Computer Algebra Independent Integration Tests

Summer 2024

6-Hyperbolic-functions/6.7-Hyperbolic-exponential/321-6.7.1

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**119**]. This is test number [321].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	98.32 (117)	1.68 (2)
Mathematica	98.32 (117)	1.68 (2)
Fricas	77.31 (92)	22.69 (27)
Giac	73.95 (88)	26.05 (31)
Maple	70.59 (84)	29.41 (35)
Maxima	70.59 (84)	29.41 (35)
Mupad	58.82 (70)	41.18 (49)
Reduce	55.46 (66)	44.54 (53)
Sympy	26.89 (32)	73.11 (87)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

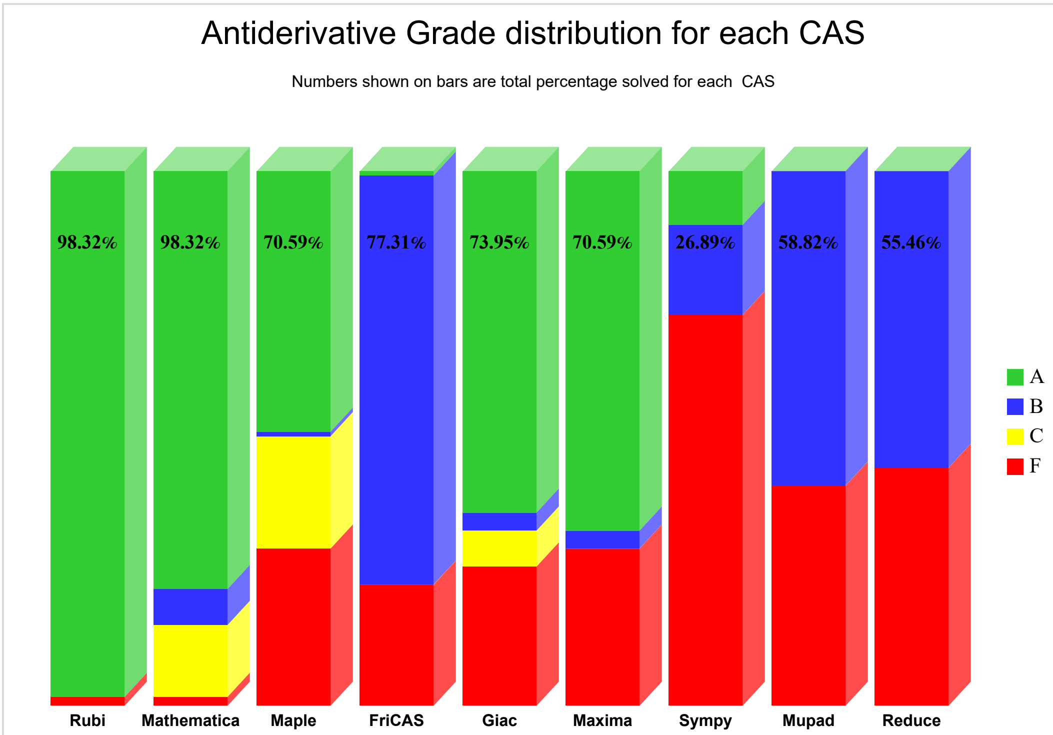
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

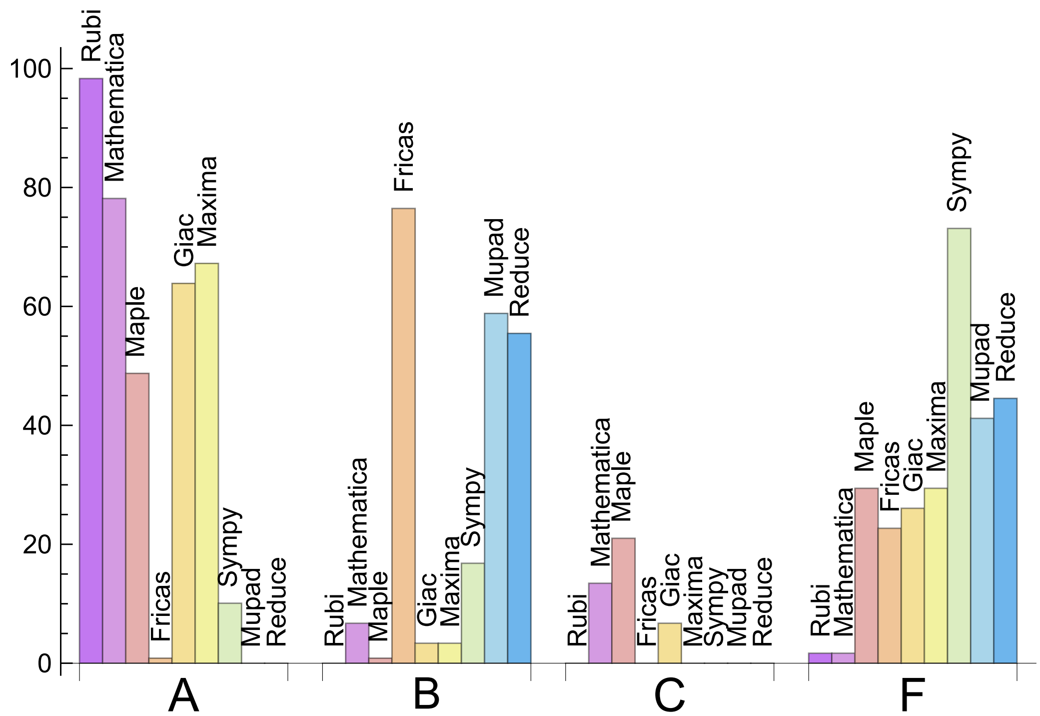
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.319	0.000	0.000	1.681
Mathematica	78.151	6.723	13.445	1.681
Maxima	67.227	3.361	0.000	29.412
Giac	63.866	3.361	6.723	26.050
Maple	48.739	0.840	21.008	29.412
Sympy	10.084	16.807	0.000	73.109
Fricas	0.840	76.471	0.000	22.689
Mupad	0.000	58.824	0.000	41.176
Reduce	0.000	55.462	0.000	44.538

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	2	100.00	0.00	0.00
Mathematica	2	100.00	0.00	0.00
Fricas	27	96.30	0.00	3.70
Giac	31	100.00	0.00	0.00
Maple	35	100.00	0.00	0.00
Maxima	35	100.00	0.00	0.00
Mupad	49	0.00	100.00	0.00
Reduce	53	100.00	0.00	0.00
Sympy	87	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.07
Fricas	0.11
Giac	0.13
Reduce	0.22
Rubi	0.31
Mathematica	0.33
Maple	0.82
Mupad	2.64
Sympy	2.68

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	93.41	0.90	75.00	0.78
Reduce	97.14	1.30	67.50	1.28
Mathematica	100.29	1.11	87.00	1.00
Maxima	109.36	1.14	89.50	1.02
Maple	109.87	1.12	85.50	0.99
Mupad	155.13	1.32	84.50	1.22
Giac	192.34	1.89	74.00	0.89
Sympy	408.22	3.89	141.00	2.05
Fricas	1405.34	8.65	402.50	5.39

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

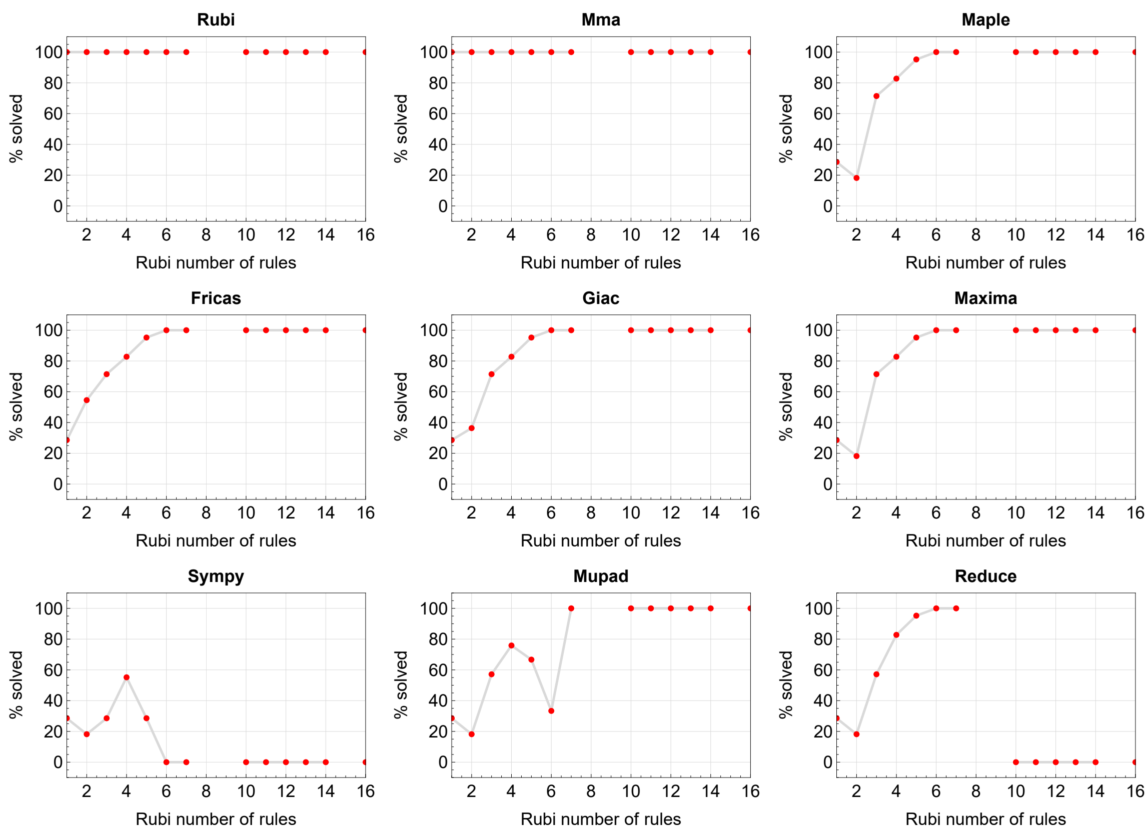


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

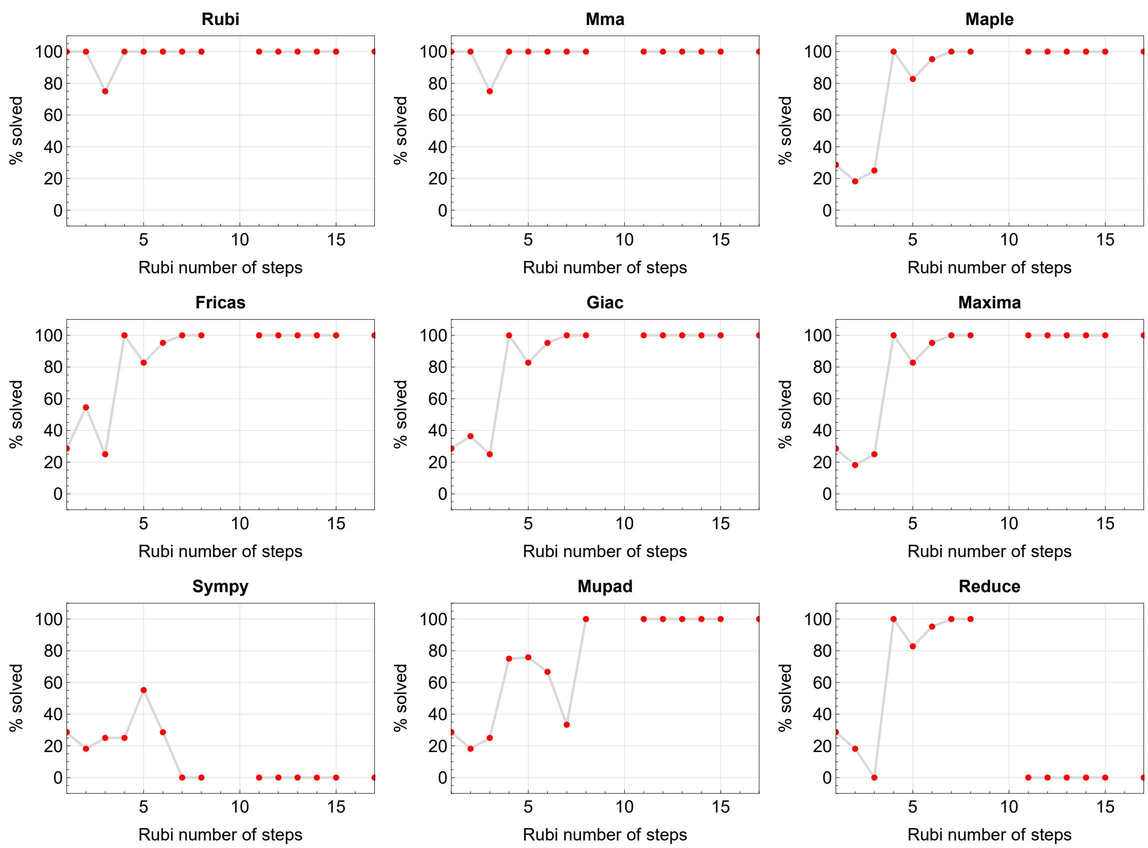


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

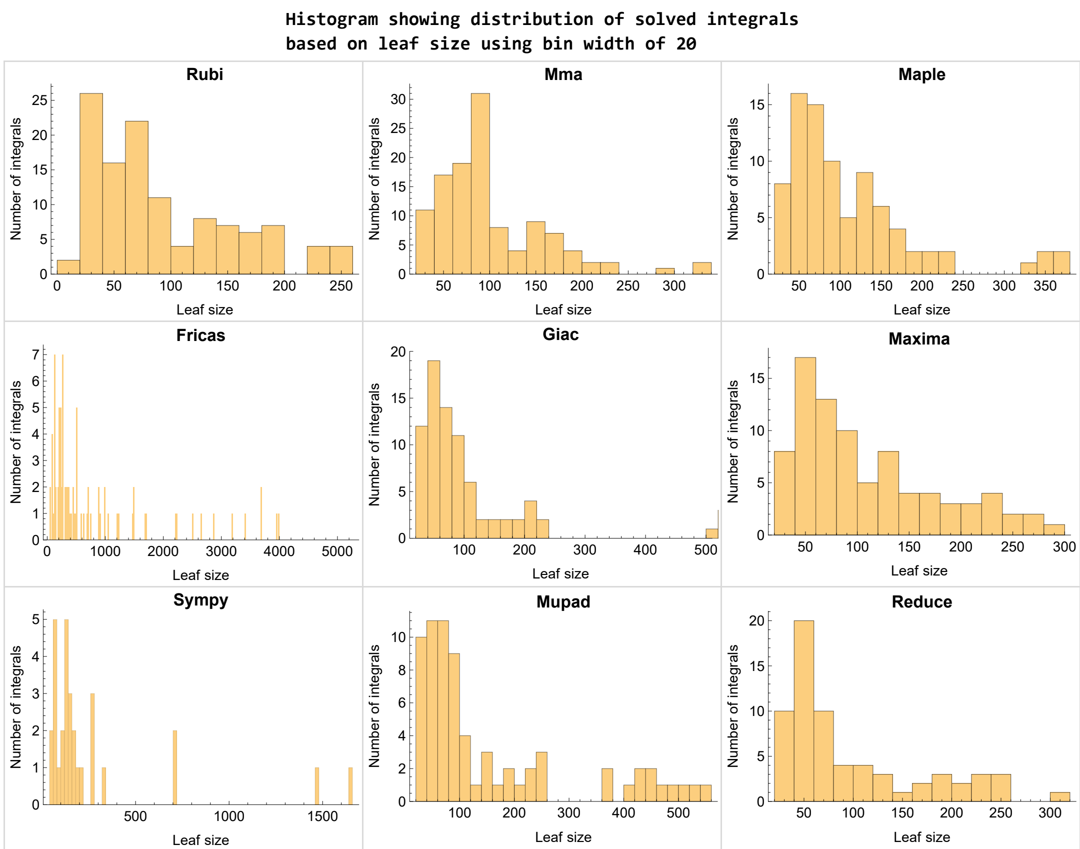


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

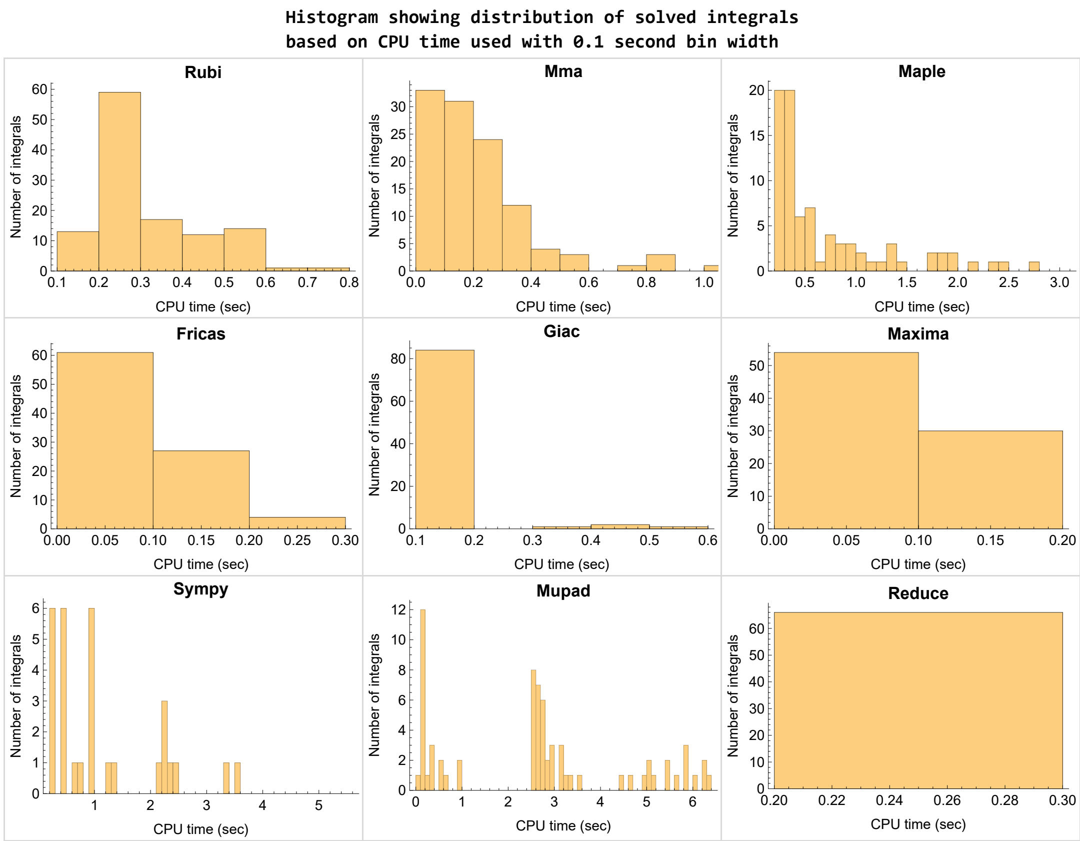


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

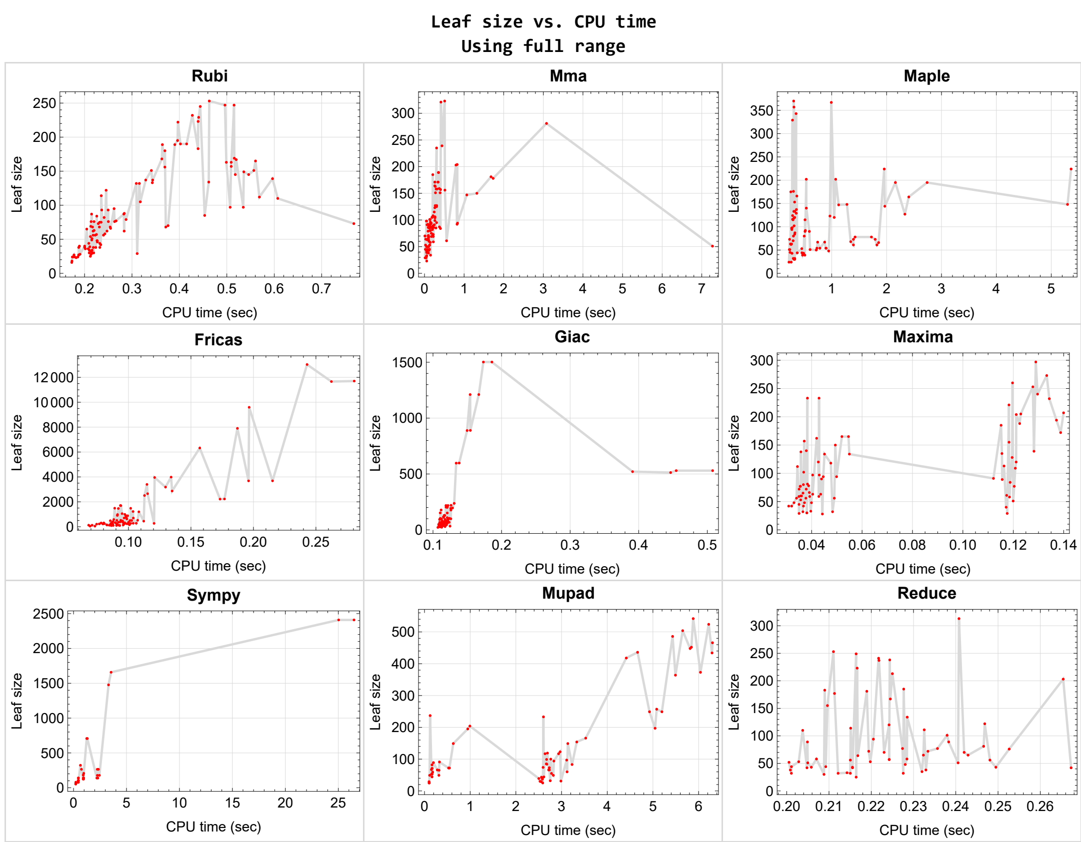


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 22, 23, 24, 25, 26, 27, 28, 29, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112}

Mathematica {116}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

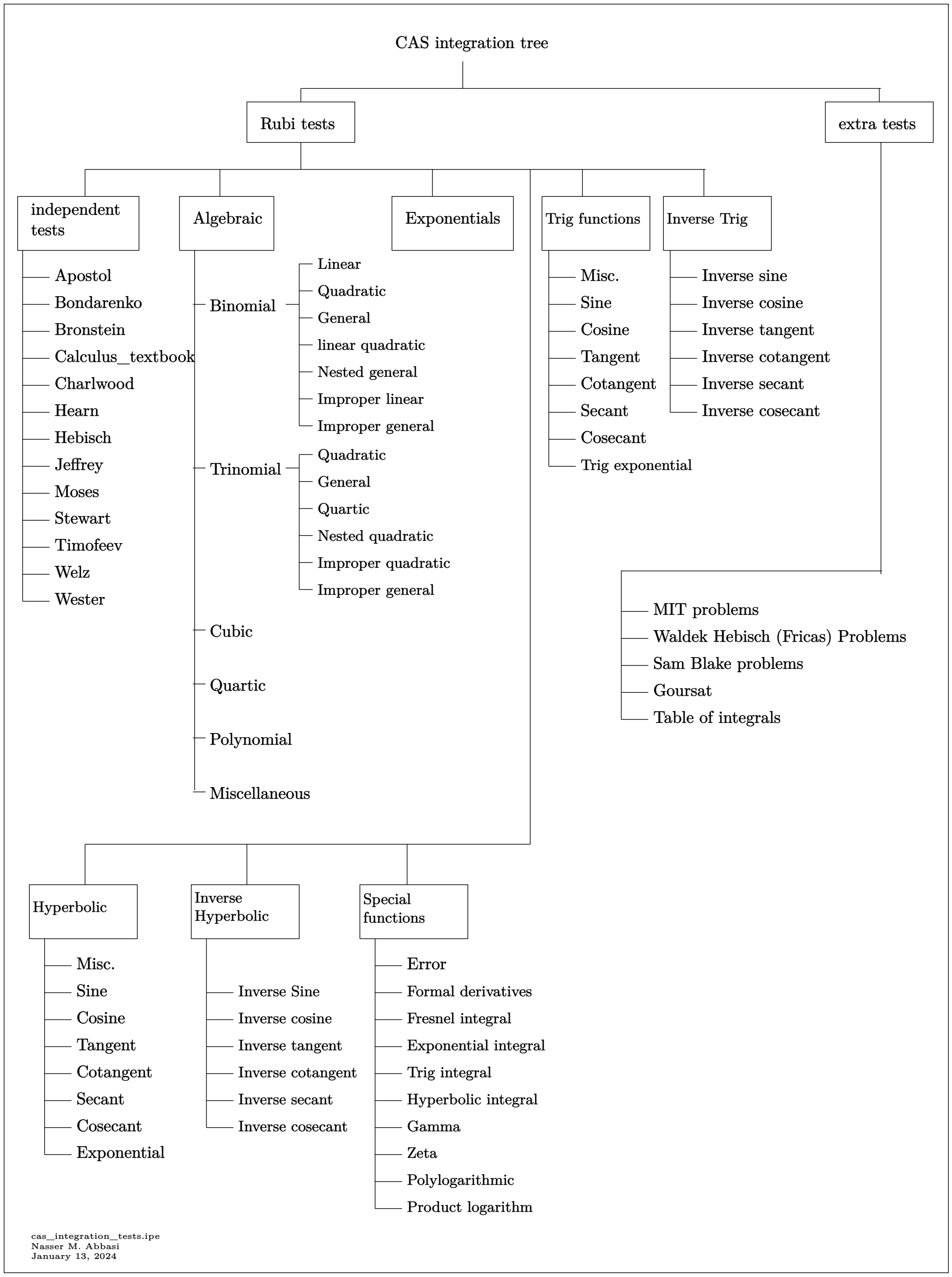
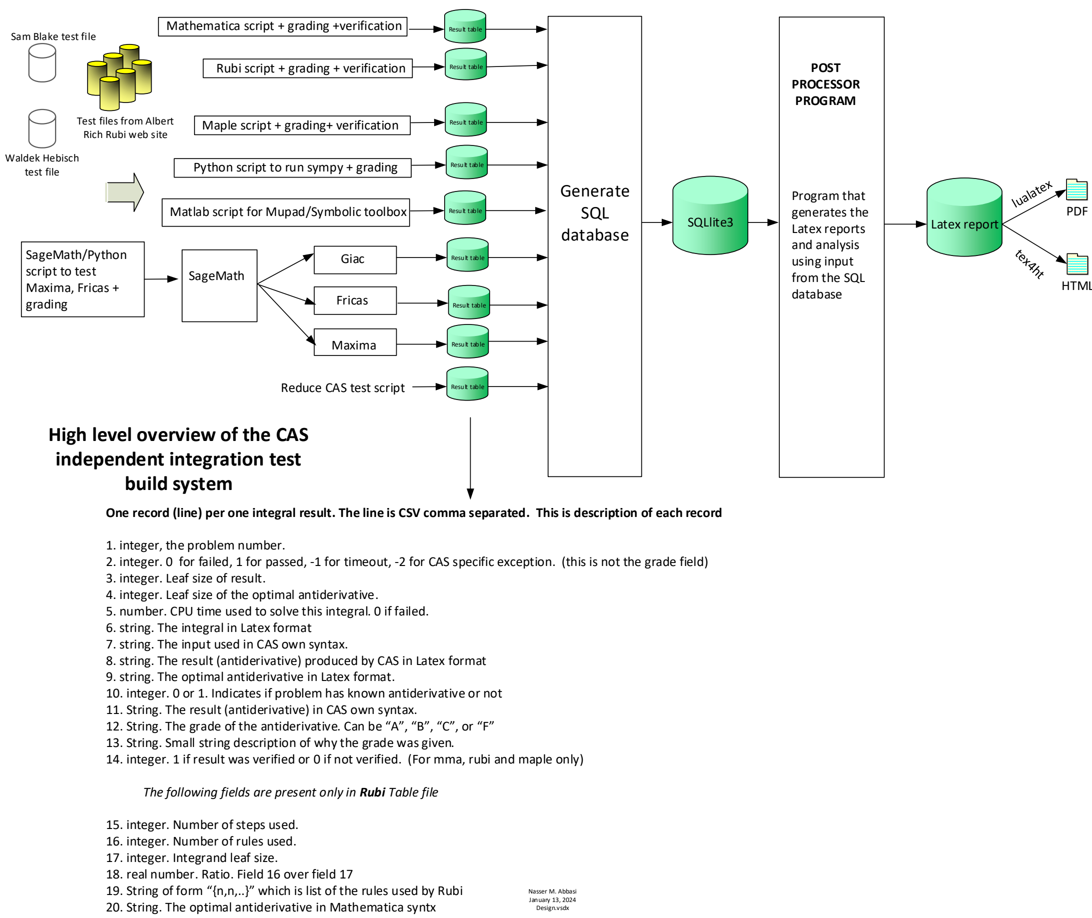


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119 }

B grade { }

C grade { }

F normal fail { 58, 75 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 48, 49, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 107, 113, 114, 116, 117, 118, 119 }

B grade { 46, 47, 64, 103, 104, 105, 106, 115 }

C grade { 50, 51, 52, 67, 68, 69, 86, 87, 88, 89, 90, 108, 109, 110, 111, 112 }

F normal fail { 58, 75 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 46, 47, 48, 49, 60, 61, 62, 63, 64, 65, 66, 76, 78, 80, 82, 84, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107 }

B grade { 59 }

C grade { 42, 43, 44, 45, 50, 51, 52, 67, 68, 69, 77, 79, 81, 83, 85, 86, 87, 88, 89, 90, 108, 109, 110, 111, 112 }

F normal fail { 17, 18, 19, 20, 21, 38, 39, 40, 41, 53, 54, 55, 56, 57, 58, 70, 71, 72, 73, 74, 75, 91, 92, 93, 94, 95, 96, 97, 113, 114, 115, 116, 117, 118, 119 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 98 }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 96, 97, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 118, 119 }

C grade { }

F normal fail { 18, 19, 38, 39, 53, 54, 55, 56, 57, 58, 70, 71, 72, 73, 74, 75, 91, 92, 93, 94, 95, 113, 114, 115, 116, 117 }

F(-1) timedout fail { }

F(-2) exception fail { 17 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 76, 77, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 98, 99, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112 }

B grade { 78, 80, 100, 102 }

C grade { }

F normal fail { 17, 18, 19, 20, 21, 38, 39, 40, 41, 53, 54, 55, 56, 57, 58, 70, 71, 72, 73, 74, 75, 91, 92, 93, 94, 95, 96, 97, 113, 114, 115, 116, 117, 118, 119 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112 }

B grade { 20, 21, 40, 41 }

C grade { 13, 14, 15, 16, 34, 35, 36, 37 }

F normal fail { 17, 18, 19, 38, 39, 53, 54, 55, 56, 57, 58, 70, 71, 72, 73, 74, 75, 91, 92, 93, 94, 95, 96, 97, 113, 114, 115, 116, 117, 118, 119 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 42, 43, 46, 47, 50, 51, 52, 59, 60, 63, 64, 67, 68, 69, 76, 77, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 98, 99, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112 }

C grade { }

F normal fail { }

F(-1) timedout fail { 17, 18, 19, 20, 21, 38, 39, 40, 41, 44, 45, 48, 49, 53, 54, 55, 56, 57, 58, 61, 62, 65, 66, 70, 71, 72, 73, 74, 75, 78, 79, 80, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 113, 114, 115, 116, 117, 118, 119 }

F(-2) exception fail { }

Sympy

A grade { 4, 5, 9, 10, 11, 12, 25, 26, 30, 31, 32, 33 }

B grade { 1, 2, 3, 6, 7, 8, 13, 14, 15, 16, 22, 23, 24, 27, 28, 29, 34, 35, 36, 37 }

C grade { }

F normal fail { 17, 18, 19, 20, 21, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119 }

F(-1) timedout fail { }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 59, 60, 61, 62, 63, 64, 65, 66, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107 }

C grade { }

F normal fail { 14, 15, 16, 17, 18, 19, 35, 36, 37, 38, 39, 50, 51, 52, 53, 54, 55, 56, 57, 58, 67, 68, 69, 70, 71, 72, 73, 74, 75, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	27	39	24	30	98	80	28	38	25
N.S.	1	0.93	1.34	0.83	1.03	3.38	2.76	0.97	1.31	0.86
time (sec)	N/A	0.178	0.055	0.221	0.038	0.090	0.215	0.110	0.233	0.104

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	36	50	43	42	124	78	40	43	39
N.S.	1	0.71	0.98	0.84	0.82	2.43	1.53	0.78	0.84	0.76
time (sec)	N/A	0.202	0.099	0.488	0.031	0.092	0.416	0.110	0.206	2.503

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	38	89	54	60	213	182	66	58	65
N.S.	1	0.58	1.37	0.83	0.92	3.28	2.80	1.02	0.89	1.00
time (sec)	N/A	0.224	0.119	0.730	0.036	0.087	0.913	0.114	0.207	0.318

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	56	101	73	72	270	139	69	70	72
N.S.	1	0.64	1.16	0.84	0.83	3.10	1.60	0.79	0.80	0.83
time (sec)	N/A	0.222	0.142	1.395	0.035	0.097	2.198	0.116	0.223	0.527

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	28	39	30	32	138	54	33	32	29
N.S.	1	0.74	1.03	0.79	0.84	3.63	1.42	0.87	0.84	0.76
time (sec)	N/A	0.214	0.066	0.313	0.037	0.086	0.201	0.110	0.228	2.556

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	30	53	44	56	194	139	47	44	44
N.S.	1	0.59	1.04	0.86	1.10	3.80	2.73	0.92	0.86	0.86
time (sec)	N/A	0.217	0.129	0.371	0.049	0.087	0.450	0.108	0.209	2.613

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	44	86	67	59	234	124	61	57	66
N.S.	1	0.56	1.10	0.86	0.76	3.00	1.59	0.78	0.73	0.85
time (sec)	N/A	0.232	0.181	0.743	0.035	0.099	0.908	0.116	0.224	2.733

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	48	105	78	77	352	265	93	72	149
N.S.	1	0.52	1.13	0.84	0.83	3.78	2.85	1.00	0.77	1.60
time (sec)	N/A	0.226	0.194	1.425	0.035	0.087	2.242	0.110	0.233	0.630

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	39	44	32	33	415	65	33	32	30
N.S.	1	0.68	0.77	0.56	0.58	7.28	1.14	0.58	0.56	0.53
time (sec)	N/A	0.211	0.071	0.326	0.040	0.088	0.212	0.125	0.201	2.530

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	49	58	39	48	486	104	49	53	41
N.S.	1	0.56	0.66	0.44	0.55	5.52	1.18	0.56	0.60	0.47
time (sec)	N/A	0.215	0.146	0.468	0.040	0.085	0.433	0.114	0.203	2.580

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	63	94	54	59	759	141	63	65	91
N.S.	1	0.56	0.83	0.48	0.52	6.72	1.25	0.56	0.58	0.81
time (sec)	N/A	0.220	0.170	0.784	0.039	0.098	0.918	0.114	0.243	0.327

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	73	108	61	78	1051	178	80	77	83
N.S.	1	0.51	0.75	0.42	0.54	7.30	1.24	0.56	0.53	0.58
time (sec)	N/A	0.229	0.216	1.395	0.039	0.095	2.248	0.113	0.227	3.230

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	75	50	51	63	244	323	598	51	73
N.S.	1	1.19	0.79	0.81	1.00	3.87	5.13	9.49	0.81	1.16
time (sec)	N/A	0.235	0.054	0.228	0.040	0.100	0.650	0.138	0.241	2.732

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	132	86	90	94	703	707	890	23	97
N.S.	1	1.42	0.92	0.97	1.01	7.56	7.60	9.57	0.25	1.04
time (sec)	N/A	0.309	0.094	0.513	0.045	0.098	1.244	0.154	0.224	3.112

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	190	157	147	134	2228	1477	1211	23	166
N.S.	1	1.44	1.19	1.11	1.02	16.88	11.19	9.17	0.17	1.26
time (sec)	N/A	0.415	0.406	1.126	0.045	0.177	3.303	0.167	0.248	3.529

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	247	203	195	165	3686	2409	1502	23	257
N.S.	1	1.52	1.25	1.20	1.02	22.75	14.87	9.27	0.14	1.59
time (sec)	N/A	0.496	0.795	2.160	0.052	0.215	25.034	0.186	0.203	5.083

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	51	0	0	0	0	0	150	0
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	2.05	0.00
time (sec)	N/A	0.768	7.270	0.000	0.000	0.000	0.000	0.000	0.276	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	95	86	0	0	0	0	0	305	0
N.S.	1	1.25	1.13	0.00	0.00	0.00	0.00	0.00	4.01	0.00
time (sec)	N/A	0.262	0.232	0.000	0.000	0.000	0.000	0.000	0.227	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	97	98	0	0	0	0	0	69	0
N.S.	1	0.99	1.00	0.00	0.00	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	0.535	0.052	0.000	0.000	0.000	0.000	0.000	0.224	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	145	80	0	0	267	0	530	238	0
N.S.	1	1.77	0.98	0.00	0.00	3.26	0.00	6.46	2.90	0.00
time (sec)	N/A	0.518	0.147	0.000	0.000	0.120	0.000	0.455	0.224	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	151	71	0	0	277	0	530	223	0
N.S.	1	1.80	0.85	0.00	0.00	3.30	0.00	6.31	2.65	0.00
time (sec)	N/A	0.557	0.098	0.000	0.000	0.106	0.000	0.508	0.217	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	27	39	24	30	98	63	26	38	25
N.S.	1	0.93	1.34	0.83	1.03	3.38	2.17	0.90	1.31	0.86
time (sec)	N/A	0.188	0.056	0.261	0.038	0.088	0.211	0.110	0.201	0.100

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	36	50	43	42	124	78	40	43	37
N.S.	1	0.71	0.98	0.84	0.82	2.43	1.53	0.78	0.84	0.73
time (sec)	N/A	0.208	0.101	0.481	0.032	0.087	0.460	0.110	0.216	2.580

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	38	89	54	60	214	207	64	58	66
N.S.	1	0.58	1.37	0.83	0.92	3.29	3.18	0.98	0.89	1.02
time (sec)	N/A	0.219	0.128	0.893	0.036	0.091	0.964	0.118	0.228	2.717

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	56	101	73	72	270	139	67	70	72
N.S.	1	0.64	1.16	0.84	0.83	3.10	1.60	0.77	0.80	0.83
time (sec)	N/A	0.218	0.132	1.792	0.038	0.086	2.298	0.110	0.242	0.546

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	28	39	32	32	138	54	33	32	29
N.S.	1	0.74	1.03	0.84	0.84	3.63	1.42	0.87	0.84	0.76
time (sec)	N/A	0.190	0.067	0.299	0.048	0.096	0.213	0.110	0.215	0.100

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	30	53	44	56	194	141	47	44	44
N.S.	1	0.59	1.04	0.86	1.10	3.80	2.76	0.92	0.86	0.86
time (sec)	N/A	0.219	0.131	0.456	0.038	0.086	0.456	0.110	0.201	0.165

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	44	86	67	59	232	124	61	57	66
N.S.	1	0.56	1.10	0.86	0.76	2.97	1.59	0.78	0.73	0.85
time (sec)	N/A	0.209	0.184	0.871	0.043	0.086	0.950	0.115	0.224	0.280

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	48	105	78	77	352	265	93	72	149
N.S.	1	0.52	1.13	0.84	0.83	3.78	2.85	1.00	0.77	1.60
time (sec)	N/A	0.227	0.189	1.723	0.036	0.096	2.348	0.118	0.219	3.137

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	38	29	30	29	222	65	30	32	25
N.S.	1	1.03	0.78	0.81	0.78	6.00	1.76	0.81	0.86	0.68
time (sec)	N/A	0.200	0.015	0.326	0.035	0.080	0.220	0.112	0.212	2.588

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	54	45	39	42	272	104	43	53	43
N.S.	1	0.93	0.78	0.67	0.72	4.69	1.79	0.74	0.91	0.74
time (sec)	N/A	0.212	0.023	0.510	0.037	0.090	0.473	0.119	0.220	2.560

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	68	53	54	53	390	141	54	65	49
N.S.	1	0.93	0.73	0.74	0.73	5.34	1.93	0.74	0.89	0.67
time (sec)	N/A	0.217	0.026	0.897	0.036	0.094	0.957	0.112	0.232	0.315

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	84	64	61	65	511	178	66	77	60
N.S.	1	0.89	0.68	0.65	0.69	5.44	1.89	0.70	0.82	0.64
time (sec)	N/A	0.226	0.029	1.823	0.037	0.091	2.441	0.111	0.236	3.121

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	75	50	51	63	246	265	597	51	74
N.S.	1	1.19	0.79	0.81	1.00	3.90	4.21	9.48	0.81	1.17
time (sec)	N/A	0.232	0.054	0.237	0.044	0.081	0.745	0.134	0.205	2.596

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	132	85	90	94	699	707	889	23	100
N.S.	1	1.42	0.91	0.97	1.01	7.52	7.60	9.56	0.25	1.08
time (sec)	N/A	0.316	0.115	0.592	0.050	0.104	1.301	0.150	0.213	2.764

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	190	159	148	134	2218	1658	1211	23	154
N.S.	1	1.44	1.20	1.12	1.02	16.80	12.56	9.17	0.17	1.17
time (sec)	N/A	0.402	0.373	1.277	0.055	0.173	3.544	0.154	0.220	3.333

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	247	204	195	165	3682	2409	1502	23	197
N.S.	1	1.52	1.26	1.20	1.02	22.73	14.87	9.27	0.14	1.22
time (sec)	N/A	0.515	0.824	2.738	0.055	0.196	26.484	0.173	0.246	5.047

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	93	89	0	0	0	0	0	299	0
N.S.	1	1.22	1.17	0.00	0.00	0.00	0.00	0.00	3.93	0.00
time (sec)	N/A	0.236	0.214	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	97	98	0	0	0	0	0	69	0
N.S.	1	0.99	1.00	0.00	0.00	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	0.507	0.049	0.000	0.000	0.000	0.000	0.000	0.227	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	145	80	0	0	267	0	513	237	0
N.S.	1	1.77	0.98	0.00	0.00	3.26	0.00	6.26	2.89	0.00
time (sec)	N/A	0.546	0.141	0.000	0.000	0.102	0.000	0.447	0.222	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	149	70	0	0	275	0	521	213	0
N.S.	1	1.80	0.84	0.00	0.00	3.31	0.00	6.28	2.57	0.00
time (sec)	N/A	0.536	0.096	0.000	0.000	0.104	0.000	0.391	0.225	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	24	45	70	29	87	0	29	30	55
N.S.	1	0.75	1.41	2.19	0.91	2.72	0.00	0.91	0.94	1.72
time (sec)	N/A	0.186	0.128	0.296	0.118	0.086	0.000	0.118	0.209	2.779

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	38	84	102	61	321	0	58	76	94
N.S.	1	0.66	1.45	1.76	1.05	5.53	0.00	1.00	1.31	1.62
time (sec)	N/A	0.232	0.403	0.314	0.117	0.097	0.000	0.121	0.253	2.832

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	58	125	120	89	702	0	73	120	0
N.S.	1	0.69	1.49	1.43	1.06	8.36	0.00	0.87	1.43	0.00
time (sec)	N/A	0.243	0.306	0.325	0.116	0.097	0.000	0.122	0.224	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	76	171	137	120	1223	0	89	167	0
N.S.	1	0.67	1.50	1.20	1.05	10.73	0.00	0.78	1.46	0.00
time (sec)	N/A	0.263	0.261	0.356	0.121	0.104	0.000	0.122	0.225	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	32	92	62	58	156	0	40	43	49
N.S.	1	0.71	2.04	1.38	1.29	3.47	0.00	0.89	0.96	1.09
time (sec)	N/A	0.214	0.154	0.258	0.118	0.101	0.000	0.109	0.249	2.820

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	42	158	94	91	453	0	67	94	82
N.S.	1	0.58	2.16	1.29	1.25	6.21	0.00	0.92	1.29	1.12
time (sec)	N/A	0.234	0.317	0.273	0.112	0.095	0.000	0.117	0.220	0.164

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	56	151	115	113	897	0	95	134	0
N.S.	1	0.55	1.50	1.14	1.12	8.88	0.00	0.94	1.33	0.00
time (sec)	N/A	0.240	0.394	0.295	0.116	0.098	0.000	0.110	0.229	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	68	189	129	135	1482	0	109	183	0
N.S.	1	0.52	1.44	0.98	1.03	11.31	0.00	0.83	1.40	0.00
time (sec)	N/A	0.254	0.352	0.353	0.115	0.102	0.000	0.121	0.209	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	156	87	124	139	1200	0	177	19	436
N.S.	1	0.86	0.48	0.69	0.77	6.63	0.00	0.98	0.10	2.41
time (sec)	N/A	0.369	0.154	0.286	0.128	0.108	0.000	0.126	0.223	4.664

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	183	127	153	172	3184	0	201	21	466
N.S.	1	0.80	0.56	0.67	0.75	13.96	0.00	0.88	0.09	2.04
time (sec)	N/A	0.439	0.308	0.323	0.139	0.130	0.000	0.121	0.221	6.304

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	232	171	166	194	6325	0	218	21	504
N.S.	1	0.86	0.63	0.61	0.72	23.34	0.00	0.80	0.08	1.86
time (sec)	N/A	0.427	0.340	0.364	0.137	0.157	0.000	0.122	0.213	5.652

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	79	61	0	0	0	0	0	21	0
N.S.	1	0.99	0.76	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.287	0.557	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	137	94	0	0	0	0	0	23	0
N.S.	1	1.36	0.93	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.329	0.834	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	195	150	0	0	0	0	0	23	0
N.S.	1	1.26	0.97	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.396	1.322	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	215	253	181	0	0	0	0	0	23	0
N.S.	1	1.18	0.84	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.463	1.679	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	104	0	0	0	0	0	20	0
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.220	0.265	0.000	0.000	0.000	0.000	0.000	0.265	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	0	0	0	0	0	0	0	25	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	24	34	63	56	117	0	54	42	59
N.S.	1	0.75	1.06	1.97	1.75	3.66	0.00	1.69	1.31	1.84
time (sec)	N/A	0.174	0.153	0.248	0.034	0.085	0.000	0.114	0.205	0.179

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	40	99	97	90	446	0	85	111	99
N.S.	1	0.67	1.65	1.62	1.50	7.43	0.00	1.42	1.85	1.65
time (sec)	N/A	0.200	0.263	0.269	0.044	0.112	0.000	0.118	0.233	2.673

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	62	95	116	120	980	0	103	185	0
N.S.	1	0.70	1.08	1.32	1.36	11.14	0.00	1.17	2.10	0.00
time (sec)	N/A	0.218	0.306	0.286	0.043	0.091	0.000	0.124	0.228	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	82	171	133	150	1693	0	117	253	0
N.S.	1	0.68	1.42	1.11	1.25	14.11	0.00	0.98	2.11	0.00
time (sec)	N/A	0.241	0.261	0.326	0.049	0.094	0.000	0.115	0.211	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	34	92	63	81	156	0	40	52	49
N.S.	1	0.74	2.00	1.37	1.76	3.39	0.00	0.87	1.13	1.07
time (sec)	N/A	0.210	0.143	0.268	0.038	0.083	0.000	0.109	0.201	0.112

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	46	158	98	118	465	0	68	122	85
N.S.	1	0.60	2.05	1.27	1.53	6.04	0.00	0.88	1.58	1.10
time (sec)	N/A	0.227	0.294	0.279	0.048	0.095	0.000	0.124	0.247	2.667

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	62	151	119	138	905	0	96	181	0
N.S.	1	0.58	1.41	1.11	1.29	8.46	0.00	0.90	1.69	0.00
time (sec)	N/A	0.247	0.277	0.314	0.036	0.091	0.000	0.122	0.219	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	76	189	133	162	1490	0	112	249	0
N.S.	1	0.55	1.36	0.96	1.17	10.72	0.00	0.81	1.79	0.00
time (sec)	N/A	0.250	0.355	0.309	0.042	0.089	0.000	0.116	0.216	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	151	87	329	204	622	0	196	19	418
N.S.	1	0.81	0.47	1.77	1.10	3.34	0.00	1.05	0.10	2.25
time (sec)	N/A	0.341	0.139	0.285	0.121	0.099	0.000	0.129	0.240	4.417

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	180	127	357	240	3400	0	220	21	448
N.S.	1	0.78	0.55	1.54	1.03	14.66	0.00	0.95	0.09	1.93
time (sec)	N/A	0.369	0.271	0.311	0.130	0.115	0.000	0.126	0.223	5.814

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	229	171	370	260	7903	0	237	21	486
N.S.	1	0.82	0.61	1.33	0.93	28.33	0.00	0.85	0.08	1.74
time (sec)	N/A	0.441	0.343	0.306	0.120	0.187	0.000	0.131	0.208	5.431

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	77	59	0	0	0	0	0	21	0
N.S.	1	0.99	0.76	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.266	0.223	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	133	92	0	0	0	0	0	23	0
N.S.	1	1.32	0.91	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.343	0.822	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	189	147	0	0	0	0	0	23	0
N.S.	1	1.20	0.94	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.390	1.068	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	219	245	178	0	0	0	0	0	23	0
N.S.	1	1.12	0.81	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.444	1.733	0.000	0.000	0.000	0.000	0.000	0.213	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	76	106	0	0	0	0	0	20	0
N.S.	1	0.90	1.26	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.214	0.265	0.000	0.000	0.000	0.000	0.000	0.214	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	0	0	0	0	0	0	0	25	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	16	23	43	28	45	0	22	25	31
N.S.	1	0.67	0.96	1.79	1.17	1.88	0.00	0.92	1.04	1.29
time (sec)	N/A	0.173	0.056	0.276	0.044	0.073	0.000	0.107	0.216	2.986

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	38	43	92	51	207	0	50	64	84
N.S.	1	0.81	0.91	1.96	1.09	4.40	0.00	1.06	1.36	1.79
time (sec)	N/A	0.188	0.113	0.519	0.120	0.075	0.000	0.113	0.217	2.678

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	23	29	51	97	122	0	37	42	0
N.S.	1	0.77	0.97	1.70	3.23	4.07	0.00	1.23	1.40	0.00
time (sec)	N/A	0.182	0.039	0.605	0.043	0.071	0.000	0.113	0.267	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	87	71	127	109	996	0	75	155	0
N.S.	1	0.84	0.69	1.23	1.06	9.67	0.00	0.73	1.50	0.00
time (sec)	N/A	0.215	0.210	2.333	0.121	0.097	0.000	0.113	0.210	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	51	42	66	233	303	0	48	89	0
N.S.	1	0.63	0.52	0.81	2.88	3.74	0.00	0.59	1.10	0.00
time (sec)	N/A	0.216	0.062	1.854	0.038	0.078	0.000	0.115	0.205	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	24	32	80	40	98	0	32	35	67
N.S.	1	0.60	0.80	2.00	1.00	2.45	0.00	0.80	0.88	1.68
time (sec)	N/A	0.175	0.049	0.302	0.117	0.093	0.000	0.109	0.232	0.151

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	30	63	80	77	201	0	45	81	53
N.S.	1	0.54	1.12	1.43	1.38	3.59	0.00	0.80	1.45	0.95
time (sec)	N/A	0.210	0.063	0.498	0.120	0.086	0.000	0.120	0.247	0.156

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	65	60	120	84	581	0	61	114	119
N.S.	1	0.77	0.71	1.43	1.00	6.92	0.00	0.73	1.36	1.42
time (sec)	N/A	0.205	0.113	1.045	0.119	0.088	0.000	0.111	0.215	2.691

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	27	33	48	48	228	0	50	56	117
N.S.	1	0.77	0.94	1.37	1.37	6.51	0.00	1.43	1.60	3.34
time (sec)	N/A	0.187	0.057	0.944	0.033	0.077	0.000	0.113	0.215	2.639

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	114	91	148	128	1710	0	83	203	233
N.S.	1	0.74	0.59	0.95	0.83	11.03	0.00	0.54	1.31	1.50
time (sec)	N/A	0.235	0.248	5.297	0.120	0.094	0.000	0.119	0.265	2.604

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	62	42	176	185	362	0	101	19	204
N.S.	1	0.43	0.29	1.22	1.28	2.51	0.00	0.70	0.13	1.42
time (sec)	N/A	0.283	0.050	0.302	0.115	0.092	0.000	0.110	0.225	0.993

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	168	45	140	155	2654	0	178	21	452
N.S.	1	0.84	0.22	0.70	0.78	13.27	0.00	0.89	0.10	2.26
time (sec)	N/A	0.363	0.074	0.537	0.118	0.115	0.000	0.112	0.221	5.842

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	105	64	202	188	3955	0	130	21	249
N.S.	1	0.52	0.32	1.00	0.93	19.48	0.00	0.64	0.10	1.23
time (sec)	N/A	0.318	0.085	1.072	0.123	0.121	0.000	0.116	0.202	4.925

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	222	68	164	207	9595	0	203	21	542
N.S.	1	0.78	0.24	0.57	0.72	33.55	0.00	0.71	0.07	1.90
time (sec)	N/A	0.397	0.105	2.404	0.140	0.197	0.000	0.119	0.238	5.882

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	157	85	224	232	11662	0	152	21	364
N.S.	1	0.55	0.30	0.79	0.81	40.92	0.00	0.53	0.07	1.28
time (sec)	N/A	0.509	0.289	5.362	0.134	0.262	0.000	0.110	0.217	5.493

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	68	70	0	0	0	0	0	21	0
N.S.	1	0.99	1.01	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.371	0.014	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	70	70	0	0	0	0	0	23	0
N.S.	1	0.95	0.95	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.376	0.011	0.000	0.000	0.000	0.000	0.000	0.257	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	139	96	0	0	0	0	0	23	0
N.S.	1	1.88	1.30	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.596	0.148	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	85	64	0	0	0	0	0	20	0
N.S.	1	1.29	0.97	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.453	0.086	0.000	0.000	0.000	0.000	0.000	0.217	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	110	91	0	0	0	0	0	25	0
N.S.	1	1.25	1.03	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.607	0.063	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	167	69	0	0	500	0	0	39	0
N.S.	1	1.27	0.52	0.00	0.00	3.79	0.00	0.00	0.30	0.00
time (sec)	N/A	0.520	0.174	0.000	0.000	0.103	0.000	0.000	0.239	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	163	73	0	0	518	0	0	40	0
N.S.	1	1.25	0.56	0.00	0.00	3.98	0.00	0.00	0.31	0.00
time (sec)	N/A	0.499	0.219	0.000	0.000	0.094	0.000	0.000	0.219	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	18	38	45	45	45	0	23	33	32
N.S.	1	0.69	1.46	1.73	1.73	1.73	0.00	0.88	1.27	1.23
time (sec)	N/A	0.173	0.100	0.241	0.035	0.069	0.000	0.113	0.214	2.752

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	40	82	87	80	332	0	71	101	89
N.S.	1	0.82	1.67	1.78	1.63	6.78	0.00	1.45	2.06	1.82
time (sec)	N/A	0.189	0.202	0.329	0.037	0.089	0.000	0.120	0.238	0.195

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	25	63	53	97	124	0	37	42	0
N.S.	1	0.78	1.97	1.66	3.03	3.88	0.00	1.16	1.31	0.00
time (sec)	N/A	0.212	0.102	0.454	0.040	0.068	0.000	0.115	0.216	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	93	164	123	140	1466	0	97	241	0
N.S.	1	0.85	1.50	1.13	1.28	13.45	0.00	0.89	2.21	0.00
time (sec)	N/A	0.250	0.260	0.967	0.038	0.092	0.000	0.112	0.222	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	57	85	68	233	303	0	48	89	0
N.S.	1	0.66	0.98	0.78	2.68	3.48	0.00	0.55	1.02	0.00
time (sec)	N/A	0.225	0.136	1.346	0.043	0.080	0.000	0.113	0.238	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	24	163	73	66	135	0	50	48	71
N.S.	1	0.60	4.08	1.82	1.65	3.38	0.00	1.25	1.20	1.78
time (sec)	N/A	0.179	0.270	0.232	0.039	0.083	0.000	0.108	0.228	0.166

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	34	123	84	102	205	0	47	110	53
N.S.	1	0.57	2.05	1.40	1.70	3.42	0.00	0.78	1.83	0.88
time (sec)	N/A	0.216	0.261	0.325	0.037	0.096	0.000	0.112	0.204	0.167

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	69	239	115	112	883	0	80	177	123
N.S.	1	0.78	2.72	1.31	1.27	10.03	0.00	0.91	2.01	1.40
time (sec)	N/A	0.213	0.442	0.513	0.034	0.099	0.000	0.118	0.211	2.968

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	29	87	50	48	229	0	50	56	117
N.S.	1	0.78	2.35	1.35	1.30	6.19	0.00	1.35	1.51	3.16
time (sec)	N/A	0.311	0.214	0.718	0.037	0.086	0.000	0.116	0.248	2.935

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	122	321	144	157	2509	0	102	313	237
N.S.	1	0.75	1.97	0.88	0.96	15.39	0.00	0.63	1.92	1.45
time (sec)	N/A	0.245	0.415	1.967	0.037	0.113	0.000	0.111	0.241	0.123

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	87	185	175	205	363	0	99	19	195
N.S.	1	0.60	1.28	1.22	1.42	2.52	0.00	0.69	0.13	1.35
time (sec)	N/A	0.283	0.209	0.254	0.123	0.100	0.000	0.126	0.204	0.947

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	165	115	343	221	2870	0	193	21	434
N.S.	1	0.81	0.56	1.68	1.08	14.07	0.00	0.95	0.10	2.13
time (sec)	N/A	0.560	0.245	0.350	0.118	0.135	0.000	0.126	0.211	6.294

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	134	235	202	253	3987	0	129	21	249
N.S.	1	0.65	1.14	0.98	1.22	19.26	0.00	0.62	0.10	1.20
time (sec)	N/A	0.462	0.305	0.536	0.128	0.134	0.000	0.117	0.231	5.197

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	223	156	367	273	13031	0	218	21	524
N.S.	1	0.75	0.53	1.24	0.92	44.02	0.00	0.74	0.07	1.77
time (sec)	N/A	0.439	0.515	0.992	0.133	0.243	0.000	0.119	0.209	6.221

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	189	323	224	297	11702	0	151	21	373
N.S.	1	0.65	1.10	0.76	1.01	39.94	0.00	0.52	0.07	1.27
time (sec)	N/A	0.364	0.509	1.957	0.129	0.281	0.000	0.121	0.201	6.041

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	66	79	0	0	0	0	0	21	0
N.S.	1	0.99	1.18	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.255	0.090	0.000	0.000	0.000	0.000	0.000	0.214	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	68	87	0	0	0	0	0	23	0
N.S.	1	0.94	1.21	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.231	0.377	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	137	281	0	0	0	0	0	23	0
N.S.	1	1.90	3.90	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.344	3.080	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	88	66	0	0	0	0	0	20	0
N.S.	1	1.33	1.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.284	0.099	0.000	0.000	0.000	0.000	0.000	0.217	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	112	92	0	0	0	0	0	25	0
N.S.	1	1.27	1.05	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.569	0.085	0.000	0.000	0.000	0.000	0.000	0.245	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	169	69	0	0	501	0	0	39	0
N.S.	1	1.26	0.51	0.00	0.00	3.74	0.00	0.00	0.29	0.00
time (sec)	N/A	0.516	0.220	0.000	0.000	0.101	0.000	0.000	0.221	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	163	75	0	0	519	0	0	43	0
N.S.	1	1.25	0.58	0.00	0.00	3.99	0.00	0.00	0.33	0.00
time (sec)	N/A	0.509	0.172	0.000	0.000	0.107	0.000	0.000	0.232	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [52] had the largest ratio of [.8000000000000044]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	0.93	14	0.286
2	A	5	4	0.71	16	0.250
3	A	6	5	0.58	16	0.312
4	A	5	4	0.64	16	0.250
5	A	4	3	0.74	16	0.188
6	A	6	5	0.59	18	0.278
7	A	5	4	0.56	18	0.222
8	A	6	5	0.52	18	0.278
9	A	5	4	0.68	18	0.222
10	A	5	4	0.56	20	0.200
11	A	5	4	0.56	20	0.200
12	A	5	4	0.51	20	0.200
13	A	1	1	1.19	16	0.062
14	A	2	2	1.42	18	0.111
15	A	2	2	1.44	18	0.111
16	A	3	3	1.52	18	0.167
17	A	1	1	1.00	56	0.018
18	A	6	5	1.25	16	0.312
19	A	3	3	0.99	20	0.150
20	A	2	2	1.77	28	0.071
21	A	2	2	1.80	29	0.069

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	5	4	0.93	14	0.286
23	A	5	4	0.71	16	0.250
24	A	6	5	0.58	16	0.312
25	A	5	4	0.64	16	0.250
26	A	4	3	0.74	16	0.188
27	A	6	5	0.59	18	0.278
28	A	5	4	0.56	18	0.222
29	A	6	5	0.52	18	0.278
30	A	5	4	1.03	18	0.222
31	A	5	4	0.93	20	0.200
32	A	5	4	0.93	20	0.200
33	A	5	4	0.89	20	0.200
34	A	1	1	1.19	16	0.062
35	A	2	2	1.42	18	0.111
36	A	2	2	1.44	18	0.111
37	A	3	3	1.52	18	0.167
38	A	5	4	1.22	16	0.250
39	A	3	3	0.99	20	0.150
40	A	2	2	1.77	28	0.071
41	A	2	2	1.80	29	0.069
42	A	6	5	0.75	14	0.357
43	A	5	4	0.66	16	0.250
44	A	6	5	0.69	16	0.312
45	A	5	4	0.67	16	0.250
46	A	7	6	0.71	16	0.375
47	A	6	5	0.58	18	0.278
48	A	7	6	0.55	18	0.333
49	A	6	5	0.52	18	0.278
50	A	13	12	0.86	18	0.667
51	A	14	13	0.80	20	0.650
52	A	17	16	0.86	20	0.800
53	A	2	2	0.99	16	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	1.36	18	0.111
55	A	2	2	1.26	18	0.111
56	A	2	2	1.18	18	0.111
57	A	5	4	1.00	16	0.250
58	F	0	0	N/A	0.000	N/A
59	A	6	5	0.75	14	0.357
60	A	5	4	0.67	16	0.250
61	A	6	5	0.70	16	0.312
62	A	5	4	0.68	16	0.250
63	A	7	6	0.74	16	0.375
64	A	6	5	0.60	18	0.278
65	A	7	6	0.58	18	0.333
66	A	6	5	0.55	18	0.278
67	A	13	12	0.81	18	0.667
68	A	14	13	0.78	20	0.650
69	A	17	16	0.82	20	0.800
70	A	2	2	0.99	16	0.125
71	A	2	2	1.32	18	0.111
72	A	2	2	1.20	18	0.111
73	A	2	2	1.12	18	0.111
74	A	5	4	0.90	16	0.250
75	F	0	0	N/A	0.000	N/A
76	A	4	3	0.67	14	0.214
77	A	5	4	0.81	16	0.250
78	A	4	3	0.77	16	0.188
79	A	7	6	0.84	16	0.375
80	A	6	5	0.63	16	0.312
81	A	5	4	0.60	16	0.250
82	A	6	5	0.54	18	0.278
83	A	6	5	0.77	18	0.278
84	A	4	3	0.77	18	0.167
85	A	8	7	0.74	18	0.389

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	12	11	0.43	18	0.611
87	A	12	11	0.84	20	0.550
88	A	13	12	0.52	20	0.600
89	A	14	13	0.78	20	0.650
90	A	15	14	0.55	20	0.700
91	A	1	1	0.99	16	0.062
92	A	1	1	0.95	18	0.056
93	A	2	2	1.88	18	0.111
94	A	5	4	1.29	16	0.250
95	A	3	3	1.25	20	0.150
96	A	2	2	1.27	28	0.071
97	A	2	2	1.25	29	0.069
98	A	4	3	0.69	14	0.214
99	A	5	4	0.82	16	0.250
100	A	4	3	0.78	16	0.188
101	A	7	6	0.85	16	0.375
102	A	6	5	0.66	16	0.312
103	A	5	4	0.60	16	0.250
104	A	6	5	0.57	18	0.278
105	A	6	5	0.78	18	0.278
106	A	4	3	0.78	18	0.167
107	A	8	7	0.75	18	0.389
108	A	11	10	0.60	18	0.556
109	A	12	11	0.81	20	0.550
110	A	12	11	0.65	20	0.550
111	A	14	13	0.75	20	0.650
112	A	14	13	0.65	20	0.650
113	A	1	1	0.99	16	0.062
114	A	1	1	0.94	18	0.056
115	A	2	2	1.90	18	0.111
116	A	5	4	1.33	16	0.250
117	A	3	3	1.27	20	0.150

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	2	2	1.26	28	0.071
119	A	2	2	1.25	29	0.069

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int e^{a+bx} \sinh(d+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 29

$$\int e^{a+bx} \sinh(d+bx)\,dx = \frac{e^{a+d+2bx}}{4b} - \frac{1}{2}e^{a-d}x$$

output
```
1/4*exp(2*b*x+a+d)/b-1/2*exp(a-d)*x
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int e^{a+bx} \sinh(d+bx)\,dx = \frac{e^a\left(\left(e^{2bx}-2bx\right)\cosh(d)+\left(e^{2bx}+2bx\right)\sinh(d)\right)}{4b}$$

input
```
Integrate[E^(a + b*x)*Sinh[d + b*x],x]
```

output
```
(E^a*((E^(2*b*x) - 2*b*x)*Cosh[d] + (E^(2*b*x) + 2*b*x)*Sinh[d]))/(4*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{1}{2}e^{a-bx}\left(1-e^{2bx}\right)de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^a\int e^{-bx}\left(1-e^{2bx}\right)de^{bx}}{2b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{e^a\int \left(e^{-bx}-e^{bx}\right)de^{bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^a\left(\log\left(e^{bx}\right)-\frac{1}{2}e^{2bx}\right)}{2b}$$

input
```
Int[E^(a + b*x)*Sinh[d + b*x],x]
```

output
```
-1/2*(E^a*(-1/2*E^(2*b*x) + Log[E^(b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{{\mathrm e}^{2 b x +a +d}}{4 b}-\frac{{\mathrm e}^{a -d} x}{2}$	24
default	$-\frac{\cosh \left(a -d \right) x}{2}+\frac{\sinh \left(2 b x +a +d \right)}{4 b}-\frac{\sinh \left(a -d \right) x}{2}+\frac{\cosh \left(2 b x +a +d \right)}{4 b}$	46
orering	$\frac{\left(2 b x +1\right) {\mathrm e}^{b x +a} \sinh \left(b x +d \right)}{2 b}-\frac{x \left(b \,{\mathrm e}^{b x +a} \sinh \left(b x +d \right)+{\mathrm e}^{b x +a} b \cosh \left(b x +d \right)\right)}{2 b}$	60

input
```
int(exp(b*x+a)*sinh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(2*b*x+a+d)/b-1/2*exp(a-d)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(23) = 46$.

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.38

$$\int e^{a+bx}\sinh(d+bx)\,dx = -\frac{(2\,bx-1)\cosh\left(bx+d\right)\cosh\left(-a+d\right)-(2\,bx-1)\cosh\left(bx+d\right)\sinh\left(-a+d\right)-((2\,bx+1)\cosh\left(-a\right.}{4\left(b\cosh\left(bx+d\right)-b\sinh\left(bx+d\right)\right)}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d),x, algorithm="fricas")
```

output
```
-1/4*((2*b*x - 1)*cosh(b*x + d)*cosh(-a + d) - (2*b*x - 1)*cosh(b*x + d)*s
inh(-a + d) - ((2*b*x + 1)*cosh(-a + d) - (2*b*x + 1)*sinh(-a + d))*sinh(b
*x + d))/(b*cosh(b*x + d) - b*sinh(b*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(20) = 40$.

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.76

$$\int e^{a+bx}\sinh(d+bx)\,dx = \begin{cases} \frac{xe^{a}e^{bx}\sinh\left(bx+d\right)}{2} - \frac{xe^{a}e^{bx}\cosh\left(bx+d\right)}{2} - \frac{e^{a}e^{bx}\sinh\left(bx+d\right)}{2b} + \frac{e^{a}e^{bx}\cosh\left(bx+d\right)}{b} & \text{for } b \neq 0 \\ xe^{a}\sinh\left(d\right) & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d),x)
```

output
```
Piecewise((x*exp(a)*exp(b*x)*sinh(b*x + d)/2 - x*exp(a)*exp(b*x)*cosh(b*x
+ d)/2 - exp(a)*exp(b*x)*sinh(b*x + d)/(2*b) + exp(a)*exp(b*x)*cosh(b*x +
d)/b, Ne(b, 0)), (x*exp(a)*sinh(d), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int e^{a+bx}\sinh(d+bx)\,dx = -\frac{(bx+a)e^{(a-d)}}{2\,b} + \frac{e^{(2\,bx+a+d)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d),x, algorithm="maxima")
```

output
```
-1/2*(b*x + a)*e^(a - d)/b + 1/4*e^(2*b*x + a + d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int e^{a+bx}\sinh(d+bx)\,dx = -\frac{\left(2\,bxe^{a} - e^{(2\,bx+a+2\,d)}\right)e^{(-d)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d),x, algorithm="giac")
```

output
```
-1/4*(2*b*x*e^a - e^(2*b*x + a + 2*d))*e^(-d)/b
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int e^{a+bx}\sinh(d+bx)\,dx = \frac{\mathrm{e}^{a-d}\left(\mathrm{e}^{2\,d+2\,b\,x} - 2\,b\,x\right)}{4\,b}$$

input
```
int(exp(a + b*x)*sinh(d + b*x),x)
```

output
```
(exp(a - d)*(exp(2*d + 2*b*x) - 2*b*x))/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.31

$$\int e^{a+bx}\sinh(d+bx)\,dx = \frac{e^{bx+a}(-\cosh(bx+d)\,bx+\cosh(bx+d)+\sinh(bx+d)\,bx)}{2b}$$

input
```
int(exp(b*x+a)*sinh(b*x+d),x)
```

output
```
(e**(a + b*x)*( - cosh(b*x + d)*b*x + cosh(b*x + d) + sinh(b*x + d)*b*x))/
(2*b)
```

3.2 $\int e^{a+bx} \sinh^2(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 51

$$\int e^{a+bx} \sinh^2(d+bx)\,dx = -\frac{e^{a-2d-bx}}{4b} - \frac{e^{a+bx}}{2b} + \frac{e^{a+2d+3bx}}{12b}$$

output
```
-1/4*exp(-b*x+a-2*d)/b-1/2*exp(b*x+a)/b+1/12*exp(3*b*x+a+2*d)/b
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int e^{a+bx} \sinh^2(d+bx)\,dx = \frac{e^{a-bx}\left(-6e^{2bx} + \left(-3+e^{4bx}\right)\cosh(2d) + \left(3+e^{4bx}\right)\sinh(2d)\right)}{12b}$$

input
```
Integrate[E^(a + b*x)*Sinh[d + b*x]^2,x]
```

output
```
(E^(a - b*x)*(-6*E^(2*b*x) + (-3 + E^(4*b*x))*Cosh[2*d] + (3 + E^(4*b*x))*
Sinh[2*d]))/(12*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx} \sinh^2(bx+d) \, dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int \frac{1}{4} e^{a-2bx} \left(1-e^{2bx}\right)^2 de^{bx}}{b}$$

$$\Big\downarrow \ 27$$

$$\frac{e^a \int e^{-2bx} \left(1-e^{2bx}\right)^2 de^{bx}}{4b}$$

$$\Big\downarrow \ 244$$

$$\frac{e^a \int \left(-2+e^{-2bx}+e^{2bx}\right) de^{bx}}{4b}$$

$$\Big\downarrow \ 2009$$

$$\frac{e^a \left(-e^{-bx}-2e^{bx}+\frac{1}{3}e^{3bx}\right)}{4b}$$

input
```
Int[E^(a + b*x)*Sinh[d + b*x]^2,x]
```

output
```
(E^a*(-E^(-(b*x)) - 2*E^(b*x) + E^(3*b*x)/3))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{{\mathrm e}^{-b x +a -2 d}}{4 b}-\frac{{\mathrm e}^{b x +a}}{2 b}+\frac{{\mathrm e}^{3 b x +a +2 d}}{12 b}$
parallelrisch	$\frac{4 \,{\mathrm e}^{b x +a} \left(-1+2 \tanh \left(\frac{b x}{2}+\frac{d}{2}\right)\right)}{3 b \left(-1+\tanh \left(\frac{b x}{2}+\frac{d}{2}\right)\right)^{2} \left(1+\tanh \left(\frac{b x}{2}+\frac{d}{2}\right)\right)^{2}}$
default	$-\frac{\sinh \left(b x +a \right)}{2 b}-\frac{\sinh \left(-b x +a -2 d \right)}{4 b}+\frac{\sinh \left(3 b x +a +2 d \right)}{12 b}-\frac{\cosh \left(b x +a \right)}{2 b}-\frac{\cosh \left(-b x +a -2 d \right)}{4 b}+\frac{\cosh \left(3 b x +a +2 d \right)}{12 b}$
orering	$\frac{{\mathrm e}^{b x +a} \sinh \left(b x +d \right)^{2}}{3 b}+\frac{b \,{\mathrm e}^{b x +a} \sinh \left(b x +d \right)^{2}+2 \,{\mathrm e}^{b x +a} \sinh \left(b x +d \right) b \cosh \left(b x +d \right)}{b^{2}}-\frac{3 b^{2} {\mathrm e}^{b x +a} \sinh \left(b x +d \right)^{2}+4 b^{2} {\mathrm e}^{b x +a} \sinh (b x}{3}$

input
```
int(exp(b*x+a)*sinh(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*exp(-b*x+a-2*d)/b-1/2*exp(b*x+a)/b+1/12*exp(3*b*x+a+2*d)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(42) = 84$.

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.43

$$\int e^{a+bx}\sinh^2(d+bx)\,dx = -\frac{\cosh(bx+d)^2\cosh(-a+d)+(\cosh(-a+d)-\sinh(-a+d))\sinh(bx+d)^2-4(\cosh(bx+d)\cosh}{6(b\cosh(bx}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)^2,x, algorithm="fricas")
```

output
```
-1/6*(cosh(b*x + d)^2*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*
x + d)^2 - 4*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sin
h(b*x + d) - (cosh(b*x + d)^2 + 3)*sinh(-a + d) + 3*cosh(-a + d))/(b*cosh(
b*x + d) - b*sinh(b*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(37) = 74$.

Time = 0.42 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.53

$$\int e^{a+bx}\sinh^2(d+bx)\,dx = \begin{cases} \frac{e^a e^{bx}\sinh^2(bx+d)}{3b} + \frac{2e^a e^{bx}\sinh(bx+d)\cosh(bx+d)}{3b} - \frac{2e^a e^{bx}\cosh^2(bx+d)}{3b} & \text{for } b \neq 0 \\ xe^a\sinh^2(d) & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)**2,x)
```

output
```
Piecewise((exp(a)*exp(b*x)*sinh(b*x + d)**2/(3*b) + 2*exp(a)*exp(b*x)*sinh
(b*x + d)*cosh(b*x + d)/(3*b) - 2*exp(a)*exp(b*x)*cosh(b*x + d)**2/(3*b),
Ne(b, 0)), (x*exp(a)*sinh(d)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int e^{a+bx}\sinh^2(d+bx)\,dx=\frac{e^{(3\,bx+a+2\,d)}}{12\,b}-\frac{e^{(bx+a)}}{2\,b}-\frac{e^{(-bx+a-2\,d)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)^2,x, algorithm="maxima")
```

output
```
1/12*e^(3*b*x + a + 2*d)/b - 1/2*e^(b*x + a)/b - 1/4*e^(-b*x + a - 2*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int e^{a+bx}\sinh^2(d+bx)\,dx=\frac{\left(e^{(3\,bx+a+4\,d)}-6\,e^{(bx+a+2\,d)}-3\,e^{(-bx+a)}\right)e^{(-2\,d)}}{12\,b}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)^2,x, algorithm="giac")
```

output
```
1/12*(e^(3*b*x + a + 4*d) - 6*e^(b*x + a + 2*d) - 3*e^(-b*x + a))*e^(-2*d)
/b
```

Mupad [B] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int e^{a+bx}\sinh^2(d+bx)\,dx=-\frac{\mathrm{e}^{a-2\,d-b\,x}\,\left(6\,\mathrm{e}^{2\,d+2\,b\,x}-\mathrm{e}^{4\,d+4\,b\,x}+3\right)}{12\,b}$$

input
```
int(exp(a + b*x)*sinh(d + b*x)^2,x)
```

output
```
-(exp(a - 2*d - b*x)*(6*exp(2*d + 2*b*x) - exp(4*d + 4*b*x) + 3))/(12*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int e^{a+bx}\sinh^2(d+bx)\,dx=\frac{e^a\left(e^{4bx+4d}-6e^{2bx+2d}-3\right)}{12e^{bx+2d}b}$$

input
```
int(exp(b*x+a)*sinh(b*x+d)^2,x)
```

output
```
(e**a*(e**(4*b*x + 4*d) - 6*e**(2*b*x + 2*d) - 3))/(12*e**(b*x + 2*d)*b)
```

3.3 $\int e^{a+bx} \sinh^3(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 65

$$\int e^{a+bx} \sinh^3(d+bx)\,dx = \frac{e^{a-3d-2bx}}{16b} - \frac{3e^{a+d+2bx}}{16b} + \frac{e^{a+3d+4bx}}{32b} + \frac{3}{8}e^{a-d}x$$

output
```
1/16*exp(-2*b*x+a-3*d)/b-3/16*exp(2*b*x+a+d)/b+1/32*exp(4*b*x+a+3*d)/b+3/8
*exp(a-d)*x
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.37

$$\int e^{a+bx} \sinh^3(d+bx)\,dx$$
$$= \frac{e^{a-2bx}\left(-6e^{2bx}\left(e^{2bx}-2bx\right)\cosh(d) + \left(2+e^{6bx}\right)\cosh(3d) - 6e^{4bx}\sinh(d) - 12be^{2bx}x\sinh(d) - 2\sinh(3d}{32b}$$

input
```
Integrate[E^(a + b*x)*Sinh[d + b*x]^3,x]
```

output
```
(E^(a - 2*b*x)*(-6*E^(2*b*x)*(E^(2*b*x) - 2*b*x)*Cosh[d] + (2 + E^(6*b*x))
*Cosh[3*d] - 6*E^(4*b*x)*Sinh[d] - 12*b*E^(2*b*x)*x*Sinh[d] - 2*Sinh[3*d]
+ E^(6*b*x)*Sinh[3*d]))/(32*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{1}{8}e^{a-3bx}\left(1-e^{2bx}\right)^3de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^a\int e^{-3bx}\left(1-e^{2bx}\right)^3de^{bx}}{8b}$$

$$\Big\downarrow \text{243}$$

$$-\frac{e^a\int e^{-2bx}\left(1-e^{2bx}\right)^3de^{2bx}}{16b}$$

$$\Big\downarrow \text{49}$$

$$-\frac{e^a\int \left(3+e^{-2bx}-3e^{-bx}-e^{2bx}\right)de^{2bx}}{16b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^a\left(-e^{-bx}+\frac{5}{2}e^{2bx}-3\log\left(e^{2bx}\right)\right)}{16b}$$

input
```
Int[E^(a + b*x)*Sinh[d + b*x]^3,x]
```

output
```
-1/16*(E^a*(-E^(-(b*x)) + (5*E^(2*b*x))/2 - 3*Log[E^(2*b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result
risch	$\frac{{\mathrm e}^{-2 b x +a -3 d}}{16 b}-\frac{3 \,{\mathrm e}^{2 b x +a +d}}{16 b}+\frac{{\mathrm e}^{4 b x +a +3 d}}{32 b}+\frac{3 \,{\mathrm e}^{a -d} x}{8}$
default	$\frac{3 \cosh \left(a -d \right) x}{8}+\frac{\sinh \left(-2 b x +a -3 d \right)}{16 b}-\frac{3 \sinh \left(2 b x +a +d \right)}{16 b}+\frac{\sinh \left(4 b x +a +3 d \right)}{32 b}+\frac{3 \sinh \left(a -d \right) x}{8}+\frac{\cosh \left(-2 b x +a -3 d \right)}{16 b}-$ 3 co
orering	$\frac{\left(4 b x +1\right) {\mathrm e}^{b x +a} \sinh \left(b x +d \right)^{3}}{4 b}-\frac{\left(b x -1\right) \left(b \,{\mathrm e}^{b x +a} \sinh \left(b x +d \right)^{3}+3 \,{\mathrm e}^{b x +a} \sinh \left(b x +d \right)^{2} b \cosh \left(b x +d \right)\right)}{4 b^{2}}-$ (4bx+1)(4 e^{bx+a} sinh(bx+

input
```
int(exp(b*x+a)*sinh(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*exp(-2*b*x+a-3*d)/b-3/16*exp(2*b*x+a+d)/b+1/32*exp(4*b*x+a+3*d)/b+3/8
*exp(a-d)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(53) = 106$.

Time = 0.09 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.28

$$\int e^{a+bx}\sinh^3(d+bx)\,dx$$
$$= \frac{3\cosh(bx+d)^3\cosh(-a+d) - (\cosh(-a+d) - \sinh(-a+d))\sinh(bx+d)^3 + 6(2bx-1)\cosh(bx}{}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)^3,x, algorithm="fricas")
```

output
```
1/32*(3*cosh(b*x + d)^3*cosh(-a + d) - (cosh(-a + d) - sinh(-a + d))*sinh(
b*x + d)^3 + 6*(2*b*x - 1)*cosh(b*x + d)*cosh(-a + d) + 9*(cosh(b*x + d)*c
osh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d)^2 - 3*(cosh(b*x +
d)^2*cosh(-a + d) + 2*(2*b*x + 1)*cosh(-a + d) - (4*b*x + cosh(b*x + d)^2
+ 2)*sinh(-a + d))*sinh(b*x + d) - 3*(cosh(b*x + d)^3 + 2*(2*b*x - 1)*cosh
(b*x + d))*sinh(-a + d))/(b*cosh(b*x + d) - b*sinh(b*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(54) = 108$.

Time = 0.91 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.80

$$\int e^{a+bx}\sinh^3(d+bx)\,dx$$
$$= \begin{cases} \frac{3xe^ae^{bx}\sinh^3(bx+d)}{8} - \frac{3xe^ae^{bx}\sinh^2(bx+d)\cosh(bx+d)}{8} - \frac{3xe^ae^{bx}\sinh(bx+d)\cosh^2(bx+d)}{8} + \frac{3xe^ae^{bx}\cosh^3(bx+d)}{8} - \frac{e^ae^{bx}\sinh}{8} \\ xe^a\sinh^3(d) \end{cases}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)**3,x)
```

output
```
Piecewise((3*x*exp(a)*exp(b*x)*sinh(b*x + d)**3/8 - 3*x*exp(a)*exp(b*x)*si
nh(b*x + d)**2*cosh(b*x + d)/8 - 3*x*exp(a)*exp(b*x)*sinh(b*x + d)*cosh(b*
x + d)**2/8 + 3*x*exp(a)*exp(b*x)*cosh(b*x + d)**3/8 - exp(a)*exp(b*x)*sin
h(b*x + d)**3/(8*b) + 3*exp(a)*exp(b*x)*sinh(b*x + d)**2*cosh(b*x + d)/(4*
b) - 3*exp(a)*exp(b*x)*cosh(b*x + d)**3/(8*b), Ne(b, 0)), (x*exp(a)*sinh(d
)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int e^{a+bx}\sinh^3(d+bx)\,dx = \frac{3\,(bx+a)e^{(a-d)}}{8\,b} + \frac{e^{(4\,bx+a+3\,d)}}{32\,b} - \frac{3\,e^{(2\,bx+a+d)}}{16\,b} + \frac{e^{(-2\,bx+a-3\,d)}}{16\,b}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)^3,x, algorithm="maxima")
```

output
```
3/8*(b*x + a)*e^(a - d)/b + 1/32*e^(4*b*x + a + 3*d)/b - 3/16*e^(2*b*x + a
 + d)/b + 1/16*e^(-2*b*x + a - 3*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int e^{a+bx}\sinh^3(d+bx)\,dx = \frac{\left(12\,bxe^{(a+2\,d)} - 2\left(3\,e^{(2\,bx+a+2\,d)} - e^{a}\right)e^{(-2\,bx)} + e^{(4\,bx+a+6\,d)} - 6\,e^{(2\,bx+a+4\,d)}\right)e^{(-3\,d)}}{32\,b}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)^3,x, algorithm="giac")
```

output
```
1/32*(12*b*x*e^(a + 2*d) - 2*(3*e^(2*b*x + a + 2*d) - e^a)*e^(-2*b*x) + e^
(4*b*x + a + 6*d) - 6*e^(2*b*x + a + 4*d))*e^(-3*d)/b
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\sinh^3(d+bx)\,dx =$$
$$-\frac{e^{a+b\,x}\,\left(5\cosh(d+b\,x)+\cosh(d+b\,x)^2\sinh(d+b\,x)-3\cosh(d+b\,x)^3-3\,b\,x\cosh(d+b\,x)+3\,b\,x\,\mathrm{si}\right.}{8\,b}$$

input
```
int(exp(a + b*x)*sinh(d + b*x)^3,x)
```

output
```
-(exp(a + b*x)*(5*cosh(d + b*x) + cosh(d + b*x)^2*sinh(d + b*x) - 3*cosh(d
 + b*x)^3 - 3*b*x*cosh(d + b*x) + 3*b*x*sinh(d + b*x)))/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int e^{a+bx}\sinh^3(d+bx)\,dx = \frac{e^a\left(e^{6bx+6d}-6e^{4bx+4d}+12e^{2bx+2d}bx+2\right)}{32e^{2bx+3d}b}$$

input
```
int(exp(b*x+a)*sinh(b*x+d)^3,x)
```

output
```
(e**a*(e**(6*b*x + 6*d) - 6*e**(4*b*x + 4*d) + 12*e**(2*b*x + 2*d)*b*x + 2
))/(32*e**(2*b*x + 3*d)*b)
```

3.4 $\int e^{a+bx} \sinh^4(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 87

$$\int e^{a+bx} \sinh^4(d+bx)\,dx = -\frac{e^{a-4d-3bx}}{48b} + \frac{e^{a-2d-bx}}{4b} + \frac{3e^{a+bx}}{8b} - \frac{e^{a+2d+3bx}}{12b} + \frac{e^{a+4d+5bx}}{80b}$$

output
```
-1/48*exp(-3*b*x+a-4*d)/b+1/4*exp(-b*x+a-2*d)/b+3/8*exp(b*x+a)/b-1/12*exp(
3*b*x+a+2*d)/b+1/80*exp(5*b*x+a+4*d)/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.16

$$\int e^{a+bx} \sinh^4(d+bx)\,dx$$
$$= \frac{e^{a-3bx}\left(90e^{4bx} - 20e^{2bx}\left(-3+e^{4bx}\right)\cosh(2d) + \left(-5+3e^{8bx}\right)\cosh(4d) - 60e^{2bx}\sinh(2d) - 20e^{6bx}\sinh(2d\right.}{240b}$$

input
```
Integrate[E^(a + b*x)*Sinh[d + b*x]^4,x]
```

output
```
(E^(a - 3*b*x)*(90*E^(4*b*x) - 20*E^(2*b*x)*(-3 + E^(4*b*x))*Cosh[2*d] + (
-5 + 3*E^(8*b*x))*Cosh[4*d] - 60*E^(2*b*x)*Sinh[2*d] - 20*E^(6*b*x)*Sinh[2
*d] + 5*Sinh[4*d] + 3*E^(8*b*x)*Sinh[4*d]))/(240*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx} \sinh^4(bx+d)\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int \frac{1}{16} e^{a-4bx} \left(1-e^{2bx}\right)^4 de^{bx}}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{e^a \int e^{-4bx} \left(1-e^{2bx}\right)^4 de^{bx}}{16b}$$

$$\Big\downarrow \; 244$$

$$\frac{e^a \int \left(6+e^{-4bx}-4e^{-2bx}-4e^{2bx}+e^{4bx}\right) de^{bx}}{16b}$$

$$\Big\downarrow \; 2009$$

$$\frac{e^a \left(-\frac{1}{3}e^{-3bx}+4e^{-bx}+6e^{bx}-\frac{4}{3}e^{3bx}+\frac{1}{5}e^{5bx}\right)}{16b}$$

input
```
Int[E^(a + b*x)*Sinh[d + b*x]^4,x]
```

output
```
(E^a*(-1/3*1/E^(3*b*x) + 4/E^(b*x) + 6*E^(b*x) - (4*E^(3*b*x))/3 + E^(5*b*
x)/5))/(16*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{{\mathrm e}^{-3 b x +a -4 d}}{48 b}+\frac{{\mathrm e}^{-b x +a -2 d}}{4 b}+\frac{3 \,{\mathrm e}^{b x +a}}{8 b}-\frac{{\mathrm e}^{3 b x +a +2 d}}{12 b}+\frac{{\mathrm e}^{5 b x +a +4 d}}{80 b}$
parallelrisch	$\frac{16 \,{\mathrm e}^{b x +a} \left(6 \tanh \left(\frac{b x}{2}+\frac{d}{2}\right)^{3}-2 \tanh \left(\frac{b x}{2}+\frac{d}{2}\right)^{2}-2 \tanh \left(\frac{b x}{2}+\frac{d}{2}\right)+1\right)}{15 b \left(-1+\tanh \left(\frac{b x}{2}+\frac{d}{2}\right)\right)^{4} \left(1+\tanh \left(\frac{b x}{2}+\frac{d}{2}\right)\right)^{4}}$
default	$\frac{3 \sinh \left(b x +a \right)}{8 b}-\frac{\sinh \left(-3 b x +a -4 d \right)}{48 b}+\frac{\sinh \left(-b x +a -2 d \right)}{4 b}-\frac{\sinh \left(3 b x +a +2 d \right)}{12 b}+\frac{\sinh \left(5 b x +a +4 d \right)}{80 b}+\frac{3 \cosh \left(b x +a \right)}{8 b}-$
orering	$\frac{{\mathrm e}^{b x +a} \sinh \left(b x +d \right)^{4}}{5 b}+\frac{\frac{10 b \,{\mathrm e}^{b x +a} \sinh \left(b x +d \right)^{4}}{9}+\frac{40 \,{\mathrm e}^{b x +a} \sinh \left(b x +d \right)^{3} b \cosh \left(b x +d \right)}{9}}{b^{2}}-\frac{2\left(5 b^{2} {\mathrm e}^{b x +a} \sinh \left(b x +d \right)^{4}+8 b^{2} {\mathrm e}^{b x +a} \sinh\right.}{}$

input
```
int(exp(b*x+a)*sinh(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/48*exp(-3*b*x+a-4*d)/b+1/4*exp(-b*x+a-2*d)/b+3/8*exp(b*x+a)/b-1/12*exp(
3*b*x+a+2*d)/b+1/80*exp(5*b*x+a+4*d)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(72) = 144$.

Time = 0.10 (sec) , antiderivative size = 270, normalized size of antiderivative = 3.10

$$\int e^{a+bx}\sinh^4(d+bx)\,dx = -\frac{\cosh(bx+d)^4\cosh(-a+d) + (\cosh(-a+d) - \sinh(-a+d))\sinh(bx+d)^4 - 16(\cosh(bx+d)\cos$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)^4,x, algorithm="fricas")
```

output
```
-1/120*(cosh(b*x + d)^4*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(
b*x + d)^4 - 16*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*
sinh(b*x + d)^3 - 20*cosh(b*x + d)^2*cosh(-a + d) + 2*(3*cosh(b*x + d)^2*c
osh(-a + d) - (3*cosh(b*x + d)^2 - 10)*sinh(-a + d) - 10*cosh(-a + d))*sin
h(b*x + d)^2 - 16*(cosh(b*x + d)^3*cosh(-a + d) - 5*cosh(b*x + d)*cosh(-a
+ d) - (cosh(b*x + d)^3 - 5*cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d) - (
cosh(b*x + d)^4 - 20*cosh(b*x + d)^2 - 45)*sinh(-a + d) - 45*cosh(-a + d))
/(b*cosh(b*x + d) - b*sinh(b*x + d))
```

Sympy [A] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.60

$$\int e^{a+bx}\sinh^4(d+bx)\,dx$$

$$= \begin{cases} \frac{e^a e^{bx}\sinh^4(bx+d)}{5b} + \frac{4e^a e^{bx}\sinh^3(bx+d)\cosh(bx+d)}{5b} - \frac{4e^a e^{bx}\sinh^2(bx+d)\cosh^2(bx+d)}{5b} - \frac{8e^a e^{bx}\sinh(bx+d)\cosh^3(bx+d)}{15b} + \frac{8e^a}{} \\ xe^a\sinh^4(d) \end{cases}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)**4,x)
```

output
```
Piecewise((exp(a)*exp(b*x)*sinh(b*x + d)**4/(5*b) + 4*exp(a)*exp(b*x)*sinh
(b*x + d)**3*cosh(b*x + d)/(5*b) - 4*exp(a)*exp(b*x)*sinh(b*x + d)**2*cosh
(b*x + d)**2/(5*b) - 8*exp(a)*exp(b*x)*sinh(b*x + d)*cosh(b*x + d)**3/(15*
b) + 8*exp(a)*exp(b*x)*cosh(b*x + d)**4/(15*b), Ne(b, 0)), (x*exp(a)*sinh(
d)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int e^{a+bx}\sinh^4(d+bx)\,dx = \frac{e^{(5\,bx+a+4\,d)}}{80\,b} - \frac{e^{(3\,bx+a+2\,d)}}{12\,b} + \frac{3\,e^{(bx+a)}}{8\,b} + \frac{e^{(-bx+a-2\,d)}}{4\,b} - \frac{e^{(-3\,bx+a-4\,d)}}{48\,b}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)^4,x, algorithm="maxima")
```

output
```
1/80*e^(5*b*x + a + 4*d)/b - 1/12*e^(3*b*x + a + 2*d)/b + 3/8*e^(b*x + a)/
b + 1/4*e^(-b*x + a - 2*d)/b - 1/48*e^(-3*b*x + a - 4*d)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

$$\int e^{a+bx}\sinh^4(d+bx)\,dx = \frac{\left(5\left(12\,e^{(2\,bx+a+2\,d)} - e^{a}\right)e^{(-3\,bx)} + 3\,e^{(5\,bx+a+8\,d)} - 20\,e^{(3\,bx+a+6\,d)} + 90\,e^{(bx+a+4\,d)}\right)e^{(-4\,d)}}{240\,b}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)^4,x, algorithm="giac")
```

output
```
1/240*(5*(12*e^(2*b*x + a + 2*d) - e^a)*e^(-3*b*x) + 3*e^(5*b*x + a + 8*d)
 - 20*e^(3*b*x + a + 6*d) + 90*e^(b*x + a + 4*d))*e^(-4*d)/b
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int e^{a+bx}\sinh^4(d+bx)\,dx=\frac{\mathrm{e}^{a-2\,d-b\,x}}{4\,b}-\frac{\mathrm{e}^{a+2\,d+3\,b\,x}}{12\,b}-\frac{\mathrm{e}^{a-4\,d-3\,b\,x}}{48\,b}+\frac{\mathrm{e}^{a+4\,d+5\,b\,x}}{80\,b}+\frac{3\,\mathrm{e}^{a+b\,x}}{8\,b}$$

input
```
int(exp(a + b*x)*sinh(d + b*x)^4,x)
```

output
```
exp(a - 2*d - b*x)/(4*b) - exp(a + 2*d + 3*b*x)/(12*b) - exp(a - 4*d - 3*b
*x)/(48*b) + exp(a + 4*d + 5*b*x)/(80*b) + (3*exp(a + b*x))/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int e^{a+bx}\sinh^4(d+bx)\,dx=\frac{e^{a}\left(3e^{8bx+8d}-20e^{6bx+6d}+90e^{4bx+4d}+60e^{2bx+2d}-5\right)}{240e^{3bx+4d}b}$$

input
```
int(exp(b*x+a)*sinh(b*x+d)^4,x)
```

output
```
(e**a*(3*e**(8*b*x + 8*d) - 20*e**(6*b*x + 6*d) + 90*e**(4*b*x + 4*d) + 60
*e**(2*b*x + 2*d) - 5))/(240*e**(3*b*x + 4*d)*b)
```

3.5 $\int e^{2(a+bx)} \sinh(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 38

$$\int e^{2(a+bx)} \sinh(d+bx)\,dx = -\frac{e^{2a-d+bx}}{2b} + \frac{e^{2a+d+3bx}}{6b}$$

output
```
-1/2*exp(b*x+2*a-d)/b+1/6*exp(3*b*x+2*a+d)/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int e^{2(a+bx)} \sinh(d+bx)\,dx = \frac{e^{2a+bx}\left(\left(-3+e^{2bx}\right)\cosh(d)+\left(3+e^{2bx}\right)\sinh(d)\right)}{6b}$$

input
```
Integrate[E^(2*(a + b*x))*Sinh[d + b*x],x]
```

output
```
(E^(2*a + b*x)*((-3 + E^(2*b*x))*Cosh[d] + (3 + E^(2*b*x))*Sinh[d]))/(6*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2720, 27, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\sinh(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{1}{2}e^{2a}\left(1-e^{2bx}\right)de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^{2a}\int \left(1-e^{2bx}\right)de^{bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{2a}\left(e^{bx}-\frac{1}{3}e^{3bx}\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Sinh[d + b*x],x]
```

output
```
-1/2*(E^(2*a)*(E^(b*x) - E^(3*b*x)/3))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

method	result	size
parallelrisch	$-\frac{{\mathrm e}^{2 b x +2 a} \left(\cosh \left(b x +d \right)-2 \sinh \left(b x +d \right)\right)}{3 b}$	30
risch	$-\frac{{\mathrm e}^{b x +2 a -d}}{2 b}+\frac{{\mathrm e}^{3 b x +2 a +d}}{6 b}$	33
orering	$\frac{4 \,{\mathrm e}^{2 b x +2 a} \sinh \left(b x +d \right)}{3 b}-\frac{2 \,{\mathrm e}^{2 b x +2 a} b \sinh \left(b x +d \right)+{\mathrm e}^{2 b x +2 a} b \cosh \left(b x +d \right)}{3 b^{2}}$	63
default	$-\frac{\sinh \left(b x +2 a -d \right)}{2 b}+\frac{\sinh \left(3 b x +2 a +d \right)}{6 b}-\frac{\cosh \left(b x +2 a -d \right)}{2 b}+\frac{\cosh \left(3 b x +2 a +d \right)}{6 b}$	64

input
```
int(exp(2*b*x+2*a)*sinh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/3*exp(2*b*x+2*a)*(cosh(b*x+d)-2*sinh(b*x+d))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(32) = 64.

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.63

$$\int e^{2(a+bx)} \sinh(d+bx)\,dx$$
$$= \frac{\cosh\left(bx+d\right)^2 \cosh\left(-2\,a+2\,d\right) + \left(\cosh\left(-2\,a+2\,d\right) - \sinh\left(-2\,a+2\,d\right)\right)\sinh\left(bx+d\right)^2 + 2\left(\cosh\left(bx+\right.\right.}{6\left(b\right.}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d),x, algorithm="fricas")
```

output
```
1/6*(cosh(b*x + d)^2*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*
d))*sinh(b*x + d)^2 + 2*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*si
nh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^2 - 3)*sinh(-2*a + 2*d) - 3
*cosh(-2*a + 2*d))/(b*cosh(b*x + d) - b*sinh(b*x + d))
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int e^{2(a+bx)}\sinh(d+bx)\,dx=\begin{cases}\frac{2e^{2a}e^{2bx}\sinh{(bx+d)}}{3b}-\frac{e^{2a}e^{2bx}\cosh{(bx+d)}}{3b} & \text{for } b\neq 0\\ xe^{2a}\sinh{(d)} & \text{otherwise}\end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d),x)
```

output
```
Piecewise((2*exp(2*a)*exp(2*b*x)*sinh(b*x + d)/(3*b) - exp(2*a)*exp(2*b*x)
*cosh(b*x + d)/(3*b), Ne(b, 0)), (x*exp(2*a)*sinh(d), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int e^{2(a+bx)}\sinh(d+bx)\,dx=\frac{e^{(3\,bx+2\,a+d)}}{6\,b}-\frac{e^{(bx+2\,a-d)}}{2\,b}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d),x, algorithm="maxima")
```

output
```
1/6*e^(3*b*x + 2*a + d)/b - 1/2*e^(b*x + 2*a - d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int e^{2(a+bx)}\sinh(d+bx)\,dx=\frac{\left(e^{(3\,bx+2\,a+3\,d)}-3\,e^{(bx+2\,a+d)}\right)e^{(-2\,d)}}{6\,b}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d),x, algorithm="giac")
```

output
```
1/6*(e^(3*b*x + 2*a + 3*d) - 3*e^(b*x + 2*a + d))*e^(-2*d)/b
```

Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int e^{2(a+bx)}\sinh(d+bx)\,dx=\frac{e^{2\,a}\,e^{-d}\,e^{b\,x}\,\left(e^{2\,d}\,e^{2\,b\,x}-3\right)}{6\,b}$$

input
```
int(exp(2*a + 2*b*x)*sinh(d + b*x),x)
```

output
```
(exp(2*a)*exp(-d)*exp(b*x)*(exp(2*d)*exp(2*b*x) - 3))/(6*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int e^{2(a+bx)}\sinh(d+bx)\,dx=\frac{e^{2bx+2a}(-\cosh(bx+d)+2\sinh(bx+d))}{3b}$$

input
```
int(exp(2*b*x+2*a)*sinh(b*x+d),x)
```

output
```
(e**(2*a + 2*b*x)*( - cosh(b*x + d) + 2*sinh(b*x + d)))/(3*b)
```

3.6 $\int e^{2(a+bx)} \sinh^2(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 51

$$\int e^{2(a+bx)} \sinh^2(d+bx)\,dx = -\frac{e^{2a+2bx}}{4b} + \frac{e^{2(a+d)+4bx}}{16b} + \frac{1}{4}e^{2a-2d}x$$

output
```
-1/4*exp(2*b*x+2*a)/b+1/16*exp(4*b*x+2*a+2*d)/b+1/4*exp(2*a-2*d)*x
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int e^{2(a+bx)} \sinh^2(d+bx)\,dx = \frac{e^{2a}\left(-4e^{2bx} + \left(e^{4bx}+4bx\right)\cosh(2d) + \left(e^{4bx}-4bx\right)\sinh(2d)\right)}{16b}$$

input
```
Integrate[E^(2*(a + b*x))*Sinh[d + b*x]^2,x]
```

output
```
(E^(2*a)*(-4*E^(2*b*x) + (E^(4*b*x) + 4*b*x)*Cosh[2*d] + (E^(4*b*x) - 4*b*
x)*Sinh[2*d]))/(16*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.59, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\sinh^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{4}e^{2a-bx}\left(1-e^{2bx}\right)^2de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{2a}\int e^{-bx}\left(1-e^{2bx}\right)^2de^{bx}}{4b}$$

$$\Big\downarrow \text{243}$$

$$\frac{e^{2a}\int e^{-bx}\left(1-e^{2bx}\right)^2de^{2bx}}{8b}$$

$$\Big\downarrow \text{49}$$

$$\frac{e^{2a}\int \left(-2+e^{-bx}+e^{2bx}\right)de^{2bx}}{8b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2a}\left(\log\left(e^{2bx}\right)-\frac{3}{2}e^{2bx}\right)}{8b}$$

input
```
Int[E^(2*(a + b*x))*Sinh[d + b*x]^2,x]
```

output
```
(E^(2*a)*((-3*E^(2*b*x))/2 + Log[E^(2*b*x)]))/(8*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{e^{2bx+2a}}{4b}+\frac{e^{4bx+2a+2d}}{16b}+\frac{e^{2a-2d}x}{4}$
default	$\frac{x\cosh(2a-2d)}{4}-\frac{\sinh(2bx+2a)}{4b}+\frac{\sinh(4bx+2a+2d)}{16b}+\frac{x\sinh(2a-2d)}{4}-\frac{\cosh(2bx+2a)}{4b}+\frac{\cosh(4bx+2a+2d)}{16b}$
orering	$\frac{(4bx+3)e^{2bx+2a}\sinh(bx+d)^2}{4b}-\frac{(6bx+1)\left(2\,e^{2bx+2a}b\sinh(bx+d)^2+2\,e^{2bx+2a}\sinh(bx+d)b\cosh(bx+d)\right)}{8b^2}+\frac{x\left(6\,e^{2bx+2a}\sinh(bx\right.}{}$

input
```
int(exp(2*b*x+2*a)*sinh(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*exp(2*b*x+2*a)/b+1/16*exp(4*b*x+2*a+2*d)/b+1/4*exp(2*a-2*d)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(43) = 86$.

Time = 0.09 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.80

$$\int e^{2(a+bx)}\sinh^2(d+bx)\,dx$$
$$= \frac{(4\,bx+1)\cosh\left(bx+d\right)^2\cosh\left(-2\,a+2\,d\right)+((4\,bx+1)\cosh\left(-2\,a+2\,d\right)-(4\,bx+1)\sinh\left(-2\,a+2\,d\right)}{}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)^2,x, algorithm="fricas")
```

output
```
1/16*((4*b*x + 1)*cosh(b*x + d)^2*cosh(-2*a + 2*d) + ((4*b*x + 1)*cosh(-2*
a + 2*d) - (4*b*x + 1)*sinh(-2*a + 2*d))*sinh(b*x + d)^2 - 2*((4*b*x - 1)*
cosh(b*x + d)*cosh(-2*a + 2*d) - (4*b*x - 1)*cosh(b*x + d)*sinh(-2*a + 2*d
))*sinh(b*x + d) - ((4*b*x + 1)*cosh(b*x + d)^2 - 4)*sinh(-2*a + 2*d) - 4*
cosh(-2*a + 2*d))/(b*cosh(b*x + d)^2 - 2*b*cosh(b*x + d)*sinh(b*x + d) + b
*sinh(b*x + d)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(41) = 82$.

Time = 0.45 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.73

$$\int e^{2(a+bx)}\sinh^2(d+bx)\,dx$$
$$= \begin{cases} \frac{xe^{2a}e^{2bx}\sinh^2\left(bx+d\right)}{4} - \frac{xe^{2a}e^{2bx}\sinh\left(bx+d\right)\cosh\left(bx+d\right)}{2} + \frac{xe^{2a}e^{2bx}\cosh^2\left(bx+d\right)}{4} + \frac{e^{2a}e^{2bx}\sinh^2\left(bx+d\right)}{2b} - \frac{e^{2a}e^{2bx}\sinh\left(bx+d\right)\mathrm{c}}{4b} \\ xe^{2a}\sinh^2\left(d\right) \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)**2,x)
```

output
```
Piecewise((x*exp(2*a)*exp(2*b*x)*sinh(b*x + d)**2/4 - x*exp(2*a)*exp(2*b*x
)*sinh(b*x + d)*cosh(b*x + d)/2 + x*exp(2*a)*exp(2*b*x)*cosh(b*x + d)**2/4
 + exp(2*a)*exp(2*b*x)*sinh(b*x + d)**2/(2*b) - exp(2*a)*exp(2*b*x)*sinh(b
*x + d)*cosh(b*x + d)/(4*b), Ne(b, 0)), (x*exp(2*a)*sinh(d)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int e^{2(a+bx)}\sinh^2(d+bx)\,dx = \frac{1}{4}\,xe^{(2\,a-2\,d)} - \frac{\left(4\,e^{(-2\,bx-2\,d)}-1\right)e^{(4\,bx+2\,a+2\,d)}}{16\,b} + \frac{de^{(2\,a-2\,d)}}{4\,b}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)^2,x, algorithm="maxima")
```

output
```
1/4*x*e^(2*a - 2*d) - 1/16*(4*e^(-2*b*x - 2*d) - 1)*e^(4*b*x + 2*a + 2*d)/
b + 1/4*d*e^(2*a - 2*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int e^{2(a+bx)}\sinh^2(d+bx)\,dx = \frac{\left(4\,(bx+d)e^{(2\,a)}+e^{(4\,bx+2\,a+4\,d)}-4\,e^{(2\,bx+2\,a+2\,d)}\right)e^{(-2\,d)}}{16\,b}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)^2,x, algorithm="giac")
```

output
```
1/16*(4*(b*x + d)*e^(2*a) + e^(4*b*x + 2*a + 4*d) - 4*e^(2*b*x + 2*a + 2*d
))*e^(-2*d)/b
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int e^{2(a+bx)}\sinh^2(d+bx)\,dx = \frac{x\,e^{2\,a}\,e^{-2\,d}}{4} - \frac{e^{2\,a}\,e^{2\,b\,x}}{4\,b} + \frac{e^{2\,a}\,e^{2\,d}\,e^{4\,b\,x}}{16\,b}$$

input
```
int(exp(2*a + 2*b*x)*sinh(d + b*x)^2,x)
```

output
```
(x*exp(2*a)*exp(-2*d))/4 - (exp(2*a)*exp(2*b*x))/(4*b) + (exp(2*a)*exp(2*d
)*exp(4*b*x))/(16*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int e^{2(a+bx)}\sinh^2(d+bx)\,dx = \frac{e^{2a}\left(e^{4bx+4d} - 4e^{2bx+2d} + 4bx\right)}{16e^{2d}b}$$

input
```
int(exp(2*b*x+2*a)*sinh(b*x+d)^2,x)
```

output
```
(e**(2*a)*(e**(4*b*x + 4*d) - 4*e**(2*b*x + 2*d) + 4*b*x))/(16*e**(2*d)*b)
```

3.7 $\int e^{2(a+bx)} \sinh^3(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 78

$$\int e^{2(a+bx)} \sinh^3(d+bx)\,dx = \frac{e^{2a-3d-bx}}{8b} + \frac{3e^{2a-d+bx}}{8b} - \frac{e^{2a+d+3bx}}{8b} + \frac{e^{2a+3d+5bx}}{40b}$$

output
```
1/8*exp(-b*x+2*a-3*d)/b+3/8*exp(b*x+2*a-d)/b-1/8*exp(3*b*x+2*a+d)/b+1/40*e
xp(5*b*x+2*a+3*d)/b
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10

$$\int e^{2(a+bx)} \sinh^3(d+bx)\,dx$$
$$= \frac{e^{2a-bx}\left(-5e^{2bx}\left(-3+e^{2bx}\right)\cosh(d) + \left(5+e^{6bx}\right)\cosh(3d) - 15e^{2bx}\sinh(d) - 5e^{4bx}\sinh(d) - 5\sinh(3d) + \right.}{40b}$$

input
```
Integrate[E^(2*(a + b*x))*Sinh[d + b*x]^3,x]
```

output
```
(E^(2*a - b*x)*(-5*E^(2*b*x)*(-3 + E^(2*b*x))*Cosh[d] + (5 + E^(6*b*x))*Co
sh[3*d] - 15*E^(2*b*x)*Sinh[d] - 5*E^(4*b*x)*Sinh[d] - 5*Sinh[3*d] + E^(6*
b*x)*Sinh[3*d]))/(40*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.56, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)} \sinh^3(bx+d) \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int -\frac{1}{8} e^{2a-2bx} \left(1-e^{2bx}\right)^3 de^{bx}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{e^{2a} \int e^{-2bx} \left(1-e^{2bx}\right)^3 de^{bx}}{8b}$$

$$\Big\downarrow \text{ 244}$$

$$-\frac{e^{2a} \int \left(-3+e^{-2bx}+3e^{2bx}-e^{4bx}\right) de^{bx}}{8b}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{e^{2a} \left(-e^{-bx}-3e^{bx}+e^{3bx}-\frac{1}{5}e^{5bx}\right)}{8b}$$

input
```
Int[E^(2*(a + b*x))*Sinh[d + b*x]^3,x]
```

output
```
-1/8*(E^(2*a)*(-E^(-(b*x)) - 3*E^(b*x) + E^(3*b*x) - E^(5*b*x)/5))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

method	result
risch	$\frac{e^{-bx+2a-3d}}{8b}+\frac{3\,e^{bx+2a-d}}{8b}-\frac{e^{3bx+2a+d}}{8b}+\frac{e^{5bx+2a+3d}}{40b}$
parallelrisch	$-\frac{4\,e^{2bx+2a}\left(5\tanh\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}-4\tanh\left(\frac{bx}{2}+\frac{d}{2}\right)+1\right)}{5b\left(-1+\tanh\left(\frac{bx}{2}+\frac{d}{2}\right)\right)^{3}\left(1+\tanh\left(\frac{bx}{2}+\frac{d}{2}\right)\right)^{3}}$
default	$\frac{\sinh(-bx+2a-3d)}{8b}+\frac{3\sinh(bx+2a-d)}{8b}-\frac{\sinh(3bx+2a+d)}{8b}+\frac{\sinh(5bx+2a+3d)}{40b}+\frac{\cosh(-bx+2a-3d)}{8b}+\frac{3\cosh(bx+}{8b}$
orering	$\frac{8\,e^{2bx+2a}\sinh(bx+d)^{3}}{15b}+\frac{\frac{28\,e^{2bx+2a}b\sinh(bx+d)^{3}}{15}+\frac{14\,e^{2bx+2a}\sinh(bx+d)^{2}b\cosh(bx+d)}{5}}{b^{2}}-\frac{8\left(7\,e^{2bx+2a}b^{2}\sinh(bx+d)^{3}+12\,e\right.}{}$

input
```
int(exp(2*b*x+2*a)*sinh(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*exp(-b*x+2*a-3*d)/b+3/8*exp(b*x+2*a-d)/b-1/8*exp(3*b*x+2*a+d)/b+1/40*e
xp(5*b*x+2*a+3*d)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(66) = 132$.

Time = 0.10 (sec) , antiderivative size = 234, normalized size of antiderivative = 3.00

$$\int e^{2(a+bx)}\sinh^3(d+bx)\,dx$$
$$= \frac{3\cosh(bx+d)^3\cosh(-2a+2d) - 2(\cosh(-2a+2d) - \sinh(-2a+2d))\sinh(bx+d)^3 + 9(\cosh(b}{}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)^3,x, algorithm="fricas")
```

output
```
1/20*(3*cosh(b*x + d)^3*cosh(-2*a + 2*d) - 2*(cosh(-2*a + 2*d) - sinh(-2*a
 + 2*d))*sinh(b*x + d)^3 + 9*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x +
d)*sinh(-2*a + 2*d))*sinh(b*x + d)^2 + 5*cosh(b*x + d)*cosh(-2*a + 2*d) -
2*(3*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (3*cosh(b*x + d)^2 + 5)*sinh(-2*a
+ 2*d) + 5*cosh(-2*a + 2*d))*sinh(b*x + d) - (3*cosh(b*x + d)^3 + 5*cosh(b
*x + d))*sinh(-2*a + 2*d))/(b*cosh(b*x + d)^2 - 2*b*cosh(b*x + d)*sinh(b*x
 + d) + b*sinh(b*x + d)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(61) = 122$.

Time = 0.91 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.59

$$\int e^{2(a+bx)}\sinh^3(d+bx)\,dx$$
$$= \begin{cases} \frac{2e^{2a}e^{2bx}\sinh^3(bx+d)}{5b} + \frac{e^{2a}e^{2bx}\sinh^2(bx+d)\cosh(bx+d)}{5b} - \frac{4e^{2a}e^{2bx}\sinh(bx+d)\cosh^2(bx+d)}{5b} + \frac{2e^{2a}e^{2bx}\cosh^3(bx+d)}{5b} & \text{for } b \neq \\ xe^{2a}\sinh^3(d) & \text{otherw} \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)**3,x)
```

output
```
Piecewise((2*exp(2*a)*exp(2*b*x)*sinh(b*x + d)**3/(5*b) + exp(2*a)*exp(2*b
*x)*sinh(b*x + d)**2*cosh(b*x + d)/(5*b) - 4*exp(2*a)*exp(2*b*x)*sinh(b*x
+ d)*cosh(b*x + d)**2/(5*b) + 2*exp(2*a)*exp(2*b*x)*cosh(b*x + d)**3/(5*b)
, Ne(b, 0)), (x*exp(2*a)*sinh(d)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.76

$$\int e^{2(a+bx)}\sinh^3(d+bx)\,dx = -\frac{\left(5\,e^{(-2\,bx-2\,d)}-15\,e^{(-4\,bx-4\,d)}-1\right)e^{(5\,bx+2\,a+3\,d)}}{40\,b} + \frac{e^{(-bx+2\,a-3\,d)}}{8\,b}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)^3,x, algorithm="maxima")
```

output
```
-1/40*(5*e^(-2*b*x - 2*d) - 15*e^(-4*b*x - 4*d) - 1)*e^(5*b*x + 2*a + 3*d)
/b + 1/8*e^(-b*x + 2*a - 3*d)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int e^{2(a+bx)}\sinh^3(d+bx)\,dx = \frac{\left(e^{(5\,bx+2\,a+5\,d)}-5\,e^{(3\,bx+2\,a+3\,d)}+15\,e^{(bx+2\,a+d)}+5\,e^{(-bx+2\,a-d)}\right)e^{(-2\,d)}}{40\,b}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)^3,x, algorithm="giac")
```

output
```
1/40*(e^(5*b*x + 2*a + 5*d) - 5*e^(3*b*x + 2*a + 3*d) + 15*e^(b*x + 2*a +
d) + 5*e^(-b*x + 2*a - d))*e^(-2*d)/b
```

Mupad [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int e^{2(a+bx)}\sinh^3(d+bx)\,dx=\frac{3\,\mathrm{e}^{2\,a-d+b\,x}}{8\,b}-\frac{\mathrm{e}^{2\,a+d+3\,b\,x}}{8\,b}+\frac{\mathrm{e}^{2\,a-3\,d-b\,x}}{8\,b}+\frac{\mathrm{e}^{2\,a+3\,d+5\,b\,x}}{40\,b}$$

input
```
int(exp(2*a + 2*b*x)*sinh(d + b*x)^3,x)
```

output
```
(3*exp(2*a - d + b*x))/(8*b) - exp(2*a + d + 3*b*x)/(8*b) + exp(2*a - 3*d
- b*x)/(8*b) + exp(2*a + 3*d + 5*b*x)/(40*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.73

$$\int e^{2(a+bx)}\sinh^3(d+bx)\,dx=\frac{e^{2a}\left(e^{6bx+6d}-5e^{4bx+4d}+15e^{2bx+2d}+5\right)}{40e^{bx+3d}b}$$

input
```
int(exp(2*b*x+2*a)*sinh(b*x+d)^3,x)
```

output
```
(e**(2*a)*(e**(6*b*x + 6*d) - 5*e**(4*b*x + 4*d) + 15*e**(2*b*x + 2*d) + 5
))/(40*e**(b*x + 3*d)*b)
```

3.8 $\int e^{2(a+bx)} \sinh^4(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 93

$$\int e^{2(a+bx)} \sinh^4(d+bx)\,dx = -\frac{e^{2(a-2d)-2bx}}{32b} + \frac{3e^{2a+2bx}}{16b} - \frac{e^{2(a+d)+4bx}}{16b} + \frac{e^{2(a+2d)+6bx}}{96b} - \frac{1}{4}e^{2a-2d}x$$

output
```
-1/32*exp(-2*b*x+2*a-4*d)/b+3/16*exp(2*b*x+2*a)/b-1/16*exp(4*b*x+2*a+2*d)/
b+1/96*exp(6*b*x+2*a+4*d)/b-1/4*exp(2*a-2*d)*x
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.13

$$\int e^{2(a+bx)} \sinh^4(d+bx)\,dx = \frac{e^{2a-2bx}\left(18e^{4bx} - 6e^{2bx}\left(e^{4bx}+4bx\right)\cosh(2d) + \left(-3+e^{8bx}\right)\cosh(4d) - 6e^{6bx}\sinh(2d) + 24be^{2bx}x\sinh(2d}{96b}$$

input
```
Integrate[E^(2*(a + b*x))*Sinh[d + b*x]^4,x]
```

output
```
(E^(2*a - 2*b*x)*(18*E^(4*b*x) - 6*E^(2*b*x)*(E^(4*b*x) + 4*b*x)*Cosh[2*d]
 + (-3 + E^(8*b*x))*Cosh[4*d] - 6*E^(6*b*x)*Sinh[2*d] + 24*b*E^(2*b*x)*x*S
inh[2*d] + 3*Sinh[4*d] + E^(8*b*x)*Sinh[4*d]))/(96*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.52, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\sinh^4(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{16}e^{2a-3bx}\left(1-e^{2bx}\right)^4de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{2a}\int e^{-3bx}\left(1-e^{2bx}\right)^4de^{bx}}{16b}$$

$$\Big\downarrow \text{243}$$

$$\frac{e^{2a}\int e^{-2bx}\left(1-e^{2bx}\right)^4de^{2bx}}{32b}$$

$$\Big\downarrow \text{49}$$

$$\frac{e^{2a}\int \left(6+e^{-2bx}-4e^{-bx}-3e^{2bx}\right)de^{2bx}}{32b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2a}\left(-e^{-bx}+4e^{2bx}+\frac{1}{3}e^{3bx}-4\log\left(e^{2bx}\right)\right)}{32b}$$

input
```
Int[E^(2*(a + b*x))*Sinh[d + b*x]^4,x]
```

output
```
(E^(2*a)*(-E^(-(b*x)) + 4*E^(2*b*x) + E^(3*b*x)/3 - 4*Log[E^(2*b*x)]))/(32
*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{{\mathrm e}^{-2 b x +2 a -4 d}}{32 b}+\frac{3 \,{\mathrm e}^{2 b x +2 a}}{16 b}-\frac{{\mathrm e}^{4 b x +2 a +2 d}}{16 b}+\frac{{\mathrm e}^{6 b x +2 a +4 d}}{96 b}-\frac{{\mathrm e}^{2 a -2 d} x}{4}$
default	$-\frac{x \cosh \left(2 a -2 d \right)}{4}+\frac{3 \sinh \left(2 b x +2 a \right)}{16 b}-\frac{\sinh \left(-2 b x +2 a -4 d \right)}{32 b}-\frac{\sinh \left(4 b x +2 a +2 d \right)}{16 b}+\frac{\sinh \left(6 b x +2 a +4 d \right)}{96 b}-\frac{x \sinh \left(2 a -2 d \right)}{4}+$
orering	$\frac{\left(12 b x +5\right) {\mathrm e}^{2 b x +2 a} \sinh \left(b x +d \right)^{4}}{12 b}-\frac{5 \left(2 b x -1\right) \left(2 \,{\mathrm e}^{2 b x +2 a} b \sinh \left(b x +d \right)^{4}+4 \,{\mathrm e}^{2 b x +2 a} \sinh \left(b x +d \right)^{3} b \cosh \left(b x +d \right)\right)}{24 b^{2}}-\frac{5 \left(2 b x +1\right) \left(8 \,{\mathrm e}^{2 b x}\right.}{}$

input
```
int(exp(2*b*x+2*a)*sinh(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/32*exp(-2*b*x+2*a-4*d)/b+3/16*exp(2*b*x+2*a)/b-1/16*exp(4*b*x+2*a+2*d)/
b+1/96*exp(6*b*x+2*a+4*d)/b-1/4*exp(2*a-2*d)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(77) = 154.

Time = 0.09 (sec) , antiderivative size = 352, normalized size of antiderivative = 3.78

$$\int e^{2(a+bx)} \sinh^4(d+bx)\,dx =$$

$$-\frac{\cosh(bx+d)^4\cosh(-2a+2d)+(\cosh(-2a+2d)-\sinh(-2a+2d))\sinh(bx+d)^4+3(4bx+1$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)^4,x, algorithm="fricas")
```

output
```
-1/48*(cosh(b*x + d)^4*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a +
2*d))*sinh(b*x + d)^4 + 3*(4*b*x + 1)*cosh(b*x + d)^2*cosh(-2*a + 2*d) - 8
*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*
x + d)^3 + 3*(2*cosh(b*x + d)^2*cosh(-2*a + 2*d) + (4*b*x + 1)*cosh(-2*a +
 2*d) - (4*b*x + 2*cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d))*sinh(b*x + d)^2
- 2*(4*cosh(b*x + d)^3*cosh(-2*a + 2*d) + 3*(4*b*x - 1)*cosh(b*x + d)*cosh
(-2*a + 2*d) - (4*cosh(b*x + d)^3 + 3*(4*b*x - 1)*cosh(b*x + d))*sinh(-2*a
 + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^4 + 3*(4*b*x + 1)*cosh(b*x + d)^2
- 9)*sinh(-2*a + 2*d) - 9*cosh(-2*a + 2*d))/(b*cosh(b*x + d)^2 - 2*b*cosh(
b*x + d)*sinh(b*x + d) + b*sinh(b*x + d)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(76) = 152$.

Time = 2.24 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.85

$$\int e^{2(a+bx)}\sinh^4(d+bx)\,dx$$
$$= \begin{cases} \frac{xe^{2a}e^{2bx}\sinh^4(bx+d)}{4} - \frac{xe^{2a}e^{2bx}\sinh^3(bx+d)\cosh(bx+d)}{2} + \frac{xe^{2a}e^{2bx}\sinh(bx+d)\cosh^3(bx+d)}{2} - \frac{xe^{2a}e^{2bx}\cosh^4(bx+d)}{4} + \frac{e^{2a}e^{2b}}{} \\ xe^{2a}\sinh^4(d) \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)**4,x)
```

output
```
Piecewise((x*exp(2*a)*exp(2*b*x)*sinh(b*x + d)**4/4 - x*exp(2*a)*exp(2*b*x
)*sinh(b*x + d)**3*cosh(b*x + d)/2 + x*exp(2*a)*exp(2*b*x)*sinh(b*x + d)*c
osh(b*x + d)**3/2 - x*exp(2*a)*exp(2*b*x)*cosh(b*x + d)**4/4 + exp(2*a)*ex
p(2*b*x)*sinh(b*x + d)**4/(48*b) + 17*exp(2*a)*exp(2*b*x)*sinh(b*x + d)**3
*cosh(b*x + d)/(24*b) - exp(2*a)*exp(2*b*x)*sinh(b*x + d)**2*cosh(b*x + d)
**2/(2*b) - 3*exp(2*a)*exp(2*b*x)*sinh(b*x + d)*cosh(b*x + d)**3/(8*b) + 5
*exp(2*a)*exp(2*b*x)*cosh(b*x + d)**4/(16*b), Ne(b, 0)), (x*exp(2*a)*sinh(
d)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

$$\int e^{2(a+bx)}\sinh^4(d+bx)\,dx = -\frac{\left(6\,e^{(-2\,bx-2\,d)} - 18\,e^{(-4\,bx-4\,d)} - 1\right)e^{(6\,bx+2\,a+4\,d)}}{96\,b} - \frac{(bx+d)e^{(2\,a-2\,d)}}{4\,b} - \frac{e^{(-2\,bx+2\,a-4\,d)}}{32\,b}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)^4,x, algorithm="maxima")
```

output
```
-1/96*(6*e^(-2*b*x - 2*d) - 18*e^(-4*b*x - 4*d) - 1)*e^(6*b*x + 2*a + 4*d)
/b - 1/4*(b*x + d)*e^(2*a - 2*d)/b - 1/32*e^(-2*b*x + 2*a - 4*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int e^{2(a+bx)}\sinh^4(d+bx)\,dx$$
$$= \frac{\left(3\left(4\,e^{(2\,bx+2\,a+2\,d)} - e^{(2\,a)}\right)e^{(-2\,bx-2\,d)} - 24\,(bx+d)e^{(2\,a)} + e^{(6\,bx+2\,a+6\,d)} - 6\,e^{(4\,bx+2\,a+4\,d)} + 18\,e^{(2\,bx+2\,a+2\,d)}}{96\,b}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)^4,x, algorithm="giac")
```

output
```
1/96*(3*(4*e^(2*b*x + 2*a + 2*d) - e^(2*a))*e^(-2*b*x - 2*d) - 24*(b*x + d
)*e^(2*a) + e^(6*b*x + 2*a + 6*d) - 6*e^(4*b*x + 2*a + 4*d) + 18*e^(2*b*x
+ 2*a + 2*d))*e^(-2*d)/b
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.60

$$\int e^{2(a+bx)}\sinh^4(d+bx)\,dx = \frac{3\,e^{2\,a+2\,b\,x}}{16\,b} - \frac{x\,e^{2\,a+2\,b\,x}\cosh(2\,d+2\,b\,x)}{4} + \frac{x\,e^{2\,a+2\,b\,x}\sinh(2\,d+2\,b\,x)}{4} + \frac{e^{2\,a+2\,b\,x}\cosh(2\,d+2\,b\,x)}{6\,b} - \frac{e^{2\,a+2\,b\,x}\cosh(4\,d+4\,b\,x)}{48\,b} - \frac{7\,e^{2\,a+2\,b\,x}\sinh(2\,d+2\,b\,x)}{24\,b} + \frac{e^{2\,a+2\,b\,x}\sinh(4\,d+4\,b\,x)}{24\,b}$$

input
```
int(exp(2*a + 2*b*x)*sinh(d + b*x)^4,x)
```

output
```
(3*exp(2*a + 2*b*x))/(16*b) - (x*exp(2*a + 2*b*x)*cosh(2*d + 2*b*x))/4 + (
x*exp(2*a + 2*b*x)*sinh(2*d + 2*b*x))/4 + (exp(2*a + 2*b*x)*cosh(2*d + 2*b
*x))/(6*b) - (exp(2*a + 2*b*x)*cosh(4*d + 4*b*x))/(48*b) - (7*exp(2*a + 2*
b*x)*sinh(2*d + 2*b*x))/(24*b) + (exp(2*a + 2*b*x)*sinh(4*d + 4*b*x))/(24*
b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.77

$$\int e^{2(a+bx)}\sinh^4(d+bx)\,dx = \frac{e^{2a}\left(e^{8bx+8d}-6e^{6bx+6d}+18e^{4bx+4d}-24e^{2bx+2d}bx-3\right)}{96e^{2bx+4d}b}$$

input
```
int(exp(2*b*x+2*a)*sinh(b*x+d)^4,x)
```

output
```
(e**(2*a)*(e**(8*b*x + 8*d) - 6*e**(6*b*x + 6*d) + 18*e**(4*b*x + 4*d) - 2
4*e**(2*b*x + 2*d)*b*x - 3))/(96*e**(2*b*x + 4*d)*b)
```

3.9 $\int e^{\frac{5}{3}(a+bx)} \sinh(d+bx)\, dx$

Optimal result

Integrand size = 18, antiderivative size = 57

$$\int e^{\frac{5}{3}(a+bx)} \sinh(d+bx)\, dx = -\frac{3e^{\frac{5(a-d)}{3}+\frac{2}{3}(d+bx)}}{4b} + \frac{3e^{\frac{5(a-d)}{3}+\frac{8}{3}(d+bx)}}{16b}$$

output
```
-3/4*exp(5/3*a-d+2/3*b*x)/b+3/16*exp(5/3*a+d+8/3*b*x)/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int e^{\frac{5}{3}(a+bx)} \sinh(d+bx)\, dx = \frac{3e^{\frac{5a}{3}+\frac{2bx}{3}}\left(\left(-4+e^{2bx}\right)\cosh(d)+\left(4+e^{2bx}\right)\sinh(d)\right)}{16b}$$

input
```
Integrate[E^((5*(a + b*x))/3)*Sinh[d + b*x],x]
```

output
```
(3*E^((5*a)/3 + (2*b*x)/3)*((-4 + E^(2*b*x))*Cosh[d] + (4 + E^(2*b*x))*Sin
h[d]))/(16*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.68, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)} \sinh(bx+d) \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{3\int -\frac{1}{2}e^{\frac{5a}{3}+\frac{bx}{3}}\left(1-e^{2bx}\right)de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3e^{5a/3}\int e^{\frac{bx}{3}}\left(1-e^{2bx}\right)de^{\frac{bx}{3}}}{2b}$$

$$\Big\downarrow \text{ 802}$$

$$-\frac{3e^{5a/3}\int \left(e^{\frac{bx}{3}}-e^{\frac{7bx}{3}}\right)de^{\frac{bx}{3}}}{2b}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{3e^{5a/3}\left(\frac{1}{2}e^{\frac{2bx}{3}}-\frac{1}{8}e^{\frac{8bx}{3}}\right)}{2b}$$

input
```
Int[E^((5*(a + b*x))/3)*Sinh[d + b*x],x]
```

output
```
(-3*E^((5*a)/3)*(E^((2*b*x)/3)/2 - E^((8*b*x)/3)/8))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.56

method	result	size
parallelrisch	$\frac{3\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \left(-3 \cosh \left(b x +d \right)+5 \sinh \left(b x +d \right)\right)}{16 b}$	32
risch	$-\frac{3\,{\mathrm e}^{\frac{5 a}{3}-d +\frac{2 b x}{3}}}{4 b}+\frac{3\,{\mathrm e}^{\frac{5 a}{3}+d +\frac{8 b x}{3}}}{16 b}$	34
orering	$\frac{15\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \sinh \left(b x +d \right)}{8 b}-\frac{9 \left(\frac{5 b\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \sinh \left(b x +d \right)}{3}+{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} b \cosh \left(b x +d \right)\right)}{16 b^{2}}$	63
default	$-\frac{3 \sinh \left(\frac{5 a}{3}-d +\frac{2 b x}{3}\right)}{4 b}+\frac{3 \sinh \left(\frac{5 a}{3}+d +\frac{8 b x}{3}\right)}{16 b}-\frac{3 \cosh \left(\frac{5 a}{3}-d +\frac{2 b x}{3}\right)}{4 b}+\frac{3 \cosh \left(\frac{5 a}{3}+d +\frac{8 b x}{3}\right)}{16 b}$	66

input
```
int(exp(5/3*b*x+5/3*a)*sinh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
3/16*exp(5/3*b*x+5/3*a)*(-3*cosh(b*x+d)+5*sinh(b*x+d))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(33) = 66$.

Time = 0.09 (sec) , antiderivative size = 415, normalized size of antiderivative = 7.28

$$\int e^{\frac{5}{3}(a+bx)}\sinh(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sinh(b*x+d),x, algorithm="fricas")
```

output
```
3/16*(cosh(1/3*b*x + 1/3*d)^4*cosh(-5/3*a + 5/3*d) + (cosh(-5/3*a + 5/3*d)
 - sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^4 + 4*(cosh(1/3*b*x + 1/3*d
)*cosh(-5/3*a + 5/3*d) - cosh(1/3*b*x + 1/3*d)*sinh(-5/3*a + 5/3*d))*sinh(
1/3*b*x + 1/3*d)^3 - 4*cosh(1/3*b*x + 1/3*d)^2*cosh(-5/3*a + 5/3*d) + 2*(3
*cosh(1/3*b*x + 1/3*d)^2*cosh(-5/3*a + 5/3*d) - (3*cosh(1/3*b*x + 1/3*d)^2
 - 2)*sinh(-5/3*a + 5/3*d) - 2*cosh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)
^2 + 4*(cosh(1/3*b*x + 1/3*d)^3*cosh(-5/3*a + 5/3*d) + 2*cosh(1/3*b*x + 1/
3*d)*cosh(-5/3*a + 5/3*d) - (cosh(1/3*b*x + 1/3*d)^3 + 2*cosh(1/3*b*x + 1/
3*d))*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d) - (cosh(1/3*b*x + 1/3*d)
^4 - 4*cosh(1/3*b*x + 1/3*d)^2)*sinh(-5/3*a + 5/3*d))/(b*cosh(1/3*b*x + 1/
3*d)^4 - 4*b*cosh(1/3*b*x + 1/3*d)^3*sinh(1/3*b*x + 1/3*d) + 6*b*cosh(1/3*
b*x + 1/3*d)^2*sinh(1/3*b*x + 1/3*d)^2 - 4*b*cosh(1/3*b*x + 1/3*d)*sinh(1/
3*b*x + 1/3*d)^3 + b*sinh(1/3*b*x + 1/3*d)^4)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14

$$\int e^{\frac{5}{3}(a+bx)}\sinh(d+bx)\,dx = \begin{cases} \frac{15e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\sinh{(bx+d)}}{16b} - \frac{9e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\cosh{(bx+d)}}{16b} & \text{for } b \neq 0 \\ xe^{\frac{5a}{3}}\sinh{(d)} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sinh(b*x+d),x)
```

output
```
Piecewise((15*exp(5*a/3)*exp(5*b*x/3)*sinh(b*x + d)/(16*b) - 9*exp(5*a/3)*
exp(5*b*x/3)*cosh(b*x + d)/(16*b), Ne(b, 0)), (x*exp(5*a/3)*sinh(d), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.58

$$\int e^{\frac{5}{3}(a+bx)}\sinh(d+bx)\,dx=\frac{3\,e^{\left(\frac{8}{3}\,bx+\frac{5}{3}\,a+d\right)}}{16\,b}-\frac{3\,e^{\left(\frac{2}{3}\,bx+\frac{5}{3}\,a-d\right)}}{4\,b}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sinh(b*x+d),x, algorithm="maxima")
```

output
```
3/16*e^(8/3*b*x + 5/3*a + d)/b - 3/4*e^(2/3*b*x + 5/3*a - d)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.58

$$\int e^{\frac{5}{3}(a+bx)}\sinh(d+bx)\,dx=\frac{3\left(e^{\left(\frac{8}{3}\,bx+\frac{5}{3}\,a+2\,d\right)}-4\,e^{\left(\frac{2}{3}\,bx+\frac{5}{3}\,a\right)}\right)e^{(-d)}}{16\,b}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sinh(b*x+d),x, algorithm="giac")
```

output
```
3/16*(e^(8/3*b*x + 5/3*a + 2*d) - 4*e^(2/3*b*x + 5/3*a))*e^(-d)/b
```

Mupad [B] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.53

$$\int e^{\frac{5}{3}(a+bx)}\sinh(d+bx)\,dx=\frac{3\,\mathrm{e}^{\frac{5\,a}{3}}\,\mathrm{e}^{-d}\,\mathrm{e}^{\frac{2\,b\,x}{3}}\,\left(\mathrm{e}^{2\,d}\,\mathrm{e}^{2\,b\,x}-4\right)}{16\,b}$$

input
```
int(exp((5*a)/3 + (5*b*x)/3)*sinh(d + b*x),x)
```

output
```
(3*exp((5*a)/3)*exp(-d)*exp((2*b*x)/3)*(exp(2*d)*exp(2*b*x) - 4))/(16*b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.56

$$\int e^{\frac{5}{3}(a+bx)}\sinh(d+bx)\,dx=\frac{3e^{\frac{5bx}{3}+\frac{5a}{3}}(-3\cosh(bx+d)+5\sinh(bx+d))}{16b}$$

input
```
int(exp(5/3*b*x+5/3*a)*sinh(b*x+d),x)
```

output
```
(3*e**((5*a + 5*b*x)/3)*( - 3*cosh(b*x + d) + 5*sinh(b*x + d)))/(16*b)
```

3.10 $\int e^{\frac{5}{3}(a+bx)} \sinh^2(d+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int e^{\frac{5}{3}(a+bx)} \sinh^2(d+bx)\,dx = -\frac{3e^{\frac{5(a-d)}{3}+\frac{1}{3}(-d-bx)}}{4b} - \frac{3e^{\frac{5(a-d)}{3}+\frac{5}{3}(d+bx)}}{10b} + \frac{3e^{\frac{5(a-d)}{3}+\frac{11}{3}(d+bx)}}{44b}$$

output
```
-3/4*exp(5/3*a-2*d-1/3*b*x)/b-3/10*exp(5/3*b*x+5/3*a)/b+3/44*exp(5/3*a+2*d
+11/3*b*x)/b
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int e^{\frac{5}{3}(a+bx)} \sinh^2(d+bx)\,dx$$
$$= \frac{3e^{\frac{5a}{3}-\frac{bx}{3}}\left(-22e^{2bx}+5\left(-11+e^{4bx}\right)\cosh(2d)+5\left(11+e^{4bx}\right)\sinh(2d)\right)}{220b}$$

input
```
Integrate[E^((5*(a + b*x))/3)*Sinh[d + b*x]^2,x]
```

output
```
(3*E^((5*a)/3 - (b*x)/3)*(-22*E^(2*b*x) + 5*(-11 + E^(4*b*x))*Cosh[2*d] +
5*(11 + E^(4*b*x))*Sinh[2*d]))/(220*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.56, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)}\sinh^2(bx+d)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{3\int \frac{1}{4}e^{\frac{5a}{3}-\frac{2bx}{3}}\left(1-e^{2bx}\right)^2de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3e^{5a/3}\int e^{-\frac{2bx}{3}}\left(1-e^{2bx}\right)^2de^{\frac{bx}{3}}}{4b}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{3e^{5a/3}\int \left(e^{-\frac{2bx}{3}}-2e^{\frac{4bx}{3}}+e^{\frac{10bx}{3}}\right)de^{\frac{bx}{3}}}{4b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3e^{5a/3}\left(-e^{-\frac{bx}{3}}-\frac{2}{5}e^{\frac{5bx}{3}}+\frac{1}{11}e^{\frac{11bx}{3}}\right)}{4b}$$

input
```
Int[E^((5*(a + b*x))/3)*Sinh[d + b*x]^2,x]
```

output
```
(3*E^((5*a)/3)*(-E^(-1/3*(b*x)) - (2*E^((5*b*x)/3))/5 + E^((11*b*x)/3)/11)
)/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.44

method	result
parallelrisch	$-\frac{3\,{\mathrm e}^{\frac{5bx}{3}+\frac{5a}{3}} \left(11+25\cosh \left(2bx+2d\right)-30\sinh \left(2bx+2d\right)\right)}{110b}$
risch	$-\frac{3\,{\mathrm e}^{\frac{5a}{3}-2d-\frac{bx}{3}}}{4b}-\frac{3\,{\mathrm e}^{\frac{5bx}{3}+\frac{5a}{3}}}{10b}+\frac{3\,{\mathrm e}^{\frac{5a}{3}+2d+\frac{11bx}{3}}}{44b}$
default	$-\frac{3\sinh\left(\frac{5bx}{3}+\frac{5a}{3}\right)}{10b}-\frac{3\sinh\left(\frac{5a}{3}-2d-\frac{bx}{3}\right)}{4b}+\frac{3\sinh\left(\frac{5a}{3}+2d+\frac{11bx}{3}\right)}{44b}-\frac{3\cosh\left(\frac{5bx}{3}+\frac{5a}{3}\right)}{10b}-\frac{3\cosh\left(\frac{5a}{3}-2d-\frac{bx}{3}\right)}{4b}+$
orering	$-\frac{117\,{\mathrm e}^{\frac{5bx}{3}+\frac{5a}{3}}\sinh\left(bx+d\right)^{2}}{55b}+\frac{\frac{45b\,{\mathrm e}^{\frac{5bx}{3}+\frac{5a}{3}}\sinh\left(bx+d\right)^{2}}{11}+\frac{54\,{\mathrm e}^{\frac{5bx}{3}+\frac{5a}{3}}\sinh\left(bx+d\right)b\cosh\left(bx+d\right)}{11}}{b^{2}}-\frac{27\left(\frac{43b^{2}{\mathrm e}^{\frac{5bx}{3}+\frac{5a}{3}}\sinh(bx}{9}\right.}{}$

input
```
int(exp(5/3*b*x+5/3*a)*sinh(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/110*exp(5/3*b*x+5/3*a)*(11+25*cosh(2*b*x+2*d)-30*sinh(2*b*x+2*d))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(49) = 98$.

Time = 0.09 (sec) , antiderivative size = 486, normalized size of antiderivative = 5.52

$$\int e^{\frac{5}{3}(a+bx)}\sinh^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sinh(b*x+d)^2,x, algorithm="fricas")
```

output
```
-3/110*(25*cosh(1/3*b*x + 1/3*d)^6*cosh(-5/3*a + 5/3*d) + 25*(cosh(-5/3*a
+ 5/3*d) - sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^6 - 180*(cosh(1/3*b
*x + 1/3*d)*cosh(-5/3*a + 5/3*d) - cosh(1/3*b*x + 1/3*d)*sinh(-5/3*a + 5/3
*d))*sinh(1/3*b*x + 1/3*d)^5 + 375*(cosh(1/3*b*x + 1/3*d)^2*cosh(-5/3*a +
5/3*d) - cosh(1/3*b*x + 1/3*d)^2*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*
d)^4 - 600*(cosh(1/3*b*x + 1/3*d)^3*cosh(-5/3*a + 5/3*d) - cosh(1/3*b*x +
1/3*d)^3*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^3 + 375*(cosh(1/3*b*x
 + 1/3*d)^4*cosh(-5/3*a + 5/3*d) - cosh(1/3*b*x + 1/3*d)^4*sinh(-5/3*a + 5
/3*d))*sinh(1/3*b*x + 1/3*d)^2 - 180*(cosh(1/3*b*x + 1/3*d)^5*cosh(-5/3*a
+ 5/3*d) - cosh(1/3*b*x + 1/3*d)^5*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/
3*d) - (25*cosh(1/3*b*x + 1/3*d)^6 + 11)*sinh(-5/3*a + 5/3*d) + 11*cosh(-5
/3*a + 5/3*d))/(b*cosh(1/3*b*x + 1/3*d)^5 - 5*b*cosh(1/3*b*x + 1/3*d)^4*si
nh(1/3*b*x + 1/3*d) + 10*b*cosh(1/3*b*x + 1/3*d)^3*sinh(1/3*b*x + 1/3*d)^2
 - 10*b*cosh(1/3*b*x + 1/3*d)^2*sinh(1/3*b*x + 1/3*d)^3 + 5*b*cosh(1/3*b*x
 + 1/3*d)*sinh(1/3*b*x + 1/3*d)^4 - b*sinh(1/3*b*x + 1/3*d)^5)
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.18

$$\int e^{\frac{5}{3}(a+bx)}\sinh^2(d+bx)\,dx = \begin{cases} -\frac{21e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\sinh^2(bx+d)}{55b} + \frac{18e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\sinh(bx+d)\cosh(bx+d)}{11b} - \frac{54e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\cosh^2(bx+d)}{55b} & \text{for } b \neq 0 \\ xe^{\frac{5a}{3}}\sinh^2(d) & \text{otherwise} \end{cases}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sinh(b*x+d)**2,x)
```

output
```
Piecewise((-21*exp(5*a/3)*exp(5*b*x/3)*sinh(b*x + d)**2/(55*b) + 18*exp(5*
a/3)*exp(5*b*x/3)*sinh(b*x + d)*cosh(b*x + d)/(11*b) - 54*exp(5*a/3)*exp(5
*b*x/3)*cosh(b*x + d)**2/(55*b), Ne(b, 0)), (x*exp(5*a/3)*sinh(d)**2, True
))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.55

$$\int e^{\frac{5}{3}(a+bx)} \sinh^2(d+bx)\,dx = -\frac{3\left(22\,e^{(-2\,bx-2\,d)}-5\right)e^{\left(\frac{11}{3}\,bx+\frac{5}{3}\,a+2\,d\right)}}{220\,b} - \frac{3\,e^{\left(-\frac{1}{3}\,bx+\frac{5}{3}\,a-2\,d\right)}}{4\,b}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sinh(b*x+d)^2,x, algorithm="maxima")
```

output
```
-3/220*(22*e^(-2*b*x - 2*d) - 5)*e^(11/3*b*x + 5/3*a + 2*d)/b - 3/4*e^(-1/
3*b*x + 5/3*a - 2*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.56

$$\int e^{\frac{5}{3}(a+bx)} \sinh^2(d+bx)\,dx = \frac{3\left(5\,e^{\left(\frac{11}{3}\,bx+\frac{5}{3}\,a+4\,d\right)}-22\,e^{\left(\frac{5}{3}\,bx+\frac{5}{3}\,a+2\,d\right)}-55\,e^{\left(-\frac{1}{3}\,bx+\frac{5}{3}\,a\right)}\right)e^{(-2\,d)}}{220\,b}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sinh(b*x+d)^2,x, algorithm="giac")
```

output
```
3/220*(5*e^(11/3*b*x + 5/3*a + 4*d) - 22*e^(5/3*b*x + 5/3*a + 2*d) - 55*e^
(-1/3*b*x + 5/3*a))*e^(-2*d)/b
```

Mupad [B] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.47

$$\int e^{\frac{5}{3}(a+bx)}\sinh^2(d+bx)\,dx=-\frac{3\,\mathrm{e}^{\frac{5\,a}{3}-2\,d-\frac{b\,x}{3}}\,\left(22\,\mathrm{e}^{2\,d+2\,b\,x}-5\,\mathrm{e}^{4\,d+4\,b\,x}+55\right)}{220\,b}$$

input
```
int(exp((5*a)/3 + (5*b*x)/3)*sinh(d + b*x)^2,x)
```

output
```
-(3*exp((5*a)/3 - 2*d - (b*x)/3)*(22*exp(2*d + 2*b*x) - 5*exp(4*d + 4*b*x)
 + 55))/(220*b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int e^{\frac{5}{3}(a+bx)}\sinh^2(d+bx)\,dx=\frac{3e^{\frac{5bx}{3}+\frac{5a}{3}}\left(5e^{4bx+4d}-22e^{2bx+2d}-55\right)}{220e^{2bx+2d}b}$$

input
```
int(exp(5/3*b*x+5/3*a)*sinh(b*x+d)^2,x)
```

output
```
(3*e**((5*a + 5*b*x)/3)*(5*e**(4*b*x + 4*d) - 22*e**(2*b*x + 2*d) - 55))/(
220*e**(2*b*x + 2*d)*b)
```

3.11 $\int e^{\frac{5}{3}(a+bx)} \sinh^3(d+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int e^{\frac{5}{3}(a+bx)} \sinh^3(d+bx)\,dx = \frac{3e^{\frac{5(a-d)}{3}-\frac{4}{3}(d+bx)}}{32b} + \frac{9e^{\frac{5(a-d)}{3}+\frac{2}{3}(d+bx)}}{16b} - \frac{9e^{\frac{5(a-d)}{3}+\frac{8}{3}(d+bx)}}{64b} + \frac{3e^{\frac{5(a-d)}{3}+\frac{14}{3}(d+bx)}}{112b}$$

output
```
3/32*exp(5/3*a-3*d-4/3*b*x)/b+9/16*exp(5/3*a-d+2/3*b*x)/b-9/64*exp(5/3*a+d
+8/3*b*x)/b+3/112*exp(5/3*a+3*d+14/3*b*x)/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83

$$\int e^{\frac{5}{3}(a+bx)} \sinh^3(d+bx)\,dx$$
$$= \frac{3e^{\frac{5a}{3}-\frac{4bx}{3}}\left(-21e^{2bx}\left(-4+e^{2bx}\right)\cosh(d)+2\left(7+2e^{6bx}\right)\cosh(3d)-84e^{2bx}\sinh(d)-21e^{4bx}\sinh(d)-14\sin}{448b}$$

input
```
Integrate[E^((5*(a + b*x))/3)*Sinh[d + b*x]^3,x]
```

output
```
(3*E^((5*a)/3 - (4*b*x)/3)*(-21*E^(2*b*x)*(-4 + E^(2*b*x))*Cosh[d] + 2*(7
+ 2*E^(6*b*x))*Cosh[3*d] - 84*E^(2*b*x)*Sinh[d] - 21*E^(4*b*x)*Sinh[d] - 1
4*Sinh[3*d] + 4*E^(6*b*x)*Sinh[3*d]))/(448*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.56, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)}\sinh^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{3\int -\frac{1}{8}e^{\frac{5a}{3}-\frac{5bx}{3}}\left(1-e^{2bx}\right)^3de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3e^{5a/3}\int e^{-\frac{5bx}{3}}\left(1-e^{2bx}\right)^3de^{\frac{bx}{3}}}{8b}$$

$$\Big\downarrow \text{802}$$

$$-\frac{3e^{5a/3}\int \left(e^{-\frac{5bx}{3}}-3e^{\frac{bx}{3}}+3e^{\frac{7bx}{3}}-e^{\frac{13bx}{3}}\right)de^{\frac{bx}{3}}}{8b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3e^{5a/3}\left(-\frac{1}{4}e^{-\frac{4bx}{3}}-\frac{3}{2}e^{\frac{2bx}{3}}+\frac{3}{8}e^{\frac{8bx}{3}}-\frac{1}{14}e^{\frac{14bx}{3}}\right)}{8b}$$

input
```
Int[E^((5*(a + b*x))/3)*Sinh[d + b*x]^3,x]
```

output
```
(-3*E^((5*a)/3)*(-1/4*1/E^((4*b*x)/3) - (3*E^((2*b*x)/3))/2 + (3*E^((8*b*x
)/3))/8 - E^((14*b*x)/3)/14))/(8*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.48

method	result
parallelrisch	$-\frac{3\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \left(-18 \cosh \left(3 b x +3 d \right)-63 \cosh \left(b x +d \right)+10 \sinh \left(3 b x +3 d \right)+105 \sinh \left(b x +d \right)\right)}{448 b}$
risch	$\frac{3\,{\mathrm e}^{\frac{5 a}{3}-3 d -\frac{4 b x}{3}}}{32 b}+\frac{9\,{\mathrm e}^{\frac{5 a}{3}-d +\frac{2 b x}{3}}}{16 b}-\frac{9\,{\mathrm e}^{\frac{5 a}{3}+d +\frac{8 b x}{3}}}{64 b}+\frac{3\,{\mathrm e}^{\frac{5 a}{3}+3 d +\frac{14 b x}{3}}}{112 b}$
default	$\frac{3 \sinh \left(\frac{5 a}{3}-3 d -\frac{4 b x}{3}\right)}{32 b}+\frac{9 \sinh \left(\frac{5 a}{3}-d +\frac{2 b x}{3}\right)}{16 b}-\frac{9 \sinh \left(\frac{5 a}{3}+d +\frac{8 b x}{3}\right)}{64 b}+\frac{3 \sinh \left(\frac{5 a}{3}+3 d +\frac{14 b x}{3}\right)}{112 b}+\frac{3 \cosh \left(\frac{5 a}{3}-3 d -\frac{4 b x}{3}\right)}{32 b}$
orering	$\frac{75\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \sinh \left(b x +d \right)^{3}}{56 b}+\frac{\frac{225 b\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \sinh \left(b x +d \right)^{3}}{224}+\frac{405\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \sinh \left(b x +d \right)^{2} b \cosh \left(b x +d \right)}{224}}{b^{2}}-\frac{135\left(\frac{52 b^{2} {\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \sinh(bx}{9}\right.}{}$

input
```
int(exp(5/3*b*x+5/3*a)*sinh(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/448*exp(5/3*b*x+5/3*a)/b*(-18*cosh(3*b*x+3*d)-63*cosh(b*x+d)+10*sinh(3*
b*x+3*d)+105*sinh(b*x+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 759 vs. $2(67) = 134$.

Time = 0.10 (sec) , antiderivative size = 759, normalized size of antiderivative = 6.72

$$\int e^{\frac{5}{3}(a+bx)}\sinh^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sinh(b*x+d)^3,x, algorithm="fricas")
```

output
```
3/448*(18*cosh(1/3*b*x + 1/3*d)^9*cosh(-5/3*a + 5/3*d) - 10*(cosh(-5/3*a +
 5/3*d) - sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^9 + 162*(cosh(1/3*b*
x + 1/3*d)*cosh(-5/3*a + 5/3*d) - cosh(1/3*b*x + 1/3*d)*sinh(-5/3*a + 5/3*
d))*sinh(1/3*b*x + 1/3*d)^8 - 360*(cosh(1/3*b*x + 1/3*d)^2*cosh(-5/3*a + 5
/3*d) - cosh(1/3*b*x + 1/3*d)^2*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d
)^7 + 1512*(cosh(1/3*b*x + 1/3*d)^3*cosh(-5/3*a + 5/3*d) - cosh(1/3*b*x +
1/3*d)^3*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^6 - 1260*(cosh(1/3*b*
x + 1/3*d)^4*cosh(-5/3*a + 5/3*d) - cosh(1/3*b*x + 1/3*d)^4*sinh(-5/3*a +
5/3*d))*sinh(1/3*b*x + 1/3*d)^5 + 2268*(cosh(1/3*b*x + 1/3*d)^5*cosh(-5/3*
a + 5/3*d) - cosh(1/3*b*x + 1/3*d)^5*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x +
1/3*d)^4 + 63*cosh(1/3*b*x + 1/3*d)^3*cosh(-5/3*a + 5/3*d) - 105*(8*cosh(1
/3*b*x + 1/3*d)^6*cosh(-5/3*a + 5/3*d) - (8*cosh(1/3*b*x + 1/3*d)^6 + 1)*s
inh(-5/3*a + 5/3*d) + cosh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^3 + 27*(
24*cosh(1/3*b*x + 1/3*d)^7*cosh(-5/3*a + 5/3*d) + 7*cosh(1/3*b*x + 1/3*d)*
cosh(-5/3*a + 5/3*d) - (24*cosh(1/3*b*x + 1/3*d)^7 + 7*cosh(1/3*b*x + 1/3*
d))*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^2 - 45*(2*cosh(1/3*b*x + 1
/3*d)^8*cosh(-5/3*a + 5/3*d) + 7*cosh(1/3*b*x + 1/3*d)^2*cosh(-5/3*a + 5/3
*d) - (2*cosh(1/3*b*x + 1/3*d)^8 + 7*cosh(1/3*b*x + 1/3*d)^2)*sinh(-5/3*a
+ 5/3*d))*sinh(1/3*b*x + 1/3*d) - 9*(2*cosh(1/3*b*x + 1/3*d)^9 + 7*cosh(1/
3*b*x + 1/3*d)^3)*sinh(-5/3*a + 5/3*d))/(b*cosh(1/3*b*x + 1/3*d)^5 - 5*...
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.25

$$\int e^{\frac{5}{3}(a+bx)}\sinh^3(d+bx)\,dx$$
$$=\begin{cases}\frac{285e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\sinh^3(bx+d)}{448b}-\frac{27e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\sinh^2(bx+d)\cosh(bx+d)}{448b}-\frac{405e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\sinh(bx+d)\cosh^2(bx+d)}{448b}+\frac{243e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\cosh^3(bx+d)}{448b} \\ xe^{\frac{5a}{3}}\sinh^3(d)\end{cases}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sinh(b*x+d)**3,x)
```

output
```
Piecewise((285*exp(5*a/3)*exp(5*b*x/3)*sinh(b*x + d)**3/(448*b) - 27*exp(5
*a/3)*exp(5*b*x/3)*sinh(b*x + d)**2*cosh(b*x + d)/(448*b) - 405*exp(5*a/3)
*exp(5*b*x/3)*sinh(b*x + d)*cosh(b*x + d)**2/(448*b) + 243*exp(5*a/3)*exp(
5*b*x/3)*cosh(b*x + d)**3/(448*b), Ne(b, 0)), (x*exp(5*a/3)*sinh(d)**3, Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.52

$$\int e^{\frac{5}{3}(a+bx)}\sinh^3(d+bx)\,dx=-\frac{3\left(21\,e^{(-2\,bx-2\,d)}-84\,e^{(-4\,bx-4\,d)}-4\right)e^{\left(\frac{14}{3}\,bx+\frac{5}{3}\,a+3\,d\right)}}{448\,b}+\frac{3\,e^{\left(-\frac{4}{3}\,bx+\frac{5}{3}\,a-3\,d\right)}}{32\,b}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sinh(b*x+d)^3,x, algorithm="maxima")
```

output
```
-3/448*(21*e^(-2*b*x - 2*d) - 84*e^(-4*b*x - 4*d) - 4)*e^(14/3*b*x + 5/3*a
 + 3*d)/b + 3/32*e^(-4/3*b*x + 5/3*a - 3*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.56

$$\int e^{\frac{5}{3}(a+bx)}\sinh^3(d+bx)\,dx = \frac{3\left(4\,e^{\left(\frac{14}{3}\,bx+\frac{5}{3}\,a+6\,d\right)}-21\,e^{\left(\frac{8}{3}\,bx+\frac{5}{3}\,a+4\,d\right)}+84\,e^{\left(\frac{2}{3}\,bx+\frac{5}{3}\,a+2\,d\right)}+14\,e^{\left(-\frac{4}{3}\,bx+\frac{5}{3}\,a\right)}\right)e^{(-3\,d)}}{448\,b}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sinh(b*x+d)^3,x, algorithm="giac")
```

output
```
3/448*(4*e^(14/3*b*x + 5/3*a + 6*d) - 21*e^(8/3*b*x + 5/3*a + 4*d) + 84*e^
(2/3*b*x + 5/3*a + 2*d) + 14*e^(-4/3*b*x + 5/3*a))*e^(-3*d)/b
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81

$$\int e^{\frac{5}{3}(a+bx)}\sinh^3(d+bx)\,dx = \frac{3\,e^{\frac{5\,a}{3}}\,e^{-3\,d}\,e^{-3\,b\,x}\,e^{\frac{5\,b\,x}{3}}}{32\,b}-\frac{9\,e^{\frac{5\,a}{3}}\,e^{-3\,b\,x}\,e^{\frac{17\,b\,x}{3}}\,e^{d}}{64\,b}+\frac{9\,e^{\frac{5\,a}{3}}\,e^{-d}\,e^{-3\,b\,x}\,e^{\frac{11\,b\,x}{3}}}{16\,b}+\frac{3\,e^{\frac{5\,a}{3}}\,e^{3\,d}\,e^{-3\,b\,x}\,e^{\frac{23\,b\,x}{3}}}{112\,b}$$

input
```
int(exp((5*a)/3 + (5*b*x)/3)*sinh(d + b*x)^3,x)
```

output
```
(3*exp((5*a)/3)*exp(-3*d)*exp(-3*b*x)*exp((5*b*x)/3))/(32*b) - (9*exp((5*a
)/3)*exp(-3*b*x)*exp((17*b*x)/3)*exp(d))/(64*b) + (9*exp((5*a)/3)*exp(-d)*
exp(-3*b*x)*exp((11*b*x)/3))/(16*b) + (3*exp((5*a)/3)*exp(3*d)*exp(-3*b*x)
*exp((23*b*x)/3))/(112*b)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.58

$$\int e^{\frac{5}{3}(a+bx)}\sinh^3(d+bx)\,dx=\frac{3e^{\frac{5bx}{3}+\frac{5a}{3}}\left(4e^{6bx+6d}-21e^{4bx+4d}+84e^{2bx+2d}+14\right)}{448e^{3bx+3d}b}$$

input
```
int(exp(5/3*b*x+5/3*a)*sinh(b*x+d)^3,x)
```

output
```
(3*e**((5*a + 5*b*x)/3)*(4*e**(6*b*x + 6*d) - 21*e**(4*b*x + 4*d) + 84*e**
(2*b*x + 2*d) + 14))/(448*e**(3*b*x + 3*d)*b)
```

3.12 $\int e^{\frac{5}{3}(a+bx)} \sinh^4(d+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 144

$$\int e^{\frac{5}{3}(a+bx)} \sinh^4(d+bx)\,dx = \frac{3e^{\frac{5(a-d)}{3}+\frac{1}{3}(-d-bx)}}{4b} - \frac{3e^{\frac{5(a-d)}{3}-\frac{7}{3}(d+bx)}}{112b} + \frac{9e^{\frac{5(a-d)}{3}+\frac{5}{3}(d+bx)}}{40b} - \frac{3e^{\frac{5(a-d)}{3}+\frac{11}{3}(d+bx)}}{44b} + \frac{3e^{\frac{5(a-d)}{3}+\frac{17}{3}(d+bx)}}{272b}$$

output
```
3/4*exp(5/3*a-2*d-1/3*b*x)/b-3/112*exp(5/3*a-4*d-7/3*b*x)/b+9/40*exp(5/3*b
*x+5/3*a)/b-3/44*exp(5/3*a+2*d+11/3*b*x)/b+3/272*exp(5/3*a+4*d+17/3*b*x)/b
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.75

$$\int e^{\frac{5}{3}(a+bx)} \sinh^4(d+bx)\,dx = \frac{3e^{\frac{5a}{3}-\frac{7bx}{3}}\left(7854e^{4bx} - 2380e^{2bx}\left(-11+e^{4bx}\right)\cosh(2d) + 55\left(-17+7e^{8bx}\right)\cosh(4d) - 26180e^{2bx}\sinh(2d) - \right.}{104720b}$$

input
```
Integrate[E^((5*(a + b*x))/3)*Sinh[d + b*x]^4,x]
```

output
```
(3*E^((5*a)/3 - (7*b*x)/3)*(7854*E^(4*b*x) - 2380*E^(2*b*x)*(-11 + E^(4*b*
x))*Cosh[2*d] + 55*(-17 + 7*E^(8*b*x))*Cosh[4*d] - 26180*E^(2*b*x)*Sinh[2*
d] - 2380*E^(6*b*x)*Sinh[2*d] + 935*Sinh[4*d] + 385*E^(8*b*x)*Sinh[4*d]))/
(104720*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.51, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)}\sinh^4(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{3\int \frac{1}{16}e^{\frac{5a}{3}-\frac{8bx}{3}}\left(1-e^{2bx}\right)^4de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{3e^{5a/3}\int e^{-\frac{8bx}{3}}\left(1-e^{2bx}\right)^4de^{\frac{bx}{3}}}{16b}$$

$$\Big\downarrow \text{802}$$

$$\frac{3e^{5a/3}\int \left(e^{-\frac{8bx}{3}}-4e^{-\frac{2bx}{3}}+6e^{\frac{4bx}{3}}-4e^{\frac{10bx}{3}}+e^{\frac{16bx}{3}}\right)de^{\frac{bx}{3}}}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3e^{5a/3}\left(-\frac{1}{7}e^{-\frac{7bx}{3}}+4e^{-\frac{bx}{3}}+\frac{6}{5}e^{\frac{5bx}{3}}-\frac{4}{11}e^{\frac{11bx}{3}}+\frac{1}{17}e^{\frac{17bx}{3}}\right)}{16b}$$

input
```
Int[E^((5*(a + b*x))/3)*Sinh[d + b*x]^4,x]
```

output
```
(3*E^((5*a)/3)*(-1/7*1/E^((7*b*x)/3) + 4/E^((b*x)/3) + (6*E^((5*b*x)/3))/5
 - (4*E^((11*b*x)/3))/11 + E^((17*b*x)/3)/17))/(16*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.42

method	result
parallelrisch	$-\frac{3\,\mathrm{e}^{\frac{5bx}{3}+\frac{5a}{3}}\left(-11900\cosh\left(2bx+2d\right)+275\cosh\left(4bx+4d\right)+14280\sinh\left(2bx+2d\right)-660\sinh\left(4bx+4d\right)-3927\right)}{52360b}$
risch	$\frac{3\,\mathrm{e}^{\frac{5a}{3}-2d-\frac{bx}{3}}}{4b}-\frac{3\,\mathrm{e}^{\frac{5a}{3}-4d-\frac{7bx}{3}}}{112b}+\frac{9\,\mathrm{e}^{\frac{5bx}{3}+\frac{5a}{3}}}{40b}-\frac{3\,\mathrm{e}^{\frac{5a}{3}+2d+\frac{11bx}{3}}}{44b}+\frac{3\,\mathrm{e}^{\frac{5a}{3}+4d+\frac{17bx}{3}}}{272b}$
default	$\frac{9\sinh\left(\frac{5bx}{3}+\frac{5a}{3}\right)}{40b}-\frac{3\sinh\left(\frac{5a}{3}-4d-\frac{7bx}{3}\right)}{112b}+\frac{3\sinh\left(\frac{5a}{3}-2d-\frac{bx}{3}\right)}{4b}-\frac{3\sinh\left(\frac{5a}{3}+2d+\frac{11bx}{3}\right)}{44b}+\frac{3\sinh\left(\frac{5a}{3}+4d+\frac{17bx}{3}\right)}{272b}$ −
orering	$-\frac{15573\,\mathrm{e}^{\frac{5bx}{3}+\frac{5a}{3}}\sinh\left(bx+d\right)^{4}}{6545b}+\frac{\frac{4350b\,\mathrm{e}^{\frac{5bx}{3}+\frac{5a}{3}}\sinh\left(bx+d\right)^{4}}{1309}+\frac{10440\,\mathrm{e}^{\frac{5bx}{3}+\frac{5a}{3}}\sinh\left(bx+d\right)^{3}b\cosh\left(bx+d\right)}{1309}}{b^{2}}+\frac{366b^{2}\mathrm{e}^{\frac{5bx}{3}+\frac{5a}{3}}\,\mathrm{si}}{187}$

input
```
int(exp(5/3*b*x+5/3*a)*sinh(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-3/52360*exp(5/3*b*x+5/3*a)*(-11900*cosh(2*b*x+2*d)+275*cosh(4*b*x+4*d)+14
280*sinh(2*b*x+2*d)-660*sinh(4*b*x+4*d)-3927)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1051 vs. $2(83) = 166$.

