Computer Algebra
Independent Integration Tests

Summer 2024

6-Hyperbolic-functions/6.7-Hyperbolic-exponential/322-6.7.2

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**178**]. This is test number [322].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	87.64 (156)	12.36 (22)
Rubi	82.02 (146)	17.98 (32)
Maple	62.36 (111)	37.64 (67)
Fricas	62.36 (111)	37.64 (67)
Maxima	61.24 (109)	38.76 (69)
Giac	59.55 (106)	40.45 (72)
Reduce	55.06 (98)	44.94 (80)
Mupad	53.37 (95)	46.63 (83)
Sympy	19.66 (35)	80.34 (143)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

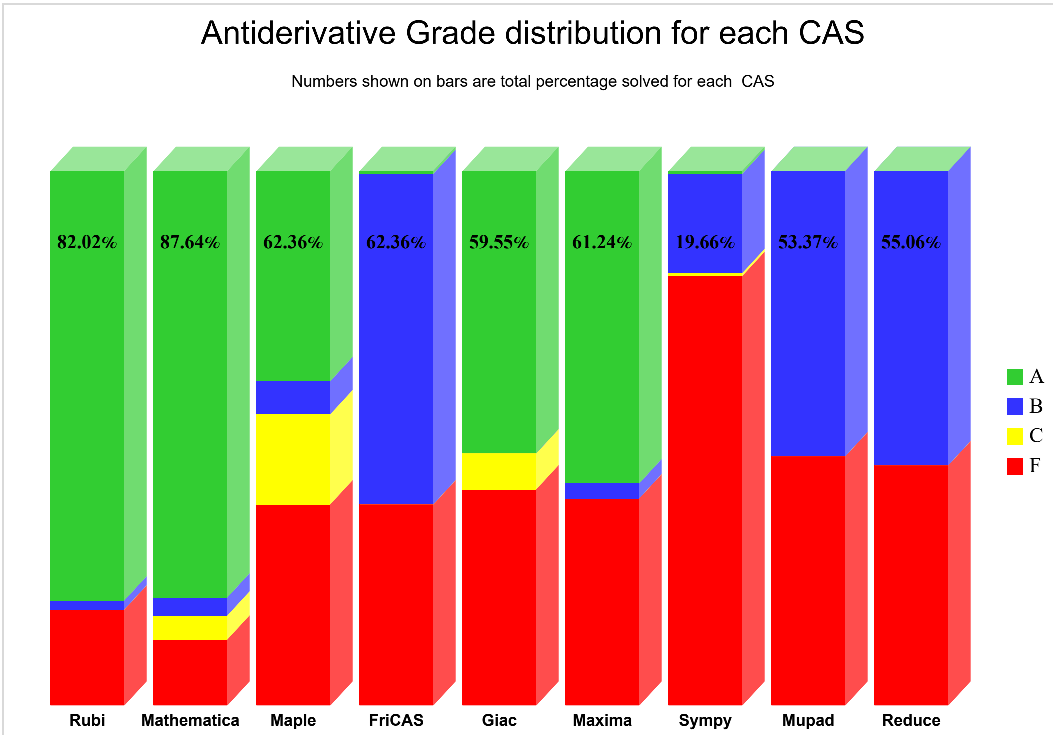
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

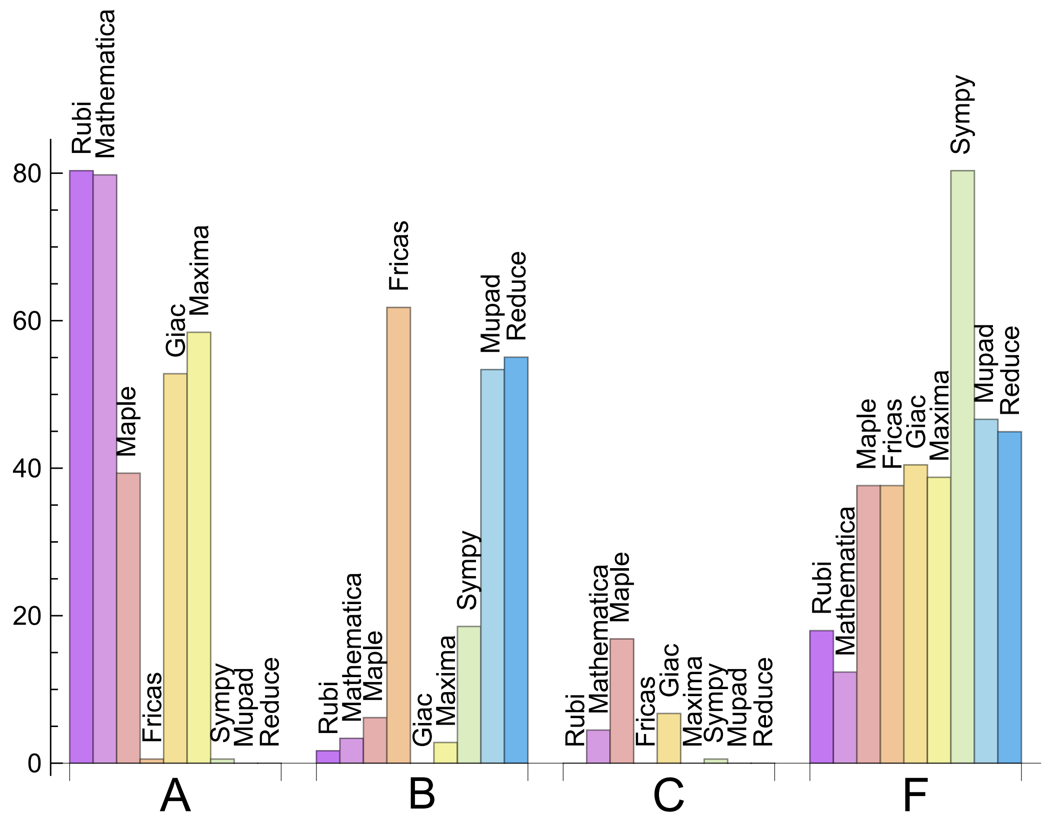
System	% A grade	% B grade	% C grade	% F grade
Rubi	80.337	1.685	0.000	17.978
Mathematica	79.775	3.371	4.494	12.360
Maxima	58.427	2.809	0.000	38.764
Giac	52.809	0.000	6.742	40.449
Maple	39.326	6.180	16.854	37.640
Fricas	0.562	61.798	0.000	37.640
Sympy	0.562	18.539	0.562	80.337
Mupad	0.000	53.371	0.000	46.629
Reduce	0.000	55.056	0.000	44.944

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	22	90.91	9.09	0.00
Rubi	32	100.00	0.00	0.00
Fricas	67	100.00	0.00	0.00
Maple	67	100.00	0.00	0.00
Maxima	69	100.00	0.00	0.00
Giac	72	100.00	0.00	0.00
Reduce	80	100.00	0.00	0.00
Mupad	83	0.00	100.00	0.00
Sympy	143	81.12	18.88	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.07
Fricas	0.11
Giac	0.12
Reduce	0.22
Rubi	0.38
Mathematica	0.74
Mupad	1.90
Sympy	5.11
Maple	24.37

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	89.62	1.17	89.00	1.03
Rubi	94.03	0.96	64.50	0.77
Mathematica	98.19	1.09	89.00	0.96
Mupad	107.12	1.31	86.00	1.24
Maple	119.63	1.41	97.00	1.37
Reduce	120.14	1.50	84.00	1.34
Giac	194.93	1.90	76.00	0.97
Sympy	471.46	4.71	202.00	2.91
Fricas	823.84	7.82	424.00	5.87

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

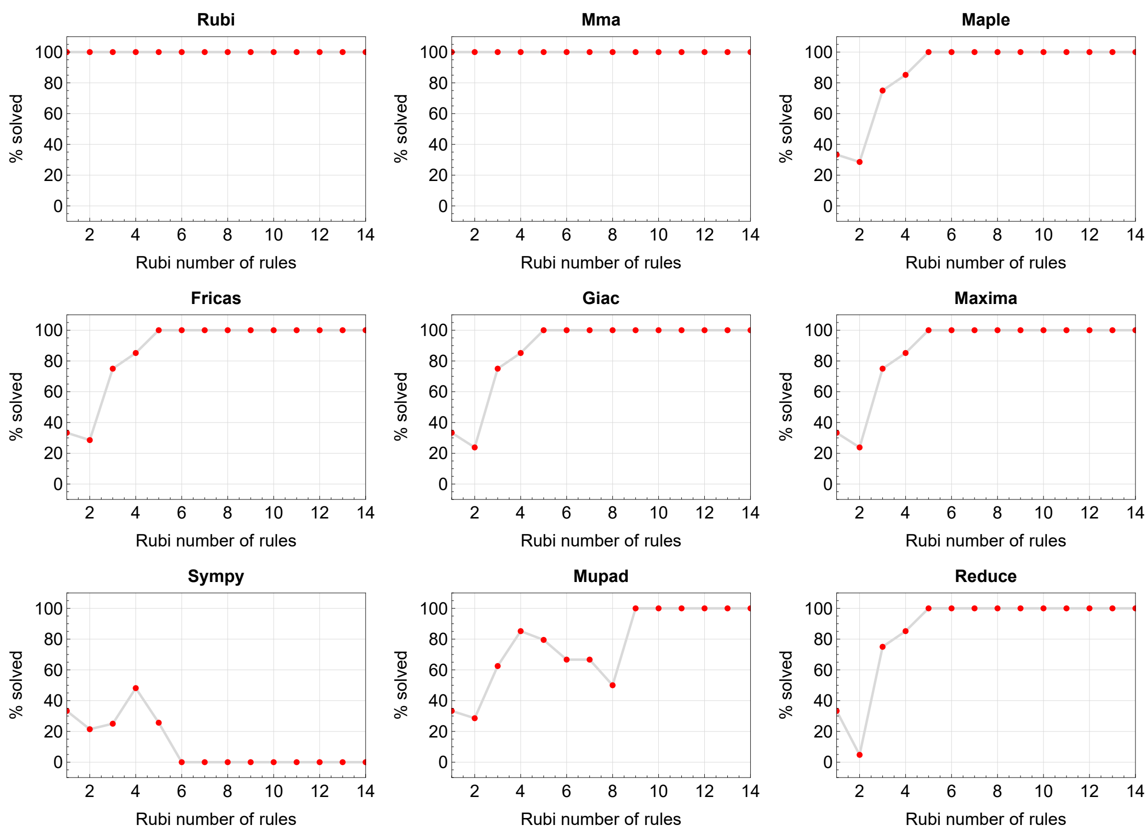


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

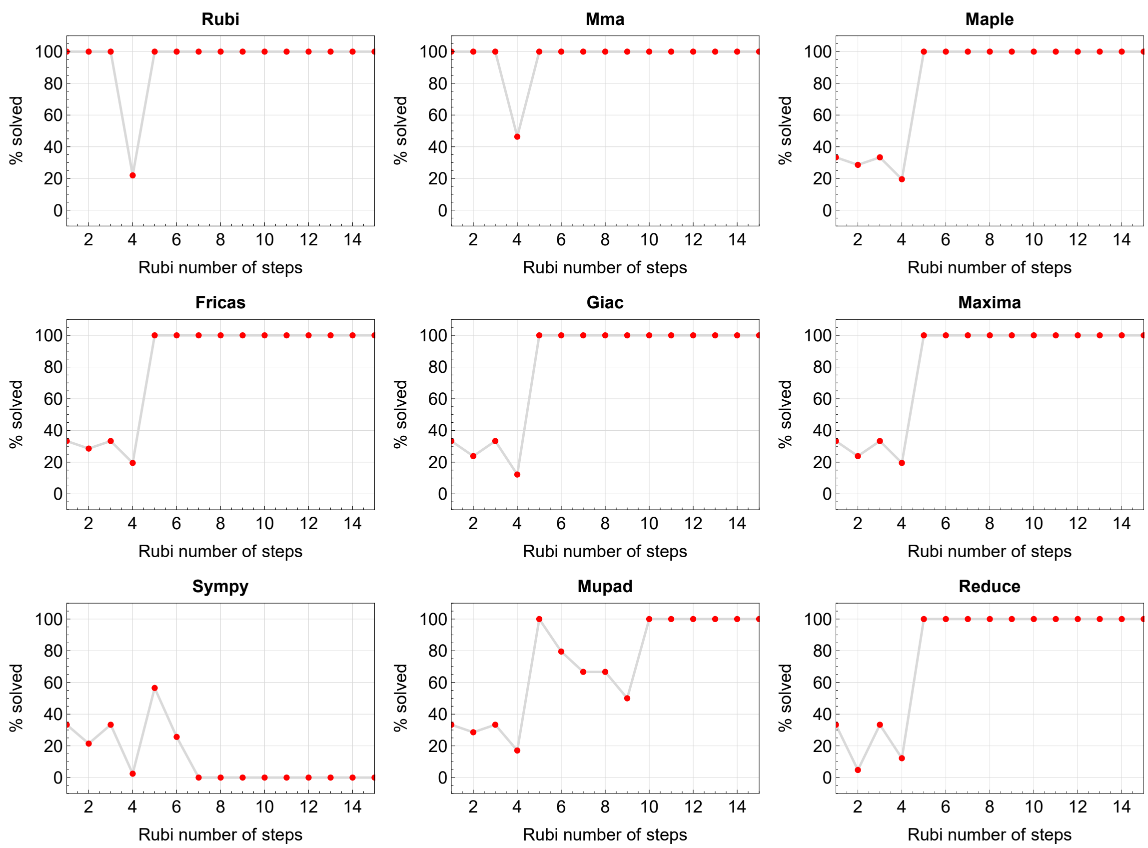


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

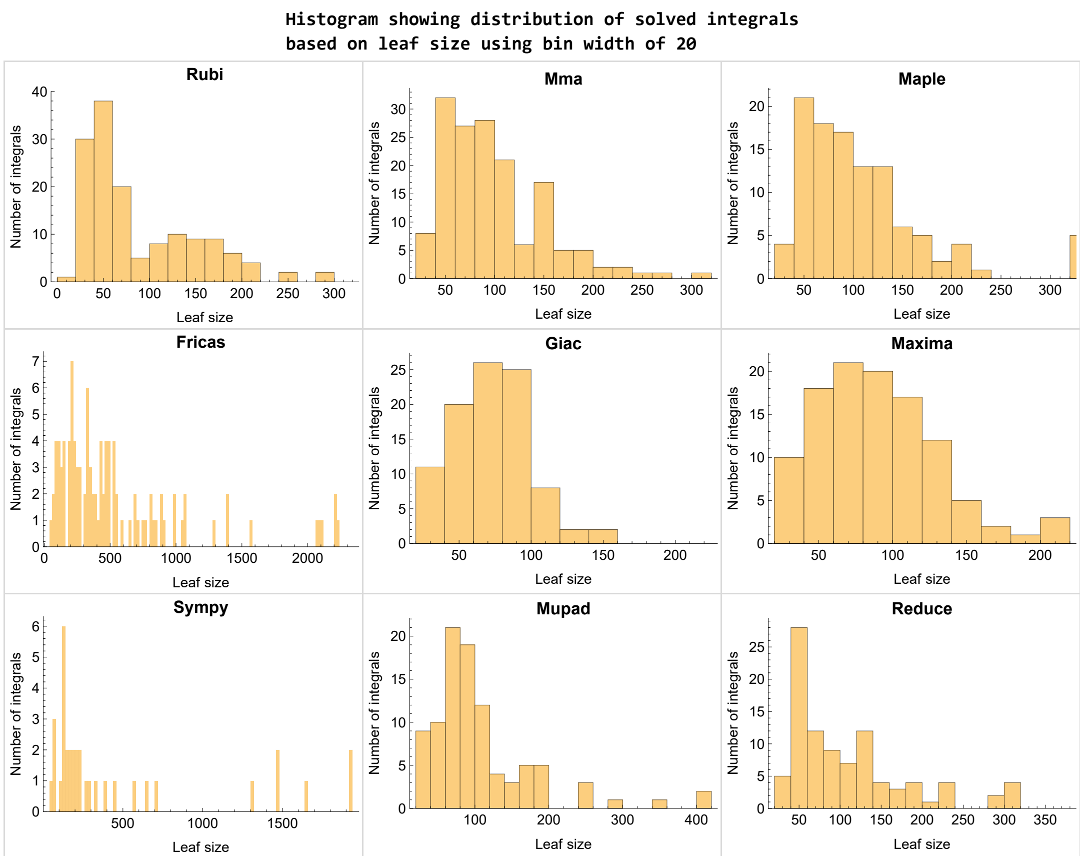


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

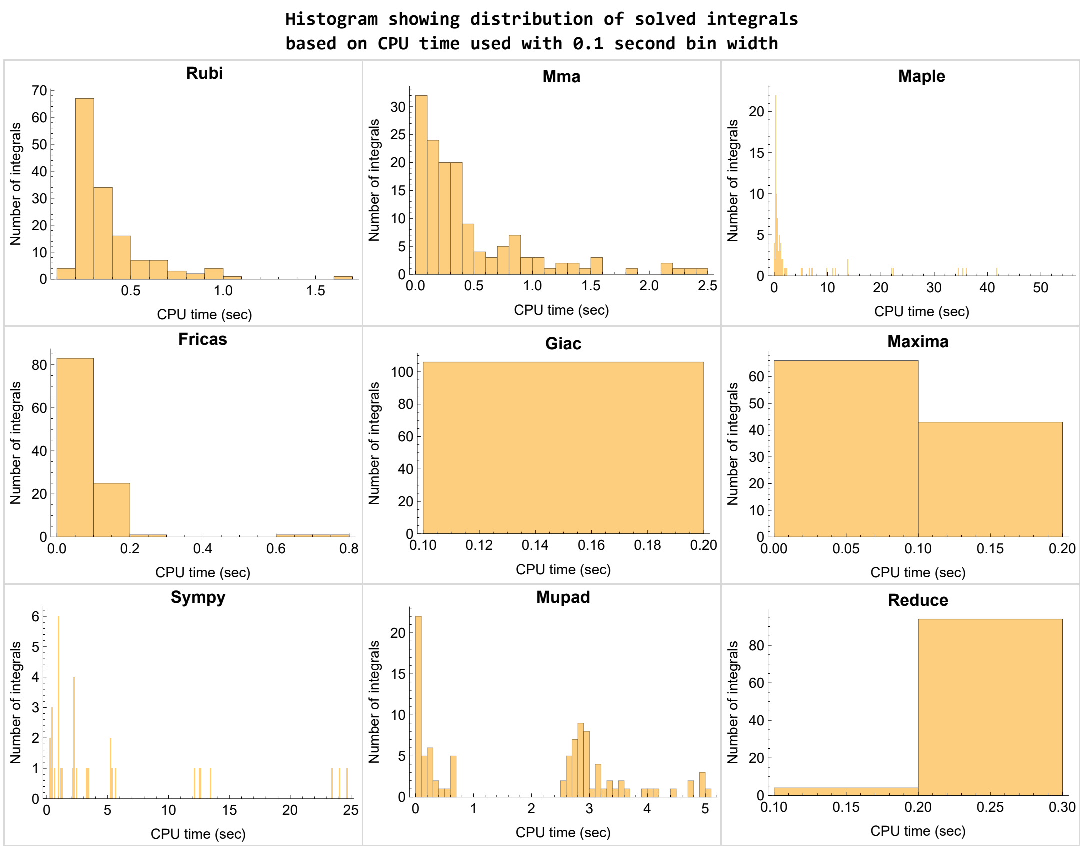


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

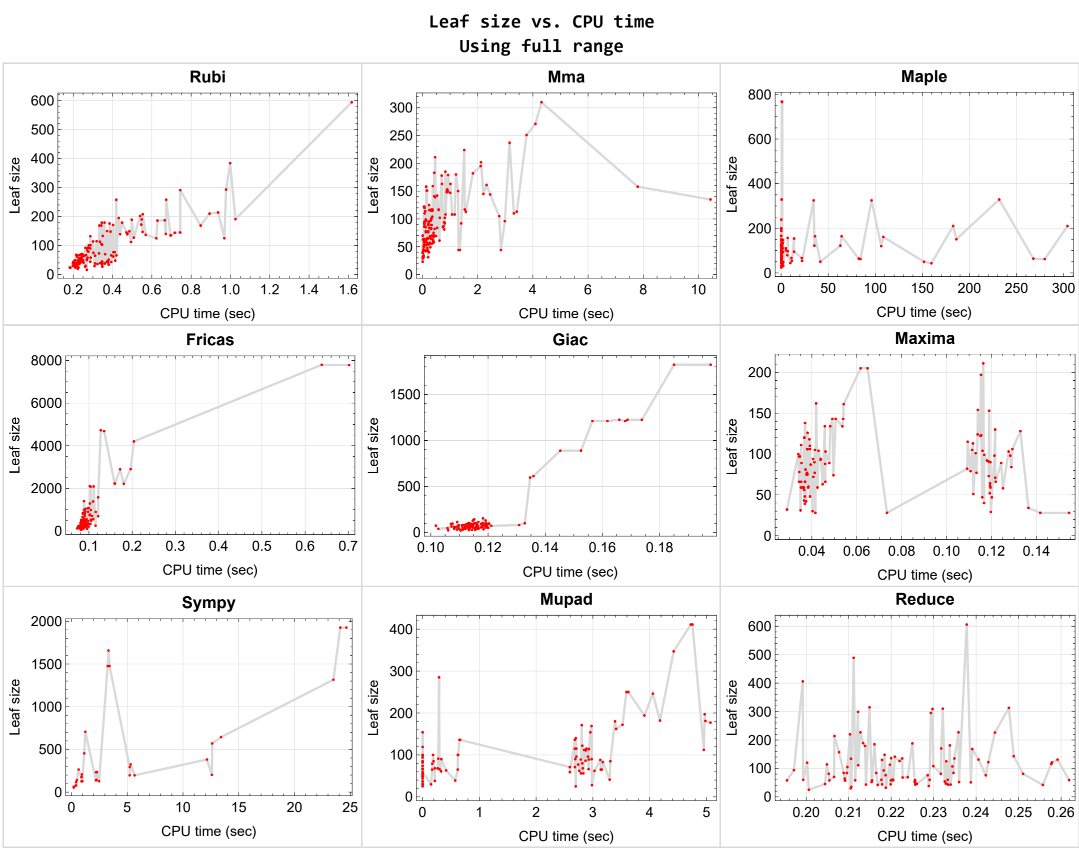


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86}

Mathematica $\{56, 150, 152, 155, 157\}$

Maple $\{\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals.

These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

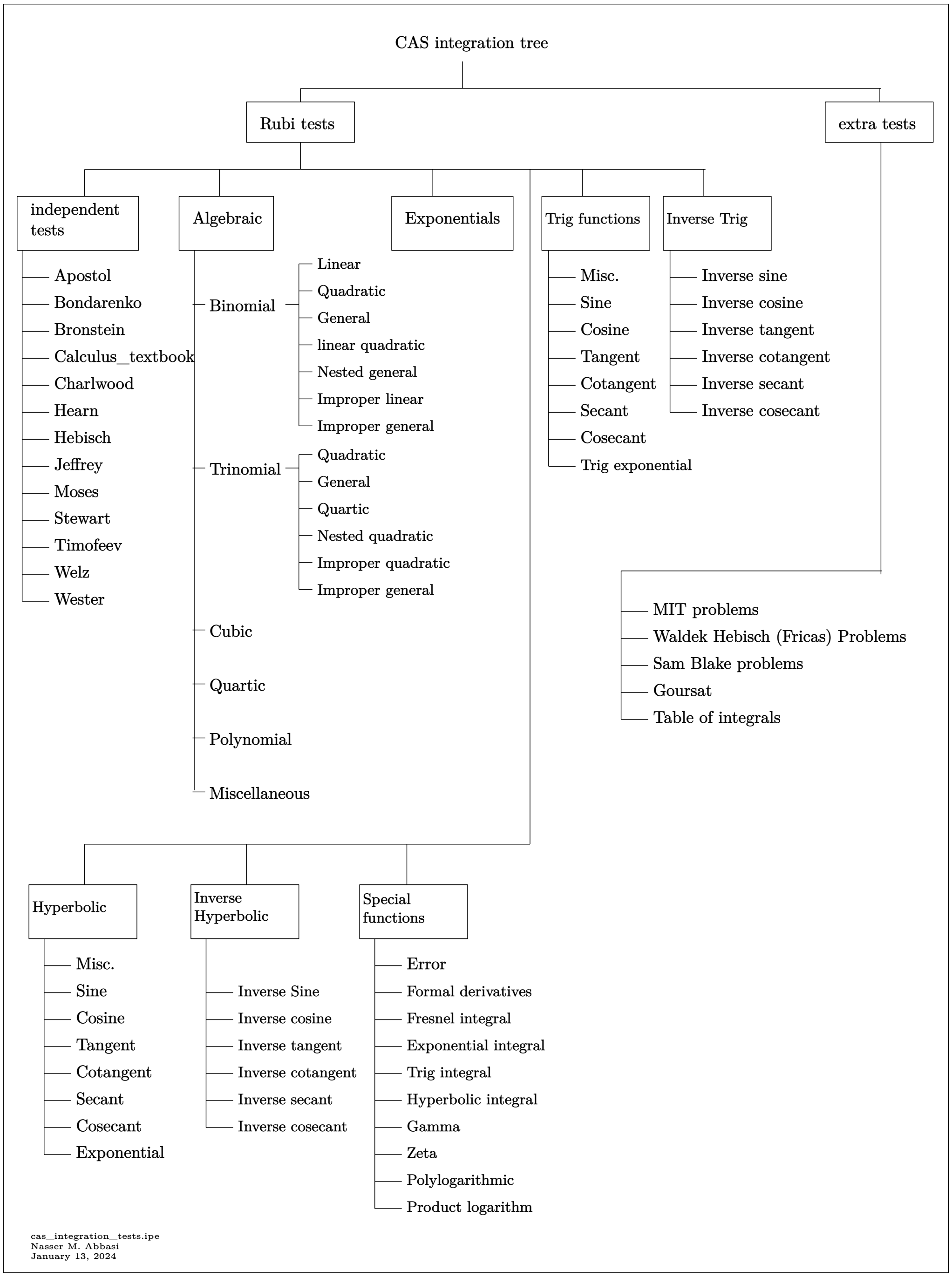
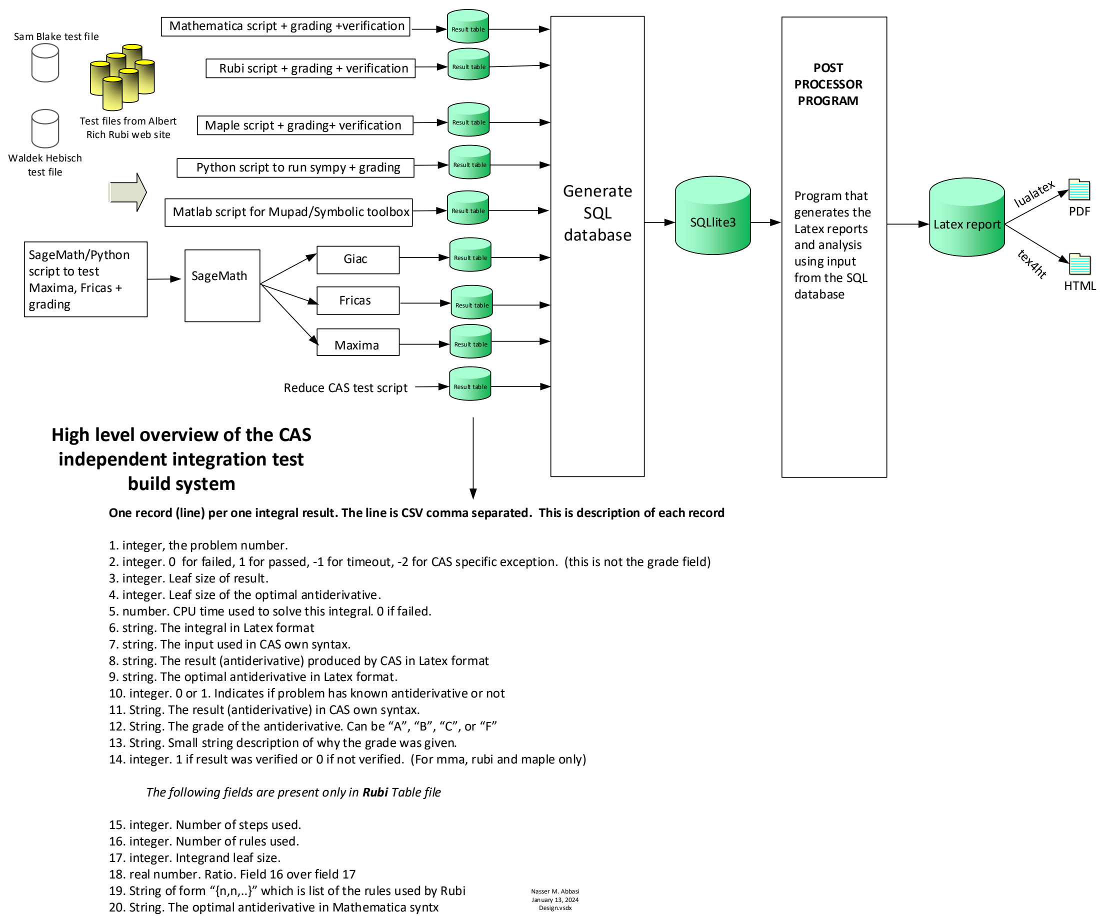


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 127, 129, 130, 131, 132, 133, 134, 135, 137, 139, 140, 142, 143, 145, 158, 160, 161, 163 }

B grade { 120, 128, 136 }

C grade { }

F normal fail { 138, 141, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 57, 58, 59, 60, 61, 63, 64, 65, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 91, 92, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 150, 151, 152, 154, 155, 157, 158, 160, 161, 163 }

B grade { 6, 48, 54, 55, 66, 72 }

C grade { 42, 56, 62, 87, 89, 90, 93, 94 }

F normal fail { 147, 148, 149, 153, 156, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178 }

F(-1) timedout fail { 159, 162 }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 25, 27, 29, 31, 35, 37, 39, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 68, 70, 72, 74, 78, 80, 82, 86, 97, 98, 103, 105, 112, 137, 138, 140, 141, 144 }

B grade { 5, 33, 76, 84, 95, 96, 102, 104, 109, 110, 111 }

C grade { 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 65, 67, 69, 71, 73, 75, 77, 79, 81, 83, 85, 87, 88, 89, 90, 91, 92, 93, 94 }

F normal fail { 99, 100, 101, 106, 107, 108, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 144 }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 102, 103, 104, 105, 109, 110, 111, 112, 137, 138, 140, 141 }

C grade { }

F normal fail { 99, 100, 101, 106, 107, 108, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 102, 103, 104, 105, 109, 110, 111, 112, 138, 141, 144 }

B grade { 27, 33, 39, 68, 76 }

C grade { }

F normal fail { 99, 100, 101, 106, 107, 108, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 140, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94 }

B grade { }

C grade { 95, 96, 97, 98, 102, 103, 104, 105, 109, 110, 111, 112 }

F normal fail { 99, 100, 101, 106, 107, 108, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 29, 30, 31, 32, 34, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 102, 103, 104, 105, 109, 110, 111, 112, 137, 138, 140, 141, 144 }

C grade { }

F normal fail { }

F(-1) timedout fail { 7, 14, 21, 27, 28, 33, 35, 36, 37, 38, 39, 40, 41, 42, 63, 78, 99, 100, 101, 106, 107, 108, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178 }

F(-2) exception fail { }

Sympy

A grade { 46 }

B grade { 1, 2, 3, 4, 8, 9, 10, 11, 15, 16, 17, 18, 43, 44, 45, 50, 51, 52, 53, 57, 58, 59, 60, 95, 96, 97, 98, 103, 104, 105, 111, 112, 137 }

C grade { 140 }

F normal fail { 5, 6, 7, 12, 13, 14, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 47, 48, 49, 54, 55, 56, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 99, 100, 101, 106, 107, 108, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 142, 143, 146, 149, 152, 156, 158, 160, 161, 163, 164, 167, 170, 173, 177 }

F(-1) timedout fail { 102, 109, 110, 138, 141, 144, 145, 147, 148, 150, 151, 153, 154, 155, 157, 159, 162, 165, 166, 168, 169, 171, 172, 174, 175, 176, 178 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 97, 98, 137, 140 }

C grade { }

F normal fail { 95, 96, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	49	89	62	77	260	139	58	58	65
N.S.	1	0.67	1.22	0.85	1.05	3.56	1.90	0.79	0.79	0.89
time (sec)	N/A	0.208	0.315	279.983	0.041	0.090	2.191	0.114	0.195	0.621

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	38	76	54	66	215	177	66	58	61
N.S.	1	0.58	1.17	0.83	1.02	3.31	2.72	1.02	0.89	0.94
time (sec)	N/A	0.361	0.210	22.301	0.034	0.092	0.906	0.107	0.216	0.327

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	29	42	32	31	114	76	29	43	28
N.S.	1	0.78	1.14	0.86	0.84	3.08	2.05	0.78	1.16	0.76
time (sec)	N/A	0.308	0.019	2.216	0.035	0.079	0.389	0.114	0.214	2.984

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	27	39	24	30	98	63	26	38	25
N.S.	1	0.93	1.34	0.83	1.03	3.38	2.17	0.90	1.31	0.86
time (sec)	N/A	0.261	0.006	0.214	0.040	0.087	0.202	0.113	0.229	0.002

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	24	34	63	56	117	0	54	42	59
N.S.	1	0.75	1.06	1.97	1.75	3.66	0.00	1.69	1.31	1.84
time (sec)	N/A	0.211	0.038	0.239	0.037	0.085	0.000	0.119	0.256	0.003

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	33	116	79	77	188	0	77	105	68
N.S.	1	0.60	2.11	1.44	1.40	3.42	0.00	1.40	1.91	1.24
time (sec)	N/A	0.218	0.249	0.312	0.038	0.080	0.000	0.111	0.210	0.217

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	66	55	106	110	806	0	81	170	0
N.S.	1	0.86	0.71	1.38	1.43	10.47	0.00	1.05	2.21	0.00
time (sec)	N/A	0.220	0.264	0.600	0.039	0.086	0.000	0.112	0.232	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	56	115	166	105	392	294	92	84	136
N.S.	1	0.55	1.14	1.64	1.04	3.88	2.91	0.91	0.83	1.35
time (sec)	N/A	0.243	0.365	0.053	0.041	0.081	5.222	0.115	0.235	0.662

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	38	43	43	48	208	144	42	46	39
N.S.	1	0.75	0.84	0.84	0.94	4.08	2.82	0.82	0.90	0.76
time (sec)	N/A	0.227	0.117	159.610	0.037	0.075	2.247	0.112	0.226	0.576

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	36	76	54	66	217	175	64	58	62
N.S.	1	0.55	1.17	0.83	1.02	3.34	2.69	0.98	0.89	0.95
time (sec)	N/A	0.224	0.122	11.464	0.037	0.087	0.904	0.113	0.212	3.028

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	36	50	43	42	124	78	40	43	37
N.S.	1	0.71	0.98	0.84	0.82	2.43	1.53	0.78	0.84	0.73
time (sec)	N/A	0.323	0.043	0.455	0.037	0.086	0.422	0.108	0.226	0.002

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	39	91	67	74	159	0	50	52	56
N.S.	1	0.71	1.65	1.22	1.35	2.89	0.00	0.91	0.95	1.02
time (sec)	N/A	0.336	0.218	0.409	0.050	0.076	0.000	0.111	0.236	2.832

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	40	99	97	90	446	0	85	111	99
N.S.	1	0.67	1.65	1.62	1.50	7.43	0.00	1.42	1.85	1.65
time (sec)	N/A	0.215	0.118	0.244	0.041	0.092	0.000	0.119	0.212	0.002

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	50	65	98	106	487	0	97	157	0
N.S.	1	0.60	0.78	1.18	1.28	5.87	0.00	1.17	1.89	0.00
time (sec)	N/A	0.236	0.168	0.384	0.038	0.087	0.000	0.120	0.208	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	45	53	122	75	352	202	58	58	65
N.S.	1	0.62	0.73	1.67	1.03	4.82	2.77	0.79	0.79	0.89
time (sec)	N/A	0.347	0.124	0.043	0.038	0.081	12.590	0.113	0.220	3.138

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	58	115	166	103	389	325	92	84	136
N.S.	1	0.57	1.14	1.64	1.02	3.85	3.22	0.91	0.83	1.35
time (sec)	N/A	0.273	0.309	0.204	0.046	0.084	5.315	0.116	0.216	0.651

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	49	89	62	75	260	139	56	58	65
N.S.	1	0.67	1.22	0.85	1.03	3.56	1.90	0.77	0.79	0.89
time (sec)	N/A	0.222	0.206	84.307	0.038	0.080	2.241	0.118	0.205	3.157

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	38	89	54	60	214	207	64	58	66
N.S.	1	0.58	1.37	0.83	0.92	3.29	3.18	0.98	0.89	1.02
time (sec)	N/A	0.223	0.071	0.768	0.038	0.082	0.939	0.109	0.220	0.002

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	44	69	93	93	400	0	80	81	89
N.S.	1	0.65	1.01	1.37	1.37	5.88	0.00	1.18	1.19	1.31
time (sec)	N/A	0.287	0.267	1.250	0.044	0.081	0.000	0.111	0.251	2.928

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	51	65	101	106	461	0	80	136	90
N.S.	1	0.61	0.78	1.22	1.28	5.55	0.00	0.96	1.64	1.08
time (sec)	N/A	0.239	0.194	0.595	0.039	0.087	0.000	0.119	0.220	0.332

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	62	95	116	120	980	0	103	185	0
N.S.	1	0.70	1.08	1.32	1.36	11.14	0.00	1.17	2.10	0.00
time (sec)	N/A	0.404	0.237	0.245	0.037	0.104	0.000	0.118	0.216	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	44	55	101	66	332	0	60	60	86
N.S.	1	0.64	0.80	1.46	0.96	4.81	0.00	0.87	0.87	1.25
time (sec)	N/A	0.371	0.285	1.008	0.122	0.087	0.000	0.108	0.218	2.871

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	35	91	66	59	155	0	47	43	56
N.S.	1	0.65	1.69	1.22	1.09	2.87	0.00	0.87	0.80	1.04
time (sec)	N/A	0.221	0.214	0.402	0.036	0.081	0.000	0.114	0.233	2.790

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	24	45	70	29	87	0	29	30	55
N.S.	1	0.75	1.41	2.19	0.91	2.72	0.00	0.91	0.94	1.72
time (sec)	N/A	0.227	0.033	0.162	0.120	0.084	0.000	0.115	0.210	0.003

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	16	23	43	28	45	0	22	25	31
N.S.	1	0.67	0.96	1.79	1.17	1.88	0.00	0.92	1.04	1.29
time (sec)	N/A	0.269	0.005	0.250	0.042	0.082	0.000	0.110	0.201	0.003

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	29	42	111	64	99	0	60	45	181
N.S.	1	0.71	1.02	2.71	1.56	2.41	0.00	1.46	1.10	4.41
time (sec)	N/A	0.199	0.122	1.687	0.119	0.081	0.000	0.118	0.204	4.980

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	36	40	97	106	319	0	73	143	0
N.S.	1	0.69	0.77	1.87	2.04	6.13	0.00	1.40	2.75	0.00
time (sec)	N/A	0.231	0.236	7.185	0.038	0.086	0.000	0.119	0.249	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	77	101	164	128	1068	0	101	220	0
N.S.	1	0.80	1.05	1.71	1.33	11.12	0.00	1.05	2.29	0.00
time (sec)	N/A	0.411	0.323	36.079	0.133	0.098	0.000	0.133	0.210	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	49	64	98	89	461	0	83	108	88
N.S.	1	0.61	0.80	1.22	1.11	5.76	0.00	1.04	1.35	1.10
time (sec)	N/A	0.250	0.188	0.566	0.048	0.082	0.000	0.112	0.230	3.131

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	38	84	102	61	321	0	58	76	94
N.S.	1	0.66	1.45	1.76	1.05	5.53	0.00	1.00	1.31	1.62
time (sec)	N/A	0.198	0.244	0.245	0.119	0.094	0.000	0.120	0.242	0.002

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	32	99	75	59	184	0	74	76	65
N.S.	1	0.63	1.94	1.47	1.16	3.61	0.00	1.45	1.49	1.27
time (sec)	N/A	0.219	0.316	0.922	0.038	0.093	0.000	0.114	0.219	0.170

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	38	43	92	51	207	0	50	64	84
N.S.	1	0.81	0.91	1.96	1.09	4.40	0.00	1.06	1.36	1.79
time (sec)	N/A	0.311	0.046	0.460	0.112	0.080	0.000	0.120	0.209	0.003

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	34	44	95	104	313	0	73	141	0
N.S.	1	0.68	0.88	1.90	2.08	6.26	0.00	1.46	2.82	0.00
time (sec)	N/A	0.380	0.185	13.779	0.043	0.088	0.000	0.121	0.221	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	55	55	164	98	740	0	87	131	194
N.S.	1	0.80	0.80	2.38	1.42	10.72	0.00	1.26	1.90	2.81
time (sec)	N/A	0.234	0.270	64.271	0.128	0.101	0.000	0.112	0.259	3.907

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	66	66	151	162	1397	0	130	315	0
N.S.	1	0.62	0.62	1.42	1.53	13.18	0.00	1.23	2.97	0.00
time (sec)	N/A	0.394	0.274	186.143	0.042	0.089	0.000	0.115	0.215	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	58	125	120	89	702	0	73	120	0
N.S.	1	0.69	1.49	1.43	1.06	8.36	0.00	0.87	1.43	0.00
time (sec)	N/A	0.289	0.238	0.280	0.124	0.107	0.000	0.117	0.200	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	41	56	94	88	477	0	92	113	0
N.S.	1	0.53	0.73	1.22	1.14	6.19	0.00	1.19	1.47	0.00
time (sec)	N/A	0.378	0.195	1.957	0.046	0.085	0.000	0.120	0.220	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	62	55	111	79	529	0	58	107	0
N.S.	1	0.86	0.76	1.54	1.10	7.35	0.00	0.81	1.49	0.00
time (sec)	N/A	0.219	0.172	5.010	0.111	0.105	0.000	0.108	0.234	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	23	29	51	97	122	0	37	42	0
N.S.	1	0.77	0.97	1.70	3.23	4.07	0.00	1.23	1.40	0.00
time (sec)	N/A	0.185	0.009	0.514	0.034	0.073	0.000	0.116	0.217	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	73	97	161	123	1042	0	96	214	0
N.S.	1	0.79	1.05	1.75	1.34	11.33	0.00	1.04	2.33	0.00
time (sec)	N/A	0.331	0.298	108.597	0.115	0.092	0.000	0.108	0.207	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	64	70	151	161	1384	0	131	313	0
N.S.	1	0.62	0.67	1.45	1.55	13.31	0.00	1.26	3.01	0.00
time (sec)	N/A	0.256	0.347	0.255	0.054	0.110	0.000	0.119	0.248	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	83	64	188	130	2075	0	101	226	0
N.S.	1	0.79	0.61	1.79	1.24	19.76	0.00	0.96	2.15	0.00
time (sec)	N/A	0.272	0.142	0.252	0.122	0.104	0.000	0.119	0.244	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	46	82	64	66	345	235	81	60	85
N.S.	1	0.61	1.08	0.84	0.87	4.54	3.09	1.07	0.79	1.12
time (sec)	N/A	0.250	0.440	267.807	0.037	0.099	2.288	0.114	0.229	3.308

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	48	78	67	59	234	128	63	59	66
N.S.	1	0.62	1.00	0.86	0.76	3.00	1.64	0.81	0.76	0.85
time (sec)	N/A	0.236	0.210	22.024	0.043	0.076	0.912	0.115	0.262	0.299

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	29	45	30	43	181	117	35	69	30
N.S.	1	0.85	1.32	0.88	1.26	5.32	3.44	1.03	2.03	0.88
time (sec)	N/A	0.208	0.038	2.118	0.037	0.086	0.418	0.111	0.224	0.151

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	28	39	32	32	138	54	33	32	29
N.S.	1	0.74	1.03	0.84	0.84	3.63	1.42	0.87	0.84	0.76
time (sec)	N/A	0.220	0.006	0.227	0.029	0.079	0.208	0.109	0.219	0.002

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	34	92	63	81	156	0	40	52	49
N.S.	1	0.74	2.00	1.37	1.76	3.39	0.00	0.87	1.13	1.07
time (sec)	N/A	0.347	0.070	0.208	0.037	0.089	0.000	0.116	0.215	0.002

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	42	211	111	100	497	0	82	116	100
N.S.	1	0.58	2.89	1.52	1.37	6.81	0.00	1.12	1.59	1.37
time (sec)	N/A	0.344	0.457	0.302	0.034	0.091	0.000	0.115	0.258	0.187

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	48	56	105	126	529	0	68	168	90
N.S.	1	0.53	0.62	1.17	1.40	5.88	0.00	0.76	1.87	1.00
time (sec)	N/A	0.243	0.363	0.573	0.038	0.090	0.000	0.113	0.239	2.979

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	66	117	200	89	424	197	95	84	100
N.S.	1	0.56	0.99	1.69	0.75	3.59	1.67	0.81	0.71	0.85
time (sec)	N/A	0.420	0.361	0.050	0.035	0.086	5.662	0.107	0.209	0.627

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	41	42	50	48	248	128	50	46	41
N.S.	1	0.68	0.70	0.83	0.80	4.13	2.13	0.83	0.77	0.68
time (sec)	N/A	0.237	0.187	151.749	0.039	0.082	2.480	0.116	0.233	3.295

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	46	78	67	59	234	128	63	59	66
N.S.	1	0.59	1.00	0.86	0.76	3.00	1.64	0.81	0.76	0.85
time (sec)	N/A	0.232	0.119	10.986	0.035	0.088	0.980	0.116	0.220	2.924

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	30	53	44	56	194	141	47	44	44
N.S.	1	0.59	1.04	0.86	1.10	3.80	2.76	0.92	0.86	0.86
time (sec)	N/A	0.202	0.049	0.387	0.037	0.081	0.491	0.116	0.220	0.002

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	38	183	88	78	264	0	76	61	89
N.S.	1	0.63	3.05	1.47	1.30	4.40	0.00	1.27	1.02	1.48
time (sec)	N/A	0.211	0.428	0.366	0.034	0.088	0.000	0.118	0.220	2.786

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	46	158	98	118	465	0	68	122	85
N.S.	1	0.60	2.05	1.27	1.53	6.04	0.00	0.88	1.58	1.10
time (sec)	N/A	0.376	0.133	0.236	0.039	0.098	0.000	0.110	0.243	0.002

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	71	251	131	120	1074	0	103	189	140
N.S.	1	0.67	2.37	1.24	1.13	10.13	0.00	0.97	1.78	1.32
time (sec)	N/A	0.244	3.770	0.347	0.037	0.099	0.000	0.116	0.213	2.700

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	48	57	126	66	433	382	81	60	63
N.S.	1	0.63	0.75	1.66	0.87	5.70	5.03	1.07	0.79	0.83
time (sec)	N/A	0.414	0.264	0.048	0.046	0.093	12.154	0.110	0.199	0.416

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	68	117	200	87	425	197	93	84	100
N.S.	1	0.58	0.99	1.69	0.74	3.60	1.67	0.79	0.71	0.85
time (sec)	N/A	0.250	0.326	0.237	0.040	0.082	5.222	0.118	0.210	0.623

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	48	82	64	66	345	233	77	60	83
N.S.	1	0.63	1.08	0.84	0.87	4.54	3.07	1.01	0.79	1.09
time (sec)	N/A	0.236	0.309	82.449	0.035	0.093	2.203	0.119	0.233	3.174

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	44	86	67	59	232	124	61	57	66
N.S.	1	0.56	1.10	0.86	0.76	2.97	1.59	0.78	0.73	0.85
time (sec)	N/A	0.229	0.066	0.770	0.036	0.078	0.919	0.112	0.209	0.002

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	43	60	91	97	333	0	77	68	68
N.S.	1	0.54	0.76	1.15	1.23	4.22	0.00	0.97	0.86	0.86
time (sec)	N/A	0.393	0.390	1.211	0.035	0.099	0.000	0.114	0.223	2.822

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	57	224	126	111	689	0	102	131	115
N.S.	1	0.61	2.41	1.35	1.19	7.41	0.00	1.10	1.41	1.24
time (sec)	N/A	0.236	1.513	0.587	0.035	0.121	0.000	0.113	0.241	2.885

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	62	151	119	138	905	0	96	181	0
N.S.	1	0.58	1.41	1.11	1.29	8.46	0.00	0.90	1.69	0.00
time (sec)	N/A	0.244	0.165	0.250	0.037	0.091	0.000	0.120	0.234	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	41	58	89	73	333	0	76	59	68
N.S.	1	0.53	0.75	1.16	0.95	4.32	0.00	0.99	0.77	0.88
time (sec)	N/A	0.252	0.414	0.979	0.119	0.083	0.000	0.115	0.226	0.269

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	38	83	95	52	227	0	54	47	85
N.S.	1	0.63	1.38	1.58	0.87	3.78	0.00	0.90	0.78	1.42
time (sec)	N/A	0.348	0.321	0.395	0.119	0.096	0.000	0.115	0.233	0.195

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	32	92	62	58	156	0	40	43	49
N.S.	1	0.71	2.04	1.38	1.29	3.47	0.00	0.89	0.96	1.09
time (sec)	N/A	0.208	0.069	0.162	0.125	0.087	0.000	0.103	0.234	0.002

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	24	32	80	40	98	0	32	35	67
N.S.	1	0.60	0.80	2.00	1.00	2.45	0.00	0.80	0.88	1.68
time (sec)	N/A	0.182	0.008	0.250	0.117	0.096	0.000	0.111	0.211	0.002

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	20	48	49	98	71	0	25	50	25
N.S.	1	0.71	1.71	1.75	3.50	2.54	0.00	0.89	1.79	0.89
time (sec)	N/A	0.223	0.173	1.640	0.121	0.082	0.000	0.106	0.226	2.698

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	51	69	157	104	498	0	75	135	182
N.S.	1	0.65	0.88	2.01	1.33	6.38	0.00	0.96	1.73	2.33
time (sec)	N/A	0.380	0.494	7.030	0.116	0.104	0.000	0.114	0.223	4.181

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	54	64	122	124	830	0	89	227	115
N.S.	1	0.63	0.74	1.42	1.44	9.65	0.00	1.03	2.64	1.34
time (sec)	N/A	0.270	0.318	35.367	0.114	0.088	0.000	0.120	0.236	2.709

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	55	70	130	82	551	0	80	93	111
N.S.	1	0.60	0.77	1.43	0.90	6.05	0.00	0.88	1.02	1.22
time (sec)	N/A	0.241	0.432	0.571	0.109	0.110	0.000	0.113	0.218	2.877

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	42	158	94	91	453	0	67	94	82
N.S.	1	0.58	2.16	1.29	1.25	6.21	0.00	0.92	1.29	1.12
time (sec)	N/A	0.225	0.133	0.230	0.119	0.084	0.000	0.119	0.197	0.002

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	43	104	115	71	360	0	62	81	96
N.S.	1	0.61	1.46	1.62	1.00	5.07	0.00	0.87	1.14	1.35
time (sec)	N/A	0.205	0.355	0.907	0.122	0.094	0.000	0.114	0.232	2.709

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	30	63	80	77	201	0	45	81	53
N.S.	1	0.54	1.12	1.43	1.38	3.59	0.00	0.80	1.45	0.95
time (sec)	N/A	0.263	0.018	0.470	0.113	0.099	0.000	0.117	0.205	0.002

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	49	70	154	103	491	0	75	135	180
N.S.	1	0.64	0.92	2.03	1.36	6.46	0.00	0.99	1.78	2.37
time (sec)	N/A	0.237	0.378	13.742	0.128	0.106	0.000	0.102	0.235	3.389

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	44	49	122	106	658	0	66	148	79
N.S.	1	0.77	0.86	2.14	1.86	11.54	0.00	1.16	2.60	1.39
time (sec)	N/A	0.237	0.287	62.958	0.129	0.085	0.000	0.116	0.218	0.172

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	115	117	210	153	2109	0	115	310	250
N.S.	1	0.84	0.85	1.53	1.12	15.39	0.00	0.84	2.26	1.82
time (sec)	N/A	0.319	0.583	182.686	0.119	0.102	0.000	0.117	0.232	3.625

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	56	151	115	113	897	0	95	134	0
N.S.	1	0.55	1.50	1.14	1.12	8.88	0.00	0.94	1.33	0.00
time (sec)	N/A	0.230	0.155	0.263	0.112	0.098	0.000	0.119	0.211	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	67	69	135	92	770	0	81	126	136
N.S.	1	0.66	0.68	1.32	0.90	7.55	0.00	0.79	1.24	1.33
time (sec)	N/A	0.265	0.354	1.882	0.118	0.102	0.000	0.131	0.222	2.682

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	44	54	101	99	520	0	67	121	90
N.S.	1	0.52	0.64	1.20	1.18	6.19	0.00	0.80	1.44	1.07
time (sec)	N/A	0.250	0.212	5.253	0.117	0.082	0.000	0.114	0.258	2.690

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	65	60	120	84	581	0	61	114	119
N.S.	1	0.77	0.71	1.43	1.00	6.92	0.00	0.73	1.36	1.42
time (sec)	N/A	0.311	0.079	0.984	0.129	0.088	0.000	0.118	0.205	0.002

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	50	63	120	122	817	0	90	227	115
N.S.	1	0.61	0.77	1.46	1.49	9.96	0.00	1.10	2.77	1.40
time (sec)	N/A	0.270	0.367	106.346	0.115	0.091	0.000	0.114	0.213	2.683

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	112	116	210	154	2090	0	115	309	250
N.S.	1	0.73	0.76	1.37	1.01	13.66	0.00	0.75	2.02	1.63
time (sec)	N/A	0.494	0.586	304.066	0.114	0.111	0.000	0.109	0.230	3.591

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	27	31	73	39	247	0	39	44	59
N.S.	1	0.77	0.89	2.09	1.11	7.06	0.00	1.11	1.26	1.69
time (sec)	N/A	0.348	0.205	0.260	0.037	0.084	0.000	0.113	0.218	2.595

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	176	163	239	197	4685	0	143	489	347
N.S.	1	0.91	0.84	1.24	1.02	24.27	0.00	0.74	2.53	1.80
time (sec)	N/A	0.385	0.708	0.266	0.115	0.135	0.000	0.115	0.211	4.422

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	114	139	197	211	4726	0	154	606	285
N.S.	1	0.56	0.68	0.97	1.03	23.17	0.00	0.75	2.97	1.40
time (sec)	N/A	0.301	0.457	0.279	0.116	0.127	0.000	0.118	0.238	0.292

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	127	42	40	90	365	0	90	188	91
N.S.	1	1.44	0.48	0.45	1.02	4.15	0.00	1.02	2.14	1.03
time (sec)	N/A	0.508	0.023	1.217	0.119	0.094	0.000	0.113	0.225	0.278

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	148	120	44	101	854	0	95	295	122
N.S.	1	1.47	1.19	0.44	1.00	8.46	0.00	0.94	2.92	1.21
time (sec)	N/A	0.390	0.072	9.872	0.113	0.086	0.000	0.114	0.229	2.814

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	150	58	48	105	893	0	99	299	112
N.S.	1	1.43	0.55	0.46	1.00	8.50	0.00	0.94	2.85	1.07
time (sec)	N/A	0.372	0.053	2.365	0.111	0.116	0.000	0.115	0.212	2.846

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	171	64	50	115	1577	0	103	406	154
N.S.	1	1.42	0.53	0.42	0.96	13.14	0.00	0.86	3.38	1.28
time (sec)	N/A	0.403	0.055	41.721	0.109	0.121	0.000	0.113	0.199	2.956

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	44	31	48	34	202	0	35	76	38
N.S.	1	1.29	0.91	1.41	1.00	5.94	0.00	1.03	2.24	1.12
time (sec)	N/A	0.201	0.042	0.321	0.136	0.090	0.000	0.118	0.229	0.217

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	67	54	54	47	522	0	42	125	80
N.S.	1	1.31	1.06	1.06	0.92	10.24	0.00	0.82	2.45	1.57
time (sec)	N/A	0.231	0.065	0.782	0.116	0.084	0.000	0.112	0.233	2.813

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	69	161	56	47	557	0	42	130	62
N.S.	1	1.25	2.93	1.02	0.85	10.13	0.00	0.76	2.36	1.13
time (sec)	N/A	0.238	2.323	0.393	0.120	0.094	0.000	0.110	0.218	2.760

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	92	310	60	59	992	0	48	179	114
N.S.	1	1.30	4.37	0.85	0.83	13.97	0.00	0.68	2.52	1.61
time (sec)	N/A	0.262	4.312	0.650	0.120	0.089	0.000	0.106	0.214	2.930

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	179	108	329	143	2900	1926	1225	29	197
N.S.	1	1.29	0.78	2.37	1.03	20.86	13.86	8.81	0.21	1.42
time (sec)	N/A	0.449	1.166	0.662	0.051	0.196	24.099	0.166	0.285	4.968

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	169	102	325	134	2224	1476	1211	29	172
N.S.	1	1.28	0.77	2.46	1.02	16.85	11.18	9.17	0.22	1.30
time (sec)	N/A	0.343	0.808	96.036	0.046	0.159	3.249	0.162	0.224	3.521

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	90	59	79	72	433	454	613	70	96
N.S.	1	1.29	0.84	1.13	1.03	6.19	6.49	8.76	1.00	1.37
time (sec)	N/A	0.286	0.057	6.520	0.041	0.090	1.147	0.136	0.207	2.914

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	75	50	51	63	246	265	597	51	74
N.S.	1	1.19	0.79	0.81	1.00	3.90	4.21	9.48	0.81	1.17
time (sec)	N/A	0.303	0.053	0.339	0.045	0.115	0.647	0.135	0.239	0.002

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	77	59	0	0	0	0	0	21	0
N.S.	1	0.99	0.76	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.404	0.136	0.000	0.000	0.000	0.000	0.000	0.224	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	135	135	0	0	0	0	0	27	0
N.S.	1	1.21	1.21	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.697	10.436	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	145	81	0	0	0	0	0	29	0
N.S.	1	1.21	0.68	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.745	0.790	0.000	0.000	0.000	0.000	0.000	0.218	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	258	149	768	205	7798	0	1823	31	411
N.S.	1	1.28	0.74	3.82	1.02	38.80	0.00	9.07	0.15	2.04
time (sec)	N/A	0.673	0.915	0.813	0.065	0.638	0.000	0.198	0.219	4.723

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	113	89	143	94	1289	1316	890	31	112
N.S.	1	1.22	0.96	1.54	1.01	13.86	14.15	9.57	0.33	1.20
time (sec)	N/A	0.336	0.333	0.279	0.041	0.102	23.483	0.152	0.219	4.953

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	169	102	325	134	2216	1476	1211	29	162
N.S.	1	1.28	0.77	2.46	1.02	16.79	11.18	9.17	0.22	1.23
time (sec)	N/A	0.333	0.628	34.589	0.048	0.181	3.417	0.168	0.221	3.409

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	132	85	90	94	699	707	889	23	100
N.S.	1	1.42	0.91	0.97	1.01	7.52	7.60	9.56	0.25	1.08
time (sec)	N/A	0.287	0.118	0.988	0.044	0.090	1.246	0.145	0.221	0.003

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	139	158	0	0	0	0	0	27	0
N.S.	1	1.04	1.18	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.475	7.800	0.000	0.000	0.000	0.000	0.000	0.233	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	133	92	0	0	0	0	0	23	0
N.S.	1	1.32	0.91	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.350	0.811	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	202	145	0	0	0	0	0	29	0
N.S.	1	1.22	0.87	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.540	2.207	0.000	0.000	0.000	0.000	0.000	0.221	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	179	163	329	143	4202	0	1225	31	246
N.S.	1	1.29	1.17	2.37	1.03	30.23	0.00	8.81	0.22	1.77
time (sec)	N/A	0.359	1.014	0.566	0.054	0.204	0.000	0.169	0.268	4.055

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	258	150	766	205	7786	0	1823	31	411
N.S.	1	1.28	0.75	3.81	1.02	38.74	0.00	9.07	0.15	2.04
time (sec)	N/A	0.419	0.880	1.033	0.062	0.702	0.000	0.185	0.240	4.752

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	179	108	329	143	2892	1926	1225	29	177
N.S.	1	1.29	0.78	2.37	1.03	20.81	13.86	8.81	0.21	1.27
time (sec)	N/A	0.345	0.835	231.615	0.049	0.172	24.649	0.174	0.210	5.070

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	190	159	148	134	2218	1658	1211	23	154
N.S.	1	1.44	1.20	1.12	1.02	16.80	12.56	9.17	0.17	1.17
time (sec)	N/A	0.549	0.374	2.094	0.054	0.160	3.320	0.156	0.234	0.002

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	169	141	0	0	0	0	0	29	0
N.S.	1	1.06	0.88	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.848	0.424	0.000	0.000	0.000	0.000	0.000	0.232	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	181	210	182	0	0	0	0	0	29	0
N.S.	1	1.16	1.01	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.894	1.827	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	189	147	0	0	0	0	0	23	0
N.S.	1	1.20	0.94	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.498	1.004	0.000	0.000	0.000	0.000	0.000	0.224	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	172	142	0	0	0	0	0	29	0
N.S.	1	1.06	0.87	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.546	0.532	0.000	0.000	0.000	0.000	0.000	0.229	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	136	141	108	0	0	0	0	0	27	0
N.S.	1	1.04	0.79	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.475	1.071	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	79	61	0	0	0	0	0	21	0
N.S.	1	0.99	0.76	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.277	0.116	0.000	0.000	0.000	0.000	0.000	0.226	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	68	70	0	0	0	0	0	21	0
N.S.	1	0.99	1.01	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.215	0.008	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	186	96	0	0	0	0	0	27	0
N.S.	1	2.58	1.33	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.629	2.983	0.000	0.000	0.000	0.000	0.000	0.244	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	145	180	0	0	0	0	0	29	0
N.S.	1	0.75	0.93	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.471	1.213	0.000	0.000	0.000	0.000	0.000	0.261	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	253	187	152	0	0	0	0	0	29	0
N.S.	1	0.74	0.60	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.666	0.904	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	181	214	185	0	0	0	0	0	29	0
N.S.	1	1.18	1.02	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.937	0.830	0.000	0.000	0.000	0.000	0.000	0.270	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	137	94	0	0	0	0	0	23	0
N.S.	1	1.36	0.93	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.569	0.782	0.000	0.000	0.000	0.000	0.000	0.229	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	144	92	0	0	0	0	0	27	0
N.S.	1	1.29	0.82	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.717	1.403	0.000	0.000	0.000	0.000	0.000	0.268	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	70	70	0	0	0	0	0	23	0
N.S.	1	0.95	0.95	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.363	0.009	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	192	147	179	0	0	0	0	0	29	0
N.S.	1	0.77	0.93	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.553	0.927	0.000	0.000	0.000	0.000	0.000	0.233	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	384	61	0	0	0	0	0	31	0
N.S.	1	5.33	0.85	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.998	0.616	0.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	301	291	237	0	0	0	0	0	31	0
N.S.	1	0.97	0.79	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.745	3.156	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	195	150	0	0	0	0	0	23	0
N.S.	1	1.26	0.97	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.432	1.290	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	164	208	148	0	0	0	0	0	29	0
N.S.	1	1.27	0.90	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.554	0.878	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	149	84	0	0	0	0	0	29	0
N.S.	1	1.24	0.70	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.486	0.531	0.000	0.000	0.000	0.000	0.000	0.213	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	139	96	0	0	0	0	0	23	0
N.S.	1	1.88	1.30	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.439	0.144	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	249	191	153	0	0	0	0	0	29	0
N.S.	1	0.77	0.61	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.025	0.895	0.000	0.000	0.000	0.000	0.000	0.234	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	312	293	271	0	0	0	0	0	31	0
N.S.	1	0.94	0.87	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.978	4.093	0.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	594	144	0	0	0	0	0	31	0
N.S.	1	8.25	2.00	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	1.617	2.454	0.000	0.000	0.000	0.000	0.000	0.214	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	101	68	129	0	322	570	0	54	169
N.S.	1	0.89	0.60	1.14	0.00	2.85	5.04	0.00	0.48	1.50
time (sec)	N/A	0.288	0.408	1.167	0.000	0.095	12.620	0.000	0.233	2.976

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	B	F(-1)	F	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	0	44	41	28	103	0	0	33	71
N.S.	1	0.00	1.00	0.93	0.64	2.34	0.00	0.00	0.75	1.61
time (sec)	N/A	0.000	1.306	1.480	0.155	0.081	0.000	0.000	0.216	2.683

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	91	178	0	0	0	0	0	25	0
N.S.	1	0.99	1.93	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.269	0.695	0.000	0.000	0.000	0.000	0.000	0.211	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	101	75	126	0	321	644	0	54	171
N.S.	1	0.89	0.66	1.12	0.00	2.84	5.70	0.00	0.48	1.51
time (sec)	N/A	0.279	0.400	1.239	0.000	0.103	13.413	0.000	0.226	2.804

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	B	F(-1)	F	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	0	44	41	28	103	0	0	33	71
N.S.	1	0.00	1.00	0.93	0.64	2.34	0.00	0.00	0.75	1.61
time (sec)	N/A	0.000	1.354	1.481	0.142	0.080	0.000	0.000	0.246	2.594

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	93	95	0	0	0	0	0	25	0
N.S.	1	0.99	1.01	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.277	0.067	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	105	87	0	0	0	0	0	25	0
N.S.	1	0.81	0.67	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.343	0.762	0.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	F(-1)	F	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	0	44	41	28	72	0	0	33	77
N.S.	1	0.00	1.00	0.93	0.64	1.64	0.00	0.00	0.75	1.75
time (sec)	N/A	0.000	2.842	1.075	0.073	0.082	0.000	0.000	0.318	2.742

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	103	85	0	0	0	0	0	25	0
N.S.	1	0.80	0.66	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.347	0.737	0.000	0.000	0.000	0.000	0.000	0.205	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	0	105	0	0	0	0	0	124	0
N.S.	1	0.00	0.87	0.00	0.00	0.00	0.00	0.00	1.02	0.00
time (sec)	N/A	0.000	2.781	0.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	249	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	244	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.232	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.226	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	209	0	202	0	0	0	0	0	33	0
N.S.	1	0.00	0.97	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.000	2.121	0.000	0.000	0.000	0.000	0.000	0.238	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	0	117	0	0	0	0	0	33	0
N.S.	1	0.00	1.09	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.000	1.530	0.000	0.000	0.000	0.000	0.000	0.246	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	0	113	0	0	0	0	0	33	0
N.S.	1	0.00	1.06	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.000	3.417	0.000	0.000	0.000	0.000	0.000	0.244	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	0	0	0	0	0	0	0	110	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.262	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	0	113	0	0	0	0	0	149	0
N.S.	1	0.00	1.06	0.00	0.00	0.00	0.00	0.00	1.39	0.00
time (sec)	N/A	0.000	1.572	0.000	0.000	0.000	0.000	0.000	0.286	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	207	0	195	0	0	0	0	0	149	0
N.S.	1	0.00	0.94	0.00	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.000	2.107	0.000	0.000	0.000	0.000	0.000	0.294	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	0	0	0	0	0	0	0	110	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.75	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.266	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	0	110	0	0	0	0	0	33	0
N.S.	1	0.00	1.03	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.000	3.311	0.000	0.000	0.000	0.000	0.000	0.247	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	125	122	0	0	0	0	0	113	0
N.S.	1	0.98	0.96	0.00	0.00	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	0.969	0.070	0.000	0.000	0.000	0.000	0.000	0.224	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F(-1)	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.229	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	140	114	0	0	0	0	0	30	0
N.S.	1	1.32	1.08	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.671	0.101	0.000	0.000	0.000	0.000	0.000	0.221	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	125	122	0	0	0	0	0	113	0
N.S.	1	0.98	0.96	0.00	0.00	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	0.623	0.053	0.000	0.000	0.000	0.000	0.000	0.232	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F(-1)	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	135	110	0	0	0	0	0	30	0
N.S.	1	1.27	1.04	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.695	0.077	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.247	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.226	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.244	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	170	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.239	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.279	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.268	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	176	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.273	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	165	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.257	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.234	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.266	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [89] had the largest ratio of [.9285710000000000035]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	0.67	22	0.182
2	A	6	5	0.58	22	0.227
3	A	5	4	0.78	20	0.200
4	A	5	4	0.93	14	0.286
5	A	6	5	0.75	14	0.357
6	A	6	5	0.60	20	0.250
7	A	7	6	0.86	22	0.273
8	A	6	5	0.55	24	0.208
9	A	5	4	0.75	24	0.167
10	A	6	5	0.55	22	0.227
11	A	5	4	0.71	16	0.250
12	A	6	5	0.71	20	0.250
13	A	5	4	0.67	16	0.250
14	A	6	5	0.60	22	0.227
15	A	5	4	0.62	24	0.167
16	A	6	5	0.57	24	0.208
17	A	5	4	0.67	22	0.182
18	A	6	5	0.58	16	0.312
19	A	5	4	0.65	22	0.182
20	A	6	5	0.61	22	0.227
21	A	6	5	0.70	16	0.312

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	5	4	0.64	22	0.182
23	A	6	5	0.65	20	0.250
24	A	6	5	0.75	14	0.357
25	A	4	3	0.67	14	0.214
26	A	6	5	0.71	20	0.250
27	A	6	5	0.69	22	0.227
28	A	9	8	0.80	22	0.364
29	A	6	5	0.61	22	0.227
30	A	5	4	0.66	16	0.250
31	A	6	5	0.63	20	0.250
32	A	5	4	0.81	16	0.250
33	A	6	5	0.68	22	0.227
34	A	7	6	0.80	24	0.250
35	A	6	5	0.62	24	0.208
36	A	6	5	0.69	16	0.312
37	A	6	5	0.53	22	0.227
38	A	7	6	0.86	22	0.273
39	A	4	3	0.77	16	0.188
40	A	9	8	0.79	22	0.364
41	A	6	5	0.62	24	0.208
42	A	8	7	0.79	24	0.292
43	A	6	5	0.61	24	0.208
44	A	5	4	0.62	24	0.167
45	A	6	5	0.85	22	0.227
46	A	4	3	0.74	16	0.188
47	A	7	6	0.74	16	0.375
48	A	7	6	0.58	22	0.273
49	A	6	5	0.53	24	0.208
50	A	5	4	0.56	26	0.154
51	A	5	4	0.68	26	0.154
52	A	5	4	0.59	24	0.167
53	A	6	5	0.59	18	0.278

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	5	4	0.63	22	0.182
55	A	6	5	0.60	18	0.278
56	A	8	7	0.67	24	0.292
57	A	6	5	0.63	26	0.192
58	A	5	4	0.58	26	0.154
59	A	6	5	0.63	24	0.208
60	A	5	4	0.56	18	0.222
61	A	6	5	0.54	24	0.208
62	A	5	4	0.61	24	0.167
63	A	7	6	0.58	18	0.333
64	A	6	5	0.53	24	0.208
65	A	5	4	0.63	22	0.182
66	A	7	6	0.71	16	0.375
67	A	5	4	0.60	16	0.250
68	A	4	3	0.71	22	0.136
69	A	7	6	0.65	24	0.250
70	A	6	5	0.63	24	0.208
71	A	5	4	0.60	24	0.167
72	A	6	5	0.58	18	0.278
73	A	7	6	0.61	22	0.273
74	A	6	5	0.54	18	0.278
75	A	7	6	0.64	24	0.250
76	A	6	5	0.77	26	0.192
77	A	11	10	0.84	26	0.385
78	A	7	6	0.55	18	0.333
79	A	9	8	0.66	24	0.333
80	A	6	5	0.52	24	0.208
81	A	6	5	0.77	18	0.278
82	A	6	5	0.61	24	0.208
83	A	11	10	0.73	26	0.385
84	A	4	3	0.77	26	0.115
85	A	15	14	0.91	26	0.538

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	6	5	0.56	26	0.192
87	A	12	11	1.44	12	0.917
88	A	13	12	1.47	14	0.857
89	A	14	13	1.43	14	0.929
90	A	15	14	1.42	16	0.875
91	A	7	6	1.29	12	0.500
92	A	8	7	1.31	14	0.500
93	A	9	8	1.25	14	0.571
94	A	10	9	1.30	16	0.562
95	A	2	2	1.29	24	0.083
96	A	2	2	1.28	24	0.083
97	A	3	3	1.29	22	0.136
98	A	1	1	1.19	16	0.062
99	A	2	2	0.99	16	0.125
100	A	2	2	1.21	22	0.091
101	A	2	2	1.21	24	0.083
102	A	2	2	1.28	26	0.077
103	A	2	2	1.22	26	0.077
104	A	2	2	1.28	24	0.083
105	A	2	2	1.42	18	0.111
106	A	2	2	1.04	22	0.091
107	A	2	2	1.32	18	0.111
108	A	2	2	1.22	24	0.083
109	A	2	2	1.29	26	0.077
110	A	2	2	1.28	26	0.077
111	A	2	2	1.29	24	0.083
112	A	2	2	1.44	18	0.111
113	A	2	2	1.06	24	0.083
114	A	2	2	1.16	24	0.083
115	A	2	2	1.20	18	0.111
116	A	2	2	1.06	24	0.083
117	A	2	2	1.04	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	2	2	0.99	16	0.125
119	A	1	1	0.99	16	0.062
120	B	2	2	2.58	22	0.091
121	A	2	2	0.75	24	0.083
122	A	2	2	0.74	24	0.083
123	A	2	2	1.18	24	0.083
124	A	2	2	1.36	18	0.111
125	A	2	2	1.29	22	0.091
126	A	1	1	0.95	18	0.056
127	A	2	2	0.77	24	0.083
128	B	2	2	5.33	26	0.077
129	A	2	2	0.97	26	0.077
130	A	2	2	1.26	18	0.111
131	A	2	2	1.27	24	0.083
132	A	2	2	1.24	24	0.083
133	A	2	2	1.88	18	0.111
134	A	2	2	0.77	24	0.083
135	A	2	2	0.94	26	0.077
136	B	2	2	8.25	26	0.077
137	A	2	2	0.89	34	0.059
138	F	0	0	N/A	0.000	N/A
139	A	2	2	0.99	34	0.059
140	A	2	2	0.89	34	0.059
141	F	0	0	N/A	0.000	N/A
142	A	2	2	0.99	34	0.059
143	A	3	3	0.81	34	0.088
144	F	0	0	N/A	0.000	N/A
145	A	3	3	0.80	34	0.088
146	F	0	0	N/A	0.000	N/A
147	F	0	0	N/A	0.000	N/A
148	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
149	F	0	0	N/A	0.000	N/A
150	F	0	0	N/A	0.000	N/A
151	F	0	0	N/A	0.000	N/A
152	F	0	0	N/A	0.000	N/A
153	F	0	0	N/A	0.000	N/A
154	F	0	0	N/A	0.000	N/A
155	F	0	0	N/A	0.000	N/A
156	F	0	0	N/A	0.000	N/A
157	F	0	0	N/A	0.000	N/A
158	A	4	4	0.98	30	0.133
159	F	0	0	N/A	0.000	N/A
160	A	4	4	1.32	30	0.133
161	A	4	4	0.98	30	0.133
162	F	0	0	N/A	0.000	N/A
163	A	4	4	1.27	30	0.133
164	F	0	0	N/A	0.000	N/A
165	F	0	0	N/A	0.000	N/A
166	F	0	0	N/A	0.000	N/A
167	F	0	0	N/A	0.000	N/A
168	F	0	0	N/A	0.000	N/A
169	F	0	0	N/A	0.000	N/A
170	F	0	0	N/A	0.000	N/A
171	F	0	0	N/A	0.000	N/A
172	F	0	0	N/A	0.000	N/A
173	F	0	0	N/A	0.000	N/A
174	F	0	0	N/A	0.000	N/A
175	F	0	0	N/A	0.000	N/A
176	F	0	0	N/A	0.000	N/A
177	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
178	F	0	0	N/A	0.000	N/A

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int e^{a+bx}\cosh(d+bx)\sinh^3(d+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int e^{a+bx}\cosh(d+bx)\sinh^3(d+bx)\,dx=\frac{e^{a-4d-3bx}}{48b}-\frac{e^{a-2d-bx}}{8b}-\frac{e^{a+2d+3bx}}{24b}+\frac{e^{a+4d+5bx}}{80b}$$

output
```
1/48*exp(-3*b*x+a-4*d)/b-1/8*exp(-b*x+a-2*d)/b-1/24*exp(3*b*x+a+2*d)/b+1/8
0*exp(5*b*x+a+4*d)/b
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.22

$$\int e^{a+bx}\cosh(d+bx)\sinh^3(d+bx)\,dx$$
$$=-\frac{e^{a-bx}\left(\left(3+e^{4bx}\right)\cosh(2d)+\left(-3+e^{4bx}\right)\sinh(2d)\right)}{24b}+\frac{e^{a-3bx}\left(\left(5+3e^{8bx}\right)\cosh(4d)+\left(-5+3e^{8bx}\right)\sinh(4d)\right)}{240b}$$

input
```
Integrate[E^(a + b*x)*Cosh[d + b*x]*Sinh[d + b*x]^3,x]
```

output
```
-1/24*(E^(a - b*x)*((3 + E^(4*b*x))*Cosh[2*d] + (-3 + E^(4*b*x))*Sinh[2*d]
))/b + (E^(a - 3*b*x)*((5 + 3*E^(8*b*x))*Cosh[4*d] + (-5 + 3*E^(8*b*x))*Si
nh[4*d]))/(240*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2720, 27, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh^3(bx+d)\cosh(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{1}{16}e^{a-4bx}\left(1-e^{2bx}\right)^3\left(1+e^{2bx}\right)de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^a\int e^{-4bx}\left(1-e^{2bx}\right)^3\left(1+e^{2bx}\right)de^{bx}}{16b}$$

$$\Big\downarrow \text{355}$$

$$-\frac{e^a\int \left(e^{-4bx}-2e^{-2bx}+2e^{2bx}-e^{4bx}\right)de^{bx}}{16b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^a\left(-\frac{1}{3}e^{-3bx}+2e^{-bx}+\frac{2}{3}e^{3bx}-\frac{1}{5}e^{5bx}\right)}{16b}$$

input
```
Int[E^(a + b*x)*Cosh[d + b*x]*Sinh[d + b*x]^3,x]
```

output
```
-1/16*(E^a*(-1/3*1/E^(3*b*x) + 2/E^(b*x) + (2*E^(3*b*x))/3 - E^(5*b*x)/5))
/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 279.98 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

method	result
risch	$\frac{e^{-3bx+a-4d}}{48b}-\frac{e^{-bx+a-2d}}{8b}-\frac{e^{3bx+a+2d}}{24b}+\frac{e^{5bx+a+4d}}{80b}$
default	$\frac{\sinh(-3bx+a-4d)}{48b}-\frac{\sinh(-bx+a-2d)}{8b}-\frac{\sinh(3bx+a+2d)}{24b}+\frac{\sinh(5bx+a+4d)}{80b}+\frac{\cosh(-3bx+a-4d)}{48b}-\frac{\cosh(-bx+a-2d)}{8b}$
orering	$-\frac{4\,e^{bx+a}\cosh(bx+d)\sinh(bx+d)^3}{5b}+\frac{\frac{14b\,e^{bx+a}\cosh(bx+d)\sinh(bx+d)^3}{45}+\frac{14\,e^{bx+a}b\sinh(bx+d)^4}{45}+\frac{14\,e^{bx+a}\cosh(bx+d)^2\sinh(bx+d)^2b}{15}}{b^2}$

input
```
int(exp(b*x+a)*cosh(b*x+d)*sinh(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/48*exp(-3*b*x+a-4*d)/b-1/8*exp(-b*x+a-2*d)/b-1/24*exp(3*b*x+a+2*d)/b+1/8
0*exp(5*b*x+a+4*d)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(61) = 122$.

Time = 0.09 (sec) , antiderivative size = 260, normalized size of antiderivative = 3.56

$$\int e^{a+bx}\cosh(d+bx)\sinh^3(d+bx)\,dx$$
$$= \frac{\cosh(bx+d)^4\cosh(-a+d) + (\cosh(-a+d) - \sinh(-a+d))\sinh(bx+d)^4 - (\cosh(bx+d)\cosh(-a}{}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)*sinh(b*x+d)^3,x, algorithm="fricas")
```

output
```
1/30*(cosh(b*x + d)^4*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*
x + d)^4 - (cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(
b*x + d)^3 - 5*cosh(b*x + d)^2*cosh(-a + d) + (6*cosh(b*x + d)^2*cosh(-a +
 d) - (6*cosh(b*x + d)^2 - 5)*sinh(-a + d) - 5*cosh(-a + d))*sinh(b*x + d)
^2 - (cosh(b*x + d)^3*cosh(-a + d) - 5*cosh(b*x + d)*cosh(-a + d) - (cosh(
b*x + d)^3 - 5*cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)
^4 - 5*cosh(b*x + d)^2)*sinh(-a + d))/(b*cosh(b*x + d) - b*sinh(b*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(58) = 116$.

Time = 2.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.90

$$\int e^{a+bx}\cosh(d+bx)\sinh^3(d+bx)\,dx$$
$$= \begin{cases} \frac{e^a e^{bx}\sinh^4(bx+d)}{5b} - \frac{e^a e^{bx}\sinh^3(bx+d)\cosh(bx+d)}{5b} + \frac{e^a e^{bx}\sinh^2(bx+d)\cosh^2(bx+d)}{5b} + \frac{2e^a e^{bx}\sinh(bx+d)\cosh^3(bx+d)}{15b} - \frac{2e^a e^{b}}{} \\ xe^a\sinh^3(d)\cosh(d) \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)*sinh(b*x+d)**3,x)
```

output
```
Piecewise((exp(a)*exp(b*x)*sinh(b*x + d)**4/(5*b) - exp(a)*exp(b*x)*sinh(b
*x + d)**3*cosh(b*x + d)/(5*b) + exp(a)*exp(b*x)*sinh(b*x + d)**2*cosh(b*x
 + d)**2/(5*b) + 2*exp(a)*exp(b*x)*sinh(b*x + d)*cosh(b*x + d)**3/(15*b) -
 2*exp(a)*exp(b*x)*cosh(b*x + d)**4/(15*b), Ne(b, 0)), (x*exp(a)*sinh(d)**
3*cosh(d), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05

$$\int e^{a+bx}\cosh(d+bx)\sinh^3(d+bx)\,dx = -\frac{\left(6\,e^{(2\,bx+4\,a+2\,d)}-e^{(4\,a)}\right)e^{(-3\,bx-3\,a-4\,d)}}{48\,b} + \frac{\left(3\,e^{(5\,bx+5\,a+4\,d)}-10\,e^{(3\,bx+5\,a+2\,d)}\right)e^{(-4\,a)}}{240\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)*sinh(b*x+d)^3,x, algorithm="maxima")
```

output
```
-1/48*(6*e^(2*b*x + 4*a + 2*d) - e^(4*a))*e^(-3*b*x - 3*a - 4*d)/b + 1/240
*(3*e^(5*b*x + 5*a + 4*d) - 10*e^(3*b*x + 5*a + 2*d))*e^(-4*a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int e^{a+bx}\cosh(d+bx)\sinh^3(d+bx)\,dx = -\frac{\left(5\left(6\,e^{(2\,bx+a+2\,d)}-e^{a}\right)e^{(-3\,bx)}-3\,e^{(5\,bx+a+8\,d)}+10\,e^{(3\,bx+a+6\,d)}\right)e^{(-4\,d)}}{240\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)*sinh(b*x+d)^3,x, algorithm="giac")
```

output
```
-1/240*(5*(6*e^(2*b*x + a + 2*d) - e^a)*e^(-3*b*x) - 3*e^(5*b*x + a + 8*d)
 + 10*e^(3*b*x + a + 6*d))*e^(-4*d)/b
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int e^{a+bx}\cosh(d+bx)\sinh^3(d+bx)\,dx=\frac{e^{-4\,d}\,e^{-3\,b\,x}\,e^{a}}{48\,b}-\frac{e^{2\,d}\,e^{3\,b\,x}\,e^{a}}{24\,b}-\frac{e^{-2\,d}\,e^{-b\,x}\,e^{a}}{8\,b}+\frac{e^{4\,d}\,e^{5\,b\,x}\,e^{a}}{80\,b}$$

input
```
int(cosh(d + b*x)*exp(a + b*x)*sinh(d + b*x)^3,x)
```

output
```
(exp(-4*d)*exp(-3*b*x)*exp(a))/(48*b) - (exp(2*d)*exp(3*b*x)*exp(a))/(24*b
) - (exp(-2*d)*exp(-b*x)*exp(a))/(8*b) + (exp(4*d)*exp(5*b*x)*exp(a))/(80*
b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int e^{a+bx}\cosh(d+bx)\sinh^3(d+bx)\,dx=\frac{e^{a}\left(3e^{8bx+8d}-10e^{6bx+6d}-30e^{2bx+2d}+5\right)}{240e^{3bx+4d}b}$$

input
```
int(exp(b*x+a)*cosh(b*x+d)*sinh(b*x+d)^3,x)
```

output
```
(e**a*(3*e**(8*b*x + 8*d) - 10*e**(6*b*x + 6*d) - 30*e**(2*b*x + 2*d) + 5)
)/(240*e**(3*b*x + 4*d)*b)
```

3.2 $\int e^{a+bx}\cosh(d+bx)\sinh^2(d+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 65

$$\int e^{a+bx}\cosh(d+bx)\sinh^2(d+bx)\,dx = -\frac{e^{a-3d-2bx}}{16b}-\frac{e^{a+d+2bx}}{16b}+\frac{e^{a+3d+4bx}}{32b}-\frac{1}{8}e^{a-d}x$$

output
```
-1/16*exp(-2*b*x+a-3*d)/b-1/16*exp(2*b*x+a+d)/b+1/32*exp(4*b*x+a+3*d)/b-1/
8*exp(a-d)*x
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.17

$$\int e^{a+bx}\cosh(d+bx)\sinh^2(d+bx)\,dx = \frac{e^a\left(-2\left(\left(e^{2bx}+2bx\right)\cosh(d)+\left(e^{2bx}-2bx\right)\sinh(d)\right)+e^{-2bx}\left(\left(-2+e^{6bx}\right)\cosh(3d)+\left(2+e^{6bx}\right)\sinh(3d)\right)\right)}{32b}$$

input
```
Integrate[E^(a + b*x)*Cosh[d + b*x]*Sinh[d + b*x]^2,x]
```

output
```
(E^a*(-2*((E^(2*b*x) + 2*b*x)*Cosh[d] + (E^(2*b*x) - 2*b*x)*Sinh[d]) + ((-
2 + E^(6*b*x))*Cosh[3*d] + (2 + E^(6*b*x))*Sinh[3*d])/E^(2*b*x)))/(32*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2720, 27, 354, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh^2(bx+d)\cosh(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{8}e^{a-3bx}\left(1-e^{2bx}\right)^2\left(1+e^{2bx}\right)de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^a\int e^{-3bx}\left(1-e^{2bx}\right)^2\left(1+e^{2bx}\right)de^{bx}}{8b}$$

$$\Big\downarrow \text{354}$$

$$\frac{e^a\int e^{-2bx}\left(1-e^{2bx}\right)^2\left(1+e^{2bx}\right)de^{2bx}}{16b}$$

$$\Big\downarrow \text{84}$$

$$\frac{e^a\int \left(-1+e^{-2bx}-e^{-bx}+e^{2bx}\right)de^{2bx}}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^a\left(-e^{-bx}-\frac{1}{2}e^{2bx}-\log\left(e^{2bx}\right)\right)}{16b}$$

input
```
Int[E^(a + b*x)*Cosh[d + b*x]*Sinh[d + b*x]^2,x]
```

output
```
(E^a*(-E^(-(b*x)) - E^(2*b*x)/2 - Log[E^(2*b*x)]))/(16*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 22.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{{\mathrm e}^{-2 b x +a -3 d}}{16 b}-\frac{{\mathrm e}^{2 b x +a +d}}{16 b}+\frac{{\mathrm e}^{4 b x +a +3 d}}{32 b}-\frac{{\mathrm e}^{a -d} x}{8}$
default	$-\frac{\cosh \left(a -d \right) x}{8}-\frac{\sinh \left(-2 b x +a -3 d \right)}{16 b}-\frac{\sinh \left(2 b x +a +d \right)}{16 b}+\frac{\sinh \left(4 b x +a +3 d \right)}{32 b}-\frac{\sinh \left(a -d \right) x}{8}-\frac{\cosh \left(-2 b x +a -3 d \right)}{16 b}-\cosh($
orering	$\frac{\left(4 b x +1\right) {\mathrm e}^{b x +a} \cosh \left(b x +d \right) \sinh \left(b x +d \right)^{2}}{4 b}-\frac{\left(b x -1\right) \left(b \,{\mathrm e}^{b x +a} \cosh \left(b x +d \right) \sinh \left(b x +d \right)^{2}+{\mathrm e}^{b x +a} b \sinh \left(b x +d \right)^{3}+2 \,{\mathrm e}^{b x +a} \cosh \left(b x +d \right)^{2}\right.}{4 b^{2}}$

input
```
int(exp(b*x+a)*cosh(b*x+d)*sinh(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/16*exp(-2*b*x+a-3*d)/b-1/16*exp(2*b*x+a+d)/b+1/32*exp(4*b*x+a+3*d)/b-1/
8*exp(a-d)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(53) = 106$.

Time = 0.09 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.31

$$\int e^{a+bx}\cosh(d+bx)\sinh^2(d+bx)\,dx = -\frac{\cosh(bx+d)^3\cosh(-a+d) - 3(\cosh(-a+d) - \sinh(-a+d))\sinh(bx+d)^3 + 2(2bx+1)\cosh(b}{}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)*sinh(b*x+d)^2,x, algorithm="fricas")
```

output
```
-1/32*(cosh(b*x + d)^3*cosh(-a + d) - 3*(cosh(-a + d) - sinh(-a + d))*sinh
(b*x + d)^3 + 2*(2*b*x + 1)*cosh(b*x + d)*cosh(-a + d) + 3*(cosh(b*x + d)*
cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d)^2 - (9*cosh(b*x +
 d)^2*cosh(-a + d) + 2*(2*b*x - 1)*cosh(-a + d) - (4*b*x + 9*cosh(b*x + d)
^2 - 2)*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^3 + 2*(2*b*x + 1)*cos
h(b*x + d))*sinh(-a + d))/(b*cosh(b*x + d) - b*sinh(b*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(51) = 102$.

Time = 0.91 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.72

$$\int e^{a+bx}\cosh(d+bx)\sinh^2(d+bx)\,dx = \begin{cases} -\frac{xe^ae^{bx}\sinh^3(bx+d)}{8} + \frac{xe^ae^{bx}\sinh^2(bx+d)\cosh(bx+d)}{8} + \frac{xe^ae^{bx}\sinh(bx+d)\cosh^2(bx+d)}{8} - \frac{xe^ae^{bx}\cosh^3(bx+d)}{8} + \frac{3e^ae^{bx}\sinh^3}{8b} \\ xe^a\sinh^2(d)\cosh(d) \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)*sinh(b*x+d)**2,x)
```

output
```
Piecewise((-x*exp(a)*exp(b*x)*sinh(b*x + d)**3/8 + x*exp(a)*exp(b*x)*sinh(
b*x + d)**2*cosh(b*x + d)/8 + x*exp(a)*exp(b*x)*sinh(b*x + d)*cosh(b*x + d
)**2/8 - x*exp(a)*exp(b*x)*cosh(b*x + d)**3/8 + 3*exp(a)*exp(b*x)*sinh(b*x
 + d)**3/(8*b) - exp(a)*exp(b*x)*sinh(b*x + d)**2*cosh(b*x + d)/(4*b) + ex
p(a)*exp(b*x)*cosh(b*x + d)**3/(8*b), Ne(b, 0)), (x*exp(a)*sinh(d)**2*cosh
(d), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int e^{a+bx}\cosh(d+bx)\sinh^2(d+bx)\,dx = \frac{\left(e^{(4\,bx+4\,a+3\,d)}-2\,e^{(2\,bx+4\,a+d)}\right)e^{(-3\,a)}}{32\,b} - \frac{(bx+a)e^{(a-d)}}{8\,b} - \frac{e^{(-2\,bx+a-3\,d)}}{16\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)*sinh(b*x+d)^2,x, algorithm="maxima")
```

output
```
1/32*(e^(4*b*x + 4*a + 3*d) - 2*e^(2*b*x + 4*a + d))*e^(-3*a)/b - 1/8*(b*x
 + a)*e^(a - d)/b - 1/16*e^(-2*b*x + a - 3*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int e^{a+bx}\cosh(d+bx)\sinh^2(d+bx)\,dx = -\frac{\left(4\,bxe^{(a+2\,d)}-2\left(e^{(2\,bx+a+2\,d)}-e^{a}\right)e^{(-2\,bx)}-e^{(4\,bx+a+6\,d)}+2\,e^{(2\,bx+a+4\,d)}\right)e^{(-3\,d)}}{32\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)*sinh(b*x+d)^2,x, algorithm="giac")
```

output
```
-1/32*(4*b*x*e^(a + 2*d) - 2*(e^(2*b*x + a + 2*d) - e^a)*e^(-2*b*x) - e^(4
*b*x + a + 6*d) + 2*e^(2*b*x + a + 4*d))*e^(-3*d)/b
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int e^{a+bx}\cosh(d+bx)\sinh^2(d+bx)\,dx = -\frac{e^{a+b\,x}\left(\cosh(d+b\,x)-3\cosh(d+b\,x)^2\sinh(d+b\,x)+\cosh(d+b\,x)^3+b\,x\cosh(d+b\,x)-b\,x\sinh(d\right.}{8\,b}$$

input
```
int(cosh(d + b*x)*exp(a + b*x)*sinh(d + b*x)^2,x)
```

output
```
-(exp(a + b*x)*(cosh(d + b*x) - 3*cosh(d + b*x)^2*sinh(d + b*x) + cosh(d +
 b*x)^3 + b*x*cosh(d + b*x) - b*x*sinh(d + b*x)))/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int e^{a+bx}\cosh(d+bx)\sinh^2(d+bx)\,dx = \frac{e^a\left(e^{6bx+6d}-2e^{4bx+4d}-4e^{2bx+2d}bx-2\right)}{32e^{2bx+3d}b}$$

input
```
int(exp(b*x+a)*cosh(b*x+d)*sinh(b*x+d)^2,x)
```

output
```
(e**a*(e**(6*b*x + 6*d) - 2*e**(4*b*x + 4*d) - 4*e**(2*b*x + 2*d)*b*x - 2)
)/(32*e**(2*b*x + 3*d)*b)
```

3.3 $\int e^{a+bx} \cosh(d+bx) \sinh(d+bx) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 37

$$\int e^{a+bx} \cosh(d+bx) \sinh(d+bx) \, dx = \frac{e^{a-2d-bx}}{4b} + \frac{e^{a+2d+3bx}}{12b}$$

output
```
1/4*exp(-b*x+a-2*d)/b+1/12*exp(3*b*x+a+2*d)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int e^{a+bx} \cosh(d+bx) \sinh(d+bx) \, dx = \frac{e^{a-bx} \left((3+e^{4bx}) \cosh(2d) + (-3+e^{4bx}) \sinh(2d) \right)}{12b}$$

input
```
Integrate[E^(a + b*x)*Cosh[d + b*x]*Sinh[d + b*x],x]
```

output
```
(E^(a - b*x)*((3 + E^(4*b*x))*Cosh[2*d] + (-3 + E^(4*b*x))*Sinh[2*d]))/(12
*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh(bx+d)\cosh(bx+d)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int -\frac{1}{4}e^{a-2bx}\left(1-e^{4bx}\right)de^{bx}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{e^a\int e^{-2bx}\left(1-e^{4bx}\right)de^{bx}}{4b}$$

$$\Big\downarrow \text{ 802}$$

$$-\frac{e^a\int \left(e^{-2bx}-e^{2bx}\right)de^{bx}}{4b}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{e^a\left(-e^{-bx}-\frac{1}{3}e^{3bx}\right)}{4b}$$

input
```
Int[E^(a + b*x)*Cosh[d + b*x]*Sinh[d + b*x],x]
```

output
```
-1/4*(E^a*(-E^(-(b*x)) - E^(3*b*x)/3))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{{\mathrm e}^{-b x +a -2 d}}{4 b}+\frac{{\mathrm e}^{3 b x +a +2 d}}{12 b}$	32
default	$\frac{\sinh \left(-b x +a -2 d \right)}{4 b}+\frac{\sinh \left(3 b x +a +2 d \right)}{12 b}+\frac{\cosh \left(-b x +a -2 d \right)}{4 b}+\frac{\cosh \left(3 b x +a +2 d \right)}{12 b}$	62
orering	$-\frac{2 \,{\mathrm e}^{b x +a} \cosh \left(b x +d \right) \sinh \left(b x +d \right)}{3 b}+\frac{{\mathrm e}^{b x +a} \sinh \left(b x +d \right) b \cosh \left(b x +d \right)+{\mathrm e}^{b x +a} b \sinh \left(b x +d \right)^{2}+{\mathrm e}^{b x +a} \cosh \left(b x +d \right)^{2} b}{3 b^{2}}$	83

input
```
int(exp(b*x+a)*cosh(b*x+d)*sinh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(-b*x+a-2*d)/b+1/12*exp(3*b*x+a+2*d)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(31) = 62$.

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.08

$$\int e^{a+bx}\cosh(d+bx)\sinh(d+bx)\,dx = \frac{\cosh(bx+d)^2\cosh(-a+d) + (\cosh(-a+d) - \sinh(-a+d))\sinh(bx+d)^2 - \cosh(bx+d)^2\sinh(-a \ldots}{3\,(b\cosh(bx+d) - b\sinh(b\ldots}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)*sinh(b*x+d),x, algorithm="fricas")
```

output
```
1/3*(cosh(b*x + d)^2*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x
 + d)^2 - cosh(b*x + d)^2*sinh(-a + d) - (cosh(b*x + d)*cosh(-a + d) - cos
h(b*x + d)*sinh(-a + d))*sinh(b*x + d))/(b*cosh(b*x + d) - b*sinh(b*x + d)
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(27) = 54$.

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.05

$$\int e^{a+bx}\cosh(d+bx)\sinh(d+bx)\,dx = \begin{cases} \frac{e^a e^{bx}\sinh^2(bx+d)}{3b} - \frac{e^a e^{bx}\sinh(bx+d)\cosh(bx+d)}{3b} + \frac{e^a e^{bx}\cosh^2(bx+d)}{3b} & \text{for } b \neq 0 \\ xe^a\sinh(d)\cosh(d) & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)*sinh(b*x+d),x)
```

output
```
Piecewise((exp(a)*exp(b*x)*sinh(b*x + d)**2/(3*b) - exp(a)*exp(b*x)*sinh(b
*x + d)*cosh(b*x + d)/(3*b) + exp(a)*exp(b*x)*cosh(b*x + d)**2/(3*b), Ne(b
, 0)), (x*exp(a)*sinh(d)*cosh(d), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int e^{a+bx}\cosh(d+bx)\sinh(d+bx)\,dx = \frac{e^{(3\,bx+a+2\,d)}}{12\,b} + \frac{e^{(-bx+a-2\,d)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)*sinh(b*x+d),x, algorithm="maxima")
```

output
```
1/12*e^(3*b*x + a + 2*d)/b + 1/4*e^(-b*x + a - 2*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int e^{a+bx}\cosh(d+bx)\sinh(d+bx)\,dx = \frac{\left(e^{(3\,bx+a+4\,d)} + 3\,e^{(-bx+a)}\right)e^{(-2\,d)}}{12\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)*sinh(b*x+d),x, algorithm="giac")
```

output
```
1/12*(e^(3*b*x + a + 4*d) + 3*e^(-b*x + a))*e^(-2*d)/b
```

Mupad [B] (verification not implemented)

Time = 2.98 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int e^{a+bx}\cosh(d+bx)\sinh(d+bx)\,dx = \frac{e^{-2\,d}\,e^{-b\,x}\,e^{a}\,\left(e^{4\,d}\,e^{4\,b\,x}+3\right)}{12\,b}$$

input
```
int(cosh(d + b*x)*exp(a + b*x)*sinh(d + b*x),x)
```

output
```
(exp(-2*d)*exp(-b*x)*exp(a)*(exp(4*d)*exp(4*b*x) + 3))/(12*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

$$\int e^{a+bx}\cosh(d+bx)\sinh(d+bx)\,dx = \frac{e^{bx+a}\left(\cosh\left(bx+d\right)^2-\cosh\left(bx+d\right)\sinh\left(bx+d\right)+\sinh\left(bx+d\right)^2\right)}{3b}$$

input
```
int(exp(b*x+a)*cosh(b*x+d)*sinh(b*x+d),x)
```

output
```
(e**(a + b*x)*(cosh(b*x + d)**2 - cosh(b*x + d)*sinh(b*x + d) + sinh(b*x +
 d)**2))/(3*b)
```

3.4 $\int e^{a+bx}\cosh(d+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 29

$$\int e^{a+bx}\cosh(d+bx)\,dx=\frac{e^{a+d+2bx}}{4b}+\frac{1}{2}e^{a-d}x$$

output
```
1/4*exp(2*b*x+a+d)/b+1/2*exp(a-d)*x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int e^{a+bx}\cosh(d+bx)\,dx=\frac{e^a\left(\left(e^{2bx}+2bx\right)\cosh(d)+\left(e^{2bx}-2bx\right)\sinh(d)\right)}{4b}$$

input
```
Integrate[E^(a + b*x)*Cosh[d + b*x],x]
```

output
```
(E^a*((E^(2*b*x) + 2*b*x)*Cosh[d] + (E^(2*b*x) - 2*b*x)*Sinh[d]))/(4*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\cosh(bx+d)\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int \frac{1}{2}e^{a-bx}\left(1+e^{2bx}\right)de^{bx}}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{e^a\int e^{-bx}\left(1+e^{2bx}\right)de^{bx}}{2b}$$

$$\Big\downarrow \; 244$$

$$\frac{e^a\int \left(e^{-bx}+e^{bx}\right)de^{bx}}{2b}$$

$$\Big\downarrow \; 2009$$

$$\frac{e^a\left(\frac{1}{2}e^{2bx}+\log\left(e^{bx}\right)\right)}{2b}$$

input
```
Int[E^(a + b*x)*Cosh[d + b*x],x]
```

output
```
(E^a*(E^(2*b*x)/2 + Log[E^(b*x)]))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{{\rm e}^{2 b x +a +d}}{4 b}+\frac{{\rm e}^{a -d} x}{2}$	24
parallelrisch	$\frac{{\rm e}^{b x +a} \left(b x \cosh \left(b x +d \right)-x b \sinh \left(b x +d \right)+\sinh \left(b x +d \right)\right)}{2 b}$	38
default	$\frac{\cosh \left(a -d \right) x}{2}+\frac{\sinh \left(2 b x +a +d \right)}{4 b}+\frac{\sinh \left(a -d \right) x}{2}+\frac{\cosh \left(2 b x +a +d \right)}{4 b}$	46
orering	$\frac{\left(2 b x +1\right) {\rm e}^{b x +a} \cosh \left(b x +d \right)}{2 b}-\frac{x \left(b \,{\rm e}^{b x +a} \cosh \left(b x +d \right)+{\rm e}^{b x +a} b \sinh \left(b x +d \right)\right)}{2 b}$	60

input
```
int(exp(b*x+a)*cosh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(2*b*x+a+d)/b+1/2*exp(a-d)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(23) = 46$.

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.38

$$\int e^{a+bx}\cosh(d+bx)\,dx = \frac{(2\,bx+1)\cosh(bx+d)\cosh(-a+d)-(2\,bx+1)\cosh(bx+d)\sinh(-a+d)-((2\,bx-1)\cosh(-a+ }{4\,(b\cosh(bx+d)-b\sinh(bx+d))}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d),x, algorithm="fricas")
```

output
```
1/4*((2*b*x + 1)*cosh(b*x + d)*cosh(-a + d) - (2*b*x + 1)*cosh(b*x + d)*si
nh(-a + d) - ((2*b*x - 1)*cosh(-a + d) - (2*b*x - 1)*sinh(-a + d))*sinh(b*
x + d))/(b*cosh(b*x + d) - b*sinh(b*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(20) = 40$.

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.17

$$\int e^{a+bx}\cosh(d+bx)\,dx = \begin{cases} -\frac{xe^{a}e^{bx}\sinh(bx+d)}{2}+\frac{xe^{a}e^{bx}\cosh(bx+d)}{2}+\frac{e^{a}e^{bx}\sinh(bx+d)}{2b} & \text{for } b\neq 0\\ xe^{a}\cosh(d) & \text{otherwise}\end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d),x)
```

output
```
Piecewise((-x*exp(a)*exp(b*x)*sinh(b*x + d)/2 + x*exp(a)*exp(b*x)*cosh(b*x
 + d)/2 + exp(a)*exp(b*x)*sinh(b*x + d)/(2*b), Ne(b, 0)), (x*exp(a)*cosh(d
), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int e^{a+bx}\cosh(d+bx)\,dx = \frac{(bx+a)e^{(a-d)}}{2\,b} + \frac{e^{(2\,bx+a+d)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d),x, algorithm="maxima")
```

output
```
1/2*(b*x + a)*e^(a - d)/b + 1/4*e^(2*b*x + a + d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int e^{a+bx}\cosh(d+bx)\,dx = \frac{\left(2\,bxe^{a} + e^{(2\,bx+a+2\,d)}\right)e^{(-d)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d),x, algorithm="giac")
```

output
```
1/4*(2*b*x*e^a + e^(2*b*x + a + 2*d))*e^(-d)/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int e^{a+bx}\cosh(d+bx)\,dx = \frac{e^{a-d}\left(e^{2\,d+2\,b\,x} + 2\,b\,x\right)}{4\,b}$$

input
```
int(cosh(d + b*x)*exp(a + b*x),x)
```

output
```
(exp(a - d)*(exp(2*d + 2*b*x) + 2*b*x))/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.31

$$\int e^{a+bx}\cosh(d+bx)\,dx = \frac{e^{bx+a}(\cosh(bx+d)\,bx+\cosh(bx+d)-\sinh(bx+d)\,bx)}{2b}$$

input
```
int(exp(b*x+a)*cosh(b*x+d),x)
```

output
```
(e**(a + b*x)*(cosh(b*x + d)*b*x + cosh(b*x + d) - sinh(b*x + d)*b*x))/(2*
b)
```

3.5 $\int e^{a+bx}\coth(d+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 32

$$\int e^{a+bx}\coth(d+bx)\,dx=\frac{e^{a+bx}}{b}-\frac{2e^{a-d}\text{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
exp(b*x+a)/b-2*exp(a-d)*arctanh(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int e^{a+bx}\coth(d+bx)\,dx$$
$$=\frac{e^a\left(e^{bx}-2\text{arctanh}\left(e^{bx}(\cosh(d)+\sinh(d))\right)(\cosh(d)-\sinh(d))\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Coth[d + b*x],x]
```

output
```
(E^a*(E^(b*x) - 2*ArcTanh[E^(b*x)*(Cosh[d] + Sinh[d])]*(Cosh[d] - Sinh[d])
))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {2720, 25, 27, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx} \coth(bx+d) \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int -\frac{e^a(1+e^{2bx})}{1-e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{e^a(1+e^{2bx})}{1-e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{e^a \int \frac{1+e^{2bx}}{1-e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{ 299}$$

$$-\frac{e^a \left(2 \int \frac{1}{1-e^{2bx}}de^{bx} - e^{bx}\right)}{b}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{e^a \left(2\text{arctanh}\left(e^{bx}\right) - e^{bx}\right)}{b}$$

input
```
Int[E^(a + b*x)*Coth[d + b*x],x]
```

output
```
-((E^a*(-E^(b*x) + 2*ArcTanh[E^(b*x)]))/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(29) = 58$.

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.97

method	result	size
risch	$\frac{e^{bx+a}}{b}+\frac{\ln\left(e^{bx+a}-e^{a-d}\right)e^{a-d}}{b}-\frac{\ln\left(e^{bx+a}+e^{a-d}\right)e^{a-d}}{b}$	63

input
```
int(exp(b*x+a)*coth(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
exp(b*x+a)/b+ln(exp(b*x+a)-exp(a-d))/b*exp(a-d)-ln(exp(b*x+a)+exp(a-d))/b*
exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(29) = 58$.

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.66

$$\int e^{a+bx}\coth(d+bx)\,dx = \frac{\cosh(bx+d)\cosh(-a+d) - (\cosh(-a+d) - \sinh(-a+d))\log(\cosh(bx+d) + \sinh(bx+d) + 1) +}{}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d),x, algorithm="fricas")
```

output
```
(cosh(b*x + d)*cosh(-a + d) - (cosh(-a + d) - sinh(-a + d))*log(cosh(b*x +
 d) + sinh(b*x + d) + 1) + (cosh(-a + d) - sinh(-a + d))*log(cosh(b*x + d)
 + sinh(b*x + d) - 1) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d) - cosh
(b*x + d)*sinh(-a + d))/b
```

Sympy [F]

$$\int e^{a+bx}\coth(d+bx)\,dx = e^{a}\int e^{bx}\coth(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*coth(b*x+d),x)
```

output
```
exp(a)*Integral(exp(b*x)*coth(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.75

$$\int e^{a+bx}\coth(d+bx)\,dx = -\frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^{a}\right)}{b} + \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}-e^{a}\right)}{b} + \frac{e^{(bx+a)}}{b}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d),x, algorithm="maxima")
```

output
```
-e^(a - d)*log(e^(b*x + a + d) + e^a)/b + e^(a - d)*log(e^(b*x + a + d) -
e^a)/b + e^(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int e^{a+bx}\coth(d+bx)\,dx = -\frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^{a}\right) - e^{(a-d)}\log\left(\left|e^{(bx+a+d)}-e^{a}\right|\right) - e^{(bx+a)}}{b}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d),x, algorithm="giac")
```

output
```
-(e^(a - d)*log(e^(b*x + a + d) + e^a) - e^(a - d)*log(abs(e^(b*x + a + d)
 - e^a)) - e^(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.84

$$\int e^{a+bx}\coth(d+bx)\,dx=\frac{\mathrm{e}^{a+b\,x}}{b}-\frac{2\,\sqrt{\mathrm{e}^{2\,a-2\,d}}\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b\,\sqrt{\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}}}\right)}{\sqrt{-b^2}}$$

input
```
int(coth(d + b*x)*exp(a + b*x),x)
```

output
```
exp(a + b*x)/b - (2*exp(2*a - 2*d)^(1/2)*atan((exp(b*x)*exp(a)*(-b^2)^(1/2
))/(b*(exp(2*a)*exp(-2*d))^(1/2))))/(-b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int e^{a+bx}\coth(d+bx)\,dx=\frac{e^{a}\left(e^{bx+d}+\log\left(e^{bx+d}-1\right)-\log\left(e^{bx+d}+1\right)\right)}{e^{d}b}$$

input
```
int(exp(b*x+a)*coth(b*x+d),x)
```

output
```
(e**a*(e**(b*x + d) + log(e**(b*x + d) - 1) - log(e**(b*x + d) + 1)))/(e**
d*b)
```

3.6 $\int e^{a+bx}\coth(d+bx)\operatorname{csch}(d+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int e^{a+bx}\coth(d+bx)\operatorname{csch}(d+bx)\,dx=\frac{2e^{a-d}}{b\left(1-e^{2d+2bx}\right)}+\frac{e^{a-d}\log\left(1-e^{2d+2bx}\right)}{b}$$

output
```
2*exp(a-d)/b/(1-exp(2*b*x+2*d))+exp(a-d)*ln(1-exp(2*b*x+2*d))/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 116 vs. 2(55) = 110.

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.11

$$\int e^{a+bx}\coth(d+bx)\operatorname{csch}(d+bx)\,dx$$
$$=\frac{e^a(\cosh(d)-\sinh(d))\left(\cosh(d)\left(-2+\left(-1+e^{2bx}\right)\log\left(\left(-1+e^{2bx}\right)\cosh(d)+\left(1+e^{2bx}\right)\sinh(d)\right)\right)+(2}{b\left(\left(-1+e^{2bx}\right)\cosh(d)+\left(1+e^{2bx}\right)\sinh\right.}$$

input
```
Integrate[E^(a + b*x)*Coth[d + b*x]*Csch[d + b*x],x]
```

output
```
(E^a*(Cosh[d] - Sinh[d])*(Cosh[d]*(-2 + (-1 + E^(2*b*x))*Log[(-1 + E^(2*b*
x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d]]) + (2 + (1 + E^(2*b*x))*Log[(-1 + E
^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d]])*Sinh[d]))/(b*((-1 + E^(2*b*x
))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d]))
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.60, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\coth(bx+d)\text{csch}(bx+d)\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int \frac{2e^{a+bx}\left(1+e^{2bx}\right)}{\left(1-e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{2e^a\int \frac{e^{bx}\left(1+e^{2bx}\right)}{\left(1-e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \; 353$$

$$\frac{e^a\int \frac{1+e^{2bx}}{\left(1-e^{2bx}\right)^2}de^{2bx}}{b}$$

$$\Big\downarrow \; 49$$

$$\frac{e^a\int \left(\frac{1}{-1+e^{2bx}}+\frac{2}{\left(-1+e^{2bx}\right)^2}\right)de^{2bx}}{b}$$

$$\Big\downarrow \; 2009$$

$$\frac{e^a\left(\frac{2}{1-e^{2bx}}+\log\left(1-e^{2bx}\right)\right)}{b}$$

input
```
Int[E^(a + b*x)*Coth[d + b*x]*Csch[d + b*x],x]
```

output
```
(E^a*(2/(1 - E^(2*b*x)) + Log[1 - E^(2*b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.44

method	result	size
risch	$-\frac{2\,e^{a-d}a}{b}+\frac{2\,e^{3a-d}}{\left(-e^{2bx+2a+2d}+e^{2a}\right)b}+\frac{\ln\left(e^{2bx+2a}-e^{2a-2d}\right)e^{a-d}}{b}$	79

input
```
int(exp(b*x+a)*coth(b*x+d)*csch(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
-2/b*exp(a-d)*a+2/(-exp(2*b*x+2*a+2*d)+exp(2*a))/b*exp(3*a-d)+ln(exp(2*b*x
+2*a)-exp(2*a-2*d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(49) = 98$.

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.42

$$\int e^{a+bx}\coth(d+bx)\text{csch}(d+bx)\,dx$$
$$=\frac{(\cosh(bx+d)^2\cosh(-a+d)+(\cosh(-a+d)-\sinh(-a+d))\sinh(bx+d)^2+2(\cosh(bx+d)\cosh(}{b}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)*csch(b*x+d),x, algorithm="fricas")
```

output
```
((cosh(b*x + d)^2*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x +
d)^2 + 2*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*
x + d) - (cosh(b*x + d)^2 - 1)*sinh(-a + d) - cosh(-a + d))*log(2*sinh(b*x
 + d)/(cosh(b*x + d) - sinh(b*x + d))) - 2*cosh(-a + d) + 2*sinh(-a + d))/
(b*cosh(b*x + d)^2 + 2*b*cosh(b*x + d)*sinh(b*x + d) + b*sinh(b*x + d)^2 -
 b)
```

Sympy [F]

$$\int e^{a+bx}\coth(d+bx)\operatorname{csch}(d+bx)\,dx = e^a\int e^{bx}\coth(bx+d)\operatorname{csch}(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)*csch(b*x+d),x)
```

output
```
exp(a)*Integral(exp(b*x)*coth(b*x + d)*csch(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int e^{a+bx}\coth(d+bx)\operatorname{csch}(d+bx)\,dx = \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^a\right)}{b} + \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}-e^a\right)}{b} - \frac{2\,e^{(3\,a)}}{b\left(e^{(2\,bx+2\,a+3\,d)}-e^{(2\,a+d)}\right)}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)*csch(b*x+d),x, algorithm="maxima")
```

output
```
e^(a - d)*log(e^(b*x + a + d) + e^a)/b + e^(a - d)*log(e^(b*x + a + d) - e
^a)/b - 2*e^(3*a)/(b*(e^(2*b*x + 2*a + 3*d) - e^(2*a + d)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int e^{a+bx}\coth(d+bx)\operatorname{csch}(d+bx)\,dx = \frac{e^{(a-d)}\log\left(\left|e^{(2\,bx+2\,a+2\,d)}-e^{(2\,a)}\right|\right) - \frac{\left(e^{(2\,bx+3\,a+2\,d)}+e^{(3\,a)}\right)e^{(-d)}}{e^{(2\,bx+2\,a+2\,d)}-e^{(2\,a)}}}{b}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)*csch(b*x+d),x, algorithm="giac")
```

output
```
(e^(a - d)*log(abs(e^(2*b*x + 2*a + 2*d) - e^(2*a))) - (e^(2*b*x + 3*a + 2
*d) + e^(3*a))*e^(-d)/(e^(2*b*x + 2*a + 2*d) - e^(2*a)))/b
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.24

$$\int e^{a+bx}\coth(d+bx)\mathrm{csch}(d+bx)\,dx = \frac{2\,\mathrm{e}^{3\,a-3\,d}}{b\,\left(\mathrm{e}^{2\,a-2\,d}-\mathrm{e}^{2\,a+2\,b\,x}\right)}+\frac{\mathrm{e}^{a-d}\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}\right)}{b}$$

input
```
int((coth(d + b*x)*exp(a + b*x))/sinh(d + b*x),x)
```

output
```
(2*exp(3*a - 3*d))/(b*(exp(2*a - 2*d) - exp(2*a + 2*b*x))) + (exp(a - d)*l
og(exp(2*a)*exp(2*b*x) - exp(2*a)*exp(-2*d)))/b
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.91

$$\int e^{a+bx}\coth(d+bx)\mathrm{csch}(d+bx)\,dx = \frac{e^{a}\left(e^{2bx+2d}\log\left(e^{bx+d}-1\right)+e^{2bx+2d}\log\left(e^{bx+d}+1\right)-2e^{2bx+2d}-\log\left(e^{bx+d}-1\right)-\log\left(e^{bx+d}+1\right)\right)}{e^{d}b\left(e^{2bx+2d}-1\right)}$$

input
```
int(exp(b*x+a)*coth(b*x+d)*csch(b*x+d),x)
```

output
```
(e**a*(e**(2*b*x + 2*d)*log(e**(b*x + d) - 1) + e**(2*b*x + 2*d)*log(e**(b
*x + d) + 1) - 2*e**(2*b*x + 2*d) - log(e**(b*x + d) - 1) - log(e**(b*x +
d) + 1)))/(e**d*b*(e**(2*b*x + 2*d) - 1))
```

3.7 $\int e^{a+bx}\coth(d+bx)\text{csch}^2(d+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int e^{a+bx}\coth(d+bx)\text{csch}^2(d+bx)\,dx = -\frac{2e^{a+bx}}{b\left(1-e^{2d+2bx}\right)^2}+\frac{3e^{a+bx}}{b\left(1-e^{2d+2bx}\right)} - \frac{e^{a-d}\text{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
-2*exp(b*x+a)/b/(1-exp(2*b*x+2*d))^2+3*exp(b*x+a)/b/(1-exp(2*b*x+2*d))-exp
(a-d)*arctanh(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int e^{a+bx}\coth(d+bx)\text{csch}^2(d+bx)\,dx = \frac{e^a\left(\frac{e^{bx}-3e^{2d+3bx}}{\left(-1+e^{2(d+bx)}\right)^2}-e^{-d}\text{arctanh}\left(e^{d+bx}\right)\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Coth[d + b*x]*Csch[d + b*x]^2,x]
```

output
```
(E^a*((E^(b*x) - 3*E^(2*d + 3*b*x))/(-1 + E^(2*(d + b*x)))^2 - ArcTanh[E^(
d + b*x)]/E^d))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2720, 27, 360, 27, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\coth(bx+d)\text{csch}^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{4e^{a+2bx}\left(1+e^{2bx}\right)}{\left(1-e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4e^a\int \frac{e^{2bx}\left(1+e^{2bx}\right)}{\left(1-e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{360}$$

$$-\frac{4e^a\left(\frac{e^{bx}}{2\left(1-e^{2bx}\right)^2}-\frac{1}{4}\int \frac{2\left(1+2e^{2bx}\right)}{\left(1-e^{2bx}\right)^2}de^{bx}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4e^a\left(\frac{e^{bx}}{2\left(1-e^{2bx}\right)^2}-\frac{1}{2}\int \frac{1+2e^{2bx}}{\left(1-e^{2bx}\right)^2}de^{bx}\right)}{b}$$

$$\Big\downarrow \text{298}$$

$$-\frac{4e^a\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1-e^{2bx}}de^{bx}-\frac{3e^{bx}}{2\left(1-e^{2bx}\right)}\right)+\frac{e^{bx}}{2\left(1-e^{2bx}\right)^2}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{4e^a\left(\frac{1}{2}\left(\frac{1}{2}\text{arctanh}\left(e^{bx}\right)-\frac{3e^{bx}}{2\left(1-e^{2bx}\right)}\right)+\frac{e^{bx}}{2\left(1-e^{2bx}\right)^2}\right)}{b}$$

input
```
Int[E^(a + b*x)*Coth[d + b*x]*Csch[d + b*x]^2,x]
```

output
```
(-4*E^a*(E^(b*x)/(2*(1 - E^(2*b*x))^2) + ((-3*E^(b*x))/(2*(1 - E^(2*b*x)))
 + ArcTanh[E^(b*x)]/2)/2))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.38

method	result	size
risch	$\frac{(-3\,\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a})\mathrm{e}^{bx+3a}}{(-\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a})^2 b}+\frac{\ln(\mathrm{e}^{bx+a}-\mathrm{e}^{a-d})\mathrm{e}^{a-d}}{2b}-\frac{\ln(\mathrm{e}^{bx+a}+\mathrm{e}^{a-d})\mathrm{e}^{a-d}}{2b}$	106

input
```
int(exp(b*x+a)*coth(b*x+d)*csch(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(-exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(-3*exp(2*b*x+2*a+2*d)+exp(2*a))*exp(
b*x+3*a)+1/2*ln(exp(b*x+a)-exp(a-d))/b*exp(a-d)-1/2*ln(exp(b*x+a)+exp(a-d)
)/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. 2(67) = 134.

Time = 0.09 (sec) , antiderivative size = 806, normalized size of antiderivative = 10.47

$$\int e^{a+bx}\coth(d+bx)\text{csch}^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)*csch(b*x+d)^2,x, algorithm="fricas")
```

output
```
-1/2*(6*cosh(b*x + d)^3*cosh(-a + d) + 6*(cosh(-a + d) - sinh(-a + d))*sin
h(b*x + d)^3 + 18*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d)
)*sinh(b*x + d)^2 - 2*cosh(b*x + d)*cosh(-a + d) + (cosh(b*x + d)^4*cosh(-
a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*
cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d)^3 - 2*cosh(b*x +
d)^2*cosh(-a + d) + 2*(3*cosh(b*x + d)^2*cosh(-a + d) - (3*cosh(b*x + d)^2
 - 1)*sinh(-a + d) - cosh(-a + d))*sinh(b*x + d)^2 + 4*(cosh(b*x + d)^3*co
sh(-a + d) - cosh(b*x + d)*cosh(-a + d) - (cosh(b*x + d)^3 - cosh(b*x + d)
)*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^4 - 2*cosh(b*x + d)^2 + 1)*
sinh(-a + d) + cosh(-a + d))*log(cosh(b*x + d) + sinh(b*x + d) + 1) - (cos
h(b*x + d)^4*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d)^4
+ 4*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d
)^3 - 2*cosh(b*x + d)^2*cosh(-a + d) + 2*(3*cosh(b*x + d)^2*cosh(-a + d) -
 (3*cosh(b*x + d)^2 - 1)*sinh(-a + d) - cosh(-a + d))*sinh(b*x + d)^2 + 4*
(cosh(b*x + d)^3*cosh(-a + d) - cosh(b*x + d)*cosh(-a + d) - (cosh(b*x + d
)^3 - cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^4 - 2*co
sh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*log(cosh(b*x + d) + sinh(b
*x + d) - 1) + 2*(9*cosh(b*x + d)^2*cosh(-a + d) - (9*cosh(b*x + d)^2 - 1)
*sinh(-a + d) - cosh(-a + d))*sinh(b*x + d) - 2*(3*cosh(b*x + d)^3 - cosh(
b*x + d))*sinh(-a + d))/(b*cosh(b*x + d)^4 + 4*b*cosh(b*x + d)*sinh(b*x...
```

Sympy [F]

$$\int e^{a+bx}\coth(d+bx)\text{csch}^2(d+bx)\,dx = e^a\int e^{bx}\coth{(bx+d)}\,\text{csch}^2{(bx+d)}\,dx$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)*csch(b*x+d)**2,x)
```

output
```
exp(a)*Integral(exp(b*x)*coth(b*x + d)*csch(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.43

$$\int e^{a+bx}\coth(d+bx)\operatorname{csch}^2(d+bx)\,dx = -\frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^a\right)}{2\,b} + \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}-e^a\right)}{2\,b} - \frac{3\,e^{(3\,bx+5\,a+2\,d)}-e^{(bx+5\,a)}}{b(e^{(4\,bx+4\,a+4\,d)}-2\,e^{(2\,bx+4\,a+2\,d)}+e^{(4\,a)})}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)*csch(b*x+d)^2,x, algorithm="maxima")
```

output
```
-1/2*e^(a - d)*log(e^(b*x + a + d) + e^a)/b + 1/2*e^(a - d)*log(e^(b*x + a
 + d) - e^a)/b - (3*e^(3*b*x + 5*a + 2*d) - e^(b*x + 5*a))/(b*(e^(4*b*x +
4*a + 4*d) - 2*e^(2*b*x + 4*a + 2*d) + e^(4*a)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

$$\int e^{a+bx}\coth(d+bx)\operatorname{csch}^2(d+bx)\,dx = -\frac{e^{(a-d)}\log\left(e^{(bx+d)}+1\right)-e^{(a-d)}\log\left(\left|e^{(bx+d)}-1\right|\right)+\frac{2\left(3\,e^{(3\,bx+a+3\,d)}-e^{(bx+a+d)}\right)e^{(-d)}}{\left(e^{(2\,bx+2\,d)}-1\right)^2}}{2\,b}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)*csch(b*x+d)^2,x, algorithm="giac")
```

output
```
-1/2*(e^(a - d)*log(e^(b*x + d) + 1) - e^(a - d)*log(abs(e^(b*x + d) - 1))
 + 2*(3*e^(3*b*x + a + 3*d) - e^(b*x + a + d))*e^(-d)/(e^(2*b*x + 2*d) - 1
)^2)/b
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\coth(d+bx)\operatorname{csch}^2(d+bx)\,dx = \int \frac{\coth(d+b\,x)\,\mathrm{e}^{a+b\,x}}{\sinh\left(d+b\,x\right)^2}\,dx$$

input
```
int((coth(d + b*x)*exp(a + b*x))/sinh(d + b*x)^2,x)
```

output
```
int((coth(d + b*x)*exp(a + b*x))/sinh(d + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.21

$$\int e^{a+bx}\coth(d+bx)\operatorname{csch}^2(d+bx)\,dx = \frac{e^a\left(e^{4bx+4d}\log\left(e^{bx+d}-1\right)-e^{4bx+4d}\log\left(e^{bx+d}+1\right)-6e^{3bx+3d}-2e^{2bx+2d}\log\left(e^{bx+d}-1\right)+2e^{2bx+2d}\log\left(e^{bx+\cdots\right.\right.}{2e^d b\left(e^{4bx+4d}-2e^{2bx+2d}+1\right)}$$

input
```
int(exp(b*x+a)*coth(b*x+d)*csch(b*x+d)^2,x)
```

output
```
(e**a*(e**(4*b*x + 4*d)*log(e**(b*x + d) - 1) - e**(4*b*x + 4*d)*log(e**(b
*x + d) + 1) - 6*e**(3*b*x + 3*d) - 2*e**(2*b*x + 2*d)*log(e**(b*x + d) -
1) + 2*e**(2*b*x + 2*d)*log(e**(b*x + d) + 1) + 2*e**(b*x + d) + log(e**(b
*x + d) - 1) - log(e**(b*x + d) + 1)))/(2*e**d*b*(e**(4*b*x + 4*d) - 2*e**
(2*b*x + 2*d) + 1))
```

3.8 $\int e^{a+bx}\cosh^2(d+bx)\sinh^3(d+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 101

$$\int e^{a+bx}\cosh^2(d+bx)\sinh^3(d+bx)\,dx=\frac{e^{a-5d-4bx}}{128b}-\frac{e^{a-3d-2bx}}{64b}-\frac{e^{a+d+2bx}}{32b}-\frac{e^{a+3d+4bx}}{128b}+\frac{e^{a+5d+6bx}}{192b}+\frac{1}{16}e^{a-d}x$$

output
```
1/128*exp(-4*b*x+a-5*d)/b-1/64*exp(-2*b*x+a-3*d)/b-1/32*exp(2*b*x+a+d)/b-1
/128*exp(4*b*x+a+3*d)/b+1/192*exp(6*b*x+a+5*d)/b+1/16*exp(a-d)*x
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.14

$$\int e^{a+bx}\cosh^2(d+bx)\sinh^3(d+bx)\,dx$$
$$=\frac{e^a\left(-12\left(\left(e^{2bx}-2bx\right)\cosh(d)+\left(e^{2bx}+2bx\right)\sinh(d)\right)-3e^{-2bx}\left(\left(2+e^{6bx}\right)\cosh(3d)+\left(-2+e^{6bx}\right)\sinh(3\right.\right.}{384b}$$

input
```
Integrate[E^(a + b*x)*Cosh[d + b*x]^2*Sinh[d + b*x]^3,x]
```

output
```
(E^a*(-12*((E^(2*b*x) - 2*b*x)*Cosh[d] + (E^(2*b*x) + 2*b*x)*Sinh[d]) - (3
*((2 + E^(6*b*x))*Cosh[3*d] + (-2 + E^(6*b*x))*Sinh[3*d]))/E^(2*b*x) + ((3
 + 2*E^(10*b*x))*Cosh[5*d] + (-3 + 2*E^(10*b*x))*Sinh[5*d])/E^(4*b*x)))/(3
84*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.55, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2720, 27, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh^3(bx+d)\cosh^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{1}{32}e^{a-5bx}\left(1-e^{2bx}\right)^3\left(1+e^{2bx}\right)^2de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^a\int e^{-5bx}\left(1-e^{2bx}\right)^3\left(1+e^{2bx}\right)^2de^{bx}}{32b}$$

$$\Big\downarrow \text{354}$$

$$-\frac{e^a\int e^{-3bx}\left(1-e^{2bx}\right)^3\left(1+e^{2bx}\right)^2de^{2bx}}{64b}$$

$$\Big\downarrow \text{99}$$

$$-\frac{e^a\int \left(2+e^{-3bx}-e^{-2bx}-2e^{-bx}\right)de^{2bx}}{64b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^a\left(-\frac{1}{2}e^{-2bx}+e^{-bx}+\frac{5}{2}e^{2bx}-\frac{1}{3}e^{3bx}-2\log\left(e^{2bx}\right)\right)}{64b}$$

input
```
Int[E^(a + b*x)*Cosh[d + b*x]^2*Sinh[d + b*x]^3,x]
```

output
```
-1/64*(E^a*(-1/2*1/E^(2*b*x) + E^(-(b*x)) + (5*E^(2*b*x))/2 - E^(3*b*x)/3
- 2*Log[E^(2*b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.64

$$\frac{\cosh(a-d)\,x}{16}+\frac{\sinh(-4bx+a-5d)}{128b}-\frac{\sinh(-2bx+a-3d)}{64b}-\frac{\sinh(2bx+a+d)}{32b}-\frac{\sinh(4bx+a+3d)}{128b}+$$

input
```
int(exp(b*x+a)*cosh(b*x+d)^2*sinh(b*x+d)^3,x)
```

output
```
1/16*cosh(a-d)*x+1/128/b*sinh(-4*b*x+a-5*d)-1/64/b*sinh(-2*b*x+a-3*d)-1/32
/b*sinh(2*b*x+a+d)-1/128/b*sinh(4*b*x+a+3*d)+1/192/b*sinh(6*b*x+a+5*d)+1/1
6*sinh(a-d)*x+1/128*cosh(-4*b*x+a-5*d)/b-1/64*cosh(-2*b*x+a-3*d)/b-1/32*co
sh(2*b*x+a+d)/b-1/128*cosh(4*b*x+a+3*d)/b+1/192*cosh(6*b*x+a+5*d)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. 2(83) = 166.

Time = 0.08 (sec) , antiderivative size = 392, normalized size of antiderivative = 3.88

$$\int e^{a+bx}\cosh^2(d+bx)\sinh^3(d+bx)\,dx$$
$$=\frac{5\cosh(bx+d)^5\cosh(-a+d)-(\cosh(-a+d)-\sinh(-a+d))\sinh(bx+d)^5+25(\cosh(bx+d)\cos}{}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^2*sinh(b*x+d)^3,x, algorithm="fricas")
```

output
```
1/384*(5*cosh(b*x + d)^5*cosh(-a + d) - (cosh(-a + d) - sinh(-a + d))*sinh
(b*x + d)^5 + 25*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))
*sinh(b*x + d)^4 - 9*cosh(b*x + d)^3*cosh(-a + d) - (10*cosh(b*x + d)^2*co
sh(-a + d) - (10*cosh(b*x + d)^2 - 3)*sinh(-a + d) - 3*cosh(-a + d))*sinh(
b*x + d)^3 + 12*(2*b*x - 1)*cosh(b*x + d)*cosh(-a + d) + (50*cosh(b*x + d)
^3*cosh(-a + d) - 27*cosh(b*x + d)*cosh(-a + d) - (50*cosh(b*x + d)^3 - 27
*cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d)^2 - (5*cosh(b*x + d)^4*cosh(-a
 + d) - 9*cosh(b*x + d)^2*cosh(-a + d) + 12*(2*b*x + 1)*cosh(-a + d) - (5*
cosh(b*x + d)^4 + 24*b*x - 9*cosh(b*x + d)^2 + 12)*sinh(-a + d))*sinh(b*x
+ d) - (5*cosh(b*x + d)^5 - 9*cosh(b*x + d)^3 + 12*(2*b*x - 1)*cosh(b*x +
d))*sinh(-a + d))/(b*cosh(b*x + d) - b*sinh(b*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(82) = 164$.

Time = 5.22 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.91

$$\int e^{a+bx}\cosh^2(d+bx)\sinh^3(d+bx)\,dx$$
$$= \begin{cases} -\frac{xe^ae^{bx}\sinh^5(bx+d)}{16} + \frac{xe^ae^{bx}\sinh^4(bx+d)\cosh(bx+d)}{16} + \frac{xe^ae^{bx}\sinh^3(bx+d)\cosh^2(bx+d)}{8} - \frac{xe^ae^{bx}\sinh^2(bx+d)\cosh^3(bx+d)}{8} - \\ xe^a\sinh^3(d)\cosh^2(d) \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)**2*sinh(b*x+d)**3,x)
```

output
```
Piecewise((-x*exp(a)*exp(b*x)*sinh(b*x + d)**5/16 + x*exp(a)*exp(b*x)*sinh
(b*x + d)**4*cosh(b*x + d)/16 + x*exp(a)*exp(b*x)*sinh(b*x + d)**3*cosh(b*
x + d)**2/8 - x*exp(a)*exp(b*x)*sinh(b*x + d)**2*cosh(b*x + d)**3/8 - x*ex
p(a)*exp(b*x)*sinh(b*x + d)*cosh(b*x + d)**4/16 + x*exp(a)*exp(b*x)*cosh(b
*x + d)**5/16 - exp(a)*exp(b*x)*sinh(b*x + d)**5/(32*b) + 3*exp(a)*exp(b*x
)*sinh(b*x + d)**4*cosh(b*x + d)/(32*b) + exp(a)*exp(b*x)*sinh(b*x + d)**2
*cosh(b*x + d)**3/(6*b) - exp(a)*exp(b*x)*sinh(b*x + d)*cosh(b*x + d)**4/(
96*b) - 5*exp(a)*exp(b*x)*cosh(b*x + d)**5/(96*b), Ne(b, 0)), (x*exp(a)*si
nh(d)**3*cosh(d)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04

$$\int e^{a+bx}\cosh^2(d+bx)\sinh^3(d+bx)\,dx$$
$$= -\frac{\left(2\,e^{(2\,bx+5\,a+2\,d)} - e^{(5\,a)}\right)e^{(-4\,bx-4\,a-5\,d)}}{128\,b} + \frac{\left(2\,e^{(6\,bx+6\,a+5\,d)} - 3\,e^{(4\,bx+6\,a+3\,d)} - 12\,e^{(2\,bx+6\,a+d)}\right)e^{(-5\,a)}}{384\,b} + \frac{(bx+a)e^{(a-d)}}{16\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^2*sinh(b*x+d)^3,x, algorithm="maxima")
```

output
```
-1/128*(2*e^(2*b*x + 5*a + 2*d) - e^(5*a))*e^(-4*b*x - 4*a - 5*d)/b + 1/38
4*(2*e^(6*b*x + 6*a + 5*d) - 3*e^(4*b*x + 6*a + 3*d) - 12*e^(2*b*x + 6*a +
 d))*e^(-5*a)/b + 1/16*(b*x + a)*e^(a - d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

$$\int e^{a+bx}\cosh^2(d+bx)\sinh^3(d+bx)\,dx$$
$$= \frac{\left(24\,bxe^{(a+4\,d)} - 3\,\left(6\,e^{(4\,bx+a+4\,d)} + 2\,e^{(2\,bx+a+2\,d)} - e^{a}\right)e^{(-4\,bx)} + 2\,e^{(6\,bx+a+10\,d)} - 3\,e^{(4\,bx+a+8\,d)} - 12\,e^{(2\,bx+a}\right.}{384\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^2*sinh(b*x+d)^3,x, algorithm="giac")
```

output
```
1/384*(24*b*x*e^(a + 4*d) - 3*(6*e^(4*b*x + a + 4*d) + 2*e^(2*b*x + a + 2*
d) - e^a)*e^(-4*b*x) + 2*e^(6*b*x + a + 10*d) - 3*e^(4*b*x + a + 8*d) - 12
*e^(2*b*x + a + 6*d))*e^(-5*d)/b
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.35

$$\int e^{a+bx}\cosh^2(d+bx)\sinh^3(d+bx)\,dx = \frac{5\cosh(d+b\,x)^5\,\mathrm{e}^{a+b\,x}}{24\,b} - \frac{17\cosh(d+b\,x)^3\,\mathrm{e}^{a+b\,x}}{48\,b} + \frac{x\cosh(d+b\,x)\,\mathrm{e}^{a+b\,x}}{16} - \frac{x\,\mathrm{e}^{a+b\,x}\sinh(d+b\,x)}{16} + \frac{\cosh(d+b\,x)\,\mathrm{e}^{a+b\,x}}{16\,b} + \frac{\cosh(d+b\,x)^2\,\mathrm{e}^{a+b\,x}\sinh(d+b\,x)}{16\,b} - \frac{\cosh(d+b\,x)^4\,\mathrm{e}^{a+b\,x}\sinh(d+b\,x)}{24\,b}$$

input
```
int(cosh(d + b*x)^2*exp(a + b*x)*sinh(d + b*x)^3,x)
```

output
```
(5*cosh(d + b*x)^5*exp(a + b*x))/(24*b) - (17*cosh(d + b*x)^3*exp(a + b*x)
)/(48*b) + (x*cosh(d + b*x)*exp(a + b*x))/16 - (x*exp(a + b*x)*sinh(d + b*
x))/16 + (cosh(d + b*x)*exp(a + b*x))/(16*b) + (cosh(d + b*x)^2*exp(a + b*
x)*sinh(d + b*x))/(16*b) - (cosh(d + b*x)^4*exp(a + b*x)*sinh(d + b*x))/(2
4*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83

$$\int e^{a+bx}\cosh^2(d+bx)\sinh^3(d+bx)\,dx = \frac{e^a\left(2e^{10bx+10d}-3e^{8bx+8d}-12e^{6bx+6d}+24e^{4bx+4d}bx-6e^{2bx+2d}+3\right)}{384e^{4bx+5d}b}$$

input
```
int(exp(b*x+a)*cosh(b*x+d)^2*sinh(b*x+d)^3,x)
```

output
```
(e**a*(2*e**(10*b*x + 10*d) - 3*e**(8*b*x + 8*d) - 12*e**(6*b*x + 6*d) + 2
4*e**(4*b*x + 4*d)*b*x - 6*e**(2*b*x + 2*d) + 3))/(384*e**(4*b*x + 5*d)*b)
```

3.9 $\int e^{a+bx}\cosh^2(d+bx)\sinh^2(d+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int e^{a+bx}\cosh^2(d+bx)\sinh^2(d+bx)\,dx=-\frac{e^{a-4d-3bx}}{48b}-\frac{e^{a+bx}}{8b}+\frac{e^{a+4d+5bx}}{80b}$$

output
```
-1/48*exp(-3*b*x+a-4*d)/b-1/8*exp(b*x+a)/b+1/80*exp(5*b*x+a+4*d)/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int e^{a+bx}\cosh^2(d+bx)\sinh^2(d+bx)\,dx=-\frac{5e^{a-4d-3bx}+30e^{a+bx}-3e^{a+4d+5bx}}{240b}$$

input
```
Integrate[E^(a + b*x)*Cosh[d + b*x]^2*Sinh[d + b*x]^2,x]
```

output
```
-1/240*(5*E^(a - 4*d - 3*b*x) + 30*E^(a + b*x) - 3*E^(a + 4*d + 5*b*x))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh^2(bx+d)\cosh^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{16}e^{a-4bx}\left(1-e^{4bx}\right)^2de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^a\int e^{-4bx}\left(1-e^{4bx}\right)^2de^{bx}}{16b}$$

$$\Big\downarrow \text{802}$$

$$\frac{e^a\int \left(-2+e^{-4bx}+e^{4bx}\right)de^{bx}}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^a\left(-\frac{1}{3}e^{-3bx}-2e^{bx}+\frac{1}{5}e^{5bx}\right)}{16b}$$

input
```
Int[E^(a + b*x)*Cosh[d + b*x]^2*Sinh[d + b*x]^2,x]
```

output
```
(E^a*(-1/3*1/E^(3*b*x) - 2*E^(b*x) + E^(5*b*x)/5))/(16*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 159.61 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{e^{-3bx+a-4d}}{48b}-\frac{e^{bx+a}}{8b}+\frac{e^{5bx+a+4d}}{80b}$
default	$-\frac{\sinh(bx+a)}{8b}-\frac{\sinh(-3bx+a-4d)}{48b}+\frac{\sinh(5bx+a+4d)}{80b}-\frac{\cosh(bx+a)}{8b}-\frac{\cosh(-3bx+a-4d)}{48b}+\frac{\cosh(5bx+a+4d)}{80b}$
orering	$\frac{13\,e^{bx+a}\cosh(bx+d)^2\sinh(bx+d)^2}{15b}+\frac{e^{bx+a}\cosh(bx+d)^2\sinh(bx+d)^2 b+2b\,e^{bx+a}\cosh(bx+d)\sinh(bx+d)^3+2\,e^{bx+a}\cosh(bx+d)^3}{5b^2}$

input
```
int(exp(b*x+a)*cosh(b*x+d)^2*sinh(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/48*exp(-3*b*x+a-4*d)/b-1/8*exp(b*x+a)/b+1/80*exp(5*b*x+a+4*d)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(42) = 84$.

Time = 0.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 4.08

$$\int e^{a+bx}\cosh^2(d+bx)\sinh^2(d+bx)\,dx = -\frac{\cosh(bx+d)^4\cosh(-a+d)+(\cosh(-a+d)-\sinh(-a+d))\sinh(bx+d)^4-16\,(\cosh(bx+d)\cos$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^2*sinh(b*x+d)^2,x, algorithm="fricas")
```

output
```
-1/120*(cosh(b*x + d)^4*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(
b*x + d)^4 - 16*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*
sinh(b*x + d)^3 + 6*(cosh(b*x + d)^2*cosh(-a + d) - cosh(b*x + d)^2*sinh(-
a + d))*sinh(b*x + d)^2 - 16*(cosh(b*x + d)^3*cosh(-a + d) - cosh(b*x + d)
^3*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^4 + 15)*sinh(-a + d) + 15*
cosh(-a + d))/(b*cosh(b*x + d) - b*sinh(b*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(39) = 78$.

