downarrow \text{25}$$

$$a^4\left(-\frac{\int \frac{b-3a\sinh(x)}{(b+a\sinh(x))\left(\sinh^2(x)a^2+a^2\right)^2}d(a\sinh(x))}{4\left(a^2+b^2\right)}-\frac{b-a\sinh(x)}{4\left(a^2+b^2\right)\left(a^2\sinh^2(x)+a^2\right)^2}\right)$$

$$\Big\downarrow \text{686}$$

$$a^4\left(-\frac{\frac{4a^2b-a(3a^2-b^2)\sinh(x)}{2a^2(a^2+b^2)(a^2\sinh^2(x)+a^2)}-\frac{\int -\frac{b(5a^2+b^2)-a(3a^2-b^2)\sinh(x)}{(b+a\sinh(x))(\sinh^2(x)a^2+a^2)}d(a\sinh(x))}{2a^2(a^2+b^2)}}{4(a^2+b^2)}-\frac{b-a\sinh(x)}{4(a^2+b^2)(a^2\sinh^2(x)+a^2)^2}\right)$$

$$\Big\downarrow \; 25$$

$$a^4\left(-\frac{\frac{\int \frac{b(5a^2+b^2)-a(3a^2-b^2)\sinh(x)}{(b+a\sinh(x))(\sinh^2(x)a^2+a^2)}d(a\sinh(x))}{2a^2(a^2+b^2)}+\frac{4a^2b-a(3a^2-b^2)\sinh(x)}{2a^2(a^2+b^2)(a^2\sinh^2(x)+a^2)}}{4(a^2+b^2)}-\frac{b-a\sinh(x)}{4(a^2+b^2)(a^2\sinh^2(x)+a^2)^2}\right)$$

$$\Big\downarrow \; 657$$

$$a^4\left(-\frac{\frac{\int \left(\frac{8ba^2}{(a^2+b^2)(b+a\sinh(x))}+\frac{-3a^4-8b\sinh(x)a^3+6b^2a^2+b^4}{(a^2+b^2)(\sinh^2(x)a^2+a^2)}\right)d(a\sinh(x))}{2a^2(a^2+b^2)}+\frac{4a^2b-a(3a^2-b^2)\sinh(x)}{2a^2(a^2+b^2)(a^2\sinh^2(x)+a^2)}}{4(a^2+b^2)}-\frac{b-a\sinh(x)}{4(a^2+b^2)(a^2\sinh^2(x$$

$$\Big\downarrow \; 2009$$

$$a^4\left(-\frac{b-a\sinh(x)}{4(a^2+b^2)(a^2\sinh^2(x)+a^2)^2}-\frac{\frac{4a^2b-a(3a^2-b^2)\sinh(x)}{2a^2(a^2+b^2)(a^2\sinh^2(x)+a^2)}+\frac{-\frac{4a^2b\log(a^2\sinh^2(x)+a^2)}{a^2+b^2}+\frac{8a^2b\log(a\sinh(x)+b)}{a^2+b^2}-\frac{(3a^4-6a^2}{}}{2a^2(a^2+b^2)}}{4(a^2+b^2)}\right.$$

input
```
Int[Sech[x]^5/(a + b*Csch[x]),x]
```

output
```
a^4*(-1/4*(b - a*Sinh[x])/((a^2 + b^2)*(a^2 + a^2*Sinh[x]^2)^2) - ((-(((3*
a^4 - 6*a^2*b^2 - b^4)*ArcTan[Sinh[x]])/(a*(a^2 + b^2))) + (8*a^2*b*Log[b
+ a*Sinh[x]])/(a^2 + b^2) - (4*a^2*b*Log[a^2 + a^2*Sinh[x]^2])/(a^2 + b^2)
)/(2*a^2*(a^2 + b^2)) + (4*a^2*b - a*(3*a^2 - b^2)*Sinh[x])/(2*a^2*(a^2 +
b^2)*(a^2 + a^2*Sinh[x]^2)))/(4*(a^2 + b^2)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4360
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(146) = 292$.

Time = 22.98 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.11

method	result
default	$-\frac{a^4 b \ln\left(-b \tanh\left(\frac{x}{2}\right)^2+2a\tanh\left(\frac{x}{2}\right)+b\right)}{(a^4+2a^2b^2+b^4)(a^2+b^2)}+\frac{2\left(\left(-\frac{5}{8}a^5-\frac{3}{4}a^3b^2-\frac{1}{8}b^4a\right)\tanh\left(\frac{x}{2}\right)^7+\left(2a^4b+3a^2b^3+b^5\right)\tanh\left(\frac{x}{2}\right)^6+\left(\frac{3}{8}a^5+\frac{5}{4}a^3b^2+\frac{7}{8}b^4a\right)\right.}{}$
risch	$\frac{(3a^3e^{6x}-a b^2e^{6x}-8a^2b e^{5x}+11a^3e^{4x}+7a b^2e^{4x}-32a^2b e^{3x}-16b^3e^{3x}-11a^3e^{2x}-7a b^2e^{2x}-8b e^x a^2-3a^3+a b^2)e^x}{4(a^2+b^2)^2(e^{2x}+1)^4}-\frac{3ia^5\ln(}{8(a^6+3a^4b^2+}$

input
```
int(sech(x)^5/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-a^4*b/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*ln(-b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b
)+2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*(((-5/8*a^5-3/4*a^3*b^2-1/8*b^4*a)*tanh(
1/2*x)^7+(2*a^4*b+3*a^2*b^3+b^5)*tanh(1/2*x)^6+(3/8*a^5+5/4*a^3*b^2+7/8*b^
4*a)*tanh(1/2*x)^5+(2*a^4*b+2*a^2*b^3)*tanh(1/2*x)^4+(-3/8*a^5-5/4*a^3*b^2
-7/8*b^4*a)*tanh(1/2*x)^3+(2*a^4*b+3*a^2*b^3+b^5)*tanh(1/2*x)^2+(5/8*a^5+3
/4*a^3*b^2+1/8*b^4*a)*tanh(1/2*x))/(tanh(1/2*x)^2+1)^4+1/8*a*(4*a^3*b*ln(t
anh(1/2*x)^2+1)+(3*a^4-6*a^2*b^2-b^4)*arctan(tanh(1/2*x))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2778 vs. $2(143) = 286$.

Time = 0.14 (sec) , antiderivative size = 2778, normalized size of antiderivative = 18.52

$$\int \frac{\operatorname{sech}^5(x)}{a + b\operatorname{csch}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^5/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
1/4*((3*a^5 + 2*a^3*b^2 - a*b^4)*cosh(x)^7 + (3*a^5 + 2*a^3*b^2 - a*b^4)*s
inh(x)^7 - 8*(a^4*b + a^2*b^3)*cosh(x)^6 - (8*a^4*b + 8*a^2*b^3 - 7*(3*a^5
 + 2*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^6 + (11*a^5 + 18*a^3*b^2 + 7*a*b^4)
*cosh(x)^5 + (11*a^5 + 18*a^3*b^2 + 7*a*b^4 + 21*(3*a^5 + 2*a^3*b^2 - a*b^
4)*cosh(x)^2 - 48*(a^4*b + a^2*b^3)*cosh(x))*sinh(x)^5 - 16*(2*a^4*b + 3*a
^2*b^3 + b^5)*cosh(x)^4 - (32*a^4*b + 48*a^2*b^3 + 16*b^5 - 35*(3*a^5 + 2*
a^3*b^2 - a*b^4)*cosh(x)^3 + 120*(a^4*b + a^2*b^3)*cosh(x)^2 - 5*(11*a^5 +
 18*a^3*b^2 + 7*a*b^4)*cosh(x))*sinh(x)^4 - (11*a^5 + 18*a^3*b^2 + 7*a*b^4
)*cosh(x)^3 - (11*a^5 + 18*a^3*b^2 + 7*a*b^4 - 35*(3*a^5 + 2*a^3*b^2 - a*b
^4)*cosh(x)^4 + 160*(a^4*b + a^2*b^3)*cosh(x)^3 - 10*(11*a^5 + 18*a^3*b^2
+ 7*a*b^4)*cosh(x)^2 + 64*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x))*sinh(x)^3 -
 8*(a^4*b + a^2*b^3)*cosh(x)^2 + (21*(3*a^5 + 2*a^3*b^2 - a*b^4)*cosh(x)^5
 - 8*a^4*b - 8*a^2*b^3 - 120*(a^4*b + a^2*b^3)*cosh(x)^4 + 10*(11*a^5 + 18
*a^3*b^2 + 7*a*b^4)*cosh(x)^3 - 96*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^2 -
 3*(11*a^5 + 18*a^3*b^2 + 7*a*b^4)*cosh(x))*sinh(x)^2 + ((3*a^5 - 6*a^3*b^
2 - a*b^4)*cosh(x)^8 + 8*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)*sinh(x)^7 + (
3*a^5 - 6*a^3*b^2 - a*b^4)*sinh(x)^8 + 4*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(
x)^6 + 4*(3*a^5 - 6*a^3*b^2 - a*b^4 + 7*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x
)^2)*sinh(x)^6 + 8*(7*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^3 + 3*(3*a^5 - 6
*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^5 + 3*a^5 - 6*a^3*b^2 - a*b^4 + 6*(3...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^5(x)}{a + b\operatorname{csch}(x)}\,dx = \int \frac{\operatorname{sech}^5(x)}{a + b\operatorname{csch}(x)}\,dx$$

input
```
integrate(sech(x)**5/(a+b*csch(x)),x)
```

output
```
Integral(sech(x)**5/(a + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(143) = 286.

Time = 0.13 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.32

$$\int \frac{\operatorname{sech}^5(x)}{a + b\operatorname{csch}(x)}\,dx = -\frac{a^4 b \log\left(-2\,be^{(-x)} + ae^{(-2\,x)} - a\right)}{a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6} + \frac{a^4 b \log\left(e^{(-2\,x)} + 1\right)}{a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6} - \frac{(3\,a^5 - 6\,a^3b^2 - ab^4)\arctan\left(e^{(-x)}\right)}{4\,(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6)} - \frac{8\,a^2be^{(-2\,x)} + 8\,a^2be^{(-6\,x)} - (3\,a^3 - ab^2)e^{(-x)} - (11\,a^3 + 7\,ab^2)e^{(-3\,x)} + 16\,(2\,a^2b + b^3)e^{(-4\,x)} + (11\,a^3 +}{4\,(a^4 + 2\,a^2b^2 + b^4 + 4\,(a^4 + 2\,a^2b^2 + b^4)e^{(-2\,x)} + 6\,(a^4 + 2\,a^2b^2 + b^4)e^{(-4\,x)} + 4\,(a^4 + 2\,a^2b^2 + b^4)e^{(-}}$$

input
```
integrate(sech(x)^5/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
-a^4*b*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6) + a^4*b*log(e^(-2*x) + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(3*
a^5 - 6*a^3*b^2 - a*b^4)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
) - 1/4*(8*a^2*b*e^(-2*x) + 8*a^2*b*e^(-6*x) - (3*a^3 - a*b^2)*e^(-x) - (1
1*a^3 + 7*a*b^2)*e^(-3*x) + 16*(2*a^2*b + b^3)*e^(-4*x) + (11*a^3 + 7*a*b^
2)*e^(-5*x) + (3*a^3 - a*b^2)*e^(-7*x))/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 +
2*a^2*b^2 + b^4)*e^(-2*x) + 6*(a^4 + 2*a^2*b^2 + b^4)*e^(-4*x) + 4*(a^4 +
2*a^2*b^2 + b^4)*e^(-6*x) + (a^4 + 2*a^2*b^2 + b^4)*e^(-8*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(143) = 286$.

Time = 0.13 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.49

$$\int \frac{\operatorname{sech}^5(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{a^5 b \log\left(\left|-a\left(e^{(-x)}-e^x\right)+2\,b\right|\right)}{a^7+3\,a^5b^2+3\,a^3b^4+ab^6}+\frac{a^4 b \log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)} + \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)(3\,a^5-6\,a^3b^2-ab^4)}{16\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)} - \frac{3\,a^4b\left(e^{(-x)}-e^x\right)^4+3\,a^5\left(e^{(-x)}-e^x\right)^3+2\,a^3b^2\left(e^{(-x)}-e^x\right)^3-ab^4\left(e^{(-x)}-e^x\right)^3+32\,a^4b\left(e^{(-x)}-e^x\right)^2 \cdots}{4\,(a^6+3\,a^4b^2+3\,a^2 \cdots}$$

input
```
integrate(sech(x)^5/(a+b*csch(x)),x, algorithm="giac")
```

output
```
-a^5*b*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*
b^6) + 1/2*a^4*b*log((e^(-x) - e^x)^2 + 4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6) + 1/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(3*a^5 - 6*a^3*b^2 -
 a*b^4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(3*a^4*b*(e^(-x) - e^x)^
4 + 3*a^5*(e^(-x) - e^x)^3 + 2*a^3*b^2*(e^(-x) - e^x)^3 - a*b^4*(e^(-x) -
e^x)^3 + 32*a^4*b*(e^(-x) - e^x)^2 + 8*a^2*b^3*(e^(-x) - e^x)^2 + 20*a^5*(
e^(-x) - e^x) + 24*a^3*b^2*(e^(-x) - e^x) + 4*a*b^4*(e^(-x) - e^x) + 96*a^
4*b + 64*a^2*b^3 + 16*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*((e^(-x) -
 e^x)^2 + 4)^2)
```

Mupad [B] (verification not implemented)

Time = 6.87 (sec) , antiderivative size = 513, normalized size of antiderivative = 3.42

$$\int \frac{\operatorname{sech}^5(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\frac{8\,(a^2\,b+b^3)}{(a^2+b^2)^2}-\frac{6\,e^x\,(a^3+a\,b^2)}{(a^2+b^2)^2}}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1}-\frac{\frac{2\,(a^2\,b+2\,b^3)}{(a^2+b^2)^2}-\frac{e^x\,(a^3+5\,a\,b^2)}{2\,(a^2+b^2)^2}}{2\,e^{2\,x}+e^{4\,x}+1} - \frac{\frac{4\,b}{a^2+b^2}-\frac{4\,a\,e^x}{a^2+b^2}}{4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1}-\frac{\frac{2\,(a^4\,b+a^2\,b^3)}{(a^2+b^2)^3}-\frac{e^x\,(3\,a^5+2\,a^3\,b^2-a\,b^4)}{4\,(a^2+b^2)^3}}{e^{2\,x}+1} + \frac{\ln\left(e^x+1i\right)\,(a\,b-a^2\,3i)}{8\,(-a^3-a^2\,b\,3i+3\,a\,b^2+b^3\,1i)}+\frac{\ln\left(1+e^x\,1i\right)\,(-3\,a^2+a\,b\,1i)}{8\,(-a^3\,1i-3\,a^2\,b+a\,b^2\,3i+b^3)} - \frac{a^4\,b\,\ln\left(9\,a^{10}\,e^{2\,x}-9\,a^{10}-a^2\,b^8-12\,a^4\,b^6-30\,a^6\,b^4-220\,a^8\,b^2+a^2\,b^8\,e^{2\,x}+12\,a^4\,b^6\,e^{2\,x}+30\,a^6\,b^4\,e^{2\,x} \cdots\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b \cdots}$$

input
```
int(1/(cosh(x)^5*(a + b/sinh(x))),x)
```

output
```
((8*(a^2*b + b^3))/(a^2 + b^2)^2 - (6*exp(x)*(a*b^2 + a^3))/(a^2 + b^2)^2)
/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - ((2*(a^2*b + 2*b^3))/(a^2 + b^
2)^2 - (exp(x)*(5*a*b^2 + a^3))/(2*(a^2 + b^2)^2))/(2*exp(2*x) + exp(4*x)
+ 1) - ((4*b)/(a^2 + b^2) - (4*a*exp(x))/(a^2 + b^2))/(4*exp(2*x) + 6*exp(
4*x) + 4*exp(6*x) + exp(8*x) + 1) - ((2*(a^4*b + a^2*b^3))/(a^2 + b^2)^3 -
 (exp(x)*(3*a^5 - a*b^4 + 2*a^3*b^2))/(4*(a^2 + b^2)^3))/(exp(2*x) + 1) +
(log(exp(x) + 1i)*(a*b - a^2*3i))/(8*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i))
+ (log(exp(x)*1i + 1)*(a*b*1i - 3*a^2))/(8*(a*b^2*3i - 3*a^2*b - a^3*1i +
b^3)) - (a^4*b*log(9*a^10*exp(2*x) - 9*a^10 - a^2*b^8 - 12*a^4*b^6 - 30*a^
6*b^4 - 220*a^8*b^2 + a^2*b^8*exp(2*x) + 12*a^4*b^6*exp(2*x) + 30*a^6*b^4*
exp(2*x) + 220*a^8*b^2*exp(2*x) + 2*a*b^9*exp(x) + 18*a^9*b*exp(x) + 24*a^
3*b^7*exp(x) + 60*a^5*b^5*exp(x) + 440*a^7*b^3*exp(x)))/(a^6 + b^6 + 3*a^2
*b^4 + 3*a^4*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 861, normalized size of antiderivative = 5.74

$$\int \frac{\operatorname{sech}^5(x)}{a + b\operatorname{csch}(x)}\,dx = \text{Too large to display}$$

input
```
int(sech(x)^5/(a+b*csch(x)),x)
```

output
```
(3*e**(8*x)*atan(e**x)*a**5 - 6*e**(8*x)*atan(e**x)*a**3*b**2 - e**(8*x)*a
tan(e**x)*a*b**4 + 12*e**(6*x)*atan(e**x)*a**5 - 24*e**(6*x)*atan(e**x)*a*
*3*b**2 - 4*e**(6*x)*atan(e**x)*a*b**4 + 18*e**(4*x)*atan(e**x)*a**5 - 36*
e**(4*x)*atan(e**x)*a**3*b**2 - 6*e**(4*x)*atan(e**x)*a*b**4 + 12*e**(2*x)
*atan(e**x)*a**5 - 24*e**(2*x)*atan(e**x)*a**3*b**2 - 4*e**(2*x)*atan(e**x
)*a*b**4 + 3*atan(e**x)*a**5 - 6*atan(e**x)*a**3*b**2 - atan(e**x)*a*b**4
+ 4*e**(8*x)*log(e**(2*x) + 1)*a**4*b - 4*e**(8*x)*log(e**(2*x)*a + 2*e**x
*b - a)*a**4*b + 2*e**(8*x)*a**4*b + 2*e**(8*x)*a**2*b**3 + 3*e**(7*x)*a**
5 + 2*e**(7*x)*a**3*b**2 - e**(7*x)*a*b**4 + 16*e**(6*x)*log(e**(2*x) + 1)
*a**4*b - 16*e**(6*x)*log(e**(2*x)*a + 2*e**x*b - a)*a**4*b + 11*e**(5*x)*
a**5 + 18*e**(5*x)*a**3*b**2 + 7*e**(5*x)*a*b**4 + 24*e**(4*x)*log(e**(2*x
) + 1)*a**4*b - 24*e**(4*x)*log(e**(2*x)*a + 2*e**x*b - a)*a**4*b - 20*e**
(4*x)*a**4*b - 36*e**(4*x)*a**2*b**3 - 16*e**(4*x)*b**5 - 11*e**(3*x)*a**5
 - 18*e**(3*x)*a**3*b**2 - 7*e**(3*x)*a*b**4 + 16*e**(2*x)*log(e**(2*x) +
1)*a**4*b - 16*e**(2*x)*log(e**(2*x)*a + 2*e**x*b - a)*a**4*b - 3*e**x*a**
5 - 2*e**x*a**3*b**2 + e**x*a*b**4 + 4*log(e**(2*x) + 1)*a**4*b - 4*log(e*
*(2*x)*a + 2*e**x*b - a)*a**4*b + 2*a**4*b + 2*a**2*b**3)/(4*(e**(8*x)*a**
6 + 3*e**(8*x)*a**4*b**2 + 3*e**(8*x)*a**2*b**4 + e**(8*x)*b**6 + 4*e**(6*
x)*a**6 + 12*e**(6*x)*a**4*b**2 + 12*e**(6*x)*a**2*b**4 + 4*e**(6*x)*b**6
+ 6*e**(4*x)*a**6 + 18*e**(4*x)*a**4*b**2 + 18*e**(4*x)*a**2*b**4 + 6*e...
```

3.102 $\int \frac{\tanh^5(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 109

$$\int \frac{\tanh^5(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{21}{32}i\log(i-\sinh(x))-\frac{11}{32}i\log(i+\sinh(x)) + \frac{i}{32(1-i\sinh(x))^2}-\frac{i}{4(1-i\sinh(x))}-\frac{i}{24(1+i\sinh(x))^3} + \frac{9i}{32(1+i\sinh(x))^2}-\frac{15i}{16(1+i\sinh(x))}$$

output
```
-21/32*I*ln(I-sinh(x))-11/32*I*ln(I+sinh(x))+1/32*I/(1-I*sinh(x))^2-1/4*I/
(1-I*sinh(x))-1/24*I/(1+I*sinh(x))^3+9/32*I/(1+I*sinh(x))^2-15/16*I/(1+I*s
inh(x))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.69

$$\int \frac{\tanh^5(x)}{i+\operatorname{csch}(x)}\,dx = \frac{1}{96}\left(-63i\log(i-\sinh(x))-33i\log(i+\sinh(x)) - \frac{2\left(44+29i\sinh(x)+79\sinh^2(x)+39i\sinh^3(x)+33\sinh^4(x)\right)}{(-i+\sinh(x))^3(i+\sinh(x))^2}\right)$$

input
```
Integrate[Tanh[x]^5/(I + Csch[x]),x]
```

output
```
((-63*I)*Log[I - Sinh[x]] - (33*I)*Log[I + Sinh[x]] - (2*(44 + (29*I)*Sinh
[x] + 79*Sinh[x]^2 + (39*I)*Sinh[x]^3 + 33*Sinh[x]^4))/((-I + Sinh[x])^3*(
I + Sinh[x])^2))/96
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 26, 26, 4367, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(x)}{\text{csch}(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\cot(ix)^5(i\csc(ix)+i)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int -\frac{i}{\cot(ix)^5(\csc(ix)+1)}dx$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{1}{\cot(ix)^5(\csc(ix)+1)}dx$$

$$\Big\downarrow \text{4367}$$

$$i\int -\frac{\sinh^6(x)}{(1-i\sinh(x))^3(i\sinh(x)+1)^4}d(i\sinh(x))$$

$$\Big\downarrow \text{99}$$

$$i\int \left(-\frac{21}{32(i\sinh(x)+1)}+\frac{15}{16(i\sinh(x)+1)^2}-\frac{9}{16(i\sinh(x)+1)^3}+\frac{1}{8(i\sinh(x)+1)^4}-\frac{11}{32(i\sinh(x)-1)}-\frac{4}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{1}{4(1-i\sinh(x))}-\frac{15}{16(1+i\sinh(x))}+\frac{1}{32(1-i\sinh(x))^2}+\frac{9}{32(1+i\sinh(x))^2}-\frac{1}{24(1+i\sinh(x))^3}-\frac{11}{32}\mathrm{lo}\right.$$

input
```
Int[Tanh[x]^5/(I + Csch[x]),x]
```

output
```
I*((-11*Log[1 - I*Sinh[x]])/32 - (21*Log[1 + I*Sinh[x]])/32 + 1/(32*(1 - I
*Sinh[x])^2) - 1/(4*(1 - I*Sinh[x])) - 1/(24*(1 + I*Sinh[x])^3) + 9/(32*(1
 + I*Sinh[x])^2) - 15/(16*(1 + I*Sinh[x])))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

method	result
risch	$ix-\frac{-78i\,\mathrm{e}^{2x}+184\,\mathrm{e}^{3x}+2i\,\mathrm{e}^{4x}+270\,\mathrm{e}^{5x}+33\,\mathrm{e}^{x}-2i\,\mathrm{e}^{6x}+184\,\mathrm{e}^{7x}+78i\,\mathrm{e}^{8x}+33\,\mathrm{e}^{9x}}{24(\mathrm{e}^{x}-i)^{6}(\mathrm{e}^{x}+i)^{4}}-\frac{21i\ln(\mathrm{e}^{x}-i)}{16}-\frac{11i\ln(\mathrm{e}^{x}+i)}{16}$
default	$-\frac{21i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)}{16}+\frac{3i}{8\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}+\frac{i}{3\left(\tanh\left(\frac{x}{2}\right)-i\right)^{6}}-\frac{3i}{8\left(\tanh\left(\frac{x}{2}\right)-i\right)^{4}}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{5}}+\frac{11}{12\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}}+$

input
```
int(tanh(x)^5/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
I*x-1/24*(-78*I*exp(x)^2+184*exp(x)^3+2*I*exp(x)^4+270*exp(x)^5+33*exp(x)-
2*I*exp(x)^6+184*exp(x)^7+78*I*exp(x)^8+33*exp(x)^9)/(exp(x)-I)^6/(exp(x)+
I)^4-21/16*I*ln(exp(x)-I)-11/16*I*ln(exp(x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(67) = 134.

Time = 0.09 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.79

$$\int \frac{\tanh^5(x)}{i+\operatorname{csch}(x)}\,dx = \frac{48i\,xe^{(10\,x)}+6\,(16\,x-11)e^{(9\,x)}-12\,(-12i\,x+13i)e^{(8\,x)}+16\,(24\,x-23)e^{(7\,x)}-4\,(-24i\,x-i)e^{(6\,x)}+36}{}$$

input
```
integrate(tanh(x)^5/(I+csch(x)),x, algorithm="fricas")
```

output
```
1/48*(48*I*x*e^(10*x) + 6*(16*x - 11)*e^(9*x) - 12*(-12*I*x + 13*I)*e^(8*x
) + 16*(24*x - 23)*e^(7*x) - 4*(-24*I*x - I)*e^(6*x) + 36*(16*x - 15)*e^(5
*x) - 4*(24*I*x + I)*e^(4*x) + 16*(24*x - 23)*e^(3*x) - 12*(12*I*x - 13*I)
*e^(2*x) + 6*(16*x - 11)*e^x - 33*(I*e^(10*x) + 2*e^(9*x) + 3*I*e^(8*x) +
8*e^(7*x) + 2*I*e^(6*x) + 12*e^(5*x) - 2*I*e^(4*x) + 8*e^(3*x) - 3*I*e^(2*
x) + 2*e^x - I)*log(e^x + I) - 63*(I*e^(10*x) + 2*e^(9*x) + 3*I*e^(8*x) +
8*e^(7*x) + 2*I*e^(6*x) + 12*e^(5*x) - 2*I*e^(4*x) + 8*e^(3*x) - 3*I*e^(2*
x) + 2*e^x - I)*log(e^x - I) - 48*I*x)/(e^(10*x) - 2*I*e^(9*x) + 3*e^(8*x)
 - 8*I*e^(7*x) + 2*e^(6*x) - 12*I*e^(5*x) - 2*e^(4*x) - 8*I*e^(3*x) - 3*e^
(2*x) - 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\tanh^5(x)}{i + \operatorname{csch}(x)}\,dx = \int \frac{\tanh^5(x)}{\operatorname{csch}(x) + i}\,dx$$

input
```
integrate(tanh(x)**5/(I+csch(x)),x)
```

output
```
Integral(tanh(x)**5/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(67) = 134.

Time = 0.04 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.32

$$\int \frac{\tanh^5(x)}{i + \operatorname{csch}(x)}\,dx = -i\,x + \frac{33\,e^{(-x)} + 78i\,e^{(-2\,x)} + 184\,e^{(-3\,x)} - 2i\,e^{(-4\,x)} + 270\,e^{(-5\,x)} + 2i\,e^{(-6\,x)} + 184\,e^{(-7\,x)} - 78i\,e^{(-8\,x)} \cdots}{48i\,e^{(-x)} - 72\,e^{(-2\,x)} + 192i\,e^{(-3\,x)} - 48\,e^{(-4\,x)} + 288i\,e^{(-5\,x)} + 48\,e^{(-6\,x)} + 192i\,e^{(-7\,x)} + 72\,e^{(-8\,x)} + 48 \cdots} - \frac{11}{16}i\,\log\left(e^{(-x)} - i\right) - \frac{21}{16}i\,\log\left(i\,e^{(-x)} - 1\right)$$

input
```
integrate(tanh(x)^5/(I+csch(x)),x, algorithm="maxima")
```

output
```
-I*x + (33*e^(-x) + 78*I*e^(-2*x) + 184*e^(-3*x) - 2*I*e^(-4*x) + 270*e^(-
5*x) + 2*I*e^(-6*x) + 184*e^(-7*x) - 78*I*e^(-8*x) + 33*e^(-9*x))/(48*I*e^
(-x) - 72*e^(-2*x) + 192*I*e^(-3*x) - 48*e^(-4*x) + 288*I*e^(-5*x) + 48*e^
(-6*x) + 192*I*e^(-7*x) + 72*e^(-8*x) + 48*I*e^(-9*x) + 24*e^(-10*x) - 24)
 - 11/16*I*log(e^(-x) - I) - 21/16*I*log(I*e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.10

$$\begin{aligned}\int \frac{\tanh^5(x)}{i+\operatorname{csch}(x)}\,dx &= -\frac{33i\left(e^{(-x)}-e^{x}\right)^2+100\,e^{(-x)}-100\,e^{x}-76i}{64\left(-i\,e^{(-x)}+i\,e^{x}-2\right)^2} \\ &\quad - \frac{-231i\left(e^{(-x)}-e^{x}\right)^3+1026\left(e^{(-x)}-e^{x}\right)^2+1548i\,e^{(-x)}-1548i\,e^{x}-776}{192\left(e^{(-x)}-e^{x}+2i\right)^3} \\ &\quad - \frac{11}{32}i\,\log\left(-e^{(-x)}+e^{x}+2i\right)-\frac{21}{32}i\,\log\left(-e^{(-x)}+e^{x}-2i\right)\end{aligned}$$

input
```
integrate(tanh(x)^5/(I+csch(x)),x, algorithm="giac")
```

output
```
-1/64*(33*I*(e^(-x) - e^x)^2 + 100*e^(-x) - 100*e^x - 76*I)/(-I*e^(-x) + I
*e^x - 2)^2 - 1/192*(-231*I*(e^(-x) - e^x)^3 + 1026*(e^(-x) - e^x)^2 + 154
8*I*e^(-x) - 1548*I*e^x - 776)/(e^(-x) - e^x + 2*I)^3 - 11/32*I*log(-e^(-x
) + e^x + 2*I) - 21/32*I*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 5.38 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.51

$$\int \frac{\tanh^5(x)}{i+\operatorname{csch}(x)}\,dx = x\,1\mathrm{i} - \ln\left(\left(\frac{5\,\mathrm{e}^x}{8}-\frac{5}{8}\mathrm{i}\right)\left(\frac{5\,\mathrm{e}^x}{8}+\frac{5}{8}\mathrm{i}\right)\right)1\mathrm{i} + \frac{5\,\mathrm{atan}(\mathrm{e}^x)}{8} + \frac{1\mathrm{i}}{3\,(15\,\mathrm{e}^{4\,x}-15\,\mathrm{e}^{2\,x}-\mathrm{e}^{6\,x}+1-\mathrm{e}^{3\,x}\,20\mathrm{i}+\mathrm{e}^{5\,x}\,6\mathrm{i}+\mathrm{e}^x\,6\mathrm{i})} - \frac{1}{\mathrm{e}^{2\,x}\,10\mathrm{i}-10\,\mathrm{e}^{3\,x}-\mathrm{e}^{4\,x}\,5\mathrm{i}+\mathrm{e}^{5\,x}+5\,\mathrm{e}^x-\mathrm{i}} - \frac{31}{12\,(\mathrm{e}^{2\,x}\,3\mathrm{i}-\mathrm{e}^{3\,x}+3\,\mathrm{e}^x-\mathrm{i})} - \frac{5\mathrm{i}}{8\,(\mathrm{e}^{2\,x}-1+\mathrm{e}^x\,2\mathrm{i})} + \frac{17\mathrm{i}}{8\,(\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1-\mathrm{e}^{3\,x}\,4\mathrm{i}+\mathrm{e}^x\,4\mathrm{i})} + \frac{1\mathrm{i}}{8\,(\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1+\mathrm{e}^{3\,x}\,4\mathrm{i}-\mathrm{e}^x\,4\mathrm{i})} + \frac{3\mathrm{i}}{1-\mathrm{e}^{2\,x}+\mathrm{e}^x\,2\mathrm{i}} - \frac{15}{8\,(\mathrm{e}^x-\mathrm{i})} + \frac{1}{2\,(\mathrm{e}^x+1\mathrm{i})} - \frac{1}{4\,(\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^x-\mathrm{i})}$$

input
```
int(tanh(x)^5/(1/sinh(x) + 1i),x)
```

output
```
x*1i - log(((5*exp(x))/8 - 5i/8)*((5*exp(x))/8 + 5i/8))*1i + (5*atan(exp(x
)))/8 + 1i/(3*(15*exp(4*x) - exp(3*x)*20i - 15*exp(2*x) + exp(5*x)*6i - ex
p(6*x) + exp(x)*6i + 1)) - 1/(exp(2*x)*10i - 10*exp(3*x) - exp(4*x)*5i + e
xp(5*x) + 5*exp(x) - 1i) - 31/(12*(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i)
) - 5i/(8*(exp(2*x) + exp(x)*2i - 1)) + 17i/(8*(exp(4*x) - exp(3*x)*4i - 6
*exp(2*x) + exp(x)*4i + 1)) + 1i/(8*(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) -
 exp(x)*4i + 1)) + 3i/(exp(x)*2i - exp(2*x) + 1) - 15/(8*(exp(x) - 1i)) +
1/(2*(exp(x) + 1i)) - 1/(4*(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i))
```

Reduce [F]

$$\int \frac{\tanh^5(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\tanh(x)^5}{\operatorname{csch}(x)+i}dx$$

input
```
int(tanh(x)^5/(I+csch(x)),x)
```

output
```
int(tanh(x)**5/(csch(x) + i),x)
```

3.103 $\int \frac{\tanh^4(x)}{i+\text{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \frac{\tanh^4(x)}{i+\text{csch}(x)}\,dx = -ix+\frac{1}{15}(15i-8\text{csch}(x))\tanh(x) + \frac{1}{15}(5i-4\text{csch}(x))\tanh^3(x)+\frac{1}{5}(i-\text{csch}(x))\tanh^5(x)$$

output
```
-I*x+1/15*(15*I-8*csch(x))*tanh(x)+1/15*(5*I-4*csch(x))*tanh(x)^3+1/5*(I-c
sch(x))*tanh(x)^5
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 126 vs. 2(52) = 104.

Time = 0.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.42

$$\int \frac{\tanh^4(x)}{i+\text{csch}(x)}\,dx = \frac{-200+6(89-120ix)\cosh(x)-128\cosh(2x)+178\cosh(3x)-240ix\cosh(3x)-184\cosh(4x)+64i\,\text{si}}{960\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^3\left(\cosh\right.}$$

input
```
Integrate[Tanh[x]^4/(I + Csch[x]),x]
```

output
```
(-200 + 6*(89 - (120*I)*x)*Cosh[x] - 128*Cosh[2*x] + 178*Cosh[3*x] - (240*
I)*x*Cosh[3*x] - 184*Cosh[4*x] + (64*I)*Sinh[x] + (178*I)*Sinh[2*x] + 240*
x*Sinh[2*x] + (128*I)*Sinh[3*x] + (89*I)*Sinh[4*x] + 120*x*Sinh[4*x])/(960
*(Cosh[x/2] - I*Sinh[x/2])^3*(Cosh[x/2] + I*Sinh[x/2])^5)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4376, 3042, 25, 26, 4370, 25, 3042, 4370, 3042, 25, 4370, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(x)}{\text{csch}(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cot(ix)^4(i\csc(ix)+i)}dx$$

$$\Big\downarrow \text{4376}$$

$$-\int (i-\text{csch}(x))\tanh^6(x)dx$$

$$\Big\downarrow \text{3042}$$

$$-\int -\frac{i-i\csc(ix)}{\cot(ix)^6}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{i(1-\csc(ix))}{\cot(ix)^6}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1-\csc(ix)}{\cot(ix)^6}dx$$

$$\Big\downarrow \text{4370}$$

$$i\left(\frac{1}{5}\int -\left((4i\text{csch}(x)+5)\tanh^4(x)\right)dx+\frac{1}{5}\tanh^5(x)(1+i\text{csch}(x))\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(\frac{1}{5}\tanh^5(x)(1+i\text{csch}(x))-\frac{1}{5}\int (4i\text{csch}(x)+5)\tanh^4(x)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{1}{5}\tanh^5(x)(1+i\text{csch}(x))-\frac{1}{5}\int \frac{5-4\csc(ix)}{\cot(ix)^4}dx\right)$$

$$\Big\downarrow \text{4370}$$

$$i\left(\frac{1}{5}\left(\frac{1}{3}\tanh^3(x)(5+4i\text{csch}(x))-\frac{1}{3}\int (8i\text{csch}(x)+15)\tanh^2(x)dx\right)+\frac{1}{5}\tanh^5(x)(1+i\text{csch}(x))\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{1}{5}\left(\frac{1}{3}\tanh^3(x)(5+4i\text{csch}(x))-\frac{1}{3}\int -\frac{15-8\csc(ix)}{\cot(ix)^2}dx\right)+\frac{1}{5}\tanh^5(x)(1+i\text{csch}(x))\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{15-8\csc(ix)}{\cot(ix)^2}dx+\frac{1}{3}\tanh^3(x)(5+4i\text{csch}(x))\right)+\frac{1}{5}\tanh^5(x)(1+i\text{csch}(x))\right)$$

$$\Big\downarrow \text{4370}$$

$$i\left(\frac{1}{5}\left(\frac{1}{3}\left(\int -15dx+\tanh(x)(15+8i\text{csch}(x))\right)+\frac{1}{3}\tanh^3(x)(5+4i\text{csch}(x))\right)+\frac{1}{5}\tanh^5(x)(1+i\text{csch}(x))\right)$$

$$\Big\downarrow \text{24}$$

$$i\left(\frac{1}{5}\tanh^5(x)(1+i\text{csch}(x))+\frac{1}{5}\left(\frac{1}{3}\tanh^3(x)(5+4i\text{csch}(x))+\frac{1}{3}(-15x+\tanh(x)(15+8i\text{csch}(x)))\right)\right)$$

input
```
Int[Tanh[x]^4/(I + Csch[x]),x]
```

output
```
I*(((1 + I*Csch[x])*Tanh[x]^5)/5 + (((5 + (4*I)*Csch[x])*Tanh[x]^3)/3 + (-
15*x + (15 + (8*I)*Csch[x])*Tanh[x])/3)/5)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.31

method	result
risch	$-ix-\frac{2\left(-31i\,{\mathrm e}^{2x}+73\,{\mathrm e}^{3x}+31\,{\mathrm e}^{x}-25i\,{\mathrm e}^{4x}+65\,{\mathrm e}^{5x}-23i+15i\,{\mathrm e}^{6x}+15\,{\mathrm e}^{7x}\right)}{15\left({\mathrm e}^{x}+i\right)^{3}\left({\mathrm e}^{x}-i\right)^{5}}$
default	$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{5i}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)}+\frac{i}{6\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{11i}{8\left(\tanh\left(\frac{x}{2}\right)-i\right)}+\frac{2i}{5\left(\tanh\left(\frac{x}{2}\right)-i\right)^{5}}+$

input
```
int(tanh(x)^4/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-I*x-2/15*(-31*I*exp(x)^2+73*exp(x)^3+31*exp(x)-25*I*exp(x)^4+65*exp(x)^5-
23*I+15*I*exp(x)^6+15*exp(x)^7)/(exp(x)+I)^3/(exp(x)-I)^5
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(36) = 72$.

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.38

$$\int \frac{\tanh^4(x)}{i + \operatorname{csch}(x)}\,dx = \frac{-15i\,xe^{(8\,x)} - 30\,(x+1)e^{(7\,x)} - 30\,(i\,x+i)e^{(6\,x)} - 10\,(9\,x+13)e^{(5\,x)} - 2\,(45\,x+73)e^{(3\,x)} - 2\,(-15i\,x - }{15\,(e^{(8\,x)} - 2i\,e^{(7\,x)} + 2\,e^{(6\,x)} - 6i\,e^{(5\,x)} - 6i\,e^{(3\,x)} - 2\,e^{(2\,x)} - 2}$$

input
```
integrate(tanh(x)^4/(I+csch(x)),x, algorithm="fricas")
```

output
```
1/15*(-15*I*x*e^(8*x) - 30*(x + 1)*e^(7*x) - 30*(I*x + I)*e^(6*x) - 10*(9*
x + 13)*e^(5*x) - 2*(45*x + 73)*e^(3*x) - 2*(-15*I*x - 31*I)*e^(2*x) - 2*(
15*x + 31)*e^x + 15*I*x + 50*I*e^(4*x) + 46*I)/(e^(8*x) - 2*I*e^(7*x) + 2*
e^(6*x) - 6*I*e^(5*x) - 6*I*e^(3*x) - 2*e^(2*x) - 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{i + \operatorname{csch}(x)}\,dx = \int \frac{\tanh^4(x)}{\operatorname{csch}(x) + i}\,dx$$

input
```
integrate(tanh(x)**4/(I+csch(x)),x)
```

output
```
Integral(tanh(x)**4/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(36) = 72$.

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.85

$$\int \frac{\tanh^4(x)}{i+\operatorname{csch}(x)}\,dx = -i\,x - \frac{2\left(31\,e^{(-x)} + 31i\,e^{(-2\,x)} + 73\,e^{(-3\,x)} + 25i\,e^{(-4\,x)} + 65\,e^{(-5\,x)} - 15i\,e^{(-6\,x)} + 15\,e^{(-7\,x)} + 23i\right)}{30i\,e^{(-x)} - 30\,e^{(-2\,x)} + 90i\,e^{(-3\,x)} + 90i\,e^{(-5\,x)} + 30\,e^{(-6\,x)} + 30i\,e^{(-7\,x)} + 15\,e^{(-8\,x)} - 15}$$

input
```
integrate(tanh(x)^4/(I+csch(x)),x, algorithm="maxima")
```

output
```
-I*x - 2*(31*e^(-x) + 31*I*e^(-2*x) + 73*e^(-3*x) + 25*I*e^(-4*x) + 65*e^(
-5*x) - 15*I*e^(-6*x) + 15*e^(-7*x) + 23*I)/(30*I*e^(-x) - 30*e^(-2*x) + 9
0*I*e^(-3*x) + 90*I*e^(-5*x) + 30*e^(-6*x) + 30*I*e^(-7*x) + 15*e^(-8*x) -
 15)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^4(x)}{i+\operatorname{csch}(x)}\,dx = -i\,x - \frac{21i\,e^{(2\,x)} - 36\,e^{x} - 19i}{24\,(i\,e^{x} - 1)^3} - \frac{115\,e^{(4\,x)} - 380i\,e^{(3\,x)} - 530\,e^{(2\,x)} + 340i\,e^{x} + 91}{40\,(e^{x} - i)^5}$$

input
```
integrate(tanh(x)^4/(I+csch(x)),x, algorithm="giac")
```

output
```
-I*x - 1/24*(21*I*e^(2*x) - 36*e^x - 19*I)/(I*e^x - 1)^3 - 1/40*(115*e^(4*
x) - 380*I*e^(3*x) - 530*e^(2*x) + 340*I*e^x + 91)/(e^x - I)^5
```

Mupad [B] (verification not implemented)

Time = 3.57 (sec) , antiderivative size = 237, normalized size of antiderivative = 4.56

$$\int \frac{\tanh^4(x)}{i+\operatorname{csch}(x)}\,dx = -x\,1\mathrm{i} - \frac{1\mathrm{i}}{4\,\left(\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}\right)} + \frac{\frac{23\,\mathrm{e}^{x}}{40}-\frac{3}{8}\mathrm{i}}{1-\mathrm{e}^{2\,x}+\mathrm{e}^{x}\,2\mathrm{i}} - \frac{23}{40\,\left(\mathrm{e}^{x}-\mathrm{i}\right)} + \frac{7}{8\,\left(\mathrm{e}^{x}+1\mathrm{i}\right)} + \frac{\frac{\mathrm{e}^{2\,x}\,9\mathrm{i}}{8}-\frac{23\,\mathrm{e}^{3\,x}}{40}+\frac{9\,\mathrm{e}^{x}}{8}-\frac{3}{8}\mathrm{i}}{\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1-\mathrm{e}^{3\,x}\,4\mathrm{i}+\mathrm{e}^{x}\,4\mathrm{i}} - \frac{\frac{3}{8}-\frac{23\,\mathrm{e}^{2\,x}}{40}+\frac{\mathrm{e}^{x}\,3\mathrm{i}}{4}}{\mathrm{e}^{2\,x}\,3\mathrm{i}-\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}-\mathrm{i}} - \frac{1}{6\,\left(\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^{x}-\mathrm{i}\right)} - \frac{\frac{23\,\mathrm{e}^{4\,x}}{40}-\frac{9\,\mathrm{e}^{2\,x}}{4}+\frac{23}{40}-\frac{\mathrm{e}^{3\,x}\,3\mathrm{i}}{2}+\frac{\mathrm{e}^{x}\,3\mathrm{i}}{2}}{\mathrm{e}^{2\,x}\,10\mathrm{i}-10\,\mathrm{e}^{3\,x}-\mathrm{e}^{4\,x}\,5\mathrm{i}+\mathrm{e}^{5\,x}+5\,\mathrm{e}^{x}-\mathrm{i}}$$

input
```
int(tanh(x)^4/(1/sinh(x) + 1i),x)
```

output
```
((23*exp(x))/40 - 3i/8)/(exp(x)*2i - exp(2*x) + 1) - 1i/(4*(exp(2*x) + exp
(x)*2i - 1)) - x*1i - 23/(40*(exp(x) - 1i)) + 7/(8*(exp(x) + 1i)) + ((exp(
2*x)*9i)/8 - (23*exp(3*x))/40 + (9*exp(x))/8 - 3i/8)/(exp(4*x) - exp(3*x)*
4i - 6*exp(2*x) + exp(x)*4i + 1) - ((exp(x)*3i)/4 - (23*exp(2*x))/40 + 3/8
)/(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) - 1/(6*(exp(2*x)*3i + exp(3*x)
- 3*exp(x) - 1i)) - ((23*exp(4*x))/40 - (exp(3*x)*3i)/2 - (9*exp(2*x))/4 +
 (exp(x)*3i)/2 + 23/40)/(exp(2*x)*10i - 10*exp(3*x) - exp(4*x)*5i + exp(5*
x) + 5*exp(x) - 1i)
```

Reduce [F]

$$\int \frac{\tanh^4(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\tanh(x)^4}{\operatorname{csch}(x)+i}dx$$

input
```
int(tanh(x)^4/(I+csch(x)),x)
```

output
```
int(tanh(x)**4/(csch(x) + i),x)
```

3.104 $\int \frac{\tanh^3(x)}{i+\operatorname{csch}(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 77

$$\int \frac{\tanh^3(x)}{i+\operatorname{csch}(x)} \, dx = -\frac{11}{16} i \log(i-\sinh(x)) - \frac{5}{16} i \log(i+\sinh(x)) - \frac{i}{8(1-i\sinh(x))} + \frac{i}{8(1+i\sinh(x))^2} - \frac{3i}{4(1+i\sinh(x))}$$

output
```
-11/16*I*ln(I-sinh(x))-5/16*I*ln(I+sinh(x))-1/8*I/(1-I*sinh(x))+1/8*I/(1+I
*sinh(x))^2-3/4*I/(1+I*sinh(x))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{\tanh^3(x)}{i+\operatorname{csch}(x)} \, dx = \frac{1}{16} \left(-11i \log(i-\sinh(x)) - 5i \log(i+\sinh(x)) - \frac{2(6+3i\sinh(x)+5\sinh^2(x))}{(-i+\sinh(x))^2(i+\sinh(x))} \right)$$

input
```
Integrate[Tanh[x]^3/(I + Csch[x]),x]
```

output
```
((-11*I)*Log[I - Sinh[x]] - (5*I)*Log[I + Sinh[x]] - (2*(6 + (3*I)*Sinh[x]
 + 5*Sinh[x]^2))/((-I + Sinh[x])^2*(I + Sinh[x])))/16
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 26, 26, 4367, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{\operatorname{csch}(x)+i}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i}{\cot(ix)^3(i\csc(ix)+i)}dx$$
$$\Big\downarrow \text{26}$$
$$i\int -\frac{i}{\cot(ix)^3(\csc(ix)+1)}dx$$
$$\Big\downarrow \text{26}$$
$$\int \frac{1}{\cot(ix)^3(1+\csc(ix))}dx$$
$$\Big\downarrow \text{4367}$$
$$-i\int \frac{\sinh^4(x)}{(1-i\sinh(x))^2(i\sinh(x)+1)^3}d(i\sinh(x))$$
$$\Big\downarrow \text{99}$$
$$-i\int \left(\frac{11}{16(i\sinh(x)+1)}-\frac{3}{4(i\sinh(x)+1)^2}+\frac{1}{4(i\sinh(x)+1)^3}+\frac{5}{16(i\sinh(x)-1)}+\frac{1}{8(i\sinh(x)-1)^2}\right)d(i\sinh(x))$$
$$\Big\downarrow \text{2009}$$
$$-i\left(\frac{1}{8(1-i\sinh(x))}+\frac{3}{4(1+i\sinh(x))}-\frac{1}{8(1+i\sinh(x))^2}+\frac{5}{16}\log(1-i\sinh(x))+\frac{11}{16}\log(1+i\sinh(x))\right)$$

input
```
Int[Tanh[x]^3/(I + Csch[x]),x]
```

output
```
(-I)*((5*Log[1 - I*Sinh[x]])/16 + (11*Log[1 + I*Sinh[x]])/16 + 1/(8*(1 - I
*Sinh[x])) - 1/(8*(1 + I*Sinh[x])^2) + 3/(4*(1 + I*Sinh[x])))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

method	result
risch	$ix - \frac{5\,\mathrm{e}^{x}-6i\mathrm{e}^{2x}+14\,\mathrm{e}^{3x}+6i\mathrm{e}^{4x}+5\,\mathrm{e}^{5x}}{4(\mathrm{e}^{x}+i)^{2}(\mathrm{e}^{x}-i)^{4}} - \frac{5i\ln(\mathrm{e}^{x}+i)}{8} - \frac{11i\ln(\mathrm{e}^{x}-i)}{8}$
default	$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) - \frac{5i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{8} + \frac{i}{4\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}} - \frac{1}{4\left(\tanh\left(\frac{x}{2}\right)+i\right)} - \frac{11i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)}{8} + \frac{i}{2\left(\tanh\left(\frac{x}{2}\right)-i\right.}$

input
```
int(tanh(x)^3/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
I*x-1/4*(5*exp(x)-6*I*exp(x)^2+14*exp(x)^3+6*I*exp(x)^4+5*exp(x)^5)/(exp(x
)+I)^2/(exp(x)-I)^4-5/8*I*ln(exp(x)+I)-11/8*I*ln(exp(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(47) = 94.

Time = 0.09 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.42

$$\int \frac{\tanh^3(x)}{i+\operatorname{csch}(x)}\,dx = \frac{8i\,xe^{(6\,x)} + 2\,(8\,x-5)e^{(5\,x)} - 4\,(-2i\,x+3i)e^{(4\,x)} + 4\,(8\,x-7)e^{(3\,x)} - 4\,(2i\,x-3i)e^{(2\,x)} + 2\,(8\,x-5)e^{x} - \cdots}{8\,(e^{(6\,x} \cdots}$$

input
```
integrate(tanh(x)^3/(I+csch(x)),x, algorithm="fricas")
```

output
```
1/8*(8*I*x*e^(6*x) + 2*(8*x - 5)*e^(5*x) - 4*(-2*I*x + 3*I)*e^(4*x) + 4*(8
*x - 7)*e^(3*x) - 4*(2*I*x - 3*I)*e^(2*x) + 2*(8*x - 5)*e^x - 5*(I*e^(6*x)
 + 2*e^(5*x) + I*e^(4*x) + 4*e^(3*x) - I*e^(2*x) + 2*e^x - I)*log(e^x + I)
 - 11*(I*e^(6*x) + 2*e^(5*x) + I*e^(4*x) + 4*e^(3*x) - I*e^(2*x) + 2*e^x -
 I)*log(e^x - I) - 8*I*x)/(e^(6*x) - 2*I*e^(5*x) + e^(4*x) - 4*I*e^(3*x) -
 e^(2*x) - 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\tanh^3(x)}{\operatorname{csch}(x)+i}\,dx$$

input
```
integrate(tanh(x)**3/(I+csch(x)),x)
```

output
```
Integral(tanh(x)**3/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(47) = 94$.

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.25

$$\int \frac{\tanh^3(x)}{i+\operatorname{csch}(x)}\,dx = -i\,x + \frac{5\,e^{(-x)} + 6i\,e^{(-2\,x)} + 14\,e^{(-3\,x)} - 6i\,e^{(-4\,x)} + 5\,e^{(-5\,x)}}{8i\,e^{(-x)} - 4\,e^{(-2\,x)} + 16i\,e^{(-3\,x)} + 4\,e^{(-4\,x)} + 8i\,e^{(-5\,x)} + 4\,e^{(-6\,x)} - 4} - \frac{5}{8}i\,\log\left(e^{(-x)} - i\right) - \frac{11}{8}i\,\log\left(i\,e^{(-x)} - 1\right)$$

input
```
integrate(tanh(x)^3/(I+csch(x)),x, algorithm="maxima")
```

output
```
-I*x + (5*e^(-x) + 6*I*e^(-2*x) + 14*e^(-3*x) - 6*I*e^(-4*x) + 5*e^(-5*x))
/(8*I*e^(-x) - 4*e^(-2*x) + 16*I*e^(-3*x) + 4*e^(-4*x) + 8*I*e^(-5*x) + 4*
e^(-6*x) - 4) - 5/8*I*log(e^(-x) - I) - 11/8*I*log(I*e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.22

$$\int \frac{\tanh^3(x)}{i+\operatorname{csch}(x)}\,dx = \frac{5\,e^{(-x)}-5\,e^{x}-6i}{16\left(-i\,e^{(-x)}+i\,e^{x}-2\right)} + \frac{33i\left(e^{(-x)}-e^{x}\right)^2-84\,e^{(-x)}+84\,e^{x}-52i}{32\left(e^{(-x)}-e^{x}+2i\right)^2} - \frac{5}{16}i\,\log\left(-e^{(-x)}+e^{x}+2i\right) - \frac{11}{16}i\,\log\left(-e^{(-x)}+e^{x}-2i\right)$$

input

```
integrate(tanh(x)^3/(I+csch(x)),x, algorithm="giac")
```

output

```
1/16*(5*e^(-x) - 5*e^x - 6*I)/(-I*e^(-x) + I*e^x - 2) + 1/32*(33*I*(e^(-x)
 - e^x)^2 - 84*e^(-x) + 84*e^x - 52*I)/(e^(-x) - e^x + 2*I)^2 - 5/16*I*log
(-e^(-x) + e^x + 2*I) - 11/16*I*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.82

$$\int \frac{\tanh^3(x)}{i+\operatorname{csch}(x)}\,dx = x\,1\mathrm{i} - \ln\left(\left(\frac{3\,\mathrm{e}^{x}}{4}-\frac{3}{4}\mathrm{i}\right)\left(\frac{3\,\mathrm{e}^{x}}{4}+\frac{3}{4}\mathrm{i}\right)\right)1\mathrm{i} + \frac{3\,\mathrm{atan}(\mathrm{e}^{x})}{4} - \frac{1}{\mathrm{e}^{2\,x}\,3\mathrm{i}-\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}-\mathrm{i}} - \frac{1\mathrm{i}}{4\,\left(\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}\right)} + \frac{1\mathrm{i}}{2\,\left(\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1-\mathrm{e}^{3\,x}\,4\mathrm{i}+\mathrm{e}^{x}\,4\mathrm{i}\right)} + \frac{2\mathrm{i}}{1-\mathrm{e}^{2\,x}+\mathrm{e}^{x}\,2\mathrm{i}} - \frac{3}{2\,\left(\mathrm{e}^{x}-\mathrm{i}\right)} + \frac{1}{4\,\left(\mathrm{e}^{x}+1\mathrm{i}\right)}$$

input

```
int(tanh(x)^3/(1/sinh(x) + 1i),x)
```

output

```
x*1i - log(((3*exp(x))/4 - 3i/4)*((3*exp(x))/4 + 3i/4))*1i + (3*atan(exp(x
)))/4 - 1/(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) - 1i/(4*(exp(2*x) + exp
(x)*2i - 1)) + 1i/(2*(exp(4*x) - exp(3*x)*4i - 6*exp(2*x) + exp(x)*4i + 1)
) + 2i/(exp(x)*2i - exp(2*x) + 1) - 3/(2*(exp(x) - 1i)) + 1/(4*(exp(x) + 1
i))
```

Reduce [F]

$$\int \frac{\tanh^3(x)}{i + \operatorname{csch}(x)}\, dx = \int \frac{\tanh{(x)}^3}{\operatorname{csch}{(x)} + i} dx$$

input
```
int(tanh(x)^3/(I+csch(x)),x)
```

output
```
int(tanh(x)**3/(csch(x) + i),x)
```

3.105 $\int \frac{\tanh^2(x)}{i+\text{csch}(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{\tanh^2(x)}{i+\text{csch}(x)} \, dx = -ix+\frac{1}{3}(3i-2\text{csch}(x))\tanh(x)+\frac{1}{3}(i-\text{csch}(x))\tanh^3(x)$$

output
```
-I*x+1/3*(3*I-2*csch(x))*tanh(x)+1/3*(I-csch(x))*tanh(x)^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.97

$$\int \frac{\tanh^2(x)}{i+\text{csch}(x)} \, dx = \frac{-4\cosh(2x)+2i\sinh(x)+(5i+6x)\cosh(x)(-i+\sinh(x))}{6\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3}$$

input
```
Integrate[Tanh[x]^2/(I + Csch[x]),x]
```

output
```
(-4*Cosh[2*x] + (2*I)*Sinh[x] + (5*I + 6*x)*Cosh[x]*(-I + Sinh[x]))/(6*(Co
sh[x/2] - I*Sinh[x/2])*(Cosh[x/2] + I*Sinh[x/2])^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.846, Rules used = {3042, 25, 26, 4376, 25, 3042, 4370, 3042, 25, 4370, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)}{\operatorname{csch}(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\cot(ix)^2(i\csc(ix)+i)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{i}{\cot(ix)^2(\csc(ix)+1)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\cot(ix)^2(\csc(ix)+1)}dx$$

$$\Big\downarrow \text{4376}$$

$$i\int -\left((i\operatorname{csch}(x)+1)\tanh^4(x)\right)dx$$

$$\Big\downarrow \text{25}$$

$$-i\int (i\operatorname{csch}(x)+1)\tanh^4(x)dx$$

$$\Big\downarrow \text{3042}$$

$$-i\int \frac{1-\csc(ix)}{\cot(ix)^4}dx$$

$$\Big\downarrow \text{4370}$$

$$-i\left(\frac{1}{3}\int (2i\operatorname{csch}(x)+3)\tanh^2(x)dx-\frac{1}{3}\tanh^3(x)(1+i\operatorname{csch}(x))\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{1}{3}\int -\frac{3-2\csc(ix)}{\cot(ix)^2}dx-\frac{1}{3}\tanh^3(x)(1+i\text{csch}(x))\right)$$

$\Big\downarrow$ 25

$$-i\left(-\frac{1}{3}\int \frac{3-2\csc(ix)}{\cot(ix)^2}dx-\frac{1}{3}\tanh^3(x)(1+i\text{csch}(x))\right)$$

$\Big\downarrow$ 4370

$$-i\left(\frac{1}{3}(-\int -3dx-(\tanh(x)(3+2i\text{csch}(x))))-\frac{1}{3}\tanh^3(x)(1+i\text{csch}(x))\right)$$

$\Big\downarrow$ 24

$$-i\left(\frac{1}{3}(3x-\tanh(x)(3+2i\text{csch}(x)))-\frac{1}{3}\tanh^3(x)(1+i\text{csch}(x))\right)$$

input
```
Int[Tanh[x]^2/(I + Csch[x]),x]
```

output
```
(-I)*(-1/3*((1 + I*Csch[x])*Tanh[x]^3) + (3*x - (3 + (2*I)*Csch[x])*Tanh[x
])/3)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

method	result
risch	$-ix-\frac{2\left(-4i+5\,{\mathrm e}^{x}+3\,{\mathrm e}^{3x}\right)}{3\left({\mathrm e}^{x}+i\right)\left({\mathrm e}^{x}-i\right)^{3}}$
default	$\frac{i}{2\tanh\left(\frac{x}{2}\right)+2i}+i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{3i}{2\left(\tanh\left(\frac{x}{2}\right)-i\right)}+\frac{2i}{3\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)-\right.}$

input
```
int(tanh(x)^2/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-I*x-2/3*(-4*I+5*exp(x)+3*exp(3*x))/(exp(x)+I)/(exp(x)-I)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(24) = 48.

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.39

$$\int\frac{\tanh^2(x)}{i+\operatorname{csch}(x)}\,dx=\frac{-3i\,xe^{(4\,x)}-6\,(x+1)e^{(3\,x)}-2\,(3\,x+5)e^{x}+3i\,x+8i}{3\,\left(e^{(4\,x)}-2i\,e^{(3\,x)}-2i\,e^{x}-1\right)}$$

input
```
integrate(tanh(x)^2/(I+csch(x)),x, algorithm="fricas")
```

output
```
1/3*(-3*I*x*e^(4*x) - 6*(x + 1)*e^(3*x) - 2*(3*x + 5)*e^x + 3*I*x + 8*I)/(
e^(4*x) - 2*I*e^(3*x) - 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{i+\operatorname{csch}(x)}\,dx=\int \frac{\tanh^2{(x)}}{\operatorname{csch}{(x)}+i}\,dx$$

input
```
integrate(tanh(x)**2/(I+csch(x)),x)
```

output
```
Integral(tanh(x)**2/(csch(x) + I), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int \frac{\tanh^2(x)}{i+\operatorname{csch}(x)}\,dx=-i\,x-\frac{2\left(5\,e^{(-x)}+3\,e^{(-3\,x)}+4i\right)}{6i\,e^{(-x)}+6i\,e^{(-3\,x)}+3\,e^{(-4\,x)}-3}$$

input
```
integrate(tanh(x)^2/(I+csch(x)),x, algorithm="maxima")
```

output
```
-I*x - 2*(5*e^(-x) + 3*e^(-3*x) + 4*I)/(6*I*e^(-x) + 6*I*e^(-3*x) + 3*e^(-
4*x) - 3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{\tanh^2(x)}{i+\operatorname{csch}(x)}\,dx=-i\,x+\frac{i}{2\,(i\,e^x-1)}-\frac{15\,e^{(2\,x)}-24i\,e^x-13}{6\,(e^x-i)^3}$$

input
```
integrate(tanh(x)^2/(I+csch(x)),x, algorithm="giac")
```

output
```
-I*x + 1/2*I/(I*e^x - 1) - 1/6*(15*e^(2*x) - 24*I*e^x - 13)/(e^x - I)^3
```

Mupad [B] (verification not implemented)

Time = 3.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.36

$$\int \frac{\tanh^2(x)}{i+\text{csch}(x)}\,dx = -x\,1\text{i} + \frac{\frac{5\,e^x}{6}-\frac{1}{2}\text{i}}{1-e^{2\,x}+e^x\,2\text{i}} - \frac{5}{6\,(e^x-\text{i})} + \frac{1}{2\,(e^x+1\text{i})} - \frac{\frac{5}{6}-\frac{5\,e^{2\,x}}{6}+e^x\,1\text{i}}{e^{2\,x}\,3\text{i}-e^{3\,x}+3\,e^x-\text{i}}$$

input
```
int(tanh(x)^2/(1/sinh(x) + 1i),x)
```

output
```
((5*exp(x))/6 - 1i/2)/(exp(x)*2i - exp(2*x) + 1) - x*1i - 5/(6*(exp(x) - 1
i)) + 1/(2*(exp(x) + 1i)) - (exp(x)*1i - (5*exp(2*x))/6 + 5/6)/(exp(2*x)*3
i - exp(3*x) + 3*exp(x) - 1i)
```

Reduce [F]

$$\int \frac{\tanh^2(x)}{i+\text{csch}(x)}\,dx = \int \frac{\tanh(x)^2}{\text{csch}(x)+i}dx$$

input
```
int(tanh(x)^2/(I+csch(x)),x)
```

output
```
int(tanh(x)**2/(csch(x) + i),x)
```

3.106 $\int \frac{\tanh(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 45

$$\int \frac{\tanh(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{3}{4}i\log(i-\sinh(x))-\frac{1}{4}i\log(i+\sinh(x))-\frac{i}{2(1+i\sinh(x))}$$

output
```
-3/4*I*ln(I-sinh(x))-1/4*I*ln(I+sinh(x))-1/2*I/(1+I*sinh(x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{\tanh(x)}{i+\operatorname{csch}(x)}\,dx = \frac{1}{4}\left(-3i\log(i-\sinh(x))-i\log(i+\sinh(x))-\frac{2}{-i+\sinh(x)}\right)$$

input
```
Integrate[Tanh[x]/(I + Csch[x]),x]
```

output
```
((-3*I)*Log[I - Sinh[x]] - I*Log[I + Sinh[x]] - 2/(-I + Sinh[x]))/4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 26, 26, 4367, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\text{csch}(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\cot(ix)(i\csc(ix)+i)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int -\frac{i}{\cot(ix)(\csc(ix)+1)}dx$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{1}{\cot(ix)(\csc(ix)+1)}dx$$

$$\Big\downarrow \text{4367}$$

$$i\int -\frac{\sinh^2(x)}{(1-i\sinh(x))(i\sinh(x)+1)^2}d(i\sinh(x))$$

$$\Big\downarrow \text{99}$$

$$i\int \left(-\frac{3}{4(i\sinh(x)+1)}+\frac{1}{2(i\sinh(x)+1)^2}-\frac{1}{4(i\sinh(x)-1)}\right)d(i\sinh(x))$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{1}{2(1+i\sinh(x))}-\frac{1}{4}\log(1-i\sinh(x))-\frac{3}{4}\log(1+i\sinh(x))\right)$$

input
```
Int[Tanh[x]/(I + Csch[x]),x]
```

output
```
I*(-1/4*Log[1 - I*Sinh[x]] - (3*Log[1 + I*Sinh[x]])/4 - 1/(2*(1 + I*Sinh[x
])))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result
risch	$ix-\frac{e^{x}}{\left(e^{x}-i\right)^{2}}-\frac{i\ln\left(e^{x}+i\right)}{2}-\frac{3i\ln\left(e^{x}-i\right)}{2}$
default	$i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)}{2}+\frac{i}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}+\frac{1}{\tanh\left(\frac{x}{2}\right)-i}-\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{2}+i\ln\left(\tanh\left(\frac{x}{2}\right)-\right.$

input
```
int(tanh(x)/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
I*x-1/(exp(x)-I)^2*exp(x)-1/2*I*ln(exp(x)+I)-3/2*I*ln(exp(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(27) = 54.

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.58

$$\int \frac{\tanh(x)}{i+\operatorname{csch}(x)}\,dx = \frac{2i\,xe^{(2\,x)}+2\,(2\,x-1)e^{x}+\left(-i\,e^{(2\,x)}-2\,e^{x}+i\right)\log\left(e^{x}+i\right)-3\left(i\,e^{(2\,x)}+2\,e^{x}-i\right)\log\left(e^{x}-i\right)-2i\,x}{2\left(e^{(2\,x)}-2i\,e^{x}-1\right)}$$

input
```
integrate(tanh(x)/(I+csch(x)),x, algorithm="fricas")
```

output
```
1/2*(2*I*x*e^(2*x) + 2*(2*x - 1)*e^x + (-I*e^(2*x) - 2*e^x + I)*log(e^x +
I) - 3*(I*e^(2*x) + 2*e^x - I)*log(e^x - I) - 2*I*x)/(e^(2*x) - 2*I*e^x -
1)
```

Sympy [F]

$$\int \frac{\tanh(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\tanh(x)}{\operatorname{csch}(x)+i}\,dx$$

input
```
integrate(tanh(x)/(I+csch(x)),x)
```

output
```
Integral(tanh(x)/(csch(x) + I), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(x)}{i+\operatorname{csch}(x)}\,dx = -i\,x + \frac{e^{(-x)}}{2i\,e^{(-x)} + e^{(-2\,x)} - 1} - \frac{1}{2}i\,\log\left(i\,e^{(-x)} + 1\right) - \frac{3}{2}i\,\log\left(i\,e^{(-x)} - 1\right)$$

input
```
integrate(tanh(x)/(I+csch(x)),x, algorithm="maxima")
```

output
```
-I*x + e^(-x)/(2*I*e^(-x) + e^(-2*x) - 1) - 1/2*I*log(I*e^(-x) + 1) - 3/2*
I*log(I*e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \frac{\tanh(x)}{i+\operatorname{csch}(x)}\,dx = \frac{3i\,e^{(-x)} - 3i\,e^{x} - 2}{4\left(e^{(-x)} - e^{x} + 2i\right)} - \frac{1}{4}i\,\log\left(-e^{(-x)} + e^{x} + 2i\right) - \frac{3}{4}i\,\log\left(-e^{(-x)} + e^{x} - 2i\right)$$

input
```
integrate(tanh(x)/(I+csch(x)),x, algorithm="giac")
```

output
```
1/4*(3*I*e^(-x) - 3*I*e^x - 2)/(e^(-x) - e^x + 2*I) - 1/4*I*log(-e^(-x) +
e^x + 2*I) - 3/4*I*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int \frac{\tanh(x)}{i+\operatorname{csch}(x)}\,dx = x\,1\mathrm{i} + \operatorname{atan}(\mathrm{e}^{x}) - \ln\left(\left(\mathrm{e}^{x} - \mathrm{i}\right)\,\left(\mathrm{e}^{x} + 1\mathrm{i}\right)\right)\,1\mathrm{i} + \frac{1\mathrm{i}}{1 - \mathrm{e}^{2\,x} + \mathrm{e}^{x}\,2\mathrm{i}} - \frac{1}{\mathrm{e}^{x} - \mathrm{i}}$$

input
```
int(tanh(x)/(1/sinh(x) + 1i),x)
```

output
```
x*1i + atan(exp(x)) - log((exp(x) - 1i)*(exp(x) + 1i))*1i + 1i/(exp(x)*2i
- exp(2*x) + 1) - 1/(exp(x) - 1i)
```

Reduce [F]

$$\int \frac{\tanh(x)}{i + \operatorname{csch}(x)}\,dx = \int \frac{\tanh(x)}{\operatorname{csch}(x) + i}dx$$

input
```
int(tanh(x)/(I+csch(x)),x)
```

output
```
int(tanh(x)/(csch(x) + i),x)
```

3.107 $\int \frac{\coth(x)}{i+\text{csch}(x)} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{\coth(x)}{i + \text{csch}(x)} \, dx = -i \log(i - \sinh(x))$$

output
```
-I*ln(I-sinh(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\coth(x)}{i + \text{csch}(x)} \, dx = -i \log(i - \sinh(x))$$

input
```
Integrate[Coth[x]/(I + Csch[x]),x]
```

output
```
(-I)*Log[I - Sinh[x]]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 26, 26, 4367, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{\operatorname{csch}(x)+i}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{i\cot(ix)}{i\csc(ix)+i}dx$$
$$\Big\downarrow \text{ 26}$$
$$i\int -\frac{i\cot(ix)}{\csc(ix)+1}dx$$
$$\Big\downarrow \text{ 26}$$
$$\int \frac{\cot(ix)}{1+\csc(ix)}dx$$
$$\Big\downarrow \text{ 4367}$$
$$-i\int \frac{1}{i\sinh(x)+1}d(i\sinh(x))$$
$$\Big\downarrow \text{ 16}$$
$$-i\log(1+i\sinh(x))$$

input
```
Int[Coth[x]/(I + Csch[x]),x]
```

output
```
(-I)*Log[1 + I*Sinh[x]]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

method	result	size
risch	$ix - 2i\ln\left(\mathrm{e}^{x} - i\right)$	15
derivativedivides	$-\frac{i\ln\left(1+\operatorname{csch}(x)^{2}\right)}{2} - \arctan\left(\operatorname{csch}\left(x\right)\right) + i\ln\left(\operatorname{csch}\left(x\right)\right)$	23
default	$-\frac{i\ln\left(1+\operatorname{csch}(x)^{2}\right)}{2} - \arctan\left(\operatorname{csch}\left(x\right)\right) + i\ln\left(\operatorname{csch}\left(x\right)\right)$	23

input
```
int(coth(x)/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
I*x-2*I*ln(exp(x)-I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\coth(x)}{i + \operatorname{csch}(x)} \, dx = i\, x - 2i\, \log\left(e^{x} - i\right)$$

input
```
integrate(coth(x)/(I+csch(x)),x, algorithm="fricas")
```

output
```
I*x - 2*I*log(e^x - I)
```

Sympy [F]

$$\int \frac{\coth(x)}{i + \operatorname{csch}(x)} \, dx = \int \frac{\coth{\left(x \right)}}{\operatorname{csch}{\left(x \right)} + i}\, dx$$

input
```
integrate(coth(x)/(I+csch(x)),x)
```

output
```
Integral(coth(x)/(csch(x) + I), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\coth(x)}{i + \operatorname{csch}(x)} \, dx = -i\, x - 2i\, \log\left(i\, e^{(-x)} - 1\right)$$

input
```
integrate(coth(x)/(I+csch(x)),x, algorithm="maxima")
```

output
```
-I*x - 2*I*log(I*e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\coth(x)}{i+\operatorname{csch}(x)}\,dx = i\,x - 2i\,\log\left(e^{x} - i\right)$$

input
```
integrate(coth(x)/(I+csch(x)),x, algorithm="giac")
```

output
```
I*x - 2*I*log(e^x - I)
```

Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\coth(x)}{i+\operatorname{csch}(x)}\,dx = x\,1\mathrm{i} - \ln\left(\mathrm{e}^{x} - \mathrm{i}\right)\,2\mathrm{i}$$

input
```
int(coth(x)/(1/sinh(x) + 1i),x)
```

output
```
x*1i - log(exp(x) - 1i)*2i
```

Reduce [F]

$$\int \frac{\coth(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\coth(x)}{\operatorname{csch}(x)+i}dx$$

input
```
int(coth(x)/(I+csch(x)),x)
```

output
```
int(coth(x)/(csch(x) + i),x)
```

3.108 $\int \frac{\coth^2(x)}{i+\text{csch}(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \frac{\coth^2(x)}{i+\text{csch}(x)} dx = -ix - \text{arctanh}(\cosh(x))$$

output
```
-I*x-arctanh(cosh(x))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{\coth^2(x)}{i+\text{csch}(x)} dx = -ix - \log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Coth[x]^2/(I + Csch[x]),x]
```

output
```
(-I)*x - Log[Cosh[x/2]] + Log[Sinh[x/2]]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 25, 26, 4376, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{\text{csch}(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cot(ix)^2}{i\csc(ix)+i}dx$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{i\cot(ix)^2}{\csc(ix)+1}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cot(ix)^2}{\csc(ix)+1}dx$$

$$\Big\downarrow \text{4376}$$

$$i\int (-i\text{csch}(x)-1)dx$$

$$\Big\downarrow \text{2009}$$

$$i(-x+i\text{arctanh}(\cosh(x)))$$

input
```
Int[Coth[x]^2/(I + Csch[x]),x]
```

output
```
I*(-x + I*ArcTanh[Cosh[x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

method	result	size
risch	$-ix+\ln\left(\mathrm{e}^{x}-1\right)-\ln\left(1+\mathrm{e}^{x}\right)$	18
default	$-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	27

input
```
int(coth(x)^2/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-I*x+ln(exp(x)-1)-ln(1+exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

$$\int \frac{\coth^2(x)}{i+\operatorname{csch}(x)}\,dx = -i\,x - \log\left(e^x+1\right) + \log\left(e^x-1\right)$$

input
```
integrate(coth(x)^2/(I+csch(x)),x, algorithm="fricas")
```

output
```
-I*x - log(e^x + 1) + log(e^x - 1)
```

Sympy [F]

$$\int \frac{\coth^2(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\coth^2(x)}{\operatorname{csch}(x)+i}\,dx$$

input
```
integrate(coth(x)**2/(I+csch(x)),x)
```

output
```
Integral(coth(x)**2/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(9) = 18$.