Time = 0.09 (sec) , antiderivative size = 1051, normalized size of antiderivative = 7.30

$$\int e^{\frac{5}{3}(a+bx)}\sinh^4(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sinh(b*x+d)^4,x, algorithm="fricas")
```

output
```
-3/52360*(275*cosh(1/3*b*x + 1/3*d)^12*cosh(-5/3*a + 5/3*d) + 275*(cosh(-5
/3*a + 5/3*d) - sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^12 - 7920*(cos
h(1/3*b*x + 1/3*d)*cosh(-5/3*a + 5/3*d) - cosh(1/3*b*x + 1/3*d)*sinh(-5/3*
a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^11 + 18150*(cosh(1/3*b*x + 1/3*d)^2*cosh
(-5/3*a + 5/3*d) - cosh(1/3*b*x + 1/3*d)^2*sinh(-5/3*a + 5/3*d))*sinh(1/3*
b*x + 1/3*d)^10 - 145200*(cosh(1/3*b*x + 1/3*d)^3*cosh(-5/3*a + 5/3*d) - c
osh(1/3*b*x + 1/3*d)^3*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^9 + 136
125*(cosh(1/3*b*x + 1/3*d)^4*cosh(-5/3*a + 5/3*d) - cosh(1/3*b*x + 1/3*d)^
4*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^8 - 522720*(cosh(1/3*b*x + 1
/3*d)^5*cosh(-5/3*a + 5/3*d) - cosh(1/3*b*x + 1/3*d)^5*sinh(-5/3*a + 5/3*d
))*sinh(1/3*b*x + 1/3*d)^7 - 11900*cosh(1/3*b*x + 1/3*d)^6*cosh(-5/3*a + 5
/3*d) + 700*(363*cosh(1/3*b*x + 1/3*d)^6*cosh(-5/3*a + 5/3*d) - (363*cosh(
1/3*b*x + 1/3*d)^6 - 17)*sinh(-5/3*a + 5/3*d) - 17*cosh(-5/3*a + 5/3*d))*s
inh(1/3*b*x + 1/3*d)^6 - 720*(726*cosh(1/3*b*x + 1/3*d)^7*cosh(-5/3*a + 5/
3*d) - 119*cosh(1/3*b*x + 1/3*d)*cosh(-5/3*a + 5/3*d) - (726*cosh(1/3*b*x
+ 1/3*d)^7 - 119*cosh(1/3*b*x + 1/3*d))*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x
 + 1/3*d)^5 + 375*(363*cosh(1/3*b*x + 1/3*d)^8*cosh(-5/3*a + 5/3*d) - 476*
cosh(1/3*b*x + 1/3*d)^2*cosh(-5/3*a + 5/3*d) - (363*cosh(1/3*b*x + 1/3*d)^
8 - 476*cosh(1/3*b*x + 1/3*d)^2)*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*
d)^4 - 1200*(121*cosh(1/3*b*x + 1/3*d)^9*cosh(-5/3*a + 5/3*d) - 238*cos...
```

Sympy [A] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.24

$$\int e^{\frac{5}{3}(a+bx)}\sinh^4(d+bx)\,dx$$
$$=\begin{cases} -\frac{3093e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\sinh^4(bx+d)}{6545b}+\frac{2340e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\sinh^3(bx+d)\cosh(bx+d)}{1309b}-\frac{324e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\sinh^2(bx+d)\cosh^2(bx+d)}{595b}-\frac{1944e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\sin \\ xe^{\frac{5a}{3}}\sinh^4(d) \end{cases}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sinh(b*x+d)**4,x)
```

output
```
Piecewise((-3093*exp(5*a/3)*exp(5*b*x/3)*sinh(b*x + d)**4/(6545*b) + 2340*
exp(5*a/3)*exp(5*b*x/3)*sinh(b*x + d)**3*cosh(b*x + d)/(1309*b) - 324*exp(
5*a/3)*exp(5*b*x/3)*sinh(b*x + d)**2*cosh(b*x + d)**2/(595*b) - 1944*exp(5
*a/3)*exp(5*b*x/3)*sinh(b*x + d)*cosh(b*x + d)**3/(1309*b) + 5832*exp(5*a/
3)*exp(5*b*x/3)*cosh(b*x + d)**4/(6545*b), Ne(b, 0)), (x*exp(5*a/3)*sinh(d
)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.54

$$\int e^{\frac{5}{3}(a+bx)}\sinh^4(d+bx)\,dx = -\frac{3\left(340\,e^{(-2\,bx-2\,d)}-1122\,e^{(-4\,bx-4\,d)}-55\right)e^{\left(\frac{17}{3}\,bx+\frac{5}{3}\,a+4\,d\right)}}{14960\,b} + \frac{3\left(28\,e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}-e^{\left(-\frac{7}{3}\,bx-\frac{7}{3}\,d\right)}\right)e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,d\right)}}{112\,b}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sinh(b*x+d)^4,x, algorithm="maxima")
```

output
```
-3/14960*(340*e^(-2*b*x - 2*d) - 1122*e^(-4*b*x - 4*d) - 55)*e^(17/3*b*x +
 5/3*a + 4*d)/b + 3/112*(28*e^(-1/3*b*x - 1/3*d) - e^(-7/3*b*x - 7/3*d))*e
^(5/3*a - 5/3*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.56

$$\int e^{\frac{5}{3}(a+bx)}\sinh^4(d+bx)\,dx = \frac{3\left(935\left(28\,e^{\left(2\,bx+\frac{5}{3}\,a+2\,d\right)}-e^{\left(\frac{5}{3}\,a\right)}\right)e^{\left(-\frac{7}{3}\,bx\right)}+385\,e^{\left(\frac{17}{3}\,bx+\frac{5}{3}\,a+8\,d\right)}-2380\,e^{\left(\frac{11}{3}\,bx+\frac{5}{3}\,a+6\,d\right)}+7854\,e^{\left(\frac{5}{3}\,bx+\frac{5}{3}\,a+4\,d\right.}\right.}{104720\,b}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sinh(b*x+d)^4,x, algorithm="giac")
```

output
```
3/104720*(935*(28*e^(2*b*x + 5/3*a + 2*d) - e^(5/3*a))*e^(-7/3*b*x) + 385*
e^(17/3*b*x + 5/3*a + 8*d) - 2380*e^(11/3*b*x + 5/3*a + 6*d) + 7854*e^(5/3
*b*x + 5/3*a + 4*d))*e^(-4*d)/b
```

Mupad [B] (verification not implemented)

Time = 3.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.58

$$\int e^{\frac{5}{3}(a+bx)}\sinh^4(d+bx)\,dx = \frac{3\,{\mathrm{e}}^{\frac{5\,a}{3}-2\,d-\frac{b\,x}{3}}}{4\,b}-\frac{3\,{\mathrm{e}}^{\frac{5\,a}{3}-4\,d-\frac{7\,b\,x}{3}}}{112\,b}-\frac{3\,{\mathrm{e}}^{\frac{5\,a}{3}+2\,d+\frac{11\,b\,x}{3}}}{44\,b}+\frac{3\,{\mathrm{e}}^{\frac{5\,a}{3}+4\,d+\frac{17\,b\,x}{3}}}{272\,b}+\frac{9\,{\mathrm{e}}^{\frac{5\,a}{3}+\frac{5\,b\,x}{3}}}{40\,b}$$

input
```
int(exp((5*a)/3 + (5*b*x)/3)*sinh(d + b*x)^4,x)
```

output
```
(3*exp((5*a)/3 - 2*d - (b*x)/3))/(4*b) - (3*exp((5*a)/3 - 4*d - (7*b*x)/3)
)/(112*b) - (3*exp((5*a)/3 + 2*d + (11*b*x)/3))/(44*b) + (3*exp((5*a)/3 +
4*d + (17*b*x)/3))/(272*b) + (9*exp((5*a)/3 + (5*b*x)/3))/(40*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.53

$$\int e^{\frac{5}{3}(a+bx)}\sinh^4(d+bx)\,dx = \frac{3e^{\frac{5bx}{3}+\frac{5a}{3}}\left(385e^{8bx+8d}-2380e^{6bx+6d}+7854e^{4bx+4d}+26180e^{2bx+2d}-935\right)}{104720e^{4bx+4d}b}$$

input
```
int(exp(5/3*b*x+5/3*a)*sinh(b*x+d)^4,x)
```

output
```
(3*e**((5*a + 5*b*x)/3)*(385*e**(8*b*x + 8*d) - 2380*e**(6*b*x + 6*d) + 78
54*e**(4*b*x + 4*d) + 26180*e**(2*b*x + 2*d) - 935))/(104720*e**(4*b*x + 4
*d)*b)
```

3.13 $\int F^{c(a+bx)} \sinh(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 63

$$\int F^{c(a+bx)} \sinh(d+ex)\,dx = \frac{e^{-d-ex}F^{c(a+bx)}}{2(e-bc\log(F))} + \frac{e^{d+ex}F^{c(a+bx)}}{2(e+bc\log(F))}$$

output
```
exp(-e*x-d)*F^(c*(b*x+a))/(2*e-2*b*c*ln(F))+exp(e*x+d)*F^(c*(b*x+a))/(2*e+
2*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int F^{c(a+bx)} \sinh(d+ex)\,dx = \frac{F^{c(a+bx)}(e\cosh(d+ex) - bc\log(F)\sinh(d+ex))}{(e-bc\log(F))(e+bc\log(F))}$$

input
```
Integrate[F^(c*(a + b*x))*Sinh[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*(e*Cosh[d + e*x] - b*c*Log[F]*Sinh[d + e*x]))/((e - b*c*L
og[F])*(e + b*c*Log[F]))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5997}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{ 5997}$$

$$\frac{e\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}$$

input
```
Int[F^(c*(a + b*x))*Sinh[d + e*x],x]
```

output
```
(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2) - (b*c*F^(c*(a
+ b*x))*Log[F]*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 5997
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

method	result	size
parallelrisch	$\frac{(bc\ln(F)\sinh(ex+d)-e\cosh(ex+d))F^{c(bx+a)}}{b^2c^2\ln(F)^2-e^2}$	51
risch	$\frac{(\ln(F)bc\,\mathrm{e}^{2ex+2d}-bc\ln(F)-\mathrm{e}^{2ex+2d}e-e)\mathrm{e}^{-ex-d}F^{c(bx+a)}}{2(bc\ln(F)-e)(e+bc\ln(F))}$	77
orering	$\frac{2\ln(F)bc\sinh(ex+d)F^{c(bx+a)}}{b^2c^2\ln(F)^2-e^2}-\frac{e\cosh(ex+d)F^{c(bx+a)}+\sinh(ex+d)F^{c(bx+a)}bc\ln(F)}{b^2c^2\ln(F)^2-e^2}$	101

input
```
int(sinh(e*x+d)*F^(c*(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
(b*c*ln(F)*sinh(e*x+d)-e*cosh(e*x+d))*F^(c*(b*x+a))/(b^2*c^2*ln(F)^2-e^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(58) = 116$.

Time = 0.10 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.87

$$\int F^{c(a+bx)}\sinh(d+ex)\,dx =$$

$$-\frac{\left(e\cosh\left(ex+d\right)^2-\left(bc\log\left(F\right)-e\right)\sinh\left(ex+d\right)^2-\left(bc\cosh\left(ex+d\right)^2-bc\right)\log\left(F\right)-2\left(bc\cosh\left(ex+\right.\right.\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d),x, algorithm="fricas")
```

output
```
-1/2*((e*cosh(e*x + d)^2 - (b*c*log(F) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*
x + d)^2 - b*c)*log(F) - 2*(b*c*cosh(e*x + d)*log(F) - e*cosh(e*x + d))*si
nh(e*x + d) + e)*cosh((b*c*x + a*c)*log(F)) + (e*cosh(e*x + d)^2 - (b*c*lo
g(F) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*x + d)^2 - b*c)*log(F) - 2*(b*c*co
sh(e*x + d)*log(F) - e*cosh(e*x + d))*sinh(e*x + d) + e)*sinh((b*c*x + a*c
)*log(F)))/(b^2*c^2*cosh(e*x + d)*log(F)^2 - e^2*cosh(e*x + d) + (b^2*c^2*
log(F)^2 - e^2)*sinh(e*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(54) = 108$.

Time = 0.65 (sec) , antiderivative size = 323, normalized size of antiderivative = 5.13

$$\int F^{c(a+bx)} \sinh(d+ex)\,dx$$
$$= \begin{cases} x\sinh(d) \\ F^{ac}x\sinh(d) \\ x\sinh(d) \\ -\frac{F^{ac+bcx}x\sinh(bcx\log(F)-d)}{2} + \frac{F^{ac+bcx}x\cosh(bcx\log(F)-d)}{2} + \frac{F^{ac+bcx}\sinh(bcx\log(F)-d)}{2bc\log(F)} - \frac{F^{ac+bcx}\cosh(bcx\log(F)-d)}{bc\log(F)} \\ \frac{F^{ac+bcx}x\sinh(bcx\log(F)+d)}{2} - \frac{F^{ac+bcx}x\cosh(bcx\log(F)+d)}{2} - \frac{F^{ac+bcx}\sinh(bcx\log(F)+d)}{2bc\log(F)} + \frac{F^{ac+bcx}\cosh(bcx\log(F)+d)}{bc\log(F)} \\ \frac{F^{ac+bcx}bc\log(F)\sinh(d+ex)}{b^2c^2\log(F)^2-e^2} - \frac{F^{ac+bcx}e\cosh(d+ex)}{b^2c^2\log(F)^2-e^2} \end{cases}$$

input
```
integrate(F**(c*(b*x+a))*sinh(e*x+d),x)
```

output
```
Piecewise((x*sinh(d), Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*sinh(d), Eq(b, 0)
& Eq(e, 0)), (x*sinh(d), Eq(c, 0) & Eq(e, 0)), (-F**(a*c + b*c*x)*x*sinh(b
*c*x*log(F) - d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F) - d)/2 + F**(a*c
 + b*c*x)*sinh(b*c*x*log(F) - d)/(2*b*c*log(F)) - F**(a*c + b*c*x)*cosh(b*
c*x*log(F) - d)/(b*c*log(F)), Eq(e, -b*c*log(F))), (F**(a*c + b*c*x)*x*sin
h(b*c*x*log(F) + d)/2 - F**(a*c + b*c*x)*x*cosh(b*c*x*log(F) + d)/2 - F**(
a*c + b*c*x)*sinh(b*c*x*log(F) + d)/(2*b*c*log(F)) + F**(a*c + b*c*x)*cosh
(b*c*x*log(F) + d)/(b*c*log(F)), Eq(e, b*c*log(F))), (F**(a*c + b*c*x)*b*c
*log(F)*sinh(d + e*x)/(b**2*c**2*log(F)**2 - e**2) - F**(a*c + b*c*x)*e*co
sh(d + e*x)/(b**2*c**2*log(F)**2 - e**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)} \sinh(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{2\,(bc\log(F)+e)} - \frac{F^{ac}e^{(bcx\log(F)-ex)}}{2\,(bce^d\log(F)-ee^d)}$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d),x, algorithm="maxima")
```

output
```
1/2*F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) - 1/2*F^(a*c)*e^(b
*c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 598, normalized size of antiderivative = 9.49

$$\int F^{c(a+bx)}\sinh(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d),x, algorithm="giac")
```

output
```
(2*(b*c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*
a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F))
+ e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x
 - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*lo
g(abs(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 1/2
*I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/
2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) + 2*e) - I*e^(
-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a
*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F)) + 2*e))*e^(a*c*log(ab
s(F)) + (b*c*log(abs(F)) + e)*x + d) - (2*(b*c*log(abs(F)) - e)*cos(-1/2*p
i*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*s
gn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sgn(F) - pi*b*c)*
sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/
((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(F
)) + (b*c*log(abs(F)) - e)*x - d) + 1/2*I*(-I*e^(1/2*I*pi*b*c*x*sgn(F) - 1
/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*p
i*b*c + 2*b*c*log(abs(F)) - 2*e) + I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*
b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c +
 2*b*c*log(abs(F)) - 2*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x -
d)
```

Mupad [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16

$$\int F^{c(a+bx)}\sinh(d+ex)\,dx = \frac{F^{a\,c+b\,c\,x}\,\mathrm{e}^{-d-e\,x}\,\left(e+e\,\mathrm{e}^{2\,d+2\,e\,x}+b\,c\,\ln\left(F\right)-b\,c\,\mathrm{e}^{2\,d+2\,e\,x}\,\ln\left(F\right)\right)}{2\,\left(e^2-b^2\,c^2\,\ln\left(F\right)^2\right)}$$

input
```
int(F^(c*(a + b*x))*sinh(d + e*x),x)
```

output
```
(F^(a*c + b*c*x)*exp(- d - e*x)*(e + e*exp(2*d + 2*e*x) + b*c*log(F) - b*c
*exp(2*d + 2*e*x)*log(F)))/(2*(e^2 - b^2*c^2*log(F)^2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int F^{c(a+bx)}\sinh(d+ex)\,dx = \frac{f^{bcx+ac}(-\cosh\left(ex+d\right)e+\log(f)\sinh\left(ex+d\right)bc)}{\log\left(f\right)^2b^2c^2-e^2}$$

input
```
int(F^(c*(b*x+a))*sinh(e*x+d),x)
```

output
```
(f**(a*c + b*c*x)*( - cosh(d + e*x)*e + log(f)*sinh(d + e*x)*b*c))/(log(f)
**2*b**2*c**2 - e**2)
```

3.14 $\int F^{c(a+bx)}\sinh^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 93

$$\int F^{c(a+bx)}\sinh^2(d+ex)\,dx=-\frac{F^{c(a+bx)}}{2bc\log(F)}-\frac{e^{-2d-2ex}F^{c(a+bx)}}{4(2e-bc\log(F))}+\frac{e^{2d+2ex}F^{c(a+bx)}}{4(2e+bc\log(F))}$$

output
```
-1/2*F^(c*(b*x+a))/b/c/ln(F)-exp(-2*e*x-2*d)*F^(c*(b*x+a))/(8*e-4*b*c*ln(F
))+exp(2*e*x+2*d)*F^(c*(b*x+a))/(8*e+4*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.92

$$\int F^{c(a+bx)}\sinh^2(d+ex)\,dx$$
$$=\frac{F^{c(a+bx)}\left(4e^2-b^2c^2\log^2(F)+b^2c^2\cosh(2(d+ex))\log^2(F)-2bce\log(F)\sinh(2(d+ex))\right)}{-8bce^2\log(F)+2b^3c^3\log^3(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sinh[d + e*x]^2,x]
```

output
```
(F^(c*(a + b*x))*(4*e^2 - b^2*c^2*Log[F]^2 + b^2*c^2*Cosh[2*(d + e*x)]*Log
[F]^2 - 2*b*c*e*Log[F]*Sinh[2*(d + e*x)]))/(-8*b*c*e^2*Log[F] + 2*b^3*c^3*
Log[F]^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.42, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5999, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{5999}$$

$$-\frac{2e^2\int F^{c(a+bx)}dx}{4e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\sinh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}$$

$$\Big\downarrow \text{2624}$$

$$-\frac{bc\log(F)\sinh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}-\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}$$

input
```
Int[F^(c*(a + b*x))*Sinh[d + e*x]^2,x]
```

output
```
(-2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 - b^2*c^2*Log[F]^2)) + (2*e*F^
(c*(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x])/(4*e^2 - b^2*c^2*Log[F]^2) - (b
*c*F^(c*(a + b*x))*Log[F]*Sinh[d + e*x]^2)/(4*e^2 - b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 5999
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Simp[e*n*F^(c*(a + b*x))*Cosh[d + e*x]*(Sinh[d + e*x]^
(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x] - Simp[n*(n - 1)*(e^2/(e^2*n^2 -
b^2*c^2*Log[F]^2))   Int[F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97

method	result
parallelrisch	$-\frac{2\left(-\frac{c^{2} b^{2} \ln \left(F\right)^{2} \cosh \left(2 e x+2 d\right)}{2}+\frac{b^{2} c^{2} \ln \left(F\right)^{2}}{2}+\ln \left(F\right) b c e \sinh \left(2 e x+2 d\right)-2 e^{2}\right) F^{c \left(b x+a\right)}}{2 c^{3} b^{3} \ln \left(F\right)^{3}-8 e^{2} b c \ln \left(F\right)}$
risch	$\frac{\left(\ln \left(F\right)^{2} b^{2} c^{2} {\mathrm e}^{4 e x+4 d}-2 \ln \left(F\right)^{2} b^{2} c^{2} {\mathrm e}^{2 e x+2 d}-2 \ln \left(F\right) b c e \,{\mathrm e}^{4 e x+4 d}+b^{2} c^{2} \ln \left(F\right)^{2}+2 \ln \left(F\right) b c e+8 e^{2} {\mathrm e}^{2 e x+2 d}\right) {\mathrm e}^{-2 e x-2 d} F^{c \left(b x+a\right)}}{4 \ln \left(F\right) b c \left(b c \ln \left(F\right)-2 e\right) \left(2 e+b c \ln \left(F\right)\right)}$
orering	$\frac{\left(3 b^{2} c^{2} \ln \left(F\right)^{2}-4 e^{2}\right) F^{c \left(b x+a\right)} \sinh \left(e x+d\right)^{2}}{\ln \left(F\right) b c \left(b^{2} c^{2} \ln \left(F\right)^{2}-4 e^{2}\right)}-\frac{3\left(F^{c \left(b x+a\right)} b c \ln \left(F\right) \sinh \left(e x+d\right)^{2}+2 F^{c \left(b x+a\right)} \sinh \left(e x+d\right) e \cosh \left(e x+d\right)\right)}{b^{2} c^{2} \ln \left(F\right)^{2}-4 e^{2}}+$

input
```
int(F^(c*(b*x+a))*sinh(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(-1/2*c^2*b^2*ln(F)^2*cosh(2*e*x+2*d)+1/2*b^2*c^2*ln(F)^2+ln(F)*b*c*e*s
inh(2*e*x+2*d)-2*e^2)*F^(c*(b*x+a))/(2*c^3*b^3*ln(F)^3-8*e^2*b*c*ln(F))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(84) = 168$.

Time = 0.10 (sec) , antiderivative size = 703, normalized size of antiderivative = 7.56

$$\int F^{c(a+bx)} \sinh^2(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/4*(((b^2*c^2*log(F)^2 - 2*b*c*e*log(F))*sinh(e*x + d)^4 + 8*e^2*cosh(e*x
 + d)^2 + 4*(b^2*c^2*cosh(e*x + d)*log(F)^2 - 2*b*c*e*cosh(e*x + d)*log(F)
)*sinh(e*x + d)^3 + (b^2*c^2*cosh(e*x + d)^4 - 2*b^2*c^2*cosh(e*x + d)^2 +
 b^2*c^2)*log(F)^2 - 2*(6*b*c*e*cosh(e*x + d)^2*log(F) - (3*b^2*c^2*cosh(e
*x + d)^2 - b^2*c^2)*log(F)^2 - 4*e^2)*sinh(e*x + d)^2 - 2*(b*c*e*cosh(e*x
 + d)^4 - b*c*e)*log(F) - 4*(2*b*c*e*cosh(e*x + d)^3*log(F) - 4*e^2*cosh(e
*x + d) - (b^2*c^2*cosh(e*x + d)^3 - b^2*c^2*cosh(e*x + d))*log(F)^2)*sinh
(e*x + d))*cosh((b*c*x + a*c)*log(F)) + ((b^2*c^2*log(F)^2 - 2*b*c*e*log(F
))*sinh(e*x + d)^4 + 8*e^2*cosh(e*x + d)^2 + 4*(b^2*c^2*cosh(e*x + d)*log(
F)^2 - 2*b*c*e*cosh(e*x + d)*log(F))*sinh(e*x + d)^3 + (b^2*c^2*cosh(e*x +
 d)^4 - 2*b^2*c^2*cosh(e*x + d)^2 + b^2*c^2)*log(F)^2 - 2*(6*b*c*e*cosh(e*
x + d)^2*log(F) - (3*b^2*c^2*cosh(e*x + d)^2 - b^2*c^2)*log(F)^2 - 4*e^2)*
sinh(e*x + d)^2 - 2*(b*c*e*cosh(e*x + d)^4 - b*c*e)*log(F) - 4*(2*b*c*e*co
sh(e*x + d)^3*log(F) - 4*e^2*cosh(e*x + d) - (b^2*c^2*cosh(e*x + d)^3 - b^
2*c^2*cosh(e*x + d))*log(F)^2)*sinh(e*x + d))*sinh((b*c*x + a*c)*log(F)))/
(b^3*c^3*cosh(e*x + d)^2*log(F)^3 - 4*b*c*e^2*cosh(e*x + d)^2*log(F) + (b^
3*c^3*log(F)^3 - 4*b*c*e^2*log(F))*sinh(e*x + d)^2 + 2*(b^3*c^3*cosh(e*x +
 d)*log(F)^3 - 4*b*c*e^2*cosh(e*x + d)*log(F))*sinh(e*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(78) = 156$.

Time = 1.24 (sec) , antiderivative size = 707, normalized size of antiderivative = 7.60

$$\int F^{c(a+bx)} \sinh^2(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*sinh(e*x+d)**2,x)
```

output
```
Piecewise((x*sinh(d)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)), (x*si
nh(d + e*x)**2/2 - x*cosh(d + e*x)**2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*e
), Eq(F, 1)), (F**(a*c)*(x*sinh(d + e*x)**2/2 - x*cosh(d + e*x)**2/2 + sin
h(d + e*x)*cosh(d + e*x)/(2*e)), Eq(b, 0)), (x*sinh(d + e*x)**2/2 - x*cosh
(d + e*x)**2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*e), Eq(c, 0)), (F**(a*c +
b*c*x)*x*sinh(b*c*x*log(F)/2 - d)**2/4 - F**(a*c + b*c*x)*x*sinh(b*c*x*log
(F)/2 - d)*cosh(b*c*x*log(F)/2 - d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(
F)/2 - d)**2/4 + F**(a*c + b*c*x)*sinh(b*c*x*log(F)/2 - d)**2/(b*c*log(F))
 - F**(a*c + b*c*x)*sinh(b*c*x*log(F)/2 - d)*cosh(b*c*x*log(F)/2 - d)/(2*b
*c*log(F)), Eq(e, -b*c*log(F)/2)), (F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/2
 + d)**2/4 - F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/2 + d)*cosh(b*c*x*log(F)
/2 + d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F)/2 + d)**2/4 + 3*F**(a*c +
 b*c*x)*sinh(b*c*x*log(F)/2 + d)*cosh(b*c*x*log(F)/2 + d)/(2*b*c*log(F)) -
 F**(a*c + b*c*x)*cosh(b*c*x*log(F)/2 + d)**2/(b*c*log(F)), Eq(e, b*c*log(
F)/2)), (F**(a*c + b*c*x)*b**2*c**2*log(F)**2*sinh(d + e*x)**2/(b**3*c**3*
log(F)**3 - 4*b*c*e**2*log(F)) - 2*F**(a*c + b*c*x)*b*c*e*log(F)*sinh(d +
e*x)*cosh(d + e*x)/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F)) - 2*F**(a*c +
 b*c*x)*e**2*sinh(d + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F)) +
2*F**(a*c + b*c*x)*e**2*cosh(d + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e**2
*log(F)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01

$$\int F^{c(a+bx)}\sinh^2(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+2\,ex+2\,d)}}{4\,(bc\log(F)+2\,e)} + \frac{F^{ac}e^{(bcx\log(F)-2\,ex)}}{4\,(bce^{(2\,d)}\log(F)-2\,ee^{(2\,d)})} - \frac{F^{bcx+ac}}{2\,bc\log(F)}$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)^2,x, algorithm="maxima")
```

output
```
1/4*F^(a*c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) + 1/4*F^(a*c
)*e^(b*c*x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) - 1/2*F^(b*c
*x + a*c)/(b*c*log(F))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 890, normalized size of antiderivative = 9.57

$$\int F^{c(a+bx)} \sinh^2(d+ex)\, dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)^2,x, algorithm="giac")
```

output
```
-(2*b*c*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*
pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2)
 - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*
pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi
*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(-I*e^(1/2*I*pi*b*c*
x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*I*pi*b*
c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) + I*e^(-1/2*I*pi*b*c*x*sgn(F) +
 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F)
+ 2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
 + 1/2*(2*(b*c*log(abs(F)) + 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x
- 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log
(abs(F)) + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1
/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2
+ 4*(b*c*log(abs(F)) + 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*
e)*x + 2*d) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*
c*sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F
)) + 16*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*s
gn(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F))
 + 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x + 2*d) + 1/2*(2*(
b*c*log(abs(F)) - 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi...
```

Mupad [B] (verification not implemented)

Time = 3.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04

$$\int F^{c(a+bx)}\sinh^2(d+ex)\,dx$$
$$=-\frac{F^{a\,c+b\,c\,x}\,\left(2\,e^2-\frac{b^2\,c^2\,\ln(F)^2}{2}+\frac{b^2\,c^2\,\ln(F)^2\,\cosh(2\,d+2\,e\,x)}{2}-b\,c\,e\,\ln\left(F\right)\,\sinh(2\,d+2\,e\,x)\right)}{b\,c\,\ln\left(F\right)\,\left(4\,e^2-b^2\,c^2\,\ln\left(F\right)^2\right)}$$

input
```
int(F^(c*(a + b*x))*sinh(d + e*x)^2,x)
```

output
```
-(F^(a*c + b*c*x)*(2*e^2 - (b^2*c^2*log(F)^2)/2 + (b^2*c^2*log(F)^2*cosh(2
*d + 2*e*x))/2 - b*c*e*log(F)*sinh(2*d + 2*e*x)))/(b*c*log(F)*(4*e^2 - b^2
*c^2*log(F)^2))
```

Reduce [F]

$$\int F^{c(a+bx)}\sinh^2(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\sinh\left(ex+d\right)^2dx\right)$$

input
```
int(F^(c*(b*x+a))*sinh(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sinh(d + e*x)**2,x)
```

3.15 $\int F^{c(a+bx)} \sinh^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 132

$$\int F^{c(a+bx)} \sinh^3(d+ex)\,dx = -\frac{3e^{-d-ex}F^{c(a+bx)}}{8(e-bc\log(F))}+\frac{e^{-3d-3ex}F^{c(a+bx)}}{8(3e-bc\log(F))} - \frac{3e^{d+ex}F^{c(a+bx)}}{8(e+bc\log(F))}+\frac{e^{3d+3ex}F^{c(a+bx)}}{8(3e+bc\log(F))}$$

output
```
-3*exp(-e*x-d)*F^(c*(b*x+a))/(8*e-8*b*c*ln(F))+exp(-3*e*x-3*d)*F^(c*(b*x+a
))/(24*e-8*b*c*ln(F))-3*exp(e*x+d)*F^(c*(b*x+a))/(8*e+8*b*c*ln(F))+exp(3*e
*x+3*d)*F^(c*(b*x+a))/(24*e+8*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.19

$$\int F^{c(a+bx)} \sinh^3(d+ex)\,dx = \frac{F^{c(a+bx)}\left(3\cosh(3(d+ex))\left(e^3-b^2c^2e\log^2(F)\right)+3\cosh(d+ex)\left(-9e^3+b^2c^2e\log^2(F)\right)+2bc\log(F)\left(\right.\right.}{4\left(9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log\right.}$$

input
```
Integrate[F^(c*(a + b*x))*Sinh[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(3*Cosh[3*(d + e*x)]*(e^3 - b^2*c^2*e*Log[F]^2) + 3*Cosh[
d + e*x]*(-9*e^3 + b^2*c^2*e*Log[F]^2) + 2*b*c*Log[F]*(13*e^2 - b^2*c^2*Lo
g[F]^2 + Cosh[2*(d + e*x)]*(-e^2 + b^2*c^2*Log[F]^2))*Sinh[d + e*x]))/(4*(
9*e^4 - 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.44, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5999, 5997}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{5999}$$

$$-\frac{6e^2\int F^{c(a+bx)}\sinh(d+ex)dx}{9e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\sinh^3(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{3e\sinh^2(d+ex)\cosh(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}$$

$$\Big\downarrow \text{5997}$$

$$-\frac{bc\log(F)\sinh^3(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{3e\sinh^2(d+ex)\cosh(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}-\frac{6e^2\left(\frac{e\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}\right)}{9e^2-b^2c^2\log^2(F)}$$

input
```
Int[F^(c*(a + b*x))*Sinh[d + e*x]^3,x]
```

output
```
(3*e*F^(c*(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x]^2)/(9*e^2 - b^2*c^2*Log[F
]^2) - (b*c*F^(c*(a + b*x))*Log[F]*Sinh[d + e*x]^3)/(9*e^2 - b^2*c^2*Log[F
]^2) - (6*e^2*((e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
- (b*c*F^(c*(a + b*x))*Log[F]*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)))/(9
*e^2 - b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 5997
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

rule 5999
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Simp[e*n*F^(c*(a + b*x))*Cosh[d + e*x]*(Sinh[d + e*x]^
(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x] - Simp[n*(n - 1)*(e^2/(e^2*n^2 -
b^2*c^2*Log[F]^2))   Int[F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.11

method	result
parallelrisch	$-\frac{3F^{c(bx+a)}\left(\left(c^{2}b^{2}\ln(F)^{2}e-e^{3}\right)\cosh(3ex+3d)+\frac{\left(-c^{3}b^{3}\ln(F)^{3}+e^{2}bc\ln(F)\right)\sinh(3ex+3d)}{3}+(bc\ln(F)-3e)(bc\ln(F)+3e)(bc\ln}{4\left(\ln(F)^{4}b^{4}c^{4}-10\ln(F)^{2}b^{2}c^{2}e^{2}+9e^{4}\right)}$
risch	$\frac{\left(\ln(F)^{3}b^{3}c^{3}\mathrm{e}^{6ex+6d}-3\ln(F)^{3}b^{3}c^{3}\mathrm{e}^{4ex+4d}-3\ln(F)^{2}b^{2}c^{2}e\,\mathrm{e}^{6ex+6d}+3\ln(F)^{3}b^{3}c^{3}\mathrm{e}^{2ex+2d}+3\ln(F)^{2}b^{2}c^{2}e\,\mathrm{e}^{4ex+4d}-\ln(F)bc}{}$
orering	$\frac{4\ln(F)bc\left(b^{2}c^{2}\ln(F)^{2}-5e^{2}\right)F^{c(bx+a)}\sinh(ex+d)^{3}}{\ln(F)^{4}b^{4}c^{4}-10\ln(F)^{2}b^{2}c^{2}e^{2}+9e^{4}}-\frac{2\left(3b^{2}c^{2}\ln(F)^{2}-5e^{2}\right)\left(F^{c(bx+a)}bc\ln(F)\sinh(ex+d)^{3}+3F^{c(bx+a)}\mathrm{si}}{\ln(F)^{4}b^{4}c^{4}-10\ln(F)^{2}b^{2}c^{2}e^{2}+9e^{4}}$

input
```
int(F^(c*(b*x+a))*sinh(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/4*F^(c*(b*x+a))*((c^2*b^2*ln(F)^2*e-e^3)*cosh(3*e*x+3*d)+1/3*(-c^3*b^3*
ln(F)^3+e^2*b*c*ln(F))*sinh(3*e*x+3*d)+(b*c*ln(F)-3*e)*(b*c*ln(F)+3*e)*(b*
c*ln(F)*sinh(e*x+d)-e*cosh(e*x+d)))/(ln(F)^4*b^4*c^4-10*ln(F)^2*b^2*c^2*e^
2+9*e^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2228 vs. $2(120) = 240$.

Time = 0.18 (sec) , antiderivative size = 2228, normalized size of antiderivative = 16.88

$$\int F^{c(a+bx)} \sinh^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)^3,x, algorithm="fricas")
```

output
```
1/8*((3*e^3*cosh(e*x + d)^6 - 27*e^3*cosh(e*x + d)^4 + (b^3*c^3*log(F)^3 -
 3*b^2*c^2*e*log(F)^2 - b*c*e^2*log(F) + 3*e^3)*sinh(e*x + d)^6 + 6*(b^3*c
^3*cosh(e*x + d)*log(F)^3 - 3*b^2*c^2*e*cosh(e*x + d)*log(F)^2 - b*c*e^2*c
osh(e*x + d)*log(F) + 3*e^3*cosh(e*x + d))*sinh(e*x + d)^5 - 27*e^3*cosh(e
*x + d)^2 + 3*(15*e^3*cosh(e*x + d)^2 + (5*b^3*c^3*cosh(e*x + d)^2 - b^3*c
^3)*log(F)^3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^2 - b^2*c^2*e)*log(F)^2
 - (5*b*c*e^2*cosh(e*x + d)^2 - 9*b*c*e^2)*log(F))*sinh(e*x + d)^4 + (b^3*
c^3*cosh(e*x + d)^6 - 3*b^3*c^3*cosh(e*x + d)^4 + 3*b^3*c^3*cosh(e*x + d)^
2 - b^3*c^3)*log(F)^3 + 4*(15*e^3*cosh(e*x + d)^3 - 27*e^3*cosh(e*x + d) +
 (5*b^3*c^3*cosh(e*x + d)^3 - 3*b^3*c^3*cosh(e*x + d))*log(F)^3 - 3*(5*b^2
*c^2*e*cosh(e*x + d)^3 - b^2*c^2*e*cosh(e*x + d))*log(F)^2 - (5*b*c*e^2*co
sh(e*x + d)^3 - 27*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d)^3 + 3*e^3
- 3*(b^2*c^2*e*cosh(e*x + d)^6 - b^2*c^2*e*cosh(e*x + d)^4 - b^2*c^2*e*cos
h(e*x + d)^2 + b^2*c^2*e)*log(F)^2 + 3*(15*e^3*cosh(e*x + d)^4 - 54*e^3*co
sh(e*x + d)^2 + (5*b^3*c^3*cosh(e*x + d)^4 - 6*b^3*c^3*cosh(e*x + d)^2 + b
^3*c^3)*log(F)^3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^4 - 6*b^2*c^2*e*cos
h(e*x + d)^2 - b^2*c^2*e)*log(F)^2 - (5*b*c*e^2*cosh(e*x + d)^4 - 54*b*c*e
^2*cosh(e*x + d)^2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^2 - (b*c*e^2*cosh(e*
x + d)^6 - 27*b*c*e^2*cosh(e*x + d)^4 + 27*b*c*e^2*cosh(e*x + d)^2 - b*c*e
^2)*log(F) + 6*(3*e^3*cosh(e*x + d)^5 - 18*e^3*cosh(e*x + d)^3 - 9*e^3*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1477 vs. $2(121) = 242$.

Time = 3.30 (sec) , antiderivative size = 1477, normalized size of antiderivative = 11.19

$$\int F^{c(a+bx)} \sinh^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*sinh(e*x+d)**3,x)
```

output
```
Piecewise((x*sinh(d)**3, Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*sinh(d)**3, Eq(
b, 0) & Eq(e, 0)), (x*sinh(d)**3, Eq(c, 0) & Eq(e, 0)), (-3*F**(a*c + b*c*
x)*x*sinh(b*c*x*log(F) - d)**3/8 + 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)
- d)**2*cosh(b*c*x*log(F) - d)/8 + 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)
- d)*cosh(b*c*x*log(F) - d)**2/8 - 3*F**(a*c + b*c*x)*x*cosh(b*c*x*log(F)
- d)**3/8 + F**(a*c + b*c*x)*sinh(b*c*x*log(F) - d)**3/(8*b*c*log(F)) - 3*
F**(a*c + b*c*x)*sinh(b*c*x*log(F) - d)**2*cosh(b*c*x*log(F) - d)/(4*b*c*l
og(F)) + 3*F**(a*c + b*c*x)*cosh(b*c*x*log(F) - d)**3/(8*b*c*log(F)), Eq(e
, -b*c*log(F))), (-F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)**3/8 + 3*F*
*(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)**2*cosh(b*c*x*log(F)/3 - d)/8 -
3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)*cosh(b*c*x*log(F)/3 - d)**2/
8 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F)/3 - d)**3/8 - 9*F**(a*c + b*c*x)*
sinh(b*c*x*log(F)/3 - d)**3/(8*b*c*log(F)) + 3*F**(a*c + b*c*x)*sinh(b*c*x
*log(F)/3 - d)**2*cosh(b*c*x*log(F)/3 - d)/(4*b*c*log(F)) - F**(a*c + b*c*
x)*cosh(b*c*x*log(F)/3 - d)**3/(8*b*c*log(F)), Eq(e, -b*c*log(F)/3)), (F**
(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 + d)**3/8 - 3*F**(a*c + b*c*x)*x*sinh(
b*c*x*log(F)/3 + d)**2*cosh(b*c*x*log(F)/3 + d)/8 + 3*F**(a*c + b*c*x)*x*s
inh(b*c*x*log(F)/3 + d)*cosh(b*c*x*log(F)/3 + d)**2/8 - F**(a*c + b*c*x)*x
*cosh(b*c*x*log(F)/3 + d)**3/8 + 9*F**(a*c + b*c*x)*sinh(b*c*x*log(F)/3 +
d)**3/(8*b*c*log(F)) - 3*F**(a*c + b*c*x)*sinh(b*c*x*log(F)/3 + d)**2*c...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02

$$\int F^{c(a+bx)}\sinh^3(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+3\,ex+3\,d)}}{8\,(bc\log(F)+3\,e)} - \frac{3\,F^{ac}e^{(bcx\log(F)+ex+d)}}{8\,(bc\log(F)+e)} + \frac{3\,F^{ac}e^{(bcx\log(F)-ex)}}{8\,(bce^{d}\log(F)-ee^{d})} - \frac{F^{ac}e^{(bcx\log(F)-3\,ex)}}{8\,(bce^{(3\,d)}\log(F)-3\,ee^{(3\,d)})}$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/8*F^(a*c)*e^(b*c*x*log(F) + 3*e*x + 3*d)/(b*c*log(F) + 3*e) - 3/8*F^(a*c
)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) + 3/8*F^(a*c)*e^(b*c*x*log(F
) - e*x)/(b*c*e^d*log(F) - e*e^d) - 1/8*F^(a*c)*e^(b*c*x*log(F) - 3*e*x)/(
b*c*e^(3*d)*log(F) - 3*e*e^(3*d))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 1211, normalized size of antiderivative = 9.17

$$\int F^{c(a+bx)}\sinh^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)^3,x, algorithm="giac")
```

output
```
1/4*(2*(b*c*log(abs(F)) + 3*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1
/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(ab
s(F)) + 3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*
pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4
*(b*c*log(abs(F)) + 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*
x + 3*d) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*s
gn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F))
 + 48*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn
(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F))
+ 48*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3*d) - 3/4*(2*(b
*c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*s
gn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e)^
2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs
(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 3*I*(-I*
e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi
*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e) + I*e^(
-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a
*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e))*e^(a*c*
log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 3/4*(2*(b*c*log(abs(F)) - ...
```

Mupad [B] (verification not implemented)

Time = 3.53 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.26

$$\int F^{c(a+bx)}\sinh^3(d+ex)\,dx = -\frac{F^{a\,c+b\,c\,x}\,\left(-b^3\,c^3\,\sinh(d+e\,x)^3\,\ln\left(F\right)^3+3\,b^2\,c^2\,e\cosh(d+e\,x)\,\sinh(d+e\,x)^2\ln\left(F\right)^2-6\,b\,c\,e^2\cosh(d\right.}{b^4\,c^4\ln\left(F\right)^4}$$

input
```
int(F^(c*(a + b*x))*sinh(d + e*x)^3,x)
```

output
```
-(F^(a*c + b*c*x)*(6*e^3*cosh(d + e*x)^3 - 9*e^3*cosh(d + e*x)*sinh(d + e*
x)^2 - b^3*c^3*sinh(d + e*x)^3*log(F)^3 + 7*b*c*e^2*sinh(d + e*x)^3*log(F)
 + 3*b^2*c^2*e*cosh(d + e*x)*sinh(d + e*x)^2*log(F)^2 - 6*b*c*e^2*cosh(d +
 e*x)^2*sinh(d + e*x)*log(F)))/(9*e^4 + b^4*c^4*log(F)^4 - 10*b^2*c^2*e^2*
log(F)^2)
```

Reduce [F]

$$\int F^{c(a+bx)} \sinh^3(d+ex)\,dx = f^{ac}\left(\int f^{bcx} \sinh\left(ex+d\right)^3 dx\right)$$

input
```
int(F^(c*(b*x+a))*sinh(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sinh(d + e*x)**3,x)
```

3.16 $\int F^{c(a+bx)} \sinh^4(d+ex)\, dx$

Optimal result

Integrand size = 18, antiderivative size = 162

$$\int F^{c(a+bx)} \sinh^4(d+ex)\, dx = \frac{3F^{c(a+bx)}}{8bc\log(F)} + \frac{e^{-2d-2ex}F^{c(a+bx)}}{4(2e-bc\log(F))} - \frac{e^{-4d-4ex}F^{c(a+bx)}}{16(4e-bc\log(F))} - \frac{e^{2d+2ex}F^{c(a+bx)}}{4(2e+bc\log(F))} + \frac{e^{4d+4ex}F^{c(a+bx)}}{16(4e+bc\log(F))}$$

output
```
3/8*F^(c*(b*x+a))/b/c/ln(F)+exp(-2*e*x-2*d)*F^(c*(b*x+a))/(8*e-4*b*c*ln(F)
)-exp(-4*e*x-4*d)*F^(c*(b*x+a))/(64*e-16*b*c*ln(F))-exp(2*e*x+2*d)*F^(c*(b
*x+a))/(8*e+4*b*c*ln(F))+exp(4*e*x+4*d)*F^(c*(b*x+a))/(64*e+16*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.25

$$\int F^{c(a+bx)}\sinh^4(d+ex)\,dx=\frac{1}{8}F^{c(a+bx)}\left(\frac{3}{bc\log(F)}+\frac{4\cosh(2ex)(bc\cosh(2d)\log(F)-2e\sinh(2d))}{4e^2-b^2c^2\log^2(F)}+\frac{\cosh(4ex)(-bc\cosh(4d)\log(F)+4e\sinh(4d))}{16e^2-b^2c^2\log^2(F)}-\frac{4(2e\cosh(2d)-bc\log(F)\sinh(2d))\sinh(2ex)}{4e^2-b^2c^2\log^2(F)}+\frac{(4e\cosh(4d)-bc\log(F)\sinh(4d))\sinh(4ex)}{16e^2-b^2c^2\log^2(F)}\right)$$

input
```
Integrate[F^(c*(a + b*x))*Sinh[d + e*x]^4,x]
```

output
```
(F^(c*(a + b*x))*(3/(b*c*Log[F]) + (4*Cosh[2*e*x]*(b*c*Cosh[2*d]*Log[F] -
2*e*Sinh[2*d]))/(4*e^2 - b^2*c^2*Log[F]^2) + (Cosh[4*e*x]*(-(b*c*Cosh[4*d]
*Log[F]) + 4*e*Sinh[4*d]))/(16*e^2 - b^2*c^2*Log[F]^2) - (4*(2*e*Cosh[2*d]
 - b*c*Log[F]*Sinh[2*d])*Sinh[2*e*x])/(4*e^2 - b^2*c^2*Log[F]^2) + ((4*e*C
osh[4*d] - b*c*Log[F]*Sinh[4*d])*Sinh[4*e*x])/(16*e^2 - b^2*c^2*Log[F]^2))
)/8
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.52, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5999, 5999, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \; 5999$$

$$-\frac{12e^2\int F^{c(a+bx)}\sinh^2(d+ex)dx}{16e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\sinh^4(d+ex)F^{c(a+bx)}}{16e^2-b^2c^2\log^2(F)}+\frac{4e\sinh^3(d+ex)\cosh(d+ex)F^{c(a+bx)}}{16e^2-b^2c^2\log^2(F)}$$

$$\Big\downarrow \text{5999}$$

$$-\frac{12e^2\left(-\frac{2e^2\int F^{c(a+bx)}dx}{4e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\sinh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}\right)}{16e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\sinh^4(d+ex)F^{c(a+bx)}}{16e^2-b^2c^2\log^2(F)}+\frac{4e\sinh^3(d+ex)\cosh(d+ex)F^{c(a+bx)}}{16e^2-b^2c^2\log^2(F)}$$

$$\Big\downarrow \text{2624}$$

$$-\frac{bc\log(F)\sinh^4(d+ex)F^{c(a+bx)}}{16e^2-b^2c^2\log^2(F)}+\frac{4e\sinh^3(d+ex)\cosh(d+ex)F^{c(a+bx)}}{16e^2-b^2c^2\log^2(F)}-\frac{12e^2\left(-\frac{bc\log(F)\sinh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}-\frac{2e^2F^{c(a+bx)}}{bc\log(F)(4e^2-b^2c^2\log^2(F))}\right)}{16e^2-b^2c^2\log^2(F)}$$

input

```
Int[F^(c*(a + b*x))*Sinh[d + e*x]^4,x]
```

output

```
(4*e*F^(c*(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x]^3)/(16*e^2 - b^2*c^2*Log[
F]^2) - (b*c*F^(c*(a + b*x))*Log[F]*Sinh[d + e*x]^4)/(16*e^2 - b^2*c^2*Log
[F]^2) - (12*e^2*((-2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 - b^2*c^2*Lo
g[F]^2)) + (2*e*F^(c*(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x])/(4*e^2 - b^2*
c^2*Log[F]^2) - (b*c*F^(c*(a + b*x))*Log[F]*Sinh[d + e*x]^2)/(4*e^2 - b^2*
c^2*Log[F]^2)))/(16*e^2 - b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 5999
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Simp[e*n*F^(c*(a + b*x))*Cosh[d + e*x]*(Sinh[d + e*x]^
(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x] - Simp[n*(n - 1)*(e^2/(e^2*n^2 -
b^2*c^2*Log[F]^2))   Int[F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.20

method	result
parallelrisch	$\frac{F^{c(bx+a)} ((-4\ln(F)^3 b^3 c^3 e + 16\ln(F) bc\, e^3) \sinh(4ex+4d) + (\ln(F)^4 b^4 c^4 - 4\ln(F)^2 b^2 c^2 e^2) \cosh(4ex+4d) + 8(bc\ln(F)+4e) (}{8c^5 b^5 \ln(F)^5 - 160 c^3 b^3 \ln(F)^3 e^2 + 512 cb \ln}$
risch	$\frac{(\ln(F)^4 b^4 c^4 \mathrm{e}^{8ex+8d} - 4\ln(F)^4 b^4 c^4 \mathrm{e}^{6ex+6d} - 4\ln(F)^3 b^3 c^3 e\, \mathrm{e}^{8ex+8d} + 6\ln(F)^4 b^4 c^4 \mathrm{e}^{4ex+4d} + 8\ln(F)^3 b^3 c^3 e\, \mathrm{e}^{6ex+6d} - 4\ln(F)^2}{}$
orering	Expression too large to display

input
```
int(F^(c*(b*x+a))*sinh(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
F^(c*(b*x+a))*((-4*ln(F)^3*b^3*c^3*e+16*ln(F)*b*c*e^3)*sinh(4*e*x+4*d)+(ln
(F)^4*b^4*c^4-4*ln(F)^2*b^2*c^2*e^2)*cosh(4*e*x+4*d)+8*(b*c*ln(F)+4*e)*(-1
/2*c^2*b^2*ln(F)^2*cosh(2*e*x+2*d)+3/8*b^2*c^2*ln(F)^2+ln(F)*b*c*e*sinh(2*
e*x+2*d)-3/2*e^2)*(b*c*ln(F)-4*e))/(8*c^5*b^5*ln(F)^5-160*c^3*b^3*ln(F)^3*
e^2+512*c*b*ln(F)*e^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3686 vs. 2(146) = 292.

Time = 0.22 (sec) , antiderivative size = 3686, normalized size of antiderivative = 22.75

$$\int F^{c(a+bx)} \sinh^4(d+ex) \, dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2409 vs. 2(143) = 286.

Time = 25.03 (sec) , antiderivative size = 2409, normalized size of antiderivative = 14.87

$$\int F^{c(a+bx)} \sinh^4(d+ex) \, dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*sinh(e*x+d)**4,x)
```

output
```
Piecewise((x*sinh(d)**4, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)), (3*x*
sinh(d + e*x)**4/8 - 3*x*sinh(d + e*x)**2*cosh(d + e*x)**2/4 + 3*x*cosh(d
+ e*x)**4/8 + 5*sinh(d + e*x)**3*cosh(d + e*x)/(8*e) - 3*sinh(d + e*x)*cos
h(d + e*x)**3/(8*e), Eq(F, 1)), (F**(a*c)*(3*x*sinh(d + e*x)**4/8 - 3*x*si
nh(d + e*x)**2*cosh(d + e*x)**2/4 + 3*x*cosh(d + e*x)**4/8 + 5*sinh(d + e*
x)**3*cosh(d + e*x)/(8*e) - 3*sinh(d + e*x)*cosh(d + e*x)**3/(8*e)), Eq(b,
 0)), (3*x*sinh(d + e*x)**4/8 - 3*x*sinh(d + e*x)**2*cosh(d + e*x)**2/4 +
3*x*cosh(d + e*x)**4/8 + 5*sinh(d + e*x)**3*cosh(d + e*x)/(8*e) - 3*sinh(d
 + e*x)*cosh(d + e*x)**3/(8*e), Eq(c, 0)), (F**(a*c + b*c*x)*x*sinh(b*c*x*
log(F)/2 - d)**4/4 - F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/2 - d)**3*cosh(b
*c*x*log(F)/2 - d)/2 + F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/2 - d)*cosh(b*
c*x*log(F)/2 - d)**3/2 - F**(a*c + b*c*x)*x*cosh(b*c*x*log(F)/2 - d)**4/4
+ F**(a*c + b*c*x)*sinh(b*c*x*log(F)/2 - d)**4/(24*b*c*log(F)) + 17*F**(a*
c + b*c*x)*sinh(b*c*x*log(F)/2 - d)**3*cosh(b*c*x*log(F)/2 - d)/(12*b*c*lo
g(F)) - F**(a*c + b*c*x)*sinh(b*c*x*log(F)/2 - d)**2*cosh(b*c*x*log(F)/2 -
 d)**2/(b*c*log(F)) - 3*F**(a*c + b*c*x)*sinh(b*c*x*log(F)/2 - d)*cosh(b*c
*x*log(F)/2 - d)**3/(4*b*c*log(F)) + 5*F**(a*c + b*c*x)*cosh(b*c*x*log(F)/
2 - d)**4/(8*b*c*log(F)), Eq(e, -b*c*log(F)/2)), (F**(a*c + b*c*x)*x*sinh(
b*c*x*log(F)/4 - d)**4/16 - F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/4 - d)**3
*cosh(b*c*x*log(F)/4 - d)/4 + 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/4 ...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.02

$$\int F^{c(a+bx)}\sinh^4(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+4\,ex+4\,d)}}{16\,(bc\log(F)+4\,e)} - \frac{F^{ac}e^{(bcx\log(F)+2\,ex+2\,d)}}{4\,(bc\log(F)+2\,e)} - \frac{F^{ac}e^{(bcx\log(F)-2\,ex)}}{4\,(bce^{(2\,d)}\log(F)-2\,ee^{(2\,d)})} + \frac{F^{ac}e^{(bcx\log(F)-4\,ex)}}{16\,(bce^{(4\,d)}\log(F)-4\,ee^{(4\,d)})} + \frac{3\,F^{bcx+ac}}{8\,bc\log(F)}$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)^4,x, algorithm="maxima")
```

output
```
1/16*F^(a*c)*e^(b*c*x*log(F) + 4*e*x + 4*d)/(b*c*log(F) + 4*e) - 1/4*F^(a*
c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) - 1/4*F^(a*c)*e^(b*c*
x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) + 1/16*F^(a*c)*e^(b*c
*x*log(F) - 4*e*x)/(b*c*e^(4*d)*log(F) - 4*e*e^(4*d)) + 3/8*F^(b*c*x + a*c
)/(b*c*log(F))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 1502, normalized size of antiderivative = 9.27

$$\int F^{c(a+bx)} \sinh^4(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)^4,x, algorithm="giac")
```

output
```
3/4*(2*b*c*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1
/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)
^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1
/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) -
 pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 3*I*(I*e^(1/2*I*pi*
b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(8*I*p
i*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F))) - I*e^(-1/2*I*pi*b*c*x*sgn
(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sg
n(F) + 8*I*pi*b*c + 16*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(ab
s(F))) + 1/8*(2*(b*c*log(abs(F)) + 4*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*
b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b
*c*log(abs(F)) + 4*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(
F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b
*c)^2 + 4*(b*c*log(abs(F)) + 4*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)
) + 4*e)*x + 4*d) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I
*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(16*I*pi*b*c*sgn(F) - 16*I*pi*b*c + 32*b*c*
log(abs(F)) + 128*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*
I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-16*I*pi*b*c*sgn(F) + 16*I*pi*b*c + 32*b*
c*log(abs(F)) + 128*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 4*e)*x + 4
*d) - 1/2*(2*(b*c*log(abs(F)) + 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*...
```

Mupad [B] (verification not implemented)

Time = 5.08 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.59

$$\int F^{c(a+bx)}\sinh^4(d+ex)\,dx = \frac{F^{a\,c+b\,c\,x}\left(24\,e^4+\frac{3\,b^4\,c^4\,\ln(F)^4}{8}-\frac{b^4\,c^4\,\ln(F)^4\cosh(2\,d+2\,e\,x)}{2}+\frac{b^4\,c^4\,\ln(F)^4\cosh(4\,d+4\,e\,x)}{8}-\frac{15\,b^2\,c^2\,e^2\,\ln(F)^2}{2}+b^3\,c^3\,e\ln(\ldots\right.}{}$$

input
```
int(F^(c*(a + b*x))*sinh(d + e*x)^4,x)
```

output
```
(F^(a*c + b*c*x)*(24*e^4 + (3*b^4*c^4*log(F)^4)/8 - (b^4*c^4*log(F)^4*cosh
(2*d + 2*e*x))/2 + (b^4*c^4*log(F)^4*cosh(4*d + 4*e*x))/8 - (15*b^2*c^2*e^
2*log(F)^2)/2 + b^3*c^3*e*log(F)^3*sinh(2*d + 2*e*x) - (b^3*c^3*e*log(F)^3
*sinh(4*d + 4*e*x))/2 - 16*b*c*e^3*log(F)*sinh(2*d + 2*e*x) + 2*b*c*e^3*lo
g(F)*sinh(4*d + 4*e*x) + 8*b^2*c^2*e^2*log(F)^2*cosh(2*d + 2*e*x) - (b^2*c
^2*e^2*log(F)^2*cosh(4*d + 4*e*x))/2))/(b*c*log(F)*(64*e^4 + b^4*c^4*log(F
)^4 - 20*b^2*c^2*e^2*log(F)^2))
```

Reduce [F]

$$\int F^{c(a+bx)}\sinh^4(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\sinh(ex+d)^4\,dx\right)$$

input
```
int(F^(c*(b*x+a))*sinh(e*x+d)^4,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sinh(d + e*x)**4,x)
```

3.17 $\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)\,dx$

Optimal result

Integrand size = 56, antiderivative size = 73

$$\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)\,dx = -\frac{6de^{a+bx}\cosh(c+dx)\sqrt{\sinh(c+dx)}}{4b^2-9d^2}+\frac{4be^{a+bx}\sinh^{\frac{3}{2}}(c+dx)}{4b^2-9d^2}$$

output
```
-6*d*exp(b*x+a)*cosh(d*x+c)*sinh(d*x+c)^(1/2)/(4*b^2-9*d^2)+4*b*exp(b*x+a)
*sinh(d*x+c)^(3/2)/(4*b^2-9*d^2)
```

Mathematica [A] (verified)

Time = 7.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)\,dx = \frac{2e^{a+bx}\sqrt{\sinh(c+dx)}(-3d\cosh(c+dx)+2b\sinh(c+dx))}{4b^2-9d^2}$$

input
```
Integrate[(-3*d^2*E^(a + b*x))/(4*(b^2 - (9*d^2)/4)*Sqrt[Sinh[c + d*x]]) +
 E^(a + b*x)*Sinh[c + d*x]^(3/2),x]
```

output
```
(2*E^(a + b*x)*Sqrt[Sinh[c + d*x]]*(-3*d*Cosh[c + d*x] + 2*b*Sinh[c + d*x]
))/(4*b^2 - 9*d^2)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.018, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{4be^{a+bx}\sinh^{\frac{3}{2}}(c+dx)}{4b^2-9d^2}-\frac{6de^{a+bx}\sqrt{\sinh(c+dx)}\cosh(c+dx)}{4b^2-9d^2}$$

input
```
Int[(-3*d^2*E^(a + b*x))/(4*(b^2 - (9*d^2)/4)*Sqrt[Sinh[c + d*x]]) + E^(a
+ b*x)*Sinh[c + d*x]^(3/2),x]
```

output
```
(-6*d*E^(a + b*x)*Cosh[c + d*x]*Sqrt[Sinh[c + d*x]])/(4*b^2 - 9*d^2) + (4*
b*E^(a + b*x)*Sinh[c + d*x]^(3/2))/(4*b^2 - 9*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(-\frac{3d^2 e^{bx+a}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh\left(dx+c\right)}}+e^{bx+a}\sinh\left(dx+c\right)^{\frac{3}{2}}\right)dx$$

input
```
int(-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/sinh(d*x+c)^(1/2)+exp(b*x+a)*sinh(d*
x+c)^(3/2),x)
```

output
```
int(-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/sinh(d*x+c)^(1/2)+exp(b*x+a)*sinh(d*
x+c)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)dx$$
$$= \text{Exception raised: TypeError}$$

input
```
integrate(-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/sinh(d*x+c)^(1/2)+exp(b*x+a)*s
inh(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)dx$$
$$=\frac{\left(\int 4b^2e^{bx}\sinh^{\frac{3}{2}}(c+dx)\,dx+\int\left(-\frac{3d^2e^{bx}}{\sqrt{\sinh(c+dx)}}\right)dx+\int\left(-9d^2e^{bx}\sinh^{\frac{3}{2}}(c+dx)\right)dx\right)e^a}{(2b-3d)\,(2b+3d)}$$

input
```
integrate(-3/4*d**2*exp(b*x+a)/(b**2-9/4*d**2)/sinh(d*x+c)**(1/2)+exp(b*x+
a)*sinh(d*x+c)**(3/2),x)
```

output
```
(Integral(4*b**2*exp(b*x)*sinh(c + d*x)**(3/2), x) + Integral(-3*d**2*exp(
b*x)/sqrt(sinh(c + d*x)), x) + Integral(-9*d**2*exp(b*x)*sinh(c + d*x)**(3
/2), x))*exp(a)/((2*b - 3*d)*(2*b + 3*d))
```

Maxima [F]

$$\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)dx$$
$$=\int e^{(bx+a)}\sinh(dx+c)^{\frac{3}{2}}-\frac{3\,d^2e^{(bx+a)}}{(4\,b^2-9\,d^2)\sqrt{\sinh(dx+c)}}\,dx$$

input
```
integrate(-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/sinh(d*x+c)^(1/2)+exp(b*x+a)*s
inh(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(e^(b*x + a)*sinh(d*x + c)^(3/2) - 3*d^2*e^(b*x + a)/((4*b^2 - 9*
d^2)*sqrt(sinh(d*x + c))), x)
```

Giac [F]

$$\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)dx$$
$$=\int e^{(bx+a)}\sinh\left(dx+c\right)^{\frac{3}{2}}-\frac{3\,d^2e^{(bx+a)}}{(4\,b^2-9\,d^2)\sqrt{\sinh\left(dx+c\right)}}\,dx$$

input
```
integrate(-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/sinh(d*x+c)^(1/2)+exp(b*x+a)*s
inh(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(e^(b*x + a)*sinh(d*x + c)^(3/2) - 3*d^2*e^(b*x + a)/((4*b^2 - 9*
d^2)*sqrt(sinh(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)dx$$
$$=\int \mathrm{e}^{a+b\,x}\sinh(c+d\,x)^{3/2}-\frac{3\,d^2\,\mathrm{e}^{a+b\,x}}{4\,\sqrt{\sinh\left(c+d\,x\right)}\,\left(b^2-\frac{9\,d^2}{4}\right)}\,dx$$

input
```
int(exp(a + b*x)*sinh(c + d*x)^(3/2) - (3*d^2*exp(a + b*x))/(4*sinh(c + d*
x)^(1/2)*(b^2 - (9*d^2)/4)),x)
```

output
```
int(exp(a + b*x)*sinh(c + d*x)^(3/2) - (3*d^2*exp(a + b*x))/(4*sinh(c + d*
x)^(1/2)*(b^2 - (9*d^2)/4)), x)
```

Reduce [F]

$$\int \left(-\frac{3d^2e^{a+bx}}{4\left(b^2-\frac{9d^2}{4}\right)\sqrt{\sinh(c+dx)}}+e^{a+bx}\sinh^{\frac{3}{2}}(c+dx)\right)dx$$
$$=\frac{e^a\left(8e^{bx}\sqrt{\sinh\left(dx+c\right)}\,\sinh\left(dx+c\right)b^2-18e^{bx}\sqrt{\sinh\left(dx+c\right)}\,\sinh\left(dx+c\right)d^2-6\left(\int\frac{e^{bx}\sqrt{\sinh(dx+c)}}{\sinh(dx+c)}dx\right)\right.}{2b\left(4b^2\right.}$$

input
```
int(-3/4*d^2*exp(b*x+a)/(b^2-9/4*d^2)/sinh(d*x+c)^(1/2)+exp(b*x+a)*sinh(d*
x+c)^(3/2),x)
```

output
```
(e**a*(8*e**(b*x)*sqrt(sinh(c + d*x))*sinh(c + d*x)*b**2 - 18*e**(b*x)*sqr
t(sinh(c + d*x))*sinh(c + d*x)*d**2 - 6*int((e**(b*x)*sqrt(sinh(c + d*x)))
/sinh(c + d*x),x)*b*d**2 - 12*int(e**(b*x)*sqrt(sinh(c + d*x))*cosh(c + d*
x),x)*b**2*d + 27*int(e**(b*x)*sqrt(sinh(c + d*x))*cosh(c + d*x),x)*d**3))
/(2*b*(4*b**2 - 9*d**2))
```

3.18 $\int e^{a+bx}\sinh^n(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 76

$$\int e^{a+bx}\sinh^n(a+bx)\,dx = \frac{e^{a+bx}\left(1-e^{2a+2bx}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1-n}{2},-n,\frac{3-n}{2},e^{2a+2bx}\right)\sinh^n(a+bx)}{b(1-n)}$$

output
```
exp(b*x+a)*hypergeom([-n, 1/2-1/2*n],[3/2-1/2*n],exp(2*b*x+2*a))*sinh(b*x+
a)^n/b/((1-exp(2*b*x+2*a))^n)/(1-n)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13

$$\int e^{a+bx}\sinh^n(a+bx)\,dx = \frac{e^{a+bx}\left(2-2e^{-2(a+bx)}\right)^{-n}\left(-e^{-a-bx}+e^{a+bx}\right)^n\text{Hypergeometric2F1}\left(\frac{1}{2}(-1-n),-n,\frac{1-n}{2},e^{-2(a+bx)}\right)}{b(1+n)}$$

input
```
Integrate[E^(a + b*x)*Sinh[a + b*x]^n,x]
```

output
```
(E^(a + b*x)*(-E^(-a - b*x) + E^(a + b*x))^n*Hypergeometric2F1[(-1 - n)/2,
 -n, (1 - n)/2, E^(-2*(a + b*x))])/(b*(2 - 2/E^(2*(a + b*x)))^n*(1 + n))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2720, 27, 1917, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx} \sinh^n(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int 2^{-n}\left(-e^{-a-bx}+e^{a+bx}\right)^n de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2^{-n}\int \left(-e^{-a-bx}+e^{a+bx}\right)^n de^{a+bx}}{b}$$

$$\Big\downarrow \text{1917}$$

$$\frac{2^{-n}\left(e^{a+bx}\right)^n\left(e^{a+bx}-e^{-a-bx}\right)^n\left(e^{2a+2bx}-1\right)^{-n}\int \left(e^{a+bx}\right)^{-n}\left(-1+e^{2a+2bx}\right)^n de^{a+bx}}{b}$$

$$\Big\downarrow \text{279}$$

$$\frac{2^{-n}\left(e^{a+bx}\right)^n\left(e^{a+bx}-e^{-a-bx}\right)^n\left(1-e^{2a+2bx}\right)^{-n}\int \left(e^{a+bx}\right)^{-n}\left(1-e^{2a+2bx}\right)^n de^{a+bx}}{b}$$

$$\Big\downarrow \text{278}$$

$$\frac{2^{-n}e^{a+bx}\left(e^{a+bx}-e^{-a-bx}\right)^n\left(1-e^{2a+2bx}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1-n}{2},-n,\frac{3-n}{2},e^{2a+2bx}\right)}{b(1-n)}$$

input
```
Int[E^(a + b*x)*Sinh[a + b*x]^n,x]
```

output
```
(E^(a + b*x)*(-E^(-a - b*x) + E^(a + b*x))^n*Hypergeometric2F1[(1 - n)/2,
-n, (3 - n)/2, E^(2*a + 2*b*x)])/(2^n*b*(1 - E^(2*a + 2*b*x))^n*(1 - n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [F]

$$\int e^{bx+a} \sinh(bx+a)^n \, dx$$

input
```
int(exp(b*x+a)*sinh(b*x+a)^n,x)
```

output
```
int(exp(b*x+a)*sinh(b*x+a)^n,x)
```

Fricas [F]

$$\int e^{a+bx} \sinh^n(a+bx) \, dx = \int \sinh(bx+a)^n e^{(bx+a)} \, dx$$

input
```
integrate(exp(b*x+a)*sinh(b*x+a)^n,x, algorithm="fricas")
```

output
```
integral(sinh(b*x + a)^n*e^(b*x + a), x)
```

Sympy [F]

$$\int e^{a+bx} \sinh^n(a+bx) \, dx = e^a \int e^{bx} \sinh^n(a+bx) \, dx$$

input
```
integrate(exp(b*x+a)*sinh(b*x+a)**n,x)
```

output
```
exp(a)*Integral(exp(b*x)*sinh(a + b*x)**n, x)
```

Maxima [F]

$$\int e^{a+bx}\sinh^n(a+bx)\,dx=\int \sinh\left(bx+a\right)^n e^{(bx+a)}\,dx$$

input
```
integrate(exp(b*x+a)*sinh(b*x+a)^n,x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)^n*e^(b*x + a), x)
```

Giac [F]

$$\int e^{a+bx}\sinh^n(a+bx)\,dx=\int \sinh\left(bx+a\right)^n e^{(bx+a)}\,dx$$

input
```
integrate(exp(b*x+a)*sinh(b*x+a)^n,x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^n*e^(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\sinh^n(a+bx)\,dx=\int \mathrm{e}^{a+b\,x}\,\sinh(a+b\,x)^n\,dx$$

input
```
int(exp(a + b*x)*sinh(a + b*x)^n,x)
```

output
```
int(exp(a + b*x)*sinh(a + b*x)^n, x)
```

Reduce [F]

$$\int e^{a+bx}\sinh^n(a+bx)\,dx$$

$$= \frac{e^a\left(-e^{bx}\left(e^{2bx+2a}-1\right)^n n+e^{bnx+an+bx}\sinh\left(bx+a\right)^n 2^n n+e^{bnx+an+bx}\sinh\left(bx+a\right)^n 2^n-2e^{bnx+an}\left(\int \frac{}{e^{bn}}\right.\right.}{e^{bn}}$$

input
```
int(exp(b*x+a)*sinh(b*x+a)^n,x)
```

output
```
(e**a*( - e**(b*x)*(e**(2*a + 2*b*x) - 1)**n*n + e**(a*n + b*n*x + b*x)*si
nh(a + b*x)**n*2**n*n + e**(a*n + b*n*x + b*x)*sinh(a + b*x)**n*2**n - 2*e
**(a*n + b*n*x)*int((e**(b*x)*(e**(2*a + 2*b*x) - 1)**n)/(e**(a*n + 2*a +
b*n*x + 2*b*x)*n + e**(a*n + 2*a + b*n*x + 2*b*x) - e**(a*n + b*n*x)*n - e
**(a*n + b*n*x)),x)*b*n**2 - 2*e**(a*n + b*n*x)*int((e**(b*x)*(e**(2*a + 2
*b*x) - 1)**n)/(e**(a*n + 2*a + b*n*x + 2*b*x)*n + e**(a*n + 2*a + b*n*x +
 2*b*x) - e**(a*n + b*n*x)*n - e**(a*n + b*n*x)),x)*b*n))/(e**(a*n + b*n*x
)*2**n*b*(n + 1))
```

3.19 $\int F^{c(a+bx)}(f\sinh(d+ex))^n\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int F^{c(a+bx)}(f\sinh(d+ex))^n\,dx = -\frac{\left(1-e^{2d+2ex}\right)^{-n}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(-n,\frac{1}{2}\left(-n+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(2-n+\frac{bc\log(F)}{e}\right),e^{2d+2ex}\right)(}{en-bc\log(F)}$$

output
```
-F^(c*(b*x+a))*hypergeom([-n, -1/2*n+1/2*b*c*ln(F)/e],[1-1/2*n+1/2*b*c*ln(
F)/e],exp(2*e*x+2*d))*(f*sinh(e*x+d))^n/((1-exp(2*e*x+2*d))^n)/(e*n-b*c*ln
(F))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(f\sinh(d+ex))^n\,dx = \frac{\left(1-e^{2(d+ex)}\right)^{-n}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(-n,\frac{-en+bc\log(F)}{2e},1+\frac{-en+bc\log(F)}{2e},e^{2(d+ex)}\right)(f\sinh(d+e}{-en+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Sinh[d + e*x])^n,x]
```

output
```
(F^(c*(a + b*x))*Hypergeometric2F1[-n, (-(e*n) + b*c*Log[F])/(2*e), 1 + (-
(e*n) + b*c*Log[F])/(2*e), E^(2*(d + e*x))]*(f*Sinh[d + e*x])^n)/((1 - E^(
2*(d + e*x)))^n*(-(e*n) + b*c*Log[F]))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {7271, 6005, 2689}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\sinh(d+ex))^n\,dx$$

$$\Big\downarrow \; 7271$$

$$\sinh^{-n}(d+ex)(f\sinh(d+ex))^n\int F^{c(a+bx)}\sinh^n(d+ex)dx$$

$$\Big\downarrow \; 6005$$

$$e^{n(d+ex)}\left(e^{2(d+ex)}-1\right)^{-n}(f\sinh(d+ex))^n\int e^{-n(d+ex)}\left(-1+e^{2(d+ex)}\right)^nF^{c(a+bx)}dx$$

$$\Big\downarrow \; 2689$$

$$-\frac{\left(1-e^{2(d+ex)}\right)^{-n}F^{c(a+bx)}(f\sinh(d+ex))^n\,\text{Hypergeometric2F1}\left(-n,-\frac{en-bc\log(F)}{2e},\frac{1}{2}\left(-n+\frac{bc\log(F)}{e}+2\right),e^{2(d+ex)}\right)}{en-bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*(f*Sinh[d + e*x])^n,x]
```

output
```
-((F^(c*(a + b*x))*Hypergeometric2F1[-n, -1/2*(e*n - b*c*Log[F])/e, (2 - n
 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))]*(f*Sinh[d + e*x])^n)/((1 - E^(2*(d
+ e*x)))^n*(e*n - b*c*Log[F])))
```

Defintions of rubi rules used

rule 2689
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

rule 6005
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[E^(n*(d + e*x))*(Sinh[d + e*x]^n/(-1 + E^(2*(d + e*x)))^n)   In
t[F^(c*(a + b*x))*((-1 + E^(2*(d + e*x)))^n/E^(n*(d + e*x))), x], x] /; Fre
eQ[{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}(f\sinh(ex+d))^n\,dx$$

input
```
int(F^(c*(b*x+a))*(f*sinh(e*x+d))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*sinh(e*x+d))^n,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\sinh(d+ex))^n\,dx=\int (f\sinh(ex+d))^nF^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sinh(e*x+d))^n,x, algorithm="fricas")
```

output
```
integral((f*sinh(e*x + d))^n*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}(f\sinh(d+ex))^n\,dx=\int F^{c(a+bx)}(f\sinh(d+ex))^n\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*sinh(e*x+d))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*sinh(d + e*x))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}(f\sinh(d+ex))^n\,dx=\int (f\sinh(ex+d))^nF^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sinh(e*x+d))^n,x, algorithm="maxima")
```

output
```
integrate((f*sinh(e*x + d))^n*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\sinh(d+ex))^n\,dx=\int (f\sinh(ex+d))^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sinh(e*x+d))^n,x, algorithm="giac")
```

output
```
integrate((f*sinh(e*x + d))^n*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\sinh(d+ex))^n\,dx=\int F^{c\,(a+b\,x)}\,(f\sinh(d+e\,x))^n\,dx$$

input
```
int(F^(c*(a + b*x))*(f*sinh(d + e*x))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f*sinh(d + e*x))^n, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\sinh(d+ex))^n\,dx$$
$$=\frac{f^{ac+n}\left(f^{bcx}\sinh(ex+d)^n-\left(\int\frac{f^{bcx}\sinh(ex+d)^n\cosh(ex+d)}{\sinh(ex+d)}dx\right)en\right)}{\log(f)\,bc}$$

input
```
int(F^(c*(b*x+a))*(f*sinh(e*x+d))^n,x)
```

output
```
(f**(a*c + n)*(f**(b*c*x)*sinh(d + e*x)**n - int((f**(b*c*x)*sinh(d + e*x)
**n*cosh(d + e*x))/sinh(d + e*x),x)*e*n))/(log(f)*b*c)
```

3.20 $\int F^{c(a+bx)}\left(f\sinh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)^n\,dx$

Optimal result

Integrand size = 28, antiderivative size = 82

$$\int F^{c(a+bx)}\left(f\sinh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)^n\,dx = \frac{F^{c(a+bx)}(2+n)\left(\cosh\left(d+\frac{bcx\log(F)}{2+n}\right)-\sinh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)\left(f\sinh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)^{1+n}}{bcf(1+n)\log(F)}$$

output
```
F^(c*(b*x+a))*(2+n)*(cosh(d+b*c*x*ln(F)/(2+n))-sinh(d+b*c*x*ln(F)/(2+n)))*
(f*sinh(d+b*c*x*ln(F)/(2+n)))^(1+n)/b/c/f/(1+n)/ln(F)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int F^{c(a+bx)}\left(f\sinh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)^n\,dx = \frac{e^{-2d}F^{c\left(a+\frac{bnx}{2+n}\right)}\left(-1+e^{2d}F^{\frac{2bcx}{2+n}}\right)(2+n)\left(f\sinh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)^n}{2bc(1+n)\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Sinh[d + (b*c*x*Log[F])/(2 + n)])^n,x]
```

output
```
(F^(c*(a + (b*n*x)/(2 + n)))*(-1 + E^(2*d)*F^((2*b*c*x)/(2 + n)))*(2 + n)*
(f*Sinh[d + (b*c*x*Log[F])/(2 + n)])^n)/(2*b*c*E^(2*d)*(1 + n)*Log[F])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.77, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {7271, 6001}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\left(f\sinh\left(\frac{bcx\log(F)}{n+2}+d\right)\right)^n dx$$

$$\Big\downarrow \text{7271}$$

$$\sinh^{-n}\left(\frac{bcx\log(F)}{n+2}+d\right)\left(f\sinh\left(\frac{bcx\log(F)}{n+2}+d\right)\right)^n\int F^{c(a+bx)}\sinh^n\left(d+\frac{bcx\log(F)}{n+2}\right)dx$$

$$\Big\downarrow \text{6001}$$

$$\sinh^{-n}\left(\frac{bcx\log(F)}{n+2}+d\right)\left(f\sinh\left(\frac{bcx\log(F)}{n+2}+d\right)\right)^n\left(\frac{(n+2)F^{c(a+bx)}\sinh^{n+1}\left(\frac{bcx\log(F)}{n+2}+d\right)\cosh\left(\frac{bcx\log(F}{n+2}\right.}{bc(n+1)\log(F)}\right.$$

input
```
Int[F^(c*(a + b*x))*(f*Sinh[d + (b*c*x*Log[F])/(2 + n)])^n,x]
```

output
```
((f*Sinh[d + (b*c*x*Log[F])/(2 + n)])^n*((F^(c*(a + b*x))*(2 + n)*Cosh[d +
 (b*c*x*Log[F])/(2 + n)]*Sinh[d + (b*c*x*Log[F])/(2 + n)]^(1 + n))/(b*c*(1
 + n)*Log[F]) - (F^(c*(a + b*x))*(2 + n)*Sinh[d + (b*c*x*Log[F])/(2 + n)]^
(2 + n))/(b*c*(1 + n)*Log[F])))/Sinh[d + (b*c*x*Log[F])/(2 + n)]^n
```

Defintions of rubi rules used

rule 6001
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]^(n + 2)/(e^2*(n +
1)*(n + 2))), x] + Simp[F^(c*(a + b*x))*Cosh[d + e*x]*(Sinh[d + e*x]^(n + 1
)/(e*(n + 1))), x] /; FreeQ[{F, a, b, c, d, e, n}, x] && EqQ[e^2*(n + 2)^2
- b^2*c^2*Log[F]^2, 0] && NeQ[n, -1] && NeQ[n, -2]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}\left(f\sinh\left(d+\frac{bcx\ln(F)}{2+n}\right)\right)^{n}dx$$

input
```
int(F^(c*(b*x+a))*(f*sinh(d+b*c*x*ln(F)/(2+n)))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*sinh(d+b*c*x*ln(F)/(2+n)))^n,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(82) = 164.

Time = 0.12 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.26

$$\int F^{c(a+bx)}\left(f\sinh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)^{n}dx$$
$$=\frac{\left((n+2)\cosh\left((bcx+ac)\log(F)\right)\sinh\left(\frac{bcx\log(F)+dn+2d}{n+2}\right)+(n+2)\sinh\left((bcx+ac)\log(F)\right)\sinh\left(\frac{bcx\log(\right.\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*(f*sinh(d+b*c*x*log(F)/(2+n)))^n,x, algorithm="fri
cas")
```

output
```
(((n + 2)*cosh((b*c*x + a*c)*log(F))*sinh((b*c*x*log(F) + d*n + 2*d)/(n +
2)) + (n + 2)*sinh((b*c*x + a*c)*log(F))*sinh((b*c*x*log(F) + d*n + 2*d)/(
n + 2)))*cosh(n*log(f*sinh((b*c*x*log(F) + d*n + 2*d)/(n + 2)))) + ((n + 2
)*cosh((b*c*x + a*c)*log(F))*sinh((b*c*x*log(F) + d*n + 2*d)/(n + 2)) + (n
 + 2)*sinh((b*c*x + a*c)*log(F))*sinh((b*c*x*log(F) + d*n + 2*d)/(n + 2)))
*sinh(n*log(f*sinh((b*c*x*log(F) + d*n + 2*d)/(n + 2)))))/((b*c*n + b*c)*c
osh((b*c*x*log(F) + d*n + 2*d)/(n + 2))*log(F) + (b*c*n + b*c)*log(F)*sinh
((b*c*x*log(F) + d*n + 2*d)/(n + 2)))
```

Sympy [F]

$$\int F^{c(a+bx)}\left(f\sinh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)^n dx = \int F^{c(a+bx)}\left(f\sinh\left(\frac{bcx\log(F)}{n+2}+d\right)\right)^n dx$$

input
```
integrate(F**(c*(b*x+a))*(f*sinh(d+b*c*x*ln(F)/(2+n)))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*sinh(b*c*x*log(F)/(n + 2) + d))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\left(f\sinh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)^n dx = \int \left(f\sinh\left(\frac{bcx\log(F)}{n+2}+d\right)\right)^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sinh(d+b*c*x*log(F)/(2+n)))^n,x, algorithm="max
ima")
```

output
```
integrate((f*sinh(b*c*x*log(F)/(n + 2) + d))^n*F^((b*x + a)*c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(82) = 164$.

Time $= 0.45$ (sec) , antiderivative size $= 530$, normalized size of antiderivative $= 6.46$

$$\int F^{c(a+bx)}\left(f\sinh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)^n dx$$

$$= \frac{F^{ac}ne^{\left(\frac{2\,bcx\log(F)-dn^2-n^2\log(2)+n^2\log\left(fe^{\left(\frac{2\,(bcx\log(F)+dn+2\,d)}{n+2}\right)}-f\right)-2\,dn-2\,n\log(2)+2\,n\log\left(fe^{\left(\frac{2\,(bcx\log(F)+dn+2\,d)}{n+2}\right)}-f\right)}{n+2}+\frac{2\,(bcx\log(F)}{n+}\right.}}{}$$

input
```
integrate(F^(c*(b*x+a))*(f*sinh(d+b*c*x*log(F)/(2+n)))^n,x, algorithm="gia
c")
```

output
```
1/2*(F^(a*c)*n*e^((2*b*c*x*log(F) - d*n^2 - n^2*log(2) + n^2*log(f*e^(2*(b
*c*x*log(F) + d*n + 2*d)/(n + 2)) - f) - 2*d*n - 2*n*log(2) + 2*n*log(f*e^
(2*(b*c*x*log(F) + d*n + 2*d)/(n + 2)) - f))/(n + 2) + 2*(b*c*x*log(F) + d
*n + 2*d)/(n + 2)) - F^(a*c)*n*e^((2*b*c*x*log(F) - d*n^2 - n^2*log(2) + n
^2*log(f*e^(2*(b*c*x*log(F) + d*n + 2*d)/(n + 2)) - f) - 2*d*n - 2*n*log(2
) + 2*n*log(f*e^(2*(b*c*x*log(F) + d*n + 2*d)/(n + 2)) - f))/(n + 2)) + 2*
F^(a*c)*e^((2*b*c*x*log(F) - d*n^2 - n^2*log(2) + n^2*log(f*e^(2*(b*c*x*lo
g(F) + d*n + 2*d)/(n + 2)) - f) - 2*d*n - 2*n*log(2) + 2*n*log(f*e^(2*(b*c
*x*log(F) + d*n + 2*d)/(n + 2)) - f))/(n + 2) + 2*(b*c*x*log(F) + d*n + 2*
d)/(n + 2)) - 2*F^(a*c)*e^((2*b*c*x*log(F) - d*n^2 - n^2*log(2) + n^2*log(
f*e^(2*(b*c*x*log(F) + d*n + 2*d)/(n + 2)) - f) - 2*d*n - 2*n*log(2) + 2*n
*log(f*e^(2*(b*c*x*log(F) + d*n + 2*d)/(n + 2)) - f))/(n + 2)))/(b*c*n*e^(
2*(b*c*x*log(F) + d*n + 2*d)/(n + 2))*log(F) + b*c*e^(2*(b*c*x*log(F) + d*
n + 2*d)/(n + 2))*log(F))
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\left(f\sinh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)^n\,dx$$
$$=\int F^{c\,(a+b\,x)}\,\left(f\sinh\left(d+\frac{b\,c\,x\,\ln(F)}{n+2}\right)\right)^n\,dx$$

input
```
int(F^(c*(a + b*x))*(f*sinh(d + (b*c*x*log(F))/(n + 2)))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f*sinh(d + (b*c*x*log(F))/(n + 2)))^n, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.90

$$\int F^{c(a+bx)}\left(f\sinh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)^n\,dx$$
$$=\frac{f^{ac+n}\left(-e^{\frac{2\log(f)bcx+2dn+4d}{n+2}}\left(e^{\frac{2\log(f)bcx+2dn+4d}{n+2}}-1\right)^n n-\left(e^{\frac{2\log(f)bcx+2dn+4d}{n+2}}-1\right)^n n-2\left(e^{\frac{2\log(f)bcx+2dn+4d}{n+2}}-1\right)^n}{2e^{dn+2d}2^n\log(f)\,bc\,(n+}$$

input
```
int(F^(c*(b*x+a))*(f*sinh(d+b*c*x*log(F)/(2+n)))^n,x)
```

output
```
(f**(a*c + n)*( - e**((2*log(f)*b*c*x + 2*d*n + 4*d)/(n + 2))*(e**((2*log(
f)*b*c*x + 2*d*n + 4*d)/(n + 2)) - 1)**n*n - (e**((2*log(f)*b*c*x + 2*d*n
+ 4*d)/(n + 2)) - 1)**n*n - 2*(e**((2*log(f)*b*c*x + 2*d*n + 4*d)/(n + 2))
 - 1)**n + 2*f**(b*c*x)*e**(d*n + 2*d)*sinh((log(f)*b*c*x + d*n + 2*d)/(n
+ 2))**n*2**n*n + 2*f**(b*c*x)*e**(d*n + 2*d)*sinh((log(f)*b*c*x + d*n + 2
*d)/(n + 2))**n*2**n))/(2*e**(d*n + 2*d)*2**n*log(f)*b*c*(n + 1))
```

3.21 $\int F^{c(a+bx)}\left(f\sinh\left(d-\frac{bcx\log(F)}{2+n}\right)\right)^n\,dx$

Optimal result

Integrand size = 29, antiderivative size = 84

$$\int F^{c(a+bx)}\left(f\sinh\left(d-\frac{bcx\log(F)}{2+n}\right)\right)^n\,dx = -\frac{F^{c(a+bx)}(2+n)\left(f\sinh\left(d-\frac{bcx\log(F)}{2+n}\right)\right)^{1+n}\left(\cosh\left(d-\frac{bcx\log(F)}{2+n}\right)+\sinh\left(d-\frac{bcx\log(F)}{2+n}\right)\right)}{bcf(1+n)\log(F)}$$

output
```
-F^(c*(b*x+a))*(2+n)*(-f*sinh(-d+b*c*x*ln(F)/(2+n)))^(1+n)*(cosh(-d+b*c*x*
ln(F)/(2+n))-sinh(-d+b*c*x*ln(F)/(2+n)))/b/c/f/(1+n)/ln(F)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85

$$\int F^{c(a+bx)}\left(f\sinh\left(d-\frac{bcx\log(F)}{2+n}\right)\right)^n\,dx = \frac{F^{c(a+bx)}\left(1-e^{2d}F^{-\frac{2bcx}{2+n}}\right)(2+n)\left(f\sinh\left(d-\frac{bcx\log(F)}{2+n}\right)\right)^n}{2bc(1+n)\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Sinh[d - (b*c*x*Log[F])/(2 + n)])^n,x]
```

output
```
(F^(c*(a + b*x))*(1 - E^(2*d)/F^((2*b*c*x)/(2 + n)))*(2 + n)*(f*Sinh[d - (
b*c*x*Log[F])/(2 + n)])^n)/(2*b*c*(1 + n)*Log[F])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.80, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {7271, 6001}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\left(f\sinh\left(d-\frac{bcx\log(F)}{n+2}\right)\right)^n dx$$

$$\Big\downarrow \text{7271}$$

$$\sinh^{-n}\left(d-\frac{bcx\log(F)}{n+2}\right)\left(f\sinh\left(d-\frac{bcx\log(F)}{n+2}\right)\right)^n\int F^{c(a+bx)}\sinh^n\left(d-\frac{bcx\log(F)}{n+2}\right)dx$$

$$\Big\downarrow \text{6001}$$

$$\sinh^{-n}\left(d-\frac{bcx\log(F)}{n+2}\right)\left(f\sinh\left(d-\frac{bcx\log(F)}{n+2}\right)\right)^n\left(-\frac{(n+2)F^{c(a+bx)}\sinh^{n+2}\left(d-\frac{bcx\log(F)}{n+2}\right)}{bc(n+1)\log(F)}-\frac{(n+2)F\right.$$

input
```
Int[F^(c*(a + b*x))*(f*Sinh[d - (b*c*x*Log[F])/(2 + n)])^n,x]
```

output
```
((f*Sinh[d - (b*c*x*Log[F])/(2 + n)])^n*(-((F^(c*(a + b*x))*(2 + n)*Cosh[d
 - (b*c*x*Log[F])/(2 + n)]*Sinh[d - (b*c*x*Log[F])/(2 + n)]^(1 + n))/(b*c*
(1 + n)*Log[F])) - (F^(c*(a + b*x))*(2 + n)*Sinh[d - (b*c*x*Log[F])/(2 + n
)]^(2 + n))/(b*c*(1 + n)*Log[F])))/Sinh[d - (b*c*x*Log[F])/(2 + n)]^n
```

Defintions of rubi rules used

rule 6001
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]^(n + 2)/(e^2*(n +
1)*(n + 2))), x] + Simp[F^(c*(a + b*x))*Cosh[d + e*x]*(Sinh[d + e*x]^(n + 1
)/(e*(n + 1))), x] /; FreeQ[{F, a, b, c, d, e, n}, x] && EqQ[e^2*(n + 2)^2
- b^2*c^2*Log[F]^2, 0] && NeQ[n, -1] && NeQ[n, -2]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}\left(-f\sinh\left(-d+\frac{bcx\ln(F)}{2+n}\right)\right)^n dx$$

input
```
int(F^(c*(b*x+a))*(-f*sinh(-d+b*c*x*ln(F)/(2+n)))^n,x)
```

output
```
int(F^(c*(b*x+a))*(-f*sinh(-d+b*c*x*ln(F)/(2+n)))^n,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(90) = 180$.

Time $= 0.11$ (sec) , antiderivative size $= 277$, normalized size of antiderivative $= 3.30$

$$\int F^{c(a+bx)}\left(f\sinh\left(d-\frac{bcx\log(F)}{2+n}\right)\right)^n dx$$
$$= \frac{\left((n+2)\cosh\left((bcx+ac)\log(F)\right)\sinh\left(\frac{bcx\log(F)-dn-2d}{n+2}\right)+(n+2)\sinh\left((bcx+ac)\log(F)\right)\sinh\left(\frac{bcx\log(\right.\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*(-f*sinh(-d+b*c*x*log(F)/(2+n)))^n,x, algorithm="f
ricas")
```

output
```
(((n + 2)*cosh((b*c*x + a*c)*log(F))*sinh((b*c*x*log(F) - d*n - 2*d)/(n +
2)) + (n + 2)*sinh((b*c*x + a*c)*log(F))*sinh((b*c*x*log(F) - d*n - 2*d)/(
n + 2)))*cosh(n*log(-f*sinh((b*c*x*log(F) - d*n - 2*d)/(n + 2)))) + ((n +
2)*cosh((b*c*x + a*c)*log(F))*sinh((b*c*x*log(F) - d*n - 2*d)/(n + 2)) + (
n + 2)*sinh((b*c*x + a*c)*log(F))*sinh((b*c*x*log(F) - d*n - 2*d)/(n + 2))
)*sinh(n*log(-f*sinh((b*c*x*log(F) - d*n - 2*d)/(n + 2)))))/((b*c*n + b*c)
*cosh((b*c*x*log(F) - d*n - 2*d)/(n + 2))*log(F) + (b*c*n + b*c)*log(F)*si
nh((b*c*x*log(F) - d*n - 2*d)/(n + 2)))
```

Sympy [F]

$$\int F^{c(a+bx)}\left(f\sinh\left(d-\frac{bcx\log(F)}{2+n}\right)\right)^n dx = \int F^{c(a+bx)}\left(-f\sinh\left(\frac{bcx\log(F)}{n+2}-d\right)\right)^n dx$$

input
```
integrate(F**(c*(b*x+a))*(-f*sinh(-d+b*c*x*ln(F)/(2+n)))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(-f*sinh(b*c*x*log(F)/(n + 2) - d))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\left(f\sinh\left(d-\frac{bcx\log(F)}{2+n}\right)\right)^n dx = \int \left(-f\sinh\left(\frac{bcx\log(F)}{n+2}-d\right)\right)^n F^{(bx+a)c}\, dx$$

input
```
integrate(F^(c*(b*x+a))*(-f*sinh(-d+b*c*x*log(F)/(2+n)))^n,x, algorithm="m
axima")
```

output
```
integrate((-f*sinh(b*c*x*log(F)/(n + 2) - d))^n*F^((b*x + a)*c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(90) = 180$.

Time = 0.51 (sec) , antiderivative size = 530, normalized size of antiderivative = 6.31

$$\int F^{c(a+bx)}\left(f\sinh\left(d-\frac{bcx\log(F)}{2+n}\right)\right)^n dx$$

$$= \frac{F^{ac}ne^{\left(\frac{2\,bcx\log(F)+dn^2-n^2\log(2)+n^2\log\left(-fe^{\left(\frac{2\,(bcx\log(F)-dn-2\,d)}{n+2}\right)}+f\right)+2\,dn-2\,n\log(2)+2\,n\log\left(-fe^{\left(\frac{2\,(bcx\log(F)-dn-2\,d)}{n+2}\right)}+f\right)}{n+2}+\frac{2\,(bcx\log\cdots}{\cdots}\right)}}{\cdots}$$

input
```
integrate(F^(c*(b*x+a))*(-f*sinh(-d+b*c*x*log(F)/(2+n)))^n,x, algorithm="g
iac")
```

output
```
1/2*(F^(a*c)*n*e^((2*b*c*x*log(F) + d*n^2 - n^2*log(2) + n^2*log(-f*e^(2*(
b*c*x*log(F) - d*n - 2*d)/(n + 2)) + f) + 2*d*n - 2*n*log(2) + 2*n*log(-f*
e^(2*(b*c*x*log(F) - d*n - 2*d)/(n + 2)) + f))/(n + 2) + 2*(b*c*x*log(F) -
 d*n - 2*d)/(n + 2)) - F^(a*c)*n*e^((2*b*c*x*log(F) + d*n^2 - n^2*log(2) +
 n^2*log(-f*e^(2*(b*c*x*log(F) - d*n - 2*d)/(n + 2)) + f) + 2*d*n - 2*n*lo
g(2) + 2*n*log(-f*e^(2*(b*c*x*log(F) - d*n - 2*d)/(n + 2)) + f))/(n + 2))
+ 2*F^(a*c)*e^((2*b*c*x*log(F) + d*n^2 - n^2*log(2) + n^2*log(-f*e^(2*(b*c
*x*log(F) - d*n - 2*d)/(n + 2)) + f) + 2*d*n - 2*n*log(2) + 2*n*log(-f*e^(
2*(b*c*x*log(F) - d*n - 2*d)/(n + 2)) + f))/(n + 2) + 2*(b*c*x*log(F) - d*
n - 2*d)/(n + 2)) - 2*F^(a*c)*e^((2*b*c*x*log(F) + d*n^2 - n^2*log(2) + n^
2*log(-f*e^(2*(b*c*x*log(F) - d*n - 2*d)/(n + 2)) + f) + 2*d*n - 2*n*log(2
) + 2*n*log(-f*e^(2*(b*c*x*log(F) - d*n - 2*d)/(n + 2)) + f))/(n + 2)))/(b
*c*n*e^(2*(b*c*x*log(F) - d*n - 2*d)/(n + 2))*log(F) + b*c*e^(2*(b*c*x*log
(F) - d*n - 2*d)/(n + 2))*log(F))
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\left(f\sinh\left(d-\frac{bcx\log(F)}{2+n}\right)\right)^n\,dx$$
$$=\int F^{c\,(a+b\,x)}\,\left(f\,\sinh\left(d-\frac{b\,c\,x\,\ln\left(F\right)}{n+2}\right)\right)^n\,dx$$

input
```
int(F^(c*(a + b*x))*(f*sinh(d - (b*c*x*log(F))/(n + 2)))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f*sinh(d - (b*c*x*log(F))/(n + 2)))^n, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.65

$$\int F^{c(a+bx)}\left(f\sinh\left(d-\frac{bcx\log(F)}{2+n}\right)\right)^n\,dx$$
$$=\frac{f^{ac+n}(-1)^n\left(-e^{\frac{2\log(f)bcx}{n+2}}\left(e^{\frac{2\log(f)bcx}{n+2}}-e^{2d}\right)^n n-e^{2d}\left(e^{\frac{2\log(f)bcx}{n+2}}-e^{2d}\right)^n n-2e^{2d}\left(e^{\frac{2\log(f)bcx}{n+2}}-e^{2d}\right)^n+2f^{bc}\right.}{2e^{dn}2^n\log\left(f\right)bc\left(n+1\right)}$$

input
```
int(F^(c*(b*x+a))*(-f*sinh(-d+b*c*x*log(F)/(2+n)))^n,x)
```

output
```
(f**(a*c + n)*( - 1)**n*( - e**((2*log(f)*b*c*x)/(n + 2))*(e**((2*log(f)*b
*c*x)/(n + 2)) - e**(2*d))**n*n - e**(2*d)*(e**((2*log(f)*b*c*x)/(n + 2))
- e**(2*d))**n*n - 2*e**(2*d)*(e**((2*log(f)*b*c*x)/(n + 2)) - e**(2*d))**
n + 2*f**(b*c*x)*e**(d*n)*sinh((log(f)*b*c*x - d*n - 2*d)/(n + 2))**n*2**n
*n + 2*f**(b*c*x)*e**(d*n)*sinh((log(f)*b*c*x - d*n - 2*d)/(n + 2))**n*2**
n))/(2*e**(d*n)*2**n*log(f)*b*c*(n + 1))
```

3.22 $\int e^{a+bx}\cosh(d+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 29

$$\int e^{a+bx}\cosh(d+bx)\,dx=\frac{e^{a+d+2bx}}{4b}+\frac{1}{2}e^{a-d}x$$

output
```
1/4*exp(2*b*x+a+d)/b+1/2*exp(a-d)*x
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int e^{a+bx}\cosh(d+bx)\,dx=\frac{e^a\left(\left(e^{2bx}+2bx\right)\cosh(d)+\left(e^{2bx}-2bx\right)\sinh(d)\right)}{4b}$$

input
```
Integrate[E^(a + b*x)*Cosh[d + b*x],x]
```

output
```
(E^a*((E^(2*b*x) + 2*b*x)*Cosh[d] + (E^(2*b*x) - 2*b*x)*Sinh[d]))/(4*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\cosh(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{2}e^{a-bx}\left(1+e^{2bx}\right)de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^a\int e^{-bx}\left(1+e^{2bx}\right)de^{bx}}{2b}$$

$$\Big\downarrow \text{244}$$

$$\frac{e^a\int \left(e^{-bx}+e^{bx}\right)de^{bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^a\left(\frac{1}{2}e^{2bx}+\log\left(e^{bx}\right)\right)}{2b}$$

input
```
Int[E^(a + b*x)*Cosh[d + b*x],x]
```

output
```
(E^a*(E^(2*b*x)/2 + Log[E^(b*x)]))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{{\mathrm e}^{2 b x +a +d}}{4 b}+\frac{{\mathrm e}^{a -d} x}{2}$	24
parallelrisch	$\frac{{\mathrm e}^{b x +a} \left(b x \cosh \left(b x +d \right)-x b \sinh \left(b x +d \right)+\sinh \left(b x +d \right)\right)}{2 b}$	38
default	$\frac{\cosh \left(a -d \right) x}{2}+\frac{\sinh \left(2 b x +a +d \right)}{4 b}+\frac{\sinh \left(a -d \right) x}{2}+\frac{\cosh \left(2 b x +a +d \right)}{4 b}$	46
orering	$\frac{\left(2 b x +1\right) {\mathrm e}^{b x +a} \cosh \left(b x +d \right)}{2 b}-\frac{x \left(b \,{\mathrm e}^{b x +a} \cosh \left(b x +d \right)+{\mathrm e}^{b x +a} b \sinh \left(b x +d \right)\right)}{2 b}$	60

input
```
int(exp(b*x+a)*cosh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(2*b*x+a+d)/b+1/2*exp(a-d)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(23) = 46$.

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.38

$$\int e^{a+bx}\cosh(d+bx)\,dx$$
$$= \frac{(2\,bx+1)\cosh{(bx+d)}\cosh{(-a+d)} - (2\,bx+1)\cosh{(bx+d)}\sinh{(-a+d)} - ((2\,bx-1)\cosh{(-a+}}{4\,(b\cosh{(bx+d)} - b\sinh{(bx+d)})}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d),x, algorithm="fricas")
```

output
```
1/4*((2*b*x + 1)*cosh(b*x + d)*cosh(-a + d) - (2*b*x + 1)*cosh(b*x + d)*si
nh(-a + d) - ((2*b*x - 1)*cosh(-a + d) - (2*b*x - 1)*sinh(-a + d))*sinh(b*
x + d))/(b*cosh(b*x + d) - b*sinh(b*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(20) = 40$.

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.17

$$\int e^{a+bx}\cosh(d+bx)\,dx$$
$$= \begin{cases} -\frac{xe^{a}e^{bx}\sinh{(bx+d)}}{2} + \frac{xe^{a}e^{bx}\cosh{(bx+d)}}{2} + \frac{e^{a}e^{bx}\sinh{(bx+d)}}{2b} & \text{for } b \neq 0 \\ xe^{a}\cosh{(d)} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d),x)
```

output
```
Piecewise((-x*exp(a)*exp(b*x)*sinh(b*x + d)/2 + x*exp(a)*exp(b*x)*cosh(b*x
 + d)/2 + exp(a)*exp(b*x)*sinh(b*x + d)/(2*b), Ne(b, 0)), (x*exp(a)*cosh(d
), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int e^{a+bx}\cosh(d+bx)\,dx = \frac{(bx+a)e^{(a-d)}}{2\,b} + \frac{e^{(2\,bx+a+d)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d),x, algorithm="maxima")
```

output
```
1/2*(b*x + a)*e^(a - d)/b + 1/4*e^(2*b*x + a + d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int e^{a+bx}\cosh(d+bx)\,dx = \frac{\left(2\,bxe^{a} + e^{(2\,bx+a+2\,d)}\right)e^{(-d)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d),x, algorithm="giac")
```

output
```
1/4*(2*b*x*e^a + e^(2*b*x + a + 2*d))*e^(-d)/b
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int e^{a+bx}\cosh(d+bx)\,dx = \frac{\mathrm{e}^{a-d}\left(\mathrm{e}^{2\,d+2\,b\,x} + 2\,b\,x\right)}{4\,b}$$

input
```
int(cosh(d + b*x)*exp(a + b*x),x)
```

output
```
(exp(a - d)*(exp(2*d + 2*b*x) + 2*b*x))/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.31

$$\int e^{a+bx}\cosh(d+bx)\,dx = \frac{e^{bx+a}(\cosh(bx+d)\,bx+\cosh(bx+d)-\sinh(bx+d)\,bx)}{2b}$$

input
```
int(exp(b*x+a)*cosh(b*x+d),x)
```

output
```
(e**(a + b*x)*(cosh(b*x + d)*b*x + cosh(b*x + d) - sinh(b*x + d)*b*x))/(2*
b)
```

3.23 $\int e^{a+bx}\cosh^2(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 51

$$\int e^{a+bx}\cosh^2(d+bx)\,dx=-\frac{e^{a-2d-bx}}{4b}+\frac{e^{a+bx}}{2b}+\frac{e^{a+2d+3bx}}{12b}$$

output
```
-1/4*exp(-b*x+a-2*d)/b+1/2*exp(b*x+a)/b+1/12*exp(3*b*x+a+2*d)/b
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int e^{a+bx}\cosh^2(d+bx)\,dx=\frac{e^{a-bx}\left(6e^{2bx}+\left(-3+e^{4bx}\right)\cosh(2d)+\left(3+e^{4bx}\right)\sinh(2d)\right)}{12b}$$

input
```
Integrate[E^(a + b*x)*Cosh[d + b*x]^2,x]
```

output
```
(E^(a - b*x)*(6*E^(2*b*x) + (-3 + E^(4*b*x))*Cosh[2*d] + (3 + E^(4*b*x))*S
inh[2*d]))/(12*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\cosh^2(bx+d)\,dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int \frac{1}{4}e^{a-2bx}\left(1+e^{2bx}\right)^2de^{bx}}{b}$$

$$\Big\downarrow \ 27$$

$$\frac{e^a\int e^{-2bx}\left(1+e^{2bx}\right)^2de^{bx}}{4b}$$

$$\Big\downarrow \ 244$$

$$\frac{e^a\int \left(2+e^{-2bx}+e^{2bx}\right)de^{bx}}{4b}$$

$$\Big\downarrow \ 2009$$

$$\frac{e^a\left(-e^{-bx}+2e^{bx}+\frac{1}{3}e^{3bx}\right)}{4b}$$

input
```
Int[E^(a + b*x)*Cosh[d + b*x]^2,x]
```

output
```
(E^a*(-E^(-(b*x)) + 2*E^(b*x) + E^(3*b*x)/3))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{{\mathrm e}^{-b x +a -2 d}}{4 b}+\frac{{\mathrm e}^{b x +a}}{2 b}+\frac{{\mathrm e}^{3 b x +a +2 d}}{12 b}$
parallelrisch	$\frac{2 \,{\mathrm e}^{b x +a} \left(3 \tanh \left(\frac{b x}{2}+\frac{d}{2}\right)^{3}-3 \tanh \left(\frac{b x}{2}+\frac{d}{2}\right)^{2}+\tanh \left(\frac{b x}{2}+\frac{d}{2}\right)+1\right)}{3 b \left(-1+\tanh \left(\frac{b x}{2}+\frac{d}{2}\right)\right)^{2} \left(1+\tanh \left(\frac{b x}{2}+\frac{d}{2}\right)\right)^{2}}$
default	$\frac{\sinh \left(b x +a \right)}{2 b}-\frac{\sinh \left(-b x +a -2 d \right)}{4 b}+\frac{\sinh \left(3 b x +a +2 d \right)}{12 b}+\frac{\cosh \left(b x +a \right)}{2 b}-\frac{\cosh \left(-b x +a -2 d \right)}{4 b}+\frac{\cosh \left(3 b x +a +2 d \right)}{12 b}$
orering	$\frac{{\mathrm e}^{b x +a} \cosh \left(b x +d \right)^{2}}{3 b}+\frac{b \,{\mathrm e}^{b x +a} \cosh \left(b x +d \right)^{2}+2 \,{\mathrm e}^{b x +a} \sinh \left(b x +d \right) b \cosh \left(b x +d \right)}{b^{2}}-\frac{3 \,{\mathrm e}^{b x +a} b^{2} \cosh \left(b x +d \right)^{2}+4 b^{2} {\mathrm e}^{b x +a} \sinh(b}{3}$

input
```
int(exp(b*x+a)*cosh(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*exp(-b*x+a-2*d)/b+1/2*exp(b*x+a)/b+1/12*exp(3*b*x+a+2*d)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(42) = 84$.

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.43

$$\int e^{a+bx}\cosh^2(d+bx)\,dx = -\frac{\cosh(bx+d)^2\cosh(-a+d) + (\cosh(-a+d) - \sinh(-a+d))\sinh(bx+d)^2 - 4(\cosh(bx+d)\cosh}{6(b\cosh(bx}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^2,x, algorithm="fricas")
```

output
```
-1/6*(cosh(b*x + d)^2*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*
x + d)^2 - 4*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sin
h(b*x + d) - (cosh(b*x + d)^2 - 3)*sinh(-a + d) - 3*cosh(-a + d))/(b*cosh(
b*x + d) - b*sinh(b*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(37) = 74$.

Time = 0.46 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.53

$$\int e^{a+bx}\cosh^2(d+bx)\,dx = \begin{cases} -\frac{2e^a e^{bx}\sinh^2(bx+d)}{3b} + \frac{2e^a e^{bx}\sinh(bx+d)\cosh(bx+d)}{3b} + \frac{e^a e^{bx}\cosh^2(bx+d)}{3b} & \text{for } b \neq 0 \\ xe^a\cosh^2(d) & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)**2,x)
```

output
```
Piecewise((-2*exp(a)*exp(b*x)*sinh(b*x + d)**2/(3*b) + 2*exp(a)*exp(b*x)*s
inh(b*x + d)*cosh(b*x + d)/(3*b) + exp(a)*exp(b*x)*cosh(b*x + d)**2/(3*b),
 Ne(b, 0)), (x*exp(a)*cosh(d)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int e^{a+bx}\cosh^2(d+bx)\,dx=\frac{e^{(3\,bx+a+2\,d)}}{12\,b}+\frac{e^{(bx+a)}}{2\,b}-\frac{e^{(-bx+a-2\,d)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^2,x, algorithm="maxima")
```

output
```
1/12*e^(3*b*x + a + 2*d)/b + 1/2*e^(b*x + a)/b - 1/4*e^(-b*x + a - 2*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int e^{a+bx}\cosh^2(d+bx)\,dx=\frac{\left(e^{(3\,bx+a+4\,d)}+6\,e^{(bx+a+2\,d)}-3\,e^{(-bx+a)}\right)e^{(-2\,d)}}{12\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^2,x, algorithm="giac")
```

output
```
1/12*(e^(3*b*x + a + 4*d) + 6*e^(b*x + a + 2*d) - 3*e^(-b*x + a))*e^(-2*d)
/b
```

Mupad [B] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int e^{a+bx}\cosh^2(d+bx)\,dx=\frac{\mathrm{e}^{a-2\,d-b\,x}\,\left(6\,\mathrm{e}^{2\,d+2\,b\,x}+\mathrm{e}^{4\,d+4\,b\,x}-3\right)}{12\,b}$$

input
```
int(cosh(d + b*x)^2*exp(a + b*x),x)
```

output
```
(exp(a - 2*d - b*x)*(6*exp(2*d + 2*b*x) + exp(4*d + 4*b*x) - 3))/(12*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int e^{a+bx}\cosh^2(d+bx)\,dx = \frac{e^a\left(e^{4bx+4d}+6e^{2bx+2d}-3\right)}{12e^{bx+2d}b}$$

input
```
int(exp(b*x+a)*cosh(b*x+d)^2,x)
```

output
```
(e**a*(e**(4*b*x + 4*d) + 6*e**(2*b*x + 2*d) - 3))/(12*e**(b*x + 2*d)*b)
```

3.24 $\int e^{a+bx}\cosh^3(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 65

$$\int e^{a+bx}\cosh^3(d+bx)\,dx=-\frac{e^{a-3d-2bx}}{16b}+\frac{3e^{a+d+2bx}}{16b}+\frac{e^{a+3d+4bx}}{32b}+\frac{3}{8}e^{a-d}x$$

output
```
-1/16*exp(-2*b*x+a-3*d)/b+3/16*exp(2*b*x+a+d)/b+1/32*exp(4*b*x+a+3*d)/b+3/
8*exp(a-d)*x
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.37

$$\int e^{a+bx}\cosh^3(d+bx)\,dx$$
$$=\frac{e^{a-2bx}\left(6e^{2bx}\left(e^{2bx}+2bx\right)\cosh(d)+\left(-2+e^{6bx}\right)\cosh(3d)+6e^{4bx}\sinh(d)-12be^{2bx}x\sinh(d)+2\sinh(3d)}{32b}$$

input
```
Integrate[E^(a + b*x)*Cosh[d + b*x]^3,x]
```

output
```
(E^(a - 2*b*x)*(6*E^(2*b*x)*(E^(2*b*x) + 2*b*x)*Cosh[d] + (-2 + E^(6*b*x))
*Cosh[3*d] + 6*E^(4*b*x)*Sinh[d] - 12*b*E^(2*b*x)*x*Sinh[d] + 2*Sinh[3*d]
+ E^(6*b*x)*Sinh[3*d]))/(32*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx} \cosh^3(bx+d) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{8} e^{a-3bx} \left(1+e^{2bx}\right)^3 de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^a \int e^{-3bx} \left(1+e^{2bx}\right)^3 de^{bx}}{8b}$$

$$\Big\downarrow \text{243}$$

$$\frac{e^a \int e^{-2bx} \left(1+e^{2bx}\right)^3 de^{2bx}}{16b}$$

$$\Big\downarrow \text{49}$$

$$\frac{e^a \int \left(3+e^{-2bx}+3e^{-bx}+e^{2bx}\right) de^{2bx}}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^a \left(-e^{-bx}+\frac{7}{2} e^{2bx}+3 \log \left(e^{2bx}\right)\right)}{16b}$$

input
```
Int[E^(a + b*x)*Cosh[d + b*x]^3,x]
```

output
```
(E^a*(-E^(-(b*x)) + (7*E^(2*b*x))/2 + 3*Log[E^(2*b*x)]))/(16*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{e^{-2bx+a-3d}}{16b}+\frac{3\,e^{2bx+a+d}}{16b}+\frac{e^{4bx+a+3d}}{32b}+\frac{3\,e^{a-d}x}{8}$
parallelrisch	$\frac{e^{bx+a}\left(12bx\cosh(bx+d)-12xb\sinh(bx+d)+\cosh(bx+d)+11\sinh(bx+d)+3\sinh(3bx+3d)-\cosh(3bx+3d)\right)}{32b}$
default	$\frac{3\cosh(a-d)x}{8}-\frac{\sinh(-2bx+a-3d)}{16b}+\frac{3\sinh(2bx+a+d)}{16b}+\frac{\sinh(4bx+a+3d)}{32b}+\frac{3\sinh(a-d)x}{8}-\frac{\cosh(-2bx+a-3d)}{16b}$
orering	$\frac{(4bx+1)e^{bx+a}\cosh(bx+d)^3}{4b}-\frac{(bx-1)\left(b\,e^{bx+a}\cosh(bx+d)^3+3\,e^{bx+a}\cosh(bx+d)^2 b\sinh(bx+d)\right)}{4b^2}-\frac{(4bx+1)\left(4\,e^{bx+a}\cos\right.}{}$

input
```
int(exp(b*x+a)*cosh(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*exp(-2*b*x+a-3*d)/b+3/16*exp(2*b*x+a+d)/b+1/32*exp(4*b*x+a+3*d)/b+3/
8*exp(a-d)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(53) = 106.

Time = 0.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 3.29

$$\int e^{a+bx}\cosh^3(d+bx)\,dx = -\frac{\cosh(bx+d)^3\cosh(-a+d) - 3(\cosh(-a+d) - \sinh(-a+d))\sinh(bx+d)^3 - 6(2bx+1)\cosh(b$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^3,x, algorithm="fricas")
```

output
```
-1/32*(cosh(b*x + d)^3*cosh(-a + d) - 3*(cosh(-a + d) - sinh(-a + d))*sinh
(b*x + d)^3 - 6*(2*b*x + 1)*cosh(b*x + d)*cosh(-a + d) + 3*(cosh(b*x + d)*
cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d)^2 - 3*(3*cosh(b*x
 + d)^2*cosh(-a + d) - 2*(2*b*x - 1)*cosh(-a + d) + (4*b*x - 3*cosh(b*x +
d)^2 - 2)*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^3 - 6*(2*b*x + 1)*c
osh(b*x + d))*sinh(-a + d))/(b*cosh(b*x + d) - b*sinh(b*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(54) = 108.

Time = 0.96 (sec) , antiderivative size = 207, normalized size of antiderivative = 3.18

$$\int e^{a+bx}\cosh^3(d+bx)\,dx = \begin{cases} \frac{3xe^ae^{bx}\sinh^3(bx+d)}{8} - \frac{3xe^ae^{bx}\sinh^2(bx+d)\cosh(bx+d)}{8} - \frac{3xe^ae^{bx}\sinh(bx+d)\cosh^2(bx+d)}{8} + \frac{3xe^ae^{bx}\cosh^3(bx+d)}{8} - \frac{5e^ae^{bx}\sin}{8} \\ xe^a\cosh^3(d) \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)**3,x)
```

output
```
Piecewise((3*x*exp(a)*exp(b*x)*sinh(b*x + d)**3/8 - 3*x*exp(a)*exp(b*x)*si
nh(b*x + d)**2*cosh(b*x + d)/8 - 3*x*exp(a)*exp(b*x)*sinh(b*x + d)*cosh(b*
x + d)**2/8 + 3*x*exp(a)*exp(b*x)*cosh(b*x + d)**3/8 - 5*exp(a)*exp(b*x)*s
inh(b*x + d)**3/(8*b) + exp(a)*exp(b*x)*sinh(b*x + d)**2*cosh(b*x + d)/(4*
b) + exp(a)*exp(b*x)*sinh(b*x + d)*cosh(b*x + d)**2/b - 3*exp(a)*exp(b*x)*
cosh(b*x + d)**3/(8*b), Ne(b, 0)), (x*exp(a)*cosh(d)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int e^{a+bx}\cosh^3(d+bx)\,dx = \frac{3\,(bx+a)e^{(a-d)}}{8\,b} + \frac{e^{(4\,bx+a+3\,d)}}{32\,b} + \frac{3\,e^{(2\,bx+a+d)}}{16\,b} - \frac{e^{(-2\,bx+a-3\,d)}}{16\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^3,x, algorithm="maxima")
```

output
```
3/8*(b*x + a)*e^(a - d)/b + 1/32*e^(4*b*x + a + 3*d)/b + 3/16*e^(2*b*x + a
 + d)/b - 1/16*e^(-2*b*x + a - 3*d)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int e^{a+bx}\cosh^3(d+bx)\,dx = \frac{\left(12\,bxe^{(a+2\,d)} - 2\left(3\,e^{(2\,bx+a+2\,d)} + e^{a}\right)e^{(-2\,bx)} + e^{(4\,bx+a+6\,d)} + 6\,e^{(2\,bx+a+4\,d)}\right)e^{(-3\,d)}}{32\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^3,x, algorithm="giac")
```

output
```
1/32*(12*b*x*e^(a + 2*d) - 2*(3*e^(2*b*x + a + 2*d) + e^a)*e^(-2*b*x) + e^
(4*b*x + a + 6*d) + 6*e^(2*b*x + a + 4*d))*e^(-3*d)/b
```

Mupad [B] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int e^{a+bx}\cosh^3(d+bx)\,dx$$
$$=\frac{e^{a+b\,x}\,\left(3\cosh(d+b\,x)+3\cosh(d+b\,x)^2\sinh(d+b\,x)-\cosh(d+b\,x)^3+3\,b\,x\cosh(d+b\,x)-3\,b\,x\sinh\right.}{8\,b}$$

input
```
int(cosh(d + b*x)^3*exp(a + b*x),x)
```

output
```
(exp(a + b*x)*(3*cosh(d + b*x) + 3*cosh(d + b*x)^2*sinh(d + b*x) - cosh(d
+ b*x)^3 + 3*b*x*cosh(d + b*x) - 3*b*x*sinh(d + b*x)))/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int e^{a+bx}\cosh^3(d+bx)\,dx=\frac{e^a\left(e^{6bx+6d}+6e^{4bx+4d}+12e^{2bx+2d}bx-2\right)}{32e^{2bx+3d}b}$$

input
```
int(exp(b*x+a)*cosh(b*x+d)^3,x)
```

output
```
(e**a*(e**(6*b*x + 6*d) + 6*e**(4*b*x + 4*d) + 12*e**(2*b*x + 2*d)*b*x - 2
))/(32*e**(2*b*x + 3*d)*b)
```

3.25 $\int e^{a+bx}\cosh^4(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 87

$$\int e^{a+bx}\cosh^4(d+bx)\,dx=-\frac{e^{a-4d-3bx}}{48b}-\frac{e^{a-2d-bx}}{4b}+\frac{3e^{a+bx}}{8b}+\frac{e^{a+2d+3bx}}{12b}+\frac{e^{a+4d+5bx}}{80b}$$

output
```
-1/48*exp(-3*b*x+a-4*d)/b-1/4*exp(-b*x+a-2*d)/b+3/8*exp(b*x+a)/b+1/12*exp(
3*b*x+a+2*d)/b+1/80*exp(5*b*x+a+4*d)/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.16

$$\int e^{a+bx}\cosh^4(d+bx)\,dx$$
$$=\frac{e^{a-3bx}\left(90e^{4bx}+20e^{2bx}\left(-3+e^{4bx}\right)\cosh(2d)+\left(-5+3e^{8bx}\right)\cosh(4d)+60e^{2bx}\sinh(2d)+20e^{6bx}\sinh(2d)}{240b}$$

input
```
Integrate[E^(a + b*x)*Cosh[d + b*x]^4,x]
```

output
```
(E^(a - 3*b*x)*(90*E^(4*b*x) + 20*E^(2*b*x)*(-3 + E^(4*b*x))*Cosh[2*d] + (
-5 + 3*E^(8*b*x))*Cosh[4*d] + 60*E^(2*b*x)*Sinh[2*d] + 20*E^(6*b*x)*Sinh[2
*d] + 5*Sinh[4*d] + 3*E^(8*b*x)*Sinh[4*d]))/(240*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\cosh^4(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{16}e^{a-4bx}\left(1+e^{2bx}\right)^4de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^a\int e^{-4bx}\left(1+e^{2bx}\right)^4de^{bx}}{16b}$$

$$\Big\downarrow \text{244}$$

$$\frac{e^a\int \left(6+e^{-4bx}+4e^{-2bx}+4e^{2bx}+e^{4bx}\right)de^{bx}}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^a\left(-\frac{1}{3}e^{-3bx}-4e^{-bx}+6e^{bx}+\frac{4}{3}e^{3bx}+\frac{1}{5}e^{5bx}\right)}{16b}$$

input
```
Int[E^(a + b*x)*Cosh[d + b*x]^4,x]
```

output
```
(E^a*(-1/3*1/E^(3*b*x) - 4/E^(b*x) + 6*E^(b*x) + (4*E^(3*b*x))/3 + E^(5*b*
x)/5))/(16*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{{\mathrm e}^{-3 b x +a -4 d}}{48 b}-\frac{{\mathrm e}^{-b x +a -2 d}}{4 b}+\frac{3 \,{\mathrm e}^{b x +a}}{8 b}+\frac{{\mathrm e}^{3 b x +a +2 d}}{12 b}+\frac{{\mathrm e}^{5 b x +a +4 d}}{80 b}$
parallelrisch	$\frac{2 \,{\mathrm e}^{b x +a} \left(15 \tanh \left(\frac{b x}{2}+\frac{d}{2}\right)^{7}-15 \tanh \left(\frac{b x}{2}+\frac{d}{2}\right)^{6}-5 \tanh \left(\frac{b x}{2}+\frac{d}{2}\right)^{5}+25 \tanh \left(\frac{b x}{2}+\frac{d}{2}\right)^{4}+13 \tanh \left(\frac{b x}{2}+\frac{d}{2}\right)^{3}-21 \tanh \left(\frac{b x}{2}+\frac{d}{2}\right)^{2}}{15 b \left(-1+\tanh \left(\frac{b x}{2}+\frac{d}{2}\right)\right)^{4} \left(1+\tanh \left(\frac{b x}{2}+\frac{d}{2}\right)\right)^{4}}$
default	$\frac{3 \sinh \left(b x +a \right)}{8 b}-\frac{\sinh \left(-3 b x +a -4 d \right)}{48 b}-\frac{\sinh \left(-b x +a -2 d \right)}{4 b}+\frac{\sinh \left(3 b x +a +2 d \right)}{12 b}+\frac{\sinh \left(5 b x +a +4 d \right)}{80 b}+\frac{3 \cosh \left(b x +a \right)}{8 b}-$
orering	$\frac{{\mathrm e}^{b x +a} \cosh \left(b x +d \right)^{4}}{5 b}+\frac{\frac{10 b \,{\mathrm e}^{b x +a} \cosh \left(b x +d \right)^{4}}{9}+\frac{40 \,{\mathrm e}^{b x +a} \cosh \left(b x +d \right)^{3} b \sinh \left(b x +d \right)}{9}}{b^{2}}-\frac{2 \left(5 b^{2} {\mathrm e}^{b x +a} \cosh \left(b x +d \right)^{4}+8 b^{2} {\mathrm e}^{b x +a} \cosh\right.}{}$

input
```
int(exp(b*x+a)*cosh(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/48*exp(-3*b*x+a-4*d)/b-1/4*exp(-b*x+a-2*d)/b+3/8*exp(b*x+a)/b+1/12*exp(
3*b*x+a+2*d)/b+1/80*exp(5*b*x+a+4*d)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(72) = 144$.

Time = 0.09 (sec) , antiderivative size = 270, normalized size of antiderivative = 3.10

$$\int e^{a+bx}\cosh^4(d+bx)\,dx = -\frac{\cosh(bx+d)^4\cosh(-a+d)+(\cosh(-a+d)-\sinh(-a+d))\sinh(bx+d)^4-16\,(\cosh(bx+d)\cos$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^4,x, algorithm="fricas")
```

output
```
-1/120*(cosh(b*x + d)^4*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(
b*x + d)^4 - 16*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*
sinh(b*x + d)^3 + 20*cosh(b*x + d)^2*cosh(-a + d) + 2*(3*cosh(b*x + d)^2*c
osh(-a + d) - (3*cosh(b*x + d)^2 + 10)*sinh(-a + d) + 10*cosh(-a + d))*sin
h(b*x + d)^2 - 16*(cosh(b*x + d)^3*cosh(-a + d) + 5*cosh(b*x + d)*cosh(-a
+ d) - (cosh(b*x + d)^3 + 5*cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d) - (
cosh(b*x + d)^4 + 20*cosh(b*x + d)^2 - 45)*sinh(-a + d) - 45*cosh(-a + d))
/(b*cosh(b*x + d) - b*sinh(b*x + d))
```

Sympy [A] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.60

$$\int e^{a+bx}\cosh^4(d+bx)\,dx$$

$$= \begin{cases} \frac{8e^ae^{bx}\sinh^4(bx+d)}{15b} - \frac{8e^ae^{bx}\sinh^3(bx+d)\cosh(bx+d)}{15b} - \frac{4e^ae^{bx}\sinh^2(bx+d)\cosh^2(bx+d)}{5b} + \frac{4e^ae^{bx}\sinh(bx+d)\cosh^3(bx+d)}{5b} + \frac{e^a}{} \\ xe^a\cosh^4(d) \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)**4,x)
```

output
```
Piecewise((8*exp(a)*exp(b*x)*sinh(b*x + d)**4/(15*b) - 8*exp(a)*exp(b*x)*s
inh(b*x + d)**3*cosh(b*x + d)/(15*b) - 4*exp(a)*exp(b*x)*sinh(b*x + d)**2*
cosh(b*x + d)**2/(5*b) + 4*exp(a)*exp(b*x)*sinh(b*x + d)*cosh(b*x + d)**3/
(5*b) + exp(a)*exp(b*x)*cosh(b*x + d)**4/(5*b), Ne(b, 0)), (x*exp(a)*cosh(
d)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int e^{a+bx}\cosh^4(d+bx)\,dx = \frac{e^{(5\,bx+a+4\,d)}}{80\,b} + \frac{e^{(3\,bx+a+2\,d)}}{12\,b} + \frac{3\,e^{(bx+a)}}{8\,b} - \frac{e^{(-bx+a-2\,d)}}{4\,b} - \frac{e^{(-3\,bx+a-4\,d)}}{48\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^4,x, algorithm="maxima")
```

output
```
1/80*e^(5*b*x + a + 4*d)/b + 1/12*e^(3*b*x + a + 2*d)/b + 3/8*e^(b*x + a)/
b - 1/4*e^(-b*x + a - 2*d)/b - 1/48*e^(-3*b*x + a - 4*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.77

$$\int e^{a+bx}\cosh^4(d+bx)\,dx = -\frac{\left(5\left(12\,e^{(2\,bx+a+2\,d)}+e^{a}\right)e^{(-3\,bx)} - 3\,e^{(5\,bx+a+8\,d)} - 20\,e^{(3\,bx+a+6\,d)} - 90\,e^{(bx+a+4\,d)}\right)e^{(-4\,d)}}{240\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^4,x, algorithm="giac")
```

output
```
-1/240*(5*(12*e^(2*b*x + a + 2*d) + e^a)*e^(-3*b*x) - 3*e^(5*b*x + a + 8*d
) - 20*e^(3*b*x + a + 6*d) - 90*e^(b*x + a + 4*d))*e^(-4*d)/b
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int e^{a+bx}\cosh^4(d+bx)\,dx=\frac{e^{a+2\,d+3\,b\,x}}{12\,b}-\frac{e^{a-2\,d-b\,x}}{4\,b}-\frac{e^{a-4\,d-3\,b\,x}}{48\,b}+\frac{e^{a+4\,d+5\,b\,x}}{80\,b}+\frac{3\,e^{a+b\,x}}{8\,b}$$

input
```
int(cosh(d + b*x)^4*exp(a + b*x),x)
```

output
```
exp(a + 2*d + 3*b*x)/(12*b) - exp(a - 2*d - b*x)/(4*b) - exp(a - 4*d - 3*b
*x)/(48*b) + exp(a + 4*d + 5*b*x)/(80*b) + (3*exp(a + b*x))/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int e^{a+bx}\cosh^4(d+bx)\,dx=\frac{e^a\left(3e^{8bx+8d}+20e^{6bx+6d}+90e^{4bx+4d}-60e^{2bx+2d}-5\right)}{240e^{3bx+4d}b}$$

input
```
int(exp(b*x+a)*cosh(b*x+d)^4,x)
```

output
```
(e**a*(3*e**(8*b*x + 8*d) + 20*e**(6*b*x + 6*d) + 90*e**(4*b*x + 4*d) - 60
*e**(2*b*x + 2*d) - 5))/(240*e**(3*b*x + 4*d)*b)
```

3.26 $\int e^{2(a+bx)}\cosh(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 38

$$\int e^{2(a+bx)}\cosh(d+bx)\,dx=\frac{e^{2a-d+bx}}{2b}+\frac{e^{2a+d+3bx}}{6b}$$

output
```
1/2*exp(b*x+2*a-d)/b+1/6*exp(3*b*x+2*a+d)/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int e^{2(a+bx)}\cosh(d+bx)\,dx=\frac{e^{2a+bx}\left(\left(3+e^{2bx}\right)\cosh(d)+\left(-3+e^{2bx}\right)\sinh(d)\right)}{6b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[d + b*x],x]
```

output
```
(E^(2*a + b*x)*((3 + E^(2*b*x))*Cosh[d] + (-3 + E^(2*b*x))*Sinh[d]))/(6*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2720, 27, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)} \cosh(bx+d) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{2} e^{2a} \left(1+e^{2bx}\right) de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{2a} \int \left(1+e^{2bx}\right) de^{bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2a} \left(e^{bx}+\frac{1}{3} e^{3bx}\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[d + b*x],x]
```

output
```
(E^(2*a)*(E^(b*x) + E^(3*b*x)/3))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

method	result	size
parallelrisch	$\frac{e^{2bx+2a}(2\cosh(bx+d)-\sinh(bx+d))}{3b}$	32
risch	$\frac{e^{bx+2a-d}}{2b}+\frac{e^{3bx+2a+d}}{6b}$	33
orering	$\frac{4\,e^{2bx+2a}\cosh(bx+d)}{3b}-\frac{2\,e^{2bx+2a}b\cosh(bx+d)+e^{2bx+2a}b\sinh(bx+d)}{3b^2}$	63
default	$\frac{\sinh(bx+2a-d)}{2b}+\frac{\sinh(3bx+2a+d)}{6b}+\frac{\cosh(bx+2a-d)}{2b}+\frac{\cosh(3bx+2a+d)}{6b}$	64

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/3*exp(2*b*x+2*a)*(2*cosh(b*x+d)-sinh(b*x+d))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(32) = 64$.