Time = 2.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.82

$$\int e^{a+bx}\cosh^2(d+bx)\sinh^2(d+bx)\,dx = \begin{cases} -\frac{2e^ae^{bx}\sinh^4(bx+d)}{15b}+\frac{2e^ae^{bx}\sinh^3(bx+d)\cosh(bx+d)}{15b}+\frac{e^ae^{bx}\sinh^2(bx+d)\cosh^2(bx+d)}{5b}+\frac{2e^ae^{bx}\sinh(bx+d)\cosh^3(bx+d)}{15b}-2 \\ xe^a\sinh^2(d)\cosh^2(d) \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)**2*sinh(b*x+d)**2,x)
```

output
```
Piecewise((-2*exp(a)*exp(b*x)*sinh(b*x + d)**4/(15*b) + 2*exp(a)*exp(b*x)*
sinh(b*x + d)**3*cosh(b*x + d)/(15*b) + exp(a)*exp(b*x)*sinh(b*x + d)**2*c
osh(b*x + d)**2/(5*b) + 2*exp(a)*exp(b*x)*sinh(b*x + d)*cosh(b*x + d)**3/(
15*b) - 2*exp(a)*exp(b*x)*cosh(b*x + d)**4/(15*b), Ne(b, 0)), (x*exp(a)*si
nh(d)**2*cosh(d)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int e^{a+bx}\cosh^2(d+bx)\sinh^2(d+bx)\,dx = \frac{\left(e^{(5\,bx+5\,a+4\,d)}-10\,e^{(bx+5\,a)}\right)e^{(-4\,a)}}{80\,b}-\frac{e^{(-3\,bx+a-4\,d)}}{48\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^2*sinh(b*x+d)^2,x, algorithm="maxima")
```

output
```
1/80*(e^(5*b*x + 5*a + 4*d) - 10*e^(b*x + 5*a))*e^(-4*a)/b - 1/48*e^(-3*b*
x + a - 4*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int e^{a+bx}\cosh^2(d+bx)\sinh^2(d+bx)\,dx = \frac{\left(3\,e^{(5\,bx+a+8\,d)}-30\,e^{(bx+a+4\,d)}-5\,e^{(-3\,bx+a)}\right)e^{(-4\,d)}}{240\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^2*sinh(b*x+d)^2,x, algorithm="giac")
```

output
```
1/240*(3*e^(5*b*x + a + 8*d) - 30*e^(b*x + a + 4*d) - 5*e^(-3*b*x + a))*e^
(-4*d)/b
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int e^{a+bx}\cosh^2(d+bx)\sinh^2(d+bx)\,dx = -\frac{e^{a-4\,d-3\,b\,x}\,\left(30\,e^{4\,d+4\,b\,x}-3\,e^{8\,d+8\,b\,x}+5\right)}{240\,b}$$

input
```
int(cosh(d + b*x)^2*exp(a + b*x)*sinh(d + b*x)^2,x)
```

output
```
-(exp(a - 4*d - 3*b*x)*(30*exp(4*d + 4*b*x) - 3*exp(8*d + 8*b*x) + 5))/(24
0*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int e^{a+bx}\cosh^2(d+bx)\sinh^2(d+bx)\,dx = \frac{e^a\left(3e^{8bx+8d}-30e^{4bx+4d}-5\right)}{240e^{3bx+4d}b}$$

input
```
int(exp(b*x+a)*cosh(b*x+d)^2*sinh(b*x+d)^2,x)
```

output
```
(e**a*(3*e**(8*b*x + 8*d) - 30*e**(4*b*x + 4*d) - 5))/(240*e**(3*b*x + 4*d
)*b)
```

3.10 $\int e^{a+bx}\cosh^2(d+bx)\sinh(d+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 65

$$\int e^{a+bx}\cosh^2(d+bx)\sinh(d+bx)\,dx=\frac{e^{a-3d-2bx}}{16b}+\frac{e^{a+d+2bx}}{16b}+\frac{e^{a+3d+4bx}}{32b}-\frac{1}{8}e^{a-d}x$$

output
```
1/16*exp(-2*b*x+a-3*d)/b+1/16*exp(2*b*x+a+d)/b+1/32*exp(4*b*x+a+3*d)/b-1/8
*exp(a-d)*x
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.17

$$\int e^{a+bx}\cosh^2(d+bx)\sinh(d+bx)\,dx$$
$$=\frac{e^a\left(2\left(\left(e^{2bx}-2bx\right)\cosh(d)+\left(e^{2bx}+2bx\right)\sinh(d)\right)+e^{-2bx}\left(\left(2+e^{6bx}\right)\cosh(3d)+\left(-2+e^{6bx}\right)\sinh(3d)\right)\right)}{32b}$$

input
```
Integrate[E^(a + b*x)*Cosh[d + b*x]^2*Sinh[d + b*x],x]
```

output
```
(E^a*(2*((E^(2*b*x) - 2*b*x)*Cosh[d] + (E^(2*b*x) + 2*b*x)*Sinh[d]) + ((2
+ E^(6*b*x))*Cosh[3*d] + (-2 + E^(6*b*x))*Sinh[3*d])/E^(2*b*x)))/(32*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2720, 27, 354, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh(bx+d)\cosh^2(bx+d)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int -\frac{1}{8}e^{a-3bx}\left(1-e^{2bx}\right)\left(1+e^{2bx}\right)^2de^{bx}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{e^a\int e^{-3bx}\left(1-e^{2bx}\right)\left(1+e^{2bx}\right)^2de^{bx}}{8b}$$

$$\Big\downarrow \text{ 354}$$

$$-\frac{e^a\int e^{-2bx}\left(1-e^{2bx}\right)\left(1+e^{2bx}\right)^2de^{2bx}}{16b}$$

$$\Big\downarrow \text{ 84}$$

$$-\frac{e^a\int \left(-1+e^{-2bx}+e^{-bx}-e^{2bx}\right)de^{2bx}}{16b}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{e^a\left(-e^{-bx}-\frac{3}{2}e^{2bx}+\log\left(e^{2bx}\right)\right)}{16b}$$

input
```
Int[E^(a + b*x)*Cosh[d + b*x]^2*Sinh[d + b*x],x]
```

output
```
-1/16*(E^a*(-E^(-(b*x)) - (3*E^(2*b*x))/2 + Log[E^(2*b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 11.46 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result
risch	$\frac{e^{-2bx+a-3d}}{16b}+\frac{e^{2bx+a+d}}{16b}+\frac{e^{4bx+a+3d}}{32b}-\frac{e^{a-d}x}{8}$
default	$-\frac{\cosh(a-d)x}{8}+\frac{\sinh(-2bx+a-3d)}{16b}+\frac{\sinh(2bx+a+d)}{16b}+\frac{\sinh(4bx+a+3d)}{32b}-\frac{\sinh(a-d)x}{8}+\frac{\cosh(-2bx+a-3d)}{16b}+\cosh($
orering	$\frac{(4bx+1)e^{bx+a}\cosh(bx+d)^2\sinh(bx+d)}{4b}-\frac{(bx-1)\left(e^{bx+a}\cosh(bx+d)^2\sinh(bx+d)b+2b\,e^{bx+a}\cosh(bx+d)\sinh(bx+d)^2+e^{bx+a}\,\mathrm{co}\right.}{4b^2}$

input
```
int(exp(b*x+a)*cosh(b*x+d)^2*sinh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/16*exp(-2*b*x+a-3*d)/b+1/16*exp(2*b*x+a+d)/b+1/32*exp(4*b*x+a+3*d)/b-1/8
*exp(a-d)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(53) = 106$.

Time = 0.09 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.34

$$\int e^{a+bx}\cosh^2(d+bx)\sinh(d+bx)\,dx$$
$$= \frac{3\,\cosh\left(bx+d\right)^3\cosh\left(-a+d\right) - \left(\cosh\left(-a+d\right) - \sinh\left(-a+d\right)\right)\sinh\left(bx+d\right)^3 - 2\left(2\,bx-1\right)\cosh\left(bx\right.}{}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^2*sinh(b*x+d),x, algorithm="fricas")
```

output
```
1/32*(3*cosh(b*x + d)^3*cosh(-a + d) - (cosh(-a + d) - sinh(-a + d))*sinh(
b*x + d)^3 - 2*(2*b*x - 1)*cosh(b*x + d)*cosh(-a + d) + 9*(cosh(b*x + d)*c
osh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d)^2 - (3*cosh(b*x +
d)^2*cosh(-a + d) - 2*(2*b*x + 1)*cosh(-a + d) + (4*b*x - 3*cosh(b*x + d)^
2 + 2)*sinh(-a + d))*sinh(b*x + d) - (3*cosh(b*x + d)^3 - 2*(2*b*x - 1)*co
sh(b*x + d))*sinh(-a + d))/(b*cosh(b*x + d) - b*sinh(b*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(51) = 102$.

Time = 0.90 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.69

$$\int e^{a+bx}\cosh^2(d+bx)\sinh(d+bx)\,dx$$
$$= \begin{cases} -\frac{xe^a e^{bx}\sinh^3(bx+d)}{8} + \frac{xe^a e^{bx}\sinh^2(bx+d)\cosh(bx+d)}{8} + \frac{xe^a e^{bx}\sinh(bx+d)\cosh^2(bx+d)}{8} - \frac{xe^a e^{bx}\cosh^3(bx+d)}{8} - \frac{e^a e^{bx}\sinh^3}{8b} \\ xe^a\sinh(d)\cosh^2(d) \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)**2*sinh(b*x+d),x)
```

output
```
Piecewise((-x*exp(a)*exp(b*x)*sinh(b*x + d)**3/8 + x*exp(a)*exp(b*x)*sinh(
b*x + d)**2*cosh(b*x + d)/8 + x*exp(a)*exp(b*x)*sinh(b*x + d)*cosh(b*x + d
)**2/8 - x*exp(a)*exp(b*x)*cosh(b*x + d)**3/8 - exp(a)*exp(b*x)*sinh(b*x +
 d)**3/(8*b) + exp(a)*exp(b*x)*sinh(b*x + d)**2*cosh(b*x + d)/(4*b) + exp(
a)*exp(b*x)*cosh(b*x + d)**3/(8*b), Ne(b, 0)), (x*exp(a)*sinh(d)*cosh(d)**
2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int e^{a+bx}\cosh^2(d+bx)\sinh(d+bx)\,dx = \frac{\left(e^{(4\,bx+4\,a+3\,d)}+2\,e^{(2\,bx+4\,a+d)}\right)e^{(-3\,a)}}{32\,b} - \frac{(bx+a)e^{(a-d)}}{8\,b} + \frac{e^{(-2\,bx+a-3\,d)}}{16\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^2*sinh(b*x+d),x, algorithm="maxima")
```

output
```
1/32*(e^(4*b*x + 4*a + 3*d) + 2*e^(2*b*x + 4*a + d))*e^(-3*a)/b - 1/8*(b*x
 + a)*e^(a - d)/b + 1/16*e^(-2*b*x + a - 3*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int e^{a+bx}\cosh^2(d+bx)\sinh(d+bx)\,dx = -\frac{\left(4\,bxe^{(a+2\,d)}-2\,\left(e^{(2\,bx+a+2\,d)}+e^{a}\right)e^{(-2\,bx)}-e^{(4\,bx+a+6\,d)}-2\,e^{(2\,bx+a+4\,d)}\right)e^{(-3\,d)}}{32\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^2*sinh(b*x+d),x, algorithm="giac")
```

output
```
-1/32*(4*b*x*e^(a + 2*d) - 2*(e^(2*b*x + a + 2*d) + e^a)*e^(-2*b*x) - e^(4
*b*x + a + 6*d) - 2*e^(2*b*x + a + 4*d))*e^(-3*d)/b
```

Mupad [B] (verification not implemented)

Time = 3.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int e^{a+bx}\cosh^2(d+bx)\sinh(d+bx)\,dx = -\frac{e^{a+b\,x}\,\left(\cosh(d+b\,x)+\cosh(d+b\,x)^2\sinh(d+b\,x)-3\cosh(d+b\,x)^3+b\,x\cosh(d+b\,x)-b\,x\sinh(d$$

$$8\,b$$

input
```
int(cosh(d + b*x)^2*exp(a + b*x)*sinh(d + b*x),x)
```

output
```
-(exp(a + b*x)*(cosh(d + b*x) + cosh(d + b*x)^2*sinh(d + b*x) - 3*cosh(d +
 b*x)^3 + b*x*cosh(d + b*x) - b*x*sinh(d + b*x)))/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int e^{a+bx}\cosh^2(d+bx)\sinh(d+bx)\,dx = \frac{e^a\left(e^{6bx+6d}+2e^{4bx+4d}-4e^{2bx+2d}bx+2\right)}{32e^{2bx+3d}b}$$

input
```
int(exp(b*x+a)*cosh(b*x+d)^2*sinh(b*x+d),x)
```

output
```
(e**a*(e**(6*b*x + 6*d) + 2*e**(4*b*x + 4*d) - 4*e**(2*b*x + 2*d)*b*x + 2)
)/(32*e**(2*b*x + 3*d)*b)
```

3.11 $\int e^{a+bx}\cosh^2(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 51

$$\int e^{a+bx}\cosh^2(d+bx)\,dx=-\frac{e^{a-2d-bx}}{4b}+\frac{e^{a+bx}}{2b}+\frac{e^{a+2d+3bx}}{12b}$$

output
```
-1/4*exp(-b*x+a-2*d)/b+1/2*exp(b*x+a)/b+1/12*exp(3*b*x+a+2*d)/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int e^{a+bx}\cosh^2(d+bx)\,dx=\frac{e^{a-bx}\left(6e^{2bx}+\left(-3+e^{4bx}\right)\cosh(2d)+\left(3+e^{4bx}\right)\sinh(2d)\right)}{12b}$$

input
```
Integrate[E^(a + b*x)*Cosh[d + b*x]^2,x]
```

output
```
(E^(a - b*x)*(6*E^(2*b*x) + (-3 + E^(4*b*x))*Cosh[2*d] + (3 + E^(4*b*x))*S
inh[2*d]))/(12*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\cosh^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{4}e^{a-2bx}\left(1+e^{2bx}\right)^2de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^a\int e^{-2bx}\left(1+e^{2bx}\right)^2de^{bx}}{4b}$$

$$\Big\downarrow \text{244}$$

$$\frac{e^a\int\left(2+e^{-2bx}+e^{2bx}\right)de^{bx}}{4b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^a\left(-e^{-bx}+2e^{bx}+\frac{1}{3}e^{3bx}\right)}{4b}$$

input
```
Int[E^(a + b*x)*Cosh[d + b*x]^2,x]
```

output
```
(E^a*(-E^(-(b*x)) + 2*E^(b*x) + E^(3*b*x)/3))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{e^{-bx+a-2d}}{4b}+\frac{e^{bx+a}}{2b}+\frac{e^{3bx+a+2d}}{12b}$
parallelrisch	$\frac{2\,e^{bx+a}\left(3\tanh\left(\frac{bx}{2}+\frac{d}{2}\right)^{3}-3\tanh\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}+\tanh\left(\frac{bx}{2}+\frac{d}{2}\right)+1\right)}{3b\left(-1+\tanh\left(\frac{bx}{2}+\frac{d}{2}\right)\right)^{2}\left(1+\tanh\left(\frac{bx}{2}+\frac{d}{2}\right)\right)^{2}}$
default	$\frac{\sinh(bx+a)}{2b}-\frac{\sinh(-bx+a-2d)}{4b}+\frac{\sinh(3bx+a+2d)}{12b}+\frac{\cosh(bx+a)}{2b}-\frac{\cosh(-bx+a-2d)}{4b}+\frac{\cosh(3bx+a+2d)}{12b}$
orering	$\frac{e^{bx+a}\cosh(bx+d)^{2}}{3b}+\frac{e^{bx+a}\cosh(bx+d)^{2}b+2\,e^{bx+a}\sinh(bx+d)b\cosh(bx+d)}{b^{2}}-\frac{3b^{2}e^{bx+a}\cosh(bx+d)^{2}+4\,e^{bx+a}\cosh(bx-}{3}$

input
```
int(exp(b*x+a)*cosh(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*exp(-b*x+a-2*d)/b+1/2*exp(b*x+a)/b+1/12*exp(3*b*x+a+2*d)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(42) = 84.

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.43

$$\int e^{a+bx}\cosh^2(d+bx)\,dx = -\frac{\cosh(bx+d)^2\cosh(-a+d) + (\cosh(-a+d) - \sinh(-a+d))\sinh(bx+d)^2 - 4(\cosh(bx+d)\cosh}{6(b\cosh(bx}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^2,x, algorithm="fricas")
```

output
```
-1/6*(cosh(b*x + d)^2*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*
x + d)^2 - 4*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sin
h(b*x + d) - (cosh(b*x + d)^2 - 3)*sinh(-a + d) - 3*cosh(-a + d))/(b*cosh(
b*x + d) - b*sinh(b*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(37) = 74.

Time = 0.42 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.53

$$\int e^{a+bx}\cosh^2(d+bx)\,dx = \begin{cases} -\frac{2e^a e^{bx}\sinh^2(bx+d)}{3b} + \frac{2e^a e^{bx}\sinh(bx+d)\cosh(bx+d)}{3b} + \frac{e^a e^{bx}\cosh^2(bx+d)}{3b} & \text{for } b \neq 0 \\ xe^a\cosh^2(d) & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)**2,x)
```

output
```
Piecewise((-2*exp(a)*exp(b*x)*sinh(b*x + d)**2/(3*b) + 2*exp(a)*exp(b*x)*s
inh(b*x + d)*cosh(b*x + d)/(3*b) + exp(a)*exp(b*x)*cosh(b*x + d)**2/(3*b),
 Ne(b, 0)), (x*exp(a)*cosh(d)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int e^{a+bx}\cosh^2(d+bx)\,dx = \frac{e^{(3\,bx+a+2\,d)}}{12\,b} + \frac{e^{(bx+a)}}{2\,b} - \frac{e^{(-bx+a-2\,d)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^2,x, algorithm="maxima")
```

output
```
1/12*e^(3*b*x + a + 2*d)/b + 1/2*e^(b*x + a)/b - 1/4*e^(-b*x + a - 2*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int e^{a+bx}\cosh^2(d+bx)\,dx = \frac{\left(e^{(3\,bx+a+4\,d)} + 6\,e^{(bx+a+2\,d)} - 3\,e^{(-bx+a)}\right)e^{(-2\,d)}}{12\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^2,x, algorithm="giac")
```

output
```
1/12*(e^(3*b*x + a + 4*d) + 6*e^(b*x + a + 2*d) - 3*e^(-b*x + a))*e^(-2*d)
/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int e^{a+bx}\cosh^2(d+bx)\,dx = \frac{\mathrm{e}^{a-2\,d-b\,x}\,\left(6\,\mathrm{e}^{2\,d+2\,b\,x}+\mathrm{e}^{4\,d+4\,b\,x}-3\right)}{12\,b}$$

input
```
int(cosh(d + b*x)^2*exp(a + b*x),x)
```

output
```
(exp(a - 2*d - b*x)*(6*exp(2*d + 2*b*x) + exp(4*d + 4*b*x) - 3))/(12*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int e^{a+bx}\cosh^2(d+bx)\,dx = \frac{e^a\left(e^{4bx+4d}+6e^{2bx+2d}-3\right)}{12e^{bx+2d}b}$$

input
```
int(exp(b*x+a)*cosh(b*x+d)^2,x)
```

output
```
(e**a*(e**(4*b*x + 4*d) + 6*e**(2*b*x + 2*d) - 3))/(12*e**(b*x + 2*d)*b)
```

3.12 $\int e^{a+bx} \cosh(d+bx) \coth(d+bx) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int e^{a+bx} \cosh(d+bx) \coth(d+bx) \, dx = \frac{e^{a+d+2bx}}{4b} - \frac{1}{2} e^{a-d} x + \frac{e^{a-d} \log\left(1-e^{2d+2bx}\right)}{b}$$

output
```
1/4*exp(2*b*x+a+d)/b-1/2*exp(a-d)*x+exp(a-d)*ln(1-exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.65

$$\int e^{a+bx} \cosh(d+bx) \coth(d+bx) \, dx$$
$$= \frac{e^a \left(\cosh(d) \left(e^{2bx} - 2bx + 4\log\left(\left(-1+e^{2bx}\right)\cosh(d) + \left(1+e^{2bx}\right)\sinh(d)\right)\right) + \left(e^{2bx} + 2bx - 4\log\left(\left(-1+\right.\right.\right.}{4b}$$

input
```
Integrate[E^(a + b*x)*Cosh[d + b*x]*Coth[d + b*x],x]
```

output
```
(E^a*(Cosh[d]*(E^(2*b*x) - 2*b*x + 4*Log[(-1 + E^(2*b*x))*Cosh[d] + (1 + E
^(2*b*x))*Sinh[d]]) + (E^(2*b*x) + 2*b*x - 4*Log[(-1 + E^(2*b*x))*Cosh[d]
+ (1 + E^(2*b*x))*Sinh[d]])*Sinh[d]))/(4*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 27, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\cosh(bx+d)\coth(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{e^{a-bx}\left(1+e^{2bx}\right)^2}{2\left(1-e^{2bx}\right)}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^a\int \frac{e^{-bx}\left(1+e^{2bx}\right)^2}{1-e^{2bx}}de^{bx}}{2b}$$

$$\Big\downarrow \text{354}$$

$$-\frac{e^a\int \frac{e^{-bx}\left(1+e^{2bx}\right)^2}{1-e^{2bx}}de^{2bx}}{4b}$$

$$\Big\downarrow \text{93}$$

$$-\frac{e^a\int \left(e^{-bx}-1-\frac{4}{-1+e^{2bx}}\right)de^{2bx}}{4b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^a\left(-e^{2bx}+\log\left(e^{2bx}\right)-4\log\left(1-e^{2bx}\right)\right)}{4b}$$

input
```
Int[E^(a + b*x)*Cosh[d + b*x]*Coth[d + b*x],x]
```

output
```
-1/4*(E^a*(-E^(2*b*x) + Log[E^(2*b*x)] - 4*Log[1 - E^(2*b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

method	result	size
risch	$-\frac{{\mathrm e}^{a -d} x}{2}+\frac{{\mathrm e}^{2 b x +a +d}}{4 b}-\frac{2 \,{\mathrm e}^{a -d} a}{b}+\frac{\ln \left({\mathrm e}^{2 b x +2 a}-{\mathrm e}^{2 a -2 d}\right) {\mathrm e}^{a -d}}{b}$	67

input
```
int(exp(b*x+a)*cosh(b*x+d)*coth(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(a-d)*x+1/4*exp(2*b*x+a+d)/b-2/b*exp(a-d)*a+ln(exp(2*b*x+2*a)-exp(
2*a-2*d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(47) = 94$.

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.89

$$\int e^{a+bx}\cosh(d+bx)\coth(d+bx)\,dx =$$
$$-\frac{2\,bx\cosh\left(-a+d\right)-\cosh\left(bx+d\right)^{2}\cosh\left(-a+d\right)-\left(\cosh\left(-a+d\right)-\sinh\left(-a+d\right)\right)\sinh\left(bx+d\right)^{2}-}{}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)*coth(b*x+d),x, algorithm="fricas")
```

output
```
-1/4*(2*b*x*cosh(-a + d) - cosh(b*x + d)^2*cosh(-a + d) - (cosh(-a + d) -
sinh(-a + d))*sinh(b*x + d)^2 - 4*(cosh(-a + d) - sinh(-a + d))*log(2*sinh
(b*x + d)/(cosh(b*x + d) - sinh(b*x + d))) - 2*(cosh(b*x + d)*cosh(-a + d)
 - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d) - (2*b*x - cosh(b*x + d)^2)*s
inh(-a + d))/b
```

Sympy [F]

$$\int e^{a+bx}\cosh(d+bx)\coth(d+bx)\,dx = e^{a}\int e^{bx}\cosh\left(bx+d\right)\coth\left(bx+d\right)\,dx$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)*coth(b*x+d),x)
```

output
```
exp(a)*Integral(exp(b*x)*cosh(b*x + d)*coth(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

$$\int e^{a+bx}\cosh(d+bx)\coth(d+bx)\,dx = -\frac{(bx+a)e^{(a-d)}}{2\,b} + \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^{a}\right)}{b} + \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}-e^{a}\right)}{b} + \frac{e^{(2\,bx+a+d)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)*coth(b*x+d),x, algorithm="maxima")
```

output
```
-1/2*(b*x + a)*e^(a - d)/b + e^(a - d)*log(e^(b*x + a + d) + e^a)/b + e^(a
 - d)*log(e^(b*x + a + d) - e^a)/b + 1/4*e^(2*b*x + a + d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int e^{a+bx}\cosh(d+bx)\coth(d+bx)\,dx = -\frac{2\,(bx+d)e^{(a-d)} - 4\,e^{(a-d)}\log\left(\left|e^{(2\,bx+2\,d)}-1\right|\right) - e^{(2\,bx+a+d)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)*coth(b*x+d),x, algorithm="giac")
```

output
```
-1/4*(2*(b*x + d)*e^(a - d) - 4*e^(a - d)*log(abs(e^(2*b*x + 2*d) - 1)) -
e^(2*b*x + a + d))/b
```

Mupad [B] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int e^{a+bx}\cosh(d+bx)\coth(d+bx)\,dx = \frac{e^{2\,b\,x}\,e^{a}\,e^{d}}{4\,b}-\frac{x\,e^{-d}\,e^{a}}{2}+\frac{e^{-d}\,e^{a}\,\ln\left(e^{2\,a}\,e^{2\,b\,x}-e^{2\,a}\,e^{-2\,d}\right)}{b}$$

input
```
int(cosh(d + b*x)*coth(d + b*x)*exp(a + b*x),x)
```

output
```
(exp(2*b*x)*exp(a)*exp(d))/(4*b) - (x*exp(-d)*exp(a))/2 + (exp(-d)*exp(a)*
log(exp(2*a)*exp(2*b*x) - exp(2*a)*exp(-2*d)))/b
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int e^{a+bx}\cosh(d+bx)\coth(d+bx)\,dx = \frac{e^{a}\left(e^{2bx+2d}+4\log\left(e^{bx+d}-1\right)+4\log\left(e^{bx+d}+1\right)-2bx\right)}{4e^{d}b}$$

input
```
int(exp(b*x+a)*cosh(b*x+d)*coth(b*x+d),x)
```

output
```
(e**a*(e**(2*b*x + 2*d) + 4*log(e**(b*x + d) - 1) + 4*log(e**(b*x + d) + 1
) - 2*b*x))/(4*e**d*b)
```

3.13 $\int e^{a+bx}\coth^2(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 60

$$\int e^{a+bx}\coth^2(d+bx)\,dx=\frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2d+2bx}\right)}-\frac{2e^{a-d}\text{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
exp(b*x+a)/b+2*exp(b*x+a)/b/(1-exp(2*b*x+2*d))-2*exp(a-d)*arctanh(exp(b*x+
d))/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.65

$$\int e^{a+bx}\coth^2(d+bx)\,dx$$
$$=\frac{e^a\left(e^{bx}\left(\left(-3+e^{2bx}\right)\cosh(d)+\left(3+e^{2bx}\right)\sinh(d)\right)-2\text{arctanh}\left(e^{bx}(\cosh(d)+\sinh(d))\right)\left(e^{2bx}-\cosh^2(d)-\right.\right.}{b\left(\left(-1+e^{2bx}\right)\cosh(d)+\left(1+e^{2bx}\right)\sinh(d)\right)}$$

input
```
Integrate[E^(a + b*x)*Coth[d + b*x]^2,x]
```

output
```
(E^a*(E^(b*x)*((-3 + E^(2*b*x))*Cosh[d] + (3 + E^(2*b*x))*Sinh[d]) - 2*Arc
Tanh[E^(b*x)*(Cosh[d] + Sinh[d])]*(E^(2*b*x) - Cosh[d]^2 - Sinh[d]^2 + Sin
h[2*d])))/(b*((-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d]))
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 27, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\coth^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{e^a(1+e^{2bx})^2}{(1-e^{2bx})^2}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^a\int \frac{(1+e^{2bx})^2}{(1-e^{2bx})^2}de^{bx}}{b}$$

$$\Big\downarrow \text{300}$$

$$\frac{e^a\int \left(1+\frac{4e^{2bx}}{(1-e^{2bx})^2}\right)de^{bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^a\left(-2\text{arctanh}\left(e^{bx}\right)+e^{bx}+\frac{2e^{bx}}{1-e^{2bx}}\right)}{b}$$

input
```
Int[E^(a + b*x)*Coth[d + b*x]^2,x]
```

output
```
(E^a*(E^(b*x) + (2*E^(b*x))/(1 - E^(2*b*x)) - 2*ArcTanh[E^(b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

method	result	size
risch	$\frac{e^{bx+a}}{b}+\frac{2\,e^{bx+3a}}{\left(-e^{2bx+2a+2d}+e^{2a}\right)b}+\frac{\ln\left(e^{bx+a}-e^{a-d}\right)e^{a-d}}{b}-\frac{\ln\left(e^{bx+a}+e^{a-d}\right)e^{a-d}}{b}$	97

input
```
int(exp(b*x+a)*coth(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
exp(b*x+a)/b+2/(-exp(2*b*x+2*a+2*d)+exp(2*a))/b*exp(b*x+3*a)+ln(exp(b*x+a)
-exp(a-d))/b*exp(a-d)-ln(exp(b*x+a)+exp(a-d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(53) = 106$.

Time = 0.09 (sec) , antiderivative size = 446, normalized size of antiderivative = 7.43

$$\int e^{a+bx}\coth^2(d+bx)\,dx$$
$$= \frac{\cosh(bx+d)^3\cosh(-a+d) + (\cosh(-a+d) - \sinh(-a+d))\sinh(bx+d)^3 + 3(\cosh(bx+d)\cosh(-$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)^2,x, algorithm="fricas")
```

output
```
(cosh(b*x + d)^3*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d
)^3 + 3*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x
 + d)^2 - 3*cosh(b*x + d)*cosh(-a + d) - (cosh(b*x + d)^2*cosh(-a + d) + (
cosh(-a + d) - sinh(-a + d))*sinh(b*x + d)^2 + 2*(cosh(b*x + d)*cosh(-a +
d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^2 - 1)*sin
h(-a + d) - cosh(-a + d))*log(cosh(b*x + d) + sinh(b*x + d) + 1) + (cosh(b
*x + d)^2*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d)^2 + 2
*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d) -
 (cosh(b*x + d)^2 - 1)*sinh(-a + d) - cosh(-a + d))*log(cosh(b*x + d) + si
nh(b*x + d) - 1) + 3*(cosh(b*x + d)^2*cosh(-a + d) - (cosh(b*x + d)^2 - 1)
*sinh(-a + d) - cosh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^3 - 3*cosh(b*
x + d))*sinh(-a + d))/(b*cosh(b*x + d)^2 + 2*b*cosh(b*x + d)*sinh(b*x + d)
 + b*sinh(b*x + d)^2 - b)
```

Sympy [F]

$$\int e^{a+bx}\coth^2(d+bx)\,dx = e^a\int e^{bx}\coth^2(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)**2,x)
```

output
```
exp(a)*Integral(exp(b*x)*coth(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.50

$$\int e^{a+bx}\coth^2(d+bx)\,dx = -\frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^a\right)}{b} + \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}-e^a\right)}{b} + \frac{e^{(bx+a)}}{b} - \frac{2\,e^{(bx+3\,a)}}{b\left(e^{(2\,bx+2\,a+2\,d)}-e^{(2\,a)}\right)}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)^2,x, algorithm="maxima")
```

output
```
-e^(a - d)*log(e^(b*x + a + d) + e^a)/b + e^(a - d)*log(e^(b*x + a + d) -
e^a)/b + e^(b*x + a)/b - 2*e^(b*x + 3*a)/(b*(e^(2*b*x + 2*a + 2*d) - e^(2*
a)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.42

$$\int e^{a+bx}\coth^2(d+bx)\,dx = -\frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^a\right) - e^{(a-d)}\log\left(\left|e^{(bx+a+d)}-e^a\right|\right) + \frac{2\,e^{(bx+3\,a)}}{e^{(2\,bx+2\,a+2\,d)}-e^{(2\,a)}} - e^{(bx+a)}}{b}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)^2,x, algorithm="giac")
```

output
```
-(e^(a - d)*log(e^(b*x + a + d) + e^a) - e^(a - d)*log(abs(e^(b*x + a + d)
 - e^a)) + 2*e^(b*x + 3*a)/(e^(2*b*x + 2*a + 2*d) - e^(2*a)) - e^(b*x + a)
)/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.65

$$\int e^{a+bx}\coth^2(d+bx)\,dx = \frac{e^{bx}\,e^{a}}{b} - \frac{2\,\mathrm{atan}\left(\frac{e^{bx}\,e^{a}\,\sqrt{-b^2}}{b\,\sqrt{e^{2a}\,e^{-2d}}}\right)\,\sqrt{e^{2a}\,e^{-2d}}}{\sqrt{-b^2}} + \frac{2\,e^{3a}\,e^{-2d}\,e^{bx}}{b\,e^{2a}\,e^{-2d} - b\,e^{2a}\,e^{2bx}}$$

input
```
int(coth(d + b*x)^2*exp(a + b*x),x)
```

output
```
(exp(b*x)*exp(a))/b - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/(b*(exp(2*a)*
exp(-2*d))^(1/2)))*(exp(2*a)*exp(-2*d))^(1/2))/(-b^2)^(1/2) + (2*exp(3*a)*
exp(-2*d)*exp(b*x))/(b*exp(2*a)*exp(-2*d) - b*exp(2*a)*exp(2*b*x))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.85

$$\int e^{a+bx}\coth^2(d+bx)\,dx$$
$$= \frac{e^{a}\left(e^{3bx+3d} + e^{2bx+2d}\log\left(e^{bx+d}-1\right) - e^{2bx+2d}\log\left(e^{bx+d}+1\right) - 3e^{bx+d} - \log\left(e^{bx+d}-1\right) + \log\left(e^{bx+d}+1\right)\right)}{e^{d}b\left(e^{2bx+2d}-1\right)}$$

input
```
int(exp(b*x+a)*coth(b*x+d)^2,x)
```

output
```
(e**a*(e**(3*b*x + 3*d) + e**(2*b*x + 2*d)*log(e**(b*x + d) - 1) - e**(2*b
*x + 2*d)*log(e**(b*x + d) + 1) - 3*e**(b*x + d) - log(e**(b*x + d) - 1) +
 log(e**(b*x + d) + 1)))/(e**d*b*(e**(2*b*x + 2*d) - 1))
```

3.14 $\int e^{a+bx} \coth^2(d+bx)\mathbf{csch}(d+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int e^{a+bx}\coth^2(d+bx)\text{csch}(d+bx)\,dx=-\frac{2e^{a-d}}{b\left(1-e^{2d+2bx}\right)^2}+\frac{4e^{a-d}}{b\left(1-e^{2d+2bx}\right)}+\frac{e^{a-d}\log\left(1-e^{2d+2bx}\right)}{b}$$

output
```
-2*exp(a-d)/b/(1-exp(2*b*x+2*d))^2+4*exp(a-d)/b/(1-exp(2*b*x+2*d))+exp(a-d
)*ln(1-exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int e^{a+bx}\coth^2(d+bx)\text{csch}(d+bx)\,dx=\frac{2e^{a-d}\left(-\frac{1}{\left(1-e^{2(d+bx)}\right)^2}+\frac{2}{1-e^{2(d+bx)}}+\frac{1}{2}\log\left(1-e^{2(d+bx)}\right)\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Coth[d + b*x]^2*Csch[d + b*x],x]
```

output
```
(2*E^(a - d)*(-(1 - E^(2*(d + b*x)))^(-2) + 2/(1 - E^(2*(d + b*x))) + Log[
1 - E^(2*(d + b*x))]/2))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.60, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2720, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx} \coth^2(bx+d)\text{csch}(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{2e^{a+bx}(1+e^{2bx})^2}{(1-e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2e^a \int \frac{e^{bx}(1+e^{2bx})^2}{(1-e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{353}$$

$$-\frac{e^a \int \frac{(1+e^{2bx})^2}{(1-e^{2bx})^3}de^{2bx}}{b}$$

$$\Big\downarrow \text{49}$$

$$-\frac{e^a \int \left(-\frac{4}{(-1+e^{2bx})^2}-\frac{4}{(-1+e^{2bx})^3}+\frac{1}{1-e^{2bx}}\right)de^{2bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^a\left(-\frac{4}{1-e^{2bx}}+\frac{2}{(1-e^{2bx})^2}-\log\left(1-e^{2bx}\right)\right)}{b}$$

input
```
Int[E^(a + b*x)*Coth[d + b*x]^2*Csch[d + b*x],x]
```

output
```
-((E^a*(2/(1 - E^(2*b*x))^2 - 4/(1 - E^(2*b*x)) - Log[1 - E^(2*b*x)]))/b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18

method	result	size
risch	$-\frac{2\,\mathrm{e}^{a-d} a}{b}+\frac{2\left(-2\,\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)\mathrm{e}^{3a-d}}{\left(-\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)^{2} b}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-\mathrm{e}^{2a-2d}\right)\mathrm{e}^{a-d}}{b}$	98

input
```
int(exp(b*x+a)*coth(b*x+d)^2*csch(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
-2/b*exp(a-d)*a+2/(-exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(-2*exp(2*b*x+2*a+2*d
)+exp(2*a))*exp(3*a-d)+ln(exp(2*b*x+2*a)-exp(2*a-2*d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(73) = 146$.

Time = 0.09 (sec) , antiderivative size = 487, normalized size of antiderivative = 5.87

$$\int e^{a+bx}\coth^2(d+bx)\operatorname{csch}(d+bx)\,dx =$$
$$-\frac{4\cosh(bx+d)^2\cosh(-a+d)+4\left(\cosh(-a+d)-\sinh(-a+d)\right)\sinh(bx+d)^2-\left(\cosh(bx+d)^4\right.}{}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)^2*csch(b*x+d),x, algorithm="fricas")
```

output
```
-(4*cosh(b*x + d)^2*cosh(-a + d) + 4*(cosh(-a + d) - sinh(-a + d))*sinh(b*
x + d)^2 - (cosh(b*x + d)^4*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*s
inh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d
))*sinh(b*x + d)^3 - 2*cosh(b*x + d)^2*cosh(-a + d) + 2*(3*cosh(b*x + d)^2
*cosh(-a + d) - (3*cosh(b*x + d)^2 - 1)*sinh(-a + d) - cosh(-a + d))*sinh(
b*x + d)^2 + 4*(cosh(b*x + d)^3*cosh(-a + d) - cosh(b*x + d)*cosh(-a + d)
- (cosh(b*x + d)^3 - cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d) - (cosh(b*
x + d)^4 - 2*cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*log(2*sinh(
b*x + d)/(cosh(b*x + d) - sinh(b*x + d))) + 8*(cosh(b*x + d)*cosh(-a + d)
- cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d) - 2*(2*cosh(b*x + d)^2 - 1)*si
nh(-a + d) - 2*cosh(-a + d))/(b*cosh(b*x + d)^4 + 4*b*cosh(b*x + d)*sinh(b
*x + d)^3 + b*sinh(b*x + d)^4 - 2*b*cosh(b*x + d)^2 + 2*(3*b*cosh(b*x + d)
^2 - b)*sinh(b*x + d)^2 + 4*(b*cosh(b*x + d)^3 - b*cosh(b*x + d))*sinh(b*x
 + d) + b)
```

Sympy [F]

$$\int e^{a+bx}\coth^2(d+bx)\text{csch}(d+bx)\,dx = e^{a}\int e^{bx}\coth^{2}\left(bx+d\right)\text{csch}\left(bx+d\right)dx$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)**2*csch(b*x+d),x)
```

output
```
exp(a)*Integral(exp(b*x)*coth(b*x + d)**2*csch(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.28

$$\int e^{a+bx}\coth^2(d+bx)\text{csch}(d+bx)\,dx = \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^{a}\right)}{b} + \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}-e^{a}\right)}{b} - \frac{2\left(2\,e^{(2\,bx+5\,a+2\,d)}-e^{(5\,a)}\right)}{b(e^{(4\,bx+4\,a+5\,d)}-2\,e^{(2\,bx+4\,a+3\,d)}+e^{(4\,a+d)})}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)^2*csch(b*x+d),x, algorithm="maxima")
```

output
```
e^(a - d)*log(e^(b*x + a + d) + e^a)/b + e^(a - d)*log(e^(b*x + a + d) - e
^a)/b - 2*(2*e^(2*b*x + 5*a + 2*d) - e^(5*a))/(b*(e^(4*b*x + 4*a + 5*d) -
2*e^(2*b*x + 4*a + 3*d) + e^(4*a + d)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17

$$\int e^{a+bx}\coth^2(d+bx)\text{csch}(d+bx)\,dx$$
$$=\frac{2\,e^{(a-d)}\log\left(\left|e^{(2\,bx+2\,a+2\,d)}-e^{(2\,a)}\right|\right)-\frac{\left(3\,e^{(4\,bx+5\,a+4\,d)}+2\,e^{(2\,bx+5\,a+2\,d)}-e^{(5\,a)}\right)e^{(-d)}}{\left(e^{(2\,bx+2\,a+2\,d)}-e^{(2\,a)}\right)^2}}{2\,b}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)^2*csch(b*x+d),x, algorithm="giac")
```

output
```
1/2*(2*e^(a - d)*log(abs(e^(2*b*x + 2*a + 2*d) - e^(2*a))) - (3*e^(4*b*x +
 5*a + 4*d) + 2*e^(2*b*x + 5*a + 2*d) - e^(5*a))*e^(-d)/(e^(2*b*x + 2*a +
2*d) - e^(2*a))^2)/b
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\coth^2(d+bx)\text{csch}(d+bx)\,dx=\int \frac{{\mathrm{coth}\left(d+b\,x\right)}^2\,{\mathrm{e}}^{a+b\,x}}{\mathrm{sinh}\left(d+b\,x\right)}\,dx$$

input
```
int((coth(d + b*x)^2*exp(a + b*x))/sinh(d + b*x),x)
```

output
```
int((coth(d + b*x)^2*exp(a + b*x))/sinh(d + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.89

$$\int e^{a+bx}\coth^2(d+bx)\text{csch}(d+bx)\,dx$$
$$= \frac{e^a\left(e^{4bx+4d}\log\left(e^{bx+d}-1\right)+e^{4bx+4d}\log\left(e^{bx+d}+1\right)-2e^{4bx+4d}-2e^{2bx+2d}\log\left(e^{bx+d}-1\right)-2e^{2bx+2d}\log\left(e^{bx+\cdots}\right.\right.}{e^d b\left(e^{4bx+4d}-2e^{2bx+2d}+1\right)}$$

input
```
int(exp(b*x+a)*coth(b*x+d)^2*csch(b*x+d),x)
```

output
```
(e**a*(e**(4*b*x + 4*d)*log(e**(b*x + d) - 1) + e**(4*b*x + 4*d)*log(e**(b
*x + d) + 1) - 2*e**(4*b*x + 4*d) - 2*e**(2*b*x + 2*d)*log(e**(b*x + d) -
1) - 2*e**(2*b*x + 2*d)*log(e**(b*x + d) + 1) + log(e**(b*x + d) - 1) + lo
g(e**(b*x + d) + 1)))/(e**d*b*(e**(4*b*x + 4*d) - 2*e**(2*b*x + 2*d) + 1))
```

3.15 $\int e^{a+bx}\cosh^3(d+bx)\sinh^3(d+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 73

$$\int e^{a+bx}\cosh^3(d+bx)\sinh^3(d+bx)\,dx=\frac{e^{a-6d-5bx}}{320b}-\frac{3e^{a-2d-bx}}{64b}-\frac{e^{a+2d+3bx}}{64b}+\frac{e^{a+6d+7bx}}{448b}$$

output
```
1/320*exp(-5*b*x+a-6*d)/b-3/64*exp(-b*x+a-2*d)/b-1/64*exp(3*b*x+a+2*d)/b+1
/448*exp(7*b*x+a+6*d)/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.73

$$\int e^{a+bx}\cosh^3(d+bx)\sinh^3(d+bx)\,dx=\frac{e^{a-6d-5bx}\left(7-105e^{4(d+bx)}-35e^{8(d+bx)}+5e^{12(d+bx)}\right)}{2240b}$$

input
```
Integrate[E^(a + b*x)*Cosh[d + b*x]^3*Sinh[d + b*x]^3,x]
```

output
```
(E^(a - 6*d - 5*b*x)*(7 - 105*E^(4*(d + b*x)) - 35*E^(8*(d + b*x)) + 5*E^(
12*(d + b*x))))/(2240*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh^3(bx+d)\cosh^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{1}{64}e^{a-6bx}\left(1-e^{4bx}\right)^3de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^a\int e^{-6bx}\left(1-e^{4bx}\right)^3de^{bx}}{64b}$$

$$\Big\downarrow \text{802}$$

$$-\frac{e^a\int \left(e^{-6bx}-3e^{-2bx}+3e^{2bx}-e^{6bx}\right)de^{bx}}{64b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^a\left(-\frac{1}{5}e^{-5bx}+3e^{-bx}+e^{3bx}-\frac{1}{7}e^{7bx}\right)}{64b}$$

input
```
Int[E^(a + b*x)*Cosh[d + b*x]^3*Sinh[d + b*x]^3,x]
```

output
```
-1/64*(E^a*(-1/5*1/E^(5*b*x) + 3/E^(b*x) + E^(3*b*x) - E^(7*b*x)/7))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.67

$$\frac{\sinh\left(-5bx+a-6d\right)}{320b}-\frac{3\sinh\left(-bx+a-2d\right)}{64b}-\frac{\sinh\left(3bx+a+2d\right)}{64b}+\frac{\sinh\left(7bx+a+6d\right)}{448b}+\frac{\cosh\left(-5bx+\right.}{320b}$$

input
```
int(exp(b*x+a)*cosh(b*x+d)^3*sinh(b*x+d)^3,x)
```

output
```
1/320/b*sinh(-5*b*x+a-6*d)-3/64/b*sinh(-b*x+a-2*d)-1/64/b*sinh(3*b*x+a+2*d
)+1/448/b*sinh(7*b*x+a+6*d)+1/320*cosh(-5*b*x+a-6*d)/b-3/64*cosh(-b*x+a-2*
d)/b-1/64*cosh(3*b*x+a+2*d)/b+1/448*cosh(7*b*x+a+6*d)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(61) = 122$.

Time = 0.08 (sec) , antiderivative size = 352, normalized size of antiderivative = 4.82

$$\int e^{a+bx}\cosh^3(d+bx)\sinh^3(d+bx)\,dx$$
$$= \frac{3\,\cosh\left(bx+d\right)^{6}\cosh\left(-a+d\right)+3\left(\cosh\left(-a+d\right)-\sinh\left(-a+d\right)\right)\sinh\left(bx+d\right)^{6}-3\left(\cosh\left(bx+d\right)\cos\right.}{}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^3*sinh(b*x+d)^3,x, algorithm="fricas")
```

output
```
1/560*(3*cosh(b*x + d)^6*cosh(-a + d) + 3*(cosh(-a + d) - sinh(-a + d))*si
nh(b*x + d)^6 - 3*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d)
)*sinh(b*x + d)^5 + 45*(cosh(b*x + d)^2*cosh(-a + d) - cosh(b*x + d)^2*sin
h(-a + d))*sinh(b*x + d)^4 - 10*(cosh(b*x + d)^3*cosh(-a + d) - cosh(b*x +
 d)^3*sinh(-a + d))*sinh(b*x + d)^3 - 35*cosh(b*x + d)^2*cosh(-a + d) + 5*
(9*cosh(b*x + d)^4*cosh(-a + d) - (9*cosh(b*x + d)^4 - 7)*sinh(-a + d) - 7
*cosh(-a + d))*sinh(b*x + d)^2 - (3*cosh(b*x + d)^5*cosh(-a + d) - 35*cosh
(b*x + d)*cosh(-a + d) - (3*cosh(b*x + d)^5 - 35*cosh(b*x + d))*sinh(-a +
d))*sinh(b*x + d) - (3*cosh(b*x + d)^6 - 35*cosh(b*x + d)^2)*sinh(-a + d))
/(b*cosh(b*x + d) - b*sinh(b*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(60) = 120$.

Time = 12.59 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.77

$$\int e^{a+bx}\cosh^3(d+bx)\sinh^3(d+bx)\,dx$$
$$= \begin{cases} -\frac{2e^{a}e^{bx}\sinh^{6}\left(bx+d\right)}{35b}+\frac{2e^{a}e^{bx}\sinh^{5}\left(bx+d\right)\cosh\left(bx+d\right)}{35b}+\frac{e^{a}e^{bx}\sinh^{4}\left(bx+d\right)\cosh^{2}\left(bx+d\right)}{7b}-\frac{e^{a}e^{bx}\sinh^{3}\left(bx+d\right)\cosh^{3}\left(bx+d\right)}{7b}+ \\ xe^{a}\sinh^{3}\left(d\right)\cosh^{3}\left(d\right) \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)**3*sinh(b*x+d)**3,x)
```

output
```
Piecewise((-2*exp(a)*exp(b*x)*sinh(b*x + d)**6/(35*b) + 2*exp(a)*exp(b*x)*
sinh(b*x + d)**5*cosh(b*x + d)/(35*b) + exp(a)*exp(b*x)*sinh(b*x + d)**4*c
osh(b*x + d)**2/(7*b) - exp(a)*exp(b*x)*sinh(b*x + d)**3*cosh(b*x + d)**3/
(7*b) + exp(a)*exp(b*x)*sinh(b*x + d)**2*cosh(b*x + d)**4/(7*b) + 2*exp(a)
*exp(b*x)*sinh(b*x + d)*cosh(b*x + d)**5/(35*b) - 2*exp(a)*exp(b*x)*cosh(b
*x + d)**6/(35*b), Ne(b, 0)), (x*exp(a)*sinh(d)**3*cosh(d)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int e^{a+bx}\cosh^3(d+bx)\sinh^3(d+bx)\,dx = -\frac{\left(15\,e^{(4\,bx+6\,a+4\,d)}-e^{(6\,a)}\right)e^{(-5\,bx-5\,a-6\,d)}}{320\,b} + \frac{\left(e^{(7\,bx+7\,a+6\,d)}-7\,e^{(3\,bx+7\,a+2\,d)}\right)e^{(-6\,a)}}{448\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^3*sinh(b*x+d)^3,x, algorithm="maxima")
```

output
```
-1/320*(15*e^(4*b*x + 6*a + 4*d) - e^(6*a))*e^(-5*b*x - 5*a - 6*d)/b + 1/4
48*(e^(7*b*x + 7*a + 6*d) - 7*e^(3*b*x + 7*a + 2*d))*e^(-6*a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int e^{a+bx}\cosh^3(d+bx)\sinh^3(d+bx)\,dx = -\frac{\left(7\left(15\,e^{(4\,bx+a+4\,d)}-e^{a}\right)e^{(-5\,bx)}-5\,e^{(7\,bx+a+12\,d)}+35\,e^{(3\,bx+a+8\,d)}\right)e^{(-6\,d)}}{2240\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^3*sinh(b*x+d)^3,x, algorithm="giac")
```

output
```
-1/2240*(7*(15*e^(4*b*x + a + 4*d) - e^a)*e^(-5*b*x) - 5*e^(7*b*x + a + 12
*d) + 35*e^(3*b*x + a + 8*d))*e^(-6*d)/b
```

Mupad [B] (verification not implemented)

Time = 3.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int e^{a+bx}\cosh^3(d+bx)\sinh^3(d+bx)\,dx = \frac{e^{-6d}\,e^{-5bx}\,e^a}{320\,b} - \frac{e^{2d}\,e^{3bx}\,e^a}{64\,b} - \frac{3\,e^{-2d}\,e^{-bx}\,e^a}{64\,b} + \frac{e^{6d}\,e^{7bx}\,e^a}{448\,b}$$

input
```
int(cosh(d + b*x)^3*exp(a + b*x)*sinh(d + b*x)^3,x)
```

output
```
(exp(-6*d)*exp(-5*b*x)*exp(a))/(320*b) - (exp(2*d)*exp(3*b*x)*exp(a))/(64*
b) - (3*exp(-2*d)*exp(-b*x)*exp(a))/(64*b) + (exp(6*d)*exp(7*b*x)*exp(a))/
(448*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int e^{a+bx}\cosh^3(d+bx)\sinh^3(d+bx)\,dx = \frac{e^a\left(5e^{12bx+12d}-35e^{8bx+8d}-105e^{4bx+4d}+7\right)}{2240e^{5bx+6d}b}$$

input
```
int(exp(b*x+a)*cosh(b*x+d)^3*sinh(b*x+d)^3,x)
```

output
```
(e**a*(5*e**(12*b*x + 12*d) - 35*e**(8*b*x + 8*d) - 105*e**(4*b*x + 4*d) +
 7))/(2240*e**(5*b*x + 6*d)*b)
```

3.16 $\int e^{a+bx}\cosh^3(d+bx)\sinh^2(d+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 101

$$\int e^{a+bx}\cosh^3(d+bx)\sinh^2(d+bx)\,dx = -\frac{e^{a-5d-4bx}}{128b}-\frac{e^{a-3d-2bx}}{64b}-\frac{e^{a+d+2bx}}{32b}+\frac{e^{a+3d+4bx}}{128b}+\frac{e^{a+5d+6bx}}{192b}-\frac{1}{16}e^{a-d}x$$

output
```
-1/128*exp(-4*b*x+a-5*d)/b-1/64*exp(-2*b*x+a-3*d)/b-1/32*exp(2*b*x+a+d)/b+
1/128*exp(4*b*x+a+3*d)/b+1/192*exp(6*b*x+a+5*d)/b-1/16*exp(a-d)*x
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.14

$$\int e^{a+bx}\cosh^3(d+bx)\sinh^2(d+bx)\,dx$$
$$= \frac{e^a\left(-12\left(\left(e^{2bx}+2bx\right)\cosh(d)+\left(e^{2bx}-2bx\right)\sinh(d)\right)+3e^{-2bx}\left(\left(-2+e^{6bx}\right)\cosh(3d)+\left(2+e^{6bx}\right)\sinh(3\right.\right.}{384b}$$

input
```
Integrate[E^(a + b*x)*Cosh[d + b*x]^3*Sinh[d + b*x]^2,x]
```

output
```
(E^a*(-12*((E^(2*b*x) + 2*b*x)*Cosh[d] + (E^(2*b*x) - 2*b*x)*Sinh[d]) + (3
*((-2 + E^(6*b*x))*Cosh[3*d] + (2 + E^(6*b*x))*Sinh[3*d]))/E^(2*b*x) + ((-
3 + 2*E^(10*b*x))*Cosh[5*d] + (3 + 2*E^(10*b*x))*Sinh[5*d])/E^(4*b*x)))/(3
84*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.57, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2720, 27, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh^2(bx+d)\cosh^3(bx+d)\,dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int \frac{1}{32}e^{a-5bx}\left(1-e^{2bx}\right)^2\left(1+e^{2bx}\right)^3de^{bx}}{b}$$

$$\Big\downarrow \ 27$$

$$\frac{e^a\int e^{-5bx}\left(1-e^{2bx}\right)^2\left(1+e^{2bx}\right)^3de^{bx}}{32b}$$

$$\Big\downarrow \ 354$$

$$\frac{e^a\int e^{-3bx}\left(1-e^{2bx}\right)^2\left(1+e^{2bx}\right)^3de^{2bx}}{64b}$$

$$\Big\downarrow \ 99$$

$$\frac{e^a\int \left(-2+e^{-3bx}+e^{-2bx}-2e^{-bx}+2e^{2bx}\right)de^{2bx}}{64b}$$

$$\Big\downarrow \ 2009$$

$$\frac{e^a\left(-\frac{1}{2}e^{-2bx}-e^{-bx}-\frac{3}{2}e^{2bx}+\frac{1}{3}e^{3bx}-2\log\left(e^{2bx}\right)\right)}{64b}$$

input
```
Int[E^(a + b*x)*Cosh[d + b*x]^3*Sinh[d + b*x]^2,x]
```

output
```
(E^a*(-1/2*1/E^(2*b*x) - E^(-(b*x)) - (3*E^(2*b*x))/2 + E^(3*b*x)/3 - 2*Lo
g[E^(2*b*x)]))/(64*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.64

$$-\frac{\cosh \left(a-d\right) x}{16}-\frac{\sinh \left(-4bx+a-5d\right)}{128b}-\frac{\sinh \left(-2bx+a-3d\right)}{64b}-\frac{\sinh \left(2bx+a+d\right)}{32b}+\frac{\sinh \left(4bx+a+3d\right)}{128b}$$

input
```
int(exp(b*x+a)*cosh(b*x+d)^3*sinh(b*x+d)^2,x)
```

output
```
-1/16*cosh(a-d)*x-1/128/b*sinh(-4*b*x+a-5*d)-1/64/b*sinh(-2*b*x+a-3*d)-1/3
2/b*sinh(2*b*x+a+d)+1/128/b*sinh(4*b*x+a+3*d)+1/192/b*sinh(6*b*x+a+5*d)-1/
16*sinh(a-d)*x-1/128*cosh(-4*b*x+a-5*d)/b-1/64*cosh(-2*b*x+a-3*d)/b-1/32*c
osh(2*b*x+a+d)/b+1/128*cosh(4*b*x+a+3*d)/b+1/192*cosh(6*b*x+a+5*d)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(83) = 166$.

Time = 0.08 (sec) , antiderivative size = 389, normalized size of antiderivative = 3.85

$$\int e^{a+bx}\cosh^3(d+bx)\sinh^2(d+bx)\,dx =$$

$$-\frac{\cosh \left(bx+d\right)^5 \cosh \left(-a+d\right)-5\left(\cosh \left(-a+d\right)-\sinh \left(-a+d\right)\right)\sinh \left(bx+d\right)^5+5\left(\cosh \left(bx+d\right)\mathrm{co}\right.}{}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^3*sinh(b*x+d)^2,x, algorithm="fricas")
```

output
```
-1/384*(cosh(b*x + d)^5*cosh(-a + d) - 5*(cosh(-a + d) - sinh(-a + d))*sin
h(b*x + d)^5 + 5*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))
*sinh(b*x + d)^4 + 3*cosh(b*x + d)^3*cosh(-a + d) - (50*cosh(b*x + d)^2*co
sh(-a + d) - (50*cosh(b*x + d)^2 + 9)*sinh(-a + d) + 9*cosh(-a + d))*sinh(
b*x + d)^3 + 12*(2*b*x + 1)*cosh(b*x + d)*cosh(-a + d) + (10*cosh(b*x + d)
^3*cosh(-a + d) + 9*cosh(b*x + d)*cosh(-a + d) - (10*cosh(b*x + d)^3 + 9*c
osh(b*x + d))*sinh(-a + d))*sinh(b*x + d)^2 - (25*cosh(b*x + d)^4*cosh(-a
+ d) + 27*cosh(b*x + d)^2*cosh(-a + d) + 12*(2*b*x - 1)*cosh(-a + d) - (25
*cosh(b*x + d)^4 + 24*b*x + 27*cosh(b*x + d)^2 - 12)*sinh(-a + d))*sinh(b*
x + d) - (cosh(b*x + d)^5 + 3*cosh(b*x + d)^3 + 12*(2*b*x + 1)*cosh(b*x +
d))*sinh(-a + d))/(b*cosh(b*x + d) - b*sinh(b*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(82) = 164$.

Time = 5.31 (sec) , antiderivative size = 325, normalized size of antiderivative = 3.22

$$\int e^{a+bx}\cosh^3(d+bx)\sinh^2(d+bx)\,dx$$
$$= \begin{cases} \frac{xe^ae^{bx}\sinh^5(bx+d)}{16} - \frac{xe^ae^{bx}\sinh^4(bx+d)\cosh(bx+d)}{16} - \frac{xe^ae^{bx}\sinh^3(bx+d)\cosh^2(bx+d)}{8} + \frac{xe^ae^{bx}\sinh^2(bx+d)\cosh^3(bx+d)}{8} + \\ xe^a\sinh^2(d)\cosh^3(d) \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)**3*sinh(b*x+d)**2,x)
```

output
```
Piecewise((x*exp(a)*exp(b*x)*sinh(b*x + d)**5/16 - x*exp(a)*exp(b*x)*sinh(
b*x + d)**4*cosh(b*x + d)/16 - x*exp(a)*exp(b*x)*sinh(b*x + d)**3*cosh(b*x
 + d)**2/8 + x*exp(a)*exp(b*x)*sinh(b*x + d)**2*cosh(b*x + d)**3/8 + x*exp
(a)*exp(b*x)*sinh(b*x + d)*cosh(b*x + d)**4/16 - x*exp(a)*exp(b*x)*cosh(b*
x + d)**5/16 - 13*exp(a)*exp(b*x)*sinh(b*x + d)**5/(96*b) + 7*exp(a)*exp(b
*x)*sinh(b*x + d)**4*cosh(b*x + d)/(96*b) + exp(a)*exp(b*x)*sinh(b*x + d)*
*3*cosh(b*x + d)**2/(3*b) - exp(a)*exp(b*x)*sinh(b*x + d)**2*cosh(b*x + d)
**3/(6*b) + exp(a)*exp(b*x)*sinh(b*x + d)*cosh(b*x + d)**4/(96*b) + 5*exp(
a)*exp(b*x)*cosh(b*x + d)**5/(96*b), Ne(b, 0)), (x*exp(a)*sinh(d)**2*cosh(
d)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02

$$\int e^{a+bx}\cosh^3(d+bx)\sinh^2(d+bx)\,dx$$
$$= -\frac{\left(2\,e^{(2\,bx+5\,a+2\,d)} + e^{(5\,a)}\right)e^{(-4\,bx-4\,a-5\,d)}}{128\,b} + \frac{\left(2\,e^{(6\,bx+6\,a+5\,d)} + 3\,e^{(4\,bx+6\,a+3\,d)} - 12\,e^{(2\,bx+6\,a+d)}\right)e^{(-5\,a)}}{384\,b} - \frac{(bx+a)e^{(a-d)}}{16\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^3*sinh(b*x+d)^2,x, algorithm="maxima")
```

output
```
-1/128*(2*e^(2*b*x + 5*a + 2*d) + e^(5*a))*e^(-4*b*x - 4*a - 5*d)/b + 1/38
4*(2*e^(6*b*x + 6*a + 5*d) + 3*e^(4*b*x + 6*a + 3*d) - 12*e^(2*b*x + 6*a +
 d))*e^(-5*a)/b - 1/16*(b*x + a)*e^(a - d)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

$$\int e^{a+bx}\cosh^3(d+bx)\sinh^2(d+bx)\,dx = -\frac{\left(24\,bxe^{(a+4\,d)}-3\left(6\,e^{(4\,bx+a+4\,d)}-2\,e^{(2\,bx+a+2\,d)}-e^{a}\right)e^{(-4\,bx)}-2\,e^{(6\,bx+a+10\,d)}-3\,e^{(4\,bx+a+8\,d)}+12\,e^{(2\,bx}\right.}{384\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^3*sinh(b*x+d)^2,x, algorithm="giac")
```

output
```
-1/384*(24*b*x*e^(a + 4*d) - 3*(6*e^(4*b*x + a + 4*d) - 2*e^(2*b*x + a + 2
*d) - e^a)*e^(-4*b*x) - 2*e^(6*b*x + a + 10*d) - 3*e^(4*b*x + a + 8*d) + 1
2*e^(2*b*x + a + 6*d))*e^(-5*d)/b
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.35

$$\int e^{a+bx}\cosh^3(d+bx)\sinh^2(d+bx)\,dx = \frac{\cosh(d+b\,x)^3\,\mathrm{e}^{a+b\,x}}{48\,b}-\frac{\cosh(d+b\,x)^5\,\mathrm{e}^{a+b\,x}}{24\,b}-\frac{x\cosh(d+b\,x)\,\mathrm{e}^{a+b\,x}}{16}+\frac{x\,\mathrm{e}^{a+b\,x}\sinh(d+b\,x)}{16}-\frac{\cosh(d+b\,x)\,\mathrm{e}^{a+b\,x}}{16\,b}-\frac{\cosh(d+b\,x)^2\,\mathrm{e}^{a+b\,x}\sinh(d+b\,x)}{16\,b}+\frac{5\cosh(d+b\,x)^4\,\mathrm{e}^{a+b\,x}\sinh(d+b\,x)}{24\,b}$$

input
```
int(cosh(d + b*x)^3*exp(a + b*x)*sinh(d + b*x)^2,x)
```

output
```
(cosh(d + b*x)^3*exp(a + b*x))/(48*b) - (cosh(d + b*x)^5*exp(a + b*x))/(24
*b) - (x*cosh(d + b*x)*exp(a + b*x))/16 + (x*exp(a + b*x)*sinh(d + b*x))/1
6 - (cosh(d + b*x)*exp(a + b*x))/(16*b) - (cosh(d + b*x)^2*exp(a + b*x)*si
nh(d + b*x))/(16*b) + (5*cosh(d + b*x)^4*exp(a + b*x)*sinh(d + b*x))/(24*b
)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83

$$\int e^{a+bx}\cosh^3(d+bx)\sinh^2(d+bx)\,dx = \frac{e^a\left(2e^{10bx+10d}+3e^{8bx+8d}-12e^{6bx+6d}-24e^{4bx+4d}bx-6e^{2bx+2d}-3\right)}{384e^{4bx+5d}b}$$

input
```
int(exp(b*x+a)*cosh(b*x+d)^3*sinh(b*x+d)^2,x)
```

output
```
(e**a*(2*e**(10*b*x + 10*d) + 3*e**(8*b*x + 8*d) - 12*e**(6*b*x + 6*d) - 2
4*e**(4*b*x + 4*d)*b*x - 6*e**(2*b*x + 2*d) - 3))/(384*e**(4*b*x + 5*d)*b)
```

3.17 $\int e^{a+bx}\cosh^3(d+bx)\sinh(d+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int e^{a+bx}\cosh^3(d+bx)\sinh(d+bx)\,dx=\frac{e^{a-4d-3bx}}{48b}+\frac{e^{a-2d-bx}}{8b}+\frac{e^{a+2d+3bx}}{24b}+\frac{e^{a+4d+5bx}}{80b}$$

output
```
1/48*exp(-3*b*x+a-4*d)/b+1/8*exp(-b*x+a-2*d)/b+1/24*exp(3*b*x+a+2*d)/b+1/8
0*exp(5*b*x+a+4*d)/b
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.22

$$\int e^{a+bx}\cosh^3(d+bx)\sinh(d+bx)\,dx$$
$$=\frac{e^{a-bx}\left((3+e^{4bx})\cosh(2d)+(-3+e^{4bx})\sinh(2d)\right)}{24b}+\frac{e^{a-3bx}\left((5+3e^{8bx})\cosh(4d)+(-5+3e^{8bx})\sinh(4d)\right)}{240b}$$

input
```
Integrate[E^(a + b*x)*Cosh[d + b*x]^3*Sinh[d + b*x],x]
```

output
```
(E^(a - b*x)*((3 + E^(4*b*x))*Cosh[2*d] + (-3 + E^(4*b*x))*Sinh[2*d]))/(24
*b) + (E^(a - 3*b*x)*((5 + 3*E^(8*b*x))*Cosh[4*d] + (-5 + 3*E^(8*b*x))*Sin
h[4*d]))/(240*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2720, 27, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh(bx+d)\cosh^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{1}{16}e^{a-4bx}\left(1-e^{2bx}\right)\left(1+e^{2bx}\right)^3de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^a\int e^{-4bx}\left(1-e^{2bx}\right)\left(1+e^{2bx}\right)^3de^{bx}}{16b}$$

$$\Big\downarrow \text{355}$$

$$-\frac{e^a\int \left(e^{-4bx}+2e^{-2bx}-2e^{2bx}-e^{4bx}\right)de^{bx}}{16b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^a\left(-\frac{1}{3}e^{-3bx}-2e^{-bx}-\frac{2}{3}e^{3bx}-\frac{1}{5}e^{5bx}\right)}{16b}$$

input
```
Int[E^(a + b*x)*Cosh[d + b*x]^3*Sinh[d + b*x],x]
```

output
```
-1/16*(E^a*(-1/3*1/E^(3*b*x) - 2/E^(b*x) - (2*E^(3*b*x))/3 - E^(5*b*x)/5))
/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 84.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

method	result
risch	$\frac{e^{-3bx+a-4d}}{48b}+\frac{e^{-bx+a-2d}}{8b}+\frac{e^{3bx+a+2d}}{24b}+\frac{e^{5bx+a+4d}}{80b}$
default	$\frac{\sinh(-3bx+a-4d)}{48b}+\frac{\sinh(-bx+a-2d)}{8b}+\frac{\sinh(3bx+a+2d)}{24b}+\frac{\sinh(5bx+a+4d)}{80b}+\frac{\cosh(-3bx+a-4d)}{48b}+\frac{\cosh(-bx+a-2d)}{8b}$
orering	$-\frac{4\,e^{bx+a}\cosh(bx+d)^3\sinh(bx+d)}{5b}+\frac{\frac{14\,e^{bx+a}\cosh(bx+d)^3\sinh(bx+d)b}{45}+\frac{14\,e^{bx+a}\cosh(bx+d)^2\sinh(bx+d)^2 b}{15}+\frac{14\,e^{bx+a}\cosh(bx+d)^4 b}{45}}{b^2}$

input
```
int(exp(b*x+a)*cosh(b*x+d)^3*sinh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/48*exp(-3*b*x+a-4*d)/b+1/8*exp(-b*x+a-2*d)/b+1/24*exp(3*b*x+a+2*d)/b+1/8
0*exp(5*b*x+a+4*d)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(61) = 122$.

Time = 0.08 (sec) , antiderivative size = 260, normalized size of antiderivative = 3.56

$$\int e^{a+bx}\cosh^3(d+bx)\sinh(d+bx)\,dx$$
$$= \frac{\cosh{(bx+d)}^4\cosh{(-a+d)} + (\cosh{(-a+d)} - \sinh{(-a+d)})\sinh{(bx+d)}^4 - (\cosh{(bx+d)}\cosh{(-a}}{}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^3*sinh(b*x+d),x, algorithm="fricas")
```

output
```
1/30*(cosh(b*x + d)^4*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*
x + d)^4 - (cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(
b*x + d)^3 + 5*cosh(b*x + d)^2*cosh(-a + d) + (6*cosh(b*x + d)^2*cosh(-a +
 d) - (6*cosh(b*x + d)^2 + 5)*sinh(-a + d) + 5*cosh(-a + d))*sinh(b*x + d)
^2 - (cosh(b*x + d)^3*cosh(-a + d) + 5*cosh(b*x + d)*cosh(-a + d) - (cosh(
b*x + d)^3 + 5*cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)
^4 + 5*cosh(b*x + d)^2)*sinh(-a + d))/(b*cosh(b*x + d) - b*sinh(b*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(58) = 116$.

Time = 2.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.90

$$\int e^{a+bx}\cosh^3(d+bx)\sinh(d+bx)\,dx$$
$$= \begin{cases} -\frac{2e^ae^{bx}\sinh^4{(bx+d)}}{15b} + \frac{2e^ae^{bx}\sinh^3{(bx+d)}\cosh{(bx+d)}}{15b} + \frac{e^ae^{bx}\sinh^2{(bx+d)}\cosh^2{(bx+d)}}{5b} - \frac{e^ae^{bx}\sinh{(bx+d)}\cosh^3{(bx+d)}}{5b} + \frac{e^a}{} \\ xe^a\sinh{(d)}\cosh^3{(d)} \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)**3*sinh(b*x+d),x)
```

output
```
Piecewise((-2*exp(a)*exp(b*x)*sinh(b*x + d)**4/(15*b) + 2*exp(a)*exp(b*x)*
sinh(b*x + d)**3*cosh(b*x + d)/(15*b) + exp(a)*exp(b*x)*sinh(b*x + d)**2*c
osh(b*x + d)**2/(5*b) - exp(a)*exp(b*x)*sinh(b*x + d)*cosh(b*x + d)**3/(5*
b) + exp(a)*exp(b*x)*cosh(b*x + d)**4/(5*b), Ne(b, 0)), (x*exp(a)*sinh(d)*
cosh(d)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int e^{a+bx}\cosh^3(d+bx)\sinh(d+bx)\,dx = \frac{\left(6\,e^{(2\,bx+4\,a+2\,d)}+e^{(4\,a)}\right)e^{(-3\,bx-3\,a-4\,d)}}{48\,b} + \frac{\left(3\,e^{(5\,bx+5\,a+4\,d)}+10\,e^{(3\,bx+5\,a+2\,d)}\right)e^{(-4\,a)}}{240\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^3*sinh(b*x+d),x, algorithm="maxima")
```

output
```
1/48*(6*e^(2*b*x + 4*a + 2*d) + e^(4*a))*e^(-3*b*x - 3*a - 4*d)/b + 1/240*
(3*e^(5*b*x + 5*a + 4*d) + 10*e^(3*b*x + 5*a + 2*d))*e^(-4*a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int e^{a+bx}\cosh^3(d+bx)\sinh(d+bx)\,dx = \frac{\left(5\,\left(6\,e^{(2\,bx+a+2\,d)}+e^{a}\right)e^{(-3\,bx)}+3\,e^{(5\,bx+a+8\,d)}+10\,e^{(3\,bx+a+6\,d)}\right)e^{(-4\,d)}}{240\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^3*sinh(b*x+d),x, algorithm="giac")
```

output
```
1/240*(5*(6*e^(2*b*x + a + 2*d) + e^a)*e^(-3*b*x) + 3*e^(5*b*x + a + 8*d)
+ 10*e^(3*b*x + a + 6*d))*e^(-4*d)/b
```

Mupad [B] (verification not implemented)

Time = 3.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int e^{a+bx}\cosh^3(d+bx)\sinh(d+bx)\,dx = \frac{e^{-2\,d}\,e^{-b\,x}\,e^{a}}{8\,b}+\frac{e^{2\,d}\,e^{3\,b\,x}\,e^{a}}{24\,b}+\frac{e^{-4\,d}\,e^{-3\,b\,x}\,e^{a}}{48\,b}+\frac{e^{4\,d}\,e^{5\,b\,x}\,e^{a}}{80\,b}$$

input
```
int(cosh(d + b*x)^3*exp(a + b*x)*sinh(d + b*x),x)
```

output
```
(exp(-2*d)*exp(-b*x)*exp(a))/(8*b) + (exp(2*d)*exp(3*b*x)*exp(a))/(24*b) +
 (exp(-4*d)*exp(-3*b*x)*exp(a))/(48*b) + (exp(4*d)*exp(5*b*x)*exp(a))/(80*
b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int e^{a+bx}\cosh^3(d+bx)\sinh(d+bx)\,dx = \frac{e^{a}\left(3e^{8bx+8d}+10e^{6bx+6d}+30e^{2bx+2d}+5\right)}{240e^{3bx+4d}b}$$

input
```
int(exp(b*x+a)*cosh(b*x+d)^3*sinh(b*x+d),x)
```

output
```
(e**a*(3*e**(8*b*x + 8*d) + 10*e**(6*b*x + 6*d) + 30*e**(2*b*x + 2*d) + 5)
)/(240*e**(3*b*x + 4*d)*b)
```

3.18 $\int e^{a+bx} \cosh^3(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 65

$$\int e^{a+bx}\cosh^3(d+bx)\,dx=-\frac{e^{a-3d-2bx}}{16b}+\frac{3e^{a+d+2bx}}{16b}+\frac{e^{a+3d+4bx}}{32b}+\frac{3}{8}e^{a-d}x$$

output
```
-1/16*exp(-2*b*x+a-3*d)/b+3/16*exp(2*b*x+a+d)/b+1/32*exp(4*b*x+a+3*d)/b+3/
8*exp(a-d)*x
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.37

$$\int e^{a+bx}\cosh^3(d+bx)\,dx$$
$$=\frac{e^{a-2bx}\left(6e^{2bx}\left(e^{2bx}+2bx\right)\cosh(d)+\left(-2+e^{6bx}\right)\cosh(3d)+6e^{4bx}\sinh(d)-12be^{2bx}x\sinh(d)+2\sinh(3d)}{32b}$$

input
```
Integrate[E^(a + b*x)*Cosh[d + b*x]^3,x]
```

output
```
(E^(a - 2*b*x)*(6*E^(2*b*x)*(E^(2*b*x) + 2*b*x)*Cosh[d] + (-2 + E^(6*b*x))
*Cosh[3*d] + 6*E^(4*b*x)*Sinh[d] - 12*b*E^(2*b*x)*x*Sinh[d] + 2*Sinh[3*d]
+ E^(6*b*x)*Sinh[3*d]))/(32*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\cosh^3(bx+d)\,dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int \frac{1}{8}e^{a-3bx}\left(1+e^{2bx}\right)^3de^{bx}}{b}$$

$$\Big\downarrow \ 27$$

$$\frac{e^a\int e^{-3bx}\left(1+e^{2bx}\right)^3de^{bx}}{8b}$$

$$\Big\downarrow \ 243$$

$$\frac{e^a\int e^{-2bx}\left(1+e^{2bx}\right)^3de^{2bx}}{16b}$$

$$\Big\downarrow \ 49$$

$$\frac{e^a\int\left(3+e^{-2bx}+3e^{-bx}+e^{2bx}\right)de^{2bx}}{16b}$$

$$\Big\downarrow \ 2009$$

$$\frac{e^a\left(-e^{-bx}+\frac{7}{2}e^{2bx}+3\log\left(e^{2bx}\right)\right)}{16b}$$

input
```
Int[E^(a + b*x)*Cosh[d + b*x]^3,x]
```

output
```
(E^a*(-E^(-(b*x)) + (7*E^(2*b*x))/2 + 3*Log[E^(2*b*x)]))/(16*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{e^{-2bx+a-3d}}{16b}+\frac{3\,e^{2bx+a+d}}{16b}+\frac{e^{4bx+a+3d}}{32b}+\frac{3\,e^{a-d}x}{8}$
parallelrisch	$\frac{e^{bx+a}(12bx\cosh(bx+d)-12xb\sinh(bx+d)+\cosh(bx+d)+11\sinh(bx+d)-\cosh(3bx+3d)+3\sinh(3bx+3d))}{32b}$
default	$\frac{3\cosh(a-d)x}{8}-\frac{\sinh(-2bx+a-3d)}{16b}+\frac{3\sinh(2bx+a+d)}{16b}+\frac{\sinh(4bx+a+3d)}{32b}+\frac{3\sinh(a-d)x}{8}-\frac{\cosh(-2bx+a-3d)}{16b}$
orering	$\frac{(4bx+1)e^{bx+a}\cosh(bx+d)^3}{4b}-\frac{(bx-1)\left(e^{bx+a}\cosh(bx+d)^3b+3\,e^{bx+a}\cosh(bx+d)^2\sinh(bx+d)b\right)}{4b^2}-\frac{(4bx+1)\left(4\,e^{bx+a}\cos\right.}{}$

input
```
int(exp(b*x+a)*cosh(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*exp(-2*b*x+a-3*d)/b+3/16*exp(2*b*x+a+d)/b+1/32*exp(4*b*x+a+3*d)/b+3/
8*exp(a-d)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(53) = 106.

Time = 0.08 (sec) , antiderivative size = 214, normalized size of antiderivative = 3.29

$$\int e^{a+bx}\cosh^3(d+bx)\,dx =$$

$$-\frac{\cosh\left(bx+d\right)^3\cosh\left(-a+d\right)-3\left(\cosh\left(-a+d\right)-\sinh\left(-a+d\right)\right)\sinh\left(bx+d\right)^3-6\left(2\,bx+1\right)\cosh\left(b\right.}{}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^3,x, algorithm="fricas")
```

output
```
-1/32*(cosh(b*x + d)^3*cosh(-a + d) - 3*(cosh(-a + d) - sinh(-a + d))*sinh
(b*x + d)^3 - 6*(2*b*x + 1)*cosh(b*x + d)*cosh(-a + d) + 3*(cosh(b*x + d)*
cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d)^2 - 3*(3*cosh(b*x
 + d)^2*cosh(-a + d) - 2*(2*b*x - 1)*cosh(-a + d) + (4*b*x - 3*cosh(b*x +
d)^2 - 2)*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^3 - 6*(2*b*x + 1)*c
osh(b*x + d))*sinh(-a + d))/(b*cosh(b*x + d) - b*sinh(b*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(54) = 108.

Time = 0.94 (sec) , antiderivative size = 207, normalized size of antiderivative = 3.18

$$\int e^{a+bx}\cosh^3(d+bx)\,dx$$

$$=\begin{cases}\frac{3xe^ae^{bx}\sinh^3\left(bx+d\right)}{8}-\frac{3xe^ae^{bx}\sinh^2\left(bx+d\right)\cosh\left(bx+d\right)}{8}-\frac{3xe^ae^{bx}\sinh\left(bx+d\right)\cosh^2\left(bx+d\right)}{8}+\frac{3xe^ae^{bx}\cosh^3\left(bx+d\right)}{8}-\frac{5e^ae^{bx}\sin}{8} \\ xe^a\cosh^3\left(d\right)\end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)**3,x)
```

output
```
Piecewise((3*x*exp(a)*exp(b*x)*sinh(b*x + d)**3/8 - 3*x*exp(a)*exp(b*x)*si
nh(b*x + d)**2*cosh(b*x + d)/8 - 3*x*exp(a)*exp(b*x)*sinh(b*x + d)*cosh(b*
x + d)**2/8 + 3*x*exp(a)*exp(b*x)*cosh(b*x + d)**3/8 - 5*exp(a)*exp(b*x)*s
inh(b*x + d)**3/(8*b) + exp(a)*exp(b*x)*sinh(b*x + d)**2*cosh(b*x + d)/(4*
b) + exp(a)*exp(b*x)*sinh(b*x + d)*cosh(b*x + d)**2/b - 3*exp(a)*exp(b*x)*
cosh(b*x + d)**3/(8*b), Ne(b, 0)), (x*exp(a)*cosh(d)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int e^{a+bx}\cosh^3(d+bx)\,dx = \frac{3\,(bx+a)e^{(a-d)}}{8\,b} + \frac{e^{(4\,bx+a+3\,d)}}{32\,b} + \frac{3\,e^{(2\,bx+a+d)}}{16\,b} - \frac{e^{(-2\,bx+a-3\,d)}}{16\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^3,x, algorithm="maxima")
```

output
```
3/8*(b*x + a)*e^(a - d)/b + 1/32*e^(4*b*x + a + 3*d)/b + 3/16*e^(2*b*x + a
 + d)/b - 1/16*e^(-2*b*x + a - 3*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int e^{a+bx}\cosh^3(d+bx)\,dx = \frac{\left(12\,bxe^{(a+2\,d)} - 2\left(3\,e^{(2\,bx+a+2\,d)} + e^{a}\right)e^{(-2\,bx)} + e^{(4\,bx+a+6\,d)} + 6\,e^{(2\,bx+a+4\,d)}\right)e^{(-3\,d)}}{32\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^3,x, algorithm="giac")
```

output
```
1/32*(12*b*x*e^(a + 2*d) - 2*(3*e^(2*b*x + a + 2*d) + e^a)*e^(-2*b*x) + e^
(4*b*x + a + 6*d) + 6*e^(2*b*x + a + 4*d))*e^(-3*d)/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int e^{a+bx}\cosh^3(d+bx)\,dx$$
$$= \frac{e^{a+b\,x}\,\left(3\cosh(d+b\,x)+3\cosh(d+b\,x)^2\sinh(d+b\,x)-\cosh(d+b\,x)^3+3\,b\,x\cosh(d+b\,x)-3\,b\,x\sinh}{8\,b}$$

input
```
int(cosh(d + b*x)^3*exp(a + b*x),x)
```

output
```
(exp(a + b*x)*(3*cosh(d + b*x) + 3*cosh(d + b*x)^2*sinh(d + b*x) - cosh(d
+ b*x)^3 + 3*b*x*cosh(d + b*x) - 3*b*x*sinh(d + b*x)))/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int e^{a+bx}\cosh^3(d+bx)\,dx = \frac{e^a\left(e^{6bx+6d}+6e^{4bx+4d}+12e^{2bx+2d}bx-2\right)}{32e^{2bx+3d}b}$$

input
```
int(exp(b*x+a)*cosh(b*x+d)^3,x)
```

output
```
(e**a*(e**(6*b*x + 6*d) + 6*e**(4*b*x + 4*d) + 12*e**(2*b*x + 2*d)*b*x - 2
))/(32*e**(2*b*x + 3*d)*b)
```

3.19 $\int e^{a+bx}\cosh^2(d+bx)\coth(d+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 68

$$\int e^{a+bx}\cosh^2(d+bx)\coth(d+bx)\,dx=\frac{e^{a-2d-bx}}{4b}+\frac{e^{a+bx}}{b}+\frac{e^{a+2d+3bx}}{12b}-\frac{2e^{a-d}\text{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
1/4*exp(-b*x+a-2*d)/b+exp(b*x+a)/b+1/12*exp(3*b*x+a+2*d)/b-2*exp(a-d)*arct
anh(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int e^{a+bx}\cosh^2(d+bx)\coth(d+bx)\,dx=\frac{e^{a-2d-bx}\left(3+12e^{2(d+bx)}+e^{4(d+bx)}-24\sqrt{e^{2(d+bx)}}\text{arctanh}\left(\sqrt{e^{2(d+bx)}}\right)\right)}{12b}$$

input
```
Integrate[E^(a + b*x)*Cosh[d + b*x]^2*Coth[d + b*x],x]
```

output
```
(E^(a - 2*d - b*x)*(3 + 12*E^(2*(d + b*x)) + E^(4*(d + b*x)) - 24*Sqrt[E^(
2*(d + b*x))]*ArcTanh[Sqrt[E^(2*(d + b*x))]]))/(12*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.65, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2720, 27, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\cosh^2(bx+d)\coth(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{e^{a-2bx}\left(1+e^{2bx}\right)^3}{4\left(1-e^{2bx}\right)}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^a\int \frac{e^{-2bx}\left(1+e^{2bx}\right)^3}{1-e^{2bx}}de^{bx}}{4b}$$

$$\Big\downarrow \text{364}$$

$$-\frac{e^a\int \left(e^{-2bx}-e^{2bx}-4-\frac{8}{-1+e^{2bx}}\right)de^{bx}}{4b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^a\left(8\text{arctanh}\left(e^{bx}\right)-e^{-bx}-4e^{bx}-\frac{1}{3}e^{3bx}\right)}{4b}$$

input
```
Int[E^(a + b*x)*Cosh[d + b*x]^2*Coth[d + b*x],x]
```

output
```
-1/4*(E^a*(-E^(-(b*x)) - 4*E^(b*x) - E^(3*b*x)/3 + 8*ArcTanh[E^(b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.37

method	result	size
risch	$\frac{e^{3bx+a+2d}}{12b}+\frac{e^{bx+a}}{b}+\frac{e^{-bx+a-2d}}{4b}+\frac{\ln\left(e^{bx+a}-e^{a-d}\right)e^{a-d}}{b}-\frac{\ln\left(e^{bx+a}+e^{a-d}\right)e^{a-d}}{b}$	93

input
```
int(exp(b*x+a)*cosh(b*x+d)^2*coth(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/12*exp(3*b*x+a+2*d)/b+exp(b*x+a)/b+1/4*exp(-b*x+a-2*d)/b+ln(exp(b*x+a)-e
xp(a-d))/b*exp(a-d)-ln(exp(b*x+a)+exp(a-d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(59) = 118$.

Time = 0.08 (sec) , antiderivative size = 400, normalized size of antiderivative = 5.88

$$\int e^{a+bx}\cosh^2(d+bx)\coth(d+bx)\,dx$$
$$= \frac{\cosh(bx+d)^4\cosh(-a+d) + (\cosh(-a+d) - \sinh(-a+d))\sinh(bx+d)^4 + 4(\cosh(bx+d)\cosh(-a+d)}{}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^2*coth(b*x+d),x, algorithm="fricas")
```

output
```
1/12*(cosh(b*x + d)^4*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*
x + d)^4 + 4*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sin
h(b*x + d)^3 + 12*cosh(b*x + d)^2*cosh(-a + d) + 6*(cosh(b*x + d)^2*cosh(-
a + d) - (cosh(b*x + d)^2 + 2)*sinh(-a + d) + 2*cosh(-a + d))*sinh(b*x + d
)^2 - 12*(cosh(b*x + d)*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(
b*x + d) - cosh(b*x + d)*sinh(-a + d))*log(cosh(b*x + d) + sinh(b*x + d) +
 1) + 12*(cosh(b*x + d)*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(
b*x + d) - cosh(b*x + d)*sinh(-a + d))*log(cosh(b*x + d) + sinh(b*x + d) -
 1) + 4*(cosh(b*x + d)^3*cosh(-a + d) + 6*cosh(b*x + d)*cosh(-a + d) - (co
sh(b*x + d)^3 + 6*cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x +
 d)^4 + 12*cosh(b*x + d)^2 + 3)*sinh(-a + d) + 3*cosh(-a + d))/(b*cosh(b*x
 + d) + b*sinh(b*x + d))
```

Sympy [F]

$$\int e^{a+bx}\cosh^2(d+bx)\coth(d+bx)\,dx = e^a\int e^{bx}\cosh^2(bx+d)\coth(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)**2*coth(b*x+d),x)
```

output
```
exp(a)*Integral(exp(b*x)*cosh(b*x + d)**2*coth(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.37

$$\int e^{a+bx}\cosh^2(d+bx)\coth(d+bx)\,dx = \frac{\left(e^{(3\,bx+3\,a+2\,d)}+12\,e^{(bx+3\,a)}\right)e^{(-2\,a)}}{12\,b} - \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^{a}\right)}{b} + \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}-e^{a}\right)}{b} + \frac{e^{(-bx+a-2\,d)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^2*coth(b*x+d),x, algorithm="maxima")
```

output
```
1/12*(e^(3*b*x + 3*a + 2*d) + 12*e^(b*x + 3*a))*e^(-2*a)/b - e^(a - d)*log
(e^(b*x + a + d) + e^a)/b + e^(a - d)*log(e^(b*x + a + d) - e^a)/b + 1/4*e
^(-b*x + a - 2*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

$$\int e^{a+bx}\cosh^2(d+bx)\coth(d+bx)\,dx = \frac{\left(e^{(3\,bx+a+5\,d)}+12\,e^{(bx+a+3\,d)}\right)e^{(-3\,d)} - 12\,e^{(a-d)}\log\left(e^{(bx+d)}+1\right) + 12\,e^{(a-d)}\log\left(\left|e^{(bx+d)}-1\right|\right) + 3\,e^{(-bx+a}}{12\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)^2*coth(b*x+d),x, algorithm="giac")
```

output
```
1/12*((e^(3*b*x + a + 5*d) + 12*e^(b*x + a + 3*d))*e^(-3*d) - 12*e^(a - d)
*log(e^(b*x + d) + 1) + 12*e^(a - d)*log(abs(e^(b*x + d) - 1)) + 3*e^(-b*x
 + a - 2*d))/b
```

Mupad [B] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.31

$$\int e^{a+bx}\cosh^2(d+bx)\coth(d+bx)\,dx=\frac{\mathrm{e}^{a-2\,d-b\,x}}{4\,b}+\frac{\mathrm{e}^{a+2\,d+3\,b\,x}}{12\,b}+\frac{\mathrm{e}^{a+b\,x}}{b}-\frac{2\,\sqrt{\mathrm{e}^{2\,a-2\,d}}\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b\,\sqrt{\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}}}\right)}{\sqrt{-b^2}}$$

input
```
int(cosh(d + b*x)^2*coth(d + b*x)*exp(a + b*x),x)
```

output
```
exp(a - 2*d - b*x)/(4*b) + exp(a + 2*d + 3*b*x)/(12*b) + exp(a + b*x)/b -
(2*exp(2*a - 2*d)^(1/2)*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/(b*(exp(2*a)*e
xp(-2*d))^(1/2))))/(-b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

$$\int e^{a+bx}\cosh^2(d+bx)\coth(d+bx)\,dx=\frac{e^{a}\left(e^{4bx+4d}+12e^{2bx+2d}+12e^{bx+d}\log\left(e^{bx+d}-1\right)-12e^{bx+d}\log\left(e^{bx+d}+1\right)+3\right)}{12e^{bx+2d}b}$$

input
```
int(exp(b*x+a)*cosh(b*x+d)^2*coth(b*x+d),x)
```

output
```
(e**a*(e**(4*b*x + 4*d) + 12*e**(2*b*x + 2*d) + 12*e**(b*x + d)*log(e**(b*
x + d) - 1) - 12*e**(b*x + d)*log(e**(b*x + d) + 1) + 3))/(12*e**(b*x + 2*
d)*b)
```

3.20 $\int e^{a+bx}\cosh(d+bx)\coth^2(d+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int e^{a+bx}\cosh(d+bx)\coth^2(d+bx)\,dx=\frac{e^{a+d+2bx}}{4b}+\frac{2e^{a-d}}{b\left(1-e^{2d+2bx}\right)}+\frac{1}{2}e^{a-d}x+\frac{e^{a-d}\log\left(1-e^{2d+2bx}\right)}{b}$$

output
```
1/4*exp(2*b*x+a+d)/b+2*exp(a-d)/b/(1-exp(2*b*x+2*d))+1/2*exp(a-d)*x+exp(a-
d)*ln(1-exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int e^{a+bx}\cosh(d+bx)\coth^2(d+bx)\,dx=\frac{e^{a-d}\left(d+\frac{1}{2}e^{2(d+bx)}+\frac{4}{1-e^{2(d+bx)}}+bx+2\log\left(1-e^{2(d+bx)}\right)\right)}{2b}$$

input
```
Integrate[E^(a + b*x)*Cosh[d + b*x]*Coth[d + b*x]^2,x]
```

output
```
(E^(a - d)*(d + E^(2*(d + b*x))/2 + 4/(1 - E^(2*(d + b*x))) + b*x + 2*Log[
1 - E^(2*(d + b*x))]))/(2*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.61, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2720, 27, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\cosh(bx+d)\coth^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{e^{a-bx}\left(1+e^{2bx}\right)^3}{2\left(1-e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^a\int \frac{e^{-bx}\left(1+e^{2bx}\right)^3}{\left(1-e^{2bx}\right)^2}de^{bx}}{2b}$$

$$\Big\downarrow \text{354}$$

$$\frac{e^a\int \frac{e^{-bx}\left(1+e^{2bx}\right)^3}{\left(1-e^{2bx}\right)^2}de^{2bx}}{4b}$$

$$\Big\downarrow \text{99}$$

$$\frac{e^a\int \left(e^{-bx}+1+\frac{4}{-1+e^{2bx}}+\frac{8}{\left(-1+e^{2bx}\right)^2}\right)de^{2bx}}{4b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^a\left(e^{2bx}+\frac{8}{1-e^{2bx}}+\log\left(e^{2bx}\right)+4\log\left(1-e^{2bx}\right)\right)}{4b}$$

input
```
Int[E^(a + b*x)*Cosh[d + b*x]*Coth[d + b*x]^2,x]
```

output
```
(E^a*(E^(2*b*x) + 8/(1 - E^(2*b*x)) + Log[E^(2*b*x)] + 4*Log[1 - E^(2*b*x)
]))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.22

method	result	size
risch	$\frac{e^{a-d}x}{2}+\frac{e^{2bx+a+d}}{4b}-\frac{2\,e^{a-d}a}{b}+\frac{2\,e^{3a-d}}{\left(-e^{2bx+2a+2d}+e^{2a}\right)b}+\frac{\ln\left(e^{2bx+2a}-e^{2a-2d}\right)e^{a-d}}{b}$	101

input
```
int(exp(b*x+a)*cosh(b*x+d)*coth(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(a-d)*x+1/4*exp(2*b*x+a+d)/b-2/b*exp(a-d)*a+2/(-exp(2*b*x+2*a+2*d)+
exp(2*a))/b*exp(3*a-d)+ln(exp(2*b*x+2*a)-exp(2*a-2*d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(71) = 142$.

Time = 0.09 (sec) , antiderivative size = 461, normalized size of antiderivative = 5.55

$$\int e^{a+bx}\cosh(d+bx)\coth^2(d+bx)\,dx$$
$$=\frac{\cosh\left(bx+d\right)^4\cosh\left(-a+d\right)+\left(\cosh\left(-a+d\right)-\sinh\left(-a+d\right)\right)\sinh\left(bx+d\right)^4+\left(2\,bx-1\right)\cosh\left(bx+d\right.}{}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)*coth(b*x+d)^2,x, algorithm="fricas")
```

output
```
1/4*(cosh(b*x + d)^4*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x
 + d)^4 + (2*b*x - 1)*cosh(b*x + d)^2*cosh(-a + d) + 4*(cosh(b*x + d)*cosh
(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d)^3 + (6*cosh(b*x + d)^
2*cosh(-a + d) + (2*b*x - 1)*cosh(-a + d) - (2*b*x + 6*cosh(b*x + d)^2 - 1
)*sinh(-a + d))*sinh(b*x + d)^2 - 2*(b*x + 4)*cosh(-a + d) + 4*(cosh(b*x +
 d)^2*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d)^2 + 2*(co
sh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d) - (co
sh(b*x + d)^2 - 1)*sinh(-a + d) - cosh(-a + d))*log(2*sinh(b*x + d)/(cosh(
b*x + d) - sinh(b*x + d))) + 2*(2*cosh(b*x + d)^3*cosh(-a + d) + (2*b*x -
1)*cosh(b*x + d)*cosh(-a + d) - (2*cosh(b*x + d)^3 + (2*b*x - 1)*cosh(b*x
+ d))*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^4 + (2*b*x - 1)*cosh(b*
x + d)^2 - 2*b*x - 8)*sinh(-a + d))/(b*cosh(b*x + d)^2 + 2*b*cosh(b*x + d)
*sinh(b*x + d) + b*sinh(b*x + d)^2 - b)
```

Sympy [F]

$$\int e^{a+bx}\cosh(d+bx)\coth^2(d+bx)\,dx = e^{a}\int e^{bx}\cosh\left(bx+d\right)\coth^2\left(bx+d\right)dx$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)*coth(b*x+d)**2,x)
```

output
```
exp(a)*Integral(exp(b*x)*cosh(b*x + d)*coth(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.28

$$\int e^{a+bx}\cosh(d+bx)\coth^2(d+bx)\,dx = \frac{(bx+a)e^{(a-d)}}{2\,b} + \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^{a}\right)}{b} + \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}-e^{a}\right)}{b} + \frac{e^{(2\,bx+a+d)}}{4\,b} - \frac{2\,e^{(3\,a)}}{b\left(e^{(2\,bx+2\,a+3\,d)}-e^{(2\,a+d)}\right)}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)*coth(b*x+d)^2,x, algorithm="maxima")
```

output
```
1/2*(b*x + a)*e^(a - d)/b + e^(a - d)*log(e^(b*x + a + d) + e^a)/b + e^(a
- d)*log(e^(b*x + a + d) - e^a)/b + 1/4*e^(2*b*x + a + d)/b - 2*e^(3*a)/(b
*(e^(2*b*x + 2*a + 3*d) - e^(2*a + d)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int e^{a+bx}\cosh(d+bx)\coth^2(d+bx)\,dx = \frac{2\,(bx+d)e^{(a-d)}+4\,e^{(a-d)}\log\left(\left|e^{(2\,bx+2\,d)}-1\right|\right)-\frac{4\,\left(e^{(2\,bx+a+2\,d)}+e^{a}\right)e^{(-d)}}{e^{(2\,bx+2\,d)}-1}+e^{(2\,bx+a+d)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+d)*coth(b*x+d)^2,x, algorithm="giac")
```

output
```
1/4*(2*(b*x + d)*e^(a - d) + 4*e^(a - d)*log(abs(e^(2*b*x + 2*d) - 1)) - 4
*(e^(2*b*x + a + 2*d) + e^a)*e^(-d)/(e^(2*b*x + 2*d) - 1) + e^(2*b*x + a +
 d))/b
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.08

$$\int e^{a+bx}\cosh(d+bx)\coth^2(d+bx)\,dx = \frac{x\,\mathrm{e}^{a-d}}{2}+\frac{\mathrm{e}^{a+d+2\,b\,x}}{4\,b}+\frac{2\,\mathrm{e}^{3\,a-3\,d}}{b\,\left(\mathrm{e}^{2\,a-2\,d}-\mathrm{e}^{2\,a+2\,b\,x}\right)} + \frac{\mathrm{e}^{a-d}\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}\right)}{b}$$

input
```
int(cosh(d + b*x)*coth(d + b*x)^2*exp(a + b*x),x)
```

output
```
(x*exp(a - d))/2 + exp(a + d + 2*b*x)/(4*b) + (2*exp(3*a - 3*d))/(b*(exp(2
*a - 2*d) - exp(2*a + 2*b*x))) + (exp(a - d)*log(exp(2*a)*exp(2*b*x) - exp
(2*a)*exp(-2*d)))/b
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.64

$$\int e^{a+bx}\cosh(d+bx)\coth^2(d+bx)\,dx$$
$$=\frac{e^a\left(e^{4bx+4d}+4e^{2bx+2d}\log\left(e^{bx+d}-1\right)+4e^{2bx+2d}\log\left(e^{bx+d}+1\right)+2e^{2bx+2d}bx-9e^{2bx+2d}-4\log\left(e^{bx+d}-1\right)\right.}{4e^d b\left(e^{2bx+2d}-1\right)}$$

input
```
int(exp(b*x+a)*cosh(b*x+d)*coth(b*x+d)^2,x)
```

output
```
(e**a*(e**(4*b*x + 4*d) + 4*e**(2*b*x + 2*d)*log(e**(b*x + d) - 1) + 4*e**
(2*b*x + 2*d)*log(e**(b*x + d) + 1) + 2*e**(2*b*x + 2*d)*b*x - 9*e**(2*b*x
 + 2*d) - 4*log(e**(b*x + d) - 1) - 4*log(e**(b*x + d) + 1) - 2*b*x))/(4*e
**d*b*(e**(2*b*x + 2*d) - 1))
```

3.21 $\int e^{a+bx} \coth^3(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 88

$$\int e^{a+bx}\coth^3(d+bx)\,dx=\frac{e^{a+bx}}{b}-\frac{2e^{a+bx}}{b\left(1-e^{2d+2bx}\right)^2}+\frac{3e^{a+bx}}{b\left(1-e^{2d+2bx}\right)}-\frac{3e^{a-d}\operatorname{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
exp(b*x+a)/b-2*exp(b*x+a)/b/(1-exp(2*b*x+2*d))^2+3*exp(b*x+a)/b/(1-exp(2*b
*x+2*d))-3*exp(a-d)*arctanh(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

$$\int e^{a+bx}\coth^3(d+bx)\,dx=\frac{e^a\left(-3\operatorname{arctanh}\left(e^{bx}(\cosh(d)+\sinh(d))\right)(\cosh(d)-\sinh(d))+\frac{e^{bx}\left(-5e^{2bx}+\left(2+e^{4bx}\right)\cosh(2d)+\left(-2+e^{4bx}\right)\sinh(2d)\right)}{\left(\left(-1+e^{2bx}\right)\cosh(d)+\left(1+e^{2bx}\right)\sinh(d)\right)^2}\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Coth[d + b*x]^3,x]
```

output
```
(E^a*(-3*ArcTanh[E^(b*x)*(Cosh[d] + Sinh[d])]*(Cosh[d] - Sinh[d]) + (E^(b*
x)*(-5*E^(2*b*x) + (2 + E^(4*b*x))*Cosh[2*d] + (-2 + E^(4*b*x))*Sinh[2*d])
)/((-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d])^2))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2720, 25, 27, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\coth^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{e^a\left(1+e^{2bx}\right)^3}{\left(1-e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{e^a\left(1+e^{2bx}\right)^3}{\left(1-e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^a\int \frac{\left(1+e^{2bx}\right)^3}{\left(1-e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{300}$$

$$-\frac{e^a\int \left(\frac{2\left(1+3e^{4bx}\right)}{\left(1-e^{2bx}\right)^3}-1\right)de^{bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^a\left(3\text{arctanh}\left(e^{bx}\right)-e^{bx}-\frac{3e^{bx}}{1-e^{2bx}}+\frac{2e^{bx}}{\left(1-e^{2bx}\right)^2}\right)}{b}$$

input
```
Int[E^(a + b*x)*Coth[d + b*x]^3,x]
```

output
```
-((E^a*(-E^(b*x) + (2*E^(b*x))/(1 - E^(2*b*x))^2 - (3*E^(b*x))/(1 - E^(2*b
*x)) + 3*ArcTanh[E^(b*x)]))/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.32

method	result	size
risch	$\frac{{\mathrm e}^{b x +a}}{b}+\frac{\left(-3 \,{\mathrm e}^{2 b x +2 a +2 d}+{\mathrm e}^{2 a}\right) {\mathrm e}^{b x +3 a}}{\left(-{\mathrm e}^{2 b x +2 a +2 d}+{\mathrm e}^{2 a}\right)^{2} b}-\frac{3 \ln \left({\mathrm e}^{b x +a}+{\mathrm e}^{a -d}\right) {\mathrm e}^{a -d}}{2 b}+\frac{3 \ln \left({\mathrm e}^{b x +a}-{\mathrm e}^{a -d}\right) {\mathrm e}^{a -d}}{2 b}$	116

input
```
int(exp(b*x+a)*coth(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
exp(b*x+a)/b+1/(-exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(-3*exp(2*b*x+2*a+2*d)+e
xp(2*a))*exp(b*x+3*a)-3/2*ln(exp(b*x+a)+exp(a-d))/b*exp(a-d)+3/2*ln(exp(b*
x+a)-exp(a-d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 980 vs. $2(77) = 154$.

Time = 0.10 (sec) , antiderivative size = 980, normalized size of antiderivative = 11.14

$$\int e^{a+bx}\coth^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)^3,x, algorithm="fricas")
```

output
```
1/2*(2*cosh(b*x + d)^5*cosh(-a + d) + 2*(cosh(-a + d) - sinh(-a + d))*sinh
(b*x + d)^5 + 10*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))
*sinh(b*x + d)^4 - 10*cosh(b*x + d)^3*cosh(-a + d) + 10*(2*cosh(b*x + d)^2
*cosh(-a + d) - (2*cosh(b*x + d)^2 - 1)*sinh(-a + d) - cosh(-a + d))*sinh(
b*x + d)^3 + 10*(2*cosh(b*x + d)^3*cosh(-a + d) - 3*cosh(b*x + d)*cosh(-a
+ d) - (2*cosh(b*x + d)^3 - 3*cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d)^2
 + 4*cosh(b*x + d)*cosh(-a + d) - 3*(cosh(b*x + d)^4*cosh(-a + d) + (cosh(
-a + d) - sinh(-a + d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-a + d) -
cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d)^3 - 2*cosh(b*x + d)^2*cosh(-a +
d) + 2*(3*cosh(b*x + d)^2*cosh(-a + d) - (3*cosh(b*x + d)^2 - 1)*sinh(-a +
 d) - cosh(-a + d))*sinh(b*x + d)^2 + 4*(cosh(b*x + d)^3*cosh(-a + d) - co
sh(b*x + d)*cosh(-a + d) - (cosh(b*x + d)^3 - cosh(b*x + d))*sinh(-a + d))
*sinh(b*x + d) - (cosh(b*x + d)^4 - 2*cosh(b*x + d)^2 + 1)*sinh(-a + d) +
cosh(-a + d))*log(cosh(b*x + d) + sinh(b*x + d) + 1) + 3*(cosh(b*x + d)^4*
cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d)^4 + 4*(cosh(b*x
 + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d)^3 - 2*cosh(
b*x + d)^2*cosh(-a + d) + 2*(3*cosh(b*x + d)^2*cosh(-a + d) - (3*cosh(b*x
+ d)^2 - 1)*sinh(-a + d) - cosh(-a + d))*sinh(b*x + d)^2 + 4*(cosh(b*x + d
)^3*cosh(-a + d) - cosh(b*x + d)*cosh(-a + d) - (cosh(b*x + d)^3 - cosh(b*
x + d))*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^4 - 2*cosh(b*x + d...
```

Sympy [F]

$$\int e^{a+bx}\coth^3(d+bx)\,dx = e^a\int e^{bx}\coth^3(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)**3,x)
```

output
```
exp(a)*Integral(exp(b*x)*coth(b*x + d)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.36

$$\int e^{a+bx}\coth^3(d+bx)\,dx = -\frac{3\,e^{(a-d)}\log\left(e^{(bx+a+d)}+e^a\right)}{2\,b} + \frac{3\,e^{(a-d)}\log\left(e^{(bx+a+d)}-e^a\right)}{2\,b} + \frac{e^{(bx+a)}}{b} - \frac{3\,e^{(3\,bx+5\,a+2\,d)}-e^{(bx+5\,a)}}{b\left(e^{(4\,bx+4\,a+4\,d)}-2\,e^{(2\,bx+4\,a+2\,d)}+e^{(4\,a)}\right)}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)^3,x, algorithm="maxima")
```

output
```
-3/2*e^(a - d)*log(e^(b*x + a + d) + e^a)/b + 3/2*e^(a - d)*log(e^(b*x + a
 + d) - e^a)/b + e^(b*x + a)/b - (3*e^(3*b*x + 5*a + 2*d) - e^(b*x + 5*a))
/(b*(e^(4*b*x + 4*a + 4*d) - 2*e^(2*b*x + 4*a + 2*d) + e^(4*a)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.17

$$\int e^{a+bx}\coth^3(d+bx)\,dx = -\frac{3\,e^{(a-d)}\log\left(e^{(bx+a+d)}+e^a\right) - 3\,e^{(a-d)}\log\left(\left|e^{(bx+a+d)}-e^a\right|\right) + \frac{2\,\left(3\,e^{(3\,bx+5\,a+2\,d)}-e^{(bx+5\,a)}\right)}{\left(e^{(2\,bx+2\,a+2\,d)}-e^{(2\,a)}\right)^2} - 2\,e^{(bx+a)}}{2\,b}$$

input
```
integrate(exp(b*x+a)*coth(b*x+d)^3,x, algorithm="giac")
```

output
```
-1/2*(3*e^(a - d)*log(e^(b*x + a + d) + e^a) - 3*e^(a - d)*log(abs(e^(b*x
+ a + d) - e^a)) + 2*(3*e^(3*b*x + 5*a + 2*d) - e^(b*x + 5*a))/(e^(2*b*x +
 2*a + 2*d) - e^(2*a))^2 - 2*e^(b*x + a))/b
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\coth^3(d+bx)\,dx=\int \coth(d+b\,x)^3\,\mathrm{e}^{a+b\,x}\,dx$$

input
```
int(coth(d + b*x)^3*exp(a + b*x),x)
```

output
```
int(coth(d + b*x)^3*exp(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.10

$$\int e^{a+bx}\coth^3(d+bx)\,dx$$
$$=\frac{e^{a}\left(2e^{5bx+5d}+3e^{4bx+4d}\log\left(e^{bx+d}-1\right)-3e^{4bx+4d}\log\left(e^{bx+d}+1\right)-10e^{3bx+3d}-6e^{2bx+2d}\log\left(e^{bx+d}-1\right)+6\right.}{2e^{d}b\left(e^{4bx+4d}-2e^{2bx+2d}+1\right)}$$

input
```
int(exp(b*x+a)*coth(b*x+d)^3,x)
```

output
```
(e**a*(2*e**(5*b*x + 5*d) + 3*e**(4*b*x + 4*d)*log(e**(b*x + d) - 1) - 3*e
**(4*b*x + 4*d)*log(e**(b*x + d) + 1) - 10*e**(3*b*x + 3*d) - 6*e**(2*b*x
+ 2*d)*log(e**(b*x + d) - 1) + 6*e**(2*b*x + 2*d)*log(e**(b*x + d) + 1) +
4*e**(b*x + d) + 3*log(e**(b*x + d) - 1) - 3*log(e**(b*x + d) + 1)))/(2*e*
*d*b*(e**(4*b*x + 4*d) - 2*e**(2*b*x + 2*d) + 1))
```

3.22 $\int e^{a+bx}\sinh^2(d+bx)\tanh(d+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 69

$$\int e^{a+bx}\sinh^2(d+bx)\tanh(d+bx)\,dx=\frac{e^{a-2d-bx}}{4b}-\frac{e^{a+bx}}{b}+\frac{e^{a+2d+3bx}}{12b}+\frac{2e^{a-d}\arctan\left(e^{d+bx}\right)}{b}$$

output
```
1/4*exp(-b*x+a-2*d)/b-exp(b*x+a)/b+1/12*exp(3*b*x+a+2*d)/b+2*exp(a-d)*arct
an(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int e^{a+bx}\sinh^2(d+bx)\tanh(d+bx)\,dx=\frac{e^a\left(-12e^{bx}+3e^{-2d-bx}+e^{2d+3bx}+24e^{-d}\arctan\left(e^{d+bx}\right)\right)}{12b}$$

input
```
Integrate[E^(a + b*x)*Sinh[d + b*x]^2*Tanh[d + b*x],x]
```

output
```
(E^a*(-12*E^(b*x) + 3*E^(-2*d - b*x) + E^(2*d + 3*b*x) + (24*ArcTan[E^(d +
 b*x)])/E^d))/(12*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2720, 27, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh^2(bx+d)\tanh(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{e^{a-2bx}\left(1-e^{2bx}\right)^3}{4\left(1+e^{2bx}\right)}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^a\int \frac{e^{-2bx}\left(1-e^{2bx}\right)^3}{1+e^{2bx}}de^{bx}}{4b}$$

$$\Big\downarrow \text{364}$$

$$-\frac{e^a\int \left(e^{-2bx}-e^{2bx}+4-\frac{8}{1+e^{2bx}}\right)de^{bx}}{4b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^a\left(-8\arctan\left(e^{bx}\right)-e^{-bx}+4e^{bx}-\frac{1}{3}e^{3bx}\right)}{4b}$$

input
```
Int[E^(a + b*x)*Sinh[d + b*x]^2*Tanh[d + b*x],x]
```

output
```
-1/4*(E^a*(-E^(-(b*x)) + 4*E^(b*x) - E^(3*b*x)/3 - 8*ArcTan[E^(b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.46

method	result	size
risch	$\frac{e^{3bx+a+2d}}{12b}-\frac{e^{bx+a}}{b}+\frac{e^{-bx+a-2d}}{4b}+\frac{i\ln\left(e^{bx+a}+ie^{a-d}\right)e^{a-d}}{b}-\frac{i\ln\left(e^{bx+a}-ie^{a-d}\right)e^{a-d}}{b}$	101

input
```
int(exp(b*x+a)*sinh(b*x+d)^2*tanh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/12*exp(3*b*x+a+2*d)/b-exp(b*x+a)/b+1/4*exp(-b*x+a-2*d)/b+I*ln(exp(b*x+a)
+I*exp(a-d))/b*exp(a-d)-I*ln(exp(b*x+a)-I*exp(a-d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(60) = 120$.

Time = 0.09 (sec) , antiderivative size = 332, normalized size of antiderivative = 4.81

$$\int e^{a+bx}\sinh^2(d+bx)\tanh(d+bx)\,dx$$
$$= \frac{\cosh(bx+d)^4\cosh(-a+d) + (\cosh(-a+d) - \sinh(-a+d))\sinh(bx+d)^4 + 4(\cosh(bx+d)\cosh(-a+d)\cdots}{}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)^2*tanh(b*x+d),x, algorithm="fricas")
```

output
```
1/12*(cosh(b*x + d)^4*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*
x + d)^4 + 4*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sin
h(b*x + d)^3 - 12*cosh(b*x + d)^2*cosh(-a + d) + 6*(cosh(b*x + d)^2*cosh(-
a + d) - (cosh(b*x + d)^2 - 2)*sinh(-a + d) - 2*cosh(-a + d))*sinh(b*x + d
)^2 + 24*(cosh(b*x + d)*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(
b*x + d) - cosh(b*x + d)*sinh(-a + d))*arctan(cosh(b*x + d) + sinh(b*x + d
)) + 4*(cosh(b*x + d)^3*cosh(-a + d) - 6*cosh(b*x + d)*cosh(-a + d) - (cos
h(b*x + d)^3 - 6*cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x +
d)^4 - 12*cosh(b*x + d)^2 + 3)*sinh(-a + d) + 3*cosh(-a + d))/(b*cosh(b*x
+ d) + b*sinh(b*x + d))
```

Sympy [F]

$$\int e^{a+bx}\sinh^2(d+bx)\tanh(d+bx)\,dx = e^a\int e^{bx}\sinh^2(bx+d)\tanh(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)**2*tanh(b*x+d),x)
```

output
```
exp(a)*Integral(exp(b*x)*sinh(b*x + d)**2*tanh(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int e^{a+bx}\sinh^2(d+bx)\tanh(d+bx)\,dx = \frac{\left(e^{(3\,bx+3\,a+2\,d)}-12\,e^{(bx+3\,a)}\right)e^{(-2\,a)}}{12\,b} + \frac{2\,\arctan\left(e^{(bx+d)}\right)e^{(a-d)}}{b} + \frac{e^{(-bx+a-2\,d)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)^2*tanh(b*x+d),x, algorithm="maxima")
```

output
```
1/12*(e^(3*b*x + 3*a + 2*d) - 12*e^(b*x + 3*a))*e^(-2*a)/b + 2*arctan(e^(b
*x + d))*e^(a - d)/b + 1/4*e^(-b*x + a - 2*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int e^{a+bx}\sinh^2(d+bx)\tanh(d+bx)\,dx = \frac{24\,\arctan\left(e^{(bx+d)}\right)e^{(a-d)}+\left(e^{(3\,bx+a+5\,d)}-12\,e^{(bx+a+3\,d)}\right)e^{(-3\,d)}+3\,e^{(-bx+a-2\,d)}}{12\,b}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)^2*tanh(b*x+d),x, algorithm="giac")
```

output
```
1/12*(24*arctan(e^(b*x + d))*e^(a - d) + (e^(3*b*x + a + 5*d) - 12*e^(b*x
+ a + 3*d))*e^(-3*d) + 3*e^(-b*x + a - 2*d))/b
```

Mupad [B] (verification not implemented)

Time = 2.87 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.25

$$\int e^{a+bx}\sinh^2(d+bx)\tanh(d+bx)\,dx=\frac{\mathrm{e}^{a-2\,d-b\,x}}{4\,b}+\frac{\mathrm{e}^{a+2\,d+3\,b\,x}}{12\,b}-\frac{\mathrm{e}^{a+b\,x}}{b}+\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b\,\sqrt{\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}}}\right)\,\sqrt{\mathrm{e}^{2\,a-2\,d}}}{\sqrt{b^2}}$$

input
```
int(exp(a + b*x)*sinh(d + b*x)^2*tanh(d + b*x),x)
```

output
```
exp(a - 2*d - b*x)/(4*b) + exp(a + 2*d + 3*b*x)/(12*b) - exp(a + b*x)/b +
(2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/(b*(exp(2*a)*exp(-2*d))^(1/2)))*exp(
2*a - 2*d)^(1/2))/(b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int e^{a+bx}\sinh^2(d+bx)\tanh(d+bx)\,dx=\frac{e^a\left(24e^{bx+d}atan\left(e^{bx+d}\right)+e^{4bx+4d}-12e^{2bx+2d}+3\right)}{12e^{bx+2d}b}$$

input
```
int(exp(b*x+a)*sinh(b*x+d)^2*tanh(b*x+d),x)
```

output
```
(e**a*(24*e**(b*x + d)*atan(e**(b*x + d)) + e**(4*b*x + 4*d) - 12*e**(2*b*
x + 2*d) + 3))/(12*e**(b*x + 2*d)*b)
```

3.23 $\int e^{a+bx}\sinh(d+bx)\tanh(d+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 54

$$\int e^{a+bx}\sinh(d+bx)\tanh(d+bx)\,dx=\frac{e^{a+d+2bx}}{4b}+\frac{1}{2}e^{a-d}x-\frac{e^{a-d}\log\left(1+e^{2d+2bx}\right)}{b}$$

output
```
1/4*exp(2*b*x+a+d)/b+1/2*exp(a-d)*x-exp(a-d)*ln(1+exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.69

$$\int e^{a+bx}\sinh(d+bx)\tanh(d+bx)\,dx$$
$$=\frac{e^{a}\left(\cosh(d)\left(e^{2bx}+2bx-4\log\left(\left(1+e^{2bx}\right)\cosh(d)+\left(-1+e^{2bx}\right)\sinh(d)\right)\right)+\left(e^{2bx}-2bx+4\log\left(\left(1+e\right.\right.\right.\right.}{4b}$$

input
```
Integrate[E^(a + b*x)*Sinh[d + b*x]*Tanh[d + b*x],x]
```

output
```
(E^a*(Cosh[d]*(E^(2*b*x) + 2*b*x - 4*Log[(1 + E^(2*b*x))*Cosh[d] + (-1 + E
^(2*b*x))*Sinh[d]]) + (E^(2*b*x) - 2*b*x + 4*Log[(1 + E^(2*b*x))*Cosh[d] +
 (-1 + E^(2*b*x))*Sinh[d]])*Sinh[d]))/(4*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.65, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 27, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh(bx+d)\tanh(bx+d)\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int \frac{e^{a-bx}\left(1-e^{2bx}\right)^2}{2\left(1+e^{2bx}\right)}de^{bx}}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{e^a\int \frac{e^{-bx}\left(1-e^{2bx}\right)^2}{1+e^{2bx}}de^{bx}}{2b}$$

$$\Big\downarrow \; 354$$

$$\frac{e^a\int \frac{e^{-bx}\left(1-e^{2bx}\right)^2}{1+e^{2bx}}de^{2bx}}{4b}$$

$$\Big\downarrow \; 93$$

$$\frac{e^a\int \left(e^{-bx}+1-\frac{4}{1+e^{2bx}}\right)de^{2bx}}{4b}$$

$$\Big\downarrow \; 2009$$

$$\frac{e^a\left(e^{2bx}+\log\left(e^{2bx}\right)-4\log\left(e^{2bx}+1\right)\right)}{4b}$$

input
```
Int[E^(a + b*x)*Sinh[d + b*x]*Tanh[d + b*x],x]
```

output
```
(E^a*(E^(2*b*x) + Log[E^(2*b*x)] - 4*Log[1 + E^(2*b*x)]))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22

method	result	size
risch	$\frac{e^{a-d} x}{2}+\frac{e^{2 b x+a+d}}{4 b}+\frac{2 e^{a-d} a}{b}-\frac{\ln \left(e^{2 b x+2 a}+e^{2 a-2 d}\right) e^{a-d}}{b}$	66

input
```
int(exp(b*x+a)*sinh(b*x+d)*tanh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(a-d)*x+1/4*exp(2*b*x+a+d)/b+2/b*exp(a-d)*a-ln(exp(2*b*x+2*a)+exp(2
*a-2*d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(46) = 92$.

Time = 0.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.87

$$\int e^{a+bx}\sinh(d+bx)\tanh(d+bx)\,dx$$
$$= \frac{2\,bx\cosh\left(-a+d\right)+\cosh\left(bx+d\right)^{2}\cosh\left(-a+d\right)+\left(\cosh\left(-a+d\right)-\sinh\left(-a+d\right)\right)\sinh\left(bx+d\right)^{2}-4}{}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)*tanh(b*x+d),x, algorithm="fricas")
```

output
```
1/4*(2*b*x*cosh(-a + d) + cosh(b*x + d)^2*cosh(-a + d) + (cosh(-a + d) - s
inh(-a + d))*sinh(b*x + d)^2 - 4*(cosh(-a + d) - sinh(-a + d))*log(2*cosh(
b*x + d)/(cosh(b*x + d) - sinh(b*x + d))) + 2*(cosh(b*x + d)*cosh(-a + d)
- cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d) - (2*b*x + cosh(b*x + d)^2)*si
nh(-a + d))/b
```

Sympy [F]

$$\int e^{a+bx}\sinh(d+bx)\tanh(d+bx)\,dx = e^{a}\int e^{bx}\sinh\left(bx+d\right)\tanh\left(bx+d\right)dx$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)*tanh(b*x+d),x)
```

output
```
exp(a)*Integral(exp(b*x)*sinh(b*x + d)*tanh(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int e^{a+bx}\sinh(d+bx)\tanh(d+bx)\,dx = \frac{(bx+a)e^{(a-d)}}{2\,b} - \frac{e^{(a-d)}\log\left(e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right)}{b} + \frac{e^{(2\,bx+a+d)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)*tanh(b*x+d),x, algorithm="maxima")
```

output
```
1/2*(b*x + a)*e^(a - d)/b - e^(a - d)*log(e^(2*b*x + 2*a + 2*d) + e^(2*a))
/b + 1/4*e^(2*b*x + a + d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int e^{a+bx}\sinh(d+bx)\tanh(d+bx)\,dx = \frac{2\,(bx+d)e^{(a-d)} - 4\,e^{(a-d)}\log\left(e^{(2\,bx+2\,d)}+1\right) + e^{(2\,bx+a+d)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)*tanh(b*x+d),x, algorithm="giac")
```

output
```
1/4*(2*(b*x + d)*e^(a - d) - 4*e^(a - d)*log(e^(2*b*x + 2*d) + 1) + e^(2*b
*x + a + d))/b
```

Mupad [B] (verification not implemented)

Time = 2.79 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int e^{a+bx} \sinh(d+bx) \tanh(d+bx)\, dx = \frac{x\, e^{-d}\, e^{a}}{2} + \frac{e^{2\,b\,x}\, e^{a}\, e^{d}}{4\,b} - \frac{e^{-d}\, e^{a}\, \ln\left(e^{2\,a}\, e^{2\,b\,x} + e^{2\,a}\, e^{-2\,d}\right)}{b}$$

input
```
int(exp(a + b*x)*sinh(d + b*x)*tanh(d + b*x),x)
```

output
```
(x*exp(-d)*exp(a))/2 + (exp(2*b*x)*exp(a)*exp(d))/(4*b) - (exp(-d)*exp(a)*
log(exp(2*a)*exp(2*b*x) + exp(2*a)*exp(-2*d)))/b
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int e^{a+bx} \sinh(d+bx) \tanh(d+bx)\, dx = \frac{e^{a}\left(e^{2bx+2d} - 4\log\left(e^{2bx+2d} + 1\right) + 2bx\right)}{4e^{d}b}$$

input
```
int(exp(b*x+a)*sinh(b*x+d)*tanh(b*x+d),x)
```

output
```
(e**a*(e**(2*b*x + 2*d) - 4*log(e**(2*b*x + 2*d) + 1) + 2*b*x))/(4*e**d*b)
```

3.24 $\int e^{a+bx} \tanh(d+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 32

$$\int e^{a+bx} \tanh(d+bx)\,dx = \frac{e^{a+bx}}{b} - \frac{2e^{a-d}\arctan\left(e^{d+bx}\right)}{b}$$

output
```
exp(b*x+a)/b-2*exp(a-d)*arctan(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.41

$$\int e^{a+bx} \tanh(d+bx)\,dx = \frac{e^a\left(e^{bx} - 2\arctan\left(e^{bx}(\cosh(d)+\sinh(d))\right)\cosh(d) + 2\arctan\left(e^{bx}(\cosh(d)+\sinh(d))\right)\sinh(d)\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Tanh[d + b*x],x]
```

output
```
(E^a*(E^(b*x) - 2*ArcTan[E^(b*x)*(Cosh[d] + Sinh[d])]*Cosh[d] + 2*ArcTan[E
^(b*x)*(Cosh[d] + Sinh[d])]*Sinh[d]))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {2720, 25, 27, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\tanh(bx+d)\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int -\frac{e^a(1-e^{2bx})}{1+e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{e^a(1-e^{2bx})}{1+e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{e^a\int \frac{1-e^{2bx}}{1+e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \; 299$$

$$-\frac{e^a\left(2\int \frac{1}{1+e^{2bx}}de^{bx}-e^{bx}\right)}{b}$$

$$\Big\downarrow \; 216$$

$$-\frac{e^a\left(2\arctan\left(e^{bx}\right)-e^{bx}\right)}{b}$$

input
```
Int[E^(a + b*x)*Tanh[d + b*x],x]
```

output
```
-((E^a*(-E^(b*x) + 2*ArcTan[E^(b*x)]))/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.19

method	result	size
risch	$\frac{e^{bx+a}}{b}+\frac{i\ln\left(e^{bx+a}-ie^{a-d}\right)e^{a-d}}{b}-\frac{i\ln\left(e^{bx+a}+ie^{a-d}\right)e^{a-d}}{b}$	70

input
```
int(exp(b*x+a)*tanh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
exp(b*x+a)/b+I*ln(exp(b*x+a)-I*exp(a-d))/b*exp(a-d)-I*ln(exp(b*x+a)+I*exp(
a-d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(29) = 58$.

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.72

$$\int e^{a+bx}\tanh(d+bx)\,dx = -\frac{2\left(\cosh\left(-a+d\right)-\sinh\left(-a+d\right)\right)\arctan\left(\cosh\left(bx+d\right)+\sinh\left(bx+d\right)\right)-\cosh\left(bx+d\right)\cosh\left(-a+d\right)}{b}$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d),x, algorithm="fricas")
```

output
```
-(2*(cosh(-a + d) - sinh(-a + d))*arctan(cosh(b*x + d) + sinh(b*x + d)) -
cosh(b*x + d)*cosh(-a + d) - (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d) +
 cosh(b*x + d)*sinh(-a + d))/b
```

Sympy [F]

$$\int e^{a+bx}\tanh(d+bx)\,dx = e^{a}\int e^{bx}\tanh\left(bx+d\right)dx$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d),x)
```

output
```
exp(a)*Integral(exp(b*x)*tanh(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int e^{a+bx}\tanh(d+bx)\,dx=-\frac{2\arctan\left(e^{(bx+d)}\right)e^{(a-d)}}{b}+\frac{e^{(bx+a)}}{b}$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d),x, algorithm="maxima")
```

output
```
-2*arctan(e^(b*x + d))*e^(a - d)/b + e^(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int e^{a+bx}\tanh(d+bx)\,dx=-\frac{2\arctan\left(e^{(bx+d)}\right)e^{(a-d)}-e^{(bx+a)}}{b}$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d),x, algorithm="giac")
```

output
```
-(2*arctan(e^(b*x + d))*e^(a - d) - e^(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.72

$$\int e^{a+bx}\tanh(d+bx)\,dx=\frac{\mathrm{e}^{a+b\,x}}{b}-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b\,\sqrt{\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}}}\right)\,\sqrt{\mathrm{e}^{2\,a-2\,d}}}{\sqrt{b^2}}$$

input
```
int(exp(a + b*x)*tanh(d + b*x),x)
```

output
```
exp(a + b*x)/b - (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/(b*(exp(2*a)*exp(-2
*d))^(1/2)))*exp(2*a - 2*d)^(1/2))/(b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int e^{a+bx}\tanh(d+bx)\,dx = \frac{e^{a}\left(-2atan\left(e^{bx+d}\right)+e^{bx+d}\right)}{e^{d}b}$$

input
```
int(exp(b*x+a)*tanh(b*x+d),x)
```

output
```
(e**a*( - 2*atan(e**(b*x + d)) + e**(b*x + d)))/(e**d*b)
```

3.25 $\int e^{a+bx}\textbf{sech}(d+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 24

$$\int e^{a+bx}\text{sech}(d+bx)\,dx=\frac{e^{a-d}\log\left(1+e^{2d+2bx}\right)}{b}$$

output
```
exp(a-d)*ln(1+exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int e^{a+bx}\text{sech}(d+bx)\,dx=\frac{e^{a-d}\log\left(1+e^{2(d+bx)}\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Sech[d + b*x],x]
```

output
```
(E^(a - d)*Log[1 + E^(2*(d + b*x))])/b
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2720, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{sech}(bx+d)\,dx$$
$$\Big\downarrow \text{2720}$$
$$\frac{\int \frac{2e^{a+bx}}{1+e^{2bx}}de^{bx}}{b}$$
$$\Big\downarrow \text{27}$$
$$\frac{2e^a\int \frac{e^{bx}}{1+e^{2bx}}de^{bx}}{b}$$
$$\Big\downarrow \text{240}$$
$$\frac{e^a\log\left(e^{2bx}+1\right)}{b}$$

input
```
Int[E^(a + b*x)*Sech[d + b*x],x]
```

output
```
(E^a*Log[1 + E^(2*b*x)])/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

method	result	size
risch	$-\frac{2\,\mathrm{e}^{a-d} a}{b}+\frac{\ln\left(\mathrm{e}^{2 b x+2 a}+\mathrm{e}^{2 a-2 d}\right) \mathrm{e}^{a-d}}{b}$	43

input
```
int(exp(b*x+a)*sech(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
-2/b*exp(a-d)*a+ln(exp(2*b*x+2*a)+exp(2*a-2*d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(22) = 44.

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.88

$$\int e^{a+bx}\mathrm{sech}(d+bx)\,dx=\frac{(\cosh(-a+d)-\sinh(-a+d))\log\left(\frac{2\cosh(bx+d)}{\cosh(bx+d)-\sinh(bx+d)}\right)}{b}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d),x, algorithm="fricas")
```

output
```
(cosh(-a + d) - sinh(-a + d))*log(2*cosh(b*x + d)/(cosh(b*x + d) - sinh(b*
x + d)))/b
```

Sympy [F]

$$\int e^{a+bx}\text{sech}(d+bx)\,dx = e^{a}\int e^{bx}\,\text{sech}\,(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+d),x)
```

output
```
exp(a)*Integral(exp(b*x)*sech(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int e^{a+bx}\text{sech}(d+bx)\,dx = \frac{e^{(a-d)}\log\left(e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right)}{b}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d),x, algorithm="maxima")
```

output
```
e^(a - d)*log(e^(2*b*x + 2*a + 2*d) + e^(2*a))/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int e^{a+bx}\text{sech}(d+bx)\,dx = \frac{e^{(a-d)}\log\left(e^{(2\,bx+2\,d)}+1\right)}{b}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d),x, algorithm="giac")
```

output
```
e^(a - d)*log(e^(2*b*x + 2*d) + 1)/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int e^{a+bx}\operatorname{sech}(d+bx)\,dx=\frac{\mathrm{e}^{a-d}\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}\right)}{b}$$

input
```
int(exp(a + b*x)/cosh(d + b*x),x)
```

output
```
(exp(a - d)*log(exp(2*a)*exp(2*b*x) + exp(2*a)*exp(-2*d)))/b
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int e^{a+bx}\operatorname{sech}(d+bx)\,dx=\frac{e^{a}\log\left(e^{2bx+2d}+1\right)}{e^{d}b}$$

input
```
int(exp(b*x+a)*sech(b*x+d),x)
```

output
```
(e**a*log(e**(2*b*x + 2*d) + 1))/(e**d*b)
```

3.26 $\int e^{a+bx}\mathbf{csch}(d+bx)\mathbf{sech}(d+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 41

$$\int e^{a+bx}\text{csch}(d+bx)\text{sech}(d+bx)\,dx=\frac{2e^{a-d}\arctan\left(e^{d+bx}\right)}{b}-\frac{2e^{a-d}\text{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
2*exp(a-d)*arctan(exp(b*x+d))/b-2*exp(a-d)*arctanh(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int e^{a+bx}\text{csch}(d+bx)\text{sech}(d+bx)\,dx$$
$$=\frac{2e^a\left(\arctan\left(e^{bx}(\cosh(d)+\sinh(d))\right)-\text{arctanh}\left(e^{bx}(\cosh(d)+\sinh(d))\right)\right)(\cosh(d)-\sinh(d))}{b}$$

input
```
Integrate[E^(a + b*x)*Csch[d + b*x]*Sech[d + b*x],x]
```

output
```
(2*E^a*(ArcTan[E^(b*x)*(Cosh[d] + Sinh[d])] - ArcTanh[E^(b*x)*(Cosh[d] + S
inh[d])])*(Cosh[d] - Sinh[d]))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{csch}(bx+d)\text{sech}(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{4e^{a+2bx}}{1-e^{4bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4e^a\int \frac{e^{2bx}}{1-e^{4bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{827}$$

$$-\frac{4e^a\left(\frac{1}{2}\int \frac{1}{1-e^{2bx}}de^{bx}-\frac{1}{2}\int \frac{1}{1+e^{2bx}}de^{bx}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$-\frac{4e^a\left(\frac{1}{2}\int \frac{1}{1-e^{2bx}}de^{bx}-\frac{1}{2}\arctan\left(e^{bx}\right)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{4e^a\left(\frac{1}{2}\text{arctanh}\left(e^{bx}\right)-\frac{1}{2}\arctan\left(e^{bx}\right)\right)}{b}$$

input
```
Int[E^(a + b*x)*Csch[d + b*x]*Sech[d + b*x],x]
```

output
```
(-4*E^a*(-1/2*ArcTan[E^(b*x)] + ArcTanh[E^(b*x)]/2))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.69 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.71

method	result	size
risch	$\frac{\ln\left(e^{bx+a}-e^{a-d}\right)e^{a-d}}{b}-\frac{\ln\left(e^{bx+a}+e^{a-d}\right)e^{a-d}}{b}+\frac{i\ln\left(e^{bx+a}+ie^{a-d}\right)e^{a-d}}{b}-\frac{i\ln\left(e^{bx+a}-ie^{a-d}\right)e^{a-d}}{b}$	111

input
```
int(exp(b*x+a)*csch(b*x+d)*sech(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
ln(exp(b*x+a)-exp(a-d))/b*exp(a-d)-ln(exp(b*x+a)+exp(a-d))/b*exp(a-d)+I*ln
(exp(b*x+a)+I*exp(a-d))/b*exp(a-d)-I*ln(exp(b*x+a)-I*exp(a-d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(37) = 74$.

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.41

$$\int e^{a+bx}\operatorname{csch}(d+bx)\operatorname{sech}(d+bx)\,dx = \frac{2\left(\cosh\left(-a+d\right)-\sinh\left(-a+d\right)\right)\arctan\left(\cosh\left(bx+d\right)+\sinh\left(bx+d\right)\right)-\left(\cosh\left(-a+d\right)-\sinh\left(-a+\right.\right.}{}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)*sech(b*x+d),x, algorithm="fricas")
```

output
```
(2*(cosh(-a + d) - sinh(-a + d))*arctan(cosh(b*x + d) + sinh(b*x + d)) - (
cosh(-a + d) - sinh(-a + d))*log(cosh(b*x + d) + sinh(b*x + d) + 1) + (cos
h(-a + d) - sinh(-a + d))*log(cosh(b*x + d) + sinh(b*x + d) - 1))/b
```

Sympy [F]

$$\int e^{a+bx}\operatorname{csch}(d+bx)\operatorname{sech}(d+bx)\,dx = e^{a}\int e^{bx}\operatorname{csch}\left(bx+d\right)\operatorname{sech}\left(bx+d\right)dx$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)*sech(b*x+d),x)
```

output
```
exp(a)*Integral(exp(b*x)*csch(b*x + d)*sech(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.56

$$\int e^{a+bx}\text{csch}(d+bx)\text{sech}(d+bx)\,dx = \frac{2\arctan\left(e^{(bx+d)}\right)e^{(a-d)}}{b} - \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^{a}\right)}{b} + \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}-e^{a}\right)}{b}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)*sech(b*x+d),x, algorithm="maxima")
```

output
```
2*arctan(e^(b*x + d))*e^(a - d)/b - e^(a - d)*log(e^(b*x + a + d) + e^a)/b
 + e^(a - d)*log(e^(b*x + a + d) - e^a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.46

$$\int e^{a+bx}\text{csch}(d+bx)\text{sech}(d+bx)\,dx = \left(\frac{2\arctan\left(e^{(bx+d)}\right)e^{(-3d)}}{b} - \frac{e^{(-3d)}\log\left(e^{(bx+d)}+1\right)}{b} + \frac{e^{(-3d)}\log\left(\left|e^{(bx+d)}-1\right|\right)}{b}\right)e^{(a+2d)}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)*sech(b*x+d),x, algorithm="giac")
```

output
```
(2*arctan(e^(b*x + d))*e^(-3*d)/b - e^(-3*d)*log(e^(b*x + d) + 1)/b + e^(-
3*d)*log(abs(e^(b*x + d) - 1))/b)*e^(a + 2*d)
```

Mupad [B] (verification not implemented)

Time = 4.98 (sec) , antiderivative size = 181, normalized size of antiderivative = 4.41

$$\int e^{a+bx}\text{csch}(d+bx)\text{sech}(d+bx)\,dx =$$
$$-\frac{\left(e^{4a-4d}\right)^{1/4}\left(\ln\left(e^{12a}\,e^{-12d}+e^{11a}\,e^{-10d}\,e^{bx}\,\left(e^{4a}\,e^{-4d}\right)^{1/4}\right)-\ln\left(e^{12a}\,e^{-12d}-e^{11a}\,e^{-10d}\,e^{bx}\,\left(e^{4a}\,e^{-4d}\right)\right.\right.}{}$$

input
```
int(exp(a + b*x)/(cosh(d + b*x)*sinh(d + b*x)),x)
```

output
```
-(exp(4*a - 4*d)^(1/4)*(log(exp(12*a)*exp(-12*d) + exp(11*a)*exp(-10*d)*ex
p(b*x)*(exp(4*a)*exp(-4*d))^(1/4)) - log(exp(12*a)*exp(-12*d) - exp(11*a)*
exp(-10*d)*exp(b*x)*(exp(4*a)*exp(-4*d))^(1/4)) - log(exp(12*a)*exp(-12*d)
*1i + exp(11*a)*exp(-10*d)*exp(b*x)*(exp(4*a)*exp(-4*d))^(1/4))*1i + log(6
4*exp(11*a)*exp(-10*d)*exp(b*x)*(exp(4*a)*exp(-4*d))^(1/2) - exp(12*a)*exp
(-12*d)*(exp(4*a)*exp(-4*d))^(1/4)*64i)*1i))/b
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int e^{a+bx}\text{csch}(d+bx)\text{sech}(d+bx)\,dx = \frac{e^{a}\left(2atan\left(e^{bx+d}\right)+\log\left(e^{bx+d}-1\right)-\log\left(e^{bx+d}+1\right)\right)}{e^{d}b}$$

input
```
int(exp(b*x+a)*csch(b*x+d)*sech(b*x+d),x)
```

output
```
(e**a*(2*atan(e**(b*x + d)) + log(e**(b*x + d) - 1) - log(e**(b*x + d) + 1
)))/(e**d*b)
```

3.27 $\int e^{a+bx}\mathrm{csch}^2(d+bx)\mathrm{sech}(d+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 52

$$\int e^{a+bx}\mathrm{csch}^2(d+bx)\mathrm{sech}(d+bx)\,dx=\frac{2e^{a-d}}{b\left(1-e^{2d+2bx}\right)}-\frac{2e^{a-d}\mathrm{arctanh}\left(e^{2d+2bx}\right)}{b}$$

output
```
2*exp(a-d)/b/(1-exp(2*b*x+2*d))-2*exp(a-d)*arctanh(exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int e^{a+bx}\mathrm{csch}^2(d+bx)\mathrm{sech}(d+bx)\,dx=\frac{2e^{a-d}\left(\frac{1}{1-e^{2(d+bx)}}-\mathrm{arctanh}\left(e^{2(d+bx)}\right)\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Csch[d + b*x]^2*Sech[d + b*x],x]
```

output
```
(2*E^(a - d)*((1 - E^(2*(d + b*x)))^(-1) - ArcTanh[E^(2*(d + b*x))]))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2720, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{csch}^2(bx+d)\text{sech}(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{8e^{a+3bx}}{(1-e^{2bx})^2(1+e^{2bx})}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{8e^a\int \frac{e^{3bx}}{(1-e^{2bx})^2(1+e^{2bx})}de^{bx}}{b}$$

$$\Big\downarrow \text{354}$$

$$\frac{4e^a\int \frac{e^{2bx}}{(1-e^{2bx})^2(1+e^{2bx})}de^{2bx}}{b}$$

$$\Big\downarrow \text{86}$$

$$\frac{4e^a\int \left(\frac{1}{2(-1+e^{2bx})}+\frac{1}{2(-1+e^{2bx})^2}\right)de^{2bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4e^a\left(\frac{1}{2(1-e^{2bx})}-\frac{1}{2}\text{arctanh}\left(e^{2bx}\right)\right)}{b}$$

input
```
Int[E^(a + b*x)*Csch[d + b*x]^2*Sech[d + b*x],x]
```

output
```
(4*E^a*(1/(2*(1 - E^(2*b*x))) - ArcTanh[E^(2*b*x)]/2))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 7.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.87

method	result	size
risch	$\frac{2\,{\mathrm e}^{3a-d}}{\left(-{\mathrm e}^{2bx+2a+2d}+{\mathrm e}^{2a}\right)b}-\frac{\ln\left({\mathrm e}^{2bx+2a}+{\mathrm e}^{2a-2d}\right){\mathrm e}^{a-d}}{b}+\frac{\ln\left({\mathrm e}^{2bx+2a}-{\mathrm e}^{2a-2d}\right){\mathrm e}^{a-d}}{b}$	97

input
```
int(exp(b*x+a)*csch(b*x+d)^2*sech(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
2/(-exp(2*b*x+2*a+2*d)+exp(2*a))/b*exp(3*a-d)-ln(exp(2*b*x+2*a)+exp(2*a-2*
d))/b*exp(a-d)+ln(exp(2*b*x+2*a)-exp(2*a-2*d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(46) = 92$.

Time = 0.09 (sec) , antiderivative size = 319, normalized size of antiderivative = 6.13

$$\int e^{a+bx}\text{csch}^2(d+bx)\text{sech}(d+bx)\,dx =$$

$$-\frac{(\cosh(bx+d)^2\cosh(-a+d) + (\cosh(-a+d) - \sinh(-a+d))\sinh(bx+d)^2 + 2(\cosh(bx+d)\cos}{}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^2*sech(b*x+d),x, algorithm="fricas")
```

output
```
-((cosh(b*x + d)^2*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x +
 d)^2 + 2*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b
*x + d) - (cosh(b*x + d)^2 - 1)*sinh(-a + d) - cosh(-a + d))*log(2*cosh(b*
x + d)/(cosh(b*x + d) - sinh(b*x + d))) - (cosh(b*x + d)^2*cosh(-a + d) +
(cosh(-a + d) - sinh(-a + d))*sinh(b*x + d)^2 + 2*(cosh(b*x + d)*cosh(-a +
 d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^2 - 1)*si
nh(-a + d) - cosh(-a + d))*log(2*sinh(b*x + d)/(cosh(b*x + d) - sinh(b*x +
 d))) + 2*cosh(-a + d) - 2*sinh(-a + d))/(b*cosh(b*x + d)^2 + 2*b*cosh(b*x
 + d)*sinh(b*x + d) + b*sinh(b*x + d)^2 - b)
```

Sympy [F]

$$\int e^{a+bx}\text{csch}^2(d+bx)\text{sech}(d+bx)\,dx = e^a\int e^{bx}\,\text{csch}^2(bx+d)\,\text{sech}(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)**2*sech(b*x+d),x)
```

output
```
exp(a)*Integral(exp(b*x)*csch(b*x + d)**2*sech(b*x + d), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(46) = 92$.

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.04

$$\int e^{a+bx}\operatorname{csch}^2(d+bx)\operatorname{sech}(d+bx)\,dx = -\frac{e^{(a-d)}\log\left(e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right)}{b} + \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^{a}\right)}{b} + \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}-e^{a}\right)}{b} - \frac{2\,e^{(3\,a)}}{b\left(e^{(2\,bx+2\,a+3\,d)}-e^{(2\,a+d)}\right)}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^2*sech(b*x+d),x, algorithm="maxima")
```

output
```
-e^(a - d)*log(e^(2*b*x + 2*a + 2*d) + e^(2*a))/b + e^(a - d)*log(e^(b*x +
 a + d) + e^a)/b + e^(a - d)*log(e^(b*x + a + d) - e^a)/b - 2*e^(3*a)/(b*(
e^(2*b*x + 2*a + 3*d) - e^(2*a + d)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.40

$$\int e^{a+bx}\operatorname{csch}^2(d+bx)\operatorname{sech}(d+bx)\,dx = -\left(\frac{e^{(-4\,d)}\log\left(e^{(2\,bx+2\,d)}+1\right)}{b} - \frac{e^{(-4\,d)}\log\left(\left|e^{(2\,bx+2\,d)}-1\right|\right)}{b} + \frac{2\,e^{(-4\,d)}}{b\left(e^{(2\,bx+2\,d)}-1\right)}\right)e^{(a+3\,d)}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^2*sech(b*x+d),x, algorithm="giac")
```

output
```
-(e^(-4*d)*log(e^(2*b*x + 2*d) + 1)/b - e^(-4*d)*log(abs(e^(2*b*x + 2*d) -
 1))/b + 2*e^(-4*d)/(b*(e^(2*b*x + 2*d) - 1)))*e^(a + 3*d)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\operatorname{csch}^2(d+bx)\operatorname{sech}(d+bx)\,dx=\int \frac{\mathrm{e}^{a+b\,x}}{\cosh\left(d+b\,x\right)\,{\sinh\left(d+b\,x\right)}^2}\,dx$$

input
```
int(exp(a + b*x)/(cosh(d + b*x)*sinh(d + b*x)^2),x)
```

output
```
int(exp(a + b*x)/(cosh(d + b*x)*sinh(d + b*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.75

$$\int e^{a+bx}\operatorname{csch}^2(d+bx)\operatorname{sech}(d+bx)\,dx$$
$$=\frac{e^{a}\left(e^{2bx+2d}\log\left(e^{bx+d}-1\right)+e^{2bx+2d}\log\left(e^{bx+d}+1\right)-e^{2bx+2d}\log\left(e^{2bx+2d}+1\right)-2e^{2bx+2d}-\log\left(e^{bx+d}-1\right)\right.}{e^{d}b\left(e^{2bx+2d}-1\right)}$$

input
```
int(exp(b*x+a)*csch(b*x+d)^2*sech(b*x+d),x)
```

output
```
(e**a*(e**(2*b*x + 2*d)*log(e**(b*x + d) - 1) + e**(2*b*x + 2*d)*log(e**(b
*x + d) + 1) - e**(2*b*x + 2*d)*log(e**(2*b*x + 2*d) + 1) - 2*e**(2*b*x +
2*d) - log(e**(b*x + d) - 1) - log(e**(b*x + d) + 1) + log(e**(2*b*x + 2*d
) + 1)))/(e**d*b*(e**(2*b*x + 2*d) - 1))
```

3.28 $\int e^{a+bx}\mathrm{csch}^3(d+bx)\mathrm{sech}(d+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 96

$$\int e^{a+bx}\mathrm{csch}^3(d+bx)\mathrm{sech}(d+bx)\,dx=-\frac{2e^{a+bx}}{b\left(1-e^{2d+2bx}\right)^2}+\frac{3e^{a+bx}}{b\left(1-e^{2d+2bx}\right)}-\frac{2e^{a-d}\arctan\left(e^{d+bx}\right)}{b}+\frac{e^{a-d}\operatorname{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
-2*exp(b*x+a)/b/(1-exp(2*b*x+2*d))^2+3*exp(b*x+a)/b/(1-exp(2*b*x+2*d))-2*e
xp(a-d)*arctan(exp(b*x+d))/b+exp(a-d)*arctanh(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.05

$$\int e^{a+bx}\mathrm{csch}^3(d+bx)\mathrm{sech}(d+bx)\,dx$$
$$=\frac{e^a\left(-\frac{4e^{bx}}{\left(-1+e^{2(d+bx)}\right)^2}-\frac{6e^{bx}}{-1+e^{2(d+bx)}}-4e^{-d}\arctan\left(e^{d+bx}\right)-e^{-d}\log\left(1-e^{d+bx}\right)+e^{-d}\log\left(1+e^{d+bx}\right)\right)}{2b}$$

input
```
Integrate[E^(a + b*x)*Csch[d + b*x]^3*Sech[d + b*x],x]
```

output
```
(E^a*((-4*E^(b*x))/(-1 + E^(2*(d + b*x)))^2 - (6*E^(b*x))/(-1 + E^(2*(d +
b*x))) - (4*ArcTan[E^(d + b*x)])/E^d - Log[1 - E^(d + b*x)]/E^d + Log[1 +
E^(d + b*x)]/E^d))/(2*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.80, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2720, 27, 372, 402, 27, 397, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{csch}^3(bx+d)\text{sech}(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{16e^{a+4bx}}{\left(1-e^{2bx}\right)^3\left(1+e^{2bx}\right)}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{16e^a\int \frac{e^{4bx}}{\left(1-e^{2bx}\right)^3\left(1+e^{2bx}\right)}de^{bx}}{b}$$

$$\Big\downarrow \text{372}$$

$$-\frac{16e^a\left(\frac{e^{bx}}{8\left(1-e^{2bx}\right)^2}-\frac{1}{8}\int \frac{1+5e^{2bx}}{\left(1-e^{2bx}\right)^2\left(1+e^{2bx}\right)}de^{bx}\right)}{b}$$

$$\Big\downarrow \text{402}$$

$$-\frac{16e^a\left(\frac{1}{8}\left(-\frac{1}{4}\int -\frac{2\left(1-3e^{2bx}\right)}{\left(1-e^{2bx}\right)\left(1+e^{2bx}\right)}de^{bx}-\frac{3e^{bx}}{2\left(1-e^{2bx}\right)}\right)+\frac{e^{bx}}{8\left(1-e^{2bx}\right)^2}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{16e^a\left(\frac{1}{8}\left(\frac{1}{2}\int \frac{1-3e^{2bx}}{\left(1-e^{2bx}\right)\left(1+e^{2bx}\right)}de^{bx}-\frac{3e^{bx}}{2\left(1-e^{2bx}\right)}\right)+\frac{e^{bx}}{8\left(1-e^{2bx}\right)^2}\right)}{b}$$

$$\Big\downarrow \text{397}$$

$$-\frac{16e^a\left(\frac{1}{8}\left(\frac{1}{2}\left(2\int\frac{1}{1+e^{2bx}}de^{bx}-\int\frac{1}{1-e^{2bx}}de^{bx}\right)-\frac{3e^{bx}}{2(1-e^{2bx})}\right)+\frac{e^{bx}}{8(1-e^{2bx})^2}\right)}{b}$$

$$\Big\downarrow \ 216$$

$$-\frac{16e^a\left(\frac{1}{8}\left(\frac{1}{2}\left(2\arctan\left(e^{bx}\right)-\int\frac{1}{1-e^{2bx}}de^{bx}\right)-\frac{3e^{bx}}{2(1-e^{2bx})}\right)+\frac{e^{bx}}{8(1-e^{2bx})^2}\right)}{b}$$

$$\Big\downarrow \ 219$$

$$-\frac{16e^a\left(\frac{1}{8}\left(\frac{1}{2}\left(2\arctan\left(e^{bx}\right)-\text{arctanh}\left(e^{bx}\right)\right)-\frac{3e^{bx}}{2(1-e^{2bx})}\right)+\frac{e^{bx}}{8(1-e^{2bx})^2}\right)}{b}$$

input
```
Int[E^(a + b*x)*Csch[d + b*x]^3*Sech[d + b*x],x]
```

output
```
(-16*E^a*(E^(b*x)/(8*(1 - E^(2*b*x))^2) + ((-3*E^(b*x))/(2*(1 - E^(2*b*x))
) + (2*ArcTan[E^(b*x)] - ArcTanh[E^(b*x)])/2)/8))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 36.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.71

method	result
risch	$\frac{\left(-3\,{e}^{2bx+2a+2d}+{e}^{2a}\right){e}^{bx+3a}}{\left(-{e}^{2bx+2a+2d}+{e}^{2a}\right)^{2} b}+\frac{i \ln\left({e}^{bx+a}-i\,{e}^{a-d}\right){e}^{a-d}}{b}-\frac{i \ln\left({e}^{bx+a}+i\,{e}^{a-d}\right){e}^{a-d}}{b}+\frac{\ln\left({e}^{bx+a}+{e}^{a-d}\right){e}^{a-d}}{2b}-\frac{\ln\left({e}^{bx+a}-{e}\right.}{2b}$

input
```
int(exp(b*x+a)*csch(b*x+d)^3*sech(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/(-exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(-3*exp(2*b*x+2*a+2*d)+exp(2*a))*exp(
b*x+3*a)+I*ln(exp(b*x+a)-I*exp(a-d))/b*exp(a-d)-I*ln(exp(b*x+a)+I*exp(a-d)
)/b*exp(a-d)+1/2*ln(exp(b*x+a)+exp(a-d))/b*exp(a-d)-1/2*ln(exp(b*x+a)-exp(
a-d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1068 vs. $2(84) = 168$.