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82

$$\int \frac{\coth^2(x)}{i+\operatorname{csch}(x)}\,dx = -i\,x - \log\left(e^{(-x)}+1\right) + \log\left(e^{(-x)}-1\right)$$

input
```
integrate(coth(x)^2/(I+csch(x)),x, algorithm="maxima")
```

output
```
-I*x - log(e^(-x) + 1) + log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{\coth^2(x)}{i + \operatorname{csch}(x)}\,dx = -i\,x - \log\left(e^x + 1\right) + \log\left(|e^x - 1|\right)$$

input
```
integrate(coth(x)^2/(I+csch(x)),x, algorithm="giac")
```

output
```
-I*x - log(e^x + 1) + log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.91

$$\int \frac{\coth^2(x)}{i + \operatorname{csch}(x)}\,dx = \ln\left(2 - 2\,\mathrm{e}^x\right) - \ln\left(-2\,\mathrm{e}^x - 2\right) - x\,1\mathrm{i}$$

input
```
int(coth(x)^2/(1/sinh(x) + 1i),x)
```

output
```
log(2 - 2*exp(x)) - log(- 2*exp(x) - 2) - x*1i
```

Reduce [F]

$$\int \frac{\coth^2(x)}{i + \operatorname{csch}(x)}\,dx = \int \frac{\coth(x)^2}{\operatorname{csch}(x) + i}dx$$

input
```
int(coth(x)^2/(I+csch(x)),x)
```

output
```
int(coth(x)**2/(csch(x) + i),x)
```

3.109 $\int \frac{\coth^3(x)}{i+\mathbf{csch}(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{\coth^3(x)}{i+\operatorname{csch}(x)} \, dx = -\operatorname{csch}(x) - i\log(\sinh(x))$$

output
```
-csch(x)-I*ln(sinh(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\coth^3(x)}{i+\operatorname{csch}(x)} \, dx = -\operatorname{csch}(x) - i\log(\sinh(x))$$

input
```
Integrate[Coth[x]^3/(I + Csch[x]),x]
```

output
```
-Csch[x] - I*Log[Sinh[x]]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 26, 26, 4367, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(x)}{\text{csch}(x)+i}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int -\frac{i\cot(ix)^3}{i\csc(ix)+i}dx$$
$$\Big\downarrow \text{ 26}$$
$$-i\int -\frac{i\cot(ix)^3}{\csc(ix)+1}dx$$
$$\Big\downarrow \text{ 26}$$
$$-\int \frac{\cot(ix)^3}{\csc(ix)+1}dx$$
$$\Big\downarrow \text{ 4367}$$
$$i\int -\text{csch}^2(x)(1-i\sinh(x))d(i\sinh(x))$$
$$\Big\downarrow \text{ 49}$$
$$i\int \left(i\text{csch}(x)-\text{csch}^2(x)\right)d(i\sinh(x))$$
$$\Big\downarrow \text{ 2009}$$
$$i(i\text{csch}(x)-\log(i\sinh(x)))$$

input
```
Int[Coth[x]^3/(I + Csch[x]),x]
```

output
```
I*(I*Csch[x] - Log[I*Sinh[x]])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\operatorname{csch}(x)+i\ln(\operatorname{csch}(x))$	12
default	$-\operatorname{csch}(x)+i\ln(\operatorname{csch}(x))$	12
risch	$ix-\frac{2\,\mathrm{e}^{x}}{\mathrm{e}^{2x}-1}-i\ln(\mathrm{e}^{2x}-1)$	28

input
```
int(coth(x)^3/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-csch(x)+I*ln(csch(x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(10) = 20.

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.33

$$\int \frac{\coth^3(x)}{i+\operatorname{csch}(x)}\,dx = \frac{i\,xe^{(2\,x)} + \left(-i\,e^{(2\,x)} + i\right)\log\left(e^{(2\,x)} - 1\right) - i\,x - 2\,e^{x}}{e^{(2\,x)} - 1}$$

input
```
integrate(coth(x)^3/(I+csch(x)),x, algorithm="fricas")
```

output
```
(I*x*e^(2*x) + (-I*e^(2*x) + I)*log(e^(2*x) - 1) - I*x - 2*e^x)/(e^(2*x) -
 1)
```

Sympy [F]

$$\int \frac{\coth^3(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\coth^3(x)}{\operatorname{csch}(x)+i}\,dx$$

input
```
integrate(coth(x)**3/(I+csch(x)),x)
```

output
```
Integral(coth(x)**3/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(10) = 20.

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 3.00

$$\int \frac{\coth^3(x)}{i+\operatorname{csch}(x)}\,dx = -i\,x + \frac{2\,e^{(-x)}}{e^{(-2\,x)} - 1} - i\,\log\left(e^{(-x)} + 1\right) - i\,\log\left(e^{(-x)} - 1\right)$$

input
```
integrate(coth(x)^3/(I+csch(x)),x, algorithm="maxima")
```

output
```
-I*x + 2*e^(-x)/(e^(-2*x) - 1) - I*log(e^(-x) + 1) - I*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(10) = 20.

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 3.25

$$\int \frac{\coth^3(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{-i\,e^{(-x)}+i\,e^{x}-2}{e^{(-x)}-e^{x}} - i\,\log\left(\left|-e^{(-x)}+e^{x}\right|\right)$$

input
```
integrate(coth(x)^3/(I+csch(x)),x, algorithm="giac")
```

output
```
-(-I*e^(-x) + I*e^x - 2)/(e^(-x) - e^x) - I*log(abs(-e^(-x) + e^x))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int \frac{\coth^3(x)}{i+\operatorname{csch}(x)}\,dx = -\frac{2\,\mathrm{e}^{x}}{\mathrm{e}^{2\,x}-1} + x\,1\mathrm{i} - \ln\left(\mathrm{e}^{2\,x}-1\right)\,1\mathrm{i}$$

input
```
int(coth(x)^3/(1/sinh(x) + 1i),x)
```

output
```
x*1i - log(exp(2*x) - 1)*1i - (2*exp(x))/(exp(2*x) - 1)
```

Reduce [F]

$$\int \frac{\coth^3(x)}{i + \operatorname{csch}(x)}\, dx = \int \frac{\coth{(x)}^3}{\operatorname{csch}{(x)} + i} dx$$

input
```
int(coth(x)^3/(I+csch(x)),x)
```

output
```
int(coth(x)**3/(csch(x) + i),x)
```

3.110 $\int \frac{\coth^4(x)}{i+\operatorname{csch}(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{\coth^4(x)}{i+\operatorname{csch}(x)} \, dx = -ix - \frac{1}{2}\operatorname{arctanh}(\cosh(x)) + \frac{1}{2}\coth(x)(2i - \operatorname{csch}(x))$$

output
```
-I*x-1/2*arctanh(cosh(x))+1/2*coth(x)*(2*I-csch(x))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 76 vs. 2(27) = 54.

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.81

$$\int \frac{\coth^4(x)}{i+\operatorname{csch}(x)} \, dx = -ix + \frac{1}{2}i\coth\left(\frac{x}{2}\right) - \frac{1}{8}\operatorname{csch}^2\left(\frac{x}{2}\right) - \frac{1}{2}\log\left(\cosh\left(\frac{x}{2}\right)\right) + \frac{1}{2}\log\left(\sinh\left(\frac{x}{2}\right)\right) - \frac{1}{8}\operatorname{sech}^2\left(\frac{x}{2}\right) + \frac{1}{2}i\tanh\left(\frac{x}{2}\right)$$

input
```
Integrate[Coth[x]^4/(I + Csch[x]),x]
```

output
```
(-I)*x + (I/2)*Coth[x/2] - Csch[x/2]^2/8 - Log[Cosh[x/2]]/2 + Log[Sinh[x/2
]]/2 - Sech[x/2]^2/8 + (I/2)*Tanh[x/2]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3042, 4376, 3042, 25, 26, 4369, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(x)}{\text{csch}(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(ix)^4}{i\csc(ix)+i}dx$$

$$\Big\downarrow \text{4376}$$

$$-\int \coth^2(x)(i-\text{csch}(x))dx$$

$$\Big\downarrow \text{3042}$$

$$-\int -\cot(ix)^2(i-i\csc(ix))dx$$

$$\Big\downarrow \text{25}$$

$$\int i\cot(ix)^2(1-\csc(ix))dx$$

$$\Big\downarrow \text{26}$$

$$i\int \cot(ix)^2(1-\csc(ix))dx$$

$$\Big\downarrow \text{4369}$$

$$i\left(\frac{1}{2}\coth(x)(2+i\text{csch}(x))-\frac{1}{2}\int (i\text{csch}(x)+2)dx\right)$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{1}{2}(-2x+i\text{arctanh}(\cosh(x)))+\frac{1}{2}\coth(x)(2+i\text{csch}(x))\right)$$

input
```
Int[Coth[x]^4/(I + Csch[x]),x]
```

output
```
I*((-2*x + I*ArcTanh[Cosh[x]])/2 + (Coth[x]*(2 + I*Csch[x]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(21) = 42.

Time = 2.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

method	result
risch	$-ix-\frac{-2i\mathrm{e}^{2x}+\mathrm{e}^{3x}+2i+\mathrm{e}^{x}}{\left(\mathrm{e}^{2x}-1\right)^{2}}-\frac{\ln\left(1+\mathrm{e}^{x}\right)}{2}+\frac{\ln\left(\mathrm{e}^{x}-1\right)}{2}$
default	$\frac{i\tanh\left(\frac{x}{2}\right)}{2}+\frac{\tanh\left(\frac{x}{2}\right)^{2}}{8}+i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{1}{8\tanh\left(\frac{x}{2}\right)^{2}}+\frac{i}{2\tanh\left(\frac{x}{2}\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2}-i\ln\left(\tanh\left(\frac{x}{2}\right)+\right.$

input
```
int(coth(x)^4/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-I*x-(-2*I*exp(x)^2+exp(x)^3+2*I+exp(x))/(exp(x)^2-1)^2-1/2*ln(1+exp(x))+1
/2*ln(exp(x)-1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(17) = 34.

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.19

$$\int \frac{\coth^4(x)}{i+\operatorname{csch}(x)}\,dx = \frac{-2i\,xe^{(4\,x)}-4\left(-i\,x-i\right)e^{(2\,x)}-\left(e^{(4\,x)}-2\,e^{(2\,x)}+1\right)\log\left(e^{x}+1\right)+\left(e^{(4\,x)}-2\,e^{(2\,x)}+1\right)\log\left(e^{x}-1\right)-2}{2\left(e^{(4\,x)}-2\,e^{(2\,x)}+1\right)}$$

input
```
integrate(coth(x)^4/(I+csch(x)),x, algorithm="fricas")
```

output
```
1/2*(-2*I*x*e^(4*x) - 4*(-I*x - I)*e^(2*x) - (e^(4*x) - 2*e^(2*x) + 1)*log
(e^x + 1) + (e^(4*x) - 2*e^(2*x) + 1)*log(e^x - 1) - 2*I*x - 2*e^(3*x) - 2
*e^x - 4*I)/(e^(4*x) - 2*e^(2*x) + 1)
```

Sympy [F]

$$\int \frac{\coth^4(x)}{i+\operatorname{csch}(x)}\,dx=\int \frac{\coth^4(x)}{\operatorname{csch}(x)+i}\,dx$$

input
```
integrate(coth(x)**4/(I+csch(x)),x)
```

output
```
Integral(coth(x)**4/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int \frac{\coth^4(x)}{i+\operatorname{csch}(x)}\,dx=-i\,x+\frac{e^{(-x)}+2i\,e^{(-2\,x)}+e^{(-3\,x)}-2i}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}-\frac{1}{2}\log\left(e^{(-x)}+1\right)+\frac{1}{2}\log\left(e^{(-x)}-1\right)$$

input
```
integrate(coth(x)^4/(I+csch(x)),x, algorithm="maxima")
```

output
```
-I*x + (e^(-x) + 2*I*e^(-2*x) + e^(-3*x) - 2*I)/(2*e^(-2*x) - e^(-4*x) - 1
) - 1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(17) = 34$.

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{\coth^4(x)}{i+\operatorname{csch}(x)}\,dx=-i\,x-\frac{e^{(3\,x)}-2i\,e^{(2\,x)}+e^{x}+2i}{\left(e^{(2\,x)}-1\right)^2}-\frac{1}{2}\log\left(e^{x}+1\right)+\frac{1}{2}\log\left(|e^{x}-1|\right)$$

input
```
integrate(coth(x)^4/(I+csch(x)),x, algorithm="giac")
```

output
```
-I*x - (e^(3*x) - 2*I*e^(2*x) + e^x + 2*I)/(e^(2*x) - 1)^2 - 1/2*log(e^x +
 1) + 1/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 2.84 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

$$\int \frac{\coth^4(x)}{i+\operatorname{csch}(x)}\,dx = \frac{\ln\left(1-e^x\right)}{2} - \frac{\ln\left(-e^x-1\right)}{2} - \frac{e^x-2i}{e^{2x}-1} - \frac{2\,e^x}{e^{4x}-2\,e^{2x}+1} - x\,1i$$

input
```
int(coth(x)^4/(1/sinh(x) + 1i),x)
```

output
```
log(1 - exp(x))/2 - log(- exp(x) - 1)/2 - x*1i - (exp(x) - 2i)/(exp(2*x) -
 1) - (2*exp(x))/(exp(4*x) - 2*exp(2*x) + 1)
```

Reduce [F]

$$\int \frac{\coth^4(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\coth\left(x\right)^4}{\operatorname{csch}\left(x\right)+i}dx$$

input
```
int(coth(x)^4/(I+csch(x)),x)
```

output
```
int(coth(x)**4/(csch(x) + i),x)
```

3.111 $\int \frac{\coth^5(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{\coth^5(x)}{i+\operatorname{csch}(x)}\,dx=-\operatorname{csch}(x)+\frac{1}{2}i\operatorname{csch}^2(x)-\frac{\operatorname{csch}^3(x)}{3}-i\log(\sinh(x))$$

output
```
-csch(x)+1/2*I*csch(x)^2-1/3*csch(x)^3-I*ln(sinh(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\coth^5(x)}{i+\operatorname{csch}(x)}\,dx=-\operatorname{csch}(x)+\frac{1}{2}i\operatorname{csch}^2(x)-\frac{\operatorname{csch}^3(x)}{3}-i\log(\sinh(x))$$

input
```
Integrate[Coth[x]^5/(I + Csch[x]),x]
```

output
```
-Csch[x] + (I/2)*Csch[x]^2 - Csch[x]^3/3 - I*Log[Sinh[x]]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 26, 26, 4367, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^5(x)}{\text{csch}(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cot(ix)^5}{i\csc(ix)+i}dx$$

$$\Big\downarrow \text{26}$$

$$i\int -\frac{i\cot(ix)^5}{\csc(ix)+1}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\cot(ix)^5}{1+\csc(ix)}dx$$

$$\Big\downarrow \text{4367}$$

$$-i\int \text{csch}^4(x)(1-i\sinh(x))^2(i\sinh(x)+1)d(i\sinh(x))$$

$$\Big\downarrow \text{84}$$

$$-i\int \left(\text{csch}^4(x)-i\text{csch}^3(x)+\text{csch}^2(x)-i\text{csch}(x)\right)d(i\sinh(x))$$

$$\Big\downarrow \text{2009}$$

$$-i\left(-\frac{1}{3}i\text{csch}^3(x)-\frac{\text{csch}^2(x)}{2}-i\text{csch}(x)+\log(i\sinh(x))\right)$$

input
```
Int[Coth[x]^5/(I + Csch[x]),x]
```

output
```
(-I)*((-I)*Csch[x] - Csch[x]^2/2 - (I/3)*Csch[x]^3 + Log[I*Sinh[x]])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4367
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[1/(a^(m - n - 1)*b^n*d)   Subst[Int[(a - b*x)^((m -
1)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fr
eeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && Integer
Q[n]
```

Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\operatorname{csch}\left(x\right)-\frac{\operatorname{csch}\left(x\right)^{3}}{3}+\frac{i \operatorname{csch}\left(x\right)^{2}}{2}+i \ln\left(\operatorname{csch}\left(x\right)\right)$	25
default	$-\operatorname{csch}\left(x\right)-\frac{\operatorname{csch}\left(x\right)^{3}}{3}+\frac{i \operatorname{csch}\left(x\right)^{2}}{2}+i \ln\left(\operatorname{csch}\left(x\right)\right)$	25
risch	$ix-\frac{2 \,{\mathrm e}^{x} \left(-3 i {\mathrm e}^{3 x}+3 \,{\mathrm e}^{4 x}+3 i {\mathrm e}^{x}-2 \,{\mathrm e}^{2 x}+3\right)}{3 \left({\mathrm e}^{2 x}-1\right)^{3}}-i \ln\left({\mathrm e}^{2 x}-1\right)$	54

input
```
int(coth(x)^5/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-csch(x)-1/3*csch(x)^3+1/2*I*csch(x)^2+I*ln(csch(x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(22) = 44$.

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 3.33

$$\int \frac{\coth^5(x)}{i+\operatorname{csch}(x)}\,dx = \frac{3i\,xe^{(6\,x)} - 3\,(3i\,x - 2i)e^{(4\,x)} - 3\,(-3i\,x + 2i)e^{(2\,x)} - 3\,\left(i\,e^{(6\,x)} - 3i\,e^{(4\,x)} + 3i\,e^{(2\,x)} - i\right)\log\left(e^{(2\,x)} - 1\right) -}{3\,\left(e^{(6\,x)} - 3\,e^{(4\,x)} + 3\,e^{(2\,x)} - 1\right)}$$

input
```
integrate(coth(x)^5/(I+csch(x)),x, algorithm="fricas")
```

output
```
1/3*(3*I*x*e^(6*x) - 3*(3*I*x - 2*I)*e^(4*x) - 3*(-3*I*x + 2*I)*e^(2*x) -
3*(I*e^(6*x) - 3*I*e^(4*x) + 3*I*e^(2*x) - I)*log(e^(2*x) - 1) - 3*I*x - 6
*e^(5*x) + 4*e^(3*x) - 6*e^x)/(e^(6*x) - 3*e^(4*x) + 3*e^(2*x) - 1)
```

Sympy [F]

$$\int \frac{\coth^5(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\coth^5(x)}{\operatorname{csch}(x)+i}\,dx$$

input
```
integrate(coth(x)**5/(I+csch(x)),x)
```

output
```
Integral(coth(x)**5/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(22) = 44$.

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.50

$$\int \frac{\coth^5(x)}{i + \operatorname{csch}(x)}\,dx = -i\,x + \frac{2\left(3\,e^{(-x)} - 3i\,e^{(-2\,x)} - 2\,e^{(-3\,x)} + 3i\,e^{(-4\,x)} + 3\,e^{(-5\,x)}\right)}{3\left(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1\right)} - i\,\log\left(e^{(-x)} + 1\right) - i\,\log\left(e^{(-x)} - 1\right)$$

input
```
integrate(coth(x)^5/(I+csch(x)),x, algorithm="maxima")
```

output
```
-I*x + 2/3*(3*e^(-x) - 3*I*e^(-2*x) - 2*e^(-3*x) + 3*I*e^(-4*x) + 3*e^(-5*
x))/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) - I*log(e^(-x) + 1) - I*log(e
^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(22) = 44$.

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.17

$$\int \frac{\coth^5(x)}{i + \operatorname{csch}(x)}\,dx = \frac{11i\left(e^{(-x)} - e^{x}\right)^3 + 12\left(e^{(-x)} - e^{x}\right)^2 + 12i\,e^{(-x)} - 12i\,e^{x} + 16}{6\left(e^{(-x)} - e^{x}\right)^3} - i\,\log\left(\left|-e^{(-x)} + e^{x}\right|\right)$$

input
```
integrate(coth(x)^5/(I+csch(x)),x, algorithm="giac")
```

output
```
1/6*(11*I*(e^(-x) - e^x)^3 + 12*(e^(-x) - e^x)^2 + 12*I*e^(-x) - 12*I*e^x
+ 16)/(e^(-x) - e^x)^3 - I*log(abs(-e^(-x) + e^x))
```

Mupad [B] (verification not implemented)

Time = 2.94 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.70

$$\int \frac{\coth^5(x)}{i + \operatorname{csch}(x)}\,dx = x\,1\mathrm{i} - \ln\left(\mathrm{e}^{2\,x} - 1\right)\,1\mathrm{i} - \frac{8\,\mathrm{e}^{x}}{3\,\left(3\,\mathrm{e}^{2\,x} - 3\,\mathrm{e}^{4\,x} + \mathrm{e}^{6\,x} - 1\right)} - \frac{\frac{8\,\mathrm{e}^{x}}{3} - 2\mathrm{i}}{\mathrm{e}^{4\,x} - 2\,\mathrm{e}^{2\,x} + 1} - \frac{2\,\mathrm{e}^{x} - 2\mathrm{i}}{\mathrm{e}^{2\,x} - 1}$$

input
```
int(coth(x)^5/(1/sinh(x) + 1i),x)
```

output
```
x*1i - log(exp(2*x) - 1)*1i - (8*exp(x))/(3*(3*exp(2*x) - 3*exp(4*x) + exp
(6*x) - 1)) - ((8*exp(x))/3 - 2i)/(exp(4*x) - 2*exp(2*x) + 1) - (2*exp(x)
- 2i)/(exp(2*x) - 1)
```

Reduce [F]

$$\int \frac{\coth^5(x)}{i + \operatorname{csch}(x)}\,dx = \int \frac{\coth\left(x\right)^5}{\operatorname{csch}\left(x\right) + i}dx$$

input
```
int(coth(x)^5/(I+csch(x)),x)
```

output
```
int(coth(x)**5/(csch(x) + i),x)
```

3.112 $\int \frac{\coth^6(x)}{i+\operatorname{csch}(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{\coth^6(x)}{i+\operatorname{csch}(x)} \, dx = -ix - \frac{3}{8}\operatorname{arctanh}(\cosh(x)) + \frac{1}{12}\coth^3(x)(4i - 3\operatorname{csch}(x)) + \frac{1}{8}\coth(x)(8i - 3\operatorname{csch}(x))$$

output
```
-I*x-3/8*arctanh(cosh(x))+1/12*coth(x)^3*(4*I-3*csch(x))+1/8*coth(x)*(8*I-
3*csch(x))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 140 vs. $2(43) = 86$.

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.26

$$\int \frac{\coth^6(x)}{i+\operatorname{csch}(x)}\,dx = -ix + \frac{2}{3}i\coth\left(\frac{x}{2}\right) - \frac{5}{32}\operatorname{csch}^2\left(\frac{x}{2}\right) + \frac{1}{24}i\coth\left(\frac{x}{2}\right)\operatorname{csch}^2\left(\frac{x}{2}\right) - \frac{1}{64}\operatorname{csch}^4\left(\frac{x}{2}\right) - \frac{3}{8}\log\left(\cosh\left(\frac{x}{2}\right)\right) + \frac{3}{8}\log\left(\sinh\left(\frac{x}{2}\right)\right) - \frac{5}{32}\operatorname{sech}^2\left(\frac{x}{2}\right) + \frac{1}{64}\operatorname{sech}^4\left(\frac{x}{2}\right) + \frac{2}{3}i\tanh\left(\frac{x}{2}\right) - \frac{1}{24}i\operatorname{sech}^2\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)$$

input
```
Integrate[Coth[x]^6/(I + Csch[x]),x]
```

output
```
(-I)*x + ((2*I)/3)*Coth[x/2] - (5*Csch[x/2]^2)/32 + (I/24)*Coth[x/2]*Csch[
x/2]^2 - Csch[x/2]^4/64 - (3*Log[Cosh[x/2]])/8 + (3*Log[Sinh[x/2]])/8 - (5
*Sech[x/2]^2)/32 + Sech[x/2]^4/64 + ((2*I)/3)*Tanh[x/2] - (I/24)*Sech[x/2]
^2*Tanh[x/2]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.28, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$, Rules used = {3042, 25, 26, 4376, 25, 3042, 4369, 25, 3042, 25, 4369, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^6(x)}{\operatorname{csch}(x)+i}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{\cot(ix)^6}{i\csc(ix)+i}dx$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{i\cot(ix)^6}{\csc(ix)+1}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cot(ix)^6}{\csc(ix)+1}dx$$

$$\Big\downarrow \text{4376}$$

$$i\int -\coth^4(x)(i\text{csch}(x)+1)dx$$

$$\Big\downarrow \text{25}$$

$$-i\int \coth^4(x)(i\text{csch}(x)+1)dx$$

$$\Big\downarrow \text{3042}$$

$$-i\int \cot(ix)^4(1-\csc(ix))dx$$

$$\Big\downarrow \text{4369}$$

$$-i\left(-\frac{1}{4}\int -\coth^2(x)(3i\text{csch}(x)+4)dx-\frac{1}{12}\coth^3(x)(4+3i\text{csch}(x))\right)$$

$$\Big\downarrow \text{25}$$

$$-i\left(\frac{1}{4}\int \coth^2(x)(3i\text{csch}(x)+4)dx-\frac{1}{12}\coth^3(x)(4+3i\text{csch}(x))\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{1}{4}\int -\cot(ix)^2(4-3\csc(ix))dx-\frac{1}{12}\coth^3(x)(4+3i\text{csch}(x))\right)$$

$$\Big\downarrow \text{25}$$

$$-i\left(-\frac{1}{4}\int \cot(ix)^2(4-3\csc(ix))dx-\frac{1}{12}\coth^3(x)(4+3i\text{csch}(x))\right)$$

$$\Big\downarrow \text{4369}$$

$$-i\left(\frac{1}{4}\left(\frac{1}{2}\int (3i\text{csch}(x)+8)dx-\frac{1}{2}\coth(x)(8+3i\text{csch}(x))\right)-\frac{1}{12}\coth^3(x)(4+3i\text{csch}(x))\right)$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{1}{4}\left(\frac{1}{2}(8x-3i\text{arctanh}(\cosh(x)))-\frac{1}{2}\coth(x)(8+3i\text{csch}(x))\right)-\frac{1}{12}\coth^3(x)(4+3i\text{csch}(x))\right)$$

input
```
Int[Coth[x]^6/(I + Csch[x]),x]
```

output
```
(-I)*(((8*x - (3*I)*ArcTanh[Cosh[x]])/2 - (Coth[x]*(8 + (3*I)*Csch[x]))/2)
/4 - (Coth[x]^3*(4 + (3*I)*Csch[x]))/12)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4376
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[a^(2*n)/e^(2*n)   Int[(e*Cot[c + d*x])^(m + 2*
n)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a
^2 - b^2, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(34) = 68.

Time = 6.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.77

method	result
risch	$-ix-\frac{-48i\mathrm{e}^{6x}+15\,\mathrm{e}^{7x}+96i\mathrm{e}^{4x}+9\,\mathrm{e}^{5x}-80i\mathrm{e}^{2x}+9\,\mathrm{e}^{3x}+32i+15\,\mathrm{e}^{x}}{12(\mathrm{e}^{2x}-1)^{4}}-\frac{3\ln(1+\mathrm{e}^{x})}{8}+\frac{3\ln(\mathrm{e}^{x}-1)}{8}$
default	$\frac{5i\tanh(\frac{x}{2})}{8}+\frac{\tanh(\frac{x}{2})^{4}}{64}+\frac{i\tanh(\frac{x}{2})^{3}}{24}+\frac{\tanh(\frac{x}{2})^{2}}{8}-\frac{1}{64\tanh(\frac{x}{2})^{4}}+\frac{5i}{8\tanh(\frac{x}{2})}+\frac{i}{24\tanh(\frac{x}{2})^{3}}-\frac{1}{8\tanh(\frac{x}{2})^{2}}+\frac{3\ln}{}$

input
```
int(coth(x)^6/(I+csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-I*x-1/12*(-48*I*exp(x)^6+15*exp(x)^7+96*I*exp(x)^4+9*exp(x)^5-80*I*exp(x)
^2+9*exp(x)^3+32*I+15*exp(x))/(exp(x)^2-1)^4-3/8*ln(1+exp(x))+3/8*ln(exp(x
)-1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(31) = 62.

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 3.65

$$\int \frac{\coth^6(x)}{i+\operatorname{csch}(x)}\,dx = \frac{-24i\,xe^{(8\,x)}-96\,(-i\,x-i)e^{(6\,x)}-48\,(3i\,x+4i)e^{(4\,x)}-32\,(-3i\,x-5i)e^{(2\,x)}-9\,\left(e^{(8\,x)}-4\,e^{(6\,x)}+6\,e^{(4\,x)}\right.}{24\,(e^{}}$$

input
```
integrate(coth(x)^6/(I+csch(x)),x, algorithm="fricas")
```

output
```
1/24*(-24*I*x*e^(8*x) - 96*(-I*x - I)*e^(6*x) - 48*(3*I*x + 4*I)*e^(4*x) -
 32*(-3*I*x - 5*I)*e^(2*x) - 9*(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x
) + 1)*log(e^x + 1) + 9*(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)*
log(e^x - 1) - 24*I*x - 30*e^(7*x) - 18*e^(5*x) - 18*e^(3*x) - 30*e^x - 64
*I)/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)
```

Sympy [F]

$$\int \frac{\coth^6(x)}{i+\operatorname{csch}(x)}\,dx = \int \frac{\coth^6(x)}{\operatorname{csch}(x)+i}\,dx$$

input
```
integrate(coth(x)**6/(I+csch(x)),x)
```

output
```
Integral(coth(x)**6/(csch(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(31) = 62$.

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.23

$$\int \frac{\coth^6(x)}{i+\operatorname{csch}(x)}\,dx = -i\,x + \frac{15\,e^{(-x)} + 80i\,e^{(-2\,x)} + 9\,e^{(-3\,x)} - 96i\,e^{(-4\,x)} + 9\,e^{(-5\,x)} + 48i\,e^{(-6\,x)} + 15\,e^{(-7\,x)} - 32i}{12\,(4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1)} - \frac{3}{8}\,\log\left(e^{(-x)}+1\right) + \frac{3}{8}\,\log\left(e^{(-x)}-1\right)$$

input
```
integrate(coth(x)^6/(I+csch(x)),x, algorithm="maxima")
```

output
```
-I*x + 1/12*(15*e^(-x) + 80*I*e^(-2*x) + 9*e^(-3*x) - 96*I*e^(-4*x) + 9*e^
(-5*x) + 48*I*e^(-6*x) + 15*e^(-7*x) - 32*I)/(4*e^(-2*x) - 6*e^(-4*x) + 4*
e^(-6*x) - e^(-8*x) - 1) - 3/8*log(e^(-x) + 1) + 3/8*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(31) = 62.

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{\coth^6(x)}{i+\operatorname{csch}(x)}\,dx = -i\,x - \frac{15\,e^{(7\,x)} - 48i\,e^{(6\,x)} + 9\,e^{(5\,x)} + 96i\,e^{(4\,x)} + 9\,e^{(3\,x)} - 80i\,e^{(2\,x)} + 15\,e^{x} + 32i}{12\,\left(e^{(2\,x)} - 1\right)^4} - \frac{3}{8}\,\log\left(e^{x} + 1\right) + \frac{3}{8}\,\log\left(|e^{x} - 1|\right)$$

input
```
integrate(coth(x)^6/(I+csch(x)),x, algorithm="giac")
```

output
```
-I*x - 1/12*(15*e^(7*x) - 48*I*e^(6*x) + 9*e^(5*x) + 96*I*e^(4*x) + 9*e^(3
*x) - 80*I*e^(2*x) + 15*e^x + 32*I)/(e^(2*x) - 1)^4 - 3/8*log(e^x + 1) + 3
/8*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 3.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.47

$$\int \frac{\coth^6(x)}{i+\operatorname{csch}(x)}\,dx = \frac{3\,\ln\left(\frac{3}{4} - \frac{3\,\mathrm{e}^{x}}{4}\right)}{8} - x\,1\mathrm{i} - \frac{3\,\ln\left(\frac{3\,\mathrm{e}^{x}}{4} + \frac{3}{4}\right)}{8} - \frac{5\,\mathrm{e}^{x}}{4\,\left(\mathrm{e}^{2\,x} - 1\right)} - \frac{9\,\mathrm{e}^{x}}{2\,\left(\mathrm{e}^{2\,x} - 1\right)^2} - \frac{6\,\mathrm{e}^{x}}{\left(\mathrm{e}^{2\,x} - 1\right)^3} - \frac{4\,\mathrm{e}^{x}}{\left(\mathrm{e}^{2\,x} - 1\right)^4} + \frac{4\mathrm{i}}{\mathrm{e}^{2\,x} - 1} + \frac{4\mathrm{i}}{\left(\mathrm{e}^{2\,x} - 1\right)^2} + \frac{8\mathrm{i}}{3\,\left(\mathrm{e}^{2\,x} - 1\right)^3}$$

input
```
int(coth(x)^6/(1/sinh(x) + 1i),x)
```

output
```
(3*log(3/4 - (3*exp(x))/4))/8 - x*1i - (3*log((3*exp(x))/4 + 3/4))/8 - (5*
exp(x))/(4*(exp(2*x) - 1)) - (9*exp(x))/(2*(exp(2*x) - 1)^2) - (6*exp(x))/
(exp(2*x) - 1)^3 - (4*exp(x))/(exp(2*x) - 1)^4 + 4i/(exp(2*x) - 1) + 4i/(e
xp(2*x) - 1)^2 + 8i/(3*(exp(2*x) - 1)^3)
```

Reduce [F]

$$\int \frac{\coth^6(x)}{i + \operatorname{csch}(x)}\, dx = \int \frac{\coth(x)^6}{\operatorname{csch}(x) + i} dx$$

input
```
int(coth(x)^6/(I+csch(x)),x)
```

output
```
int(coth(x)**6/(csch(x) + i),x)
```

3.113 $\int \frac{\tanh^5(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 194

$$\begin{aligned}\int \frac{\tanh^5(x)}{a+b\operatorname{csch}(x)}\,dx = & -\frac{b^5\arctan(\sinh(x))}{(a^2+b^2)^3}-\frac{b^3\arctan(\sinh(x))}{2(a^2+b^2)^2} \\ & -\frac{3b\arctan(\sinh(x))}{8(a^2+b^2)}+\frac{b^6\log(a+b\operatorname{csch}(x))}{a(a^2+b^2)^3}+\frac{\log(\sinh(x))}{a} \\ & -\frac{a(a^4+3a^2b^2+3b^4)\log(\tanh(x))}{(a^2+b^2)^3}+\frac{3b\operatorname{sech}(x)\tanh(x)}{8(a^2+b^2)} \\ & -\frac{(a(a^2+2b^2)-b^3\operatorname{csch}(x))\tanh^2(x)}{2(a^2+b^2)^2}-\frac{(a-b\operatorname{csch}(x))\tanh^4(x)}{4(a^2+b^2)}\end{aligned}$$

output
```
-b^5*arctan(sinh(x))/(a^2+b^2)^3-1/2*b^3*arctan(sinh(x))/(a^2+b^2)^2-3*b*a
rctan(sinh(x))/(8*a^2+8*b^2)+b^6*ln(a+b*csch(x))/a/(a^2+b^2)^3+ln(sinh(x))
/a-a*(a^4+3*a^2*b^2+3*b^4)*ln(tanh(x))/(a^2+b^2)^3+3*b*sech(x)*tanh(x)/(8*
a^2+8*b^2)-1/2*(a*(a^2+2*b^2)-b^3*csch(x))*tanh(x)^2/(a^2+b^2)^2-(a-b*csch
(x))*tanh(x)^4/(4*a^2+4*b^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.30

$$\int \frac{\tanh^5(x)}{a+b\text{csch}(x)}\,dx$$
$$= \frac{ab(5a^4+14a^2b^2+9b^4)\arctan(\sinh(x))+4a(a^5+ia^4b+3a^3b^2+3ia^2b^3+3ab^4+3ib^5)\log(i-\sinh(x))}{}$$

input
```
Integrate[Tanh[x]^5/(a + b*Csch[x]),x]
```

output
```
(a*b*(5*a^4 + 14*a^2*b^2 + 9*b^4)*ArcTan[Sinh[x]] + 4*a*(a^5 + I*a^4*b + 3
*a^3*b^2 + (3*I)*a^2*b^3 + 3*a*b^4 + (3*I)*b^5)*Log[I - Sinh[x]] + 4*a*(a^
5 - I*a^4*b + 3*a^3*b^2 - (3*I)*a^2*b^3 + 3*a*b^4 - (3*I)*b^5)*Log[I + Sin
h[x]] + 8*b^6*Log[b + a*Sinh[x]] + 4*a^2*(2*a^4 + 5*a^2*b^2 + 3*b^4)*Sech[
x]^2 - 2*a^2*(a^2 + b^2)^2*Sech[x]^4 + a*b*(5*a^4 + 14*a^2*b^2 + 9*b^4)*Se
ch[x]*Tanh[x] - 2*a*b*(a^2 + b^2)^2*Sech[x]^3*Tanh[x])/(8*a*(a^2 + b^2)^3)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 26, 4373, 25, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(x)}{a+b\text{csch}(x)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int -\frac{i}{\cot(ix)^5(a+ib\csc(ix))}dx$$
$$\Big\downarrow \; 26$$

$$-i\int \frac{1}{\cot(ix)^5(a+ib\csc(ix))}dx$$

$\Big\downarrow$ 4373

$$b^6\int -\frac{\sinh(x)}{b(a+b\text{csch}(x))\left(\text{csch}^2(x)b^2+b^2\right)^3}d(b\text{csch}(x))$$

$\Big\downarrow$ 25

$$-b^6\int \frac{\sinh(x)}{b(a+b\text{csch}(x))\left(\text{csch}^2(x)b^2+b^2\right)^3}d(b\text{csch}(x))$$

$\Big\downarrow$ 615

$$-b^6\int \left(\frac{-b^2-a\text{csch}(x)b}{b^2\left(a^2+b^2\right)\left(\text{csch}^2(x)b^2+b^2\right)^3}+\frac{\sinh(x)}{ab^7}-\frac{1}{a\left(a^2+b^2\right)^3(a+b\text{csch}(x))}+\frac{-b^6-a\left(a^4+3b^2a^2+3b^4\right)\text{cs}}{b^6\left(a^2+b^2\right)^3\left(\text{csch}^2(x)b^2+\right.}\right.$$

$\Big\downarrow$ 2009

$$b^6\left(\frac{\arctan(\text{csch}(x))}{b\left(a^2+b^2\right)^3}+\frac{3\arctan(\text{csch}(x))}{8b^5\left(a^2+b^2\right)}+\frac{\arctan(\text{csch}(x))}{2b^3\left(a^2+b^2\right)^2}-\frac{a-b\text{csch}(x)}{4b^2\left(a^2+b^2\right)\left(b^2\text{csch}^2(x)+b^2\right)^2}+\frac{\log(a+b\text{csch}(}{a\left(a^2+b^2\right)^{}}\right.$$

input
```
Int[Tanh[x]^5/(a + b*Csch[x]),x]
```

output
```
b^6*(ArcTan[Csch[x]]/(b*(a^2 + b^2)^3) + ArcTan[Csch[x]]/(2*b^3*(a^2 + b^2
)^2) + (3*ArcTan[Csch[x]])/(8*b^5*(a^2 + b^2)) - (a - b*Csch[x])/(4*b^2*(a
^2 + b^2)*(b^2 + b^2*Csch[x]^2)^2) + (3*Csch[x])/(8*b^3*(a^2 + b^2)*(b^2 +
 b^2*Csch[x]^2)) - (a*(a^2 + 2*b^2) - b^3*Csch[x])/(2*b^4*(a^2 + b^2)^2*(b
^2 + b^2*Csch[x]^2)) - Log[b*Csch[x]]/(a*b^6) + Log[a + b*Csch[x]]/(a*(a^2
 + b^2)^3) + (a*(a^4 + 3*a^2*b^2 + 3*b^4)*Log[b^2 + b^2*Csch[x]^2])/(2*b^6
*(a^2 + b^2)^3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.88

method	result
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{2\left(\left(-\frac{3}{8}a^{4}b-\frac{5}{4}a^{2}b^{3}-\frac{7}{8}b^{5}\right)\tanh\left(\frac{x}{2}\right)^{7}+\left(-a^{5}-3a^{3}b^{2}-2b^{4}a\right)\tanh\left(\frac{x}{2}\right)^{6}+\left(-\frac{13}{4}a^{2}b^{3}-\frac{15}{8}b^{5}-\frac{11}{8}a^{4}b\right)\tanh\left(\frac{x}{2}\right)^{5}+\left(-4a\right.\right.}{}$
risch	$\frac{x}{a}-\frac{2x\,a^{5}}{a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}}-\frac{6x\,a^{3}b^{2}}{a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}}-\frac{6x\,b^{4}a}{a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}}-\frac{2x\,b^{6}}{a\left(a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}\right)}+\frac{(5a^{2}b\,e^{6x}+9b^{3}e}{}$

input
```
int(tanh(x)^5/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/a*ln(tanh(1/2*x)-1)+2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*(((-3/8*a^4*b-5/4*a
^2*b^3-7/8*b^5)*tanh(1/2*x)^7+(-a^5-3*a^3*b^2-2*a*b^4)*tanh(1/2*x)^6+(-13/
4*a^2*b^3-15/8*b^5-11/8*a^4*b)*tanh(1/2*x)^5+(-4*a^5-10*a^3*b^2-6*a*b^4)*t
anh(1/2*x)^4+(13/4*a^2*b^3+15/8*b^5+11/8*a^4*b)*tanh(1/2*x)^3+(-a^5-3*a^3*
b^2-2*a*b^4)*tanh(1/2*x)^2+(3/8*a^4*b+5/4*a^2*b^3+7/8*b^5)*tanh(1/2*x))/(t
anh(1/2*x)^2+1)^4+1/16*(8*a^5+24*a^3*b^2+24*a*b^4)*ln(tanh(1/2*x)^2+1)+1/8
*(-3*a^4*b-10*a^2*b^3-15*b^5)*arctan(tanh(1/2*x)))-1/a*ln(tanh(1/2*x)+1)+b
^6/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/a*ln(-b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4025 vs. $2(185) = 370$.

Time = 0.20 (sec) , antiderivative size = 4025, normalized size of antiderivative = 20.75

$$\int \frac{\tanh^5(x)}{a + b\text{csch}(x)}\, dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^5/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh^5(x)}{a + b\text{csch}(x)}\, dx = \int \frac{\tanh^5(x)}{a + b\,\text{csch}\,(x)}\, dx$$

input
```
integrate(tanh(x)**5/(a+b*csch(x)),x)
```

output
```
Integral(tanh(x)**5/(a + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(185) = 370$.

Time = 0.14 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.97

$$\int \frac{\tanh^5(x)}{a + b\text{csch}(x)}\,dx = \frac{b^6 \log\left(-2\,be^{(-x)} + ae^{(-2\,x)} - a\right)}{a^7 + 3\,a^5b^2 + 3\,a^3b^4 + ab^6} + \frac{(3\,a^4b + 10\,a^2b^3 + 15\,b^5)\arctan\left(e^{(-x)}\right)}{4\,(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6)} + \frac{(a^5 + 3\,a^3b^2 + 3\,ab^4)\log\left(e^{(-2\,x)} + 1\right)}{a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6} + \frac{(5\,a^2b + 9\,b^3)e^{(-x)} + 8\,(2\,a^3 + 3\,ab^2)e^{(-2\,x)} - (3\,a^2b - b^3)e^{(-3\,x)} + 16\,(a^3 + 2\,ab^2)e^{(-4\,x)} + (3\,a^2b - b^3)e^{(}}{4\,(a^4 + 2\,a^2b^2 + b^4 + 4\,(a^4 + 2\,a^2b^2 + b^4)e^{(-2\,x)} + 6\,(a^4 + 2\,a^2b^2 + b^4)e^{(-4\,x)} + 4\,(a^4 + 2\,a^2b^2} + \frac{x}{a}$$

input
```
integrate(tanh(x)^5/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
b^6*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6
) + 1/4*(3*a^4*b + 10*a^2*b^3 + 15*b^5)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 +
3*a^2*b^4 + b^6) + (a^5 + 3*a^3*b^2 + 3*a*b^4)*log(e^(-2*x) + 1)/(a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6) + 1/4*((5*a^2*b + 9*b^3)*e^(-x) + 8*(2*a^3 + 3*
a*b^2)*e^(-2*x) - (3*a^2*b - b^3)*e^(-3*x) + 16*(a^3 + 2*a*b^2)*e^(-4*x) +
 (3*a^2*b - b^3)*e^(-5*x) + 8*(2*a^3 + 3*a*b^2)*e^(-6*x) - (5*a^2*b + 9*b^
3)*e^(-7*x))/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*e^(-2*x) +
 6*(a^4 + 2*a^2*b^2 + b^4)*e^(-4*x) + 4*(a^4 + 2*a^2*b^2 + b^4)*e^(-6*x) +
 (a^4 + 2*a^2*b^2 + b^4)*e^(-8*x)) + x/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(185) = 370$.

Time = 0.13 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.23

$$\int \frac{\tanh^5(x)}{a + b\text{csch}(x)}\,dx = \frac{b^6 \log\left(\left|-a\left(e^{(-x)} - e^x\right) + 2\,b\right|\right)}{a^7 + 3\,a^5b^2 + 3\,a^3b^4 + ab^6} - \frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)\left(3\,a^4b + 10\,a^2b^3 + 15\,b^5\right)}{16\left(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6\right)} + \frac{\left(a^5 + 3\,a^3b^2 + 3\,ab^4\right)\log\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}{2\left(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6\right)} - \frac{3\,a^5\left(e^{(-x)} - e^x\right)^4 + 9\,a^3b^2\left(e^{(-x)} - e^x\right)^4 + 9\,ab^4\left(e^{(-x)} - e^x\right)^4 + 5\,a^4b\left(e^{(-x)} - e^x\right)^3 + 14\,a^2b^3\left(e^{(-x)} - e^x\right)}{}$$

input
```
integrate(tanh(x)^5/(a+b*csch(x)),x, algorithm="giac")
```

output
```
b^6*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6
) - 1/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(3*a^4*b + 10*a^2*b^3 +
 15*b^5)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(a^5 + 3*a^3*b^2 + 3*a*
b^4)*log((e^(-x) - e^x)^2 + 4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(
3*a^5*(e^(-x) - e^x)^4 + 9*a^3*b^2*(e^(-x) - e^x)^4 + 9*a*b^4*(e^(-x) - e^
x)^4 + 5*a^4*b*(e^(-x) - e^x)^3 + 14*a^2*b^3*(e^(-x) - e^x)^3 + 9*b^5*(e^(
-x) - e^x)^3 + 8*a^5*(e^(-x) - e^x)^2 + 32*a^3*b^2*(e^(-x) - e^x)^2 + 48*a
*b^4*(e^(-x) - e^x)^2 + 12*a^4*b*(e^(-x) - e^x) + 40*a^2*b^3*(e^(-x) - e^x
) + 28*b^5*(e^(-x) - e^x) + 16*a^3*b^2 + 64*a*b^4)/((a^6 + 3*a^4*b^2 + 3*a
^2*b^4 + b^6)*((e^(-x) - e^x)^2 + 4)^2)
```

Mupad [B] (verification not implemented)

Time = 8.11 (sec) , antiderivative size = 611, normalized size of antiderivative = 3.15

$$\int \frac{\tanh^5(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\frac{6\,e^x\,(a^2\,b+b^3)}{(a^2+b^2)^2}+\frac{8\,(a^4+a^2\,b^2)}{a\,(a^2+b^2)^2}}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1} - \frac{\frac{4\,a}{a^2+b^2}+\frac{4\,b\,e^x}{a^2+b^2}}{4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1} - \frac{x}{a} - \frac{\frac{e^x\,(9\,a^2\,b+13\,b^3)}{2\,(a^2+b^2)^2}+\frac{2\,(4\,a^4+5\,a^2\,b^2)}{a\,(a^2+b^2)^2}}{2\,e^{2\,x}+e^{4\,x}+1} + \frac{\frac{e^x\,(5\,a^4\,b+14\,a^2\,b^3+9\,b^5)}{4\,(a^2+b^2)^3}+\frac{2\,(2\,a^6+5\,a^4\,b^2+3\,a^2\,b^4)}{a\,(a^2+b^2)^3}}{e^{2\,x}+1} + \frac{\ln\left(1+e^x\,1\mathrm{i}\right)\,\left(-8\,a^2+a\,b\,21\mathrm{i}+15\,b^2\right)}{8\,\left(-a^3+a^2\,b\,3\mathrm{i}+3\,a\,b^2-b^3\,1\mathrm{i}\right)} + \frac{b^6\,\ln\left(64\,a^{13}\,e^{2\,x}-64\,a\,b^{12}-64\,a^{13}+159\,a^3\,b^{10}-492\,a^5\,b^8-1214\,a^7\,b^6-1020\,a^9\,b^4-393\,a^{11}\,b^2+128\cdots\right.}{\cdots} + \frac{\ln\left(e^x+1\mathrm{i}\right)\,\left(-a^2\,8\mathrm{i}+21\,a\,b+b^2\,15\mathrm{i}\right)}{8\,\left(-a^3\,1\mathrm{i}+3\,a^2\,b+a\,b^2\,3\mathrm{i}-b^3\right)}$$

input
```
int(tanh(x)^5/(a + b/sinh(x)),x)
```

output
```
((6*exp(x)*(a^2*b + b^3))/(a^2 + b^2)^2 + (8*(a^4 + a^2*b^2))/(a*(a^2 + b^
2)^2))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - ((4*a)/(a^2 + b^2) + (4*
b*exp(x))/(a^2 + b^2))/(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) +
1) - x/a - ((exp(x)*(9*a^2*b + 13*b^3))/(2*(a^2 + b^2)^2) + (2*(4*a^4 + 5*
a^2*b^2))/(a*(a^2 + b^2)^2))/(2*exp(2*x) + exp(4*x) + 1) + ((exp(x)*(5*a^4
*b + 9*b^5 + 14*a^2*b^3))/(4*(a^2 + b^2)^3) + (2*(2*a^6 + 3*a^2*b^4 + 5*a^
4*b^2))/(a*(a^2 + b^2)^3))/(exp(2*x) + 1) + (log(exp(x)*1i + 1)*(a*b*21i -
 8*a^2 + 15*b^2))/(8*(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)) + (b^6*log(64*a^
13*exp(2*x) - 64*a*b^12 - 64*a^13 + 159*a^3*b^10 - 492*a^5*b^8 - 1214*a^7*
b^6 - 1020*a^9*b^4 - 393*a^11*b^2 + 128*b^13*exp(x) - 159*a^3*b^10*exp(2*x
) + 492*a^5*b^8*exp(2*x) + 1214*a^7*b^6*exp(2*x) + 1020*a^9*b^4*exp(2*x) +
 393*a^11*b^2*exp(2*x) + 128*a^12*b*exp(x) + 64*a*b^12*exp(2*x) - 318*a^2*
b^11*exp(x) + 984*a^4*b^9*exp(x) + 2428*a^6*b^7*exp(x) + 2040*a^8*b^5*exp(
x) + 786*a^10*b^3*exp(x)))/(a*b^6 + a^7 + 3*a^3*b^4 + 3*a^5*b^2) + (log(ex
p(x) + 1i)*(21*a*b - a^2*8i + b^2*15i))/(8*(a*b^2*3i + 3*a^2*b - a^3*1i -
b^3))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1309, normalized size of antiderivative = 6.75

$$\int \frac{\tanh^5(x)}{a + b\text{csch}(x)}\, dx = \text{Too large to display}$$

input
```
int(tanh(x)^5/(a+b*csch(x)),x)
```

output
```
( - 3*e**(8*x)*atan(e**x)*a**5*b - 10*e**(8*x)*atan(e**x)*a**3*b**3 - 15*e
**(8*x)*atan(e**x)*a*b**5 - 12*e**(6*x)*atan(e**x)*a**5*b - 40*e**(6*x)*at
an(e**x)*a**3*b**3 - 60*e**(6*x)*atan(e**x)*a*b**5 - 18*e**(4*x)*atan(e**x
)*a**5*b - 60*e**(4*x)*atan(e**x)*a**3*b**3 - 90*e**(4*x)*atan(e**x)*a*b**
5 - 12*e**(2*x)*atan(e**x)*a**5*b - 40*e**(2*x)*atan(e**x)*a**3*b**3 - 60*
e**(2*x)*atan(e**x)*a*b**5 - 3*atan(e**x)*a**5*b - 10*atan(e**x)*a**3*b**3
 - 15*atan(e**x)*a*b**5 + 4*e**(8*x)*log(e**(2*x) + 1)*a**6 + 12*e**(8*x)*
log(e**(2*x) + 1)*a**4*b**2 + 12*e**(8*x)*log(e**(2*x) + 1)*a**2*b**4 + 4*
e**(8*x)*log(e**(2*x)*a + 2*e**x*b - a)*b**6 - 4*e**(8*x)*a**6*x - 4*e**(8
*x)*a**6 - 12*e**(8*x)*a**4*b**2*x - 10*e**(8*x)*a**4*b**2 - 12*e**(8*x)*a
**2*b**4*x - 6*e**(8*x)*a**2*b**4 - 4*e**(8*x)*b**6*x + 5*e**(7*x)*a**5*b
+ 14*e**(7*x)*a**3*b**3 + 9*e**(7*x)*a*b**5 + 16*e**(6*x)*log(e**(2*x) + 1
)*a**6 + 48*e**(6*x)*log(e**(2*x) + 1)*a**4*b**2 + 48*e**(6*x)*log(e**(2*x
) + 1)*a**2*b**4 + 16*e**(6*x)*log(e**(2*x)*a + 2*e**x*b - a)*b**6 - 16*e*
*(6*x)*a**6*x - 48*e**(6*x)*a**4*b**2*x - 48*e**(6*x)*a**2*b**4*x - 16*e**
(6*x)*b**6*x - 3*e**(5*x)*a**5*b - 2*e**(5*x)*a**3*b**3 + e**(5*x)*a*b**5
+ 24*e**(4*x)*log(e**(2*x) + 1)*a**6 + 72*e**(4*x)*log(e**(2*x) + 1)*a**4*
b**2 + 72*e**(4*x)*log(e**(2*x) + 1)*a**2*b**4 + 24*e**(4*x)*log(e**(2*x)*
a + 2*e**x*b - a)*b**6 - 24*e**(4*x)*a**6*x - 8*e**(4*x)*a**6 - 72*e**(4*x
)*a**4*b**2*x - 12*e**(4*x)*a**4*b**2 - 72*e**(4*x)*a**2*b**4*x - 4*e**...
```

3.114 $\int \frac{\tanh^4(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 183

$$\begin{aligned}\int \frac{\tanh^4(x)}{a+b\operatorname{csch}(x)}\,dx =& \frac{ab^2x}{(a^2+b^2)^2}+\frac{b^4x}{a(a^2+b^2)^2}+\frac{ax}{a^2+b^2} \\ &+\frac{2b^5\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^{5/2}}+\frac{b^3\operatorname{sech}(x)}{(a^2+b^2)^2}+\frac{b\operatorname{sech}(x)}{a^2+b^2} \\ &-\frac{b\operatorname{sech}^3(x)}{3(a^2+b^2)}-\frac{ab^2\tanh(x)}{(a^2+b^2)^2}-\frac{a\tanh(x)}{a^2+b^2}-\frac{a\tanh^3(x)}{3(a^2+b^2)}\end{aligned}$$

output
```
a*b^2*x/(a^2+b^2)^2+b^4*x/a/(a^2+b^2)^2+a*x/(a^2+b^2)+2*b^5*arctanh((a-b*t
anh(1/2*x))/(a^2+b^2)^(1/2))/a/(a^2+b^2)^(5/2)+b^3*sech(x)/(a^2+b^2)^2+b*s
ech(x)/(a^2+b^2)-b*sech(x)^3/(3*a^2+3*b^2)-a*b^2*tanh(x)/(a^2+b^2)^2-a*tan
h(x)/(a^2+b^2)-a*tanh(x)^3/(3*a^2+3*b^2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.77

$$\int \frac{\tanh^4(x)}{a+b\text{csch}(x)}\,dx=\frac{1}{3}\left(\frac{3\left(x-\frac{2b^5\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{5/2}}\right)}{a}+\frac{3b(a^2+2b^2)\,\text{sech}(x)}{(a^2+b^2)^2}-\frac{b\text{sech}^3(x)}{a^2+b^2}-\frac{a(4a^2+7b^2)\tanh(x)}{(a^2+b^2)^2}+\frac{a\text{sech}^2(x)\tanh(x)}{a^2+b^2}\right)$$

input
```
Integrate[Tanh[x]^4/(a + b*Csch[x]),x]
```

output
```
((3*(x - (2*b^5*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(
5/2)))/a + (3*b*(a^2 + 2*b^2)*Sech[x])/(a^2 + b^2)^2 - (b*Sech[x]^3)/(a^2
+ b^2) - (a*(4*a^2 + 7*b^2)*Tanh[x])/(a^2 + b^2)^2 + (a*Sech[x]^2*Tanh[x])
/(a^2 + b^2))/3
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(x)}{a+b\text{csch}(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cot(ix)^4(a+ib\csc(ix))}dx$$

$$\Big\downarrow \ 4386$$

$$\int \frac{i\sinh(x)\tanh^4(x)}{ia\sinh(x)+ib}dx$$

$$\Big\downarrow \ 26$$

$$i\int -\frac{i\sinh(x)\tanh^4(x)}{b+a\sinh(x)}dx$$

$$\Big\downarrow \ 26$$

$$\int \frac{\sinh(x)\tanh^4(x)}{a\sinh(x)+b}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{i\sin(ix)^5}{\cos(ix)^4(b-ia\sin(ix))}dx$$

$$\Big\downarrow \ 26$$

$$-i\int \frac{\sin(ix)^5}{\cos(ix)^4(b-ia\sin(ix))}dx$$

$$\Big\downarrow \ 3381$$

$$-i\left(\frac{ia\int \tanh^4(x)dx}{a^2+b^2}-\frac{b^2\int -\frac{i\sinh(x)\tanh^2(x)}{b+a\sinh(x)}dx}{a^2+b^2}+\frac{b\int -i\text{sech}(x)\tanh^3(x)dx}{a^2+b^2}\right)$$

$$\Big\downarrow \ 26$$

$$-i\left(\frac{ia\int \tanh^4(x)dx}{a^2+b^2}+\frac{ib^2\int \frac{\sinh(x)\tanh^2(x)}{b+a\sinh(x)}dx}{a^2+b^2}-\frac{ib\int \text{sech}(x)\tanh^3(x)dx}{a^2+b^2}\right)$$

$$\Big\downarrow \ 3042$$

$$-i\left(\frac{ia\int \tan(ix)^4dx}{a^2+b^2}+\frac{ib^2\int \frac{i\sin(ix)^3}{\cos(ix)^2(b-ia\sin(ix))}dx}{a^2+b^2}-\frac{ib\int i\sec(ix)\tan(ix)^3dx}{a^2+b^2}\right)$$

$$\Big\downarrow \ 26$$

$$-i\left(\frac{ia\int \tan(ix)^4dx}{a^2+b^2}-\frac{b^2\int \frac{\sin(ix)^3}{\cos(ix)^2(b-ia\sin(ix))}dx}{a^2+b^2}+\frac{b\int \sec(ix)\tan(ix)^3dx}{a^2+b^2}\right)$$

$$\Big\downarrow \ 3086$$

$$-i\left(\frac{ia\int \tan(ix)^4dx}{a^2+b^2}-\frac{ib\int\left(\text{sech}^2(x)-1\right)d\text{sech}(x)}{a^2+b^2}-\frac{b^2\int\frac{\sin(ix)^3}{\cos(ix)^2(b-ia\sin(ix))}dx}{a^2+b^2}\right)$$

$$\Big\downarrow \; 2009$$

$$-i\left(\frac{ia\int \tan(ix)^4dx}{a^2+b^2}-\frac{b^2\int\frac{\sin(ix)^3}{\cos(ix)^2(b-ia\sin(ix))}dx}{a^2+b^2}-\frac{ib\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a^2+b^2}\right)$$

$$\Big\downarrow \; 3381$$

$$-i\left(\frac{ia\int \tan(ix)^4dx}{a^2+b^2}-\frac{b^2\left(-\frac{b^2\int\frac{i\sinh(x)}{b+a\sinh(x)}dx}{a^2+b^2}+\frac{ia\int-\tanh^2(x)dx}{a^2+b^2}+\frac{b\int i\text{sech}(x)\tanh(x)dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{ib\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a^2+b^2}\right)$$

$$\Big\downarrow \; 25$$

$$-i\left(\frac{ia\int \tan(ix)^4dx}{a^2+b^2}-\frac{b^2\left(-\frac{b^2\int\frac{i\sinh(x)}{b+a\sinh(x)}dx}{a^2+b^2}-\frac{ia\int\tanh^2(x)dx}{a^2+b^2}+\frac{b\int i\text{sech}(x)\tanh(x)dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{ib\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a^2+b^2}\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(\frac{ia\int \tan(ix)^4dx}{a^2+b^2}-\frac{b^2\left(-\frac{ib^2\int\frac{\sinh(x)}{b+a\sinh(x)}dx}{a^2+b^2}-\frac{ia\int\tanh^2(x)dx}{a^2+b^2}+\frac{ib\int\text{sech}(x)\tanh(x)dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{ib\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a^2+b^2}\right)$$

$$\Big\downarrow \; 3042$$

$$-i\left(\frac{ia\int \tan(ix)^4dx}{a^2+b^2}-\frac{b^2\left(-\frac{ib^2\int-\frac{i\sin(ix)}{b-ia\sin(ix)}dx}{a^2+b^2}-\frac{ia\int-\tan(ix)^2dx}{a^2+b^2}+\frac{ib\int-i\sec(ix)\tan(ix)dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{ib\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a^2+b^2}\right)$$

$$\Big\downarrow \; 25$$

$$-i\left(\frac{ia\int \tan(ix)^4dx}{a^2+b^2}-\frac{b^2\left(-\frac{ib^2\int-\frac{i\sin(ix)}{b-ia\sin(ix)}dx}{a^2+b^2}+\frac{ia\int\tan(ix)^2dx}{a^2+b^2}+\frac{ib\int-i\sec(ix)\tan(ix)dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{ib\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a^2+b^2}\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(\frac{ia\int \tan(ix)^4dx}{a^2+b^2}-\frac{b^2\left(-\frac{b^2\int \frac{\sin(ix)}{b-ia\sin(ix)}dx}{a^2+b^2}+\frac{ia\int \tan(ix)^2dx}{a^2+b^2}+\frac{b\int \sec(ix)\tan(ix)dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{ib\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a^2+b^2}\right)$$

$$\Big\downarrow \; 3086$$

$$-i\left(\frac{ia\int \tan(ix)^4dx}{a^2+b^2}-\frac{b^2\left(-\frac{b^2\int \frac{\sin(ix)}{b-ia\sin(ix)}dx}{a^2+b^2}+\frac{ia\int \tan(ix)^2dx}{a^2+b^2}-\frac{ib\int 1d\text{sech}(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ib\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a^2+b^2}\right)$$

$$\Big\downarrow \; 24$$

$$-i\left(\frac{ia\int \tan(ix)^4dx}{a^2+b^2}-\frac{b^2\left(-\frac{b^2\int \frac{\sin(ix)}{b-ia\sin(ix)}dx}{a^2+b^2}+\frac{ia\int \tan(ix)^2dx}{a^2+b^2}-\frac{ib\text{sech}(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ib\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a^2+b^2}\right)$$

$$\Big\downarrow \; 3214$$

$$-i\left(\frac{ia\int \tan(ix)^4dx}{a^2+b^2}-\frac{b^2\left(\frac{ia\int \tan(ix)^2dx}{a^2+b^2}-\frac{b^2\left(\frac{ix}{a}-\frac{ib\int \frac{1}{b+a\sinh(x)}dx}{a}\right)}{a^2+b^2}-\frac{ib\text{sech}(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ib\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a^2+b^2}\right)$$

$$\Big\downarrow \; 3042$$

$$-i\left(\frac{ia\int \tan(ix)^4dx}{a^2+b^2}-\frac{b^2\left(-\frac{b^2\left(\frac{ix}{a}-\frac{ib\int \frac{1}{b-ia\sin(ix)}dx}{a}\right)}{a^2+b^2}+\frac{ia\int \tan(ix)^2dx}{a^2+b^2}-\frac{ib\text{sech}(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ib\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a^2+b^2}\right)$$

$$\Big\downarrow \; 3139$$

$$-i\left(\frac{ia\int \tan(ix)^4dx}{a^2+b^2}-\frac{b^2\left(\frac{ia\int \tan(ix)^2dx}{a^2+b^2}-\frac{b^2\left(\frac{ix}{a}-\frac{2ib\int \frac{1}{-b\tanh^2\left(\frac{x}{2}\right)+2a\tanh\left(\frac{x}{2}\right)+b}d\tanh\left(\frac{x}{2}\right)}{a}\right)}{a^2+b^2}-\frac{ib\text{sech}(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ib\left(\frac{\text{sech}^3(x)}{3}\right.}{a^2}\right.$$

$\Big\downarrow$ 1083

$$-i\left(\frac{ia\int \tan(ix)^4dx}{a^2+b^2}-\frac{b^2\left(\frac{ia\int \tan(ix)^2dx}{a^2+b^2}-\frac{b^2\left(\frac{4ib\int \frac{1}{4\left(a^2+b^2\right)-\left(2a-2b\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2a-2b\tanh\left(\frac{x}{2}\right)\right)}{a}+\frac{ix}{a}\right)}{a^2+b^2}-\frac{ib\text{sech}(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ib\left(\right.}{}\right.$$

$\Big\downarrow$ 219

$$-i\left(-\frac{b^2\left(\frac{ia\int \tan(ix)^2dx}{a^2+b^2}-\frac{b^2\left(\frac{2ib\,\text{arctanh}\left(\frac{2a-2b\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{ix}{a}\right)}{a^2+b^2}-\frac{ib\text{sech}(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{ia\int \tan(ix)^4dx}{a^2+b^2}-\frac{ib\left(\frac{\text{sech}^3(x)}{3}-\text{se}\right.}{a^2+b^2}\right.$$

$\Big\downarrow$ 3954

$$-i\left(-\frac{b^2\left(\frac{ia(\tanh(x)-\int 1dx)}{a^2+b^2}-\frac{b^2\left(\frac{2ib\text{arctanh}\left(\frac{2a-2b\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{ix}{a}\right)}{a^2+b^2}-\frac{ib\text{sech}(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{ia\left(-\int-\tanh^2(x)dx-\frac{1}{3}\tanh^3(\right.}{a^2+b^2}\right.$$

$\downarrow$ 24

$$-i\left(\frac{ia\left(-\int-\tanh^2(x)dx-\frac{1}{3}\tanh^3(x)\right)}{a^2+b^2}-\frac{b^2\left(-\frac{b^2\left(\frac{2ib\text{arctanh}\left(\frac{2a-2b\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{ix}{a}\right)}{a^2+b^2}+\frac{ia(\tanh(x)-x)}{a^2+b^2}-\frac{ib\text{sech}(x)}{a^2+b^2}\right)}{a^2+b^2}\right.$$

$\downarrow$ 25

$$-i\left(\frac{ia\left(\int \tanh^2(x)dx-\frac{\tanh^3(x)}{3}\right)}{a^2+b^2}-\frac{b^2\left(-\frac{b^2\left(\frac{2ib\text{arctanh}\left(\frac{2a-2b\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{ix}{a}\right)}{a^2+b^2}+\frac{ia(\tanh(x)-x)}{a^2+b^2}-\frac{ib\text{sech}(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ib\left(\right.}{}\right.$$

$\downarrow$ 3042

$$-i\left(\frac{ia\left(-\frac{\tanh^3(x)}{3}+\int -\tan(ix)^2dx\right)}{a^2+b^2}-\frac{b^2\left(-\frac{b^2\left(\frac{2ib\text{arctanh}\left(\frac{2a-2b\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{ix}{a}\right)}{a^2+b^2}+\frac{ia(\tanh(x)-x)}{a^2+b^2}-\frac{ib\text{sech}(x)}{a^2+b^2}\right)}{a^2+b^2}-\right.$$

$\downarrow$ 25

$$-i\left(\frac{ia\left(-\frac{1}{3}\tanh^3(x)-\int\tan(ix)^2dx\right)}{a^2+b^2}-\frac{b^2\left(-\frac{b^2\left(\frac{2ib\,\mathrm{arctanh}\left(\frac{2a-2b\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{ix}{a}\right)}{a^2+b^2}+\frac{ia(\tanh(x)-x)}{a^2+b^2}-\frac{ib\,\mathrm{sech}(x)}{a^2+b^2}\right)}{a^2+b^2}-\right.$$

input
```
Int[Tanh[x]^4/(a + b*Csch[x]),x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.13

method	result
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{2\left(-a^3-2a\,b^2\right)\tanh\left(\frac{x}{2}\right)^5+2\tanh\left(\frac{x}{2}\right)^4b^3+2\left(-\frac{10}{3}a^3-\frac{16}{3}a\,b^2\right)\tanh\left(\frac{x}{2}\right)^3+2\left(2a^2b+4b^3\right)\tanh\left(\frac{x}{2}\right)^2+2\left(-a^3-2\right.}{\left(a^4+2a^2b^2+b^4\right)\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^3}$
risch	$\frac{x}{a}+\frac{2a^2b\,e^{5x}+4\,e^{5x}b^3+4a^3e^{4x}+6a\,b^2e^{4x}+\frac{4a^2b\,e^{3x}}{3}+\frac{16b^3e^{3x}}{3}+4a^3e^{2x}+8a\,b^2e^{2x}+2b\,e^xa^2+4b^3e^x+\frac{8a^3}{3}+\frac{14a\,b^2}{3}}{\left(a^4+2a^2b^2+b^4\right)\left(e^{2x}+1\right)^3}+\frac{b^5\ln\left(e^x+\frac{(a}{}\right.}{}$

input
```
int(tanh(x)^4/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/a*ln(tanh(1/2*x)-1)+2/(a^4+2*a^2*b^2+b^4)*((-a^3-2*a*b^2)*tanh(1/2*x)^5
+tanh(1/2*x)^4*b^3+(-10/3*a^3-16/3*a*b^2)*tanh(1/2*x)^3+(2*a^2*b+4*b^3)*ta
nh(1/2*x)^2+(-a^3-2*a*b^2)*tanh(1/2*x)+2/3*a^2*b+5/3*b^3)/(tanh(1/2*x)^2+1
)^3+2/a*b^5/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*tanh(1/2*x
)*b+2*a)/(a^2+b^2)^(1/2))+1/a*ln(tanh(1/2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1746 vs. $2(175) = 350$.

Time = 0.12 (sec) , antiderivative size = 1746, normalized size of antiderivative = 9.54

$$\int \frac{\tanh^4(x)}{a + b\text{csch}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
1/3*(3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^6 + 3*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*x*sinh(x)^6 + 8*a^6 + 22*a^4*b^2 + 14*a^2*b^4 + 6*(a^
5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x)^5 + 6*(a^5*b + 3*a^3*b^3 + 2*a*b^5 + 3*
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x))*sinh(x)^5 + 3*(4*a^6 + 10*a
^4*b^2 + 6*a^2*b^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^4 +
3*(4*a^6 + 10*a^4*b^2 + 6*a^2*b^4 + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
*x*cosh(x)^2 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x + 10*(a^5*b + 3*a^3
*b^3 + 2*a*b^5)*cosh(x))*sinh(x)^4 + 4*(a^5*b + 5*a^3*b^3 + 4*a*b^5)*cosh(
x)^3 + 4*(a^5*b + 5*a^3*b^3 + 4*a*b^5 + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6)*x*cosh(x)^3 + 15*(a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x)^2 + 3*(4*a^6 +
 10*a^4*b^2 + 6*a^2*b^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)
)*sinh(x)^3 + 3*(4*a^6 + 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 + 3*a^4*b^2 + 3*a
^2*b^4 + b^6)*x)*cosh(x)^2 + 3*(4*a^6 + 12*a^4*b^2 + 8*a^2*b^4 + 15*(a^6 +
 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^4 + 20*(a^5*b + 3*a^3*b^3 + 2*a*b^
5)*cosh(x)^3 + 6*(4*a^6 + 10*a^4*b^2 + 6*a^2*b^4 + 3*(a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6)*x)*cosh(x)^2 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x + 4*
(a^5*b + 5*a^3*b^3 + 4*a*b^5)*cosh(x))*sinh(x)^2 + 3*(b^5*cosh(x)^6 + 6*b^
5*cosh(x)*sinh(x)^5 + b^5*sinh(x)^6 + 3*b^5*cosh(x)^4 + 3*b^5*cosh(x)^2 +
b^5 + 3*(5*b^5*cosh(x)^2 + b^5)*sinh(x)^4 + 4*(5*b^5*cosh(x)^3 + 3*b^5*cos
h(x))*sinh(x)^3 + 3*(5*b^5*cosh(x)^4 + 6*b^5*cosh(x)^2 + b^5)*sinh(x)^2...
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{a + b\operatorname{csch}(x)}\,dx = \int \frac{\tanh^4(x)}{a + b\operatorname{csch}(x)}\,dx$$

input
```
integrate(tanh(x)**4/(a+b*csch(x)),x)
```

output
```
Integral(tanh(x)**4/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.43

$$\int \frac{\tanh^4(x)}{a + b\operatorname{csch}(x)}\,dx = -\frac{b^5 \log\left(\frac{ae^{(-x)} - b - \sqrt{a^2+b^2}}{ae^{(-x)} - b + \sqrt{a^2+b^2}}\right)}{(a^5 + 2\,a^3b^2 + ab^4)\sqrt{a^2+b^2}} - \frac{2\,(4\,a^3 + 7\,ab^2 - 3\,(a^2b + 2\,b^3)e^{(-x)} + 6\,(a^3 + 2\,ab^2)e^{(-2\,x)} - 2\,(a^2b + 4\,b^3)e^{(-3\,x)} + 3\,(2\,a^3 + 3\,ab^2)e^{(-4}}{3\,(a^4 + 2\,a^2b^2 + b^4 + 3\,(a^4 + 2\,a^2b^2 + b^4)e^{(-2\,x)} + 3\,(a^4 + 2\,a^2b^2 + b^4)e^{(-4\,x)} + (a^4 + 2\,a^2b^2} + \frac{x}{a}$$

input
```
integrate(tanh(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
-b^5*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2))
)/((a^5 + 2*a^3*b^2 + a*b^4)*sqrt(a^2 + b^2)) - 2/3*(4*a^3 + 7*a*b^2 - 3*(
a^2*b + 2*b^3)*e^(-x) + 6*(a^3 + 2*a*b^2)*e^(-2*x) - 2*(a^2*b + 4*b^3)*e^(
-3*x) + 3*(2*a^3 + 3*a*b^2)*e^(-4*x) - 3*(a^2*b + 2*b^3)*e^(-5*x))/(a^4 +
2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*e^(-2*x) + 3*(a^4 + 2*a^2*b^2
+ b^4)*e^(-4*x) + (a^4 + 2*a^2*b^2 + b^4)*e^(-6*x)) + x/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.17

$$\int \frac{\tanh^4(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{b^5 \log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^5+2\,a^3b^2+ab^4)\sqrt{a^2+b^2}} + \frac{x}{a} + \frac{2\left(3\,a^2be^{(5\,x)}+6\,b^3e^{(5\,x)}+6\,a^3e^{(4\,x)}+9\,ab^2e^{(4\,x)}+2\,a^2be^{(3\,x)}+8\,b^3e^{(3\,x)}+6\,a^3e^{(2\,x)}+12\,ab^2e^{(2\,x)}+3\,a^2\right.}{3\,(a^4+2\,a^2b^2+b^4)(e^{(2\,x)}+1)^3}$$

input
```
integrate(tanh(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

output
```
-b^5*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt
(a^2 + b^2)))/((a^5 + 2*a^3*b^2 + a*b^4)*sqrt(a^2 + b^2)) + x/a + 2/3*(3*a
^2*b*e^(5*x) + 6*b^3*e^(5*x) + 6*a^3*e^(4*x) + 9*a*b^2*e^(4*x) + 2*a^2*b*e
^(3*x) + 8*b^3*e^(3*x) + 6*a^3*e^(2*x) + 12*a*b^2*e^(2*x) + 3*a^2*b*e^x +
6*b^3*e^x + 4*a^3 + 7*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(e^(2*x) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 707, normalized size of antiderivative = 3.86

$$\int \frac{\tanh^4(x)}{a+b\operatorname{csch}(x)}\,dx$$

$$= \frac{x}{a} + \frac{\frac{8\,a}{3\,(a^2+b^2)}+\frac{8\,b\,e^x}{3\,(a^2+b^2)}}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1} - \frac{\frac{8\,e^x\,(a^2\,b+b^3)}{3\,(a^2+b^2)^2}+\frac{4\,(a^4+a^2\,b^2)}{a\,(a^2+b^2)^2}}{2\,e^{2\,x}+e^{4\,x}+1} + \frac{\frac{2\,e^x\,(a^2\,b+2\,b^3)}{(a^2+b^2)^2}+\frac{2\,(2\,a^4+3\,a^2\,b^2)}{a\,(a^2+b^2)^2}}{e^{2\,x}+1} + \frac{2\,\mathrm{atan}\left(\left(\left(e^x\left(\frac{2\,b^5}{a^3\,\sqrt{b^{10}}\,(a^2+b^2)^2\,(a^5+2\,a^3\,b^2+a\,b^4)}+\frac{2\left(2\,a^3\,b^3\,\sqrt{b^{10}}+a\,b^5\,\sqrt{b^{10}}+a^5\,b\,\sqrt{b^{10}}\right)}{a^2\,b^4\,\sqrt{-a^2\,(a^2+b^2)^5}\,(a^5+2\,a^3\,b^2+a\,b^4)\,\sqrt{-a^{12}-5\,a^{10}\,b^2-10\,a^8\,b^4-10\,a^6\,b^6-5}}\right.\right.\right.\right.}{}$$

input
```
int(tanh(x)^4/(a + b/sinh(x)),x)
```

output
```
x/a + ((8*a)/(3*(a^2 + b^2)) + (8*b*exp(x))/(3*(a^2 + b^2)))/(3*exp(2*x) +
 3*exp(4*x) + exp(6*x) + 1) - ((8*exp(x)*(a^2*b + b^3))/(3*(a^2 + b^2)^2)
+ (4*(a^4 + a^2*b^2))/(a*(a^2 + b^2)^2))/(2*exp(2*x) + exp(4*x) + 1) + ((2
*exp(x)*(a^2*b + 2*b^3))/(a^2 + b^2)^2 + (2*(2*a^4 + 3*a^2*b^2))/(a*(a^2 +
 b^2)^2))/(exp(2*x) + 1) + (2*atan((exp(x)*((2*b^5)/(a^3*(b^10)^(1/2)*(a^2
 + b^2)^2*(a*b^4 + a^5 + 2*a^3*b^2)) + (2*(2*a^3*b^3*(b^10)^(1/2) + a*b^5*
(b^10)^(1/2) + a^5*b*(b^10)^(1/2)))/(a^2*b^4*(-a^2*(a^2 + b^2)^5)^(1/2)*(a
*b^4 + a^5 + 2*a^3*b^2)*(- a^12 - a^2*b^10 - 5*a^4*b^8 - 10*a^6*b^6 - 10*a
^8*b^4 - 5*a^10*b^2)^(1/2))) - (2*(a^6*(b^10)^(1/2) + a^2*b^4*(b^10)^(1/2)
 + 2*a^4*b^2*(b^10)^(1/2)))/(a^2*b^4*(-a^2*(a^2 + b^2)^5)^(1/2)*(a*b^4 + a
^5 + 2*a^3*b^2)*(- a^12 - a^2*b^10 - 5*a^4*b^8 - 10*a^6*b^6 - 10*a^8*b^4 -
 5*a^10*b^2)^(1/2)))*((a^6*(- a^12 - a^2*b^10 - 5*a^4*b^8 - 10*a^6*b^6 - 1
0*a^8*b^4 - 5*a^10*b^2)^(1/2))/2 + (a^2*b^4*(- a^12 - a^2*b^10 - 5*a^4*b^8
 - 10*a^6*b^6 - 10*a^8*b^4 - 5*a^10*b^2)^(1/2))/2 + a^4*b^2*(- a^12 - a^2*
b^10 - 5*a^4*b^8 - 10*a^6*b^6 - 10*a^8*b^4 - 5*a^10*b^2)^(1/2)))*(b^10)^(1
/2))/(- a^12 - a^2*b^10 - 5*a^4*b^8 - 10*a^6*b^6 - 10*a^8*b^4 - 5*a^10*b^2
)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 688, normalized size of antiderivative = 3.76

$$\int \frac{\tanh^4(x)}{a + b\text{csch}(x)}\,dx$$

$$= \frac{-4e^{6x}a^6 - 6\sqrt{a^2+b^2}\,atan\left(\frac{e^x ai+bi}{\sqrt{a^2+b^2}}\right)b^5 i + 9e^{6x}a^4b^2x + 9e^{6x}a^2b^4x + 27e^{2x}a^4b^2x + 27e^{2x}a^2b^4x + 6e^{5x}a^5b +}{}$$

input
```
int(tanh(x)^4/(a+b*csch(x)),x)
```

output
```
( - 6*e**(6*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2))*
b**5*i - 18*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b
**2))*b**5*i - 18*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a*
*2 + b**2))*b**5*i - 6*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 +
 b**2))*b**5*i + 3*e**(6*x)*a**6*x - 4*e**(6*x)*a**6 + 9*e**(6*x)*a**4*b**
2*x - 10*e**(6*x)*a**4*b**2 + 9*e**(6*x)*a**2*b**4*x - 6*e**(6*x)*a**2*b**
4 + 3*e**(6*x)*b**6*x + 6*e**(5*x)*a**5*b + 18*e**(5*x)*a**3*b**3 + 12*e**
(5*x)*a*b**5 + 9*e**(4*x)*a**6*x + 27*e**(4*x)*a**4*b**2*x + 27*e**(4*x)*a
**2*b**4*x + 9*e**(4*x)*b**6*x + 4*e**(3*x)*a**5*b + 20*e**(3*x)*a**3*b**3
 + 16*e**(3*x)*a*b**5 + 9*e**(2*x)*a**6*x + 27*e**(2*x)*a**4*b**2*x + 6*e*
*(2*x)*a**4*b**2 + 27*e**(2*x)*a**2*b**4*x + 6*e**(2*x)*a**2*b**4 + 9*e**(
2*x)*b**6*x + 6*e**x*a**5*b + 18*e**x*a**3*b**3 + 12*e**x*a*b**5 + 3*a**6*
x + 4*a**6 + 9*a**4*b**2*x + 12*a**4*b**2 + 9*a**2*b**4*x + 8*a**2*b**4 +
3*b**6*x)/(3*a*(e**(6*x)*a**6 + 3*e**(6*x)*a**4*b**2 + 3*e**(6*x)*a**2*b**
4 + e**(6*x)*b**6 + 3*e**(4*x)*a**6 + 9*e**(4*x)*a**4*b**2 + 9*e**(4*x)*a*
*2*b**4 + 3*e**(4*x)*b**6 + 3*e**(2*x)*a**6 + 9*e**(2*x)*a**4*b**2 + 9*e**
(2*x)*a**2*b**4 + 3*e**(2*x)*b**6 + a**6 + 3*a**4*b**2 + 3*a**2*b**4 + b**
6))
```