Time = 0.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.63

$$\int e^{2(a+bx)}\cosh(d+bx)\,dx$$
$$=\frac{\cosh(bx+d)^2\cosh(-2a+2d)+(\cosh(-2a+2d)-\sinh(-2a+2d))\sinh(bx+d)^2+2(\cosh(bx+\ldots}{6(b\ldots}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d),x, algorithm="fricas")
```

output
```
1/6*(cosh(b*x + d)^2*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*
d))*sinh(b*x + d)^2 + 2*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*si
nh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^2 + 3)*sinh(-2*a + 2*d) + 3
*cosh(-2*a + 2*d))/(b*cosh(b*x + d) - b*sinh(b*x + d))
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int e^{2(a+bx)}\cosh(d+bx)\,dx = \begin{cases} -\frac{e^{2a}e^{2bx}\sinh(bx+d)}{3b} + \frac{2e^{2a}e^{2bx}\cosh(bx+d)}{3b} & \text{for } b \neq 0 \\ xe^{2a}\cosh(d) & \text{otherwise} \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d),x)
```

output
```
Piecewise((-exp(2*a)*exp(2*b*x)*sinh(b*x + d)/(3*b) + 2*exp(2*a)*exp(2*b*x
)*cosh(b*x + d)/(3*b), Ne(b, 0)), (x*exp(2*a)*cosh(d), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int e^{2(a+bx)}\cosh(d+bx)\,dx = \frac{e^{(3\,bx+2\,a+d)}}{6\,b} + \frac{e^{(bx+2\,a-d)}}{2\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d),x, algorithm="maxima")
```

output
```
1/6*e^(3*b*x + 2*a + d)/b + 1/2*e^(b*x + 2*a - d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int e^{2(a+bx)}\cosh(d+bx)\,dx=\frac{\left(e^{(3\,bx+2\,a+3\,d)}+3\,e^{(bx+2\,a+d)}\right)e^{(-2\,d)}}{6\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d),x, algorithm="giac")
```

output
```
1/6*(e^(3*b*x + 2*a + 3*d) + 3*e^(b*x + 2*a + d))*e^(-2*d)/b
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int e^{2(a+bx)}\cosh(d+bx)\,dx=\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{-d}\,\mathrm{e}^{b\,x}\,\left(\mathrm{e}^{2\,d}\,\mathrm{e}^{2\,b\,x}+3\right)}{6\,b}$$

input
```
int(cosh(d + b*x)*exp(2*a + 2*b*x),x)
```

output
```
(exp(2*a)*exp(-d)*exp(b*x)*(exp(2*d)*exp(2*b*x) + 3))/(6*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int e^{2(a+bx)}\cosh(d+bx)\,dx=\frac{e^{2bx+2a}(2\cosh(bx+d)-\sinh(bx+d))}{3b}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d),x)
```

output
```
(e**(2*a + 2*b*x)*(2*cosh(b*x + d) - sinh(b*x + d)))/(3*b)
```

3.27 $\int e^{2(a+bx)}\cosh^2(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 51

$$\int e^{2(a+bx)}\cosh^2(d+bx)\,dx=\frac{e^{2a+2bx}}{4b}+\frac{e^{2(a+d)+4bx}}{16b}+\frac{1}{4}e^{2a-2d}x$$

output
```
1/4*exp(2*b*x+2*a)/b+1/16*exp(4*b*x+2*a+2*d)/b+1/4*exp(2*a-2*d)*x
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int e^{2(a+bx)}\cosh^2(d+bx)\,dx=\frac{e^{2a}\left(4e^{2bx}+\left(e^{4bx}+4bx\right)\cosh(2d)+\left(e^{4bx}-4bx\right)\sinh(2d)\right)}{16b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[d + b*x]^2,x]
```

output
```
(E^(2*a)*(4*E^(2*b*x) + (E^(4*b*x) + 4*b*x)*Cosh[2*d] + (E^(4*b*x) - 4*b*x
)*Sinh[2*d]))/(16*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.59, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\cosh^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{4}e^{2a-bx}\left(1+e^{2bx}\right)^2de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{2a}\int e^{-bx}\left(1+e^{2bx}\right)^2de^{bx}}{4b}$$

$$\Big\downarrow \text{243}$$

$$\frac{e^{2a}\int e^{-bx}\left(1+e^{2bx}\right)^2de^{2bx}}{8b}$$

$$\Big\downarrow \text{49}$$

$$\frac{e^{2a}\int \left(2+e^{-bx}+e^{2bx}\right)de^{2bx}}{8b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2a}\left(\frac{5}{2}e^{2bx}+\log\left(e^{2bx}\right)\right)}{8b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[d + b*x]^2,x]
```

output
```
(E^(2*a)*((5*E^(2*b*x))/2 + Log[E^(2*b*x)]))/(8*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

method	result
risch	$\frac{{\mathrm e}^{2 b x +2 a}}{4 b}+\frac{{\mathrm e}^{4 b x +2 a +2 d}}{16 b}+\frac{{\mathrm e}^{2 a -2 d} x}{4}$
parallelrisch	$-\frac{{\mathrm e}^{2 b x +2 a} \left(2 b x \sinh \left(2 b x +2 d \right)-2 b x \cosh \left(2 b x +2 d \right)-3 \sinh \left(2 b x +2 d \right)+2 \cosh \left(2 b x +2 d \right)-2\right)}{8 b}$
default	$\frac{x \cosh \left(2 a -2 d \right)}{4}+\frac{\sinh \left(2 b x +2 a \right)}{4 b}+\frac{\sinh \left(4 b x +2 a +2 d \right)}{16 b}+\frac{x \sinh \left(2 a -2 d \right)}{4}+\frac{\cosh \left(2 b x +2 a \right)}{4 b}+\frac{\cosh \left(4 b x +2 a +2 d \right)}{16 b}$
orering	$\frac{\left(4 b x +3\right) {\mathrm e}^{2 b x +2 a} \cosh \left(b x +d \right)^{2}}{4 b}-\frac{\left(6 b x +1\right) \left(2 \,{\mathrm e}^{2 b x +2 a} b \cosh \left(b x +d \right)^{2}+2 \,{\mathrm e}^{2 b x +2 a} \sinh \left(b x +d \right) b \cosh \left(b x +d \right)\right)}{8 b^{2}}+ \frac{x \left(6 \,{\mathrm e}^{2 b x +2 a} \mathrm{c}\right.}{}$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(2*b*x+2*a)/b+1/16*exp(4*b*x+2*a+2*d)/b+1/4*exp(2*a-2*d)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(43) = 86$.

Time = 0.09 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.80

$$\int e^{2(a+bx)}\cosh^2(d+bx)\,dx$$
$$= \frac{(4\,bx+1)\cosh\left(bx+d\right)^2\cosh\left(-2\,a+2\,d\right)+\left((4\,bx+1)\cosh\left(-2\,a+2\,d\right)-(4\,bx+1)\sinh\left(-2\,a+2\,d\right)\right)}{}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^2,x, algorithm="fricas")
```

output
```
1/16*((4*b*x + 1)*cosh(b*x + d)^2*cosh(-2*a + 2*d) + ((4*b*x + 1)*cosh(-2*
a + 2*d) - (4*b*x + 1)*sinh(-2*a + 2*d))*sinh(b*x + d)^2 - 2*((4*b*x - 1)*
cosh(b*x + d)*cosh(-2*a + 2*d) - (4*b*x - 1)*cosh(b*x + d)*sinh(-2*a + 2*d
))*sinh(b*x + d) - ((4*b*x + 1)*cosh(b*x + d)^2 + 4)*sinh(-2*a + 2*d) + 4*
cosh(-2*a + 2*d))/(b*cosh(b*x + d)^2 - 2*b*cosh(b*x + d)*sinh(b*x + d) + b
*sinh(b*x + d)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(41) = 82$.

Time = 0.46 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.76

$$\int e^{2(a+bx)}\cosh^2(d+bx)\,dx$$
$$= \begin{cases} \frac{xe^{2a}e^{2bx}\sinh^2\left(bx+d\right)}{4} - \frac{xe^{2a}e^{2bx}\sinh\left(bx+d\right)\cosh\left(bx+d\right)}{2} + \frac{xe^{2a}e^{2bx}\cosh^2\left(bx+d\right)}{4} - \frac{e^{2a}e^{2bx}\sinh^2\left(bx+d\right)}{2b} + \frac{3e^{2a}e^{2bx}\sinh\left(bx+d\right)}{4b} \\ xe^{2a}\cosh^2\left(d\right) \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)**2,x)
```

output
```
Piecewise((x*exp(2*a)*exp(2*b*x)*sinh(b*x + d)**2/4 - x*exp(2*a)*exp(2*b*x
)*sinh(b*x + d)*cosh(b*x + d)/2 + x*exp(2*a)*exp(2*b*x)*cosh(b*x + d)**2/4
 - exp(2*a)*exp(2*b*x)*sinh(b*x + d)**2/(2*b) + 3*exp(2*a)*exp(2*b*x)*sinh
(b*x + d)*cosh(b*x + d)/(4*b), Ne(b, 0)), (x*exp(2*a)*cosh(d)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int e^{2(a+bx)}\cosh^2(d+bx)\,dx = \frac{1}{4}\,xe^{(2\,a-2\,d)} + \frac{\left(4\,e^{(-2\,bx-2\,d)}+1\right)e^{(4\,bx+2\,a+2\,d)}}{16\,b} + \frac{de^{(2\,a-2\,d)}}{4\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^2,x, algorithm="maxima")
```

output
```
1/4*x*e^(2*a - 2*d) + 1/16*(4*e^(-2*b*x - 2*d) + 1)*e^(4*b*x + 2*a + 2*d)/
b + 1/4*d*e^(2*a - 2*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int e^{2(a+bx)}\cosh^2(d+bx)\,dx = \frac{\left(4\,(bx+d)e^{(2\,a)} + e^{(4\,bx+2\,a+4\,d)} + 4\,e^{(2\,bx+2\,a+2\,d)}\right)e^{(-2\,d)}}{16\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^2,x, algorithm="giac")
```

output
```
1/16*(4*(b*x + d)*e^(2*a) + e^(4*b*x + 2*a + 4*d) + 4*e^(2*b*x + 2*a + 2*d
))*e^(-2*d)/b
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int e^{2(a+bx)}\cosh^2(d+bx)\,dx=\frac{x\,\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}}{4}+\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{4\,b}+\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,d}\,\mathrm{e}^{4\,b\,x}}{16\,b}$$

input
```
int(cosh(d + b*x)^2*exp(2*a + 2*b*x),x)
```

output
```
(x*exp(2*a)*exp(-2*d))/4 + (exp(2*a)*exp(2*b*x))/(4*b) + (exp(2*a)*exp(2*d
)*exp(4*b*x))/(16*b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int e^{2(a+bx)}\cosh^2(d+bx)\,dx=\frac{e^{2a}\left(e^{4bx+4d}+4e^{2bx+2d}+4bx\right)}{16e^{2d}b}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^2,x)
```

output
```
(e**(2*a)*(e**(4*b*x + 4*d) + 4*e**(2*b*x + 2*d) + 4*b*x))/(16*e**(2*d)*b)
```

3.28 $\int e^{2(a+bx)} \cosh^3(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 78

$$\int e^{2(a+bx)} \cosh^3(d+bx)\,dx = -\frac{e^{2a-3d-bx}}{8b} + \frac{3e^{2a-d+bx}}{8b} + \frac{e^{2a+d+3bx}}{8b} + \frac{e^{2a+3d+5bx}}{40b}$$

output
```
-1/8*exp(-b*x+2*a-3*d)/b+3/8*exp(b*x+2*a-d)/b+1/8*exp(3*b*x+2*a+d)/b+1/40*
exp(5*b*x+2*a+3*d)/b
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10

$$\int e^{2(a+bx)} \cosh^3(d+bx)\,dx$$
$$= \frac{e^{2a-bx}\left(5e^{2bx}\left(3+e^{2bx}\right)\cosh(d) + \left(-5+e^{6bx}\right)\cosh(3d) - 15e^{2bx}\sinh(d) + 5e^{4bx}\sinh(d) + 5\sinh(3d) + e^{6}\right.}{40b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[d + b*x]^3,x]
```

output
```
(E^(2*a - b*x)*(5*E^(2*b*x)*(3 + E^(2*b*x))*Cosh[d] + (-5 + E^(6*b*x))*Cos
h[3*d] - 15*E^(2*b*x)*Sinh[d] + 5*E^(4*b*x)*Sinh[d] + 5*Sinh[3*d] + E^(6*b
*x)*Sinh[3*d]))/(40*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.56, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)} \cosh^3(bx+d) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{8} e^{2a-2bx} \left(1+e^{2bx}\right)^3 de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{2a} \int e^{-2bx} \left(1+e^{2bx}\right)^3 de^{bx}}{8b}$$

$$\Big\downarrow \text{244}$$

$$\frac{e^{2a} \int \left(3+e^{-2bx}+3e^{2bx}+e^{4bx}\right) de^{bx}}{8b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2a} \left(-e^{-bx}+3e^{bx}+e^{3bx}+\frac{1}{5}e^{5bx}\right)}{8b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[d + b*x]^3,x]
```

output
```
(E^(2*a)*(-E^(-(b*x)) + 3*E^(b*x) + E^(3*b*x) + E^(5*b*x)/5))/(8*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{{\mathrm e}^{-b x +2 a -3 d}}{8 b}+\frac{3 \,{\mathrm e}^{b x +2 a -d}}{8 b}+\frac{{\mathrm e}^{3 b x +2 a +d}}{8 b}+\frac{{\mathrm e}^{5 b x +2 a +3 d}}{40 b}$
parallelrisch	$-\frac{2 \,{\mathrm e}^{2 b x +2 a} \left(5 \tanh \left(\frac{b x}{2}+\frac{d}{2}\right)^{5}-10 \tanh \left(\frac{b x}{2}+\frac{d}{2}\right)^{4}+10 \tanh \left(\frac{b x}{2}+\frac{d}{2}\right)^{3}-3 \tanh \left(\frac{b x}{2}+\frac{d}{2}\right)+2\right)}{5 b \left(-1+\tanh \left(\frac{b x}{2}+\frac{d}{2}\right)\right)^{3} \left(1+\tanh \left(\frac{b x}{2}+\frac{d}{2}\right)\right)^{3}}$
default	$-\frac{\sinh \left(-b x +2 a -3 d \right)}{8 b}+\frac{3 \sinh \left(b x +2 a -d \right)}{8 b}+\frac{\sinh \left(3 b x +2 a +d \right)}{8 b}+\frac{\sinh \left(5 b x +2 a +3 d \right)}{40 b}-\frac{\cosh \left(-b x +2 a -3 d \right)}{8 b}+\frac{3 \cosh (b}{}$
orering	$\frac{8 \,{\mathrm e}^{2 b x +2 a} \cosh \left(b x +d \right)^{3}}{15 b}+\frac{\frac{28 \,{\mathrm e}^{2 b x +2 a} b \cosh \left(b x +d \right)^{3}}{15}+\frac{14 \,{\mathrm e}^{2 b x +2 a} \cosh \left(b x +d \right)^{2} b \sinh \left(b x +d \right)}{5}}{b^{2}}-\frac{8 \left(7 \,{\mathrm e}^{2 b x +2 a} b^{2} \cosh \left(b x +d \right)^{3}+12 \,{\mathrm e}\right.}{}$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*exp(-b*x+2*a-3*d)/b+3/8*exp(b*x+2*a-d)/b+1/8*exp(3*b*x+2*a+d)/b+1/40*
exp(5*b*x+2*a+3*d)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(66) = 132$.

Time = 0.09 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.97

$$\int e^{2(a+bx)}\cosh^3(d+bx)\,dx =$$
$$-\frac{2\cosh(bx+d)^3\cosh(-2a+2d)-3(\cosh(-2a+2d)-\sinh(-2a+2d))\sinh(bx+d)^3+6(\cosh$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^3,x, algorithm="fricas")
```

output
```
-1/20*(2*cosh(b*x + d)^3*cosh(-2*a + 2*d) - 3*(cosh(-2*a + 2*d) - sinh(-2*
a + 2*d))*sinh(b*x + d)^3 + 6*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x +
 d)*sinh(-2*a + 2*d))*sinh(b*x + d)^2 - 10*cosh(b*x + d)*cosh(-2*a + 2*d)
- (9*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (9*cosh(b*x + d)^2 - 5)*sinh(-2*a
+ 2*d) - 5*cosh(-2*a + 2*d))*sinh(b*x + d) - 2*(cosh(b*x + d)^3 - 5*cosh(b
*x + d))*sinh(-2*a + 2*d))/(b*cosh(b*x + d)^2 - 2*b*cosh(b*x + d)*sinh(b*x
 + d) + b*sinh(b*x + d)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(61) = 122$.

Time = 0.95 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.59

$$\int e^{2(a+bx)}\cosh^3(d+bx)\,dx$$
$$= \begin{cases} \frac{2e^{2a}e^{2bx}\sinh^3(bx+d)}{5b} - \frac{4e^{2a}e^{2bx}\sinh^2(bx+d)\cosh(bx+d)}{5b} + \frac{e^{2a}e^{2bx}\sinh(bx+d)\cosh^2(bx+d)}{5b} + \frac{2e^{2a}e^{2bx}\cosh^3(bx+d)}{5b} & \text{for } b \neq \\ xe^{2a}\cosh^3(d) & \text{otherw} \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)**3,x)
```

output
```
Piecewise((2*exp(2*a)*exp(2*b*x)*sinh(b*x + d)**3/(5*b) - 4*exp(2*a)*exp(2
*b*x)*sinh(b*x + d)**2*cosh(b*x + d)/(5*b) + exp(2*a)*exp(2*b*x)*sinh(b*x
+ d)*cosh(b*x + d)**2/(5*b) + 2*exp(2*a)*exp(2*b*x)*cosh(b*x + d)**3/(5*b)
, Ne(b, 0)), (x*exp(2*a)*cosh(d)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.76

$$\int e^{2(a+bx)}\cosh^3(d+bx)\,dx = \frac{\left(5\,e^{(-2\,bx-2\,d)}+15\,e^{(-4\,bx-4\,d)}+1\right)e^{(5\,bx+2\,a+3\,d)}}{40\,b}-\frac{e^{(-bx+2\,a-3\,d)}}{8\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^3,x, algorithm="maxima")
```

output
```
1/40*(5*e^(-2*b*x - 2*d) + 15*e^(-4*b*x - 4*d) + 1)*e^(5*b*x + 2*a + 3*d)/
b - 1/8*e^(-b*x + 2*a - 3*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int e^{2(a+bx)}\cosh^3(d+bx)\,dx = \frac{\left(e^{(5\,bx+2\,a+5\,d)}+5\,e^{(3\,bx+2\,a+3\,d)}+15\,e^{(bx+2\,a+d)}-5\,e^{(-bx+2\,a-d)}\right)e^{(-2\,d)}}{40\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^3,x, algorithm="giac")
```

output
```
1/40*(e^(5*b*x + 2*a + 5*d) + 5*e^(3*b*x + 2*a + 3*d) + 15*e^(b*x + 2*a +
d) - 5*e^(-b*x + 2*a - d))*e^(-2*d)/b
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int e^{2(a+bx)}\cosh^3(d+bx)\,dx = \frac{e^{2\,a+d+3\,b\,x}}{8\,b} + \frac{3\,e^{2\,a-d+b\,x}}{8\,b} - \frac{e^{2\,a-3\,d-b\,x}}{8\,b} + \frac{e^{2\,a+3\,d+5\,b\,x}}{40\,b}$$

input
```
int(cosh(d + b*x)^3*exp(2*a + 2*b*x),x)
```

output
```
exp(2*a + d + 3*b*x)/(8*b) + (3*exp(2*a - d + b*x))/(8*b) - exp(2*a - 3*d
- b*x)/(8*b) + exp(2*a + 3*d + 5*b*x)/(40*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.73

$$\int e^{2(a+bx)}\cosh^3(d+bx)\,dx = \frac{e^{2a}\left(e^{6bx+6d}+5e^{4bx+4d}+15e^{2bx+2d}-5\right)}{40e^{bx+3d}b}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^3,x)
```

output
```
(e**(2*a)*(e**(6*b*x + 6*d) + 5*e**(4*b*x + 4*d) + 15*e**(2*b*x + 2*d) - 5
))/(40*e**(b*x + 3*d)*b)
```

3.29 $\int e^{2(a+bx)} \cosh^4(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 93

$$\int e^{2(a+bx)} \cosh^4(d+bx)\,dx = -\frac{e^{2(a-2d)-2bx}}{32b} + \frac{3e^{2a+2bx}}{16b} + \frac{e^{2(a+d)+4bx}}{16b} + \frac{e^{2(a+2d)+6bx}}{96b} + \frac{1}{4}e^{2a-2d}x$$

output
```
-1/32*exp(-2*b*x+2*a-4*d)/b+3/16*exp(2*b*x+2*a)/b+1/16*exp(4*b*x+2*a+2*d)/
b+1/96*exp(6*b*x+2*a+4*d)/b+1/4*exp(2*a-2*d)*x
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.13

$$\int e^{2(a+bx)} \cosh^4(d+bx)\,dx = \frac{e^{2a-2bx}\left(18e^{4bx} + 6e^{2bx}\left(e^{4bx}+4bx\right)\cosh(2d) + \left(-3+e^{8bx}\right)\cosh(4d) + 6e^{6bx}\sinh(2d) - 24be^{2bx}x\sinh(2d\right.}{96b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[d + b*x]^4,x]
```

output
```
(E^(2*a - 2*b*x)*(18*E^(4*b*x) + 6*E^(2*b*x)*(E^(4*b*x) + 4*b*x)*Cosh[2*d]
 + (-3 + E^(8*b*x))*Cosh[4*d] + 6*E^(6*b*x)*Sinh[2*d] - 24*b*E^(2*b*x)*x*S
inh[2*d] + 3*Sinh[4*d] + E^(8*b*x)*Sinh[4*d]))/(96*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.52, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)} \cosh^4(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{16} e^{2a-3bx} \left(1+e^{2bx}\right)^4 de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{2a} \int e^{-3bx} \left(1+e^{2bx}\right)^4 de^{bx}}{16b}$$

$$\Big\downarrow \text{243}$$

$$\frac{e^{2a} \int e^{-2bx} \left(1+e^{2bx}\right)^4 de^{2bx}}{32b}$$

$$\Big\downarrow \text{49}$$

$$\frac{e^{2a} \int \left(6+e^{-2bx}+4e^{-bx}+5e^{2bx}\right) de^{2bx}}{32b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2a} \left(-e^{-bx}+8e^{2bx}+\frac{1}{3}e^{3bx}+4\log\left(e^{2bx}\right)\right)}{32b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[d + b*x]^4,x]
```

output
```
(E^(2*a)*(-E^(-(b*x)) + 8*E^(2*b*x) + E^(3*b*x)/3 + 4*Log[E^(2*b*x)]))/(32
*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{e^{-2bx+2a-4d}}{32b}+\frac{3\,e^{2bx+2a}}{16b}+\frac{e^{4bx+2a+2d}}{16b}+\frac{e^{6bx+2a+4d}}{96b}+\frac{e^{2a-2d}x}{4}$
parallelrisch	$\frac{e^{2bx+2a}(-12bx\sinh(2bx+2d)+12bx\cosh(2bx+2d)+2\sinh(4bx+4d)+14\sinh(2bx+2d)-\cosh(4bx+4d)-8\cosh(2bx+2d)+9)}{48b}$
default	$\frac{x\cosh(2a-2d)}{4}+\frac{3\sinh(2bx+2a)}{16b}-\frac{\sinh(-2bx+2a-4d)}{32b}+\frac{\sinh(4bx+2a+2d)}{16b}+\frac{\sinh(6bx+2a+4d)}{96b}+\frac{x\sinh(2a-2d)}{4}$
orering	$\frac{(12bx+5)e^{2bx+2a}\cosh(bx+d)^4}{12b}-\frac{5(2bx-1)\left(2\,e^{2bx+2a}b\cosh(bx+d)^4+4\,e^{2bx+2a}\cosh(bx+d)^3b\sinh(bx+d)\right)}{24b^2}-\frac{5(2bx+1)}{}$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/32*exp(-2*b*x+2*a-4*d)/b+3/16*exp(2*b*x+2*a)/b+1/16*exp(4*b*x+2*a+2*d)/
b+1/96*exp(6*b*x+2*a+4*d)/b+1/4*exp(2*a-2*d)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(77) = 154.

Time = 0.10 (sec) , antiderivative size = 352, normalized size of antiderivative = 3.78

$$\int e^{2(a+bx)}\cosh^4(d+bx)\,dx =$$
$$-\frac{\cosh\left(bx+d\right)^4\cosh\left(-2\,a+2\,d\right)+\left(\cosh\left(-2\,a+2\,d\right)-\sinh\left(-2\,a+2\,d\right)\right)\sinh\left(bx+d\right)^4-3\left(4\,bx+1\right.}{}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^4,x, algorithm="fricas")
```

output
```
-1/48*(cosh(b*x + d)^4*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a +
2*d))*sinh(b*x + d)^4 - 3*(4*b*x + 1)*cosh(b*x + d)^2*cosh(-2*a + 2*d) - 8
*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*
x + d)^3 + 3*(2*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (4*b*x + 1)*cosh(-2*a +
 2*d) + (4*b*x - 2*cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d))*sinh(b*x + d)^2
- 2*(4*cosh(b*x + d)^3*cosh(-2*a + 2*d) - 3*(4*b*x - 1)*cosh(b*x + d)*cosh
(-2*a + 2*d) - (4*cosh(b*x + d)^3 - 3*(4*b*x - 1)*cosh(b*x + d))*sinh(-2*a
 + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^4 - 3*(4*b*x + 1)*cosh(b*x + d)^2
- 9)*sinh(-2*a + 2*d) - 9*cosh(-2*a + 2*d))/(b*cosh(b*x + d)^2 - 2*b*cosh(
b*x + d)*sinh(b*x + d) + b*sinh(b*x + d)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(76) = 152$.

Time = 2.35 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.85

$$\int e^{2(a+bx)}\cosh^4(d+bx)\,dx$$
$$= \begin{cases} -\frac{xe^{2a}e^{2bx}\sinh^4(bx+d)}{4} + \frac{xe^{2a}e^{2bx}\sinh^3(bx+d)\cosh(bx+d)}{2} - \frac{xe^{2a}e^{2bx}\sinh(bx+d)\cosh^3(bx+d)}{2} + \frac{xe^{2a}e^{2bx}\cosh^4(bx+d)}{4} + \frac{13e^{2}}{} \\ xe^{2a}\cosh^4(d) \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)**4,x)
```

output
```
Piecewise((-x*exp(2*a)*exp(2*b*x)*sinh(b*x + d)**4/4 + x*exp(2*a)*exp(2*b*
x)*sinh(b*x + d)**3*cosh(b*x + d)/2 - x*exp(2*a)*exp(2*b*x)*sinh(b*x + d)*
cosh(b*x + d)**3/2 + x*exp(2*a)*exp(2*b*x)*cosh(b*x + d)**4/4 + 13*exp(2*a
)*exp(2*b*x)*sinh(b*x + d)**4/(48*b) - 7*exp(2*a)*exp(2*b*x)*sinh(b*x + d)
**3*cosh(b*x + d)/(24*b) - exp(2*a)*exp(2*b*x)*sinh(b*x + d)**2*cosh(b*x +
 d)**2/(2*b) + 5*exp(2*a)*exp(2*b*x)*sinh(b*x + d)*cosh(b*x + d)**3/(8*b)
+ exp(2*a)*exp(2*b*x)*cosh(b*x + d)**4/(16*b), Ne(b, 0)), (x*exp(2*a)*cosh
(d)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

$$\int e^{2(a+bx)}\cosh^4(d+bx)\,dx = \frac{\left(6\,e^{(-2\,bx-2\,d)}+18\,e^{(-4\,bx-4\,d)}+1\right)e^{(6\,bx+2\,a+4\,d)}}{96\,b} + \frac{(bx+d)e^{(2\,a-2\,d)}}{4\,b} - \frac{e^{(-2\,bx+2\,a-4\,d)}}{32\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^4,x, algorithm="maxima")
```

output
```
1/96*(6*e^(-2*b*x - 2*d) + 18*e^(-4*b*x - 4*d) + 1)*e^(6*b*x + 2*a + 4*d)/
b + 1/4*(b*x + d)*e^(2*a - 2*d)/b - 1/32*e^(-2*b*x + 2*a - 4*d)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int e^{2(a+bx)}\cosh^4(d+bx)\,dx =$$
$$-\frac{\left(3\left(4\,e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right)e^{(-2\,bx-2\,d)}-24\,(bx+d)e^{(2\,a)}-e^{(6\,bx+2\,a+6\,d)}-6\,e^{(4\,bx+2\,a+4\,d)}-18\,e^{(2\,bx+2\,a+}\cdots}{96\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^4,x, algorithm="giac")
```

output
```
-1/96*(3*(4*e^(2*b*x + 2*a + 2*d) + e^(2*a))*e^(-2*b*x - 2*d) - 24*(b*x +
d)*e^(2*a) - e^(6*b*x + 2*a + 6*d) - 6*e^(4*b*x + 2*a + 4*d) - 18*e^(2*b*x
 + 2*a + 2*d))*e^(-2*d)/b
```

Mupad [B] (verification not implemented)

Time = 3.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.60

$$\int e^{2(a+bx)}\cosh^4(d+bx)\,dx = \frac{3\,\mathrm{e}^{2\,a+2\,b\,x}}{16\,b}+\frac{x\,\mathrm{e}^{2\,a+2\,b\,x}\cosh(2\,d+2\,b\,x)}{4} - \frac{x\,\mathrm{e}^{2\,a+2\,b\,x}\sinh(2\,d+2\,b\,x)}{4} + \frac{\mathrm{e}^{2\,a+2\,b\,x}\cosh(2\,d+2\,b\,x)}{6\,b}-\frac{\mathrm{e}^{2\,a+2\,b\,x}\cosh(4\,d+4\,b\,x)}{48\,b} - \frac{\mathrm{e}^{2\,a+2\,b\,x}\sinh(2\,d+2\,b\,x)}{24\,b}+\frac{\mathrm{e}^{2\,a+2\,b\,x}\sinh(4\,d+4\,b\,x)}{24\,b}$$

input
```
int(cosh(d + b*x)^4*exp(2*a + 2*b*x),x)
```

output
```
(3*exp(2*a + 2*b*x))/(16*b) + (x*exp(2*a + 2*b*x)*cosh(2*d + 2*b*x))/4 - (
x*exp(2*a + 2*b*x)*sinh(2*d + 2*b*x))/4 + (exp(2*a + 2*b*x)*cosh(2*d + 2*b
*x))/(6*b) - (exp(2*a + 2*b*x)*cosh(4*d + 4*b*x))/(48*b) - (exp(2*a + 2*b*
x)*sinh(2*d + 2*b*x))/(24*b) + (exp(2*a + 2*b*x)*sinh(4*d + 4*b*x))/(24*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.77

$$\int e^{2(a+bx)}\cosh^4(d+bx)\,dx = \frac{e^{2a}\left(e^{8bx+8d}+6e^{6bx+6d}+18e^{4bx+4d}+24e^{2bx+2d}bx-3\right)}{96e^{2bx+4d}b}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^4,x)
```

output
```
(e**(2*a)*(e**(8*b*x + 8*d) + 6*e**(6*b*x + 6*d) + 18*e**(4*b*x + 4*d) + 2
4*e**(2*b*x + 2*d)*b*x - 3))/(96*e**(2*b*x + 4*d)*b)
```

3.30 $\int e^{\frac{5}{3}(a+bx)} \cosh(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int e^{\frac{5}{3}(a+bx)} \cosh(a+bx)\,dx = \frac{3e^{\frac{2}{3}(a+bx)}}{4b} + \frac{3e^{\frac{8}{3}(a+bx)}}{16b}$$

output
```
3/4*exp(2/3*b*x+2/3*a)/b+3/16*exp(8/3*b*x+8/3*a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int e^{\frac{5}{3}(a+bx)} \cosh(a+bx)\,dx = \frac{3e^{\frac{2}{3}(a+bx)}\left(4+e^{2(a+bx)}\right)}{16b}$$

input
```
Integrate[E^((5*(a + b*x))/3)*Cosh[a + b*x],x]
```

output
```
(3*E^((2*(a + b*x))/3)*(4 + E^(2*(a + b*x))))/(16*b)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)}\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{3\int \frac{1}{2}e^{\frac{1}{3}(a+bx)}\left(1+e^{2(a+bx)}\right)de^{\frac{1}{3}(a+bx)}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int e^{\frac{1}{3}(a+bx)}\left(1+e^{2(a+bx)}\right)de^{\frac{1}{3}(a+bx)}}{2b}$$

$$\Big\downarrow \text{802}$$

$$\frac{3\int \left(e^{\frac{1}{3}(a+bx)}+e^{\frac{7}{3}(a+bx)}\right)de^{\frac{1}{3}(a+bx)}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(\frac{1}{2}e^{\frac{2}{3}(a+bx)}+\frac{1}{8}e^{\frac{8}{3}(a+bx)}\right)}{2b}$$

input
```
Int[E^((5*(a + b*x))/3)*Cosh[a + b*x],x]
```

output
```
(3*(E^((2*(a + b*x))/3)/2 + E^((8*(a + b*x))/3)/8))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{3\,{\mathrm e}^{\frac{2 b x}{3}+\frac{2 a}{3}}}{4 b}+\frac{3\,{\mathrm e}^{\frac{8 b x}{3}+\frac{8 a}{3}}}{16 b}$	30
parallelrisch	$\frac{3\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \left(5 \cosh \left(b x +a \right)-3 \sinh \left(b x +a \right)\right)}{16 b}$	32
default	$\frac{3 \sinh \left(\frac{2 b x}{3}+\frac{2 a}{3}\right)}{4 b}+\frac{3 \sinh \left(\frac{8 b x}{3}+\frac{8 a}{3}\right)}{16 b}+\frac{3 \cosh \left(\frac{2 b x}{3}+\frac{2 a}{3}\right)}{4 b}+\frac{3 \cosh \left(\frac{8 b x}{3}+\frac{8 a}{3}\right)}{16 b}$	58
orering	$\frac{15\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \cosh \left(b x +a \right)}{8 b}-\frac{9 \left(\frac{5 b\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \cosh \left(b x +a \right)}{3}+{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} b \sinh \left(b x +a \right)\right)}{16 b^{2}}$	63

input
```
int(exp(5/3*b*x+5/3*a)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
3/4*exp(2/3*b*x+2/3*a)/b+3/16*exp(8/3*b*x+8/3*a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(29) = 58$.

Time = 0.08 (sec) , antiderivative size = 222, normalized size of antiderivative = 6.00

$$\int e^{\frac{5}{3}(a+bx)}\cosh(a+bx)\,dx$$
$$= \frac{3\left(\cosh\left(\frac{1}{3}bx+\frac{1}{3}a\right)^4+4\cosh\left(\frac{1}{3}bx+\frac{1}{3}a\right)\sinh\left(\frac{1}{3}bx+\frac{1}{3}a\right)^3+\sinh\left(\frac{1}{3}bx+\frac{1}{3}a\right)^4+2\left(3\cosh\left(\frac{1}{3}bx+\frac{1}{3}\right.\right.\right.}{16\left(b\cosh\left(\frac{1}{3}bx+\frac{1}{3}a\right)^4-4b\cosh\left(\frac{1}{3}bx+\frac{1}{3}a\right)^3\sinh\left(\frac{1}{3}bx+\frac{1}{3}a\right)+6b\cosh\left(\frac{1}{3}b\right.\right.}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*cosh(b*x+a),x, algorithm="fricas")
```

output
```
3/16*(cosh(1/3*b*x + 1/3*a)^4 + 4*cosh(1/3*b*x + 1/3*a)*sinh(1/3*b*x + 1/3
*a)^3 + sinh(1/3*b*x + 1/3*a)^4 + 2*(3*cosh(1/3*b*x + 1/3*a)^2 + 2)*sinh(1
/3*b*x + 1/3*a)^2 + 4*cosh(1/3*b*x + 1/3*a)^2 + 4*(cosh(1/3*b*x + 1/3*a)^3
 - 2*cosh(1/3*b*x + 1/3*a))*sinh(1/3*b*x + 1/3*a))/(b*cosh(1/3*b*x + 1/3*a
)^4 - 4*b*cosh(1/3*b*x + 1/3*a)^3*sinh(1/3*b*x + 1/3*a) + 6*b*cosh(1/3*b*x
 + 1/3*a)^2*sinh(1/3*b*x + 1/3*a)^2 - 4*b*cosh(1/3*b*x + 1/3*a)*sinh(1/3*b
*x + 1/3*a)^3 + b*sinh(1/3*b*x + 1/3*a)^4)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.76

$$\int e^{\frac{5}{3}(a+bx)}\cosh(a+bx)\,dx = \begin{cases} -\frac{9e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\sinh(a+bx)}{16b}+\frac{15e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\cosh(a+bx)}{16b} & \text{for } b\neq 0 \\ xe^{\frac{5a}{3}}\cosh(a) & \text{otherwise} \end{cases}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*cosh(b*x+a),x)
```

output
```
Piecewise((-9*exp(5*a/3)*exp(5*b*x/3)*sinh(a + b*x)/(16*b) + 15*exp(5*a/3)
*exp(5*b*x/3)*cosh(a + b*x)/(16*b), Ne(b, 0)), (x*exp(5*a/3)*cosh(a), True
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int e^{\frac{5}{3}(a+bx)}\cosh(a+bx)\,dx = \frac{3\,e^{\left(\frac{8}{3}\,bx+\frac{8}{3}\,a\right)}}{16\,b} + \frac{3\,e^{\left(\frac{2}{3}\,bx+\frac{2}{3}\,a\right)}}{4\,b}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*cosh(b*x+a),x, algorithm="maxima")
```

output
```
3/16*e^(8/3*b*x + 8/3*a)/b + 3/4*e^(2/3*b*x + 2/3*a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int e^{\frac{5}{3}(a+bx)}\cosh(a+bx)\,dx = \frac{3\left(e^{\left(\frac{8}{3}\,bx+\frac{11}{3}\,a\right)}+4\,e^{\left(\frac{2}{3}\,bx+\frac{5}{3}\,a\right)}\right)e^{(-a)}}{16\,b}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*cosh(b*x+a),x, algorithm="giac")
```

output
```
3/16*(e^(8/3*b*x + 11/3*a) + 4*e^(2/3*b*x + 5/3*a))*e^(-a)/b
```

Mupad [B] (verification not implemented)

Time = 2.59 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int e^{\frac{5}{3}(a+bx)}\cosh(a+bx)\,dx = \frac{3\,\mathrm{e}^{\frac{2\,a}{3}+\frac{2\,b\,x}{3}}\,\left(\mathrm{e}^{2\,a+2\,b\,x}+4\right)}{16\,b}$$

input
```
int(cosh(a + b*x)*exp((5*a)/3 + (5*b*x)/3),x)
```

output
```
(3*exp((2*a)/3 + (2*b*x)/3)*(exp(2*a + 2*b*x) + 4))/(16*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int e^{\frac{5}{3}(a+bx)}\cosh(a+bx)\,dx=\frac{3e^{\frac{5bx}{3}+\frac{5a}{3}}\left(5\cosh\left(bx+a\right)-3\sinh\left(bx+a\right)\right)}{16b}$$

input
```
int(exp(5/3*b*x+5/3*a)*cosh(b*x+a),x)
```

output
```
(3*e**((5*a + 5*b*x)/3)*(5*cosh(a + b*x) - 3*sinh(a + b*x)))/(16*b)
```

3.31 $\int e^{\frac{5}{3}(a+bx)} \cosh^2(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 58

$$\int e^{\frac{5}{3}(a+bx)} \cosh^2(a+bx)\,dx = -\frac{3e^{\frac{1}{3}(-a-bx)}}{4b} + \frac{3e^{\frac{5}{3}(a+bx)}}{10b} + \frac{3e^{\frac{11}{3}(a+bx)}}{44b}$$

output
```
-3/4*exp(-1/3*b*x-1/3*a)/b+3/10*exp(5/3*b*x+5/3*a)/b+3/44*exp(11/3*b*x+11/
3*a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

$$\int e^{\frac{5}{3}(a+bx)} \cosh^2(a+bx)\,dx = \frac{3e^{\frac{1}{3}(-a-bx)}\left(-55+22e^{2(a+bx)}+5e^{4(a+bx)}\right)}{220b}$$

input
```
Integrate[E^((5*(a + b*x))/3)*Cosh[a + b*x]^2,x]
```

output
```
(3*E^((-a - b*x)/3)*(-55 + 22*E^(2*(a + b*x)) + 5*E^(4*(a + b*x))))/(220*b
)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)}\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{3\int \frac{1}{4}e^{-\frac{2}{3}(a+bx)}\left(1+e^{2(a+bx)}\right)^2de^{\frac{1}{3}(a+bx)}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3\int e^{-\frac{2}{3}(a+bx)}\left(1+e^{2(a+bx)}\right)^2de^{\frac{1}{3}(a+bx)}}{4b}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{3\int \left(e^{-\frac{2}{3}(a+bx)}+2e^{\frac{4}{3}(a+bx)}+e^{\frac{10}{3}(a+bx)}\right)de^{\frac{1}{3}(a+bx)}}{4b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3\left(-e^{\frac{1}{3}(-a-bx)}+\frac{2}{5}e^{\frac{5}{3}(a+bx)}+\frac{1}{11}e^{\frac{11}{3}(a+bx)}\right)}{4b}$$

input
```
Int[E^((5*(a + b*x))/3)*Cosh[a + b*x]^2,x]
```

output
```
(3*(-E^((-a - b*x)/3) + (2*E^((5*(a + b*x))/3))/5 + E^((11*(a + b*x))/3)/1
1))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

method	result
parallelrisch	$-\frac{3\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \left(-11+25 \cosh \left(2 b x +2 a \right)-30 \sinh \left(2 b x +2 a \right)\right)}{110 b}$
risch	$-\frac{3\,{\mathrm e}^{-\frac{b x}{3}-\frac{a}{3}}}{4 b}+\frac{3\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}}}{10 b}+\frac{3\,{\mathrm e}^{\frac{11 b x}{3}+\frac{11 a}{3}}}{44 b}$
default	$\frac{3 \sinh \left(\frac{b x}{3}+\frac{a}{3}\right)}{4 b}+\frac{3 \sinh \left(\frac{5 b x}{3}+\frac{5 a}{3}\right)}{10 b}+\frac{3 \sinh \left(\frac{11 b x}{3}+\frac{11 a}{3}\right)}{44 b}-\frac{3 \cosh \left(\frac{b x}{3}+\frac{a}{3}\right)}{4 b}+\frac{3 \cosh \left(\frac{5 b x}{3}+\frac{5 a}{3}\right)}{10 b}+\frac{3 \cosh \left(\frac{11 b x}{3}+\frac{11 a}{3}\right)}{44 b}$
orering	$-\frac{117\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \cosh \left(b x +a \right)^{2}}{55 b}+\frac{\frac{45 b\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \cosh \left(b x +a \right)^{2}}{11}+\frac{54\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \cosh \left(b x +a \right) b \sinh \left(b x +a \right)}{11}}{b^{2}}-\frac{27 \left(\frac{43 b^{2} {\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \cosh (b x}{9}\right.}{}$

input
```
int(exp(5/3*b*x+5/3*a)*cosh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/110*exp(5/3*b*x+5/3*a)*(-11+25*cosh(2*b*x+2*a)-30*sinh(2*b*x+2*a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(43) = 86.

Time = 0.09 (sec) , antiderivative size = 272, normalized size of antiderivative = 4.69

$$\int e^{\frac{5}{3}(a+bx)}\cosh^2(a+bx)\,dx =$$
$$-\frac{3\left(25\cosh\left(\frac{1}{3}bx+\frac{1}{3}a\right)^6-180\cosh\left(\frac{1}{3}bx+\frac{1}{3}a\right)^5\sinh\left(\frac{1}{3}bx+\frac{1}{3}a\right)+375\cosh\left(\frac{1}{3}bx+\frac{1}{3}a\right)^4\sinh\left(\frac{1}{3}b\right.\right.}{110\left(b\cosh\left(\frac{1}{3}bx+\frac{1}{3}a\right)^5-5\,b\cosh\left(\frac{1}{3}bx+\frac{1}{3}a\right)^4\sinh\left(\frac{1}{3}bx+\frac{1}{3}a\right)+10\,b\,\mathrm{co}\right.}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*cosh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-3/110*(25*cosh(1/3*b*x + 1/3*a)^6 - 180*cosh(1/3*b*x + 1/3*a)^5*sinh(1/3*
b*x + 1/3*a) + 375*cosh(1/3*b*x + 1/3*a)^4*sinh(1/3*b*x + 1/3*a)^2 - 600*c
osh(1/3*b*x + 1/3*a)^3*sinh(1/3*b*x + 1/3*a)^3 + 375*cosh(1/3*b*x + 1/3*a)
^2*sinh(1/3*b*x + 1/3*a)^4 - 180*cosh(1/3*b*x + 1/3*a)*sinh(1/3*b*x + 1/3*
a)^5 + 25*sinh(1/3*b*x + 1/3*a)^6 - 11)/(b*cosh(1/3*b*x + 1/3*a)^5 - 5*b*c
osh(1/3*b*x + 1/3*a)^4*sinh(1/3*b*x + 1/3*a) + 10*b*cosh(1/3*b*x + 1/3*a)^
3*sinh(1/3*b*x + 1/3*a)^2 - 10*b*cosh(1/3*b*x + 1/3*a)^2*sinh(1/3*b*x + 1/
3*a)^3 + 5*b*cosh(1/3*b*x + 1/3*a)*sinh(1/3*b*x + 1/3*a)^4 - b*sinh(1/3*b*
x + 1/3*a)^5)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.79

$$\int e^{\frac{5}{3}(a+bx)}\cosh^2(a+bx)\,dx$$
$$=\begin{cases}-\frac{54e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\sinh^2(a+bx)}{55b}+\frac{18e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\sinh(a+bx)\cosh(a+bx)}{11b}-\frac{21e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\cosh^2(a+bx)}{55b} & \text{for } b\neq 0\\ xe^{\frac{5a}{3}}\cosh^2(a) & \text{otherwise}\end{cases}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*cosh(b*x+a)**2,x)
```

output
```
Piecewise((-54*exp(5*a/3)*exp(5*b*x/3)*sinh(a + b*x)**2/(55*b) + 18*exp(5*
a/3)*exp(5*b*x/3)*sinh(a + b*x)*cosh(a + b*x)/(11*b) - 21*exp(5*a/3)*exp(5
*b*x/3)*cosh(a + b*x)**2/(55*b), Ne(b, 0)), (x*exp(5*a/3)*cosh(a)**2, True
))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.72

$$\int e^{\frac{5}{3}(a+bx)}\cosh^2(a+bx)\,dx=\frac{3\left(22\,e^{(-2\,bx-2\,a)}+5\right)e^{\left(\frac{11}{3}\,bx+\frac{11}{3}\,a\right)}}{220\,b}-\frac{3\,e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,a\right)}}{4\,b}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*cosh(b*x+a)^2,x, algorithm="maxima")
```

output
```
3/220*(22*e^(-2*b*x - 2*a) + 5)*e^(11/3*b*x + 11/3*a)/b - 3/4*e^(-1/3*b*x
- 1/3*a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.74

$$\int e^{\frac{5}{3}(a+bx)}\cosh^2(a+bx)\,dx=\frac{3\left(5\,e^{\left(\frac{11}{3}\,bx+\frac{17}{3}\,a\right)}+22\,e^{\left(\frac{5}{3}\,bx+\frac{11}{3}\,a\right)}-55\,e^{\left(-\frac{1}{3}\,bx+\frac{5}{3}\,a\right)}\right)e^{(-2\,a)}}{220\,b}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*cosh(b*x+a)^2,x, algorithm="giac")
```

output
```
3/220*(5*e^(11/3*b*x + 17/3*a) + 22*e^(5/3*b*x + 11/3*a) - 55*e^(-1/3*b*x
+ 5/3*a))*e^(-2*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.74

$$\int e^{\frac{5}{3}(a+bx)}\cosh^2(a+bx)\,dx=\frac{3\,\mathrm{e}^{\frac{5\,a}{3}+\frac{5\,b\,x}{3}}}{10\,b}-\frac{3\,\mathrm{e}^{-\frac{a}{3}-\frac{b\,x}{3}}}{4\,b}+\frac{3\,\mathrm{e}^{\frac{11\,a}{3}+\frac{11\,b\,x}{3}}}{44\,b}$$

input
```
int(cosh(a + b*x)^2*exp((5*a)/3 + (5*b*x)/3),x)
```

output
```
(3*exp((5*a)/3 + (5*b*x)/3))/(10*b) - (3*exp(- a/3 - (b*x)/3))/(4*b) + (3*
exp((11*a)/3 + (11*b*x)/3))/(44*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int e^{\frac{5}{3}(a+bx)}\cosh^2(a+bx)\,dx=\frac{3e^{\frac{5bx}{3}+\frac{5a}{3}}\left(5e^{4bx+4a}+22e^{2bx+2a}-55\right)}{220e^{2bx+2a}b}$$

input
```
int(exp(5/3*b*x+5/3*a)*cosh(b*x+a)^2,x)
```

output
```
(3*e**((5*a + 5*b*x)/3)*(5*e**(4*a + 4*b*x) + 22*e**(2*a + 2*b*x) - 55))/(
220*e**(2*a + 2*b*x)*b)
```

3.32 $\int e^{\frac{5}{3}(a+bx)} \cosh^3(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 73

$$\int e^{\frac{5}{3}(a+bx)} \cosh^3(a+bx)\,dx=-\frac{3e^{-\frac{4}{3}(a+bx)}}{32b}+\frac{9e^{\frac{2}{3}(a+bx)}}{16b}+\frac{9e^{\frac{8}{3}(a+bx)}}{64b}+\frac{3e^{\frac{14}{3}(a+bx)}}{112b}$$

output
```
-3/32/b/exp(4/3*b*x+4/3*a)+9/16*exp(2/3*b*x+2/3*a)/b+9/64*exp(8/3*b*x+8/3*
a)/b+3/112*exp(14/3*b*x+14/3*a)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.73

$$\int e^{\frac{5}{3}(a+bx)} \cosh^3(a+bx)\,dx=\frac{3e^{-\frac{4}{3}(a+bx)}\left(-14+84e^{2(a+bx)}+21e^{4(a+bx)}+4e^{6(a+bx)}\right)}{448b}$$

input
```
Integrate[E^((5*(a + b*x))/3)*Cosh[a + b*x]^3,x]
```

output
```
(3*(-14 + 84*E^(2*(a + b*x)) + 21*E^(4*(a + b*x)) + 4*E^(6*(a + b*x))))/(4
48*b*E^((4*(a + b*x))/3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)}\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{3\int \frac{1}{8}e^{-\frac{5}{3}(a+bx)}\left(1+e^{2(a+bx)}\right)^3de^{\frac{1}{3}(a+bx)}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int e^{-\frac{5}{3}(a+bx)}\left(1+e^{2(a+bx)}\right)^3de^{\frac{1}{3}(a+bx)}}{8b}$$

$$\Big\downarrow \text{802}$$

$$\frac{3\int \left(e^{-\frac{5}{3}(a+bx)}+3e^{\frac{1}{3}(a+bx)}+3e^{\frac{7}{3}(a+bx)}+e^{\frac{13}{3}(a+bx)}\right)de^{\frac{1}{3}(a+bx)}}{8b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(-\frac{1}{4}e^{-\frac{4}{3}(a+bx)}+\frac{3}{2}e^{\frac{2}{3}(a+bx)}+\frac{3}{8}e^{\frac{8}{3}(a+bx)}+\frac{1}{14}e^{\frac{14}{3}(a+bx)}\right)}{8b}$$

input
```
Int[E^((5*(a + b*x))/3)*Cosh[a + b*x]^3,x]
```

output
```
(3*(-1/4*1/E^((4*(a + b*x))/3) + (3*E^((2*(a + b*x))/3))/2 + (3*E^((8*(a +
 b*x))/3))/8 + E^((14*(a + b*x))/3)/14))/(8*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{3\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \left(-10 \cosh \left(3 b x +3 a \right)+105 \cosh \left(b x +a \right)-63 \sinh \left(b x +a \right)+18 \sinh \left(3 b x +3 a \right)\right)}{448 b}$
risch	$\frac{3\,{\mathrm e}^{\frac{14 b x}{3}+\frac{14 a}{3}}}{112 b}+\frac{9\,{\mathrm e}^{\frac{8 b x}{3}+\frac{8 a}{3}}}{64 b}+\frac{9\,{\mathrm e}^{\frac{2 b x}{3}+\frac{2 a}{3}}}{16 b}-\frac{3\,{\mathrm e}^{-\frac{4 b x}{3}-\frac{4 a}{3}}}{32 b}$
default	$\frac{9 \sinh \left(\frac{2 b x}{3}+\frac{2 a}{3}\right)}{16 b}+\frac{3 \sinh \left(\frac{4 b x}{3}+\frac{4 a}{3}\right)}{32 b}+\frac{9 \sinh \left(\frac{8 b x}{3}+\frac{8 a}{3}\right)}{64 b}+\frac{3 \sinh \left(\frac{14 b x}{3}+\frac{14 a}{3}\right)}{112 b}+\frac{9 \cosh \left(\frac{2 b x}{3}+\frac{2 a}{3}\right)}{16 b}-\frac{3 \cosh \left(\frac{4 b x}{3}+\right.}{32 b}$
orering	$\frac{75\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \cosh \left(b x +a \right)^{3}}{56 b}+\frac{\frac{225 b\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \cosh \left(b x +a \right)^{3}}{224}+\frac{405\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \cosh \left(b x +a \right)^{2} b \sinh \left(b x +a \right)}{224}}{b^{2}}-\frac{135 \left(\frac{52 b^{2} {\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \cosh (b\right.}{9}}{}$

input
```
int(exp(5/3*b*x+5/3*a)*cosh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
3/448*exp(5/3*b*x+5/3*a)/b*(-10*cosh(3*b*x+3*a)+105*cosh(b*x+a)-63*sinh(b*
x+a)+18*sinh(3*b*x+3*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(57) = 114$.

Time = 0.09 (sec) , antiderivative size = 390, normalized size of antiderivative = 5.34

$$\int e^{\frac{5}{3}(a+bx)} \cosh^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*cosh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-3/448*(10*cosh(1/3*b*x + 1/3*a)^9 + 1260*cosh(1/3*b*x + 1/3*a)^5*sinh(1/3
*b*x + 1/3*a)^4 - 2268*cosh(1/3*b*x + 1/3*a)^4*sinh(1/3*b*x + 1/3*a)^5 + 8
40*cosh(1/3*b*x + 1/3*a)^3*sinh(1/3*b*x + 1/3*a)^6 - 648*cosh(1/3*b*x + 1/
3*a)^2*sinh(1/3*b*x + 1/3*a)^7 + 90*cosh(1/3*b*x + 1/3*a)*sinh(1/3*b*x + 1
/3*a)^8 - 18*sinh(1/3*b*x + 1/3*a)^9 - 63*(24*cosh(1/3*b*x + 1/3*a)^6 - 1)
*sinh(1/3*b*x + 1/3*a)^3 - 105*cosh(1/3*b*x + 1/3*a)^3 + 45*(8*cosh(1/3*b*
x + 1/3*a)^7 - 7*cosh(1/3*b*x + 1/3*a))*sinh(1/3*b*x + 1/3*a)^2 - 27*(6*co
sh(1/3*b*x + 1/3*a)^8 - 7*cosh(1/3*b*x + 1/3*a)^2)*sinh(1/3*b*x + 1/3*a))/
(b*cosh(1/3*b*x + 1/3*a)^5 - 5*b*cosh(1/3*b*x + 1/3*a)^4*sinh(1/3*b*x + 1/
3*a) + 10*b*cosh(1/3*b*x + 1/3*a)^3*sinh(1/3*b*x + 1/3*a)^2 - 10*b*cosh(1/
3*b*x + 1/3*a)^2*sinh(1/3*b*x + 1/3*a)^3 + 5*b*cosh(1/3*b*x + 1/3*a)*sinh(
1/3*b*x + 1/3*a)^4 - b*sinh(1/3*b*x + 1/3*a)^5)
```

Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.93

$$\int e^{\frac{5}{3}(a+bx)} \cosh^3(a+bx)\,dx$$

$$= \begin{cases} \frac{243e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\sinh^3(a+bx)}{448b} - \frac{405e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\sinh^2(a+bx)\cosh(a+bx)}{448b} - \frac{27e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\sinh(a+bx)\cosh^2(a+bx)}{448b} + \frac{285e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\cosh^3(a+b}{448b} \\ xe^{\frac{5a}{3}}\cosh^3(a) \end{cases}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*cosh(b*x+a)**3,x)
```

output
```
Piecewise((243*exp(5*a/3)*exp(5*b*x/3)*sinh(a + b*x)**3/(448*b) - 405*exp(
5*a/3)*exp(5*b*x/3)*sinh(a + b*x)**2*cosh(a + b*x)/(448*b) - 27*exp(5*a/3)
*exp(5*b*x/3)*sinh(a + b*x)*cosh(a + b*x)**2/(448*b) + 285*exp(5*a/3)*exp(
5*b*x/3)*cosh(a + b*x)**3/(448*b), Ne(b, 0)), (x*exp(5*a/3)*cosh(a)**3, Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.73

$$\int e^{\frac{5}{3}(a+bx)}\cosh^3(a+bx)\,dx = \frac{3\left(21\,e^{(-2\,bx-2\,a)}+84\,e^{(-4\,bx-4\,a)}+4\right)e^{\left(\frac{14}{3}\,bx+\frac{14}{3}\,a\right)}}{448\,b} - \frac{3\,e^{\left(-\frac{4}{3}\,bx-\frac{4}{3}\,a\right)}}{32\,b}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*cosh(b*x+a)^3,x, algorithm="maxima")
```

output
```
3/448*(21*e^(-2*b*x - 2*a) + 84*e^(-4*b*x - 4*a) + 4)*e^(14/3*b*x + 14/3*a
)/b - 3/32*e^(-4/3*b*x - 4/3*a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

$$\int e^{\frac{5}{3}(a+bx)}\cosh^3(a+bx)\,dx = \frac{3\left(4\,e^{\left(\frac{14}{3}\,bx+\frac{23}{3}\,a\right)}+21\,e^{\left(\frac{8}{3}\,bx+\frac{17}{3}\,a\right)}+84\,e^{\left(\frac{2}{3}\,bx+\frac{11}{3}\,a\right)}-14\,e^{\left(-\frac{4}{3}\,bx+\frac{5}{3}\,a\right)}\right)e^{(-3\,a)}}{448\,b}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*cosh(b*x+a)^3,x, algorithm="giac")
```

output
```
3/448*(4*e^(14/3*b*x + 23/3*a) + 21*e^(8/3*b*x + 17/3*a) + 84*e^(2/3*b*x +
 11/3*a) - 14*e^(-4/3*b*x + 5/3*a))*e^(-3*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int e^{\frac{5}{3}(a+bx)}\cosh^3(a+bx)\,dx=\frac{3\,\mathrm{e}^{-\frac{4\,a}{3}-\frac{4\,b\,x}{3}}\,\left(84\,\mathrm{e}^{2\,a+2\,b\,x}+21\,\mathrm{e}^{4\,a+4\,b\,x}+4\,\mathrm{e}^{6\,a+6\,b\,x}-14\right)}{448\,b}$$

input
```
int(cosh(a + b*x)^3*exp((5*a)/3 + (5*b*x)/3),x)
```

output
```
(3*exp(- (4*a)/3 - (4*b*x)/3)*(84*exp(2*a + 2*b*x) + 21*exp(4*a + 4*b*x) +
 4*exp(6*a + 6*b*x) - 14))/(448*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int e^{\frac{5}{3}(a+bx)}\cosh^3(a+bx)\,dx=\frac{3e^{\frac{5bx}{3}+\frac{5a}{3}}\left(4e^{6bx+6a}+21e^{4bx+4a}+84e^{2bx+2a}-14\right)}{448e^{3bx+3a}b}$$

input
```
int(exp(5/3*b*x+5/3*a)*cosh(b*x+a)^3,x)
```

output
```
(3*e**((5*a + 5*b*x)/3)*(4*e**(6*a + 6*b*x) + 21*e**(4*a + 4*b*x) + 84*e**
(2*a + 2*b*x) - 14))/(448*e**(3*a + 3*b*x)*b)
```

3.33 $\int e^{\frac{5}{3}(a+bx)} \cosh^4(a+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 94

$$\int e^{\frac{5}{3}(a+bx)} \cosh^4(a+bx)\,dx = -\frac{3e^{\frac{1}{3}(-a-bx)}}{4b} - \frac{3e^{-\frac{7}{3}(a+bx)}}{112b} + \frac{9e^{\frac{5}{3}(a+bx)}}{40b} + \frac{3e^{\frac{11}{3}(a+bx)}}{44b} + \frac{3e^{\frac{17}{3}(a+bx)}}{272b}$$

output
```
-3/4*exp(-1/3*b*x-1/3*a)/b-3/112/b/exp(7/3*b*x+7/3*a)+9/40*exp(5/3*b*x+5/3
*a)/b+3/44*exp(11/3*b*x+11/3*a)/b+3/272*exp(17/3*b*x+17/3*a)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.68

$$\int e^{\frac{5}{3}(a+bx)} \cosh^4(a+bx)\,dx = \frac{3e^{-\frac{7}{3}(a+bx)}\left(-935 - 26180e^{2(a+bx)} + 7854e^{4(a+bx)} + 2380e^{6(a+bx)} + 385e^{8(a+bx)}\right)}{104720b}$$

input
```
Integrate[E^((5*(a + b*x))/3)*Cosh[a + b*x]^4,x]
```

output
```
(3*(-935 - 26180*E^(2*(a + b*x)) + 7854*E^(4*(a + b*x)) + 2380*E^(6*(a + b
*x)) + 385*E^(8*(a + b*x))))/(104720*b*E^((7*(a + b*x))/3))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)}\cosh^4(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{3\int \frac{1}{16}e^{-\frac{8}{3}(a+bx)}\left(1+e^{2(a+bx)}\right)^4de^{\frac{1}{3}(a+bx)}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int e^{-\frac{8}{3}(a+bx)}\left(1+e^{2(a+bx)}\right)^4de^{\frac{1}{3}(a+bx)}}{16b}$$

$$\Big\downarrow \text{802}$$

$$\frac{3\int \left(e^{-\frac{8}{3}(a+bx)}+4e^{-\frac{2}{3}(a+bx)}+6e^{\frac{4}{3}(a+bx)}+4e^{\frac{10}{3}(a+bx)}+e^{\frac{16}{3}(a+bx)}\right)de^{\frac{1}{3}(a+bx)}}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(-4e^{\frac{1}{3}(-a-bx)}-\frac{1}{7}e^{-\frac{7}{3}(a+bx)}+\frac{6}{5}e^{\frac{5}{3}(a+bx)}+\frac{4}{11}e^{\frac{11}{3}(a+bx)}+\frac{1}{17}e^{\frac{17}{3}(a+bx)}\right)}{16b}$$

input
```
Int[E^((5*(a + b*x))/3)*Cosh[a + b*x]^4,x]
```

output
```
(3*(-4*E^((-a - b*x)/3) - 1/(7*E^((7*(a + b*x))/3)) + (6*E^((5*(a + b*x))/
3))/5 + (4*E^((11*(a + b*x))/3))/11 + E^((17*(a + b*x))/3)/17))/(16*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.65

method	result
parallelrisch	$-\frac{3\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \left(275 \cosh \left(4 b x +4 a \right)-3927-14280 \sinh \left(2 b x +2 a \right)+11900 \cosh \left(2 b x +2 a \right)-660 \sinh \left(4 b x +4 a \right)\right)}{52360 b}$
risch	$\frac{3\,{\mathrm e}^{\frac{17 b x}{3}+\frac{17 a}{3}}}{272 b}+\frac{3\,{\mathrm e}^{\frac{11 b x}{3}+\frac{11 a}{3}}}{44 b}+\frac{9\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}}}{40 b}-\frac{3\,{\mathrm e}^{-\frac{b x}{3}-\frac{a}{3}}}{4 b}-\frac{3\,{\mathrm e}^{-\frac{7 b x}{3}-\frac{7 a}{3}}}{112 b}$
default	$\frac{3 \sinh \left(\frac{b x}{3}+\frac{a}{3}\right)}{4 b}+\frac{9 \sinh \left(\frac{5 b x}{3}+\frac{5 a}{3}\right)}{40 b}+\frac{3 \sinh \left(\frac{7 b x}{3}+\frac{7 a}{3}\right)}{112 b}+\frac{3 \sinh \left(\frac{11 b x}{3}+\frac{11 a}{3}\right)}{44 b}+\frac{3 \sinh \left(\frac{17 b x}{3}+\frac{17 a}{3}\right)}{272 b}-\frac{3 \cosh \left(\frac{b x}{3}+\frac{a}{3}\right)}{4 b}$
orering	$-\frac{15573\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \cosh \left(b x +a \right)^{4}}{6545 b}+\frac{\frac{4350 b\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \cosh \left(b x +a \right)^{4}}{1309}+\frac{10440\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} \cosh \left(b x +a \right)^{3} b \sinh \left(b x +a \right)}{1309}}{b^{2}}+\frac{366 b^{2} {\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}} c}{187}$

input
```
int(exp(5/3*b*x+5/3*a)*cosh(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-3/52360*exp(5/3*b*x+5/3*a)*(275*cosh(4*b*x+4*a)-3927-14280*sinh(2*b*x+2*a
)+11900*cosh(2*b*x+2*a)-660*sinh(4*b*x+4*a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(71) = 142$.

Time = 0.09 (sec) , antiderivative size = 511, normalized size of antiderivative = 5.44

$$\int e^{\frac{5}{3}(a+bx)}\cosh^4(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*cosh(b*x+a)^4,x, algorithm="fricas")
```

output
```
-3/52360*(275*cosh(1/3*b*x + 1/3*a)^12 - 522720*cosh(1/3*b*x + 1/3*a)^5*si
nh(1/3*b*x + 1/3*a)^7 + 136125*cosh(1/3*b*x + 1/3*a)^4*sinh(1/3*b*x + 1/3*
a)^8 - 145200*cosh(1/3*b*x + 1/3*a)^3*sinh(1/3*b*x + 1/3*a)^9 + 18150*cosh
(1/3*b*x + 1/3*a)^2*sinh(1/3*b*x + 1/3*a)^10 - 7920*cosh(1/3*b*x + 1/3*a)*
sinh(1/3*b*x + 1/3*a)^11 + 275*sinh(1/3*b*x + 1/3*a)^12 + 700*(363*cosh(1/
3*b*x + 1/3*a)^6 + 17)*sinh(1/3*b*x + 1/3*a)^6 + 11900*cosh(1/3*b*x + 1/3*
a)^6 - 720*(726*cosh(1/3*b*x + 1/3*a)^7 + 119*cosh(1/3*b*x + 1/3*a))*sinh(
1/3*b*x + 1/3*a)^5 + 375*(363*cosh(1/3*b*x + 1/3*a)^8 + 476*cosh(1/3*b*x +
 1/3*a)^2)*sinh(1/3*b*x + 1/3*a)^4 - 1200*(121*cosh(1/3*b*x + 1/3*a)^9 + 2
38*cosh(1/3*b*x + 1/3*a)^3)*sinh(1/3*b*x + 1/3*a)^3 + 150*(121*cosh(1/3*b*
x + 1/3*a)^10 + 1190*cosh(1/3*b*x + 1/3*a)^4)*sinh(1/3*b*x + 1/3*a)^2 - 72
0*(11*cosh(1/3*b*x + 1/3*a)^11 + 119*cosh(1/3*b*x + 1/3*a)^5)*sinh(1/3*b*x
 + 1/3*a) - 3927)/(b*cosh(1/3*b*x + 1/3*a)^5 - 5*b*cosh(1/3*b*x + 1/3*a)^4
*sinh(1/3*b*x + 1/3*a) + 10*b*cosh(1/3*b*x + 1/3*a)^3*sinh(1/3*b*x + 1/3*a
)^2 - 10*b*cosh(1/3*b*x + 1/3*a)^2*sinh(1/3*b*x + 1/3*a)^3 + 5*b*cosh(1/3*
b*x + 1/3*a)*sinh(1/3*b*x + 1/3*a)^4 - b*sinh(1/3*b*x + 1/3*a)^5)
```

Sympy [A] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.89

$$\int e^{\frac{5}{3}(a+bx)}\cosh^4(a+bx)\,dx$$

$$= \begin{cases} \frac{5832e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\sinh^4(a+bx)}{6545b} - \frac{1944e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\sinh^3(a+bx)\cosh(a+bx)}{1309b} - \frac{324e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\sinh^2(a+bx)\cosh^2(a+bx)}{595b} + \frac{2340e^{\frac{5a}{3}}e^{\frac{5bx}{3}}\sinh(}{13} \\ xe^{\frac{5a}{3}}\cosh^4(a) \end{cases}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*cosh(b*x+a)**4,x)
```

output
```
Piecewise((5832*exp(5*a/3)*exp(5*b*x/3)*sinh(a + b*x)**4/(6545*b) - 1944*e
xp(5*a/3)*exp(5*b*x/3)*sinh(a + b*x)**3*cosh(a + b*x)/(1309*b) - 324*exp(5
*a/3)*exp(5*b*x/3)*sinh(a + b*x)**2*cosh(a + b*x)**2/(595*b) + 2340*exp(5*
a/3)*exp(5*b*x/3)*sinh(a + b*x)*cosh(a + b*x)**3/(1309*b) - 3093*exp(5*a/3
)*exp(5*b*x/3)*cosh(a + b*x)**4/(6545*b), Ne(b, 0)), (x*exp(5*a/3)*cosh(a)
**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.69

$$\int e^{\frac{5}{3}(a+bx)}\cosh^4(a+bx)\,dx = \frac{3\left(340\,e^{(-2\,bx-2\,a)}+1122\,e^{(-4\,bx-4\,a)}+55\right)e^{\left(\frac{17}{3}\,bx+\frac{17}{3}\,a\right)}}{14960\,b} - \frac{3\left(28\,e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,a\right)}+e^{\left(-\frac{7}{3}\,bx-\frac{7}{3}\,a\right)}\right)}{112\,b}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*cosh(b*x+a)^4,x, algorithm="maxima")
```

output
```
3/14960*(340*e^(-2*b*x - 2*a) + 1122*e^(-4*b*x - 4*a) + 55)*e^(17/3*b*x +
17/3*a)/b - 3/112*(28*e^(-1/3*b*x - 1/3*a) + e^(-7/3*b*x - 7/3*a))/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.70

$$\int e^{\frac{5}{3}(a+bx)}\cosh^4(a+bx)\,dx = -\frac{3\left(935\left(28\,e^{\left(2\,bx+\frac{11}{3}\,a\right)}+e^{\left(\frac{5}{3}\,a\right)}\right)e^{\left(-\frac{7}{3}\,bx\right)}-385\,e^{\left(\frac{17}{3}\,bx+\frac{29}{3}\,a\right)}-2380\,e^{\left(\frac{11}{3}\,bx+\frac{23}{3}\,a\right)}-7854\,e^{\left(\frac{5}{3}\,bx+\frac{17}{3}\,a\right)}\right)e^{(-4\,a)}}{104720\,b}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*cosh(b*x+a)^4,x, algorithm="giac")
```

output
```
-3/104720*(935*(28*e^(2*b*x + 11/3*a) + e^(5/3*a))*e^(-7/3*b*x) - 385*e^(1
7/3*b*x + 29/3*a) - 2380*e^(11/3*b*x + 23/3*a) - 7854*e^(5/3*b*x + 17/3*a)
)*e^(-4*a)/b
```

Mupad [B] (verification not implemented)

Time = 3.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.64

$$\int e^{\frac{5}{3}(a+bx)} \cosh^4(a+bx) \, dx = \frac{3 \, e^{-\frac{7a}{3}-\frac{7bx}{3}} \left(7854 \, e^{4a+4bx} - 26180 \, e^{2a+2bx} + 2380 \, e^{6a+6bx} + 385 \, e^{8a+8bx} - 935\right)}{104720 \, b}$$

input
```
int(cosh(a + b*x)^4*exp((5*a)/3 + (5*b*x)/3),x)
```

output
```
(3*exp(- (7*a)/3 - (7*b*x)/3)*(7854*exp(4*a + 4*b*x) - 26180*exp(2*a + 2*b
*x) + 2380*exp(6*a + 6*b*x) + 385*exp(8*a + 8*b*x) - 935))/(104720*b)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.82

$$\int e^{\frac{5}{3}(a+bx)} \cosh^4(a+bx) \, dx = \frac{3e^{\frac{5bx}{3}+\frac{5a}{3}} \left(385e^{8bx+8a} + 2380e^{6bx+6a} + 7854e^{4bx+4a} - 26180e^{2bx+2a} - 935\right)}{104720e^{4bx+4a}b}$$

input
```
int(exp(5/3*b*x+5/3*a)*cosh(b*x+a)^4,x)
```

output
```
(3*e**((5*a + 5*b*x)/3)*(385*e**(8*a + 8*b*x) + 2380*e**(6*a + 6*b*x) + 78
54*e**(4*a + 4*b*x) - 26180*e**(2*a + 2*b*x) - 935))/(104720*e**(4*a + 4*b
*x)*b)
```

3.34 $\int F^{c(a+bx)}\cosh(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 63

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx=-\frac{e^{-d-ex}F^{c(a+bx)}}{2(e-bc\log(F))}+\frac{e^{d+ex}F^{c(a+bx)}}{2(e+bc\log(F))}$$

output
```
-1/2*exp(-e*x-d)*F^(c*(b*x+a))/(e-b*c*ln(F))+exp(e*x+d)*F^(c*(b*x+a))/(2*e
+2*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx=\frac{F^{c(a+bx)}(-bc\cosh(d+ex)\log(F)+e\sinh(d+ex))}{(e-bc\log(F))(e+bc\log(F))}$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*(-(b*c*Cosh[d + e*x]*Log[F]) + e*Sinh[d + e*x]))/((e - b*
c*Log[F])*(e + b*c*Log[F]))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5998}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{5998}$$

$$\frac{e\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x],x]
```

output
```
-((b*c*F^(c*(a + b*x))*Cosh[d + e*x]*Log[F])/(e^2 - b^2*c^2*Log[F]^2)) + (
e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 5998
```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

method	result	size
parallelrisch	$\frac{(bc\cosh(ex+d)\ln(F)-e\sinh(ex+d))F^{c(bx+a)}}{b^2c^2\ln(F)^2-e^2}$	51
risch	$\frac{(\ln(F)bc\,\mathrm{e}^{2ex+2d}+bc\ln(F)-\mathrm{e}^{2ex+2d}e+e)\mathrm{e}^{-ex-d}F^{c(bx+a)}}{2(bc\ln(F)-e)(e+bc\ln(F))}$	74
orering	$\frac{2\ln(F)bc\cosh(ex+d)F^{c(bx+a)}}{b^2c^2\ln(F)^2-e^2}-\frac{e\sinh(ex+d)F^{c(bx+a)}+\cosh(ex+d)F^{c(bx+a)}bc\ln(F)}{b^2c^2\ln(F)^2-e^2}$	101

input
```
int(cosh(e*x+d)*F^(c*(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
(b*c*cosh(e*x+d)*ln(F)-e*sinh(e*x+d))*F^(c*(b*x+a))/(b^2*c^2*ln(F)^2-e^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(58) = 116$.

Time = 0.08 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.90

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx = -\frac{\left(e\cosh\left(ex+d\right)^2-\left(bc\log\left(F\right)-e\right)\sinh\left(ex+d\right)^2-\left(bc\cosh\left(ex+d\right)^2+bc\right)\log\left(F\right)-2\left(bc\cosh\left(ex+\right.\right.\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d),x, algorithm="fricas")
```

output
```
-1/2*((e*cosh(e*x + d)^2 - (b*c*log(F) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*
x + d)^2 + b*c)*log(F) - 2*(b*c*cosh(e*x + d)*log(F) - e*cosh(e*x + d))*si
nh(e*x + d) - e)*cosh((b*c*x + a*c)*log(F)) + (e*cosh(e*x + d)^2 - (b*c*lo
g(F) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*x + d)^2 + b*c)*log(F) - 2*(b*c*co
sh(e*x + d)*log(F) - e*cosh(e*x + d))*sinh(e*x + d) - e)*sinh((b*c*x + a*c
)*log(F)))/(b^2*c^2*cosh(e*x + d)*log(F)^2 - e^2*cosh(e*x + d) + (b^2*c^2*
log(F)^2 - e^2)*sinh(e*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(54) = 108$.

Time = 0.75 (sec) , antiderivative size = 265, normalized size of antiderivative = 4.21

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx$$
$$= \begin{cases} x\cosh(d) & \text{for } F=1\wedge e=0 \\ F^{ac}x\cosh(d) & \text{for } b=0\wedge e=0 \\ x\cosh(d) & \text{for } c=0\wedge e=0 \\ -\frac{F^{ac+bcx}x\sinh(bcx\log(F)-d)}{2}+\frac{F^{ac+bcx}x\cosh(bcx\log(F)-d)}{2}+\frac{F^{ac+bcx}\sinh(bcx\log(F)-d)}{2bc\log(F)} & \text{for } e=-bc\log(F) \\ -\frac{F^{ac+bcx}x\sinh(bcx\log(F)+d)}{2}+\frac{F^{ac+bcx}x\cosh(bcx\log(F)+d)}{2}+\frac{F^{ac+bcx}\sinh(bcx\log(F)+d)}{2bc\log(F)} & \text{for } e=bc\log(F) \\ \frac{F^{ac+bcx}bc\log(F)\cosh(d+ex)}{b^2c^2\log(F)^2-e^2}-\frac{F^{ac+bcx}e\sinh(d+ex)}{b^2c^2\log(F)^2-e^2} & \text{otherwise} \end{cases}$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d),x)
```

output
```
Piecewise((x*cosh(d), Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*cosh(d), Eq(b, 0)
& Eq(e, 0)), (x*cosh(d), Eq(c, 0) & Eq(e, 0)), (-F**(a*c + b*c*x)*x*sinh(b
*c*x*log(F) - d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F) - d)/2 + F**(a*c
 + b*c*x)*sinh(b*c*x*log(F) - d)/(2*b*c*log(F)), Eq(e, -b*c*log(F))), (-F*
*(a*c + b*c*x)*x*sinh(b*c*x*log(F) + d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*
log(F) + d)/2 + F**(a*c + b*c*x)*sinh(b*c*x*log(F) + d)/(2*b*c*log(F)), Eq
(e, b*c*log(F))), (F**(a*c + b*c*x)*b*c*log(F)*cosh(d + e*x)/(b**2*c**2*lo
g(F)**2 - e**2) - F**(a*c + b*c*x)*e*sinh(d + e*x)/(b**2*c**2*log(F)**2 -
e**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{2\,(bc\log(F)+e)}+\frac{F^{ac}e^{(bcx\log(F)-ex)}}{2\,(bce^d\log(F)-ee^d)}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d),x, algorithm="maxima")
```

output
```
1/2*F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) + 1/2*F^(a*c)*e^(b
*c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 597, normalized size of antiderivative = 9.48

$$\int F^{c(a+bx)} \cosh(d+ex)\, dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d),x, algorithm="giac")
```

output
```
(2*(b*c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*
a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F))
+ e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x
 - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*lo
g(abs(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 1/2
*I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/
2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) + 2*e) - I*e^(
-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a
*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F)) + 2*e))*e^(a*c*log(ab
s(F)) + (b*c*log(abs(F)) + e)*x + d) + (2*(b*c*log(abs(F)) - e)*cos(-1/2*p
i*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*s
gn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sgn(F) - pi*b*c)*
sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/
((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(F
)) + (b*c*log(abs(F)) - e)*x - d) + 1/2*I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/
2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi
*b*c + 2*b*c*log(abs(F)) - 2*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b
*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c +
2*b*c*log(abs(F)) - 2*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d
)
```

Mupad [B] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.17

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx = -\frac{F^{a\,c+b\,c\,x}\,\mathrm{e}^{-d-e\,x}\,\left(e-e\,\mathrm{e}^{2\,d+2\,e\,x}+b\,c\,\ln\left(F\right)+b\,c\,\mathrm{e}^{2\,d+2\,e\,x}\,\ln\left(F\right)\right)}{2\,\left(e^2-b^2\,c^2\,\ln\left(F\right)^2\right)}$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x),x)
```

output
```
-(F^(a*c + b*c*x)*exp(- d - e*x)*(e - e*exp(2*d + 2*e*x) + b*c*log(F) + b*
c*exp(2*d + 2*e*x)*log(F)))/(2*(e^2 - b^2*c^2*log(F)^2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx = \frac{f^{bcx+ac}(\cosh(ex+d)\log(f)\,bc-\sinh(ex+d)\,e)}{\log(f)^2\,b^2c^2-e^2}$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d),x)
```

output
```
(f**(a*c + b*c*x)*(cosh(d + e*x)*log(f)*b*c - sinh(d + e*x)*e))/(log(f)**2
*b**2*c**2 - e**2)
```

3.35 $\int F^{c(a+bx)}\cosh^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 93

$$\int F^{c(a+bx)}\cosh^2(d+ex)\,dx=\frac{F^{c(a+bx)}}{2bc\log(F)}-\frac{e^{-2d-2ex}F^{c(a+bx)}}{4(2e-bc\log(F))}+\frac{e^{2d+2ex}F^{c(a+bx)}}{4(2e+bc\log(F))}$$

output
```
1/2*F^(c*(b*x+a))/b/c/ln(F)-exp(-2*e*x-2*d)*F^(c*(b*x+a))/(8*e-4*b*c*ln(F)
)+exp(2*e*x+2*d)*F^(c*(b*x+a))/(8*e+4*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91

$$\int F^{c(a+bx)}\cosh^2(d+ex)\,dx=\frac{F^{c(a+bx)}\left(-4e^2+b^2c^2\log^2(F)+b^2c^2\cosh(2(d+ex))\log^2(F)-2bce\log(F)\sinh(2(d+ex))\right)}{-8bce^2\log(F)+2b^3c^3\log^3(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^2,x]
```

output
```
(F^(c*(a + b*x))*(-4*e^2 + b^2*c^2*Log[F]^2 + b^2*c^2*Cosh[2*(d + e*x)]*Lo
g[F]^2 - 2*b*c*e*Log[F]*Sinh[2*(d + e*x)]))/(-8*b*c*e^2*Log[F] + 2*b^3*c^3
*Log[F]^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.42, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6000, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6000}$$

$$\frac{2e^2\int F^{c(a+bx)}dx}{4e^2-b^2c^2\log^2(F)} - \frac{bc\log(F)\cosh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)} + \frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}$$

$$\Big\downarrow \text{2624}$$

$$-\frac{bc\log(F)\cosh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)} + \frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)} + \frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x]^2,x]
```

output
```
(2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 - b^2*c^2*Log[F]^2)) - (b*c*F^(
c*(a + b*x))*Cosh[d + e*x]^2*Log[F])/(4*e^2 - b^2*c^2*Log[F]^2) + (2*e*F^(
c*(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x])/(4*e^2 - b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 6000
```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Simp[e*n*F^(c*(a + b*x))*Sinh[d + e*x]*(Cosh[d + e*x]^
(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x] + Simp[n*(n - 1)*(e^2/(e^2*n^2 -
b^2*c^2*Log[F]^2))   Int[F^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97

method	result
parallelrisch	$-\frac{2\left(-\frac{c^{2} b^{2} \ln \left(F\right)^{2} \cosh \left(2 e x+2 d\right)}{2}-\frac{b^{2} c^{2} \ln \left(F\right)^{2}}{2}+\ln \left(F\right) b c e \sinh \left(2 e x+2 d\right)+2 e^{2}\right) F^{c \left(b x+a\right)}}{2 c^{3} b^{3} \ln \left(F\right)^{3}-8 e^{2} b c \ln \left(F\right)}$
risch	$\frac{\left(\ln \left(F\right)^{2} b^{2} c^{2} {\mathrm e}^{4 e x+4 d}+2 \ln \left(F\right)^{2} b^{2} c^{2} {\mathrm e}^{2 e x+2 d}-2 \ln \left(F\right) b c e\,{\mathrm e}^{4 e x+4 d}+b^{2} c^{2} \ln \left(F\right)^{2}+2 \ln \left(F\right) b c e-8 e^{2} {\mathrm e}^{2 e x+2 d}\right) {\mathrm e}^{-2 e x-2 d} F^{c \left(b x+a\right)}}{4 \ln \left(F\right) b c \left(b c \ln \left(F\right)-2 e\right) \left(2 e+b c \ln \left(F\right)\right)}$
orering	$\frac{\left(3 b^{2} c^{2} \ln \left(F\right)^{2}-4 e^{2}\right) F^{c \left(b x+a\right)} \cosh \left(e x+d\right)^{2}}{\left(b^{2} c^{2} \ln \left(F\right)^{2}-4 e^{2}\right) \ln \left(F\right) b c}-\frac{3\left(F^{c \left(b x+a\right)} b c \ln \left(F\right) \cosh \left(e x+d\right)^{2}+2 F^{c \left(b x+a\right)} \cosh \left(e x+d\right) e \sinh \left(e x+d\right)\right)}{b^{2} c^{2} \ln \left(F\right)^{2}-4 e^{2}}+$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(-1/2*c^2*b^2*ln(F)^2*cosh(2*e*x+2*d)-1/2*b^2*c^2*ln(F)^2+ln(F)*b*c*e*s
inh(2*e*x+2*d)+2*e^2)*F^(c*(b*x+a))/(2*c^3*b^3*ln(F)^3-8*e^2*b*c*ln(F))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(84) = 168$.

Time = 0.10 (sec) , antiderivative size = 699, normalized size of antiderivative = 7.52

$$\int F^{c(a+bx)} \cosh^2(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/4*(((b^2*c^2*log(F)^2 - 2*b*c*e*log(F))*sinh(e*x + d)^4 - 8*e^2*cosh(e*x
 + d)^2 + 4*(b^2*c^2*cosh(e*x + d)*log(F)^2 - 2*b*c*e*cosh(e*x + d)*log(F)
)*sinh(e*x + d)^3 + (b^2*c^2*cosh(e*x + d)^4 + 2*b^2*c^2*cosh(e*x + d)^2 +
 b^2*c^2)*log(F)^2 - 2*(6*b*c*e*cosh(e*x + d)^2*log(F) - (3*b^2*c^2*cosh(e
*x + d)^2 + b^2*c^2)*log(F)^2 + 4*e^2)*sinh(e*x + d)^2 - 2*(b*c*e*cosh(e*x
 + d)^4 - b*c*e)*log(F) - 4*(2*b*c*e*cosh(e*x + d)^3*log(F) + 4*e^2*cosh(e
*x + d) - (b^2*c^2*cosh(e*x + d)^3 + b^2*c^2*cosh(e*x + d))*log(F)^2)*sinh
(e*x + d))*cosh((b*c*x + a*c)*log(F)) + ((b^2*c^2*log(F)^2 - 2*b*c*e*log(F
))*sinh(e*x + d)^4 - 8*e^2*cosh(e*x + d)^2 + 4*(b^2*c^2*cosh(e*x + d)*log(
F)^2 - 2*b*c*e*cosh(e*x + d)*log(F))*sinh(e*x + d)^3 + (b^2*c^2*cosh(e*x +
 d)^4 + 2*b^2*c^2*cosh(e*x + d)^2 + b^2*c^2)*log(F)^2 - 2*(6*b*c*e*cosh(e*
x + d)^2*log(F) - (3*b^2*c^2*cosh(e*x + d)^2 + b^2*c^2)*log(F)^2 + 4*e^2)*
sinh(e*x + d)^2 - 2*(b*c*e*cosh(e*x + d)^4 - b*c*e)*log(F) - 4*(2*b*c*e*co
sh(e*x + d)^3*log(F) + 4*e^2*cosh(e*x + d) - (b^2*c^2*cosh(e*x + d)^3 + b^
2*c^2*cosh(e*x + d))*log(F)^2)*sinh(e*x + d))*sinh((b*c*x + a*c)*log(F)))/
(b^3*c^3*cosh(e*x + d)^2*log(F)^3 - 4*b*c*e^2*cosh(e*x + d)^2*log(F) + (b^
3*c^3*log(F)^3 - 4*b*c*e^2*log(F))*sinh(e*x + d)^2 + 2*(b^3*c^3*cosh(e*x +
 d)*log(F)^3 - 4*b*c*e^2*cosh(e*x + d)*log(F))*sinh(e*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(78) = 156$.

Time = 1.30 (sec) , antiderivative size = 707, normalized size of antiderivative = 7.60

$$\int F^{c(a+bx)} \cosh^2(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d)**2,x)
```

output
```
Piecewise((x*cosh(d)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)), (-x*s
inh(d + e*x)**2/2 + x*cosh(d + e*x)**2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*
e), Eq(F, 1)), (F**(a*c)*(-x*sinh(d + e*x)**2/2 + x*cosh(d + e*x)**2/2 + s
inh(d + e*x)*cosh(d + e*x)/(2*e)), Eq(b, 0)), (-x*sinh(d + e*x)**2/2 + x*c
osh(d + e*x)**2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*e), Eq(c, 0)), (F**(a*c
 + b*c*x)*x*sinh(b*c*x*log(F)/2 - d)**2/4 - F**(a*c + b*c*x)*x*sinh(b*c*x*
log(F)/2 - d)*cosh(b*c*x*log(F)/2 - d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*l
og(F)/2 - d)**2/4 - F**(a*c + b*c*x)*sinh(b*c*x*log(F)/2 - d)**2/(b*c*log(
F)) + 3*F**(a*c + b*c*x)*sinh(b*c*x*log(F)/2 - d)*cosh(b*c*x*log(F)/2 - d)
/(2*b*c*log(F)), Eq(e, -b*c*log(F)/2)), (F**(a*c + b*c*x)*x*sinh(b*c*x*log
(F)/2 + d)**2/4 - F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/2 + d)*cosh(b*c*x*l
og(F)/2 + d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F)/2 + d)**2/4 - F**(a*
c + b*c*x)*sinh(b*c*x*log(F)/2 + d)*cosh(b*c*x*log(F)/2 + d)/(2*b*c*log(F)
) + F**(a*c + b*c*x)*cosh(b*c*x*log(F)/2 + d)**2/(b*c*log(F)), Eq(e, b*c*l
og(F)/2)), (F**(a*c + b*c*x)*b**2*c**2*log(F)**2*cosh(d + e*x)**2/(b**3*c*
*3*log(F)**3 - 4*b*c*e**2*log(F)) - 2*F**(a*c + b*c*x)*b*c*e*log(F)*sinh(d
 + e*x)*cosh(d + e*x)/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F)) + 2*F**(a*
c + b*c*x)*e**2*sinh(d + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F))
 - 2*F**(a*c + b*c*x)*e**2*cosh(d + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e
**2*log(F)), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01

$$\int F^{c(a+bx)}\cosh^2(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+2\,ex+2\,d)}}{4\,(bc\log{(F)}+2\,e)} + \frac{F^{ac}e^{(bcx\log(F)-2\,ex)}}{4\,(bce^{(2\,d)}\log{(F)}-2\,ee^{(2\,d)})} + \frac{F^{bcx+ac}}{2\,bc\log{(F)}}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2,x, algorithm="maxima")
```

output
```
1/4*F^(a*c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) + 1/4*F^(a*c
)*e^(b*c*x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) + 1/2*F^(b*c
*x + a*c)/(b*c*log(F))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 889, normalized size of antiderivative = 9.56

$$\int F^{c(a+bx)}\cosh^2(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2,x, algorithm="giac")
```

output
```
(2*b*c*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*p
i*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2)
- (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*p
i*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*
b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*e^(1/2*I*pi*b*c*x*
sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*I*pi*b*c*
sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1
/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) +
2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) +
 1/2*(2*(b*c*log(abs(F)) + 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x -
1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(a
bs(F)) + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2
*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 +
4*(b*c*log(abs(F)) + 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)
*x + 2*d) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*
sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F))
 + 16*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn
(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) +
 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x + 2*d) + 1/2*(2*(b*
c*log(abs(F)) - 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a...
```

Mupad [B] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.08

$$\int F^{c(a+bx)}\cosh^2(d+ex)\,dx = -\frac{2\,F^{a\,c+b\,c\,x}\,e^2-F^{a\,c+b\,c\,x}\,b^2\,c^2\,\cosh(d+e\,x)^2\,\ln\left(F\right)^2+2\,F^{a\,c+b\,c\,x}\,b\,c\,e\,\cosh(d+e\,x)\,\sinh(d+e\,x)\,\ln\left(F\right)}{b^3\,c^3\,\ln\left(F\right)^3-4\,b\,c\,e^2\,\ln\left(F\right)}$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x)^2,x)
```

output
```
-(2*F^(a*c + b*c*x)*e^2 - F^(a*c + b*c*x)*b^2*c^2*cosh(d + e*x)^2*log(F)^2
 + 2*F^(a*c + b*c*x)*b*c*e*cosh(d + e*x)*sinh(d + e*x)*log(F))/(b^3*c^3*lo
g(F)^3 - 4*b*c*e^2*log(F))
```

Reduce [F]

$$\int F^{c(a+bx)}\cosh^2(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\cosh\left(ex+d\right)^2dx\right)$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cosh(d + e*x)**2,x)
```

3.36 $\int F^{c(a+bx)} \cosh^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 132

$$\int F^{c(a+bx)} \cosh^3(d+ex)\,dx = -\frac{3e^{-d-ex}F^{c(a+bx)}}{8(e-bc\log(F))} - \frac{e^{-3d-3ex}F^{c(a+bx)}}{8(3e-bc\log(F))} + \frac{3e^{d+ex}F^{c(a+bx)}}{8(e+bc\log(F))} + \frac{e^{3d+3ex}F^{c(a+bx)}}{8(3e+bc\log(F))}$$

output
```
-3*exp(-e*x-d)*F^(c*(b*x+a))/(8*e-8*b*c*ln(F))-exp(-3*e*x-3*d)*F^(c*(b*x+a
))/(24*e-8*b*c*ln(F))+3*exp(e*x+d)*F^(c*(b*x+a))/(8*e+8*b*c*ln(F))+exp(3*e
*x+3*d)*F^(c*(b*x+a))/(24*e+8*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.20

$$\int F^{c(a+bx)} \cosh^3(d+ex)\,dx$$
$$= \frac{F^{c(a+bx)}\left(3\cosh(d+ex)\left(-9bce^2\log(F)+b^3c^3\log^3(F)\right)+\cosh(3(d+ex))\left(-bce^2\log(F)+b^3c^3\log^3(F)\right.\right.}{4\left(9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\right.}$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(3*Cosh[d + e*x]*(-9*b*c*e^2*Log[F] + b^3*c^3*Log[F]^3) +
 Cosh[3*(d + e*x)]*(-(b*c*e^2*Log[F]) + b^3*c^3*Log[F]^3) + 6*e*(5*e^2 - b
^2*c^2*Log[F]^2 + Cosh[2*(d + e*x)]*(e^2 - b^2*c^2*Log[F]^2))*Sinh[d + e*x
]))/(4*(9*e^4 - 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.44, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6000, 5998}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{ 6000}$$

$$\frac{6e^2\int F^{c(a+bx)}\cosh(d+ex)dx}{9e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh^3(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{3e\sinh(d+ex)\cosh^2(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}$$

$$\Big\downarrow \text{ 5998}$$

$$-\frac{bc\log(F)\cosh^3(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{3e\sinh(d+ex)\cosh^2(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{6e^2\left(\frac{e\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}\right)}{9e^2-b^2c^2\log^2(F)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x]^3,x]
```

output
```
-((b*c*F^(c*(a + b*x))*Cosh[d + e*x]^3*Log[F])/(9*e^2 - b^2*c^2*Log[F]^2))
 + (3*e*F^(c*(a + b*x))*Cosh[d + e*x]^2*Sinh[d + e*x])/(9*e^2 - b^2*c^2*Lo
g[F]^2) + (6*e^2*(-((b*c*F^(c*(a + b*x))*Cosh[d + e*x]*Log[F])/(e^2 - b^2*
c^2*Log[F]^2)) + (e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2
)))/(9*e^2 - b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 5998
```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

rule 6000
```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Simp[e*n*F^(c*(a + b*x))*Sinh[d + e*x]*(Cosh[d + e*x]^
(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x] + Simp[n*(n - 1)*(e^2/(e^2*n^2 -
b^2*c^2*Log[F]^2))   Int[F^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.12

method	result
parallelrisch	$\frac{F^{c(bx+a)} ((c^3 b^3 \ln(F)^3 - e^2 bc \ln(F)) \cosh(3ex+3d) + (-3c^2 b^2 \ln(F)^2 e + 3e^3) \sinh(3ex+3d) - 3(bc\ln(F)-3e)(bc\ln(F)+3e)(}{4\ln(F)^4 b^4 c^4 - 40\ln(F)^2 b^2 c^2 e^2 + 36e^4}$
risch	$(\ln(F)^3 b^3 c^3 \mathrm{e}^{6ex+6d} + 3\ln(F)^3 b^3 c^3 \mathrm{e}^{4ex+4d} - 3\ln(F)^2 b^2 c^2 e\, \mathrm{e}^{6ex+6d} + 3\ln(F)^3 b^3 c^3 \mathrm{e}^{2ex+2d} - 3\ln(F)^2 b^2 c^2 e\, \mathrm{e}^{4ex+4d} - \ln(F) bc$
orering	$\frac{4\ln(F) bc (b^2 c^2 \ln(F)^2 - 5e^2) F^{c(bx+a)} \cosh(ex+d)^3}{\ln(F)^4 b^4 c^4 - 10\ln(F)^2 b^2 c^2 e^2 + 9e^4} - \frac{2 (3b^2 c^2 \ln(F)^2 - 5e^2) (F^{c(bx+a)} bc \ln(F) \cosh(ex+d)^3 + 3F^{c(bx+a)}}{\ln(F)^4 b^4 c^4 - 10\ln(F)^2 b^2 c^2 e^2 + 9e^4}$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
F^(c*(b*x+a))*((c^3*b^3*ln(F)^3-e^2*b*c*ln(F))*cosh(3*e*x+3*d)+(-3*c^2*b^2
*ln(F)^2*e+3*e^3)*sinh(3*e*x+3*d)-3*(b*c*ln(F)-3*e)*(b*c*ln(F)+3*e)*(-b*c*
cosh(e*x+d)*ln(F)+e*sinh(e*x+d)))/(4*ln(F)^4*b^4*c^4-40*ln(F)^2*b^2*c^2*e^
2+36*e^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2218 vs. $2(120) = 240$.

Time = 0.17 (sec) , antiderivative size = 2218, normalized size of antiderivative = 16.80

$$\int F^{c(a+bx)} \cosh^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^3,x, algorithm="fricas")
```

output
```
1/8*((3*e^3*cosh(e*x + d)^6 + 27*e^3*cosh(e*x + d)^4 + (b^3*c^3*log(F)^3 -
 3*b^2*c^2*e*log(F)^2 - b*c*e^2*log(F) + 3*e^3)*sinh(e*x + d)^6 + 6*(b^3*c
^3*cosh(e*x + d)*log(F)^3 - 3*b^2*c^2*e*cosh(e*x + d)*log(F)^2 - b*c*e^2*c
osh(e*x + d)*log(F) + 3*e^3*cosh(e*x + d))*sinh(e*x + d)^5 - 27*e^3*cosh(e
*x + d)^2 + 3*(15*e^3*cosh(e*x + d)^2 + (5*b^3*c^3*cosh(e*x + d)^2 + b^3*c
^3)*log(F)^3 + 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^2 + b^2*c^2*e)*log(F)^2
 - (5*b*c*e^2*cosh(e*x + d)^2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^4 + (b^3*
c^3*cosh(e*x + d)^6 + 3*b^3*c^3*cosh(e*x + d)^4 + 3*b^3*c^3*cosh(e*x + d)^
2 + b^3*c^3)*log(F)^3 + 4*(15*e^3*cosh(e*x + d)^3 + 27*e^3*cosh(e*x + d) +
 (5*b^3*c^3*cosh(e*x + d)^3 + 3*b^3*c^3*cosh(e*x + d))*log(F)^3 - 3*(5*b^2
*c^2*e*cosh(e*x + d)^3 + b^2*c^2*e*cosh(e*x + d))*log(F)^2 - (5*b*c*e^2*co
sh(e*x + d)^3 + 27*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d)^3 - 3*e^3
- 3*(b^2*c^2*e*cosh(e*x + d)^6 + b^2*c^2*e*cosh(e*x + d)^4 - b^2*c^2*e*cos
h(e*x + d)^2 - b^2*c^2*e)*log(F)^2 + 3*(15*e^3*cosh(e*x + d)^4 + 54*e^3*co
sh(e*x + d)^2 + (5*b^3*c^3*cosh(e*x + d)^4 + 6*b^3*c^3*cosh(e*x + d)^2 + b
^3*c^3)*log(F)^3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^4 + 6*b^2*c^2*e*cos
h(e*x + d)^2 - b^2*c^2*e)*log(F)^2 - (5*b*c*e^2*cosh(e*x + d)^4 + 54*b*c*e
^2*cosh(e*x + d)^2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^2 - (b*c*e^2*cosh(e*
x + d)^6 + 27*b*c*e^2*cosh(e*x + d)^4 + 27*b*c*e^2*cosh(e*x + d)^2 + b*c*e
^2)*log(F) + 6*(3*e^3*cosh(e*x + d)^5 + 18*e^3*cosh(e*x + d)^3 - 9*e^3*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1658 vs. $2(121) = 242$.

Time = 3.54 (sec) , antiderivative size = 1658, normalized size of antiderivative = 12.56

$$\int F^{c(a+bx)}\cosh^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d)**3,x)
```

output
```
Piecewise((x*cosh(d)**3, Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*cosh(d)**3, Eq(
b, 0) & Eq(e, 0)), (x*cosh(d)**3, Eq(c, 0) & Eq(e, 0)), (3*F**(a*c + b*c*x
)*x*sinh(b*c*x*log(F) - d)**3/8 - 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F) -
 d)**2*cosh(b*c*x*log(F) - d)/8 - 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F) -
 d)*cosh(b*c*x*log(F) - d)**2/8 + 3*F**(a*c + b*c*x)*x*cosh(b*c*x*log(F) -
 d)**3/8 - 5*F**(a*c + b*c*x)*sinh(b*c*x*log(F) - d)**3/(8*b*c*log(F)) + F
**(a*c + b*c*x)*sinh(b*c*x*log(F) - d)**2*cosh(b*c*x*log(F) - d)/(4*b*c*lo
g(F)) + F**(a*c + b*c*x)*sinh(b*c*x*log(F) - d)*cosh(b*c*x*log(F) - d)**2/
(b*c*log(F)) - 3*F**(a*c + b*c*x)*cosh(b*c*x*log(F) - d)**3/(8*b*c*log(F))
, Eq(e, -b*c*log(F))), (-F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)**3/8
+ 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)**2*cosh(b*c*x*log(F)/3 - d
)/8 - 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)*cosh(b*c*x*log(F)/3 -
d)**2/8 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F)/3 - d)**3/8 + 11*F**(a*c +
b*c*x)*sinh(b*c*x*log(F)/3 - d)**3/(8*b*c*log(F)) - 15*F**(a*c + b*c*x)*si
nh(b*c*x*log(F)/3 - d)**2*cosh(b*c*x*log(F)/3 - d)/(4*b*c*log(F)) + 3*F**(
a*c + b*c*x)*sinh(b*c*x*log(F)/3 - d)*cosh(b*c*x*log(F)/3 - d)**2/(b*c*log
(F)) - F**(a*c + b*c*x)*cosh(b*c*x*log(F)/3 - d)**3/(8*b*c*log(F)), Eq(e,
-b*c*log(F)/3)), (-F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 + d)**3/8 + 3*F*
*(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 + d)**2*cosh(b*c*x*log(F)/3 + d)/8 -
3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 + d)*cosh(b*c*x*log(F)/3 + d)*...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02

$$\int F^{c(a+bx)}\cosh^3(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+3\,ex+3\,d)}}{8\,(bc\log(F)+3\,e)} + \frac{3\,F^{ac}e^{(bcx\log(F)+ex+d)}}{8\,(bc\log(F)+e)} + \frac{3\,F^{ac}e^{(bcx\log(F)-ex)}}{8\,(bce^{d}\log(F)-ee^{d})} + \frac{F^{ac}e^{(bcx\log(F)-3\,ex)}}{8\,(bce^{(3\,d)}\log(F)-3\,ee^{(3\,d)})}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/8*F^(a*c)*e^(b*c*x*log(F) + 3*e*x + 3*d)/(b*c*log(F) + 3*e) + 3/8*F^(a*c
)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) + 3/8*F^(a*c)*e^(b*c*x*log(F
) - e*x)/(b*c*e^d*log(F) - e*e^d) + 1/8*F^(a*c)*e^(b*c*x*log(F) - 3*e*x)/(
b*c*e^(3*d)*log(F) - 3*e*e^(3*d))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 1211, normalized size of antiderivative = 9.17

$$\int F^{c(a+bx)}\cosh^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^3,x, algorithm="giac")
```

output
```
1/4*(2*(b*c*log(abs(F)) + 3*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1
/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(ab
s(F)) + 3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*
pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4
*(b*c*log(abs(F)) + 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*
x + 3*d) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*s
gn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F))
 + 48*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn
(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F))
+ 48*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3*d) + 3/4*(2*(b
*c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*s
gn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e)^
2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs
(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 3*I*(I*e
^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*
a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e) - I*e^(-
1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*
c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e))*e^(a*c*l
og(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 3/4*(2*(b*c*log(abs(F)) - e...
```

Mupad [B] (verification not implemented)

Time = 3.33 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.17

$$\int F^{c(a+bx)}\cosh^3(d+ex)\,dx = \frac{F^{a\,c+b\,c\,x}\left(6\,e^3\sinh(d+e\,x)+3\,e^3\cosh(d+e\,x)^2\sinh(d+e\,x)+b^3\,c^3\cosh(d+e\,x)^3\ln\left(F\right)^3-b\,c\,e^2\cosh\right.}{b^4\,c^4\ln\left(F\right)^4-10\,b^2\,c^2\,e^2}$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x)^3,x)
```

output
```
(F^(a*c + b*c*x)*(6*e^3*sinh(d + e*x) + 3*e^3*cosh(d + e*x)^2*sinh(d + e*x
) + b^3*c^3*cosh(d + e*x)^3*log(F)^3 - b*c*e^2*cosh(d + e*x)^3*log(F) - 6*
b*c*e^2*cosh(d + e*x)*log(F) - 3*b^2*c^2*e*cosh(d + e*x)^2*sinh(d + e*x)*l
og(F)^2))/(9*e^4 + b^4*c^4*log(F)^4 - 10*b^2*c^2*e^2*log(F)^2)
```

Reduce [F]

$$\int F^{c(a+bx)}\cosh^3(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\cosh\left(ex+d\right)^3 dx\right)$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cosh(d + e*x)**3,x)
```

3.37 $\int F^{c(a+bx)} \cosh^4(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 162

$$\int F^{c(a+bx)} \cosh^4(d+ex)\,dx = \frac{3F^{c(a+bx)}}{8bc\log(F)} - \frac{e^{-2d-2ex}F^{c(a+bx)}}{4(2e-bc\log(F))} - \frac{e^{-4d-4ex}F^{c(a+bx)}}{16(4e-bc\log(F))} + \frac{e^{2d+2ex}F^{c(a+bx)}}{4(2e+bc\log(F))} + \frac{e^{4d+4ex}F^{c(a+bx)}}{16(4e+bc\log(F))}$$

output
```
3/8*F^(c*(b*x+a))/b/c/ln(F)-exp(-2*e*x-2*d)*F^(c*(b*x+a))/(8*e-4*b*c*ln(F)
)-exp(-4*e*x-4*d)*F^(c*(b*x+a))/(64*e-16*b*c*ln(F))+exp(2*e*x+2*d)*F^(c*(b
*x+a))/(8*e+4*b*c*ln(F))+exp(4*e*x+4*d)*F^(c*(b*x+a))/(64*e+16*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.26

$$\int F^{c(a+bx)}\cosh^4(d+ex)\,dx=\frac{1}{8}F^{c(a+bx)}\left(\frac{3}{bc\log(F)}+\frac{4\cosh(2ex)(-bc\cosh(2d)\log(F)+2e\sinh(2d))}{4e^2-b^2c^2\log^2(F)}+\frac{\cosh(4ex)(-bc\cosh(4d)\log(F)+4e\sinh(4d))}{16e^2-b^2c^2\log^2(F)}+\frac{4(2e\cosh(2d)-bc\log(F)\sinh(2d))\sinh(2ex)}{4e^2-b^2c^2\log^2(F)}+\frac{(4e\cosh(4d)-bc\log(F)\sinh(4d))\sinh(4ex)}{16e^2-b^2c^2\log^2(F)}\right)$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^4,x]
```

output
```
(F^(c*(a + b*x))*(3/(b*c*Log[F]) + (4*Cosh[2*e*x]*(-(b*c*Cosh[2*d]*Log[F])
 + 2*e*Sinh[2*d]))/(4*e^2 - b^2*c^2*Log[F]^2) + (Cosh[4*e*x]*(-(b*c*Cosh[4
*d]*Log[F]) + 4*e*Sinh[4*d]))/(16*e^2 - b^2*c^2*Log[F]^2) + (4*(2*e*Cosh[2
*d] - b*c*Log[F]*Sinh[2*d])*Sinh[2*e*x])/(4*e^2 - b^2*c^2*Log[F]^2) + ((4*
e*Cosh[4*d] - b*c*Log[F]*Sinh[4*d])*Sinh[4*e*x])/(16*e^2 - b^2*c^2*Log[F]^
2)))/8
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.52, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6000, 6000, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^4(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{ 6000}$$

$$\frac{12e^2\int F^{c(a+bx)}\cosh^2(d+ex)dx}{16e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh^4(d+ex)F^{c(a+bx)}}{16e^2-b^2c^2\log^2(F)}+\frac{4e\sinh(d+ex)\cosh^3(d+ex)F^{c(a+bx)}}{16e^2-b^2c^2\log^2(F)}$$

$$\Big\downarrow \text{6000}$$

$$\frac{12e^2\left(\frac{2e^2\int F^{c(a+bx)}dx}{4e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}\right)}{16e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh^4(d+ex)F^{c(a+bx)}}{16e^2-b^2c^2\log^2(F)}+\frac{4e\sinh(d+ex)\cosh^3(d+ex)F^{c(a+bx)}}{16e^2-b^2c^2\log^2(F)}$$

$$\Big\downarrow \text{2624}$$

$$-\frac{bc\log(F)\cosh^4(d+ex)F^{c(a+bx)}}{16e^2-b^2c^2\log^2(F)}+\frac{4e\sinh(d+ex)\cosh^3(d+ex)F^{c(a+bx)}}{16e^2-b^2c^2\log^2(F)}+\frac{12e^2\left(-\frac{bc\log(F)\cosh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}\right)}{16e^2-b^2c^2\log^2(F)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x]^4,x]
```

output
```
-((b*c*F^(c*(a + b*x))*Cosh[d + e*x]^4*Log[F])/(16*e^2 - b^2*c^2*Log[F]^2)
) + (4*e*F^(c*(a + b*x))*Cosh[d + e*x]^3*Sinh[d + e*x])/(16*e^2 - b^2*c^2*
Log[F]^2) + (12*e^2*((2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 - b^2*c^2*
Log[F]^2)) - (b*c*F^(c*(a + b*x))*Cosh[d + e*x]^2*Log[F])/(4*e^2 - b^2*c^2
*Log[F]^2) + (2*e*F^(c*(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x])/(4*e^2 - b^
2*c^2*Log[F]^2)))/(16*e^2 - b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 6000
```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Simp[e*n*F^(c*(a + b*x))*Sinh[d + e*x]*(Cosh[d + e*x]^
(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x] + Simp[n*(n - 1)*(e^2/(e^2*n^2 -
b^2*c^2*Log[F]^2))   Int[F^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.20

method	result
parallelrisch	$\frac{F^{c(bx+a)}\left(\left(-4\ln(F)^{3}b^{3}c^{3}e+16\ln(F)bc\,e^{3}\right)\sinh(4ex+4d)+\left(\ln(F)^{4}b^{4}c^{4}-4\ln(F)^{2}b^{2}c^{2}e^{2}\right)\cosh(4ex+4d)-8(bc\ln(F)+4e)\left(\right.}{8c^{5}b^{5}\ln(F)^{5}-160c^{3}b^{3}\ln(F)^{3}e^{2}+512cb\ln}$
risch	$\frac{\left(\ln(F)^{4}b^{4}c^{4}e^{8ex+8d}+4\ln(F)^{4}b^{4}c^{4}e^{6ex+6d}-4\ln(F)^{3}b^{3}c^{3}e\,e^{8ex+8d}+6\ln(F)^{4}b^{4}c^{4}e^{4ex+4d}-8\ln(F)^{3}b^{3}c^{3}e\,e^{6ex+6d}-4\ln(F)^{2}\right.}{}$
orering	Expression too large to display

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
F^(c*(b*x+a))*((-4*ln(F)^3*b^3*c^3*e+16*ln(F)*b*c*e^3)*sinh(4*e*x+4*d)+(ln
(F)^4*b^4*c^4-4*ln(F)^2*b^2*c^2*e^2)*cosh(4*e*x+4*d)-8*(b*c*ln(F)+4*e)*(-1
/2*c^2*b^2*ln(F)^2*cosh(2*e*x+2*d)-3/8*b^2*c^2*ln(F)^2+ln(F)*b*c*e*sinh(2*
e*x+2*d)+3/2*e^2)*(b*c*ln(F)-4*e))/(8*c^5*b^5*ln(F)^5-160*c^3*b^3*ln(F)^3*
e^2+512*c*b*ln(F)*e^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3682 vs. $2(146) = 292$.

Time = 0.20 (sec) , antiderivative size = 3682, normalized size of antiderivative = 22.73

$$\int F^{c(a+bx)} \cosh^4(d+ex)\, dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2409 vs. $2(143) = 286$.

Time = 26.48 (sec) , antiderivative size = 2409, normalized size of antiderivative = 14.87

$$\int F^{c(a+bx)} \cosh^4(d+ex)\, dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d)**4,x)
```

output
```
Piecewise((x*cosh(d)**4, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)), (3*x*
sinh(d + e*x)**4/8 - 3*x*sinh(d + e*x)**2*cosh(d + e*x)**2/4 + 3*x*cosh(d
+ e*x)**4/8 - 3*sinh(d + e*x)**3*cosh(d + e*x)/(8*e) + 5*sinh(d + e*x)*cos
h(d + e*x)**3/(8*e), Eq(F, 1)), (F**(a*c)*(3*x*sinh(d + e*x)**4/8 - 3*x*si
nh(d + e*x)**2*cosh(d + e*x)**2/4 + 3*x*cosh(d + e*x)**4/8 - 3*sinh(d + e*
x)**3*cosh(d + e*x)/(8*e) + 5*sinh(d + e*x)*cosh(d + e*x)**3/(8*e)), Eq(b,
 0)), (3*x*sinh(d + e*x)**4/8 - 3*x*sinh(d + e*x)**2*cosh(d + e*x)**2/4 +
3*x*cosh(d + e*x)**4/8 - 3*sinh(d + e*x)**3*cosh(d + e*x)/(8*e) + 5*sinh(d
 + e*x)*cosh(d + e*x)**3/(8*e), Eq(c, 0)), (-F**(a*c + b*c*x)*x*sinh(b*c*x
*log(F)/2 - d)**4/4 + F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/2 - d)**3*cosh(
b*c*x*log(F)/2 - d)/2 - F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/2 - d)*cosh(b
*c*x*log(F)/2 - d)**3/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F)/2 - d)**4/4
 + 13*F**(a*c + b*c*x)*sinh(b*c*x*log(F)/2 - d)**4/(24*b*c*log(F)) - 7*F**
(a*c + b*c*x)*sinh(b*c*x*log(F)/2 - d)**3*cosh(b*c*x*log(F)/2 - d)/(12*b*c
*log(F)) - F**(a*c + b*c*x)*sinh(b*c*x*log(F)/2 - d)**2*cosh(b*c*x*log(F)/
2 - d)**2/(b*c*log(F)) + 5*F**(a*c + b*c*x)*sinh(b*c*x*log(F)/2 - d)*cosh(
b*c*x*log(F)/2 - d)**3/(4*b*c*log(F)) + F**(a*c + b*c*x)*cosh(b*c*x*log(F)
/2 - d)**4/(8*b*c*log(F)), Eq(e, -b*c*log(F)/2)), (F**(a*c + b*c*x)*x*sinh
(b*c*x*log(F)/4 - d)**4/16 - F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/4 - d)**
3*cosh(b*c*x*log(F)/4 - d)/4 + 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/4...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.02

$$\int F^{c(a+bx)}\cosh^4(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+4\,ex+4\,d)}}{16\,(bc\log(F)+4\,e)} + \frac{F^{ac}e^{(bcx\log(F)+2\,ex+2\,d)}}{4\,(bc\log(F)+2\,e)} + \frac{F^{ac}e^{(bcx\log(F)-2\,ex)}}{4\,(bce^{(2\,d)}\log(F)-2\,ee^{(2\,d)})} + \frac{F^{ac}e^{(bcx\log(F)-4\,ex)}}{16\,(bce^{(4\,d)}\log(F)-4\,ee^{(4\,d)})} + \frac{3\,F^{bcx+ac}}{8\,bc\log(F)}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^4,x, algorithm="maxima")
```

output
```
1/16*F^(a*c)*e^(b*c*x*log(F) + 4*e*x + 4*d)/(b*c*log(F) + 4*e) + 1/4*F^(a*
c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) + 1/4*F^(a*c)*e^(b*c*
x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) + 1/16*F^(a*c)*e^(b*c
*x*log(F) - 4*e*x)/(b*c*e^(4*d)*log(F) - 4*e*e^(4*d)) + 3/8*F^(b*c*x + a*c
)/(b*c*log(F))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 1502, normalized size of antiderivative = 9.27

$$\int F^{c(a+bx)} \cosh^4(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^4,x, algorithm="giac")
```

output
```
3/4*(2*b*c*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1
/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)
^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1
/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) -
 pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 3*I*(I*e^(1/2*I*pi*
b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(8*I*p
i*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F))) - I*e^(-1/2*I*pi*b*c*x*sgn
(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sg
n(F) + 8*I*pi*b*c + 16*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(ab
s(F))) + 1/8*(2*(b*c*log(abs(F)) + 4*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*
b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b
*c*log(abs(F)) + 4*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(
F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b
*c)^2 + 4*(b*c*log(abs(F)) + 4*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)
) + 4*e)*x + 4*d) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I
*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(16*I*pi*b*c*sgn(F) - 16*I*pi*b*c + 32*b*c*
log(abs(F)) + 128*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*
I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-16*I*pi*b*c*sgn(F) + 16*I*pi*b*c + 32*b*
c*log(abs(F)) + 128*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 4*e)*x + 4
*d) + 1/2*(2*(b*c*log(abs(F)) + 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*...
```

Mupad [B] (verification not implemented)

Time = 5.05 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.22

$$\int F^{c(a+bx)}\cosh^4(d+ex)\,dx$$
$$= \frac{F^{a\,c+b\,c\,x}\,\left(24\,e^4+b^4\,c^4\,\cosh(d+e\,x)^4\,\ln\left(F\right)^4-12\,b^2\,c^2\,e^2\,\cosh(d+e\,x)^2\,\ln\left(F\right)^2-4\,b^2\,c^2\,e^2\,\cosh(d+e\,x)\right.}{b\,c\,\ln}$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x)^4,x)
```

output
```
(F^(a*c + b*c*x)*(24*e^4 + b^4*c^4*cosh(d + e*x)^4*log(F)^4 - 12*b^2*c^2*e
^2*cosh(d + e*x)^2*log(F)^2 - 4*b^2*c^2*e^2*cosh(d + e*x)^4*log(F)^2 + 24*
b*c*e^3*cosh(d + e*x)*sinh(d + e*x)*log(F) - 4*b^3*c^3*e*cosh(d + e*x)^3*s
inh(d + e*x)*log(F)^3 + 16*b*c*e^3*cosh(d + e*x)^3*sinh(d + e*x)*log(F)))/
(b*c*log(F)*(64*e^4 + b^4*c^4*log(F)^4 - 20*b^2*c^2*e^2*log(F)^2))
```

Reduce [F]

$$\int F^{c(a+bx)}\cosh^4(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\cosh\left(ex+d\right)^4dx\right)$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^4,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cosh(d + e*x)**4,x)
```

3.38 $\int e^{a+bx}\cosh^n(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 76

$$\int e^{a+bx}\cosh^n(a+bx)\,dx = \frac{e^{a+bx}\left(1+e^{2a+2bx}\right)^{-n}\cosh^n(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},-n,\frac{3-n}{2},-e^{2a+2bx}\right)}{b(1-n)}$$

output
```
exp(b*x+a)*cosh(b*x+a)^n*hypergeom([-n, 1/2-1/2*n],[3/2-1/2*n],-exp(2*b*x+
2*a))/b/((1+exp(2*b*x+2*a))^n)/(1-n)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.17

$$\int e^{a+bx}\cosh^n(a+bx)\,dx = \frac{2^{-n}e^{a+bx}\left(1+e^{-2(a+bx)}\right)^{-n}\left(e^{-a-bx}+e^{a+bx}\right)^n\text{Hypergeometric2F1}\left(\frac{1}{2}(-1-n),-n,\frac{1-n}{2},-e^{-2(a+bx)}\right)}{b(1+n)}$$

input
```
Integrate[E^(a + b*x)*Cosh[a + b*x]^n,x]
```

output
```
(E^(a + b*x)*(E^(-a - b*x) + E^(a + b*x))^n*Hypergeometric2F1[(-1 - n)/2,
-n, (1 - n)/2, -E^(-2*(a + b*x))])/(2^n*b*(1 + E^(-2*(a + b*x)))^n*(1 + n)
)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 27, 1917, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx} \cosh^n(a+bx) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int 2^{-n} \left(e^{-a-bx}+e^{a+bx}\right)^n de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2^{-n} \int \left(e^{-a-bx}+e^{a+bx}\right)^n de^{a+bx}}{b}$$

$$\Big\downarrow \text{1917}$$

$$\frac{2^{-n} \left(e^{a+bx}\right)^n \left(e^{-a-bx}+e^{a+bx}\right)^n \left(e^{2a+2bx}+1\right)^{-n} \int \left(e^{a+bx}\right)^{-n} \left(1+e^{2a+2bx}\right)^n de^{a+bx}}{b}$$

$$\Big\downarrow \text{278}$$

$$\frac{2^{-n} e^{a+bx} \left(e^{-a-bx}+e^{a+bx}\right)^n \left(e^{2a+2bx}+1\right)^{-n} \text{Hypergeometric2F1}\left(\frac{1-n}{2},-n,\frac{3-n}{2},-e^{2a+2bx}\right)}{b(1-n)}$$

input
```
Int[E^(a + b*x)*Cosh[a + b*x]^n,x]
```

output
```
(E^(a + b*x)*(E^(-a - b*x) + E^(a + b*x))^n*Hypergeometric2F1[(1 - n)/2, -
n, (3 - n)/2, -E^(2*a + 2*b*x)])/(2^n*b*(1 + E^(2*a + 2*b*x))^n*(1 - n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [F]

$$\int e^{bx+a} \cosh(bx+a)^n \, dx$$

input
```
int(exp(b*x+a)*cosh(b*x+a)^n,x)
```

output
```
int(exp(b*x+a)*cosh(b*x+a)^n,x)
```

Fricas [F]

$$\int e^{a+bx}\cosh^n(a+bx)\,dx = \int \cosh{(bx+a)}^n e^{(bx+a)}\,dx$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^n,x, algorithm="fricas")
```

output
```
integral(cosh(b*x + a)^n*e^(b*x + a), x)
```

Sympy [F]

$$\int e^{a+bx}\cosh^n(a+bx)\,dx = e^a\int e^{bx}\cosh^n{(a+bx)}\,dx$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)**n,x)
```

output
```
exp(a)*Integral(exp(b*x)*cosh(a + b*x)**n, x)
```

Maxima [F]

$$\int e^{a+bx}\cosh^n(a+bx)\,dx = \int \cosh{(bx+a)}^n e^{(bx+a)}\,dx$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^n,x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^n*e^(b*x + a), x)
```

Giac [F]

$$\int e^{a+bx}\cosh^n(a+bx)\,dx=\int \cosh\left(bx+a\right)^n e^{(bx+a)}\,dx$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^n,x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^n*e^(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\cosh^n(a+bx)\,dx=\int \cosh(a+b\,x)^n\,\mathrm{e}^{a+b\,x}\,dx$$

input
```
int(cosh(a + b*x)^n*exp(a + b*x),x)
```

output
```
int(cosh(a + b*x)^n*exp(a + b*x), x)
```

Reduce [F]

$$\int e^{a+bx}\cosh^n(a+bx)\,dx$$
$$=\frac{e^a\left(e^{bnx+an+bx}\cosh\left(bx+a\right)^n 2^n n+e^{bnx+an+bx}\cosh\left(bx+a\right)^n 2^n-e^{bx}\left(e^{2bx+2a}+1\right)^n n+2e^{bnx+an}\left(\int \frac{\cdots}{e^{bnx\cdots}}\right.\cdots}{e^{bnx\cdots}}$$

input
```
int(exp(b*x+a)*cosh(b*x+a)^n,x)
```

output
```
(e**a*(e**(a*n + b*n*x + b*x)*cosh(a + b*x)**n*2**n*n + e**(a*n + b*n*x +
b*x)*cosh(a + b*x)**n*2**n - e**(b*x)*(e**(2*a + 2*b*x) + 1)**n*n + 2*e**(
a*n + b*n*x)*int((e**(b*x)*(e**(2*a + 2*b*x) + 1)**n)/(e**(a*n + 2*a + b*n
*x + 2*b*x)*n + e**(a*n + 2*a + b*n*x + 2*b*x) + e**(a*n + b*n*x)*n + e**(
a*n + b*n*x)),x)*b*n**2 + 2*e**(a*n + b*n*x)*int((e**(b*x)*(e**(2*a + 2*b*
x) + 1)**n)/(e**(a*n + 2*a + b*n*x + 2*b*x)*n + e**(a*n + 2*a + b*n*x + 2*
b*x) + e**(a*n + b*n*x)*n + e**(a*n + b*n*x)),x)*b*n))/(e**(a*n + b*n*x)*2
**n*b*(n + 1))
```

3.39 $\int F^{c(a+bx)}(f\cosh(d+ex))^n\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int F^{c(a+bx)}(f\cosh(d+ex))^n\,dx = -\frac{\left(1+e^{2d+2ex}\right)^{-n}F^{c(a+bx)}(f\cosh(d+ex))^n\,\text{Hypergeometric2F1}\left(-n,\frac{1}{2}\left(-n+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(2-n+\frac{bc\log(F)}{e}\right)\ldots\right)}{en-bc\log(F)}$$

output
```
-F^(c*(b*x+a))*(f*cosh(e*x+d))^n*hypergeom([-n, -1/2*n+1/2*b*c*ln(F)/e],[1
-1/2*n+1/2*b*c*ln(F)/e],-exp(2*e*x+2*d))/((1+exp(2*e*x+2*d))^n)/(e*n-b*c*l
n(F))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(f\cosh(d+ex))^n\,dx = \frac{\left(1+e^{2(d+ex)}\right)^{-n}F^{c(a+bx)}(f\cosh(d+ex))^n\,\text{Hypergeometric2F1}\left(-n,\frac{-en+bc\log(F)}{2e},1+\frac{-en+bc\log(F)}{2e},-e^{2(d\ldots}\right)}{-en+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Cosh[d + e*x])^n,x]
```

output
```
(F^(c*(a + b*x))*(f*Cosh[d + e*x])^n*Hypergeometric2F1[-n, (-(e*n) + b*c*L
og[F])/(2*e), 1 + (-(e*n) + b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/((1 + E^
(2*(d + e*x)))^n*(-(e*n) + b*c*Log[F]))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {7271, 6006, 2689}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\cosh(d+ex))^n\,dx$$

$$\Big\downarrow \text{7271}$$

$$\cosh^{-n}(d+ex)(f\cosh(d+ex))^n\int F^{c(a+bx)}\cosh^n(d+ex)dx$$

$$\Big\downarrow \text{6006}$$

$$e^{n(d+ex)}\left(e^{2(d+ex)}+1\right)^{-n}(f\cosh(d+ex))^n\int e^{-n(d+ex)}\left(1+e^{2(d+ex)}\right)^nF^{c(a+bx)}dx$$

$$\Big\downarrow \text{2689}$$

$$-\frac{\left(e^{2(d+ex)}+1\right)^{-n}F^{c(a+bx)}(f\cosh(d+ex))^n\,\text{Hypergeometric2F1}\left(-n,-\frac{en-bc\log(F)}{2e},\frac{1}{2}\left(-n+\frac{bc\log(F)}{e}+2\right),-e\right.}{en-bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*(f*Cosh[d + e*x])^n,x]
```

output
```
-((F^(c*(a + b*x))*(f*Cosh[d + e*x])^n*Hypergeometric2F1[-n, -1/2*(e*n - b
*c*Log[F])/e, (2 - n + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/((1 + E^(2*(d
 + e*x)))^n*(e*n - b*c*Log[F])))
```

Defintions of rubi rules used

rule 2689
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

rule 6006
```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[E^(n*(d + e*x))*(Cosh[d + e*x]^n/(1 + E^(2*(d + e*x)))^n)   Int
[F^(c*(a + b*x))*((1 + E^(2*(d + e*x)))^n/E^(n*(d + e*x))), x], x] /; FreeQ
[{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}(f\cosh(ex+d))^n\,dx$$

input
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^n,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\cosh(d+ex))^n\,dx = \int (f\cosh(ex+d))^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^n,x, algorithm="fricas")
```

output
```
integral((f*cosh(e*x + d))^n*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}(f\cosh(d+ex))^n\,dx = \int F^{c(a+bx)}(f\cosh(d+ex))^n\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*cosh(e*x+d))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*cosh(d + e*x))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}(f\cosh(d+ex))^n\,dx = \int (f\cosh(ex+d))^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^n,x, algorithm="maxima")
```

output
```
integrate((f*cosh(e*x + d))^n*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\cosh(d+ex))^n\,dx=\int (f\cosh{(ex+d)})^nF^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^n,x, algorithm="giac")
```

output
```
integrate((f*cosh(e*x + d))^n*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\cosh(d+ex))^n\,dx=\int F^{c\,(a+b\,x)}\,(f\cosh(d+e\,x))^n\,dx$$

input
```
int(F^(c*(a + b*x))*(f*cosh(d + e*x))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f*cosh(d + e*x))^n, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\cosh(d+ex))^n\,dx$$
$$=\frac{f^{ac+n}\left(f^{bcx}\cosh{(ex+d)}^n-\left(\int\frac{f^{bcx}\cosh(ex+d)^n\sinh(ex+d)}{\cosh(ex+d)}dx\right)en\right)}{\log{(f)}\,bc}$$

input
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^n,x)
```

output
```
(f**(a*c + n)*(f**(b*c*x)*cosh(d + e*x)**n - int((f**(b*c*x)*cosh(d + e*x)
**n*sinh(d + e*x))/cosh(d + e*x),x)*e*n))/(log(f)*b*c)
```

3.40 $\int F^{c(a+bx)}\left(f\cosh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)^n\,dx$

Optimal result

Integrand size = 28, antiderivative size = 82

$$\int F^{c(a+bx)}\left(f\cosh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)^n\,dx = \frac{F^{c(a+bx)}(2+n)\left(f\cosh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)^{1+n}\left(\cosh\left(d+\frac{bcx\log(F)}{2+n}\right)-\sinh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)}{bcf(1+n)\log(F)}$$

output
```
F^(c*(b*x+a))*(2+n)*(f*cosh(d+b*c*x*ln(F)/(2+n)))^(1+n)*(cosh(d+b*c*x*ln(F
)/(2+n))-sinh(d+b*c*x*ln(F)/(2+n)))/b/c/f/(1+n)/ln(F)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int F^{c(a+bx)}\left(f\cosh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)^n\,dx = \frac{e^{-2d}F^{c\left(a+\frac{bnx}{2+n}\right)}\left(1+e^{2d}F^{\frac{2bcx}{2+n}}\right)(2+n)\left(f\cosh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)^n}{2bc(1+n)\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Cosh[d + (b*c*x*Log[F])/(2 + n)])^n,x]
```

output
```
(F^(c*(a + (b*n*x)/(2 + n)))*(1 + E^(2*d)*F^((2*b*c*x)/(2 + n)))*(2 + n)*(
f*Cosh[d + (b*c*x*Log[F])/(2 + n)])^n)/(2*b*c*E^(2*d)*(1 + n)*Log[F])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.77, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {7271, 6002}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\left(f\cosh\left(\frac{bcx\log(F)}{n+2}+d\right)\right)^n\,dx$$

$$\Big\downarrow \; 7271$$

$$\cosh^{-n}\left(\frac{bcx\log(F)}{n+2}+d\right)\left(f\cosh\left(\frac{bcx\log(F)}{n+2}+d\right)\right)^n\int F^{c(a+bx)}\cosh^n\left(d+\frac{bcx\log(F)}{n+2}\right)dx$$

$$\Big\downarrow \; 6002$$

$$\cosh^{-n}\left(\frac{bcx\log(F)}{n+2}+d\right)\left(f\cosh\left(\frac{bcx\log(F)}{n+2}+d\right)\right)^n\left(\frac{(n+2)F^{c(a+bx)}\cosh^{n+2}\left(\frac{bcx\log(F)}{n+2}+d\right)}{bc(n+1)\log(F)}-\frac{(n+2)F^{c}}{}\right.$$

input
```
Int[F^(c*(a + b*x))*(f*Cosh[d + (b*c*x*Log[F])/(2 + n)])^n,x]
```

output
```
((f*Cosh[d + (b*c*x*Log[F])/(2 + n)])^n*((F^(c*(a + b*x))*(2 + n)*Cosh[d +
 (b*c*x*Log[F])/(2 + n)]^(2 + n))/(b*c*(1 + n)*Log[F]) - (F^(c*(a + b*x))*
(2 + n)*Cosh[d + (b*c*x*Log[F])/(2 + n)]^(1 + n)*Sinh[d + (b*c*x*Log[F])/(
2 + n)])/(b*c*(1 + n)*Log[F])))/Cosh[d + (b*c*x*Log[F])/(2 + n)]^n
```

Defintions of rubi rules used

rule 6002
```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]^(n + 2)/(e^2*(n + 1)*
(n + 2))), x] - Simp[F^(c*(a + b*x))*Sinh[d + e*x]*(Cosh[d + e*x]^(n + 1)/(
e*(n + 1))), x] /; FreeQ[{F, a, b, c, d, e, n}, x] && EqQ[e^2*(n + 2)^2 - b
^2*c^2*Log[F]^2, 0] && NeQ[n, -1] && NeQ[n, -2]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}\left(f\cosh\left(d+\frac{bcx\ln(F)}{2+n}\right)\right)^n dx$$

input
```
int(F^(c*(b*x+a))*(f*cosh(d+b*c*x*ln(F)/(2+n)))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*cosh(d+b*c*x*ln(F)/(2+n)))^n,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(82) = 164.

Time = 0.10 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.26

$$\int F^{c(a+bx)}\left(f\cosh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)^n dx$$
$$= \frac{\left((n+2)\cosh\left((bcx+ac)\log(F)\right)\cosh\left(\frac{bcx\log(F)+dn+2d}{n+2}\right)+(n+2)\cosh\left(\frac{bcx\log(F)+dn+2d}{n+2}\right)\sinh\left((bcx+a\right.\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(d+b*c*x*log(F)/(2+n)))^n,x, algorithm="fri
cas")
```

output
```
(((n + 2)*cosh((b*c*x + a*c)*log(F))*cosh((b*c*x*log(F) + d*n + 2*d)/(n +
2)) + (n + 2)*cosh((b*c*x*log(F) + d*n + 2*d)/(n + 2))*sinh((b*c*x + a*c)*
log(F)))*cosh(n*log(f*cosh((b*c*x*log(F) + d*n + 2*d)/(n + 2)))) + ((n + 2
)*cosh((b*c*x + a*c)*log(F))*cosh((b*c*x*log(F) + d*n + 2*d)/(n + 2)) + (n
 + 2)*cosh((b*c*x*log(F) + d*n + 2*d)/(n + 2))*sinh((b*c*x + a*c)*log(F)))
*sinh(n*log(f*cosh((b*c*x*log(F) + d*n + 2*d)/(n + 2)))))/((b*c*n + b*c)*c
osh((b*c*x*log(F) + d*n + 2*d)/(n + 2))*log(F) + (b*c*n + b*c)*log(F)*sinh
((b*c*x*log(F) + d*n + 2*d)/(n + 2)))
```

Sympy [F]

$$\int F^{c(a+bx)}\left(f\cosh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)^n dx = \int F^{c(a+bx)}\left(f\cosh\left(\frac{bcx\log(F)}{n+2}+d\right)\right)^n dx$$

input
```
integrate(F**(c*(b*x+a))*(f*cosh(d+b*c*x*ln(F)/(2+n)))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*cosh(b*c*x*log(F)/(n + 2) + d))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\left(f\cosh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)^n dx = \int \left(f\cosh\left(\frac{bcx\log(F)}{n+2}+d\right)\right)^n F^{(bx+a)c}\, dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(d+b*c*x*log(F)/(2+n)))^n,x, algorithm="max
ima")
```

output
```
integrate((f*cosh(b*c*x*log(F)/(n + 2) + d))^n*F^((b*x + a)*c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(82) = 164$.

Time = 0.45 (sec) , antiderivative size = 513, normalized size of antiderivative = 6.26

$$\int F^{c(a+bx)}\left(f\cosh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)^n\,dx$$

$$= \frac{F^{ac}ne^{\left(\frac{2\,bcx\log(F)-dn^2-n^2\log(2)+n^2\log\left(fe^{\left(\frac{2\,(bcx\log(F)+dn+2\,d)}{n+2}\right)}+f\right)-2\,dn-2\,n\log(2)+2\,n\log\left(fe^{\left(\frac{2\,(bcx\log(F)+dn+2\,d)}{n+2}\right)}+f\right)}{n+2}+\frac{2\,(bcx\log(F)}{n+2}\right.}}{}$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(d+b*c*x*log(F)/(2+n)))^n,x, algorithm="gia
c")
```

output
```
1/2*(F^(a*c)*n*e^((2*b*c*x*log(F) - d*n^2 - n^2*log(2) + n^2*log(f*e^(2*(b
*c*x*log(F) + d*n + 2*d)/(n + 2)) + f) - 2*d*n - 2*n*log(2) + 2*n*log(f*e^
(2*(b*c*x*log(F) + d*n + 2*d)/(n + 2)) + f))/(n + 2) + 2*(b*c*x*log(F) + d
*n + 2*d)/(n + 2)) + F^(a*c)*n*e^((2*b*c*x*log(F) - d*n^2 - n^2*log(2) + n
^2*log(f*e^(2*(b*c*x*log(F) + d*n + 2*d)/(n + 2)) + f) - 2*d*n - 2*n*log(2
) + 2*n*log(f*e^(2*(b*c*x*log(F) + d*n + 2*d)/(n + 2)) + f))/(n + 2)) + 2*
F^(a*c)*e^((2*b*c*x*log(F) - d*n^2 - n^2*log(2) + n^2*log(f*e^(2*(b*c*x*lo
g(F) + d*n + 2*d)/(n + 2)) + f) - 2*d*n - 2*n*log(2) + 2*n*log(f*e^(2*(b*c
*x*log(F) + d*n + 2*d)/(n + 2)) + f))/(n + 2) + 2*(b*c*x*log(F) + d*n + 2*
d)/(n + 2)) + 2*F^(a*c)*e^((2*b*c*x*log(F) - d*n^2 - n^2*log(2) + n^2*log(
f*e^(2*(b*c*x*log(F) + d*n + 2*d)/(n + 2)) + f) - 2*d*n - 2*n*log(2) + 2*n
*log(f*e^(2*(b*c*x*log(F) + d*n + 2*d)/(n + 2)) + f))/(n + 2)))/(b*c*n*e^(
2*(b*c*x*log(F) + d*n + 2*d)/(n + 2))*log(F) + b*c*e^(2*(b*c*x*log(F) + d*
n + 2*d)/(n + 2))*log(F))
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\left(f\cosh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)^n dx = \int F^{c\,(a+b\,x)}\,\left(f\cosh\left(d+\frac{b\,c\,x\,\ln(F)}{n+2}\right)\right)^n dx$$

input
```
int(F^(c*(a + b*x))*(f*cosh(d + (b*c*x*log(F))/(n + 2)))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f*cosh(d + (b*c*x*log(F))/(n + 2)))^n, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.89

$$\int F^{c(a+bx)}\left(f\cosh\left(d+\frac{bcx\log(F)}{2+n}\right)\right)^n dx = \frac{f^{ac+n}\left(2f^{bcx}e^{dn+2d}\cosh\left(\frac{\log(f)bcx+dn+2d}{n+2}\right)^n 2^n n+2f^{bcx}e^{dn+2d}\cosh\left(\frac{\log(f)bcx+dn+2d}{n+2}\right)^n 2^n-e^{\frac{2\log(f)bcx+2dn+4d}{n+2}}\right.}{2e^{dn+2d}2^n\log(f)\,bc\,(n+}$$

input
```
int(F^(c*(b*x+a))*(f*cosh(d+b*c*x*log(F)/(2+n)))^n,x)
```

output
```
(f**(a*c + n)*(2*f**(b*c*x)*e**(d*n + 2*d)*cosh((log(f)*b*c*x + d*n + 2*d)
/(n + 2))**n*2**n*n + 2*f**(b*c*x)*e**(d*n + 2*d)*cosh((log(f)*b*c*x + d*n
 + 2*d)/(n + 2))**n*2**n - e**((2*log(f)*b*c*x + 2*d*n + 4*d)/(n + 2))*(e*
*((2*log(f)*b*c*x + 2*d*n + 4*d)/(n + 2)) + 1)**n*n + (e**((2*log(f)*b*c*x
 + 2*d*n + 4*d)/(n + 2)) + 1)**n*n + 2*(e**((2*log(f)*b*c*x + 2*d*n + 4*d)
/(n + 2)) + 1)**n))/(2*e**(d*n + 2*d)*2**n*log(f)*b*c*(n + 1))
```

3.41 $\int F^{c(a+bx)} \left(f \cosh \left(d-\frac{bcx \log(F)}{2+n}\right)\right)^n \, dx$

Optimal result

Integrand size = 29, antiderivative size = 83

$$\int F^{c(a+bx)} \left(f \cosh \left(d-\frac{bcx \log(F)}{2+n}\right)\right)^n \, dx = \frac{F^{c(a+bx)}(2+n) \left(f \cosh \left(d-\frac{bcx \log(F)}{2+n}\right)\right)^{1+n} \left(\cosh \left(d-\frac{bcx \log(F)}{2+n}\right)+\sinh \left(d-\frac{bcx \log(F)}{2+n}\right)\right)}{bcf(1+n) \log(F)}$$

output
```
F^(c*(b*x+a))*(2+n)*(f*cosh(-d+b*c*x*ln(F)/(2+n)))^(1+n)*(cosh(-d+b*c*x*ln
(F)/(2+n))-sinh(-d+b*c*x*ln(F)/(2+n)))/b/c/f/(1+n)/ln(F)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int F^{c(a+bx)} \left(f \cosh \left(d-\frac{bcx \log(F)}{2+n}\right)\right)^n \, dx = \frac{F^{c(a+bx)} \left(1+e^{2d} F^{-\frac{2bcx}{2+n}}\right) (2+n) \left(f \cosh \left(d-\frac{bcx \log(F)}{2+n}\right)\right)^n}{2bc(1+n) \log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Cosh[d - (b*c*x*Log[F])/(2 + n)])^n,x]
```

output
```
(F^(c*(a + b*x))*(1 + E^(2*d)/F^((2*b*c*x)/(2 + n)))*(2 + n)*(f*Cosh[d - (
b*c*x*Log[F])/(2 + n)])^n)/(2*b*c*(1 + n)*Log[F])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.80, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {7271, 6002}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\left(f\cosh\left(d-\frac{bcx\log(F)}{n+2}\right)\right)^n dx$$

$$\Big\downarrow \text{ 7271}$$

$$\cosh^{-n}\left(d-\frac{bcx\log(F)}{n+2}\right)\left(f\cosh\left(d-\frac{bcx\log(F)}{n+2}\right)\right)^n\int F^{c(a+bx)}\cosh^n\left(d-\frac{bcx\log(F)}{n+2}\right)dx$$

$$\Big\downarrow \text{ 6002}$$

$$\cosh^{-n}\left(d-\frac{bcx\log(F)}{n+2}\right)\left(f\cosh\left(d-\frac{bcx\log(F)}{n+2}\right)\right)^n\left(\frac{(n+2)F^{c(a+bx)}\cosh^{n+2}\left(d-\frac{bcx\log(F)}{n+2}\right)}{bc(n+1)\log(F)}+\frac{(n+2)F^{c}}{}\right.$$

input
```
Int[F^(c*(a + b*x))*(f*Cosh[d - (b*c*x*Log[F])/(2 + n)])^n,x]
```

output
```
((f*Cosh[d - (b*c*x*Log[F])/(2 + n)])^n*((F^(c*(a + b*x))*(2 + n)*Cosh[d -
 (b*c*x*Log[F])/(2 + n)]^(2 + n))/(b*c*(1 + n)*Log[F]) + (F^(c*(a + b*x))*
(2 + n)*Cosh[d - (b*c*x*Log[F])/(2 + n)]^(1 + n)*Sinh[d - (b*c*x*Log[F])/(
2 + n)])/(b*c*(1 + n)*Log[F])))/Cosh[d - (b*c*x*Log[F])/(2 + n)]^n
```

Defintions of rubi rules used

rule 6002
```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]^(n + 2)/(e^2*(n + 1)*
(n + 2))), x] - Simp[F^(c*(a + b*x))*Sinh[d + e*x]*(Cosh[d + e*x]^(n + 1)/(
e*(n + 1))), x] /; FreeQ[{F, a, b, c, d, e, n}, x] && EqQ[e^2*(n + 2)^2 - b
^2*c^2*Log[F]^2, 0] && NeQ[n, -1] && NeQ[n, -2]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}\left(f\cosh\left(-d+\frac{bcx\ln(F)}{2+n}\right)\right)^n dx$$

input
```
int(F^(c*(b*x+a))*(f*cosh(-d+b*c*x*ln(F)/(2+n)))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*cosh(-d+b*c*x*ln(F)/(2+n)))^n,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(88) = 176.

Time = 0.10 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.31

$$\int F^{c(a+bx)}\left(f\cosh\left(d-\frac{bcx\log(F)}{2+n}\right)\right)^n dx = \frac{\left((n+2)\cosh\left((bcx+ac)\log\left(F\right)\right)\cosh\left(\frac{bcx\log(F)-dn-2\,d}{n+2}\right)+(n+2)\cosh\left(\frac{bcx\log(F)-dn-2\,d}{n+2}\right)\sinh\left((bcx+a\right.\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(-d+b*c*x*log(F)/(2+n)))^n,x, algorithm="fr
icas")
```

output
```
(((n + 2)*cosh((b*c*x + a*c)*log(F))*cosh((b*c*x*log(F) - d*n - 2*d)/(n +
2)) + (n + 2)*cosh((b*c*x*log(F) - d*n - 2*d)/(n + 2))*sinh((b*c*x + a*c)*
log(F)))*cosh(n*log(f*cosh((b*c*x*log(F) - d*n - 2*d)/(n + 2)))) + ((n + 2
)*cosh((b*c*x + a*c)*log(F))*cosh((b*c*x*log(F) - d*n - 2*d)/(n + 2)) + (n
 + 2)*cosh((b*c*x*log(F) - d*n - 2*d)/(n + 2))*sinh((b*c*x + a*c)*log(F)))
*sinh(n*log(f*cosh((b*c*x*log(F) - d*n - 2*d)/(n + 2)))))/((b*c*n + b*c)*c
osh((b*c*x*log(F) - d*n - 2*d)/(n + 2))*log(F) + (b*c*n + b*c)*log(F)*sinh
((b*c*x*log(F) - d*n - 2*d)/(n + 2)))
```

Sympy [F]

$$\int F^{c(a+bx)}\left(f\cosh\left(d-\frac{bcx\log(F)}{2+n}\right)\right)^n\,dx = \int F^{c(a+bx)}\left(f\cosh\left(\frac{bcx\log\left(F\right)}{n+2}-d\right)\right)^n\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*cosh(-d+b*c*x*ln(F)/(2+n)))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*cosh(b*c*x*log(F)/(n + 2) - d))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\left(f\cosh\left(d-\frac{bcx\log(F)}{2+n}\right)\right)^n\,dx = \int \left(f\cosh\left(\frac{bcx\log\left(F\right)}{n+2}-d\right)\right)^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(-d+b*c*x*log(F)/(2+n)))^n,x, algorithm="ma
xima")
```

output
```
integrate((f*cosh(b*c*x*log(F)/(n + 2) - d))^n*F^((b*x + a)*c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(88) = 176$.