Time = 0.10 (sec) , antiderivative size = 1068, normalized size of antiderivative = 11.12

$$\int e^{a+bx}\operatorname{csch}^3(d+bx)\operatorname{sech}(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^3*sech(b*x+d),x, algorithm="fricas")
```

output
```
-1/2*(6*cosh(b*x + d)^3*cosh(-a + d) + 6*(cosh(-a + d) - sinh(-a + d))*sin
h(b*x + d)^3 + 18*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d)
)*sinh(b*x + d)^2 + 4*(cosh(b*x + d)^4*cosh(-a + d) + (cosh(-a + d) - sinh
(-a + d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*
sinh(-a + d))*sinh(b*x + d)^3 - 2*cosh(b*x + d)^2*cosh(-a + d) + 2*(3*cosh
(b*x + d)^2*cosh(-a + d) - (3*cosh(b*x + d)^2 - 1)*sinh(-a + d) - cosh(-a
+ d))*sinh(b*x + d)^2 + 4*(cosh(b*x + d)^3*cosh(-a + d) - cosh(b*x + d)*co
sh(-a + d) - (cosh(b*x + d)^3 - cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d)
 - (cosh(b*x + d)^4 - 2*cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*
arctan(cosh(b*x + d) + sinh(b*x + d)) - 2*cosh(b*x + d)*cosh(-a + d) - (co
sh(b*x + d)^4*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d)^4
 + 4*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x +
d)^3 - 2*cosh(b*x + d)^2*cosh(-a + d) + 2*(3*cosh(b*x + d)^2*cosh(-a + d)
- (3*cosh(b*x + d)^2 - 1)*sinh(-a + d) - cosh(-a + d))*sinh(b*x + d)^2 + 4
*(cosh(b*x + d)^3*cosh(-a + d) - cosh(b*x + d)*cosh(-a + d) - (cosh(b*x +
d)^3 - cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^4 - 2*c
osh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*log(cosh(b*x + d) + sinh(
b*x + d) + 1) + (cosh(b*x + d)^4*cosh(-a + d) + (cosh(-a + d) - sinh(-a +
d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-
a + d))*sinh(b*x + d)^3 - 2*cosh(b*x + d)^2*cosh(-a + d) + 2*(3*cosh(b*...
```

Sympy [F]

$$\int e^{a+bx}\text{csch}^3(d+bx)\text{sech}(d+bx)\,dx = e^a\int e^{bx}\,\text{csch}^3\,(bx+d)\,\text{sech}\,(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)**3*sech(b*x+d),x)
```

output
```
exp(a)*Integral(exp(b*x)*csch(b*x + d)**3*sech(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.33

$$\int e^{a+bx}\text{csch}^3(d+bx)\text{sech}(d+bx)\,dx = -\frac{2\,\arctan\left(e^{(bx+d)}\right)e^{(a-d)}}{b} + \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^{a}\right)}{2\,b} - \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}-e^{a}\right)}{2\,b} - \frac{3\,e^{(3\,bx+5\,a+2\,d)}-e^{(bx+5\,a)}}{b\left(e^{(4\,bx+4\,a+4\,d)}-2\,e^{(2\,bx+4\,a+2\,d)}+e^{(4\,a)}\right)}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^3*sech(b*x+d),x, algorithm="maxima")
```

output
```
-2*arctan(e^(b*x + d))*e^(a - d)/b + 1/2*e^(a - d)*log(e^(b*x + a + d) + e
^a)/b - 1/2*e^(a - d)*log(e^(b*x + a + d) - e^a)/b - (3*e^(3*b*x + 5*a + 2
*d) - e^(b*x + 5*a))/(b*(e^(4*b*x + 4*a + 4*d) - 2*e^(2*b*x + 4*a + 2*d) +
 e^(4*a)))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.05

$$\int e^{a+bx}\text{csch}^3(d+bx)\text{sech}(d+bx)\,dx = -\frac{1}{2}\left(\frac{4\,\arctan\left(e^{(bx+d)}\right)e^{(-5\,d)}}{b} - \frac{e^{(-5\,d)}\log\left(e^{(bx+d)}+1\right)}{b} + \frac{e^{(-5\,d)}\log\left(\left|e^{(bx+d)}-1\right|\right)}{b} + \frac{2\left(3\,e^{(3\,bx+2\,d)}-e\right.}{b(e^{(2\,bx+2\,d)}}\right.$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^3*sech(b*x+d),x, algorithm="giac")
```

output
```
-1/2*(4*arctan(e^(b*x + d))*e^(-5*d)/b - e^(-5*d)*log(e^(b*x + d) + 1)/b +
 e^(-5*d)*log(abs(e^(b*x + d) - 1))/b + 2*(3*e^(3*b*x + 2*d) - e^(b*x))*e^
(-4*d)/(b*(e^(2*b*x + 2*d) - 1)^2))*e^(a + 4*d)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\operatorname{csch}^3(d+bx)\operatorname{sech}(d+bx)\,dx=\int \frac{e^{a+b\,x}}{\cosh\left(d+b\,x\right)\,\sinh\left(d+b\,x\right)^3}\,dx$$

input
```
int(exp(a + b*x)/(cosh(d + b*x)*sinh(d + b*x)^3),x)
```

output
```
int(exp(a + b*x)/(cosh(d + b*x)*sinh(d + b*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.29

$$\int e^{a+bx}\operatorname{csch}^3(d+bx)\operatorname{sech}(d+bx)\,dx$$
$$=\frac{e^{a}\left(-4e^{4bx+4d}atan\left(e^{bx+d}\right)+8e^{2bx+2d}atan\left(e^{bx+d}\right)-4atan\left(e^{bx+d}\right)-e^{4bx+4d}\log\left(e^{bx+d}-1\right)+e^{4bx+4d}\log\left(e^{b}\right.\right.}{2e^{d}b\left(e^{4b}\right.}$$

input
```
int(exp(b*x+a)*csch(b*x+d)^3*sech(b*x+d),x)
```

output
```
(e**a*( - 4*e**(4*b*x + 4*d)*atan(e**(b*x + d)) + 8*e**(2*b*x + 2*d)*atan(
e**(b*x + d)) - 4*atan(e**(b*x + d)) - e**(4*b*x + 4*d)*log(e**(b*x + d) -
 1) + e**(4*b*x + 4*d)*log(e**(b*x + d) + 1) - 6*e**(3*b*x + 3*d) + 2*e**(
2*b*x + 2*d)*log(e**(b*x + d) - 1) - 2*e**(2*b*x + 2*d)*log(e**(b*x + d) +
 1) + 2*e**(b*x + d) - log(e**(b*x + d) - 1) + log(e**(b*x + d) + 1)))/(2*
e**d*b*(e**(4*b*x + 4*d) - 2*e**(2*b*x + 2*d) + 1))
```

3.29 $\int e^{a+bx} \sinh(d+bx) \tanh^2(d+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 80

$$\int e^{a+bx} \sinh(d+bx) \tanh^2(d+bx)\,dx = \frac{e^{a+d+2bx}}{4b} - \frac{2e^{a-d}}{b\left(1+e^{2d+2bx}\right)} - \frac{1}{2}e^{a-d}x - \frac{e^{a-d}\log\left(1+e^{2d+2bx}\right)}{b}$$

output
```
1/4*exp(2*b*x+a+d)/b-2*exp(a-d)/b/(1+exp(2*b*x+2*d))-1/2*exp(a-d)*x-exp(a-
d)*ln(1+exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int e^{a+bx} \sinh(d+bx) \tanh^2(d+bx)\,dx = \frac{e^{a-d}\left(-d+\frac{1}{2}e^{2(d+bx)} - \frac{4}{1+e^{2(d+bx)}} - bx - 2\log\left(1+e^{2(d+bx)}\right)\right)}{2b}$$

input
```
Integrate[E^(a + b*x)*Sinh[d + b*x]*Tanh[d + b*x]^2,x]
```

output
```
(E^(a - d)*(-d + E^(2*(d + b*x))/2 - 4/(1 + E^(2*(d + b*x))) - b*x - 2*Log
[1 + E^(2*(d + b*x))]))/(2*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.61, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2720, 27, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh(bx+d)\tanh^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{e^{a-bx}\left(1-e^{2bx}\right)^3}{2\left(1+e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^a\int \frac{e^{-bx}\left(1-e^{2bx}\right)^3}{\left(1+e^{2bx}\right)^2}de^{bx}}{2b}$$

$$\Big\downarrow \text{354}$$

$$-\frac{e^a\int \frac{e^{-bx}\left(1-e^{2bx}\right)^3}{\left(1+e^{2bx}\right)^2}de^{2bx}}{4b}$$

$$\Big\downarrow \text{99}$$

$$-\frac{e^a\int \left(e^{-bx}-1+\frac{4}{1+e^{2bx}}-\frac{8}{\left(1+e^{2bx}\right)^2}\right)de^{2bx}}{4b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^a\left(-e^{2bx}+\frac{8}{e^{2bx}+1}+\log\left(e^{2bx}\right)+4\log\left(e^{2bx}+1\right)\right)}{4b}$$

input
```
Int[E^(a + b*x)*Sinh[d + b*x]*Tanh[d + b*x]^2,x]
```

output
```
-1/4*(E^a*(-E^(2*b*x) + 8/(1 + E^(2*b*x)) + Log[E^(2*b*x)] + 4*Log[1 + E^(
2*b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.22

method	result	size
risch	$-\frac{e^{a-d}x}{2}+\frac{e^{2bx+a+d}}{4b}+\frac{2\,e^{a-d}a}{b}-\frac{2\,e^{3a-d}}{(e^{2bx+2a+2d}+e^{2a})b}-\frac{\ln(e^{2bx+2a}+e^{2a-2d})e^{a-d}}{b}$	98

input
```
int(exp(b*x+a)*sinh(b*x+d)*tanh(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(a-d)*x+1/4*exp(2*b*x+a+d)/b+2/b*exp(a-d)*a-2/(exp(2*b*x+2*a+2*d)+
exp(2*a))/b*exp(3*a-d)-ln(exp(2*b*x+2*a)+exp(2*a-2*d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(70) = 140.

Time = 0.08 (sec) , antiderivative size = 461, normalized size of antiderivative = 5.76

$$\int e^{a+bx}\sinh(d+bx)\tanh^2(d+bx)\,dx$$
$$=\frac{\cosh(bx+d)^4\cosh(-a+d)+(\cosh(-a+d)-\sinh(-a+d))\sinh(bx+d)^4-(2\,bx-1)\cosh(bx+d$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)*tanh(b*x+d)^2,x, algorithm="fricas")
```

output
```
1/4*(cosh(b*x + d)^4*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x
 + d)^4 - (2*b*x - 1)*cosh(b*x + d)^2*cosh(-a + d) + 4*(cosh(b*x + d)*cosh
(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d)^3 + (6*cosh(b*x + d)^
2*cosh(-a + d) - (2*b*x - 1)*cosh(-a + d) + (2*b*x - 6*cosh(b*x + d)^2 - 1
)*sinh(-a + d))*sinh(b*x + d)^2 - 2*(b*x + 4)*cosh(-a + d) - 4*(cosh(b*x +
 d)^2*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d)^2 + 2*(co
sh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d) - (co
sh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*log(2*cosh(b*x + d)/(cosh(
b*x + d) - sinh(b*x + d))) + 2*(2*cosh(b*x + d)^3*cosh(-a + d) - (2*b*x -
1)*cosh(b*x + d)*cosh(-a + d) - (2*cosh(b*x + d)^3 - (2*b*x - 1)*cosh(b*x
+ d))*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^4 - (2*b*x - 1)*cosh(b*
x + d)^2 - 2*b*x - 8)*sinh(-a + d))/(b*cosh(b*x + d)^2 + 2*b*cosh(b*x + d)
*sinh(b*x + d) + b*sinh(b*x + d)^2 + b)
```

Sympy [F]

$$\int e^{a+bx}\sinh(d+bx)\tanh^2(d+bx)\,dx = e^a\int e^{bx}\sinh\left(bx+d\right)\tanh^2\left(bx+d\right)dx$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)*tanh(b*x+d)**2,x)
```

output
```
exp(a)*Integral(exp(b*x)*sinh(b*x + d)*tanh(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11

$$\int e^{a+bx}\sinh(d+bx)\tanh^2(d+bx)\,dx = -\frac{(bx+a)e^{(a-d)}}{2\,b} - \frac{e^{(a-d)}\log\left(e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right)}{b} + \frac{e^{(2\,bx+a+d)}}{4\,b} - \frac{2\,e^{(3\,a)}}{b\left(e^{(2\,bx+2\,a+3\,d)}+e^{(2\,a+d)}\right)}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)*tanh(b*x+d)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*x + a)*e^(a - d)/b - e^(a - d)*log(e^(2*b*x + 2*a + 2*d) + e^(2*a)
)/b + 1/4*e^(2*b*x + a + d)/b - 2*e^(3*a)/(b*(e^(2*b*x + 2*a + 3*d) + e^(2
*a + d)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

$$\int e^{a+bx}\sinh(d+bx)\tanh^2(d+bx)\,dx$$
$$= -\frac{2\,(bx+d)e^{(a-d)}+4\,e^{(a-d)}\log\left(e^{(2\,bx+2\,d)}+1\right)-\frac{4\,\left(e^{(2\,bx+a+2\,d)}-e^{a}\right)e^{(-d)}}{e^{(2\,bx+2\,d)}+1}-e^{(2\,bx+a+d)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+d)*tanh(b*x+d)^2,x, algorithm="giac")
```

output
```
-1/4*(2*(b*x + d)*e^(a - d) + 4*e^(a - d)*log(e^(2*b*x + 2*d) + 1) - 4*(e^
(2*b*x + a + 2*d) - e^a)*e^(-d)/(e^(2*b*x + 2*d) + 1) - e^(2*b*x + a + d))
/b
```

Mupad [B] (verification not implemented)

Time = 3.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10

$$\int e^{a+bx}\sinh(d+bx)\tanh^2(d+bx)\,dx = \frac{e^{a+d+2\,b\,x}}{4\,b}-\frac{x\,e^{a-d}}{2}-\frac{2\,e^{3\,a-3\,d}}{b\,\left(e^{2\,a-2\,d}+e^{2\,a+2\,b\,x}\right)} - \frac{e^{a-d}\,\ln\left(e^{2\,a}\,e^{2\,b\,x}+e^{2\,a}\,e^{-2\,d}\right)}{b}$$

input
```
int(exp(a + b*x)*sinh(d + b*x)*tanh(d + b*x)^2,x)
```

output
```
exp(a + d + 2*b*x)/(4*b) - (x*exp(a - d))/2 - (2*exp(3*a - 3*d))/(b*(exp(2
*a - 2*d) + exp(2*a + 2*b*x))) - (exp(a - d)*log(exp(2*a)*exp(2*b*x) + exp
(2*a)*exp(-2*d)))/b
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.35

$$\int e^{a+bx}\sinh(d+bx)\tanh^2(d+bx)\,dx$$
$$=\frac{e^{a}\left(e^{4bx+4d}-4e^{2bx+2d}\log\left(e^{2bx+2d}+1\right)-2e^{2bx+2d}bx+9e^{2bx+2d}-4\log\left(e^{2bx+2d}+1\right)-2bx\right)}{4e^{d}b\left(e^{2bx+2d}+1\right)}$$

input
```
int(exp(b*x+a)*sinh(b*x+d)*tanh(b*x+d)^2,x)
```

output
```
(e**a*(e**(4*b*x + 4*d) - 4*e**(2*b*x + 2*d)*log(e**(2*b*x + 2*d) + 1) - 2
*e**(2*b*x + 2*d)*b*x + 9*e**(2*b*x + 2*d) - 4*log(e**(2*b*x + 2*d) + 1) -
 2*b*x))/(4*e**d*b*(e**(2*b*x + 2*d) + 1))
```

3.30 $\int e^{a+bx}\tanh^2(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 58

$$\int e^{a+bx}\tanh^2(d+bx)\,dx=\frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1+e^{2d+2bx}\right)}-\frac{2e^{a-d}\arctan\left(e^{d+bx}\right)}{b}$$

output
```
exp(b*x+a)/b+2*exp(b*x+a)/b/(1+exp(2*b*x+2*d))-2*exp(a-d)*arctan(exp(b*x+d
))/b
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.45

$$\int e^{a+bx}\tanh^2(d+bx)\,dx$$
$$=\frac{e^a\left(e^{bx}-2\arctan\left(e^{bx}(\cosh(d)+\sinh(d))\right)\cosh(d)+2\arctan\left(e^{bx}(\cosh(d)+\sinh(d))\right)\sinh(d)+\frac{2}{(1+e^{2bx}}\right.}{b}$$

input
```
Integrate[E^(a + b*x)*Tanh[d + b*x]^2,x]
```

output
```
(E^a*(E^(b*x) - 2*ArcTan[E^(b*x)*(Cosh[d] + Sinh[d])]*Cosh[d] + 2*ArcTan[E
^(b*x)*(Cosh[d] + Sinh[d])]*Sinh[d] + (2*E^(b*x)*(Cosh[d] - Sinh[d]))/((1
+ E^(2*b*x))*Cosh[d] + (-1 + E^(2*b*x))*Sinh[d])))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 27, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\tanh^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{e^a\left(1-e^{2bx}\right)^2}{\left(1+e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^a\int \frac{\left(1-e^{2bx}\right)^2}{\left(1+e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{300}$$

$$\frac{e^a\int \left(1-\frac{4e^{2bx}}{\left(1+e^{2bx}\right)^2}\right)de^{bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^a\left(-2\arctan\left(e^{bx}\right)+e^{bx}+\frac{2e^{bx}}{e^{2bx}+1}\right)}{b}$$

input
```
Int[E^(a + b*x)*Tanh[d + b*x]^2,x]
```

output
```
(E^a*(E^(b*x) + (2*E^(b*x))/(1 + E^(2*b*x)) - 2*ArcTan[E^(b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.76

method	result	size
risch	$\frac{\mathrm{e}^{b x +a}}{b}+\frac{2 \,\mathrm{e}^{b x +3 a}}{\left(\mathrm{e}^{2 b x +2 a +2 d}+\mathrm{e}^{2 a}\right) b}+\frac{i \ln \left(\mathrm{e}^{b x +a}-i \mathrm{e}^{a -d}\right) \mathrm{e}^{a -d}}{b}-\frac{i \ln \left(\mathrm{e}^{b x +a}+i \mathrm{e}^{a -d}\right) \mathrm{e}^{a -d}}{b}$	102

input
```
int(exp(b*x+a)*tanh(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
exp(b*x+a)/b+2/(exp(2*b*x+2*a+2*d)+exp(2*a))/b*exp(b*x+3*a)+I*ln(exp(b*x+a
)-I*exp(a-d))/b*exp(a-d)-I*ln(exp(b*x+a)+I*exp(a-d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(53) = 106$.

Time = 0.09 (sec) , antiderivative size = 321, normalized size of antiderivative = 5.53

$$\int e^{a+bx}\tanh^2(d+bx)\,dx = \frac{\cosh(bx+d)^3\cosh(-a+d)+(\cosh(-a+d)-\sinh(-a+d))\sinh(bx+d)^3+3(\cosh(bx+d)\cosh(\cdots}{}$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d)^2,x, algorithm="fricas")
```

output
```
(cosh(b*x + d)^3*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d
)^3 + 3*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x
 + d)^2 - 2*(cosh(b*x + d)^2*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*
sinh(b*x + d)^2 + 2*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a +
d))*sinh(b*x + d) - (cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*arc
tan(cosh(b*x + d) + sinh(b*x + d)) + 3*cosh(b*x + d)*cosh(-a + d) + 3*(cos
h(b*x + d)^2*cosh(-a + d) - (cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a +
 d))*sinh(b*x + d) - (cosh(b*x + d)^3 + 3*cosh(b*x + d))*sinh(-a + d))/(b*
cosh(b*x + d)^2 + 2*b*cosh(b*x + d)*sinh(b*x + d) + b*sinh(b*x + d)^2 + b)
```

Sympy [F]

$$\int e^{a+bx}\tanh^2(d+bx)\,dx = e^a\int e^{bx}\tanh^2{(bx+d)}\,dx$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d)**2,x)
```

output
```
exp(a)*Integral(exp(b*x)*tanh(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

$$\int e^{a+bx}\tanh^2(d+bx)\,dx=-\frac{2\arctan\left(e^{(bx+d)}\right)e^{(a-d)}}{b}+\frac{e^{(bx+a)}}{b}+\frac{2\,e^{(bx+3\,a)}}{b\left(e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right)}$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d)^2,x, algorithm="maxima")
```

output
```
-2*arctan(e^(b*x + d))*e^(a - d)/b + e^(b*x + a)/b + 2*e^(b*x + 3*a)/(b*(e
^(2*b*x + 2*a + 2*d) + e^(2*a)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\tanh^2(d+bx)\,dx=-\frac{2\arctan\left(e^{(bx+d)}\right)e^{(a-d)}-\frac{2\,e^{(bx+3\,a)}}{e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}}-e^{(bx+a)}}{b}$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d)^2,x, algorithm="giac")
```

output
```
-(2*arctan(e^(b*x + d))*e^(a - d) - 2*e^(b*x + 3*a)/(e^(2*b*x + 2*a + 2*d)
 + e^(2*a)) - e^(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.62

$$\int e^{a+bx}\tanh^2(d+bx)\,dx$$
$$=\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}}{b}-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b\,\sqrt{\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}}}\right)\,\sqrt{\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}}}{\sqrt{b^2}}+\frac{2\,\mathrm{e}^{3\,a}\,\mathrm{e}^{-2\,d}\,\mathrm{e}^{b\,x}}{b\,\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}+b\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}$$

input
```
int(exp(a + b*x)*tanh(d + b*x)^2,x)
```

output
```
(exp(b*x)*exp(a))/b - (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/(b*(exp(2*a)*e
xp(-2*d))^(1/2)))*(exp(2*a)*exp(-2*d))^(1/2))/(b^2)^(1/2) + (2*exp(3*a)*ex
p(-2*d)*exp(b*x))/(b*exp(2*a)*exp(-2*d) + b*exp(2*a)*exp(2*b*x))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.31

$$\int e^{a+bx}\tanh^2(d+bx)\,dx=\frac{e^a\left(-2e^{2bx+2d}atan\left(e^{bx+d}\right)-2atan\left(e^{bx+d}\right)+e^{3bx+3d}+3e^{bx+d}\right)}{e^d b\left(e^{2bx+2d}+1\right)}$$

input
```
int(exp(b*x+a)*tanh(b*x+d)^2,x)
```

output
```
(e**a*( - 2*e**(2*b*x + 2*d)*atan(e**(b*x + d)) - 2*atan(e**(b*x + d)) + e
**(3*b*x + 3*d) + 3*e**(b*x + d)))/(e**d*b*(e**(2*b*x + 2*d) + 1))
```

3.31 $\int e^{a+bx}\mathbf{sech}(d+bx)\tanh(d+bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 51

$$\int e^{a+bx}\text{sech}(d+bx)\tanh(d+bx)\,dx=\frac{2e^{a-d}}{b\left(1+e^{2d+2bx}\right)}+\frac{e^{a-d}\log\left(1+e^{2d+2bx}\right)}{b}$$

output
```
2*exp(a-d)/b/(1+exp(2*b*x+2*d))+exp(a-d)*ln(1+exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.94

$$\int e^{a+bx}\text{sech}(d+bx)\tanh(d+bx)\,dx$$
$$=\frac{e^a\Big(\cosh(d)\log\left(\left(1+e^{2bx}\right)\cosh(d)+\left(-1+e^{2bx}\right)\sinh(d)\right)-\log\left(\left(1+e^{2bx}\right)\cosh(d)+\left(-1+e^{2bx}\right)\sinh(\right.}{b}$$

input
```
Integrate[E^(a + b*x)*Sech[d + b*x]*Tanh[d + b*x],x]
```

output
```
(E^a*(Cosh[d]*Log[(1 + E^(2*b*x))*Cosh[d] + (-1 + E^(2*b*x))*Sinh[d]] - Lo
g[(1 + E^(2*b*x))*Cosh[d] + (-1 + E^(2*b*x))*Sinh[d]]*Sinh[d] + (2*(Cosh[d
] - Sinh[d])^2)/((1 + E^(2*b*x))*Cosh[d] + (-1 + E^(2*b*x))*Sinh[d])))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\tanh(bx+d)\text{sech}(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{2e^{a+bx}\left(1-e^{2bx}\right)}{\left(1+e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2e^a\int \frac{e^{bx}\left(1-e^{2bx}\right)}{\left(1+e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{353}$$

$$-\frac{e^a\int \frac{1-e^{2bx}}{\left(1+e^{2bx}\right)^2}de^{2bx}}{b}$$

$$\Big\downarrow \text{49}$$

$$-\frac{e^a\int \left(\frac{2}{\left(1+e^{2bx}\right)^2}+\frac{1}{-1-e^{2bx}}\right)de^{2bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^a\left(-\frac{2}{e^{2bx}+1}-\log\left(e^{2bx}+1\right)\right)}{b}$$

input
```
Int[E^(a + b*x)*Sech[d + b*x]*Tanh[d + b*x],x]
```

output
```
-((E^a*(-2/(1 + E^(2*b*x)) - Log[1 + E^(2*b*x)]))/b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.47

method	result	size
risch	$-\frac{2\,{\mathrm e}^{a-d} a}{b}+\frac{2\,{\mathrm e}^{3 a-d}}{\left({\mathrm e}^{2 b x+2 a+2 d}+{\mathrm e}^{2 a}\right) b}+\frac{\ln\left({\mathrm e}^{2 b x+2 a}+{\mathrm e}^{2 a-2 d}\right) {\mathrm e}^{a-d}}{b}$	75

input
```
int(exp(b*x+a)*sech(b*x+d)*tanh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
-2/b*exp(a-d)*a+2/(exp(2*b*x+2*a+2*d)+exp(2*a))/b*exp(3*a-d)+ln(exp(2*b*x+
2*a)+exp(2*a-2*d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(47) = 94$.

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.61

$$\int e^{a+bx}\text{sech}(d+bx)\tanh(d+bx)\,dx$$
$$= \frac{\left(\cosh\left(bx+d\right)^2\cosh\left(-a+d\right)+\left(\cosh\left(-a+d\right)-\sinh\left(-a+d\right)\right)\sinh\left(bx+d\right)^2+2\left(\cosh\left(bx+d\right)\cosh\left(\right.\right.}{b}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)*tanh(b*x+d),x, algorithm="fricas")
```

output
```
((cosh(b*x + d)^2*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x +
d)^2 + 2*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*
x + d) - (cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*log(2*cosh(b*x
 + d)/(cosh(b*x + d) - sinh(b*x + d))) + 2*cosh(-a + d) - 2*sinh(-a + d))/
(b*cosh(b*x + d)^2 + 2*b*cosh(b*x + d)*sinh(b*x + d) + b*sinh(b*x + d)^2 +
 b)
```

Sympy [F]

$$\int e^{a+bx}\text{sech}(d+bx)\tanh(d+bx)\,dx = e^a\int e^{bx}\tanh\left(bx+d\right)\text{sech}\left(bx+d\right)dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)*tanh(b*x+d),x)
```

output
```
exp(a)*Integral(exp(b*x)*tanh(b*x + d)*sech(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int e^{a+bx}\text{sech}(d+bx)\tanh(d+bx)\,dx = \frac{e^{(a-d)}\log\left(e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right)}{b} + \frac{2\,e^{(3\,a)}}{b\left(e^{(2\,bx+2\,a+3\,d)}+e^{(2\,a+d)}\right)}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)*tanh(b*x+d),x, algorithm="maxima")
```

output
```
e^(a - d)*log(e^(2*b*x + 2*a + 2*d) + e^(2*a))/b + 2*e^(3*a)/(b*(e^(2*b*x
+ 2*a + 3*d) + e^(2*a + d)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.45

$$\int e^{a+bx}\text{sech}(d+bx)\tanh(d+bx)\,dx = \frac{e^{(a-d)}\log\left(e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right) - \frac{\left(e^{(2\,bx+3\,a+2\,d)}-e^{(3\,a)}\right)e^{(-d)}}{e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}}}{b}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)*tanh(b*x+d),x, algorithm="giac")
```

output
```
(e^(a - d)*log(e^(2*b*x + 2*a + 2*d) + e^(2*a)) - (e^(2*b*x + 3*a + 2*d) -
 e^(3*a))*e^(-d)/(e^(2*b*x + 2*a + 2*d) + e^(2*a)))/b
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

$$\int e^{a+bx}\text{sech}(d+bx)\tanh(d+bx)\,dx = \frac{2\,\mathrm{e}^{3\,a-3\,d}}{b\,\left(\mathrm{e}^{2\,a-2\,d}+\mathrm{e}^{2\,a+2\,b\,x}\right)} + \frac{\mathrm{e}^{a-d}\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}\right)}{b}$$

input
```
int((exp(a + b*x)*tanh(d + b*x))/cosh(d + b*x),x)
```

output
```
(2*exp(3*a - 3*d))/(b*(exp(2*a - 2*d) + exp(2*a + 2*b*x))) + (exp(a - d)*l
og(exp(2*a)*exp(2*b*x) + exp(2*a)*exp(-2*d)))/b
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.49

$$\int e^{a+bx}\text{sech}(d+bx)\tanh(d+bx)\,dx = \frac{e^{a}\left(e^{2bx+2d}\log\left(e^{2bx+2d}+1\right)-2e^{2bx+2d}+\log\left(e^{2bx+2d}+1\right)\right)}{e^{d}b\left(e^{2bx+2d}+1\right)}$$

input
```
int(exp(b*x+a)*sech(b*x+d)*tanh(b*x+d),x)
```

output
```
(e**a*(e**(2*b*x + 2*d)*log(e**(2*b*x + 2*d) + 1) - 2*e**(2*b*x + 2*d) + l
og(e**(2*b*x + 2*d) + 1)))/(e**d*b*(e**(2*b*x + 2*d) + 1))
```

3.32 $\int e^{a+bx}\operatorname{sech}^2(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 47

$$\int e^{a+bx}\operatorname{sech}^2(d+bx)\,dx=-\frac{2e^{a+bx}}{b\left(1+e^{2d+2bx}\right)}+\frac{2e^{a-d}\arctan\left(e^{d+bx}\right)}{b}$$

output
```
-2*exp(b*x+a)/b/(1+exp(2*b*x+2*d))+2*exp(a-d)*arctan(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int e^{a+bx}\operatorname{sech}^2(d+bx)\,dx=\frac{2e^a\left(-\frac{e^{bx}}{1+e^{2(d+bx)}}+e^{-d}\arctan\left(e^{d+bx}\right)\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Sech[d + b*x]^2,x]
```

output
```
(2*E^a*(-(E^(b*x)/(1 + E^(2*(d + b*x)))) + ArcTan[E^(d + b*x)]/E^d))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 27, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{sech}^2(bx+d)\,dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int \frac{4e^{a+2bx}}{(1+e^{2bx})^2}de^{bx}}{b}$$

$$\Big\downarrow \ 27$$

$$\frac{4e^a\int \frac{e^{2bx}}{(1+e^{2bx})^2}de^{bx}}{b}$$

$$\Big\downarrow \ 252$$

$$\frac{4e^a\left(\frac{1}{2}\int \frac{1}{1+e^{2bx}}de^{bx}-\frac{e^{bx}}{2(e^{2bx}+1)}\right)}{b}$$

$$\Big\downarrow \ 216$$

$$\frac{4e^a\left(\frac{1}{2}\arctan\left(e^{bx}\right)-\frac{e^{bx}}{2(e^{2bx}+1)}\right)}{b}$$

input
```
Int[E^(a + b*x)*Sech[d + b*x]^2,x]
```

output
```
(4*E^a*(-1/2*E^(b*x)/(1 + E^(2*b*x)) + ArcTan[E^(b*x)]/2))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.96

method	result	size
risch	$-\frac{2\,{\mathrm e}^{b x +3 a}}{\left({\mathrm e}^{2 b x +2 a +2 d}+{\mathrm e}^{2 a}\right) b}+\frac{i \ln\left({\mathrm e}^{b x +a}+i {\mathrm e}^{a -d}\right) {\mathrm e}^{a -d}}{b}-\frac{i \ln\left({\mathrm e}^{b x +a}-i {\mathrm e}^{a -d}\right) {\mathrm e}^{a -d}}{b}$	92

input
```
int(exp(b*x+a)*sech(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/(exp(2*b*x+2*a+2*d)+exp(2*a))/b*exp(b*x+3*a)+I*ln(exp(b*x+a)+I*exp(a-d)
)/b*exp(a-d)-I*ln(exp(b*x+a)-I*exp(a-d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(43) = 86$.

Time = 0.08 (sec) , antiderivative size = 207, normalized size of antiderivative = 4.40

$$\int e^{a+bx}\text{sech}^2(d+bx)\,dx = \frac{2\left(\left(\cosh\left(bx+d\right)^2\cosh\left(-a+d\right)+\left(\cosh\left(-a+d\right)-\sinh\left(-a+d\right)\right)\sinh\left(bx+d\right)^2+2\left(\cosh\left(bx+d\right)\cos\right.\right.\right.}{}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)^2,x, algorithm="fricas")
```

output
```
2*((cosh(b*x + d)^2*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x
+ d)^2 + 2*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(
b*x + d) - (cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*arctan(cosh(
b*x + d) + sinh(b*x + d)) - cosh(b*x + d)*cosh(-a + d) - (cosh(-a + d) - s
inh(-a + d))*sinh(b*x + d) + cosh(b*x + d)*sinh(-a + d))/(b*cosh(b*x + d)^
2 + 2*b*cosh(b*x + d)*sinh(b*x + d) + b*sinh(b*x + d)^2 + b)
```

Sympy [F]

$$\int e^{a+bx}\text{sech}^2(d+bx)\,dx = e^{a}\int e^{bx}\operatorname{sech}^2\left(bx+d\right)dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)**2,x)
```

output
```
exp(a)*Integral(exp(b*x)*sech(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int e^{a+bx}\text{sech}^2(d+bx)\,dx = \frac{2\,\arctan\left(e^{(bx+d)}\right)e^{(a-d)}}{b} - \frac{2\,e^{(bx+3\,a)}}{b\left(e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right)}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)^2,x, algorithm="maxima")
```

output
```
2*arctan(e^(b*x + d))*e^(a - d)/b - 2*e^(b*x + 3*a)/(b*(e^(2*b*x + 2*a + 2
*d) + e^(2*a)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int e^{a+bx}\text{sech}^2(d+bx)\,dx = 2\left(\frac{\arctan\left(e^{(bx+d)}\right)e^{(-3\,d)}}{b} - \frac{e^{(bx-2\,d)}}{b\left(e^{(2\,bx+2\,d)}+1\right)}\right)e^{(a+2\,d)}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)^2,x, algorithm="giac")
```

output
```
2*(arctan(e^(b*x + d))*e^(-3*d)/b - e^(b*x - 2*d)/(b*(e^(2*b*x + 2*d) + 1)
))*e^(a + 2*d)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.79

$$\int e^{a+bx}\text{sech}^2(d+bx)\,dx = \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b\,\sqrt{\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}}}\right)\sqrt{\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}}}{\sqrt{b^2}} - \frac{2\,\mathrm{e}^{3\,a}\,\mathrm{e}^{-2\,d}\,\mathrm{e}^{b\,x}}{b\,\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}+b\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}$$

input
```
int(exp(a + b*x)/cosh(d + b*x)^2,x)
```

output
```
(2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/(b*(exp(2*a)*exp(-2*d))^(1/2)))*(exp
(2*a)*exp(-2*d))^(1/2))/(b^2)^(1/2) - (2*exp(3*a)*exp(-2*d)*exp(b*x))/(b*e
xp(2*a)*exp(-2*d) + b*exp(2*a)*exp(2*b*x))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.36

$$\int e^{a+bx}\text{sech}^2(d+bx)\,dx = \frac{2e^a\left(e^{2bx+2d}atan\left(e^{bx+d}\right)+atan\left(e^{bx+d}\right)-e^{bx+d}\right)}{e^d b\left(e^{2bx+2d}+1\right)}$$

input
```
int(exp(b*x+a)*sech(b*x+d)^2,x)
```

output
```
(2*e**a*(e**(2*b*x + 2*d)*atan(e**(b*x + d)) + atan(e**(b*x + d)) - e**(b*
x + d)))/(e**d*b*(e**(2*b*x + 2*d) + 1))
```

3.33 $\int e^{a+bx}\operatorname{csch}(d+bx)\operatorname{sech}^2(d+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 50

$$\int e^{a+bx}\operatorname{csch}(d+bx)\operatorname{sech}^2(d+bx)\,dx=-\frac{2e^{a-d}}{b(1+e^{2d+2bx})}-\frac{2e^{a-d}\operatorname{arctanh}\left(e^{2d+2bx}\right)}{b}$$

output
```
-2*exp(a-d)/b/(1+exp(2*b*x+2*d))-2*exp(a-d)*arctanh(exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int e^{a+bx}\operatorname{csch}(d+bx)\operatorname{sech}^2(d+bx)\,dx=\frac{8e^{a-d}\left(-\frac{1}{4(1+e^{2(d+bx)})}-\frac{1}{4}\operatorname{arctanh}\left(e^{2(d+bx)}\right)\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Csch[d + b*x]*Sech[d + b*x]^2,x]
```

output
```
(8*E^(a - d)*(-1/4*1/(1 + E^(2*(d + b*x))) - ArcTanh[E^(2*(d + b*x))]/4))/
b
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2720, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{csch}(bx+d)\text{sech}^2(bx+d)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int -\frac{8e^{a+3bx}}{\left(1-e^{2bx}\right)\left(1+e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{8e^a\int \frac{e^{3bx}}{\left(1-e^{2bx}\right)\left(1+e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{ 354}$$

$$-\frac{4e^a\int \frac{e^{2bx}}{\left(1-e^{2bx}\right)\left(1+e^{2bx}\right)^2}de^{2bx}}{b}$$

$$\Big\downarrow \text{ 86}$$

$$-\frac{4e^a\int \left(-\frac{1}{2\left(1+e^{2bx}\right)^2}-\frac{1}{2\left(-1+e^{2bx}\right)}\right)de^{2bx}}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{4e^a\left(\frac{1}{2}\text{arctanh}\left(e^{2bx}\right)+\frac{1}{2\left(e^{2bx}+1\right)}\right)}{b}$$

input
```
Int[E^(a + b*x)*Csch[d + b*x]*Sech[d + b*x]^2,x]
```

output
```
(-4*E^a*(1/(2*(1 + E^(2*b*x))) + ArcTanh[E^(2*b*x)]/2))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(46) = 92$.

Time = 13.78 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.90

method	result	size
risch	$-\frac{2\,{\mathrm e}^{3 a -d}}{\left({\mathrm e}^{2 b x +2 a +2 d}+{\mathrm e}^{2 a}\right) b}+\frac{\ln\left({\mathrm e}^{2 b x +2 a}-{\mathrm e}^{2 a -2 d}\right) {\mathrm e}^{a -d}}{b}-\frac{\ln\left({\mathrm e}^{2 b x +2 a}+{\mathrm e}^{2 a -2 d}\right) {\mathrm e}^{a -d}}{b}$	95

input
```
int(exp(b*x+a)*csch(b*x+d)*sech(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/(exp(2*b*x+2*a+2*d)+exp(2*a))/b*exp(3*a-d)+ln(exp(2*b*x+2*a)-exp(2*a-2*
d))/b*exp(a-d)-ln(exp(2*b*x+2*a)+exp(2*a-2*d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(46) = 92$.

Time = 0.09 (sec) , antiderivative size = 313, normalized size of antiderivative = 6.26

$$\int e^{a+bx}\text{csch}(d+bx)\text{sech}^2(d+bx)\,dx =$$

$$-\frac{(\cosh(bx+d)^2\cosh(-a+d)+(\cosh(-a+d)-\sinh(-a+d))\sinh(bx+d)^2+2(\cosh(bx+d)\cos}{}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)*sech(b*x+d)^2,x, algorithm="fricas")
```

output
```
-((cosh(b*x + d)^2*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x +
 d)^2 + 2*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b
*x + d) - (cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*log(2*cosh(b*
x + d)/(cosh(b*x + d) - sinh(b*x + d))) - (cosh(b*x + d)^2*cosh(-a + d) +
(cosh(-a + d) - sinh(-a + d))*sinh(b*x + d)^2 + 2*(cosh(b*x + d)*cosh(-a +
 d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^2 + 1)*si
nh(-a + d) + cosh(-a + d))*log(2*sinh(b*x + d)/(cosh(b*x + d) - sinh(b*x +
 d))) + 2*cosh(-a + d) - 2*sinh(-a + d))/(b*cosh(b*x + d)^2 + 2*b*cosh(b*x
 + d)*sinh(b*x + d) + b*sinh(b*x + d)^2 + b)
```

Sympy [F]

$$\int e^{a+bx}\text{csch}(d+bx)\text{sech}^2(d+bx)\,dx = e^{a}\int e^{bx}\operatorname{csch}(bx+d)\operatorname{sech}^2(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)*sech(b*x+d)**2,x)
```

output
```
exp(a)*Integral(exp(b*x)*csch(b*x + d)*sech(b*x + d)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(46) = 92.

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.08

$$\int e^{a+bx}\operatorname{csch}(d+bx)\operatorname{sech}^2(d+bx)\,dx = -\frac{e^{(a-d)}\log\left(e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right)}{b} + \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^{a}\right)}{b} + \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}-e^{a}\right)}{b} - \frac{2\,e^{(3\,a)}}{b\left(e^{(2\,bx+2\,a+3\,d)}+e^{(2\,a+d)}\right)}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)*sech(b*x+d)^2,x, algorithm="maxima")
```

output
```
-e^(a - d)*log(e^(2*b*x + 2*a + 2*d) + e^(2*a))/b + e^(a - d)*log(e^(b*x +
 a + d) + e^a)/b + e^(a - d)*log(e^(b*x + a + d) - e^a)/b - 2*e^(3*a)/(b*(
e^(2*b*x + 2*a + 3*d) + e^(2*a + d)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.46

$$\int e^{a+bx}\operatorname{csch}(d+bx)\operatorname{sech}^2(d+bx)\,dx = -\left(\frac{e^{(-4\,d)}\log\left(e^{(2\,bx+2\,d)}+1\right)}{b} - \frac{e^{(-4\,d)}\log\left(\left|e^{(2\,bx+2\,d)}-1\right|\right)}{b} + \frac{2\,e^{(-4\,d)}}{b\left(e^{(2\,bx+2\,d)}+1\right)}\right)e^{(a+3\,d)}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)*sech(b*x+d)^2,x, algorithm="giac")
```

output
```
-(e^(-4*d)*log(e^(2*b*x + 2*d) + 1)/b - e^(-4*d)*log(abs(e^(2*b*x + 2*d) -
 1))/b + 2*e^(-4*d)/(b*(e^(2*b*x + 2*d) + 1)))*e^(a + 3*d)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\text{csch}(d+bx)\text{sech}^2(d+bx)\,dx=\int \frac{e^{a+b\,x}}{{\cosh\left(d+b\,x\right)}^2\,\sinh\left(d+b\,x\right)}\,dx$$

input
```
int(exp(a + b*x)/(cosh(d + b*x)^2*sinh(d + b*x)),x)
```

output
```
int(exp(a + b*x)/(cosh(d + b*x)^2*sinh(d + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.82

$$\int e^{a+bx}\text{csch}(d+bx)\text{sech}^2(d+bx)\,dx = \frac{e^a\left(e^{2bx+2d}\log\left(e^{bx+d}-1\right)+e^{2bx+2d}\log\left(e^{bx+d}+1\right)-e^{2bx+2d}\log\left(e^{2bx+2d}+1\right)+2e^{2bx+2d}+\log\left(e^{bx+d}-1\right)\right.}{e^d b\left(e^{2bx+2d}+1\right)}$$

input
```
int(exp(b*x+a)*csch(b*x+d)*sech(b*x+d)^2,x)
```

output
```
(e**a*(e**(2*b*x + 2*d)*log(e**(b*x + d) - 1) + e**(2*b*x + 2*d)*log(e**(b
*x + d) + 1) - e**(2*b*x + 2*d)*log(e**(2*b*x + 2*d) + 1) + 2*e**(2*b*x +
2*d) + log(e**(b*x + d) - 1) + log(e**(b*x + d) + 1) - log(e**(2*b*x + 2*d
) + 1)))/(e**d*b*(e**(2*b*x + 2*d) + 1))
```

3.34 $\int e^{a+bx}\text{csch}^2(d+bx)\text{sech}^2(d+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 69

$$\int e^{a+bx}\text{csch}^2(d+bx)\text{sech}^2(d+bx)\,dx = \frac{4e^{a+bx}}{b(1-e^{4d+4bx})} - \frac{2e^{a-d}\arctan\left(e^{d+bx}\right)}{b} - \frac{2e^{a-d}\text{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
4*exp(b*x+a)/b/(1-exp(4*b*x+4*d))-2*exp(a-d)*arctan(exp(b*x+d))/b-2*exp(a-
d)*arctanh(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int e^{a+bx}\text{csch}^2(d+bx)\text{sech}^2(d+bx)\,dx = \frac{2e^{a-d}\left(-\frac{2e^{d+bx}}{-1+e^{4(d+bx)}} - \arctan\left(e^{d+bx}\right) - \text{arctanh}\left(e^{d+bx}\right)\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Csch[d + b*x]^2*Sech[d + b*x]^2,x]
```

output
```
(2*E^(a - d)*((-2*E^(d + b*x))/(-1 + E^(4*(d + b*x))) - ArcTan[E^(d + b*x)
] - ArcTanh[E^(d + b*x)]))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 27, 817, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\operatorname{csch}^2(bx+d)\operatorname{sech}^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{16e^{a+4bx}}{\left(1-e^{4bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{16e^a\int \frac{e^{4bx}}{\left(1-e^{4bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{817}$$

$$\frac{16e^a\left(\frac{e^{bx}}{4\left(1-e^{4bx}\right)}-\frac{1}{4}\int \frac{1}{1-e^{4bx}}de^{bx}\right)}{b}$$

$$\Big\downarrow \text{756}$$

$$\frac{16e^a\left(\frac{1}{4}\left(-\frac{1}{2}\int \frac{1}{1-e^{2bx}}de^{bx}-\frac{1}{2}\int \frac{1}{1+e^{2bx}}de^{bx}\right)+\frac{e^{bx}}{4\left(1-e^{4bx}\right)}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{16e^a\left(\frac{1}{4}\left(-\frac{1}{2}\int \frac{1}{1-e^{2bx}}de^{bx}-\frac{1}{2}\arctan\left(e^{bx}\right)\right)+\frac{e^{bx}}{4\left(1-e^{4bx}\right)}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{16e^a\left(\frac{1}{4}\left(-\frac{1}{2}\arctan\left(e^{bx}\right)-\frac{1}{2}\text{arctanh}\left(e^{bx}\right)\right)+\frac{e^{bx}}{4\left(1-e^{4bx}\right)}\right)}{b}$$

input
```
Int[E^(a + b*x)*Csch[d + b*x]^2*Sech[d + b*x]^2,x]
```

output
```
(16*E^a*(E^(b*x)/(4*(1 - E^(4*b*x))) + (-1/2*ArcTan[E^(b*x)] - ArcTanh[E^(
b*x)]/2)/4))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 64.27 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.38

method	result
risch	$\frac{4\,\mathrm{e}^{bx+5a}}{\left(\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)\left(-\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)b}+\frac{\ln\left(\mathrm{e}^{bx+a}-\mathrm{e}^{a-d}\right)\mathrm{e}^{a-d}}{b}-\frac{\ln\left(\mathrm{e}^{bx+a}+\mathrm{e}^{a-d}\right)\mathrm{e}^{a-d}}{b}+\frac{i\ln\left(\mathrm{e}^{bx+a}-i\mathrm{e}^{a-d}\right)\mathrm{e}^{a-d}}{b}-\frac{i\ln$

input
```
int(exp(b*x+a)*csch(b*x+d)^2*sech(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
4/(exp(2*b*x+2*a+2*d)+exp(2*a))/(-exp(2*b*x+2*a+2*d)+exp(2*a))/b*exp(b*x+5
*a)+ln(exp(b*x+a)-exp(a-d))/b*exp(a-d)-ln(exp(b*x+a)+exp(a-d))/b*exp(a-d)+
I*ln(exp(b*x+a)-I*exp(a-d))/b*exp(a-d)-I*ln(exp(b*x+a)+I*exp(a-d))/b*exp(a
-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(61) = 122$.

Time = 0.10 (sec) , antiderivative size = 740, normalized size of antiderivative = 10.72

$$\int e^{a+bx}\mathrm{csch}^2(d+bx)\mathrm{sech}^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^2*sech(b*x+d)^2,x, algorithm="fricas")
```

output
```
-(2*(cosh(b*x + d)^4*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x
 + d)^4 + 4*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh
(b*x + d)^3 + 6*(cosh(b*x + d)^2*cosh(-a + d) - cosh(b*x + d)^2*sinh(-a +
d))*sinh(b*x + d)^2 + 4*(cosh(b*x + d)^3*cosh(-a + d) - cosh(b*x + d)^3*si
nh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^4 - 1)*sinh(-a + d) - cosh(-a +
 d))*arctan(cosh(b*x + d) + sinh(b*x + d)) + 4*cosh(b*x + d)*cosh(-a + d)
+ (cosh(b*x + d)^4*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x +
 d)^4 + 4*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b
*x + d)^3 + 6*(cosh(b*x + d)^2*cosh(-a + d) - cosh(b*x + d)^2*sinh(-a + d)
)*sinh(b*x + d)^2 + 4*(cosh(b*x + d)^3*cosh(-a + d) - cosh(b*x + d)^3*sinh
(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^4 - 1)*sinh(-a + d) - cosh(-a + d
))*log(cosh(b*x + d) + sinh(b*x + d) + 1) - (cosh(b*x + d)^4*cosh(-a + d)
+ (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-a
 + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d)^3 + 6*(cosh(b*x + d)^2*c
osh(-a + d) - cosh(b*x + d)^2*sinh(-a + d))*sinh(b*x + d)^2 + 4*(cosh(b*x
+ d)^3*cosh(-a + d) - cosh(b*x + d)^3*sinh(-a + d))*sinh(b*x + d) - (cosh(
b*x + d)^4 - 1)*sinh(-a + d) - cosh(-a + d))*log(cosh(b*x + d) + sinh(b*x
+ d) - 1) + 4*(cosh(-a + d) - sinh(-a + d))*sinh(b*x + d) - 4*cosh(b*x + d
)*sinh(-a + d))/(b*cosh(b*x + d)^4 + 4*b*cosh(b*x + d)^3*sinh(b*x + d) + 6
*b*cosh(b*x + d)^2*sinh(b*x + d)^2 + 4*b*cosh(b*x + d)*sinh(b*x + d)^3 ...
```

Sympy [F]

$$\int e^{a+bx}\text{csch}^2(d+bx)\text{sech}^2(d+bx)\,dx = e^a\int e^{bx}\,\text{csch}^2\,(bx+d)\,\text{sech}^2\,(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)**2*sech(b*x+d)**2,x)
```

output
```
exp(a)*Integral(exp(b*x)*csch(b*x + d)**2*sech(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.42

$$\int e^{a+bx}\operatorname{csch}^2(d+bx)\operatorname{sech}^2(d+bx)\,dx = -\frac{2\arctan\left(e^{(bx+d)}\right)e^{(a-d)}}{b} - \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^{a}\right)}{b} + \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}-e^{a}\right)}{b} - \frac{4\,e^{(bx+5\,a)}}{b\left(e^{(4\,bx+4\,a+4\,d)}-e^{(4\,a)}\right)}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^2*sech(b*x+d)^2,x, algorithm="maxima")
```

output
```
-2*arctan(e^(b*x + d))*e^(a - d)/b - e^(a - d)*log(e^(b*x + a + d) + e^a)/
b + e^(a - d)*log(e^(b*x + a + d) - e^a)/b - 4*e^(b*x + 5*a)/(b*(e^(4*b*x
+ 4*a + 4*d) - e^(4*a)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

$$\int e^{a+bx}\operatorname{csch}^2(d+bx)\operatorname{sech}^2(d+bx)\,dx = -\left(\frac{2\arctan\left(e^{(bx+d)}\right)e^{(-5\,d)}}{b} + \frac{e^{(-5\,d)}\log\left(e^{(bx+d)}+1\right)}{b} - \frac{e^{(-5\,d)}\log\left(\left|e^{(bx+d)}-1\right|\right)}{b} + \frac{4\,e^{(bx-4\,d)}}{b\left(e^{(4\,bx+4\,d)}-1\right)}\right)e$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^2*sech(b*x+d)^2,x, algorithm="giac")
```

output
```
-(2*arctan(e^(b*x + d))*e^(-5*d)/b + e^(-5*d)*log(e^(b*x + d) + 1)/b - e^(
-5*d)*log(abs(e^(b*x + d) - 1))/b + 4*e^(b*x - 4*d)/(b*(e^(4*b*x + 4*d) -
1)))*e^(a + 4*d)
```

Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.81

$$\int e^{a+bx}\operatorname{csch}^2(d+bx)\operatorname{sech}^2(d+bx)\,dx$$
$$= \frac{4\,e^{a+b\,x}}{b-b\,e^{4\,d+4\,b\,x}} - \frac{\left(e^{4\,a-4\,d}\right)^{1/4}\ln\left(-4\,\left(e^{4\,a}\,e^{-4\,d}\right)^{5/4}-4\,e^{5\,a}\,e^{-4\,d}\,e^{b\,x}\right)}{b} + \frac{\left(e^{4\,a-4\,d}\right)^{1/4}\ln\left(4\,\left(e^{4\,a}\,e^{-4\,d}\right)^{5/4}-4\,e^{5\,a}\,e^{-4\,d}\,e^{b\,x}\right)}{b} - \frac{\left(e^{4\,a-4\,d}\right)^{1/4}\left(\ln\left(-4\,e^{5\,a}\,e^{-4\,d}\,e^{b\,x}-\left(e^{4\,a}\,e^{-4\,d}\right)^{5/4}4\mathrm{i}\right)1\mathrm{i}-\ln\left(-4\,e^{5\,a}\,e^{-4\,d}\,e^{b\,x}+\left(e^{4\,a}\,e^{-4\,d}\right)^{5/4}4\mathrm{i}\right)1\mathrm{i}\right)}{b}$$

input
```
int(exp(a + b*x)/(cosh(d + b*x)^2*sinh(d + b*x)^2),x)
```

output
```
(4*exp(a + b*x))/(b - b*exp(4*d + 4*b*x)) - (exp(4*a - 4*d)^(1/4)*log(- 4*
(exp(4*a)*exp(-4*d))^(5/4) - 4*exp(5*a)*exp(-4*d)*exp(b*x)))/b + (exp(4*a
- 4*d)^(1/4)*log(4*(exp(4*a)*exp(-4*d))^(5/4) - 4*exp(5*a)*exp(-4*d)*exp(b
*x)))/b - (exp(4*a - 4*d)^(1/4)*(log(- (exp(4*a)*exp(-4*d))^(5/4)*4i - 4*e
xp(5*a)*exp(-4*d)*exp(b*x))*1i - log((exp(4*a)*exp(-4*d))^(5/4)*4i - 4*exp
(5*a)*exp(-4*d)*exp(b*x))*1i))/b
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.90

$$\int e^{a+bx}\operatorname{csch}^2(d+bx)\operatorname{sech}^2(d+bx)\,dx$$
$$= \frac{e^{a}\left(-2e^{4bx+4d}atan\left(e^{bx+d}\right)+2atan\left(e^{bx+d}\right)+e^{4bx+4d}\log\left(e^{bx+d}-1\right)-e^{4bx+4d}\log\left(e^{bx+d}+1\right)-4e^{bx+d}-\mathrm{lo}\right.}{e^{d}b\left(e^{4bx+4d}-1\right)}$$

input
```
int(exp(b*x+a)*csch(b*x+d)^2*sech(b*x+d)^2,x)
```

output
```
(e**a*( - 2*e**(4*b*x + 4*d)*atan(e**(b*x + d)) + 2*atan(e**(b*x + d)) + e
**(4*b*x + 4*d)*log(e**(b*x + d) - 1) - e**(4*b*x + 4*d)*log(e**(b*x + d)
+ 1) - 4*e**(b*x + d) - log(e**(b*x + d) - 1) + log(e**(b*x + d) + 1)))/(e
**d*b*(e**(4*b*x + 4*d) - 1))
```

3.35 $\int e^{a+bx}\mathrm{csch}^3(d+bx)\mathrm{sech}^2(d+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 106

$$\int e^{a+bx}\mathrm{csch}^3(d+bx)\mathrm{sech}^2(d+bx)\,dx=-\frac{2e^{a-d}}{b\left(1-e^{2d+2bx}\right)^2}+\frac{4e^{a-d}}{b\left(1-e^{2d+2bx}\right)}+\frac{2e^{a-d}}{b\left(1+e^{2d+2bx}\right)}+\frac{2e^{a-d}\mathrm{arctanh}\left(e^{2d+2bx}\right)}{b}$$

output
```
-2*exp(a-d)/b/(1-exp(2*b*x+2*d))^2+4*exp(a-d)/b/(1-exp(2*b*x+2*d))+2*exp(a
-d)/b/(1+exp(2*b*x+2*d))+2*exp(a-d)*arctanh(exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.62

$$\int e^{a+bx}\mathrm{csch}^3(d+bx)\mathrm{sech}^2(d+bx)\,dx=\frac{2e^{a-d}\left(-\frac{1}{\left(-1+e^{2(d+bx)}\right)^2}-\frac{2}{-1+e^{2(d+bx)}}+\frac{1}{1+e^{2(d+bx)}}+\mathrm{arctanh}\left(e^{2(d+bx)}\right)\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Csch[d + b*x]^3*Sech[d + b*x]^2,x]
```

output
```
(2*E^(a - d)*(-(-1 + E^(2*(d + b*x)))^(-2) - 2/(-1 + E^(2*(d + b*x))) + (1
 + E^(2*(d + b*x)))^(-1) + ArcTanh[E^(2*(d + b*x))]))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.62, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2720, 27, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{csch}^3(bx+d)\text{sech}^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{32e^{a+5bx}}{(1-e^{2bx})^3(1+e^{2bx})^2}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{32e^a\int \frac{e^{5bx}}{(1-e^{2bx})^3(1+e^{2bx})^2}de^{bx}}{b}$$

$$\Big\downarrow \text{354}$$

$$-\frac{16e^a\int \frac{e^{2bx}}{(1-e^{2bx})^3(1+e^{2bx})^2}de^{2bx}}{b}$$

$$\Big\downarrow \text{99}$$

$$-\frac{16e^a\int \left(\frac{1}{8(1+e^{2bx})^2}+\frac{1}{8(-1+e^{2bx})}-\frac{1}{4(-1+e^{2bx})^2}-\frac{1}{4(-1+e^{2bx})^3}\right)de^{2bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{16e^a\left(-\frac{1}{8}\text{arctanh}\left(e^{2bx}\right)-\frac{1}{4(1-e^{2bx})}-\frac{1}{8(e^{2bx}+1)}+\frac{1}{8(1-e^{2bx})^2}\right)}{b}$$

input
```
Int[E^(a + b*x)*Csch[d + b*x]^3*Sech[d + b*x]^2,x]
```

output
```
(-16*E^a*(1/(8*(1 - E^(2*b*x))^2) - 1/(4*(1 - E^(2*b*x))) - 1/(8*(1 + E^(2
*b*x))) - ArcTanh[E^(2*b*x)]/8))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 186.14 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.42

method	result	size
risch	$\frac{2\left(-\mathrm{e}^{4bx+4a+4d}-3\,\mathrm{e}^{2bx+4a+2d}+2\,\mathrm{e}^{4a}\right)\mathrm{e}^{3a-d}}{\left(-\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)^{2}\left(\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)b}+\frac{\ln\left(\mathrm{e}^{2bx+2a}+\mathrm{e}^{2a-2d}\right)\mathrm{e}^{a-d}}{b}-\frac{\ln\left(\mathrm{e}^{2bx+2a}-\mathrm{e}^{2a-2d}\right)\mathrm{e}^{a-d}}{b}$	151

input
```
int(exp(b*x+a)*csch(b*x+d)^3*sech(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
2/(-exp(2*b*x+2*a+2*d)+exp(2*a))^2/(exp(2*b*x+2*a+2*d)+exp(2*a))/b*(-exp(4
*b*x+4*a+4*d)-3*exp(2*b*x+4*a+2*d)+2*exp(4*a))*exp(3*a-d)+ln(exp(2*b*x+2*a
)+exp(2*a-2*d))/b*exp(a-d)-ln(exp(2*b*x+2*a)-exp(2*a-2*d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1397 vs. 2(94) = 188.

Time = 0.09 (sec) , antiderivative size = 1397, normalized size of antiderivative = 13.18

$$\int e^{a+bx}\operatorname{csch}^3(d+bx)\operatorname{sech}^2(d+bx)\,dx=\text{Too large to display}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^3*sech(b*x+d)^2,x, algorithm="fricas")
```

output
```
-(2*cosh(b*x + d)^4*cosh(-a + d) + 2*(cosh(-a + d) - sinh(-a + d))*sinh(b*
x + d)^4 + 8*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sin
h(b*x + d)^3 + 6*cosh(b*x + d)^2*cosh(-a + d) + 6*(2*cosh(b*x + d)^2*cosh(
-a + d) - (2*cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*sinh(b*x +
d)^2 - (cosh(b*x + d)^6*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(
b*x + d)^6 + 6*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*s
inh(b*x + d)^5 - cosh(b*x + d)^4*cosh(-a + d) + (15*cosh(b*x + d)^2*cosh(-
a + d) - (15*cosh(b*x + d)^2 - 1)*sinh(-a + d) - cosh(-a + d))*sinh(b*x +
d)^4 + 4*(5*cosh(b*x + d)^3*cosh(-a + d) - cosh(b*x + d)*cosh(-a + d) - (5
*cosh(b*x + d)^3 - cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d)^3 - cosh(b*x
 + d)^2*cosh(-a + d) + (15*cosh(b*x + d)^4*cosh(-a + d) - 6*cosh(b*x + d)^
2*cosh(-a + d) - (15*cosh(b*x + d)^4 - 6*cosh(b*x + d)^2 - 1)*sinh(-a + d)
 - cosh(-a + d))*sinh(b*x + d)^2 + 2*(3*cosh(b*x + d)^5*cosh(-a + d) - 2*c
osh(b*x + d)^3*cosh(-a + d) - cosh(b*x + d)*cosh(-a + d) - (3*cosh(b*x + d
)^5 - 2*cosh(b*x + d)^3 - cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d) - (co
sh(b*x + d)^6 - cosh(b*x + d)^4 - cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh
(-a + d))*log(2*cosh(b*x + d)/(cosh(b*x + d) - sinh(b*x + d))) + (cosh(b*x
 + d)^6*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d)^6 + 6*(
cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d)^5 -
 cosh(b*x + d)^4*cosh(-a + d) + (15*cosh(b*x + d)^2*cosh(-a + d) - (15*...
```

Sympy [F]

$$\int e^{a+bx}\text{csch}^3(d+bx)\text{sech}^2(d+bx)\,dx = e^a \int e^{bx}\,\text{csch}^3\,(bx+d)\,\text{sech}^2\,(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)**3*sech(b*x+d)**2,x)
```

output
```
exp(a)*Integral(exp(b*x)*csch(b*x + d)**3*sech(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.53

$$\int e^{a+bx}\mathrm{csch}^3(d+bx)\mathrm{sech}^2(d+bx)\,dx = \frac{e^{(a-d)}\log\left(e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right)}{b} - \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^{a}\right)}{b} - \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}-e^{a}\right)}{b} - \frac{2\left(e^{(4\,bx+7\,a+4\,d)}+3\,e^{(2\,bx+7\,a+2\,d)}-2\,e^{(7\,a)}\right)}{b\left(e^{(6\,bx+6\,a+7\,d)}-e^{(4\,bx+6\,a+5\,d)}-e^{(2\,bx+6\,a+3\,d)}+e^{(6\,a+d)}\right)}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^3*sech(b*x+d)^2,x, algorithm="maxima")
```

output
```
e^(a - d)*log(e^(2*b*x + 2*a + 2*d) + e^(2*a))/b - e^(a - d)*log(e^(b*x +
a + d) + e^a)/b - e^(a - d)*log(e^(b*x + a + d) - e^a)/b - 2*(e^(4*b*x + 7
*a + 4*d) + 3*e^(2*b*x + 7*a + 2*d) - 2*e^(7*a))/(b*(e^(6*b*x + 6*a + 7*d)
 - e^(4*b*x + 6*a + 5*d) - e^(2*b*x + 6*a + 3*d) + e^(6*a + d)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.23

$$\int e^{a+bx}\mathrm{csch}^3(d+bx)\mathrm{sech}^2(d+bx)\,dx = \frac{1}{2}\left(\frac{2\,e^{(-6\,d)}\log\left(e^{(2\,bx+2\,d)}+1\right)}{b} - \frac{2\,e^{(-6\,d)}\log\left(\left|e^{(2\,bx+2\,d)}-1\right|\right)}{b} - \frac{2\left(e^{(2\,bx+2\,d)}-1\right)e^{(-6\,d)}}{b\left(e^{(2\,bx+2\,d)}+1\right)} + \frac{\left(3\,e^{(4\,bx+4\,d)}\right.}{b(e}\right.$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^3*sech(b*x+d)^2,x, algorithm="giac")
```

output
```
1/2*(2*e^(-6*d)*log(e^(2*b*x + 2*d) + 1)/b - 2*e^(-6*d)*log(abs(e^(2*b*x +
 2*d) - 1))/b - 2*(e^(2*b*x + 2*d) - 1)*e^(-6*d)/(b*(e^(2*b*x + 2*d) + 1))
 + (3*e^(4*b*x + 4*d) - 14*e^(2*b*x + 2*d) + 7)*e^(-6*d)/(b*(e^(2*b*x + 2*
d) - 1)^2))*e^(a + 5*d)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\operatorname{csch}^3(d+bx)\operatorname{sech}^2(d+bx)\,dx=\int \frac{\mathrm{e}^{a+b\,x}}{{\mathrm{cosh}\left(d+b\,x\right)}^2\,{\mathrm{sinh}\left(d+b\,x\right)}^3}\,dx$$

input
```
int(exp(a + b*x)/(cosh(d + b*x)^2*sinh(d + b*x)^3),x)
```

output
```
int(exp(a + b*x)/(cosh(d + b*x)^2*sinh(d + b*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.97

$$\int e^{a+bx}\operatorname{csch}^3(d+bx)\operatorname{sech}^2(d+bx)\,dx$$
$$=\frac{e^a\left(-e^{6bx+6d}\log\left(e^{bx+d}-1\right)-e^{6bx+6d}\log\left(e^{bx+d}+1\right)+e^{6bx+6d}\log\left(e^{2bx+2d}+1\right)-2e^{6bx+6d}+e^{4bx+4d}\log\left(e^{bx\cdots}\right.\right.}{}$$

input
```
int(exp(b*x+a)*csch(b*x+d)^3*sech(b*x+d)^2,x)
```

output
```
(e**a*( - e**(6*b*x + 6*d)*log(e**(b*x + d) - 1) - e**(6*b*x + 6*d)*log(e*
*(b*x + d) + 1) + e**(6*b*x + 6*d)*log(e**(2*b*x + 2*d) + 1) - 2*e**(6*b*x
 + 6*d) + e**(4*b*x + 4*d)*log(e**(b*x + d) - 1) + e**(4*b*x + 4*d)*log(e*
*(b*x + d) + 1) - e**(4*b*x + 4*d)*log(e**(2*b*x + 2*d) + 1) + e**(2*b*x +
 2*d)*log(e**(b*x + d) - 1) + e**(2*b*x + 2*d)*log(e**(b*x + d) + 1) - e**
(2*b*x + 2*d)*log(e**(2*b*x + 2*d) + 1) - 4*e**(2*b*x + 2*d) - log(e**(b*x
 + d) - 1) - log(e**(b*x + d) + 1) + log(e**(2*b*x + 2*d) + 1) + 2))/(e**d
*b*(e**(6*b*x + 6*d) - e**(4*b*x + 4*d) - e**(2*b*x + 2*d) + 1))
```

3.36 $\int e^{a+bx} \tanh^3(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 84

$$\int e^{a+bx} \tanh^3(d+bx)\,dx = \frac{e^{a+bx}}{b} - \frac{2e^{a+bx}}{b\left(1+e^{2d+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1+e^{2d+2bx}\right)} - \frac{3e^{a-d}\arctan\left(e^{d+bx}\right)}{b}$$

output
```
exp(b*x+a)/b-2*exp(b*x+a)/b/(1+exp(2*b*x+2*d))^2+3*exp(b*x+a)/b/(1+exp(2*b
*x+2*d))-3*exp(a-d)*arctan(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.49

$$\int e^{a+bx} \tanh^3(d+bx)\,dx = \frac{e^a\left(e^{bx} - 3\arctan\left(e^{bx}(\cosh(d)+\sinh(d))\right)\cosh(d) + 3\arctan\left(e^{bx}(\cosh(d)+\sinh(d))\right)\sinh(d) - \frac{\ }{((1+e^{2b}}\right.}{b}$$

input
```
Integrate[E^(a + b*x)*Tanh[d + b*x]^3,x]
```

output
```
(E^a*(E^(b*x) - 3*ArcTan[E^(b*x)*(Cosh[d] + Sinh[d])]*Cosh[d] + 3*ArcTan[E
^(b*x)*(Cosh[d] + Sinh[d])]*Sinh[d] - (2*E^(b*x)*(Cosh[d] - Sinh[d])^2)/((
1 + E^(2*b*x))*Cosh[d] + (-1 + E^(2*b*x))*Sinh[d])^2 + (3*E^(b*x)*(Cosh[d]
 - Sinh[d]))/((1 + E^(2*b*x))*Cosh[d] + (-1 + E^(2*b*x))*Sinh[d])))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2720, 25, 27, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx} \tanh^3(bx+d)\,dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int -\frac{e^a(1-e^{2bx})^3}{(1+e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{e^a(1-e^{2bx})^3}{(1+e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \ 27$$

$$-\frac{e^a\int \frac{(1-e^{2bx})^3}{(1+e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \ 300$$

$$-\frac{e^a\int \left(\frac{2(1+3e^{4bx})}{(1+e^{2bx})^3}-1\right)de^{bx}}{b}$$

$$\Big\downarrow \ 2009$$

$$-\frac{e^a\left(3\arctan\left(e^{bx}\right)-e^{bx}-\frac{3e^{bx}}{e^{2bx}+1}+\frac{2e^{bx}}{(e^{2bx}+1)^2}\right)}{b}$$

input
```
Int[E^(a + b*x)*Tanh[d + b*x]^3,x]
```

output
```
-((E^a*(-E^(b*x) + (2*E^(b*x))/(1 + E^(2*b*x))^2 - (3*E^(b*x))/(1 + E^(2*b
*x)) + 3*ArcTan[E^(b*x)]))/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.43

method	result	size
risch	$\frac{e^{bx+a}}{b}+\frac{\left(3\,e^{2bx+2a+2d}+e^{2a}\right)e^{bx+3a}}{\left(e^{2bx+2a+2d}+e^{2a}\right)^{2}b}+\frac{3i\ln\left(e^{bx+a}-ie^{a-d}\right)e^{a-d}}{2b}-\frac{3i\ln\left(e^{bx+a}+ie^{a-d}\right)e^{a-d}}{2b}$	120

input
```
int(exp(b*x+a)*tanh(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
exp(b*x+a)/b+1/(exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(3*exp(2*b*x+2*a+2*d)+exp
(2*a))*exp(b*x+3*a)+3/2*I*ln(exp(b*x+a)-I*exp(a-d))/b*exp(a-d)-3/2*I*ln(ex
p(b*x+a)+I*exp(a-d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 702 vs. $2(77) = 154$.

Time = 0.11 (sec) , antiderivative size = 702, normalized size of antiderivative = 8.36

$$\int e^{a+bx}\tanh^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d)^3,x, algorithm="fricas")
```

output
```
(cosh(b*x + d)^5*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d
)^5 + 5*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x
 + d)^4 + 5*cosh(b*x + d)^3*cosh(-a + d) + 5*(2*cosh(b*x + d)^2*cosh(-a +
d) - (2*cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*sinh(b*x + d)^3
+ 5*(2*cosh(b*x + d)^3*cosh(-a + d) + 3*cosh(b*x + d)*cosh(-a + d) - (2*co
sh(b*x + d)^3 + 3*cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d)^2 - 3*(cosh(b
*x + d)^4*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d)^4 + 4
*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d)^3
 + 2*cosh(b*x + d)^2*cosh(-a + d) + 2*(3*cosh(b*x + d)^2*cosh(-a + d) - (3
*cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*sinh(b*x + d)^2 + 4*(co
sh(b*x + d)^3*cosh(-a + d) + cosh(b*x + d)*cosh(-a + d) - (cosh(b*x + d)^3
 + cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^4 + 2*cosh(
b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*arctan(cosh(b*x + d) + sinh(b
*x + d)) + 2*cosh(b*x + d)*cosh(-a + d) + (5*cosh(b*x + d)^4*cosh(-a + d)
+ 15*cosh(b*x + d)^2*cosh(-a + d) - (5*cosh(b*x + d)^4 + 15*cosh(b*x + d)^
2 + 2)*sinh(-a + d) + 2*cosh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^5 + 5
*cosh(b*x + d)^3 + 2*cosh(b*x + d))*sinh(-a + d))/(b*cosh(b*x + d)^4 + 4*b
*cosh(b*x + d)*sinh(b*x + d)^3 + b*sinh(b*x + d)^4 + 2*b*cosh(b*x + d)^2 +
 2*(3*b*cosh(b*x + d)^2 + b)*sinh(b*x + d)^2 + 4*(b*cosh(b*x + d)^3 + b*co
sh(b*x + d))*sinh(b*x + d) + b)
```

Sympy [F]

$$\int e^{a+bx} \tanh^3(d+bx)\,dx = e^a \int e^{bx} \tanh^3(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d)**3,x)
```

output
```
exp(a)*Integral(exp(b*x)*tanh(b*x + d)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.06

$$\int e^{a+bx} \tanh^3(d+bx)\,dx = -\frac{3 \arctan\left(e^{(bx+d)}\right) e^{(a-d)}}{b} + \frac{e^{(bx+a)}}{b} + \frac{3 e^{(3bx+5a+2d)} + e^{(bx+5a)}}{b(e^{(4bx+4a+4d)} + 2 e^{(2bx+4a+2d)} + e^{(4a)})}$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d)^3,x, algorithm="maxima")
```

output
```
-3*arctan(e^(b*x + d))*e^(a - d)/b + e^(b*x + a)/b + (3*e^(3*b*x + 5*a + 2
*d) + e^(b*x + 5*a))/(b*(e^(4*b*x + 4*a + 4*d) + 2*e^(2*b*x + 4*a + 2*d) +
 e^(4*a)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int e^{a+bx} \tanh^3(d+bx)\,dx = -\frac{3 \arctan\left(e^{(bx+d)}\right) e^{(a-d)} - \frac{3 e^{(3bx+5a+2d)} + e^{(bx+5a)}}{\left(e^{(2bx+2a+2d)} + e^{(2a)}\right)^2} - e^{(bx+a)}}{b}$$

input
```
integrate(exp(b*x+a)*tanh(b*x+d)^3,x, algorithm="giac")
```

output
```
-(3*arctan(e^(b*x + d))*e^(a - d) - (3*e^(3*b*x + 5*a + 2*d) + e^(b*x + 5*
a))/(e^(2*b*x + 2*a + 2*d) + e^(2*a))^2 - e^(b*x + a))/b
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\tanh^3(d+bx)\,dx=\int e^{a+b\,x}\,\tanh(d+b\,x)^3\,dx$$

input
```
int(exp(a + b*x)*tanh(d + b*x)^3,x)
```

output
```
int(exp(a + b*x)*tanh(d + b*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.43

$$\int e^{a+bx}\tanh^3(d+bx)\,dx = \frac{e^a\left(-3e^{4bx+4d}atan\left(e^{bx+d}\right)-6e^{2bx+2d}atan\left(e^{bx+d}\right)-3atan\left(e^{bx+d}\right)+e^{5bx+5d}+5e^{3bx+3d}+2e^{bx+d}\right)}{e^d b\left(e^{4bx+4d}+2e^{2bx+2d}+1\right)}$$

input
```
int(exp(b*x+a)*tanh(b*x+d)^3,x)
```

output
```
(e**a*( - 3*e**(4*b*x + 4*d)*atan(e**(b*x + d)) - 6*e**(2*b*x + 2*d)*atan(
e**(b*x + d)) - 3*atan(e**(b*x + d)) + e**(5*b*x + 5*d) + 5*e**(3*b*x + 3*
d) + 2*e**(b*x + d)))/(e**d*b*(e**(4*b*x + 4*d) + 2*e**(2*b*x + 2*d) + 1))
```

3.37 $\int e^{a+bx}\mathbf{sech}(d+bx)\tanh^2(d+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int e^{a+bx}\text{sech}(d+bx)\tanh^2(d+bx)\,dx=-\frac{2e^{a-d}}{b\left(1+e^{2d+2bx}\right)^2}+\frac{4e^{a-d}}{b\left(1+e^{2d+2bx}\right)}+\frac{e^{a-d}\log\left(1+e^{2d+2bx}\right)}{b}$$

output
```
-2*exp(a-d)/b/(1+exp(2*b*x+2*d))^2+4*exp(a-d)/b/(1+exp(2*b*x+2*d))+exp(a-d
)*ln(1+exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.73

$$\int e^{a+bx}\text{sech}(d+bx)\tanh^2(d+bx)\,dx=\frac{2e^{a-d}\left(\frac{1+2e^{2(d+bx)}}{\left(1+e^{2(d+bx)}\right)^2}+\frac{1}{2}\log\left(1+e^{2(d+bx)}\right)\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Sech[d + b*x]*Tanh[d + b*x]^2,x]
```

output
```
(2*E^(a - d)*((1 + 2*E^(2*(d + b*x)))/(1 + E^(2*(d + b*x)))^2 + Log[1 + E^
(2*(d + b*x))]/2))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.53, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2720, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\tanh^2(bx+d)\text{sech}(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{2e^{a+bx}\left(1-e^{2bx}\right)^2}{\left(1+e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2e^a\int \frac{e^{bx}\left(1-e^{2bx}\right)^2}{\left(1+e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{353}$$

$$\frac{e^a\int \frac{\left(1-e^{2bx}\right)^2}{\left(1+e^{2bx}\right)^3}de^{2bx}}{b}$$

$$\Big\downarrow \text{49}$$

$$\frac{e^a\int \left(\frac{1}{1+e^{2bx}}-\frac{4}{\left(1+e^{2bx}\right)^2}+\frac{4}{\left(1+e^{2bx}\right)^3}\right)de^{2bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^a\left(\frac{4}{e^{2bx}+1}-\frac{2}{\left(e^{2bx}+1\right)^2}+\log\left(e^{2bx}+1\right)\right)}{b}$$

input
```
Int[E^(a + b*x)*Sech[d + b*x]*Tanh[d + b*x]^2,x]
```

output
```
(E^a*(-2/(1 + E^(2*b*x))^2 + 4/(1 + E^(2*b*x)) + Log[1 + E^(2*b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.22

method	result	size
risch	$-\frac{2\,\mathrm{e}^{a-d} a}{b}+\frac{2\left(2\,\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)\mathrm{e}^{3a-d}}{\left(\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)^{2} b}+\frac{\ln\left(\mathrm{e}^{2bx+2a}+\mathrm{e}^{2a-2d}\right)\mathrm{e}^{a-d}}{b}$	94

input
```
int(exp(b*x+a)*sech(b*x+d)*tanh(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/b*exp(a-d)*a+2/(exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(2*exp(2*b*x+2*a+2*d)+
exp(2*a))*exp(3*a-d)+ln(exp(2*b*x+2*a)+exp(2*a-2*d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(71) = 142$.

Time = 0.09 (sec) , antiderivative size = 477, normalized size of antiderivative = 6.19

$$\int e^{a+bx}\operatorname{sech}(d+bx)\tanh^2(d+bx)\,dx$$
$$= \frac{4\cosh(bx+d)^2\cosh(-a+d) + 4(\cosh(-a+d) - \sinh(-a+d))\sinh(bx+d)^2 + \left(\cosh(bx+d)^4\cos\right.}{}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)*tanh(b*x+d)^2,x, algorithm="fricas")
```

output
```
(4*cosh(b*x + d)^2*cosh(-a + d) + 4*(cosh(-a + d) - sinh(-a + d))*sinh(b*x
 + d)^2 + (cosh(b*x + d)^4*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*si
nh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d)
)*sinh(b*x + d)^3 + 2*cosh(b*x + d)^2*cosh(-a + d) + 2*(3*cosh(b*x + d)^2*
cosh(-a + d) - (3*cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*sinh(b
*x + d)^2 + 4*(cosh(b*x + d)^3*cosh(-a + d) + cosh(b*x + d)*cosh(-a + d) -
 (cosh(b*x + d)^3 + cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x
 + d)^4 + 2*cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*log(2*cosh(b
*x + d)/(cosh(b*x + d) - sinh(b*x + d))) + 8*(cosh(b*x + d)*cosh(-a + d) -
 cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d) - 2*(2*cosh(b*x + d)^2 + 1)*sin
h(-a + d) + 2*cosh(-a + d))/(b*cosh(b*x + d)^4 + 4*b*cosh(b*x + d)*sinh(b*
x + d)^3 + b*sinh(b*x + d)^4 + 2*b*cosh(b*x + d)^2 + 2*(3*b*cosh(b*x + d)^
2 + b)*sinh(b*x + d)^2 + 4*(b*cosh(b*x + d)^3 + b*cosh(b*x + d))*sinh(b*x
+ d) + b)
```

Sympy [F]

$$\int e^{a+bx}\operatorname{sech}(d+bx)\tanh^2(d+bx)\,dx = e^a\int e^{bx}\tanh^2(bx+d)\operatorname{sech}(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)*tanh(b*x+d)**2,x)
```

output
```
exp(a)*Integral(exp(b*x)*tanh(b*x + d)**2*sech(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

$$\int e^{a+bx}\operatorname{sech}(d+bx)\tanh^2(d+bx)\,dx = \frac{e^{(a-d)}\log\left(e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right)}{b} + \frac{2\left(2\,e^{(2\,bx+5\,a+2\,d)}+e^{(5\,a)}\right)}{b\left(e^{(4\,bx+4\,a+5\,d)}+2\,e^{(2\,bx+4\,a+3\,d)}+e^{(4\,a+d)}\right)}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)*tanh(b*x+d)^2,x, algorithm="maxima")
```

output
```
e^(a - d)*log(e^(2*b*x + 2*a + 2*d) + e^(2*a))/b + 2*(2*e^(2*b*x + 5*a + 2
*d) + e^(5*a))/(b*(e^(4*b*x + 4*a + 5*d) + 2*e^(2*b*x + 4*a + 3*d) + e^(4*
a + d)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.19

$$\int e^{a+bx}\operatorname{sech}(d+bx)\tanh^2(d+bx)\,dx = \frac{2\,e^{(a-d)}\log\left(e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right) - \frac{\left(3\,e^{(4\,bx+5\,a+4\,d)}-2\,e^{(2\,bx+5\,a+2\,d)}-e^{(5\,a)}\right)e^{(-d)}}{\left(e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right)^2}}{2\,b}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)*tanh(b*x+d)^2,x, algorithm="giac")
```

output
```
1/2*(2*e^(a - d)*log(e^(2*b*x + 2*a + 2*d) + e^(2*a)) - (3*e^(4*b*x + 5*a
+ 4*d) - 2*e^(2*b*x + 5*a + 2*d) - e^(5*a))*e^(-d)/(e^(2*b*x + 2*a + 2*d)
+ e^(2*a))^2)/b
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\text{sech}(d+bx)\tanh^2(d+bx)\,dx = \int \frac{e^{a+b\,x}\tanh(d+b\,x)^2}{\cosh(d+b\,x)}\,dx$$

input
```
int((exp(a + b*x)*tanh(d + b*x)^2)/cosh(d + b*x),x)
```

output
```
int((exp(a + b*x)*tanh(d + b*x)^2)/cosh(d + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.47

$$\int e^{a+bx}\text{sech}(d+bx)\tanh^2(d+bx)\,dx = \frac{e^a\left(e^{4bx+4d}\log\left(e^{2bx+2d}+1\right)-2e^{4bx+4d}+2e^{2bx+2d}\log\left(e^{2bx+2d}+1\right)+\log\left(e^{2bx+2d}+1\right)\right)}{e^d b\left(e^{4bx+4d}+2e^{2bx+2d}+1\right)}$$

input
```
int(exp(b*x+a)*sech(b*x+d)*tanh(b*x+d)^2,x)
```

output
```
(e**a*(e**(4*b*x + 4*d)*log(e**(2*b*x + 2*d) + 1) - 2*e**(4*b*x + 4*d) + 2
*e**(2*b*x + 2*d)*log(e**(2*b*x + 2*d) + 1) + log(e**(2*b*x + 2*d) + 1)))/
(e**d*b*(e**(4*b*x + 4*d) + 2*e**(2*b*x + 2*d) + 1))
```

3.38 $\int e^{a+bx}\mathrm{sech}^2(d+bx)\tanh(d+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int e^{a+bx}\mathrm{sech}^2(d+bx)\tanh(d+bx)\,dx=\frac{2e^{a+bx}}{b\left(1+e^{2d+2bx}\right)^2}-\frac{3e^{a+bx}}{b\left(1+e^{2d+2bx}\right)}+\frac{e^{a-d}\arctan\left(e^{d+bx}\right)}{b}$$

output
```
2*exp(b*x+a)/b/(1+exp(2*b*x+2*d))^2-3*exp(b*x+a)/b/(1+exp(2*b*x+2*d))+exp(
a-d)*arctan(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$\int e^{a+bx}\mathrm{sech}^2(d+bx)\tanh(d+bx)\,dx=\frac{e^a\left(-\frac{e^{bx}\left(1+3e^{2(d+bx)}\right)}{\left(1+e^{2(d+bx)}\right)^2}+e^{-d}\arctan\left(e^{d+bx}\right)\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Sech[d + b*x]^2*Tanh[d + b*x],x]
```

output
```
(E^a*(-((E^(b*x)*(1 + 3*E^(2*(d + b*x))))/(1 + E^(2*(d + b*x)))^2) + ArcTa
n[E^(d + b*x)]/E^d))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2720, 27, 360, 27, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\tanh(bx+d)\text{sech}^2(bx+d)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int -\frac{4e^{a+2bx}(1-e^{2bx})}{(1+e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{4e^a\int \frac{e^{2bx}(1-e^{2bx})}{(1+e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{ 360}$$

$$-\frac{4e^a\left(-\frac{1}{4}\int -\frac{2(1-2e^{2bx})}{(1+e^{2bx})^2}de^{bx}-\frac{e^{bx}}{2(e^{2bx}+1)^2}\right)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{4e^a\left(\frac{1}{2}\int \frac{1-2e^{2bx}}{(1+e^{2bx})^2}de^{bx}-\frac{e^{bx}}{2(e^{2bx}+1)^2}\right)}{b}$$

$$\Big\downarrow \text{ 298}$$

$$-\frac{4e^a\left(\frac{1}{2}\left(\frac{3e^{bx}}{2(e^{2bx}+1)}-\frac{1}{2}\int \frac{1}{1+e^{2bx}}de^{bx}\right)-\frac{e^{bx}}{2(e^{2bx}+1)^2}\right)}{b}$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{4e^a\left(\frac{1}{2}\left(\frac{3e^{bx}}{2(e^{2bx}+1)}-\frac{1}{2}\arctan\left(e^{bx}\right)\right)-\frac{e^{bx}}{2(e^{2bx}+1)^2}\right)}{b}$$

input
```
Int[E^(a + b*x)*Sech[d + b*x]^2*Tanh[d + b*x],x]
```

output
```
(-4*E^a*(-1/2*E^(b*x)/(1 + E^(2*b*x))^2 + ((3*E^(b*x))/(2*(1 + E^(2*b*x)))
 - ArcTan[E^(b*x)]/2)/2))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.01 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.54

method	result	size
risch	$-\frac{\left(3\,\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)\mathrm{e}^{bx+3a}}{\left(\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)^{2}b}+\frac{i\ln\left(\mathrm{e}^{bx+a}+i\mathrm{e}^{a-d}\right)\mathrm{e}^{a-d}}{2b}-\frac{i\ln\left(\mathrm{e}^{bx+a}-i\mathrm{e}^{a-d}\right)\mathrm{e}^{a-d}}{2b}$	111

input
```
int(exp(b*x+a)*sech(b*x+d)^2*tanh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/(exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(3*exp(2*b*x+2*a+2*d)+exp(2*a))*exp(b
*x+3*a)+1/2*I*ln(exp(b*x+a)+I*exp(a-d))/b*exp(a-d)-1/2*I*ln(exp(b*x+a)-I*e
xp(a-d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(66) = 132.

Time = 0.10 (sec) , antiderivative size = 529, normalized size of antiderivative = 7.35

$$\int e^{a+bx}\text{sech}^2(d+bx)\tanh(d+bx)\,dx=\text{Too large to display}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)^2*tanh(b*x+d),x, algorithm="fricas")
```

output
```
-(3*cosh(b*x + d)^3*cosh(-a + d) + 3*(cosh(-a + d) - sinh(-a + d))*sinh(b*
x + d)^3 + 9*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sin
h(b*x + d)^2 - (cosh(b*x + d)^4*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d
))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a
 + d))*sinh(b*x + d)^3 + 2*cosh(b*x + d)^2*cosh(-a + d) + 2*(3*cosh(b*x +
d)^2*cosh(-a + d) - (3*cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*s
inh(b*x + d)^2 + 4*(cosh(b*x + d)^3*cosh(-a + d) + cosh(b*x + d)*cosh(-a +
 d) - (cosh(b*x + d)^3 + cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d) - (cos
h(b*x + d)^4 + 2*cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*arctan(
cosh(b*x + d) + sinh(b*x + d)) + cosh(b*x + d)*cosh(-a + d) + (9*cosh(b*x
+ d)^2*cosh(-a + d) - (9*cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))
*sinh(b*x + d) - (3*cosh(b*x + d)^3 + cosh(b*x + d))*sinh(-a + d))/(b*cosh
(b*x + d)^4 + 4*b*cosh(b*x + d)*sinh(b*x + d)^3 + b*sinh(b*x + d)^4 + 2*b*
cosh(b*x + d)^2 + 2*(3*b*cosh(b*x + d)^2 + b)*sinh(b*x + d)^2 + 4*(b*cosh(
b*x + d)^3 + b*cosh(b*x + d))*sinh(b*x + d) + b)
```

Sympy [F]

$$\int e^{a+bx}\text{sech}^2(d+bx)\tanh(d+bx)\,dx = e^a\int e^{bx}\tanh{(bx+d)}\,\text{sech}^2\,(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)**2*tanh(b*x+d),x)
```

output
```
exp(a)*Integral(exp(b*x)*tanh(b*x + d)*sech(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int e^{a+bx}\text{sech}^2(d+bx)\tanh(d+bx)\,dx = \frac{\arctan\left(e^{(bx+d)}\right)e^{(a-d)}}{b} - \frac{3\,e^{(3\,bx+5\,a+2\,d)}+e^{(bx+5\,a)}}{b(e^{(4\,bx+4\,a+4\,d)}+2\,e^{(2\,bx+4\,a+2\,d)}+e^{(4\,a)})}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)^2*tanh(b*x+d),x, algorithm="maxima")
```

output
```
arctan(e^(b*x + d))*e^(a - d)/b - (3*e^(3*b*x + 5*a + 2*d) + e^(b*x + 5*a)
)/(b*(e^(4*b*x + 4*a + 4*d) + 2*e^(2*b*x + 4*a + 2*d) + e^(4*a)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int e^{a+bx}\text{sech}^2(d+bx)\tanh(d+bx)\,dx = \frac{\arctan\left(e^{(bx+d)}\right)e^{(a-d)} - \frac{\left(3\,e^{(3\,bx+a+3\,d)}+e^{(bx+a+d)}\right)e^{(-d)}}{\left(e^{(2\,bx+2\,d)}+1\right)^2}}{b}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)^2*tanh(b*x+d),x, algorithm="giac")
```

output
```
(arctan(e^(b*x + d))*e^(a - d) - (3*e^(3*b*x + a + 3*d) + e^(b*x + a + d))
*e^(-d)/(e^(2*b*x + 2*d) + 1)^2)/b
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\text{sech}^2(d+bx)\tanh(d+bx)\,dx = \int \frac{\mathrm{e}^{a+b\,x}\,\tanh(d+b\,x)}{\cosh\left(d+b\,x\right)^2}\,dx$$

input
```
int((exp(a + b*x)*tanh(d + b*x))/cosh(d + b*x)^2,x)
```

output
```
int((exp(a + b*x)*tanh(d + b*x))/cosh(d + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.49

$$\int e^{a+bx}\text{sech}^2(d+bx)\tanh(d+bx)\,dx$$
$$=\frac{e^a\left(e^{4bx+4d}atan\left(e^{bx+d}\right)+2e^{2bx+2d}atan\left(e^{bx+d}\right)+atan\left(e^{bx+d}\right)-3e^{3bx+3d}-e^{bx+d}\right)}{e^d b\left(e^{4bx+4d}+2e^{2bx+2d}+1\right)}$$

input
```
int(exp(b*x+a)*sech(b*x+d)^2*tanh(b*x+d),x)
```

output
```
(e**a*(e**(4*b*x + 4*d)*atan(e**(b*x + d)) + 2*e**(2*b*x + 2*d)*atan(e**(b
*x + d)) + atan(e**(b*x + d)) - 3*e**(3*b*x + 3*d) - e**(b*x + d)))/(e**d*
b*(e**(4*b*x + 4*d) + 2*e**(2*b*x + 2*d) + 1))
```

3.39 $\int e^{a+bx}\operatorname{sech}^3(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 30

$$\int e^{a+bx}\operatorname{sech}^3(d+bx)\,dx=\frac{2e^{a+3d+4bx}}{b\,(1+e^{2d+2bx})^2}$$

output
```
2*exp(4*b*x+a+3*d)/b/(1+exp(2*b*x+2*d))^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int e^{a+bx}\operatorname{sech}^3(d+bx)\,dx=\frac{2e^{a+3d+4bx}}{b\,(1+e^{2(d+bx)})^2}$$

input
```
Integrate[E^(a + b*x)*Sech[d + b*x]^3,x]
```

output
```
(2*E^(a + 3*d + 4*b*x))/(b*(1 + E^(2*(d + b*x)))^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2720, 27, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{sech}^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{8e^{a+3bx}}{(1+e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{8e^a\int \frac{e^{3bx}}{(1+e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{242}$$

$$\frac{2e^{a+4bx}}{b\,(e^{2bx}+1)^2}$$

input
```
Int[E^(a + b*x)*Sech[d + b*x]^3,x]
```

output
```
(2*E^(a + 4*b*x))/(b*(1 + E^(2*b*x))^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.70

method	result	size
parallelrisch	$-\frac{e^{bx+a}\left(-1+\tanh\left(\frac{bx}{2}+\frac{d}{2}\right)\right)\left(1+\tanh\left(\frac{bx}{2}+\frac{d}{2}\right)\right)^{3}}{2b\left(1+\tanh\left(\frac{bx}{2}+\frac{d}{2}\right)^{2}\right)^{2}}$	51
risch	$-\frac{2\left(2\,e^{2bx+2a+2d}+e^{2a}\right)e^{3a-d}}{\left(e^{2bx+2a+2d}+e^{2a}\right)^{2}b}$	52

input
```
int(exp(b*x+a)*sech(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(b*x+a)*(-1+tanh(1/2*b*x+1/2*d))*(1+tanh(1/2*b*x+1/2*d))^3/b/(1+ta
nh(1/2*b*x+1/2*d)^2)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(28) = 56.

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 4.07

$$\int e^{a+bx}\text{sech}^3(d+bx)\,dx =$$
$$-\frac{2\left(3\,\cosh\left(bx+d\right)\cosh\left(-a+d\right)+\left(\cosh\left(-a+d\right)-\sinh\left(-a+d\right)\right)\sinh\left(bx+d\right)-3\,\cosh\left(bx\right.\right.}{b\cosh\left(bx+d\right)^{3}+3\,b\cosh\left(bx+d\right)\sinh\left(bx+d\right)^{2}+b\sinh\left(bx+d\right)^{3}+3\,b\cosh\left(bx+d\right)+\left(3\,b\cosh\left(bx\right.\right.}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)^3,x, algorithm="fricas")
```

output
```
-2*(3*cosh(b*x + d)*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x
+ d) - 3*cosh(b*x + d)*sinh(-a + d))/(b*cosh(b*x + d)^3 + 3*b*cosh(b*x + d
)*sinh(b*x + d)^2 + b*sinh(b*x + d)^3 + 3*b*cosh(b*x + d) + (3*b*cosh(b*x
+ d)^2 + b)*sinh(b*x + d))
```

Sympy [F]

$$\int e^{a+bx}\mathrm{sech}^3(d+bx)\,dx = e^a\int e^{bx}\,\mathrm{sech}^3\,(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)**3,x)
```

output
```
exp(a)*Integral(exp(b*x)*sech(b*x + d)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(28) = 56$.

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.23

$$\int e^{a+bx}\mathrm{sech}^3(d+bx)\,dx = -\frac{4\,e^{(2\,bx+5\,a+2\,d)}}{b\big(e^{(4\,bx+4\,a+5\,d)}+2\,e^{(2\,bx+4\,a+3\,d)}+e^{(4\,a+d)}\big)} - \frac{2\,e^{(5\,a)}}{b\big(e^{(4\,bx+4\,a+5\,d)}+2\,e^{(2\,bx+4\,a+3\,d)}+e^{(4\,a+d)}\big)}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)^3,x, algorithm="maxima")
```

output
```
-4*e^(2*b*x + 5*a + 2*d)/(b*(e^(4*b*x + 4*a + 5*d) + 2*e^(2*b*x + 4*a + 3*
d) + e^(4*a + d))) - 2*e^(5*a)/(b*(e^(4*b*x + 4*a + 5*d) + 2*e^(2*b*x + 4*
a + 3*d) + e^(4*a + d)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int e^{a+bx}\operatorname{sech}^3(d+bx)\,dx = -\frac{2\left(2\,e^{(2\,bx+2\,d)}+1\right)e^{(a-d)}}{b\left(e^{(2\,bx+2\,d)}+1\right)^2}$$

input
```
integrate(exp(b*x+a)*sech(b*x+d)^3,x, algorithm="giac")
```

output
```
-2*(2*e^(2*b*x + 2*d) + 1)*e^(a - d)/(b*(e^(2*b*x + 2*d) + 1)^2)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\operatorname{sech}^3(d+bx)\,dx = \int \frac{e^{a+b\,x}}{\cosh\left(d+b\,x\right)^3}\,dx$$

input
```
int(exp(a + b*x)/cosh(d + b*x)^3,x)
```

output
```
int(exp(a + b*x)/cosh(d + b*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int e^{a+bx}\operatorname{sech}^3(d+bx)\,dx = \frac{2e^{4bx+a+3d}}{b\left(e^{4bx+4d}+2e^{2bx+2d}+1\right)}$$

input
```
int(exp(b*x+a)*sech(b*x+d)^3,x)
```

output
```
(2*e**(a + 4*b*x + 3*d))/(b*(e**(4*b*x + 4*d) + 2*e**(2*b*x + 2*d) + 1))
```

3.40 $\int e^{a+bx}\operatorname{csch}(d+bx)\operatorname{sech}^3(d+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 92

$$\int e^{a+bx}\operatorname{csch}(d+bx)\operatorname{sech}^3(d+bx)\,dx = -\frac{2e^{a+bx}}{b\,(1+e^{2d+2bx})^2}+\frac{3e^{a+bx}}{b\,(1+e^{2d+2bx})}+\frac{e^{a-d}\arctan\left(e^{d+bx}\right)}{b}-\frac{2e^{a-d}\operatorname{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
-2*exp(b*x+a)/b/(1+exp(2*b*x+2*d))^2+3*exp(b*x+a)/b/(1+exp(2*b*x+2*d))+exp
(a-d)*arctan(exp(b*x+d))/b-2*exp(a-d)*arctanh(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05

$$\int e^{a+bx}\operatorname{csch}(d+bx)\operatorname{sech}^3(d+bx)\,dx = \frac{e^a\left(-\frac{2e^{bx}}{(1+e^{2(d+bx)})^2}+\frac{3e^{bx}}{1+e^{2(d+bx)}}+e^{-d}\arctan\left(e^{d+bx}\right)+e^{-d}\log\left(1-e^{d+bx}\right)-e^{-d}\log\left(1+e^{d+bx}\right)\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Csch[d + b*x]*Sech[d + b*x]^3,x]
```

output
```
(E^a*((-2*E^(b*x))/(1 + E^(2*(d + b*x)))^2 + (3*E^(b*x))/(1 + E^(2*(d + b*
x))) + ArcTan[E^(d + b*x)]/E^d + Log[1 - E^(d + b*x)]/E^d - Log[1 + E^(d +
 b*x)]/E^d))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.79, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2720, 27, 372, 402, 27, 397, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{csch}(bx+d)\text{sech}^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{16e^{a+4bx}}{(1-e^{2bx})(1+e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{16e^a\int \frac{e^{4bx}}{(1-e^{2bx})(1+e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{372}$$

$$-\frac{16e^a\left(\frac{e^{bx}}{8(e^{2bx}+1)^2}-\frac{1}{8}\int \frac{1-5e^{2bx}}{(1-e^{2bx})(1+e^{2bx})^2}de^{bx}\right)}{b}$$

$$\Big\downarrow \text{402}$$

$$-\frac{16e^a\left(\frac{1}{8}\left(\frac{1}{4}\int \frac{2(1+3e^{2bx})}{(1-e^{2bx})(1+e^{2bx})}de^{bx}-\frac{3e^{bx}}{2(e^{2bx}+1)}\right)+\frac{e^{bx}}{8(e^{2bx}+1)^2}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{16e^a\left(\frac{1}{8}\left(\frac{1}{2}\int \frac{1+3e^{2bx}}{(1-e^{2bx})(1+e^{2bx})}de^{bx}-\frac{3e^{bx}}{2(e^{2bx}+1)}\right)+\frac{e^{bx}}{8(e^{2bx}+1)^2}\right)}{b}$$

$$\Big\downarrow \text{397}$$

$$-\frac{16e^a\left(\frac{1}{8}\left(\frac{1}{2}\left(2\int \frac{1}{1-e^{2bx}}de^{bx}-\int \frac{1}{1+e^{2bx}}de^{bx}\right)-\frac{3e^{bx}}{2\left(e^{2bx}+1\right)}\right)+\frac{e^{bx}}{8\left(e^{2bx}+1\right)^2}\right)}{b}$$

$$\Big\downarrow \; 216$$

$$-\frac{16e^a\left(\frac{1}{8}\left(\frac{1}{2}\left(2\int \frac{1}{1-e^{2bx}}de^{bx}-\arctan\left(e^{bx}\right)\right)-\frac{3e^{bx}}{2\left(e^{2bx}+1\right)}\right)+\frac{e^{bx}}{8\left(e^{2bx}+1\right)^2}\right)}{b}$$

$$\Big\downarrow \; 219$$

$$-\frac{16e^a\left(\frac{1}{8}\left(\frac{1}{2}\left(2\text{arctanh}\left(e^{bx}\right)-\arctan\left(e^{bx}\right)\right)-\frac{3e^{bx}}{2\left(e^{2bx}+1\right)}\right)+\frac{e^{bx}}{8\left(e^{2bx}+1\right)^2}\right)}{b}$$

input
```
Int[E^(a + b*x)*Csch[d + b*x]*Sech[d + b*x]^3,x]
```

output
```
(-16*E^a*(E^(b*x)/(8*(1 + E^(2*b*x))^2) + ((-3*E^(b*x))/(2*(1 + E^(2*b*x))
) + (-ArcTan[E^(b*x)] + 2*ArcTanh[E^(b*x)])/2)/8))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 108.60 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.75

method	result
risch	$\frac{\left(3\,{\rm e}^{2bx+2a+2d}+{\rm e}^{2a}\right){\rm e}^{bx+3a}}{\left({\rm e}^{2bx+2a+2d}+{\rm e}^{2a}\right)^{2} b}+\frac{i\ln\left({\rm e}^{bx+a}+i\,{\rm e}^{a-d}\right){\rm e}^{a-d}}{2b}-\frac{i\ln\left({\rm e}^{bx+a}-i\,{\rm e}^{a-d}\right){\rm e}^{a-d}}{2b}+\frac{\ln\left({\rm e}^{bx+a}-{\rm e}^{a-d}\right){\rm e}^{a-d}}{b}-\frac{\ln\left({\rm e}^{bx+a}+{\rm e}^{a-}\right.}{b}$

input
```
int(exp(b*x+a)*csch(b*x+d)*sech(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/(exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(3*exp(2*b*x+2*a+2*d)+exp(2*a))*exp(b*
x+3*a)+1/2*I*ln(exp(b*x+a)+I*exp(a-d))/b*exp(a-d)-1/2*I*ln(exp(b*x+a)-I*ex
p(a-d))/b*exp(a-d)+ln(exp(b*x+a)-exp(a-d))/b*exp(a-d)-ln(exp(b*x+a)+exp(a-
d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1042 vs. $2(84) = 168$.

Time = 0.09 (sec) , antiderivative size = 1042, normalized size of antiderivative = 11.33

$$\int e^{a+bx}\text{csch}(d+bx)\text{sech}^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)*sech(b*x+d)^3,x, algorithm="fricas")
```

output
```
(3*cosh(b*x + d)^3*cosh(-a + d) + 3*(cosh(-a + d) - sinh(-a + d))*sinh(b*x
 + d)^3 + 9*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh
(b*x + d)^2 + (cosh(b*x + d)^4*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d)
)*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a
+ d))*sinh(b*x + d)^3 + 2*cosh(b*x + d)^2*cosh(-a + d) + 2*(3*cosh(b*x + d
)^2*cosh(-a + d) - (3*cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*si
nh(b*x + d)^2 + 4*(cosh(b*x + d)^3*cosh(-a + d) + cosh(b*x + d)*cosh(-a +
d) - (cosh(b*x + d)^3 + cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d) - (cosh
(b*x + d)^4 + 2*cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*arctan(c
osh(b*x + d) + sinh(b*x + d)) + cosh(b*x + d)*cosh(-a + d) - (cosh(b*x + d
)^4*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d)^4 + 4*(cosh
(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d)^3 + 2*c
osh(b*x + d)^2*cosh(-a + d) + 2*(3*cosh(b*x + d)^2*cosh(-a + d) - (3*cosh(
b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*sinh(b*x + d)^2 + 4*(cosh(b*x
 + d)^3*cosh(-a + d) + cosh(b*x + d)*cosh(-a + d) - (cosh(b*x + d)^3 + cos
h(b*x + d))*sinh(-a + d))*sinh(b*x + d) - (cosh(b*x + d)^4 + 2*cosh(b*x +
d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*log(cosh(b*x + d) + sinh(b*x + d) +
 1) + (cosh(b*x + d)^4*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b
*x + d)^4 + 4*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*si
nh(b*x + d)^3 + 2*cosh(b*x + d)^2*cosh(-a + d) + 2*(3*cosh(b*x + d)^2*c...
```

Sympy [F]

$$\int e^{a+bx}\text{csch}(d+bx)\text{sech}^3(d+bx)\,dx = e^{a}\int e^{bx}\operatorname{csch}{\left(bx+d\right)}\operatorname{sech}^{3}{\left(bx+d\right)}\,dx$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)*sech(b*x+d)**3,x)
```

output
```
exp(a)*Integral(exp(b*x)*csch(b*x + d)*sech(b*x + d)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.34

$$\int e^{a+bx}\operatorname{csch}(d+bx)\operatorname{sech}^3(d+bx)\,dx = \frac{\arctan\left(e^{(bx+d)}\right)e^{(a-d)}}{b} - \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^{a}\right)}{b} + \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}-e^{a}\right)}{b} + \frac{3\,e^{(3\,bx+5\,a+2\,d)}+e^{(bx+5\,a)}}{b(e^{(4\,bx+4\,a+4\,d)}+2\,e^{(2\,bx+4\,a+2\,d)}+e^{(4\,a)})}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)*sech(b*x+d)^3,x, algorithm="maxima")
```

output
```
arctan(e^(b*x + d))*e^(a - d)/b - e^(a - d)*log(e^(b*x + a + d) + e^a)/b +
 e^(a - d)*log(e^(b*x + a + d) - e^a)/b + (3*e^(3*b*x + 5*a + 2*d) + e^(b*
x + 5*a))/(b*(e^(4*b*x + 4*a + 4*d) + 2*e^(2*b*x + 4*a + 2*d) + e^(4*a)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04

$$\int e^{a+bx}\operatorname{csch}(d+bx)\operatorname{sech}^3(d+bx)\,dx = \left(\frac{\arctan\left(e^{(bx+d)}\right)e^{(-5\,d)}}{b} - \frac{e^{(-5\,d)}\log\left(e^{(bx+d)}+1\right)}{b} + \frac{e^{(-5\,d)}\log\left(\left|e^{(bx+d)}-1\right|\right)}{b} + \frac{\left(3\,e^{(3\,bx+2\,d)}+e^{(bx)}\right)e^{(-4}}{b\left(e^{(2\,bx+2\,d)}+1\right)^2}\right.$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)*sech(b*x+d)^3,x, algorithm="giac")
```

output
```
(arctan(e^(b*x + d))*e^(-5*d)/b - e^(-5*d)*log(e^(b*x + d) + 1)/b + e^(-5*
d)*log(abs(e^(b*x + d) - 1))/b + (3*e^(3*b*x + 2*d) + e^(b*x))*e^(-4*d)/(b
*(e^(2*b*x + 2*d) + 1)^2))*e^(a + 4*d)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\text{csch}(d+bx)\text{sech}^3(d+bx)\,dx=\int \frac{e^{a+b\,x}}{\cosh\left(d+b\,x\right)^3\,\sinh\left(d+b\,x\right)}\,dx$$

input
```
int(exp(a + b*x)/(cosh(d + b*x)^3*sinh(d + b*x)),x)
```

output
```
int(exp(a + b*x)/(cosh(d + b*x)^3*sinh(d + b*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.33

$$\int e^{a+bx}\text{csch}(d+bx)\text{sech}^3(d+bx)\,dx$$
$$=\frac{e^{a}\left(e^{4bx+4d}atan\left(e^{bx+d}\right)+2e^{2bx+2d}atan\left(e^{bx+d}\right)+atan\left(e^{bx+d}\right)+e^{4bx+4d}\log\left(e^{bx+d}-1\right)-e^{4bx+4d}\log\left(e^{bx+d}\right.\right.}{e^{d}b\left(e^{4bx+4\cdots}\right.}$$

input
```
int(exp(b*x+a)*csch(b*x+d)*sech(b*x+d)^3,x)
```

output
```
(e**a*(e**(4*b*x + 4*d)*atan(e**(b*x + d)) + 2*e**(2*b*x + 2*d)*atan(e**(b
*x + d)) + atan(e**(b*x + d)) + e**(4*b*x + 4*d)*log(e**(b*x + d) - 1) - e
**(4*b*x + 4*d)*log(e**(b*x + d) + 1) + 3*e**(3*b*x + 3*d) + 2*e**(2*b*x +
 2*d)*log(e**(b*x + d) - 1) - 2*e**(2*b*x + 2*d)*log(e**(b*x + d) + 1) + e
**(b*x + d) + log(e**(b*x + d) - 1) - log(e**(b*x + d) + 1)))/(e**d*b*(e**
(4*b*x + 4*d) + 2*e**(2*b*x + 2*d) + 1))
```

3.41 $\int e^{a+bx}\operatorname{csch}^2(d+bx)\operatorname{sech}^3(d+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int e^{a+bx}\operatorname{csch}^2(d+bx)\operatorname{sech}^3(d+bx)\,dx=\frac{2e^{a-d}}{b\left(1-e^{2d+2bx}\right)}-\frac{2e^{a-d}}{b\left(1+e^{2d+2bx}\right)^2}+\frac{4e^{a-d}}{b\left(1+e^{2d+2bx}\right)}-\frac{2e^{a-d}\operatorname{arctanh}\left(e^{2d+2bx}\right)}{b}$$

output
```
2*exp(a-d)/b/(1-exp(2*b*x+2*d))-2*exp(a-d)/b/(1+exp(2*b*x+2*d))^2+4*exp(a-
d)/b/(1+exp(2*b*x+2*d))-2*exp(a-d)*arctanh(exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.67

$$\int e^{a+bx}\operatorname{csch}^2(d+bx)\operatorname{sech}^3(d+bx)\,dx=\frac{2e^{a-d}\left(\frac{1}{1-e^{2(d+bx)}}-\frac{1}{\left(1+e^{2(d+bx)}\right)^2}+\frac{2}{1+e^{2(d+bx)}}-\operatorname{arctanh}\left(e^{2(d+bx)}\right)\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Csch[d + b*x]^2*Sech[d + b*x]^3,x]
```

output
```
(2*E^(a - d)*((1 - E^(2*(d + b*x)))^(-1) - (1 + E^(2*(d + b*x)))^(-2) + 2/
(1 + E^(2*(d + b*x))) - ArcTanh[E^(2*(d + b*x))]))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.62, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2720, 27, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{csch}^2(bx+d)\text{sech}^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{32e^{a+5bx}}{\left(1-e^{2bx}\right)^2\left(1+e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{32e^a\int \frac{e^{5bx}}{\left(1-e^{2bx}\right)^2\left(1+e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{354}$$

$$\frac{16e^a\int \frac{e^{2bx}}{\left(1-e^{2bx}\right)^2\left(1+e^{2bx}\right)^3}de^{2bx}}{b}$$

$$\Big\downarrow \text{99}$$

$$\frac{16e^a\int \left(-\frac{1}{4\left(1+e^{2bx}\right)^2}+\frac{1}{4\left(1+e^{2bx}\right)^3}+\frac{1}{8\left(-1+e^{2bx}\right)}+\frac{1}{8\left(-1+e^{2bx}\right)^2}\right)de^{2bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{16e^a\left(-\frac{1}{8}\text{arctanh}\left(e^{2bx}\right)+\frac{1}{8\left(1-e^{2bx}\right)}+\frac{1}{4\left(e^{2bx}+1\right)}-\frac{1}{8\left(e^{2bx}+1\right)^2}\right)}{b}$$

input
```
Int[E^(a + b*x)*Csch[d + b*x]^2*Sech[d + b*x]^3,x]
```

output
```
(16*E^a*(1/(8*(1 - E^(2*b*x))) - 1/(8*(1 + E^(2*b*x))^2) + 1/(4*(1 + E^(2*
b*x))) - ArcTanh[E^(2*b*x)]/8))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.45

$$\frac{2\left(-\mathrm{e}^{4bx+4a+4d}+3\,\mathrm{e}^{2bx+4a+2d}+2\,\mathrm{e}^{4a}\right)\mathrm{e}^{3a-d}}{\left(-\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)\left(\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)^{2}b}-\frac{\ln\left(\mathrm{e}^{2bx+2a}+\mathrm{e}^{2a-2d}\right)\mathrm{e}^{a-d}}{b}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-\mathrm{e}^{2a-2d}\right)\mathrm{e}^{a-d}}{b}$$

input
```
int(exp(b*x+a)*csch(b*x+d)^2*sech(b*x+d)^3,x)
```

output
```
2/(-exp(2*b*x+2*a+2*d)+exp(2*a))/(exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(-exp(4
*b*x+4*a+4*d)+3*exp(2*b*x+4*a+2*d)+2*exp(4*a))*exp(3*a-d)-ln(exp(2*b*x+2*a
)+exp(2*a-2*d))/b*exp(a-d)+ln(exp(2*b*x+2*a)-exp(2*a-2*d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1384 vs. 2(94) = 188.

Time = 0.11 (sec) , antiderivative size = 1384, normalized size of antiderivative = 13.31

$$\int e^{a+bx}\operatorname{csch}^2(d+bx)\operatorname{sech}^3(d+bx)\,dx=\text{Too large to display}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^2*sech(b*x+d)^3,x, algorithm="fricas")
```

output
```
(2*cosh(b*x + d)^4*cosh(-a + d) + 2*(cosh(-a + d) - sinh(-a + d))*sinh(b*x
 + d)^4 + 8*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh
(b*x + d)^3 - 6*cosh(b*x + d)^2*cosh(-a + d) + 6*(2*cosh(b*x + d)^2*cosh(-
a + d) - (2*cosh(b*x + d)^2 - 1)*sinh(-a + d) - cosh(-a + d))*sinh(b*x + d
)^2 - (cosh(b*x + d)^6*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b
*x + d)^6 + 6*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*si
nh(b*x + d)^5 + cosh(b*x + d)^4*cosh(-a + d) + (15*cosh(b*x + d)^2*cosh(-a
 + d) - (15*cosh(b*x + d)^2 + 1)*sinh(-a + d) + cosh(-a + d))*sinh(b*x + d
)^4 + 4*(5*cosh(b*x + d)^3*cosh(-a + d) + cosh(b*x + d)*cosh(-a + d) - (5*
cosh(b*x + d)^3 + cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d)^3 - cosh(b*x
+ d)^2*cosh(-a + d) + (15*cosh(b*x + d)^4*cosh(-a + d) + 6*cosh(b*x + d)^2
*cosh(-a + d) - (15*cosh(b*x + d)^4 + 6*cosh(b*x + d)^2 - 1)*sinh(-a + d)
- cosh(-a + d))*sinh(b*x + d)^2 + 2*(3*cosh(b*x + d)^5*cosh(-a + d) + 2*co
sh(b*x + d)^3*cosh(-a + d) - cosh(b*x + d)*cosh(-a + d) - (3*cosh(b*x + d)
^5 + 2*cosh(b*x + d)^3 - cosh(b*x + d))*sinh(-a + d))*sinh(b*x + d) - (cos
h(b*x + d)^6 + cosh(b*x + d)^4 - cosh(b*x + d)^2 - 1)*sinh(-a + d) - cosh(
-a + d))*log(2*cosh(b*x + d)/(cosh(b*x + d) - sinh(b*x + d))) + (cosh(b*x
+ d)^6*cosh(-a + d) + (cosh(-a + d) - sinh(-a + d))*sinh(b*x + d)^6 + 6*(c
osh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a + d))*sinh(b*x + d)^5 +
cosh(b*x + d)^4*cosh(-a + d) + (15*cosh(b*x + d)^2*cosh(-a + d) - (15*c...
```

Sympy [F]

$$\int e^{a+bx}\text{csch}^2(d+bx)\text{sech}^3(d+bx)\,dx=e^a\int e^{bx}\,\text{csch}^2\,(bx+d)\,\text{sech}^3\,(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)**2*sech(b*x+d)**3,x)
```

output
```
exp(a)*Integral(exp(b*x)*csch(b*x + d)**2*sech(b*x + d)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.55

$$\int e^{a+bx}\operatorname{csch}^2(d+bx)\operatorname{sech}^3(d+bx)\,dx$$
$$= -\frac{e^{(a-d)}\log\left(e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right)}{b} + \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}+e^{a}\right)}{b} + \frac{e^{(a-d)}\log\left(e^{(bx+a+d)}-e^{a}\right)}{b} + \frac{2\left(e^{(4\,bx+7\,a+4\,d)}-3\,e^{(2\,bx+7\,a+2\,d)}-2\,e^{(7\,a)}\right)}{b\left(e^{(6\,bx+6\,a+7\,d)}+e^{(4\,bx+6\,a+5\,d)}-e^{(2\,bx+6\,a+3\,d)}-e^{(6\,a+d)}\right)}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^2*sech(b*x+d)^3,x, algorithm="maxima")
```

output
```
-e^(a - d)*log(e^(2*b*x + 2*a + 2*d) + e^(2*a))/b + e^(a - d)*log(e^(b*x +
 a + d) + e^a)/b + e^(a - d)*log(e^(b*x + a + d) - e^a)/b + 2*(e^(4*b*x +
7*a + 4*d) - 3*e^(2*b*x + 7*a + 2*d) - 2*e^(7*a))/(b*(e^(6*b*x + 6*a + 7*d
) + e^(4*b*x + 6*a + 5*d) - e^(2*b*x + 6*a + 3*d) - e^(6*a + d)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.26

$$\int e^{a+bx}\operatorname{csch}^2(d+bx)\operatorname{sech}^3(d+bx)\,dx =$$
$$-\frac{1}{2}\left(\frac{2\,e^{(-6\,d)}\log\left(e^{(2\,bx+2\,d)}+1\right)}{b} - \frac{2\,e^{(-6\,d)}\log\left(\left|e^{(2\,bx+2\,d)}-1\right|\right)}{b} + \frac{2\left(e^{(2\,bx+2\,d)}+1\right)e^{(-6\,d)}}{b\left(e^{(2\,bx+2\,d)}-1\right)} - \frac{\left(3\,e^{(4\,bx+4\,d)}\right.}{}\right.$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^2*sech(b*x+d)^3,x, algorithm="giac")
```

output
```
-1/2*(2*e^(-6*d)*log(e^(2*b*x + 2*d) + 1)/b - 2*e^(-6*d)*log(abs(e^(2*b*x
+ 2*d) - 1))/b + 2*(e^(2*b*x + 2*d) + 1)*e^(-6*d)/(b*(e^(2*b*x + 2*d) - 1)
) - (3*e^(4*b*x + 4*d) + 14*e^(2*b*x + 2*d) + 7)*e^(-6*d)/(b*(e^(2*b*x + 2
*d) + 1)^2))*e^(a + 5*d)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\operatorname{csch}^2(d+bx)\operatorname{sech}^3(d+bx)\,dx=\int \frac{e^{a+b\,x}}{\cosh\left(d+b\,x\right)^3\,\sinh\left(d+b\,x\right)^2}\,dx$$

input
```
int(exp(a + b*x)/(cosh(d + b*x)^3*sinh(d + b*x)^2),x)
```

output
```
int(exp(a + b*x)/(cosh(d + b*x)^3*sinh(d + b*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.01

$$\int e^{a+bx}\operatorname{csch}^2(d+bx)\operatorname{sech}^3(d+bx)\,dx$$
$$=\frac{e^{a}\left(e^{6bx+6d}\log\left(e^{bx+d}-1\right)+e^{6bx+6d}\log\left(e^{bx+d}+1\right)-e^{6bx+6d}\log\left(e^{2bx+2d}+1\right)-2e^{6bx+6d}+e^{4bx+4d}\log\left(e^{bx+\cdots}\right.\right.}{}$$

input
```
int(exp(b*x+a)*csch(b*x+d)^2*sech(b*x+d)^3,x)
```

output
```
(e**a*(e**(6*b*x + 6*d)*log(e**(b*x + d) - 1) + e**(6*b*x + 6*d)*log(e**(b
*x + d) + 1) - e**(6*b*x + 6*d)*log(e**(2*b*x + 2*d) + 1) - 2*e**(6*b*x +
6*d) + e**(4*b*x + 4*d)*log(e**(b*x + d) - 1) + e**(4*b*x + 4*d)*log(e**(b
*x + d) + 1) - e**(4*b*x + 4*d)*log(e**(2*b*x + 2*d) + 1) - e**(2*b*x + 2*
d)*log(e**(b*x + d) - 1) - e**(2*b*x + 2*d)*log(e**(b*x + d) + 1) + e**(2*
b*x + 2*d)*log(e**(2*b*x + 2*d) + 1) - 4*e**(2*b*x + 2*d) - log(e**(b*x +
d) - 1) - log(e**(b*x + d) + 1) + log(e**(2*b*x + 2*d) + 1) - 2))/(e**d*b*
(e**(6*b*x + 6*d) + e**(4*b*x + 4*d) - e**(2*b*x + 2*d) - 1))
```

3.42 $\int e^{a+bx}\text{csch}^3(d+bx)\text{sech}^3(d+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 105

$$\int e^{a+bx}\text{csch}^3(d+bx)\text{sech}^3(d+bx)\,dx=-\frac{8e^{a+2d+3bx}}{b\left(1-e^{4d+4bx}\right)^2}+\frac{6e^{a+2d+3bx}}{b\left(1-e^{4d+4bx}\right)}-\frac{3e^{a-d}\arctan\left(e^{d+bx}\right)}{b}+\frac{3e^{a-d}\text{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
-8*exp(3*b*x+a+2*d)/b/(1-exp(4*b*x+4*d))^2+6*exp(3*b*x+a+2*d)/b/(1-exp(4*b
*x+4*d))-3*exp(a-d)*arctan(exp(b*x+d))/b+3*exp(a-d)*arctanh(exp(b*x+d))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.61

$$\int e^{a+bx}\text{csch}^3(d+bx)\text{sech}^3(d+bx)\,dx=\frac{64e^{a+2d+3bx}\left(-1+\left(-1+e^{4(d+bx)}\right)^2\text{Hypergeometric2F1}\left(\frac{3}{4},3,\frac{7}{4},e^{4(d+bx)}\right)\right)}{5b\left(-1+e^{4(d+bx)}\right)^2}$$

input
```
Integrate[E^(a + b*x)*Csch[d + b*x]^3*Sech[d + b*x]^3,x]
```

output
```
(64*E^(a + 2*d + 3*b*x)*(-1 + (-1 + E^(4*(d + b*x)))^2*Hypergeometric2F1[3
/4, 3, 7/4, E^(4*(d + b*x))]))/(5*b*(-1 + E^(4*(d + b*x)))^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.79, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2720, 27, 817, 819, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\operatorname{csch}^3(bx+d)\operatorname{sech}^3(bx+d)\,dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int -\frac{64e^{a+6bx}}{(1-e^{4bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \ 27$$

$$-\frac{64e^a\int \frac{e^{6bx}}{(1-e^{4bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \ 817$$

$$-\frac{64e^a\left(\frac{e^{3bx}}{8(1-e^{4bx})^2}-\frac{3}{8}\int \frac{e^{2bx}}{(1-e^{4bx})^2}de^{bx}\right)}{b}$$

$$\Big\downarrow \ 819$$

$$-\frac{64e^a\left(\frac{e^{3bx}}{8(1-e^{4bx})^2}-\frac{3}{8}\left(\frac{1}{4}\int \frac{e^{2bx}}{1-e^{4bx}}de^{bx}+\frac{e^{3bx}}{4(1-e^{4bx})}\right)\right)}{b}$$

$$\Big\downarrow \ 827$$

$$-\frac{64e^a\left(\frac{e^{3bx}}{8(1-e^{4bx})^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{1-e^{2bx}}de^{bx}-\frac{1}{2}\int \frac{1}{1+e^{2bx}}de^{bx}\right)+\frac{e^{3bx}}{4(1-e^{4bx})}\right)\right)}{b}$$

$$\Big\downarrow \ 216$$

$$-\frac{64e^a\left(\frac{e^{3bx}}{8(1-e^{4bx})^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{1}{2}\int\frac{1}{1-e^{2bx}}de^{bx}-\frac{1}{2}\arctan\left(e^{bx}\right)\right)+\frac{e^{3bx}}{4(1-e^{4bx})}\right)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{64e^a\left(\frac{e^{3bx}}{8(1-e^{4bx})^2}-\frac{3}{8}\left(\frac{1}{4}\left(\frac{1}{2}\text{arctanh}(e^{bx})-\frac{1}{2}\arctan\left(e^{bx}\right)\right)+\frac{e^{3bx}}{4(1-e^{4bx})}\right)\right)}{b}$$

input
```
Int[E^(a + b*x)*Csch[d + b*x]^3*Sech[d + b*x]^3,x]
```

output
```
(-64*E^a*(E^(3*b*x)/(8*(1 - E^(4*b*x))^2) - (3*(E^(3*b*x)/(4*(1 - E^(4*b*x
))) + (-1/2*ArcTan[E^(b*x)] + ArcTanh[E^(b*x)]/2)/4))/8))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.79

$$-\frac{2\left(3\,e^{4bx+4a+4d}+e^{4a}\right)e^{3bx+5a+2d}}{\left(e^{2bx+2a+2d}+e^{2a}\right)^{2}\left(-e^{2bx+2a+2d}+e^{2a}\right)^{2}b}+\frac{3\ln\left(e^{bx+a}+e^{a-d}\right)e^{a-d}}{2b}-\frac{3\ln\left(e^{bx+a}-e^{a-d}\right)e^{a-d}}{2b}+\frac{3i\ln\left(e^{bx+\cdots}\right.}{}$$

input
```
int(exp(b*x+a)*csch(b*x+d)^3*sech(b*x+d)^3,x)
```

output
```
-2/(exp(2*b*x+2*a+2*d)+exp(2*a))^2/(-exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(3*e
xp(4*b*x+4*a+4*d)+exp(4*a))*exp(3*b*x+5*a+2*d)+3/2*ln(exp(b*x+a)+exp(a-d))
/b*exp(a-d)-3/2*ln(exp(b*x+a)-exp(a-d))/b*exp(a-d)+3/2*I*ln(exp(b*x+a)-I*e
xp(a-d))/b*exp(a-d)-3/2*I*ln(exp(b*x+a)+I*exp(a-d))/b*exp(a-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2075 vs. $2(93) = 186$.

Time = 0.10 (sec) , antiderivative size = 2075, normalized size of antiderivative = 19.76

$$\int e^{a+bx}\operatorname{csch}^3(d+bx)\operatorname{sech}^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^3*sech(b*x+d)^3,x, algorithm="fricas")
```

output
```
-1/2*(12*cosh(b*x + d)^7*cosh(-a + d) + 12*(cosh(-a + d) - sinh(-a + d))*s
inh(b*x + d)^7 + 84*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x + d)*sinh(-a +
d))*sinh(b*x + d)^6 + 252*(cosh(b*x + d)^2*cosh(-a + d) - cosh(b*x + d)^2*
sinh(-a + d))*sinh(b*x + d)^5 + 420*(cosh(b*x + d)^3*cosh(-a + d) - cosh(b
*x + d)^3*sinh(-a + d))*sinh(b*x + d)^4 + 4*cosh(b*x + d)^3*cosh(-a + d) +
 4*(105*cosh(b*x + d)^4*cosh(-a + d) - (105*cosh(b*x + d)^4 + 1)*sinh(-a +
 d) + cosh(-a + d))*sinh(b*x + d)^3 + 12*(21*cosh(b*x + d)^5*cosh(-a + d)
+ cosh(b*x + d)*cosh(-a + d) - (21*cosh(b*x + d)^5 + cosh(b*x + d))*sinh(-
a + d))*sinh(b*x + d)^2 + 6*(cosh(b*x + d)^8*cosh(-a + d) + (cosh(-a + d)
- sinh(-a + d))*sinh(b*x + d)^8 + 8*(cosh(b*x + d)*cosh(-a + d) - cosh(b*x
 + d)*sinh(-a + d))*sinh(b*x + d)^7 + 28*(cosh(b*x + d)^2*cosh(-a + d) - c
osh(b*x + d)^2*sinh(-a + d))*sinh(b*x + d)^6 + 56*(cosh(b*x + d)^3*cosh(-a
 + d) - cosh(b*x + d)^3*sinh(-a + d))*sinh(b*x + d)^5 - 2*cosh(b*x + d)^4*
cosh(-a + d) + 2*(35*cosh(b*x + d)^4*cosh(-a + d) - (35*cosh(b*x + d)^4 -
1)*sinh(-a + d) - cosh(-a + d))*sinh(b*x + d)^4 + 8*(7*cosh(b*x + d)^5*cos
h(-a + d) - cosh(b*x + d)*cosh(-a + d) - (7*cosh(b*x + d)^5 - cosh(b*x + d
))*sinh(-a + d))*sinh(b*x + d)^3 + 4*(7*cosh(b*x + d)^6*cosh(-a + d) - 3*c
osh(b*x + d)^2*cosh(-a + d) - (7*cosh(b*x + d)^6 - 3*cosh(b*x + d)^2)*sinh
(-a + d))*sinh(b*x + d)^2 + 8*(cosh(b*x + d)^7*cosh(-a + d) - cosh(b*x + d
)^3*cosh(-a + d) - (cosh(b*x + d)^7 - cosh(b*x + d)^3)*sinh(-a + d))*si...
```

Sympy [F]

$$\int e^{a+bx}\text{csch}^3(d+bx)\text{sech}^3(d+bx)\,dx = e^{a}\int e^{bx}\,\text{csch}^3\,(bx+d)\,\text{sech}^3\,(bx+d)\,dx$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)**3*sech(b*x+d)**3,x)
```

output
```
exp(a)*Integral(exp(b*x)*csch(b*x + d)**3*sech(b*x + d)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.24

$$\int e^{a+bx}\text{csch}^3(d+bx)\text{sech}^3(d+bx)\,dx = -\frac{3\,\arctan\left(e^{(bx+d)}\right)e^{(a-d)}}{b} + \frac{3\,e^{(a-d)}\log\left(e^{(bx+a+d)}+e^{a}\right)}{2\,b} - \frac{3\,e^{(a-d)}\log\left(e^{(bx+a+d)}-e^{a}\right)}{2\,b} - \frac{2\left(3\,e^{(7\,bx+9\,a+6\,d)}+e^{(3\,bx+9\,a+2\,d)}\right)}{b\left(e^{(8\,bx+8\,a+8\,d)}-2\,e^{(4\,bx+8\,a+4\,d)}+e^{(8\,a)}\right)}$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^3*sech(b*x+d)^3,x, algorithm="maxima")
```

output
```
-3*arctan(e^(b*x + d))*e^(a - d)/b + 3/2*e^(a - d)*log(e^(b*x + a + d) + e
^a)/b - 3/2*e^(a - d)*log(e^(b*x + a + d) - e^a)/b - 2*(3*e^(7*b*x + 9*a +
 6*d) + e^(3*b*x + 9*a + 2*d))/(b*(e^(8*b*x + 8*a + 8*d) - 2*e^(4*b*x + 8*
a + 4*d) + e^(8*a)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.96

$$\int e^{a+bx}\mathrm{csch}^3(d+bx)\mathrm{sech}^3(d+bx)\,dx = -\frac{1}{2}\left(\frac{6\arctan\left(e^{(bx+d)}\right)e^{(-7\,d)}}{b}-\frac{3\,e^{(-7\,d)}\log\left(e^{(bx+d)}+1\right)}{b}+\frac{3\,e^{(-7\,d)}\log\left(\left|e^{(bx+d)}-1\right|\right)}{b}+\frac{4\left(e^{(3\,bx)}+3\,e^{\cdots}\right.}{b(e^{(4\,bx+\cdots}}\right.$$

input
```
integrate(exp(b*x+a)*csch(b*x+d)^3*sech(b*x+d)^3,x, algorithm="giac")
```

output
```
-1/2*(6*arctan(e^(b*x + d))*e^(-7*d)/b - 3*e^(-7*d)*log(e^(b*x + d) + 1)/b
 + 3*e^(-7*d)*log(abs(e^(b*x + d) - 1))/b + 4*(e^(3*b*x) + 3*e^(7*b*x + 4*
d))*e^(-4*d)/(b*(e^(4*b*x + 4*d) - 1)^2))*e^(a + 6*d)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+bx}\mathrm{csch}^3(d+bx)\mathrm{sech}^3(d+bx)\,dx = \int \frac{e^{a+b\,x}}{\cosh\left(d+b\,x\right)^3\sinh\left(d+b\,x\right)^3}\,dx$$

input
```
int(exp(a + b*x)/(cosh(d + b*x)^3*sinh(d + b*x)^3),x)
```

output
```
int(exp(a + b*x)/(cosh(d + b*x)^3*sinh(d + b*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.15

$$\int e^{a+bx}\mathrm{csch}^3(d+bx)\mathrm{sech}^3(d+bx)\,dx = \frac{e^a\left(-6e^{8bx+8d}atan\left(e^{bx+d}\right)+12e^{4bx+4d}atan\left(e^{bx+d}\right)-6atan\left(e^{bx+d}\right)-3e^{8bx+8d}\log\left(e^{bx+d}-1\right)+3e^{8bx+8d}\mathrm{lo}\cdots\right.}{2e^{d}\cdots}$$

input
```
int(exp(b*x+a)*csch(b*x+d)^3*sech(b*x+d)^3,x)
```

output
```
(e**a*( - 6*e**(8*b*x + 8*d)*atan(e**(b*x + d)) + 12*e**(4*b*x + 4*d)*atan
(e**(b*x + d)) - 6*atan(e**(b*x + d)) - 3*e**(8*b*x + 8*d)*log(e**(b*x + d
) - 1) + 3*e**(8*b*x + 8*d)*log(e**(b*x + d) + 1) - 12*e**(7*b*x + 7*d) +
6*e**(4*b*x + 4*d)*log(e**(b*x + d) - 1) - 6*e**(4*b*x + 4*d)*log(e**(b*x
+ d) + 1) - 4*e**(3*b*x + 3*d) - 3*log(e**(b*x + d) - 1) + 3*log(e**(b*x +
 d) + 1)))/(2*e**d*b*(e**(8*b*x + 8*d) - 2*e**(4*b*x + 4*d) + 1))
```

3.43 $\int e^{2(a+bx)} \cosh(d+bx)\sinh^3(d+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 76

$$\int e^{2(a+bx)} \cosh(d+bx)\sinh^3(d+bx)\,dx = \frac{e^{2(a-2d)-2bx}}{32b} - \frac{e^{2(a+d)+4bx}}{32b} + \frac{e^{2(a+2d)+6bx}}{96b} + \frac{1}{8}e^{2a-2d}x$$

output
```
1/32*exp(-2*b*x+2*a-4*d)/b-1/32*exp(4*b*x+2*a+2*d)/b+1/96*exp(6*b*x+2*a+4*
d)/b+1/8*exp(2*a-2*d)*x
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int e^{2(a+bx)} \cosh(d+bx)\sinh^3(d+bx)\,dx = \frac{e^{2a}\left(-3\left(\left(e^{4bx}-4bx\right)\cosh(2d)+\left(e^{4bx}+4bx\right)\sinh(2d)\right)+e^{-2bx}\left(\left(3+e^{8bx}\right)\cosh(4d)+\left(-3+e^{8bx}\right)\sinh(\right.\right.}{96b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[d + b*x]*Sinh[d + b*x]^3,x]
```

output
```
(E^(2*a)*(-3*((E^(4*b*x) - 4*b*x)*Cosh[2*d] + (E^(4*b*x) + 4*b*x)*Sinh[2*d
]) + ((3 + E^(8*b*x))*Cosh[4*d] + (-3 + E^(8*b*x))*Sinh[4*d])/E^(2*b*x)))/
(96*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2720, 27, 354, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)} \sinh^3(bx+d) \cosh(bx+d) \, dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int -\frac{1}{16} e^{2a-3bx} \left(1-e^{2bx}\right)^3 \left(1+e^{2bx}\right) de^{bx}}{b}$$

$$\Big\downarrow \ 27$$

$$-\frac{e^{2a} \int e^{-3bx} \left(1-e^{2bx}\right)^3 \left(1+e^{2bx}\right) de^{bx}}{16b}$$

$$\Big\downarrow \ 354$$

$$-\frac{e^{2a} \int e^{-2bx} \left(1-e^{2bx}\right)^3 \left(1+e^{2bx}\right) de^{2bx}}{32b}$$

$$\Big\downarrow \ 84$$

$$-\frac{e^{2a} \int \left(e^{-2bx} - 2e^{-bx} + e^{2bx}\right) de^{2bx}}{32b}$$

$$\Big\downarrow \ 2009$$

$$-\frac{e^{2a} \left(-e^{-bx} + e^{2bx} - \frac{1}{3} e^{3bx} - 2 \log\left(e^{2bx}\right)\right)}{32b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[d + b*x]*Sinh[d + b*x]^3,x]
```

output
```
-1/32*(E^(2*a)*(-E^(-(b*x)) + E^(2*b*x) - E^(3*b*x)/3 - 2*Log[E^(2*b*x)]))
/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 267.81 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

method	result
risch	$\frac{e^{-2bx+2a-4d}}{32b}-\frac{e^{4bx+2a+2d}}{32b}+\frac{e^{6bx+2a+4d}}{96b}+\frac{e^{2a-2d}x}{8}$
default	$\frac{x\cosh(2a-2d)}{8}+\frac{\sinh(-2bx+2a-4d)}{32b}-\frac{\sinh(4bx+2a+2d)}{32b}+\frac{\sinh(6bx+2a+4d)}{96b}+\frac{x\sinh(2a-2d)}{8}+\frac{\cosh(-2bx+2a-4d)}{32b}$
orering	$\frac{(12bx-1)e^{2bx+2a}\cosh(bx+d)\sinh(bx+d)^3}{12b}+\frac{(bx+2)\left(2\,e^{2bx+2a}b\cosh(bx+d)\sinh(bx+d)^3+e^{2bx+2a}b\sinh(bx+d)^4+3\,e^{2bx+2a}\cos\right.}{12b^2}$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)*sinh(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*exp(-2*b*x+2*a-4*d)/b-1/32*exp(4*b*x+2*a+2*d)/b+1/96*exp(6*b*x+2*a+4*
d)/b+1/8*exp(2*a-2*d)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(63) = 126$.

Time = 0.10 (sec) , antiderivative size = 345, normalized size of antiderivative = 4.54

$$\int e^{2(a+bx)}\cosh(d+bx)\sinh^3(d+bx)\,dx$$
$$=\frac{4\,\cosh\left(bx+d\right)^4\cosh\left(-2\,a+2\,d\right)+4\left(\cosh\left(-2\,a+2\,d\right)-\sinh\left(-2\,a+2\,d\right)\right)\sinh\left(bx+d\right)^4+3\left(4\,bx-\right.}{}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)*sinh(b*x+d)^3,x, algorithm="fricas")
```

output
```
1/96*(4*cosh(b*x + d)^4*cosh(-2*a + 2*d) + 4*(cosh(-2*a + 2*d) - sinh(-2*a
 + 2*d))*sinh(b*x + d)^4 + 3*(4*b*x - 1)*cosh(b*x + d)^2*cosh(-2*a + 2*d)
- 8*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh
(b*x + d)^3 + 3*(8*cosh(b*x + d)^2*cosh(-2*a + 2*d) + (4*b*x - 1)*cosh(-2*
a + 2*d) - (4*b*x + 8*cosh(b*x + d)^2 - 1)*sinh(-2*a + 2*d))*sinh(b*x + d)
^2 - 2*(4*cosh(b*x + d)^3*cosh(-2*a + 2*d) + 3*(4*b*x + 1)*cosh(b*x + d)*c
osh(-2*a + 2*d) - (4*cosh(b*x + d)^3 + 3*(4*b*x + 1)*cosh(b*x + d))*sinh(-
2*a + 2*d))*sinh(b*x + d) - (4*cosh(b*x + d)^4 + 3*(4*b*x - 1)*cosh(b*x +
d)^2)*sinh(-2*a + 2*d))/(b*cosh(b*x + d)^2 - 2*b*cosh(b*x + d)*sinh(b*x +
d) + b*sinh(b*x + d)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(61) = 122$.

Time = 2.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.09

$$\int e^{2(a+bx)}\cosh(d+bx)\sinh^3(d+bx)\,dx$$
$$= \begin{cases} -\frac{xe^{2a}e^{2bx}\sinh^4(bx+d)}{8} + \frac{xe^{2a}e^{2bx}\sinh^3(bx+d)\cosh(bx+d)}{4} - \frac{xe^{2a}e^{2bx}\sinh(bx+d)\cosh^3(bx+d)}{4} + \frac{xe^{2a}e^{2bx}\cosh^4(bx+d)}{8} + \frac{7e^{2c}}{} \\ xe^{2a}\sinh^3(d)\cosh(d) \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)*sinh(b*x+d)**3,x)
```

output
```
Piecewise((-x*exp(2*a)*exp(2*b*x)*sinh(b*x + d)**4/8 + x*exp(2*a)*exp(2*b*
x)*sinh(b*x + d)**3*cosh(b*x + d)/4 - x*exp(2*a)*exp(2*b*x)*sinh(b*x + d)*
cosh(b*x + d)**3/4 + x*exp(2*a)*exp(2*b*x)*cosh(b*x + d)**4/8 + 7*exp(2*a)
*exp(2*b*x)*sinh(b*x + d)**4/(48*b) - exp(2*a)*exp(2*b*x)*sinh(b*x + d)**3
*cosh(b*x + d)/(6*b) + exp(2*a)*exp(2*b*x)*sinh(b*x + d)**2*cosh(b*x + d)*
*2/(4*b) - exp(2*a)*exp(2*b*x)*cosh(b*x + d)**4/(16*b), Ne(b, 0)), (x*exp(
2*a)*sinh(d)**3*cosh(d), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int e^{2(a+bx)}\cosh(d+bx)\sinh^3(d+bx)\,dx = -\frac{\left(3\,e^{(-2\,bx-2\,d)}-1\right)e^{(6\,bx+2\,a+4\,d)}}{96\,b} + \frac{(bx+d)e^{(2\,a-2\,d)}}{8\,b} + \frac{e^{(-2\,bx+2\,a-4\,d)}}{32\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)*sinh(b*x+d)^3,x, algorithm="maxima")
```

output
```
-1/96*(3*e^(-2*b*x - 2*d) - 1)*e^(6*b*x + 2*a + 4*d)/b + 1/8*(b*x + d)*e^(
2*a - 2*d)/b + 1/32*e^(-2*b*x + 2*a - 4*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07

$$\int e^{2(a+bx)}\cosh(d+bx)\sinh^3(d+bx)\,dx = -\frac{\left(3\left(2\,e^{(2\,bx+2\,a+2\,d)}-e^{(2\,a)}\right)e^{(-2\,bx-2\,d)}-12\,(bx+d)e^{(2\,a)}-e^{(6\,bx+2\,a+6\,d)}+3\,e^{(4\,bx+2\,a+4\,d)}\right)e^{(-2\,d)}}{96\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)*sinh(b*x+d)^3,x, algorithm="giac")
```

output
```
-1/96*(3*(2*e^(2*b*x + 2*a + 2*d) - e^(2*a))*e^(-2*b*x - 2*d) - 12*(b*x +
d)*e^(2*a) - e^(6*b*x + 2*a + 6*d) + 3*e^(4*b*x + 2*a + 4*d))*e^(-2*d)/b
```

Mupad [B] (verification not implemented)

Time = 3.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.12

$$\int e^{2(a+bx)}\cosh(d+bx)\sinh^3(d+bx)\,dx$$
$$= \frac{e^{2a+2bx}\,(2\cosh(2d+2bx)+2\cosh(4d+4bx)-5\sinh(2d+2bx)-\sinh(4d+4bx)-6bx\sinh(2d$$
$$48b$$

input
```
int(cosh(d + b*x)*exp(2*a + 2*b*x)*sinh(d + b*x)^3,x)
```

output
```
(exp(2*a + 2*b*x)*(2*cosh(2*d + 2*b*x) + 2*cosh(4*d + 4*b*x) - 5*sinh(2*d
+ 2*b*x) - sinh(4*d + 4*b*x) - 6*b*x*sinh(2*d + 2*b*x) + 6*b*x*cosh(2*d +
2*b*x)))/(48*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int e^{2(a+bx)}\cosh(d+bx)\sinh^3(d+bx)\,dx = \frac{e^{2a}\left(e^{8bx+8d}-3e^{6bx+6d}+12e^{2bx+2d}bx+3\right)}{96e^{2bx+4d}b}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)*sinh(b*x+d)^3,x)
```

output
```
(e**(2*a)*(e**(8*b*x + 8*d) - 3*e**(6*b*x + 6*d) + 12*e**(2*b*x + 2*d)*b*x
 + 3))/(96*e**(2*b*x + 4*d)*b)
```

3.44 $\int e^{2(a+bx)}\cosh(d+bx)\sinh^2(d+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 78

$$\int e^{2(a+bx)}\cosh(d+bx)\sinh^2(d+bx)\,dx=-\frac{e^{2a-3d-bx}}{8b}-\frac{e^{2a-d+bx}}{8b}-\frac{e^{2a+d+3bx}}{24b}+\frac{e^{2a+3d+5bx}}{40b}$$

output
```
-1/8*exp(-b*x+2*a-3*d)/b-1/8*exp(b*x+2*a-d)/b-1/24*exp(3*b*x+2*a+d)/b+1/40
*exp(5*b*x+2*a+3*d)/b
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int e^{2(a+bx)}\cosh(d+bx)\sinh^2(d+bx)\,dx$$
$$=\frac{e^{2a-bx}\left(-5e^{2bx}\left(\left(3+e^{2bx}\right)\cosh(d)+\left(-3+e^{2bx}\right)\sinh(d)\right)+3\left(\left(-5+e^{6bx}\right)\cosh(3d)+\left(5+e^{6bx}\right)\sinh(3d\right.\right.}{120b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[d + b*x]*Sinh[d + b*x]^2,x]
```

output
```
(E^(2*a - b*x)*(-5*E^(2*b*x)*((3 + E^(2*b*x))*Cosh[d] + (-3 + E^(2*b*x))*S
inh[d]) + 3*((-5 + E^(6*b*x))*Cosh[3*d] + (5 + E^(6*b*x))*Sinh[3*d])))/(12
0*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2720, 27, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\sinh^2(bx+d)\cosh(bx+d)\,dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int \frac{1}{8}e^{2a-2bx}\left(1-e^{2bx}\right)^2\left(1+e^{2bx}\right)de^{bx}}{b}$$

$$\Big\downarrow \ 27$$

$$\frac{e^{2a}\int e^{-2bx}\left(1-e^{2bx}\right)^2\left(1+e^{2bx}\right)de^{bx}}{8b}$$

$$\Big\downarrow \ 355$$

$$\frac{e^{2a}\int \left(-1+e^{-2bx}-e^{2bx}+e^{4bx}\right)de^{bx}}{8b}$$

$$\Big\downarrow \ 2009$$

$$\frac{e^{2a}\left(-e^{-bx}-e^{bx}-\frac{1}{3}e^{3bx}+\frac{1}{5}e^{5bx}\right)}{8b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[d + b*x]*Sinh[d + b*x]^2,x]
```

output
```
(E^(2*a)*(-E^(-(b*x)) - E^(b*x) - E^(3*b*x)/3 + E^(5*b*x)/5))/(8*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 22.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{e^{-bx+2a-3d}}{8b}-\frac{e^{bx+2a-d}}{8b}-\frac{e^{3bx+2a+d}}{24b}+\frac{e^{5bx+2a+3d}}{40b}$
default	$-\frac{\sinh(-bx+2a-3d)}{8b}-\frac{\sinh(bx+2a-d)}{8b}-\frac{\sinh(3bx+2a+d)}{24b}+\frac{\sinh(5bx+2a+3d)}{40b}-\frac{\cosh(-bx+2a-3d)}{8b}-\frac{\cosh(bx+2a-d)}{8b}$
orering	$\frac{8\,e^{2bx+2a}\cosh(bx+d)\sinh(bx+d)^2}{15b}+\frac{\frac{28\,e^{2bx+2a}b\cosh(bx+d)\sinh(bx+d)^2}{15}+\frac{14\,e^{2bx+2a}b\sinh(bx+d)^3}{15}+\frac{28\,e^{2bx+2a}\cosh(bx+d)^2\sinh(bx+\cdots}{15}}{b^2}$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)*sinh(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*exp(-b*x+2*a-3*d)/b-1/8*exp(b*x+2*a-d)/b-1/24*exp(3*b*x+2*a+d)/b+1/40
*exp(5*b*x+2*a+3*d)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(66) = 132$.

Time = 0.08 (sec) , antiderivative size = 234, normalized size of antiderivative = 3.00

$$\int e^{2(a+bx)}\cosh(d+bx)\sinh^2(d+bx)\,dx =$$

$$-\frac{6\cosh(bx+d)^3\cosh(-2a+2d) - 9(\cosh(-2a+2d) - \sinh(-2a+2d))\sinh(bx+d)^3 + 18(\cos}{}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)*sinh(b*x+d)^2,x, algorithm="fricas")
```

output
```
-1/60*(6*cosh(b*x + d)^3*cosh(-2*a + 2*d) - 9*(cosh(-2*a + 2*d) - sinh(-2*
a + 2*d))*sinh(b*x + d)^3 + 18*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x
+ d)*sinh(-2*a + 2*d))*sinh(b*x + d)^2 + 10*cosh(b*x + d)*cosh(-2*a + 2*d)
 - (27*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (27*cosh(b*x + d)^2 + 5)*sinh(-2
*a + 2*d) + 5*cosh(-2*a + 2*d))*sinh(b*x + d) - 2*(3*cosh(b*x + d)^3 + 5*c
osh(b*x + d))*sinh(-2*a + 2*d))/(b*cosh(b*x + d)^2 - 2*b*cosh(b*x + d)*sin
h(b*x + d) + b*sinh(b*x + d)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(60) = 120$.

Time = 0.91 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.64

$$\int e^{2(a+bx)}\cosh(d+bx)\sinh^2(d+bx)\,dx$$

$$= \begin{cases} \frac{e^{2a}e^{2bx}\sinh^3(bx+d)}{15b} - \frac{2e^{2a}e^{2bx}\sinh^2(bx+d)\cosh(bx+d)}{15b} + \frac{8e^{2a}e^{2bx}\sinh(bx+d)\cosh^2(bx+d)}{15b} - \frac{4e^{2a}e^{2bx}\cosh^3(bx+d)}{15b} & \text{for } b \neq \\ xe^{2a}\sinh^2(d)\cosh(d) & \text{otherw} \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)*sinh(b*x+d)**2,x)
```

output
```
Piecewise((exp(2*a)*exp(2*b*x)*sinh(b*x + d)**3/(15*b) - 2*exp(2*a)*exp(2*
b*x)*sinh(b*x + d)**2*cosh(b*x + d)/(15*b) + 8*exp(2*a)*exp(2*b*x)*sinh(b*
x + d)*cosh(b*x + d)**2/(15*b) - 4*exp(2*a)*exp(2*b*x)*cosh(b*x + d)**3/(1
5*b), Ne(b, 0)), (x*exp(2*a)*sinh(d)**2*cosh(d), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.76

$$\int e^{2(a+bx)}\cosh(d+bx)\sinh^2(d+bx)\,dx = -\frac{\left(5\,e^{(-2\,bx-2\,d)}+15\,e^{(-4\,bx-4\,d)}-3\right)e^{(5\,bx+2\,a+3\,d)}}{120\,b}-\frac{e^{(-bx+2\,a-3\,d)}}{8\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)*sinh(b*x+d)^2,x, algorithm="maxima")
```

output
```
-1/120*(5*e^(-2*b*x - 2*d) + 15*e^(-4*b*x - 4*d) - 3)*e^(5*b*x + 2*a + 3*d
)/b - 1/8*e^(-b*x + 2*a - 3*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)}\cosh(d+bx)\sinh^2(d+bx)\,dx = \frac{\left(3\,e^{(5\,bx+2\,a+5\,d)}-5\,e^{(3\,bx+2\,a+3\,d)}-15\,e^{(bx+2\,a+d)}-15\,e^{(-bx+2\,a-d)}\right)e^{(-2\,d)}}{120\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)*sinh(b*x+d)^2,x, algorithm="giac")
```

output
```
1/120*(3*e^(5*b*x + 2*a + 5*d) - 5*e^(3*b*x + 2*a + 3*d) - 15*e^(b*x + 2*a
 + d) - 15*e^(-b*x + 2*a - d))*e^(-2*d)/b
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int e^{2(a+bx)}\cosh(d+bx)\sinh^2(d+bx)\,dx = \frac{e^{2\,a+3\,d+5\,b\,x}}{40\,b} - \frac{e^{2\,a-d+b\,x}}{8\,b} - \frac{e^{2\,a-3\,d-b\,x}}{8\,b} - \frac{e^{2\,a+d+3\,b\,x}}{24\,b}$$

input
```
int(cosh(d + b*x)*exp(2*a + 2*b*x)*sinh(d + b*x)^2,x)
```

output
```
exp(2*a + 3*d + 5*b*x)/(40*b) - exp(2*a - d + b*x)/(8*b) - exp(2*a - 3*d -
 b*x)/(8*b) - exp(2*a + d + 3*b*x)/(24*b)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.76

$$\int e^{2(a+bx)}\cosh(d+bx)\sinh^2(d+bx)\,dx = \frac{e^{2a}\left(3e^{6bx+6d}-5e^{4bx+4d}-15e^{2bx+2d}-15\right)}{120e^{bx+3d}b}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)*sinh(b*x+d)^2,x)
```

output
```
(e**(2*a)*(3*e**(6*b*x + 6*d) - 5*e**(4*b*x + 4*d) - 15*e**(2*b*x + 2*d) -
 15))/(120*e**(b*x + 3*d)*b)
```

3.45 $\int e^{2(a+bx)} \cosh(d+bx) \sinh(d+bx) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 34

$$\int e^{2(a+bx)} \cosh(d+bx) \sinh(d+bx) \, dx = \frac{e^{2(a+d)+4bx}}{16b} - \frac{1}{4} e^{2a-2d} x$$

output
```
1/16*exp(4*b*x+2*a+2*d)/b-1/4*exp(2*a-2*d)*x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.32

$$\int e^{2(a+bx)} \cosh(d+bx) \sinh(d+bx) \, dx = \frac{e^{2a}\left(\left(e^{4bx} - 4bx\right)\cosh(2d) + \left(e^{4bx} + 4bx\right)\sinh(2d)\right)}{16b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[d + b*x]*Sinh[d + b*x],x]
```

output
```
(E^(2*a)*((E^(4*b*x) - 4*b*x)*Cosh[2*d] + (E^(4*b*x) + 4*b*x)*Sinh[2*d]))/
(16*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2720, 27, 335, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)} \sinh(bx+d)\cosh(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{1}{4}e^{2a-bx}\left(1-e^{2bx}\right)\left(1+e^{2bx}\right)de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^{2a}\int e^{-bx}\left(1-e^{2bx}\right)\left(1+e^{2bx}\right)de^{bx}}{4b}$$

$$\Big\downarrow \text{335}$$

$$-\frac{e^{2a}\int e^{-bx}\left(1-e^{4bx}\right)de^{bx}}{4b}$$

$$\Big\downarrow \text{802}$$

$$-\frac{e^{2a}\int \left(e^{-bx}-e^{3bx}\right)de^{bx}}{4b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{2a}\left(\log\left(e^{bx}\right)-\frac{1}{4}e^{4bx}\right)}{4b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[d + b*x]*Sinh[d + b*x],x]
```

output
```
-1/4*(E^(2*a)*(-1/4*E^(4*b*x) + Log[E^(b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 335
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(e*x)^m*(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]
))
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

method	result
risch	$\frac{{\mathrm e}^{4 b x +2 a +2 d}}{16 b}-\frac{{\mathrm e}^{2 a -2 d} x}{4}$
default	$-\frac{x \cosh \left(2 a -2 d \right)}{4}+\frac{\sinh \left(4 b x +2 a +2 d \right)}{16 b}-\frac{x \sinh \left(2 a -2 d \right)}{4}+\frac{\cosh \left(4 b x +2 a +2 d \right)}{16 b}$
orering	$\frac{\left(4 b x +1\right) {\mathrm e}^{2 b x +2 a} \cosh \left(b x +d \right) \sinh \left(b x +d \right)}{4 b}-\frac{x \left(2 \,{\mathrm e}^{2 b x +2 a} b \cosh \left(b x +d \right) \sinh \left(b x +d \right)+{\mathrm e}^{2 b x +2 a} b \sinh \left(b x +d \right)^{2}+{\mathrm e}^{2 b x +2 a} \cosh \left(b x +d \right)^{2} b\right.}{4 b}$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)*sinh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/16*exp(4*b*x+2*a+2*d)/b-1/4*exp(2*a-2*d)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(29) = 58.