3.115 $\int \frac{\tanh^3(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 113

$$\begin{aligned}\int \frac{\tanh^3(x)}{a+b\operatorname{csch}(x)}\,dx = &-\frac{b^3\arctan(\sinh(x))}{(a^2+b^2)^2}-\frac{b\arctan(\sinh(x))}{2(a^2+b^2)}\\&+\frac{b^4\log(a+b\operatorname{csch}(x))}{a(a^2+b^2)^2}+\frac{\log(\sinh(x))}{a}\\&-\frac{a(a^2+2b^2)\log(\tanh(x))}{(a^2+b^2)^2}-\frac{(a-b\operatorname{csch}(x))\tanh^2(x)}{2(a^2+b^2)}\end{aligned}$$

output
```
-b^3*arctan(sinh(x))/(a^2+b^2)^2-b*arctan(sinh(x))/(2*a^2+2*b^2)+b^4*ln(a+
b*csch(x))/a/(a^2+b^2)^2+ln(sinh(x))/a-a*(a^2+2*b^2)*ln(tanh(x))/(a^2+b^2)
^2-(a-b*csch(x))*tanh(x)^2/(2*a^2+2*b^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.69

$$\int \frac{\tanh^3(x)}{a+b\text{csch}(x)}\,dx$$
$$= \frac{ab(a^2+b^2)\arctan(\sinh(x))+a^4\log(i-\sinh(x))+ia^3b\log(i-\sinh(x))+2a^2b^2\log(i-\sinh(x))+2iab}{}$$

input
```
Integrate[Tanh[x]^3/(a + b*Csch[x]),x]
```

output
```
(a*b*(a^2 + b^2)*ArcTan[Sinh[x]] + a^4*Log[I - Sinh[x]] + I*a^3*b*Log[I -
Sinh[x]] + 2*a^2*b^2*Log[I - Sinh[x]] + (2*I)*a*b^3*Log[I - Sinh[x]] + a^4
*Log[I + Sinh[x]] - I*a^3*b*Log[I + Sinh[x]] + 2*a^2*b^2*Log[I + Sinh[x]]
- (2*I)*a*b^3*Log[I + Sinh[x]] + 2*b^4*Log[b + a*Sinh[x]] + a^2*(a^2 + b^2
)*Sech[x]^2 + a*b*(a^2 + b^2)*Sech[x]*Tanh[x])/(2*a*(a^2 + b^2)^2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.34, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 26, 4373, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{a+b\text{csch}(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i}{\cot(ix)^3(a+ib\csc(ix))}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{1}{\cot(ix)^3(a+ib\csc(ix))}dx$$

$$\Big\downarrow \text{4373}$$

$$-b^4 \int \frac{\sinh(x)}{b(a+b\text{csch}(x))\left(\text{csch}^2(x)b^2+b^2\right)^2}d(b\text{csch}(x))$$

$$\Big\downarrow \text{615}$$

$$-b^4 \int \left(\frac{-b^2-a\text{csch}(x)b}{b^2\left(a^2+b^2\right)\left(\text{csch}^2(x)b^2+b^2\right)^2}+\frac{\sinh(x)}{ab^5}-\frac{1}{a\left(a^2+b^2\right)^2(a+b\text{csch}(x))}+\frac{-b^4-a\left(a^2+2b^2\right)\text{csch}(x)b}{b^4\left(a^2+b^2\right)^2\left(\text{csch}^2(x)b^2+b\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$-b^4\left(-\frac{\arctan(\text{csch}(x))}{b\left(a^2+b^2\right)^2}-\frac{\arctan(\text{csch}(x))}{2b^3\left(a^2+b^2\right)}+\frac{a-b\text{csch}(x)}{2b^2\left(a^2+b^2\right)\left(b^2\text{csch}^2(x)+b^2\right)}-\frac{\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)^2}-\frac{a\left(a^2+2b^2\right)}{2}\right.$$

input
```
Int[Tanh[x]^3/(a + b*Csch[x]),x]
```

output
```
-(b^4*(-(ArcTan[Csch[x]]/(b*(a^2 + b^2)^2)) - ArcTan[Csch[x]]/(2*b^3*(a^2
+ b^2)) + (a - b*Csch[x])/(2*b^2*(a^2 + b^2)*(b^2 + b^2*Csch[x]^2)) + Log[
b*Csch[x]]/(a*b^4) - Log[a + b*Csch[x]]/(a*(a^2 + b^2)^2) - (a*(a^2 + 2*b^
2)*Log[b^2 + b^2*Csch[x]^2])/(2*b^4*(a^2 + b^2)^2)))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.61

method	result
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{\frac{2\left(\left(-\frac{1}{2}a^{2}b-\frac{1}{2}b^{3}\right)\tanh\left(\frac{x}{2}\right)^{3}+\left(-a^{3}-a\,b^{2}\right)\tanh\left(\frac{x}{2}\right)^{2}+\left(\frac{1}{2}a^{2}b+\frac{1}{2}b^{3}\right)\tanh\left(\frac{x}{2}\right)\right)}{\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{2}}+\frac{\left(2a^{3}+4a\,b^{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)}{2}+(}{\left(a^{2}+b^{2}\right)^{2}}$
risch	$\frac{x}{a}-\frac{2x\,a^{3}}{a^{4}+2a^{2}b^{2}+b^{4}}-\frac{4xa\,b^{2}}{a^{4}+2a^{2}b^{2}+b^{4}}-\frac{2x\,b^{4}}{a\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}+\frac{\mathrm{e}^{x}\left(\mathrm{e}^{2x}b+2\,\mathrm{e}^{x}a-b\right)}{\left(\mathrm{e}^{2x}+1\right)^{2}\left(a^{2}+b^{2}\right)}+\frac{i\ln\left(\mathrm{e}^{x}-i\right)a^{2}b}{2a^{4}+4a^{2}b^{2}+2b^{4}}+\frac{3i\ln\left(\mathrm{e}^{x}-i\right)b^{3}}{2\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}+$

input
```
int(tanh(x)^3/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/a*ln(tanh(1/2*x)-1)+2/(a^2+b^2)^2*(((-1/2*a^2*b-1/2*b^3)*tanh(1/2*x)^3+
(-a^3-a*b^2)*tanh(1/2*x)^2+(1/2*a^2*b+1/2*b^3)*tanh(1/2*x))/(tanh(1/2*x)^2
+1)^2+1/4*(2*a^3+4*a*b^2)*ln(tanh(1/2*x)^2+1)+1/2*(-a^2*b-3*b^3)*arctan(ta
nh(1/2*x)))+b^4/a/(a^2+b^2)^2*ln(-b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b)-1/a*l
n(tanh(1/2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 965 vs. 2(110) = 220.

Time = 0.13 (sec) , antiderivative size = 965, normalized size of antiderivative = 8.54

$$\int \frac{\tanh^3(x)}{a+b\text{csch}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
-((a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^4 + (a^4 + 2*a^2*b^2 + b^4)*x*sinh(x)^
4 - (a^3*b + a*b^3)*cosh(x)^3 - (a^3*b + a*b^3 - 4*(a^4 + 2*a^2*b^2 + b^4)
*x*cosh(x))*sinh(x)^3 - 2*(a^4 + a^2*b^2 - (a^4 + 2*a^2*b^2 + b^4)*x)*cosh
(x)^2 - (2*a^4 + 2*a^2*b^2 - 6*(a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^2 - 2*(a^
4 + 2*a^2*b^2 + b^4)*x + 3*(a^3*b + a*b^3)*cosh(x))*sinh(x)^2 + (a^4 + 2*a
^2*b^2 + b^4)*x + ((a^3*b + 3*a*b^3)*cosh(x)^4 + 4*(a^3*b + 3*a*b^3)*cosh(
x)*sinh(x)^3 + (a^3*b + 3*a*b^3)*sinh(x)^4 + a^3*b + 3*a*b^3 + 2*(a^3*b +
3*a*b^3)*cosh(x)^2 + 2*(a^3*b + 3*a*b^3 + 3*(a^3*b + 3*a*b^3)*cosh(x)^2)*s
inh(x)^2 + 4*((a^3*b + 3*a*b^3)*cosh(x)^3 + (a^3*b + 3*a*b^3)*cosh(x))*sin
h(x))*arctan(cosh(x) + sinh(x)) + (a^3*b + a*b^3)*cosh(x) - (b^4*cosh(x)^4
 + 4*b^4*cosh(x)*sinh(x)^3 + b^4*sinh(x)^4 + 2*b^4*cosh(x)^2 + b^4 + 2*(3*
b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 + b^4*cosh(x))*sinh(x))*
log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) - ((a^4 + 2*a^2*b^2)*cosh(x)^4
+ 4*(a^4 + 2*a^2*b^2)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^2*b^2)*sinh(x)^4 + a^
4 + 2*a^2*b^2 + 2*(a^4 + 2*a^2*b^2)*cosh(x)^2 + 2*(a^4 + 2*a^2*b^2 + 3*(a^
4 + 2*a^2*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 + 2*a^2*b^2)*cosh(x)^3 + (a^
4 + 2*a^2*b^2)*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + (4*(
a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^3 + a^3*b + a*b^3 - 3*(a^3*b + a*b^3)*cos
h(x)^2 - 4*(a^4 + a^2*b^2 - (a^4 + 2*a^2*b^2 + b^4)*x)*cosh(x))*sinh(x))/(
a^5 + 2*a^3*b^2 + a*b^4 + (a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^4 + 4*(a^5 ...
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{a + b\text{csch}(x)}\, dx = \int \frac{\tanh^3{(x)}}{a + b\operatorname{csch}{(x)}}\, dx$$

input
```
integrate(tanh(x)**3/(a+b*csch(x)),x)
```

output
```
Integral(tanh(x)**3/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.52

$$\int \frac{\tanh^3(x)}{a + b\operatorname{csch}(x)}\,dx = \frac{b^4 \log\left(-2\,be^{(-x)} + ae^{(-2\,x)} - a\right)}{a^5 + 2\,a^3b^2 + ab^4} + \frac{(a^2b + 3\,b^3)\arctan\left(e^{(-x)}\right)}{a^4 + 2\,a^2b^2 + b^4} + \frac{(a^3 + 2\,ab^2)\log\left(e^{(-2\,x)} + 1\right)}{a^4 + 2\,a^2b^2 + b^4} + \frac{be^{(-x)} + 2\,ae^{(-2\,x)} - be^{(-3\,x)}}{a^2 + b^2 + 2\,(a^2 + b^2)e^{(-2\,x)} + (a^2 + b^2)e^{(-4\,x)}} + \frac{x}{a}$$

input
```
integrate(tanh(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
b^4*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^5 + 2*a^3*b^2 + a*b^4) + (a^2*b +
 3*b^3)*arctan(e^(-x))/(a^4 + 2*a^2*b^2 + b^4) + (a^3 + 2*a*b^2)*log(e^(-2
*x) + 1)/(a^4 + 2*a^2*b^2 + b^4) + (b*e^(-x) + 2*a*e^(-2*x) - b*e^(-3*x))/
(a^2 + b^2 + 2*(a^2 + b^2)*e^(-2*x) + (a^2 + b^2)*e^(-4*x)) + x/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(110) = 220.

Time = 0.12 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.07

$$\int \frac{\tanh^3(x)}{a + b\operatorname{csch}(x)}\,dx = \frac{b^4 \log\left(\left|-a\left(e^{(-x)} - e^x\right) + 2\,b\right|\right)}{a^5 + 2\,a^3b^2 + ab^4} - \frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)(a^2b + 3\,b^3)}{4\,(a^4 + 2\,a^2b^2 + b^4)} + \frac{(a^3 + 2\,ab^2)\log\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}{2\,(a^4 + 2\,a^2b^2 + b^4)} - \frac{a^3\left(e^{(-x)} - e^x\right)^2 + 2\,ab^2\left(e^{(-x)} - e^x\right)^2 + 2\,a^2b\left(e^{(-x)} - e^x\right) + 2\,b^3\left(e^{(-x)} - e^x\right) + 4\,ab^2}{2\,(a^4 + 2\,a^2b^2 + b^4)\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}$$

input
```
integrate(tanh(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

output
```
b^4*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^5 + 2*a^3*b^2 + a*b^4) - 1/4*(pi
+ 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(a^2*b + 3*b^3)/(a^4 + 2*a^2*b^2 + b
^4) + 1/2*(a^3 + 2*a*b^2)*log((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4
) - 1/2*(a^3*(e^(-x) - e^x)^2 + 2*a*b^2*(e^(-x) - e^x)^2 + 2*a^2*b*(e^(-x)
 - e^x) + 2*b^3*(e^(-x) - e^x) + 4*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*((e^(-x
) - e^x)^2 + 4))
```

Mupad [B] (verification not implemented)

Time = 4.94 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.96

$$\int \frac{\tanh^3(x)}{a+b\mathrm{csch}(x)}\,dx = \frac{\frac{\mathrm{e}^x\,\left(a^2\,b+b^3\right)}{{\left(a^2+b^2\right)}^2}+\frac{2\,\left(a^4+a^2\,b^2\right)}{a\,{\left(a^2+b^2\right)}^2}}{\mathrm{e}^{2\,x}+1}-\frac{\frac{2\,a}{a^2+b^2}+\frac{2\,b\,\mathrm{e}^x}{a^2+b^2}}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1}-\frac{x}{a} + \frac{b^4\,\ln\left(4\,a^9\,\mathrm{e}^{2\,x}-4\,a\,b^8-4\,a^9+7\,a^3\,b^6-14\,a^5\,b^4-17\,a^7\,b^2+8\,b^9\,\mathrm{e}^x-7\,a^3\,b^6\,\mathrm{e}^{2\,x}+14\,a^5\,b^4\,\mathrm{e}^{2\,x}+17\,a^7}{a^5+2\,a^3\,b^2+a\,b^4} + \frac{\ln\left(1+\mathrm{e}^x\,1{}\mathrm{i}\right)\,\left(3\,b+a\,2{}\mathrm{i}\right)}{2\,\left(a^2\,1{}\mathrm{i}+2\,a\,b-b^2\,1{}\mathrm{i}\right)}+\frac{\ln\left(\mathrm{e}^x+1{}\mathrm{i}\right)\,\left(2\,a+b\,3{}\mathrm{i}\right)}{2\,\left(a^2+a\,b\,2{}\mathrm{i}-b^2\right)}$$

input
```
int(tanh(x)^3/(a + b/sinh(x)),x)
```

output
```
((exp(x)*(a^2*b + b^3))/(a^2 + b^2)^2 + (2*(a^4 + a^2*b^2))/(a*(a^2 + b^2)
^2))/(exp(2*x) + 1) - ((2*a)/(a^2 + b^2) + (2*b*exp(x))/(a^2 + b^2))/(2*ex
p(2*x) + exp(4*x) + 1) - x/a + (b^4*log(4*a^9*exp(2*x) - 4*a*b^8 - 4*a^9 +
 7*a^3*b^6 - 14*a^5*b^4 - 17*a^7*b^2 + 8*b^9*exp(x) - 7*a^3*b^6*exp(2*x) +
 14*a^5*b^4*exp(2*x) + 17*a^7*b^2*exp(2*x) + 8*a^8*b*exp(x) + 4*a*b^8*exp(
2*x) - 14*a^2*b^7*exp(x) + 28*a^4*b^5*exp(x) + 34*a^6*b^3*exp(x)))/(a*b^4
+ a^5 + 2*a^3*b^2) + (log(exp(x)*1i + 1)*(a*2i + 3*b))/(2*(2*a*b + a^2*1i
- b^2*1i)) + (log(exp(x) + 1i)*(2*a + b*3i))/(2*(a*b*2i + a^2 - b^2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 515, normalized size of antiderivative = 4.56

$$\int \frac{\tanh^3(x)}{a + b\operatorname{csch}(x)}\,dx$$
$$= \frac{-2e^{4x}a^2b^2x - e^{4x}a^4 + e^{3x}a^3b + e^{3x}a\,b^3 - e^{x}a^3b - e^{4x}a^4x - e^{4x}b^4x - e^{4x}atan(e^x)\,a^3b - atan(e^x)\,a^3b + e^{4}}{}$$

input
```
int(tanh(x)^3/(a+b*csch(x)),x)
```

output
```
( - e**(4*x)*atan(e**x)*a**3*b - 3*e**(4*x)*atan(e**x)*a*b**3 - 2*e**(2*x)
*atan(e**x)*a**3*b - 6*e**(2*x)*atan(e**x)*a*b**3 - atan(e**x)*a**3*b - 3*
atan(e**x)*a*b**3 + e**(4*x)*log(e**(2*x) + 1)*a**4 + 2*e**(4*x)*log(e**(2
*x) + 1)*a**2*b**2 + e**(4*x)*log(e**(2*x)*a + 2*e**x*b - a)*b**4 - e**(4*
x)*a**4*x - e**(4*x)*a**4 - 2*e**(4*x)*a**2*b**2*x - e**(4*x)*a**2*b**2 -
e**(4*x)*b**4*x + e**(3*x)*a**3*b + e**(3*x)*a*b**3 + 2*e**(2*x)*log(e**(2
*x) + 1)*a**4 + 4*e**(2*x)*log(e**(2*x) + 1)*a**2*b**2 + 2*e**(2*x)*log(e*
*(2*x)*a + 2*e**x*b - a)*b**4 - 2*e**(2*x)*a**4*x - 4*e**(2*x)*a**2*b**2*x
 - 2*e**(2*x)*b**4*x - e**x*a**3*b - e**x*a*b**3 + log(e**(2*x) + 1)*a**4
+ 2*log(e**(2*x) + 1)*a**2*b**2 + log(e**(2*x)*a + 2*e**x*b - a)*b**4 - a*
*4*x - a**4 - 2*a**2*b**2*x - a**2*b**2 - b**4*x)/(a*(e**(4*x)*a**4 + 2*e*
*(4*x)*a**2*b**2 + e**(4*x)*b**4 + 2*e**(2*x)*a**4 + 4*e**(2*x)*a**2*b**2
+ 2*e**(2*x)*b**4 + a**4 + 2*a**2*b**2 + b**4))
```

3.116 $\int \frac{\tanh^2(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 100

$$\int \frac{\tanh^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{ax}{a^2+b^2}+\frac{b^2x}{a(a^2+b^2)}+\frac{2b^3\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^{3/2}}+\frac{b\operatorname{sech}(x)}{a^2+b^2}-\frac{a\tanh(x)}{a^2+b^2}$$

output
```
a*x/(a^2+b^2)+b^2*x/a/(a^2+b^2)+2*b^3*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^
(1/2))/a/(a^2+b^2)^(3/2)+b*sech(x)/(a^2+b^2)-a*tanh(x)/(a^2+b^2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

$$\int \frac{\tanh^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{x+\frac{2b^3\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}}}{a}+\frac{b\operatorname{sech}(x)}{a^2+b^2}-\frac{a\tanh(x)}{a^2+b^2}$$

input
```
Integrate[Tanh[x]^2/(a + b*Csch[x]),x]
```

output
```
(x + (2*b^3*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3/2)
)/a + (b*Sech[x])/(a^2 + b^2) - (a*Tanh[x])/(a^2 + b^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.692, Rules used = {3042, 25, 4386, 26, 26, 3042, 26, 3381, 25, 26, 3042, 25, 26, 3086, 24, 3214, 3042, 3139, 1083, 219, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)}{a+b\text{csch}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\cot(ix)^2(a+ib\csc(ix))}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\cot(ix)^2(a+ib\csc(ix))}dx$$

$$\Big\downarrow \text{4386}$$

$$-\int -\frac{i\sinh(x)\tanh^2(x)}{ib+ia\sinh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int -\frac{i\sinh(x)\tanh^2(x)}{b+a\sinh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\sinh(x)\tanh^2(x)}{a\sinh(x)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ix)^3}{\cos(ix)^2(b-ia\sin(ix))}dx$$

$$\Big\downarrow \ 26$$

$$i\int \frac{\sin(ix)^3}{\cos(ix)^2(b-ia\sin(ix))}dx$$

$$\Big\downarrow \ 3381$$

$$i\left(-\frac{b^2\int \frac{i\sinh(x)}{b+a\sinh(x)}dx}{a^2+b^2}+\frac{ia\int -\tanh^2(x)dx}{a^2+b^2}+\frac{b\int i\text{sech}(x)\tanh(x)dx}{a^2+b^2}\right)$$

$$\Big\downarrow \ 25$$

$$i\left(-\frac{b^2\int \frac{i\sinh(x)}{b+a\sinh(x)}dx}{a^2+b^2}-\frac{ia\int \tanh^2(x)dx}{a^2+b^2}+\frac{b\int i\text{sech}(x)\tanh(x)dx}{a^2+b^2}\right)$$

$$\Big\downarrow \ 26$$

$$i\left(-\frac{ib^2\int \frac{\sinh(x)}{b+a\sinh(x)}dx}{a^2+b^2}-\frac{ia\int \tanh^2(x)dx}{a^2+b^2}+\frac{ib\int \text{sech}(x)\tanh(x)dx}{a^2+b^2}\right)$$

$$\Big\downarrow \ 3042$$

$$i\left(-\frac{ib^2\int -\frac{i\sin(ix)}{b-ia\sin(ix)}dx}{a^2+b^2}-\frac{ia\int -\tan(ix)^2dx}{a^2+b^2}+\frac{ib\int -i\sec(ix)\tan(ix)dx}{a^2+b^2}\right)$$

$$\Big\downarrow \ 25$$

$$i\left(-\frac{ib^2\int -\frac{i\sin(ix)}{b-ia\sin(ix)}dx}{a^2+b^2}+\frac{ia\int \tan(ix)^2dx}{a^2+b^2}+\frac{ib\int -i\sec(ix)\tan(ix)dx}{a^2+b^2}\right)$$

$$\Big\downarrow \ 26$$

$$i\left(-\frac{b^2\int \frac{\sin(ix)}{b-ia\sin(ix)}dx}{a^2+b^2}+\frac{ia\int \tan(ix)^2dx}{a^2+b^2}+\frac{b\int \sec(ix)\tan(ix)dx}{a^2+b^2}\right)$$

$$\Big\downarrow \ 3086$$

$$i\left(-\frac{b^2\int \frac{\sin(ix)}{b-ia\sin(ix)}dx}{a^2+b^2}+\frac{ia\int \tan(ix)^2dx}{a^2+b^2}-\frac{ib\int 1d\text{sech}(x)}{a^2+b^2}\right)$$

$$\Big\downarrow \ 24$$

$$i\left(-\frac{b^2\int \frac{\sin(ix)}{b-ia\sin(ix)}dx}{a^2+b^2}+\frac{ia\int \tan(ix)^2dx}{a^2+b^2}-\frac{ib\text{sech}(x)}{a^2+b^2}\right)$$

$$\Big\downarrow \text{3214}$$

$$i\left(\frac{ia\int \tan(ix)^2dx}{a^2+b^2}-\frac{b^2\left(\frac{ix}{a}-\frac{ib\int \frac{1}{b+a\sinh(x)}dx}{a}\right)}{a^2+b^2}-\frac{ib\text{sech}(x)}{a^2+b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-\frac{b^2\left(\frac{ix}{a}-\frac{ib\int \frac{1}{b-ia\sin(ix)}dx}{a}\right)}{a^2+b^2}+\frac{ia\int \tan(ix)^2dx}{a^2+b^2}-\frac{ib\text{sech}(x)}{a^2+b^2}\right)$$

$$\Big\downarrow \text{3139}$$

$$i\left(\frac{ia\int \tan(ix)^2dx}{a^2+b^2}-\frac{b^2\left(\frac{ix}{a}-\frac{2ib\int \frac{1}{-b\tanh^2\left(\frac{x}{2}\right)+2a\tanh\left(\frac{x}{2}\right)+b}d\tanh\left(\frac{x}{2}\right)}{a}\right)}{a^2+b^2}-\frac{ib\text{sech}(x)}{a^2+b^2}\right)$$

$$\Big\downarrow \text{1083}$$

$$i\left(\frac{ia\int \tan(ix)^2dx}{a^2+b^2}-\frac{b^2\left(\frac{4ib\int \frac{1}{4\left(a^2+b^2\right)-\left(2a-2b\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2a-2b\tanh\left(\frac{x}{2}\right)\right)}{a}+\frac{ix}{a}\right)}{a^2+b^2}-\frac{ib\text{sech}(x)}{a^2+b^2}\right)$$

$$\Big\downarrow \text{219}$$

$$i\left(\frac{ia\int \tan(ix)^2dx}{a^2+b^2}-\frac{b^2\left(\frac{2ib\text{arctanh}\left(\frac{2a-2b\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{ix}{a}\right)}{a^2+b^2}-\frac{ib\text{sech}(x)}{a^2+b^2}\right)$$

$$\Big\downarrow \text{3954}$$

$$i\left(\frac{ia(\tanh(x)-\int 1dx)}{a^2+b^2}-\frac{b^2\left(\frac{2ib\text{arctanh}\left(\frac{2a-2b\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{ix}{a}\right)}{a^2+b^2}-\frac{ib\text{sech}(x)}{a^2+b^2}\right)$$

$$\Big\downarrow \text{24}$$

$$i\left(-\frac{b^2\left(\frac{2ib\text{arctanh}\left(\frac{2a-2b\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{ix}{a}\right)}{a^2+b^2}+\frac{ia(\tanh(x)-x)}{a^2+b^2}-\frac{ib\text{sech}(x)}{a^2+b^2}\right)$$

input
```
Int[Tanh[x]^2/(a + b*Csch[x]),x]
```

output
```
I*(-((b^2*((I*x)/a + ((2*I)*b*ArcTanh[(2*a - 2*b*Tanh[x/2])/(2*Sqrt[a^2 +
b^2])])/(a*Sqrt[a^2 + b^2])))/(a^2 + b^2)) - (I*b*Sech[x])/(a^2 + b^2) + (
I*a*(-x + Tanh[x]))/(a^2 + b^2))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3381
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[a*(d^2/(a^2
- b^2))   Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Simp[
b*(d/(a^2 - b^2))   Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x]
 - Simp[a^2*(d^2/(g^2*(a^2 - b^2)))   Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[
e + f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1
]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4386
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{-2a\tanh\left(\frac{x}{2}\right)+2b}{\left(a^{2}+b^{2}\right)\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{2b^{3}\operatorname{arctanh}\left(\frac{-2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{a^{2}+b^{2}}}\right)}{a\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}$	95
risch	$\frac{x}{a}+\frac{2b\,\mathrm{e}^{x}+2a}{\left(\mathrm{e}^{2x}+1\right)\left(a^{2}+b^{2}\right)}+\frac{b^{3}\ln\left(\mathrm{e}^{x}+\frac{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}b+a^{4}+2a^{2}b^{2}+b^{4}}{a\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}a}-\frac{b^{3}\ln\left(\mathrm{e}^{x}+\frac{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}b-a^{4}-2a^{2}b^{2}-b^{4}}{a\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}a}$	155

input
```
int(tanh(x)^2/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(a^2+b^2)*(-a*tanh(1/2*x)+b)/(tanh(1/2*x)^2+1)+1/a*ln(tanh(1/2*x)+1)-1/a
*ln(tanh(1/2*x)-1)+2/a*b^3/(a^2+b^2)^(3/2)*arctanh(1/2*(-2*tanh(1/2*x)*b+2
*a)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(96) = 192.

Time = 0.10 (sec) , antiderivative size = 349, normalized size of antiderivative = 3.49

$$\int \frac{\tanh^2(x)}{a+b\operatorname{csch}(x)}\,dx$$
$$= \frac{2a^4+2a^2b^2+(a^4+2a^2b^2+b^4)x\cosh(x)^2+(a^4+2a^2b^2+b^4)x\sinh(x)^2+\left(b^3\cosh(x)^2+2b^3\cosh(x\right.}{}$$

input
```
integrate(tanh(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
(2*a^4 + 2*a^2*b^2 + (a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^2 + (a^4 + 2*a^2*b^
2 + b^4)*x*sinh(x)^2 + (b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x
)^2 + b^3)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh
(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*c
osh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*c
osh(x) + b)*sinh(x) - a)) + (a^4 + 2*a^2*b^2 + b^4)*x + 2*(a^3*b + a*b^3)*
cosh(x) + 2*(a^3*b + a*b^3 + (a^4 + 2*a^2*b^2 + b^4)*x*cosh(x))*sinh(x))/(
a^5 + 2*a^3*b^2 + a*b^4 + (a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^2 + 2*(a^5 + 2
*a^3*b^2 + a*b^4)*cosh(x)*sinh(x) + (a^5 + 2*a^3*b^2 + a*b^4)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{a + b\text{csch}(x)}\,dx = \int \frac{\tanh^2(x)}{a + b\,\text{csch}\,(x)}\,dx$$

input
```
integrate(tanh(x)**2/(a+b*csch(x)),x)
```

output
```
Integral(tanh(x)**2/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08

$$\int \frac{\tanh^2(x)}{a + b\text{csch}(x)}\,dx = -\frac{b^3\log\left(\frac{ae^{(-x)} - b - \sqrt{a^2+b^2}}{ae^{(-x)} - b + \sqrt{a^2+b^2}}\right)}{(a^3 + ab^2)\sqrt{a^2+b^2}} + \frac{2\left(be^{(-x)} - a\right)}{a^2 + b^2 + (a^2+b^2)e^{(-2x)}} + \frac{x}{a}$$

input
```
integrate(tanh(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
-b^3*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2))
)/((a^3 + a*b^2)*sqrt(a^2 + b^2)) + 2*(b*e^(-x) - a)/(a^2 + b^2 + (a^2 + b
^2)*e^(-2*x)) + x/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.02

$$\int \frac{\tanh^2(x)}{a+b\text{csch}(x)}\,dx = -\frac{b^3\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^3+ab^2)\sqrt{a^2+b^2}} + \frac{x}{a} + \frac{2\,(be^x+a)}{(a^2+b^2)(e^{(2\,x)}+1)}$$

input
```
integrate(tanh(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

output
```
-b^3*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt
(a^2 + b^2)))/((a^3 + a*b^2)*sqrt(a^2 + b^2)) + x/a + 2*(b*e^x + a)/((a^2
+ b^2)*(e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 3.70 (sec) , antiderivative size = 376, normalized size of antiderivative = 3.76

$$\int \frac{\tanh^2(x)}{a+b\text{csch}(x)}\,dx = \frac{x}{a} + \frac{\frac{2\,a}{a^2+b^2}+\frac{2\,b\,\mathrm{e}^x}{a^2+b^2}}{\mathrm{e}^{2\,x}+1} + \frac{2\,\mathrm{atan}\left(\left(\frac{a^4\,\sqrt{-a^8-3\,a^6\,b^2-3\,a^4\,b^4-a^2\,b^6}}{2}+\frac{a^2\,b^2\,\sqrt{-a^8-3\,a^6\,b^2-3\,a^4\,b^4-a^2\,b^6}}{2}\right)\left(\mathrm{e}^x\left(\frac{2\,b^3}{a^3\,(a^3+a\,b^2)\,\sqrt{b^6}\,(a^2+b^2)}+\frac{\cdots}{a^2\,b^2\,\sqrt{-a^2\cdots}}\right.\right.\right.}{\sqrt{-a^8-3\,a^6\,b^2-3\cdots}}$$

input
```
int(tanh(x)^2/(a + b/sinh(x)),x)
```

output
```
x/a + ((2*a)/(a^2 + b^2) + (2*b*exp(x))/(a^2 + b^2))/(exp(2*x) + 1) + (2*a
tan(((a^4*(- a^8 - a^2*b^6 - 3*a^4*b^4 - 3*a^6*b^2)^(1/2))/2 + (a^2*b^2*(-
 a^8 - a^2*b^6 - 3*a^4*b^4 - 3*a^6*b^2)^(1/2))/2)*(exp(x)*((2*b^3)/(a^3*(a
*b^2 + a^3)*(b^6)^(1/2)*(a^2 + b^2)) + (2*(a*b^3*(b^6)^(1/2) + a^3*b*(b^6)
^(1/2)))/(a^2*b^2*(-a^2*(a^2 + b^2)^3)^(1/2)*(a*b^2 + a^3)*(- a^8 - a^2*b^
6 - 3*a^4*b^4 - 3*a^6*b^2)^(1/2))) - (2*(a^4*(b^6)^(1/2) + a^2*b^2*(b^6)^(
1/2)))/(a^2*b^2*(-a^2*(a^2 + b^2)^3)^(1/2)*(a*b^2 + a^3)*(- a^8 - a^2*b^6
- 3*a^4*b^4 - 3*a^6*b^2)^(1/2))))*(b^6)^(1/2))/(- a^8 - a^2*b^6 - 3*a^4*b^
4 - 3*a^6*b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.24

$$\int \frac{\tanh^2(x)}{a+b\operatorname{csch}(x)}\,dx$$
$$= \frac{-2e^{2x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x ai+bi}{\sqrt{a^2+b^2}}\right)b^3i-2\sqrt{a^2+b^2}\,atan\left(\frac{e^x ai+bi}{\sqrt{a^2+b^2}}\right)b^3i+e^{2x}a^4x-2e^{2x}a^4+2e^{2x}a^2b^2x-2e^{2x}a}{a\left(e^{2x}a^4+2e^{2x}a^2b^2+e^{2x}b^4+a^4+2a^2b^2+b^4\right)}$$

input
```
int(tanh(x)^2/(a+b*csch(x)),x)
```

output
```
( - 2*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2))*
b**3*i - 2*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2))*b**3
*i + e**(2*x)*a**4*x - 2*e**(2*x)*a**4 + 2*e**(2*x)*a**2*b**2*x - 2*e**(2*
x)*a**2*b**2 + e**(2*x)*b**4*x + 2*e**x*a**3*b + 2*e**x*a*b**3 + a**4*x +
2*a**2*b**2*x + b**4*x)/(a*(e**(2*x)*a**4 + 2*e**(2*x)*a**2*b**2 + e**(2*x
)*b**4 + a**4 + 2*a**2*b**2 + b**4))
```

3.117 $\int \frac{\tanh(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 61

$$\int \frac{\tanh(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{b\arctan(\sinh(x))}{a^2+b^2}+\frac{b^2\log(a+b\operatorname{csch}(x))}{a(a^2+b^2)}+\frac{\log(\sinh(x))}{a}-\frac{a\log(\tanh(x))}{a^2+b^2}$$

output
```
-b*arctan(sinh(x))/(a^2+b^2)+b^2*ln(a+b*csch(x))/a/(a^2+b^2)+ln(sinh(x))/a
-a*ln(tanh(x))/(a^2+b^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int \frac{\tanh(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{a(a+ib)\log(i-\sinh(x))+a(a-ib)\log(i+\sinh(x))+2b^2\log(b+a\sinh(x))}{2a(a^2+b^2)}$$

input
```
Integrate[Tanh[x]/(a + b*Csch[x]),x]
```

output
```
(a*(a + I*b)*Log[I - Sinh[x]] + a*(a - I*b)*Log[I + Sinh[x]] + 2*b^2*Log[b
 + a*Sinh[x]])/(2*a*(a^2 + b^2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.38, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 26, 4373, 25, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{a+b\text{csch}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\cot(ix)(a+ib\csc(ix))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\cot(ix)(a+ib\csc(ix))}dx$$

$$\Big\downarrow \text{4373}$$

$$b^2\int -\frac{\sinh(x)}{b(a+b\text{csch}(x))\left(\text{csch}^2(x)b^2+b^2\right)}d(b\text{csch}(x))$$

$$\Big\downarrow \text{25}$$

$$-b^2\int \frac{\sinh(x)}{b(a+b\text{csch}(x))\left(\text{csch}^2(x)b^2+b^2\right)}d(b\text{csch}(x))$$

$$\Big\downarrow \text{615}$$

$$-b^2\int \left(\frac{-b^2-a\text{csch}(x)b}{b^2\left(a^2+b^2\right)\left(\text{csch}^2(x)b^2+b^2\right)}+\frac{\sinh(x)}{ab^3}-\frac{1}{a\left(a^2+b^2\right)(a+b\text{csch}(x))}\right)d(b\text{csch}(x))$$

$$\Big\downarrow \text{2009}$$

$$b^2\left(\frac{\arctan(\text{csch}(x))}{b\left(a^2+b^2\right)}+\frac{a\log\left(b^2\text{csch}^2(x)+b^2\right)}{2b^2\left(a^2+b^2\right)}+\frac{\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)}-\frac{\log(b\text{csch}(x))}{ab^2}\right)$$

input
```
Int[Tanh[x]/(a + b*Csch[x]),x]
```

output
```
b^2*(ArcTan[Csch[x]]/(b*(a^2 + b^2)) - Log[b*Csch[x]]/(a*b^2) + Log[a + b*
Csch[x]]/(a*(a^2 + b^2)) + (a*Log[b^2 + b^2*Csch[x]^2])/(2*b^2*(a^2 + b^2)
))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.59

method	result	si
default	$\frac{4a\ln\left(\tanh\left(\frac{x}{2}\right)^2+1\right)-8b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{4a^2+4b^2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}+\frac{b^2\ln\left(-b\tanh\left(\frac{x}{2}\right)^2+2a\tanh\left(\frac{x}{2}\right)+b\right)}{a(a^2+b^2)}$	9
risch	$\frac{x}{a}-\frac{2ax}{a^2+b^2}-\frac{2b^2x}{a(a^2+b^2)}+\frac{i\ln(e^x-i)b}{a^2+b^2}+\frac{\ln(e^x-i)a}{a^2+b^2}-\frac{i\ln(e^x+i)b}{a^2+b^2}+\frac{\ln(e^x+i)a}{a^2+b^2}+\frac{b^2\ln\left(e^{2x}+\frac{2b\,e^x}{a}-1\right)}{a(a^2+b^2)}$	1

input
```
int(tanh(x)/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
8/(4*a^2+4*b^2)*(1/2*a*ln(tanh(1/2*x)^2+1)-b*arctan(tanh(1/2*x)))-1/a*ln(t
anh(1/2*x)-1)-1/a*ln(tanh(1/2*x)+1)+b^2/a/(a^2+b^2)*ln(-b*tanh(1/2*x)^2+2*
a*tanh(1/2*x)+b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int \frac{\tanh(x)}{a+b\text{csch}(x)}\,dx = -\frac{2\,ab\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)-b^2\log\left(\frac{2\,(a\sinh(x)+b)}{\cosh(x)-\sinh(x)}\right)-a^2\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)+(a^2+b^2)x}{a^3+ab^2}$$

input
```
integrate(tanh(x)/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
-(2*a*b*arctan(cosh(x) + sinh(x)) - b^2*log(2*(a*sinh(x) + b)/(cosh(x) - s
inh(x))) - a^2*log(2*cosh(x)/(cosh(x) - sinh(x))) + (a^2 + b^2)*x)/(a^3 +
a*b^2)
```

Sympy [F]

$$\int \frac{\tanh(x)}{a + b\text{csch}(x)}\,dx = \int \frac{\tanh{(x)}}{a + b\,\text{csch}\,(x)}\,dx$$

input
```
integrate(tanh(x)/(a+b*csch(x)),x)
```

output
```
Integral(tanh(x)/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.21

$$\int \frac{\tanh(x)}{a + b\text{csch}(x)}\,dx = \frac{b^2\log\left(-2\,be^{(-x)} + ae^{(-2\,x)} - a\right)}{a^3 + ab^2} + \frac{2\,b\arctan\left(e^{(-x)}\right)}{a^2 + b^2} + \frac{a\log\left(e^{(-2\,x)} + 1\right)}{a^2 + b^2} + \frac{x}{a}$$

input
```
integrate(tanh(x)/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
b^2*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^3 + a*b^2) + 2*b*arctan(e^(-x))/(
a^2 + b^2) + a*log(e^(-2*x) + 1)/(a^2 + b^2) + x/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.46

$$\int \frac{\tanh(x)}{a + b\text{csch}(x)}\,dx = \frac{b^2\log\left(\left|-a\left(e^{(-x)} - e^{x}\right) + 2\,b\right|\right)}{a^3 + ab^2} - \frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)b}{2\left(a^2 + b^2\right)} + \frac{a\log\left(\left(e^{(-x)} - e^{x}\right)^2 + 4\right)}{2\left(a^2 + b^2\right)}$$

input
```
integrate(tanh(x)/(a+b*csch(x)),x, algorithm="giac")
```

output
```
b^2*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^3 + a*b^2) - 1/2*(pi + 2*arctan(1
/2*(e^(2*x) - 1)*e^(-x)))*b/(a^2 + b^2) + 1/2*a*log((e^(-x) - e^x)^2 + 4)/
(a^2 + b^2)
```

Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.16

$$\int \frac{\tanh(x)}{a + b\text{csch}(x)}\,dx = \frac{\ln(1 + e^x\,1i)}{a - b\,1i} - \frac{x}{a} + \frac{b^2\,\ln(a^5\,e^{2\,x} - a\,b^4 - a^5 + a^3\,b^2 + 2\,b^5\,e^x - a^3\,b^2\,e^{2\,x} + 2\,a^4\,b\,e^x + a\,b^4\,e^{2\,x} - 2\,a^2\,b^3\,e^x)}{a^3 + a\,b^2} + \frac{\ln(e^x + 1i)\,1i}{-b + a\,1i}$$

input
```
int(tanh(x)/(a + b/sinh(x)),x)
```

output
```
log(exp(x)*1i + 1)/(a - b*1i) + (log(exp(x) + 1i)*1i)/(a*1i - b) - x/a + (
b^2*log(a^5*exp(2*x) - a*b^4 - a^5 + a^3*b^2 + 2*b^5*exp(x) - a^3*b^2*exp(
2*x) + 2*a^4*b*exp(x) + a*b^4*exp(2*x) - 2*a^2*b^3*exp(x)))/(a*b^2 + a^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \frac{\tanh(x)}{a + b\text{csch}(x)}\,dx = \frac{-2atan(e^x)\,ab + \log(e^{2x} + 1)\,a^2 + \log(e^{2x}a + 2e^xb - a)\,b^2 - a^2x - b^2x}{a\,(a^2 + b^2)}$$

input
```
int(tanh(x)/(a+b*csch(x)),x)
```

output
```
( - 2*atan(e**x)*a*b + log(e**(2*x) + 1)*a**2 + log(e**(2*x)*a + 2*e**x*b
- a)*b**2 - a**2*x - b**2*x)/(a*(a**2 + b**2))
```

3.118 $\int \frac{\coth(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{\coth(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\log(a+b\operatorname{csch}(x))}{a} + \frac{\log(\sinh(x))}{a}$$

output
```
ln(a+b*csch(x))/a+ln(sinh(x))/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{\coth(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\log(b+a\sinh(x))}{a}$$

input
```
Integrate[Coth[x]/(a + b*Csch[x]),x]
```

output
```
Log[b + a*Sinh[x]]/a
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 26, 4373, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{a+b\text{csch}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cot(ix)}{a+ib\csc(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cot(ix)}{a+ib\csc(ix)}dx$$

$$\Big\downarrow \text{4373}$$

$$-\int \frac{\sinh(x)}{b(a+b\text{csch}(x))}d(b\text{csch}(x))$$

$$\Big\downarrow \text{47}$$

$$\frac{\int \frac{1}{a+b\text{csch}(x)}d(b\text{csch}(x))}{a}-\frac{\int \frac{\sinh(x)}{b}d(b\text{csch}(x))}{a}$$

$$\Big\downarrow \text{14}$$

$$\frac{\int \frac{1}{a+b\text{csch}(x)}d(b\text{csch}(x))}{a}-\frac{\log(b\text{csch}(x))}{a}$$

$$\Big\downarrow \text{16}$$

$$\frac{\log(a+b\text{csch}(x))}{a}-\frac{\log(b\text{csch}(x))}{a}$$

input
```
Int[Coth[x]/(a + b*Csch[x]),x]
```

output
```
-(Log[b*Csch[x]]/a) + Log[a + b*Csch[x]]/a
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{\ln(a+b\,\mathrm{csch}(x))}{a}-\frac{\ln(\mathrm{csch}(x))}{a}$	21
default	$\frac{\ln(a+b\,\mathrm{csch}(x))}{a}-\frac{\ln(\mathrm{csch}(x))}{a}$	21
risch	$-\frac{x}{a}+\frac{\ln\left(\mathrm{e}^{2x}+\frac{2b\,\mathrm{e}^{x}}{a}-1\right)}{a}$	27

input
```
int(coth(x)/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*csch(x))/a-1/a*ln(csch(x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{\coth(x)}{a+b\text{csch}(x)}\,dx = -\frac{x-\log\left(\frac{2\,(a\sinh(x)+b)}{\cosh(x)-\sinh(x)}\right)}{a}$$

input
```
integrate(coth(x)/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
-(x - log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))))/a
```

Sympy [F]

$$\int \frac{\coth(x)}{a+b\text{csch}(x)}\,dx = \int \frac{\coth{(x)}}{a+b\,\text{csch}{(x)}}\,dx$$

input
```
integrate(coth(x)/(a+b*csch(x)),x)
```

output
```
Integral(coth(x)/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{\coth(x)}{a+b\text{csch}(x)}\,dx = \frac{x}{a}+\frac{\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{a}$$

input
```
integrate(coth(x)/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
x/a + log(-2*b*e^(-x) + a*e^(-2*x) - a)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{\coth(x)}{a+b\text{csch}(x)}\,dx=\frac{\log\left(\left|-a\left(e^{(-x)}-e^{x}\right)+2\,b\right|\right)}{a}$$

input
```
integrate(coth(x)/(a+b*csch(x)),x, algorithm="giac")
```

output
```
log(abs(-a*(e^(-x) - e^x) + 2*b))/a
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{\coth(x)}{a+b\text{csch}(x)}\,dx=-\frac{x-\ln\left(2\,b\,\mathrm{e}^{x}-a+a\,\mathrm{e}^{2\,x}\right)}{a}$$

input
```
int(coth(x)/(a + b/sinh(x)),x)
```

output
```
-(x - log(2*b*exp(x) - a + a*exp(2*x)))/a
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{\coth(x)}{a+b\text{csch}(x)}\,dx=\frac{\log(e^{2x}a+2e^{x}b-a)-x}{a}$$

input
```
int(coth(x)/(a+b*csch(x)),x)
```

output
```
(log(e**(2*x)*a + 2*e**x*b - a) - x)/a
```

3.119 $\int \frac{\coth^2(x)}{a+b\text{csch}(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 57

$$\int \frac{\coth^2(x)}{a+b\text{csch}(x)} \, dx = \frac{x}{a} - \frac{\text{arctanh}(\cosh(x))}{b} + \frac{2\sqrt{a^2+b^2}\text{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab}$$

output
```
x/a-arctanh(cosh(x))/b+2*(a^2+b^2)^(1/2)*arctanh((a-b*tanh(1/2*x))/(a^2+b^
2)^(1/2))/a/b
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32

$$\int \frac{\coth^2(x)}{a+b\text{csch}(x)} \, dx = \frac{bx + 2\sqrt{-a^2-b^2}\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right) - a\log\left(\cosh\left(\frac{x}{2}\right)\right) + a\log\left(\sinh\left(\frac{x}{2}\right)\right)}{ab}$$

input
```
Integrate[Coth[x]^2/(a + b*Csch[x]),x]
```

output
```
(b*x + 2*Sqrt[-a^2 - b^2]*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]] - a*L
og[Cosh[x/2]] + a*Log[Sinh[x/2]])/(a*b)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.385, Rules used = {3042, 25, 4382, 3042, 4539, 26, 3042, 26, 4257, 4407, 26, 3042, 26, 4318, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{a+b\text{csch}(x)} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cot(ix)^2}{a+ib\csc(ix)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cot(ix)^2}{a+ib\csc(ix)}dx$$

$$\Big\downarrow \text{4382}$$

$$-\int \frac{-\text{csch}^2(x)-1}{a+b\text{csch}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\csc(ix)^2-1}{a+ib\csc(ix)}dx$$

$$\Big\downarrow \text{4539}$$

$$\frac{i\int -\frac{i(b-a\text{csch}(x))}{a+b\text{csch}(x)}dx}{b}+\frac{i\int -i\text{csch}(x)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{b-a\text{csch}(x)}{a+b\text{csch}(x)}dx}{b}+\frac{\int \text{csch}(x)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b-ia\csc(ix)}{a+ib\csc(ix)}dx}{b}+\frac{\int i\csc(ix)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{b-ia\csc(ix)}{a+ib\csc(ix)}dx}{b}+\frac{i\int \csc(ix)dx}{b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\text{arctanh}(\cosh(x))}{b}+\frac{\int \frac{b-ia\csc(ix)}{a+ib\csc(ix)}dx}{b}$$

$$\Big\downarrow \text{4407}$$

$$-\frac{\text{arctanh}(\cosh(x))}{b}+\frac{\frac{bx}{a}-\frac{i(a^2+b^2)\int -\frac{i\text{csch}(x)}{a+b\text{csch}(x)}dx}{a}}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{bx}{a}-\frac{(a^2+b^2)\int \frac{\text{csch}(x)}{a+b\text{csch}(x)}dx}{a}}{b}-\frac{\text{arctanh}(\cosh(x))}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\text{arctanh}(\cosh(x))}{b}+\frac{\frac{bx}{a}-\frac{(a^2+b^2)\int \frac{i\csc(ix)}{a+ib\csc(ix)}dx}{a}}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\text{arctanh}(\cosh(x))}{b}+\frac{\frac{bx}{a}-\frac{i(a^2+b^2)\int \frac{\csc(ix)}{a+ib\csc(ix)}dx}{a}}{b}$$

$$\Big\downarrow \text{4318}$$

$$\frac{\frac{bx}{a}-\frac{(a^2+b^2)\int \frac{1}{\frac{a\sinh(x)}{b}+1}dx}{ab}}{b}-\frac{\text{arctanh}(\cosh(x))}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\text{arctanh}(\cosh(x))}{b}+\frac{\frac{bx}{a}-\frac{(a^2+b^2)\int \frac{1}{1-\frac{ia\sin(ix)}{b}}dx}{ab}}{b}$$

$$\Big\downarrow \text{3139}$$

$$\frac{\frac{bx}{a}-\frac{2(a^2+b^2)\int \frac{1}{-\tanh^2\left(\frac{x}{2}\right)+\frac{2a\tanh\left(\frac{x}{2}\right)}{b}+1}d\tanh\left(\frac{x}{2}\right)}{ab}}{b}-\frac{\operatorname{arctanh}(\cosh(x))}{b}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{4(a^2+b^2)\int \frac{1}{4\left(\frac{a^2}{b^2}+1\right)-\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)^2}d\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{ab}+\frac{bx}{a}}{b}-\frac{\operatorname{arctanh}(\cosh(x))}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{2\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b\left(\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{2\sqrt{a^2+b^2}}\right)}{a}+\frac{bx}{a}}{b}-\frac{\operatorname{arctanh}(\cosh(x))}{b}$$

input
```
Int[Coth[x]^2/(a + b*Csch[x]),x]
```

output
```
-(ArcTanh[Cosh[x]]/b) + ((b*x)/a + (2*Sqrt[a^2 + b^2]*ArcTanh[(b*((2*a)/b
- 2*Tanh[x/2]))/(2*Sqrt[a^2 + b^2])])/a)/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4382
```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4539
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)), x_Symbol] :> Simp[C/b   Int[Csc[e + f*x], x], x] + Simp[1/b   In
t[(A*b - a*C*Csc[e + f*x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e,
f, A, C}, x]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.47

method	result	size
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{\left(2a^{2}+2b^{2}\right)\operatorname{arctanh}\left(\frac{-2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{a^{2}+b^{2}}}\right)}{ab\sqrt{a^{2}+b^{2}}}$	84
risch	$\frac{x}{a}+\frac{\sqrt{a^{2}+b^{2}}\,\ln\left(\mathrm{e}^{x}+\frac{b+\sqrt{a^{2}+b^{2}}}{a}\right)}{ba}-\frac{\sqrt{a^{2}+b^{2}}\,\ln\left(\mathrm{e}^{x}-\frac{-b+\sqrt{a^{2}+b^{2}}}{a}\right)}{ba}+\frac{\ln\left(\mathrm{e}^{x}-1\right)}{b}-\frac{\ln\left(1+\mathrm{e}^{x}\right)}{b}$	100

input
```
int(coth(x)^2/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a*ln(tanh(1/2*x)+1)+1/b*ln(tanh(1/2*x))-1/a*ln(tanh(1/2*x)-1)+(2*a^2+2*b
^2)/a/b/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*tanh(1/2*x)*b+2*a)/(a^2+b^2)^(1/2)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(53) = 106.

Time = 0.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.47

$$\int \frac{\coth^2(x)}{a+b\operatorname{csch}(x)}\,dx$$
$$=\frac{bx-a\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+a\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)+\sqrt{a^{2}+b^{2}}\log\left(\frac{a^{2}\cosh(x)^{2}+a^{2}\sinh(x)^{2}\cdots}{\cdots}\right.}{ab}$$

input
```
integrate(coth(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
(b*x - a*log(cosh(x) + sinh(x) + 1) + a*log(cosh(x) + sinh(x) - 1) + sqrt(
a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^
2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(
x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh
(x) - a)))/(a*b)
```

Sympy [F]

$$\int \frac{\coth^2(x)}{a+b\operatorname{csch}(x)}\,dx=\int \frac{\coth^2(x)}{a+b\operatorname{csch}(x)}\,dx$$

input
```
integrate(coth(x)**2/(a+b*csch(x)),x)
```

output
```
Integral(coth(x)**2/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.58

$$\int \frac{\coth^2(x)}{a+b\operatorname{csch}(x)}\,dx=\frac{x}{a}-\frac{\log\left(e^{(-x)}+1\right)}{b}+\frac{\log\left(e^{(-x)}-1\right)}{b}-\frac{\sqrt{a^2+b^2}\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{ab}$$

input
```
integrate(coth(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
x/a - log(e^(-x) + 1)/b + log(e^(-x) - 1)/b - sqrt(a^2 + b^2)*log((a*e^(-x
) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/(a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.56

$$\int \frac{\coth^2(x)}{a+b\operatorname{csch}(x)}\,dx=\frac{x}{a}-\frac{\log\left(e^x+1\right)}{b}+\frac{\log\left(|e^x-1|\right)}{b}-\frac{\sqrt{a^2+b^2}\log\left(\frac{|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}|}{|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}|}\right)}{ab}$$

input
```
integrate(coth(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

output
```
x/a - log(e^x + 1)/b + log(abs(e^x - 1))/b - sqrt(a^2 + b^2)*log(abs(2*a*e
^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^2 + b^2)))/(a*b
)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 316, normalized size of antiderivative = 5.54

$$\int \frac{\coth^2(x)}{a+b\text{csch}(x)}\,dx = \frac{x}{a} + \frac{\ln\left(32\,a^2\,b+32\,b^3-32\,b^3\,\mathrm{e}^x-32\,a^2\,b\,\mathrm{e}^x\right)}{b} - \frac{\ln\left(32\,a^2\,b+32\,b^3+32\,b^3\,\mathrm{e}^x+32\,a^2\,b\,\mathrm{e}^x\right)}{b} + \frac{\ln\left(128\,b^5\,\mathrm{e}^x-64\,a^3\,b^2-64\,a\,b^4-128\,b^4\,\mathrm{e}^x\,\sqrt{a^2+b^2}+32\,a^4\,b\,\mathrm{e}^x+160\,a^2\,b^3\,\mathrm{e}^x+64\,a\,b^3\,\sqrt{a^2+b^2}+32\right.}{a\,b} - \frac{\ln\left(128\,b^5\,\mathrm{e}^x-64\,a^3\,b^2-64\,a\,b^4+128\,b^4\,\mathrm{e}^x\,\sqrt{a^2+b^2}+32\,a^4\,b\,\mathrm{e}^x+160\,a^2\,b^3\,\mathrm{e}^x-64\,a\,b^3\,\sqrt{a^2+b^2}-32\right.}{a\,b}$$

input
```
int(coth(x)^2/(a + b/sinh(x)),x)
```

output
```
x/a + log(32*a^2*b + 32*b^3 - 32*b^3*exp(x) - 32*a^2*b*exp(x))/b - log(32*
a^2*b + 32*b^3 + 32*b^3*exp(x) + 32*a^2*b*exp(x))/b + (log(128*b^5*exp(x)
- 64*a^3*b^2 - 64*a*b^4 - 128*b^4*exp(x)*(a^2 + b^2)^(1/2) + 32*a^4*b*exp(
x) + 160*a^2*b^3*exp(x) + 64*a*b^3*(a^2 + b^2)^(1/2) + 32*a^3*b*(a^2 + b^2
)^(1/2) - 96*a^2*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a*b) -
(log(128*b^5*exp(x) - 64*a^3*b^2 - 64*a*b^4 + 128*b^4*exp(x)*(a^2 + b^2)^(
1/2) + 32*a^4*b*exp(x) + 160*a^2*b^3*exp(x) - 64*a*b^3*(a^2 + b^2)^(1/2) -
 32*a^3*b*(a^2 + b^2)^(1/2) + 96*a^2*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 +
b^2)^(1/2))/(a*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{\coth^2(x)}{a+b\text{csch}(x)}\,dx = \frac{-2\sqrt{a^2+b^2}\,atan\left(\frac{e^x ai+bi}{\sqrt{a^2+b^2}}\right)i+\log(e^x-1)\,a-\log(e^x+1)\,a+bx}{ab}$$

input
```
int(coth(x)^2/(a+b*csch(x)),x)
```

output
```
( - 2*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2))*i + log(e
**x - 1)*a - log(e**x + 1)*a + b*x)/(a*b)
```

3.120 $\int \frac{\coth^3(x)}{a+b\text{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{\coth^3(x)}{a+b\text{csch}(x)}\,dx = -\frac{\text{csch}(x)}{b}+\left(\frac{1}{a}+\frac{a}{b^2}\right)\log(a+b\text{csch}(x))+\frac{\log(\sinh(x))}{a}$$

output
```
-csch(x)/b+(1/a+a/b^2)*ln(a+b*csch(x))+ln(sinh(x))/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{\coth^3(x)}{a+b\text{csch}(x)}\,dx = \frac{-ab\text{csch}(x)-a^2\log(\sinh(x))+(a^2+b^2)\log(b+a\sinh(x))}{ab^2}$$

input
```
Integrate[Coth[x]^3/(a + b*Csch[x]),x]
```

output
```
(-(a*b*Csch[x]) - a^2*Log[Sinh[x]] + (a^2 + b^2)*Log[b + a*Sinh[x]])/(a*b^
2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 26, 4373, 25, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(x)}{a+b\text{csch}(x)} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\cot(ix)^3}{a+ib\csc(ix)}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\cot(ix)^3}{a+ib\csc(ix)}dx$$

$$\Big\downarrow \text{ 4373}$$

$$\frac{\int -\frac{\left(\text{csch}^2(x)b^2+b^2\right)\sinh(x)}{b(a+b\text{csch}(x))}d(b\text{csch}(x))}{b^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{\left(\text{csch}^2(x)b^2+b^2\right)\sinh(x)}{b(a+b\text{csch}(x))}d(b\text{csch}(x))}{b^2}$$

$$\Big\downarrow \text{ 522}$$

$$-\frac{\int \left(\frac{-a^2-b^2}{a(a+b\text{csch}(x))}+\frac{b\sinh(x)}{a}+1\right)d(b\text{csch}(x))}{b^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{(a^2+b^2)\log(a+b\text{csch}(x))}{a}-\frac{b^2\log(b\text{csch}(x))}{a}-b\text{csch}(x)}{b^2}$$

input
```
Int[Coth[x]^3/(a + b*Csch[x]),x]
```

output
```
(-(b*Csch[x]) - (b^2*Log[b*Csch[x]])/a + ((a^2 + b^2)*Log[a + b*Csch[x]])/
a)/b^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(32) = 64$.

Time = 0.64 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.34

method	result
risch	$-\frac{x}{a}-\frac{2\,{\mathrm e}^{x}}{b\left({\mathrm e}^{2 x}-1\right)}+\frac{a \ln\left({\mathrm e}^{2 x}+\frac{2 b\,{\mathrm e}^{x}}{a}-1\right)}{b^{2}}+\frac{\ln\left({\mathrm e}^{2 x}+\frac{2 b\,{\mathrm e}^{x}}{a}-1\right)}{a}-\frac{a \ln\left({\mathrm e}^{2 x}-1\right)}{b^{2}}$
default	$\frac{\tanh\left(\frac{x}{2}\right)}{2 b}-\frac{1}{2 b \tanh\left(\frac{x}{2}\right)}-\frac{a \ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b^{2}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{\left(2 a^{2}+2 b^{2}\right) \ln\left(-b \tanh\left(\frac{x}{2}\right)^{2}+2 a \tanh\left(\frac{x}{2}\right.\right.}{2 a\,b^{2}}$

input
```
int(coth(x)^3/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-x/a-2*exp(x)/b/(exp(2*x)-1)+a/b^2*ln(exp(2*x)+2*b/a*exp(x)-1)+1/a*ln(exp(
2*x)+2*b/a*exp(x)-1)-1/b^2*a*ln(exp(2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(32) = 64$.

Time = 0.11 (sec) , antiderivative size = 199, normalized size of antiderivative = 6.22

$$\int \frac{\coth^3(x)}{a+b\text{csch}(x)}\,dx =$$
$$-\frac{b^2 x \cosh\left(x\right)^2 + b^2 x \sinh\left(x\right)^2 - b^2 x + 2\,ab\cosh\left(x\right) - \left(\left(a^2+b^2\right)\cosh\left(x\right)^2 + 2\left(a^2+b^2\right)\cosh\left(x\right)\sinh\left(x\right.\right.}{}$$

input
```
integrate(coth(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
-(b^2*x*cosh(x)^2 + b^2*x*sinh(x)^2 - b^2*x + 2*a*b*cosh(x) - ((a^2 + b^2)
*cosh(x)^2 + 2*(a^2 + b^2)*cosh(x)*sinh(x) + (a^2 + b^2)*sinh(x)^2 - a^2 -
 b^2)*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + (a^2*cosh(x)^2 + 2*a^2*
cosh(x)*sinh(x) + a^2*sinh(x)^2 - a^2)*log(2*sinh(x)/(cosh(x) - sinh(x)))
+ 2*(b^2*x*cosh(x) + a*b)*sinh(x))/(a*b^2*cosh(x)^2 + 2*a*b^2*cosh(x)*sinh
(x) + a*b^2*sinh(x)^2 - a*b^2)
```

Sympy [F]

$$\int \frac{\coth^3(x)}{a+b\text{csch}(x)}\,dx=\int \frac{\coth^3{(x)}}{a+b\operatorname{csch}{(x)}}\,dx$$

input
```
integrate(coth(x)**3/(a+b*csch(x)),x)
```

output
```
Integral(coth(x)**3/(a + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(32) = 64$.