Time = 0.39 (sec) , antiderivative size = 521, normalized size of antiderivative = 6.28

$$\int F^{c(a+bx)}\left(f\cosh\left(d-\frac{bcx\log(F)}{2+n}\right)\right)^n dx = \frac{F^{ac}ne^{\left(\frac{2\,bcx\log(F)+dn^2-n^2\log(2)+n^2\log\left(fe^{\left(\frac{2\,(bcx\log(F)-dn-2\,d)}{n+2}\right)}+f\right)+2\,dn-2\,n\log(2)+2\,n\log\left(fe^{\left(\frac{2\,(bcx\log(F)-dn-2\,d)}{n+2}\right)}+f\right)}{n+2}+\frac{2\,(bcx\log(F)}{n+\cdots}\right.}}{\cdots}$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(-d+b*c*x*log(F)/(2+n)))^n,x, algorithm="gi
ac")
```

output
```
1/2*(F^(a*c)*n*e^((2*b*c*x*log(F) + d*n^2 - n^2*log(2) + n^2*log(f*e^(2*(b
*c*x*log(F) - d*n - 2*d)/(n + 2)) + f) + 2*d*n - 2*n*log(2) + 2*n*log(f*e^
(2*(b*c*x*log(F) - d*n - 2*d)/(n + 2)) + f))/(n + 2) + 2*(b*c*x*log(F) - d
*n - 2*d)/(n + 2)) + F^(a*c)*n*e^((2*b*c*x*log(F) + d*n^2 - n^2*log(2) + n
^2*log(f*e^(2*(b*c*x*log(F) - d*n - 2*d)/(n + 2)) + f) + 2*d*n - 2*n*log(2
) + 2*n*log(f*e^(2*(b*c*x*log(F) - d*n - 2*d)/(n + 2)) + f))/(n + 2)) + 2*
F^(a*c)*e^((2*b*c*x*log(F) + d*n^2 - n^2*log(2) + n^2*log(f*e^(2*(b*c*x*lo
g(F) - d*n - 2*d)/(n + 2)) + f) + 2*d*n - 2*n*log(2) + 2*n*log(f*e^(2*(b*c
*x*log(F) - d*n - 2*d)/(n + 2)) + f))/(n + 2) + 2*(b*c*x*log(F) - d*n - 2*
d)/(n + 2)) + 2*F^(a*c)*e^((2*b*c*x*log(F) + d*n^2 - n^2*log(2) + n^2*log(
f*e^(2*(b*c*x*log(F) - d*n - 2*d)/(n + 2)) + f) + 2*d*n - 2*n*log(2) + 2*n
*log(f*e^(2*(b*c*x*log(F) - d*n - 2*d)/(n + 2)) + f))/(n + 2)))/(b*c*n*e^(
2*(b*c*x*log(F) - d*n - 2*d)/(n + 2))*log(F) + b*c*e^(2*(b*c*x*log(F) - d*
n - 2*d)/(n + 2))*log(F))
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\left(f\cosh\left(d-\frac{bcx\log(F)}{2+n}\right)\right)^n\,dx = \int F^{c\,(a+b\,x)}\,\left(f\cosh\left(d-\frac{b\,c\,x\,\ln\left(F\right)}{n+2}\right)\right)^n\,dx$$

input
```
int(F^(c*(a + b*x))*(f*cosh(d - (b*c*x*log(F))/(n + 2)))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f*cosh(d - (b*c*x*log(F))/(n + 2)))^n, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.57

$$\int F^{c(a+bx)}\left(f\cosh\left(d-\frac{bcx\log(F)}{2+n}\right)\right)^n\,dx = \frac{f^{ac+n}\left(2f^{bcx}e^{dn}\cosh\left(\frac{\log(f)bcx-dn-2d}{n+2}\right)^n 2^n n+2f^{bcx}e^{dn}\cosh\left(\frac{\log(f)bcx-dn-2d}{n+2}\right)^n 2^n-e^{\frac{2\log(f)bcx}{n+2}}\left(e^{\frac{2\log(f)bcx}{n+2}}+\right.\right.}{2e^{dn}2^n\log\left(f\right)bc\left(n+1\right)}$$

input
```
int(F^(c*(b*x+a))*(f*cosh(-d+b*c*x*log(F)/(2+n)))^n,x)
```

output
```
(f**(a*c + n)*(2*f**(b*c*x)*e**(d*n)*cosh((log(f)*b*c*x - d*n - 2*d)/(n +
2))**n*2**n*n + 2*f**(b*c*x)*e**(d*n)*cosh((log(f)*b*c*x - d*n - 2*d)/(n +
 2))**n*2**n - e**((2*log(f)*b*c*x)/(n + 2))*(e**((2*log(f)*b*c*x)/(n + 2)
) + e**(2*d))**n*n + e**(2*d)*(e**((2*log(f)*b*c*x)/(n + 2)) + e**(2*d))**
n*n + 2*e**(2*d)*(e**((2*log(f)*b*c*x)/(n + 2)) + e**(2*d))**n))/(2*e**(d*
n)*2**n*log(f)*b*c*(n + 1))
```

3.42 $\int e^{a+bx} \tanh(d+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 32

$$\int e^{a+bx}\tanh(d+bx)\,dx=\frac{e^{a+bx}}{b}-\frac{2e^{a-d}\arctan\left(e^{d+bx}\right)}{b}$$

output
```
exp(b*x+a)/b-2*exp(a-d)*arctan(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.41

$$\int e^{a+bx}\tanh(d+bx)\,dx =\frac{e^a\left(e^{bx}-2\arctan\left(e^{bx}(\cosh(d)+\sinh(d))\right)\cosh(d)+2\arctan\left(e^{bx}(\cosh(d)+\sinh(d))\right)\sinh(d)\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Tanh[d + b*x],x]
```

output
```
(E^a*(E^(b*x) - 2*ArcTan[E^(b*x)*(Cosh[d] + Sinh[d])]*Cosh[d] + 2*ArcTan[E
^(b*x)*(Cosh[d] + Sinh[d])]*Sinh[d]))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {2720, 25, 27, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\tanh(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{e^a(1-e^{2bx})}{1+e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{e^a(1-e^{2bx})}{1+e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^a\int \frac{1-e^{2bx}}{1+e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{299}$$

$$-\frac{e^a\left(2\int \frac{1}{1+e^{2bx}}de^{bx}-e^{bx}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$-\frac{e^a\left(2\arctan\left(e^{bx}\right)-e^{bx}\right)}{b}$$

input
```
Int[E^(a + b*x)*Tanh[d + b*x],x]
```

output
```
-((E^a*(-E^(b*x) + 2*ArcTan[E^(b*x)]))/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.19

method	result	size
risch	$\frac{\mathrm{e}^{b x+a}}{b}+\frac{i \ln\left(\mathrm{e}^{b x+a}-i \mathrm{e}^{a-d}\right) \mathrm{e}^{a-d}}{b}-\frac{i \ln\left(\mathrm{e}^{b x+a}+i \mathrm{e}^{a-d}\right) \mathrm{e}^{a-d}}{b}$	70

input
```
int(exp(b*x+a)*tanh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
exp(b*x+a)/b+I*ln(exp(b*x+a)-I*exp(a-d))/b*exp(a-d)-I*ln(exp(b*x+a)+I*exp(
a-d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(29) = 58$.

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.72

$$\int e^{a+bx}\tanh(d+bx)\,dx = -\frac{2\left(\cosh\left(-a+d\right)-\sinh\left(-a+d\right)\right)\arctan\left(\cosh\left(bx+d\right)+\sinh\left(bx+d\right)\right)-\cosh\left(bx+d\right)\cosh\left(-a+d\right)}{b}$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d),x, algorithm="fricas")
```

output
```
-(2*(cosh(-a + d) - sinh(-a + d))*arctan(cosh(b*x + d) + sinh(b*x + d)) -
cosh(b*x + d)*cosh(-a + d) - (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d) +
 cosh(b*x + d)*sinh(-a + d))/b
```

Sympy [F]

$$\int e^{a+bx}\tanh(d+bx)\,dx = e^{a}\int e^{bx}\tanh\left(bx+d\right)dx$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d),x)
```

output
```
exp(a)*Integral(exp(b*x)*tanh(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int e^{a+bx}\tanh(d+bx)\,dx = -\frac{2\,\arctan\left(e^{(bx+d)}\right)e^{(a-d)}}{b} + \frac{e^{(bx+a)}}{b}$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d),x, algorithm="maxima")
```

output
```
-2*arctan(e^(b*x + d))*e^(a - d)/b + e^(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int e^{a+bx}\tanh(d+bx)\,dx = -\frac{2\,\arctan\left(e^{(bx+d)}\right)e^{(a-d)} - e^{(bx+a)}}{b}$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d),x, algorithm="giac")
```

output
```
-(2*arctan(e^(b*x + d))*e^(a - d) - e^(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 2.78 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.72

$$\int e^{a+bx}\tanh(d+bx)\,dx = \frac{\mathrm{e}^{a+b\,x}}{b} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b\,\sqrt{\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}}}\right)\,\sqrt{\mathrm{e}^{2\,a-2\,d}}}{\sqrt{b^2}}$$

input
```
int(exp(a + b*x)*tanh(d + b*x),x)
```

output
```
exp(a + b*x)/b - (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/(b*(exp(2*a)*exp(-2
*d))^(1/2)))*exp(2*a - 2*d)^(1/2))/(b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int e^{a+bx}\tanh(d+bx)\,dx = \frac{e^{a}\left(-2atan\left(e^{bx+d}\right)+e^{bx+d}\right)}{e^{d}b}$$

input
```
int(exp(b*x+a)*tanh(b*x+d),x)
```

output
```
(e**a*( - 2*atan(e**(b*x + d)) + e**(b*x + d)))/(e**d*b)
```

3.43 $\int e^{a+bx}\tanh^2(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 58

$$\int e^{a+bx}\tanh^2(d+bx)\,dx=\frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1+e^{2d+2bx}\right)}-\frac{2e^{a-d}\arctan\left(e^{d+bx}\right)}{b}$$

output
```
exp(b*x+a)/b+2*exp(b*x+a)/b/(1+exp(2*b*x+2*d))-2*exp(a-d)*arctan(exp(b*x+d
))/b
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.45

$$\int e^{a+bx}\tanh^2(d+bx)\,dx$$
$$=\frac{e^a\left(e^{bx}-2\arctan\left(e^{bx}(\cosh(d)+\sinh(d))\right)\cosh(d)+2\arctan\left(e^{bx}(\cosh(d)+\sinh(d))\right)\sinh(d)+\frac{2}{(1+e^{2bx}}\right.}{b}$$

input
```
Integrate[E^(a + b*x)*Tanh[d + b*x]^2,x]
```

output
```
(E^a*(E^(b*x) - 2*ArcTan[E^(b*x)*(Cosh[d] + Sinh[d])]*Cosh[d] + 2*ArcTan[E
^(b*x)*(Cosh[d] + Sinh[d])]*Sinh[d] + (2*E^(b*x)*(Cosh[d] - Sinh[d]))/((1
+ E^(2*b*x))*Cosh[d] + (-1 + E^(2*b*x))*Sinh[d])))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 27, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\tanh^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{e^a\left(1-e^{2bx}\right)^2}{\left(1+e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^a\int \frac{\left(1-e^{2bx}\right)^2}{\left(1+e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{300}$$

$$\frac{e^a\int \left(1-\frac{4e^{2bx}}{\left(1+e^{2bx}\right)^2}\right)de^{bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^a\left(-2\arctan\left(e^{bx}\right)+e^{bx}+\frac{2e^{bx}}{e^{2bx}+1}\right)}{b}$$

input
```
Int[E^(a + b*x)*Tanh[d + b*x]^2,x]
```

output
```
(E^a*(E^(b*x) + (2*E^(b*x))/(1 + E^(2*b*x)) - 2*ArcTan[E^(b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.76

method	result	size
risch	$\frac{{\mathrm e}^{b x +a}}{b}+\frac{2 \,{\mathrm e}^{b x +3 a}}{\left({\mathrm e}^{2 b x +2 a +2 d}+{\mathrm e}^{2 a}\right) b}+\frac{i \ln \left({\mathrm e}^{b x +a}-i {\mathrm e}^{a -d}\right) {\mathrm e}^{a -d}}{b}-\frac{i \ln \left({\mathrm e}^{b x +a}+i {\mathrm e}^{a -d}\right) {\mathrm e}^{a -d}}{b}$	102

input
```
int(exp(b*x+a)*tanh(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
exp(b*x+a)/b+2/(exp(2*b*x+2*a+2*d)+exp(2*a))/b*exp(b*x+3*a)+I*ln(exp(b*x+a
)-I*exp(a-d))/b*exp(a-d)-I*ln(exp(b*x+a)+I*exp(a-d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(53) = 106$.

Time = 0.10 (sec) , antiderivative size = 321, normalized size of antiderivative = 5.53

$$\int e^{a+bx}\tanh^2(d+bx)\,dx = \frac{\cosh(bx+d)^3\cosh(-a+d) + (\cosh(-a+d)-\sinh(-a+d))\sinh(bx+d)^3 + 3(\cosh(bx+d)\cosh(-\dots}{}$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d)^2,x, algorithm="fricas")
```

output
```
(cosh(b*x + d)^3*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d
)^3 + 3*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x
 + d)^2 - 2*(cosh(b*x + d)^2*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*
sinh(b*x + d)^2 + 2*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a +
d))*sinh(b*x + d) - (cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*arc
tan(cosh(b*x + d) + sinh(b*x + d)) + 3*cosh(b*x + d)*cosh(-a + d) + 3*(cos
h(b*x + d)^2*cosh(-a + d) - (cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a +
 d))*sinh(b*x + d) - (cosh(b*x + d)^3 + 3*cosh(b*x + d))*sinh(-a + d))/(b*
cosh(b*x + d)^2 + 2*b*cosh(b*x + d)*sinh(b*x + d) + b*sinh(b*x + d)^2 + b)
```

Sympy [F]

$$\int e^{a+bx}\tanh^2(d+bx)\,dx = e^a\int e^{bx}\tanh^2(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d)**2,x)
```

output
```
exp(a)*Integral(exp(b*x)*tanh(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

$$\int e^{a+bx}\tanh^2(d+bx)\,dx = -\frac{2\arctan\left(e^{(bx+d)}\right)e^{(a-d)}}{b} + \frac{e^{(bx+a)}}{b} + \frac{2\,e^{(bx+3\,a)}}{b\left(e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right)}$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d)^2,x, algorithm="maxima")
```

output
```
-2*arctan(e^(b*x + d))*e^(a - d)/b + e^(b*x + a)/b + 2*e^(b*x + 3*a)/(b*(e
^(2*b*x + 2*a + 2*d) + e^(2*a)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\tanh^2(d+bx)\,dx = -\frac{2\arctan\left(e^{(bx+d)}\right)e^{(a-d)} - \frac{2\,e^{(bx+3\,a)}}{e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}} - e^{(bx+a)}}{b}$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d)^2,x, algorithm="giac")
```

output
```
-(2*arctan(e^(b*x + d))*e^(a - d) - 2*e^(b*x + 3*a)/(e^(2*b*x + 2*a + 2*d)
 + e^(2*a)) - e^(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.62

$$\int e^{a+bx}\tanh^2(d+bx)\,dx = \frac{e^{b\,x}\,e^{a}}{b} - \frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b\,\sqrt{e^{2\,a}\,e^{-2\,d}}}\right)\sqrt{e^{2\,a}\,e^{-2\,d}}}{\sqrt{b^2}} + \frac{2\,e^{3\,a}\,e^{-2\,d}\,e^{b\,x}}{b\,e^{2\,a}\,e^{-2\,d}+b\,e^{2\,a}\,e^{2\,b\,x}}$$

input
```
int(exp(a + b*x)*tanh(d + b*x)^2,x)
```

output
```
(exp(b*x)*exp(a))/b - (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/(b*(exp(2*a)*e
xp(-2*d))^(1/2)))*(exp(2*a)*exp(-2*d))^(1/2))/(b^2)^(1/2) + (2*exp(3*a)*ex
p(-2*d)*exp(b*x))/(b*exp(2*a)*exp(-2*d) + b*exp(2*a)*exp(2*b*x))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.31

$$\int e^{a+bx}\tanh^2(d+bx)\,dx = \frac{e^a\left(-2e^{2bx+2d}atan\left(e^{bx+d}\right)-2atan\left(e^{bx+d}\right)+e^{3bx+3d}+3e^{bx+d}\right)}{e^d b\left(e^{2bx+2d}+1\right)}$$

input
```
int(exp(b*x+a)*tanh(b*x+d)^2,x)
```

output
```
(e**a*( - 2*e**(2*b*x + 2*d)*atan(e**(b*x + d)) - 2*atan(e**(b*x + d)) + e
**(3*b*x + 3*d) + 3*e**(b*x + d)))/(e**d*b*(e**(2*b*x + 2*d) + 1))
```

3.44 $\int e^{a+bx}\tanh^3(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 84

$$\int e^{a+bx}\tanh^3(d+bx)\,dx = \frac{e^{a+bx}}{b} - \frac{2e^{a+bx}}{b\left(1+e^{2d+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1+e^{2d+2bx}\right)} - \frac{3e^{a-d}\arctan\left(e^{d+bx}\right)}{b}$$

output
```
exp(b*x+a)/b-2*exp(b*x+a)/b/(1+exp(2*b*x+2*d))^2+3*exp(b*x+a)/b/(1+exp(2*b
*x+2*d))-3*exp(a-d)*arctan(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.49

$$\int e^{a+bx}\tanh^3(d+bx)\,dx = \frac{e^a\left(e^{bx} - 3\arctan\left(e^{bx}(\cosh(d)+\sinh(d))\right)\cosh(d) + 3\arctan\left(e^{bx}(\cosh(d)+\sinh(d))\right)\sinh(d) - \frac{\cdots}{((1+e^{2b}\cdots}\right.}{b}$$

input
```
Integrate[E^(a + b*x)*Tanh[d + b*x]^3,x]
```

output
```
(E^a*(E^(b*x) - 3*ArcTan[E^(b*x)*(Cosh[d] + Sinh[d])]*Cosh[d] + 3*ArcTan[E
^(b*x)*(Cosh[d] + Sinh[d])]*Sinh[d] - (2*E^(b*x)*(Cosh[d] - Sinh[d])^2)/((
1 + E^(2*b*x))*Cosh[d] + (-1 + E^(2*b*x))*Sinh[d])^2 + (3*E^(b*x)*(Cosh[d]
 - Sinh[d]))/((1 + E^(2*b*x))*Cosh[d] + (-1 + E^(2*b*x))*Sinh[d])))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2720, 25, 27, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\tanh^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{e^a\left(1-e^{2bx}\right)^3}{\left(1+e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{e^a\left(1-e^{2bx}\right)^3}{\left(1+e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^a\int \frac{\left(1-e^{2bx}\right)^3}{\left(1+e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{300}$$

$$-\frac{e^a\int \left(\frac{2\left(1+3e^{4bx}\right)}{\left(1+e^{2bx}\right)^3}-1\right)de^{bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^a\left(3\arctan\left(e^{bx}\right)-e^{bx}-\frac{3e^{bx}}{e^{2bx}+1}+\frac{2e^{bx}}{\left(e^{2bx}+1\right)^2}\right)}{b}$$

input
```
Int[E^(a + b*x)*Tanh[d + b*x]^3,x]
```

output
```
-((E^a*(-E^(b*x) + (2*E^(b*x))/(1 + E^(2*b*x))^2 - (3*E^(b*x))/(1 + E^(2*b
*x)) + 3*ArcTan[E^(b*x)]))/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.43

method	result	size
risch	$\frac{{\mathrm e}^{b x +a}}{b}+\frac{\left(3 \,{\mathrm e}^{2 b x +2 a +2 d}+{\mathrm e}^{2 a}\right) {\mathrm e}^{b x +3 a}}{\left({\mathrm e}^{2 b x +2 a +2 d}+{\mathrm e}^{2 a}\right)^{2} b}+\frac{3 i \ln \left({\mathrm e}^{b x +a}-i {\mathrm e}^{a -d}\right) {\mathrm e}^{a -d}}{2 b}-\frac{3 i \ln \left({\mathrm e}^{b x +a}+i {\mathrm e}^{a -d}\right) {\mathrm e}^{a -d}}{2 b}$	120

input
```
int(exp(b*x+a)*tanh(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
exp(b*x+a)/b+1/(exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(3*exp(2*b*x+2*a+2*d)+exp
(2*a))*exp(b*x+3*a)+3/2*I*ln(exp(b*x+a)-I*exp(a-d))/b*exp(a-d)-3/2*I*ln(ex
p(b*x+a)+I*exp(a-d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 702 vs. $2(77) = 154$.

Time = 0.10 (sec) , antiderivative size = 702, normalized size of antiderivative = 8.36

$$\int e^{a+bx}\tanh^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d)^3,x, algorithm="fricas")
```

output
```
(cosh(b*x + d)^5*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d
)^5 + 5*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x
 + d)^4 + 5*cosh(b*x + d)^3*cosh(-a + d) + 5*(2*cosh(b*x + d)^2*cosh(-a +
d) - (2*cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*sinh(b*x + d)^3
+ 5*(2*cosh(b*x + d)^3*cosh(-a + d) + 3*cosh(b*x + d)*cosh(-a + d) - (2*co
sh(b*x + d)^3 + 3*cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d)^2 - 3*(cosh(b
*x + d)^4*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d)^4 + 4
*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d)^3
 + 2*cosh(b*x + d)^2*cosh(-a + d) + 2*(3*cosh(b*x + d)^2*cosh(-a + d) - (3
*cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*sinh(b*x + d)^2 + 4*(co
sh(b*x + d)^3*cosh(-a + d) + cosh(b*x + d)*cosh(-a + d) - (cosh(b*x + d)^3
 + cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^4 + 2*cosh(
b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*arctan(cosh(b*x + d) + sinh(b
*x + d)) + 2*cosh(b*x + d)*cosh(-a + d) + (5*cosh(b*x + d)^4*cosh(-a + d)
+ 15*cosh(b*x + d)^2*cosh(-a + d) - (5*cosh(b*x + d)^4 + 15*cosh(b*x + d)^
2 + 2)*sinh(-a + d) + 2*cosh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^5 + 5
*cosh(b*x + d)^3 + 2*cosh(b*x + d))*sinh(-a + d))/(b*cosh(b*x + d)^4 + 4*b
*cosh(b*x + d)*sinh(b*x + d)^3 + b*sinh(b*x + d)^4 + 2*b*cosh(b*x + d)^2 +
 2*(3*b*cosh(b*x + d)^2 + b)*sinh(b*x + d)^2 + 4*(b*cosh(b*x + d)^3 + b*co
sh(b*x + d))*sinh(b*x + d) + b)
```

Sympy [F]

$$\int e^{a+bx} \tanh^3(d+bx)\,dx = e^a \int e^{bx} \tanh^3(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d)**3,x)
```

output
```
exp(a)*Integral(exp(b*x)*tanh(b*x + d)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.06

$$\int e^{a+bx} \tanh^3(d+bx)\,dx = -\frac{3 \arctan\left(e^{(bx+d)}\right) e^{(a-d)}}{b} + \frac{e^{(bx+a)}}{b} + \frac{3 e^{(3 bx+5 a+2 d)} + e^{(bx+5 a)}}{b(e^{(4 bx+4 a+4 d)} + 2 e^{(2 bx+4 a+2 d)} + e^{(4 a)})}$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d)^3,x, algorithm="maxima")
```

output
```
-3*arctan(e^(b*x + d))*e^(a - d)/b + e^(b*x + a)/b + (3*e^(3*b*x + 5*a + 2
*d) + e^(b*x + 5*a))/(b*(e^(4*b*x + 4*a + 4*d) + 2*e^(2*b*x + 4*a + 2*d) +
 e^(4*a)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int e^{a+bx} \tanh^3(d+bx)\,dx = -\frac{3 \arctan\left(e^{(bx+d)}\right) e^{(a-d)} - \frac{3 e^{(3 bx+5 a+2 d)}+e^{(bx+5 a)}}{\left(e^{(2 bx+2 a+2 d)}+e^{(2 a)}\right)^2} - e^{(bx+a)}}{b}$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d)^3,x, algorithm="giac")
```

output
```
-(3*arctan(e^(b*x + d))*e^(a - d) - (3*e^(3*b*x + 5*a + 2*d) + e^(b*x + 5*
a))/(e^(2*b*x + 2*a + 2*d) + e^(2*a))^2 - e^(b*x + a))/b
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\tanh^3(d+bx)\,dx=\int e^{a+b\,x}\,\tanh(d+b\,x)^3\,dx$$

input
```
int(exp(a + b*x)*tanh(d + b*x)^3,x)
```

output
```
int(exp(a + b*x)*tanh(d + b*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.43

$$\int e^{a+bx}\tanh^3(d+bx)\,dx$$
$$=\frac{e^a\left(-3e^{4bx+4d}atan\left(e^{bx+d}\right)-6e^{2bx+2d}atan\left(e^{bx+d}\right)-3atan\left(e^{bx+d}\right)+e^{5bx+5d}+5e^{3bx+3d}+2e^{bx+d}\right)}{e^d b\left(e^{4bx+4d}+2e^{2bx+2d}+1\right)}$$

input
```
int(exp(b*x+a)*tanh(b*x+d)^3,x)
```

output
```
(e**a*( - 3*e**(4*b*x + 4*d)*atan(e**(b*x + d)) - 6*e**(2*b*x + 2*d)*atan(
e**(b*x + d)) - 3*atan(e**(b*x + d)) + e**(5*b*x + 5*d) + 5*e**(3*b*x + 3*
d) + 2*e**(b*x + d)))/(e**d*b*(e**(4*b*x + 4*d) + 2*e**(2*b*x + 2*d) + 1))
```

3.45 $\int e^{a+bx}\tanh^4(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 114

$$\int e^{a+bx}\tanh^4(d+bx)\,dx = \frac{e^{a+bx}}{b}+\frac{8e^{a+bx}}{3b\left(1+e^{2d+2bx}\right)^3}-\frac{14e^{a+bx}}{3b\left(1+e^{2d+2bx}\right)^2}+\frac{5e^{a+bx}}{b\left(1+e^{2d+2bx}\right)}-\frac{3e^{a-d}\arctan\left(e^{d+bx}\right)}{b}$$

output
```
exp(b*x+a)/b+8/3*exp(b*x+a)/b/(1+exp(2*b*x+2*d))^3-14/3*exp(b*x+a)/b/(1+ex
p(2*b*x+2*d))^2+5*exp(b*x+a)/b/(1+exp(2*b*x+2*d))-3*exp(a-d)*arctan(exp(b*
x+d))/b
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.50

$$\int e^{a+bx}\tanh^4(d+bx)\,dx = \frac{e^a\left(3e^{bx}-9\arctan\left(e^{bx}(\cosh(d)+\sinh(d))\right)\cosh(d)+9\arctan\left(e^{bx}(\cosh(d)+\sinh(d))\right)\sinh(d)+\frac{\cdots}{((1+e^{\cdots}}\right.}{3b}$$

input
```
Integrate[E^(a + b*x)*Tanh[d + b*x]^4,x]
```

output
```
(E^a*(3*E^(b*x) - 9*ArcTan[E^(b*x)*(Cosh[d] + Sinh[d])]*Cosh[d] + 9*ArcTan
[E^(b*x)*(Cosh[d] + Sinh[d])]*Sinh[d] + (8*E^(b*x)*(Cosh[d] - Sinh[d])^3)/
((1 + E^(2*b*x))*Cosh[d] + (-1 + E^(2*b*x))*Sinh[d])^3 - (14*E^(b*x)*(Cosh
[d] - Sinh[d])^2)/((1 + E^(2*b*x))*Cosh[d] + (-1 + E^(2*b*x))*Sinh[d])^2 +
 (15*E^(b*x)*(Cosh[d] - Sinh[d]))/((1 + E^(2*b*x))*Cosh[d] + (-1 + E^(2*b*
x))*Sinh[d])))/(3*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 27, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx} \tanh^4(bx+d) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{e^a (1-e^{2bx})^4}{(1+e^{2bx})^4} de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^a \int \frac{(1-e^{2bx})^4}{(1+e^{2bx})^4} de^{bx}}{b}$$

$$\Big\downarrow \text{300}$$

$$\frac{e^a \int \left(1-\frac{8e^{2bx}(1+e^{4bx})}{(1+e^{2bx})^4}\right) de^{bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^a \left(-3\arctan\left(e^{bx}\right)+e^{bx}+\frac{5e^{bx}}{e^{2bx}+1}-\frac{14e^{bx}}{3(e^{2bx}+1)^2}+\frac{8e^{bx}}{3(e^{2bx}+1)^3}\right)}{b}$$

input
```
Int[E^(a + b*x)*Tanh[d + b*x]^4,x]
```

output
```
(E^a*(E^(b*x) + (8*E^(b*x))/(3*(1 + E^(2*b*x))^3) - (14*E^(b*x))/(3*(1 + E
^(2*b*x))^2) + (5*E^(b*x))/(1 + E^(2*b*x)) - 3*ArcTan[E^(b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.20

method	result	size
risch	$\frac{{\mathrm e}^{b x +a}}{b}+\frac{\left(15 \,{\mathrm e}^{4 b x +4 a +4 d}+16 \,{\mathrm e}^{2 b x +4 a +2 d}+9 \,{\mathrm e}^{4 a}\right) {\mathrm e}^{b x +3 a}}{3 \left({\mathrm e}^{2 b x +2 a +2 d}+{\mathrm e}^{2 a}\right)^{3} b}+\frac{3 i \ln \left({\mathrm e}^{b x +a}-i {\mathrm e}^{a -d}\right) {\mathrm e}^{a -d}}{2 b}-\frac{3 i \ln \left({\mathrm e}^{b x +a}+i {\mathrm e}^{a -d}\right) {\mathrm e}^{a -d}}{2 b}$	137

input
```
int(exp(b*x+a)*tanh(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
exp(b*x+a)/b+1/3/(exp(2*b*x+2*a+2*d)+exp(2*a))^3/b*(15*exp(4*b*x+4*a+4*d)+
16*exp(2*b*x+4*a+2*d)+9*exp(4*a))*exp(b*x+3*a)+3/2*I*ln(exp(b*x+a)-I*exp(a
-d))/b*exp(a-d)-3/2*I*ln(exp(b*x+a)+I*exp(a-d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1223 vs. $2(101) = 202$.

Time = 0.10 (sec) , antiderivative size = 1223, normalized size of antiderivative = 10.73

$$\int e^{a+bx}\tanh^4(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d)^4,x, algorithm="fricas")
```

output
```
1/3*(3*cosh(b*x + d)^7*cosh(-a + d) + 3*(cosh(-a + d) - sinh(-a + d))*sinh
(b*x + d)^7 + 21*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))
*sinh(b*x + d)^6 + 24*cosh(b*x + d)^5*cosh(-a + d) + 3*(21*cosh(b*x + d)^2
*cosh(-a + d) - (21*cosh(b*x + d)^2 + 8)*sinh(-a + d) + 8*cosh(-a + d))*si
nh(b*x + d)^5 + 15*(7*cosh(b*x + d)^3*cosh(-a + d) + 8*cosh(b*x + d)*cosh(
-a + d) - (7*cosh(b*x + d)^3 + 8*cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d
)^4 + 25*cosh(b*x + d)^3*cosh(-a + d) + 5*(21*cosh(b*x + d)^4*cosh(-a + d)
 + 48*cosh(b*x + d)^2*cosh(-a + d) - (21*cosh(b*x + d)^4 + 48*cosh(b*x + d
)^2 + 5)*sinh(-a + d) + 5*cosh(-a + d))*sinh(b*x + d)^3 + 3*(21*cosh(b*x +
 d)^5*cosh(-a + d) + 80*cosh(b*x + d)^3*cosh(-a + d) + 25*cosh(b*x + d)*co
sh(-a + d) - (21*cosh(b*x + d)^5 + 80*cosh(b*x + d)^3 + 25*cosh(b*x + d))*
sinh(-a + d))*sinh(b*x + d)^2 - 9*(cosh(b*x + d)^6*cosh(-a + d) + (cosh(-a
 + d) - sinh(-a + d))*sinh(b*x + d)^6 + 6*(cosh(b*x + d)*cosh(-a + d) - co
sh(b*x + d)*sinh(-a + d))*sinh(b*x + d)^5 + 3*cosh(b*x + d)^4*cosh(-a + d)
 + 3*(5*cosh(b*x + d)^2*cosh(-a + d) - (5*cosh(b*x + d)^2 + 1)*sinh(-a + d
) + cosh(-a + d))*sinh(b*x + d)^4 + 4*(5*cosh(b*x + d)^3*cosh(-a + d) + 3*
cosh(b*x + d)*cosh(-a + d) - (5*cosh(b*x + d)^3 + 3*cosh(b*x + d))*sinh(-a
 + d))*sinh(b*x + d)^3 + 3*cosh(b*x + d)^2*cosh(-a + d) + 3*(5*cosh(b*x +
d)^4*cosh(-a + d) + 6*cosh(b*x + d)^2*cosh(-a + d) - (5*cosh(b*x + d)^4 +
6*cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*sinh(b*x + d)^2 + 6...
```

Sympy [F]

$$\int e^{a+bx}\tanh^4(d+bx)\,dx = e^a\int e^{bx}\tanh^4(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d)**4,x)
```

output
```
exp(a)*Integral(exp(b*x)*tanh(b*x + d)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.05

$$\int e^{a+bx}\tanh^4(d+bx)\,dx = -\frac{3\arctan\left(e^{(bx+d)}\right)e^{(a-d)}}{b}+\frac{e^{(bx+a)}}{b}+\frac{15\,e^{(5\,bx+7\,a+4\,d)}+16\,e^{(3\,bx+7\,a+2\,d)}+9\,e^{(bx+7\,a)}}{3\,b\left(e^{(6\,bx+6\,a+6\,d)}+3\,e^{(4\,bx+6\,a+4\,d)}+3\,e^{(2\,bx+6\,a+2\,d)}+e^{(6\,a)}\right)}$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d)^4,x, algorithm="maxima")
```

output
```
-3*arctan(e^(b*x + d))*e^(a - d)/b + e^(b*x + a)/b + 1/3*(15*e^(5*b*x + 7*
a + 4*d) + 16*e^(3*b*x + 7*a + 2*d) + 9*e^(b*x + 7*a))/(b*(e^(6*b*x + 6*a
+ 6*d) + 3*e^(4*b*x + 6*a + 4*d) + 3*e^(2*b*x + 6*a + 2*d) + e^(6*a)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.78

$$\int e^{a+bx}\tanh^4(d+bx)\,dx = -\frac{9\arctan\left(e^{(bx+d)}\right)e^{(a-d)}-\frac{15\,e^{(5\,bx+7\,a+4\,d)}+16\,e^{(3\,bx+7\,a+2\,d)}+9\,e^{(bx+7\,a)}}{\left(e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right)^3}-3\,e^{(bx+a)}}{3\,b}$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d)^4,x, algorithm="giac")
```

output
```
-1/3*(9*arctan(e^(b*x + d))*e^(a - d) - (15*e^(5*b*x + 7*a + 4*d) + 16*e^(
3*b*x + 7*a + 2*d) + 9*e^(b*x + 7*a))/(e^(2*b*x + 2*a + 2*d) + e^(2*a))^3
- 3*e^(b*x + a))/b
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\tanh^4(d+bx)\,dx = \int e^{a+b\,x}\tanh(d+b\,x)^4\,dx$$

input
```
int(exp(a + b*x)*tanh(d + b*x)^4,x)
```

output
```
int(exp(a + b*x)*tanh(d + b*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.46

$$\int e^{a+bx}\tanh^4(d+bx)\,dx$$
$$= \frac{e^{a}\left(-9e^{6bx+6d}atan\left(e^{bx+d}\right)-27e^{4bx+4d}atan\left(e^{bx+d}\right)-27e^{2bx+2d}atan\left(e^{bx+d}\right)-9atan\left(e^{bx+d}\right)+3e^{7bx+7d}+2\right.}{3e^{d}b\left(e^{6bx+6d}+3e^{4bx+4d}+3e^{2bx+2d}+1\right)}$$

input
```
int(exp(b*x+a)*tanh(b*x+d)^4,x)
```

output
```
(e**a*( - 9*e**(6*b*x + 6*d)*atan(e**(b*x + d)) - 27*e**(4*b*x + 4*d)*atan
(e**(b*x + d)) - 27*e**(2*b*x + 2*d)*atan(e**(b*x + d)) - 9*atan(e**(b*x +
 d)) + 3*e**(7*b*x + 7*d) + 24*e**(5*b*x + 5*d) + 25*e**(3*b*x + 3*d) + 12
*e**(b*x + d)))/(3*e**d*b*(e**(6*b*x + 6*d) + 3*e**(4*b*x + 4*d) + 3*e**(2
*b*x + 2*d) + 1))
```

3.46 $\int e^{2(a+bx)} \tanh(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int e^{2(a+bx)} \tanh(d+bx)\,dx = \frac{e^{2a+2bx}}{2b} - \frac{e^{2a-2d}\log\left(1+e^{2d+2bx}\right)}{b}$$

output
```
1/2*exp(2*b*x+2*a)/b-exp(2*a-2*d)*ln(1+exp(2*b*x+2*d))/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 92 vs. 2(45) = 90.

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.04

$$\int e^{2(a+bx)} \tanh(d+bx)\,dx$$
$$= \frac{e^{2a}(\cosh(d)-\sinh(d))\left(\cosh(d)\left(e^{2bx}-2\log\left(\left(1+e^{2bx}\right)\cosh(d)+\left(-1+e^{2bx}\right)\sinh(d)\right)\right)+\left(e^{2bx}+2\log\right.\right.}{2b}$$

input
```
Integrate[E^(2*(a + b*x))*Tanh[d + b*x],x]
```

output
```
(E^(2*a)*(Cosh[d] - Sinh[d])*(Cosh[d]*(E^(2*b*x) - 2*Log[(1 + E^(2*b*x))*C
osh[d] + (-1 + E^(2*b*x))*Sinh[d]]) + (E^(2*b*x) + 2*Log[(1 + E^(2*b*x))*C
osh[d] + (-1 + E^(2*b*x))*Sinh[d]])*Sinh[d]))/(2*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2720, 25, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\tanh(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{e^{2a+bx}(1-e^{2bx})}{1+e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{e^{2a+bx}(1-e^{2bx})}{1+e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^{2a}\int \frac{e^{bx}(1-e^{2bx})}{1+e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{353}$$

$$-\frac{e^{2a}\int \frac{1-e^{2bx}}{1+e^{2bx}}de^{2bx}}{2b}$$

$$\Big\downarrow \text{49}$$

$$-\frac{e^{2a}\int \left(\frac{2}{1+e^{2bx}}-1\right)de^{2bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{2a}\left(2\log\left(e^{2bx}+1\right)-e^{2bx}\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Tanh[d + b*x],x]
```

output
```
-1/2*(E^(2*a)*(-E^(2*b*x) + 2*Log[1 + E^(2*b*x)]))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.38

method	result	size
risch	$\frac{e^{2bx+2a}}{2b}+\frac{2\,e^{2a-2d}a}{b}-\frac{\ln\left(e^{2bx+2a}+e^{2a-2d}\right)e^{2a-2d}}{b}$	62

input
```
int(exp(2*b*x+2*a)*tanh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*b*x+2*a)/b+2/b*exp(2*a-2*d)*a-ln(exp(2*b*x+2*a)+exp(2*a-2*d))/b*
exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(40) = 80.

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.47

$$\int e^{2(a+bx)}\tanh(d+bx)\,dx$$
$$=\frac{\cosh\left(bx+d\right)^2\cosh\left(-2\,a+2\,d\right)+\left(\cosh\left(-2\,a+2\,d\right)-\sinh\left(-2\,a+2\,d\right)\right)\sinh\left(bx+d\right)^2-\cosh\left(bx+d\right)}{}$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d),x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + d)^2*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*
d))*sinh(b*x + d)^2 - cosh(b*x + d)^2*sinh(-2*a + 2*d) - 2*(cosh(-2*a + 2*
d) - sinh(-2*a + 2*d))*log(2*cosh(b*x + d)/(cosh(b*x + d) - sinh(b*x + d))
) + 2*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*si
nh(b*x + d))/b
```

Sympy [F]

$$\int e^{2(a+bx)}\tanh(d+bx)\,dx = e^{2a}\int e^{2bx}\tanh(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d),x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*tanh(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int e^{2(a+bx)}\tanh(d+bx)\,dx = -\frac{2\,(bx+d)e^{(2\,a-2\,d)}}{b} - \frac{e^{(2\,a-2\,d)}\log\left(e^{(-2\,bx-2\,d)}+1\right)}{b} + \frac{e^{(2\,bx+2\,a)}}{2\,b}$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d),x, algorithm="maxima")
```

output
```
-2*(b*x + d)*e^(2*a - 2*d)/b - e^(2*a - 2*d)*log(e^(-2*b*x - 2*d) + 1)/b +
 1/2*e^(2*b*x + 2*a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int e^{2(a+bx)}\tanh(d+bx)\,dx = -\frac{e^{(2\,a-2\,d)}\log\left(e^{(2\,bx+2\,d)}+1\right)}{b} + \frac{e^{(2\,bx+2\,a)}}{2\,b}$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d),x, algorithm="giac")
```

output
```
-e^(2*a - 2*d)*log(e^(2*b*x + 2*d) + 1)/b + 1/2*e^(2*b*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.82 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int e^{2(a+bx)}\tanh(d+bx)\,dx = \frac{e^{2\,a}\,e^{2\,b\,x}}{2\,b} - \frac{e^{2\,a}\,e^{-2\,d}\,\ln\left(e^{2\,a}\,e^{2\,b\,x}+e^{2\,a}\,e^{-2\,d}\right)}{b}$$

input
```
int(exp(2*a + 2*b*x)*tanh(d + b*x),x)
```

output
```
(exp(2*a)*exp(2*b*x))/(2*b) - (exp(2*a)*exp(-2*d)*log(exp(2*a)*exp(2*b*x)
+ exp(2*a)*exp(-2*d)))/b
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int e^{2(a+bx)}\tanh(d+bx)\,dx = \frac{e^{2a}\left(e^{2bx+2d}-2\log\left(e^{2bx+2d}+1\right)\right)}{2e^{2d}b}$$

input
```
int(exp(2*b*x+2*a)*tanh(b*x+d),x)
```

output
```
(e**(2*a)*(e**(2*b*x + 2*d) - 2*log(e**(2*b*x + 2*d) + 1)))/(2*e**(2*d)*b)
```

3.47 $\int e^{2(a+bx)} \tanh^2(d+bx) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 73

$$\int e^{2(a+bx)} \tanh^2(d+bx) \, dx = \frac{e^{2a+2bx}}{2b} - \frac{2e^{2a-2d}}{b(1+e^{2d+2bx})} - \frac{2e^{2a-2d} \log(1+e^{2d+2bx})}{b}$$

output
```
1/2*exp(2*b*x+2*a)/b-2*exp(2*a-2*d)/b/(1+exp(2*b*x+2*d))-2*exp(2*a-2*d)*ln
(1+exp(2*b*x+2*d))/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 158 vs. 2(73) = 146.

Time = 0.32 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.16

$$\int e^{2(a+bx)} \tanh^2(d+bx) \, dx$$
$$= \frac{e^{2a}(\cosh(d)-\sinh(d)) \left(e^{2bx}\left(1-4\log\left(\left(1+e^{2bx}\right)\cosh(d)+\left(-1+e^{2bx}\right)\sinh(d)\right)\right)+\cosh(2d)\left(-4+e^{4b}\right.\right.}{2b\left(\left(1+e^{2\ldots}\right.\right.}$$

input
```
Integrate[E^(2*(a + b*x))*Tanh[d + b*x]^2,x]
```

output
```
(E^(2*a)*(Cosh[d] - Sinh[d])*(E^(2*b*x)*(1 - 4*Log[(1 + E^(2*b*x))*Cosh[d]
 + (-1 + E^(2*b*x))*Sinh[d]]) + Cosh[2*d]*(-4 + E^(4*b*x) - 4*Log[(1 + E^(
2*b*x))*Cosh[d] + (-1 + E^(2*b*x))*Sinh[d]]) + (4 + E^(4*b*x) + 4*Log[(1 +
 E^(2*b*x))*Cosh[d] + (-1 + E^(2*b*x))*Sinh[d]])*Sinh[2*d]))/(2*b*((1 + E^
(2*b*x))*Cosh[d] + (-1 + E^(2*b*x))*Sinh[d]))
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.58, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2720, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\tanh^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{e^{2a+bx}\left(1-e^{2bx}\right)^2}{\left(1+e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{2a}\int \frac{e^{bx}\left(1-e^{2bx}\right)^2}{\left(1+e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{353}$$

$$\frac{e^{2a}\int \frac{\left(1-e^{2bx}\right)^2}{\left(1+e^{2bx}\right)^2}de^{2bx}}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{e^{2a}\int \left(1-\frac{4}{1+e^{2bx}}+\frac{4}{\left(1+e^{2bx}\right)^2}\right)de^{2bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2a}\left(e^{2bx}-\frac{4}{e^{2bx}+1}-4\log\left(e^{2bx}+1\right)\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Tanh[d + b*x]^2,x]
```

output
```
(E^(2*a)*(E^(2*b*x) - 4/(1 + E^(2*b*x)) - 4*Log[1 + E^(2*b*x)]))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.29

method	result	size
risch	$\frac{e^{2bx+2a}}{2b}+\frac{4\,e^{2a-2d}a}{b}-\frac{2\,e^{4a-2d}}{\left(e^{2bx+2a+2d}+e^{2a}\right)b}-\frac{2\ln\left(e^{2bx+2a}+e^{2a-2d}\right)e^{2a-2d}}{b}$	94

input
```
int(exp(2*b*x+2*a)*tanh(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*b*x+2*a)/b+4/b*exp(2*a-2*d)*a-2/(exp(2*b*x+2*a+2*d)+exp(2*a))/b*
exp(4*a-2*d)-2*ln(exp(2*b*x+2*a)+exp(2*a-2*d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. 2(66) = 132.

Time = 0.10 (sec) , antiderivative size = 453, normalized size of antiderivative = 6.21

$$\int e^{2(a+bx)}\tanh^2(d+bx)\,dx$$

$$=\frac{\cosh(bx+d)^4\cosh(-2a+2d)+(\cosh(-2a+2d)-\sinh(-2a+2d))\sinh(bx+d)^4+4(\cosh(bx+}{}$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d)^2,x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + d)^4*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*
d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*si
nh(-2*a + 2*d))*sinh(b*x + d)^3 + cosh(b*x + d)^2*cosh(-2*a + 2*d) + (6*co
sh(b*x + d)^2*cosh(-2*a + 2*d) - (6*cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d)
+ cosh(-2*a + 2*d))*sinh(b*x + d)^2 - 4*(cosh(b*x + d)^2*cosh(-2*a + 2*d)
+ (cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^2 + 2*(cosh(b*x + d)
*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d) - (cosh(
b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*log(2*cosh(b*x + d)/(
cosh(b*x + d) - sinh(b*x + d))) + 2*(2*cosh(b*x + d)^3*cosh(-2*a + 2*d) +
cosh(b*x + d)*cosh(-2*a + 2*d) - (2*cosh(b*x + d)^3 + cosh(b*x + d))*sinh(
-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^4 + cosh(b*x + d)^2 - 4)*sinh(
-2*a + 2*d) - 4*cosh(-2*a + 2*d))/(b*cosh(b*x + d)^2 + 2*b*cosh(b*x + d)*s
inh(b*x + d) + b*sinh(b*x + d)^2 + b)
```

Sympy [F]

$$\int e^{2(a+bx)}\tanh^2(d+bx)\,dx = e^{2a}\int e^{2bx}\tanh^2{(bx+d)}\,dx$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d)**2,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*tanh(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int e^{2(a+bx)}\tanh^2(d+bx)\,dx = -\frac{4\,(bx+d)e^{(2\,a-2\,d)}}{b} - \frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(-2\,bx-2\,d)}+1\right)}{b} + \frac{\left(5\,e^{(-2\,bx-2\,d)}+1\right)e^{(2\,a-2\,d)}}{2\,b\left(e^{(-2\,bx-2\,d)}+e^{(-4\,bx-4\,d)}\right)}$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d)^2,x, algorithm="maxima")
```

output
```
-4*(b*x + d)*e^(2*a - 2*d)/b - 2*e^(2*a - 2*d)*log(e^(-2*b*x - 2*d) + 1)/b
 + 1/2*(5*e^(-2*b*x - 2*d) + 1)*e^(2*a - 2*d)/(b*(e^(-2*b*x - 2*d) + e^(-4
*b*x - 4*d)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int e^{2(a+bx)}\tanh^2(d+bx)\,dx = -\frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(2\,bx+2\,d)}+1\right)}{b}+\frac{e^{(2\,bx+2\,a)}}{2\,b}+\frac{2\,e^{(2\,bx+2\,a)}}{b(e^{(2\,bx+2\,d)}+1)}$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d)^2,x, algorithm="giac")
```

output
```
-2*e^(2*a - 2*d)*log(e^(2*b*x + 2*d) + 1)/b + 1/2*e^(2*b*x + 2*a)/b + 2*e^
(2*b*x + 2*a)/(b*(e^(2*b*x + 2*d) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12

$$\int e^{2(a+bx)}\tanh^2(d+bx)\,dx = \frac{\mathrm{e}^{2\,a+2\,b\,x}}{2\,b}-\frac{2\,\mathrm{e}^{2\,a-2\,d}\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}\right)}{b} - \frac{2\,\mathrm{e}^{4\,a-4\,d}}{b\,\left(\mathrm{e}^{2\,a-2\,d}+\mathrm{e}^{2\,a+2\,b\,x}\right)}$$

input
```
int(exp(2*a + 2*b*x)*tanh(d + b*x)^2,x)
```

output
```
exp(2*a + 2*b*x)/(2*b) - (2*exp(2*a - 2*d)*log(exp(2*a)*exp(2*b*x) + exp(2
*a)*exp(-2*d)))/b - (2*exp(4*a - 4*d))/(b*(exp(2*a - 2*d) + exp(2*a + 2*b*
x)))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.29

$$\int e^{2(a+bx)}\tanh^2(d+bx)\,dx$$
$$=\frac{e^{2a}\left(e^{4bx+4d}-4e^{2bx+2d}\log\left(e^{2bx+2d}+1\right)+5e^{2bx+2d}-4\log\left(e^{2bx+2d}+1\right)\right)}{2e^{2d}b\left(e^{2bx+2d}+1\right)}$$

input
```
int(exp(2*b*x+2*a)*tanh(b*x+d)^2,x)
```

output
```
(e**(2*a)*(e**(4*b*x + 4*d) - 4*e**(2*b*x + 2*d)*log(e**(2*b*x + 2*d) + 1)
 + 5*e**(2*b*x + 2*d) - 4*log(e**(2*b*x + 2*d) + 1)))/(2*e**(2*d)*b*(e**(2
*b*x + 2*d) + 1))
```

3.48 $\int e^{2(a+bx)}\tanh^3(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 101

$$\int e^{2(a+bx)}\tanh^3(d+bx)\,dx=\frac{e^{2a+2bx}}{2b}+\frac{2e^{2a-2d}}{b\left(1+e^{2d+2bx}\right)^2}-\frac{6e^{2a-2d}}{b\left(1+e^{2d+2bx}\right)}-\frac{3e^{2a-2d}\log\left(1+e^{2d+2bx}\right)}{b}$$

output
```
1/2*exp(2*b*x+2*a)/b+2*exp(2*a-2*d)/b/(1+exp(2*b*x+2*d))^2-6*exp(2*a-2*d)/
b/(1+exp(2*b*x+2*d))-3*exp(2*a-2*d)*ln(1+exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.50

$$\int e^{2(a+bx)}\tanh^3(d+bx)\,dx$$

$$=\frac{e^{2a}\left(e^{2bx}-6\cosh(2d)\log\left(\left(1+e^{2bx}\right)\cosh(d)+\left(-1+e^{2bx}\right)\sinh(d)\right)+\frac{4(\cosh(d)-\sinh(d))^4}{\left(\left(1+e^{2bx}\right)\cosh(d)+\left(-1+e^{2bx}\right)\sinh(d)\right)^2}-\cdots}{2b}$$

input
```
Integrate[E^(2*(a + b*x))*Tanh[d + b*x]^3,x]
```

output
```
(E^(2*a)*(E^(2*b*x) - 6*Cosh[2*d]*Log[(1 + E^(2*b*x))*Cosh[d] + (-1 + E^(2
*b*x))*Sinh[d]] + (4*(Cosh[d] - Sinh[d])^4)/((1 + E^(2*b*x))*Cosh[d] + (-1
 + E^(2*b*x))*Sinh[d])^2 - (12*(Cosh[d] - Sinh[d])^3)/((1 + E^(2*b*x))*Cos
h[d] + (-1 + E^(2*b*x))*Sinh[d]) + 6*Log[(1 + E^(2*b*x))*Cosh[d] + (-1 + E
^(2*b*x))*Sinh[d]]*Sinh[2*d]))/(2*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.55, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2720, 25, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\tanh^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{e^{2a+bx}\left(1-e^{2bx}\right)^3}{\left(1+e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{e^{2a+bx}\left(1-e^{2bx}\right)^3}{\left(1+e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^{2a}\int \frac{e^{bx}\left(1-e^{2bx}\right)^3}{\left(1+e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{353}$$

$$-\frac{e^{2a}\int \frac{\left(1-e^{2bx}\right)^3}{\left(1+e^{2bx}\right)^3}de^{2bx}}{2b}$$

$$\Big\downarrow \text{49}$$

$$-\frac{e^{2a}\int \left(-1+\frac{6}{1+e^{2bx}}-\frac{12}{\left(1+e^{2bx}\right)^2}+\frac{8}{\left(1+e^{2bx}\right)^3}\right)de^{2bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{2a}\left(-e^{2bx}+\frac{12}{e^{2bx}+1}-\frac{4}{\left(e^{2bx}+1\right)^2}+6\log\left(e^{2bx}+1\right)\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Tanh[d + b*x]^3,x]
```

output
```
-1/2*(E^(2*a)*(-E^(2*b*x) - 4/(1 + E^(2*b*x))^2 + 12/(1 + E^(2*b*x)) + 6*L
og[1 + E^(2*b*x)]))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.14

method	result	size
risch	$\frac{\mathrm{e}^{2bx+2a}}{2b}+\frac{6\,\mathrm{e}^{2a-2d}a}{b}-\frac{2\left(3\,\mathrm{e}^{2bx+2a+2d}+2\,\mathrm{e}^{2a}\right)\mathrm{e}^{4a-2d}}{\left(\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)^{2}b}-\frac{3\ln\left(\mathrm{e}^{2bx+2a}+\mathrm{e}^{2a-2d}\right)\mathrm{e}^{2a-2d}}{b}$	115

input
```
int(exp(2*b*x+2*a)*tanh(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*b*x+2*a)/b+6/b*exp(2*a-2*d)*a-2/(exp(2*b*x+2*a+2*d)+exp(2*a))^2/
b*(3*exp(2*b*x+2*a+2*d)+2*exp(2*a))*exp(4*a-2*d)-3*ln(exp(2*b*x+2*a)+exp(2
*a-2*d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 897 vs. 2(92) = 184.

Time = 0.10 (sec) , antiderivative size = 897, normalized size of antiderivative = 8.88

$$\int e^{2(a+bx)}\tanh^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d)^3,x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + d)^6*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*
d))*sinh(b*x + d)^6 + 6*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*si
nh(-2*a + 2*d))*sinh(b*x + d)^5 + 2*cosh(b*x + d)^4*cosh(-2*a + 2*d) + (15
*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (15*cosh(b*x + d)^2 + 2)*sinh(-2*a + 2
*d) + 2*cosh(-2*a + 2*d))*sinh(b*x + d)^4 + 4*(5*cosh(b*x + d)^3*cosh(-2*a
 + 2*d) + 2*cosh(b*x + d)*cosh(-2*a + 2*d) - (5*cosh(b*x + d)^3 + 2*cosh(b
*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d)^3 - 11*cosh(b*x + d)^2*cosh(-2*a
+ 2*d) + (15*cosh(b*x + d)^4*cosh(-2*a + 2*d) + 12*cosh(b*x + d)^2*cosh(-2
*a + 2*d) - (15*cosh(b*x + d)^4 + 12*cosh(b*x + d)^2 - 11)*sinh(-2*a + 2*d
) - 11*cosh(-2*a + 2*d))*sinh(b*x + d)^2 - 6*(cosh(b*x + d)^4*cosh(-2*a +
2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^4 + 4*(cosh(b*x
 + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^3 +
 2*cosh(b*x + d)^2*cosh(-2*a + 2*d) + 2*(3*cosh(b*x + d)^2*cosh(-2*a + 2*d
) - (3*cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*sinh(b*x
+ d)^2 + 4*(cosh(b*x + d)^3*cosh(-2*a + 2*d) + cosh(b*x + d)*cosh(-2*a + 2
*d) - (cosh(b*x + d)^3 + cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d) -
(cosh(b*x + d)^4 + 2*cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2
*d))*log(2*cosh(b*x + d)/(cosh(b*x + d) - sinh(b*x + d))) + 2*(3*cosh(b*x
+ d)^5*cosh(-2*a + 2*d) + 4*cosh(b*x + d)^3*cosh(-2*a + 2*d) - 11*cosh(b*x
 + d)*cosh(-2*a + 2*d) - (3*cosh(b*x + d)^5 + 4*cosh(b*x + d)^3 - 11*co...
```

Sympy [F]

$$\int e^{2(a+bx)} \tanh^3(d+bx)\,dx = e^{2a} \int e^{2bx} \tanh^3{(bx+d)}\,dx$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d)**3,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*tanh(b*x + d)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.12

$$\int e^{2(a+bx)}\tanh^3(d+bx)\,dx = -\frac{6\,(bx+d)e^{(2\,a-2\,d)}}{b} - \frac{3\,e^{(2\,a-2\,d)}\log\left(e^{(-2\,bx-2\,d)}+1\right)}{b} + \frac{\left(10\,e^{(-2\,bx-2\,d)}+5\,e^{(-4\,bx-4\,d)}+1\right)e^{(2\,a-2\,d)}}{2\,b\left(e^{(-2\,bx-2\,d)}+2\,e^{(-4\,bx-4\,d)}+e^{(-6\,bx-6\,d)}\right)}$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d)^3,x, algorithm="maxima")
```

output
```
-6*(b*x + d)*e^(2*a - 2*d)/b - 3*e^(2*a - 2*d)*log(e^(-2*b*x - 2*d) + 1)/b
 + 1/2*(10*e^(-2*b*x - 2*d) + 5*e^(-4*b*x - 4*d) + 1)*e^(2*a - 2*d)/(b*(e^
(-2*b*x - 2*d) + 2*e^(-4*b*x - 4*d) + e^(-6*b*x - 6*d)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\int e^{2(a+bx)}\tanh^3(d+bx)\,dx = -\frac{3\,e^{(2\,a-2\,d)}\log\left(e^{(2\,bx+2\,d)}+1\right)}{b} + \frac{e^{(2\,bx+2\,a)}}{2\,b} + \frac{\left(9\,e^{(4\,bx+2\,a+4\,d)}+6\,e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right)e^{(-2\,d)}}{2\,b\left(e^{(2\,bx+2\,d)}+1\right)^2}$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d)^3,x, algorithm="giac")
```

output
```
-3*e^(2*a - 2*d)*log(e^(2*b*x + 2*d) + 1)/b + 1/2*e^(2*b*x + 2*a)/b + 1/2*
(9*e^(4*b*x + 2*a + 4*d) + 6*e^(2*b*x + 2*a + 2*d) + e^(2*a))*e^(-2*d)/(b*
(e^(2*b*x + 2*d) + 1)^2)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2(a+bx)}\tanh^3(d+bx)\,dx=\int \mathrm{e}^{2\,a+2\,b\,x}\,\tanh(d+b\,x)^3\,dx$$

input
```
int(exp(2*a + 2*b*x)*tanh(d + b*x)^3,x)
```

output
```
int(exp(2*a + 2*b*x)*tanh(d + b*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.33

$$\int e^{2(a+bx)}\tanh^3(d+bx)\,dx \\ =\frac{e^{2a}\left(2e^{6bx+6d}-12e^{4bx+4d}\log\left(e^{2bx+2d}+1\right)+15e^{4bx+4d}-24e^{2bx+2d}\log\left(e^{2bx+2d}+1\right)-12\log\left(e^{2bx+2d}+1\right)-\right.}{4e^{2d}b\left(e^{4bx+4d}+2e^{2bx+2d}+1\right)}$$

input
```
int(exp(2*b*x+2*a)*tanh(b*x+d)^3,x)
```

output
```
(e**(2*a)*(2*e**(6*b*x + 6*d) - 12*e**(4*b*x + 4*d)*log(e**(2*b*x + 2*d) +
 1) + 15*e**(4*b*x + 4*d) - 24*e**(2*b*x + 2*d)*log(e**(2*b*x + 2*d) + 1)
- 12*log(e**(2*b*x + 2*d) + 1) - 5))/(4*e**(2*d)*b*(e**(4*b*x + 4*d) + 2*e
**(2*b*x + 2*d) + 1))
```

3.49 $\int e^{2(a+bx)} \tanh^4(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 131

$$\int e^{2(a+bx)} \tanh^4(d+bx)\,dx = \frac{e^{2a+2bx}}{2b} - \frac{8e^{2a-2d}}{3b\left(1+e^{2d+2bx}\right)^3} + \frac{8e^{2a-2d}}{b\left(1+e^{2d+2bx}\right)^2} - \frac{12e^{2a-2d}}{b\left(1+e^{2d+2bx}\right)} - \frac{4e^{2a-2d}\log\left(1+e^{2d+2bx}\right)}{b}$$

output
```
1/2*exp(2*b*x+2*a)/b-8/3*exp(2*a-2*d)/b/(1+exp(2*b*x+2*d))^3+8*exp(2*a-2*d
)/b/(1+exp(2*b*x+2*d))^2-12*exp(2*a-2*d)/b/(1+exp(2*b*x+2*d))-4*exp(2*a-2*
d)*ln(1+exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.44

$$\int e^{2(a+bx)} \tanh^4(d+bx)\,dx$$
$$= \frac{e^{2a}\left(3e^{2bx} - 24\cosh(2d)\log\left(\left(1+e^{2bx}\right)\cosh(d) + \left(-1+e^{2bx}\right)\sinh(d)\right) - \frac{16(\cosh(d)-\sinh(d))^5}{\left(\left(1+e^{2bx}\right)\cosh(d)+\left(-1+e^{2bx}\right)\sinh(d)\right)^3}\right.}{}$$

input
```
Integrate[E^(2*(a + b*x))*Tanh[d + b*x]^4,x]
```

output
```
(E^(2*a)*(3*E^(2*b*x) - 24*Cosh[2*d]*Log[(1 + E^(2*b*x))*Cosh[d] + (-1 + E
^(2*b*x))*Sinh[d]] - (16*(Cosh[d] - Sinh[d])^5)/((1 + E^(2*b*x))*Cosh[d] +
 (-1 + E^(2*b*x))*Sinh[d])^3 + (48*(Cosh[d] - Sinh[d])^4)/((1 + E^(2*b*x))
*Cosh[d] + (-1 + E^(2*b*x))*Sinh[d])^2 - (72*(Cosh[d] - Sinh[d])^3)/((1 +
E^(2*b*x))*Cosh[d] + (-1 + E^(2*b*x))*Sinh[d]) + 24*Log[(1 + E^(2*b*x))*Co
sh[d] + (-1 + E^(2*b*x))*Sinh[d]]*Sinh[2*d]))/(6*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.52, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2720, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)} \tanh^4(bx+d) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{e^{2a+bx}\left(1-e^{2bx}\right)^4}{\left(1+e^{2bx}\right)^4}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{2a}\int \frac{e^{bx}\left(1-e^{2bx}\right)^4}{\left(1+e^{2bx}\right)^4}de^{bx}}{b}$$

$$\Big\downarrow \text{353}$$

$$\frac{e^{2a}\int \frac{\left(1-e^{2bx}\right)^4}{\left(1+e^{2bx}\right)^4}de^{2bx}}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{e^{2a}\int \left(1-\frac{8}{1+e^{2bx}}+\frac{24}{\left(1+e^{2bx}\right)^2}-\frac{32}{\left(1+e^{2bx}\right)^3}+\frac{16}{\left(1+e^{2bx}\right)^4}\right)de^{2bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2a}\left(e^{2bx}-\frac{24}{e^{2bx}+1}+\frac{16}{(e^{2bx}+1)^2}-\frac{16}{3(e^{2bx}+1)^3}-8\log\left(e^{2bx}+1\right)\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Tanh[d + b*x]^4,x]
```

output
```
(E^(2*a)*(E^(2*b*x) - 16/(3*(1 + E^(2*b*x))^3) + 16/(1 + E^(2*b*x))^2 - 24
/(1 + E^(2*b*x)) - 8*Log[1 + E^(2*b*x)]))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

method	result	size
risch	$\frac{e^{2bx+2a}}{2b}+\frac{8\,e^{2a-2d}a}{b}-\frac{4\left(9\,e^{4bx+4a+4d}+12\,e^{2bx+4a+2d}+5\,e^{4a}\right)e^{4a-2d}}{3\left(e^{2bx+2a+2d}+e^{2a}\right)^{3}b}-\frac{4\ln\left(e^{2bx+2a}+e^{2a-2d}\right)e^{2a-2d}}{b}$	129

input
```
int(exp(2*b*x+2*a)*tanh(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*b*x+2*a)/b+8/b*exp(2*a-2*d)*a-4/3/(exp(2*b*x+2*a+2*d)+exp(2*a))^
3/b*(9*exp(4*b*x+4*a+4*d)+12*exp(2*b*x+4*a+2*d)+5*exp(4*a))*exp(4*a-2*d)-4
*ln(exp(2*b*x+2*a)+exp(2*a-2*d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1482 vs. 2(118) = 236.

Time = 0.10 (sec) , antiderivative size = 1482, normalized size of antiderivative = 11.31

$$\int e^{2(a+bx)}\tanh^4(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d)^4,x, algorithm="fricas")
```

output
```
1/6*(3*cosh(b*x + d)^8*cosh(-2*a + 2*d) + 3*(cosh(-2*a + 2*d) - sinh(-2*a
+ 2*d))*sinh(b*x + d)^8 + 24*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x +
d)*sinh(-2*a + 2*d))*sinh(b*x + d)^7 + 9*cosh(b*x + d)^6*cosh(-2*a + 2*d)
+ 3*(28*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (28*cosh(b*x + d)^2 + 3)*sinh(-
2*a + 2*d) + 3*cosh(-2*a + 2*d))*sinh(b*x + d)^6 + 6*(28*cosh(b*x + d)^3*c
osh(-2*a + 2*d) + 9*cosh(b*x + d)*cosh(-2*a + 2*d) - (28*cosh(b*x + d)^3 +
 9*cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d)^5 - 63*cosh(b*x + d)^4*c
osh(-2*a + 2*d) + 3*(70*cosh(b*x + d)^4*cosh(-2*a + 2*d) + 45*cosh(b*x + d
)^2*cosh(-2*a + 2*d) - (70*cosh(b*x + d)^4 + 45*cosh(b*x + d)^2 - 21)*sinh
(-2*a + 2*d) - 21*cosh(-2*a + 2*d))*sinh(b*x + d)^4 + 12*(14*cosh(b*x + d)
^5*cosh(-2*a + 2*d) + 15*cosh(b*x + d)^3*cosh(-2*a + 2*d) - 21*cosh(b*x +
d)*cosh(-2*a + 2*d) - (14*cosh(b*x + d)^5 + 15*cosh(b*x + d)^3 - 21*cosh(b
*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d)^3 - 93*cosh(b*x + d)^2*cosh(-2*a
+ 2*d) + 3*(28*cosh(b*x + d)^6*cosh(-2*a + 2*d) + 45*cosh(b*x + d)^4*cosh(
-2*a + 2*d) - 126*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (28*cosh(b*x + d)^6 +
 45*cosh(b*x + d)^4 - 126*cosh(b*x + d)^2 - 31)*sinh(-2*a + 2*d) - 31*cosh
(-2*a + 2*d))*sinh(b*x + d)^2 - 24*(cosh(b*x + d)^6*cosh(-2*a + 2*d) + (co
sh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^6 + 6*(cosh(b*x + d)*cosh
(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^5 + 3*cosh(b*
x + d)^4*cosh(-2*a + 2*d) + 3*(5*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (5*...
```

Sympy [F]

$$\int e^{2(a+bx)}\tanh^4(d+bx)\,dx = e^{2a}\int e^{2bx}\tanh^4(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d)**4,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*tanh(b*x + d)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.03

$$\int e^{2(a+bx)} \tanh^4(d+bx)\,dx = -\frac{8\,(bx+d)e^{(2\,a-2\,d)}}{b} - \frac{4\,e^{(2\,a-2\,d)}\log\left(e^{(-2\,bx-2\,d)}+1\right)}{b} + \frac{\left(49\,e^{(-2\,bx-2\,d)}+57\,e^{(-4\,bx-4\,d)}+27\,e^{(-6\,bx-6\,d)}+3\right)e^{(2\,a-2\,d)}}{6\,b\left(e^{(-2\,bx-2\,d)}+3\,e^{(-4\,bx-4\,d)}+3\,e^{(-6\,bx-6\,d)}+e^{(-8\,bx-8\,d)}\right)}$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d)^4,x, algorithm="maxima")
```

output
```
-8*(b*x + d)*e^(2*a - 2*d)/b - 4*e^(2*a - 2*d)*log(e^(-2*b*x - 2*d) + 1)/b
 + 1/6*(49*e^(-2*b*x - 2*d) + 57*e^(-4*b*x - 4*d) + 27*e^(-6*b*x - 6*d) +
3)*e^(2*a - 2*d)/(b*(e^(-2*b*x - 2*d) + 3*e^(-4*b*x - 4*d) + 3*e^(-6*b*x -
 6*d) + e^(-8*b*x - 8*d)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.83

$$\int e^{2(a+bx)} \tanh^4(d+bx)\,dx = -\frac{4\,e^{(2\,a-2\,d)}\log\left(e^{(2\,bx+2\,d)}+1\right)}{b} + \frac{e^{(2\,bx+2\,a)}}{2\,b} + \frac{2\left(11\,e^{(6\,bx+2\,a+6\,d)}+15\,e^{(4\,bx+2\,a+4\,d)}+9\,e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right)e^{(-2\,d)}}{3\,b\left(e^{(2\,bx+2\,d)}+1\right)^3}$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d)^4,x, algorithm="giac")
```

output
```
-4*e^(2*a - 2*d)*log(e^(2*b*x + 2*d) + 1)/b + 1/2*e^(2*b*x + 2*a)/b + 2/3*
(11*e^(6*b*x + 2*a + 6*d) + 15*e^(4*b*x + 2*a + 4*d) + 9*e^(2*b*x + 2*a +
2*d) + e^(2*a))*e^(-2*d)/(b*(e^(2*b*x + 2*d) + 1)^3)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2(a+bx)}\tanh^4(d+bx)\,dx=\int e^{2\,a+2\,b\,x}\tanh(d+b\,x)^4\,dx$$

input
```
int(exp(2*a + 2*b*x)*tanh(d + b*x)^4,x)
```

output
```
int(exp(2*a + 2*b*x)*tanh(d + b*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.40

$$\int e^{2(a+bx)}\tanh^4(d+bx)\,dx$$
$$=\frac{e^{2a}\left(3e^{8bx+8d}-24e^{6bx+6d}\log\left(e^{2bx+2d}+1\right)+30e^{6bx+6d}-72e^{4bx+4d}\log\left(e^{2bx+2d}+1\right)-72e^{2bx+2d}\log\left(e^{2bx+2d}\right.\right.}{6e^{2d}b\left(e^{6bx+6d}+3e^{4bx+4d}+3e^{2bx+2d}+1\right)}$$

input
```
int(exp(2*b*x+2*a)*tanh(b*x+d)^4,x)
```

output
```
(e**(2*a)*(3*e**(8*b*x + 8*d) - 24*e**(6*b*x + 6*d)*log(e**(2*b*x + 2*d) +
 1) + 30*e**(6*b*x + 6*d) - 72*e**(4*b*x + 4*d)*log(e**(2*b*x + 2*d) + 1)
- 72*e**(2*b*x + 2*d)*log(e**(2*b*x + 2*d) + 1) - 30*e**(2*b*x + 2*d) - 24
*log(e**(2*b*x + 2*d) + 1) - 19))/(6*e**(2*d)*b*(e**(6*b*x + 6*d) + 3*e**(
4*b*x + 4*d) + 3*e**(2*b*x + 2*d) + 1))
```

3.50 $\int e^{\frac{5}{3}(a+bx)} \tanh(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 181

$$\int e^{\frac{5}{3}(a+bx)} \tanh(d+bx)\,dx = \frac{3e^{\frac{5(a-d)}{3}+\frac{5}{3}(d+bx)}}{5b} - \frac{2e^{\frac{5(a-d)}{3}} \arctan\left(e^{\frac{1}{3}(d+bx)}\right)}{b} + \frac{e^{\frac{5(a-d)}{3}} \arctan\left(\sqrt{3}-2e^{\frac{1}{3}(d+bx)}\right)}{b} - \frac{e^{\frac{5(a-d)}{3}} \arctan\left(\sqrt{3}+2e^{\frac{1}{3}(d+bx)}\right)}{b} + \frac{\sqrt{3}e^{\frac{5(a-d)}{3}} \operatorname{arctanh}\left(\frac{\sqrt{3}e^{\frac{1}{3}(d+bx)}}{1+e^{\frac{2}{3}(d+bx)}}\right)}{b}$$

output
```
3/5*exp(5/3*b*x+5/3*a)/b-2*exp(5/3*a-5/3*d)*arctan(exp(1/3*b*x+1/3*d))/b-e
xp(5/3*a-5/3*d)*arctan(-3^(1/2)+2*exp(1/3*b*x+1/3*d))/b-exp(5/3*a-5/3*d)*a
rctan(3^(1/2)+2*exp(1/3*b*x+1/3*d))/b+3^(1/2)*exp(5/3*a-5/3*d)*arctanh(3^(
1/2)*exp(1/3*b*x+1/3*d)/(1+exp(2/3*b*x+2/3*d)))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.48

$$\int e^{\frac{5}{3}(a+bx)}\tanh(d+bx)\,dx$$
$$= \frac{e^{5a/3}\left(9e^{\frac{5bx}{3}}+5\text{RootSum}\left[\cosh(d)-\sinh(d)+\cosh(d)\#1^6+\sinh(d)\#1^6\&,\frac{bx-3\log\left(e^{\frac{bx}{3}}-\#1\right)}{\#1}\&\right](\cosh(2}{15b}$$

input
```
Integrate[E^((5*(a + b*x))/3)*Tanh[d + b*x],x]
```

output
```
(E^((5*a)/3)*(9*E^((5*b*x)/3) + 5*RootSum[Cosh[d] - Sinh[d] + Cosh[d]*#1^6
 + Sinh[d]*#1^6 & , (b*x - 3*Log[E^((b*x)/3) - #1])/#1 & ]*(Cosh[2*d] - Si
nh[2*d])))/(15*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.86, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2720, 25, 27, 959, 824, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)}\tanh(bx+d)\,dx$$
$$\Big\downarrow \text{2720}$$
$$\frac{3\int -\frac{e^{\frac{5a}{3}+\frac{4bx}{3}}\left(1-e^{2bx}\right)}{1+e^{2bx}}de^{\frac{bx}{3}}}{b}$$
$$\Big\downarrow \text{25}$$
$$-\frac{3\int \frac{e^{\frac{5a}{3}+\frac{4bx}{3}}\left(1-e^{2bx}\right)}{1+e^{2bx}}de^{\frac{bx}{3}}}{b}$$

$\downarrow$ 27

$$-\frac{3e^{5a/3}\int \frac{e^{\frac{4bx}{3}}\left(1-e^{2bx}\right)}{1+e^{2bx}}de^{\frac{bx}{3}}}{b}$$

$\downarrow$ 959

$$-\frac{3e^{5a/3}\left(2\int \frac{e^{\frac{4bx}{3}}}{1+e^{2bx}}de^{\frac{bx}{3}}-\frac{1}{5}e^{\frac{5bx}{3}}\right)}{b}$$

$\downarrow$ 824

$$-\frac{3e^{5a/3}\left(2\left(\frac{1}{3}\int \frac{1}{1+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{3}\int -\frac{1-\sqrt{3}e^{\frac{bx}{3}}}{2\left(1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}\right)}de^{\frac{bx}{3}}+\frac{1}{3}\int -\frac{1+\sqrt{3}e^{\frac{bx}{3}}}{2\left(1+\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}\right)}de^{\frac{bx}{3}}\right)-\frac{1}{5}e^{\frac{5bx}{3}}\right)}{b}$$

$\downarrow$ 27

$$-\frac{3e^{5a/3}\left(2\left(\frac{1}{3}\int \frac{1}{1+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1-\sqrt{3}e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1+\sqrt{3}e^{\frac{bx}{3}}}{1+\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)-\frac{1}{5}e^{\frac{5bx}{3}}\right)}{b}$$

$\downarrow$ 216

$$-\frac{3e^{5a/3}\left(2\left(-\frac{1}{6}\int \frac{1-\sqrt{3}e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1+\sqrt{3}e^{\frac{bx}{3}}}{1+\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{3}\arctan\left(e^{\frac{bx}{3}}\right)\right)-\frac{1}{5}e^{\frac{5bx}{3}}\right)}{b}$$

$\downarrow$ 1142

$$-\frac{3e^{5a/3}\left(2\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}-2e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{1+\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{2}\sqrt{3}\int \cdots\right.\right.\right.}{b}$$

$\downarrow$ 25

$$-\frac{3e^{5a/3}\left(2\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-2e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{1+\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{2}\sqrt{3}\int \cdots\right.\right.\right.}{b}$$

$\downarrow$ 1083

$$-\frac{3e^{5a/3}\left(2\left(\frac{1}{6}\left(-\int \frac{1}{-1-e^{\frac{2bx}{3}}}d\left(-\sqrt{3}+2e^{\frac{bx}{3}}\right)-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-2e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(-\int \frac{1}{-1-e^{\frac{2bx}{3}}}d\left(\sqrt{3}+2e^{\frac{bx}{3}}\right)-\frac{1}{2}\cdots\right.\right.\right.}{b}$$

$$\Big\downarrow \; 217$$

$$-\frac{3e^{5a/3}\left(2\left(\frac{1}{6}\left(-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-2e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\arctan\left(\sqrt{3}-2e^{\frac{bx}{3}}\right)\right)+\frac{1}{6}\left(\arctan\left(2e^{\frac{bx}{3}}+\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}+2e}{1+\sqrt{3}e^{\frac{bx}{3}}}\right.\right.\right.}{b}$$

$$\Big\downarrow \; 1103$$

$$-\frac{3e^{5a/3}\left(2\left(\frac{1}{3}\arctan\left(e^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\log\left(-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}+1\right)-\arctan\left(\sqrt{3}-2e^{\frac{bx}{3}}\right)\right)+\frac{1}{6}\left(\arctan\left(2e^{\frac{bx}{3}}+\sqrt{3}\right.\right.\right.\right.}{b}$$

input
```
Int[E^((5*(a + b*x))/3)*Tanh[d + b*x],x]
```

output
```
(-3*E^((5*a)/3)*(-1/5*E^((5*b*x)/3) + 2*(ArcTan[E^((b*x)/3)]/3 + (-ArcTan[
Sqrt[3] - 2*E^((b*x)/3)] + (Sqrt[3]*Log[1 - Sqrt[3]*E^((b*x)/3) + E^((2*b*
x)/3)])/2)/6 + (ArcTan[Sqrt[3] + 2*E^((b*x)/3)] - (Sqrt[3]*Log[1 + Sqrt[3]
*E^((b*x)/3) + E^((2*b*x)/3)])/2)/6)))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.69

method	result
risch	$\frac{3\,\mathrm{e}^{\frac{5bx}{3}+\frac{5a}{3}}}{5b}+\frac{i\ln\left(\mathrm{e}^{\frac{bx}{3}+\frac{d}{3}}-i\right)\mathrm{e}^{\frac{5a}{3}-\frac{5d}{3}}}{b}-\frac{i\ln\left(\mathrm{e}^{\frac{bx}{3}+\frac{d}{3}}+i\right)\mathrm{e}^{\frac{5a}{3}-\frac{5d}{3}}}{b}+\left(\sum_{\_R=\mathrm{RootOf}\left(b^4\_Z^4-b^2\_Z^2+1\right)}\_R\ln\left(-b^3\_\right.\right.$

input
```
int(exp(5/3*b*x+5/3*a)*tanh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
3/5*exp(5/3*b*x+5/3*a)/b+I/b*ln(exp(1/3*b*x+1/3*d)-I)*exp(5/3*a-5/3*d)-I/b
*ln(exp(1/3*b*x+1/3*d)+I)*exp(5/3*a-5/3*d)+sum(_R*ln(-b^3*_R^3+b*_R+exp(1/
3*b*x+1/3*d)),_R=RootOf(_Z^4*b^4-_Z^2*b^2+1))*exp(5/3*a-5/3*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1200 vs. 2(140) = 280.

Time = 0.11 (sec) , antiderivative size = 1200, normalized size of antiderivative = 6.63

$$\int e^{\frac{5}{3}(a+bx)}\tanh(d+bx)\,dx=\text{Too large to display}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*tanh(b*x+d),x, algorithm="fricas")
```

output
```
1/10*(6*cosh(1/3*b*x + 1/3*d)^5*cosh(-5/3*a + 5/3*d) + 6*(cosh(-5/3*a + 5/
3*d) - sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^5 - 6*cosh(1/3*b*x + 1/
3*d)^5*sinh(-5/3*a + 5/3*d) + 30*(cosh(1/3*b*x + 1/3*d)*cosh(-5/3*a + 5/3*
d) - cosh(1/3*b*x + 1/3*d)*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^4 +
 60*(cosh(1/3*b*x + 1/3*d)^2*cosh(-5/3*a + 5/3*d) - cosh(1/3*b*x + 1/3*d)^
2*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^3 + 5*sqrt(3)*b*sqrt((cosh(-
5/3*a + 5/3*d) - sinh(-5/3*a + 5/3*d))/(b^2*cosh(-5/3*a + 5/3*d) + b^2*sin
h(-5/3*a + 5/3*d)))*log(cosh(1/3*b*x + 1/3*d)^2*cosh(-5/3*a + 5/3*d) + (co
sh(-5/3*a + 5/3*d) - sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^2 + sqrt(
3)*(b*cosh(1/3*b*x + 1/3*d) + b*sinh(1/3*b*x + 1/3*d))*sqrt((cosh(-5/3*a +
 5/3*d) - sinh(-5/3*a + 5/3*d))/(b^2*cosh(-5/3*a + 5/3*d) + b^2*sinh(-5/3*
a + 5/3*d))) + 2*(cosh(1/3*b*x + 1/3*d)*cosh(-5/3*a + 5/3*d) - cosh(1/3*b*
x + 1/3*d)*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d) - (cosh(1/3*b*x + 1
/3*d)^2 + 1)*sinh(-5/3*a + 5/3*d) + cosh(-5/3*a + 5/3*d)) - 5*sqrt(3)*b*sq
rt((cosh(-5/3*a + 5/3*d) - sinh(-5/3*a + 5/3*d))/(b^2*cosh(-5/3*a + 5/3*d)
 + b^2*sinh(-5/3*a + 5/3*d)))*log(cosh(1/3*b*x + 1/3*d)^2*cosh(-5/3*a + 5/
3*d) + (cosh(-5/3*a + 5/3*d) - sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)
^2 - sqrt(3)*(b*cosh(1/3*b*x + 1/3*d) + b*sinh(1/3*b*x + 1/3*d))*sqrt((cos
h(-5/3*a + 5/3*d) - sinh(-5/3*a + 5/3*d))/(b^2*cosh(-5/3*a + 5/3*d) + b^2*
sinh(-5/3*a + 5/3*d))) + 2*(cosh(1/3*b*x + 1/3*d)*cosh(-5/3*a + 5/3*d) ...
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+bx)}\tanh(d+bx)\,dx=e^{\frac{5a}{3}}\int e^{\frac{5bx}{3}}\tanh\left(bx+d\right)dx$$

input
```
integrate(exp(5/3*b*x+5/3*a)*tanh(b*x+d),x)
```

output
```
exp(5*a/3)*Integral(exp(5*b*x/3)*tanh(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.77

$$\int e^{\frac{5}{3}(a+bx)}\tanh(d+bx)\,dx$$
$$=\frac{\left(\sqrt{3}\log\left(\sqrt{3}e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}+e^{\left(-\frac{2}{3}\,bx-\frac{2}{3}\,d\right)}+1\right)-\sqrt{3}\log\left(-\sqrt{3}e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}+e^{\left(-\frac{2}{3}\,bx-\frac{2}{3}\,d\right)}+1\right)+2\arctan\left(\right.}{2\,b}$$
$$+\frac{3\,e^{\left(\frac{5}{3}\,bx+\frac{5}{3}\,a\right)}}{5\,b}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*tanh(b*x+d),x, algorithm="maxima")
```

output
```
1/2*(sqrt(3)*log(sqrt(3)*e^(-1/3*b*x - 1/3*d) + e^(-2/3*b*x - 2/3*d) + 1)
- sqrt(3)*log(-sqrt(3)*e^(-1/3*b*x - 1/3*d) + e^(-2/3*b*x - 2/3*d) + 1) +
2*arctan(sqrt(3) + 2*e^(-1/3*b*x - 1/3*d)) + 2*arctan(-sqrt(3) + 2*e^(-1/3
*b*x - 1/3*d)) + 4*arctan(e^(-1/3*b*x - 1/3*d)))*e^(5/3*a - 5/3*d)/b + 3/5
*e^(5/3*b*x + 5/3*a)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.98

$$\int e^{\frac{5}{3}(a+bx)}\tanh(d+bx)\,dx$$
$$=\frac{5\,\sqrt{3}e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,d\right)}\log\left(\sqrt{3}e^{\left(\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}+e^{\left(\frac{2}{3}\,bx\right)}+e^{\left(-\frac{2}{3}\,d\right)}\right)-5\,\sqrt{3}e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,d\right)}\log\left(-\sqrt{3}e^{\left(\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}+e^{\left(\frac{2}{3}\,bx\right)}+e^{\left(-\right.}\right.}{}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*tanh(b*x+d),x, algorithm="giac")
```

output
```
1/10*(5*sqrt(3)*e^(5/3*a - 5/3*d)*log(sqrt(3)*e^(1/3*b*x - 1/3*d) + e^(2/3
*b*x) + e^(-2/3*d)) - 5*sqrt(3)*e^(5/3*a - 5/3*d)*log(-sqrt(3)*e^(1/3*b*x
- 1/3*d) + e^(2/3*b*x) + e^(-2/3*d)) - 10*arctan((sqrt(3)*e^(-1/3*d) + 2*e
^(1/3*b*x))*e^(1/3*d))*e^(5/3*a - 5/3*d) - 10*arctan(-(sqrt(3)*e^(-1/3*d)
- 2*e^(1/3*b*x))*e^(1/3*d))*e^(5/3*a - 5/3*d) - 20*arctan(e^(1/3*b*x + 1/3
*d))*e^(5/3*a - 5/3*d) + 6*e^(5/3*b*x + 5/3*a))/b
```

Mupad [B] (verification not implemented)

Time = 4.66 (sec) , antiderivative size = 436, normalized size of antiderivative = 2.41

$$\int e^{\frac{5}{3}(a+bx)} \tanh(d+bx)\,dx = \text{Too large to display}$$

input
```
int(exp((5*a)/3 + (5*b*x)/3)*tanh(d + b*x),x)
```

output
```
(3*exp((5*a)/3 + (5*b*x)/3))/(5*b) - ((-exp(10*a - 10*d))^(1/6)*log(4*exp(
(10*a)/3)*exp(-(10*d)/3) - 4*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)
/3)*(-exp(10*a)*exp(-10*d))^(1/6)))/b + ((-exp(10*a - 10*d))^(1/6)*log(4*e
xp((10*a)/3)*exp(-(10*d)/3) + 4*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b
*x)/3)*(-exp(10*a)*exp(-10*d))^(1/6)))/b - (log(4*exp((10*a)/3)*exp(-(10*d
)/3) - 4*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)/2
- 1/2)*(-exp(10*a)*exp(-10*d))^(1/6))*(-exp(10*a - 10*d))^(1/6)*((3^(1/2)*
1i)/2 - 1/2))/b + (log(4*exp((10*a)/3)*exp(-(10*d)/3) + 4*exp((5*a)/3)*exp
(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)/2 - 1/2)*(-exp(10*a)*exp(-1
0*d))^(1/6))*(-exp(10*a - 10*d))^(1/6)*((3^(1/2)*1i)/2 - 1/2))/b - (log(4*
exp((10*a)/3)*exp(-(10*d)/3) - 4*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((
b*x)/3)*((3^(1/2)*1i)/2 + 1/2)*(-exp(10*a)*exp(-10*d))^(1/6))*(-exp(10*a -
 10*d))^(1/6)*((3^(1/2)*1i)/2 + 1/2))/b + (log(4*exp((10*a)/3)*exp(-(10*d)
/3) + 4*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)/2 +
 1/2)*(-exp(10*a)*exp(-10*d))^(1/6))*(-exp(10*a - 10*d))^(1/6)*((3^(1/2)*1
i)/2 + 1/2))/b
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+bx)} \tanh(d+bx)\,dx = \int e^{\frac{5bx}{3}+\frac{5a}{3}} \tanh(bx+d)\,dx$$

input
```
int(exp(5/3*b*x+5/3*a)*tanh(b*x+d),x)
```

output
```
int(e**((5*a + 5*b*x)/3)*tanh(b*x + d),x)
```

3.51 $\int e^{\frac{5}{3}(a+bx)} \tanh^2(d+bx)\, dx$

Optimal result

Integrand size = 20, antiderivative size = 228

$$\int e^{\frac{5}{3}(a+bx)} \tanh^2(d+bx)\, dx = \frac{3e^{\frac{5(a-d)}{3}+\frac{5}{3}(d+bx)}}{5b} + \frac{2e^{\frac{5(a-d)}{3}+\frac{5}{3}(d+bx)}}{b\left(1+e^{2(d+bx)}\right)} - \frac{10e^{\frac{5(a-d)}{3}} \arctan\left(e^{\frac{1}{3}(d+bx)}\right)}{3b} + \frac{5e^{\frac{5(a-d)}{3}} \arctan\left(\sqrt{3}-2e^{\frac{1}{3}(d+bx)}\right)}{3b} - \frac{5e^{\frac{5(a-d)}{3}} \arctan\left(\sqrt{3}+2e^{\frac{1}{3}(d+bx)}\right)}{3b} + \frac{5e^{\frac{5(a-d)}{3}} \operatorname{arctanh}\left(\frac{\sqrt{3}e^{\frac{1}{3}(d+bx)}}{1+e^{\frac{2}{3}(d+bx)}}\right)}{\sqrt{3}b}$$

output
```
3/5*exp(5/3*b*x+5/3*a)/b+2*exp(5/3*b*x+5/3*a)/b/(1+exp(2*b*x+2*d))-10/3*ex
p(5/3*a-5/3*d)*arctan(exp(1/3*b*x+1/3*d))/b-5/3*exp(5/3*a-5/3*d)*arctan(-3
^(1/2)+2*exp(1/3*b*x+1/3*d))/b-5/3*exp(5/3*a-5/3*d)*arctan(3^(1/2)+2*exp(1
/3*b*x+1/3*d))/b+5/3*3^(1/2)*exp(5/3*a-5/3*d)*arctanh(3^(1/2)*exp(1/3*b*x+
1/3*d)/(1+exp(2/3*b*x+2/3*d)))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.31 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.56

$$\int e^{\frac{5}{3}(a+bx)}\tanh^2(d+bx)\,dx$$
$$=\frac{e^{5a/3}\left(27e^{\frac{5bx}{3}}+25\text{RootSum}\left[\cosh(d)-\sinh(d)+\cosh(d)\#1^6+\sinh(d)\#1^6\&,\frac{bx-3\log\left(e^{\frac{bx}{3}}-\#1\right)}{\#1}\&\right](\cosh$$
$$45b$$

input
```
Integrate[E^((5*(a + b*x))/3)*Tanh[d + b*x]^2,x]
```

output
```
(E^((5*a)/3)*(27*E^((5*b*x)/3) + 25*RootSum[Cosh[d] - Sinh[d] + Cosh[d]*#1
^6 + Sinh[d]*#1^6 & , (b*x - 3*Log[E^((b*x)/3) - #1])/#1 & ]*(Cosh[d] - Si
nh[d])^2 + (90*E^((5*b*x)/3)*(Cosh[d] - Sinh[d]))/((1 + E^(2*b*x))*Cosh[d]
 + (-1 + E^(2*b*x))*Sinh[d])))/(45*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.80, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.650, Rules used = {2720, 27, 963, 27, 959, 824, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)}\tanh^2(bx+d)\,dx$$
$$\Big\downarrow \text{2720}$$
$$\frac{3\int \frac{e^{\frac{5a}{3}+\frac{4bx}{3}}\left(1-e^{2bx}\right)^2}{\left(1+e^{2bx}\right)^2}de^{\frac{bx}{3}}}{b}$$
$$\Big\downarrow \text{27}$$

$$\frac{3e^{5a/3}\int \frac{e^{\frac{4bx}{3}}\left(1-e^{2bx}\right)^2}{\left(1+e^{2bx}\right)^2}de^{\frac{bx}{3}}}{b}$$

$\big\downarrow$ 963

$$\frac{3e^{5a/3}\left(\frac{2e^{\frac{5bx}{3}}}{3\left(e^{2bx}+1\right)}-\frac{1}{6}\int \frac{2e^{\frac{4bx}{3}}\left(7-3e^{2bx}\right)}{1+e^{2bx}}de^{\frac{bx}{3}}\right)}{b}$$

$\big\downarrow$ 27

$$\frac{3e^{5a/3}\left(\frac{2e^{\frac{5bx}{3}}}{3\left(e^{2bx}+1\right)}-\frac{1}{3}\int \frac{e^{\frac{4bx}{3}}\left(7-3e^{2bx}\right)}{1+e^{2bx}}de^{\frac{bx}{3}}\right)}{b}$$

$\big\downarrow$ 959

$$\frac{3e^{5a/3}\left(\frac{1}{3}\left(\frac{3}{5}e^{\frac{5bx}{3}}-10\int \frac{e^{\frac{4bx}{3}}}{1+e^{2bx}}de^{\frac{bx}{3}}\right)+\frac{2e^{\frac{5bx}{3}}}{3\left(e^{2bx}+1\right)}\right)}{b}$$

$\big\downarrow$ 824

$$\frac{3e^{5a/3}\left(\frac{1}{3}\left(\frac{3}{5}e^{\frac{5bx}{3}}-10\left(\frac{1}{3}\int \frac{1}{1+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{3}\int -\frac{1-\sqrt{3}e^{\frac{bx}{3}}}{2\left(1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}\right)}de^{\frac{bx}{3}}+\frac{1}{3}\int -\frac{1+\sqrt{3}e^{\frac{bx}{3}}}{2\left(1+\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}\right)}de^{\frac{bx}{3}}\right)\right)+\frac{2e^{5\cdots}}{3\left(e^{2b\cdots}\right.}\cdots\right.}{b}$$

$\big\downarrow$ 27

$$\frac{3e^{5a/3}\left(\frac{1}{3}\left(\frac{3}{5}e^{\frac{5bx}{3}}-10\left(\frac{1}{3}\int \frac{1}{1+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1-\sqrt{3}e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1+\sqrt{3}e^{\frac{bx}{3}}}{1+\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)\right)+\frac{2e^{\frac{5bx}{3}}}{3\left(e^{2bx}+1\right)}\right)}{b}$$

$\big\downarrow$ 216

$$\frac{3e^{5a/3}\left(\frac{1}{3}\left(\frac{3}{5}e^{\frac{5bx}{3}}-10\left(-\frac{1}{6}\int \frac{1-\sqrt{3}e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1+\sqrt{3}e^{\frac{bx}{3}}}{1+\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{3}\arctan\left(e^{\frac{bx}{3}}\right)\right)\right)+\frac{2e^{\frac{5bx}{3}}}{3\left(e^{2bx}+1\right)}\right)}{b}$$

$\big\downarrow$ 1142

$$\frac{3e^{5a/3}\left(\frac{1}{3}\left(\frac{3}{5}e^{\frac{5bx}{3}}-10\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}-2e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{1+\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\cdots\right.\right.\right.\right.}{b}$$

$\big\downarrow$ 25

$$\frac{3e^{5a/3}\left(\frac{1}{3}\left(\frac{3}{5}e^{\frac{5bx}{3}}-10\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{1+\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{2}\sqrt{}\right.\right.\right.\right.}{b}$$

$$\Big\downarrow \text{1083}$$

$$\frac{3e^{5a/3}\left(\frac{1}{3}\left(\frac{3}{5}e^{\frac{5bx}{3}}-10\left(\frac{1}{6}\left(-\int\frac{1}{-1-e^{\frac{2bx}{3}}}d\left(-\sqrt{3}+2e^{\frac{bx}{3}}\right)-\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(-\int\frac{1}{-1-e^{\frac{2bx}{3}}}d\left(\sqrt{3}+\right.\right.\right.\right.\right.}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{3e^{5a/3}\left(\frac{1}{3}\left(\frac{3}{5}e^{\frac{5bx}{3}}-10\left(\frac{1}{6}\left(-\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\arctan\left(\sqrt{3}-2e^{\frac{bx}{3}}\right)\right)+\frac{1}{6}\left(\arctan\left(2e^{\frac{bx}{3}}+\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\right.\right.\right.\right.}{b}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3e^{5a/3}\left(\frac{1}{3}\left(\frac{3}{5}e^{\frac{5bx}{3}}-10\left(\frac{1}{3}\arctan\left(e^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\log\left(-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}+1\right)-\arctan\left(\sqrt{3}-2e^{\frac{bx}{3}}\right)\right)+\frac{1}{6}\left(\arctan\left(\right.\right.\right.\right.\right.}{b}$$

input
```
Int[E^((5*(a + b*x))/3)*Tanh[d + b*x]^2,x]
```

output
```
(3*E^((5*a)/3)*((2*E^((5*b*x)/3))/(3*(1 + E^(2*b*x))) + ((3*E^((5*b*x)/3))
/5 - 10*(ArcTan[E^((b*x)/3)]/3 + (-ArcTan[Sqrt[3] - 2*E^((b*x)/3)] + (Sqrt
[3]*Log[1 - Sqrt[3]*E^((b*x)/3) + E^((2*b*x)/3)])/2)/6 + (ArcTan[Sqrt[3] +
 2*E^((b*x)/3)] - (Sqrt[3]*Log[1 + Sqrt[3]*E^((b*x)/3) + E^((2*b*x)/3)])/2
)/6))/3))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 963
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Simp[1/(a*b^2*n*(p + 1))   Int[(e*x)^m*(a + b*x
^n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.32 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.67

method	result
risch	$\frac{3\,{e}^{\frac{5bx}{3}+\frac{5a}{3}}}{5b}+\frac{2\,{e}^{\frac{5bx}{3}+\frac{5a}{3}}}{b\left(1+{e}^{2bx+2d}\right)}+\frac{5i\ln\left({e}^{\frac{bx}{3}+\frac{d}{3}}-i\right){e}^{\frac{5a}{3}-\frac{5d}{3}}}{3b}-\frac{5i\ln\left({e}^{\frac{bx}{3}+\frac{d}{3}}+i\right){e}^{\frac{5a}{3}-\frac{5d}{3}}}{3b}+\left(\sum_{\_R=\operatorname{RootOf}\left(81b^{4}\_Z^{4}-225b^{2}\_\right.}\right.$

input
```
int(exp(5/3*b*x+5/3*a)*tanh(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
3/5*exp(5/3*b*x+5/3*a)/b+2*exp(5/3*b*x+5/3*a)/b/(1+exp(2*b*x+2*d))+5/3*I/b
*ln(exp(1/3*b*x+1/3*d)-I)*exp(5/3*a-5/3*d)-5/3*I/b*ln(exp(1/3*b*x+1/3*d)+I
)*exp(5/3*a-5/3*d)+sum(_R*ln(exp(1/3*b*x+1/3*d)-27/125*b^3*_R^3+3/5*b*_R),
_R=RootOf(81*_Z^4*b^4-225*_Z^2*b^2+625))*exp(5/3*a-5/3*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3184 vs. 2(168) = 336.

Time = 0.13 (sec) , antiderivative size = 3184, normalized size of antiderivative = 13.96

$$\int e^{\frac{5}{3}(a+bx)}\tanh^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*tanh(b*x+d)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+bx)}\tanh^2(d+bx)\,dx = e^{\frac{5a}{3}}\int e^{\frac{5bx}{3}}\tanh^2(bx+d)\,dx$$

input
```
integrate(exp(5/3*b*x+5/3*a)*tanh(b*x+d)**2,x)
```

output
```
exp(5*a/3)*Integral(exp(5*b*x/3)*tanh(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.75

$$\int e^{\frac{5}{3}(a+bx)}\tanh^2(d+bx)\,dx$$
$$= \frac{5\left(\sqrt{3}\log\left(\sqrt{3}e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{\left(-\frac{2}{3}bx-\frac{2}{3}d\right)}+1\right)-\sqrt{3}\log\left(-\sqrt{3}e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{\left(-\frac{2}{3}bx-\frac{2}{3}d\right)}+1\right)+2\arctan}{6b}$$
$$+\frac{\left(13e^{(-2bx-2d)}+3\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}}{5b\left(e^{\left(-\frac{5}{3}bx-\frac{5}{3}d\right)}+e^{\left(-\frac{11}{3}bx-\frac{11}{3}d\right)}\right)}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*tanh(b*x+d)^2,x, algorithm="maxima")
```

output
```
5/6*(sqrt(3)*log(sqrt(3)*e^(-1/3*b*x - 1/3*d) + e^(-2/3*b*x - 2/3*d) + 1)
- sqrt(3)*log(-sqrt(3)*e^(-1/3*b*x - 1/3*d) + e^(-2/3*b*x - 2/3*d) + 1) +
2*arctan(sqrt(3) + 2*e^(-1/3*b*x - 1/3*d)) + 2*arctan(-sqrt(3) + 2*e^(-1/3
*b*x - 1/3*d)) + 4*arctan(e^(-1/3*b*x - 1/3*d)))*e^(5/3*a - 5/3*d)/b + 1/5
*(13*e^(-2*b*x - 2*d) + 3)*e^(5/3*a - 5/3*d)/(b*(e^(-5/3*b*x - 5/3*d) + e^
(-11/3*b*x - 11/3*d)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.88

$$\int e^{\frac{5}{3}(a+bx)} \tanh^2(d+bx)\,dx = \frac{25\sqrt{3}e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)} \log\left(\sqrt{3}e^{\left(\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{\left(\frac{2}{3}bx\right)}+e^{\left(-\frac{2}{3}d\right)}\right) - 25\sqrt{3}e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)} \log\left(-\sqrt{3}e^{\left(\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{\left(\frac{2}{3}bx\right)}+e\right.}{}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*tanh(b*x+d)^2,x, algorithm="giac")
```

output
```
1/30*(25*sqrt(3)*e^(5/3*a - 5/3*d)*log(sqrt(3)*e^(1/3*b*x - 1/3*d) + e^(2/
3*b*x) + e^(-2/3*d)) - 25*sqrt(3)*e^(5/3*a - 5/3*d)*log(-sqrt(3)*e^(1/3*b*
x - 1/3*d) + e^(2/3*b*x) + e^(-2/3*d)) - 50*arctan((sqrt(3)*e^(-1/3*d) + 2
*e^(1/3*b*x))*e^(1/3*d))*e^(5/3*a - 5/3*d) - 50*arctan(-(sqrt(3)*e^(-1/3*d
) - 2*e^(1/3*b*x))*e^(1/3*d))*e^(5/3*a - 5/3*d) - 100*arctan(e^(1/3*b*x +
1/3*d))*e^(5/3*a - 5/3*d) + 60*e^(5/3*b*x + 5/3*a)/(e^(2*b*x + 2*d) + 1) +
 18*e^(5/3*b*x + 5/3*a))/b
```

Mupad [B] (verification not implemented)

Time = 6.30 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.04

$$\int e^{\frac{5}{3}(a+bx)} \tanh^2(d+bx)\,dx = \text{Too large to display}$$

input
```
int(exp((5*a)/3 + (5*b*x)/3)*tanh(d + b*x)^2,x)
```

output
```
(3*exp((5*a)/3 + (5*b*x)/3))/(5*b) - (5*(-exp(10*a - 10*d))^(1/6)*log((100
*exp((10*a)/3)*exp(-(10*d)/3))/9 - (100*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3
)*exp((b*x)/3)*(-exp(10*a)*exp(-10*d))^(1/6))/9))/(3*b) + (5*(-exp(10*a -
10*d))^(1/6)*log((100*exp((10*a)/3)*exp(-(10*d)/3))/9 + (100*exp((5*a)/3)*
exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*(-exp(10*a)*exp(-10*d))^(1/6))/9))/(3*
b) + (2*exp((5*a)/3 + (5*b*x)/3))/(b*(exp(2*d + 2*b*x) + 1)) - (5*log((100
*exp((10*a)/3)*exp(-(10*d)/3))/9 - (100*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3
)*exp((b*x)/3)*((3^(1/2)*1i)/2 - 1/2)*(-exp(10*a)*exp(-10*d))^(1/6))/9)*(-
exp(10*a - 10*d))^(1/6)*((3^(1/2)*1i)/2 - 1/2))/(3*b) + (5*log((100*exp((1
0*a)/3)*exp(-(10*d)/3))/9 + (100*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((
b*x)/3)*((3^(1/2)*1i)/2 - 1/2)*(-exp(10*a)*exp(-10*d))^(1/6))/9)*(-exp(10*
a - 10*d))^(1/6)*((3^(1/2)*1i)/2 - 1/2))/(3*b) - (5*log((100*exp((10*a)/3)
*exp(-(10*d)/3))/9 - (100*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)
*((3^(1/2)*1i)/2 + 1/2)*(-exp(10*a)*exp(-10*d))^(1/6))/9)*(-exp(10*a - 10*
d))^(1/6)*((3^(1/2)*1i)/2 + 1/2))/(3*b) + (5*log((100*exp((10*a)/3)*exp(-(
10*d)/3))/9 + (100*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1
/2)*1i)/2 + 1/2)*(-exp(10*a)*exp(-10*d))^(1/6))/9)*(-exp(10*a - 10*d))^(1/
6)*((3^(1/2)*1i)/2 + 1/2))/(3*b)
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+bx)}\tanh^2(d+bx)\,dx=\int e^{\frac{5bx}{3}+\frac{5a}{3}}\tanh\left(bx+d\right)^2dx$$

input
```
int(exp(5/3*b*x+5/3*a)*tanh(b*x+d)^2,x)
```

output
```
int(e**((5*a + 5*b*x)/3)*tanh(b*x + d)**2,x)
```

3.52 $\int e^{\frac{5}{3}(a+bx)}\tanh^3(d+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 271

$$\begin{aligned}\int e^{\frac{5}{3}(a+bx)}\tanh^3(d+bx)\,dx &= \frac{3e^{\frac{5(a-d)}{3}+\frac{5}{3}(d+bx)}}{5b}-\frac{2e^{\frac{5(a-d)}{3}+\frac{5}{3}(d+bx)}}{b\left(1+e^{2(d+bx)}\right)^2}\\&\quad+\frac{11e^{\frac{5(a-d)}{3}+\frac{5}{3}(d+bx)}}{3b\left(1+e^{2(d+bx)}\right)}-\frac{43e^{\frac{5(a-d)}{3}}\arctan\left(e^{\frac{1}{3}(d+bx)}\right)}{9b}\\&\quad+\frac{43e^{\frac{5(a-d)}{3}}\arctan\left(\sqrt{3}-2e^{\frac{1}{3}(d+bx)}\right)}{18b}\\&\quad-\frac{43e^{\frac{5(a-d)}{3}}\arctan\left(\sqrt{3}+2e^{\frac{1}{3}(d+bx)}\right)}{18b}\\&\quad+\frac{43e^{\frac{5(a-d)}{3}}\operatorname{arctanh}\left(\frac{\sqrt{3}e^{\frac{1}{3}(d+bx)}}{1+e^{\frac{2}{3}(d+bx)}}\right)}{6\sqrt{3}b}\end{aligned}$$

output

```
3/5*exp(5/3*b*x+5/3*a)/b-2*exp(5/3*b*x+5/3*a)/b/(1+exp(2*b*x+2*d))^2+11/3*
exp(5/3*b*x+5/3*a)/b/(1+exp(2*b*x+2*d))-43/9*exp(5/3*a-5/3*d)*arctan(exp(1
/3*b*x+1/3*d))/b-43/18*exp(5/3*a-5/3*d)*arctan(-3^(1/2)+2*exp(1/3*b*x+1/3*
d))/b-43/18*exp(5/3*a-5/3*d)*arctan(3^(1/2)+2*exp(1/3*b*x+1/3*d))/b+43/18*
3^(1/2)*exp(5/3*a-5/3*d)*arctanh(3^(1/2)*exp(1/3*b*x+1/3*d)/(1+exp(2/3*b*x
+2/3*d)))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.63

$$\int e^{\frac{5}{3}(a+bx)}\tanh^3(d+bx)\,dx$$

$$= \frac{e^{5a/3}\left(162e^{\frac{5bx}{3}}+215\text{RootSum}\left[\cosh(d)-\sinh(d)+\cosh(d)\#1^6+\sinh(d)\#1^6\&,\frac{bx-3\log\left(e^{\frac{bx}{3}}-\#1\right)}{\#1}\&\right](c\right.}{270b}$$

input
```
Integrate[E^((5*(a + b*x))/3)*Tanh[d + b*x]^3,x]
```

output
```
(E^((5*a)/3)*(162*E^((5*b*x)/3) + 215*RootSum[Cosh[d] - Sinh[d] + Cosh[d]*
#1^6 + Sinh[d]*#1^6 & , (b*x - 3*Log[E^((b*x)/3) - #1])/#1 & ]*(Cosh[d] -
Sinh[d])^2 - (540*E^((5*b*x)/3)*(Cosh[d] - Sinh[d])^2)/((1 + E^(2*b*x))*Co
sh[d] + (-1 + E^(2*b*x))*Sinh[d])^2 + (990*E^((5*b*x)/3)*(Cosh[d] - Sinh[d
]))/((1 + E^(2*b*x))*Cosh[d] + (-1 + E^(2*b*x))*Sinh[d])))/(270*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.86, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {2720, 25, 27, 968, 27, 1047, 27, 959, 824, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)}\tanh^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{3\int -\frac{e^{\frac{5a}{3}+\frac{4bx}{3}}\left(1-e^{2bx}\right)^3}{\left(1+e^{2bx}\right)^3}de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3\int \frac{e^{\frac{5a}{3}+\frac{4bx}{3}}\left(1-e^{2bx}\right)^3}{\left(1+e^{2bx}\right)^3}de^{\frac{bx}{3}}}{b}$$

$\big\downarrow$ 27

$$-\frac{3e^{5a/3}\int \frac{e^{\frac{4bx}{3}}\left(1-e^{2bx}\right)^3}{\left(1+e^{2bx}\right)^3}de^{\frac{bx}{3}}}{b}$$

$\big\downarrow$ 968

$$-\frac{3e^{5a/3}\left(\frac{e^{\frac{5bx}{3}}\left(1-e^{2bx}\right)^2}{6\left(e^{2bx}+1\right)^2}-\frac{1}{12}\int -\frac{2e^{\frac{4bx}{3}}\left(1-e^{2bx}\right)\left(1+11e^{2bx}\right)}{\left(1+e^{2bx}\right)^2}de^{\frac{bx}{3}}\right)}{b}$$

$\big\downarrow$ 27

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\int \frac{e^{\frac{4bx}{3}}\left(1-e^{2bx}\right)\left(1+11e^{2bx}\right)}{\left(1+e^{2bx}\right)^2}de^{\frac{bx}{3}}+\frac{e^{\frac{5bx}{3}}\left(1-e^{2bx}\right)^2}{6\left(e^{2bx}+1\right)^2}\right)}{b}$$

$\big\downarrow$ 1047

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\left(\frac{e^{\frac{5bx}{3}}\left(11e^{2bx}+1\right)}{3\left(e^{2bx}+1\right)}-\frac{1}{6}\int \frac{4e^{\frac{4bx}{3}}\left(1+44e^{2bx}\right)}{1+e^{2bx}}de^{\frac{bx}{3}}\right)+\frac{e^{\frac{5bx}{3}}\left(1-e^{2bx}\right)^2}{6\left(e^{2bx}+1\right)^2}\right)}{b}$$

$\big\downarrow$ 27

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\left(\frac{e^{\frac{5bx}{3}}\left(11e^{2bx}+1\right)}{3\left(e^{2bx}+1\right)}-\frac{2}{3}\int \frac{e^{\frac{4bx}{3}}\left(1+44e^{2bx}\right)}{1+e^{2bx}}de^{\frac{bx}{3}}\right)+\frac{e^{\frac{5bx}{3}}\left(1-e^{2bx}\right)^2}{6\left(e^{2bx}+1\right)^2}\right)}{b}$$

$\big\downarrow$ 959

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\left(\frac{e^{\frac{5bx}{3}}\left(11e^{2bx}+1\right)}{3\left(e^{2bx}+1\right)}-\frac{2}{3}\left(\frac{44}{5}e^{\frac{5bx}{3}}-43\int \frac{e^{\frac{4bx}{3}}}{1+e^{2bx}}de^{\frac{bx}{3}}\right)\right)+\frac{e^{\frac{5bx}{3}}\left(1-e^{2bx}\right)^2}{6\left(e^{2bx}+1\right)^2}\right)}{b}$$

$\big\downarrow$ 824

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\left(\frac{e^{\frac{5bx}{3}}\left(11e^{2bx}+1\right)}{3\left(e^{2bx}+1\right)}-\frac{2}{3}\left(\frac{44}{5}e^{\frac{5bx}{3}}-43\left(\frac{1}{3}\int \frac{1}{1+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{3}\int -\frac{1-\sqrt{3}e^{\frac{bx}{3}}}{2\left(1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}\right)}de^{\frac{bx}{3}}+\frac{1}{3}\int -\frac{1+\sqrt{3}e^{\frac{bx}{3}}}{2\left(1+\sqrt{3}e^{\frac{bx}{3}}\right.}\right.\right.\right.\right.}{b}$$

$\big\downarrow$ 27

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\left(\frac{e^{\frac{5bx}{3}}(11e^{2bx}+1)}{3(e^{2bx}+1)}-\frac{2}{3}\left(\frac{44}{5}e^{\frac{5bx}{3}}-43\left(\frac{1}{3}\int\frac{1}{1+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int\frac{1-\sqrt{3}e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int\frac{1+\sqrt{3}e^{\frac{bx}{3}}}{1+\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)\right.\right.\right.}{b}$$

$\downarrow$ 216

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\left(\frac{e^{\frac{5bx}{3}}(11e^{2bx}+1)}{3(e^{2bx}+1)}-\frac{2}{3}\left(\frac{44}{5}e^{\frac{5bx}{3}}-43\left(-\frac{1}{6}\int\frac{1-\sqrt{3}e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int\frac{1+\sqrt{3}e^{\frac{bx}{3}}}{1+\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{3}\arctan\left(e^{\frac{bx}{3}}\right)\right.\right.\right.}{b}$$

$\downarrow$ 1142

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\left(\frac{e^{\frac{5bx}{3}}(11e^{2bx}+1)}{3(e^{2bx}+1)}-\frac{2}{3}\left(\frac{44}{5}e^{\frac{5bx}{3}}-43\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{2}\sqrt{3}\int-\frac{\sqrt{3}-2e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\int\right.\right.\right.\right.}{b}$$

$\downarrow$ 25

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\left(\frac{e^{\frac{5bx}{3}}(11e^{2bx}+1)}{3(e^{2bx}+1)}-\frac{2}{3}\left(\frac{44}{5}e^{\frac{5bx}{3}}-43\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\int-\right.\right.\right.\right.}{b}$$

$\downarrow$ 1083

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\left(\frac{e^{\frac{5bx}{3}}(11e^{2bx}+1)}{3(e^{2bx}+1)}-\frac{2}{3}\left(\frac{44}{5}e^{\frac{5bx}{3}}-43\left(\frac{1}{6}\left(-\int\frac{1}{-1-e^{\frac{2bx}{3}}}d\left(-\sqrt{3}+2e^{\frac{bx}{3}}\right)-\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\right.\right.\right.}{b}$$

$\downarrow$ 217

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\left(\frac{e^{\frac{5bx}{3}}(11e^{2bx}+1)}{3(e^{2bx}+1)}-\frac{2}{3}\left(\frac{44}{5}e^{\frac{5bx}{3}}-43\left(\frac{1}{6}\left(-\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\arctan\left(\sqrt{3}-2e^{\frac{bx}{3}}\right)\right)+\frac{1}{6}\left(\text{arcta}\right.\right.\right.\right.}{b}$$

$\downarrow$ 1103

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\left(\frac{e^{\frac{5bx}{3}}(11e^{2bx}+1)}{3(e^{2bx}+1)}-\frac{2}{3}\left(\frac{44}{5}e^{\frac{5bx}{3}}-43\left(\frac{1}{3}\arctan\left(e^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\log\left(-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}+1\right)-\arctan\left(\sqrt{3}-\right.\right.\right.\right.\right.}{b}$$

input
```
Int[E^((5*(a + b*x))/3)*Tanh[d + b*x]^3,x]
```

output
```
(-3*E^((5*a)/3)*((E^((5*b*x)/3)*(1 - E^(2*b*x))^2)/(6*(1 + E^(2*b*x))^2) +
 ((E^((5*b*x)/3)*(1 + 11*E^(2*b*x)))/(3*(1 + E^(2*b*x))) - (2*((44*E^((5*b
*x)/3))/5 - 43*(ArcTan[E^((b*x)/3)]/3 + (-ArcTan[Sqrt[3] - 2*E^((b*x)/3)]
+ (Sqrt[3]*Log[1 - Sqrt[3]*E^((b*x)/3) + E^((2*b*x)/3)])/2)/6 + (ArcTan[Sq
rt[3] + 2*E^((b*x)/3)] - (Sqrt[3]*Log[1 + Sqrt[3]*E^((b*x)/3) + E^((2*b*x)
/3)])/2)/6)))/3)/6))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 968
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
 x]
```

rule 1047
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Simp[1/(
a*b*n*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c
*(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m
+ n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e
- a*f])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.36 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.61

method	result
risch	$\frac{3\,\mathrm{e}^{\frac{5bx}{3}+\frac{5a}{3}}}{5b}+\frac{(11\,\mathrm{e}^{2bx+2d}+5)\mathrm{e}^{\frac{5bx}{3}+\frac{5a}{3}}}{3(1+\mathrm{e}^{2bx+2d})^{2}b}+\frac{43i\ln\left(\mathrm{e}^{\frac{bx}{3}+\frac{d}{3}}-i\right)\mathrm{e}^{\frac{5a}{3}-\frac{5d}{3}}}{18b}-\frac{43i\ln\left(\mathrm{e}^{\frac{bx}{3}+\frac{d}{3}}+i\right)\mathrm{e}^{\frac{5a}{3}-\frac{5d}{3}}}{18b}+\Big(\_R=\mathrm{RootOf}(1049$

input
```
int(exp(5/3*b*x+5/3*a)*tanh(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
3/5*exp(5/3*b*x+5/3*a)/b+1/3/(1+exp(2*b*x+2*d))^2/b*(11*exp(2*b*x+2*d)+5)*
exp(5/3*b*x+5/3*a)+43/18*I/b*ln(exp(1/3*b*x+1/3*d)-I)*exp(5/3*a-5/3*d)-43/
18*I/b*ln(exp(1/3*b*x+1/3*d)+I)*exp(5/3*a-5/3*d)+sum(_R*ln(exp(1/3*b*x+1/3
*d)-5832/79507*b^3*_R^3+18/43*b*_R),_R=RootOf(104976*_Z^4*b^4-599076*_Z^2*
b^2+3418801))*exp(5/3*a-5/3*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6325 vs. 2(195) = 390.

Time = 0.16 (sec) , antiderivative size = 6325, normalized size of antiderivative = 23.34

$$\int e^{\frac{5}{3}(a+bx)}\tanh^3(d+bx)\,dx=\text{Too large to display}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*tanh(b*x+d)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+bx)}\tanh^3(d+bx)\,dx=e^{\frac{5a}{3}}\int e^{\frac{5bx}{3}}\tanh^3{(bx+d)}\,dx$$

input
```
integrate(exp(5/3*b*x+5/3*a)*tanh(b*x+d)**3,x)
```

output
```
exp(5*a/3)*Integral(exp(5*b*x/3)*tanh(b*x + d)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.72

$$\int e^{\frac{5}{3}(a+bx)}\tanh^3(d+bx)\,dx = \frac{43\left(\sqrt{3}\log\left(\sqrt{3}e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}+e^{\left(-\frac{2}{3}\,bx-\frac{2}{3}\,d\right)}+1\right)-\sqrt{3}\log\left(-\sqrt{3}e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}+e^{\left(-\frac{2}{3}\,bx-\frac{2}{3}\,d\right)}+1\right)+2\arctan\dots}{36\,b} + \frac{\left(73\,e^{(-2\,bx-2\,d)}+34\,e^{(-4\,bx-4\,d)}+9\right)e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,d\right)}}{15\,b\left(e^{\left(-\frac{5}{3}\,bx-\frac{5}{3}\,d\right)}+2\,e^{\left(-\frac{11}{3}\,bx-\frac{11}{3}\,d\right)}+e^{\left(-\frac{17}{3}\,bx-\frac{17}{3}\,d\right)}\right)}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*tanh(b*x+d)^3,x, algorithm="maxima")
```

output
```
43/36*(sqrt(3)*log(sqrt(3)*e^(-1/3*b*x - 1/3*d) + e^(-2/3*b*x - 2/3*d) + 1
) - sqrt(3)*log(-sqrt(3)*e^(-1/3*b*x - 1/3*d) + e^(-2/3*b*x - 2/3*d) + 1)
+ 2*arctan(sqrt(3) + 2*e^(-1/3*b*x - 1/3*d)) + 2*arctan(-sqrt(3) + 2*e^(-1
/3*b*x - 1/3*d)) + 4*arctan(e^(-1/3*b*x - 1/3*d)))*e^(5/3*a - 5/3*d)/b + 1
/15*(73*e^(-2*b*x - 2*d) + 34*e^(-4*b*x - 4*d) + 9)*e^(5/3*a - 5/3*d)/(b*(
e^(-5/3*b*x - 5/3*d) + 2*e^(-11/3*b*x - 11/3*d) + e^(-17/3*b*x - 17/3*d)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.80

$$\int e^{\frac{5}{3}(a+bx)}\tanh^3(d+bx)\,dx$$

$$= \frac{215\sqrt{3}e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(\sqrt{3}e^{\left(\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{\left(\frac{2}{3}bx\right)}+e^{\left(-\frac{2}{3}d\right)}\right)-215\sqrt{3}e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(-\sqrt{3}e^{\left(\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{\left(\frac{2}{3}bx\right)}+\right.}{}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*tanh(b*x+d)^3,x, algorithm="giac")
```

output
```
1/180*(215*sqrt(3)*e^(5/3*a - 5/3*d)*log(sqrt(3)*e^(1/3*b*x - 1/3*d) + e^(
2/3*b*x) + e^(-2/3*d)) - 215*sqrt(3)*e^(5/3*a - 5/3*d)*log(-sqrt(3)*e^(1/3
*b*x - 1/3*d) + e^(2/3*b*x) + e^(-2/3*d)) - 430*arctan((sqrt(3)*e^(-1/3*d)
 + 2*e^(1/3*b*x))*e^(1/3*d))*e^(5/3*a - 5/3*d) - 430*arctan(-(sqrt(3)*e^(-
1/3*d) - 2*e^(1/3*b*x))*e^(1/3*d))*e^(5/3*a - 5/3*d) - 860*arctan(e^(1/3*b
*x + 1/3*d))*e^(5/3*a - 5/3*d) + 60*(11*e^(11/3*b*x + 5/3*a + 2*d) + 5*e^(
5/3*b*x + 5/3*a))/(e^(2*b*x + 2*d) + 1)^2 + 108*e^(5/3*b*x + 5/3*a))/b
```

Mupad [B] (verification not implemented)

Time = 5.65 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.86

$$\int e^{\frac{5}{3}(a+bx)}\tanh^3(d+bx)\,dx = \text{Too large to display}$$

input
```
int(exp((5*a)/3 + (5*b*x)/3)*tanh(d + b*x)^3,x)
```

output
```
(3*exp((5*a)/3 + (5*b*x)/3))/(5*b) - (43*(-exp(10*a - 10*d))^(1/6)*log((18
49*exp((10*a)/3)*exp(-(10*d)/3))/81 - (1849*exp((5*a)/3)*exp(d/3)*exp(-(5*
d)/3)*exp((b*x)/3)*(-exp(10*a)*exp(-10*d))^(1/6))/81))/(18*b) + (43*(-exp(
10*a - 10*d))^(1/6)*log((1849*exp((10*a)/3)*exp(-(10*d)/3))/81 + (1849*exp
((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*(-exp(10*a)*exp(-10*d))^(1/6
))/81))/(18*b) + (11*exp((5*a)/3 + (5*b*x)/3))/(3*b*(exp(2*d + 2*b*x) + 1)
) - (2*exp((5*a)/3 + (5*b*x)/3))/(b*(2*exp(2*d + 2*b*x) + exp(4*d + 4*b*x)
 + 1)) - (43*log((1849*exp((10*a)/3)*exp(-(10*d)/3))/81 - (1849*exp((5*a)/
3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)/2 - 1/2)*(-exp(10*a)*
exp(-10*d))^(1/6))/81)*(-exp(10*a - 10*d))^(1/6)*((3^(1/2)*1i)/2 - 1/2))/(
18*b) + (43*log((1849*exp((10*a)/3)*exp(-(10*d)/3))/81 + (1849*exp((5*a)/3
)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)/2 - 1/2)*(-exp(10*a)*e
xp(-10*d))^(1/6))/81)*(-exp(10*a - 10*d))^(1/6)*((3^(1/2)*1i)/2 - 1/2))/(1
8*b) - (43*log((1849*exp((10*a)/3)*exp(-(10*d)/3))/81 - (1849*exp((5*a)/3)
*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)/2 + 1/2)*(-exp(10*a)*ex
p(-10*d))^(1/6))/81)*(-exp(10*a - 10*d))^(1/6)*((3^(1/2)*1i)/2 + 1/2))/(18
*b) + (43*log((1849*exp((10*a)/3)*exp(-(10*d)/3))/81 + (1849*exp((5*a)/3)*
exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)/2 + 1/2)*(-exp(10*a)*exp
(-10*d))^(1/6))/81)*(-exp(10*a - 10*d))^(1/6)*((3^(1/2)*1i)/2 + 1/2))/(18*
b)
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+bx)}\tanh^3(d+bx)\,dx=\int e^{\frac{5bx}{3}+\frac{5a}{3}}\tanh\left(bx+d\right)^3dx$$

input
```
int(exp(5/3*b*x+5/3*a)*tanh(b*x+d)^3,x)
```

output
```
int(e**((5*a + 5*b*x)/3)*tanh(b*x + d)**3,x)
```

3.53 $\int F^{c(a+bx)} \tanh(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 80

$$\int F^{c(a+bx)}\tanh(d+ex)\,dx = \frac{F^{c(a+bx)}}{bc\log(F)} - \frac{2F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},-e^{2d+2ex}\right)}{bc\log(F)}$$

output
```
F^(c*(b*x+a))/b/c/ln(F)-2*F^(c*(b*x+a))*hypergeom([1, 1/2*b*c*ln(F)/e],[1+
1/2*b*c*ln(F)/e],-exp(2*e*x+2*d))/b/c/ln(F)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int F^{c(a+bx)}\tanh(d+ex)\,dx = \frac{F^{c(a+bx)}\left(1-2\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)\right)}{bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Tanh[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*(1 - 2*Hypergeometric2F1[1, (b*c*Log[F])/(2*e), 1 + (b*c*
Log[F])/(2*e), -E^(2*(d + e*x))]))/(b*c*Log[F])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6007, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \ 6007$$

$$\int \left(F^{c(a+bx)} - \frac{2F^{c(a+bx)}}{e^{2(d+ex)}+1}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{F^{c(a+bx)}}{bc\log(F)} - \frac{2F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1, \frac{bc\log(F)}{2e}, \frac{bc\log(F)}{2e}+1, -e^{2(d+ex)}\right)}{bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Tanh[d + e*x],x]
```

output
```
F^(c*(a + b*x))/(b*c*Log[F]) - (2*F^(c*(a + b*x))*Hypergeometric2F1[1, (b*
c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6007
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tanh[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((-1 + E^(2*(d + e*x)))^n/(1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \tanh (ex+d)\, dx$$

input
```
int(F^(c*(b*x+a))*tanh(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*tanh(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)} \tanh(d+ex)\, dx = \int F^{(bx+a)c} \tanh (ex+d)\ dx$$

input
```
integrate(F^(c*(b*x+a))*tanh(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*tanh(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\tanh(d+ex)\,dx=\int F^{c(a+bx)}\tanh\left(d+ex\right)dx$$

input
```
integrate(F**(c*(b*x+a))*tanh(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*tanh(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\tanh(d+ex)\,dx=\int F^{(bx+a)c}\tanh\left(ex+d\right)\,dx$$

input
```
integrate(F^(c*(b*x+a))*tanh(e*x+d),x, algorithm="maxima")
```

output
```
4*F^(a*c)*e*integrate(F^(b*c*x)/(b*c*log(F) + (b*c*e^(4*d)*log(F) - 2*e*e^
(4*d))*e^(4*e*x) + 2*(b*c*e^(2*d)*log(F) - 2*e*e^(2*d))*e^(2*e*x) - 2*e),
x) - (F^(a*c)*b*c*log(F) + 2*F^(a*c)*e - (F^(a*c)*b*c*e^(2*d)*log(F) - 2*F
^(a*c)*e*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 2*b*c*e*log(F)
+ (b^2*c^2*e^(2*d)*log(F)^2 - 2*b*c*e*e^(2*d)*log(F))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\tanh(d+ex)\,dx=\int F^{(bx+a)c}\tanh\left(ex+d\right)\,dx$$

input
```
integrate(F^(c*(b*x+a))*tanh(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*tanh(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\tanh(d+ex)\,dx=\int F^{c\,(a+b\,x)}\tanh(d+e\,x)\;dx$$

input
```
int(F^(c*(a + b*x))*tanh(d + e*x),x)
```

output
```
int(F^(c*(a + b*x))*tanh(d + e*x), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\tanh(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\tanh\left(ex+d\right)dx\right)$$

input
```
int(F^(c*(b*x+a))*tanh(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*tanh(d + e*x),x)
```

3.54 $\int F^{c(a+bx)} \tanh^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 101

$$\int F^{c(a+bx)} \tanh^2(d+ex)\,dx = \frac{2F^{c(a+bx)}}{e\left(1+e^{2d+2ex}\right)} - \frac{2F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},-e^{2d+2ex}\right)}{e} + \frac{F^{c(a+bx)}}{bc\log(F)}$$

output
```
2*F^(c*(b*x+a))/e/(1+exp(2*e*x+2*d))-2*F^(c*(b*x+a))*hypergeom([1, 1/2*b*c
*ln(F)/e],[1+1/2*b*c*ln(F)/e],-exp(2*e*x+2*d))/e+F^(c*(b*x+a))/b/c/ln(F)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93

$$\int F^{c(a+bx)}\tanh^2(d+ex)\,dx$$
$$= F^{c(a+bx)}\left(\frac{2}{e+ee^{2d}}-\frac{2\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)}{e}+\frac{1}{bc\log(F)}-\frac{\text{sech}(d)\text{sech}(d+ex)\sinh(ex)}{e}\right)$$

input
```
Integrate[F^(c*(a + b*x))*Tanh[d + e*x]^2,x]
```

output
```
F^(c*(a + b*x))*(2/(e + e*E^(2*d)) - (2*Hypergeometric2F1[1, (b*c*Log[F])/
(2*e), 1 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/e + 1/(b*c*Log[F]) - (Se
ch[d]*Sech[d + e*x]*Sinh[e*x])/e)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.36, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6007, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(d+ex)F^{c(a+bx)}\,dx$$
$$\Big\downarrow \text{6007}$$
$$\int \left(-\frac{4F^{c(a+bx)}}{e^{2(d+ex)}+1}+\frac{4F^{c(a+bx)}}{\left(e^{2(d+ex)}+1\right)^2}+F^{c(a+bx)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{4F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,-e^{2(d+ex)}\right)}{bc\log(F)}+\frac{4F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,-e^{2(d+ex)}\right)}{bc\log(F)}+\frac{F^{c(a+bx)}}{bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Tanh[d + e*x]^2,x]
```

output
```
F^(c*(a + b*x))/(b*c*Log[F]) - (4*F^(c*(a + b*x))*Hypergeometric2F1[1, (b*
c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(b*c*Log[F]) +
 (4*F^(c*(a + b*x))*Hypergeometric2F1[2, (b*c*Log[F])/(2*e), 1 + (b*c*Log[
F])/(2*e), -E^(2*(d + e*x))])/(b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6007
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tanh[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((-1 + E^(2*(d + e*x)))^n/(1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)}\tanh\left(ex+d\right)^2dx$$

input
```
int(F^(c*(b*x+a))*tanh(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*tanh(e*x+d)^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\tanh^2(d+ex)\,dx = \int F^{(bx+a)c}\tanh\left(ex+d\right)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*tanh(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*tanh(e*x + d)^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\tanh^2(d+ex)\,dx = \int F^{c(a+bx)}\tanh^2\left(d+ex\right)dx$$

input
```
integrate(F**(c*(b*x+a))*tanh(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*tanh(d + e*x)**2, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\tanh^2(d+ex)\,dx = \int F^{(bx+a)c}\tanh\left(ex+d\right)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*tanh(e*x+d)^2,x, algorithm="maxima")
```

output
```
-16*F^(a*c)*b*c*e*integrate(F^(b*c*x)/(b^2*c^2*log(F)^2 - 6*b*c*e*log(F) +
 8*e^2 + (b^2*c^2*e^(6*d)*log(F)^2 - 6*b*c*e*e^(6*d)*log(F) + 8*e^2*e^(6*d
))*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*log(F) + 8*e^
2*e^(4*d))*e^(4*e*x) + 3*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^(2*d)*log(F
) + 8*e^2*e^(2*d))*e^(2*e*x)), x)*log(F) + (F^(a*c)*b^2*c^2*log(F)^2 + 10*
F^(a*c)*b*c*e*log(F) + 8*F^(a*c)*e^2 + (F^(a*c)*b^2*c^2*e^(4*d)*log(F)^2 -
 6*F^(a*c)*b*c*e*e^(4*d)*log(F) + 8*F^(a*c)*e^2*e^(4*d))*e^(4*e*x) - 2*(F^
(a*c)*b^2*c^2*e^(2*d)*log(F)^2 - 2*F^(a*c)*b*c*e*e^(2*d)*log(F) - 8*F^(a*c
)*e^2*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^3*c^3*log(F)^3 - 6*b^2*c^2*e*log(F)
^2 + 8*b*c*e^2*log(F) + (b^3*c^3*e^(4*d)*log(F)^3 - 6*b^2*c^2*e*e^(4*d)*lo
g(F)^2 + 8*b*c*e^2*e^(4*d)*log(F))*e^(4*e*x) + 2*(b^3*c^3*e^(2*d)*log(F)^3
 - 6*b^2*c^2*e*e^(2*d)*log(F)^2 + 8*b*c*e^2*e^(2*d)*log(F))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)} \tanh^2(d+ex)\,dx = \int F^{(bx+a)c} \tanh(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*tanh(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*tanh(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)} \tanh^2(d+ex)\,dx = \int F^{c\,(a+b\,x)} \tanh(d+e\,x)^2\,dx$$

input
```
int(F^(c*(a + b*x))*tanh(d + e*x)^2,x)
```

output
```
int(F^(c*(a + b*x))*tanh(d + e*x)^2, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\tanh^2(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\tanh(ex+d)^2\,dx\right)$$

input
```
int(F^(c*(b*x+a))*tanh(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*tanh(d + e*x)**2,x)
```

3.55 $\int F^{c(a+bx)}\tanh^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 155

$$\int F^{c(a+bx)}\tanh^3(d+ex)\,dx = -\frac{2F^{c(a+bx)}}{e\left(1+e^{2d+2ex}\right)^2}+\frac{F^{c(a+bx)}}{bc\log(F)}+\frac{F^{c(a+bx)}(2e+bc\log(F))}{e^2\left(1+e^{2d+2ex}\right)} - F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},-e^{2d+2ex}\right)\left(\frac{2}{bc\log(F)}+\frac{bc\log(F)}{e^2}\right)$$

output
```
-2*F^(c*(b*x+a))/e/(1+exp(2*e*x+2*d))^2+F^(c*(b*x+a))/b/c/ln(F)+F^(c*(b*x+
a))*(2*e+b*c*ln(F))/e^2/(1+exp(2*e*x+2*d))-F^(c*(b*x+a))*hypergeom([1, 1/2
*b*c*ln(F)/e],[1+1/2*b*c*ln(F)/e],-exp(2*e*x+2*d))*(2/b/c/ln(F)+b*c*ln(F)/
e^2)
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.97

$$\int F^{c(a+bx)}\tanh^3(d+ex)\,dx$$
$$=\frac{1}{2}F^{c(a+bx)}\left(\frac{2\left(1-\left(1+e^{2d}\right)\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)\right)\left(2e^2+b^2c^2\log^2(F\right.}{bce^2\left(1+e^{2d}\right)\log(F)}\right.$$
$$\left.+\frac{\text{sech}^2(d+ex)}{e}-\frac{bc\log(F)\text{sech}(d)\text{sech}(d+ex)\sinh(ex)}{e^2}+\frac{2\tanh(d)}{bc\log(F)}\right)$$

input
```
Integrate[F^(c*(a + b*x))*Tanh[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*((2*(1 - (1 + E^(2*d))*Hypergeometric2F1[1, (b*c*Log[F])/
(2*e), 1 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])*(2*e^2 + b^2*c^2*Log[F]^
2))/(b*c*e^2*(1 + E^(2*d))*Log[F]) + Sech[d + e*x]^2/e - (b*c*Log[F]*Sech[
d]*Sech[d + e*x]*Sinh[e*x])/e^2 + (2*Tanh[d])/(b*c*Log[F])))/2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6007, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(d+ex)F^{c(a+bx)}\,dx$$
$$\Big\downarrow \text{ 6007}$$
$$\int \left(-\frac{6F^{c(a+bx)}}{e^{2(d+ex)}+1}+\frac{12F^{c(a+bx)}}{\left(e^{2(d+ex)}+1\right)^2}-\frac{8F^{c(a+bx)}}{\left(e^{2(d+ex)}+1\right)^3}+F^{c(a+bx)}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{6F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,-e^{2(d+ex)}\right)}{bc\log(F)}+\frac{12F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,-e^{2(d+ex)}\right)}{bc\log(F)}-\frac{8F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(3,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,-e^{2(d+ex)}\right)}{bc\log(F)}+\frac{F^{c(a+bx)}}{bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Tanh[d + e*x]^3,x]
```

output
```
F^(c*(a + b*x))/(b*c*Log[F]) - (6*F^(c*(a + b*x))*Hypergeometric2F1[1, (b*
c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(b*c*Log[F]) +
 (12*F^(c*(a + b*x))*Hypergeometric2F1[2, (b*c*Log[F])/(2*e), 1 + (b*c*Log
[F])/(2*e), -E^(2*(d + e*x))])/(b*c*Log[F]) - (8*F^(c*(a + b*x))*Hypergeom
etric2F1[3, (b*c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])
/(b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6007
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tanh[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((-1 + E^(2*(d + e*x)))^n/(1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)}\tanh(ex+d)^3\,dx$$

input
```
int(F^(c*(b*x+a))*tanh(e*x+d)^3,x)
```

output
```
int(F^(c*(b*x+a))*tanh(e*x+d)^3,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\tanh^3(d+ex)\,dx = \int F^{(bx+a)c}\tanh\left(ex+d\right)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*tanh(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*tanh(e*x + d)^3, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\tanh^3(d+ex)\,dx = \int F^{c(a+bx)}\tanh^3\left(d+ex\right)dx$$

input
```
integrate(F**(c*(b*x+a))*tanh(e*x+d)**3,x)
```

output
```
Integral(F**(c*(a + b*x))*tanh(d + e*x)**3, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\tanh^3(d+ex)\,dx = \int F^{(bx+a)c}\tanh\left(ex+d\right)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*tanh(e*x+d)^3,x, algorithm="maxima")
```

output
```
48*(F^(a*c)*b^2*c^2*e*log(F)^2 + 2*F^(a*c)*e^3)*integrate(F^(b*c*x)/(b^3*c
^3*log(F)^3 - 12*b^2*c^2*e*log(F)^2 + 44*b*c*e^2*log(F) - 48*e^3 + (b^3*c^
3*e^(8*d)*log(F)^3 - 12*b^2*c^2*e*e^(8*d)*log(F)^2 + 44*b*c*e^2*e^(8*d)*lo
g(F) - 48*e^3*e^(8*d))*e^(8*e*x) + 4*(b^3*c^3*e^(6*d)*log(F)^3 - 12*b^2*c^
2*e*e^(6*d)*log(F)^2 + 44*b*c*e^2*e^(6*d)*log(F) - 48*e^3*e^(6*d))*e^(6*e*
x) + 6*(b^3*c^3*e^(4*d)*log(F)^3 - 12*b^2*c^2*e*e^(4*d)*log(F)^2 + 44*b*c*
e^2*e^(4*d)*log(F) - 48*e^3*e^(4*d))*e^(4*e*x) + 4*(b^3*c^3*e^(2*d)*log(F)
^3 - 12*b^2*c^2*e*e^(2*d)*log(F)^2 + 44*b*c*e^2*e^(2*d)*log(F) - 48*e^3*e^
(2*d))*e^(2*e*x)), x) - (F^(a*c)*b^3*c^3*log(F)^3 + 36*F^(a*c)*b^2*c^2*e*l
og(F)^2 + 44*F^(a*c)*b*c*e^2*log(F) + 48*F^(a*c)*e^3 - (F^(a*c)*b^3*c^3*e^
(6*d)*log(F)^3 - 12*F^(a*c)*b^2*c^2*e*e^(6*d)*log(F)^2 + 44*F^(a*c)*b*c*e^
2*e^(6*d)*log(F) - 48*F^(a*c)*e^3*e^(6*d))*e^(6*e*x) + 3*(F^(a*c)*b^3*c^3*
e^(4*d)*log(F)^3 - 8*F^(a*c)*b^2*c^2*e*e^(4*d)*log(F)^2 + 4*F^(a*c)*b*c*e^
2*e^(4*d)*log(F) + 48*F^(a*c)*e^3*e^(4*d))*e^(4*e*x) - 3*(F^(a*c)*b^3*c^3*
e^(2*d)*log(F)^3 - 28*F^(a*c)*b*c*e^2*e^(2*d)*log(F) - 48*F^(a*c)*e^3*e^(2
*d))*e^(2*e*x))*F^(b*c*x)/(b^4*c^4*log(F)^4 - 12*b^3*c^3*e*log(F)^3 + 44*b
^2*c^2*e^2*log(F)^2 - 48*b*c*e^3*log(F) + (b^4*c^4*e^(6*d)*log(F)^4 - 12*b
^3*c^3*e*e^(6*d)*log(F)^3 + 44*b^2*c^2*e^2*e^(6*d)*log(F)^2 - 48*b*c*e^3*e
^(6*d)*log(F))*e^(6*e*x) + 3*(b^4*c^4*e^(4*d)*log(F)^4 - 12*b^3*c^3*e*e^(4
*d)*log(F)^3 + 44*b^2*c^2*e^2*e^(4*d)*log(F)^2 - 48*b*c*e^3*e^(4*d)*log...
```

Giac [F]

$$\int F^{c(a+bx)} \tanh^3(d+ex) \, dx = \int F^{(bx+a)c} \tanh{(ex+d)}^3 \, dx$$

input
```
integrate(F^(c*(b*x+a))*tanh(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*tanh(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\tanh^3(d+ex)\,dx=\int F^{c\,(a+b\,x)}\,\tanh(d+e\,x)^3\,dx$$

input
```
int(F^(c*(a + b*x))*tanh(d + e*x)^3,x)
```

output
```
int(F^(c*(a + b*x))*tanh(d + e*x)^3, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\tanh^3(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\tanh\left(ex+d\right)^3dx\right)$$

input
```
int(F^(c*(b*x+a))*tanh(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*tanh(d + e*x)**3,x)
```

3.56 $\int F^{c(a+bx)} \tanh^4(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 215

$$\int F^{c(a+bx)} \tanh^4(d+ex)\,dx = \frac{8F^{c(a+bx)}}{3e\left(1+e^{2d+2ex}\right)^3} + \frac{F^{c(a+bx)}}{bc\log(F)} - \frac{2F^{c(a+bx)}(6e+bc\log(F))}{3e^2\left(1+e^{2d+2ex}\right)^2} - \frac{F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},-e^{2d+2ex}\right)\left(8e^2+b^2c^2\log^2(F)\right)}{3e^3} + \frac{F^{c(a+bx)}\left(12e^2+2bce\log(F)+b^2c^2\log^2(F)\right)}{3e^3\left(1+e^{2d+2ex}\right)}$$

output

```
8/3*F^(c*(b*x+a))/e/(1+exp(2*e*x+2*d))^3+F^(c*(b*x+a))/b/c/ln(F)-2/3*F^(c*
(b*x+a))*(6*e+b*c*ln(F))/e^2/(1+exp(2*e*x+2*d))^2-1/3*F^(c*(b*x+a))*hyperg
eom([1, 1/2*b*c*ln(F)/e],[1+1/2*b*c*ln(F)/e],-exp(2*e*x+2*d))*(8*e^2+b^2*c
^2*ln(F)^2)/e^3+1/3*F^(c*(b*x+a))*(12*e^2+2*b*c*e*ln(F)+b^2*c^2*ln(F)^2)/e
^3/(1+exp(2*e*x+2*d))
```

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.84

$$\int F^{c(a+bx)}\tanh^4(d+ex)\,dx=\frac{1}{6}F^{c(a+bx)}\left(\frac{6}{bc\log(F)}+\frac{2\left(1-\left(1+e^{2d}\right)\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)\right)\left(8e^2+b^2c^2\log^2(F)\right)}{e^3\left(1+e^{2d}\right)}-\frac{\left(8e^2+b^2c^2\log^2(F)\right)\text{sech}(d)\text{sech}(d+ex)\sinh(ex)}{e^3}+\frac{2\text{sech}(d)\text{sech}^3(d+ex)\sinh(ex)}{e}+\frac{\text{sech}^2(d+ex)(bc\log(F)+2e\tanh(d))}{e^2}\right)$$

input
```
Integrate[F^(c*(a + b*x))*Tanh[d + e*x]^4,x]
```

output
```
(F^(c*(a + b*x))*(6/(b*c*Log[F]) + (2*(1 - (1 + E^(2*d))*Hypergeometric2F1
[1, (b*c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])*(8*e^2
+ b^2*c^2*Log[F]^2))/(e^3*(1 + E^(2*d))) - ((8*e^2 + b^2*c^2*Log[F]^2)*Sec
h[d]*Sech[d + e*x]*Sinh[e*x])/e^3 + (2*Sech[d]*Sech[d + e*x]^3*Sinh[e*x])/
e + (Sech[d + e*x]^2*(b*c*Log[F] + 2*e*Tanh[d]))/e^2))/6
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6007, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^4(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6007}$$

$$\int \left(-\frac{8F^{c(a+bx)}}{e^{2(d+ex)}+1}+\frac{24F^{c(a+bx)}}{\left(e^{2(d+ex)}+1\right)^2}-\frac{32F^{c(a+bx)}}{\left(e^{2(d+ex)}+1\right)^3}+\frac{16F^{c(a+bx)}}{\left(e^{2(d+ex)}+1\right)^4}+F^{c(a+bx)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{8F^{c(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,-e^{2(d+ex)}\right)}{bc\log(F)}+\frac{24F^{c(a+bx)}\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,-e^{2(d+ex)}\right)}{bc\log(F)}-\frac{32F^{c(a+bx)}\text{Hypergeometric2F1}\left(3,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,-e^{2(d+ex)}\right)}{bc\log(F)}+\frac{16F^{c(a+bx)}\text{Hypergeometric2F1}\left(4,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,-e^{2(d+ex)}\right)}{bc\log(F)}+\frac{F^{c(a+bx)}}{bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Tanh[d + e*x]^4,x]
```

output
```
F^(c*(a + b*x))/(b*c*Log[F]) - (8*F^(c*(a + b*x))*Hypergeometric2F1[1, (b*
c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(b*c*Log[F]) +
 (24*F^(c*(a + b*x))*Hypergeometric2F1[2, (b*c*Log[F])/(2*e), 1 + (b*c*Log
[F])/(2*e), -E^(2*(d + e*x))])/(b*c*Log[F]) - (32*F^(c*(a + b*x))*Hypergeo
metric2F1[3, (b*c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))]
)/(b*c*Log[F]) + (16*F^(c*(a + b*x))*Hypergeometric2F1[4, (b*c*Log[F])/(2*
e), 1 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6007
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tanh[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((-1 + E^(2*(d + e*x)))^n/(1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \tanh(ex+d)^4 \, dx$$

input
```
int(F^(c*(b*x+a))*tanh(e*x+d)^4,x)
```

output
```
int(F^(c*(b*x+a))*tanh(e*x+d)^4,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \tanh^4(d+ex) \, dx = \int F^{(bx+a)c} \tanh(ex+d)^4 \, dx$$

input
```
integrate(F^(c*(b*x+a))*tanh(e*x+d)^4,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*tanh(e*x + d)^4, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \tanh^4(d+ex) \, dx = \int F^{c(a+bx)} \tanh^4(d+ex) \, dx$$

input
```
integrate(F**(c*(b*x+a))*tanh(e*x+d)**4,x)
```

output
```
Integral(F**(c*(a + b*x))*tanh(d + e*x)**4, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\tanh^4(d+ex)\,dx=\int F^{(bx+a)c}\tanh\left(ex+d\right)^4\,dx$$

input
```
integrate(F^(c*(b*x+a))*tanh(e*x+d)^4,x, algorithm="maxima")
```

output
```
-128*(F^(a*c)*b^3*c^3*e*log(F)^3 + 8*F^(a*c)*b*c*e^3*log(F))*integrate(F^(
b*c*x)/(b^4*c^4*log(F)^4 - 20*b^3*c^3*e*log(F)^3 + 140*b^2*c^2*e^2*log(F)^
2 - 400*b*c*e^3*log(F) + 384*e^4 + (b^4*c^4*e^(10*d)*log(F)^4 - 20*b^3*c^3
*e*e^(10*d)*log(F)^3 + 140*b^2*c^2*e^2*e^(10*d)*log(F)^2 - 400*b*c*e^3*e^(
10*d)*log(F) + 384*e^4*e^(10*d))*e^(10*e*x) + 5*(b^4*c^4*e^(8*d)*log(F)^4
- 20*b^3*c^3*e*e^(8*d)*log(F)^3 + 140*b^2*c^2*e^2*e^(8*d)*log(F)^2 - 400*b
*c*e^3*e^(8*d)*log(F) + 384*e^4*e^(8*d))*e^(8*e*x) + 10*(b^4*c^4*e^(6*d)*l
og(F)^4 - 20*b^3*c^3*e*e^(6*d)*log(F)^3 + 140*b^2*c^2*e^2*e^(6*d)*log(F)^2
 - 400*b*c*e^3*e^(6*d)*log(F) + 384*e^4*e^(6*d))*e^(6*e*x) + 10*(b^4*c^4*e
^(4*d)*log(F)^4 - 20*b^3*c^3*e*e^(4*d)*log(F)^3 + 140*b^2*c^2*e^2*e^(4*d)*
log(F)^2 - 400*b*c*e^3*e^(4*d)*log(F) + 384*e^4*e^(4*d))*e^(4*e*x) + 5*(b^
4*c^4*e^(2*d)*log(F)^4 - 20*b^3*c^3*e*e^(2*d)*log(F)^3 + 140*b^2*c^2*e^2*e
^(2*d)*log(F)^2 - 400*b*c*e^3*e^(2*d)*log(F) + 384*e^4*e^(2*d))*e^(2*e*x))
, x) + (F^(a*c)*b^4*c^4*log(F)^4 + 108*F^(a*c)*b^3*c^3*e*log(F)^3 + 140*F^
(a*c)*b^2*c^2*e^2*log(F)^2 + 624*F^(a*c)*b*c*e^3*log(F) + 384*F^(a*c)*e^4
+ (F^(a*c)*b^4*c^4*e^(8*d)*log(F)^4 - 20*F^(a*c)*b^3*c^3*e*e^(8*d)*log(F)^
3 + 140*F^(a*c)*b^2*c^2*e^2*e^(8*d)*log(F)^2 - 400*F^(a*c)*b*c*e^3*e^(8*d)
*log(F) + 384*F^(a*c)*e^4*e^(8*d))*e^(8*e*x) - 4*(F^(a*c)*b^4*c^4*e^(6*d)*
log(F)^4 - 16*F^(a*c)*b^3*c^3*e*e^(6*d)*log(F)^3 + 68*F^(a*c)*b^2*c^2*e^2*
e^(6*d)*log(F)^2 + 16*F^(a*c)*b*c*e^3*e^(6*d)*log(F) - 384*F^(a*c)*e^4*...
```

Giac [F]

$$\int F^{c(a+bx)}\tanh^4(d+ex)\,dx=\int F^{(bx+a)c}\tanh\left(ex+d\right)^4\,dx$$

input
```
integrate(F^(c*(b*x+a))*tanh(e*x+d)^4,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*tanh(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\tanh^4(d+ex)\,dx=\int F^{c\,(a+b\,x)}\,\tanh(d+e\,x)^4\,dx$$

input
```
int(F^(c*(a + b*x))*tanh(d + e*x)^4,x)
```

output
```
int(F^(c*(a + b*x))*tanh(d + e*x)^4, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\tanh^4(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\tanh\left(ex+d\right)^4dx\right)$$

input
```
int(F^(c*(b*x+a))*tanh(e*x+d)^4,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*tanh(d + e*x)**4,x)
```

3.57 $\int e^{a+bx}\tanh^n(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 76

$$\int e^{a+bx}\tanh^n(a+bx)\,dx = \frac{e^{a+bx}\left(1-e^{2a+2bx}\right)^{-n}\left(-1+e^{2a+2bx}\right)^{n}\operatorname{AppellF1}\left(\frac{1}{2},-n,n,\frac{3}{2},e^{2a+2bx},-e^{2a+2bx}\right)}{b}$$

output
```
exp(b*x+a)*(-1+exp(2*b*x+2*a))^n*AppellF1(1/2,n,-n,3/2,-exp(2*b*x+2*a),exp
(2*b*x+2*a))/b/((1-exp(2*b*x+2*a))^n)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.37

$$\int e^{a+bx}\tanh^n(a+bx)\,dx = \frac{e^{a+bx}\left(-1+e^{-2(a+bx)}\right)^{n}\left(1+e^{-2(a+bx)}\right)^{n}\left(-e^{-4(a+bx)}\left(-1+e^{2(a+bx)}\right)^{2}\right)^{-n}\operatorname{AppellF1}\left(-\frac{1}{2},n,-n,\frac{1}{2},-e^{-2(a}\right.}{b}$$

input
```
Integrate[E^(a + b*x)*Tanh[a + b*x]^n,x]
```

output
```
(E^(a + b*x)*(-1 + E^(-2*(a + b*x)))^n*(1 + E^(-2*(a + b*x)))^n*AppellF1[-
1/2, n, -n, 1/2, -E^(-2*(a + b*x)), E^(-2*(a + b*x))]*Tanh[a + b*x]^n)/(b*
(-((-1 + E^(2*(a + b*x)))^2/E^(4*(a + b*x))))^n)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 2050, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\tanh^n(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \left(-\frac{1-e^{2a+2bx}}{1+e^{2a+2bx}}\right)^n de^{a+bx}}{b}$$

$$\Big\downarrow \text{2050}$$

$$\frac{\int \left(-1+e^{2a+2bx}\right)^n \left(1+e^{2a+2bx}\right)^{-n} de^{a+bx}}{b}$$

$$\Big\downarrow \text{334}$$

$$\frac{\left(1-e^{2a+2bx}\right)^{-n} \left(e^{2a+2bx}-1\right)^n \int \left(1-e^{2a+2bx}\right)^n \left(1+e^{2a+2bx}\right)^{-n} de^{a+bx}}{b}$$

$$\Big\downarrow \text{333}$$

$$\frac{e^{a+bx}\left(1-e^{2a+2bx}\right)^{-n} \left(e^{2a+2bx}-1\right)^n \operatorname{AppellF1}\left(\frac{1}{2},-n,n,\frac{3}{2},e^{2a+2bx},-e^{2a+2bx}\right)}{b}$$

input
```
Int[E^(a + b*x)*Tanh[a + b*x]^n,x]
```

output
```
(E^(a + b*x)*(-1 + E^(2*a + 2*b*x))^n*AppellF1[1/2, -n, n, 3/2, E^(2*a + 2
*b*x), -E^(2*a + 2*b*x)])/(b*(1 - E^(2*a + 2*b*x))^n)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [F]

$$\int e^{bx+a} \tanh (bx + a)^n \, dx$$

input
```
int(exp(b*x+a)*tanh(b*x+a)^n,x)
```

output
```
int(exp(b*x+a)*tanh(b*x+a)^n,x)
```

Fricas [F]

$$\int e^{a+bx}\tanh^n(a+bx)\,dx=\int \tanh\left(bx+a\right)^n e^{(bx+a)}\,dx$$

input
```
integrate(exp(b*x+a)*tanh(b*x+a)^n,x, algorithm="fricas")
```

output
```
integral(tanh(b*x + a)^n*e^(b*x + a), x)
```

Sympy [F]

$$\int e^{a+bx}\tanh^n(a+bx)\,dx=e^a\int e^{bx}\tanh^n\left(a+bx\right)dx$$

input
```
integrate(exp(b*x+a)*tanh(b*x+a)**n,x)
```

output
```
exp(a)*Integral(exp(b*x)*tanh(a + b*x)**n, x)
```

Maxima [F]

$$\int e^{a+bx}\tanh^n(a+bx)\,dx=\int \tanh\left(bx+a\right)^n e^{(bx+a)}\,dx$$

input
```
integrate(exp(b*x+a)*tanh(b*x+a)^n,x, algorithm="maxima")
```

output
```
integrate(tanh(b*x + a)^n*e^(b*x + a), x)
```

Giac [F]

$$\int e^{a+bx}\tanh^n(a+bx)\,dx=\int\tanh\left(bx+a\right)^n e^{(bx+a)}\,dx$$

input
```
integrate(exp(b*x+a)*tanh(b*x+a)^n,x, algorithm="giac")
```

output
```
integrate(tanh(b*x + a)^n*e^(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\tanh^n(a+bx)\,dx=\int e^{a+b\,x}\,\tanh(a+b\,x)^n\,dx$$

input
```
int(exp(a + b*x)*tanh(a + b*x)^n,x)
```

output
```
int(exp(a + b*x)*tanh(a + b*x)^n, x)
```

Reduce [F]

$$\int e^{a+bx}\tanh^n(a+bx)\,dx=e^a\left(\int e^{bx}\tanh\left(bx+a\right)^n dx\right)$$

input
```
int(exp(b*x+a)*tanh(b*x+a)^n,x)
```

output
```
e**a*int(e**(b*x)*tanh(a + b*x)**n,x)
```

3.58 $\int F^{c(a+bx)}(f\tanh(d+ex))^n\,dx$

Optimal result

Integrand size = 20, antiderivative size = 139

$$\int F^{c(a+bx)}(f\tanh(d+ex))^n\,dx = -\frac{2^{-1-n}\left(e^{2d+2ex}\right)^{-\frac{bc\log(F)}{2e}}\left(1-e^{2d+2ex}\right)\left(1+e^{2d+2ex}\right)^n F^{c(a+bx)}\operatorname{AppellF1}\left(1+n,1-\frac{bc\log(F)}{2e},n,2+n,1-\right.}{e(1+n)}$$

output
```
-2^(-1-n)*(1-exp(2*e*x+2*d))*(1+exp(2*e*x+2*d))^n*F^(c*(b*x+a))*AppellF1(1
+n,1-1/2*b*c*ln(F)/e,n,2+n,1-exp(2*e*x+2*d),1/2-1/2*exp(2*e*x+2*d))*(f*tan
h(e*x+d))^n/e/(exp(2*e*x+2*d)^(1/2*b*c*ln(F)/e))/(1+n)
```

Mathematica [F]

$$\int F^{c(a+bx)}(f\tanh(d+ex))^n\,dx = \int F^{c(a+bx)}(f\tanh(d+ex))^n\,dx$$

input
```
Integrate[F^(c*(a + b*x))*(f*Tanh[d + e*x])^n,x]
```

output
```
Integrate[F^(c*(a + b*x))*(f*Tanh[d + e*x])^n, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\tanh(d+ex))^n\,dx$$

$$\Big\downarrow \text{7271}$$

$$\tanh^{-n}(d+ex)(f\tanh(d+ex))^n\int F^{c(a+bx)}\tanh^n(d+ex)dx$$

$$\Big\downarrow \text{6030}$$

$$\tanh^{-n}(d+ex)(f\tanh(d+ex))^n\int F^{ac+bxc}\tanh^n(d+ex)dx$$

$$\Big\downarrow \text{7299}$$

$$\tanh^{-n}(d+ex)(f\tanh(d+ex))^n\int F^{ac+bxc}\tanh^n(d+ex)dx$$

input
```
Int[F^(c*(a + b*x))*(f*Tanh[d + e*x])^n,x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}(f\tanh(ex+d))^n\,dx$$

input
```
int(F^(c*(b*x+a))*(f*tanh(e*x+d))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*tanh(e*x+d))^n,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\tanh(d+ex))^n\,dx = \int (f\tanh{(ex+d)})^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*tanh(e*x+d))^n,x, algorithm="fricas")
```

output
```
integral((f*tanh(e*x + d))^n*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}(f\tanh(d+ex))^n\,dx = \int F^{c(a+bx)}(f\tanh{(d+ex)})^n\;dx$$

input
```
integrate(F**(c*(b*x+a))*(f*tanh(e*x+d))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*tanh(d + e*x))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}(f\tanh(d+ex))^n\,dx = \int (f\tanh{(ex+d)})^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*tanh(e*x+d))^n,x, algorithm="maxima")
```

output
```
integrate((f*tanh(e*x + d))^n*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\tanh(d+ex))^n\,dx=\int(f\tanh(ex+d))^nF^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*tanh(e*x+d))^n,x, algorithm="giac")
```

output
```
integrate((f*tanh(e*x + d))^n*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\tanh(d+ex))^n\,dx=\int F^{c\,(a+b\,x)}\,(f\tanh(d+e\,x))^n\,dx$$

input
```
int(F^(c*(a + b*x))*(f*tanh(d + e*x))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f*tanh(d + e*x))^n, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\tanh(d+ex))^n\,dx=f^{ac+n}\left(\int f^{bcx}\tanh(ex+d)^n\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*tanh(e*x+d))^n,x)
```

output
```
f**(a*c + n)*int(f**(b*c*x)*tanh(d + e*x)**n,x)
```

3.59 $\int e^{a+bx}\coth(d+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 32

$$\int e^{a+bx}\coth(d+bx)\,dx=\frac{e^{a+bx}}{b}-\frac{2e^{a-d}\text{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
exp(b*x+a)/b-2*exp(a-d)*arctanh(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int e^{a+bx}\coth(d+bx)\,dx$$
$$=\frac{e^a\left(e^{bx}-2\text{arctanh}\left(e^{bx}(\cosh(d)+\sinh(d))\right)(\cosh(d)-\sinh(d))\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Coth[d + b*x],x]
```

output
```
(E^a*(E^(b*x) - 2*ArcTanh[E^(b*x)*(Cosh[d] + Sinh[d])]*(Cosh[d] - Sinh[d])
))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {2720, 25, 27, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\coth(bx+d)\,dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int -\frac{e^a\left(1+e^{2bx}\right)}{1-e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{e^a\left(1+e^{2bx}\right)}{1-e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \ 27$$

$$-\frac{e^a\int \frac{1+e^{2bx}}{1-e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \ 299$$

$$-\frac{e^a\left(2\int \frac{1}{1-e^{2bx}}de^{bx}-e^{bx}\right)}{b}$$

$$\Big\downarrow \ 219$$

$$-\frac{e^a\left(2\text{arctanh}\left(e^{bx}\right)-e^{bx}\right)}{b}$$

input
```
Int[E^(a + b*x)*Coth[d + b*x],x]
```

output
```
-((E^a*(-E^(b*x) + 2*ArcTanh[E^(b*x)]))/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(29) = 58$.