Time = 0.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 5.32

$$\int e^{2(a+bx)}\cosh(d+bx)\sinh(d+bx)\,dx = -\frac{(4\,bx-1)\cosh{(bx+d)}^2\cosh{(-2\,a+2\,d)} - (4\,bx-1)\cosh{(bx+d)}^2\sinh{(-2\,a+2\,d)} + ((4\,bx-1)\,c}{16}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)*sinh(b*x+d),x, algorithm="fricas")
```

output
```
-1/16*((4*b*x - 1)*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (4*b*x - 1)*cosh(b*x
 + d)^2*sinh(-2*a + 2*d) + ((4*b*x - 1)*cosh(-2*a + 2*d) - (4*b*x - 1)*sin
h(-2*a + 2*d))*sinh(b*x + d)^2 - 2*((4*b*x + 1)*cosh(b*x + d)*cosh(-2*a +
2*d) - (4*b*x + 1)*cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d))/(b*cosh(
b*x + d)^2 - 2*b*cosh(b*x + d)*sinh(b*x + d) + b*sinh(b*x + d)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(27) = 54.

Time = 0.42 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.44

$$\int e^{2(a+bx)}\cosh(d+bx)\sinh(d+bx)\,dx = \begin{cases} -\frac{xe^{2a}e^{2bx}\sinh^2{(bx+d)}}{4} + \frac{xe^{2a}e^{2bx}\sinh{(bx+d)}\cosh{(bx+d)}}{2} - \frac{xe^{2a}e^{2bx}\cosh^2{(bx+d)}}{4} + \frac{e^{2a}e^{2bx}\sinh{(bx+d)}\cosh{(bx+d)}}{4b} & \text{for } b \neq \\ xe^{2a}\sinh{(d)}\cosh{(d)} & \text{otherw} \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)*sinh(b*x+d),x)
```

output
```
Piecewise((-x*exp(2*a)*exp(2*b*x)*sinh(b*x + d)**2/4 + x*exp(2*a)*exp(2*b*
x)*sinh(b*x + d)*cosh(b*x + d)/2 - x*exp(2*a)*exp(2*b*x)*cosh(b*x + d)**2/
4 + exp(2*a)*exp(2*b*x)*sinh(b*x + d)*cosh(b*x + d)/(4*b), Ne(b, 0)), (x*e
xp(2*a)*sinh(d)*cosh(d), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int e^{2(a+bx)}\cosh(d+bx)\sinh(d+bx)\,dx = -\frac{1}{4}\,xe^{(2\,a-2\,d)} - \frac{de^{(2\,a-2\,d)}}{4\,b} + \frac{e^{(4\,bx+2\,a+2\,d)}}{16\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)*sinh(b*x+d),x, algorithm="maxima")
```

output
```
-1/4*x*e^(2*a - 2*d) - 1/4*d*e^(2*a - 2*d)/b + 1/16*e^(4*b*x + 2*a + 2*d)/
b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int e^{2(a+bx)}\cosh(d+bx)\sinh(d+bx)\,dx = -\frac{\left(4\,(bx+d)e^{(2\,a)} - e^{(4\,bx+2\,a+4\,d)}\right)e^{(-2\,d)}}{16\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)*sinh(b*x+d),x, algorithm="giac")
```

output
```
-1/16*(4*(b*x + d)*e^(2*a) - e^(4*b*x + 2*a + 4*d))*e^(-2*d)/b
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int e^{2(a+bx)}\cosh(d+bx)\sinh(d+bx)\,dx = \frac{e^{2a}\,e^{2d}\,e^{4bx}}{16\,b} - \frac{x\,e^{2a}\,e^{-2d}}{4}$$

input
```
int(cosh(d + b*x)*exp(2*a + 2*b*x)*sinh(d + b*x),x)
```

output
```
(exp(2*a)*exp(2*d)*exp(4*b*x))/(16*b) - (x*exp(2*a)*exp(-2*d))/4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.03

$$\int e^{2(a+bx)}\cosh(d+bx)\sinh(d+bx)\,dx$$
$$= \frac{e^{2bx+2a}\left(-\cosh\left(bx+d\right)^{2}bx+2\cosh\left(bx+d\right)\sinh\left(bx+d\right)bx+\cosh\left(bx+d\right)\sinh\left(bx+d\right)-\sinh\left(bx+\right.\right.}{4b}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)*sinh(b*x+d),x)
```

output
```
(e**(2*a + 2*b*x)*( - cosh(b*x + d)**2*b*x + 2*cosh(b*x + d)*sinh(b*x + d)
*b*x + cosh(b*x + d)*sinh(b*x + d) - sinh(b*x + d)**2*b*x))/(4*b)
```

3.46 $\int e^{2(a+bx)} \cosh(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 38

$$\int e^{2(a+bx)}\cosh(d+bx)\,dx=\frac{e^{2a-d+bx}}{2b}+\frac{e^{2a+d+3bx}}{6b}$$

output
```
1/2*exp(b*x+2*a-d)/b+1/6*exp(3*b*x+2*a+d)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int e^{2(a+bx)}\cosh(d+bx)\,dx=\frac{e^{2a+bx}\left(\left(3+e^{2bx}\right)\cosh(d)+\left(-3+e^{2bx}\right)\sinh(d)\right)}{6b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[d + b*x],x]
```

output
```
(E^(2*a + b*x)*((3 + E^(2*b*x))*Cosh[d] + (-3 + E^(2*b*x))*Sinh[d]))/(6*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2720, 27, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\cosh(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{2}e^{2a}\left(1+e^{2bx}\right)de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{2a}\int \left(1+e^{2bx}\right)de^{bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2a}\left(e^{bx}+\frac{1}{3}e^{3bx}\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[d + b*x],x]
```

output
```
(E^(2*a)*(E^(b*x) + E^(3*b*x)/3))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

method	result	size
parallelrisch	$\frac{e^{2bx+2a}(2\cosh(bx+d)-\sinh(bx+d))}{3b}$	32
risch	$\frac{e^{bx+2a-d}}{2b}+\frac{e^{3bx+2a+d}}{6b}$	33
orering	$\frac{4\,e^{2bx+2a}\cosh(bx+d)}{3b}-\frac{2\,e^{2bx+2a}b\cosh(bx+d)+e^{2bx+2a}b\sinh(bx+d)}{3b^2}$	63
default	$\frac{\sinh(bx+2a-d)}{2b}+\frac{\sinh(3bx+2a+d)}{6b}+\frac{\cosh(bx+2a-d)}{2b}+\frac{\cosh(3bx+2a+d)}{6b}$	64

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/3*exp(2*b*x+2*a)*(2*cosh(b*x+d)-sinh(b*x+d))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(32) = 64.

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.63

$$\int e^{2(a+bx)}\cosh(d+bx)\,dx$$
$$=\frac{\cosh(bx+d)^2\cosh(-2a+2d)+(\cosh(-2a+2d)-\sinh(-2a+2d))\sinh(bx+d)^2+2(\cosh(bx+\ldots}{6(b\ldots}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d),x, algorithm="fricas")
```

output
```
1/6*(cosh(b*x + d)^2*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*
d))*sinh(b*x + d)^2 + 2*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*si
nh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^2 + 3)*sinh(-2*a + 2*d) + 3
*cosh(-2*a + 2*d))/(b*cosh(b*x + d) - b*sinh(b*x + d))
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int e^{2(a+bx)}\cosh(d+bx)\,dx = \begin{cases} -\frac{e^{2a}e^{2bx}\sinh{(bx+d)}}{3b} + \frac{2e^{2a}e^{2bx}\cosh{(bx+d)}}{3b} & \text{for } b \neq 0 \\ xe^{2a}\cosh{(d)} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d),x)
```

output
```
Piecewise((-exp(2*a)*exp(2*b*x)*sinh(b*x + d)/(3*b) + 2*exp(2*a)*exp(2*b*x
)*cosh(b*x + d)/(3*b), Ne(b, 0)), (x*exp(2*a)*cosh(d), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int e^{2(a+bx)}\cosh(d+bx)\,dx = \frac{e^{(3\,bx+2\,a+d)}}{6\,b} + \frac{e^{(bx+2\,a-d)}}{2\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d),x, algorithm="maxima")
```

output
```
1/6*e^(3*b*x + 2*a + d)/b + 1/2*e^(b*x + 2*a - d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int e^{2(a+bx)}\cosh(d+bx)\,dx = \frac{\left(e^{(3\,bx+2\,a+3\,d)} + 3\,e^{(bx+2\,a+d)}\right)e^{(-2\,d)}}{6\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d),x, algorithm="giac")
```

output
```
1/6*(e^(3*b*x + 2*a + 3*d) + 3*e^(b*x + 2*a + d))*e^(-2*d)/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int e^{2(a+bx)}\cosh(d+bx)\,dx = \frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{-d}\,\mathrm{e}^{b\,x}\,\left(\mathrm{e}^{2\,d}\,\mathrm{e}^{2\,b\,x}+3\right)}{6\,b}$$

input
```
int(cosh(d + b*x)*exp(2*a + 2*b*x),x)
```

output
```
(exp(2*a)*exp(-d)*exp(b*x)*(exp(2*d)*exp(2*b*x) + 3))/(6*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int e^{2(a+bx)}\cosh(d+bx)\,dx = \frac{e^{2bx+2a}(2\cosh(bx+d)-\sinh(bx+d))}{3b}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d),x)
```

output
```
(e**(2*a + 2*b*x)*(2*cosh(b*x + d) - sinh(b*x + d)))/(3*b)
```

3.47 $\int e^{2(a+bx)}\coth(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int e^{2(a+bx)}\coth(d+bx)\,dx=\frac{e^{2a+2bx}}{2b}+\frac{e^{2a-2d}\log\left(1-e^{2d+2bx}\right)}{b}$$

output
```
1/2*exp(2*b*x+2*a)/b+exp(2*a-2*d)*ln(1-exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.00

$$\int e^{2(a+bx)}\coth(d+bx)\,dx$$
$$=\frac{e^{2a}(\cosh(d)-\sinh(d))\left(\cosh(d)\left(e^{2bx}+2\log\left(\left(-1+e^{2bx}\right)\cosh(d)+\left(1+e^{2bx}\right)\sinh(d)\right)\right)+\left(e^{2bx}-2\log\right.\right.}{2b}$$

input
```
Integrate[E^(2*(a + b*x))*Coth[d + b*x],x]
```

output
```
(E^(2*a)*(Cosh[d] - Sinh[d])*(Cosh[d]*(E^(2*b*x) + 2*Log[(-1 + E^(2*b*x))*
Cosh[d] + (1 + E^(2*b*x))*Sinh[d]]) + (E^(2*b*x) - 2*Log[(-1 + E^(2*b*x))*
Cosh[d] + (1 + E^(2*b*x))*Sinh[d]])*Sinh[d]))/(2*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2720, 25, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\coth(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{e^{2a+bx}\left(1+e^{2bx}\right)}{1-e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{e^{2a+bx}\left(1+e^{2bx}\right)}{1-e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^{2a}\int \frac{e^{bx}\left(1+e^{2bx}\right)}{1-e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{353}$$

$$-\frac{e^{2a}\int \frac{1+e^{2bx}}{1-e^{2bx}}de^{2bx}}{2b}$$

$$\Big\downarrow \text{49}$$

$$-\frac{e^{2a}\int \left(-1-\frac{2}{-1+e^{2bx}}\right)de^{2bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{2a}\left(-e^{2bx}-2\log\left(1-e^{2bx}\right)\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Coth[d + b*x],x]
```

output
```
-1/2*(E^(2*a)*(-E^(2*b*x) - 2*Log[1 - E^(2*b*x)]))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

method	result	size
risch	$\frac{e^{2bx+2a}}{2b}-\frac{2\,e^{2a-2d}a}{b}+\frac{\ln\left(e^{2bx+2a}-e^{2a-2d}\right)e^{2a-2d}}{b}$	63

input
```
int(exp(2*b*x+2*a)*coth(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*b*x+2*a)/b-2/b*exp(2*a-2*d)*a+ln(exp(2*b*x+2*a)-exp(2*a-2*d))/b*
exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(41) = 82.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.39

$$\int e^{2(a+bx)}\coth(d+bx)\,dx$$
$$=\frac{\cosh\left(bx+d\right)^2\cosh\left(-2\,a+2\,d\right)+\left(\cosh\left(-2\,a+2\,d\right)-\sinh\left(-2\,a+2\,d\right)\right)\sinh\left(bx+d\right)^2-\cosh\left(bx+d\right)}{}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d),x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + d)^2*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*
d))*sinh(b*x + d)^2 - cosh(b*x + d)^2*sinh(-2*a + 2*d) + 2*(cosh(-2*a + 2*
d) - sinh(-2*a + 2*d))*log(2*sinh(b*x + d)/(cosh(b*x + d) - sinh(b*x + d))
) + 2*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*si
nh(b*x + d))/b
```

Sympy [F]

$$\int e^{2(a+bx)}\coth(d+bx)\,dx = e^{2a}\int e^{2bx}\coth(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d),x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*coth(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76

$$\int e^{2(a+bx)}\coth(d+bx)\,dx = \frac{2\,(bx+d)e^{(2\,a-2\,d)}}{b} + \frac{e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}+1\right)}{b} + \frac{e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}-1\right)}{b} + \frac{e^{(2\,bx+2\,a)}}{2\,b}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d),x, algorithm="maxima")
```

output
```
2*(b*x + d)*e^(2*a - 2*d)/b + e^(2*a - 2*d)*log(e^(-b*x - d) + 1)/b + e^(2
*a - 2*d)*log(e^(-b*x - d) - 1)/b + 1/2*e^(2*b*x + 2*a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int e^{2(a+bx)}\coth(d+bx)\,dx = \frac{e^{(2\,a-2\,d)}\log\left(\left|e^{(2\,bx+2\,d)}-1\right|\right)}{b} + \frac{e^{(2\,bx+2\,a)}}{2\,b}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d),x, algorithm="giac")
```

output
```
e^(2*a - 2*d)*log(abs(e^(2*b*x + 2*d) - 1))/b + 1/2*e^(2*b*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int e^{2(a+bx)}\coth(d+bx)\,dx=\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{2\,b}+\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}\right)}{b}$$

input
```
int(coth(d + b*x)*exp(2*a + 2*b*x),x)
```

output
```
(exp(2*a)*exp(2*b*x))/(2*b) + (exp(2*a)*exp(-2*d)*log(exp(2*a)*exp(2*b*x)
- exp(2*a)*exp(-2*d)))/b
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int e^{2(a+bx)}\coth(d+bx)\,dx=\frac{e^{2a}\left(e^{2bx+2d}+2\log\left(e^{bx+d}-1\right)+2\log\left(e^{bx+d}+1\right)\right)}{2e^{2d}b}$$

input
```
int(exp(2*b*x+2*a)*coth(b*x+d),x)
```

output
```
(e**(2*a)*(e**(2*b*x + 2*d) + 2*log(e**(b*x + d) - 1) + 2*log(e**(b*x + d)
 + 1)))/(2*e**(2*d)*b)
```

3.48 $\int e^{2(a+bx)} \coth(d+bx)\mathbf{csch}(d+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int e^{2(a+bx)} \coth(d+bx)\operatorname{csch}(d+bx)\,dx = \frac{2e^{2a-d+bx}}{b} + \frac{2e^{2a-d+bx}}{b\left(1-e^{2d+2bx}\right)} - \frac{4e^{2a-2d}\operatorname{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
2*exp(b*x+2*a-d)/b+2*exp(b*x+2*a-d)/b/(1-exp(2*b*x+2*d))-4*exp(2*a-2*d)*ar
ctanh(exp(b*x+d))/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 211 vs. 2(73) = 146.

Time = 0.46 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.89

$$\int e^{2(a+bx)} \coth(d+bx)\operatorname{csch}(d+bx)\,dx$$
$$= \frac{2e^{2a}\left(e^{bx}\cosh(d) - \cosh(2d)\log\left(\left(1+e^{bx}\right)\cosh\left(\frac{d}{2}\right) + \left(-1+e^{bx}\right)\sinh\left(\frac{d}{2}\right)\right) + \cosh(2d)\log\left(\left(-1+e^{bx}\right)\mathrm{c}\right.\right.}{}$$

input
```
Integrate[E^(2*(a + b*x))*Coth[d + b*x]*Csch[d + b*x],x]
```

output
```
(2*E^(2*a)*(E^(b*x)*Cosh[d] - Cosh[2*d]*Log[(1 + E^(b*x))*Cosh[d/2] + (-1
+ E^(b*x))*Sinh[d/2]] + Cosh[2*d]*Log[(-1 + E^(b*x))*Cosh[d/2] + (1 + E^(b
*x))*Sinh[d/2]] - E^(b*x)*Sinh[d] - (E^(b*x)*(Cosh[d] - Sinh[d])^2)/((-1 +
 E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d]) + Log[(1 + E^(b*x))*Cosh[d/
2] + (-1 + E^(b*x))*Sinh[d/2]]*Sinh[2*d] - Log[(-1 + E^(b*x))*Cosh[d/2] +
(1 + E^(b*x))*Sinh[d/2]]*Sinh[2*d]))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.58, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2720, 27, 360, 27, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\coth(bx+d)\text{csch}(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{2e^{2a+2bx}\left(1+e^{2bx}\right)}{\left(1-e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2e^{2a}\int \frac{e^{2bx}\left(1+e^{2bx}\right)}{\left(1-e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{360}$$

$$\frac{2e^{2a}\left(\frac{e^{bx}}{1-e^{2bx}}-\frac{1}{2}\int \frac{2\left(1+e^{2bx}\right)}{1-e^{2bx}}de^{bx}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2e^{2a}\left(\frac{e^{bx}}{1-e^{2bx}}-\int \frac{1+e^{2bx}}{1-e^{2bx}}de^{bx}\right)}{b}$$

$$\Big\downarrow \text{299}$$

$$\frac{2e^{2a}\left(-2\int \frac{1}{1-e^{2bx}}de^{bx}+e^{bx}+\frac{e^{bx}}{1-e^{2bx}}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{2e^{2a}\left(-2\text{arctanh}\left(e^{bx}\right)+e^{bx}+\frac{e^{bx}}{1-e^{2bx}}\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Coth[d + b*x]*Csch[d + b*x],x]
```

output
```
(2*E^(2*a)*(E^(b*x) + E^(b*x)/(1 - E^(2*b*x)) - 2*ArcTanh[E^(b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.52

method	result	size
risch	$\frac{2\,\mathrm{e}^{bx+2a-d}}{b}+\frac{2\,\mathrm{e}^{bx+4a-d}}{\left(-\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)b}+\frac{2\ln\left(\mathrm{e}^{bx+a}-\mathrm{e}^{a-d}\right)\mathrm{e}^{2a-2d}}{b}-\frac{2\ln\left(\mathrm{e}^{bx+a}+\mathrm{e}^{a-d}\right)\mathrm{e}^{2a-2d}}{b}$	111

input
```
int(exp(2*b*x+2*a)*coth(b*x+d)*csch(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
2*exp(b*x+2*a-d)/b+2/(-exp(2*b*x+2*a+2*d)+exp(2*a))/b*exp(b*x+4*a-d)+2*ln(
exp(b*x+a)-exp(a-d))/b*exp(2*a-2*d)-2*ln(exp(b*x+a)+exp(a-d))/b*exp(2*a-2*
d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. 2(66) = 132.

Time = 0.09 (sec) , antiderivative size = 497, normalized size of antiderivative = 6.81

$$\int e^{2(a+bx)}\coth(d+bx)\text{csch}(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)*csch(b*x+d),x, algorithm="fricas")
```

output
```
2*(cosh(b*x + d)^3*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*d)
)*sinh(b*x + d)^3 + 3*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh
(-2*a + 2*d))*sinh(b*x + d)^2 - 2*cosh(b*x + d)*cosh(-2*a + 2*d) - (cosh(b
*x + d)^2*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*
x + d)^2 + 2*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2
*d))*sinh(b*x + d) - (cosh(b*x + d)^2 - 1)*sinh(-2*a + 2*d) - cosh(-2*a +
2*d))*log(cosh(b*x + d) + sinh(b*x + d) + 1) + (cosh(b*x + d)^2*cosh(-2*a
+ 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^2 + 2*(cosh(b
*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d) -
 (cosh(b*x + d)^2 - 1)*sinh(-2*a + 2*d) - cosh(-2*a + 2*d))*log(cosh(b*x +
 d) + sinh(b*x + d) - 1) + (3*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (3*cosh(b
*x + d)^2 - 2)*sinh(-2*a + 2*d) - 2*cosh(-2*a + 2*d))*sinh(b*x + d) - (cos
h(b*x + d)^3 - 2*cosh(b*x + d))*sinh(-2*a + 2*d))/(b*cosh(b*x + d)^2 + 2*b
*cosh(b*x + d)*sinh(b*x + d) + b*sinh(b*x + d)^2 - b)
```

Sympy [F]

$$\int e^{2(a+bx)}\coth(d+bx)\text{csch}(d+bx)\,dx = e^{2a}\int e^{2bx}\coth\left(bx+d\right)\text{csch}\left(bx+d\right)dx$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)*csch(b*x+d),x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*coth(b*x + d)*csch(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.37

$$\int e^{2(a+bx)}\coth(d+bx)\text{csch}(d+bx)\,dx = -\frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}+1\right)}{b} + \frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}-1\right)}{b} - \frac{2\left(2\,e^{(-2\,bx-2\,d)}-1\right)e^{(2\,a-2\,d)}}{b\left(e^{(-bx-d)}-e^{(-3\,bx-3\,d)}\right)}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)*csch(b*x+d),x, algorithm="maxima")
```

output
```
-2*e^(2*a - 2*d)*log(e^(-b*x - d) + 1)/b + 2*e^(2*a - 2*d)*log(e^(-b*x - d
) - 1)/b - 2*(2*e^(-2*b*x - 2*d) - 1)*e^(2*a - 2*d)/(b*(e^(-b*x - d) - e^(
-3*b*x - 3*d)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12

$$\int e^{2(a+bx)}\coth(d+bx)\text{csch}(d+bx)\,dx$$
$$= -\frac{2\left(e^{(2a-2d)}\log\left(e^{(bx+d)}+1\right)-e^{(2a-2d)}\log\left(\left|e^{(bx+d)}-1\right|\right)+\frac{e^{(bx+2a-d)}}{e^{(2bx+2d)}-1}-e^{(bx+2a-d)}\right)}{b}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)*csch(b*x+d),x, algorithm="giac")
```

output
```
-2*(e^(2*a - 2*d)*log(e^(b*x + d) + 1) - e^(2*a - 2*d)*log(abs(e^(b*x + d)
 - 1)) + e^(b*x + 2*a - d)/(e^(2*b*x + 2*d) - 1) - e^(b*x + 2*a - d))/b
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.37

$$\int e^{2(a+bx)}\coth(d+bx)\text{csch}(d+bx)\,dx = \frac{2\,e^{2\,a-d+b\,x}}{b}-\frac{2\,e^{2\,a-d+b\,x}}{b\,\left(e^{2\,d+2\,b\,x}-1\right)}-\frac{4\,\sqrt{e^{4\,a-4\,d}}\,\text{atan}\left(\frac{e^{2\,a}\,e^{-d}\,e^{b\,x}\,\sqrt{-b^2}}{b\,\sqrt{e^{4\,a}\,e^{-4\,d}}}\right)}{\sqrt{-b^2}}$$

input
```
int((coth(d + b*x)*exp(2*a + 2*b*x))/sinh(d + b*x),x)
```

output
```
(2*exp(2*a - d + b*x))/b - (2*exp(2*a - d + b*x))/(b*(exp(2*d + 2*b*x) - 1
)) - (4*exp(4*a - 4*d)^(1/2)*atan((exp(2*a)*exp(-d)*exp(b*x)*(-b^2)^(1/2))
/(b*(exp(4*a)*exp(-4*d))^(1/2))))/(-b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.59

$$\int e^{2(a+bx)}\coth(d+bx)\mathrm{csch}(d+bx)\,dx$$
$$= \frac{2e^{2a}\left(e^{3bx+3d}+e^{2bx+2d}\log\left(e^{bx+d}-1\right)-e^{2bx+2d}\log\left(e^{bx+d}+1\right)-2e^{bx+d}-\log\left(e^{bx+d}-1\right)+\log\left(e^{bx+d}+1\right)\right)}{e^{2d}b\left(e^{2bx+2d}-1\right)}$$

input
```
int(exp(2*b*x+2*a)*coth(b*x+d)*csch(b*x+d),x)
```

output
```
(2*e**(2*a)*(e**(3*b*x + 3*d) + e**(2*b*x + 2*d)*log(e**(b*x + d) - 1) - e
**(2*b*x + 2*d)*log(e**(b*x + d) + 1) - 2*e**(b*x + d) - log(e**(b*x + d)
- 1) + log(e**(b*x + d) + 1)))/(e**(2*d)*b*(e**(2*b*x + 2*d) - 1))
```

3.49 $\int e^{2(a+bx)} \coth(d+bx)\mathbf{csch}^2(d+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 90

$$\int e^{2(a+bx)} \coth(d+bx)\text{csch}^2(d+bx)\,dx = -\frac{2e^{2a-2d}}{b(1-e^{2d+2bx})^2} + \frac{6e^{2a-2d}}{b(1-e^{2d+2bx})} + \frac{2e^{2a-2d}\log(1-e^{2d+2bx})}{b}$$

output
```
-2*exp(2*a-2*d)/b/(1-exp(2*b*x+2*d))^2+6*exp(2*a-2*d)/b/(1-exp(2*b*x+2*d))
+2*exp(2*a-2*d)*ln(1-exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int e^{2(a+bx)} \coth(d+bx)\text{csch}^2(d+bx)\,dx = \frac{2e^{2a-2d}\left(\frac{2-3e^{2(d+bx)}}{(-1+e^{2(d+bx)})^2} + \log\left(1-e^{2(d+bx)}\right)\right)}{b}$$

input
```
Integrate[E^(2*(a + b*x))*Coth[d + b*x]*Csch[d + b*x]^2,x]
```

output
```
(2*E^(2*a - 2*d)*((2 - 3*E^(2*(d + b*x)))/(-1 + E^(2*(d + b*x)))^2 + Log[1
 - E^(2*(d + b*x))]))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.53, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2720, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\coth(bx+d)\text{csch}^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{4e^{2a+3bx}(1+e^{2bx})}{(1-e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4e^{2a}\int \frac{e^{3bx}(1+e^{2bx})}{(1-e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{354}$$

$$-\frac{2e^{2a}\int \frac{e^{2bx}(1+e^{2bx})}{(1-e^{2bx})^3}de^{2bx}}{b}$$

$$\Big\downarrow \text{86}$$

$$-\frac{2e^{2a}\int \left(-\frac{3}{(-1+e^{2bx})^2}-\frac{2}{(-1+e^{2bx})^3}+\frac{1}{1-e^{2bx}}\right)de^{2bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2e^{2a}\left(-\frac{3}{1-e^{2bx}}+\frac{1}{(e^{2bx}-1)^2}-\log\left(1-e^{2bx}\right)\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Coth[d + b*x]*Csch[d + b*x]^2,x]
```

output
```
(-2*E^(2*a)*(-3/(1 - E^(2*b*x)) + (-1 + E^(2*b*x))^(-2) - Log[1 - E^(2*b*x
)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17

method	result	size
risch	$-\frac{4\,{\mathrm e}^{2 a -2 d} a}{b}+\frac{2 \left(-3\,{\mathrm e}^{2 b x +2 a +2 d}+2\,{\mathrm e}^{2 a}\right) {\mathrm e}^{4 a -2 d}}{\left(-{\mathrm e}^{2 b x +2 a +2 d}+{\mathrm e}^{2 a}\right)^{2} b}+\frac{2 \ln \left({\mathrm e}^{2 b x +2 a}-{\mathrm e}^{2 a -2 d}\right) {\mathrm e}^{2 a -2 d}}{b}$	105

input
```
int(exp(2*b*x+2*a)*coth(b*x+d)*csch(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-4/b*exp(2*a-2*d)*a+2/(-exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(-3*exp(2*b*x+2*a
+2*d)+2*exp(2*a))*exp(4*a-2*d)+2*ln(exp(2*b*x+2*a)-exp(2*a-2*d))/b*exp(2*a
-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(80) = 160$.

Time = 0.09 (sec) , antiderivative size = 529, normalized size of antiderivative = 5.88

$$\int e^{2(a+bx)} \coth(d+bx)\text{csch}^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)*csch(b*x+d)^2,x, algorithm="fricas")
```

output
```
-2*(3*cosh(b*x + d)^2*cosh(-2*a + 2*d) + 3*(cosh(-2*a + 2*d) - sinh(-2*a +
 2*d))*sinh(b*x + d)^2 - (cosh(b*x + d)^4*cosh(-2*a + 2*d) + (cosh(-2*a +
2*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-2*a + 2*
d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^3 - 2*cosh(b*x + d)^2*c
osh(-2*a + 2*d) + 2*(3*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (3*cosh(b*x + d)
^2 - 1)*sinh(-2*a + 2*d) - cosh(-2*a + 2*d))*sinh(b*x + d)^2 + 4*(cosh(b*x
 + d)^3*cosh(-2*a + 2*d) - cosh(b*x + d)*cosh(-2*a + 2*d) - (cosh(b*x + d)
^3 - cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^4 - 2
*cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*log(2*sinh(b*x
+ d)/(cosh(b*x + d) - sinh(b*x + d))) + 6*(cosh(b*x + d)*cosh(-2*a + 2*d)
- cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d) - (3*cosh(b*x + d)^2 - 2)*
sinh(-2*a + 2*d) - 2*cosh(-2*a + 2*d))/(b*cosh(b*x + d)^4 + 4*b*cosh(b*x +
 d)*sinh(b*x + d)^3 + b*sinh(b*x + d)^4 - 2*b*cosh(b*x + d)^2 + 2*(3*b*cos
h(b*x + d)^2 - b)*sinh(b*x + d)^2 + 4*(b*cosh(b*x + d)^3 - b*cosh(b*x + d)
)*sinh(b*x + d) + b)
```

Sympy [F]

$$\int e^{2(a+bx)}\coth(d+bx)\text{csch}^2(d+bx)\,dx = e^{2a}\int e^{2bx}\coth(bx+d)\,\text{csch}^2(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)*csch(b*x+d)**2,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*coth(b*x + d)*csch(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.40

$$\int e^{2(a+bx)}\coth(d+bx)\text{csch}^2(d+bx)\,dx = 4\,xe^{(2\,a-2\,d)} + \frac{4\,de^{(2\,a-2\,d)}}{b} + \frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}+1\right)}{b} + \frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}-1\right)}{b} - \frac{2\left(e^{(-2\,bx-2\,d)}-2\right)e^{(2\,a-2\,d)}}{b(2\,e^{(-2\,bx-2\,d)}-e^{(-4\,bx-4\,d)}-1)}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)*csch(b*x+d)^2,x, algorithm="maxima")
```

output
```
4*x*e^(2*a - 2*d) + 4*d*e^(2*a - 2*d)/b + 2*e^(2*a - 2*d)*log(e^(-b*x - d)
 + 1)/b + 2*e^(2*a - 2*d)*log(e^(-b*x - d) - 1)/b - 2*(e^(-2*b*x - 2*d) -
2)*e^(2*a - 2*d)/(b*(2*e^(-2*b*x - 2*d) - e^(-4*b*x - 4*d) - 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int e^{2(a+bx)}\coth(d+bx)\text{csch}^2(d+bx)\,dx = \frac{2\,e^{(2\,a-2\,d)}\log\left(\left|e^{(2\,bx+2\,d)}-1\right|\right)-\frac{\left(3\,e^{(4\,bx+2\,a+4\,d)}-e^{(2\,a)}\right)e^{(-2\,d)}}{\left(e^{(2\,bx+2\,d)}-1\right)^2}}{b}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)*csch(b*x+d)^2,x, algorithm="giac")
```

output
```
(2*e^(2*a - 2*d)*log(abs(e^(2*b*x + 2*d) - 1)) - (3*e^(4*b*x + 2*a + 4*d)
- e^(2*a))*e^(-2*d)/(e^(2*b*x + 2*d) - 1)^2)/b
```

Mupad [B] (verification not implemented)

Time = 2.98 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int e^{2(a+bx)}\coth(d+bx)\text{csch}^2(d+bx)\,dx = \frac{2\,e^{2\,a-2\,d}\,\ln\left(e^{2\,d}\,e^{2\,b\,x}-1\right)}{b} - \frac{2\,e^{2\,a-2\,d}}{b\,\left(e^{4\,d+4\,b\,x}-2\,e^{2\,d+2\,b\,x}+1\right)} - \frac{6\,e^{2\,a-2\,d}}{b\,\left(e^{2\,d+2\,b\,x}-1\right)}$$

input
```
int((coth(d + b*x)*exp(2*a + 2*b*x))/sinh(d + b*x)^2,x)
```

output
```
(2*exp(2*a - 2*d)*log(exp(2*d)*exp(2*b*x) - 1))/b - (2*exp(2*a - 2*d))/(b*
(exp(4*d + 4*b*x) - 2*exp(2*d + 2*b*x) + 1)) - (6*exp(2*a - 2*d))/(b*(exp(
2*d + 2*b*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.87

$$\int e^{2(a+bx)}\coth(d+bx)\text{csch}^2(d+bx)\,dx$$
$$= \frac{e^{2a}\left(2e^{4bx+4d}\log\left(e^{bx+d}-1\right)+2e^{4bx+4d}\log\left(e^{bx+d}+1\right)-3e^{4bx+4d}-4e^{2bx+2d}\log\left(e^{bx+d}-1\right)-4e^{2bx+2d}\log\left(e\right.\right.}{e^{2d}b\left(e^{4bx+4d}-2e^{2bx+2d}+1\right)}$$

input
```
int(exp(2*b*x+2*a)*coth(b*x+d)*csch(b*x+d)^2,x)
```

output
```
(e**(2*a)*(2*e**(4*b*x + 4*d)*log(e**(b*x + d) - 1) + 2*e**(4*b*x + 4*d)*l
og(e**(b*x + d) + 1) - 3*e**(4*b*x + 4*d) - 4*e**(2*b*x + 2*d)*log(e**(b*x
 + d) - 1) - 4*e**(2*b*x + 2*d)*log(e**(b*x + d) + 1) + 2*log(e**(b*x + d)
 - 1) + 2*log(e**(b*x + d) + 1) + 1))/(e**(2*d)*b*(e**(4*b*x + 4*d) - 2*e*
*(2*b*x + 2*d) + 1))
```

3.50 $\int e^{2(a+bx)}\cosh^2(d+bx)\sinh^3(d+bx)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 118

$$\int e^{2(a+bx)}\cosh^2(d+bx)\sinh^3(d+bx)\,dx=\frac{e^{2a-5d-3bx}}{96b}-\frac{e^{2a-3d-bx}}{32b}+\frac{e^{2a-d+bx}}{16b}-\frac{e^{2a+d+3bx}}{48b}-\frac{e^{2a+3d+5bx}}{160b}+\frac{e^{2a+5d+7bx}}{224b}$$

output
```
1/96*exp(-3*b*x+2*a-5*d)/b-1/32*exp(-b*x+2*a-3*d)/b+1/16*exp(b*x+2*a-d)/b-
1/48*exp(3*b*x+2*a+d)/b-1/160*exp(5*b*x+2*a+3*d)/b+1/224*exp(7*b*x+2*a+5*d
)/b
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.99

$$\int e^{2(a+bx)}\cosh^2(d+bx)\sinh^3(d+bx)\,dx$$
$$=\frac{e^{2a-3bx}\left(-70e^{4bx}\left(\left(-3+e^{2bx}\right)\cosh(d)+\left(3+e^{2bx}\right)\sinh(d)\right)-21e^{2bx}\left(\left(5+e^{6bx}\right)\cosh(3d)+\left(-5+e^{6bx}\right)\mathrm{s}\right.}{3360b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[d + b*x]^2*Sinh[d + b*x]^3,x]
```

output
```
(E^(2*a - 3*b*x)*(-70*E^(4*b*x)*((-3 + E^(2*b*x))*Cosh[d] + (3 + E^(2*b*x)
)*Sinh[d]) - 21*E^(2*b*x)*((5 + E^(6*b*x))*Cosh[3*d] + (-5 + E^(6*b*x))*Si
nh[3*d]) + 5*((7 + 3*E^(10*b*x))*Cosh[5*d] + (-7 + 3*E^(10*b*x))*Sinh[5*d]
)))/(3360*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.56, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2720, 27, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\sinh^3(bx+d)\cosh^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{1}{32}e^{2a-4bx}\left(1-e^{2bx}\right)^3\left(1+e^{2bx}\right)^2de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^{2a}\int e^{-4bx}\left(1-e^{2bx}\right)^3\left(1+e^{2bx}\right)^2de^{bx}}{32b}$$

$$\Big\downarrow \text{355}$$

$$-\frac{e^{2a}\int \left(-2+e^{-4bx}-e^{-2bx}+2e^{2bx}+e^{4bx}-e^{6bx}\right)de^{bx}}{32b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{2a}\left(-\frac{1}{3}e^{-3bx}+e^{-bx}-2e^{bx}+\frac{2}{3}e^{3bx}+\frac{1}{5}e^{5bx}-\frac{1}{7}e^{7bx}\right)}{32b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[d + b*x]^2*Sinh[d + b*x]^3,x]
```

output
```
-1/32*(E^(2*a)*(-1/3*1/E^(3*b*x) + E^(-(b*x)) - 2*E^(b*x) + (2*E^(3*b*x))/
3 + E^(5*b*x)/5 - E^(7*b*x)/7))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.69

$$\frac{\sinh\left(-3bx+2a-5d\right)}{96b}-\frac{\sinh\left(-bx+2a-3d\right)}{32b}+\frac{\sinh\left(bx+2a-d\right)}{16b}-\frac{\sinh\left(3bx+2a+d\right)}{48b}-\frac{\sinh\left(5bx+2a\right.}{160b}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^2*sinh(b*x+d)^3,x)
```

output
```
1/96/b*sinh(-3*b*x+2*a-5*d)-1/32/b*sinh(-b*x+2*a-3*d)+1/16/b*sinh(b*x+2*a-
d)-1/48/b*sinh(3*b*x+2*a+d)-1/160/b*sinh(5*b*x+2*a+3*d)+1/224/b*sinh(7*b*x
+2*a+5*d)+1/96*cosh(-3*b*x+2*a-5*d)/b-1/32*cosh(-b*x+2*a-3*d)/b+1/16*cosh(
b*x+2*a-d)/b-1/48*cosh(3*b*x+2*a+d)/b-1/160*cosh(5*b*x+2*a+3*d)/b+1/224*co
sh(7*b*x+2*a+5*d)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(100) = 200.

Time = 0.09 (sec) , antiderivative size = 424, normalized size of antiderivative = 3.59

$$\int e^{2(a+bx)}\cosh^2(d+bx)\sinh^3(d+bx)\,dx = \frac{25\cosh(bx+d)^5\cosh(-2a+2d)-10(\cosh(-2a+2d)-\sinh(-2a+2d))\sinh(bx+d)^5+125(\mathrm{co}}{}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^2*sinh(b*x+d)^3,x, algorithm="fricas"
)
```

output
```
1/1680*(25*cosh(b*x + d)^5*cosh(-2*a + 2*d) - 10*(cosh(-2*a + 2*d) - sinh(
-2*a + 2*d))*sinh(b*x + d)^5 + 125*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(
b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^4 - 63*cosh(b*x + d)^3*cosh(-2*a
+ 2*d) - 2*(50*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (50*cosh(b*x + d)^2 - 21
)*sinh(-2*a + 2*d) - 21*cosh(-2*a + 2*d))*sinh(b*x + d)^3 + (250*cosh(b*x
+ d)^3*cosh(-2*a + 2*d) - 189*cosh(b*x + d)*cosh(-2*a + 2*d) - (250*cosh(b
*x + d)^3 - 189*cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d)^2 + 70*cosh
(b*x + d)*cosh(-2*a + 2*d) - 2*(25*cosh(b*x + d)^4*cosh(-2*a + 2*d) - 63*c
osh(b*x + d)^2*cosh(-2*a + 2*d) - (25*cosh(b*x + d)^4 - 63*cosh(b*x + d)^2
 + 70)*sinh(-2*a + 2*d) + 70*cosh(-2*a + 2*d))*sinh(b*x + d) - (25*cosh(b*
x + d)^5 - 63*cosh(b*x + d)^3 + 70*cosh(b*x + d))*sinh(-2*a + 2*d))/(b*cos
h(b*x + d)^2 - 2*b*cosh(b*x + d)*sinh(b*x + d) + b*sinh(b*x + d)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(94) = 188.

Time = 5.66 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.67

$$\int e^{2(a+bx)}\cosh^2(d+bx)\sinh^3(d+bx)\,dx = \begin{cases} -\frac{4e^{2a}e^{2bx}\sinh^5(bx+d)}{35b}+\frac{8e^{2a}e^{2bx}\sinh^4(bx+d)\cosh(bx+d)}{35b}+\frac{2e^{2a}e^{2bx}\sinh^3(bx+d)\cosh^2(bx+d)}{35b}-\frac{e^{2a}e^{2bx}\sinh^2(bx+d)\cosh^3(bx}{105b} \\ xe^{2a}\sinh^3(d)\cosh^2(d) \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)**2*sinh(b*x+d)**3,x)
```

output
```
Piecewise((-4*exp(2*a)*exp(2*b*x)*sinh(b*x + d)**5/(35*b) + 8*exp(2*a)*exp
(2*b*x)*sinh(b*x + d)**4*cosh(b*x + d)/(35*b) + 2*exp(2*a)*exp(2*b*x)*sinh
(b*x + d)**3*cosh(b*x + d)**2/(35*b) - exp(2*a)*exp(2*b*x)*sinh(b*x + d)**
2*cosh(b*x + d)**3/(105*b) - 4*exp(2*a)*exp(2*b*x)*sinh(b*x + d)*cosh(b*x
+ d)**4/(105*b) + 2*exp(2*a)*exp(2*b*x)*cosh(b*x + d)**5/(105*b), Ne(b, 0)
), (x*exp(2*a)*sinh(d)**3*cosh(d)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int e^{2(a+bx)}\cosh^2(d+bx)\sinh^3(d+bx)\,dx$$
$$= -\frac{\left(21\,e^{(-2\,bx-2\,d)}+70\,e^{(-4\,bx-4\,d)}-210\,e^{(-6\,bx-6\,d)}-15\right)e^{(7\,bx+2\,a+5\,d)}}{3360\,b} - \frac{\left(3\,e^{(-bx-d)}-e^{(-3\,bx-3\,d)}\right)e^{(2\,a-2\,d)}}{96\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^2*sinh(b*x+d)^3,x, algorithm="maxima"
)
```

output
```
-1/3360*(21*e^(-2*b*x - 2*d) + 70*e^(-4*b*x - 4*d) - 210*e^(-6*b*x - 6*d)
- 15)*e^(7*b*x + 2*a + 5*d)/b - 1/96*(3*e^(-b*x - d) - e^(-3*b*x - 3*d))*e
^(2*a - 2*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)}\cosh^2(d+bx)\sinh^3(d+bx)\,dx =$$
$$-\frac{\left(35\left(3\,e^{(2\,bx+2\,a+2\,d)}-e^{(2\,a)}\right)e^{(-3\,bx-3\,d)}-15\,e^{(7\,bx+2\,a+7\,d)}+21\,e^{(5\,bx+2\,a+5\,d)}+70\,e^{(3\,bx+2\,a+3\,d)}-210\,e^{(bx}}{3360\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^2*sinh(b*x+d)^3,x, algorithm="giac")
```

output
```
-1/3360*(35*(3*e^(2*b*x + 2*a + 2*d) - e^(2*a))*e^(-3*b*x - 3*d) - 15*e^(7
*b*x + 2*a + 7*d) + 21*e^(5*b*x + 2*a + 5*d) + 70*e^(3*b*x + 2*a + 3*d) -
210*e^(b*x + 2*a + d))*e^(-2*d)/b
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.85

$$\int e^{2(a+bx)}\cosh^2(d+bx)\sinh^3(d+bx)\,dx = \frac{e^{2a-d+bx}}{16b}-\frac{e^{2a+d+3bx}}{48b}-\frac{e^{2a-3d-bx}}{32b}-\frac{e^{2a+3d+5bx}}{160b}+\frac{e^{2a-5d-3bx}}{96b}+\frac{e^{2a+5d+7bx}}{224b}$$

input
```
int(cosh(d + b*x)^2*exp(2*a + 2*b*x)*sinh(d + b*x)^3,x)
```

output
```
exp(2*a - d + b*x)/(16*b) - exp(2*a + d + 3*b*x)/(48*b) - exp(2*a - 3*d -
b*x)/(32*b) - exp(2*a + 3*d + 5*b*x)/(160*b) + exp(2*a - 5*d - 3*b*x)/(96*
b) + exp(2*a + 5*d + 7*b*x)/(224*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.71

$$\int e^{2(a+bx)}\cosh^2(d+bx)\sinh^3(d+bx)\,dx = \frac{e^{2a}\left(15e^{10bx+10d}-21e^{8bx+8d}-70e^{6bx+6d}+210e^{4bx+4d}-105e^{2bx+2d}+35\right)}{3360e^{3bx+5d}b}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^2*sinh(b*x+d)^3,x)
```

output
```
(e**(2*a)*(15*e**(10*b*x + 10*d) - 21*e**(8*b*x + 8*d) - 70*e**(6*b*x + 6*
d) + 210*e**(4*b*x + 4*d) - 105*e**(2*b*x + 2*d) + 35))/(3360*e**(3*b*x +
5*d)*b)
```

3.51 $\int e^{2(a+bx)} \cosh^2(d+bx) \sinh^2(d+bx)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 60

$$\int e^{2(a+bx)} \cosh^2(d+bx) \sinh^2(d+bx)\,dx = -\frac{e^{2(a-2d)-2bx}}{32b} - \frac{e^{2a+2bx}}{16b} + \frac{e^{2(a+2d)+6bx}}{96b}$$

output
```
-1/32*exp(-2*b*x+2*a-4*d)/b-1/16*exp(2*b*x+2*a)/b+1/96*exp(6*b*x+2*a+4*d)/
b
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int e^{2(a+bx)} \cosh^2(d+bx) \sinh^2(d+bx)\,dx = \frac{e^{2a-4d-2bx}\left(-3-6e^{4(d+bx)}+e^{8(d+bx)}\right)}{96b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[d + b*x]^2*Sinh[d + b*x]^2,x]
```

output
```
(E^(2*a - 4*d - 2*b*x)*(-3 - 6*E^(4*(d + b*x)) + E^(8*(d + b*x))))/(96*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.68, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\sinh^2(bx+d)\cosh^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{16}e^{2a-3bx}\left(1-e^{4bx}\right)^2de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{2a}\int e^{-3bx}\left(1-e^{4bx}\right)^2de^{bx}}{16b}$$

$$\Big\downarrow \text{802}$$

$$\frac{e^{2a}\int \left(e^{-3bx}-2e^{bx}+e^{5bx}\right)de^{bx}}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2a}\left(-\frac{1}{2}e^{-2bx}-e^{2bx}+\frac{1}{6}e^{6bx}\right)}{16b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[d + b*x]^2*Sinh[d + b*x]^2,x]
```

output
```
(E^(2*a)*(-1/2*1/E^(2*b*x) - E^(2*b*x) + E^(6*b*x)/6))/(16*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 151.75 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{e^{-2bx+2a-4d}}{32b}-\frac{e^{2bx+2a}}{16b}+\frac{e^{6bx+2a+4d}}{96b}$
default	$-\frac{\sinh(2bx+2a)}{16b}-\frac{\sinh(-2bx+2a-4d)}{32b}+\frac{\sinh(6bx+2a+4d)}{96b}-\frac{\cosh(2bx+2a)}{16b}-\frac{\cosh(-2bx+2a-4d)}{32b}+\frac{\cosh(6bx+2a+4d)}{96b}$
orering	$\frac{e^{2bx+2a}\cosh(bx+d)^2\sinh(bx+d)^2}{6b}+\frac{2\,e^{2bx+2a}\cosh(bx+d)^2\sinh(bx+d)^2 b+2\,e^{2bx+2a} b\cosh(bx+d)\sinh(bx+d)^3+2\,e^{2bx+2a}\cosh(}{4b^2}$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^2*sinh(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/32*exp(-2*b*x+2*a-4*d)/b-1/16*exp(2*b*x+2*a)/b+1/96*exp(6*b*x+2*a+4*d)/
b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(49) = 98$.

Time = 0.08 (sec) , antiderivative size = 248, normalized size of antiderivative = 4.13

$$\int e^{2(a+bx)}\cosh^2(d+bx)\sinh^2(d+bx)\,dx = -\frac{\cosh(bx+d)^4\cosh(-2a+2d)+(\cosh(-2a+2d)-\sinh(-2a+2d))\sinh(bx+d)^4-8(\cosh(bx}{}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^2*sinh(b*x+d)^2,x, algorithm="fricas"
)
```

output
```
-1/48*(cosh(b*x + d)^4*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a +
2*d))*sinh(b*x + d)^4 - 8*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*
sinh(-2*a + 2*d))*sinh(b*x + d)^3 + 6*(cosh(b*x + d)^2*cosh(-2*a + 2*d) -
cosh(b*x + d)^2*sinh(-2*a + 2*d))*sinh(b*x + d)^2 - 8*(cosh(b*x + d)^3*cos
h(-2*a + 2*d) - cosh(b*x + d)^3*sinh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*
x + d)^4 + 3)*sinh(-2*a + 2*d) + 3*cosh(-2*a + 2*d))/(b*cosh(b*x + d)^2 -
2*b*cosh(b*x + d)*sinh(b*x + d) + b*sinh(b*x + d)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(46) = 92$.

Time = 2.48 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.13

$$\int e^{2(a+bx)}\cosh^2(d+bx)\sinh^2(d+bx)\,dx = \begin{cases} -\frac{5e^{2a}e^{2bx}\sinh^4(bx+d)}{48b}+\frac{5e^{2a}e^{2bx}\sinh^3(bx+d)\cosh(bx+d)}{24b}+\frac{e^{2a}e^{2bx}\sinh(bx+d)\cosh^3(bx+d)}{8b}-\frac{e^{2a}e^{2bx}\cosh^4(bx+d)}{16b} & \text{for } b \neq \\ xe^{2a}\sinh^2(d)\cosh^2(d) & \text{other} \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)**2*sinh(b*x+d)**2,x)
```

output
```
Piecewise((-5*exp(2*a)*exp(2*b*x)*sinh(b*x + d)**4/(48*b) + 5*exp(2*a)*exp
(2*b*x)*sinh(b*x + d)**3*cosh(b*x + d)/(24*b) + exp(2*a)*exp(2*b*x)*sinh(b
*x + d)*cosh(b*x + d)**3/(8*b) - exp(2*a)*exp(2*b*x)*cosh(b*x + d)**4/(16*
b), Ne(b, 0)), (x*exp(2*a)*sinh(d)**2*cosh(d)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int e^{2(a+bx)}\cosh^2(d+bx)\sinh^2(d+bx)\,dx = -\frac{\left(6\,e^{(-4\,bx-4\,d)}-1\right)e^{(6\,bx+2\,a+4\,d)}}{96\,b} - \frac{e^{(-2\,bx+2\,a-4\,d)}}{32\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^2*sinh(b*x+d)^2,x, algorithm="maxima"
)
```

output
```
-1/96*(6*e^(-4*b*x - 4*d) - 1)*e^(6*b*x + 2*a + 4*d)/b - 1/32*e^(-2*b*x +
2*a - 4*d)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int e^{2(a+bx)}\cosh^2(d+bx)\sinh^2(d+bx)\,dx = \frac{\left(e^{(6\,bx+2\,a+6\,d)}-6\,e^{(2\,bx+2\,a+2\,d)}-3\,e^{(-2\,bx+2\,a-2\,d)}\right)e^{(-2\,d)}}{96\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^2*sinh(b*x+d)^2,x, algorithm="giac")
```

output
```
1/96*(e^(6*b*x + 2*a + 6*d) - 6*e^(2*b*x + 2*a + 2*d) - 3*e^(-2*b*x + 2*a
- 2*d))*e^(-2*d)/b
```

Mupad [B] (verification not implemented)

Time = 3.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.68

$$\int e^{2(a+bx)}\cosh^2(d+bx)\sinh^2(d+bx)\,dx = \frac{e^{2a-4d-2bx}\left(e^{2d+2bx}+1\right)^3\left(e^{2d+2bx}-3\right)}{96\,b}$$

input
```
int(cosh(d + b*x)^2*exp(2*a + 2*b*x)*sinh(d + b*x)^2,x)
```

output
```
(exp(2*a - 4*d - 2*b*x)*(exp(2*d + 2*b*x) + 1)^3*(exp(2*d + 2*b*x) - 3))/(
96*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int e^{2(a+bx)}\cosh^2(d+bx)\sinh^2(d+bx)\,dx = \frac{e^{2a}\left(e^{8bx+8d}-6e^{4bx+4d}-3\right)}{96e^{2bx+4d}b}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^2*sinh(b*x+d)^2,x)
```

output
```
(e**(2*a)*(e**(8*b*x + 8*d) - 6*e**(4*b*x + 4*d) - 3))/(96*e**(2*b*x + 4*d
)*b)
```

3.52 $\int e^{2(a+bx)}\cosh^2(d+bx)\sinh(d+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 78

$$\int e^{2(a+bx)}\cosh^2(d+bx)\sinh(d+bx)\,dx = \frac{e^{2a-3d-bx}}{8b} - \frac{e^{2a-d+bx}}{8b} + \frac{e^{2a+d+3bx}}{24b} + \frac{e^{2a+3d+5bx}}{40b}$$

output
```
1/8*exp(-b*x+2*a-3*d)/b-1/8*exp(b*x+2*a-d)/b+1/24*exp(3*b*x+2*a+d)/b+1/40*
exp(5*b*x+2*a+3*d)/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int e^{2(a+bx)}\cosh^2(d+bx)\sinh(d+bx)\,dx = \frac{e^{2a-bx}\left(5e^{2bx}\left(\left(-3+e^{2bx}\right)\cosh(d)+\left(3+e^{2bx}\right)\sinh(d)\right)+3\left(\left(5+e^{6bx}\right)\cosh(3d)+\left(-5+e^{6bx}\right)\sinh(3d)\right)}{120b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[d + b*x]^2*Sinh[d + b*x],x]
```

output
```
(E^(2*a - b*x)*(5*E^(2*b*x)*((-3 + E^(2*b*x))*Cosh[d] + (3 + E^(2*b*x))*Si
nh[d]) + 3*((5 + E^(6*b*x))*Cosh[3*d] + (-5 + E^(6*b*x))*Sinh[3*d])))/(120
*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.59, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2720, 27, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)} \sinh(bx+d) \cosh^2(bx+d) \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int -\frac{1}{8} e^{2a-2bx} \left(1-e^{2bx}\right) \left(1+e^{2bx}\right)^2 de^{bx}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{e^{2a} \int e^{-2bx} \left(1-e^{2bx}\right) \left(1+e^{2bx}\right)^2 de^{bx}}{8b}$$

$$\Big\downarrow \text{ 355}$$

$$-\frac{e^{2a} \int \left(1+e^{-2bx}-e^{2bx}-e^{4bx}\right) de^{bx}}{8b}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{e^{2a} \left(-e^{-bx}+e^{bx}-\frac{1}{3} e^{3bx}-\frac{1}{5} e^{5bx}\right)}{8b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[d + b*x]^2*Sinh[d + b*x],x]
```

output
```
-1/8*(E^(2*a)*(-E^(-(b*x)) + E^(b*x) - E^(3*b*x)/3 - E^(5*b*x)/5))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 10.99 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

method	result
risch	$\frac{e^{-bx+2a-3d}}{8b}-\frac{e^{bx+2a-d}}{8b}+\frac{e^{3bx+2a+d}}{24b}+\frac{e^{5bx+2a+3d}}{40b}$
default	$\frac{\sinh(-bx+2a-3d)}{8b}-\frac{\sinh(bx+2a-d)}{8b}+\frac{\sinh(3bx+2a+d)}{24b}+\frac{\sinh(5bx+2a+3d)}{40b}+\frac{\cosh(-bx+2a-3d)}{8b}-\frac{\cosh(bx+2a-d)}{8b}+$
orering	$\frac{8\,e^{2bx+2a}\cosh(bx+d)^2\sinh(bx+d)}{15b}+\frac{\frac{28\,e^{2bx+2a}\cosh(bx+d)^2\sinh(bx+d)b}{15}+\frac{28\,e^{2bx+2a}b\cosh(bx+d)\sinh(bx+d)^2}{15}+\frac{14\,e^{2bx+2a}\cosh(bx+}{15}}{b^2}$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^2*sinh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/8*exp(-b*x+2*a-3*d)/b-1/8*exp(b*x+2*a-d)/b+1/24*exp(3*b*x+2*a+d)/b+1/40*
exp(5*b*x+2*a+3*d)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(66) = 132$.

Time = 0.09 (sec) , antiderivative size = 234, normalized size of antiderivative = 3.00

$$\int e^{2(a+bx)}\cosh^2(d+bx)\sinh(d+bx)\,dx$$
$$= \frac{9\,\cosh\left(bx+d\right)^{3}\cosh\left(-2\,a+2\,d\right)-6\left(\cosh\left(-2\,a+2\,d\right)-\sinh\left(-2\,a+2\,d\right)\right)\sinh\left(bx+d\right)^{3}+27\left(\cosh\left(\right.\right.}{}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^2*sinh(b*x+d),x, algorithm="fricas")
```

output
```
1/60*(9*cosh(b*x + d)^3*cosh(-2*a + 2*d) - 6*(cosh(-2*a + 2*d) - sinh(-2*a
 + 2*d))*sinh(b*x + d)^3 + 27*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x +
 d)*sinh(-2*a + 2*d))*sinh(b*x + d)^2 - 5*cosh(b*x + d)*cosh(-2*a + 2*d) -
 2*(9*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (9*cosh(b*x + d)^2 - 5)*sinh(-2*a
 + 2*d) - 5*cosh(-2*a + 2*d))*sinh(b*x + d) - (9*cosh(b*x + d)^3 - 5*cosh(
b*x + d))*sinh(-2*a + 2*d))/(b*cosh(b*x + d)^2 - 2*b*cosh(b*x + d)*sinh(b*
x + d) + b*sinh(b*x + d)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(60) = 120$.

Time = 0.98 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.64

$$\int e^{2(a+bx)}\cosh^2(d+bx)\sinh(d+bx)\,dx$$
$$= \begin{cases} -\frac{4e^{2a}e^{2bx}\sinh^3\left(bx+d\right)}{15b}+\frac{8e^{2a}e^{2bx}\sinh^2\left(bx+d\right)\cosh\left(bx+d\right)}{15b}-\frac{2e^{2a}e^{2bx}\sinh\left(bx+d\right)\cosh^2\left(bx+d\right)}{15b}+\frac{e^{2a}e^{2bx}\cosh^3\left(bx+d\right)}{15b} & \text{for } b \\ xe^{2a}\sinh\left(d\right)\cosh^2\left(d\right) & \text{other} \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)**2*sinh(b*x+d),x)
```

output
```
Piecewise((-4*exp(2*a)*exp(2*b*x)*sinh(b*x + d)**3/(15*b) + 8*exp(2*a)*exp
(2*b*x)*sinh(b*x + d)**2*cosh(b*x + d)/(15*b) - 2*exp(2*a)*exp(2*b*x)*sinh
(b*x + d)*cosh(b*x + d)**2/(15*b) + exp(2*a)*exp(2*b*x)*cosh(b*x + d)**3/(
15*b), Ne(b, 0)), (x*exp(2*a)*sinh(d)*cosh(d)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.76

$$\int e^{2(a+bx)}\cosh^2(d+bx)\sinh(d+bx)\,dx = \frac{\left(5\,e^{(-2\,bx-2\,d)}-15\,e^{(-4\,bx-4\,d)}+3\right)e^{(5\,bx+2\,a+3\,d)}}{120\,b} + \frac{e^{(-bx+2\,a-3\,d)}}{8\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^2*sinh(b*x+d),x, algorithm="maxima")
```

output
```
1/120*(5*e^(-2*b*x - 2*d) - 15*e^(-4*b*x - 4*d) + 3)*e^(5*b*x + 2*a + 3*d)
/b + 1/8*e^(-b*x + 2*a - 3*d)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.81

$$\int e^{2(a+bx)}\cosh^2(d+bx)\sinh(d+bx)\,dx = \frac{\left(3\,e^{(5\,bx+2\,a+5\,d)}+5\,e^{(3\,bx+2\,a+3\,d)}-15\,e^{(bx+2\,a+d)}+15\,e^{(-bx+2\,a-d)}\right)e^{(-2\,d)}}{120\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^2*sinh(b*x+d),x, algorithm="giac")
```

output
```
1/120*(3*e^(5*b*x + 2*a + 5*d) + 5*e^(3*b*x + 2*a + 3*d) - 15*e^(b*x + 2*a
 + d) + 15*e^(-b*x + 2*a - d))*e^(-2*d)/b
```

Mupad [B] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int e^{2(a+bx)}\cosh^2(d+bx)\sinh(d+bx)\,dx = \frac{e^{2\,a+d+3\,b\,x}}{24\,b}-\frac{e^{2\,a-d+b\,x}}{8\,b}+\frac{e^{2\,a-3\,d-b\,x}}{8\,b}+\frac{e^{2\,a+3\,d+5\,b\,x}}{40\,b}$$

input
```
int(cosh(d + b*x)^2*exp(2*a + 2*b*x)*sinh(d + b*x),x)
```

output
```
exp(2*a + d + 3*b*x)/(24*b) - exp(2*a - d + b*x)/(8*b) + exp(2*a - 3*d - b
*x)/(8*b) + exp(2*a + 3*d + 5*b*x)/(40*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.76

$$\int e^{2(a+bx)}\cosh^2(d+bx)\sinh(d+bx)\,dx = \frac{e^{2a}\left(3e^{6bx+6d}+5e^{4bx+4d}-15e^{2bx+2d}+15\right)}{120e^{bx+3d}b}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^2*sinh(b*x+d),x)
```

output
```
(e**(2*a)*(3*e**(6*b*x + 6*d) + 5*e**(4*b*x + 4*d) - 15*e**(2*b*x + 2*d) +
 15))/(120*e**(b*x + 3*d)*b)
```

3.53 $\int e^{2(a+bx)}\cosh^2(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 51

$$\int e^{2(a+bx)}\cosh^2(d+bx)\,dx=\frac{e^{2a+2bx}}{4b}+\frac{e^{2(a+d)+4bx}}{16b}+\frac{1}{4}e^{2a-2d}x$$

output
```
1/4*exp(2*b*x+2*a)/b+1/16*exp(4*b*x+2*a+2*d)/b+1/4*exp(2*a-2*d)*x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int e^{2(a+bx)}\cosh^2(d+bx)\,dx=\frac{e^{2a}\left(4e^{2bx}+\left(e^{4bx}+4bx\right)\cosh(2d)+\left(e^{4bx}-4bx\right)\sinh(2d)\right)}{16b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[d + b*x]^2,x]
```

output
```
(E^(2*a)*(4*E^(2*b*x) + (E^(4*b*x) + 4*b*x)*Cosh[2*d] + (E^(4*b*x) - 4*b*x
)*Sinh[2*d]))/(16*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.59, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\cosh^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{4}e^{2a-bx}\left(1+e^{2bx}\right)^2de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{2a}\int e^{-bx}\left(1+e^{2bx}\right)^2de^{bx}}{4b}$$

$$\Big\downarrow \text{243}$$

$$\frac{e^{2a}\int e^{-bx}\left(1+e^{2bx}\right)^2de^{2bx}}{8b}$$

$$\Big\downarrow \text{49}$$

$$\frac{e^{2a}\int \left(2+e^{-bx}+e^{2bx}\right)de^{2bx}}{8b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2a}\left(\frac{5}{2}e^{2bx}+\log\left(e^{2bx}\right)\right)}{8b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[d + b*x]^2,x]
```

output
```
(E^(2*a)*((5*E^(2*b*x))/2 + Log[E^(2*b*x)]))/(8*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

method	result
risch	$\frac{e^{2bx+2a}}{4b}+\frac{e^{4bx+2a+2d}}{16b}+\frac{e^{2a-2d}x}{4}$
parallelrisch	$\frac{e^{2bx+2a}\left(-2bx\sinh\left(2bx+2d\right)+2bx\cosh\left(2bx+2d\right)+3\sinh\left(2bx+2d\right)-2\cosh\left(2bx+2d\right)+2\right)}{8b}$
default	$\frac{x\cosh\left(2a-2d\right)}{4}+\frac{\sinh\left(2bx+2a\right)}{4b}+\frac{\sinh\left(4bx+2a+2d\right)}{16b}+\frac{x\sinh\left(2a-2d\right)}{4}+\frac{\cosh\left(2bx+2a\right)}{4b}+\frac{\cosh\left(4bx+2a+2d\right)}{16b}$
orering	$\frac{(4bx+3)e^{2bx+2a}\cosh\left(bx+d\right)^{2}}{4b}-\frac{(6bx+1)\left(2\,e^{2bx+2a}\cosh\left(bx+d\right)^{2}b+2\,e^{2bx+2a}b\cosh\left(bx+d\right)\sinh\left(bx+d\right)\right)}{8b^{2}}+\frac{x\left(6\,e^{2bx+2a}\,\mathrm{c}\right.}{}$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(2*b*x+2*a)/b+1/16*exp(4*b*x+2*a+2*d)/b+1/4*exp(2*a-2*d)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(43) = 86$.

Time = 0.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.80

$$\int e^{2(a+bx)}\cosh^2(d+bx)\,dx = \frac{(4\,bx+1)\cosh\left(bx+d\right)^2\cosh\left(-2\,a+2\,d\right)+\left((4\,bx+1)\cosh\left(-2\,a+2\,d\right)-(4\,bx+1)\sinh\left(-2\,a+2\,d\right)\right)}{}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^2,x, algorithm="fricas")
```

output
```
1/16*((4*b*x + 1)*cosh(b*x + d)^2*cosh(-2*a + 2*d) + ((4*b*x + 1)*cosh(-2*
a + 2*d) - (4*b*x + 1)*sinh(-2*a + 2*d))*sinh(b*x + d)^2 - 2*((4*b*x - 1)*
cosh(b*x + d)*cosh(-2*a + 2*d) - (4*b*x - 1)*cosh(b*x + d)*sinh(-2*a + 2*d
))*sinh(b*x + d) - ((4*b*x + 1)*cosh(b*x + d)^2 + 4)*sinh(-2*a + 2*d) + 4*
cosh(-2*a + 2*d))/(b*cosh(b*x + d)^2 - 2*b*cosh(b*x + d)*sinh(b*x + d) + b
*sinh(b*x + d)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(41) = 82$.

Time = 0.49 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.76

$$\int e^{2(a+bx)}\cosh^2(d+bx)\,dx = \begin{cases} \frac{xe^{2a}e^{2bx}\sinh^2\left(bx+d\right)}{4}-\frac{xe^{2a}e^{2bx}\sinh\left(bx+d\right)\cosh\left(bx+d\right)}{2}+\frac{xe^{2a}e^{2bx}\cosh^2\left(bx+d\right)}{4}-\frac{e^{2a}e^{2bx}\sinh^2\left(bx+d\right)}{2b}+\frac{3e^{2a}e^{2bx}\sinh\left(bx+d\right)}{4b} \\ xe^{2a}\cosh^2\left(d\right) \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)**2,x)
```

output
```
Piecewise((x*exp(2*a)*exp(2*b*x)*sinh(b*x + d)**2/4 - x*exp(2*a)*exp(2*b*x
)*sinh(b*x + d)*cosh(b*x + d)/2 + x*exp(2*a)*exp(2*b*x)*cosh(b*x + d)**2/4
 - exp(2*a)*exp(2*b*x)*sinh(b*x + d)**2/(2*b) + 3*exp(2*a)*exp(2*b*x)*sinh
(b*x + d)*cosh(b*x + d)/(4*b), Ne(b, 0)), (x*exp(2*a)*cosh(d)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int e^{2(a+bx)}\cosh^2(d+bx)\,dx=\frac{1}{4}\,xe^{(2\,a-2\,d)}+\frac{\left(4\,e^{(-2\,bx-2\,d)}+1\right)e^{(4\,bx+2\,a+2\,d)}}{16\,b}+\frac{de^{(2\,a-2\,d)}}{4\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^2,x, algorithm="maxima")
```

output
```
1/4*x*e^(2*a - 2*d) + 1/16*(4*e^(-2*b*x - 2*d) + 1)*e^(4*b*x + 2*a + 2*d)/
b + 1/4*d*e^(2*a - 2*d)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int e^{2(a+bx)}\cosh^2(d+bx)\,dx=\frac{\left(4\,(bx+d)e^{(2\,a)}+e^{(4\,bx+2\,a+4\,d)}+4\,e^{(2\,bx+2\,a+2\,d)}\right)e^{(-2\,d)}}{16\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^2,x, algorithm="giac")
```

output
```
1/16*(4*(b*x + d)*e^(2*a) + e^(4*b*x + 2*a + 4*d) + 4*e^(2*b*x + 2*a + 2*d
))*e^(-2*d)/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int e^{2(a+bx)}\cosh^2(d+bx)\,dx=\frac{x\,e^{2\,a}\,e^{-2\,d}}{4}+\frac{e^{2\,a}\,e^{2\,b\,x}}{4\,b}+\frac{e^{2\,a}\,e^{2\,d}\,e^{4\,b\,x}}{16\,b}$$

input
```
int(cosh(d + b*x)^2*exp(2*a + 2*b*x),x)
```

output
```
(x*exp(2*a)*exp(-2*d))/4 + (exp(2*a)*exp(2*b*x))/(4*b) + (exp(2*a)*exp(2*d
)*exp(4*b*x))/(16*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int e^{2(a+bx)}\cosh^2(d+bx)\,dx=\frac{e^{2a}\left(e^{4bx+4d}+4e^{2bx+2d}+4bx\right)}{16e^{2d}b}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^2,x)
```

output
```
(e**(2*a)*(e**(4*b*x + 4*d) + 4*e**(2*b*x + 2*d) + 4*b*x))/(16*e**(2*d)*b)
```

3.54 $\int e^{2(a+bx)}\cosh(d+bx)\coth(d+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 60

$$\int e^{2(a+bx)}\cosh(d+bx)\coth(d+bx)\,dx=\frac{3e^{2a-d+bx}}{2b}+\frac{e^{2a+d+3bx}}{6b}-\frac{2e^{2a-2d}\text{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
3/2*exp(b*x+2*a-d)/b+1/6*exp(3*b*x+2*a+d)/b-2*exp(2*a-2*d)*arctanh(exp(b*x
+d))/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 183 vs. 2(60) = 120.

Time = 0.43 (sec) , antiderivative size = 183, normalized size of antiderivative = 3.05

$$\int e^{2(a+bx)}\cosh(d+bx)\coth(d+bx)\,dx$$
$$=\frac{e^{2a}\left(-6\cosh(2d)\left(\log\left(\left(1+e^{bx}\right)\cosh\left(\frac{d}{2}\right)+\left(-1+e^{bx}\right)\sinh\left(\frac{d}{2}\right)\right)-\log\left(\left(-1+e^{bx}\right)\cosh\left(\frac{d}{2}\right)+\left(1+e^{bx}\right)\right.\right.\right.}{}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[d + b*x]*Coth[d + b*x],x]
```

output
```
(E^(2*a)*(-6*Cosh[2*d]*(Log[(1 + E^(b*x))*Cosh[d/2] + (-1 + E^(b*x))*Sinh[
d/2]] - Log[(-1 + E^(b*x))*Cosh[d/2] + (1 + E^(b*x))*Sinh[d/2]]) + E^(b*x)
*(-9 + E^(2*b*x))*Sinh[d] + Cosh[d]*(E^(b*x)*(9 + E^(2*b*x)) + 12*(Log[(1
+ E^(b*x))*Cosh[d/2] + (-1 + E^(b*x))*Sinh[d/2]] - Log[(-1 + E^(b*x))*Cosh
[d/2] + (1 + E^(b*x))*Sinh[d/2]])*Sinh[d])))/(6*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.63, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2720, 27, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)} \cosh(bx+d) \coth(bx+d) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{e^{2a}\left(1+e^{2bx}\right)^2}{2\left(1-e^{2bx}\right)}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^{2a}\int \frac{\left(1+e^{2bx}\right)^2}{1-e^{2bx}}de^{bx}}{2b}$$

$$\Big\downarrow \text{300}$$

$$-\frac{e^{2a}\int \left(-e^{2bx}-3+\frac{4}{1-e^{2bx}}\right)de^{bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{2a}\left(4\text{arctanh}\left(e^{bx}\right)-3e^{bx}-\frac{1}{3}e^{3bx}\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[d + b*x]*Coth[d + b*x],x]
```

output
```
-1/2*(E^(2*a)*(-3*E^(b*x) - E^(3*b*x)/3 + 4*ArcTanh[E^(b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.47

method	result	size
risch	$\frac{e^{3bx+2a+d}}{6b}+\frac{3\,e^{bx+2a-d}}{2b}+\frac{\ln\left(e^{bx+a}-e^{a-d}\right)e^{2a-2d}}{b}-\frac{\ln\left(e^{bx+a}+e^{a-d}\right)e^{2a-2d}}{b}$	88

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)*coth(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/6*exp(3*b*x+2*a+d)/b+3/2*exp(b*x+2*a-d)/b+ln(exp(b*x+a)-exp(a-d))/b*exp(
2*a-2*d)-ln(exp(b*x+a)+exp(a-d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(52) = 104$.

Time = 0.09 (sec) , antiderivative size = 264, normalized size of antiderivative = 4.40

$$\int e^{2(a+bx)}\cosh(d+bx)\coth(d+bx)\,dx = \frac{\cosh(bx+d)^3\cosh(-2a+2d)+(\cosh(-2a+2d)-\sinh(-2a+2d))\sinh(bx+d)^3+3(\cosh(bx+$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)*coth(b*x+d),x, algorithm="fricas")
```

output
```
1/6*(cosh(b*x + d)^3*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*
d))*sinh(b*x + d)^3 + 3*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*si
nh(-2*a + 2*d))*sinh(b*x + d)^2 + 9*cosh(b*x + d)*cosh(-2*a + 2*d) - 6*(co
sh(-2*a + 2*d) - sinh(-2*a + 2*d))*log(cosh(b*x + d) + sinh(b*x + d) + 1)
+ 6*(cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*log(cosh(b*x + d) + sinh(b*x + d
) - 1) + 3*(cosh(b*x + d)^2*cosh(-2*a + 2*d) - (cosh(b*x + d)^2 + 3)*sinh(
-2*a + 2*d) + 3*cosh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^3 + 9*cos
h(b*x + d))*sinh(-2*a + 2*d))/b
```

Sympy [F]

$$\int e^{2(a+bx)}\cosh(d+bx)\coth(d+bx)\,dx = e^{2a}\int e^{2bx}\cosh(bx+d)\coth(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)*coth(b*x+d),x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*cosh(b*x + d)*coth(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

$$\int e^{2(a+bx)}\cosh(d+bx)\coth(d+bx)\,dx = \frac{\left(9\,e^{(-2\,bx-2\,d)}+1\right)e^{(3\,bx+2\,a+d)}}{6\,b} - \frac{e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}+1\right)}{b} + \frac{e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}-1\right)}{b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)*coth(b*x+d),x, algorithm="maxima")
```

output
```
1/6*(9*e^(-2*b*x - 2*d) + 1)*e^(3*b*x + 2*a + d)/b - e^(2*a - 2*d)*log(e^(
-b*x - d) + 1)/b + e^(2*a - 2*d)*log(e^(-b*x - d) - 1)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.27

$$\int e^{2(a+bx)}\cosh(d+bx)\coth(d+bx)\,dx = \frac{\left(e^{(3\,bx+2\,a+7\,d)}+9\,e^{(bx+2\,a+5\,d)}\right)e^{(-6\,d)} - 6\,e^{(2\,a-2\,d)}\log\left(e^{(bx+d)}+1\right) + 6\,e^{(2\,a-2\,d)}\log\left(\left|e^{(bx+d)}-1\right|\right)}{6\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)*coth(b*x+d),x, algorithm="giac")
```

output
```
1/6*((e^(3*b*x + 2*a + 7*d) + 9*e^(b*x + 2*a + 5*d))*e^(-6*d) - 6*e^(2*a -
 2*d)*log(e^(b*x + d) + 1) + 6*e^(2*a - 2*d)*log(abs(e^(b*x + d) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 2.79 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.48

$$\int e^{2(a+bx)}\cosh(d+bx)\coth(d+bx)\,dx$$
$$=\frac{3\,\mathrm{e}^{2\,a}\,\mathrm{e}^{-d}\,\mathrm{e}^{b\,x}}{2\,b}-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{-d}\,\mathrm{e}^{b\,x}\,\sqrt{-b^2}}{b\,\sqrt{\mathrm{e}^{4\,a}\,\mathrm{e}^{-4\,d}}}\right)\,\sqrt{\mathrm{e}^{4\,a}\,\mathrm{e}^{-4\,d}}}{\sqrt{-b^2}}+\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{3\,b\,x}\,\mathrm{e}^{d}}{6\,b}$$

input
```
int(cosh(d + b*x)*coth(d + b*x)*exp(2*a + 2*b*x),x)
```

output
```
(3*exp(2*a)*exp(-d)*exp(b*x))/(2*b) - (2*atan((exp(2*a)*exp(-d)*exp(b*x)*(
-b^2)^(1/2))/(b*(exp(4*a)*exp(-4*d))^(1/2)))*(exp(4*a)*exp(-4*d))^(1/2))/(
-b^2)^(1/2) + (exp(2*a)*exp(3*b*x)*exp(d))/(6*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int e^{2(a+bx)}\cosh(d+bx)\coth(d+bx)\,dx$$
$$=\frac{e^{2a}\left(e^{3bx+3d}+9e^{bx+d}+6\log\left(e^{bx+d}-1\right)-6\log\left(e^{bx+d}+1\right)\right)}{6e^{2d}b}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)*coth(b*x+d),x)
```

output
```
(e**(2*a)*(e**(3*b*x + 3*d) + 9*e**(b*x + d) + 6*log(e**(b*x + d) - 1) - 6
*log(e**(b*x + d) + 1)))/(6*e**(2*d)*b)
```

3.55 $\int e^{2(a+bx)} \coth^2(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 77

$$\int e^{2(a+bx)} \coth^2(d+bx)\,dx = \frac{e^{2a+2bx}}{2b} + \frac{2e^{2a-2d}}{b\left(1-e^{2d+2bx}\right)} + \frac{2e^{2a-2d}\log\left(1-e^{2d+2bx}\right)}{b}$$

output
```
1/2*exp(2*b*x+2*a)/b+2*exp(2*a-2*d)/b/(1-exp(2*b*x+2*d))+2*exp(2*a-2*d)*ln
(1-exp(2*b*x+2*d))/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 158 vs. 2(77) = 154.

Time = 0.13 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.05

$$\int e^{2(a+bx)} \coth^2(d+bx)\,dx$$
$$= \frac{e^{2a}(\cosh(d)-\sinh(d))\left(\cosh(2d)\left(-4+e^{4bx}-4\log\left(\left(-1+e^{2bx}\right)\cosh(d)+\left(1+e^{2bx}\right)\sinh(d)\right)\right)+e^{2bx}\left(\cdots\right.}{2b\left(\left(-1+\cdots\right.\right.}$$

input
```
Integrate[E^(2*(a + b*x))*Coth[d + b*x]^2,x]
```

output
```
(E^(2*a)*(Cosh[d] - Sinh[d])*(Cosh[2*d]*(-4 + E^(4*b*x) - 4*Log[(-1 + E^(2
*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d]]) + E^(2*b*x)*(-1 + 4*Log[(-1 + E
^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d]]) + (4 + E^(4*b*x) + 4*Log[(-1
 + E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d]])*Sinh[2*d]))/(2*b*((-1 +
E^(2*b*x))*Cosh[d] + (1 + E^(2*b*x))*Sinh[d]))
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.60, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2720, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\coth^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{e^{2a+bx}\left(1+e^{2bx}\right)^2}{\left(1-e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{2a}\int \frac{e^{bx}\left(1+e^{2bx}\right)^2}{\left(1-e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{353}$$

$$\frac{e^{2a}\int \frac{\left(1+e^{2bx}\right)^2}{\left(1-e^{2bx}\right)^2}de^{2bx}}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{e^{2a}\int \left(1+\frac{4}{-1+e^{2bx}}+\frac{4}{\left(-1+e^{2bx}\right)^2}\right)de^{2bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2a}\left(e^{2bx}+\frac{4}{1-e^{2bx}}+4\log\left(1-e^{2bx}\right)\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Coth[d + b*x]^2,x]
```

output
```
(E^(2*a)*(E^(2*b*x) + 4/(1 - E^(2*b*x)) + 4*Log[1 - E^(2*b*x)]))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27

method	result	size
risch	$\frac{e^{2bx+2a}}{2b}-\frac{4\,e^{2a-2d}a}{b}+\frac{2\,e^{4a-2d}}{\left(-e^{2bx+2a+2d}+e^{2a}\right)b}+\frac{2\ln\left(e^{2bx+2a}-e^{2a-2d}\right)e^{2a-2d}}{b}$	98

input
```
int(exp(2*b*x+2*a)*coth(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*b*x+2*a)/b-4/b*exp(2*a-2*d)*a+2/(-exp(2*b*x+2*a+2*d)+exp(2*a))/b
*exp(4*a-2*d)+2*ln(exp(2*b*x+2*a)-exp(2*a-2*d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. 2(68) = 136.

Time = 0.10 (sec) , antiderivative size = 465, normalized size of antiderivative = 6.04

$$\int e^{2(a+bx)}\coth^2(d+bx)\,dx$$
$$=\frac{\cosh\left(bx+d\right)^4\cosh\left(-2\,a+2\,d\right)+\left(\cosh\left(-2\,a+2\,d\right)-\sinh\left(-2\,a+2\,d\right)\right)\sinh\left(bx+d\right)^4+4\left(\cosh\left(bx+\right.\right.}{}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)^2,x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + d)^4*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*
d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*si
nh(-2*a + 2*d))*sinh(b*x + d)^3 - cosh(b*x + d)^2*cosh(-2*a + 2*d) + (6*co
sh(b*x + d)^2*cosh(-2*a + 2*d) - (6*cosh(b*x + d)^2 - 1)*sinh(-2*a + 2*d)
- cosh(-2*a + 2*d))*sinh(b*x + d)^2 + 4*(cosh(b*x + d)^2*cosh(-2*a + 2*d)
+ (cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^2 + 2*(cosh(b*x + d)
*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d) - (cosh(
b*x + d)^2 - 1)*sinh(-2*a + 2*d) - cosh(-2*a + 2*d))*log(2*sinh(b*x + d)/(
cosh(b*x + d) - sinh(b*x + d))) + 2*(2*cosh(b*x + d)^3*cosh(-2*a + 2*d) -
cosh(b*x + d)*cosh(-2*a + 2*d) - (2*cosh(b*x + d)^3 - cosh(b*x + d))*sinh(
-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^4 - cosh(b*x + d)^2 - 4)*sinh(
-2*a + 2*d) - 4*cosh(-2*a + 2*d))/(b*cosh(b*x + d)^2 + 2*b*cosh(b*x + d)*s
inh(b*x + d) + b*sinh(b*x + d)^2 - b)
```

Sympy [F]

$$\int e^{2(a+bx)}\coth^2(d+bx)\,dx = e^{2a}\int e^{2bx}\coth^2{(bx+d)}\,dx$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)**2,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*coth(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.53

$$\int e^{2(a+bx)}\coth^2(d+bx)\,dx = \frac{4\,(bx+d)e^{(2\,a-2\,d)}}{b} + \frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}+1\right)}{b} + \frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}-1\right)}{b} - \frac{\left(5\,e^{(-2\,bx-2\,d)}-1\right)e^{(2\,a-2\,d)}}{2\,b\left(e^{(-2\,bx-2\,d)}-e^{(-4\,bx-4\,d)}\right)}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)^2,x, algorithm="maxima")
```

output
```
4*(b*x + d)*e^(2*a - 2*d)/b + 2*e^(2*a - 2*d)*log(e^(-b*x - d) + 1)/b + 2*
e^(2*a - 2*d)*log(e^(-b*x - d) - 1)/b - 1/2*(5*e^(-2*b*x - 2*d) - 1)*e^(2*
a - 2*d)/(b*(e^(-2*b*x - 2*d) - e^(-4*b*x - 4*d)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

$$\int e^{2(a+bx)} \coth^2(d+bx)\,dx = \frac{2\,e^{(2\,a-2\,d)} \log\left(\left|e^{(2\,bx+2\,d)}-1\right|\right)}{b} + \frac{e^{(2\,bx+2\,a)}}{2\,b} - \frac{2\,e^{(2\,bx+2\,a)}}{b(e^{(2\,bx+2\,d)}-1)}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)^2,x, algorithm="giac")
```

output
```
2*e^(2*a - 2*d)*log(abs(e^(2*b*x + 2*d) - 1))/b + 1/2*e^(2*b*x + 2*a)/b -
2*e^(2*b*x + 2*a)/(b*(e^(2*b*x + 2*d) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int e^{2(a+bx)} \coth^2(d+bx)\,dx = \frac{e^{2\,a+2\,b\,x}}{2\,b} + \frac{2\,e^{2\,a-2\,d}\,\ln\left(e^{2\,a}\,e^{2\,b\,x}-e^{2\,a}\,e^{-2\,d}\right)}{b} + \frac{2\,e^{4\,a-4\,d}}{b\,\left(e^{2\,a-2\,d}-e^{2\,a+2\,b\,x}\right)}$$

input
```
int(coth(d + b*x)^2*exp(2*a + 2*b*x),x)
```

output
```
exp(2*a + 2*b*x)/(2*b) + (2*exp(2*a - 2*d)*log(exp(2*a)*exp(2*b*x) - exp(2
*a)*exp(-2*d)))/b + (2*exp(4*a - 4*d))/(b*(exp(2*a - 2*d) - exp(2*a + 2*b*
x)))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.58

$$\int e^{2(a+bx)}\coth^2(d+bx)\,dx$$
$$=\frac{e^{2a}\left(e^{4bx+4d}+4e^{2bx+2d}\log\left(e^{bx+d}-1\right)+4e^{2bx+2d}\log\left(e^{bx+d}+1\right)-5e^{2bx+2d}-4\log\left(e^{bx+d}-1\right)-4\log\left(e^{bx-}\right.\right.}{2e^{2d}b\left(e^{2bx+2d}-1\right)}$$

input
```
int(exp(2*b*x+2*a)*coth(b*x+d)^2,x)
```

output
```
(e**(2*a)*(e**(4*b*x + 4*d) + 4*e**(2*b*x + 2*d)*log(e**(b*x + d) - 1) + 4
*e**(2*b*x + 2*d)*log(e**(b*x + d) + 1) - 5*e**(2*b*x + 2*d) - 4*log(e**(b
*x + d) - 1) - 4*log(e**(b*x + d) + 1)))/(2*e**(2*d)*b*(e**(2*b*x + 2*d) -
 1))
```

3.56 $\int e^{2(a+bx)}\coth^2(d+bx)\mathbf{csch}(d+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 106

$$\int e^{2(a+bx)}\coth^2(d+bx)\operatorname{csch}(d+bx)\,dx=\frac{2e^{2a-d+bx}}{b}-\frac{2e^{2a-d+bx}}{b\left(1-e^{2d+2bx}\right)^2}+\frac{5e^{2a-d+bx}}{b\left(1-e^{2d+2bx}\right)}-\frac{5e^{2a-2d}\operatorname{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
2*exp(b*x+2*a-d)/b-2*exp(b*x+2*a-d)/b/(1-exp(2*b*x+2*d))^2+5*exp(b*x+2*a-d
)/b/(1-exp(2*b*x+2*d))-5*exp(2*a-2*d)*arctanh(exp(b*x+d))/b
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.77 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.37

$$\int e^{2(a+bx)}\coth^2(d+bx)\operatorname{csch}(d+bx)\,dx=$$

$$-\frac{e^{2a-5d-3bx}\left(-21\left(56595+62725e^{2(d+bx)}-12071e^{4(d+bx)}-19353e^{6(d+bx)}+768e^{8(d+bx)}\right)+\frac{315(3773+2924e}{}\right.}{}$$

input
```
Integrate[E^(2*(a + b*x))*Coth[d + b*x]^2*Csch[d + b*x],x]
```

output
```
-1/10080*(E^(2*a - 5*d - 3*b*x)*(-21*(56595 + 62725*E^(2*(d + b*x)) - 1207
1*E^(4*(d + b*x)) - 19353*E^(6*(d + b*x)) + 768*E^(8*(d + b*x))) + (315*(3
773 + 2924*E^(2*(d + b*x)) - 2534*E^(4*(d + b*x)) - 1548*E^(6*(d + b*x)) +
 297*E^(8*(d + b*x)))*ArcTanh[Sqrt[E^(2*(d + b*x))]])/Sqrt[E^(2*(d + b*x))
] + 128*E^(8*(d + b*x))*(9 + 16*E^(2*(d + b*x)) + 7*E^(4*(d + b*x)))*Hyper
geometricPFQ[{2, 2, 2, 5/2}, {1, 1, 11/2}, E^(2*(d + b*x))] + 128*E^(8*(d
+ b*x))*(1 + E^(2*(d + b*x)))^2*HypergeometricPFQ[{2, 2, 2, 2, 5/2}, {1, 1
, 1, 11/2}, E^(2*(d + b*x))]))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.67, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2720, 27, 366, 27, 360, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\coth^2(bx+d)\text{csch}(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{2e^{2a+2bx}\left(1+e^{2bx}\right)^2}{\left(1-e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2e^{2a}\int \frac{e^{2bx}\left(1+e^{2bx}\right)^2}{\left(1-e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{366}$$

$$-\frac{2e^{2a}\left(\frac{e^{3bx}}{\left(1-e^{2bx}\right)^2}-\frac{1}{4}\int \frac{4e^{2bx}\left(2+e^{2bx}\right)}{\left(1-e^{2bx}\right)^2}de^{bx}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2e^{2a}\left(\frac{e^{3bx}}{\left(1-e^{2bx}\right)^2}-\int \frac{e^{2bx}\left(2+e^{2bx}\right)}{\left(1-e^{2bx}\right)^2}de^{bx}\right)}{b}$$

$$\Big\downarrow \text{360}$$

$$-\frac{2e^{2a}\left(\frac{1}{2}\int\frac{3+2e^{2bx}}{1-e^{2bx}}de^{bx}-\frac{3e^{bx}}{2(1-e^{2bx})}+\frac{e^{3bx}}{(1-e^{2bx})^2}\right)}{b}$$

$$\Big\downarrow \text{299}$$

$$-\frac{2e^{2a}\left(\frac{1}{2}\left(5\int\frac{1}{1-e^{2bx}}de^{bx}-2e^{bx}\right)-\frac{3e^{bx}}{2(1-e^{2bx})}+\frac{e^{3bx}}{(1-e^{2bx})^2}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2e^{2a}\left(\frac{1}{2}\left(5\text{arctanh}\left(e^{bx}\right)-2e^{bx}\right)-\frac{3e^{bx}}{2(1-e^{2bx})}+\frac{e^{3bx}}{(1-e^{2bx})^2}\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Coth[d + b*x]^2*Csch[d + b*x],x]
```

output
```
(-2*E^(2*a)*(E^(3*b*x)/(1 - E^(2*b*x))^2 - (3*E^(b*x))/(2*(1 - E^(2*b*x)))
 + (-2*E^(b*x) + 5*ArcTanh[E^(b*x)])/2))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.24

method	result	size
risch	$\frac{2\,\mathrm{e}^{bx+2a-d}}{b}+\frac{\left(-5\,\mathrm{e}^{2bx+2a+2d}+3\,\mathrm{e}^{2a}\right)\mathrm{e}^{bx+4a-d}}{\left(-\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)^{2}b}-\frac{5\ln\left(\mathrm{e}^{bx+a}+\mathrm{e}^{a-d}\right)\mathrm{e}^{2a-2d}}{2b}+\frac{5\ln\left(\mathrm{e}^{bx+a}-\mathrm{e}^{a-d}\right)\mathrm{e}^{2a-2d}}{2b}$	131

input
```
int(exp(2*b*x+2*a)*coth(b*x+d)^2*csch(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
2*exp(b*x+2*a-d)/b+1/(-exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(-5*exp(2*b*x+2*a+
2*d)+3*exp(2*a))*exp(b*x+4*a-d)-5/2*ln(exp(b*x+a)+exp(a-d))/b*exp(2*a-2*d)
+5/2*ln(exp(b*x+a)-exp(a-d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1074 vs. $2(95) = 190$.

Time = 0.10 (sec) , antiderivative size = 1074, normalized size of antiderivative = 10.13

$$\int e^{2(a+bx)}\coth^2(d+bx)\text{csch}(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)^2*csch(b*x+d),x, algorithm="fricas")
```

output
```
1/2*(4*cosh(b*x + d)^5*cosh(-2*a + 2*d) + 4*(cosh(-2*a + 2*d) - sinh(-2*a
+ 2*d))*sinh(b*x + d)^5 + 20*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x +
d)*sinh(-2*a + 2*d))*sinh(b*x + d)^4 - 18*cosh(b*x + d)^3*cosh(-2*a + 2*d)
 + 2*(20*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (20*cosh(b*x + d)^2 - 9)*sinh(
-2*a + 2*d) - 9*cosh(-2*a + 2*d))*sinh(b*x + d)^3 + 2*(20*cosh(b*x + d)^3*
cosh(-2*a + 2*d) - 27*cosh(b*x + d)*cosh(-2*a + 2*d) - (20*cosh(b*x + d)^3
 - 27*cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d)^2 + 10*cosh(b*x + d)*
cosh(-2*a + 2*d) - 5*(cosh(b*x + d)^4*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d)
 - sinh(-2*a + 2*d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-2*a + 2*d) -
 cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^3 - 2*cosh(b*x + d)^2*cosh(
-2*a + 2*d) + 2*(3*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (3*cosh(b*x + d)^2 -
 1)*sinh(-2*a + 2*d) - cosh(-2*a + 2*d))*sinh(b*x + d)^2 + 4*(cosh(b*x + d
)^3*cosh(-2*a + 2*d) - cosh(b*x + d)*cosh(-2*a + 2*d) - (cosh(b*x + d)^3 -
 cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^4 - 2*cos
h(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*log(cosh(b*x + d) +
 sinh(b*x + d) + 1) + 5*(cosh(b*x + d)^4*cosh(-2*a + 2*d) + (cosh(-2*a + 2
*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-2*a + 2*d
) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^3 - 2*cosh(b*x + d)^2*co
sh(-2*a + 2*d) + 2*(3*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (3*cosh(b*x + d)^
2 - 1)*sinh(-2*a + 2*d) - cosh(-2*a + 2*d))*sinh(b*x + d)^2 + 4*(cosh(b...
```

Sympy [F]

$$\int e^{2(a+bx)}\coth^2(d+bx)\text{csch}(d+bx)\,dx = e^{2a}\int e^{2bx}\coth^2(bx+d)\,\text{csch}\,(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)**2*csch(b*x+d),x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*coth(b*x + d)**2*csch(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.13

$$\int e^{2(a+bx)}\coth^2(d+bx)\text{csch}(d+bx)\,dx = -\frac{5\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}+1\right)}{2\,b} + \frac{5\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}-1\right)}{2\,b} - \frac{\left(9\,e^{(-2\,bx-2\,d)}-5\,e^{(-4\,bx-4\,d)}-2\right)e^{(2\,a-2\,d)}}{b\left(e^{(-bx-d)}-2\,e^{(-3\,bx-3\,d)}+e^{(-5\,bx-5\,d)}\right)}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)^2*csch(b*x+d),x, algorithm="maxima")
```

output
```
-5/2*e^(2*a - 2*d)*log(e^(-b*x - d) + 1)/b + 5/2*e^(2*a - 2*d)*log(e^(-b*x
 - d) - 1)/b - (9*e^(-2*b*x - 2*d) - 5*e^(-4*b*x - 4*d) - 2)*e^(2*a - 2*d)
/(b*(e^(-b*x - d) - 2*e^(-3*b*x - 3*d) + e^(-5*b*x - 5*d)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.97

$$\int e^{2(a+bx)}\coth^2(d+bx)\text{csch}(d+bx)\,dx =$$

$$-\frac{5\,e^{(2\,a-2\,d)}\log\left(e^{(bx+d)}+1\right)-5\,e^{(2\,a-2\,d)}\log\left(\left|e^{(bx+d)}-1\right|\right)+\frac{2\left(5\,e^{(3\,bx+2\,a+3\,d)}-3\,e^{(bx+2\,a+d)}\right)e^{(-2\,d)}}{\left(e^{(2\,bx+2\,d)}-1\right)^2}-4\,e^{(bx+2\,a}}{2\,b}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)^2*csch(b*x+d),x, algorithm="giac")
```

output
```
-1/2*(5*e^(2*a - 2*d)*log(e^(b*x + d) + 1) - 5*e^(2*a - 2*d)*log(abs(e^(b*
x + d) - 1)) + 2*(5*e^(3*b*x + 2*a + 3*d) - 3*e^(b*x + 2*a + d))*e^(-2*d)/
(e^(2*b*x + 2*d) - 1)^2 - 4*e^(b*x + 2*a - d))/b
```

Mupad [B] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.32

$$\int e^{2(a+bx)}\coth^2(d+bx)\mathrm{csch}(d+bx)\,dx=\frac{2\,\mathrm{e}^{2\,a-d+b\,x}}{b}-\frac{5\,\mathrm{e}^{2\,a-d+b\,x}}{b\,\left(\mathrm{e}^{2\,d+2\,b\,x}-1\right)}-\frac{2\,\mathrm{e}^{2\,a-d+b\,x}}{b\,\left(\mathrm{e}^{4\,d+4\,b\,x}-2\,\mathrm{e}^{2\,d+2\,b\,x}+1\right)}-\frac{5\,\sqrt{\mathrm{e}^{4\,a-4\,d}}\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{-d}\,\mathrm{e}^{b\,x}\,\sqrt{-b^2}}{b\,\sqrt{\mathrm{e}^{4\,a}\,\mathrm{e}^{-4\,d}}}\right)}{\sqrt{-b^2}}$$

input
```
int((coth(d + b*x)^2*exp(2*a + 2*b*x))/sinh(d + b*x),x)
```

output
```
(2*exp(2*a - d + b*x))/b - (5*exp(2*a - d + b*x))/(b*(exp(2*d + 2*b*x) - 1
)) - (2*exp(2*a - d + b*x))/(b*(exp(4*d + 4*b*x) - 2*exp(2*d + 2*b*x) + 1)
) - (5*exp(4*a - 4*d)^(1/2)*atan((exp(2*a)*exp(-d)*exp(b*x)*(-b^2)^(1/2))/
(b*(exp(4*a)*exp(-4*d))^(1/2))))/(-b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.78

$$\int e^{2(a+bx)}\coth^2(d+bx)\mathrm{csch}(d+bx)\,dx$$
$$=\frac{e^{2a}\left(4e^{5bx+5d}+5e^{4bx+4d}\log\left(e^{bx+d}-1\right)-5e^{4bx+4d}\log\left(e^{bx+d}+1\right)-18e^{3bx+3d}-10e^{2bx+2d}\log\left(e^{bx+d}-1\right)+\right.}{2e^{2d}b\left(e^{4bx+4d}-2e^{2bx+2d}+1\right)}$$

input
```
int(exp(2*b*x+2*a)*coth(b*x+d)^2*csch(b*x+d),x)
```

output
```
(e**(2*a)*(4*e**(5*b*x + 5*d) + 5*e**(4*b*x + 4*d)*log(e**(b*x + d) - 1) -
 5*e**(4*b*x + 4*d)*log(e**(b*x + d) + 1) - 18*e**(3*b*x + 3*d) - 10*e**(2
*b*x + 2*d)*log(e**(b*x + d) - 1) + 10*e**(2*b*x + 2*d)*log(e**(b*x + d) +
 1) + 10*e**(b*x + d) + 5*log(e**(b*x + d) - 1) - 5*log(e**(b*x + d) + 1))
)/(2*e**(2*d)*b*(e**(4*b*x + 4*d) - 2*e**(2*b*x + 2*d) + 1))
```

3.57 $\int e^{2(a+bx)} \cosh^3(d+bx) \sinh^3(d+bx)\, dx$

Optimal result

Integrand size = 26, antiderivative size = 76

$$\int e^{2(a+bx)} \cosh^3(d+bx) \sinh^3(d+bx)\, dx = \frac{e^{2(a-3d)-4bx}}{256b} - \frac{3e^{2(a+d)+4bx}}{256b} + \frac{e^{2(a+3d)+8bx}}{512b} + \frac{3}{64}e^{2a-2d}x$$

output
```
1/256*exp(-4*b*x+2*a-6*d)/b-3/256*exp(4*b*x+2*a+2*d)/b+1/512*exp(8*b*x+2*a
+6*d)/b+3/64*exp(2*a-2*d)*x
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$\int e^{2(a+bx)} \cosh^3(d+bx) \sinh^3(d+bx)\, dx = \frac{e^{2a-2d}\left(e^{-4(d+bx)} - 3e^{4(d+bx)} + \frac{1}{2}e^{8(d+bx)} + 12(d+bx)\right)}{256b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[d + b*x]^3*Sinh[d + b*x]^3,x]
```

output
```
(E^(2*a - 2*d)*(E^(-4*(d + b*x)) - 3*E^(4*(d + b*x)) + E^(8*(d + b*x))/2 +
 12*(d + b*x)))/(256*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.63, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2720, 27, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)} \sinh^3(bx+d) \cosh^3(bx+d) \, dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int -\frac{1}{64} e^{2a-5bx} \left(1-e^{4bx}\right)^3 de^{bx}}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{e^{2a} \int e^{-5bx} \left(1-e^{4bx}\right)^3 de^{bx}}{64b}$$

$$\Big\downarrow \; 798$$

$$-\frac{e^{2a} \int e^{-2bx} \left(1-e^{4bx}\right)^3 de^{4bx}}{256b}$$

$$\Big\downarrow \; 49$$

$$-\frac{e^{2a} \int \left(3+e^{-2bx}-3e^{-bx}-e^{4bx}\right) de^{4bx}}{256b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{e^{2a} \left(-e^{-bx}-\frac{1}{2} e^{2bx}+3e^{4bx}-3\log\left(e^{4bx}\right)\right)}{256b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[d + b*x]^3*Sinh[d + b*x]^3,x]
```

output
```
-1/256*(E^(2*a)*(-E^(-(b*x)) - E^(2*b*x)/2 + 3*E^(4*b*x) - 3*Log[E^(4*b*x)
]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.66

$$\frac{3x\cosh\left(2a-2d\right)}{64}+\frac{\sinh\left(-4bx+2a-6d\right)}{256b}-\frac{3\sinh\left(4bx+2a+2d\right)}{256b}+\frac{\sinh\left(8bx+2a+6d\right)}{512b}+\frac{3x\sinh\left(2a-\right.}{64}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^3*sinh(b*x+d)^3,x)
```

output
```
3/64*x*cosh(2*a-2*d)+1/256/b*sinh(-4*b*x+2*a-6*d)-3/256/b*sinh(4*b*x+2*a+2
*d)+1/512/b*sinh(8*b*x+2*a+6*d)+3/64*x*sinh(2*a-2*d)+1/256*cosh(-4*b*x+2*a
-6*d)/b-3/256*cosh(4*b*x+2*a+2*d)/b+1/512*cosh(8*b*x+2*a+6*d)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(63) = 126$.

Time = 0.09 (sec) , antiderivative size = 433, normalized size of antiderivative = 5.70

$$\int e^{2(a+bx)}\cosh^3(d+bx)\sinh^3(d+bx)\,dx$$
$$= \frac{3\,\cosh\left(bx+d\right)^{6}\cosh\left(-2\,a+2\,d\right)+3\left(\cosh\left(-2\,a+2\,d\right)-\sinh\left(-2\,a+2\,d\right)\right)\sinh\left(bx+d\right)^{6}-6\left(\cosh\left(b\right.\right.}{}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^3*sinh(b*x+d)^3,x, algorithm="fricas"
)
```

output
```
1/512*(3*cosh(b*x + d)^6*cosh(-2*a + 2*d) + 3*(cosh(-2*a + 2*d) - sinh(-2*
a + 2*d))*sinh(b*x + d)^6 - 6*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x +
 d)*sinh(-2*a + 2*d))*sinh(b*x + d)^5 + 45*(cosh(b*x + d)^2*cosh(-2*a + 2*
d) - cosh(b*x + d)^2*sinh(-2*a + 2*d))*sinh(b*x + d)^4 + 6*(4*b*x - 1)*cos
h(b*x + d)^2*cosh(-2*a + 2*d) - 20*(cosh(b*x + d)^3*cosh(-2*a + 2*d) - cos
h(b*x + d)^3*sinh(-2*a + 2*d))*sinh(b*x + d)^3 + 3*(15*cosh(b*x + d)^4*cos
h(-2*a + 2*d) + 2*(4*b*x - 1)*cosh(-2*a + 2*d) - (15*cosh(b*x + d)^4 + 8*b
*x - 2)*sinh(-2*a + 2*d))*sinh(b*x + d)^2 - 6*(cosh(b*x + d)^5*cosh(-2*a +
 2*d) + 2*(4*b*x + 1)*cosh(b*x + d)*cosh(-2*a + 2*d) - (cosh(b*x + d)^5 +
2*(4*b*x + 1)*cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d) - 3*(cosh(b*x
 + d)^6 + 2*(4*b*x - 1)*cosh(b*x + d)^2)*sinh(-2*a + 2*d))/(b*cosh(b*x + d
)^2 - 2*b*cosh(b*x + d)*sinh(b*x + d) + b*sinh(b*x + d)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(65) = 130$.

Time = 12.15 (sec) , antiderivative size = 382, normalized size of antiderivative = 5.03

$$\int e^{2(a+bx)}\cosh^3(d+bx)\sinh^3(d+bx)\,dx$$
$$= \begin{cases} \frac{3xe^{2a}e^{2bx}\sinh^6(bx+d)}{64} - \frac{3xe^{2a}e^{2bx}\sinh^5(bx+d)\cosh(bx+d)}{32} - \frac{3xe^{2a}e^{2bx}\sinh^4(bx+d)\cosh^2(bx+d)}{64} + \frac{3xe^{2a}e^{2bx}\sinh^3(bx+d)\cosh^3}{16} \\ xe^{2a}\sinh^3(d)\cosh^3(d) \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)**3*sinh(b*x+d)**3,x)
```

output
```
Piecewise((3*x*exp(2*a)*exp(2*b*x)*sinh(b*x + d)**6/64 - 3*x*exp(2*a)*exp(
2*b*x)*sinh(b*x + d)**5*cosh(b*x + d)/32 - 3*x*exp(2*a)*exp(2*b*x)*sinh(b*
x + d)**4*cosh(b*x + d)**2/64 + 3*x*exp(2*a)*exp(2*b*x)*sinh(b*x + d)**3*c
osh(b*x + d)**3/16 - 3*x*exp(2*a)*exp(2*b*x)*sinh(b*x + d)**2*cosh(b*x + d
)**4/64 - 3*x*exp(2*a)*exp(2*b*x)*sinh(b*x + d)*cosh(b*x + d)**5/32 + 3*x*
exp(2*a)*exp(2*b*x)*cosh(b*x + d)**6/64 + 3*exp(2*a)*exp(2*b*x)*sinh(b*x +
 d)**6/(32*b) - 15*exp(2*a)*exp(2*b*x)*sinh(b*x + d)**5*cosh(b*x + d)/(64*
b) + 13*exp(2*a)*exp(2*b*x)*sinh(b*x + d)**3*cosh(b*x + d)**3/(32*b) - 15*
exp(2*a)*exp(2*b*x)*sinh(b*x + d)*cosh(b*x + d)**5/(64*b) + 3*exp(2*a)*exp
(2*b*x)*cosh(b*x + d)**6/(32*b), Ne(b, 0)), (x*exp(2*a)*sinh(d)**3*cosh(d)
**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int e^{2(a+bx)}\cosh^3(d+bx)\sinh^3(d+bx)\,dx = -\frac{\left(6\,e^{(-4\,bx-4\,d)}-1\right)e^{(8\,bx+2\,a+6\,d)}}{512\,b} + \frac{3\,(bx+d)e^{(2\,a-2\,d)}}{64\,b} + \frac{e^{(-4\,bx+2\,a-6\,d)}}{256\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^3*sinh(b*x+d)^3,x, algorithm="maxima"
)
```

output
```
-1/512*(6*e^(-4*b*x - 4*d) - 1)*e^(8*b*x + 2*a + 6*d)/b + 3/64*(b*x + d)*e
^(2*a - 2*d)/b + 1/256*e^(-4*b*x + 2*a - 6*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07

$$\int e^{2(a+bx)}\cosh^3(d+bx)\sinh^3(d+bx)\,dx = -\frac{\left(2\left(3e^{(4bx+2a+4d)}-e^{(2a)}\right)e^{(-4bx-4d)}-24(bx+d)e^{(2a)}-e^{(8bx+2a+8d)}+6e^{(4bx+2a+4d)}\right)e^{(-2d)}}{512b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^3*sinh(b*x+d)^3,x, algorithm="giac")
```

output
```
-1/512*(2*(3*e^(4*b*x + 2*a + 4*d) - e^(2*a))*e^(-4*b*x - 4*d) - 24*(b*x +
 d)*e^(2*a) - e^(8*b*x + 2*a + 8*d) + 6*e^(4*b*x + 2*a + 4*d))*e^(-2*d)/b
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

$$\int e^{2(a+bx)}\cosh^3(d+bx)\sinh^3(d+bx)\,dx = \frac{3x\,e^{2a-2d}}{64}-\frac{3\,e^{2a+2d+4bx}}{256b}+\frac{e^{2a-6d-4bx}}{256b}+\frac{e^{2a+6d+8bx}}{512b}$$

input
```
int(cosh(d + b*x)^3*exp(2*a + 2*b*x)*sinh(d + b*x)^3,x)
```

output
```
(3*x*exp(2*a - 2*d))/64 - (3*exp(2*a + 2*d + 4*b*x))/(256*b) + exp(2*a - 6
*d - 4*b*x)/(256*b) + exp(2*a + 6*d + 8*b*x)/(512*b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int e^{2(a+bx)}\cosh^3(d+bx)\sinh^3(d+bx)\,dx = \frac{e^{2a}\left(e^{12bx+12d}-6e^{8bx+8d}+24e^{4bx+4d}bx+2\right)}{512e^{4bx+6d}b}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^3*sinh(b*x+d)^3,x)
```

output
```
(e**(2*a)*(e**(12*b*x + 12*d) - 6*e**(8*b*x + 8*d) + 24*e**(4*b*x + 4*d)*b
*x + 2))/(512*e**(4*b*x + 6*d)*b)
```

3.58 $\int e^{2(a+bx)}\cosh^3(d+bx)\sinh^2(d+bx)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 118

$$\int e^{2(a+bx)}\cosh^3(d+bx)\sinh^2(d+bx)\,dx = -\frac{e^{2a-5d-3bx}}{96b}-\frac{e^{2a-3d-bx}}{32b}-\frac{e^{2a-d+bx}}{16b}-\frac{e^{2a+d+3bx}}{48b}+\frac{e^{2a+3d+5bx}}{160b}+\frac{e^{2a+5d+7bx}}{224b}$$

output
```
-1/96*exp(-3*b*x+2*a-5*d)/b-1/32*exp(-b*x+2*a-3*d)/b-1/16*exp(b*x+2*a-d)/b
-1/48*exp(3*b*x+2*a+d)/b+1/160*exp(5*b*x+2*a+3*d)/b+1/224*exp(7*b*x+2*a+5*
d)/b
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.99

$$\int e^{2(a+bx)}\cosh^3(d+bx)\sinh^2(d+bx)\,dx = \frac{e^{2a-3bx}\left(-70e^{4bx}\left(\left(3+e^{2bx}\right)\cosh(d)+\left(-3+e^{2bx}\right)\sinh(d)\right)+21e^{2bx}\left(\left(-5+e^{6bx}\right)\cosh(3d)+\left(5+e^{6bx}\right)\mathrm{s}\right.\right.}{3360b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[d + b*x]^3*Sinh[d + b*x]^2,x]
```

output
```
(E^(2*a - 3*b*x)*(-70*E^(4*b*x)*((3 + E^(2*b*x))*Cosh[d] + (-3 + E^(2*b*x)
)*Sinh[d]) + 21*E^(2*b*x)*((-5 + E^(6*b*x))*Cosh[3*d] + (5 + E^(6*b*x))*Si
nh[3*d]) + 5*((-7 + 3*E^(10*b*x))*Cosh[5*d] + (7 + 3*E^(10*b*x))*Sinh[5*d]
)))/(3360*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.58, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2720, 27, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\sinh^2(bx+d)\cosh^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{32}e^{2a-4bx}\left(1-e^{2bx}\right)^2\left(1+e^{2bx}\right)^3de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{2a}\int e^{-4bx}\left(1-e^{2bx}\right)^2\left(1+e^{2bx}\right)^3de^{bx}}{32b}$$

$$\Big\downarrow \text{355}$$

$$\frac{e^{2a}\int \left(-2+e^{-4bx}+e^{-2bx}-2e^{2bx}+e^{4bx}+e^{6bx}\right)de^{bx}}{32b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2a}\left(-\frac{1}{3}e^{-3bx}-e^{-bx}-2e^{bx}-\frac{2}{3}e^{3bx}+\frac{1}{5}e^{5bx}+\frac{1}{7}e^{7bx}\right)}{32b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[d + b*x]^3*Sinh[d + b*x]^2,x]
```

output
```
(E^(2*a)*(-1/3*1/E^(3*b*x) - E^(-(b*x)) - 2*E^(b*x) - (2*E^(3*b*x))/3 + E^
(5*b*x)/5 + E^(7*b*x)/7))/(32*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.69

$$-\frac{\sinh\left(-3bx+2a-5d\right)}{96b}-\frac{\sinh\left(-bx+2a-3d\right)}{32b}-\frac{\sinh\left(bx+2a-d\right)}{16b}-\frac{\sinh\left(3bx+2a+d\right)}{48b}+\frac{\sinh\left(5bx+2\right.}{160b}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^3*sinh(b*x+d)^2,x)
```

output
```
-1/96/b*sinh(-3*b*x+2*a-5*d)-1/32/b*sinh(-b*x+2*a-3*d)-1/16/b*sinh(b*x+2*a
-d)-1/48/b*sinh(3*b*x+2*a+d)+1/160/b*sinh(5*b*x+2*a+3*d)+1/224/b*sinh(7*b*
x+2*a+5*d)-1/96*cosh(-3*b*x+2*a-5*d)/b-1/32*cosh(-b*x+2*a-3*d)/b-1/16*cosh
(b*x+2*a-d)/b-1/48*cosh(3*b*x+2*a+d)/b+1/160*cosh(5*b*x+2*a+3*d)/b+1/224*c
osh(7*b*x+2*a+5*d)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(100) = 200$.

Time = 0.08 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.60

$$\int e^{2(a+bx)}\cosh^3(d+bx)\sinh^2(d+bx)\,dx =$$
$$-\frac{10\,\cosh\left(bx+d\right)^5\cosh\left(-2\,a+2\,d\right)-25\left(\cosh\left(-2\,a+2\,d\right)-\sinh\left(-2\,a+2\,d\right)\right)\sinh\left(bx+d\right)^5+50\,(\mathrm{c}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^3*sinh(b*x+d)^2,x, algorithm="fricas"
)
```

output
```
-1/1680*(10*cosh(b*x + d)^5*cosh(-2*a + 2*d) - 25*(cosh(-2*a + 2*d) - sinh
(-2*a + 2*d))*sinh(b*x + d)^5 + 50*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(
b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^4 + 42*cosh(b*x + d)^3*cosh(-2*a
+ 2*d) - (250*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (250*cosh(b*x + d)^2 + 63
)*sinh(-2*a + 2*d) + 63*cosh(-2*a + 2*d))*sinh(b*x + d)^3 + 2*(50*cosh(b*x
 + d)^3*cosh(-2*a + 2*d) + 63*cosh(b*x + d)*cosh(-2*a + 2*d) - (50*cosh(b*
x + d)^3 + 63*cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d)^2 + 140*cosh(
b*x + d)*cosh(-2*a + 2*d) - (125*cosh(b*x + d)^4*cosh(-2*a + 2*d) + 189*co
sh(b*x + d)^2*cosh(-2*a + 2*d) - (125*cosh(b*x + d)^4 + 189*cosh(b*x + d)^
2 + 70)*sinh(-2*a + 2*d) + 70*cosh(-2*a + 2*d))*sinh(b*x + d) - 2*(5*cosh(
b*x + d)^5 + 21*cosh(b*x + d)^3 + 70*cosh(b*x + d))*sinh(-2*a + 2*d))/(b*c
osh(b*x + d)^2 - 2*b*cosh(b*x + d)*sinh(b*x + d) + b*sinh(b*x + d)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(94) = 188$.

Time = 5.22 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.67

$$\int e^{2(a+bx)}\cosh^3(d+bx)\sinh^2(d+bx)\,dx$$
$$=\begin{cases}\frac{2e^{2a}e^{2bx}\sinh^5(bx+d)}{105b}-\frac{4e^{2a}e^{2bx}\sinh^4(bx+d)\cosh(bx+d)}{105b}-\frac{e^{2a}e^{2bx}\sinh^3(bx+d)\cosh^2(bx+d)}{105b}+\frac{2e^{2a}e^{2bx}\sinh^2(bx+d)\cosh^3(bx+\mathrm{c}}{35b}\\ xe^{2a}\sinh^2(d)\cosh^3(d)\end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)**3*sinh(b*x+d)**2,x)
```

output
```
Piecewise((2*exp(2*a)*exp(2*b*x)*sinh(b*x + d)**5/(105*b) - 4*exp(2*a)*exp
(2*b*x)*sinh(b*x + d)**4*cosh(b*x + d)/(105*b) - exp(2*a)*exp(2*b*x)*sinh(
b*x + d)**3*cosh(b*x + d)**2/(105*b) + 2*exp(2*a)*exp(2*b*x)*sinh(b*x + d)
**2*cosh(b*x + d)**3/(35*b) + 8*exp(2*a)*exp(2*b*x)*sinh(b*x + d)*cosh(b*x
 + d)**4/(35*b) - 4*exp(2*a)*exp(2*b*x)*cosh(b*x + d)**5/(35*b), Ne(b, 0))
, (x*exp(2*a)*sinh(d)**2*cosh(d)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.74

$$\int e^{2(a+bx)}\cosh^3(d+bx)\sinh^2(d+bx)\,dx = \frac{\left(21\,e^{(-2\,bx-2\,d)}-70\,e^{(-4\,bx-4\,d)}-210\,e^{(-6\,bx-6\,d)}+15\right)e^{(7\,bx+2\,a+5\,d)}}{3360\,b} - \frac{\left(3\,e^{(-bx-d)}+e^{(-3\,bx-3\,d)}\right)e^{(2\,a-2\,d)}}{96\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^3*sinh(b*x+d)^2,x, algorithm="maxima"
)
```

output
```
1/3360*(21*e^(-2*b*x - 2*d) - 70*e^(-4*b*x - 4*d) - 210*e^(-6*b*x - 6*d) +
 15)*e^(7*b*x + 2*a + 5*d)/b - 1/96*(3*e^(-b*x - d) + e^(-3*b*x - 3*d))*e^
(2*a - 2*d)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.79

$$\int e^{2(a+bx)}\cosh^3(d+bx)\sinh^2(d+bx)\,dx = -\frac{\left(35\left(3\,e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right)e^{(-3\,bx-3\,d)}-15\,e^{(7\,bx+2\,a+7\,d)}-21\,e^{(5\,bx+2\,a+5\,d)}+70\,e^{(3\,bx+2\,a+3\,d)}+210\,e^{(bx}}{3360\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^3*sinh(b*x+d)^2,x, algorithm="giac")
```

output
```
-1/3360*(35*(3*e^(2*b*x + 2*a + 2*d) + e^(2*a))*e^(-3*b*x - 3*d) - 15*e^(7
*b*x + 2*a + 7*d) - 21*e^(5*b*x + 2*a + 5*d) + 70*e^(3*b*x + 2*a + 3*d) +
210*e^(b*x + 2*a + d))*e^(-2*d)/b
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.85

$$\int e^{2(a+bx)}\cosh^3(d+bx)\sinh^2(d+bx)\,dx = \frac{e^{2a+3d+5bx}}{160b}-\frac{e^{2a-d+bx}}{16b}-\frac{e^{2a-3d-bx}}{32b}-\frac{e^{2a+d+3bx}}{48b}-\frac{e^{2a-5d-3bx}}{96b}+\frac{e^{2a+5d+7bx}}{224b}$$

input
```
int(cosh(d + b*x)^3*exp(2*a + 2*b*x)*sinh(d + b*x)^2,x)
```

output
```
exp(2*a + 3*d + 5*b*x)/(160*b) - exp(2*a - d + b*x)/(16*b) - exp(2*a - 3*d
 - b*x)/(32*b) - exp(2*a + d + 3*b*x)/(48*b) - exp(2*a - 5*d - 3*b*x)/(96*
b) + exp(2*a + 5*d + 7*b*x)/(224*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.71

$$\int e^{2(a+bx)}\cosh^3(d+bx)\sinh^2(d+bx)\,dx = \frac{e^{2a}\left(15e^{10bx+10d}+21e^{8bx+8d}-70e^{6bx+6d}-210e^{4bx+4d}-105e^{2bx+2d}-35\right)}{3360e^{3bx+5d}b}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^3*sinh(b*x+d)^2,x)
```

output
```
(e**(2*a)*(15*e**(10*b*x + 10*d) + 21*e**(8*b*x + 8*d) - 70*e**(6*b*x + 6*
d) - 210*e**(4*b*x + 4*d) - 105*e**(2*b*x + 2*d) - 35))/(3360*e**(3*b*x +
5*d)*b)
```

3.59 $\int e^{2(a+bx)} \cosh^3(d+bx) \sinh(d+bx) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 76

$$\int e^{2(a+bx)} \cosh^3(d+bx) \sinh(d+bx) \, dx = \frac{e^{2(a-2d)-2bx}}{32b} + \frac{e^{2(a+d)+4bx}}{32b} + \frac{e^{2(a+2d)+6bx}}{96b} - \frac{1}{8}e^{2a-2d}x$$

output
```
1/32*exp(-2*b*x+2*a-4*d)/b+1/32*exp(4*b*x+2*a+2*d)/b+1/96*exp(6*b*x+2*a+4*
d)/b-1/8*exp(2*a-2*d)*x
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int e^{2(a+bx)} \cosh^3(d+bx) \sinh(d+bx) \, dx = \frac{e^{2a}\left(3\left(\left(e^{4bx}-4bx\right)\cosh(2d)+\left(e^{4bx}+4bx\right)\sinh(2d)\right)+e^{-2bx}\left(\left(3+e^{8bx}\right)\cosh(4d)+\left(-3+e^{8bx}\right)\sinh(4c\right.\right.}{96b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[d + b*x]^3*Sinh[d + b*x],x]
```

output
```
(E^(2*a)*(3*((E^(4*b*x) - 4*b*x)*Cosh[2*d] + (E^(4*b*x) + 4*b*x)*Sinh[2*d]
) + ((3 + E^(8*b*x))*Cosh[4*d] + (-3 + E^(8*b*x))*Sinh[4*d])/E^(2*b*x)))/(
96*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.63, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2720, 27, 354, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)} \sinh(bx+d)\cosh^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{1}{16}e^{2a-3bx}\left(1-e^{2bx}\right)\left(1+e^{2bx}\right)^3de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^{2a}\int e^{-3bx}\left(1-e^{2bx}\right)\left(1+e^{2bx}\right)^3de^{bx}}{16b}$$

$$\Big\downarrow \text{354}$$

$$-\frac{e^{2a}\int e^{-2bx}\left(1-e^{2bx}\right)\left(1+e^{2bx}\right)^3de^{2bx}}{32b}$$

$$\Big\downarrow \text{84}$$

$$-\frac{e^{2a}\int \left(e^{-2bx}+2e^{-bx}-3e^{2bx}\right)de^{2bx}}{32b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{2a}\left(-e^{-bx}-e^{2bx}-\frac{1}{3}e^{3bx}+2\log\left(e^{2bx}\right)\right)}{32b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[d + b*x]^3*Sinh[d + b*x],x]
```

output
```
-1/32*(E^(2*a)*(-E^(-(b*x)) - E^(2*b*x) - E^(3*b*x)/3 + 2*Log[E^(2*b*x)]))
/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 82.45 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

method	result
risch	$\frac{e^{-2bx+2a-4d}}{32b}+\frac{e^{4bx+2a+2d}}{32b}+\frac{e^{6bx+2a+4d}}{96b}-\frac{e^{2a-2d}x}{8}$
default	$-\frac{x\cosh(2a-2d)}{8}+\frac{\sinh(-2bx+2a-4d)}{32b}+\frac{\sinh(4bx+2a+2d)}{32b}+\frac{\sinh(6bx+2a+4d)}{96b}-\frac{x\sinh(2a-2d)}{8}+\frac{\cosh(-2bx+2a-4d}{32b}$
orering	$\frac{(12bx-1)e^{2bx+2a}\cosh(bx+d)^3\sinh(bx+d)}{12b}+\frac{(bx+2)\left(2\,e^{2bx+2a}\cosh(bx+d)^3\sinh(bx+d)b+3\,e^{2bx+2a}\cosh(bx+d)^2\sinh(bx+d)^2b\right.}{12b^2}$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^3*sinh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/32*exp(-2*b*x+2*a-4*d)/b+1/32*exp(4*b*x+2*a+2*d)/b+1/96*exp(6*b*x+2*a+4*
d)/b-1/8*exp(2*a-2*d)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(63) = 126$.

Time = 0.09 (sec) , antiderivative size = 345, normalized size of antiderivative = 4.54

$$\int e^{2(a+bx)}\cosh^3(d+bx)\sinh(d+bx)\,dx$$
$$=\frac{4\cosh(bx+d)^4\cosh(-2a+2d)+4(\cosh(-2a+2d)-\sinh(-2a+2d))\sinh(bx+d)^4-3(4bx-}{}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^3*sinh(b*x+d),x, algorithm="fricas")
```

output
```
1/96*(4*cosh(b*x + d)^4*cosh(-2*a + 2*d) + 4*(cosh(-2*a + 2*d) - sinh(-2*a
 + 2*d))*sinh(b*x + d)^4 - 3*(4*b*x - 1)*cosh(b*x + d)^2*cosh(-2*a + 2*d)
- 8*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh
(b*x + d)^3 + 3*(8*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (4*b*x - 1)*cosh(-2*
a + 2*d) + (4*b*x - 8*cosh(b*x + d)^2 - 1)*sinh(-2*a + 2*d))*sinh(b*x + d)
^2 - 2*(4*cosh(b*x + d)^3*cosh(-2*a + 2*d) - 3*(4*b*x + 1)*cosh(b*x + d)*c
osh(-2*a + 2*d) - (4*cosh(b*x + d)^3 - 3*(4*b*x + 1)*cosh(b*x + d))*sinh(-
2*a + 2*d))*sinh(b*x + d) - (4*cosh(b*x + d)^4 - 3*(4*b*x - 1)*cosh(b*x +
d)^2)*sinh(-2*a + 2*d))/(b*cosh(b*x + d)^2 - 2*b*cosh(b*x + d)*sinh(b*x +
d) + b*sinh(b*x + d)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(61) = 122$.

Time = 2.20 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.07

$$\int e^{2(a+bx)}\cosh^3(d+bx)\sinh(d+bx)\,dx$$
$$= \begin{cases} \frac{xe^{2a}e^{2bx}\sinh^4(bx+d)}{8} - \frac{xe^{2a}e^{2bx}\sinh^3(bx+d)\cosh(bx+d)}{4} + \frac{xe^{2a}e^{2bx}\sinh(bx+d)\cosh^3(bx+d)}{4} - \frac{xe^{2a}e^{2bx}\cosh^4(bx+d)}{8} + \frac{e^{2a}e^{2b}}{} \\ xe^{2a}\sinh(d)\cosh^3(d) \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)**3*sinh(b*x+d),x)
```

output
```
Piecewise((x*exp(2*a)*exp(2*b*x)*sinh(b*x + d)**4/8 - x*exp(2*a)*exp(2*b*x
)*sinh(b*x + d)**3*cosh(b*x + d)/4 + x*exp(2*a)*exp(2*b*x)*sinh(b*x + d)*c
osh(b*x + d)**3/4 - x*exp(2*a)*exp(2*b*x)*cosh(b*x + d)**4/8 + exp(2*a)*ex
p(2*b*x)*sinh(b*x + d)**4/(48*b) - exp(2*a)*exp(2*b*x)*sinh(b*x + d)**3*co
sh(b*x + d)/(6*b) + exp(2*a)*exp(2*b*x)*sinh(b*x + d)**2*cosh(b*x + d)**2/
(4*b) + exp(2*a)*exp(2*b*x)*cosh(b*x + d)**4/(16*b), Ne(b, 0)), (x*exp(2*a
)*sinh(d)*cosh(d)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int e^{2(a+bx)} \cosh^3(d+bx) \sinh(d+bx) \, dx = \frac{\left(3\, e^{(-2\, bx-2\, d)}+1\right)e^{(6\, bx+2\, a+4\, d)}}{96\, b} - \frac{(bx+d)e^{(2\, a-2\, d)}}{8\, b} + \frac{e^{(-2\, bx+2\, a-4\, d)}}{32\, b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^3*sinh(b*x+d),x, algorithm="maxima")
```

output
```
1/96*(3*e^(-2*b*x - 2*d) + 1)*e^(6*b*x + 2*a + 4*d)/b - 1/8*(b*x + d)*e^(2
*a - 2*d)/b + 1/32*e^(-2*b*x + 2*a - 4*d)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int e^{2(a+bx)} \cosh^3(d+bx) \sinh(d+bx) \, dx = \frac{\left(3\left(2\, e^{(2\, bx+2\, a+2\, d)}+e^{(2\, a)}\right)e^{(-2\, bx-2\, d)}-12\,(bx+d)e^{(2\, a)}+e^{(6\, bx+2\, a+6\, d)}+3\, e^{(4\, bx+2\, a+4\, d)}\right)e^{(-2\, d)}}{96\, b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^3*sinh(b*x+d),x, algorithm="giac")
```

output
```
1/96*(3*(2*e^(2*b*x + 2*a + 2*d) + e^(2*a))*e^(-2*b*x - 2*d) - 12*(b*x + d
)*e^(2*a) + e^(6*b*x + 2*a + 6*d) + 3*e^(4*b*x + 2*a + 4*d))*e^(-2*d)/b
```

Mupad [B] (verification not implemented)

Time = 3.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09

$$\int e^{2(a+bx)}\cosh^3(d+bx)\sinh(d+bx)\,dx$$
$$= \frac{e^{2\,a+2\,b\,x}\,\left(2\cosh(2\,d+2\,b\,x)+2\cosh(4\,d+4\,b\,x)+\sinh(2\,d+2\,b\,x)-\sinh(4\,d+4\,b\,x)+6\,b\,x\sinh(2\,d+\right.}{48\,b}$$

input
```
int(cosh(d + b*x)^3*exp(2*a + 2*b*x)*sinh(d + b*x),x)
```

output
```
(exp(2*a + 2*b*x)*(2*cosh(2*d + 2*b*x) + 2*cosh(4*d + 4*b*x) + sinh(2*d +
2*b*x) - sinh(4*d + 4*b*x) + 6*b*x*sinh(2*d + 2*b*x) - 6*b*x*cosh(2*d + 2*
b*x)))/(48*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int e^{2(a+bx)}\cosh^3(d+bx)\sinh(d+bx)\,dx = \frac{e^{2a}\left(e^{8bx+8d}+3e^{6bx+6d}-12e^{2bx+2d}bx+3\right)}{96e^{2bx+4d}b}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^3*sinh(b*x+d),x)
```

output
```
(e**(2*a)*(e**(8*b*x + 8*d) + 3*e**(6*b*x + 6*d) - 12*e**(2*b*x + 2*d)*b*x
 + 3))/(96*e**(2*b*x + 4*d)*b)
```

3.60 $\int e^{2(a+bx)} \cosh^3(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 78

$$\int e^{2(a+bx)} \cosh^3(d+bx)\,dx = -\frac{e^{2a-3d-bx}}{8b} + \frac{3e^{2a-d+bx}}{8b} + \frac{e^{2a+d+3bx}}{8b} + \frac{e^{2a+3d+5bx}}{40b}$$

output
```
-1/8*exp(-b*x+2*a-3*d)/b+3/8*exp(b*x+2*a-d)/b+1/8*exp(3*b*x+2*a+d)/b+1/40*
exp(5*b*x+2*a+3*d)/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10

$$\int e^{2(a+bx)} \cosh^3(d+bx)\,dx$$
$$= \frac{e^{2a-bx}\left(5e^{2bx}\left(3+e^{2bx}\right)\cosh(d) + \left(-5+e^{6bx}\right)\cosh(3d) - 15e^{2bx}\sinh(d) + 5e^{4bx}\sinh(d) + 5\sinh(3d) + e^{6}\right.}{40b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[d + b*x]^3,x]
```

output
```
(E^(2*a - b*x)*(5*E^(2*b*x)*(3 + E^(2*b*x))*Cosh[d] + (-5 + E^(6*b*x))*Cos
h[3*d] - 15*E^(2*b*x)*Sinh[d] + 5*E^(4*b*x)*Sinh[d] + 5*Sinh[3*d] + E^(6*b
*x)*Sinh[3*d]))/(40*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.56, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\cosh^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{8}e^{2a-2bx}\left(1+e^{2bx}\right)^3de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{2a}\int e^{-2bx}\left(1+e^{2bx}\right)^3de^{bx}}{8b}$$

$$\Big\downarrow \text{244}$$

$$\frac{e^{2a}\int \left(3+e^{-2bx}+3e^{2bx}+e^{4bx}\right)de^{bx}}{8b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2a}\left(-e^{-bx}+3e^{bx}+e^{3bx}+\frac{1}{5}e^{5bx}\right)}{8b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[d + b*x]^3,x]
```

output
```
(E^(2*a)*(-E^(-(b*x)) + 3*E^(b*x) + E^(3*b*x) + E^(5*b*x)/5))/(8*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{e^{-bx+2a-3d}}{8b}+\frac{3\,e^{bx+2a-d}}{8b}+\frac{e^{3bx+2a+d}}{8b}+\frac{e^{5bx+2a+3d}}{40b}$
parallelrisch	$-\frac{2\,e^{2bx+2a}\left(5\tanh\left(\frac{bx}{2}+\frac{d}{2}\right)^{5}-10\tanh\left(\frac{bx}{2}+\frac{d}{2}\right)^{4}+10\tanh\left(\frac{bx}{2}+\frac{d}{2}\right)^{3}-3\tanh\left(\frac{bx}{2}+\frac{d}{2}\right)+2\right)}{5b\left(-1+\tanh\left(\frac{bx}{2}+\frac{d}{2}\right)\right)^{3}\left(1+\tanh\left(\frac{bx}{2}+\frac{d}{2}\right)\right)^{3}}$
default	$-\frac{\sinh(-bx+2a-3d)}{8b}+\frac{3\sinh(bx+2a-d)}{8b}+\frac{\sinh(3bx+2a+d)}{8b}+\frac{\sinh(5bx+2a+3d)}{40b}-\frac{\cosh(-bx+2a-3d)}{8b}+\frac{3\cosh(b$
orering	$\frac{8\,e^{2bx+2a}\cosh(bx+d)^{3}}{15b}+\frac{\frac{28\,e^{2bx+2a}\cosh(bx+d)^{3}b}{15}+\frac{14\,e^{2bx+2a}\cosh(bx+d)^{2}\sinh(bx+d)b}{5}}{b^{2}}-\frac{8\left(7\,e^{2bx+2a}\cosh(bx+d)^{3}b^{2}+12\,e\right.}{}$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*exp(-b*x+2*a-3*d)/b+3/8*exp(b*x+2*a-d)/b+1/8*exp(3*b*x+2*a+d)/b+1/40*
exp(5*b*x+2*a+3*d)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(66) = 132$.

Time = 0.08 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.97

$$\int e^{2(a+bx)}\cosh^3(d+bx)\,dx = -\frac{2\cosh(bx+d)^3\cosh(-2a+2d) - 3(\cosh(-2a+2d) - \sinh(-2a+2d))\sinh(bx+d)^3 + 6(\cosh$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^3,x, algorithm="fricas")
```

output
```
-1/20*(2*cosh(b*x + d)^3*cosh(-2*a + 2*d) - 3*(cosh(-2*a + 2*d) - sinh(-2*
a + 2*d))*sinh(b*x + d)^3 + 6*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x +
 d)*sinh(-2*a + 2*d))*sinh(b*x + d)^2 - 10*cosh(b*x + d)*cosh(-2*a + 2*d)
- (9*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (9*cosh(b*x + d)^2 - 5)*sinh(-2*a
+ 2*d) - 5*cosh(-2*a + 2*d))*sinh(b*x + d) - 2*(cosh(b*x + d)^3 - 5*cosh(b
*x + d))*sinh(-2*a + 2*d))/(b*cosh(b*x + d)^2 - 2*b*cosh(b*x + d)*sinh(b*x
 + d) + b*sinh(b*x + d)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(61) = 122$.