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.56

$$\int \frac{\coth^3(x)}{a+b\text{csch}(x)}\,dx=\frac{x}{a}+\frac{2\,e^{(-x)}}{be^{(-2\,x)}-b}-\frac{a\log\left(e^{(-x)}+1\right)}{b^2}-\frac{a\log\left(e^{(-x)}-1\right)}{b^2}+\frac{(a^2+b^2)\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{ab^2}$$

input
```
integrate(coth(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
x/a + 2*e^(-x)/(b*e^(-2*x) - b) - a*log(e^(-x) + 1)/b^2 - a*log(e^(-x) - 1
)/b^2 + (a^2 + b^2)*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a*b^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(32) = 64$.

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.50

$$\int \frac{\coth^3(x)}{a+b\text{csch}(x)}\,dx=-\frac{a\log\left(\left|-e^{(-x)}+e^x\right|\right)}{b^2}+\frac{(a^2+b^2)\log\left(\left|-a\left(e^{(-x)}-e^x\right)+2\,b\right|\right)}{ab^2}+\frac{a\left(e^{(-x)}-e^x\right)+2\,b}{b^2\left(e^{(-x)}-e^x\right)}$$

input
```
integrate(coth(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

output
```
-a*log(abs(-e^(-x) + e^x))/b^2 + (a^2 + b^2)*log(abs(-a*(e^(-x) - e^x) + 2
*b))/(a*b^2) + (a*(e^(-x) - e^x) + 2*b)/(b^2*(e^(-x) - e^x))
```

Mupad [B] (verification not implemented)

Time = 3.14 (sec) , antiderivative size = 261, normalized size of antiderivative = 8.16

$$\int \frac{\coth^3(x)}{a + b\text{csch}(x)}\,dx = \frac{2\,\mathrm{e}^{x}}{b - b\,\mathrm{e}^{2\,x}} - \frac{x}{a} + \frac{\ln\left(16\,a^5\,\mathrm{e}^{2\,x} - 4\,a\,b^4 - 16\,a^5 - 16\,a^3\,b^2 + 8\,b^5\,\mathrm{e}^{x} + 16\,a^3\,b^2\,\mathrm{e}^{2\,x} + 32\,a^4\,b\,\mathrm{e}^{x} + 4\,a\,b^4\,\mathrm{e}^{2\,x} + 32\,a^2\,b^3\,\mathrm{e}^{x}\right)}{a} + \frac{a\,\ln\left(16\,a^5\,\mathrm{e}^{2\,x} - 4\,a\,b^4 - 16\,a^5 - 16\,a^3\,b^2 + 8\,b^5\,\mathrm{e}^{x} + 16\,a^3\,b^2\,\mathrm{e}^{2\,x} + 32\,a^4\,b\,\mathrm{e}^{x} + 4\,a\,b^4\,\mathrm{e}^{2\,x} + 32\,a^2\,b^3\,\mathrm{e}^{x}\right)}{b^2} - \frac{a\,\ln\left(16\,a^6\,\mathrm{e}^{2\,x} + 4\,b^6\,\mathrm{e}^{2\,x} - 16\,a^6 - 4\,b^6 - 20\,a^2\,b^4 - 32\,a^4\,b^2 + 20\,a^2\,b^4\,\mathrm{e}^{2\,x} + 32\,a^4\,b^2\,\mathrm{e}^{2\,x}\right)}{b^2}$$

input
```
int(coth(x)^3/(a + b/sinh(x)),x)
```

output
```
(2*exp(x))/(b - b*exp(2*x)) - x/a + log(16*a^5*exp(2*x) - 4*a*b^4 - 16*a^5
 - 16*a^3*b^2 + 8*b^5*exp(x) + 16*a^3*b^2*exp(2*x) + 32*a^4*b*exp(x) + 4*a
*b^4*exp(2*x) + 32*a^2*b^3*exp(x))/a + (a*log(16*a^5*exp(2*x) - 4*a*b^4 -
16*a^5 - 16*a^3*b^2 + 8*b^5*exp(x) + 16*a^3*b^2*exp(2*x) + 32*a^4*b*exp(x)
 + 4*a*b^4*exp(2*x) + 32*a^2*b^3*exp(x)))/b^2 - (a*log(16*a^6*exp(2*x) + 4
*b^6*exp(2*x) - 16*a^6 - 4*b^6 - 20*a^2*b^4 - 32*a^4*b^2 + 20*a^2*b^4*exp(
2*x) + 32*a^4*b^2*exp(2*x)))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 192, normalized size of antiderivative = 6.00

$$\int \frac{\coth^3(x)}{a+b\text{csch}(x)}\,dx = \frac{-e^{2x}\log(e^x-1)\,a^2-e^{2x}\log(e^x+1)\,a^2+e^{2x}\log(e^{2x}a+2e^xb-a)\,a^2+e^{2x}\log(e^{2x}a+2e^xb-a)\,b^2-e^{2x}b^2}{a\,b^2\,(e^{2x}}$$

input
```
int(coth(x)^3/(a+b*csch(x)),x)
```

output
```
( - e**(2*x)*log(e**x - 1)*a**2 - e**(2*x)*log(e**x + 1)*a**2 + e**(2*x)*l
og(e**(2*x)*a + 2*e**x*b - a)*a**2 + e**(2*x)*log(e**(2*x)*a + 2*e**x*b -
a)*b**2 - e**(2*x)*b**2*x - 2*e**x*a*b + log(e**x - 1)*a**2 + log(e**x + 1
)*a**2 - log(e**(2*x)*a + 2*e**x*b - a)*a**2 - log(e**(2*x)*a + 2*e**x*b -
 a)*b**2 + b**2*x)/(a*b**2*(e**(2*x) - 1))
```

3.121 $\int \frac{\coth^4(x)}{a+b\text{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 88

$$\int \frac{\coth^4(x)}{a+b\text{csch}(x)}\,dx = \frac{x}{a}-\frac{(2a^2+3b^2)\,\text{arctanh}(\cosh(x))}{2b^3}+\frac{2(a^2+b^2)^{3/2}\,\text{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab^3}+\frac{a\coth(x)}{b^2}-\frac{\coth(x)\text{csch}(x)}{2b}$$

output
```
x/a-1/2*(2*a^2+3*b^2)*arctanh(cosh(x))/b^3+2*(a^2+b^2)^(3/2)*arctanh((a-b*
tanh(1/2*x))/(a^2+b^2)^(1/2))/a/b^3+a*coth(x)/b^2-1/2*coth(x)*csch(x)/b
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.95

$$\int \frac{\coth^4(x)}{a+b\text{csch}(x)}\,dx = \frac{\text{csch}(x)(b+a\sinh(x))\left(8b^3x-16(-a^2-b^2)^{3/2}\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+4a^2b\coth\left(\frac{x}{2}\right)-ab^2\text{csch}^2\left(\frac{x}{2}\right)-4a\cdots\right.}{8ab^3(a+b\text{csch}(\cdots}$$

input
```
Integrate[Coth[x]^4/(a + b*Csch[x]),x]
```

output
```
(Csch[x]*(b + a*Sinh[x])*(8*b^3*x - 16*(-a^2 - b^2)^(3/2)*ArcTan[(a - b*Ta
nh[x/2])/Sqrt[-a^2 - b^2]] + 4*a^2*b*Coth[x/2] - a*b^2*Csch[x/2]^2 - 4*a*(
2*a^2 + 3*b^2)*Log[Cosh[x/2]] + 4*a*(2*a^2 + 3*b^2)*Log[Sinh[x/2]] - a*b^2
*Sech[x/2]^2 + 4*a^2*b*Tanh[x/2]))/(8*a*b^3*(a + b*Csch[x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.34, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.385$, Rules used = {3042, 4386, 26, 26, 3042, 26, 3372, 26, 3042, 26, 3536, 26, 3042, 26, 3139, 1083, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(x)}{a+b\text{csch}(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(ix)^4}{a+ib\csc(ix)}dx$$

$$\Big\downarrow \text{4386}$$

$$\int \frac{i\cosh(x)\coth^3(x)}{ia\sinh(x)+ib}dx$$

$$\Big\downarrow \text{26}$$

$$i\int -\frac{i\cosh(x)\coth^3(x)}{b+a\sinh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\cosh(x)\coth^3(x)}{a\sinh(x)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cos(ix)^4}{\sin(ix)^3(b-ia\sin(ix))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos(ix)^4}{\sin(ix)^3(b-ia\sin(ix))}dx$$

$$\Big\downarrow \text{3372}$$

$$-i\left(-\frac{\int -\frac{i\text{csch}(x)\left(2a^2-b\sinh(x)a+3b^2+2b^2\sinh^2(x)\right)}{b+a\sinh(x)}dx}{2b^2}+\frac{ia\coth(x)}{b^2}-\frac{i\coth(x)\text{csch}(x)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\int \frac{\text{csch}(x)\left(2a^2-b\sinh(x)a+3b^2+2b^2\sinh^2(x)\right)}{b+a\sinh(x)}dx}{2b^2}+\frac{ia\coth(x)}{b^2}-\frac{i\coth(x)\text{csch}(x)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\int \frac{i\left(2a^2+ib\sin(ix)a+3b^2-2b^2\sin(ix)^2\right)}{\sin(ix)(b-ia\sin(ix))}dx}{2b^2}+\frac{ia\coth(x)}{b^2}-\frac{i\coth(x)\text{csch}(x)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{\int \frac{2a^2+ib\sin(ix)a+3b^2-2b^2\sin(ix)^2}{\sin(ix)(b-ia\sin(ix))}dx}{2b^2}+\frac{ia\coth(x)}{b^2}-\frac{i\coth(x)\text{csch}(x)}{2b}\right)$$

$$\Big\downarrow \text{3536}$$

$$-i\left(-\frac{\frac{2i\left(a^2+b^2\right)^2\int \frac{1}{b+a\sinh(x)}dx}{ab}+\frac{\left(2a^2+3b^2\right)\int -i\text{csch}(x)dx}{b}-\frac{2ib^2x}{a}}{2b^2}+\frac{ia\coth(x)}{b^2}-\frac{i\coth(x)\text{csch}(x)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{\frac{2i\left(a^2+b^2\right)^2\int \frac{1}{b+a\sinh(x)}dx}{ab}-\frac{i\left(2a^2+3b^2\right)\int \text{csch}(x)dx}{b}-\frac{2ib^2x}{a}}{2b^2}+\frac{ia\coth(x)}{b^2}-\frac{i\coth(x)\text{csch}(x)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{\frac{2i\left(a^2+b^2\right)^2\int \frac{1}{b-ia\sin(ix)}dx}{ab}-\frac{i\left(2a^2+3b^2\right)\int i\csc(ix)dx}{b}-\frac{2ib^2x}{a}}{2b^2}+\frac{ia\coth(x)}{b^2}-\frac{i\coth(x)\text{csch}(x)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{\frac{2i(a^2+b^2)^2\int\frac{1}{b-ia\sin(ix)}dx}{ab}+\frac{(2a^2+3b^2)\int\csc(ix)dx}{b}-\frac{2ib^2x}{a}}{2b^2}+\frac{ia\coth(x)}{b^2}-\frac{i\coth(x)\text{csch}(x)}{2b}\right)$$

$\Big\downarrow$ 3139

$$-i\left(-\frac{\frac{(2a^2+3b^2)\int\csc(ix)dx}{b}+\frac{4i(a^2+b^2)^2\int\frac{1}{-b\tanh^2\left(\frac{x}{2}\right)+2a\tanh\left(\frac{x}{2}\right)+b}d\tanh\left(\frac{x}{2}\right)}{ab}-\frac{2ib^2x}{a}}{2b^2}+\frac{ia\coth(x)}{b^2}-\frac{i\coth(x)\text{csch}(x)}{2b}\right)$$

$\Big\downarrow$ 1083

$$-i\left(-\frac{\frac{(2a^2+3b^2)\int\csc(ix)dx}{b}-\frac{8i(a^2+b^2)^2\int\frac{1}{4(a^2+b^2)-\left(2a-2b\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2a-2b\tanh\left(\frac{x}{2}\right)\right)}{ab}-\frac{2ib^2x}{a}}{2b^2}+\frac{ia\coth(x)}{b^2}-\frac{i\coth(x)\text{cs}}{2b}\right.$$

$\Big\downarrow$ 219

$$-i\left(-\frac{\frac{(2a^2+3b^2)\int\csc(ix)dx}{b}-\frac{4i(a^2+b^2)^{3/2}\text{arctanh}\left(\frac{2a-2b\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{ab}-\frac{2ib^2x}{a}}{2b^2}+\frac{ia\coth(x)}{b^2}-\frac{i\coth(x)\text{csch}(x)}{2b}\right)$$

$\Big\downarrow$ 4257

$$-i\left(-\frac{\frac{i(2a^2+3b^2)\text{arctanh}(\cosh(x))}{b}-\frac{4i(a^2+b^2)^{3/2}\text{arctanh}\left(\frac{2a-2b\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{ab}-\frac{2ib^2x}{a}}{2b^2}+\frac{ia\coth(x)}{b^2}-\frac{i\coth(x)\text{csch}(x)}{2b}\right)$$

input
```
Int[Coth[x]^4/(a + b*Csch[x]),x]
```

output
```
(-I)*(-1/2*(((-2*I)*b^2*x)/a + (I*(2*a^2 + 3*b^2)*ArcTanh[Cosh[x]])/b - ((
4*I)*(a^2 + b^2)^(3/2)*ArcTanh[(2*a - 2*b*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])
/(a*b))/b^2 + (I*a*Coth[x])/b^2 - ((I/2)*Coth[x]*Csch[x])/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3372
```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Si
mp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x]
)^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x] - Simp[1/(a^2*d^2*(n + 1)*(n + 2
))   Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])^(n + 2)*Simp[a^2*n*(n + 2)
 - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f*x] - (a^2*(n + 1)*(n + 2)
- b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x], x], x]) /; FreeQ[{a, b,
d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
 &&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

rule 3536
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[C*(x/(b*d)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d))   Int[1/(a + b*Sin[e + f*x]), x], x] - Simp[(c^2*C - B*c*d
 + A*d^2)/(d*(b*c - a*d))   Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a
, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4386
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.70

method	result
default	$\frac{\frac{b \tanh\left(\frac{x}{2}\right)^{2}}{2}+2 a \tanh\left(\frac{x}{2}\right)}{4 b^{2}}-\frac{1}{8 b \tanh\left(\frac{x}{2}\right)^{2}}+\frac{\left(4 a^{2}+6 b^{2}\right) \ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4 b^{3}}+\frac{a}{2 b^{2} \tanh\left(\frac{x}{2}\right)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$
risch	$\frac{x}{a}+\frac{-b\,e^{3 x}+2\,e^{2 x} a-b\,e^{x}-2 a}{\left(e^{2 x}-1\right)^{2} b^{2}}+\frac{\ln\left(e^{x}-1\right) a^{2}}{b^{3}}+\frac{3 \ln\left(e^{x}-1\right)}{2 b}-\frac{\ln\left(1+e^{x}\right) a^{2}}{b^{3}}-\frac{3 \ln\left(1+e^{x}\right)}{2 b}+\frac{\left(a^{2}+b^{2}\right)^{\frac{3}{2}} \ln\left(e^{x}+\frac{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}+a}{\left(a^{2}+b^{2}\right)}\right.}{b^{3} a}$

input
```
int(coth(x)^4/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
1/4/b^2*(1/2*b*tanh(1/2*x)^2+2*a*tanh(1/2*x))-1/8/b/tanh(1/2*x)^2+1/4/b^3*
(4*a^2+6*b^2)*ln(tanh(1/2*x))+1/2/b^2*a/tanh(1/2*x)-1/a*ln(tanh(1/2*x)-1)+
1/a*ln(tanh(1/2*x)+1)+1/4*(8*a^4+16*a^2*b^2+8*b^4)/a/b^3/(a^2+b^2)^(1/2)*a
rctanh(1/2*(-2*tanh(1/2*x)*b+2*a)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 831 vs. $2(80) = 160$.

Time = 0.16 (sec) , antiderivative size = 831, normalized size of antiderivative = 9.44

$$\int \frac{\coth^4(x)}{a + b\text{csch}(x)} \, dx = \text{Too large to display}$$

input
```
integrate(coth(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
1/2*(2*b^3*x*cosh(x)^4 + 2*b^3*x*sinh(x)^4 - 2*a*b^2*cosh(x)^3 + 2*b^3*x -
 2*a*b^2*cosh(x) + 2*(4*b^3*x*cosh(x) - a*b^2)*sinh(x)^3 - 4*a^2*b - 4*(b^
3*x - a^2*b)*cosh(x)^2 + 2*(6*b^3*x*cosh(x)^2 - 2*b^3*x - 3*a*b^2*cosh(x)
+ 2*a^2*b)*sinh(x)^2 + 2*((a^2 + b^2)*cosh(x)^4 + 4*(a^2 + b^2)*cosh(x)*si
nh(x)^3 + (a^2 + b^2)*sinh(x)^4 - 2*(a^2 + b^2)*cosh(x)^2 + 2*(3*(a^2 + b^
2)*cosh(x)^2 - a^2 - b^2)*sinh(x)^2 + a^2 + b^2 + 4*((a^2 + b^2)*cosh(x)^3
 - (a^2 + b^2)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*
sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) +
2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2
+ 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) - ((2*a^3 + 3*a*b^2)*cosh(
x)^4 + 4*(2*a^3 + 3*a*b^2)*cosh(x)*sinh(x)^3 + (2*a^3 + 3*a*b^2)*sinh(x)^4
 + 2*a^3 + 3*a*b^2 - 2*(2*a^3 + 3*a*b^2)*cosh(x)^2 - 2*(2*a^3 + 3*a*b^2 -
3*(2*a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 + 3*a*b^2)*cosh(x)^3
- (2*a^3 + 3*a*b^2)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + ((2*a^3
 + 3*a*b^2)*cosh(x)^4 + 4*(2*a^3 + 3*a*b^2)*cosh(x)*sinh(x)^3 + (2*a^3 + 3
*a*b^2)*sinh(x)^4 + 2*a^3 + 3*a*b^2 - 2*(2*a^3 + 3*a*b^2)*cosh(x)^2 - 2*(2
*a^3 + 3*a*b^2 - 3*(2*a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 + 3*
a*b^2)*cosh(x)^3 - (2*a^3 + 3*a*b^2)*cosh(x))*sinh(x))*log(cosh(x) + sinh(
x) - 1) + 2*(4*b^3*x*cosh(x)^3 - 3*a*b^2*cosh(x)^2 - a*b^2 - 4*(b^3*x - a^
2*b)*cosh(x))*sinh(x))/(a*b^3*cosh(x)^4 + 4*a*b^3*cosh(x)*sinh(x)^3 + a...
```

Sympy [F]

$$\int \frac{\coth^4(x)}{a + b\text{csch}(x)}\, dx = \int \frac{\coth^4{(x)}}{a + b\operatorname{csch}{(x)}}\, dx$$

input
```
integrate(coth(x)**4/(a+b*csch(x)),x)
```

output
```
Integral(coth(x)**4/(a + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(80) = 160.

Time = 0.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.02

$$\int \frac{\coth^4(x)}{a + b\text{csch}(x)}\, dx = \frac{be^{(-x)} + 2\, ae^{(-2\, x)} + be^{(-3\, x)} - 2\, a}{2\, b^2 e^{(-2\, x)} - b^2 e^{(-4\, x)} - b^2} + \frac{x}{a} - \frac{(2\, a^2 + 3\, b^2)\log\left(e^{(-x)} + 1\right)}{2\, b^3} + \frac{(2\, a^2 + 3\, b^2)\log\left(e^{(-x)} - 1\right)}{2\, b^3} - \frac{(a^4 + 2\, a^2 b^2 + b^4)\log\left(\frac{ae^{(-x)} - b - \sqrt{a^2 + b^2}}{ae^{(-x)} - b + \sqrt{a^2 + b^2}}\right)}{\sqrt{a^2 + b^2} a b^3}$$

input
```
integrate(coth(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
(b*e^(-x) + 2*a*e^(-2*x) + b*e^(-3*x) - 2*a)/(2*b^2*e^(-2*x) - b^2*e^(-4*x
) - b^2) + x/a - 1/2*(2*a^2 + 3*b^2)*log(e^(-x) + 1)/b^3 + 1/2*(2*a^2 + 3*
b^2)*log(e^(-x) - 1)/b^3 - (a^4 + 2*a^2*b^2 + b^4)*log((a*e^(-x) - b - sqr
t(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(80) = 160$.

Time = 0.13 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.83

$$\int \frac{\coth^4(x)}{a+b\text{csch}(x)}\,dx = \frac{x}{a} - \frac{(2\,a^2+3\,b^2)\log(e^x+1)}{2\,b^3} + \frac{(2\,a^2+3\,b^2)\log(|e^x-1|)}{2\,b^3} - \frac{(a^4+2\,a^2b^2+b^4)\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}ab^3} - \frac{be^{(3\,x)}-2\,ae^{(2\,x)}+be^x+2\,a}{b^2(e^{(2\,x)}-1)^2}$$

input
```
integrate(coth(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

output
```
x/a - 1/2*(2*a^2 + 3*b^2)*log(e^x + 1)/b^3 + 1/2*(2*a^2 + 3*b^2)*log(abs(e
^x - 1))/b^3 - (a^4 + 2*a^2*b^2 + b^4)*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2
+ b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*b^3) -
(b*e^(3*x) - 2*a*e^(2*x) + b*e^x + 2*a)/(b^2*(e^(2*x) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 378, normalized size of antiderivative = 4.30

$$\int \frac{\coth^4(x)}{a+b\text{csch}(x)}\,dx = \frac{\frac{2\,a}{b^2}-\frac{e^x}{b}}{e^{2\,x}-1} + \frac{x}{a} + \frac{\ln(e^x-1)\,(2\,a^2+3\,b^2)}{2\,b^3} - \frac{\ln(e^x+1)\,(2\,a^2+3\,b^2)}{2\,b^3} - \frac{2\,e^x}{b\,(e^{4\,x}-2\,e^{2\,x}+1)} + \frac{\ln\Big(a^3\,\sqrt{(a^2+b^2)^3}-2\,a^5\,b-2\,a\,b^5-4\,a^3\,b^3+a^6\,e^x+4\,b^6\,e^x+2\,a\,b^2\,\sqrt{(a^2+b^2)^3}-4\,b^3\,e^x\,\sqrt{(a^2+b^2}}{a\,b^3} - \frac{\ln\Big(a^6\,e^x-2\,a^5\,b-a^3\,\sqrt{(a^2+b^2)^3}-4\,a^3\,b^3-2\,a\,b^5+4\,b^6\,e^x-2\,a\,b^2\,\sqrt{(a^2+b^2)^3}+4\,b^3\,e^x\,\sqrt{(a^2+b^2}}{a\,b^3}$$

input
```
int(coth(x)^4/(a + b/sinh(x)),x)
```

output
```
((2*a)/b^2 - exp(x)/b)/(exp(2*x) - 1) + x/a + (log(exp(x) - 1)*(2*a^2 + 3*
b^2))/(2*b^3) - (log(exp(x) + 1)*(2*a^2 + 3*b^2))/(2*b^3) - (2*exp(x))/(b*
(exp(4*x) - 2*exp(2*x) + 1)) + (log(a^3*((a^2 + b^2)^3)^(1/2) - 2*a^5*b -
2*a*b^5 - 4*a^3*b^3 + a^6*exp(x) + 4*b^6*exp(x) + 2*a*b^2*((a^2 + b^2)^3)^
(1/2) - 4*b^3*exp(x)*((a^2 + b^2)^3)^(1/2) + 9*a^2*b^4*exp(x) + 6*a^4*b^2*
exp(x) - 3*a^2*b*exp(x)*((a^2 + b^2)^3)^(1/2))*((a^2 + b^2)^3)^(1/2))/(a*b
^3) - (log(a^6*exp(x) - 2*a^5*b - a^3*((a^2 + b^2)^3)^(1/2) - 4*a^3*b^3 -
2*a*b^5 + 4*b^6*exp(x) - 2*a*b^2*((a^2 + b^2)^3)^(1/2) + 4*b^3*exp(x)*((a^
2 + b^2)^3)^(1/2) + 9*a^2*b^4*exp(x) + 6*a^4*b^2*exp(x) + 3*a^2*b*exp(x)*(
(a^2 + b^2)^3)^(1/2))*((a^2 + b^2)^3)^(1/2))/(a*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 504, normalized size of antiderivative = 5.73

$$\int \frac{\coth^4(x)}{a+b\text{csch}(x)}\,dx$$
$$= \frac{2e^{4x}a^2b - 2e^{3x}a\,b^2 - 2e^xa\,b^2 + 2b^3x - 4\sqrt{a^2+b^2}\,atan\left(\frac{e^xai+bi}{\sqrt{a^2+b^2}}\right)b^2i + 3e^{4x}\log(e^x-1)\,a\,b^2 - 3e^{4x}\log(e^x+}{}$$

input
```
int(coth(x)^4/(a+b*csch(x)),x)
```

output
```
( - 4*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2))*
a**2*i - 4*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b*
*2))*b**2*i + 8*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2
 + b**2))*a**2*i + 8*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt
(a**2 + b**2))*b**2*i - 4*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**
2 + b**2))*a**2*i - 4*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 +
b**2))*b**2*i + 2*e**(4*x)*log(e**x - 1)*a**3 + 3*e**(4*x)*log(e**x - 1)*a
*b**2 - 2*e**(4*x)*log(e**x + 1)*a**3 - 3*e**(4*x)*log(e**x + 1)*a*b**2 +
2*e**(4*x)*a**2*b + 2*e**(4*x)*b**3*x - 2*e**(3*x)*a*b**2 - 4*e**(2*x)*log
(e**x - 1)*a**3 - 6*e**(2*x)*log(e**x - 1)*a*b**2 + 4*e**(2*x)*log(e**x +
1)*a**3 + 6*e**(2*x)*log(e**x + 1)*a*b**2 - 4*e**(2*x)*b**3*x - 2*e**x*a*b
**2 + 2*log(e**x - 1)*a**3 + 3*log(e**x - 1)*a*b**2 - 2*log(e**x + 1)*a**3
 - 3*log(e**x + 1)*a*b**2 - 2*a**2*b + 2*b**3*x)/(2*a*b**3*(e**(4*x) - 2*e
**(2*x) + 1))
```

3.122 $\int \frac{\coth^5(x)}{a+b\,\text{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 70

$$\int \frac{\coth^5(x)}{a+b\text{csch}(x)}\,dx = -\frac{(a^2+2b^2)\,\text{csch}(x)}{b^3}+\frac{a\text{csch}^2(x)}{2b^2}-\frac{\text{csch}^3(x)}{3b} + \frac{(a^2+b^2)^2\log(a+b\text{csch}(x))}{ab^4}+\frac{\log(\sinh(x))}{a}$$

output
```
-(a^2+2*b^2)*csch(x)/b^3+1/2*a*csch(x)^2/b^2-1/3*csch(x)^3/b+(a^2+b^2)^2*l
n(a+b*csch(x))/a/b^4+ln(sinh(x))/a
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.19

$$\int \frac{\coth^5(x)}{a+b\text{csch}(x)}\,dx = \frac{-6ab(a^2+2b^2)\,\text{csch}(x)+3a^2b^2\text{csch}^2(x)-2ab^3\text{csch}^3(x)-6a^2(a^2+2b^2)\log(\sinh(x))+6(a^2+b^2)^2\log(b}{6ab^4}$$

input
```
Integrate[Coth[x]^5/(a + b*Csch[x]),x]
```

output
```
(-6*a*b*(a^2 + 2*b^2)*Csch[x] + 3*a^2*b^2*Csch[x]^2 - 2*a*b^3*Csch[x]^3 -
6*a^2*(a^2 + 2*b^2)*Log[Sinh[x]] + 6*(a^2 + b^2)^2*Log[b + a*Sinh[x]])/(6*
a*b^4)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 26, 4373, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^5(x)}{a+b\text{csch}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cot(ix)^5}{a+ib\csc(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cot(ix)^5}{a+ib\csc(ix)}dx$$

$$\Big\downarrow \text{4373}$$

$$-\frac{\int \frac{\left(\text{csch}^2(x)b^2+b^2\right)^2\sinh(x)}{b(a+b\text{csch}(x))}d(b\text{csch}(x))}{b^4}$$

$$\Big\downarrow \text{522}$$

$$-\frac{\int \left(\frac{\sinh(x)b^3}{a}+\text{csch}^2(x)b^2-a\text{csch}(x)b+a^2\left(\frac{2b^2}{a^2}+1\right)-\frac{\left(a^2+b^2\right)^2}{a(a+b\text{csch}(x))}\right)d(b\text{csch}(x))}{b^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(a^2+2b^2\right)\text{csch}(x)-\frac{\left(a^2+b^2\right)^2\log(a+b\text{csch}(x))}{a}+\frac{b^4\log(b\text{csch}(x))}{a}-\frac{1}{2}ab^2\text{csch}^2(x)+\frac{1}{3}b^3\text{csch}^3(x)}{b^4}$$

input
```
Int[Coth[x]^5/(a + b*Csch[x]),x]
```

output
```
-((b*(a^2 + 2*b^2)*Csch[x] - (a*b^2*Csch[x]^2)/2 + (b^3*Csch[x]^3)/3 + (b^
4*Log[b*Csch[x]])/a - ((a^2 + b^2)^2*Log[a + b*Csch[x]])/a)/b^4)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(66) = 132$.

Time = 1.60 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.49

method	result
risch	$-\frac{x}{a}-\frac{2\,\mathrm{e}^{x}\left(3\,\mathrm{e}^{4x}a^{2}+6b^{2}\mathrm{e}^{4x}-3\,\mathrm{e}^{3x}ab-6\,\mathrm{e}^{2x}a^{2}-8b^{2}\mathrm{e}^{2x}+3a\,\mathrm{e}^{x}b+3a^{2}+6b^{2}\right)}{3b^{3}\left(\mathrm{e}^{2x}-1\right)^{3}}-\frac{a^{3}\ln\left(\mathrm{e}^{2x}-1\right)}{b^{4}}-\frac{2a\ln\left(\mathrm{e}^{2x}-1\right)}{b^{2}}+\frac{a^{3}\ln\left(\mathrm{e}^{2x}+\frac{2b}{}\right.}{b^{4}}$
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^{3}b^{2}}{3}+\tanh\left(\frac{x}{2}\right)^{2}ab+4\tanh\left(\frac{x}{2}\right)a^{2}+7\tanh\left(\frac{x}{2}\right)b^{2}}{8b^{3}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{1}{24b\tanh\left(\frac{x}{2}\right)^{3}}-\frac{4a^{2}+7b^{2}}{8b^{3}\tanh\left(\frac{x}{2}\right.}$

input
```
int(coth(x)^5/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
-x/a-2/3*exp(x)*(3*exp(4*x)*a^2+6*b^2*exp(4*x)-3*exp(3*x)*a*b-6*exp(2*x)*a
^2-8*b^2*exp(2*x)+3*a*exp(x)*b+3*a^2+6*b^2)/b^3/(exp(2*x)-1)^3-1/b^4*a^3*l
n(exp(2*x)-1)-2/b^2*a*ln(exp(2*x)-1)+1/b^4*a^3*ln(exp(2*x)+2*b/a*exp(x)-1)
+2*a/b^2*ln(exp(2*x)+2*b/a*exp(x)-1)+1/a*ln(exp(2*x)+2*b/a*exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1288 vs. $2(66) = 132$.

Time = 0.10 (sec) , antiderivative size = 1288, normalized size of antiderivative = 18.40

$$\int \frac{\coth^5(x)}{a + b\text{csch}(x)}\, dx = \text{Too large to display}$$

input
```
integrate(coth(x)^5/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
-1/3*(3*b^4*x*cosh(x)^6 + 3*b^4*x*sinh(x)^6 + 6*(a^3*b + 2*a*b^3)*cosh(x)^
5 + 6*(3*b^4*x*cosh(x) + a^3*b + 2*a*b^3)*sinh(x)^5 - 3*b^4*x - 3*(3*b^4*x
 + 2*a^2*b^2)*cosh(x)^4 + 3*(15*b^4*x*cosh(x)^2 - 3*b^4*x - 2*a^2*b^2 + 10
*(a^3*b + 2*a*b^3)*cosh(x))*sinh(x)^4 - 4*(3*a^3*b + 4*a*b^3)*cosh(x)^3 +
4*(15*b^4*x*cosh(x)^3 - 3*a^3*b - 4*a*b^3 + 15*(a^3*b + 2*a*b^3)*cosh(x)^2
 - 3*(3*b^4*x + 2*a^2*b^2)*cosh(x))*sinh(x)^3 + 3*(3*b^4*x + 2*a^2*b^2)*co
sh(x)^2 + 3*(15*b^4*x*cosh(x)^4 + 3*b^4*x + 2*a^2*b^2 + 20*(a^3*b + 2*a*b^
3)*cosh(x)^3 - 6*(3*b^4*x + 2*a^2*b^2)*cosh(x)^2 - 4*(3*a^3*b + 4*a*b^3)*c
osh(x))*sinh(x)^2 + 6*(a^3*b + 2*a*b^3)*cosh(x) - 3*((a^4 + 2*a^2*b^2 + b^
4)*cosh(x)^6 + 6*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^5 + (a^4 + 2*a^2*
b^2 + b^4)*sinh(x)^6 - 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^4 - 3*(a^4 + 2*a^
2*b^2 + b^4 - 5*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 - a^4 - 2*a^2
*b^2 - b^4 + 4*(5*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 - 3*(a^4 + 2*a^2*b^2 +
 b^4)*cosh(x))*sinh(x)^3 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 3*(5*(a^4
 + 2*a^2*b^2 + b^4)*cosh(x)^4 + a^4 + 2*a^2*b^2 + b^4 - 6*(a^4 + 2*a^2*b^2
 + b^4)*cosh(x)^2)*sinh(x)^2 + 6*((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^5 - 2*(a
^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)
)*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + 3*((a^4 + 2*a^2*b^2)*cosh(x
)^6 + 6*(a^4 + 2*a^2*b^2)*cosh(x)*sinh(x)^5 + (a^4 + 2*a^2*b^2)*sinh(x)^6
- 3*(a^4 + 2*a^2*b^2)*cosh(x)^4 - 3*(a^4 + 2*a^2*b^2 - 5*(a^4 + 2*a^2*b...
```

Sympy [F]

$$\int \frac{\coth^5(x)}{a + b\text{csch}(x)}\,dx = \int \frac{\coth^5(x)}{a + b\,\text{csch}(x)}\,dx$$

input
```
integrate(coth(x)**5/(a+b*csch(x)),x)
```

output
```
Integral(coth(x)**5/(a + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(66) = 132$.

Time = 0.04 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.71

$$\int \frac{\coth^5(x)}{a + b\operatorname{csch}(x)}\,dx = -\frac{2\left(3\,abe^{(-2\,x)} - 3\,abe^{(-4\,x)} - 3\,(a^2 + 2\,b^2)e^{(-x)} + 2\,(3\,a^2 + 4\,b^2)e^{(-3\,x)} - 3\,(a^2 + 2\,b^2)e^{(-5\,x)}\right)}{3\left(3\,b^3e^{(-2\,x)} - 3\,b^3e^{(-4\,x)} + b^3e^{(-6\,x)} - b^3\right)} + \frac{x}{a} - \frac{(a^3 + 2\,ab^2)\log\left(e^{(-x)} + 1\right)}{b^4} - \frac{(a^3 + 2\,ab^2)\log\left(e^{(-x)} - 1\right)}{b^4} + \frac{(a^4 + 2\,a^2b^2 + b^4)\log\left(-2\,be^{(-x)} + ae^{(-2\,x)} - a\right)}{ab^4}$$

input
```
integrate(coth(x)^5/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
-2/3*(3*a*b*e^(-2*x) - 3*a*b*e^(-4*x) - 3*(a^2 + 2*b^2)*e^(-x) + 2*(3*a^2
+ 4*b^2)*e^(-3*x) - 3*(a^2 + 2*b^2)*e^(-5*x))/(3*b^3*e^(-2*x) - 3*b^3*e^(-
4*x) + b^3*e^(-6*x) - b^3) + x/a - (a^3 + 2*a*b^2)*log(e^(-x) + 1)/b^4 - (
a^3 + 2*a*b^2)*log(e^(-x) - 1)/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log(-2*b*e^(-
x) + a*e^(-2*x) - a)/(a*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(66) = 132$.

Time = 0.13 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.43

$$\int \frac{\coth^5(x)}{a + b\operatorname{csch}(x)}\,dx = -\frac{(a^3 + 2\,ab^2)\log\left(\left|-e^{(-x)} + e^x\right|\right)}{b^4} + \frac{(a^4 + 2\,a^2b^2 + b^4)\log\left(\left|-a\left(e^{(-x)} - e^x\right) + 2\,b\right|\right)}{ab^4} + \frac{11\,a^3\left(e^{(-x)} - e^x\right)^3 + 22\,ab^2\left(e^{(-x)} - e^x\right)^3 + 12\,a^2b\left(e^{(-x)} - e^x\right)^2 + 24\,b^3\left(e^{(-x)} - e^x\right)^2 + 12\,ab^2\left(e^{(-x)} - e\right.}{6\,b^4\left(e^{(-x)} - e^x\right)^3}$$

input
```
integrate(coth(x)^5/(a+b*csch(x)),x, algorithm="giac")
```

output
```
-(a^3 + 2*a*b^2)*log(abs(-e^(-x) + e^x))/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log
(abs(-a*(e^(-x) - e^x) + 2*b))/(a*b^4) + 1/6*(11*a^3*(e^(-x) - e^x)^3 + 22
*a*b^2*(e^(-x) - e^x)^3 + 12*a^2*b*(e^(-x) - e^x)^2 + 24*b^3*(e^(-x) - e^x
)^2 + 12*a*b^2*(e^(-x) - e^x) + 16*b^3)/(b^4*(e^(-x) - e^x)^3)
```

Mupad [B] (verification not implemented)

Time = 3.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.21

$$\int \frac{\coth^5(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\frac{2\,a}{b^2}-\frac{2\,\mathrm{e}^x\,\left(a^2+2\,b^2\right)}{b^3}}{\mathrm{e}^{2\,x}-1}-\frac{x}{a}+\frac{\frac{2\,a}{b^2}-\frac{8\,\mathrm{e}^x}{3\,b}}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1} - \frac{8\,\mathrm{e}^x}{3\,b\,\left(3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1\right)}-\frac{\ln\left(\mathrm{e}^{2\,x}-1\right)\,\left(a^3+2\,a\,b^2\right)}{b^4} + \frac{\ln\left(2\,b\,\mathrm{e}^x-a+a\,\mathrm{e}^{2\,x}\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}{a\,b^4}$$

input
```
int(coth(x)^5/(a + b/sinh(x)),x)
```

output
```
((2*a)/b^2 - (2*exp(x)*(a^2 + 2*b^2))/b^3)/(exp(2*x) - 1) - x/a + ((2*a)/b
^2 - (8*exp(x))/(3*b))/(exp(4*x) - 2*exp(2*x) + 1) - (8*exp(x))/(3*b*(3*ex
p(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - (log(exp(2*x) - 1)*(2*a*b^2 + a^3))
/b^4 + (log(2*b*exp(x) - a + a*exp(2*x))*(a^4 + b^4 + 2*a^2*b^2))/(a*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 747, normalized size of antiderivative = 10.67

$$\int \frac{\coth^5(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{12e^{3x}a^3b+16e^{3x}a\,b^3-6e^{x}a^3b+2e^{6x}a^2b^2-6e^{5x}a^3b-12e^{5x}a\,b^3+9e^{4x}b^4x-9e^{4x}\log(e^{2x}a+2e^{x}b-a)\,b^4}{}$$

input
```
int(coth(x)^5/(a+b*csch(x)),x)
```

output

```
( - 3*e**(6*x)*log(e**x - 1)*a**4 - 6*e**(6*x)*log(e**x - 1)*a**2*b**2 - 3
*e**(6*x)*log(e**x + 1)*a**4 - 6*e**(6*x)*log(e**x + 1)*a**2*b**2 + 3*e**(
6*x)*log(e**(2*x)*a + 2*e**x*b - a)*a**4 + 6*e**(6*x)*log(e**(2*x)*a + 2*e
**x*b - a)*a**2*b**2 + 3*e**(6*x)*log(e**(2*x)*a + 2*e**x*b - a)*b**4 + 2*
e**(6*x)*a**2*b**2 - 3*e**(6*x)*b**4*x - 6*e**(5*x)*a**3*b - 12*e**(5*x)*a
*b**3 + 9*e**(4*x)*log(e**x - 1)*a**4 + 18*e**(4*x)*log(e**x - 1)*a**2*b**
2 + 9*e**(4*x)*log(e**x + 1)*a**4 + 18*e**(4*x)*log(e**x + 1)*a**2*b**2 -
9*e**(4*x)*log(e**(2*x)*a + 2*e**x*b - a)*a**4 - 18*e**(4*x)*log(e**(2*x)*
a + 2*e**x*b - a)*a**2*b**2 - 9*e**(4*x)*log(e**(2*x)*a + 2*e**x*b - a)*b*
*4 + 9*e**(4*x)*b**4*x + 12*e**(3*x)*a**3*b + 16*e**(3*x)*a*b**3 - 9*e**(2
*x)*log(e**x - 1)*a**4 - 18*e**(2*x)*log(e**x - 1)*a**2*b**2 - 9*e**(2*x)*
log(e**x + 1)*a**4 - 18*e**(2*x)*log(e**x + 1)*a**2*b**2 + 9*e**(2*x)*log(
e**(2*x)*a + 2*e**x*b - a)*a**4 + 18*e**(2*x)*log(e**(2*x)*a + 2*e**x*b -
a)*a**2*b**2 + 9*e**(2*x)*log(e**(2*x)*a + 2*e**x*b - a)*b**4 - 9*e**(2*x)
*b**4*x - 6*e**x*a**3*b - 12*e**x*a*b**3 + 3*log(e**x - 1)*a**4 + 6*log(e*
*x - 1)*a**2*b**2 + 3*log(e**x + 1)*a**4 + 6*log(e**x + 1)*a**2*b**2 - 3*l
og(e**(2*x)*a + 2*e**x*b - a)*a**4 - 6*log(e**(2*x)*a + 2*e**x*b - a)*a**2
*b**2 - 3*log(e**(2*x)*a + 2*e**x*b - a)*b**4 - 2*a**2*b**2 + 3*b**4*x)/(3
*a*b**4*(e**(6*x) - 3*e**(4*x) + 3*e**(2*x) - 1))
```

3.123 $\int \frac{\coth^6(x)}{a+b\text{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 183

$$\begin{aligned}\int \frac{\coth^6(x)}{a+b\text{csch}(x)}\,dx &= \frac{x}{a}-\frac{3\text{arctanh}(\cosh(x))}{8b}+\frac{(a^2+3b^2)\,\text{arctanh}(\cosh(x))}{2b^3}\\ &\quad-\frac{(a^4+3a^2b^2+3b^4)\,\text{arctanh}(\cosh(x))}{b^5}\\ &\quad+\frac{2(a^2+b^2)^{5/2}\,\text{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab^5}-\frac{a\coth(x)}{b^2}\\ &\quad+\frac{a(a^2+3b^2)\coth(x)}{b^4}+\frac{a\coth^3(x)}{3b^2}+\frac{3\coth(x)\text{csch}(x)}{8b}\\ &\quad-\frac{(a^2+3b^2)\coth(x)\text{csch}(x)}{2b^3}-\frac{\coth(x)\text{csch}^3(x)}{4b}\end{aligned}$$

output
```
x/a-3/8*arctanh(cosh(x))/b+1/2*(a^2+3*b^2)*arctanh(cosh(x))/b^3-(a^4+3*a^2
*b^2+3*b^4)*arctanh(cosh(x))/b^5+2*(a^2+b^2)^(5/2)*arctanh((a-b*tanh(1/2*x
))/(a^2+b^2)^(1/2))/a/b^5-a*coth(x)/b^2+a*(a^2+3*b^2)*coth(x)/b^4+1/3*a*co
th(x)^3/b^2+3/8*coth(x)*csch(x)/b-1/2*(a^2+3*b^2)*coth(x)*csch(x)/b^3-1/4*
coth(x)*csch(x)^3/b
```

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.63

$$\int \frac{\coth^6(x)}{a+b\operatorname{csch}(x)}\,dx$$
$$= \frac{\operatorname{csch}(x)(b+a\sinh(x))\left(192b^5x+384(-a^2-b^2)^{5/2}\arctan\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+32a^2b(3a^2+7b^2)\coth\left(\frac{x}{2}\right)-6\right.}{}$$

input
```
Integrate[Coth[x]^6/(a + b*Csch[x]),x]
```

output
```
(Csch[x]*(b + a*Sinh[x])*(192*b^5*x + 384*(-a^2 - b^2)^(5/2)*ArcTan[(a - b
*Tanh[x/2])/Sqrt[-a^2 - b^2]] + 32*a^2*b*(3*a^2 + 7*b^2)*Coth[x/2] - 6*a*b
^2*(4*a^2 + 9*b^2)*Csch[x/2]^2 - 3*a*b^4*Csch[x/2]^4 - 24*a*(8*a^4 + 20*a^
2*b^2 + 15*b^4)*Log[Cosh[x/2]] + 24*a*(8*a^4 + 20*a^2*b^2 + 15*b^4)*Log[Si
nh[x/2]] - 6*a*b^2*(4*a^2 + 9*b^2)*Sech[x/2]^2 + 3*a*b^4*Sech[x/2]^4 - 64*
a^2*b^3*Csch[x]^3*Sinh[x/2]^4 + 4*a^2*b^3*Csch[x/2]^4*Sinh[x] + 32*a^2*b*(
3*a^2 + 7*b^2)*Tanh[x/2]))/(192*a*b^5*(a + b*Csch[x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {3042, 25, 4386, 26, 26, 3042, 26, 3376, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^6(x)}{a+b\operatorname{csch}(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{\cot(ix)^6}{a+ib\csc(ix)}dx$$
$$\Big\downarrow \text{25}$$

$$-\int \frac{\cot(ix)^6}{a+ib\csc(ix)}dx$$

$\big\downarrow$ 4386

$$-\int -\frac{i\cosh(x)\coth^5(x)}{ib+ia\sinh(x)}dx$$

$\big\downarrow$ 26

$$i\int -\frac{i\cosh(x)\coth^5(x)}{b+a\sinh(x)}dx$$

$\big\downarrow$ 26

$$\int \frac{\cosh(x)\coth^5(x)}{a\sinh(x)+b}dx$$

$\big\downarrow$ 3042

$$\int \frac{i\cos(ix)^6}{\sin(ix)^5(b-ia\sin(ix))}dx$$

$\big\downarrow$ 26

$$i\int \frac{\cos(ix)^6}{\sin(ix)^5(b-ia\sin(ix))}dx$$

$\big\downarrow$ 3376

$$i\int \left(-\frac{i\text{csch}^5(x)}{b}+\frac{ia\text{csch}^4(x)}{b^2}+\frac{i(-a^2-3b^2)\,\text{csch}^3(x)}{b^3}+\frac{i(a^3+3b^2a)\,\text{csch}^2(x)}{b^4}-\frac{i(a^4+3b^2a^2+3b^4)\,\text{csch}(x)}{b^5}\right.$$

$\big\downarrow$ 2009

$$i\left(-\frac{2i(a^2+b^2)^{5/2}\operatorname{arctanh}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab^5}-\frac{i(a^2+3b^2)\operatorname{arctanh}(\cosh(x))}{2b^3}-\frac{ia(a^2+3b^2)\coth(x)}{b^4}+\frac{i(a^2+3b^2)}{}\right.$$

input
```
Int[Coth[x]^6/(a + b*Csch[x]),x]
```

output
```
I*(((-I)*x)/a + (((3*I)/8)*ArcTanh[Cosh[x]])/b - ((I/2)*(a^2 + 3*b^2)*ArcT
anh[Cosh[x]])/b^3 + (I*(a^4 + 3*a^2*b^2 + 3*b^4)*ArcTanh[Cosh[x]])/b^5 - (
(2*I)*(a^2 + b^2)^(5/2)*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a*b^5
) + (I*a*Coth[x])/b^2 - (I*a*(a^2 + 3*b^2)*Coth[x])/b^4 - ((I/3)*a*Coth[x]
^3)/b^2 - (((3*I)/8)*Coth[x]*Csch[x])/b + ((I/2)*(a^2 + 3*b^2)*Coth[x]*Csc
h[x])/b^3 + ((I/4)*Coth[x]*Csch[x]^3)/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3376
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

rule 4386
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.42

method	result
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^{4} b^{3}}{4}+\frac{2a\tanh\left(\frac{x}{2}\right)^{3} b^{2}}{3}+2a^{2}b\tanh\left(\frac{x}{2}\right)^{2}+4b^{3}\tanh\left(\frac{x}{2}\right)^{2}+8\tanh\left(\frac{x}{2}\right)a^{3}+18a\,b^{2}\tanh\left(\frac{x}{2}\right)}{16b^{4}}-\frac{1}{64b\tanh\left(\frac{x}{2}\right)^{4}}-\frac{4a^{2}+8b^{2}}{32b^{3}\tanh\left(\frac{x}{2}\right)^{2}}$
risch	$\frac{x}{a}+\frac{-12a^{2}b\,e^{7x}-27b^{3}e^{7x}+24a^{3}e^{6x}+72a\,b^{2}e^{6x}+12a^{2}b\,e^{5x}+3\,e^{5x}b^{3}-72a^{3}e^{4x}-168a\,b^{2}e^{4x}+12a^{2}b\,e^{3x}+3b^{3}e^{3x}+72a^{3}e^{2x}+152a\,b^{2}}{12b^{4}\left(e^{2x}-1\right)^{4}}$

input
```
int(coth(x)^6/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
1/16/b^4*(1/4*tanh(1/2*x)^4*b^3+2/3*a*tanh(1/2*x)^3*b^2+2*a^2*b*tanh(1/2*x
)^2+4*b^3*tanh(1/2*x)^2+8*tanh(1/2*x)*a^3+18*a*b^2*tanh(1/2*x))-1/64/b/tan
h(1/2*x)^4-1/32*(4*a^2+8*b^2)/b^3/tanh(1/2*x)^2+1/16/b^5*(16*a^4+40*a^2*b^
2+30*b^4)*ln(tanh(1/2*x))+1/24/b^2*a/tanh(1/2*x)^3+1/8*a*(4*a^2+9*b^2)/b^4
/tanh(1/2*x)+1/16*(32*a^6+96*a^4*b^2+96*a^2*b^4+32*b^6)/b^5/a/(a^2+b^2)^(1
/2)*arctanh(1/2*(-2*tanh(1/2*x)*b+2*a)/(a^2+b^2)^(1/2))+1/a*ln(tanh(1/2*x)
+1)-1/a*ln(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3160 vs. $2(167) = 334$.

Time = 0.27 (sec) , antiderivative size = 3160, normalized size of antiderivative = 17.27

$$\int \frac{\coth^6(x)}{a+b\text{csch}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^6/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth^6(x)}{a + b\text{csch}(x)}\,dx = \int \frac{\coth^6(x)}{a + b\,\text{csch}\,(x)}\,dx$$

input
```
integrate(coth(x)**6/(a+b*csch(x)),x)
```

output
```
Integral(coth(x)**6/(a + b*csch(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.80

$$\int \frac{\coth^6(x)}{a + b\text{csch}(x)}\,dx = -\frac{24\,a^3 + 56\,ab^2 - 3\,(4\,a^2b + 9\,b^3)e^{(-x)} - 8\,(9\,a^3 + 19\,ab^2)e^{(-2\,x)} + 3\,(4\,a^2b + b^3)e^{(-3\,x)} + 24\,(3\,a^3 + 7\,ab^2}{12\,(4\,b^4e^{(-2\,x)} - 6\,b^4e^{(-4\,x)} + 4\,b^4e^{(-6\,x)} -} + \frac{x}{a} - \frac{(8\,a^4 + 20\,a^2b^2 + 15\,b^4)\log\left(e^{(-x)} + 1\right)}{8\,b^5} + \frac{(8\,a^4 + 20\,a^2b^2 + 15\,b^4)\log\left(e^{(-x)} - 1\right)}{8\,b^5} - \frac{(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6)\log\left(\frac{ae^{(-x)} - b - \sqrt{a^2+b^2}}{ae^{(-x)} - b + \sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}ab^5}$$

input
```
integrate(coth(x)^6/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
-1/12*(24*a^3 + 56*a*b^2 - 3*(4*a^2*b + 9*b^3)*e^(-x) - 8*(9*a^3 + 19*a*b^
2)*e^(-2*x) + 3*(4*a^2*b + b^3)*e^(-3*x) + 24*(3*a^3 + 7*a*b^2)*e^(-4*x) +
 3*(4*a^2*b + b^3)*e^(-5*x) - 24*(a^3 + 3*a*b^2)*e^(-6*x) - 3*(4*a^2*b + 9
*b^3)*e^(-7*x))/(4*b^4*e^(-2*x) - 6*b^4*e^(-4*x) + 4*b^4*e^(-6*x) - b^4*e^
(-8*x) - b^4) + x/a - 1/8*(8*a^4 + 20*a^2*b^2 + 15*b^4)*log(e^(-x) + 1)/b^
5 + 1/8*(8*a^4 + 20*a^2*b^2 + 15*b^4)*log(e^(-x) - 1)/b^5 - (a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6)*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b +
 sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*b^5)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.67

$$\int \frac{\coth^6(x)}{a+b\text{csch}(x)}\,dx$$
$$= \frac{x}{a} - \frac{(8\,a^4+20\,a^2b^2+15\,b^4)\log(e^x+1)}{8\,b^5} + \frac{(8\,a^4+20\,a^2b^2+15\,b^4)\log(|e^x-1|)}{8\,b^5}$$
$$- \frac{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\log\left(\frac{|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}|}{|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}|}\right)}{\sqrt{a^2+b^2}ab^5}$$
$$- \frac{12\,a^2be^{(7\,x)}+27\,b^3e^{(7\,x)}-24\,a^3e^{(6\,x)}-72\,ab^2e^{(6\,x)}-12\,a^2be^{(5\,x)}-3\,b^3e^{(5\,x)}+72\,a^3e^{(4\,x)}+168\,ab^2e^{(4\,x)}}{12\,b^4(e^{(2\,x)}-1}$$

input
```
integrate(coth(x)^6/(a+b*csch(x)),x, algorithm="giac")
```

output
```
x/a - 1/8*(8*a^4 + 20*a^2*b^2 + 15*b^4)*log(e^x + 1)/b^5 + 1/8*(8*a^4 + 20
*a^2*b^2 + 15*b^4)*log(abs(e^x - 1))/b^5 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6)*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt
(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*b^5) - 1/12*(12*a^2*b*e^(7*x) + 27*b^3*e^
(7*x) - 24*a^3*e^(6*x) - 72*a*b^2*e^(6*x) - 12*a^2*b*e^(5*x) - 3*b^3*e^(5*
x) + 72*a^3*e^(4*x) + 168*a*b^2*e^(4*x) - 12*a^2*b*e^(3*x) - 3*b^3*e^(3*x)
 - 72*a^3*e^(2*x) - 152*a*b^2*e^(2*x) + 12*a^2*b*e^x + 27*b^3*e^x + 24*a^3
 + 56*a*b^2)/(b^4*(e^(2*x) - 1)^4)
```

Mupad [B] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 543, normalized size of antiderivative = 2.97

$$\int \frac{\coth^6(x)}{a+b\mathrm{csch}(x)}\,dx = \frac{\frac{8\,a}{3\,b^2}-\frac{6\,\mathrm{e}^x}{b}}{3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1}-\frac{\frac{\mathrm{e}^x\,\left(4\,a^2+9\,b^2\right)}{4\,b^3}-\frac{2\,\left(a^4+3\,a^2\,b^2\right)}{a\,b^4}}{\mathrm{e}^{2\,x}-1}$$
$$+\frac{\frac{4\,a}{b^2}-\frac{\mathrm{e}^x\,\left(4\,a^2+13\,b^2\right)}{2\,b^3}}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1}+\frac{x}{a}+\frac{\ln\left(\mathrm{e}^x-1\right)\,\left(8\,a^4+20\,a^2\,b^2+15\,b^4\right)}{8\,b^5}$$
$$-\frac{\ln\left(\mathrm{e}^x+1\right)\,\left(8\,a^4+20\,a^2\,b^2+15\,b^4\right)}{8\,b^5}-\frac{4\,\mathrm{e}^x}{b\,\left(6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}$$
$$+\frac{\ln\Bigg(a^3\,\sqrt{{\left(a^2+b^2\right)}^5}-2\,a^7\,b-2\,a\,b^7-6\,a^3\,b^5-6\,a^5\,b^3+a^8\,\mathrm{e}^x+4\,b^8\,\mathrm{e}^x+2\,a\,b^2\,\sqrt{{\left(a^2+b^2\right)}^5}-4\,b^3\,\mathrm{e}^x\,\sqrt{}}{a\,b^5}$$
$$-\frac{\ln\Bigg(a^8\,\mathrm{e}^x-2\,a^7\,b-a^3\,\sqrt{{\left(a^2+b^2\right)}^5}-6\,a^3\,b^5-6\,a^5\,b^3-2\,a\,b^7+4\,b^8\,\mathrm{e}^x-2\,a\,b^2\,\sqrt{{\left(a^2+b^2\right)}^5}+4\,b^3\,\mathrm{e}^x\,\sqrt{}}{a\,b^5}$$

input
```
int(coth(x)^6/(a + b/sinh(x)),x)
```

output
```
((8*a)/(3*b^2) - (6*exp(x))/b)/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) -
((exp(x)*(4*a^2 + 9*b^2))/(4*b^3) - (2*(a^4 + 3*a^2*b^2))/(a*b^4))/(exp(2*
x) - 1) + ((4*a)/b^2 - (exp(x)*(4*a^2 + 13*b^2))/(2*b^3))/(exp(4*x) - 2*ex
p(2*x) + 1) + x/a + (log(exp(x) - 1)*(8*a^4 + 15*b^4 + 20*a^2*b^2))/(8*b^5
) - (log(exp(x) + 1)*(8*a^4 + 15*b^4 + 20*a^2*b^2))/(8*b^5) - (4*exp(x))/(
b*(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1)) + (log(a^3*((a^2
+ b^2)^5)^(1/2) - 2*a^7*b - 2*a*b^7 - 6*a^3*b^5 - 6*a^5*b^3 + a^8*exp(x) +
 4*b^8*exp(x) + 2*a*b^2*((a^2 + b^2)^5)^(1/2) - 4*b^3*exp(x)*((a^2 + b^2)^
5)^(1/2) + 13*a^2*b^6*exp(x) + 15*a^4*b^4*exp(x) + 7*a^6*b^2*exp(x) - 3*a^
2*b*exp(x)*((a^2 + b^2)^5)^(1/2))*((a^2 + b^2)^5)^(1/2))/(a*b^5) - (log(a^
8*exp(x) - 2*a^7*b - a^3*((a^2 + b^2)^5)^(1/2) - 6*a^3*b^5 - 6*a^5*b^3 - 2
*a*b^7 + 4*b^8*exp(x) - 2*a*b^2*((a^2 + b^2)^5)^(1/2) + 4*b^3*exp(x)*((a^2
 + b^2)^5)^(1/2) + 13*a^2*b^6*exp(x) + 15*a^4*b^4*exp(x) + 7*a^6*b^2*exp(x
) + 3*a^2*b*exp(x)*((a^2 + b^2)^5)^(1/2))*((a^2 + b^2)^5)^(1/2))/(a*b^5)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1372, normalized size of antiderivative = 7.50

$$\int \frac{\coth^6(x)}{a + b\text{csch}(x)}\, dx = \text{Too large to display}$$

input
```
int(coth(x)^6/(a+b*csch(x)),x)
```

output
```
( - 48*e**(8*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2))
*a**4*i - 96*e**(8*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 +
b**2))*a**2*b**2*i - 48*e**(8*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/s
qrt(a**2 + b**2))*b**4*i + 192*e**(6*x)*sqrt(a**2 + b**2)*atan((e**x*a*i +
 b*i)/sqrt(a**2 + b**2))*a**4*i + 384*e**(6*x)*sqrt(a**2 + b**2)*atan((e**
x*a*i + b*i)/sqrt(a**2 + b**2))*a**2*b**2*i + 192*e**(6*x)*sqrt(a**2 + b**
2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2))*b**4*i - 288*e**(4*x)*sqrt(a**
2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2))*a**4*i - 576*e**(4*x)*s
qrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2))*a**2*b**2*i - 28
8*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2))*b**4
*i + 192*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2 + b**2
))*a**4*i + 384*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqrt(a**2
 + b**2))*a**2*b**2*i + 192*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*a*i + b*
i)/sqrt(a**2 + b**2))*b**4*i - 48*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/
sqrt(a**2 + b**2))*a**4*i - 96*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/sqr
t(a**2 + b**2))*a**2*b**2*i - 48*sqrt(a**2 + b**2)*atan((e**x*a*i + b*i)/s
qrt(a**2 + b**2))*b**4*i + 24*e**(8*x)*log(e**x - 1)*a**5 + 60*e**(8*x)*lo
g(e**x - 1)*a**3*b**2 + 45*e**(8*x)*log(e**x - 1)*a*b**4 - 24*e**(8*x)*log
(e**x + 1)*a**5 - 60*e**(8*x)*log(e**x + 1)*a**3*b**2 - 45*e**(8*x)*log(e*
*x + 1)*a*b**4 + 12*e**(8*x)*a**4*b + 36*e**(8*x)*a**2*b**3 + 24*e**(8*...
```

3.124 $\int \frac{\coth^7(x)}{a+b\,\mathrm{csch}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 119

$$\begin{aligned}\int \frac{\coth^7(x)}{a+b\mathrm{csch}(x)}\,dx = &-\frac{(a^4+3a^2b^2+3b^4)\,\mathrm{csch}(x)}{b^5}+\frac{a(a^2+3b^2)\,\mathrm{csch}^2(x)}{2b^4}\\ &-\frac{(a^2+3b^2)\,\mathrm{csch}^3(x)}{3b^3}+\frac{a\mathrm{csch}^4(x)}{4b^2}-\frac{\mathrm{csch}^5(x)}{5b}\\ &+\frac{(a^2+b^2)^3\log(a+b\mathrm{csch}(x))}{ab^6}+\frac{\log(\sinh(x))}{a}\end{aligned}$$

output
```
-(a^4+3*a^2*b^2+3*b^4)*csch(x)/b^5+1/2*a*(a^2+3*b^2)*csch(x)^2/b^4-1/3*(a^
2+3*b^2)*csch(x)^3/b^3+1/4*a*csch(x)^4/b^2-1/5*csch(x)^5/b+(a^2+b^2)^3*ln(
a+b*csch(x))/a/b^6+ln(sinh(x))/a
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.09

$$\int \frac{\coth^7(x)}{a+b\mathrm{csch}(x)}\,dx$$
$$=\frac{-60b(a^4+3a^2b^2+3b^4)\,\mathrm{csch}(x)+30ab^2(a^2+3b^2)\,\mathrm{csch}^2(x)-20b^3(a^2+3b^2)\,\mathrm{csch}^3(x)+15ab^4\mathrm{csch}^4(x)-}{60b^6}$$

input
```
Integrate[Coth[x]^7/(a + b*Csch[x]),x]
```

output
```
(-60*b*(a^4 + 3*a^2*b^2 + 3*b^4)*Csch[x] + 30*a*b^2*(a^2 + 3*b^2)*Csch[x]^
2 - 20*b^3*(a^2 + 3*b^2)*Csch[x]^3 + 15*a*b^4*Csch[x]^4 - 12*b^5*Csch[x]^5
 - 60*a*(a^4 + 3*a^2*b^2 + 3*b^4)*Log[Sinh[x]] + (60*(a^2 + b^2)^3*Log[b +
 a*Sinh[x]])/a)/(60*b^6)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 26, 4373, 25, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^7(x)}{a+b\text{csch}(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\cot(ix)^7}{a+ib\csc(ix)}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\cot(ix)^7}{a+ib\csc(ix)}dx$$

$$\Big\downarrow \text{ 4373}$$

$$\frac{\int -\frac{\left(\text{csch}^2(x)b^2+b^2\right)^3\sinh(x)}{b(a+b\text{csch}(x))}d(b\text{csch}(x))}{b^6}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{\left(\text{csch}^2(x)b^2+b^2\right)^3\sinh(x)}{b(a+b\text{csch}(x))}d(b\text{csch}(x))}{b^6}$$

$$\Big\downarrow \text{ 522}$$

$$-\frac{\int \left(\frac{\sinh(x)b^5}{a} + \operatorname{csch}^4(x)b^4 - a\operatorname{csch}^3(x)b^3 + (a^2+3b^2)\operatorname{csch}^2(x)b^2 - a(a^2+3b^2)\operatorname{csch}(x)b + a^4\left(\frac{3(a^2+b^2)b^2}{a^4}+1\right)\right.}{b^6}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}ab^2(a^2+3b^2)\operatorname{csch}^2(x) + \frac{(a^2+b^2)^3\log(a+b\operatorname{csch}(x))}{a} - \frac{1}{3}b^3(a^2+3b^2)\operatorname{csch}^3(x) - b(a^4+3a^2b^2+3b^4)\operatorname{csch}(x) - \frac{b^6\log}{}}{b^6}$$

input
```
Int[Coth[x]^7/(a + b*Csch[x]),x]
```

output
```
(-(b*(a^4 + 3*a^2*b^2 + 3*b^4)*Csch[x]) + (a*b^2*(a^2 + 3*b^2)*Csch[x]^2)/
2 - (b^3*(a^2 + 3*b^2)*Csch[x]^3)/3 + (a*b^4*Csch[x]^4)/4 - (b^5*Csch[x]^5
)/5 - (b^6*Log[b*Csch[x]])/a + ((a^2 + b^2)^3*Log[a + b*Csch[x]])/a)/b^6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4373
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Simp[-(-1)^((m - 1)/2)/(d*b^(m - 1))   Subst[Int[(b^2 - x^
2)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(111) = 222$.

Time = 3.71 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.64

method	result
default	$\frac{\frac{b^4 \tanh\left(\frac{x}{2}\right)^5}{5}+\frac{a \tanh\left(\frac{x}{2}\right)^4 b^3}{2}+\frac{4a^2b^2 \tanh\left(\frac{x}{2}\right)^3}{3}+3\tanh\left(\frac{x}{2}\right)^3 b^4+4a^3b\tanh\left(\frac{x}{2}\right)^2+10b^3\tanh\left(\frac{x}{2}\right)^2 a+16a^4\tanh\left(\frac{x}{2}\right)+44\tanh\left(\frac{x}{2}\right)a^2b^2+}{32b^5}$
risch	$-\frac{x}{a}-\frac{2\,\mathrm{e}^{x}\left(15a^4\mathrm{e}^{8x}+45a^2b^2\mathrm{e}^{8x}+45b^4\mathrm{e}^{8x}-15a^3b\,\mathrm{e}^{7x}-45a\,b^3\mathrm{e}^{7x}-60a^4\mathrm{e}^{6x}-160a^2b^2\mathrm{e}^{6x}-120b^4\mathrm{e}^{6x}+45a^3b\,\mathrm{e}^{5x}+105a\,b^3\mathrm{e}^{5x}+90\,\mathrm{e}\right.}{15}$

input
```
int(coth(x)^7/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

output
```
1/32/b^5*(1/5*b^4*tanh(1/2*x)^5+1/2*a*tanh(1/2*x)^4*b^3+4/3*a^2*b^2*tanh(1
/2*x)^3+3*tanh(1/2*x)^3*b^4+4*a^3*b*tanh(1/2*x)^2+10*b^3*tanh(1/2*x)^2*a+1
6*a^4*tanh(1/2*x)+44*tanh(1/2*x)*a^2*b^2+38*tanh(1/2*x)*b^4)-1/a*ln(tanh(1
/2*x)-1)-1/a*ln(tanh(1/2*x)+1)+1/32/a/b^6*(32*a^6+96*a^4*b^2+96*a^2*b^4+32
*b^6)*ln(-b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b)-1/160/b/tanh(1/2*x)^5-1/96/b^
3*(4*a^2+9*b^2)/tanh(1/2*x)^3-1/32*(16*a^4+44*a^2*b^2+38*b^4)/b^5/tanh(1/2
*x)+1/64/b^2*a/tanh(1/2*x)^4+1/16*a/b^4*(2*a^2+5*b^2)/tanh(1/2*x)^2-1/b^6*
a*(a^4+3*a^2*b^2+3*b^4)*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4024 vs. $2(111) = 222$.

Time = 0.15 (sec) , antiderivative size = 4024, normalized size of antiderivative = 33.82

$$\int \frac{\coth^7(x)}{a+b\operatorname{csch}(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^7/(a+b*csch(x)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth^7(x)}{a + b\text{csch}(x)}\,dx = \int \frac{\coth^7(x)}{a + b\,\text{csch}(x)}\,dx$$

input
```
integrate(coth(x)**7/(a+b*csch(x)),x)
```

output
```
Integral(coth(x)**7/(a + b*csch(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(111) = 222$.

Time = 0.05 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.06

$$\int \frac{\coth^7(x)}{a + b\text{csch}(x)}\,dx = \frac{2\left(15\left(a^4 + 3a^2b^2 + 3b^4\right)e^{(-x)} - 15\left(a^3b + 3ab^3\right)e^{(-2x)} - 20\left(3a^4 + 8a^2b^2 + 6b^4\right)e^{(-3x)} + 15\left(3a^3b + 7a\right.\right.}{15\left(5b^5\right.} + \frac{x}{a} - \frac{\left(a^5 + 3a^3b^2 + 3ab^4\right)\log\left(e^{(-x)} + 1\right)}{b^6} - \frac{\left(a^5 + 3a^3b^2 + 3ab^4\right)\log\left(e^{(-x)} - 1\right)}{b^6} + \frac{\left(a^6 + 3a^4b^2 + 3a^2b^4 + b^6\right)\log\left(-2be^{(-x)} + ae^{(-2x)} - a\right)}{ab^6}$$

input
```
integrate(coth(x)^7/(a+b*csch(x)),x, algorithm="maxima")
```

output
```
2/15*(15*(a^4 + 3*a^2*b^2 + 3*b^4)*e^(-x) - 15*(a^3*b + 3*a*b^3)*e^(-2*x)
- 20*(3*a^4 + 8*a^2*b^2 + 6*b^4)*e^(-3*x) + 15*(3*a^3*b + 7*a*b^3)*e^(-4*x
) + 2*(45*a^4 + 115*a^2*b^2 + 99*b^4)*e^(-5*x) - 15*(3*a^3*b + 7*a*b^3)*e^
(-6*x) - 20*(3*a^4 + 8*a^2*b^2 + 6*b^4)*e^(-7*x) + 15*(a^3*b + 3*a*b^3)*e^
(-8*x) + 15*(a^4 + 3*a^2*b^2 + 3*b^4)*e^(-9*x))/(5*b^5*e^(-2*x) - 10*b^5*e
^(-4*x) + 10*b^5*e^(-6*x) - 5*b^5*e^(-8*x) + b^5*e^(-10*x) - b^5) + x/a -
(a^5 + 3*a^3*b^2 + 3*a*b^4)*log(e^(-x) + 1)/b^6 - (a^5 + 3*a^3*b^2 + 3*a*b
^4)*log(e^(-x) - 1)/b^6 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*log(-2*b*e^(
-x) + a*e^(-2*x) - a)/(a*b^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(111) = 222$.

Time = 0.13 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.48

$$\int \frac{\coth^7(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{(a^5+3\,a^3b^2+3\,ab^4)\log\left(\left|-e^{(-x)}+e^x\right|\right)}{b^6} + \frac{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\log\left(\left|-a\left(e^{(-x)}-e^x\right)+2\,b\right|\right)}{ab^6} + \frac{137\,a^5\left(e^{(-x)}-e^x\right)^5+411\,a^3b^2\left(e^{(-x)}-e^x\right)^5+411\,ab^4\left(e^{(-x)}-e^x\right)^5+120\,a^4b\left(e^{(-x)}-e^x\right)^4+360\,a^2b^3\left(\right.}{}$$

input
```
integrate(coth(x)^7/(a+b*csch(x)),x, algorithm="giac")
```

output
```
-(a^5 + 3*a^3*b^2 + 3*a*b^4)*log(abs(-e^(-x) + e^x))/b^6 + (a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a*b^6) + 1/60*(137
*a^5*(e^(-x) - e^x)^5 + 411*a^3*b^2*(e^(-x) - e^x)^5 + 411*a*b^4*(e^(-x) -
 e^x)^5 + 120*a^4*b*(e^(-x) - e^x)^4 + 360*a^2*b^3*(e^(-x) - e^x)^4 + 360*
b^5*(e^(-x) - e^x)^4 + 120*a^3*b^2*(e^(-x) - e^x)^3 + 360*a*b^4*(e^(-x) -
e^x)^3 + 160*a^2*b^3*(e^(-x) - e^x)^2 + 480*b^5*(e^(-x) - e^x)^2 + 240*a*b
^4*(e^(-x) - e^x) + 384*b^5)/(b^6*(e^(-x) - e^x)^5)
```

Mupad [B] (verification not implemented)

Time = 3.44 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.66

$$\int \frac{\coth^7(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\frac{4a}{b^2}-\frac{64\,e^x}{5b}}{6\,e^{4x}-4\,e^{2x}-4\,e^{6x}+e^{8x}+1}+\frac{\frac{8a}{b^2}-\frac{8\,e^x\,(5a^2+27b^2)}{15b^3}}{3\,e^{2x}-3\,e^{4x}+e^{6x}-1} - \frac{\frac{8\,e^x\,(a^2+3b^2)}{3b^3}-\frac{2\,(a^4+5a^2b^2)}{a\,b^4}}{e^{4x}-2\,e^{2x}+1}-\frac{x}{a}-\frac{\frac{2\,e^x\,(a^4+3a^2b^2+3b^4)}{b^5}-\frac{2\,(a^4+3a^2b^2)}{a\,b^4}}{e^{2x}-1} - \frac{32\,e^x}{5b\,(5\,e^{2x}-10\,e^{4x}+10\,e^{6x}-5\,e^{8x}+e^{10x}-1)} - \frac{\ln(e^{2x}-1)\,(a^5+3a^3b^2+3a\,b^4)}{b^6} + \frac{\ln(2b\,e^x-a+a\,e^{2x})\,(a^6+3a^4b^2+3a^2b^4+b^6)}{a\,b^6}$$

input
```
int(coth(x)^7/(a + b/sinh(x)),x)
```

output
```
((4*a)/b^2 - (64*exp(x))/(5*b))/(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + ex
p(8*x) + 1) + ((8*a)/b^2 - (8*exp(x)*(5*a^2 + 27*b^2))/(15*b^3))/(3*exp(2*
x) - 3*exp(4*x) + exp(6*x) - 1) - ((8*exp(x)*(a^2 + 3*b^2))/(3*b^3) - (2*(
a^4 + 5*a^2*b^2))/(a*b^4))/(exp(4*x) - 2*exp(2*x) + 1) - x/a - ((2*exp(x)*
(a^4 + 3*b^4 + 3*a^2*b^2))/b^5 - (2*(a^4 + 3*a^2*b^2))/(a*b^4))/(exp(2*x)
- 1) - (32*exp(x))/(5*b*(5*exp(2*x) - 10*exp(4*x) + 10*exp(6*x) - 5*exp(8*
x) + exp(10*x) - 1)) - (log(exp(2*x) - 1)*(3*a*b^4 + a^5 + 3*a^3*b^2))/b^6
 + (log(2*b*exp(x) - a + a*exp(2*x))*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))/
(a*b^6)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1676, normalized size of antiderivative = 14.08

$$\int \frac{\coth^7(x)}{a+b\operatorname{csch}(x)}\,dx = \text{Too large to display}$$

input
```
int(coth(x)^7/(a+b*csch(x)),x)
```

output
```
( - 15*e**(10*x)*log(e**x - 1)*a**6 - 45*e**(10*x)*log(e**x - 1)*a**4*b**2
 - 45*e**(10*x)*log(e**x - 1)*a**2*b**4 - 15*e**(10*x)*log(e**x + 1)*a**6
- 45*e**(10*x)*log(e**x + 1)*a**4*b**2 - 45*e**(10*x)*log(e**x + 1)*a**2*b
**4 + 15*e**(10*x)*log(e**(2*x)*a + 2*e**x*b - a)*a**6 + 45*e**(10*x)*log(
e**(2*x)*a + 2*e**x*b - a)*a**4*b**2 + 45*e**(10*x)*log(e**(2*x)*a + 2*e**
x*b - a)*a**2*b**4 + 15*e**(10*x)*log(e**(2*x)*a + 2*e**x*b - a)*b**6 + 6*
e**(10*x)*a**4*b**2 + 18*e**(10*x)*a**2*b**4 - 15*e**(10*x)*b**6*x - 30*e*
*(9*x)*a**5*b - 90*e**(9*x)*a**3*b**3 - 90*e**(9*x)*a*b**5 + 75*e**(8*x)*l
og(e**x - 1)*a**6 + 225*e**(8*x)*log(e**x - 1)*a**4*b**2 + 225*e**(8*x)*lo
g(e**x - 1)*a**2*b**4 + 75*e**(8*x)*log(e**x + 1)*a**6 + 225*e**(8*x)*log(
e**x + 1)*a**4*b**2 + 225*e**(8*x)*log(e**x + 1)*a**2*b**4 - 75*e**(8*x)*l
og(e**(2*x)*a + 2*e**x*b - a)*a**6 - 225*e**(8*x)*log(e**(2*x)*a + 2*e**x*
b - a)*a**4*b**2 - 225*e**(8*x)*log(e**(2*x)*a + 2*e**x*b - a)*a**2*b**4 -
 75*e**(8*x)*log(e**(2*x)*a + 2*e**x*b - a)*b**6 + 75*e**(8*x)*b**6*x + 12
0*e**(7*x)*a**5*b + 320*e**(7*x)*a**3*b**3 + 240*e**(7*x)*a*b**5 - 150*e**
(6*x)*log(e**x - 1)*a**6 - 450*e**(6*x)*log(e**x - 1)*a**4*b**2 - 450*e**(
6*x)*log(e**x - 1)*a**2*b**4 - 150*e**(6*x)*log(e**x + 1)*a**6 - 450*e**(6
*x)*log(e**x + 1)*a**4*b**2 - 450*e**(6*x)*log(e**x + 1)*a**2*b**4 + 150*e
**(6*x)*log(e**(2*x)*a + 2*e**x*b - a)*a**6 + 450*e**(6*x)*log(e**(2*x)*a
+ 2*e**x*b - a)*a**4*b**2 + 450*e**(6*x)*log(e**(2*x)*a + 2*e**x*b - a)...
```

3.125 $\int e^{c(a+bx)}\mathbf{csch}^2(ac+bcx)^{7/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 199

$$\int e^{c(a+bx)}\text{csch}^2(ac+bcx)^{7/2}\,dx = -\frac{32\sqrt{\text{csch}^2(ac+bcx)}\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^6} + \frac{192\sqrt{\text{csch}^2(ac+bcx)}\sinh(ac+bcx)}{5bc\left(1-e^{2c(a+bx)}\right)^5} - \frac{48\sqrt{\text{csch}^2(ac+bcx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4} + \frac{64\sqrt{\text{csch}^2(ac+bcx)}\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3}$$

output
```
-32/3*(csch(b*c*x+a*c)^2)^(1/2)*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^6
+192/5*(csch(b*c*x+a*c)^2)^(1/2)*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^
5-48*(csch(b*c*x+a*c)^2)^(1/2)*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^4+
64/3*(csch(b*c*x+a*c)^2)^(1/2)*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.42

$$\int e^{c(a+bx)}\text{csch}^2(ac+bcx)^{7/2}\,dx = -\frac{16\left(-1+6e^{2c(a+bx)}-15e^{4c(a+bx)}+20e^{6c(a+bx)}\right)\sqrt{\text{csch}^2(c(a+bx))}\sinh(c(a+bx))}{15bc\left(-1+e^{2c(a+bx)}\right)^6}$$

input
```
Integrate[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(7/2),x]
```

output
```
(-16*(-1 + 6*E^(2*c*(a + b*x)) - 15*E^(4*c*(a + b*x)) + 20*E^(6*c*(a + b*x
)))*Sqrt[Csch[c*(a + b*x)]^2]*Sinh[c*(a + b*x)])/(15*b*c*(-1 + E^(2*c*(a +
 b*x)))^6)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.57, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {7271, 2720, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\text{csch}^2(ac+bcx)^{7/2}\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$\sinh(ac+bcx)\sqrt{\text{csch}^2(ac+bcx)}\int e^{c(a+bx)}\text{csch}^7(ac+bxc)dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\sinh(ac+bcx)\sqrt{\text{csch}^2(ac+bcx)}\int -\frac{128e^{7c(a+bx)}}{\left(1-e^{2c(a+bx)}\right)^7}de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{128\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}\int \frac{e^{7c(a+bx)}}{\left(1-e^{2c(a+bx)}\right)^7}de^{c(a+bx)}}{bc}$$

$\big\downarrow$ 243

$$-\frac{64\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}\int \frac{e^{3c(a+bx)}}{\left(1-e^{2c(a+bx)}\right)^7}de^{2c(a+bx)}}{bc}$$

$\big\downarrow$ 53

$$-\frac{64\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}\int \left(-\frac{1}{\left(-1+e^{2c(a+bx)}\right)^4}-\frac{3}{\left(-1+e^{2c(a+bx)}\right)^5}-\frac{3}{\left(-1+e^{2c(a+bx)}\right)^6}-\frac{1}{\left(-1+e^{2c(a+bx)}\right)^7}\right)d}{bc}$$

$\big\downarrow$ 2009

$$-\frac{64\left(-\frac{1}{3\left(1-e^{2c(a+bx)}\right)^3}+\frac{3}{4\left(1-e^{2c(a+bx)}\right)^4}-\frac{3}{5\left(1-e^{2c(a+bx)}\right)^5}+\frac{1}{6\left(1-e^{2c(a+bx)}\right)^6}\right)\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc}$$

input
```
Int[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(7/2),x]
```

output
```
(-64*(1/(6*(1 - E^(2*c*(a + b*x)))^6) - 3/(5*(1 - E^(2*c*(a + b*x)))^5) +
3/(4*(1 - E^(2*c*(a + b*x)))^4) - 1/(3*(1 - E^(2*c*(a + b*x)))^3))*Sqrt[Cs
ch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(b*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.44

method	result	si
default	$-\frac{\operatorname{csgn}\left(\operatorname{csch}\left(c\left(bx+a\right)\right)\right)\left(\frac{\coth\left(c\left(bx+a\right)\right)^{6}}{6}+\frac{\coth\left(c\left(bx+a\right)\right)^{5}}{5}-\frac{\coth\left(c\left(bx+a\right)\right)^{4}}{2}-\frac{2\coth\left(c\left(bx+a\right)\right)^{3}}{3}+\frac{\coth\left(c\left(bx+a\right)\right)^{2}}{2}+\coth\left(c\left(bx+a\right)\right)\right)}{bc}$	8
risch	$-\frac{16\left(20\,{\mathrm e}^{6c\left(bx+a\right)}-15\,{\mathrm e}^{4c\left(bx+a\right)}+6\,{\mathrm e}^{2c\left(bx+a\right)}-1\right)\sqrt{\frac{{\mathrm e}^{2c\left(bx+a\right)}}{\left({\mathrm e}^{2c\left(bx+a\right)}-1\right)^{2}}}\,{\mathrm e}^{-c\left(bx+a\right)}}{15bc\left({\mathrm e}^{2c\left(bx+a\right)}-1\right)^{5}}$	9

input
```
int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-csgn(csch(c*(b*x+a)))/b/c*(1/6*coth(c*(b*x+a))^6+1/5*coth(c*(b*x+a))^5-1/
2*coth(c*(b*x+a))^4-2/3*coth(c*(b*x+a))^3+1/2*coth(c*(b*x+a))^2+coth(c*(b*
x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. 2(173) = 346.