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.97

method	result	size
risch	$\frac{e^{bx+a}}{b}+\frac{\ln\left(e^{bx+a}-e^{a-d}\right)e^{a-d}}{b}-\frac{\ln\left(e^{bx+a}+e^{a-d}\right)e^{a-d}}{b}$	63

input
```
int(exp(b*x+a)*coth(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
exp(b*x+a)/b+ln(exp(b*x+a)-exp(a-d))/b*exp(a-d)-ln(exp(b*x+a)+exp(a-d))/b*
exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(29) = 58$.

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.66

$$\int e^{a+bx}\coth(d+bx)\,dx = \frac{\cosh(bx+d)\cosh(-a+d) - (\cosh(-a+d) - \sinh(-a+d))\log(\cosh(bx+d) + \sinh(bx+d) + 1) +}{}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d),x, algorithm="fricas")
```

output
```
(cosh(b*x + d)*cosh(-a + d) - (cosh(-a + d) - sinh(-a + d))*log(cosh(b*x +
 d) + sinh(b*x + d) + 1) + (cosh(-a + d) - sinh(-a + d))*log(cosh(b*x + d)
 + sinh(b*x + d) - 1) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d) - cosh
(b*x + d)*sinh(-a + d))/b
```

Sympy [F]

$$\int e^{a+bx}\coth(d+bx)\,dx = e^a\int e^{bx}\coth(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*coth(b*x+d),x)
```

output
```
exp(a)*Integral(exp(b*x)*coth(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.75

$$\int e^{a+bx}\coth(d+bx)\,dx = -\frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^a\right)}{b} + \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}-e^a\right)}{b} + \frac{e^{(bx+a)}}{b}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d),x, algorithm="maxima")
```

output
```
-e^(a - d)*log(e^(b*x + a + d) + e^a)/b + e^(a - d)*log(e^(b*x + a + d) -
e^a)/b + e^(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int e^{a+bx}\coth(d+bx)\,dx = -\frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^a\right) - e^{(a-d)}\log\left(\left|e^{(bx+a+d)}-e^a\right|\right) - e^{(bx+a)}}{b}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d),x, algorithm="giac")
```

output
```
-(e^(a - d)*log(e^(b*x + a + d) + e^a) - e^(a - d)*log(abs(e^(b*x + a + d)
 - e^a)) - e^(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.84

$$\int e^{a+bx}\coth(d+bx)\,dx=\frac{\mathrm{e}^{a+b\,x}}{b}-\frac{2\,\sqrt{\mathrm{e}^{2\,a-2\,d}}\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b\,\sqrt{\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}}}\right)}{\sqrt{-b^2}}$$

input
```
int(coth(d + b*x)*exp(a + b*x),x)
```

output
```
exp(a + b*x)/b - (2*exp(2*a - 2*d)^(1/2)*atan((exp(b*x)*exp(a)*(-b^2)^(1/2
))/(b*(exp(2*a)*exp(-2*d))^(1/2))))/(-b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int e^{a+bx}\coth(d+bx)\,dx=\frac{e^{a}\left(e^{bx+d}+\log\left(e^{bx+d}-1\right)-\log\left(e^{bx+d}+1\right)\right)}{e^{d}b}$$

input
```
int(exp(b*x+a)*coth(b*x+d),x)
```

output
```
(e**a*(e**(b*x + d) + log(e**(b*x + d) - 1) - log(e**(b*x + d) + 1)))/(e**
d*b)
```

3.60 $\int e^{a+bx}\coth^2(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 60

$$\int e^{a+bx}\coth^2(d+bx)\,dx=\frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2d+2bx}\right)}-\frac{2e^{a-d}\text{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
exp(b*x+a)/b+2*exp(b*x+a)/b/(1-exp(2*b*x+2*d))-2*exp(a-d)*arctanh(exp(b*x+
d))/b
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.65

$$\int e^{a+bx}\coth^2(d+bx)\,dx$$
$$=\frac{e^{a}\left(e^{bx}\left(\left(-3+e^{2bx}\right)\cosh(d)+\left(3+e^{2bx}\right)\sinh(d)\right)-2\text{arctanh}\left(e^{bx}(\cosh(d)+\sinh(d))\right)\left(e^{2bx}-\cosh^2(d)\right.\right.}{b\left(\left(-1+e^{2bx}\right)\cosh(d)+\left(1+e^{2bx}\right)\sinh(d)\right)}$$

input
```
Integrate[E^(a + b*x)*Coth[d + b*x]^2,x]
```

output
```
(E^a*(E^(b*x)*((-3 + E^(2*b*x))*Cosh[d] + (3 + E^(2*b*x))*Sinh[d]) - 2*Arc
Tanh[E^(b*x)*(Cosh[d] + Sinh[d])]*(E^(2*b*x) - Cosh[d]^2 - Sinh[d]^2 + Sin
h[2*d])))/(b*((-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d]))
```

Rubi [A] (warning: unable to verify)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 27, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\coth^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{e^a\left(1+e^{2bx}\right)^2}{\left(1-e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^a\int \frac{\left(1+e^{2bx}\right)^2}{\left(1-e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{300}$$

$$\frac{e^a\int \left(1+\frac{4e^{2bx}}{\left(1-e^{2bx}\right)^2}\right)de^{bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^a\left(-2\text{arctanh}\left(e^{bx}\right)+e^{bx}+\frac{2e^{bx}}{1-e^{2bx}}\right)}{b}$$

input
```
Int[E^(a + b*x)*Coth[d + b*x]^2,x]
```

output
```
(E^a*(E^(b*x) + (2*E^(b*x))/(1 - E^(2*b*x)) - 2*ArcTanh[E^(b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

method	result	size
risch	$\frac{{\mathrm e}^{b x +a}}{b}+\frac{2 \,{\mathrm e}^{b x +3 a}}{\left(-{\mathrm e}^{2 b x +2 a +2 d}+{\mathrm e}^{2 a}\right) b}+\frac{\ln\left({\mathrm e}^{b x +a}-{\mathrm e}^{a -d}\right) {\mathrm e}^{a -d}}{b}-\frac{\ln\left({\mathrm e}^{b x +a}+{\mathrm e}^{a -d}\right) {\mathrm e}^{a -d}}{b}$	97

input
```
int(exp(b*x+a)*coth(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
exp(b*x+a)/b+2/(-exp(2*b*x+2*a+2*d)+exp(2*a))/b*exp(b*x+3*a)+ln(exp(b*x+a)
-exp(a-d))/b*exp(a-d)-ln(exp(b*x+a)+exp(a-d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(53) = 106$.

Time = 0.11 (sec) , antiderivative size = 446, normalized size of antiderivative = 7.43

$$\int e^{a+bx}\coth^2(d+bx)\,dx = \frac{\cosh(bx+d)^3\cosh(-a+d)+(\cosh(-a+d)-\sinh(-a+d))\sinh(bx+d)^3+3(\cosh(bx+d)\cosh(\cdots}{}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)^2,x, algorithm="fricas")
```

output
```
(cosh(b*x + d)^3*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d
)^3 + 3*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x
 + d)^2 - 3*cosh(b*x + d)*cosh(-a + d) - (cosh(b*x + d)^2*cosh(-a + d) + (
cosh(-a + d) - sinh(-a + d))*sinh(b*x + d)^2 + 2*(cosh(b*x + d)*cosh(-a +
d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^2 - 1)*sin
h(-a + d) - cosh(-a + d))*log(cosh(b*x + d) + sinh(b*x + d) + 1) + (cosh(b
*x + d)^2*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d)^2 + 2
*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d) -
 (cosh(b*x + d)^2 - 1)*sinh(-a + d) - cosh(-a + d))*log(cosh(b*x + d) + si
nh(b*x + d) - 1) + 3*(cosh(b*x + d)^2*cosh(-a + d) - (cosh(b*x + d)^2 - 1)
*sinh(-a + d) - cosh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^3 - 3*cosh(b*
x + d))*sinh(-a + d))/(b*cosh(b*x + d)^2 + 2*b*cosh(b*x + d)*sinh(b*x + d)
 + b*sinh(b*x + d)^2 - b)
```

Sympy [F]

$$\int e^{a+bx}\coth^2(d+bx)\,dx = e^a\int e^{bx}\coth^2(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)**2,x)
```

output
```
exp(a)*Integral(exp(b*x)*coth(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.50

$$\int e^{a+bx}\coth^2(d+bx)\,dx = -\frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^a\right)}{b}+\frac{e^{(a-d)}\log\left(e^{(bx+a+d)}-e^a\right)}{b} + \frac{e^{(bx+a)}}{b}-\frac{2\,e^{(bx+3\,a)}}{b\left(e^{(2\,bx+2\,a+2\,d)}-e^{(2\,a)}\right)}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)^2,x, algorithm="maxima")
```

output
```
-e^(a - d)*log(e^(b*x + a + d) + e^a)/b + e^(a - d)*log(e^(b*x + a + d) -
e^a)/b + e^(b*x + a)/b - 2*e^(b*x + 3*a)/(b*(e^(2*b*x + 2*a + 2*d) - e^(2*
a)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.42

$$\int e^{a+bx}\coth^2(d+bx)\,dx = -\frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^a\right)-e^{(a-d)}\log\left(\left|e^{(bx+a+d)}-e^a\right|\right)+\frac{2\,e^{(bx+3\,a)}}{e^{(2\,bx+2\,a+2\,d)}-e^{(2\,a)}}-e^{(bx+a)}}{b}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)^2,x, algorithm="giac")
```

output
```
-(e^(a - d)*log(e^(b*x + a + d) + e^a) - e^(a - d)*log(abs(e^(b*x + a + d)
 - e^a)) + 2*e^(b*x + 3*a)/(e^(2*b*x + 2*a + 2*d) - e^(2*a)) - e^(b*x + a)
)/b
```

Mupad [B] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.65

$$\int e^{a+bx}\coth^2(d+bx)\,dx=\frac{e^{b\,x}\,e^a}{b}-\frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^a\,\sqrt{-b^2}}{b\,\sqrt{e^{2\,a}\,e^{-2\,d}}}\right)\,\sqrt{e^{2\,a}\,e^{-2\,d}}}{\sqrt{-b^2}}+\frac{2\,e^{3\,a}\,e^{-2\,d}\,e^{b\,x}}{b\,e^{2\,a}\,e^{-2\,d}-b\,e^{2\,a}\,e^{2\,b\,x}}$$

input
```
int(coth(d + b*x)^2*exp(a + b*x),x)
```

output
```
(exp(b*x)*exp(a))/b - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/(b*(exp(2*a)*
exp(-2*d))^(1/2)))*(exp(2*a)*exp(-2*d))^(1/2))/(-b^2)^(1/2) + (2*exp(3*a)*
exp(-2*d)*exp(b*x))/(b*exp(2*a)*exp(-2*d) - b*exp(2*a)*exp(2*b*x))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.85

$$\int e^{a+bx}\coth^2(d+bx)\,dx$$
$$=\frac{e^a\left(e^{3bx+3d}+e^{2bx+2d}\log\left(e^{bx+d}-1\right)-e^{2bx+2d}\log\left(e^{bx+d}+1\right)-3e^{bx+d}-\log\left(e^{bx+d}-1\right)+\log\left(e^{bx+d}+1\right)\right)}{e^d b\left(e^{2bx+2d}-1\right)}$$

input
```
int(exp(b*x+a)*coth(b*x+d)^2,x)
```

output
```
(e**a*(e**(3*b*x + 3*d) + e**(2*b*x + 2*d)*log(e**(b*x + d) - 1) - e**(2*b
*x + 2*d)*log(e**(b*x + d) + 1) - 3*e**(b*x + d) - log(e**(b*x + d) - 1) +
 log(e**(b*x + d) + 1)))/(e**d*b*(e**(2*b*x + 2*d) - 1))
```

3.61 $\int e^{a+bx}\coth^3(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 88

$$\int e^{a+bx}\coth^3(d+bx)\,dx=\frac{e^{a+bx}}{b}-\frac{2e^{a+bx}}{b\left(1-e^{2d+2bx}\right)^2}+\frac{3e^{a+bx}}{b\left(1-e^{2d+2bx}\right)}-\frac{3e^{a-d}\operatorname{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
exp(b*x+a)/b-2*exp(b*x+a)/b/(1-exp(2*b*x+2*d))^2+3*exp(b*x+a)/b/(1-exp(2*b
*x+2*d))-3*exp(a-d)*arctanh(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

$$\int e^{a+bx}\coth^3(d+bx)\,dx=\frac{e^a\left(-3\operatorname{arctanh}\left(e^{bx}(\cosh(d)+\sinh(d))\right)(\cosh(d)-\sinh(d))+\frac{e^{bx}\left(-5e^{2bx}+\left(2+e^{4bx}\right)\cosh(2d)+\left(-2+e^{4bx}\right)\sinh(2d)\right)}{\left(\left(-1+e^{2bx}\right)\cosh(d)+\left(1+e^{2bx}\right)\sinh(d)\right)^2}\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Coth[d + b*x]^3,x]
```

output
```
(E^a*(-3*ArcTanh[E^(b*x)*(Cosh[d] + Sinh[d])]*(Cosh[d] - Sinh[d]) + (E^(b*
x)*(-5*E^(2*b*x) + (2 + E^(4*b*x))*Cosh[2*d] + (-2 + E^(4*b*x))*Sinh[2*d])
)/((-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d])^2))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2720, 25, 27, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx} \coth^3(bx+d) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{e^a \left(1+e^{2bx}\right)^3}{\left(1-e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{e^a \left(1+e^{2bx}\right)^3}{\left(1-e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^a \int \frac{\left(1+e^{2bx}\right)^3}{\left(1-e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{300}$$

$$-\frac{e^a \int \left(\frac{2\left(1+3e^{4bx}\right)}{\left(1-e^{2bx}\right)^3}-1\right)de^{bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^a \left(3\text{arctanh}\left(e^{bx}\right)-e^{bx}-\frac{3e^{bx}}{1-e^{2bx}}+\frac{2e^{bx}}{\left(1-e^{2bx}\right)^2}\right)}{b}$$

input
```
Int[E^(a + b*x)*Coth[d + b*x]^3,x]
```

output
```
-((E^a*(-E^(b*x) + (2*E^(b*x))/(1 - E^(2*b*x))^2 - (3*E^(b*x))/(1 - E^(2*b
*x)) + 3*ArcTanh[E^(b*x)]))/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.32

method	result	size
risch	$\frac{{\mathrm e}^{b x +a}}{b}+\frac{\left(-3 \,{\mathrm e}^{2 b x +2 a +2 d}+{\mathrm e}^{2 a}\right) {\mathrm e}^{b x +3 a}}{\left(-{\mathrm e}^{2 b x +2 a +2 d}+{\mathrm e}^{2 a}\right)^{2} b}-\frac{3 \ln \left({\mathrm e}^{b x +a}+{\mathrm e}^{a -d}\right) {\mathrm e}^{a -d}}{2 b}+\frac{3 \ln \left({\mathrm e}^{b x +a}-{\mathrm e}^{a -d}\right) {\mathrm e}^{a -d}}{2 b}$	116

input
```
int(exp(b*x+a)*coth(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
exp(b*x+a)/b+1/(-exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(-3*exp(2*b*x+2*a+2*d)+e
xp(2*a))*exp(b*x+3*a)-3/2*ln(exp(b*x+a)+exp(a-d))/b*exp(a-d)+3/2*ln(exp(b*
x+a)-exp(a-d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 980 vs. $2(77) = 154$.

Time = 0.09 (sec) , antiderivative size = 980, normalized size of antiderivative = 11.14

$$\int e^{a+bx}\coth^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)^3,x, algorithm="fricas")
```

output
```
1/2*(2*cosh(b*x + d)^5*cosh(-a + d) + 2*(cosh(-a + d) - sinh(-a + d))*sinh
(b*x + d)^5 + 10*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))
*sinh(b*x + d)^4 - 10*cosh(b*x + d)^3*cosh(-a + d) + 10*(2*cosh(b*x + d)^2
*cosh(-a + d) - (2*cosh(b*x + d)^2 - 1)*sinh(-a + d) - cosh(-a + d))*sinh(
b*x + d)^3 + 10*(2*cosh(b*x + d)^3*cosh(-a + d) - 3*cosh(b*x + d)*cosh(-a
+ d) - (2*cosh(b*x + d)^3 - 3*cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d)^2
 + 4*cosh(b*x + d)*cosh(-a + d) - 3*(cosh(b*x + d)^4*cosh(-a + d) + (cosh(
-a + d) - sinh(-a + d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-a + d) -
cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d)^3 - 2*cosh(b*x + d)^2*cosh(-a +
d) + 2*(3*cosh(b*x + d)^2*cosh(-a + d) - (3*cosh(b*x + d)^2 - 1)*sinh(-a +
 d) - cosh(-a + d))*sinh(b*x + d)^2 + 4*(cosh(b*x + d)^3*cosh(-a + d) - co
sh(b*x + d)*cosh(-a + d) - (cosh(b*x + d)^3 - cosh(b*x + d))*sinh(-a + d))
*sinh(b*x + d) - (cosh(b*x + d)^4 - 2*cosh(b*x + d)^2 + 1)*sinh(-a + d) +
cosh(-a + d))*log(cosh(b*x + d) + sinh(b*x + d) + 1) + 3*(cosh(b*x + d)^4*
cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d)^4 + 4*(cosh(b*x
 + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d)^3 - 2*cosh(
b*x + d)^2*cosh(-a + d) + 2*(3*cosh(b*x + d)^2*cosh(-a + d) - (3*cosh(b*x
+ d)^2 - 1)*sinh(-a + d) - cosh(-a + d))*sinh(b*x + d)^2 + 4*(cosh(b*x + d
)^3*cosh(-a + d) - cosh(b*x + d)*cosh(-a + d) - (cosh(b*x + d)^3 - cosh(b*
x + d))*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^4 - 2*cosh(b*x + d...
```

Sympy [F]

$$\int e^{a+bx}\coth^3(d+bx)\,dx = e^a\int e^{bx}\coth^3\left(bx+d\right)dx$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)**3,x)
```

output
```
exp(a)*Integral(exp(b*x)*coth(b*x + d)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.36

$$\int e^{a+bx}\coth^3(d+bx)\,dx = -\frac{3\,e^{(a-d)}\log\left(e^{(bx+a+d)}+e^a\right)}{2\,b}+\frac{3\,e^{(a-d)}\log\left(e^{(bx+a+d)}-e^a\right)}{2\,b}+\frac{e^{(bx+a)}}{b}-\frac{3\,e^{(3\,bx+5\,a+2\,d)}-e^{(bx+5\,a)}}{b\left(e^{(4\,bx+4\,a+4\,d)}-2\,e^{(2\,bx+4\,a+2\,d)}+e^{(4\,a)}\right)}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)^3,x, algorithm="maxima")
```

output
```
-3/2*e^(a - d)*log(e^(b*x + a + d) + e^a)/b + 3/2*e^(a - d)*log(e^(b*x + a
 + d) - e^a)/b + e^(b*x + a)/b - (3*e^(3*b*x + 5*a + 2*d) - e^(b*x + 5*a))
/(b*(e^(4*b*x + 4*a + 4*d) - 2*e^(2*b*x + 4*a + 2*d) + e^(4*a)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.17

$$\int e^{a+bx}\coth^3(d+bx)\,dx = -\frac{3\,e^{(a-d)}\log\left(e^{(bx+a+d)}+e^a\right)-3\,e^{(a-d)}\log\left(\left|e^{(bx+a+d)}-e^a\right|\right)+\frac{2\left(3\,e^{(3\,bx+5\,a+2\,d)}-e^{(bx+5\,a)}\right)}{\left(e^{(2\,bx+2\,a+2\,d)}-e^{(2\,a)}\right)^2}-2\,e^{(bx+a)}}{2\,b}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)^3,x, algorithm="giac")
```

output
```
-1/2*(3*e^(a - d)*log(e^(b*x + a + d) + e^a) - 3*e^(a - d)*log(abs(e^(b*x
+ a + d) - e^a)) + 2*(3*e^(3*b*x + 5*a + 2*d) - e^(b*x + 5*a))/(e^(2*b*x +
 2*a + 2*d) - e^(2*a))^2 - 2*e^(b*x + a))/b
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\coth^3(d+bx)\,dx=\int \coth(d+b\,x)^3\,e^{a+b\,x}\,dx$$

input
```
int(coth(d + b*x)^3*exp(a + b*x),x)
```

output
```
int(coth(d + b*x)^3*exp(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.10

$$\int e^{a+bx}\coth^3(d+bx)\,dx = \frac{e^a\left(2e^{5bx+5d}+3e^{4bx+4d}\log\left(e^{bx+d}-1\right)-3e^{4bx+4d}\log\left(e^{bx+d}+1\right)-10e^{3bx+3d}-6e^{2bx+2d}\log\left(e^{bx+d}-1\right)+6\right.}{2e^d b\left(e^{4bx+4d}-2e^{2bx+2d}+1\right)}$$

input
```
int(exp(b*x+a)*coth(b*x+d)^3,x)
```

output
```
(e**a*(2*e**(5*b*x + 5*d) + 3*e**(4*b*x + 4*d)*log(e**(b*x + d) - 1) - 3*e
**(4*b*x + 4*d)*log(e**(b*x + d) + 1) - 10*e**(3*b*x + 3*d) - 6*e**(2*b*x
+ 2*d)*log(e**(b*x + d) - 1) + 6*e**(2*b*x + 2*d)*log(e**(b*x + d) + 1) +
4*e**(b*x + d) + 3*log(e**(b*x + d) - 1) - 3*log(e**(b*x + d) + 1)))/(2*e*
*d*b*(e**(4*b*x + 4*d) - 2*e**(2*b*x + 2*d) + 1))
```

3.62 $\int e^{a+bx}\coth^4(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 120

$$\int e^{a+bx}\coth^4(d+bx)\,dx=\frac{e^{a+bx}}{b}+\frac{8e^{a+bx}}{3b\left(1-e^{2d+2bx}\right)^3}-\frac{14e^{a+bx}}{3b\left(1-e^{2d+2bx}\right)^2}+\frac{5e^{a+bx}}{b\left(1-e^{2d+2bx}\right)}-\frac{3e^{a-d}\text{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
exp(b*x+a)/b+8/3*exp(b*x+a)/b/(1-exp(2*b*x+2*d))^3-14/3*exp(b*x+a)/b/(1-ex
p(2*b*x+2*d))^2+5*exp(b*x+a)/b/(1-exp(2*b*x+2*d))-3*exp(a-d)*arctanh(exp(b
*x+d))/b
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.42

$$\int e^{a+bx}\coth^4(d+bx)\,dx$$
$$=\frac{e^a\left(3e^{bx}-9\text{arctanh}\left(e^{bx}(\cosh(d)+\sinh(d))\right)\cosh(d)+9\text{arctanh}\left(e^{bx}(\cosh(d)+\sinh(d))\right)\sinh(d)-\frac{\ldots}{((-1\ldots}\right.}{3b}$$

input
```
Integrate[E^(a + b*x)*Coth[d + b*x]^4,x]
```

output
```
(E^a*(3*E^(b*x) - 9*ArcTanh[E^(b*x)*(Cosh[d] + Sinh[d])]*Cosh[d] + 9*ArcTa
nh[E^(b*x)*(Cosh[d] + Sinh[d])]*Sinh[d] - (8*E^(b*x)*(Cosh[d] - Sinh[d])^3
)/((-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d])^3 - (14*E^(b*x)*(Co
sh[d] - Sinh[d])^2)/((-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d])^2
 - (15*E^(b*x)*(Cosh[d] - Sinh[d]))/((-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2*
b*x))*Sinh[d])))/(3*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.68, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 27, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\coth^4(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{e^a\left(1+e^{2bx}\right)^4}{\left(1-e^{2bx}\right)^4}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^a\int \frac{\left(1+e^{2bx}\right)^4}{\left(1-e^{2bx}\right)^4}de^{bx}}{b}$$

$$\Big\downarrow \text{300}$$

$$\frac{e^a\int \left(\frac{8e^{2bx}\left(1+e^{4bx}\right)}{\left(1-e^{2bx}\right)^4}+1\right)de^{bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^a\left(-3\text{arctanh}\left(e^{bx}\right)+e^{bx}+\frac{5e^{bx}}{1-e^{2bx}}-\frac{14e^{bx}}{3\left(1-e^{2bx}\right)^2}+\frac{8e^{bx}}{3\left(1-e^{2bx}\right)^3}\right)}{b}$$

input
```
Int[E^(a + b*x)*Coth[d + b*x]^4,x]
```

output
```
(E^a*(E^(b*x) + (8*E^(b*x))/(3*(1 - E^(2*b*x))^3) - (14*E^(b*x))/(3*(1 - E
^(2*b*x))^2) + (5*E^(b*x))/(1 - E^(2*b*x)) - 3*ArcTanh[E^(b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.11

method	result	size
risch	$\frac{e^{bx+a}}{b}+\frac{\left(15\,e^{4bx+4a+4d}-16\,e^{2bx+4a+2d}+9\,e^{4a}\right)e^{bx+3a}}{3\left(-e^{2bx+2a+2d}+e^{2a}\right)^{3}b}+\frac{3\ln\left(e^{bx+a}-e^{a-d}\right)e^{a-d}}{2b}-\frac{3\ln\left(e^{bx+a}+e^{a-d}\right)e^{a-d}}{2b}$	133

input
```
int(exp(b*x+a)*coth(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
exp(b*x+a)/b+1/3/(-exp(2*b*x+2*a+2*d)+exp(2*a))^3/b*(15*exp(4*b*x+4*a+4*d)
-16*exp(2*b*x+4*a+2*d)+9*exp(4*a))*exp(b*x+3*a)+3/2*ln(exp(b*x+a)-exp(a-d)
)/b*exp(a-d)-3/2*ln(exp(b*x+a)+exp(a-d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1693 vs. $2(101) = 202$.

Time = 0.09 (sec) , antiderivative size = 1693, normalized size of antiderivative = 14.11

$$\int e^{a+bx} \coth^4(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)^4,x, algorithm="fricas")
```

output
```
1/6*(6*cosh(b*x + d)^7*cosh(-a + d) + 6*(cosh(-a + d) - sinh(-a + d))*sinh
(b*x + d)^7 + 42*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))
*sinh(b*x + d)^6 - 48*cosh(b*x + d)^5*cosh(-a + d) + 6*(21*cosh(b*x + d)^2
*cosh(-a + d) - (21*cosh(b*x + d)^2 - 8)*sinh(-a + d) - 8*cosh(-a + d))*si
nh(b*x + d)^5 + 30*(7*cosh(b*x + d)^3*cosh(-a + d) - 8*cosh(b*x + d)*cosh(
-a + d) - (7*cosh(b*x + d)^3 - 8*cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d
)^4 + 50*cosh(b*x + d)^3*cosh(-a + d) + 10*(21*cosh(b*x + d)^4*cosh(-a + d
) - 48*cosh(b*x + d)^2*cosh(-a + d) - (21*cosh(b*x + d)^4 - 48*cosh(b*x +
d)^2 + 5)*sinh(-a + d) + 5*cosh(-a + d))*sinh(b*x + d)^3 + 6*(21*cosh(b*x
+ d)^5*cosh(-a + d) - 80*cosh(b*x + d)^3*cosh(-a + d) + 25*cosh(b*x + d)*c
osh(-a + d) - (21*cosh(b*x + d)^5 - 80*cosh(b*x + d)^3 + 25*cosh(b*x + d))
*sinh(-a + d))*sinh(b*x + d)^2 - 24*cosh(b*x + d)*cosh(-a + d) - 9*(cosh(b
*x + d)^6*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d)^6 + 6
*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d)^5
 - 3*cosh(b*x + d)^4*cosh(-a + d) + 3*(5*cosh(b*x + d)^2*cosh(-a + d) - (5
*cosh(b*x + d)^2 - 1)*sinh(-a + d) - cosh(-a + d))*sinh(b*x + d)^4 + 4*(5*
cosh(b*x + d)^3*cosh(-a + d) - 3*cosh(b*x + d)*cosh(-a + d) - (5*cosh(b*x
+ d)^3 - 3*cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d)^3 + 3*cosh(b*x + d)^
2*cosh(-a + d) + 3*(5*cosh(b*x + d)^4*cosh(-a + d) - 6*cosh(b*x + d)^2*cos
h(-a + d) - (5*cosh(b*x + d)^4 - 6*cosh(b*x + d)^2 + 1)*sinh(-a + d) + ...
```

Sympy [F]

$$\int e^{a+bx}\coth^4(d+bx)\,dx=e^a\int e^{bx}\coth^4{(bx+d)}\,dx$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)**4,x)
```

output
```
exp(a)*Integral(exp(b*x)*coth(b*x + d)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.25

$$\int e^{a+bx}\coth^4(d+bx)\,dx=-\frac{3\,e^{(a-d)}\log\left(e^{(bx+a+d)}+e^a\right)}{2\,b}+\frac{3\,e^{(a-d)}\log\left(e^{(bx+a+d)}-e^a\right)}{2\,b}+\frac{e^{(bx+a)}}{b}-\frac{15\,e^{(5\,bx+7\,a+4\,d)}-16\,e^{(3\,bx+7\,a+2\,d)}+9\,e^{(bx+7\,a)}}{3\,b(e^{(6\,bx+6\,a+6\,d)}-3\,e^{(4\,bx+6\,a+4\,d)}+3\,e^{(2\,bx+6\,a+2\,d)}-e^{(6\,a)})}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)^4,x, algorithm="maxima")
```

output
```
-3/2*e^(a - d)*log(e^(b*x + a + d) + e^a)/b + 3/2*e^(a - d)*log(e^(b*x + a
 + d) - e^a)/b + e^(b*x + a)/b - 1/3*(15*e^(5*b*x + 7*a + 4*d) - 16*e^(3*b
*x + 7*a + 2*d) + 9*e^(b*x + 7*a))/(b*(e^(6*b*x + 6*a + 6*d) - 3*e^(4*b*x
+ 6*a + 4*d) + 3*e^(2*b*x + 6*a + 2*d) - e^(6*a)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int e^{a+bx}\coth^4(d+bx)\,dx =$$
$$-\frac{9\,e^{(a-d)}\log\left(e^{(bx+a+d)}+e^{a}\right)-9\,e^{(a-d)}\log\left(\left|e^{(bx+a+d)}-e^{a}\right|\right)+\frac{2\left(15\,e^{(5\,bx+7\,a+4\,d)}-16\,e^{(3\,bx+7\,a+2\,d)}+9\,e^{(bx+7\,a)}\right)}{\left(e^{(2\,bx+2\,a+2\,d)}-e^{(2\,a)}\right)^3}-}{6\,b}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)^4,x, algorithm="giac")
```

output
```
-1/6*(9*e^(a - d)*log(e^(b*x + a + d) + e^a) - 9*e^(a - d)*log(abs(e^(b*x
+ a + d) - e^a)) + 2*(15*e^(5*b*x + 7*a + 4*d) - 16*e^(3*b*x + 7*a + 2*d)
+ 9*e^(b*x + 7*a))/(e^(2*b*x + 2*a + 2*d) - e^(2*a))^3 - 6*e^(b*x + a))/b
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\coth^4(d+bx)\,dx = \int \coth(d+b\,x)^4\,\mathrm{e}^{a+b\,x}\,dx$$

input
```
int(coth(d + b*x)^4*exp(a + b*x),x)
```

output
```
int(coth(d + b*x)^4*exp(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.11

$$\int e^{a+bx}\coth^4(d+bx)\,dx$$
$$= \frac{e^{a}\left(6e^{7bx+7d}+9e^{6bx+6d}\log\left(e^{bx+d}-1\right)-9e^{6bx+6d}\log\left(e^{bx+d}+1\right)-48e^{5bx+5d}-27e^{4bx+4d}\log\left(e^{bx+d}-1\right)+\right.}{6e}$$

input
```
int(exp(b*x+a)*coth(b*x+d)^4,x)
```

output
```
(e**a*(6*e**(7*b*x + 7*d) + 9*e**(6*b*x + 6*d)*log(e**(b*x + d) - 1) - 9*e
**(6*b*x + 6*d)*log(e**(b*x + d) + 1) - 48*e**(5*b*x + 5*d) - 27*e**(4*b*x
 + 4*d)*log(e**(b*x + d) - 1) + 27*e**(4*b*x + 4*d)*log(e**(b*x + d) + 1)
+ 50*e**(3*b*x + 3*d) + 27*e**(2*b*x + 2*d)*log(e**(b*x + d) - 1) - 27*e**
(2*b*x + 2*d)*log(e**(b*x + d) + 1) - 24*e**(b*x + d) - 9*log(e**(b*x + d)
 - 1) + 9*log(e**(b*x + d) + 1)))/(6*e**d*b*(e**(6*b*x + 6*d) - 3*e**(4*b*
x + 4*d) + 3*e**(2*b*x + 2*d) - 1))
```

3.63 $\int e^{2(a+bx)} \coth(d+bx)\, dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int e^{2(a+bx)} \coth(d+bx)\, dx = \frac{e^{2a+2bx}}{2b} + \frac{e^{2a-2d} \log\left(1-e^{2d+2bx}\right)}{b}$$

output
```
1/2*exp(2*b*x+2*a)/b+exp(2*a-2*d)*ln(1-exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.00

$$\int e^{2(a+bx)} \coth(d+bx)\, dx$$
$$= \frac{e^{2a}(\cosh(d)-\sinh(d)) \left(\cosh(d) \left(e^{2bx} + 2\log\left(\left(-1+e^{2bx}\right)\cosh(d) + \left(1+e^{2bx}\right)\sinh(d)\right)\right) + \left(e^{2bx} - 2\log\right.\right.}{2b}$$

input
```
Integrate[E^(2*(a + b*x))*Coth[d + b*x],x]
```

output
```
(E^(2*a)*(Cosh[d] - Sinh[d])*(Cosh[d]*(E^(2*b*x) + 2*Log[(-1 + E^(2*b*x))*
Cosh[d] + (1 + E^(2*b*x))*Sinh[d]]) + (E^(2*b*x) - 2*Log[(-1 + E^(2*b*x))*
Cosh[d] + (1 + E^(2*b*x))*Sinh[d]])*Sinh[d]))/(2*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2720, 25, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\coth(bx+d)\,dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int -\frac{e^{2a+bx}(1+e^{2bx})}{1-e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{e^{2a+bx}(1+e^{2bx})}{1-e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \ 27$$

$$-\frac{e^{2a}\int \frac{e^{bx}(1+e^{2bx})}{1-e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \ 353$$

$$-\frac{e^{2a}\int \frac{1+e^{2bx}}{1-e^{2bx}}de^{2bx}}{2b}$$

$$\Big\downarrow \ 49$$

$$-\frac{e^{2a}\int \left(-1-\frac{2}{-1+e^{2bx}}\right)de^{2bx}}{2b}$$

$$\Big\downarrow \ 2009$$

$$-\frac{e^{2a}\left(-e^{2bx}-2\log\left(1-e^{2bx}\right)\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Coth[d + b*x],x]
```

output
```
-1/2*(E^(2*a)*(-E^(2*b*x) - 2*Log[1 - E^(2*b*x)]))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

method	result	size
risch	$\frac{e^{2bx+2a}}{2b}-\frac{2\,e^{2a-2d}a}{b}+\frac{\ln\left(e^{2bx+2a}-e^{2a-2d}\right)e^{2a-2d}}{b}$	63

input
```
int(exp(2*b*x+2*a)*coth(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*b*x+2*a)/b-2/b*exp(2*a-2*d)*a+ln(exp(2*b*x+2*a)-exp(2*a-2*d))/b*
exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(41) = 82$.

Time = 0.08 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.39

$$\int e^{2(a+bx)}\coth(d+bx)\,dx$$
$$=\frac{\cosh\left(bx+d\right)^{2}\cosh\left(-2\,a+2\,d\right)+\left(\cosh\left(-2\,a+2\,d\right)-\sinh\left(-2\,a+2\,d\right)\right)\sinh\left(bx+d\right)^{2}-\cosh\left(bx+d\right)}{}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d),x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + d)^2*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*
d))*sinh(b*x + d)^2 - cosh(b*x + d)^2*sinh(-2*a + 2*d) + 2*(cosh(-2*a + 2*
d) - sinh(-2*a + 2*d))*log(2*sinh(b*x + d)/(cosh(b*x + d) - sinh(b*x + d))
) + 2*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*si
nh(b*x + d))/b
```

Sympy [F]

$$\int e^{2(a+bx)}\coth(d+bx)\,dx = e^{2a}\int e^{2bx}\coth(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d),x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*coth(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76

$$\int e^{2(a+bx)}\coth(d+bx)\,dx = \frac{2\,(bx+d)e^{(2\,a-2\,d)}}{b} + \frac{e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}+1\right)}{b} + \frac{e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}-1\right)}{b} + \frac{e^{(2\,bx+2\,a)}}{2\,b}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d),x, algorithm="maxima")
```

output
```
2*(b*x + d)*e^(2*a - 2*d)/b + e^(2*a - 2*d)*log(e^(-b*x - d) + 1)/b + e^(2
*a - 2*d)*log(e^(-b*x - d) - 1)/b + 1/2*e^(2*b*x + 2*a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int e^{2(a+bx)}\coth(d+bx)\,dx = \frac{e^{(2\,a-2\,d)}\log\left(\left|e^{(2\,bx+2\,d)}-1\right|\right)}{b} + \frac{e^{(2\,bx+2\,a)}}{2\,b}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d),x, algorithm="giac")
```

output
```
e^(2*a - 2*d)*log(abs(e^(2*b*x + 2*d) - 1))/b + 1/2*e^(2*b*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int e^{2(a+bx)}\coth(d+bx)\,dx = \frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{2\,b} + \frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x} - \mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}\right)}{b}$$

input
```
int(coth(d + b*x)*exp(2*a + 2*b*x),x)
```

output
```
(exp(2*a)*exp(2*b*x))/(2*b) + (exp(2*a)*exp(-2*d)*log(exp(2*a)*exp(2*b*x)
- exp(2*a)*exp(-2*d)))/b
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int e^{2(a+bx)}\coth(d+bx)\,dx = \frac{e^{2a}\left(e^{2bx+2d} + 2\log\left(e^{bx+d}-1\right) + 2\log\left(e^{bx+d}+1\right)\right)}{2e^{2d}b}$$

input
```
int(exp(2*b*x+2*a)*coth(b*x+d),x)
```

output
```
(e**(2*a)*(e**(2*b*x + 2*d) + 2*log(e**(b*x + d) - 1) + 2*log(e**(b*x + d)
 + 1)))/(2*e**(2*d)*b)
```

3.64 $\int e^{2(a+bx)}\coth^2(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 77

$$\int e^{2(a+bx)}\coth^2(d+bx)\,dx=\frac{e^{2a+2bx}}{2b}+\frac{2e^{2a-2d}}{b\left(1-e^{2d+2bx}\right)}+\frac{2e^{2a-2d}\log\left(1-e^{2d+2bx}\right)}{b}$$

output
```
1/2*exp(2*b*x+2*a)/b+2*exp(2*a-2*d)/b/(1-exp(2*b*x+2*d))+2*exp(2*a-2*d)*ln
(1-exp(2*b*x+2*d))/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 158 vs. 2(77) = 154.

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.05

$$\int e^{2(a+bx)}\coth^2(d+bx)\,dx$$
$$=\frac{e^{2a}(\cosh(d)-\sinh(d))\left(\cosh(2d)\left(-4+e^{4bx}-4\log\left(\left(-1+e^{2bx}\right)\cosh(d)+\left(1+e^{2bx}\right)\sinh(d)\right)\right)+e^{2bx}\left(\right.}{2b\left(\left(-1+\right.\right.}$$

input
```
Integrate[E^(2*(a + b*x))*Coth[d + b*x]^2,x]
```

output
```
(E^(2*a)*(Cosh[d] - Sinh[d])*(Cosh[2*d]*(-4 + E^(4*b*x) - 4*Log[(-1 + E^(2
*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d]]) + E^(2*b*x)*(-1 + 4*Log[(-1 + E
^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d]]) + (4 + E^(4*b*x) + 4*Log[(-1
 + E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d]])*Sinh[2*d]))/(2*b*((-1 +
E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d]))
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.60, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2720, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\coth^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{e^{2a+bx}\left(1+e^{2bx}\right)^2}{\left(1-e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{2a}\int \frac{e^{bx}\left(1+e^{2bx}\right)^2}{\left(1-e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{353}$$

$$\frac{e^{2a}\int \frac{\left(1+e^{2bx}\right)^2}{\left(1-e^{2bx}\right)^2}de^{2bx}}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{e^{2a}\int \left(1+\frac{4}{-1+e^{2bx}}+\frac{4}{\left(-1+e^{2bx}\right)^2}\right)de^{2bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2a}\left(e^{2bx}+\frac{4}{1-e^{2bx}}+4\log\left(1-e^{2bx}\right)\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Coth[d + b*x]^2,x]
```

output
```
(E^(2*a)*(E^(2*b*x) + 4/(1 - E^(2*b*x)) + 4*Log[1 - E^(2*b*x)]))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27

method	result	size
risch	$\frac{e^{2bx+2a}}{2b}-\frac{4\,e^{2a-2d}a}{b}+\frac{2\,e^{4a-2d}}{\left(-e^{2bx+2a+2d}+e^{2a}\right)b}+\frac{2\ln\left(e^{2bx+2a}-e^{2a-2d}\right)e^{2a-2d}}{b}$	98

input
```
int(exp(2*b*x+2*a)*coth(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*b*x+2*a)/b-4/b*exp(2*a-2*d)*a+2/(-exp(2*b*x+2*a+2*d)+exp(2*a))/b
*exp(4*a-2*d)+2*ln(exp(2*b*x+2*a)-exp(2*a-2*d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. 2(68) = 136.

Time = 0.09 (sec) , antiderivative size = 465, normalized size of antiderivative = 6.04

$$\int e^{2(a+bx)}\coth^2(d+bx)\,dx$$

$$=\frac{\cosh\left(bx+d\right)^4\cosh\left(-2\,a+2\,d\right)+\left(\cosh\left(-2\,a+2\,d\right)-\sinh\left(-2\,a+2\,d\right)\right)\sinh\left(bx+d\right)^4+4\left(\cosh\left(bx+\right.\right.}{}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)^2,x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + d)^4*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*
d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*si
nh(-2*a + 2*d))*sinh(b*x + d)^3 - cosh(b*x + d)^2*cosh(-2*a + 2*d) + (6*co
sh(b*x + d)^2*cosh(-2*a + 2*d) - (6*cosh(b*x + d)^2 - 1)*sinh(-2*a + 2*d)
- cosh(-2*a + 2*d))*sinh(b*x + d)^2 + 4*(cosh(b*x + d)^2*cosh(-2*a + 2*d)
+ (cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^2 + 2*(cosh(b*x + d)
*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d) - (cosh(
b*x + d)^2 - 1)*sinh(-2*a + 2*d) - cosh(-2*a + 2*d))*log(2*sinh(b*x + d)/(
cosh(b*x + d) - sinh(b*x + d))) + 2*(2*cosh(b*x + d)^3*cosh(-2*a + 2*d) -
cosh(b*x + d)*cosh(-2*a + 2*d) - (2*cosh(b*x + d)^3 - cosh(b*x + d))*sinh(
-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^4 - cosh(b*x + d)^2 - 4)*sinh(
-2*a + 2*d) - 4*cosh(-2*a + 2*d))/(b*cosh(b*x + d)^2 + 2*b*cosh(b*x + d)*s
inh(b*x + d) + b*sinh(b*x + d)^2 - b)
```

Sympy [F]

$$\int e^{2(a+bx)}\coth^2(d+bx)\,dx = e^{2a}\int e^{2bx}\coth^2{(bx+d)}\,dx$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)**2,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*coth(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.53

$$\int e^{2(a+bx)}\coth^2(d+bx)\,dx = \frac{4\,(bx+d)e^{(2\,a-2\,d)}}{b} + \frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}+1\right)}{b} + \frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}-1\right)}{b} - \frac{\left(5\,e^{(-2\,bx-2\,d)}-1\right)e^{(2\,a-2\,d)}}{2\,b\left(e^{(-2\,bx-2\,d)}-e^{(-4\,bx-4\,d)}\right)}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)^2,x, algorithm="maxima")
```

output
```
4*(b*x + d)*e^(2*a - 2*d)/b + 2*e^(2*a - 2*d)*log(e^(-b*x - d) + 1)/b + 2*
e^(2*a - 2*d)*log(e^(-b*x - d) - 1)/b - 1/2*(5*e^(-2*b*x - 2*d) - 1)*e^(2*
a - 2*d)/(b*(e^(-2*b*x - 2*d) - e^(-4*b*x - 4*d)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

$$\int e^{2(a+bx)}\coth^2(d+bx)\,dx = \frac{2\,e^{(2\,a-2\,d)}\log\left(\left|e^{(2\,bx+2\,d)}-1\right|\right)}{b}+\frac{e^{(2\,bx+2\,a)}}{2\,b}-\frac{2\,e^{(2\,bx+2\,a)}}{b(e^{(2\,bx+2\,d)}-1)}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)^2,x, algorithm="giac")
```

output
```
2*e^(2*a - 2*d)*log(abs(e^(2*b*x + 2*d) - 1))/b + 1/2*e^(2*b*x + 2*a)/b -
2*e^(2*b*x + 2*a)/(b*(e^(2*b*x + 2*d) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int e^{2(a+bx)}\coth^2(d+bx)\,dx = \frac{\mathrm{e}^{2\,a+2\,b\,x}}{2\,b}+\frac{2\,\mathrm{e}^{2\,a-2\,d}\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}\right)}{b}+\frac{2\,\mathrm{e}^{4\,a-4\,d}}{b\,\left(\mathrm{e}^{2\,a-2\,d}-\mathrm{e}^{2\,a+2\,b\,x}\right)}$$

input
```
int(coth(d + b*x)^2*exp(2*a + 2*b*x),x)
```

output
```
exp(2*a + 2*b*x)/(2*b) + (2*exp(2*a - 2*d)*log(exp(2*a)*exp(2*b*x) - exp(2
*a)*exp(-2*d)))/b + (2*exp(4*a - 4*d))/(b*(exp(2*a - 2*d) - exp(2*a + 2*b*
x)))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.58

$$\int e^{2(a+bx)}\coth^2(d+bx)\,dx = \frac{e^{2a}\left(e^{4bx+4d}+4e^{2bx+2d}\log\left(e^{bx+d}-1\right)+4e^{2bx+2d}\log\left(e^{bx+d}+1\right)-5e^{2bx+2d}-4\log\left(e^{bx+d}-1\right)-4\log\left(e^{bx-}\right.\right.}{2e^{2d}b\left(e^{2bx+2d}-1\right)}$$

input
```
int(exp(2*b*x+2*a)*coth(b*x+d)^2,x)
```

output
```
(e**(2*a)*(e**(4*b*x + 4*d) + 4*e**(2*b*x + 2*d)*log(e**(b*x + d) - 1) + 4
*e**(2*b*x + 2*d)*log(e**(b*x + d) + 1) - 5*e**(2*b*x + 2*d) - 4*log(e**(b
*x + d) - 1) - 4*log(e**(b*x + d) + 1)))/(2*e**(2*d)*b*(e**(2*b*x + 2*d) -
 1))
```

3.65 $\int e^{2(a+bx)} \coth^3(d+bx) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int e^{2(a+bx)} \coth^3(d+bx) \, dx = \frac{e^{2a+2bx}}{2b} - \frac{2e^{2a-2d}}{b\left(1-e^{2d+2bx}\right)^2} + \frac{6e^{2a-2d}}{b\left(1-e^{2d+2bx}\right)} + \frac{3e^{2a-2d}\log\left(1-e^{2d+2bx}\right)}{b}$$

output
```
1/2*exp(2*b*x+2*a)/b-2*exp(2*a-2*d)/b/(1-exp(2*b*x+2*d))^2+6*exp(2*a-2*d)/
b/(1-exp(2*b*x+2*d))+3*exp(2*a-2*d)*ln(1-exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.41

$$\int e^{2(a+bx)} \coth^3(d+bx) \, dx = \frac{e^{2a}\left(e^{2bx} + 6\cosh(2d)\log\left(\left(-1+e^{2bx}\right)\cosh(d) + \left(1+e^{2bx}\right)\sinh(d)\right) - \frac{4(\cosh(d)-\sinh(d))^4}{\left(\left(-1+e^{2bx}\right)\cosh(d)+\left(1+e^{2bx}\right)\sinh(d)\right)^2} - \right.}{2b}$$

input
```
Integrate[E^(2*(a + b*x))*Coth[d + b*x]^3,x]
```

output
```
(E^(2*a)*(E^(2*b*x) + 6*Cosh[2*d]*Log[(-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2
*b*x))*Sinh[d]] - (4*(Cosh[d] - Sinh[d])^4)/((-1 + E^(2*b*x))*Cosh[d] + (1
 + E^(2*b*x))*Sinh[d])^2 - (12*(Cosh[d] - Sinh[d])^3)/((-1 + E^(2*b*x))*Co
sh[d] + (1 + E^(2*b*x))*Sinh[d]) - 6*Log[(-1 + E^(2*b*x))*Cosh[d] + (1 + E
^(2*b*x))*Sinh[d]]*Sinh[2*d]))/(2*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.58, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2720, 25, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)} \coth^3(bx+d) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{e^{2a+bx}\left(1+e^{2bx}\right)^3}{\left(1-e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{e^{2a+bx}\left(1+e^{2bx}\right)^3}{\left(1-e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^{2a}\int \frac{e^{bx}\left(1+e^{2bx}\right)^3}{\left(1-e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{353}$$

$$-\frac{e^{2a}\int \frac{\left(1+e^{2bx}\right)^3}{\left(1-e^{2bx}\right)^3}de^{2bx}}{2b}$$

$$\Big\downarrow \text{49}$$

$$-\frac{e^{2a}\int \left(-1-\frac{6}{-1+e^{2bx}}-\frac{12}{\left(-1+e^{2bx}\right)^2}-\frac{8}{\left(-1+e^{2bx}\right)^3}\right)de^{2bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{2a}\left(-e^{2bx}-\frac{12}{1-e^{2bx}}+\frac{4}{(1-e^{2bx})^2}-6\log\left(1-e^{2bx}\right)\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Coth[d + b*x]^3,x]
```

output
```
-1/2*(E^(2*a)*(-E^(2*b*x) + 4/(1 - E^(2*b*x))^2 - 12/(1 - E^(2*b*x)) - 6*L
og[1 - E^(2*b*x)]))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.11

method	result	size
risch	$\frac{\mathrm{e}^{2bx+2a}}{2b}-\frac{6\,\mathrm{e}^{2a-2d}a}{b}+\frac{2\left(-3\,\mathrm{e}^{2bx+2a+2d}+2\,\mathrm{e}^{2a}\right)\mathrm{e}^{4a-2d}}{\left(-\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)^{2}b}+\frac{3\ln\left(\mathrm{e}^{2bx+2a}-\mathrm{e}^{2a-2d}\right)\mathrm{e}^{2a-2d}}{b}$	119

input
```
int(exp(2*b*x+2*a)*coth(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*b*x+2*a)/b-6/b*exp(2*a-2*d)*a+2/(-exp(2*b*x+2*a+2*d)+exp(2*a))^2
/b*(-3*exp(2*b*x+2*a+2*d)+2*exp(2*a))*exp(4*a-2*d)+3*ln(exp(2*b*x+2*a)-exp
(2*a-2*d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 905 vs. 2(94) = 188.

Time = 0.09 (sec) , antiderivative size = 905, normalized size of antiderivative = 8.46

$$\int e^{2(a+bx)}\coth^3(d+bx)\,dx=\text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)^3,x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + d)^6*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*
d))*sinh(b*x + d)^6 + 6*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*si
nh(-2*a + 2*d))*sinh(b*x + d)^5 - 2*cosh(b*x + d)^4*cosh(-2*a + 2*d) + (15
*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (15*cosh(b*x + d)^2 - 2)*sinh(-2*a + 2
*d) - 2*cosh(-2*a + 2*d))*sinh(b*x + d)^4 + 4*(5*cosh(b*x + d)^3*cosh(-2*a
 + 2*d) - 2*cosh(b*x + d)*cosh(-2*a + 2*d) - (5*cosh(b*x + d)^3 - 2*cosh(b
*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d)^3 - 11*cosh(b*x + d)^2*cosh(-2*a
+ 2*d) + (15*cosh(b*x + d)^4*cosh(-2*a + 2*d) - 12*cosh(b*x + d)^2*cosh(-2
*a + 2*d) - (15*cosh(b*x + d)^4 - 12*cosh(b*x + d)^2 - 11)*sinh(-2*a + 2*d
) - 11*cosh(-2*a + 2*d))*sinh(b*x + d)^2 + 6*(cosh(b*x + d)^4*cosh(-2*a +
2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^4 + 4*(cosh(b*x
 + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^3 -
 2*cosh(b*x + d)^2*cosh(-2*a + 2*d) + 2*(3*cosh(b*x + d)^2*cosh(-2*a + 2*d
) - (3*cosh(b*x + d)^2 - 1)*sinh(-2*a + 2*d) - cosh(-2*a + 2*d))*sinh(b*x
+ d)^2 + 4*(cosh(b*x + d)^3*cosh(-2*a + 2*d) - cosh(b*x + d)*cosh(-2*a + 2
*d) - (cosh(b*x + d)^3 - cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d) -
(cosh(b*x + d)^4 - 2*cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2
*d))*log(2*sinh(b*x + d)/(cosh(b*x + d) - sinh(b*x + d))) + 2*(3*cosh(b*x
+ d)^5*cosh(-2*a + 2*d) - 4*cosh(b*x + d)^3*cosh(-2*a + 2*d) - 11*cosh(b*x
 + d)*cosh(-2*a + 2*d) - (3*cosh(b*x + d)^5 - 4*cosh(b*x + d)^3 - 11*co...
```

Sympy [F]

$$\int e^{2(a+bx)}\coth^3(d+bx)\,dx=e^{2a}\int e^{2bx}\coth^3{(bx+d)}\,dx$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)**3,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*coth(b*x + d)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.29

$$\int e^{2(a+bx)}\coth^3(d+bx)\,dx = \frac{6\,(bx+d)e^{(2\,a-2\,d)}}{b}+\frac{3\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}+1\right)}{b}+\frac{3\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}-1\right)}{b}-\frac{\left(10\,e^{(-2\,bx-2\,d)}-5\,e^{(-4\,bx-4\,d)}-1\right)e^{(2\,a-2\,d)}}{2\,b\left(e^{(-2\,bx-2\,d)}-2\,e^{(-4\,bx-4\,d)}+e^{(-6\,bx-6\,d)}\right)}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)^3,x, algorithm="maxima")
```

output
```
6*(b*x + d)*e^(2*a - 2*d)/b + 3*e^(2*a - 2*d)*log(e^(-b*x - d) + 1)/b + 3*
e^(2*a - 2*d)*log(e^(-b*x - d) - 1)/b - 1/2*(10*e^(-2*b*x - 2*d) - 5*e^(-4
*b*x - 4*d) - 1)*e^(2*a - 2*d)/(b*(e^(-2*b*x - 2*d) - 2*e^(-4*b*x - 4*d) +
 e^(-6*b*x - 6*d)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90

$$\int e^{2(a+bx)}\coth^3(d+bx)\,dx = \frac{3\,e^{(2\,a-2\,d)}\log\left(\left|e^{(2\,bx+2\,d)}-1\right|\right)}{b}+\frac{e^{(2\,bx+2\,a)}}{2\,b}-\frac{\left(9\,e^{(4\,bx+2\,a+4\,d)}-6\,e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right)e^{(-2\,d)}}{2\,b\left(e^{(2\,bx+2\,d)}-1\right)^2}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)^3,x, algorithm="giac")
```

output
```
3*e^(2*a - 2*d)*log(abs(e^(2*b*x + 2*d) - 1))/b + 1/2*e^(2*b*x + 2*a)/b -
1/2*(9*e^(4*b*x + 2*a + 4*d) - 6*e^(2*b*x + 2*a + 2*d) + e^(2*a))*e^(-2*d)
/(b*(e^(2*b*x + 2*d) - 1)^2)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2(a+bx)}\coth^3(d+bx)\,dx=\int \coth(d+b\,x)^3\,\mathrm{e}^{2\,a+2\,b\,x}\,dx$$

input
```
int(coth(d + b*x)^3*exp(2*a + 2*b*x),x)
```

output
```
int(coth(d + b*x)^3*exp(2*a + 2*b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.69

$$\int e^{2(a+bx)}\coth^3(d+bx)\,dx$$
$$=\frac{e^{2a}\left(2e^{6bx+6d}+12e^{4bx+4d}\log\left(e^{bx+d}-1\right)+12e^{4bx+4d}\log\left(e^{bx+d}+1\right)-15e^{4bx+4d}-24e^{2bx+2d}\log\left(e^{bx+d}-1\right)\right.}{4e^{2d}b\left(e^{4bx+4d}-2e^{2bx+2d}+1\right)}$$

input
```
int(exp(2*b*x+2*a)*coth(b*x+d)^3,x)
```

output
```
(e**(2*a)*(2*e**(6*b*x + 6*d) + 12*e**(4*b*x + 4*d)*log(e**(b*x + d) - 1)
+ 12*e**(4*b*x + 4*d)*log(e**(b*x + d) + 1) - 15*e**(4*b*x + 4*d) - 24*e**
(2*b*x + 2*d)*log(e**(b*x + d) - 1) - 24*e**(2*b*x + 2*d)*log(e**(b*x + d)
 + 1) + 12*log(e**(b*x + d) - 1) + 12*log(e**(b*x + d) + 1) + 5))/(4*e**(2
*d)*b*(e**(4*b*x + 4*d) - 2*e**(2*b*x + 2*d) + 1))
```

3.66 $\int e^{2(a+bx)} \coth^4(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 139

$$\int e^{2(a+bx)} \coth^4(d+bx)\,dx = \frac{e^{2a+2bx}}{2b} + \frac{8e^{2a-2d}}{3b\left(1-e^{2d+2bx}\right)^3} - \frac{8e^{2a-2d}}{b\left(1-e^{2d+2bx}\right)^2} + \frac{12e^{2a-2d}}{b\left(1-e^{2d+2bx}\right)} + \frac{4e^{2a-2d}\log\left(1-e^{2d+2bx}\right)}{b}$$

output
```
1/2*exp(2*b*x+2*a)/b+8/3*exp(2*a-2*d)/b/(1-exp(2*b*x+2*d))^3-8*exp(2*a-2*d
)/b/(1-exp(2*b*x+2*d))^2+12*exp(2*a-2*d)/b/(1-exp(2*b*x+2*d))+4*exp(2*a-2*
d)*ln(1-exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.36

$$\int e^{2(a+bx)} \coth^4(d+bx)\,dx$$

$$= \frac{e^{2a}\left(3e^{2bx} + 24\cosh(2d)\log\left(\left(-1+e^{2bx}\right)\cosh(d) + \left(1+e^{2bx}\right)\sinh(d)\right) - \frac{16(\cosh(d)-\sinh(d))^5}{\left(\left(-1+e^{2bx}\right)\cosh(d)+\left(1+e^{2bx}\right)\sinh(d)\right)^3}\right.}{}$$

input
```
Integrate[E^(2*(a + b*x))*Coth[d + b*x]^4,x]
```

output
```
(E^(2*a)*(3*E^(2*b*x) + 24*Cosh[2*d]*Log[(-1 + E^(2*b*x))*Cosh[d] + (1 + E
^(2*b*x))*Sinh[d]] - (16*(Cosh[d] - Sinh[d])^5)/((-1 + E^(2*b*x))*Cosh[d]
+ (1 + E^(2*b*x))*Sinh[d])^3 - (48*(Cosh[d] - Sinh[d])^4)/((-1 + E^(2*b*x)
)*Cosh[d] + (1 + E^(2*b*x))*Sinh[d])^2 - (72*(Cosh[d] - Sinh[d])^3)/((-1 +
 E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d]) - 24*Log[(-1 + E^(2*b*x))*C
osh[d] + (1 + E^(2*b*x))*Sinh[d]]*Sinh[2*d]))/(6*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.55, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2720, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\coth^4(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{e^{2a+bx}\left(1+e^{2bx}\right)^4}{\left(1-e^{2bx}\right)^4}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{2a}\int \frac{e^{bx}\left(1+e^{2bx}\right)^4}{\left(1-e^{2bx}\right)^4}de^{bx}}{b}$$

$$\Big\downarrow \text{353}$$

$$\frac{e^{2a}\int \frac{\left(1+e^{2bx}\right)^4}{\left(1-e^{2bx}\right)^4}de^{2bx}}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{e^{2a}\int \left(1+\frac{8}{-1+e^{2bx}}+\frac{24}{\left(-1+e^{2bx}\right)^2}+\frac{32}{\left(-1+e^{2bx}\right)^3}+\frac{16}{\left(-1+e^{2bx}\right)^4}\right)de^{2bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2a}\left(e^{2bx}+\frac{24}{1-e^{2bx}}-\frac{16}{(1-e^{2bx})^2}+\frac{16}{3(1-e^{2bx})^3}+8\log\left(1-e^{2bx}\right)\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Coth[d + b*x]^4,x]
```

output
```
(E^(2*a)*(E^(2*b*x) + 16/(3*(1 - E^(2*b*x))^3) - 16/(1 - E^(2*b*x))^2 + 24
/(1 - E^(2*b*x)) + 8*Log[1 - E^(2*b*x)]))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

method	result	size
risch	$\frac{e^{2bx+2a}}{2b}-\frac{8\,e^{2a-2d}a}{b}+\frac{4\left(9\,e^{4bx+4a+4d}-12\,e^{2bx+4a+2d}+5\,e^{4a}\right)e^{4a-2d}}{3\left(-e^{2bx+2a+2d}+e^{2a}\right)^{3}b}+\frac{4\ln\left(e^{2bx+2a}-e^{2a-2d}\right)e^{2a-2d}}{b}$	133

input
```
int(exp(2*b*x+2*a)*coth(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*b*x+2*a)/b-8/b*exp(2*a-2*d)*a+4/3/(-exp(2*b*x+2*a+2*d)+exp(2*a))
^3/b*(9*exp(4*b*x+4*a+4*d)-12*exp(2*b*x+4*a+2*d)+5*exp(4*a))*exp(4*a-2*d)+
4*ln(exp(2*b*x+2*a)-exp(2*a-2*d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1490 vs. 2(120) = 240.

Time = 0.09 (sec) , antiderivative size = 1490, normalized size of antiderivative = 10.72

$$\int e^{2(a+bx)}\coth^4(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)^4,x, algorithm="fricas")
```

output
```
1/6*(3*cosh(b*x + d)^8*cosh(-2*a + 2*d) + 3*(cosh(-2*a + 2*d) - sinh(-2*a
+ 2*d))*sinh(b*x + d)^8 + 24*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x +
d)*sinh(-2*a + 2*d))*sinh(b*x + d)^7 - 9*cosh(b*x + d)^6*cosh(-2*a + 2*d)
+ 3*(28*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (28*cosh(b*x + d)^2 - 3)*sinh(-
2*a + 2*d) - 3*cosh(-2*a + 2*d))*sinh(b*x + d)^6 + 6*(28*cosh(b*x + d)^3*c
osh(-2*a + 2*d) - 9*cosh(b*x + d)*cosh(-2*a + 2*d) - (28*cosh(b*x + d)^3 -
 9*cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d)^5 - 63*cosh(b*x + d)^4*c
osh(-2*a + 2*d) + 3*(70*cosh(b*x + d)^4*cosh(-2*a + 2*d) - 45*cosh(b*x + d
)^2*cosh(-2*a + 2*d) - (70*cosh(b*x + d)^4 - 45*cosh(b*x + d)^2 - 21)*sinh
(-2*a + 2*d) - 21*cosh(-2*a + 2*d))*sinh(b*x + d)^4 + 12*(14*cosh(b*x + d)
^5*cosh(-2*a + 2*d) - 15*cosh(b*x + d)^3*cosh(-2*a + 2*d) - 21*cosh(b*x +
d)*cosh(-2*a + 2*d) - (14*cosh(b*x + d)^5 - 15*cosh(b*x + d)^3 - 21*cosh(b
*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d)^3 + 93*cosh(b*x + d)^2*cosh(-2*a
+ 2*d) + 3*(28*cosh(b*x + d)^6*cosh(-2*a + 2*d) - 45*cosh(b*x + d)^4*cosh(
-2*a + 2*d) - 126*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (28*cosh(b*x + d)^6 -
 45*cosh(b*x + d)^4 - 126*cosh(b*x + d)^2 + 31)*sinh(-2*a + 2*d) + 31*cosh
(-2*a + 2*d))*sinh(b*x + d)^2 + 24*(cosh(b*x + d)^6*cosh(-2*a + 2*d) + (co
sh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^6 + 6*(cosh(b*x + d)*cosh
(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^5 - 3*cosh(b*
x + d)^4*cosh(-2*a + 2*d) + 3*(5*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (5*...
```

Sympy [F]

$$\int e^{2(a+bx)}\coth^4(d+bx)\,dx=e^{2a}\int e^{2bx}\coth^4{(bx+d)}\,dx$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)**4,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*coth(b*x + d)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.17

$$\int e^{2(a+bx)}\coth^4(d+bx)\,dx$$
$$= \frac{8\,(bx+d)e^{(2\,a-2\,d)}}{b} + \frac{4\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}+1\right)}{b} + \frac{4\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}-1\right)}{b} - \frac{\left(49\,e^{(-2\,bx-2\,d)} - 57\,e^{(-4\,bx-4\,d)} + 27\,e^{(-6\,bx-6\,d)} - 3\right)e^{(2\,a-2\,d)}}{6\,b\left(e^{(-2\,bx-2\,d)} - 3\,e^{(-4\,bx-4\,d)} + 3\,e^{(-6\,bx-6\,d)} - e^{(-8\,bx-8\,d)}\right)}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)^4,x, algorithm="maxima")
```

output
```
8*(b*x + d)*e^(2*a - 2*d)/b + 4*e^(2*a - 2*d)*log(e^(-b*x - d) + 1)/b + 4*
e^(2*a - 2*d)*log(e^(-b*x - d) - 1)/b - 1/6*(49*e^(-2*b*x - 2*d) - 57*e^(-
4*b*x - 4*d) + 27*e^(-6*b*x - 6*d) - 3)*e^(2*a - 2*d)/(b*(e^(-2*b*x - 2*d)
 - 3*e^(-4*b*x - 4*d) + 3*e^(-6*b*x - 6*d) - e^(-8*b*x - 8*d)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)}\coth^4(d+bx)\,dx$$
$$= \frac{4\,e^{(2\,a-2\,d)}\log\left(\left|e^{(2\,bx+2\,d)}-1\right|\right)}{b} + \frac{e^{(2\,bx+2\,a)}}{2\,b} - \frac{2\left(11\,e^{(6\,bx+2\,a+6\,d)} - 15\,e^{(4\,bx+2\,a+4\,d)} + 9\,e^{(2\,bx+2\,a+2\,d)} - e^{(2\,a)}\right)e^{(-2\,d)}}{3\,b\left(e^{(2\,bx+2\,d)}-1\right)^3}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)^4,x, algorithm="giac")
```

output
```
4*e^(2*a - 2*d)*log(abs(e^(2*b*x + 2*d) - 1))/b + 1/2*e^(2*b*x + 2*a)/b -
2/3*(11*e^(6*b*x + 2*a + 6*d) - 15*e^(4*b*x + 2*a + 4*d) + 9*e^(2*b*x + 2*
a + 2*d) - e^(2*a))*e^(-2*d)/(b*(e^(2*b*x + 2*d) - 1)^3)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2(a+bx)}\coth^4(d+bx)\,dx=\int \coth(d+b\,x)^4\,e^{2\,a+2\,b\,x}\,dx$$

input
```
int(coth(d + b*x)^4*exp(2*a + 2*b*x),x)
```

output
```
int(coth(d + b*x)^4*exp(2*a + 2*b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.79

$$\int e^{2(a+bx)}\coth^4(d+bx)\,dx$$
$$=\frac{e^{2a}\left(3e^{8bx+8d}+24e^{6bx+6d}\log\left(e^{bx+d}-1\right)+24e^{6bx+6d}\log\left(e^{bx+d}+1\right)-30e^{6bx+6d}-72e^{4bx+4d}\log\left(e^{bx+d}-1\right)\right.}{6e^{}}$$

input
```
int(exp(2*b*x+2*a)*coth(b*x+d)^4,x)
```

output
```
(e**(2*a)*(3*e**(8*b*x + 8*d) + 24*e**(6*b*x + 6*d)*log(e**(b*x + d) - 1)
+ 24*e**(6*b*x + 6*d)*log(e**(b*x + d) + 1) - 30*e**(6*b*x + 6*d) - 72*e**
(4*b*x + 4*d)*log(e**(b*x + d) - 1) - 72*e**(4*b*x + 4*d)*log(e**(b*x + d)
 + 1) + 72*e**(2*b*x + 2*d)*log(e**(b*x + d) - 1) + 72*e**(2*b*x + 2*d)*lo
g(e**(b*x + d) + 1) + 30*e**(2*b*x + 2*d) - 24*log(e**(b*x + d) - 1) - 24*
log(e**(b*x + d) + 1) - 19))/(6*e**(2*d)*b*(e**(6*b*x + 6*d) - 3*e**(4*b*x
 + 4*d) + 3*e**(2*b*x + 2*d) - 1))
```

3.67 $\int e^{\frac{5}{3}(a+bx)} \coth(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 186

$$\int e^{\frac{5}{3}(a+bx)} \coth(d+bx)\,dx = \frac{3e^{\frac{5(a-d)}{3}+\frac{5}{3}(d+bx)}}{5b} - \frac{\sqrt{3}e^{\frac{5(a-d)}{3}}\arctan\left(\frac{1-2e^{\frac{1}{3}(d+bx)}}{\sqrt{3}}\right)}{b} + \frac{\sqrt{3}e^{\frac{5(a-d)}{3}}\arctan\left(\frac{1+2e^{\frac{1}{3}(d+bx)}}{\sqrt{3}}\right)}{b} - \frac{2e^{\frac{5(a-d)}{3}}\operatorname{arctanh}\left(e^{\frac{1}{3}(d+bx)}\right)}{b} - \frac{e^{\frac{5(a-d)}{3}}\operatorname{arctanh}\left(\frac{e^{\frac{1}{3}(d+bx)}}{1+e^{\frac{2}{3}(d+bx)}}\right)}{b}$$

output
```
3/5*exp(5/3*b*x+5/3*a)/b-3^(1/2)*exp(5/3*a-5/3*d)*arctan(1/3*(1-2*exp(1/3*
b*x+1/3*d))*3^(1/2))/b+3^(1/2)*exp(5/3*a-5/3*d)*arctan(1/3*(1+2*exp(1/3*b*
x+1/3*d))*3^(1/2))/b-2*exp(5/3*a-5/3*d)*arctanh(exp(1/3*b*x+1/3*d))/b-exp(
5/3*a-5/3*d)*arctanh(exp(1/3*b*x+1/3*d)/(1+exp(2/3*b*x+2/3*d)))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.47

$$\int e^{\frac{5}{3}(a+bx)}\coth(d+bx)\,dx$$
$$= \frac{e^{5a/3}\left(9e^{\frac{5bx}{3}} - 5\text{RootSum}\left[-\cosh(d)+\sinh(d)+\cosh(d)\#1^6+\sinh(d)\#1^6\&, \frac{bx-3\log\left(e^{\frac{bx}{3}}-\#1\right)}{\#1}\&\right](\cosh$$
$$15b$$

input
```
Integrate[E^((5*(a + b*x))/3)*Coth[d + b*x],x]
```

output
```
(E^((5*a)/3)*(9*E^((5*b*x)/3) - 5*RootSum[-Cosh[d] + Sinh[d] + Cosh[d]*#1^
6 + Sinh[d]*#1^6 & , (b*x - 3*Log[E^((b*x)/3) - #1])/#1 & ]*(Cosh[2*d] - S
inh[2*d])))/(15*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.81, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2720, 25, 27, 959, 825, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)}\coth(bx+d)\,dx$$
$$\Big\downarrow \text{2720}$$
$$\frac{3\int -\frac{e^{\frac{5a}{3}+\frac{4bx}{3}}\left(1+e^{2bx}\right)}{1-e^{2bx}}de^{\frac{bx}{3}}}{b}$$
$$\Big\downarrow \text{25}$$
$$-\frac{3\int \frac{e^{\frac{5a}{3}+\frac{4bx}{3}}\left(1+e^{2bx}\right)}{1-e^{2bx}}de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \ 27$$

$$-\frac{3e^{5a/3}\int \frac{e^{\frac{4bx}{3}}\left(1+e^{2bx}\right)}{1-e^{2bx}}de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \ 959$$

$$-\frac{3e^{5a/3}\left(2\int \frac{e^{\frac{4bx}{3}}}{1-e^{2bx}}de^{\frac{bx}{3}}-\frac{1}{5}e^{\frac{5bx}{3}}\right)}{b}$$

$$\Big\downarrow \ 825$$

$$-\frac{3e^{5a/3}\left(2\left(\frac{1}{3}\int \frac{1}{1-e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{3}\int -\frac{1+e^{\frac{bx}{3}}}{2\left(1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}\right)}de^{\frac{bx}{3}}+\frac{1}{3}\int -\frac{1-e^{\frac{bx}{3}}}{2\left(1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}\right)}de^{\frac{bx}{3}}\right)-\frac{1}{5}e^{\frac{5bx}{3}}\right)}{b}$$

$$\Big\downarrow \ 27$$

$$-\frac{3e^{5a/3}\left(2\left(\frac{1}{3}\int \frac{1}{1-e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1+e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1-e^{\frac{bx}{3}}}{1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)-\frac{1}{5}e^{\frac{5bx}{3}}\right)}{b}$$

$$\Big\downarrow \ 219$$

$$-\frac{3e^{5a/3}\left(2\left(-\frac{1}{6}\int \frac{1+e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1-e^{\frac{bx}{3}}}{1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{3}\text{arctanh}\left(e^{\frac{bx}{3}}\right)\right)-\frac{1}{5}e^{\frac{5bx}{3}}\right)}{b}$$

$$\Big\downarrow \ 1142$$

$$-\frac{3e^{5a/3}\left(2\left(\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{2}\int -\frac{1-2e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1+2e^{\frac{bx}{3}}}{1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{3}{2}\int \frac{1}{1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right.\right.\right.}{b}$$

$$\Big\downarrow \ 25$$

$$-\frac{3e^{5a/3}\left(2\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1-2e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{3}{2}\int \frac{1}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1+2e^{\frac{bx}{3}}}{1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{3}{2}\int \frac{1}{1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\right.\right.}{b}$$

$$\Big\downarrow \ 1083$$

$$-\frac{3e^{5a/3}\left(2\left(\frac{1}{6}\left(3\int \frac{1}{-3-e^{\frac{2bx}{3}}}d\left(-1+2e^{\frac{bx}{3}}\right)+\frac{1}{2}\int \frac{1-2e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(3\int \frac{1}{-3-e^{\frac{2bx}{3}}}d\left(1+2e^{\frac{bx}{3}}\right)+\frac{1}{2}\int \frac{1+2e^{\frac{bx}{3}}}{1+e^{\frac{bx}{3}}+e^{2}}\right.\right.\right.}{b}$$

$\Big\downarrow$ 217

$$-\frac{3e^{5a/3}\left(2\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\sqrt{3}\arctan\left(\frac{2e^{\frac{bx}{3}}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{1+2e^{\frac{bx}{3}}}{1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\sqrt{3}\arctan\left(\frac{2e^{\frac{bx}{3}}+1}{\sqrt{3}}\right)\right)\right.\right.}{b}$$

$\Big\downarrow$ 1103

$$-\frac{3e^{5a/3}\left(2\left(\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{2e^{\frac{bx}{3}}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}+1\right)\right)+\frac{1}{6}\left(\frac{1}{2}\log\left(e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}+1\right)-\sqrt{3}\arctan\left(2\right.\right.\right.\right.}{b}$$

input
```
Int[E^((5*(a + b*x))/3)*Coth[d + b*x],x]
```

output
```
(-3*E^((5*a)/3)*(-1/5*E^((5*b*x)/3) + 2*(ArcTanh[E^((b*x)/3)]/3 + (-(Sqrt[
3]*ArcTan[(-1 + 2*E^((b*x)/3))/Sqrt[3]]) - Log[1 - E^((b*x)/3) + E^((2*b*x
)/3)]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + 2*E^((b*x)/3))/Sqrt[3]]) + Log[1 + E^(
(b*x)/3) + E^((2*b*x)/3)]/2)/6)))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.77

method	result
risch	$\frac{3\,e^{\frac{5bx}{3}+\frac{5a}{3}}}{5b}+\frac{\ln\left(e^{\frac{bx}{3}+\frac{d}{3}}-1\right)e^{\frac{5a}{3}-\frac{5d}{3}}}{b}-\frac{\ln\left(1+e^{\frac{bx}{3}+\frac{d}{3}}\right)e^{\frac{5a}{3}-\frac{5d}{3}}}{b}-\frac{\ln\left(e^{\frac{bx}{3}+\frac{d}{3}}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)e^{\frac{5a}{3}-\frac{5d}{3}}}{2b}+\frac{i\ln\left(e^{\frac{bx}{3}+\frac{d}{3}}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right.}{2b}$

input
```
int(exp(5/3*b*x+5/3*a)*coth(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
3/5*exp(5/3*b*x+5/3*a)/b+1/b*ln(exp(1/3*b*x+1/3*d)-1)*exp(5/3*a-5/3*d)-1/b
*ln(1+exp(1/3*b*x+1/3*d))*exp(5/3*a-5/3*d)-1/2/b*ln(exp(1/3*b*x+1/3*d)+1/2
+1/2*I*3^(1/2))*exp(5/3*a-5/3*d)+1/2*I/b*ln(exp(1/3*b*x+1/3*d)+1/2+1/2*I*3
^(1/2))*exp(5/3*a-5/3*d)*3^(1/2)-1/2/b*ln(exp(1/3*b*x+1/3*d)+1/2-1/2*I*3^(
1/2))*exp(5/3*a-5/3*d)-1/2*I/b*ln(exp(1/3*b*x+1/3*d)+1/2-1/2*I*3^(1/2))*ex
p(5/3*a-5/3*d)*3^(1/2)+1/2/b*ln(exp(1/3*b*x+1/3*d)-1/2+1/2*I*3^(1/2))*exp(
5/3*a-5/3*d)+1/2*I/b*ln(exp(1/3*b*x+1/3*d)-1/2+1/2*I*3^(1/2))*exp(5/3*a-5/
3*d)*3^(1/2)+1/2/b*ln(exp(1/3*b*x+1/3*d)-1/2-1/2*I*3^(1/2))*exp(5/3*a-5/3*
d)-1/2*I/b*ln(exp(1/3*b*x+1/3*d)-1/2-1/2*I*3^(1/2))*exp(5/3*a-5/3*d)*3^(1/
2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. 2(144) = 288.

Time = 0.10 (sec) , antiderivative size = 622, normalized size of antiderivative = 3.34

$$\int e^{\frac{5}{3}(a+bx)}\coth(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*coth(b*x+d),x, algorithm="fricas")
```

output
```
1/10*(6*cosh(1/3*b*x + 1/3*d)^5*cosh(-5/3*a + 5/3*d) + 6*(cosh(-5/3*a + 5/
3*d) - sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^5 - 6*cosh(1/3*b*x + 1/
3*d)^5*sinh(-5/3*a + 5/3*d) + 30*(cosh(1/3*b*x + 1/3*d)*cosh(-5/3*a + 5/3*
d) - cosh(1/3*b*x + 1/3*d)*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^4 +
 60*(cosh(1/3*b*x + 1/3*d)^2*cosh(-5/3*a + 5/3*d) - cosh(1/3*b*x + 1/3*d)^
2*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^3 + 60*(cosh(1/3*b*x + 1/3*d
)^3*cosh(-5/3*a + 5/3*d) - cosh(1/3*b*x + 1/3*d)^3*sinh(-5/3*a + 5/3*d))*s
inh(1/3*b*x + 1/3*d)^2 + 10*(sqrt(3)*cosh(-5/3*a + 5/3*d) - sqrt(3)*sinh(-
5/3*a + 5/3*d))*arctan(2/3*sqrt(3)*cosh(1/3*b*x + 1/3*d) + 2/3*sqrt(3)*sin
h(1/3*b*x + 1/3*d) + 1/3*sqrt(3)) + 10*(sqrt(3)*cosh(-5/3*a + 5/3*d) - sqr
t(3)*sinh(-5/3*a + 5/3*d))*arctan(2/3*sqrt(3)*cosh(1/3*b*x + 1/3*d) + 2/3*
sqrt(3)*sinh(1/3*b*x + 1/3*d) - 1/3*sqrt(3)) - 5*(cosh(-5/3*a + 5/3*d) - s
inh(-5/3*a + 5/3*d))*log((2*cosh(1/3*b*x + 1/3*d) + 1)/(cosh(1/3*b*x + 1/3
*d) - sinh(1/3*b*x + 1/3*d))) + 5*(cosh(-5/3*a + 5/3*d) - sinh(-5/3*a + 5/
3*d))*log((2*cosh(1/3*b*x + 1/3*d) - 1)/(cosh(1/3*b*x + 1/3*d) - sinh(1/3*
b*x + 1/3*d))) - 10*(cosh(-5/3*a + 5/3*d) - sinh(-5/3*a + 5/3*d))*log(cosh
(1/3*b*x + 1/3*d) + sinh(1/3*b*x + 1/3*d) + 1) + 10*(cosh(-5/3*a + 5/3*d)
- sinh(-5/3*a + 5/3*d))*log(cosh(1/3*b*x + 1/3*d) + sinh(1/3*b*x + 1/3*d)
- 1) + 30*(cosh(1/3*b*x + 1/3*d)^4*cosh(-5/3*a + 5/3*d) - cosh(1/3*b*x + 1
/3*d)^4*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d))/b
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+bx)}\coth(d+bx)\,dx = e^{\frac{5a}{3}}\int e^{\frac{5bx}{3}}\coth{(bx+d)}\,dx$$

input
```
integrate(exp(5/3*b*x+5/3*a)*coth(b*x+d),x)
```

output
```
exp(5*a/3)*Integral(exp(5*b*x/3)*coth(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.10

$$\int e^{\frac{5}{3}(a+bx)}\coth(d+bx)\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}+1\right)\right)e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,d\right)}}{b} - \frac{\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}-1\right)\right)e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,d\right)}}{b} - \frac{e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,d\right)}\log\left(e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}+e^{\left(-\frac{2}{3}\,bx-\frac{2}{3}\,d\right)}+1\right)}{2\,b} - \frac{e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,d\right)}\log\left(e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}+1\right)}{b} + \frac{e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,d\right)}\log\left(e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}-1\right)}{b} + \frac{e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,d\right)}\log\left(-e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}+e^{\left(-\frac{2}{3}\,bx-\frac{2}{3}\,d\right)}+1\right)}{2\,b} + \frac{3\,e^{\left(\frac{5}{3}\,bx+\frac{5}{3}\,a\right)}}{5\,b}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*coth(b*x+d),x, algorithm="maxima")
```

output
```
-sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-1/3*b*x - 1/3*d) + 1))*e^(5/3*a - 5/3*d
)/b - sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-1/3*b*x - 1/3*d) - 1))*e^(5/3*a -
5/3*d)/b - 1/2*e^(5/3*a - 5/3*d)*log(e^(-1/3*b*x - 1/3*d) + e^(-2/3*b*x -
2/3*d) + 1)/b - e^(5/3*a - 5/3*d)*log(e^(-1/3*b*x - 1/3*d) + 1)/b + e^(5/3
*a - 5/3*d)*log(e^(-1/3*b*x - 1/3*d) - 1)/b + 1/2*e^(5/3*a - 5/3*d)*log(-e
^(-1/3*b*x - 1/3*d) + e^(-2/3*b*x - 2/3*d) + 1)/b + 3/5*e^(5/3*b*x + 5/3*a
)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.05

$$\int e^{\frac{5}{3}(a+bx)}\coth(d+bx)\,dx$$
$$= \frac{10\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(\frac{1}{3}bx\right)}+e^{\left(-\frac{1}{3}d\right)}\right)e^{\left(\frac{1}{3}d\right)}\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}+10\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(\frac{1}{3}bx\right)}-e^{\left(-\frac{1}{3}d\right)}\right)e^{\left(\frac{1}{3}d\right)}\right.}{}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*coth(b*x+d),x, algorithm="giac")
```

output
```
1/10*(10*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(1/3*b*x) + e^(-1/3*d))*e^(1/3*d)
)*e^(5/3*a - 5/3*d) + 10*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(1/3*b*x) - e^(-1
/3*d))*e^(1/3*d))*e^(5/3*a - 5/3*d) - 5*e^(5/3*a - 5/3*d)*log(e^(1/3*b*x -
 1/3*d) + e^(2/3*b*x) + e^(-2/3*d)) + 5*e^(5/3*a - 5/3*d)*log(-e^(1/3*b*x
- 1/3*d) + e^(2/3*b*x) + e^(-2/3*d)) - 10*e^(5/3*a - 5/3*d)*log(e^(1/3*b*x
) + e^(-1/3*d)) + 10*e^(5/3*a - 5/3*d)*log(abs(e^(1/3*b*x) - e^(-1/3*d)))
+ 6*e^(5/3*b*x + 5/3*a))/b
```

Mupad [B] (verification not implemented)

Time = 4.42 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.25

$$\int e^{\frac{5}{3}(a+bx)}\coth(d+bx)\,dx = \text{Too large to display}$$

input
```
int(coth(d + b*x)*exp((5*a)/3 + (5*b*x)/3),x)
```

output
```
(3*exp((5*a)/3 + (5*b*x)/3))/(5*b) - (exp(10*a - 10*d)^(1/6)*log(- 4*exp((
10*a)/3)*exp(-(10*d)/3) - 4*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/
3)*(exp(10*a)*exp(-10*d))^(1/6)))/b + (exp(10*a - 10*d)^(1/6)*log(4*exp((5
*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*(exp(10*a)*exp(-10*d))^(1/6) -
4*exp((10*a)/3)*exp(-(10*d)/3)))/b - (log(- 4*exp((10*a)/3)*exp(-(10*d)/3)
 - 4*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)/2 - 1/
2)*(exp(10*a)*exp(-10*d))^(1/6))*exp(10*a - 10*d)^(1/6)*((3^(1/2)*1i)/2 -
1/2))/b + (log(4*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2
)*1i)/2 - 1/2)*(exp(10*a)*exp(-10*d))^(1/6) - 4*exp((10*a)/3)*exp(-(10*d)/
3))*exp(10*a - 10*d)^(1/6)*((3^(1/2)*1i)/2 - 1/2))/b - (log(- 4*exp((10*a)
/3)*exp(-(10*d)/3) - 4*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((
3^(1/2)*1i)/2 + 1/2)*(exp(10*a)*exp(-10*d))^(1/6))*exp(10*a - 10*d)^(1/6)*
((3^(1/2)*1i)/2 + 1/2))/b + (log(4*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp
((b*x)/3)*((3^(1/2)*1i)/2 + 1/2)*(exp(10*a)*exp(-10*d))^(1/6) - 4*exp((10*
a)/3)*exp(-(10*d)/3))*exp(10*a - 10*d)^(1/6)*((3^(1/2)*1i)/2 + 1/2))/b
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+bx)}\coth(d+bx)\,dx=\int e^{\frac{5bx}{3}+\frac{5a}{3}}\coth\left(bx+d\right)dx$$

input
```
int(exp(5/3*b*x+5/3*a)*coth(b*x+d),x)
```

output
```
int(e**((5*a + 5*b*x)/3)*coth(b*x + d),x)
```

3.68 $\int e^{\frac{5}{3}(a+bx)} \coth^2(d+bx)\, dx$

Optimal result

Integrand size = 20, antiderivative size = 232

$$\int e^{\frac{5}{3}(a+bx)} \coth^2(d+bx)\, dx = \frac{3e^{\frac{5(a-d)}{3}+\frac{5}{3}(d+bx)}}{5b} + \frac{2e^{\frac{5(a-d)}{3}+\frac{5}{3}(d+bx)}}{b\left(1-e^{2(d+bx)}\right)} - \frac{5e^{\frac{5(a-d)}{3}}\arctan\left(\frac{1-2e^{\frac{1}{3}(d+bx)}}{\sqrt{3}}\right)}{\sqrt{3}b} + \frac{5e^{\frac{5(a-d)}{3}}\arctan\left(\frac{1+2e^{\frac{1}{3}(d+bx)}}{\sqrt{3}}\right)}{\sqrt{3}b} - \frac{10e^{\frac{5(a-d)}{3}}\text{arctanh}\left(e^{\frac{1}{3}(d+bx)}\right)}{3b} - \frac{5e^{\frac{5(a-d)}{3}}\text{arctanh}\left(\frac{e^{\frac{1}{3}(d+bx)}}{1+e^{\frac{2}{3}(d+bx)}}\right)}{3b}$$

output
```
3/5*exp(5/3*b*x+5/3*a)/b+2*exp(5/3*b*x+5/3*a)/b/(1-exp(2*b*x+2*d))-5/3*3^(
1/2)*exp(5/3*a-5/3*d)*arctan(1/3*(1-2*exp(1/3*b*x+1/3*d))*3^(1/2))/b+5/3*3
^(1/2)*exp(5/3*a-5/3*d)*arctan(1/3*(1+2*exp(1/3*b*x+1/3*d))*3^(1/2))/b-10/
3*exp(5/3*a-5/3*d)*arctanh(exp(1/3*b*x+1/3*d))/b-5/3*exp(5/3*a-5/3*d)*arct
anh(exp(1/3*b*x+1/3*d)/(1+exp(2/3*b*x+2/3*d)))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.55

$$\int e^{\frac{5}{3}(a+bx)}\coth^2(d+bx)\,dx$$
$$= \frac{e^{5a/3}\left(27e^{\frac{5bx}{3}} - 25\text{RootSum}\left[-\cosh(d)+\sinh(d)+\cosh(d)\#1^6+\sinh(d)\#1^6\&, \frac{bx-3\log\left(e^{\frac{bx}{3}}-\#1\right)}{\#1}\&\right](\text{co}}{45b}$$

input
```
Integrate[E^((5*(a + b*x))/3)*Coth[d + b*x]^2,x]
```

output
```
(E^((5*a)/3)*(27*E^((5*b*x)/3) - 25*RootSum[-Cosh[d] + Sinh[d] + Cosh[d]*#
1^6 + Sinh[d]*#1^6 & , (b*x - 3*Log[E^((b*x)/3) - #1])/#1 & ]*(Cosh[d] - S
inh[d])^2 - (90*E^((5*b*x)/3)*(Cosh[d] - Sinh[d]))/((-1 + E^(2*b*x))*Cosh[
d] + (1 + E^(2*b*x))*Sinh[d])))/(45*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.78, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {2720, 27, 963, 27, 959, 825, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)}\coth^2(bx+d)\,dx$$
$$\Big\downarrow \text{2720}$$
$$\frac{3\int \frac{e^{\frac{5a}{3}+\frac{4bx}{3}}\left(1+e^{2bx}\right)^2}{\left(1-e^{2bx}\right)^2}de^{\frac{bx}{3}}}{b}$$
$$\Big\downarrow \text{27}$$

$$\frac{3e^{5a/3}\int \frac{e^{\frac{4bx}{3}}\left(1+e^{2bx}\right)^2}{\left(1-e^{2bx}\right)^2}de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \text{963}$$

$$\frac{3e^{5a/3}\left(\frac{2e^{\frac{5bx}{3}}}{3\left(1-e^{2bx}\right)}-\frac{1}{6}\int \frac{2e^{\frac{4bx}{3}}\left(7+3e^{2bx}\right)}{1-e^{2bx}}de^{\frac{bx}{3}}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{3e^{5a/3}\left(\frac{2e^{\frac{5bx}{3}}}{3\left(1-e^{2bx}\right)}-\frac{1}{3}\int \frac{e^{\frac{4bx}{3}}\left(7+3e^{2bx}\right)}{1-e^{2bx}}de^{\frac{bx}{3}}\right)}{b}$$

$$\Big\downarrow \text{959}$$

$$\frac{3e^{5a/3}\left(\frac{1}{3}\left(\frac{3}{5}e^{\frac{5bx}{3}}-10\int \frac{e^{\frac{4bx}{3}}}{1-e^{2bx}}de^{\frac{bx}{3}}\right)+\frac{2e^{\frac{5bx}{3}}}{3\left(1-e^{2bx}\right)}\right)}{b}$$

$$\Big\downarrow \text{825}$$

$$\frac{3e^{5a/3}\left(\frac{1}{3}\left(\frac{3}{5}e^{\frac{5bx}{3}}-10\left(\frac{1}{3}\int \frac{1}{1-e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{3}\int -\frac{1+e^{\frac{bx}{3}}}{2\left(1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}\right)}de^{\frac{bx}{3}}+\frac{1}{3}\int -\frac{1-e^{\frac{bx}{3}}}{2\left(1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}\right)}de^{\frac{bx}{3}}\right)\right)+\frac{2e^{\frac{5bx}{3}}}{3\left(1-e^{2bx}\right)}\right.}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{3e^{5a/3}\left(\frac{1}{3}\left(\frac{3}{5}e^{\frac{5bx}{3}}-10\left(\frac{1}{3}\int \frac{1}{1-e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1+e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1-e^{\frac{bx}{3}}}{1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)\right)+\frac{2e^{\frac{5bx}{3}}}{3\left(1-e^{2bx}\right)}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{3e^{5a/3}\left(\frac{1}{3}\left(\frac{3}{5}e^{\frac{5bx}{3}}-10\left(-\frac{1}{6}\int \frac{1+e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1-e^{\frac{bx}{3}}}{1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{3}\text{arctanh}\left(e^{\frac{bx}{3}}\right)\right)\right)+\frac{2e^{\frac{5bx}{3}}}{3\left(1-e^{2bx}\right)}\right)}{b}$$

$$\Big\downarrow \text{1142}$$

$$\frac{3e^{5a/3}\left(\frac{1}{3}\left(\frac{3}{5}e^{\frac{5bx}{3}}-10\left(\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{2}\int -\frac{1-2e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1+2e^{\frac{bx}{3}}}{1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{3}{2}\int \frac{1}{1+e^{\frac{bx}{3}}}\right.\right.\right.\right.}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{3e^{5a/3}\Big(\frac{1}{3}\Big(\frac{3}{5}e^{\frac{5bx}{3}}-10\Big(\frac{1}{6}\Big(\frac{1}{2}\int\frac{1-2e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{3}{2}\int\frac{1}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\Big)+\frac{1}{6}\Big(\frac{1}{2}\int\frac{1+2e^{\frac{bx}{3}}}{1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{3}{2}\int\frac{1}{1+e^{\frac{bx}{3}}+e^{\frac{2b}{}}}}{b}$$

$$\Big\downarrow \; 1083$$

$$\frac{3e^{5a/3}\Big(\frac{1}{3}\Big(\frac{3}{5}e^{\frac{5bx}{3}}-10\Big(\frac{1}{6}\Big(3\int\frac{1}{-3-e^{\frac{2bx}{3}}}d\Big(-1+2e^{\frac{bx}{3}}\Big)+\frac{1}{2}\int\frac{1-2e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\Big)+\frac{1}{6}\Big(3\int\frac{1}{-3-e^{\frac{2bx}{3}}}d\Big(1+2e^{\frac{bx}{3}}\Big)+\frac{1}{2}}{b}$$

$$\Big\downarrow \; 217$$

$$\frac{3e^{5a/3}\Big(\frac{1}{3}\Big(\frac{3}{5}e^{\frac{5bx}{3}}-10\Big(\frac{1}{6}\Big(\frac{1}{2}\int\frac{1-2e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\sqrt{3}\arctan\Big(\frac{2e^{\frac{bx}{3}}-1}{\sqrt{3}}\Big)\Big)+\frac{1}{6}\Big(\frac{1}{2}\int\frac{1+2e^{\frac{bx}{3}}}{1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\sqrt{3}\arctan\Big(}{b}$$

$$\Big\downarrow \; 1103$$

$$\frac{3e^{5a/3}\Big(\frac{1}{3}\Big(\frac{3}{5}e^{\frac{5bx}{3}}-10\Big(\frac{1}{6}\Big(-\sqrt{3}\arctan\Big(\frac{2e^{\frac{bx}{3}}-1}{\sqrt{3}}\Big)-\frac{1}{2}\log\Big(-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}+1\Big)\Big)+\frac{1}{6}\Big(\frac{1}{2}\log\Big(e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}+1\Big)-\sqrt{3}}{b}$$

input
```
Int[E^((5*(a + b*x))/3)*Coth[d + b*x]^2,x]
```

output
```
(3*E^((5*a)/3)*((2*E^((5*b*x)/3))/(3*(1 - E^(2*b*x))) + ((3*E^((5*b*x)/3))
/5 - 10*(ArcTanh[E^((b*x)/3)]/3 + (-(Sqrt[3]*ArcTan[(-1 + 2*E^((b*x)/3))/S
qrt[3]]) - Log[1 - E^((b*x)/3) + E^((2*b*x)/3)]/2)/6 + (-(Sqrt[3]*ArcTan[(
1 + 2*E^((b*x)/3))/Sqrt[3]]) + Log[1 + E^((b*x)/3) + E^((2*b*x)/3)]/2)/6))
/3))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 963
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Simp[1/(a*b^2*n*(p + 1))   Int[(e*x)^m*(a + b*x
^n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.54

method	result
risch	$\frac{3\,e^{\frac{5bx}{3}+\frac{5a}{3}}}{5b}-\frac{2\,e^{\frac{5bx}{3}+\frac{5a}{3}}}{\left(e^{2bx+2d}-1\right)b}-\frac{5\ln\left(1+e^{\frac{bx}{3}+\frac{d}{3}}\right)e^{\frac{5a}{3}-\frac{5d}{3}}}{3b}+\frac{5\ln\left(e^{\frac{bx}{3}+\frac{d}{3}}-1\right)e^{\frac{5a}{3}-\frac{5d}{3}}}{3b}+\frac{5\ln\left(e^{\frac{bx}{3}+\frac{d}{3}}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)e^{\frac{5a}{3}-\frac{5d}{3}}}{6b}+$

input
```
int(exp(5/3*b*x+5/3*a)*coth(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
3/5*exp(5/3*b*x+5/3*a)/b-2/(exp(2*b*x+2*d)-1)/b*exp(5/3*b*x+5/3*a)-5/3/b*l
n(1+exp(1/3*b*x+1/3*d))*exp(5/3*a-5/3*d)+5/3/b*ln(exp(1/3*b*x+1/3*d)-1)*ex
p(5/3*a-5/3*d)+5/6/b*ln(exp(1/3*b*x+1/3*d)-1/2+1/2*I*3^(1/2))*exp(5/3*a-5/
3*d)+5/6*I/b*ln(exp(1/3*b*x+1/3*d)-1/2+1/2*I*3^(1/2))*exp(5/3*a-5/3*d)*3^(
1/2)+5/6/b*ln(exp(1/3*b*x+1/3*d)-1/2-1/2*I*3^(1/2))*exp(5/3*a-5/3*d)-5/6*I
/b*ln(exp(1/3*b*x+1/3*d)-1/2-1/2*I*3^(1/2))*exp(5/3*a-5/3*d)*3^(1/2)-5/6/b
*ln(exp(1/3*b*x+1/3*d)+1/2+1/2*I*3^(1/2))*exp(5/3*a-5/3*d)+5/6*I/b*ln(exp(
1/3*b*x+1/3*d)+1/2+1/2*I*3^(1/2))*exp(5/3*a-5/3*d)*3^(1/2)-5/6/b*ln(exp(1/
3*b*x+1/3*d)+1/2-1/2*I*3^(1/2))*exp(5/3*a-5/3*d)-5/6*I/b*ln(exp(1/3*b*x+1/
3*d)+1/2-1/2*I*3^(1/2))*exp(5/3*a-5/3*d)*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3400 vs. $2(172) = 344$.

Time = 0.11 (sec) , antiderivative size = 3400, normalized size of antiderivative = 14.66

$$\int e^{\frac{5}{3}(a+bx)} \coth^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*coth(b*x+d)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+bx)} \coth^2(d+bx)\,dx = e^{\frac{5a}{3}} \int e^{\frac{5bx}{3}} \coth^2(bx+d)\,dx$$

input
```
integrate(exp(5/3*b*x+5/3*a)*coth(b*x+d)**2,x)
```

output
```
exp(5*a/3)*Integral(exp(5*b*x/3)*coth(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.03

$$\int e^{\frac{5}{3}(a+bx)}\coth^2(d+bx)\,dx = -\frac{5\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+1\right)\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}}{3b} - \frac{5\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}-1\right)\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}}{3b} - \frac{5e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{\left(-\frac{2}{3}bx-\frac{2}{3}d\right)}+1\right)}{6b} - \frac{5e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+1\right)}{3b} + \frac{5e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}-1\right)}{3b} + \frac{5e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(-e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{\left(-\frac{2}{3}bx-\frac{2}{3}d\right)}+1\right)}{6b} - \frac{\left(13e^{(-2bx-2d)}-3\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}}{5b\left(e^{\left(-\frac{5}{3}bx-\frac{5}{3}d\right)}-e^{\left(-\frac{11}{3}bx-\frac{11}{3}d\right)}\right)}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*coth(b*x+d)^2,x, algorithm="maxima")
```

output
```
-5/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-1/3*b*x - 1/3*d) + 1))*e^(5/3*a - 5
/3*d)/b - 5/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-1/3*b*x - 1/3*d) - 1))*e^(
5/3*a - 5/3*d)/b - 5/6*e^(5/3*a - 5/3*d)*log(e^(-1/3*b*x - 1/3*d) + e^(-2/
3*b*x - 2/3*d) + 1)/b - 5/3*e^(5/3*a - 5/3*d)*log(e^(-1/3*b*x - 1/3*d) + 1
)/b + 5/3*e^(5/3*a - 5/3*d)*log(e^(-1/3*b*x - 1/3*d) - 1)/b + 5/6*e^(5/3*a
 - 5/3*d)*log(-e^(-1/3*b*x - 1/3*d) + e^(-2/3*b*x - 2/3*d) + 1)/b - 1/5*(1
3*e^(-2*b*x - 2*d) - 3)*e^(5/3*a - 5/3*d)/(b*(e^(-5/3*b*x - 5/3*d) - e^(-1
1/3*b*x - 11/3*d)))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.95

$$\int e^{\frac{5}{3}(a+bx)}\coth^2(d+bx)\,dx$$
$$= \frac{50\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(\frac{1}{3}bx\right)}+e^{\left(-\frac{1}{3}d\right)}\right)e^{\left(\frac{1}{3}d\right)}\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}+50\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(\frac{1}{3}bx\right)}-e^{\left(-\frac{1}{3}d\right)}\right)e^{\left(\frac{1}{3}d\right)}\right.}{}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*coth(b*x+d)^2,x, algorithm="giac")
```

output
```
1/30*(50*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(1/3*b*x) + e^(-1/3*d))*e^(1/3*d)
)*e^(5/3*a - 5/3*d) + 50*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(1/3*b*x) - e^(-1
/3*d))*e^(1/3*d))*e^(5/3*a - 5/3*d) - 25*e^(5/3*a - 5/3*d)*log(e^(1/3*b*x
- 1/3*d) + e^(2/3*b*x) + e^(-2/3*d)) + 25*e^(5/3*a - 5/3*d)*log(-e^(1/3*b*
x - 1/3*d) + e^(2/3*b*x) + e^(-2/3*d)) - 50*e^(5/3*a - 5/3*d)*log(e^(1/3*b
*x) + e^(-1/3*d)) + 50*e^(5/3*a - 5/3*d)*log(abs(e^(1/3*b*x) - e^(-1/3*d))
) - 60*e^(5/3*b*x + 5/3*a)/(e^(2*b*x + 2*d) - 1) + 18*e^(5/3*b*x + 5/3*a))
/b
```

Mupad [B] (verification not implemented)

Time = 5.81 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.93

$$\int e^{\frac{5}{3}(a+bx)}\coth^2(d+bx)\,dx = \text{Too large to display}$$

input
```
int(coth(d + b*x)^2*exp((5*a)/3 + (5*b*x)/3),x)
```

output
```
(3*exp((5*a)/3 + (5*b*x)/3))/(5*b) - (2*exp((5*a)/3 + (5*b*x)/3))/(b*(exp(
2*d + 2*b*x) - 1)) - (5*exp(10*a - 10*d)^(1/6)*log(- (100*exp((10*a)/3)*ex
p(-(10*d)/3))/9 - (100*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*(e
xp(10*a)*exp(-10*d))^(1/6))/9))/(3*b) + (5*exp(10*a - 10*d)^(1/6)*log((100
*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*(exp(10*a)*exp(-10*d))^(
1/6))/9 - (100*exp((10*a)/3)*exp(-(10*d)/3))/9))/(3*b) - (5*log(- (100*exp
((10*a)/3)*exp(-(10*d)/3))/9 - (100*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*ex
p((b*x)/3)*((3^(1/2)*1i)/2 - 1/2)*(exp(10*a)*exp(-10*d))^(1/6))/9)*exp(10*
a - 10*d)^(1/6)*((3^(1/2)*1i)/2 - 1/2))/(3*b) + (5*log((100*exp((5*a)/3)*e
xp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)/2 - 1/2)*(exp(10*a)*exp(-
10*d))^(1/6))/9 - (100*exp((10*a)/3)*exp(-(10*d)/3))/9)*exp(10*a - 10*d)^(
1/6)*((3^(1/2)*1i)/2 - 1/2))/(3*b) - (5*log(- (100*exp((10*a)/3)*exp(-(10*
d)/3))/9 - (100*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)
*1i)/2 + 1/2)*(exp(10*a)*exp(-10*d))^(1/6))/9)*exp(10*a - 10*d)^(1/6)*((3^
(1/2)*1i)/2 + 1/2))/(3*b) + (5*log((100*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3
)*exp((b*x)/3)*((3^(1/2)*1i)/2 + 1/2)*(exp(10*a)*exp(-10*d))^(1/6))/9 - (1
00*exp((10*a)/3)*exp(-(10*d)/3))/9)*exp(10*a - 10*d)^(1/6)*((3^(1/2)*1i)/2
 + 1/2))/(3*b)
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+bx)}\coth^2(d+bx)\,dx = \int e^{\frac{5bx}{3}+\frac{5a}{3}}\coth\left(bx+d\right)^2dx$$

input
```
int(exp(5/3*b*x+5/3*a)*coth(b*x+d)^2,x)
```

output
```
int(e**((5*a + 5*b*x)/3)*coth(b*x + d)**2,x)
```

3.69 $\int e^{\frac{5}{3}(a+bx)}\coth^3(d+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 279

$$\int e^{\frac{5}{3}(a+bx)}\coth^3(d+bx)\,dx = \frac{3e^{\frac{5(a-d)}{3}+\frac{5}{3}(d+bx)}}{5b} - \frac{2e^{\frac{5(a-d)}{3}+\frac{5}{3}(d+bx)}}{b\left(1-e^{2(d+bx)}\right)^2} + \frac{11e^{\frac{5(a-d)}{3}+\frac{5}{3}(d+bx)}}{3b\left(1-e^{2(d+bx)}\right)} - \frac{43e^{\frac{5(a-d)}{3}}\arctan\left(\frac{1-2e^{\frac{1}{3}(d+bx)}}{\sqrt{3}}\right)}{6\sqrt{3}b} + \frac{43e^{\frac{5(a-d)}{3}}\arctan\left(\frac{1+2e^{\frac{1}{3}(d+bx)}}{\sqrt{3}}\right)}{6\sqrt{3}b} - \frac{43e^{\frac{5(a-d)}{3}}\operatorname{arctanh}\left(e^{\frac{1}{3}(d+bx)}\right)}{9b} - \frac{43e^{\frac{5(a-d)}{3}}\operatorname{arctanh}\left(\frac{e^{\frac{1}{3}(d+bx)}}{1+e^{\frac{2}{3}(d+bx)}}\right)}{18b}$$

output
```
3/5*exp(5/3*b*x+5/3*a)/b-2*exp(5/3*b*x+5/3*a)/b/(1-exp(2*b*x+2*d))^2+11/3*
exp(5/3*b*x+5/3*a)/b/(1-exp(2*b*x+2*d))-43/18*3^(1/2)*exp(5/3*a-5/3*d)*arc
tan(1/3*(1-2*exp(1/3*b*x+1/3*d))*3^(1/2))/b+43/18*3^(1/2)*exp(5/3*a-5/3*d)
*arctan(1/3*(1+2*exp(1/3*b*x+1/3*d))*3^(1/2))/b-43/9*exp(5/3*a-5/3*d)*arct
anh(exp(1/3*b*x+1/3*d))/b-43/18*exp(5/3*a-5/3*d)*arctanh(exp(1/3*b*x+1/3*d
)/(1+exp(2/3*b*x+2/3*d)))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.61

$$\int e^{\frac{5}{3}(a+bx)}\coth^3(d+bx)\,dx$$
$$=\frac{e^{5a/3}\left(162e^{\frac{5bx}{3}}-215\text{RootSum}\left[-\cosh(d)+\sinh(d)+\cosh(d)\#1^6+\sinh(d)\#1^6\&,\frac{bx-3\log\left(e^{\frac{bx}{3}}-\#1\right)}{\#1}\&\right]\right.}{270b}$$

input
```
Integrate[E^((5*(a + b*x))/3)*Coth[d + b*x]^3,x]
```

output
```
(E^((5*a)/3)*(162*E^((5*b*x)/3) - 215*RootSum[-Cosh[d] + Sinh[d] + Cosh[d]
*#1^6 + Sinh[d]*#1^6 & , (b*x - 3*Log[E^((b*x)/3) - #1])/#1 & ]*(Cosh[d] -
 Sinh[d])^2 - (540*E^((5*b*x)/3)*(Cosh[d] - Sinh[d])^2)/((-1 + E^(2*b*x))*
Cosh[d] + (1 + E^(2*b*x))*Sinh[d])^2 - (990*E^((5*b*x)/3)*(Cosh[d] - Sinh[
d]))/((-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d])))/(270*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.82, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.800$, Rules used = {2720, 25, 27, 968, 27, 1047, 27, 959, 825, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)}\coth^3(bx+d)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{3\int -\frac{e^{\frac{5a}{3}+\frac{4bx}{3}}\left(1+e^{2bx}\right)^3}{\left(1-e^{2bx}\right)^3}de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{3\int \frac{e^{\frac{5a}{3}+\frac{4bx}{3}}\left(1+e^{2bx}\right)^3}{\left(1-e^{2bx}\right)^3}de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3e^{5a/3}\int \frac{e^{\frac{4bx}{3}}\left(1+e^{2bx}\right)^3}{\left(1-e^{2bx}\right)^3}de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \text{968}$$

$$-\frac{3e^{5a/3}\left(\frac{1}{12}\int \frac{2e^{\frac{4bx}{3}}\left(1-11e^{2bx}\right)\left(1+e^{2bx}\right)}{\left(1-e^{2bx}\right)^2}de^{\frac{bx}{3}}+\frac{e^{\frac{5bx}{3}}\left(e^{2bx}+1\right)^2}{6\left(1-e^{2bx}\right)^2}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\int \frac{e^{\frac{4bx}{3}}\left(1-11e^{2bx}\right)\left(1+e^{2bx}\right)}{\left(1-e^{2bx}\right)^2}de^{\frac{bx}{3}}+\frac{e^{\frac{5bx}{3}}\left(e^{2bx}+1\right)^2}{6\left(1-e^{2bx}\right)^2}\right)}{b}$$

$$\Big\downarrow \text{1047}$$

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\left(\frac{1}{6}\int -\frac{4e^{\frac{4bx}{3}}\left(1-44e^{2bx}\right)}{1-e^{2bx}}de^{\frac{bx}{3}}+\frac{e^{\frac{5bx}{3}}\left(1-11e^{2bx}\right)}{3\left(1-e^{2bx}\right)}\right)+\frac{e^{\frac{5bx}{3}}\left(e^{2bx}+1\right)^2}{6\left(1-e^{2bx}\right)^2}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\left(\frac{e^{\frac{5bx}{3}}\left(1-11e^{2bx}\right)}{3\left(1-e^{2bx}\right)}-\frac{2}{3}\int \frac{e^{\frac{4bx}{3}}\left(1-44e^{2bx}\right)}{1-e^{2bx}}de^{\frac{bx}{3}}\right)+\frac{e^{\frac{5bx}{3}}\left(e^{2bx}+1\right)^2}{6\left(1-e^{2bx}\right)^2}\right)}{b}$$

$$\Big\downarrow \text{959}$$

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\left(\frac{e^{\frac{5bx}{3}}\left(1-11e^{2bx}\right)}{3\left(1-e^{2bx}\right)}-\frac{2}{3}\left(\frac{44}{5}e^{\frac{5bx}{3}}-43\int \frac{e^{\frac{4bx}{3}}}{1-e^{2bx}}de^{\frac{bx}{3}}\right)\right)+\frac{e^{\frac{5bx}{3}}\left(e^{2bx}+1\right)^2}{6\left(1-e^{2bx}\right)^2}\right)}{b}$$

$$\Big\downarrow \text{825}$$

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\left(\frac{e^{\frac{5bx}{3}}\left(1-11e^{2bx}\right)}{3\left(1-e^{2bx}\right)}-\frac{2}{3}\left(\frac{44}{5}e^{\frac{5bx}{3}}-43\left(\frac{1}{3}\int \frac{1}{1-e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{3}\int -\frac{1+e^{\frac{bx}{3}}}{2\left(1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}\right)}de^{\frac{bx}{3}}+\frac{1}{3}\int -\frac{1-e^{\frac{bx}{3}}}{2\left(1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}\right.}\right.\right.\right.}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\left(\frac{e^{\frac{5bx}{3}}(1-11e^{2bx})}{3(1-e^{2bx})}-\frac{2}{3}\left(\frac{44}{5}e^{\frac{5bx}{3}}-43\left(\frac{1}{3}\int\frac{1}{1-e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int\frac{1+e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int\frac{1-e^{\frac{bx}{3}}}{1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)\right)\right)+}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\left(\frac{e^{\frac{5bx}{3}}(1-11e^{2bx})}{3(1-e^{2bx})}-\frac{2}{3}\left(\frac{44}{5}e^{\frac{5bx}{3}}-43\left(-\frac{1}{6}\int\frac{1+e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int\frac{1-e^{\frac{bx}{3}}}{1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{3}\text{arctanh}\left(e^{\frac{bx}{3}}\right)\right)\right)\right)}{b}$$

$$\Big\downarrow \text{1142}$$

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\left(\frac{e^{\frac{5bx}{3}}(1-11e^{2bx})}{3(1-e^{2bx})}-\frac{2}{3}\left(\frac{44}{5}e^{\frac{5bx}{3}}-43\left(\frac{1}{6}\left(-\frac{3}{2}\int\frac{1}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{2}\int-\frac{1-2e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{1+2}{1+e^{\frac{bx}{3}}}\right.\right.\right.\right.\right.}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\left(\frac{e^{\frac{5bx}{3}}(1-11e^{2bx})}{3(1-e^{2bx})}-\frac{2}{3}\left(\frac{44}{5}e^{\frac{5bx}{3}}-43\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{3}{2}\int\frac{1}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{1+2e^{\frac{bx}{3}}}{1+e^{\frac{bx}{3}}+e^{\frac{2}{}}}\right.\right.\right.\right.\right.}{b}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\left(\frac{e^{\frac{5bx}{3}}(1-11e^{2bx})}{3(1-e^{2bx})}-\frac{2}{3}\left(\frac{44}{5}e^{\frac{5bx}{3}}-43\left(\frac{1}{6}\left(3\int\frac{1}{-3-e^{\frac{2bx}{3}}}d\left(-1+2e^{\frac{bx}{3}}\right)+\frac{1}{2}\int\frac{1-2e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(3\int\frac{}{-3-}\right.\right.\right.\right.\right.}{b}$$

$$\Big\downarrow \text{217}$$

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\left(\frac{e^{\frac{5bx}{3}}(1-11e^{2bx})}{3(1-e^{2bx})}-\frac{2}{3}\left(\frac{44}{5}e^{\frac{5bx}{3}}-43\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\sqrt{3}\arctan\left(\frac{2e^{\frac{bx}{3}}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{1+2e^{\frac{bx}{3}}}{1+e^{\frac{bx}{3}}+e}\right.\right.\right.\right.\right.}{b}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{3e^{5a/3}\left(\frac{1}{6}\left(\frac{e^{\frac{5bx}{3}}(1-11e^{2bx})}{3(1-e^{2bx})}-\frac{2}{3}\left(\frac{44}{5}e^{\frac{5bx}{3}}-43\left(\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{2e^{\frac{bx}{3}}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}+1\right)\right)+\frac{1}{6}\left(\frac{1}{2}\log\right.\right.\right.\right.\right.}{b}$$

input

```
Int[E^((5*(a + b*x))/3)*Coth[d + b*x]^3,x]
```

output
```
(-3*E^((5*a)/3)*((E^((5*b*x)/3)*(1 + E^(2*b*x))^2)/(6*(1 - E^(2*b*x))^2) +
 ((E^((5*b*x)/3)*(1 - 11*E^(2*b*x)))/(3*(1 - E^(2*b*x))) - (2*((44*E^((5*b
*x)/3))/5 - 43*(ArcTanh[E^((b*x)/3)]/3 + (-(Sqrt[3]*ArcTan[(-1 + 2*E^((b*x
)/3))/Sqrt[3]]) - Log[1 - E^((b*x)/3) + E^((2*b*x)/3)]/2)/6 + (-(Sqrt[3]*A
rcTan[(1 + 2*E^((b*x)/3))/Sqrt[3]]) + Log[1 + E^((b*x)/3) + E^((2*b*x)/3)]
/2)/6)))/3)/6))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 968
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
 x]
```

rule 1047
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Simp[1/(
a*b*n*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c
*(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m
+ n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e
- a*f])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.33

method	result
risch	$\frac{3\,\mathrm{e}^{\frac{5bx}{3}+\frac{5a}{3}}}{5b}-\frac{\left(11\,\mathrm{e}^{2bx+2d}-5\right)\mathrm{e}^{\frac{5bx}{3}+\frac{5a}{3}}}{3\left(\mathrm{e}^{2bx+2d}-1\right)^{2}b}+\frac{43\ln\left(\mathrm{e}^{\frac{bx}{3}+\frac{d}{3}}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\mathrm{e}^{\frac{5a}{3}-\frac{5d}{3}}}{36b}+\frac{43i\ln\left(\mathrm{e}^{\frac{bx}{3}+\frac{d}{3}}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\mathrm{e}^{\frac{5a}{3}-\frac{5d}{3}}\sqrt{3}}{36b}+\frac{43\ln}{}$

input
```
int(exp(5/3*b*x+5/3*a)*coth(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
3/5*exp(5/3*b*x+5/3*a)/b-1/3/(exp(2*b*x+2*d)-1)^2/b*(11*exp(2*b*x+2*d)-5)*
exp(5/3*b*x+5/3*a)+43/36/b*ln(exp(1/3*b*x+1/3*d)-1/2+1/2*I*3^(1/2))*exp(5/
3*a-5/3*d)+43/36*I/b*ln(exp(1/3*b*x+1/3*d)-1/2+1/2*I*3^(1/2))*exp(5/3*a-5/
3*d)*3^(1/2)+43/36/b*ln(exp(1/3*b*x+1/3*d)-1/2-1/2*I*3^(1/2))*exp(5/3*a-5/
3*d)-43/36*I/b*ln(exp(1/3*b*x+1/3*d)-1/2-1/2*I*3^(1/2))*exp(5/3*a-5/3*d)*3
^(1/2)+43/18/b*ln(exp(1/3*b*x+1/3*d)-1)*exp(5/3*a-5/3*d)-43/18/b*ln(1+exp(
1/3*b*x+1/3*d))*exp(5/3*a-5/3*d)-43/36/b*ln(exp(1/3*b*x+1/3*d)+1/2+1/2*I*3
^(1/2))*exp(5/3*a-5/3*d)+43/36*I/b*ln(exp(1/3*b*x+1/3*d)+1/2+1/2*I*3^(1/2)
)*exp(5/3*a-5/3*d)*3^(1/2)-43/36/b*ln(exp(1/3*b*x+1/3*d)+1/2-1/2*I*3^(1/2)
)*exp(5/3*a-5/3*d)-43/36*I/b*ln(exp(1/3*b*x+1/3*d)+1/2-1/2*I*3^(1/2))*exp(
5/3*a-5/3*d)*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7903 vs. $2(199) = 398$.