Time = 0.92 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.59

$$\int e^{2(a+bx)}\cosh^3(d+bx)\,dx = \begin{cases} \frac{2e^{2a}e^{2bx}\sinh^3(bx+d)}{5b} - \frac{4e^{2a}e^{2bx}\sinh^2(bx+d)\cosh(bx+d)}{5b} + \frac{e^{2a}e^{2bx}\sinh(bx+d)\cosh^2(bx+d)}{5b} + \frac{2e^{2a}e^{2bx}\cosh^3(bx+d)}{5b} & \text{for } b \neq \\ xe^{2a}\cosh^3(d) & \text{otherw} \end{cases}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)**3,x)
```

output
```
Piecewise((2*exp(2*a)*exp(2*b*x)*sinh(b*x + d)**3/(5*b) - 4*exp(2*a)*exp(2
*b*x)*sinh(b*x + d)**2*cosh(b*x + d)/(5*b) + exp(2*a)*exp(2*b*x)*sinh(b*x
+ d)*cosh(b*x + d)**2/(5*b) + 2*exp(2*a)*exp(2*b*x)*cosh(b*x + d)**3/(5*b)
, Ne(b, 0)), (x*exp(2*a)*cosh(d)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.76

$$\int e^{2(a+bx)}\cosh^3(d+bx)\,dx = \frac{\left(5\,e^{(-2\,bx-2\,d)}+15\,e^{(-4\,bx-4\,d)}+1\right)e^{(5\,bx+2\,a+3\,d)}}{40\,b}-\frac{e^{(-bx+2\,a-3\,d)}}{8\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^3,x, algorithm="maxima")
```

output
```
1/40*(5*e^(-2*b*x - 2*d) + 15*e^(-4*b*x - 4*d) + 1)*e^(5*b*x + 2*a + 3*d)/
b - 1/8*e^(-b*x + 2*a - 3*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int e^{2(a+bx)}\cosh^3(d+bx)\,dx = \frac{\left(e^{(5\,bx+2\,a+5\,d)}+5\,e^{(3\,bx+2\,a+3\,d)}+15\,e^{(bx+2\,a+d)}-5\,e^{(-bx+2\,a-d)}\right)e^{(-2\,d)}}{40\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^3,x, algorithm="giac")
```

output
```
1/40*(e^(5*b*x + 2*a + 5*d) + 5*e^(3*b*x + 2*a + 3*d) + 15*e^(b*x + 2*a +
d) - 5*e^(-b*x + 2*a - d))*e^(-2*d)/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int e^{2(a+bx)}\cosh^3(d+bx)\,dx=\frac{e^{2\,a+d+3\,b\,x}}{8\,b}+\frac{3\,e^{2\,a-d+b\,x}}{8\,b}-\frac{e^{2\,a-3\,d-b\,x}}{8\,b}+\frac{e^{2\,a+3\,d+5\,b\,x}}{40\,b}$$

input
```
int(cosh(d + b*x)^3*exp(2*a + 2*b*x),x)
```

output
```
exp(2*a + d + 3*b*x)/(8*b) + (3*exp(2*a - d + b*x))/(8*b) - exp(2*a - 3*d
- b*x)/(8*b) + exp(2*a + 3*d + 5*b*x)/(40*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.73

$$\int e^{2(a+bx)}\cosh^3(d+bx)\,dx=\frac{e^{2a}\left(e^{6bx+6d}+5e^{4bx+4d}+15e^{2bx+2d}-5\right)}{40e^{bx+3d}b}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^3,x)
```

output
```
(e**(2*a)*(e**(6*b*x + 6*d) + 5*e**(4*b*x + 4*d) + 15*e**(2*b*x + 2*d) - 5
))/(40*e**(b*x + 3*d)*b)
```

3.61 $\int e^{2(a+bx)}\cosh^2(d+bx)\coth(d+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int e^{2(a+bx)}\cosh^2(d+bx)\coth(d+bx)\,dx=\frac{e^{2a+2bx}}{2b}+\frac{e^{2(a+d)+4bx}}{16b}-\frac{1}{4}e^{2a-2d}x+\frac{e^{2a-2d}\log\left(1-e^{2d+2bx}\right)}{b}$$

output
```
1/2*exp(2*b*x+2*a)/b+1/16*exp(4*b*x+2*a+2*d)/b-1/4*exp(2*a-2*d)*x+exp(2*a-
2*d)*ln(1-exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.76

$$\int e^{2(a+bx)}\cosh^2(d+bx)\coth(d+bx)\,dx=\frac{e^{2a-2d}\left(-4d+8e^{2(d+bx)}+e^{4(d+bx)}-4bx+16\log\left(1-e^{2(d+bx)}\right)\right)}{16b}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[d + b*x]^2*Coth[d + b*x],x]
```

output
```
(E^(2*a - 2*d)*(-4*d + 8*E^(2*(d + b*x)) + E^(4*(d + b*x)) - 4*b*x + 16*Lo
g[1 - E^(2*(d + b*x))]))/(16*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.54, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2720, 27, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\cosh^2(bx+d)\coth(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{e^{2a-bx}(1+e^{2bx})^3}{4(1-e^{2bx})}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^{2a}\int \frac{e^{-bx}(1+e^{2bx})^3}{1-e^{2bx}}de^{bx}}{4b}$$

$$\Big\downarrow \text{354}$$

$$-\frac{e^{2a}\int \frac{e^{-bx}(1+e^{2bx})^3}{1-e^{2bx}}de^{2bx}}{8b}$$

$$\Big\downarrow \text{93}$$

$$-\frac{e^{2a}\int \left(e^{-bx}-e^{2bx}-4-\frac{8}{-1+e^{2bx}}\right)de^{2bx}}{8b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{2a}\left(-\frac{9}{2}e^{2bx}+\log\left(e^{2bx}\right)-8\log\left(1-e^{2bx}\right)\right)}{8b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[d + b*x]^2*Coth[d + b*x],x]
```

output
```
-1/8*(E^(2*a)*((-9*E^(2*b*x))/2 + Log[E^(2*b*x)] - 8*Log[1 - E^(2*b*x)]))/
b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.15

method	result	size
risch	$-\frac{\mathrm{e}^{2a-2d} x}{4}+\frac{\mathrm{e}^{4bx+2a+2d}}{16b}+\frac{\mathrm{e}^{2bx+2a}}{2b}-\frac{2\,\mathrm{e}^{2a-2d} a}{b}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-\mathrm{e}^{2a-2d}\right) \mathrm{e}^{2a-2d}}{b}$	91

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^2*coth(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*exp(2*a-2*d)*x+1/16*exp(4*b*x+2*a+2*d)/b+1/2*exp(2*b*x+2*a)/b-2/b*exp
(2*a-2*d)*a+ln(exp(2*b*x+2*a)-exp(2*a-2*d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(69) = 138.

Time = 0.10 (sec) , antiderivative size = 333, normalized size of antiderivative = 4.22

$$\int e^{2(a+bx)}\cosh^2(d+bx)\coth(d+bx)\,dx$$

$$= \frac{\cosh(bx+d)^4\cosh(-2a+2d) + (\cosh(-2a+2d) - \sinh(-2a+2d))\sinh(bx+d)^4 + 4(\cosh(bx+}{}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^2*coth(b*x+d),x, algorithm="fricas")
```

output
```
1/16*(cosh(b*x + d)^4*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2
*d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*s
inh(-2*a + 2*d))*sinh(b*x + d)^3 - 4*b*x*cosh(-2*a + 2*d) + 8*cosh(b*x + d
)^2*cosh(-2*a + 2*d) + 2*(3*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (3*cosh(b*x
 + d)^2 + 4)*sinh(-2*a + 2*d) + 4*cosh(-2*a + 2*d))*sinh(b*x + d)^2 + 16*(
cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*log(2*sinh(b*x + d)/(cosh(b*x + d) -
sinh(b*x + d))) + 4*(cosh(b*x + d)^3*cosh(-2*a + 2*d) + 4*cosh(b*x + d)*co
sh(-2*a + 2*d) - (cosh(b*x + d)^3 + 4*cosh(b*x + d))*sinh(-2*a + 2*d))*sin
h(b*x + d) - (cosh(b*x + d)^4 - 4*b*x + 8*cosh(b*x + d)^2)*sinh(-2*a + 2*d
))/b
```

Sympy [F]

$$\int e^{2(a+bx)}\cosh^2(d+bx)\coth(d+bx)\,dx = e^{2a}\int e^{2bx}\cosh^2(bx+d)\coth(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)**2*coth(b*x+d),x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*cosh(b*x + d)**2*coth(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.23

$$\int e^{2(a+bx)}\cosh^2(d+bx)\coth(d+bx)\,dx = \frac{\left(8\,e^{(-2\,bx-2\,d)}+1\right)e^{(4\,bx+2\,a+2\,d)}}{16\,b} + \frac{7\,(bx+d)e^{(2\,a-2\,d)}}{4\,b} + \frac{e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}+1\right)}{b} + \frac{e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}-1\right)}{b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^2*coth(b*x+d),x, algorithm="maxima")
```

output
```
1/16*(8*e^(-2*b*x - 2*d) + 1)*e^(4*b*x + 2*a + 2*d)/b + 7/4*(b*x + d)*e^(2
*a - 2*d)/b + e^(2*a - 2*d)*log(e^(-b*x - d) + 1)/b + e^(2*a - 2*d)*log(e^
(-b*x - d) - 1)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\int e^{2(a+bx)}\cosh^2(d+bx)\coth(d+bx)\,dx = -\frac{4\,(bx+d)e^{(2\,a-2\,d)}-\left(e^{(4\,bx+2\,a+6\,d)}+8\,e^{(2\,bx+2\,a+4\,d)}\right)e^{(-4\,d)}-16\,e^{(2\,a-2\,d)}\log\left(\left|e^{(2\,bx+2\,d)}-1\right|\right)}{16\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)^2*coth(b*x+d),x, algorithm="giac")
```

output
```
-1/16*(4*(b*x + d)*e^(2*a - 2*d) - (e^(4*b*x + 2*a + 6*d) + 8*e^(2*b*x + 2
*a + 4*d))*e^(-4*d) - 16*e^(2*a - 2*d)*log(abs(e^(2*b*x + 2*d) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 2.82 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int e^{2(a+bx)}\cosh^2(d+bx)\coth(d+bx)\,dx = \frac{e^{2\,a+2\,d+4\,b\,x}}{16\,b}-\frac{x\,e^{2\,a-2\,d}}{4}+\frac{e^{2\,a+2\,b\,x}}{2\,b}+\frac{e^{2\,a-2\,d}\,\ln\left(e^{2\,d}\,e^{2\,b\,x}-1\right)}{b}$$

input
```
int(cosh(d + b*x)^2*coth(d + b*x)*exp(2*a + 2*b*x),x)
```

output
```
exp(2*a + 2*d + 4*b*x)/(16*b) - (x*exp(2*a - 2*d))/4 + exp(2*a + 2*b*x)/(2
*b) + (exp(2*a - 2*d)*log(exp(2*d)*exp(2*b*x) - 1))/b
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int e^{2(a+bx)}\cosh^2(d+bx)\coth(d+bx)\,dx$$
$$=\frac{e^{2a}\left(e^{4bx+4d}+8e^{2bx+2d}+16\log\left(e^{bx+d}-1\right)+16\log\left(e^{bx+d}+1\right)-4bx\right)}{16e^{2d}b}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)^2*coth(b*x+d),x)
```

output
```
(e**(2*a)*(e**(4*b*x + 4*d) + 8*e**(2*b*x + 2*d) + 16*log(e**(b*x + d) - 1
) + 16*log(e**(b*x + d) + 1) - 4*b*x))/(16*e**(2*d)*b)
```

3.62 $\int e^{2(a+bx)} \cosh(d+bx) \coth^2(d+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int e^{2(a+bx)} \cosh(d+bx) \coth^2(d+bx)\,dx = \frac{5e^{2a-d+bx}}{2b} + \frac{e^{2a+d+3bx}}{6b} + \frac{2e^{2a-d+bx}}{b\left(1-e^{2d+2bx}\right)} - \frac{4e^{2a-2d}\text{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
5/2*exp(b*x+2*a-d)/b+1/6*exp(3*b*x+2*a+d)/b+2*exp(b*x+2*a-d)/b/(1-exp(2*b*
x+2*d))-4*exp(2*a-2*d)*arctanh(exp(b*x+d))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.51 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.41

$$\int e^{2(a+bx)} \cosh(d+bx) \coth^2(d+bx)\,dx = \frac{e^{2a-7d-5bx}\left(-21\left(36015+91925e^{2(d+bx)}+61158e^{4(d+bx)}-20166e^{6(d+bx)}-15061e^{8(d+bx)}+753e^{10(d+bx)}\right)\right.}{}$$

input
```
Integrate[E^(2*(a + b*x))*Cosh[d + b*x]*Coth[d + b*x]^2,x]
```

output
```
(E^(2*a - 7*d - 5*b*x)*(-21*(36015 + 91925*E^(2*(d + b*x)) + 61158*E^(4*(d
 + b*x)) - 20166*E^(6*(d + b*x)) - 15061*E^(8*(d + b*x)) + 753*E^(10*(d +
b*x))) - (315*(-2401 - 5328*E^(2*(d + b*x)) - 1821*E^(4*(d + b*x)) + 3264*
E^(6*(d + b*x)) + 1149*E^(8*(d + b*x)) - 240*E^(10*(d + b*x)) + E^(12*(d +
 b*x)))*ArcTanh[Sqrt[E^(2*(d + b*x))]])/Sqrt[E^(2*(d + b*x))] + 256*E^(8*(
d + b*x))*(1 + E^(2*(d + b*x)))^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1,
 1, 1, 11/2}, E^(2*(d + b*x))]))/(60480*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2720, 27, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\cosh(bx+d)\coth^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{e^{2a}\left(1+e^{2bx}\right)^3}{2\left(1-e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{2a}\int \frac{\left(1+e^{2bx}\right)^3}{\left(1-e^{2bx}\right)^2}de^{bx}}{2b}$$

$$\Big\downarrow \text{300}$$

$$\frac{e^{2a}\int \left(-\frac{4\left(1-3e^{2bx}\right)}{\left(1-e^{2bx}\right)^2}+e^{2bx}+5\right)de^{bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2a}\left(-8\text{arctanh}\left(e^{bx}\right)+5e^{bx}+\frac{1}{3}e^{3bx}+\frac{4e^{bx}}{1-e^{2bx}}\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Cosh[d + b*x]*Coth[d + b*x]^2,x]
```

output
```
(E^(2*a)*(5*E^(b*x) + E^(3*b*x)/3 + (4*E^(b*x))/(1 - E^(2*b*x)) - 8*ArcTan
h[E^(b*x)]))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.35

method	result	size
risch	$\frac{{\mathrm e}^{3 b x +2 a +d}}{6 b}+\frac{5 \,{\mathrm e}^{b x +2 a -d}}{2 b}+\frac{2 \,{\mathrm e}^{b x +4 a -d}}{\left(-{\mathrm e}^{2 b x +2 a +2 d}+{\mathrm e}^{2 a}\right) b}-\frac{2 \ln \left({\mathrm e}^{b x +a}+{\mathrm e}^{a -d}\right) {\mathrm e}^{2 a -2 d}}{b}+\frac{2 \ln \left({\mathrm e}^{b x +a}-{\mathrm e}^{a -d}\right) {\mathrm e}^{2 a -2 d}}{b}$	126

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)*coth(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*exp(3*b*x+2*a+d)/b+5/2*exp(b*x+2*a-d)/b+2/(-exp(2*b*x+2*a+2*d)+exp(2*a
))/b*exp(b*x+4*a-d)-2*ln(exp(b*x+a)+exp(a-d))/b*exp(2*a-2*d)+2*ln(exp(b*x+
a)-exp(a-d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(81) = 162$.

Time = 0.12 (sec) , antiderivative size = 689, normalized size of antiderivative = 7.41

$$\int e^{2(a+bx)}\cosh(d+bx)\coth^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)*coth(b*x+d)^2,x, algorithm="fricas")
```

output
```
1/6*(cosh(b*x + d)^5*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*
d))*sinh(b*x + d)^5 + 5*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*si
nh(-2*a + 2*d))*sinh(b*x + d)^4 + 14*cosh(b*x + d)^3*cosh(-2*a + 2*d) + 2*
(5*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (5*cosh(b*x + d)^2 + 7)*sinh(-2*a +
2*d) + 7*cosh(-2*a + 2*d))*sinh(b*x + d)^3 + 2*(5*cosh(b*x + d)^3*cosh(-2*
a + 2*d) + 21*cosh(b*x + d)*cosh(-2*a + 2*d) - (5*cosh(b*x + d)^3 + 21*cos
h(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d)^2 - 27*cosh(b*x + d)*cosh(-2*a
 + 2*d) - 12*(cosh(b*x + d)^2*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(
-2*a + 2*d))*sinh(b*x + d)^2 + 2*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*
x + d)*sinh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^2 - 1)*sinh(-2*a +
 2*d) - cosh(-2*a + 2*d))*log(cosh(b*x + d) + sinh(b*x + d) + 1) + 12*(cos
h(b*x + d)^2*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh
(b*x + d)^2 + 2*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a
+ 2*d))*sinh(b*x + d) - (cosh(b*x + d)^2 - 1)*sinh(-2*a + 2*d) - cosh(-2*a
 + 2*d))*log(cosh(b*x + d) + sinh(b*x + d) - 1) + (5*cosh(b*x + d)^4*cosh(
-2*a + 2*d) + 42*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (5*cosh(b*x + d)^4 + 4
2*cosh(b*x + d)^2 - 27)*sinh(-2*a + 2*d) - 27*cosh(-2*a + 2*d))*sinh(b*x +
 d) - (cosh(b*x + d)^5 + 14*cosh(b*x + d)^3 - 27*cosh(b*x + d))*sinh(-2*a
+ 2*d))/(b*cosh(b*x + d)^2 + 2*b*cosh(b*x + d)*sinh(b*x + d) + b*sinh(b*x
+ d)^2 - b)
```

Sympy [F]

$$\int e^{2(a+bx)}\cosh(d+bx)\coth^2(d+bx)\,dx = e^{2a}\int e^{2bx}\cosh(bx+d)\coth^2(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)*coth(b*x+d)**2,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*cosh(b*x + d)*coth(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.19

$$\int e^{2(a+bx)}\cosh(d+bx)\coth^2(d+bx)\,dx = -\frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}+1\right)}{b} + \frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}-1\right)}{b} + \frac{\left(14\,e^{(-2\,bx-2\,d)}-27\,e^{(-4\,bx-4\,d)}+1\right)e^{(2\,a-2\,d)}}{6\,b\left(e^{(-3\,bx-3\,d)}-e^{(-5\,bx-5\,d)}\right)}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)*coth(b*x+d)^2,x, algorithm="maxima")
```

output
```
-2*e^(2*a - 2*d)*log(e^(-b*x - d) + 1)/b + 2*e^(2*a - 2*d)*log(e^(-b*x - d
) - 1)/b + 1/6*(14*e^(-2*b*x - 2*d) - 27*e^(-4*b*x - 4*d) + 1)*e^(2*a - 2*
d)/(b*(e^(-3*b*x - 3*d) - e^(-5*b*x - 5*d)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.10

$$\int e^{2(a+bx)}\cosh(d+bx)\coth^2(d+bx)\,dx = \frac{\left(e^{(3\,bx+2\,a+7\,d)}+15\,e^{(bx+2\,a+5\,d)}\right)e^{(-6\,d)}-12\,e^{(2\,a-2\,d)}\log\left(e^{(bx+d)}+1\right)+12\,e^{(2\,a-2\,d)}\log\left(\left|e^{(bx+d)}-1\right|\right)-\frac{12}{e^{(}}}{6\,b}$$

input
```
integrate(exp(2*b*x+2*a)*cosh(b*x+d)*coth(b*x+d)^2,x, algorithm="giac")
```

output
```
1/6*((e^(3*b*x + 2*a + 7*d) + 15*e^(b*x + 2*a + 5*d))*e^(-6*d) - 12*e^(2*a
 - 2*d)*log(e^(b*x + d) + 1) + 12*e^(2*a - 2*d)*log(abs(e^(b*x + d) - 1))
- 12*e^(b*x + 2*a - d)/(e^(2*b*x + 2*d) - 1))/b
```

Mupad [B] (verification not implemented)

Time = 2.89 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.24

$$\int e^{2(a+bx)}\cosh(d+bx)\coth^2(d+bx)\,dx=\frac{e^{2\,a+d+3\,b\,x}}{6\,b}+\frac{5\,e^{2\,a-d+b\,x}}{2\,b}-\frac{2\,e^{2\,a-d+b\,x}}{b\,\left(e^{2\,d+2\,b\,x}-1\right)}-\frac{4\,\sqrt{e^{4\,a-4\,d}}\,\mathrm{atan}\left(\frac{e^{2\,a}\,e^{-d}\,e^{b\,x}\,\sqrt{-b^2}}{b\,\sqrt{e^{4\,a}\,e^{-4\,d}}}\right)}{\sqrt{-b^2}}$$

input
```
int(cosh(d + b*x)*coth(d + b*x)^2*exp(2*a + 2*b*x),x)
```

output
```
exp(2*a + d + 3*b*x)/(6*b) + (5*exp(2*a - d + b*x))/(2*b) - (2*exp(2*a - d
 + b*x))/(b*(exp(2*d + 2*b*x) - 1)) - (4*exp(4*a - 4*d)^(1/2)*atan((exp(2*
a)*exp(-d)*exp(b*x)*(-b^2)^(1/2))/(b*(exp(4*a)*exp(-4*d))^(1/2))))/(-b^2)^
(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.41

$$\int e^{2(a+bx)}\cosh(d+bx)\coth^2(d+bx)\,dx=\frac{e^{2a}\left(e^{5bx+5d}+14e^{3bx+3d}+12e^{2bx+2d}\log\left(e^{bx+d}-1\right)-12e^{2bx+2d}\log\left(e^{bx+d}+1\right)-27e^{bx+d}-12\log\left(e^{bx+d}-\right.\right.}{6e^{2d}b\left(e^{2bx+2d}-1\right)}$$

input
```
int(exp(2*b*x+2*a)*cosh(b*x+d)*coth(b*x+d)^2,x)
```

output
```
(e**(2*a)*(e**(5*b*x + 5*d) + 14*e**(3*b*x + 3*d) + 12*e**(2*b*x + 2*d)*lo
g(e**(b*x + d) - 1) - 12*e**(2*b*x + 2*d)*log(e**(b*x + d) + 1) - 27*e**(b
*x + d) - 12*log(e**(b*x + d) - 1) + 12*log(e**(b*x + d) + 1)))/(6*e**(2*d
)*b*(e**(2*b*x + 2*d) - 1))
```

3.63 $\int e^{2(a+bx)} \coth^3(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int e^{2(a+bx)}\coth^3(d+bx)\,dx = \frac{e^{2a+2bx}}{2b} - \frac{2e^{2a-2d}}{b\left(1-e^{2d+2bx}\right)^2} + \frac{6e^{2a-2d}}{b\left(1-e^{2d+2bx}\right)} + \frac{3e^{2a-2d}\log\left(1-e^{2d+2bx}\right)}{b}$$

output
```
1/2*exp(2*b*x+2*a)/b-2*exp(2*a-2*d)/b/(1-exp(2*b*x+2*d))^2+6*exp(2*a-2*d)/
b/(1-exp(2*b*x+2*d))+3*exp(2*a-2*d)*ln(1-exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.41

$$\int e^{2(a+bx)}\coth^3(d+bx)\,dx = \frac{e^{2a}\left(e^{2bx}+6\cosh(2d)\log\left(\left(-1+e^{2bx}\right)\cosh(d)+\left(1+e^{2bx}\right)\sinh(d)\right)-\frac{4(\cosh(d)-\sinh(d))^4}{\left(\left(-1+e^{2bx}\right)\cosh(d)+\left(1+e^{2bx}\right)\sinh(d)\right)^2}-\right.}{2b}$$

input
```
Integrate[E^(2*(a + b*x))*Coth[d + b*x]^3,x]
```

output
```
(E^(2*a)*(E^(2*b*x) + 6*Cosh[2*d]*Log[(-1 + E^(2*b*x))*Cosh[d] + (1 + E^(2
*b*x))*Sinh[d]] - (4*(Cosh[d] - Sinh[d])^4)/((-1 + E^(2*b*x))*Cosh[d] + (1
 + E^(2*b*x))*Sinh[d])^2 - (12*(Cosh[d] - Sinh[d])^3)/((-1 + E^(2*b*x))*Co
sh[d] + (1 + E^(2*b*x))*Sinh[d]) - 6*Log[(-1 + E^(2*b*x))*Cosh[d] + (1 + E
^(2*b*x))*Sinh[d]]*Sinh[2*d]))/(2*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.58, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2720, 25, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)} \coth^3(bx+d) \, dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int -\frac{e^{2a+bx}\left(1+e^{2bx}\right)^3}{\left(1-e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{e^{2a+bx}\left(1+e^{2bx}\right)^3}{\left(1-e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{e^{2a}\int \frac{e^{bx}\left(1+e^{2bx}\right)^3}{\left(1-e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \; 353$$

$$-\frac{e^{2a}\int \frac{\left(1+e^{2bx}\right)^3}{\left(1-e^{2bx}\right)^3}de^{2bx}}{2b}$$

$$\Big\downarrow \; 49$$

$$-\frac{e^{2a}\int \left(-1-\frac{6}{-1+e^{2bx}}-\frac{12}{\left(-1+e^{2bx}\right)^2}-\frac{8}{\left(-1+e^{2bx}\right)^3}\right)de^{2bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{2a}\left(-e^{2bx}-\frac{12}{1-e^{2bx}}+\frac{4}{(1-e^{2bx})^2}-6\log\left(1-e^{2bx}\right)\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Coth[d + b*x]^3,x]
```

output
```
-1/2*(E^(2*a)*(-E^(2*b*x) + 4/(1 - E^(2*b*x))^2 - 12/(1 - E^(2*b*x)) - 6*L
og[1 - E^(2*b*x)]))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.11

method	result	size
risch	$\frac{e^{2bx+2a}}{2b}-\frac{6\,e^{2a-2d}a}{b}+\frac{2\left(-3\,e^{2bx+2a+2d}+2\,e^{2a}\right)e^{4a-2d}}{\left(-e^{2bx+2a+2d}+e^{2a}\right)^{2}b}+\frac{3\ln\left(e^{2bx+2a}-e^{2a-2d}\right)e^{2a-2d}}{b}$	119

input
```
int(exp(2*b*x+2*a)*coth(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*b*x+2*a)/b-6/b*exp(2*a-2*d)*a+2/(-exp(2*b*x+2*a+2*d)+exp(2*a))^2
/b*(-3*exp(2*b*x+2*a+2*d)+2*exp(2*a))*exp(4*a-2*d)+3*ln(exp(2*b*x+2*a)-exp
(2*a-2*d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 905 vs. 2(94) = 188.

Time = 0.09 (sec) , antiderivative size = 905, normalized size of antiderivative = 8.46

$$\int e^{2(a+bx)}\coth^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)^3,x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + d)^6*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*
d))*sinh(b*x + d)^6 + 6*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*si
nh(-2*a + 2*d))*sinh(b*x + d)^5 - 2*cosh(b*x + d)^4*cosh(-2*a + 2*d) + (15
*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (15*cosh(b*x + d)^2 - 2)*sinh(-2*a + 2
*d) - 2*cosh(-2*a + 2*d))*sinh(b*x + d)^4 + 4*(5*cosh(b*x + d)^3*cosh(-2*a
 + 2*d) - 2*cosh(b*x + d)*cosh(-2*a + 2*d) - (5*cosh(b*x + d)^3 - 2*cosh(b
*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d)^3 - 11*cosh(b*x + d)^2*cosh(-2*a
+ 2*d) + (15*cosh(b*x + d)^4*cosh(-2*a + 2*d) - 12*cosh(b*x + d)^2*cosh(-2
*a + 2*d) - (15*cosh(b*x + d)^4 - 12*cosh(b*x + d)^2 - 11)*sinh(-2*a + 2*d
) - 11*cosh(-2*a + 2*d))*sinh(b*x + d)^2 + 6*(cosh(b*x + d)^4*cosh(-2*a +
2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^4 + 4*(cosh(b*x
 + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^3 -
 2*cosh(b*x + d)^2*cosh(-2*a + 2*d) + 2*(3*cosh(b*x + d)^2*cosh(-2*a + 2*d
) - (3*cosh(b*x + d)^2 - 1)*sinh(-2*a + 2*d) - cosh(-2*a + 2*d))*sinh(b*x
+ d)^2 + 4*(cosh(b*x + d)^3*cosh(-2*a + 2*d) - cosh(b*x + d)*cosh(-2*a + 2
*d) - (cosh(b*x + d)^3 - cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d) -
(cosh(b*x + d)^4 - 2*cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2
*d))*log(2*sinh(b*x + d)/(cosh(b*x + d) - sinh(b*x + d))) + 2*(3*cosh(b*x
+ d)^5*cosh(-2*a + 2*d) - 4*cosh(b*x + d)^3*cosh(-2*a + 2*d) - 11*cosh(b*x
 + d)*cosh(-2*a + 2*d) - (3*cosh(b*x + d)^5 - 4*cosh(b*x + d)^3 - 11*co...
```

Sympy [F]

$$\int e^{2(a+bx)}\coth^3(d+bx)\,dx=e^{2a}\int e^{2bx}\coth^3{(bx+d)}\,dx$$

input
```
integrate(exp(2*b*x+2*a)*coth(b*x+d)**3,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*coth(b*x + d)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.29

$$\int e^{2(a+bx)}\coth^3(d+bx)\,dx = \frac{6\,(bx+d)e^{(2\,a-2\,d)}}{b} + \frac{3\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}+1\right)}{b} + \frac{3\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}-1\right)}{b} - \frac{\left(10\,e^{(-2\,bx-2\,d)}-5\,e^{(-4\,bx-4\,d)}-1\right)e^{(2\,a-2\,d)}}{2\,b\left(e^{(-2\,bx-2\,d)}-2\,e^{(-4\,bx-4\,d)}+e^{(-6\,bx-6\,d)}\right)}$$

input

```
integrate(exp(2*b*x+2*a)*coth(b*x+d)^3,x, algorithm="maxima")
```

output

```
6*(b*x + d)*e^(2*a - 2*d)/b + 3*e^(2*a - 2*d)*log(e^(-b*x - d) + 1)/b + 3*
e^(2*a - 2*d)*log(e^(-b*x - d) - 1)/b - 1/2*(10*e^(-2*b*x - 2*d) - 5*e^(-4
*b*x - 4*d) - 1)*e^(2*a - 2*d)/(b*(e^(-2*b*x - 2*d) - 2*e^(-4*b*x - 4*d) +
 e^(-6*b*x - 6*d)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90

$$\int e^{2(a+bx)}\coth^3(d+bx)\,dx = \frac{3\,e^{(2\,a-2\,d)}\log\left(\left|e^{(2\,bx+2\,d)}-1\right|\right)}{b} + \frac{e^{(2\,bx+2\,a)}}{2\,b} - \frac{\left(9\,e^{(4\,bx+2\,a+4\,d)}-6\,e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right)e^{(-2\,d)}}{2\,b\left(e^{(2\,bx+2\,d)}-1\right)^2}$$

input

```
integrate(exp(2*b*x+2*a)*coth(b*x+d)^3,x, algorithm="giac")
```

output

```
3*e^(2*a - 2*d)*log(abs(e^(2*b*x + 2*d) - 1))/b + 1/2*e^(2*b*x + 2*a)/b -
1/2*(9*e^(4*b*x + 2*a + 4*d) - 6*e^(2*b*x + 2*a + 2*d) + e^(2*a))*e^(-2*d)
/(b*(e^(2*b*x + 2*d) - 1)^2)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2(a+bx)} \coth^3(d+bx)\,dx = \int \coth(d+b\,x)^3\,e^{2\,a+2\,b\,x}\,dx$$

input
```
int(coth(d + b*x)^3*exp(2*a + 2*b*x),x)
```

output
```
int(coth(d + b*x)^3*exp(2*a + 2*b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.69

$$\int e^{2(a+bx)} \coth^3(d+bx)\,dx = \frac{e^{2a}\left(2e^{6bx+6d}+12e^{4bx+4d}\log\left(e^{bx+d}-1\right)+12e^{4bx+4d}\log\left(e^{bx+d}+1\right)-15e^{4bx+4d}-24e^{2bx+2d}\log\left(e^{bx+d}-1\right)\right.}{4e^{2d}b\left(e^{4bx+4d}-2e^{2bx+2d}+1\right)}$$

input
```
int(exp(2*b*x+2*a)*coth(b*x+d)^3,x)
```

output
```
(e**(2*a)*(2*e**(6*b*x + 6*d) + 12*e**(4*b*x + 4*d)*log(e**(b*x + d) - 1)
+ 12*e**(4*b*x + 4*d)*log(e**(b*x + d) + 1) - 15*e**(4*b*x + 4*d) - 24*e**
(2*b*x + 2*d)*log(e**(b*x + d) - 1) - 24*e**(2*b*x + 2*d)*log(e**(b*x + d)
 + 1) + 12*log(e**(b*x + d) - 1) + 12*log(e**(b*x + d) + 1) + 5))/(4*e**(2
*d)*b*(e**(4*b*x + 4*d) - 2*e**(2*b*x + 2*d) + 1))
```

3.64 $\int e^{2(a+bx)} \sinh^2(d+bx) \tanh(d+bx) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int e^{2(a+bx)} \sinh^2(d+bx) \tanh(d+bx) \, dx = -\frac{e^{2a+2bx}}{2b} + \frac{e^{2(a+d)+4bx}}{16b} - \frac{1}{4}e^{2a-2d}x + \frac{e^{2a-2d} \log\left(1+e^{2d+2bx}\right)}{b}$$

output
```
-1/2*exp(2*b*x+2*a)/b+1/16*exp(4*b*x+2*a+2*d)/b-1/4*exp(2*a-2*d)*x+exp(2*a
-2*d)*ln(1+exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

$$\int e^{2(a+bx)} \sinh^2(d+bx) \tanh(d+bx) \, dx = \frac{e^{2a-2d}\left(-4d-8e^{2(d+bx)}+e^{4(d+bx)}-4bx+16\log\left(1+e^{2(d+bx)}\right)\right)}{16b}$$

input
```
Integrate[E^(2*(a + b*x))*Sinh[d + b*x]^2*Tanh[d + b*x],x]
```

output
```
(E^(2*a - 2*d)*(-4*d - 8*E^(2*(d + b*x)) + E^(4*(d + b*x)) - 4*b*x + 16*Lo
g[1 + E^(2*(d + b*x))]))/(16*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.53, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2720, 27, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\sinh^2(bx+d)\tanh(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{e^{2a-bx}\left(1-e^{2bx}\right)^3}{4\left(1+e^{2bx}\right)}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^{2a}\int \frac{e^{-bx}\left(1-e^{2bx}\right)^3}{1+e^{2bx}}de^{bx}}{4b}$$

$$\Big\downarrow \text{354}$$

$$-\frac{e^{2a}\int \frac{e^{-bx}\left(1-e^{2bx}\right)^3}{1+e^{2bx}}de^{2bx}}{8b}$$

$$\Big\downarrow \text{93}$$

$$-\frac{e^{2a}\int \left(e^{-bx}-e^{2bx}+4-\frac{8}{1+e^{2bx}}\right)de^{2bx}}{8b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{2a}\left(\frac{7}{2}e^{2bx}+\log\left(e^{2bx}\right)-8\log\left(e^{2bx}+1\right)\right)}{8b}$$

input
```
Int[E^(2*(a + b*x))*Sinh[d + b*x]^2*Tanh[d + b*x],x]
```

output
```
-1/8*(E^(2*a)*((7*E^(2*b*x))/2 + Log[E^(2*b*x)] - 8*Log[1 + E^(2*b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.16

method	result	size
risch	$-\frac{\mathrm{e}^{2a-2d} x}{4}+\frac{\mathrm{e}^{4bx+2a+2d}}{16b}-\frac{\mathrm{e}^{2bx+2a}}{2b}-\frac{2\,\mathrm{e}^{2a-2d} a}{b}+\frac{\ln\left(\mathrm{e}^{2bx+2a}+\mathrm{e}^{2a-2d}\right) \mathrm{e}^{2a-2d}}{b}$	89

input
```
int(exp(2*b*x+2*a)*sinh(b*x+d)^2*tanh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*exp(2*a-2*d)*x+1/16*exp(4*b*x+2*a+2*d)/b-1/2*exp(2*b*x+2*a)/b-2/b*exp
(2*a-2*d)*a+ln(exp(2*b*x+2*a)+exp(2*a-2*d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(67) = 134$.

Time = 0.08 (sec) , antiderivative size = 333, normalized size of antiderivative = 4.32

$$\int e^{2(a+bx)}\sinh^2(d+bx)\tanh(d+bx)\,dx = \frac{\cosh(bx+d)^4\cosh(-2a+2d) + (\cosh(-2a+2d) - \sinh(-2a+2d))\sinh(bx+d)^4 + 4(\cosh(bx+d)}{}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)^2*tanh(b*x+d),x, algorithm="fricas")
```

output
```
1/16*(cosh(b*x + d)^4*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2
*d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*s
inh(-2*a + 2*d))*sinh(b*x + d)^3 - 4*b*x*cosh(-2*a + 2*d) - 8*cosh(b*x + d
)^2*cosh(-2*a + 2*d) + 2*(3*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (3*cosh(b*x
 + d)^2 - 4)*sinh(-2*a + 2*d) - 4*cosh(-2*a + 2*d))*sinh(b*x + d)^2 + 16*(
cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*log(2*cosh(b*x + d)/(cosh(b*x + d) -
sinh(b*x + d))) + 4*(cosh(b*x + d)^3*cosh(-2*a + 2*d) - 4*cosh(b*x + d)*co
sh(-2*a + 2*d) - (cosh(b*x + d)^3 - 4*cosh(b*x + d))*sinh(-2*a + 2*d))*sin
h(b*x + d) - (cosh(b*x + d)^4 - 4*b*x - 8*cosh(b*x + d)^2)*sinh(-2*a + 2*d
))/b
```

Sympy [F]

$$\int e^{2(a+bx)}\sinh^2(d+bx)\tanh(d+bx)\,dx = e^{2a}\int e^{2bx}\sinh^2(bx+d)\tanh(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)**2*tanh(b*x+d),x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*sinh(b*x + d)**2*tanh(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int e^{2(a+bx)}\sinh^2(d+bx)\tanh(d+bx)\,dx = -\frac{\left(8\,e^{(-2\,bx-2\,d)}-1\right)e^{(4\,bx+2\,a+2\,d)}}{16\,b} + \frac{7\,(bx+d)e^{(2\,a-2\,d)}}{4\,b} + \frac{e^{(2\,a-2\,d)}\log\left(e^{(-2\,bx-2\,d)}+1\right)}{b}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)^2*tanh(b*x+d),x, algorithm="maxima")
```

output
```
-1/16*(8*e^(-2*b*x - 2*d) - 1)*e^(4*b*x + 2*a + 2*d)/b + 7/4*(b*x + d)*e^(
2*a - 2*d)/b + e^(2*a - 2*d)*log(e^(-2*b*x - 2*d) + 1)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int e^{2(a+bx)}\sinh^2(d+bx)\tanh(d+bx)\,dx = -\frac{4\,(bx+d)e^{(2\,a-2\,d)}-\left(e^{(4\,bx+2\,a+6\,d)}-8\,e^{(2\,bx+2\,a+4\,d)}\right)e^{(-4\,d)}-16\,e^{(2\,a-2\,d)}\log\left(e^{(2\,bx+2\,d)}+1\right)}{16\,b}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)^2*tanh(b*x+d),x, algorithm="giac")
```

output
```
-1/16*(4*(b*x + d)*e^(2*a - 2*d) - (e^(4*b*x + 2*a + 6*d) - 8*e^(2*b*x + 2
*a + 4*d))*e^(-4*d) - 16*e^(2*a - 2*d)*log(e^(2*b*x + 2*d) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

$$\int e^{2(a+bx)}\sinh^2(d+bx)\tanh(d+bx)\,dx = \frac{e^{2a+2d+4bx}}{16b} - \frac{x\,e^{2a-2d}}{4} - \frac{e^{2a+2bx}}{2b} + \frac{e^{2a-2d}\ln\left(e^{2d}\,e^{2bx}+1\right)}{b}$$

input
```
int(exp(2*a + 2*b*x)*sinh(d + b*x)^2*tanh(d + b*x),x)
```

output
```
exp(2*a + 2*d + 4*b*x)/(16*b) - (x*exp(2*a - 2*d))/4 - exp(2*a + 2*b*x)/(2
*b) + (exp(2*a - 2*d)*log(exp(2*d)*exp(2*b*x) + 1))/b
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.77

$$\int e^{2(a+bx)}\sinh^2(d+bx)\tanh(d+bx)\,dx = \frac{e^{2a}\left(e^{4bx+4d}-8e^{2bx+2d}+16\log\left(e^{2bx+2d}+1\right)-4bx\right)}{16e^{2d}b}$$

input
```
int(exp(2*b*x+2*a)*sinh(b*x+d)^2*tanh(b*x+d),x)
```

output
```
(e**(2*a)*(e**(4*b*x + 4*d) - 8*e**(2*b*x + 2*d) + 16*log(e**(2*b*x + 2*d)
 + 1) - 4*b*x))/(16*e**(2*d)*b)
```

3.65 $\int e^{2(a+bx)} \sinh(d+bx) \tanh(d+bx) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 60

$$\int e^{2(a+bx)} \sinh(d+bx) \tanh(d+bx) \, dx = -\frac{3e^{2a-d+bx}}{2b} + \frac{e^{2a+d+3bx}}{6b} + \frac{2e^{2a-2d} \arctan\left(e^{d+bx}\right)}{b}$$

output
```
-3/2*exp(b*x+2*a-d)/b+1/6*exp(3*b*x+2*a+d)/b+2*exp(2*a-2*d)*arctan(exp(b*x
+d))/b
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.38

$$\int e^{2(a+bx)} \sinh(d+bx) \tanh(d+bx) \, dx$$
$$= \frac{e^{2a}\left(e^{bx}\left(-9+e^{2bx}\right)\cosh(d) + 12\arctan\left(e^{bx}(\cosh(d)+\sinh(d))\right)\cosh(2d) + 9e^{bx}\sinh(d) + e^{3bx}\sinh(d) -}{6b}$$

input
```
Integrate[E^(2*(a + b*x))*Sinh[d + b*x]*Tanh[d + b*x],x]
```

output
```
(E^(2*a)*(E^(b*x)*(-9 + E^(2*b*x))*Cosh[d] + 12*ArcTan[E^(b*x)*(Cosh[d] +
Sinh[d])]*Cosh[2*d] + 9*E^(b*x)*Sinh[d] + E^(3*b*x)*Sinh[d] - 12*ArcTan[E^
(b*x)*(Cosh[d] + Sinh[d])]*Sinh[2*d]))/(6*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.63, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2720, 27, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)} \sinh(bx+d)\tanh(bx+d)\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int \frac{e^{2a}\left(1-e^{2bx}\right)^2}{2\left(1+e^{2bx}\right)}de^{bx}}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{e^{2a}\int \frac{\left(1-e^{2bx}\right)^2}{1+e^{2bx}}de^{bx}}{2b}$$

$$\Big\downarrow \; 300$$

$$\frac{e^{2a}\int \left(e^{2bx}-3+\frac{4}{1+e^{2bx}}\right)de^{bx}}{2b}$$

$$\Big\downarrow \; 2009$$

$$\frac{e^{2a}\left(4\arctan\left(e^{bx}\right)-3e^{bx}+\frac{1}{3}e^{3bx}\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Sinh[d + b*x]*Tanh[d + b*x],x]
```

output
```
(E^(2*a)*(-3*E^(b*x) + E^(3*b*x)/3 + 4*ArcTan[E^(b*x)]))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.58

method	result	size
risch	$\frac{e^{3bx+2a+d}}{6b}-\frac{3\,e^{bx+2a-d}}{2b}+\frac{i\ln\left(e^{bx+a}+ie^{a-d}\right)e^{2a-2d}}{b}-\frac{i\ln\left(e^{bx+a}-ie^{a-d}\right)e^{2a-2d}}{b}$	95

input
```
int(exp(2*b*x+2*a)*sinh(b*x+d)*tanh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/6*exp(3*b*x+2*a+d)/b-3/2*exp(b*x+2*a-d)/b+I*ln(exp(b*x+a)+I*exp(a-d))/b*
exp(2*a-2*d)-I*ln(exp(b*x+a)-I*exp(a-d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(52) = 104$.

Time = 0.10 (sec) , antiderivative size = 227, normalized size of antiderivative = 3.78

$$\int e^{2(a+bx)}\sinh(d+bx)\tanh(d+bx)\,dx$$
$$= \frac{\cosh\left(bx+d\right)^3\cosh\left(-2\,a+2\,d\right)+\left(\cosh\left(-2\,a+2\,d\right)-\sinh\left(-2\,a+2\,d\right)\right)\sinh\left(bx+d\right)^3+3\left(\cosh\left(bx+\right.\right.}{}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)*tanh(b*x+d),x, algorithm="fricas")
```

output
```
1/6*(cosh(b*x + d)^3*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*
d))*sinh(b*x + d)^3 + 3*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*si
nh(-2*a + 2*d))*sinh(b*x + d)^2 + 12*(cosh(-2*a + 2*d) - sinh(-2*a + 2*d))
*arctan(cosh(b*x + d) + sinh(b*x + d)) - 9*cosh(b*x + d)*cosh(-2*a + 2*d)
+ 3*(cosh(b*x + d)^2*cosh(-2*a + 2*d) - (cosh(b*x + d)^2 - 3)*sinh(-2*a +
2*d) - 3*cosh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^3 - 9*cosh(b*x +
 d))*sinh(-2*a + 2*d))/b
```

Sympy [F]

$$\int e^{2(a+bx)}\sinh(d+bx)\tanh(d+bx)\,dx = e^{2a}\int e^{2bx}\sinh\left(bx+d\right)\tanh\left(bx+d\right)dx$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)*tanh(b*x+d),x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*sinh(b*x + d)*tanh(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int e^{2(a+bx)} \sinh(d+bx) \tanh(d+bx) \, dx = -\frac{\left(9 e^{(-2 bx-2 d)} - 1\right)e^{(3 bx+2 a+d)}}{6 b} - \frac{2 \arctan\left(e^{(-bx-d)}\right) e^{(2 a-2 d)}}{b}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)*tanh(b*x+d),x, algorithm="maxima")
```

output
```
-1/6*(9*e^(-2*b*x - 2*d) - 1)*e^(3*b*x + 2*a + d)/b - 2*arctan(e^(-b*x - d
))*e^(2*a - 2*d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int e^{2(a+bx)} \sinh(d+bx) \tanh(d+bx) \, dx = \frac{12 \arctan\left(e^{(bx+d)}\right) e^{(2 a-2 d)} + \left(e^{(3 bx+2 a+7 d)} - 9 e^{(bx+2 a+5 d)}\right)e^{(-6 d)}}{6 b}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)*tanh(b*x+d),x, algorithm="giac")
```

output
```
1/6*(12*arctan(e^(b*x + d))*e^(2*a - 2*d) + (e^(3*b*x + 2*a + 7*d) - 9*e^(
b*x + 2*a + 5*d))*e^(-6*d))/b
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.42

$$\int e^{2(a+bx)}\sinh(d+bx)\tanh(d+bx)\,dx = \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{-d}\,\mathrm{e}^{b\,x}\,\sqrt{b^2}}{b\,\sqrt{\mathrm{e}^{4\,a}\,\mathrm{e}^{-4\,d}}}\right)\,\sqrt{\mathrm{e}^{4\,a}\,\mathrm{e}^{-4\,d}}}{\sqrt{b^2}} - \frac{3\,\mathrm{e}^{2\,a}\,\mathrm{e}^{-d}\,\mathrm{e}^{b\,x}}{2\,b} + \frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{3\,b\,x}\,\mathrm{e}^{d}}{6\,b}$$

input
```
int(exp(2*a + 2*b*x)*sinh(d + b*x)*tanh(d + b*x),x)
```

output
```
(2*atan((exp(2*a)*exp(-d)*exp(b*x)*(b^2)^(1/2))/(b*(exp(4*a)*exp(-4*d))^(1
/2)))*(exp(4*a)*exp(-4*d))^(1/2))/(b^2)^(1/2) - (3*exp(2*a)*exp(-d)*exp(b*
x))/(2*b) + (exp(2*a)*exp(3*b*x)*exp(d))/(6*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

$$\int e^{2(a+bx)}\sinh(d+bx)\tanh(d+bx)\,dx = \frac{e^{2a}\left(12atan\left(e^{bx+d}\right)+e^{3bx+3d}-9e^{bx+d}\right)}{6e^{2d}b}$$

input
```
int(exp(2*b*x+2*a)*sinh(b*x+d)*tanh(b*x+d),x)
```

output
```
(e**(2*a)*(12*atan(e**(b*x + d)) + e**(3*b*x + 3*d) - 9*e**(b*x + d)))/(6*
e**(2*d)*b)
```

3.66 $\int e^{2(a+bx)} \tanh(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int e^{2(a+bx)} \tanh(d+bx)\,dx = \frac{e^{2a+2bx}}{2b} - \frac{e^{2a-2d}\log\left(1+e^{2d+2bx}\right)}{b}$$

output
```
1/2*exp(2*b*x+2*a)/b-exp(2*a-2*d)*ln(1+exp(2*b*x+2*d))/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 92 vs. 2(45) = 90.

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.04

$$\int e^{2(a+bx)} \tanh(d+bx)\,dx$$
$$= \frac{e^{2a}(\cosh(d)-\sinh(d))\left(\cosh(d)\left(e^{2bx}-2\log\left(\left(1+e^{2bx}\right)\cosh(d)+\left(-1+e^{2bx}\right)\sinh(d)\right)\right)+\left(e^{2bx}+2\log\right.\right.}{2b}$$

input
```
Integrate[E^(2*(a + b*x))*Tanh[d + b*x],x]
```

output
```
(E^(2*a)*(Cosh[d] - Sinh[d])*(Cosh[d]*(E^(2*b*x) - 2*Log[(1 + E^(2*b*x))*C
osh[d] + (-1 + E^(2*b*x))*Sinh[d]]) + (E^(2*b*x) + 2*Log[(1 + E^(2*b*x))*C
osh[d] + (-1 + E^(2*b*x))*Sinh[d]])*Sinh[d]))/(2*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2720, 25, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)} \tanh(bx+d) \, dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int -\frac{e^{2a+bx}(1-e^{2bx})}{1+e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{e^{2a+bx}(1-e^{2bx})}{1+e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \ 27$$

$$-\frac{e^{2a}\int \frac{e^{bx}(1-e^{2bx})}{1+e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \ 353$$

$$-\frac{e^{2a}\int \frac{1-e^{2bx}}{1+e^{2bx}}de^{2bx}}{2b}$$

$$\Big\downarrow \ 49$$

$$-\frac{e^{2a}\int \left(\frac{2}{1+e^{2bx}}-1\right)de^{2bx}}{2b}$$

$$\Big\downarrow \ 2009$$

$$-\frac{e^{2a}\left(2\log\left(e^{2bx}+1\right)-e^{2bx}\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Tanh[d + b*x],x]
```

output
```
-1/2*(E^(2*a)*(-E^(2*b*x) + 2*Log[1 + E^(2*b*x)]))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.38

method	result	size
risch	$\frac{e^{2bx+2a}}{2b}+\frac{2\,e^{2a-2d}a}{b}-\frac{\ln\left(e^{2bx+2a}+e^{2a-2d}\right)e^{2a-2d}}{b}$	62

input
```
int(exp(2*b*x+2*a)*tanh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*b*x+2*a)/b+2/b*exp(2*a-2*d)*a-ln(exp(2*b*x+2*a)+exp(2*a-2*d))/b*
exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(40) = 80.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.47

$$\int e^{2(a+bx)}\tanh(d+bx)\,dx$$
$$= \frac{\cosh(bx+d)^2\cosh(-2\,a+2\,d)+(\cosh(-2\,a+2\,d)-\sinh(-2\,a+2\,d))\sinh(bx+d)^2-\cosh(bx+d)}{}$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d),x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + d)^2*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*
d))*sinh(b*x + d)^2 - cosh(b*x + d)^2*sinh(-2*a + 2*d) - 2*(cosh(-2*a + 2*
d) - sinh(-2*a + 2*d))*log(2*cosh(b*x + d)/(cosh(b*x + d) - sinh(b*x + d))
) + 2*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*si
nh(b*x + d))/b
```

Sympy [F]

$$\int e^{2(a+bx)}\tanh(d+bx)\,dx = e^{2a}\int e^{2bx}\tanh(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d),x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*tanh(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int e^{2(a+bx)}\tanh(d+bx)\,dx = -\frac{2\,(bx+d)e^{(2\,a-2\,d)}}{b} - \frac{e^{(2\,a-2\,d)}\log\left(e^{(-2\,bx-2\,d)}+1\right)}{b} + \frac{e^{(2\,bx+2\,a)}}{2\,b}$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d),x, algorithm="maxima")
```

output
```
-2*(b*x + d)*e^(2*a - 2*d)/b - e^(2*a - 2*d)*log(e^(-2*b*x - 2*d) + 1)/b +
 1/2*e^(2*b*x + 2*a)/b
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int e^{2(a+bx)}\tanh(d+bx)\,dx = -\frac{e^{(2\,a-2\,d)}\log\left(e^{(2\,bx+2\,d)}+1\right)}{b} + \frac{e^{(2\,bx+2\,a)}}{2\,b}$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d),x, algorithm="giac")
```

output
```
-e^(2*a - 2*d)*log(e^(2*b*x + 2*d) + 1)/b + 1/2*e^(2*b*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int e^{2(a+bx)}\tanh(d+bx)\,dx = \frac{e^{2a}\,e^{2bx}}{2b} - \frac{e^{2a}\,e^{-2d}\,\ln\left(e^{2a}\,e^{2bx}+e^{2a}\,e^{-2d}\right)}{b}$$

input
```
int(exp(2*a + 2*b*x)*tanh(d + b*x),x)
```

output
```
(exp(2*a)*exp(2*b*x))/(2*b) - (exp(2*a)*exp(-2*d)*log(exp(2*a)*exp(2*b*x)
+ exp(2*a)*exp(-2*d)))/b
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int e^{2(a+bx)}\tanh(d+bx)\,dx = \frac{e^{2a}\left(e^{2bx+2d}-2\log\left(e^{2bx+2d}+1\right)\right)}{2e^{2d}b}$$

input
```
int(exp(2*b*x+2*a)*tanh(b*x+d),x)
```

output
```
(e**(2*a)*(e**(2*b*x + 2*d) - 2*log(e**(2*b*x + 2*d) + 1)))/(2*e**(2*d)*b)
```

3.67 $\int e^{2(a+bx)}\mathbf{sech}(d+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int e^{2(a+bx)}\mathrm{sech}(d+bx)\,dx=\frac{2e^{2a-d+bx}}{b}-\frac{2e^{2a-2d}\arctan\left(e^{d+bx}\right)}{b}$$

output
```
2*exp(b*x+2*a-d)/b-2*exp(2*a-2*d)*arctan(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2(a+bx)}\mathrm{sech}(d+bx)\,dx=\frac{2e^{2a-2d}\left(e^{d+bx}-\arctan\left(e^{d+bx}\right)\right)}{b}$$

input
```
Integrate[E^(2*(a + b*x))*Sech[d + b*x],x]
```

output
```
(2*E^(2*a - 2*d)*(E^(d + b*x) - ArcTan[E^(d + b*x)]))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 27, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{sech}(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{2e^{2a+2bx}}{1+e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2e^{2a}\int \frac{e^{2bx}}{1+e^{2bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{2e^{2a}\left(e^{bx}-\int \frac{1}{1+e^{2bx}}de^{bx}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{2e^{2a}\left(e^{bx}-\arctan\left(e^{bx}\right)\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Sech[d + b*x],x]
```

output
```
(2*E^(2*a)*(E^(b*x) - ArcTan[E^(b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.00

method	result	size
risch	$\frac{2\,\mathrm{e}^{b x+2 a-d}}{b}+\frac{i \ln\left(\mathrm{e}^{b x+a}-i \mathrm{e}^{a-d}\right) \mathrm{e}^{2 a-2 d}}{b}-\frac{i \ln\left(\mathrm{e}^{b x+a}+i \mathrm{e}^{a-d}\right) \mathrm{e}^{2 a-2 d}}{b}$	80

input
```
int(exp(2*b*x+2*a)*sech(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
2*exp(b*x+2*a-d)/b+I*ln(exp(b*x+a)-I*exp(a-d))/b*exp(2*a-2*d)-I*ln(exp(b*x
+a)+I*exp(a-d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(37) = 74$.

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.45

$$\int e^{2(a+bx)}\operatorname{sech}(d+bx)\,dx = -\frac{2\left((\cosh(-2a+2d)-\sinh(-2a+2d))\arctan(\cosh(bx+d)+\sinh(bx+d))-\cosh(bx+d)\cosh\right.}{}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d),x, algorithm="fricas")
```

output
```
-2*((cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*arctan(cosh(b*x + d) + sinh(b*x
+ d)) - cosh(b*x + d)*cosh(-2*a + 2*d) - (cosh(-2*a + 2*d) - sinh(-2*a + 2
*d))*sinh(b*x + d) + cosh(b*x + d)*sinh(-2*a + 2*d))/b
```

Sympy [F]

$$\int e^{2(a+bx)}\operatorname{sech}(d+bx)\,dx = e^{2a}\int e^{2bx}\operatorname{sech}(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d),x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*sech(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int e^{2(a+bx)}\operatorname{sech}(d+bx)\,dx = \frac{2\arctan\left(e^{(-bx-d)}\right)e^{(2a-2d)}}{b} + \frac{2e^{(bx+2a-d)}}{b}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d),x, algorithm="maxima")
```

output
```
2*arctan(e^(-b*x - d))*e^(2*a - 2*d)/b + 2*e^(b*x + 2*a - d)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int e^{2(a+bx)}\text{sech}(d+bx)\,dx = -\frac{2\left(\arctan\left(e^{(bx+d)}\right)e^{(-2d)}-e^{(bx-d)}\right)e^{(2a)}}{b}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d),x, algorithm="giac")
```

output
```
-2*(arctan(e^(b*x + d))*e^(-2*d) - e^(b*x - d))*e^(2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.68

$$\int e^{2(a+bx)}\text{sech}(d+bx)\,dx = \frac{2\,e^{2\,a-d+b\,x}}{b}-\frac{2\,\sqrt{e^{4\,a-4\,d}}\,\text{atan}\left(\frac{e^{2\,a}\,e^{-d}\,e^{b\,x}\,\sqrt{b^2}}{b\,\sqrt{e^{4\,a}\,e^{-4\,d}}}\right)}{\sqrt{b^2}}$$

input
```
int(exp(2*a + 2*b*x)/cosh(d + b*x),x)
```

output
```
(2*exp(2*a - d + b*x))/b - (2*exp(4*a - 4*d)^(1/2)*atan((exp(2*a)*exp(-d)*
exp(b*x)*(b^2)^(1/2))/(b*(exp(4*a)*exp(-4*d))^(1/2))))/(b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int e^{2(a+bx)}\text{sech}(d+bx)\,dx = \frac{2e^{2a}\left(-atan\left(e^{bx+d}\right)+e^{bx+d}\right)}{e^{2d}b}$$

input
```
int(exp(2*b*x+2*a)*sech(b*x+d),x)
```

output
```
(2*e**(2*a)*( - atan(e**(b*x + d)) + e**(b*x + d)))/(e**(2*d)*b)
```

3.68 $\int e^{2(a+bx)}\text{csch}(d+bx)\text{sech}(d+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 28

$$\int e^{2(a+bx)}\text{csch}(d+bx)\text{sech}(d+bx)\,dx=\frac{e^{2a-2d}\log\left(1-e^{4d+4bx}\right)}{b}$$

output
```
exp(2*a-2*d)*ln(1-exp(4*b*x+4*d))/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int e^{2(a+bx)}\text{csch}(d+bx)\text{sech}(d+bx)\,dx$$
$$=\frac{e^{2a}\log\left(\left(-1+e^{4bx}\right)\cosh(2d)+\left(1+e^{4bx}\right)\sinh(2d)\right)\left(\cosh(2d)-\sinh(2d)\right)}{b}$$

input
```
Integrate[E^(2*(a + b*x))*Csch[d + b*x]*Sech[d + b*x],x]
```

output
```
(E^(2*a)*Log[(-1 + E^(4*b*x))*Cosh[2*d] + (1 + E^(4*b*x))*Sinh[2*d]]*(Cosh
[2*d] - Sinh[2*d]))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2720, 27, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{csch}(bx+d)\text{sech}(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{4e^{2a+3bx}}{1-e^{4bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4e^{2a}\int \frac{e^{3bx}}{1-e^{4bx}}de^{bx}}{b}$$

$$\Big\downarrow \text{792}$$

$$\frac{e^{2a}\log\left(1-e^{4bx}\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Csch[d + b*x]*Sech[d + b*x],x]
```

output
```
(E^(2*a)*Log[1 - E^(4*b*x)])/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

method	result	size
risch	$-\frac{4\,\mathrm{e}^{2a-2d} a}{b}+\frac{\ln\left(\mathrm{e}^{4bx+4a}-\mathrm{e}^{4a-4d}\right)\mathrm{e}^{2a-2d}}{b}$	49

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)*sech(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
-4/b*exp(2*a-2*d)*a+ln(exp(4*b*x+4*a)-exp(4*a-4*d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(26) = 52.

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.54

$$\int e^{2(a+bx)}\mathrm{csch}(d+bx)\mathrm{sech}(d+bx)\,dx = \frac{(\cosh(-2\,a+2\,d)-\sinh(-2\,a+2\,d))\log\left(\frac{4\,\cosh(bx+d)\sinh(bx+d)}{\cosh(bx+d)^2-2\,\cosh(bx+d)\sinh(bx+d)+\sinh(bx+d)^2}\right)}{b}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)*sech(b*x+d),x, algorithm="fricas")
```

output
```
(cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*log(4*cosh(b*x + d)*sinh(b*x + d)/(c
osh(b*x + d)^2 - 2*cosh(b*x + d)*sinh(b*x + d) + sinh(b*x + d)^2))/b
```

Sympy [F]

$$\int e^{2(a+bx)}\text{csch}(d+bx)\text{sech}(d+bx)\,dx = e^{2a}\int e^{2bx}\,\text{csch}\,(bx+d)\,\text{sech}\,(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)*sech(b*x+d),x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*csch(b*x + d)*sech(b*x + d), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(26) = 52$.

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.50

$$\int e^{2(a+bx)}\text{csch}(d+bx)\text{sech}(d+bx)\,dx = 4\,xe^{(2\,a-2\,d)} + \frac{4\,de^{(2\,a-2\,d)}}{b} + \frac{e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}+1\right)}{b} + \frac{e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}-1\right)}{b} + \frac{e^{(2\,a-2\,d)}\log\left(e^{(-2\,bx-2\,d)}+1\right)}{b}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)*sech(b*x+d),x, algorithm="maxima")
```

output
```
4*x*e^(2*a - 2*d) + 4*d*e^(2*a - 2*d)/b + e^(2*a - 2*d)*log(e^(-b*x - d) +
 1)/b + e^(2*a - 2*d)*log(e^(-b*x - d) - 1)/b + e^(2*a - 2*d)*log(e^(-2*b*
x - 2*d) + 1)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int e^{2(a+bx)}\text{csch}(d+bx)\text{sech}(d+bx)\,dx=\frac{e^{(2\,a-2\,d)}\log\left(\left|e^{(4\,bx+4\,d)}-1\right|\right)}{b}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)*sech(b*x+d),x, algorithm="giac")
```

output
```
e^(2*a - 2*d)*log(abs(e^(4*b*x + 4*d) - 1))/b
```

Mupad [B] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int e^{2(a+bx)}\text{csch}(d+bx)\text{sech}(d+bx)\,dx=\frac{e^{2\,a-2\,d}\,\ln\left(e^{4\,d}\,e^{4\,b\,x}-1\right)}{b}$$

input
```
int(exp(2*a + 2*b*x)/(cosh(d + b*x)*sinh(d + b*x)),x)
```

output
```
(exp(2*a - 2*d)*log(exp(4*d)*exp(4*b*x) - 1))/b
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int e^{2(a+bx)}\text{csch}(d+bx)\text{sech}(d+bx)\,dx$$
$$=\frac{e^{2a}\left(\log\left(e^{bx+d}-1\right)+\log\left(e^{bx+d}+1\right)+\log\left(e^{2bx+2d}+1\right)\right)}{e^{2d}b}$$

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)*sech(b*x+d),x)
```

output
```
(e**(2*a)*(log(e**(b*x + d) - 1) + log(e**(b*x + d) + 1) + log(e**(2*b*x +
 2*d) + 1)))/(e**(2*d)*b)
```

3.69 $\int e^{2(a+bx)}\mathbf{csch}^2(d+bx)\mathbf{sech}(d+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 78

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\text{sech}(d+bx)\,dx=\frac{2e^{2a-d+bx}}{b\left(1-e^{2d+2bx}\right)}+\frac{2e^{2a-2d}\arctan\left(e^{d+bx}\right)}{b}-\frac{4e^{2a-2d}\text{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
2*exp(b*x+2*a-d)/b/(1-exp(2*b*x+2*d))+2*exp(2*a-2*d)*arctan(exp(b*x+d))/b-
4*exp(2*a-2*d)*arctanh(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\text{sech}(d+bx)\,dx=\frac{2e^{2a-2d}\left(-\frac{e^{d+bx}}{-1+e^{2(d+bx)}}+\arctan\left(e^{d+bx}\right)+\log\left(1-e^{d+bx}\right)-\log\left(1+e^{d+bx}\right)\right)}{b}$$

input
```
Integrate[E^(2*(a + b*x))*Csch[d + b*x]^2*Sech[d + b*x],x]
```

output
```
(2*E^(2*a - 2*d)*(-(E^(d + b*x)/(-1 + E^(2*(d + b*x)))) + ArcTan[E^(d + b*
x)] + Log[1 - E^(d + b*x)] - Log[1 + E^(d + b*x)]))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 27, 372, 397, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{csch}^2(bx+d)\text{sech}(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{8e^{2a+4bx}}{\left(1-e^{2bx}\right)^2\left(1+e^{2bx}\right)}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{8e^{2a}\int \frac{e^{4bx}}{\left(1-e^{2bx}\right)^2\left(1+e^{2bx}\right)}de^{bx}}{b}$$

$$\Big\downarrow \text{372}$$

$$\frac{8e^{2a}\left(\frac{e^{bx}}{4\left(1-e^{2bx}\right)}-\frac{1}{4}\int \frac{1+3e^{2bx}}{\left(1-e^{2bx}\right)\left(1+e^{2bx}\right)}de^{bx}\right)}{b}$$

$$\Big\downarrow \text{397}$$

$$\frac{8e^{2a}\left(\frac{1}{4}\left(\int \frac{1}{1+e^{2bx}}de^{bx}-2\int \frac{1}{1-e^{2bx}}de^{bx}\right)+\frac{e^{bx}}{4\left(1-e^{2bx}\right)}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{8e^{2a}\left(\frac{1}{4}\left(\arctan\left(e^{bx}\right)-2\int \frac{1}{1-e^{2bx}}de^{bx}\right)+\frac{e^{bx}}{4\left(1-e^{2bx}\right)}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{8e^{2a}\left(\frac{1}{4}\left(\arctan\left(e^{bx}\right)-2\text{arctanh}\left(e^{bx}\right)\right)+\frac{e^{bx}}{4\left(1-e^{2bx}\right)}\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Csch[d + b*x]^2*Sech[d + b*x],x]
```

output
```
(8*E^(2*a)*(E^(b*x)/(4*(1 - E^(2*b*x))) + (ArcTan[E^(b*x)] - 2*ArcTanh[E^(
b*x)])/4))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.03 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.01

method	result
risch	$\frac{2\,\mathrm{e}^{bx+4a-d}}{\left(-\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)b}+\frac{i\ln\left(\mathrm{e}^{bx+a}+i\mathrm{e}^{a-d}\right)\mathrm{e}^{2a-2d}}{b}-\frac{i\ln\left(\mathrm{e}^{bx+a}-i\mathrm{e}^{a-d}\right)\mathrm{e}^{2a-2d}}{b}+\frac{2\ln\left(\mathrm{e}^{bx+a}-\mathrm{e}^{a-d}\right)\mathrm{e}^{2a-2d}}{b}-\frac{2\ln\left(\mathrm{e}^{bx+a}+\right.}{}$

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)^2*sech(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
2/(-exp(2*b*x+2*a+2*d)+exp(2*a))/b*exp(b*x+4*a-d)+I*ln(exp(b*x+a)+I*exp(a-
d))/b*exp(2*a-2*d)-I*ln(exp(b*x+a)-I*exp(a-d))/b*exp(2*a-2*d)+2*ln(exp(b*x
+a)-exp(a-d))/b*exp(2*a-2*d)-2*ln(exp(b*x+a)+exp(a-d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(70) = 140$.

Time = 0.10 (sec) , antiderivative size = 498, normalized size of antiderivative = 6.38

$$\int e^{2(a+bx)}\operatorname{csch}^2(d+bx)\operatorname{sech}(d+bx)\,dx=\text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^2*sech(b*x+d),x, algorithm="fricas")
```

output
```
2*((cosh(b*x + d)^2*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*d
))*sinh(b*x + d)^2 + 2*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sin
h(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^2 - 1)*sinh(-2*a + 2*d) - co
sh(-2*a + 2*d))*arctan(cosh(b*x + d) + sinh(b*x + d)) - cosh(b*x + d)*cosh
(-2*a + 2*d) - (cosh(b*x + d)^2*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sin
h(-2*a + 2*d))*sinh(b*x + d)^2 + 2*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(
b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^2 - 1)*sinh(-2*a
 + 2*d) - cosh(-2*a + 2*d))*log(cosh(b*x + d) + sinh(b*x + d) + 1) + (cosh
(b*x + d)^2*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(
b*x + d)^2 + 2*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a +
 2*d))*sinh(b*x + d) - (cosh(b*x + d)^2 - 1)*sinh(-2*a + 2*d) - cosh(-2*a
+ 2*d))*log(cosh(b*x + d) + sinh(b*x + d) - 1) - (cosh(-2*a + 2*d) - sinh(
-2*a + 2*d))*sinh(b*x + d) + cosh(b*x + d)*sinh(-2*a + 2*d))/(b*cosh(b*x +
 d)^2 + 2*b*cosh(b*x + d)*sinh(b*x + d) + b*sinh(b*x + d)^2 - b)
```

Sympy [F]

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\text{sech}(d+bx)\,dx = e^{2a}\int e^{2bx}\,\text{csch}^2\left(bx+d\right)\text{sech}\left(bx+d\right)dx$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)**2*sech(b*x+d),x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*csch(b*x + d)**2*sech(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.33

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\text{sech}(d+bx)\,dx = -\frac{2\arctan\left(e^{(-bx-d)}\right)e^{(2a-2d)}}{b} - \frac{2e^{(2a-2d)}\log\left(e^{(-bx-d)}+1\right)}{b} + \frac{2e^{(2a-2d)}\log\left(e^{(-bx-d)}-1\right)}{b} + \frac{2e^{(-bx+2a-3d)}}{b(e^{(-2bx-2d)}-1)}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^2*sech(b*x+d),x, algorithm="maxima")
```

output
```
-2*arctan(e^(-b*x - d))*e^(2*a - 2*d)/b - 2*e^(2*a - 2*d)*log(e^(-b*x - d)
 + 1)/b + 2*e^(2*a - 2*d)*log(e^(-b*x - d) - 1)/b + 2*e^(-b*x + 2*a - 3*d)
/(b*(e^(-2*b*x - 2*d) - 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\text{sech}(d+bx)\,dx = \frac{2\left(\arctan\left(e^{(bx+d)}\right)e^{(-2d)} - e^{(-2d)}\log\left(e^{(bx+d)}+1\right) + e^{(-2d)}\log\left(\left|e^{(bx+d)}-1\right|\right) - \frac{e^{(bx-d)}}{e^{(2bx+2d)}-1}\right)e^{(2a)}}{b}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^2*sech(b*x+d),x, algorithm="giac")
```

output
```
2*(arctan(e^(b*x + d))*e^(-2*d) - e^(-2*d)*log(e^(b*x + d) + 1) + e^(-2*d)
*log(abs(e^(b*x + d) - 1)) - e^(b*x - d)/(e^(2*b*x + 2*d) - 1))*e^(2*a)/b
```

Mupad [B] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.33

$$\begin{aligned}\int e^{2(a+bx)}\text{csch}^2(d+bx)\text{sech}(d+bx)\,dx &= \frac{2\,e^{2a-d+bx}}{b-b\,e^{2d+2bx}} \\ &\quad - \frac{e^{2a-2d}\left(\ln\left(160\,e^{6a}\,e^{-5d}\,e^{bx} - e^{6a}\,e^{-6d}\,160\mathrm{i}\right)\,1\mathrm{i} - \ln\left(160\,e^{6a}\,e^{-5d}\,e^{bx} + e^{6a}\,e^{-6d}\,160\mathrm{i}\right)\,1\mathrm{i}\right)}{b} \\ &\quad - \frac{2\,e^{2a-2d}\,\ln\left(-320\,e^{6a}\,e^{-6d} - 320\,e^{6a}\,e^{-5d}\,e^{bx}\right)}{b} \\ &\quad + \frac{2\,e^{2a-2d}\,\ln\left(320\,e^{6a}\,e^{-6d} - 320\,e^{6a}\,e^{-5d}\,e^{bx}\right)}{b}\end{aligned}$$

input
```
int(exp(2*a + 2*b*x)/(cosh(d + b*x)*sinh(d + b*x)^2),x)
```

output
```
(2*exp(2*a - d + b*x))/(b - b*exp(2*d + 2*b*x)) - (exp(2*a - 2*d)*(log(160
*exp(6*a)*exp(-5*d)*exp(b*x) - exp(6*a)*exp(-6*d)*160i)*1i - log(exp(6*a)*
exp(-6*d)*160i + 160*exp(6*a)*exp(-5*d)*exp(b*x))*1i))/b - (2*exp(2*a - 2*
d)*log(- 320*exp(6*a)*exp(-6*d) - 320*exp(6*a)*exp(-5*d)*exp(b*x)))/b + (2
*exp(2*a - 2*d)*log(320*exp(6*a)*exp(-6*d) - 320*exp(6*a)*exp(-5*d)*exp(b*
x)))/b
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.73

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\text{sech}(d+bx)\,dx = \frac{2e^{2a}\left(e^{2bx+2d}atan\left(e^{bx+d}\right)-atan\left(e^{bx+d}\right)+e^{2bx+2d}\log\left(e^{bx+d}-1\right)-e^{2bx+2d}\log\left(e^{bx+d}+1\right)-e^{bx+d}-\log\left(e\right.\right.}{e^{2d}b\left(e^{2bx+2d}-1\right)}$$

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)^2*sech(b*x+d),x)
```

output
```
(2*e**(2*a)*(e**(2*b*x + 2*d)*atan(e**(b*x + d)) - atan(e**(b*x + d)) + e*
*(2*b*x + 2*d)*log(e**(b*x + d) - 1) - e**(2*b*x + 2*d)*log(e**(b*x + d) +
 1) - e**(b*x + d) - log(e**(b*x + d) - 1) + log(e**(b*x + d) + 1)))/(e**(
2*d)*b*(e**(2*b*x + 2*d) - 1))
```

3.70 $\int e^{2(a+bx)}\operatorname{csch}^3(d+bx)\operatorname{sech}(d+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 86

$$\int e^{2(a+bx)}\operatorname{csch}^3(d+bx)\operatorname{sech}(d+bx)\,dx=-\frac{2e^{2a-2d}}{b\left(1-e^{2d+2bx}\right)^2}+\frac{6e^{2a-2d}}{b\left(1-e^{2d+2bx}\right)}-\frac{2e^{2a-2d}\operatorname{arctanh}\left(e^{2d+2bx}\right)}{b}$$

output
```
-2*exp(2*a-2*d)/b/(1-exp(2*b*x+2*d))^2+6*exp(2*a-2*d)/b/(1-exp(2*b*x+2*d))
-2*exp(2*a-2*d)*arctanh(exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.74

$$\int e^{2(a+bx)}\operatorname{csch}^3(d+bx)\operatorname{sech}(d+bx)\,dx=-\frac{2e^{2a-2d}\left(-2+3e^{2(d+bx)}+\left(-1+e^{2(d+bx)}\right)^2\operatorname{arctanh}\left(e^{2(d+bx)}\right)\right)}{b\left(-1+e^{2(d+bx)}\right)^2}$$

input
```
Integrate[E^(2*(a + b*x))*Csch[d + b*x]^3*Sech[d + b*x],x]
```

output
```
(-2*E^(2*a - 2*d)*(-2 + 3*E^(2*(d + b*x)) + (-1 + E^(2*(d + b*x)))^2*ArcTa
nh[E^(2*(d + b*x))]))/(b*(-1 + E^(2*(d + b*x)))^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.63, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2720, 27, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{csch}^3(bx+d)\text{sech}(bx+d)\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int -\frac{16e^{2a+5bx}}{\left(1-e^{2bx}\right)^3\left(1+e^{2bx}\right)}de^{bx}}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{16e^{2a}\int \frac{e^{5bx}}{\left(1-e^{2bx}\right)^3\left(1+e^{2bx}\right)}de^{bx}}{b}$$

$$\Big\downarrow \; 354$$

$$-\frac{8e^{2a}\int \frac{e^{2bx}}{\left(1-e^{2bx}\right)^3\left(1+e^{2bx}\right)}de^{2bx}}{b}$$

$$\Big\downarrow \; 99$$

$$-\frac{8e^{2a}\int \left(-\frac{1}{4\left(-1+e^{2bx}\right)}-\frac{3}{4\left(-1+e^{2bx}\right)^2}-\frac{1}{2\left(-1+e^{2bx}\right)^3}\right)de^{2bx}}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{8e^{2a}\left(\frac{1}{4}\text{arctanh}\left(e^{2bx}\right)-\frac{3}{4\left(1-e^{2bx}\right)}+\frac{1}{4\left(1-e^{2bx}\right)^2}\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Csch[d + b*x]^3*Sech[d + b*x],x]
```

output
```
(-8*E^(2*a)*(1/(4*(1 - E^(2*b*x))^2) - 3/(4*(1 - E^(2*b*x))) + ArcTanh[E^(
2*b*x)]/4))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 35.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.42

method	result	size
risch	$\frac{2\left(-3\,\mathrm{e}^{2bx+2a+2d}+2\,\mathrm{e}^{2a}\right)\mathrm{e}^{4a-2d}}{\left(-\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)^{2}b}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-\mathrm{e}^{2a-2d}\right)\mathrm{e}^{2a-2d}}{b}-\frac{\ln\left(\mathrm{e}^{2bx+2a}+\mathrm{e}^{2a-2d}\right)\mathrm{e}^{2a-2d}}{b}$	122

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)^3*sech(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
2/(-exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(-3*exp(2*b*x+2*a+2*d)+2*exp(2*a))*ex
p(4*a-2*d)+ln(exp(2*b*x+2*a)-exp(2*a-2*d))/b*exp(2*a-2*d)-ln(exp(2*b*x+2*a
)+exp(2*a-2*d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 830 vs. 2(76) = 152.

Time = 0.09 (sec) , antiderivative size = 830, normalized size of antiderivative = 9.65

$$\int e^{2(a+bx)}\mathrm{csch}^3(d+bx)\mathrm{sech}(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^3*sech(b*x+d),x, algorithm="fricas")
```

output
```
-(6*cosh(b*x + d)^2*cosh(-2*a + 2*d) + 6*(cosh(-2*a + 2*d) - sinh(-2*a + 2
*d))*sinh(b*x + d)^2 + (cosh(b*x + d)^4*cosh(-2*a + 2*d) + (cosh(-2*a + 2*
d) - sinh(-2*a + 2*d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-2*a + 2*d)
 - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^3 - 2*cosh(b*x + d)^2*cos
h(-2*a + 2*d) + 2*(3*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (3*cosh(b*x + d)^2
 - 1)*sinh(-2*a + 2*d) - cosh(-2*a + 2*d))*sinh(b*x + d)^2 + 4*(cosh(b*x +
 d)^3*cosh(-2*a + 2*d) - cosh(b*x + d)*cosh(-2*a + 2*d) - (cosh(b*x + d)^3
 - cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^4 - 2*c
osh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*log(2*cosh(b*x +
d)/(cosh(b*x + d) - sinh(b*x + d))) - (cosh(b*x + d)^4*cosh(-2*a + 2*d) +
(cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*c
osh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^3 - 2*cosh
(b*x + d)^2*cosh(-2*a + 2*d) + 2*(3*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (3*
cosh(b*x + d)^2 - 1)*sinh(-2*a + 2*d) - cosh(-2*a + 2*d))*sinh(b*x + d)^2
+ 4*(cosh(b*x + d)^3*cosh(-2*a + 2*d) - cosh(b*x + d)*cosh(-2*a + 2*d) - (
cosh(b*x + d)^3 - cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b
*x + d)^4 - 2*cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*lo
g(2*sinh(b*x + d)/(cosh(b*x + d) - sinh(b*x + d))) + 12*(cosh(b*x + d)*cos
h(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d) - 2*(3*cosh(
b*x + d)^2 - 2)*sinh(-2*a + 2*d) - 4*cosh(-2*a + 2*d))/(b*cosh(b*x + d)...
```

Sympy [F]

$$\int e^{2(a+bx)}\text{csch}^3(d+bx)\text{sech}(d+bx)\,dx = e^{2a}\int e^{2bx}\,\text{csch}^3\,(bx+d)\,\text{sech}\,(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)**3*sech(b*x+d),x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*csch(b*x + d)**3*sech(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.44

$$\int e^{2(a+bx)}\operatorname{csch}^3(d+bx)\operatorname{sech}(d+bx)\,dx=\frac{e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}+1\right)}{b}+\frac{e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}-1\right)}{b}-\frac{e^{(2\,a-2\,d)}\log\left(e^{(-2\,bx-2\,d)}+1\right)}{b}-\frac{2\left(e^{(-2\,bx-2\,d)}-2\right)e^{(2\,a-2\,d)}}{b(2\,e^{(-2\,bx-2\,d)}-e^{(-4\,bx-4\,d)}-1)}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^3*sech(b*x+d),x, algorithm="maxima")
```

output
```
e^(2*a - 2*d)*log(e^(-b*x - d) + 1)/b + e^(2*a - 2*d)*log(e^(-b*x - d) - 1
)/b - e^(2*a - 2*d)*log(e^(-2*b*x - 2*d) + 1)/b - 2*(e^(-2*b*x - 2*d) - 2)
*e^(2*a - 2*d)/(b*(2*e^(-2*b*x - 2*d) - e^(-4*b*x - 4*d) - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.03

$$\int e^{2(a+bx)}\operatorname{csch}^3(d+bx)\operatorname{sech}(d+bx)\,dx=-\frac{\left(2\,e^{(-2\,d)}\log\left(e^{(2\,bx+2\,d)}+1\right)-2\,e^{(-2\,d)}\log\left(\left|e^{(2\,bx+2\,d)}-1\right|\right)+\frac{\left(3\,e^{(4\,bx+4\,d)}+6\,e^{(2\,bx+2\,d)}-5\right)e^{(-2\,d)}}{\left(e^{(2\,bx+2\,d)}-1\right)^2}\right)e^{(2\,a)}}{2\,b}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^3*sech(b*x+d),x, algorithm="giac")
```

output
```
-1/2*(2*e^(-2*d)*log(e^(2*b*x + 2*d) + 1) - 2*e^(-2*d)*log(abs(e^(2*b*x +
2*d) - 1)) + (3*e^(4*b*x + 4*d) + 6*e^(2*b*x + 2*d) - 5)*e^(-2*d)/(e^(2*b*
x + 2*d) - 1)^2)*e^(2*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.34

$$\int e^{2(a+bx)}\operatorname{csch}^3(d+bx)\operatorname{sech}(d+bx)\,dx=-\frac{2\,e^{2\,a-2\,d}}{b\,\left(e^{4\,d+4\,b\,x}-2\,e^{2\,d+2\,b\,x}+1\right)}-\frac{2\,\sqrt{e^{4\,a-4\,d}}\,\operatorname{atan}\left(\frac{e^{2\,a}\,e^{2\,b\,x}\,\sqrt{-b^2}}{b\,\sqrt{e^{4\,a}\,e^{-4\,d}}}\right)}{\sqrt{-b^2}}-\frac{6\,e^{2\,a-2\,d}}{b\,\left(e^{2\,d+2\,b\,x}-1\right)}$$

input
```
int(exp(2*a + 2*b*x)/(cosh(d + b*x)*sinh(d + b*x)^3),x)
```

output
```
- (2*exp(2*a - 2*d))/(b*(exp(4*d + 4*b*x) - 2*exp(2*d + 2*b*x) + 1)) - (2*
exp(4*a - 4*d)^(1/2)*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/(b*(exp(4*a)*
exp(-4*d))^(1/2))))/(-b^2)^(1/2) - (6*exp(2*a - 2*d))/(b*(exp(2*d + 2*b*x)
 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.64

$$\int e^{2(a+bx)}\operatorname{csch}^3(d+bx)\operatorname{sech}(d+bx)\,dx$$
$$=\frac{e^{2a}\left(e^{4bx+4d}\log\left(e^{bx+d}-1\right)+e^{4bx+4d}\log\left(e^{bx+d}+1\right)-e^{4bx+4d}\log\left(e^{2bx+2d}+1\right)-3e^{4bx+4d}-2e^{2bx+2d}\log\left(e^{b\cdots}\right.\right.}{e^{2d}b\left(e\cdots\right.}$$

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)^3*sech(b*x+d),x)
```

output
```
(e**(2*a)*(e**(4*b*x + 4*d)*log(e**(b*x + d) - 1) + e**(4*b*x + 4*d)*log(e
**(b*x + d) + 1) - e**(4*b*x + 4*d)*log(e**(2*b*x + 2*d) + 1) - 3*e**(4*b*
x + 4*d) - 2*e**(2*b*x + 2*d)*log(e**(b*x + d) - 1) - 2*e**(2*b*x + 2*d)*l
og(e**(b*x + d) + 1) + 2*e**(2*b*x + 2*d)*log(e**(2*b*x + 2*d) + 1) + log(
e**(b*x + d) - 1) + log(e**(b*x + d) + 1) - log(e**(2*b*x + 2*d) + 1) + 1)
)/(e**(2*d)*b*(e**(4*b*x + 4*d) - 2*e**(2*b*x + 2*d) + 1))
```

3.71 $\int e^{2(a+bx)} \sinh(d+bx) \tanh^2(d+bx) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 91

$$\int e^{2(a+bx)} \sinh(d+bx) \tanh^2(d+bx) \, dx = -\frac{5e^{2a-d+bx}}{2b} + \frac{e^{2a+d+3bx}}{6b} - \frac{2e^{2a-d+bx}}{b(1+e^{2d+2bx})} + \frac{4e^{2a-2d} \arctan\left(e^{d+bx}\right)}{b}$$

output
```
-5/2*exp(b*x+2*a-d)/b+1/6*exp(3*b*x+2*a+d)/b-2*exp(b*x+2*a-d)/b/(1+exp(2*b
*x+2*d))+4*exp(2*a-2*d)*arctan(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

$$\int e^{2(a+bx)} \sinh(d+bx) \tanh^2(d+bx) \, dx = \frac{e^{2a-2d}\left(\frac{e^{d+bx}\left(-27-14e^{2(d+bx)}+e^{4(d+bx)}\right)}{1+e^{2(d+bx)}} + 24\arctan\left(e^{d+bx}\right)\right)}{6b}$$

input
```
Integrate[E^(2*(a + b*x))*Sinh[d + b*x]*Tanh[d + b*x]^2,x]
```

output
```
(E^(2*a - 2*d)*((E^(d + b*x)*(-27 - 14*E^(2*(d + b*x)) + E^(4*(d + b*x))))
/(1 + E^(2*(d + b*x))) + 24*ArcTan[E^(d + b*x)]))/(6*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.60, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2720, 27, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\sinh(bx+d)\tanh^2(bx+d)\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int -\frac{e^{2a}\left(1-e^{2bx}\right)^3}{2\left(1+e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{e^{2a}\int \frac{\left(1-e^{2bx}\right)^3}{\left(1+e^{2bx}\right)^2}de^{bx}}{2b}$$

$$\Big\downarrow \; 300$$

$$-\frac{e^{2a}\int \left(-\frac{4\left(1+3e^{2bx}\right)}{\left(1+e^{2bx}\right)^2}-e^{2bx}+5\right)de^{bx}}{2b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{e^{2a}\left(-8\arctan\left(e^{bx}\right)+5e^{bx}-\frac{1}{3}e^{3bx}+\frac{4e^{bx}}{e^{2bx}+1}\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Sinh[d + b*x]*Tanh[d + b*x]^2,x]
```

output
```
-1/2*(E^(2*a)*(5*E^(b*x) - E^(3*b*x)/3 + (4*E^(b*x))/(1 + E^(2*b*x)) - 8*A
rcTan[E^(b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.43

method	result	size
risch	$\frac{e^{3bx+2a+d}}{6b}-\frac{5\,e^{bx+2a-d}}{2b}-\frac{2\,e^{bx+4a-d}}{\left(e^{2bx+2a+2d}+e^{2a}\right)b}+\frac{2i\ln\left(e^{bx+a}+i e^{a-d}\right)e^{2a-2d}}{b}-\frac{2i\ln\left(e^{bx+a}-i e^{a-d}\right)e^{2a-2d}}{b}$	130

input
```
int(exp(2*b*x+2*a)*sinh(b*x+d)*tanh(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*exp(3*b*x+2*a+d)/b-5/2*exp(b*x+2*a-d)/b-2/(exp(2*b*x+2*a+2*d)+exp(2*a)
)/b*exp(b*x+4*a-d)+2*I*ln(exp(b*x+a)+I*exp(a-d))/b*exp(2*a-2*d)-2*I*ln(exp
(b*x+a)-I*exp(a-d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(81) = 162$.

Time = 0.11 (sec) , antiderivative size = 551, normalized size of antiderivative = 6.05

$$\int e^{2(a+bx)} \sinh(d+bx) \tanh^2(d+bx) \, dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)*tanh(b*x+d)^2,x, algorithm="fricas")
```

output
```
1/6*(cosh(b*x + d)^5*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*
d))*sinh(b*x + d)^5 + 5*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*si
nh(-2*a + 2*d))*sinh(b*x + d)^4 - 14*cosh(b*x + d)^3*cosh(-2*a + 2*d) + 2*
(5*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (5*cosh(b*x + d)^2 - 7)*sinh(-2*a +
2*d) - 7*cosh(-2*a + 2*d))*sinh(b*x + d)^3 + 2*(5*cosh(b*x + d)^3*cosh(-2*
a + 2*d) - 21*cosh(b*x + d)*cosh(-2*a + 2*d) - (5*cosh(b*x + d)^3 - 21*cos
h(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d)^2 + 24*(cosh(b*x + d)^2*cosh(-
2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^2 + 2*(co
sh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x +
d) - (cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*arctan(cos
h(b*x + d) + sinh(b*x + d)) - 27*cosh(b*x + d)*cosh(-2*a + 2*d) + (5*cosh(
b*x + d)^4*cosh(-2*a + 2*d) - 42*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (5*cos
h(b*x + d)^4 - 42*cosh(b*x + d)^2 - 27)*sinh(-2*a + 2*d) - 27*cosh(-2*a +
2*d))*sinh(b*x + d) - (cosh(b*x + d)^5 - 14*cosh(b*x + d)^3 - 27*cosh(b*x
+ d))*sinh(-2*a + 2*d))/(b*cosh(b*x + d)^2 + 2*b*cosh(b*x + d)*sinh(b*x +
d) + b*sinh(b*x + d)^2 + b)
```

Sympy [F]

$$\int e^{2(a+bx)} \sinh(d+bx) \tanh^2(d+bx) \, dx = e^{2a} \int e^{2bx} \sinh(bx+d) \tanh^2(bx+d) \, dx$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)*tanh(b*x+d)**2,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*sinh(b*x + d)*tanh(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90

$$\int e^{2(a+bx)}\sinh(d+bx)\tanh^2(d+bx)\,dx = -\frac{4\,\arctan\left(e^{(-bx-d)}\right)e^{(2\,a-2\,d)}}{b} - \frac{\left(14\,e^{(-2\,bx-2\,d)}+27\,e^{(-4\,bx-4\,d)}-1\right)e^{(2\,a-2\,d)}}{6\,b\left(e^{(-3\,bx-3\,d)}+e^{(-5\,bx-5\,d)}\right)}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)*tanh(b*x+d)^2,x, algorithm="maxima")
```

output
```
-4*arctan(e^(-b*x - d))*e^(2*a - 2*d)/b - 1/6*(14*e^(-2*b*x - 2*d) + 27*e^
(-4*b*x - 4*d) - 1)*e^(2*a - 2*d)/(b*(e^(-3*b*x - 3*d) + e^(-5*b*x - 5*d))
)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.88

$$\int e^{2(a+bx)}\sinh(d+bx)\tanh^2(d+bx)\,dx = \frac{24\,\arctan\left(e^{(bx+d)}\right)e^{(2\,a-2\,d)}+\left(e^{(3\,bx+2\,a+7\,d)}-15\,e^{(bx+2\,a+5\,d)}\right)e^{(-6\,d)}-\frac{12\,e^{(bx+2\,a-d)}}{e^{(2\,bx+2\,d)}+1}}{6\,b}$$

input
```
integrate(exp(2*b*x+2*a)*sinh(b*x+d)*tanh(b*x+d)^2,x, algorithm="giac")
```

output
```
1/6*(24*arctan(e^(b*x + d))*e^(2*a - 2*d) + (e^(3*b*x + 2*a + 7*d) - 15*e^
(b*x + 2*a + 5*d))*e^(-6*d) - 12*e^(b*x + 2*a - d)/(e^(2*b*x + 2*d) + 1))/
b
```

Mupad [B] (verification not implemented)

Time = 2.88 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

$$\int e^{2(a+bx)}\sinh(d+bx)\tanh^2(d+bx)\,dx=\frac{e^{2\,a+d+3\,b\,x}}{6\,b}-\frac{5\,e^{2\,a-d+b\,x}}{2\,b}-\frac{2\,e^{2\,a-d+b\,x}}{b\,\left(e^{2\,d+2\,b\,x}+1\right)}+\frac{4\,\sqrt{e^{4\,a-4\,d}}\,\mathrm{atan}\left(\frac{e^{2\,a}\,e^{-d}\,e^{b\,x}\,\sqrt{b^2}}{b\,\sqrt{e^{4\,a}\,e^{-4\,d}}}\right)}{\sqrt{b^2}}$$

input
```
int(exp(2*a + 2*b*x)*sinh(d + b*x)*tanh(d + b*x)^2,x)
```

output
```
exp(2*a + d + 3*b*x)/(6*b) - (5*exp(2*a - d + b*x))/(2*b) - (2*exp(2*a - d
 + b*x))/(b*(exp(2*d + 2*b*x) + 1)) + (4*exp(4*a - 4*d)^(1/2)*atan((exp(2*
a)*exp(-d)*exp(b*x)*(b^2)^(1/2))/(b*(exp(4*a)*exp(-4*d))^(1/2))))/(b^2)^(1
/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.02

$$\int e^{2(a+bx)}\sinh(d+bx)\tanh^2(d+bx)\,dx=\frac{e^{2a}\left(24e^{2bx+2d}atan\left(e^{bx+d}\right)+24atan\left(e^{bx+d}\right)+e^{5bx+5d}-14e^{3bx+3d}-27e^{bx+d}\right)}{6e^{2d}b\left(e^{2bx+2d}+1\right)}$$

input
```
int(exp(2*b*x+2*a)*sinh(b*x+d)*tanh(b*x+d)^2,x)
```

output
```
(e**(2*a)*(24*e**(2*b*x + 2*d)*atan(e**(b*x + d)) + 24*atan(e**(b*x + d))
+ e**(5*b*x + 5*d) - 14*e**(3*b*x + 3*d) - 27*e**(b*x + d)))/(6*e**(2*d)*b
*(e**(2*b*x + 2*d) + 1))
```

3.72 $\int e^{2(a+bx)}\tanh^2(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 73

$$\int e^{2(a+bx)}\tanh^2(d+bx)\,dx=\frac{e^{2a+2bx}}{2b}-\frac{2e^{2a-2d}}{b\left(1+e^{2d+2bx}\right)}-\frac{2e^{2a-2d}\log\left(1+e^{2d+2bx}\right)}{b}$$

output
```
1/2*exp(2*b*x+2*a)/b-2*exp(2*a-2*d)/b/(1+exp(2*b*x+2*d))-2*exp(2*a-2*d)*ln
(1+exp(2*b*x+2*d))/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 158 vs. 2(73) = 146.

Time = 0.13 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.16

$$\int e^{2(a+bx)}\tanh^2(d+bx)\,dx$$
$$=\frac{e^{2a}(\cosh(d)-\sinh(d))\left(e^{2bx}\left(1-4\log\left(\left(1+e^{2bx}\right)\cosh(d)+\left(-1+e^{2bx}\right)\sinh(d)\right)\right)+\cosh(2d)\left(-4+e^{4b}\right.\right.}{2b\left(\left(1+e^{2\right.\right.}}$$

input
```
Integrate[E^(2*(a + b*x))*Tanh[d + b*x]^2,x]
```

output
```
(E^(2*a)*(Cosh[d] - Sinh[d])*(E^(2*b*x)*(1 - 4*Log[(1 + E^(2*b*x))*Cosh[d]
 + (-1 + E^(2*b*x))*Sinh[d]]) + Cosh[2*d]*(-4 + E^(4*b*x) - 4*Log[(1 + E^(
2*b*x))*Cosh[d] + (-1 + E^(2*b*x))*Sinh[d]]) + (4 + E^(4*b*x) + 4*Log[(1 +
 E^(2*b*x))*Cosh[d] + (-1 + E^(2*b*x))*Sinh[d]])*Sinh[2*d]))/(2*b*((1 + E^
(2*b*x))*Cosh[d] + (-1 + E^(2*b*x))*Sinh[d]))
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.58, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2720, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)} \tanh^2(bx+d) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{e^{2a+bx}\left(1-e^{2bx}\right)^2}{\left(1+e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{2a}\int \frac{e^{bx}\left(1-e^{2bx}\right)^2}{\left(1+e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{353}$$

$$\frac{e^{2a}\int \frac{\left(1-e^{2bx}\right)^2}{\left(1+e^{2bx}\right)^2}de^{2bx}}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{e^{2a}\int \left(1-\frac{4}{1+e^{2bx}}+\frac{4}{\left(1+e^{2bx}\right)^2}\right)de^{2bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2a}\left(e^{2bx}-\frac{4}{e^{2bx}+1}-4\log\left(e^{2bx}+1\right)\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Tanh[d + b*x]^2,x]
```

output
```
(E^(2*a)*(E^(2*b*x) - 4/(1 + E^(2*b*x)) - 4*Log[1 + E^(2*b*x)]))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.29

method	result	size
risch	$\frac{e^{2bx+2a}}{2b}+\frac{4\,e^{2a-2d}a}{b}-\frac{2\,e^{4a-2d}}{\left(e^{2bx+2a+2d}+e^{2a}\right)b}-\frac{2\ln\left(e^{2bx+2a}+e^{2a-2d}\right)e^{2a-2d}}{b}$	94

input
```
int(exp(2*b*x+2*a)*tanh(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*b*x+2*a)/b+4/b*exp(2*a-2*d)*a-2/(exp(2*b*x+2*a+2*d)+exp(2*a))/b*
exp(4*a-2*d)-2*ln(exp(2*b*x+2*a)+exp(2*a-2*d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. 2(66) = 132.

Time = 0.08 (sec) , antiderivative size = 453, normalized size of antiderivative = 6.21

$$\int e^{2(a+bx)}\tanh^2(d+bx)\,dx$$
$$=\frac{\cosh(bx+d)^4\cosh(-2a+2d)+(\cosh(-2a+2d)-\sinh(-2a+2d))\sinh(bx+d)^4+4(\cosh(bx+}{}$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d)^2,x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + d)^4*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*
d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*si
nh(-2*a + 2*d))*sinh(b*x + d)^3 + cosh(b*x + d)^2*cosh(-2*a + 2*d) + (6*co
sh(b*x + d)^2*cosh(-2*a + 2*d) - (6*cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d)
+ cosh(-2*a + 2*d))*sinh(b*x + d)^2 - 4*(cosh(b*x + d)^2*cosh(-2*a + 2*d)
+ (cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^2 + 2*(cosh(b*x + d)
*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d) - (cosh(
b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*log(2*cosh(b*x + d)/(
cosh(b*x + d) - sinh(b*x + d))) + 2*(2*cosh(b*x + d)^3*cosh(-2*a + 2*d) +
cosh(b*x + d)*cosh(-2*a + 2*d) - (2*cosh(b*x + d)^3 + cosh(b*x + d))*sinh(
-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^4 + cosh(b*x + d)^2 - 4)*sinh(
-2*a + 2*d) - 4*cosh(-2*a + 2*d))/(b*cosh(b*x + d)^2 + 2*b*cosh(b*x + d)*s
inh(b*x + d) + b*sinh(b*x + d)^2 + b)
```

Sympy [F]

$$\int e^{2(a+bx)}\tanh^2(d+bx)\,dx = e^{2a}\int e^{2bx}\tanh^2(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d)**2,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*tanh(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int e^{2(a+bx)}\tanh^2(d+bx)\,dx = -\frac{4\,(bx+d)e^{(2\,a-2\,d)}}{b} - \frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(-2\,bx-2\,d)}+1\right)}{b} + \frac{\left(5\,e^{(-2\,bx-2\,d)}+1\right)e^{(2\,a-2\,d)}}{2\,b\left(e^{(-2\,bx-2\,d)}+e^{(-4\,bx-4\,d)}\right)}$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d)^2,x, algorithm="maxima")
```

output
```
-4*(b*x + d)*e^(2*a - 2*d)/b - 2*e^(2*a - 2*d)*log(e^(-2*b*x - 2*d) + 1)/b
 + 1/2*(5*e^(-2*b*x - 2*d) + 1)*e^(2*a - 2*d)/(b*(e^(-2*b*x - 2*d) + e^(-4
*b*x - 4*d)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int e^{2(a+bx)}\tanh^2(d+bx)\,dx = -\frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(2\,bx+2\,d)}+1\right)}{b}+\frac{e^{(2\,bx+2\,a)}}{2\,b}+\frac{2\,e^{(2\,bx+2\,a)}}{b(e^{(2\,bx+2\,d)}+1)}$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d)^2,x, algorithm="giac")
```

output
```
-2*e^(2*a - 2*d)*log(e^(2*b*x + 2*d) + 1)/b + 1/2*e^(2*b*x + 2*a)/b + 2*e^
(2*b*x + 2*a)/(b*(e^(2*b*x + 2*d) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12

$$\int e^{2(a+bx)}\tanh^2(d+bx)\,dx = \frac{\mathrm{e}^{2\,a+2\,b\,x}}{2\,b}-\frac{2\,\mathrm{e}^{2\,a-2\,d}\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+\mathrm{e}^{2\,a}\,\mathrm{e}^{-2\,d}\right)}{b} - \frac{2\,\mathrm{e}^{4\,a-4\,d}}{b\,\left(\mathrm{e}^{2\,a-2\,d}+\mathrm{e}^{2\,a+2\,b\,x}\right)}$$

input
```
int(exp(2*a + 2*b*x)*tanh(d + b*x)^2,x)
```

output
```
exp(2*a + 2*b*x)/(2*b) - (2*exp(2*a - 2*d)*log(exp(2*a)*exp(2*b*x) + exp(2
*a)*exp(-2*d)))/b - (2*exp(4*a - 4*d))/(b*(exp(2*a - 2*d) + exp(2*a + 2*b*
x)))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.29

$$\int e^{2(a+bx)}\tanh^2(d+bx)\,dx$$
$$=\frac{e^{2a}\left(e^{4bx+4d}-4e^{2bx+2d}\log\left(e^{2bx+2d}+1\right)+5e^{2bx+2d}-4\log\left(e^{2bx+2d}+1\right)\right)}{2e^{2d}b\left(e^{2bx+2d}+1\right)}$$

input
```
int(exp(2*b*x+2*a)*tanh(b*x+d)^2,x)
```

output
```
(e**(2*a)*(e**(4*b*x + 4*d) - 4*e**(2*b*x + 2*d)*log(e**(2*b*x + 2*d) + 1)
 + 5*e**(2*b*x + 2*d) - 4*log(e**(2*b*x + 2*d) + 1)))/(2*e**(2*d)*b*(e**(2
*b*x + 2*d) + 1))
```

3.73 $\int e^{2(a+bx)}\mathbf{sech}(d+bx)\tanh(d+bx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int e^{2(a+bx)}\text{sech}(d+bx)\tanh(d+bx)\,dx=\frac{2e^{2a-d+bx}}{b}+\frac{2e^{2a-d+bx}}{b\left(1+e^{2d+2bx}\right)}-\frac{4e^{2a-2d}\arctan\left(e^{d+bx}\right)}{b}$$

output
```
2*exp(b*x+2*a-d)/b+2*exp(b*x+2*a-d)/b/(1+exp(2*b*x+2*d))-4*exp(2*a-2*d)*ar
ctan(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.46

$$\int e^{2(a+bx)}\text{sech}(d+bx)\tanh(d+bx)\,dx$$
$$=\frac{2e^{2a}\left(e^{bx}\cosh(d)-2\arctan\left(e^{bx}(\cosh(d)+\sinh(d))\right)\cosh(2d)-e^{bx}\sinh(d)+\frac{e^{bx}(\cosh(d)-\sinh(d))^2}{(1+e^{2bx})\cosh(d)+(-1+e^{2bx})\sinh}\right.}{b}$$

input
```
Integrate[E^(2*(a + b*x))*Sech[d + b*x]*Tanh[d + b*x],x]
```

output
```
(2*E^(2*a)*(E^(b*x)*Cosh[d] - 2*ArcTan[E^(b*x)*(Cosh[d] + Sinh[d])]*Cosh[2
*d] - E^(b*x)*Sinh[d] + (E^(b*x)*(Cosh[d] - Sinh[d])^2)/((1 + E^(2*b*x))*C
osh[d] + (-1 + E^(2*b*x))*Sinh[d]) + 2*ArcTan[E^(b*x)*(Cosh[d] + Sinh[d])]
*Sinh[2*d]))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2720, 27, 360, 27, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\tanh(bx+d)\text{sech}(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{2e^{2a+2bx}\left(1-e^{2bx}\right)}{\left(1+e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2e^{2a}\int \frac{e^{2bx}\left(1-e^{2bx}\right)}{\left(1+e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \text{360}$$

$$-\frac{2e^{2a}\left(-\frac{1}{2}\int -\frac{2\left(1-e^{2bx}\right)}{1+e^{2bx}}de^{bx}-\frac{e^{bx}}{e^{2bx}+1}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2e^{2a}\left(\int \frac{1-e^{2bx}}{1+e^{2bx}}de^{bx}-\frac{e^{bx}}{e^{2bx}+1}\right)}{b}$$

$$\Big\downarrow \text{299}$$

$$-\frac{2e^{2a}\left(2\int \frac{1}{1+e^{2bx}}de^{bx}-e^{bx}-\frac{e^{bx}}{e^{2bx}+1}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2e^{2a}\left(2\arctan\left(e^{bx}\right)-e^{bx}-\frac{e^{bx}}{e^{2bx}+1}\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Sech[d + b*x]*Tanh[d + b*x],x]
```

output
```
(-2*E^(2*a)*(-E^(b*x) - E^(b*x)/(1 + E^(2*b*x)) + 2*ArcTan[E^(b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.62

method	result	size
risch	$\frac{2\,\mathrm{e}^{bx+2a-d}}{b}+\frac{2\,\mathrm{e}^{bx+4a-d}}{\left(\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)b}+\frac{2i\ln\left(\mathrm{e}^{bx+a}-i\mathrm{e}^{a-d}\right)\mathrm{e}^{2a-2d}}{b}-\frac{2i\ln\left(\mathrm{e}^{bx+a}+i\mathrm{e}^{a-d}\right)\mathrm{e}^{2a-2d}}{b}$	115

input
```
int(exp(2*b*x+2*a)*sech(b*x+d)*tanh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
2*exp(b*x+2*a-d)/b+2/(exp(2*b*x+2*a+2*d)+exp(2*a))/b*exp(b*x+4*a-d)+2*I*ln
(exp(b*x+a)-I*exp(a-d))/b*exp(2*a-2*d)-2*I*ln(exp(b*x+a)+I*exp(a-d))/b*exp
(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(66) = 132.

Time = 0.09 (sec) , antiderivative size = 360, normalized size of antiderivative = 5.07

$$\int e^{2(a+bx)}\mathrm{sech}(d+bx)\tanh(d+bx)\,dx$$
$$=\frac{2\left(\cosh\left(bx+d\right)^{3}\cosh\left(-2\,a+2\,d\right)+\left(\cosh\left(-2\,a+2\,d\right)-\sinh\left(-2\,a+2\,d\right)\right)\sinh\left(bx+d\right)^{3}+3\left(\cosh\left(bx\right.\right.}{}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)*tanh(b*x+d),x, algorithm="fricas")
```

output
```
2*(cosh(b*x + d)^3*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*d)
)*sinh(b*x + d)^3 + 3*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh
(-2*a + 2*d))*sinh(b*x + d)^2 - 2*(cosh(b*x + d)^2*cosh(-2*a + 2*d) + (cos
h(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^2 + 2*(cosh(b*x + d)*cosh(
-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x +
d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*arctan(cosh(b*x + d) + sinh
(b*x + d)) + 2*cosh(b*x + d)*cosh(-2*a + 2*d) + (3*cosh(b*x + d)^2*cosh(-2
*a + 2*d) - (3*cosh(b*x + d)^2 + 2)*sinh(-2*a + 2*d) + 2*cosh(-2*a + 2*d))
*sinh(b*x + d) - (cosh(b*x + d)^3 + 2*cosh(b*x + d))*sinh(-2*a + 2*d))/(b*
cosh(b*x + d)^2 + 2*b*cosh(b*x + d)*sinh(b*x + d) + b*sinh(b*x + d)^2 + b)
```

Sympy [F]

$$\int e^{2(a+bx)}\operatorname{sech}(d+bx)\tanh(d+bx)\,dx = e^{2a}\int e^{2bx}\tanh\left(bx+d\right)\operatorname{sech}\left(bx+d\right)dx$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)*tanh(b*x+d),x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*tanh(b*x + d)*sech(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int e^{2(a+bx)}\operatorname{sech}(d+bx)\tanh(d+bx)\,dx = \frac{4\arctan\left(e^{(-bx-d)}\right)e^{(2a-2d)}}{b} + \frac{2\left(2e^{(-2bx-2d)}+1\right)e^{(2a-2d)}}{b\left(e^{(-bx-d)}+e^{(-3bx-3d)}\right)}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)*tanh(b*x+d),x, algorithm="maxima")
```

output
```
4*arctan(e^(-b*x - d))*e^(2*a - 2*d)/b + 2*(2*e^(-2*b*x - 2*d) + 1)*e^(2*a
 - 2*d)/(b*(e^(-b*x - d) + e^(-3*b*x - 3*d)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

$$\int e^{2(a+bx)}\text{sech}(d+bx)\tanh(d+bx)\,dx = -\frac{2\left(2\,\arctan\left(e^{(bx+d)}\right)e^{(2\,a-2\,d)}-\frac{e^{(bx+2\,a-d)}}{e^{(2\,bx+2\,d)}+1}-e^{(bx+2\,a-d)}\right)}{b}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)*tanh(b*x+d),x, algorithm="giac")
```

output
```
-2*(2*arctan(e^(b*x + d))*e^(2*a - 2*d) - e^(b*x + 2*a - d)/(e^(2*b*x + 2*
d) + 1) - e^(b*x + 2*a - d))/b
```

Mupad [B] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.35

$$\int e^{2(a+bx)}\text{sech}(d+bx)\tanh(d+bx)\,dx = \frac{2\,e^{2\,a-d+b\,x}}{b}+\frac{2\,e^{2\,a-d+b\,x}}{b\,\left(e^{2\,d+2\,b\,x}+1\right)} - \frac{4\,\sqrt{e^{4\,a-4\,d}}\,\mathrm{atan}\left(\frac{e^{2\,a}\,e^{-d}\,e^{b\,x}\,\sqrt{b^2}}{b\,\sqrt{e^{4\,a}\,e^{-4\,d}}}\right)}{\sqrt{b^2}}$$

input
```
int((exp(2*a + 2*b*x)*tanh(d + b*x))/cosh(d + b*x),x)
```

output
```
(2*exp(2*a - d + b*x))/b + (2*exp(2*a - d + b*x))/(b*(exp(2*d + 2*b*x) + 1
)) - (4*exp(4*a - 4*d)^(1/2)*atan((exp(2*a)*exp(-d)*exp(b*x)*(b^2)^(1/2))/
(b*(exp(4*a)*exp(-4*d))^(1/2))))/(b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14

$$\int e^{2(a+bx)}\operatorname{sech}(d+bx)\tanh(d+bx)\,dx$$
$$= \frac{2e^{2a}\left(-2e^{2bx+2d}atan\left(e^{bx+d}\right)-2atan\left(e^{bx+d}\right)+e^{3bx+3d}+2e^{bx+d}\right)}{e^{2d}b\left(e^{2bx+2d}+1\right)}$$

input
```
int(exp(2*b*x+2*a)*sech(b*x+d)*tanh(b*x+d),x)
```

output
```
(2*e**(2*a)*( - 2*e**(2*b*x + 2*d)*atan(e**(b*x + d)) - 2*atan(e**(b*x + d
)) + e**(3*b*x + 3*d) + 2*e**(b*x + d)))/(e**(2*d)*b*(e**(2*b*x + 2*d) + 1
))
```

3.74 $\int e^{2(a+bx)}\operatorname{sech}^2(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 56

$$\int e^{2(a+bx)}\operatorname{sech}^2(d+bx)\,dx=\frac{2e^{2a-2d}}{b\left(1+e^{2d+2bx}\right)}+\frac{2e^{2a-2d}\log\left(1+e^{2d+2bx}\right)}{b}$$

output
```
2*exp(2*a-2*d)/b/(1+exp(2*b*x+2*d))+2*exp(2*a-2*d)*ln(1+exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

$$\int e^{2(a+bx)}\operatorname{sech}^2(d+bx)\,dx=\frac{2e^{2a-2d}\left(-e^{2(d+bx)}+\left(1+e^{2(d+bx)}\right)\log\left(1+e^{2(d+bx)}\right)\right)}{b\left(1+e^{2(d+bx)}\right)}$$

input
```
Integrate[E^(2*(a + b*x))*Sech[d + b*x]^2,x]
```

output
```
(2*E^(2*a - 2*d)*(-E^(2*(d + b*x)) + (1 + E^(2*(d + b*x)))*Log[1 + E^(2*(d
 + b*x))]))/(b*(1 + E^(2*(d + b*x))))
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.54, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{sech}^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{4e^{2a+3bx}}{(1+e^{2bx})^2}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{4e^{2a}\int \frac{e^{3bx}}{(1+e^{2bx})^2}de^{bx}}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{2e^{2a}\int \frac{e^{2bx}}{(1+e^{2bx})^2}de^{2bx}}{b}$$

$$\Big\downarrow \text{49}$$

$$\frac{2e^{2a}\int \left(\frac{1}{1+e^{2bx}}-\frac{1}{(1+e^{2bx})^2}\right)de^{2bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2e^{2a}\left(\frac{1}{e^{2bx}+1}+\log\left(e^{2bx}+1\right)\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Sech[d + b*x]^2,x]
```

output
```
(2*E^(2*a)*((1 + E^(2*b*x))^(-1) + Log[1 + E^(2*b*x)]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.43

method	result	size
risch	$-\frac{4\,\mathrm{e}^{2a-2d} a}{b}+\frac{2\,\mathrm{e}^{4a-2d}}{\left(\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right) b}+\frac{2 \ln \left(\mathrm{e}^{2bx+2a}+\mathrm{e}^{2a-2d}\right) \mathrm{e}^{2a-2d}}{b}$	80

input
```
int(exp(2*b*x+2*a)*sech(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-4/b*exp(2*a-2*d)*a+2/(exp(2*b*x+2*a+2*d)+exp(2*a))/b*exp(4*a-2*d)+2*ln(ex
p(2*b*x+2*a)+exp(2*a-2*d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(52) = 104$.

Time = 0.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 3.59

$$\int e^{2(a+bx)}\text{sech}^2(d+bx)\,dx$$
$$= \frac{2\left(\left(\cosh\left(bx+d\right)^2\cosh\left(-2a+2d\right)+\left(\cosh\left(-2a+2d\right)-\sinh\left(-2a+2d\right)\right)\sinh\left(bx+d\right)^2+2\left(\cosh\left(b\right.\right.\right.}{}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)^2,x, algorithm="fricas")
```

output
```
2*((cosh(b*x + d)^2*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*d
))*sinh(b*x + d)^2 + 2*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sin
h(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + co
sh(-2*a + 2*d))*log(2*cosh(b*x + d)/(cosh(b*x + d) - sinh(b*x + d))) + cos
h(-2*a + 2*d) - sinh(-2*a + 2*d))/(b*cosh(b*x + d)^2 + 2*b*cosh(b*x + d)*s
inh(b*x + d) + b*sinh(b*x + d)^2 + b)
```

Sympy [F]

$$\int e^{2(a+bx)}\text{sech}^2(d+bx)\,dx = e^{2a}\int e^{2bx}\,\text{sech}^2\left(bx+d\right)dx$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)**2,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*sech(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.38

$$\int e^{2(a+bx)}\text{sech}^2(d+bx)\,dx = 4\,xe^{(2\,a-2\,d)} + \frac{4\,de^{(2\,a-2\,d)}}{b} + \frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(-2\,bx-2\,d)}+1\right)}{b} - \frac{2\,e^{(2\,a-2\,d)}}{b(e^{(-2\,bx-2\,d)}+1)}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)^2,x, algorithm="maxima")
```

output
```
4*x*e^(2*a - 2*d) + 4*d*e^(2*a - 2*d)/b + 2*e^(2*a - 2*d)*log(e^(-2*b*x -
2*d) + 1)/b - 2*e^(2*a - 2*d)/(b*(e^(-2*b*x - 2*d) + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int e^{2(a+bx)}\text{sech}^2(d+bx)\,dx = \frac{2\left(e^{(-2\,d)}\log\left(e^{(2\,bx+2\,d)}+1\right)+\frac{e^{(-2\,d)}}{e^{(2\,bx+2\,d)}+1}\right)e^{(2\,a)}}{b}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)^2,x, algorithm="giac")
```

output
```
2*(e^(-2*d)*log(e^(2*b*x + 2*d) + 1) + e^(-2*d)/(e^(2*b*x + 2*d) + 1))*e^(
2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int e^{2(a+bx)}\text{sech}^2(d+bx)\,dx = \frac{2\,\mathrm{e}^{2\,a-2\,d}\,\ln\left(\mathrm{e}^{2\,d}\,\mathrm{e}^{2\,b\,x}+1\right)}{b} + \frac{2\,\mathrm{e}^{2\,a-2\,d}}{b\,\left(\mathrm{e}^{2\,d+2\,b\,x}+1\right)}$$

input
```
int(exp(2*a + 2*b*x)/cosh(d + b*x)^2,x)
```

output
```
(2*exp(2*a - 2*d)*log(exp(2*d)*exp(2*b*x) + 1))/b + (2*exp(2*a - 2*d))/(b*
(exp(2*d + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

$$\int e^{2(a+bx)}\text{sech}^2(d+bx)\,dx = \frac{2e^{2a}\left(e^{2bx+2d}\log\left(e^{2bx+2d}+1\right)-e^{2bx+2d}+\log\left(e^{2bx+2d}+1\right)\right)}{e^{2d}b\left(e^{2bx+2d}+1\right)}$$

input
```
int(exp(2*b*x+2*a)*sech(b*x+d)^2,x)
```

output
```
(2*e**(2*a)*(e**(2*b*x + 2*d)*log(e**(2*b*x + 2*d) + 1) - e**(2*b*x + 2*d)
 + log(e**(2*b*x + 2*d) + 1)))/(e**(2*d)*b*(e**(2*b*x + 2*d) + 1))
```

3.75 $\int e^{2(a+bx)}\mathrm{csch}(d+bx)\mathrm{sech}^2(d+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 76

$$\int e^{2(a+bx)}\mathrm{csch}(d+bx)\mathrm{sech}^2(d+bx)\,dx = -\frac{2e^{2a-d+bx}}{b\left(1+e^{2d+2bx}\right)}+\frac{4e^{2a-2d}\arctan\left(e^{d+bx}\right)}{b} - \frac{2e^{2a-2d}\mathrm{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
-2*exp(b*x+2*a-d)/b/(1+exp(2*b*x+2*d))+4*exp(2*a-2*d)*arctan(exp(b*x+d))/b
-2*exp(2*a-2*d)*arctanh(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int e^{2(a+bx)}\mathrm{csch}(d+bx)\mathrm{sech}^2(d+bx)\,dx = \frac{e^{2a-2d}\left(-\frac{2e^{d+bx}}{1+e^{2(d+bx)}}+4\arctan\left(e^{d+bx}\right)+\log\left(1-e^{d+bx}\right)-\log\left(1+e^{d+bx}\right)\right)}{b}$$

input
```
Integrate[E^(2*(a + b*x))*Csch[d + b*x]*Sech[d + b*x]^2,x]
```

output
```
(E^(2*a - 2*d)*((-2*E^(d + b*x))/(1 + E^(2*(d + b*x))) + 4*ArcTan[E^(d + b
*x)] + Log[1 - E^(d + b*x)] - Log[1 + E^(d + b*x)]))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 27, 372, 397, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{csch}(bx+d)\text{sech}^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{8e^{2a+4bx}}{(1-e^{2bx})(1+e^{2bx})^2}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{8e^{2a}\int \frac{e^{4bx}}{(1-e^{2bx})(1+e^{2bx})^2}de^{bx}}{b}$$

$$\Big\downarrow \text{372}$$

$$-\frac{8e^{2a}\left(\frac{e^{bx}}{4(e^{2bx}+1)}-\frac{1}{4}\int \frac{1-3e^{2bx}}{(1-e^{2bx})(1+e^{2bx})}de^{bx}\right)}{b}$$

$$\Big\downarrow \text{397}$$

$$-\frac{8e^{2a}\left(\frac{1}{4}\left(\int \frac{1}{1-e^{2bx}}de^{bx}-2\int \frac{1}{1+e^{2bx}}de^{bx}\right)+\frac{e^{bx}}{4(e^{2bx}+1)}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$-\frac{8e^{2a}\left(\frac{1}{4}\left(\int \frac{1}{1-e^{2bx}}de^{bx}-2\arctan\left(e^{bx}\right)\right)+\frac{e^{bx}}{4(e^{2bx}+1)}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{8e^{2a}\left(\frac{1}{4}\left(\text{arctanh}(e^{bx})-2\arctan\left(e^{bx}\right)\right)+\frac{e^{bx}}{4(e^{2bx}+1)}\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Csch[d + b*x]*Sech[d + b*x]^2,x]
```

output
```
(-8*E^(2*a)*(E^(b*x)/(4*(1 + E^(2*b*x))) + (-2*ArcTan[E^(b*x)] + ArcTanh[E
^(b*x)])/4))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 13.74 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.03

method	result
risch	$-\frac{2\,\mathrm{e}^{bx+4a-d}}{\left(\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)b}+\frac{2i\ln\left(\mathrm{e}^{bx+a}+i\mathrm{e}^{a-d}\right)\mathrm{e}^{2a-2d}}{b}-\frac{2i\ln\left(\mathrm{e}^{bx+a}-i\mathrm{e}^{a-d}\right)\mathrm{e}^{2a-2d}}{b}+\frac{\ln\left(\mathrm{e}^{bx+a}-\mathrm{e}^{a-d}\right)\mathrm{e}^{2a-2d}}{b}-\frac{\ln\left(\mathrm{e}^{bx+a}+\right.}{}$

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)*sech(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/(exp(2*b*x+2*a+2*d)+exp(2*a))/b*exp(b*x+4*a-d)+2*I*ln(exp(b*x+a)+I*exp(
a-d))/b*exp(2*a-2*d)-2*I*ln(exp(b*x+a)-I*exp(a-d))/b*exp(2*a-2*d)+ln(exp(b
*x+a)-exp(a-d))/b*exp(2*a-2*d)-ln(exp(b*x+a)+exp(a-d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(70) = 140$.

Time = 0.11 (sec) , antiderivative size = 491, normalized size of antiderivative = 6.46

$$\int e^{2(a+bx)}\mathrm{csch}(d+bx)\mathrm{sech}^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)*sech(b*x+d)^2,x, algorithm="fricas")
```

output
```
(4*(cosh(b*x + d)^2*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*d
))*sinh(b*x + d)^2 + 2*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sin
h(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + co
sh(-2*a + 2*d))*arctan(cosh(b*x + d) + sinh(b*x + d)) - 2*cosh(b*x + d)*co
sh(-2*a + 2*d) - (cosh(b*x + d)^2*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - s
inh(-2*a + 2*d))*sinh(b*x + d)^2 + 2*(cosh(b*x + d)*cosh(-2*a + 2*d) - cos
h(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^2 + 1)*sinh(-2
*a + 2*d) + cosh(-2*a + 2*d))*log(cosh(b*x + d) + sinh(b*x + d) + 1) + (co
sh(b*x + d)^2*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*sin
h(b*x + d)^2 + 2*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a
 + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*
a + 2*d))*log(cosh(b*x + d) + sinh(b*x + d) - 1) - 2*(cosh(-2*a + 2*d) - s
inh(-2*a + 2*d))*sinh(b*x + d) + 2*cosh(b*x + d)*sinh(-2*a + 2*d))/(b*cosh
(b*x + d)^2 + 2*b*cosh(b*x + d)*sinh(b*x + d) + b*sinh(b*x + d)^2 + b)
```

Sympy [F]

$$\int e^{2(a+bx)}\operatorname{csch}(d+bx)\operatorname{sech}^2(d+bx)\,dx = e^{2a}\int e^{2bx}\operatorname{csch}(bx+d)\operatorname{sech}^2(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)*sech(b*x+d)**2,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*csch(b*x + d)*sech(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.36

$$\begin{aligned}\int e^{2(a+bx)}\operatorname{csch}(d+bx)\operatorname{sech}^2(d+bx)\,dx = &-\frac{4\arctan\left(e^{(-bx-d)}\right)e^{(2a-2d)}}{b}\\&-\frac{e^{(2a-2d)}\log\left(e^{(-bx-d)}+1\right)}{b}\\&+\frac{e^{(2a-2d)}\log\left(e^{(-bx-d)}-1\right)}{b}\\&-\frac{2e^{(-bx+2a-3d)}}{b(e^{(-2bx-2d)}+1)}\end{aligned}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)*sech(b*x+d)^2,x, algorithm="maxima")
```

output
```
-4*arctan(e^(-b*x - d))*e^(2*a - 2*d)/b - e^(2*a - 2*d)*log(e^(-b*x - d) +
 1)/b + e^(2*a - 2*d)*log(e^(-b*x - d) - 1)/b - 2*e^(-b*x + 2*a - 3*d)/(b*
(e^(-2*b*x - 2*d) + 1))
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int e^{2(a+bx)}\text{csch}(d+bx)\text{sech}^2(d+bx)\,dx = \frac{\left(4\arctan\left(e^{(bx+d)}\right)e^{(-2d)} - e^{(-2d)}\log\left(e^{(bx+d)}+1\right) + e^{(-2d)}\log\left(\left|e^{(bx+d)}-1\right|\right) - \frac{2e^{(bx-d)}}{e^{(2bx+2d)}+1}\right)e^{(2a)}}{b}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)*sech(b*x+d)^2,x, algorithm="giac")
```

output
```
(4*arctan(e^(b*x + d))*e^(-2*d) - e^(-2*d)*log(e^(b*x + d) + 1) + e^(-2*d)
*log(abs(e^(b*x + d) - 1)) - 2*e^(b*x - d)/(e^(2*b*x + 2*d) + 1))*e^(2*a)/
b
```

Mupad [B] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.37

$$\int e^{2(a+bx)}\text{csch}(d+bx)\text{sech}^2(d+bx)\,dx = \frac{e^{2a-2d}\ln\left(160\,e^{6a}\,e^{-6d} - 160\,e^{6a}\,e^{-5d}\,e^{bx}\right)}{b} - \frac{2\,e^{2a-2d}\left(\ln\left(320\,e^{6a}\,e^{-5d}\,e^{bx} - e^{6a}\,e^{-6d}\,320\mathrm{i}\right)\,1\mathrm{i} - \ln\left(320\,e^{6a}\,e^{-5d}\,e^{bx} + e^{6a}\,e^{-6d}\,320\mathrm{i}\right)\,1\mathrm{i}\right)}{b} - \frac{e^{2a-2d}\ln\left(-160\,e^{6a}\,e^{-6d} - 160\,e^{6a}\,e^{-5d}\,e^{bx}\right)}{b} - \frac{2\,e^{2a-d+bx}}{b+b\,e^{2d+2bx}}$$

input
```
int(exp(2*a + 2*b*x)/(cosh(d + b*x)^2*sinh(d + b*x)),x)
```

output
```
(exp(2*a - 2*d)*log(160*exp(6*a)*exp(-6*d) - 160*exp(6*a)*exp(-5*d)*exp(b*
x)))/b - (2*exp(2*a - 2*d)*(log(320*exp(6*a)*exp(-5*d)*exp(b*x) - exp(6*a)
*exp(-6*d)*320i)*1i - log(exp(6*a)*exp(-6*d)*320i + 320*exp(6*a)*exp(-5*d)
*exp(b*x))*1i))/b - (exp(2*a - 2*d)*log(- 160*exp(6*a)*exp(-6*d) - 160*exp
(6*a)*exp(-5*d)*exp(b*x)))/b - (2*exp(2*a - d + b*x))/(b + b*exp(2*d + 2*b
*x))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.78

$$\int e^{2(a+bx)}\mathrm{csch}(d+bx)\mathrm{sech}^2(d+bx)\,dx$$
$$= \frac{e^{2a}\left(4e^{2bx+2d}atan\left(e^{bx+d}\right)+4atan\left(e^{bx+d}\right)+e^{2bx+2d}\log\left(e^{bx+d}-1\right)-e^{2bx+2d}\log\left(e^{bx+d}+1\right)-2e^{bx+d}+\log}{e^{2d}b\left(e^{2bx+2d}+1\right)}$$

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)*sech(b*x+d)^2,x)
```

output
```
(e**(2*a)*(4*e**(2*b*x + 2*d)*atan(e**(b*x + d)) + 4*atan(e**(b*x + d)) +
e**(2*b*x + 2*d)*log(e**(b*x + d) - 1) - e**(2*b*x + 2*d)*log(e**(b*x + d)
 + 1) - 2*e**(b*x + d) + log(e**(b*x + d) - 1) - log(e**(b*x + d) + 1)))/(
e**(2*d)*b*(e**(2*b*x + 2*d) + 1))
```

3.76 $\int e^{2(a+bx)}\mathbf{csch}^2(d+bx)\mathbf{sech}^2(d+bx)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 57

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\text{sech}^2(d+bx)\,dx=\frac{4e^{2a+2bx}}{b\left(1-e^{4d+4bx}\right)}-\frac{4e^{2a-2d}\text{arctanh}\left(e^{2d+2bx}\right)}{b}$$

output
```
4*exp(2*b*x+2*a)/b/(1-exp(4*b*x+4*d))-4*exp(2*a-2*d)*arctanh(exp(2*b*x+2*d
))/b
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\text{sech}^2(d+bx)\,dx=\frac{4e^{2a}\left(-\frac{e^{2bx}}{-1+e^{4(d+bx)}}-e^{-2d}\text{arctanh}\left(e^{2(d+bx)}\right)\right)}{b}$$

input
```
Integrate[E^(2*(a + b*x))*Csch[d + b*x]^2*Sech[d + b*x]^2,x]
```

output
```
(4*E^(2*a)*(-(E^(2*b*x)/(-1 + E^(4*(d + b*x)))) - ArcTanh[E^(2*(d + b*x))]
/E^(2*d)))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2720, 27, 807, 252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{csch}^2(bx+d)\text{sech}^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{16e^{2a+5bx}}{(1-e^{4bx})^2}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{16e^{2a}\int \frac{e^{5bx}}{(1-e^{4bx})^2}de^{bx}}{b}$$

$$\Big\downarrow \text{807}$$

$$\frac{8e^{2a}\int \frac{e^{2bx}}{(1-e^{2bx})^2}de^{2bx}}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{8e^{2a}\left(\frac{e^{2bx}}{2(1-e^{2bx})}-\frac{1}{2}\int \frac{1}{1-e^{2bx}}de^{2bx}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{8e^{2a}\left(\frac{e^{2bx}}{2(1-e^{2bx})}-\frac{1}{2}\text{arctanh}\left(e^{2bx}\right)\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Csch[d + b*x]^2*Sech[d + b*x]^2,x]
```

output
```
(8*E^(2*a)*(E^(2*b*x)/(2*(1 - E^(2*b*x))) - ArcTanh[E^(2*b*x)]/2))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(53) = 106$.

Time = 62.96 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.14

method	result	size
risch	$\frac{4\,\mathrm{e}^{2bx+6a}}{\left(-\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)\left(\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)b}-\frac{2\ln\left(\mathrm{e}^{2bx+2a}+\mathrm{e}^{2a-2d}\right)\mathrm{e}^{2a-2d}}{b}+\frac{2\ln\left(\mathrm{e}^{2bx+2a}-\mathrm{e}^{2a-2d}\right)\mathrm{e}^{2a-2d}}{b}$	122

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)^2*sech(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
4/(-exp(2*b*x+2*a+2*d)+exp(2*a))/(exp(2*b*x+2*a+2*d)+exp(2*a))/b*exp(2*b*x
+6*a)-2*ln(exp(2*b*x+2*a)+exp(2*a-2*d))/b*exp(2*a-2*d)+2*ln(exp(2*b*x+2*a)
-exp(2*a-2*d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(51) = 102$.

Time = 0.09 (sec) , antiderivative size = 658, normalized size of antiderivative = 11.54

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\text{sech}^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^2*sech(b*x+d)^2,x, algorithm="fricas"
)
```

output
```
-2*(2*cosh(b*x + d)^2*cosh(-2*a + 2*d) + 2*(cosh(-2*a + 2*d) - sinh(-2*a +
 2*d))*sinh(b*x + d)^2 - 2*cosh(b*x + d)^2*sinh(-2*a + 2*d) + (cosh(b*x +
d)^4*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*x + d
)^4 + 4*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*
sinh(b*x + d)^3 + 6*(cosh(b*x + d)^2*cosh(-2*a + 2*d) - cosh(b*x + d)^2*si
nh(-2*a + 2*d))*sinh(b*x + d)^2 + 4*(cosh(b*x + d)^3*cosh(-2*a + 2*d) - co
sh(b*x + d)^3*sinh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^4 - 1)*sinh
(-2*a + 2*d) - cosh(-2*a + 2*d))*log(2*cosh(b*x + d)/(cosh(b*x + d) - sinh
(b*x + d))) - (cosh(b*x + d)^4*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh
(-2*a + 2*d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b
*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^3 + 6*(cosh(b*x + d)^2*cosh(-2*a +
 2*d) - cosh(b*x + d)^2*sinh(-2*a + 2*d))*sinh(b*x + d)^2 + 4*(cosh(b*x +
d)^3*cosh(-2*a + 2*d) - cosh(b*x + d)^3*sinh(-2*a + 2*d))*sinh(b*x + d) -
(cosh(b*x + d)^4 - 1)*sinh(-2*a + 2*d) - cosh(-2*a + 2*d))*log(2*sinh(b*x
+ d)/(cosh(b*x + d) - sinh(b*x + d))) + 4*(cosh(b*x + d)*cosh(-2*a + 2*d)
- cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d))/(b*cosh(b*x + d)^4 + 4*b*
cosh(b*x + d)^3*sinh(b*x + d) + 6*b*cosh(b*x + d)^2*sinh(b*x + d)^2 + 4*b*
cosh(b*x + d)*sinh(b*x + d)^3 + b*sinh(b*x + d)^4 - b)
```

Sympy [F]

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\text{sech}^2(d+bx)\,dx = e^{2a}\int e^{2bx}\,\text{csch}^2\left(bx+d\right)\text{sech}^2\left(bx+d\right)dx$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)**2*sech(b*x+d)**2,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*csch(b*x + d)**2*sech(b*x + d)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(51) = 102$.

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.86

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\text{sech}^2(d+bx)\,dx = \frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}+1\right)}{b} + \frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}-1\right)}{b} - \frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(-2\,bx-2\,d)}+1\right)}{b} + \frac{4\,e^{(-2\,bx+2\,a-4\,d)}}{b(e^{(-4\,bx-4\,d)}-1)}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^2*sech(b*x+d)^2,x, algorithm="maxima"
)
```

output
```
2*e^(2*a - 2*d)*log(e^(-b*x - d) + 1)/b + 2*e^(2*a - 2*d)*log(e^(-b*x - d)
 - 1)/b - 2*e^(2*a - 2*d)*log(e^(-2*b*x - 2*d) + 1)/b + 4*e^(-2*b*x + 2*a
- 4*d)/(b*(e^(-4*b*x - 4*d) - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\text{sech}^2(d+bx)\,dx$$
$$= -\frac{2\left(e^{(-2\,d)}\log\left(e^{(2\,bx+2\,d)}+1\right)-e^{(-2\,d)}\log\left(\left|e^{(2\,bx+2\,d)}-1\right|\right)+\frac{2\,e^{(2\,bx)}}{e^{(4\,bx+4\,d)}-1}\right)e^{(2\,a)}}{b}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^2*sech(b*x+d)^2,x, algorithm="giac")
```

output
```
-2*(e^(-2*d)*log(e^(2*b*x + 2*d) + 1) - e^(-2*d)*log(abs(e^(2*b*x + 2*d) -
 1)) + 2*e^(2*b*x)/(e^(4*b*x + 4*d) - 1))*e^(2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.39

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\text{sech}^2(d+bx)\,dx = \frac{4\,e^{2\,a+2\,b\,x}}{b-b\,e^{4\,d+4\,b\,x}} - \frac{4\,\sqrt{e^{4\,a-4\,d}}\,\text{atan}\left(\frac{e^{2\,a}\,e^{2\,b\,x}\,\sqrt{-b^2}}{b\,\sqrt{e^{4\,a}\,e^{-4\,d}}}\right)}{\sqrt{-b^2}}$$

input
```
int(exp(2*a + 2*b*x)/(cosh(d + b*x)^2*sinh(d + b*x)^2),x)
```

output
```
(4*exp(2*a + 2*b*x))/(b - b*exp(4*d + 4*b*x)) - (4*exp(4*a - 4*d)^(1/2)*at
an((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/(b*(exp(4*a)*exp(-4*d))^(1/2))))/(-b
^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.60

$$\int e^{2(a+bx)}\operatorname{csch}^2(d+bx)\operatorname{sech}^2(d+bx)\,dx$$
$$= \frac{2e^{2a}\left(e^{4bx+4d}\log\left(e^{bx+d}-1\right)+e^{4bx+4d}\log\left(e^{bx+d}+1\right)-e^{4bx+4d}\log\left(e^{2bx+2d}+1\right)-2e^{2bx+2d}-\log\left(e^{bx+d}-1\right.\right.}{e^{2d}b\left(e^{4bx+4d}-1\right)}$$

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)^2*sech(b*x+d)^2,x)
```

output
```
(2*e**(2*a)*(e**(4*b*x + 4*d)*log(e**(b*x + d) - 1) + e**(4*b*x + 4*d)*log
(e**(b*x + d) + 1) - e**(4*b*x + 4*d)*log(e**(2*b*x + 2*d) + 1) - 2*e**(2*
b*x + 2*d) - log(e**(b*x + d) - 1) - log(e**(b*x + d) + 1) + log(e**(2*b*x
 + 2*d) + 1)))/(e**(2*d)*b*(e**(4*b*x + 4*d) - 1))
```

3.77 $\int e^{2(a+bx)}\mathrm{csch}^3(d+bx)\mathrm{sech}^2(d+bx)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 137

$$\int e^{2(a+bx)}\mathrm{csch}^3(d+bx)\mathrm{sech}^2(d+bx)\,dx = -\frac{4e^{2a+d+3bx}}{b\left(1-e^{2d+2bx}\right)^2\left(1+e^{2d+2bx}\right)} + \frac{e^{2a-d+bx}\left(5+3e^{2d+2bx}\right)}{b\left(1-e^{4d+4bx}\right)} - \frac{4e^{2a-2d}\arctan\left(e^{d+bx}\right)}{b} - \frac{e^{2a-2d}\mathrm{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
-4*exp(3*b*x+2*a+d)/b/(1-exp(2*b*x+2*d))^2/(1+exp(2*b*x+2*d))+exp(b*x+2*a-
d)*(5+3*exp(2*b*x+2*d))/b/(1-exp(4*b*x+4*d))-4*exp(2*a-2*d)*arctan(exp(b*x
+d))/b-exp(2*a-2*d)*arctanh(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int e^{2(a+bx)}\text{csch}^3(d+bx)\text{sech}^2(d+bx)\,dx = \frac{e^{2a-2d}\left(-\frac{4e^{d+bx}}{\left(-1+e^{2(d+bx)}\right)^2}-\frac{10e^{d+bx}}{-1+e^{2(d+bx)}}+\frac{4e^{d+bx}}{1+e^{2(d+bx)}}-8\arctan\left(e^{d+bx}\right)+\log\left(1-e^{d+bx}\right)-\log\left(1+e^{d+bx}\right)\right)}{2b}$$

input
```
Integrate[E^(2*(a + b*x))*Csch[d + b*x]^3*Sech[d + b*x]^2,x]
```

output
```
(E^(2*a - 2*d)*((-4*E^(d + b*x))/(-1 + E^(2*(d + b*x)))^2 - (10*E^(d + b*x
))/(-1 + E^(2*(d + b*x))) + (4*E^(d + b*x))/(1 + E^(2*(d + b*x))) - 8*ArcT
an[E^(d + b*x)] + Log[1 - E^(d + b*x)] - Log[1 + E^(d + b*x)]))/(2*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.84, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2720, 27, 372, 440, 27, 402, 25, 397, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{csch}^3(bx+d)\text{sech}^2(bx+d)\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int -\frac{32e^{2a+6bx}}{\left(1-e^{2bx}\right)^3\left(1+e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{32e^{2a}\int \frac{e^{6bx}}{\left(1-e^{2bx}\right)^3\left(1+e^{2bx}\right)^2}de^{bx}}{b}$$

$$\Big\downarrow \; 372$$

$$-\frac{32e^{2a}\left(\frac{e^{3bx}}{8(1-e^{2bx})^2(e^{2bx}+1)}-\frac{1}{8}\int\frac{e^{2bx}(3+5e^{2bx})}{(1-e^{2bx})^2(1+e^{2bx})^2}de^{bx}\right)}{b}$$

$\downarrow$ 440

$$-\frac{32e^{2a}\left(\frac{1}{8}\left(-\frac{1}{4}\int-\frac{4(2-e^{2bx})}{(1-e^{2bx})(1+e^{2bx})^2}de^{bx}-\frac{2e^{bx}}{(1-e^{2bx})(e^{2bx}+1)}\right)+\frac{e^{3bx}}{8(1-e^{2bx})^2(e^{2bx}+1)}\right)}{b}$$

$\downarrow$ 27

$$-\frac{32e^{2a}\left(\frac{1}{8}\left(\int\frac{2-e^{2bx}}{(1-e^{2bx})(1+e^{2bx})^2}de^{bx}-\frac{2e^{bx}}{(1-e^{2bx})(e^{2bx}+1)}\right)+\frac{e^{3bx}}{8(1-e^{2bx})^2(e^{2bx}+1)}\right)}{b}$$

$\downarrow$ 402

$$-\frac{32e^{2a}\left(\frac{1}{8}\left(-\frac{1}{4}\int-\frac{5-3e^{2bx}}{(1-e^{2bx})(1+e^{2bx})}de^{bx}+\frac{3e^{bx}}{4(e^{2bx}+1)}-\frac{2e^{bx}}{(1-e^{2bx})(e^{2bx}+1)}\right)+\frac{e^{3bx}}{8(1-e^{2bx})^2(e^{2bx}+1)}\right)}{b}$$

$\downarrow$ 25

$$-\frac{32e^{2a}\left(\frac{1}{8}\left(\frac{1}{4}\int\frac{5-3e^{2bx}}{(1-e^{2bx})(1+e^{2bx})}de^{bx}+\frac{3e^{bx}}{4(e^{2bx}+1)}-\frac{2e^{bx}}{(1-e^{2bx})(e^{2bx}+1)}\right)+\frac{e^{3bx}}{8(1-e^{2bx})^2(e^{2bx}+1)}\right)}{b}$$

$\downarrow$ 397

$$-\frac{32e^{2a}\left(\frac{1}{8}\left(\frac{1}{4}\left(\int\frac{1}{1-e^{2bx}}de^{bx}+4\int\frac{1}{1+e^{2bx}}de^{bx}\right)+\frac{3e^{bx}}{4(e^{2bx}+1)}-\frac{2e^{bx}}{(1-e^{2bx})(e^{2bx}+1)}\right)+\frac{e^{3bx}}{8(1-e^{2bx})^2(e^{2bx}+1)}\right)}{b}$$

$\downarrow$ 216

$$-\frac{32e^{2a}\left(\frac{1}{8}\left(\frac{1}{4}\left(\int\frac{1}{1-e^{2bx}}de^{bx}+4\arctan\left(e^{bx}\right)\right)+\frac{3e^{bx}}{4(e^{2bx}+1)}-\frac{2e^{bx}}{(1-e^{2bx})(e^{2bx}+1)}\right)+\frac{e^{3bx}}{8(1-e^{2bx})^2(e^{2bx}+1)}\right)}{b}$$

$\downarrow$ 219

$$-\frac{32e^{2a}\left(\frac{1}{8}\left(\frac{1}{4}\left(4\arctan\left(e^{bx}\right)+\text{arctanh}\left(e^{bx}\right)\right)+\frac{3e^{bx}}{4(e^{2bx}+1)}-\frac{2e^{bx}}{(1-e^{2bx})(e^{2bx}+1)}\right)+\frac{e^{3bx}}{8(1-e^{2bx})^2(e^{2bx}+1)}\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Csch[d + b*x]^3*Sech[d + b*x]^2,x]
```

output
```
(-32*E^(2*a)*(E^(3*b*x)/(8*(1 - E^(2*b*x))^2*(1 + E^(2*b*x))) + ((3*E^(b*x
))/(4*(1 + E^(2*b*x))) - (2*E^(b*x))/((1 - E^(2*b*x))*(1 + E^(2*b*x))) + (
4*ArcTan[E^(b*x)] + ArcTanh[E^(b*x)])/4)/8))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 182.69 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.53

method	result
risch	$\frac{\left(-3\,\mathrm{e}^{4bx+4a+4d}-6\,\mathrm{e}^{2bx+4a+2d}+5\,\mathrm{e}^{4a}\right)\mathrm{e}^{bx+4a-d}}{\left(\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)\left(-\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)^{2}b}+\frac{\ln\left(\mathrm{e}^{bx+a}-\mathrm{e}^{a-d}\right)\mathrm{e}^{2a-2d}}{2b}-\frac{\ln\left(\mathrm{e}^{bx+a}+\mathrm{e}^{a-d}\right)\mathrm{e}^{2a-2d}}{2b}+\frac{2i\ln\left(\mathrm{e}^{bx+a}-i\mathrm{e}^{a-d}\right)}{b}$

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)^3*sech(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(exp(2*b*x+2*a+2*d)+exp(2*a))/(-exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(-3*exp
(4*b*x+4*a+4*d)-6*exp(2*b*x+4*a+2*d)+5*exp(4*a))*exp(b*x+4*a-d)+1/2*ln(exp
(b*x+a)-exp(a-d))/b*exp(2*a-2*d)-1/2*ln(exp(b*x+a)+exp(a-d))/b*exp(2*a-2*d
)+2*I*ln(exp(b*x+a)-I*exp(a-d))/b*exp(2*a-2*d)-2*I*ln(exp(b*x+a)+I*exp(a-d
))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2109 vs. $2(124) = 248$.

Time = 0.10 (sec) , antiderivative size = 2109, normalized size of antiderivative = 15.39

$$\int e^{2(a+bx)}\operatorname{csch}^3(d+bx)\operatorname{sech}^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^3*sech(b*x+d)^2,x, algorithm="fricas"
)
```

output
```
-1/2*(6*cosh(b*x + d)^5*cosh(-2*a + 2*d) + 6*(cosh(-2*a + 2*d) - sinh(-2*a
 + 2*d))*sinh(b*x + d)^5 + 30*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x +
 d)*sinh(-2*a + 2*d))*sinh(b*x + d)^4 + 12*cosh(b*x + d)^3*cosh(-2*a + 2*d
) + 12*(5*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (5*cosh(b*x + d)^2 + 1)*sinh(
-2*a + 2*d) + cosh(-2*a + 2*d))*sinh(b*x + d)^3 + 12*(5*cosh(b*x + d)^3*co
sh(-2*a + 2*d) + 3*cosh(b*x + d)*cosh(-2*a + 2*d) - (5*cosh(b*x + d)^3 + 3
*cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d)^2 + 8*(cosh(b*x + d)^6*cos
h(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^6 + 6*
(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x
 + d)^5 - cosh(b*x + d)^4*cosh(-2*a + 2*d) + (15*cosh(b*x + d)^2*cosh(-2*a
 + 2*d) - (15*cosh(b*x + d)^2 - 1)*sinh(-2*a + 2*d) - cosh(-2*a + 2*d))*si
nh(b*x + d)^4 + 4*(5*cosh(b*x + d)^3*cosh(-2*a + 2*d) - cosh(b*x + d)*cosh
(-2*a + 2*d) - (5*cosh(b*x + d)^3 - cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(
b*x + d)^3 - cosh(b*x + d)^2*cosh(-2*a + 2*d) + (15*cosh(b*x + d)^4*cosh(-
2*a + 2*d) - 6*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (15*cosh(b*x + d)^4 - 6*
cosh(b*x + d)^2 - 1)*sinh(-2*a + 2*d) - cosh(-2*a + 2*d))*sinh(b*x + d)^2
+ 2*(3*cosh(b*x + d)^5*cosh(-2*a + 2*d) - 2*cosh(b*x + d)^3*cosh(-2*a + 2*
d) - cosh(b*x + d)*cosh(-2*a + 2*d) - (3*cosh(b*x + d)^5 - 2*cosh(b*x + d)
^3 - cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^6 - c
osh(b*x + d)^4 - cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*...
```

Sympy [F]

$$\int e^{2(a+bx)}\text{csch}^3(d+bx)\text{sech}^2(d+bx)\,dx = e^{2a}\int e^{2bx}\,\text{csch}^3\,(bx+d)\,\text{sech}^2\,(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)**3*sech(b*x+d)**2,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*csch(b*x + d)**3*sech(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12

$$\int e^{2(a+bx)}\text{csch}^3(d+bx)\text{sech}^2(d+bx)\,dx = \frac{4\,\arctan\left(e^{(-bx-d)}\right)e^{(2\,a-2\,d)}}{b} - \frac{e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}+1\right)}{2\,b} + \frac{e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}-1\right)}{2\,b} + \frac{\left(3\,e^{(-bx-d)}+6\,e^{(-3\,bx-3\,d)}-5\,e^{(-5\,bx-5\,d)}\right)e^{(2\,a-2\,d)}}{b\left(e^{(-2\,bx-2\,d)}+e^{(-4\,bx-4\,d)}-e^{(-6\,bx-6\,d)}-1\right)}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^3*sech(b*x+d)^2,x, algorithm="maxima"
)
```

output
```
4*arctan(e^(-b*x - d))*e^(2*a - 2*d)/b - 1/2*e^(2*a - 2*d)*log(e^(-b*x - d
) + 1)/b + 1/2*e^(2*a - 2*d)*log(e^(-b*x - d) - 1)/b + (3*e^(-b*x - d) + 6
*e^(-3*b*x - 3*d) - 5*e^(-5*b*x - 5*d))*e^(2*a - 2*d)/(b*(e^(-2*b*x - 2*d)
 + e^(-4*b*x - 4*d) - e^(-6*b*x - 6*d) - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.84

$$\int e^{2(a+bx)}\operatorname{csch}^3(d+bx)\operatorname{sech}^2(d+bx)\,dx = -\frac{\left(8\arctan\left(e^{(bx+d)}\right)e^{(-2d)}+e^{(-2d)}\log\left(e^{(bx+d)}+1\right)-e^{(-2d)}\log\left(\left|e^{(bx+d)}-1\right|\right)-\frac{4e^{(bx-d)}}{e^{(2bx+2d)}+1}+\frac{2\,(5\,e^{(3\,bx}}{(e}\right.}{2b}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^3*sech(b*x+d)^2,x, algorithm="giac")
```

output
```
-1/2*(8*arctan(e^(b*x + d))*e^(-2*d) + e^(-2*d)*log(e^(b*x + d) + 1) - e^(
-2*d)*log(abs(e^(b*x + d) - 1)) - 4*e^(b*x - d)/(e^(2*b*x + 2*d) + 1) + 2*
(5*e^(3*b*x + 3*d) - 3*e^(b*x + d))*e^(-2*d)/(e^(2*b*x + 2*d) - 1)^2)*e^(2
*a)/b
```

Mupad [B] (verification not implemented)

Time = 3.62 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.82

$$\int e^{2(a+bx)}\operatorname{csch}^3(d+bx)\operatorname{sech}^2(d+bx)\,dx$$
$$= \frac{2\,e^{2\,a-d+b\,x}}{b+b\,e^{2\,d+2\,b\,x}} - \frac{2\,e^{2\,a-d+b\,x}}{b-2\,b\,e^{2\,d+2\,b\,x}+b\,e^{4\,d+4\,b\,x}} + \frac{5\,e^{2\,a-d+b\,x}}{b-b\,e^{2\,d+2\,b\,x}}$$
$$- \frac{2\,e^{2\,a-2\,d}\left(\ln\left(-272\,e^{6\,a}\,e^{-5\,d}\,e^{b\,x}-e^{6\,a}\,e^{-6\,d}\,272\mathrm{i}\right)\,1\mathrm{i}-\ln\left(-272\,e^{6\,a}\,e^{-5\,d}\,e^{b\,x}+e^{6\,a}\,e^{-6\,d}\,272\mathrm{i}\right)\,1\mathrm{i}\right)}{b}$$
$$- \frac{e^{2\,a-2\,d}\,\ln\left(-68\,e^{6\,a}\,e^{-6\,d}-68\,e^{6\,a}\,e^{-5\,d}\,e^{b\,x}\right)}{2\,b}$$
$$+ \frac{e^{2\,a-2\,d}\,\ln\left(68\,e^{6\,a}\,e^{-6\,d}-68\,e^{6\,a}\,e^{-5\,d}\,e^{b\,x}\right)}{2\,b}$$

input
```
int(exp(2*a + 2*b*x)/(cosh(d + b*x)^2*sinh(d + b*x)^3),x)
```

output
```
(2*exp(2*a - d + b*x))/(b + b*exp(2*d + 2*b*x)) - (2*exp(2*a - d + b*x))/(
b - 2*b*exp(2*d + 2*b*x) + b*exp(4*d + 4*b*x)) + (5*exp(2*a - d + b*x))/(b
 - b*exp(2*d + 2*b*x)) - (2*exp(2*a - 2*d)*(log(- exp(6*a)*exp(-6*d)*272i
- 272*exp(6*a)*exp(-5*d)*exp(b*x))*1i - log(exp(6*a)*exp(-6*d)*272i - 272*
exp(6*a)*exp(-5*d)*exp(b*x))*1i))/b - (exp(2*a - 2*d)*log(- 68*exp(6*a)*ex
p(-6*d) - 68*exp(6*a)*exp(-5*d)*exp(b*x)))/(2*b) + (exp(2*a - 2*d)*log(68*
exp(6*a)*exp(-6*d) - 68*exp(6*a)*exp(-5*d)*exp(b*x)))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.26

$$\int e^{2(a+bx)}\operatorname{csch}^3(d+bx)\operatorname{sech}^2(d+bx)\,dx$$
$$= \frac{e^{2a}\left(-8e^{6bx+6d}atan\left(e^{bx+d}\right)+8e^{4bx+4d}atan\left(e^{bx+d}\right)+8e^{2bx+2d}atan\left(e^{bx+d}\right)-8atan\left(e^{bx+d}\right)+e^{6bx+6d}\log\left(e^{b\cdots}\right.\right.}{}$$

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)^3*sech(b*x+d)^2,x)
```

output
```
(e**(2*a)*( - 8*e**(6*b*x + 6*d)*atan(e**(b*x + d)) + 8*e**(4*b*x + 4*d)*a
tan(e**(b*x + d)) + 8*e**(2*b*x + 2*d)*atan(e**(b*x + d)) - 8*atan(e**(b*x
 + d)) + e**(6*b*x + 6*d)*log(e**(b*x + d) - 1) - e**(6*b*x + 6*d)*log(e**
(b*x + d) + 1) - 6*e**(5*b*x + 5*d) - e**(4*b*x + 4*d)*log(e**(b*x + d) -
1) + e**(4*b*x + 4*d)*log(e**(b*x + d) + 1) - 12*e**(3*b*x + 3*d) - e**(2*
b*x + 2*d)*log(e**(b*x + d) - 1) + e**(2*b*x + 2*d)*log(e**(b*x + d) + 1)
+ 10*e**(b*x + d) + log(e**(b*x + d) - 1) - log(e**(b*x + d) + 1)))/(2*e**
(2*d)*b*(e**(6*b*x + 6*d) - e**(4*b*x + 4*d) - e**(2*b*x + 2*d) + 1))
```

3.78 $\int e^{2(a+bx)} \tanh^3(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 101

$$\int e^{2(a+bx)} \tanh^3(d+bx)\,dx = \frac{e^{2a+2bx}}{2b} + \frac{2e^{2a-2d}}{b\left(1+e^{2d+2bx}\right)^2} - \frac{6e^{2a-2d}}{b\left(1+e^{2d+2bx}\right)} - \frac{3e^{2a-2d}\log\left(1+e^{2d+2bx}\right)}{b}$$

output
```
1/2*exp(2*b*x+2*a)/b+2*exp(2*a-2*d)/b/(1+exp(2*b*x+2*d))^2-6*exp(2*a-2*d)/
b/(1+exp(2*b*x+2*d))-3*exp(2*a-2*d)*ln(1+exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.50

$$\int e^{2(a+bx)} \tanh^3(d+bx)\,dx$$
$$= \frac{e^{2a}\left(e^{2bx} - 6\cosh(2d)\log\left(\left(1+e^{2bx}\right)\cosh(d) + \left(-1+e^{2bx}\right)\sinh(d)\right) + \frac{4(\cosh(d)-\sinh(d))^4}{\left(\left(1+e^{2bx}\right)\cosh(d)+\left(-1+e^{2bx}\right)\sinh(d)\right)^2} -}{2b}$$

input
```
Integrate[E^(2*(a + b*x))*Tanh[d + b*x]^3,x]
```

output
```
(E^(2*a)*(E^(2*b*x) - 6*Cosh[2*d]*Log[(1 + E^(2*b*x))*Cosh[d] + (-1 + E^(2
*b*x))*Sinh[d]] + (4*(Cosh[d] - Sinh[d])^4)/((1 + E^(2*b*x))*Cosh[d] + (-1
 + E^(2*b*x))*Sinh[d])^2 - (12*(Cosh[d] - Sinh[d])^3)/((1 + E^(2*b*x))*Cos
h[d] + (-1 + E^(2*b*x))*Sinh[d]) + 6*Log[(1 + E^(2*b*x))*Cosh[d] + (-1 + E
^(2*b*x))*Sinh[d]]*Sinh[2*d]))/(2*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.55, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2720, 25, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\tanh^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{e^{2a+bx}\left(1-e^{2bx}\right)^3}{\left(1+e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{e^{2a+bx}\left(1-e^{2bx}\right)^3}{\left(1+e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e^{2a}\int \frac{e^{bx}\left(1-e^{2bx}\right)^3}{\left(1+e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{353}$$

$$-\frac{e^{2a}\int \frac{\left(1-e^{2bx}\right)^3}{\left(1+e^{2bx}\right)^3}de^{2bx}}{2b}$$

$$\Big\downarrow \text{49}$$

$$-\frac{e^{2a}\int \left(-1+\frac{6}{1+e^{2bx}}-\frac{12}{\left(1+e^{2bx}\right)^2}+\frac{8}{\left(1+e^{2bx}\right)^3}\right)de^{2bx}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{2a}\left(-e^{2bx}+\frac{12}{e^{2bx}+1}-\frac{4}{(e^{2bx}+1)^2}+6\log\left(e^{2bx}+1\right)\right)}{2b}$$

input
```
Int[E^(2*(a + b*x))*Tanh[d + b*x]^3,x]
```

output
```
-1/2*(E^(2*a)*(-E^(2*b*x) - 4/(1 + E^(2*b*x))^2 + 12/(1 + E^(2*b*x)) + 6*L
og[1 + E^(2*b*x)]))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.14

method	result	size
risch	$\frac{\mathrm{e}^{2bx+2a}}{2b}+\frac{6\,\mathrm{e}^{2a-2d}a}{b}-\frac{2\left(3\,\mathrm{e}^{2bx+2a+2d}+2\,\mathrm{e}^{2a}\right)\mathrm{e}^{4a-2d}}{\left(\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)^{2}b}-\frac{3\ln\left(\mathrm{e}^{2bx+2a}+\mathrm{e}^{2a-2d}\right)\mathrm{e}^{2a-2d}}{b}$	115

input
```
int(exp(2*b*x+2*a)*tanh(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*b*x+2*a)/b+6/b*exp(2*a-2*d)*a-2/(exp(2*b*x+2*a+2*d)+exp(2*a))^2/
b*(3*exp(2*b*x+2*a+2*d)+2*exp(2*a))*exp(4*a-2*d)-3*ln(exp(2*b*x+2*a)+exp(2
*a-2*d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 897 vs. 2(92) = 184.