Time = 0.08 (sec) , antiderivative size = 592, normalized size of antiderivative = 2.97

$$\int e^{c(a+bx)}\text{csch}^2(ac+bcx)^{7/2}\,dx =$$

$$-\frac{}{15\left(bc\cosh\left(bcx+ac\right)^9+9\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^8+bc\sinh\left(bcx+ac\right)^9-6\,bc\cosh\left(bcx+ac\right)\right.}$$

input
```
integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(7/2),x, algorithm="fricas")
```

output
```
-16/15*(19*cosh(b*c*x + a*c)^3 + 57*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2
+ 21*sinh(b*c*x + a*c)^3 + 21*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c
) - 9*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^9 + 9*b*c*cosh(b*c*x + a*c
)*sinh(b*c*x + a*c)^8 + b*c*sinh(b*c*x + a*c)^9 - 6*b*c*cosh(b*c*x + a*c)^
7 + 6*(6*b*c*cosh(b*c*x + a*c)^2 - b*c)*sinh(b*c*x + a*c)^7 + 15*b*c*cosh(
b*c*x + a*c)^5 + 42*(2*b*c*cosh(b*c*x + a*c)^3 - b*c*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c)^6 + 3*(42*b*c*cosh(b*c*x + a*c)^4 - 42*b*c*cosh(b*c*x + a*
c)^2 + 5*b*c)*sinh(b*c*x + a*c)^5 - 19*b*c*cosh(b*c*x + a*c)^3 + 3*(42*b*c
*cosh(b*c*x + a*c)^5 - 70*b*c*cosh(b*c*x + a*c)^3 + 25*b*c*cosh(b*c*x + a*
c))*sinh(b*c*x + a*c)^4 + 3*(28*b*c*cosh(b*c*x + a*c)^6 - 70*b*c*cosh(b*c*
x + a*c)^4 + 50*b*c*cosh(b*c*x + a*c)^2 - 7*b*c)*sinh(b*c*x + a*c)^3 + 9*b
*c*cosh(b*c*x + a*c) + 3*(12*b*c*cosh(b*c*x + a*c)^7 - 42*b*c*cosh(b*c*x +
 a*c)^5 + 50*b*c*cosh(b*c*x + a*c)^3 - 19*b*c*cosh(b*c*x + a*c))*sinh(b*c*
x + a*c)^2 + 3*(3*b*c*cosh(b*c*x + a*c)^8 - 14*b*c*cosh(b*c*x + a*c)^6 + 2
5*b*c*cosh(b*c*x + a*c)^4 - 21*b*c*cosh(b*c*x + a*c)^2 + 7*b*c)*sinh(b*c*x
 + a*c))
```

Sympy [F]

$$\int e^{c(a+bx)}\text{csch}^2(ac+bcx)^{7/2}\,dx = e^{ac}\int\left(\text{csch}^2\left(ac+bcx\right)\right)^{\frac{7}{2}}e^{bcx}\,dx$$

input
```
integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)**2)**(7/2),x)
```

output
```
exp(a*c)*Integral((csch(a*c + b*c*x)**2)**(7/2)*exp(b*c*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(173) = 346.

Time = 0.14 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.94

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{7/2}\,dx =$$

$$-\frac{64\,e^{(6\,bcx+6\,ac)}}{3\,bc(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1)}$$
$$+\frac{16\,e^{(4\,bcx+4\,ac)}}{bc(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1)}$$
$$-\frac{32\,e^{(2\,bcx+2\,ac)}}{5\,bc(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1)}$$
$$+\frac{16}{15\,bc(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1)}$$

input
```
integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(7/2),x, algorithm="maxima")
```

output
```
-64/3*e^(6*b*c*x + 6*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10*b*c*x + 10
*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x +
4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)) + 16*e^(4*b*c*x + 4*a*c)/(b*c*(e^(12*
b*c*x + 12*a*c) - 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^
(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)) -
 32/5*e^(2*b*c*x + 2*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10*b*c*x + 10
*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x +
4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)) + 16/15/(b*c*(e^(12*b*c*x + 12*a*c) -
 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c)
 + 15*e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.45

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{7/2}\,dx = -\frac{16\left(20\,e^{(6\,bcx+6\,ac)}-15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}-1\right)}{15\,bc(e^{(2\,bcx+2\,ac)}-1)^6\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}$$

input
```
integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(7/2),x, algorithm="giac")
```

output
```
-16/15*(20*e^(6*b*c*x + 6*a*c) - 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2
*a*c) - 1)/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^6*sgn(e^(b*c*x + a*c) - e^(-b*c*
x - a*c)))
```

Mupad [B] (verification not implemented)

Time = 2.95 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.08

$$\begin{aligned}\int e^{c(a+bx)}\operatorname{csch}^2(ac&\\ +bcx)^{7/2}\,dx &= \frac{32\sqrt{\frac{1}{{\left(\frac{{\mathrm{e}}^{a\,c+b\,c\,x}}{2}-\frac{{\mathrm{e}}^{-a\,c-b\,c\,x}}{2}\right)}^2}}\,\left({\mathrm{e}}^{4\,a\,c+4\,b\,c\,x}-2\,{\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}+1\right)}{3\,b\,c\,\left({\mathrm{e}}^{a\,c+b\,c\,x}-{\mathrm{e}}^{3\,a\,c+3\,b\,c\,x}\right)\,{\left({\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}-1\right)}^3}\\ &+\frac{24\sqrt{\frac{1}{{\left(\frac{{\mathrm{e}}^{a\,c+b\,c\,x}}{2}-\frac{{\mathrm{e}}^{-a\,c-b\,c\,x}}{2}\right)}^2}}\,\left({\mathrm{e}}^{4\,a\,c+4\,b\,c\,x}-2\,{\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}+1\right)}{b\,c\,\left({\mathrm{e}}^{a\,c+b\,c\,x}-{\mathrm{e}}^{3\,a\,c+3\,b\,c\,x}\right)\,{\left({\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}-1\right)}^4}\\ &+\frac{96\sqrt{\frac{1}{{\left(\frac{{\mathrm{e}}^{a\,c+b\,c\,x}}{2}-\frac{{\mathrm{e}}^{-a\,c-b\,c\,x}}{2}\right)}^2}}\,\left({\mathrm{e}}^{4\,a\,c+4\,b\,c\,x}-2\,{\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}+1\right)}{5\,b\,c\,\left({\mathrm{e}}^{a\,c+b\,c\,x}-{\mathrm{e}}^{3\,a\,c+3\,b\,c\,x}\right)\,{\left({\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}-1\right)}^5}\\ &+\frac{16\sqrt{\frac{1}{{\left(\frac{{\mathrm{e}}^{a\,c+b\,c\,x}}{2}-\frac{{\mathrm{e}}^{-a\,c-b\,c\,x}}{2}\right)}^2}}\,\left({\mathrm{e}}^{4\,a\,c+4\,b\,c\,x}-2\,{\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}+1\right)}{3\,b\,c\,\left({\mathrm{e}}^{a\,c+b\,c\,x}-{\mathrm{e}}^{3\,a\,c+3\,b\,c\,x}\right)\,{\left({\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}-1\right)}^6}\end{aligned}$$

input
```
int(exp(c*(a + b*x))*(1/sinh(a*c + b*c*x)^2)^(7/2),x)
```

output
```
(32*(1/(exp(a*c + b*c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(1/2)*(exp(4*a*c + 4
*b*c*x) - 2*exp(2*a*c + 2*b*c*x) + 1))/(3*b*c*(exp(a*c + b*c*x) - exp(3*a*
c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x) - 1)^3) + (24*(1/(exp(a*c + b*c*x)/2 -
 exp(- a*c - b*c*x)/2)^2)^(1/2)*(exp(4*a*c + 4*b*c*x) - 2*exp(2*a*c + 2*b*
c*x) + 1))/(b*c*(exp(a*c + b*c*x) - exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b
*c*x) - 1)^4) + (96*(1/(exp(a*c + b*c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(1/2
)*(exp(4*a*c + 4*b*c*x) - 2*exp(2*a*c + 2*b*c*x) + 1))/(5*b*c*(exp(a*c + b
*c*x) - exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x) - 1)^5) + (16*(1/(exp(
a*c + b*c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(1/2)*(exp(4*a*c + 4*b*c*x) - 2*
exp(2*a*c + 2*b*c*x) + 1))/(3*b*c*(exp(a*c + b*c*x) - exp(3*a*c + 3*b*c*x)
)*(exp(2*a*c + 2*b*c*x) - 1)^6)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.69

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{7/2}\,dx=\frac{-\frac{64e^{6bcx+6ac}}{3}+16e^{4bcx+4ac}-\frac{32e^{2bcx+2ac}}{5}+\frac{16}{15}}{bc\left(e^{12bcx+12ac}-6e^{10bcx+10ac}+15e^{8bcx+8ac}-20e^{6bcx+6ac}+15e^{4bcx+4ac}-6e^{2bcx+2ac}+1\right)}$$

input
```
int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(7/2),x)
```

output
```
(16*( - 20*e**(6*a*c + 6*b*c*x) + 15*e**(4*a*c + 4*b*c*x) - 6*e**(2*a*c +
2*b*c*x) + 1))/(15*b*c*(e**(12*a*c + 12*b*c*x) - 6*e**(10*a*c + 10*b*c*x)
+ 15*e**(8*a*c + 8*b*c*x) - 20*e**(6*a*c + 6*b*c*x) + 15*e**(4*a*c + 4*b*c
*x) - 6*e**(2*a*c + 2*b*c*x) + 1))
```

3.126 $\int e^{c(a+bx)}\mathbf{csch}^2(ac+bcx)^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 147

$$\int e^{c(a+bx)}\text{csch}^2(ac+bcx)^{5/2}\,dx=-\frac{4\sqrt{\text{csch}^2(ac+bcx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4}+\frac{32\sqrt{\text{csch}^2(ac+bcx)}\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3}-\frac{8\sqrt{\text{csch}^2(ac+bcx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

output
```
-4*(csch(b*c*x+a*c)^2)^(1/2)*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^4+32
/3*(csch(b*c*x+a*c)^2)^(1/2)*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^3-8*
(csch(b*c*x+a*c)^2)^(1/2)*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.49

$$\int e^{c(a+bx)}\text{csch}^2(ac+bcx)^{5/2}\,dx=-\frac{4\left(1-4e^{2c(a+bx)}+6e^{4c(a+bx)}\right)\sqrt{\text{csch}^2(c(a+bx))}\sinh(c(a+bx))}{3bc\left(-1+e^{2c(a+bx)}\right)^4}$$

input
```
Integrate[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(5/2),x]
```

output
```
(-4*(1 - 4*E^(2*c*(a + b*x)) + 6*E^(4*c*(a + b*x)))*Sqrt[Csch[c*(a + b*x)]
^2]*Sinh[c*(a + b*x)])/(3*b*c*(-1 + E^(2*c*(a + b*x)))^4)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.63, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {7271, 2720, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{5/2}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}\int e^{c(a+bx)}\operatorname{csch}^5(ac+bxc)dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}\int -\frac{32e^{5c(a+bx)}}{\left(1-e^{2c(a+bx)}\right)^5}de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{27}$$

$$-\frac{32\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}\int \frac{e^{5c(a+bx)}}{\left(1-e^{2c(a+bx)}\right)^5}de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{243}$$

$$-\frac{16\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}\int \frac{e^{2c(a+bx)}}{\left(1-e^{2c(a+bx)}\right)^5}de^{2c(a+bx)}}{bc}$$

$$\Big\downarrow \text{53}$$

$$-\frac{16\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}\int \left(-\frac{1}{\left(-1+e^{2c(a+bx)}\right)^3}-\frac{2}{\left(-1+e^{2c(a+bx)}\right)^4}-\frac{1}{\left(-1+e^{2c(a+bx)}\right)^5}\right)de^{2c(a+bx)}}{bc}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{16\left(\frac{1}{2(1-e^{2c(a+bx)})^2}-\frac{2}{3(1-e^{2c(a+bx)})^3}+\frac{1}{4(1-e^{2c(a+bx)})^4}\right)\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc}$$

input
```
Int[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(5/2),x]
```

output
```
(-16*(1/(4*(1 - E^(2*c*(a + b*x)))^4) - 2/(3*(1 - E^(2*c*(a + b*x)))^3) +
1/(2*(1 - E^(2*c*(a + b*x)))^2))*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*
x])/(b*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.44

method	result	size
default	$-\frac{\operatorname{csgn}(\operatorname{csch}(c(bx+a)))\left(\frac{\coth(c(bx+a))^{4}}{4}+\frac{\coth(c(bx+a))^{3}}{3}-\frac{\coth(c(bx+a))^{2}}{2}-\coth(c(bx+a))\right)}{bc}$	65
risch	$-\frac{4\left(6\,{\mathrm e}^{4c(bx+a)}-4\,{\mathrm e}^{2c(bx+a)}+1\right)\sqrt{\frac{{\mathrm e}^{2c(bx+a)}}{\left({\mathrm e}^{2c(bx+a)}-1\right)^{2}}}\,{\mathrm e}^{-c(bx+a)}}{3bc\left({\mathrm e}^{2c(bx+a)}-1\right)^{3}}$	80

input
```
int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-csgn(csch(c*(b*x+a)))/b/c*(1/4*coth(c*(b*x+a))^4+1/3*coth(c*(b*x+a))^3-1/
2*coth(c*(b*x+a))^2-coth(c*(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(130) = 260.

Time = 0.10 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.14

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{5/2}\,dx =$$

$$-\frac{}{3\left(bc\cosh\left(bcx+ac\right)^{6}+6\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{5}+bc\sinh\left(bcx+ac\right)^{6}-4\,bc\cosh\left(bcx+ac\right)^{4}\right.}$$

input
```
integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

output
```
-4/3*(7*cosh(b*c*x + a*c)^2 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 7*s
inh(b*c*x + a*c)^2 - 4)/(b*c*cosh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)
*sinh(b*c*x + a*c)^5 + b*c*sinh(b*c*x + a*c)^6 - 4*b*c*cosh(b*c*x + a*c)^4
 + (15*b*c*cosh(b*c*x + a*c)^2 - 4*b*c)*sinh(b*c*x + a*c)^4 + 7*b*c*cosh(b
*c*x + a*c)^2 + 4*(5*b*c*cosh(b*c*x + a*c)^3 - 4*b*c*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c)^3 + (15*b*c*cosh(b*c*x + a*c)^4 - 24*b*c*cosh(b*c*x + a*c)
^2 + 7*b*c)*sinh(b*c*x + a*c)^2 - 4*b*c + 2*(3*b*c*cosh(b*c*x + a*c)^5 - 8
*b*c*cosh(b*c*x + a*c)^3 + 5*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F]

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{5/2}\,dx = e^{ac}\int\left(\operatorname{csch}^2\left(ac+bcx\right)\right)^{\frac{5}{2}}e^{bcx}\,dx$$

input
```
integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)**2)**(5/2),x)
```

output
```
exp(a*c)*Integral((csch(a*c + b*c*x)**2)**(5/2)*exp(b*c*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.42

$$\begin{aligned}\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{5/2}\,dx =& -\frac{8\,e^{(4\,bcx+4\,ac)}}{bc(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1)}\\&+\frac{16\,e^{(2\,bcx+2\,ac)}}{3\,bc(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1)}\\&-\frac{4}{3\,bc(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1)}\end{aligned}$$

input
```
integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

output
```
-8*e^(4*b*c*x + 4*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b*c*x + 6*a*c) +
 6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1)) + 16/3*e^(2*b*c*x + 2
*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4
*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1)) - 4/3/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^
(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.52

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{5/2}\,dx = -\frac{4\left(6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1\right)}{3\,bc(e^{(2\,bcx+2\,ac)}-1)^4\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}$$

input
```
integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

output
```
-4/3*(6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x
+ 2*a*c) - 1)^4*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)))
```

Mupad [B] (verification not implemented)

Time = 2.95 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.62

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{5/2}\,dx = -\frac{2\,e^{-a\,c-b\,c\,x}\,\sqrt{\frac{1}{{\left(\frac{e^{a\,c+b\,c\,x}}{2}-\frac{e^{-a\,c-b\,c\,x}}{2}\right)}^2}}\,\left(6\,e^{4\,a\,c+4\,b\,c\,x}-4\,e^{2\,a\,c+2\,b\,c\,x}+1\right)}{3\,b\,c\,{\left(e^{2\,a\,c+2\,b\,c\,x}-1\right)}^3}$$

input
```
int(exp(c*(a + b*x))*(1/sinh(a*c + b*c*x)^2)^(5/2),x)
```

output
```
-(2*exp(- a*c - b*c*x)*(1/(exp(a*c + b*c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(
1/2)*(6*exp(4*a*c + 4*b*c*x) - 4*exp(2*a*c + 2*b*c*x) + 1))/(3*b*c*(exp(2*
a*c + 2*b*c*x) - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.65

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{5/2}\,dx = \frac{-8e^{4bcx+4ac}+\frac{16e^{2bcx+2ac}}{3}-\frac{4}{3}}{bc\left(e^{8bcx+8ac}-4e^{6bcx+6ac}+6e^{4bcx+4ac}-4e^{2bcx+2ac}+1\right)}$$

input
```
int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(5/2),x)
```

output
```
(4*( - 6*e**(4*a*c + 4*b*c*x) + 4*e**(2*a*c + 2*b*c*x) - 1))/(3*b*c*(e**(8
*a*c + 8*b*c*x) - 4*e**(6*a*c + 6*b*c*x) + 6*e**(4*a*c + 4*b*c*x) - 4*e**(
2*a*c + 2*b*c*x) + 1))
```

3.127 $\int e^{c(a+bx)}\mathbf{csch}^2(ac+bcx)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 58

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{3/2}\,dx=-\frac{2e^{4c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

output
```
-2*exp(4*c*(b*x+a))*(csch(b*c*x+a*c)^2)^(1/2)*sinh(b*c*x+a*c)/b/c/(1-exp(2
*c*(b*x+a)))^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{3/2}\,dx=-\frac{2e^{4c(a+bx)}\operatorname{csch}^2(c(a+bx))^{3/2}\sinh^3(c(a+bx))}{bc\left(-1+e^{2c(a+bx)}\right)^2}$$

input
```
Integrate[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(3/2),x]
```

output
```
(-2*E^(4*c*(a + b*x))*(Csch[c*(a + b*x)]^2)^(3/2)*Sinh[c*(a + b*x)]^3)/(b*
c*(-1 + E^(2*c*(a + b*x)))^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {7271, 2720, 27, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\text{csch}^2(ac+bcx)^{3/2}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\sinh(ac+bcx)\sqrt{\text{csch}^2(ac+bcx)}\int e^{c(a+bx)}\text{csch}^3(ac+bxc)dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\sinh(ac+bcx)\sqrt{\text{csch}^2(ac+bcx)}\int -\frac{8e^{3c(a+bx)}}{\left(1-e^{2c(a+bx)}\right)^3}de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{27}$$

$$-\frac{8\sinh(ac+bcx)\sqrt{\text{csch}^2(ac+bcx)}\int \frac{e^{3c(a+bx)}}{\left(1-e^{2c(a+bx)}\right)^3}de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{242}$$

$$-\frac{2e^{4c(a+bx)}\sinh(ac+bcx)\sqrt{\text{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

input
```
Int[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(3/2),x]
```

output
```
(-2*E^(4*c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(b*c*(1
 - E^(2*c*(a + b*x)))^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.33 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\operatorname{csgn}\left(\operatorname{csch}\left(c\left(bx+a\right)\right)\right)\left(-\frac{\coth\left(c\left(bx+a\right)\right)^{2}}{2}-\coth\left(c\left(bx+a\right)\right)\right)}{cb}$	40
risch	$-\frac{2\left(2\,{\mathrm e}^{2c\left(bx+a\right)}-1\right)\sqrt{\frac{{\mathrm e}^{2c\left(bx+a\right)}}{\left({\mathrm e}^{2c\left(bx+a\right)}-1\right)^{2}}}\,{\mathrm e}^{-c\left(bx+a\right)}}{bc\left({\mathrm e}^{2c\left(bx+a\right)}-1\right)}$	69

input
```
int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(csch(c*(b*x+a)))/c/b*(-1/2*coth(c*(b*x+a))^2-coth(c*(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(52) = 104.

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.09

$$\int e^{c(a+bx)}\text{csch}^2(ac+bcx)^{3/2}\,dx =$$
$$-\frac{2\left(\cosh\left(bcx+ac\right)+3\,\sinh\left(bcx+ac\right)\right)}{bc\cosh\left(bcx+ac\right)^3+3\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^2+bc\sinh\left(bcx+ac\right)^3-bc\cosh\left(bcx+ac\right)+3}$$

input
```
integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

output
```
-2*(cosh(b*c*x + a*c) + 3*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^3 + 3*
b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 + b*c*sinh(b*c*x + a*c)^3 - b*c*
cosh(b*c*x + a*c) + 3*(b*c*cosh(b*c*x + a*c)^2 - b*c)*sinh(b*c*x + a*c))
```

Sympy [F]

$$\int e^{c(a+bx)}\text{csch}^2(ac+bcx)^{3/2}\,dx = e^{ac}\int\left(\text{csch}^2\left(ac+bcx\right)\right)^{\frac{3}{2}}e^{bcx}\,dx$$

input
```
integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)**2)**(3/2),x)
```

output
```
exp(a*c)*Integral((csch(a*c + b*c*x)**2)**(3/2)*exp(b*c*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.45

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{3/2}\,dx = -\frac{4\,e^{(2\,bcx+2\,ac)}}{bc(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1)} + \frac{2}{bc(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1)}$$

input
```
integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

output
```
-4*e^(2*b*c*x + 2*a*c)/(b*c*(e^(4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) +
 1)) + 2/(b*c*(e^(4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{3/2}\,dx = -\frac{2\left(2\,e^{(2\,bcx+2\,ac)}-1\right)}{bc(e^{(2\,bcx+2\,ac)}-1)^2\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}$$

input
```
integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

output
```
-2*(2*e^(2*b*c*x + 2*a*c) - 1)/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^2*sgn(e^(b*c
*x + a*c) - e^(-b*c*x - a*c)))
```

Mupad [B] (verification not implemented)

Time = 2.98 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.34

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{3/2}\,dx = -\frac{\mathrm{e}^{-a\,c-b\,c\,x}\,\left(2\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}-1\right)\,\sqrt{\frac{1}{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}-\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}}{b\,c\,\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}-1\right)}$$

input
```
int(exp(c*(a + b*x))*(1/sinh(a*c + b*c*x)^2)^(3/2),x)
```

output
```
-(exp(- a*c - b*c*x)*(2*exp(2*a*c + 2*b*c*x) - 1)*(1/(exp(a*c + b*c*x)/2 -
 exp(- a*c - b*c*x)/2)^2)^(1/2))/(b*c*(exp(2*a*c + 2*b*c*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.86

$$\int e^{c(a+bx)}\text{csch}^2(ac+bcx)^{3/2}\,dx = -\frac{2e^{4bcx+4ac}}{bc\,(e^{4bcx+4ac}-2e^{2bcx+2ac}+1)}$$

input
```
int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(3/2),x)
```

output
```
( - 2*e**(4*a*c + 4*b*c*x))/(b*c*(e**(4*a*c + 4*b*c*x) - 2*e**(2*a*c + 2*b
*c*x) + 1))
```

3.128 $\int e^{c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 46

$$\int e^{c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,dx=\frac{\sqrt{\operatorname{csch}^2(ac+bcx)}\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)}{bc}$$

output
```
(csch(b*c*x+a*c)^2)^(1/2)*ln(1-exp(2*c*(b*x+a)))*sinh(b*c*x+a*c)/b/c
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int e^{c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,dx=\frac{\sqrt{\operatorname{csch}^2(c(a+bx))}\log\left(1-e^{2c(a+bx)}\right)\sinh(c(a+bx))}{bc}$$

input
```
Integrate[E^(c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2],x]
```

output
```
(Sqrt[Csch[c*(a + b*x)]^2]*Log[1 - E^(2*c*(a + b*x))]*Sinh[c*(a + b*x)])/(
b*c)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {7271, 2720, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\sqrt{\text{csch}^2(ac+bcx)}dx$$

$$\Big\downarrow \text{7271}$$

$$\sinh(ac+bcx)\sqrt{\text{csch}^2(ac+bcx)}\int e^{c(a+bx)}\text{csch}(ac+bxc)dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\sinh(ac+bcx)\sqrt{\text{csch}^2(ac+bcx)}\int -\frac{2e^{c(a+bx)}}{1-e^{2c(a+bx)}}de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\sinh(ac+bcx)\sqrt{\text{csch}^2(ac+bcx)}\int \frac{e^{c(a+bx)}}{1-e^{2c(a+bx)}}de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{240}$$

$$\frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)\sqrt{\text{csch}^2(ac+bcx)}}{bc}$$

input
```
Int[E^(c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2],x]
```

output
```
(Sqrt[Csch[a*c + b*c*x]^2]*Log[1 - E^(2*c*(a + b*x))]*Sinh[a*c + b*c*x])/(
b*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.33 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

method	result	size
default	$\operatorname{csgn}\left(\operatorname{csch}\left(c(bx+a)\right)\right)\left(x+\frac{\ln\left(\sinh\left(c(bx+a)\right)\right)}{cb}\right)$	29
risch	$\frac{\ln\left({\mathrm e}^{2bxc}-{\mathrm e}^{-2ac}\right)\left({\mathrm e}^{2c(bx+a)}-1\right)\sqrt{\frac{{\mathrm e}^{2c(bx+a)}}{\left({\mathrm e}^{2c(bx+a)}-1\right)^{2}}}\,{\mathrm e}^{-c(bx+a)}}{bc}$	68

input
```
int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(csch(c*(b*x+a)))*(x+1/c/b*ln(sinh(c*(b*x+a))))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int e^{c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,dx=\frac{\log\left(\frac{2\,\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

output
```
log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c)))/(b*c)
```

Sympy [F]

$$\int e^{c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,dx=e^{ac}\int\sqrt{\operatorname{csch}^2(ac+bcx)}e^{bcx}\,dx$$

input
```
integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)**2)**(1/2),x)
```

output
```
exp(a*c)*Integral(sqrt(csch(a*c + b*c*x)**2)*exp(b*c*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int e^{c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,dx=\frac{\log\left(e^{(bcx+ac)}+1\right)}{bc}+\frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

output
```
log(e^(b*c*x + a*c) + 1)/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int e^{c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,dx = \frac{\log\left(\left|e^{(2\,bcx+2\,ac)}-1\right|\right)}{bc\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}$$

input
```
integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

output
```
log(abs(e^(2*b*c*x + 2*a*c) - 1))/(b*c*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a
*c)))
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,dx = \int e^{c\,(a+b\,x)}\,\sqrt{\frac{1}{{\sinh\left(a\,c+b\,c\,x\right)}^2}}\,dx$$

input
```
int(exp(c*(a + b*x))*(1/sinh(a*c + b*c*x)^2)^(1/2),x)
```

output
```
int(exp(c*(a + b*x))*(1/sinh(a*c + b*c*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int e^{c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,dx = \frac{\log\left(e^{bcx+2ac}+e^{ac}\right)+\log\left(e^{bcx+2ac}-e^{ac}\right)}{bc}$$

input
```
int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(1/2),x)
```

output
```
(log(e**(2*a*c + b*c*x) + e**(a*c)) + log(e**(2*a*c + b*c*x) - e**(a*c)))/
(b*c)
```

3.129 $\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{csch}^2(ac+bcx)}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{csch}^2(ac+bcx)}} \, dx = \frac{e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{4bc\sqrt{\operatorname{csch}^2(ac+bcx)}} - \frac{x\operatorname{csch}(ac+bcx)}{2\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

output
```
1/4*exp(2*c*(b*x+a))*csch(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)-1/2*x*c
sch(b*c*x+a*c)/(csch(b*c*x+a*c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{csch}^2(ac+bcx)}} \, dx = \frac{\left(e^{2c(a+bx)} - 2bcx\right)\operatorname{csch}(c(a+bx))}{4bc\sqrt{\operatorname{csch}^2(c(a+bx))}}$$

input
```
Integrate[E^(c*(a + b*x))/Sqrt[Csch[a*c + b*c*x]^2],x]
```

output
```
((E^(2*c*(a + b*x)) - 2*b*c*x)*Csch[c*(a + b*x)])/(4*b*c*Sqrt[Csch[c*(a +
b*x)]^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7271, 2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{csch}^2(ac+bcx)}}dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\operatorname{csch}(ac+bcx)\int e^{c(a+bx)}\sinh(ac+bxc)dx}{\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\operatorname{csch}(ac+bcx)\int -\frac{1}{2}e^{-c(a+bx)}\left(1-e^{2c(a+bx)}\right)de^{c(a+bx)}}{bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\operatorname{csch}(ac+bcx)\int e^{-c(a+bx)}\left(1-e^{2c(a+bx)}\right)de^{c(a+bx)}}{2bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\operatorname{csch}(ac+bcx)\int \left(e^{-c(a+bx)}-e^{c(a+bx)}\right)de^{c(a+bx)}}{2bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{1}{2}e^{2c(a+bx)}-\log\left(e^{c(a+bx)}\right)\right)\operatorname{csch}(ac+bcx)}{2bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

input
```
Int[E^(c*(a + b*x))/Sqrt[Csch[a*c + b*c*x]^2],x]
```

output
```
(Csch[a*c + b*c*x]*(E^(2*c*(a + b*x))/2 - Log[E^(c*(a + b*x))]))/(2*b*c*Sq
rt[Csch[a*c + b*c*x]^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.43

method	result	size
risch	$-\frac{x\,{\mathrm e}^{c(bx+a)}}{2\left({\mathrm e}^{2c(bx+a)}-1\right)\sqrt{\frac{{\mathrm e}^{2c(bx+a)}}{\left({\mathrm e}^{2c(bx+a)}-1\right)^{2}}}}+\frac{{\mathrm e}^{3c(bx+a)}}{4bc\left({\mathrm e}^{2c(bx+a)}-1\right)\sqrt{\frac{{\mathrm e}^{2c(bx+a)}}{\left({\mathrm e}^{2c(bx+a)}-1\right)^{2}}}}$	106

input
```
int(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1
/2)*exp(c*(b*x+a))+1/4/b/c/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*
exp(2*c*(b*x+a)))^(1/2)*exp(3*c*(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\,dx = -\frac{(2\,bcx-1)\cosh\left(bcx+ac\right)-(2\,bcx+1)\sinh\left(bcx+ac\right)}{4\left(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right)\right)}$$

input
```
integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*((2*b*c*x - 1)*cosh(b*c*x + a*c) - (2*b*c*x + 1)*sinh(b*c*x + a*c))/(
b*c*cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [F]

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\,dx = e^{ac}\int \frac{e^{bcx}}{\sqrt{\operatorname{csch}^2\left(ac+bcx\right)}}\,dx$$

input
```
integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)**2)**(1/2),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)/sqrt(csch(a*c + b*c*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.49

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\,dx = -\frac{bcx+ac}{2\,bc} + \frac{e^{(2\,bcx+2\,ac)}}{4\,bc}$$

input
```
integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(b*c*x + a*c)/(b*c) + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\,dx = -\frac{2\,bcx\operatorname{sgn}\left(e^{(bcx+ac)} - e^{(-bcx-ac)}\right) - e^{(2\,bcx+2\,ac)}\operatorname{sgn}\left(e^{(bcx+ac)} - e^{(-bcx-ac)}\right)}{4\,bc}$$

input
```
integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(2*b*c*x*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - e^(2*b*c*x + 2*a*c
)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)))/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\,dx = \int \frac{e^{c\,(a+b\,x)}}{\sqrt{\frac{1}{{\sinh\left(a\,c+b\,c\,x\right)}^2}}}\,dx$$

input
```
int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(1/2),x)
```

output
```
int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.35

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\,dx=\frac{e^{2bcx+2ac}-2bcx}{4bc}$$

input
```
int(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2),x)
```

output
```
(e**(2*a*c + 2*b*c*x) - 2*b*c*x)/(4*b*c)
```

3.130 $\int \frac{e^{c(a+bx)}}{\mathrm{csch}^2(ac+bcx)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 162

$$\int \frac{e^{c(a+bx)}}{\mathrm{csch}^2(ac+bcx)^{3/2}}\,dx = \frac{e^{-2c(a+bx)}\mathrm{csch}(ac+bcx)}{16bc\sqrt{\mathrm{csch}^2(ac+bcx)}} - \frac{3e^{2c(a+bx)}\mathrm{csch}(ac+bcx)}{16bc\sqrt{\mathrm{csch}^2(ac+bcx)}} + \frac{e^{4c(a+bx)}\mathrm{csch}(ac+bcx)}{32bc\sqrt{\mathrm{csch}^2(ac+bcx)}} + \frac{3x\mathrm{csch}(ac+bcx)}{8\sqrt{\mathrm{csch}^2(ac+bcx)}}$$

output
```
1/16*csch(b*c*x+a*c)/b/c/exp(2*c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2)-3/16*e
xp(2*c*(b*x+a))*csch(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)+1/32*exp(4*c
*(b*x+a))*csch(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)+3/8*x*csch(b*c*x+a
*c)/(csch(b*c*x+a*c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.47

$$\int \frac{e^{c(a+bx)}}{\mathrm{csch}^2(ac+bcx)^{3/2}}\,dx = \frac{\left(e^{-2c(a+bx)} - 3e^{2c(a+bx)} + \frac{1}{2}e^{4c(a+bx)} + 6bcx\right)\mathrm{csch}^3(c(a+bx))}{16bc\,\mathrm{csch}^2(c(a+bx))^{3/2}}$$

input
```
Integrate[E^(c*(a + b*x))/(Csch[a*c + b*c*x]^2)^(3/2),x]
```

output
```
((E^(-2*c*(a + b*x)) - 3*E^(2*c*(a + b*x)) + E^(4*c*(a + b*x))/2 + 6*b*c*x
)*Csch[c*(a + b*x)]^3)/(16*b*c*(Csch[c*(a + b*x)]^2)^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.46, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {7271, 2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{3/2}}dx$$

$$\Big\downarrow \text{ 7271}$$

$$\frac{\operatorname{csch}(ac+bcx)\int e^{c(a+bx)}\sinh^3(ac+bxc)dx}{\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\operatorname{csch}(ac+bcx)\int -\frac{1}{8}e^{-3c(a+bx)}\left(1-e^{2c(a+bx)}\right)^3de^{c(a+bx)}}{bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\operatorname{csch}(ac+bcx)\int e^{-3c(a+bx)}\left(1-e^{2c(a+bx)}\right)^3de^{c(a+bx)}}{8bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{\operatorname{csch}(ac+bcx)\int e^{-2c(a+bx)}\left(1-e^{2c(a+bx)}\right)^3de^{2c(a+bx)}}{16bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$\Big\downarrow \text{ 49}$$

$$-\frac{\operatorname{csch}(ac+bcx)\int \left(3+e^{-2c(a+bx)}-3e^{-c(a+bx)}-e^{2c(a+bx)}\right)de^{2c(a+bx)}}{16bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\left(-e^{-c(a+bx)}+\frac{5}{2}e^{2c(a+bx)}-3\log\left(e^{2c(a+bx)}\right)\right)\operatorname{csch}(ac+bcx)}{16bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

input
```
Int[E^(c*(a + b*x))/(Csch[a*c + b*c*x]^2)^(3/2),x]
```

output
```
-1/16*(Csch[a*c + b*c*x]*(-E^(-(c*(a + b*x))) + (5*E^(2*c*(a + b*x)))/2 -
3*Log[E^(2*c*(a + b*x))]))/(b*c*Sqrt[Csch[a*c + b*c*x]^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.33

method	result
risch	$\frac{3x\,\mathrm{e}^{c(bx+a)}}{8\left(\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}}}}+\frac{\mathrm{e}^{5c(bx+a)}}{32bc\left(\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}}}}-\frac{3\,\mathrm{e}^{3c(bx+a)}}{16bc\left(\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}}}}+\frac{}{16bc(\mathrm{e}}$

input
```
int(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/8*x/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/
2)*exp(c*(b*x+a))+1/32/b/c/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*
exp(2*c*(b*x+a)))^(1/2)*exp(5*c*(b*x+a))-3/16/b/c/(exp(2*c*(b*x+a))-1)/(1/
(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)*exp(3*c*(b*x+a))+1/16/b/c/(
exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)*exp(
-c*(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.78

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{3/2}}\,dx=\frac{3\,\cosh\left(bcx+ac\right)^{3}+9\,\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{2}-\sinh\left(bcx+ac\right)^{3}+6\,(}{32\left(bc\cosh\left(bcx+ac\right)\right.}$$

input
```
integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/32*(3*cosh(b*c*x + a*c)^3 + 9*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 - si
nh(b*c*x + a*c)^3 + 6*(2*b*c*x - 1)*cosh(b*c*x + a*c) - 3*(4*b*c*x + cosh(
b*c*x + a*c)^2 + 2)*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c) - b*c*sinh(b
*c*x + a*c))
```

Sympy [F]

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{3/2}}\,dx = e^{ac}\int \frac{e^{bcx}}{\left(\operatorname{csch}^2\left(ac+bcx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)**2)**(3/2),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)/(csch(a*c + b*c*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.38

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{3/2}}\,dx = \frac{\left(e^{(6\,bcx+6\,ac)}-6\,e^{(4\,bcx+4\,ac)}+2\right)e^{(-2\,bcx-2\,ac)}}{32\,bc}+\frac{3\,(bcx+ac)}{8\,bc}$$

input
```
integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/32*(e^(6*b*c*x + 6*a*c) - 6*e^(4*b*c*x + 4*a*c) + 2)*e^(-2*b*c*x - 2*a*c
)/(b*c) + 3/8*(b*c*x + a*c)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.19

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{3/2}}\,dx = \frac{\left(12\,bcxe^{(2\,ac)}\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-2\left(3\,e^{(2\,bcx+2\,ac)}\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-a}\right.\right.}{}$$

input
```
integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

output
```
1/32*(12*b*c*x*e^(2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 2*(3*e^
(2*b*c*x + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - sgn(e^(b*c*x +
 a*c) - e^(-b*c*x - a*c)))*e^(-2*b*c*x) + e^(4*b*c*x + 6*a*c)*sgn(e^(b*c*x
 + a*c) - e^(-b*c*x - a*c)) - 6*e^(2*b*c*x + 4*a*c)*sgn(e^(b*c*x + a*c) -
e^(-b*c*x - a*c)))*e^(-2*a*c)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{3/2}}\,dx = \int \frac{e^{c\,(a+b\,x)}}{{\left(\frac{1}{{\sinh\left(a\,c+b\,c\,x\right)}^2}\right)}^{3/2}}\,dx$$

input
```
int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(3/2),x)
```

output
```
int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.41

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{3/2}}\,dx = \frac{e^{6bcx+6ac}-6e^{4bcx+4ac}+12e^{2bcx+2ac}bcx+2}{32e^{2bcx+2ac}bc}$$

input
```
int(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(3/2),x)
```

output
```
(e**(6*a*c + 6*b*c*x) - 6*e**(4*a*c + 4*b*c*x) + 12*e**(2*a*c + 2*b*c*x)*b
*c*x + 2)/(32*e**(2*a*c + 2*b*c*x)*b*c)
```

3.131 $\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 250

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{5/2}}\,dx = \frac{e^{-4c(a+bx)}\operatorname{csch}(ac+bcx)}{128bc\sqrt{\operatorname{csch}^2(ac+bcx)}} - \frac{5e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)}{64bc\sqrt{\operatorname{csch}^2(ac+bcx)}} + \frac{5e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{32bc\sqrt{\operatorname{csch}^2(ac+bcx)}} - \frac{5e^{4c(a+bx)}\operatorname{csch}(ac+bcx)}{128bc\sqrt{\operatorname{csch}^2(ac+bcx)}} + \frac{e^{6c(a+bx)}\operatorname{csch}(ac+bcx)}{192bc\sqrt{\operatorname{csch}^2(ac+bcx)}} - \frac{5x\operatorname{csch}(ac+bcx)}{16\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

output
```
1/128*csch(b*c*x+a*c)/b/c/exp(4*c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2)-5/64*
csch(b*c*x+a*c)/b/c/exp(2*c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2)+5/32*exp(2*
c*(b*x+a))*csch(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)-5/128*exp(4*c*(b*
x+a))*csch(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)+1/192*exp(6*c*(b*x+a))
*csch(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)-5/16*x*csch(b*c*x+a*c)/(csc
h(b*c*x+a*c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.44

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{5/2}}\,dx = \frac{\left(\frac{1}{128}e^{-4c(a+bx)} - \frac{5}{64}e^{-2c(a+bx)} + \frac{5}{32}e^{2c(a+bx)} - \frac{5}{128}e^{4c(a+bx)} + \frac{1}{192}e^{6c(a+bx)} - \frac{5bcx}{16}\right) \mathrm{c}}{bc\operatorname{csch}^2(c(a+bx))^{5/2}}$$

input
```
Integrate[E^(c*(a + b*x))/(Csch[a*c + b*c*x]^2)^(5/2),x]
```

output
```
((1/(128*E^(4*c*(a + b*x))) - 5/(64*E^(2*c*(a + b*x))) + (5*E^(2*c*(a + b*
x)))/32 - (5*E^(4*c*(a + b*x)))/128 + E^(6*c*(a + b*x))/192 - (5*b*c*x)/16
)*Csch[c*(a + b*x)]^5)/(b*c*(Csch[c*(a + b*x)]^2)^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.41, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {7271, 2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{5/2}}\,dx$$

$$\Big\downarrow \; 7271$$

$$\frac{\operatorname{csch}(ac+bcx)\int e^{c(a+bx)}\sinh^5(ac+bxc)dx}{\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$\Big\downarrow \; 2720$$

$$\frac{\operatorname{csch}(ac+bcx)\int -\frac{1}{32}e^{-5c(a+bx)}\left(1-e^{2c(a+bx)}\right)^5 de^{c(a+bx)}}{bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\operatorname{csch}(ac+bcx)\int e^{-5c(a+bx)}\left(1-e^{2c(a+bx)}\right)^5 de^{c(a+bx)}}{32bc\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

$$\downarrow \text{243}$$

$$-\frac{\text{csch}(ac+bcx)\int e^{-3c(a+bx)}\left(1-e^{2c(a+bx)}\right)^5de^{2c(a+bx)}}{64bc\sqrt{\text{csch}^2(ac+bcx)}}$$

$$\downarrow \text{49}$$

$$-\frac{\text{csch}(ac+bcx)\int \left(-10+e^{-3c(a+bx)}-5e^{-2c(a+bx)}+10e^{-c(a+bx)}+4e^{2c(a+bx)}\right)de^{2c(a+bx)}}{64bc\sqrt{\text{csch}^2(ac+bcx)}}$$

$$\downarrow \text{2009}$$

$$-\frac{\left(-\frac{1}{2}e^{-2c(a+bx)}+5e^{-c(a+bx)}-\frac{15}{2}e^{2c(a+bx)}-\frac{1}{3}e^{3c(a+bx)}+10\log\left(e^{2c(a+bx)}\right)\right)\text{csch}(ac+bcx)}{64bc\sqrt{\text{csch}^2(ac+bcx)}}$$

input
```
Int[E^(c*(a + b*x))/(Csch[a*c + b*c*x]^2)^(5/2),x]
```

output
```
-1/64*(Csch[a*c + b*c*x]*(-1/2*1/E^(2*c*(a + b*x)) + 5/E^(c*(a + b*x)) - (
15*E^(2*c*(a + b*x)))/2 - E^(3*c*(a + b*x))/3 + 10*Log[E^(2*c*(a + b*x))])
)/(b*c*Sqrt[Csch[a*c + b*c*x]^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.30

method	result
risch	$-\frac{5x\,e^{c(bx+a)}}{16\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)^{2}}}}+\frac{e^{7c(bx+a)}}{192bc\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)^{2}}}}-\frac{5\,e^{5c(bx+a)}}{128bc\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)^{2}}}}+$

input
```
int(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-5/16*x/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(
1/2)*exp(c*(b*x+a))+1/192/b/c/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)
^2*exp(2*c*(b*x+a)))^(1/2)*exp(7*c*(b*x+a))-5/128/b/c/(exp(2*c*(b*x+a))-1)
/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)*exp(5*c*(b*x+a))+5/32/b
/c/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)*
exp(3*c*(b*x+a))-5/64/b/c/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*e
xp(2*c*(b*x+a)))^(1/2)*exp(-c*(b*x+a))+1/128/b/c/(exp(2*c*(b*x+a))-1)/(1/(
exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)*exp(-3*c*(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.87

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{5/2}}\,dx = \frac{5\cosh(bcx+ac)^5+25\cosh(bcx+ac)\sinh(bcx+ac)^4-\sinh(bcx+ac)^5-5$$

input
```
integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/384*(5*cosh(b*c*x + a*c)^5 + 25*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 -
sinh(b*c*x + a*c)^5 - 5*(2*cosh(b*c*x + a*c)^2 - 3)*sinh(b*c*x + a*c)^3 -
45*cosh(b*c*x + a*c)^3 + 5*(10*cosh(b*c*x + a*c)^3 - 27*cosh(b*c*x + a*c))
*sinh(b*c*x + a*c)^2 - 60*(2*b*c*x - 1)*cosh(b*c*x + a*c) - 5*(cosh(b*c*x
+ a*c)^4 - 24*b*c*x - 9*cosh(b*c*x + a*c)^2 - 12)*sinh(b*c*x + a*c))/(b*c*
cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [F]

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{5/2}}\,dx = e^{ac}\int \frac{e^{bcx}}{\left(\operatorname{csch}^2(ac+bcx)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)**2)**(5/2),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)/(csch(a*c + b*c*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.36

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{5/2}}\,dx = \frac{\left(2e^{(10bcx+10ac)}-15e^{(8bcx+8ac)}+60e^{(6bcx+6ac)}-30e^{(2bcx+2ac)}+3\right)e^{(-4bcx-4ac)}}{384bc} - \frac{5(bcx+ac)}{16bc}$$

input
```
integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/384*(2*e^(10*b*c*x + 10*a*c) - 15*e^(8*b*c*x + 8*a*c) + 60*e^(6*b*c*x +
6*a*c) - 30*e^(2*b*c*x + 2*a*c) + 3)*e^(-4*b*c*x - 4*a*c)/(b*c) - 5/16*(b*
c*x + a*c)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.06

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{5/2}}\,dx = -\frac{\left(120\,bcxe^{(4\,ac)}\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-3\left(30\,e^{(4\,bcx+4\,ac)}\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-10\,e^{(2\,bcx+2\,ac)}\operatorname{sgn}(\right.}{}$$

input
```
integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/384*(120*b*c*x*e^(4*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 3*(3
0*e^(4*b*c*x + 4*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 10*e^(2*b*
c*x + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + sgn(e^(b*c*x + a*c)
 - e^(-b*c*x - a*c)))*e^(-4*b*c*x) - 2*e^(6*b*c*x + 10*a*c)*sgn(e^(b*c*x +
 a*c) - e^(-b*c*x - a*c)) + 15*e^(4*b*c*x + 8*a*c)*sgn(e^(b*c*x + a*c) - e
^(-b*c*x - a*c)) - 60*e^(2*b*c*x + 6*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x
- a*c)))*e^(-4*a*c)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{5/2}}\,dx = \int \frac{e^{c\,(a+b\,x)}}{{\left(\frac{1}{{\mathrm{sinh}\left(a\,c+b\,c\,x\right)}^2}\right)}^{5/2}}\,dx$$

input
```
int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(5/2),x)
```

output
```
int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.39

$$\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{5/2}}\,dx = \frac{2e^{10bcx+10ac}-15e^{8bcx+8ac}+60e^{6bcx+6ac}-120e^{4bcx+4ac}bcx-30e^{2bcx+2ac}+3}{384e^{4bcx+4ac}bc}$$

input
```
int(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(5/2),x)
```

output
```
(2*e**(10*a*c + 10*b*c*x) - 15*e**(8*a*c + 8*b*c*x) + 60*e**(6*a*c + 6*b*c
*x) - 120*e**(4*a*c + 4*b*c*x)*b*c*x - 30*e**(2*a*c + 2*b*c*x) + 3)/(384*e
**(4*a*c + 4*b*c*x)*b*c)
```

3.132 $\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 81

$$\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = -\frac{2x^2}{21c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^6}{7\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2\,\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{21c^7\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

output
```
-2/21*x^2/c^4/csch(2*ln(c*x))^(1/2)+1/7*x^6/csch(2*ln(c*x))^(1/2)+2/21*Inv
erseJacobiAM(arccsc(c*x),I)/c^7/(1-1/c^4/x^4)^(1/2)/x/csch(2*ln(c*x))^(1/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{x^2\left(-(1-c^4x^4)^{3/2} + \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},c^4x^4\right)\right)}{7c^4\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

input
```
Integrate[x^5/Sqrt[Csch[2*Log[c*x]]],x]
```

output
```
(x^2*(-(1 - c^4*x^4)^(3/2) + Hypergeometric2F1[-1/2, 1/4, 5/4, c^4*x^4]))/
(7*c^4*Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6086, 6084, 858, 809, 847, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}dx$$

$$\Big\downarrow \text{6086}$$

$$\frac{\int \frac{c^5x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}d(cx)}{c^6}$$

$$\Big\downarrow \text{6084}$$

$$\frac{\int c^6\sqrt{1-\frac{1}{c^4x^4}}x^6d(cx)}{c^7x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{\int \frac{\sqrt{1-c^4x^4}}{c^8x^8}d\frac{1}{cx}}{c^7x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$\Big\downarrow \text{809}$$

$$-\frac{-\frac{2}{7}\int \frac{1}{c^4x^4\sqrt{1-c^4x^4}}d\frac{1}{cx}-\frac{\sqrt{1-c^4x^4}}{7c^7x^7}}{c^7x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{-\frac{2}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{1-c^4x^4}}d\frac{1}{cx}-\frac{\sqrt{1-c^4x^4}}{3c^3x^3}\right)-\frac{\sqrt{1-c^4x^4}}{7c^7x^7}}{c^7x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$\Big\downarrow \text{ 762}$$

$$-\frac{-\frac{2}{7}\left(\frac{1}{3}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{cx}\right),-1\right)-\frac{\sqrt{1-c^4x^4}}{3c^3x^3}\right)-\frac{\sqrt{1-c^4x^4}}{7c^7x^7}}{c^7x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}$$

input
```
Int[x^5/Sqrt[Csch[2*Log[c*x]]],x]
```

output
```
-((-1/7*Sqrt[1 - c^4*x^4]/(c^7*x^7) - (2*(-1/3*Sqrt[1 - c^4*x^4]/(c^3*x^3)
 + EllipticF[ArcSin[1/(c*x)], -1]/3))/7)/(c^7*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt
[Csch[2*Log[c*x]]]))
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6084
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6086
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.54

method	result	size
risch	$\frac{x^{2}\left(3c^{4}x^{4}-2\right)\sqrt{2}}{42c^{4}\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}-1}}}-\frac{\sqrt{c^{2}x^{2}+1}\,\sqrt{-c^{2}x^{2}+1}\,\operatorname{EllipticF}\left(x\sqrt{-c^{2}}, i\right)\sqrt{2}\,x}{21c^{4}\sqrt{-c^{2}}\,\left(c^{4}x^{4}-1\right)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}-1}}}$	125

input
```
int(x^5/csch(2*ln(x*c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/42*x^2*(3*c^4*x^4-2)/c^4*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)-1/21/c^4/(-
c^2)^(1/2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*EllipticF(x*(-
c^2)^(1/2),I)*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{\sqrt{2}(3\,c^{10}x^8 - 5\,c^6x^4 + 2\,c^2)\sqrt{\frac{c^2x^2}{c^4x^4-1}} + 4\,\sqrt{\frac{1}{2}}\sqrt{c^4}F\left(\arcsin\left(\frac{1}{cx}\right)\,|-1\right)}{42\,c^8}$$

input
```
integrate(x^5/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

output
```
1/42*(sqrt(2)*(3*c^10*x^8 - 5*c^6*x^4 + 2*c^2)*sqrt(c^2*x^2/(c^4*x^4 - 1))
 + 4*sqrt(1/2)*sqrt(c^4)*elliptic_f(arcsin(1/(c*x)), -1))/c^8
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^5}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(x**5/csch(2*ln(c*x))**(1/2),x)
```

output
```
Integral(x**5/sqrt(csch(2*log(c*x))), x)
```

Maxima [F]

$$\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^5}{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(x^5/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^5/sqrt(csch(2*log(c*x))), x)
```

Giac [F]

$$\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^5}{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(x^5/csch(2*log(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^5/sqrt(csch(2*log(c*x))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^5}{\sqrt{\frac{1}{\sinh(2\,\ln(c\,x))}}}\,dx$$

input
```
int(x^5/(1/sinh(2*log(c*x)))^(1/2),x)
```

output
```
int(x^5/(1/sinh(2*log(c*x)))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}\,x^5}{\operatorname{csch}\left(2\log\left(cx\right)\right)}dx$$

input
```
int(x^5/csch(2*log(c*x))^(1/2),x)
```

output
```
int((sqrt(csch(2*log(c*x)))*x**5)/csch(2*log(c*x)),x)
```

3.133 $\int \frac{x^4}{\sqrt{\mathrm{csch}(2\log(cx))}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{x^4}{\sqrt{\mathrm{csch}(2\log(cx))}}\,dx=\frac{\left(c^4-\frac{1}{x^4}\right)x^5}{6c^4\sqrt{\mathrm{csch}(2\log(cx))}}$$

output
```
1/6*(c^4-1/x^4)*x^5/c^4/csch(2*ln(c*x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \frac{x^4}{\sqrt{\mathrm{csch}(2\log(cx))}}\,dx=\frac{(-1+c^4x^4)^2\sqrt{\frac{c^2x^2}{-2+2c^4x^4}}}{6c^6x}$$

input
```
Integrate[x^4/Sqrt[Csch[2*Log[c*x]]],x]
```

output
```
((-1 + c^4*x^4)^2*Sqrt[(c^2*x^2)/(-2 + 2*c^4*x^4)])/(6*c^6*x)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6086, 6084, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(cx))}}dx$$

$$\Big\downarrow \text{ 6086}$$

$$\frac{\int \frac{c^4x^4}{\sqrt{\operatorname{csch}(2\log(cx))}}d(cx)}{c^5}$$

$$\Big\downarrow \text{ 6084}$$

$$\frac{\int c^5\sqrt{1-\frac{1}{c^4x^4}}x^5d(cx)}{c^6x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{x^5\left(1-\frac{1}{c^4x^4}\right)}{6\sqrt{\operatorname{csch}(2\log(cx))}}$$

input
```
Int[x^4/Sqrt[Csch[2*Log[c*x]]],x]
```

output
```
((1 - 1/(c^4*x^4))*x^5)/(6*Sqrt[Csch[2*Log[c*x]]])
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 6084
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6086
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30

method	result	size
risch	$\frac{\sqrt{2}\, x \left(c^{4} x^{4}-1\right)}{12 \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}\, c^{4}}$	39

input
```
int(x^4/csch(2*ln(x*c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)*(c^4*x^4-1)/c^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.60

$$\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{\sqrt{2}(c^8x^8 - 2\,c^4x^4 + 1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{12\,c^6x}$$

input
```
integrate(x^4/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(2)*(c^8*x^8 - 2*c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c^6*x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^4}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(x**4/csch(2*ln(c*x))**(1/2),x)
```

output
```
Integral(x**4/sqrt(csch(2*log(c*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{\left(\sqrt{2}c^4x^4 - \sqrt{2}\right)\sqrt{c^2x^2+1}\sqrt{cx+1}\sqrt{cx-1}}{12\,c^5}$$

input
```
integrate(x^4/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

output
```
1/12*(sqrt(2)*c^4*x^4 - sqrt(2))*sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(c*x
- 1)/c^5
```

Giac [F]

$$\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^4}{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(x^4/csch(2*log(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(csch(2*log(c*x))), x)
```

Mupad [B] (verification not implemented)

Time = 3.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{\left(c^4\,x^4-1\right)^2\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4-1}}}{12\,c^6\,x}$$

input
```
int(x^4/(1/sinh(2*log(c*x)))^(1/2),x)
```

output
```
((c^4*x^4 - 1)^2*((2*c^2*x^2)/(c^4*x^4 - 1))^(1/2))/(12*c^6*x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}\,x^4}{\operatorname{csch}\left(2\log\left(cx\right)\right)}dx$$

input
```
int(x^4/csch(2*log(c*x))^(1/2),x)
```

output
```
int((sqrt(csch(2*log(c*x)))*x**4)/csch(2*log(c*x)),x)
```

3.134 $\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 119

$$\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = -\frac{2}{5c^4\sqrt{\operatorname{csch}(2\log(cx))}}+\frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}} - \frac{2E(\csc^{-1}(cx)|-1)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2\,\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

output
```
-2/5/c^4/csch(2*ln(c*x))^(1/2)+1/5*x^4/csch(2*ln(c*x))^(1/2)-2/5*EllipticE
(1/c/x,I)/c^5/(1-1/c^4/x^4)^(1/2)/x/csch(2*ln(c*x))^(1/2)+2/5*InverseJacob
iAM(arccsc(c*x),I)/c^5/(1-1/c^4/x^4)^(1/2)/x/csch(2*ln(c*x))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.50

$$\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx=\frac{x^4\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},c^4x^4\right)}{3\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

input
```
Integrate[x^3/Sqrt[Csch[2*Log[c*x]]],x]
```

output
```
(x^4*Hypergeometric2F1[-1/2, 3/4, 7/4, c^4*x^4])/(3*Sqrt[2 - 2*c^4*x^4]*Sq
rt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.600$, Rules used = {6086, 6084, 858, 809, 847, 836, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$$

$$\Big\downarrow \text{6086}$$

$$\frac{\int \frac{c^3x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}d(cx)}{c^4}$$

$$\Big\downarrow \text{6084}$$

$$\frac{\int c^4\sqrt{1-\frac{1}{c^4x^4}}x^4d(cx)}{c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{\int \frac{\sqrt{1-c^4x^4}}{c^6x^6}d\frac{1}{cx}}{c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}}$$

$\Big\downarrow$ 809

$$-\frac{-\frac{2}{5}\int \frac{1}{c^2x^2\sqrt{1-c^4x^4}}d\frac{1}{cx}-\frac{\sqrt{1-c^4x^4}}{5c^5x^5}}{c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}}$$

$\Big\downarrow$ 847

$$-\frac{-\frac{2}{5}\left(-\int \frac{c^2x^2}{\sqrt{1-c^4x^4}}d\frac{1}{cx}-\frac{\sqrt{1-c^4x^4}}{cx}\right)-\frac{\sqrt{1-c^4x^4}}{5c^5x^5}}{c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}}$$

$\Big\downarrow$ 836

$$-\frac{-\frac{2}{5}\left(\int \frac{1}{\sqrt{1-c^4x^4}}d\frac{1}{cx}-\int \frac{c^2x^2+1}{\sqrt{1-c^4x^4}}d\frac{1}{cx}-\frac{\sqrt{1-c^4x^4}}{cx}\right)-\frac{\sqrt{1-c^4x^4}}{5c^5x^5}}{c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}}$$

$\Big\downarrow$ 762

$$-\frac{-\frac{2}{5}\left(-\int \frac{c^2x^2+1}{\sqrt{1-c^4x^4}}d\frac{1}{cx}+\text{EllipticF}\left(\arcsin\left(\frac{1}{cx}\right),-1\right)-\frac{\sqrt{1-c^4x^4}}{cx}\right)-\frac{\sqrt{1-c^4x^4}}{5c^5x^5}}{c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}}$$

$\Big\downarrow$ 1388

$$-\frac{-\frac{2}{5}\left(-\int \frac{\sqrt{c^2x^2+1}}{\sqrt{1-c^2x^2}}d\frac{1}{cx}+\text{EllipticF}\left(\arcsin\left(\frac{1}{cx}\right),-1\right)-\frac{\sqrt{1-c^4x^4}}{cx}\right)-\frac{\sqrt{1-c^4x^4}}{5c^5x^5}}{c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}}$$

$\Big\downarrow$ 327

$$-\frac{-\frac{2}{5}\left(\text{EllipticF}\left(\arcsin\left(\frac{1}{cx}\right),-1\right)-E\left(\arcsin\left(\frac{1}{cx}\right)\middle|-1\right)-\frac{\sqrt{1-c^4x^4}}{cx}\right)-\frac{\sqrt{1-c^4x^4}}{5c^5x^5}}{c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}}$$

input
```
Int[x^3/Sqrt[Csch[2*Log[c*x]]],x]
```

output
```
-((-1/5*Sqrt[1 - c^4*x^4]/(c^5*x^5) - (2*(-(Sqrt[1 - c^4*x^4]/(c*x)) - Ell
ipticE[ArcSin[1/(c*x)], -1] + EllipticF[ArcSin[1/(c*x)], -1]))/5)/(c^5*Sqr
t[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]]))
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 6084
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6086
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.07

method	result	size
risch	$\frac{\sqrt{2}\,x^{4}}{10\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}-1}}}-\frac{\sqrt{c^{2}x^{2}+1}\,\sqrt{-c^{2}x^{2}+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{-c^{2}},i\right)-\operatorname{EllipticE}\left(x\sqrt{-c^{2}},i\right)\right)\sqrt{2}\,x}{5\sqrt{-c^{2}}\,\left(c^{4}x^{4}-1\right)c^{2}\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}-1}}}$	127

input
```
int(x^3/csch(2*ln(x*c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*2^(1/2)*x^4/(c^2*x^2/(c^4*x^4-1))^(1/2)-1/5/(-c^2)^(1/2)*(c^2*x^2+1)^
(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)/c^2*(EllipticF(x*(-c^2)^(1/2),I)-Elli
pticE(x*(-c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.79

$$\int \frac{x^{3}}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{\sqrt{2}(c^{10}x^{8}-3\,c^{6}x^{4}+2\,c^{2})\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}-1}}-4\,\sqrt{\frac{1}{2}}\sqrt{c^{4}}\left(x^{2}E(\arcsin\left(\frac{1}{cx}\right)|-1)-x^{2}F(\arcsin\left(\frac{1}{cx}\right)|-1)\right)}{10\,c^{8}x^{2}}$$

input
```
integrate(x^3/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

output
```
1/10*(sqrt(2)*(c^10*x^8 - 3*c^6*x^4 + 2*c^2)*sqrt(c^2*x^2/(c^4*x^4 - 1)) -
 4*sqrt(1/2)*sqrt(c^4)*(x^2*elliptic_e(arcsin(1/(c*x)), -1) - x^2*elliptic
_f(arcsin(1/(c*x)), -1)))/(c^8*x^2)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^3}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(x**3/csch(2*ln(c*x))**(1/2),x)
```

output
```
Integral(x**3/sqrt(csch(2*log(c*x))), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^3}{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(x^3/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(csch(2*log(c*x))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/csch(2*log(c*x))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly exception caught Unable to
 convert to real %%{poly1[1.00000000000000000000000000000,0.000000000000
000000000
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^3}{\sqrt{\frac{1}{\sinh(2\ln(c\,x))}}}\,dx$$

input
```
int(x^3/(1/sinh(2*log(c*x)))^(1/2),x)
```

output
```
int(x^3/(1/sinh(2*log(c*x)))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}\,x^3}{\operatorname{csch}\left(2\log\left(cx\right)\right)}dx$$

input
```
int(x^3/csch(2*log(c*x))^(1/2),x)
```

output
```
int((sqrt(csch(2*log(c*x)))*x**3)/csch(2*log(c*x)),x)
```

3.135 $\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx=\frac{x^3}{4\sqrt{\operatorname{csch}(2\log(cx))}}-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

output
```
1/4*x^3/csch(2*ln(c*x))^(1/2)-1/4*arctanh((1-1/c^4/x^4)^(1/2))/c^4/(1-1/c^
4/x^4)^(1/2)/x/csch(2*ln(c*x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx=\frac{x\left(c^2x^2\sqrt{1-c^4x^4}+\arcsin\left(c^2x^2\right)\right)}{4c^2\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

input
```
Integrate[x^2/Sqrt[Csch[2*Log[c*x]]],x]
```

output
```
(x*(c^2*x^2*Sqrt[1 - c^4*x^4] + ArcSin[c^2*x^2]))/(4*c^2*Sqrt[2 - 2*c^4*x^
4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6086, 6084, 798, 51, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{\text{csch}(2\log(cx))}}dx$$

$$\Big\downarrow \text{ 6086}$$

$$\frac{\int \frac{c^2x^2}{\sqrt{\text{csch}(2\log(cx))}}d(cx)}{c^3}$$

$$\Big\downarrow \text{ 6084}$$

$$\frac{\int c^3\sqrt{1-\frac{1}{c^4x^4}}x^3d(cx)}{c^4x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}}$$

$$\Big\downarrow \text{ 798}$$

$$-\frac{\int \frac{\sqrt{1-\frac{1}{c^4x^4}}}{c^2x^2}d\frac{1}{c^4x^4}}{4c^4x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}}$$

$$\Big\downarrow \text{ 51}$$

$$-\frac{-\frac{1}{2}\int \frac{1}{c\sqrt{1-\frac{1}{c^4x^4}}x}d\frac{1}{c^4x^4}-\frac{\sqrt{1-\frac{1}{c^4x^4}}}{cx}}{4c^4x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{\int \frac{1}{1-c^2x^2}d\sqrt{1-\frac{1}{c^4x^4}}-\frac{\sqrt{1-\frac{1}{c^4x^4}}}{cx}}{4c^4x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)-\frac{\sqrt{1-\frac{1}{c^4x^4}}}{cx}}{4c^4x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}$$

input
```
Int[x^2/Sqrt[Csch[2*Log[c*x]]],x]
```

output
```
-1/4*(-(Sqrt[1 - 1/(c^4*x^4)]/(c*x)) + ArcTanh[Sqrt[1 - 1/(c^4*x^4)]])/(c^
4*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]])
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6084
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6086
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.41

method	result	size
risch	$\frac{\sqrt{2}\, x^{3}}{8 \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}-\frac{\ln \left(\frac{c^{4} x^{2}}{\sqrt{c^{4}}}+\sqrt{c^{4} x^{4}-1}\right) \sqrt{2}\, x}{8 \sqrt{c^{4}}\, \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}\, \sqrt{c^{4} x^{4}-1}}$	97

input
```
int(x^2/csch(2*ln(x*c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*2^(1/2)*x^3/(c^2*x^2/(c^4*x^4-1))^(1/2)-1/8*ln(c^4*x^2/(c^4)^(1/2)+(c^
4*x^4-1)^(1/2))/(c^4)^(1/2)*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)/(c^4*x^4
-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.33

$$\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{2\sqrt{2}(c^5x^5-cx)\sqrt{\frac{c^2x^2}{c^4x^4-1}}+\sqrt{2}\log\left(2\,c^4x^4-2\,(c^5x^5-cx)\sqrt{\frac{c^2x^2}{c^4x^4-1}}-1\right)}{16\,c^3}$$

input
```
integrate(x^2/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

output
```
1/16*(2*sqrt(2)*(c^5*x^5 - c*x)*sqrt(c^2*x^2/(c^4*x^4 - 1)) + sqrt(2)*log(
2*c^4*x^4 - 2*(c^5*x^5 - c*x)*sqrt(c^2*x^2/(c^4*x^4 - 1)) - 1))/c^3
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^2}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(x**2/csch(2*ln(c*x))**(1/2),x)
```

output
```
Integral(x**2/sqrt(csch(2*log(c*x))), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^2}{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(x^2/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(csch(2*log(c*x))), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^2}{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(x^2/csch(2*log(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(csch(2*log(c*x))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x^2}{\sqrt{\frac{1}{\sinh(2\,\ln(c\,x))}}}\,dx$$

input
```
int(x^2/(1/sinh(2*log(c*x)))^(1/2),x)
```

output
```
int(x^2/(1/sinh(2*log(c*x)))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}\,x^2}{\operatorname{csch}\left(2\log\left(cx\right)\right)}dx$$

input
```
int(x^2/csch(2*log(c*x))^(1/2),x)
```

output
```
int((sqrt(csch(2*log(c*x)))*x**2)/csch(2*log(c*x)),x)
```

3.136 $\int \frac{x}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{x}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx=\frac{x^2}{3\sqrt{\operatorname{csch}(2\log(cx))}}+\frac{2\,\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{3c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

output
```
1/3*x^2/csch(2*ln(c*x))^(1/2)+2/3*InverseJacobiAM(arccsc(c*x),I)/c^3/(1-1/
c^4/x^4)^(1/2)/x/csch(2*ln(c*x))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \frac{x}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx=\frac{x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},c^4x^4\right)}{\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

input
```
Integrate[x/Sqrt[Csch[2*Log[c*x]]],x]
```

output
```
(x^2*Hypergeometric2F1[-1/2, 1/4, 5/4, c^4*x^4])/(Sqrt[2 - 2*c^4*x^4]*Sqrt
[(c^2*x^2)/(-1 + c^4*x^4)])
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6086, 6084, 858, 809, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\operatorname{csch}(2\log(cx))}}dx$$

$$\Big\downarrow \text{6086}$$

$$\frac{\int \frac{cx}{\sqrt{\operatorname{csch}(2\log(cx))}}d(cx)}{c^2}$$

$$\Big\downarrow \text{6084}$$

$$\frac{\int c^2\sqrt{1-\frac{1}{c^4x^4}}x^2d(cx)}{c^3x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{\int \frac{\sqrt{1-c^4x^4}}{c^4x^4}d\frac{1}{cx}}{c^3x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$\Big\downarrow \text{809}$$

$$-\frac{-\frac{2}{3}\int \frac{1}{\sqrt{1-c^4x^4}}d\frac{1}{cx}-\frac{\sqrt{1-c^4x^4}}{3c^3x^3}}{c^3x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$\Big\downarrow \text{762}$$

$$-\frac{-\frac{2}{3}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{cx}\right),-1\right)-\frac{\sqrt{1-c^4x^4}}{3c^3x^3}}{c^3x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}$$

input
```
Int[x/Sqrt[Csch[2*Log[c*x]]],x]
```

output
```
-((-1/3*Sqrt[1 - c^4*x^4]/(c^3*x^3) - (2*EllipticF[ArcSin[1/(c*x)], -1])/3
)/(c^3*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]]))
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6084
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6086
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(51) = 102$.

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.82

method	result	size
risch	$\frac{\sqrt{2}\,x^{2}}{6\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}-1}}}-\frac{\sqrt{c^{2}x^{2}+1}\,\sqrt{-c^{2}x^{2}+1}\,\operatorname{EllipticF}\left(x\sqrt{-c^{2}},i\right)\sqrt{2}\,x}{3\sqrt{-c^{2}}\,\left(c^{4}x^{4}-1\right)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}-1}}}$	109

input
```
int(x/csch(2*ln(x*c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/2)*x^2/(c^2*x^2/(c^4*x^4-1))^(1/2)-1/3/(-c^2)^(1/2)*(c^2*x^2+1)^(
1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*EllipticF(x*(-c^2)^(1/2),I)*2^(1/2)*x/
(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

$$\int \frac{x}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx=\frac{\sqrt{2}(c^{6}x^{4}-c^{2})\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}-1}}+4\sqrt{\frac{1}{2}}\sqrt{c^{4}}F(\arcsin\left(\frac{1}{cx}\right)\mid-1)}{6\,c^{4}}$$

input
```
integrate(x/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(2)*(c^6*x^4 - c^2)*sqrt(c^2*x^2/(c^4*x^4 - 1)) + 4*sqrt(1/2)*sqr
t(c^4)*elliptic_f(arcsin(1/(c*x)), -1))/c^4
```

Sympy [F]

$$\int \frac{x}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(x/csch(2*ln(c*x))**(1/2),x)
```

output
```
Integral(x/sqrt(csch(2*log(c*x))), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x}{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(x/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(csch(2*log(c*x))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/csch(2*log(c*x))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly exception caught Unable to
 convert to real %%{poly1[1.00000000000000000000000000000,0.000000000000
000000000
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{x}{\sqrt{\frac{1}{\sinh(2\ln(c\,x))}}}\,dx$$

input
```
int(x/(1/sinh(2*log(c*x)))^(1/2),x)
```

output
```
int(x/(1/sinh(2*log(c*x)))^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{\sqrt{\operatorname{csch}(2\log(cx))}\,x}{\operatorname{csch}(2\log(cx))}dx$$

input
```
int(x/csch(2*log(c*x))^(1/2),x)
```

output
```
int((sqrt(csch(2*log(c*x)))*x)/csch(2*log(c*x)),x)
```

3.137 $\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 60

$$\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx=\frac{x}{2\sqrt{\operatorname{csch}(2\log(cx))}}+\frac{\csc^{-1}\left(c^2x^2\right)}{2c^2\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

output
```
1/2*x/csch(2*ln(c*x))^(1/2)+1/2*arccsc(c^2*x^2)/c^2/(1-1/c^4/x^4)^(1/2)/x/
csch(2*ln(c*x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx=\frac{x\left(\sqrt{-1+c^4x^4}-\arctan\left(\sqrt{-1+c^4x^4}\right)\right)}{2\sqrt{2}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}\sqrt{-1+c^4x^4}}$$

input
```
Integrate[1/Sqrt[Csch[2*Log[c*x]]],x]
```

output
```
(x*(Sqrt[-1 + c^4*x^4] - ArcTan[Sqrt[-1 + c^4*x^4]]))/(2*Sqrt[2]*Sqrt[(c^2
*x^2)/(-1 + c^4*x^4)]*Sqrt[-1 + c^4*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {6080, 6078, 858, 807, 247, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}dx$$

$$\Big\downarrow \text{6080}$$

$$\frac{\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}d(cx)}{c}$$

$$\Big\downarrow \text{6078}$$

$$\frac{\int c\sqrt{1-\frac{1}{c^4x^4}}xd(cx)}{c^2x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{\int \frac{\sqrt{1-c^4x^4}}{c^3x^3}d\frac{1}{cx}}{c^2x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{\int \frac{\sqrt{1-c^2x^2}}{c^2x^2}d(c^2x^2)}{2c^2x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$\Big\downarrow \text{247}$$

$$-\frac{-\int \frac{1}{\sqrt{1-c^2x^2}}d(c^2x^2)-c^2x^2\sqrt{1-c^2x^2}}{2c^2x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{-\arcsin\left(c^2x^2\right)-c^2x^2\sqrt{1-c^2x^2}}{2c^2x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}$$

input
```
Int[1/Sqrt[Csch[2*Log[c*x]]],x]
```

output
```
-1/2*(-(c^2*x^2*Sqrt[1 - c^2*x^2]) - ArcSin[c^2*x^2])/(c^2*Sqrt[1 - 1/(c^4
*x^4)]*x*Sqrt[Csch[2*Log[c*x]]])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6078
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Simp[Csch[d*(a
+ b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*d*p))   Int[1/(x^(b
*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; FreeQ[{a, b, d, p}, x] &&
  !IntegerQ[p]
```

rule 6080
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

Maple [F]

$$\int \frac{1}{\sqrt{\operatorname{csch}\left(2\ln\left(xc\right)\right)}}dx$$

input
```
int(1/csch(2*ln(x*c))^(1/2),x)
```

output
```
int(1/csch(2*ln(x*c))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

$$\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = -\frac{\sqrt{2}cx\arctan\left(\frac{(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{cx}\right)-\sqrt{2}(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{4\,c^2x}$$

input
```
integrate(1/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(2)*c*x*arctan((c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c*x))
- sqrt(2)*(c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1)))/(c^2*x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{1}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(1/csch(2*ln(c*x))**(1/2),x)
```

output
```
Integral(1/sqrt(csch(2*log(c*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{1}{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}\,dx$$

input
```
integrate(1/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(csch(2*log(c*x))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \text{Timed out}$$

input
```
integrate(1/csch(2*log(c*x))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\sinh(2\ln(c\,x))}}}\,dx$$

input
```
int(1/(1/sinh(2*log(c*x)))^(1/2),x)
```

output
```
int(1/(1/sinh(2*log(c*x)))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{\operatorname{csch}(2\log(cx))}dx$$

input
```
int(1/csch(2*log(c*x))^(1/2),x)
```

output
```
int(sqrt(csch(2*log(c*x)))/csch(2*log(c*x)),x)
```

3.138 $\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx = i\sqrt{\operatorname{csch}(2\log(cx))}\,\operatorname{EllipticF}\left(\frac{\pi}{4}-i\log(cx),2\right)\sqrt{i\sinh(2\log(cx))}$$

output
```
-I*csch(2*ln(c*x))^(1/2)*InverseJacobiAM(-1/4*Pi+I*ln(c*x),2^(1/2))*(I*sin
h(2*ln(c*x)))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx = \operatorname{csch}^{\frac{3}{2}}(2\log(cx))\,\operatorname{EllipticF}\left(\frac{\pi}{4}-i\log(cx),2\right)(i\sinh(2\log(cx)))^{3/2}$$

input
```
Integrate[Sqrt[Csch[2*Log[c*x]]]/x,x]
```

output
```
Csch[2*Log[c*x]]^(3/2)*EllipticF[Pi/4 - I*Log[c*x], 2]*(I*Sinh[2*Log[c*x]]
)^(3/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3039, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\text{csch}(2\log(cx))}}{x} \, dx$$

$$\Big\downarrow \text{3039}$$

$$\int \sqrt{\text{csch}(2\log(cx))}d\log(cx)$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{i\csc(2i\log(cx))}d\log(cx)$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{i\sinh(2\log(cx))}\sqrt{\text{csch}(2\log(cx))}\int \frac{1}{\sqrt{i\sinh(2\log(cx))}}d\log(cx)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{i\sinh(2\log(cx))}\sqrt{\text{csch}(2\log(cx))}\int \frac{1}{\sqrt{\sin(2i\log(cx))}}d\log(cx)$$

$$\Big\downarrow \text{3120}$$

$$i\sqrt{i\sinh(2\log(cx))}\sqrt{\text{csch}(2\log(cx))}\,\text{EllipticF}\left(\frac{\pi}{4}-i\log(cx),2\right)$$

input
```
Int[Sqrt[Csch[2*Log[c*x]]]/x,x]
```

output
```
I*Sqrt[Csch[2*Log[c*x]]]*EllipticF[Pi/4 - I*Log[c*x], 2]*Sqrt[I*Sinh[2*Log
[c*x]]]
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(39) = 78$.