Time = 0.19 (sec) , antiderivative size = 7903, normalized size of antiderivative = 28.33

$$\int e^{\frac{5}{3}(a+bx)} \coth^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*coth(b*x+d)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+bx)} \coth^3(d+bx)\,dx = e^{\frac{5a}{3}} \int e^{\frac{5bx}{3}} \coth^3(bx+d)\,dx$$

input
```
integrate(exp(5/3*b*x+5/3*a)*coth(b*x+d)**3,x)
```

output
```
exp(5*a/3)*Integral(exp(5*b*x/3)*coth(b*x + d)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.93

$$\int e^{\frac{5}{3}(a+bx)}\coth^3(d+bx)\,dx = -\frac{43\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+1\right)\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}}{18b} - \frac{43\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}-1\right)\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}}{18b} - \frac{43e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{\left(-\frac{2}{3}bx-\frac{2}{3}d\right)}+1\right)}{36b} - \frac{43e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+1\right)}{18b} + \frac{43e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}-1\right)}{18b} + \frac{43e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(-e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{\left(-\frac{2}{3}bx-\frac{2}{3}d\right)}+1\right)}{36b} - \frac{\left(73e^{(-2bx-2d)}-34e^{(-4bx-4d)}-9\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}}{15b\left(e^{\left(-\frac{5}{3}bx-\frac{5}{3}d\right)}-2e^{\left(-\frac{11}{3}bx-\frac{11}{3}d\right)}+e^{\left(-\frac{17}{3}bx-\frac{17}{3}d\right)}\right)}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*coth(b*x+d)^3,x, algorithm="maxima")
```

output
```
-43/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-1/3*b*x - 1/3*d) + 1))*e^(5/3*a -
 5/3*d)/b - 43/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-1/3*b*x - 1/3*d) - 1))
*e^(5/3*a - 5/3*d)/b - 43/36*e^(5/3*a - 5/3*d)*log(e^(-1/3*b*x - 1/3*d) +
e^(-2/3*b*x - 2/3*d) + 1)/b - 43/18*e^(5/3*a - 5/3*d)*log(e^(-1/3*b*x - 1/
3*d) + 1)/b + 43/18*e^(5/3*a - 5/3*d)*log(e^(-1/3*b*x - 1/3*d) - 1)/b + 43
/36*e^(5/3*a - 5/3*d)*log(-e^(-1/3*b*x - 1/3*d) + e^(-2/3*b*x - 2/3*d) + 1
)/b - 1/15*(73*e^(-2*b*x - 2*d) - 34*e^(-4*b*x - 4*d) - 9)*e^(5/3*a - 5/3*
d)/(b*(e^(-5/3*b*x - 5/3*d) - 2*e^(-11/3*b*x - 11/3*d) + e^(-17/3*b*x - 17
/3*d)))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.85

$$\int e^{\frac{5}{3}(a+bx)}\coth^3(d+bx)\,dx$$

$$= \frac{430\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(\frac{1}{3}bx\right)}+e^{\left(-\frac{1}{3}d\right)}\right)e^{\left(\frac{1}{3}d\right)}\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}+430\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(\frac{1}{3}bx\right)}-e^{\left(-\frac{1}{3}d\right)}\right)e^{\left(\frac{1}{3}\right.}\right.}{}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*coth(b*x+d)^3,x, algorithm="giac")
```

output
```
1/180*(430*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(1/3*b*x) + e^(-1/3*d))*e^(1/3*
d))*e^(5/3*a - 5/3*d) + 430*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(1/3*b*x) - e^
(-1/3*d))*e^(1/3*d))*e^(5/3*a - 5/3*d) - 215*e^(5/3*a - 5/3*d)*log(e^(1/3*
b*x - 1/3*d) + e^(2/3*b*x) + e^(-2/3*d)) + 215*e^(5/3*a - 5/3*d)*log(-e^(1
/3*b*x - 1/3*d) + e^(2/3*b*x) + e^(-2/3*d)) - 430*e^(5/3*a - 5/3*d)*log(e^
(1/3*b*x) + e^(-1/3*d)) + 430*e^(5/3*a - 5/3*d)*log(abs(e^(1/3*b*x) - e^(-
1/3*d))) - 60*(11*e^(11/3*b*x + 5/3*a + 2*d) - 5*e^(5/3*b*x + 5/3*a))/(e^(
2*b*x + 2*d) - 1)^2 + 108*e^(5/3*b*x + 5/3*a))/b
```

Mupad [B] (verification not implemented)

Time = 5.43 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.74

$$\int e^{\frac{5}{3}(a+bx)}\coth^3(d+bx)\,dx = \text{Too large to display}$$

input
```
int(coth(d + b*x)^3*exp((5*a)/3 + (5*b*x)/3),x)
```

output
```
(3*exp((5*a)/3 + (5*b*x)/3))/(5*b) - (11*exp((5*a)/3 + (5*b*x)/3))/(3*b*(e
xp(2*d + 2*b*x) - 1)) - (2*exp((5*a)/3 + (5*b*x)/3))/(b*(exp(4*d + 4*b*x)
- 2*exp(2*d + 2*b*x) + 1)) - (43*exp(10*a - 10*d)^(1/6)*log(- (1849*exp((1
0*a)/3)*exp(-(10*d)/3))/81 - (1849*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp
((b*x)/3)*(exp(10*a)*exp(-10*d))^(1/6))/81))/(18*b) + (43*exp(10*a - 10*d)
^(1/6)*log((1849*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*(exp(10*
a)*exp(-10*d))^(1/6))/81 - (1849*exp((10*a)/3)*exp(-(10*d)/3))/81))/(18*b)
 - (43*log(- (1849*exp((10*a)/3)*exp(-(10*d)/3))/81 - (1849*exp((5*a)/3)*e
xp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)/2 - 1/2)*(exp(10*a)*exp(-
10*d))^(1/6))/81)*exp(10*a - 10*d)^(1/6)*((3^(1/2)*1i)/2 - 1/2))/(18*b) +
(43*log((1849*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1
i)/2 - 1/2)*(exp(10*a)*exp(-10*d))^(1/6))/81 - (1849*exp((10*a)/3)*exp(-(1
0*d)/3))/81)*exp(10*a - 10*d)^(1/6)*((3^(1/2)*1i)/2 - 1/2))/(18*b) - (43*l
og(- (1849*exp((10*a)/3)*exp(-(10*d)/3))/81 - (1849*exp((5*a)/3)*exp(d/3)*
exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)/2 + 1/2)*(exp(10*a)*exp(-10*d))^(
1/6))/81)*exp(10*a - 10*d)^(1/6)*((3^(1/2)*1i)/2 + 1/2))/(18*b) + (43*log(
(1849*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)/2 + 1
/2)*(exp(10*a)*exp(-10*d))^(1/6))/81 - (1849*exp((10*a)/3)*exp(-(10*d)/3))
/81)*exp(10*a - 10*d)^(1/6)*((3^(1/2)*1i)/2 + 1/2))/(18*b)
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+bx)} \coth^3(d+bx)\,dx = \int e^{\frac{5bx}{3}+\frac{5a}{3}} \coth\left(bx+d\right)^3 dx$$

input
```
int(exp(5/3*b*x+5/3*a)*coth(b*x+d)^3,x)
```

output
```
int(e**((5*a + 5*b*x)/3)*coth(b*x + d)**3,x)
```

3.70 $\int F^{c(a+bx)}\coth(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 78

$$\int F^{c(a+bx)}\coth(d+ex)\,dx = \frac{F^{c(a+bx)}}{bc\log(F)} - \frac{2F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},e^{2d+2ex}\right)}{bc\log(F)}$$

output
```
F^(c*(b*x+a))/b/c/ln(F)-2*F^(c*(b*x+a))*hypergeom([1, 1/2*b*c*ln(F)/e],[1+
1/2*b*c*ln(F)/e],exp(2*e*x+2*d))/b/c/ln(F)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.76

$$\int F^{c(a+bx)}\coth(d+ex)\,dx = \frac{F^{c(a+bx)}\left(1-2\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},e^{2(d+ex)}\right)\right)}{bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Coth[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*(1 - 2*Hypergeometric2F1[1, (b*c*Log[F])/(2*e), 1 + (b*c*
Log[F])/(2*e), E^(2*(d + e*x))]))/(b*c*Log[F])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6008}$$

$$\int \left(\frac{2F^{c(a+bx)}}{e^{2(d+ex)}-1}+F^{c(a+bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{F^{c(a+bx)}}{bc\log(F)}-\frac{2F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,e^{2(d+ex)}\right)}{bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Coth[d + e*x],x]
```

output
```
F^(c*(a + b*x))/(b*c*Log[F]) - (2*F^(c*(a + b*x))*Hypergeometric2F1[1, (b*
c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6008
```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*(d + e*x)))^n/(-1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \coth(ex+d)\,dx$$

input
```
int(F^(c*(b*x+a))*coth(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*coth(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)} \coth(d+ex)\,dx = \int F^{(bx+a)c} \coth(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*coth(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\coth(d+ex)\,dx = \int F^{c(a+bx)}\coth\left(d+ex\right)dx$$

input
```
integrate(F**(c*(b*x+a))*coth(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*coth(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\coth(d+ex)\,dx = \int F^{(bx+a)c}\coth\left(ex+d\right)\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d),x, algorithm="maxima")
```

output
```
4*F^(a*c)*e*integrate(F^(b*c*x)/(b*c*log(F) + (b*c*e^(4*d)*log(F) - 2*e*e^
(4*d))*e^(4*e*x) - 2*(b*c*e^(2*d)*log(F) - 2*e*e^(2*d))*e^(2*e*x) - 2*e),
x) - (F^(a*c)*b*c*log(F) + 2*F^(a*c)*e + (F^(a*c)*b*c*e^(2*d)*log(F) - 2*F
^(a*c)*e*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 2*b*c*e*log(F)
- (b^2*c^2*e^(2*d)*log(F)^2 - 2*b*c*e*e^(2*d)*log(F))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\coth(d+ex)\,dx = \int F^{(bx+a)c}\coth\left(ex+d\right)\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*coth(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\coth(d+ex)\,dx = \int F^{c\,(a+b\,x)}\coth(d+e\,x)\;dx$$

input
```
int(F^(c*(a + b*x))*coth(d + e*x),x)
```

output
```
int(F^(c*(a + b*x))*coth(d + e*x), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\coth(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\coth\left(ex+d\right)dx\right)$$

input
```
int(F^(c*(b*x+a))*coth(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*coth(d + e*x),x)
```

3.71 $\int F^{c(a+bx)} \coth^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 101

$$\int F^{c(a+bx)} \coth^2(d+ex)\,dx = \frac{2F^{c(a+bx)}}{e(1-e^{2d+2ex})} - \frac{2F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},e^{2d+2ex}\right)}{e} + \frac{F^{c(a+bx)}}{bc\log(F)}$$

output
```
2*F^(c*(b*x+a))/e/(1-exp(2*e*x+2*d))-2*F^(c*(b*x+a))*hypergeom([1, 1/2*b*c
*ln(F)/e],[1+1/2*b*c*ln(F)/e],exp(2*e*x+2*d))/e+F^(c*(b*x+a))/b/c/ln(F)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

$$\int F^{c(a+bx)}\coth^2(d+ex)\,dx = F^{c(a+bx)}\left(\frac{2}{e-ee^{2d}} - \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},e^{2(d+ex)}\right)}{e} + \frac{1}{bc\log(F)} + \frac{\text{csch}(d)\text{csch}(d+ex)\sinh(ex)}{e}\right)$$

input
```
Integrate[F^(c*(a + b*x))*Coth[d + e*x]^2,x]
```

output
```
F^(c*(a + b*x))*(2/(e - e*E^(2*d)) - (2*Hypergeometric2F1[1, (b*c*Log[F])/
(2*e), 1 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/e + 1/(b*c*Log[F]) + (Csc
h[d]*Csch[d + e*x]*Sinh[e*x])/e)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6008}$$

$$\int \left(\frac{4F^{c(a+bx)}}{e^{2(d+ex)}-1}+\frac{4F^{c(a+bx)}}{\left(e^{2(d+ex)}-1\right)^2}+F^{c(a+bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,e^{2(d+ex)}\right)}{bc\log(F)}+\frac{4F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,e^{2(d+ex)}\right)}{bc\log(F)}+\frac{F^{c(a+bx)}}{bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Coth[d + e*x]^2,x]
```

output
```
F^(c*(a + b*x))/(b*c*Log[F]) - (4*F^(c*(a + b*x))*Hypergeometric2F1[1, (b*
c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(b*c*Log[F]) +
(4*F^(c*(a + b*x))*Hypergeometric2F1[2, (b*c*Log[F])/(2*e), 1 + (b*c*Log[F
])/(2*e), E^(2*(d + e*x))])/(b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6008
```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*(d + e*x)))^n/(-1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)}\coth\left(ex+d\right)^{2}dx$$

input
```
int(F^(c*(b*x+a))*coth(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*coth(e*x+d)^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\coth^2(d+ex)\,dx = \int F^{(bx+a)c}\coth\left(ex+d\right)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*coth(e*x + d)^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\coth^2(d+ex)\,dx = \int F^{c(a+bx)}\coth^2\left(d+ex\right)dx$$

input
```
integrate(F**(c*(b*x+a))*coth(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*coth(d + e*x)**2, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\coth^2(d+ex)\,dx = \int F^{(bx+a)c}\coth\left(ex+d\right)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)^2,x, algorithm="maxima")
```

output
```
16*F^(a*c)*b*c*e*integrate(-F^(b*c*x)/(b^2*c^2*log(F)^2 - 6*b*c*e*log(F) +
 8*e^2 - (b^2*c^2*e^(6*d)*log(F)^2 - 6*b*c*e*e^(6*d)*log(F) + 8*e^2*e^(6*d
))*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*log(F) + 8*e^
2*e^(4*d))*e^(4*e*x) - 3*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^(2*d)*log(F
) + 8*e^2*e^(2*d))*e^(2*e*x)), x)*log(F) + (F^(a*c)*b^2*c^2*log(F)^2 + 10*
F^(a*c)*b*c*e*log(F) + 8*F^(a*c)*e^2 + (F^(a*c)*b^2*c^2*e^(4*d)*log(F)^2 -
 6*F^(a*c)*b*c*e*e^(4*d)*log(F) + 8*F^(a*c)*e^2*e^(4*d))*e^(4*e*x) + 2*(F^
(a*c)*b^2*c^2*e^(2*d)*log(F)^2 - 2*F^(a*c)*b*c*e*e^(2*d)*log(F) - 8*F^(a*c
)*e^2*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^3*c^3*log(F)^3 - 6*b^2*c^2*e*log(F)
^2 + 8*b*c*e^2*log(F) + (b^3*c^3*e^(4*d)*log(F)^3 - 6*b^2*c^2*e*e^(4*d)*lo
g(F)^2 + 8*b*c*e^2*e^(4*d)*log(F))*e^(4*e*x) - 2*(b^3*c^3*e^(2*d)*log(F)^3
 - 6*b^2*c^2*e*e^(2*d)*log(F)^2 + 8*b*c*e^2*e^(2*d)*log(F))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\coth^2(d+ex)\,dx = \int F^{(bx+a)c}\coth(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*coth(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\coth^2(d+ex)\,dx = \int F^{c\,(a+b\,x)}\,\mathrm{coth}(d+e\,x)^2\,dx$$

input
```
int(F^(c*(a + b*x))*coth(d + e*x)^2,x)
```

output
```
int(F^(c*(a + b*x))*coth(d + e*x)^2, x)
```

Reduce [F]

$$\int F^{c(a+bx)} \coth^2(d+ex)\,dx = f^{ac}\left(\int f^{bcx} \coth(ex+d)^2\,dx\right)$$

input
```
int(F^(c*(b*x+a))*coth(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*coth(d + e*x)**2,x)
```

3.72 $\int F^{c(a+bx)} \coth^3(d+ex)\, dx$

Optimal result

Integrand size = 18, antiderivative size = 157

$$\int F^{c(a+bx)} \coth^3(d+ex)\, dx = -\frac{2F^{c(a+bx)}}{e\left(1-e^{2d+2ex}\right)^2} + \frac{F^{c(a+bx)}}{bc\log(F)} + \frac{F^{c(a+bx)}(2e+bc\log(F))}{e^2\left(1-e^{2d+2ex}\right)} - F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},e^{2d+2ex}\right)\left(\frac{2}{bc\log(F)}+\frac{bc\log(F)}{e^2}\right)$$

output
```
-2*F^(c*(b*x+a))/e/(1-exp(2*e*x+2*d))^2+F^(c*(b*x+a))/b/c/ln(F)+F^(c*(b*x+
a))*(2*e+b*c*ln(F))/e^2/(1-exp(2*e*x+2*d))-F^(c*(b*x+a))*hypergeom([1, 1/2
*b*c*ln(F)/e],[1+1/2*b*c*ln(F)/e],exp(2*e*x+2*d))*(2/b/c/ln(F)+b*c*ln(F)/e
^2)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94

$$\int F^{c(a+bx)}\coth^3(d+ex)\,dx = \frac{1}{2}F^{c(a+bx)}\left(-\frac{\operatorname{csch}^2(d+ex)}{e}+\frac{2\coth(d)}{bc\log(F)} - \frac{2\left(1+\left(-1+e^{2d}\right)\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},e^{2(d+ex)}\right)\right)\left(2e^2+b^2c^2\log^2(F)\right)}{bce^2\left(-1+e^{2d}\right)\log(F)} + \frac{bc\operatorname{csch}(d)\operatorname{csch}(d+ex)\log(F)\sinh(ex)}{e^2}\right)$$

input
```
Integrate[F^(c*(a + b*x))*Coth[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(-(Csch[d + e*x]^2/e) + (2*Coth[d])/(b*c*Log[F]) - (2*(1
+ (-1 + E^(2*d))*Hypergeometric2F1[1, (b*c*Log[F])/(2*e), 1 + (b*c*Log[F])
/(2*e), E^(2*(d + e*x))])*(2*e^2 + b^2*c^2*Log[F]^2))/(b*c*e^2*(-1 + E^(2*
d))*Log[F]) + (b*c*Csch[d]*Csch[d + e*x]*Log[F]*Sinh[e*x])/e^2))/2
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.20, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6008}$$

$$\int \left(\frac{6F^{c(a+bx)}}{e^{2(d+ex)}-1}+\frac{12F^{c(a+bx)}}{\left(e^{2(d+ex)}-1\right)^2}+\frac{8F^{c(a+bx)}}{\left(e^{2(d+ex)}-1\right)^3}+F^{c(a+bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6F^{c(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,e^{2(d+ex)}\right)}{bc\log(F)}+\frac{12F^{c(a+bx)}\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,e^{2(d+ex)}\right)}{bc\log(F)}-\frac{8F^{c(a+bx)}\text{Hypergeometric2F1}\left(3,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,e^{2(d+ex)}\right)}{bc\log(F)}+\frac{F^{c(a+bx)}}{bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Coth[d + e*x]^3,x]
```

output
```
F^(c*(a + b*x))/(b*c*Log[F]) - (6*F^(c*(a + b*x))*Hypergeometric2F1[1, (b*
c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(b*c*Log[F]) +
(12*F^(c*(a + b*x))*Hypergeometric2F1[2, (b*c*Log[F])/(2*e), 1 + (b*c*Log[
F])/(2*e), E^(2*(d + e*x))])/(b*c*Log[F]) - (8*F^(c*(a + b*x))*Hypergeomet
ric2F1[3, (b*c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(b
*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6008
```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*(d + e*x)))^n/(-1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)}\coth(ex+d)^3\,dx$$

input
```
int(F^(c*(b*x+a))*coth(e*x+d)^3,x)
```

output
```
int(F^(c*(b*x+a))*coth(e*x+d)^3,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\coth^3(d+ex)\,dx = \int F^{(bx+a)c}\coth(ex+d)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*coth(e*x + d)^3, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\coth^3(d+ex)\,dx = \int F^{c(a+bx)}\coth^3(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*coth(e*x+d)**3,x)
```

output
```
Integral(F**(c*(a + b*x))*coth(d + e*x)**3, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\coth^3(d+ex)\,dx = \int F^{(bx+a)c}\coth(ex+d)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)^3,x, algorithm="maxima")
```

output
```
48*(F^(a*c)*b^2*c^2*e*log(F)^2 + 2*F^(a*c)*e^3)*integrate(F^(b*c*x)/(b^3*c
^3*log(F)^3 - 12*b^2*c^2*e*log(F)^2 + 44*b*c*e^2*log(F) - 48*e^3 + (b^3*c^
3*e^(8*d)*log(F)^3 - 12*b^2*c^2*e*e^(8*d)*log(F)^2 + 44*b*c*e^2*e^(8*d)*lo
g(F) - 48*e^3*e^(8*d))*e^(8*e*x) - 4*(b^3*c^3*e^(6*d)*log(F)^3 - 12*b^2*c^
2*e*e^(6*d)*log(F)^2 + 44*b*c*e^2*e^(6*d)*log(F) - 48*e^3*e^(6*d))*e^(6*e*
x) + 6*(b^3*c^3*e^(4*d)*log(F)^3 - 12*b^2*c^2*e*e^(4*d)*log(F)^2 + 44*b*c*
e^2*e^(4*d)*log(F) - 48*e^3*e^(4*d))*e^(4*e*x) - 4*(b^3*c^3*e^(2*d)*log(F)
^3 - 12*b^2*c^2*e*e^(2*d)*log(F)^2 + 44*b*c*e^2*e^(2*d)*log(F) - 48*e^3*e^
(2*d))*e^(2*e*x)), x) - (F^(a*c)*b^3*c^3*log(F)^3 + 36*F^(a*c)*b^2*c^2*e*l
og(F)^2 + 44*F^(a*c)*b*c*e^2*log(F) + 48*F^(a*c)*e^3 + (F^(a*c)*b^3*c^3*e^
(6*d)*log(F)^3 - 12*F^(a*c)*b^2*c^2*e*e^(6*d)*log(F)^2 + 44*F^(a*c)*b*c*e^
2*e^(6*d)*log(F) - 48*F^(a*c)*e^3*e^(6*d))*e^(6*e*x) + 3*(F^(a*c)*b^3*c^3*
e^(4*d)*log(F)^3 - 8*F^(a*c)*b^2*c^2*e*e^(4*d)*log(F)^2 + 4*F^(a*c)*b*c*e^
2*e^(4*d)*log(F) + 48*F^(a*c)*e^3*e^(4*d))*e^(4*e*x) + 3*(F^(a*c)*b^3*c^3*
e^(2*d)*log(F)^3 - 28*F^(a*c)*b*c*e^2*e^(2*d)*log(F) - 48*F^(a*c)*e^3*e^(2
*d))*e^(2*e*x))*F^(b*c*x)/(b^4*c^4*log(F)^4 - 12*b^3*c^3*e*log(F)^3 + 44*b
^2*c^2*e^2*log(F)^2 - 48*b*c*e^3*log(F) - (b^4*c^4*e^(6*d)*log(F)^4 - 12*b
^3*c^3*e*e^(6*d)*log(F)^3 + 44*b^2*c^2*e^2*e^(6*d)*log(F)^2 - 48*b*c*e^3*e
^(6*d)*log(F))*e^(6*e*x) + 3*(b^4*c^4*e^(4*d)*log(F)^4 - 12*b^3*c^3*e*e^(4
*d)*log(F)^3 + 44*b^2*c^2*e^2*e^(4*d)*log(F)^2 - 48*b*c*e^3*e^(4*d)*log...
```

Giac [F]

$$\int F^{c(a+bx)}\coth^3(d+ex)\,dx=\int F^{(bx+a)c}\coth\left(ex+d\right)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*coth(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\coth^3(d+ex)\,dx=\int F^{c\,(a+b\,x)}\,\coth(d+e\,x)^3\,dx$$

input
```
int(F^(c*(a + b*x))*coth(d + e*x)^3,x)
```

output
```
int(F^(c*(a + b*x))*coth(d + e*x)^3, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\coth^3(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\coth(ex+d)^3\,dx\right)$$

input
```
int(F^(c*(b*x+a))*coth(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*coth(d + e*x)**3,x)
```

3.73 $\int F^{c(a+bx)} \coth^4(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 219

$$\begin{aligned}\int F^{c(a+bx)}\coth^4(d+ex)\,dx &= \frac{8F^{c(a+bx)}}{3e\left(1-e^{2d+2ex}\right)^3}+\frac{F^{c(a+bx)}}{bc\log(F)}-\frac{2F^{c(a+bx)}(6e+bc\log(F))}{3e^2\left(1-e^{2d+2ex}\right)^2}\\ &\quad-\frac{F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},e^{2d+2ex}\right)\left(8e^2+b^2c^2\log^2(F)\right)}{3e^3}\\ &\quad+\frac{F^{c(a+bx)}\left(12e^2+2bce\log(F)+b^2c^2\log^2(F)\right)}{3e^3\left(1-e^{2d+2ex}\right)}\end{aligned}$$

output
```
8/3*F^(c*(b*x+a))/e/(1-exp(2*e*x+2*d))^3+F^(c*(b*x+a))/b/c/ln(F)-2/3*F^(c*
(b*x+a))*(6*e+b*c*ln(F))/e^2/(1-exp(2*e*x+2*d))^2-1/3*F^(c*(b*x+a))*hyperg
eom([1, 1/2*b*c*ln(F)/e],[1+1/2*b*c*ln(F)/e],exp(2*e*x+2*d))*(8*e^2+b^2*c^
2*ln(F)^2)/e^3+1/3*F^(c*(b*x+a))*(12*e^2+2*b*c*e*ln(F)+b^2*c^2*ln(F)^2)/e^
3/(1-exp(2*e*x+2*d))
```

Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.81

$$\int F^{c(a+bx)}\coth^4(d+ex)\,dx$$
$$=\frac{1}{6}F^{c(a+bx)}\left(\frac{6}{bc\log(F)}-\frac{\operatorname{csch}^2(d+ex)(2e\coth(d)+bc\log(F))}{e^2}-\frac{2\left(1+(-1+e^{2d})\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},e^{2(d+ex)}\right)\right)\left(8e^2+b^2c^2\log^2(F)\right)}{e^3\left(-1+e^{2d}\right)}+\frac{2\operatorname{csch}(d)\operatorname{csch}^3(d+ex)\sinh(ex)}{e}+\frac{\operatorname{csch}(d)\operatorname{csch}(d+ex)\left(8e^2+b^2c^2\log^2(F)\right)\sinh(ex)}{e^3}\right)$$

input
```
Integrate[F^(c*(a + b*x))*Coth[d + e*x]^4,x]
```

output
```
(F^(c*(a + b*x))*(6/(b*c*Log[F]) - (Csch[d + e*x]^2*(2*e*Coth[d] + b*c*Log
[F]))/e^2 - (2*(1 + (-1 + E^(2*d))*Hypergeometric2F1[1, (b*c*Log[F])/(2*e)
, 1 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])*(8*e^2 + b^2*c^2*Log[F]^2))/(e
^3*(-1 + E^(2*d))) + (2*Csch[d]*Csch[d + e*x]^3*Sinh[e*x])/e + (Csch[d]*Cs
ch[d + e*x]*(8*e^2 + b^2*c^2*Log[F]^2)*Sinh[e*x])/e^3))/6
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^4(d+ex)F^{c(a+bx)}\,dx$$
$$\Big\downarrow \; 6008$$

$$\int \left(\frac{8F^{c(a+bx)}}{e^{2(d+ex)}-1}+\frac{24F^{c(a+bx)}}{\left(e^{2(d+ex)}-1\right)^2}+\frac{32F^{c(a+bx)}}{\left(e^{2(d+ex)}-1\right)^3}+\frac{16F^{c(a+bx)}}{\left(e^{2(d+ex)}-1\right)^4}+F^{c(a+bx)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{8F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,e^{2(d+ex)}\right)}{bc\log(F)}+\frac{24F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,e^{2(d+ex)}\right)}{bc\log(F)}-\frac{32F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(3,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,e^{2(d+ex)}\right)}{bc\log(F)}+\frac{16F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(4,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,e^{2(d+ex)}\right)}{bc\log(F)}+\frac{F^{c(a+bx)}}{bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Coth[d + e*x]^4,x]
```

output
```
F^(c*(a + b*x))/(b*c*Log[F]) - (8*F^(c*(a + b*x))*Hypergeometric2F1[1, (b*
c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(b*c*Log[F]) +
(24*F^(c*(a + b*x))*Hypergeometric2F1[2, (b*c*Log[F])/(2*e), 1 + (b*c*Log[
F])/(2*e), E^(2*(d + e*x))])/(b*c*Log[F]) - (32*F^(c*(a + b*x))*Hypergeome
tric2F1[3, (b*c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(
b*c*Log[F]) + (16*F^(c*(a + b*x))*Hypergeometric2F1[4, (b*c*Log[F])/(2*e),
 1 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6008
```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*(d + e*x)))^n/(-1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \coth(ex+d)^4 \, dx$$

input
```
int(F^(c*(b*x+a))*coth(e*x+d)^4,x)
```

output
```
int(F^(c*(b*x+a))*coth(e*x+d)^4,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \coth^4(d+ex) \, dx = \int F^{(bx+a)c} \coth(ex+d)^4 \, dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)^4,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*coth(e*x + d)^4, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \coth^4(d+ex) \, dx = \int F^{c(a+bx)} \coth^4(d+ex) \, dx$$

input
```
integrate(F**(c*(b*x+a))*coth(e*x+d)**4,x)
```

output
```
Integral(F**(c*(a + b*x))*coth(d + e*x)**4, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\coth^4(d+ex)\,dx=\int F^{(bx+a)c}\coth\left(ex+d\right)^4\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)^4,x, algorithm="maxima")
```

output
```
128*(F^(a*c)*b^3*c^3*e*log(F)^3 + 8*F^(a*c)*b*c*e^3*log(F))*integrate(-F^(
b*c*x)/(b^4*c^4*log(F)^4 - 20*b^3*c^3*e*log(F)^3 + 140*b^2*c^2*e^2*log(F)^
2 - 400*b*c*e^3*log(F) + 384*e^4 - (b^4*c^4*e^(10*d)*log(F)^4 - 20*b^3*c^3
*e*e^(10*d)*log(F)^3 + 140*b^2*c^2*e^2*e^(10*d)*log(F)^2 - 400*b*c*e^3*e^(
10*d)*log(F) + 384*e^4*e^(10*d))*e^(10*e*x) + 5*(b^4*c^4*e^(8*d)*log(F)^4
- 20*b^3*c^3*e*e^(8*d)*log(F)^3 + 140*b^2*c^2*e^2*e^(8*d)*log(F)^2 - 400*b
*c*e^3*e^(8*d)*log(F) + 384*e^4*e^(8*d))*e^(8*e*x) - 10*(b^4*c^4*e^(6*d)*l
og(F)^4 - 20*b^3*c^3*e*e^(6*d)*log(F)^3 + 140*b^2*c^2*e^2*e^(6*d)*log(F)^2
 - 400*b*c*e^3*e^(6*d)*log(F) + 384*e^4*e^(6*d))*e^(6*e*x) + 10*(b^4*c^4*e
^(4*d)*log(F)^4 - 20*b^3*c^3*e*e^(4*d)*log(F)^3 + 140*b^2*c^2*e^2*e^(4*d)*
log(F)^2 - 400*b*c*e^3*e^(4*d)*log(F) + 384*e^4*e^(4*d))*e^(4*e*x) - 5*(b^
4*c^4*e^(2*d)*log(F)^4 - 20*b^3*c^3*e*e^(2*d)*log(F)^3 + 140*b^2*c^2*e^2*e
^(2*d)*log(F)^2 - 400*b*c*e^3*e^(2*d)*log(F) + 384*e^4*e^(2*d))*e^(2*e*x))
, x) + (F^(a*c)*b^4*c^4*log(F)^4 + 108*F^(a*c)*b^3*c^3*e*log(F)^3 + 140*F^
(a*c)*b^2*c^2*e^2*log(F)^2 + 624*F^(a*c)*b*c*e^3*log(F) + 384*F^(a*c)*e^4
+ (F^(a*c)*b^4*c^4*e^(8*d)*log(F)^4 - 20*F^(a*c)*b^3*c^3*e*e^(8*d)*log(F)^
3 + 140*F^(a*c)*b^2*c^2*e^2*e^(8*d)*log(F)^2 - 400*F^(a*c)*b*c*e^3*e^(8*d)
*log(F) + 384*F^(a*c)*e^4*e^(8*d))*e^(8*e*x) + 4*(F^(a*c)*b^4*c^4*e^(6*d)*
log(F)^4 - 16*F^(a*c)*b^3*c^3*e*e^(6*d)*log(F)^3 + 68*F^(a*c)*b^2*c^2*e^2*
e^(6*d)*log(F)^2 + 16*F^(a*c)*b*c*e^3*e^(6*d)*log(F) - 384*F^(a*c)*e^4*...
```

Giac [F]

$$\int F^{c(a+bx)}\coth^4(d+ex)\,dx=\int F^{(bx+a)c}\coth\left(ex+d\right)^4\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)^4,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*coth(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\coth^4(d+ex)\,dx = \int F^{c\,(a+b\,x)}\coth(d+e\,x)^4\,dx$$

input
```
int(F^(c*(a + b*x))*coth(d + e*x)^4,x)
```

output
```
int(F^(c*(a + b*x))*coth(d + e*x)^4, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\coth^4(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\coth\left(ex+d\right)^4dx\right)$$

input
```
int(F^(c*(b*x+a))*coth(e*x+d)^4,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*coth(d + e*x)**4,x)
```

3.74 $\int e^{a+bx} \coth^n(a+bx) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 84

$$\int e^{a+bx} \coth^n(a+bx) \, dx = \frac{e^{a+bx}\left(1-e^{2a+2bx}\right)^n \left(1+e^{2a+2bx}\right)^{-n} \text{AppellF1}\left(\frac{1}{2},n,-n,\frac{3}{2},e^{2a+2bx},-e^{2a+2bx}\right)\coth^n(a+bx)}{b}$$

output
```
exp(b*x+a)*(1-exp(2*b*x+2*a))^n*AppellF1(1/2,-n,n,3/2,-exp(2*b*x+2*a),exp(
2*b*x+2*a))*coth(b*x+a)^n/b/((1+exp(2*b*x+2*a))^n)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.26

$$\int e^{a+bx} \coth^n(a+bx) \, dx = \frac{e^{a+bx}\left(-1+e^{-2(a+bx)}\right)^{-n}\left(1+e^{-2(a+bx)}\right)^{-n}\left(-e^{-4(a+bx)}\left(-1+e^{2(a+bx)}\right)^2\right)^n \text{AppellF1}\left(-\frac{1}{2},-n,n,\frac{1}{2},-e^{-2 \cdots}\right.}{b}$$

input
```
Integrate[E^(a + b*x)*Coth[a + b*x]^n,x]
```

output
```
(E^(a + b*x)*(-((-1 + E^(2*(a + b*x)))^2/E^(4*(a + b*x))))^n*AppellF1[-1/2
, -n, n, 1/2, -E^(-2*(a + b*x)), E^(-2*(a + b*x))]*Coth[a + b*x]^n)/(b*(-1
 + E^(-2*(a + b*x)))^n*(1 + E^(-2*(a + b*x)))^n)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 2050, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\coth^n(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \left(-\frac{1+e^{2a+2bx}}{1-e^{2a+2bx}}\right)^n de^{a+bx}}{b}$$

$$\Big\downarrow \text{2050}$$

$$\frac{\int \left(-1-e^{2a+2bx}\right)^n \left(1-e^{2a+2bx}\right)^{-n} de^{a+bx}}{b}$$

$$\Big\downarrow \text{334}$$

$$\frac{\left(-e^{2a+2bx}-1\right)^n \left(e^{2a+2bx}+1\right)^{-n}\int \left(1-e^{2a+2bx}\right)^{-n} \left(1+e^{2a+2bx}\right)^n de^{a+bx}}{b}$$

$$\Big\downarrow \text{333}$$

$$\frac{e^{a+bx}\left(-e^{2a+2bx}-1\right)^n \left(e^{2a+2bx}+1\right)^{-n}\text{AppellF1}\left(\frac{1}{2},n,-n,\frac{3}{2},e^{2a+2bx},-e^{2a+2bx}\right)}{b}$$

input
```
Int[E^(a + b*x)*Coth[a + b*x]^n,x]
```

output
```
(E^(a + b*x)*(-1 - E^(2*a + 2*b*x))^n*AppellF1[1/2, n, -n, 3/2, E^(2*a + 2
*b*x), -E^(2*a + 2*b*x)])/(b*(1 + E^(2*a + 2*b*x))^n)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [F]

$$\int e^{bx+a}\coth(bx+a)^n\,dx$$

input
```
int(exp(b*x+a)*coth(b*x+a)^n,x)
```

output
```
int(exp(b*x+a)*coth(b*x+a)^n,x)
```

Fricas [F]

$$\int e^{a+bx}\coth^n(a+bx)\,dx=\int \coth{(bx+a)}^n e^{(bx+a)}\,dx$$

input
```
integrate(exp(b*x+a)*coth(b*x+a)^n,x, algorithm="fricas")
```

output
```
integral(coth(b*x + a)^n*e^(b*x + a), x)
```

Sympy [F]

$$\int e^{a+bx}\coth^n(a+bx)\,dx=e^a\int e^{bx}\coth^n{(a+bx)}\,dx$$

input
```
integrate(exp(b*x+a)*coth(b*x+a)**n,x)
```

output
```
exp(a)*Integral(exp(b*x)*coth(a + b*x)**n, x)
```

Maxima [F]

$$\int e^{a+bx}\coth^n(a+bx)\,dx=\int \coth{(bx+a)}^n e^{(bx+a)}\,dx$$

input
```
integrate(exp(b*x+a)*coth(b*x+a)^n,x, algorithm="maxima")
```

output
```
integrate(coth(b*x + a)^n*e^(b*x + a), x)
```

Giac [F]

$$\int e^{a+bx}\coth^n(a+bx)\,dx=\int \coth\left(bx+a\right)^n e^{(bx+a)}\,dx$$

input
```
integrate(exp(b*x+a)*coth(b*x+a)^n,x, algorithm="giac")
```

output
```
integrate(coth(b*x + a)^n*e^(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\coth^n(a+bx)\,dx=\int \coth(a+b\,x)^n\,\mathrm{e}^{a+b\,x}\,dx$$

input
```
int(coth(a + b*x)^n*exp(a + b*x),x)
```

output
```
int(coth(a + b*x)^n*exp(a + b*x), x)
```

Reduce [F]

$$\int e^{a+bx}\coth^n(a+bx)\,dx=e^a\left(\int e^{bx}\coth\left(bx+a\right)^n dx\right)$$

input
```
int(exp(b*x+a)*coth(b*x+a)^n,x)
```

output
```
e**a*int(e**(b*x)*coth(a + b*x)**n,x)
```

3.75 $\int F^{c(a+bx)}(f\coth(d+ex))^n\,dx$

Optimal result

Integrand size = 20, antiderivative size = 147

$$\int F^{c(a+bx)}(f\coth(d+ex))^n\,dx = -\frac{2^{-1+n}\left(e^{2d+2ex}\right)^{-\frac{bc\log(F)}{2e}}\left(1-e^{2d+2ex}\right)\left(1+e^{2d+2ex}\right)^{-n}F^{c(a+bx)}\,\text{AppellF1}\left(1-n,1-\frac{bc\log(F)}{2e},-n,2-n\right.}{e(1-n)}$$

output
```
-2^(-1+n)*(1-exp(2*e*x+2*d))*F^(c*(b*x+a))*AppellF1(1-n,1-1/2*b*c*ln(F)/e,
-n,2-n,1-exp(2*e*x+2*d),1/2-1/2*exp(2*e*x+2*d))*(f*coth(e*x+d))^n/e/(exp(2
*e*x+2*d)^(1/2*b*c*ln(F)/e))/((1+exp(2*e*x+2*d))^n)/(1-n)
```

Mathematica [F]

$$\int F^{c(a+bx)}(f\coth(d+ex))^n\,dx = \int F^{c(a+bx)}(f\coth(d+ex))^n\,dx$$

input
```
Integrate[F^(c*(a + b*x))*(f*Coth[d + e*x])^n,x]
```

output
```
Integrate[F^(c*(a + b*x))*(f*Coth[d + e*x])^n, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\coth(d+ex))^n\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$\coth^{-n}(d+ex)(f\coth(d+ex))^n\int F^{c(a+bx)}\coth^n(d+ex)dx$$

$$\Big\downarrow \text{ 6030}$$

$$\coth^{-n}(d+ex)(f\coth(d+ex))^n\int F^{ac+bxc}\coth^n(d+ex)dx$$

$$\Big\downarrow \text{ 7299}$$

$$\coth^{-n}(d+ex)(f\coth(d+ex))^n\int F^{ac+bxc}\coth^n(d+ex)dx$$

input
```
Int[F^(c*(a + b*x))*(f*Coth[d + e*x])^n,x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}(f\coth(ex+d))^n\,dx$$

input
```
int(F^(c*(b*x+a))*(f*coth(e*x+d))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*coth(e*x+d))^n,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\coth(d+ex))^n\,dx = \int (f\coth(ex+d))^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*coth(e*x+d))^n,x, algorithm="fricas")
```

output
```
integral((f*coth(e*x + d))^n*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}(f\coth(d+ex))^n\,dx = \int F^{c(a+bx)}(f\coth(d+ex))^n\;dx$$

input
```
integrate(F**(c*(b*x+a))*(f*coth(e*x+d))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*coth(d + e*x))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}(f\coth(d+ex))^n\,dx = \int (f\coth(ex+d))^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*coth(e*x+d))^n,x, algorithm="maxima")
```

output
```
integrate((f*coth(e*x + d))^n*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\coth(d+ex))^n\,dx=\int(f\coth(ex+d))^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*coth(e*x+d))^n,x, algorithm="giac")
```

output
```
integrate((f*coth(e*x + d))^n*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\coth(d+ex))^n\,dx=\int F^{c\,(a+b\,x)}\,(f\coth(d+e\,x))^n\,dx$$

input
```
int(F^(c*(a + b*x))*(f*coth(d + e*x))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f*coth(d + e*x))^n, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\coth(d+ex))^n\,dx=f^{ac+n}\left(\int f^{bcx}\coth(ex+d)^n\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*coth(e*x+d))^n,x)
```

output
```
f**(a*c + n)*int(f**(b*c*x)*coth(d + e*x)**n,x)
```

3.76 $\int e^{a+bx}\mathbf{sech}(d+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 24

$$\int e^{a+bx}\text{sech}(d+bx)\,dx=\frac{e^{a-d}\log\left(1+e^{2d+2bx}\right)}{b}$$

output
```
exp(a-d)*ln(1+exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int e^{a+bx}\text{sech}(d+bx)\,dx=\frac{e^{a-d}\log\left(1+e^{2(d+bx)}\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Sech[d + b*x],x]
```

output
```
(E^(a - d)*Log[1 + E^(2*(d + b*x))])/b
```

Rubi [A] (warning: unable to verify)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2720, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{sech}(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{2e^{a+bx}}{1+e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2e^a\int \frac{e^{bx}}{1+e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{240}$$

$$\frac{e^a\log\left(e^{2bx}+1\right)}{b}$$

input
```
Int[E^(a + b*x)*Sech[d + b*x],x]
```

output
```
(E^a*Log[1 + E^(2*b*x)])/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

method	result	size
risch	$-\frac{2\,e^{a-d}a}{b}+\frac{\ln\left(e^{2bx+2a}+e^{2a-2d}\right)e^{a-d}}{b}$	43

input
```
int(exp(b*x+a)*sech(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
-2/b*exp(a-d)*a+ln(exp(2*b*x+2*a)+exp(2*a-2*d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(22) = 44$.

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.88

$$\int e^{a+bx}\mathrm{sech}(d+bx)\,dx=\frac{(\cosh(-a+d)-\sinh(-a+d))\log\left(\frac{2\cosh(bx+d)}{\cosh(bx+d)-\sinh(bx+d)}\right)}{b}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d),x, algorithm="fricas")
```

output
```
(cosh(-a + d) - sinh(-a + d))*log(2*cosh(b*x + d)/(cosh(b*x + d) - sinh(b*
x + d)))/b
```

Sympy [F]

$$\int e^{a+bx}\text{sech}(d+bx)\,dx = e^{a}\int e^{bx}\,\text{sech}\,(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+d),x)
```

output
```
exp(a)*Integral(exp(b*x)*sech(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int e^{a+bx}\text{sech}(d+bx)\,dx = \frac{e^{(a-d)}\log\left(e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right)}{b}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d),x, algorithm="maxima")
```

output
```
e^(a - d)*log(e^(2*b*x + 2*a + 2*d) + e^(2*a))/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int e^{a+bx}\text{sech}(d+bx)\,dx = \frac{e^{(a-d)}\log\left(e^{(2\,bx+2\,d)}+1\right)}{b}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d),x, algorithm="giac")
```

output
```
e^(a - d)*log(e^(2*b*x + 2*d) + 1)/b
```

Mupad [B] (verification not implemented)

Time = 2.99 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int e^{a+bx}\mathrm{sech}(d+bx)\,dx=\frac{\mathrm{e}^{a-d}\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}\right)}{b}$$

input
```
int(exp(a + b*x)/cosh(d + b*x),x)
```

output
```
(exp(a - d)*log(exp(2*a)*exp(2*b*x) + exp(2*a)*exp(-2*d)))/b
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int e^{a+bx}\mathrm{sech}(d+bx)\,dx=\frac{e^{a}\log\left(e^{2bx+2d}+1\right)}{e^{d}b}$$

input
```
int(exp(b*x+a)*sech(b*x+d),x)
```

output
```
(e**a*log(e**(2*b*x + 2*d) + 1))/(e**d*b)
```

3.77 $\int e^{a+bx}\text{sech}^2(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 47

$$\int e^{a+bx}\text{sech}^2(d+bx)\,dx=-\frac{2e^{a+bx}}{b\left(1+e^{2d+2bx}\right)}+\frac{2e^{a-d}\arctan\left(e^{d+bx}\right)}{b}$$

output
```
-2*exp(b*x+a)/b/(1+exp(2*b*x+2*d))+2*exp(a-d)*arctan(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int e^{a+bx}\text{sech}^2(d+bx)\,dx=\frac{2e^a\left(-\frac{e^{bx}}{1+e^{2(d+bx)}}+e^{-d}\arctan\left(e^{d+bx}\right)\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Sech[d + b*x]^2,x]
```

output
```
(2*E^a*(-(E^(b*x)/(1 + E^(2*(d + b*x)))) + ArcTan[E^(d + b*x)]/E^d))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 27, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{sech}^2(bx+d)\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int \frac{4e^{a+2bx}}{(1+e^{2bx})^2}de^{bx}}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{4e^a\int \frac{e^{2bx}}{(1+e^{2bx})^2}de^{bx}}{b}$$

$$\Big\downarrow \; 252$$

$$\frac{4e^a\left(\frac{1}{2}\int \frac{1}{1+e^{2bx}}de^{bx}-\frac{e^{bx}}{2(e^{2bx}+1)}\right)}{b}$$

$$\Big\downarrow \; 216$$

$$\frac{4e^a\left(\frac{1}{2}\arctan\left(e^{bx}\right)-\frac{e^{bx}}{2(e^{2bx}+1)}\right)}{b}$$

input
```
Int[E^(a + b*x)*Sech[d + b*x]^2,x]
```

output
```
(4*E^a*(-1/2*E^(b*x)/(1 + E^(2*b*x)) + ArcTan[E^(b*x)]/2))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.96

method	result	size
risch	$-\frac{2\,{\mathrm e}^{b x +3 a}}{\left({\mathrm e}^{2 b x +2 a +2 d}+{\mathrm e}^{2 a}\right) b}+\frac{i \ln\left({\mathrm e}^{b x +a}+i {\mathrm e}^{a -d}\right) {\mathrm e}^{a -d}}{b}-\frac{i \ln\left({\mathrm e}^{b x +a}-i {\mathrm e}^{a -d}\right) {\mathrm e}^{a -d}}{b}$	92

input
```
int(exp(b*x+a)*sech(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/(exp(2*b*x+2*a+2*d)+exp(2*a))/b*exp(b*x+3*a)+I*ln(exp(b*x+a)+I*exp(a-d)
)/b*exp(a-d)-I*ln(exp(b*x+a)-I*exp(a-d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(43) = 86$.

Time = 0.07 (sec) , antiderivative size = 207, normalized size of antiderivative = 4.40

$$\int e^{a+bx}\text{sech}^2(d+bx)\,dx = \frac{2\left(\left(\cosh(bx+d)^2\cosh(-a+d)+(\cosh(-a+d)-\sinh(-a+d))\sinh(bx+d)^2+2\left(\cosh(bx+d)\cos\right.\right.\right.}{}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)^2,x, algorithm="fricas")
```

output
```
2*((cosh(b*x + d)^2*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x
+ d)^2 + 2*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(
b*x + d) - (cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*arctan(cosh(
b*x + d) + sinh(b*x + d)) - cosh(b*x + d)*cosh(-a + d) - (cosh(-a + d) - s
inh(-a + d))*sinh(b*x + d) + cosh(b*x + d)*sinh(-a + d))/(b*cosh(b*x + d)^
2 + 2*b*cosh(b*x + d)*sinh(b*x + d) + b*sinh(b*x + d)^2 + b)
```

Sympy [F]

$$\int e^{a+bx}\text{sech}^2(d+bx)\,dx = e^a\int e^{bx}\,\text{sech}^2\,(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)**2,x)
```

output
```
exp(a)*Integral(exp(b*x)*sech(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int e^{a+bx}\operatorname{sech}^2(d+bx)\,dx = \frac{2\,\arctan\left(e^{(bx+d)}\right)e^{(a-d)}}{b} - \frac{2\,e^{(bx+3\,a)}}{b\left(e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right)}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)^2,x, algorithm="maxima")
```

output
```
2*arctan(e^(b*x + d))*e^(a - d)/b - 2*e^(b*x + 3*a)/(b*(e^(2*b*x + 2*a + 2
*d) + e^(2*a)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int e^{a+bx}\operatorname{sech}^2(d+bx)\,dx = 2\left(\frac{\arctan\left(e^{(bx+d)}\right)e^{(-3\,d)}}{b} - \frac{e^{(bx-2\,d)}}{b\left(e^{(2\,bx+2\,d)}+1\right)}\right)e^{(a+2\,d)}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)^2,x, algorithm="giac")
```

output
```
2*(arctan(e^(b*x + d))*e^(-3*d)/b - e^(b*x - 2*d)/(b*(e^(2*b*x + 2*d) + 1)
))*e^(a + 2*d)
```

Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.79

$$\int e^{a+bx}\operatorname{sech}^2(d+bx)\,dx = \frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b\,\sqrt{e^{2\,a}\,e^{-2\,d}}}\right)\,\sqrt{e^{2\,a}\,e^{-2\,d}}}{\sqrt{b^2}} - \frac{2\,e^{3\,a}\,e^{-2\,d}\,e^{b\,x}}{b\,e^{2\,a}\,e^{-2\,d}+b\,e^{2\,a}\,e^{2\,b\,x}}$$

input
```
int(exp(a + b*x)/cosh(d + b*x)^2,x)
```

output
```
(2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/(b*(exp(2*a)*exp(-2*d))^(1/2)))*(exp
(2*a)*exp(-2*d))^(1/2))/(b^2)^(1/2) - (2*exp(3*a)*exp(-2*d)*exp(b*x))/(b*e
xp(2*a)*exp(-2*d) + b*exp(2*a)*exp(2*b*x))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.36

$$\int e^{a+bx}\mathrm{sech}^2(d+bx)\,dx = \frac{2e^a\left(e^{2bx+2d}atan\left(e^{bx+d}\right)+atan\left(e^{bx+d}\right)-e^{bx+d}\right)}{e^d b\left(e^{2bx+2d}+1\right)}$$

input
```
int(exp(b*x+a)*sech(b*x+d)^2,x)
```

output
```
(2*e**a*(e**(2*b*x + 2*d)*atan(e**(b*x + d)) + atan(e**(b*x + d)) - e**(b*
x + d)))/(e**d*b*(e**(2*b*x + 2*d) + 1))
```

3.78 $\int e^{a+bx}\mathbf{sech}^3(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 30

$$\int e^{a+bx}\text{sech}^3(d+bx)\,dx=\frac{2e^{a+3d+4bx}}{b\left(1+e^{2d+2bx}\right)^2}$$

output
```
2*exp(4*b*x+a+3*d)/b/(1+exp(2*b*x+2*d))^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int e^{a+bx}\text{sech}^3(d+bx)\,dx=\frac{2e^{a+3d+4bx}}{b\left(1+e^{2(d+bx)}\right)^2}$$

input
```
Integrate[E^(a + b*x)*Sech[d + b*x]^3,x]
```

output
```
(2*E^(a + 3*d + 4*b*x))/(b*(1 + E^(2*(d + b*x)))^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2720, 27, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{sech}^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{8e^{a+3bx}}{\left(1+e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{8e^a\int \frac{e^{3bx}}{\left(1+e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{242}$$

$$\frac{2e^{a+4bx}}{b\left(e^{2bx}+1\right)^2}$$

input
```
Int[E^(a + b*x)*Sech[d + b*x]^3,x]
```

output
```
(2*E^(a + 4*b*x))/(b*(1 + E^(2*b*x))^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.70

method	result	size
parallelrisch	$-\frac{\mathrm{e}^{b x +a} \left(-1+\tanh \left(\frac{b x}{2}+\frac{d}{2}\right)\right) \left(1+\tanh \left(\frac{b x}{2}+\frac{d}{2}\right)\right)^{3}}{2 b \left(1+\tanh \left(\frac{b x}{2}+\frac{d}{2}\right)^{2}\right)^{2}}$	51
risch	$-\frac{2 \left(2 \,\mathrm{e}^{2 b x +2 a +2 d}+\mathrm{e}^{2 a}\right) \mathrm{e}^{3 a -d}}{\left(\mathrm{e}^{2 b x +2 a +2 d}+\mathrm{e}^{2 a}\right)^{2} b}$	52

input
```
int(exp(b*x+a)*sech(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(b*x+a)*(-1+tanh(1/2*b*x+1/2*d))*(1+tanh(1/2*b*x+1/2*d))^3/b/(1+ta
nh(1/2*b*x+1/2*d)^2)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(28) = 56.

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 4.07

$$\int e^{a+bx}\operatorname{sech}^3(d+bx)\,dx =$$
$$-\frac{2\left(3\cosh\left(bx+d\right)\cosh\left(-a+d\right)+\left(\cosh\left(-a+d\right)-\sinh\left(-a+d\right)\right)\sinh\left(bx+d\right)-3\cosh\left(bx-\right.}{b\cosh\left(bx+d\right)^{3}+3\,b\cosh\left(bx+d\right)\sinh\left(bx+d\right)^{2}+b\sinh\left(bx+d\right)^{3}+3\,b\cosh\left(bx+d\right)+\left(3\,b\cosh\left(bx\right.\right.}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)^3,x, algorithm="fricas")
```

output
```
-2*(3*cosh(b*x + d)*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x
+ d) - 3*cosh(b*x + d)*sinh(-a + d))/(b*cosh(b*x + d)^3 + 3*b*cosh(b*x + d
)*sinh(b*x + d)^2 + b*sinh(b*x + d)^3 + 3*b*cosh(b*x + d) + (3*b*cosh(b*x
+ d)^2 + b)*sinh(b*x + d))
```

Sympy [F]

$$\int e^{a+bx}\text{sech}^3(d+bx)\,dx = e^a\int e^{bx}\,\text{sech}^3\,(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)**3,x)
```

output
```
exp(a)*Integral(exp(b*x)*sech(b*x + d)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(28) = 56$.

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.23

$$\int e^{a+bx}\text{sech}^3(d+bx)\,dx = -\frac{4\,e^{(2\,bx+5\,a+2\,d)}}{b(e^{(4\,bx+4\,a+5\,d)}+2\,e^{(2\,bx+4\,a+3\,d)}+e^{(4\,a+d)})} - \frac{2\,e^{(5\,a)}}{b(e^{(4\,bx+4\,a+5\,d)}+2\,e^{(2\,bx+4\,a+3\,d)}+e^{(4\,a+d)})}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)^3,x, algorithm="maxima")
```

output
```
-4*e^(2*b*x + 5*a + 2*d)/(b*(e^(4*b*x + 4*a + 5*d) + 2*e^(2*b*x + 4*a + 3*
d) + e^(4*a + d))) - 2*e^(5*a)/(b*(e^(4*b*x + 4*a + 5*d) + 2*e^(2*b*x + 4*
a + 3*d) + e^(4*a + d)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int e^{a+bx}\text{sech}^3(d+bx)\,dx = -\frac{2\left(2\,e^{(2\,bx+2\,d)}+1\right)e^{(a-d)}}{b\left(e^{(2\,bx+2\,d)}+1\right)^2}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)^3,x, algorithm="giac")
```

output
```
-2*(2*e^(2*b*x + 2*d) + 1)*e^(a - d)/(b*(e^(2*b*x + 2*d) + 1)^2)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\text{sech}^3(d+bx)\,dx = \int \frac{e^{a+b\,x}}{\cosh\left(d+b\,x\right)^3}\,dx$$

input
```
int(exp(a + b*x)/cosh(d + b*x)^3,x)
```

output
```
int(exp(a + b*x)/cosh(d + b*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int e^{a+bx}\text{sech}^3(d+bx)\,dx = \frac{2e^{4bx+a+3d}}{b\left(e^{4bx+4d}+2e^{2bx+2d}+1\right)}$$

input
```
int(exp(b*x+a)*sech(b*x+d)^3,x)
```

output
```
(2*e**(a + 4*b*x + 3*d))/(b*(e**(4*b*x + 4*d) + 2*e**(2*b*x + 2*d) + 1))
```

3.79 $\int e^{a+bx}\text{sech}^4(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 103

$$\int e^{a+bx}\text{sech}^4(d+bx)\,dx = -\frac{8e^{a+2d+3bx}}{3b\left(1+e^{2d+2bx}\right)^3}-\frac{2e^{a+bx}}{b\left(1+e^{2d+2bx}\right)^2}+\frac{e^{a+bx}}{b\left(1+e^{2d+2bx}\right)}+\frac{e^{a-d}\arctan\left(e^{d+bx}\right)}{b}$$

output
```
-8/3*exp(3*b*x+a+2*d)/b/(1+exp(2*b*x+2*d))^3-2*exp(b*x+a)/b/(1+exp(2*b*x+2
*d))^2+exp(b*x+a)/b/(1+exp(2*b*x+2*d))+exp(a-d)*arctan(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.69

$$\int e^{a+bx}\text{sech}^4(d+bx)\,dx = \frac{e^{a+bx}\left(-3-8e^{2(d+bx)}+3e^{4(d+bx)}\right)}{3b\left(1+e^{2(d+bx)}\right)^3}+\frac{e^{a-d}\arctan\left(e^{d+bx}\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Sech[d + b*x]^4,x]
```

output
```
(E^(a + b*x)*(-3 - 8*E^(2*(d + b*x)) + 3*E^(4*(d + b*x))))/(3*b*(1 + E^(2*
(d + b*x)))^3) + (E^(a - d)*ArcTan[E^(d + b*x)])/b
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2720, 27, 252, 252, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{sech}^4(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{16e^{a+4bx}}{(1+e^{2bx})^4}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{16e^a\int \frac{e^{4bx}}{(1+e^{2bx})^4}de^{bx}}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{16e^a\left(\frac{1}{2}\int \frac{e^{2bx}}{(1+e^{2bx})^3}de^{bx}-\frac{e^{3bx}}{6(e^{2bx}+1)^3}\right)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{16e^a\left(\frac{1}{2}\left(\frac{1}{4}\int \frac{1}{(1+e^{2bx})^2}de^{bx}-\frac{e^{bx}}{4(e^{2bx}+1)^2}\right)-\frac{e^{3bx}}{6(e^{2bx}+1)^3}\right)}{b}$$

$$\Big\downarrow \text{215}$$

$$\frac{16e^a\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{1+e^{2bx}}de^{bx}+\frac{e^{bx}}{2(e^{2bx}+1)}\right)-\frac{e^{bx}}{4(e^{2bx}+1)^2}\right)-\frac{e^{3bx}}{6(e^{2bx}+1)^3}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{16e^a\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\arctan\left(e^{bx}\right)+\frac{e^{bx}}{2(e^{2bx}+1)}\right)-\frac{e^{bx}}{4(e^{2bx}+1)^2}\right)-\frac{e^{3bx}}{6(e^{2bx}+1)^3}\right)}{b}$$

input
```
Int[E^(a + b*x)*Sech[d + b*x]^4,x]
```

output
```
(16*E^a*(-1/6*E^(3*b*x)/(1 + E^(2*b*x))^3 + (-1/4*E^(b*x)/(1 + E^(2*b*x))^
2 + (E^(b*x)/(2*(1 + E^(2*b*x))) + ArcTan[E^(b*x)]/2)/4)/2))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.33 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.23

method	result	size
risch	$-\frac{\left(-3\,\mathrm{e}^{4bx+4a+4d}+8\,\mathrm{e}^{2bx+4a+2d}+3\,\mathrm{e}^{4a}\right)\mathrm{e}^{bx+3a}}{3\left(\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)^{3}b}+\frac{i\ln\left(\mathrm{e}^{bx+a}+i\mathrm{e}^{a-d}\right)\mathrm{e}^{a-d}}{2b}-\frac{i\ln\left(\mathrm{e}^{bx+a}-i\mathrm{e}^{a-d}\right)\mathrm{e}^{a-d}}{2b}$	127

input
```
int(exp(b*x+a)*sech(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3/(exp(2*b*x+2*a+2*d)+exp(2*a))^3/b*(-3*exp(4*b*x+4*a+4*d)+8*exp(2*b*x+
4*a+2*d)+3*exp(4*a))*exp(b*x+3*a)+1/2*I*ln(exp(b*x+a)+I*exp(a-d))/b*exp(a-
d)-1/2*I*ln(exp(b*x+a)-I*exp(a-d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 996 vs. 2(93) = 186.

Time = 0.10 (sec) , antiderivative size = 996, normalized size of antiderivative = 9.67

$$\int e^{a+bx}\mathrm{sech}^4(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)^4,x, algorithm="fricas")
```

output
```
1/3*(3*cosh(b*x + d)^5*cosh(-a + d) + 3*(cosh(-a + d) - sinh(-a + d))*sinh
(b*x + d)^5 + 15*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))
*sinh(b*x + d)^4 - 8*cosh(b*x + d)^3*cosh(-a + d) + 2*(15*cosh(b*x + d)^2*
cosh(-a + d) - (15*cosh(b*x + d)^2 - 4)*sinh(-a + d) - 4*cosh(-a + d))*sin
h(b*x + d)^3 + 6*(5*cosh(b*x + d)^3*cosh(-a + d) - 4*cosh(b*x + d)*cosh(-a
 + d) - (5*cosh(b*x + d)^3 - 4*cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d)^
2 + 3*(cosh(b*x + d)^6*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b
*x + d)^6 + 6*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*si
nh(b*x + d)^5 + 3*cosh(b*x + d)^4*cosh(-a + d) + 3*(5*cosh(b*x + d)^2*cosh
(-a + d) - (5*cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*sinh(b*x +
 d)^4 + 4*(5*cosh(b*x + d)^3*cosh(-a + d) + 3*cosh(b*x + d)*cosh(-a + d) -
 (5*cosh(b*x + d)^3 + 3*cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d)^3 + 3*c
osh(b*x + d)^2*cosh(-a + d) + 3*(5*cosh(b*x + d)^4*cosh(-a + d) + 6*cosh(b
*x + d)^2*cosh(-a + d) - (5*cosh(b*x + d)^4 + 6*cosh(b*x + d)^2 + 1)*sinh(
-a + d) + cosh(-a + d))*sinh(b*x + d)^2 + 6*(cosh(b*x + d)^5*cosh(-a + d)
+ 2*cosh(b*x + d)^3*cosh(-a + d) + cosh(b*x + d)*cosh(-a + d) - (cosh(b*x
+ d)^5 + 2*cosh(b*x + d)^3 + cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d) -
(cosh(b*x + d)^6 + 3*cosh(b*x + d)^4 + 3*cosh(b*x + d)^2 + 1)*sinh(-a + d)
 + cosh(-a + d))*arctan(cosh(b*x + d) + sinh(b*x + d)) - 3*cosh(b*x + d)*c
osh(-a + d) + 3*(5*cosh(b*x + d)^4*cosh(-a + d) - 8*cosh(b*x + d)^2*cos...
```

Sympy [F]

$$\int e^{a+bx}\mathrm{sech}^4(d+bx)\,dx = e^a\int e^{bx}\,\mathrm{sech}^4\left(bx+d\right)dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)**4,x)
```

output
```
exp(a)*Integral(exp(b*x)*sech(b*x + d)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.06

$$\int e^{a+bx}\text{sech}^4(d+bx)\,dx = \frac{\arctan\left(e^{(bx+d)}\right)e^{(a-d)}}{b} + \frac{3\,e^{(5\,bx+7\,a+4\,d)} - 8\,e^{(3\,bx+7\,a+2\,d)} - 3\,e^{(bx+7\,a)}}{3\,b\left(e^{(6\,bx+6\,a+6\,d)} + 3\,e^{(4\,bx+6\,a+4\,d)} + 3\,e^{(2\,bx+6\,a+2\,d)} + e^{(6\,a)}\right)}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)^4,x, algorithm="maxima")
```

output
```
arctan(e^(b*x + d))*e^(a - d)/b + 1/3*(3*e^(5*b*x + 7*a + 4*d) - 8*e^(3*b*
x + 7*a + 2*d) - 3*e^(b*x + 7*a))/(b*(e^(6*b*x + 6*a + 6*d) + 3*e^(4*b*x +
 6*a + 4*d) + 3*e^(2*b*x + 6*a + 2*d) + e^(6*a)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.73

$$\int e^{a+bx}\text{sech}^4(d+bx)\,dx = \frac{1}{3}\left(\frac{3\,\arctan\left(e^{(bx+d)}\right)e^{(-5\,d)}}{b} + \frac{\left(3\,e^{(5\,bx+4\,d)} - 8\,e^{(3\,bx+2\,d)} - 3\,e^{(bx)}\right)e^{(-4\,d)}}{b\left(e^{(2\,bx+2\,d)} + 1\right)^3}\right)e^{(a+4\,d)}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)^4,x, algorithm="giac")
```

output
```
1/3*(3*arctan(e^(b*x + d))*e^(-5*d)/b + (3*e^(5*b*x + 4*d) - 8*e^(3*b*x +
2*d) - 3*e^(b*x))*e^(-4*d)/(b*(e^(2*b*x + 2*d) + 1)^3))*e^(a + 4*d)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\operatorname{sech}^4(d+bx)\,dx=\int \frac{e^{a+b\,x}}{\cosh\left(d+b\,x\right)^4}\,dx$$

input
```
int(exp(a + b*x)/cosh(d + b*x)^4,x)
```

output
```
int(exp(a + b*x)/cosh(d + b*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.50

$$\int e^{a+bx}\operatorname{sech}^4(d+bx)\,dx$$
$$=\frac{e^{a}\left(3e^{6bx+6d}atan\left(e^{bx+d}\right)+9e^{4bx+4d}atan\left(e^{bx+d}\right)+9e^{2bx+2d}atan\left(e^{bx+d}\right)+3atan\left(e^{bx+d}\right)+3e^{5bx+5d}-8e^{3bx}\right.}{3e^{d}b\left(e^{6bx+6d}+3e^{4bx+4d}+3e^{2bx+2d}+1\right)}$$

input
```
int(exp(b*x+a)*sech(b*x+d)^4,x)
```

output
```
(e**a*(3*e**(6*b*x + 6*d)*atan(e**(b*x + d)) + 9*e**(4*b*x + 4*d)*atan(e**
(b*x + d)) + 9*e**(2*b*x + 2*d)*atan(e**(b*x + d)) + 3*atan(e**(b*x + d))
+ 3*e**(5*b*x + 5*d) - 8*e**(3*b*x + 3*d) - 3*e**(b*x + d)))/(3*e**d*b*(e*
*(6*b*x + 6*d) + 3*e**(4*b*x + 4*d) + 3*e**(2*b*x + 2*d) + 1))
```

3.80 $\int e^{a+bx}\text{sech}^5(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int e^{a+bx}\text{sech}^5(d+bx)\,dx=-\frac{4e^{a-d}}{b\left(1+e^{2d+2bx}\right)^4}+\frac{32e^{a-d}}{3b\left(1+e^{2d+2bx}\right)^3}-\frac{8e^{a-d}}{b\left(1+e^{2d+2bx}\right)^2}$$

output
```
-4*exp(a-d)/b/(1+exp(2*b*x+2*d))^4+32/3*exp(a-d)/b/(1+exp(2*b*x+2*d))^3-8*
exp(a-d)/b/(1+exp(2*b*x+2*d))^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int e^{a+bx}\text{sech}^5(d+bx)\,dx=\frac{4e^{a+5d+6bx}\left(4+e^{2(d+bx)}\right)}{3b\left(1+e^{2(d+bx)}\right)^4}$$

input
```
Integrate[E^(a + b*x)*Sech[d + b*x]^5,x]
```

output
```
(4*E^(a + 5*d + 6*b*x)*(4 + E^(2*(d + b*x))))/(3*b*(1 + E^(2*(d + b*x)))^4
)
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.63, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2720, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{sech}^5(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{32e^{a+5bx}}{(1+e^{2bx})^5}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{32e^a\int \frac{e^{5bx}}{(1+e^{2bx})^5}de^{bx}}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{16e^a\int \frac{e^{2bx}}{(1+e^{2bx})^5}de^{2bx}}{b}$$

$$\Big\downarrow \text{53}$$

$$\frac{16e^a\int \left(\frac{1}{(1+e^{2bx})^3}-\frac{2}{(1+e^{2bx})^4}+\frac{1}{(1+e^{2bx})^5}\right)de^{2bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{16e^a\left(-\frac{1}{2(e^{2bx}+1)^2}+\frac{2}{3(e^{2bx}+1)^3}-\frac{1}{4(e^{2bx}+1)^4}\right)}{b}$$

input
```
Int[E^(a + b*x)*Sech[d + b*x]^5,x]
```

output
```
(16*E^a*(-1/4*1/(1 + E^(2*b*x))^4 + 2/(3*(1 + E^(2*b*x))^3) - 1/(2*(1 + E^
(2*b*x))^2)))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

method	result	size
risch	$-\frac{4\left(6\,e^{4bx+4a+4d}+4\,e^{2bx+4a+2d}+e^{4a}\right)e^{5a-d}}{3\left(e^{2bx+2a+2d}+e^{2a}\right)^{4}b}$	66
parallelrisch	$\frac{2\,e^{bx+a}\left(\sinh\left(3bx+3d\right)+4\sinh\left(bx+d\right)+4\cosh\left(bx+d\right)+\cosh\left(3bx+3d\right)\right)}{3b\left(\cosh\left(4bx+4d\right)+4\cosh\left(2bx+2d\right)+3\right)}$	71

input
```
int(exp(b*x+a)*sech(b*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
-4/3/(exp(2*b*x+2*a+2*d)+exp(2*a))^4/b*(6*exp(4*b*x+4*a+4*d)+4*exp(2*b*x+4
*a+2*d)+exp(4*a))*exp(5*a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(73) = 146$.

Time = 0.08 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.74

$$\int e^{a+bx}\text{sech}^5(d+bx)\,dx =$$

$$-\frac{4\left(7\cosh\left(bx+d\right)^2\cosh\left(-a+d\right)\right.}{3\left(b\cosh\left(bx+d\right)^6+6\,b\cosh\left(bx+d\right)\sinh\left(bx+d\right)^5+b\sinh\left(bx+d\right)^6+4\,b\cosh\left(bx+d\right)^4+\left(15\,b\cos\right.\right.}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)^5,x, algorithm="fricas")
```

output
```
-4/3*(7*cosh(b*x + d)^2*cosh(-a + d) + 7*(cosh(-a + d) - sinh(-a + d))*sin
h(b*x + d)^2 + 10*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d)
)*sinh(b*x + d) - (7*cosh(b*x + d)^2 + 4)*sinh(-a + d) + 4*cosh(-a + d))/(
b*cosh(b*x + d)^6 + 6*b*cosh(b*x + d)*sinh(b*x + d)^5 + b*sinh(b*x + d)^6
+ 4*b*cosh(b*x + d)^4 + (15*b*cosh(b*x + d)^2 + 4*b)*sinh(b*x + d)^4 + 4*(
5*b*cosh(b*x + d)^3 + 4*b*cosh(b*x + d))*sinh(b*x + d)^3 + 7*b*cosh(b*x +
d)^2 + (15*b*cosh(b*x + d)^4 + 24*b*cosh(b*x + d)^2 + 7*b)*sinh(b*x + d)^2
 + 2*(3*b*cosh(b*x + d)^5 + 8*b*cosh(b*x + d)^3 + 5*b*cosh(b*x + d))*sinh(
b*x + d) + 4*b)
```

Sympy [F]

$$\int e^{a+bx}\text{sech}^5(d+bx)\,dx = e^a\int e^{bx}\operatorname{sech}^5\left(bx+d\right)dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)**5,x)
```

output
```
exp(a)*Integral(exp(b*x)*sech(b*x + d)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(73) = 146$.

Time = 0.04 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.88

$$\int e^{a+bx}\mathrm{sech}^5(d+bx)\,dx = -\frac{8\,e^{(4\,bx+9\,a+4\,d)}}{b(e^{(8\,bx+8\,a+9\,d)}+4\,e^{(6\,bx+8\,a+7\,d)}+6\,e^{(4\,bx+8\,a+5\,d)}+4\,e^{(2\,bx+8\,a+3\,d)}+e^{(8\,a+d)})} - \frac{16\,e^{(2\,bx+9\,a+2\,d)}}{3\,b(e^{(8\,bx+8\,a+9\,d)}+4\,e^{(6\,bx+8\,a+7\,d)}+6\,e^{(4\,bx+8\,a+5\,d)}+4\,e^{(2\,bx+8\,a+3\,d)}+e^{(8\,a+d)})} - \frac{4\,e^{(9\,a)}}{3\,b(e^{(8\,bx+8\,a+9\,d)}+4\,e^{(6\,bx+8\,a+7\,d)}+6\,e^{(4\,bx+8\,a+5\,d)}+4\,e^{(2\,bx+8\,a+3\,d)}+e^{(8\,a+d)})}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)^5,x, algorithm="maxima")
```

output
```
-8*e^(4*b*x + 9*a + 4*d)/(b*(e^(8*b*x + 8*a + 9*d) + 4*e^(6*b*x + 8*a + 7*
d) + 6*e^(4*b*x + 8*a + 5*d) + 4*e^(2*b*x + 8*a + 3*d) + e^(8*a + d))) - 1
6/3*e^(2*b*x + 9*a + 2*d)/(b*(e^(8*b*x + 8*a + 9*d) + 4*e^(6*b*x + 8*a + 7
*d) + 6*e^(4*b*x + 8*a + 5*d) + 4*e^(2*b*x + 8*a + 3*d) + e^(8*a + d))) -
4/3*e^(9*a)/(b*(e^(8*b*x + 8*a + 9*d) + 4*e^(6*b*x + 8*a + 7*d) + 6*e^(4*b
*x + 8*a + 5*d) + 4*e^(2*b*x + 8*a + 3*d) + e^(8*a + d)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.59

$$\int e^{a+bx}\mathrm{sech}^5(d+bx)\,dx = -\frac{4\left(6\,e^{(4\,bx+4\,d)}+4\,e^{(2\,bx+2\,d)}+1\right)e^{(a-d)}}{3\,b\left(e^{(2\,bx+2\,d)}+1\right)^4}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)^5,x, algorithm="giac")
```

output
```
-4/3*(6*e^(4*b*x + 4*d) + 4*e^(2*b*x + 2*d) + 1)*e^(a - d)/(b*(e^(2*b*x +
2*d) + 1)^4)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\operatorname{sech}^5(d+bx)\,dx=\int \frac{e^{a+b\,x}}{\cosh\left(d+b\,x\right)^5}\,dx$$

input
```
int(exp(a + b*x)/cosh(d + b*x)^5,x)
```

output
```
int(exp(a + b*x)/cosh(d + b*x)^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10

$$\int e^{a+bx}\operatorname{sech}^5(d+bx)\,dx=\frac{4e^a\left(-6e^{4bx+4d}-4e^{2bx+2d}-1\right)}{3e^db\left(e^{8bx+8d}+4e^{6bx+6d}+6e^{4bx+4d}+4e^{2bx+2d}+1\right)}$$

input
```
int(exp(b*x+a)*sech(b*x+d)^5,x)
```

output
```
(4*e**a*( - 6*e**(4*b*x + 4*d) - 4*e**(2*b*x + 2*d) - 1))/(3*e**d*b*(e**(8
*b*x + 8*d) + 4*e**(6*b*x + 6*d) + 6*e**(4*b*x + 4*d) + 4*e**(2*b*x + 2*d)
 + 1))
```

3.81 $\int e^{2(a+bx)}\operatorname{sech}(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int e^{2(a+bx)}\operatorname{sech}(d+bx)\,dx=\frac{2e^{2a-d+bx}}{b}-\frac{2e^{2a-2d}\arctan\left(e^{d+bx}\right)}{b}$$

output
```
2*exp(b*x+2*a-d)/b-2*exp(2*a-2*d)*arctan(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2(a+bx)}\operatorname{sech}(d+bx)\,dx=\frac{2e^{2a-2d}\left(e^{d+bx}-\arctan\left(e^{d+bx}\right)\right)}{b}$$

input
```
Integrate[E^(2*(a + b*x))*Sech[d + b*x],x]
```

output
```
(2*E^(2*a - 2*d)*(E^(d + b*x) - ArcTan[E^(d + b*x)]))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 27, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{sech}(bx+d)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int \frac{2e^{2a+2bx}}{1+e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2e^{2a}\int \frac{e^{2bx}}{1+e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{2e^{2a}\left(e^{bx}-\int \frac{1}{1+e^{2bx}}de^{bx}\right)}{b}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{2e^{2a}\left(e^{bx}-\arctan\left(e^{bx}\right)\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Sech[d + b*x],x]
```

output
```
(2*E^(2*a)*(E^(b*x) - ArcTan[E^(b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.00

method	result	size
risch	$\frac{2\,\mathrm{e}^{b x+2 a-d}}{b}+\frac{i \ln\left(\mathrm{e}^{b x+a}-i \mathrm{e}^{a-d}\right) \mathrm{e}^{2 a-2 d}}{b}-\frac{i \ln\left(\mathrm{e}^{b x+a}+i \mathrm{e}^{a-d}\right) \mathrm{e}^{2 a-2 d}}{b}$	80

input
```
int(exp(2*b*x+2*a)*sech(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
2*exp(b*x+2*a-d)/b+I*ln(exp(b*x+a)-I*exp(a-d))/b*exp(2*a-2*d)-I*ln(exp(b*x
+a)+I*exp(a-d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(37) = 74$.

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.45

$$\int e^{2(a+bx)}\text{sech}(d+bx)\,dx = -\frac{2\left(\left(\cosh\left(-2\,a+2\,d\right)-\sinh\left(-2\,a+2\,d\right)\right)\arctan\left(\cosh\left(bx+d\right)+\sinh\left(bx+d\right)\right)-\cosh\left(bx+d\right)\cosh}{b}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d),x, algorithm="fricas")
```

output
```
-2*((cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*arctan(cosh(b*x + d) + sinh(b*x
+ d)) - cosh(b*x + d)*cosh(-2*a + 2*d) - (cosh(-2*a + 2*d) - sinh(-2*a + 2
*d))*sinh(b*x + d) + cosh(b*x + d)*sinh(-2*a + 2*d))/b
```

Sympy [F]

$$\int e^{2(a+bx)}\text{sech}(d+bx)\,dx = e^{2a}\int e^{2bx}\,\text{sech}\left(bx+d\right)dx$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d),x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*sech(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int e^{2(a+bx)}\text{sech}(d+bx)\,dx = \frac{2\,\arctan\left(e^{(-bx-d)}\right)e^{(2\,a-2\,d)}}{b}+\frac{2\,e^{(bx+2\,a-d)}}{b}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d),x, algorithm="maxima")
```

output
```
2*arctan(e^(-b*x - d))*e^(2*a - 2*d)/b + 2*e^(b*x + 2*a - d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2(a+bx)}\operatorname{sech}(d+bx)\,dx = -\frac{2\left(\arctan\left(e^{(bx+d)}\right)e^{(-2\,d)}-e^{(bx-d)}\right)e^{(2\,a)}}{b}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d),x, algorithm="giac")
```

output
```
-2*(arctan(e^(b*x + d))*e^(-2*d) - e^(b*x - d))*e^(2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.68

$$\int e^{2(a+bx)}\operatorname{sech}(d+bx)\,dx = \frac{2\,\mathrm{e}^{2\,a-d+b\,x}}{b}-\frac{2\,\sqrt{\mathrm{e}^{4\,a-4\,d}}\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{-d}\,\mathrm{e}^{b\,x}\,\sqrt{b^2}}{b\,\sqrt{\mathrm{e}^{4\,a}\,\mathrm{e}^{-4\,d}}}\right)}{\sqrt{b^2}}$$

input
```
int(exp(2*a + 2*b*x)/cosh(d + b*x),x)
```

output
```
(2*exp(2*a - d + b*x))/b - (2*exp(4*a - 4*d)^(1/2)*atan((exp(2*a)*exp(-d)*
exp(b*x)*(b^2)^(1/2))/(b*(exp(4*a)*exp(-4*d))^(1/2))))/(b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int e^{2(a+bx)}\operatorname{sech}(d+bx)\,dx = \frac{2e^{2a}\left(-atan\left(e^{bx+d}\right)+e^{bx+d}\right)}{e^{2d}b}$$

input
```
int(exp(2*b*x+2*a)*sech(b*x+d),x)
```

output
```
(2*e**(2*a)*( - atan(e**(b*x + d)) + e**(b*x + d)))/(e**(2*d)*b)
```

3.82 $\int e^{2(a+bx)}\text{sech}^2(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 56

$$\int e^{2(a+bx)}\text{sech}^2(d+bx)\,dx=\frac{2e^{2a-2d}}{b\left(1+e^{2d+2bx}\right)}+\frac{2e^{2a-2d}\log\left(1+e^{2d+2bx}\right)}{b}$$

output
```
2*exp(2*a-2*d)/b/(1+exp(2*b*x+2*d))+2*exp(2*a-2*d)*ln(1+exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

$$\int e^{2(a+bx)}\text{sech}^2(d+bx)\,dx=\frac{2e^{2a-2d}\left(-e^{2(d+bx)}+\left(1+e^{2(d+bx)}\right)\log\left(1+e^{2(d+bx)}\right)\right)}{b\left(1+e^{2(d+bx)}\right)}$$

input
```
Integrate[E^(2*(a + b*x))*Sech[d + b*x]^2,x]
```

output
```
(2*E^(2*a - 2*d)*(-E^(2*(d + b*x)) + (1 + E^(2*(d + b*x)))*Log[1 + E^(2*(d
 + b*x))]))/(b*(1 + E^(2*(d + b*x))))
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.54, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{sech}^2(bx+d)\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int \frac{4e^{2a+3bx}}{(1+e^{2bx})^2}de^{bx}}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{4e^{2a}\int \frac{e^{3bx}}{(1+e^{2bx})^2}de^{bx}}{b}$$

$$\Big\downarrow \; 243$$

$$\frac{2e^{2a}\int \frac{e^{2bx}}{(1+e^{2bx})^2}de^{2bx}}{b}$$

$$\Big\downarrow \; 49$$

$$\frac{2e^{2a}\int \left(\frac{1}{1+e^{2bx}}-\frac{1}{(1+e^{2bx})^2}\right)de^{2bx}}{b}$$

$$\Big\downarrow \; 2009$$

$$\frac{2e^{2a}\left(\frac{1}{e^{2bx}+1}+\log\left(e^{2bx}+1\right)\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Sech[d + b*x]^2,x]
```

output
```
(2*E^(2*a)*((1 + E^(2*b*x))^(-1) + Log[1 + E^(2*b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.43

method	result	size
risch	$-\frac{4\,{\mathrm e}^{2 a -2 d} a}{b}+\frac{2\,{\mathrm e}^{4 a -2 d}}{\left({\mathrm e}^{2 b x +2 a +2 d}+{\mathrm e}^{2 a}\right) b}+\frac{2 \ln \left({\mathrm e}^{2 b x +2 a}+{\mathrm e}^{2 a -2 d}\right) {\mathrm e}^{2 a -2 d}}{b}$	80

input
```
int(exp(2*b*x+2*a)*sech(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-4/b*exp(2*a-2*d)*a+2/(exp(2*b*x+2*a+2*d)+exp(2*a))/b*exp(4*a-2*d)+2*ln(ex
p(2*b*x+2*a)+exp(2*a-2*d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(52) = 104$.

Time = 0.09 (sec) , antiderivative size = 201, normalized size of antiderivative = 3.59

$$\int e^{2(a+bx)}\text{sech}^2(d+bx)\,dx = \frac{2\left(\left(\cosh\left(bx+d\right)^2\cosh\left(-2\,a+2\,d\right)+\left(\cosh\left(-2\,a+2\,d\right)-\sinh\left(-2\,a+2\,d\right)\right)\sinh\left(bx+d\right)^2+2\left(\cosh\left(b\right.\right.\right.}{}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)^2,x, algorithm="fricas")
```

output
```
2*((cosh(b*x + d)^2*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*d
))*sinh(b*x + d)^2 + 2*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sin
h(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + co
sh(-2*a + 2*d))*log(2*cosh(b*x + d)/(cosh(b*x + d) - sinh(b*x + d))) + cos
h(-2*a + 2*d) - sinh(-2*a + 2*d))/(b*cosh(b*x + d)^2 + 2*b*cosh(b*x + d)*s
inh(b*x + d) + b*sinh(b*x + d)^2 + b)
```

Sympy [F]

$$\int e^{2(a+bx)}\text{sech}^2(d+bx)\,dx = e^{2a}\int e^{2bx}\,\text{sech}^2\left(bx+d\right)dx$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)**2,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*sech(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.38

$$\int e^{2(a+bx)}\operatorname{sech}^2(d+bx)\,dx = 4\,xe^{(2\,a-2\,d)} + \frac{4\,de^{(2\,a-2\,d)}}{b} + \frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(-2\,bx-2\,d)}+1\right)}{b} - \frac{2\,e^{(2\,a-2\,d)}}{b(e^{(-2\,bx-2\,d)}+1)}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)^2,x, algorithm="maxima")
```

output
```
4*x*e^(2*a - 2*d) + 4*d*e^(2*a - 2*d)/b + 2*e^(2*a - 2*d)*log(e^(-2*b*x -
2*d) + 1)/b - 2*e^(2*a - 2*d)/(b*(e^(-2*b*x - 2*d) + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int e^{2(a+bx)}\operatorname{sech}^2(d+bx)\,dx = \frac{2\left(e^{(-2\,d)}\log\left(e^{(2\,bx+2\,d)}+1\right) + \frac{e^{(-2\,d)}}{e^{(2\,bx+2\,d)}+1}\right)e^{(2\,a)}}{b}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)^2,x, algorithm="giac")
```

output
```
2*(e^(-2*d)*log(e^(2*b*x + 2*d) + 1) + e^(-2*d)/(e^(2*b*x + 2*d) + 1))*e^(
2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int e^{2(a+bx)}\operatorname{sech}^2(d+bx)\,dx = \frac{2\,\mathrm{e}^{2\,a-2\,d}\,\ln\left(\mathrm{e}^{2\,d}\,\mathrm{e}^{2\,b\,x}+1\right)}{b} + \frac{2\,\mathrm{e}^{2\,a-2\,d}}{b\,\left(\mathrm{e}^{2\,d+2\,b\,x}+1\right)}$$

input
```
int(exp(2*a + 2*b*x)/cosh(d + b*x)^2,x)
```

output
```
(2*exp(2*a - 2*d)*log(exp(2*d)*exp(2*b*x) + 1))/b + (2*exp(2*a - 2*d))/(b*
(exp(2*d + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

$$\int e^{2(a+bx)}\operatorname{sech}^2(d+bx)\,dx=\frac{2e^{2a}\left(e^{2bx+2d}\log\left(e^{2bx+2d}+1\right)-e^{2bx+2d}+\log\left(e^{2bx+2d}+1\right)\right)}{e^{2d}b\left(e^{2bx+2d}+1\right)}$$

input
```
int(exp(2*b*x+2*a)*sech(b*x+d)^2,x)
```

output
```
(2*e**(2*a)*(e**(2*b*x + 2*d)*log(e**(2*b*x + 2*d) + 1) - e**(2*b*x + 2*d)
 + log(e**(2*b*x + 2*d) + 1)))/(e**(2*d)*b*(e**(2*b*x + 2*d) + 1))
```

3.83 $\int e^{2(a+bx)}\text{sech}^3(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 84

$$\int e^{2(a+bx)}\text{sech}^3(d+bx)\,dx=-\frac{2e^{2a+d+3bx}}{b\left(1+e^{2d+2bx}\right)^2}-\frac{3e^{2a-d+bx}}{b\left(1+e^{2d+2bx}\right)}+\frac{3e^{2a-2d}\arctan\left(e^{d+bx}\right)}{b}$$

output
```
-2*exp(3*b*x+2*a+d)/b/(1+exp(2*b*x+2*d))^2-3*exp(b*x+2*a-d)/b/(1+exp(2*b*x
+2*d))+3*exp(2*a-2*d)*arctan(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int e^{2(a+bx)}\text{sech}^3(d+bx)\,dx = \frac{e^{2a-2d}\left(-6e^{d+bx}+6\arctan\left(e^{d+bx}\right)+e^{2(d+bx)}\text{sech}(d+bx)(2+\tanh(d+bx))\right)}{2b}$$

input
```
Integrate[E^(2*(a + b*x))*Sech[d + b*x]^3,x]
```

output
```
(E^(2*a - 2*d)*(-6*E^(d + b*x) + 6*ArcTan[E^(d + b*x)] + E^(2*(d + b*x))*S
ech[d + b*x]*(2 + Tanh[d + b*x])))/(2*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2720, 27, 252, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{sech}^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{8e^{2a+4bx}}{(1+e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{8e^{2a}\int \frac{e^{4bx}}{(1+e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{8e^{2a}\left(\frac{3}{4}\int \frac{e^{2bx}}{(1+e^{2bx})^2}de^{bx}-\frac{e^{3bx}}{4(e^{2bx}+1)^2}\right)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{8e^{2a}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1+e^{2bx}}de^{bx}-\frac{e^{bx}}{2(e^{2bx}+1)}\right)-\frac{e^{3bx}}{4(e^{2bx}+1)^2}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{8e^{2a}\left(\frac{3}{4}\left(\frac{1}{2}\arctan\left(e^{bx}\right)-\frac{e^{bx}}{2(e^{2bx}+1)}\right)-\frac{e^{3bx}}{4(e^{2bx}+1)^2}\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Sech[d + b*x]^3,x]
```

output
```
(8*E^(2*a)*(-1/4*E^(3*b*x)/(1 + E^(2*b*x))^2 + (3*(-1/2*E^(b*x)/(1 + E^(2*
b*x)) + ArcTan[E^(b*x)]/2))/4))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.43

method	result	size
risch	$-\frac{\left(5\,{\mathrm e}^{2 b x +2 a +2 d}+3\,{\mathrm e}^{2 a}\right) {\mathrm e}^{b x +4 a -d}}{\left({\mathrm e}^{2 b x +2 a +2 d}+{\mathrm e}^{2 a}\right)^{2} b}+\frac{3 i \ln \left({\mathrm e}^{b x +a}+i {\mathrm e}^{a -d}\right) {\mathrm e}^{2 a -2 d}}{2 b}-\frac{3 i \ln \left({\mathrm e}^{b x +a}-i {\mathrm e}^{a -d}\right) {\mathrm e}^{2 a -2 d}}{2 b}$	120

input
```
int(exp(2*b*x+2*a)*sech(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/(exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(5*exp(2*b*x+2*a+2*d)+3*exp(2*a))*exp
(b*x+4*a-d)+3/2*I*ln(exp(b*x+a)+I*exp(a-d))/b*exp(2*a-2*d)-3/2*I*ln(exp(b*
x+a)-I*exp(a-d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(78) = 156$.

Time = 0.09 (sec) , antiderivative size = 581, normalized size of antiderivative = 6.92

$$\int e^{2(a+bx)}\text{sech}^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)^3,x, algorithm="fricas")
```

output
```
-(5*cosh(b*x + d)^3*cosh(-2*a + 2*d) + 5*(cosh(-2*a + 2*d) - sinh(-2*a + 2
*d))*sinh(b*x + d)^3 + 15*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*
sinh(-2*a + 2*d))*sinh(b*x + d)^2 - 3*(cosh(b*x + d)^4*cosh(-2*a + 2*d) +
(cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*c
osh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^3 + 2*cosh
(b*x + d)^2*cosh(-2*a + 2*d) + 2*(3*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (3*
cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*sinh(b*x + d)^2
+ 4*(cosh(b*x + d)^3*cosh(-2*a + 2*d) + cosh(b*x + d)*cosh(-2*a + 2*d) - (
cosh(b*x + d)^3 + cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b
*x + d)^4 + 2*cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*ar
ctan(cosh(b*x + d) + sinh(b*x + d)) + 3*cosh(b*x + d)*cosh(-2*a + 2*d) + 3
*(5*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (5*cosh(b*x + d)^2 + 1)*sinh(-2*a +
 2*d) + cosh(-2*a + 2*d))*sinh(b*x + d) - (5*cosh(b*x + d)^3 + 3*cosh(b*x
+ d))*sinh(-2*a + 2*d))/(b*cosh(b*x + d)^4 + 4*b*cosh(b*x + d)*sinh(b*x +
d)^3 + b*sinh(b*x + d)^4 + 2*b*cosh(b*x + d)^2 + 2*(3*b*cosh(b*x + d)^2 +
b)*sinh(b*x + d)^2 + 4*(b*cosh(b*x + d)^3 + b*cosh(b*x + d))*sinh(b*x + d)
 + b)
```

Sympy [F]

$$\int e^{2(a+bx)}\text{sech}^3(d+bx)\,dx = e^{2a}\int e^{2bx}\,\text{sech}^3\,(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)**3,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*sech(b*x + d)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int e^{2(a+bx)}\text{sech}^3(d+bx)\,dx = -\frac{3\,\arctan\left(e^{(-bx-d)}\right)e^{(2\,a-2\,d)}}{b} - \frac{\left(5\,e^{(-bx-d)}+3\,e^{(-3\,bx-3\,d)}\right)e^{(2\,a-2\,d)}}{b(2\,e^{(-2\,bx-2\,d)}+e^{(-4\,bx-4\,d)}+1)}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)^3,x, algorithm="maxima")
```

output
```
-3*arctan(e^(-b*x - d))*e^(2*a - 2*d)/b - (5*e^(-b*x - d) + 3*e^(-3*b*x -
3*d))*e^(2*a - 2*d)/(b*(2*e^(-2*b*x - 2*d) + e^(-4*b*x - 4*d) + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int e^{2(a+bx)}\text{sech}^3(d+bx)\,dx = \frac{\left(3\,\arctan\left(e^{(bx+d)}\right)e^{(-2\,d)} - \frac{\left(5\,e^{(3\,bx+3\,d)}+3\,e^{(bx+d)}\right)e^{(-2\,d)}}{\left(e^{(2\,bx+2\,d)}+1\right)^2}\right)e^{(2\,a)}}{b}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)^3,x, algorithm="giac")
```

output
```
(3*arctan(e^(b*x + d))*e^(-2*d) - (5*e^(3*b*x + 3*d) + 3*e^(b*x + d))*e^(-
2*d)/(e^(2*b*x + 2*d) + 1)^2)*e^(2*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.42

$$\int e^{2(a+bx)}\operatorname{sech}^3(d+bx)\,dx = \frac{3\sqrt{e^{4a-4d}}\operatorname{atan}\left(\frac{e^{2a}\,e^{-d}\,e^{bx}\,\sqrt{b^2}}{b\sqrt{e^{4a}\,e^{-4d}}}\right)}{\sqrt{b^2}} - \frac{3\,e^{2a-d+bx}}{b\,\left(e^{2d+2bx}+1\right)} - \frac{2\,e^{2a+d+3bx}}{b\,\left(2\,e^{2d+2bx}+e^{4d+4bx}+1\right)}$$

input
```
int(exp(2*a + 2*b*x)/cosh(d + b*x)^3,x)
```

output
```
(3*exp(4*a - 4*d)^(1/2)*atan((exp(2*a)*exp(-d)*exp(b*x)*(b^2)^(1/2))/(b*(e
xp(4*a)*exp(-4*d))^(1/2))))/(b^2)^(1/2) - (3*exp(2*a - d + b*x))/(b*(exp(2
*d + 2*b*x) + 1)) - (2*exp(2*a + d + 3*b*x))/(b*(2*exp(2*d + 2*b*x) + exp(
4*d + 4*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.36

$$\int e^{2(a+bx)}\operatorname{sech}^3(d+bx)\,dx = \frac{e^{2a}\left(3e^{4bx+4d}atan\left(e^{bx+d}\right)+6e^{2bx+2d}atan\left(e^{bx+d}\right)+3atan\left(e^{bx+d}\right)-5e^{3bx+3d}-3e^{bx+d}\right)}{e^{2d}b\left(e^{4bx+4d}+2e^{2bx+2d}+1\right)}$$

input
```
int(exp(2*b*x+2*a)*sech(b*x+d)^3,x)
```

output
```
(e**(2*a)*(3*e**(4*b*x + 4*d)*atan(e**(b*x + d)) + 6*e**(2*b*x + 2*d)*atan
(e**(b*x + d)) + 3*atan(e**(b*x + d)) - 5*e**(3*b*x + 3*d) - 3*e**(b*x + d
)))/(e**(2*d)*b*(e**(4*b*x + 4*d) + 2*e**(2*b*x + 2*d) + 1))
```

3.84 $\int e^{2(a+bx)}\mathrm{sech}^4(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 35

$$\int e^{2(a+bx)}\mathrm{sech}^4(d+bx)\,dx=\frac{8e^{2(a+2d)+6bx}}{3b\left(1+e^{2d+2bx}\right)^3}$$

output
```
8/3*exp(6*b*x+2*a+4*d)/b/(1+exp(2*b*x+2*d))^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int e^{2(a+bx)}\mathrm{sech}^4(d+bx)\,dx=\frac{8e^{2a+4d+6bx}}{3b\left(1+e^{2(d+bx)}\right)^3}$$

input
```
Integrate[E^(2*(a + b*x))*Sech[d + b*x]^4,x]
```

output
```
(8*E^(2*a + 4*d + 6*b*x))/(3*b*(1 + E^(2*(d + b*x)))^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2720, 27, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{sech}^4(bx+d)\,dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int \frac{16e^{2a+5bx}}{(1+e^{2bx})^4}de^{bx}}{b}$$

$$\Big\downarrow \ 27$$

$$\frac{16e^{2a}\int \frac{e^{5bx}}{(1+e^{2bx})^4}de^{bx}}{b}$$

$$\Big\downarrow \ 242$$

$$\frac{8e^{2a+6bx}}{3b\left(e^{2bx}+1\right)^3}$$

input
```
Int[E^(2*(a + b*x))*Sech[d + b*x]^4,x]
```

output
```
(8*E^(2*a + 6*b*x))/(3*b*(1 + E^(2*b*x))^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

method	result	size
parallelrisch	$\frac{4\,{\mathrm e}^{2 b x+2 a} \left(\sinh\left(b x+d\right)+\cosh\left(b x+d\right)\right)}{3 b \left(\cosh\left(3 b x+3 d\right)+3 \cosh\left(b x+d\right)\right)}$	48
risch	$-\frac{8 \left(3\,{\mathrm e}^{4 b x+4 a+4 d}+3\,{\mathrm e}^{2 b x+4 a+2 d}+{\mathrm e}^{4 a}\right) {\mathrm e}^{4 a-2 d}}{3 \left({\mathrm e}^{2 b x+2 a+2 d}+{\mathrm e}^{2 a}\right)^{3} b}$	66

input
```
int(exp(2*b*x+2*a)*sech(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
4/3*exp(2*b*x+2*a)/b/(cosh(3*b*x+3*d)+3*cosh(b*x+d))*(sinh(b*x+d)+cosh(b*x
+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(30) = 60.

Time = 0.08 (sec) , antiderivative size = 228, normalized size of antiderivative = 6.51

$$\int e^{2(a+bx)}\text{sech}^4(d+bx)\,dx = -\frac{8\left(4\cosh\left(bx+d\right)^2\cosh\left(-2\,a+2\,d\right)+4\left(\cosh\left(-2\,a+2\,d\right)-\sinh\left(-2\,a+2\,d\right)\right)\sinh\left(bx+d\right)^2+4\,(\mathrm{co}}{3\left(b\cosh\left(bx+d\right)^4+4\,b\cosh\left(bx+d\right)\sinh\left(bx+d\right)^3+b\sinh\left(bx+d\right)^4+4\,b}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)^4,x, algorithm="fricas")
```

output
```
-8/3*(4*cosh(b*x + d)^2*cosh(-2*a + 2*d) + 4*(cosh(-2*a + 2*d) - sinh(-2*a
 + 2*d))*sinh(b*x + d)^2 + 4*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x +
d)*sinh(-2*a + 2*d))*sinh(b*x + d) - (4*cosh(b*x + d)^2 + 3)*sinh(-2*a + 2
*d) + 3*cosh(-2*a + 2*d))/(b*cosh(b*x + d)^4 + 4*b*cosh(b*x + d)*sinh(b*x
+ d)^3 + b*sinh(b*x + d)^4 + 4*b*cosh(b*x + d)^2 + 2*(3*b*cosh(b*x + d)^2
+ 2*b)*sinh(b*x + d)^2 + 4*(b*cosh(b*x + d)^3 + b*cosh(b*x + d))*sinh(b*x
+ d) + 3*b)
```

Sympy [F]

$$\int e^{2(a+bx)}\text{sech}^4(d+bx)\,dx=e^{2a}\int e^{2bx}\,\text{sech}^4\,(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)**4,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*sech(b*x + d)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int e^{2(a+bx)}\text{sech}^4(d+bx)\,dx=\frac{8\,e^{(2\,a-2\,d)}}{3\,b(3\,e^{(-2\,bx-2\,d)}+3\,e^{(-4\,bx-4\,d)}+e^{(-6\,bx-6\,d)}+1)}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)^4,x, algorithm="maxima")
```

output
```
8/3*e^(2*a - 2*d)/(b*(3*e^(-2*b*x - 2*d) + 3*e^(-4*b*x - 4*d) + e^(-6*b*x
- 6*d) + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.43

$$\int e^{2(a+bx)}\text{sech}^4(d+bx)\,dx = -\frac{8\left(3\,e^{(4\,bx+4\,d)}+3\,e^{(2\,bx+2\,d)}+1\right)e^{(2\,a-2\,d)}}{3\,b\left(e^{(2\,bx+2\,d)}+1\right)^3}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)^4,x, algorithm="giac")
```

output
```
-8/3*(3*e^(4*b*x + 4*d) + 3*e^(2*b*x + 2*d) + 1)*e^(2*a - 2*d)/(b*(e^(2*b*
x + 2*d) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.34

$$\int e^{2(a+bx)}\text{sech}^4(d+bx)\,dx = -\frac{8\,e^{2\,a+2\,d+4\,b\,x}}{3\,b\,\left(3\,e^{2\,d+2\,b\,x}+3\,e^{4\,d+4\,b\,x}+e^{6\,d+6\,b\,x}+1\right)} - \frac{8\,e^{2\,a+2\,b\,x}}{3\,b\,\left(2\,e^{2\,d+2\,b\,x}+e^{4\,d+4\,b\,x}+1\right)} - \frac{8\,e^{2\,a-2\,d}}{3\,b\,\left(e^{2\,d+2\,b\,x}+1\right)}$$

input
```
int(exp(2*a + 2*b*x)/cosh(d + b*x)^4,x)
```

output
```
- (8*exp(2*a + 2*d + 4*b*x))/(3*b*(3*exp(2*d + 2*b*x) + 3*exp(4*d + 4*b*x)
 + exp(6*d + 6*b*x) + 1)) - (8*exp(2*a + 2*b*x))/(3*b*(2*exp(2*d + 2*b*x)
+ exp(4*d + 4*b*x) + 1)) - (8*exp(2*a - 2*d))/(3*b*(exp(2*d + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.60

$$\int e^{2(a+bx)}\text{sech}^4(d+bx)\,dx = \frac{8e^{6bx+2a+4d}}{3b\left(e^{6bx+6d}+3e^{4bx+4d}+3e^{2bx+2d}+1\right)}$$

input
```
int(exp(2*b*x+2*a)*sech(b*x+d)^4,x)
```

output

```
(8*e**(2*a + 6*b*x + 4*d))/(3*b*(e**(6*b*x + 6*d) + 3*e**(4*b*x + 4*d) + 3
*e**(2*b*x + 2*d) + 1))
```

3.85 $\int e^{2(a+bx)}\text{sech}^5(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 155

$$\int e^{2(a+bx)}\text{sech}^5(d+bx)\,dx=-\frac{4e^{2a+3d+5bx}}{b\left(1+e^{2d+2bx}\right)^4}-\frac{10e^{2a+d+3bx}}{3b\left(1+e^{2d+2bx}\right)^3}-\frac{5e^{2a-d+bx}}{2b\left(1+e^{2d+2bx}\right)^2}+\frac{5e^{2a-d+bx}}{4b\left(1+e^{2d+2bx}\right)}+\frac{5e^{2a-2d}\arctan\left(e^{d+bx}\right)}{4b}$$

output
```
-4*exp(5*b*x+2*a+3*d)/b/(1+exp(2*b*x+2*d))^4-10/3*exp(3*b*x+2*a+d)/b/(1+ex
p(2*b*x+2*d))^3-5/2*exp(b*x+2*a-d)/b/(1+exp(2*b*x+2*d))^2+5/4*exp(b*x+2*a-
d)/b/(1+exp(2*b*x+2*d))+5/4*exp(2*a-2*d)*arctan(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.59

$$\int e^{2(a+bx)}\text{sech}^5(d+bx)\,dx=\frac{e^{2a-2d}\left(30\left(-e^{d+bx}+\arctan\left(e^{d+bx}\right)\right)+5e^{2(d+bx)}\text{sech}(d+bx)(2+\tanh(d+bx))+2e^{2(d+bx)}\text{sech}^3(d+bx)\right.}{24b}$$

input
```
Integrate[E^(2*(a + b*x))*Sech[d + b*x]^5,x]
```

output
```
(E^(2*a - 2*d)*(30*(-E^(d + b*x) + ArcTan[E^(d + b*x)]) + 5*E^(2*(d + b*x)
)*Sech[d + b*x]*(2 + Tanh[d + b*x]) + 2*E^(2*(d + b*x))*Sech[d + b*x]^3*(2
 + 3*Tanh[d + b*x])))/(24*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.74, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2720, 27, 252, 252, 252, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{sech}^5(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{32e^{2a+6bx}}{(1+e^{2bx})^5}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{32e^{2a}\int \frac{e^{6bx}}{(1+e^{2bx})^5}de^{bx}}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{32e^{2a}\left(\frac{5}{8}\int \frac{e^{4bx}}{(1+e^{2bx})^4}de^{bx}-\frac{e^{5bx}}{8(e^{2bx}+1)^4}\right)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{32e^{2a}\left(\frac{5}{8}\left(\frac{1}{2}\int \frac{e^{2bx}}{(1+e^{2bx})^3}de^{bx}-\frac{e^{3bx}}{6(e^{2bx}+1)^3}\right)-\frac{e^{5bx}}{8(e^{2bx}+1)^4}\right)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{32e^{2a}\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\int \frac{1}{(1+e^{2bx})^2}de^{bx}-\frac{e^{bx}}{4(e^{2bx}+1)^2}\right)-\frac{e^{3bx}}{6(e^{2bx}+1)^3}\right)-\frac{e^{5bx}}{8(e^{2bx}+1)^4}\right)}{b}$$

$$\Big\downarrow \text{215}$$

$$\frac{32e^{2a}\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{1+e^{2bx}}de^{bx}+\frac{e^{bx}}{2(e^{2bx}+1)}\right)-\frac{e^{bx}}{4(e^{2bx}+1)^2}\right)-\frac{e^{3bx}}{6(e^{2bx}+1)^3}\right)-\frac{e^{5bx}}{8(e^{2bx}+1)^4}\right)}{b}$$

$$\Big\downarrow \; 216$$

$$\frac{32e^{2a}\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\arctan\left(e^{bx}\right)+\frac{e^{bx}}{2(e^{2bx}+1)}\right)-\frac{e^{bx}}{4(e^{2bx}+1)^2}\right)-\frac{e^{3bx}}{6(e^{2bx}+1)^3}\right)-\frac{e^{5bx}}{8(e^{2bx}+1)^4}\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Sech[d + b*x]^5,x]
```

output
```
(32*E^(2*a)*(-1/8*E^(5*b*x)/(1 + E^(2*b*x))^4 + (5*(-1/6*E^(3*b*x)/(1 + E^
(2*b*x))^3 + (-1/4*E^(b*x)/(1 + E^(2*b*x))^2 + (E^(b*x)/(2*(1 + E^(2*b*x))
) + ArcTan[E^(b*x)]/2)/4)/2))/8))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{\left(-15\,\mathrm{e}^{6bx+6a+6d}+73\,\mathrm{e}^{4bx+6a+4d}+55\,\mathrm{e}^{2bx+6a+2d}+15\,\mathrm{e}^{6a}\right)\mathrm{e}^{bx+4a-d}}{12\left(\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)^{4}b}+\frac{5i\ln\left(\mathrm{e}^{bx+a}+i\mathrm{e}^{a-d}\right)\mathrm{e}^{2a-2d}}{8b}-\frac{5i\ln\left(\mathrm{e}^{bx+a}-i\mathrm{e}^{a-d}\right)\mathrm{e}^{2c}}{8b}$

input
```
int(exp(2*b*x+2*a)*sech(b*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12/(exp(2*b*x+2*a+2*d)+exp(2*a))^4/b*(-15*exp(6*b*x+6*a+6*d)+73*exp(4*b
*x+6*a+4*d)+55*exp(2*b*x+6*a+2*d)+15*exp(6*a))*exp(b*x+4*a-d)+5/8*I*ln(exp
(b*x+a)+I*exp(a-d))/b*exp(2*a-2*d)-5/8*I*ln(exp(b*x+a)-I*exp(a-d))/b*exp(2
*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1710 vs. 2(137) = 274.

Time = 0.09 (sec) , antiderivative size = 1710, normalized size of antiderivative = 11.03

$$\int e^{2(a+bx)}\text{sech}^5(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)^5,x, algorithm="fricas")
```

output
```
1/12*(15*cosh(b*x + d)^7*cosh(-2*a + 2*d) + 15*(cosh(-2*a + 2*d) - sinh(-2
*a + 2*d))*sinh(b*x + d)^7 + 105*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*
x + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^6 - 73*cosh(b*x + d)^5*cosh(-2*a +
2*d) + (315*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (315*cosh(b*x + d)^2 - 73)*
sinh(-2*a + 2*d) - 73*cosh(-2*a + 2*d))*sinh(b*x + d)^5 + 5*(105*cosh(b*x
+ d)^3*cosh(-2*a + 2*d) - 73*cosh(b*x + d)*cosh(-2*a + 2*d) - (105*cosh(b*
x + d)^3 - 73*cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d)^4 - 55*cosh(b
*x + d)^3*cosh(-2*a + 2*d) + 5*(105*cosh(b*x + d)^4*cosh(-2*a + 2*d) - 146
*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (105*cosh(b*x + d)^4 - 146*cosh(b*x +
d)^2 - 11)*sinh(-2*a + 2*d) - 11*cosh(-2*a + 2*d))*sinh(b*x + d)^3 + 5*(63
*cosh(b*x + d)^5*cosh(-2*a + 2*d) - 146*cosh(b*x + d)^3*cosh(-2*a + 2*d) -
 33*cosh(b*x + d)*cosh(-2*a + 2*d) - (63*cosh(b*x + d)^5 - 146*cosh(b*x +
d)^3 - 33*cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d)^2 + 15*(cosh(b*x
+ d)^8*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*x +
 d)^8 + 8*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d)
)*sinh(b*x + d)^7 + 4*cosh(b*x + d)^6*cosh(-2*a + 2*d) + 4*(7*cosh(b*x + d
)^2*cosh(-2*a + 2*d) - (7*cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*
a + 2*d))*sinh(b*x + d)^6 + 8*(7*cosh(b*x + d)^3*cosh(-2*a + 2*d) + 3*cosh
(b*x + d)*cosh(-2*a + 2*d) - (7*cosh(b*x + d)^3 + 3*cosh(b*x + d))*sinh(-2
*a + 2*d))*sinh(b*x + d)^5 + 6*cosh(b*x + d)^4*cosh(-2*a + 2*d) + 2*(35...
```

Sympy [F]

$$\int e^{2(a+bx)}\text{sech}^5(d+bx)\,dx = e^{2a}\int e^{2bx}\,\text{sech}^5\,(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)**5,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*sech(b*x + d)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.83

$$\int e^{2(a+bx)}\operatorname{sech}^5(d+bx)\,dx = -\frac{5\arctan\left(e^{(-bx-d)}\right)e^{(2a-2d)}}{4b} + \frac{\left(15e^{(-bx-d)} - 73e^{(-3bx-3d)} - 55e^{(-5bx-5d)} - 15e^{(-7bx-7d)}\right)e^{(2a-2d)}}{12b(4e^{(-2bx-2d)} + 6e^{(-4bx-4d)} + 4e^{(-6bx-6d)} + e^{(-8bx-8d)} + 1)}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)^5,x, algorithm="maxima")
```

output
```
-5/4*arctan(e^(-b*x - d))*e^(2*a - 2*d)/b + 1/12*(15*e^(-b*x - d) - 73*e^(
-3*b*x - 3*d) - 55*e^(-5*b*x - 5*d) - 15*e^(-7*b*x - 7*d))*e^(2*a - 2*d)/(
b*(4*e^(-2*b*x - 2*d) + 6*e^(-4*b*x - 4*d) + 4*e^(-6*b*x - 6*d) + e^(-8*b*
x - 8*d) + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.54

$$\int e^{2(a+bx)}\operatorname{sech}^5(d+bx)\,dx = \frac{\left(15\arctan\left(e^{(bx+d)}\right)e^{(-2d)} + \frac{\left(15e^{(7bx+7d)} - 73e^{(5bx+5d)} - 55e^{(3bx+3d)} - 15e^{(bx+d)}\right)e^{(-2d)}}{\left(e^{(2bx+2d)}+1\right)^4}\right)e^{(2a)}}{12b}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)^5,x, algorithm="giac")
```

output
```
1/12*(15*arctan(e^(b*x + d))*e^(-2*d) + (15*e^(7*b*x + 7*d) - 73*e^(5*b*x
+ 5*d) - 55*e^(3*b*x + 3*d) - 15*e^(b*x + d))*e^(-2*d)/(e^(2*b*x + 2*d) +
1)^4)*e^(2*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.50

$$\int e^{2(a+bx)}\mathrm{sech}^5(d+bx)\,dx = \frac{5\,\mathrm{e}^{2\,a-d+b\,x}}{4\,b\,\left(\mathrm{e}^{2\,d+2\,b\,x}+1\right)} - \frac{10\,\mathrm{e}^{2\,a+d+3\,b\,x}}{3\,b\,\left(3\,\mathrm{e}^{2\,d+2\,b\,x}+3\,\mathrm{e}^{4\,d+4\,b\,x}+\mathrm{e}^{6\,d+6\,b\,x}+1\right)} - \frac{5\,\mathrm{e}^{2\,a-d+b\,x}}{2\,b\,\left(2\,\mathrm{e}^{2\,d+2\,b\,x}+\mathrm{e}^{4\,d+4\,b\,x}+1\right)} - \frac{4\,\mathrm{e}^{2\,a+3\,d+5\,b\,x}}{b\,\left(4\,\mathrm{e}^{2\,d+2\,b\,x}+6\,\mathrm{e}^{4\,d+4\,b\,x}+4\,\mathrm{e}^{6\,d+6\,b\,x}+\mathrm{e}^{8\,d+8\,b\,x}+1\right)} + \frac{5\,\sqrt{\mathrm{e}^{4\,a-4\,d}}\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{-d}\,\mathrm{e}^{b\,x}\,\sqrt{b^2}}{b\,\sqrt{\mathrm{e}^{4\,a}\,\mathrm{e}^{-4\,d}}}\right)}{4\,\sqrt{b^2}}$$

input
```
int(exp(2*a + 2*b*x)/cosh(d + b*x)^5,x)
```

output
```
(5*exp(2*a - d + b*x))/(4*b*(exp(2*d + 2*b*x) + 1)) - (10*exp(2*a + d + 3*
b*x))/(3*b*(3*exp(2*d + 2*b*x) + 3*exp(4*d + 4*b*x) + exp(6*d + 6*b*x) + 1
)) - (5*exp(2*a - d + b*x))/(2*b*(2*exp(2*d + 2*b*x) + exp(4*d + 4*b*x) +
1)) - (4*exp(2*a + 3*d + 5*b*x))/(b*(4*exp(2*d + 2*b*x) + 6*exp(4*d + 4*b*
x) + 4*exp(6*d + 6*b*x) + exp(8*d + 8*b*x) + 1)) + (5*exp(4*a - 4*d)^(1/2)
*atan((exp(2*a)*exp(-d)*exp(b*x)*(b^2)^(1/2))/(b*(exp(4*a)*exp(-4*d))^(1/2
))))/(4*(b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.31

$$\int e^{2(a+bx)}\mathrm{sech}^5(d+bx)\,dx = \frac{e^{2a}\left(15e^{8bx+8d}atan\left(e^{bx+d}\right)+60e^{6bx+6d}atan\left(e^{bx+d}\right)+90e^{4bx+4d}atan\left(e^{bx+d}\right)+60e^{2bx+2d}atan\left(e^{bx+d}\right)+15a\right.}{12e^{2d}b\left(e^{8bx+8d}+4e^{6bx+6d}+6e^{4bx+4d}+4e^{2bx+2d}\right.}$$

input
```
int(exp(2*b*x+2*a)*sech(b*x+d)^5,x)
```

output
```
(e**(2*a)*(15*e**(8*b*x + 8*d)*atan(e**(b*x + d)) + 60*e**(6*b*x + 6*d)*at
an(e**(b*x + d)) + 90*e**(4*b*x + 4*d)*atan(e**(b*x + d)) + 60*e**(2*b*x +
 2*d)*atan(e**(b*x + d)) + 15*atan(e**(b*x + d)) + 15*e**(7*b*x + 7*d) - 7
3*e**(5*b*x + 5*d) - 55*e**(3*b*x + 3*d) - 15*e**(b*x + d)))/(12*e**(2*d)*
b*(e**(8*b*x + 8*d) + 4*e**(6*b*x + 6*d) + 6*e**(4*b*x + 4*d) + 4*e**(2*b*
x + 2*d) + 1))
```

3.86 $\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 144

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}(d+bx)\,dx = \frac{3e^{\frac{5(a-d)}{3}+\frac{2}{3}(d+bx)}}{b} + \frac{\sqrt{3}e^{\frac{5(a-d)}{3}}\arctan\left(\frac{1-2e^{\frac{2}{3}(d+bx)}}{\sqrt{3}}\right)}{b} - \frac{e^{\frac{5(a-d)}{3}}\log\left(1+e^{\frac{2}{3}(d+bx)}\right)}{b} + \frac{e^{\frac{5(a-d)}{3}}\log\left(1-e^{\frac{2}{3}(d+bx)}+e^{\frac{4}{3}(d+bx)}\right)}{2b}$$

output
```
3*exp(5/3*a-d+2/3*b*x)/b+3^(1/2)*exp(5/3*a-5/3*d)*arctan(1/3*(1-2*exp(2/3*
b*x+2/3*d))*3^(1/2))/b-exp(5/3*a-5/3*d)*ln(1+exp(2/3*b*x+2/3*d))/b+1/2*exp
(5/3*a-5/3*d)*ln(1-exp(2/3*b*x+2/3*d)+exp(4/3*b*x+4/3*d))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.29

$$\int e^{\frac{5}{3}(a+bx)}\text{sech}(d+bx)\,dx=\frac{3e^{d+bx+\frac{5}{3}(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{4}{3},\frac{7}{3},-e^{2(d+bx)}\right)}{4b}$$

input
```
Integrate[E^((5*(a + b*x))/3)*Sech[d + b*x],x]
```

output
```
(3*E^(d + b*x + (5*(a + b*x))/3)*Hypergeometric2F1[1, 4/3, 7/3, -E^(2*(d +
 b*x))])/(4*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.43, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.611$, Rules used = {2720, 27, 807, 843, 750, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)}\text{sech}(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{3\int \frac{2e^{\frac{5a}{3}+\frac{7bx}{3}}}{1+e^{2bx}}de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{6e^{5a/3}\int \frac{e^{\frac{7bx}{3}}}{1+e^{2bx}}de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \text{807}$$

$$\frac{3e^{5a/3}\int \frac{e^{bx}}{1+e^{bx}}de^{\frac{2bx}{3}}}{b}$$

$$\Big\downarrow \text{843}$$

$$\frac{3e^{5a/3}\left(e^{\frac{2bx}{3}}-\int \frac{1}{1+e^{bx}}de^{\frac{2bx}{3}}\right)}{b}$$

$$\Big\downarrow \text{750}$$

$$\frac{3e^{5a/3}\left(-\frac{1}{3}\int \left(2-e^{\frac{2bx}{3}}\right)de^{\frac{2bx}{3}}-\frac{1}{3}\int \frac{1}{1+e^{\frac{2bx}{3}}}de^{\frac{2bx}{3}}+e^{\frac{2bx}{3}}\right)}{b}$$

$$\Big\downarrow \text{16}$$

$$\frac{3e^{5a/3}\left(-\frac{1}{3}\int \left(2-e^{\frac{2bx}{3}}\right)de^{\frac{2bx}{3}}+e^{\frac{2bx}{3}}-\frac{1}{3}\log\left(e^{\frac{2bx}{3}}+1\right)\right)}{b}$$

$$\Big\downarrow \text{1142}$$

$$\frac{3e^{5a/3}\left(\frac{1}{3}\left(\frac{1}{2}\int \left(-1+2e^{\frac{2bx}{3}}\right)de^{\frac{2bx}{3}}-\frac{3}{2}\int 1de^{\frac{2bx}{3}}\right)+e^{\frac{2bx}{3}}-\frac{1}{3}\log\left(e^{\frac{2bx}{3}}+1\right)\right)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{3e^{5a/3}\left(\frac{1}{3}\left(-\frac{3}{2}\int 1de^{\frac{2bx}{3}}-\frac{1}{2}\int \left(1-2e^{\frac{2bx}{3}}\right)de^{\frac{2bx}{3}}\right)+e^{\frac{2bx}{3}}-\frac{1}{3}\log\left(e^{\frac{2bx}{3}}+1\right)\right)}{b}$$

$$\Big\downarrow \text{1083}$$

$$\frac{3e^{5a/3}\left(\frac{1}{3}\left(3\int \frac{1}{-2-2e^{\frac{2bx}{3}}}d\left(-1+2e^{\frac{2bx}{3}}\right)-\frac{1}{2}\int \left(1-2e^{\frac{2bx}{3}}\right)de^{\frac{2bx}{3}}\right)+e^{\frac{2bx}{3}}-\frac{1}{3}\log\left(e^{\frac{2bx}{3}}+1\right)\right)}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{3e^{5a/3}\left(\frac{1}{3}\left(-\frac{1}{2}\int \left(1-2e^{\frac{2bx}{3}}\right)de^{\frac{2bx}{3}}-\sqrt{3}\arctan\left(\frac{2e^{\frac{2bx}{3}}-1}{\sqrt{3}}\right)\right)+e^{\frac{2bx}{3}}-\frac{1}{3}\log\left(e^{\frac{2bx}{3}}+1\right)\right)}{b}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3e^{5a/3}\left(-\frac{\arctan\left(\frac{2e^{\frac{2bx}{3}}-1}{\sqrt{3}}\right)}{\sqrt{3}}+e^{\frac{2bx}{3}}-\frac{1}{3}\log\left(e^{\frac{2bx}{3}}+1\right)\right)}{b}$$

input
```
Int[E^((5*(a + b*x))/3)*Sech[d + b*x],x]
```

output
```
(3*E^((5*a)/3)*(E^((2*b*x)/3) - ArcTan[(-1 + 2*E^((2*b*x)/3))/Sqrt[3]]/Sqr
t[3] - Log[1 + E^((2*b*x)/3)]/3))/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.22

method	result
risch	$\frac{3\,{\mathrm e}^{\frac{5 a}{3}-d+\frac{2 b x}{3}}}{b}+\frac{\ln\left({\mathrm e}^{\frac{2 b x}{3}+\frac{2 d}{3}}-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right){\mathrm e}^{\frac{5 a}{3}-\frac{5 d}{3}}}{2 b}+\frac{i\ln\left({\mathrm e}^{\frac{2 b x}{3}+\frac{2 d}{3}}-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right){\mathrm e}^{\frac{5 a}{3}-\frac{5 d}{3}}\sqrt{3}}{2 b}+\frac{\ln\left({\mathrm e}^{\frac{2 b x}{3}+\frac{2 d}{3}}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right){\mathrm e}^{\frac{5 a}{3}-\frac{5 d}{3}}}{2 b}$

input
```
int(exp(5/3*b*x+5/3*a)*sech(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
3*exp(5/3*a-d+2/3*b*x)/b+1/2*ln(exp(2/3*b*x+2/3*d)-1/2-1/2*I*3^(1/2))/b*ex
p(5/3*a-5/3*d)+1/2*I*ln(exp(2/3*b*x+2/3*d)-1/2-1/2*I*3^(1/2))/b*exp(5/3*a-
5/3*d)*3^(1/2)+1/2*ln(exp(2/3*b*x+2/3*d)-1/2+1/2*I*3^(1/2))/b*exp(5/3*a-5/
3*d)-1/2*I*ln(exp(2/3*b*x+2/3*d)-1/2+1/2*I*3^(1/2))/b*exp(5/3*a-5/3*d)*3^(
1/2)-exp(5/3*a-5/3*d)*ln(1+exp(2/3*b*x+2/3*d))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(113) = 226$.

Time = 0.09 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.51

$$\int e^{\frac{5}{3}(a+bx)}\text{sech}(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sech(b*x+d),x, algorithm="fricas")
```

output
```
1/2*(6*cosh(1/3*b*x + 1/3*d)^2*cosh(-5/3*a + 5/3*d) + 6*(cosh(-5/3*a + 5/3
*d) - sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^2 - 6*cosh(1/3*b*x + 1/3
*d)^2*sinh(-5/3*a + 5/3*d) + 2*(sqrt(3)*cosh(-5/3*a + 5/3*d) - sqrt(3)*sin
h(-5/3*a + 5/3*d))*arctan(-1/3*(sqrt(3)*cosh(1/3*b*x + 1/3*d) + 3*sqrt(3)*
sinh(1/3*b*x + 1/3*d))/(cosh(1/3*b*x + 1/3*d) - sinh(1/3*b*x + 1/3*d))) +
(cosh(-5/3*a + 5/3*d) - sinh(-5/3*a + 5/3*d))*log((2*cosh(1/3*b*x + 1/3*d)
^2 + 2*sinh(1/3*b*x + 1/3*d)^2 - 1)/(cosh(1/3*b*x + 1/3*d)^2 - 2*cosh(1/3*
b*x + 1/3*d)*sinh(1/3*b*x + 1/3*d) + sinh(1/3*b*x + 1/3*d)^2)) - 2*(cosh(-
5/3*a + 5/3*d) - sinh(-5/3*a + 5/3*d))*log(2*cosh(1/3*b*x + 1/3*d)/(cosh(1
/3*b*x + 1/3*d) - sinh(1/3*b*x + 1/3*d))) + 12*(cosh(1/3*b*x + 1/3*d)*cosh
(-5/3*a + 5/3*d) - cosh(1/3*b*x + 1/3*d)*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*
x + 1/3*d))/b
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+bx)}\text{sech}(d+bx)\,dx = e^{\frac{5a}{3}}\int e^{\frac{5bx}{3}}\,\text{sech}\,(bx+d)\,dx$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sech(b*x+d),x)
```

output
```
exp(5*a/3)*Integral(exp(5*b*x/3)*sech(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.28

$$\int e^{\frac{5}{3}(a+bx)}\text{sech}(d+bx)\,dx = -\frac{\sqrt{3}\arctan\left(\sqrt{3}+2\,e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}\right)e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,d\right)}}{b} + \frac{\sqrt{3}\arctan\left(-\sqrt{3}+2\,e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}\right)e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,d\right)}}{b} + \frac{e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,d\right)}\log\left(\sqrt{3}e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}+e^{\left(-\frac{2}{3}\,bx-\frac{2}{3}\,d\right)}+1\right)}{2\,b} + \frac{e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,d\right)}\log\left(-\sqrt{3}e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}+e^{\left(-\frac{2}{3}\,bx-\frac{2}{3}\,d\right)}+1\right)}{2\,b} - \frac{e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,d\right)}\log\left(e^{\left(-\frac{2}{3}\,bx-\frac{2}{3}\,d\right)}+1\right)}{b} + \frac{3\,e^{\left(\frac{2}{3}\,bx+\frac{5}{3}\,a-d\right)}}{b}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sech(b*x+d),x, algorithm="maxima")
```

output
```
-sqrt(3)*arctan(sqrt(3) + 2*e^(-1/3*b*x - 1/3*d))*e^(5/3*a - 5/3*d)/b + sq
rt(3)*arctan(-sqrt(3) + 2*e^(-1/3*b*x - 1/3*d))*e^(5/3*a - 5/3*d)/b + 1/2*
e^(5/3*a - 5/3*d)*log(sqrt(3)*e^(-1/3*b*x - 1/3*d) + e^(-2/3*b*x - 2/3*d)
+ 1)/b + 1/2*e^(5/3*a - 5/3*d)*log(-sqrt(3)*e^(-1/3*b*x - 1/3*d) + e^(-2/3
*b*x - 2/3*d) + 1)/b - e^(5/3*a - 5/3*d)*log(e^(-2/3*b*x - 2/3*d) + 1)/b +
 3*e^(2/3*b*x + 5/3*a - d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.70

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}(d+bx)\,dx = -\frac{\left(2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(\frac{2}{3}bx\right)}-e^{\left(-\frac{2}{3}d\right)}\right)e^{\left(\frac{2}{3}d\right)}\right)e^{\left(-\frac{8}{3}d\right)}-e^{\left(-\frac{8}{3}d\right)}\log\left(e^{\left(\frac{4}{3}bx\right)}-e^{\left(\frac{2}{3}bx-\frac{2}{3}d\right)}+e^{\left(-\frac{4}{3}d\right)}\right)+2\right.}{2b}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sech(b*x+d),x, algorithm="giac")
```

output
```
-1/2*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2/3*b*x) - e^(-2/3*d))*e^(2/3*d))
*e^(-8/3*d) - e^(-8/3*d)*log(e^(4/3*b*x) - e^(2/3*b*x - 2/3*d) + e^(-4/3*d
)) + 2*e^(-8/3*d)*log(e^(2/3*b*x) + e^(-2/3*d)) - 6*e^(2/3*b*x - 2*d))*e^(
5/3*a + d)/b
```

Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.42

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}(d+bx)\,dx$$

$$= \frac{3\,e^{\frac{5a}{3}-d+\frac{2bx}{3}}}{b} + \frac{\left(-e^{5a-5d}\right)^{1/3}\ln\left(2\,e^{\frac{5a}{3}}\,e^{\frac{2d}{3}}\,e^{-\frac{5d}{3}}\,e^{\frac{2bx}{3}}-2\left(-e^{5a}\,e^{-5d}\right)^{1/3}\right)}{b}$$
$$+\frac{\left(-e^{5a-5d}\right)^{1/3}\ln\left(2\,e^{\frac{5a}{3}}\,e^{\frac{2d}{3}}\,e^{-\frac{5d}{3}}\,e^{\frac{2bx}{3}}-2\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-e^{5a}\,e^{-5d}\right)^{1/3}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{b}$$
$$-\frac{\left(-e^{5a-5d}\right)^{1/3}\ln\left(2\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-e^{5a}\,e^{-5d}\right)^{1/3}+2\,e^{\frac{5a}{3}}\,e^{\frac{2d}{3}}\,e^{-\frac{5d}{3}}\,e^{\frac{2bx}{3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{b}$$

input
```
int(exp((5*a)/3 + (5*b*x)/3)/cosh(d + b*x),x)
```

output
```
(3*exp((5*a)/3 - d + (2*b*x)/3))/b + ((-exp(5*a - 5*d))^(1/3)*log(2*exp((5
*a)/3)*exp((2*d)/3)*exp(-(5*d)/3)*exp((2*b*x)/3) - 2*(-exp(5*a)*exp(-5*d))
^(1/3)))/b + ((-exp(5*a - 5*d))^(1/3)*log(2*exp((5*a)/3)*exp((2*d)/3)*exp(
-(5*d)/3)*exp((2*b*x)/3) - 2*((3^(1/2)*1i)/2 - 1/2)*(-exp(5*a)*exp(-5*d))^
(1/3))*((3^(1/2)*1i)/2 - 1/2))/b - ((-exp(5*a - 5*d))^(1/3)*log(2*((3^(1/2
)*1i)/2 + 1/2)*(-exp(5*a)*exp(-5*d))^(1/3) + 2*exp((5*a)/3)*exp((2*d)/3)*e
xp(-(5*d)/3)*exp((2*b*x)/3))*((3^(1/2)*1i)/2 + 1/2))/b
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}(d+bx)\,dx=\int e^{\frac{5bx}{3}+\frac{5a}{3}}\operatorname{sech}(bx+d)\,dx$$

input
```
int(exp(5/3*b*x+5/3*a)*sech(b*x+d),x)
```

output
```
int(e**((5*a + 5*b*x)/3)*sech(b*x + d),x)
```

3.87 $\int e^{\frac{5}{3}(a+bx)}\text{sech}^2(d+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 200

$$\int e^{\frac{5}{3}(a+bx)}\text{sech}^2(d+bx)\,dx = -\frac{2e^{\frac{5(a-d)}{3}+\frac{5}{3}(d+bx)}}{b\left(1+e^{2(d+bx)}\right)}+\frac{10e^{\frac{5(a-d)}{3}}\arctan\left(e^{\frac{1}{3}(d+bx)}\right)}{3b} - \frac{5e^{\frac{5(a-d)}{3}}\arctan\left(\sqrt{3}-2e^{\frac{1}{3}(d+bx)}\right)}{3b} + \frac{5e^{\frac{5(a-d)}{3}}\arctan\left(\sqrt{3}+2e^{\frac{1}{3}(d+bx)}\right)}{3b} - \frac{5e^{\frac{5(a-d)}{3}}\text{arctanh}\left(\frac{\sqrt{3}e^{\frac{1}{3}(d+bx)}}{1+e^{\frac{2}{3}(d+bx)}}\right)}{\sqrt{3}b}$$

output
```
-2*exp(5/3*b*x+5/3*a)/b/(1+exp(2*b*x+2*d))+10/3*exp(5/3*a-5/3*d)*arctan(ex
p(1/3*b*x+1/3*d))/b+5/3*exp(5/3*a-5/3*d)*arctan(-3^(1/2)+2*exp(1/3*b*x+1/3
*d))/b+5/3*exp(5/3*a-5/3*d)*arctan(3^(1/2)+2*exp(1/3*b*x+1/3*d))/b-5/3*3^(
1/2)*exp(5/3*a-5/3*d)*arctanh(3^(1/2)*exp(1/3*b*x+1/3*d)/(1+exp(2/3*b*x+2/
3*d)))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.22

$$\int e^{\frac{5}{3}(a+bx)}\text{sech}^2(d+bx)\,dx=\frac{12e^{\frac{5}{3}(a+bx)+2(d+bx)}\,\text{Hypergeometric2F1}\left(\frac{11}{6},2,\frac{17}{6},-e^{2(d+bx)}\right)}{11b}$$

input
```
Integrate[E^((5*(a + b*x))/3)*Sech[d + b*x]^2,x]
```

output
```
(12*E^((5*(a + b*x))/3 + 2*(d + b*x))*Hypergeometric2F1[11/6, 2, 17/6, -E^
(2*(d + b*x))])/(11*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.84, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.550$, Rules used = {2720, 27, 817, 824, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)}\text{sech}^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{3\int \frac{4e^{\frac{5a}{3}+\frac{10bx}{3}}}{\left(1+e^{2bx}\right)^2}de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{12e^{5a/3}\int \frac{e^{\frac{10bx}{3}}}{\left(1+e^{2bx}\right)^2}de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \text{817}$$

$$\frac{12e^{5a/3}\left(\frac{5}{6}\int \frac{e^{\frac{4bx}{3}}}{1+e^{2bx}}de^{\frac{bx}{3}}-\frac{e^{\frac{5bx}{3}}}{6\left(e^{2bx}+1\right)}\right)}{b}$$

$$\Big\downarrow 824$$

$$\frac{12e^{5a/3}\left(\frac{5}{6}\left(\frac{1}{3}\int \frac{1}{1+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{3}\int -\frac{1-\sqrt{3}e^{\frac{bx}{3}}}{2\left(1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}\right)}de^{\frac{bx}{3}}+\frac{1}{3}\int -\frac{1+\sqrt{3}e^{\frac{bx}{3}}}{2\left(1+\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}\right)}de^{\frac{bx}{3}}\right)-\frac{e^{\frac{5bx}{3}}}{6(e^{2bx}+1)}\right)}{b}$$

$$\Big\downarrow 27$$

$$\frac{12e^{5a/3}\left(\frac{5}{6}\left(\frac{1}{3}\int \frac{1}{1+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1-\sqrt{3}e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1+\sqrt{3}e^{\frac{bx}{3}}}{1+\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)-\frac{e^{\frac{5bx}{3}}}{6(e^{2bx}+1)}\right)}{b}$$

$$\Big\downarrow 216$$

$$\frac{12e^{5a/3}\left(\frac{5}{6}\left(-\frac{1}{6}\int \frac{1-\sqrt{3}e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1+\sqrt{3}e^{\frac{bx}{3}}}{1+\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{3}\arctan\left(e^{\frac{bx}{3}}\right)\right)-\frac{e^{\frac{5bx}{3}}}{6(e^{2bx}+1)}\right)}{b}$$

$$\Big\downarrow 1142$$

$$\frac{12e^{5a/3}\left(\frac{5}{6}\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}-2e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{1+\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{2}\sqrt{3}\int \cdots\right.\right.\right.}{b}$$

$$\Big\downarrow 25$$

$$\frac{12e^{5a/3}\left(\frac{5}{6}\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-2e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{1+\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{2}\sqrt{3}\int \cdots\right.\right.\right.}{b}$$

$$\Big\downarrow 1083$$

$$\frac{12e^{5a/3}\left(\frac{5}{6}\left(\frac{1}{6}\left(-\int \frac{1}{-1-e^{\frac{2bx}{3}}}d\left(-\sqrt{3}+2e^{\frac{bx}{3}}\right)-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-2e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(-\int \frac{1}{-1-e^{\frac{2bx}{3}}}d\left(\sqrt{3}+2e^{\frac{bx}{3}}\right)-\frac{1}{2}\cdots\right.\right.\right.}{b}$$

$$\Big\downarrow 217$$

$$\frac{12e^{5a/3}\left(\frac{5}{6}\left(\frac{1}{6}\left(-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-2e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\arctan\left(\sqrt{3}-2e^{\frac{bx}{3}}\right)\right)+\frac{1}{6}\left(\arctan\left(2e^{\frac{bx}{3}}+\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\int \cdots\right.\right.\right.}{b}$$

$$\Big\downarrow 1103$$

$$\frac{12e^{5a/3}\left(\frac{5}{6}\left(\frac{1}{3}\arctan\left(e^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\log\left(-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}+1\right)-\arctan\left(\sqrt{3}-2e^{\frac{bx}{3}}\right)\right)+\frac{1}{6}\left(\arctan\left(2e^{\frac{bx}{3}}+\sqrt{3}\right.\right.\right.}{b}$$

input
```
Int[E^((5*(a + b*x))/3)*Sech[d + b*x]^2,x]
```

output
```
(12*E^((5*a)/3)*(-1/6*E^((5*b*x)/3)/(1 + E^(2*b*x)) + (5*(ArcTan[E^((b*x)/
3)]/3 + (-ArcTan[Sqrt[3] - 2*E^((b*x)/3)] + (Sqrt[3]*Log[1 - Sqrt[3]*E^((b
*x)/3) + E^((2*b*x)/3)])/2)/6 + (ArcTan[Sqrt[3] + 2*E^((b*x)/3)] - (Sqrt[3
]*Log[1 + Sqrt[3]*E^((b*x)/3) + E^((2*b*x)/3)])/2)/6))/6))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.54 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{2\,\mathrm{e}^{\frac{5bx}{3}+\frac{5a}{3}}}{b\left(1+\mathrm{e}^{2bx+2d}\right)}+\frac{5i\ln\left(\mathrm{e}^{\frac{bx}{3}+\frac{d}{3}}+i\right)\mathrm{e}^{\frac{5a}{3}-\frac{5d}{3}}}{3b}-\frac{5i\ln\left(\mathrm{e}^{\frac{bx}{3}+\frac{d}{3}}-i\right)\mathrm{e}^{\frac{5a}{3}-\frac{5d}{3}}}{3b}+4\left(\sum_{\_R=\mathrm{RootOf}\left(20736b^4\_Z^4-3600b^2\_Z^2+62\right.}\right.$

input
```
int(exp(5/3*b*x+5/3*a)*sech(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*exp(5/3*b*x+5/3*a)/b/(1+exp(2*b*x+2*d))+5/3*I*ln(exp(1/3*b*x+1/3*d)+I)/
b*exp(5/3*a-5/3*d)-5/3*I*ln(exp(1/3*b*x+1/3*d)-I)/b*exp(5/3*a-5/3*d)+4*sum
(_R*ln(exp(1/3*b*x+1/3*d)+1728/125*b^3*_R^3-12/5*b*_R),_R=RootOf(20736*_Z^
4*b^4-3600*_Z^2*b^2+625))*exp(5/3*a-5/3*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2654 vs. $2(154) = 308$.

Time = 0.12 (sec) , antiderivative size = 2654, normalized size of antiderivative = 13.27

$$\int e^{\frac{5}{3}(a+bx)}\mathrm{sech}^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sech(b*x+d)^2,x, algorithm="fricas")
```

output
```
-1/6*(12*cosh(1/3*b*x + 1/3*d)^5*cosh(-5/3*a + 5/3*d) + 12*(cosh(-5/3*a +
5/3*d) - sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^5 - 12*cosh(1/3*b*x +
 1/3*d)^5*sinh(-5/3*a + 5/3*d) + 60*(cosh(1/3*b*x + 1/3*d)*cosh(-5/3*a + 5
/3*d) - cosh(1/3*b*x + 1/3*d)*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^
4 + 120*(cosh(1/3*b*x + 1/3*d)^2*cosh(-5/3*a + 5/3*d) - cosh(1/3*b*x + 1/3
*d)^2*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^3 + 15*sqrt(1/3)*(b*cosh
(1/3*b*x + 1/3*d)^6 + 6*b*cosh(1/3*b*x + 1/3*d)^5*sinh(1/3*b*x + 1/3*d) +
15*b*cosh(1/3*b*x + 1/3*d)^4*sinh(1/3*b*x + 1/3*d)^2 + 20*b*cosh(1/3*b*x +
 1/3*d)^3*sinh(1/3*b*x + 1/3*d)^3 + 15*b*cosh(1/3*b*x + 1/3*d)^2*sinh(1/3*
b*x + 1/3*d)^4 + 6*b*cosh(1/3*b*x + 1/3*d)*sinh(1/3*b*x + 1/3*d)^5 + b*sin
h(1/3*b*x + 1/3*d)^6 + b)*sqrt((cosh(-5/3*a + 5/3*d) - sinh(-5/3*a + 5/3*d
))/(b^2*cosh(-5/3*a + 5/3*d) + b^2*sinh(-5/3*a + 5/3*d)))*log(5*cosh(1/3*b
*x + 1/3*d)^2*cosh(-5/3*a + 5/3*d) + 5*(cosh(-5/3*a + 5/3*d) - sinh(-5/3*a
 + 5/3*d))*sinh(1/3*b*x + 1/3*d)^2 + 15*sqrt(1/3)*(b*cosh(1/3*b*x + 1/3*d)
 + b*sinh(1/3*b*x + 1/3*d))*sqrt((cosh(-5/3*a + 5/3*d) - sinh(-5/3*a + 5/3
*d))/(b^2*cosh(-5/3*a + 5/3*d) + b^2*sinh(-5/3*a + 5/3*d))) + 10*(cosh(1/3
*b*x + 1/3*d)*cosh(-5/3*a + 5/3*d) - cosh(1/3*b*x + 1/3*d)*sinh(-5/3*a + 5
/3*d))*sinh(1/3*b*x + 1/3*d) - 5*(cosh(1/3*b*x + 1/3*d)^2 + 1)*sinh(-5/3*a
 + 5/3*d) + 5*cosh(-5/3*a + 5/3*d)) - 15*sqrt(1/3)*(b*cosh(1/3*b*x + 1/3*d
)^6 + 6*b*cosh(1/3*b*x + 1/3*d)^5*sinh(1/3*b*x + 1/3*d) + 15*b*cosh(1/3...
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+bx)}\text{sech}^2(d+bx)\,dx = e^{\frac{5a}{3}}\int e^{\frac{5bx}{3}}\operatorname{sech}^2(bx+d)\,dx$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sech(b*x+d)**2,x)
```

output
```
exp(5*a/3)*Integral(exp(5*b*x/3)*sech(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.78

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}^2(d+bx)\,dx = -\frac{5\left(\sqrt{3}\log\left(\sqrt{3}e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}+e^{\left(-\frac{2}{3}\,bx-\frac{2}{3}\,d\right)}+1\right)-\sqrt{3}\log\left(-\sqrt{3}e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}+e^{\left(-\frac{2}{3}\,bx-\frac{2}{3}\,d\right)}+1\right)+2\,\mathrm{arct}\right.}{6\,b} - \frac{2\,e^{\left(-\frac{1}{3}\,bx+\frac{5}{3}\,a-2\,d\right)}}{b\left(e^{\left(-2\,bx-2\,d\right)}+1\right)}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sech(b*x+d)^2,x, algorithm="maxima")
```

output
```
-5/6*(sqrt(3)*log(sqrt(3)*e^(-1/3*b*x - 1/3*d) + e^(-2/3*b*x - 2/3*d) + 1)
 - sqrt(3)*log(-sqrt(3)*e^(-1/3*b*x - 1/3*d) + e^(-2/3*b*x - 2/3*d) + 1) +
 2*arctan(sqrt(3) + 2*e^(-1/3*b*x - 1/3*d)) + 2*arctan(-sqrt(3) + 2*e^(-1/
3*b*x - 1/3*d)) + 4*arctan(e^(-1/3*b*x - 1/3*d)))*e^(5/3*a - 5/3*d)/b - 2*
e^(-1/3*b*x + 5/3*a - 2*d)/(b*(e^(-2*b*x - 2*d) + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.89

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}^2(d+bx)\,dx = -\frac{\left(5\sqrt{3}e^{\left(-\frac{11}{3}\,d\right)}\log\left(\sqrt{3}e^{\left(\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}+e^{\left(\frac{2}{3}\,bx\right)}+e^{\left(-\frac{2}{3}\,d\right)}\right)-5\sqrt{3}e^{\left(-\frac{11}{3}\,d\right)}\log\left(-\sqrt{3}e^{\left(\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}+e^{\left(\frac{2}{3}\,bx\right)}+e^{(-}\right.\right.}{}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sech(b*x+d)^2,x, algorithm="giac")
```

output
```
-1/6*(5*sqrt(3)*e^(-11/3*d)*log(sqrt(3)*e^(1/3*b*x - 1/3*d) + e^(2/3*b*x)
+ e^(-2/3*d)) - 5*sqrt(3)*e^(-11/3*d)*log(-sqrt(3)*e^(1/3*b*x - 1/3*d) + e
^(2/3*b*x) + e^(-2/3*d)) - 10*arctan((sqrt(3)*e^(-1/3*d) + 2*e^(1/3*b*x))*
e^(1/3*d))*e^(-11/3*d) - 10*arctan(-(sqrt(3)*e^(-1/3*d) - 2*e^(1/3*b*x))*e
^(1/3*d))*e^(-11/3*d) - 20*arctan(e^(1/3*b*x + 1/3*d))*e^(-11/3*d) + 12*e^
(5/3*b*x - 2*d)/(e^(2*b*x + 2*d) + 1))*e^(5/3*a + 2*d)/b
```

Mupad [B] (verification not implemented)

Time = 5.84 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.26

$$\int e^{\frac{5}{3}(a+bx)}\text{sech}^2(d+bx)\,dx = \text{Too large to display}$$

input
```
int(exp((5*a)/3 + (5*b*x)/3)/cosh(d + b*x)^2,x)
```

output
```
(5*(-exp(10*a - 10*d))^(1/6)*log((100*exp((10*a)/3)*exp(-(10*d)/3))/9 - (1
00*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*(-exp(10*a)*exp(-10*d)
)^(1/6))/9))/(3*b) - (5*(-exp(10*a - 10*d))^(1/6)*log((100*exp((10*a)/3)*e
xp(-(10*d)/3))/9 + (100*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*(
-exp(10*a)*exp(-10*d))^(1/6))/9))/(3*b) - (2*exp((5*a)/3 + (5*b*x)/3))/(b*
(exp(2*d + 2*b*x) + 1)) + (5*log((100*exp((10*a)/3)*exp(-(10*d)/3))/9 - (1
00*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)/2 - 1/2)
*(-exp(10*a)*exp(-10*d))^(1/6))/9)*(-exp(10*a - 10*d))^(1/6)*((3^(1/2)*1i)
/2 - 1/2))/(3*b) - (5*log((100*exp((10*a)/3)*exp(-(10*d)/3))/9 + (100*exp(
(5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)/2 - 1/2)*(-exp(
10*a)*exp(-10*d))^(1/6))/9)*(-exp(10*a - 10*d))^(1/6)*((3^(1/2)*1i)/2 - 1/
2))/(3*b) + (5*log((100*exp((10*a)/3)*exp(-(10*d)/3))/9 - (100*exp((5*a)/3
)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)/2 + 1/2)*(-exp(10*a)*e
xp(-10*d))^(1/6))/9)*(-exp(10*a - 10*d))^(1/6)*((3^(1/2)*1i)/2 + 1/2))/(3*
b) - (5*log((100*exp((10*a)/3)*exp(-(10*d)/3))/9 + (100*exp((5*a)/3)*exp(d
/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)/2 + 1/2)*(-exp(10*a)*exp(-10*
d))^(1/6))/9)*(-exp(10*a - 10*d))^(1/6)*((3^(1/2)*1i)/2 + 1/2))/(3*b)
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+bx)}\text{sech}^2(d+bx)\,dx = \int e^{\frac{5bx}{3}+\frac{5a}{3}}\text{sech}(bx+d)^2\,dx$$

input
```
int(exp(5/3*b*x+5/3*a)*sech(b*x+d)^2,x)
```

output
```
int(e**((5*a + 5*b*x)/3)*sech(b*x + d)**2,x)
```

3.88 $\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}^3(d+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 203

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}^3(d+bx)\,dx=-\frac{2e^{\frac{5(a-d)}{3}+\frac{8}{3}(d+bx)}}{b\left(1+e^{2(d+bx)}\right)^2}-\frac{8e^{\frac{5(a-d)}{3}+\frac{2}{3}(d+bx)}}{3b\left(1+e^{2(d+bx)}\right)}-\frac{8e^{\frac{5(a-d)}{3}}\arctan\left(\frac{1-2e^{\frac{2}{3}(d+bx)}}{\sqrt{3}}\right)}{3\sqrt{3}b}+\frac{8e^{\frac{5(a-d)}{3}}\log\left(1+e^{\frac{2}{3}(d+bx)}\right)}{9b}-\frac{4e^{\frac{5(a-d)}{3}}\log\left(1-e^{\frac{2}{3}(d+bx)}+e^{\frac{4}{3}(d+bx)}\right)}{9b}$$

output

```
-2*exp(5/3*a+d+8/3*b*x)/b/(1+exp(2*b*x+2*d))^2-8/3*exp(5/3*a-d+2/3*b*x)/b/
(1+exp(2*b*x+2*d))-8/9*3^(1/2)*exp(5/3*a-5/3*d)*arctan(1/3*(1-2*exp(2/3*b*
x+2/3*d))*3^(1/2))/b+8/9*exp(5/3*a-5/3*d)*ln(1+exp(2/3*b*x+2/3*d))/b-4/9*e
xp(5/3*a-5/3*d)*ln(1-exp(2/3*b*x+2/3*d)+exp(4/3*b*x+4/3*d))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.32

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}^3(d+bx)\,dx = \frac{e^{\frac{5}{3}(a+bx)}\left(-4e^{d+bx}\,\text{Hypergeometric2F1}\left(1,\frac{4}{3},\frac{7}{3},-e^{2(d+bx)}\right)+\operatorname{sech}(d+bx)(5+3\tanh(d+bx))\right)}{6b}$$

input
```
Integrate[E^((5*(a + b*x))/3)*Sech[d + b*x]^3,x]
```

output
```
(E^((5*(a + b*x))/3)*(-4*E^(d + b*x)*Hypergeometric2F1[1, 4/3, 7/3, -E^(2*
(d + b*x))] + Sech[d + b*x]*(5 + 3*Tanh[d + b*x])))/(6*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.52, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2720, 27, 807, 817, 817, 750, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}^3(bx+d)\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{3\int \frac{8e^{\frac{5a}{3}+\frac{13bx}{3}}}{\left(1+e^{2bx}\right)^3}de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{24e^{5a/3}\int \frac{e^{\frac{13bx}{3}}}{\left(1+e^{2bx}\right)^3}de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \; 807$$

$$\frac{12e^{5a/3}\int \frac{e^{2bx}}{(1+e^{bx})^3}de^{\frac{2bx}{3}}}{b}$$

$\downarrow$ 817

$$\frac{12e^{5a/3}\left(\frac{2}{3}\int \frac{e^{bx}}{(1+e^{bx})^2}de^{\frac{2bx}{3}}-\frac{e^{\frac{4bx}{3}}}{6(e^{bx}+1)^2}\right)}{b}$$

$\downarrow$ 817

$$\frac{12e^{5a/3}\left(\frac{2}{3}\left(\frac{1}{3}\int \frac{1}{1+e^{bx}}de^{\frac{2bx}{3}}-\frac{e^{\frac{2bx}{3}}}{3(e^{bx}+1)}\right)-\frac{e^{\frac{4bx}{3}}}{6(e^{bx}+1)^2}\right)}{b}$$

$\downarrow$ 750

$$\frac{12e^{5a/3}\left(\frac{2}{3}\left(\frac{1}{3}\left(\frac{1}{3}\int \left(2-e^{\frac{2bx}{3}}\right)de^{\frac{2bx}{3}}+\frac{1}{3}\int \frac{1}{1+e^{\frac{2bx}{3}}}de^{\frac{2bx}{3}}\right)-\frac{e^{\frac{2bx}{3}}}{3(e^{bx}+1)}\right)-\frac{e^{\frac{4bx}{3}}}{6(e^{bx}+1)^2}\right)}{b}$$

$\downarrow$ 16

$$\frac{12e^{5a/3}\left(\frac{2}{3}\left(\frac{1}{3}\left(\frac{1}{3}\int \left(2-e^{\frac{2bx}{3}}\right)de^{\frac{2bx}{3}}+\frac{1}{3}\log\left(e^{\frac{2bx}{3}}+1\right)\right)-\frac{e^{\frac{2bx}{3}}}{3(e^{bx}+1)}\right)-\frac{e^{\frac{4bx}{3}}}{6(e^{bx}+1)^2}\right)}{b}$$

$\downarrow$ 1142

$$\frac{12e^{5a/3}\left(\frac{2}{3}\left(\frac{1}{3}\left(\frac{1}{3}\left(\frac{3}{2}\int 1de^{\frac{2bx}{3}}-\frac{1}{2}\int \left(-1+2e^{\frac{2bx}{3}}\right)de^{\frac{2bx}{3}}\right)+\frac{1}{3}\log\left(e^{\frac{2bx}{3}}+1\right)\right)-\frac{e^{\frac{2bx}{3}}}{3(e^{bx}+1)}\right)-\frac{e^{\frac{4bx}{3}}}{6(e^{bx}+1)^2}\right)}{b}$$

$\downarrow$ 25

$$\frac{12e^{5a/3}\left(\frac{2}{3}\left(\frac{1}{3}\left(\frac{1}{3}\left(\frac{3}{2}\int 1de^{\frac{2bx}{3}}+\frac{1}{2}\int \left(1-2e^{\frac{2bx}{3}}\right)de^{\frac{2bx}{3}}\right)+\frac{1}{3}\log\left(e^{\frac{2bx}{3}}+1\right)\right)-\frac{e^{\frac{2bx}{3}}}{3(e^{bx}+1)}\right)-\frac{e^{\frac{4bx}{3}}}{6(e^{bx}+1)^2}\right)}{b}$$

$\downarrow$ 1083

$$\frac{12e^{5a/3}\left(\frac{2}{3}\left(\frac{1}{3}\left(\frac{1}{3}\left(\frac{1}{2}\int \left(1-2e^{\frac{2bx}{3}}\right)de^{\frac{2bx}{3}}-3\int \frac{1}{-2-2e^{\frac{2bx}{3}}}d\left(-1+2e^{\frac{2bx}{3}}\right)\right)+\frac{1}{3}\log\left(e^{\frac{2bx}{3}}+1\right)\right)-\frac{e^{\frac{2bx}{3}}}{3(e^{bx}+1)}\right)-\frac{e^{\frac{4bx}{3}}}{6(e^{bx}+1)^2}\right)}{b}$$

$\downarrow$ 217

$$\frac{12e^{5a/3}\left(\frac{2}{3}\left(\frac{1}{3}\left(\frac{1}{3}\left(\frac{1}{2}\int\left(1-2e^{\frac{2bx}{3}}\right)de^{\frac{2bx}{3}}+\sqrt{3}\arctan\left(\frac{2e^{\frac{2bx}{3}}-1}{\sqrt{3}}\right)\right)+\frac{1}{3}\log\left(e^{\frac{2bx}{3}}+1\right)\right)-\frac{e^{\frac{2bx}{3}}}{3(e^{bx}+1)}\right)-\frac{e^{\frac{4bx}{3}}}{6(e^{bx}+1)^2}\right)}{b}$$

$$\Big\downarrow \ 1103$$

$$\frac{12e^{5a/3}\left(\frac{2}{3}\left(\frac{1}{3}\left(\frac{\arctan\left(\frac{2e^{\frac{2bx}{3}}-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log\left(e^{\frac{2bx}{3}}+1\right)\right)-\frac{e^{\frac{2bx}{3}}}{3(e^{bx}+1)}\right)-\frac{e^{\frac{4bx}{3}}}{6(e^{bx}+1)^2}\right)}{b}$$

input
```
Int[E^((5*(a + b*x))/3)*Sech[d + b*x]^3,x]
```

output
```
(12*E^((5*a)/3)*(-1/6*E^((4*b*x)/3)/(1 + E^(b*x))^2 + (2*(-1/3*E^((2*b*x)/
3)/(1 + E^(b*x)) + (ArcTan[(-1 + 2*E^((2*b*x)/3))/Sqrt[3]]/Sqrt[3] + Log[1
 + E^((2*b*x)/3)]/3)/3))/3))/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{2(7\,e^{2bx+2d}+4)e^{\frac{5a}{3}-d+\frac{2bx}{3}}}{3(1+e^{2bx+2d})^2 b}-\frac{4\ln\left(e^{\frac{2bx}{3}+\frac{2d}{3}}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)e^{\frac{5a}{3}-\frac{5d}{3}}}{9b}+\frac{4i\ln\left(e^{\frac{2bx}{3}+\frac{2d}{3}}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)e^{\frac{5a}{3}-\frac{5d}{3}}\sqrt{3}}{9b}-\frac{4\ln\left(e^{\frac{2bx}{3}+\frac{2d}{3}}\right.}{}$

input
```
int(exp(5/3*b*x+5/3*a)*sech(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3/(1+exp(2*b*x+2*d))^2/b*(7*exp(2*b*x+2*d)+4)*exp(5/3*a-d+2/3*b*x)-4/9*
ln(exp(2/3*b*x+2/3*d)-1/2+1/2*I*3^(1/2))/b*exp(5/3*a-5/3*d)+4/9*I*ln(exp(2
/3*b*x+2/3*d)-1/2+1/2*I*3^(1/2))/b*exp(5/3*a-5/3*d)*3^(1/2)-4/9*ln(exp(2/3
*b*x+2/3*d)-1/2-1/2*I*3^(1/2))/b*exp(5/3*a-5/3*d)-4/9*I*ln(exp(2/3*b*x+2/3
*d)-1/2-1/2*I*3^(1/2))/b*exp(5/3*a-5/3*d)*3^(1/2)+8/9*exp(5/3*a-5/3*d)*ln(
1+exp(2/3*b*x+2/3*d))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3955 vs. 2(155) = 310.

Time = 0.12 (sec) , antiderivative size = 3955, normalized size of antiderivative = 19.48

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sech(b*x+d)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}^3(d+bx)\,dx = e^{\frac{5a}{3}}\int e^{\frac{5bx}{3}}\operatorname{sech}^3(bx+d)\,dx$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sech(b*x+d)**3,x)
```

output
```
exp(5*a/3)*Integral(exp(5*b*x/3)*sech(b*x + d)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.93

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}^3(d+bx)\,dx$$
$$= \frac{4\left(2\sqrt{3}\arctan\left(\sqrt{3}+2e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}\right)-2\sqrt{3}\arctan\left(-\sqrt{3}+2e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}\right)-\log\left(\sqrt{3}e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{(-\right.}\right.}{9b}$$
$$-\frac{2\left(7e^{\left(-\frac{4}{3}bx-\frac{4}{3}d\right)}+4e^{\left(-\frac{10}{3}bx-\frac{10}{3}d\right)}\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}}{3b\left(2e^{(-2bx-2d)}+e^{(-4bx-4d)}+1\right)}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sech(b*x+d)^3,x, algorithm="maxima")
```

output
```
4/9*(2*sqrt(3)*arctan(sqrt(3) + 2*e^(-1/3*b*x - 1/3*d)) - 2*sqrt(3)*arctan
(-sqrt(3) + 2*e^(-1/3*b*x - 1/3*d)) - log(sqrt(3)*e^(-1/3*b*x - 1/3*d) + e
^(-2/3*b*x - 2/3*d) + 1) - log(-sqrt(3)*e^(-1/3*b*x - 1/3*d) + e^(-2/3*b*x
 - 2/3*d) + 1) + 2*log(e^(-2/3*b*x - 2/3*d) + 1))*e^(5/3*a - 5/3*d)/b - 2/
3*(7*e^(-4/3*b*x - 4/3*d) + 4*e^(-10/3*b*x - 10/3*d))*e^(5/3*a - 5/3*d)/(b
*(2*e^(-2*b*x - 2*d) + e^(-4*b*x - 4*d) + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.64

$$\int e^{\frac{5}{3}(a+bx)}\text{sech}^3(d+bx)\,dx$$
$$= \frac{2\left(4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(\frac{2}{3}bx\right)}-e^{\left(-\frac{2}{3}d\right)}\right)e^{\left(\frac{2}{3}d\right)}\right)e^{\left(-\frac{14}{3}d\right)}-2e^{\left(-\frac{14}{3}d\right)}\log\left(e^{\left(\frac{4}{3}bx\right)}-e^{\left(\frac{2}{3}bx-\frac{2}{3}d\right)}+e^{\left(-\frac{4}{3}d\right)}\right)\cdots}{9b}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sech(b*x+d)^3,x, algorithm="giac")
```

output
```
2/9*(4*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2/3*b*x) - e^(-2/3*d))*e^(2/3*d))*
e^(-14/3*d) - 2*e^(-14/3*d)*log(e^(4/3*b*x) - e^(2/3*b*x - 2/3*d) + e^(-4/
3*d)) + 4*e^(-14/3*d)*log(e^(2/3*b*x) + e^(-2/3*d)) - 3*(7*e^(8/3*b*x + 2*
d) + 4*e^(2/3*b*x))*e^(-4*d)/(e^(2*b*x + 2*d) + 1)^2)*e^(5/3*a + 3*d)/b
```

Mupad [B] (verification not implemented)

Time = 4.93 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.23

$$\int e^{\frac{5}{3}(a+bx)}\text{sech}^3(d+bx)\,dx$$
$$= \frac{8\left(e^{5a-5d}\right)^{1/3}\ln\left(-\frac{16\left(e^{5a}e^{-5d}\right)^{1/3}}{9}-\frac{16e^{\frac{5a}{3}}e^{\frac{2d}{3}}e^{-\frac{5d}{3}}e^{\frac{2bx}{3}}}{9}\right)}{9b} - \frac{8e^{\frac{5a}{3}-d+\frac{2bx}{3}}}{3b\left(e^{2d+2bx}+1\right)} - \frac{2e^{\frac{5a}{3}+d+\frac{8bx}{3}}}{b\left(2e^{2d+2bx}+e^{4d+4bx}+1\right)}$$
$$+ \frac{8\left(e^{5a-5d}\right)^{1/3}\ln\left(-\frac{16\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(e^{5a}e^{-5d}\right)^{1/3}}{9}-\frac{16e^{\frac{5a}{3}}e^{\frac{2d}{3}}e^{-\frac{5d}{3}}e^{\frac{2bx}{3}}}{9}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{9b}$$
$$- \frac{8\left(e^{5a-5d}\right)^{1/3}\ln\left(\frac{16\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(e^{5a}e^{-5d}\right)^{1/3}}{9}-\frac{16e^{\frac{5a}{3}}e^{\frac{2d}{3}}e^{-\frac{5d}{3}}e^{\frac{2bx}{3}}}{9}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{9b}$$

input
```
int(exp((5*a)/3 + (5*b*x)/3)/cosh(d + b*x)^3,x)
```

output
```
(8*exp(5*a - 5*d)^(1/3)*log(- (16*(exp(5*a)*exp(-5*d))^(1/3))/9 - (16*exp(
(5*a)/3)*exp((2*d)/3)*exp(-(5*d)/3)*exp((2*b*x)/3))/9))/(9*b) - (8*exp((5*
a)/3 - d + (2*b*x)/3))/(3*b*(exp(2*d + 2*b*x) + 1)) - (2*exp((5*a)/3 + d +
 (8*b*x)/3))/(b*(2*exp(2*d + 2*b*x) + exp(4*d + 4*b*x) + 1)) + (8*exp(5*a
- 5*d)^(1/3)*log(- (16*((3^(1/2)*1i)/2 - 1/2)*(exp(5*a)*exp(-5*d))^(1/3))/
9 - (16*exp((5*a)/3)*exp((2*d)/3)*exp(-(5*d)/3)*exp((2*b*x)/3))/9)*((3^(1/
2)*1i)/2 - 1/2))/(9*b) - (8*exp(5*a - 5*d)^(1/3)*log((16*((3^(1/2)*1i)/2 +
 1/2)*(exp(5*a)*exp(-5*d))^(1/3))/9 - (16*exp((5*a)/3)*exp((2*d)/3)*exp(-(
5*d)/3)*exp((2*b*x)/3))/9)*((3^(1/2)*1i)/2 + 1/2))/(9*b)
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}^3(d+bx)\,dx = \int e^{\frac{5bx}{3}+\frac{5a}{3}}\operatorname{sech}(bx+d)^3\,dx$$

input
```
int(exp(5/3*b*x+5/3*a)*sech(b*x+d)^3,x)
```

output
```
int(e**((5*a + 5*b*x)/3)*sech(b*x + d)**3,x)
```

3.89 $\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}^4(d+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 286

$$\begin{aligned}\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}^4(d+bx)\,dx &= -\frac{8e^{\frac{5(a-d)}{3}+\frac{11}{3}(d+bx)}}{3b\left(1+e^{2(d+bx)}\right)^3}-\frac{22e^{\frac{5(a-d)}{3}+\frac{5}{3}(d+bx)}}{9b\left(1+e^{2(d+bx)}\right)^2}\\&+\frac{55e^{\frac{5(a-d)}{3}+\frac{5}{3}(d+bx)}}{27b\left(1+e^{2(d+bx)}\right)}+\frac{55e^{\frac{5(a-d)}{3}}\arctan\left(e^{\frac{1}{3}(d+bx)}\right)}{81b}\\&-\frac{55e^{\frac{5(a-d)}{3}}\arctan\left(\sqrt{3}-2e^{\frac{1}{3}(d+bx)}\right)}{162b}\\&+\frac{55e^{\frac{5(a-d)}{3}}\arctan\left(\sqrt{3}+2e^{\frac{1}{3}(d+bx)}\right)}{162b}\\&-\frac{55e^{\frac{5(a-d)}{3}}\operatorname{arctanh}\left(\frac{\sqrt{3}e^{\frac{1}{3}(d+bx)}}{1+e^{\frac{2}{3}(d+bx)}}\right)}{54\sqrt{3}b}\end{aligned}$$

output

```
-8/3*exp(5/3*a+2*d+11/3*b*x)/b/(1+exp(2*b*x+2*d))^3-22/9*exp(5/3*b*x+5/3*a
)/b/(1+exp(2*b*x+2*d))^2+55/27*exp(5/3*b*x+5/3*a)/b/(1+exp(2*b*x+2*d))+55/
81*exp(5/3*a-5/3*d)*arctan(exp(1/3*b*x+1/3*d))/b+55/162*exp(5/3*a-5/3*d)*a
rctan(-3^(1/2)+2*exp(1/3*b*x+1/3*d))/b+55/162*exp(5/3*a-5/3*d)*arctan(3^(1
/2)+2*exp(1/3*b*x+1/3*d))/b-55/162*3^(1/2)*exp(5/3*a-5/3*d)*arctanh(3^(1/2
)*exp(1/3*b*x+1/3*d)/(1+exp(2/3*b*x+2/3*d)))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.24