Time = 0.10 (sec) , antiderivative size = 897, normalized size of antiderivative = 8.88

$$\int e^{2(a+bx)}\tanh^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d)^3,x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + d)^6*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*
d))*sinh(b*x + d)^6 + 6*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*si
nh(-2*a + 2*d))*sinh(b*x + d)^5 + 2*cosh(b*x + d)^4*cosh(-2*a + 2*d) + (15
*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (15*cosh(b*x + d)^2 + 2)*sinh(-2*a + 2
*d) + 2*cosh(-2*a + 2*d))*sinh(b*x + d)^4 + 4*(5*cosh(b*x + d)^3*cosh(-2*a
 + 2*d) + 2*cosh(b*x + d)*cosh(-2*a + 2*d) - (5*cosh(b*x + d)^3 + 2*cosh(b
*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d)^3 - 11*cosh(b*x + d)^2*cosh(-2*a
+ 2*d) + (15*cosh(b*x + d)^4*cosh(-2*a + 2*d) + 12*cosh(b*x + d)^2*cosh(-2
*a + 2*d) - (15*cosh(b*x + d)^4 + 12*cosh(b*x + d)^2 - 11)*sinh(-2*a + 2*d
) - 11*cosh(-2*a + 2*d))*sinh(b*x + d)^2 - 6*(cosh(b*x + d)^4*cosh(-2*a +
2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^4 + 4*(cosh(b*x
 + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^3 +
 2*cosh(b*x + d)^2*cosh(-2*a + 2*d) + 2*(3*cosh(b*x + d)^2*cosh(-2*a + 2*d
) - (3*cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*sinh(b*x
+ d)^2 + 4*(cosh(b*x + d)^3*cosh(-2*a + 2*d) + cosh(b*x + d)*cosh(-2*a + 2
*d) - (cosh(b*x + d)^3 + cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d) -
(cosh(b*x + d)^4 + 2*cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2
*d))*log(2*cosh(b*x + d)/(cosh(b*x + d) - sinh(b*x + d))) + 2*(3*cosh(b*x
+ d)^5*cosh(-2*a + 2*d) + 4*cosh(b*x + d)^3*cosh(-2*a + 2*d) - 11*cosh(b*x
 + d)*cosh(-2*a + 2*d) - (3*cosh(b*x + d)^5 + 4*cosh(b*x + d)^3 - 11*co...
```

Sympy [F]

$$\int e^{2(a+bx)}\tanh^3(d+bx)\,dx = e^{2a}\int e^{2bx}\tanh^3{(bx+d)}\,dx$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d)**3,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*tanh(b*x + d)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.12

$$\int e^{2(a+bx)}\tanh^3(d+bx)\,dx = -\frac{6\,(bx+d)e^{(2\,a-2\,d)}}{b} - \frac{3\,e^{(2\,a-2\,d)}\log\left(e^{(-2\,bx-2\,d)}+1\right)}{b} + \frac{\left(10\,e^{(-2\,bx-2\,d)}+5\,e^{(-4\,bx-4\,d)}+1\right)e^{(2\,a-2\,d)}}{2\,b\left(e^{(-2\,bx-2\,d)}+2\,e^{(-4\,bx-4\,d)}+e^{(-6\,bx-6\,d)}\right)}$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d)^3,x, algorithm="maxima")
```

output
```
-6*(b*x + d)*e^(2*a - 2*d)/b - 3*e^(2*a - 2*d)*log(e^(-2*b*x - 2*d) + 1)/b
 + 1/2*(10*e^(-2*b*x - 2*d) + 5*e^(-4*b*x - 4*d) + 1)*e^(2*a - 2*d)/(b*(e^
(-2*b*x - 2*d) + 2*e^(-4*b*x - 4*d) + e^(-6*b*x - 6*d)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\int e^{2(a+bx)}\tanh^3(d+bx)\,dx = -\frac{3\,e^{(2\,a-2\,d)}\log\left(e^{(2\,bx+2\,d)}+1\right)}{b} + \frac{e^{(2\,bx+2\,a)}}{2\,b} + \frac{\left(9\,e^{(4\,bx+2\,a+4\,d)}+6\,e^{(2\,bx+2\,a+2\,d)}+e^{(2\,a)}\right)e^{(-2\,d)}}{2\,b\left(e^{(2\,bx+2\,d)}+1\right)^2}$$

input
```
integrate(exp(2*b*x+2*a)*tanh(b*x+d)^3,x, algorithm="giac")
```

output
```
-3*e^(2*a - 2*d)*log(e^(2*b*x + 2*d) + 1)/b + 1/2*e^(2*b*x + 2*a)/b + 1/2*
(9*e^(4*b*x + 2*a + 4*d) + 6*e^(2*b*x + 2*a + 2*d) + e^(2*a))*e^(-2*d)/(b*
(e^(2*b*x + 2*d) + 1)^2)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2(a+bx)}\tanh^3(d+bx)\,dx=\int \mathrm{e}^{2\,a+2\,b\,x}\,\tanh(d+b\,x)^3\,dx$$

input
```
int(exp(2*a + 2*b*x)*tanh(d + b*x)^3,x)
```

output
```
int(exp(2*a + 2*b*x)*tanh(d + b*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.33

$$\int e^{2(a+bx)}\tanh^3(d+bx)\,dx$$
$$=\frac{e^{2a}\left(2e^{6bx+6d}-12e^{4bx+4d}\log\left(e^{2bx+2d}+1\right)+15e^{4bx+4d}-24e^{2bx+2d}\log\left(e^{2bx+2d}+1\right)-12\log\left(e^{2bx+2d}+1\right)\right.}{4e^{2d}b\left(e^{4bx+4d}+2e^{2bx+2d}+1\right)}$$

input
```
int(exp(2*b*x+2*a)*tanh(b*x+d)^3,x)
```

output
```
(e**(2*a)*(2*e**(6*b*x + 6*d) - 12*e**(4*b*x + 4*d)*log(e**(2*b*x + 2*d) +
 1) + 15*e**(4*b*x + 4*d) - 24*e**(2*b*x + 2*d)*log(e**(2*b*x + 2*d) + 1)
- 12*log(e**(2*b*x + 2*d) + 1) - 5))/(4*e**(2*d)*b*(e**(4*b*x + 4*d) + 2*e
**(2*b*x + 2*d) + 1))
```

3.79 $\int e^{2(a+bx)}\mathbf{sech}(d+bx)\tanh^2(d+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 102

$$\int e^{2(a+bx)}\text{sech}(d+bx)\tanh^2(d+bx)\,dx=\frac{2e^{2a-d+bx}}{b}-\frac{2e^{2a-d+bx}}{b\left(1+e^{2d+2bx}\right)^2}+\frac{5e^{2a-d+bx}}{b\left(1+e^{2d+2bx}\right)}-\frac{5e^{2a-2d}\arctan\left(e^{d+bx}\right)}{b}$$

output
```
2*exp(b*x+2*a-d)/b-2*exp(b*x+2*a-d)/b/(1+exp(2*b*x+2*d))^2+5*exp(b*x+2*a-d
)/b/(1+exp(2*b*x+2*d))-5*exp(2*a-2*d)*arctan(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.68

$$\int e^{2(a+bx)}\text{sech}(d+bx)\tanh^2(d+bx)\,dx=\frac{e^{2a-2d}\left(\frac{e^{d+bx}\left(5+9e^{2(d+bx)}+2e^{4(d+bx)}\right)}{\left(1+e^{2(d+bx)}\right)^2}-5\arctan\left(e^{d+bx}\right)\right)}{b}$$

input
```
Integrate[E^(2*(a + b*x))*Sech[d + b*x]*Tanh[d + b*x]^2,x]
```

output
```
(E^(2*a - 2*d)*((E^(d + b*x)*(5 + 9*E^(2*(d + b*x)) + 2*E^(4*(d + b*x))))/
(1 + E^(2*(d + b*x)))^2 - 5*ArcTan[E^(d + b*x)]))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.66, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2720, 27, 366, 27, 360, 25, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\tanh^2(bx+d)\text{sech}(bx+d)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int \frac{2e^{2a+2bx}(1-e^{2bx})^2}{(1+e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2e^{2a}\int \frac{e^{2bx}(1-e^{2bx})^2}{(1+e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{ 366}$$

$$\frac{2e^{2a}\left(\frac{e^{3bx}}{(e^{2bx}+1)^2}-\frac{1}{4}\int \frac{4e^{2bx}(2-e^{2bx})}{(1+e^{2bx})^2}de^{bx}\right)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2e^{2a}\left(\frac{e^{3bx}}{(e^{2bx}+1)^2}-\int \frac{e^{2bx}(2-e^{2bx})}{(1+e^{2bx})^2}de^{bx}\right)}{b}$$

$$\Big\downarrow \text{ 360}$$

$$\frac{2e^{2a}\left(\frac{1}{2}\int -\frac{3-2e^{2bx}}{1+e^{2bx}}de^{bx}+\frac{3e^{bx}}{2(e^{2bx}+1)}+\frac{e^{3bx}}{(e^{2bx}+1)^2}\right)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2e^{2a}\left(-\frac{1}{2}\int\frac{3-2e^{2bx}}{1+e^{2bx}}de^{bx}+\frac{3e^{bx}}{2(e^{2bx}+1)}+\frac{e^{3bx}}{(e^{2bx}+1)^2}\right)}{b}$$

$$\Big\downarrow \text{299}$$

$$\frac{2e^{2a}\left(\frac{1}{2}\left(2e^{bx}-5\int\frac{1}{1+e^{2bx}}de^{bx}\right)+\frac{3e^{bx}}{2(e^{2bx}+1)}+\frac{e^{3bx}}{(e^{2bx}+1)^2}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{2e^{2a}\left(\frac{1}{2}\left(2e^{bx}-5\arctan\left(e^{bx}\right)\right)+\frac{3e^{bx}}{2(e^{2bx}+1)}+\frac{e^{3bx}}{(e^{2bx}+1)^2}\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Sech[d + b*x]*Tanh[d + b*x]^2,x]
```

output
```
(2*E^(2*a)*(E^(3*b*x)/(1 + E^(2*b*x))^2 + (3*E^(b*x))/(2*(1 + E^(2*b*x)))
+ (2*E^(b*x) - 5*ArcTan[E^(b*x)])/2))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.88 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.32

method	result	size
risch	$\frac{2\,{\rm e}^{b x +2 a -d}}{b}+\frac{\left(5\,{\rm e}^{2 b x +2 a +2 d}+3\,{\rm e}^{2 a}\right) {\rm e}^{b x +4 a -d}}{\left({\rm e}^{2 b x +2 a +2 d}+{\rm e}^{2 a}\right)^{2} b}+\frac{5 i \ln\left({\rm e}^{b x +a}-i {\rm e}^{a -d}\right) {\rm e}^{2 a -2 d}}{2 b}-\frac{5 i \ln\left({\rm e}^{b x +a}+i {\rm e}^{a -d}\right) {\rm e}^{2 a -2 d}}{2 b}$	135

input
```
int(exp(2*b*x+2*a)*sech(b*x+d)*tanh(b*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
2*exp(b*x+2*a-d)/b+1/(exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(5*exp(2*b*x+2*a+2*
d)+3*exp(2*a))*exp(b*x+4*a-d)+5/2*I*ln(exp(b*x+a)-I*exp(a-d))/b*exp(2*a-2*
d)-5/2*I*ln(exp(b*x+a)+I*exp(a-d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(95) = 190$.

Time = 0.10 (sec) , antiderivative size = 770, normalized size of antiderivative = 7.55

$$\int e^{2(a+bx)}\text{sech}(d+bx)\tanh^2(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)*tanh(b*x+d)^2,x, algorithm="fricas")
```

output
```
(2*cosh(b*x + d)^5*cosh(-2*a + 2*d) + 2*(cosh(-2*a + 2*d) - sinh(-2*a + 2*
d))*sinh(b*x + d)^5 + 10*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*s
inh(-2*a + 2*d))*sinh(b*x + d)^4 + 9*cosh(b*x + d)^3*cosh(-2*a + 2*d) + (2
0*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (20*cosh(b*x + d)^2 + 9)*sinh(-2*a +
2*d) + 9*cosh(-2*a + 2*d))*sinh(b*x + d)^3 + (20*cosh(b*x + d)^3*cosh(-2*a
 + 2*d) + 27*cosh(b*x + d)*cosh(-2*a + 2*d) - (20*cosh(b*x + d)^3 + 27*cos
h(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d)^2 - 5*(cosh(b*x + d)^4*cosh(-2
*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^4 + 4*(cos
h(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d
)^3 + 2*cosh(b*x + d)^2*cosh(-2*a + 2*d) + 2*(3*cosh(b*x + d)^2*cosh(-2*a
+ 2*d) - (3*cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*sinh
(b*x + d)^2 + 4*(cosh(b*x + d)^3*cosh(-2*a + 2*d) + cosh(b*x + d)*cosh(-2*
a + 2*d) - (cosh(b*x + d)^3 + cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x +
d) - (cosh(b*x + d)^4 + 2*cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*
a + 2*d))*arctan(cosh(b*x + d) + sinh(b*x + d)) + 5*cosh(b*x + d)*cosh(-2*
a + 2*d) + (10*cosh(b*x + d)^4*cosh(-2*a + 2*d) + 27*cosh(b*x + d)^2*cosh(
-2*a + 2*d) - (10*cosh(b*x + d)^4 + 27*cosh(b*x + d)^2 + 5)*sinh(-2*a + 2*
d) + 5*cosh(-2*a + 2*d))*sinh(b*x + d) - (2*cosh(b*x + d)^5 + 9*cosh(b*x +
 d)^3 + 5*cosh(b*x + d))*sinh(-2*a + 2*d))/(b*cosh(b*x + d)^4 + 4*b*cosh(b
*x + d)*sinh(b*x + d)^3 + b*sinh(b*x + d)^4 + 2*b*cosh(b*x + d)^2 + 2*(...
```

Sympy [F]

$$\int e^{2(a+bx)}\text{sech}(d+bx)\tanh^2(d+bx)\,dx = e^{2a}\int e^{2bx}\tanh^2(bx+d)\,\text{sech}\,(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)*tanh(b*x+d)**2,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*tanh(b*x + d)**2*sech(b*x + d), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.90

$$\int e^{2(a+bx)}\text{sech}(d+bx)\tanh^2(d+bx)\,dx = \frac{5\arctan\left(e^{(-bx-d)}\right)e^{(2a-2d)}}{b} + \frac{\left(9e^{(-2bx-2d)}+5e^{(-4bx-4d)}+2\right)e^{(2a-2d)}}{b\left(e^{(-bx-d)}+2e^{(-3bx-3d)}+e^{(-5bx-5d)}\right)}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)*tanh(b*x+d)^2,x, algorithm="maxima")
```

output
```
5*arctan(e^(-b*x - d))*e^(2*a - 2*d)/b + (9*e^(-2*b*x - 2*d) + 5*e^(-4*b*x
 - 4*d) + 2)*e^(2*a - 2*d)/(b*(e^(-b*x - d) + 2*e^(-3*b*x - 3*d) + e^(-5*b
*x - 5*d)))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int e^{2(a+bx)}\text{sech}(d+bx)\tanh^2(d+bx)\,dx = -\frac{5\arctan\left(e^{(bx+d)}\right)e^{(2a-2d)} - \frac{\left(5e^{(3bx+2a+3d)}+3e^{(bx+2a+d)}\right)e^{(-2d)}}{\left(e^{(2bx+2d)}+1\right)^2} - 2e^{(bx+2a-d)}}{b}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)*tanh(b*x+d)^2,x, algorithm="giac")
```

output
```
-(5*arctan(e^(b*x + d))*e^(2*a - 2*d) - (5*e^(3*b*x + 2*a + 3*d) + 3*e^(b*
x + 2*a + d))*e^(-2*d)/(e^(2*b*x + 2*d) + 1)^2 - 2*e^(b*x + 2*a - d))/b
```

Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.33

$$\int e^{2(a+bx)}\operatorname{sech}(d+bx)\tanh^2(d+bx)\,dx = \frac{2\,\mathrm{e}^{2\,a-d+b\,x}}{b}+\frac{5\,\mathrm{e}^{2\,a-d+b\,x}}{b\,\left(\mathrm{e}^{2\,d+2\,b\,x}+1\right)} - \frac{2\,\mathrm{e}^{2\,a-d+b\,x}}{b\,\left(2\,\mathrm{e}^{2\,d+2\,b\,x}+\mathrm{e}^{4\,d+4\,b\,x}+1\right)} - \frac{5\,\sqrt{\mathrm{e}^{4\,a-4\,d}}\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{-d}\,\mathrm{e}^{b\,x}\,\sqrt{b^2}}{b\,\sqrt{\mathrm{e}^{4\,a}\,\mathrm{e}^{-4\,d}}}\right)}{\sqrt{b^2}}$$

input
```
int((exp(2*a + 2*b*x)*tanh(d + b*x)^2)/cosh(d + b*x),x)
```

output
```
(2*exp(2*a - d + b*x))/b + (5*exp(2*a - d + b*x))/(b*(exp(2*d + 2*b*x) + 1
)) - (2*exp(2*a - d + b*x))/(b*(2*exp(2*d + 2*b*x) + exp(4*d + 4*b*x) + 1)
) - (5*exp(4*a - 4*d)^(1/2)*atan((exp(2*a)*exp(-d)*exp(b*x)*(b^2)^(1/2))/(
b*(exp(4*a)*exp(-4*d))^(1/2))))/(b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.24

$$\int e^{2(a+bx)}\operatorname{sech}(d+bx)\tanh^2(d+bx)\,dx = \frac{e^{2a}\left(-5e^{4bx+4d}atan\left(e^{bx+d}\right)-10e^{2bx+2d}atan\left(e^{bx+d}\right)-5atan\left(e^{bx+d}\right)+2e^{5bx+5d}+9e^{3bx+3d}+5e^{bx+d}\right)}{e^{2d}b\left(e^{4bx+4d}+2e^{2bx+2d}+1\right)}$$

input
```
int(exp(2*b*x+2*a)*sech(b*x+d)*tanh(b*x+d)^2,x)
```

output
```
(e**(2*a)*( - 5*e**(4*b*x + 4*d)*atan(e**(b*x + d)) - 10*e**(2*b*x + 2*d)*
atan(e**(b*x + d)) - 5*atan(e**(b*x + d)) + 2*e**(5*b*x + 5*d) + 9*e**(3*b
*x + 3*d) + 5*e**(b*x + d)))/(e**(2*d)*b*(e**(4*b*x + 4*d) + 2*e**(2*b*x +
 2*d) + 1))
```

3.80 $\int e^{2(a+bx)}\operatorname{sech}^2(d+bx)\tanh(d+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 84

$$\int e^{2(a+bx)}\operatorname{sech}^2(d+bx)\tanh(d+bx)\,dx=-\frac{2e^{2a-2d}}{b\left(1+e^{2d+2bx}\right)^2}+\frac{6e^{2a-2d}}{b\left(1+e^{2d+2bx}\right)}+\frac{2e^{2a-2d}\log\left(1+e^{2d+2bx}\right)}{b}$$

output
```
-2*exp(2*a-2*d)/b/(1+exp(2*b*x+2*d))^2+6*exp(2*a-2*d)/b/(1+exp(2*b*x+2*d))
+2*exp(2*a-2*d)*ln(1+exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

$$\int e^{2(a+bx)}\operatorname{sech}^2(d+bx)\tanh(d+bx)\,dx=\frac{2e^{2a-2d}\left(\frac{2+3e^{2(d+bx)}}{\left(1+e^{2(d+bx)}\right)^2}+\log\left(1+e^{2(d+bx)}\right)\right)}{b}$$

input
```
Integrate[E^(2*(a + b*x))*Sech[d + b*x]^2*Tanh[d + b*x],x]
```

output
```
(2*E^(2*a - 2*d)*((2 + 3*E^(2*(d + b*x)))/(1 + E^(2*(d + b*x)))^2 + Log[1
+ E^(2*(d + b*x))]))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.52, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2720, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\tanh(bx+d)\text{sech}^2(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{4e^{2a+3bx}(1-e^{2bx})}{(1+e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4e^{2a}\int \frac{e^{3bx}(1-e^{2bx})}{(1+e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{354}$$

$$-\frac{2e^{2a}\int \frac{e^{2bx}(1-e^{2bx})}{(1+e^{2bx})^3}de^{2bx}}{b}$$

$$\Big\downarrow \text{86}$$

$$-\frac{2e^{2a}\int \left(\frac{3}{(1+e^{2bx})^2}-\frac{2}{(1+e^{2bx})^3}+\frac{1}{-1-e^{2bx}}\right)de^{2bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2e^{2a}\left(-\frac{3}{e^{2bx}+1}+\frac{1}{(e^{2bx}+1)^2}-\log\left(e^{2bx}+1\right)\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Sech[d + b*x]^2*Tanh[d + b*x],x]
```

output
```
(-2*E^(2*a)*((1 + E^(2*b*x))^(-2) - 3/(1 + E^(2*b*x)) - Log[1 + E^(2*b*x)]
))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 5.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.20

method	result	size
risch	$-\frac{4\,{\mathrm e}^{2a-2d} a}{b}+\frac{2\left(3\,{\mathrm e}^{2bx+2a+2d}+2\,{\mathrm e}^{2a}\right){\mathrm e}^{4a-2d}}{\left({\mathrm e}^{2bx+2a+2d}+{\mathrm e}^{2a}\right)^{2} b}+\frac{2\ln\left({\mathrm e}^{2bx+2a}+{\mathrm e}^{2a-2d}\right){\mathrm e}^{2a-2d}}{b}$	101

input
```
int(exp(2*b*x+2*a)*sech(b*x+d)^2*tanh(b*x+d),x,method=_RETURNVERBOSE)
```

output
```
-4/b*exp(2*a-2*d)*a+2/(exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(3*exp(2*b*x+2*a+2
*d)+2*exp(2*a))*exp(4*a-2*d)+2*ln(exp(2*b*x+2*a)+exp(2*a-2*d))/b*exp(2*a-2
*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(78) = 156.

Time = 0.08 (sec) , antiderivative size = 520, normalized size of antiderivative = 6.19

$$\int e^{2(a+bx)}\text{sech}^2(d+bx)\tanh(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)^2*tanh(b*x+d),x, algorithm="fricas")
```

output
```
2*(3*cosh(b*x + d)^2*cosh(-2*a + 2*d) + 3*(cosh(-2*a + 2*d) - sinh(-2*a +
2*d))*sinh(b*x + d)^2 + (cosh(b*x + d)^4*cosh(-2*a + 2*d) + (cosh(-2*a + 2
*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-2*a + 2*d
) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^3 + 2*cosh(b*x + d)^2*co
sh(-2*a + 2*d) + 2*(3*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (3*cosh(b*x + d)^
2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*sinh(b*x + d)^2 + 4*(cosh(b*x
+ d)^3*cosh(-2*a + 2*d) + cosh(b*x + d)*cosh(-2*a + 2*d) - (cosh(b*x + d)^
3 + cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^4 + 2*
cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*log(2*cosh(b*x +
 d)/(cosh(b*x + d) - sinh(b*x + d))) + 6*(cosh(b*x + d)*cosh(-2*a + 2*d) -
 cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d) - (3*cosh(b*x + d)^2 + 2)*s
inh(-2*a + 2*d) + 2*cosh(-2*a + 2*d))/(b*cosh(b*x + d)^4 + 4*b*cosh(b*x +
d)*sinh(b*x + d)^3 + b*sinh(b*x + d)^4 + 2*b*cosh(b*x + d)^2 + 2*(3*b*cosh
(b*x + d)^2 + b)*sinh(b*x + d)^2 + 4*(b*cosh(b*x + d)^3 + b*cosh(b*x + d))
*sinh(b*x + d) + b)
```

Sympy [F]

$$\int e^{2(a+bx)}\text{sech}^2(d+bx)\tanh(d+bx)\,dx = e^{2a}\int e^{2bx}\tanh\left(bx+d\right)\text{sech}^2\left(bx+d\right)dx$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)**2*tanh(b*x+d),x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*tanh(b*x + d)*sech(b*x + d)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.18

$$\int e^{2(a+bx)}\text{sech}^2(d+bx)\tanh(d+bx)\,dx = 4\,xe^{(2\,a-2\,d)} + \frac{4\,de^{(2\,a-2\,d)}}{b} + \frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(-2\,bx-2\,d)}+1\right)}{b} - \frac{2\,\left(e^{(-2\,bx-2\,d)}+2\right)e^{(2\,a-2\,d)}}{b(2\,e^{(-2\,bx-2\,d)}+e^{(-4\,bx-4\,d)}+1)}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)^2*tanh(b*x+d),x, algorithm="maxima")
```

output
```
4*x*e^(2*a - 2*d) + 4*d*e^(2*a - 2*d)/b + 2*e^(2*a - 2*d)*log(e^(-2*b*x -
2*d) + 1)/b - 2*(e^(-2*b*x - 2*d) + 2)*e^(2*a - 2*d)/(b*(2*e^(-2*b*x - 2*d
) + e^(-4*b*x - 4*d) + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.80

$$\int e^{2(a+bx)}\text{sech}^2(d+bx)\tanh(d+bx)\,dx = \frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(2\,bx+2\,d)}+1\right) - \frac{\left(3\,e^{(4\,bx+2\,a+4\,d)}-e^{(2\,a)}\right)e^{(-2\,d)}}{\left(e^{(2\,bx+2\,d)}+1\right)^2}}{b}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)^2*tanh(b*x+d),x, algorithm="giac")
```

output
```
(2*e^(2*a - 2*d)*log(e^(2*b*x + 2*d) + 1) - (3*e^(4*b*x + 2*a + 4*d) - e^(
2*a))*e^(-2*d)/(e^(2*b*x + 2*d) + 1)^2)/b
```

Mupad [B] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07

$$\int e^{2(a+bx)}\text{sech}^2(d+bx)\tanh(d+bx)\,dx = \frac{2\,e^{2\,a-2\,d}\,\ln\left(e^{2\,d}\,e^{2\,b\,x}+1\right)}{b} - \frac{2\,e^{2\,a-2\,d}}{b\,\left(2\,e^{2\,d+2\,b\,x}+e^{4\,d+4\,b\,x}+1\right)} + \frac{6\,e^{2\,a-2\,d}}{b\,\left(e^{2\,d+2\,b\,x}+1\right)}$$

input
```
int((exp(2*a + 2*b*x)*tanh(d + b*x))/cosh(d + b*x)^2,x)
```

output
```
(2*exp(2*a - 2*d)*log(exp(2*d)*exp(2*b*x) + 1))/b - (2*exp(2*a - 2*d))/(b*
(2*exp(2*d + 2*b*x) + exp(4*d + 4*b*x) + 1)) + (6*exp(2*a - 2*d))/(b*(exp(
2*d + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.44

$$\int e^{2(a+bx)}\text{sech}^2(d+bx)\tanh(d+bx)\,dx = \frac{e^{2a}\left(2e^{4bx+4d}\log\left(e^{2bx+2d}+1\right)-3e^{4bx+4d}+4e^{2bx+2d}\log\left(e^{2bx+2d}+1\right)+2\log\left(e^{2bx+2d}+1\right)+1\right)}{e^{2d}b\left(e^{4bx+4d}+2e^{2bx+2d}+1\right)}$$

input
```
int(exp(2*b*x+2*a)*sech(b*x+d)^2*tanh(b*x+d),x)
```

output
```
(e**(2*a)*(2*e**(4*b*x + 4*d)*log(e**(2*b*x + 2*d) + 1) - 3*e**(4*b*x + 4*
d) + 4*e**(2*b*x + 2*d)*log(e**(2*b*x + 2*d) + 1) + 2*log(e**(2*b*x + 2*d)
 + 1) + 1))/(e**(2*d)*b*(e**(4*b*x + 4*d) + 2*e**(2*b*x + 2*d) + 1))
```

3.81 $\int e^{2(a+bx)}\text{sech}^3(d+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 84

$$\int e^{2(a+bx)}\text{sech}^3(d+bx)\,dx=-\frac{2e^{2a+d+3bx}}{b\left(1+e^{2d+2bx}\right)^2}-\frac{3e^{2a-d+bx}}{b\left(1+e^{2d+2bx}\right)}+\frac{3e^{2a-2d}\arctan\left(e^{d+bx}\right)}{b}$$

output
```
-2*exp(3*b*x+2*a+d)/b/(1+exp(2*b*x+2*d))^2-3*exp(b*x+2*a-d)/b/(1+exp(2*b*x
+2*d))+3*exp(2*a-2*d)*arctan(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int e^{2(a+bx)}\text{sech}^3(d+bx)\,dx$$
$$=\frac{e^{2a-2d}\left(-6e^{d+bx}+6\arctan\left(e^{d+bx}\right)+e^{2(d+bx)}\text{sech}(d+bx)(2+\tanh(d+bx))\right)}{2b}$$

input
```
Integrate[E^(2*(a + b*x))*Sech[d + b*x]^3,x]
```

output
```
(E^(2*a - 2*d)*(-6*E^(d + b*x) + 6*ArcTan[E^(d + b*x)] + E^(2*(d + b*x))*S
ech[d + b*x]*(2 + Tanh[d + b*x])))/(2*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2720, 27, 252, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{sech}^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{8e^{2a+4bx}}{(1+e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{8e^{2a}\int \frac{e^{4bx}}{(1+e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{8e^{2a}\left(\frac{3}{4}\int \frac{e^{2bx}}{(1+e^{2bx})^2}de^{bx}-\frac{e^{3bx}}{4(e^{2bx}+1)^2}\right)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{8e^{2a}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1+e^{2bx}}de^{bx}-\frac{e^{bx}}{2(e^{2bx}+1)}\right)-\frac{e^{3bx}}{4(e^{2bx}+1)^2}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{8e^{2a}\left(\frac{3}{4}\left(\frac{1}{2}\arctan\left(e^{bx}\right)-\frac{e^{bx}}{2(e^{2bx}+1)}\right)-\frac{e^{3bx}}{4(e^{2bx}+1)^2}\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Sech[d + b*x]^3,x]
```

output
```
(8*E^(2*a)*(-1/4*E^(3*b*x)/(1 + E^(2*b*x))^2 + (3*(-1/2*E^(b*x)/(1 + E^(2*
b*x)) + ArcTan[E^(b*x)]/2))/4))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.43

method	result	size
risch	$-\frac{\left(5\,{\mathrm e}^{2 b x +2 a +2 d}+3\,{\mathrm e}^{2 a}\right) {\mathrm e}^{b x +4 a -d}}{\left({\mathrm e}^{2 b x +2 a +2 d}+{\mathrm e}^{2 a}\right)^{2} b}+\frac{3 i \ln\left({\mathrm e}^{b x +a}+i {\mathrm e}^{a -d}\right) {\mathrm e}^{2 a -2 d}}{2 b}-\frac{3 i \ln\left({\mathrm e}^{b x +a}-i {\mathrm e}^{a -d}\right) {\mathrm e}^{2 a -2 d}}{2 b}$	120

input
```
int(exp(2*b*x+2*a)*sech(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/(exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(5*exp(2*b*x+2*a+2*d)+3*exp(2*a))*exp
(b*x+4*a-d)+3/2*I*ln(exp(b*x+a)+I*exp(a-d))/b*exp(2*a-2*d)-3/2*I*ln(exp(b*
x+a)-I*exp(a-d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(78) = 156$.

Time = 0.09 (sec) , antiderivative size = 581, normalized size of antiderivative = 6.92

$$\int e^{2(a+bx)}\text{sech}^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)^3,x, algorithm="fricas")
```

output
```
-(5*cosh(b*x + d)^3*cosh(-2*a + 2*d) + 5*(cosh(-2*a + 2*d) - sinh(-2*a + 2
*d))*sinh(b*x + d)^3 + 15*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*
sinh(-2*a + 2*d))*sinh(b*x + d)^2 - 3*(cosh(b*x + d)^4*cosh(-2*a + 2*d) +
(cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*c
osh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^3 + 2*cosh
(b*x + d)^2*cosh(-2*a + 2*d) + 2*(3*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (3*
cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*sinh(b*x + d)^2
+ 4*(cosh(b*x + d)^3*cosh(-2*a + 2*d) + cosh(b*x + d)*cosh(-2*a + 2*d) - (
cosh(b*x + d)^3 + cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b
*x + d)^4 + 2*cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*ar
ctan(cosh(b*x + d) + sinh(b*x + d)) + 3*cosh(b*x + d)*cosh(-2*a + 2*d) + 3
*(5*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (5*cosh(b*x + d)^2 + 1)*sinh(-2*a +
 2*d) + cosh(-2*a + 2*d))*sinh(b*x + d) - (5*cosh(b*x + d)^3 + 3*cosh(b*x
+ d))*sinh(-2*a + 2*d))/(b*cosh(b*x + d)^4 + 4*b*cosh(b*x + d)*sinh(b*x +
d)^3 + b*sinh(b*x + d)^4 + 2*b*cosh(b*x + d)^2 + 2*(3*b*cosh(b*x + d)^2 +
b)*sinh(b*x + d)^2 + 4*(b*cosh(b*x + d)^3 + b*cosh(b*x + d))*sinh(b*x + d)
 + b)
```

Sympy [F]

$$\int e^{2(a+bx)}\operatorname{sech}^3(d+bx)\,dx = e^{2a}\int e^{2bx}\operatorname{sech}^3(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)**3,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*sech(b*x + d)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int e^{2(a+bx)}\operatorname{sech}^3(d+bx)\,dx = -\frac{3\arctan\left(e^{(-bx-d)}\right)e^{(2a-2d)}}{b} - \frac{\left(5e^{(-bx-d)}+3e^{(-3bx-3d)}\right)e^{(2a-2d)}}{b(2e^{(-2bx-2d)}+e^{(-4bx-4d)}+1)}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)^3,x, algorithm="maxima")
```

output
```
-3*arctan(e^(-b*x - d))*e^(2*a - 2*d)/b - (5*e^(-b*x - d) + 3*e^(-3*b*x -
3*d))*e^(2*a - 2*d)/(b*(2*e^(-2*b*x - 2*d) + e^(-4*b*x - 4*d) + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int e^{2(a+bx)}\operatorname{sech}^3(d+bx)\,dx = \frac{\left(3\arctan\left(e^{(bx+d)}\right)e^{(-2d)} - \frac{\left(5e^{(3bx+3d)}+3e^{(bx+d)}\right)e^{(-2d)}}{\left(e^{(2bx+2d)}+1\right)^2}\right)e^{(2a)}}{b}$$

input
```
integrate(exp(2*b*x+2*a)*sech(b*x+d)^3,x, algorithm="giac")
```

output
```
(3*arctan(e^(b*x + d))*e^(-2*d) - (5*e^(3*b*x + 3*d) + 3*e^(b*x + d))*e^(-
2*d)/(e^(2*b*x + 2*d) + 1)^2)*e^(2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.42

$$\int e^{2(a+bx)}\operatorname{sech}^3(d+bx)\,dx=\frac{3\,\sqrt{e^{4\,a-4\,d}}\,\mathrm{atan}\left(\frac{e^{2\,a}\,e^{-d}\,e^{b\,x}\,\sqrt{b^2}}{b\,\sqrt{e^{4\,a}\,e^{-4\,d}}}\right)}{\sqrt{b^2}}-\frac{3\,e^{2\,a-d+b\,x}}{b\,\left(e^{2\,d+2\,b\,x}+1\right)}-\frac{2\,e^{2\,a+d+3\,b\,x}}{b\,\left(2\,e^{2\,d+2\,b\,x}+e^{4\,d+4\,b\,x}+1\right)}$$

input
```
int(exp(2*a + 2*b*x)/cosh(d + b*x)^3,x)
```

output
```
(3*exp(4*a - 4*d)^(1/2)*atan((exp(2*a)*exp(-d)*exp(b*x)*(b^2)^(1/2))/(b*(e
xp(4*a)*exp(-4*d))^(1/2))))/(b^2)^(1/2) - (3*exp(2*a - d + b*x))/(b*(exp(2
*d + 2*b*x) + 1)) - (2*exp(2*a + d + 3*b*x))/(b*(2*exp(2*d + 2*b*x) + exp(
4*d + 4*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.36

$$\int e^{2(a+bx)}\operatorname{sech}^3(d+bx)\,dx=\frac{e^{2a}\left(3e^{4bx+4d}atan\left(e^{bx+d}\right)+6e^{2bx+2d}atan\left(e^{bx+d}\right)+3atan\left(e^{bx+d}\right)-5e^{3bx+3d}-3e^{bx+d}\right)}{e^{2d}b\left(e^{4bx+4d}+2e^{2bx+2d}+1\right)}$$

input
```
int(exp(2*b*x+2*a)*sech(b*x+d)^3,x)
```

output
```
(e**(2*a)*(3*e**(4*b*x + 4*d)*atan(e**(b*x + d)) + 6*e**(2*b*x + 2*d)*atan
(e**(b*x + d)) + 3*atan(e**(b*x + d)) - 5*e**(3*b*x + 3*d) - 3*e**(b*x + d
)))/(e**(2*d)*b*(e**(4*b*x + 4*d) + 2*e**(2*b*x + 2*d) + 1))
```

3.82 $\int e^{2(a+bx)}\mathbf{csch}(d+bx)\mathbf{sech}^3(d+bx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int e^{2(a+bx)}\text{csch}(d+bx)\text{sech}^3(d+bx)\,dx = \frac{2e^{2a-2d}}{b\left(1+e^{2d+2bx}\right)^2}-\frac{6e^{2a-2d}}{b\left(1+e^{2d+2bx}\right)}-\frac{2e^{2a-2d}\text{arctanh}\left(e^{2d+2bx}\right)}{b}$$

output
```
2*exp(2*a-2*d)/b/(1+exp(2*b*x+2*d))^2-6*exp(2*a-2*d)/b/(1+exp(2*b*x+2*d))-
2*exp(2*a-2*d)*arctanh(exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int e^{2(a+bx)}\text{csch}(d+bx)\text{sech}^3(d+bx)\,dx = \frac{16e^{2a-2d}\left(\frac{1}{8\left(1+e^{2(d+bx)}\right)^2}-\frac{3}{8\left(1+e^{2(d+bx)}\right)}-\frac{1}{8}\text{arctanh}\left(e^{2(d+bx)}\right)\right)}{b}$$

input
```
Integrate[E^(2*(a + b*x))*Csch[d + b*x]*Sech[d + b*x]^3,x]
```

output
```
(16*E^(2*a - 2*d)*(1/(8*(1 + E^(2*(d + b*x)))^2) - 3/(8*(1 + E^(2*(d + b*x
)))) - ArcTanh[E^(2*(d + b*x))]/8))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.61, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2720, 27, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{csch}(bx+d)\text{sech}^3(bx+d)\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int -\frac{16e^{2a+5bx}}{(1-e^{2bx})(1+e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{16e^{2a}\int \frac{e^{5bx}}{(1-e^{2bx})(1+e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \; 354$$

$$-\frac{8e^{2a}\int \frac{e^{2bx}}{(1-e^{2bx})(1+e^{2bx})^3}de^{2bx}}{b}$$

$$\Big\downarrow \; 99$$

$$-\frac{8e^{2a}\int \left(-\frac{3}{4(1+e^{2bx})^2}+\frac{1}{2(1+e^{2bx})^3}-\frac{1}{4(-1+e^{2bx})}\right)de^{2bx}}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{8e^{2a}\left(\frac{1}{4}\text{arctanh}\left(e^{2bx}\right)+\frac{3}{4(e^{2bx}+1)}-\frac{1}{4(e^{2bx}+1)^2}\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Csch[d + b*x]*Sech[d + b*x]^3,x]
```

output
```
(-8*E^(2*a)*(-1/4*1/(1 + E^(2*b*x))^2 + 3/(4*(1 + E^(2*b*x))) + ArcTanh[E^
(2*b*x)]/4))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 106.35 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.46

method	result	size
risch	$-\frac{2\left(3\,\mathrm{e}^{2bx+2a+2d}+2\,\mathrm{e}^{2a}\right)\mathrm{e}^{4a-2d}}{\left(\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)^{2}b}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-\mathrm{e}^{2a-2d}\right)\mathrm{e}^{2a-2d}}{b}-\frac{\ln\left(\mathrm{e}^{2bx+2a}+\mathrm{e}^{2a-2d}\right)\mathrm{e}^{2a-2d}}{b}$	120

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)*sech(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/(exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(3*exp(2*b*x+2*a+2*d)+2*exp(2*a))*exp
(4*a-2*d)+ln(exp(2*b*x+2*a)-exp(2*a-2*d))/b*exp(2*a-2*d)-ln(exp(2*b*x+2*a)
+exp(2*a-2*d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 817 vs. 2(76) = 152.

Time = 0.09 (sec) , antiderivative size = 817, normalized size of antiderivative = 9.96

$$\int e^{2(a+bx)}\mathrm{csch}(d+bx)\mathrm{sech}^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)*sech(b*x+d)^3,x, algorithm="fricas")
```

output
```
-(6*cosh(b*x + d)^2*cosh(-2*a + 2*d) + 6*(cosh(-2*a + 2*d) - sinh(-2*a + 2
*d))*sinh(b*x + d)^2 + (cosh(b*x + d)^4*cosh(-2*a + 2*d) + (cosh(-2*a + 2*
d) - sinh(-2*a + 2*d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*cosh(-2*a + 2*d)
 - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^3 + 2*cosh(b*x + d)^2*cos
h(-2*a + 2*d) + 2*(3*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (3*cosh(b*x + d)^2
 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*sinh(b*x + d)^2 + 4*(cosh(b*x +
 d)^3*cosh(-2*a + 2*d) + cosh(b*x + d)*cosh(-2*a + 2*d) - (cosh(b*x + d)^3
 + cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^4 + 2*c
osh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*log(2*cosh(b*x +
d)/(cosh(b*x + d) - sinh(b*x + d))) - (cosh(b*x + d)^4*cosh(-2*a + 2*d) +
(cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^4 + 4*(cosh(b*x + d)*c
osh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^3 + 2*cosh
(b*x + d)^2*cosh(-2*a + 2*d) + 2*(3*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (3*
cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*sinh(b*x + d)^2
+ 4*(cosh(b*x + d)^3*cosh(-2*a + 2*d) + cosh(b*x + d)*cosh(-2*a + 2*d) - (
cosh(b*x + d)^3 + cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b
*x + d)^4 + 2*cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*lo
g(2*sinh(b*x + d)/(cosh(b*x + d) - sinh(b*x + d))) + 12*(cosh(b*x + d)*cos
h(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d) - 2*(3*cosh(
b*x + d)^2 + 2)*sinh(-2*a + 2*d) + 4*cosh(-2*a + 2*d))/(b*cosh(b*x + d)...
```

Sympy [F]

$$\int e^{2(a+bx)}\operatorname{csch}(d+bx)\operatorname{sech}^3(d+bx)\,dx = e^{2a}\int e^{2bx}\operatorname{csch}(bx+d)\operatorname{sech}^3(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)*sech(b*x+d)**3,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*csch(b*x + d)*sech(b*x + d)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.49

$$\int e^{2(a+bx)}\operatorname{csch}(d+bx)\operatorname{sech}^3(d+bx)\,dx = \frac{e^{(2a-2d)}\log\left(e^{(-bx-d)}+1\right)}{b} + \frac{e^{(2a-2d)}\log\left(e^{(-bx-d)}-1\right)}{b} - \frac{e^{(2a-2d)}\log\left(e^{(-2bx-2d)}+1\right)}{b} + \frac{2\left(e^{(-2bx-2d)}+2\right)e^{(2a-2d)}}{b(2e^{(-2bx-2d)}+e^{(-4bx-4d)}+1)}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)*sech(b*x+d)^3,x, algorithm="maxima")
```

output
```
e^(2*a - 2*d)*log(e^(-b*x - d) + 1)/b + e^(2*a - 2*d)*log(e^(-b*x - d) - 1
)/b - e^(2*a - 2*d)*log(e^(-2*b*x - 2*d) + 1)/b + 2*(e^(-2*b*x - 2*d) + 2)
*e^(2*a - 2*d)/(b*(2*e^(-2*b*x - 2*d) + e^(-4*b*x - 4*d) + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10

$$\int e^{2(a+bx)}\operatorname{csch}(d+bx)\operatorname{sech}^3(d+bx)\,dx = -\frac{\left(2e^{(-2d)}\log\left(e^{(2bx+2d)}+1\right)-2e^{(-2d)}\log\left(\left|e^{(2bx+2d)}-1\right|\right)-\frac{\left(3e^{(4bx+4d)}-6e^{(2bx+2d)}-5\right)e^{(-2d)}}{\left(e^{(2bx+2d)}+1\right)^2}\right)e^{(2a)}}{2b}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)*sech(b*x+d)^3,x, algorithm="giac")
```

output
```
-1/2*(2*e^(-2*d)*log(e^(2*b*x + 2*d) + 1) - 2*e^(-2*d)*log(abs(e^(2*b*x +
2*d) - 1)) - (3*e^(4*b*x + 4*d) - 6*e^(2*b*x + 2*d) - 5)*e^(-2*d)/(e^(2*b*
x + 2*d) + 1)^2)*e^(2*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.40

$$\int e^{2(a+bx)}\text{csch}(d+bx)\text{sech}^3(d+bx)\,dx = \frac{2\,e^{2\,a-2\,d}}{b\,\left(2\,e^{2\,d+2\,b\,x}+e^{4\,d+4\,b\,x}+1\right)} - \frac{2\,\sqrt{e^{4\,a-4\,d}}\,\text{atan}\left(\frac{e^{2\,a}\,e^{2\,b\,x}\,\sqrt{-b^2}}{b\,\sqrt{e^{4\,a}\,e^{-4\,d}}}\right)}{\sqrt{-b^2}} - \frac{6\,e^{2\,a-2\,d}}{b\,\left(e^{2\,d+2\,b\,x}+1\right)}$$

input
```
int(exp(2*a + 2*b*x)/(cosh(d + b*x)^3*sinh(d + b*x)),x)
```

output
```
(2*exp(2*a - 2*d))/(b*(2*exp(2*d + 2*b*x) + exp(4*d + 4*b*x) + 1)) - (2*ex
p(4*a - 4*d)^(1/2)*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/(b*(exp(4*a)*ex
p(-4*d))^(1/2))))/(-b^2)^(1/2) - (6*exp(2*a - 2*d))/(b*(exp(2*d + 2*b*x) +
 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.77

$$\int e^{2(a+bx)}\text{csch}(d+bx)\text{sech}^3(d+bx)\,dx = \frac{e^{2a}\left(e^{4bx+4d}\log\left(e^{bx+d}-1\right)+e^{4bx+4d}\log\left(e^{bx+d}+1\right)-e^{4bx+4d}\log\left(e^{2bx+2d}+1\right)+3e^{4bx+4d}+2e^{2bx+2d}\log\left(e^{bx}\right.\right.}{e^{2d}b\left(e\right.}$$

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)*sech(b*x+d)^3,x)
```

output
```
(e**(2*a)*(e**(4*b*x + 4*d)*log(e**(b*x + d) - 1) + e**(4*b*x + 4*d)*log(e
**(b*x + d) + 1) - e**(4*b*x + 4*d)*log(e**(2*b*x + 2*d) + 1) + 3*e**(4*b*
x + 4*d) + 2*e**(2*b*x + 2*d)*log(e**(b*x + d) - 1) + 2*e**(2*b*x + 2*d)*l
og(e**(b*x + d) + 1) - 2*e**(2*b*x + 2*d)*log(e**(2*b*x + 2*d) + 1) + log(
e**(b*x + d) - 1) + log(e**(b*x + d) + 1) - log(e**(2*b*x + 2*d) + 1) - 1)
)/(e**(2*d)*b*(e**(4*b*x + 4*d) + 2*e**(2*b*x + 2*d) + 1))
```

3.83 $\int e^{2(a+bx)}\textbf{csch}^2(d+bx)\textbf{sech}^3(d+bx)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 153

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\text{sech}^3(d+bx)\,dx = \frac{2e^{2a-d+bx}}{b(1+e^{2d+2bx})^2} + \frac{8e^{2a+d+3bx}}{b(1-e^{2d+2bx})(1+e^{2d+2bx})^2} + \frac{3e^{2a-d+bx}}{b(1+e^{2d+2bx})} - \frac{e^{2a-2d}\arctan\left(e^{d+bx}\right)}{b} - \frac{4e^{2a-2d}\text{arctanh}\left(e^{d+bx}\right)}{b}$$

output
```
2*exp(b*x+2*a-d)/b/(1+exp(2*b*x+2*d))^2+8*exp(3*b*x+2*a+d)/b/(1-exp(2*b*x+
2*d))/(1+exp(2*b*x+2*d))^2+3*exp(b*x+2*a-d)/b/(1+exp(2*b*x+2*d))-exp(2*a-2
*d)*arctan(exp(b*x+d))/b-4*exp(2*a-2*d)*arctanh(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.76

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\text{sech}^3(d+bx)\,dx = \frac{e^{2a-2d}\left(-\frac{2e^{d+bx}}{-1+e^{2(d+bx)}}-\frac{2e^{d+bx}}{\left(1+e^{2(d+bx)}\right)^2}+\frac{5e^{d+bx}}{1+e^{2(d+bx)}}-\arctan\left(e^{d+bx}\right)+2\log\left(1-e^{d+bx}\right)-2\log\left(1+e^{d+bx}\right)\right)}{b}$$

input
```
Integrate[E^(2*(a + b*x))*Csch[d + b*x]^2*Sech[d + b*x]^3,x]
```

output
```
(E^(2*a - 2*d)*((-2*E^(d + b*x))/(-1 + E^(2*(d + b*x))) - (2*E^(d + b*x))/
(1 + E^(2*(d + b*x)))^2 + (5*E^(d + b*x))/(1 + E^(2*(d + b*x))) - ArcTan[E
^(d + b*x)] + 2*Log[1 - E^(d + b*x)] - 2*Log[1 + E^(d + b*x)]))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.73, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2720, 27, 372, 440, 27, 402, 27, 397, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{csch}^2(bx+d)\text{sech}^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{32e^{2a+6bx}}{\left(1-e^{2bx}\right)^2\left(1+e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{32e^{2a}\int \frac{e^{6bx}}{\left(1-e^{2bx}\right)^2\left(1+e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{372}$$

$$\frac{32e^{2a}\left(\frac{e^{3bx}}{4(1-e^{2bx})(e^{2bx}+1)^2}-\frac{1}{4}\int\frac{e^{2bx}(3+e^{2bx})}{(1-e^{2bx})(1+e^{2bx})^3}de^{bx}\right)}{b}$$

$$\Big\downarrow \text{440}$$

$$\frac{32e^{2a}\left(\frac{1}{4}\left(\frac{e^{bx}}{4(e^{2bx}+1)^2}-\frac{1}{8}\int\frac{2(1+7e^{2bx})}{(1-e^{2bx})(1+e^{2bx})^2}de^{bx}\right)+\frac{e^{3bx}}{4(1-e^{2bx})(e^{2bx}+1)^2}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{32e^{2a}\left(\frac{1}{4}\left(\frac{e^{bx}}{4(e^{2bx}+1)^2}-\frac{1}{4}\int\frac{1+7e^{2bx}}{(1-e^{2bx})(1+e^{2bx})^2}de^{bx}\right)+\frac{e^{3bx}}{4(1-e^{2bx})(e^{2bx}+1)^2}\right)}{b}$$

$$\Big\downarrow \text{402}$$

$$\frac{32e^{2a}\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{4}\int-\frac{2(5+3e^{2bx})}{(1-e^{2bx})(1+e^{2bx})}de^{bx}+\frac{3e^{bx}}{2(e^{2bx}+1)}\right)+\frac{e^{bx}}{4(e^{2bx}+1)^2}\right)+\frac{e^{3bx}}{4(1-e^{2bx})(e^{2bx}+1)^2}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{32e^{2a}\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{3e^{bx}}{2(e^{2bx}+1)}-\frac{1}{2}\int\frac{5+3e^{2bx}}{(1-e^{2bx})(1+e^{2bx})}de^{bx}\right)+\frac{e^{bx}}{4(e^{2bx}+1)^2}\right)+\frac{e^{3bx}}{4(1-e^{2bx})(e^{2bx}+1)^2}\right)}{b}$$

$$\Big\downarrow \text{397}$$

$$\frac{32e^{2a}\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{2}\left(-4\int\frac{1}{1-e^{2bx}}de^{bx}-\int\frac{1}{1+e^{2bx}}de^{bx}\right)+\frac{3e^{bx}}{2(e^{2bx}+1)}\right)+\frac{e^{bx}}{4(e^{2bx}+1)^2}\right)+\frac{e^{3bx}}{4(1-e^{2bx})(e^{2bx}+1)^2}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{32e^{2a}\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{2}\left(-4\int\frac{1}{1-e^{2bx}}de^{bx}-\arctan\left(e^{bx}\right)\right)+\frac{3e^{bx}}{2(e^{2bx}+1)}\right)+\frac{e^{bx}}{4(e^{2bx}+1)^2}\right)+\frac{e^{3bx}}{4(1-e^{2bx})(e^{2bx}+1)^2}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{32e^{2a}\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{2}\left(-\arctan\left(e^{bx}\right)-4\text{arctanh}\left(e^{bx}\right)\right)+\frac{3e^{bx}}{2(e^{2bx}+1)}\right)+\frac{e^{bx}}{4(e^{2bx}+1)^2}\right)+\frac{e^{3bx}}{4(1-e^{2bx})(e^{2bx}+1)^2}\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Csch[d + b*x]^2*Sech[d + b*x]^3,x]
```

output
```
(32*E^(2*a)*(E^(3*b*x)/(4*(1 - E^(2*b*x))*(1 + E^(2*b*x))^2) + (E^(b*x)/(4
*(1 + E^(2*b*x))^2) + ((3*E^(b*x))/(2*(1 + E^(2*b*x))) + (-ArcTan[E^(b*x)]
 - 4*ArcTanh[E^(b*x)])/2)/4)/4))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 304.07 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.37

method	result
risch	$\frac{\left(-3\,\mathrm{e}^{4bx+4a+4d}+6\,\mathrm{e}^{2bx+4a+2d}+5\,\mathrm{e}^{4a}\right)\mathrm{e}^{bx+4a-d}}{\left(-\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)\left(\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)^{2}b}+\frac{i\ln\left(\mathrm{e}^{bx+a}-i\mathrm{e}^{a-d}\right)\mathrm{e}^{2a-2d}}{2b}-\frac{i\ln\left(\mathrm{e}^{bx+a}+i\mathrm{e}^{a-d}\right)\mathrm{e}^{2a-2d}}{2b}+\frac{2\ln\left(\mathrm{e}^{bx+a}-\mathrm{e}^{a-}\right.}{b}$

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)^2*sech(b*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/(-exp(2*b*x+2*a+2*d)+exp(2*a))/(exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(-3*exp
(4*b*x+4*a+4*d)+6*exp(2*b*x+4*a+2*d)+5*exp(4*a))*exp(b*x+4*a-d)+1/2*I*ln(e
xp(b*x+a)-I*exp(a-d))/b*exp(2*a-2*d)-1/2*I*ln(exp(b*x+a)+I*exp(a-d))/b*exp
(2*a-2*d)+2*ln(exp(b*x+a)-exp(a-d))/b*exp(2*a-2*d)-2*ln(exp(b*x+a)+exp(a-d
))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2090 vs. $2(140) = 280$.

Time $= 0.11$ (sec) , antiderivative size $= 2090$, normalized size of antiderivative $= 13.66$

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\text{sech}^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^2*sech(b*x+d)^3,x, algorithm="fricas"
)
```

output
```
(3*cosh(b*x + d)^5*cosh(-2*a + 2*d) + 3*(cosh(-2*a + 2*d) - sinh(-2*a + 2*
d))*sinh(b*x + d)^5 + 15*(cosh(b*x + d)*cosh(-2*a + 2*d) - cosh(b*x + d)*s
inh(-2*a + 2*d))*sinh(b*x + d)^4 - 6*cosh(b*x + d)^3*cosh(-2*a + 2*d) + 6*
(5*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (5*cosh(b*x + d)^2 - 1)*sinh(-2*a +
2*d) - cosh(-2*a + 2*d))*sinh(b*x + d)^3 + 6*(5*cosh(b*x + d)^3*cosh(-2*a
+ 2*d) - 3*cosh(b*x + d)*cosh(-2*a + 2*d) - (5*cosh(b*x + d)^3 - 3*cosh(b*
x + d))*sinh(-2*a + 2*d))*sinh(b*x + d)^2 - (cosh(b*x + d)^6*cosh(-2*a + 2
*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*d))*sinh(b*x + d)^6 + 6*(cosh(b*x
+ d)*cosh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d)^5 +
cosh(b*x + d)^4*cosh(-2*a + 2*d) + (15*cosh(b*x + d)^2*cosh(-2*a + 2*d) -
(15*cosh(b*x + d)^2 + 1)*sinh(-2*a + 2*d) + cosh(-2*a + 2*d))*sinh(b*x + d
)^4 + 4*(5*cosh(b*x + d)^3*cosh(-2*a + 2*d) + cosh(b*x + d)*cosh(-2*a + 2*
d) - (5*cosh(b*x + d)^3 + cosh(b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d)^3
 - cosh(b*x + d)^2*cosh(-2*a + 2*d) + (15*cosh(b*x + d)^4*cosh(-2*a + 2*d)
 + 6*cosh(b*x + d)^2*cosh(-2*a + 2*d) - (15*cosh(b*x + d)^4 + 6*cosh(b*x +
 d)^2 - 1)*sinh(-2*a + 2*d) - cosh(-2*a + 2*d))*sinh(b*x + d)^2 + 2*(3*cos
h(b*x + d)^5*cosh(-2*a + 2*d) + 2*cosh(b*x + d)^3*cosh(-2*a + 2*d) - cosh(
b*x + d)*cosh(-2*a + 2*d) - (3*cosh(b*x + d)^5 + 2*cosh(b*x + d)^3 - cosh(
b*x + d))*sinh(-2*a + 2*d))*sinh(b*x + d) - (cosh(b*x + d)^6 + cosh(b*x +
d)^4 - cosh(b*x + d)^2 - 1)*sinh(-2*a + 2*d) - cosh(-2*a + 2*d))*arctan...
```

Sympy [F]

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\text{sech}^3(d+bx)\,dx = e^{2a}\int e^{2bx}\,\text{csch}^2\,(bx+d)\,\text{sech}^3\,(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)**2*sech(b*x+d)**3,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*csch(b*x + d)**2*sech(b*x + d)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\text{sech}^3(d+bx)\,dx = \frac{\arctan\left(e^{(-bx-d)}\right)e^{(2\,a-2\,d)}}{b} - \frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}+1\right)}{b} + \frac{2\,e^{(2\,a-2\,d)}\log\left(e^{(-bx-d)}-1\right)}{b} + \frac{\left(3\,e^{(-bx-d)}-6\,e^{(-3\,bx-3\,d)}-5\,e^{(-5\,bx-5\,d)}\right)e^{(2\,a-2\,d)}}{b\left(e^{(-2\,bx-2\,d)}-e^{(-4\,bx-4\,d)}-e^{(-6\,bx-6\,d)}+1\right)}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^2*sech(b*x+d)^3,x, algorithm="maxima"
)
```

output
```
arctan(e^(-b*x - d))*e^(2*a - 2*d)/b - 2*e^(2*a - 2*d)*log(e^(-b*x - d) +
1)/b + 2*e^(2*a - 2*d)*log(e^(-b*x - d) - 1)/b + (3*e^(-b*x - d) - 6*e^(-3
*b*x - 3*d) - 5*e^(-5*b*x - 5*d))*e^(2*a - 2*d)/(b*(e^(-2*b*x - 2*d) - e^(
-4*b*x - 4*d) - e^(-6*b*x - 6*d) + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.75

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\text{sech}^3(d+bx)\,dx = -\frac{\left(\arctan\left(e^{(bx+d)}\right)e^{(-2d)} + 2\,e^{(-2d)}\log\left(e^{(bx+d)}+1\right) - 2\,e^{(-2d)}\log\left(\left|e^{(bx+d)}-1\right|\right) + \frac{2\,e^{(bx-d)}}{e^{(2bx+2d)}-1} - \frac{(5\,e^{(3bx}}{(}}{b}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^2*sech(b*x+d)^3,x, algorithm="giac")
```

output
```
-(arctan(e^(b*x + d))*e^(-2*d) + 2*e^(-2*d)*log(e^(b*x + d) + 1) - 2*e^(-2
*d)*log(abs(e^(b*x + d) - 1)) + 2*e^(b*x - d)/(e^(2*b*x + 2*d) - 1) - (5*e
^(3*b*x + 3*d) + 3*e^(b*x + d))*e^(-2*d)/(e^(2*b*x + 2*d) + 1)^2)*e^(2*a)/
b
```

Mupad [B] (verification not implemented)

Time = 3.59 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.63

$$\int e^{2(a+bx)}\text{csch}^2(d+bx)\text{sech}^3(d+bx)\,dx$$

$$= \frac{5\,e^{2a-d+bx}}{b+b\,e^{2d+2bx}} - \frac{2\,e^{2a-d+bx}}{b+2\,b\,e^{2d+2bx}+b\,e^{4d+4bx}} + \frac{2\,e^{2a-d+bx}}{b-b\,e^{2d+2bx}} - \frac{e^{2a-2d}\left(\ln\left(-68\,e^{6a}\,e^{-5d}\,e^{bx} - e^{6a}\,e^{-6d}\,68\mathrm{i}\right)\,1\mathrm{i} - \ln\left(-68\,e^{6a}\,e^{-5d}\,e^{bx} + e^{6a}\,e^{-6d}\,68\mathrm{i}\right)\,1\mathrm{i}\right)}{2\,b} - \frac{2\,e^{2a-2d}\,\ln\left(-272\,e^{6a}\,e^{-6d} - 272\,e^{6a}\,e^{-5d}\,e^{bx}\right)}{b} + \frac{2\,e^{2a-2d}\,\ln\left(272\,e^{6a}\,e^{-6d} - 272\,e^{6a}\,e^{-5d}\,e^{bx}\right)}{b}$$

input
```
int(exp(2*a + 2*b*x)/(cosh(d + b*x)^3*sinh(d + b*x)^2),x)
```

output
```
(5*exp(2*a - d + b*x))/(b + b*exp(2*d + 2*b*x)) - (2*exp(2*a - d + b*x))/(
b + 2*b*exp(2*d + 2*b*x) + b*exp(4*d + 4*b*x)) + (2*exp(2*a - d + b*x))/(b
 - b*exp(2*d + 2*b*x)) - (exp(2*a - 2*d)*(log(- exp(6*a)*exp(-6*d)*68i - 6
8*exp(6*a)*exp(-5*d)*exp(b*x))*1i - log(exp(6*a)*exp(-6*d)*68i - 68*exp(6*
a)*exp(-5*d)*exp(b*x))*1i))/(2*b) - (2*exp(2*a - 2*d)*log(- 272*exp(6*a)*e
xp(-6*d) - 272*exp(6*a)*exp(-5*d)*exp(b*x)))/b + (2*exp(2*a - 2*d)*log(272
*exp(6*a)*exp(-6*d) - 272*exp(6*a)*exp(-5*d)*exp(b*x)))/b
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.02

$$\int e^{2(a+bx)}\operatorname{csch}^2(d+bx)\operatorname{sech}^3(d+bx)\,dx = \frac{e^{2a}\left(-e^{6bx+6d}atan\left(e^{bx+d}\right)-e^{4bx+4d}atan\left(e^{bx+d}\right)+e^{2bx+2d}atan\left(e^{bx+d}\right)+atan\left(e^{bx+d}\right)+2e^{6bx+6d}\log\left(e^{bx+d}\right.\right.}{}$$

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)^2*sech(b*x+d)^3,x)
```

output
```
(e**(2*a)*( - e**(6*b*x + 6*d)*atan(e**(b*x + d)) - e**(4*b*x + 4*d)*atan(
e**(b*x + d)) + e**(2*b*x + 2*d)*atan(e**(b*x + d)) + atan(e**(b*x + d)) +
 2*e**(6*b*x + 6*d)*log(e**(b*x + d) - 1) - 2*e**(6*b*x + 6*d)*log(e**(b*x
 + d) + 1) + 3*e**(5*b*x + 5*d) + 2*e**(4*b*x + 4*d)*log(e**(b*x + d) - 1)
 - 2*e**(4*b*x + 4*d)*log(e**(b*x + d) + 1) - 6*e**(3*b*x + 3*d) - 2*e**(2
*b*x + 2*d)*log(e**(b*x + d) - 1) + 2*e**(2*b*x + 2*d)*log(e**(b*x + d) +
1) - 5*e**(b*x + d) - 2*log(e**(b*x + d) - 1) + 2*log(e**(b*x + d) + 1)))/
(e**(2*d)*b*(e**(6*b*x + 6*d) + e**(4*b*x + 4*d) - e**(2*b*x + 2*d) - 1))
```

3.84 $\int e^{2(a+bx)}\text{csch}^3(d+bx)\text{sech}^3(d+bx)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 35

$$\int e^{2(a+bx)}\text{csch}^3(d+bx)\text{sech}^3(d+bx)\,dx = -\frac{8e^{2(a+3d)+8bx}}{b\left(1-e^{4d+4bx}\right)^2}$$

output
```
-8*exp(8*b*x+2*a+6*d)/b/(1-exp(4*b*x+4*d))^2
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int e^{2(a+bx)}\text{csch}^3(d+bx)\text{sech}^3(d+bx)\,dx = -\frac{8e^{2a+6d+8bx}}{b\left(-1+e^{4(d+bx)}\right)^2}$$

input
```
Integrate[E^(2*(a + b*x))*Csch[d + b*x]^3*Sech[d + b*x]^3,x]
```

output
```
(-8*E^(2*a + 6*d + 8*b*x))/(b*(-1 + E^(4*(d + b*x)))^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2720, 27, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{csch}^3(bx+d)\text{sech}^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{64e^{2a+7bx}}{(1-e^{4bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{64e^{2a}\int \frac{e^{7bx}}{(1-e^{4bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{796}$$

$$-\frac{8e^{2a+8bx}}{b\left(1-e^{4bx}\right)^2}$$

input
```
Int[E^(2*(a + b*x))*Csch[d + b*x]^3*Sech[d + b*x]^3,x]
```

output
```
(-8*E^(2*a + 8*b*x))/(b*(1 - E^(4*b*x))^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(32) = 64$.

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.09

$$\frac{8\left(-2\,\mathrm{e}^{4bx+4a+4d}+\mathrm{e}^{4a}\right)\mathrm{e}^{6a-2d}}{\left(-\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)^{2}\left(\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)^{2}b}$$

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)^3*sech(b*x+d)^3,x)
```

output
```
8/(-exp(2*b*x+2*a+2*d)+exp(2*a))^2/(exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(-2*e
xp(4*b*x+4*a+4*d)+exp(4*a))*exp(6*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(30) = 60$.

Time = 0.08 (sec) , antiderivative size = 247, normalized size of antiderivative = 7.06

$$\int e^{2(a+bx)}\operatorname{csch}^3(d+bx)\operatorname{sech}^3(d+bx)\,dx = -\frac{8\left(\cosh\left(bx+d\right)^{2}\cosh\left(-2\,a+2\,d\right)+\left(\cosh\left(-2\,a+2\,d\right)-\sinh\left(-2\,a+2\,d\right)\right)\sinh}{b\cosh\left(bx+d\right)^{6}+20\,b\cosh\left(bx+d\right)^{3}\sinh\left(bx+d\right)^{3}+15\,b\cosh\left(bx+d\right)^{2}\sinh\left(bx+d\right)^{4}+6\,b\cosh\left(bx\right.}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^3*sech(b*x+d)^3,x, algorithm="fricas"
)
```

output
```
-8*(cosh(b*x + d)^2*cosh(-2*a + 2*d) + (cosh(-2*a + 2*d) - sinh(-2*a + 2*d
))*sinh(b*x + d)^2 - cosh(b*x + d)^2*sinh(-2*a + 2*d) + 6*(cosh(b*x + d)*c
osh(-2*a + 2*d) - cosh(b*x + d)*sinh(-2*a + 2*d))*sinh(b*x + d))/(b*cosh(b
*x + d)^6 + 20*b*cosh(b*x + d)^3*sinh(b*x + d)^3 + 15*b*cosh(b*x + d)^2*si
nh(b*x + d)^4 + 6*b*cosh(b*x + d)*sinh(b*x + d)^5 + b*sinh(b*x + d)^6 - b*
cosh(b*x + d)^2 + (15*b*cosh(b*x + d)^4 - b)*sinh(b*x + d)^2 + 6*(b*cosh(b
*x + d)^5 - b*cosh(b*x + d))*sinh(b*x + d))
```

Sympy [F]

$$\int e^{2(a+bx)}\text{csch}^3(d+bx)\text{sech}^3(d+bx)\,dx = e^{2a}\int e^{2bx}\,\text{csch}^3\left(bx+d\right)\text{sech}^3\left(bx+d\right)dx$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)**3*sech(b*x+d)**3,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*csch(b*x + d)**3*sech(b*x + d)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int e^{2(a+bx)}\text{csch}^3(d+bx)\text{sech}^3(d+bx)\,dx = \frac{8\,e^{(2\,a-2\,d)}}{b(2\,e^{(-4\,bx-4\,d)}-e^{(-8\,bx-8\,d)}-1)}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^3*sech(b*x+d)^3,x, algorithm="maxima"
)
```

output
```
8*e^(2*a - 2*d)/(b*(2*e^(-4*b*x - 4*d) - e^(-8*b*x - 8*d) - 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int e^{2(a+bx)}\operatorname{csch}^3(d+bx)\operatorname{sech}^3(d+bx)\,dx = -\frac{8\left(2\,e^{(4\,bx+4\,d)}-1\right)e^{(2\,a-2\,d)}}{b\left(e^{(4\,bx+4\,d)}-1\right)^2}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^3*sech(b*x+d)^3,x, algorithm="giac")
```

output
```
-8*(2*e^(4*b*x + 4*d) - 1)*e^(2*a - 2*d)/(b*(e^(4*b*x + 4*d) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 2.59 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int e^{2(a+bx)}\operatorname{csch}^3(d+bx)\operatorname{sech}^3(d+bx)\,dx = -\frac{8\left(e^{2\,a+2\,d+4\,b\,x}-e^{2\,a-2\,d}+e^{2\,a-2\,d}\,e^{4\,d+4\,b\,x}\right)}{b\left(e^{4\,d+4\,b\,x}-1\right)^2}$$

input
```
int(exp(2*a + 2*b*x)/(cosh(d + b*x)^3*sinh(d + b*x)^3),x)
```

output
```
-(8*(exp(2*a + 2*d + 4*b*x) - exp(2*a - 2*d) + exp(2*a - 2*d)*exp(4*d + 4*
b*x)))/(b*(exp(4*d + 4*b*x) - 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int e^{2(a+bx)}\operatorname{csch}^3(d+bx)\operatorname{sech}^3(d+bx)\,dx = -\frac{8e^{8bx+2a+6d}}{b\left(e^{8bx+8d}-2e^{4bx+4d}+1\right)}$$

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)^3*sech(b*x+d)^3,x)
```

output
```
( - 8*e**(2*a + 8*b*x + 6*d))/(b*(e**(8*b*x + 8*d) - 2*e**(4*b*x + 4*d) +
1))
```

3.85 $\int e^{2(a+bx)}\mathrm{csch}^4(d+bx)\mathrm{sech}^3(d+bx)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 193

$$\int e^{2(a+bx)}\mathrm{csch}^4(d+bx)\mathrm{sech}^3(d+bx)\,dx = \frac{32e^{2a+3d+5bx}}{3b\left(1-e^{2d+2bx}\right)^3\left(1+e^{2d+2bx}\right)^2} - \frac{4e^{2a-d+bx}\left(5+7e^{2d+2bx}\right)}{3b\left(1-e^{4d+4bx}\right)^2} + \frac{e^{2a-d+bx}\left(5+21e^{2d+2bx}\right)}{3b\left(1-e^{4d+4bx}\right)} - \frac{e^{2a-2d}\arctan\left(e^{d+bx}\right)}{b} + \frac{6e^{2a-2d}\mathrm{arctanh}\left(e^{d+bx}\right)}{b}$$

output

```
32/3*exp(5*b*x+2*a+3*d)/b/(1-exp(2*b*x+2*d))^3/(1+exp(2*b*x+2*d))^2-4/3*ex
p(b*x+2*a-d)*(5+7*exp(2*b*x+2*d))/b/(1-exp(4*b*x+4*d))^2+1/3*exp(b*x+2*a-d
)*(5+21*exp(2*b*x+2*d))/b/(1-exp(4*b*x+4*d))-exp(2*a-2*d)*arctan(exp(b*x+d
))/b+6*exp(2*a-2*d)*arctanh(exp(b*x+d))/b
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.84

$$\int e^{2(a+bx)}\operatorname{csch}^4(d+bx)\operatorname{sech}^3(d+bx)\,dx$$
$$= \frac{e^{2a-2d}\left(-\frac{8e^{d+bx}}{(-1+e^{2(d+bx)})^3}-\frac{20e^{d+bx}}{(-1+e^{2(d+bx)})^2}-\frac{6e^{d+bx}}{-1+e^{2(d+bx)}}+\frac{6e^{d+bx}}{(1+e^{2(d+bx)})^2}-\frac{15e^{d+bx}}{1+e^{2(d+bx)}}-3\arctan\left(e^{d+bx}\right)-9\log\right.}{3b}$$

input
```
Integrate[E^(2*(a + b*x))*Csch[d + b*x]^4*Sech[d + b*x]^3,x]
```

output
```
(E^(2*a - 2*d)*((-8*E^(d + b*x))/(-1 + E^(2*(d + b*x)))^3 - (20*E^(d + b*x
))/(-1 + E^(2*(d + b*x)))^2 - (6*E^(d + b*x))/(-1 + E^(2*(d + b*x))) + (6*
E^(d + b*x))/(1 + E^(2*(d + b*x)))^2 - (15*E^(d + b*x))/(1 + E^(2*(d + b*x
))) - 3*ArcTan[E^(d + b*x)] - 9*Log[1 - E^(d + b*x)] + 9*Log[1 + E^(d + b*
x)]))/(3*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.91, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2720, 27, 372, 440, 27, 440, 27, 402, 27, 402, 27, 397, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\operatorname{csch}^4(bx+d)\operatorname{sech}^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{128e^{2a+8bx}}{(1-e^{2bx})^4(1+e^{2bx})^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{128e^{2a}\int \frac{e^{8bx}}{(1-e^{2bx})^4(1+e^{2bx})^3}de^{bx}}{b}$$

$$\downarrow 372$$

$$\frac{128e^{2a}\left(\frac{e^{5bx}}{12(1-e^{2bx})^3(e^{2bx}+1)^2}-\frac{1}{12}\int \frac{e^{4bx}(5+7e^{2bx})}{(1-e^{2bx})^3(1+e^{2bx})^3}de^{bx}\right)}{b}$$

$$\downarrow 440$$

$$\frac{128e^{2a}\left(\frac{1}{12}\left(-\frac{1}{8}\int -\frac{4e^{2bx}(9-e^{2bx})}{(1-e^{2bx})^2(1+e^{2bx})^3}de^{bx}-\frac{3e^{3bx}}{2(1-e^{2bx})^2(e^{2bx}+1)^2}\right)+\frac{e^{5bx}}{12(1-e^{2bx})^3(e^{2bx}+1)^2}\right)}{b}$$

$$\downarrow 27$$

$$\frac{128e^{2a}\left(\frac{1}{12}\left(\frac{1}{2}\int \frac{e^{2bx}(9-e^{2bx})}{(1-e^{2bx})^2(1+e^{2bx})^3}de^{bx}-\frac{3e^{3bx}}{2(1-e^{2bx})^2(e^{2bx}+1)^2}\right)+\frac{e^{5bx}}{12(1-e^{2bx})^3(e^{2bx}+1)^2}\right)}{b}$$

$$\downarrow 440$$

$$\frac{128e^{2a}\left(\frac{1}{12}\left(\frac{1}{2}\left(\frac{1}{4}\int -\frac{4(2-11e^{2bx})}{(1-e^{2bx})(1+e^{2bx})^3}de^{bx}+\frac{2e^{bx}}{(1-e^{2bx})(e^{2bx}+1)^2}\right)-\frac{3e^{3bx}}{2(1-e^{2bx})^2(e^{2bx}+1)^2}\right)+\frac{e^{5bx}}{12(1-e^{2bx})^3(e^{2bx}+1)^2}\right)}{b}$$

$$\downarrow 27$$

$$\frac{128e^{2a}\left(\frac{1}{12}\left(\frac{1}{2}\left(\frac{2e^{bx}}{(1-e^{2bx})(e^{2bx}+1)^2}-\int \frac{2-11e^{2bx}}{(1-e^{2bx})(1+e^{2bx})^3}de^{bx}\right)-\frac{3e^{3bx}}{2(1-e^{2bx})^2(e^{2bx}+1)^2}\right)+\frac{e^{5bx}}{12(1-e^{2bx})^3(e^{2bx}+1)^2}\right)}{b}$$

$$\downarrow 402$$

$$\frac{128e^{2a}\left(\frac{1}{12}\left(\frac{1}{2}\left(\frac{1}{8}\int -\frac{3(1-13e^{2bx})}{(1-e^{2bx})(1+e^{2bx})^2}de^{bx}-\frac{13e^{bx}}{8(e^{2bx}+1)^2}+\frac{2e^{bx}}{(1-e^{2bx})(e^{2bx}+1)^2}\right)-\frac{3e^{3bx}}{2(1-e^{2bx})^2(e^{2bx}+1)^2}\right)+\frac{e^{5bx}}{12(1-e^{2bx})^3(e^{2bx}+1)^2}\right)}{b}$$

$$\downarrow 27$$

$$\frac{128e^{2a}\left(\frac{1}{12}\left(\frac{1}{2}\left(-\frac{3}{8}\int \frac{1-13e^{2bx}}{(1-e^{2bx})(1+e^{2bx})^2}de^{bx}-\frac{13e^{bx}}{8(e^{2bx}+1)^2}+\frac{2e^{bx}}{(1-e^{2bx})(e^{2bx}+1)^2}\right)-\frac{3e^{3bx}}{2(1-e^{2bx})^2(e^{2bx}+1)^2}\right)+\frac{e^{5bx}}{12(1-e^{2bx})^3(e^{2bx}+1)^2}\right)}{b}$$

$$\downarrow 402$$

$$\frac{128e^{2a}\left(\frac{1}{12}\left(\frac{1}{2}\left(-\frac{3}{8}\left(\frac{7e^{bx}}{2(e^{2bx}+1)}-\frac{1}{4}\int \frac{2(5+7e^{2bx})}{(1-e^{2bx})(1+e^{2bx})}de^{bx}\right)-\frac{13e^{bx}}{8(e^{2bx}+1)^2}+\frac{2e^{bx}}{(1-e^{2bx})(e^{2bx}+1)^2}\right)-\frac{3e^{3bx}}{2(1-e^{2bx})^2(e^{2bx}+1)^2}\right)+\frac{e^{5bx}}{12(1-e^{2bx})^3(e^{2bx}+1)^2}\right)}{b}$$

$\downarrow$ 27

$$\frac{128e^{2a}\left(\frac{1}{12}\left(\frac{1}{2}\left(-\frac{3}{8}\left(\frac{7e^{bx}}{2(e^{2bx}+1)}-\frac{1}{2}\int\frac{5+7e^{2bx}}{(1-e^{2bx})(1+e^{2bx})}de^{bx}\right)-\frac{13e^{bx}}{8(e^{2bx}+1)^2}+\frac{2e^{bx}}{(1-e^{2bx})(e^{2bx}+1)^2}\right)-\frac{3e^{3bx}}{2(1-e^{2bx})^2(e^{2bx}+1)^2}\right)\right.}{b}$$

$\downarrow$ 397

$$\frac{128e^{2a}\left(\frac{1}{12}\left(\frac{1}{2}\left(-\frac{3}{8}\left(\frac{1}{2}\left(\int\frac{1}{1+e^{2bx}}de^{bx}-6\int\frac{1}{1-e^{2bx}}de^{bx}\right)+\frac{7e^{bx}}{2(e^{2bx}+1)}\right)-\frac{13e^{bx}}{8(e^{2bx}+1)^2}+\frac{2e^{bx}}{(1-e^{2bx})(e^{2bx}+1)^2}\right)-\frac{3e^{3}}{2(1-e^{2bx})^2}\right.\right.}{b}$$

$\downarrow$ 216

$$\frac{128e^{2a}\left(\frac{1}{12}\left(\frac{1}{2}\left(-\frac{3}{8}\left(\frac{1}{2}\left(\arctan\left(e^{bx}\right)-6\int\frac{1}{1-e^{2bx}}de^{bx}\right)+\frac{7e^{bx}}{2(e^{2bx}+1)}\right)-\frac{13e^{bx}}{8(e^{2bx}+1)^2}+\frac{2e^{bx}}{(1-e^{2bx})(e^{2bx}+1)^2}\right)-\frac{3e^{3}}{2(1-e^{2bx})^2}\right.\right.}{b}$$

$\downarrow$ 219

$$\frac{128e^{2a}\left(\frac{1}{12}\left(\frac{1}{2}\left(-\frac{3}{8}\left(\frac{1}{2}\left(\arctan\left(e^{bx}\right)-6\text{arctanh}\left(e^{bx}\right)\right)+\frac{7e^{bx}}{2(e^{2bx}+1)}\right)-\frac{13e^{bx}}{8(e^{2bx}+1)^2}+\frac{2e^{bx}}{(1-e^{2bx})(e^{2bx}+1)^2}\right)-\frac{3e^{3}}{2(1-e^{2bx})^2}\right.\right.}{b}$$

input

```
Int[E^(2*(a + b*x))*Csch[d + b*x]^4*Sech[d + b*x]^3,x]
```

output

```
(128*E^(2*a)*(E^(5*b*x)/(12*(1 - E^(2*b*x))^3*(1 + E^(2*b*x))^2) + ((-3*E^
(3*b*x))/(2*(1 - E^(2*b*x))^2*(1 + E^(2*b*x))^2) + ((-13*E^(b*x))/(8*(1 +
E^(2*b*x))^2) + (2*E^(b*x))/((1 - E^(2*b*x))*(1 + E^(2*b*x))^2) - (3*((7*E
^(b*x))/(2*(1 + E^(2*b*x))) + (ArcTan[E^(b*x)] - 6*ArcTanh[E^(b*x)])/2))/8
)/2)/12))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.24

$$-\frac{\left(-21\,\mathrm{e}^{8bx+8a+8d}+16\,\mathrm{e}^{6bx+8a+6d}-34\,\mathrm{e}^{4bx+8a+4d}-8\,\mathrm{e}^{2bx+8a+2d}+15\,\mathrm{e}^{8a}\right)\mathrm{e}^{bx+4a-d}}{3\left(-\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)^{3}\left(\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)^{2}b}-\frac{3\ln\left(\mathrm{e}^{bx+a}-\mathrm{e}^{a-d}\right)\mathrm{e}^{2a-}}{b}$$

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)^4*sech(b*x+d)^3,x)
```

output
```
-1/3/(-exp(2*b*x+2*a+2*d)+exp(2*a))^3/(exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(-
21*exp(8*b*x+8*a+8*d)+16*exp(6*b*x+8*a+6*d)-34*exp(4*b*x+8*a+4*d)-8*exp(2*
b*x+8*a+2*d)+15*exp(8*a))*exp(b*x+4*a-d)-3*ln(exp(b*x+a)-exp(a-d))/b*exp(2
*a-2*d)+1/2*I*ln(exp(b*x+a)-I*exp(a-d))/b*exp(2*a-2*d)-1/2*I*ln(exp(b*x+a)
+I*exp(a-d))/b*exp(2*a-2*d)+3*ln(exp(b*x+a)+exp(a-d))/b*exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4685 vs. $2(168) = 336$.

Time = 0.14 (sec) , antiderivative size = 4685, normalized size of antiderivative = 24.27

$$\int e^{2(a+bx)}\operatorname{csch}^4(d+bx)\operatorname{sech}^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^4*sech(b*x+d)^3,x, algorithm="fricas"
)
```

output
```
Too large to include
```

Sympy [F]

$$\int e^{2(a+bx)}\operatorname{csch}^4(d+bx)\operatorname{sech}^3(d+bx)\,dx = e^{2a}\int e^{2bx}\operatorname{csch}^4(bx+d)\operatorname{sech}^3(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)**4*sech(b*x+d)**3,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*csch(b*x + d)**4*sech(b*x + d)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.02

$$\begin{aligned}&\int e^{2(a+bx)}\operatorname{csch}^4(d+bx)\operatorname{sech}^3(d+bx)\,dx\\&=\frac{\arctan\left(e^{(-bx-d)}\right)e^{(2a-2d)}}{b}+\frac{3e^{(2a-2d)}\log\left(e^{(-bx-d)}+1\right)}{b}\\&\quad-\frac{3e^{(2a-2d)}\log\left(e^{(-bx-d)}-1\right)}{b}\\&\quad+\frac{\left(21e^{(-bx-d)}-16e^{(-3bx-3d)}+34e^{(-5bx-5d)}+8e^{(-7bx-7d)}-15e^{(-9bx-9d)}\right)e^{(2a-2d)}}{3b\left(e^{(-2bx-2d)}+2e^{(-4bx-4d)}-2e^{(-6bx-6d)}-e^{(-8bx-8d)}+e^{(-10bx-10d)}-1\right)}\end{aligned}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^4*sech(b*x+d)^3,x, algorithm="maxima"
)
```

output
```
arctan(e^(-b*x - d))*e^(2*a - 2*d)/b + 3*e^(2*a - 2*d)*log(e^(-b*x - d) +
1)/b - 3*e^(2*a - 2*d)*log(e^(-b*x - d) - 1)/b + 1/3*(21*e^(-b*x - d) - 16
*e^(-3*b*x - 3*d) + 34*e^(-5*b*x - 5*d) + 8*e^(-7*b*x - 7*d) - 15*e^(-9*b*
x - 9*d))*e^(2*a - 2*d)/(b*(e^(-2*b*x - 2*d) + 2*e^(-4*b*x - 4*d) - 2*e^(-
6*b*x - 6*d) - e^(-8*b*x - 8*d) + e^(-10*b*x - 10*d) - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.74

$$\int e^{2(a+bx)}\text{csch}^4(d+bx)\text{sech}^3(d+bx)\,dx =$$
$$-\frac{\left(3\arctan\left(e^{(bx+d)}\right)e^{(-2d)} - 9e^{(-2d)}\log\left(e^{(bx+d)}+1\right) + 9e^{(-2d)}\log\left(\left|e^{(bx+d)}-1\right|\right) + \frac{3\left(5e^{(3bx+3d)}+3e^{(bx+}\right.}{\left(e^{(2bx+2d)}+1\right)}}{3b}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^4*sech(b*x+d)^3,x, algorithm="giac")
```

output
```
-1/3*(3*arctan(e^(b*x + d))*e^(-2*d) - 9*e^(-2*d)*log(e^(b*x + d) + 1) + 9
*e^(-2*d)*log(abs(e^(b*x + d) - 1)) + 3*(5*e^(3*b*x + 3*d) + 3*e^(b*x + d)
)*e^(-2*d)/(e^(2*b*x + 2*d) + 1)^2 + 2*(3*e^(5*b*x + 5*d) + 4*e^(3*b*x + 3
*d) - 3*e^(b*x + d))*e^(-2*d)/(e^(2*b*x + 2*d) - 1)^3)*e^(2*a)/b
```

Mupad [B] (verification not implemented)

Time = 4.42 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.80

$$\int e^{2(a+bx)}\operatorname{csch}^4(d+bx)\operatorname{sech}^3(d+bx)\,dx$$
$$= \frac{2\,e^{2\,a+b\,x}}{2\,b\,e^{3\,d+2\,b\,x}+b\,e^{5\,d+4\,b\,x}+b\,e^{d}} - \frac{20\,e^{2\,a+b\,x}}{3\,\left(b\,e^{5\,d+4\,b\,x}-2\,b\,e^{3\,d+2\,b\,x}+b\,e^{d}\right)}$$
$$+ \frac{8\,e^{2\,a-d+b\,x}}{3\,\left(b-3\,b\,e^{2\,d+2\,b\,x}+3\,b\,e^{4\,d+4\,b\,x}-b\,e^{6\,d+6\,b\,x}\right)} - \frac{5\,e^{2\,a+b\,x}}{b\,e^{3\,d+2\,b\,x}+b\,e^{d}} - \frac{2\,e^{2\,a+b\,x}}{b\,e^{3\,d+2\,b\,x}-b\,e^{d}}$$
$$- \frac{e^{2\,a-2\,d}\,\left(\ln\left(-148\,e^{6\,a}\,e^{-5\,d}\,e^{b\,x}-e^{6\,a}\,e^{-6\,d}\,148\mathrm{i}\right)\,1\mathrm{i}-\ln\left(-148\,e^{6\,a}\,e^{-5\,d}\,e^{b\,x}+e^{6\,a}\,e^{-6\,d}\,148\mathrm{i}\right)\,1\mathrm{i}\right)}{2\,b}$$
$$- \frac{3\,e^{2\,a-2\,d}\,\ln\left(888\,e^{6\,a}\,e^{-5\,d}\,e^{b\,x}-888\,e^{6\,a}\,e^{-6\,d}\right)}{b}$$
$$+ \frac{3\,e^{2\,a-2\,d}\,\ln\left(888\,e^{6\,a}\,e^{-6\,d}+888\,e^{6\,a}\,e^{-5\,d}\,e^{b\,x}\right)}{b}$$

input
```
int(exp(2*a + 2*b*x)/(cosh(d + b*x)^3*sinh(d + b*x)^4),x)
```

output
```
(2*exp(2*a + b*x))/(2*b*exp(3*d + 2*b*x) + b*exp(5*d + 4*b*x) + b*exp(d))
- (20*exp(2*a + b*x))/(3*(b*exp(5*d + 4*b*x) - 2*b*exp(3*d + 2*b*x) + b*ex
p(d))) + (8*exp(2*a - d + b*x))/(3*(b - 3*b*exp(2*d + 2*b*x) + 3*b*exp(4*d
 + 4*b*x) - b*exp(6*d + 6*b*x))) - (5*exp(2*a + b*x))/(b*exp(3*d + 2*b*x)
+ b*exp(d)) - (2*exp(2*a + b*x))/(b*exp(3*d + 2*b*x) - b*exp(d)) - (exp(2*
a - 2*d)*(log(- exp(6*a)*exp(-6*d)*148i - 148*exp(6*a)*exp(-5*d)*exp(b*x))
*1i - log(exp(6*a)*exp(-6*d)*148i - 148*exp(6*a)*exp(-5*d)*exp(b*x))*1i))/
(2*b) - (3*exp(2*a - 2*d)*log(888*exp(6*a)*exp(-5*d)*exp(b*x) - 888*exp(6*
a)*exp(-6*d)))/b + (3*exp(2*a - 2*d)*log(888*exp(6*a)*exp(-6*d) + 888*exp(
6*a)*exp(-5*d)*exp(b*x)))/b
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.53

$$\int e^{2(a+bx)}\mathrm{csch}^4(d+bx)\mathrm{sech}^3(d+bx)\,dx$$
$$= \frac{e^{2a}\left(-3e^{10bx+10d}atan\left(e^{bx+d}\right)+3e^{8bx+8d}atan\left(e^{bx+d}\right)+6e^{6bx+6d}atan\left(e^{bx+d}\right)-6e^{4bx+4d}atan\left(e^{bx+d}\right)-3e^{2bx}\right.}{}$$

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)^4*sech(b*x+d)^3,x)
```

output
```
(e**(2*a)*( - 3*e**(10*b*x + 10*d)*atan(e**(b*x + d)) + 3*e**(8*b*x + 8*d)
*atan(e**(b*x + d)) + 6*e**(6*b*x + 6*d)*atan(e**(b*x + d)) - 6*e**(4*b*x
+ 4*d)*atan(e**(b*x + d)) - 3*e**(2*b*x + 2*d)*atan(e**(b*x + d)) + 3*atan
(e**(b*x + d)) - 9*e**(10*b*x + 10*d)*log(e**(b*x + d) - 1) + 9*e**(10*b*x
 + 10*d)*log(e**(b*x + d) + 1) - 21*e**(9*b*x + 9*d) + 9*e**(8*b*x + 8*d)*
log(e**(b*x + d) - 1) - 9*e**(8*b*x + 8*d)*log(e**(b*x + d) + 1) + 16*e**(
7*b*x + 7*d) + 18*e**(6*b*x + 6*d)*log(e**(b*x + d) - 1) - 18*e**(6*b*x +
6*d)*log(e**(b*x + d) + 1) - 34*e**(5*b*x + 5*d) - 18*e**(4*b*x + 4*d)*log
(e**(b*x + d) - 1) + 18*e**(4*b*x + 4*d)*log(e**(b*x + d) + 1) - 8*e**(3*b
*x + 3*d) - 9*e**(2*b*x + 2*d)*log(e**(b*x + d) - 1) + 9*e**(2*b*x + 2*d)*
log(e**(b*x + d) + 1) + 15*e**(b*x + d) + 9*log(e**(b*x + d) - 1) - 9*log(
e**(b*x + d) + 1)))/(3*e**(2*d)*b*(e**(10*b*x + 10*d) - e**(8*b*x + 8*d) -
 2*e**(6*b*x + 6*d) + 2*e**(4*b*x + 4*d) + e**(2*b*x + 2*d) - 1))
```

3.86 $\int e^{2(a+bx)}\operatorname{csch}^5(d+bx)\operatorname{sech}^3(d+bx)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 204

$$\int e^{2(a+bx)}\text{csch}^5(d+bx)\text{sech}^3(d+bx)\,dx = -\frac{4e^{2a-2d}}{b\left(1-e^{2d+2bx}\right)^4}+\frac{40e^{2a-2d}}{3b\left(1-e^{2d+2bx}\right)^3} - \frac{12e^{2a-2d}}{b\left(1-e^{2d+2bx}\right)^2}-\frac{4e^{2a-2d}}{b\left(1-e^{2d+2bx}\right)} + \frac{2e^{2a-2d}}{b\left(1+e^{2d+2bx}\right)^2}-\frac{6e^{2a-2d}}{b\left(1+e^{2d+2bx}\right)} + \frac{2e^{2a-2d}\text{arctanh}\left(e^{2d+2bx}\right)}{b}$$

output
```
-4*exp(2*a-2*d)/b/(1-exp(2*b*x+2*d))^4+40/3*exp(2*a-2*d)/b/(1-exp(2*b*x+2*
d))^3-12*exp(2*a-2*d)/b/(1-exp(2*b*x+2*d))^2-4*exp(2*a-2*d)/b/(1-exp(2*b*x
+2*d))+2*exp(2*a-2*d)/b/(1+exp(2*b*x+2*d))^2-6*exp(2*a-2*d)/b/(1+exp(2*b*x
+2*d))+2*exp(2*a-2*d)*arctanh(exp(2*b*x+2*d))/b
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.68

$$\int e^{2(a+bx)}\text{csch}^5(d+bx)\text{sech}^3(d+bx)\,dx$$
$$= \frac{256e^{2a-2d}\left(-\frac{1}{64\left(1-e^{2(d+bx)}\right)^4}+\frac{5}{96\left(1-e^{2(d+bx)}\right)^3}-\frac{3}{64\left(1-e^{2(d+bx)}\right)^2}-\frac{1}{64\left(1-e^{2(d+bx)}\right)}+\frac{1}{128\left(1+e^{2(d+bx)}\right)^2}-\frac{3}{128\left(1+e^{2(d+bx}\right.}\right.}{b}$$

input
```
Integrate[E^(2*(a + b*x))*Csch[d + b*x]^5*Sech[d + b*x]^3,x]
```

output
```
(256*E^(2*a - 2*d)*(-1/64*1/(1 - E^(2*(d + b*x)))^4 + 5/(96*(1 - E^(2*(d +
 b*x)))^3) - 3/(64*(1 - E^(2*(d + b*x)))^2) - 1/(64*(1 - E^(2*(d + b*x))))
 + 1/(128*(1 + E^(2*(d + b*x)))^2) - 3/(128*(1 + E^(2*(d + b*x)))) + ArcTa
nh[E^(2*(d + b*x))]/128))/b
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.56, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2720, 27, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2(a+bx)}\text{csch}^5(bx+d)\text{sech}^3(bx+d)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{256e^{2a+9bx}}{\left(1-e^{2bx}\right)^5\left(1+e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{256e^{2a}\int \frac{e^{9bx}}{\left(1-e^{2bx}\right)^5\left(1+e^{2bx}\right)^3}de^{bx}}{b}$$

$$\Big\downarrow \text{354}$$

$$-\frac{128e^{2a}\int \frac{e^{4bx}}{(1-e^{2bx})^5(1+e^{2bx})^3}de^{2bx}}{b}$$

$$\Big\downarrow \text{99}$$

$$-\frac{128e^{2a}\int \left(-\frac{3}{64(1+e^{2bx})^2}+\frac{1}{32(1+e^{2bx})^3}+\frac{1}{64(-1+e^{2bx})}+\frac{1}{32(-1+e^{2bx})^2}-\frac{3}{16(-1+e^{2bx})^3}-\frac{5}{16(-1+e^{2bx})^4}-\frac{1}{8(-1+e^{2bx})^5}\right)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{128e^{2a}\left(-\frac{1}{64}\text{arctanh}\left(e^{2bx}\right)+\frac{1}{32(1-e^{2bx})}+\frac{3}{64(e^{2bx}+1)}+\frac{3}{32(1-e^{2bx})^2}-\frac{1}{64(e^{2bx}+1)^2}-\frac{5}{48(1-e^{2bx})^3}+\frac{1}{32(1-e^{2bx})^4}\right)}{b}$$

input
```
Int[E^(2*(a + b*x))*Csch[d + b*x]^5*Sech[d + b*x]^3,x]
```

output
```
(-128*E^(2*a)*(1/(32*(1 - E^(2*b*x))^4) - 5/(48*(1 - E^(2*b*x))^3) + 3/(32
*(1 - E^(2*b*x))^2) + 1/(32*(1 - E^(2*b*x))) - 1/(64*(1 + E^(2*b*x))^2) +
3/(64*(1 + E^(2*b*x))) - ArcTanh[E^(2*b*x)]/64))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.97

$$-\frac{2\left(3\,\mathrm{e}^{10bx+10a+10d}-6\,\mathrm{e}^{8bx+10a+8d}+62\,\mathrm{e}^{6bx+10a+6d}-22\,\mathrm{e}^{4bx+10a+4d}-29\,\mathrm{e}^{2bx+10a+2d}+16\,\mathrm{e}^{10a}\right)\mathrm{e}^{4a-2d}}{3\left(-\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)^{4}\left(\mathrm{e}^{2bx+2a+2d}+\mathrm{e}^{2a}\right)^{2}b}+\frac{\ln\left(\mathrm{e}\right.}{}$$

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)^5*sech(b*x+d)^3,x)
```

output
```
-2/3/(-exp(2*b*x+2*a+2*d)+exp(2*a))^4/(exp(2*b*x+2*a+2*d)+exp(2*a))^2/b*(3
*exp(10*b*x+10*a+10*d)-6*exp(8*b*x+10*a+8*d)+62*exp(6*b*x+10*a+6*d)-22*exp
(4*b*x+10*a+4*d)-29*exp(2*b*x+10*a+2*d)+16*exp(10*a))*exp(4*a-2*d)+ln(exp(
2*b*x+2*a)+exp(2*a-2*d))/b*exp(2*a-2*d)-ln(exp(2*b*x+2*a)-exp(2*a-2*d))/b*
exp(2*a-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4726 vs. $2(180) = 360$.

Time = 0.13 (sec) , antiderivative size = 4726, normalized size of antiderivative = 23.17

$$\int e^{2(a+bx)}\text{csch}^5(d+bx)\text{sech}^3(d+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^5*sech(b*x+d)^3,x, algorithm="fricas"
)
```

output
```
Too large to include
```

Sympy [F]

$$\int e^{2(a+bx)}\mathrm{csch}^5(d+bx)\mathrm{sech}^3(d+bx)\,dx = e^{2a}\int e^{2bx}\,\mathrm{csch}^5\,(bx+d)\,\mathrm{sech}^3\,(bx+d)\,dx$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)**5*sech(b*x+d)**3,x)
```

output
```
exp(2*a)*Integral(exp(2*b*x)*csch(b*x + d)**5*sech(b*x + d)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.03

$$\int e^{2(a+bx)}\mathrm{csch}^5(d+bx)\mathrm{sech}^3(d+bx)\,dx = -\frac{e^{(2a-2d)}\log\left(e^{(-bx-d)}+1\right)}{b} - \frac{e^{(2a-2d)}\log\left(e^{(-bx-d)}-1\right)}{b} + \frac{e^{(2a-2d)}\log\left(e^{(-2bx-2d)}+1\right)}{b} + \frac{2\left(35e^{(-2bx-2d)}+10e^{(-4bx-4d)}-2e^{(-6bx-6d)}-6e^{(-8bx-8d)}+3e^{(-10bx-10d)}-16\right)e^{(2a-2d)}}{3b(2e^{(-2bx-2d)}+e^{(-4bx-4d)}-4e^{(-6bx-6d)}+e^{(-8bx-8d)}+2e^{(-10bx-10d)}-e^{(-12bx-12d)}-1)}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^5*sech(b*x+d)^3,x, algorithm="maxima"
)
```

output
```
-e^(2*a - 2*d)*log(e^(-b*x - d) + 1)/b - e^(2*a - 2*d)*log(e^(-b*x - d) -
1)/b + e^(2*a - 2*d)*log(e^(-2*b*x - 2*d) + 1)/b + 2/3*(35*e^(-2*b*x - 2*d
) + 10*e^(-4*b*x - 4*d) - 2*e^(-6*b*x - 6*d) - 6*e^(-8*b*x - 8*d) + 3*e^(-
10*b*x - 10*d) - 16)*e^(2*a - 2*d)/(b*(2*e^(-2*b*x - 2*d) + e^(-4*b*x - 4*
d) - 4*e^(-6*b*x - 6*d) + e^(-8*b*x - 8*d) + 2*e^(-10*b*x - 10*d) - e^(-12
*b*x - 12*d) - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.75

$$\int e^{2(a+bx)}\operatorname{csch}^5(d+bx)\operatorname{sech}^3(d+bx)\,dx$$
$$= \frac{\left(12\,e^{(-2\,d)}\log\left(e^{(2\,bx+2\,d)}+1\right)-12\,e^{(-2\,d)}\log\left(\left|e^{(2\,bx+2\,d)}-1\right|\right)-\frac{6\,\left(3\,e^{(4\,bx+4\,d)}+18\,e^{(2\,bx+2\,d)}+11\right)e^{(-2\,d)}}{\left(e^{(2\,bx+2\,d)}+1\right)^2}+\frac{(25\,e^{(8\,b}{}\right.}{12\,b}$$

input
```
integrate(exp(2*b*x+2*a)*csch(b*x+d)^5*sech(b*x+d)^3,x, algorithm="giac")
```

output
```
1/12*(12*e^(-2*d)*log(e^(2*b*x + 2*d) + 1) - 12*e^(-2*d)*log(abs(e^(2*b*x
+ 2*d) - 1)) - 6*(3*e^(4*b*x + 4*d) + 18*e^(2*b*x + 2*d) + 11)*e^(-2*d)/(e
^(2*b*x + 2*d) + 1)^2 + (25*e^(8*b*x + 8*d) - 52*e^(6*b*x + 6*d) - 138*e^(
4*b*x + 4*d) + 172*e^(2*b*x + 2*d) - 55)*e^(-2*d)/(e^(2*b*x + 2*d) - 1)^4)
*e^(2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.40

$$\int e^{2(a+bx)}\operatorname{csch}^5(d+bx)\operatorname{sech}^3(d+bx)\,dx$$
$$= \frac{2\,{\mathrm{e}}^{2\,a-2\,d}}{b\,\left(2\,{\mathrm{e}}^{2\,d+2\,b\,x}+{\mathrm{e}}^{4\,d+4\,b\,x}+1\right)}-\frac{12\,{\mathrm{e}}^{2\,a-2\,d}}{b\,\left({\mathrm{e}}^{4\,d+4\,b\,x}-2\,{\mathrm{e}}^{2\,d+2\,b\,x}+1\right)}$$
$$-\frac{40\,{\mathrm{e}}^{2\,a-2\,d}}{3\,b\,\left(3\,{\mathrm{e}}^{2\,d+2\,b\,x}-3\,{\mathrm{e}}^{4\,d+4\,b\,x}+{\mathrm{e}}^{6\,d+6\,b\,x}-1\right)}$$
$$-\frac{4\,{\mathrm{e}}^{2\,a-2\,d}}{b\,\left(6\,{\mathrm{e}}^{4\,d+4\,b\,x}-4\,{\mathrm{e}}^{2\,d+2\,b\,x}-4\,{\mathrm{e}}^{6\,d+6\,b\,x}+{\mathrm{e}}^{8\,d+8\,b\,x}+1\right)}$$
$$+\frac{2\,\sqrt{{\mathrm{e}}^{4\,a-4\,d}}\,\mathrm{atan}\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}\,\sqrt{-b^2}}{b\,\sqrt{{\mathrm{e}}^{4\,a}\,{\mathrm{e}}^{-4\,d}}}\right)}{\sqrt{-b^2}}+\frac{4\,{\mathrm{e}}^{2\,a-2\,d}}{b\,\left({\mathrm{e}}^{2\,d+2\,b\,x}-1\right)}-\frac{6\,{\mathrm{e}}^{2\,a-2\,d}}{b\,\left({\mathrm{e}}^{2\,d+2\,b\,x}+1\right)}$$

input
```
int(exp(2*a + 2*b*x)/(cosh(d + b*x)^3*sinh(d + b*x)^5),x)
```

output
```
(2*exp(2*a - 2*d))/(b*(2*exp(2*d + 2*b*x) + exp(4*d + 4*b*x) + 1)) - (12*e
xp(2*a - 2*d))/(b*(exp(4*d + 4*b*x) - 2*exp(2*d + 2*b*x) + 1)) - (40*exp(2
*a - 2*d))/(3*b*(3*exp(2*d + 2*b*x) - 3*exp(4*d + 4*b*x) + exp(6*d + 6*b*x
) - 1)) - (4*exp(2*a - 2*d))/(b*(6*exp(4*d + 4*b*x) - 4*exp(2*d + 2*b*x) -
 4*exp(6*d + 6*b*x) + exp(8*d + 8*b*x) + 1)) + (2*exp(4*a - 4*d)^(1/2)*ata
n((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/(b*(exp(4*a)*exp(-4*d))^(1/2))))/(-b^
2)^(1/2) + (4*exp(2*a - 2*d))/(b*(exp(2*d + 2*b*x) - 1)) - (6*exp(2*a - 2*
d))/(b*(exp(2*d + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 606, normalized size of antiderivative = 2.97

$$\int e^{2(a+bx)}\text{csch}^5(d+bx)\text{sech}^3(d+bx)\,dx = \frac{e^{2a}\left(-35-3\log\left(e^{bx+d}-1\right)-3\log\left(e^{bx+d}+1\right)+6e^{2bx+2d}\log\left(e^{bx+d}-1\right)+6e^{2bx+2d}\log\left(e^{bx+d}+1\right)-3e^{8b}\right.}{}$$

input
```
int(exp(2*b*x+2*a)*csch(b*x+d)^5*sech(b*x+d)^3,x)
```

output
```
(e**(2*a)*( - 3*e**(12*b*x + 12*d)*log(e**(b*x + d) - 1) - 3*e**(12*b*x +
12*d)*log(e**(b*x + d) + 1) + 3*e**(12*b*x + 12*d)*log(e**(2*b*x + 2*d) +
1) - 3*e**(12*b*x + 12*d) + 6*e**(10*b*x + 10*d)*log(e**(b*x + d) - 1) + 6
*e**(10*b*x + 10*d)*log(e**(b*x + d) + 1) - 6*e**(10*b*x + 10*d)*log(e**(2
*b*x + 2*d) + 1) + 3*e**(8*b*x + 8*d)*log(e**(b*x + d) - 1) + 3*e**(8*b*x
+ 8*d)*log(e**(b*x + d) + 1) - 3*e**(8*b*x + 8*d)*log(e**(2*b*x + 2*d) + 1
) + 15*e**(8*b*x + 8*d) - 12*e**(6*b*x + 6*d)*log(e**(b*x + d) - 1) - 12*e
**(6*b*x + 6*d)*log(e**(b*x + d) + 1) + 12*e**(6*b*x + 6*d)*log(e**(2*b*x
+ 2*d) + 1) - 136*e**(6*b*x + 6*d) + 3*e**(4*b*x + 4*d)*log(e**(b*x + d) -
 1) + 3*e**(4*b*x + 4*d)*log(e**(b*x + d) + 1) - 3*e**(4*b*x + 4*d)*log(e*
*(2*b*x + 2*d) + 1) + 47*e**(4*b*x + 4*d) + 6*e**(2*b*x + 2*d)*log(e**(b*x
 + d) - 1) + 6*e**(2*b*x + 2*d)*log(e**(b*x + d) + 1) - 6*e**(2*b*x + 2*d)
*log(e**(2*b*x + 2*d) + 1) + 64*e**(2*b*x + 2*d) - 3*log(e**(b*x + d) - 1)
 - 3*log(e**(b*x + d) + 1) + 3*log(e**(2*b*x + 2*d) + 1) - 35))/(3*e**(2*d
)*b*(e**(12*b*x + 12*d) - 2*e**(10*b*x + 10*d) - e**(8*b*x + 8*d) + 4*e**(
6*b*x + 6*d) - e**(4*b*x + 4*d) - 2*e**(2*b*x + 2*d) + 1))
```

3.87 $\int e^x \mathbf{sech}(2x) \tanh(2x) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 88

$$\int e^x \text{sech}(2x) \tanh(2x) \, dx = -\frac{e^{3x}}{1+e^{4x}} - \frac{\arctan\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}e^x\right)}{2\sqrt{2}} - \frac{\text{arctanh}\left(\frac{\sqrt{2}e^x}{1+e^{2x}}\right)}{2\sqrt{2}}$$

output
```
-exp(3*x)/(1+exp(4*x))+1/4*arctan(-1+2^(1/2)*exp(x))*2^(1/2)+1/4*arctan(1+
2^(1/2)*exp(x))*2^(1/2)-1/4*arctanh(2^(1/2)*exp(x)/(1+exp(2*x)))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.48

$$\int e^x \text{sech}(2x) \tanh(2x) \, dx = \frac{2}{3} e^{3x} \left(\text{Hypergeometric2F1}\left(\frac{3}{4}, 1, \frac{7}{4}, -e^{4x}\right) - 2\,\text{Hypergeometric2F1}\left(\frac{3}{4}, 2, \frac{7}{4}, -e^{4x}\right)\right)$$

input
```
Integrate[E^x*Sech[2*x]*Tanh[2*x],x]
```

output
```
(2*E^(3*x)*(Hypergeometric2F1[3/4, 1, 7/4, -E^(4*x)] - 2*Hypergeometric2F1
[3/4, 2, 7/4, -E^(4*x)]))/3
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.44, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {2720, 27, 957, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \tanh(2x)\text{sech}(2x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int -\frac{2e^{2x}(1-e^{4x})}{(e^{4x}+1)^2}de^x$$

$$\Big\downarrow \text{27}$$

$$-2\int \frac{e^{2x}(1-e^{4x})}{(1+e^{4x})^2}de^x$$

$$\Big\downarrow \text{957}$$

$$-2\left(\frac{e^{3x}}{2(e^{4x}+1)}-\frac{1}{2}\int \frac{e^{2x}}{1+e^{4x}}de^x\right)$$

$$\Big\downarrow \text{826}$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x-\frac{1}{2}\int \frac{1+e^{2x}}{1+e^{4x}}de^x\right)+\frac{e^{3x}}{2(e^{4x}+1)}\right)$$

$$\Big\downarrow \text{1476}$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{1-\sqrt{2}e^x+e^{2x}}de^x-\frac{1}{2}\int \frac{1}{1+\sqrt{2}e^x+e^{2x}}de^x\right)+\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x\right)+\frac{e^{3x}}{2(e^{4x}+1)}\right)$$

$$\Big\downarrow \text{1082}$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-e^{2x}}d(1+\sqrt{2}e^x)}{\sqrt{2}}-\frac{\int \frac{1}{-1-e^{2x}}d(1-\sqrt{2}e^x)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x\right)+\frac{e^{3x}}{2(e^{4x}+1)}\right)$$

$$\Big\downarrow \text{ 217}$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)\right)+\frac{e^{3x}}{2\left(e^{4x}+1\right)}\right)$$

$$\Big\downarrow \text{ 1479}$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)\right)+\right.$$

$$\Big\downarrow \text{ 25}$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)\right)+\frac{e^{3}}{2\left(e^{4x}\right.}\right.$$

$$\Big\downarrow \text{ 27}$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}+\frac{1}{2}\int \frac{1+\sqrt{2}e^x}{1+\sqrt{2}e^x+e^{2x}}de^x\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)\right)+\right.$$

$$\Big\downarrow \text{ 1103}$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}-\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}\right)\right)\right.$$

input
```
Int[E^x*Sech[2*x]*Tanh[2*x],x]
```

output
```
-2*(E^(3*x)/(2*(1 + E^(4*x))) + ((ArcTan[1 - Sqrt[2]*E^x]/Sqrt[2] - ArcTan
[1 + Sqrt[2]*E^x]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*E^x + E^(2*x)]/Sqrt[2
] + Log[1 + Sqrt[2]*E^x + E^(2*x)]/(2*Sqrt[2]))/2)/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.45

method	result
risch	$-\frac{e^{3x}}{e^{4x}+1}+2\left(\sum_{\_R=\operatorname{RootOf}\left(4096\_Z^4+1\right)}\_R\ln\left(512\_R^3+e^x\right)\right)$
default	$\frac{\tanh\left(\frac{x}{2}\right)^3-3\tanh\left(\frac{x}{2}\right)^2-\tanh\left(\frac{x}{2}\right)-1}{\tanh\left(\frac{x}{2}\right)^4+6\tanh\left(\frac{x}{2}\right)^2+1}+\frac{\sqrt{2}\,\ln\left(\tanh\left(\frac{x}{2}\right)^2+3-2\sqrt{2}\right)}{8}-\frac{\left(\sqrt{2}-2\right)\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{2\sqrt{2}-2}\right)}{2\left(2\sqrt{2}-2\right)}-\frac{\sqrt{2}\,\ln\left(\tanh\left(\frac{x}{2}\right)^2+3+\right.}{8}$

input
```
int(exp(x)*sech(2*x)*tanh(2*x),x,method=_RETURNVERBOSE)
```

output
```
-exp(x)^3/(exp(x)^4+1)+2*sum(_R*ln(512*_R^3+exp(x)),_R=RootOf(4096*_Z^4+1)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(63) = 126$.

Time = 0.09 (sec) , antiderivative size = 365, normalized size of antiderivative = 4.15

$$\int e^x \operatorname{sech}(2x)\tanh(2x)\,dx =$$

$$-\frac{8\,\cosh\left(x\right)^{3}+24\,\cosh\left(x\right)^{2}\sinh\left(x\right)+24\,\cosh\left(x\right)\sinh\left(x\right)^{2}+8\,\sinh\left(x\right)^{3}-2\,\left(\sqrt{2}\cosh\left(x\right)^{4}+4\,\sqrt{2}\cosh}{}$$

input
```
integrate(exp(x)*sech(2*x)*tanh(2*x),x, algorithm="fricas")
```

output
```
-1/8*(8*cosh(x)^3 + 24*cosh(x)^2*sinh(x) + 24*cosh(x)*sinh(x)^2 + 8*sinh(x
)^3 - 2*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)^3*sinh(x) + 6*sqrt(2)*cosh(
x)^2*sinh(x)^2 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + sqrt(2)
)*arctan(sqrt(2)*cosh(x) + sqrt(2)*sinh(x) + 1) - 2*(sqrt(2)*cosh(x)^4 + 4
*sqrt(2)*cosh(x)^3*sinh(x) + 6*sqrt(2)*cosh(x)^2*sinh(x)^2 + 4*sqrt(2)*cos
h(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + sqrt(2))*arctan(sqrt(2)*cosh(x) + sqr
t(2)*sinh(x) - 1) + (sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)^3*sinh(x) + 6*s
qrt(2)*cosh(x)^2*sinh(x)^2 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)
^4 + sqrt(2))*log((sqrt(2) + 2*cosh(x))/(cosh(x) - sinh(x))) - (sqrt(2)*co
sh(x)^4 + 4*sqrt(2)*cosh(x)^3*sinh(x) + 6*sqrt(2)*cosh(x)^2*sinh(x)^2 + 4*
sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + sqrt(2))*log(-(sqrt(2) - 2
*cosh(x))/(cosh(x) - sinh(x))))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(
x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 1)
```

Sympy [F]

$$\int e^x \operatorname{sech}(2x)\tanh(2x)\,dx = \int e^{x}\tanh\left(2x\right)\operatorname{sech}\left(2x\right)dx$$

input
```
integrate(exp(x)*sech(2*x)*tanh(2*x),x)
```

output
```
Integral(exp(x)*tanh(2*x)*sech(2*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int e^x \operatorname{sech}(2x)\tanh(2x)\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) + \frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^x + e^{(2x)} + 1\right) + \frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^x + e^{(2x)} + 1\right) - \frac{e^{(3x)}}{e^{(4x)} + 1}$$

input
```
integrate(exp(x)*sech(2*x)*tanh(2*x),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
+ 1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - e^(3*x)/(e^(4*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int e^x \operatorname{sech}(2x)\tanh(2x)\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) + \frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^x + e^{(2x)} + 1\right) + \frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^x + e^{(2x)} + 1\right) - \frac{e^{(3x)}}{e^{(4x)} + 1}$$

input
```
integrate(exp(x)*sech(2*x)*tanh(2*x),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
+ 1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - e^(3*x)/(e^(4*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.03

$$\int e^x \operatorname{sech}(2x)\tanh(2x)\,dx = -\frac{e^{3x}}{e^{4x}+1} + \sqrt{2}\ln\left(1+\sqrt{2}\,e^x\left(-\frac{1}{2}-\frac{1}{2}i\right)\right)\left(\frac{1}{8}+\frac{1}{8}i\right) + \sqrt{2}\ln\left(1+\sqrt{2}\,e^x\left(-\frac{1}{2}+\frac{1}{2}i\right)\right)\left(\frac{1}{8}-\frac{1}{8}i\right) + \sqrt{2}\ln\left(1+\sqrt{2}\,e^x\left(\frac{1}{2}-\frac{1}{2}i\right)\right)\left(-\frac{1}{8}+\frac{1}{8}i\right) + \sqrt{2}\ln\left(1+\sqrt{2}\,e^x\left(\frac{1}{2}+\frac{1}{2}i\right)\right)\left(-\frac{1}{8}-\frac{1}{8}i\right)$$

input
```
int((tanh(2*x)*exp(x))/cosh(2*x),x)
```

output
```
2^(1/2)*log(1 - 2^(1/2)*exp(x)*(1/2 + 1i/2))*(1/8 + 1i/8) + 2^(1/2)*log(1
- 2^(1/2)*exp(x)*(1/2 - 1i/2))*(1/8 - 1i/8) - 2^(1/2)*log(2^(1/2)*exp(x)*(
1/2 - 1i/2) + 1)*(1/8 - 1i/8) - 2^(1/2)*log(2^(1/2)*exp(x)*(1/2 + 1i/2) +
1)*(1/8 + 1i/8) - exp(3*x)/(exp(4*x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.14

$$\int e^x \operatorname{sech}(2x)\tanh(2x)\,dx = \frac{2e^{4x}\sqrt{2}\,atan\left(\frac{2e^x-\sqrt{2}}{\sqrt{2}}\right) + 2\sqrt{2}\,atan\left(\frac{2e^x-\sqrt{2}}{\sqrt{2}}\right) + 2e^{4x}\sqrt{2}\,atan\left(\frac{2e^x+\sqrt{2}}{\sqrt{2}}\right) + 2\sqrt{2}\,atan\left(\frac{2e^x+\sqrt{2}}{\sqrt{2}}\right) + e^{4x}\sqrt{2}\log}{8}$$

input
```
int(exp(x)*sech(2*x)*tanh(2*x),x)
```

output
```
(2*e**(4*x)*sqrt(2)*atan((2*e**x - sqrt(2))/sqrt(2)) + 2*sqrt(2)*atan((2*e
**x - sqrt(2))/sqrt(2)) + 2*e**(4*x)*sqrt(2)*atan((2*e**x + sqrt(2))/sqrt(
2)) + 2*sqrt(2)*atan((2*e**x + sqrt(2))/sqrt(2)) + e**(4*x)*sqrt(2)*log(e*
*(2*x) - e**x*sqrt(2) + 1) - e**(4*x)*sqrt(2)*log(e**(2*x) + e**x*sqrt(2)
+ 1) - 8*e**(3*x) + sqrt(2)*log(e**(2*x) - e**x*sqrt(2) + 1) - sqrt(2)*log
(e**(2*x) + e**x*sqrt(2) + 1))/(8*(e**(4*x) + 1))
```

3.88 $\int e^x \operatorname{sech}^2(2x)\tanh(2x)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 101

$$\int e^x \operatorname{sech}^2(2x)\tanh(2x)\,dx = \frac{e^x}{(1+e^{4x})^2} - \frac{5e^x}{4(1+e^{4x})} - \frac{\arctan\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}e^x\right)}{8\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}e^x}{1+e^{2x}}\right)}{8\sqrt{2}}$$

output
```
exp(x)/(1+exp(4*x))^2-5*exp(x)/(4+4*exp(4*x))+1/16*arctan(-1+2^(1/2)*exp(x
))*2^(1/2)+1/16*arctan(1+2^(1/2)*exp(x))*2^(1/2)+1/16*arctanh(2^(1/2)*exp(
x)/(1+exp(2*x)))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.19

$$\int e^x \operatorname{sech}^2(2x)\tanh(2x)\,dx = \frac{1}{32}\left(\frac{32e^x}{(1+e^{4x})^2} - \frac{40e^x}{1+e^{4x}} - 2\sqrt{2}\arctan\left(1-\sqrt{2}e^x\right) + 2\sqrt{2}\arctan\left(1+\sqrt{2}e^x\right) - \sqrt{2}\log\left(1-\sqrt{2}e^x+e^{2x}\right) + \sqrt{2}\log\left(1+\sqrt{2}e^x+e^{2x}\right)\right)$$

input
```
Integrate[E^x*Sech[2*x]^2*Tanh[2*x],x]
```

output
```
((32*E^x)/(1 + E^(4*x))^2 - (40*E^x)/(1 + E^(4*x)) - 2*Sqrt[2]*ArcTan[1 -
Sqrt[2]*E^x] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^x] - Sqrt[2]*Log[1 - Sqrt[2]
*E^x + E^(2*x)] + Sqrt[2]*Log[1 + Sqrt[2]*E^x + E^(2*x)])/32
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.47, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2720, 27, 957, 817, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \tanh(2x)\text{sech}^2(2x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int -\frac{4e^{4x}(1-e^{4x})}{(e^{4x}+1)^3}de^x$$

$$\Big\downarrow \text{27}$$

$$-4\int \frac{e^{4x}(1-e^{4x})}{(1+e^{4x})^3}de^x$$

$$\Big\downarrow \text{957}$$

$$-4\left(\frac{e^{5x}}{4(e^{4x}+1)^2}-\frac{1}{4}\int \frac{e^{4x}}{(1+e^{4x})^2}de^x\right)$$

$$\Big\downarrow \text{817}$$

$$-4\left(\frac{1}{4}\left(\frac{e^x}{4(e^{4x}+1)}-\frac{1}{4}\int \frac{1}{1+e^{4x}}de^x\right)+\frac{e^{5x}}{4(e^{4x}+1)^2}\right)$$

$$\Big\downarrow \text{755}$$

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\left(-\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x-\frac{1}{2}\int \frac{1+e^{2x}}{1+e^{4x}}de^x\right)+\frac{e^x}{4(e^{4x}+1)}\right)+\frac{e^{5x}}{4(e^{4x}+1)^2}\right)$$

$$\Big\downarrow \text{1476}$$

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{1-\sqrt{2}e^x+e^{2x}}de^x-\frac{1}{2}\int\frac{1}{1+\sqrt{2}e^x+e^{2x}}de^x\right)-\frac{1}{2}\int\frac{1-e^{2x}}{1+e^{4x}}de^x\right)+\frac{e^x}{4\left(e^{4x}+1\right)}\right)+\frac{}{4(}\right.$$

$$\Big\downarrow \text{1082}$$

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-e^{2x}}d\left(1+\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-e^{2x}}d\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-e^{2x}}{1+e^{4x}}de^x\right)+\frac{e^x}{4\left(e^{4x}+1\right)}\right)+\frac{e^{5x}}{4\left(e^{4x}+\right.}\right.$$

$$\Big\downarrow \text{217}$$

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-e^{2x}}{1+e^{4x}}de^x\right)+\frac{e^x}{4\left(e^{4x}+1\right)}\right)+\frac{e^{5x}}{4\left(e^{4x}+1\right)^2}\right)$$

$$\Big\downarrow \text{1479}$$

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)\right)\right.\right.$$

$$\Big\downarrow \text{25}$$

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)\right)+\right.\right.$$

$$\Big\downarrow \text{27}$$

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}-\frac{1}{2}\int\frac{1+\sqrt{2}e^x}{1+\sqrt{2}e^x+e^{2x}}de^x\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{1103}$$

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}\right.\right.\right.\right.$$

input
```
Int[E^x*Sech[2*x]^2*Tanh[2*x],x]
```

output
```
-4*(E^(5*x)/(4*(1 + E^(4*x))^2) + (E^x/(4*(1 + E^(4*x))) + ((ArcTan[1 - Sq
rt[2]*E^x]/Sqrt[2] - ArcTan[1 + Sqrt[2]*E^x]/Sqrt[2])/2 + (Log[1 - Sqrt[2]
*E^x + E^(2*x)]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/(2*Sqrt[2]))/
2)/4)/4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 9.87 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.44

method	result
risch	$-\frac{e^{x} \left(5 e^{4 x}+1\right)}{4 \left(e^{4 x}+1\right)^{2}}+4 \left(\sum_{\_R =\operatorname{RootOf}\left(16777216 \_Z^{4}+1\right)}\_R \ln \left(e^{x}+64 \_R \right)\right)$
default	$\frac{\frac{\tanh \left(\frac{x}{2}\right)^{7}}{4}-\frac{17 \tanh \left(\frac{x}{2}\right)^{6}}{4}-\frac{11 \tanh \left(\frac{x}{2}\right)^{5}}{4}-\frac{57 \tanh \left(\frac{x}{2}\right)^{4}}{4}+\frac{11 \tanh \left(\frac{x}{2}\right)^{3}}{4}-\frac{19 \tanh \left(\frac{x}{2}\right)^{2}}{4}-\frac{\tanh \left(\frac{x}{2}\right)}{4}-\frac{3}{4}}{\left(\tanh \left(\frac{x}{2}\right)^{4}+6 \tanh \left(\frac{x}{2}\right)^{2}+1\right)^{2}}+\frac{\sqrt{2}\, \ln \left(\tanh \left(\frac{x}{2}\right)^{2}+3+2 \sqrt{2}\right)}{32}$ -

input
```
int(exp(x)*sech(2*x)^2*tanh(2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*exp(x)*(5*exp(4*x)+1)/(exp(4*x)+1)^2+4*sum(_R*ln(exp(x)+64*_R),_R=Roo
tOf(16777216*_Z^4+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 854 vs. $2(72) = 144$.

Time = 0.09 (sec) , antiderivative size = 854, normalized size of antiderivative = 8.46

$$\int e^{x}\operatorname{sech}^{2}(2x)\tanh(2x)\,dx = \text{Too large to display}$$

input
```
integrate(exp(x)*sech(2*x)^2*tanh(2*x),x, algorithm="fricas")
```

output
```
-1/32*(40*cosh(x)^5 + 400*cosh(x)^3*sinh(x)^2 + 400*cosh(x)^2*sinh(x)^3 +
200*cosh(x)*sinh(x)^4 + 40*sinh(x)^5 - 2*(sqrt(2)*cosh(x)^8 + 56*sqrt(2)*c
osh(x)^3*sinh(x)^5 + 28*sqrt(2)*cosh(x)^2*sinh(x)^6 + 8*sqrt(2)*cosh(x)*si
nh(x)^7 + sqrt(2)*sinh(x)^8 + 2*(35*sqrt(2)*cosh(x)^4 + sqrt(2))*sinh(x)^4
 + 2*sqrt(2)*cosh(x)^4 + 8*(7*sqrt(2)*cosh(x)^5 + sqrt(2)*cosh(x))*sinh(x)
^3 + 4*(7*sqrt(2)*cosh(x)^6 + 3*sqrt(2)*cosh(x)^2)*sinh(x)^2 + 8*(sqrt(2)*
cosh(x)^7 + sqrt(2)*cosh(x)^3)*sinh(x) + sqrt(2))*arctan(sqrt(2)*cosh(x) +
 sqrt(2)*sinh(x) + 1) - 2*(sqrt(2)*cosh(x)^8 + 56*sqrt(2)*cosh(x)^3*sinh(x
)^5 + 28*sqrt(2)*cosh(x)^2*sinh(x)^6 + 8*sqrt(2)*cosh(x)*sinh(x)^7 + sqrt(
2)*sinh(x)^8 + 2*(35*sqrt(2)*cosh(x)^4 + sqrt(2))*sinh(x)^4 + 2*sqrt(2)*co
sh(x)^4 + 8*(7*sqrt(2)*cosh(x)^5 + sqrt(2)*cosh(x))*sinh(x)^3 + 4*(7*sqrt(
2)*cosh(x)^6 + 3*sqrt(2)*cosh(x)^2)*sinh(x)^2 + 8*(sqrt(2)*cosh(x)^7 + sqr
t(2)*cosh(x)^3)*sinh(x) + sqrt(2))*arctan(sqrt(2)*cosh(x) + sqrt(2)*sinh(x
) - 1) - (sqrt(2)*cosh(x)^8 + 56*sqrt(2)*cosh(x)^3*sinh(x)^5 + 28*sqrt(2)*
cosh(x)^2*sinh(x)^6 + 8*sqrt(2)*cosh(x)*sinh(x)^7 + sqrt(2)*sinh(x)^8 + 2*
(35*sqrt(2)*cosh(x)^4 + sqrt(2))*sinh(x)^4 + 2*sqrt(2)*cosh(x)^4 + 8*(7*sq
rt(2)*cosh(x)^5 + sqrt(2)*cosh(x))*sinh(x)^3 + 4*(7*sqrt(2)*cosh(x)^6 + 3*
sqrt(2)*cosh(x)^2)*sinh(x)^2 + 8*(sqrt(2)*cosh(x)^7 + sqrt(2)*cosh(x)^3)*s
inh(x) + sqrt(2))*log((sqrt(2) + 2*cosh(x))/(cosh(x) - sinh(x))) + (sqrt(2
)*cosh(x)^8 + 56*sqrt(2)*cosh(x)^3*sinh(x)^5 + 28*sqrt(2)*cosh(x)^2*sin...
```

Sympy [F]

$$\int e^x \text{sech}^2(2x) \tanh(2x)\,dx = \int e^x \tanh(2x) \operatorname{sech}^2(2x)\,dx$$

input
```
integrate(exp(x)*sech(2*x)**2*tanh(2*x),x)
```

output
```
Integral(exp(x)*tanh(2*x)*sech(2*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\int e^x \operatorname{sech}^2(2x)\tanh(2x)\,dx = \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,e^x\right)\right) + \frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,e^x\right)\right) + \frac{1}{32}\sqrt{2}\log\left(\sqrt{2}e^x + e^{(2\,x)} + 1\right) - \frac{1}{32}\sqrt{2}\log\left(-\sqrt{2}e^x + e^{(2\,x)} + 1\right) - \frac{5\,e^{(5\,x)} + e^x}{4\left(e^{(8\,x)} + 2\,e^{(4\,x)} + 1\right)}$$

input
```
integrate(exp(x)*sech(2*x)^2*tanh(2*x),x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/16*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) +
1) - 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/4*(5*e^(5*x) + e^x)/
(e^(8*x) + 2*e^(4*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\int e^x \operatorname{sech}^2(2x)\tanh(2x)\,dx = \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,e^x\right)\right) + \frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,e^x\right)\right) + \frac{1}{32}\sqrt{2}\log\left(\sqrt{2}e^x + e^{(2\,x)} + 1\right) - \frac{1}{32}\sqrt{2}\log\left(-\sqrt{2}e^x + e^{(2\,x)} + 1\right) - \frac{5\,e^{(5\,x)} + e^x}{4\left(e^{(4\,x)} + 1\right)^2}$$

input
```
integrate(exp(x)*sech(2*x)^2*tanh(2*x),x, algorithm="giac")
```

output
```
1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/16*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) +
1) - 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/4*(5*e^(5*x) + e^x)/
(e^(4*x) + 1)^2
```

Mupad [B] (verification not implemented)

Time = 2.81 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.21

$$\begin{aligned}\int e^x \operatorname{sech}^2(2x)\tanh(2x)\,dx = &-\frac{\frac{e^{5x}}{2}-\frac{e^x}{2}}{2\,e^{4x}+e^{8x}+1}-\frac{3\,e^x}{4\,(e^{4x}+1)}\\&+\sqrt{2}\,\ln\left(-\frac{e^x}{4}+\sqrt{2}\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)\right)\left(\frac{1}{32}+\frac{1}{32}\mathrm{i}\right)\\&+\sqrt{2}\,\ln\left(-\frac{e^x}{4}+\sqrt{2}\left(-\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)\right)\left(\frac{1}{32}-\frac{1}{32}\mathrm{i}\right)\\&+\sqrt{2}\,\ln\left(-\frac{e^x}{4}+\sqrt{2}\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)\right)\left(-\frac{1}{32}+\frac{1}{32}\mathrm{i}\right)\\&+\sqrt{2}\,\ln\left(-\frac{e^x}{4}+\sqrt{2}\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)\right)\left(-\frac{1}{32}-\frac{1}{32}\mathrm{i}\right)\end{aligned}$$

input
```
int((tanh(2*x)*exp(x))/cosh(2*x)^2,x)
```

output
```
2^(1/2)*log(- exp(x)/4 - 2^(1/2)*(1/8 + 1i/8))*(1/32 + 1i/32) - (3*exp(x))
/(4*(exp(4*x) + 1)) - (exp(5*x)/2 - exp(x)/2)/(2*exp(4*x) + exp(8*x) + 1)
+ 2^(1/2)*log(- exp(x)/4 - 2^(1/2)*(1/8 - 1i/8))*(1/32 - 1i/32) - 2^(1/2)*
log(2^(1/2)*(1/8 - 1i/8) - exp(x)/4)*(1/32 - 1i/32) - 2^(1/2)*log(2^(1/2)*
(1/8 + 1i/8) - exp(x)/4)*(1/32 + 1i/32)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.92

$$\int e^x \operatorname{sech}^2(2x) \tanh(2x)\, dx$$

$$= \frac{2e^{8x}\sqrt{2}\, atan\left(\frac{2e^x-\sqrt{2}}{\sqrt{2}}\right) + 4e^{4x}\sqrt{2}\, atan\left(\frac{2e^x-\sqrt{2}}{\sqrt{2}}\right) + 2\sqrt{2}\, atan\left(\frac{2e^x-\sqrt{2}}{\sqrt{2}}\right) + 2e^{8x}\sqrt{2}\, atan\left(\frac{2e^x+\sqrt{2}}{\sqrt{2}}\right) + 4e^{4x}\sqrt{2}}{}$$

input
```
int(exp(x)*sech(2*x)^2*tanh(2*x),x)
```

output
```
(2*e**(8*x)*sqrt(2)*atan((2*e**x - sqrt(2))/sqrt(2)) + 4*e**(4*x)*sqrt(2)*
atan((2*e**x - sqrt(2))/sqrt(2)) + 2*sqrt(2)*atan((2*e**x - sqrt(2))/sqrt(
2)) + 2*e**(8*x)*sqrt(2)*atan((2*e**x + sqrt(2))/sqrt(2)) + 4*e**(4*x)*sqr
t(2)*atan((2*e**x + sqrt(2))/sqrt(2)) + 2*sqrt(2)*atan((2*e**x + sqrt(2))/
sqrt(2)) - e**(8*x)*sqrt(2)*log(e**(2*x) - e**x*sqrt(2) + 1) + e**(8*x)*sq
rt(2)*log(e**(2*x) + e**x*sqrt(2) + 1) - 40*e**(5*x) - 2*e**(4*x)*sqrt(2)*
log(e**(2*x) - e**x*sqrt(2) + 1) + 2*e**(4*x)*sqrt(2)*log(e**(2*x) + e**x*
sqrt(2) + 1) - 8*e**x - sqrt(2)*log(e**(2*x) - e**x*sqrt(2) + 1) + sqrt(2)
*log(e**(2*x) + e**x*sqrt(2) + 1))/(32*(e**(8*x) + 2*e**(4*x) + 1))
```

3.89 $\int e^x \mathbf{sech}(2x) \tanh^2(2x) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 105

$$\int e^x \operatorname{sech}(2x) \tanh^2(2x) \, dx = \frac{e^{3x}}{(1+e^{4x})^2} - \frac{3e^{3x}}{4(1+e^{4x})} - \frac{5\arctan\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}} + \frac{5\arctan\left(1+\sqrt{2}e^x\right)}{8\sqrt{2}} - \frac{5\operatorname{arctanh}\left(\frac{\sqrt{2}e^x}{1+e^{2x}}\right)}{8\sqrt{2}}$$

output
```
exp(3*x)/(1+exp(4*x))^2-3*exp(3*x)/(4+4*exp(4*x))+5/16*arctan(-1+2^(1/2)*e
xp(x))*2^(1/2)+5/16*arctan(1+2^(1/2)*exp(x))*2^(1/2)-5/16*arctanh(2^(1/2)*
exp(x)/(1+exp(2*x)))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.55

$$\int e^x \operatorname{sech}(2x) \tanh^2(2x) \, dx = \frac{e^{3x} - 3e^{7x}}{4(1+e^{4x})^2} - \frac{5}{16} \text{RootSum}\left[1+\#1^4\&, \frac{x - \log(e^x - \#1)}{\#1}\&\right]$$

input
```
Integrate[E^x*Sech[2*x]*Tanh[2*x]^2,x]
```

output
```
(E^(3*x) - 3*E^(7*x))/(4*(1 + E^(4*x))^2) - (5*RootSum[1 + #1^4 & , (x - L
og[E^x - #1])/#1 & ])/16
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.43, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {2720, 27, 963, 27, 957, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \tanh^2(2x)\text{sech}(2x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{2e^{2x}(1-e^{4x})^2}{(e^{4x}+1)^3}de^x$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{e^{2x}(1-e^{4x})^2}{(1+e^{4x})^3}de^x$$

$$\Big\downarrow \text{963}$$

$$2\left(\frac{e^{3x}}{2(e^{4x}+1)^2}-\frac{1}{8}\int \frac{4e^{2x}(1-2e^{4x})}{(1+e^{4x})^2}de^x\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{e^{3x}}{2(e^{4x}+1)^2}-\frac{1}{2}\int \frac{e^{2x}(1-2e^{4x})}{(1+e^{4x})^2}de^x\right)$$

$$\Big\downarrow \text{957}$$

$$2\left(\frac{1}{2}\left(\frac{5}{4}\int \frac{e^{2x}}{1+e^{4x}}de^x-\frac{3e^{3x}}{4(e^{4x}+1)}\right)+\frac{e^{3x}}{2(e^{4x}+1)^2}\right)$$

$$\Big\downarrow \text{826}$$

$$2\left(\frac{1}{2}\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{1+e^{2x}}{1+e^{4x}}de^x-\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x\right)-\frac{3e^{3x}}{4(e^{4x}+1)}\right)+\frac{e^{3x}}{2(e^{4x}+1)^2}\right)$$

$$\Big\downarrow \text{1476}$$

$$2\left(\frac{1}{2}\left(\frac{5}{4}\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{1-\sqrt{2}e^x+e^{2x}}de^x+\frac{1}{2}\int\frac{1}{1+\sqrt{2}e^x+e^{2x}}de^x\right)-\frac{1}{2}\int\frac{1-e^{2x}}{1+e^{4x}}de^x\right)-\frac{3e^{3x}}{4\left(e^{4x}+1\right)}\right)+\frac{e^{3x}}{2\left(e^{4x}\right.}$$

$$\Big\downarrow \text{1082}$$

$$2\left(\frac{1}{2}\left(\frac{5}{4}\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-e^{2x}}d\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-e^{2x}}d\left(1+\sqrt{2}e^x\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-e^{2x}}{1+e^{4x}}de^x\right)-\frac{3e^{3x}}{4\left(e^{4x}+1\right)}\right)+\frac{e^{3x}}{2\left(e^{4x}+1\right.}$$

$$\Big\downarrow \text{217}$$

$$2\left(\frac{1}{2}\left(\frac{5}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-e^{2x}}{1+e^{4x}}de^x\right)-\frac{3e^{3x}}{4\left(e^{4x}+1\right)}\right)+\frac{e^{3x}}{2\left(e^{4x}+1\right)^2}\right)$$

$$\Big\downarrow \text{1479}$$

$$2\left(\frac{1}{2}\left(\frac{5}{4}\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)-\right.\right.$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{1}{2}\left(\frac{5}{4}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)-\frac{}{4}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{1}{2}\left(\frac{5}{4}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}-\frac{1}{2}\int\frac{1+\sqrt{2}e^x}{1+\sqrt{2}e^x+e^{2x}}de^x\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right.\right.\right.$$

$$\Big\downarrow \text{1103}$$

$$2\left(\frac{1}{2}\left(\frac{5}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}\right)\right.\right.\right.$$

input
```
Int[E^x*Sech[2*x]*Tanh[2*x]^2,x]
```

output
```
2*(E^(3*x)/(2*(1 + E^(4*x))^2) + ((-3*E^(3*x))/(4*(1 + E^(4*x))) + (5*((-(
ArcTan[1 - Sqrt[2]*E^x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*E^x]/Sqrt[2])/2 + (L
og[1 - Sqrt[2]*E^x + E^(2*x)]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]
/(2*Sqrt[2]))/2))/4)/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 963
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Simp[1/(a*b^2*n*(p + 1))   Int[(e*x)^m*(a + b*x
^n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.46

method	result
risch	$-\frac{e^{3x}(3e^{4x}-1)}{4(e^{4x}+1)^2}+2\left(\sum_{\_R=\mathrm{RootOf}(1048576\_Z^4+625)}\_R\ln\left(e^x+\frac{32768\_R^3}{125}\right)\right)$
default	$\frac{\frac{5\tanh(\frac{x}{2})^7}{4}+\frac{5\tanh(\frac{x}{2})^6}{4}+\frac{9\tanh(\frac{x}{2})^5}{4}-\frac{19\tanh(\frac{x}{2})^4}{4}-\frac{9\tanh(\frac{x}{2})^3}{4}-\frac{17\tanh(\frac{x}{2})^2}{4}-\frac{5\tanh(\frac{x}{2})}{4}-\frac{1}{4}}{\left(\tanh(\frac{x}{2})^4+6\tanh(\frac{x}{2})^2+1\right)^2}+\frac{5\sqrt{2}\ln\left(\tanh(\frac{x}{2})^2+3-2\sqrt{2}\right)}{32}$

input
```
int(exp(x)*sech(2*x)*tanh(2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*exp(x)^3*(3*exp(x)^4-1)/(exp(x)^4+1)^2+2*sum(_R*ln(exp(x)+32768/125*_
R^3),_R=RootOf(1048576*_Z^4+625))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 893 vs. 2(76) = 152.