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.96

method	result	si
derivativedivides	$\frac{i\sqrt{-i(\sinh(2\ln(xc))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(2\ln(xc))+i)}\,\sqrt{i\sinh(2\ln(xc))}\,\operatorname{EllipticF}\left(\sqrt{-i(\sinh(2\ln(xc))+i)},\frac{\sqrt{2}}{2}\right)}{2\cosh(2\ln(xc))\sqrt{\sinh(2\ln(xc))}}$	90
default	$\frac{i\sqrt{-i(\sinh(2\ln(xc))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(2\ln(xc))+i)}\,\sqrt{i\sinh(2\ln(xc))}\,\operatorname{EllipticF}\left(\sqrt{-i(\sinh(2\ln(xc))+i)},\frac{\sqrt{2}}{2}\right)}{2\cosh(2\ln(xc))\sqrt{\sinh(2\ln(xc))}}$	90

input
```
int(csch(2*ln(x*c))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*(-I*(sinh(2*ln(x*c))+I))^(1/2)*2^(1/2)*(-I*(-sinh(2*ln(x*c))+I))^(1/
2)*(I*sinh(2*ln(x*c)))^(1/2)*EllipticF((-I*(sinh(2*ln(x*c))+I))^(1/2),1/2*
2^(1/2))/cosh(2*ln(x*c))/sinh(2*ln(x*c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx = -2\sqrt{-\frac{1}{2}}F(\arcsin{(cx)}\,|-1)$$

input
```
integrate(csch(2*log(c*x))^(1/2)/x,x, algorithm="fricas")
```

output
```
-2*sqrt(-1/2)*elliptic_f(arcsin(c*x), -1)
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx = \int \frac{\sqrt{\operatorname{csch}{(2\log{(cx)})}}}{x}\,dx$$

input
```
integrate(csch(2*ln(c*x))**(1/2)/x,x)
```

output
```
Integral(sqrt(csch(2*log(c*x)))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx = \int \frac{\sqrt{\operatorname{csch}{(2\ \log{(cx)})}}}{x}\,dx$$

input
```
integrate(csch(2*log(c*x))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(csch(2*log(c*x)))/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx = \text{Timed out}$$

input
```
integrate(csch(2*log(c*x))^(1/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx = \int \frac{\sqrt{\frac{1}{\sinh(2\ln(c\,x))}}}{x}\,dx$$

input
```
int((1/sinh(2*log(c*x)))^(1/2)/x,x)
```

output
```
int((1/sinh(2*log(c*x)))^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x}dx$$

input
```
int(csch(2*log(c*x))^(1/2)/x,x)
```

output
```
int(sqrt(csch(2*log(c*x)))/x,x)
```

3.139 $\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx = -\frac{1}{2}c^2\sqrt{1-\frac{1}{c^4x^4}}\,x\csc^{-1}\left(c^2x^2\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

output
```
-1/2*c^2*(1-1/c^4/x^4)^(1/2)*x*arccsc(c^2*x^2)*csch(2*ln(c*x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx = \frac{\sqrt{-1+c^4x^4}\sqrt{\frac{c^2x^2}{-2+2c^4x^4}}\arctan\left(\sqrt{-1+c^4x^4}\right)}{x}$$

input
```
Integrate[Sqrt[Csch[2*Log[c*x]]]/x^2,x]
```

output
```
(Sqrt[-1 + c^4*x^4]*Sqrt[(c^2*x^2)/(-2 + 2*c^4*x^4)]*ArcTan[Sqrt[-1 + c^4*
x^4]])/x
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6086, 6084, 858, 807, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx$$

$$\Big\downarrow \text{ 6086}$$

$$c\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{c^2x^2}d(cx)$$

$$\Big\downarrow \text{ 6084}$$

$$c^2x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}\int \frac{1}{c^3\sqrt{1-\frac{1}{c^4x^4}}x^3}d(cx)$$

$$\Big\downarrow \text{ 858}$$

$$-c^2x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}\int \frac{1}{cx\sqrt{1-c^4x^4}}d\frac{1}{cx}$$

$$\Big\downarrow \text{ 807}$$

$$-\frac{1}{2}c^2x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}\int \frac{1}{\sqrt{1-c^2x^2}}d(c^2x^2)$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{1}{2}c^2x\sqrt{1-\frac{1}{c^4x^4}}\arcsin\left(c^2x^2\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

input
```
Int[Sqrt[Csch[2*Log[c*x]]]/x^2,x]
```

output
```
-1/2*(c^2*Sqrt[1 - 1/(c^4*x^4)]*x*ArcSin[c^2*x^2]*Sqrt[Csch[2*Log[c*x]]])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6084
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6086
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\ln(xc))}}{x^2}dx$$

input
```
int(csch(2*ln(x*c))^(1/2)/x^2,x)
```

output
```
int(csch(2*ln(x*c))^(1/2)/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx = \frac{1}{2}\sqrt{2}c\arctan\left(\frac{(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{cx}\right)$$

input
```
integrate(csch(2*log(c*x))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*c*arctan((c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c*x))
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx = \int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx$$

input
```
integrate(csch(2*ln(c*x))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(csch(2*log(c*x)))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx = \int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx$$

input
```
integrate(csch(2*log(c*x))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(csch(2*log(c*x)))/x^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate(csch(2*log(c*x))^(1/2)/x^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx = \int \frac{\sqrt{\frac{1}{\sinh(2\ln(c\,x))}}}{x^2}\,dx$$

input
```
int((1/sinh(2*log(c*x)))^(1/2)/x^2,x)
```

output
```
int((1/sinh(2*log(c*x)))^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x^2}dx$$

input
```
int(csch(2*log(c*x))^(1/2)/x^2,x)
```

output
```
int(sqrt(csch(2*log(c*x)))/x**2,x)
```

3.140 $\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx = -c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}E\left(\csc^{-1}(cx)\middle|-1\right) + c^3\sqrt{1-\frac{1}{c^4x^4}}x\sqrt{\operatorname{csch}(2\log(cx))}\operatorname{EllipticF}\left(\csc^{-1}(cx),-1\right)$$

output
```
-c^3*(1-1/c^4/x^4)^(1/2)*x*csch(2*ln(c*x))^(1/2)*EllipticE(1/c/x,I)+c^3*(1
-1/c^4/x^4)^(1/2)*x*csch(2*ln(c*x))^(1/2)*InverseJacobiAM(arccsc(c*x),I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx = -\frac{\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},c^4x^4\right)}{x^2}$$

input
```
Integrate[Sqrt[Csch[2*Log[c*x]]]/x^3,x]
```

output
```
-((Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Hypergeometric2F1[-1
/4, 1/2, 3/4, c^4*x^4])/x^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.73, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {6086, 6084, 858, 836, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\text{csch}(2\log(cx))}}{x^3}\,dx$$

$$\Big\downarrow \ 6086$$

$$c^2\int \frac{\sqrt{\text{csch}(2\log(cx))}}{c^3x^3}d(cx)$$

$$\Big\downarrow \ 6084$$

$$c^3x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}\int \frac{1}{c^4\sqrt{1-\frac{1}{c^4x^4}}x^4}d(cx)$$

$$\Big\downarrow \ 858$$

$$-c^3x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}\int \frac{c^2x^2}{\sqrt{1-c^4x^4}}d\frac{1}{cx}$$

$$\Big\downarrow \ 836$$

$$-c^3x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}\left(\int \frac{c^2x^2+1}{\sqrt{1-c^4x^4}}d\frac{1}{cx}-\int \frac{1}{\sqrt{1-c^4x^4}}d\frac{1}{cx}\right)$$

$$\Big\downarrow \ 762$$

$$-c^3x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}\left(\int \frac{c^2x^2+1}{\sqrt{1-c^4x^4}}d\frac{1}{cx}-\text{EllipticF}\left(\arcsin\left(\frac{1}{cx}\right),-1\right)\right)$$

$$\Big\downarrow \ 1388$$

$$-c^3x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}\left(\int \frac{\sqrt{c^2x^2+1}}{\sqrt{1-c^2x^2}}d\frac{1}{cx}-\text{EllipticF}\left(\arcsin\left(\frac{1}{cx}\right),-1\right)\right)$$

$$\Big\downarrow \text{327}$$

$$-c^3x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}\left(E\left(\arcsin\left(\frac{1}{cx}\right)\middle|-1\right)-\text{EllipticF}\left(\arcsin\left(\frac{1}{cx}\right),-1\right)\right)$$

input
```
Int[Sqrt[Csch[2*Log[c*x]]]/x^3,x]
```

output
```
-(c^3*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]]*(EllipticE[ArcSin[1/(
c*x)], -1] - EllipticF[ArcSin[1/(c*x)], -1]))
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 6084
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6086
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.70

method	result	size
risch	$\frac{(c^4x^4-1)\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{x^2}-\frac{c^2\sqrt{c^2x^2+1}\,\sqrt{-c^2x^2+1}\left(\operatorname{EllipticF}\left(x\sqrt{-c^2},i\right)-\operatorname{EllipticE}\left(x\sqrt{-c^2},i\right)\right)\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{\sqrt{-c^2}\,x}$	126

input
```
int(csch(2*ln(x*c))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
(c^4*x^4-1)/x^2*2^(1/2)*(c^2*x^2/(c^4*x^4-1))^(1/2)-c^2/(-c^2)^(1/2)*(c^2*
x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(EllipticF(x*(-c^2)^(1/2),I)-EllipticE(x*(
-c^2)^(1/2),I))*2^(1/2)*(c^2*x^2/(c^4*x^4-1))^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx = \frac{2\sqrt{-\frac{1}{2}}c^4x^2E(\arcsin(cx)\,|-1)-2\sqrt{-\frac{1}{2}}c^4x^2F(\arcsin(cx)\,|-1)+\sqrt{2}(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{x^2}$$

input
```
integrate(csch(2*log(c*x))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
(2*sqrt(-1/2)*c^4*x^2*elliptic_e(arcsin(c*x), -1) - 2*sqrt(-1/2)*c^4*x^2*e
lliptic_f(arcsin(c*x), -1) + sqrt(2)*(c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 -
 1)))/x^2
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x^3}\,dx$$

input
```
integrate(csch(2*ln(c*x))**(1/2)/x**3,x)
```

output
```
Integral(sqrt(csch(2*log(c*x)))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}{x^3}\,dx$$

input
```
integrate(csch(2*log(c*x))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(csch(2*log(c*x)))/x^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(csch(2*log(c*x))^(1/2)/x^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx = \int \frac{\sqrt{\frac{1}{\sinh(2\,\ln(c\,x))}}}{x^3}\,dx$$

input
```
int((1/sinh(2*log(c*x)))^(1/2)/x^3,x)
```

output
```
int((1/sinh(2*log(c*x)))^(1/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x^3}dx$$

input
```
int(csch(2*log(c*x))^(1/2)/x^3,x)
```

output
```
int(sqrt(csch(2*log(c*x)))/x**3,x)
```

3.141 $\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4} dx = \frac{1}{2}\left(c^4 - \frac{1}{x^4}\right) x\sqrt{\operatorname{csch}(2\log(cx))}$$

output
```
1/2*(c^4-1/x^4)*x*csch(2*ln(c*x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4} dx = \frac{c^2}{2x\sqrt{\frac{c^2x^2}{-2+2c^4x^4}}}$$

input
```
Integrate[Sqrt[Csch[2*Log[c*x]]]/x^4,x]
```

output
```
c^2/(2*x*Sqrt[(c^2*x^2)/(-2 + 2*c^4*x^4)])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6086, 6084, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4}\,dx$$

$$\Big\downarrow \text{ 6086}$$

$$c^3\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{c^4x^4}d(cx)$$

$$\Big\downarrow \text{ 6084}$$

$$c^4x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}\int \frac{1}{c^5\sqrt{1-\frac{1}{c^4x^4}}x^5}d(cx)$$

$$\Big\downarrow \text{ 793}$$

$$\frac{1}{2}c^4x\left(1-\frac{1}{c^4x^4}\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

input
```
Int[Sqrt[Csch[2*Log[c*x]]]/x^4,x]
```

output
```
(c^4*(1 - 1/(c^4*x^4))*x*Sqrt[Csch[2*Log[c*x]]])/2
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 6084
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6086
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

method	result	size
risch	$\frac{(c^4x^4-1)\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{2x^3}$	38

input
```
int(csch(2*ln(x*c))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/2*(c^4*x^4-1)/x^3*2^(1/2)*(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4}\,dx = \frac{\sqrt{2}(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{2\,x^3}$$

input
```
integrate(csch(2*log(c*x))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*(c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/x^3
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x^4}\,dx$$

input
```
integrate(csch(2*ln(c*x))**(1/2)/x**4,x)
```

output
```
Integral(sqrt(csch(2*log(c*x)))/x**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(21) = 42$.

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.56

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4}\,dx = \frac{1}{2}c^3\left(\frac{\sqrt{2}}{\sqrt{\frac{1}{cx}+1}\sqrt{-\frac{1}{cx}+1}\sqrt{\frac{1}{c^2x^2}+1}} - \frac{\sqrt{2}}{c^4x^4\sqrt{\frac{1}{cx}+1}\sqrt{-\frac{1}{cx}+1}\sqrt{\frac{1}{c^2x^2}+1}}\right)$$

input
```
integrate(csch(2*log(c*x))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
1/2*c^3*(sqrt(2)/(sqrt(1/(c*x) + 1)*sqrt(-1/(c*x) + 1)*sqrt(1/(c^2*x^2) +
1)) - sqrt(2)/(c^4*x^4*sqrt(1/(c*x) + 1)*sqrt(-1/(c*x) + 1)*sqrt(1/(c^2*x^
2) + 1)))
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(csch(2*log(c*x))^(1/2)/x^4,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 2.81 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4}\,dx = \frac{c^4\,x\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4-1}}}{2} - \frac{\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4-1}}}{2\,x^3}$$

input
```
int((1/sinh(2*log(c*x)))^(1/2)/x^4,x)
```

output
```
(c^4*x*((2*c^2*x^2)/(c^4*x^4 - 1))^(1/2))/2 - ((2*c^2*x^2)/(c^4*x^4 - 1))^
(1/2)/(2*x^3)
```

Reduce [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x^4}dx$$

input
```
int(csch(2*log(c*x))^(1/2)/x^4,x)
```

output
```
int(sqrt(csch(2*log(c*x)))/x**4,x)
```

3.142 $\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 64

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^5}\,dx = \frac{1}{3}\left(c^4-\frac{1}{x^4}\right)\sqrt{\operatorname{csch}(2\log(cx))} - \frac{1}{3}c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\operatorname{EllipticF}\left(\csc^{-1}(cx),-1\right)$$

output
```
1/3*(c^4-1/x^4)*csch(2*ln(c*x))^(1/2)-1/3*c^5*(1-1/c^4/x^4)^(1/2)*x*csch(2
*ln(c*x))^(1/2)*InverseJacobiAM(arccsc(c*x),I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^5}\,dx = -\frac{\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},c^4x^4\right)}{3x^4}$$

input
```
Integrate[Sqrt[Csch[2*Log[c*x]]]/x^5,x]
```

output
```
-1/3*(Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Hypergeometric2F1
[-3/4, 1/2, 1/4, c^4*x^4])/x^4
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6086, 6084, 858, 843, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\text{csch}(2\log(cx))}}{x^5}dx$$

$$\Big\downarrow \text{6086}$$

$$c^4\int \frac{\sqrt{\text{csch}(2\log(cx))}}{c^5x^5}d(cx)$$

$$\Big\downarrow \text{6084}$$

$$c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}\int \frac{1}{c^6\sqrt{1-\frac{1}{c^4x^4}}x^6}d(cx)$$

$$\Big\downarrow \text{858}$$

$$-c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}\int \frac{c^4x^4}{\sqrt{1-c^4x^4}}d\frac{1}{cx}$$

$$\Big\downarrow \text{843}$$

$$-c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}\left(\frac{1}{3}\int \frac{1}{\sqrt{1-c^4x^4}}d\frac{1}{cx}-\frac{\sqrt{1-c^4x^4}}{3cx}\right)$$

$$\Big\downarrow \text{762}$$

$$-c^5x\sqrt{1-\frac{1}{c^4x^4}}\left(\frac{1}{3}\text{EllipticF}\left(\arcsin\left(\frac{1}{cx}\right),-1\right)-\frac{\sqrt{1-c^4x^4}}{3cx}\right)\sqrt{\text{csch}(2\log(cx))}$$

input
```
Int[Sqrt[Csch[2*Log[c*x]]]/x^5,x]
```

output
```
-(c^5*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]]*(-1/3*Sqrt[1 - c^4*x^
4]/(c*x) + EllipticF[ArcSin[1/(c*x)], -1]/3))
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6084
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6086
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(55) = 110$.

Time = 0.35 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.75

method	result	size
risch	$\frac{\left(c^{4} x^{4}-1\right) \sqrt{2}\, \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}{3 x^{4}}+\frac{c^{4} \sqrt{c^{2} x^{2}+1}\, \sqrt{-c^{2} x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{-c^{2}}, i\right) \sqrt{2}\, \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}{3 \sqrt{-c^{2}}\, x}$	112

input
```
int(csch(2*ln(x*c))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/3*(c^4*x^4-1)/x^4*2^(1/2)*(c^2*x^2/(c^4*x^4-1))^(1/2)+1/3*c^4/(-c^2)^(1/
2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)*EllipticF(x*(-c^2)^(1/2),I)*2^(1/2
)*(c^2*x^2/(c^4*x^4-1))^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^5}\,dx = -\frac{2\,\sqrt{-\frac{1}{2}}c^4x^4F(\arcsin{(cx)}\mid -1) - \sqrt{2}(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{3\,x^4}$$

input
```
integrate(csch(2*log(c*x))^(1/2)/x^5,x, algorithm="fricas")
```

output
```
-1/3*(2*sqrt(-1/2)*c^4*x^4*elliptic_f(arcsin(c*x), -1) - sqrt(2)*(c^4*x^4
- 1)*sqrt(c^2*x^2/(c^4*x^4 - 1)))/x^4
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^5}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x^5}\,dx$$

input
```
integrate(csch(2*ln(c*x))**(1/2)/x**5,x)
```

output
```
Integral(sqrt(csch(2*log(c*x)))/x**5, x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^5}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\,\log\left(cx\right)\right)}}{x^5}\,dx$$

input
```
integrate(csch(2*log(c*x))^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(csch(2*log(c*x)))/x^5, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate(csch(2*log(c*x))^(1/2)/x^5,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^5}\,dx = \int \frac{\sqrt{\frac{1}{\sinh(2\ln(c\,x))}}}{x^5}\,dx$$

input
```
int((1/sinh(2*log(c*x)))^(1/2)/x^5,x)
```

output
```
int((1/sinh(2*log(c*x)))^(1/2)/x^5, x)
```

Reduce [F]

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^5}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x^5}dx$$

input
```
int(csch(2*log(c*x))^(1/2)/x^5,x)
```

output
```
int(sqrt(csch(2*log(c*x)))/x**5,x)
```

3.143 $\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 128

$$\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{x}{32c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{32c^{12}\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

output
```
1/32*x/c^4/(c^4-1/x^4)/csch(2*ln(c*x))^(3/2)-1/16*x^5/(c^4-1/x^4)/csch(2*l
n(c*x))^(3/2)+1/12*x^9/csch(2*ln(c*x))^(3/2)+1/32*arctanh((1-1/c^4/x^4)^(1
/2))/c^12/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.74

$$\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{c^3x^3\sqrt{1-c^4x^4}(3-14c^4x^4+8c^8x^8)-3cx\arcsin(c^2x^2)}{192c^9\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

input
```
Integrate[x^8/Csch[2*Log[c*x]]^(3/2),x]
```

output
```
(c^3*x^3*Sqrt[1 - c^4*x^4]*(3 - 14*c^4*x^4 + 8*c^8*x^8) - 3*c*x*ArcSin[c^2
*x^2])/(192*c^9*Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {6086, 6084, 798, 51, 51, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}dx$$

$$\Big\downarrow \text{6086}$$

$$\frac{\int \frac{c^8x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}d(cx)}{c^9}$$

$$\Big\downarrow \text{6084}$$

$$\frac{\int c^{11}\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^{11}d(cx)}{c^{12}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{798}$$

$$-\frac{\int \frac{\left(1-\frac{1}{c^4x^4}\right)^{3/2}}{c^4x^4}d\frac{1}{c^4x^4}}{4c^{12}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{51}$$

$$-\frac{-\frac{1}{2}\int \frac{\sqrt{1-\frac{1}{c^4x^4}}}{c^3x^3}d\frac{1}{c^4x^4}-\frac{\left(1-\frac{1}{c^4x^4}\right)^{3/2}}{3c^3x^3}}{4c^{12}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{51}$$

$$-\frac{\frac{1}{2}\left(\frac{1}{4}\int \frac{1}{c^2\sqrt{1-\frac{1}{c^4x^4}}x^2}d\frac{1}{c^4x^4}+\frac{\sqrt{1-\frac{1}{c^4x^4}}}{2c^2x^2}\right)-\frac{\left(1-\frac{1}{c^4x^4}\right)^{3/2}}{3c^3x^3}}{4c^{12}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{52}$$

$$-\frac{\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{c\sqrt{1-\frac{1}{c^4x^4}}x}d\frac{1}{c^4x^4}-\frac{\sqrt{1-\frac{1}{c^4x^4}}}{cx}\right)+\frac{\sqrt{1-\frac{1}{c^4x^4}}}{2c^2x^2}\right)-\frac{\left(1-\frac{1}{c^4x^4}\right)^{3/2}}{3c^3x^3}}{4c^{12}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{1}{2}\left(\frac{1}{4}\left(-\int \frac{1}{1-c^2x^2}d\sqrt{1-\frac{1}{c^4x^4}}-\frac{\sqrt{1-\frac{1}{c^4x^4}}}{cx}\right)+\frac{\sqrt{1-\frac{1}{c^4x^4}}}{2c^2x^2}\right)-\frac{\left(1-\frac{1}{c^4x^4}\right)^{3/2}}{3c^3x^3}}{4c^{12}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{1}{2}\left(\frac{1}{4}\left(-\text{arctanh}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)-\frac{\sqrt{1-\frac{1}{c^4x^4}}}{cx}\right)+\frac{\sqrt{1-\frac{1}{c^4x^4}}}{2c^2x^2}\right)-\frac{\left(1-\frac{1}{c^4x^4}\right)^{3/2}}{3c^3x^3}}{4c^{12}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

input
```
Int[x^8/Csch[2*Log[c*x]]^(3/2),x]
```

output
```
-1/4*(-1/3*(1 - 1/(c^4*x^4))^(3/2)/(c^3*x^3) + (Sqrt[1 - 1/(c^4*x^4)]/(2*c
^2*x^2) + (-(Sqrt[1 - 1/(c^4*x^4)]/(c*x)) - ArcTanh[Sqrt[1 - 1/(c^4*x^4)]]
)/4)/2)/(c^12*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^(3/2))
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6084
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6086
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{x^{3}\left(8 c^{8} x^{8}-14 c^{4} x^{4}+3\right) \sqrt{2}}{384 c^{6} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}+\frac{\ln \left(\frac{c^{4} x^{2}}{\sqrt{c^{4}}}+\sqrt{c^{4} x^{4}-1}\right) \sqrt{2}\, x}{128 c^{6} \sqrt{c^{4}}\, \sqrt{c^{4} x^{4}-1}\, \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}$	121

input
```
int(x^8/csch(2*ln(x*c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/384*x^3*(8*c^8*x^8-14*c^4*x^4+3)/c^6*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)
+1/128/c^6*ln(c^4*x^2/(c^4)^(1/2)+(c^4*x^4-1)^(1/2))/(c^4)^(1/2)*2^(1/2)*x
/(c^4*x^4-1)^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.86

$$\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{2\sqrt{2}(8\,c^{13}x^{13}-22\,c^9x^9+17\,c^5x^5-3\,cx)\sqrt{\frac{c^2x^2}{c^4x^4-1}}+3\sqrt{2}\log\left(2\,c^4x^4+2\,(c^5x^5-cx)\sqrt{\frac{c^2x^2}{c^4x^4-1}}-1\right)}{768\,c^9}$$

input
```
integrate(x^8/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

output
```
1/768*(2*sqrt(2)*(8*c^13*x^13 - 22*c^9*x^9 + 17*c^5*x^5 - 3*c*x)*sqrt(c^2*
x^2/(c^4*x^4 - 1)) + 3*sqrt(2)*log(2*c^4*x^4 + 2*(c^5*x^5 - c*x)*sqrt(c^2*
x^2/(c^4*x^4 - 1)) - 1))/c^9
```

Sympy [F]

$$\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$$

input
```
integrate(x**8/csch(2*ln(c*x))**(3/2),x)
```

output
```
Integral(x**8/csch(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^8}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^8/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^8/csch(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

input
```
integrate(x^8/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^8}{{\left(\frac{1}{\sinh\left(2\,\ln\left(c\,x\right)\right)}\right)}^{3/2}}\,dx$$

input
```
int(x^8/(1/sinh(2*log(c*x)))^(3/2),x)
```

output
```
int(x^8/(1/sinh(2*log(c*x)))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}\,x^8}{\operatorname{csch}\left(2\log\left(cx\right)\right)^2}dx$$

input
```
int(x^8/csch(2*log(c*x))^(3/2),x)
```

output
```
int((sqrt(csch(2*log(c*x)))*x**8)/csch(2*log(c*x))**2,x)
```

3.144 $\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 118

$$\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{4}{77c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{6x^4}{77\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{4\operatorname{EllipticF}\left(\csc^{-1}(cx),-1\right)}{77c^{11}\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

output
```
4/77/c^4/(c^4-1/x^4)/csch(2*ln(c*x))^(3/2)-6/77*x^4/(c^4-1/x^4)/csch(2*ln(
c*x))^(3/2)+1/11*x^8/csch(2*ln(c*x))^(3/2)-4/77*InverseJacobiAM(arccsc(c*x
),I)/c^11/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.68

$$\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{x^2\left(\left(1-c^4x^4\right)^{5/2}-\operatorname{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},c^4x^4\right)\right)}{22c^6\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

input
```
Integrate[x^7/Csch[2*Log[c*x]]^(3/2),x]
```

output
```
(x^2*((1 - c^4*x^4)^(5/2) - Hypergeometric2F1[-3/2, 1/4, 5/4, c^4*x^4]))/(
22*c^6*Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {6086, 6084, 858, 809, 809, 847, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}dx$$

$$\Big\downarrow \text{6086}$$

$$\frac{\int \frac{c^7x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}d(cx)}{c^8}$$

$$\Big\downarrow \text{6084}$$

$$\frac{\int c^{10}\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^{10}d(cx)}{c^{11}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{858}$$

$$-\frac{\int \frac{\left(1-c^4x^4\right)^{3/2}}{c^{12}x^{12}}d\frac{1}{cx}}{c^{11}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{809}$$

$$-\frac{-\frac{6}{11}\int \frac{\sqrt{1-c^4x^4}}{c^8x^8}d\frac{1}{cx}-\frac{\left(1-c^4x^4\right)^{3/2}}{11c^{11}x^{11}}}{c^{11}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{809}$$

$$-\frac{-\frac{6}{11}\left(-\frac{2}{7}\int \frac{1}{c^4x^4\sqrt{1-c^4x^4}}d\frac{1}{cx}-\frac{\sqrt{1-c^4x^4}}{7c^7x^7}\right)-\frac{\left(1-c^4x^4\right)^{3/2}}{11c^{11}x^{11}}}{c^{11}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \ 847$$

$$-\frac{-\frac{6}{11}\left(-\frac{2}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{1-c^4x^4}}d\frac{1}{cx}-\frac{\sqrt{1-c^4x^4}}{3c^3x^3}\right)-\frac{\sqrt{1-c^4x^4}}{7c^7x^7}\right)-\frac{\left(1-c^4x^4\right)^{3/2}}{11c^{11}x^{11}}}{c^{11}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \ 762$$

$$-\frac{-\frac{6}{11}\left(-\frac{2}{7}\left(\frac{1}{3}\text{EllipticF}\left(\arcsin\left(\frac{1}{cx}\right),-1\right)-\frac{\sqrt{1-c^4x^4}}{3c^3x^3}\right)-\frac{\sqrt{1-c^4x^4}}{7c^7x^7}\right)-\frac{\left(1-c^4x^4\right)^{3/2}}{11c^{11}x^{11}}}{c^{11}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

input
```
Int[x^7/Csch[2*Log[c*x]]^(3/2),x]
```

output
```
-((-1/11*(1 - c^4*x^4)^(3/2)/(c^11*x^11) - (6*(-1/7*Sqrt[1 - c^4*x^4]/(c^7
*x^7) - (2*(-1/3*Sqrt[1 - c^4*x^4]/(c^3*x^3) + EllipticF[ArcSin[1/(c*x)],
-1]/3))/7))/11)/(c^11*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^(3/2)))
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6084
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6086
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.13

method	result	size
risch	$\frac{x^{2} \left(7 c^{8} x^{8}-13 c^{4} x^{4}+4\right) \sqrt{2}}{308 c^{6} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}+\frac{\sqrt{c^{2} x^{2}+1}\, \sqrt{-c^{2} x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{-c^{2}}, i\right) \sqrt{2}\, x}{77 c^{6} \sqrt{-c^{2}}\, \left(c^{4} x^{4}-1\right) \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}$	133

input
```
int(x^7/csch(2*ln(x*c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/308*x^2*(7*c^8*x^8-13*c^4*x^4+4)/c^6*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)
+1/77/c^6/(-c^2)^(1/2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*El
lipticF(x*(-c^2)^(1/2),I)*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.68

$$\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\sqrt{2}(7\,c^{14}x^{12} - 20\,c^{10}x^8 + 17\,c^6x^4 - 4\,c^2)\sqrt{\frac{c^2x^2}{c^4x^4-1}} - 8\,\sqrt{\frac{1}{2}}\sqrt{c^4}F(\arcsin\left(\frac{1}{cx}\right)\,|-1)}{308\,c^{10}}$$

input
```
integrate(x^7/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

output
```
1/308*(sqrt(2)*(7*c^14*x^12 - 20*c^10*x^8 + 17*c^6*x^4 - 4*c^2)*sqrt(c^2*x
^2/(c^4*x^4 - 1)) - 8*sqrt(1/2)*sqrt(c^4)*elliptic_f(arcsin(1/(c*x)), -1))
/c^10
```

Sympy [F]

$$\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$$

input
```
integrate(x**7/csch(2*ln(c*x))**(3/2),x)
```

output
```
Integral(x**7/csch(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^7}{\operatorname{csch}(2\log(cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^7/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^7/csch(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

input
```
integrate(x^7/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^7}{\left(\frac{1}{\sinh(2\ln(cx))}\right)^{3/2}}\,dx$$

input
```
int(x^7/(1/sinh(2*log(c*x)))^(3/2),x)
```

output
```
int(x^7/(1/sinh(2*log(c*x)))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}\,x^7}{\operatorname{csch}\left(2\log\left(cx\right)\right)^2}dx$$

input
```
int(x^7/csch(2*log(c*x))^(3/2),x)
```

output
```
int((sqrt(csch(2*log(c*x)))*x**7)/csch(2*log(c*x))**2,x)
```

3.145 $\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} \, dx = \frac{\left(c^4 - \frac{1}{x^4}\right) x^7}{10c^4 \operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

output
```
1/10*(c^4-1/x^4)*x^7/c^4/csch(2*ln(c*x))^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} \, dx = \frac{(-1 + c^4x^4)^3 \sqrt{\frac{c^2x^2}{-2+2c^4x^4}}}{20c^8x}$$

input
```
Integrate[x^6/Csch[2*Log[c*x]]^(3/2),x]
```

output
```
((-1 + c^4*x^4)^3*Sqrt[(c^2*x^2)/(-2 + 2*c^4*x^4)])/(20*c^8*x)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6086, 6084, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}dx$$

$$\Big\downarrow \text{6086}$$

$$\frac{\int \frac{c^6x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}d(cx)}{c^7}$$

$$\Big\downarrow \text{6084}$$

$$\frac{\int c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^9d(cx)}{c^{10}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{796}$$

$$\frac{x^7\left(1-\frac{1}{c^4x^4}\right)}{10\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

input
```
Int[x^6/Csch[2*Log[c*x]]^(3/2),x]
```

output
```
((1 - 1/(c^4*x^4))*x^7)/(10*Csch[2*Log[c*x]]^(3/2))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 6084
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6086
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.57

method	result	size
risch	$\frac{\sqrt{2}\, x \left(c^{8} x^{8}-2 c^{4} x^{4}+1\right)}{40 c^{6} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}$	47

input
```
int(x^6/csch(2*ln(x*c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/40*2^(1/2)/c^6*x/(c^2*x^2/(c^4*x^4-1))^(1/2)*(c^8*x^8-2*c^4*x^4+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(26) = 52.

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.87

$$\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\, dx = \frac{\sqrt{2}(c^{12}x^{12} - 3\, c^8x^8 + 3\, c^4x^4 - 1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{40\, c^8x}$$

input
```
integrate(x^6/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

output
```
1/40*sqrt(2)*(c^12*x^12 - 3*c^8*x^8 + 3*c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4
 - 1))/(c^8*x)
```

Sympy [F]

$$\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$$

input
```
integrate(x**6/csch(2*ln(c*x))**(3/2),x)
```

output
```
Integral(x**6/csch(2*log(c*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\left(\sqrt{2}c^4x^4-\sqrt{2}\right)(c^2x^2+1)^{\frac{3}{2}}(cx+1)^{\frac{3}{2}}(cx-1)^{\frac{3}{2}}}{40\,c^7}$$

input
```
integrate(x^6/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

output
```
1/40*(sqrt(2)*c^4*x^4 - sqrt(2))*(c^2*x^2 + 1)^(3/2)*(c*x + 1)^(3/2)*(c*x
- 1)^(3/2)/c^7
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

input
```
integrate(x^6/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{(c^4x^4-1)^3\sqrt{\frac{2c^2x^2}{c^4x^4-1}}}{40c^8x}$$

input
```
int(x^6/(1/sinh(2*log(c*x)))^(3/2),x)
```

output
```
((c^4*x^4 - 1)^3*((2*c^2*x^2)/(c^4*x^4 - 1))^(1/2))/(40*c^8*x)
```

Reduce [F]

$$\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{\sqrt{\operatorname{csch}(2\log(cx))}\,x^6}{\operatorname{csch}(2\log(cx))^2}dx$$

input
```
int(x^6/csch(2*log(c*x))^(3/2),x)
```

output
```
int((sqrt(csch(2*log(c*x)))*x**6)/csch(2*log(c*x))**2,x)
```

3.146 $\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 162

$$\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{4}{15c^4\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{4E(\csc^{-1}(cx)|-1)}{15c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{4\operatorname{EllipticF}\left(\csc^{-1}(cx),-1\right)}{15c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

output
```
4/15/c^4/(c^4-1/x^4)/x^2/csch(2*ln(c*x))^(3/2)-2/15*x^2/(c^4-1/x^4)/csch(2
*ln(c*x))^(3/2)+1/9*x^6/csch(2*ln(c*x))^(3/2)+4/15*EllipticE(1/c/x,I)/c^9/
(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)-4/15*InverseJacobiAM(arccsc(
c*x),I)/c^9/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.39

$$\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx=-\frac{x^4\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},c^4x^4\right)}{6c^2\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

input
```
Integrate[x^5/Csch[2*Log[c*x]]^(3/2),x]
```

output
```
-1/6*(x^4*Hypergeometric2F1[-3/2, 3/4, 7/4, c^4*x^4])/(c^2*Sqrt[2 - 2*c^4*
x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6086, 6084, 858, 809, 809, 847, 836, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}dx$$

$$\Big\downarrow \text{ 6086}$$

$$\frac{\int \frac{c^5x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}d(cx)}{c^6}$$

$$\Big\downarrow \text{ 6084}$$

$$\frac{\int c^8\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^8d(cx)}{c^9x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{ 858}$$

$$-\frac{\int \frac{\left(1-c^4x^4\right)^{3/2}}{c^{10}x^{10}}d\frac{1}{cx}}{c^9x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{809}$$

$$-\frac{-\frac{2}{3}\int \frac{\sqrt{1-c^4x^4}}{c^6x^6}d\frac{1}{cx}-\frac{\left(1-c^4x^4\right)^{3/2}}{9c^9x^9}}{c^9x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{809}$$

$$-\frac{-\frac{2}{3}\left(-\frac{2}{5}\int \frac{1}{c^2x^2\sqrt{1-c^4x^4}}d\frac{1}{cx}-\frac{\sqrt{1-c^4x^4}}{5c^5x^5}\right)-\frac{\left(1-c^4x^4\right)^{3/2}}{9c^9x^9}}{c^9x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{847}$$

$$-\frac{-\frac{2}{3}\left(-\frac{2}{5}\left(-\int \frac{c^2x^2}{\sqrt{1-c^4x^4}}d\frac{1}{cx}-\frac{\sqrt{1-c^4x^4}}{cx}\right)-\frac{\sqrt{1-c^4x^4}}{5c^5x^5}\right)-\frac{\left(1-c^4x^4\right)^{3/2}}{9c^9x^9}}{c^9x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{836}$$

$$-\frac{-\frac{2}{3}\left(-\frac{2}{5}\left(\int \frac{1}{\sqrt{1-c^4x^4}}d\frac{1}{cx}-\int \frac{c^2x^2+1}{\sqrt{1-c^4x^4}}d\frac{1}{cx}-\frac{\sqrt{1-c^4x^4}}{cx}\right)-\frac{\sqrt{1-c^4x^4}}{5c^5x^5}\right)-\frac{\left(1-c^4x^4\right)^{3/2}}{9c^9x^9}}{c^9x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{762}$$

$$-\frac{-\frac{2}{3}\left(-\frac{2}{5}\left(-\int \frac{c^2x^2+1}{\sqrt{1-c^4x^4}}d\frac{1}{cx}+\text{EllipticF}\left(\arcsin\left(\frac{1}{cx}\right),-1\right)-\frac{\sqrt{1-c^4x^4}}{cx}\right)-\frac{\sqrt{1-c^4x^4}}{5c^5x^5}\right)-\frac{\left(1-c^4x^4\right)^{3/2}}{9c^9x^9}}{c^9x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{1388}$$

$$-\frac{-\frac{2}{3}\left(-\frac{2}{5}\left(-\int \frac{\sqrt{c^2x^2+1}}{\sqrt{1-c^2x^2}}d\frac{1}{cx}+\text{EllipticF}\left(\arcsin\left(\frac{1}{cx}\right),-1\right)-\frac{\sqrt{1-c^4x^4}}{cx}\right)-\frac{\sqrt{1-c^4x^4}}{5c^5x^5}\right)-\frac{\left(1-c^4x^4\right)^{3/2}}{9c^9x^9}}{c^9x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{327}$$

$$-\frac{-\frac{2}{3}\left(-\frac{2}{5}\left(\text{EllipticF}\left(\arcsin\left(\frac{1}{cx}\right),-1\right)-E\left(\arcsin\left(\frac{1}{cx}\right)\middle|-1\right)-\frac{\sqrt{1-c^4x^4}}{cx}\right)-\frac{\sqrt{1-c^4x^4}}{5c^5x^5}\right)-\frac{\left(1-c^4x^4\right)^{3/2}}{9c^9x^9}}{c^9x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

input
```
Int[x^5/Csch[2*Log[c*x]]^(3/2),x]
```

output
```
-((-1/9*(1 - c^4*x^4)^(3/2)/(c^9*x^9) - (2*(-1/5*Sqrt[1 - c^4*x^4]/(c^5*x^
5) - (2*(-(Sqrt[1 - c^4*x^4]/(c*x)) - EllipticE[ArcSin[1/(c*x)], -1] + Ell
ipticF[ArcSin[1/(c*x)], -1]))/5))/3)/(c^9*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch
[2*Log[c*x]]^(3/2)))
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 6084
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6086
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{x^{4}\left(5 c^{4} x^{4}-11\right) \sqrt{2}}{180 c^{2} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}+\frac{\sqrt{c^{2} x^{2}+1}\, \sqrt{-c^{2} x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \sqrt{-c^{2}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{-c^{2}}, i\right)\right) \sqrt{2}\, x}{15 \sqrt{-c^{2}}\, \left(c^{4} x^{4}-1\right) c^{4} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}$	140

input
```
int(x^5/csch(2*ln(x*c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/180*x^4*(5*c^4*x^4-11)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(1/2)+1/15/(-c^
2)^(1/2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)/c^4*(EllipticF(x
*(-c^2)^(1/2),I)-EllipticE(x*(-c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^4*x^4-
1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.64

$$\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\sqrt{2}(5\,c^{14}x^{12} - 16\,c^{10}x^8 + 23\,c^6x^4 - 12\,c^2)\sqrt{\frac{c^2x^2}{c^4x^4-1}} + 24\,\sqrt{\frac{1}{2}}\sqrt{c^4}\left(x^2E\left(\arcsin\left(\frac{1}{cx}\right)|-1\right) - x^2F\left(\arcsin\left(\frac{1}{cx}\right)|-1\right)\right)}{180\,c^{10}x^2}$$

input
```
integrate(x^5/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

output
```
1/180*(sqrt(2)*(5*c^14*x^12 - 16*c^10*x^8 + 23*c^6*x^4 - 12*c^2)*sqrt(c^2*
x^2/(c^4*x^4 - 1)) + 24*sqrt(1/2)*sqrt(c^4)*(x^2*elliptic_e(arcsin(1/(c*x)
), -1) - x^2*elliptic_f(arcsin(1/(c*x)), -1)))/(c^10*x^2)
```

Sympy [F]

$$\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$$

input
```
integrate(x**5/csch(2*ln(c*x))**(3/2),x)
```

output
```
Integral(x**5/csch(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^5}{\operatorname{csch}(2\log(cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^5/csch(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

input
```
integrate(x^5/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^5}{\left(\frac{1}{\sinh(2\ln(cx))}\right)^{3/2}}\,dx$$

input
```
int(x^5/(1/sinh(2*log(c*x)))^(3/2),x)
```

output
```
int(x^5/(1/sinh(2*log(c*x)))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{\sqrt{\operatorname{csch}(2\log(cx))}\,x^5}{\operatorname{csch}(2\log(cx))^2}dx$$

input
```
int(x^5/csch(2*log(c*x))^(3/2),x)
```

output
```
int((sqrt(csch(2*log(c*x)))*x**5)/csch(2*log(c*x))**2,x)
```

3.147 $\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 96

$$\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{3x}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^5}{8\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{16c^8\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

output
```
-3/16*x/(c^4-1/x^4)/csch(2*ln(c*x))^(3/2)+1/8*x^5/csch(2*ln(c*x))^(3/2)+3/
16*arctanh((1-1/c^4/x^4)^(1/2))/c^8/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x)
)^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{c^3x^3\sqrt{1-c^4x^4}(-5+2c^4x^4)-3cx\arcsin\left(c^2x^2\right)}{32c^5\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

input
```
Integrate[x^4/Csch[2*Log[c*x]]^(3/2),x]
```

output
```
(c^3*x^3*Sqrt[1 - c^4*x^4]*(-5 + 2*c^4*x^4) - 3*c*x*ArcSin[c^2*x^2])/(32*c
^5*Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {6086, 6084, 798, 51, 51, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}dx$$

$$\Big\downarrow \text{6086}$$

$$\frac{\int \frac{c^4x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}d(cx)}{c^5}$$

$$\Big\downarrow \text{6084}$$

$$\frac{\int c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^7d(cx)}{c^8x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{798}$$

$$-\frac{\int \frac{\left(1-\frac{1}{c^4x^4}\right)^{3/2}}{c^3x^3}d\frac{1}{c^4x^4}}{4c^8x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{51}$$

$$-\frac{-\frac{3}{4}\int \frac{\sqrt{1-\frac{1}{c^4x^4}}}{c^2x^2}d\frac{1}{c^4x^4}-\frac{\left(1-\frac{1}{c^4x^4}\right)^{3/2}}{2c^2x^2}}{4c^8x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{51}$$

$$-\frac{-\frac{3}{4}\left(-\frac{1}{2}\int \frac{1}{c\sqrt{1-\frac{1}{c^4x^4}}x}d\frac{1}{c^4x^4}-\frac{\sqrt{1-\frac{1}{c^4x^4}}}{cx}\right)-\frac{\left(1-\frac{1}{c^4x^4}\right)^{3/2}}{2c^2x^2}}{4c^8x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{73}$$

$$-\frac{-\frac{3}{4}\left(\int \frac{1}{1-c^2x^2}d\sqrt{1-\frac{1}{c^4x^4}}-\frac{\sqrt{1-\frac{1}{c^4x^4}}}{cx}\right)-\frac{\left(1-\frac{1}{c^4x^4}\right)^{3/2}}{2c^2x^2}}{4c^8x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-\frac{3}{4}\left(\text{arctanh}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)-\frac{\sqrt{1-\frac{1}{c^4x^4}}}{cx}\right)-\frac{\left(1-\frac{1}{c^4x^4}\right)^{3/2}}{2c^2x^2}}{4c^8x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

input
```
Int[x^4/Csch[2*Log[c*x]]^(3/2),x]
```

output
```
-1/4*(-1/2*(1 - 1/(c^4*x^4))^(3/2)/(c^2*x^2) - (3*(-(Sqrt[1 - 1/(c^4*x^4)]
/(c*x)) + ArcTanh[Sqrt[1 - 1/(c^4*x^4)]]))/4)/(c^8*(1 - 1/(c^4*x^4))^(3/2)
*x^3*Csch[2*Log[c*x]]^(3/2))
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6084
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6086
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.18

method	result	size
risch	$\frac{x^{3} \left(2 c^{4} x^{4}-5\right) \sqrt{2}}{64 c^{2} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}+\frac{3 \ln \left(\frac{c^{4} x^{2}}{\sqrt{c^{4}}}+\sqrt{c^{4} x^{4}-1}\right) \sqrt{2}\, x}{64 \sqrt{c^{4}}\, c^{2} \sqrt{c^{4} x^{4}-1}\, \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}$	113

input
```
int(x^4/csch(2*ln(x*c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/64*x^3*(2*c^4*x^4-5)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(1/2)+3/64*ln(c^4
*x^2/(c^4)^(1/2)+(c^4*x^4-1)^(1/2))/(c^4)^(1/2)*2^(1/2)/c^2*x/(c^4*x^4-1)^
(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06

$$\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{2\sqrt{2}(2c^9x^9 - 7c^5x^5 + 5cx)\sqrt{\frac{c^2x^2}{c^4x^4-1}} + 3\sqrt{2}\log\left(2c^4x^4 + 2(c^5x^5 - cx)\sqrt{\frac{c^2x^2}{c^4x^4-1}} - 1\right)}{128c^5}$$

input
```
integrate(x^4/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

output
```
1/128*(2*sqrt(2)*(2*c^9*x^9 - 7*c^5*x^5 + 5*c*x)*sqrt(c^2*x^2/(c^4*x^4 - 1
)) + 3*sqrt(2)*log(2*c^4*x^4 + 2*(c^5*x^5 - c*x)*sqrt(c^2*x^2/(c^4*x^4 - 1
)) - 1))/c^5
```

Sympy [F]

$$\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$$

input
```
integrate(x**4/csch(2*ln(c*x))**(3/2),x)
```

output
```
Integral(x**4/csch(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^4}{\operatorname{csch}(2\log(cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/csch(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

input
```
integrate(x^4/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^4}{\left(\frac{1}{\sinh(2\ln(c\,x))}\right)^{3/2}}\,dx$$

input
```
int(x^4/(1/sinh(2*log(c*x)))^(3/2),x)
```

output
```
int(x^4/(1/sinh(2*log(c*x)))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{\sqrt{\operatorname{csch}(2\log(cx))}\,x^4}{\operatorname{csch}(2\log(cx))^2}dx$$

input
```
int(x^4/csch(2*log(c*x))^(3/2),x)
```

output
```
int((sqrt(csch(2*log(c*x)))*x**4)/csch(2*log(c*x))**2,x)
```

3.148 $\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 86

$$\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{2}{7\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^4}{7\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{4\,\text{EllipticF}\left(\csc^{-1}(cx),-1\right)}{7c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

output
```
-2/7/(c^4-1/x^4)/csch(2*ln(c*x))^(3/2)+1/7*x^4/csch(2*ln(c*x))^(3/2)-4/7*I
nverseJacobiAM(arccsc(c*x),I)/c^7/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^
(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\sqrt{1-c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},c^4x^4\right)}{2\sqrt{2}c^4}$$

input
```
Integrate[x^3/Csch[2*Log[c*x]]^(3/2),x]
```

output
```
(Sqrt[1 - c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Hypergeometric2F1[-3/2,
1/4, 5/4, c^4*x^4])/(2*Sqrt[2]*c^4)
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6086, 6084, 858, 809, 809, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\text{csch}^{\frac{3}{2}}(2\log(cx))}dx$$

$$\Big\downarrow \text{6086}$$

$$\frac{\int \frac{c^3x^3}{\text{csch}^{\frac{3}{2}}(2\log(cx))}d(cx)}{c^4}$$

$$\Big\downarrow \text{6084}$$

$$\frac{\int c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^6d(cx)}{c^7x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{858}$$

$$-\frac{\int \frac{\left(1-c^4x^4\right)^{3/2}}{c^8x^8}d\frac{1}{cx}}{c^7x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{809}$$

$$-\frac{-\frac{6}{7}\int \frac{\sqrt{1-c^4x^4}}{c^4x^4}d\frac{1}{cx}-\frac{\left(1-c^4x^4\right)^{3/2}}{7c^7x^7}}{c^7x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{809}$$

$$-\frac{-\frac{6}{7}\left(-\frac{2}{3}\int \frac{1}{\sqrt{1-c^4x^4}}d\frac{1}{cx}-\frac{\sqrt{1-c^4x^4}}{3c^3x^3}\right)-\frac{\left(1-c^4x^4\right)^{3/2}}{7c^7x^7}}{c^7x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{762}$$

$$-\frac{-\frac{6}{7}\left(-\frac{2}{3}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{cx}\right),-1\right)-\frac{\sqrt{1-c^4x^4}}{3c^3x^3}\right)-\frac{\left(1-c^4x^4\right)^{3/2}}{7c^7x^7}}{c^7x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

input
```
Int[x^3/Csch[2*Log[c*x]]^(3/2),x]
```

output
```
-((-1/7*(1 - c^4*x^4)^(3/2)/(c^7*x^7) - (6*(-1/3*Sqrt[1 - c^4*x^4]/(c^3*x^
3) - (2*EllipticF[ArcSin[1/(c*x)], -1])/3))/7)/(c^7*(1 - 1/(c^4*x^4))^(3/2
)*x^3*Csch[2*Log[c*x]]^(3/2)))
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6084
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6086
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.44

method	result	size
risch	$\frac{x^{2} \left(c^{4} x^{4}-3\right) \sqrt{2}}{28 c^{2} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}+\frac{\sqrt{c^{2} x^{2}+1}\, \sqrt{-c^{2} x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{-c^{2}}, i\right) \sqrt{2}\, x}{7 \sqrt{-c^{2}}\, \left(c^{4} x^{4}-1\right) c^{2} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}$	124

input
```
int(x^3/csch(2*ln(x*c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/28*x^2*(c^4*x^4-3)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(1/2)+1/7/(-c^2)^(1
/2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*EllipticF(x*(-c^2)^(1
/2),I)*2^(1/2)/c^2*x/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2 \log(cx))}\, dx = \frac{\sqrt{2}(c^{10}x^8 - 4\,c^6x^4 + 3\,c^2)\sqrt{\frac{c^2x^2}{c^4x^4-1}} - 8\,\sqrt{\frac{1}{2}}\sqrt{c^4}F(\arcsin\left(\frac{1}{cx}\right)\,|\,-1)}{28\,c^6}$$

input
```
integrate(x^3/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

output
```
1/28*(sqrt(2)*(c^10*x^8 - 4*c^6*x^4 + 3*c^2)*sqrt(c^2*x^2/(c^4*x^4 - 1)) -
 8*sqrt(1/2)*sqrt(c^4)*elliptic_f(arcsin(1/(c*x)), -1))/c^6
```

Sympy [F]

$$\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

input
```
integrate(x**3/csch(2*ln(c*x))**(3/2),x)
```

output
```
Integral(x**3/csch(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^3}{\operatorname{csch}\left(2\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/csch(2*log(c*x))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly exception caught Unable to
 convert to real %%{poly1[1.00000000000000000000000000000000,0.000000000000
000000000
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^3}{\left(\frac{1}{\sinh(2\ln(cx))}\right)^{3/2}}\,dx$$

input
```
int(x^3/(1/sinh(2*log(c*x)))^(3/2),x)
```

output
```
int(x^3/(1/sinh(2*log(c*x)))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{\sqrt{\operatorname{csch}(2\log(cx))}\,x^3}{\operatorname{csch}(2\log(cx))^2}dx$$

input
```
int(x^3/csch(2*log(c*x))^(3/2),x)
```

output
```
int((sqrt(csch(2*log(c*x)))*x**3)/csch(2*log(c*x))**2,x)
```

3.149 $\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{1}{2\left(c^4-\frac{1}{x^4}\right)x\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^3}{6\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{\csc^{-1}\left(c^2x^2\right)}{2c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

output
```
-1/2/(c^4-1/x^4)/x/csch(2*ln(c*x))^(3/2)+1/6*x^3/csch(2*ln(c*x))^(3/2)-1/2
*arccsc(c^2*x^2)/c^6/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{x\left(\left(-4+c^4x^4\right)\sqrt{-1+c^4x^4}+3\arctan\left(\sqrt{-1+c^4x^4}\right)\right)}{12\sqrt{2}c^2\sqrt{\frac{c^2x^2}{-1+c^4x^4}}\sqrt{-1+c^4x^4}}$$

input
```
Integrate[x^2/Csch[2*Log[c*x]]^(3/2),x]
```

output
```
(x*((-4 + c^4*x^4)*Sqrt[-1 + c^4*x^4] + 3*ArcTan[Sqrt[-1 + c^4*x^4]]))/(12
*Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Sqrt[-1 + c^4*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {6086, 6084, 858, 807, 247, 247, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}dx$$

$$\Big\downarrow \text{6086}$$

$$\frac{\int \frac{c^2x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}d(cx)}{c^3}$$

$$\Big\downarrow \text{6084}$$

$$\frac{\int c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^5d(cx)}{c^6x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{858}$$

$$-\frac{\int \frac{\left(1-c^4x^4\right)^{3/2}}{c^7x^7}d\frac{1}{cx}}{c^6x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{807}$$

$$-\frac{\int \frac{\left(1-c^2x^2\right)^{3/2}}{c^4x^4}d\left(c^2x^2\right)}{2c^6x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{247}$$

$$-\frac{-\int \frac{\sqrt{1-c^2x^2}}{c^2x^2}d\left(c^2x^2\right)-\frac{\left(1-c^2x^2\right)^{3/2}}{3c^3x^3}}{2c^6x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{247}$$

$$-\frac{\int \frac{1}{\sqrt{1-c^2x^2}}d(c^2x^2)+c^2x^2\sqrt{1-c^2x^2}-\frac{(1-c^2x^2)^{3/2}}{3c^3x^3}}{2c^6x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{223}$$

$$-\frac{\arcsin\left(c^2x^2\right)+c^2x^2\sqrt{1-c^2x^2}-\frac{(1-c^2x^2)^{3/2}}{3c^3x^3}}{2c^6x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

input
```
Int[x^2/Csch[2*Log[c*x]]^(3/2),x]
```

output
```
-1/2*(c^2*x^2*Sqrt[1 - c^2*x^2] - (1 - c^2*x^2)^(3/2)/(3*c^3*x^3) + ArcSin
[c^2*x^2])/(c^6*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^(3/2))
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6084
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6086
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{x^2}{\operatorname{csch}\left(2\ln\left(xc\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2/csch(2*ln(x*c))^(3/2),x)
```

output
```
int(x^2/csch(2*ln(x*c))^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.03

$$\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{3\sqrt{2}cx\arctan\left(\frac{(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{cx}\right)+\sqrt{2}(c^8x^8-5\,c^4x^4+4)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{24\,c^4x}$$

input
```
integrate(x^2/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

output
```
1/24*(3*sqrt(2)*c*x*arctan((c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c*x)
) + sqrt(2)*(c^8*x^8 - 5*c^4*x^4 + 4)*sqrt(c^2*x^2/(c^4*x^4 - 1)))/(c^4*x)
```

Sympy [F]

$$\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

input
```
integrate(x**2/csch(2*ln(c*x))**(3/2),x)
```

output
```
Integral(x**2/csch(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^2}{\operatorname{csch}\left(2\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/csch(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

input
```
integrate(x^2/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^2}{\left(\frac{1}{\sinh(2\ln(c\,x))}\right)^{3/2}}\,dx$$

input
```
int(x^2/(1/sinh(2*log(c*x)))^(3/2),x)
```

output
```
int(x^2/(1/sinh(2*log(c*x)))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{\sqrt{\operatorname{csch}(2\log(cx))}\,x^2}{\operatorname{csch}(2\log(cx))^2}dx$$

input
```
int(x^2/csch(2*log(c*x))^(3/2),x)
```

output
```
int((sqrt(csch(2*log(c*x)))*x**2)/csch(2*log(c*x))**2,x)
```

3.150 $\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 130

$$\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{6}{5\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^2}{5\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{12E(\csc^{-1}(cx)|-1)}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{12\operatorname{EllipticF}\left(\csc^{-1}(cx),-1\right)}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

output
```
-6/5/(c^4-1/x^4)/x^2/csch(2*ln(c*x))^(3/2)+1/5*x^2/csch(2*ln(c*x))^(3/2)-1
2/5*EllipticE(1/c/x,I)/c^5/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)+1
2/5*InverseJacobiAM(arccsc(c*x),I)/c^5/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c
*x))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.46

$$\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},\frac{3}{4},c^4x^4\right)}{2c^2\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

input
```
Integrate[x/Csch[2*Log[c*x]]^(3/2),x]
```

output
```
Hypergeometric2F1[-3/2, -1/4, 3/4, c^4*x^4]/(2*c^2*Sqrt[2 - 2*c^4*x^4]*Sqr
t[(c^2*x^2)/(-1 + c^4*x^4)])
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.85, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {6086, 6084, 858, 809, 809, 836, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}dx$$

$$\Big\downarrow \text{6086}$$

$$\frac{\int \frac{cx}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}d(cx)}{c^2}$$

$$\Big\downarrow \text{6084}$$

$$\frac{\int c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^4d(cx)}{c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{858}$$

$$-\frac{\int \frac{\left(1-c^4x^4\right)^{3/2}}{c^6x^6}d\frac{1}{cx}}{c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{809}$$

$$-\frac{-\frac{6}{5}\int \frac{\sqrt{1-c^4x^4}}{c^2x^2}d\frac{1}{cx}-\frac{\left(1-c^4x^4\right)^{3/2}}{5c^5x^5}}{c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{809}$$

$$-\frac{-\frac{6}{5}\left(-2\int \frac{c^2x^2}{\sqrt{1-c^4x^4}}d\frac{1}{cx}-\frac{\sqrt{1-c^4x^4}}{cx}\right)-\frac{\left(1-c^4x^4\right)^{3/2}}{5c^5x^5}}{c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{836}$$

$$-\frac{-\frac{6}{5}\left(-2\left(\int \frac{c^2x^2+1}{\sqrt{1-c^4x^4}}d\frac{1}{cx}-\int \frac{1}{\sqrt{1-c^4x^4}}d\frac{1}{cx}\right)-\frac{\sqrt{1-c^4x^4}}{cx}\right)-\frac{\left(1-c^4x^4\right)^{3/2}}{5c^5x^5}}{c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{762}$$

$$-\frac{-\frac{6}{5}\left(-2\left(\int \frac{c^2x^2+1}{\sqrt{1-c^4x^4}}d\frac{1}{cx}-\text{EllipticF}\left(\arcsin\left(\frac{1}{cx}\right),-1\right)\right)-\frac{\sqrt{1-c^4x^4}}{cx}\right)-\frac{\left(1-c^4x^4\right)^{3/2}}{5c^5x^5}}{c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{1388}$$

$$-\frac{-\frac{6}{5}\left(-2\left(\int \frac{\sqrt{c^2x^2+1}}{\sqrt{1-c^2x^2}}d\frac{1}{cx}-\text{EllipticF}\left(\arcsin\left(\frac{1}{cx}\right),-1\right)\right)-\frac{\sqrt{1-c^4x^4}}{cx}\right)-\frac{\left(1-c^4x^4\right)^{3/2}}{5c^5x^5}}{c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{327}$$

$$-\frac{-\frac{6}{5}\left(-2\left(E\left(\arcsin\left(\frac{1}{cx}\right)\middle|-1\right)-\text{EllipticF}\left(\arcsin\left(\frac{1}{cx}\right),-1\right)\right)-\frac{\sqrt{1-c^4x^4}}{cx}\right)-\frac{\left(1-c^4x^4\right)^{3/2}}{5c^5x^5}}{c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

input
```
Int[x/Csch[2*Log[c*x]]^(3/2),x]
```

output
```
-((-1/5*(1 - c^4*x^4)^(3/2)/(c^5*x^5) - (6*(-(Sqrt[1 - c^4*x^4]/(c*x)) - 2
*(EllipticE[ArcSin[1/(c*x)], -1] - EllipticF[ArcSin[1/(c*x)], -1])))/5)/(c
^5*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^(3/2)))
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 6084
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6086
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.17

method	result	size
risch	$\frac{(c^8x^8+4c^4x^4-5)\sqrt{2}}{20(c^4x^4-1)c^2\sqrt{\frac{c^2x^2}{c^4x^4-1}}}-\frac{3\sqrt{c^2x^2+1}\,\sqrt{-c^2x^2+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{-c^2},i\right)-\operatorname{EllipticE}\left(x\sqrt{-c^2},i\right)\right)\sqrt{2}\,x}{5\sqrt{-c^2}\,(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$	152

input
```
int(x/csch(2*ln(x*c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*(c^8*x^8+4*c^4*x^4-5)/(c^4*x^4-1)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(
1/2)-3/5/(-c^2)^(1/2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*(El
lipticF(x*(-c^2)^(1/2),I)-EllipticE(x*(-c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/
(c^4*x^4-1))^(1/2)
```

Fricas [F]

$$\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx=\int \frac{x}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

output
```
integral(x/csch(2*log(c*x))^(3/2), x)
```

Sympy [F]

$$\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$$

input
```
integrate(x/csch(2*ln(c*x))**(3/2),x)
```

output
```
Integral(x/csch(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x}{\operatorname{csch}(2\log(cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/csch(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

input
```
integrate(x/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x}{\left(\frac{1}{\sinh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

input
```
int(x/(1/sinh(2*log(c*x)))^(3/2),x)
```

output
```
int(x/(1/sinh(2*log(c*x)))^(3/2), x)
```

Reduce [F]

$$\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}\,x}{\operatorname{csch}\left(2\log\left(cx\right)\right)^{2}}dx$$

input
```
int(x/csch(2*log(c*x))^(3/2),x)
```

output
```
int((sqrt(csch(2*log(c*x)))*x)/csch(2*log(c*x))**2,x)
```

3.151 $\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 96

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{3}{4\left(c^4-\frac{1}{x^4}\right)x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

output
```
3/4/(c^4-1/x^4)/x^3/csch(2*ln(c*x))^(3/2)+1/4*x/csch(2*ln(c*x))^(3/2)-3/4*
arctanh((1-1/c^4/x^4)^(1/2))/c^4/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(
3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{2},\frac{1}{2},c^4x^4\right)}{4c^2x\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

input
```
Integrate[Csch[2*Log[c*x]]^(-3/2),x]
```

output
```
Hypergeometric2F1[-3/2, -1/2, 1/2, c^4*x^4]/(4*c^2*x*Sqrt[2 - 2*c^4*x^4]*S
qrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {6080, 6078, 798, 51, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}dx$$

$$\Big\downarrow \text{6080}$$

$$\frac{\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}d(cx)}{c}$$

$$\Big\downarrow \text{6078}$$

$$\frac{\int c^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3d(cx)}{c^4x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{798}$$

$$-\frac{\int \frac{\left(1-\frac{1}{c^4x^4}\right)^{3/2}}{c^2x^2}d\frac{1}{c^4x^4}}{4c^4x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{51}$$

$$-\frac{-\frac{3}{2}\int \frac{\sqrt{1-\frac{1}{c^4x^4}}}{cx}d\frac{1}{c^4x^4}-\frac{\left(1-\frac{1}{c^4x^4}\right)^{3/2}}{cx}}{4c^4x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{60}$$

$$-\frac{-\frac{3}{2}\left(\int \frac{1}{c\sqrt{1-\frac{1}{c^4x^4}}x}d\frac{1}{c^4x^4}+2\sqrt{1-\frac{1}{c^4x^4}}\right)-\frac{\left(1-\frac{1}{c^4x^4}\right)^{3/2}}{cx}}{4c^4x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{-\frac{3}{2}\left(2\sqrt{1-\frac{1}{c^4x^4}}-2\int \frac{1}{1-c^2x^2}d\sqrt{1-\frac{1}{c^4x^4}}\right)-\frac{\left(1-\frac{1}{c^4x^4}\right)^{3/2}}{cx}}{4c^4x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{-\frac{3}{2}\left(2\sqrt{1-\frac{1}{c^4x^4}}-2\text{arctanh}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)\right)-\frac{\left(1-\frac{1}{c^4x^4}\right)^{3/2}}{cx}}{4c^4x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

input
```
Int[Csch[2*Log[c*x]]^(-3/2),x]
```

output
```
-1/4*(-((1 - 1/(c^4*x^4))^(3/2)/(c*x)) - (3*(2*Sqrt[1 - 1/(c^4*x^4)] - 2*A
rcTanh[Sqrt[1 - 1/(c^4*x^4)]]))/2)/(c^4*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2
*Log[c*x]]^(3/2))
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6078
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Simp[Csch[d*(a
+ b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*d*p))   Int[1/(x^(b
*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; FreeQ[{a, b, d, p}, x] &&
  !IntegerQ[p]
```

rule 6080
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.35

method	result	size
risch	$\frac{\left(c^{8} x^{8}+c^{4} x^{4}-2\right) \sqrt{2}}{16 x \left(c^{4} x^{4}-1\right) c^{2} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}-\frac{3 c^{2} \ln \left(\frac{c^{4} x^{2}}{\sqrt{c^{4}}}+\sqrt{c^{4} x^{4}-1}\right) \sqrt{2}\, x}{16 \sqrt{c^{4}}\, \sqrt{c^{4} x^{4}-1}\, \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}$	130

input
```
int(1/csch(2*ln(x*c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*(c^8*x^8+c^4*x^4-2)/x/(c^4*x^4-1)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(
1/2)-3/16*c^2*ln(c^4*x^2/(c^4)^(1/2)+(c^4*x^4-1)^(1/2))/(c^4)^(1/2)*2^(1/2
)*x/(c^4*x^4-1)^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.10

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{3\sqrt{2}c^3x^3\log\left(2\,c^4x^4 - 2\,(c^5x^5 - cx)\sqrt{\frac{c^2x^2}{c^4x^4-1}} - 1\right) + 2\sqrt{2}(c^8x^8 + c^4x^4 - 2)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{32\,c^4x^3}$$

input
```
integrate(1/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

output
```
1/32*(3*sqrt(2)*c^3*x^3*log(2*c^4*x^4 - 2*(c^5*x^5 - c*x)*sqrt(c^2*x^2/(c^
4*x^4 - 1)) - 1) + 2*sqrt(2)*(c^8*x^8 + c^4*x^4 - 2)*sqrt(c^2*x^2/(c^4*x^4
 - 1)))/(c^4*x^3)
```

Sympy [F]

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$$

input
```
integrate(1/csch(2*ln(c*x))**(3/2),x)
```

output
```
Integral(csch(2*log(c*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{1}{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(csch(2*log(c*x))^(-3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

input
```
integrate(1/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{1}{\left(\frac{1}{\sinh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

input
```
int(1/(1/sinh(2*log(c*x)))^(3/2),x)
```

output
```
int(1/(1/sinh(2*log(c*x)))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{\operatorname{csch}\left(2\log\left(cx\right)\right)^{2}}dx$$

input
```
int(1/csch(2*log(c*x))^(3/2),x)
```

output
```
int(sqrt(csch(2*log(c*x)))/csch(2*log(c*x))**2,x)
```

3.152 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = -\cosh(2\log(cx))\sqrt{\operatorname{csch}(2\log(cx))} + \frac{iE\left(\frac{\pi}{4}-i\log(cx)\middle|2\right)}{\sqrt{\operatorname{csch}(2\log(cx))}\sqrt{i\sinh(2\log(cx))}}$$

output
```
-cosh(2*ln(c*x))*csch(2*ln(c*x))^(1/2)+I*EllipticE(cos(1/4*Pi+I*ln(c*x)),2
^(1/2))/csch(2*ln(c*x))^(1/2)/(I*sinh(2*ln(c*x)))^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \sqrt{\operatorname{csch}(2\log(cx))}\left(-\cosh(2\log(cx)) + E\left(\frac{\pi}{4}-i\log(cx)\middle|2\right)\sqrt{i\sinh(2\log(cx))}\right)$$

input
```
Integrate[Csch[2*Log[c*x]]^(3/2)/x,x]
```

output
```
Sqrt[Csch[2*Log[c*x]]]*(-Cosh[2*Log[c*x]] + EllipticE[Pi/4 - I*Log[c*x], 2
]*Sqrt[I*Sinh[2*Log[c*x]]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3039, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\int \operatorname{csch}^{\frac{3}{2}}(2\log(cx))d\log(cx)$$

$$\Big\downarrow \text{3042}$$

$$\int (i\csc(2i\log(cx)))^{3/2}d\log(cx)$$

$$\Big\downarrow \text{4255}$$

$$\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}d\log(cx)-\cosh(2\log(cx))\sqrt{\operatorname{csch}(2\log(cx))}$$

$$\Big\downarrow \text{3042}$$

$$-\cosh(2\log(cx))\sqrt{\operatorname{csch}(2\log(cx))}+\int \frac{1}{\sqrt{i\csc(2i\log(cx))}}d\log(cx)$$

$$\Big\downarrow \text{4258}$$

$$-\cosh(2\log(cx))\sqrt{\operatorname{csch}(2\log(cx))}+\frac{\int \sqrt{i\sinh(2\log(cx))}d\log(cx)}{\sqrt{i\sinh(2\log(cx))}\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$\Big\downarrow \text{3042}$$

$$-\cosh(2\log(cx))\sqrt{\operatorname{csch}(2\log(cx))}+\frac{\int \sqrt{\sin(2i\log(cx))}d\log(cx)}{\sqrt{i\sinh(2\log(cx))}\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$\Big\downarrow \text{3119}$$

$$-\cosh(2\log(cx))\sqrt{\operatorname{csch}(2\log(cx))}+\frac{iE\left(\frac{\pi}{4}-i\log(cx)\middle|2\right)}{\sqrt{i\sinh(2\log(cx))}\sqrt{\operatorname{csch}(2\log(cx))}}$$

input
```
Int[Csch[2*Log[c*x]]^(3/2)/x,x]
```

output
```
-(Cosh[2*Log[c*x]]*Sqrt[Csch[2*Log[c*x]]]) + (I*EllipticE[Pi/4 - I*Log[c*x
], 2])/(Sqrt[Csch[2*Log[c*x]]]*Sqrt[I*Sinh[2*Log[c*x]]])
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(59) = 118.

Time = 0.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.43

method	result
derivativedivides	$\frac{2\sqrt{1-i\sinh(2\ln(xc))}\,\sqrt{2}\,\sqrt{i\sinh(2\ln(xc))+1}\,\sqrt{i\sinh(2\ln(xc))}\ \mathrm{EllipticE}\left(\sqrt{1-i\sinh(2\ln(xc))},\frac{\sqrt{2}}{2}\right)-\sqrt{1-i\sinh(2\ln}}{2\cosh(2\ln(xc))\sqrt{\sinh(}}$
default	$\frac{2\sqrt{1-i\sinh(2\ln(xc))}\,\sqrt{2}\,\sqrt{i\sinh(2\ln(xc))+1}\,\sqrt{i\sinh(2\ln(xc))}\ \mathrm{EllipticE}\left(\sqrt{1-i\sinh(2\ln(xc))},\frac{\sqrt{2}}{2}\right)-\sqrt{1-i\sinh(2\ln}}{2\cosh(2\ln(xc))\sqrt{\sinh(}}$

input
```
int(csch(2*ln(x*c))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*(1-I*sinh(2*ln(x*c)))^(1/2)*2^(1/2)*(I*sinh(2*ln(x*c))+1)^(1/2)*(I*
sinh(2*ln(x*c)))^(1/2)*EllipticE((1-I*sinh(2*ln(x*c)))^(1/2),1/2*2^(1/2))-
(1-I*sinh(2*ln(x*c)))^(1/2)*2^(1/2)*(I*sinh(2*ln(x*c))+1)^(1/2)*(I*sinh(2*
ln(x*c)))^(1/2)*EllipticF((1-I*sinh(2*ln(x*c)))^(1/2),1/2*2^(1/2))-2*cosh(
2*ln(x*c))^2)/cosh(2*ln(x*c))/sinh(2*ln(x*c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = -\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}c^2x^2 - 2\sqrt{-\frac{1}{2}}c^2E(\arcsin(cx)\,|-1) + 2\sqrt{-\frac{1}{2}}c^2F(\arcsin(cx)\,|-1)$$

input
```
integrate(csch(2*log(c*x))^(3/2)/x,x, algorithm="fricas")
```

output
```
-sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 - 1))*c^2*x^2 - 2*sqrt(-1/2)*c^2*elliptic_e
(arcsin(c*x), -1) + 2*sqrt(-1/2)*c^2*elliptic_f(arcsin(c*x), -1)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log{(cx)})}{x}\,dx$$

input
```
integrate(csch(2*ln(c*x))**(3/2)/x,x)
```

output
```
Integral(csch(2*log(c*x))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \int \frac{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(csch(2*log(c*x))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(csch(2*log(c*x))^(3/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \text{Timed out}$$

input
```
integrate(csch(2*log(c*x))^(3/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \int \frac{\left(\frac{1}{\sinh(2\ln(cx))}\right)^{3/2}}{x}\,dx$$

input
```
int((1/sinh(2*log(c*x)))^(3/2)/x,x)
```

output
```
int((1/sinh(2*log(c*x)))^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \int \frac{\sqrt{\operatorname{csch}(2\log(cx))}\,\operatorname{csch}(2\log(cx))}{x}\,dx$$

input
```
int(csch(2*log(c*x))^(3/2)/x,x)
```

output
```
int((sqrt(csch(2*log(c*x)))*csch(2*log(c*x)))/x,x)
```

3.153 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

output
```
-1/2*(c^4-1/x^4)*x^3*csch(2*ln(c*x))^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = -\sqrt{2}c^2x\sqrt{\frac{c^2x^2}{-1+c^4x^4}}$$

input
```
Integrate[Csch[2*Log[c*x]]^(3/2)/x^2,x]
```

output
```
-(Sqrt[2]*c^2*x*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6086, 6084, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}dx$$

$$\Big\downarrow \text{ 6086}$$

$$c\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{c^2x^2}d(cx)$$

$$\Big\downarrow \text{ 6084}$$

$$c^4x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\int \frac{1}{c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^5}d(cx)$$

$$\Big\downarrow \text{ 793}$$

$$-\frac{1}{2}c^4x^3\left(1-\frac{1}{c^4x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

input
```
Int[Csch[2*Log[c*x]]^(3/2)/x^2,x]
```

output
```
-1/2*(c^4*(1 - 1/(c^4*x^4))*x^3*Csch[2*Log[c*x]]^(3/2))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 6084
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6086
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\operatorname{csch}\left(2\ln\left(xc\right)\right)^{\frac{3}{2}}}{x^{2}}dx$$

input
```
int(csch(2*ln(x*c))^(3/2)/x^2,x)
```

output
```
int(csch(2*ln(x*c))^(3/2)/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = -\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}c^2x$$

input
```
integrate(csch(2*log(c*x))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
-sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 - 1))*c^2*x
```

Sympy [F]

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = \int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx$$

input
```
integrate(csch(2*ln(c*x))**(3/2)/x**2,x)
```

output
```
Integral(csch(2*log(c*x))**(3/2)/x**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(23) = 46$.