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}^4(d+bx)\,dx$$
$$=\frac{e^{\frac{5}{3}(a+bx)}\left(4e^{2(d+bx)}\,\text{Hypergeometric2F1}\left(\frac{11}{6},2,\frac{17}{6},-e^{2(d+bx)}\right)+\operatorname{sech}^2(d+bx)(5+6\tanh(d+bx))\right)}{18b}$$

input
```
Integrate[E^((5*(a + b*x))/3)*Sech[d + b*x]^4,x]
```

output
```
(E^((5*(a + b*x))/3)*(4*E^(2*(d + b*x))*Hypergeometric2F1[11/6, 2, 17/6, -
E^(2*(d + b*x))] + Sech[d + b*x]^2*(5 + 6*Tanh[d + b*x])))/(18*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.78, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.650, Rules used = {2720, 27, 817, 817, 819, 824, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}^4(bx+d)\,dx$$
$$\Big\downarrow \text{2720}$$
$$\frac{3\int \frac{16e^{\frac{5a}{3}+\frac{16bx}{3}}}{\left(1+e^{2bx}\right)^4}de^{\frac{bx}{3}}}{b}$$
$$\Big\downarrow \text{27}$$
$$\frac{48e^{5a/3}\int \frac{e^{\frac{16bx}{3}}}{\left(1+e^{2bx}\right)^4}de^{\frac{bx}{3}}}{b}$$
$$\Big\downarrow \text{817}$$

$$\frac{48e^{5a/3}\left(\frac{11}{18}\int \frac{e^{\frac{10bx}{3}}}{\left(1+e^{2bx}\right)^3}de^{\frac{bx}{3}}-\frac{e^{\frac{11bx}{3}}}{18\left(e^{2bx}+1\right)^3}\right)}{b}$$

$\downarrow$ 817

$$\frac{48e^{5a/3}\left(\frac{11}{18}\left(\frac{5}{12}\int \frac{e^{\frac{4bx}{3}}}{\left(1+e^{2bx}\right)^2}de^{\frac{bx}{3}}-\frac{e^{\frac{5bx}{3}}}{12\left(e^{2bx}+1\right)^2}\right)-\frac{e^{\frac{11bx}{3}}}{18\left(e^{2bx}+1\right)^3}\right)}{b}$$

$\downarrow$ 819

$$\frac{48e^{5a/3}\left(\frac{11}{18}\left(\frac{5}{12}\left(\frac{1}{6}\int \frac{e^{\frac{4bx}{3}}}{1+e^{2bx}}de^{\frac{bx}{3}}+\frac{e^{\frac{5bx}{3}}}{6\left(e^{2bx}+1\right)}\right)-\frac{e^{\frac{5bx}{3}}}{12\left(e^{2bx}+1\right)^2}\right)-\frac{e^{\frac{11bx}{3}}}{18\left(e^{2bx}+1\right)^3}\right)}{b}$$

$\downarrow$ 824

$$\frac{48e^{5a/3}\left(\frac{11}{18}\left(\frac{5}{12}\left(\frac{1}{6}\left(\frac{1}{3}\int \frac{1}{1+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{3}\int -\frac{1-\sqrt{3}e^{\frac{bx}{3}}}{2\left(1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}\right)}de^{\frac{bx}{3}}+\frac{1}{3}\int -\frac{1+\sqrt{3}e^{\frac{bx}{3}}}{2\left(1+\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}\right)}de^{\frac{bx}{3}}\right)+\frac{e^{\frac{5bx}{3}}}{6\left(e^{2bx}+1\right)}\cdots\right.\right.\right.}{b}$$

$\downarrow$ 27

$$\frac{48e^{5a/3}\left(\frac{11}{18}\left(\frac{5}{12}\left(\frac{1}{6}\left(\frac{1}{3}\int \frac{1}{1+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1-\sqrt{3}e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1+\sqrt{3}e^{\frac{bx}{3}}}{1+\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{e^{\frac{5bx}{3}}}{6\left(e^{2bx}+1\right)}\right)-\frac{e^{\frac{5bx}{3}}}{12\left(e^{2bx}+1\cdots\right.\right.\right.}{b}$$

$\downarrow$ 216

$$\frac{48e^{5a/3}\left(\frac{11}{18}\left(\frac{5}{12}\left(\frac{1}{6}\left(-\frac{1}{6}\int \frac{1-\sqrt{3}e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1+\sqrt{3}e^{\frac{bx}{3}}}{1+\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{3}\arctan\left(e^{\frac{bx}{3}}\right)\right)+\frac{e^{\frac{5bx}{3}}}{6\left(e^{2bx}+1\right)}\right)-\frac{e^{\frac{5bx}{3}}}{12\left(e^{2bx}+\cdots\right.\right.\right.}{b}$$

$\downarrow$ 1142

$$\frac{48e^{5a/3}\left(\frac{11}{18}\left(\frac{5}{12}\left(\frac{1}{6}\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}-2e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{1+\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{2}\sqrt{3}\cdots\right.\right.\right.\right.}{b}$$

$\downarrow$ 25

$$\frac{48e^{5a/3}\left(\frac{11}{18}\left(\frac{5}{12}\left(\frac{1}{6}\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-2e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{1+\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{2}\sqrt{3}\int\cdots\right.\right.\right.\right.}{b}$$

$\downarrow$ 1083

$$\frac{48e^{5a/3}\left(\frac{11}{18}\left(\frac{5}{12}\left(\frac{1}{6}\left(\frac{1}{6}\left(-\int \frac{1}{-1-e^{\frac{2bx}{3}}}d\left(-\sqrt{3}+2e^{\frac{bx}{3}}\right)-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-2e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(-\int \frac{1}{-1-e^{\frac{2bx}{3}}}d\left(\sqrt{3}+2e^{\frac{b}{}}\right.\right.\right.\right.\right.\right.}{b}$$

$\downarrow$ 217

$$\frac{48e^{5a/3}\left(\frac{11}{18}\left(\frac{5}{12}\left(\frac{1}{6}\left(\frac{1}{6}\left(-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-2e^{\frac{bx}{3}}}{1-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\arctan\left(\sqrt{3}-2e^{\frac{bx}{3}}\right)\right)+\frac{1}{6}\left(\arctan\left(2e^{\frac{bx}{3}}+\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\int\right.\right.\right.\right.\right.}{b}$$

$\downarrow$ 1103

$$\frac{48e^{5a/3}\left(\frac{11}{18}\left(\frac{5}{12}\left(\frac{1}{6}\left(\frac{1}{3}\arctan\left(e^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\log\left(-\sqrt{3}e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}+1\right)-\arctan\left(\sqrt{3}-2e^{\frac{bx}{3}}\right)\right)+\frac{1}{6}\left(\arctan\left(2e\right.\right.\right.\right.\right.\right.}{b}$$

input
```
Int[E^((5*(a + b*x))/3)*Sech[d + b*x]^4,x]
```

output
```
(48*E^((5*a)/3)*(-1/18*E^((11*b*x)/3)/(1 + E^(2*b*x))^3 + (11*(-1/12*E^((5
*b*x)/3)/(1 + E^(2*b*x))^2 + (5*(E^((5*b*x)/3)/(6*(1 + E^(2*b*x))) + (ArcT
an[E^((b*x)/3)]/3 + (-ArcTan[Sqrt[3] - 2*E^((b*x)/3)] + (Sqrt[3]*Log[1 - S
qrt[3]*E^((b*x)/3) + E^((2*b*x)/3)])/2)/6 + (ArcTan[Sqrt[3] + 2*E^((b*x)/3
)] - (Sqrt[3]*Log[1 + Sqrt[3]*E^((b*x)/3) + E^((2*b*x)/3)])/2)/6)/6))/12))
/18))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.40 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\left(55\,\mathrm{e}^{4bx+4d}-28\,\mathrm{e}^{2bx+2d}-11\right)\mathrm{e}^{\frac{5bx}{3}+\frac{5a}{3}}}{27\left(1+\mathrm{e}^{2bx+2d}\right)^{3}b}+16\left(\sum_{\_R=\mathrm{RootOf}\left(45137758519296b^{4}\_Z^{4}-20323353600b^{2}\_Z^{2}+9150625\right)}\_R\ln\left(\right.\right.$

input
```
int(exp(5/3*b*x+5/3*a)*sech(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
1/27/(1+exp(2*b*x+2*d))^3/b*(55*exp(4*b*x+4*d)-28*exp(2*b*x+2*d)-11)*exp(5
/3*b*x+5/3*a)+16*sum(_R*ln(exp(1/3*b*x+1/3*d)+17414258688/166375*b^3*_R^3-
2592/55*b*_R),_R=RootOf(45137758519296*_Z^4*b^4-20323353600*_Z^2*b^2+91506
25))*exp(5/3*a-5/3*d)+55/162*I*ln(exp(1/3*b*x+1/3*d)+I)/b*exp(5/3*a-5/3*d)
-55/162*I*ln(exp(1/3*b*x+1/3*d)-I)/b*exp(5/3*a-5/3*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9595 vs. 2(211) = 422.

Time = 0.20 (sec) , antiderivative size = 9595, normalized size of antiderivative = 33.55

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}^4(d+bx)\,dx=\text{Too large to display}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sech(b*x+d)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}^4(d+bx)\,dx=e^{\frac{5a}{3}}\int e^{\frac{5bx}{3}}\operatorname{sech}^4(bx+d)\,dx$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sech(b*x+d)**4,x)
```

output
```
exp(5*a/3)*Integral(exp(5*b*x/3)*sech(b*x + d)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.72

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}^4(d+bx)\,dx=$$
$$-\frac{55\left(\sqrt{3}\log\left(\sqrt{3}e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{\left(-\frac{2}{3}bx-\frac{2}{3}d\right)}+1\right)-\sqrt{3}\log\left(-\sqrt{3}e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{\left(-\frac{2}{3}bx-\frac{2}{3}d\right)}+1\right)+2\operatorname{arc}}{324\,b}$$
$$+\frac{\left(55\,e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}-28\,e^{\left(-\frac{7}{3}bx-\frac{7}{3}d\right)}-11\,e^{\left(-\frac{13}{3}bx-\frac{13}{3}d\right)}\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}}{27\,b\left(3\,e^{(-2\,bx-2\,d)}+3\,e^{(-4\,bx-4\,d)}+e^{(-6\,bx-6\,d)}+1\right)}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sech(b*x+d)^4,x, algorithm="maxima")
```

output
```
-55/324*(sqrt(3)*log(sqrt(3)*e^(-1/3*b*x - 1/3*d) + e^(-2/3*b*x - 2/3*d) +
 1) - sqrt(3)*log(-sqrt(3)*e^(-1/3*b*x - 1/3*d) + e^(-2/3*b*x - 2/3*d) + 1
) + 2*arctan(sqrt(3) + 2*e^(-1/3*b*x - 1/3*d)) + 2*arctan(-sqrt(3) + 2*e^(
-1/3*b*x - 1/3*d)) + 4*arctan(e^(-1/3*b*x - 1/3*d)))*e^(5/3*a - 5/3*d)/b +
 1/27*(55*e^(-1/3*b*x - 1/3*d) - 28*e^(-7/3*b*x - 7/3*d) - 11*e^(-13/3*b*x
 - 13/3*d))*e^(5/3*a - 5/3*d)/(b*(3*e^(-2*b*x - 2*d) + 3*e^(-4*b*x - 4*d)
+ e^(-6*b*x - 6*d) + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.71

$$\int e^{\frac{5}{3}(a+bx)}\text{sech}^4(d+bx)\,dx =$$

$$-\frac{\left(55\sqrt{3}e^{\left(-\frac{17}{3}d\right)}\log\left(\sqrt{3}e^{\left(\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{\left(\frac{2}{3}bx\right)}+e^{\left(-\frac{2}{3}d\right)}\right)-55\sqrt{3}e^{\left(-\frac{17}{3}d\right)}\log\left(-\sqrt{3}e^{\left(\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{\left(\frac{2}{3}bx\right)}+\cdots\right.\right.}{\cdots}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sech(b*x+d)^4,x, algorithm="giac")
```

output
```
-1/324*(55*sqrt(3)*e^(-17/3*d)*log(sqrt(3)*e^(1/3*b*x - 1/3*d) + e^(2/3*b*
x) + e^(-2/3*d)) - 55*sqrt(3)*e^(-17/3*d)*log(-sqrt(3)*e^(1/3*b*x - 1/3*d)
 + e^(2/3*b*x) + e^(-2/3*d)) - 110*arctan((sqrt(3)*e^(-1/3*d) + 2*e^(1/3*b
*x))*e^(1/3*d))*e^(-17/3*d) - 110*arctan(-(sqrt(3)*e^(-1/3*d) - 2*e^(1/3*b
*x))*e^(1/3*d))*e^(-17/3*d) - 220*arctan(e^(1/3*b*x + 1/3*d))*e^(-17/3*d)
+ 12*(11*e^(5/3*b*x) - 55*e^(17/3*b*x + 4*d) + 28*e^(11/3*b*x + 2*d))*e^(-
4*d)/(e^(2*b*x + 2*d) + 1)^3)*e^(5/3*a + 4*d)/b
```

Mupad [B] (verification not implemented)

Time = 5.88 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.90

$$\int e^{\frac{5}{3}(a+bx)}\text{sech}^4(d+bx)\,dx = \text{Too large to display}$$

input
```
int(exp((5*a)/3 + (5*b*x)/3)/cosh(d + b*x)^4,x)
```

output
```
(55*(-exp(10*a - 10*d))^(1/6)*log((3025*exp((10*a)/3)*exp(-(10*d)/3))/6561
 - (3025*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*(-exp(10*a)*exp(
-10*d))^(1/6))/6561))/(162*b) - (55*(-exp(10*a - 10*d))^(1/6)*log((3025*ex
p((10*a)/3)*exp(-(10*d)/3))/6561 + (3025*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/
3)*exp((b*x)/3)*(-exp(10*a)*exp(-10*d))^(1/6))/6561))/(162*b) + (55*exp((5
*a)/3 + (5*b*x)/3))/(27*b*(exp(2*d + 2*b*x) + 1)) - (8*exp((5*a)/3 + 2*d +
 (11*b*x)/3))/(3*b*(3*exp(2*d + 2*b*x) + 3*exp(4*d + 4*b*x) + exp(6*d + 6*
b*x) + 1)) - (22*exp((5*a)/3 + (5*b*x)/3))/(9*b*(2*exp(2*d + 2*b*x) + exp(
4*d + 4*b*x) + 1)) + (55*log((3025*exp((10*a)/3)*exp(-(10*d)/3))/6561 - (3
025*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)/2 - 1/2
)*(-exp(10*a)*exp(-10*d))^(1/6))/6561)*(-exp(10*a - 10*d))^(1/6)*((3^(1/2)
*1i)/2 - 1/2))/(162*b) - (55*log((3025*exp((10*a)/3)*exp(-(10*d)/3))/6561
+ (3025*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)/2 -
 1/2)*(-exp(10*a)*exp(-10*d))^(1/6))/6561)*(-exp(10*a - 10*d))^(1/6)*((3^(
1/2)*1i)/2 - 1/2))/(162*b) + (55*log((3025*exp((10*a)/3)*exp(-(10*d)/3))/6
561 - (3025*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)
/2 + 1/2)*(-exp(10*a)*exp(-10*d))^(1/6))/6561)*(-exp(10*a - 10*d))^(1/6)*(
(3^(1/2)*1i)/2 + 1/2))/(162*b) - (55*log((3025*exp((10*a)/3)*exp(-(10*d)/3
))/6561 + (3025*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)
*1i)/2 + 1/2)*(-exp(10*a)*exp(-10*d))^(1/6))/6561)*(-exp(10*a - 10*d))^...
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}^4(d+bx)\,dx = \int e^{\frac{5bx}{3}+\frac{5a}{3}}\operatorname{sech}(bx+d)^4\,dx$$

input
```
int(exp(5/3*b*x+5/3*a)*sech(b*x+d)^4,x)
```

output
```
int(e**((5*a + 5*b*x)/3)*sech(b*x + d)**4,x)
```

3.90 $\int e^{\frac{5}{3}(a+bx)}\text{sech}^5(d+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 285

$$\int e^{\frac{5}{3}(a+bx)}\text{sech}^5(d+bx)\,dx = -\frac{4e^{\frac{5(a-d)}{3}+\frac{14}{3}(d+bx)}}{b\left(1+e^{2(d+bx)}\right)^4}-\frac{28e^{\frac{5(a-d)}{3}+\frac{8}{3}(d+bx)}}{9b\left(1+e^{2(d+bx)}\right)^3}-\frac{56e^{\frac{5(a-d)}{3}+\frac{2}{3}(d+bx)}}{27b\left(1+e^{2(d+bx)}\right)^2}+\frac{56e^{\frac{5(a-d)}{3}+\frac{2}{3}(d+bx)}}{81b\left(1+e^{2(d+bx)}\right)}-\frac{112e^{\frac{5(a-d)}{3}}\arctan\left(\frac{1-2e^{\frac{2}{3}(d+bx)}}{\sqrt{3}}\right)}{81\sqrt{3}b}+\frac{112e^{\frac{5(a-d)}{3}}\log\left(1+e^{\frac{2}{3}(d+bx)}\right)}{243b}-\frac{56e^{\frac{5(a-d)}{3}}\log\left(1-e^{\frac{2}{3}(d+bx)}+e^{\frac{4}{3}(d+bx)}\right)}{243b}$$

output

```
-4*exp(5/3*a+3*d+14/3*b*x)/b/(1+exp(2*b*x+2*d))^4-28/9*exp(5/3*a+d+8/3*b*x
)/b/(1+exp(2*b*x+2*d))^3-56/27*exp(5/3*a-d+2/3*b*x)/b/(1+exp(2*b*x+2*d))^2
+56/81*exp(5/3*a-d+2/3*b*x)/b/(1+exp(2*b*x+2*d))-112/243*3^(1/2)*exp(5/3*a
-5/3*d)*arctan(1/3*(1-2*exp(2/3*b*x+2/3*d))*3^(1/2))/b+112/243*exp(5/3*a-5
/3*d)*ln(1+exp(2/3*b*x+2/3*d))/b-56/243*exp(5/3*a-5/3*d)*ln(1-exp(2/3*b*x+
2/3*d)+exp(4/3*b*x+4/3*d))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.30

$$\int e^{\frac{5}{3}(a+bx)}\text{sech}^5(d+bx)\,dx$$
$$=\frac{e^{\frac{5}{3}(a+bx)}\left(-112e^{d+bx}\,\text{Hypergeometric2F1}\left(1,\frac{4}{3},\frac{7}{3},-e^{2(d+bx)}\right)+28\text{sech}(d+bx)(5+3\tanh(d+bx))+9\text{sec}\right.}{324b}$$

input
```
Integrate[E^((5*(a + b*x))/3)*Sech[d + b*x]^5,x]
```

output
```
(E^((5*(a + b*x))/3)*(-112*E^(d + b*x)*Hypergeometric2F1[1, 4/3, 7/3, -E^(
2*(d + b*x))] + 28*Sech[d + b*x]*(5 + 3*Tanh[d + b*x]) + 9*Sech[d + b*x]^3
*(5 + 9*Tanh[d + b*x])))/(324*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.55, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {2720, 27, 807, 817, 817, 817, 749, 750, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)}\text{sech}^5(bx+d)\,dx$$
$$\Big\downarrow \; 2720$$
$$\frac{3\int \frac{32e^{\frac{5a}{3}+\frac{19bx}{3}}}{\left(1+e^{2bx}\right)^5}de^{\frac{bx}{3}}}{b}$$
$$\Big\downarrow \; 27$$
$$\frac{96e^{5a/3}\int \frac{e^{\frac{19bx}{3}}}{\left(1+e^{2bx}\right)^5}de^{\frac{bx}{3}}}{b}$$
$$\Big\downarrow \; 807$$

$$\frac{48e^{5a/3}\int \frac{e^{3bx}}{(1+e^{bx})^5}de^{\frac{2bx}{3}}}{b}$$

$\Big\downarrow$ 817

$$\frac{48e^{5a/3}\left(\frac{7}{12}\int \frac{e^{2bx}}{(1+e^{bx})^4}de^{\frac{2bx}{3}}-\frac{e^{\frac{7bx}{3}}}{12(e^{bx}+1)^4}\right)}{b}$$

$\Big\downarrow$ 817

$$\frac{48e^{5a/3}\left(\frac{7}{12}\left(\frac{4}{9}\int \frac{e^{bx}}{(1+e^{bx})^3}de^{\frac{2bx}{3}}-\frac{e^{\frac{4bx}{3}}}{9(e^{bx}+1)^3}\right)-\frac{e^{\frac{7bx}{3}}}{12(e^{bx}+1)^4}\right)}{b}$$

$\Big\downarrow$ 817

$$\frac{48e^{5a/3}\left(\frac{7}{12}\left(\frac{4}{9}\left(\frac{1}{6}\int \frac{1}{(1+e^{bx})^2}de^{\frac{2bx}{3}}-\frac{e^{\frac{2bx}{3}}}{6(e^{bx}+1)^2}\right)-\frac{e^{\frac{4bx}{3}}}{9(e^{bx}+1)^3}\right)-\frac{e^{\frac{7bx}{3}}}{12(e^{bx}+1)^4}\right)}{b}$$

$\Big\downarrow$ 749

$$\frac{48e^{5a/3}\left(\frac{7}{12}\left(\frac{4}{9}\left(\frac{1}{6}\left(\frac{2}{3}\int \frac{1}{1+e^{bx}}de^{\frac{2bx}{3}}+\frac{e^{\frac{2bx}{3}}}{3(e^{bx}+1)}\right)-\frac{e^{\frac{2bx}{3}}}{6(e^{bx}+1)^2}\right)-\frac{e^{\frac{4bx}{3}}}{9(e^{bx}+1)^3}\right)-\frac{e^{\frac{7bx}{3}}}{12(e^{bx}+1)^4}\right)}{b}$$

$\Big\downarrow$ 750

$$\frac{48e^{5a/3}\left(\frac{7}{12}\left(\frac{4}{9}\left(\frac{1}{6}\left(\frac{2}{3}\left(\frac{1}{3}\int \left(2-e^{\frac{2bx}{3}}\right)de^{\frac{2bx}{3}}+\frac{1}{3}\int \frac{1}{1+e^{\frac{2bx}{3}}}de^{\frac{2bx}{3}}\right)+\frac{e^{\frac{2bx}{3}}}{3(e^{bx}+1)}\right)-\frac{e^{\frac{2bx}{3}}}{6(e^{bx}+1)^2}\right)-\frac{e^{\frac{4bx}{3}}}{9(e^{bx}+1)^3}\right)-\frac{e^{\frac{7bx}{3}}}{12(e^{bx}+1)^4}\right)}{b}$$

$\Big\downarrow$ 16

$$\frac{48e^{5a/3}\left(\frac{7}{12}\left(\frac{4}{9}\left(\frac{1}{6}\left(\frac{2}{3}\left(\frac{1}{3}\int \left(2-e^{\frac{2bx}{3}}\right)de^{\frac{2bx}{3}}+\frac{1}{3}\log\left(e^{\frac{2bx}{3}}+1\right)\right)+\frac{e^{\frac{2bx}{3}}}{3(e^{bx}+1)}\right)-\frac{e^{\frac{2bx}{3}}}{6(e^{bx}+1)^2}\right)-\frac{e^{\frac{4bx}{3}}}{9(e^{bx}+1)^3}\right)-\frac{e^{\frac{7bx}{3}}}{12(e^{bx}+1)^4}\right)}{b}$$

$\Big\downarrow$ 1142

$$\frac{48e^{5a/3}\left(\frac{7}{12}\left(\frac{4}{9}\left(\frac{1}{6}\left(\frac{2}{3}\left(\frac{1}{3}\left(\frac{3}{2}\int 1de^{\frac{2bx}{3}}-\frac{1}{2}\int \left(-1+2e^{\frac{2bx}{3}}\right)de^{\frac{2bx}{3}}\right)+\frac{1}{3}\log\left(e^{\frac{2bx}{3}}+1\right)\right)+\frac{e^{\frac{2bx}{3}}}{3(e^{bx}+1)}\right)-\frac{e^{\frac{2bx}{3}}}{6(e^{bx}+1)^2}\right)-\cdots}{b}$$

$\Big\downarrow$ 25

$$\frac{48e^{5a/3}\left(\frac{7}{12}\left(\frac{4}{9}\left(\frac{1}{6}\left(\frac{2}{3}\left(\frac{1}{3}\left(\frac{3}{2}\int 1de^{\frac{2bx}{3}}+\frac{1}{2}\int\left(1-2e^{\frac{2bx}{3}}\right)de^{\frac{2bx}{3}}\right)+\frac{1}{3}\log\left(e^{\frac{2bx}{3}}+1\right)\right)+\frac{e^{\frac{2bx}{3}}}{3(e^{bx}+1)}\right)-\frac{e^{\frac{2bx}{3}}}{6(e^{bx}+1)^2}\right)-\frac{}{9}}{b}$$

$\Big\downarrow$ 1083

$$\frac{48e^{5a/3}\left(\frac{7}{12}\left(\frac{4}{9}\left(\frac{1}{6}\left(\frac{2}{3}\left(\frac{1}{3}\left(\frac{1}{2}\int\left(1-2e^{\frac{2bx}{3}}\right)de^{\frac{2bx}{3}}-3\int\frac{1}{-2-2e^{\frac{2bx}{3}}}d\left(-1+2e^{\frac{2bx}{3}}\right)\right)+\frac{1}{3}\log\left(e^{\frac{2bx}{3}}+1\right)\right)+\frac{e^{\frac{2bx}{3}}}{3(e^{bx}+1)}\right.}{b}$$

$\Big\downarrow$ 217

$$\frac{48e^{5a/3}\left(\frac{7}{12}\left(\frac{4}{9}\left(\frac{1}{6}\left(\frac{2}{3}\left(\frac{1}{3}\left(\frac{1}{2}\int\left(1-2e^{\frac{2bx}{3}}\right)de^{\frac{2bx}{3}}+\sqrt{3}\arctan\left(\frac{2e^{\frac{2bx}{3}}-1}{\sqrt{3}}\right)\right)+\frac{1}{3}\log\left(e^{\frac{2bx}{3}}+1\right)\right)+\frac{e^{\frac{2bx}{3}}}{3(e^{bx}+1)}\right)-\frac{}{6(}}{b}$$

$\Big\downarrow$ 1103

$$\frac{48e^{5a/3}\left(\frac{7}{12}\left(\frac{4}{9}\left(\frac{1}{6}\left(\frac{2}{3}\left(\frac{\arctan\left(\frac{2e^{\frac{2bx}{3}}-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log\left(e^{\frac{2bx}{3}}+1\right)\right)+\frac{e^{\frac{2bx}{3}}}{3(e^{bx}+1)}\right)-\frac{e^{\frac{2bx}{3}}}{6(e^{bx}+1)^2}\right)-\frac{e^{\frac{4bx}{3}}}{9(e^{bx}+1)^3}\right)-\frac{e^{\frac{7bx}{3}}}{12(e^{bx}+1}}{b}$$

input
```
Int[E^((5*(a + b*x))/3)*Sech[d + b*x]^5,x]
```

output
```
(48*E^((5*a)/3)*(-1/12*E^((7*b*x)/3)/(1 + E^(b*x))^4 + (7*(-1/9*E^((4*b*x)
/3)/(1 + E^(b*x))^3 + (4*(-1/6*E^((2*b*x)/3)/(1 + E^(b*x))^2 + (E^((2*b*x)
/3)/(3*(1 + E^(b*x))) + (2*(ArcTan[(-1 + 2*E^((2*b*x)/3))/Sqrt[3]]/Sqrt[3]
 + Log[1 + E^((2*b*x)/3)]/3))/3)/6))/9))/12))/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.36 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.79

method	result
risch	$\frac{4\left(14\,\mathrm{e}^{6bx+6d}-144\,\mathrm{e}^{4bx+4d}-105\,\mathrm{e}^{2bx+2d}-28\right)\mathrm{e}^{\frac{5a}{3}-d+\frac{2bx}{3}}}{81\left(1+\mathrm{e}^{2bx+2d}\right)^{4} b}-\frac{56\ln\left(\mathrm{e}^{\frac{2bx}{3}+\frac{2d}{3}}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\mathrm{e}^{\frac{5a}{3}-\frac{5d}{3}}}{243b}+\frac{56i\ln\left(\mathrm{e}^{\frac{2bx}{3}+\frac{2d}{3}}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{243b}$

input
```
int(exp(5/3*b*x+5/3*a)*sech(b*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
4/81/(1+exp(2*b*x+2*d))^4/b*(14*exp(6*b*x+6*d)-144*exp(4*b*x+4*d)-105*exp(
2*b*x+2*d)-28)*exp(5/3*a-d+2/3*b*x)-56/243*ln(exp(2/3*b*x+2/3*d)-1/2+1/2*I
*3^(1/2))/b*exp(5/3*a-5/3*d)+56/243*I*ln(exp(2/3*b*x+2/3*d)-1/2+1/2*I*3^(1
/2))/b*exp(5/3*a-5/3*d)*3^(1/2)-56/243*ln(exp(2/3*b*x+2/3*d)-1/2-1/2*I*3^(
1/2))/b*exp(5/3*a-5/3*d)-56/243*I*ln(exp(2/3*b*x+2/3*d)-1/2-1/2*I*3^(1/2))
/b*exp(5/3*a-5/3*d)*3^(1/2)+112/243*exp(5/3*a-5/3*d)*ln(1+exp(2/3*b*x+2/3*
d))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11662 vs. $2(215) = 430$.

Time = 0.26 (sec) , antiderivative size = 11662, normalized size of antiderivative = 40.92

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}^5(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sech(b*x+d)^5,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}^5(d+bx)\,dx = e^{\frac{5a}{3}}\int e^{\frac{5bx}{3}}\operatorname{sech}^5(bx+d)\,dx$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sech(b*x+d)**5,x)
```

output
```
exp(5*a/3)*Integral(exp(5*b*x/3)*sech(b*x + d)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.81

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}^5(d+bx)\,dx = \frac{56\left(2\sqrt{3}\arctan\left(\sqrt{3}+2e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}\right)-2\sqrt{3}\arctan\left(-\sqrt{3}+2e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}\right)-\log\left(\sqrt{3}e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{\left(\cdot\right.}\right.\right.}{243\,b} + \frac{4\left(14\,e^{\left(-\frac{4}{3}bx-\frac{4}{3}d\right)}-144\,e^{\left(-\frac{10}{3}bx-\frac{10}{3}d\right)}-105\,e^{\left(-\frac{16}{3}bx-\frac{16}{3}d\right)}-28\,e^{\left(-\frac{22}{3}bx-\frac{22}{3}d\right)}\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}}{81\,b\left(4\,e^{\left(-2\,bx-2\,d\right)}+6\,e^{\left(-4\,bx-4\,d\right)}+4\,e^{\left(-6\,bx-6\,d\right)}+e^{\left(-8\,bx-8\,d\right)}+1\right)}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sech(b*x+d)^5,x, algorithm="maxima")
```

output
```
56/243*(2*sqrt(3)*arctan(sqrt(3) + 2*e^(-1/3*b*x - 1/3*d)) - 2*sqrt(3)*arc
tan(-sqrt(3) + 2*e^(-1/3*b*x - 1/3*d)) - log(sqrt(3)*e^(-1/3*b*x - 1/3*d)
+ e^(-2/3*b*x - 2/3*d) + 1) - log(-sqrt(3)*e^(-1/3*b*x - 1/3*d) + e^(-2/3*
b*x - 2/3*d) + 1) + 2*log(e^(-2/3*b*x - 2/3*d) + 1))*e^(5/3*a - 5/3*d)/b +
 4/81*(14*e^(-4/3*b*x - 4/3*d) - 144*e^(-10/3*b*x - 10/3*d) - 105*e^(-16/3
*b*x - 16/3*d) - 28*e^(-22/3*b*x - 22/3*d))*e^(5/3*a - 5/3*d)/(b*(4*e^(-2*
b*x - 2*d) + 6*e^(-4*b*x - 4*d) + 4*e^(-6*b*x - 6*d) + e^(-8*b*x - 8*d) +
1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.53

$$\int e^{\frac{5}{3}(a+bx)}\text{sech}^5(d+bx)\,dx$$

$$= \frac{4\left(28\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(\frac{2}{3}bx\right)}-e^{\left(-\frac{2}{3}d\right)}\right)e^{\left(\frac{2}{3}d\right)}\right)e^{\left(-\frac{20}{3}d\right)}-14e^{\left(-\frac{20}{3}d\right)}\log\left(e^{\left(\frac{4}{3}bx\right)}-e^{\left(\frac{2}{3}bx-\frac{2}{3}d\right)}+e^{\left(-\frac{4}{3}d\right)}\right.\right.}{24}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*sech(b*x+d)^5,x, algorithm="giac")
```

output
```
4/243*(28*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2/3*b*x) - e^(-2/3*d))*e^(2/3*d
))*e^(-20/3*d) - 14*e^(-20/3*d)*log(e^(4/3*b*x) - e^(2/3*b*x - 2/3*d) + e^
(-4/3*d)) + 28*e^(-20/3*d)*log(e^(2/3*b*x) + e^(-2/3*d)) + 3*(14*e^(20/3*b
*x + 6*d) - 144*e^(14/3*b*x + 4*d) - 105*e^(8/3*b*x + 2*d) - 28*e^(2/3*b*x
))*e^(-6*d)/(e^(2*b*x + 2*d) + 1)^4)*e^(5/3*a + 5*d)/b
```

Mupad [B] (verification not implemented)

Time = 5.49 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.28

$$\int e^{\frac{5}{3}(a+bx)}\mathrm{sech}^5(d+bx)\,dx = \frac{56\,\mathrm{e}^{\frac{5\,a}{3}-d+\frac{2\,b\,x}{3}}}{81\,b\,\left(\mathrm{e}^{2\,d+2\,b\,x}+1\right)} - \frac{28\,\mathrm{e}^{\frac{5\,a}{3}+d+\frac{8\,b\,x}{3}}}{9\,b\,\left(3\,\mathrm{e}^{2\,d+2\,b\,x}+3\,\mathrm{e}^{4\,d+4\,b\,x}+\mathrm{e}^{6\,d+6\,b\,x}+1\right)} - \frac{56\,\mathrm{e}^{\frac{5\,a}{3}-d+\frac{2\,b\,x}{3}}}{27\,b\,\left(2\,\mathrm{e}^{2\,d+2\,b\,x}+\mathrm{e}^{4\,d+4\,b\,x}+1\right)} + \frac{112\,\left(\mathrm{e}^{5\,a-5\,d}\right)^{1/3}\,\ln\left(-\frac{224\,\left(\mathrm{e}^{5\,a}\,\mathrm{e}^{-5\,d}\right)^{1/3}}{243}-\frac{224\,\mathrm{e}^{\frac{5\,a}{3}}\,\mathrm{e}^{\frac{2\,d}{3}}\,\mathrm{e}^{-\frac{5\,d}{3}}\,\mathrm{e}^{\frac{2\,b\,x}{3}}}{243}\right)}{243\,b} - \frac{4\,\mathrm{e}^{\frac{5\,a}{3}+3\,d+\frac{14\,b\,x}{3}}}{b\,\left(4\,\mathrm{e}^{2\,d+2\,b\,x}+6\,\mathrm{e}^{4\,d+4\,b\,x}+4\,\mathrm{e}^{6\,d+6\,b\,x}+\mathrm{e}^{8\,d+8\,b\,x}+1\right)} + \frac{112\,\left(\mathrm{e}^{5\,a-5\,d}\right)^{1/3}\,\ln\left(-\frac{224\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\mathrm{e}^{5\,a}\,\mathrm{e}^{-5\,d}\right)^{1/3}}{243}-\frac{224\,\mathrm{e}^{\frac{5\,a}{3}}\,\mathrm{e}^{\frac{2\,d}{3}}\,\mathrm{e}^{-\frac{5\,d}{3}}\,\mathrm{e}^{\frac{2\,b\,x}{3}}}{243}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{243\,b} - \frac{112\,\left(\mathrm{e}^{5\,a-5\,d}\right)^{1/3}\,\ln\left(\frac{224\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\mathrm{e}^{5\,a}\,\mathrm{e}^{-5\,d}\right)^{1/3}}{243}-\frac{224\,\mathrm{e}^{\frac{5\,a}{3}}\,\mathrm{e}^{\frac{2\,d}{3}}\,\mathrm{e}^{-\frac{5\,d}{3}}\,\mathrm{e}^{\frac{2\,b\,x}{3}}}{243}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{243\,b}$$

input
```
int(exp((5*a)/3 + (5*b*x)/3)/cosh(d + b*x)^5,x)
```

output
```
(56*exp((5*a)/3 - d + (2*b*x)/3))/(81*b*(exp(2*d + 2*b*x) + 1)) - (28*exp(
(5*a)/3 + d + (8*b*x)/3))/(9*b*(3*exp(2*d + 2*b*x) + 3*exp(4*d + 4*b*x) +
exp(6*d + 6*b*x) + 1)) - (56*exp((5*a)/3 - d + (2*b*x)/3))/(27*b*(2*exp(2*
d + 2*b*x) + exp(4*d + 4*b*x) + 1)) + (112*exp(5*a - 5*d)^(1/3)*log(- (224
*(exp(5*a)*exp(-5*d))^(1/3))/243 - (224*exp((5*a)/3)*exp((2*d)/3)*exp(-(5*
d)/3)*exp((2*b*x)/3))/243))/(243*b) - (4*exp((5*a)/3 + 3*d + (14*b*x)/3))/
(b*(4*exp(2*d + 2*b*x) + 6*exp(4*d + 4*b*x) + 4*exp(6*d + 6*b*x) + exp(8*d
 + 8*b*x) + 1)) + (112*exp(5*a - 5*d)^(1/3)*log(- (224*((3^(1/2)*1i)/2 - 1
/2)*(exp(5*a)*exp(-5*d))^(1/3))/243 - (224*exp((5*a)/3)*exp((2*d)/3)*exp(-
(5*d)/3)*exp((2*b*x)/3))/243)*((3^(1/2)*1i)/2 - 1/2))/(243*b) - (112*exp(5
*a - 5*d)^(1/3)*log((224*((3^(1/2)*1i)/2 + 1/2)*(exp(5*a)*exp(-5*d))^(1/3)
)/243 - (224*exp((5*a)/3)*exp((2*d)/3)*exp(-(5*d)/3)*exp((2*b*x)/3))/243)*
((3^(1/2)*1i)/2 + 1/2))/(243*b)
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{sech}^5(d+bx)\,dx=\int e^{\frac{5bx}{3}+\frac{5a}{3}}\operatorname{sech}(bx+d)^5\,dx$$

input
```
int(exp(5/3*b*x+5/3*a)*sech(b*x+d)^5,x)
```

output
```
int(e**((5*a + 5*b*x)/3)*sech(b*x + d)**5,x)
```

3.91 $\int F^{c(a+bx)}\mathrm{sech}(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 69

$$\int F^{c(a+bx)}\mathrm{sech}(d+ex)\,dx = \frac{2e^{d+ex}F^{c(a+bx)}\,\mathrm{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),-e^{2d+2ex}\right)}{e+bc\log(F)}$$

output
```
2*exp(e*x+d)*F^(c*(b*x+a))*hypergeom([1, 1/2*(e+b*c*ln(F))/e],[3/2+1/2*b*c
*ln(F)/e],-exp(2*e*x+2*d))/(e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int F^{c(a+bx)}\mathrm{sech}(d+ex)\,dx = \frac{2e^{d+ex}F^{c(a+bx)}\,\mathrm{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{bc\log(F)}{2e},\frac{3}{2}+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)}{e+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sech[d + e*x],x]
```

output
```
(2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, 1/2 + (b*c*Log[F])/(2*
e), 3/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e + b*c*Log[F])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {6015}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{ 6015}$$

$$\frac{2e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),-e^{2(d+ex)}\right)}{bc\log(F)+e}$$

input
```
Int[F^(c*(a + b*x))*Sech[d + e*x],x]
```

output
```
(2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e)
, (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/(e + b*c*Log[F])
```

Defintions of rubi rules used

rule 6015
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \operatorname{sech}(ex+d) \, dx$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*sech(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\operatorname{sech}(d+ex) \, dx = \int F^{(bx+a)c} \operatorname{sech}(ex+d) \, dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sech(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\operatorname{sech}(d+ex) \, dx = \int F^{c(a+bx)} \operatorname{sech}(d+ex) \, dx$$

input
```
integrate(F**(c*(b*x+a))*sech(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*sech(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\text{sech}(d+ex)\,dx = \int F^{(bx+a)c}\,\text{sech}\,(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d),x, algorithm="maxima")
```

output
```
-4*F^(a*c)*e*integrate(e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + (b*c*e^(4*
d)*log(F) - e*e^(4*d))*e^(4*e*x) + 2*(b*c*e^(2*d)*log(F) - e*e^(2*d))*e^(2
*e*x) - e), x) + 2*F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + (b*c*e
^(2*d)*log(F) - e*e^(2*d))*e^(2*e*x) - e)
```

Giac [F]

$$\int F^{c(a+bx)}\text{sech}(d+ex)\,dx = \int F^{(bx+a)c}\,\text{sech}\,(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sech(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{sech}(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}}{\cosh\,(d+e\,x)}\,dx$$

input
```
int(F^(c*(a + b*x))/cosh(d + e*x),x)
```

output
```
int(F^(c*(a + b*x))/cosh(d + e*x), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\text{sech}(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\text{sech}(ex+d)\,dx\right)$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sech(d + e*x),x)
```

3.92 $\int F^{c(a+bx)}\text{sech}^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 74

$$\int F^{c(a+bx)}\text{sech}^2(d+ex)\,dx = \frac{4e^{2d+2ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(2+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(4+\frac{bc\log(F)}{e}\right),-e^{2d+2ex}\right)}{2e+bc\log(F)}$$

output
```
4*exp(2*e*x+2*d)*F^(c*(b*x+a))*hypergeom([2, 1+1/2*b*c*ln(F)/e],[2+1/2*b*c
*ln(F)/e],-exp(2*e*x+2*d))/(2*e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int F^{c(a+bx)}\text{sech}^2(d+ex)\,dx = \frac{4e^{2(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{2e},2+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)}{2e+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sech[d + e*x]^2,x]
```

output
```
(4*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(
2*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(2*e + b*c*Log[F])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {6015}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \; 6015$$

$$\frac{4e^{2(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{2e}+1,\frac{bc\log(F)}{2e}+2,-e^{2(d+ex)}\right)}{bc\log(F)+2e}$$

input
```
Int[F^(c*(a + b*x))*Sech[d + e*x]^2,x]
```

output

```
(4*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(
2*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(2*e + b*c*Log[F])
```


Defintions of rubi rules used

rule 6015
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)}\operatorname{sech}(ex+d)^2\,dx$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*sech(e*x+d)^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx = \int F^{(bx+a)c}\operatorname{sech}(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sech(e*x + d)^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx = \int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*sech(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*sech(d + e*x)**2, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{sech}(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^2,x, algorithm="maxima")
```

output
```
16*F^(a*c)*b*c*e*integrate(F^(b*c*x)/(b^2*c^2*log(F)^2 - 6*b*c*e*log(F) +
8*e^2 + (b^2*c^2*e^(6*d)*log(F)^2 - 6*b*c*e*e^(6*d)*log(F) + 8*e^2*e^(6*d)
)*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*log(F) + 8*e^2
*e^(4*d))*e^(4*e*x) + 3*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^(2*d)*log(F)
 + 8*e^2*e^(2*d))*e^(2*e*x)), x)*log(F) - 4*(4*F^(a*c)*e - (F^(a*c)*b*c*e^
(2*d)*log(F) - 4*F^(a*c)*e*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2
 - 6*b*c*e*log(F) + 8*e^2 + (b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*lo
g(F) + 8*e^2*e^(4*d))*e^(4*e*x) + 2*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^
(2*d)*log(F) + 8*e^2*e^(2*d))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{sech}(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sech(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\cosh\left(d+e\,x\right)^2}\,dx$$

input
```
int(F^(c*(a + b*x))/cosh(d + e*x)^2,x)
```

output
```
int(F^(c*(a + b*x))/cosh(d + e*x)^2, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\operatorname{sech}(ex+d)^2\,dx\right)$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sech(d + e*x)**2,x)
```

3.93 $\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 74

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx = \frac{8e^{3d+3ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(5+\frac{bc\log(F)}{e}\right),-e^{2d+2ex}\right)}{3e+bc\log(F)}$$

output
```
8*exp(3*e*x+3*d)*F^(c*(b*x+a))*hypergeom([3, 3/2+1/2*b*c*ln(F)/e],[5/2+1/2
*b*c*ln(F)/e],-exp(2*e*x+2*d))/(b*c*ln(F)+3*e)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.30

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx = \frac{F^{c(a+bx)}\left(2e^{d+ex}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),-e^{2(d+ex)}\right)(e-bc\log(F))+\text{sech}(d\right.}{2e^2}$$

input
```
Integrate[F^(c*(a + b*x))*Sech[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(2*E^(d + e*x)*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e
), (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))]*(e - b*c*Log[F]) + Sech[d + e
*x]*(b*c*Log[F] + e*Tanh[d + e*x])))/(2*e^2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.88, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6013, 6015}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^3(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6013}$$

$$\frac{1}{2}\left(1-\frac{b^2c^2\log^2(F)}{e^2}\right)\int F^{c(a+bx)}\operatorname{sech}(d+ex)dx+\frac{bc\log(F)\operatorname{sech}(d+ex)F^{c(a+bx)}}{2e^2}+\frac{\tanh(d+ex)\operatorname{sech}(d+ex)F^{c(a+bx)}}{2e}$$

$$\Big\downarrow \text{6015}$$

$$\frac{e^{d+ex}F^{c(a+bx)}\left(1-\frac{b^2c^2\log^2(F)}{e^2}\right)\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),-e^{2(d+ex)}\right)}{bc\log(F)+e}+\frac{bc\log(F)\operatorname{sech}(d+ex)F^{c(a+bx)}}{2e^2}+\frac{\tanh(d+ex)\operatorname{sech}(d+ex)F^{c(a+bx)}}{2e}$$

input
```
Int[F^(c*(a + b*x))*Sech[d + e*x]^3,x]
```

output
```
(E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e),
(3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))]*(1 - (b^2*c^2*Log[F]^2)/e^2))/(e
 + b*c*Log[F]) + (b*c*F^(c*(a + b*x))*Log[F]*Sech[d + e*x])/(2*e^2) + (F^(
c*(a + b*x))*Sech[d + e*x]*Tanh[d + e*x])/(2*e)
```

Defintions of rubi rules used

rule 6013
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sech[d + e*x]^(n - 2)/(e^2*(n - 1)*
(n - 2))), x] + (Simp[F^(c*(a + b*x))*Sech[d + e*x]^(n - 1)*(Sinh[d + e*x]/
(e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2))   Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a,
b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
 NeQ[n, 2]
```

rule 6015
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \operatorname{sech}(ex+d)^3 \, dx$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)^3,x)
```

output
```
int(F^(c*(b*x+a))*sech(e*x+d)^3,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\operatorname{sech}^3(d+ex)\,dx = \int F^{(bx+a)c} \operatorname{sech}(ex+d)^3 \, dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sech(e*x + d)^3, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx = \int F^{c(a+bx)}\,\text{sech}^3\,(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*sech(e*x+d)**3,x)
```

output
```
Integral(F**(c*(a + b*x))*sech(d + e*x)**3, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx = \int F^{(bx+a)c}\,\text{sech}\,(ex+d)^3\;dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^3,x, algorithm="maxima")
```

output
```
48*(F^(a*c)*b*c*e*e^d*log(F) + F^(a*c)*e^2*e^d)*integrate(e^(b*c*x*log(F)
+ e*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*log(F) + 15*e^2 + (b^2*c^2*e^(8*d)*log(
F)^2 - 8*b*c*e*e^(8*d)*log(F) + 15*e^2*e^(8*d))*e^(8*e*x) + 4*(b^2*c^2*e^(
6*d)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e^2*e^(6*d))*e^(6*e*x) + 6*(b^
2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(F) + 15*e^2*e^(4*d))*e^(4*e*x
) + 4*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(2*d)*log(F) + 15*e^2*e^(2*d))
*e^(2*e*x)), x) - 8*(6*F^(a*c)*e*e^(e*x + d) - (F^(a*c)*b*c*e^(3*d)*log(F)
 - 5*F^(a*c)*e*e^(3*d))*e^(3*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*l
og(F) + 15*e^2 + (b^2*c^2*e^(6*d)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e
^2*e^(6*d))*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(
F) + 15*e^2*e^(4*d))*e^(4*e*x) + 3*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(
2*d)*log(F) + 15*e^2*e^(2*d))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx=\int F^{(bx+a)c}\,\text{sech}\,(ex+d)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sech(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\cosh\,(d+e\,x)^3}\,dx$$

input
```
int(F^(c*(a + b*x))/cosh(d + e*x)^3,x)
```

output
```
int(F^(c*(a + b*x))/cosh(d + e*x)^3, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\text{sech}(ex+d)^3\,dx\right)$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sech(d + e*x)**3,x)
```

3.94 $\int e^{a+bx}\text{sech}^n(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 66

$$\int e^{a+bx}\text{sech}^n(a+bx)\,dx = \frac{e^{a+bx}\left(1+e^{2a+2bx}\right)^n \text{Hypergeometric2F1}\left(n,\frac{1+n}{2},\frac{3+n}{2},-e^{2a+2bx}\right)\text{sech}^n(a+bx)}{b(1+n)}$$

output
```
exp(b*x+a)*(1+exp(2*b*x+2*a))^n*hypergeom([n, 1/2+1/2*n],[3/2+1/2*n],-exp(
2*b*x+2*a))*sech(b*x+a)^n/b/(1+n)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int e^{a+bx}\text{sech}^n(a+bx)\,dx = \frac{e^{a+bx}\left(1+e^{2(a+bx)}\right)\text{Hypergeometric2F1}\left(1,\frac{3-n}{2},\frac{3+n}{2},-e^{2(a+bx)}\right)\text{sech}^n(a+bx)}{b(1+n)}$$

input
```
Integrate[E^(a + b*x)*Sech[a + b*x]^n,x]
```

output
```
(E^(a + b*x)*(1 + E^(2*(a + b*x)))*Hypergeometric2F1[1, (3 - n)/2, (3 + n)
/2, -E^(2*(a + b*x))]*Sech[a + b*x]^n)/(b*(1 + n))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 27, 7270, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{sech}^n(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int 2^n\left(\frac{e^{a+bx}}{1+e^{2a+2bx}}\right)^n de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2^n\int\left(\frac{e^{a+bx}}{1+e^{2a+2bx}}\right)^n de^{a+bx}}{b}$$

$$\Big\downarrow \text{7270}$$

$$\frac{2^n\left(e^{a+bx}\right)^{-n}\left(\frac{e^{a+bx}}{e^{2a+2bx}+1}\right)^n\left(e^{2a+2bx}+1\right)^n\int\left(e^{a+bx}\right)^n\left(1+e^{2a+2bx}\right)^{-n}de^{a+bx}}{b}$$

$$\Big\downarrow \text{278}$$

$$\frac{2^n e^{a+bx}\left(\frac{e^{a+bx}}{e^{2a+2bx}+1}\right)^n\left(e^{2a+2bx}+1\right)^n\text{Hypergeometric2F1}\left(n,\frac{n+1}{2},\frac{n+3}{2},-e^{2a+2bx}\right)}{b(n+1)}$$

input
```
Int[E^(a + b*x)*Sech[a + b*x]^n,x]
```

output
```
(2^n*E^(a + b*x)*(E^(a + b*x)/(1 + E^(2*a + 2*b*x)))^n*(1 + E^(2*a + 2*b*x
))^n*Hypergeometric2F1[n, (1 + n)/2, (3 + n)/2, -E^(2*a + 2*b*x)])/(b*(1 +
 n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

Maple [F]

$$\int e^{bx+a}\operatorname{sech}(bx+a)^n\,dx$$

input
```
int(exp(b*x+a)*sech(b*x+a)^n,x)
```

output
```
int(exp(b*x+a)*sech(b*x+a)^n,x)
```

Fricas [F]

$$\int e^{a+bx}\operatorname{sech}^n(a+bx)\,dx = \int \operatorname{sech}(bx+a)^n e^{(bx+a)}\,dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^n,x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)^n*e^(b*x + a), x)
```

Sympy [F]

$$\int e^{a+bx}\operatorname{sech}^n(a+bx)\,dx = e^a \int e^{bx}\operatorname{sech}^n(a+bx)\,dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)**n,x)
```

output
```
exp(a)*Integral(exp(b*x)*sech(a + b*x)**n, x)
```

Maxima [F]

$$\int e^{a+bx}\operatorname{sech}^n(a+bx)\,dx = \int \operatorname{sech}(bx+a)^n e^{(bx+a)}\,dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^n,x, algorithm="maxima")
```

output
```
integrate(sech(b*x + a)^n*e^(b*x + a), x)
```

Giac [F]

$$\int e^{a+bx}\text{sech}^n(a+bx)\,dx=\int \text{sech}\,(bx+a)^n\,e^{(bx+a)}\,dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^n,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^n*e^(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\text{sech}^n(a+bx)\,dx=\int \mathrm{e}^{a+b\,x}\,\left(\frac{1}{\cosh\left(a+b\,x\right)}\right)^n\,dx$$

input
```
int(exp(a + b*x)*(1/cosh(a + b*x))^n,x)
```

output
```
int(exp(a + b*x)*(1/cosh(a + b*x))^n, x)
```

Reduce [F]

$$\int e^{a+bx}\text{sech}^n(a+bx)\,dx=e^a\left(\int e^{bx}\text{sech}(bx+a)^n\,dx\right)$$

input
```
int(exp(b*x+a)*sech(b*x+a)^n,x)
```

output
```
e**a*int(e**(b*x)*sech(a + b*x)**n,x)
```

3.95 $\int F^{c(a+bx)}(f\,\mathrm{sech}(d+ex))^n\,dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int F^{c(a+bx)}(f\,\mathrm{sech}(d+ex))^n\,dx = \frac{\left(1+e^{2d+2ex}\right)^n F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(n,\frac{1}{2}\left(n+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(2+n+\frac{bc\log(F)}{e}\right),-e^{2d+2ex}\right)(f\,\mathrm{sech}(}{en+bc\log(F)}$$

output
```
(1+exp(2*e*x+2*d))^n*F^(c*(b*x+a))*hypergeom([n, 1/2*n+1/2*b*c*ln(F)/e],[1
+1/2*n+1/2*b*c*ln(F)/e],-exp(2*e*x+2*d))*(f*sech(e*x+d))^n/(e*n+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.03

$$\int F^{c(a+bx)}(f\,\mathrm{sech}(d+ex))^n\,dx = \frac{\left(1+e^{2(d+ex)}\right)^n F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(n,\frac{en+bc\log(F)}{2e},1+\frac{en+bc\log(F)}{2e},-e^{2(d+ex)}\right)(f\,\mathrm{sech}(d+ex))^n}{en+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Sech[d + e*x])^n,x]
```

output
```
((1 + E^(2*(d + e*x)))^n*F^(c*(a + b*x))*Hypergeometric2F1[n, (e*n + b*c*L
og[F])/(2*e), 1 + (e*n + b*c*Log[F])/(2*e), -E^(2*(d + e*x))]*(f*Sech[d +
e*x])^n)/(e*n + b*c*Log[F])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.25, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {7271, 6017, 2689}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\text{sech}(d+ex))^n\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$\text{sech}^{-n}(d+ex)(f\text{sech}(d+ex))^n\int F^{c(a+bx)}\text{sech}^n(d+ex)dx$$

$$\Big\downarrow \text{ 6017}$$

$$e^{-n(d+ex)}\left(e^{2(d+ex)}+1\right)^n(f\text{sech}(d+ex))^n\int e^{dn+exn}\left(1+e^{2(d+ex)}\right)^{-n}F^{ac+bxc}dx$$

$$\Big\downarrow \text{ 2689}$$

$$\frac{e^{-n(d+ex)+dn+enx}\left(e^{2(d+ex)}+1\right)^nF^{ac+bcx}(f\text{sech}(d+ex))^n\,\text{Hypergeometric2F1}\left(n,\frac{en+bc\log(F)}{2e},\frac{en+bc\log(F)}{2e}+1,-\right.}{bc\log(F)+en}$$

input
```
Int[F^(c*(a + b*x))*(f*Sech[d + e*x])^n,x]
```

output
```
(E^(d*n + e*n*x - n*(d + e*x))*(1 + E^(2*(d + e*x)))^n*F^(a*c + b*c*x)*Hyp
ergeometric2F1[n, (e*n + b*c*Log[F])/(2*e), 1 + (e*n + b*c*Log[F])/(2*e),
-E^(2*(d + e*x))]*(f*Sech[d + e*x])^n)/(e*n + b*c*Log[F])
```

Defintions of rubi rules used

rule 2689
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] && !IntegerQ[p]
```

rule 6017
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(1 + E^(2*(d + e*x)))^n*(Sech[d + e*x]^n/E^(n*(d + e*x)))   In
t[SimplifyIntegrand[F^(c*(a + b*x))*(E^(n*(d + e*x))/(1 + E^(2*(d + e*x)))^
n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] && !IntegerQ[n]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}(f\ \mathrm{sech}\,(ex+d))^n\,dx$$

input
```
int(F^(c*(b*x+a))*(f*sech(e*x+d))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*sech(e*x+d))^n,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\mathrm{sech}(d+ex))^n\,dx = \int (f\,\mathrm{sech}\,(ex+d))^n\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sech(e*x+d))^n,x, algorithm="fricas")
```

output
```
integral((f*sech(e*x + d))^n*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}(f\mathrm{sech}(d+ex))^n\,dx = \int F^{c(a+bx)}(f\,\mathrm{sech}\,(d+ex))^n\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*sech(e*x+d))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*sech(d + e*x))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}(f\mathrm{sech}(d+ex))^n\,dx = \int (f\,\mathrm{sech}\,(ex+d))^n\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sech(e*x+d))^n,x, algorithm="maxima")
```

output
```
integrate((f*sech(e*x + d))^n*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\text{sech}(d+ex))^n\,dx=\int (f\,\text{sech}\,(ex+d))^n\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sech(e*x+d))^n,x, algorithm="giac")
```

output
```
integrate((f*sech(e*x + d))^n*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\text{sech}(d+ex))^n\,dx=\int F^{c\,(a+b\,x)}\,\left(\frac{f}{\cosh\left(d+e\,x\right)}\right)^n\,dx$$

input
```
int(F^(c*(a + b*x))*(f/cosh(d + e*x))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f/cosh(d + e*x))^n, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\text{sech}(d+ex))^n\,dx=f^{ac+n}\left(\int f^{bcx}\text{sech}(ex+d)^n\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*sech(e*x+d))^n,x)
```

output
```
f**(a*c + n)*int(f**(b*c*x)*sech(d + e*x)**n,x)
```

3.96 $\int F^{c(a+bx)}\left(f\operatorname{sech}\left(d+\frac{bcx\log(F)}{-2+n}\right)\right)^n dx$

Optimal result

Integrand size = 28, antiderivative size = 132

$$\int F^{c(a+bx)}\left(f\operatorname{sech}\left(d+\frac{bcx\log(F)}{-2+n}\right)\right)^n dx = \frac{f^2F^{c(a+bx)}(2-n)\left(f\operatorname{sech}\left(d-\frac{bcx\log(F)}{2-n}\right)\right)^{-2+n}}{bc(1-n)\log(F)} + \frac{fF^{c(a+bx)}(2-n)\left(f\operatorname{sech}\left(d-\frac{bcx\log(F)}{2-n}\right)\right)^{-1+n}\sinh\left(d-\frac{bcx\log(F)}{2-n}\right)}{bc(1-n)\log(F)}$$

output
```
f^2*F^(c*(b*x+a))*(2-n)*(f*sech(-d+b*c*x*ln(F)/(2-n)))^(-2+n)/b/c/(1-n)/ln
(F)-f*F^(c*(b*x+a))*(2-n)*(f*sech(-d+b*c*x*ln(F)/(2-n)))^(-1+n)*sinh(-d+b*
c*x*ln(F)/(2-n))/b/c/(1-n)/ln(F)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.52

$$\int F^{c(a+bx)}\left(f\operatorname{sech}\left(d+\frac{bcx\log(F)}{-2+n}\right)\right)^n\,dx = \frac{F^{c(a+bx)}\left(1+e^{2d}F^{\frac{2bcx}{-2+n}}\right)(-2+n)\left(f\operatorname{sech}\left(d+\frac{bcx\log(F)}{-2+n}\right)\right)^n}{2bc(-1+n)\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Sech[d + (b*c*x*Log[F])/(-2 + n)])^n,x]
```

output
```
(F^(c*(a + b*x))*(1 + E^(2*d)*F^((2*b*c*x)/(-2 + n)))*(-2 + n)*(f*Sech[d +
 (b*c*x*Log[F])/(-2 + n)])^n)/(2*b*c*(-1 + n)*Log[F])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {7271, 6011}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\left(f\operatorname{sech}\left(\frac{bcx\log(F)}{n-2}+d\right)\right)^n\,dx$$

$$\Big\downarrow \ 7271$$

$$\operatorname{sech}^{-n}\left(d-\frac{bcx\log(F)}{2-n}\right)\left(f\operatorname{sech}\left(d-\frac{bcx\log(F)}{2-n}\right)\right)^n\int F^{c(a+bx)}\operatorname{sech}^n\left(d-\frac{bcx\log(F)}{2-n}\right)dx$$

$$\Big\downarrow \ 6011$$

$$\operatorname{sech}^{-n}\left(d-\frac{bcx\log(F)}{2-n}\right)\left(f\operatorname{sech}\left(d-\frac{bcx\log(F)}{2-n}\right)\right)^n\left(\frac{(2-n)F^{c(a+bx)}\operatorname{sech}^{n-2}\left(d-\frac{bcx\log(F)}{2-n}\right)}{bc(1-n)\log(F)}+\frac{(2-n)F^{c(a+}\cdots\right.$$

input
```
Int[F^(c*(a + b*x))*(f*Sech[d + (b*c*x*Log[F])/(-2 + n)])^n,x]
```

output
```
((f*Sech[d - (b*c*x*Log[F])/(2 - n)])^n*((F^(c*(a + b*x))*(2 - n)*Sech[d -
 (b*c*x*Log[F])/(2 - n)]^(-2 + n))/(b*c*(1 - n)*Log[F]) + (F^(c*(a + b*x))
*(2 - n)*Sech[d - (b*c*x*Log[F])/(2 - n)]^(-1 + n)*Sinh[d - (b*c*x*Log[F])
/(2 - n)])/(b*c*(1 - n)*Log[F])))/Sech[d - (b*c*x*Log[F])/(2 - n)]^n
```

Defintions of rubi rules used

rule 6011
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sech[d + e*x]^(n - 2)/(e^2*(n - 1)*
(n - 2))), x] + Simp[F^(c*(a + b*x))*Sech[d + e*x]^(n - 1)*(Sinh[d + e*x]/(
e*(n - 1))), x] /; FreeQ[{F, a, b, c, d, e, n}, x] && EqQ[e^2*(n - 2)^2 - b
^2*c^2*Log[F]^2, 0] && NeQ[n, 1] && NeQ[n, 2]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}\left(f\ \mathrm{sech}\left(d+\frac{bcx\ln(F)}{n-2}\right)\right)^{n}dx$$

input
```
int(F^(c*(b*x+a))*(f*sech(d+b*c*x*ln(F)/(n-2)))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*sech(d+b*c*x*ln(F)/(n-2)))^n,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. 2(125) = 250.

Time = 0.10 (sec) , antiderivative size = 500, normalized size of antiderivative = 3.79

$$\int F^{c(a+bx)}\left(f\text{sech}\left(d+\frac{bcx\log(F)}{-2+n}\right)\right)^n dx$$

$$= \frac{\left((n-2)\cosh\left((bcx+ac)\log\left(F\right)\right)\cosh\left(\frac{bcx\log(F)+dn-2\,d}{n-2}\right)+(n-2)\cosh\left(\frac{bcx\log(F)+dn-2\,d}{n-2}\right)\sinh\left((bcx+a\right.\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*(f*sech(d+b*c*x*log(F)/(-2+n)))^n,x, algorithm="fr
icas")
```

output
```
(((n - 2)*cosh((b*c*x + a*c)*log(F))*cosh((b*c*x*log(F) + d*n - 2*d)/(n -
2)) + (n - 2)*cosh((b*c*x*log(F) + d*n - 2*d)/(n - 2))*sinh((b*c*x + a*c)*
log(F)))*cosh(n*log(2*(f*cosh((b*c*x*log(F) + d*n - 2*d)/(n - 2)) + f*sinh
((b*c*x*log(F) + d*n - 2*d)/(n - 2)))/(cosh((b*c*x*log(F) + d*n - 2*d)/(n
- 2))^2 + 2*cosh((b*c*x*log(F) + d*n - 2*d)/(n - 2))*sinh((b*c*x*log(F) +
d*n - 2*d)/(n - 2)) + sinh((b*c*x*log(F) + d*n - 2*d)/(n - 2))^2 + 1))) +
((n - 2)*cosh((b*c*x + a*c)*log(F))*cosh((b*c*x*log(F) + d*n - 2*d)/(n - 2
)) + (n - 2)*cosh((b*c*x*log(F) + d*n - 2*d)/(n - 2))*sinh((b*c*x + a*c)*l
og(F)))*sinh(n*log(2*(f*cosh((b*c*x*log(F) + d*n - 2*d)/(n - 2)) + f*sinh(
(b*c*x*log(F) + d*n - 2*d)/(n - 2)))/(cosh((b*c*x*log(F) + d*n - 2*d)/(n -
 2))^2 + 2*cosh((b*c*x*log(F) + d*n - 2*d)/(n - 2))*sinh((b*c*x*log(F) + d
*n - 2*d)/(n - 2)) + sinh((b*c*x*log(F) + d*n - 2*d)/(n - 2))^2 + 1))))/((
b*c*n - b*c)*cosh((b*c*x*log(F) + d*n - 2*d)/(n - 2))*log(F) - (b*c*n - b*
c)*log(F)*sinh((b*c*x*log(F) + d*n - 2*d)/(n - 2)))
```

Sympy [F]

$$\int F^{c(a+bx)}\left(f\operatorname{sech}\left(d+\frac{bcx\log(F)}{-2+n}\right)\right)^n dx=\int F^{c(a+bx)}\left(f\operatorname{sech}\left(\frac{bcx\log(F)}{n-2}+d\right)\right)^n dx$$

input
```
integrate(F**(c*(b*x+a))*(f*sech(d+b*c*x*ln(F)/(-2+n)))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*sech(b*c*x*log(F)/(n - 2) + d))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\left(f\operatorname{sech}\left(d+\frac{bcx\log(F)}{-2+n}\right)\right)^n dx=\int\left(f\operatorname{sech}\left(\frac{bcx\log(F)}{n-2}+d\right)\right)^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sech(d+b*c*x*log(F)/(-2+n)))^n,x, algorithm="ma
xima")
```

output
```
integrate((f*sech(b*c*x*log(F)/(n - 2) + d))^n*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\left(f\operatorname{sech}\left(d+\frac{bcx\log(F)}{-2+n}\right)\right)^n dx=\int\left(f\operatorname{sech}\left(\frac{bcx\log(F)}{n-2}+d\right)\right)^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sech(d+b*c*x*log(F)/(-2+n)))^n,x, algorithm="gi
ac")
```

output
```
integrate((f*sech(b*c*x*log(F)/(n - 2) + d))^n*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\left(f\mathrm{sech}\left(d+\frac{bcx\log(F)}{-2+n}\right)\right)^n\,dx=\int F^{c\,(a+b\,x)}\left(\frac{f}{\cosh\left(d+\frac{b\,c\,x\,\ln(F)}{n-2}\right)}\right)^n\,dx$$

input
```
int(F^(c*(a + b*x))*(f/cosh(d + (b*c*x*log(F))/(n - 2)))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f/cosh(d + (b*c*x*log(F))/(n - 2)))^n, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\left(f\mathrm{sech}\left(d+\frac{bcx\log(F)}{-2+n}\right)\right)^n\,dx$$
$$=f^{ac+n}\left(\int f^{bcx}\mathrm{sech}\left(\frac{\log(f)\,bcx+dn-2d}{n-2}\right)^n\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*sech(d+b*c*x*log(F)/(-2+n)))^n,x)
```

output
```
f**(a*c + n)*int(f**(b*c*x)*sech((log(f)*b*c*x + d*n - 2*d)/(n - 2))**n,x)
```

3.97 $\int F^{c(a+bx)}\left(f\text{sech}\left(d-\frac{bcx\log(F)}{-2+n}\right)\right)^n\,dx$

Optimal result

Integrand size = 29, antiderivative size = 130

$$\int F^{c(a+bx)}\left(f\text{sech}\left(d-\frac{bcx\log(F)}{-2+n}\right)\right)^n\,dx = \frac{f^2F^{c(a+bx)}(2-n)\left(f\text{sech}\left(d+\frac{bcx\log(F)}{2-n}\right)\right)^{-2+n}}{bc(1-n)\log(F)} - \frac{fF^{c(a+bx)}(2-n)\left(f\text{sech}\left(d+\frac{bcx\log(F)}{2-n}\right)\right)^{-1+n}\sinh\left(d+\frac{bcx\log(F)}{2-n}\right)}{bc(1-n)\log(F)}$$

output
```
f^2*F^(c*(b*x+a))*(2-n)*(f*sech(d+b*c*x*ln(F)/(2-n)))^(-2+n)/b/c/(1-n)/ln(
F)-f*F^(c*(b*x+a))*(2-n)*(f*sech(d+b*c*x*ln(F)/(2-n)))^(-1+n)*sinh(d+b*c*x
*ln(F)/(2-n))/b/c/(1-n)/ln(F)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.56

$$\int F^{c(a+bx)}\left(f\text{sech}\left(d-\frac{bcx\log(F)}{-2+n}\right)\right)^n\,dx = \frac{e^{-2d}F^{c(a+bx)}\left(e^{2d}+F^{\frac{2bcx}{-2+n}}\right)(-2+n)\left(f\text{sech}\left(d-\frac{bcx\log(F)}{-2+n}\right)\right)^n}{2bc(-1+n)\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Sech[d - (b*c*x*Log[F])/(-2 + n)])^n,x]
```

output
```
(F^(c*(a + b*x))*(E^(2*d) + F^((2*b*c*x)/(-2 + n)))*(-2 + n)*(f*Sech[d - (
b*c*x*Log[F])/(-2 + n)])^n)/(2*b*c*E^(2*d)*(-1 + n)*Log[F])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {7271, 6011}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\left(f\text{sech}\left(d-\frac{bcx\log(F)}{n-2}\right)\right)^n\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$\text{sech}^{-n}\left(\frac{bcx\log(F)}{2-n}+d\right)\left(f\text{sech}\left(\frac{bcx\log(F)}{2-n}+d\right)\right)^n\int F^{c(a+bx)}\text{sech}^n\left(d+\frac{bcx\log(F)}{2-n}\right)dx$$

$$\Big\downarrow \text{ 6011}$$

$$\text{sech}^{-n}\left(\frac{bcx\log(F)}{2-n}+d\right)\left(f\text{sech}\left(\frac{bcx\log(F)}{2-n}+d\right)\right)^n\left(\frac{(2-n)F^{c(a+bx)}\text{sech}^{n-2}\left(\frac{bcx\log(F)}{2-n}+d\right)}{bc(1-n)\log(F)}-\frac{(2-n)F^{c(a-}}{}\right.$$

input
```
Int[F^(c*(a + b*x))*(f*Sech[d - (b*c*x*Log[F])/(-2 + n)])^n,x]
```

output
```
((f*Sech[d + (b*c*x*Log[F])/(2 - n)])^n*((F^(c*(a + b*x))*(2 - n)*Sech[d +
 (b*c*x*Log[F])/(2 - n)]^(-2 + n))/(b*c*(1 - n)*Log[F]) - (F^(c*(a + b*x))
*(2 - n)*Sech[d + (b*c*x*Log[F])/(2 - n)]^(-1 + n)*Sinh[d + (b*c*x*Log[F])
/(2 - n)])/(b*c*(1 - n)*Log[F])))/Sech[d + (b*c*x*Log[F])/(2 - n)]^n
```

Defintions of rubi rules used

rule 6011
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sech[d + e*x]^(n - 2)/(e^2*(n - 1)*
(n - 2))), x] + Simp[F^(c*(a + b*x))*Sech[d + e*x]^(n - 1)*(Sinh[d + e*x]/(
e*(n - 1))), x] /; FreeQ[{F, a, b, c, d, e, n}, x] && EqQ[e^2*(n - 2)^2 - b
^2*c^2*Log[F]^2, 0] && NeQ[n, 1] && NeQ[n, 2]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}\left(f\,\mathrm{sech}\left(-d+\frac{bcx\ln\left(F\right)}{n-2}\right)\right)^{n}dx$$

input
```
int(F^(c*(b*x+a))*(f*sech(-d+b*c*x*ln(F)/(n-2)))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*sech(-d+b*c*x*ln(F)/(n-2)))^n,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(119) = 238$.

Time = 0.09 (sec) , antiderivative size = 518, normalized size of antiderivative = 3.98

$$\int F^{c(a+bx)}\left(f\text{sech}\left(d-\frac{bcx\log(F)}{-2+n}\right)\right)^n\,dx$$

$$= \frac{\left((n-2)\cosh\left((bcx+ac)\log\left(F\right)\right)\cosh\left(\frac{bcx\log(F)-dn+2\,d}{n-2}\right)+(n-2)\cosh\left(\frac{bcx\log(F)-dn+2\,d}{n-2}\right)\sinh\left((bcx+a\right.\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*(f*sech(-d+b*c*x*log(F)/(-2+n)))^n,x, algorithm="f
ricas")
```

output
```
(((n - 2)*cosh((b*c*x + a*c)*log(F))*cosh((b*c*x*log(F) - d*n + 2*d)/(n -
2)) + (n - 2)*cosh((b*c*x*log(F) - d*n + 2*d)/(n - 2))*sinh((b*c*x + a*c)*
log(F)))*cosh(n*log(2*(f*cosh((b*c*x*log(F) - d*n + 2*d)/(n - 2)) + f*sinh
((b*c*x*log(F) - d*n + 2*d)/(n - 2)))/(cosh((b*c*x*log(F) - d*n + 2*d)/(n
- 2))^2 + 2*cosh((b*c*x*log(F) - d*n + 2*d)/(n - 2))*sinh((b*c*x*log(F) -
d*n + 2*d)/(n - 2)) + sinh((b*c*x*log(F) - d*n + 2*d)/(n - 2))^2 + 1))) +
((n - 2)*cosh((b*c*x + a*c)*log(F))*cosh((b*c*x*log(F) - d*n + 2*d)/(n - 2
)) + (n - 2)*cosh((b*c*x*log(F) - d*n + 2*d)/(n - 2))*sinh((b*c*x + a*c)*l
og(F)))*sinh(n*log(2*(f*cosh((b*c*x*log(F) - d*n + 2*d)/(n - 2)) + f*sinh(
(b*c*x*log(F) - d*n + 2*d)/(n - 2)))/(cosh((b*c*x*log(F) - d*n + 2*d)/(n -
 2))^2 + 2*cosh((b*c*x*log(F) - d*n + 2*d)/(n - 2))*sinh((b*c*x*log(F) - d
*n + 2*d)/(n - 2)) + sinh((b*c*x*log(F) - d*n + 2*d)/(n - 2))^2 + 1))))/((
b*c*n - b*c)*cosh((b*c*x*log(F) - d*n + 2*d)/(n - 2))*log(F) - (b*c*n - b*
c)*log(F)*sinh((b*c*x*log(F) - d*n + 2*d)/(n - 2)))
```

Sympy [F]

$$\int F^{c(a+bx)}\left(f\text{sech}\left(d-\frac{bcx\log(F)}{-2+n}\right)\right)^n dx = \int F^{c(a+bx)}\left(f\operatorname{sech}\left(\frac{bcx\log(F)}{n-2}-d\right)\right)^n dx$$

input
```
integrate(F**(c*(b*x+a))*(f*sech(-d+b*c*x*ln(F)/(-2+n)))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*sech(b*c*x*log(F)/(n - 2) - d))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\left(f\text{sech}\left(d-\frac{bcx\log(F)}{-2+n}\right)\right)^n dx = \int \left(f\operatorname{sech}\left(\frac{bcx\log(F)}{n-2}-d\right)\right)^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sech(-d+b*c*x*log(F)/(-2+n)))^n,x, algorithm="m
axima")
```

output
```
integrate((f*sech(b*c*x*log(F)/(n - 2) - d))^n*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\left(f\text{sech}\left(d-\frac{bcx\log(F)}{-2+n}\right)\right)^n dx = \int \left(f\operatorname{sech}\left(\frac{bcx\log(F)}{n-2}-d\right)\right)^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sech(-d+b*c*x*log(F)/(-2+n)))^n,x, algorithm="g
iac")
```

output
```
integrate((f*sech(b*c*x*log(F)/(n - 2) - d))^n*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\left(f\operatorname{sech}\left(d-\frac{bcx\log(F)}{-2+n}\right)\right)^n dx=\int F^{c\,(a+b\,x)}\,{\left(\frac{f}{\cosh\left(d-\frac{b\,c\,x\,\ln(F)}{n-2}\right)}\right)}^n\,dx$$

input
```
int(F^(c*(a + b*x))*(f/cosh(d - (b*c*x*log(F))/(n - 2)))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f/cosh(d - (b*c*x*log(F))/(n - 2)))^n, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\left(f\operatorname{sech}\left(d-\frac{bcx\log(F)}{-2+n}\right)\right)^n dx$$
$$=f^{ac+n}\left(\int f^{bcx}\operatorname{sech}\left(\frac{\log(f)\,bcx-dn+2d}{n-2}\right)^n dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*sech(-d+b*c*x*log(F)/(-2+n)))^n,x)
```

output
```
f**(a*c + n)*int(f**(b*c*x)*sech((log(f)*b*c*x - d*n + 2*d)/(n - 2))**n,x)
```

3.98 $\int e^{a+bx}\operatorname{csch}(d+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int e^{a+bx}\operatorname{csch}(d+bx)\,dx=\frac{e^{a-d}\log\left(1-e^{2d+2bx}\right)}{b}$$

output
```
exp(a-d)*ln(1-exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int e^{a+bx}\operatorname{csch}(d+bx)\,dx=\frac{e^{a}\log\left(\left(-1+e^{2bx}\right)\cosh(d)+\left(1+e^{2bx}\right)\sinh(d)\right)\left(\cosh(d)-\sinh(d)\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Csch[d + b*x],x]
```

output
```
(E^a*Log[(-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d]]*(Cosh[d] - Si
nh[d]))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2720, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{csch}(bx+d)\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int -\frac{2e^{a+bx}}{1-e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{2e^a\int \frac{e^{bx}}{1-e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \; 240$$

$$\frac{e^a\log\left(1-e^{2bx}\right)}{b}$$

input
```
Int[E^(a + b*x)*Csch[d + b*x],x]
```

output
```
(E^a*Log[1 - E^(2*b*x)])/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.73

method	result	size
risch	$-\frac{2\,{\mathrm e}^{a-d} a}{b}+\frac{\ln\left({\mathrm e}^{2 b x+2 a}-{\mathrm e}^{2 a-2 d}\right) {\mathrm e}^{a-d}}{b}$	45

input
```
int(exp(b*x+a)*csch(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
-2/b*exp(a-d)*a+ln(exp(2*b*x+2*a)-exp(2*a-2*d))/b*exp(a-d)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.73

$$\int e^{a+bx}\mathrm{csch}(d+bx)\,dx=\frac{(\cosh\left(-a+d\right)-\sinh\left(-a+d\right))\log\left(\frac{2\,\sinh(bx+d)}{\cosh(bx+d)-\sinh(bx+d)}\right)}{b}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d),x, algorithm="fricas")
```

output
```
(cosh(-a + d) - sinh(-a + d))*log(2*sinh(b*x + d)/(cosh(b*x + d) - sinh(b*
x + d)))/b
```

Sympy [F]

$$\int e^{a+bx}\operatorname{csch}(d+bx)\,dx = e^a\int e^{bx}\operatorname{csch}(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*csch(b*x+d),x)
```

output
```
exp(a)*Integral(exp(b*x)*csch(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.73

$$\int e^{a+bx}\operatorname{csch}(d+bx)\,dx = \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^a\right)}{b} + \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}-e^a\right)}{b}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d),x, algorithm="maxima")
```

output
```
e^(a - d)*log(e^(b*x + a + d) + e^a)/b + e^(a - d)*log(e^(b*x + a + d) - e
^a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int e^{a+bx}\operatorname{csch}(d+bx)\,dx = \frac{e^{(a-d)}\log\left(\left|e^{(2\,bx+2\,d)}-1\right|\right)}{b}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d),x, algorithm="giac")
```

output
```
e^(a - d)*log(abs(e^(2*b*x + 2*d) - 1))/b
```

Mupad [B] (verification not implemented)

Time = 2.75 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int e^{a+bx}\text{csch}(d+bx)\,dx=\frac{\mathrm{e}^{a-d}\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}\right)}{b}$$

input
```
int(exp(a + b*x)/sinh(d + b*x),x)
```

output
```
(exp(a - d)*log(exp(2*a)*exp(2*b*x) - exp(2*a)*exp(-2*d)))/b
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int e^{a+bx}\text{csch}(d+bx)\,dx=\frac{e^{a}\left(\log\left(e^{bx+d}-1\right)+\log\left(e^{bx+d}+1\right)\right)}{e^{d}b}$$

input
```
int(exp(b*x+a)*csch(b*x+d),x)
```

output
```
(e**a*(log(e**(b*x + d) - 1) + log(e**(b*x + d) + 1)))/(e**d*b)
```

3.99 $\int e^{a+bx}\mathbf{csch}^2(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 49

$$\int e^{a+bx}\operatorname{csch}^2(d+bx)\,dx = \frac{2e^{a+bx}}{b\left(1-e^{2d+2bx}\right)} - \frac{2e^{a-d}\operatorname{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
2*exp(b*x+a)/b/(1-exp(2*b*x+2*d))-2*exp(a-d)*arctanh(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.67

$$\int e^{a+bx}\operatorname{csch}^2(d+bx)\,dx = -\frac{2e^a(\cosh(d)-\sinh(d))\left(e^{bx}+\operatorname{arctanh}\left(e^{bx}(\cosh(d)+\sinh(d))\right)\left(\left(-1+e^{2bx}\right)\cosh(d)+\left(1+e^{2bx}\right)\sinh\right.\right.}{b\left(\left(-1+e^{2bx}\right)\cosh(d)+\left(1+e^{2bx}\right)\sinh(d)\right)}$$

input
```
Integrate[E^(a + b*x)*Csch[d + b*x]^2,x]
```

output
```
(-2*E^a*(Cosh[d] - Sinh[d])*(E^(b*x) + ArcTanh[E^(b*x)*(Cosh[d] + Sinh[d])
]*((-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d])))/(b*((-1 + E^(2*b*
x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d]))
```

Rubi [A] (warning: unable to verify)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 27, 252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\operatorname{csch}^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{4e^{a+2bx}}{\left(1-e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{4e^a\int \frac{e^{2bx}}{\left(1-e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{4e^a\left(\frac{e^{bx}}{2\left(1-e^{2bx}\right)}-\frac{1}{2}\int \frac{1}{1-e^{2bx}}de^{bx}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{4e^a\left(\frac{e^{bx}}{2\left(1-e^{2bx}\right)}-\frac{1}{2}\text{arctanh}\left(e^{bx}\right)\right)}{b}$$

input
```
Int[E^(a + b*x)*Csch[d + b*x]^2,x]
```

output
```
(4*E^a*(E^(b*x)/(2*(1 - E^(2*b*x))) - ArcTanh[E^(b*x)]/2))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.78

method	result	size
risch	$\frac{2\,\mathrm{e}^{bx+3a}}{\left(-\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)b}+\frac{\ln\left(\mathrm{e}^{bx+a}-\mathrm{e}^{a-d}\right)\mathrm{e}^{a-d}}{b}-\frac{\ln\left(\mathrm{e}^{bx+a}+\mathrm{e}^{a-d}\right)\mathrm{e}^{a-d}}{b}$	87

input
```
int(exp(b*x+a)*csch(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
2/(-exp(2*b*x+2*a+2*d)+exp(2*a))/b*exp(b*x+3*a)+ln(exp(b*x+a)-exp(a-d))/b*
exp(a-d)-ln(exp(b*x+a)+exp(a-d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(43) = 86$.

Time = 0.09 (sec) , antiderivative size = 332, normalized size of antiderivative = 6.78

$$\int e^{a+bx}\operatorname{csch}^2(d+bx)\,dx =$$
$$-\frac{2\cosh(bx+d)\cosh(-a+d)+\left(\cosh(bx+d)^2\cosh(-a+d)+\left(\cosh(-a+d)-\sinh(-a+d)\right)\sin}{}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^2,x, algorithm="fricas")
```

output
```
-(2*cosh(b*x + d)*cosh(-a + d) + (cosh(b*x + d)^2*cosh(-a + d) + (cosh(-a
+ d) - sinh(-a + d))*sinh(b*x + d)^2 + 2*(cosh(b*x + d)*cosh(-a + d) - cos
h(b*x + d)*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^2 - 1)*sinh(-a + d
) - cosh(-a + d))*log(cosh(b*x + d) + sinh(b*x + d) + 1) - (cosh(b*x + d)^
2*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d)^2 + 2*(cosh(b
*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d) - (cosh(b
*x + d)^2 - 1)*sinh(-a + d) - cosh(-a + d))*log(cosh(b*x + d) + sinh(b*x +
 d) - 1) + 2*(cosh(-a + d) - sinh(-a + d))*sinh(b*x + d) - 2*cosh(b*x + d)
*sinh(-a + d))/(b*cosh(b*x + d)^2 + 2*b*cosh(b*x + d)*sinh(b*x + d) + b*si
nh(b*x + d)^2 - b)
```

Sympy [F]

$$\int e^{a+bx}\operatorname{csch}^2(d+bx)\,dx = e^a\int e^{bx}\operatorname{csch}^2(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)**2,x)
```

output
```
exp(a)*Integral(exp(b*x)*csch(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.63

$$\int e^{a+bx}\operatorname{csch}^2(d+bx)\,dx = -\frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^a\right)}{b} + \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}-e^a\right)}{b} - \frac{2\,e^{(bx+3\,a)}}{b\left(e^{(2\,bx+2\,a+2\,d)}-e^{(2\,a)}\right)}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^2,x, algorithm="maxima")
```

output
```
-e^(a - d)*log(e^(b*x + a + d) + e^a)/b + e^(a - d)*log(e^(b*x + a + d) -
e^a)/b - 2*e^(b*x + 3*a)/(b*(e^(2*b*x + 2*a + 2*d) - e^(2*a)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.45

$$\int e^{a+bx}\operatorname{csch}^2(d+bx)\,dx = -\left(\frac{e^{(-3\,d)}\log\left(e^{(bx+d)}+1\right)}{b} - \frac{e^{(-3\,d)}\log\left(\left|e^{(bx+d)}-1\right|\right)}{b} + \frac{2\,e^{(bx-2\,d)}}{b\left(e^{(2\,bx+2\,d)}-1\right)}\right)e^{(a+2\,d)}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^2,x, algorithm="giac")
```

output
```
-(e^(-3*d)*log(e^(b*x + d) + 1)/b - e^(-3*d)*log(abs(e^(b*x + d) - 1))/b +
 2*e^(b*x - 2*d)/(b*(e^(2*b*x + 2*d) - 1)))*e^(a + 2*d)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.82

$$\int e^{a+bx}\mathrm{csch}^2(d+bx)\,dx = \frac{2\,\mathrm{e}^{3\,a}\,\mathrm{e}^{-2\,d}\,\mathrm{e}^{b\,x}}{b\,\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}-b\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b\,\sqrt{\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}}}\right)\,\sqrt{\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}}}{\sqrt{-b^2}}$$

input
```
int(exp(a + b*x)/sinh(d + b*x)^2,x)
```

output
```
(2*exp(3*a)*exp(-2*d)*exp(b*x))/(b*exp(2*a)*exp(-2*d) - b*exp(2*a)*exp(2*b
*x)) - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/(b*(exp(2*a)*exp(-2*d))^(1/2
)))*(exp(2*a)*exp(-2*d))^(1/2))/(-b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.06

$$\int e^{a+bx}\mathrm{csch}^2(d+bx)\,dx = \frac{e^a\left(e^{2bx+2d}\log\left(e^{bx+d}-1\right)-e^{2bx+2d}\log\left(e^{bx+d}+1\right)-2e^{bx+d}-\log\left(e^{bx+d}-1\right)+\log\left(e^{bx+d}+1\right)\right)}{e^d b\left(e^{2bx+2d}-1\right)}$$

input
```
int(exp(b*x+a)*csch(b*x+d)^2,x)
```

output
```
(e**a*(e**(2*b*x + 2*d)*log(e**(b*x + d) - 1) - e**(2*b*x + 2*d)*log(e**(b
*x + d) + 1) - 2*e**(b*x + d) - log(e**(b*x + d) - 1) + log(e**(b*x + d) +
 1)))/(e**d*b*(e**(2*b*x + 2*d) - 1))
```

3.100 $\int e^{a+bx}\text{csch}^3(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 32

$$\int e^{a+bx}\text{csch}^3(d+bx)\,dx = -\frac{2e^{a+3d+4bx}}{b\left(1-e^{2d+2bx}\right)^2}$$

output
```
-2*exp(4*b*x+a+3*d)/b/(1-exp(2*b*x+2*d))^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.97

$$\int e^{a+bx}\text{csch}^3(d+bx)\,dx = \frac{2e^a(\cosh(d)-\sinh(d))\left(-2e^{2bx}+\cosh^2(d)-2\cosh(d)\sinh(d)+\sinh^2(d)\right)}{b\left((-1+e^{2bx})\cosh(d)+(1+e^{2bx})\sinh(d)\right)^2}$$

input
```
Integrate[E^(a + b*x)*Csch[d + b*x]^3,x]
```

output
```
(2*E^a*(Cosh[d] - Sinh[d])*(-2*E^(2*b*x) + Cosh[d]^2 - 2*Cosh[d]*Sinh[d] +
 Sinh[d]^2))/(b*((-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d])^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2720, 27, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{csch}^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{8e^{a+3bx}}{(1-e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{8e^a\int \frac{e^{3bx}}{(1-e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{242}$$

$$-\frac{2e^{a+4bx}}{b(1-e^{2bx})^2}$$

input
```
Int[E^(a + b*x)*Csch[d + b*x]^3,x]
```

output
```
(-2*E^(a + 4*b*x))/(b*(1 - E^(2*b*x))^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

method	result	size
parallelrisch	$-\frac{e^{bx+a}\left(\coth\left(\frac{bx}{2}+\frac{d}{2}\right)+2+\tanh\left(\frac{bx}{2}+\frac{d}{2}\right)\right)\left(\coth\left(\frac{bx}{2}+\frac{d}{2}\right)-\tanh\left(\frac{bx}{2}+\frac{d}{2}\right)\right)}{8b}$	53
risch	$\frac{2\left(-2\,e^{2bx+2a+2d}+e^{2a}\right)e^{3a-d}}{\left(-e^{2bx+2a+2d}+e^{2a}\right)^{2}b}$	54

input
```
int(exp(b*x+a)*csch(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*exp(b*x+a)*(coth(1/2*b*x+1/2*d)+2+tanh(1/2*b*x+1/2*d))*(coth(1/2*b*x+
1/2*d)-tanh(1/2*b*x+1/2*d))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(28) = 56.

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.88

$$\int e^{a+bx}\operatorname{csch}^3(d+bx)\,dx =$$
$$-\frac{2\left(\cosh\left(bx+d\right)\cosh\left(-a+d\right)+3\left(\cosh\left(-a+d\right)-\sinh\left(-a+d\right)\right)\sinh\left(bx+d\right)-\cosh\left(bx+\right.\right.}{b\cosh\left(bx+d\right)^{3}+3\,b\cosh\left(bx+d\right)\sinh\left(bx+d\right)^{2}+b\sinh\left(bx+d\right)^{3}-b\cosh\left(bx+d\right)+3\left(b\cosh\left(bx\right.\right.}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^3,x, algorithm="fricas")
```

output
```
-2*(cosh(b*x + d)*cosh(-a + d) + 3*(cosh(-a + d) - sinh(-a + d))*sinh(b*x
+ d) - cosh(b*x + d)*sinh(-a + d))/(b*cosh(b*x + d)^3 + 3*b*cosh(b*x + d)*
sinh(b*x + d)^2 + b*sinh(b*x + d)^3 - b*cosh(b*x + d) + 3*(b*cosh(b*x + d)
^2 - b)*sinh(b*x + d))
```

Sympy [F]

$$\int e^{a+bx}\operatorname{csch}^3(d+bx)\,dx = e^a \int e^{bx}\operatorname{csch}^3\left(bx+d\right)dx$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)**3,x)
```

output
```
exp(a)*Integral(exp(b*x)*csch(b*x + d)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(28) = 56$.

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.03

$$\int e^{a+bx}\operatorname{csch}^3(d+bx)\,dx = -\frac{4\,e^{(2\,bx+5\,a+2\,d)}}{b\left(e^{(4\,bx+4\,a+5\,d)}-2\,e^{(2\,bx+4\,a+3\,d)}+e^{(4\,a+d)}\right)} + \frac{2\,e^{(5\,a)}}{b\left(e^{(4\,bx+4\,a+5\,d)}-2\,e^{(2\,bx+4\,a+3\,d)}+e^{(4\,a+d)}\right)}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^3,x, algorithm="maxima")
```

output
```
-4*e^(2*b*x + 5*a + 2*d)/(b*(e^(4*b*x + 4*a + 5*d) - 2*e^(2*b*x + 4*a + 3*
d) + e^(4*a + d))) + 2*e^(5*a)/(b*(e^(4*b*x + 4*a + 5*d) - 2*e^(2*b*x + 4*
a + 3*d) + e^(4*a + d)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int e^{a+bx}\operatorname{csch}^3(d+bx)\,dx=-\frac{2\left(2\,e^{(2\,bx+2\,d)}-1\right)e^{(a-d)}}{b\left(e^{(2\,bx+2\,d)}-1\right)^2}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^3,x, algorithm="giac")
```

output
```
-2*(2*e^(2*b*x + 2*d) - 1)*e^(a - d)/(b*(e^(2*b*x + 2*d) - 1)^2)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\operatorname{csch}^3(d+bx)\,dx=\int\frac{e^{a+b\,x}}{\sinh\left(d+b\,x\right)^3}\,dx$$

input
```
int(exp(a + b*x)/sinh(d + b*x)^3,x)
```

output
```
int(exp(a + b*x)/sinh(d + b*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int e^{a+bx}\operatorname{csch}^3(d+bx)\,dx=-\frac{2e^{4bx+a+3d}}{b\left(e^{4bx+4d}-2e^{2bx+2d}+1\right)}$$

input
```
int(exp(b*x+a)*csch(b*x+d)^3,x)
```

output
```
( - 2*e**(a + 4*b*x + 3*d))/(b*(e**(4*b*x + 4*d) - 2*e**(2*b*x + 2*d) + 1)
)
```

3.101 $\int e^{a+bx}\mathbf{csch}^4(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 109

$$\int e^{a+bx}\text{csch}^4(d+bx)\,dx=\frac{8e^{a+2d+3bx}}{3b\left(1-e^{2d+2bx}\right)^3}-\frac{2e^{a+bx}}{b\left(1-e^{2d+2bx}\right)^2}+\frac{e^{a+bx}}{b\left(1-e^{2d+2bx}\right)}+\frac{e^{a-d}\text{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
8/3*exp(3*b*x+a+2*d)/b/(1-exp(2*b*x+2*d))^3-2*exp(b*x+a)/b/(1-exp(2*b*x+2*
d))^2+exp(b*x+a)/b/(1-exp(2*b*x+2*d))+exp(a-d)*arctanh(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.50

$$\int e^{a+bx}\text{csch}^4(d+bx)\,dx$$
$$=\frac{e^a\left(3\text{arctanh}\left(e^{bx}(\cosh(d)+\sinh(d))\right)\cosh(d)-3\text{arctanh}\left(e^{bx}(\cosh(d)+\sinh(d))\right)\sinh(d)-\frac{8e^{bx}(c}{((-1+e^{2bx})\text{ co}}\right.}{3b}$$

input
```
Integrate[E^(a + b*x)*Csch[d + b*x]^4,x]
```

output
```
(E^a*(3*ArcTanh[E^(b*x)*(Cosh[d] + Sinh[d])]*Cosh[d] - 3*ArcTanh[E^(b*x)*(
Cosh[d] + Sinh[d])]*Sinh[d] - (8*E^(b*x)*(Cosh[d] - Sinh[d])^3)/((-1 + E^(
2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d])^3 - (14*E^(b*x)*(Cosh[d] - Sinh
[d])^2)/((-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d])^2 + (3*E^(b*x
)*(-Cosh[d] + Sinh[d]))/((-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d
]))/(3*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2720, 27, 252, 252, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{csch}^4(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{16e^{a+4bx}}{\left(1-e^{2bx}\right)^4}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{16e^a\int \frac{e^{4bx}}{\left(1-e^{2bx}\right)^4}de^{bx}}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{16e^a\left(\frac{e^{3bx}}{6\left(1-e^{2bx}\right)^3}-\frac{1}{2}\int \frac{e^{2bx}}{\left(1-e^{2bx}\right)^3}de^{bx}\right)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{16e^a\left(\frac{1}{2}\left(\frac{1}{4}\int \frac{1}{\left(1-e^{2bx}\right)^2}de^{bx}-\frac{e^{bx}}{4\left(1-e^{2bx}\right)^2}\right)+\frac{e^{3bx}}{6\left(1-e^{2bx}\right)^3}\right)}{b}$$

$$\Big\downarrow \text{215}$$

$$\frac{16e^a\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\int\frac{1}{1-e^{2bx}}de^{bx}+\frac{e^{bx}}{2(1-e^{2bx})}\right)-\frac{e^{bx}}{4(1-e^{2bx})^2}\right)+\frac{e^{3bx}}{6(1-e^{2bx})^3}\right)}{b}$$

$$\Big\downarrow \; 219$$

$$\frac{16e^a\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\text{arctanh}(e^{bx})+\frac{e^{bx}}{2(1-e^{2bx})}\right)-\frac{e^{bx}}{4(1-e^{2bx})^2}\right)+\frac{e^{3bx}}{6(1-e^{2bx})^3}\right)}{b}$$

input
```
Int[E^(a + b*x)*Csch[d + b*x]^4,x]
```

output
```
(16*E^a*(E^(3*b*x)/(6*(1 - E^(2*b*x))^3) + (-1/4*E^(b*x)/(1 - E^(2*b*x))^2
 + (E^(b*x)/(2*(1 - E^(2*b*x))) + ArcTanh[E^(b*x)]/2)/4)/2))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.13

method	result	size
risch	$-\frac{\left(-3\,{\mathrm e}^{4 b x +4 a +4 d}-8\,{\mathrm e}^{2 b x +4 a +2 d}+3\,{\mathrm e}^{4 a}\right) {\mathrm e}^{b x +3 a}}{3 \left(-{\mathrm e}^{2 b x +2 a +2 d}+{\mathrm e}^{2 a}\right)^{3} b}+\frac{\ln \left({\mathrm e}^{b x +a}+{\mathrm e}^{a -d}\right) {\mathrm e}^{a -d}}{2 b}-\frac{\ln \left({\mathrm e}^{b x +a}-{\mathrm e}^{a -d}\right) {\mathrm e}^{a -d}}{2 b}$	123

input
```
int(exp(b*x+a)*csch(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3/(-exp(2*b*x+2*a+2*d)+exp(2*a))^3/b*(-3*exp(4*b*x+4*a+4*d)-8*exp(2*b*x
+4*a+2*d)+3*exp(4*a))*exp(b*x+3*a)+1/2*ln(exp(b*x+a)+exp(a-d))/b*exp(a-d)-
1/2*ln(exp(b*x+a)-exp(a-d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1466 vs. 2(94) = 188.

Time = 0.09 (sec) , antiderivative size = 1466, normalized size of antiderivative = 13.45

$$\int e^{a+bx}\operatorname{csch}^4(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^4,x, algorithm="fricas")
```

output
```
-1/6*(6*cosh(b*x + d)^5*cosh(-a + d) + 6*(cosh(-a + d) - sinh(-a + d))*sin
h(b*x + d)^5 + 30*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d)
)*sinh(b*x + d)^4 + 16*cosh(b*x + d)^3*cosh(-a + d) + 4*(15*cosh(b*x + d)^
2*cosh(-a + d) - (15*cosh(b*x + d)^2 + 4)*sinh(-a + d) + 4*cosh(-a + d))*s
inh(b*x + d)^3 + 12*(5*cosh(b*x + d)^3*cosh(-a + d) + 4*cosh(b*x + d)*cosh
(-a + d) - (5*cosh(b*x + d)^3 + 4*cosh(b*x + d))*sinh(-a + d))*sinh(b*x +
d)^2 - 6*cosh(b*x + d)*cosh(-a + d) - 3*(cosh(b*x + d)^6*cosh(-a + d) + (c
osh(-a + d) - sinh(-a + d))*sinh(b*x + d)^6 + 6*(cosh(b*x + d)*cosh(-a + d
) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d)^5 - 3*cosh(b*x + d)^4*cosh(-
a + d) + 3*(5*cosh(b*x + d)^2*cosh(-a + d) - (5*cosh(b*x + d)^2 - 1)*sinh(
-a + d) - cosh(-a + d))*sinh(b*x + d)^4 + 4*(5*cosh(b*x + d)^3*cosh(-a + d
) - 3*cosh(b*x + d)*cosh(-a + d) - (5*cosh(b*x + d)^3 - 3*cosh(b*x + d))*s
inh(-a + d))*sinh(b*x + d)^3 + 3*cosh(b*x + d)^2*cosh(-a + d) + 3*(5*cosh(
b*x + d)^4*cosh(-a + d) - 6*cosh(b*x + d)^2*cosh(-a + d) - (5*cosh(b*x + d
)^4 - 6*cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*sinh(b*x + d)^2
+ 6*(cosh(b*x + d)^5*cosh(-a + d) - 2*cosh(b*x + d)^3*cosh(-a + d) + cosh(
b*x + d)*cosh(-a + d) - (cosh(b*x + d)^5 - 2*cosh(b*x + d)^3 + cosh(b*x +
d))*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^6 - 3*cosh(b*x + d)^4 + 3
*cosh(b*x + d)^2 - 1)*sinh(-a + d) - cosh(-a + d))*log(cosh(b*x + d) + sin
h(b*x + d) + 1) + 3*(cosh(b*x + d)^6*cosh(-a + d) + (cosh(-a + d) - sin...
```

Sympy [F]

$$\int e^{a+bx}\text{csch}^4(d+bx)\,dx = e^a\int e^{bx}\,\text{csch}^4\left(bx+d\right)dx$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)**4,x)
```

output
```
exp(a)*Integral(exp(b*x)*csch(b*x + d)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.28

$$\int e^{a+bx}\operatorname{csch}^4(d+bx)\,dx = \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^a\right)}{2\,b} - \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}-e^a\right)}{2\,b} - \frac{3\,e^{(5\,bx+7\,a+4\,d)}+8\,e^{(3\,bx+7\,a+2\,d)}-3\,e^{(bx+7\,a)}}{3\,b\left(e^{(6\,bx+6\,a+6\,d)}-3\,e^{(4\,bx+6\,a+4\,d)}+3\,e^{(2\,bx+6\,a+2\,d)}-e^{(6\,a)}\right)}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^4,x, algorithm="maxima")
```

output
```
1/2*e^(a - d)*log(e^(b*x + a + d) + e^a)/b - 1/2*e^(a - d)*log(e^(b*x + a
+ d) - e^a)/b - 1/3*(3*e^(5*b*x + 7*a + 4*d) + 8*e^(3*b*x + 7*a + 2*d) - 3
*e^(b*x + 7*a))/(b*(e^(6*b*x + 6*a + 6*d) - 3*e^(4*b*x + 6*a + 4*d) + 3*e^
(2*b*x + 6*a + 2*d) - e^(6*a)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

$$\int e^{a+bx}\operatorname{csch}^4(d+bx)\,dx = \frac{1}{6}\left(\frac{3\,e^{(-5\,d)}\log\left(e^{(bx+d)}+1\right)}{b} - \frac{3\,e^{(-5\,d)}\log\left(\left|e^{(bx+d)}-1\right|\right)}{b} - \frac{2\left(3\,e^{(5\,bx+4\,d)}+8\,e^{(3\,bx+2\,d)}-3\,e^{(bx)}\right)e^{(-4\,d)}}{b\left(e^{(2\,bx+2\,d)}-1\right)^3}\right)$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^4,x, algorithm="giac")
```

output
```
1/6*(3*e^(-5*d)*log(e^(b*x + d) + 1)/b - 3*e^(-5*d)*log(abs(e^(b*x + d) -
1))/b - 2*(3*e^(5*b*x + 4*d) + 8*e^(3*b*x + 2*d) - 3*e^(b*x))*e^(-4*d)/(b*
(e^(2*b*x + 2*d) - 1)^3))*e^(a + 4*d)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\text{csch}^4(d+bx)\,dx=\int \frac{e^{a+b\,x}}{\sinh\left(d+b\,x\right)^4}\,dx$$

input
```
int(exp(a + b*x)/sinh(d + b*x)^4,x)
```

output
```
int(exp(a + b*x)/sinh(d + b*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.21

$$\int e^{a+bx}\text{csch}^4(d+bx)\,dx$$
$$=\frac{e^a\left(-3e^{6bx+6d}\log\left(e^{bx+d}-1\right)+3e^{6bx+6d}\log\left(e^{bx+d}+1\right)-6e^{5bx+5d}+9e^{4bx+4d}\log\left(e^{bx+d}-1\right)-9e^{4bx+4d}\log\right.}{6e^db\left(e^{6bx+\right.}}$$

input
```
int(exp(b*x+a)*csch(b*x+d)^4,x)
```

output
```
(e**a*( - 3*e**(6*b*x + 6*d)*log(e**(b*x + d) - 1) + 3*e**(6*b*x + 6*d)*lo
g(e**(b*x + d) + 1) - 6*e**(5*b*x + 5*d) + 9*e**(4*b*x + 4*d)*log(e**(b*x
+ d) - 1) - 9*e**(4*b*x + 4*d)*log(e**(b*x + d) + 1) - 16*e**(3*b*x + 3*d)
 - 9*e**(2*b*x + 2*d)*log(e**(b*x + d) - 1) + 9*e**(2*b*x + 2*d)*log(e**(b
*x + d) + 1) + 6*e**(b*x + d) + 3*log(e**(b*x + d) - 1) - 3*log(e**(b*x +
d) + 1)))/(6*e**d*b*(e**(6*b*x + 6*d) - 3*e**(4*b*x + 4*d) + 3*e**(2*b*x +
 2*d) - 1))
```

3.102 $\int e^{a+bx}\text{csch}^5(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 87

$$\int e^{a+bx}\text{csch}^5(d+bx)\,dx=-\frac{4e^{a-d}}{b\left(1-e^{2d+2bx}\right)^4}+\frac{32e^{a-d}}{3b\left(1-e^{2d+2bx}\right)^3}-\frac{8e^{a-d}}{b\left(1-e^{2d+2bx}\right)^2}$$

output
```
-4*exp(a-d)/b/(1-exp(2*b*x+2*d))^4+32/3*exp(a-d)/b/(1-exp(2*b*x+2*d))^3-8*
exp(a-d)/b/(1-exp(2*b*x+2*d))^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int e^{a+bx}\text{csch}^5(d+bx)\,dx = -\frac{4e^a\left(-4e^{2bx}+\left(1+6e^{4bx}\right)\cosh(2d)+\left(-1+6e^{4bx}\right)\sinh(2d)\right)\left(\cosh(3d)-\sinh(3d)\right)}{3b\left(\left(-1+e^{2bx}\right)\cosh(d)+\left(1+e^{2bx}\right)\sinh(d)\right)^4}$$

input
```
Integrate[E^(a + b*x)*Csch[d + b*x]^5,x]
```

output
```
(-4*E^a*(-4*E^(2*b*x) + (1 + 6*E^(4*b*x))*Cosh[2*d] + (-1 + 6*E^(4*b*x))*S
inh[2*d])*(Cosh[3*d] - Sinh[3*d]))/(3*b*((-1 + E^(2*b*x))*Cosh[d] + (1 + E
^(2*b*x))*Sinh[d])^4)
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.66, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2720, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{csch}^5(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{32e^{a+5bx}}{\left(1-e^{2bx}\right)^5}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{32e^a\int \frac{e^{5bx}}{\left(1-e^{2bx}\right)^5}de^{bx}}{b}$$

$$\Big\downarrow \text{243}$$

$$-\frac{16e^a\int \frac{e^{2bx}}{\left(1-e^{2bx}\right)^5}de^{2bx}}{b}$$

$$\Big\downarrow \text{53}$$

$$-\frac{16e^a\int \left(-\frac{1}{\left(-1+e^{2bx}\right)^3}-\frac{2}{\left(-1+e^{2bx}\right)^4}-\frac{1}{\left(-1+e^{2bx}\right)^5}\right)de^{2bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{16e^a\left(\frac{1}{2\left(1-e^{2bx}\right)^2}-\frac{2}{3\left(1-e^{2bx}\right)^3}+\frac{1}{4\left(1-e^{2bx}\right)^4}\right)}{b}$$

input
```
Int[E^(a + b*x)*Csch[d + b*x]^5,x]
```

output
```
(-16*E^a*(1/(4*(1 - E^(2*b*x))^4) - 2/(3*(1 - E^(2*b*x))^3) + 1/(2*(1 - E^
(2*b*x))^2)))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{4\left(6\,\mathrm{e}^{4bx+4a+4d}-4\,\mathrm{e}^{2bx+4a+2d}+\mathrm{e}^{4a}\right)\mathrm{e}^{5a-d}}{3\left(-\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)^{4}b}$
parallelrisch	$\frac{\mathrm{e}^{bx+a}\left(3\tanh\left(\frac{bx}{2}+\frac{d}{2}\right)^{4}-3\coth\left(\frac{bx}{2}+\frac{d}{2}\right)^{4}+2\tanh\left(\frac{bx}{2}+\frac{d}{2}\right)^{3}-2\coth\left(\frac{bx}{2}+\frac{d}{2}\right)^{3}-22\tanh\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}+22\coth\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}-38\,\mathrm{ta}\right.}{192b}$

input

```
int(exp(b*x+a)*csch(b*x+d)^5,x,method=_RETURNVERBOSE)
```

output

```
-4/3/(-exp(2*b*x+2*a+2*d)+exp(2*a))^4/b*(6*exp(4*b*x+4*a+4*d)-4*exp(2*b*x+
4*a+2*d)+exp(4*a))*exp(5*a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(73) = 146$.

Time = 0.08 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.48

$$\int e^{a+bx}\operatorname{csch}^5(d+bx)\,dx =$$

$$-\frac{4\left(7\cosh\left(bx+d\right)^{2}\cosh\left(-a+d\right)\right.}{3\left(b\cosh\left(bx+d\right)^{6}+6\,b\cosh\left(bx+d\right)\sinh\left(bx+d\right)^{5}+b\sinh\left(bx+d\right)^{6}-4\,b\cosh\left(bx+d\right)^{4}+\left(15\,b\cos\right.\right.}$$

input

```
integrate(exp(b*x+a)*csch(b*x+d)^5,x, algorithm="fricas")
```

output

```
-4/3*(7*cosh(b*x + d)^2*cosh(-a + d) + 7*(cosh(-a + d) - sinh(-a + d))*sin
h(b*x + d)^2 + 10*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d)
)*sinh(b*x + d) - (7*cosh(b*x + d)^2 - 4)*sinh(-a + d) - 4*cosh(-a + d))/(
b*cosh(b*x + d)^6 + 6*b*cosh(b*x + d)*sinh(b*x + d)^5 + b*sinh(b*x + d)^6
- 4*b*cosh(b*x + d)^4 + (15*b*cosh(b*x + d)^2 - 4*b)*sinh(b*x + d)^4 + 4*(
5*b*cosh(b*x + d)^3 - 4*b*cosh(b*x + d))*sinh(b*x + d)^3 + 7*b*cosh(b*x +
d)^2 + (15*b*cosh(b*x + d)^4 - 24*b*cosh(b*x + d)^2 + 7*b)*sinh(b*x + d)^2
 + 2*(3*b*cosh(b*x + d)^5 - 8*b*cosh(b*x + d)^3 + 5*b*cosh(b*x + d))*sinh(
b*x + d) - 4*b)
```

Sympy [F]

$$\int e^{a+bx}\operatorname{csch}^5(d+bx)\,dx = e^a\int e^{bx}\operatorname{csch}^5(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)**5,x)
```

output
```
exp(a)*Integral(exp(b*x)*csch(b*x + d)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(73) = 146.

Time = 0.04 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.68

$$\int e^{a+bx}\operatorname{csch}^5(d+bx)\,dx$$
$$= -\frac{8\,e^{(4\,bx+9\,a+4\,d)}}{b(e^{(8\,bx+8\,a+9\,d)} - 4\,e^{(6\,bx+8\,a+7\,d)} + 6\,e^{(4\,bx+8\,a+5\,d)} - 4\,e^{(2\,bx+8\,a+3\,d)} + e^{(8\,a+d)})} + \frac{16\,e^{(2\,bx+9\,a+2\,d)}}{3\,b(e^{(8\,bx+8\,a+9\,d)} - 4\,e^{(6\,bx+8\,a+7\,d)} + 6\,e^{(4\,bx+8\,a+5\,d)} - 4\,e^{(2\,bx+8\,a+3\,d)} + e^{(8\,a+d)})} - \frac{4\,e^{(9\,a)}}{3\,b(e^{(8\,bx+8\,a+9\,d)} - 4\,e^{(6\,bx+8\,a+7\,d)} + 6\,e^{(4\,bx+8\,a+5\,d)} - 4\,e^{(2\,bx+8\,a+3\,d)} + e^{(8\,a+d)})}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^5,x, algorithm="maxima")
```

output
```
-8*e^(4*b*x + 9*a + 4*d)/(b*(e^(8*b*x + 8*a + 9*d) - 4*e^(6*b*x + 8*a + 7*
d) + 6*e^(4*b*x + 8*a + 5*d) - 4*e^(2*b*x + 8*a + 3*d) + e^(8*a + d))) + 1
6/3*e^(2*b*x + 9*a + 2*d)/(b*(e^(8*b*x + 8*a + 9*d) - 4*e^(6*b*x + 8*a + 7
*d) + 6*e^(4*b*x + 8*a + 5*d) - 4*e^(2*b*x + 8*a + 3*d) + e^(8*a + d))) -
4/3*e^(9*a)/(b*(e^(8*b*x + 8*a + 9*d) - 4*e^(6*b*x + 8*a + 7*d) + 6*e^(4*b
*x + 8*a + 5*d) - 4*e^(2*b*x + 8*a + 3*d) + e^(8*a + d)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.55

$$\int e^{a+bx}\text{csch}^5(d+bx)\,dx=-\frac{4\left(6\,e^{(4\,bx+4\,d)}-4\,e^{(2\,bx+2\,d)}+1\right)e^{(a-d)}}{3\,b\left(e^{(2\,bx+2\,d)}-1\right)^4}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^5,x, algorithm="giac")
```

output
```
-4/3*(6*e^(4*b*x + 4*d) - 4*e^(2*b*x + 2*d) + 1)*e^(a - d)/(b*(e^(2*b*x +
2*d) - 1)^4)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\text{csch}^5(d+bx)\,dx=\int \frac{e^{a+b\,x}}{\sinh\left(d+b\,x\right)^5}\,dx$$

input
```
int(exp(a + b*x)/sinh(d + b*x)^5,x)
```

output
```
int(exp(a + b*x)/sinh(d + b*x)^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int e^{a+bx}\text{csch}^5(d+bx)\,dx=\frac{4e^a\left(-6e^{4bx+4d}+4e^{2bx+2d}-1\right)}{3e^d b\left(e^{8bx+8d}-4e^{6bx+6d}+6e^{4bx+4d}-4e^{2bx+2d}+1\right)}$$

input
```
int(exp(b*x+a)*csch(b*x+d)^5,x)
```

output
```
(4*e**a*( - 6*e**(4*b*x + 4*d) + 4*e**(2*b*x + 2*d) - 1))/(3*e**d*b*(e**(8
*b*x + 8*d) - 4*e**(6*b*x + 6*d) + 6*e**(4*b*x + 4*d) - 4*e**(2*b*x + 2*d)
 + 1))
```

3.103 $\int e^{2(a+bx)}\text{csch}(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int e^{2(a+bx)}\text{csch}(d+bx)\,dx=\frac{2e^{2a-d+bx}}{b}-\frac{2e^{2a-2d}\text{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
2*exp(b*x+2*a-d)/b-2*exp(2*a-2*d)*arctanh(exp(b*x+d))/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 163 vs. $2(40)=80$.

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 4.08

$$\int e^{2(a+bx)}\text{csch}(d+bx)\,dx$$
$$=\frac{e^{2a}\left(\cosh(2d)\left(-\log\left(\left(1+e^{bx}\right)\cosh\left(\frac{d}{2}\right)+\left(-1+e^{bx}\right)\sinh\left(\frac{d}{2}\right)\right)+\log\left(\left(-1+e^{bx}\right)\cosh\left(\frac{d}{2}\right)+\left(1+e^{bx}\right)\text{s}\right.\right.\right.}{}$$

input
```
Integrate[E^(2*(a + b*x))*Csch[d + b*x],x]
```

output
```
(E^(2*a)*(Cosh[2*d]*(-Log[(1 + E^(b*x))*Cosh[d/2] + (-1 + E^(b*x))*Sinh[d/
2]] + Log[(-1 + E^(b*x))*Cosh[d/2] + (1 + E^(b*x))*Sinh[d/2]]) - 2*E^(b*x)
*Sinh[d] + 2*Cosh[d]*(E^(b*x) + (Log[(1 + E^(b*x))*Cosh[d/2] + (-1 + E^(b*
x))*Sinh[d/2]] - Log[(-1 + E^(b*x))*Cosh[d/2] + (1 + E^(b*x))*Sinh[d/2]])*
Sinh[d])))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 27, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{csch}(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{2e^{2a+2bx}}{1-e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2e^{2a}\int \frac{e^{2bx}}{1-e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{2e^{2a}\left(\int \frac{1}{1-e^{2bx}}de^{bx}-e^{bx}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2e^{2a}\left(\text{arctanh}\left(e^{bx}\right)-e^{bx}\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Csch[d + b*x],x]
```

output
```
(-2*E^(2*a)*(-E^(b*x) + ArcTanh[E^(b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.82

method	result	size
risch	$\frac{2\,{\mathrm e}^{b x +2 a -d}}{b}+\frac{\ln\left({\mathrm e}^{b x +a}-{\mathrm e}^{a -d}\right) {\mathrm e}^{2 a -2 d}}{b}-\frac{\ln\left({\mathrm e}^{b x +a}+{\mathrm e}^{a -d}\right) {\mathrm e}^{2 a -2 d}}{b}$	73

input
```
int(exp(2*b*x+2*a)*csch(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
2*exp(b*x+2*a-d)/b+ln(exp(b*x+a)-exp(a-d))/b*exp(2*a-2*d)-ln(exp(b*x+a)+ex
p(a-d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(37) = 74$.

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.38

$$\int e^{2(a+bx)}\text{csch}(d+bx)\,dx$$
$$= \frac{2\cosh(bx+d)\cosh(-2a+2d) - (\cosh(-2a+2d) - \sinh(-2a+2d))\log(\cosh(bx+d) + \sinh(bx}{}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d),x, algorithm="fricas")
```

output
```
(2*cosh(b*x + d)*cosh(-2*a + 2*d) - (cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*
log(cosh(b*x + d) + sinh(b*x + d) + 1) + (cosh(-2*a + 2*d) - sinh(-2*a + 2
*d))*log(cosh(b*x + d) + sinh(b*x + d) - 1) + 2*(cosh(-2*a + 2*d) - sinh(-
2*a + 2*d))*sinh(b*x + d) - 2*cosh(b*x + d)*sinh(-2*a + 2*d))/b
```

Sympy [F]

$$\int e^{2(a+bx)}\text{csch}(d+bx)\,dx = e^{2a}\int e^{2bx}\,\text{csch}\,(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d),x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*csch(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int e^{2(a+bx)}\text{csch}(d+bx)\,dx = -\frac{e^{(2a-2d)}\log\left(e^{(-bx-d)}+1\right)}{b} + \frac{e^{(2a-2d)}\log\left(e^{(-bx-d)}-1\right)}{b} + \frac{2e^{(bx+2a-d)}}{b}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d),x, algorithm="maxima")
```

output
```
-e^(2*a - 2*d)*log(e^(-b*x - d) + 1)/b + e^(2*a - 2*d)*log(e^(-b*x - d) -
1)/b + 2*e^(b*x + 2*a - d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int e^{2(a+bx)}\operatorname{csch}(d+bx)\,dx = -\frac{\left(e^{(-2\,d)}\log\left(e^{(bx+d)}+1\right)-e^{(-2\,d)}\log\left(\left|e^{(bx+d)}-1\right|\right)-2\,e^{(bx-d)}\right)e^{(2\,a)}}{b}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d),x, algorithm="giac")
```

output
```
-(e^(-2*d)*log(e^(b*x + d) + 1) - e^(-2*d)*log(abs(e^(b*x + d) - 1)) - 2*e
^(b*x - d))*e^(2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.78

$$\int e^{2(a+bx)}\operatorname{csch}(d+bx)\,dx = \frac{2\,\mathrm{e}^{2\,a-d+b\,x}}{b}-\frac{2\,\sqrt{\mathrm{e}^{4\,a-4\,d}}\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{-d}\,\mathrm{e}^{b\,x}\,\sqrt{-b^2}}{b\,\sqrt{\mathrm{e}^{4\,a}\,\mathrm{e}^{-4\,d}}}\right)}{\sqrt{-b^2}}$$

input
```
int(exp(2*a + 2*b*x)/sinh(d + b*x),x)
```

output
```
(2*exp(2*a - d + b*x))/b - (2*exp(4*a - 4*d)^(1/2)*atan((exp(2*a)*exp(-d)*
exp(b*x)*(-b^2)^(1/2))/(b*(exp(4*a)*exp(-4*d))^(1/2))))/(-b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int e^{2(a+bx)}\text{csch}(d+bx)\,dx=\frac{e^{2a}\left(2e^{bx+d}+\log\left(e^{bx+d}-1\right)-\log\left(e^{bx+d}+1\right)\right)}{e^{2d}b}$$

input
```
int(exp(2*b*x+2*a)*csch(b*x+d),x)
```

output
```
(e**(2*a)*(2*e**(b*x + d) + log(e**(b*x + d) - 1) - log(e**(b*x + d) + 1))
)/(e**(2*d)*b)
```

3.104 $\int e^{2(a+bx)}\text{csch}^2(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 60

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\,dx=\frac{2e^{2a-2d}}{b\left(1-e^{2d+2bx}\right)}+\frac{2e^{2a-2d}\log\left(1-e^{2d+2bx}\right)}{b}$$

output
```
2*exp(2*a-2*d)/b/(1-exp(2*b*x+2*d))+2*exp(2*a-2*d)*ln(1-exp(2*b*x+2*d))/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 123 vs. 2(60) = 120.

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.05

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\,dx$$
$$=\frac{2e^{2a}(\cosh(2d)-2\cosh(d)\sinh(d))\left(\cosh(d)\left(-1+\left(-1+e^{2bx}\right)\log\left(\left(-1+e^{2bx}\right)\cosh(d)+\left(1+e^{2bx}\right)\sin\right.\right.\right.}{b\left(\left(-1+e^{2bx}\right)\cosh(d)+\left(1+e^{2b}\right.\right.}$$

input
```
Integrate[E^(2*(a + b*x))*Csch[d + b*x]^2,x]
```

output
```
(2*E^(2*a)*(Cosh[2*d] - 2*Cosh[d]*Sinh[d])*(Cosh[d]*(-1 + (-1 + E^(2*b*x))
*Log[(-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d]]) + (1 + (1 + E^(2
*b*x))*Log[(-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d]])*Sinh[d]))/
(b*((-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d]))
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{csch}^2(bx+d)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int \frac{4e^{2a+3bx}}{\left(1-e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4e^{2a}\int \frac{e^{3bx}}{\left(1-e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{2e^{2a}\int \frac{e^{2bx}}{\left(1-e^{2bx}\right)^2}de^{2bx}}{b}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{2e^{2a}\int \left(\frac{1}{-1+e^{2bx}}+\frac{1}{\left(-1+e^{2bx}\right)^2}\right)de^{2bx}}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2e^{2a}\left(\frac{1}{1-e^{2bx}}+\log\left(1-e^{2bx}\right)\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Csch[d + b*x]^2,x]
```

output
```
(2*E^(2*a)*((1 - E^(2*b*x))^(-1) + Log[1 - E^(2*b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.40

method	result	size
risch	$-\frac{4\,\mathrm{e}^{2a-2d}a}{b}+\frac{2\,\mathrm{e}^{4a-2d}}{\left(-\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)b}+\frac{2\ln\left(\mathrm{e}^{2bx+2a}-\mathrm{e}^{2a-2d}\right)\mathrm{e}^{2a-2d}}{b}$	84

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-4/b*exp(2*a-2*d)*a+2/(-exp(2*b*x+2*a+2*d)+exp(2*a))/b*exp(4*a-2*d)+2*ln(e
xp(2*b*x+2*a)-exp(2*a-2*d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(54) = 108.

Time = 0.10 (sec) , antiderivative size = 205, normalized size of antiderivative = 3.42

$$\int e^{2(a+bx)}\mathrm{csch}^2(d+bx)\,dx$$
$$= \frac{2\left(\left(\cosh\left(bx+d\right)^2\cosh\left(-2\,a+2\,d\right)+\left(\cosh\left(-2\,a+2\,d\right)-\sinh\left(-2\,a+2\,d\right)\right)\sinh\left(bx+d\right)^2+2\left(\cosh\left(b\right.\right.\right.}{}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^2,x, algorithm="fricas")
```

output
```
2*((cosh(b*x + d)^2*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*d
))*sinh(b*x + d)^2 + 2*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sin
h(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^2 - 1)*sinh(-2*a + 2*d) - co
sh(-2*a + 2*d))*log(2*sinh(b*x + d)/(cosh(b*x + d) - sinh(b*x + d))) - cos
h(-2*a + 2*d) + sinh(-2*a + 2*d))/(b*cosh(b*x + d)^2 + 2*b*cosh(b*x + d)*s
inh(b*x + d) + b*sinh(b*x + d)^2 - b)
```

Sympy [F]

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\,dx=e^{2a}\int e^{2bx}\,\text{csch}^2\,(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)**2,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*csch(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.70

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\,dx=4\,xe^{(2\,a-2\,d)}+\frac{4\,de^{(2\,a-2\,d)}}{b}+\frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}+1\right)}{b}+\frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}-1\right)}{b}+\frac{2\,e^{(2\,a-2\,d)}}{b(e^{(-2\,bx-2\,d)}-1)}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^2,x, algorithm="maxima")
```

output
```
4*x*e^(2*a - 2*d) + 4*d*e^(2*a - 2*d)/b + 2*e^(2*a - 2*d)*log(e^(-b*x - d)
 + 1)/b + 2*e^(2*a - 2*d)*log(e^(-b*x - d) - 1)/b + 2*e^(2*a - 2*d)/(b*(e^
(-2*b*x - 2*d) - 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\,dx=\frac{2\left(e^{(-2\,d)}\log\left(\left|e^{(2\,bx+2\,d)}-1\right|\right)-\frac{e^{(-2\,d)}}{e^{(2\,bx+2\,d)}-1}\right)e^{(2\,a)}}{b}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^2,x, algorithm="giac")
```

output
```
2*(e^(-2*d)*log(abs(e^(2*b*x + 2*d) - 1)) - e^(-2*d)/(e^(2*b*x + 2*d) - 1)
)*e^(2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int e^{2(a+bx)}\operatorname{csch}^2(d+bx)\,dx=\frac{2\,e^{2\,a-2\,d}\,\ln\left(e^{2\,d}\,e^{2\,b\,x}-1\right)}{b}-\frac{2\,e^{2\,a-2\,d}}{b\,\left(e^{2\,d+2\,b\,x}-1\right)}$$

input
```
int(exp(2*a + 2*b*x)/sinh(d + b*x)^2,x)
```

output
```
(2*exp(2*a - 2*d)*log(exp(2*d)*exp(2*b*x) - 1))/b - (2*exp(2*a - 2*d))/(b*
(exp(2*d + 2*b*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.83

$$\int e^{2(a+bx)}\operatorname{csch}^2(d+bx)\,dx$$
$$=\frac{2e^{2a}\left(e^{2bx+2d}\log\left(e^{bx+d}-1\right)+e^{2bx+2d}\log\left(e^{bx+d}+1\right)-e^{2bx+2d}-\log\left(e^{bx+d}-1\right)-\log\left(e^{bx+d}+1\right)\right)}{e^{2d}b\left(e^{2bx+2d}-1\right)}$$

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)^2,x)
```

output
```
(2*e**(2*a)*(e**(2*b*x + 2*d)*log(e**(b*x + d) - 1) + e**(2*b*x + 2*d)*log
(e**(b*x + d) + 1) - e**(2*b*x + 2*d) - log(e**(b*x + d) - 1) - log(e**(b*
x + d) + 1)))/(e**(2*d)*b*(e**(2*b*x + 2*d) - 1))
```

3.105 $\int e^{2(a+bx)}\text{csch}^3(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 88

$$\int e^{2(a+bx)}\text{csch}^3(d+bx)\,dx = -\frac{2e^{2a+d+3bx}}{b\left(1-e^{2d+2bx}\right)^2}+\frac{3e^{2a-d+bx}}{b\left(1-e^{2d+2bx}\right)}-\frac{3e^{2a-2d}\text{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
-2*exp(3*b*x+2*a+d)/b/(1-exp(2*b*x+2*d))^2+3*exp(b*x+2*a-d)/b/(1-exp(2*b*x
+2*d))-3*exp(2*a-2*d)*arctanh(exp(b*x+d))/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 239 vs. 2(88) = 176.

Time = 0.44 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.72

$$\int e^{2(a+bx)}\text{csch}^3(d+bx)\,dx = \frac{e^{2a}\left(-3\cosh(2d)\log\left(\left(1+e^{bx}\right)\cosh\left(\frac{d}{2}\right)+\left(-1+e^{bx}\right)\sinh\left(\frac{d}{2}\right)\right)+3\cosh(2d)\log\left(\left(-1+e^{bx}\right)\cosh\left(\frac{d}{2}\right)+\right.\right.}{}$$

input
```
Integrate[E^(2*(a + b*x))*Csch[d + b*x]^3,x]
```

output
```
(E^(2*a)*(-3*Cosh[2*d]*Log[(1 + E^(b*x))*Cosh[d/2] + (-1 + E^(b*x))*Sinh[d
/2]] + 3*Cosh[2*d]*Log[(-1 + E^(b*x))*Cosh[d/2] + (1 + E^(b*x))*Sinh[d/2]]
 - (4*E^(b*x)*(Cosh[d] - Sinh[d])^3)/((-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2
*b*x))*Sinh[d])^2 - (10*E^(b*x)*(Cosh[d] - Sinh[d])^2)/((-1 + E^(2*b*x))*C
osh[d] + (1 + E^(2*b*x))*Sinh[d]) + 3*Log[(1 + E^(b*x))*Cosh[d/2] + (-1 +
E^(b*x))*Sinh[d/2]]*Sinh[2*d] - 3*Log[(-1 + E^(b*x))*Cosh[d/2] + (1 + E^(b
*x))*Sinh[d/2]]*Sinh[2*d]))/(2*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2720, 27, 252, 252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{csch}^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{8e^{2a+4bx}}{(1-e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{8e^{2a}\int \frac{e^{4bx}}{(1-e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{8e^{2a}\left(\frac{e^{3bx}}{4(1-e^{2bx})^2}-\frac{3}{4}\int \frac{e^{2bx}}{(1-e^{2bx})^2}de^{bx}\right)}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{8e^{2a}\left(\frac{e^{3bx}}{4(1-e^{2bx})^2}-\frac{3}{4}\left(\frac{e^{bx}}{2(1-e^{2bx})}-\frac{1}{2}\int \frac{1}{1-e^{2bx}}de^{bx}\right)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{8e^{2a}\left(\frac{e^{3bx}}{4(1-e^{2bx})^2}-\frac{3}{4}\left(\frac{e^{bx}}{2(1-e^{2bx})}-\frac{1}{2}\text{arctanh}\left(e^{bx}\right)\right)\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Csch[d + b*x]^3,x]
```

output
```
(-8*E^(2*a)*(E^(3*b*x)/(4*(1 - E^(2*b*x))^2) - (3*(E^(b*x)/(2*(1 - E^(2*b*
x))) - ArcTanh[E^(b*x)]/2))/4))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.31

method	result	size
risch	$\frac{\left(-5\,\mathrm{e}^{2bx+2a+2d}+3\,\mathrm{e}^{2a}\right)\mathrm{e}^{bx+4a-d}}{\left(-\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)^{2}b}-\frac{3\ln\left(\mathrm{e}^{bx+a}+\mathrm{e}^{a-d}\right)\mathrm{e}^{2a-2d}}{2b}+\frac{3\ln\left(\mathrm{e}^{bx+a}-\mathrm{e}^{a-d}\right)\mathrm{e}^{2a-2d}}{2b}$	115

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/(-exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(-5*exp(2*b*x+2*a+2*d)+3*exp(2*a))*ex
p(b*x+4*a-d)-3/2*ln(exp(b*x+a)+exp(a-d))/b*exp(2*a-2*d)+3/2*ln(exp(b*x+a)-
exp(a-d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 883 vs. 2(78) = 156.

Time = 0.10 (sec) , antiderivative size = 883, normalized size of antiderivative = 10.03

$$\int e^{2(a+bx)}\operatorname{csch}^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^3,x, algorithm="fricas")
```

output
```
-1/2*(10*cosh(b*x + d)^3*cosh(-2*a + 2*d) + 10*(cosh(-2*a + 2*d) - sinh(-2
*a + 2*d))*sinh(b*x + d)^3 + 30*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x
 + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^2 - 6*cosh(b*x + d)*cosh(-2*a + 2*d)
 + 3*(cosh(b*x + d)^4*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2
*d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*s
inh(-2*a + 2*d))*sinh(b*x + d)^3 - 2*cosh(b*x + d)^2*cosh(-2*a + 2*d) + 2*
(3*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (3*cosh(b*x + d)^2 - 1)*sinh(-2*a +
2*d) - cosh(-2*a + 2*d))*sinh(b*x + d)^2 + 4*(cosh(b*x + d)^3*cosh(-2*a +
2*d) - cosh(b*x + d)*cosh(-2*a + 2*d) - (cosh(b*x + d)^3 - cosh(b*x + d))*
sinh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^4 - 2*cosh(b*x + d)^2 + 1
)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*log(cosh(b*x + d) + sinh(b*x + d) +
 1) - 3*(cosh(b*x + d)^4*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a
+ 2*d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d
)*sinh(-2*a + 2*d))*sinh(b*x + d)^3 - 2*cosh(b*x + d)^2*cosh(-2*a + 2*d) +
 2*(3*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (3*cosh(b*x + d)^2 - 1)*sinh(-2*a
 + 2*d) - cosh(-2*a + 2*d))*sinh(b*x + d)^2 + 4*(cosh(b*x + d)^3*cosh(-2*a
 + 2*d) - cosh(b*x + d)*cosh(-2*a + 2*d) - (cosh(b*x + d)^3 - cosh(b*x + d
))*sinh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^4 - 2*cosh(b*x + d)^2
+ 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*log(cosh(b*x + d) + sinh(b*x + d
) - 1) + 6*(5*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (5*cosh(b*x + d)^2 - 1...
```

Sympy [F]

$$\int e^{2(a+bx)}\text{csch}^3(d+bx)\,dx = e^{2a}\int e^{2bx}\,\text{csch}^3\,(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)**3,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*csch(b*x + d)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.27

$$\int e^{2(a+bx)}\operatorname{csch}^3(d+bx)\,dx = -\frac{3\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}+1\right)}{2\,b} + \frac{3\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}-1\right)}{2\,b} + \frac{\left(5\,e^{(-bx-d)}-3\,e^{(-3\,bx-3\,d)}\right)e^{(2\,a-2\,d)}}{b(2\,e^{(-2\,bx-2\,d)}-e^{(-4\,bx-4\,d)}-1)}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^3,x, algorithm="maxima")
```

output
```
-3/2*e^(2*a - 2*d)*log(e^(-b*x - d) + 1)/b + 3/2*e^(2*a - 2*d)*log(e^(-b*x
 - d) - 1)/b + (5*e^(-b*x - d) - 3*e^(-3*b*x - 3*d))*e^(2*a - 2*d)/(b*(2*e
^(-2*b*x - 2*d) - e^(-4*b*x - 4*d) - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91

$$\int e^{2(a+bx)}\operatorname{csch}^3(d+bx)\,dx = -\frac{\left(3\,e^{(-2\,d)}\log\left(e^{(bx+d)}+1\right)-3\,e^{(-2\,d)}\log\left(\left|e^{(bx+d)}-1\right|\right)+\frac{2\,\left(5\,e^{(3\,bx+3\,d)}-3\,e^{(bx+d)}\right)e^{(-2\,d)}}{\left(e^{(2\,bx+2\,d)}-1\right)^2}\right)e^{(2\,a)}}{2\,b}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^3,x, algorithm="giac")
```

output
```
-1/2*(3*e^(-2*d)*log(e^(b*x + d) + 1) - 3*e^(-2*d)*log(abs(e^(b*x + d) - 1
)) + 2*(5*e^(3*b*x + 3*d) - 3*e^(b*x + d))*e^(-2*d)/(e^(2*b*x + 2*d) - 1)^
2)*e^(2*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.97 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.40

$$\int e^{2(a+bx)}\operatorname{csch}^3(d+bx)\,dx = -\frac{3\,e^{2\,a-d+b\,x}}{b\,\left(e^{2\,d+2\,b\,x}-1\right)}-\frac{3\,\sqrt{e^{4\,a-4\,d}}\,\mathrm{atan}\left(\frac{e^{2\,a}\,e^{-d}\,e^{b\,x}\,\sqrt{-b^2}}{b\,\sqrt{e^{4\,a}\,e^{-4\,d}}}\right)}{\sqrt{-b^2}} - \frac{2\,e^{2\,a+d+3\,b\,x}}{b\,\left(e^{4\,d+4\,b\,x}-2\,e^{2\,d+2\,b\,x}+1\right)}$$

input

```
int(exp(2*a + 2*b*x)/sinh(d + b*x)^3,x)
```

output

```
- (3*exp(2*a - d + b*x))/(b*(exp(2*d + 2*b*x) - 1)) - (3*exp(4*a - 4*d)^(1
/2)*atan((exp(2*a)*exp(-d)*exp(b*x)*(-b^2)^(1/2))/(b*(exp(4*a)*exp(-4*d))^
(1/2))))/(-b^2)^(1/2) - (2*exp(2*a + d + 3*b*x))/(b*(exp(4*d + 4*b*x) - 2*
exp(2*d + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.01

$$\int e^{2(a+bx)}\operatorname{csch}^3(d+bx)\,dx = \frac{e^{2a}\left(3e^{4bx+4d}\log\left(e^{bx+d}-1\right)-3e^{4bx+4d}\log\left(e^{bx+d}+1\right)-10e^{3bx+3d}-6e^{2bx+2d}\log\left(e^{bx+d}-1\right)+6e^{2bx+2d}\log\right.}{2e^{2d}b\left(e^{4bx+4d}-2e^{2bx+2d}+1\right)}$$

input

```
int(exp(2*b*x+2*a)*csch(b*x+d)^3,x)
```

output

```
(e**(2*a)*(3*e**(4*b*x + 4*d)*log(e**(b*x + d) - 1) - 3*e**(4*b*x + 4*d)*l
og(e**(b*x + d) + 1) - 10*e**(3*b*x + 3*d) - 6*e**(2*b*x + 2*d)*log(e**(b*
x + d) - 1) + 6*e**(2*b*x + 2*d)*log(e**(b*x + d) + 1) + 6*e**(b*x + d) +
3*log(e**(b*x + d) - 1) - 3*log(e**(b*x + d) + 1)))/(2*e**(2*d)*b*(e**(4*b
*x + 4*d) - 2*e**(2*b*x + 2*d) + 1))
```

3.106 $\int e^{2(a+bx)}\operatorname{csch}^4(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int e^{2(a+bx)}\operatorname{csch}^4(d+bx)\,dx=\frac{8e^{2(a+2d)+6bx}}{3b\left(1-e^{2d+2bx}\right)^3}$$

output
```
8/3*exp(6*b*x+2*a+4*d)/b/(1-exp(2*b*x+2*d))^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(37) = 74.

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.35

$$\int e^{2(a+bx)}\operatorname{csch}^4(d+bx)\,dx$$
$$=-\frac{8e^{2a}\left(-3e^{2bx}+\left(1+3e^{4bx}\right)\cosh(2d)+\left(-1+3e^{4bx}\right)\sinh(2d)\right)\left(\cosh(3d)-\sinh(3d)\right)}{3b\left(\left(-1+e^{2bx}\right)\cosh(d)+\left(1+e^{2bx}\right)\sinh(d)\right)^3}$$

input
```
Integrate[E^(2*(a + b*x))*Csch[d + b*x]^4,x]
```

output
```
(-8*E^(2*a)*(-3*E^(2*b*x) + (1 + 3*E^(4*b*x))*Cosh[2*d] + (-1 + 3*E^(4*b*x
))*Sinh[2*d])*(Cosh[3*d] - Sinh[3*d]))/(3*b*((-1 + E^(2*b*x))*Cosh[d] + (1
 + E^(2*b*x))*Sinh[d])^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2720, 27, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\operatorname{csch}^4(bx+d)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int \frac{16e^{2a+5bx}}{(1-e^{2bx})^4}de^{bx}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{16e^{2a}\int \frac{e^{5bx}}{(1-e^{2bx})^4}de^{bx}}{b}$$

$$\Big\downarrow \text{ 242}$$

$$\frac{8e^{2a+6bx}}{3b\left(1-e^{2bx}\right)^3}$$

input
```
Int[E^(2*(a + b*x))*Csch[d + b*x]^4,x]
```

output
```
(8*E^(2*a + 6*b*x))/(3*b*(1 - E^(2*b*x))^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

method	result	size
parallelrisch	$-\frac{e^{2bx+2a}(\sinh(bx+d)+\cosh(bx+d))\operatorname{sech}\left(\frac{bx}{2}+\frac{d}{2}\right)^{3}\operatorname{csch}\left(\frac{bx}{2}+\frac{d}{2}\right)^{3}}{24b}$	50
risch	$\frac{8\left(3\,e^{4bx+4a+4d}-3\,e^{2bx+4a+2d}+e^{4a}\right)e^{4a-2d}}{3\left(-e^{2bx+2a+2d}+e^{2a}\right)^{3}b}$	68

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*exp(2*b*x+2*a)/b*(sinh(b*x+d)+cosh(b*x+d))*sech(1/2*b*x+1/2*d)^3*csc
h(1/2*b*x+1/2*d)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(30) = 60$.