Time = 0.12 (sec) , antiderivative size = 893, normalized size of antiderivative = 8.50

$$\int e^x\operatorname{sech}(2x)\tanh^2(2x)\,dx=\text{Too large to display}$$

input
```
integrate(exp(x)*sech(2*x)*tanh(2*x)^2,x, algorithm="fricas")
```

output
```
-1/32*(24*cosh(x)^7 + 840*cosh(x)^3*sinh(x)^4 + 504*cosh(x)^2*sinh(x)^5 +
168*cosh(x)*sinh(x)^6 + 24*sinh(x)^7 + 8*(105*cosh(x)^4 - 1)*sinh(x)^3 - 8
*cosh(x)^3 + 24*(21*cosh(x)^5 - cosh(x))*sinh(x)^2 - 10*(sqrt(2)*cosh(x)^8
 + 56*sqrt(2)*cosh(x)^3*sinh(x)^5 + 28*sqrt(2)*cosh(x)^2*sinh(x)^6 + 8*sqr
t(2)*cosh(x)*sinh(x)^7 + sqrt(2)*sinh(x)^8 + 2*(35*sqrt(2)*cosh(x)^4 + sqr
t(2))*sinh(x)^4 + 2*sqrt(2)*cosh(x)^4 + 8*(7*sqrt(2)*cosh(x)^5 + sqrt(2)*c
osh(x))*sinh(x)^3 + 4*(7*sqrt(2)*cosh(x)^6 + 3*sqrt(2)*cosh(x)^2)*sinh(x)^
2 + 8*(sqrt(2)*cosh(x)^7 + sqrt(2)*cosh(x)^3)*sinh(x) + sqrt(2))*arctan(sq
rt(2)*cosh(x) + sqrt(2)*sinh(x) + 1) - 10*(sqrt(2)*cosh(x)^8 + 56*sqrt(2)*
cosh(x)^3*sinh(x)^5 + 28*sqrt(2)*cosh(x)^2*sinh(x)^6 + 8*sqrt(2)*cosh(x)*s
inh(x)^7 + sqrt(2)*sinh(x)^8 + 2*(35*sqrt(2)*cosh(x)^4 + sqrt(2))*sinh(x)^
4 + 2*sqrt(2)*cosh(x)^4 + 8*(7*sqrt(2)*cosh(x)^5 + sqrt(2)*cosh(x))*sinh(x
)^3 + 4*(7*sqrt(2)*cosh(x)^6 + 3*sqrt(2)*cosh(x)^2)*sinh(x)^2 + 8*(sqrt(2)
*cosh(x)^7 + sqrt(2)*cosh(x)^3)*sinh(x) + sqrt(2))*arctan(sqrt(2)*cosh(x)
+ sqrt(2)*sinh(x) - 1) + 5*(sqrt(2)*cosh(x)^8 + 56*sqrt(2)*cosh(x)^3*sinh(
x)^5 + 28*sqrt(2)*cosh(x)^2*sinh(x)^6 + 8*sqrt(2)*cosh(x)*sinh(x)^7 + sqrt
(2)*sinh(x)^8 + 2*(35*sqrt(2)*cosh(x)^4 + sqrt(2))*sinh(x)^4 + 2*sqrt(2)*c
osh(x)^4 + 8*(7*sqrt(2)*cosh(x)^5 + sqrt(2)*cosh(x))*sinh(x)^3 + 4*(7*sqrt
(2)*cosh(x)^6 + 3*sqrt(2)*cosh(x)^2)*sinh(x)^2 + 8*(sqrt(2)*cosh(x)^7 + sq
rt(2)*cosh(x)^3)*sinh(x) + sqrt(2))*log((sqrt(2) + 2*cosh(x))/(cosh(x) ...
```

Sympy [F]

$$\int e^x \mathrm{sech}(2x)\tanh^2(2x)\,dx = \int e^x \tanh^2(2x)\,\mathrm{sech}\,(2x)\,dx$$

input
```
integrate(exp(x)*sech(2*x)*tanh(2*x)**2,x)
```

output
```
Integral(exp(x)*tanh(2*x)**2*sech(2*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int e^x \operatorname{sech}(2x)\tanh^2(2x)\,dx = \frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) + \frac{5}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{5}{32}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right) + \frac{5}{32}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right) - \frac{3e^{(7x)}-e^{(3x)}}{4\left(e^{(8x)}+2e^{(4x)}+1\right)}$$

input
```
integrate(exp(x)*sech(2*x)*tanh(2*x)^2,x, algorithm="maxima")
```

output
```
5/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 5/16*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 5/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) +
1) + 5/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/4*(3*e^(7*x) - e^(3*
x))/(e^(8*x) + 2*e^(4*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.94

$$\int e^x \operatorname{sech}(2x)\tanh^2(2x)\,dx = \frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) + \frac{5}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{5}{32}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right) + \frac{5}{32}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right) - \frac{3e^{(7x)}-e^{(3x)}}{4\left(e^{(4x)}+1\right)^2}$$

input
```
integrate(exp(x)*sech(2*x)*tanh(2*x)^2,x, algorithm="giac")
```

output
```
5/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 5/16*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 5/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) +
1) + 5/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/4*(3*e^(7*x) - e^(3*
x))/(e^(4*x) + 1)^2
```

Mupad [B] (verification not implemented)

Time = 2.85 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.07

$$\int e^x \text{sech}(2x) \tanh^2(2x)\, dx = \frac{e^{3x}}{2e^{4x} + e^{8x} + 1} - \frac{3e^{3x}}{4(e^{4x} + 1)} + \sqrt{2} \ln\left(\frac{25}{16} + \sqrt{2}\, e^x \left(-\frac{25}{32} - \frac{25}{32}i\right)\right) \left(\frac{5}{32} + \frac{5}{32}i\right) + \sqrt{2} \ln\left(\frac{25}{16} + \sqrt{2}\, e^x \left(-\frac{25}{32} + \frac{25}{32}i\right)\right) \left(\frac{5}{32} - \frac{5}{32}i\right) + \sqrt{2} \ln\left(\frac{25}{16} + \sqrt{2}\, e^x \left(\frac{25}{32} - \frac{25}{32}i\right)\right) \left(-\frac{5}{32} + \frac{5}{32}i\right) + \sqrt{2} \ln\left(\frac{25}{16} + \sqrt{2}\, e^x \left(\frac{25}{32} + \frac{25}{32}i\right)\right) \left(-\frac{5}{32} - \frac{5}{32}i\right)$$

input
```
int((tanh(2*x)^2*exp(x))/cosh(2*x),x)
```

output
```
2^(1/2)*log(25/16 - 2^(1/2)*exp(x)*(25/32 + 25i/32))*(5/32 + 5i/32) + 2^(1
/2)*log(25/16 - 2^(1/2)*exp(x)*(25/32 - 25i/32))*(5/32 - 5i/32) - 2^(1/2)*
log(2^(1/2)*exp(x)*(25/32 - 25i/32) + 25/16)*(5/32 - 5i/32) - 2^(1/2)*log(
2^(1/2)*exp(x)*(25/32 + 25i/32) + 25/16)*(5/32 + 5i/32) + exp(3*x)/(2*exp(
4*x) + exp(8*x) + 1) - (3*exp(3*x))/(4*(exp(4*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.85

$$\int e^x \operatorname{sech}(2x)\tanh^2(2x)\,dx$$
$$= \frac{10e^{8x}\sqrt{2}\,atan\left(\frac{2e^x-\sqrt{2}}{\sqrt{2}}\right)+20e^{4x}\sqrt{2}\,atan\left(\frac{2e^x-\sqrt{2}}{\sqrt{2}}\right)+10\sqrt{2}\,atan\left(\frac{2e^x-\sqrt{2}}{\sqrt{2}}\right)+10e^{8x}\sqrt{2}\,atan\left(\frac{2e^x+\sqrt{2}}{\sqrt{2}}\right)+20}{}$$

input
```
int(exp(x)*sech(2*x)*tanh(2*x)^2,x)
```

output
```
(10*e**(8*x)*sqrt(2)*atan((2*e**x - sqrt(2))/sqrt(2)) + 20*e**(4*x)*sqrt(2
)*atan((2*e**x - sqrt(2))/sqrt(2)) + 10*sqrt(2)*atan((2*e**x - sqrt(2))/sq
rt(2)) + 10*e**(8*x)*sqrt(2)*atan((2*e**x + sqrt(2))/sqrt(2)) + 20*e**(4*x
)*sqrt(2)*atan((2*e**x + sqrt(2))/sqrt(2)) + 10*sqrt(2)*atan((2*e**x + sqr
t(2))/sqrt(2)) + 5*e**(8*x)*sqrt(2)*log(e**(2*x) - e**x*sqrt(2) + 1) - 5*e
**(8*x)*sqrt(2)*log(e**(2*x) + e**x*sqrt(2) + 1) - 24*e**(7*x) + 10*e**(4*
x)*sqrt(2)*log(e**(2*x) - e**x*sqrt(2) + 1) - 10*e**(4*x)*sqrt(2)*log(e**(
2*x) + e**x*sqrt(2) + 1) + 8*e**(3*x) + 5*sqrt(2)*log(e**(2*x) - e**x*sqrt
(2) + 1) - 5*sqrt(2)*log(e**(2*x) + e**x*sqrt(2) + 1))/(32*(e**(8*x) + 2*e
**(4*x) + 1))
```

3.90 $\int e^x \mathbf{sech}^2(2x)\tanh^2(2x)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 120

$$\int e^x\operatorname{sech}^2(2x)\tanh^2(2x)\,dx = -\frac{4e^x}{3(1+e^{4x})^3}+\frac{13e^x}{6(1+e^{4x})^2} -\frac{29e^x}{24(1+e^{4x})}-\frac{3\arctan\left(1-\sqrt{2}e^x\right)}{16\sqrt{2}} +\frac{3\arctan\left(1+\sqrt{2}e^x\right)}{16\sqrt{2}}+\frac{3\operatorname{arctanh}\left(\frac{\sqrt{2}e^x}{1+e^{2x}}\right)}{16\sqrt{2}}$$

output
```
-4/3*exp(x)/(1+exp(4*x))^3+13/6*exp(x)/(1+exp(4*x))^2-29*exp(x)/(24+24*exp
(4*x))+3/32*arctan(-1+2^(1/2)*exp(x))*2^(1/2)+3/32*arctan(1+2^(1/2)*exp(x)
)*2^(1/2)+3/32*arctanh(2^(1/2)*exp(x)/(1+exp(2*x)))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.53

$$\int e^x\operatorname{sech}^2(2x)\tanh^2(2x)\,dx = \frac{1}{96}\left(-\frac{4e^x(9+6e^{4x}+29e^{8x})}{(1+e^{4x})^3} - 9\text{RootSum}\left[1+\#1^4\&,\frac{x-\log\left(e^x-\#1\right)}{\#1^3}\&\right]\right)$$

input
```
Integrate[E^x*Sech[2*x]^2*Tanh[2*x]^2,x]
```

output
```
((-4*E^x*(9 + 6*E^(4*x) + 29*E^(8*x)))/(1 + E^(4*x))^3 - 9*RootSum[1 + #1^
4 & , (x - Log[E^x - #1])/#1^3 & ])/96
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.42, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {2720, 27, 963, 27, 957, 817, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \tanh^2(2x)\text{sech}^2(2x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{4e^{4x}\left(1-e^{4x}\right)^2}{\left(e^{4x}+1\right)^4}de^x$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{e^{4x}\left(1-e^{4x}\right)^2}{\left(1+e^{4x}\right)^4}de^x$$

$$\Big\downarrow \text{963}$$

$$4\left(\frac{e^{5x}}{3\left(e^{4x}+1\right)^3}-\frac{1}{12}\int \frac{4e^{4x}\left(2-3e^{4x}\right)}{\left(1+e^{4x}\right)^3}de^x\right)$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{e^{5x}}{3\left(e^{4x}+1\right)^3}-\frac{1}{3}\int \frac{e^{4x}\left(2-3e^{4x}\right)}{\left(1+e^{4x}\right)^3}de^x\right)$$

$$\Big\downarrow \text{957}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\int \frac{e^{4x}}{\left(1+e^{4x}\right)^2}de^x-\frac{5e^{5x}}{8\left(e^{4x}+1\right)^2}\right)+\frac{e^{5x}}{3\left(e^{4x}+1\right)^3}\right)$$

$$\Big\downarrow \text{817}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\int \frac{1}{1+e^{4x}}de^x-\frac{e^x}{4\left(e^{4x}+1\right)}\right)-\frac{5e^{5x}}{8\left(e^{4x}+1\right)^2}\right)+\frac{e^{5x}}{3\left(e^{4x}+1\right)^3}\right)$$

$$\Big\downarrow \text{755}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x+\frac{1}{2}\int \frac{1+e^{2x}}{1+e^{4x}}de^x\right)-\frac{e^x}{4\left(e^{4x}+1\right)}\right)-\frac{5e^{5x}}{8\left(e^{4x}+1\right)^2}\right)+\frac{e^{5x}}{3\left(e^{4x}+1\right)^3}\right)$$

$$\Big\downarrow \text{1476}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1-\sqrt{2}e^x+e^{2x}}de^x+\frac{1}{2}\int \frac{1}{1+\sqrt{2}e^x+e^{2x}}de^x\right)+\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x\right)-\frac{e^x}{4\left(e^{4x}+1\right)}\right)-\frac{}{8\left(e\right.}$$

$$\Big\downarrow \text{1082}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-e^{2x}}d\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-e^{2x}}d\left(1+\sqrt{2}e^x\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x\right)-\frac{e^x}{4\left(e^{4x}+1\right)}\right)-\frac{5e}{8\left(e^{4x}\right.}$$

$$\Big\downarrow \text{217}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)-\frac{e^x}{4\left(e^{4x}+1\right)}\right)-\frac{5e^{5x}}{8\left(e^{4x}+1\right)^2}\right.$$

$$\Big\downarrow \text{1479}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right.\right.\right.\right.$$

$$\Big\downarrow \text{25}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)-\right.\right.\right.$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}+\frac{1}{2}\int \frac{1+\sqrt{2}e^x}{1+\sqrt{2}e^x+e^{2x}}de^x\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right.\right.\right.\right.\right.$$

$$\Big\downarrow \text{1103}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}-\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}\right.\right.\right.\right.\right.$$

input
```
Int[E^x*Sech[2*x]^2*Tanh[2*x]^2,x]
```

output
```
4*(E^(5*x)/(3*(1 + E^(4*x))^3) + ((-5*E^(5*x))/(8*(1 + E^(4*x))^2) + (9*(-
1/4*E^x/(1 + E^(4*x)) + ((-(ArcTan[1 - Sqrt[2]*E^x]/Sqrt[2]) + ArcTan[1 +
Sqrt[2]*E^x]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*E^x + E^(2*x)]/Sqrt[2] + L
og[1 + Sqrt[2]*E^x + E^(2*x)]/(2*Sqrt[2]))/2)/4))/8)/3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 963
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Simp[1/(a*b^2*n*(p + 1))   Int[(e*x)^m*(a + b*x
^n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 41.72 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.42

method	result
risch	$-\frac{e^{x}\left(29\,e^{8x}+6\,e^{4x}+9\right)}{24\left(e^{4x}+1\right)^{3}}+4\left(\sum_{\_R=\operatorname{RootOf}\left(268435456\_Z^{4}+81\right)}\_R\ln\left(e^{x}+\frac{128\_R}{3}\right)\right)$
default	$\frac{\frac{3\tanh\left(\frac{x}{2}\right)^{11}}{8}-\frac{3\tanh\left(\frac{x}{2}\right)^{10}}{8}-\frac{109\tanh\left(\frac{x}{2}\right)^{9}}{24}-\frac{173\tanh\left(\frac{x}{2}\right)^{8}}{8}-\frac{49\tanh\left(\frac{x}{2}\right)^{7}}{4}-\frac{231\tanh\left(\frac{x}{2}\right)^{6}}{4}+\frac{49\tanh\left(\frac{x}{2}\right)^{5}}{4}-\frac{117\tanh\left(\frac{x}{2}\right)^{4}}{4}+\frac{109\tanh\left(\frac{x}{2}\right)}{24}}{\left(\tanh\left(\frac{x}{2}\right)^{4}+6\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{3}}$

input
```
int(exp(x)*sech(2*x)^2*tanh(2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/24*exp(x)*(29*exp(8*x)+6*exp(4*x)+9)/(exp(4*x)+1)^3+4*sum(_R*ln(exp(x)+
128/3*_R),_R=RootOf(268435456*_Z^4+81))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1577 vs. $2(85) = 170$.

Time = 0.12 (sec) , antiderivative size = 1577, normalized size of antiderivative = 13.14

$$\int e^x \operatorname{sech}^2(2x) \tanh^2(2x)\, dx = \text{Too large to display}$$

input
```
integrate(exp(x)*sech(2*x)^2*tanh(2*x)^2,x, algorithm="fricas")
```

output
```
-1/192*(232*cosh(x)^9 + 19488*cosh(x)^3*sinh(x)^6 + 8352*cosh(x)^2*sinh(x)
^7 + 2088*cosh(x)*sinh(x)^8 + 232*sinh(x)^9 + 48*(609*cosh(x)^4 + 1)*sinh(
x)^5 + 48*cosh(x)^5 + 48*(609*cosh(x)^5 + 5*cosh(x))*sinh(x)^4 + 96*(203*c
osh(x)^6 + 5*cosh(x)^2)*sinh(x)^3 + 96*(87*cosh(x)^7 + 5*cosh(x)^3)*sinh(x
)^2 - 18*(sqrt(2)*cosh(x)^12 + 220*sqrt(2)*cosh(x)^3*sinh(x)^9 + 66*sqrt(2
)*cosh(x)^2*sinh(x)^10 + 12*sqrt(2)*cosh(x)*sinh(x)^11 + sqrt(2)*sinh(x)^1
2 + 3*(165*sqrt(2)*cosh(x)^4 + sqrt(2))*sinh(x)^8 + 3*sqrt(2)*cosh(x)^8 +
24*(33*sqrt(2)*cosh(x)^5 + sqrt(2)*cosh(x))*sinh(x)^7 + 84*(11*sqrt(2)*cos
h(x)^6 + sqrt(2)*cosh(x)^2)*sinh(x)^6 + 24*(33*sqrt(2)*cosh(x)^7 + 7*sqrt(
2)*cosh(x)^3)*sinh(x)^5 + 3*(165*sqrt(2)*cosh(x)^8 + 70*sqrt(2)*cosh(x)^4
+ sqrt(2))*sinh(x)^4 + 3*sqrt(2)*cosh(x)^4 + 4*(55*sqrt(2)*cosh(x)^9 + 42*
sqrt(2)*cosh(x)^5 + 3*sqrt(2)*cosh(x))*sinh(x)^3 + 6*(11*sqrt(2)*cosh(x)^1
0 + 14*sqrt(2)*cosh(x)^6 + 3*sqrt(2)*cosh(x)^2)*sinh(x)^2 + 12*(sqrt(2)*co
sh(x)^11 + 2*sqrt(2)*cosh(x)^7 + sqrt(2)*cosh(x)^3)*sinh(x) + sqrt(2))*arc
tan(sqrt(2)*cosh(x) + sqrt(2)*sinh(x) + 1) - 18*(sqrt(2)*cosh(x)^12 + 220*
sqrt(2)*cosh(x)^3*sinh(x)^9 + 66*sqrt(2)*cosh(x)^2*sinh(x)^10 + 12*sqrt(2)
*cosh(x)*sinh(x)^11 + sqrt(2)*sinh(x)^12 + 3*(165*sqrt(2)*cosh(x)^4 + sqrt
(2))*sinh(x)^8 + 3*sqrt(2)*cosh(x)^8 + 24*(33*sqrt(2)*cosh(x)^5 + sqrt(2)*
cosh(x))*sinh(x)^7 + 84*(11*sqrt(2)*cosh(x)^6 + sqrt(2)*cosh(x)^2)*sinh(x)
^6 + 24*(33*sqrt(2)*cosh(x)^7 + 7*sqrt(2)*cosh(x)^3)*sinh(x)^5 + 3*(165...
```

Sympy [F]

$$\int e^x \operatorname{sech}^2(2x)\tanh^2(2x)\,dx = \int e^x \tanh^2(2x)\operatorname{sech}^2(2x)\,dx$$

input
```
integrate(exp(x)*sech(2*x)**2*tanh(2*x)**2,x)
```

output
```
Integral(exp(x)*tanh(2*x)**2*sech(2*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.96

$$\begin{aligned}\int e^x \operatorname{sech}^2(2x)\tanh^2(2x)\,dx &= \frac{3}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) \\ &\quad+\frac{3}{32}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) \\ &\quad+\frac{3}{64}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right) \\ &\quad-\frac{3}{64}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right) \\ &\quad-\frac{29e^{(9x)}+6e^{(5x)}+9e^x}{24\left(e^{(12x)}+3e^{(8x)}+3e^{(4x)}+1\right)}\end{aligned}$$

input
```
integrate(exp(x)*sech(2*x)^2*tanh(2*x)^2,x, algorithm="maxima")
```

output
```
3/32*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 3/32*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) + 3/64*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) +
1) - 3/64*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/24*(29*e^(9*x) + 6*e
^(5*x) + 9*e^x)/(e^(12*x) + 3*e^(8*x) + 3*e^(4*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.86

$$\int e^x \operatorname{sech}^2(2x)\tanh^2(2x)\,dx = \frac{3}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) + \frac{3}{32}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) + \frac{3}{64}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right) - \frac{3}{64}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right) - \frac{29e^{(9x)}+6e^{(5x)}+9e^x}{24\left(e^{(4x)}+1\right)^3}$$

input
```
integrate(exp(x)*sech(2*x)^2*tanh(2*x)^2,x, algorithm="giac")
```

output
```
3/32*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 3/32*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) + 3/64*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) +
1) - 3/64*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/24*(29*e^(9*x) + 6*e
^(5*x) + 9*e^x)/(e^(4*x) + 1)^3
```

Mupad [B] (verification not implemented)

Time = 2.96 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.28

$$\int e^x \operatorname{sech}^2(2x)\tanh^2(2x)\,dx = \frac{5e^x}{6\left(2e^{4x}+e^{8x}+1\right)} - \frac{\frac{e^{9x}}{3}-\frac{2e^{5x}}{3}+\frac{e^x}{3}}{3e^{4x}+3e^{8x}+e^{12x}+1} - \frac{7e^x}{8\left(e^{4x}+1\right)} + \sqrt{2}\ln\left(-\frac{3e^x}{8}+\sqrt{2}\left(-\frac{3}{16}-\frac{3}{16}\mathrm{i}\right)\right)\left(\frac{3}{64}+\frac{3}{64}\mathrm{i}\right) + \sqrt{2}\ln\left(-\frac{3e^x}{8}+\sqrt{2}\left(-\frac{3}{16}+\frac{3}{16}\mathrm{i}\right)\right)\left(\frac{3}{64}-\frac{3}{64}\mathrm{i}\right) + \sqrt{2}\ln\left(-\frac{3e^x}{8}+\sqrt{2}\left(\frac{3}{16}-\frac{3}{16}\mathrm{i}\right)\right)\left(-\frac{3}{64}+\frac{3}{64}\mathrm{i}\right) + \sqrt{2}\ln\left(-\frac{3e^x}{8}+\sqrt{2}\left(\frac{3}{16}+\frac{3}{16}\mathrm{i}\right)\right)\left(-\frac{3}{64}-\frac{3}{64}\mathrm{i}\right)$$

input
```
int((tanh(2*x)^2*exp(x))/cosh(2*x)^2,x)
```

output
```
2^(1/2)*log(- (3*exp(x))/8 - 2^(1/2)*(3/16 + 3i/16))*(3/64 + 3i/64) - (exp
(9*x)/3 - (2*exp(5*x))/3 + exp(x)/3)/(3*exp(4*x) + 3*exp(8*x) + exp(12*x)
+ 1) - (7*exp(x))/(8*(exp(4*x) + 1)) + 2^(1/2)*log(- (3*exp(x))/8 - 2^(1/2
)*(3/16 - 3i/16))*(3/64 - 3i/64) - 2^(1/2)*log(2^(1/2)*(3/16 - 3i/16) - (3
*exp(x))/8)*(3/64 - 3i/64) - 2^(1/2)*log(2^(1/2)*(3/16 + 3i/16) - (3*exp(x
))/8)*(3/64 + 3i/64) + (5*exp(x))/(6*(2*exp(4*x) + exp(8*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 406, normalized size of antiderivative = 3.38

$$\int e^x \operatorname{sech}^2(2x)\tanh^2(2x)\,dx$$

$$= \frac{18e^{12x}\sqrt{2}\,atan\left(\frac{2e^x-\sqrt{2}}{\sqrt{2}}\right)+54e^{8x}\sqrt{2}\,atan\left(\frac{2e^x-\sqrt{2}}{\sqrt{2}}\right)+54e^{4x}\sqrt{2}\,atan\left(\frac{2e^x-\sqrt{2}}{\sqrt{2}}\right)+18\sqrt{2}\,atan\left(\frac{2e^x-\sqrt{2}}{\sqrt{2}}\right)+18}{}$$

input
```
int(exp(x)*sech(2*x)^2*tanh(2*x)^2,x)
```

output
```
(18*e**(12*x)*sqrt(2)*atan((2*e**x - sqrt(2))/sqrt(2)) + 54*e**(8*x)*sqrt(
2)*atan((2*e**x - sqrt(2))/sqrt(2)) + 54*e**(4*x)*sqrt(2)*atan((2*e**x - s
qrt(2))/sqrt(2)) + 18*sqrt(2)*atan((2*e**x - sqrt(2))/sqrt(2)) + 18*e**(12
*x)*sqrt(2)*atan((2*e**x + sqrt(2))/sqrt(2)) + 54*e**(8*x)*sqrt(2)*atan((2
*e**x + sqrt(2))/sqrt(2)) + 54*e**(4*x)*sqrt(2)*atan((2*e**x + sqrt(2))/sq
rt(2)) + 18*sqrt(2)*atan((2*e**x + sqrt(2))/sqrt(2)) - 9*e**(12*x)*sqrt(2)
*log(e**(2*x) - e**x*sqrt(2) + 1) + 9*e**(12*x)*sqrt(2)*log(e**(2*x) + e**
x*sqrt(2) + 1) - 232*e**(9*x) - 27*e**(8*x)*sqrt(2)*log(e**(2*x) - e**x*sq
rt(2) + 1) + 27*e**(8*x)*sqrt(2)*log(e**(2*x) + e**x*sqrt(2) + 1) - 48*e**
(5*x) - 27*e**(4*x)*sqrt(2)*log(e**(2*x) - e**x*sqrt(2) + 1) + 27*e**(4*x)
*sqrt(2)*log(e**(2*x) + e**x*sqrt(2) + 1) - 72*e**x - 9*sqrt(2)*log(e**(2*
x) - e**x*sqrt(2) + 1) + 9*sqrt(2)*log(e**(2*x) + e**x*sqrt(2) + 1))/(192*
(e**(12*x) + 3*e**(8*x) + 3*e**(4*x) + 1))
```

3.91 $\int e^x \coth(2x)\mathbf{csch}(2x)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 34

$$\int e^x \coth(2x)\operatorname{csch}(2x)\,dx = \frac{e^{3x}}{1-e^{4x}} + \frac{\arctan(e^x)}{2} - \frac{\operatorname{arctanh}(e^x)}{2}$$

output
```
exp(3*x)/(1-exp(4*x))+1/2*arctan(exp(x))-1/2*arctanh(exp(x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int e^x \coth(2x)\operatorname{csch}(2x)\,dx = \frac{1}{2}\left(-\frac{2e^{3x}}{-1+e^{4x}} + \arctan(e^x) - \operatorname{arctanh}(e^x)\right)$$

input
```
Integrate[E^x*Coth[2*x]*Csch[2*x],x]
```

output
```
((-2*E^(3*x))/(-1 + E^(4*x)) + ArcTan[E^x] - ArcTanh[E^x])/2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2720, 27, 957, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \coth(2x)\text{csch}(2x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{2e^{2x}\left(e^{4x}+1\right)}{\left(1-e^{4x}\right)^2}de^x$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{e^{2x}\left(1+e^{4x}\right)}{\left(1-e^{4x}\right)^2}de^x$$

$$\Big\downarrow \text{957}$$

$$2\left(\frac{e^{3x}}{2\left(1-e^{4x}\right)}-\frac{1}{2}\int \frac{e^{2x}}{1-e^{4x}}de^x\right)$$

$$\Big\downarrow \text{827}$$

$$2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1+e^{2x}}de^x-\frac{1}{2}\int \frac{1}{1-e^{2x}}de^x\right)+\frac{e^{3x}}{2\left(1-e^{4x}\right)}\right)$$

$$\Big\downarrow \text{216}$$

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(e^x\right)}{2}-\frac{1}{2}\int \frac{1}{1-e^{2x}}de^x\right)+\frac{e^{3x}}{2\left(1-e^{4x}\right)}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(e^x\right)}{2}-\frac{\text{arctanh}(e^x)}{2}\right)+\frac{e^{3x}}{2\left(1-e^{4x}\right)}\right)$$

input
```
Int[E^x*Coth[2*x]*Csch[2*x],x]
```

output
```
2*(E^(3*x)/(2*(1 - E^(4*x))) + (ArcTan[E^x]/2 - ArcTanh[E^x]/2)/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

method	result	size
risch	$-\frac{e^{3x}}{e^{4x}-1}+\frac{i\ln(e^{x}+i)}{4}-\frac{i\ln(e^{x}-i)}{4}-\frac{\ln(1+e^{x})}{4}+\frac{\ln(e^{x}-1)}{4}$	48

input
```
int(exp(x)*coth(2*x)*csch(2*x),x,method=_RETURNVERBOSE)
```

output
```
-exp(x)^3/(exp(x)^4-1)+1/4*I*ln(exp(x)+I)-1/4*I*ln(exp(x)-I)-1/4*ln(1+exp(
x))+1/4*ln(exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(25) = 50.

Time = 0.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 5.94

$$\int e^x \coth(2x)\text{csch}(2x)\,dx =$$
$$-\frac{4\cosh(x)^3+12\cosh(x)^2\sinh(x)+12\cosh(x)\sinh(x)^2+4\sinh(x)^3-2\left(\cosh(x)^4+4\cosh(x)^3 s\right.}{}$$

input
```
integrate(exp(x)*coth(2*x)*csch(2*x),x, algorithm="fricas")
```

output
```
-1/4*(4*cosh(x)^3 + 12*cosh(x)^2*sinh(x) + 12*cosh(x)*sinh(x)^2 + 4*sinh(x
)^3 - 2*(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(
x)*sinh(x)^3 + sinh(x)^4 - 1)*arctan(cosh(x) + sinh(x)) + (cosh(x)^4 + 4*c
osh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4
 - 1)*log(cosh(x) + sinh(x) + 1) - (cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*co
sh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)*log(cosh(x) + sin
h(x) - 1))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*co
sh(x)*sinh(x)^3 + sinh(x)^4 - 1)
```

Sympy [F]

$$\int e^x \coth(2x)\operatorname{csch}(2x)\,dx = \int e^x \coth(2x)\operatorname{csch}(2x)\,dx$$

input
```
integrate(exp(x)*coth(2*x)*csch(2*x),x)
```

output
```
Integral(exp(x)*coth(2*x)*csch(2*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int e^x \coth(2x)\operatorname{csch}(2x)\,dx = -\frac{e^{(3x)}}{e^{(4x)}-1} + \frac{1}{2}\arctan\left(e^x\right) - \frac{1}{4}\log\left(e^x+1\right) + \frac{1}{4}\log\left(e^x-1\right)$$

input
```
integrate(exp(x)*coth(2*x)*csch(2*x),x, algorithm="maxima")
```

output
```
-e^(3*x)/(e^(4*x) - 1) + 1/2*arctan(e^x) - 1/4*log(e^x + 1) + 1/4*log(e^x
- 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int e^x \coth(2x)\operatorname{csch}(2x)\,dx = -\frac{e^{(3x)}}{e^{(4x)}-1} + \frac{1}{2}\arctan\left(e^x\right) - \frac{1}{4}\log\left(e^x+1\right) + \frac{1}{4}\log\left(|e^x-1|\right)$$

input
```
integrate(exp(x)*coth(2*x)*csch(2*x),x, algorithm="giac")
```

output
```
-e^(3*x)/(e^(4*x) - 1) + 1/2*arctan(e^x) - 1/4*log(e^x + 1) + 1/4*log(abs(
e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int e^x \coth(2x)\operatorname{csch}(2x)\,dx = \frac{\ln\left(e^x - 1\right)}{4} - \frac{\operatorname{atan}(e^{-x})}{2} - \frac{\ln\left(-e^x - 1\right)}{4} - \frac{e^{3\,x}}{e^{4\,x} - 1}$$

input
```
int((coth(2*x)*exp(x))/sinh(2*x),x)
```

output
```
log(exp(x) - 1)/4 - atan(exp(-x))/2 - log(- exp(x) - 1)/4 - exp(3*x)/(exp(
4*x) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.24

$$\int e^x \coth(2x)\operatorname{csch}(2x)\,dx = \frac{2e^{4x}atan(e^x) - 2atan(e^x) + e^{4x}\log(e^x - 1) - e^{4x}\log(e^x + 1) - 4e^{3x} - \log(e^x - 1) + \log(e^x + 1)}{4e^{4x} - 4}$$

input
```
int(exp(x)*coth(2*x)*csch(2*x),x)
```

output
```
(2*e**(4*x)*atan(e**x) - 2*atan(e**x) + e**(4*x)*log(e**x - 1) - e**(4*x)*
log(e**x + 1) - 4*e**(3*x) - log(e**x - 1) + log(e**x + 1))/(4*(e**(4*x) -
 1))
```

3.92 $\int e^x \coth(2x)\mathbf{csch}^2(2x)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int e^x \coth(2x)\text{csch}^2(2x)\,dx = -\frac{e^x}{(1-e^{4x})^2} + \frac{5e^x}{4(1-e^{4x})} - \frac{\arctan(e^x)}{8} - \frac{\operatorname{arctanh}(e^x)}{8}$$

output
```
-exp(x)/(1-exp(4*x))^2+5*exp(x)/(4-4*exp(4*x))-1/8*arctan(exp(x))-1/8*arct
anh(exp(x))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int e^x \coth(2x)\text{csch}^2(2x)\,dx = -\frac{-2e^x + 10e^{5x} + (-1+e^{4x})^2 \arctan(e^x) + (-1+e^{4x})^2 \operatorname{arctanh}(e^x)}{8(-1+e^{4x})^2}$$

input
```
Integrate[E^x*Coth[2*x]*Csch[2*x]^2,x]
```

output
```
-1/8*(-2*E^x + 10*E^(5*x) + (-1 + E^(4*x))^2*ArcTan[E^x] + (-1 + E^(4*x))^
2*ArcTanh[E^x])/(-1 + E^(4*x))^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2720, 27, 957, 817, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \coth(2x)\text{csch}^2(2x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int -\frac{4e^{4x}\left(e^{4x}+1\right)}{\left(1-e^{4x}\right)^3}de^x$$

$$\Big\downarrow \text{27}$$

$$-4\int \frac{e^{4x}\left(1+e^{4x}\right)}{\left(1-e^{4x}\right)^3}de^x$$

$$\Big\downarrow \text{957}$$

$$-4\left(\frac{e^{5x}}{4\left(1-e^{4x}\right)^2}-\frac{1}{4}\int \frac{e^{4x}}{\left(1-e^{4x}\right)^2}de^x\right)$$

$$\Big\downarrow \text{817}$$

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\int \frac{1}{1-e^{4x}}de^x-\frac{e^x}{4\left(1-e^{4x}\right)}\right)+\frac{e^{5x}}{4\left(1-e^{4x}\right)^2}\right)$$

$$\Big\downarrow \text{756}$$

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{1-e^{2x}}de^x+\frac{1}{2}\int \frac{1}{1+e^{2x}}de^x\right)-\frac{e^x}{4\left(1-e^{4x}\right)}\right)+\frac{e^{5x}}{4\left(1-e^{4x}\right)^2}\right)$$

$$\Big\downarrow \text{216}$$

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{1-e^{2x}}de^x+\frac{\arctan\left(e^x\right)}{2}\right)-\frac{e^x}{4\left(1-e^{4x}\right)}\right)+\frac{e^{5x}}{4\left(1-e^{4x}\right)^2}\right)$$

$$\Big\downarrow \text{219}$$

$$-4\left(\frac{1}{4}\left(\frac{1}{4}\left(\frac{\arctan\left(e^x\right)}{2}+\frac{\text{arctanh}(e^x)}{2}\right)-\frac{e^x}{4\left(1-e^{4x}\right)}\right)+\frac{e^{5x}}{4\left(1-e^{4x}\right)^2}\right)$$

input
```
Int[E^x*Coth[2*x]*Csch[2*x]^2,x]
```

output
```
-4*(E^(5*x)/(4*(1 - E^(4*x))^2) + (-1/4*E^x/(1 - E^(4*x)) + (ArcTan[E^x]/2
 + ArcTanh[E^x]/2)/4)/4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

method	result
risch	$-\frac{e^{x}\left(5\,e^{4x}-1\right)}{4\left(e^{4x}-1\right)^{2}}-\frac{\ln\left(1+e^{x}\right)}{16}+\frac{i\ln\left(e^{x}-i\right)}{16}-\frac{i\ln\left(e^{x}+i\right)}{16}+\frac{\ln\left(e^{x}-1\right)}{16}$
default	$-\frac{1}{4\sinh\left(x\right)}-\frac{\arctan\left(e^{x}\right)}{8}+\frac{1}{8\sinh\left(x\right)\cosh\left(x\right)^{2}}+\frac{3\,\mathrm{sech}\left(x\right)\tanh\left(x\right)}{16}-\frac{\coth\left(x\right)\mathrm{csch}\left(x\right)}{8}-\frac{\mathrm{arctanh}\left(e^{x}\right)}{8}+\frac{1}{16\sinh\left(x\right)^{2}\cosh$

input
```
int(exp(x)*coth(2*x)*csch(2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*exp(x)*(5*exp(4*x)-1)/(exp(4*x)-1)^2-1/16*ln(1+exp(x))+1/16*I*ln(exp(
x)-I)-1/16*I*ln(exp(x)+I)+1/16*ln(exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(35) = 70$.

Time = 0.08 (sec) , antiderivative size = 522, normalized size of antiderivative = 10.24

$$\int e^x \coth(2x)\text{csch}^2(2x)\,dx = \text{Too large to display}$$

input
```
integrate(exp(x)*coth(2*x)*csch(2*x)^2,x, algorithm="fricas")
```

output
```
-1/16*(20*cosh(x)^5 + 200*cosh(x)^3*sinh(x)^2 + 200*cosh(x)^2*sinh(x)^3 +
100*cosh(x)*sinh(x)^4 + 20*sinh(x)^5 + 2*(cosh(x)^8 + 56*cosh(x)^3*sinh(x)
^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh
(x)^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*sinh(x)^3 +
 4*(7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^3)*sinh(
x) + 1)*arctan(cosh(x) + sinh(x)) + (cosh(x)^8 + 56*cosh(x)^3*sinh(x)^5 +
28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh(x)^4
 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*sinh(x)^3 + 4*(7
*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^3)*sinh(x) +
1)*log(cosh(x) + sinh(x) + 1) - (cosh(x)^8 + 56*cosh(x)^3*sinh(x)^5 + 28*c
osh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh(x)^4 - 1
)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*sinh(x)^3 + 4*(7*cos
h(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^3)*sinh(x) + 1)*l
og(cosh(x) + sinh(x) - 1) + 4*(25*cosh(x)^4 - 1)*sinh(x) - 4*cosh(x))/(cos
h(x)^8 + 56*cosh(x)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(
x)^7 + sinh(x)^8 + 2*(35*cosh(x)^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cos
h(x)^5 - cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*
(cosh(x)^7 - cosh(x)^3)*sinh(x) + 1)
```

Sympy [F]

$$\int e^x \coth(2x)\text{csch}^2(2x)\,dx = \int e^x \coth(2x)\,\text{csch}^2(2x)\,dx$$

input
```
integrate(exp(x)*coth(2*x)*csch(2*x)**2,x)
```

output
```
Integral(exp(x)*coth(2*x)*csch(2*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int e^x \coth(2x)\operatorname{csch}^2(2x)\,dx = -\frac{5\,e^{(5\,x)} - e^x}{4\,(e^{(8\,x)} - 2\,e^{(4\,x)} + 1)} - \frac{1}{8}\,\arctan\left(e^x\right) - \frac{1}{16}\,\log\left(e^x + 1\right) + \frac{1}{16}\,\log\left(e^x - 1\right)$$

input
```
integrate(exp(x)*coth(2*x)*csch(2*x)^2,x, algorithm="maxima")
```

output
```
-1/4*(5*e^(5*x) - e^x)/(e^(8*x) - 2*e^(4*x) + 1) - 1/8*arctan(e^x) - 1/16*
log(e^x + 1) + 1/16*log(e^x - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int e^x \coth(2x)\operatorname{csch}^2(2x)\,dx = -\frac{5\,e^{(5\,x)} - e^x}{4\,(e^{(4\,x)} - 1)^2} - \frac{1}{8}\,\arctan\left(e^x\right) - \frac{1}{16}\,\log\left(e^x + 1\right) + \frac{1}{16}\,\log\left(|e^x - 1|\right)$$

input
```
integrate(exp(x)*coth(2*x)*csch(2*x)^2,x, algorithm="giac")
```

output
```
-1/4*(5*e^(5*x) - e^x)/(e^(4*x) - 1)^2 - 1/8*arctan(e^x) - 1/16*log(e^x +
1) + 1/16*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 2.81 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.57

$$\int e^x \coth(2x)\operatorname{csch}^2(2x)\,dx = \frac{\ln\left(\frac{1}{4}-\frac{e^x}{4}\right)}{16} - \frac{\ln\left(\frac{e^x}{4}+\frac{1}{4}\right)}{16} - \frac{\operatorname{atan}(e^x)}{8} - \frac{e^{5x}}{2\left(e^{8x}-2e^{4x}+1\right)} - \frac{3e^x}{4\left(e^{4x}-1\right)} - \frac{e^x}{2\left(e^{8x}-2e^{4x}+1\right)}$$

input
```
int((coth(2*x)*exp(x))/sinh(2*x)^2,x)
```

output
```
log(1/4 - exp(x)/4)/16 - log(exp(x)/4 + 1/4)/16 - atan(exp(x))/8 - exp(5*x
)/(2*(exp(8*x) - 2*exp(4*x) + 1)) - (3*exp(x))/(4*(exp(4*x) - 1)) - exp(x)
/(2*(exp(8*x) - 2*exp(4*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.45

$$\int e^x \coth(2x)\operatorname{csch}^2(2x)\,dx$$
$$= \frac{-2e^{8x}atan(e^x)+4e^{4x}atan(e^x)-2atan(e^x)+e^{8x}\log(e^x-1)-e^{8x}\log(e^x+1)-20e^{5x}-2e^{4x}\log(e^x-1)}{16e^{8x}-32e^{4x}+16}$$

input
```
int(exp(x)*coth(2*x)*csch(2*x)^2,x)
```

output
```
( - 2*e**(8*x)*atan(e**x) + 4*e**(4*x)*atan(e**x) - 2*atan(e**x) + e**(8*x
)*log(e**x - 1) - e**(8*x)*log(e**x + 1) - 20*e**(5*x) - 2*e**(4*x)*log(e*
*x - 1) + 2*e**(4*x)*log(e**x + 1) + 4*e**x + log(e**x - 1) - log(e**x + 1
))/(16*(e**(8*x) - 2*e**(4*x) + 1))
```

3.93 $\int e^x \coth^2(2x)\mathbf{csch}(2x)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int e^x \coth^2(2x)\text{csch}(2x)\,dx = -\frac{e^{3x}}{(1-e^{4x})^2} + \frac{3e^{3x}}{4(1-e^{4x})} + \frac{5\arctan(e^x)}{8} - \frac{5\text{arctanh}(e^x)}{8}$$

output
```
-exp(3*x)/(1-exp(4*x))^2+3*exp(3*x)/(4-4*exp(4*x))+5/8*arctan(exp(x))-5/8*
arctanh(exp(x))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.32 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.93

$$\int e^x \coth^2(2x)\text{csch}(2x)\,dx$$
$$= \frac{e^{-5x}\left(177023 + 244931e^{4x} + 43161e^{8x} - 26091e^{12x} - 7(25289 + 24152e^{4x} - 10058e^{8x} - 9048e^{12x} + 513e\right.}{10752}$$
$$- \frac{8e^{7x}(15 + 26e^{4x} + 11e^{8x})\ {}_4F_3\left(\frac{7}{4}, 2, 2, 2; 1, 1, \frac{19}{4}; e^{4x}\right)}{1155}$$
$$- \frac{16e^{7x}(1 + e^{4x})^2\ {}_5F_4\left(\frac{7}{4}, 2, 2, 2, 2; 1, 1, 1, \frac{19}{4}; e^{4x}\right)}{1155}$$

input
```
Integrate[E^x*Coth[2*x]^2*Csch[2*x],x]
```

output
```
(177023 + 244931*E^(4*x) + 43161*E^(8*x) - 26091*E^(12*x) - 7*(25289 + 241
52*E^(4*x) - 10058*E^(8*x) - 9048*E^(12*x) + 513*E^(16*x))*Hypergeometric2
F1[3/4, 1, 7/4, E^(4*x)])/(10752*E^(5*x)) - (8*E^(7*x)*(15 + 26*E^(4*x) +
11*E^(8*x))*HypergeometricPFQ[{7/4, 2, 2, 2}, {1, 1, 19/4}, E^(4*x)])/1155
 - (16*E^(7*x)*(1 + E^(4*x))^2*HypergeometricPFQ[{7/4, 2, 2, 2, 2}, {1, 1,
 1, 19/4}, E^(4*x)])/1155
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2720, 27, 963, 27, 957, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \coth^2(2x)\text{csch}(2x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int -\frac{2e^{2x}\left(e^{4x}+1\right)^2}{\left(1-e^{4x}\right)^3}de^x$$

$$\Big\downarrow \text{27}$$

$$-2\int \frac{e^{2x}\left(1+e^{4x}\right)^2}{\left(1-e^{4x}\right)^3}de^x$$

$$\Big\downarrow \text{963}$$

$$-2\left(\frac{e^{3x}}{2\left(1-e^{4x}\right)^2}-\frac{1}{8}\int \frac{4e^{2x}\left(1+2e^{4x}\right)}{\left(1-e^{4x}\right)^2}de^x\right)$$

$$\Big\downarrow \text{27}$$

$$-2\left(\frac{e^{3x}}{2\left(1-e^{4x}\right)^2}-\frac{1}{2}\int \frac{e^{2x}\left(1+2e^{4x}\right)}{\left(1-e^{4x}\right)^2}de^x\right)$$

$$\Big\downarrow \text{957}$$

$$-2\left(\frac{1}{2}\left(\frac{5}{4}\int \frac{e^{2x}}{1-e^{4x}}de^x-\frac{3e^{3x}}{4\left(1-e^{4x}\right)}\right)+\frac{e^{3x}}{2\left(1-e^{4x}\right)^2}\right)$$

$$\Big\downarrow \textbf{827}$$

$$-2\left(\frac{1}{2}\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{1-e^{2x}}de^x-\frac{1}{2}\int \frac{1}{1+e^{2x}}de^x\right)-\frac{3e^{3x}}{4\left(1-e^{4x}\right)}\right)+\frac{e^{3x}}{2\left(1-e^{4x}\right)^2}\right)$$

$$\Big\downarrow \textbf{216}$$

$$-2\left(\frac{1}{2}\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{1-e^{2x}}de^x-\frac{\arctan\left(e^x\right)}{2}\right)-\frac{3e^{3x}}{4\left(1-e^{4x}\right)}\right)+\frac{e^{3x}}{2\left(1-e^{4x}\right)^2}\right)$$

$$\Big\downarrow \textbf{219}$$

$$-2\left(\frac{1}{2}\left(\frac{5}{4}\left(\frac{\text{arctanh}(e^x)}{2}-\frac{\arctan\left(e^x\right)}{2}\right)-\frac{3e^{3x}}{4\left(1-e^{4x}\right)}\right)+\frac{e^{3x}}{2\left(1-e^{4x}\right)^2}\right)$$

input
```
Int[E^x*Coth[2*x]^2*Csch[2*x],x]
```

output
```
-2*(E^(3*x)/(2*(1 - E^(4*x))^2) + ((-3*E^(3*x))/(4*(1 - E^(4*x))) + (5*(-1
/2*ArcTan[E^x] + ArcTanh[E^x]/2))/4)/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 963
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Simp[1/(a*b^2*n*(p + 1))   Int[(e*x)^m*(a + b*x
^n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

method	result	size
risch	$-\frac{{\mathrm e}^{3 x} \left(3 \,{\mathrm e}^{4 x}+1\right)}{4 \left({\mathrm e}^{4 x}-1\right)^{2}}+\frac{5 i \ln \left({\mathrm e}^{x}+i\right)}{16}-\frac{5 i \ln \left({\mathrm e}^{x}-i\right)}{16}+\frac{5 \ln \left({\mathrm e}^{x}-1\right)}{16}-\frac{5 \ln \left(1+{\mathrm e}^{x}\right)}{16}$	56

input
```
int(exp(x)*coth(2*x)^2*csch(2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*exp(x)^3*(3*exp(x)^4+1)/(exp(x)^4-1)^2+5/16*I*ln(exp(x)+I)-5/16*I*ln(
exp(x)-I)+5/16*ln(exp(x)-1)-5/16*ln(1+exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. $2(39) = 78$.

Time = 0.09 (sec) , antiderivative size = 557, normalized size of antiderivative = 10.13

$$\int e^x \coth^2(2x)\operatorname{csch}(2x)\,dx = \text{Too large to display}$$

input
```
integrate(exp(x)*coth(2*x)^2*csch(2*x),x, algorithm="fricas")
```

output
```
-1/16*(12*cosh(x)^7 + 420*cosh(x)^3*sinh(x)^4 + 252*cosh(x)^2*sinh(x)^5 +
84*cosh(x)*sinh(x)^6 + 12*sinh(x)^7 + 4*(105*cosh(x)^4 + 1)*sinh(x)^3 + 4*
cosh(x)^3 + 12*(21*cosh(x)^5 + cosh(x))*sinh(x)^2 - 10*(cosh(x)^8 + 56*cos
h(x)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^
8 + 2*(35*cosh(x)^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x
))*sinh(x)^3 + 4*(7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - co
sh(x)^3)*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 5*(cosh(x)^8 + 56*cosh(x
)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 +
 2*(35*cosh(x)^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*
sinh(x)^3 + 4*(7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(
x)^3)*sinh(x) + 1)*log(cosh(x) + sinh(x) + 1) - 5*(cosh(x)^8 + 56*cosh(x)^
3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2
*(35*cosh(x)^4 - 1)*sinh(x)^4 - 2*cosh(x)^4 + 8*(7*cosh(x)^5 - cosh(x))*si
nh(x)^3 + 4*(7*cosh(x)^6 - 3*cosh(x)^2)*sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)
^3)*sinh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 12*(7*cosh(x)^6 + cosh(x)^2)
*sinh(x))/(cosh(x)^8 + 56*cosh(x)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8
*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(35*cosh(x)^4 - 1)*sinh(x)^4 - 2*cosh(x
)^4 + 8*(7*cosh(x)^5 - cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 3*cosh(x)^2)*
sinh(x)^2 + 8*(cosh(x)^7 - cosh(x)^3)*sinh(x) + 1)
```

Sympy [F]

$$\int e^x \coth^2(2x)\text{csch}(2x)\,dx = \int e^x \coth^2{(2x)}\,\text{csch}\,(2x)\,dx$$

input
```
integrate(exp(x)*coth(2*x)**2*csch(2*x),x)
```

output
```
Integral(exp(x)*coth(2*x)**2*csch(2*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int e^x \coth^2(2x)\text{csch}(2x)\,dx = -\frac{3\,e^{(7\,x)} + e^{(3\,x)}}{4\,\left(e^{(8\,x)} - 2\,e^{(4\,x)} + 1\right)} + \frac{5}{8}\,\arctan\left(e^x\right) - \frac{5}{16}\,\log\left(e^x + 1\right) + \frac{5}{16}\,\log\left(e^x - 1\right)$$

input
```
integrate(exp(x)*coth(2*x)^2*csch(2*x),x, algorithm="maxima")
```

output
```
-1/4*(3*e^(7*x) + e^(3*x))/(e^(8*x) - 2*e^(4*x) + 1) + 5/8*arctan(e^x) - 5
/16*log(e^x + 1) + 5/16*log(e^x - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int e^x \coth^2(2x)\text{csch}(2x)\,dx = -\frac{3\,e^{(7\,x)} + e^{(3\,x)}}{4\,\left(e^{(4\,x)} - 1\right)^2} + \frac{5}{8}\,\arctan\left(e^x\right) - \frac{5}{16}\,\log\left(e^x + 1\right) + \frac{5}{16}\,\log\left(|e^x - 1|\right)$$

input
```
integrate(exp(x)*coth(2*x)^2*csch(2*x),x, algorithm="giac")
```

output
```
-1/4*(3*e^(7*x) + e^(3*x))/(e^(4*x) - 1)^2 + 5/8*arctan(e^x) - 5/16*log(e^
x + 1) + 5/16*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

$$\int e^x \coth^2(2x)\text{csch}(2x)\,dx = \frac{5\,\ln\left(\frac{25\,e^x}{16}-\frac{25}{16}\right)}{16} - \frac{5\,\ln\left(\frac{25\,e^x}{16}+\frac{25}{16}\right)}{16} - \frac{5\,\text{atan}(e^{-x})}{8} - \frac{e^{3\,x}}{e^{8\,x}-2\,e^{4\,x}+1} - \frac{3\,e^{3\,x}}{4\,(e^{4\,x}-1)}$$

input
```
int((coth(2*x)^2*exp(x))/sinh(2*x),x)
```

output
```
(5*log((25*exp(x))/16 - 25/16))/16 - (5*log((25*exp(x))/16 + 25/16))/16 -
(5*atan(exp(-x)))/8 - exp(3*x)/(exp(8*x) - 2*exp(4*x) + 1) - (3*exp(3*x))/
(4*(exp(4*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.36

$$\int e^x \coth^2(2x)\text{csch}(2x)\,dx = \frac{10e^{8x}atan(e^x) - 20e^{4x}atan(e^x) + 10atan(e^x) + 5e^{8x}\log(e^x-1) - 5e^{8x}\log(e^x+1) - 12e^{7x} - 10e^{4x}\log(e^{x}}{16e^{8x}-32e^{4x}+16}$$

input
```
int(exp(x)*coth(2*x)^2*csch(2*x),x)
```

output
```
(10*e**(8*x)*atan(e**x) - 20*e**(4*x)*atan(e**x) + 10*atan(e**x) + 5*e**(8
*x)*log(e**x - 1) - 5*e**(8*x)*log(e**x + 1) - 12*e**(7*x) - 10*e**(4*x)*l
og(e**x - 1) + 10*e**(4*x)*log(e**x + 1) - 4*e**(3*x) + 5*log(e**x - 1) -
5*log(e**x + 1))/(16*(e**(8*x) - 2*e**(4*x) + 1))
```

3.94 $\int e^x \coth^2(2x)\mathbf{csch}^2(2x)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 71

$$\int e^x \coth^2(2x)\text{csch}^2(2x)\,dx = \frac{4e^x}{3(1-e^{4x})^3} - \frac{13e^x}{6(1-e^{4x})^2} + \frac{29e^x}{24(1-e^{4x})} - \frac{3\arctan(e^x)}{16} - \frac{3\text{arctanh}(e^x)}{16}$$

output
```
4/3*exp(x)/(1-exp(4*x))^3-13/6*exp(x)/(1-exp(4*x))^2+29*exp(x)/(24-24*exp(
4*x))-3/16*arctan(exp(x))-3/16*arctanh(exp(x))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.31 (sec) , antiderivative size = 310, normalized size of antiderivative = 4.37

$$\int e^x \coth^2(2x)\text{csch}^2(2x)\,dx = \frac{e^{-7x}\left(-1070609085 - 946471617e^{4x} + 369641285e^{8x} + 351173641e^{12x} - 23818496e^{16x} + 1070609085\,\text{Hy}\right.}{}$$

input
```
Integrate[E^x*Coth[2*x]^2*Csch[2*x]^2,x]
```

output
```
(-1070609085 - 946471617*E^(4*x) + 369641285*E^(8*x) + 351173641*E^(12*x)
- 23818496*E^(16*x) + 1070609085*Hypergeometric2F1[1/4, 1, 5/4, E^(4*x)] +
 732349800*E^(4*x)*Hypergeometric2F1[1/4, 1, 5/4, E^(4*x)] - 635067810*E^(
8*x)*Hypergeometric2F1[1/4, 1, 5/4, E^(4*x)] - 384831720*E^(12*x)*Hypergeo
metric2F1[1/4, 1, 5/4, E^(4*x)] + 60913125*E^(16*x)*Hypergeometric2F1[1/4,
 1, 5/4, E^(4*x)] + 1280*E^(16*x)*(821 + 1346*E^(4*x) + 557*E^(8*x))*Hyper
geometricPFQ[{2, 2, 2, 9/4}, {1, 1, 21/4}, E^(4*x)] + 10240*E^(16*x)*(23 +
 42*E^(4*x) + 19*E^(8*x))*HypergeometricPFQ[{2, 2, 2, 2, 9/4}, {1, 1, 1, 2
1/4}, E^(4*x)] + 20480*E^(16*x)*HypergeometricPFQ[{2, 2, 2, 2, 2, 9/4}, {1
, 1, 1, 1, 21/4}, E^(4*x)] + 40960*E^(20*x)*HypergeometricPFQ[{2, 2, 2, 2,
 2, 9/4}, {1, 1, 1, 1, 21/4}, E^(4*x)] + 20480*E^(24*x)*HypergeometricPFQ[
{2, 2, 2, 2, 2, 9/4}, {1, 1, 1, 1, 21/4}, E^(4*x)])/(3818880*E^(7*x))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.30, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {2720, 27, 963, 27, 957, 817, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \coth^2(2x)\text{csch}^2(2x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{4e^{4x}\left(e^{4x}+1\right)^2}{\left(1-e^{4x}\right)^4}de^x$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{e^{4x}\left(1+e^{4x}\right)^2}{\left(1-e^{4x}\right)^4}de^x$$

$$\Big\downarrow \text{963}$$

$$4\left(\frac{e^{5x}}{3\left(1-e^{4x}\right)^3}-\frac{1}{12}\int \frac{4e^{4x}\left(2+3e^{4x}\right)}{\left(1-e^{4x}\right)^3}de^x\right)$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{e^{5x}}{3(1-e^{4x})^3}-\frac{1}{3}\int\frac{e^{4x}(2+3e^{4x})}{(1-e^{4x})^3}de^x\right)$$

$$\Big\downarrow \text{957}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\int\frac{e^{4x}}{(1-e^{4x})^2}de^x-\frac{5e^{5x}}{8(1-e^{4x})^2}\right)+\frac{e^{5x}}{3(1-e^{4x})^3}\right)$$

$$\Big\downarrow \text{817}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{e^x}{4(1-e^{4x})}-\frac{1}{4}\int\frac{1}{1-e^{4x}}de^x\right)-\frac{5e^{5x}}{8(1-e^{4x})^2}\right)+\frac{e^{5x}}{3(1-e^{4x})^3}\right)$$

$$\Big\downarrow \text{756}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\left(-\frac{1}{2}\int\frac{1}{1-e^{2x}}de^x-\frac{1}{2}\int\frac{1}{1+e^{2x}}de^x\right)+\frac{e^x}{4(1-e^{4x})}\right)-\frac{5e^{5x}}{8(1-e^{4x})^2}\right)+\frac{e^{5x}}{3(1-e^{4x})^3}\right)$$

$$\Big\downarrow \text{216}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\left(-\frac{1}{2}\int\frac{1}{1-e^{2x}}de^x-\frac{1}{2}\arctan(e^x)\right)+\frac{e^x}{4(1-e^{4x})}\right)-\frac{5e^{5x}}{8(1-e^{4x})^2}\right)+\frac{e^{5x}}{3(1-e^{4x})^3}\right)$$

$$\Big\downarrow \text{219}$$

$$4\left(\frac{1}{3}\left(\frac{9}{8}\left(\frac{1}{4}\left(-\frac{1}{2}\arctan(e^x)-\frac{\text{arctanh}(e^x)}{2}\right)+\frac{e^x}{4(1-e^{4x})}\right)-\frac{5e^{5x}}{8(1-e^{4x})^2}\right)+\frac{e^{5x}}{3(1-e^{4x})^3}\right)$$

input
```
Int[E^x*Coth[2*x]^2*Csch[2*x]^2,x]
```

output
```
4*(E^(5*x)/(3*(1 - E^(4*x))^3) + ((-5*E^(5*x))/(8*(1 - E^(4*x))^2) + (9*(E
^x/(4*(1 - E^(4*x))) + (-1/2*ArcTan[E^x] - ArcTanh[E^x]/2)/4))/8)/3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 963
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Simp[1/(a*b^2*n*(p + 1))   Int[(e*x)^m*(a + b*x
^n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.85

method	result	size
risch	$-\frac{e^{x}\left(29\,e^{8x}-6\,e^{4x}+9\right)}{24\left(e^{4x}-1\right)^{3}}+\frac{3\ln\left(e^{x}-1\right)}{32}-\frac{3\ln\left(1+e^{x}\right)}{32}+\frac{3i\ln\left(e^{x}-i\right)}{32}-\frac{3i\ln\left(e^{x}+i\right)}{32}$	60

input
```
int(exp(x)*coth(2*x)^2*csch(2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/24*exp(x)*(29*exp(8*x)-6*exp(4*x)+9)/(exp(4*x)-1)^3+3/32*ln(exp(x)-1)-3
/32*ln(1+exp(x))+3/32*I*ln(exp(x)-I)-3/32*I*ln(exp(x)+I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. 2(47) = 94.

Time = 0.09 (sec) , antiderivative size = 992, normalized size of antiderivative = 13.97

$$\int e^{x}\coth^{2}(2x)\operatorname{csch}^{2}(2x)\,dx=\text{Too large to display}$$

input
```
integrate(exp(x)*coth(2*x)^2*csch(2*x)^2,x, algorithm="fricas")
```

output
```
-1/96*(116*cosh(x)^9 + 9744*cosh(x)^3*sinh(x)^6 + 4176*cosh(x)^2*sinh(x)^7
 + 1044*cosh(x)*sinh(x)^8 + 116*sinh(x)^9 + 24*(609*cosh(x)^4 - 1)*sinh(x)
^5 - 24*cosh(x)^5 + 24*(609*cosh(x)^5 - 5*cosh(x))*sinh(x)^4 + 48*(203*cos
h(x)^6 - 5*cosh(x)^2)*sinh(x)^3 + 48*(87*cosh(x)^7 - 5*cosh(x)^3)*sinh(x)^
2 + 18*(cosh(x)^12 + 220*cosh(x)^3*sinh(x)^9 + 66*cosh(x)^2*sinh(x)^10 + 1
2*cosh(x)*sinh(x)^11 + sinh(x)^12 + 3*(165*cosh(x)^4 - 1)*sinh(x)^8 - 3*co
sh(x)^8 + 24*(33*cosh(x)^5 - cosh(x))*sinh(x)^7 + 84*(11*cosh(x)^6 - cosh(
x)^2)*sinh(x)^6 + 24*(33*cosh(x)^7 - 7*cosh(x)^3)*sinh(x)^5 + 3*(165*cosh(
x)^8 - 70*cosh(x)^4 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(55*cosh(x)^9 - 42*co
sh(x)^5 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 14*cosh(x)^6 + 3*cosh(
x)^2)*sinh(x)^2 + 12*(cosh(x)^11 - 2*cosh(x)^7 + cosh(x)^3)*sinh(x) - 1)*a
rctan(cosh(x) + sinh(x)) + 9*(cosh(x)^12 + 220*cosh(x)^3*sinh(x)^9 + 66*co
sh(x)^2*sinh(x)^10 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 3*(165*cosh(x)^4
 - 1)*sinh(x)^8 - 3*cosh(x)^8 + 24*(33*cosh(x)^5 - cosh(x))*sinh(x)^7 + 84
*(11*cosh(x)^6 - cosh(x)^2)*sinh(x)^6 + 24*(33*cosh(x)^7 - 7*cosh(x)^3)*si
nh(x)^5 + 3*(165*cosh(x)^8 - 70*cosh(x)^4 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4
*(55*cosh(x)^9 - 42*cosh(x)^5 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 -
14*cosh(x)^6 + 3*cosh(x)^2)*sinh(x)^2 + 12*(cosh(x)^11 - 2*cosh(x)^7 + cos
h(x)^3)*sinh(x) - 1)*log(cosh(x) + sinh(x) + 1) - 9*(cosh(x)^12 + 220*cosh
(x)^3*sinh(x)^9 + 66*cosh(x)^2*sinh(x)^10 + 12*cosh(x)*sinh(x)^11 + sin...
```

Sympy [F]

$$\int e^x \coth^2(2x)\operatorname{csch}^2(2x)\,dx = \int e^x \coth^2{(2x)}\operatorname{csch}^2{(2x)}\,dx$$

input
```
integrate(exp(x)*coth(2*x)**2*csch(2*x)**2,x)
```

output
```
Integral(exp(x)*coth(2*x)**2*csch(2*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int e^x \coth^2(2x)\operatorname{csch}^2(2x)\,dx = -\frac{29\,e^{(9\,x)} - 6\,e^{(5\,x)} + 9\,e^x}{24\,\left(e^{(12\,x)} - 3\,e^{(8\,x)} + 3\,e^{(4\,x)} - 1\right)} - \frac{3}{16}\,\arctan\left(e^x\right) - \frac{3}{32}\,\log\left(e^x + 1\right) + \frac{3}{32}\,\log\left(e^x - 1\right)$$

input
```
integrate(exp(x)*coth(2*x)^2*csch(2*x)^2,x, algorithm="maxima")
```

output
```
-1/24*(29*e^(9*x) - 6*e^(5*x) + 9*e^x)/(e^(12*x) - 3*e^(8*x) + 3*e^(4*x) -
 1) - 3/16*arctan(e^x) - 3/32*log(e^x + 1) + 3/32*log(e^x - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.68

$$\int e^x \coth^2(2x)\operatorname{csch}^2(2x)\,dx = -\frac{29\,e^{(9\,x)} - 6\,e^{(5\,x)} + 9\,e^x}{24\,\left(e^{(4\,x)} - 1\right)^3} - \frac{3}{16}\,\arctan\left(e^x\right) - \frac{3}{32}\,\log\left(e^x + 1\right) + \frac{3}{32}\,\log\left(|e^x - 1|\right)$$

input
```
integrate(exp(x)*coth(2*x)^2*csch(2*x)^2,x, algorithm="giac")
```

output
```
-1/24*(29*e^(9*x) - 6*e^(5*x) + 9*e^x)/(e^(4*x) - 1)^3 - 3/16*arctan(e^x)
- 3/32*log(e^x + 1) + 3/32*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.61

$$\int e^x \coth^2(2x)\text{csch}^2(2x)\,dx = \frac{3\ln\left(\frac{3}{8}-\frac{3\,e^x}{8}\right)}{32} - \frac{3\ln\left(-\frac{3\,e^x}{8}-\frac{3}{8}\right)}{32} - \frac{7\,e^x}{8\,(e^{4x}-1)} - \frac{\frac{2\,e^{5x}}{3}+\frac{e^{9x}}{3}+\frac{e^x}{3}}{3\,e^{4x}-3\,e^{8x}+e^{12x}-1} - \frac{5\,e^x}{6\,(e^{8x}-2\,e^{4x}+1)} - \frac{\ln\left(-\frac{3\,e^x}{8}-\frac{3}{8}\mathrm{i}\right)3\mathrm{i}}{32} + \frac{\ln\left(-\frac{3\,e^x}{8}+\frac{3}{8}\mathrm{i}\right)3\mathrm{i}}{32}$$

input
```
int((coth(2*x)^2*exp(x))/sinh(2*x)^2,x)
```

output
```
(3*log(3/8 - (3*exp(x))/8))/32 - (3*log(- (3*exp(x))/8 - 3/8))/32 - (log(-
 (3*exp(x))/8 - 3i/8)*3i)/32 + (log(3i/8 - (3*exp(x))/8)*3i)/32 - (7*exp(x
))/(8*(exp(4*x) - 1)) - ((2*exp(5*x))/3 + exp(9*x)/3 + exp(x)/3)/(3*exp(4*
x) - 3*exp(8*x) + exp(12*x) - 1) - (5*exp(x))/(6*(exp(8*x) - 2*exp(4*x) +
1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.52

$$\int e^x \coth^2(2x)\text{csch}^2(2x)\,dx = \frac{-18e^{12x}atan(e^x)+54e^{8x}atan(e^x)-54e^{4x}atan(e^x)+18atan(e^x)+9e^{12x}\log(e^x-1)-9e^{12x}\log(e^x+1)}{}$$

input
```
int(exp(x)*coth(2*x)^2*csch(2*x)^2,x)
```

output
```
( - 18*e**(12*x)*atan(e**x) + 54*e**(8*x)*atan(e**x) - 54*e**(4*x)*atan(e*
*x) + 18*atan(e**x) + 9*e**(12*x)*log(e**x - 1) - 9*e**(12*x)*log(e**x + 1
) - 116*e**(9*x) - 27*e**(8*x)*log(e**x - 1) + 27*e**(8*x)*log(e**x + 1) +
 24*e**(5*x) + 27*e**(4*x)*log(e**x - 1) - 27*e**(4*x)*log(e**x + 1) - 36*
e**x - 9*log(e**x - 1) + 9*log(e**x + 1))/(96*(e**(12*x) - 3*e**(8*x) + 3*
e**(4*x) - 1))
```

3.95 $\int F^{c(a+bx)}\cosh(d+ex)\sinh^3(d+ex)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 139

$$\int F^{c(a+bx)}\cosh(d+ex)\sinh^3(d+ex)\,dx = -\frac{e^{-2d-2ex}F^{c(a+bx)}}{8(2e-bc\log(F))}+\frac{e^{-4d-4ex}F^{c(a+bx)}}{16(4e-bc\log(F))} - \frac{e^{2d+2ex}F^{c(a+bx)}}{8(2e+bc\log(F))}+\frac{e^{4d+4ex}F^{c(a+bx)}}{16(4e+bc\log(F))}$$

output
```
-1/8*exp(-2*e*x-2*d)*F^(c*(b*x+a))/(2*e-b*c*ln(F))+exp(-4*e*x-4*d)*F^(c*(b
*x+a))/(64*e-16*b*c*ln(F))-exp(2*e*x+2*d)*F^(c*(b*x+a))/(16*e+8*b*c*ln(F))
+exp(4*e*x+4*d)*F^(c*(b*x+a))/(64*e+16*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.78

$$\int F^{c(a+bx)}\cosh(d+ex)\sinh^3(d+ex)\,dx = \frac{1}{8}F^{c(a+bx)}\left(\frac{-4e\cosh(2(d+ex))+2bc\log(F)\sinh(2(d+ex))}{4e^2-b^2c^2\log^2(F)} + \frac{4e\cosh(4(d+ex))-bc\log(F)\sinh(4(d+ex))}{16e^2-b^2c^2\log^2(F)}\right)$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*((-4*e*Cosh[2*(d + e*x)] + 2*b*c*Log[F]*Sinh[2*(d + e*x)]
)/(4*e^2 - b^2*c^2*Log[F]^2) + (4*e*Cosh[4*(d + e*x)] - b*c*Log[F]*Sinh[4*
(d + e*x)])/(16*e^2 - b^2*c^2*Log[F]^2)))/8
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6035, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(d+ex)\cosh(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \; 6035$$

$$\int \left(\frac{1}{8}\sinh(4d+4ex)F^{c(a+bx)}-\frac{1}{4}\sinh(2d+2ex)F^{c(a+bx)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{bc\log(F)\sinh(2d+2ex)F^{c(a+bx)}}{4\left(4e^2-b^2c^2\log^2(F)\right)}-\frac{bc\log(F)\sinh(4d+4ex)F^{c(a+bx)}}{8\left(16e^2-b^2c^2\log^2(F)\right)}-\frac{e\cosh(2d+2ex)F^{c(a+bx)}}{2\left(4e^2-b^2c^2\log^2(F)\right)}+\frac{e\cosh(4d+4ex)F^{c(a+bx)}}{2\left(16e^2-b^2c^2\log^2(F)\right)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x]^3,x]
```

output
```
-1/2*(e*F^(c*(a + b*x))*Cosh[2*d + 2*e*x])/(4*e^2 - b^2*c^2*Log[F]^2) + (e
*F^(c*(a + b*x))*Cosh[4*d + 4*e*x])/(2*(16*e^2 - b^2*c^2*Log[F]^2)) + (b*c
*F^(c*(a + b*x))*Log[F]*Sinh[2*d + 2*e*x])/(4*(4*e^2 - b^2*c^2*Log[F]^2))
- (b*c*F^(c*(a + b*x))*Log[F]*Sinh[4*d + 4*e*x])/(8*(16*e^2 - b^2*c^2*Log[
F]^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6035
```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(127) = 254$.

Time = 0.66 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.37

$$\frac{(\ln(F)^3 b^3c^3 \mathrm{e}^{8ex+8d} - 2\ln(F)^3 b^3c^3 \mathrm{e}^{6ex+6d} - 4\ln(F)^2 b^2c^2 e\,\mathrm{e}^{8ex+8d} + 4\ln(F)^2 b^2c^2 e\,\mathrm{e}^{6ex+6d} - 4\ln(F)\,bc\,e^2}{}$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)*sinh(e*x+d)^3,x)
```

output
```
1/16*(ln(F)^3*b^3*c^3*exp(8*e*x+8*d)-2*ln(F)^3*b^3*c^3*exp(6*e*x+6*d)-4*ln
(F)^2*b^2*c^2*e*exp(8*e*x+8*d)+4*ln(F)^2*b^2*c^2*e*exp(6*e*x+6*d)-4*ln(F)*
b*c*e^2*exp(8*e*x+8*d)+2*ln(F)^3*b^3*c^3*exp(2*e*x+2*d)+32*ln(F)*b*c*e^2*e
xp(6*e*x+6*d)+16*e^3*exp(8*e*x+8*d)-c^3*b^3*ln(F)^3+4*ln(F)^2*b^2*c^2*e*ex
p(2*e*x+2*d)-64*e^3*exp(6*e*x+6*d)-4*c^2*b^2*ln(F)^2*e-32*ln(F)*b*c*e^2*ex
p(2*e*x+2*d)+4*e^2*b*c*ln(F)-64*e^3*exp(2*e*x+2*d)+16*e^3)/(b*c*ln(F)-2*e)
*exp(-4*e*x-4*d)/(b*c*ln(F)-4*e)/(2*e+b*c*ln(F))/(b*c*ln(F)+4*e)*F^(c*(b*x
+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2900 vs. $2(125) = 250$.

Time = 0.20 (sec) , antiderivative size = 2900, normalized size of antiderivative = 20.86

$$\int F^{c(a+bx)} \cosh(d+ex) \sinh^3(d+ex)\, dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)*sinh(e*x+d)^3,x, algorithm="fricas")
```

output
```
1/16*((16*e^3*cosh(e*x + d)^8 - 64*e^3*cosh(e*x + d)^6 + (b^3*c^3*log(F)^3
 - 4*b^2*c^2*e*log(F)^2 - 4*b*c*e^2*log(F) + 16*e^3)*sinh(e*x + d)^8 + 8*(
b^3*c^3*cosh(e*x + d)*log(F)^3 - 4*b^2*c^2*e*cosh(e*x + d)*log(F)^2 - 4*b*
c*e^2*cosh(e*x + d)*log(F) + 16*e^3*cosh(e*x + d))*sinh(e*x + d)^7 + 2*(22
4*e^3*cosh(e*x + d)^2 + (14*b^3*c^3*cosh(e*x + d)^2 - b^3*c^3)*log(F)^3 -
32*e^3 - 2*(28*b^2*c^2*e*cosh(e*x + d)^2 - b^2*c^2*e)*log(F)^2 - 8*(7*b*c*
e^2*cosh(e*x + d)^2 - 2*b*c*e^2)*log(F))*sinh(e*x + d)^6 + 4*(224*e^3*cosh
(e*x + d)^3 - 96*e^3*cosh(e*x + d) + (14*b^3*c^3*cosh(e*x + d)^3 - 3*b^3*c
^3*cosh(e*x + d))*log(F)^3 - 2*(28*b^2*c^2*e*cosh(e*x + d)^3 - 3*b^2*c^2*e
*cosh(e*x + d))*log(F)^2 - 8*(7*b*c*e^2*cosh(e*x + d)^3 - 6*b*c*e^2*cosh(e
*x + d))*log(F))*sinh(e*x + d)^5 - 64*e^3*cosh(e*x + d)^2 + 10*(112*e^3*co
sh(e*x + d)^4 - 96*e^3*cosh(e*x + d)^2 + (7*b^3*c^3*cosh(e*x + d)^4 - 3*b^
3*c^3*cosh(e*x + d)^2)*log(F)^3 - 2*(14*b^2*c^2*e*cosh(e*x + d)^4 - 3*b^2*
c^2*e*cosh(e*x + d)^2)*log(F)^2 - 4*(7*b*c*e^2*cosh(e*x + d)^4 - 12*b*c*e^
2*cosh(e*x + d)^2)*log(F))*sinh(e*x + d)^4 + (b^3*c^3*cosh(e*x + d)^8 - 2*
b^3*c^3*cosh(e*x + d)^6 + 2*b^3*c^3*cosh(e*x + d)^2 - b^3*c^3)*log(F)^3 +
8*(112*e^3*cosh(e*x + d)^5 - 160*e^3*cosh(e*x + d)^3 + (7*b^3*c^3*cosh(e*x
 + d)^5 - 5*b^3*c^3*cosh(e*x + d)^3)*log(F)^3 - 2*(14*b^2*c^2*e*cosh(e*x +
 d)^5 - 5*b^2*c^2*e*cosh(e*x + d)^3)*log(F)^2 - 4*(7*b*c*e^2*cosh(e*x + d)
^5 - 20*b*c*e^2*cosh(e*x + d)^3)*log(F))*sinh(e*x + d)^3 + 16*e^3 - 4*(...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1926 vs. $2(124) = 248$.

Time = 24.10 (sec) , antiderivative size = 1926, normalized size of antiderivative = 13.86

$$\int F^{c(a+bx)} \cosh(d+ex) \sinh^3(d+ex)\, dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d)*sinh(e*x+d)**3,x)
```

output
```
Piecewise((x*sinh(d)**3*cosh(d), Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*sinh(d)
**3*cosh(d), Eq(b, 0) & Eq(e, 0)), (x*sinh(d)**3*cosh(d), Eq(c, 0) & Eq(e,
 0)), (F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/2 - d)**4/8 - F**(a*c + b*c*x)
*x*sinh(b*c*x*log(F)/2 - d)**3*cosh(b*c*x*log(F)/2 - d)/4 + F**(a*c + b*c*
x)*x*sinh(b*c*x*log(F)/2 - d)*cosh(b*c*x*log(F)/2 - d)**3/4 - F**(a*c + b*
c*x)*x*cosh(b*c*x*log(F)/2 - d)**4/8 - 7*F**(a*c + b*c*x)*sinh(b*c*x*log(F
)/2 - d)**4/(24*b*c*log(F)) + F**(a*c + b*c*x)*sinh(b*c*x*log(F)/2 - d)**3
*cosh(b*c*x*log(F)/2 - d)/(3*b*c*log(F)) - F**(a*c + b*c*x)*sinh(b*c*x*log
(F)/2 - d)**2*cosh(b*c*x*log(F)/2 - d)**2/(2*b*c*log(F)) + F**(a*c + b*c*x
)*cosh(b*c*x*log(F)/2 - d)**4/(8*b*c*log(F)), Eq(e, -b*c*log(F)/2)), (F**(
a*c + b*c*x)*x*sinh(b*c*x*log(F)/4 - d)**4/16 - F**(a*c + b*c*x)*x*sinh(b*
c*x*log(F)/4 - d)**3*cosh(b*c*x*log(F)/4 - d)/4 + 3*F**(a*c + b*c*x)*x*sin
h(b*c*x*log(F)/4 - d)**2*cosh(b*c*x*log(F)/4 - d)**2/8 - F**(a*c + b*c*x)*
x*sinh(b*c*x*log(F)/4 - d)*cosh(b*c*x*log(F)/4 - d)**3/4 + F**(a*c + b*c*x
)*x*cosh(b*c*x*log(F)/4 - d)**4/16 + F**(a*c + b*c*x)*sinh(b*c*x*log(F)/4
- d)**4/(6*b*c*log(F)) - 11*F**(a*c + b*c*x)*sinh(b*c*x*log(F)/4 - d)**3*c
osh(b*c*x*log(F)/4 - d)/(12*b*c*log(F)) + 5*F**(a*c + b*c*x)*sinh(b*c*x*lo
g(F)/4 - d)*cosh(b*c*x*log(F)/4 - d)**3/(12*b*c*log(F)) - F**(a*c + b*c*x)
*cosh(b*c*x*log(F)/4 - d)**4/(6*b*c*log(F)), Eq(e, -b*c*log(F)/4)), (-F**(
a*c + b*c*x)*x*sinh(b*c*x*log(F)/4 + d)**4/16 + F**(a*c + b*c*x)*x*sinh...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.03

$$\int F^{c(a+bx)}\cosh(d+ex)\sinh^3(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+4\,ex+4\,d)}}{16\,(bc\log(F)+4\,e)} - \frac{F^{ac}e^{(bcx\log(F)+2\,ex+2\,d)}}{8\,(bc\log(F)+2\,e)} + \frac{F^{ac}e^{(bcx\log(F)-2\,ex)}}{8\,(bce^{(2\,d)}\log(F)-2\,ee^{(2\,d)})} - \frac{F^{ac}e^{(bcx\log(F)-4\,ex)}}{16\,(bce^{(4\,d)}\log(F)-4\,ee^{(4\,d)})}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)*sinh(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/16*F^(a*c)*e^(b*c*x*log(F) + 4*e*x + 4*d)/(b*c*log(F) + 4*e) - 1/8*F^(a*
c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) + 1/8*F^(a*c)*e^(b*c*
x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) - 1/16*F^(a*c)*e^(b*c
*x*log(F) - 4*e*x)/(b*c*e^(4*d)*log(F) - 4*e*e^(4*d))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 1225, normalized size of antiderivative = 8.81

$$\int F^{c(a+bx)}\cosh(d+ex)\sinh^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)*sinh(e*x+d)^3,x, algorithm="giac")
```

output
```
1/8*(2*(b*c*log(abs(F)) + 4*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1
/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(ab
s(F)) + 4*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*
pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4
*(b*c*log(abs(F)) + 4*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 4*e)*
x + 4*d) + 1/2*I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a
*c*sgn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs
(F)) + 64*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c
*sgn(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(
F)) + 64*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 4*e)*x + 4*d) - 1/4*(
2*(b*c*log(abs(F)) + 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi
*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F))
 + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*
c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c
*log(abs(F)) + 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x + 2
*d) + 1/2*I*(-I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*s
gn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F))
+ 16*e) + I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(
F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) +
16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x + 2*d) + 1/4*(2*(...
```

Mupad [B] (verification not implemented)

Time = 4.97 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.42

$$\int F^{c(a+bx)}\cosh(d+ex)\sinh^3(d+ex)\,dx =$$

$$-\frac{F^{a\,c+b\,c\,x}\left(8\,e^3\cosh(2\,d+2\,e\,x)-2\,e^3\cosh(4\,d+4\,e\,x)+\frac{b^3\,c^3\ln(F)^3\sinh(2\,d+2\,e\,x)}{4}-\frac{b^3\,c^3\ln(F)^3\sinh(4\,d+4\,e\,x)}{8}\right.}{b^4\,c^4\ln(F)^4-20}$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x)*sinh(d + e*x)^3,x)
```

output
```
-(F^(a*c + b*c*x)*(8*e^3*cosh(2*d + 2*e*x) - 2*e^3*cosh(4*d + 4*e*x) + (b^
3*c^3*log(F)^3*sinh(2*d + 2*e*x))/4 - (b^3*c^3*log(F)^3*sinh(4*d + 4*e*x))
/8 - (b^2*c^2*e*log(F)^2*cosh(2*d + 2*e*x))/2 + (b^2*c^2*e*log(F)^2*cosh(4
*d + 4*e*x))/2 - 4*b*c*e^2*log(F)*sinh(2*d + 2*e*x) + (b*c*e^2*log(F)*sinh
(4*d + 4*e*x))/2))/(64*e^4 + b^4*c^4*log(F)^4 - 20*b^2*c^2*e^2*log(F)^2)
```

Reduce [F]

$$\int F^{c(a+bx)}\cosh(d+ex)\sinh^3(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\cosh\left(ex+d\right)\sinh\left(ex+d\right)^3 dx\right)$$

input

```
int(F^(c*(b*x+a))*cosh(e*x+d)*sinh(e*x+d)^3,x)
```

output

```
f**(a*c)*int(f**(b*c*x)*cosh(d + e*x)*sinh(d + e*x)**3,x)
```

3.96 $\int F^{c(a+bx)}\cosh(d+ex)\sinh^2(d+ex)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int F^{c(a+bx)}\cosh(d+ex)\sinh^2(d+ex)\,dx=\frac{e^{-d-ex}F^{c(a+bx)}}{8(e-bc\log(F))}-\frac{e^{-3d-3ex}F^{c(a+bx)}}{8(3e-bc\log(F))}-\frac{e^{d+ex}F^{c(a+bx)}}{8(e+bc\log(F))}+\frac{e^{3d+3ex}F^{c(a+bx)}}{8(3e+bc\log(F))}$$

output
```
exp(-e*x-d)*F^(c*(b*x+a))/(8*e-8*b*c*ln(F))-exp(-3*e*x-3*d)*F^(c*(b*x+a))/
(24*e-8*b*c*ln(F))-exp(e*x+d)*F^(c*(b*x+a))/(8*e+8*b*c*ln(F))+exp(3*e*x+3*
d)*F^(c*(b*x+a))/(24*e+8*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.77

$$\int F^{c(a+bx)}\cosh(d+ex)\sinh^2(d+ex)\,dx=\frac{1}{4}F^{c(a+bx)}\left(\frac{bc\cosh(d+ex)\log(F)-e\sinh(d+ex)}{(e-bc\log(F))(e+bc\log(F))}+\frac{-bc\cosh(3(d+ex))\log(F)+3e\sinh(3(d+ex))}{9e^2-b^2c^2\log^2(F)}\right)$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x]^2,x]
```

output
```
(F^(c*(a + b*x))*((b*c*Cosh[d + e*x]*Log[F] - e*Sinh[d + e*x])/((e - b*c*L
og[F])*(e + b*c*Log[F])) + (-(b*c*Cosh[3*(d + e*x)]*Log[F]) + 3*e*Sinh[3*(
d + e*x)])/(9*e^2 - b^2*c^2*Log[F]^2)))/4
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6035, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(d+ex)\cosh(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6035}$$

$$\int \left(\frac{1}{4}\cosh(3d+3ex)F^{c(a+bx)}-\frac{1}{4}\cosh(d+ex)F^{c(a+bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e\sinh(d+ex)F^{c(a+bx)}}{4\left(e^2-b^2c^2\log^2(F)\right)}+\frac{3e\sinh(3d+3ex)F^{c(a+bx)}}{4\left(9e^2-b^2c^2\log^2(F)\right)}+\frac{bc\log(F)\cosh(d+ex)F^{c(a+bx)}}{4\left(e^2-b^2c^2\log^2(F)\right)}-\frac{bc\log(F)\cosh(3d+3ex)F^{c(a+bx)}}{4\left(9e^2-b^2c^2\log^2(F)\right)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x]^2,x]
```

output
```
(b*c*F^(c*(a + b*x))*Cosh[d + e*x]*Log[F])/(4*(e^2 - b^2*c^2*Log[F]^2)) -
(b*c*F^(c*(a + b*x))*Cosh[3*d + 3*e*x]*Log[F])/(4*(9*e^2 - b^2*c^2*Log[F]^
2)) - (e*F^(c*(a + b*x))*Sinh[d + e*x])/(4*(e^2 - b^2*c^2*Log[F]^2)) + (3*
e*F^(c*(a + b*x))*Sinh[3*d + 3*e*x])/(4*(9*e^2 - b^2*c^2*Log[F]^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6035
```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(124) = 248$.

Time = 96.04 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.46

method	result
risch	$\frac{\left(\ln(F)^3 b^3 c^3 e^{6ex+6d} - \ln(F)^3 b^3 c^3 e^{4ex+4d} - 3\ln(F)^2 b^2 c^2 e\, e^{6ex+6d} - \ln(F)^3 b^3 c^3 e^{2ex+2d} + \ln(F)^2 b^2 c^2 e\, e^{4ex+4d} - \ln(F) bc\, e^2 e^{6ex+6d} +\right.}{8(bc\ln(}$
orering	$\frac{4bc\ln(F)\left(b^2c^2\ln(F)^2-5e^2\right)F^{c(bx+a)}\cosh(ex+d)\sinh(ex+d)^2}{\ln(F)^4b^4c^4-10\ln(F)^2b^2c^2e^2+9e^4} - \frac{2\left(3b^2c^2\ln(F)^2-5e^2\right)\left(F^{c(bx+a)}bc\ln(F)\cosh(ex+d)\sinh(ex+\right.}{\ln(F)^4b^4c^4-1}$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)*sinh(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(ln(F)^3*b^3*c^3*exp(6*e*x+6*d)-ln(F)^3*b^3*c^3*exp(4*e*x+4*d)-3*ln(F)
^2*b^2*c^2*e*exp(6*e*x+6*d)-ln(F)^3*b^3*c^3*exp(2*e*x+2*d)+ln(F)^2*b^2*c^2
*e*exp(4*e*x+4*d)-ln(F)*b*c*e^2*exp(6*e*x+6*d)+c^3*b^3*ln(F)^3-ln(F)^2*b^2
*c^2*e*exp(2*e*x+2*d)+9*ln(F)*b*c*e^2*exp(4*e*x+4*d)+3*e^3*exp(6*e*x+6*d)+
3*c^2*b^2*ln(F)^2*e+9*ln(F)*b*c*e^2*exp(2*e*x+2*d)-9*e^3*exp(4*e*x+4*d)-e^
2*b*c*ln(F)+9*e^3*exp(2*e*x+2*d)-3*e^3)/(b*c*ln(F)-e)*exp(-3*e*x-3*d)/(b*c
*ln(F)-3*e)/(e+b*c*ln(F))/(b*c*ln(F)+3*e)*F^(c*(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2224 vs. $2(120) = 240$.

Time = 0.16 (sec) , antiderivative size = 2224, normalized size of antiderivative = 16.85

$$\int F^{c(a+bx)} \cosh(d+ex) \sinh^2(d+ex) \, dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)*sinh(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/8*((3*e^3*cosh(e*x + d)^6 - 9*e^3*cosh(e*x + d)^4 + (b^3*c^3*log(F)^3 -
3*b^2*c^2*e*log(F)^2 - b*c*e^2*log(F) + 3*e^3)*sinh(e*x + d)^6 + 6*(b^3*c^
3*cosh(e*x + d)*log(F)^3 - 3*b^2*c^2*e*cosh(e*x + d)*log(F)^2 - b*c*e^2*co
sh(e*x + d)*log(F) + 3*e^3*cosh(e*x + d))*sinh(e*x + d)^5 + 9*e^3*cosh(e*x
 + d)^2 + (45*e^3*cosh(e*x + d)^2 + (15*b^3*c^3*cosh(e*x + d)^2 - b^3*c^3)
*log(F)^3 - 9*e^3 - (45*b^2*c^2*e*cosh(e*x + d)^2 - b^2*c^2*e)*log(F)^2 -
3*(5*b*c*e^2*cosh(e*x + d)^2 - 3*b*c*e^2)*log(F))*sinh(e*x + d)^4 + (b^3*c
^3*cosh(e*x + d)^6 - b^3*c^3*cosh(e*x + d)^4 - b^3*c^3*cosh(e*x + d)^2 + b
^3*c^3)*log(F)^3 + 4*(15*e^3*cosh(e*x + d)^3 - 9*e^3*cosh(e*x + d) + (5*b^
3*c^3*cosh(e*x + d)^3 - b^3*c^3*cosh(e*x + d))*log(F)^3 - (15*b^2*c^2*e*co
sh(e*x + d)^3 - b^2*c^2*e*cosh(e*x + d))*log(F)^2 - (5*b*c*e^2*cosh(e*x +
d)^3 - 9*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d)^3 - 3*e^3 - (3*b^2*c
^2*e*cosh(e*x + d)^6 - b^2*c^2*e*cosh(e*x + d)^4 + b^2*c^2*e*cosh(e*x + d)
^2 - 3*b^2*c^2*e)*log(F)^2 + (45*e^3*cosh(e*x + d)^4 - 54*e^3*cosh(e*x + d
)^2 + (15*b^3*c^3*cosh(e*x + d)^4 - 6*b^3*c^3*cosh(e*x + d)^2 - b^3*c^3)*l
og(F)^3 + 9*e^3 - (45*b^2*c^2*e*cosh(e*x + d)^4 - 6*b^2*c^2*e*cosh(e*x + d
)^2 + b^2*c^2*e)*log(F)^2 - 3*(5*b*c*e^2*cosh(e*x + d)^4 - 18*b*c*e^2*cosh
(e*x + d)^2 - 3*b*c*e^2)*log(F))*sinh(e*x + d)^2 - (b*c*e^2*cosh(e*x + d)^
6 - 9*b*c*e^2*cosh(e*x + d)^4 - 9*b*c*e^2*cosh(e*x + d)^2 + b*c*e^2)*log(F
) + 2*(9*e^3*cosh(e*x + d)^5 - 18*e^3*cosh(e*x + d)^3 + 9*e^3*cosh(e*x ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1476 vs. $2(117) = 234$.

Time = 3.25 (sec) , antiderivative size = 1476, normalized size of antiderivative = 11.18

$$\int F^{c(a+bx)} \cosh(d+ex) \sinh^2(d+ex)\, dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d)*sinh(e*x+d)**2,x)
```

output
```
Piecewise((x*sinh(d)**2*cosh(d), Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*sinh(d)
**2*cosh(d), Eq(b, 0) & Eq(e, 0)), (x*sinh(d)**2*cosh(d), Eq(c, 0) & Eq(e,
 0)), (-F**(a*c + b*c*x)*x*sinh(b*c*x*log(F) - d)**3/8 + F**(a*c + b*c*x)*
x*sinh(b*c*x*log(F) - d)**2*cosh(b*c*x*log(F) - d)/8 + F**(a*c + b*c*x)*x*
sinh(b*c*x*log(F) - d)*cosh(b*c*x*log(F) - d)**2/8 - F**(a*c + b*c*x)*x*co
sh(b*c*x*log(F) - d)**3/8 + 3*F**(a*c + b*c*x)*sinh(b*c*x*log(F) - d)**3/(
8*b*c*log(F)) - F**(a*c + b*c*x)*sinh(b*c*x*log(F) - d)**2*cosh(b*c*x*log(
F) - d)/(4*b*c*log(F)) + F**(a*c + b*c*x)*cosh(b*c*x*log(F) - d)**3/(8*b*c
*log(F)), Eq(e, -b*c*log(F))), (-F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 -
d)**3/8 + 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)**2*cosh(b*c*x*log(
F)/3 - d)/8 - 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)*cosh(b*c*x*log
(F)/3 - d)**2/8 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F)/3 - d)**3/8 - F**(a
*c + b*c*x)*sinh(b*c*x*log(F)/3 - d)**3/(8*b*c*log(F)) + 3*F**(a*c + b*c*x
)*sinh(b*c*x*log(F)/3 - d)**2*cosh(b*c*x*log(F)/3 - d)/(4*b*c*log(F)) - F*
*(a*c + b*c*x)*cosh(b*c*x*log(F)/3 - d)**3/(8*b*c*log(F)), Eq(e, -b*c*log(
F)/3)), (-F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 + d)**3/8 + 3*F**(a*c + b
*c*x)*x*sinh(b*c*x*log(F)/3 + d)**2*cosh(b*c*x*log(F)/3 + d)/8 - 3*F**(a*c
 + b*c*x)*x*sinh(b*c*x*log(F)/3 + d)*cosh(b*c*x*log(F)/3 + d)**2/8 + F**(a
*c + b*c*x)*x*cosh(b*c*x*log(F)/3 + d)**3/8 - F**(a*c + b*c*x)*sinh(b*c*x*
log(F)/3 + d)**3/(8*b*c*log(F)) + 3*F**(a*c + b*c*x)*sinh(b*c*x*log(F)/...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02

$$\int F^{c(a+bx)}\cosh(d+ex)\sinh^2(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+3\,ex+3\,d)}}{8\,(bc\log(F)+3\,e)} - \frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{8\,(bc\log(F)+e)} - \frac{F^{ac}e^{(bcx\log(F)-ex)}}{8\,(bce^{d}\log(F)-ee^{d})} + \frac{F^{ac}e^{(bcx\log(F)-3\,ex)}}{8\,(bce^{(3\,d)}\log(F)-3\,ee^{(3\,d)})}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)*sinh(e*x+d)^2,x, algorithm="maxima")
```

output
```
1/8*F^(a*c)*e^(b*c*x*log(F) + 3*e*x + 3*d)/(b*c*log(F) + 3*e) - 1/8*F^(a*c
)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) - 1/8*F^(a*c)*e^(b*c*x*log(F
) - e*x)/(b*c*e^d*log(F) - e*e^d) + 1/8*F^(a*c)*e^(b*c*x*log(F) - 3*e*x)/(
b*c*e^(3*d)*log(F) - 3*e*e^(3*d))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 1211, normalized size of antiderivative = 9.17

$$\int F^{c(a+bx)}\cosh(d+ex)\sinh^2(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)*sinh(e*x+d)^2,x, algorithm="giac")
```

output
```
1/4*(2*(b*c*log(abs(F)) + 3*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1
/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(ab
s(F)) + 3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*
pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4
*(b*c*log(abs(F)) + 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*
x + 3*d) + 1/2*I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a
*c*sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(
F)) + 24*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*
sgn(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)
) + 24*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3*d) - 1/4*(2*
(b*c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c
*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e
)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x -
1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(a
bs(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 1/2*I*
(-I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*
I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F)) + 8*e) + I*e
^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi
*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) + 8*e))*e^(a*c*
log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) - 1/4*(2*(b*c*log(abs(F)) - ...
```

Mupad [B] (verification not implemented)

Time = 3.52 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.30

$$\int F^{c(a+bx)}\cosh(d+ex)\sinh^2(d+ex)\,dx$$
$$= \frac{F^{a\,c+b\,c\,x}\left(b^3\,c^3\cosh(d+e\,x)\,\sinh(d+e\,x)^2\ln(F)^3-2\,b^2\,c^2\,e\cosh(d+e\,x)^2\sinh(d+e\,x)\,\ln(F)^2-b^2\,c^2\right.}{b^4\,c^4\ln(F)}$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x)*sinh(d + e*x)^2,x)
```

output
```
(F^(a*c + b*c*x)*(3*e^3*sinh(d + e*x)^3 + b^3*c^3*cosh(d + e*x)*sinh(d + e
*x)^2*log(F)^3 - b^2*c^2*e*sinh(d + e*x)^3*log(F)^2 + 2*b*c*e^2*cosh(d + e
*x)^3*log(F) - 2*b^2*c^2*e*cosh(d + e*x)^2*sinh(d + e*x)*log(F)^2 - 3*b*c*
e^2*cosh(d + e*x)*sinh(d + e*x)^2*log(F)))/(9*e^4 + b^4*c^4*log(F)^4 - 10*
b^2*c^2*e^2*log(F)^2)
```

Reduce [F]

$$\int F^{c(a+bx)}\cosh(d+ex)\sinh^2(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\cosh\left(ex+d\right)\sinh\left(ex+d\right)^2dx\right)$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)*sinh(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cosh(d + e*x)*sinh(d + e*x)**2,x)
```

3.97 $\int F^{c(a+bx)} \cosh(d+ex) \sinh(d+ex) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int F^{c(a+bx)} \cosh(d+ex) \sinh(d+ex) \, dx = \frac{e^{-2d-2ex} F^{c(a+bx)}}{4(2e-bc\log(F))} + \frac{e^{2d+2ex} F^{c(a+bx)}}{4(2e+bc\log(F))}$$

output
```
exp(-2*e*x-2*d)*F^(c*(b*x+a))/(8*e-4*b*c*ln(F))+exp(2*e*x+2*d)*F^(c*(b*x+a
))/(8*e+4*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int F^{c(a+bx)} \cosh(d+ex) \sinh(d+ex) \, dx = \frac{F^{c(a+bx)}(2e\cosh(2(d+ex)) - bc\log(F)\sinh(2(d+ex)))}{2\left(4e^2-b^2c^2\log^2(F)\right)}$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*(2*e*Cosh[2*(d + e*x)] - b*c*Log[F]*Sinh[2*(d + e*x)]))/(
2*(4*e^2 - b^2*c^2*Log[F]^2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6035, 27, 5997}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6035}$$

$$\int \frac{1}{2}\sinh(2d+2ex)F^{c(a+bx)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int F^{c(a+bx)}\sinh(2d+2ex)dx$$

$$\Big\downarrow \text{5997}$$

$$\frac{1}{2}\left(\frac{2e\cosh(2d+2ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\sinh(2d+2ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}\right)$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x],x]
```

output
```
((2*e*F^(c*(a + b*x))*Cosh[2*d + 2*e*x])/(4*e^2 - b^2*c^2*Log[F]^2) - (b*c
*F^(c*(a + b*x))*Log[F]*Sinh[2*d + 2*e*x])/(4*e^2 - b^2*c^2*Log[F]^2))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5997
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

rule 6035
```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 6.52 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

method	result
risch	$\frac{(\ln(F)bc\,e^{4ex+4d}-2\,e^{4ex+4d}e-bc\ln(F)-2e)\,e^{-2ex-2d}F^{c(bx+a)}}{4(bc\ln(F)-2e)(2e+bc\ln(F))}$
orering	$\frac{2bc\ln(F)F^{c(bx+a)}\cosh(ex+d)\sinh(ex+d)}{b^2c^2\ln(F)^2-4e^2}-\frac{F^{c(bx+a)}bc\ln(F)\cosh(ex+d)\sinh(ex+d)+F^{c(bx+a)}e\sinh(ex+d)^2+F^{c(bx+a)}\cosh}{b^2c^2\ln(F)^2-4e^2}$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)*sinh(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*(ln(F)*b*c*exp(4*e*x+4*d)-2*exp(4*e*x+4*d)*e-b*c*ln(F)-2*e)/(b*c*ln(F)
-2*e)*exp(-2*e*x-2*d)/(2*e+b*c*ln(F))*F^(c*(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(63) = 126$.

Time = 0.09 (sec) , antiderivative size = 433, normalized size of antiderivative = 6.19

$$\int F^{c(a+bx)}\cosh(d+ex)\sinh(d+ex)\,dx =$$
$$-\frac{\left(2\,e\cosh\left(ex+d\right)^4-\left(bc\log\left(F\right)-2\,e\right)\sinh\left(ex+d\right)^4-4\left(bc\cosh\left(ex+d\right)\log\left(F\right)-2\,e\cosh\left(ex+d\right)\right)}{}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)*sinh(e*x+d),x, algorithm="fricas")
```

output
```
-1/4*((2*e*cosh(e*x + d)^4 - (b*c*log(F) - 2*e)*sinh(e*x + d)^4 - 4*(b*c*c
osh(e*x + d)*log(F) - 2*e*cosh(e*x + d))*sinh(e*x + d)^3 - 6*(b*c*cosh(e*x
 + d)^2*log(F) - 2*e*cosh(e*x + d)^2)*sinh(e*x + d)^2 - (b*c*cosh(e*x + d)
^4 - b*c)*log(F) - 4*(b*c*cosh(e*x + d)^3*log(F) - 2*e*cosh(e*x + d)^3)*si
nh(e*x + d) + 2*e)*cosh((b*c*x + a*c)*log(F)) + (2*e*cosh(e*x + d)^4 - (b*
c*log(F) - 2*e)*sinh(e*x + d)^4 - 4*(b*c*cosh(e*x + d)*log(F) - 2*e*cosh(e
*x + d))*sinh(e*x + d)^3 - 6*(b*c*cosh(e*x + d)^2*log(F) - 2*e*cosh(e*x +
d)^2)*sinh(e*x + d)^2 - (b*c*cosh(e*x + d)^4 - b*c)*log(F) - 4*(b*c*cosh(e
*x + d)^3*log(F) - 2*e*cosh(e*x + d)^3)*sinh(e*x + d) + 2*e)*sinh((b*c*x +
 a*c)*log(F)))/(b^2*c^2*cosh(e*x + d)^2*log(F)^2 - 4*e^2*cosh(e*x + d)^2 +
 (b^2*c^2*log(F)^2 - 4*e^2)*sinh(e*x + d)^2 + 2*(b^2*c^2*cosh(e*x + d)*log
(F)^2 - 4*e^2*cosh(e*x + d))*sinh(e*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(61) = 122$.

Time = 1.15 (sec) , antiderivative size = 454, normalized size of antiderivative = 6.49

$$\int F^{c(a+bx)}\cosh(d+ex)\sinh(d+ex)\,dx$$

$$= \begin{cases} x\sinh(d)\cosh(d) \\ F^{ac}x\sinh(d)\cosh(d) \\ x\sinh(d)\cosh(d) \\ \frac{F^{ac+bcx}x\sinh^2\left(\frac{bcx\log(F)}{2}-d\right)}{4} - \frac{F^{ac+bcx}x\sinh\left(\frac{bcx\log(F)}{2}-d\right)\cosh\left(\frac{bcx\log(F)}{2}-d\right)}{2} + \frac{F^{ac+bcx}x\cosh^2\left(\frac{bcx\log(F)}{2}-d\right)}{4} - \frac{F^{ac+bcx}\ s}{} \\ -\frac{F^{ac+bcx}x\sinh^2\left(\frac{bcx\log(F)}{2}+d\right)}{4} + \frac{F^{ac+bcx}x\sinh\left(\frac{bcx\log(F)}{2}+d\right)\cosh\left(\frac{bcx\log(F)}{2}+d\right)}{2} - \frac{F^{ac+bcx}x\cosh^2\left(\frac{bcx\log(F)}{2}+d\right)}{4} + \frac{F^{ac+bc}}{} \\ \frac{F^{ac+bcx}bc\log(F)\sinh(d+ex)\cosh(d+ex)}{b^2c^2\log(F)^2-4e^2} - \frac{F^{ac+bcx}e\sinh^2(d+ex)}{b^2c^2\log(F)^2-4e^2} - \frac{F^{ac+bcx}e\cosh^2(d+ex)}{b^2c^2\log(F)^2-4e^2} \end{cases}$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d)*sinh(e*x+d),x)
```

output
```
Piecewise((x*sinh(d)*cosh(d), Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*sinh(d)*co
sh(d), Eq(b, 0) & Eq(e, 0)), (x*sinh(d)*cosh(d), Eq(c, 0) & Eq(e, 0)), (F*
*(a*c + b*c*x)*x*sinh(b*c*x*log(F)/2 - d)**2/4 - F**(a*c + b*c*x)*x*sinh(b
*c*x*log(F)/2 - d)*cosh(b*c*x*log(F)/2 - d)/2 + F**(a*c + b*c*x)*x*cosh(b*
c*x*log(F)/2 - d)**2/4 - F**(a*c + b*c*x)*sinh(b*c*x*log(F)/2 - d)*cosh(b*
c*x*log(F)/2 - d)/(2*b*c*log(F)), Eq(e, -b*c*log(F)/2)), (-F**(a*c + b*c*x
)*x*sinh(b*c*x*log(F)/2 + d)**2/4 + F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/2
 + d)*cosh(b*c*x*log(F)/2 + d)/2 - F**(a*c + b*c*x)*x*cosh(b*c*x*log(F)/2
+ d)**2/4 + F**(a*c + b*c*x)*sinh(b*c*x*log(F)/2 + d)*cosh(b*c*x*log(F)/2
+ d)/(2*b*c*log(F)), Eq(e, b*c*log(F)/2)), (F**(a*c + b*c*x)*b*c*log(F)*si
nh(d + e*x)*cosh(d + e*x)/(b**2*c**2*log(F)**2 - 4*e**2) - F**(a*c + b*c*x
)*e*sinh(d + e*x)**2/(b**2*c**2*log(F)**2 - 4*e**2) - F**(a*c + b*c*x)*e*c
osh(d + e*x)**2/(b**2*c**2*log(F)**2 - 4*e**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

$$\int F^{c(a+bx)}\cosh(d+ex)\sinh(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+2\,ex+2\,d)}}{4\,(bc\log{(F)}+2\,e)} - \frac{F^{ac}e^{(bcx\log(F)-2\,ex)}}{4\,(bce^{(2\,d)}\log{(F)}-2\,ee^{(2\,d)})}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)*sinh(e*x+d),x, algorithm="maxima")
```

output
```
1/4*F^(a*c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) - 1/4*F^(a*c
)*e^(b*c*x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 613, normalized size of antiderivative = 8.76

$$\int F^{c(a+bx)}\cosh(d+ex)\sinh(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)*sinh(e*x+d),x, algorithm="giac")
```

output
```
1/2*(2*(b*c*log(abs(F)) + 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1
/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(ab
s(F)) + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*
pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4
*(b*c*log(abs(F)) + 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*
x + 2*d) + 1/4*I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a
*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F))
+ 4*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F
) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F)) + 4*e))
*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x + 2*d) - 1/2*(2*(b*c*log(a
bs(F)) - 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F)
+ 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - 2*e)^2) -
 (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi
*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F))
 - 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - 2*e)*x - 2*d) + 1/4*I*
(-I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*
I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) - 4*e) + I*e^(-1
/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c
)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F)) - 4*e))*e^(a*c*log(abs(
F)) + (b*c*log(abs(F)) - 2*e)*x - 2*d)
```

Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.37

$$\int F^{c(a+bx)}\cosh(d+ex)\sinh(d+ex)\,dx = \frac{F^{a\,c+b\,c\,x}\,e\,\left(\frac{{\mathrm{e}}^{-2\,d-2\,e\,x}}{2}+\frac{{\mathrm{e}}^{2\,d+2\,e\,x}}{2}\right)+\frac{F^{a\,c+b\,c\,x}\,b\,c\,\ln\left(F\right)\,\left(\frac{{\mathrm{e}}^{-2\,d-2\,e\,x}}{2}-\frac{{\mathrm{e}}^{2\,d+2\,e\,x}}{2}\right)}{2}}{4\,e^2-b^2\,c^2\,{\ln\left(F\right)}^2}$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x)*sinh(d + e*x),x)
```

output
```
(F^(a*c + b*c*x)*e*(exp(- 2*d - 2*e*x)/2 + exp(2*d + 2*e*x)/2) + (F^(a*c +
 b*c*x)*b*c*log(F)*(exp(- 2*d - 2*e*x)/2 - exp(2*d + 2*e*x)/2))/2)/(4*e^2
- b^2*c^2*log(F)^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}\cosh(d+ex)\sinh(d+ex)\,dx = \frac{f^{bcx+ac}\left(-\cosh\left(ex+d\right)^{2}e+\cosh\left(ex+d\right)\log(f)\sinh\left(ex+d\right)bc-\sinh\left(ex+d\right)^{2}e\right)}{\log\left(f\right)^{2}b^{2}c^{2}-4e^{2}}$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)*sinh(e*x+d),x)
```

output
```
(f**(a*c + b*c*x)*( - cosh(d + e*x)**2*e + cosh(d + e*x)*log(f)*sinh(d + e
*x)*b*c - sinh(d + e*x)**2*e))/(log(f)**2*b**2*c**2 - 4*e**2)
```

3.98 $\int F^{c(a+bx)}\cosh(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 63

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx=-\frac{e^{-d-ex}F^{c(a+bx)}}{2(e-bc\log(F))}+\frac{e^{d+ex}F^{c(a+bx)}}{2(e+bc\log(F))}$$

output
```
-1/2*exp(-e*x-d)*F^(c*(b*x+a))/(e-b*c*ln(F))+exp(e*x+d)*F^(c*(b*x+a))/(2*e
+2*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx=\frac{F^{c(a+bx)}(-bc\cosh(d+ex)\log(F)+e\sinh(d+ex))}{(e-bc\log(F))(e+bc\log(F))}$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*(-(b*c*Cosh[d + e*x]*Log[F]) + e*Sinh[d + e*x]))/((e - b*
c*Log[F])*(e + b*c*Log[F]))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5998}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{ 5998}$$

$$\frac{e\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x],x]
```

output
```
-((b*c*F^(c*(a + b*x))*Cosh[d + e*x]*Log[F])/(e^2 - b^2*c^2*Log[F]^2)) + (
e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 5998
```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

method	result	size
parallelrisch	$\frac{(bc\cosh(ex+d)\ln(F)-e\sinh(ex+d))F^{c(bx+a)}}{b^2c^2\ln(F)^2-e^2}$	51
risch	$\frac{(\ln(F)bc\,e^{2ex+2d}+bc\ln(F)-e^{2ex+2d}e+e)\,e^{-ex-d}F^{c(bx+a)}}{2(bc\ln(F)-e)(e+bc\ln(F))}$	74
orering	$\frac{2\ln(F)bc\cosh(ex+d)F^{c(bx+a)}}{b^2c^2\ln(F)^2-e^2}-\frac{e\sinh(ex+d)F^{c(bx+a)}+\cosh(ex+d)F^{c(bx+a)}bc\ln(F)}{b^2c^2\ln(F)^2-e^2}$	101

input
```
int(cosh(e*x+d)*F^(c*(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
(b*c*cosh(e*x+d)*ln(F)-e*sinh(e*x+d))*F^(c*(b*x+a))/(b^2*c^2*ln(F)^2-e^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(58) = 116$.

Time = 0.12 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.90

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx =$$
$$-\frac{\left(e\cosh\left(ex+d\right)^2-\left(bc\log\left(F\right)-e\right)\sinh\left(ex+d\right)^2-\left(bc\cosh\left(ex+d\right)^2+bc\right)\log\left(F\right)-2\left(bc\cosh\left(ex+\right.\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d),x, algorithm="fricas")
```

output
```
-1/2*((e*cosh(e*x + d)^2 - (b*c*log(F) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*
x + d)^2 + b*c)*log(F) - 2*(b*c*cosh(e*x + d)*log(F) - e*cosh(e*x + d))*si
nh(e*x + d) - e)*cosh((b*c*x + a*c)*log(F)) + (e*cosh(e*x + d)^2 - (b*c*lo
g(F) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*x + d)^2 + b*c)*log(F) - 2*(b*c*co
sh(e*x + d)*log(F) - e*cosh(e*x + d))*sinh(e*x + d) - e)*sinh((b*c*x + a*c
)*log(F)))/(b^2*c^2*cosh(e*x + d)*log(F)^2 - e^2*cosh(e*x + d) + (b^2*c^2*
log(F)^2 - e^2)*sinh(e*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(54) = 108$.

Time = 0.65 (sec) , antiderivative size = 265, normalized size of antiderivative = 4.21

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx$$
$$= \begin{cases} x\cosh(d) & \text{for } F = 1 \wedge e = 0 \\ F^{ac}x\cosh(d) & \text{for } b = 0 \wedge e = 0 \\ x\cosh(d) & \text{for } c = 0 \wedge e = 0 \\ -\frac{F^{ac+bcx}x\sinh(bcx\log(F)-d)}{2} + \frac{F^{ac+bcx}x\cosh(bcx\log(F)-d)}{2} + \frac{F^{ac+bcx}\sinh(bcx\log(F)-d)}{2bc\log(F)} & \text{for } e = -bc\log(F) \\ -\frac{F^{ac+bcx}x\sinh(bcx\log(F)+d)}{2} + \frac{F^{ac+bcx}x\cosh(bcx\log(F)+d)}{2} + \frac{F^{ac+bcx}\sinh(bcx\log(F)+d)}{2bc\log(F)} & \text{for } e = bc\log(F) \\ \frac{F^{ac+bcx}bc\log(F)\cosh(d+ex)}{b^2c^2\log(F)^2-e^2} - \frac{F^{ac+bcx}e\sinh(d+ex)}{b^2c^2\log(F)^2-e^2} & \text{otherwise} \end{cases}$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d),x)
```

output
```
Piecewise((x*cosh(d), Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*cosh(d), Eq(b, 0)
& Eq(e, 0)), (x*cosh(d), Eq(c, 0) & Eq(e, 0)), (-F**(a*c + b*c*x)*x*sinh(b
*c*x*log(F) - d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F) - d)/2 + F**(a*c
 + b*c*x)*sinh(b*c*x*log(F) - d)/(2*b*c*log(F)), Eq(e, -b*c*log(F))), (-F*
*(a*c + b*c*x)*x*sinh(b*c*x*log(F) + d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*
log(F) + d)/2 + F**(a*c + b*c*x)*sinh(b*c*x*log(F) + d)/(2*b*c*log(F)), Eq
(e, b*c*log(F))), (F**(a*c + b*c*x)*b*c*log(F)*cosh(d + e*x)/(b**2*c**2*lo
g(F)**2 - e**2) - F**(a*c + b*c*x)*e*sinh(d + e*x)/(b**2*c**2*log(F)**2 -
e**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{2(bc\log(F)+e)} + \frac{F^{ac}e^{(bcx\log(F)-ex)}}{2(bce^d\log(F)-ee^d)}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d),x, algorithm="maxima")
```

output
```
1/2*F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) + 1/2*F^(a*c)*e^(b
*c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 597, normalized size of antiderivative = 9.48

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d),x, algorithm="giac")
```

output
```
(2*(b*c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*
a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F))
+ e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x
 - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*lo
g(abs(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 1/2
*I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/
2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) + 2*e) - I*e^(
-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a
*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F)) + 2*e))*e^(a*c*log(ab
s(F)) + (b*c*log(abs(F)) + e)*x + d) + (2*(b*c*log(abs(F)) - e)*cos(-1/2*p
i*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*s
gn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sgn(F) - pi*b*c)*
sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/
((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(F
)) + (b*c*log(abs(F)) - e)*x - d) + 1/2*I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/
2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi
*b*c + 2*b*c*log(abs(F)) - 2*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b
*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c +
2*b*c*log(abs(F)) - 2*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d
)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.17

$$\int F^{c(a+bx)} \cosh(d+ex)\,dx$$
$$= -\frac{F^{a\,c+b\,c\,x}\,\mathrm{e}^{-d-e\,x}\,\left(e-e\,\mathrm{e}^{2\,d+2\,e\,x}+b\,c\,\ln\left(F\right)+b\,c\,\mathrm{e}^{2\,d+2\,e\,x}\,\ln\left(F\right)\right)}{2\,\left(e^2-b^2\,c^2\,\ln\left(F\right)^2\right)}$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x),x)
```

output
```
-(F^(a*c + b*c*x)*exp(- d - e*x)*(e - e*exp(2*d + 2*e*x) + b*c*log(F) + b*
c*exp(2*d + 2*e*x)*log(F)))/(2*(e^2 - b^2*c^2*log(F)^2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int F^{c(a+bx)} \cosh(d+ex)\,dx = \frac{f^{bcx+ac}(\cosh\left(ex+d\right)\log(f)\,bc-\sinh\left(ex+d\right)e)}{\log\left(f\right)^2 b^2c^2-e^2}$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d),x)
```

output
```
(f**(a*c + b*c*x)*(cosh(d + e*x)*log(f)*b*c - sinh(d + e*x)*e))/(log(f)**2
*b**2*c**2 - e**2)
```

3.99 $\int F^{c(a+bx)} \coth(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 78

$$\int F^{c(a+bx)} \coth(d+ex)\,dx = \frac{F^{c(a+bx)}}{bc\log(F)} - \frac{2F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1, \frac{bc\log(F)}{2e}, 1+\frac{bc\log(F)}{2e}, e^{2d+2ex}\right)}{bc\log(F)}$$

output
```
F^(c*(b*x+a))/b/c/ln(F)-2*F^(c*(b*x+a))*hypergeom([1, 1/2*b*c*ln(F)/e],[1+
1/2*b*c*ln(F)/e],exp(2*e*x+2*d))/b/c/ln(F)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.76

$$\int F^{c(a+bx)} \coth(d+ex)\,dx = \frac{F^{c(a+bx)}\left(1-2\,\text{Hypergeometric2F1}\left(1, \frac{bc\log(F)}{2e}, 1+\frac{bc\log(F)}{2e}, e^{2(d+ex)}\right)\right)}{bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Coth[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*(1 - 2*Hypergeometric2F1[1, (b*c*Log[F])/(2*e), 1 + (b*c*
Log[F])/(2*e), E^(2*(d + e*x))]))/(b*c*Log[F])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{ 6008}$$

$$\int \left(\frac{2F^{c(a+bx)}}{e^{2(d+ex)}-1}+F^{c(a+bx)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{F^{c(a+bx)}}{bc\log(F)}-\frac{2F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,e^{2(d+ex)}\right)}{bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Coth[d + e*x],x]
```

output
```
F^(c*(a + b*x))/(b*c*Log[F]) - (2*F^(c*(a + b*x))*Hypergeometric2F1[1, (b*
c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6008
```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*(d + e*x)))^n/(-1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \coth(ex+d)\,dx$$

input
```
int(F^(c*(b*x+a))*coth(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*coth(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)} \coth(d+ex)\,dx = \int F^{(bx+a)c} \coth(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*coth(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\coth(d+ex)\,dx=\int F^{c(a+bx)}\coth\left(d+ex\right)dx$$

input
```
integrate(F**(c*(b*x+a))*coth(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*coth(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\coth(d+ex)\,dx=\int F^{(bx+a)c}\coth\left(ex+d\right)\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d),x, algorithm="maxima")
```

output
```
4*F^(a*c)*e*integrate(F^(b*c*x)/(b*c*log(F) + (b*c*e^(4*d)*log(F) - 2*e*e^
(4*d))*e^(4*e*x) - 2*(b*c*e^(2*d)*log(F) - 2*e*e^(2*d))*e^(2*e*x) - 2*e),
x) - (F^(a*c)*b*c*log(F) + 2*F^(a*c)*e + (F^(a*c)*b*c*e^(2*d)*log(F) - 2*F
^(a*c)*e*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 2*b*c*e*log(F)
- (b^2*c^2*e^(2*d)*log(F)^2 - 2*b*c*e*e^(2*d)*log(F))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\coth(d+ex)\,dx=\int F^{(bx+a)c}\coth\left(ex+d\right)\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*coth(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\coth(d+ex)\,dx = \int F^{c\,(a+b\,x)}\coth(d+e\,x)\,dx$$

input
```
int(F^(c*(a + b*x))*coth(d + e*x),x)
```

output
```
int(F^(c*(a + b*x))*coth(d + e*x), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\coth(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\coth(ex+d)\,dx\right)$$

input
```
int(F^(c*(b*x+a))*coth(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*coth(d + e*x),x)
```

3.100 $\int F^{c(a+bx)}\coth(d+ex)\operatorname{csch}(d+ex)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 112

$$\int F^{c(a+bx)}\coth(d+ex)\operatorname{csch}(d+ex)\,dx = \frac{2e^{d+ex}F^{c(a+bx)}}{e\left(1-e^{2d+2ex}\right)} - \frac{2bce^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),e^{2d+2ex}\right)\log(F)}{e(e+bc\log(F))}$$

output
```
2*exp(e*x+d)*F^(c*(b*x+a))/e/(1-exp(2*e*x+2*d))-2*b*c*exp(e*x+d)*F^(c*(b*x
+a))*hypergeom([1, 1/2*(e+b*c*ln(F))/e],[3/2+1/2*b*c*ln(F)/e],exp(2*e*x+2*
d))*ln(F)/e/(e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 10.44 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.21

$$\int F^{c(a+bx)}\coth(d+ex)\operatorname{csch}(d+ex)\,dx = \frac{F^{c(a+bx)}\left(\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{e},1+\frac{bc\log(F)}{e},-e^{d+ex}\right)-\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{e},1+\frac{bc}{}\right.\right.}{e}$$

input
```
Integrate[F^(c*(a + b*x))*Coth[d + e*x]*Csch[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*(Hypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e,
 -E^(d + e*x)] - Hypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e,
E^(d + e*x)] - (2*E^d)/(((1 + E^d)*Cosh[(e*x)/2] + (-1 + E^d)*Sinh[(e*x)/2
])*((-1 + E^d)*Cosh[(e*x)/2] + (1 + E^d)*Sinh[(e*x)/2]))))/e
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(d+ex)\text{csch}(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{ 6037}$$

$$\int \left(\frac{2e^{d+ex}F^{ac+bcx}}{e^{2(d+ex)}-1}+\frac{4e^{d+ex}F^{ac+bcx}}{\left(e^{2(d+ex)}-1\right)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4e^{d+ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(2,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),e^{2(d+ex)}\right)}{bc\log(F)+e}-\frac{2e^{d+ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),e^{2(d+ex)}\right)}{bc\log(F)+e}$$

input
```
Int[F^(c*(a + b*x))*Coth[d + e*x]*Csch[d + e*x],x]
```

output
```
(-2*E^(d + e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e
), (3 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))])/(e + b*c*Log[F]) + (4*E^(d +
e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[2, (e + b*c*Log[F])/(2*e), (3 + (b*
c*Log[F])/e)/2, E^(2*(d + e*x))])/(e + b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Maple [F]

$$\int F^{c(bx+a)} \coth(ex+d)\operatorname{csch}(ex+d)\, dx$$

input
```
int(F^(c*(b*x+a))*coth(e*x+d)*csch(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*coth(e*x+d)*csch(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)} \coth(d+ex)\operatorname{csch}(d+ex)\, dx = \int F^{(bx+a)c} \coth(ex+d)\operatorname{csch}(ex+d)\, dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)*csch(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*coth(e*x + d)*csch(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\coth(d+ex)\text{csch}(d+ex)\,dx = \int F^{c(a+bx)}\coth\left(d+ex\right)\text{csch}\left(d+ex\right)dx$$

input
```
integrate(F**(c*(b*x+a))*coth(e*x+d)*csch(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*coth(d + e*x)*csch(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\coth(d+ex)\text{csch}(d+ex)\,dx = \int F^{(bx+a)c}\coth\left(ex+d\right)\text{csch}\left(ex+d\right)\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)*csch(e*x+d),x, algorithm="maxima")
```

output
```
16*F^(a*c)*b*c*e*integrate(-e^(b*c*x*log(F) + e*x + d)/(b^2*c^2*log(F)^2 -
 4*b*c*e*log(F) + 3*e^2 - (b^2*c^2*e^(6*d)*log(F)^2 - 4*b*c*e*e^(6*d)*log(
F) + 3*e^2*e^(6*d))*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 4*b*c*e*e^(4
*d)*log(F) + 3*e^2*e^(4*d))*e^(4*e*x) - 3*(b^2*c^2*e^(2*d)*log(F)^2 - 4*b*
c*e*e^(2*d)*log(F) + 3*e^2*e^(2*d))*e^(2*e*x)), x)*log(F) + 2*((F^(a*c)*b*
c*e^(3*d)*log(F) - 3*F^(a*c)*e*e^(3*d))*e^(3*e*x) + (F^(a*c)*b*c*e^d*log(F
) + 3*F^(a*c)*e*e^d)*e^(e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 4*b*c*e*log(F)
 + 3*e^2 + (b^2*c^2*e^(4*d)*log(F)^2 - 4*b*c*e*e^(4*d)*log(F) + 3*e^2*e^(4
*d))*e^(4*e*x) - 2*(b^2*c^2*e^(2*d)*log(F)^2 - 4*b*c*e*e^(2*d)*log(F) + 3*
e^2*e^(2*d))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)} \coth(d+ex)\text{csch}(d+ex)\, dx = \int F^{(bx+a)c} \coth(ex+d) \,\text{csch}(ex+d)\, dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)*csch(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*coth(e*x + d)*csch(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)} \coth(d+ex)\text{csch}(d+ex)\, dx = \int \frac{F^{c\,(a+b\,x)} \coth(d+e\,x)}{\sinh(d+e\,x)}\, dx$$

input
```
int((F^(c*(a + b*x))*coth(d + e*x))/sinh(d + e*x),x)
```

output
```
int((F^(c*(a + b*x))*coth(d + e*x))/sinh(d + e*x), x)
```

Reduce [F]

$$\int F^{c(a+bx)} \coth(d+ex)\text{csch}(d+ex)\, dx = f^{ac}\left(\int f^{bcx} \coth(ex+d) \,\text{csch}(ex+d)\, dx\right)$$

input
```
int(F^(c*(b*x+a))*coth(e*x+d)*csch(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*coth(d + e*x)*csch(d + e*x),x)
```

3.101 $\int F^{c(a+bx)}\coth(d+ex)\operatorname{csch}^2(d+ex)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 120

$$\int F^{c(a+bx)}\coth(d+ex)\operatorname{csch}^2(d+ex)\,dx=-\frac{2e^{2d+2ex}F^{c(a+bx)}}{e\left(1-e^{2d+2ex}\right)^2}+\frac{2bce^{2d+2ex}F^{c(a+bx)}\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(2+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(4+\frac{bc\log(F)}{e}\right),e^{2d+2ex}\right)\log(F)}{e(2e+bc\log(F))}$$

output
```
-2*exp(2*e*x+2*d)*F^(c*(b*x+a))/e/(1-exp(2*e*x+2*d))^2+2*b*c*exp(2*e*x+2*d
)*F^(c*(b*x+a))*hypergeom([2, 1+1/2*b*c*ln(F)/e],[2+1/2*b*c*ln(F)/e],exp(2
*e*x+2*d))*ln(F)/e/(2*e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.68

$$\int F^{c(a+bx)}\coth(d+ex)\operatorname{csch}^2(d+ex)\,dx=-\frac{F^{c(a+bx)}\left(e\operatorname{csch}^2(d+ex)+bc(-1+\coth(d+ex))\log(F)+2bc\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{b}{}\right.\right.}{2e^2}$$

input
```
Integrate[F^(c*(a + b*x))*Coth[d + e*x]*Csch[d + e*x]^2,x]
```

output
```
-1/2*(F^(c*(a + b*x))*(e*Csch[d + e*x]^2 + b*c*(-1 + Coth[d + e*x])*Log[F]
 + 2*b*c*Hypergeometric2F1[1, (b*c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e),
E^(2*(d + e*x))]*Log[F]))/e^2
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(d+ex)\text{csch}^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6037}$$

$$\int \left(\frac{4e^{2d+2ex}F^{ac+bcx}}{\left(e^{2(d+ex)}-1\right)^2}+\frac{8e^{2d+2ex}F^{ac+bcx}}{\left(e^{2(d+ex)}-1\right)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4e^{2d+2ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(\frac{bc\log(F)}{e}+2\right),\frac{1}{2}\left(\frac{bc\log(F)}{e}+4\right),e^{2(d+ex)}\right)}{bc\log(F)+2e}-\frac{8e^{2d+2ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(\frac{bc\log(F)}{e}+2\right),\frac{1}{2}\left(\frac{bc\log(F)}{e}+4\right),e^{2(d+ex)}\right)}{bc\log(F)+2e}$$

input
```
Int[F^(c*(a + b*x))*Coth[d + e*x]*Csch[d + e*x]^2,x]
```

output
```
(4*E^(2*d + 2*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[2, (2 + (b*c*Log[F])/
e)/2, (4 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))])/(2*e + b*c*Log[F]) - (8*E^
(2*d + 2*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[3, (2 + (b*c*Log[F])/e)/2,
 (4 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))])/(2*e + b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Maple [F]

$$\int F^{c(bx+a)} \coth(ex+d) \operatorname{csch}(ex+d)^2 \, dx$$

input
```
int(F^(c*(b*x+a))*coth(e*x+d)*csch(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*coth(e*x+d)*csch(e*x+d)^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \coth(d+ex)\operatorname{csch}^2(d+ex) \, dx = \int F^{(bx+a)c} \coth(ex+d) \operatorname{csch}(ex+d)^2 \, dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)*csch(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*coth(e*x + d)*csch(e*x + d)^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\coth(d+ex)\text{csch}^2(d+ex)\,dx = \int F^{c(a+bx)}\coth{(d+ex)}\,\text{csch}^2{(d+ex)}\,dx$$

input
```
integrate(F**(c*(b*x+a))*coth(e*x+d)*csch(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*coth(d + e*x)*csch(d + e*x)**2, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\coth(d+ex)\text{csch}^2(d+ex)\,dx = \int F^{(bx+a)c}\coth{(ex+d)}\,\text{csch}{(ex+d)}^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)*csch(e*x+d)^2,x, algorithm="maxima")
```

output
```
48*F^(a*c)*b^2*c^2*e*integrate(F^(b*c*x)/(b^3*c^3*log(F)^3 - 12*b^2*c^2*e*
log(F)^2 + 44*b*c*e^2*log(F) - 48*e^3 + (b^3*c^3*e^(8*d)*log(F)^3 - 12*b^2
*c^2*e*e^(8*d)*log(F)^2 + 44*b*c*e^2*e^(8*d)*log(F) - 48*e^3*e^(8*d))*e^(8
*e*x) - 4*(b^3*c^3*e^(6*d)*log(F)^3 - 12*b^2*c^2*e*e^(6*d)*log(F)^2 + 44*b
*c*e^2*e^(6*d)*log(F) - 48*e^3*e^(6*d))*e^(6*e*x) + 6*(b^3*c^3*e^(4*d)*log
(F)^3 - 12*b^2*c^2*e*e^(4*d)*log(F)^2 + 44*b*c*e^2*e^(4*d)*log(F) - 48*e^3
*e^(4*d))*e^(4*e*x) - 4*(b^3*c^3*e^(2*d)*log(F)^3 - 12*b^2*c^2*e*e^(2*d)*l
og(F)^2 + 44*b*c*e^2*e^(2*d)*log(F) - 48*e^3*e^(2*d))*e^(2*e*x)), x)*log(F
)^2 - 4*(12*F^(a*c)*b*c*e*log(F) + (F^(a*c)*b^2*c^2*e^(4*d)*log(F)^2 - 10*
F^(a*c)*b*c*e*e^(4*d)*log(F) + 24*F^(a*c)*e^2*e^(4*d))*e^(4*e*x) + (F^(a*c
)*b^2*c^2*e^(2*d)*log(F)^2 - 2*F^(a*c)*b*c*e*e^(2*d)*log(F) - 24*F^(a*c)*e
^2*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^3*c^3*log(F)^3 - 12*b^2*c^2*e*log(F)^2
 + 44*b*c*e^2*log(F) - 48*e^3 - (b^3*c^3*e^(6*d)*log(F)^3 - 12*b^2*c^2*e*e
^(6*d)*log(F)^2 + 44*b*c*e^2*e^(6*d)*log(F) - 48*e^3*e^(6*d))*e^(6*e*x) +
3*(b^3*c^3*e^(4*d)*log(F)^3 - 12*b^2*c^2*e*e^(4*d)*log(F)^2 + 44*b*c*e^2*e
^(4*d)*log(F) - 48*e^3*e^(4*d))*e^(4*e*x) - 3*(b^3*c^3*e^(2*d)*log(F)^3 -
12*b^2*c^2*e*e^(2*d)*log(F)^2 + 44*b*c*e^2*e^(2*d)*log(F) - 48*e^3*e^(2*d)
)*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\coth(d+ex)\text{csch}^2(d+ex)\,dx=\int F^{(bx+a)c}\coth(ex+d)\,\text{csch}(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)*csch(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*coth(e*x + d)*csch(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\coth(d+ex)\text{csch}^2(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}\coth(d+e\,x)}{\sinh(d+e\,x)^2}\,dx$$

input
```
int((F^(c*(a + b*x))*coth(d + e*x))/sinh(d + e*x)^2,x)
```

output
```
int((F^(c*(a + b*x))*coth(d + e*x))/sinh(d + e*x)^2, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\coth(d+ex)\text{csch}^2(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\coth(ex+d)\,\text{csch}(ex+d)^2\,dx\right)$$

input
```
int(F^(c*(b*x+a))*coth(e*x+d)*csch(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*coth(d + e*x)*csch(d + e*x)**2,x)
```

3.102 $\int F^{c(a+bx)}\cosh^2(d+ex)\sinh^3(d+ex)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 201

$$\int F^{c(a+bx)}\cosh^2(d+ex)\sinh^3(d+ex)\,dx = -\frac{e^{-d-ex}F^{c(a+bx)}}{16(e-bc\log(F))}-\frac{e^{-3d-3ex}F^{c(a+bx)}}{32(3e-bc\log(F))}+\frac{e^{-5d-5ex}F^{c(a+bx)}}{32(5e-bc\log(F))}-\frac{e^{d+ex}F^{c(a+bx)}}{16(e+bc\log(F))}-\frac{e^{3d+3ex}F^{c(a+bx)}}{32(3e+bc\log(F))}+\frac{e^{5d+5ex}F^{c(a+bx)}}{32(5e+bc\log(F))}$$

output
```
-1/16*exp(-e*x-d)*F^(c*(b*x+a))/(e-b*c*ln(F))-exp(-3*e*x-3*d)*F^(c*(b*x+a)
)/(96*e-32*b*c*ln(F))+exp(-5*e*x-5*d)*F^(c*(b*x+a))/(160*e-32*b*c*ln(F))-e
xp(e*x+d)*F^(c*(b*x+a))/(16*e+16*b*c*ln(F))-exp(3*e*x+3*d)*F^(c*(b*x+a))/(
96*e+32*b*c*ln(F))+exp(5*e*x+5*d)*F^(c*(b*x+a))/(160*e+32*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.74

$$\int F^{c(a+bx)} \cosh^2(d+ex) \sinh^3(d+ex) \, dx$$
$$= \frac{1}{16} F^{c(a+bx)} \left(\frac{-2e \cosh(d+ex) + 2bc \log(F) \sinh(d+ex)}{(e-bc\log(F))(e+bc\log(F))} + \frac{-3e\cosh(3(d+ex)) + bc\log(F)\sinh(3(d+ex))}{9e^2 - b^2c^2\log^2(F)} + \frac{5e\cosh(5(d+ex)) - bc\log(F)\sinh(5(d+ex))}{25e^2 - b^2c^2\log^2(F)} \right)$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^2*Sinh[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*((-2*e*Cosh[d + e*x] + 2*b*c*Log[F]*Sinh[d + e*x])/((e -
b*c*Log[F])*(e + b*c*Log[F])) + (-3*e*Cosh[3*(d + e*x)] + b*c*Log[F]*Sinh[
3*(d + e*x)])/(9*e^2 - b^2*c^2*Log[F]^2) + (5*e*Cosh[5*(d + e*x)] - b*c*Lo
g[F]*Sinh[5*(d + e*x)])/(25*e^2 - b^2*c^2*Log[F]^2)))/16
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {6035, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(d+ex)\cosh^2(d+ex)F^{c(a+bx)} \, dx$$

$$\Big\downarrow \text{ 6035}$$

$$\int \left(-\frac{1}{8}\sinh(d+ex)F^{c(a+bx)} - \frac{1}{16}\sinh(3d+3ex)F^{c(a+bx)} + \frac{1}{16}\sinh(5d+5ex)F^{c(a+bx)}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{bc\log(F)\sinh(d+ex)F^{c(a+bx)}}{8\left(e^2-b^2c^2\log^2(F)\right)}+\frac{bc\log(F)\sinh(3d+3ex)F^{c(a+bx)}}{16\left(9e^2-b^2c^2\log^2(F)\right)}-\frac{bc\log(F)\sinh(5d+5ex)F^{c(a+bx)}}{16\left(25e^2-b^2c^2\log^2(F)\right)}-\frac{e\cosh(d+ex)F^{c(a+bx)}}{8\left(e^2-b^2c^2\log^2(F)\right)}-\frac{3e\cosh(3d+3ex)F^{c(a+bx)}}{16\left(9e^2-b^2c^2\log^2(F)\right)}+\frac{5e\cosh(5d+5ex)F^{c(a+bx)}}{16\left(25e^2-b^2c^2\log^2(F)\right)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x]^2*Sinh[d + e*x]^3,x]
```

output
```
-1/8*(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2) - (3*e*F^(
c*(a + b*x))*Cosh[3*d + 3*e*x])/(16*(9*e^2 - b^2*c^2*Log[F]^2)) + (5*e*F^(
c*(a + b*x))*Cosh[5*d + 5*e*x])/(16*(25*e^2 - b^2*c^2*Log[F]^2)) + (b*c*F^
(c*(a + b*x))*Log[F]*Sinh[d + e*x])/(8*(e^2 - b^2*c^2*Log[F]^2)) + (b*c*F^
(c*(a + b*x))*Log[F]*Sinh[3*d + 3*e*x])/(16*(9*e^2 - b^2*c^2*Log[F]^2)) -
(b*c*F^(c*(a + b*x))*Log[F]*Sinh[5*d + 5*e*x])/(16*(25*e^2 - b^2*c^2*Log[F
]^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6035
```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(186) = 372$.

Time = 0.81 (sec) , antiderivative size = 768, normalized size of antiderivative = 3.82

$$\left(450e^5 e^{4ex+4d}-45e^5+26\ln(F)^3 b^3c^3e^2 e^{8ex+8d}+50\ln(F)^2 b^2c^2e^3 e^{10ex+10d}+2\ln(F)^4 b^4c^4 e\, e^{4ex+4d}+68\ln\right.$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^2*sinh(e*x+d)^3,x)
```

output
```
1/32*(450*e^5*exp(4*e*x+4*d)-45*e^5+26*ln(F)^3*b^3*c^3*e^2*exp(8*e*x+8*d)+
50*ln(F)^2*b^2*c^2*e^3*exp(10*e*x+10*d)+2*ln(F)^4*b^4*c^4*e*exp(4*e*x+4*d)
+68*ln(F)^3*b^3*c^3*e^2*exp(6*e*x+6*d)-78*ln(F)^2*b^2*c^2*e^3*exp(8*e*x+8*
d)+ln(F)^5*b^5*c^5*exp(10*e*x+10*d)-ln(F)^5*b^5*c^5*exp(8*e*x+8*d)-2*ln(F)
^5*b^5*c^5*exp(6*e*x+6*d)+2*ln(F)^5*b^5*c^5*exp(4*e*x+4*d)+ln(F)^5*b^5*c^5
*exp(2*e*x+2*d)-c^5*b^5*ln(F)^5+9*ln(F)*b*c*e^4*exp(10*e*x+10*d)+3*ln(F)^4
*b^4*c^4*e*exp(2*e*x+2*d)-68*ln(F)^3*b^3*c^3*e^2*exp(4*e*x+4*d)-68*ln(F)^2
*b^2*c^2*e^3*exp(6*e*x+6*d)-25*ln(F)*b*c*e^4*exp(8*e*x+8*d)-5*ln(F)^4*b^4*
c^4*e+50*ln(F)^2*b^2*c^2*e^3-26*ln(F)^3*b^3*c^3*e^2*exp(2*e*x+2*d)-68*ln(F
)^2*b^2*c^2*e^3*exp(4*e*x+4*d)-450*ln(F)*b*c*e^4*exp(6*e*x+6*d)-78*ln(F)^2
*b^2*c^2*e^3*exp(2*e*x+2*d)+450*ln(F)*b*c*e^4*exp(4*e*x+4*d)+25*ln(F)*b*c*
e^4*exp(2*e*x+2*d)-5*ln(F)^4*b^4*c^4*e*exp(10*e*x+10*d)+3*ln(F)^4*b^4*c^4*
e*exp(8*e*x+8*d)-10*ln(F)^3*b^3*c^3*e^2*exp(10*e*x+10*d)+2*ln(F)^4*b^4*c^4
*e*exp(6*e*x+6*d)+75*e^5*exp(2*e*x+2*d)+450*e^5*exp(6*e*x+6*d)+75*e^5*exp(
8*e*x+8*d)-45*e^5*exp(10*e*x+10*d)+10*c^3*b^3*ln(F)^3*e^2-9*c*b*ln(F)*e^4)
/(b*c*ln(F)-e)*exp(-5*e*x-5*d)/(b*c*ln(F)-3*e)/(b*c*ln(F)-5*e)/(e+b*c*ln(F
))/(b*c*ln(F)+3*e)/(b*c*ln(F)+5*e)*F^(c*(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7798 vs. $2(182) = 364$.

Time = 0.64 (sec) , antiderivative size = 7798, normalized size of antiderivative = 38.80

$$\int F^{c(a+bx)} \cosh^2(d+ex) \sinh^3(d+ex)\, dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2*sinh(e*x+d)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\cosh^2(d+ex)\sinh^3(d+ex)\,dx=\text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d)**2*sinh(e*x+d)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.02

$$\begin{aligned}\int F^{c(a+bx)}\cosh^2(d+ex)\sinh^3(d+ex)\,dx &= \frac{F^{ac}e^{(bcx\log(F)+5\,ex+5\,d)}}{32\,(bc\log(F)+5\,e)}-\frac{F^{ac}e^{(bcx\log(F)+3\,ex+3\,d)}}{32\,(bc\log(F)+3\,e)} \\ &\quad -\frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{16\,(bc\log(F)+e)}+\frac{F^{ac}e^{(bcx\log(F)-ex)}}{16\,(bce^{d}\log(F)-ee^{d})} \\ &\quad +\frac{F^{ac}e^{(bcx\log(F)-3\,ex)}}{32\,(bce^{(3\,d)}\log(F)-3\,ee^{(3\,d)})}-\frac{F^{ac}e^{(bcx\log(F)-5\,ex)}}{32\,(bce^{(5\,d)}\log(F)-5\,ee^{(5\,d)})}\end{aligned}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2*sinh(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/32*F^(a*c)*e^(b*c*x*log(F) + 5*e*x + 5*d)/(b*c*log(F) + 5*e) - 1/32*F^(a
*c)*e^(b*c*x*log(F) + 3*e*x + 3*d)/(b*c*log(F) + 3*e) - 1/16*F^(a*c)*e^(b*
c*x*log(F) + e*x + d)/(b*c*log(F) + e) + 1/16*F^(a*c)*e^(b*c*x*log(F) - e*
x)/(b*c*e^d*log(F) - e*e^d) + 1/32*F^(a*c)*e^(b*c*x*log(F) - 3*e*x)/(b*c*e
^(3*d)*log(F) - 3*e*e^(3*d)) - 1/32*F^(a*c)*e^(b*c*x*log(F) - 5*e*x)/(b*c*
e^(5*d)*log(F) - 5*e*e^(5*d))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 1823, normalized size of antiderivative = 9.07

$$\int F^{c(a+bx)} \cosh^2(d+ex) \sinh^3(d+ex)\, dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2*sinh(e*x+d)^3,x, algorithm="giac")
```

output
```
1/16*(2*(b*c*log(abs(F)) + 5*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x -
1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(a
bs(F)) + 5*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2
*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 +
4*(b*c*log(abs(F)) + 5*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 5*e)
*x + 5*d) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*
sgn(F) - 1/2*I*pi*a*c)/(32*I*pi*b*c*sgn(F) - 32*I*pi*b*c + 64*b*c*log(abs(
F)) + 320*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c
*sgn(F) + 1/2*I*pi*a*c)/(-32*I*pi*b*c*sgn(F) + 32*I*pi*b*c + 64*b*c*log(ab
s(F)) + 320*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 5*e)*x + 5*d) - 1/
16*(2*(b*c*log(abs(F)) + 3*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs
(F)) + 3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*p
i*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*
(b*c*log(abs(F)) + 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x
 + 3*d) + I*(-I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*s
gn(F) - 1/2*I*pi*a*c)/(32*I*pi*b*c*sgn(F) - 32*I*pi*b*c + 64*b*c*log(abs(F
)) + 192*e) + I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*
sgn(F) + 1/2*I*pi*a*c)/(-32*I*pi*b*c*sgn(F) + 32*I*pi*b*c + 64*b*c*log(abs
(F)) + 192*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3*d) - ...
```

Mupad [B] (verification not implemented)

Time = 4.72 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.04

$$\int F^{c(a+bx)}\cosh^2(d+ex)\sinh^3(d+ex)\,dx = -\frac{F^{a\,c+b\,c\,x}\left(b^5\,c^5\cosh(d+e\,x)^2\sinh(d+e\,x)^3\ln(F)^5-3\,b^4\,c^4\,e\cosh(d+e\,x)^3\sinh(d+e\,x)^2\ln(F)^4-2\right.}{}$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x)^2*sinh(d + e*x)^3,x)
```

output
```
-(F^(a*c + b*c*x)*(30*e^5*cosh(d + e*x)^5 - 75*e^5*cosh(d + e*x)^3*sinh(d
+ e*x)^2 - 26*b*c*e^4*sinh(d + e*x)^5*log(F) - 6*b^2*c^2*e^3*cosh(d + e*x)
^5*log(F)^2 + b^5*c^5*cosh(d + e*x)^2*sinh(d + e*x)^3*log(F)^5 + 2*b^3*c^3
*e^2*sinh(d + e*x)^5*log(F)^3 + 65*b*c*e^4*cosh(d + e*x)^2*sinh(d + e*x)^3
*log(F) + 30*b^2*c^2*e^3*cosh(d + e*x)^3*sinh(d + e*x)^2*log(F)^2 - 18*b^3
*c^3*e^2*cosh(d + e*x)^2*sinh(d + e*x)^3*log(F)^3 - 2*b^4*c^4*e*cosh(d + e
*x)*sinh(d + e*x)^4*log(F)^4 - 30*b*c*e^4*cosh(d + e*x)^4*sinh(d + e*x)*lo
g(F) + 26*b^2*c^2*e^3*cosh(d + e*x)*sinh(d + e*x)^4*log(F)^2 + 6*b^3*c^3*e
^2*cosh(d + e*x)^4*sinh(d + e*x)*log(F)^3 - 3*b^4*c^4*e*cosh(d + e*x)^3*si
nh(d + e*x)^2*log(F)^4))/(225*e^6 - b^6*c^6*log(F)^6 - 259*b^2*c^2*e^4*log
(F)^2 + 35*b^4*c^4*e^2*log(F)^4)
```

Reduce [F]

$$\int F^{c(a+bx)}\cosh^2(d+ex)\sinh^3(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\cosh(ex+d)^2\sinh(ex+d)^3\,dx\right)$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^2*sinh(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cosh(d + e*x)**2*sinh(d + e*x)**3,x)
```

3.103 $\int F^{c(a+bx)} \cosh^2(d+ex) \sinh^2(d+ex)\, dx$

Optimal result

Integrand size = 26, antiderivative size = 93

$$\int F^{c(a+bx)} \cosh^2(d+ex) \sinh^2(d+ex)\, dx = -\frac{F^{c(a+bx)}}{8bc\log(F)} - \frac{e^{-4d-4ex}F^{c(a+bx)}}{16(4e-bc\log(F))} + \frac{e^{4d+4ex}F^{c(a+bx)}}{16(4e+bc\log(F))}$$

output
```
-1/8*F^(c*(b*x+a))/b/c/ln(F)-exp(-4*e*x-4*d)*F^(c*(b*x+a))/(64*e-16*b*c*ln
(F))+exp(4*e*x+4*d)*F^(c*(b*x+a))/(64*e+16*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.96

$$\int F^{c(a+bx)} \cosh^2(d+ex) \sinh^2(d+ex)\, dx = \frac{F^{c(a+bx)}\left(16e^2-b^2c^2\log^2(F)+b^2c^2\cosh(4(d+ex))\log^2(F)-4bce\log(F)\sinh(4(d+ex))\right)}{4\left(-32bce^2\log(F)+2b^3c^3\log^3(F)\right)}$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^2*Sinh[d + e*x]^2,x]
```

output
```
(F^(c*(a + b*x))*(16*e^2 - b^2*c^2*Log[F]^2 + b^2*c^2*Cosh[4*(d + e*x)]*Lo
g[F]^2 - 4*b*c*e*Log[F]*Sinh[4*(d + e*x)]))/(4*(-32*b*c*e^2*Log[F] + 2*b^3
*c^3*Log[F]^3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {6035, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(d+ex)\cosh^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6035}$$

$$\int \left(\frac{1}{8}\cosh(4d+4ex)F^{c(a+bx)}-\frac{1}{8}F^{c(a+bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e\sinh(4d+4ex)F^{c(a+bx)}}{2\left(16e^2-b^2c^2\log^2(F)\right)}-\frac{bc\log(F)\cosh(4d+4ex)F^{c(a+bx)}}{8\left(16e^2-b^2c^2\log^2(F)\right)}-\frac{F^{c(a+bx)}}{8bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x]^2*Sinh[d + e*x]^2,x]
```

output
```
-1/8*F^(c*(a + b*x))/(b*c*Log[F]) - (b*c*F^(c*(a + b*x))*Cosh[4*d + 4*e*x]
*Log[F])/(8*(16*e^2 - b^2*c^2*Log[F]^2)) + (e*F^(c*(a + b*x))*Sinh[4*d + 4
*e*x])/(2*(16*e^2 - b^2*c^2*Log[F]^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6035
```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.54

$$\frac{\left(\ln\left(F\right)^{2} b^{2} c^{2} \mathrm{e}^{8ex+8d} - 4\ln\left(F\right) bce\, \mathrm{e}^{8ex+8d} - 2\ln\left(F\right)^{2} b^{2} c^{2} \mathrm{e}^{4ex+4d} + b^{2} c^{2} \ln\left(F\right)^{2} + 32e^{2} \mathrm{e}^{4ex+4d} + 4\ln\left(F\right) bce}{16\ln\left(F\right) bc\left(bc\ln\left(F\right) - 4e\right)\left(bc\ln\left(F\right) + 4e\right)}$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^2*sinh(e*x+d)^2,x)
```

output
```
1/16*(ln(F)^2*b^2*c^2*exp(8*e*x+8*d)-4*ln(F)*b*c*e*exp(8*e*x+8*d)-2*ln(F)^
2*b^2*c^2*exp(4*e*x+4*d)+b^2*c^2*ln(F)^2+32*e^2*exp(4*e*x+4*d)+4*ln(F)*b*c
*e)/ln(F)/b/c/(b*c*ln(F)-4*e)*exp(-4*e*x-4*d)/(b*c*ln(F)+4*e)*F^(c*(b*x+a)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1289 vs. 2(84) = 168.