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.22

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = -c\left(\frac{\sqrt{2}}{\left(\frac{1}{cx}+1\right)^{\frac{3}{2}}\left(-\frac{1}{cx}+1\right)^{\frac{3}{2}}\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}} - \frac{\sqrt{2}}{c^4x^4\left(\frac{1}{cx}+1\right)^{\frac{3}{2}}\left(-\frac{1}{cx}+1\right)^{\frac{3}{2}}\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}}\right)$$

input
```
integrate(csch(2*log(c*x))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
-c*(sqrt(2)/((1/(c*x) + 1)^(3/2)*(-1/(c*x) + 1)^(3/2)*(1/(c^2*x^2) + 1)^(3
/2)) - sqrt(2)/(c^4*x^4*(1/(c*x) + 1)^(3/2)*(-1/(c*x) + 1)^(3/2)*(1/(c^2*x
^2) + 1)^(3/2)))
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = \text{Timed out}$$

input
```
integrate(csch(2*log(c*x))^(3/2)/x^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = -c^2\,x\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4-1}}$$

input
```
int((1/sinh(2*log(c*x)))^(3/2)/x^2,x)
```

output
```
-c^2*x*((2*c^2*x^2)/(c^4*x^4 - 1))^(1/2)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}\operatorname{csch}(2\log(cx))}{x^2}dx$$

input
```
int(csch(2*log(c*x))^(3/2)/x^2,x)
```

output
```
int((sqrt(csch(2*log(c*x)))*csch(2*log(c*x)))/x**2,x)
```

3.154 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))+\frac{1}{2}c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\operatorname{EllipticF}\left(\csc^{-1}(cx),-1\right)$$

output
```
-1/2*(c^4-1/x^4)*x^2*csch(2*ln(c*x))^(3/2)+1/2*c^5*(1-1/c^4/x^4)^(3/2)*x^3
*csch(2*ln(c*x))^(3/2)*InverseJacobiAM(arccsc(c*x),I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = -\sqrt{2}c^2\sqrt{\frac{c^2x^2}{-1+c^4x^4}}\left(1+\sqrt{1-c^4x^4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},c^4x^4\right)\right)$$

input
```
Integrate[Csch[2*Log[c*x]]^(3/2)/x^3,x]
```

output
```
-(Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*(1 + Sqrt[1 - c^4*x^4]*Hyperg
eometric2F1[1/4, 1/2, 5/4, c^4*x^4]))
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6086, 6084, 858, 817, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}dx$$

$$\Big\downarrow \text{6086}$$

$$c^2\int \frac{\text{csch}^{\frac{3}{2}}(2\log(cx))}{c^3x^3}d(cx)$$

$$\Big\downarrow \text{6084}$$

$$c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\text{csch}^{\frac{3}{2}}(2\log(cx))\int \frac{1}{c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^6}d(cx)$$

$$\Big\downarrow \text{858}$$

$$-c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\text{csch}^{\frac{3}{2}}(2\log(cx))\int \frac{c^4x^4}{(1-c^4x^4)^{3/2}}d\frac{1}{cx}$$

$$\Big\downarrow \text{817}$$

$$-c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\text{csch}^{\frac{3}{2}}(2\log(cx))\left(\frac{1}{2cx\sqrt{1-c^4x^4}}-\frac{1}{2}\int \frac{1}{\sqrt{1-c^4x^4}}d\frac{1}{cx}\right)$$

$$\Big\downarrow \text{762}$$

$$-c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\left(\frac{1}{2cx\sqrt{1-c^4x^4}}-\frac{1}{2}\text{EllipticF}\left(\arcsin\left(\frac{1}{cx}\right),-1\right)\right)\text{csch}^{\frac{3}{2}}(2\log(cx))$$

input
```
Int[Csch[2*Log[c*x]]^(3/2)/x^3,x]
```

output
```
-(c^5*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^(3/2)*(1/(2*c*x*Sqrt[1
- c^4*x^4]) - EllipticF[ArcSin[1/(c*x)], -1]/2))
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6084
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6086
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\operatorname{csch}(2\ln(xc))^{\frac{3}{2}}}{x^3}dx$$

input
```
int(csch(2*ln(x*c))^(3/2)/x^3,x)
```

output
```
int(csch(2*ln(x*c))^(3/2)/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.62

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = 2\sqrt{-\frac{1}{2}}c^2F(\arcsin(cx)\,|-1) - \sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}c^2$$

input
```
integrate(csch(2*log(c*x))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
2*sqrt(-1/2)*c^2*elliptic_f(arcsin(c*x), -1) - sqrt(2)*sqrt(c^2*x^2/(c^4*x
^4 - 1))*c^2
```

Sympy [F]

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx$$

input
```
integrate(csch(2*ln(c*x))**(3/2)/x**3,x)
```

output
```
Integral(csch(2*log(c*x))**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \int \frac{\operatorname{csch}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate(csch(2*log(c*x))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(csch(2*log(c*x))^(3/2)/x^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(csch(2*log(c*x))^(3/2)/x^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \int \frac{\left(\frac{1}{\sinh(2\,\ln(c\,x))}\right)^{3/2}}{x^3}\,dx$$

input
```
int((1/sinh(2*log(c*x)))^(3/2)/x^3,x)
```

output
```
int((1/sinh(2*log(c*x)))^(3/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}\operatorname{csch}(2\log(cx))}{x^3}dx$$

input
```
int(csch(2*log(c*x))^(3/2)/x^3,x)
```

output
```
int((sqrt(csch(2*log(c*x)))*csch(2*log(c*x)))/x**3,x)
```

3.155 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x\operatorname{csch}^{\frac{3}{2}}(2\log(cx)) + \frac{1}{2}c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\csc^{-1}\left(c^2x^2\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

output
```
-1/2*(c^4-1/x^4)*x*csch(2*ln(c*x))^(3/2)+1/2*c^6*(1-1/c^4/x^4)^(3/2)*x^3*a
rccsc(c^2*x^2)*csch(2*ln(c*x))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = -\frac{\sqrt{2}c^2\sqrt{\frac{c^2x^2}{-1+c^4x^4}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-c^4x^4\right)}{x}$$

input
```
Integrate[Csch[2*Log[c*x]]^(3/2)/x^4,x]
```

output
```
-((Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Hypergeometric2F1[-1/2, 1, 1
/2, 1 - c^4*x^4])/x)
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6086, 6084, 858, 807, 252, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}dx$$

$$\Big\downarrow \text{6086}$$

$$c^3\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{c^4x^4}d(cx)$$

$$\Big\downarrow \text{6084}$$

$$c^6x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\int \frac{1}{c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^7}d(cx)$$

$$\Big\downarrow \text{858}$$

$$-c^6x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\int \frac{c^5x^5}{\left(1-c^4x^4\right)^{3/2}}d\frac{1}{cx}$$

$$\Big\downarrow \text{807}$$

$$-\frac{1}{2}c^6x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\int \frac{c^2x^2}{\left(1-c^2x^2\right)^{3/2}}d\left(c^2x^2\right)$$

$$\Big\downarrow \text{252}$$

$$-\frac{1}{2}c^6x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\left(\frac{c^2x^2}{\sqrt{1-c^2x^2}}-\int \frac{1}{\sqrt{1-c^2x^2}}d\left(c^2x^2\right)\right)$$

$$\Big\downarrow \text{223}$$

$$-\frac{1}{2}c^6x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\left(\frac{c^2x^2}{\sqrt{1-c^2x^2}}-\arcsin\left(c^2x^2\right)\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

input
```
Int[Csch[2*Log[c*x]]^(3/2)/x^4,x]
```

output
```
-1/2*(c^6*(1 - 1/(c^4*x^4))^(3/2)*x^3*((c^2*x^2)/Sqrt[1 - c^2*x^2] - ArcSi
n[c^2*x^2])*Csch[2*Log[c*x]]^(3/2))
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6084
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 6086
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[
x^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\operatorname{csch}(2\ln(xc))^{\frac{3}{2}}}{x^4}dx$$

input
```
int(csch(2*ln(x*c))^(3/2)/x^4,x)
```

output
```
int(csch(2*ln(x*c))^(3/2)/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = -\frac{\sqrt{2}c^3x\arctan\left(\frac{(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{cx}\right)+\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}c^2}{x}$$

input
```
integrate(csch(2*log(c*x))^(3/2)/x^4,x, algorithm="fricas")
```

output
```
-(sqrt(2)*c^3*x*arctan((c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c*x)) +
sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 - 1))*c^2)/x
```

Sympy [F]

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx$$

input
```
integrate(csch(2*ln(c*x))**(3/2)/x**4,x)
```

output
```
Integral(csch(2*log(c*x))**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \int \frac{\operatorname{csch}(2\log(cx))^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate(csch(2*log(c*x))^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(csch(2*log(c*x))^(3/2)/x^4, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(csch(2*log(c*x))^(3/2)/x^4,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \int \frac{\left(\frac{1}{\sinh(2\,\ln(c\,x))}\right)^{3/2}}{x^4}\,dx$$

input
```
int((1/sinh(2*log(c*x)))^(3/2)/x^4,x)
```

output
```
int((1/sinh(2*log(c*x)))^(3/2)/x^4, x)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \int \frac{\sqrt{\operatorname{csch}(2\log(cx))}\,\operatorname{csch}(2\log(cx))}{x^4}dx$$

input
```
int(csch(2*log(c*x))^(3/2)/x^4,x)
```

output
```
int((sqrt(csch(2*log(c*x)))*csch(2*log(c*x)))/x**4,x)
```

3.156 $\int \operatorname{csch}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 70

$$\int \operatorname{csch}(a+b\log(cx^n))\,dx = \frac{2e^{-a}x(cx^n)^{-b}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1-\frac{1}{bn}\right),\frac{1}{2}\left(3-\frac{1}{bn}\right),e^{-2a}(cx^n)^{-2b}\right)}{1-bn}$$

output
```
2*x*hypergeom([1, 1/2-1/2/b/n],[3/2-1/2/b/n],1/exp(2*a)/((c*x^n)^(2*b)))/e
xp(a)/(-b*n+1)/((c*x^n)^b)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \operatorname{csch}(a+b\log(cx^n))\,dx = -\frac{2e^{a}x(cx^n)^{b}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{1}{bn}\right),\frac{1}{2}\left(3+\frac{1}{bn}\right),e^{2(a+b\log(cx^n))}\right)}{1+bn}$$

input
```
Integrate[Csch[a + b*Log[c*x^n]],x]
```

output
```
(-2*E^a*x*(c*x^n)^b*Hypergeometric2F1[1, (1 + 1/(b*n))/2, (3 + 1/(b*n))/2,
 E^(2*(a + b*Log[c*x^n]))])/(1 + b*n)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6080, 6082, 795, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{6080}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\operatorname{csch}(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{6082}$$

$$\frac{2e^{-a}x(cx^n)^{-1/n}\int \frac{(cx^n)^{-b+\frac{1}{n}-1}}{1-e^{-2a}(cx^n)^{-2b}}d(cx^n)}{n}$$

$$\Big\downarrow \text{795}$$

$$\frac{2e^{-a}x(cx^n)^{-1/n}\int \frac{(cx^n)^{b+\frac{1}{n}-1}}{(cx^n)^{2b}-e^{-2a}}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2e^a x(cx^n)^b \operatorname{Hypergeometric2F1}\left(1,\frac{b+\frac{1}{n}}{2b},\frac{1}{2}\left(3+\frac{1}{bn}\right),e^{2a}(cx^n)^{2b}\right)}{bn+1}$$

input
```
Int[Csch[a + b*Log[c*x^n]],x]
```

output
```
(-2*E^a*x*(c*x^n)^b*Hypergeometric2F1[1, (b + n^(-1))/(2*b), (3 + 1/(b*n))
/2, E^(2*a)*(c*x^n)^(2*b)])/(1 + b*n)
```

Defintions of rubi rules used

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6080
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6082
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[2^p/E^(a*d*p)   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b
*d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [F]

$$\int \operatorname{csch}\left(a+b\ln\left(c\,x^{n}\right)\right)dx$$

input
```
int(csch(a+b*ln(c*x^n)),x)
```

output
```
int(csch(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int \operatorname{csch}(a + b\log(cx^n)) \, dx = \int \operatorname{csch}(b\log(cx^n) + a) \, dx$$

input
```
integrate(csch(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
integral(csch(b*log(c*x^n) + a), x)
```

Sympy [F]

$$\int \operatorname{csch}(a + b\log(cx^n)) \, dx = \int \operatorname{csch}(a + b\log(cx^n)) \, dx$$

input
```
integrate(csch(a+b*ln(c*x**n)),x)
```

output
```
Integral(csch(a + b*log(c*x**n)), x)
```

Maxima [F]

$$\int \operatorname{csch}(a + b\log(cx^n)) \, dx = \int \operatorname{csch}(b\log(cx^n) + a) \, dx$$

input
```
integrate(csch(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
integrate(csch(b*log(c*x^n) + a), x)
```

Giac [F]

$$\int \operatorname{csch}(a+b\log{(cx^n)})\ dx = \int \operatorname{csch}\left(b\log{(cx^n)}+a\right)\ dx$$

input
```
integrate(csch(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
integrate(csch(b*log(c*x^n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}(a+b\log{(cx^n)})\ dx = \int \frac{1}{\sinh\left(a+b\,\ln\left(c\,x^n\right)\right)}\,dx$$

input
```
int(1/sinh(a + b*log(c*x^n)),x)
```

output
```
int(1/sinh(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int \operatorname{csch}(a+b\log{(cx^n)})\ dx = \int \operatorname{csch}(\log(x^n c)\,b+a)\,dx$$

input
```
int(csch(a+b*log(c*x^n)),x)
```

output
```
int(csch(log(x**n*c)*b + a),x)
```

3.157 $\int \operatorname{csch}^2(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 70

$$\int \operatorname{csch}^2(a+b\log(cx^n))\,dx = \frac{4e^{-2a}x(cx^n)^{-2b}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(2-\frac{1}{bn}\right),\frac{1}{2}\left(4-\frac{1}{bn}\right),e^{-2a}(cx^n)^{-2b}\right)}{1-2bn}$$

output
```
4*x*hypergeom([2, 1-1/2/b/n],[2-1/2/b/n],1/exp(2*a)/((c*x^n)^(2*b)))/exp(2
*a)/(-2*b*n+1)/((c*x^n)^(2*b))
```

Mathematica [A] (verified)

Time = 2.82 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.80

$$\int \operatorname{csch}^2(a+b\log(cx^n))\,dx = \frac{x\left(-\coth(a+b\log(cx^n)) - \frac{e^{2a}(cx^n)^{2b}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{2bn},2+\frac{1}{2bn},e^{2(a+b\log(cx^n))}\right)}{1+2bn} - \text{Hypergeometric2F1}\left(\right.\right.}{bn}$$

input
```
Integrate[Csch[a + b*Log[c*x^n]]^2,x]
```

output
```
(x*(-Coth[a + b*Log[c*x^n]] - (E^(2*a)*(c*x^n)^(2*b)*Hypergeometric2F1[1,
1 + 1/(2*b*n), 2 + 1/(2*b*n), E^(2*(a + b*Log[c*x^n]))])/(1 + 2*b*n) - Hyp
ergeometric2F1[1, 1/(2*b*n), 1 + 1/(2*b*n), E^(2*(a + b*Log[c*x^n]))]))/(b
*n)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6080, 6082, 795, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^2(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{6080}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\text{csch}^2(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{6082}$$

$$\frac{4e^{-2a}x(cx^n)^{-1/n}\int \frac{(cx^n)^{-2b+\frac{1}{n}-1}}{\left(1-e^{-2a}(cx^n)^{-2b}\right)^2}d(cx^n)}{n}$$

$$\Big\downarrow \text{795}$$

$$\frac{4e^{-2a}x(cx^n)^{-1/n}\int \frac{(cx^n)^{2b+\frac{1}{n}-1}}{\left((cx^n)^{2b}-e^{-2a}\right)^2}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{4e^{2a}x(cx^n)^{2b}\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(2+\frac{1}{bn}\right),\frac{1}{2}\left(4+\frac{1}{bn}\right),e^{2a}(cx^n)^{2b}\right)}{2bn+1}$$

input
```
Int[Csch[a + b*Log[c*x^n]]^2,x]
```

output
```
(4*E^(2*a)*x*(c*x^n)^(2*b)*Hypergeometric2F1[2, (2 + 1/(b*n))/2, (4 + 1/(b
*n))/2, E^(2*a)*(c*x^n)^(2*b)])/(1 + 2*b*n)
```

Defintions of rubi rules used

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6080
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6082
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[2^p/E^(a*d*p)   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b
*d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [F]

$$\int \operatorname{csch}\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}dx$$

input
```
int(csch(a+b*ln(c*x^n))^2,x)
```

output
```
int(csch(a+b*ln(c*x^n))^2,x)
```

Fricas [F]

$$\int \operatorname{csch}^2(a+b\log(cx^n))\,dx=\int \operatorname{csch}\left(b\log(cx^n)+a\right)^2\,dx$$

input
```
integrate(csch(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
integral(csch(b*log(c*x^n) + a)^2, x)
```

Sympy [F]

$$\int \operatorname{csch}^2(a+b\log(cx^n))\,dx=\int \operatorname{csch}^2\left(a+b\log(cx^n)\right)dx$$

input
```
integrate(csch(a+b*ln(c*x**n))**2,x)
```

output
```
Integral(csch(a + b*log(c*x**n))**2, x)
```

Maxima [F]

$$\int \operatorname{csch}^2(a+b\log(cx^n))\,dx=\int \operatorname{csch}\left(b\log(cx^n)+a\right)^2\,dx$$

input
```
integrate(csch(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
-2*x/(b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) - b*n) - 4*integrate(1/4/(b*c^b*n
*e^(b*log(x^n) + a) + b*n), x) + 4*integrate(1/4/(b*c^b*n*e^(b*log(x^n) +
a) - b*n), x)
```

Giac [F]

$$\int \operatorname{csch}^2(a+b\log(cx^n))\,dx=\int \operatorname{csch}\left(b\log(cx^n)+a\right)^2\,dx$$

input
```
integrate(csch(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
integrate(csch(b*log(c*x^n) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^2(a+b\log(cx^n))\,dx=\int \frac{1}{\sinh\left(a+b\,\ln\left(c\,x^n\right)\right)^2}\,dx$$

input
```
int(1/sinh(a + b*log(c*x^n))^2,x)
```

output
```
int(1/sinh(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int \operatorname{csch}^2(a+b\log(cx^n))\,dx=4e^{2a}c^{2b}\left(\int \frac{x^{2bn}}{x^{4bn}e^{4a}c^{4b}-2x^{2bn}e^{2a}c^{2b}+1}dx\right)$$

input
```
int(csch(a+b*log(c*x^n))^2,x)
```

output
```
4*e**(2*a)*c**(2*b)*int(x**(2*b*n)/(x**(4*b*n)*e**(4*a)*c**(4*b) - 2*x**(2
*b*n)*e**(2*a)*c**(2*b) + 1),x)
```

3.158 $\int \operatorname{csch}^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 70

$$\int \operatorname{csch}^3(a+b\log(cx^n))\,dx = \frac{8e^{-3a}x(cx^n)^{-3b}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3-\frac{1}{bn}\right),\frac{1}{2}\left(5-\frac{1}{bn}\right),e^{-2a}(cx^n)^{-2b}\right)}{1-3bn}$$

output
```
8*x*hypergeom([3, 3/2-1/2/b/n],[5/2-1/2/b/n],1/exp(2*a)/((c*x^n)^(2*b)))/e
xp(3*a)/(-3*b*n+1)/((c*x^n)^(3*b))
```

Mathematica [A] (verified)

Time = 3.92 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.44

$$\int \operatorname{csch}^3(a+b\log(cx^n))\,dx = \frac{-4x(1+bn\coth(a+b\log(cx^n)))\operatorname{csch}(a+b\log(cx^n))+8e^a(-1+bn)x(cx^n)^b\,\text{Hypergeometric2F1}\left(1,\right.}{8b^2n^2}$$

input
```
Integrate[Csch[a + b*Log[c*x^n]]^3,x]
```

output
```
(-4*x*(1 + b*n*Coth[a + b*Log[c*x^n]])*Csch[a + b*Log[c*x^n]] + 8*E^a*(-1
+ b*n)*x*(c*x^n)^b*Hypergeometric2F1[1, (1 + 1/(b*n))/2, (3 + 1/(b*n))/2,
E^(2*(a + b*Log[c*x^n]))])/(8*b^2*n^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6080, 6082, 795, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{6080}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\operatorname{csch}^3(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{6082}$$

$$\frac{8e^{-3a}x(cx^n)^{-1/n}\int \frac{(cx^n)^{-3b+\frac{1}{n}-1}}{\left(1-e^{-2a}(cx^n)^{-2b}\right)^3}d(cx^n)}{n}$$

$$\Big\downarrow \text{795}$$

$$\frac{8e^{-3a}x(cx^n)^{-1/n}\int \frac{(cx^n)^{3b+\frac{1}{n}-1}}{\left((cx^n)^{2b}-e^{-2a}\right)^3}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$-\frac{8e^{3a}x(cx^n)^{3b}\,\text{Hypergeometric2F1}\left(3,\frac{3b+\frac{1}{n}}{2b},\frac{1}{2}\left(5+\frac{1}{bn}\right),e^{2a}(cx^n)^{2b}\right)}{3bn+1}$$

input
```
Int[Csch[a + b*Log[c*x^n]]^3,x]
```

output
```
(-8*E^(3*a)*x*(c*x^n)^(3*b)*Hypergeometric2F1[3, (3*b + n^(-1))/(2*b), (5
+ 1/(b*n))/2, E^(2*a)*(c*x^n)^(2*b)])/(1 + 3*b*n)
```

Defintions of rubi rules used

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6080
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6082
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[2^p/E^(a*d*p)   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b
*d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [F]

$$\int \operatorname{csch}\left(a+b\ln\left(c\,x^{n}\right)\right)^{3}dx$$

input
```
int(csch(a+b*ln(c*x^n))^3,x)
```

output
```
int(csch(a+b*ln(c*x^n))^3,x)
```

Fricas [F]

$$\int \operatorname{csch}^3(a+b\log(cx^n))\,dx = \int \operatorname{csch}(b\log(cx^n)+a)^3\,dx$$

input
```
integrate(csch(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
integral(csch(b*log(c*x^n) + a)^3, x)
```

Sympy [F]

$$\int \operatorname{csch}^3(a+b\log(cx^n))\,dx = \int \operatorname{csch}^3(a+b\log(cx^n))\,dx$$

input
```
integrate(csch(a+b*ln(c*x**n))**3,x)
```

output
```
Integral(csch(a + b*log(c*x**n))**3, x)
```

Maxima [F]

$$\int \operatorname{csch}^3(a+b\log(cx^n))\,dx = \int \operatorname{csch}(b\log(cx^n)+a)^3\,dx$$

input
```
integrate(csch(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
-8*(b^2*n^2 - 1)*integrate(1/16/(b^2*c^b*n^2*e^(b*log(x^n) + a) + b^2*n^2)
, x) - 8*(b^2*n^2 - 1)*integrate(1/16/(b^2*c^b*n^2*e^(b*log(x^n) + a) - b^
2*n^2), x) - ((b*c^(3*b)*n + c^(3*b))*x*e^(3*b*log(x^n) + 3*a) + (b*c^b*n
- c^b)*x*e^(b*log(x^n) + a))/(b^2*c^(4*b)*n^2*e^(4*b*log(x^n) + 4*a) - 2*b
^2*c^(2*b)*n^2*e^(2*b*log(x^n) + 2*a) + b^2*n^2)
```

Giac [F]

$$\int \operatorname{csch}^3(a+b\log(cx^n))\,dx = \int \operatorname{csch}\left(b\log(cx^n)+a\right)^3\,dx$$

input
```
integrate(csch(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output
```
integrate(csch(b*log(c*x^n) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^3(a+b\log(cx^n))\,dx = \int \frac{1}{\sinh\left(a+b\,\ln\left(c\,x^n\right)\right)^3}\,dx$$

input
```
int(1/sinh(a + b*log(c*x^n))^3,x)
```

output
```
int(1/sinh(a + b*log(c*x^n))^3, x)
```

Reduce [F]

$$\int \operatorname{csch}^3(a+b\log(cx^n))\,dx = 8e^{3a}c^{3b}\left(\int \frac{x^{3bn}}{x^{6bn}e^{6a}c^{6b}-3x^{4bn}e^{4a}c^{4b}+3x^{2bn}e^{2a}c^{2b}-1}dx\right)$$

input
```
int(csch(a+b*log(c*x^n))^3,x)
```

output
```
8*e**(3*a)*c**(3*b)*int(x**(3*b*n)/(x**(6*b*n)*e**(6*a)*c**(6*b) - 3*x**(4
*b*n)*e**(4*a)*c**(4*b) + 3*x**(2*b*n)*e**(2*a)*c**(2*b) - 1),x)
```

3.159 $\int \operatorname{csch}^4(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 70

$$\int \operatorname{csch}^4(a+b\log(cx^n))\,dx = \frac{16e^{-4a}x(cx^n)^{-4b}\text{Hypergeometric2F1}\left(4,\frac{1}{2}\left(4-\frac{1}{bn}\right),\frac{1}{2}\left(6-\frac{1}{bn}\right),e^{-2a}(cx^n)^{-2b}\right)}{1-4bn}$$

output
```
16*x*hypergeom([4, 2-1/2/b/n],[3-1/2/b/n],1/exp(2*a)/((c*x^n)^(2*b)))/exp(
4*a)/(-4*b*n+1)/((c*x^n)^(4*b))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 200 vs. 2(70) = 140.

Time = 5.52 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.86

$$\int \operatorname{csch}^4(a+b\log(cx^n))\,dx = \frac{x\left(4e^{2a}(-1+2bn)(cx^n)^{2b}\text{Hypergeometric2F1}\left(1,1+\frac{1}{2bn},2+\frac{1}{2bn},e^{2(a+b\log(cx^n))}\right)+4(-1+4b^2n^2)\text{Hype}\right.}{}$$

input
```
Integrate[Csch[a + b*Log[c*x^n]]^4,x]
```

output
```
(x*(4*E^(2*a)*(-1 + 2*b*n)*(c*x^n)^(2*b)*Hypergeometric2F1[1, 1 + 1/(2*b*n
), 2 + 1/(2*b*n), E^(2*(a + b*Log[c*x^n]))] + 4*(-1 + 4*b^2*n^2)*Hypergeom
etric2F1[1, 1/(2*b*n), 1 + 1/(2*b*n), E^(2*(a + b*Log[c*x^n]))] + Csch[a +
 b*Log[c*x^n]]^3*((1 - 12*b^2*n^2)*Cosh[a + b*Log[c*x^n]] + (-1 + 4*b^2*n^
2)*Cosh[3*(a + b*Log[c*x^n])] - 4*b*n*Sinh[a + b*Log[c*x^n]])))/(24*b^3*n^
3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6080, 6082, 795, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{6080}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\operatorname{csch}^4(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{6082}$$

$$\frac{16e^{-4a}x(cx^n)^{-1/n}\int \frac{(cx^n)^{-4b+\frac{1}{n}-1}}{\left(1-e^{-2a}(cx^n)^{-2b}\right)^4}d(cx^n)}{n}$$

$$\Big\downarrow \text{795}$$

$$\frac{16e^{-4a}x(cx^n)^{-1/n}\int \frac{(cx^n)^{4b+\frac{1}{n}-1}}{\left((cx^n)^{2b}-e^{-2a}\right)^4}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{16e^{4a}x(cx^n)^{4b}\,\text{Hypergeometric2F1}\left(4,\frac{1}{2}\left(4+\frac{1}{bn}\right),\frac{1}{2}\left(6+\frac{1}{bn}\right),e^{2a}(cx^n)^{2b}\right)}{4bn+1}$$

input
```
Int[Csch[a + b*Log[c*x^n]]^4,x]
```

output
```
(16*E^(4*a)*x*(c*x^n)^(4*b)*Hypergeometric2F1[4, (4 + 1/(b*n))/2, (6 + 1/(
b*n))/2, E^(2*a)*(c*x^n)^(2*b)])/(1 + 4*b*n)
```

Defintions of rubi rules used

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6080
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6082
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[2^p/E^(a*d*p)   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b
*d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [F]

$$\int \operatorname{csch}\left(a + b\ln\left(c\,x^{n}\right)\right)^{4} dx$$

input
```
int(csch(a+b*ln(c*x^n))^4,x)
```

output
```
int(csch(a+b*ln(c*x^n))^4,x)
```

Fricas [F]

$$\int \operatorname{csch}^4(a + b\log(cx^n))\, dx = \int \operatorname{csch}(b\log(cx^n) + a)^4\, dx$$

input
```
integrate(csch(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

output
```
integral(csch(b*log(c*x^n) + a)^4, x)
```

Sympy [F]

$$\int \operatorname{csch}^4(a + b\log(cx^n))\, dx = \int \operatorname{csch}^4(a + b\log(cx^n))\, dx$$

input
```
integrate(csch(a+b*ln(c*x**n))**4,x)
```

output
```
Integral(csch(a + b*log(c*x**n))**4, x)
```

Maxima [F]

$$\int \operatorname{csch}^4(a + b\log(cx^n))\, dx = \int \operatorname{csch}(b\log(cx^n) + a)^4\, dx$$

input
```
integrate(csch(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

output
```
16*(4*b^2*n^2 - 1)*integrate(1/96/(b^3*c^b*n^3*e^(b*log(x^n) + a) + b^3*n^
3), x) - 16*(4*b^2*n^2 - 1)*integrate(1/96/(b^3*c^b*n^3*e^(b*log(x^n) + a)
 - b^3*n^3), x) - 1/3*((2*b*c^(4*b)*n + c^(4*b))*x*e^(4*b*log(x^n) + 4*a)
+ 2*(6*b^2*c^(2*b)*n^2 - b*c^(2*b)*n - c^(2*b))*x*e^(2*b*log(x^n) + 2*a) -
 (4*b^2*n^2 - 1)*x)/(b^3*c^(6*b)*n^3*e^(6*b*log(x^n) + 6*a) - 3*b^3*c^(4*b
)*n^3*e^(4*b*log(x^n) + 4*a) + 3*b^3*c^(2*b)*n^3*e^(2*b*log(x^n) + 2*a) -
b^3*n^3)
```

Giac [F]

$$\int \operatorname{csch}^4(a+b\log(cx^n))\,dx = \int \operatorname{csch}(b\log(cx^n)+a)^4\,dx$$

input
```
integrate(csch(a+b*log(c*x^n))^4,x, algorithm="giac")
```

output
```
integrate(csch(b*log(c*x^n) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^4(a+b\log(cx^n))\,dx = \int \frac{1}{\sinh(a+b\,\ln(c\,x^n))^4}\,dx$$

input
```
int(1/sinh(a + b*log(c*x^n))^4,x)
```

output
```
int(1/sinh(a + b*log(c*x^n))^4, x)
```

Reduce [F]

$$\int \operatorname{csch}^4(a+b\log(cx^n))\,dx = \text{too large to display}$$

input
```
int(csch(a+b*log(c*x^n))^4,x)
```

output

```
(16*e**(2*a)*c**(2*b)*( - 8*x**(6*b*n)*e**(6*a)*c**(6*b)*int(x**(2*b*n)/(4
*x**(8*b*n)*e**(8*a)*c**(8*b)*b*n - x**(8*b*n)*e**(8*a)*c**(8*b) - 16*x**(
6*b*n)*e**(6*a)*c**(6*b)*b*n + 4*x**(6*b*n)*e**(6*a)*c**(6*b) + 24*x**(4*b
*n)*e**(4*a)*c**(4*b)*b*n - 6*x**(4*b*n)*e**(4*a)*c**(4*b) - 16*x**(2*b*n)
*e**(2*a)*c**(2*b)*b*n + 4*x**(2*b*n)*e**(2*a)*c**(2*b) + 4*b*n - 1),x)*b*
*2*n**2 - 2*x**(6*b*n)*e**(6*a)*c**(6*b)*int(x**(2*b*n)/(4*x**(8*b*n)*e**(
8*a)*c**(8*b)*b*n - x**(8*b*n)*e**(8*a)*c**(8*b) - 16*x**(6*b*n)*e**(6*a)*
c**(6*b)*b*n + 4*x**(6*b*n)*e**(6*a)*c**(6*b) + 24*x**(4*b*n)*e**(4*a)*c**
(4*b)*b*n - 6*x**(4*b*n)*e**(4*a)*c**(4*b) - 16*x**(2*b*n)*e**(2*a)*c**(2*
b)*b*n + 4*x**(2*b*n)*e**(2*a)*c**(2*b) + 4*b*n - 1),x)*b*n + x**(6*b*n)*e
**(6*a)*c**(6*b)*int(x**(2*b*n)/(4*x**(8*b*n)*e**(8*a)*c**(8*b)*b*n - x**(
8*b*n)*e**(8*a)*c**(8*b) - 16*x**(6*b*n)*e**(6*a)*c**(6*b)*b*n + 4*x**(6*b
*n)*e**(6*a)*c**(6*b) + 24*x**(4*b*n)*e**(4*a)*c**(4*b)*b*n - 6*x**(4*b*n)
*e**(4*a)*c**(4*b) - 16*x**(2*b*n)*e**(2*a)*c**(2*b)*b*n + 4*x**(2*b*n)*e*
*(2*a)*c**(2*b) + 4*b*n - 1),x) + 24*x**(4*b*n)*e**(4*a)*c**(4*b)*int(x**(
2*b*n)/(4*x**(8*b*n)*e**(8*a)*c**(8*b)*b*n - x**(8*b*n)*e**(8*a)*c**(8*b)
- 16*x**(6*b*n)*e**(6*a)*c**(6*b)*b*n + 4*x**(6*b*n)*e**(6*a)*c**(6*b) + 2
4*x**(4*b*n)*e**(4*a)*c**(4*b)*b*n - 6*x**(4*b*n)*e**(4*a)*c**(4*b) - 16*x
**(2*b*n)*e**(2*a)*c**(2*b)*b*n + 4*x**(2*b*n)*e**(2*a)*c**(2*b) + 4*b*n -
 1),x)*b**2*n**2 + 6*x**(4*b*n)*e**(4*a)*c**(4*b)*int(x**(2*b*n)/(4*x**...
```

3.160 $\int \left(-\left(\left(1-b^2n^2\right)\operatorname{csch}(a+b\log\left(cx^n\right))\right)+2b^2n^2\operatorname{csch}$

Optimal result

Integrand size = 45, antiderivative size = 42

$$\int \left(-\left(\left(1-b^2n^2\right)\operatorname{csch}(a+b\log\left(cx^n\right))\right)+2b^2n^2\operatorname{csch}^3(a+b\log\left(cx^n\right))\right)\,dx$$
$$=-x\operatorname{csch}(a+b\log\left(cx^n\right))-bnx\coth\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}(a+b\log\left(cx^n\right))$$

output
```
-x*csch(a+b*ln(c*x^n))-b*n*x*coth(a+b*ln(c*x^n))*csch(a+b*ln(c*x^n))
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \left(-\left(\left(1-b^2n^2\right)\operatorname{csch}(a+b\log\left(cx^n\right))\right)+2b^2n^2\operatorname{csch}^3(a+b\log\left(cx^n\right))\right)\,dx$$
$$=-x(1+bn\coth\left(a+b\log\left(cx^n\right)\right))\operatorname{csch}(a+b\log\left(cx^n\right))$$

input
```
Integrate[-((1 - b^2*n^2)*Csch[a + b*Log[c*x^n]]) + 2*b^2*n^2*Csch[a + b*L
og[c*x^n]]^3,x]
```

output
```
-(x*(1 + b*n*Coth[a + b*Log[c*x^n]])*Csch[a + b*Log[c*x^n]])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.48 (sec) , antiderivative size = 137, normalized size of antiderivative = 3.26, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2b^2n^2\text{csch}^3(a+b\log(cx^n)) - (1-b^2n^2)\,\text{csch}(a+b\log(cx^n))\right) dx$$

$$\Big\downarrow \; 2009$$

$$2e^a x(1-bn)(cx^n)^b\,\text{Hypergeometric2F1}\left(1,\frac{b+\frac{1}{n}}{2b},\frac{1}{2}\left(3+\frac{1}{bn}\right),e^{2a}(cx^n)^{2b}\right) - \frac{16e^{3a}b^2n^2x(cx^n)^{3b}\,\text{Hypergeometric2F1}\left(3,\frac{3b+\frac{1}{n}}{2b},\frac{1}{2}\left(5+\frac{1}{bn}\right),e^{2a}(cx^n)^{2b}\right)}{3bn+1}$$

input
```
Int[-((1 - b^2*n^2)*Csch[a + b*Log[c*x^n]]) + 2*b^2*n^2*Csch[a + b*Log[c*x
^n]]^3,x]
```

output
```
2*E^a*(1 - b*n)*x*(c*x^n)^b*Hypergeometric2F1[1, (b + n^(-1))/(2*b), (3 +
1/(b*n))/2, E^(2*a)*(c*x^n)^(2*b)] - (16*b^2*E^(3*a)*n^2*x*(c*x^n)^(3*b)*H
ypergeometric2F1[3, (3*b + n^(-1))/(2*b), (5 + 1/(b*n))/2, E^(2*a)*(c*x^n)
^(2*b)])/(1 + 3*b*n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 21.66 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.90

method	result
parallelrisch	$\frac{x\left(-\coth\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)^{2}bn+\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)^{2}bn-2\coth\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)+2\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)\right)}{4}$
risch	$-\frac{2c^{b}(x^{n})^{b}x\left(nb(x^{n})^{2b}c^{2b}e^{3a}e^{\frac{3ib\pi\ \mathrm{csgn}(ix^{n})\ \mathrm{csgn}(ic\,x^{n})^{2}}{2}}e^{-\frac{3ib\pi\ \mathrm{csgn}(ix^{n})\ \mathrm{csgn}(ic\,x^{n})\ \mathrm{csgn}(ic)}{2}}e^{-\frac{3ib\pi\ \mathrm{csgn}(ic\,x^{n})^{3}}{2}}e^{\frac{3ib\pi\ \mathrm{csgn}(ic}{}}\right.}{}$

input
```
int(-(-b^2*n^2+1)*csch(a+b*ln(c*x^n))+2*b^2*n^2*csch(a+b*ln(c*x^n))^3,x,me
thod=_RETURNVERBOSE)
```

output
```
1/4*x*(-coth(1/2*a+b*ln((c*x^n)^(1/2)))^2*b*n+tanh(1/2*a+b*ln((c*x^n)^(1/2
)))^2*b*n-2*coth(1/2*a+b*ln((c*x^n)^(1/2)))+2*tanh(1/2*a+b*ln((c*x^n)^(1/2
))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(42) = 84.

Time = 0.10 (sec) , antiderivative size = 187, normalized size of antiderivative = 4.45

$$\int \left(-\left(\left(1-b^{2}n^{2}\right)\operatorname{csch}(a+b\log\left(cx^{n}\right))\right)+2b^{2}n^{2}\operatorname{csch}^{3}(a+b\log\left(cx^{n}\right))\right)\,dx =$$
$$-\frac{2\left((bn+1)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\left(bn+1\right)x\cosh\left(bn\log\right.\right.}{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+3\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-}$$

input
```
integrate(-(-b^2*n^2+1)*csch(a+b*log(c*x^n))+2*b^2*n^2*csch(a+b*log(c*x^n)
)^3,x, algorithm="fricas")
```

output
```
-2*((b*n + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(b*n + 1)*x*cosh(b*n
*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + (b*n + 1)*x*sinh
(b*n*log(x) + b*log(c) + a)^2 + (b*n - 1)*x)/(cosh(b*n*log(x) + b*log(c) +
 a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^
2 + sinh(b*n*log(x) + b*log(c) + a)^3 + 3*(cosh(b*n*log(x) + b*log(c) + a)
^2 - 1)*sinh(b*n*log(x) + b*log(c) + a) - cosh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]

$$\int \left(-\left(\left(1-b^2n^2\right)\operatorname{csch}(a+b\log\left(cx^n\right))\right)+2b^2n^2\operatorname{csch}^3(a+b\log\left(cx^n\right))\right)\,dx$$
$$=\int \left(2b^2n^2\operatorname{csch}^2\left(a+b\log\left(cx^n\right)\right)+b^2n^2-1\right)\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)dx$$

input
```
integrate(-(-b**2*n**2+1)*csch(a+b*ln(c*x**n))+2*b**2*n**2*csch(a+b*ln(c*x
**n))**3,x)
```

output
```
Integral((2*b**2*n**2*csch(a + b*log(c*x**n))**2 + b**2*n**2 - 1)*csch(a +
 b*log(c*x**n)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(42) = 84$.

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.26

$$\int \left(-\left(\left(1-b^2n^2\right)\operatorname{csch}(a+b\log\left(cx^n\right))\right)+2b^2n^2\operatorname{csch}^3(a+b\log\left(cx^n\right))\right)\,dx$$
$$=-\frac{2\left(\left(bc^{3\,b}n+c^{3\,b}\right)xe^{(3\,b\log(x^n)+3\,a)}+\left(bc^bn-c^b\right)xe^{(b\log(x^n)+a)}\right)}{c^{4\,b}e^{(4\,b\log(x^n)+4\,a)}-2\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)}+1}$$

input
```
integrate(-(-b^2*n^2+1)*csch(a+b*log(c*x^n))+2*b^2*n^2*csch(a+b*log(c*x^n)
)^3,x, algorithm="maxima")
```

output
```
-2*((b*c^(3*b)*n + c^(3*b))*x*e^(3*b*log(x^n) + 3*a) + (b*c^b*n - c^b)*x*e
^(b*log(x^n) + a))/(c^(4*b)*e^(4*b*log(x^n) + 4*a) - 2*c^(2*b)*e^(2*b*log(
x^n) + 2*a) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(42) = 84$.

Time = 0.28 (sec) , antiderivative size = 215, normalized size of antiderivative = 5.12

$$\int \left(-\left(\left(1-b^2n^2\right)\operatorname{csch}(a+b\log{(cx^n)})\right)+2b^2n^2\operatorname{csch}^3(a+b\log{(cx^n)})\right)\,dx$$
$$= -\frac{2\,bc^{3\,b}nxx^{3\,bn}e^{(3\,a)}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}-2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}-\frac{2\,bc^{b}nxx^{bn}e^{a}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}-2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}-\frac{2\,c^{3\,b}xx^{3\,bn}e^{(3\,a)}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}-2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}+\frac{2\,c^{b}xx^{bn}e^{a}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}-2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}$$

input
```
integrate(-(-b^2*n^2+1)*csch(a+b*log(c*x^n))+2*b^2*n^2*csch(a+b*log(c*x^n)
)^3,x, algorithm="giac")
```

output
```
-2*b*c^(3*b)*n*x*x^(3*b*n)*e^(3*a)/(c^(4*b)*x^(4*b*n)*e^(4*a) - 2*c^(2*b)*
x^(2*b*n)*e^(2*a) + 1) - 2*b*c^b*n*x*x^(b*n)*e^a/(c^(4*b)*x^(4*b*n)*e^(4*a
) - 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1) - 2*c^(3*b)*x*x^(3*b*n)*e^(3*a)/(c^(4
*b)*x^(4*b*n)*e^(4*a) - 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1) + 2*c^b*x*x^(b*n)
*e^a/(c^(4*b)*x^(4*b*n)*e^(4*a) - 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.55

$$\int \left(-\left(\left(1-b^2n^2\right)\operatorname{csch}(a+b\log{(cx^n)})\right)+2b^2n^2\operatorname{csch}^3(a+b\log{(cx^n)})\right)\,dx$$
$$= -\frac{2\,x\,\mathrm{e}^{a}\,{\left(c\,x^n\right)}^b\,\left(b\,n+\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+b\,n\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}-1\right)}{{\left(\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}-1\right)}^2}$$

input
```
int((b^2*n^2 - 1)/sinh(a + b*log(c*x^n)) + (2*b^2*n^2)/sinh(a + b*log(c*x^
n))^3,x)
```

output
```
-(2*x*exp(a)*(c*x^n)^b*(b*n + exp(2*a)*(c*x^n)^(2*b) + b*n*exp(2*a)*(c*x^n
)^(2*b) - 1))/(exp(2*a)*(c*x^n)^(2*b) - 1)^2
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.31

$$\int \left(-\left(\left(1-b^2n^2\right)\operatorname{csch}(a+b\log\left(cx^n\right))\right)+2b^2n^2\operatorname{csch}^3(a+b\log\left(cx^n\right))\right)\,dx$$
$$=\frac{2x^{bn}e^{a}c^{b}x\left(-x^{2bn}e^{2a}c^{2b}bn-x^{2bn}e^{2a}c^{2b}-bn+1\right)}{x^{4bn}e^{4a}c^{4b}-2x^{2bn}e^{2a}c^{2b}+1}$$

input
```
int(-(-b^2*n^2+1)*csch(a+b*log(c*x^n))+2*b^2*n^2*csch(a+b*log(c*x^n))^3,x)
```

output
```
(2*x**(b*n)*e**a*c**b*x*( - x**(2*b*n)*e**(2*a)*c**(2*b)*b*n - x**(2*b*n)*
e**(2*a)*c**(2*b) - b*n + 1))/(x**(4*b*n)*e**(4*a)*c**(4*b) - 2*x**(2*b*n)
*e**(2*a)*c**(2*b) + 1)
```

3.161 $\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 26

$$\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx=-\frac{2c^6e^{-a}}{\left(c^4-\frac{e^{-2a}}{x^2}\right)^2}$$

output
```
-2*c^6/exp(a)/(c^4-1/exp(2*a)/x^2)^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 62 vs. 2(26) = 52.

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.38

$$\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx=\frac{2(\cosh(a)-\sinh(a))\left(-2c^4x^2+\cosh^2(a)-2\cosh(a)\sinh(a)+\sinh^2(a)\right)}{c^2\left(\left(-1+c^4x^2\right)\cosh(a)+\left(1+c^4x^2\right)\sinh(a)\right)^2}$$

input
```
Integrate[Csch[a + 2*Log[c*Sqrt[x]]]^3,x]
```

output
```
(2*(Cosh[a] - Sinh[a])*(-2*c^4*x^2 + Cosh[a]^2 - 2*Cosh[a]*Sinh[a] + Sinh[
a]^2))/(c^2*((-1 + c^4*x^2)*Cosh[a] + (1 + c^4*x^2)*Sinh[a])^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6080, 6082, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx$$

$$\Big\downarrow \text{6080}$$

$$\frac{2\int c\sqrt{x}\operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)d\left(c\sqrt{x}\right)}{c^2}$$

$$\Big\downarrow \text{6082}$$

$$\frac{16e^{-3a}\int \frac{1}{c^5\left(1-\frac{e^{-2a}}{c^4x^2}\right)^3x^{5/2}}d\left(c\sqrt{x}\right)}{c^2}$$

$$\Big\downarrow \text{793}$$

$$-\frac{2e^{-a}}{c^2\left(1-\frac{e^{-2a}}{c^4x^2}\right)^2}$$

input
```
Int[Csch[a + 2*Log[c*Sqrt[x]]]^3,x]
```

output
```
-2/(c^2*E^a*(1 - 1/(c^4*E^(2*a))*x^2))^2)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 6080
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6082
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[2^p/E^(a*d*p)   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b
*d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [F]

$$\int \operatorname{csch}\left(a+2\ln\left(c\sqrt{x}\right)\right)^{3} dx$$

input
```
int(csch(a+2*ln(c*x^(1/2)))^3,x)
```

output
```
int(csch(a+2*ln(c*x^(1/2)))^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\, dx = -\frac{2\left(2\,c^4x^2e^{(2\,a)}-1\right)}{c^{10}x^4e^{(5\,a)}-2\,c^6x^2e^{(3\,a)}+c^2e^a}$$

input
```
integrate(csch(a+2*log(c*x^(1/2)))^3,x, algorithm="fricas")
```

output
```
-2*(2*c^4*x^2*e^(2*a) - 1)/(c^10*x^4*e^(5*a) - 2*c^6*x^2*e^(3*a) + c^2*e^a
)
```

Sympy [F]

$$\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx=\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)dx$$

input
```
integrate(csch(a+2*ln(c*x**(1/2)))**3,x)
```

output
```
Integral(csch(a + 2*log(c*sqrt(x)))**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(24) = 48$.

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.92

$$\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx=-\frac{2\left(\frac{2\,c^4x^2e^{(2\,a)}}{c^8x^4e^{(5\,a)}-2\,c^4x^2e^{(3\,a)}+e^a}-\frac{1}{c^8x^4e^{(5\,a)}-2\,c^4x^2e^{(3\,a)}+e^a}\right)}{c^2}$$

input
```
integrate(csch(a+2*log(c*x^(1/2)))^3,x, algorithm="maxima")
```

output
```
-2*(2*c^4*x^2*e^(2*a)/(c^8*x^4*e^(5*a) - 2*c^4*x^2*e^(3*a) + e^a) - 1/(c^8
*x^4*e^(5*a) - 2*c^4*x^2*e^(3*a) + e^a))/c^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx = -\frac{2\left(2\,c^4x^2e^{(2\,a)}-1\right)e^{(-a)}}{\left(c^4x^2e^{(2\,a)}-1\right)^2c^2}$$

input
```
integrate(csch(a+2*log(c*x^(1/2)))^3,x, algorithm="giac")
```

output
```
-2*(2*c^4*x^2*e^(2*a) - 1)*e^(-a)/((c^4*x^2*e^(2*a) - 1)^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx = \frac{\frac{2\,\mathrm{e}^{-a}}{c^2}-4\,c^2\,x^2\,\mathrm{e}^{a}}{\mathrm{e}^{4\,a}\,c^8\,x^4-2\,\mathrm{e}^{2\,a}\,c^4\,x^2+1}$$

input
```
int(1/sinh(a + 2*log(c*x^(1/2)))^3,x)
```

output
```
((2*exp(-a))/c^2 - 4*c^2*x^2*exp(a))/(c^8*x^4*exp(4*a) - 2*c^4*x^2*exp(2*a
) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx = -\frac{2e^{3a}c^6x^4}{e^{4a}c^8x^4-2e^{2a}c^4x^2+1}$$

input
```
int(csch(a+2*log(c*x^(1/2)))^3,x)
```

output
```
( - 2*e**(3*a)*c**6*x**4)/(e**(4*a)*c**8*x**4 - 2*e**(2*a)*c**4*x**2 + 1)
```

3.162 $\int \operatorname{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 26

$$\int \operatorname{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx=\frac{2c^2e^{-3a}}{\left(e^{-2a}-\frac{c^4}{x^2}\right)^2}$$

output
```
2*c^2/exp(3*a)/(exp(-2*a)-c^4/x^2)^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 65 vs. 2(26) = 52.

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.50

$$\int \operatorname{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx=-\frac{2c^6((c^4-2x^2)\cosh(a)+(c^4+2x^2)\sinh(a))\,(\cosh(2a)+\sinh(2a))}{((-c^4+x^2)\cosh(a)-(c^4+x^2)\sinh(a))^2}$$

input
```
Integrate[Csch[a + 2*Log[c/Sqrt[x]]]^3,x]
```

output
```
(-2*c^6*((c^4 - 2*x^2)*Cosh[a] + (c^4 + 2*x^2)*Sinh[a])*(Cosh[2*a] + Sinh[
2*a]))/((-c^4 + x^2)*Cosh[a] - (c^4 + x^2)*Sinh[a])^2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6080, 6082, 795, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)dx$$

$$\Big\downarrow \text{6080}$$

$$-2c^2\int \frac{x^{3/2}\text{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)}{c^3}d\frac{c}{\sqrt{x}}$$

$$\Big\downarrow \text{6082}$$

$$-16e^{-3a}c^2\int \frac{x^{9/2}}{c^9\left(1-\frac{e^{-2a}x^2}{c^4}\right)^3}d\frac{c}{\sqrt{x}}$$

$$\Big\downarrow \text{795}$$

$$-16e^{-3a}c^2\int \frac{c^3}{\left(\frac{c^4}{x^2}-e^{-2a}\right)^3x^{3/2}}d\frac{c}{\sqrt{x}}$$

$$\Big\downarrow \text{793}$$

$$\frac{2e^{-3a}c^2}{\left(e^{-2a}-\frac{c^4}{x^2}\right)^2}$$

input
```
Int[Csch[a + 2*Log[c/Sqrt[x]]]^3,x]
```

output
```
(2*c^2)/(E^(3*a)*(E^(-2*a) - c^4/x^2)^2)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 6080
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6082
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[2^p/E^(a*d*p)   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b
*d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [F]

$$\int \operatorname{csch}\left(a+2\ln\left(\frac{c}{\sqrt{x}}\right)\right)^3 dx$$

input
```
int(csch(a+2*ln(c/x^(1/2)))^3,x)
```

output
```
int(csch(a+2*ln(c/x^(1/2)))^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \operatorname{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx = -\frac{2\,\left(c^{10}e^{(5\,a)}-2\,c^6x^2e^{(3\,a)}\right)}{c^8e^{(4\,a)}-2\,c^4x^2e^{(2\,a)}+x^4}$$

input
```
integrate(csch(a+2*log(c/x^(1/2)))^3,x, algorithm="fricas")
```

output
```
-2*(c^10*e^(5*a) - 2*c^6*x^2*e^(3*a))/(c^8*e^(4*a) - 2*c^4*x^2*e^(2*a) + x
^4)
```

Sympy [F]

$$\int \operatorname{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx = \int \operatorname{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx$$

input
```
integrate(csch(a+2*ln(c/x**(1/2)))**3,x)
```

output
```
Integral(csch(a + 2*log(c/sqrt(x)))**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \operatorname{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx = -\frac{2\,\left(c^{10}e^{(5\,a)}-2\,c^6x^2e^{(3\,a)}\right)}{c^8e^{(4\,a)}-2\,c^4x^2e^{(2\,a)}+x^4}$$

input
```
integrate(csch(a+2*log(c/x^(1/2)))^3,x, algorithm="maxima")
```

output
```
-2*(c^10*e^(5*a) - 2*c^6*x^2*e^(3*a))/(c^8*e^(4*a) - 2*c^4*x^2*e^(2*a) + x
^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \operatorname{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx = -\frac{2\left(c^{10}e^{(5\,a)}-2\,c^6x^2e^{(3\,a)}\right)}{\left(c^4e^{(2\,a)}-x^2\right)^2}$$

input
```
integrate(csch(a+2*log(c/x^(1/2)))^3,x, algorithm="giac")
```

output
```
-2*(c^10*e^(5*a) - 2*c^6*x^2*e^(3*a))/(c^4*e^(2*a) - x^2)^2
```

Mupad [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \operatorname{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx = \frac{2\,c^2\,x^4\,\mathrm{e}^a}{\mathrm{e}^{4\,a}\,c^8-2\,\mathrm{e}^{2\,a}\,c^4\,x^2+x^4}$$

input
```
int(1/sinh(a + 2*log(c/x^(1/2)))^3,x)
```

output
```
(2*c^2*x^4*exp(a))/(c^8*exp(4*a) + x^4 - 2*c^4*x^2*exp(2*a))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \operatorname{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx = \frac{2e^ac^2x^4}{e^{4a}c^8-2e^{2a}c^4x^2+x^4}$$

input
```
int(csch(a+2*log(c/x^(1/2)))^3,x)
```

output
```
(2*e**a*c**2*x**4)/(e**(4*a)*c**8 - 2*e**(2*a)*c**4*x**2 + x**4)
```

3.163 $\int \operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 90

$$\int \operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx = -\frac{e^{2a}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}}\left(1-e^{-2a}(cx^n)^{\frac{2}{n(2-p)}}\right)\operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

output
```
-1/2*exp(2*a)*(2-p)*x*(1-(c*x^n)^(2/n/(2-p))/exp(2*a))*csch(a-ln(c*x^n)/n/
(2-p))^p/(1-p)/((c*x^n)^(2/n/(2-p)))
```

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.28

$$\int \operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx = \frac{2^{-1+p}(-2+p)x\left(\frac{e^a(cx^n)^{\frac{1}{n(-2+p)}}}{-1+e^{2a}(cx^n)^{\frac{2}{n(-2+p)}}}\right)^p\left(1+e^{2a}(cx^n)^{\frac{2}{n(-2+p)}}\left(-1+\left(1-e^{-2a}(cx^n)^{-\frac{2}{n(-2+p)}}\right)^p\right)\right)}{-1+p}$$

input
```
Integrate[Csch[a + Log[c*x^n]/(n*(-2 + p))]^p,x]
```

output
```
(2^(-1 + p)*(-2 + p)*x*((E^a*(c*x^n)^(1/(n*(-2 + p))))/(-1 + E^(2*a)*(c*x^
n)^(2/(n*(-2 + p)))))^p*(1 + E^(2*a)*(c*x^n)^(2/(n*(-2 + p)))*(-1 + (1 - 1
/(E^(2*a)*(c*x^n)^(2/(n*(-2 + p)))))^p)))/(-1 + p)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6080, 6084, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(p-2)}\right)dx$$

$$\Big\downarrow \text{ 6080}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{ 6084}$$

$$\frac{x(cx^n)^{-\frac{p}{n(2-p)}-\frac{1}{n}}\left(1-e^{-2a}(cx^n)^{\frac{2}{n(2-p)}}\right)^p\operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)\int (cx^n)^{\frac{p}{2n-np}+\frac{1}{n}-1}\left(1-e^{-2a}(cx^n)^{\frac{2}{n(2-p)}}\right)^{-p}d(cx^n)}{n}$$

$$\Big\downarrow \text{ 793}$$

$$-\frac{e^{2a}(2-p)x(cx^n)^{-\frac{p}{n(2-p)}-\frac{1}{n}}\left(1-e^{-2a}(cx^n)^{\frac{2}{n(2-p)}}\right)\operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

input
```
Int[Csch[a + Log[c*x^n]/(n*(-2 + p))]^p,x]
```

output
```
-1/2*(E^(2*a)*(2 - p)*x*(c*x^n)^(-n^(-1) - p/(n*(2 - p)))*(1 - (c*x^n)^(2/
(n*(2 - p)))/E^(2*a))*Csch[a - Log[c*x^n]/(n*(2 - p))]^p)/(1 - p)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 6080
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6084
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \operatorname{csch}\left(a+\frac{\ln\left(c\,x^{n}\right)}{n\left(-2+p\right)}\right)^{p}dx$$

input
```
int(csch(a+ln(c*x^n)/n/(-2+p))^p,x)
```

output
```
int(csch(a+ln(c*x^n)/n/(-2+p))^p,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(76) = 152$.

Time = 0.10 (sec) , antiderivative size = 475, normalized size of antiderivative = 5.28

$$\int \operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx =$$

$$-\frac{(p-2)x\cosh\left(p\log\left(\frac{2\left(\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)+\sinh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)\right)}{\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)^2+2\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)\sinh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)+\sinh\left(\right.}\right.\right.}{}$$

input
```
integrate(csch(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
```

output
```
-((p - 2)*x*cosh(p*log(2*(cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p -
2*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)))/(cosh((a*n*
p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 + 2*cosh((a*n*p - 2*a*n + n*
log(x) + log(c))/(n*p - 2*n))*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*
p - 2*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 - 1)))
*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)) + (p - 2)*x*sinh(p*
log(2*(cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)) + sinh((a*n*p
 - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)))/(cosh((a*n*p - 2*a*n + n*log(x
) + log(c))/(n*p - 2*n))^2 + 2*cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n
*p - 2*n))*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)) + sinh((a
*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 - 1)))*sinh((a*n*p - 2*a*
n + n*log(x) + log(c))/(n*p - 2*n)))/((p - 1)*cosh((a*n*p - 2*a*n + n*log(
x) + log(c))/(n*p - 2*n)) - (p - 1)*sinh((a*n*p - 2*a*n + n*log(x) + log(c
))/(n*p - 2*n)))
```

Sympy [F]

$$\int \operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\int \operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n\,(p-2)}\right)\,dx$$

input
```
integrate(csch(a+ln(c*x**n)/n/(-2+p))**p,x)
```

output
```
Integral(csch(a + log(c*x**n)/(n*(p - 2)))**p, x)
```

Maxima [F]

$$\int \operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\int \operatorname{csch}\left(a+\frac{\log(cx^n)}{n(p-2)}\right)^p\,dx$$

input
```
integrate(csch(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
```

output
```
integrate(csch(a + log(c*x^n)/(n*(p - 2)))^p, x)
```

Giac [F]

$$\int \operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\int \operatorname{csch}\left(a+\frac{\log(cx^n)}{n(p-2)}\right)^p\,dx$$

input
```
integrate(csch(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
```

output
```
integrate(csch(a + log(c*x^n)/(n*(p - 2)))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\int \left(\frac{1}{\sinh\left(a+\frac{\ln(c\,x^n)}{n\,(p-2)}\right)}\right)^p\,dx$$

input
```
int((1/sinh(a + log(c*x^n)/(n*(p - 2))))^p,x)
```

output
```
int((1/sinh(a + log(c*x^n)/(n*(p - 2))))^p, x)
```

Reduce [F]

$$\int \operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\int \operatorname{csch}\left(\frac{\log(x^nc)+anp-2an}{np-2n}\right)^p dx$$

input
```
int(csch(a+log(c*x^n)/n/(-2+p))^p,x)
```

output
```
int(csch((log(x**n*c) + a*n*p - 2*a*n)/(n*p - 2*n))**p,x)
```

3.164 $\int \operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int \operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\frac{(2-p)x\left(1-e^{-2a}(cx^n)^{-\frac{2}{n(2-p)}}\right)\operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

output
```
(2-p)*x*(1-1/exp(2*a)/((c*x^n)^(2/n/(2-p))))*csch(a+ln(c*x^n)/n/(2-p))^p/(
2-2*p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 140 vs. 2(66) = 132.

Time = 0.85 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.12

$$\int \operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\,dx =\frac{2^{-1+p}e^{-\frac{2ap}{-2+p}}(-2+p)x\left(e^{\frac{2ap}{-2+p}}-e^{\frac{4a}{-2+p}}(cx^n)^{\frac{2}{n(-2+p)}}\right)\left(-\frac{e^{\frac{a(2+p)}{-2+p}}(cx^n)^{\frac{1}{n(-2+p)}}}{-e^{\frac{2ap}{-2+p}}+e^{\frac{4a}{-2+p}}(cx^n)^{\frac{2}{n(-2+p)}}}\right)^p}{-1+p}$$

input
```
Integrate[Csch[a - Log[c*x^n]/(n*(-2 + p))]^p,x]
```

output
```
(2^(-1 + p)*(-2 + p)*x*(E^((2*a*p)/(-2 + p)) - E^((4*a)/(-2 + p))*(c*x^n)^
(2/(n*(-2 + p))))*(-((E^((a*(2 + p))/(-2 + p))*(c*x^n)^(1/(n*(-2 + p))))/(
-E^((2*a*p)/(-2 + p)) + E^((4*a)/(-2 + p))*(c*x^n)^(2/(n*(-2 + p))))))^p)/
(E^((2*a*p)/(-2 + p))*(-1 + p))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.62, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6080, 6084, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^p\left(a-\frac{\log(cx^n)}{n(p-2)}\right)dx$$

$$\Big\downarrow \text{ 6080}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\text{csch}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{ 6084}$$

$$\frac{x(cx^n)^{\frac{p}{n(2-p)}-\frac{1}{n}}\left(1-e^{-2a}(cx^n)^{-\frac{2}{n(2-p)}}\right)^p\text{csch}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)\int (cx^n)^{\frac{1-\frac{p}{2-p}}{n}-1}\left(1-e^{-2a}(cx^n)^{-\frac{2}{n(2-p)}}\right)^{-p}d(cx^n)}{n}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{(2-p)x(cx^n)^{\frac{2(1-p)}{n(2-p)}+\frac{p}{n(2-p)}-\frac{1}{n}}\left(1-e^{-2a}(cx^n)^{-\frac{2}{n(2-p)}}\right)\text{csch}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

input
```
Int[Csch[a - Log[c*x^n]/(n*(-2 + p))]^p,x]
```

output
```
((2 - p)*x*(c*x^n)^(-n^(-1) + (2*(1 - p))/(n*(2 - p)) + p/(n*(2 - p)))*(1
- 1/(E^(2*a)*(c*x^n)^(2/(n*(2 - p)))))*Csch[a + Log[c*x^n]/(n*(2 - p))]^p)
/(2*(1 - p))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 6080
```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6084
```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Simp[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p))   Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
 /; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \operatorname{csch}\left(a-\frac{\ln\left(c\,x^{n}\right)}{n\left(-2+p\right)}\right)^{p}dx$$

input
```
int(csch(a-ln(c*x^n)/n/(-2+p))^p,x)
```

output
```
int(csch(a-ln(c*x^n)/n/(-2+p))^p,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(55) = 110$.

Time = 0.10 (sec) , antiderivative size = 539, normalized size of antiderivative = 8.17

$$\int \operatorname{csch}^p\left(a-\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx =$$
$$-\frac{(p-2)x\cosh\left(p\log\left(-\frac{2\left(\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)+\sinh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)\right)}{\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)^2+2\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)\sinh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)}\right.\right.}{}$$

input
```
integrate(csch(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
```

output
```
-((p - 2)*x*cosh(p*log(-2*(cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p
- 2*n)) + sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)))/(cosh(-(
a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 + 2*cosh(-(a*n*p - 2*a*n
 - n*log(x) - log(c))/(n*p - 2*n))*sinh(-(a*n*p - 2*a*n - n*log(x) - log(c
))/(n*p - 2*n)) + sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2
 - 1)))*sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + (p - 2)*x
*sinh(p*log(-2*(cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + s
inh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)))/(cosh(-(a*n*p - 2*a
*n - n*log(x) - log(c))/(n*p - 2*n))^2 + 2*cosh(-(a*n*p - 2*a*n - n*log(x)
 - log(c))/(n*p - 2*n))*sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2
*n)) + sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 - 1)))*sin
h(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)))/((p - 1)*cosh(-(a*n*p
 - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) - (p - 1)*sinh(-(a*n*p - 2*a*n
- n*log(x) - log(c))/(n*p - 2*n)))
```

Sympy [F]

$$\int \operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\int \operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n\,(p-2)}\right)dx$$

input
```
integrate(csch(a-ln(c*x**n)/n/(-2+p))**p,x)
```

output
```
Integral(csch(a - log(c*x**n)/(n*(p - 2)))**p, x)
```

Maxima [F]

$$\int \operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\int \operatorname{csch}\left(a-\frac{\log(cx^n)}{n(p-2)}\right)^p\,dx$$

input
```
integrate(csch(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
```

output
```
integrate((-csch(-a + log(c*x^n)/(n*(p - 2))))^p, x)
```

Giac [F]

$$\int \operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\int \operatorname{csch}\left(a-\frac{\log(cx^n)}{n(p-2)}\right)^p\,dx$$

input
```
integrate(csch(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
```

output
```
integrate(csch(a - log(c*x^n)/(n*(p - 2)))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\int \left(\frac{1}{\sinh\left(a-\frac{\ln(c\,x^n)}{n\,(p-2)}\right)}\right)^p\,dx$$

input
```
int((1/sinh(a - log(c*x^n)/(n*(p - 2))))^p,x)
```

output
```
int((1/sinh(a - log(c*x^n)/(n*(p - 2))))^p, x)
```

Reduce [F]

$$\int \operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=(-1)^p\left(\int \operatorname{csch}\left(\frac{\log(x^nc)-anp+2an}{np-2n}\right)^p dx\right)$$

input
```
int(csch(a-log(c*x^n)/n/(-2+p))^p,x)
```

output
```
( - 1)**p*int(csch((log(x**n*c) - a*n*p + 2*a*n)/(n*p - 2*n))**p,x)
```

3.165 $\int \frac{\operatorname{csch}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{\operatorname{csch}(a+b\log(cx^n))}{x}\,dx = -\frac{\operatorname{arctanh}(\cosh(a+b\log(cx^n)))}{bn}$$

output
```
-arctanh(cosh(a+b*ln(c*x^n)))/b/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(a+b\log(cx^n))}{x}\,dx = -\frac{\operatorname{arctanh}(\cosh(a+b\log(cx^n)))}{bn}$$

input
```
Integrate[Csch[a + b*Log[c*x^n]]/x,x]
```

output
```
-(ArcTanh[Cosh[a + b*Log[c*x^n]]]/(b*n))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3039, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}(a+b\log(cx^n))}{x}dx$$
$$\Big\downarrow \text{3039}$$
$$\frac{\int \text{csch}(a+b\log(cx^n))d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int i\csc(ia+ib\log(cx^n))d\log(cx^n)}{n}$$
$$\Big\downarrow \text{26}$$
$$\frac{i\int \csc(ia+ib\log(cx^n))d\log(cx^n)}{n}$$
$$\Big\downarrow \text{4257}$$
$$-\frac{\text{arctanh}(\cosh(a+b\log(cx^n)))}{bn}$$

input
```
Int[Csch[a + b*Log[c*x^n]]/x,x]
```

output
```
-(ArcTanh[Cosh[a + b*Log[c*x^n]]]/(b*n))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{\ln\left(\tanh\left(\frac{a}{2}+\frac{b\ln\left(c\,x^{n}\right)}{2}\right)\right)}{nb}$
default	$\frac{\ln\left(\tanh\left(\frac{a}{2}+\frac{b\ln\left(c\,x^{n}\right)}{2}\right)\right)}{nb}$
parallelrisch	$\frac{\ln\left(\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)\right)}{nb}$
risch	$-\frac{\ln\left(c^{b}\left(x^{n}\right)^{b}\mathrm{e}^{a}\mathrm{e}^{\frac{ib\pi\ \mathrm{csgn}\left(ix^{n}\right)\mathrm{csgn}\left(ic\,x^{n}\right)^{2}}{2}}\mathrm{e}^{-\frac{ib\pi\ \mathrm{csgn}\left(ix^{n}\right)\mathrm{csgn}\left(ic\,x^{n}\right)\mathrm{csgn}\left(ic\right)}{2}}\mathrm{e}^{-\frac{ib\pi\,\mathrm{csgn}\left(ic\,x^{n}\right)^{3}}{2}}\mathrm{e}^{\frac{ib\pi\,\mathrm{csgn}\left(ic\,x^{n}\right)^{2}\mathrm{csgn}\left(i\right.}{2}} \right.}{bn}$

input
```
int(csch(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*ln(tanh(1/2*a+1/2*b*ln(c*x^n)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(20) = 40$.

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.25

$$\int \frac{\operatorname{csch}(a + b\log(cx^n))}{x}\,dx = -\frac{\log(\cosh(bn\log(x) + b\log(c) + a) + \sinh(bn\log(x) + b\log(c) + a) + 1) - \log(\cosh(bn\log(x) + b}{bn}$$

input
```
integrate(csch(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

output
```
-(log(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a) +
1) - log(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)
 - 1))/(b*n)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.10

$$\int \frac{\operatorname{csch}(a + b\log(cx^n))}{x}\,dx = -\begin{cases} -\log(x)\operatorname{csch}(a) & \text{for } b = 0 \\ -\log(x)\operatorname{csch}(a + b\log(c)) & \text{for } n = 0 \\ -\frac{\log\left(\tanh\left(\frac{a}{2} + \frac{b\log(cx^n)}{2}\right)\right)}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(csch(a+b*ln(c*x**n))/x,x)
```

output
```
-Piecewise((-log(x)*csch(a), Eq(b, 0)), (-log(x)*csch(a + b*log(c)), Eq(n,
 0)), (-log(tanh(a/2 + b*log(c*x**n)/2))/(b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}(a+b\log(cx^n))}{x}\,dx=\frac{\log\left(\tanh\left(\frac{1}{2}\,b\log\left(cx^n\right)+\frac{1}{2}\,a\right)\right)}{bn}$$

input
```
integrate(csch(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

output
```
log(tanh(1/2*b*log(c*x^n) + 1/2*a))/(b*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(20) = 40$.