Time = 0.09 (sec) , antiderivative size = 229, normalized size of antiderivative = 6.19

$$\int e^{2(a+bx)}\text{csch}^4(d+bx)\,dx = -\frac{8\left(4\cosh\left(bx+d\right)^2\cosh\left(-2\,a+2\,d\right)+4\left(\cosh\left(-2\,a+2\,d\right)-\sinh\left(-2\,a+2\,d\right)\right)\sinh\left(bx+d\right)^2+4\left(\text{co}\right.\right.}{3\left(b\cosh\left(bx+d\right)^4+4\,b\cosh\left(bx+d\right)\sinh\left(bx+d\right)^3+b\sinh\left(bx+d\right)^4-4\,b\right.}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^4,x, algorithm="fricas")
```

output
```
-8/3*(4*cosh(b*x + d)^2*cosh(-2*a + 2*d) + 4*(cosh(-2*a + 2*d) - sinh(-2*a
 + 2*d))*sinh(b*x + d)^2 + 4*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x +
d)*sinh(-2*a + 2*d))*sinh(b*x + d) - (4*cosh(b*x + d)^2 - 3)*sinh(-2*a + 2
*d) - 3*cosh(-2*a + 2*d))/(b*cosh(b*x + d)^4 + 4*b*cosh(b*x + d)*sinh(b*x
+ d)^3 + b*sinh(b*x + d)^4 - 4*b*cosh(b*x + d)^2 + 2*(3*b*cosh(b*x + d)^2
- 2*b)*sinh(b*x + d)^2 + 4*(b*cosh(b*x + d)^3 - b*cosh(b*x + d))*sinh(b*x
+ d) + 3*b)
```

Sympy [F]

$$\int e^{2(a+bx)}\text{csch}^4(d+bx)\,dx = e^{2a}\int e^{2bx}\,\text{csch}^4\left(bx+d\right)dx$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)**4,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*csch(b*x + d)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$\int e^{2(a+bx)}\operatorname{csch}^4(d+bx)\,dx = \frac{8\,e^{(2\,a-2\,d)}}{3\,b(3\,e^{(-2\,bx-2\,d)}-3\,e^{(-4\,bx-4\,d)}+e^{(-6\,bx-6\,d)}-1)}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^4,x, algorithm="maxima")
```

output
```
8/3*e^(2*a - 2*d)/(b*(3*e^(-2*b*x - 2*d) - 3*e^(-4*b*x - 4*d) + e^(-6*b*x
 - 6*d) - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int e^{2(a+bx)}\operatorname{csch}^4(d+bx)\,dx = -\frac{8\left(3\,e^{(4\,bx+4\,d)}-3\,e^{(2\,bx+2\,d)}+1\right)e^{(2\,a-2\,d)}}{3\,b(e^{(2\,bx+2\,d)}-1)^3}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^4,x, algorithm="giac")
```

output
```
-8/3*(3*e^(4*b*x + 4*d) - 3*e^(2*b*x + 2*d) + 1)*e^(2*a - 2*d)/(b*(e^(2*b*
x + 2*d) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.94 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.16

$$\int e^{2(a+bx)}\operatorname{csch}^4(d+bx)\,dx = -\frac{8\,\mathrm{e}^{2\,a+2\,d+4\,b\,x}}{3\,b\,\left(3\,\mathrm{e}^{2\,d+2\,b\,x}-3\,\mathrm{e}^{4\,d+4\,b\,x}+\mathrm{e}^{6\,d+6\,b\,x}-1\right)} - \frac{8\,\mathrm{e}^{2\,a+2\,b\,x}}{3\,b\,\left(\mathrm{e}^{4\,d+4\,b\,x}-2\,\mathrm{e}^{2\,d+2\,b\,x}+1\right)} - \frac{8\,\mathrm{e}^{2\,a-2\,d}}{3\,b\,\left(\mathrm{e}^{2\,d+2\,b\,x}-1\right)}$$

input
```
int(exp(2*a + 2*b*x)/sinh(d + b*x)^4,x)
```

output
```
- (8*exp(2*a + 2*d + 4*b*x))/(3*b*(3*exp(2*d + 2*b*x) - 3*exp(4*d + 4*b*x)
 + exp(6*d + 6*b*x) - 1)) - (8*exp(2*a + 2*b*x))/(3*b*(exp(4*d + 4*b*x) -
2*exp(2*d + 2*b*x) + 1)) - (8*exp(2*a - 2*d))/(3*b*(exp(2*d + 2*b*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.51

$$\int e^{2(a+bx)}\text{csch}^4(d+bx)\,dx = -\frac{8e^{6bx+2a+4d}}{3b\left(e^{6bx+6d}-3e^{4bx+4d}+3e^{2bx+2d}-1\right)}$$

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)^4,x)
```

output
```
( - 8*e**(2*a + 6*b*x + 4*d))/(3*b*(e**(6*b*x + 6*d) - 3*e**(4*b*x + 4*d)
+ 3*e**(2*b*x + 2*d) - 1))
```

3.107 $\int e^{2(a+bx)}\mathrm{csch}^5(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 163

$$\int e^{2(a+bx)}\mathrm{csch}^5(d+bx)\,dx = -\frac{4e^{2a+3d+5bx}}{b\left(1-e^{2d+2bx}\right)^4}+\frac{10e^{2a+d+3bx}}{3b\left(1-e^{2d+2bx}\right)^3}-\frac{5e^{2a-d+bx}}{2b\left(1-e^{2d+2bx}\right)^2}+\frac{5e^{2a-d+bx}}{4b\left(1-e^{2d+2bx}\right)}+\frac{5e^{2a-2d}\operatorname{arctanh}\left(e^{d+bx}\right)}{4b}$$

output
```
-4*exp(5*b*x+2*a+3*d)/b/(1-exp(2*b*x+2*d))^4+10/3*exp(3*b*x+2*a+d)/b/(1-ex
p(2*b*x+2*d))^3-5/2*exp(b*x+2*a-d)/b/(1-exp(2*b*x+2*d))^2+5/4*exp(b*x+2*a-
d)/b/(1-exp(2*b*x+2*d))+5/4*exp(2*a-2*d)*arctanh(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.97

$$\int e^{2(a+bx)}\mathrm{csch}^5(d+bx)\,dx$$
$$= \frac{e^{2a}\left(15\cosh(2d)\log\left(\left(1+e^{bx}\right)\cosh\left(\frac{d}{2}\right)+\left(-1+e^{bx}\right)\sinh\left(\frac{d}{2}\right)\right)-15\cosh(2d)\log\left(\left(-1+e^{bx}\right)\cosh\left(\frac{d}{2}\right)+\right.\right.}{}$$

input
```
Integrate[E^(2*(a + b*x))*Csch[d + b*x]^5,x]
```

output
```
(E^(2*a)*(15*Cosh[2*d]*Log[(1 + E^(b*x))*Cosh[d/2] + (-1 + E^(b*x))*Sinh[d
/2]] - 15*Cosh[2*d]*Log[(-1 + E^(b*x))*Cosh[d/2] + (1 + E^(b*x))*Sinh[d/2]
] - (96*E^(b*x)*(Cosh[d] - Sinh[d])^5)/((-1 + E^(2*b*x))*Cosh[d] + (1 + E^
(2*b*x))*Sinh[d])^4 - (272*E^(b*x)*(Cosh[d] - Sinh[d])^4)/((-1 + E^(2*b*x)
)*Cosh[d] + (1 + E^(2*b*x))*Sinh[d])^3 - (236*E^(b*x)*(Cosh[d] - Sinh[d])^
3)/((-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d])^2 - (30*E^(b*x)*(C
osh[d] - Sinh[d])^2)/((-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d])
- 15*Log[(1 + E^(b*x))*Cosh[d/2] + (-1 + E^(b*x))*Sinh[d/2]]*Sinh[2*d] + 1
5*Log[(-1 + E^(b*x))*Cosh[d/2] + (1 + E^(b*x))*Sinh[d/2]]*Sinh[2*d]))/(24*
b)
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.75, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2720, 27, 252, 252, 252, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{csch}^5(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{32e^{2a+6bx}}{\left(1-e^{2bx}\right)^5}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{32e^{2a}\int \frac{e^{6bx}}{\left(1-e^{2bx}\right)^5}de^{bx}}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{32e^{2a}\left(\frac{e^{5bx}}{8\left(1-e^{2bx}\right)^4}-\frac{5}{8}\int \frac{e^{4bx}}{\left(1-e^{2bx}\right)^4}de^{bx}\right)}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{32e^{2a}\left(\frac{e^{5bx}}{8\left(1-e^{2bx}\right)^4}-\frac{5}{8}\left(\frac{e^{3bx}}{6\left(1-e^{2bx}\right)^3}-\frac{1}{2}\int \frac{e^{2bx}}{\left(1-e^{2bx}\right)^3}de^{bx}\right)\right)}{b}$$

$$\Big\downarrow \; 252$$

$$-\frac{32e^{2a}\left(\frac{e^{5bx}}{8(1-e^{2bx})^4}-\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\int \frac{1}{(1-e^{2bx})^2}de^{bx}-\frac{e^{bx}}{4(1-e^{2bx})^2}\right)+\frac{e^{3bx}}{6(1-e^{2bx})^3}\right)\right)}{b}$$

$$\Big\downarrow \; 215$$

$$-\frac{32e^{2a}\left(\frac{e^{5bx}}{8(1-e^{2bx})^4}-\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{1-e^{2bx}}de^{bx}+\frac{e^{bx}}{2(1-e^{2bx})}\right)-\frac{e^{bx}}{4(1-e^{2bx})^2}\right)+\frac{e^{3bx}}{6(1-e^{2bx})^3}\right)\right)}{b}$$

$$\Big\downarrow \; 219$$

$$-\frac{32e^{2a}\left(\frac{e^{5bx}}{8(1-e^{2bx})^4}-\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\text{arctanh}\left(e^{bx}\right)+\frac{e^{bx}}{2(1-e^{2bx})}\right)-\frac{e^{bx}}{4(1-e^{2bx})^2}\right)+\frac{e^{3bx}}{6(1-e^{2bx})^3}\right)\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Csch[d + b*x]^5,x]
```

output
```
(-32*E^(2*a)*(E^(5*b*x)/(8*(1 - E^(2*b*x))^4) - (5*(E^(3*b*x)/(6*(1 - E^(2
*b*x))^3) + (-1/4*E^(b*x)/(1 - E^(2*b*x))^2 + (E^(b*x)/(2*(1 - E^(2*b*x)))
 + ArcTanh[E^(b*x)]/2)/4)/2))/8))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.88

method	result
risch	$-\frac{\left(15\,{\mathrm e}^{6 b x +6 a +6 d}+73\,{\mathrm e}^{4 b x +6 a +4 d}-55\,{\mathrm e}^{2 b x +6 a +2 d}+15\,{\mathrm e}^{6 a}\right) {\mathrm e}^{b x +4 a -d}}{12 \left(-{\mathrm e}^{2 b x +2 a +2 d}+{\mathrm e}^{2 a}\right)^{4} b}-\frac{5 \ln \left({\mathrm e}^{b x +a}-{\mathrm e}^{a -d}\right) {\mathrm e}^{2 a -2 d}}{8 b}+\frac{5 \ln \left({\mathrm e}^{b x +a}+{\mathrm e}^{a -d}\right) {\mathrm e}^{2 a -2 d}}{8 b}$

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12/(-exp(2*b*x+2*a+2*d)+exp(2*a))^4/b*(15*exp(6*b*x+6*a+6*d)+73*exp(4*b
*x+6*a+4*d)-55*exp(2*b*x+6*a+2*d)+15*exp(6*a))*exp(b*x+4*a-d)-5/8*ln(exp(b
*x+a)-exp(a-d))/b*exp(2*a-2*d)+5/8*ln(exp(b*x+a)+exp(a-d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2509 vs. $2(137) = 274$.

Time = 0.11 (sec) , antiderivative size = 2509, normalized size of antiderivative = 15.39

$$\int e^{2(a+bx)}\mathrm{csch}^5(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^5,x, algorithm="fricas")
```

output
```
-1/24*(30*cosh(b*x + d)^7*cosh(-2*a + 2*d) + 30*(cosh(-2*a + 2*d) - sinh(-
2*a + 2*d))*sinh(b*x + d)^7 + 210*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b
*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^6 + 146*cosh(b*x + d)^5*cosh(-2*a
+ 2*d) + 2*(315*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (315*cosh(b*x + d)^2 +
73)*sinh(-2*a + 2*d) + 73*cosh(-2*a + 2*d))*sinh(b*x + d)^5 + 10*(105*cosh
(b*x + d)^3*cosh(-2*a + 2*d) + 73*cosh(b*x + d)*cosh(-2*a + 2*d) - (105*co
sh(b*x + d)^3 + 73*cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d)^4 - 110*
cosh(b*x + d)^3*cosh(-2*a + 2*d) + 10*(105*cosh(b*x + d)^4*cosh(-2*a + 2*d
) + 146*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (105*cosh(b*x + d)^4 + 146*cosh
(b*x + d)^2 - 11)*sinh(-2*a + 2*d) - 11*cosh(-2*a + 2*d))*sinh(b*x + d)^3
+ 10*(63*cosh(b*x + d)^5*cosh(-2*a + 2*d) + 146*cosh(b*x + d)^3*cosh(-2*a
+ 2*d) - 33*cosh(b*x + d)*cosh(-2*a + 2*d) - (63*cosh(b*x + d)^5 + 146*cos
h(b*x + d)^3 - 33*cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d)^2 + 30*co
sh(b*x + d)*cosh(-2*a + 2*d) - 15*(cosh(b*x + d)^8*cosh(-2*a + 2*d) + (cos
h(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^8 + 8*(cosh(b*x + d)*cosh(
-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^7 - 4*cosh(b*x
 + d)^6*cosh(-2*a + 2*d) + 4*(7*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (7*cosh
(b*x + d)^2 - 1)*sinh(-2*a + 2*d) - cosh(-2*a + 2*d))*sinh(b*x + d)^6 + 8*
(7*cosh(b*x + d)^3*cosh(-2*a + 2*d) - 3*cosh(b*x + d)*cosh(-2*a + 2*d) - (
7*cosh(b*x + d)^3 - 3*cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d)^5 ...
```

Sympy [F]

$$\int e^{2(a+bx)}\operatorname{csch}^5(d+bx)\,dx = e^{2a}\int e^{2bx}\operatorname{csch}^5(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)**5,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*csch(b*x + d)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.96

$$\int e^{2(a+bx)}\operatorname{csch}^5(d+bx)\,dx$$
$$= \frac{5\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}+1\right)}{8\,b} - \frac{5\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}-1\right)}{8\,b} + \frac{\left(15\,e^{(-bx-d)}+73\,e^{(-3\,bx-3\,d)}-55\,e^{(-5\,bx-5\,d)}+15\,e^{(-7\,bx-7\,d)}\right)e^{(2\,a-2\,d)}}{12\,b(4\,e^{(-2\,bx-2\,d)}-6\,e^{(-4\,bx-4\,d)}+4\,e^{(-6\,bx-6\,d)}-e^{(-8\,bx-8\,d)}-1)}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^5,x, algorithm="maxima")
```

output
```
5/8*e^(2*a - 2*d)*log(e^(-b*x - d) + 1)/b - 5/8*e^(2*a - 2*d)*log(e^(-b*x
- d) - 1)/b + 1/12*(15*e^(-b*x - d) + 73*e^(-3*b*x - 3*d) - 55*e^(-5*b*x -
 5*d) + 15*e^(-7*b*x - 7*d))*e^(2*a - 2*d)/(b*(4*e^(-2*b*x - 2*d) - 6*e^(-
4*b*x - 4*d) + 4*e^(-6*b*x - 6*d) - e^(-8*b*x - 8*d) - 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.63

$$\int e^{2(a+bx)}\operatorname{csch}^5(d+bx)\,dx$$
$$= \frac{\left(15\,e^{(-2\,d)}\log\left(e^{(bx+d)}+1\right)-15\,e^{(-2\,d)}\log\left(\left|e^{(bx+d)}-1\right|\right)-\frac{2\left(15\,e^{(7\,bx+7\,d)}+73\,e^{(5\,bx+5\,d)}-55\,e^{(3\,bx+3\,d)}+15\,e^{(bx+d)}\right)e^{(}}{\left(e^{(2\,bx+2\,d)}-1\right)^4}}{24\,b}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^5,x, algorithm="giac")
```

output
```
1/24*(15*e^(-2*d)*log(e^(b*x + d) + 1) - 15*e^(-2*d)*log(abs(e^(b*x + d) -
 1)) - 2*(15*e^(7*b*x + 7*d) + 73*e^(5*b*x + 5*d) - 55*e^(3*b*x + 3*d) + 1
5*e^(b*x + d))*e^(-2*d)/(e^(2*b*x + 2*d) - 1)^4)*e^(2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.45

$$\int e^{2(a+bx)}\operatorname{csch}^5(d+bx)\,dx = \frac{5\sqrt{e^{4a-4d}}\operatorname{atan}\left(\frac{e^{2a}\,e^{-d}\,e^{bx}\,\sqrt{-b^2}}{b\sqrt{e^{4a}\,e^{-4d}}}\right)}{4\sqrt{-b^2}} - \frac{5\,e^{2a-d+bx}}{4b\,(e^{2d+2bx}-1)} - \frac{5\,e^{2a-d+bx}}{2b\,(e^{4d+4bx}-2\,e^{2d+2bx}+1)} - \frac{10\,e^{2a+d+3bx}}{3b\,(3\,e^{2d+2bx}-3\,e^{4d+4bx}+e^{6d+6bx}-1)} - \frac{4\,e^{2a+3d+5bx}}{b\,(6\,e^{4d+4bx}-4\,e^{2d+2bx}-4\,e^{6d+6bx}+e^{8d+8bx}+1)}$$

input
```
int(exp(2*a + 2*b*x)/sinh(d + b*x)^5,x)
```

output
```
(5*exp(4*a - 4*d)^(1/2)*atan((exp(2*a)*exp(-d)*exp(b*x)*(-b^2)^(1/2))/(b*(
exp(4*a)*exp(-4*d))^(1/2))))/(4*(-b^2)^(1/2)) - (5*exp(2*a - d + b*x))/(4*
b*(exp(2*d + 2*b*x) - 1)) - (5*exp(2*a - d + b*x))/(2*b*(exp(4*d + 4*b*x)
- 2*exp(2*d + 2*b*x) + 1)) - (10*exp(2*a + d + 3*b*x))/(3*b*(3*exp(2*d + 2
*b*x) - 3*exp(4*d + 4*b*x) + exp(6*d + 6*b*x) - 1)) - (4*exp(2*a + 3*d + 5
*b*x))/(b*(6*exp(4*d + 4*b*x) - 4*exp(2*d + 2*b*x) - 4*exp(6*d + 6*b*x) +
exp(8*d + 8*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.92

$$\int e^{2(a+bx)}\operatorname{csch}^5(d+bx)\,dx = \frac{e^{2a}\left(-15e^{8bx+8d}\log\left(e^{bx+d}-1\right)+15e^{8bx+8d}\log\left(e^{bx+d}+1\right)-30e^{7bx+7d}+60e^{6bx+6d}\log\left(e^{bx+d}-1\right)-60e^{6bx}\right.}{}$$

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)^5,x)
```

output
```
(e**(2*a)*( - 15*e**(8*b*x + 8*d)*log(e**(b*x + d) - 1) + 15*e**(8*b*x + 8
*d)*log(e**(b*x + d) + 1) - 30*e**(7*b*x + 7*d) + 60*e**(6*b*x + 6*d)*log(
e**(b*x + d) - 1) - 60*e**(6*b*x + 6*d)*log(e**(b*x + d) + 1) - 146*e**(5*
b*x + 5*d) - 90*e**(4*b*x + 4*d)*log(e**(b*x + d) - 1) + 90*e**(4*b*x + 4*
d)*log(e**(b*x + d) + 1) + 110*e**(3*b*x + 3*d) + 60*e**(2*b*x + 2*d)*log(
e**(b*x + d) - 1) - 60*e**(2*b*x + 2*d)*log(e**(b*x + d) + 1) - 30*e**(b*x
 + d) - 15*log(e**(b*x + d) - 1) + 15*log(e**(b*x + d) + 1)))/(24*e**(2*d)
*b*(e**(8*b*x + 8*d) - 4*e**(6*b*x + 6*d) + 6*e**(4*b*x + 4*d) - 4*e**(2*b
*x + 2*d) + 1))
```

3.108 $\int e^{\frac{5}{3}(a+bx)}\text{csch}(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 144

$$\int e^{\frac{5}{3}(a+bx)}\text{csch}(d+bx)\,dx = \frac{3e^{\frac{5(a-d)}{3}+\frac{2}{3}(d+bx)}}{b} - \frac{\sqrt{3}e^{\frac{5(a-d)}{3}}\arctan\left(\frac{1+2e^{\frac{2}{3}(d+bx)}}{\sqrt{3}}\right)}{b} + \frac{e^{\frac{5(a-d)}{3}}\log\left(1-e^{\frac{2}{3}(d+bx)}\right)}{b} - \frac{e^{\frac{5(a-d)}{3}}\log\left(1+e^{\frac{2}{3}(d+bx)}+e^{\frac{4}{3}(d+bx)}\right)}{2b}$$

output
```
3*exp(5/3*a-d+2/3*b*x)/b-3^(1/2)*exp(5/3*a-5/3*d)*arctan(1/3*(1+2*exp(2/3*
b*x+2/3*d))*3^(1/2))/b+exp(5/3*a-5/3*d)*ln(1-exp(2/3*b*x+2/3*d))/b-1/2*exp
(5/3*a-5/3*d)*ln(1+exp(2/3*b*x+2/3*d)+exp(4/3*b*x+4/3*d))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.21 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.28

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{csch}(d+bx)\,dx$$
$$= \frac{e^{5a/3}\left(-\left(\left(\left(-9e^{\frac{2bx}{3}}+\text{RootSum}\left[-\cosh\left(\frac{d}{2}\right)+\sinh\left(\frac{d}{2}\right)+\cosh\left(\frac{d}{2}\right)\#1^3+\sinh\left(\frac{d}{2}\right)\#1^3\&,\frac{bx-3\log\left(e^{\frac{bx}{3}}-\#\right.}{\#1}\right.\right.\right.\right.}{}$$

input
```
Integrate[E^((5*(a + b*x))/3)*Csch[d + b*x],x]
```

output
```
(E^((5*a)/3)*(-((-9*E^((2*b*x)/3) + RootSum[-Cosh[d/2] + Sinh[d/2] + Cosh[
d/2]*#1^3 + Sinh[d/2]*#1^3 & , (b*x - 3*Log[E^((b*x)/3) - #1])/#1 & ]*(Cos
h[d] - Sinh[d]))*(Cosh[d] - Sinh[d])) + RootSum[Cosh[d/2] - Sinh[d/2] + Co
sh[d/2]*#1^3 + Sinh[d/2]*#1^3 & , (b*x - 3*Log[E^((b*x)/3) - #1])/#1 & ]*(
Cosh[2*d] - 2*Cosh[d]*Sinh[d])))/(3*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.60, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {2720, 27, 807, 843, 750, 16, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{csch}(bx+d)\,dx$$

$$\Big\downarrow \ 2720$$

$$\frac{3\int -\frac{2e^{\frac{5a}{3}+\frac{7bx}{3}}}{1-e^{2bx}}de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \ 27$$

$$-\frac{6e^{5a/3}\int \frac{e^{\frac{7bx}{3}}}{1-e^{2bx}}de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \text{807}$$

$$-\frac{3e^{5a/3}\int \frac{e^{bx}}{1-e^{bx}}de^{\frac{2bx}{3}}}{b}$$

$$\Big\downarrow \text{843}$$

$$-\frac{3e^{5a/3}\left(\int \frac{1}{1-e^{bx}}de^{\frac{2bx}{3}}-e^{\frac{2bx}{3}}\right)}{b}$$

$$\Big\downarrow \text{750}$$

$$-\frac{3e^{5a/3}\left(\frac{1}{3}\int \frac{1}{1-e^{\frac{2bx}{3}}}de^{\frac{2bx}{3}}+\frac{1}{3}\int \frac{2+e^{\frac{2bx}{3}}}{1+2e^{\frac{2bx}{3}}}de^{\frac{2bx}{3}}-e^{\frac{2bx}{3}}\right)}{b}$$

$$\Big\downarrow \text{16}$$

$$-\frac{3e^{5a/3}\left(\frac{1}{3}\int \frac{2+e^{\frac{2bx}{3}}}{1+2e^{\frac{2bx}{3}}}de^{\frac{2bx}{3}}-e^{\frac{2bx}{3}}-\frac{1}{3}\log\left(1-e^{\frac{2bx}{3}}\right)\right)}{b}$$

$$\Big\downarrow \text{1142}$$

$$-\frac{3e^{5a/3}\left(\frac{1}{3}\left(\frac{1}{2}\int 1de^{\frac{2bx}{3}}+\frac{3}{2}\int \frac{1}{1+2e^{\frac{2bx}{3}}}de^{\frac{2bx}{3}}\right)-e^{\frac{2bx}{3}}-\frac{1}{3}\log\left(1-e^{\frac{2bx}{3}}\right)\right)}{b}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{3e^{5a/3}\left(\frac{1}{3}\left(\frac{1}{2}\int 1de^{\frac{2bx}{3}}-3\int \frac{1}{-4-2e^{\frac{2bx}{3}}}d\left(1+2e^{\frac{2bx}{3}}\right)\right)-e^{\frac{2bx}{3}}-\frac{1}{3}\log\left(1-e^{\frac{2bx}{3}}\right)\right)}{b}$$

$$\Big\downarrow \text{217}$$

$$-\frac{3e^{5a/3}\left(\frac{1}{3}\left(\frac{1}{2}\int 1de^{\frac{2bx}{3}}+\sqrt{3}\arctan\left(\frac{2e^{\frac{2bx}{3}}+1}{\sqrt{3}}\right)\right)-e^{\frac{2bx}{3}}-\frac{1}{3}\log\left(1-e^{\frac{2bx}{3}}\right)\right)}{b}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{3e^{5a/3}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2e^{\frac{2bx}{3}}+1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2e^{\frac{2bx}{3}}+1\right)\right)-e^{\frac{2bx}{3}}-\frac{1}{3}\log\left(1-e^{\frac{2bx}{3}}\right)\right)}{b}$$

input
```
Int[E^((5*(a + b*x))/3)*Csch[d + b*x],x]
```

output
```
(-3*E^((5*a)/3)*(-E^((2*b*x)/3) - Log[1 - E^((2*b*x)/3)]/3 + (Sqrt[3]*ArcT
an[(1 + 2*E^((2*b*x)/3))/Sqrt[3]] + Log[1 + 2*E^((2*b*x)/3)]/2)/3))/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.22

method	result
risch	$\frac{3\,\mathrm{e}^{\frac{5a}{3}-d+\frac{2bx}{3}}}{b}-\frac{\ln\left(\mathrm{e}^{\frac{2bx}{3}+\frac{2d}{3}}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{e}^{\frac{5a}{3}-\frac{5d}{3}}}{2b}+\frac{i\ln\left(\mathrm{e}^{\frac{2bx}{3}+\frac{2d}{3}}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{e}^{\frac{5a}{3}-\frac{5d}{3}}\sqrt{3}}{2b}-\frac{\ln\left(\mathrm{e}^{\frac{2bx}{3}+\frac{2d}{3}}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\mathrm{e}^{\frac{5a}{3}-\frac{5d}{3}}}{2b}$

input
```
int(exp(5/3*b*x+5/3*a)*csch(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
3*exp(5/3*a-d+2/3*b*x)/b-1/2*ln(exp(2/3*b*x+2/3*d)+1/2-1/2*I*3^(1/2))/b*ex
p(5/3*a-5/3*d)+1/2*I*ln(exp(2/3*b*x+2/3*d)+1/2-1/2*I*3^(1/2))/b*exp(5/3*a-
5/3*d)*3^(1/2)-1/2*ln(exp(2/3*b*x+2/3*d)+1/2+1/2*I*3^(1/2))/b*exp(5/3*a-5/
3*d)-1/2*I*ln(exp(2/3*b*x+2/3*d)+1/2+1/2*I*3^(1/2))/b*exp(5/3*a-5/3*d)*3^(
1/2)+ln(exp(2/3*b*x+2/3*d)-1)/b*exp(5/3*a-5/3*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(113) = 226$.

Time = 0.10 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.52

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{csch}(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*csch(b*x+d),x, algorithm="fricas")
```

output
```
1/2*(6*cosh(1/3*b*x + 1/3*d)^2*cosh(-5/3*a + 5/3*d) + 6*(cosh(-5/3*a + 5/3
*d) - sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^2 - 6*cosh(1/3*b*x + 1/3
*d)^2*sinh(-5/3*a + 5/3*d) + 2*(sqrt(3)*cosh(-5/3*a + 5/3*d) - sqrt(3)*sin
h(-5/3*a + 5/3*d))*arctan(-1/3*(3*sqrt(3)*cosh(1/3*b*x + 1/3*d) + sqrt(3)*
sinh(1/3*b*x + 1/3*d))/(cosh(1/3*b*x + 1/3*d) - sinh(1/3*b*x + 1/3*d))) -
(cosh(-5/3*a + 5/3*d) - sinh(-5/3*a + 5/3*d))*log((2*cosh(1/3*b*x + 1/3*d)
^2 + 2*sinh(1/3*b*x + 1/3*d)^2 + 1)/(cosh(1/3*b*x + 1/3*d)^2 - 2*cosh(1/3*
b*x + 1/3*d)*sinh(1/3*b*x + 1/3*d) + sinh(1/3*b*x + 1/3*d)^2)) + 2*(cosh(-
5/3*a + 5/3*d) - sinh(-5/3*a + 5/3*d))*log(2*sinh(1/3*b*x + 1/3*d)/(cosh(1
/3*b*x + 1/3*d) - sinh(1/3*b*x + 1/3*d))) + 12*(cosh(1/3*b*x + 1/3*d)*cosh
(-5/3*a + 5/3*d) - cosh(1/3*b*x + 1/3*d)*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*
x + 1/3*d))/b
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+bx)}\text{csch}(d+bx)\,dx = e^{\frac{5a}{3}}\int e^{\frac{5bx}{3}}\,\text{csch}\,(bx+d)\,dx$$

input
```
integrate(exp(5/3*b*x+5/3*a)*csch(b*x+d),x)
```

output
```
exp(5*a/3)*Integral(exp(5*b*x/3)*csch(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.42

$$\begin{aligned}\int e^{\frac{5}{3}(a+bx)}\text{csch}(d+bx)\,dx = & -\frac{\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}+1\right)\right)e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,d\right)}}{b} \\ & +\frac{\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}-1\right)\right)e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,d\right)}}{b} \\ & -\frac{e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,d\right)}\log\left(e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}+e^{\left(-\frac{2}{3}\,bx-\frac{2}{3}\,d\right)}+1\right)}{2\,b} \\ & +\frac{e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,d\right)}\log\left(e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}+1\right)}{b} \\ & +\frac{e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,d\right)}\log\left(e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}-1\right)}{b} \\ & -\frac{e^{\left(\frac{5}{3}\,a-\frac{5}{3}\,d\right)}\log\left(-e^{\left(-\frac{1}{3}\,bx-\frac{1}{3}\,d\right)}+e^{\left(-\frac{2}{3}\,bx-\frac{2}{3}\,d\right)}+1\right)}{2\,b} \\ & +\frac{3\,e^{\left(\frac{2}{3}\,bx+\frac{5}{3}\,a-d\right)}}{b}\end{aligned}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*csch(b*x+d),x, algorithm="maxima")
```

output
```
-sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-1/3*b*x - 1/3*d) + 1))*e^(5/3*a - 5/3*d
)/b + sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-1/3*b*x - 1/3*d) - 1))*e^(5/3*a -
5/3*d)/b - 1/2*e^(5/3*a - 5/3*d)*log(e^(-1/3*b*x - 1/3*d) + e^(-2/3*b*x -
2/3*d) + 1)/b + e^(5/3*a - 5/3*d)*log(e^(-1/3*b*x - 1/3*d) + 1)/b + e^(5/3
*a - 5/3*d)*log(e^(-1/3*b*x - 1/3*d) - 1)/b - 1/2*e^(5/3*a - 5/3*d)*log(-e
^(-1/3*b*x - 1/3*d) + e^(-2/3*b*x - 2/3*d) + 1)/b + 3*e^(2/3*b*x + 5/3*a -
 d)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.69

$$\int e^{\frac{5}{3}(a+bx)}\text{csch}(d+bx)\,dx =$$

$$-\frac{\left(2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(\frac{2}{3}bx\right)}+e^{\left(-\frac{2}{3}d\right)}\right)e^{\left(\frac{2}{3}d\right)}\right)e^{\left(-\frac{8}{3}d\right)}+e^{\left(-\frac{8}{3}d\right)}\log\left(e^{\left(\frac{4}{3}bx\right)}+e^{\left(\frac{2}{3}bx-\frac{2}{3}d\right)}+e^{\left(-\frac{4}{3}d\right)}\right)-2\cdots}{2b}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*csch(b*x+d),x, algorithm="giac")
```

output
```
-1/2*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2/3*b*x) + e^(-2/3*d))*e^(2/3*d))
*e^(-8/3*d) + e^(-8/3*d)*log(e^(4/3*b*x) + e^(2/3*b*x - 2/3*d) + e^(-4/3*d
)) - 2*e^(-8/3*d)*log(abs(e^(2/3*b*x) - e^(-2/3*d))) - 6*e^(2/3*b*x - 2*d)
)*e^(5/3*a + d)/b
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.35

$$\int e^{\frac{5}{3}(a+bx)}\text{csch}(d+bx)\,dx$$

$$= \frac{3\,e^{\frac{5a}{3}-d+\frac{2bx}{3}}}{b} + \frac{\left(e^{5a-5d}\right)^{1/3}\ln\left(2\left(e^{5a}\,e^{-5d}\right)^{1/3}-2\,e^{\frac{5a}{3}}\,e^{\frac{2d}{3}}\,e^{-\frac{5d}{3}}\,e^{\frac{2bx}{3}}\right)}{b}$$

$$+\frac{\left(e^{5a-5d}\right)^{1/3}\ln\left(2\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(e^{5a}\,e^{-5d}\right)^{1/3}-2\,e^{\frac{5a}{3}}\,e^{\frac{2d}{3}}\,e^{-\frac{5d}{3}}\,e^{\frac{2bx}{3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{b}$$

$$-\frac{\left(e^{5a-5d}\right)^{1/3}\ln\left(-2\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(e^{5a}\,e^{-5d}\right)^{1/3}-2\,e^{\frac{5a}{3}}\,e^{\frac{2d}{3}}\,e^{-\frac{5d}{3}}\,e^{\frac{2bx}{3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{b}$$

input
```
int(exp((5*a)/3 + (5*b*x)/3)/sinh(d + b*x),x)
```

output
```
(3*exp((5*a)/3 - d + (2*b*x)/3))/b + (exp(5*a - 5*d)^(1/3)*log(2*(exp(5*a)
*exp(-5*d))^(1/3) - 2*exp((5*a)/3)*exp((2*d)/3)*exp(-(5*d)/3)*exp((2*b*x)/
3)))/b + (exp(5*a - 5*d)^(1/3)*log(2*((3^(1/2)*1i)/2 - 1/2)*(exp(5*a)*exp(
-5*d))^(1/3) - 2*exp((5*a)/3)*exp((2*d)/3)*exp(-(5*d)/3)*exp((2*b*x)/3))*(
(3^(1/2)*1i)/2 - 1/2))/b - (exp(5*a - 5*d)^(1/3)*log(- 2*((3^(1/2)*1i)/2 +
 1/2)*(exp(5*a)*exp(-5*d))^(1/3) - 2*exp((5*a)/3)*exp((2*d)/3)*exp(-(5*d)/
3)*exp((2*b*x)/3))*((3^(1/2)*1i)/2 + 1/2))/b
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+bx)}\text{csch}(d+bx)\,dx=\int e^{\frac{5bx}{3}+\frac{5a}{3}}\text{csch}(bx+d)\,dx$$

input
```
int(exp(5/3*b*x+5/3*a)*csch(b*x+d),x)
```

output
```
int(e**((5*a + 5*b*x)/3)*csch(b*x + d),x)
```

3.109 $\int e^{\frac{5}{3}(a+bx)}\mathbf{csch}^2(d+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 204

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{csch}^2(d+bx)\,dx = \frac{2e^{\frac{5(a-d)}{3}+\frac{5}{3}(d+bx)}}{b\left(1-e^{2(d+bx)}\right)} - \frac{5e^{\frac{5(a-d)}{3}}\arctan\left(\frac{1-2e^{\frac{1}{3}(d+bx)}}{\sqrt{3}}\right)}{\sqrt{3}b} + \frac{5e^{\frac{5(a-d)}{3}}\arctan\left(\frac{1+2e^{\frac{1}{3}(d+bx)}}{\sqrt{3}}\right)}{\sqrt{3}b} - \frac{10e^{\frac{5(a-d)}{3}}\operatorname{arctanh}\left(e^{\frac{1}{3}(d+bx)}\right)}{3b} - \frac{5e^{\frac{5(a-d)}{3}}\operatorname{arctanh}\left(\frac{e^{\frac{1}{3}(d+bx)}}{1+e^{\frac{2}{3}(d+bx)}}\right)}{3b}$$

output
```
2*exp(5/3*b*x+5/3*a)/b/(1-exp(2*b*x+2*d))-5/3*3^(1/2)*exp(5/3*a-5/3*d)*arc
tan(1/3*(1-2*exp(1/3*b*x+1/3*d))*3^(1/2))/b+5/3*3^(1/2)*exp(5/3*a-5/3*d)*a
rctan(1/3*(1+2*exp(1/3*b*x+1/3*d))*3^(1/2))/b-10/3*exp(5/3*a-5/3*d)*arctan
h(exp(1/3*b*x+1/3*d))/b-5/3*exp(5/3*a-5/3*d)*arctanh(exp(1/3*b*x+1/3*d)/(1
+exp(2/3*b*x+2/3*d)))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.56

$$\int e^{\frac{5}{3}(a+bx)}\text{csch}^2(d+bx)\,dx$$
$$= \frac{e^{5a/3}(\cosh(d)-\sinh(d))\left(-5\text{RootSum}\left[-\cosh(d)+\sinh(d)+\cosh(d)\#1^6+\sinh(d)\#1^6\&,\frac{bx-3\log\left(e^{\frac{bx}{3}}\right.}{\#1}\right.\right.}{9b}$$

input
```
Integrate[E^((5*(a + b*x))/3)*Csch[d + b*x]^2,x]
```

output
```
(E^((5*a)/3)*(Cosh[d] - Sinh[d])*(-5*RootSum[-Cosh[d] + Sinh[d] + Cosh[d]*
#1^6 + Sinh[d]*#1^6 & , (b*x - 3*Log[E^((b*x)/3) - #1])/#1 & ]*(Cosh[d] -
Sinh[d]) - (18*E^((5*b*x)/3))/((-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*
Sinh[d])))/(9*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.81, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {2720, 27, 817, 825, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)}\text{csch}^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{3\int \frac{4e^{\frac{5a}{3}+\frac{10bx}{3}}}{\left(1-e^{2bx}\right)^2}de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{12e^{5a/3}\int \frac{e^{\frac{10bx}{3}}}{(1-e^{2bx})^2}de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \text{817}$$

$$\frac{12e^{5a/3}\left(\frac{e^{\frac{5bx}{3}}}{6(1-e^{2bx})}-\frac{5}{6}\int \frac{e^{\frac{4bx}{3}}}{1-e^{2bx}}de^{\frac{bx}{3}}\right)}{b}$$

$$\Big\downarrow \text{825}$$

$$\frac{12e^{5a/3}\left(\frac{e^{\frac{5bx}{3}}}{6(1-e^{2bx})}-\frac{5}{6}\left(\frac{1}{3}\int \frac{1}{1-e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{3}\int -\frac{1+e^{\frac{bx}{3}}}{2\left(1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}\right)}de^{\frac{bx}{3}}+\frac{1}{3}\int -\frac{1-e^{\frac{bx}{3}}}{2\left(1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}\right)}de^{\frac{bx}{3}}\right)\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{12e^{5a/3}\left(\frac{e^{\frac{5bx}{3}}}{6(1-e^{2bx})}-\frac{5}{6}\left(\frac{1}{3}\int \frac{1}{1-e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1+e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1-e^{\frac{bx}{3}}}{1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{12e^{5a/3}\left(\frac{e^{\frac{5bx}{3}}}{6(1-e^{2bx})}-\frac{5}{6}\left(-\frac{1}{6}\int \frac{1+e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1-e^{\frac{bx}{3}}}{1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{3}\text{arctanh}\left(e^{\frac{bx}{3}}\right)\right)\right)}{b}$$

$$\Big\downarrow \text{1142}$$

$$\frac{12e^{5a/3}\left(\frac{e^{\frac{5bx}{3}}}{6(1-e^{2bx})}-\frac{5}{6}\left(\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{2}\int -\frac{1-2e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1+2e^{\frac{bx}{3}}}{1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{3}{2}\int \frac{}{1+e^{\frac{bx}{3}}}\right.\right.\right.}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{12e^{5a/3}\left(\frac{e^{\frac{5bx}{3}}}{6(1-e^{2bx})}-\frac{5}{6}\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1-2e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{3}{2}\int \frac{1}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1+2e^{\frac{bx}{3}}}{1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{3}{2}\int \frac{1}{1+e^{\frac{bx}{3}}+e^{2}}\right.\right.\right.}{b}$$

$$\Big\downarrow \text{1083}$$

$$\frac{12e^{5a/3}\left(\frac{e^{\frac{5bx}{3}}}{6(1-e^{2bx})}-\frac{5}{6}\left(\frac{1}{6}\left(3\int \frac{1}{-3-e^{\frac{2bx}{3}}}d\left(-1+2e^{\frac{bx}{3}}\right)+\frac{1}{2}\int \frac{1-2e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(3\int \frac{1}{-3-e^{\frac{2bx}{3}}}d\left(1+2e^{\frac{bx}{3}}\right)+\frac{1}{2}\right.\right.\right.}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{12e^{5a/3}\left(\frac{e^{\frac{5bx}{3}}}{6(1-e^{2bx})}-\frac{5}{6}\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\sqrt{3}\arctan\left(\frac{2e^{\frac{bx}{3}}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{1+2e^{\frac{bx}{3}}}{1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\sqrt{3}\arctan\left(\right.\right.\right.\right.}{b}$$

$$\Big\downarrow \text{1103}$$

$$\frac{12e^{5a/3}\left(\frac{e^{\frac{5bx}{3}}}{6(1-e^{2bx})}-\frac{5}{6}\left(\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{2e^{\frac{bx}{3}}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}+1\right)\right)+\frac{1}{6}\left(\frac{1}{2}\log\left(e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}+1\right)-\sqrt{\right.\right.\right.}{b}$$

input
```
Int[E^((5*(a + b*x))/3)*Csch[d + b*x]^2,x]
```

output
```
(12*E^((5*a)/3)*(E^((5*b*x)/3)/(6*(1 - E^(2*b*x))) - (5*(ArcTanh[E^((b*x)/
3)]/3 + (-(Sqrt[3]*ArcTan[(-1 + 2*E^((b*x)/3))/Sqrt[3]]) - Log[1 - E^((b*x
)/3) + E^((2*b*x)/3)]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + 2*E^((b*x)/3))/Sqrt[3]
]) + Log[1 + E^((b*x)/3) + E^((2*b*x)/3)]/2)/6))/6))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.68

method	result
risch	$-\frac{2\,{\mathrm e}^{\frac{5 b x}{3}+\frac{5 a}{3}}}{\left({\mathrm e}^{2 b x +2 d}-1\right) b}+\frac{5 \ln \left({\mathrm e}^{\frac{b x}{3}+\frac{d}{3}}-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) {\mathrm e}^{\frac{5 a}{3}-\frac{5 d}{3}}}{6 b}+\frac{5 i \ln \left({\mathrm e}^{\frac{b x}{3}+\frac{d}{3}}-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) {\mathrm e}^{\frac{5 a}{3}-\frac{5 d}{3}} \sqrt{3}}{6 b}+\frac{5 \ln \left({\mathrm e}^{\frac{b x}{3}+\frac{d}{3}}-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) {\mathrm e}^{\frac{5 a}{3}-\frac{5 d}{3}}}{6 b}$

input
```
int(exp(5/3*b*x+5/3*a)*csch(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/(exp(2*b*x+2*d)-1)/b*exp(5/3*b*x+5/3*a)+5/6/b*ln(exp(1/3*b*x+1/3*d)-1/2
+1/2*I*3^(1/2))*exp(5/3*a-5/3*d)+5/6*I/b*ln(exp(1/3*b*x+1/3*d)-1/2+1/2*I*3
^(1/2))*exp(5/3*a-5/3*d)*3^(1/2)+5/6/b*ln(exp(1/3*b*x+1/3*d)-1/2-1/2*I*3^(
1/2))*exp(5/3*a-5/3*d)-5/6*I/b*ln(exp(1/3*b*x+1/3*d)-1/2-1/2*I*3^(1/2))*ex
p(5/3*a-5/3*d)*3^(1/2)+5/3/b*ln(exp(1/3*b*x+1/3*d)-1)*exp(5/3*a-5/3*d)-5/6
/b*ln(exp(1/3*b*x+1/3*d)+1/2+1/2*I*3^(1/2))*exp(5/3*a-5/3*d)+5/6*I/b*ln(ex
p(1/3*b*x+1/3*d)+1/2+1/2*I*3^(1/2))*exp(5/3*a-5/3*d)*3^(1/2)-5/6/b*ln(exp(
1/3*b*x+1/3*d)+1/2-1/2*I*3^(1/2))*exp(5/3*a-5/3*d)-5/6*I/b*ln(exp(1/3*b*x+
1/3*d)+1/2-1/2*I*3^(1/2))*exp(5/3*a-5/3*d)*3^(1/2)-5/3/b*ln(1+exp(1/3*b*x+
1/3*d))*exp(5/3*a-5/3*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2870 vs. 2(158) = 316.

Time = 0.13 (sec) , antiderivative size = 2870, normalized size of antiderivative = 14.07

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{csch}^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*csch(b*x+d)^2,x, algorithm="fricas")
```

output
```
-1/6*(12*cosh(1/3*b*x + 1/3*d)^5*cosh(-5/3*a + 5/3*d) + 12*(cosh(-5/3*a +
5/3*d) - sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^5 - 12*cosh(1/3*b*x +
 1/3*d)^5*sinh(-5/3*a + 5/3*d) + 60*(cosh(1/3*b*x + 1/3*d)*cosh(-5/3*a + 5
/3*d) - cosh(1/3*b*x + 1/3*d)*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^
4 + 120*(cosh(1/3*b*x + 1/3*d)^2*cosh(-5/3*a + 5/3*d) - cosh(1/3*b*x + 1/3
*d)^2*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^3 + 120*(cosh(1/3*b*x +
1/3*d)^3*cosh(-5/3*a + 5/3*d) - cosh(1/3*b*x + 1/3*d)^3*sinh(-5/3*a + 5/3*
d))*sinh(1/3*b*x + 1/3*d)^2 - 10*(sqrt(3)*cosh(1/3*b*x + 1/3*d)^6*cosh(-5/
3*a + 5/3*d) + (sqrt(3)*cosh(-5/3*a + 5/3*d) - sqrt(3)*sinh(-5/3*a + 5/3*d
))*sinh(1/3*b*x + 1/3*d)^6 + 6*(sqrt(3)*cosh(1/3*b*x + 1/3*d)*cosh(-5/3*a
+ 5/3*d) - sqrt(3)*cosh(1/3*b*x + 1/3*d)*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*
x + 1/3*d)^5 + 15*(sqrt(3)*cosh(1/3*b*x + 1/3*d)^2*cosh(-5/3*a + 5/3*d) -
sqrt(3)*cosh(1/3*b*x + 1/3*d)^2*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d
)^4 + 20*(sqrt(3)*cosh(1/3*b*x + 1/3*d)^3*cosh(-5/3*a + 5/3*d) - sqrt(3)*c
osh(1/3*b*x + 1/3*d)^3*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^3 + 15*
(sqrt(3)*cosh(1/3*b*x + 1/3*d)^4*cosh(-5/3*a + 5/3*d) - sqrt(3)*cosh(1/3*b
*x + 1/3*d)^4*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d)^2 + 6*(sqrt(3)*c
osh(1/3*b*x + 1/3*d)^5*cosh(-5/3*a + 5/3*d) - sqrt(3)*cosh(1/3*b*x + 1/3*d
)^5*sinh(-5/3*a + 5/3*d))*sinh(1/3*b*x + 1/3*d) - (sqrt(3)*cosh(1/3*b*x +
1/3*d)^6 - sqrt(3))*sinh(-5/3*a + 5/3*d) - sqrt(3)*cosh(-5/3*a + 5/3*d)...
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{csch}^2(d+bx)\,dx = e^{\frac{5a}{3}}\int e^{\frac{5bx}{3}}\operatorname{csch}^2(bx+d)\,dx$$

input
```
integrate(exp(5/3*b*x+5/3*a)*csch(b*x+d)**2,x)
```

output
```
exp(5*a/3)*Integral(exp(5*b*x/3)*csch(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.08

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{csch}^2(d+bx)\,dx = -\frac{5\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+1\right)\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}}{3b} - \frac{5\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}-1\right)\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}}{3b} - \frac{5e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{\left(-\frac{2}{3}bx-\frac{2}{3}d\right)}+1\right)}{6b} - \frac{5e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+1\right)}{3b} + \frac{5e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}-1\right)}{3b} + \frac{5e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(-e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{\left(-\frac{2}{3}bx-\frac{2}{3}d\right)}+1\right)}{6b} + \frac{2e^{\left(-\frac{1}{3}bx+\frac{5}{3}a-2d\right)}}{b\left(e^{(-2bx-2d)}-1\right)}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*csch(b*x+d)^2,x, algorithm="maxima")
```

output
```
-5/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-1/3*b*x - 1/3*d) + 1))*e^(5/3*a - 5
/3*d)/b - 5/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-1/3*b*x - 1/3*d) - 1))*e^(
5/3*a - 5/3*d)/b - 5/6*e^(5/3*a - 5/3*d)*log(e^(-1/3*b*x - 1/3*d) + e^(-2/
3*b*x - 2/3*d) + 1)/b - 5/3*e^(5/3*a - 5/3*d)*log(e^(-1/3*b*x - 1/3*d) + 1
)/b + 5/3*e^(5/3*a - 5/3*d)*log(e^(-1/3*b*x - 1/3*d) - 1)/b + 5/6*e^(5/3*a
 - 5/3*d)*log(-e^(-1/3*b*x - 1/3*d) + e^(-2/3*b*x - 2/3*d) + 1)/b + 2*e^(-
1/3*b*x + 5/3*a - 2*d)/(b*(e^(-2*b*x - 2*d) - 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.95

$$\int e^{\frac{5}{3}(a+bx)}\text{csch}^2(d+bx)\,dx$$
$$= \frac{\left(10\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,e^{\left(\frac{1}{3}\,bx\right)}+e^{\left(-\frac{1}{3}\,d\right)}\right)e^{\left(\frac{1}{3}\,d\right)}\right)e^{\left(-\frac{11}{3}\,d\right)}+10\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,e^{\left(\frac{1}{3}\,bx\right)}-e^{\left(-\frac{1}{3}\,d\right)}\right)e^{\left(\frac{1}{3}\,d\right)}\right.\right.}{}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*csch(b*x+d)^2,x, algorithm="giac")
```

output
```
1/6*(10*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(1/3*b*x) + e^(-1/3*d))*e^(1/3*d))
*e^(-11/3*d) + 10*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(1/3*b*x) - e^(-1/3*d))*
e^(1/3*d))*e^(-11/3*d) - 5*e^(-11/3*d)*log(e^(1/3*b*x - 1/3*d) + e^(2/3*b*
x) + e^(-2/3*d)) + 5*e^(-11/3*d)*log(-e^(1/3*b*x - 1/3*d) + e^(2/3*b*x) +
e^(-2/3*d)) - 10*e^(-11/3*d)*log(e^(1/3*b*x) + e^(-1/3*d)) + 10*e^(-11/3*d
)*log(abs(e^(1/3*b*x) - e^(-1/3*d))) - 12*e^(5/3*b*x - 2*d)/(e^(2*b*x + 2*
d) - 1))*e^(5/3*a + 2*d)/b
```

Mupad [B] (verification not implemented)

Time = 6.29 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.13

$$\int e^{\frac{5}{3}(a+bx)}\text{csch}^2(d+bx)\,dx = \text{Too large to display}$$

input
```
int(exp((5*a)/3 + (5*b*x)/3)/sinh(d + b*x)^2,x)
```

output
```
(5*exp(10*a - 10*d)^(1/6)*log((100*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp
((b*x)/3)*(exp(10*a)*exp(-10*d))^(1/6))/9 - (100*exp((10*a)/3)*exp(-(10*d)
/3))/9))/(3*b) - (5*exp(10*a - 10*d)^(1/6)*log(- (100*exp((10*a)/3)*exp(-(
10*d)/3))/9 - (100*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*(exp(1
0*a)*exp(-10*d))^(1/6))/9))/(3*b) - (2*exp((5*a)/3 + (5*b*x)/3))/(b*(exp(2
*d + 2*b*x) - 1)) - (5*log(- (100*exp((10*a)/3)*exp(-(10*d)/3))/9 - (100*e
xp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)/2 - 1/2)*(ex
p(10*a)*exp(-10*d))^(1/6))/9)*exp(10*a - 10*d)^(1/6)*((3^(1/2)*1i)/2 - 1/2
))/(3*b) + (5*log((100*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((
3^(1/2)*1i)/2 - 1/2)*(exp(10*a)*exp(-10*d))^(1/6))/9 - (100*exp((10*a)/3)*
exp(-(10*d)/3))/9)*exp(10*a - 10*d)^(1/6)*((3^(1/2)*1i)/2 - 1/2))/(3*b) -
(5*log(- (100*exp((10*a)/3)*exp(-(10*d)/3))/9 - (100*exp((5*a)/3)*exp(d/3)
*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)/2 + 1/2)*(exp(10*a)*exp(-10*d))^
(1/6))/9)*exp(10*a - 10*d)^(1/6)*((3^(1/2)*1i)/2 + 1/2))/(3*b) + (5*log((1
00*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)/2 + 1/2)
*(exp(10*a)*exp(-10*d))^(1/6))/9 - (100*exp((10*a)/3)*exp(-(10*d)/3))/9)*e
xp(10*a - 10*d)^(1/6)*((3^(1/2)*1i)/2 + 1/2))/(3*b)
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{csch}^2(d+bx)\,dx = \int e^{\frac{5bx}{3}+\frac{5a}{3}}\operatorname{csch}(bx+d)^2\,dx$$

input
```
int(exp(5/3*b*x+5/3*a)*csch(b*x+d)^2,x)
```

output
```
int(e**((5*a + 5*b*x)/3)*csch(b*x + d)**2,x)
```

3.110 $\int e^{\frac{5}{3}(a+bx)}\mathrm{csch}^3(d+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 207

$$\int e^{\frac{5}{3}(a+bx)}\mathrm{csch}^3(d+bx)\,dx = -\frac{2e^{\frac{5(a-d)}{3}+\frac{8}{3}(d+bx)}}{b\left(1-e^{2(d+bx)}\right)^2}+\frac{8e^{\frac{5(a-d)}{3}+\frac{2}{3}(d+bx)}}{3b\left(1-e^{2(d+bx)}\right)} - \frac{8e^{\frac{5(a-d)}{3}}\arctan\left(\frac{1+2e^{\frac{2}{3}(d+bx)}}{\sqrt{3}}\right)}{3\sqrt{3}b} + \frac{8e^{\frac{5(a-d)}{3}}\log\left(1-e^{\frac{2}{3}(d+bx)}\right)}{9b} - \frac{4e^{\frac{5(a-d)}{3}}\log\left(1+e^{\frac{2}{3}(d+bx)}+e^{\frac{4}{3}(d+bx)}\right)}{9b}$$

output
```
-2*exp(5/3*a+d+8/3*b*x)/b/(1-exp(2*b*x+2*d))^2+8/3*exp(5/3*a-d+2/3*b*x)/b/
(1-exp(2*b*x+2*d))-8/9*3^(1/2)*exp(5/3*a-5/3*d)*arctan(1/3*(1+2*exp(2/3*b*
x+2/3*d))*3^(1/2))/b+8/9*exp(5/3*a-5/3*d)*ln(1-exp(2/3*b*x+2/3*d))/b-4/9*e
xp(5/3*a-5/3*d)*ln(1+exp(2/3*b*x+2/3*d)+exp(4/3*b*x+4/3*d))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.30 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.14

$$\int e^{\frac{5}{3}(a+bx)}\text{csch}^3(d+bx)\,dx$$

$$= \frac{2e^{5a/3}(\cosh(d)-\sinh(d))^2\left(4\text{RootSum}\left[\cosh\left(\frac{d}{2}\right)-\sinh\left(\frac{d}{2}\right)+\cosh\left(\frac{d}{2}\right)\#1^3+\sinh\left(\frac{d}{2}\right)\#1^3\&,\frac{bx-3\log\left(e\right.}{\#}\right.\right.}{}$$

input
```
Integrate[E^((5*(a + b*x))/3)*Csch[d + b*x]^3,x]
```

output
```
(2*E^((5*a)/3)*(Cosh[d] - Sinh[d])^2*(4*RootSum[Cosh[d/2] - Sinh[d/2] + Co
sh[d/2]*#1^3 + Sinh[d/2]*#1^3 & , (b*x - 3*Log[E^((b*x)/3) - #1])/#1 & ] -
 4*RootSum[-Cosh[d/2] + Sinh[d/2] + Cosh[d/2]*#1^3 + Sinh[d/2]*#1^3 & , (b
*x - 3*Log[E^((b*x)/3) - #1])/#1 & ] - (27*E^((2*b*x)/3)*(Cosh[d] - Sinh[d
]))/((-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d])^2 - (63*E^((2*b*x
)/3))/((-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d])))/(27*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.65, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {2720, 27, 807, 817, 817, 750, 16, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)}\text{csch}^3(bx+d)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{3\int -\frac{8e^{\frac{5a}{3}+\frac{13bx}{3}}}{\left(1-e^{2bx}\right)^3}de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{24e^{5a/3}\int \frac{e^{\frac{13bx}{3}}}{\left(1-e^{2bx}\right)^3}de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \text{ 807}$$

$$-\frac{12e^{5a/3}\int \frac{e^{2bx}}{\left(1-e^{bx}\right)^3}de^{\frac{2bx}{3}}}{b}$$

$$\Big\downarrow \text{ 817}$$

$$-\frac{12e^{5a/3}\left(\frac{e^{\frac{4bx}{3}}}{6\left(1-e^{bx}\right)^2}-\frac{2}{3}\int \frac{e^{bx}}{\left(1-e^{bx}\right)^2}de^{\frac{2bx}{3}}\right)}{b}$$

$$\Big\downarrow \text{ 817}$$

$$-\frac{12e^{5a/3}\left(\frac{e^{\frac{4bx}{3}}}{6\left(1-e^{bx}\right)^2}-\frac{2}{3}\left(\frac{e^{\frac{2bx}{3}}}{3\left(1-e^{bx}\right)}-\frac{1}{3}\int \frac{1}{1-e^{bx}}de^{\frac{2bx}{3}}\right)\right)}{b}$$

$$\Big\downarrow \text{ 750}$$

$$-\frac{12e^{5a/3}\left(\frac{e^{\frac{4bx}{3}}}{6\left(1-e^{bx}\right)^2}-\frac{2}{3}\left(\frac{1}{3}\left(-\frac{1}{3}\int \frac{1}{1-e^{\frac{2bx}{3}}}de^{\frac{2bx}{3}}-\frac{1}{3}\int \frac{2+e^{\frac{2bx}{3}}}{1+2e^{\frac{2bx}{3}}}de^{\frac{2bx}{3}}\right)+\frac{e^{\frac{2bx}{3}}}{3\left(1-e^{bx}\right)}\right)\right)}{b}$$

$$\Big\downarrow \text{ 16}$$

$$-\frac{12e^{5a/3}\left(\frac{e^{\frac{4bx}{3}}}{6\left(1-e^{bx}\right)^2}-\frac{2}{3}\left(\frac{1}{3}\left(\frac{1}{3}\log\left(1-e^{\frac{2bx}{3}}\right)-\frac{1}{3}\int \frac{2+e^{\frac{2bx}{3}}}{1+2e^{\frac{2bx}{3}}}de^{\frac{2bx}{3}}\right)+\frac{e^{\frac{2bx}{3}}}{3\left(1-e^{bx}\right)}\right)\right)}{b}$$

$$\Big\downarrow \text{ 1142}$$

$$-\frac{12e^{5a/3}\left(\frac{e^{\frac{4bx}{3}}}{6\left(1-e^{bx}\right)^2}-\frac{2}{3}\left(\frac{1}{3}\left(\frac{1}{3}\left(-\frac{1}{2}\int 1de^{\frac{2bx}{3}}-\frac{3}{2}\int \frac{1}{1+2e^{\frac{2bx}{3}}}de^{\frac{2bx}{3}}\right)+\frac{1}{3}\log\left(1-e^{\frac{2bx}{3}}\right)\right)+\frac{e^{\frac{2bx}{3}}}{3\left(1-e^{bx}\right)}\right)\right)}{b}$$

$$\Big\downarrow \text{ 1083}$$

$$-\frac{12e^{5a/3}\left(\frac{e^{\frac{4bx}{3}}}{6\left(1-e^{bx}\right)^2}-\frac{2}{3}\left(\frac{1}{3}\left(\frac{1}{3}\left(3\int \frac{1}{-4-2e^{\frac{2bx}{3}}}d\left(1+2e^{\frac{2bx}{3}}\right)-\frac{1}{2}\int 1de^{\frac{2bx}{3}}\right)+\frac{1}{3}\log\left(1-e^{\frac{2bx}{3}}\right)\right)+\frac{e^{\frac{2bx}{3}}}{3\left(1-e^{bx}\right)}\right)\right)}{b}$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{12e^{5a/3}\left(\frac{e^{\frac{4bx}{3}}}{6\left(1-e^{bx}\right)^2}-\frac{2}{3}\left(\frac{1}{3}\left(\frac{1}{3}\left(-\frac{1}{2}\int 1de^{\frac{2bx}{3}}-\sqrt{3}\arctan\left(\frac{2e^{\frac{2bx}{3}}+1}{\sqrt{3}}\right)\right)+\frac{1}{3}\log\left(1-e^{\frac{2bx}{3}}\right)\right)+\frac{e^{\frac{2bx}{3}}}{3\left(1-e^{bx}\right)}\right)\right)}{b}$$

$$\Big\downarrow \ 1103$$

$$-\frac{12e^{5a/3}\left(\frac{e^{\frac{4bx}{3}}}{6\left(1-e^{bx}\right)^2}-\frac{2}{3}\left(\frac{1}{3}\left(\frac{1}{3}\left(-\sqrt{3}\arctan\left(\frac{2e^{\frac{2bx}{3}}+1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(2e^{\frac{2bx}{3}}+1\right)\right)+\frac{1}{3}\log\left(1-e^{\frac{2bx}{3}}\right)\right)+\frac{e^{\frac{2bx}{3}}}{3\left(1-e^{bx}\right)}\right)\right)}{b}$$

input
```
Int[E^((5*(a + b*x))/3)*Csch[d + b*x]^3,x]
```

output
```
(-12*E^((5*a)/3)*(E^((4*b*x)/3)/(6*(1 - E^(b*x))^2) - (2*(E^((2*b*x)/3)/(3
*(1 - E^(b*x))) + (Log[1 - E^((2*b*x)/3)]/3 + (-(Sqrt[3]*ArcTan[(1 + 2*E^(
(2*b*x)/3))/Sqrt[3]]) - Log[1 + 2*E^((2*b*x)/3)]/2)/3)/3))/3))/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{2\left(7\,\mathrm{e}^{2bx+2d}-4\right)\mathrm{e}^{\frac{5a}{3}-d+\frac{2bx}{3}}}{3\left(\mathrm{e}^{2bx+2d}-1\right)^{2}b}+\frac{8\ln\left(\mathrm{e}^{\frac{2bx}{3}+\frac{2d}{3}}-1\right)\mathrm{e}^{\frac{5a}{3}-\frac{5d}{3}}}{9b}-\frac{4\ln\left(\mathrm{e}^{\frac{2bx}{3}+\frac{2d}{3}}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{e}^{\frac{5a}{3}-\frac{5d}{3}}}{9b}+\frac{4i\ln\left(\mathrm{e}^{\frac{2bx}{3}+\frac{2d}{3}}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}{9b}$

input
```
int(exp(5/3*b*x+5/3*a)*csch(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3/(exp(2*b*x+2*d)-1)^2/b*(7*exp(2*b*x+2*d)-4)*exp(5/3*a-d+2/3*b*x)+8/9*
ln(exp(2/3*b*x+2/3*d)-1)/b*exp(5/3*a-5/3*d)-4/9*ln(exp(2/3*b*x+2/3*d)+1/2-
1/2*I*3^(1/2))/b*exp(5/3*a-5/3*d)+4/9*I*ln(exp(2/3*b*x+2/3*d)+1/2-1/2*I*3^
(1/2))/b*exp(5/3*a-5/3*d)*3^(1/2)-4/9*ln(exp(2/3*b*x+2/3*d)+1/2+1/2*I*3^(1
/2))/b*exp(5/3*a-5/3*d)-4/9*I*ln(exp(2/3*b*x+2/3*d)+1/2+1/2*I*3^(1/2))/b*e
xp(5/3*a-5/3*d)*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3987 vs. 2(155) = 310.

Time = 0.13 (sec) , antiderivative size = 3987, normalized size of antiderivative = 19.26

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{csch}^3(d+bx)\,dx=\text{Too large to display}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*csch(b*x+d)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{csch}^3(d+bx)\,dx = e^{\frac{5a}{3}}\int e^{\frac{5bx}{3}}\operatorname{csch}^3(bx+d)\,dx$$

input
```
integrate(exp(5/3*b*x+5/3*a)*csch(b*x+d)**3,x)
```

output
```
exp(5*a/3)*Integral(exp(5*b*x/3)*csch(b*x + d)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.22

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{csch}^3(d+bx)\,dx = -\frac{8\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+1\right)\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}}{9b} + \frac{8\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}-1\right)\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}}{9b} - \frac{4e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{\left(-\frac{2}{3}bx-\frac{2}{3}d\right)}+1\right)}{9b} + \frac{8e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+1\right)}{9b} + \frac{8e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}-1\right)}{9b} - \frac{4e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(-e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{\left(-\frac{2}{3}bx-\frac{2}{3}d\right)}+1\right)}{9b} + \frac{2\left(7e^{\left(-\frac{4}{3}bx-\frac{4}{3}d\right)}-4e^{\left(-\frac{10}{3}bx-\frac{10}{3}d\right)}\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}}{3b\left(2e^{(-2bx-2d)}-e^{(-4bx-4d)}-1\right)}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*csch(b*x+d)^3,x, algorithm="maxima")
```

output
```
-8/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-1/3*b*x - 1/3*d) + 1))*e^(5/3*a - 5
/3*d)/b + 8/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-1/3*b*x - 1/3*d) - 1))*e^(
5/3*a - 5/3*d)/b - 4/9*e^(5/3*a - 5/3*d)*log(e^(-1/3*b*x - 1/3*d) + e^(-2/
3*b*x - 2/3*d) + 1)/b + 8/9*e^(5/3*a - 5/3*d)*log(e^(-1/3*b*x - 1/3*d) + 1
)/b + 8/9*e^(5/3*a - 5/3*d)*log(e^(-1/3*b*x - 1/3*d) - 1)/b - 4/9*e^(5/3*a
 - 5/3*d)*log(-e^(-1/3*b*x - 1/3*d) + e^(-2/3*b*x - 2/3*d) + 1)/b + 2/3*(7
*e^(-4/3*b*x - 4/3*d) - 4*e^(-10/3*b*x - 10/3*d))*e^(5/3*a - 5/3*d)/(b*(2*
e^(-2*b*x - 2*d) - e^(-4*b*x - 4*d) - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.62

$$\int e^{\frac{5}{3}(a+bx)}\text{csch}^3(d+bx)\,dx =$$

$$-\frac{2\left(4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(\frac{2}{3}bx\right)}+e^{\left(-\frac{2}{3}d\right)}\right)e^{\left(\frac{2}{3}d\right)}\right)e^{\left(-\frac{14}{3}d\right)}+2e^{\left(-\frac{14}{3}d\right)}\log\left(e^{\left(\frac{4}{3}bx\right)}+e^{\left(\frac{2}{3}bx-\frac{2}{3}d\right)}+e^{\left(-\frac{4}{3}d\right)}\right.\right.}{9b}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*csch(b*x+d)^3,x, algorithm="giac")
```

output
```
-2/9*(4*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2/3*b*x) + e^(-2/3*d))*e^(2/3*d))
*e^(-14/3*d) + 2*e^(-14/3*d)*log(e^(4/3*b*x) + e^(2/3*b*x - 2/3*d) + e^(-4
/3*d)) - 4*e^(-14/3*d)*log(abs(e^(2/3*b*x) - e^(-2/3*d))) + 3*(7*e^(8/3*b*
x + 2*d) - 4*e^(2/3*b*x))*e^(-4*d)/(e^(2*b*x + 2*d) - 1)^2)*e^(5/3*a + 3*d
)/b
```

Mupad [B] (verification not implemented)

Time = 5.20 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.20

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{csch}^3(d+bx)\,dx$$
$$= \frac{8\left(e^{5a-5d}\right)^{1/3}\ln\left(\frac{16\left(e^{5a}e^{-5d}\right)^{1/3}}{9}-\frac{16e^{\frac{5a}{3}}e^{\frac{2d}{3}}e^{-\frac{5d}{3}}e^{\frac{2bx}{3}}}{9}\right)}{9b} - \frac{8e^{\frac{5a}{3}-d+\frac{2bx}{3}}}{3b\left(e^{2d+2bx}-1\right)} - \frac{2e^{\frac{5a}{3}+d+\frac{8bx}{3}}}{b\left(e^{4d+4bx}-2e^{2d+2bx}+1\right)} + \frac{8\left(e^{5a-5d}\right)^{1/3}\ln\left(\frac{16\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(e^{5a}e^{-5d}\right)^{1/3}}{9}-\frac{16e^{\frac{5a}{3}}e^{\frac{2d}{3}}e^{-\frac{5d}{3}}e^{\frac{2bx}{3}}}{9}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{9b} - \frac{8\left(e^{5a-5d}\right)^{1/3}\ln\left(-\frac{16\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(e^{5a}e^{-5d}\right)^{1/3}}{9}-\frac{16e^{\frac{5a}{3}}e^{\frac{2d}{3}}e^{-\frac{5d}{3}}e^{\frac{2bx}{3}}}{9}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{9b}$$

input
```
int(exp((5*a)/3 + (5*b*x)/3)/sinh(d + b*x)^3,x)
```

output
```
(8*exp(5*a - 5*d)^(1/3)*log((16*(exp(5*a)*exp(-5*d))^(1/3))/9 - (16*exp((5
*a)/3)*exp((2*d)/3)*exp(-(5*d)/3)*exp((2*b*x)/3))/9))/(9*b) - (8*exp((5*a)
/3 - d + (2*b*x)/3))/(3*b*(exp(2*d + 2*b*x) - 1)) - (2*exp((5*a)/3 + d + (
8*b*x)/3))/(b*(exp(4*d + 4*b*x) - 2*exp(2*d + 2*b*x) + 1)) + (8*exp(5*a -
5*d)^(1/3)*log((16*((3^(1/2)*1i)/2 - 1/2)*(exp(5*a)*exp(-5*d))^(1/3))/9 -
(16*exp((5*a)/3)*exp((2*d)/3)*exp(-(5*d)/3)*exp((2*b*x)/3))/9)*((3^(1/2)*1
i)/2 - 1/2))/(9*b) - (8*exp(5*a - 5*d)^(1/3)*log(- (16*((3^(1/2)*1i)/2 + 1
/2)*(exp(5*a)*exp(-5*d))^(1/3))/9 - (16*exp((5*a)/3)*exp((2*d)/3)*exp(-(5*
d)/3)*exp((2*b*x)/3))/9)*((3^(1/2)*1i)/2 + 1/2))/(9*b)
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{csch}^3(d+bx)\,dx = \int e^{\frac{5bx}{3}+\frac{5a}{3}}\operatorname{csch}(bx+d)^3\,dx$$

input
```
int(exp(5/3*b*x+5/3*a)*csch(b*x+d)^3,x)
```

output
```
int(e**((5*a + 5*b*x)/3)*csch(b*x + d)**3,x)
```

3.111 $\int e^{\frac{5}{3}(a+bx)}\text{csch}^4(d+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 296

$$\int e^{\frac{5}{3}(a+bx)}\text{csch}^4(d+bx)\,dx = \frac{8e^{\frac{5(a-d)}{3}+\frac{11}{3}(d+bx)}}{3b\left(1-e^{2(d+bx)}\right)^3} - \frac{22e^{\frac{5(a-d)}{3}+\frac{5}{3}(d+bx)}}{9b\left(1-e^{2(d+bx)}\right)^2} + \frac{55e^{\frac{5(a-d)}{3}+\frac{5}{3}(d+bx)}}{27b\left(1-e^{2(d+bx)}\right)} + \frac{55e^{\frac{5(a-d)}{3}}\arctan\left(\frac{1-2e^{\frac{1}{3}(d+bx)}}{\sqrt{3}}\right)}{54\sqrt{3}b} - \frac{55e^{\frac{5(a-d)}{3}}\arctan\left(\frac{1+2e^{\frac{1}{3}(d+bx)}}{\sqrt{3}}\right)}{54\sqrt{3}b} + \frac{55e^{\frac{5(a-d)}{3}}\text{arctanh}\left(e^{\frac{1}{3}(d+bx)}\right)}{81b} + \frac{55e^{\frac{5(a-d)}{3}}\text{arctanh}\left(\frac{e^{\frac{1}{3}(d+bx)}}{1+e^{\frac{2}{3}(d+bx)}}\right)}{162b}$$

output

```
8/3*exp(5/3*a+2*d+11/3*b*x)/b/(1-exp(2*b*x+2*d))^3-22/9*exp(5/3*b*x+5/3*a)
/b/(1-exp(2*b*x+2*d))^2+55/27*exp(5/3*b*x+5/3*a)/b/(1-exp(2*b*x+2*d))+55/1
62*3^(1/2)*exp(5/3*a-5/3*d)*arctan(1/3*(1-2*exp(1/3*b*x+1/3*d))*3^(1/2))/b
-55/162*3^(1/2)*exp(5/3*a-5/3*d)*arctan(1/3*(1+2*exp(1/3*b*x+1/3*d))*3^(1/
2))/b+55/81*exp(5/3*a-5/3*d)*arctanh(exp(1/3*b*x+1/3*d))/b+55/162*exp(5/3*
a-5/3*d)*arctanh(exp(1/3*b*x+1/3*d)/(1+exp(2/3*b*x+2/3*d)))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.52 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.53

$$\int e^{\frac{5}{3}(a+bx)}\text{csch}^4(d+bx)\,dx$$
$$= \frac{e^{5a/3}(\cosh(d)-\sinh(d))\left(55\text{RootSum}\left[-\cosh(d)+\sinh(d)+\cosh(d)\#1^6+\sinh(d)\#1^6\&,\frac{bx-3\log\left(e^{\frac{bx}{3}}-\right.}{\#1}\right.\right.}{486b}$$

input
```
Integrate[E^((5*(a + b*x))/3)*Csch[d + b*x]^4,x]
```

output
```
(E^((5*a)/3)*(Cosh[d] - Sinh[d])*(55*RootSum[-Cosh[d] + Sinh[d] + Cosh[d]*
#1^6 + Sinh[d]*#1^6 & , (b*x - 3*Log[E^((b*x)/3) - #1])/#1 & ]*(Cosh[d] -
Sinh[d]) - (18*E^((5*b*x)/3)*(28*E^(2*b*x) + 11*(-1 + 5*E^(4*b*x))*Cosh[2*
d] + 11*(1 + 5*E^(4*b*x))*Sinh[2*d]))/((-1 + E^(2*b*x))*Cosh[d] + (1 + E^(
2*b*x))*Sinh[d])^3))/(486*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.75, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {2720, 27, 817, 817, 819, 825, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)}\text{csch}^4(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{3\int \frac{16e^{\frac{5a}{3}+\frac{16bx}{3}}}{(1-e^{2bx})^4}de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{48e^{5a/3}\int \frac{e^{\frac{16bx}{3}}}{(1-e^{2bx})^4}de^{\frac{bx}{3}}}{b}$$

$\downarrow$ 817

$$\frac{48e^{5a/3}\left(\frac{e^{\frac{11bx}{3}}}{18(1-e^{2bx})^3}-\frac{11}{18}\int \frac{e^{\frac{10bx}{3}}}{(1-e^{2bx})^3}de^{\frac{bx}{3}}\right)}{b}$$

$\downarrow$ 817

$$\frac{48e^{5a/3}\left(\frac{e^{\frac{11bx}{3}}}{18(1-e^{2bx})^3}-\frac{11}{18}\left(\frac{e^{\frac{5bx}{3}}}{12(1-e^{2bx})^2}-\frac{5}{12}\int \frac{e^{\frac{4bx}{3}}}{(1-e^{2bx})^2}de^{\frac{bx}{3}}\right)\right)}{b}$$

$\downarrow$ 819

$$\frac{48e^{5a/3}\left(\frac{e^{\frac{11bx}{3}}}{18(1-e^{2bx})^3}-\frac{11}{18}\left(\frac{e^{\frac{5bx}{3}}}{12(1-e^{2bx})^2}-\frac{5}{12}\left(\frac{1}{6}\int \frac{e^{\frac{4bx}{3}}}{1-e^{2bx}}de^{\frac{bx}{3}}+\frac{e^{\frac{5bx}{3}}}{6(1-e^{2bx})}\right)\right)\right)}{b}$$

$\downarrow$ 825

$$48e^{5a/3}\left(\frac{e^{\frac{11bx}{3}}}{18(1-e^{2bx})^3}-\frac{11}{18}\left(\frac{e^{\frac{5bx}{3}}}{12(1-e^{2bx})^2}-\frac{5}{12}\left(\frac{1}{6}\left(\frac{1}{3}\int \frac{1}{1-e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{3}\int -\frac{1+e^{\frac{bx}{3}}}{2\left(1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}\right)}de^{\frac{bx}{3}}+\frac{1}{3}\int -\frac{1-e^{\frac{bx}{3}}}{2\left(1+e^{\frac{bx}{3}}+e^{\frac{2b}{3}}\right.}\right.\right.\right.\right.$$

$$b$$

$\downarrow$ 27

$$48e^{5a/3}\left(\frac{e^{\frac{11bx}{3}}}{18(1-e^{2bx})^3}-\frac{11}{18}\left(\frac{e^{\frac{5bx}{3}}}{12(1-e^{2bx})^2}-\frac{5}{12}\left(\frac{1}{6}\left(\frac{1}{3}\int \frac{1}{1-e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1+e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1-e^{\frac{bx}{3}}}{1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\right.\right.\right.$$

$$b$$

$\downarrow$ 219

$$48e^{5a/3}\left(\frac{e^{\frac{11bx}{3}}}{18(1-e^{2bx})^3}-\frac{11}{18}\left(\frac{e^{\frac{5bx}{3}}}{12(1-e^{2bx})^2}-\frac{5}{12}\left(\frac{1}{6}\left(-\frac{1}{6}\int \frac{1+e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{6}\int \frac{1-e^{\frac{bx}{3}}}{1+e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}+\frac{1}{3}\text{arctanh}\left(e^{\frac{bx}{3}}\right)\right)\right.\right.\right.$$

$$b$$

$\downarrow$ 1142

$$48e^{5a/3}\left(\frac{e^{\frac{11bx}{3}}}{18(1-e^{2bx})^3}-\frac{11}{18}\left(\frac{e^{\frac{5bx}{3}}}{12(1-e^{2bx})^2}-\frac{5}{12}\left(\frac{1}{6}\left(\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{1}{2}\int -\frac{1-2e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1+}{1+e}\right.\right.\right.\right.\right.$$

$$b$$

$\downarrow$ 25

$$\frac{48e^{5a/3}\left(\frac{e^{\frac{11bx}{3}}}{18(1-e^{2bx})^3}-\frac{11}{18}\left(\frac{e^{\frac{5bx}{3}}}{12(1-e^{2bx})^2}-\frac{5}{12}\left(\frac{1}{6}\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\frac{3}{2}\int\frac{1}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{1+2e^{\frac{bx}{3}}}{1+e^{\frac{bx}{3}}+e\cdots}\right.\right.\right.\right.\right.}{b}$$

$\Big\downarrow$ 1083

$$\frac{48e^{5a/3}\left(\frac{e^{\frac{11bx}{3}}}{18(1-e^{2bx})^3}-\frac{11}{18}\left(\frac{e^{\frac{5bx}{3}}}{12(1-e^{2bx})^2}-\frac{5}{12}\left(\frac{1}{6}\left(\frac{1}{6}\left(3\int\frac{1}{-3-e^{\frac{2bx}{3}}}d\left(-1+2e^{\frac{bx}{3}}\right)+\frac{1}{2}\int\frac{1-2e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}\right)+\frac{1}{6}\left(3\int\cdots\right.\right.\right.\right.\right.}{b}$$

$\Big\downarrow$ 217

$$\frac{48e^{5a/3}\left(\frac{e^{\frac{11bx}{3}}}{18(1-e^{2bx})^3}-\frac{11}{18}\left(\frac{e^{\frac{5bx}{3}}}{12(1-e^{2bx})^2}-\frac{5}{12}\left(\frac{1}{6}\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2e^{\frac{bx}{3}}}{1-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}}de^{\frac{bx}{3}}-\sqrt{3}\arctan\left(\frac{2e^{\frac{bx}{3}}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{1+2e\cdots}{1+e^{\frac{bx}{3}}+\cdots}\right.\right.\right.\right.\right.}{b}$$

$\Big\downarrow$ 1103

$$\frac{48e^{5a/3}\left(\frac{e^{\frac{11bx}{3}}}{18(1-e^{2bx})^3}-\frac{11}{18}\left(\frac{e^{\frac{5bx}{3}}}{12(1-e^{2bx})^2}-\frac{5}{12}\left(\frac{1}{6}\left(\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{2e^{\frac{bx}{3}}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(-e^{\frac{bx}{3}}+e^{\frac{2bx}{3}}+1\right)\right)+\frac{1}{6}\left(\frac{1}{2}\log\cdots\right.\right.\right.\right.\right.}{b}$$

input
```
Int[E^((5*(a + b*x))/3)*Csch[d + b*x]^4,x]
```

output
```
(48*E^((5*a)/3)*(E^((11*b*x)/3)/(18*(1 - E^(2*b*x))^3) - (11*(E^((5*b*x)/3
)/(12*(1 - E^(2*b*x))^2) - (5*(E^((5*b*x)/3)/(6*(1 - E^(2*b*x))) + (ArcTan
h[E^((b*x)/3)]/3 + (-(Sqrt[3]*ArcTan[(-1 + 2*E^((b*x)/3))/Sqrt[3]]) - Log[
1 - E^((b*x)/3) + E^((2*b*x)/3)]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + 2*E^((b*x)/
3))/Sqrt[3]]) + Log[1 + E^((b*x)/3) + E^((2*b*x)/3)]/2)/6)/6))/12))/18))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{\left(55\,\mathrm{e}^{4bx+4d}+28\,\mathrm{e}^{2bx+2d}-11\right)\mathrm{e}^{\frac{5bx}{3}+\frac{5a}{3}}}{27\left(\mathrm{e}^{2bx+2d}-1\right)^{3} b}+\frac{55\ln\left(1+\mathrm{e}^{\frac{bx}{3}+\frac{d}{3}}\right)\mathrm{e}^{\frac{5a}{3}-\frac{5d}{3}}}{162b}-\frac{55\ln\left(\mathrm{e}^{\frac{bx}{3}+\frac{d}{3}}-1\right)\mathrm{e}^{\frac{5a}{3}-\frac{5d}{3}}}{162b}+\frac{55\ln\left(\mathrm{e}^{\frac{bx}{3}+\frac{d}{3}}+\frac{1}{2}-\frac{i\sqrt{}}{2}\right.}{324b}$

input
```
int(exp(5/3*b*x+5/3*a)*csch(b*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/27/(exp(2*b*x+2*d)-1)^3/b*(55*exp(4*b*x+4*d)+28*exp(2*b*x+2*d)-11)*exp(
5/3*b*x+5/3*a)+55/162/b*ln(1+exp(1/3*b*x+1/3*d))*exp(5/3*a-5/3*d)-55/162/b
*ln(exp(1/3*b*x+1/3*d)-1)*exp(5/3*a-5/3*d)+55/324/b*ln(exp(1/3*b*x+1/3*d)+
1/2-1/2*I*3^(1/2))*exp(5/3*a-5/3*d)+55/324*I/b*ln(exp(1/3*b*x+1/3*d)+1/2-1
/2*I*3^(1/2))*exp(5/3*a-5/3*d)*3^(1/2)+55/324/b*ln(exp(1/3*b*x+1/3*d)+1/2+
1/2*I*3^(1/2))*exp(5/3*a-5/3*d)-55/324*I/b*ln(exp(1/3*b*x+1/3*d)+1/2+1/2*I
*3^(1/2))*exp(5/3*a-5/3*d)*3^(1/2)-55/324/b*ln(exp(1/3*b*x+1/3*d)-1/2-1/2*
I*3^(1/2))*exp(5/3*a-5/3*d)+55/324*I/b*ln(exp(1/3*b*x+1/3*d)-1/2-1/2*I*3^(
1/2))*exp(5/3*a-5/3*d)*3^(1/2)-55/324/b*ln(exp(1/3*b*x+1/3*d)-1/2+1/2*I*3^
(1/2))*exp(5/3*a-5/3*d)-55/324*I/b*ln(exp(1/3*b*x+1/3*d)-1/2+1/2*I*3^(1/2)
)*exp(5/3*a-5/3*d)*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13031 vs. $2(215) = 430$.

Time = 0.24 (sec) , antiderivative size = 13031, normalized size of antiderivative = 44.02

$$\int e^{\frac{5}{3}(a+bx)}\text{csch}^4(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*csch(b*x+d)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+bx)}\text{csch}^4(d+bx)\,dx = e^{\frac{5a}{3}}\int e^{\frac{5bx}{3}}\,\text{csch}^4\,(bx+d)\,dx$$

input
```
integrate(exp(5/3*b*x+5/3*a)*csch(b*x+d)**4,x)
```

output
```
exp(5*a/3)*Integral(exp(5*b*x/3)*csch(b*x + d)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.92

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{csch}^4(d+bx)\,dx$$
$$= \frac{55\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+1\right)\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}}{162b}$$
$$+\frac{55\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}-1\right)\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}}{162b}$$
$$+\frac{55e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{\left(-\frac{2}{3}bx-\frac{2}{3}d\right)}+1\right)}{324b}$$
$$+\frac{55e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+1\right)}{162b}-\frac{55e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}-1\right)}{162b}$$
$$-\frac{55e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(-e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{\left(-\frac{2}{3}bx-\frac{2}{3}d\right)}+1\right)}{324b}$$
$$+\frac{\left(55e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+28e^{\left(-\frac{7}{3}bx-\frac{7}{3}d\right)}-11e^{\left(-\frac{13}{3}bx-\frac{13}{3}d\right)}\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}}{27b(3e^{(-2bx-2d)}-3e^{(-4bx-4d)}+e^{(-6bx-6d)}-1)}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*csch(b*x+d)^4,x, algorithm="maxima")
```

output
```
55/162*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-1/3*b*x - 1/3*d) + 1))*e^(5/3*a -
 5/3*d)/b + 55/162*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-1/3*b*x - 1/3*d) - 1)
)*e^(5/3*a - 5/3*d)/b + 55/324*e^(5/3*a - 5/3*d)*log(e^(-1/3*b*x - 1/3*d)
+ e^(-2/3*b*x - 2/3*d) + 1)/b + 55/162*e^(5/3*a - 5/3*d)*log(e^(-1/3*b*x -
 1/3*d) + 1)/b - 55/162*e^(5/3*a - 5/3*d)*log(e^(-1/3*b*x - 1/3*d) - 1)/b
- 55/324*e^(5/3*a - 5/3*d)*log(-e^(-1/3*b*x - 1/3*d) + e^(-2/3*b*x - 2/3*d
) + 1)/b + 1/27*(55*e^(-1/3*b*x - 1/3*d) + 28*e^(-7/3*b*x - 7/3*d) - 11*e^
(-13/3*b*x - 13/3*d))*e^(5/3*a - 5/3*d)/(b*(3*e^(-2*b*x - 2*d) - 3*e^(-4*b
*x - 4*d) + e^(-6*b*x - 6*d) - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.74

$$\int e^{\frac{5}{3}(a+bx)}\text{csch}^4(d+bx)\,dx =$$

$$-\frac{\left(110\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(\frac{1}{3}bx\right)}+e^{\left(-\frac{1}{3}d\right)}\right)e^{\left(\frac{1}{3}d\right)}\right)e^{\left(-\frac{17}{3}d\right)}+110\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(\frac{1}{3}bx\right)}-e^{\left(-\frac{1}{3}d\right)}\right)e\right.}{}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*csch(b*x+d)^4,x, algorithm="giac")
```

output
```
-1/324*(110*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(1/3*b*x) + e^(-1/3*d))*e^(1/3
*d))*e^(-17/3*d) + 110*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(1/3*b*x) - e^(-1/3
*d))*e^(1/3*d))*e^(-17/3*d) - 55*e^(-17/3*d)*log(e^(1/3*b*x - 1/3*d) + e^(
2/3*b*x) + e^(-2/3*d)) + 55*e^(-17/3*d)*log(-e^(1/3*b*x - 1/3*d) + e^(2/3*
b*x) + e^(-2/3*d)) - 110*e^(-17/3*d)*log(e^(1/3*b*x) + e^(-1/3*d)) + 110*e
^(-17/3*d)*log(abs(e^(1/3*b*x) - e^(-1/3*d))) - 12*(11*e^(5/3*b*x) - 55*e^
(17/3*b*x + 4*d) - 28*e^(11/3*b*x + 2*d))*e^(-4*d)/(e^(2*b*x + 2*d) - 1)^3
)*e^(5/3*a + 4*d)/b
```

Mupad [B] (verification not implemented)

Time = 6.22 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.77

$$\int e^{\frac{5}{3}(a+bx)}\text{csch}^4(d+bx)\,dx = \text{Too large to display}$$

input
```
int(exp((5*a)/3 + (5*b*x)/3)/sinh(d + b*x)^4,x)
```

output
```
(55*exp(10*a - 10*d)^(1/6)*log(- (3025*exp((10*a)/3)*exp(-(10*d)/3))/6561
- (3025*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*(exp(10*a)*exp(-1
0*d))^(1/6))/6561))/(162*b) - (8*exp((5*a)/3 + 2*d + (11*b*x)/3))/(3*b*(3*
exp(2*d + 2*b*x) - 3*exp(4*d + 4*b*x) + exp(6*d + 6*b*x) - 1)) - (22*exp((
5*a)/3 + (5*b*x)/3))/(9*b*(exp(4*d + 4*b*x) - 2*exp(2*d + 2*b*x) + 1)) - (
55*exp((5*a)/3 + (5*b*x)/3))/(27*b*(exp(2*d + 2*b*x) - 1)) - (55*exp(10*a
- 10*d)^(1/6)*log((3025*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*(
exp(10*a)*exp(-10*d))^(1/6))/6561 - (3025*exp((10*a)/3)*exp(-(10*d)/3))/65
61))/(162*b) + (55*log(- (3025*exp((10*a)/3)*exp(-(10*d)/3))/6561 - (3025*
exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)/2 - 1/2)*(e
xp(10*a)*exp(-10*d))^(1/6))/6561)*exp(10*a - 10*d)^(1/6)*((3^(1/2)*1i)/2 -
 1/2))/(162*b) - (55*log((3025*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*
x)/3)*((3^(1/2)*1i)/2 - 1/2)*(exp(10*a)*exp(-10*d))^(1/6))/6561 - (3025*ex
p((10*a)/3)*exp(-(10*d)/3))/6561)*exp(10*a - 10*d)^(1/6)*((3^(1/2)*1i)/2 -
 1/2))/(162*b) + (55*log(- (3025*exp((10*a)/3)*exp(-(10*d)/3))/6561 - (302
5*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((b*x)/3)*((3^(1/2)*1i)/2 + 1/2)*
(exp(10*a)*exp(-10*d))^(1/6))/6561)*exp(10*a - 10*d)^(1/6)*((3^(1/2)*1i)/2
 + 1/2))/(162*b) - (55*log((3025*exp((5*a)/3)*exp(d/3)*exp(-(5*d)/3)*exp((
b*x)/3)*((3^(1/2)*1i)/2 + 1/2)*(exp(10*a)*exp(-10*d))^(1/6))/6561 - (3025*
exp((10*a)/3)*exp(-(10*d)/3))/6561)*exp(10*a - 10*d)^(1/6)*((3^(1/2)*1i...
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{csch}^4(d+bx)\,dx=\int e^{\frac{5bx}{3}+\frac{5a}{3}}\operatorname{csch}(bx+d)^4\,dx$$

input
```
int(exp(5/3*b*x+5/3*a)*csch(b*x+d)^4,x)
```

output
```
int(e**((5*a + 5*b*x)/3)*csch(b*x + d)**4,x)
```

3.112 $\int e^{\frac{5}{3}(a+bx)}\mathbf{csch}^5(d+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 293

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{csch}^5(d+bx)\,dx = -\frac{4e^{\frac{5(a-d)}{3}+\frac{14}{3}(d+bx)}}{b\left(1-e^{2(d+bx)}\right)^4}+\frac{28e^{\frac{5(a-d)}{3}+\frac{8}{3}(d+bx)}}{9b\left(1-e^{2(d+bx)}\right)^3}-\frac{56e^{\frac{5(a-d)}{3}+\frac{2}{3}(d+bx)}}{27b\left(1-e^{2(d+bx)}\right)^2}+\frac{56e^{\frac{5(a-d)}{3}+\frac{2}{3}(d+bx)}}{81b\left(1-e^{2(d+bx)}\right)}+\frac{112e^{\frac{5(a-d)}{3}}\arctan\left(\frac{1+2e^{\frac{2}{3}(d+bx)}}{\sqrt{3}}\right)}{81\sqrt{3}b}-\frac{112e^{\frac{5(a-d)}{3}}\log\left(1-e^{\frac{2}{3}(d+bx)}\right)}{243b}+\frac{56e^{\frac{5(a-d)}{3}}\log\left(1+e^{\frac{2}{3}(d+bx)}+e^{\frac{4}{3}(d+bx)}\right)}{243b}$$

output
```
-4*exp(5/3*a+3*d+14/3*b*x)/b/(1-exp(2*b*x+2*d))^4+28/9*exp(5/3*a+d+8/3*b*x
)/b/(1-exp(2*b*x+2*d))^3-56/27*exp(5/3*a-d+2/3*b*x)/b/(1-exp(2*b*x+2*d))^2
+56/81*exp(5/3*a-d+2/3*b*x)/b/(1-exp(2*b*x+2*d))+112/243*3^(1/2)*exp(5/3*a
-5/3*d)*arctan(1/3*(1+2*exp(2/3*b*x+2/3*d))*3^(1/2))/b-112/243*exp(5/3*a-5
/3*d)*ln(1-exp(2/3*b*x+2/3*d))/b+56/243*exp(5/3*a-5/3*d)*ln(1+exp(2/3*b*x+
2/3*d)+exp(4/3*b*x+4/3*d))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.51 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.10

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{csch}^5(d+bx)\,dx$$
$$= \frac{4e^{5a/3}(\cosh(d)-\sinh(d))^2\left(-28\text{RootSum}\left[\cosh\left(\frac{d}{2}\right)-\sinh\left(\frac{d}{2}\right)+\cosh\left(\frac{d}{2}\right)\#1^3+\sinh\left(\frac{d}{2}\right)\#1^3\&,\frac{bx-3\log\right.\right.\right.}{}$$

input
```
Integrate[E^((5*(a + b*x))/3)*Csch[d + b*x]^5,x]
```

output
```
(4*E^((5*a)/3)*(Cosh[d] - Sinh[d])^2*(-28*RootSum[Cosh[d/2] - Sinh[d/2] +
Cosh[d/2]*#1^3 + Sinh[d/2]*#1^3 & , (b*x - 3*Log[E^((b*x)/3) - #1])/#1 & ]
 + 28*RootSum[-Cosh[d/2] + Sinh[d/2] + Cosh[d/2]*#1^3 + Sinh[d/2]*#1^3 & ,
 (b*x - 3*Log[E^((b*x)/3) - #1])/#1 & ] - (729*E^((2*b*x)/3)*(Cosh[d] - Si
nh[d])^3)/((-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d])^4 - (2025*E
^((2*b*x)/3)*(Cosh[d] - Sinh[d])^2)/((-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2*
b*x))*Sinh[d])^3 - (1674*E^((2*b*x)/3)*(Cosh[d] - Sinh[d]))/((-1 + E^(2*b*
x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d])^2 - (126*E^((2*b*x)/3))/((-1 + E^(2
*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d])))/(729*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.65, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {2720, 27, 807, 817, 817, 817, 749, 750, 16, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{csch}^5(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{3\int -\frac{32e^{\frac{5a}{3}+\frac{19bx}{3}}}{\left(1-e^{2bx}\right)^5}de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{96e^{5a/3}\int \frac{e^{\frac{19bx}{3}}}{\left(1-e^{2bx}\right)^5}de^{\frac{bx}{3}}}{b}$$

$$\Big\downarrow \; 807$$

$$-\frac{48e^{5a/3}\int \frac{e^{3bx}}{\left(1-e^{bx}\right)^5}de^{\frac{2bx}{3}}}{b}$$

$$\Big\downarrow \; 817$$

$$-\frac{48e^{5a/3}\left(\frac{e^{\frac{7bx}{3}}}{12\left(1-e^{bx}\right)^4}-\frac{7}{12}\int \frac{e^{2bx}}{\left(1-e^{bx}\right)^4}de^{\frac{2bx}{3}}\right)}{b}$$

$$\Big\downarrow \; 817$$

$$-\frac{48e^{5a/3}\left(\frac{e^{\frac{7bx}{3}}}{12\left(1-e^{bx}\right)^4}-\frac{7}{12}\left(\frac{e^{\frac{4bx}{3}}}{9\left(1-e^{bx}\right)^3}-\frac{4}{9}\int \frac{e^{bx}}{\left(1-e^{bx}\right)^3}de^{\frac{2bx}{3}}\right)\right)}{b}$$

$$\Big\downarrow \; 817$$

$$-\frac{48e^{5a/3}\left(\frac{e^{\frac{7bx}{3}}}{12\left(1-e^{bx}\right)^4}-\frac{7}{12}\left(\frac{e^{\frac{4bx}{3}}}{9\left(1-e^{bx}\right)^3}-\frac{4}{9}\left(\frac{e^{\frac{2bx}{3}}}{6\left(1-e^{bx}\right)^2}-\frac{1}{6}\int \frac{1}{\left(1-e^{bx}\right)^2}de^{\frac{2bx}{3}}\right)\right)\right)}{b}$$

$$\Big\downarrow \; 749$$

$$-\frac{48e^{5a/3}\left(\frac{e^{\frac{7bx}{3}}}{12\left(1-e^{bx}\right)^4}-\frac{7}{12}\left(\frac{e^{\frac{4bx}{3}}}{9\left(1-e^{bx}\right)^3}-\frac{4}{9}\left(\frac{1}{6}\left(-\frac{2}{3}\int \frac{1}{1-e^{bx}}de^{\frac{2bx}{3}}-\frac{e^{\frac{2bx}{3}}}{3\left(1-e^{bx}\right)}\right)+\frac{e^{\frac{2bx}{3}}}{6\left(1-e^{bx}\right)^2}\right)\right)\right)}{b}$$

$$\Big\downarrow \; 750$$

$$-\frac{48e^{5a/3}\left(\frac{e^{\frac{7bx}{3}}}{12\left(1-e^{bx}\right)^4}-\frac{7}{12}\left(\frac{e^{\frac{4bx}{3}}}{9\left(1-e^{bx}\right)^3}-\frac{4}{9}\left(\frac{1}{6}\left(-\frac{2}{3}\left(\frac{1}{3}\int \frac{1}{1-e^{\frac{2bx}{3}}}de^{\frac{2bx}{3}}+\frac{1}{3}\int \frac{2+e^{\frac{2bx}{3}}}{1+2e^{\frac{2bx}{3}}}de^{\frac{2bx}{3}}\right)-\frac{e^{\frac{2bx}{3}}}{3\left(1-e^{bx}\right)}\right)+\frac{e^{\frac{2bx}{3}}}{6\left(1-e^{bx}\right)^2}\right.\right.\right.}{b}$$

$$\Big\downarrow \; 16$$

$$-\frac{48e^{5a/3}\left(\frac{e^{\frac{7bx}{3}}}{12\left(1-e^{bx}\right)^4}-\frac{7}{12}\left(\frac{e^{\frac{4bx}{3}}}{9\left(1-e^{bx}\right)^3}-\frac{4}{9}\left(\frac{1}{6}\left(-\frac{2}{3}\left(\frac{1}{3}\int \frac{2+e^{\frac{2bx}{3}}}{1+2e^{\frac{2bx}{3}}}de^{\frac{2bx}{3}}-\frac{1}{3}\log\left(1-e^{\frac{2bx}{3}}\right)\right)-\frac{e^{\frac{2bx}{3}}}{3\left(1-e^{bx}\right)}\right)+\frac{e^{\frac{2bx}{3}}}{6\left(1-e^{bx}\right)^2}\right.\right.\right.}{b}$$

$$\Big\downarrow \; 1142$$

$$-\frac{48e^{5a/3}\left(\frac{e^{\frac{7bx}{3}}}{12(1-e^{bx})^4}-\frac{7}{12}\left(\frac{e^{\frac{4bx}{3}}}{9(1-e^{bx})^3}-\frac{4}{9}\left(\frac{1}{6}\left(-\frac{2}{3}\left(\frac{1}{3}\left(\frac{1}{2}\int 1de^{\frac{2bx}{3}}+\frac{3}{2}\int \frac{1}{1+2e^{\frac{2bx}{3}}}de^{\frac{2bx}{3}}\right)-\frac{1}{3}\log\left(1-e^{\frac{2bx}{3}}\right)\right)-\frac{e}{3(1}\right.\right.\right.\right.}{b}$$

$$\Big\downarrow \; 1083$$

$$-\frac{48e^{5a/3}\left(\frac{e^{\frac{7bx}{3}}}{12(1-e^{bx})^4}-\frac{7}{12}\left(\frac{e^{\frac{4bx}{3}}}{9(1-e^{bx})^3}-\frac{4}{9}\left(\frac{1}{6}\left(-\frac{2}{3}\left(\frac{1}{3}\left(\frac{1}{2}\int 1de^{\frac{2bx}{3}}-3\int \frac{1}{-4-2e^{\frac{2bx}{3}}}d\left(1+2e^{\frac{2bx}{3}}\right)\right)-\frac{1}{3}\log\left(1-e^{\frac{2bx}{3}}\right.\right.\right.\right.\right.}{b}$$

$$\Big\downarrow \; 217$$

$$-\frac{48e^{5a/3}\left(\frac{e^{\frac{7bx}{3}}}{12(1-e^{bx})^4}-\frac{7}{12}\left(\frac{e^{\frac{4bx}{3}}}{9(1-e^{bx})^3}-\frac{4}{9}\left(\frac{1}{6}\left(-\frac{2}{3}\left(\frac{1}{3}\left(\frac{1}{2}\int 1de^{\frac{2bx}{3}}+\sqrt{3}\arctan\left(\frac{2e^{\frac{2bx}{3}}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-e^{\frac{2bx}{3}}\right)\right)\right.\right.\right.\right.}{b}$$

$$\Big\downarrow \; 1103$$

$$-\frac{48e^{5a/3}\left(\frac{e^{\frac{7bx}{3}}}{12(1-e^{bx})^4}-\frac{7}{12}\left(\frac{e^{\frac{4bx}{3}}}{9(1-e^{bx})^3}-\frac{4}{9}\left(\frac{1}{6}\left(-\frac{2}{3}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2e^{\frac{2bx}{3}}+1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(2e^{\frac{2bx}{3}}+1\right)\right)-\frac{1}{3}\log\left(1-e\right.\right.\right.\right.\right.}{b}$$

input

```
Int[E^((5*(a + b*x))/3)*Csch[d + b*x]^5,x]
```

output

```
(-48*E^((5*a)/3)*(E^((7*b*x)/3)/(12*(1 - E^(b*x))^4) - (7*(E^((4*b*x)/3)/(
9*(1 - E^(b*x))^3) - (4*(E^((2*b*x)/3)/(6*(1 - E^(b*x))^2) + (-1/3*E^((2*b
*x)/3)/(1 - E^(b*x)) - (2*(-1/3*Log[1 - E^((2*b*x)/3)] + (Sqrt[3]*ArcTan[(
1 + 2*E^((2*b*x)/3))/Sqrt[3]] + Log[1 + 2*E^((2*b*x)/3)]/2)/3))/3)/6))/9))
/12))/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.96 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{4\left(14\,\mathrm{e}^{6bx+6d}+144\,\mathrm{e}^{4bx+4d}-105\,\mathrm{e}^{2bx+2d}+28\right)\mathrm{e}^{\frac{5a}{3}-d+\frac{2bx}{3}}}{81\left(\mathrm{e}^{2bx+2d}-1\right)^{4}b}+\frac{56\ln\left(\mathrm{e}^{\frac{2bx}{3}+\frac{2d}{3}}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\mathrm{e}^{\frac{5a}{3}-\frac{5d}{3}}}{243b}+\frac{56i\ln\left(\mathrm{e}^{\frac{2bx}{3}+\frac{2d}{3}}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right.}{243b}$

input
```
int(exp(5/3*b*x+5/3*a)*csch(b*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
-4/81/(exp(2*b*x+2*d)-1)^4/b*(14*exp(6*b*x+6*d)+144*exp(4*b*x+4*d)-105*exp
(2*b*x+2*d)+28)*exp(5/3*a-d+2/3*b*x)+56/243*ln(exp(2/3*b*x+2/3*d)+1/2+1/2*
I*3^(1/2))/b*exp(5/3*a-5/3*d)+56/243*I*ln(exp(2/3*b*x+2/3*d)+1/2+1/2*I*3^(
1/2))/b*exp(5/3*a-5/3*d)*3^(1/2)+56/243*ln(exp(2/3*b*x+2/3*d)+1/2-1/2*I*3^
(1/2))/b*exp(5/3*a-5/3*d)-56/243*I*ln(exp(2/3*b*x+2/3*d)+1/2-1/2*I*3^(1/2)
)/b*exp(5/3*a-5/3*d)*3^(1/2)-112/243*ln(exp(2/3*b*x+2/3*d)-1)/b*exp(5/3*a-
5/3*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11702 vs. $2(215) = 430$.

Time = 0.28 (sec) , antiderivative size = 11702, normalized size of antiderivative = 39.94

$$\int e^{\frac{5}{3}(a+bx)}\text{csch}^5(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*csch(b*x+d)^5,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int e^{\frac{5}{3}(a+bx)}\text{csch}^5(d+bx)\,dx = e^{\frac{5a}{3}}\int e^{\frac{5bx}{3}}\,\text{csch}^5\,(bx+d)\,dx$$

input
```
integrate(exp(5/3*b*x+5/3*a)*csch(b*x+d)**5,x)
```

output
```
exp(5*a/3)*Integral(exp(5*b*x/3)*csch(b*x + d)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.01

$$\int e^{\frac{5}{3}(a+bx)}\text{csch}^5(d+bx)\,dx = \frac{112\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+1\right)\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}}{243b} - \frac{112\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}-1\right)\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}}{243b} + \frac{56e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{\left(-\frac{2}{3}bx-\frac{2}{3}d\right)}+1\right)}{243b} - \frac{112e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+1\right)}{243b} - \frac{112e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}-1\right)}{243b} + \frac{56e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}\log\left(-e^{\left(-\frac{1}{3}bx-\frac{1}{3}d\right)}+e^{\left(-\frac{2}{3}bx-\frac{2}{3}d\right)}+1\right)}{243b} + \frac{4\left(14e^{\left(-\frac{4}{3}bx-\frac{4}{3}d\right)}+144e^{\left(-\frac{10}{3}bx-\frac{10}{3}d\right)}-105e^{\left(-\frac{16}{3}bx-\frac{16}{3}d\right)}+28e^{\left(-\frac{22}{3}bx-\frac{22}{3}d\right)}\right)e^{\left(\frac{5}{3}a-\frac{5}{3}d\right)}}{81b\left(4e^{(-2bx-2d)}-6e^{(-4bx-4d)}+4e^{(-6bx-6d)}-e^{(-8bx-8d)}-1\right)}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*csch(b*x+d)^5,x, algorithm="maxima")
```

output
```
112/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-1/3*b*x - 1/3*d) + 1))*e^(5/3*a
- 5/3*d)/b - 112/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(-1/3*b*x - 1/3*d) -
1))*e^(5/3*a - 5/3*d)/b + 56/243*e^(5/3*a - 5/3*d)*log(e^(-1/3*b*x - 1/3*d
) + e^(-2/3*b*x - 2/3*d) + 1)/b - 112/243*e^(5/3*a - 5/3*d)*log(e^(-1/3*b*
x - 1/3*d) + 1)/b - 112/243*e^(5/3*a - 5/3*d)*log(e^(-1/3*b*x - 1/3*d) - 1
)/b + 56/243*e^(5/3*a - 5/3*d)*log(-e^(-1/3*b*x - 1/3*d) + e^(-2/3*b*x - 2
/3*d) + 1)/b + 4/81*(14*e^(-4/3*b*x - 4/3*d) + 144*e^(-10/3*b*x - 10/3*d)
- 105*e^(-16/3*b*x - 16/3*d) + 28*e^(-22/3*b*x - 22/3*d))*e^(5/3*a - 5/3*d
)/(b*(4*e^(-2*b*x - 2*d) - 6*e^(-4*b*x - 4*d) + 4*e^(-6*b*x - 6*d) - e^(-8
*b*x - 8*d) - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.52

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{csch}^5(d+bx)\,dx = \frac{4\left(28\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{\left(\frac{2}{3}bx\right)}+e^{\left(-\frac{2}{3}d\right)}\right)e^{\left(\frac{2}{3}d\right)}\right)e^{\left(-\frac{20}{3}d\right)}+14e^{\left(-\frac{20}{3}d\right)}\log\left(e^{\left(\frac{4}{3}bx\right)}+e^{\left(\frac{2}{3}bx-\frac{2}{3}d\right)}+e^{\left(-\frac{4}{3}d\right)}\right.}{2}$$

input
```
integrate(exp(5/3*b*x+5/3*a)*csch(b*x+d)^5,x, algorithm="giac")
```

output
```
4/243*(28*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2/3*b*x) + e^(-2/3*d))*e^(2/3*d
))*e^(-20/3*d) + 14*e^(-20/3*d)*log(e^(4/3*b*x) + e^(2/3*b*x - 2/3*d) + e^
(-4/3*d)) - 28*e^(-20/3*d)*log(abs(e^(2/3*b*x) - e^(-2/3*d))) - 3*(14*e^(2
0/3*b*x + 6*d) + 144*e^(14/3*b*x + 4*d) - 105*e^(8/3*b*x + 2*d) + 28*e^(2/
3*b*x))*e^(-6*d)/(e^(2*b*x + 2*d) - 1)^4)*e^(5/3*a + 5*d)/b
```

Mupad [B] (verification not implemented)

Time = 6.04 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.27

$$\int e^{\frac{5}{3}(a+bx)}\operatorname{csch}^5(d+bx)\,dx = \frac{112\left(-e^{5a-5d}\right)^{1/3}\ln\left(\frac{224\left(-e^{5a}e^{-5d}\right)^{1/3}}{243}+\frac{224e^{\frac{5a}{3}}e^{\frac{2d}{3}}e^{-\frac{5d}{3}}e^{\frac{2bx}{3}}}{243}\right)}{243b} - \frac{56e^{\frac{5a}{3}-d+\frac{2bx}{3}}}{81b\left(e^{2d+2bx}-1\right)} - \frac{56e^{\frac{5a}{3}-d+\frac{2bx}{3}}}{27b\left(e^{4d+4bx}-2e^{2d+2bx}+1\right)} - \frac{4e^{\frac{5a}{3}+3d+\frac{14bx}{3}}}{b\left(6e^{4d+4bx}-4e^{2d+2bx}-4e^{6d+6bx}+e^{8d+8bx}+1\right)} - \frac{28e^{\frac{5a}{3}+d+\frac{8bx}{3}}}{9b\left(3e^{2d+2bx}-3e^{4d+4bx}+e^{6d+6bx}-1\right)} + \frac{112\left(-e^{5a-5d}\right)^{1/3}\ln\left(\frac{224\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-e^{5a}e^{-5d}\right)^{1/3}}{243}+\frac{224e^{\frac{5a}{3}}e^{\frac{2d}{3}}e^{-\frac{5d}{3}}e^{\frac{2bx}{3}}}{243}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{243b} - \frac{112\left(-e^{5a-5d}\right)^{1/3}\ln\left(\frac{224e^{\frac{5a}{3}}e^{\frac{2d}{3}}e^{-\frac{5d}{3}}e^{\frac{2bx}{3}}}{243}-\frac{224\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-e^{5a}e^{-5d}\right)^{1/3}}{243}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{243b}$$

input
```
int(exp((5*a)/3 + (5*b*x)/3)/sinh(d + b*x)^5,x)
```

output
```
(112*(-exp(5*a - 5*d))^(1/3)*log((224*(-exp(5*a)*exp(-5*d))^(1/3))/243 + (
224*exp((5*a)/3)*exp((2*d)/3)*exp(-(5*d)/3)*exp((2*b*x)/3))/243))/(243*b)
- (56*exp((5*a)/3 - d + (2*b*x)/3))/(81*b*(exp(2*d + 2*b*x) - 1)) - (56*ex
p((5*a)/3 - d + (2*b*x)/3))/(27*b*(exp(4*d + 4*b*x) - 2*exp(2*d + 2*b*x) +
 1)) - (4*exp((5*a)/3 + 3*d + (14*b*x)/3))/(b*(6*exp(4*d + 4*b*x) - 4*exp(
2*d + 2*b*x) - 4*exp(6*d + 6*b*x) + exp(8*d + 8*b*x) + 1)) - (28*exp((5*a)
/3 + d + (8*b*x)/3))/(9*b*(3*exp(2*d + 2*b*x) - 3*exp(4*d + 4*b*x) + exp(6
*d + 6*b*x) - 1)) + (112*(-exp(5*a - 5*d))^(1/3)*log((224*((3^(1/2)*1i)/2
- 1/2)*(-exp(5*a)*exp(-5*d))^(1/3))/243 + (224*exp((5*a)/3)*exp((2*d)/3)*e
xp(-(5*d)/3)*exp((2*b*x)/3))/243)*((3^(1/2)*1i)/2 - 1/2))/(243*b) - (112*(
-exp(5*a - 5*d))^(1/3)*log((224*exp((5*a)/3)*exp((2*d)/3)*exp(-(5*d)/3)*ex
p((2*b*x)/3))/243 - (224*((3^(1/2)*1i)/2 + 1/2)*(-exp(5*a)*exp(-5*d))^(1/3
))/243)*((3^(1/2)*1i)/2 + 1/2))/(243*b)
```

Reduce [F]

$$\int e^{\frac{5}{3}(a+bx)}\text{csch}^5(d+bx)\,dx = \int e^{\frac{5bx}{3}+\frac{5a}{3}}\text{csch}(bx+d)^5\,dx$$

input
```
int(exp(5/3*b*x+5/3*a)*csch(b*x+d)^5,x)
```

output
```
int(e**((5*a + 5*b*x)/3)*csch(b*x + d)**5,x)
```

3.113 $\int F^{c(a+bx)}\operatorname{csch}(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 67

$$\int F^{c(a+bx)}\operatorname{csch}(d+ex)\,dx = -\frac{2e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),e^{2d+2ex}\right)}{e+bc\log(F)}$$

output
```
-2*exp(e*x+d)*F^(c*(b*x+a))*hypergeom([1, 1/2*(e+b*c*ln(F))/e],[3/2+1/2*b*
c*ln(F)/e],exp(2*e*x+2*d))/(e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.18

$$\int F^{c(a+bx)}\operatorname{csch}(d+ex)\,dx = \frac{F^{c(a+bx)}\left(\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{e},1+\frac{bc\log(F)}{e},-e^{d+ex}\right)-\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{e},1+\frac{bc}{\ldots}\right.\right.}{bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Csch[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*(Hypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e,
 -E^(d + e*x)] - Hypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e,
E^(d + e*x)]))/(b*c*Log[F])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {6016}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{ 6016}$$

$$-\frac{2e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),e^{2(d+ex)}\right)}{bc\log(F)+e}$$

input
```
Int[F^(c*(a + b*x))*Csch[d + e*x],x]
```

output
```
(-2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e
), (3 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))])/(e + b*c*Log[F])
```

Defintions of rubi rules used

rule 6016
```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(-2)^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)),
E^(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int \operatorname{csch}(ex+d)\,F^{c(bx+a)}dx$$

input
```
int(csch(e*x+d)*F^(c*(b*x+a)),x)
```

output
```
int(csch(e*x+d)*F^(c*(b*x+a)),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\operatorname{csch}(d+ex)\,dx = \int F^{(bx+a)c}\operatorname{csch}(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csch(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\operatorname{csch}(d+ex)\,dx = \int F^{c(a+bx)}\operatorname{csch}(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*csch(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*csch(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\text{csch}(d+ex)\,dx=\int F^{(bx+a)c}\,\text{csch}\,(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d),x, algorithm="maxima")
```

output
```
4*F^(a*c)*e*integrate(e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + (b*c*e^(4*d
)*log(F) - e*e^(4*d))*e^(4*e*x) - 2*(b*c*e^(2*d)*log(F) - e*e^(2*d))*e^(2*
e*x) - e), x) - 2*F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) - (b*c*e^
(2*d)*log(F) - e*e^(2*d))*e^(2*e*x) - e)
```

Giac [F]

$$\int F^{c(a+bx)}\text{csch}(d+ex)\,dx=\int F^{(bx+a)c}\,\text{csch}\,(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csch(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{csch}(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\sinh\,(d+e\,x)}\,dx$$

input
```
int(F^(c*(a + b*x))/sinh(d + e*x),x)
```

output
```
int(F^(c*(a + b*x))/sinh(d + e*x), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\text{csch}(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\text{csch}(ex+d)\,dx\right)$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*csch(d + e*x),x)
```

3.114 $\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 72

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\,dx = \frac{4e^{2d+2ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(2+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(4+\frac{bc\log(F)}{e}\right),e^{2d+2ex}\right)}{2e+bc\log(F)}$$

output
```
4*exp(2*e*x+2*d)*F^(c*(b*x+a))*hypergeom([2, 1+1/2*b*c*ln(F)/e],[2+1/2*b*c
*ln(F)/e],exp(2*e*x+2*d))/(2*e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\,dx = -\frac{2F^{c(a+bx)}\left(\left(-1+e^{2d}\right)\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},e^{2(d+ex)}\right)+\operatorname{csch}(d+ex)\sinh(d)(c\right.}{e\left(-1+e^{2d}\right)}$$

input
```
Integrate[F^(c*(a + b*x))*Csch[d + e*x]^2,x]
```

output
```
(-2*F^(c*(a + b*x))*((-1 + E^(2*d))*Hypergeometric2F1[1, (b*c*Log[F])/(2*e
), 1 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))] + Csch[d + e*x]*Sinh[d]*(Cosh[
e*x] - Sinh[e*x])))/(e*(-1 + E^(2*d)))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {6016}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6016}$$

$$\frac{4e^{2(d+ex)}F^{c(a+bx)}\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{2e}+1,\frac{bc\log(F)}{2e}+2,e^{2(d+ex)}\right)}{bc\log(F)+2e}$$

input
```
Int[F^(c*(a + b*x))*Csch[d + e*x]^2,x]
```

output
```
(4*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(
2*e), 2 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(2*e + b*c*Log[F])
```

Defintions of rubi rules used

rule 6016
```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(-2)^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)),
E^(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \operatorname{csch}(ex+d)^2 \, dx$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*csch(e*x+d)^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \operatorname{csch}^2(d+ex) \, dx = \int F^{(bx+a)c} \operatorname{csch}(ex+d)^2 \, dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csch(e*x + d)^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \operatorname{csch}^2(d+ex) \, dx = \int F^{c(a+bx)} \operatorname{csch}^2(d+ex) \, dx$$

input
```
integrate(F**(c*(b*x+a))*csch(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*csch(d + e*x)**2, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{csch}(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^2,x, algorithm="maxima")
```

output
```
16*F^(a*c)*b*c*e*integrate(-F^(b*c*x)/(b^2*c^2*log(F)^2 - 6*b*c*e*log(F) +
 8*e^2 - (b^2*c^2*e^(6*d)*log(F)^2 - 6*b*c*e*e^(6*d)*log(F) + 8*e^2*e^(6*d
))*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*log(F) + 8*e^
2*e^(4*d))*e^(4*e*x) - 3*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^(2*d)*log(F
) + 8*e^2*e^(2*d))*e^(2*e*x)), x)*log(F) + 4*(4*F^(a*c)*e + (F^(a*c)*b*c*e
^(2*d)*log(F) - 4*F^(a*c)*e*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^
2 - 6*b*c*e*log(F) + 8*e^2 + (b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*l
og(F) + 8*e^2*e^(4*d))*e^(4*e*x) - 2*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e
^(2*d)*log(F) + 8*e^2*e^(2*d))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{csch}(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csch(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\mathrm{csch}^2(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\sinh{(d+e\,x)}^2}\,dx$$

input
```
int(F^(c*(a + b*x))/sinh(d + e*x)^2,x)
```

output
```
int(F^(c*(a + b*x))/sinh(d + e*x)^2, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\mathrm{csch}^2(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\mathrm{csch}(ex+d)^2\,dx\right)$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*csch(d + e*x)**2,x)
```

3.115 $\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 72

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\,dx = -\frac{8e^{3d+3ex}F^{c(a+bx)}\operatorname{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(5+\frac{bc\log(F)}{e}\right),e^{2d+2ex}\right)}{3e+bc\log(F)}$$

output
```
-8*exp(3*e*x+3*d)*F^(c*(b*x+a))*hypergeom([3, 3/2+1/2*b*c*ln(F)/e],[5/2+1/
2*b*c*ln(F)/e],exp(2*e*x+2*d))/(b*c*ln(F)+3*e)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 281 vs. 2(72) = 144.

Time = 3.08 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.90

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\,dx = \frac{F^{c(a+bx)}\left(-e\operatorname{csch}^2\left(\frac{1}{2}(d+ex)\right)-4bc\operatorname{csch}(d)\log(F)+\operatorname{csch}(d)\left(-\frac{4e^2}{bc\log(F)}+4bc\log(F)\right)+\frac{4\left(1-(1+e^d)\operatorname{Hyper}\right.}{}\right.}{}$$

input
```
Integrate[F^(c*(a + b*x))*Csch[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(-(e*Csch[(d + e*x)/2]^2) - 4*b*c*Csch[d]*Log[F] + Csch[d
]*((-4*e^2)/(b*c*Log[F]) + 4*b*c*Log[F]) + (4*(1 - (1 + E^d)*Hypergeometri
c2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e, -E^(d + e*x)])*(e^2 - b^2*c^2*
Log[F]^2))/(b*c*(1 + E^d)*Log[F]) + (4*(1 + (-1 + E^d)*Hypergeometric2F1[1
, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e, E^(d + e*x)])*(e^2 - b^2*c^2*Log[F]^
2))/(b*c*(-1 + E^d)*Log[F]) - e*Sech[(d + e*x)/2]^2 + 2*b*c*Csch[d/2]*Csch
[(d + e*x)/2]*Log[F]*Sinh[(e*x)/2] + 2*b*c*Log[F]*Sech[d/2]*Sech[(d + e*x)
/2]*Sinh[(e*x)/2]))/(8*e^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.90, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6014, 6016}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^3(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{ 6014}$$

$$-\frac{1}{2}\left(1-\frac{b^2c^2\log^2(F)}{e^2}\right)\int F^{c(a+bx)}\text{csch}(d+ex)dx-\frac{bc\log(F)\text{csch}(d+ex)F^{c(a+bx)}}{2e^2}-\frac{\coth(d+ex)\text{csch}(d+ex)F^{c(a+bx)}}{2e}$$

$$\Big\downarrow \text{ 6016}$$

$$\frac{e^{d+ex}F^{c(a+bx)}\left(1-\frac{b^2c^2\log^2(F)}{e^2}\right)\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),e^{2(d+ex)}\right)}{bc\log(F)+e}-\frac{bc\log(F)\text{csch}(d+ex)F^{c(a+bx)}}{2e^2}-\frac{\coth(d+ex)\text{csch}(d+ex)F^{c(a+bx)}}{2e}$$

input
```
Int[F^(c*(a + b*x))*Csch[d + e*x]^3,x]
```

output
```
-1/2*(F^(c*(a + b*x))*Coth[d + e*x]*Csch[d + e*x])/e - (b*c*F^(c*(a + b*x)
)*Csch[d + e*x]*Log[F])/(2*e^2) + (E^(d + e*x)*F^(c*(a + b*x))*Hypergeomet
ric2F1[1, (e + b*c*Log[F])/(2*e), (3 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))]
*(1 - (b^2*c^2*Log[F]^2)/e^2))/(e + b*c*Log[F])
```

Defintions of rubi rules used

rule 6014
```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csch[d + e*x]^(n - 2)/(e^2*(n -
1)*(n - 2))), x] + (-Simp[F^(c*(a + b*x))*Csch[d + e*x]^(n - 1)*(Cosh[d + e
*x]/(e*(n - 1))), x] - Simp[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)
*(n - 2))   Int[F^(c*(a + b*x))*Csch[d + e*x]^(n - 2), x], x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1
] && NeQ[n, 2]
```

rule 6016
```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(-2)^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)),
E^(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \operatorname{csch}(ex+d)^3\, dx$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)^3,x)
```

output
```
int(F^(c*(b*x+a))*csch(e*x+d)^3,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\text{csch}^3(d+ex)\,dx=\int F^{(bx+a)c}\,\text{csch}\,(ex+d)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csch(e*x + d)^3, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\text{csch}^3(d+ex)\,dx=\int F^{c(a+bx)}\,\text{csch}^3\,(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*csch(e*x+d)**3,x)
```

output
```
Integral(F**(c*(a + b*x))*csch(d + e*x)**3, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\text{csch}^3(d+ex)\,dx=\int F^{(bx+a)c}\,\text{csch}\,(ex+d)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^3,x, algorithm="maxima")
```

output
```
48*(F^(a*c)*b*c*e*e^d*log(F) + F^(a*c)*e^2*e^d)*integrate(e^(b*c*x*log(F)
+ e*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*log(F) + 15*e^2 + (b^2*c^2*e^(8*d)*log(
F)^2 - 8*b*c*e*e^(8*d)*log(F) + 15*e^2*e^(8*d))*e^(8*e*x) - 4*(b^2*c^2*e^(
6*d)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e^2*e^(6*d))*e^(6*e*x) + 6*(b^
2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(F) + 15*e^2*e^(4*d))*e^(4*e*x
) - 4*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(2*d)*log(F) + 15*e^2*e^(2*d))
*e^(2*e*x)), x) - 8*(6*F^(a*c)*e*e^(e*x + d) + (F^(a*c)*b*c*e^(3*d)*log(F)
 - 5*F^(a*c)*e*e^(3*d))*e^(3*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*l
og(F) + 15*e^2 - (b^2*c^2*e^(6*d)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e
^2*e^(6*d))*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(
F) + 15*e^2*e^(4*d))*e^(4*e*x) - 3*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(
2*d)*log(F) + 15*e^2*e^(2*d))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\text{csch}^3(d+ex)\,dx=\int F^{(bx+a)c}\,\text{csch}\left(ex+d\right)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csch(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{csch}^3(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\sinh\left(d+e\,x\right)^3}\,dx$$

input
```
int(F^(c*(a + b*x))/sinh(d + e*x)^3,x)
```

output
```
int(F^(c*(a + b*x))/sinh(d + e*x)^3, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\text{csch}^3(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\text{csch}(ex+d)^3\,dx\right)$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*csch(d + e*x)**3,x)
```

3.116 $\int e^{a+bx}\text{csch}^n(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 66

$$\int e^{a+bx}\text{csch}^n(a+bx)\,dx = \frac{e^{a+bx}\left(1-e^{2a+2bx}\right)^n\text{csch}^n(a+bx)\,\text{Hypergeometric2F1}\left(n,\frac{1+n}{2},\frac{3+n}{2},e^{2a+2bx}\right)}{b(1+n)}$$

output
```
exp(b*x+a)*(1-exp(2*b*x+2*a))^n*csch(b*x+a)^n*hypergeom([n, 1/2+1/2*n],[3/
2+1/2*n],exp(2*b*x+2*a))/b/(1+n)
```

Mathematica [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\text{csch}^n(a+bx)\,dx = -\frac{e^{-a-bx}\left(-1+e^{2(a+bx)}\right)\text{csch}^n(a+bx)\,\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{1+n}{2},e^{-2(a+bx)}\right)}{b(-1+n)}$$

input
```
Integrate[E^(a + b*x)*Csch[a + b*x]^n,x]
```

output
```
-((E^(-a - b*x)*(-1 + E^(2*(a + b*x)))*Csch[a + b*x]^n*Hypergeometric2F1[1
, (1 - n)/2, (1 + n)/2, E^(-2*(a + b*x))])/(b*(-1 + n)))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 27, 7270, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{csch}^n(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int 2^n\left(-\frac{e^{a+bx}}{1-e^{2a+2bx}}\right)^n de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2^n\int \left(-\frac{e^{a+bx}}{1-e^{2a+2bx}}\right)^n de^{a+bx}}{b}$$

$$\Big\downarrow \text{7270}$$

$$\frac{2^n\left(e^{a+bx}\right)^{-n}\left(-\frac{e^{a+bx}}{1-e^{2a+2bx}}\right)^n\left(1-e^{2a+2bx}\right)^n\int \left(e^{a+bx}\right)^n\left(1-e^{2a+2bx}\right)^{-n}de^{a+bx}}{b}$$

$$\Big\downarrow \text{278}$$

$$\frac{2^n e^{a+bx}\left(-\frac{e^{a+bx}}{1-e^{2a+2bx}}\right)^n\left(1-e^{2a+2bx}\right)^n\text{Hypergeometric2F1}\left(n,\frac{n+1}{2},\frac{n+3}{2},e^{2a+2bx}\right)}{b(n+1)}$$

input
```
Int[E^(a + b*x)*Csch[a + b*x]^n,x]
```

output
```
(2^n*E^(a + b*x)*(-(E^(a + b*x)/(1 - E^(2*a + 2*b*x))))^n*(1 - E^(2*a + 2*
b*x))^n*Hypergeometric2F1[n, (1 + n)/2, (3 + n)/2, E^(2*a + 2*b*x)])/(b*(1
 + n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

Maple [F]

$$\int e^{bx+a} \operatorname{csch}(bx+a)^n \, dx$$

input
```
int(exp(b*x+a)*csch(b*x+a)^n,x)
```

output
```
int(exp(b*x+a)*csch(b*x+a)^n,x)
```

Fricas [F]

$$\int e^{a+bx}\text{csch}^n(a+bx)\,dx = \int \text{csch}\,(bx+a)^n\,e^{(bx+a)}\,dx$$

input
```
integrate(exp(b*x+a)*csch(b*x+a)^n,x, algorithm="fricas")
```

output
```
integral(csch(b*x + a)^n*e^(b*x + a), x)
```

Sympy [F]

$$\int e^{a+bx}\text{csch}^n(a+bx)\,dx = e^a \int e^{bx}\,\text{csch}^n\,(a+bx)\,dx$$

input
```
integrate(exp(b*x+a)*csch(b*x+a)**n,x)
```

output
```
exp(a)*Integral(exp(b*x)*csch(a + b*x)**n, x)
```

Maxima [F]

$$\int e^{a+bx}\text{csch}^n(a+bx)\,dx = \int \text{csch}\,(bx+a)^n\,e^{(bx+a)}\,dx$$

input
```
integrate(exp(b*x+a)*csch(b*x+a)^n,x, algorithm="maxima")
```

output
```
integrate(csch(b*x + a)^n*e^(b*x + a), x)
```

Giac [F]

$$\int e^{a+bx}\text{csch}^n(a+bx)\,dx = \int \text{csch}\,(bx+a)^n\, e^{(bx+a)}\,dx$$

input
```
integrate(exp(b*x+a)*csch(b*x+a)^n,x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^n*e^(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\text{csch}^n(a+bx)\,dx = \int e^{a+b\,x}\left(\frac{1}{\sinh\left(a+b\,x\right)}\right)^n\,dx$$

input
```
int(exp(a + b*x)*(1/sinh(a + b*x))^n,x)
```

output
```
int(exp(a + b*x)*(1/sinh(a + b*x))^n, x)
```

Reduce [F]

$$\int e^{a+bx}\text{csch}^n(a+bx)\,dx = e^a\left(\int e^{bx}\text{csch}(bx+a)^n\,dx\right)$$

input
```
int(exp(b*x+a)*csch(b*x+a)^n,x)
```

output
```
e**a*int(e**(b*x)*csch(a + b*x)**n,x)
```

3.117 $\int F^{c(a+bx)}(f\,\mathrm{csch}(d+ex))^n\,dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int F^{c(a+bx)}(f\,\mathrm{csch}(d+ex))^n\,dx = \frac{\left(1-e^{2d+2ex}\right)^n F^{c(a+bx)}(f\,\mathrm{csch}(d+ex))^n\,\mathrm{Hypergeometric2F1}\left(n,\frac{1}{2}\left(n+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(2+n+\frac{bc\log(F)}{e}\right),e\right.}{en+bc\log(F)}$$

output
```
(1-exp(2*e*x+2*d))^n*F^(c*(b*x+a))*(f*csch(e*x+d))^n*hypergeom([n, 1/2*n+1
/2*b*c*ln(F)/e],[1+1/2*n+1/2*b*c*ln(F)/e],exp(2*e*x+2*d))/(e*n+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05

$$\int F^{c(a+bx)}(f\,\mathrm{csch}(d+ex))^n\,dx = -\frac{\left(1-e^{-2(d+ex)}\right)^n F^{c(a+bx)}(f\,\mathrm{csch}(d+ex))^n\,\mathrm{Hypergeometric2F1}\left(n,\frac{en-bc\log(F)}{2e},\frac{1}{2}\left(2+n-\frac{bc\log(F)}{e}\right),e^{-}\right.}{en-bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Csch[d + e*x])^n,x]
```

output
```
-(((1 - E^(-2*(d + e*x)))^n*F^(c*(a + b*x))*(f*Csch[d + e*x])^n*Hypergeome
tric2F1[n, (e*n - b*c*Log[F])/(2*e), (2 + n - (b*c*Log[F])/e)/2, E^(-2*(d
+ e*x))])/(e*n - b*c*Log[F]))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.27, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {7271, 6018, 2689}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^n\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$\operatorname{csch}^{-n}(d+ex)(f\operatorname{csch}(d+ex))^n\int F^{c(a+bx)}\operatorname{csch}^n(d+ex)dx$$

$$\Big\downarrow \text{ 6018}$$

$$e^{n(d+ex)}\left(1-e^{-2(d+ex)}\right)^n(f\operatorname{csch}(d+ex))^n\int e^{-dn-exn}\left(1-e^{-2(d+ex)}\right)^{-n}F^{ac+bxc}dx$$

$$\Big\downarrow \text{ 2689}$$

$$-\frac{e^{n(d+ex)-dn-enx}\left(1-e^{-2(d+ex)}\right)^n F^{ac+bcx}(f\operatorname{csch}(d+ex))^n \text{Hypergeometric2F1}\left(n,\frac{en-bc\log(F)}{2e},\frac{1}{2}\left(n-\frac{bc\log(F)}{e}\right.\right.}{en-bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*(f*Csch[d + e*x])^n,x]
```

output
```
-((E^(-(d*n) - e*n*x + n*(d + e*x))*(1 - E^(-2*(d + e*x)))^n*F^(a*c + b*c*
x)*(f*Csch[d + e*x])^n*Hypergeometric2F1[n, (e*n - b*c*Log[F])/(2*e), (2 +
 n - (b*c*Log[F])/e)/2, E^(-2*(d + e*x))])/(e*n - b*c*Log[F]))
```

Defintions of rubi rules used

rule 2689
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

rule 6018
```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(1 - E^(-2*(d + e*x)))^n*(Csch[d + e*x]^n/E^((-n)*(d + e*x)))
  Int[SimplifyIntegrand[F^(c*(a + b*x))*(1/(E^(n*(d + e*x))*(1 - E^(-2*(d +
 e*x)))^n)), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] &&  !IntegerQ[n]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}(f\ \mathrm{csch}\,(ex+d))^{n}\,dx$$

input
```
int(F^(c*(b*x+a))*(f*csch(e*x+d))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*csch(e*x+d))^n,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^n\,dx = \int (f\operatorname{csch}(ex+d))^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(e*x+d))^n,x, algorithm="fricas")
```

output
```
integral((f*csch(e*x + d))^n*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^n\,dx = \int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^n\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*csch(e*x+d))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*csch(d + e*x))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^n\,dx = \int (f\operatorname{csch}(ex+d))^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(e*x+d))^n,x, algorithm="maxima")
```

output
```
integrate((f*csch(e*x + d))^n*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^n\,dx = \int (f\operatorname{csch}(ex+d))^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(e*x+d))^n,x, algorithm="giac")
```

output
```
integrate((f*csch(e*x + d))^n*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^n\,dx = \int F^{c\,(a+b\,x)}\left(\frac{f}{\sinh(d+e\,x)}\right)^n dx$$

input
```
int(F^(c*(a + b*x))*(f/sinh(d + e*x))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f/sinh(d + e*x))^n, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^n\,dx = f^{ac+n}\left(\int f^{bcx}\operatorname{csch}(ex+d)^n\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*csch(e*x+d))^n,x)
```

output
```
f**(a*c + n)*int(f**(b*c*x)*csch(d + e*x)**n,x)
```

3.118 $\int F^{c(a+bx)}\left(f\,\mathrm{csch}\left(d+\frac{bcx\log(F)}{-2+n}\right)\right)^n\,dx$

Optimal result

Integrand size = 28, antiderivative size = 134

$$\begin{aligned}\int F^{c(a+bx)}\left(f\,\mathrm{csch}\left(d+\frac{bcx\log(F)}{-2+n}\right)\right)^n\,dx &= -\frac{f^2F^{c(a+bx)}(2-n)\left(f\,\mathrm{csch}\left(d-\frac{bcx\log(F)}{2-n}\right)\right)^{-2+n}}{bc(1-n)\log(F)}\\ &\quad-\frac{fF^{c(a+bx)}(2-n)\cosh\left(d-\frac{bcx\log(F)}{2-n}\right)\left(f\,\mathrm{csch}\left(d-\frac{bcx\log(F)}{2-n}\right)\right)^{-1+n}}{bc(1-n)\log(F)}\end{aligned}$$

output
```
-f^2*F^(c*(b*x+a))*(2-n)*(-f*csch(-d+b*c*x*ln(F)/(2-n)))^(-2+n)/b/c/(1-n)/
ln(F)-f*F^(c*(b*x+a))*(2-n)*cosh(-d+b*c*x*ln(F)/(2-n))*(-f*csch(-d+b*c*x*l
n(F)/(2-n)))^(-1+n)/b/c/(1-n)/ln(F)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.51

$$\int F^{c(a+bx)}\left(f\operatorname{csch}\left(d+\frac{bcx\log(F)}{-2+n}\right)\right)^n\,dx$$
$$=-\frac{F^{c(a+bx)}\left(-1+e^{2d}F^{\frac{2bcx}{-2+n}}\right)(-2+n)\left(f\operatorname{csch}\left(d+\frac{bcx\log(F)}{-2+n}\right)\right)^n}{2bc(-1+n)\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Csch[d + (b*c*x*Log[F])/(-2 + n)])^n,x]
```

output
```
-1/2*(F^(c*(a + b*x))*(-1 + E^(2*d)*F^((2*b*c*x)/(-2 + n)))*(-2 + n)*(f*Cs
ch[d + (b*c*x*Log[F])/(-2 + n)])^n)/(b*c*(-1 + n)*Log[F])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {7271, 6012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\left(f\operatorname{csch}\left(\frac{bcx\log(F)}{n-2}+d\right)\right)^n\,dx$$
$$\Big\downarrow \text{ 7271}$$
$$\operatorname{csch}^{-n}\left(d-\frac{bcx\log(F)}{2-n}\right)\left(f\operatorname{csch}\left(d-\frac{bcx\log(F)}{2-n}\right)\right)^n\int F^{c(a+bx)}\operatorname{csch}^n\left(d-\frac{bcx\log(F)}{2-n}\right)dx$$
$$\Big\downarrow \text{ 6012}$$
$$\operatorname{csch}^{-n}\left(d-\frac{bcx\log(F)}{2-n}\right)\left(f\operatorname{csch}\left(d-\frac{bcx\log(F)}{2-n}\right)\right)^n\left(-\frac{(2-n)F^{c(a+bx)}\operatorname{csch}^{n-2}\left(d-\frac{bcx\log(F)}{2-n}\right)}{bc(1-n)\log(F)}-\frac{(2-n)F^{c(\cdots}}{\cdots}\right.$$

input
```
Int[F^(c*(a + b*x))*(f*Csch[d + (b*c*x*Log[F])/(-2 + n)])^n,x]
```

output
```
((f*Csch[d - (b*c*x*Log[F])/(2 - n)])^n*(-((F^(c*(a + b*x))*(2 - n)*Csch[d
 - (b*c*x*Log[F])/(2 - n)]^(-2 + n))/(b*c*(1 - n)*Log[F])) - (F^(c*(a + b*
x))*(2 - n)*Cosh[d - (b*c*x*Log[F])/(2 - n)]*Csch[d - (b*c*x*Log[F])/(2 -
n)]^(-1 + n))/(b*c*(1 - n)*Log[F])))/Csch[d - (b*c*x*Log[F])/(2 - n)]^n
```

Defintions of rubi rules used

rule 6012
```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csch[d + e*x]^(n - 2)/(e^2*(n -
1)*(n - 2))), x] - Simp[F^(c*(a + b*x))*Csch[d + e*x]^(n - 1)*(Cosh[d + e*x
]/(e*(n - 1))), x] /; FreeQ[{F, a, b, c, d, e, n}, x] && EqQ[e^2*(n - 2)^2
- b^2*c^2*Log[F]^2, 0] && NeQ[n, 1] && NeQ[n, 2]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}\left(f \operatorname{csch}\left(d+\frac{bcx\ln(F)}{n-2}\right)\right)^{n} dx$$

input
```
int(F^(c*(b*x+a))*(f*csch(d+b*c*x*ln(F)/(n-2)))^n,x)
```

output
```
int(F^(c*(b*x+a))*(f*csch(d+b*c*x*ln(F)/(n-2)))^n,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(128) = 256$.

Time = 0.10 (sec) , antiderivative size = 501, normalized size of antiderivative = 3.74

$$\int F^{c(a+bx)}\left(f\operatorname{csch}\left(d+\frac{bcx\log(F)}{-2+n}\right)\right)^n\,dx =$$

$$-\frac{\left((n-2)\cosh\left((bcx+ac)\log\left(F\right)\right)\sinh\left(\frac{bcx\log(F)+dn-2\,d}{n-2}\right)+(n-2)\sinh\left((bcx+ac)\log\left(F\right)\right)\sinh\left(\frac{bcx\log}{}\right.\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(d+b*c*x*log(F)/(-2+n)))^n,x, algorithm="fr
icas")
```

output
```
-(((n - 2)*cosh((b*c*x + a*c)*log(F))*sinh((b*c*x*log(F) + d*n - 2*d)/(n -
 2)) + (n - 2)*sinh((b*c*x + a*c)*log(F))*sinh((b*c*x*log(F) + d*n - 2*d)/
(n - 2)))*cosh(n*log(2*(f*cosh((b*c*x*log(F) + d*n - 2*d)/(n - 2)) + f*sin
h((b*c*x*log(F) + d*n - 2*d)/(n - 2)))/(cosh((b*c*x*log(F) + d*n - 2*d)/(n
 - 2))^2 + 2*cosh((b*c*x*log(F) + d*n - 2*d)/(n - 2))*sinh((b*c*x*log(F) +
 d*n - 2*d)/(n - 2)) + sinh((b*c*x*log(F) + d*n - 2*d)/(n - 2))^2 - 1))) +
 ((n - 2)*cosh((b*c*x + a*c)*log(F))*sinh((b*c*x*log(F) + d*n - 2*d)/(n -
2)) + (n - 2)*sinh((b*c*x + a*c)*log(F))*sinh((b*c*x*log(F) + d*n - 2*d)/(
n - 2)))*sinh(n*log(2*(f*cosh((b*c*x*log(F) + d*n - 2*d)/(n - 2)) + f*sinh
((b*c*x*log(F) + d*n - 2*d)/(n - 2)))/(cosh((b*c*x*log(F) + d*n - 2*d)/(n
- 2))^2 + 2*cosh((b*c*x*log(F) + d*n - 2*d)/(n - 2))*sinh((b*c*x*log(F) +
d*n - 2*d)/(n - 2)) + sinh((b*c*x*log(F) + d*n - 2*d)/(n - 2))^2 - 1))))/(
(b*c*n - b*c)*cosh((b*c*x*log(F) + d*n - 2*d)/(n - 2))*log(F) - (b*c*n - b
*c)*log(F)*sinh((b*c*x*log(F) + d*n - 2*d)/(n - 2)))
```

Sympy [F]

$$\int F^{c(a+bx)}\left(f\operatorname{csch}\left(d+\frac{bcx\log(F)}{-2+n}\right)\right)^n\,dx=\int F^{c(a+bx)}\left(f\operatorname{csch}\left(\frac{bcx\log(F)}{n-2}+d\right)\right)^n\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*csch(d+b*c*x*ln(F)/(-2+n)))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*csch(b*c*x*log(F)/(n - 2) + d))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\left(f\operatorname{csch}\left(d+\frac{bcx\log(F)}{-2+n}\right)\right)^n\,dx=\int\left(f\operatorname{csch}\left(\frac{bcx\log(F)}{n-2}+d\right)\right)^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(d+b*c*x*log(F)/(-2+n)))^n,x, algorithm="ma
xima")
```

output
```
integrate((f*csch(b*c*x*log(F)/(n - 2) + d))^n*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\left(f\operatorname{csch}\left(d+\frac{bcx\log(F)}{-2+n}\right)\right)^n\,dx=\int\left(f\operatorname{csch}\left(\frac{bcx\log(F)}{n-2}+d\right)\right)^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(d+b*c*x*log(F)/(-2+n)))^n,x, algorithm="gi
ac")
```

output
```
integrate((f*csch(b*c*x*log(F)/(n - 2) + d))^n*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\left(f\operatorname{csch}\left(d+\frac{bcx\log(F)}{-2+n}\right)\right)^n\,dx=\int F^{c\,(a+b\,x)}\left(\frac{f}{\sinh\left(d+\frac{b\,c\,x\,\ln(F)}{n-2}\right)}\right)^n\,dx$$

input
```
int(F^(c*(a + b*x))*(f/sinh(d + (b*c*x*log(F))/(n - 2)))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f/sinh(d + (b*c*x*log(F))/(n - 2)))^n, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\left(f\operatorname{csch}\left(d+\frac{bcx\log(F)}{-2+n}\right)\right)^n\,dx$$
$$=f^{ac+n}\left(\int f^{bcx}\operatorname{csch}\left(\frac{\log(f)\,bcx+dn-2d}{n-2}\right)^n dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*csch(d+b*c*x*log(F)/(-2+n)))^n,x)
```

output
```
f**(a*c + n)*int(f**(b*c*x)*csch((log(f)*b*c*x + d*n - 2*d)/(n - 2))**n,x)
```

3.119 $\int F^{c(a+bx)}\left(f\operatorname{csch}\left(d-\frac{bcx\log(F)}{-2+n}\right)\right)^n\,dx$

Optimal result

Integrand size = 29, antiderivative size = 130

$$\int F^{c(a+bx)}\left(f\operatorname{csch}\left(d-\frac{bcx\log(F)}{-2+n}\right)\right)^n\,dx = -\frac{f^2F^{c(a+bx)}(2-n)\left(f\operatorname{csch}\left(d+\frac{bcx\log(F)}{2-n}\right)\right)^{-2+n}}{bc(1-n)\log(F)} + \frac{fF^{c(a+bx)}(2-n)\cosh\left(d+\frac{bcx\log(F)}{2-n}\right)\left(f\operatorname{csch}\left(d+\frac{bcx\log(F)}{2-n}\right)\right)^{-1+n}}{bc(1-n)\log(F)}$$

output
```
-f^2*F^(c*(b*x+a))*(2-n)*(f*csch(d+b*c*x*ln(F)/(2-n)))^(-2+n)/b/c/(1-n)/ln
(F)+f*F^(c*(b*x+a))*(2-n)*cosh(d+b*c*x*ln(F)/(2-n))*(f*csch(d+b*c*x*ln(F)/
(2-n)))^(-1+n)/b/c/(1-n)/ln(F)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.58

$$\int F^{c(a+bx)}\left(f\operatorname{csch}\left(d-\frac{bcx\log(F)}{-2+n}\right)\right)^n dx = \frac{e^{-2d}F^{c(a+bx)}\left(e^{2d}-F^{\frac{2bcx}{-2+n}}\right)(-2+n)\left(f\operatorname{csch}\left(d-\frac{bcx\log(F)}{-2+n}\right)\right)^n}{2bc(-1+n)\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Csch[d - (b*c*x*Log[F])/(-2 + n)])^n,x]
```

output
```
(F^(c*(a + b*x))*(E^(2*d) - F^((2*b*c*x)/(-2 + n)))*(-2 + n)*(f*Csch[d - (
b*c*x*Log[F])/(-2 + n)])^n)/(2*b*c*E^(2*d)*(-1 + n)*Log[F])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {7271, 6012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\left(f\operatorname{csch}\left(d-\frac{bcx\log(F)}{n-2}\right)\right)^n dx$$

$$\Big\downarrow \text{7271}$$

$$\operatorname{csch}^{-n}\left(\frac{bcx\log(F)}{2-n}+d\right)\left(f\operatorname{csch}\left(\frac{bcx\log(F)}{2-n}+d\right)\right)^n\int F^{c(a+bx)}\operatorname{csch}^n\left(d+\frac{bcx\log(F)}{2-n}\right)dx$$

$$\Big\downarrow \text{6012}$$

$$\operatorname{csch}^{-n}\left(\frac{bcx\log(F)}{2-n}+d\right)\left(f\operatorname{csch}\left(\frac{bcx\log(F)}{2-n}+d\right)\right)^n\left(\frac{(2-n)F^{c(a+bx)}\cosh\left(\frac{bcx\log(F)}{2-n}+d\right)\operatorname{csch}^{n-1}\left(\frac{bcx\log(F)}{2-n}\right.}{bc(1-n)\log(F)}\right.$$

input
```
Int[F^(c*(a + b*x))*(f*Csch[d - (b*c*x*Log[F])/(-2 + n)])^n,x]
```

output
```
((f*Csch[d + (b*c*x*Log[F])/(2 - n)])^n*(-((F^(c*(a + b*x))*(2 - n)*Csch[d
 + (b*c*x*Log[F])/(2 - n)]^(-2 + n))/(b*c*(1 - n)*Log[F])) + (F^(c*(a + b*
x))*(2 - n)*Cosh[d + (b*c*x*Log[F])/(2 - n)]*Csch[d + (b*c*x*Log[F])/(2 -
n)]^(-1 + n))/(b*c*(1 - n)*Log[F])))/Csch[d + (b*c*x*Log[F])/(2 - n)]^n
```

Defintions of rubi rules used

rule 6012
```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csch[d + e*x]^(n - 2)/(e^2*(n -
1)*(n - 2))), x] - Simp[F^(c*(a + b*x))*Csch[d + e*x]^(n - 1)*(Cosh[d + e*x
]/(e*(n - 1))), x] /; FreeQ[{F, a, b, c, d, e, n}, x] && EqQ[e^2*(n - 2)^2
- b^2*c^2*Log[F]^2, 0] && NeQ[n, 1] && NeQ[n, 2]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)} \left(-f \operatorname{csch}\left(-d + \frac{bcx \ln(F)}{n-2}\right)\right)^n dx$$

input
```
int(F^(c*(b*x+a))*(-f*csch(-d+b*c*x*ln(F)/(n-2)))^n,x)
```

output
```
int(F^(c*(b*x+a))*(-f*csch(-d+b*c*x*ln(F)/(n-2)))^n,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(119) = 238$.

Time = 0.11 (sec) , antiderivative size = 519, normalized size of antiderivative = 3.99

$$\int F^{c(a+bx)}\left(f\text{csch}\left(d-\frac{bcx\log(F)}{-2+n}\right)\right)^n\,dx =$$

$$-\frac{\left((n-2)\cosh\left((bcx+ac)\log\left(F\right)\right)\sinh\left(\frac{bcx\log(F)-dn+2\,d}{n-2}\right)+(n-2)\sinh\left((bcx+ac)\log\left(F\right)\right)\sinh\left(\frac{bcx\log}{}\right.\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*(-f*csch(-d+b*c*x*log(F)/(-2+n)))^n,x, algorithm="
fricas")
```

output
```
-(((n - 2)*cosh((b*c*x + a*c)*log(F))*sinh((b*c*x*log(F) - d*n + 2*d)/(n -
 2)) + (n - 2)*sinh((b*c*x + a*c)*log(F))*sinh((b*c*x*log(F) - d*n + 2*d)/
(n - 2)))*cosh(n*log(-2*(f*cosh((b*c*x*log(F) - d*n + 2*d)/(n - 2)) + f*si
nh((b*c*x*log(F) - d*n + 2*d)/(n - 2)))/(cosh((b*c*x*log(F) - d*n + 2*d)/(
n - 2))^2 + 2*cosh((b*c*x*log(F) - d*n + 2*d)/(n - 2))*sinh((b*c*x*log(F)
- d*n + 2*d)/(n - 2)) + sinh((b*c*x*log(F) - d*n + 2*d)/(n - 2))^2 - 1)))
+ ((n - 2)*cosh((b*c*x + a*c)*log(F))*sinh((b*c*x*log(F) - d*n + 2*d)/(n -
 2)) + (n - 2)*sinh((b*c*x + a*c)*log(F))*sinh((b*c*x*log(F) - d*n + 2*d)/
(n - 2)))*sinh(n*log(-2*(f*cosh((b*c*x*log(F) - d*n + 2*d)/(n - 2)) + f*si
nh((b*c*x*log(F) - d*n + 2*d)/(n - 2)))/(cosh((b*c*x*log(F) - d*n + 2*d)/(
n - 2))^2 + 2*cosh((b*c*x*log(F) - d*n + 2*d)/(n - 2))*sinh((b*c*x*log(F)
- d*n + 2*d)/(n - 2)) + sinh((b*c*x*log(F) - d*n + 2*d)/(n - 2))^2 - 1))))
/((b*c*n - b*c)*cosh((b*c*x*log(F) - d*n + 2*d)/(n - 2))*log(F) - (b*c*n -
 b*c)*log(F)*sinh((b*c*x*log(F) - d*n + 2*d)/(n - 2)))
```

Sympy [F]

$$\int F^{c(a+bx)}\left(f\text{csch}\left(d-\frac{bcx\log(F)}{-2+n}\right)\right)^n dx = \int F^{c(a+bx)}\left(-f\,\text{csch}\left(\frac{bcx\log{(F)}}{n-2}-d\right)\right)^n dx$$

input
```
integrate(F**(c*(b*x+a))*(-f*csch(-d+b*c*x*ln(F)/(-2+n)))**n,x)
```

output
```
Integral(F**(c*(a + b*x))*(-f*csch(b*c*x*log(F)/(n - 2) - d))**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\left(f\text{csch}\left(d-\frac{bcx\log(F)}{-2+n}\right)\right)^n dx = \int \left(-f\,\text{csch}\left(\frac{bcx\log{(F)}}{n-2}-d\right)\right)^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(-f*csch(-d+b*c*x*log(F)/(-2+n)))^n,x, algorithm="
maxima")
```

output
```
integrate((-f*csch(b*c*x*log(F)/(n - 2) - d))^n*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\left(f\text{csch}\left(d-\frac{bcx\log(F)}{-2+n}\right)\right)^n dx = \int \left(-f\,\text{csch}\left(\frac{bcx\log{(F)}}{n-2}-d\right)\right)^n F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(-f*csch(-d+b*c*x*log(F)/(-2+n)))^n,x, algorithm="
giac")
```

output
```
integrate((-f*csch(b*c*x*log(F)/(n - 2) - d))^n*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\left(f\operatorname{csch}\left(d-\frac{bcx\log(F)}{-2+n}\right)\right)^n\,dx=\int F^{c\,(a+b\,x)}\,\left(\frac{f}{\sinh\left(d-\frac{b\,c\,x\,\ln(F)}{n-2}\right)}\right)^n\,dx$$

input
```
int(F^(c*(a + b*x))*(f/sinh(d - (b*c*x*log(F))/(n - 2)))^n,x)
```

output
```
int(F^(c*(a + b*x))*(f/sinh(d - (b*c*x*log(F))/(n - 2)))^n, x)
```

Reduce [F]

$$\begin{aligned}&\int F^{c(a+bx)}\left(f\operatorname{csch}\left(d-\frac{bcx\log(F)}{-2+n}\right)\right)^n\,dx\\&=f^{ac+n}(-1)^n\left(\int f^{bcx}\operatorname{csch}\left(\frac{\log(f)\,bcx-dn+2d}{n-2}\right)^n dx\right)\end{aligned}$$

input
```
int(F^(c*(b*x+a))*(-f*csch(-d+b*c*x*log(F)/(-2+n)))^n,x)
```

output
```
f**(a*c + n)*( - 1)**n*int(f**(b*c*x)*csch((log(f)*b*c*x - d*n + 2*d)/(n -
 2))**n,x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*   "F" if the result fails to integrate an expression that*)
(*       is integrable*)
(*   "C" if result involves higher level functions than necessary*)
(*   "B" if result is more than twice the size of the optimal*)
(*       antiderivative*)
(*   "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:    #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file