Time = 0.22 (sec) , antiderivative size = 173, normalized size of antiderivative = 8.65

$$\int \frac{\operatorname{csch}(a+b\log(cx^n))}{x}\,dx=-c^b\left(\frac{c^b e^{(-a)}\log\left(\sqrt{2\,|c|^b|x|^{bn}\cos\left(\frac{1}{2}\,\pi bn\operatorname{sgn}(x)-\frac{1}{2}\,\pi bn+\frac{1}{2}\,\pi b\operatorname{sgn}(c)-\frac{1}{2}\,\pi b\right)e^a+|c|^{2\,b}|x|^{2\,bn}e^{(2\,a)}+1}\right)}{bc^{2\,b}n}\right.$$

input
```
integrate(csch(a+b*log(c*x^n))/x,x, algorithm="giac")
```

output
```
-c^b*(c^b*e^(-a)*log(sqrt(2*abs(c)^b*abs(x)^(b*n)*cos(1/2*pi*b*n*sgn(x) -
1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*e^a + abs(c)^(2*b)*abs(x)^(2*b*n)
*e^(2*a) + 1))/(b*c^(2*b)*n) - c^b*e^(-a)*log(sqrt(-2*abs(c)^b*abs(x)^(b*n
)*cos(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*e^a + a
bs(c)^(2*b)*abs(x)^(2*b*n)*e^(2*a) + 1))/(b*c^(2*b)*n))*e^a
```

Mupad [B] (verification not implemented)

Time = 2.97 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.15

$$\int \frac{\operatorname{csch}(a+b\log(cx^n))}{x}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{-a}\,\sqrt{-b^2\,n^2}}{b\,n\,(c\,x^n)^b}\right)}{\sqrt{-b^2\,n^2}}$$

input
```
int(1/(x*sinh(a + b*log(c*x^n))),x)
```

output
```
-(2*atan((exp(-a)*(-b^2*n^2)^(1/2))/(b*n*(c*x^n)^b)))/(-b^2*n^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.50

$$\int \frac{\operatorname{csch}(a+b\log(cx^n))}{x}\,dx = \frac{-\log(x^{bn}e^a c^{2b}+c^b)+\log(x^{bn}e^a c^{2b}-c^b)}{bn}$$

input
```
int(csch(a+b*log(c*x^n))/x,x)
```

output
```
( - log(x**(b*n)*e**a*c**(2*b) + c**b) + log(x**(b*n)*e**a*c**(2*b) - c**b
))/(b*n)
```

3.166 $\int \frac{\operatorname{csch}^2(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \frac{\operatorname{csch}^2(a+b\log(cx^n))}{x}\,dx=-\frac{\coth(a+b\log(cx^n))}{bn}$$

output
```
-coth(a+b*ln(c*x^n))/b/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(a+b\log(cx^n))}{x}\,dx=-\frac{\coth(a+b\log(cx^n))}{bn}$$

input
```
Integrate[Csch[a + b*Log[c*x^n]]^2/x,x]
```

output
```
-(Coth[a + b*Log[c*x^n]]/(b*n))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3039, 3042, 25, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \operatorname{csch}^2(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\csc(ia+ib\log(cx^n))^2d\log(cx^n)}{n}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \csc(ia+ib\log(cx^n))^2d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{i\int 1d(-i\coth(a+b\log(cx^n)))}{bn}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\coth(a+b\log(cx^n))}{bn}$$

input
```
Int[Csch[a + b*Log[c*x^n]]^2/x,x]
```

output
```
-(Coth[a + b*Log[c*x^n]]/(b*n))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result
derivativedivides	$-\frac{\coth(a+b\ln(c\,x^{n}))}{bn}$
default	$-\frac{\coth(a+b\ln(c\,x^{n}))}{bn}$
parallelrisch	$\frac{-\coth\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)-\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)}{2bn}$
risch	$-\frac{2}{bn\left(\left(x^{n}\right)^{2b}c^{2b}\mathrm{e}^{2a}\mathrm{e}^{ib\pi\ \mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})^{2}}\mathrm{e}^{-ib\pi\ \mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})\,\mathrm{csgn}(ic)}\mathrm{e}^{-ib\pi\,\mathrm{csgn}(ic\,x^{n})^{3}}\mathrm{e}^{ib\pi\,\mathrm{csgn}(ic\,x^{n})^{2}\,\mathrm{csgn}(ic}\right.}$

input
```
int(csch(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
```

output
```
-coth(a+b*ln(c*x^n))/b/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(19) = 38$.

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.74

$$\int \frac{\operatorname{csch}^2(a + b\log(cx^n))}{x}\,dx =$$
$$-\frac{2}{bn\cosh(bn\log(x) + b\log(c) + a)^2 + 2\,bn\cosh(bn\log(x) + b\log(c) + a)\sinh(bn\log(x) + b\log(c) +}$$

input
```
integrate(csch(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

output
```
-2/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*b*n*cosh(b*n*log(x) + b*log(
c) + a)*sinh(b*n*log(x) + b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) +
 a)^2 - b*n)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(a + b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{csch}^2(a + b\log(cx^n))}{x}\,dx$$

input
```
integrate(csch(a+b*ln(c*x**n))**2/x,x)
```

output
```
Integral(csch(a + b*log(c*x**n))**2/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \frac{\operatorname{csch}^2(a + b\log(cx^n))}{x}\,dx = -\frac{2}{bc^{2b}ne^{(2b\log(x^n) + 2a)} - bn}$$

input
```
integrate(csch(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

output
```
-2/(b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) - b*n)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{\operatorname{csch}^2(a+b\log(cx^n))}{x}\,dx = -\frac{2}{(c^{2\,b}x^{2\,bn}e^{(2\,a)}-1)bn}$$

input
```
integrate(csch(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

output
```
-2/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)*b*n)
```

Mupad [B] (verification not implemented)

Time = 2.59 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{csch}^2(a+b\log(cx^n))}{x}\,dx = \frac{2}{b\,n-b\,n\,\mathrm{e}^{2\,a}\,(c\,x^n)^{2\,b}}$$

input
```
int(1/(x*sinh(a + b*log(c*x^n))^2),x)
```

output
```
2/(b*n - b*n*exp(2*a)*(c*x^n)^(2*b))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.37

$$\int \frac{\operatorname{csch}^2(a+b\log(cx^n))}{x}\,dx = -\frac{2x^{2bn}e^{2a}c^{2b}}{bn\,(x^{2bn}e^{2a}c^{2b}-1)}$$

input
```
int(csch(a+b*log(c*x^n))^2/x,x)
```

output
```
( - 2*x**(2*b*n)*e**(2*a)*c**(2*b))/(b*n*(x**(2*b*n)*e**(2*a)*c**(2*b) - 1
))
```

3.167 $\int \frac{\operatorname{csch}^3(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \frac{\operatorname{csch}^3(a+b\log(cx^n))}{x}\,dx = \frac{\operatorname{arctanh}(\cosh(a+b\log(cx^n)))}{2bn} - \frac{\coth(a+b\log(cx^n))\operatorname{csch}(a+b\log(cx^n))}{2bn}$$

output
```
1/2*arctanh(cosh(a+b*ln(c*x^n)))/b/n-1/2*coth(a+b*ln(c*x^n))*csch(a+b*ln(c
*x^n))/b/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.95

$$\int \frac{\operatorname{csch}^3(a+b\log(cx^n))}{x}\,dx = -\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+b\log(cx^n))\right)}{8bn} + \frac{\log\left(\cosh\left(\frac{1}{2}(a+b\log(cx^n))\right)\right)}{2bn} - \frac{\log\left(\sinh\left(\frac{1}{2}(a+b\log(cx^n))\right)\right)}{2bn} - \frac{\operatorname{sech}^2\left(\frac{1}{2}(a+b\log(cx^n))\right)}{8bn}$$

input
```
Integrate[Csch[a + b*Log[c*x^n]]^3/x,x]
```

output
```
-1/8*Csch[(a + b*Log[c*x^n])/2]^2/(b*n) + Log[Cosh[(a + b*Log[c*x^n])/2]]/
(2*b*n) - Log[Sinh[(a + b*Log[c*x^n])/2]]/(2*b*n) - Sech[(a + b*Log[c*x^n]
)/2]^2/(8*b*n)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3039, 3042, 26, 4255, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^3(a+b\log(cx^n))}{x}dx$$
$$\Big\downarrow \text{3039}$$
$$\frac{\int \text{csch}^3(a+b\log(cx^n))d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int -i\csc(ia+ib\log(cx^n))^3d\log(cx^n)}{n}$$
$$\Big\downarrow \text{26}$$
$$-\frac{i\int \csc(ia+ib\log(cx^n))^3d\log(cx^n)}{n}$$
$$\Big\downarrow \text{4255}$$
$$-\frac{i\left(\frac{1}{2}\int -i\text{csch}(a+b\log(cx^n))d\log(cx^n)-\frac{i\coth(a+b\log(cx^n))\text{csch}(a+b\log(cx^n))}{2b}\right)}{n}$$
$$\Big\downarrow \text{26}$$
$$-\frac{i\left(-\frac{1}{2}i\int \text{csch}(a+b\log(cx^n))d\log(cx^n)-\frac{i\coth(a+b\log(cx^n))\text{csch}(a+b\log(cx^n))}{2b}\right)}{n}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\left(-\frac{1}{2}i\int i\csc\left(ia+ib\log\left(cx^n\right)\right)d\log\left(cx^n\right)-\frac{i\coth(a+b\log(cx^n))\text{csch}(a+b\log(cx^n))}{2b}\right)}{n}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\left(\frac{1}{2}\int\csc\left(ia+ib\log\left(cx^n\right)\right)d\log\left(cx^n\right)-\frac{i\coth(a+b\log(cx^n))\text{csch}(a+b\log(cx^n))}{2b}\right)}{n}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{i\left(\frac{i\text{arctanh}(\cosh(a+b\log(cx^n)))}{2b}-\frac{i\coth(a+b\log(cx^n))\text{csch}(a+b\log(cx^n))}{2b}\right)}{n}$$

input
```
Int[Csch[a + b*Log[c*x^n]]^3/x,x]
```

output
```
((-I)*(((I/2)*ArcTanh[Cosh[a + b*Log[c*x^n]]])/b - ((I/2)*Coth[a + b*Log[c
*x^n]]*Csch[a + b*Log[c*x^n]])/b))/n
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{-\frac{\operatorname{csch}(a+b\ln(c\,x^n))\coth(a+b\ln(c\,x^n))}{2}+\operatorname{arctanh}\left(e^{a+b\ln(c\,x^n)}\right)}{nb}$
default	$\frac{-\frac{\operatorname{csch}(a+b\ln(c\,x^n))\coth(a+b\ln(c\,x^n))}{2}+\operatorname{arctanh}\left(e^{a+b\ln(c\,x^n)}\right)}{nb}$
parallelrisch	$\frac{-\coth\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^2+\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^2-4\ln\left(\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)\right)}{8bn}$
risch	$-\frac{c^b(x^n)^b\left((x^n)^{2b}c^{2b}e^{3a}e^{\frac{3ib\pi\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)^2}{2}}e^{-\frac{3ib\pi\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)\operatorname{csgn}(ic)}{2}}e^{-\frac{3ib\pi\operatorname{csgn}(ic\,x^n)^3}{2}}e^{\frac{3ib\pi\operatorname{csgn}(}{}}}{bn\left((x^n)^{2b}c^{2b}e^{2a}e^{ib\pi\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)^2}e^{-ib\pi\operatorname{csgn}(ix^n}\right.}$

input
```
int(csch(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(-1/2*csch(a+b*ln(c*x^n))*coth(a+b*ln(c*x^n))+arctanh(exp(a+b*ln(c*x
^n))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(51) = 102$.

Time = 0.10 (sec) , antiderivative size = 643, normalized size of antiderivative = 11.69

$$\int \frac{\operatorname{csch}^3(a+b\log(cx^n))}{x}\,dx = \text{Too large to display}$$

input
```
integrate(csch(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

output
```
-1/2*(2*cosh(b*n*log(x) + b*log(c) + a)^3 + 6*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a)^2 + 2*sinh(b*n*log(x) + b*log(c) + a)^
3 - (cosh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2
*(3*cosh(b*n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)
^2 - 2*cosh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) +
 a)^3 - cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) +
 1)*log(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)
+ 1) + (cosh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4
+ 2*(3*cosh(b*n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) +
 a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c
) + a)^3 - cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a
) + 1)*log(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) +
a) - 1) + 2*(3*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*
log(c) + a) + 2*cosh(b*n*log(x) + b*log(c) + a))/(b*n*cosh(b*n*log(x) + b*
log(c) + a)^4 + 4*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*
log(c) + a)^3 + b*n*sinh(b*n*log(x) + b*log(c) + a)^4 - 2*b*n*cosh(b*n*log
(x) + b*log(c) + a)^2 + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*
sinh(b*n*log(x) + b*log(c) + a)^2 + b*n + 4*(b*n*cosh(b*n*log(x) + b*lo...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(a + b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{csch}^3(a + b\log(cx^n))}{x}\,dx$$

input
```
integrate(csch(a+b*ln(c*x**n))**3/x,x)
```

output
```
Integral(csch(a + b*log(c*x**n))**3/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(51) = 102$.

Time = 0.05 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.73

$$\int \frac{\operatorname{csch}^3(a+b\log(cx^n))}{x}\,dx = -\frac{c^{3b}e^{(3b\log(x^n)+3a)}+c^be^{(b\log(x^n)+a)}}{bc^{4b}ne^{(4b\log(x^n)+4a)}-2bc^{2b}ne^{(2b\log(x^n)+2a)}+bn} + \frac{\log\left(\frac{\left(c^be^{(b\log(x^n)+a)}+1\right)e^{(-a)}}{c^b}\right)}{2bn} - \frac{\log\left(\frac{\left(c^be^{(b\log(x^n)+a)}-1\right)e^{(-a)}}{c^b}\right)}{2bn}$$

input
```
integrate(csch(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

output
```
-(c^(3*b)*e^(3*b*log(x^n) + 3*a) + c^b*e^(b*log(x^n) + a))/(b*c^(4*b)*n*e^
(4*b*log(x^n) + 4*a) - 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) + 1/2*l
og((c^b*e^(b*log(x^n) + a) + 1)*e^(-a)/c^b)/(b*n) - 1/2*log((c^b*e^(b*log(
x^n) + a) - 1)*e^(-a)/c^b)/(b*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(51) = 102$.

Time = 0.22 (sec) , antiderivative size = 238, normalized size of antiderivative = 4.33

$$\int \frac{\operatorname{csch}^3(a+b\log(cx^n))}{x}\,dx = \frac{1}{2}c^{3b}\left(\frac{c^be^{(-3a)}\log\left(\sqrt{2|c|^b|x|^{bn}\cos\left(\frac{1}{2}\pi bn\operatorname{sgn}(x)-\frac{1}{2}\pi bn+\frac{1}{2}\pi b\operatorname{sgn}(c)-\frac{1}{2}\pi b\right)e^a+|c|^{2b}|x|^{2bn}e^{(2a)}+\right.}{bc^{4b}n}\right.$$

input
```
integrate(csch(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

output
```
1/2*c^(3*b)*(c^b*e^(-3*a)*log(sqrt(2*abs(c)^b*abs(x)^(b*n)*cos(1/2*pi*b*n*
sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*e^a + abs(c)^(2*b)*abs(x
)^(2*b*n)*e^(2*a) + 1))/(b*c^(4*b)*n) - c^b*e^(-3*a)*log(sqrt(-2*abs(c)^b*
abs(x)^(b*n)*cos(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi
*b)*e^a + abs(c)^(2*b)*abs(x)^(2*b*n)*e^(2*a) + 1))/(b*c^(4*b)*n) - 2*(c^(
2*b)*x^(3*b*n)*e^(2*a) + x^(b*n))*e^(-2*a)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1
)^2*b*c^(2*b)*n))*e^(3*a)
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.55

$$\int \frac{\operatorname{csch}^3(a+b\log(cx^n))}{x}\,dx = \frac{\operatorname{atan}\left(\frac{\mathrm{e}^{-a}\sqrt{-b^2\,n^2}}{b\,n\,{\left(c\,x^n\right)}^b}\right)}{\sqrt{-b^2\,n^2}} + \frac{\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(b\,n-\frac{b\,n\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}}\right)} - \frac{2\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(b\,n-\frac{2\,b\,n\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}}+\frac{b\,n\,\mathrm{e}^{-4\,a}}{{\left(c\,x^n\right)}^{4\,b}}\right)}$$

input
```
int(1/(x*sinh(a + b*log(c*x^n))^3),x)
```

output
```
atan((exp(-a)*(-b^2*n^2)^(1/2))/(b*n*(c*x^n)^b))/(-b^2*n^2)^(1/2) + exp(-a
)/((c*x^n)^b*(b*n - (b*n*exp(-2*a))/(c*x^n)^(2*b))) - (2*exp(-a))/((c*x^n)
^b*(b*n - (2*b*n*exp(-2*a))/(c*x^n)^(2*b) + (b*n*exp(-4*a))/(c*x^n)^(4*b))
)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 272, normalized size of antiderivative = 4.95

$$\int \frac{\operatorname{csch}^3(a+b\log(cx^n))}{x}\,dx = \frac{x^{4bn}e^{4a}c^{4b}\log\left(x^{bn}e^{a}c^{2b}+c^{b}\right) - x^{4bn}e^{4a}c^{4b}\log\left(x^{bn}e^{a}c^{2b}-c^{b}\right) - 2x^{3bn}e^{3a}c^{3b} - 2x^{2bn}e^{2a}c^{2b}\log\left(x^{bn}e^{a}c^{2b}+c^{b}\right)}{2bn\left(x^{4bn}e^{4a}c^{4b}-2x^{2bn}e^{2a}\right.}$$

input
```
int(csch(a+b*log(c*x^n))^3/x,x)
```

output
```
(x**(4*b*n)*e**(4*a)*c**(4*b)*log(x**(b*n)*e**a*c**(2*b) + c**b) - x**(4*b
*n)*e**(4*a)*c**(4*b)*log(x**(b*n)*e**a*c**(2*b) - c**b) - 2*x**(3*b*n)*e*
*(3*a)*c**(3*b) - 2*x**(2*b*n)*e**(2*a)*c**(2*b)*log(x**(b*n)*e**a*c**(2*b
) + c**b) + 2*x**(2*b*n)*e**(2*a)*c**(2*b)*log(x**(b*n)*e**a*c**(2*b) - c*
*b) - 2*x**(b*n)*e**a*c**b + log(x**(b*n)*e**a*c**(2*b) + c**b) - log(x**(
b*n)*e**a*c**(2*b) - c**b))/(2*b*n*(x**(4*b*n)*e**(4*a)*c**(4*b) - 2*x**(2
*b*n)*e**(2*a)*c**(2*b) + 1))
```

3.168 $\int \frac{\operatorname{csch}^4(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{\operatorname{csch}^4(a+b\log(cx^n))}{x}\,dx = \frac{\coth(a+b\log(cx^n))}{bn} - \frac{\coth^3(a+b\log(cx^n))}{3bn}$$

output
```
coth(a+b*ln(c*x^n))/b/n-1/3*coth(a+b*ln(c*x^n))^3/b/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.33

$$\int \frac{\operatorname{csch}^4(a+b\log(cx^n))}{x}\,dx = \frac{2\coth(a+b\log(cx^n))}{3bn} - \frac{\coth(a+b\log(cx^n))\operatorname{csch}^2(a+b\log(cx^n))}{3bn}$$

input
```
Integrate[Csch[a + b*Log[c*x^n]]^4/x,x]
```

output
```
(2*Coth[a + b*Log[c*x^n]])/(3*b*n) - (Coth[a + b*Log[c*x^n]]*Csch[a + b*Lo
g[c*x^n]]^2)/(3*b*n)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3039, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(a+b\log(cx^n))}{x}\,dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \operatorname{csch}^4(a+b\log(cx^n))\,d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc(ia+ib\log(cx^n))^4\,d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4254}$$

$$\frac{i\int (1-\coth^2(a+b\log(cx^n)))\,d(-i\coth(a+b\log(cx^n)))}{bn}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(\frac{1}{3}i\coth^3(a+b\log(cx^n))-i\coth(a+b\log(cx^n)))}{bn}$$

input
```
Int[Csch[a + b*Log[c*x^n]]^4/x,x]
```

output
```
(I*((-I)*Coth[a + b*Log[c*x^n]] + (I/3)*Coth[a + b*Log[c*x^n]]^3))/(b*n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 6.62 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}(a+b\ln(c\,x^n))^2}{3}\right)\coth(a+b\ln(c\,x^n))}{nb}$
default	$\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}(a+b\ln(c\,x^n))^2}{3}\right)\coth(a+b\ln(c\,x^n))}{nb}$
parallelrisch	$\frac{-\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^3-\coth\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^3+9\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)+9\coth\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)}{24bn}$
risch	$-\frac{4\left(3(x^n)^{2b}c^{2b}\mathrm{e}^{2a}\mathrm{e}^{ib\pi\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)^2}\mathrm{e}^{-ib\pi\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)\operatorname{csgn}(ic)}\mathrm{e}^{-ib\pi\operatorname{csgn}(ic\,x^n)^3}\mathrm{e}^{ib\pi\operatorname{csgn}(ic\,x^n)^2\operatorname{csgn}(\cdots}}{3bn\left((x^n)^{2b}c^{2b}\mathrm{e}^{2a}\mathrm{e}^{ib\pi\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)^2}\mathrm{e}^{-ib\pi\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)\operatorname{csgn}(ic)}\mathrm{e}^{-ib\pi\operatorname{csgn}(ic\,x^n)^3}\mathrm{e}^{ib\pi\operatorname{csgn}(ic\,x^n)^2\operatorname{csgn}(\cdots}}$

input
```
int(csch(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(2/3-1/3*csch(a+b*ln(c*x^n))^2)*coth(a+b*ln(c*x^n))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(40) = 80$.

Time = 0.09 (sec) , antiderivative size = 272, normalized size of antiderivative = 6.48

$$\int \frac{\operatorname{csch}^4(a + b\log(cx^n))}{x}\,dx =$$

$$-\frac{}{3\left(bn\cosh\left(bn\log\left(x\right) + b\log\left(c\right) + a\right)^5 + 5\,bn\cosh\left(bn\log\left(x\right) + b\log\left(c\right) + a\right)\sinh\left(bn\log\left(x\right) + b\log\left(c\right)\right.\right.}$$

input
```
integrate(csch(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

output
```
-8/3*(cosh(b*n*log(x) + b*log(c) + a) + 2*sinh(b*n*log(x) + b*log(c) + a))
/(b*n*cosh(b*n*log(x) + b*log(c) + a)^5 + 5*b*n*cosh(b*n*log(x) + b*log(c)
 + a)*sinh(b*n*log(x) + b*log(c) + a)^4 + b*n*sinh(b*n*log(x) + b*log(c) +
 a)^5 - 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + (10*b*n*cosh(b*n*log(x)
+ b*log(c) + a)^2 - 3*b*n)*sinh(b*n*log(x) + b*log(c) + a)^3 + 2*b*n*cosh(
b*n*log(x) + b*log(c) + a) + (10*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 - 9
*b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^2 +
(5*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 - 9*b*n*cosh(b*n*log(x) + b*log(c
) + a)^2 + 4*b*n)*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(a + b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{csch}^4\left(a + b\log\left(cx^n\right)\right)}{x}\,dx$$

input
```
integrate(csch(a+b*ln(c*x**n))**4/x,x)
```

output
```
Integral(csch(a + b*log(c*x**n))**4/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(40) = 80$.

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.19

$$\int \frac{\operatorname{csch}^4(a + b\log(cx^n))}{x}\,dx = -\frac{4\left(3\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)} - 1\right)}{3\left(bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)} - 3\,bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)} + 3\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)} - bn\right)}$$

input
```
integrate(csch(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

output
```
-4/3*(3*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 1)/(b*c^(6*b)*n*e^(6*b*log(x^n) +
 6*a) - 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^
n) + 2*a) - b*n)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^4(a + b\log(cx^n))}{x}\,dx = -\frac{4\left(3\,c^{2\,b}x^{2\,bn}e^{(2\,a)} - 1\right)}{3\left(c^{2\,b}x^{2\,bn}e^{(2\,a)} - 1\right)^3 bn}$$

input
```
integrate(csch(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

output
```
-4/3*(3*c^(2*b)*x^(2*b*n)*e^(2*a) - 1)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^3*
b*n)
```

Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31

$$\int \frac{\operatorname{csch}^4(a+b\log(cx^n))}{x}\,dx=\frac{4\,\mathrm{e}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}\,\left(\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}-3\right)}{3\,b\,n\,{\left(\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}-1\right)}^3}$$

input
```
int(1/(x*sinh(a + b*log(c*x^n))^4),x)
```

output
```
(4*exp(4*a)*(c*x^n)^(4*b)*(exp(2*a)*(c*x^n)^(2*b) - 3))/(3*b*n*(exp(2*a)*(
c*x^n)^(2*b) - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.02

$$\int \frac{\operatorname{csch}^4(a+b\log(cx^n))}{x}\,dx=\frac{-4x^{2bn}e^{2a}c^{2b}+\frac{4}{3}}{bn\left(x^{6bn}e^{6a}c^{6b}-3x^{4bn}e^{4a}c^{4b}+3x^{2bn}e^{2a}c^{2b}-1\right)}$$

input
```
int(csch(a+b*log(c*x^n))^4/x,x)
```

output
```
(4*( - 3*x**(2*b*n)*e**(2*a)*c**(2*b) + 1))/(3*b*n*(x**(6*b*n)*e**(6*a)*c*
*(6*b) - 3*x**(4*b*n)*e**(4*a)*c**(4*b) + 3*x**(2*b*n)*e**(2*a)*c**(2*b) -
 1))
```

3.169 $\int \frac{\operatorname{csch}^5(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 89

$$\int \frac{\operatorname{csch}^5(a+b\log(cx^n))}{x}\,dx = -\frac{3\operatorname{arctanh}(\cosh(a+b\log(cx^n)))}{8bn} + \frac{3\coth(a+b\log(cx^n))\operatorname{csch}(a+b\log(cx^n))}{8bn} - \frac{\coth(a+b\log(cx^n))\operatorname{csch}^3(a+b\log(cx^n))}{4bn}$$

output
```
-3/8*arctanh(cosh(a+b*ln(c*x^n)))/b/n+3/8*coth(a+b*ln(c*x^n))*csch(a+b*ln(
c*x^n))/b/n-1/4*coth(a+b*ln(c*x^n))*csch(a+b*ln(c*x^n))^3/b/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.81

$$\int \frac{\operatorname{csch}^5(a+b\log(cx^n))}{x}\,dx = \frac{3\operatorname{csch}^2\left(\frac{1}{2}(a+b\log(cx^n))\right)}{32bn} - \frac{\operatorname{csch}^4\left(\frac{1}{2}(a+b\log(cx^n))\right)}{64bn} - \frac{3\log\left(\cosh\left(\frac{1}{2}(a+b\log(cx^n))\right)\right)}{8bn} + \frac{3\log\left(\sinh\left(\frac{1}{2}(a+b\log(cx^n))\right)\right)}{8bn} + \frac{3\operatorname{sech}^2\left(\frac{1}{2}(a+b\log(cx^n))\right)}{32bn} + \frac{\operatorname{sech}^4\left(\frac{1}{2}(a+b\log(cx^n))\right)}{64bn}$$

input
```
Integrate[Csch[a + b*Log[c*x^n]]^5/x,x]
```

output
```
(3*Csch[(a + b*Log[c*x^n])/2]^2)/(32*b*n) - Csch[(a + b*Log[c*x^n])/2]^4/(
64*b*n) - (3*Log[Cosh[(a + b*Log[c*x^n])/2]])/(8*b*n) + (3*Log[Sinh[(a + b
*Log[c*x^n])/2]])/(8*b*n) + (3*Sech[(a + b*Log[c*x^n])/2]^2)/(32*b*n) + Se
ch[(a + b*Log[c*x^n])/2]^4/(64*b*n)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {3039, 3042, 26, 4255, 26, 3042, 26, 4255, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^5(a+b\log(cx^n))}{x}\,dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \operatorname{csch}^5(a+b\log(cx^n))\,d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int i\csc(ia+ib\log(cx^n))^5\,d\log(cx^n)}{n}$$

$\Big\downarrow$ 26

$$\frac{i\int\csc(ia+ib\log(cx^n))^5\,d\log(cx^n)}{n}$$

$\Big\downarrow$ 4255

$$\frac{i\left(\frac{3}{4}\int i\text{csch}^3(a+b\log(cx^n))\,d\log(cx^n)+\frac{i\coth(a+b\log(cx^n))\text{csch}^3(a+b\log(cx^n))}{4b}\right)}{n}$$

$\Big\downarrow$ 26

$$\frac{i\left(\frac{3}{4}i\int\text{csch}^3(a+b\log(cx^n))\,d\log(cx^n)+\frac{i\coth(a+b\log(cx^n))\text{csch}^3(a+b\log(cx^n))}{4b}\right)}{n}$$

$\Big\downarrow$ 3042

$$\frac{i\left(\frac{3}{4}i\int -i\csc(ia+ib\log(cx^n))^3\,d\log(cx^n)+\frac{i\coth(a+b\log(cx^n))\text{csch}^3(a+b\log(cx^n))}{4b}\right)}{n}$$

$\Big\downarrow$ 26

$$\frac{i\left(\frac{3}{4}\int\csc(ia+ib\log(cx^n))^3\,d\log(cx^n)+\frac{i\coth(a+b\log(cx^n))\text{csch}^3(a+b\log(cx^n))}{4b}\right)}{n}$$

$\Big\downarrow$ 4255

$$\frac{i\left(\frac{3}{4}\left(\frac{1}{2}\int -i\text{csch}(a+b\log(cx^n))\,d\log(cx^n)-\frac{i\coth(a+b\log(cx^n))\text{csch}(a+b\log(cx^n))}{2b}\right)+\frac{i\coth(a+b\log(cx^n))\text{csch}^3(a+b\log(c}{4b}\right.}{n}$$

$\Big\downarrow$ 26

$$\frac{i\left(\frac{3}{4}\left(-\frac{1}{2}i\int\text{csch}(a+b\log(cx^n))\,d\log(cx^n)-\frac{i\coth(a+b\log(cx^n))\text{csch}(a+b\log(cx^n))}{2b}\right)+\frac{i\coth(a+b\log(cx^n))\text{csch}^3(a+b\log(c}{4b}\right.}{n}$$

$\Big\downarrow$ 3042

$$\frac{i\left(\frac{3}{4}\left(-\frac{1}{2}i\int i\csc(ia+ib\log(cx^n))\,d\log(cx^n)-\frac{i\coth(a+b\log(cx^n))\text{csch}(a+b\log(cx^n))}{2b}\right)+\frac{i\coth(a+b\log(cx^n))\text{csch}^3(a+b\log}{4b}\right.}{n}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(ia+ib\log\left(cx^n\right)\right)d\log\left(cx^n\right)-\frac{i\coth(a+b\log(cx^n))\text{csch}(a+b\log(cx^n))}{2b}\right)+\frac{i\coth(a+b\log(cx^n))\text{csch}^3(a+b\log(cx^n}{4b}}{n}$$

$$\Big\downarrow \text{4257}$$

$$\frac{i\left(\frac{3}{4}\left(\frac{i\text{arctanh}(\cosh(a+b\log(cx^n)))}{2b}-\frac{i\coth(a+b\log(cx^n))\text{csch}(a+b\log(cx^n))}{2b}\right)+\frac{i\coth(a+b\log(cx^n))\text{csch}^3(a+b\log(cx^n))}{4b}\right)}{n}$$

input
```
Int[Csch[a + b*Log[c*x^n]]^5/x,x]
```

output
```
(I*(((I/4)*Coth[a + b*Log[c*x^n]]*Csch[a + b*Log[c*x^n]]^3)/b + (3*(((I/2)
*ArcTanh[Cosh[a + b*Log[c*x^n]]])/b - ((I/2)*Coth[a + b*Log[c*x^n]]*Csch[a
 + b*Log[c*x^n]])/b))/4))/n
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 22.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{\left(-\frac{\operatorname{csch}(a+b\ln(c\,x^{n}))^{3}}{4}+\frac{3\,\operatorname{csch}(a+b\ln(c\,x^{n}))}{8}\right)\coth(a+b\ln(c\,x^{n}))-\frac{3\,\operatorname{arctanh}\left(\mathrm{e}^{a+b\ln(c\,x^{n})}\right)}{4}}{nb}$
default	$\frac{\left(-\frac{\operatorname{csch}(a+b\ln(c\,x^{n}))^{3}}{4}+\frac{3\,\operatorname{csch}(a+b\ln(c\,x^{n}))}{8}\right)\coth(a+b\ln(c\,x^{n}))-\frac{3\,\operatorname{arctanh}\left(\mathrm{e}^{a+b\ln(c\,x^{n})}\right)}{4}}{nb}$
parallelrisch	$\frac{-\coth\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)^{4}+\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)^{4}-8\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)^{2}+24\ln\left(\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)\right)+8\coth\left(\frac{a}{2}\right.}{64bn}$
risch	Expression too large to display

input
```
int(csch(a+b*ln(c*x^n))^5/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*((-1/4*csch(a+b*ln(c*x^n))^3+3/8*csch(a+b*ln(c*x^n)))*coth(a+b*ln(c*
x^n))-3/4*arctanh(exp(a+b*ln(c*x^n))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1806 vs. 2(83) = 166.

Time = 0.10 (sec) , antiderivative size = 1806, normalized size of antiderivative = 20.29

$$\int \frac{\operatorname{csch}^5(a+b\log(cx^n))}{x}\,dx = \text{Too large to display}$$

input
```
integrate(csch(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

output
```
1/8*(6*cosh(b*n*log(x) + b*log(c) + a)^7 + 42*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a)^6 + 6*sinh(b*n*log(x) + b*log(c) + a)^
7 + 2*(63*cosh(b*n*log(x) + b*log(c) + a)^2 - 11)*sinh(b*n*log(x) + b*log(
c) + a)^5 - 22*cosh(b*n*log(x) + b*log(c) + a)^5 + 10*(21*cosh(b*n*log(x)
+ b*log(c) + a)^3 - 11*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) +
b*log(c) + a)^4 + 2*(105*cosh(b*n*log(x) + b*log(c) + a)^4 - 110*cosh(b*n*
log(x) + b*log(c) + a)^2 - 11)*sinh(b*n*log(x) + b*log(c) + a)^3 - 22*cosh
(b*n*log(x) + b*log(c) + a)^3 + 2*(63*cosh(b*n*log(x) + b*log(c) + a)^5 -
110*cosh(b*n*log(x) + b*log(c) + a)^3 - 33*cosh(b*n*log(x) + b*log(c) + a)
)*sinh(b*n*log(x) + b*log(c) + a)^2 - 3*(cosh(b*n*log(x) + b*log(c) + a)^8
 + 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^7 + s
inh(b*n*log(x) + b*log(c) + a)^8 + 4*(7*cosh(b*n*log(x) + b*log(c) + a)^2
- 1)*sinh(b*n*log(x) + b*log(c) + a)^6 - 4*cosh(b*n*log(x) + b*log(c) + a)
^6 + 8*(7*cosh(b*n*log(x) + b*log(c) + a)^3 - 3*cosh(b*n*log(x) + b*log(c)
 + a))*sinh(b*n*log(x) + b*log(c) + a)^5 + 2*(35*cosh(b*n*log(x) + b*log(c
) + a)^4 - 30*cosh(b*n*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x) + b*l
og(c) + a)^4 + 6*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*cosh(b*n*log(x)
+ b*log(c) + a)^5 - 10*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(
x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*cosh(b*n*log(
x) + b*log(c) + a)^6 - 15*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*cosh(b*...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^5(a + b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{csch}^5(a + b\log(cx^n))}{x}\,dx$$

input
```
integrate(csch(a+b*ln(c*x**n))**5/x,x)
```

output
```
Integral(csch(a + b*log(c*x**n))**5/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(83) = 166$.

Time = 0.06 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.61

$$\int \frac{\operatorname{csch}^5(a+b\log(cx^n))}{x}\,dx = \frac{3\,c^{7\,b}e^{(7\,b\log(x^n)+7\,a)} - 11\,c^{5\,b}e^{(5\,b\log(x^n)+5\,a)} - 11\,c^{3\,b}e^{(3\,b\log(x^n)+3\,a)} + 3\,c^{b}e^{(b\log(x^n)+a)}}{4\,(bc^{8\,b}ne^{(8\,b\log(x^n)+8\,a)} - 4\,bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)} + 6\,bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)} - 4\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)} + bn)} - \frac{3\,\log\left(\frac{\left(c^{b}e^{(b\log(x^n)+a)}+1\right)e^{(-a)}}{c^{b}}\right)}{8\,bn} + \frac{3\,\log\left(\frac{\left(c^{b}e^{(b\log(x^n)+a)}-1\right)e^{(-a)}}{c^{b}}\right)}{8\,bn}$$

input
```
integrate(csch(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

output
```
1/4*(3*c^(7*b)*e^(7*b*log(x^n) + 7*a) - 11*c^(5*b)*e^(5*b*log(x^n) + 5*a)
- 11*c^(3*b)*e^(3*b*log(x^n) + 3*a) + 3*c^b*e^(b*log(x^n) + a))/(b*c^(8*b)
*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 6*b*c^(
4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n
) - 3/8*log((c^b*e^(b*log(x^n) + a) + 1)*e^(-a)/c^b)/(b*n) + 3/8*log((c^b*
e^(b*log(x^n) + a) - 1)*e^(-a)/c^b)/(b*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(83) = 166$.

Time = 0.23 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.10

$$\int \frac{\operatorname{csch}^5(a+b\log(cx^n))}{x}\,dx = -\frac{1}{8}\,c^{5\,b}\left(\frac{3\,c^{b}e^{(-5\,a)}\log\left(\sqrt{2\,|c|^{b}|x|^{bn}\cos\left(\frac{1}{2}\,\pi bn\mathrm{sgn}\left(x\right)-\frac{1}{2}\,\pi bn+\frac{1}{2}\,\pi b\mathrm{sgn}\left(c\right)-\frac{1}{2}\,\pi b\right)e^{a}+|c|^{2\,b}|x|^{2\,bn}e^{(2\,a}}\right.}{bc^{6\,b}n}\right.$$

input
```
integrate(csch(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

output
```
-1/8*c^(5*b)*(3*c^b*e^(-5*a)*log(sqrt(2*abs(c)^b*abs(x)^(b*n)*cos(1/2*pi*b
*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*e^a + abs(c)^(2*b)*ab
s(x)^(2*b*n)*e^(2*a) + 1))/(b*c^(6*b)*n) - 3*c^b*e^(-5*a)*log(sqrt(-2*abs(
c)^b*abs(x)^(b*n)*cos(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1
/2*pi*b)*e^a + abs(c)^(2*b)*abs(x)^(2*b*n)*e^(2*a) + 1))/(b*c^(6*b)*n) - 2
*(3*c^(6*b)*x^(7*b*n)*e^(6*a) - 11*c^(4*b)*x^(5*b*n)*e^(4*a) - 11*c^(2*b)*
x^(3*b*n)*e^(2*a) + 3*x^(b*n))*e^(-4*a)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^4
*b*c^(4*b)*n))*e^(5*a)
```

Mupad [B] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.57

$$\int \frac{\operatorname{csch}^5(a+b\log(cx^n))}{x}\,dx = \frac{2\,e^{-a}}{(c\,x^n)^b\,\left(b\,n-\frac{3\,b\,n\,e^{-2\,a}}{(c\,x^n)^{2\,b}}+\frac{3\,b\,n\,e^{-4\,a}}{(c\,x^n)^{4\,b}}-\frac{b\,n\,e^{-6\,a}}{(c\,x^n)^{6\,b}}\right)} - \frac{3\,\mathrm{atan}\left(\frac{e^{-a}\,\sqrt{-b^2\,n^2}}{b\,n\,(c\,x^n)^b}\right)}{4\,\sqrt{-b^2\,n^2}} - \frac{3\,e^{-a}}{4\,(c\,x^n)^b\,\left(b\,n-\frac{b\,n\,e^{-2\,a}}{(c\,x^n)^{2\,b}}\right)} - \frac{4\,e^{-3\,a}}{(c\,x^n)^{3\,b}\,\left(b\,n-\frac{4\,b\,n\,e^{-2\,a}}{(c\,x^n)^{2\,b}}+\frac{6\,b\,n\,e^{-4\,a}}{(c\,x^n)^{4\,b}}-\frac{4\,b\,n\,e^{-6\,a}}{(c\,x^n)^{6\,b}}+\frac{b\,n\,e^{-8\,a}}{(c\,x^n)^{8\,b}}\right)} - \frac{e^{-a}}{2\,(c\,x^n)^b\,\left(b\,n-\frac{2\,b\,n\,e^{-2\,a}}{(c\,x^n)^{2\,b}}+\frac{b\,n\,e^{-4\,a}}{(c\,x^n)^{4\,b}}\right)}$$

input
```
int(1/(x*sinh(a + b*log(c*x^n))^5),x)
```

output
```
(2*exp(-a))/((c*x^n)^b*(b*n - (3*b*n*exp(-2*a))/(c*x^n)^(2*b) + (3*b*n*exp
(-4*a))/(c*x^n)^(4*b) - (b*n*exp(-6*a))/(c*x^n)^(6*b))) - (3*atan((exp(-a)
*(-b^2*n^2)^(1/2))/(b*n*(c*x^n)^b)))/(4*(-b^2*n^2)^(1/2)) - (3*exp(-a))/(4
*(c*x^n)^b*(b*n - (b*n*exp(-2*a))/(c*x^n)^(2*b))) - (4*exp(-3*a))/((c*x^n)
^(3*b)*(b*n - (4*b*n*exp(-2*a))/(c*x^n)^(2*b) + (6*b*n*exp(-4*a))/(c*x^n)^
(4*b) - (4*b*n*exp(-6*a))/(c*x^n)^(6*b) + (b*n*exp(-8*a))/(c*x^n)^(8*b)))
- exp(-a)/(2*(c*x^n)^b*(b*n - (2*b*n*exp(-2*a))/(c*x^n)^(2*b) + (b*n*exp(-
4*a))/(c*x^n)^(4*b)))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 499, normalized size of antiderivative = 5.61

$$\int \frac{\operatorname{csch}^5(a + b\log(cx^n))}{x}\,dx$$
$$= \frac{-3x^{8bn}e^{8a}c^{8b}\log\left(x^{bn}e^ac^{2b} + c^b\right) + 3x^{8bn}e^{8a}c^{8b}\log\left(x^{bn}e^ac^{2b} - c^b\right) + 6x^{7bn}e^{7a}c^{7b} + 12x^{6bn}e^{6a}c^{6b}\log\left(x^{bn}e^ac^{2b}\right.}{}$$

input
```
int(csch(a+b*log(c*x^n))^5/x,x)
```

output
```
( - 3*x**(8*b*n)*e**(8*a)*c**(8*b)*log(x**(b*n)*e**a*c**(2*b) + c**b) + 3*
x**(8*b*n)*e**(8*a)*c**(8*b)*log(x**(b*n)*e**a*c**(2*b) - c**b) + 6*x**(7*
b*n)*e**(7*a)*c**(7*b) + 12*x**(6*b*n)*e**(6*a)*c**(6*b)*log(x**(b*n)*e**a
*c**(2*b) + c**b) - 12*x**(6*b*n)*e**(6*a)*c**(6*b)*log(x**(b*n)*e**a*c**(
2*b) - c**b) - 22*x**(5*b*n)*e**(5*a)*c**(5*b) - 18*x**(4*b*n)*e**(4*a)*c*
*(4*b)*log(x**(b*n)*e**a*c**(2*b) + c**b) + 18*x**(4*b*n)*e**(4*a)*c**(4*b
)*log(x**(b*n)*e**a*c**(2*b) - c**b) - 22*x**(3*b*n)*e**(3*a)*c**(3*b) + 1
2*x**(2*b*n)*e**(2*a)*c**(2*b)*log(x**(b*n)*e**a*c**(2*b) + c**b) - 12*x**
(2*b*n)*e**(2*a)*c**(2*b)*log(x**(b*n)*e**a*c**(2*b) - c**b) + 6*x**(b*n)*
e**a*c**b - 3*log(x**(b*n)*e**a*c**(2*b) + c**b) + 3*log(x**(b*n)*e**a*c**
(2*b) - c**b))/(8*b*n*(x**(8*b*n)*e**(8*a)*c**(8*b) - 4*x**(6*b*n)*e**(6*a
)*c**(6*b) + 6*x**(4*b*n)*e**(4*a)*c**(4*b) - 4*x**(2*b*n)*e**(2*a)*c**(2*
b) + 1))
```

3.170 $\int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2\cosh(a+b\log(cx^n))\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{2i\sqrt{\operatorname{csch}(a+b\log(cx^n))}\operatorname{EllipticF}\left(\frac{1}{4}(2ia-\pi+2ib\log(cx^n)),2\right)\sqrt{i\sinh(a+b\log(cx^n))}}{3bn}$$

output
```
-2/3*cosh(a+b*ln(c*x^n))*csch(a+b*ln(c*x^n))^(3/2)/b/n+2/3*I*csch(a+b*ln(c
*x^n))^(1/2)*InverseJacobiAM(1/2*I*a-1/4*Pi+1/2*I*b*ln(c*x^n),2^(1/2))*(I*
sinh(a+b*ln(c*x^n)))^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.77

$$\int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2\sqrt{\operatorname{csch}(a+b\log(cx^n))}\left(\coth(a+b\log(cx^n))+i\operatorname{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ib\log(cx^n)),2\right)\sqrt{i\sinh}\right.}{3bn}$$

input
```
Integrate[Csch[a + b*Log[c*x^n]]^(5/2)/x,x]
```

output
```
(-2*Sqrt[Csch[a + b*Log[c*x^n]]]*(Coth[a + b*Log[c*x^n]] + I*EllipticF[((-
2*I)*a + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]]))/(
3*b*n)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3039, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))\,d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (i\csc(ia+ib\log(cx^n)))^{5/2}\,d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4255}$$

$$\frac{-\frac{1}{3}\int \sqrt{\operatorname{csch}(a+b\log(cx^n))}d\log(cx^n)-\frac{2\cosh(a+b\log(cx^n))\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{3b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{2\cosh(a+b\log(cx^n))\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{3b}-\frac{1}{3}\int \sqrt{i\csc(ia+ib\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4258}$$

$$\frac{-\frac{2\cosh(a+b\log(cx^n))\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{3b}-\frac{1}{3}\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\operatorname{csch}(a+b\log(cx^n))}\int \frac{1}{\sqrt{i\sinh(a+b\log(cx^n))}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{2\cosh(a+b\log(cx^n))\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{3b}-\frac{1}{3}\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}\int\frac{1}{\sqrt{\sin(ia+ib\log(cx^n))}}d\log}{n}$$

$$\Big\downarrow \text{3120}$$

$$\frac{-\frac{2\cosh(a+b\log(cx^n))\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{3b}+\frac{2i\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\text{csch}(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right),2\right)}{3b}}{n}$$

input
```
Int[Csch[a + b*Log[c*x^n]]^(5/2)/x,x]
```

output
```
((-2*Cosh[a + b*Log[c*x^n]]*Csch[a + b*Log[c*x^n]]^(3/2))/(3*b) + (((2*I)/
3)*Sqrt[Csch[a + b*Log[c*x^n]]]*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2,
 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])/b)/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.32

method	result
derivativedivides	$-\frac{i\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\,\text{EllipticF}\left(\sqrt{1-i\sinh(a+b\ln(c\,x^n))},\right.}{3n\sinh(a+b\ln(c\,x^n))^{\frac{3}{2}}\cosh(a+b\ln(c\,x^n))b}$
default	$-\frac{i\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\,\text{EllipticF}\left(\sqrt{1-i\sinh(a+b\ln(c\,x^n))},\right.}{3n\sinh(a+b\ln(c\,x^n))^{\frac{3}{2}}\cosh(a+b\ln(c\,x^n))b}$

input
```
int(csch(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/3/n/sinh(a+b*ln(c*x^n))^(3/2)*(I*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2
)*(1+I*sinh(a+b*ln(c*x^n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF(
(1-I*sinh(a+b*ln(c*x^n)))^(1/2),1/2*2^(1/2))*sinh(a+b*ln(c*x^n))+2*cosh(a+
b*ln(c*x^n))^2)/cosh(a+b*ln(c*x^n))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(89) = 178$.

Time = 0.10 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.92

$$\int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx =$$
$$-\frac{2\left(\sqrt{2}\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+2\,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)\right.\right.\right.}{}$$

input
```
integrate(csch(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

output
```
-2/3*(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*l
og(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) +
a)^2 + 1)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(
c) + a))/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c)
 + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2
- 1)) + (sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*lo
g(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*lo
g(x) + b*log(c) + a)^2 - sqrt(2))*weierstrassPInverse(4, 0, cosh(b*n*log(x
) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x)
 + b*log(c) + a)^2 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x)
 + b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a)^2 - b*n)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$$

input
```
integrate(csch(a+b*ln(c*x**n))**(5/2)/x,x)
```

output
```
Integral(csch(a + b*log(c*x**n))**(5/2)/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{csch}(b\log(cx^n)+a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate(csch(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

output
```
integrate(csch(b*log(c*x^n) + a)^(5/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(csch(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\left(\frac{1}{\sinh(a+b\,\ln(c\,x^n))}\right)^{5/2}}{x}\,dx$$

input
```
int((1/sinh(a + b*log(c*x^n)))^(5/2)/x,x)
```

output
```
int((1/sinh(a + b*log(c*x^n)))^(5/2)/x, x)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sqrt{\operatorname{csch}(\log(x^n c)\,b+a)}\,\operatorname{csch}(\log(x^n c)\,b+a)^2}{x}dx$$

input
```
int(csch(a+b*log(c*x^n))^(5/2)/x,x)
```

output
```
int((sqrt(csch(log(x**n*c)*b + a))*csch(log(x**n*c)*b + a)**2)/x,x)
```

3.171 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 105

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2\cosh(a+b\log(cx^n))\sqrt{\operatorname{csch}(a+b\log(cx^n))}}{bn} - \frac{2iE\left(\frac{1}{4}(2ia-\pi+2ib\log(cx^n))\middle|2\right)}{bn\sqrt{\operatorname{csch}(a+b\log(cx^n))}\sqrt{i\sinh(a+b\log(cx^n))}}$$

output
```
-2*cosh(a+b*ln(c*x^n))*csch(a+b*ln(c*x^n))^(1/2)/b/n+2*I*EllipticE(cos(1/2
*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))/b/n/csch(a+b*ln(c*x^n))^(1/2)/(I*s
inh(a+b*ln(c*x^n)))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.76

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2\sqrt{\operatorname{csch}(a+b\log(cx^n))}\left(\cosh(a+b\log(cx^n)) - E\left(\frac{1}{4}(-2ia+\pi-2ib\log(cx^n))\middle|2\right)\sqrt{i\sinh(a+b\log}\right.}{bn}$$

input
```
Integrate[Csch[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
(-2*Sqrt[Csch[a + b*Log[c*x^n]]]*(Cosh[a + b*Log[c*x^n]] - EllipticE[((-2*
I)*a + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]]))/(b*
n)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3039, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (i\csc(ia+ib\log(cx^n)))^{3/2}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\int \frac{1}{\sqrt{\operatorname{csch}(a+b\log(cx^n))}}d\log(cx^n)-\frac{2\cosh(a+b\log(cx^n))\sqrt{\operatorname{csch}(a+b\log(cx^n))}}{b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{2\cosh(a+b\log(cx^n))\sqrt{\operatorname{csch}(a+b\log(cx^n))}}{b}+\int \frac{1}{\sqrt{i\csc(ia+ib\log(cx^n))}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4258}$$

$$\frac{-\frac{2\cosh(a+b\log(cx^n))\sqrt{\text{csch}(a+b\log(cx^n))}}{b}+\frac{\int\sqrt{i\sinh(a+b\log(cx^n))}d\log(cx^n)}{\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\text{csch}(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{-\frac{2\cosh(a+b\log(cx^n))\sqrt{\text{csch}(a+b\log(cx^n))}}{b}+\frac{\int\sqrt{\sin(ia+ib\log(cx^n))}d\log(cx^n)}{\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\text{csch}(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \textbf{3119}$$

$$\frac{-\frac{2\cosh(a+b\log(cx^n))\sqrt{\text{csch}(a+b\log(cx^n))}}{b}-\frac{2iE\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right)|2\right)}{b\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\text{csch}(a+b\log(cx^n))}}}{n}$$

input
```
Int[Csch[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
((-2*Cosh[a + b*Log[c*x^n]]*Sqrt[Csch[a + b*Log[c*x^n]]])/b - ((2*I)*Ellip
ticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2])/(b*Sqrt[Csch[a + b*Log[c*x^n]]]*
Sqrt[I*Sinh[a + b*Log[c*x^n]]]))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(94) = 188$.

Time = 0.47 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.02

method	result
derivativedivides	$\frac{2\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\,\operatorname{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(c\,x^n))}, \frac{\sqrt{2}}{2}\right.}{n\cosh(}$
default	$\frac{2\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\,\operatorname{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(c\,x^n))}, \frac{\sqrt{2}}{2}\right.}{n\cosh(}$

input
```
int(csch(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(2*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n)))^(
1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticE((1-I*sinh(a+b*ln(c*x^n)))^(1/
2),1/2*2^(1/2))-(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c
*x^n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(c*x
^n)))^(1/2),1/2*2^(1/2))-2*cosh(a+b*ln(c*x^n))^2)/cosh(a+b*ln(c*x^n))/sinh
(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.51

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2\left(\sqrt{2}\sqrt{\frac{\cosh(bn\log(x)+b\log(c)+a)+\sinh(bn\log(x)+b\log(c)+a)}{\cosh(bn\log(x)+b\log(c)+a)^2+2\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)+\sinh(bn\log(x)+b\log(c)+a)^2-1}}(\cos$$

input
```
integrate(csch(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

output
```
-2*(sqrt(2)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*lo
g(c) + a))/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(
c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^
2 - 1))*(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)
) + sqrt(2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(b*n*log(x
) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$$

input
```
integrate(csch(a+b*ln(c*x**n))**(3/2)/x,x)
```

output
```
Integral(csch(a + b*log(c*x**n))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{csch}(b\log(cx^n)+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(csch(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(csch(b*log(c*x^n) + a)^(3/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(csch(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\left(\frac{1}{\sinh(a+b\ln(cx^n))}\right)^{3/2}}{x}\,dx$$

input
```
int((1/sinh(a + b*log(c*x^n)))^(3/2)/x,x)
```

output
```
int((1/sinh(a + b*log(c*x^n)))^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(a + b\log(cx^n))}{x} dx = \int \frac{\sqrt{\operatorname{csch}(\log(x^n c) b + a)} \operatorname{csch}(\log(x^n c) b + a)}{x} dx$$

input
```
int(csch(a+b*log(c*x^n))^(3/2)/x,x)
```

output
```
int((sqrt(csch(log(x**n*c)*b + a))*csch(log(x**n*c)*b + a))/x,x)
```

3.172 $\int \frac{\sqrt{\operatorname{csch}(a+b\log(cx^n))}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 70

$$\int \frac{\sqrt{\operatorname{csch}(a+b\log(cx^n))}}{x}\,dx = -\frac{2i\sqrt{\operatorname{csch}(a+b\log(cx^n))}\operatorname{EllipticF}\left(\frac{1}{4}(2ia-\pi+2ib\log(cx^n)),2\right)\sqrt{i\sinh(a+b\log(cx^n))}}{bn}$$

output
```
-2*I*csch(a+b*ln(c*x^n))^(1/2)*InverseJacobiAM(1/2*I*a-1/4*Pi+1/2*I*b*ln(c
*x^n),2^(1/2))*(I*sinh(a+b*ln(c*x^n)))^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{\operatorname{csch}(a+b\log(cx^n))}}{x}\,dx = \frac{2\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))\operatorname{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ib\log(cx^n)),2\right)(i\sinh(a+b\log(cx^n)))^{3/2}}{bn}$$

input
```
Integrate[Sqrt[Csch[a + b*Log[c*x^n]]]/x,x]
```

output
```
(2*Csch[a + b*Log[c*x^n]]^(3/2)*EllipticF[((-2*I)*a + Pi - (2*I)*b*Log[c*x
^n])/4, 2]*(I*Sinh[a + b*Log[c*x^n]])^(3/2))/(b*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3039, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\operatorname{csch}(a+b\log(cx^n))}}{x}dx$$
$$\Big\downarrow \text{3039}$$
$$\frac{\int \sqrt{\operatorname{csch}(a+b\log(cx^n))}d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \sqrt{i\csc(ia+ib\log(cx^n))}d\log(cx^n)}{n}$$
$$\Big\downarrow \text{4258}$$
$$\frac{\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\operatorname{csch}(a+b\log(cx^n))}\int \frac{1}{\sqrt{i\sinh(a+b\log(cx^n))}}d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\operatorname{csch}(a+b\log(cx^n))}\int \frac{1}{\sqrt{\sin(ia+ib\log(cx^n))}}d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3120}$$
$$-\frac{2i\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\operatorname{csch}(a+b\log(cx^n))}\operatorname{EllipticF}\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right),2\right)}{bn}$$

input
```
Int[Sqrt[Csch[a + b*Log[c*x^n]]]/x,x]
```

output
```
((-2*I)*Sqrt[Csch[a + b*Log[c*x^n]]]*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n
])/2, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])/(b*n)
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.71

method	result
derivativedivides	$\frac{i\sqrt{-i(\sinh(a+b\ln(c\,x^{n}))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(a+b\ln(c\,x^{n}))+i)}\,\sqrt{i\sinh(a+b\ln(c\,x^{n}))}\,\text{EllipticF}\left(\sqrt{-i(\sinh(a+b\ln(c\,x \cdots}\right.}{n\cosh(a+b\ln(c\,x^{n}))\sqrt{\sinh(a+b\ln(c\,x^{n}))}\,b}$
default	$\frac{i\sqrt{-i(\sinh(a+b\ln(c\,x^{n}))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(a+b\ln(c\,x^{n}))+i)}\,\sqrt{i\sinh(a+b\ln(c\,x^{n}))}\,\text{EllipticF}\left(\sqrt{-i(\sinh(a+b\ln(c\,x \cdots}\right.}{n\cosh(a+b\ln(c\,x^{n}))\sqrt{\sinh(a+b\ln(c\,x^{n}))}\,b}$

input
```
int(csch(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
I/n*(-I*(sinh(a+b*ln(c*x^n))+I))^(1/2)*2^(1/2)*(-I*(-sinh(a+b*ln(c*x^n))+I
))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((-I*(sinh(a+b*ln(c*x^n))+
I))^(1/2),1/2*2^(1/2))/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = \frac{2\sqrt{2}\text{weierstrassPInverse}(4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right))}{bn}$$

input
```
integrate(csch(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

output
```
2*sqrt(2)*weierstrassPInverse(4, 0, cosh(b*n*log(x) + b*log(c) + a) + sinh
(b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx$$

input
```
integrate(csch(a+b*ln(c*x**n))**(1/2)/x,x)
```

output
```
Integral(sqrt(csch(a + b*log(c*x**n)))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{csch}\left(a+b\log\left(cx^{n}\right)\right)}}{x}\,dx=\int \frac{\sqrt{\operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)}}{x}\,dx$$

input
```
integrate(csch(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(csch(b*log(c*x^n) + a))/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{csch}\left(a+b\log\left(cx^{n}\right)\right)}}{x}\,dx=\text{Timed out}$$

input
```
integrate(csch(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{csch}\left(a+b\log\left(cx^{n}\right)\right)}}{x}\,dx=\int \frac{\sqrt{\frac{1}{\sinh(a+b\,\ln(c\,x^{n}))}}}{x}\,dx$$

input
```
int((1/sinh(a + b*log(c*x^n)))^(1/2)/x,x)
```

output
```
int((1/sinh(a + b*log(c*x^n)))^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{\operatorname{csch}\left(a + b\log\left(cx^n\right)\right)}}{x}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(\log\left(x^n c\right)b + a\right)}}{x}dx$$

input
```
int(csch(a+b*log(c*x^n))^(1/2)/x,x)
```

output
```
int(sqrt(csch(log(x**n*c)*b + a))/x,x)
```

3.173 $\int \frac{1}{x\sqrt{\operatorname{csch}(a+b\log(cx^n))}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 70

$$\int \frac{1}{x\sqrt{\operatorname{csch}(a+b\log(cx^n))}}\,dx = -\frac{2iE\left(\frac{1}{4}(2ia-\pi+2ib\log(cx^n))\middle|2\right)}{bn\sqrt{\operatorname{csch}(a+b\log(cx^n))}\sqrt{i\sinh(a+b\log(cx^n))}}$$

output
```
2*I*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))/b/n/csch(a+b*
ln(c*x^n))^(1/2)/(I*sinh(a+b*ln(c*x^n)))^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\sqrt{\operatorname{csch}(a+b\log(cx^n))}}\,dx = \frac{2\sqrt{\operatorname{csch}(a+b\log(cx^n))}E\left(\frac{1}{2}\left(\frac{\pi}{2}-i(a+b\log(cx^n))\right)\middle|2\right)\sqrt{i\sinh(a+b\log(cx^n))}}{bn}$$

input
```
Integrate[1/(x*Sqrt[Csch[a + b*Log[c*x^n]]]),x]
```

output
```
(2*Sqrt[Csch[a + b*Log[c*x^n]]]*EllipticE[(Pi/2 - I*(a + b*Log[c*x^n]))/2,
 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])/(b*n)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3039, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\operatorname{csch}(a+b\log(cx^n))}}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\sqrt{\operatorname{csch}(a+b\log(cx^n))}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{i\csc(ia+ib\log(cx^n))}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\int \sqrt{i\sinh(a+b\log(cx^n))}d\log(cx^n)}{n\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\operatorname{csch}(a+b\log(cx^n))}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\sin(ia+ib\log(cx^n))}d\log(cx^n)}{n\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\operatorname{csch}(a+b\log(cx^n))}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2iE\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\operatorname{csch}(a+b\log(cx^n))}}$$

input
```
Int[1/(x*Sqrt[Csch[a + b*Log[c*x^n]]]),x]
```

output
```
((-2*I)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2])/(b*n*Sqrt[Csch[a +
b*Log[c*x^n]]]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(61) = 122$.

Time = 0.44 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.09

method	result
derivativedivides	$\frac{\sqrt{-i(\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(\right.\right.}{n\cosh(a+b\ln(c\,x^n))\sqrt{\sinh(a+b\ln(c\,x^n))}\,b}$
default	$\frac{\sqrt{-i(\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(\right.\right.}{n\cosh(a+b\ln(c\,x^n))\sqrt{\sinh(a+b\ln(c\,x^n))}\,b}$

input
```
int(1/x/csch(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/n*(-I*(sinh(a+b*ln(c*x^n))+I))^(1/2)*2^(1/2)*(-I*(-sinh(a+b*ln(c*x^n))+I
))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*(2*EllipticE((1-I*sinh(a+b*ln(c*x^n
)))^(1/2),1/2*2^(1/2))-EllipticF((1-I*sinh(a+b*ln(c*x^n)))^(1/2),1/2*2^(1/
2)))/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(57) = 114$.

Time = 0.09 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.54

$$\int \frac{1}{x\sqrt{\operatorname{csch}(a+b\log(cx^n))}}\,dx =$$

$$-\frac{\sqrt{2}\left(\cosh(bn\log(x)+b\log(c)+a)^2+2\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+\right.}{}$$

input
```
integrate(1/x/csch(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c
) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2
 - 1)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) +
 a))/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a
)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)
) + 2*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x) +
 b*log(c) + a))*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(b*n*l
og(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))))/(b*n*cosh(b*n*l
og(x) + b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]

$$\int \frac{1}{x\sqrt{\operatorname{csch}(a+b\log(cx^n))}}\,dx = \int \frac{1}{x\sqrt{\operatorname{csch}(a+b\log(cx^n))}}\,dx$$

input
```
integrate(1/x/csch(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(csch(a + b*log(c*x**n)))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{\operatorname{csch}\left(a+b\log\left(cx^{n}\right)\right)}}\,dx=\int \frac{1}{x\sqrt{\operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)}}\,dx$$

input
```
integrate(1/x/csch(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sqrt(csch(b*log(c*x^n) + a))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\operatorname{csch}\left(a+b\log\left(cx^{n}\right)\right)}}\,dx=\text{Timed out}$$

input
```
integrate(1/x/csch(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\operatorname{csch}\left(a+b\log\left(cx^{n}\right)\right)}}\,dx=\int \frac{1}{x\,\sqrt{\frac{1}{\sinh\left(a+b\,\ln\left(c\,x^{n}\right)\right)}}}\,dx$$

input
```
int(1/(x*(1/sinh(a + b*log(c*x^n)))^(1/2)),x)
```

output
```
int(1/(x*(1/sinh(a + b*log(c*x^n)))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{\operatorname{csch}(a+b\log(cx^n))}}\,dx = \int \frac{\sqrt{\operatorname{csch}(\log(x^n c)\,b+a)}}{\operatorname{csch}(\log(x^n c)\,b+a)\,x}dx$$

input
```
int(1/x/csch(a+b*log(c*x^n))^(1/2),x)
```

output
```
int(sqrt(csch(log(x**n*c)*b + a))/(csch(log(x**n*c)*b + a)*x),x)
```

3.174 $\int \frac{1}{x\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))} dx$

Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{1}{x\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))} dx = \frac{2\cosh(a+b\log(cx^n))}{3bn\sqrt{\text{csch}(a+b\log(cx^n))}} + \frac{2i\sqrt{\text{csch}(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{4}(2ia-\pi+2ib\log(cx^n)),2\right)\sqrt{i\sinh(a+b\log(cx^n))}}{3bn}$$

output
```
2/3*cosh(a+b*ln(c*x^n))/b/n/csch(a+b*ln(c*x^n))^(1/2)+2/3*I*csch(a+b*ln(c*
x^n))^(1/2)*InverseJacobiAM(1/2*I*a-1/4*Pi+1/2*I*b*ln(c*x^n),2^(1/2))*(I*s
inh(a+b*ln(c*x^n)))^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.79

$$\int \frac{1}{x\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))} dx = \frac{\sqrt{\text{csch}(a+b\log(cx^n))}\left(-2i\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ib\log(cx^n)),2\right)\sqrt{i\sinh(a+b\log(cx^n))}+\sinh\right.}{3bn}$$

input
```
Integrate[1/(x*Csch[a + b*Log[c*x^n]]^(3/2)),x]
```

output
```
(Sqrt[Csch[a + b*Log[c*x^n]]]*((-2*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*L
og[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]] + Sinh[2*(a + b*Log[c*x^n]
)]))/(3*b*n)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3039, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{(i\csc(ia+ib\log(cx^n)))^{3/2}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4256}$$

$$\frac{\frac{2\cosh(a+b\log(cx^n))}{3b\sqrt{\text{csch}(a+b\log(cx^n))}}-\frac{1}{3}\int \sqrt{\text{csch}(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\cosh(a+b\log(cx^n))}{3b\sqrt{\text{csch}(a+b\log(cx^n))}}-\frac{1}{3}\int \sqrt{i\csc(ia+ib\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{2\cosh(a+b\log(cx^n))}{3b\sqrt{\text{csch}(a+b\log(cx^n))}}-\frac{1}{3}\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\text{csch}(a+b\log(cx^n))}\int \frac{1}{\sqrt{i\sinh(a+b\log(cx^n))}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\cosh(a+b\log(cx^n))}{3b\sqrt{\operatorname{csch}(a+b\log(cx^n))}}-\frac{1}{3}\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}\int\frac{1}{\sqrt{\sin(ia+ib\log(cx^n))}}d\log\left(cx^n\right)}{n}$$

$\Big\downarrow$ 3120

$$\frac{\frac{2\cosh(a+b\log(cx^n))}{3b\sqrt{\operatorname{csch}(a+b\log(cx^n))}}+\frac{2i\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\operatorname{csch}(a+b\log(cx^n))}\operatorname{EllipticF}\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right),2\right)}{3b}}{n}$$

input
```
Int[1/(x*Csch[a + b*Log[c*x^n]]^(3/2)),x]
```

output
```
((2*Cosh[a + b*Log[c*x^n]])/(3*b*Sqrt[Csch[a + b*Log[c*x^n]]]) + (((2*I)/3
)*Sqrt[Csch[a + b*Log[c*x^n]]]*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2,
2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])/b)/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{-\frac{i\sqrt{1-i\sinh(a+b\ln(c\,x^{n}))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^{n}))}\,\sqrt{i\sinh(a+b\ln(c\,x^{n}))}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh(a+b\ln(c\,x^{n}))},\frac{\sqrt{2}}{2}\right)}{3}+\frac{2\cosh(a\cdots}{}}{n\cosh(a+b\ln(c\,x^{n}))\sqrt{\sinh(a+b\ln(c\,x^{n}))}\,b}$
default	$\frac{-\frac{i\sqrt{1-i\sinh(a+b\ln(c\,x^{n}))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^{n}))}\,\sqrt{i\sinh(a+b\ln(c\,x^{n}))}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh(a+b\ln(c\,x^{n}))},\frac{\sqrt{2}}{2}\right)}{3}+\frac{2\cosh(a\cdots}{}}{n\cosh(a+b\ln(c\,x^{n}))\sqrt{\sinh(a+b\ln(c\,x^{n}))}\,b}$

input
```
int(1/x/csch(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/n*(-1/3*I*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n
)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(c*x^n))
)^(1/2),1/2*2^(1/2))+2/3*cosh(a+b*ln(c*x^n))^2*sinh(a+b*ln(c*x^n)))/cosh(a
+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(89) = 178$.

Time = 0.09 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.39

$$\int \frac{1}{x\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^{n}))}\,dx$$
$$= \frac{\sqrt{2}\left(\cosh(bn\log(x)+b\log(c)+a)^{4}+4\cosh(bn\log(x)+b\log(c)+a)^{3}\sinh(bn\log(x)+b\log(c)+a\cdots\right.}{}$$

input
```
integrate(1/x/csch(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*lo
g(c) + a)^3*sinh(b*n*log(x) + b*log(c) + a) + 6*cosh(b*n*log(x) + b*log(c)
 + a)^2*sinh(b*n*log(x) + b*log(c) + a)^2 + 4*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4
- 1)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) +
a))/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1))
 - 4*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*log(x
) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x
) + b*log(c) + a)^2)*weierstrassPInverse(4, 0, cosh(b*n*log(x) + b*log(c)
+ a) + sinh(b*n*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(c) +
 a)^2 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) +
 a) + b*n*sinh(b*n*log(x) + b*log(c) + a)^2)
```

Sympy [F]

$$\int \frac{1}{x\text{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}\,dx=\int \frac{1}{x\operatorname{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}\,dx$$

input
```
integrate(1/x/csch(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Integral(1/(x*csch(a + b*log(c*x**n))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\text{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}\,dx=\int \frac{1}{x\operatorname{csch}\left(b\log\left(cx^{n}\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/csch(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*csch(b*log(c*x^n) + a)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\operatorname{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x/csch(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\operatorname{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \int \frac{1}{x\left(\frac{1}{\sinh(a+b\,\ln(c\,x^n))}\right)^{3/2}}\,dx$$

input
```
int(1/(x*(1/sinh(a + b*log(c*x^n)))^(3/2)),x)
```

output
```
int(1/(x*(1/sinh(a + b*log(c*x^n)))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\operatorname{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \int \frac{\sqrt{\operatorname{csch}\left(\log\left(x^n c\right)b+a\right)}}{\operatorname{csch}\left(\log\left(x^n c\right)b+a\right)^2 x}dx$$

input
```
int(1/x/csch(a+b*log(c*x^n))^(3/2),x)
```

output
```
int(sqrt(csch(log(x**n*c)*b + a))/(csch(log(x**n*c)*b + a)**2*x),x)
```

3.175 $\int \frac{1}{x\text{csch}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{1}{x\text{csch}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{2\cosh(a+b\log(cx^n))}{5bn\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{6iE\left(\frac{1}{4}(2ia-\pi+2ib\log(cx^n))\middle|2\right)}{5bn\sqrt{\text{csch}(a+b\log(cx^n))}\sqrt{i\sinh(a+b\log(cx^n))}}$$

output
```
2/5*cosh(a+b*ln(c*x^n))/b/n/csch(a+b*ln(c*x^n))^(3/2)-6/5*I*EllipticE(cos(
1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))/b/n/csch(a+b*ln(c*x^n))^(1/2)/(
I*sinh(a+b*ln(c*x^n)))^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\text{csch}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{2\left(\cosh(a+b\log(cx^n)) - 3\text{csch}^2(a+b\log(cx^n))\,E\left(\frac{1}{4}(-2ia+\pi-2ib\log(cx^n))\middle|2\right)\sqrt{i\sinh(a+b\log(\right.}{5bn\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))}$$

input
```
Integrate[1/(x*Csch[a + b*Log[c*x^n]]^(5/2)),x]
```

output
```
(2*(Cosh[a + b*Log[c*x^n]] - 3*Csch[a + b*Log[c*x^n]]^2*EllipticE[((-2*I)*
a + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]]))/(5*b*n
*Csch[a + b*Log[c*x^n]]^(3/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3039, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\text{csch}^{\frac{5}{2}}(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\text{csch}^{\frac{5}{2}}(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{(i\csc(ia+ib\log(cx^n)))^{5/2}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4256}$$

$$\frac{\frac{2\cosh(a+b\log(cx^n))}{5b\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))}-\frac{3}{5}\int \frac{1}{\sqrt{\text{csch}(a+b\log(cx^n))}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\cosh(a+b\log(cx^n))}{5b\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))}-\frac{3}{5}\int \frac{1}{\sqrt{i\csc(ia+ib\log(cx^n))}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{2\cosh(a+b\log(cx^n))}{5b\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))}-\frac{3\int\sqrt{i\sinh(a+b\log(cx^n))}d\log(cx^n)}{5\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\text{csch}(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\cosh(a+b\log(cx^n))}{5b\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))}-\frac{3\int\sqrt{\sin(ia+ib\log(cx^n))}d\log(cx^n)}{5\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\text{csch}(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{2\cosh(a+b\log(cx^n))}{5b\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))}+\frac{6iE\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right)|2\right)}{5b\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\text{csch}(a+b\log(cx^n))}}}{n}$$

input
```
Int[1/(x*Csch[a + b*Log[c*x^n]]^(5/2)),x]
```

output
```
((2*Cosh[a + b*Log[c*x^n]])/(5*b*Csch[a + b*Log[c*x^n]]^(3/2)) + (((6*I)/5
)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2])/(b*Sqrt[Csch[a + b*Log[c*
x^n]]]*Sqrt[I*Sinh[a + b*Log[c*x^n]]]))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(94) = 188$.

Time = 0.46 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.08

method	result
derivativedivides	$\frac{-\frac{6\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\,\operatorname{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(c\,x^n))},\frac{\sqrt{2}}{2}\right)}{5}+\frac{3\sqrt{1-i\,s}}{}}{n\cosh(a-}$
default	$\frac{-\frac{6\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\,\operatorname{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(c\,x^n))},\frac{\sqrt{2}}{2}\right)}{5}+\frac{3\sqrt{1-i\,s}}{}}{n\cosh(a-}$

input
```
int(1/x/csch(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/n*(-6/5*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n))
)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticE((1-I*sinh(a+b*ln(c*x^n)))^
(1/2),1/2*2^(1/2))+3/5*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a
+b*ln(c*x^n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b
*ln(c*x^n)))^(1/2),1/2*2^(1/2))+2/5*cosh(a+b*ln(c*x^n))^4-2/5*cosh(a+b*ln(
c*x^n))^2)/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(90) = 180$.

Time = 0.10 (sec) , antiderivative size = 602, normalized size of antiderivative = 5.52

$$\int \frac{1}{x\text{csch}^{\frac{5}{2}}(a + b\log(cx^n))} dx = \text{Too large to display}$$

input
```
integrate(1/x/csch(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

output
```
1/20*(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^6 + 6*cosh(b*n*log(x) + b*l
og(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^5 + sinh(b*n*log(x) + b*log(c)
+ a)^6 + (15*cosh(b*n*log(x) + b*log(c) + a)^2 + 11)*sinh(b*n*log(x) + b*l
og(c) + a)^4 + 11*cosh(b*n*log(x) + b*log(c) + a)^4 + 4*(5*cosh(b*n*log(x)
 + b*log(c) + a)^3 + 11*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) +
 b*log(c) + a)^3 + (15*cosh(b*n*log(x) + b*log(c) + a)^4 + 66*cosh(b*n*log
(x) + b*log(c) + a)^2 - 13)*sinh(b*n*log(x) + b*log(c) + a)^2 - 13*cosh(b*
n*log(x) + b*log(c) + a)^2 + 2*(3*cosh(b*n*log(x) + b*log(c) + a)^5 + 22*c
osh(b*n*log(x) + b*log(c) + a)^3 - 13*cosh(b*n*log(x) + b*log(c) + a))*sin
h(b*n*log(x) + b*log(c) + a) + 1)*sqrt((cosh(b*n*log(x) + b*log(c) + a) +
sinh(b*n*log(x) + b*log(c) + a))/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*co
sh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*l
og(x) + b*log(c) + a)^2 - 1)) + 24*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a
)^3 + 3*sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2*sinh(b*n*log(x) + b*log(
c) + a) + 3*sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*lo
g(c) + a)^2 + sqrt(2)*sinh(b*n*log(x) + b*log(c) + a)^3)*weierstrassZeta(4
, 0, weierstrassPInverse(4, 0, cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*
log(x) + b*log(c) + a))))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*b*n*c
osh(b*n*log(x) + b*log(c) + a)^2*sinh(b*n*log(x) + b*log(c) + a) + 3*b*n*c
osh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n*...
```

Sympy [F]

$$\int \frac{1}{x\text{csch}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx=\int \frac{1}{x\,\text{csch}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$$

input
```
integrate(1/x/csch(a+b*ln(c*x**n))**(5/2),x)
```

output
```
Integral(1/(x*csch(a + b*log(c*x**n))**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\text{csch}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx=\int \frac{1}{x\,\text{csch}(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/csch(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*csch(b*log(c*x^n) + a)^(5/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\text{csch}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx=\text{Timed out}$$

input
```
integrate(1/x/csch(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\left(\frac{1}{\sinh(a+b\ln(c\,x^n))}\right)^{5/2}}\,dx$$

input
```
int(1/(x*(1/sinh(a + b*log(c*x^n)))^(5/2)),x)
```

output
```
int(1/(x*(1/sinh(a + b*log(c*x^n)))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{\sqrt{\operatorname{csch}(\log(x^n c)\,b+a)}}{\operatorname{csch}(\log(x^n c)\,b+a)^3\,x}dx$$

input
```
int(1/x/csch(a+b*log(c*x^n))^(5/2),x)
```

output
```
int(sqrt(csch(log(x**n*c)*b + a))/(csch(log(x**n*c)*b + a)**3*x),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))  #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file