Time = 0.10 (sec) , antiderivative size = 1289, normalized size of antiderivative = 13.86

$$\int F^{c(a+bx)} \cosh^2(d+ex) \sinh^2(d+ex)\, dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2*sinh(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/16*(((b^2*c^2*log(F)^2 - 4*b*c*e*log(F))*sinh(e*x + d)^8 + 8*(b^2*c^2*co
sh(e*x + d)*log(F)^2 - 4*b*c*e*cosh(e*x + d)*log(F))*sinh(e*x + d)^7 + 28*
(b^2*c^2*cosh(e*x + d)^2*log(F)^2 - 4*b*c*e*cosh(e*x + d)^2*log(F))*sinh(e
*x + d)^6 + 32*e^2*cosh(e*x + d)^4 + 56*(b^2*c^2*cosh(e*x + d)^3*log(F)^2
- 4*b*c*e*cosh(e*x + d)^3*log(F))*sinh(e*x + d)^5 - 2*(140*b*c*e*cosh(e*x
+ d)^4*log(F) - (35*b^2*c^2*cosh(e*x + d)^4 - b^2*c^2)*log(F)^2 - 16*e^2)*
sinh(e*x + d)^4 - 8*(28*b*c*e*cosh(e*x + d)^5*log(F) - 16*e^2*cosh(e*x + d
) - (7*b^2*c^2*cosh(e*x + d)^5 - b^2*c^2*cosh(e*x + d))*log(F)^2)*sinh(e*x
 + d)^3 + (b^2*c^2*cosh(e*x + d)^8 - 2*b^2*c^2*cosh(e*x + d)^4 + b^2*c^2)*
log(F)^2 - 4*(28*b*c*e*cosh(e*x + d)^6*log(F) - 48*e^2*cosh(e*x + d)^2 - (
7*b^2*c^2*cosh(e*x + d)^6 - 3*b^2*c^2*cosh(e*x + d)^2)*log(F)^2)*sinh(e*x
+ d)^2 - 4*(b*c*e*cosh(e*x + d)^8 - b*c*e)*log(F) - 8*(4*b*c*e*cosh(e*x +
d)^7*log(F) - 16*e^2*cosh(e*x + d)^3 - (b^2*c^2*cosh(e*x + d)^7 - b^2*c^2*
cosh(e*x + d)^3)*log(F)^2)*sinh(e*x + d))*cosh((b*c*x + a*c)*log(F)) + ((b
^2*c^2*log(F)^2 - 4*b*c*e*log(F))*sinh(e*x + d)^8 + 8*(b^2*c^2*cosh(e*x +
d)*log(F)^2 - 4*b*c*e*cosh(e*x + d)*log(F))*sinh(e*x + d)^7 + 28*(b^2*c^2*
cosh(e*x + d)^2*log(F)^2 - 4*b*c*e*cosh(e*x + d)^2*log(F))*sinh(e*x + d)^6
 + 32*e^2*cosh(e*x + d)^4 + 56*(b^2*c^2*cosh(e*x + d)^3*log(F)^2 - 4*b*c*e
*cosh(e*x + d)^3*log(F))*sinh(e*x + d)^5 - 2*(140*b*c*e*cosh(e*x + d)^4*lo
g(F) - (35*b^2*c^2*cosh(e*x + d)^4 - b^2*c^2)*log(F)^2 - 16*e^2)*sinh(e...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1316 vs. $2(78) = 156$.

Time = 23.48 (sec) , antiderivative size = 1316, normalized size of antiderivative = 14.15

$$\int F^{c(a+bx)} \cosh^2(d+ex) \sinh^2(d+ex)\, dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d)**2*sinh(e*x+d)**2,x)
```

output
```
Piecewise((x*sinh(d)**2*cosh(d)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e,
 0)), (-x*sinh(d + e*x)**4/8 + x*sinh(d + e*x)**2*cosh(d + e*x)**2/4 - x*c
osh(d + e*x)**4/8 + sinh(d + e*x)**3*cosh(d + e*x)/(8*e) + sinh(d + e*x)*c
osh(d + e*x)**3/(8*e), Eq(F, 1)), (F**(a*c)*(-x*sinh(d + e*x)**4/8 + x*sin
h(d + e*x)**2*cosh(d + e*x)**2/4 - x*cosh(d + e*x)**4/8 + sinh(d + e*x)**3
*cosh(d + e*x)/(8*e) + sinh(d + e*x)*cosh(d + e*x)**3/(8*e)), Eq(b, 0)), (
-x*sinh(d + e*x)**4/8 + x*sinh(d + e*x)**2*cosh(d + e*x)**2/4 - x*cosh(d +
 e*x)**4/8 + sinh(d + e*x)**3*cosh(d + e*x)/(8*e) + sinh(d + e*x)*cosh(d +
 e*x)**3/(8*e), Eq(c, 0)), (F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/4 - d)**4
/16 - F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/4 - d)**3*cosh(b*c*x*log(F)/4 -
 d)/4 + 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/4 - d)**2*cosh(b*c*x*log(F)
/4 - d)**2/8 - F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/4 - d)*cosh(b*c*x*log(
F)/4 - d)**3/4 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F)/4 - d)**4/16 - F**(a
*c + b*c*x)*sinh(b*c*x*log(F)/4 - d)**4/(6*b*c*log(F)) + 5*F**(a*c + b*c*x
)*sinh(b*c*x*log(F)/4 - d)**3*cosh(b*c*x*log(F)/4 - d)/(12*b*c*log(F)) + 5
*F**(a*c + b*c*x)*sinh(b*c*x*log(F)/4 - d)*cosh(b*c*x*log(F)/4 - d)**3/(12
*b*c*log(F)) - F**(a*c + b*c*x)*cosh(b*c*x*log(F)/4 - d)**4/(6*b*c*log(F))
, Eq(e, -b*c*log(F)/4)), (F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/4 + d)**4/1
6 - F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/4 + d)**3*cosh(b*c*x*log(F)/4 + d
)/4 + 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/4 + d)**2*cosh(b*c*x*log(F...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01

$$\int F^{c(a+bx)}\cosh^2(d+ex)\sinh^2(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+4\,ex+4\,d)}}{16\,(bc\log(F)+4\,e)} + \frac{F^{ac}e^{(bcx\log(F)-4\,ex)}}{16\,(bce^{(4\,d)}\log(F)-4\,ee^{(4\,d)})} - \frac{F^{bcx+ac}}{8\,bc\log(F)}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2*sinh(e*x+d)^2,x, algorithm="maxima")
```

output
```
1/16*F^(a*c)*e^(b*c*x*log(F) + 4*e*x + 4*d)/(b*c*log(F) + 4*e) + 1/16*F^(a
*c)*e^(b*c*x*log(F) - 4*e*x)/(b*c*e^(4*d)*log(F) - 4*e*e^(4*d)) - 1/8*F^(b
*c*x + a*c)/(b*c*log(F))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 890, normalized size of antiderivative = 9.57

$$\int F^{c(a+bx)}\cosh^2(d+ex)\sinh^2(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2*sinh(e*x+d)^2,x, algorithm="giac")
```

output
```
-1/4*(2*b*c*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) +
1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c
)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x -
1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F)
- pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(-I*e^(1/2*I*pi*
b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(8*I*p
i*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F))) + I*e^(-1/2*I*pi*b*c*x*sgn
(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sg
n(F) + 8*I*pi*b*c + 16*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(ab
s(F))) + 1/8*(2*(b*c*log(abs(F)) + 4*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*
b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b
*c*log(abs(F)) + 4*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(
F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b
*c)^2 + 4*(b*c*log(abs(F)) + 4*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)
) + 4*e)*x + 4*d) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I
*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(16*I*pi*b*c*sgn(F) - 16*I*pi*b*c + 32*b*c*
log(abs(F)) + 128*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*
I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-16*I*pi*b*c*sgn(F) + 16*I*pi*b*c + 32*b*
c*log(abs(F)) + 128*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 4*e)*x + 4
*d) + 1/8*(2*(b*c*log(abs(F)) - 4*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*...
```

Mupad [B] (verification not implemented)

Time = 4.95 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20

$$\int F^{c(a+bx)} \cosh^2(d+ex) \sinh^2(d+ex) \, dx = -\frac{b^2 c^2 \ln(F)^2 \left(\frac{F^{ac+bcx}}{8} - \frac{F^{ac+bcx} \cosh(4d+4ex)}{8}\right) - 2F^{ac+bcx} e^2 + \frac{F^{ac+bcx} bce \ln(F) \sinh(4d+4ex)}{2}}{b^3 c^3 \ln(F)^3 - 16bce^2 \ln(F)}$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x)^2*sinh(d + e*x)^2,x)
```

output
```
-(b^2*c^2*log(F)^2*(F^(a*c + b*c*x)/8 - (F^(a*c + b*c*x)*cosh(4*d + 4*e*x)
)/8) - 2*F^(a*c + b*c*x)*e^2 + (F^(a*c + b*c*x)*b*c*e*log(F)*sinh(4*d + 4*
e*x))/2)/(b^3*c^3*log(F)^3 - 16*b*c*e^2*log(F))
```

Reduce [F]

$$\int F^{c(a+bx)} \cosh^2(d+ex) \sinh^2(d+ex) \, dx = f^{ac} \left(\int f^{bcx} \cosh(ex+d)^2 \sinh(ex+d)^2 \, dx \right)$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^2*sinh(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cosh(d + e*x)**2*sinh(d + e*x)**2,x)
```

3.104 $\int F^{c(a+bx)}\cosh^2(d+ex)\sinh(d+ex)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int F^{c(a+bx)}\cosh^2(d+ex)\sinh(d+ex)\,dx=\frac{e^{-d-ex}F^{c(a+bx)}}{8(e-bc\log(F))}+\frac{e^{-3d-3ex}F^{c(a+bx)}}{8(3e-bc\log(F))}+\frac{e^{d+ex}F^{c(a+bx)}}{8(e+bc\log(F))}+\frac{e^{3d+3ex}F^{c(a+bx)}}{8(3e+bc\log(F))}$$

output
```
exp(-e*x-d)*F^(c*(b*x+a))/(8*e-8*b*c*ln(F))+exp(-3*e*x-3*d)*F^(c*(b*x+a))/
(24*e-8*b*c*ln(F))+exp(e*x+d)*F^(c*(b*x+a))/(8*e+8*b*c*ln(F))+exp(3*e*x+3*
d)*F^(c*(b*x+a))/(24*e+8*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.77

$$\int F^{c(a+bx)}\cosh^2(d+ex)\sinh(d+ex)\,dx=\frac{1}{4}F^{c(a+bx)}\left(\frac{e\cosh(d+ex)-bc\log(F)\sinh(d+ex)}{(e-bc\log(F))(e+bc\log(F))}+\frac{3e\cosh(3(d+ex))-bc\log(F)\sinh(3(d+ex))}{9e^2-b^2c^2\log^2(F)}\right)$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^2*Sinh[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*((e*Cosh[d + e*x] - b*c*Log[F]*Sinh[d + e*x])/((e - b*c*L
og[F])*(e + b*c*Log[F])) + (3*e*Cosh[3*(d + e*x)] - b*c*Log[F]*Sinh[3*(d +
 e*x)])/(9*e^2 - b^2*c^2*Log[F]^2)))/4
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6035, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(d+ex)\cosh^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{ 6035}$$

$$\int \left(\frac{1}{4}\sinh(d+ex)F^{c(a+bx)}+\frac{1}{4}\sinh(3d+3ex)F^{c(a+bx)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{bc\log(F)\sinh(d+ex)F^{c(a+bx)}}{4\left(e^2-b^2c^2\log^2(F)\right)}-\frac{bc\log(F)\sinh(3d+3ex)F^{c(a+bx)}}{4\left(9e^2-b^2c^2\log^2(F)\right)}+\frac{e\cosh(d+ex)F^{c(a+bx)}}{4\left(e^2-b^2c^2\log^2(F)\right)}+\frac{3e\cosh(3d+3ex)F^{c(a+bx)}}{4\left(9e^2-b^2c^2\log^2(F)\right)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x]^2*Sinh[d + e*x],x]
```

output
```
(e*F^(c*(a + b*x))*Cosh[d + e*x])/(4*(e^2 - b^2*c^2*Log[F]^2)) + (3*e*F^(c
*(a + b*x))*Cosh[3*d + 3*e*x])/(4*(9*e^2 - b^2*c^2*Log[F]^2)) - (b*c*F^(c*
(a + b*x))*Log[F]*Sinh[d + e*x])/(4*(e^2 - b^2*c^2*Log[F]^2)) - (b*c*F^(c*
(a + b*x))*Log[F]*Sinh[3*d + 3*e*x])/(4*(9*e^2 - b^2*c^2*Log[F]^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6035
```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(122) = 244$.

Time = 34.59 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.46

method	result
risch	$\frac{\left(\ln(F)^3 b^3 c^3 e^{6ex+6d}+\ln(F)^3 b^3 c^3 e^{4ex+4d}-3\ln(F)^2 b^2 c^2 e\, e^{6ex+6d}-\ln(F)^3 b^3 c^3 e^{2ex+2d}-\ln(F)^2 b^2 c^2 e\, e^{4ex+4d}-\ln(F) bc\, e^2 e^{6ex+6d}-\cdots}{8(bc\ln(\cdots}$
orering	$\frac{4bc\ln(F)\left(b^2c^2\ln(F)^2-5e^2\right)F^{c(bx+a)}\cosh(ex+d)^2\sinh(ex+d)}{\ln(F)^4 b^4 c^4-10\ln(F)^2 b^2 c^2 e^2+9e^4}-\frac{2\left(3b^2c^2\ln(F)^2-5e^2\right)\left(F^{c(bx+a)} bc\ln(F)\cosh(ex+d)^2\sinh(ex\cdots\right.}{\ln(F)^4 b^4 c^4-1\cdots}$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^2*sinh(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/8*(ln(F)^3*b^3*c^3*exp(6*e*x+6*d)+ln(F)^3*b^3*c^3*exp(4*e*x+4*d)-3*ln(F)
^2*b^2*c^2*e*exp(6*e*x+6*d)-ln(F)^3*b^3*c^3*exp(2*e*x+2*d)-ln(F)^2*b^2*c^2
*e*exp(4*e*x+4*d)-ln(F)*b*c*e^2*exp(6*e*x+6*d)-c^3*b^3*ln(F)^3-ln(F)^2*b^2
*c^2*e*exp(2*e*x+2*d)-9*ln(F)*b*c*e^2*exp(4*e*x+4*d)+3*e^3*exp(6*e*x+6*d)-
3*c^2*b^2*ln(F)^2*e+9*ln(F)*b*c*e^2*exp(2*e*x+2*d)+9*e^3*exp(4*e*x+4*d)+e^
2*b*c*ln(F)+9*e^3*exp(2*e*x+2*d)+3*e^3)/(b*c*ln(F)-e)*exp(-3*e*x-3*d)/(b*c
*ln(F)-3*e)/(e+b*c*ln(F))/(b*c*ln(F)+3*e)*F^(c*(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2216 vs. $2(120) = 240$.

Time = 0.18 (sec) , antiderivative size = 2216, normalized size of antiderivative = 16.79

$$\int F^{c(a+bx)}\cosh^2(d+ex)\sinh(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2*sinh(e*x+d),x, algorithm="fricas")
```

output
```
1/8*((3*e^3*cosh(e*x + d)^6 + 9*e^3*cosh(e*x + d)^4 + (b^3*c^3*log(F)^3 -
3*b^2*c^2*e*log(F)^2 - b*c*e^2*log(F) + 3*e^3)*sinh(e*x + d)^6 + 6*(b^3*c^
3*cosh(e*x + d)*log(F)^3 - 3*b^2*c^2*e*cosh(e*x + d)*log(F)^2 - b*c*e^2*co
sh(e*x + d)*log(F) + 3*e^3*cosh(e*x + d))*sinh(e*x + d)^5 + 9*e^3*cosh(e*x
 + d)^2 + (45*e^3*cosh(e*x + d)^2 + (15*b^3*c^3*cosh(e*x + d)^2 + b^3*c^3)
*log(F)^3 + 9*e^3 - (45*b^2*c^2*e*cosh(e*x + d)^2 + b^2*c^2*e)*log(F)^2 -
3*(5*b*c*e^2*cosh(e*x + d)^2 + 3*b*c*e^2)*log(F))*sinh(e*x + d)^4 + (b^3*c
^3*cosh(e*x + d)^6 + b^3*c^3*cosh(e*x + d)^4 - b^3*c^3*cosh(e*x + d)^2 - b
^3*c^3)*log(F)^3 + 4*(15*e^3*cosh(e*x + d)^3 + 9*e^3*cosh(e*x + d) + (5*b^
3*c^3*cosh(e*x + d)^3 + b^3*c^3*cosh(e*x + d))*log(F)^3 - (15*b^2*c^2*e*co
sh(e*x + d)^3 + b^2*c^2*e*cosh(e*x + d))*log(F)^2 - (5*b*c*e^2*cosh(e*x +
d)^3 + 9*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d)^3 + 3*e^3 - (3*b^2*c
^2*e*cosh(e*x + d)^6 + b^2*c^2*e*cosh(e*x + d)^4 + b^2*c^2*e*cosh(e*x + d)
^2 + 3*b^2*c^2*e)*log(F)^2 + (45*e^3*cosh(e*x + d)^4 + 54*e^3*cosh(e*x + d
)^2 + (15*b^3*c^3*cosh(e*x + d)^4 + 6*b^3*c^3*cosh(e*x + d)^2 - b^3*c^3)*l
og(F)^3 + 9*e^3 - (45*b^2*c^2*e*cosh(e*x + d)^4 + 6*b^2*c^2*e*cosh(e*x + d
)^2 + b^2*c^2*e)*log(F)^2 - 3*(5*b*c*e^2*cosh(e*x + d)^4 + 18*b*c*e^2*cosh
(e*x + d)^2 - 3*b*c*e^2)*log(F))*sinh(e*x + d)^2 - (b*c*e^2*cosh(e*x + d)^
6 + 9*b*c*e^2*cosh(e*x + d)^4 - 9*b*c*e^2*cosh(e*x + d)^2 - b*c*e^2)*log(F
) + 2*(9*e^3*cosh(e*x + d)^5 + 18*e^3*cosh(e*x + d)^3 + 9*e^3*cosh(e*x ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1476 vs. $2(117) = 234$.

Time = 3.42 (sec) , antiderivative size = 1476, normalized size of antiderivative = 11.18

$$\int F^{c(a+bx)} \cosh^2(d+ex) \sinh(d+ex) \, dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d)**2*sinh(e*x+d),x)
```

output
```
Piecewise((x*sinh(d)*cosh(d)**2, Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*sinh(d)
*cosh(d)**2, Eq(b, 0) & Eq(e, 0)), (x*sinh(d)*cosh(d)**2, Eq(c, 0) & Eq(e,
 0)), (F**(a*c + b*c*x)*x*sinh(b*c*x*log(F) - d)**3/8 - F**(a*c + b*c*x)*x
*sinh(b*c*x*log(F) - d)**2*cosh(b*c*x*log(F) - d)/8 - F**(a*c + b*c*x)*x*s
inh(b*c*x*log(F) - d)*cosh(b*c*x*log(F) - d)**2/8 + F**(a*c + b*c*x)*x*cos
h(b*c*x*log(F) - d)**3/8 + F**(a*c + b*c*x)*sinh(b*c*x*log(F) - d)**3/(8*b
*c*log(F)) - F**(a*c + b*c*x)*sinh(b*c*x*log(F) - d)**2*cosh(b*c*x*log(F)
- d)/(4*b*c*log(F)) - F**(a*c + b*c*x)*cosh(b*c*x*log(F) - d)**3/(8*b*c*lo
g(F)), Eq(e, -b*c*log(F))), (-F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)*
*3/8 + 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)**2*cosh(b*c*x*log(F)/
3 - d)/8 - 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)*cosh(b*c*x*log(F)
/3 - d)**2/8 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F)/3 - d)**3/8 + 3*F**(a*
c + b*c*x)*sinh(b*c*x*log(F)/3 - d)**3/(8*b*c*log(F)) - 3*F**(a*c + b*c*x)
*sinh(b*c*x*log(F)/3 - d)**2*cosh(b*c*x*log(F)/3 - d)/(4*b*c*log(F)) - F**
(a*c + b*c*x)*cosh(b*c*x*log(F)/3 - d)**3/(8*b*c*log(F)), Eq(e, -b*c*log(F
)/3)), (F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 + d)**3/8 - 3*F**(a*c + b*c
*x)*x*sinh(b*c*x*log(F)/3 + d)**2*cosh(b*c*x*log(F)/3 + d)/8 + 3*F**(a*c +
 b*c*x)*x*sinh(b*c*x*log(F)/3 + d)*cosh(b*c*x*log(F)/3 + d)**2/8 - F**(a*c
 + b*c*x)*x*cosh(b*c*x*log(F)/3 + d)**3/8 - 3*F**(a*c + b*c*x)*sinh(b*c*x*
log(F)/3 + d)**3/(8*b*c*log(F)) + 3*F**(a*c + b*c*x)*sinh(b*c*x*log(F)/...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02

$$\int F^{c(a+bx)}\cosh^2(d+ex)\sinh(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+3\,ex+3\,d)}}{8\,(bc\log(F)+3\,e)} + \frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{8\,(bc\log(F)+e)} - \frac{F^{ac}e^{(bcx\log(F)-ex)}}{8\,(bce^d\log(F)-ee^d)} - \frac{F^{ac}e^{(bcx\log(F)-3\,ex)}}{8\,(bce^{(3\,d)}\log(F)-3\,ee^{(3\,d)})}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2*sinh(e*x+d),x, algorithm="maxima")
```

output
```
1/8*F^(a*c)*e^(b*c*x*log(F) + 3*e*x + 3*d)/(b*c*log(F) + 3*e) + 1/8*F^(a*c
)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) - 1/8*F^(a*c)*e^(b*c*x*log(F
) - e*x)/(b*c*e^d*log(F) - e*e^d) - 1/8*F^(a*c)*e^(b*c*x*log(F) - 3*e*x)/(
b*c*e^(3*d)*log(F) - 3*e*e^(3*d))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 1211, normalized size of antiderivative = 9.17

$$\int F^{c(a+bx)}\cosh^2(d+ex)\sinh(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2*sinh(e*x+d),x, algorithm="giac")
```

output
```
1/4*(2*(b*c*log(abs(F)) + 3*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1
/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(ab
s(F)) + 3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*
pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4
*(b*c*log(abs(F)) + 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*
x + 3*d) + 1/2*I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a
*c*sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(
F)) + 24*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*
sgn(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)
) + 24*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3*d) + 1/4*(2*
(b*c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c
*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e
)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x -
1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(a
bs(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 1/2*I*
(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I
*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F)) + 8*e) - I*e^
(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*
a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) + 8*e))*e^(a*c*l
og(abs(F)) + (b*c*log(abs(F)) + e)*x + d) - 1/4*(2*(b*c*log(abs(F)) - e...
```

Mupad [B] (verification not implemented)

Time = 3.41 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.23

$$\int F^{c(a+bx)}\cosh^2(d+ex)\sinh(d+ex)\,dx = \frac{F^{a\,c+b\,c\,x}\,\left(3\,e^3\cosh(d+e\,x)^3-2\,b\,c\,e^2\sinh(d+e\,x)\,\ln\left(F\right)-3\,b^2\,c^2\,e\cosh(d+e\,x)^3\ln\left(F\right)^2+b^3\,c^3\cosh(\right.}{b^4\,c^4\ln\left(F\right)^4-10\,b}$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x)^2*sinh(d + e*x),x)
```

output
```
(F^(a*c + b*c*x)*(3*e^3*cosh(d + e*x)^3 - 2*b*c*e^2*sinh(d + e*x)*log(F) -
 3*b^2*c^2*e*cosh(d + e*x)^3*log(F)^2 + b^3*c^3*cosh(d + e*x)^2*sinh(d + e
*x)*log(F)^3 + 2*b^2*c^2*e*cosh(d + e*x)*log(F)^2 - b*c*e^2*cosh(d + e*x)^
2*sinh(d + e*x)*log(F)))/(9*e^4 + b^4*c^4*log(F)^4 - 10*b^2*c^2*e^2*log(F)
^2)
```

Reduce [F]

$$\int F^{c(a+bx)} \cosh^2(d+ex) \sinh(d+ex) \, dx = f^{ac} \left(\int f^{bcx} \cosh{(ex+d)}^2 \sinh{(ex+d)} \, dx \right)$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^2*sinh(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cosh(d + e*x)**2*sinh(d + e*x),x)
```

3.105 $\int F^{c(a+bx)} \cosh^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 93

$$\int F^{c(a+bx)} \cosh^2(d+ex)\,dx = \frac{F^{c(a+bx)}}{2bc\log(F)} - \frac{e^{-2d-2ex}F^{c(a+bx)}}{4(2e-bc\log(F))} + \frac{e^{2d+2ex}F^{c(a+bx)}}{4(2e+bc\log(F))}$$

output
```
1/2*F^(c*(b*x+a))/b/c/ln(F)-exp(-2*e*x-2*d)*F^(c*(b*x+a))/(8*e-4*b*c*ln(F)
)+exp(2*e*x+2*d)*F^(c*(b*x+a))/(8*e+4*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91

$$\int F^{c(a+bx)} \cosh^2(d+ex)\,dx = \frac{F^{c(a+bx)}\left(-4e^2+b^2c^2\log^2(F)+b^2c^2\cosh(2(d+ex))\log^2(F)-2bce\log(F)\sinh(2(d+ex))\right)}{-8bce^2\log(F)+2b^3c^3\log^3(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^2,x]
```

output
```
(F^(c*(a + b*x))*(-4*e^2 + b^2*c^2*Log[F]^2 + b^2*c^2*Cosh[2*(d + e*x)]*Lo
g[F]^2 - 2*b*c*e*Log[F]*Sinh[2*(d + e*x)]))/(-8*b*c*e^2*Log[F] + 2*b^3*c^3
*Log[F]^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.42, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6000, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6000}$$

$$\frac{2e^2\int F^{c(a+bx)}dx}{4e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}$$

$$\Big\downarrow \text{2624}$$

$$-\frac{bc\log(F)\cosh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x]^2,x]
```

output
```
(2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 - b^2*c^2*Log[F]^2)) - (b*c*F^(
c*(a + b*x))*Cosh[d + e*x]^2*Log[F])/(4*e^2 - b^2*c^2*Log[F]^2) + (2*e*F^(
c*(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x])/(4*e^2 - b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 6000
```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Simp[e*n*F^(c*(a + b*x))*Sinh[d + e*x]*(Cosh[d + e*x]^
(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x] + Simp[n*(n - 1)*(e^2/(e^2*n^2 -
b^2*c^2*Log[F]^2))   Int[F^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97

method	result
parallelrisch	$-\frac{2F^{c(bx+a)}\left(-\frac{c^2b^2\ln(F)^2\cosh(2ex+2d)}{2}-\frac{b^2c^2\ln(F)^2}{2}+\ln(F)bce\sinh(2ex+2d)+2e^2\right)}{2c^3b^3\ln(F)^3-8e^2bc\ln(F)}$
risch	$\frac{\left(\ln(F)^2b^2c^2\mathrm{e}^{4ex+4d}+2\ln(F)^2b^2c^2\mathrm{e}^{2ex+2d}-2\ln(F)bce\,\mathrm{e}^{4ex+4d}+b^2c^2\ln(F)^2+2\ln(F)bce-8e^2\mathrm{e}^{2ex+2d}\right)\mathrm{e}^{-2ex-2d}F^{c(bx+a)}}{4\ln(F)bc(bc\ln(F)-2e)(2e+bc\ln(F))}$
orering	$\frac{\left(3b^2c^2\ln(F)^2-4e^2\right)F^{c(bx+a)}\cosh(ex+d)^2}{\left(b^2c^2\ln(F)^2-4e^2\right)\ln(F)bc}-\frac{3\left(F^{c(bx+a)}bc\ln(F)\cosh(ex+d)^2+2F^{c(bx+a)}\cosh(ex+d)e\sinh(ex+d)\right)}{b^2c^2\ln(F)^2-4e^2}+$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*F^(c*(b*x+a))*(-1/2*c^2*b^2*ln(F)^2*cosh(2*e*x+2*d)-1/2*b^2*c^2*ln(F)^2
+ln(F)*b*c*e*sinh(2*e*x+2*d)+2*e^2)/(2*c^3*b^3*ln(F)^3-8*e^2*b*c*ln(F))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(84) = 168$.

Time = 0.09 (sec) , antiderivative size = 699, normalized size of antiderivative = 7.52

$$\int F^{c(a+bx)}\cosh^2(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/4*(((b^2*c^2*log(F)^2 - 2*b*c*e*log(F))*sinh(e*x + d)^4 - 8*e^2*cosh(e*x
 + d)^2 + 4*(b^2*c^2*cosh(e*x + d)*log(F)^2 - 2*b*c*e*cosh(e*x + d)*log(F)
)*sinh(e*x + d)^3 + (b^2*c^2*cosh(e*x + d)^4 + 2*b^2*c^2*cosh(e*x + d)^2 +
 b^2*c^2)*log(F)^2 - 2*(6*b*c*e*cosh(e*x + d)^2*log(F) - (3*b^2*c^2*cosh(e
*x + d)^2 + b^2*c^2)*log(F)^2 + 4*e^2)*sinh(e*x + d)^2 - 2*(b*c*e*cosh(e*x
 + d)^4 - b*c*e)*log(F) - 4*(2*b*c*e*cosh(e*x + d)^3*log(F) + 4*e^2*cosh(e
*x + d) - (b^2*c^2*cosh(e*x + d)^3 + b^2*c^2*cosh(e*x + d))*log(F)^2)*sinh
(e*x + d))*cosh((b*c*x + a*c)*log(F)) + ((b^2*c^2*log(F)^2 - 2*b*c*e*log(F
))*sinh(e*x + d)^4 - 8*e^2*cosh(e*x + d)^2 + 4*(b^2*c^2*cosh(e*x + d)*log(
F)^2 - 2*b*c*e*cosh(e*x + d)*log(F))*sinh(e*x + d)^3 + (b^2*c^2*cosh(e*x +
 d)^4 + 2*b^2*c^2*cosh(e*x + d)^2 + b^2*c^2)*log(F)^2 - 2*(6*b*c*e*cosh(e*
x + d)^2*log(F) - (3*b^2*c^2*cosh(e*x + d)^2 + b^2*c^2)*log(F)^2 + 4*e^2)*
sinh(e*x + d)^2 - 2*(b*c*e*cosh(e*x + d)^4 - b*c*e)*log(F) - 4*(2*b*c*e*co
sh(e*x + d)^3*log(F) + 4*e^2*cosh(e*x + d) - (b^2*c^2*cosh(e*x + d)^3 + b^
2*c^2*cosh(e*x + d))*log(F)^2)*sinh(e*x + d))*sinh((b*c*x + a*c)*log(F)))/
(b^3*c^3*cosh(e*x + d)^2*log(F)^3 - 4*b*c*e^2*cosh(e*x + d)^2*log(F) + (b^
3*c^3*log(F)^3 - 4*b*c*e^2*log(F))*sinh(e*x + d)^2 + 2*(b^3*c^3*cosh(e*x +
 d)*log(F)^3 - 4*b*c*e^2*cosh(e*x + d)*log(F))*sinh(e*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(78) = 156$.

Time = 1.25 (sec) , antiderivative size = 707, normalized size of antiderivative = 7.60

$$\int F^{c(a+bx)}\cosh^2(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d)**2,x)
```

output
```
Piecewise((x*cosh(d)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)), (-x*s
inh(d + e*x)**2/2 + x*cosh(d + e*x)**2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*
e), Eq(F, 1)), (F**(a*c)*(-x*sinh(d + e*x)**2/2 + x*cosh(d + e*x)**2/2 + s
inh(d + e*x)*cosh(d + e*x)/(2*e)), Eq(b, 0)), (-x*sinh(d + e*x)**2/2 + x*c
osh(d + e*x)**2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*e), Eq(c, 0)), (F**(a*c
 + b*c*x)*x*sinh(b*c*x*log(F)/2 - d)**2/4 - F**(a*c + b*c*x)*x*sinh(b*c*x*
log(F)/2 - d)*cosh(b*c*x*log(F)/2 - d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*l
og(F)/2 - d)**2/4 - F**(a*c + b*c*x)*sinh(b*c*x*log(F)/2 - d)**2/(b*c*log(
F)) + 3*F**(a*c + b*c*x)*sinh(b*c*x*log(F)/2 - d)*cosh(b*c*x*log(F)/2 - d)
/(2*b*c*log(F)), Eq(e, -b*c*log(F)/2)), (F**(a*c + b*c*x)*x*sinh(b*c*x*log
(F)/2 + d)**2/4 - F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/2 + d)*cosh(b*c*x*l
og(F)/2 + d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F)/2 + d)**2/4 - F**(a*
c + b*c*x)*sinh(b*c*x*log(F)/2 + d)*cosh(b*c*x*log(F)/2 + d)/(2*b*c*log(F)
) + F**(a*c + b*c*x)*cosh(b*c*x*log(F)/2 + d)**2/(b*c*log(F)), Eq(e, b*c*l
og(F)/2)), (F**(a*c + b*c*x)*b**2*c**2*log(F)**2*cosh(d + e*x)**2/(b**3*c*
*3*log(F)**3 - 4*b*c*e**2*log(F)) - 2*F**(a*c + b*c*x)*b*c*e*log(F)*sinh(d
 + e*x)*cosh(d + e*x)/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F)) + 2*F**(a*
c + b*c*x)*e**2*sinh(d + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F))
 - 2*F**(a*c + b*c*x)*e**2*cosh(d + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e
**2*log(F)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01

$$\int F^{c(a+bx)}\cosh^2(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+2\,ex+2\,d)}}{4\,(bc\log(F)+2\,e)} + \frac{F^{ac}e^{(bcx\log(F)-2\,ex)}}{4\,(bce^{(2\,d)}\log(F)-2\,ee^{(2\,d)})} + \frac{F^{bcx+ac}}{2\,bc\log(F)}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2,x, algorithm="maxima")
```

output
```
1/4*F^(a*c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) + 1/4*F^(a*c
)*e^(b*c*x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) + 1/2*F^(b*c
*x + a*c)/(b*c*log(F))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 889, normalized size of antiderivative = 9.56

$$\int F^{c(a+bx)}\cosh^2(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2,x, algorithm="giac")
```

output
```
(2*b*c*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*p
i*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2)
- (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*p
i*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*
b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*e^(1/2*I*pi*b*c*x*
sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*I*pi*b*c*
sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1
/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) +
2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) +
 1/2*(2*(b*c*log(abs(F)) + 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x -
1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(a
bs(F)) + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2
*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 +
4*(b*c*log(abs(F)) + 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)
*x + 2*d) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*
sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F))
 + 16*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn
(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) +
 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x + 2*d) + 1/2*(2*(b*
c*log(abs(F)) - 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a...
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.08

$$\int F^{c(a+bx)}\cosh^2(d+ex)\,dx = -\frac{2\,F^{a\,c+b\,c\,x}\,e^2-F^{a\,c+b\,c\,x}\,b^2\,c^2\cosh(d+e\,x)^2\ln\left(F\right)^2+2\,F^{a\,c+b\,c\,x}\,b\,c\,e\cosh(d+e\,x)\,\sinh(d+e\,x)\,\ln\left(F\right)}{b^3\,c^3\ln\left(F\right)^3-4\,b\,c\,e^2\,\ln\left(F\right)}$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x)^2,x)
```

output
```
-(2*F^(a*c + b*c*x)*e^2 - F^(a*c + b*c*x)*b^2*c^2*cosh(d + e*x)^2*log(F)^2
 + 2*F^(a*c + b*c*x)*b*c*e*cosh(d + e*x)*sinh(d + e*x)*log(F))/(b^3*c^3*lo
g(F)^3 - 4*b*c*e^2*log(F))
```

Reduce [F]

$$\int F^{c(a+bx)}\cosh^2(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\cosh\left(ex+d\right)^2dx\right)$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cosh(d + e*x)**2,x)
```

3.106 $\int F^{c(a+bx)} \cosh(d+ex) \coth(d+ex) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 134

$$\int F^{c(a+bx)} \cosh(d+ex) \coth(d+ex) \, dx = -\frac{3e^{-d-ex}F^{c(a+bx)}}{2(e-bc\log(F))} + \frac{2e^{-d-ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(-1+\frac{bc\log(F)}{e}\right),\frac{e+bc\log(F)}{2e},e^{2d+2ex}\right)}{e-bc\log(F)} + \frac{e^{d+ex}F^{c(a+bx)}}{2(e+bc\log(F))}$$

output
```
-3*exp(-e*x-d)*F^(c*(b*x+a))/(2*e-2*b*c*ln(F))+2*exp(-e*x-d)*F^(c*(b*x+a))
*hypergeom([1, -1/2+1/2*b*c*ln(F)/e],[1/2*(e+b*c*ln(F))/e],exp(2*e*x+2*d))
/(e-b*c*ln(F))+exp(e*x+d)*F^(c*(b*x+a))/(2*e+2*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 7.80 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.18

$$\int F^{c(a+bx)} \cosh(d+ex)\coth(d+ex)\,dx = -\frac{F^{c(a+bx)}\left(\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{e},1+\frac{bc\log(F)}{e},e^{d+ex}\right)\left(e^2-b^2c^2\log^2(F)\right)+\text{Hypergeometric2I}}{bc\log(F)(e-}$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x]*Coth[d + e*x],x]
```

output
```
-((F^(c*(a + b*x))*(Hypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/
e, E^(d + e*x)]*(e^2 - b^2*c^2*Log[F]^2) + Hypergeometric2F1[1, (b*c*Log[F
])/e, 1 + (b*c*Log[F])/e, -E^(d + e*x)]*(-e^2 + b^2*c^2*Log[F]^2) + b*c*Lo
g[F]*(-(e*Cosh[d + e*x]) + b*c*Log[F]*Sinh[d + e*x])))/(b*c*Log[F]*(e - b*
c*Log[F])*(e + b*c*Log[F])))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(d+ex)\coth(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6037}$$

$$\int \left(\frac{3}{2}e^{-d-ex}F^{ac+bcx}+\frac{1}{2}e^{2(d+ex)-d-ex}F^{ac+bcx}+\frac{2e^{-d-ex}F^{ac+bcx}}{e^{2(d+ex)}-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2e^{-d-ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(\frac{bc\log(F)}{e}-1\right),\frac{e+bc\log(F)}{2e},e^{2(d+ex)}\right)}{e-bc\log(F)}-\frac{3F^{ac}e^{-x(e-bc\log(F))-d}}{2(e-bc\log(F))}+\frac{F^{ac}e^{x(bc\log(F)+e)+d}}{2(bc\log(F)+e)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x]*Coth[d + e*x],x]
```

output
```
(-3*E^(-d - x*(e - b*c*Log[F]))*F^(a*c))/(2*(e - b*c*Log[F])) + (2*E^(-d -
 e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[1, (-1 + (b*c*Log[F])/e)/2, (e + b
*c*Log[F])/(2*e), E^(2*(d + e*x))])/(e - b*c*Log[F]) + (E^(d + x*(e + b*c*
Log[F]))*F^(a*c))/(2*(e + b*c*Log[F]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Maple [F]

$$\int F^{c(bx+a)}\cosh(ex+d)\coth(ex+d)\,dx$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)*coth(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*cosh(e*x+d)*coth(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)} \cosh(d+ex) \coth(d+ex) \, dx = \int F^{(bx+a)c} \cosh(ex+d) \coth(ex+d) \, dx$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)*coth(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*cosh(e*x + d)*coth(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)} \cosh(d+ex) \coth(d+ex) \, dx = \int F^{c(a+bx)} \cosh(d+ex) \coth(d+ex) \, dx$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d)*coth(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*cosh(d + e*x)*coth(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)} \cosh(d+ex) \coth(d+ex) \, dx = \int F^{(bx+a)c} \cosh(ex+d) \coth(ex+d) \, dx$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)*coth(e*x+d),x, algorithm="maxima")
```

output
```
4*F^(a*c)*e*integrate(F^(b*c*x)/((b*c*e^(5*d)*log(F) - 3*e*e^(5*d))*e^(5*e
*x) - 2*(b*c*e^(3*d)*log(F) - 3*e*e^(3*d))*e^(3*e*x) + (b*c*e^d*log(F) - 3
*e*e^d)*e^(e*x)), x) + 1/2*(F^(a*c)*b^2*c^2*log(F)^2 + 6*F^(a*c)*b*c*e*log
(F) + 5*F^(a*c)*e^2 + (F^(a*c)*b^2*c^2*e^(4*d)*log(F)^2 - 4*F^(a*c)*b*c*e*
e^(4*d)*log(F) + 3*F^(a*c)*e^2*e^(4*d))*e^(4*e*x) + 2*(F^(a*c)*b^2*c^2*e^(
2*d)*log(F)^2 - F^(a*c)*b*c*e*e^(2*d)*log(F) - 6*F^(a*c)*e^2*e^(2*d))*e^(2
*e*x))*F^(b*c*x)/((b^3*c^3*e^(3*d)*log(F)^3 - 3*b^2*c^2*e*e^(3*d)*log(F)^2
 - b*c*e^2*e^(3*d)*log(F) + 3*e^3*e^(3*d))*e^(3*e*x) - (b^3*c^3*e^d*log(F)
^3 - 3*b^2*c^2*e*e^d*log(F)^2 - b*c*e^2*e^d*log(F) + 3*e^3*e^d)*e^(e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\cosh(d+ex)\coth(d+ex)\,dx=\int F^{(bx+a)c}\cosh\left(ex+d\right)\coth\left(ex+d\right)\,dx$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)*coth(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*cosh(e*x + d)*coth(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\cosh(d+ex)\coth(d+ex)\,dx=\int F^{c\,(a+b\,x)}\,\cosh(d+e\,x)\,\coth(d+e\,x)\,dx$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x)*coth(d + e*x),x)
```

output
```
int(F^(c*(a + b*x))*cosh(d + e*x)*coth(d + e*x), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\cosh(d+ex)\coth(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\cosh(ex+d)\coth(ex+d)\,dx\right)$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)*coth(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cosh(d + e*x)*coth(d + e*x),x)
```

3.107 $\int F^{c(a+bx)} \coth^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 101

$$\int F^{c(a+bx)} \coth^2(d+ex)\,dx = \frac{2F^{c(a+bx)}}{e(1-e^{2d+2ex})} - \frac{2F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},e^{2d+2ex}\right)}{e} + \frac{F^{c(a+bx)}}{bc\log(F)}$$

output

```
2*F^(c*(b*x+a))/e/(1-exp(2*e*x+2*d))-2*F^(c*(b*x+a))*hypergeom([1, 1/2*b*c
*ln(F)/e],[1+1/2*b*c*ln(F)/e],exp(2*e*x+2*d))/e+F^(c*(b*x+a))/b/c/ln(F)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

$$\int F^{c(a+bx)}\coth^2(d+ex)\,dx = F^{c(a+bx)}\left(\frac{2}{e-ee^{2d}} - \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},e^{2(d+ex)}\right)}{e} + \frac{1}{bc\log(F)} + \frac{\text{csch}(d)\text{csch}(d+ex)\sinh(ex)}{e}\right)$$

input
```
Integrate[F^(c*(a + b*x))*Coth[d + e*x]^2,x]
```

output
```
F^(c*(a + b*x))*(2/(e - e*E^(2*d)) - (2*Hypergeometric2F1[1, (b*c*Log[F])/
(2*e), 1 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/e + 1/(b*c*Log[F]) + (Csc
h[d]*Csch[d + e*x]*Sinh[e*x])/e)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6008}$$

$$\int \left(\frac{4F^{c(a+bx)}}{e^{2(d+ex)}-1} + \frac{4F^{c(a+bx)}}{\left(e^{2(d+ex)}-1\right)^2} + F^{c(a+bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,e^{2(d+ex)}\right)}{bc\log(F)}+\frac{4F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,e^{2(d+ex)}\right)}{bc\log(F)}+\frac{F^{c(a+bx)}}{bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Coth[d + e*x]^2,x]
```

output
```
F^(c*(a + b*x))/(b*c*Log[F]) - (4*F^(c*(a + b*x))*Hypergeometric2F1[1, (b*
c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(b*c*Log[F]) +
(4*F^(c*(a + b*x))*Hypergeometric2F1[2, (b*c*Log[F])/(2*e), 1 + (b*c*Log[F
])/(2*e), E^(2*(d + e*x))])/(b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6008
```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*(d + e*x)))^n/(-1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)}\coth(ex+d)^2\,dx$$

input
```
int(F^(c*(b*x+a))*coth(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*coth(e*x+d)^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \coth^2(d+ex)\,dx = \int F^{(bx+a)c} \coth{(ex+d)}^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*coth(e*x + d)^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \coth^2(d+ex)\,dx = \int F^{c(a+bx)} \coth^2{(d+ex)}\,dx$$

input
```
integrate(F**(c*(b*x+a))*coth(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*coth(d + e*x)**2, x)
```

Maxima [F]

$$\int F^{c(a+bx)} \coth^2(d+ex)\,dx = \int F^{(bx+a)c} \coth{(ex+d)}^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)^2,x, algorithm="maxima")
```

output
```
16*F^(a*c)*b*c*e*integrate(-F^(b*c*x)/(b^2*c^2*log(F)^2 - 6*b*c*e*log(F) +
 8*e^2 - (b^2*c^2*e^(6*d)*log(F)^2 - 6*b*c*e*e^(6*d)*log(F) + 8*e^2*e^(6*d
))*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*log(F) + 8*e^
2*e^(4*d))*e^(4*e*x) - 3*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^(2*d)*log(F
) + 8*e^2*e^(2*d))*e^(2*e*x)), x)*log(F) + (F^(a*c)*b^2*c^2*log(F)^2 + 10*
F^(a*c)*b*c*e*log(F) + 8*F^(a*c)*e^2 + (F^(a*c)*b^2*c^2*e^(4*d)*log(F)^2 -
 6*F^(a*c)*b*c*e*e^(4*d)*log(F) + 8*F^(a*c)*e^2*e^(4*d))*e^(4*e*x) + 2*(F^
(a*c)*b^2*c^2*e^(2*d)*log(F)^2 - 2*F^(a*c)*b*c*e*e^(2*d)*log(F) - 8*F^(a*c
)*e^2*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^3*c^3*log(F)^3 - 6*b^2*c^2*e*log(F)
^2 + 8*b*c*e^2*log(F) + (b^3*c^3*e^(4*d)*log(F)^3 - 6*b^2*c^2*e*e^(4*d)*lo
g(F)^2 + 8*b*c*e^2*e^(4*d)*log(F))*e^(4*e*x) - 2*(b^3*c^3*e^(2*d)*log(F)^3
 - 6*b^2*c^2*e*e^(2*d)*log(F)^2 + 8*b*c*e^2*e^(2*d)*log(F))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)} \coth^2(d+ex)\,dx = \int F^{(bx+a)c} \coth{(ex+d)}^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*coth(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)} \coth^2(d+ex)\,dx = \int F^{c\,(a+b\,x)}\,\coth(d+e\,x)^2\,dx$$

input
```
int(F^(c*(a + b*x))*coth(d + e*x)^2,x)
```

output
```
int(F^(c*(a + b*x))*coth(d + e*x)^2, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\coth^2(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\coth{(ex+d)}^2\,dx\right)$$

input
```
int(F^(c*(b*x+a))*coth(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*coth(d + e*x)**2,x)
```

3.108 $\int F^{c(a+bx)}\coth^2(d+ex)\text{csch}(d+ex)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 166

$$\int F^{c(a+bx)}\coth^2(d+ex)\text{csch}(d+ex)\,dx = -\frac{2e^{d+ex}F^{c(a+bx)}}{e\left(1-e^{2d+2ex}\right)^2}+\frac{e^{d+ex}F^{c(a+bx)}(e+bc\log(F))}{e^2\left(1-e^{2d+2ex}\right)} - \frac{e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),e^{2d+2ex}\right)\left(e^2+b^2c^2\log^2(F)\right)}{e^2(e+bc\log(F))}$$

output
```
-2*exp(e*x+d)*F^(c*(b*x+a))/e/(1-exp(2*e*x+2*d))^2+exp(e*x+d)*F^(c*(b*x+a)
)*(e+b*c*ln(F))/e^2/(1-exp(2*e*x+2*d))-exp(e*x+d)*F^(c*(b*x+a))*hypergeom(
[1, 1/2*(e+b*c*ln(F))/e],[3/2+1/2*b*c*ln(F)/e],exp(2*e*x+2*d))*(e^2+b^2*c^
2*ln(F)^2)/e^2/(e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 2.21 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.87

$$\int F^{c(a+bx)}\coth^2(d+ex)\text{csch}(d+ex)\,dx =$$
$$-\frac{F^{c\left(a-\frac{bd}{e}\right)}\left(F^{\frac{bc(d+ex)}{e}}\text{csch}(d+ex)(e+bc\log(F))(e\coth(d+ex)+bc\log(F))+2e^{\frac{(d+ex)(e+bc\log(F))}{e}}\text{Hyperg}\right.}{2e^2(e+bc\log(F))}$$

input
```
Integrate[F^(c*(a + b*x))*Coth[d + e*x]^2*Csch[d + e*x],x]
```

output
```
-1/2*(F^(c*(a - (b*d)/e))*(F^((b*c*(d + e*x))/e)*Csch[d + e*x]*(e + b*c*Lo
g[F])*(e*Coth[d + e*x] + b*c*Log[F]) + 2*E^(((d + e*x)*(e + b*c*Log[F]))/e
)*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e), (3 + (b*c*Log[F])/e)/2, E^(
2*(d + e*x))]*(e^2 + b^2*c^2*Log[F]^2)))/(e^2*(e + b*c*Log[F]))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(d+ex)\text{csch}(d+ex)F^{c(a+bx)}\,dx$$
$$\Big\downarrow \text{6037}$$
$$\int \left(\frac{2e^{d+ex}F^{ac+bcx}}{e^{2(d+ex)}-1}+\frac{8e^{d+ex}F^{ac+bcx}}{\left(e^{2(d+ex)}-1\right)^2}+\frac{8e^{d+ex}F^{ac+bcx}}{\left(e^{2(d+ex)}-1\right)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{2e^{d+ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),e^{2(d+ex)}\right)}{bc\log(F)+e}+\frac{8e^{d+ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(2,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),e^{2(d+ex)}\right)}{bc\log(F)+e}-\frac{8e^{d+ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(3,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),e^{2(d+ex)}\right)}{bc\log(F)+e}$$

input
```
Int[F^(c*(a + b*x))*Coth[d + e*x]^2*Csch[d + e*x],x]
```

output
```
(-2*E^(d + e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e
), (3 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))])/(e + b*c*Log[F]) + (8*E^(d +
e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[2, (e + b*c*Log[F])/(2*e), (3 + (b*
c*Log[F])/e)/2, E^(2*(d + e*x))])/(e + b*c*Log[F]) - (8*E^(d + e*x)*F^(a*c
 + b*c*x)*Hypergeometric2F1[3, (e + b*c*Log[F])/(2*e), (3 + (b*c*Log[F])/e
)/2, E^(2*(d + e*x))])/(e + b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Maple [F]

$$\int F^{c(bx+a)}\coth(ex+d)^2\operatorname{csch}(ex+d)\,dx$$

input
```
int(F^(c*(b*x+a))*coth(e*x+d)^2*csch(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*coth(e*x+d)^2*csch(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)} \coth^2(d+ex)\text{csch}(d+ex)\,dx = \int F^{(bx+a)c} \coth(ex+d)^2 \,\text{csch}(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)^2*csch(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*coth(e*x + d)^2*csch(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)} \coth^2(d+ex)\text{csch}(d+ex)\,dx = \int F^{c(a+bx)} \coth^2(d+ex)\,\text{csch}(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*coth(e*x+d)**2*csch(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*coth(d + e*x)**2*csch(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)} \coth^2(d+ex)\text{csch}(d+ex)\,dx = \int F^{(bx+a)c} \coth(ex+d)^2 \,\text{csch}(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)^2*csch(e*x+d),x, algorithm="maxima")
```

output
```
48*(F^(a*c)*b^2*c^2*e*e^d*log(F)^2 + F^(a*c)*e^3*e^d)*integrate(e^(b*c*x*l
og(F) + e*x)/(b^3*c^3*log(F)^3 - 9*b^2*c^2*e*log(F)^2 + 23*b*c*e^2*log(F)
- 15*e^3 + (b^3*c^3*e^(8*d)*log(F)^3 - 9*b^2*c^2*e*e^(8*d)*log(F)^2 + 23*b
*c*e^2*e^(8*d)*log(F) - 15*e^3*e^(8*d))*e^(8*e*x) - 4*(b^3*c^3*e^(6*d)*log
(F)^3 - 9*b^2*c^2*e*e^(6*d)*log(F)^2 + 23*b*c*e^2*e^(6*d)*log(F) - 15*e^3*
e^(6*d))*e^(6*e*x) + 6*(b^3*c^3*e^(4*d)*log(F)^3 - 9*b^2*c^2*e*e^(4*d)*log
(F)^2 + 23*b*c*e^2*e^(4*d)*log(F) - 15*e^3*e^(4*d))*e^(4*e*x) - 4*(b^3*c^3
*e^(2*d)*log(F)^3 - 9*b^2*c^2*e*e^(2*d)*log(F)^2 + 23*b*c*e^2*e^(2*d)*log(
F) - 15*e^3*e^(2*d))*e^(2*e*x)), x) - 2*((F^(a*c)*b^2*c^2*e^(5*d)*log(F)^2
 - 8*F^(a*c)*b*c*e*e^(5*d)*log(F) + 15*F^(a*c)*e^2*e^(5*d))*e^(5*e*x) + 2*
(F^(a*c)*b^2*c^2*e^(3*d)*log(F)^2 - 3*F^(a*c)*b*c*e*e^(3*d)*log(F) - 10*F^
(a*c)*e^2*e^(3*d))*e^(3*e*x) + (F^(a*c)*b^2*c^2*e^d*log(F)^2 + 14*F^(a*c)*
b*c*e*e^d*log(F) + 9*F^(a*c)*e^2*e^d)*e^(e*x))*F^(b*c*x)/(b^3*c^3*log(F)^3
 - 9*b^2*c^2*e*log(F)^2 + 23*b*c*e^2*log(F) - 15*e^3 - (b^3*c^3*e^(6*d)*lo
g(F)^3 - 9*b^2*c^2*e*e^(6*d)*log(F)^2 + 23*b*c*e^2*e^(6*d)*log(F) - 15*e^3
*e^(6*d))*e^(6*e*x) + 3*(b^3*c^3*e^(4*d)*log(F)^3 - 9*b^2*c^2*e*e^(4*d)*lo
g(F)^2 + 23*b*c*e^2*e^(4*d)*log(F) - 15*e^3*e^(4*d))*e^(4*e*x) - 3*(b^3*c^
3*e^(2*d)*log(F)^3 - 9*b^2*c^2*e*e^(2*d)*log(F)^2 + 23*b*c*e^2*e^(2*d)*log
(F) - 15*e^3*e^(2*d))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\coth^2(d+ex)\text{csch}(d+ex)\,dx=\int F^{(bx+a)c}\coth{(ex+d)}^2\text{csch}\,(ex+d)\;dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)^2*csch(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*coth(e*x + d)^2*csch(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\coth^2(d+ex)\operatorname{csch}(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}\,\coth(d+e\,x)^2}{\sinh\,(d+e\,x)}\,dx$$

input
```
int((F^(c*(a + b*x))*coth(d + e*x)^2)/sinh(d + e*x),x)
```

output
```
int((F^(c*(a + b*x))*coth(d + e*x)^2)/sinh(d + e*x), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\coth^2(d+ex)\operatorname{csch}(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\coth\,(ex+d)^2\operatorname{csch}(ex+d)\,dx\right)$$

input
```
int(F^(c*(b*x+a))*coth(e*x+d)^2*csch(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*coth(d + e*x)**2*csch(d + e*x),x)
```

3.109 $\int F^{c(a+bx)}\cosh^3(d+ex)\sinh^3(d+ex)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 139

$$\int F^{c(a+bx)}\cosh^3(d+ex)\sinh^3(d+ex)\,dx=-\frac{3e^{-2d-2ex}F^{c(a+bx)}}{64(2e-bc\log(F))}+\frac{e^{-6d-6ex}F^{c(a+bx)}}{64(6e-bc\log(F))}-\frac{3e^{2d+2ex}F^{c(a+bx)}}{64(2e+bc\log(F))}+\frac{e^{6d+6ex}F^{c(a+bx)}}{64(6e+bc\log(F))}$$

output
```
-3*exp(-2*e*x-2*d)*F^(c*(b*x+a))/(128*e-64*b*c*ln(F))+exp(-6*e*x-6*d)*F^(c
*(b*x+a))/(384*e-64*b*c*ln(F))-3*exp(2*e*x+2*d)*F^(c*(b*x+a))/(128*e+64*b*
c*ln(F))+exp(6*e*x+6*d)*F^(c*(b*x+a))/(384*e+64*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.17

$$\int F^{c(a+bx)}\cosh^3(d+ex)\sinh^3(d+ex)\,dx$$

$$=\frac{F^{c(a+bx)}\left(6\cosh(6(d+ex))\left(4e^3-b^2c^2e\log^2(F)\right)+6\cosh(2(d+ex))\left(-36e^3+b^2c^2e\log^2(F)\right)+2bc\log\right.}{32\left(144e^4-40b^2c^2e^2\log^2(F)+\right.}$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^3*Sinh[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(6*Cosh[6*(d + e*x)]*(4*e^3 - b^2*c^2*e*Log[F]^2) + 6*Cos
h[2*(d + e*x)]*(-36*e^3 + b^2*c^2*e*Log[F]^2) + 2*b*c*Log[F]*(52*e^2 - b^2
*c^2*Log[F]^2 + Cosh[4*(d + e*x)]*(-4*e^2 + b^2*c^2*Log[F]^2))*Sinh[2*(d +
 e*x)]))/(32*(144*e^4 - 40*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{6035, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(d+ex)\cosh^3(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \ 6035$$

$$\int \left(\frac{1}{32}\sinh(6d+6ex)F^{c(a+bx)}-\frac{3}{32}\sinh(2d+2ex)F^{c(a+bx)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{3bc\log(F)\sinh(2d+2ex)F^{c(a+bx)}}{32\left(4e^2-b^2c^2\log^2(F)\right)}-\frac{bc\log(F)\sinh(6d+6ex)F^{c(a+bx)}}{32\left(36e^2-b^2c^2\log^2(F)\right)}-\frac{3e\cosh(2d+2ex)F^{c(a+bx)}}{16\left(4e^2-b^2c^2\log^2(F)\right)}+\frac{3e\cosh(6d+6ex)F^{c(a+bx)}}{16\left(36e^2-b^2c^2\log^2(F)\right)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x]^3*Sinh[d + e*x]^3,x]
```

output
```
(-3*e*F^(c*(a + b*x))*Cosh[2*d + 2*e*x])/(16*(4*e^2 - b^2*c^2*Log[F]^2)) +
 (3*e*F^(c*(a + b*x))*Cosh[6*d + 6*e*x])/(16*(36*e^2 - b^2*c^2*Log[F]^2))
+ (3*b*c*F^(c*(a + b*x))*Log[F]*Sinh[2*d + 2*e*x])/(32*(4*e^2 - b^2*c^2*Lo
g[F]^2)) - (b*c*F^(c*(a + b*x))*Log[F]*Sinh[6*d + 6*e*x])/(32*(36*e^2 - b^
2*c^2*Log[F]^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6035
```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(127) = 254$.

Time = 0.57 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.37

$$\frac{\left(\ln \left(F\right)^{3} b^{3} c^{3} \mathrm{e}^{12 e x +12 d}-6 \ln \left(F\right)^{2} b^{2} c^{2} e \,\mathrm{e}^{12 e x +12 d}-3 \ln \left(F\right)^{3} b^{3} c^{3} \mathrm{e}^{8 e x +8 d}-4 \ln \left(F\right) b c \,e^{2} \mathrm{e}^{12 e x +12 d}+6 \ln \left(F\right)^{2} \cdots}{}$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^3*sinh(e*x+d)^3,x)
```

output
```
1/64*(ln(F)^3*b^3*c^3*exp(12*e*x+12*d)-6*ln(F)^2*b^2*c^2*e*exp(12*e*x+12*d
)-3*ln(F)^3*b^3*c^3*exp(8*e*x+8*d)-4*ln(F)*b*c*e^2*exp(12*e*x+12*d)+6*ln(F
)^2*b^2*c^2*e*exp(8*e*x+8*d)+24*e^3*exp(12*e*x+12*d)+3*ln(F)^3*b^3*c^3*exp
(4*e*x+4*d)+108*ln(F)*b*c*e^2*exp(8*e*x+8*d)+6*ln(F)^2*b^2*c^2*e*exp(4*e*x
+4*d)-216*e^3*exp(8*e*x+8*d)-c^3*b^3*ln(F)^3-108*ln(F)*b*c*e^2*exp(4*e*x+4
*d)-6*c^2*b^2*ln(F)^2*e-216*e^3*exp(4*e*x+4*d)+4*e^2*b*c*ln(F)+24*e^3)/(b*
c*ln(F)-2*e)*exp(-6*e*x-6*d)/(b*c*ln(F)-6*e)/(2*e+b*c*ln(F))/(6*e+b*c*ln(F
))*F^(c*(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4202 vs. 2(125) = 250.

Time = 0.20 (sec) , antiderivative size = 4202, normalized size of antiderivative = 30.23

$$\int F^{c(a+bx)}\cosh^3(d+ex)\sinh^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^3*sinh(e*x+d)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\cosh^3(d+ex)\sinh^3(d+ex)\,dx = \text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d)**3*sinh(e*x+d)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.03

$$\int F^{c(a+bx)}\cosh^3(d+ex)\sinh^3(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+6\,ex+6\,d)}}{64\,(bc\log(F)+6\,e)} - \frac{3\,F^{ac}e^{(bcx\log(F)+2\,ex+2\,d)}}{64\,(bc\log(F)+2\,e)} + \frac{3\,F^{ac}e^{(bcx\log(F)-2\,ex)}}{64\,(bce^{(2\,d)}\log(F)-2\,ee^{(2\,d)})} - \frac{F^{ac}e^{(bcx\log(F)-6\,ex)}}{64\,(bce^{(6\,d)}\log(F)-6\,ee^{(6\,d)})}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^3*sinh(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/64*F^(a*c)*e^(b*c*x*log(F) + 6*e*x + 6*d)/(b*c*log(F) + 6*e) - 3/64*F^(a
*c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) + 3/64*F^(a*c)*e^(b*
c*x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) - 1/64*F^(a*c)*e^(b
*c*x*log(F) - 6*e*x)/(b*c*e^(6*d)*log(F) - 6*e*e^(6*d))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 1225, normalized size of antiderivative = 8.81

$$\int F^{c(a+bx)} \cosh^3(d+ex) \sinh^3(d+ex) \, dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^3*sinh(e*x+d)^3,x, algorithm="giac")
```

output
```
1/32*(2*(b*c*log(abs(F)) + 6*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x -
1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(a
bs(F)) + 6*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2
*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 +
4*(b*c*log(abs(F)) + 6*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 6*e)
*x + 6*d) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*
sgn(F) - 1/2*I*pi*a*c)/(64*I*pi*b*c*sgn(F) - 64*I*pi*b*c + 128*b*c*log(abs
(F)) + 768*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*
c*sgn(F) + 1/2*I*pi*a*c)/(-64*I*pi*b*c*sgn(F) + 64*I*pi*b*c + 128*b*c*log(
abs(F)) + 768*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 6*e)*x + 6*d) -
3/32*(2*(b*c*log(abs(F)) + 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x -
1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(a
bs(F)) + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2
*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 +
4*(b*c*log(abs(F)) + 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)
*x + 2*d) + 3*I*(-I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a
*c*sgn(F) - 1/2*I*pi*a*c)/(64*I*pi*b*c*sgn(F) - 64*I*pi*b*c + 128*b*c*log(
abs(F)) + 256*e) + I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi
*a*c*sgn(F) + 1/2*I*pi*a*c)/(-64*I*pi*b*c*sgn(F) + 64*I*pi*b*c + 128*b*c*l
og(abs(F)) + 256*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x + 2...
```

Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.77

$$\int F^{c(a+bx)}\cosh^3(d+ex)\sinh^3(d+ex)\,dx =$$

$$-\frac{e^3\left(\frac{27\,F^{a\,c+b\,c\,x}\cosh(2\,d+2\,e\,x)}{4}-\frac{3\,F^{a\,c+b\,c\,x}\cosh(6\,d+6\,e\,x)}{4}\right)+b^3\,c^3\ln\left(F\right)^3\left(\frac{3\,F^{a\,c+b\,c\,x}\sinh(2\,d+2\,e\,x)}{32}-\frac{F^{a\,c+b\,c\,x}\sinh(6\,d \ldots}{32}\right.}{b^4\,c^4\ln \ldots}$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x)^3*sinh(d + e*x)^3,x)
```

output
```
-(e^3*((27*F^(a*c + b*c*x)*cosh(2*d + 2*e*x))/4 - (3*F^(a*c + b*c*x)*cosh(
6*d + 6*e*x))/4) + b^3*c^3*log(F)^3*((3*F^(a*c + b*c*x)*sinh(2*d + 2*e*x))
/32 - (F^(a*c + b*c*x)*sinh(6*d + 6*e*x))/32) - b^2*c^2*e*log(F)^2*((3*F^(
a*c + b*c*x)*cosh(2*d + 2*e*x))/16 - (3*F^(a*c + b*c*x)*cosh(6*d + 6*e*x))
/16) - b*c*e^2*log(F)*((27*F^(a*c + b*c*x)*sinh(2*d + 2*e*x))/8 - (F^(a*c
+ b*c*x)*sinh(6*d + 6*e*x))/8))/(144*e^4 + b^4*c^4*log(F)^4 - 40*b^2*c^2*e
^2*log(F)^2)
```

Reduce [F]

$$\int F^{c(a+bx)}\cosh^3(d+ex)\sinh^3(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\cosh(ex+d)^3\sinh(ex+d)^3\,dx\right)$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^3*sinh(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cosh(d + e*x)**3*sinh(d + e*x)**3,x)
```

3.110 $\int F^{c(a+bx)} \cosh^3(d+ex) \sinh^2(d+ex)\, dx$

Optimal result

Integrand size = 26, antiderivative size = 201

$$\int F^{c(a+bx)} \cosh^3(d+ex) \sinh^2(d+ex)\, dx = \frac{e^{-d-ex} F^{c(a+bx)}}{16(e-bc\log(F))} - \frac{e^{-3d-3ex} F^{c(a+bx)}}{32(3e-bc\log(F))} - \frac{e^{-5d-5ex} F^{c(a+bx)}}{32(5e-bc\log(F))} - \frac{e^{d+ex} F^{c(a+bx)}}{16(e+bc\log(F))} + \frac{e^{3d+3ex} F^{c(a+bx)}}{32(3e+bc\log(F))} + \frac{e^{5d+5ex} F^{c(a+bx)}}{32(5e+bc\log(F))}$$

output
```
exp(-e*x-d)*F^(c*(b*x+a))/(16*e-16*b*c*ln(F))-exp(-3*e*x-3*d)*F^(c*(b*x+a)
)/(96*e-32*b*c*ln(F))-exp(-5*e*x-5*d)*F^(c*(b*x+a))/(160*e-32*b*c*ln(F))-e
xp(e*x+d)*F^(c*(b*x+a))/(16*e+16*b*c*ln(F))+exp(3*e*x+3*d)*F^(c*(b*x+a))/(
96*e+32*b*c*ln(F))+exp(5*e*x+5*d)*F^(c*(b*x+a))/(160*e+32*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.75

$$\int F^{c(a+bx)}\cosh^3(d+ex)\sinh^2(d+ex)\,dx$$
$$=\frac{1}{16}F^{c(a+bx)}\left(\frac{2bc\cosh(d+ex)\log(F)-2e\sinh(d+ex)}{(e-bc\log(F))(e+bc\log(F))}+\frac{-bc\cosh(3(d+ex))\log(F)+3e\sinh(3(d+ex))}{9e^2-b^2c^2\log^2(F)}+\frac{-bc\cosh(5(d+ex))\log(F)+5e\sinh(5(d+ex))}{25e^2-b^2c^2\log^2(F)}\right)$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^3*Sinh[d + e*x]^2,x]
```

output
```
(F^(c*(a + b*x))*((2*b*c*Cosh[d + e*x]*Log[F] - 2*e*Sinh[d + e*x])/((e - b
*c*Log[F])*(e + b*c*Log[F])) + (-(b*c*Cosh[3*(d + e*x)]*Log[F]) + 3*e*Sinh
[3*(d + e*x)])/(9*e^2 - b^2*c^2*Log[F]^2) + (-(b*c*Cosh[5*(d + e*x)]*Log[F
]) + 5*e*Sinh[5*(d + e*x)])/(25*e^2 - b^2*c^2*Log[F]^2)))/16
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {6035, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(d+ex)\cosh^3(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{ 6035}$$

$$\int \left(-\frac{1}{8}\cosh(d+ex)F^{c(a+bx)}+\frac{1}{16}\cosh(3d+3ex)F^{c(a+bx)}+\frac{1}{16}\cosh(5d+5ex)F^{c(a+bx)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{e\sinh(d+ex)F^{c(a+bx)}}{8\left(e^2-b^2c^2\log^2(F)\right)}+\frac{3e\sinh(3d+3ex)F^{c(a+bx)}}{16\left(9e^2-b^2c^2\log^2(F)\right)}+\frac{5e\sinh(5d+5ex)F^{c(a+bx)}}{16\left(25e^2-b^2c^2\log^2(F)\right)}+\frac{bc\log(F)\cosh(d+ex)F^{c(a+bx)}}{8\left(e^2-b^2c^2\log^2(F)\right)}-\frac{bc\log(F)\cosh(3d+3ex)F^{c(a+bx)}}{16\left(9e^2-b^2c^2\log^2(F)\right)}-\frac{bc\log(F)\cosh(5d+5ex)F^{c(a+bx)}}{16\left(25e^2-b^2c^2\log^2(F)\right)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x]^3*Sinh[d + e*x]^2,x]
```

output
```
(b*c*F^(c*(a + b*x))*Cosh[d + e*x]*Log[F])/(8*(e^2 - b^2*c^2*Log[F]^2)) -
(b*c*F^(c*(a + b*x))*Cosh[3*d + 3*e*x]*Log[F])/(16*(9*e^2 - b^2*c^2*Log[F]
^2)) - (b*c*F^(c*(a + b*x))*Cosh[5*d + 5*e*x]*Log[F])/(16*(25*e^2 - b^2*c^
2*Log[F]^2)) - (e*F^(c*(a + b*x))*Sinh[d + e*x])/(8*(e^2 - b^2*c^2*Log[F]^
2)) + (3*e*F^(c*(a + b*x))*Sinh[3*d + 3*e*x])/(16*(9*e^2 - b^2*c^2*Log[F]^
2)) + (5*e*F^(c*(a + b*x))*Sinh[5*d + 5*e*x])/(16*(25*e^2 - b^2*c^2*Log[F]
^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6035
```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(187) = 374$.

Time = 1.03 (sec) , antiderivative size = 766, normalized size of antiderivative = 3.81

$$\frac{\left(-450e^5\mathrm{e}^{4ex+4d}+45e^5-26\ln(F)^3b^3c^3e^2\mathrm{e}^{8ex+8d}+50\ln(F)^2b^2c^2e^3\mathrm{e}^{10ex+10d}-2\ln(F)^4b^4c^4e\,\mathrm{e}^{4ex+4d}+68\right.}{}$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^3*sinh(e*x+d)^2,x)
```

output
```
1/32*(-450*e^5*exp(4*e*x+4*d)+45*e^5-26*ln(F)^3*b^3*c^3*e^2*exp(8*e*x+8*d)
+50*ln(F)^2*b^2*c^2*e^3*exp(10*e*x+10*d)-2*ln(F)^4*b^4*c^4*e*exp(4*e*x+4*d
)+68*ln(F)^3*b^3*c^3*e^2*exp(6*e*x+6*d)+78*ln(F)^2*b^2*c^2*e^3*exp(8*e*x+8
*d)+ln(F)^5*b^5*c^5*exp(10*e*x+10*d)+ln(F)^5*b^5*c^5*exp(8*e*x+8*d)-2*ln(F
)^5*b^5*c^5*exp(6*e*x+6*d)-2*ln(F)^5*b^5*c^5*exp(4*e*x+4*d)+ln(F)^5*b^5*c^
5*exp(2*e*x+2*d)+c^5*b^5*ln(F)^5+9*ln(F)*b*c*e^4*exp(10*e*x+10*d)+3*ln(F)^
4*b^4*c^4*e*exp(2*e*x+2*d)+68*ln(F)^3*b^3*c^3*e^2*exp(4*e*x+4*d)-68*ln(F)^
2*b^2*c^2*e^3*exp(6*e*x+6*d)+25*ln(F)*b*c*e^4*exp(8*e*x+8*d)+5*ln(F)^4*b^4
*c^4*e-50*ln(F)^2*b^2*c^2*e^3-26*ln(F)^3*b^3*c^3*e^2*exp(2*e*x+2*d)+68*ln(
F)^2*b^2*c^2*e^3*exp(4*e*x+4*d)-450*ln(F)*b*c*e^4*exp(6*e*x+6*d)-78*ln(F)^
2*b^2*c^2*e^3*exp(2*e*x+2*d)-450*ln(F)*b*c*e^4*exp(4*e*x+4*d)+25*ln(F)*b*c
*e^4*exp(2*e*x+2*d)-5*ln(F)^4*b^4*c^4*e*exp(10*e*x+10*d)-3*ln(F)^4*b^4*c^4
*e*exp(8*e*x+8*d)-10*ln(F)^3*b^3*c^3*e^2*exp(10*e*x+10*d)+2*ln(F)^4*b^4*c^
4*e*exp(6*e*x+6*d)+75*e^5*exp(2*e*x+2*d)+450*e^5*exp(6*e*x+6*d)-75*e^5*exp
(8*e*x+8*d)-45*e^5*exp(10*e*x+10*d)-10*c^3*b^3*ln(F)^3*e^2+9*c*b*ln(F)*e^4
)/(b*c*ln(F)-e)*exp(-5*e*x-5*d)/(b*c*ln(F)-3*e)/(b*c*ln(F)-5*e)/(e+b*c*ln(
F))/(b*c*ln(F)+3*e)/(b*c*ln(F)+5*e)*F^(c*(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7786 vs. $2(182) = 364$.

Time = 0.70 (sec) , antiderivative size = 7786, normalized size of antiderivative = 38.74

$$\int F^{c(a+bx)} \cosh^3(d+ex) \sinh^2(d+ex) \, dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^3*sinh(e*x+d)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\cosh^3(d+ex)\sinh^2(d+ex)\,dx = \text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d)**3*sinh(e*x+d)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.02

$$\begin{aligned}&\int F^{c(a+bx)}\cosh^3(d+ex)\sinh^2(d+ex)\,dx\\&=\frac{F^{ac}e^{(bcx\log(F)+5ex+5d)}}{32(bc\log(F)+5e)}+\frac{F^{ac}e^{(bcx\log(F)+3ex+3d)}}{32(bc\log(F)+3e)}\\&\quad-\frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{16(bc\log(F)+e)}-\frac{F^{ac}e^{(bcx\log(F)-ex)}}{16(bce^d\log(F)-ee^d)}\\&\quad+\frac{F^{ac}e^{(bcx\log(F)-3ex)}}{32\left(bce^{(3d)}\log(F)-3ee^{(3d)}\right)}+\frac{F^{ac}e^{(bcx\log(F)-5ex)}}{32\left(bce^{(5d)}\log(F)-5ee^{(5d)}\right)}\end{aligned}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^3*sinh(e*x+d)^2,x, algorithm="maxima")
```

output
```
1/32*F^(a*c)*e^(b*c*x*log(F) + 5*e*x + 5*d)/(b*c*log(F) + 5*e) + 1/32*F^(a
*c)*e^(b*c*x*log(F) + 3*e*x + 3*d)/(b*c*log(F) + 3*e) - 1/16*F^(a*c)*e^(b*
c*x*log(F) + e*x + d)/(b*c*log(F) + e) - 1/16*F^(a*c)*e^(b*c*x*log(F) - e*
x)/(b*c*e^d*log(F) - e*e^d) + 1/32*F^(a*c)*e^(b*c*x*log(F) - 3*e*x)/(b*c*e
^(3*d)*log(F) - 3*e*e^(3*d)) + 1/32*F^(a*c)*e^(b*c*x*log(F) - 5*e*x)/(b*c*
e^(5*d)*log(F) - 5*e*e^(5*d))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 1823, normalized size of antiderivative = 9.07

$$\int F^{c(a+bx)} \cosh^3(d+ex) \sinh^2(d+ex) \, dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^3*sinh(e*x+d)^2,x, algorithm="giac")
```

output
```
1/16*(2*(b*c*log(abs(F)) + 5*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x -
1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(a
bs(F)) + 5*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2
*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 +
4*(b*c*log(abs(F)) + 5*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 5*e)
*x + 5*d) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*
sgn(F) - 1/2*I*pi*a*c)/(32*I*pi*b*c*sgn(F) - 32*I*pi*b*c + 64*b*c*log(abs(
F)) + 320*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c
*sgn(F) + 1/2*I*pi*a*c)/(-32*I*pi*b*c*sgn(F) + 32*I*pi*b*c + 64*b*c*log(ab
s(F)) + 320*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 5*e)*x + 5*d) + 1/
16*(2*(b*c*log(abs(F)) + 3*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs
(F)) + 3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*p
i*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*
(b*c*log(abs(F)) + 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x
 + 3*d) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sg
n(F) - 1/2*I*pi*a*c)/(32*I*pi*b*c*sgn(F) - 32*I*pi*b*c + 64*b*c*log(abs(F)
) + 192*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*s
gn(F) + 1/2*I*pi*a*c)/(-32*I*pi*b*c*sgn(F) + 32*I*pi*b*c + 64*b*c*log(abs(
F)) + 192*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3*d) - 1...
```

Mupad [B] (verification not implemented)

Time = 4.75 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.04

$$\int F^{c(a+bx)}\cosh^3(d+ex)\sinh^2(d+ex)\,dx = -\frac{F^{a\,c+b\,c\,x}\left(b^5\,c^5\cosh(d+e\,x)^3\sinh(d+e\,x)^2\ln(F)^5-2\,b^4\,c^4\,e\cosh(d+e\,x)^4\sinh(d+e\,x)\,\ln(F)^4-3\right.}{}$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x)^3*sinh(d + e*x)^2,x)
```

output
```
-(F^(a*c + b*c*x)*(30*e^5*sinh(d + e*x)^5 - 75*e^5*cosh(d + e*x)^2*sinh(d
+ e*x)^3 - 26*b*c*e^4*cosh(d + e*x)^5*log(F) + 2*b^3*c^3*e^2*cosh(d + e*x)
^5*log(F)^3 + b^5*c^5*cosh(d + e*x)^3*sinh(d + e*x)^2*log(F)^5 - 6*b^2*c^2
*e^3*sinh(d + e*x)^5*log(F)^2 + 65*b*c*e^4*cosh(d + e*x)^3*sinh(d + e*x)^2
*log(F) + 30*b^2*c^2*e^3*cosh(d + e*x)^2*sinh(d + e*x)^3*log(F)^2 - 18*b^3
*c^3*e^2*cosh(d + e*x)^3*sinh(d + e*x)^2*log(F)^3 - 2*b^4*c^4*e*cosh(d + e
*x)^4*sinh(d + e*x)*log(F)^4 - 30*b*c*e^4*cosh(d + e*x)*sinh(d + e*x)^4*lo
g(F) + 26*b^2*c^2*e^3*cosh(d + e*x)^4*sinh(d + e*x)*log(F)^2 + 6*b^3*c^3*e
^2*cosh(d + e*x)*sinh(d + e*x)^4*log(F)^3 - 3*b^4*c^4*e*cosh(d + e*x)^2*si
nh(d + e*x)^3*log(F)^4))/(225*e^6 - b^6*c^6*log(F)^6 - 259*b^2*c^2*e^4*log
(F)^2 + 35*b^4*c^4*e^2*log(F)^4)
```

Reduce [F]

$$\int F^{c(a+bx)}\cosh^3(d+ex)\sinh^2(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\cosh(ex+d)^3\sinh(ex+d)^2\,dx\right)$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^3*sinh(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cosh(d + e*x)**3*sinh(d + e*x)**2,x)
```

3.111 $\int F^{c(a+bx)} \cosh^3(d+ex) \sinh(d+ex)\, dx$

Optimal result

Integrand size = 24, antiderivative size = 139

$$\int F^{c(a+bx)} \cosh^3(d+ex) \sinh(d+ex)\, dx = \frac{e^{-2d-2ex}F^{c(a+bx)}}{8(2e-bc\log(F))} + \frac{e^{-4d-4ex}F^{c(a+bx)}}{16(4e-bc\log(F))} + \frac{e^{2d+2ex}F^{c(a+bx)}}{8(2e+bc\log(F))} + \frac{e^{4d+4ex}F^{c(a+bx)}}{16(4e+bc\log(F))}$$

output
```
exp(-2*e*x-2*d)*F^(c*(b*x+a))/(16*e-8*b*c*ln(F))+exp(-4*e*x-4*d)*F^(c*(b*x
+a))/(64*e-16*b*c*ln(F))+exp(2*e*x+2*d)*F^(c*(b*x+a))/(16*e+8*b*c*ln(F))+e
xp(4*e*x+4*d)*F^(c*(b*x+a))/(64*e+16*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.78

$$\int F^{c(a+bx)} \cosh^3(d+ex) \sinh(d+ex)\, dx = \frac{1}{8}F^{c(a+bx)}\left(\frac{4e\cosh(2(d+ex)) - 2bc\log(F)\sinh(2(d+ex))}{4e^2-b^2c^2\log^2(F)} + \frac{4e\cosh(4(d+ex)) - bc\log(F)\sinh(4(d+ex))}{16e^2-b^2c^2\log^2(F)}\right)$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^3*Sinh[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*((4*e*Cosh[2*(d + e*x)] - 2*b*c*Log[F]*Sinh[2*(d + e*x)])
/(4*e^2 - b^2*c^2*Log[F]^2) + (4*e*Cosh[4*(d + e*x)] - b*c*Log[F]*Sinh[4*(
d + e*x)])/(16*e^2 - b^2*c^2*Log[F]^2)))/8
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6035, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(d+ex)\cosh^3(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6035}$$

$$\int \left(\frac{1}{4}\sinh(2d+2ex)F^{c(a+bx)}+\frac{1}{8}\sinh(4d+4ex)F^{c(a+bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{bc\log(F)\sinh(2d+2ex)F^{c(a+bx)}}{4\left(4e^2-b^2c^2\log^2(F)\right)}-\frac{bc\log(F)\sinh(4d+4ex)F^{c(a+bx)}}{8\left(16e^2-b^2c^2\log^2(F)\right)}+\frac{e\cosh(2d+2ex)F^{c(a+bx)}}{2\left(4e^2-b^2c^2\log^2(F)\right)}+\frac{e\cosh(4d+4ex)F^{c(a+bx)}}{2\left(16e^2-b^2c^2\log^2(F)\right)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x]^3*Sinh[d + e*x],x]
```

output
```
(e*F^(c*(a + b*x))*Cosh[2*d + 2*e*x])/(2*(4*e^2 - b^2*c^2*Log[F]^2)) + (e*
F^(c*(a + b*x))*Cosh[4*d + 4*e*x])/(2*(16*e^2 - b^2*c^2*Log[F]^2)) - (b*c*
F^(c*(a + b*x))*Log[F]*Sinh[2*d + 2*e*x])/(4*(4*e^2 - b^2*c^2*Log[F]^2)) -
 (b*c*F^(c*(a + b*x))*Log[F]*Sinh[4*d + 4*e*x])/(8*(16*e^2 - b^2*c^2*Log[F
]^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6035
```
Int[Cosh[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(
d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x))
, Sinh[d + e*x]^m*Cosh[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(125) = 250$.

Time = 231.62 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.37

method	result
risch	$\frac{\left(\ln(F)^3 b^3 c^3 e^{8ex+8d}+2\ln(F)^3 b^3 c^3 e^{6ex+6d}-4\ln(F)^2 b^2 c^2 e\, e^{8ex+8d}-4\ln(F)^2 b^2 c^2 e\, e^{6ex+6d}-4\ln(F) bc\, e^2 e^{8ex+8d}-2\ln(F)^3 b^3 c^3 e^{2e\cdots}\right.}{1\cdots}$
orering	$\frac{4\ln(F) bc\left(b^2c^2\ln(F)^2-10e^2\right)F^{c(bx+a)}\cosh(ex+d)^3\sinh(ex+d)}{\ln(F)^4 b^4 c^4-20\ln(F)^2 b^2 c^2 e^2+64e^4}-\frac{2\left(3b^2c^2\ln(F)^2-10e^2\right)\left(F^{c(bx+a)} bc\ln(F)\cosh(ex+d)^3\sinh(\cdots\right.}{\ln(F)^4 b^4 c^4\cdots}$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^3*sinh(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/16*(ln(F)^3*b^3*c^3*exp(8*e*x+8*d)+2*ln(F)^3*b^3*c^3*exp(6*e*x+6*d)-4*ln
(F)^2*b^2*c^2*e*exp(8*e*x+8*d)-4*ln(F)^2*b^2*c^2*e*exp(6*e*x+6*d)-4*ln(F)*
b*c*e^2*exp(8*e*x+8*d)-2*ln(F)^3*b^3*c^3*exp(2*e*x+2*d)-32*ln(F)*b*c*e^2*e
xp(6*e*x+6*d)+16*e^3*exp(8*e*x+8*d)-c^3*b^3*ln(F)^3-4*ln(F)^2*b^2*c^2*e*ex
p(2*e*x+2*d)+64*e^3*exp(6*e*x+6*d)-4*c^2*b^2*ln(F)^2*e+32*ln(F)*b*c*e^2*ex
p(2*e*x+2*d)+4*e^2*b*c*ln(F)+64*e^3*exp(2*e*x+2*d)+16*e^3)/(b*c*ln(F)-2*e)
*exp(-4*e*x-4*d)/(b*c*ln(F)-4*e)/(2*e+b*c*ln(F))/(b*c*ln(F)+4*e)*F^(c*(b*x
+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2892 vs. $2(125) = 250$.

Time = 0.17 (sec) , antiderivative size = 2892, normalized size of antiderivative = 20.81

$$\int F^{c(a+bx)} \cosh^3(d+ex) \sinh(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^3*sinh(e*x+d),x, algorithm="fricas")
```

output
```
1/16*((16*e^3*cosh(e*x + d)^8 + 64*e^3*cosh(e*x + d)^6 + (b^3*c^3*log(F)^3
 - 4*b^2*c^2*e*log(F)^2 - 4*b*c*e^2*log(F) + 16*e^3)*sinh(e*x + d)^8 + 8*(
b^3*c^3*cosh(e*x + d)*log(F)^3 - 4*b^2*c^2*e*cosh(e*x + d)*log(F)^2 - 4*b*
c*e^2*cosh(e*x + d)*log(F) + 16*e^3*cosh(e*x + d))*sinh(e*x + d)^7 + 2*(22
4*e^3*cosh(e*x + d)^2 + (14*b^3*c^3*cosh(e*x + d)^2 + b^3*c^3)*log(F)^3 +
32*e^3 - 2*(28*b^2*c^2*e*cosh(e*x + d)^2 + b^2*c^2*e)*log(F)^2 - 8*(7*b*c*
e^2*cosh(e*x + d)^2 + 2*b*c*e^2)*log(F))*sinh(e*x + d)^6 + 4*(224*e^3*cosh
(e*x + d)^3 + 96*e^3*cosh(e*x + d) + (14*b^3*c^3*cosh(e*x + d)^3 + 3*b^3*c
^3*cosh(e*x + d))*log(F)^3 - 2*(28*b^2*c^2*e*cosh(e*x + d)^3 + 3*b^2*c^2*e
*cosh(e*x + d))*log(F)^2 - 8*(7*b*c*e^2*cosh(e*x + d)^3 + 6*b*c*e^2*cosh(e
*x + d))*log(F))*sinh(e*x + d)^5 + 64*e^3*cosh(e*x + d)^2 + 10*(112*e^3*co
sh(e*x + d)^4 + 96*e^3*cosh(e*x + d)^2 + (7*b^3*c^3*cosh(e*x + d)^4 + 3*b^
3*c^3*cosh(e*x + d)^2)*log(F)^3 - 2*(14*b^2*c^2*e*cosh(e*x + d)^4 + 3*b^2*
c^2*e*cosh(e*x + d)^2)*log(F)^2 - 4*(7*b*c*e^2*cosh(e*x + d)^4 + 12*b*c*e^
2*cosh(e*x + d)^2)*log(F))*sinh(e*x + d)^4 + (b^3*c^3*cosh(e*x + d)^8 + 2*
b^3*c^3*cosh(e*x + d)^6 - 2*b^3*c^3*cosh(e*x + d)^2 - b^3*c^3)*log(F)^3 +
8*(112*e^3*cosh(e*x + d)^5 + 160*e^3*cosh(e*x + d)^3 + (7*b^3*c^3*cosh(e*x
 + d)^5 + 5*b^3*c^3*cosh(e*x + d)^3)*log(F)^3 - 2*(14*b^2*c^2*e*cosh(e*x +
 d)^5 + 5*b^2*c^2*e*cosh(e*x + d)^3)*log(F)^2 - 4*(7*b*c*e^2*cosh(e*x + d)
^5 + 20*b*c*e^2*cosh(e*x + d)^3)*log(F))*sinh(e*x + d)^3 + 16*e^3 - 4*(...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1926 vs. $2(124) = 248$.

Time = 24.65 (sec) , antiderivative size = 1926, normalized size of antiderivative = 13.86

$$\int F^{c(a+bx)} \cosh^3(d+ex) \sinh(d+ex)\, dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d)**3*sinh(e*x+d),x)
```

output
```
Piecewise((x*sinh(d)*cosh(d)**3, Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*sinh(d)
*cosh(d)**3, Eq(b, 0) & Eq(e, 0)), (x*sinh(d)*cosh(d)**3, Eq(c, 0) & Eq(e,
 0)), (-F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/2 - d)**4/8 + F**(a*c + b*c*x
)*x*sinh(b*c*x*log(F)/2 - d)**3*cosh(b*c*x*log(F)/2 - d)/4 - F**(a*c + b*c
*x)*x*sinh(b*c*x*log(F)/2 - d)*cosh(b*c*x*log(F)/2 - d)**3/4 + F**(a*c + b
*c*x)*x*cosh(b*c*x*log(F)/2 - d)**4/8 - F**(a*c + b*c*x)*sinh(b*c*x*log(F)
/2 - d)**4/(24*b*c*log(F)) + F**(a*c + b*c*x)*sinh(b*c*x*log(F)/2 - d)**3*
cosh(b*c*x*log(F)/2 - d)/(3*b*c*log(F)) - F**(a*c + b*c*x)*sinh(b*c*x*log(
F)/2 - d)**2*cosh(b*c*x*log(F)/2 - d)**2/(2*b*c*log(F)) - F**(a*c + b*c*x)
*cosh(b*c*x*log(F)/2 - d)**4/(8*b*c*log(F)), Eq(e, -b*c*log(F)/2)), (F**(a
*c + b*c*x)*x*sinh(b*c*x*log(F)/4 - d)**4/16 - F**(a*c + b*c*x)*x*sinh(b*c
*x*log(F)/4 - d)**3*cosh(b*c*x*log(F)/4 - d)/4 + 3*F**(a*c + b*c*x)*x*sinh
(b*c*x*log(F)/4 - d)**2*cosh(b*c*x*log(F)/4 - d)**2/8 - F**(a*c + b*c*x)*x
*sinh(b*c*x*log(F)/4 - d)*cosh(b*c*x*log(F)/4 - d)**3/4 + F**(a*c + b*c*x)
*x*cosh(b*c*x*log(F)/4 - d)**4/16 - F**(a*c + b*c*x)*sinh(b*c*x*log(F)/4 -
 d)**4/(6*b*c*log(F)) + 5*F**(a*c + b*c*x)*sinh(b*c*x*log(F)/4 - d)**3*cos
h(b*c*x*log(F)/4 - d)/(12*b*c*log(F)) - 11*F**(a*c + b*c*x)*sinh(b*c*x*log
(F)/4 - d)*cosh(b*c*x*log(F)/4 - d)**3/(12*b*c*log(F)) + F**(a*c + b*c*x)*
cosh(b*c*x*log(F)/4 - d)**4/(6*b*c*log(F)), Eq(e, -b*c*log(F)/4)), (-F**(a
*c + b*c*x)*x*sinh(b*c*x*log(F)/4 + d)**4/16 + F**(a*c + b*c*x)*x*sinh(...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.03

$$\int F^{c(a+bx)}\cosh^3(d+ex)\sinh(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+4\,ex+4\,d)}}{16\,(bc\log{(F)}+4\,e)} + \frac{F^{ac}e^{(bcx\log(F)+2\,ex+2\,d)}}{8\,(bc\log{(F)}+2\,e)} - \frac{F^{ac}e^{(bcx\log(F)-2\,ex)}}{8\,(bce^{(2\,d)}\log{(F)}-2\,ee^{(2\,d)})} - \frac{F^{ac}e^{(bcx\log(F)-4\,ex)}}{16\,(bce^{(4\,d)}\log{(F)}-4\,ee^{(4\,d)})}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^3*sinh(e*x+d),x, algorithm="maxima")
```

output
```
1/16*F^(a*c)*e^(b*c*x*log(F) + 4*e*x + 4*d)/(b*c*log(F) + 4*e) + 1/8*F^(a*
c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) - 1/8*F^(a*c)*e^(b*c*
x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) - 1/16*F^(a*c)*e^(b*c
*x*log(F) - 4*e*x)/(b*c*e^(4*d)*log(F) - 4*e*e^(4*d))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 1225, normalized size of antiderivative = 8.81

$$\int F^{c(a+bx)}\cosh^3(d+ex)\sinh(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^3*sinh(e*x+d),x, algorithm="giac")
```

output
```
1/8*(2*(b*c*log(abs(F)) + 4*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1
/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(ab
s(F)) + 4*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*
pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4
*(b*c*log(abs(F)) + 4*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 4*e)*
x + 4*d) + 1/2*I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a
*c*sgn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs
(F)) + 64*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c
*sgn(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(
F)) + 64*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 4*e)*x + 4*d) + 1/4*(
2*(b*c*log(abs(F)) + 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi
*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F))
 + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*
c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c
*log(abs(F)) + 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x + 2
*d) + 1/2*I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sg
n(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F)) +
 16*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F
) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) + 1
6*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x + 2*d) - 1/4*(2*(b...
```

Mupad [B] (verification not implemented)

Time = 5.07 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.27

$$\int F^{c(a+bx)}\cosh^3(d+ex)\sinh(d+ex)\,dx = -\frac{F^{a\,c+b\,c\,x}\,\left(6\,e^3-16\,e^3\cosh(d+e\,x)^4-3\,b^2\,c^2\,e\cosh(d+e\,x)^2\ln\left(F\right)^2+4\,b^2\,c^2\,e\cosh(d+e\,x)^4\ln\left(F\right)^2\right.}{b^4\,c^4}$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x)^3*sinh(d + e*x),x)
```

output
```
-(F^(a*c + b*c*x)*(6*e^3 - 16*e^3*cosh(d + e*x)^4 - 3*b^2*c^2*e*cosh(d + e
*x)^2*log(F)^2 + 4*b^2*c^2*e*cosh(d + e*x)^4*log(F)^2 - b^3*c^3*cosh(d + e
*x)^3*sinh(d + e*x)*log(F)^3 + 6*b*c*e^2*cosh(d + e*x)*sinh(d + e*x)*log(F
) + 4*b*c*e^2*cosh(d + e*x)^3*sinh(d + e*x)*log(F)))/(64*e^4 + b^4*c^4*log
(F)^4 - 20*b^2*c^2*e^2*log(F)^2)
```

Reduce [F]

$$\int F^{c(a+bx)}\cosh^3(d+ex)\sinh(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\cosh\left(ex+d\right)^3\sinh\left(ex+d\right)dx\right)$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^3*sinh(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cosh(d + e*x)**3*sinh(d + e*x),x)
```

3.112 $\int F^{c(a+bx)} \cosh^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 132

$$\int F^{c(a+bx)} \cosh^3(d+ex)\,dx = -\frac{3e^{-d-ex}F^{c(a+bx)}}{8(e-bc\log(F))} - \frac{e^{-3d-3ex}F^{c(a+bx)}}{8(3e-bc\log(F))} + \frac{3e^{d+ex}F^{c(a+bx)}}{8(e+bc\log(F))} + \frac{e^{3d+3ex}F^{c(a+bx)}}{8(3e+bc\log(F))}$$

output
```
-3*exp(-e*x-d)*F^(c*(b*x+a))/(8*e-8*b*c*ln(F))-exp(-3*e*x-3*d)*F^(c*(b*x+a
))/(24*e-8*b*c*ln(F))+3*exp(e*x+d)*F^(c*(b*x+a))/(8*e+8*b*c*ln(F))+exp(3*e
*x+3*d)*F^(c*(b*x+a))/(24*e+8*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.20

$$\int F^{c(a+bx)} \cosh^3(d+ex)\,dx = \frac{F^{c(a+bx)}\left(3\cosh(d+ex)\left(-9bce^2\log(F)+b^3c^3\log^3(F)\right)+\cosh(3(d+ex))\left(-bce^2\log(F)+b^3c^3\log^3(F)\right.\right.}{4\left(9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\right.}$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(3*Cosh[d + e*x]*(-9*b*c*e^2*Log[F] + b^3*c^3*Log[F]^3) +
 Cosh[3*(d + e*x)]*(-(b*c*e^2*Log[F]) + b^3*c^3*Log[F]^3) + 6*e*(5*e^2 - b
^2*c^2*Log[F]^2 + Cosh[2*(d + e*x)]*(e^2 - b^2*c^2*Log[F]^2))*Sinh[d + e*x
]))/(4*(9*e^4 - 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.44, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6000, 5998}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{ 6000}$$

$$\frac{6e^2\int F^{c(a+bx)}\cosh(d+ex)dx}{9e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh^3(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{3e\sinh(d+ex)\cosh^2(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}$$

$$\Big\downarrow \text{ 5998}$$

$$-\frac{bc\log(F)\cosh^3(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{3e\sinh(d+ex)\cosh^2(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{6e^2\left(\frac{e\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}\right)}{9e^2-b^2c^2\log^2(F)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x]^3,x]
```

output
```
-((b*c*F^(c*(a + b*x))*Cosh[d + e*x]^3*Log[F])/(9*e^2 - b^2*c^2*Log[F]^2))
 + (3*e*F^(c*(a + b*x))*Cosh[d + e*x]^2*Sinh[d + e*x])/(9*e^2 - b^2*c^2*Lo
g[F]^2) + (6*e^2*(-((b*c*F^(c*(a + b*x))*Cosh[d + e*x]*Log[F])/(e^2 - b^2*
c^2*Log[F]^2)) + (e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2
)))/(9*e^2 - b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 5998
```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

rule 6000
```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Simp[e*n*F^(c*(a + b*x))*Sinh[d + e*x]*(Cosh[d + e*x]^
(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x] + Simp[n*(n - 1)*(e^2/(e^2*n^2 -
b^2*c^2*Log[F]^2))   Int[F^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.12

method	result
parallelrisch	$\frac{F^{c(bx+a)}\left(\left(c^3b^3\ln(F)^3-e^2bc\ln(F)\right)\cosh(3ex+3d)+\left(-3c^2b^2\ln(F)^2e+3e^3\right)\sinh(3ex+3d)-3(bc\ln(F)-3e)(bc\ln(F)+3e)(\cdots}{4\ln(F)^4b^4c^4-40\ln(F)^2b^2c^2e^2+36e^4}$
risch	$\left(\ln(F)^3b^3c^3e^{6ex+6d}+3\ln(F)^3b^3c^3e^{4ex+4d}-3\ln(F)^2b^2c^2e\,e^{6ex+6d}+3\ln(F)^3b^3c^3e^{2ex+2d}-3\ln(F)^2b^2c^2e\,e^{4ex+4d}-\ln(F)bc\cdots\right.$
orering	$\frac{4\ln(F)bc\left(b^2c^2\ln(F)^2-5e^2\right)F^{c(bx+a)}\cosh(ex+d)^3}{\ln(F)^4b^4c^4-10\ln(F)^2b^2c^2e^2+9e^4}-\frac{2\left(3b^2c^2\ln(F)^2-5e^2\right)\left(F^{c(bx+a)}bc\ln(F)\cosh(ex+d)^3+3F^{c(bx+a)}\,c\cdots\right.}{\ln(F)^4b^4c^4-10\ln(F)^2b^2c^2e^2+9e^4}$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
F^(c*(b*x+a))*((c^3*b^3*ln(F)^3-e^2*b*c*ln(F))*cosh(3*e*x+3*d)+(-3*c^2*b^2
*ln(F)^2*e+3*e^3)*sinh(3*e*x+3*d)-3*(b*c*ln(F)-3*e)*(b*c*ln(F)+3*e)*(-b*c*
cosh(e*x+d)*ln(F)+e*sinh(e*x+d)))/(4*ln(F)^4*b^4*c^4-40*ln(F)^2*b^2*c^2*e^
2+36*e^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2218 vs. $2(120) = 240$.

Time = 0.16 (sec) , antiderivative size = 2218, normalized size of antiderivative = 16.80

$$\int F^{c(a+bx)}\cosh^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^3,x, algorithm="fricas")
```

output
```
1/8*((3*e^3*cosh(e*x + d)^6 + 27*e^3*cosh(e*x + d)^4 + (b^3*c^3*log(F)^3 -
 3*b^2*c^2*e*log(F)^2 - b*c*e^2*log(F) + 3*e^3)*sinh(e*x + d)^6 + 6*(b^3*c
^3*cosh(e*x + d)*log(F)^3 - 3*b^2*c^2*e*cosh(e*x + d)*log(F)^2 - b*c*e^2*c
osh(e*x + d)*log(F) + 3*e^3*cosh(e*x + d))*sinh(e*x + d)^5 - 27*e^3*cosh(e
*x + d)^2 + 3*(15*e^3*cosh(e*x + d)^2 + (5*b^3*c^3*cosh(e*x + d)^2 + b^3*c
^3)*log(F)^3 + 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^2 + b^2*c^2*e)*log(F)^2
 - (5*b*c*e^2*cosh(e*x + d)^2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^4 + (b^3*
c^3*cosh(e*x + d)^6 + 3*b^3*c^3*cosh(e*x + d)^4 + 3*b^3*c^3*cosh(e*x + d)^
2 + b^3*c^3)*log(F)^3 + 4*(15*e^3*cosh(e*x + d)^3 + 27*e^3*cosh(e*x + d) +
 (5*b^3*c^3*cosh(e*x + d)^3 + 3*b^3*c^3*cosh(e*x + d))*log(F)^3 - 3*(5*b^2
*c^2*e*cosh(e*x + d)^3 + b^2*c^2*e*cosh(e*x + d))*log(F)^2 - (5*b*c*e^2*co
sh(e*x + d)^3 + 27*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d)^3 - 3*e^3
- 3*(b^2*c^2*e*cosh(e*x + d)^6 + b^2*c^2*e*cosh(e*x + d)^4 - b^2*c^2*e*cos
h(e*x + d)^2 - b^2*c^2*e)*log(F)^2 + 3*(15*e^3*cosh(e*x + d)^4 + 54*e^3*co
sh(e*x + d)^2 + (5*b^3*c^3*cosh(e*x + d)^4 + 6*b^3*c^3*cosh(e*x + d)^2 + b
^3*c^3)*log(F)^3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^4 + 6*b^2*c^2*e*cos
h(e*x + d)^2 - b^2*c^2*e)*log(F)^2 - (5*b*c*e^2*cosh(e*x + d)^4 + 54*b*c*e
^2*cosh(e*x + d)^2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^2 - (b*c*e^2*cosh(e*
x + d)^6 + 27*b*c*e^2*cosh(e*x + d)^4 + 27*b*c*e^2*cosh(e*x + d)^2 + b*c*e
^2)*log(F) + 6*(3*e^3*cosh(e*x + d)^5 + 18*e^3*cosh(e*x + d)^3 - 9*e^3*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1658 vs. $2(121) = 242$.

Time = 3.32 (sec) , antiderivative size = 1658, normalized size of antiderivative = 12.56

$$\int F^{c(a+bx)} \cosh^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d)**3,x)
```

output
```
Piecewise((x*cosh(d)**3, Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*cosh(d)**3, Eq(
b, 0) & Eq(e, 0)), (x*cosh(d)**3, Eq(c, 0) & Eq(e, 0)), (3*F**(a*c + b*c*x
)*x*sinh(b*c*x*log(F) - d)**3/8 - 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F) -
 d)**2*cosh(b*c*x*log(F) - d)/8 - 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F) -
 d)*cosh(b*c*x*log(F) - d)**2/8 + 3*F**(a*c + b*c*x)*x*cosh(b*c*x*log(F) -
 d)**3/8 - 5*F**(a*c + b*c*x)*sinh(b*c*x*log(F) - d)**3/(8*b*c*log(F)) + F
**(a*c + b*c*x)*sinh(b*c*x*log(F) - d)**2*cosh(b*c*x*log(F) - d)/(4*b*c*lo
g(F)) + F**(a*c + b*c*x)*sinh(b*c*x*log(F) - d)*cosh(b*c*x*log(F) - d)**2/
(b*c*log(F)) - 3*F**(a*c + b*c*x)*cosh(b*c*x*log(F) - d)**3/(8*b*c*log(F))
, Eq(e, -b*c*log(F))), (-F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)**3/8
+ 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)**2*cosh(b*c*x*log(F)/3 - d
)/8 - 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)*cosh(b*c*x*log(F)/3 -
d)**2/8 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F)/3 - d)**3/8 + 11*F**(a*c +
b*c*x)*sinh(b*c*x*log(F)/3 - d)**3/(8*b*c*log(F)) - 15*F**(a*c + b*c*x)*si
nh(b*c*x*log(F)/3 - d)**2*cosh(b*c*x*log(F)/3 - d)/(4*b*c*log(F)) + 3*F**(
a*c + b*c*x)*sinh(b*c*x*log(F)/3 - d)*cosh(b*c*x*log(F)/3 - d)**2/(b*c*log
(F)) - F**(a*c + b*c*x)*cosh(b*c*x*log(F)/3 - d)**3/(8*b*c*log(F)), Eq(e,
-b*c*log(F)/3)), (-F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 + d)**3/8 + 3*F*
*(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 + d)**2*cosh(b*c*x*log(F)/3 + d)/8 -
3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 + d)*cosh(b*c*x*log(F)/3 + d)*...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02

$$\int F^{c(a+bx)}\cosh^3(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+3\,ex+3\,d)}}{8\,(bc\log(F)+3\,e)} + \frac{3\,F^{ac}e^{(bcx\log(F)+ex+d)}}{8\,(bc\log(F)+e)} + \frac{3\,F^{ac}e^{(bcx\log(F)-ex)}}{8\,(bce^{d}\log(F)-ee^{d})} + \frac{F^{ac}e^{(bcx\log(F)-3\,ex)}}{8\,(bce^{(3\,d)}\log(F)-3\,ee^{(3\,d)})}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/8*F^(a*c)*e^(b*c*x*log(F) + 3*e*x + 3*d)/(b*c*log(F) + 3*e) + 3/8*F^(a*c
)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) + 3/8*F^(a*c)*e^(b*c*x*log(F
) - e*x)/(b*c*e^d*log(F) - e*e^d) + 1/8*F^(a*c)*e^(b*c*x*log(F) - 3*e*x)/(
b*c*e^(3*d)*log(F) - 3*e*e^(3*d))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 1211, normalized size of antiderivative = 9.17

$$\int F^{c(a+bx)}\cosh^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^3,x, algorithm="giac")
```

output
```
1/4*(2*(b*c*log(abs(F)) + 3*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1
/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(ab
s(F)) + 3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*
pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4
*(b*c*log(abs(F)) + 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*
x + 3*d) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*s
gn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F))
 + 48*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn
(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F))
+ 48*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3*d) + 3/4*(2*(b
*c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*s
gn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e)^
2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs
(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 3*I*(I*e
^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*
a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e) - I*e^(-
1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*
c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e))*e^(a*c*l
og(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 3/4*(2*(b*c*log(abs(F)) - e...
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.17

$$\int F^{c(a+bx)} \cosh^3(d+ex)\,dx$$
$$= \frac{F^{a\,c+b\,c\,x}\,\left(6\,e^3\sinh(d+e\,x)+3\,e^3\cosh(d+e\,x)^2\sinh(d+e\,x)+b^3\,c^3\cosh(d+e\,x)^3\ln\left(F\right)^3-b\,c\,e^2\cosh}{b^4\,c^4\ln\left(F\right)^4-10\,b^2\,c^2\,e^2}$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x)^3,x)
```

output
```
(F^(a*c + b*c*x)*(6*e^3*sinh(d + e*x) + 3*e^3*cosh(d + e*x)^2*sinh(d + e*x
) + b^3*c^3*cosh(d + e*x)^3*log(F)^3 - b*c*e^2*cosh(d + e*x)^3*log(F) - 6*
b*c*e^2*cosh(d + e*x)*log(F) - 3*b^2*c^2*e*cosh(d + e*x)^2*sinh(d + e*x)*l
og(F)^2))/(9*e^4 + b^4*c^4*log(F)^4 - 10*b^2*c^2*e^2*log(F)^2)
```

Reduce [F]

$$\int F^{c(a+bx)} \cosh^3(d+ex)\,dx = f^{ac}\left(\int f^{bcx} \cosh\left(ex+d\right)^3 dx\right)$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cosh(d + e*x)**3,x)
```

3.113 $\int F^{c(a+bx)}\cosh^2(d+ex)\coth(d+ex)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 160

$$\begin{aligned}\int F^{c(a+bx)}\cosh^2(d+ex)\coth(d+ex)\,dx &= \frac{F^{c(a+bx)}}{bc\log(F)}-\frac{7e^{-2d-2ex}F^{c(a+bx)}}{4(2e-bc\log(F))}\\ &+\frac{2e^{-2d-2ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(-2+\frac{bc\log(F)}{e}\right),\frac{bc\log(F)}{2e},e^{2d+2ex}\right)}{2e-bc\log(F)}\\ &+\frac{e^{2d+2ex}F^{c(a+bx)}}{4(2e+bc\log(F))}\end{aligned}$$

output
```
F^(c*(b*x+a))/b/c/ln(F)-7*exp(-2*e*x-2*d)*F^(c*(b*x+a))/(8*e-4*b*c*ln(F))+
2*exp(-2*e*x-2*d)*F^(c*(b*x+a))*hypergeom([1, -1+1/2*b*c*ln(F)/e],[1/2*b*c
*ln(F)/e],exp(2*e*x+2*d))/(2*e-b*c*ln(F))+exp(2*e*x+2*d)*F^(c*(b*x+a))/(8*
e+4*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.88

$$\int F^{c(a+bx)}\cosh^2(d+ex)\coth(d+ex)\,dx$$
$$=\frac{F^{c(a+bx)}\left(-8e^2-2bce\cosh(2(d+ex))\log(F)+2b^2c^2\log^2(F)+4\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc}{\ }\right.\right.}{-8bce^2\log(F)+2b^3c^3\log^3(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^2*Coth[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*(-8*e^2 - 2*b*c*e*Cosh[2*(d + e*x)]*Log[F] + 2*b^2*c^2*Lo
g[F]^2 + 4*Hypergeometric2F1[1, (b*c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e)
, E^(2*(d + e*x))]*(4*e^2 - b^2*c^2*Log[F]^2) + b^2*c^2*Log[F]^2*Sinh[2*(d
 + e*x)]))/(-8*b*c*e^2*Log[F] + 2*b^3*c^3*Log[F]^3)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(d+ex)\coth(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6037}$$

$$\int \left(\frac{7}{4}e^{-2d-2ex}F^{ac+bcx}+e^{2(d+ex)-2d-2ex}F^{ac+bcx}+\frac{1}{4}e^{4(d+ex)-2d-2ex}F^{ac+bcx}+\frac{2e^{-2d-2ex}F^{ac+bcx}}{e^{2(d+ex)}-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2e^{-2d-2ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(\frac{bc\log(F)}{e}-2\right),\frac{bc\log(F)}{2e},e^{2(d+ex)}\right)}{2e-bc\log(F)}-\frac{7F^{ac}e^{-x(2e-bc\log(F))-2d}}{4(2e-bc\log(F))}+\frac{F^{ac}e^{x(bc\log(F)+2e)+2d}}{4(bc\log(F)+2e)}+\frac{F^{ac+bcx}}{bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x]^2*Coth[d + e*x],x]
```

output
```
F^(a*c + b*c*x)/(b*c*Log[F]) - (7*E^(-2*d - x*(2*e - b*c*Log[F]))*F^(a*c))
/(4*(2*e - b*c*Log[F])) + (2*E^(-2*d - 2*e*x)*F^(a*c + b*c*x)*Hypergeometr
ic2F1[1, (-2 + (b*c*Log[F])/e)/2, (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(2
*e - b*c*Log[F]) + (E^(2*d + x*(2*e + b*c*Log[F]))*F^(a*c))/(4*(2*e + b*c*
Log[F]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Maple [F]

$$\int F^{c(bx+a)}\cosh(ex+d)^2\coth(ex+d)\,dx$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^2*coth(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*cosh(e*x+d)^2*coth(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\cosh^2(d+ex)\coth(d+ex)\,dx = \int F^{(bx+a)c}\cosh(ex+d)^2\coth(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2*coth(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*cosh(e*x + d)^2*coth(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\cosh^2(d+ex)\coth(d+ex)\,dx = \int F^{c(a+bx)}\cosh^2(d+ex)\coth(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d)**2*coth(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*cosh(d + e*x)**2*coth(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\cosh^2(d+ex)\coth(d+ex)\,dx = \int F^{(bx+a)c}\cosh(ex+d)^2\coth(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2*coth(e*x+d),x, algorithm="maxima")
```

output
```
4*F^(a*c)*e*integrate(F^(b*c*x)/((b*c*e^(6*d)*log(F) - 4*e*e^(6*d))*e^(6*e
*x) - 2*(b*c*e^(4*d)*log(F) - 4*e*e^(4*d))*e^(4*e*x) + (b*c*e^(2*d)*log(F)
 - 4*e*e^(2*d))*e^(2*e*x)), x) + 1/4*(F^(a*c)*b^3*c^3*log(F)^3 + 14*F^(a*c
)*b^2*c^2*e*log(F)^2 + 24*F^(a*c)*b*c*e^2*log(F) + (F^(a*c)*b^3*c^3*e^(6*d
)*log(F)^3 - 6*F^(a*c)*b^2*c^2*e*e^(6*d)*log(F)^2 + 8*F^(a*c)*b*c*e^2*e^(6
*d)*log(F))*e^(6*e*x) + (3*F^(a*c)*b^3*c^3*e^(4*d)*log(F)^3 - 10*F^(a*c)*b
^2*c^2*e*e^(4*d)*log(F)^2 - 24*F^(a*c)*b*c*e^2*e^(4*d)*log(F) + 64*F^(a*c)
*e^3*e^(4*d))*e^(4*e*x) + (3*F^(a*c)*b^3*c^3*e^(2*d)*log(F)^3 + 2*F^(a*c)*
b^2*c^2*e*e^(2*d)*log(F)^2 - 40*F^(a*c)*b*c*e^2*e^(2*d)*log(F) - 64*F^(a*c
)*e^3*e^(2*d))*e^(2*e*x))*F^(b*c*x)/((b^4*c^4*e^(4*d)*log(F)^4 - 4*b^3*c^3
*e*e^(4*d)*log(F)^3 - 4*b^2*c^2*e^2*e^(4*d)*log(F)^2 + 16*b*c*e^3*e^(4*d)*
log(F))*e^(4*e*x) - (b^4*c^4*e^(2*d)*log(F)^4 - 4*b^3*c^3*e*e^(2*d)*log(F)
^3 - 4*b^2*c^2*e^2*e^(2*d)*log(F)^2 + 16*b*c*e^3*e^(2*d)*log(F))*e^(2*e*x)
)
```

Giac [F]

$$\int F^{c(a+bx)}\cosh^2(d+ex)\coth(d+ex)\,dx=\int F^{(bx+a)c}\cosh\left(ex+d\right)^2\coth\left(ex+d\right)\,dx$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2*coth(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*cosh(e*x + d)^2*coth(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\cosh^2(d+ex)\coth(d+ex)\,dx=\int F^{c\,(a+b\,x)}\,{\cosh\left(d+e\,x\right)}^2\,\mathrm{coth}\left(d+e\,x\right)\,dx$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x)^2*coth(d + e*x),x)
```

output
```
int(F^(c*(a + b*x))*cosh(d + e*x)^2*coth(d + e*x), x)
```

Reduce [F]

$$\int F^{c(a+bx)} \cosh^2(d+ex) \coth(d+ex)\,dx = f^{ac}\left(\int f^{bcx} \cosh(ex+d)^2 \coth(ex+d)\,dx\right)$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^2*coth(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cosh(d + e*x)**2*coth(d + e*x),x)
```

3.114 $\int F^{c(a+bx)} \cosh(d+ex) \coth^2(d+ex) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 181

$$\int F^{c(a+bx)} \cosh(d+ex) \coth^2(d+ex) \, dx = \frac{2e^{-d-ex} F^{c(a+bx)}}{e\left(1-e^{2d+2ex}\right)} - \frac{5e^{-d-ex} F^{c(a+bx)}}{2(e-bc\log(F))} + \frac{2bce^{-d-ex} F^{c(a+bx)} \text{Hypergeometric2F1}\left(1, \frac{1}{2}\left(-1+\frac{bc\log(F)}{e}\right), \frac{e+bc\log(F)}{2e}, e^{2d+2ex}\right) \log(F)}{e(e-bc\log(F))} + \frac{e^{d+ex} F^{c(a+bx)}}{2(e+bc\log(F))}$$

output

```
2*exp(-e*x-d)*F^(c*(b*x+a))/e/(1-exp(2*e*x+2*d))-5*exp(-e*x-d)*F^(c*(b*x+a
))/(2*e-2*b*c*ln(F))+2*b*c*exp(-e*x-d)*F^(c*(b*x+a))*hypergeom([1, -1/2+1/
2*b*c*ln(F)/e],[1/2*(e+b*c*ln(F))/e],exp(2*e*x+2*d))*ln(F)/e/(e-b*c*ln(F))
+exp(e*x+d)*F^(c*(b*x+a))/(2*e+2*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.01

$$\int F^{c(a+bx)} \cosh(d+ex) \coth^2(d+ex)\, dx$$
$$= \frac{F^{c\left(a-\frac{bd}{e}\right)} \operatorname{csch}(d+ex) \left(-4bce^{\frac{(d+ex)(e+bc\log(F))}{e}} \text{Hypergeometric2F1}\left(1, \frac{e+bc\log(F)}{2e}, \frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right), e^{2(d+ex)}\right.\right.}{2e(e-}$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x]*Coth[d + e*x]^2,x]
```

output
```
(F^(c*(a - (b*d)/e))*Csch[d + e*x]*(-4*b*c*E^(((d + e*x)*(e + b*c*Log[F]))
/e)*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e), (3 + (b*c*Log[F])/e)/2, E
^(2*(d + e*x))]*Log[F]*(e - b*c*Log[F])*Sinh[d + e*x] + F^((b*c*(d + e*x))
/e)*(-3*e^2 + e^2*Cosh[2*(d + e*x)] + 2*b^2*c^2*Log[F]^2 - b*c*e*Log[F]*Si
nh[2*(d + e*x)])))/(2*e*(e - b*c*Log[F])*(e + b*c*Log[F]))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(d+ex) \coth^2(d+ex) F^{c(a+bx)}\, dx$$

$$\Big\downarrow \text{ 6037}$$

$$\int \left(\frac{5}{2}e^{-d-ex}F^{ac+bcx}+\frac{1}{2}e^{2(d+ex)-d-ex}F^{ac+bcx}+\frac{6e^{-d-ex}F^{ac+bcx}}{e^{2(d+ex)}-1}+\frac{4e^{-d-ex}F^{ac+bcx}}{\left(e^{2(d+ex)}-1\right)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{6e^{-d-ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(\frac{bc\log(F)}{e}-1\right),\frac{e+bc\log(F)}{2e},e^{2(d+ex)}\right)}{e-bc\log(F)}-\frac{4e^{-d-ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(\frac{bc\log(F)}{e}-1\right),\frac{e+bc\log(F)}{2e},e^{2(d+ex)}\right)}{e-bc\log(F)}-\frac{5F^{ac}e^{-x(e-bc\log(F))-d}}{2(e-bc\log(F))}+\frac{F^{ac}e^{x(bc\log(F)+e)+d}}{2(bc\log(F)+e)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x]*Coth[d + e*x]^2,x]
```

output
```
(-5*E^(-d - x*(e - b*c*Log[F]))*F^(a*c))/(2*(e - b*c*Log[F])) + (6*E^(-d -
 e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[1, (-1 + (b*c*Log[F])/e)/2, (e + b
*c*Log[F])/(2*e), E^(2*(d + e*x))])/(e - b*c*Log[F]) - (4*E^(-d - e*x)*F^(
a*c + b*c*x)*Hypergeometric2F1[2, (-1 + (b*c*Log[F])/e)/2, (e + b*c*Log[F]
)/(2*e), E^(2*(d + e*x))])/(e - b*c*Log[F]) + (E^(d + x*(e + b*c*Log[F]))*
F^(a*c))/(2*(e + b*c*Log[F]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Maple [F]

$$\int F^{c(bx+a)}\cosh(ex+d)\coth(ex+d)^2\,dx$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)*coth(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*cosh(e*x+d)*coth(e*x+d)^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\cosh(d+ex)\coth^2(d+ex)\,dx=\int F^{(bx+a)c}\cosh\left(ex+d\right)\coth\left(ex+d\right)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)*coth(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*cosh(e*x + d)*coth(e*x + d)^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\cosh(d+ex)\coth^2(d+ex)\,dx=\int F^{c(a+bx)}\cosh\left(d+ex\right)\coth^2\left(d+ex\right)dx$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d)*coth(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*cosh(d + e*x)*coth(d + e*x)**2, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\cosh(d+ex)\coth^2(d+ex)\,dx=\int F^{(bx+a)c}\cosh\left(ex+d\right)\coth\left(ex+d\right)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)*coth(e*x+d)^2,x, algorithm="maxima")
```

output
```
16*F^(a*c)*b*c*e*integrate(F^(b*c*x)/((b^2*c^2*e^(7*d)*log(F)^2 - 8*b*c*e*
e^(7*d)*log(F) + 15*e^2*e^(7*d))*e^(7*e*x) - 3*(b^2*c^2*e^(5*d)*log(F)^2 -
 8*b*c*e*e^(5*d)*log(F) + 15*e^2*e^(5*d))*e^(5*e*x) + 3*(b^2*c^2*e^(3*d)*l
og(F)^2 - 8*b*c*e*e^(3*d)*log(F) + 15*e^2*e^(3*d))*e^(3*e*x) - (b^2*c^2*e^
d*log(F)^2 - 8*b*c*e*e^d*log(F) + 15*e^2*e^d)*e^(e*x)), x)*log(F) + 1/2*(F
^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b^2*c^2*e*log(F)^2 + 39*F^(a*c)*b*c*e
^2*log(F) + 15*F^(a*c)*e^3 + (F^(a*c)*b^3*c^3*e^(6*d)*log(F)^3 - 9*F^(a*c)
*b^2*c^2*e*e^(6*d)*log(F)^2 + 23*F^(a*c)*b*c*e^2*e^(6*d)*log(F) - 15*F^(a*
c)*e^3*e^(6*d))*e^(6*e*x) + (3*F^(a*c)*b^3*c^3*e^(4*d)*log(F)^3 - 17*F^(a*
c)*b^2*c^2*e*e^(4*d)*log(F)^2 - 11*F^(a*c)*b*c*e^2*e^(4*d)*log(F) + 105*F^
(a*c)*e^3*e^(4*d))*e^(4*e*x) + (3*F^(a*c)*b^3*c^3*e^(2*d)*log(F)^3 + F^(a*
c)*b^2*c^2*e*e^(2*d)*log(F)^2 - 59*F^(a*c)*b*c*e^2*e^(2*d)*log(F) - 105*F^
(a*c)*e^3*e^(2*d))*e^(2*e*x))*F^(b*c*x)/((b^4*c^4*e^(5*d)*log(F)^4 - 8*b^3
*c^3*e*e^(5*d)*log(F)^3 + 14*b^2*c^2*e^2*e^(5*d)*log(F)^2 + 8*b*c*e^3*e^(5
*d)*log(F) - 15*e^4*e^(5*d))*e^(5*e*x) - 2*(b^4*c^4*e^(3*d)*log(F)^4 - 8*b
^3*c^3*e*e^(3*d)*log(F)^3 + 14*b^2*c^2*e^2*e^(3*d)*log(F)^2 + 8*b*c*e^3*e^
(3*d)*log(F) - 15*e^4*e^(3*d))*e^(3*e*x) + (b^4*c^4*e^d*log(F)^4 - 8*b^3*c
^3*e*e^d*log(F)^3 + 14*b^2*c^2*e^2*e^d*log(F)^2 + 8*b*c*e^3*e^d*log(F) - 1
5*e^4*e^d)*e^(e*x))
```

Giac [F]

$$\int F^{c(a+bx)} \cosh(d+ex) \coth^2(d+ex) \, dx = \int F^{(bx+a)c} \cosh(ex+d) \coth(ex+d)^2 \, dx$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)*coth(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*cosh(e*x + d)*coth(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\cosh(d+ex)\coth^2(d+ex)\,dx=\int F^{c\,(a+b\,x)}\cosh(d+e\,x)\,\coth(d+e\,x)^2\,dx$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x)*coth(d + e*x)^2,x)
```

output
```
int(F^(c*(a + b*x))*cosh(d + e*x)*coth(d + e*x)^2, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\cosh(d+ex)\coth^2(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\cosh(ex+d)\coth(ex+d)^2\,dx\right)$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)*coth(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cosh(d + e*x)*coth(d + e*x)**2,x)
```

3.115 $\int F^{c(a+bx)} \coth^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 157

$$\int F^{c(a+bx)} \coth^3(d+ex)\,dx = -\frac{2F^{c(a+bx)}}{e\left(1-e^{2d+2ex}\right)^2}+\frac{F^{c(a+bx)}}{bc\log(F)}+\frac{F^{c(a+bx)}(2e+bc\log(F))}{e^2\left(1-e^{2d+2ex}\right)} - F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},e^{2d+2ex}\right)\left(\frac{2}{bc\log(F)}+\frac{bc\log(F)}{e^2}\right)$$

output
```
-2*F^(c*(b*x+a))/e/(1-exp(2*e*x+2*d))^2+F^(c*(b*x+a))/b/c/ln(F)+F^(c*(b*x+
a))*(2*e+b*c*ln(F))/e^2/(1-exp(2*e*x+2*d))-F^(c*(b*x+a))*hypergeom([1, 1/2
*b*c*ln(F)/e],[1+1/2*b*c*ln(F)/e],exp(2*e*x+2*d))*(2/b/c/ln(F)+b*c*ln(F)/e
^2)
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94

$$\int F^{c(a+bx)}\coth^3(d+ex)\,dx = \frac{1}{2}F^{c(a+bx)}\left(-\frac{\operatorname{csch}^2(d+ex)}{e}+\frac{2\coth(d)}{bc\log(F)} - \frac{2\left(1+\left(-1+e^{2d}\right)\operatorname{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},e^{2(d+ex)}\right)\right)\left(2e^2+b^2c^2\log^2(F)\right)}{bce^2\left(-1+e^{2d}\right)\log(F)} + \frac{bc\operatorname{csch}(d)\operatorname{csch}(d+ex)\log(F)\sinh(ex)}{e^2}\right)$$

input
```
Integrate[F^(c*(a + b*x))*Coth[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(-(Csch[d + e*x]^2/e) + (2*Coth[d])/(b*c*Log[F]) - (2*(1
+ (-1 + E^(2*d))*Hypergeometric2F1[1, (b*c*Log[F])/(2*e), 1 + (b*c*Log[F])
/(2*e), E^(2*(d + e*x))])*(2*e^2 + b^2*c^2*Log[F]^2))/(b*c*e^2*(-1 + E^(2*
d))*Log[F]) + (b*c*Csch[d]*Csch[d + e*x]*Log[F]*Sinh[e*x])/e^2))/2
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.20, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{ 6008}$$

$$\int \left(\frac{6F^{c(a+bx)}}{e^{2(d+ex)}-1}+\frac{12F^{c(a+bx)}}{\left(e^{2(d+ex)}-1\right)^2}+\frac{8F^{c(a+bx)}}{\left(e^{2(d+ex)}-1\right)^3}+F^{c(a+bx)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{6F^{c(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,e^{2(d+ex)}\right)}{bc\log(F)}+\frac{12F^{c(a+bx)}\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,e^{2(d+ex)}\right)}{bc\log(F)}-\frac{8F^{c(a+bx)}\text{Hypergeometric2F1}\left(3,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,e^{2(d+ex)}\right)}{bc\log(F)}+\frac{F^{c(a+bx)}}{bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Coth[d + e*x]^3,x]
```

output
```
F^(c*(a + b*x))/(b*c*Log[F]) - (6*F^(c*(a + b*x))*Hypergeometric2F1[1, (b*
c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))]])/(b*c*Log[F]) +
(12*F^(c*(a + b*x))*Hypergeometric2F1[2, (b*c*Log[F])/(2*e), 1 + (b*c*Log[
F])/(2*e), E^(2*(d + e*x))]])/(b*c*Log[F]) - (8*F^(c*(a + b*x))*Hypergeomet
ric2F1[3, (b*c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))]])/(b
*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6008
```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*(d + e*x)))^n/(-1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \coth(ex+d)^3 \, dx$$

input
```
int(F^(c*(b*x+a))*coth(e*x+d)^3,x)
```

output
```
int(F^(c*(b*x+a))*coth(e*x+d)^3,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \coth^3(d+ex)\,dx = \int F^{(bx+a)c} \coth{(ex+d)}^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*coth(e*x + d)^3, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \coth^3(d+ex)\,dx = \int F^{c(a+bx)} \coth^3{(d+ex)}\,dx$$

input
```
integrate(F**(c*(b*x+a))*coth(e*x+d)**3,x)
```

output
```
Integral(F**(c*(a + b*x))*coth(d + e*x)**3, x)
```

Maxima [F]

$$\int F^{c(a+bx)} \coth^3(d+ex)\,dx = \int F^{(bx+a)c} \coth{(ex+d)}^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)^3,x, algorithm="maxima")
```

output
```
48*(F^(a*c)*b^2*c^2*e*log(F)^2 + 2*F^(a*c)*e^3)*integrate(F^(b*c*x)/(b^3*c
^3*log(F)^3 - 12*b^2*c^2*e*log(F)^2 + 44*b*c*e^2*log(F) - 48*e^3 + (b^3*c^
3*e^(8*d)*log(F)^3 - 12*b^2*c^2*e*e^(8*d)*log(F)^2 + 44*b*c*e^2*e^(8*d)*lo
g(F) - 48*e^3*e^(8*d))*e^(8*e*x) - 4*(b^3*c^3*e^(6*d)*log(F)^3 - 12*b^2*c^
2*e*e^(6*d)*log(F)^2 + 44*b*c*e^2*e^(6*d)*log(F) - 48*e^3*e^(6*d))*e^(6*e*
x) + 6*(b^3*c^3*e^(4*d)*log(F)^3 - 12*b^2*c^2*e*e^(4*d)*log(F)^2 + 44*b*c*
e^2*e^(4*d)*log(F) - 48*e^3*e^(4*d))*e^(4*e*x) - 4*(b^3*c^3*e^(2*d)*log(F)
^3 - 12*b^2*c^2*e*e^(2*d)*log(F)^2 + 44*b*c*e^2*e^(2*d)*log(F) - 48*e^3*e^
(2*d))*e^(2*e*x)), x) - (F^(a*c)*b^3*c^3*log(F)^3 + 36*F^(a*c)*b^2*c^2*e*l
og(F)^2 + 44*F^(a*c)*b*c*e^2*log(F) + 48*F^(a*c)*e^3 + (F^(a*c)*b^3*c^3*e^
(6*d)*log(F)^3 - 12*F^(a*c)*b^2*c^2*e*e^(6*d)*log(F)^2 + 44*F^(a*c)*b*c*e^
2*e^(6*d)*log(F) - 48*F^(a*c)*e^3*e^(6*d))*e^(6*e*x) + 3*(F^(a*c)*b^3*c^3*
e^(4*d)*log(F)^3 - 8*F^(a*c)*b^2*c^2*e*e^(4*d)*log(F)^2 + 4*F^(a*c)*b*c*e^
2*e^(4*d)*log(F) + 48*F^(a*c)*e^3*e^(4*d))*e^(4*e*x) + 3*(F^(a*c)*b^3*c^3*
e^(2*d)*log(F)^3 - 28*F^(a*c)*b*c*e^2*e^(2*d)*log(F) - 48*F^(a*c)*e^3*e^(2
*d))*e^(2*e*x))*F^(b*c*x)/(b^4*c^4*log(F)^4 - 12*b^3*c^3*e*log(F)^3 + 44*b
^2*c^2*e^2*log(F)^2 - 48*b*c*e^3*log(F) - (b^4*c^4*e^(6*d)*log(F)^4 - 12*b
^3*c^3*e*e^(6*d)*log(F)^3 + 44*b^2*c^2*e^2*e^(6*d)*log(F)^2 - 48*b*c*e^3*e
^(6*d)*log(F))*e^(6*e*x) + 3*(b^4*c^4*e^(4*d)*log(F)^4 - 12*b^3*c^3*e*e^(4
*d)*log(F)^3 + 44*b^2*c^2*e^2*e^(4*d)*log(F)^2 - 48*b*c*e^3*e^(4*d)*log...
```

Giac [F]

$$\int F^{c(a+bx)} \coth^3(d+ex)\,dx = \int F^{(bx+a)c} \coth{(ex+d)}^3 \,dx$$

input
```
integrate(F^(c*(b*x+a))*coth(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*coth(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)} \coth^3(d+ex)\,dx = \int F^{c\,(a+b\,x)} \coth(d+e\,x)^3\,dx$$

input
```
int(F^(c*(a + b*x))*coth(d + e*x)^3,x)
```

output
```
int(F^(c*(a + b*x))*coth(d + e*x)^3, x)
```

Reduce [F]

$$\int F^{c(a+bx)} \coth^3(d+ex)\,dx = f^{ac}\left(\int f^{bcx} \coth(ex+d)^3\,dx\right)$$

input
```
int(F^(c*(b*x+a))*coth(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*coth(d + e*x)**3,x)
```

3.116 $\int F^{c(a+bx)} \sinh^2(d+ex) \tanh(d+ex)\, dx$

Optimal result

Integrand size = 24, antiderivative size = 163

$$\begin{aligned}
&\int F^{c(a+bx)} \sinh^2(d+ex) \tanh(d+ex)\, dx \\
&= -\frac{F^{c(a+bx)}}{bc\log(F)} - \frac{7e^{-2d-2ex}F^{c(a+bx)}}{4(2e-bc\log(F))} \\
&\quad + \frac{2e^{-2d-2ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(-2+\frac{bc\log(F)}{e}\right),\frac{bc\log(F)}{2e},-e^{2d+2ex}\right)}{2e-bc\log(F)} \\
&\quad + \frac{e^{2d+2ex}F^{c(a+bx)}}{4(2e+bc\log(F))}
\end{aligned}$$

output

```
-F^(c*(b*x+a))/b/c/ln(F)-7*exp(-2*e*x-2*d)*F^(c*(b*x+a))/(8*e-4*b*c*ln(F))
+2*exp(-2*e*x-2*d)*F^(c*(b*x+a))*hypergeom([1, -1+1/2*b*c*ln(F)/e],[1/2*b*
c*ln(F)/e],-exp(2*e*x+2*d))/(2*e-b*c*ln(F))+exp(2*e*x+2*d)*F^(c*(b*x+a))/(
8*e+4*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.87

$$\int F^{c(a+bx)}\sinh^2(d+ex)\tanh(d+ex)\,dx$$
$$=\frac{F^{c(a+bx)}\left(8e^2-2bce\cosh(2(d+ex))\log(F)-2b^2c^2\log^2(F)+4\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log\ldots}{\ldots}\right.\right.}{-8bce^2\log(F)+2b^3c^3\log^3(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sinh[d + e*x]^2*Tanh[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*(8*e^2 - 2*b*c*e*Cosh[2*(d + e*x)]*Log[F] - 2*b^2*c^2*Log
[F]^2 + 4*Hypergeometric2F1[1, (b*c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e),
 -E^(2*(d + e*x))]*(-4*e^2 + b^2*c^2*Log[F]^2) + b^2*c^2*Log[F]^2*Sinh[2*(
d + e*x)]))/(-8*b*c*e^2*Log[F] + 2*b^3*c^3*Log[F]^3)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(d+ex)\tanh(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{ 6037}$$

$$\int \left(\frac{7}{4}e^{-2d-2ex}F^{ac+bcx}-e^{2(d+ex)-2d-2ex}F^{ac+bcx}+\frac{1}{4}e^{4(d+ex)-2d-2ex}F^{ac+bcx}-\frac{2e^{-2d-2ex}F^{ac+bcx}}{e^{2(d+ex)}+1}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2e^{-2d-2ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(\frac{bc\log(F)}{e}-2\right),\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)}{2e-bc\log(F)}-\frac{7F^{ac}e^{-x(2e-bc\log(F))-2d}}{4(2e-bc\log(F))}+\frac{F^{ac}e^{x(bc\log(F)+2e)+2d}}{4(bc\log(F)+2e)}-\frac{F^{ac+bcx}}{bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Sinh[d + e*x]^2*Tanh[d + e*x],x]
```

output
```
-(F^(a*c + b*c*x)/(b*c*Log[F])) - (7*E^(-2*d - x*(2*e - b*c*Log[F]))*F^(a*
c))/(4*(2*e - b*c*Log[F])) + (2*E^(-2*d - 2*e*x)*F^(a*c + b*c*x)*Hypergeom
etric2F1[1, (-2 + (b*c*Log[F])/e)/2, (b*c*Log[F])/(2*e), -E^(2*(d + e*x))]
)/(2*e - b*c*Log[F]) + (E^(2*d + x*(2*e + b*c*Log[F]))*F^(a*c))/(4*(2*e +
b*c*Log[F]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Maple [F]

$$\int F^{c(bx+a)}\sinh\left(ex+d\right)^{2}\tanh\left(ex+d\right)dx$$

input
```
int(F^(c*(b*x+a))*sinh(e*x+d)^2*tanh(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*sinh(e*x+d)^2*tanh(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\sinh^2(d+ex)\tanh(d+ex)\,dx = \int F^{(bx+a)c}\sinh(ex+d)^2\tanh(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)^2*tanh(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sinh(e*x + d)^2*tanh(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\sinh^2(d+ex)\tanh(d+ex)\,dx = \int F^{c(a+bx)}\sinh^2(d+ex)\tanh(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*sinh(e*x+d)**2*tanh(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*sinh(d + e*x)**2*tanh(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\sinh^2(d+ex)\tanh(d+ex)\,dx = \int F^{(bx+a)c}\sinh(ex+d)^2\tanh(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)^2*tanh(e*x+d),x, algorithm="maxima")
```

output
```
4*F^(a*c)*e*integrate(F^(b*c*x)/((b*c*e^(6*d)*log(F) - 4*e*e^(6*d))*e^(6*e
*x) + 2*(b*c*e^(4*d)*log(F) - 4*e*e^(4*d))*e^(4*e*x) + (b*c*e^(2*d)*log(F)
 - 4*e*e^(2*d))*e^(2*e*x)), x) - 1/4*(F^(a*c)*b^3*c^3*log(F)^3 + 14*F^(a*c
)*b^2*c^2*e*log(F)^2 + 24*F^(a*c)*b*c*e^2*log(F) - (F^(a*c)*b^3*c^3*e^(6*d
)*log(F)^3 - 6*F^(a*c)*b^2*c^2*e*e^(6*d)*log(F)^2 + 8*F^(a*c)*b*c*e^2*e^(6
*d)*log(F))*e^(6*e*x) + (3*F^(a*c)*b^3*c^3*e^(4*d)*log(F)^3 - 10*F^(a*c)*b
^2*c^2*e*e^(4*d)*log(F)^2 - 24*F^(a*c)*b*c*e^2*e^(4*d)*log(F) + 64*F^(a*c)
*e^3*e^(4*d))*e^(4*e*x) - (3*F^(a*c)*b^3*c^3*e^(2*d)*log(F)^3 + 2*F^(a*c)*
b^2*c^2*e*e^(2*d)*log(F)^2 - 40*F^(a*c)*b*c*e^2*e^(2*d)*log(F) - 64*F^(a*c
)*e^3*e^(2*d))*e^(2*e*x))*F^(b*c*x)/((b^4*c^4*e^(4*d)*log(F)^4 - 4*b^3*c^3
*e*e^(4*d)*log(F)^3 - 4*b^2*c^2*e^2*e^(4*d)*log(F)^2 + 16*b*c*e^3*e^(4*d)*
log(F))*e^(4*e*x) + (b^4*c^4*e^(2*d)*log(F)^4 - 4*b^3*c^3*e*e^(2*d)*log(F)
^3 - 4*b^2*c^2*e^2*e^(2*d)*log(F)^2 + 16*b*c*e^3*e^(2*d)*log(F))*e^(2*e*x)
)
```

Giac [F]

$$\int F^{c(a+bx)} \sinh^2(d+ex) \tanh(d+ex)\, dx = \int F^{(bx+a)c} \sinh\left(ex+d\right)^2 \tanh\left(ex+d\right)\, dx$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)^2*tanh(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sinh(e*x + d)^2*tanh(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)} \sinh^2(d+ex) \tanh(d+ex)\, dx = \int F^{c\,(a+b\,x)}\,\sinh(d+e\,x)^2\,\tanh(d+e\,x)\, dx$$

input
```
int(F^(c*(a + b*x))*sinh(d + e*x)^2*tanh(d + e*x),x)
```

output
```
int(F^(c*(a + b*x))*sinh(d + e*x)^2*tanh(d + e*x), x)
```

Reduce [F]

$$\int F^{c(a+bx)} \sinh^2(d+ex)\tanh(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\sinh\left(ex+d\right)^2\tanh\left(ex+d\right)dx\right)$$

input
```
int(F^(c*(b*x+a))*sinh(e*x+d)^2*tanh(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sinh(d + e*x)**2*tanh(d + e*x),x)
```

3.117 $\int F^{c(a+bx)} \sinh(d+ex) \tanh(d+ex)\, dx$

Optimal result

Integrand size = 22, antiderivative size = 136

$$\int F^{c(a+bx)} \sinh(d+ex) \tanh(d+ex)\, dx = \frac{3e^{-d-ex}F^{c(a+bx)}}{2(e-bc\log(F))} - \frac{2e^{-d-ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(-1+\frac{bc\log(F)}{e}\right),\frac{e+bc\log(F)}{2e},-e^{2d+2ex}\right)}{e-bc\log(F)} + \frac{e^{d+ex}F^{c(a+bx)}}{2(e+bc\log(F))}$$

output
```
3*exp(-e*x-d)*F^(c*(b*x+a))/(2*e-2*b*c*ln(F))-2*exp(-e*x-d)*F^(c*(b*x+a))*
hypergeom([1, -1/2+1/2*b*c*ln(F)/e],[1/2*(e+b*c*ln(F))/e],-exp(2*e*x+2*d))
/(e-b*c*ln(F))+exp(e*x+d)*F^(c*(b*x+a))/(2*e+2*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79

$$\int F^{c(a+bx)}\sinh(d+ex)\tanh(d+ex)\,dx$$
$$=\frac{F^{c(a+bx)}\left(-bc\cosh(d+ex)\log(F)-2e^{d+ex}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),-e^{2(d+ex)}\right.\right.}{(e-bc\log(F))(e+bc\log(F))}$$

input
```
Integrate[F^(c*(a + b*x))*Sinh[d + e*x]*Tanh[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*(-(b*c*Cosh[d + e*x]*Log[F]) - 2*E^(d + e*x)*Hypergeometr
ic2F1[1, (e + b*c*Log[F])/(2*e), (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))]
*(e - b*c*Log[F]) + e*Sinh[d + e*x]))/((e - b*c*Log[F])*(e + b*c*Log[F]))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(d+ex)\tanh(d+ex)F^{c(a+bx)}\,dx$$
$$\Big\downarrow \text{6037}$$
$$\int \left(-\frac{3}{2}e^{-d-ex}F^{ac+bcx}+\frac{1}{2}e^{2(d+ex)-d-ex}F^{ac+bcx}+\frac{2e^{-d-ex}F^{ac+bcx}}{e^{2(d+ex)}+1}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{2e^{-d-ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(\frac{bc\log(F)}{e}-1\right),\frac{e+bc\log(F)}{2e},-e^{2(d+ex)}\right)}{e-bc\log(F)}+\frac{3F^{ac}e^{-x(e-bc\log(F))-d}}{2(e-bc\log(F))}+\frac{F^{ac}e^{x(bc\log(F)+e)+d}}{2(bc\log(F)+e)}$$

input
```
Int[F^(c*(a + b*x))*Sinh[d + e*x]*Tanh[d + e*x],x]
```

output
```
(3*E^(-d - x*(e - b*c*Log[F]))*F^(a*c))/(2*(e - b*c*Log[F])) - (2*E^(-d -
e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[1, (-1 + (b*c*Log[F])/e)/2, (e + b*
c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e - b*c*Log[F]) + (E^(d + x*(e + b*c*
Log[F]))*F^(a*c))/(2*(e + b*c*Log[F]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Maple [F]

$$\int F^{c(bx+a)} \sinh(ex+d) \tanh(ex+d)\, dx$$

input
```
int(F^(c*(b*x+a))*sinh(e*x+d)*tanh(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*sinh(e*x+d)*tanh(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)} \sinh(d+ex)\tanh(d+ex)\, dx = \int F^{(bx+a)c} \sinh(ex+d) \tanh(ex+d)\, dx$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)*tanh(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sinh(e*x + d)*tanh(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)} \sinh(d+ex) \tanh(d+ex) \, dx = \int F^{c(a+bx)} \sinh (d+ex) \tanh (d+ex) \, dx$$

input
```
integrate(F**(c*(b*x+a))*sinh(e*x+d)*tanh(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*sinh(d + e*x)*tanh(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)} \sinh(d+ex) \tanh(d+ex) \, dx = \int F^{(bx+a)c} \sinh (ex+d) \tanh (ex+d) \, dx$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)*tanh(e*x+d),x, algorithm="maxima")
```

output
```
-4*F^(a*c)*e*integrate(F^(b*c*x)/((b*c*e^(5*d)*log(F) - 3*e*e^(5*d))*e^(5*
e*x) + 2*(b*c*e^(3*d)*log(F) - 3*e*e^(3*d))*e^(3*e*x) + (b*c*e^d*log(F) -
3*e*e^d)*e^(e*x)), x) + 1/2*(F^(a*c)*b^2*c^2*log(F)^2 + 6*F^(a*c)*b*c*e*lo
g(F) + 5*F^(a*c)*e^2 + (F^(a*c)*b^2*c^2*e^(4*d)*log(F)^2 - 4*F^(a*c)*b*c*e
*e^(4*d)*log(F) + 3*F^(a*c)*e^2*e^(4*d))*e^(4*e*x) - 2*(F^(a*c)*b^2*c^2*e^
(2*d)*log(F)^2 - F^(a*c)*b*c*e*e^(2*d)*log(F) - 6*F^(a*c)*e^2*e^(2*d))*e^(
2*e*x))*F^(b*c*x)/((b^3*c^3*e^(3*d)*log(F)^3 - 3*b^2*c^2*e*e^(3*d)*log(F)^
2 - b*c*e^2*e^(3*d)*log(F) + 3*e^3*e^(3*d))*e^(3*e*x) + (b^3*c^3*e^d*log(F
)^3 - 3*b^2*c^2*e*e^d*log(F)^2 - b*c*e^2*e^d*log(F) + 3*e^3*e^d)*e^(e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\sinh(d+ex)\tanh(d+ex)\,dx=\int F^{(bx+a)c}\sinh\left(ex+d\right)\tanh\left(ex+d\right)\,dx$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)*tanh(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sinh(e*x + d)*tanh(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sinh(d+ex)\tanh(d+ex)\,dx=\int F^{c\,(a+b\,x)}\,\sinh(d+e\,x)\,\tanh(d+e\,x)\,dx$$

input
```
int(F^(c*(a + b*x))*sinh(d + e*x)*tanh(d + e*x),x)
```

output
```
int(F^(c*(a + b*x))*sinh(d + e*x)*tanh(d + e*x), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\sinh(d+ex)\tanh(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\sinh\left(ex+d\right)\tanh\left(ex+d\right)dx\right)$$

input
```
int(F^(c*(b*x+a))*sinh(e*x+d)*tanh(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sinh(d + e*x)*tanh(d + e*x),x)
```

3.118 $\int F^{c(a+bx)} \tanh(d+ex) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 80

$$\int F^{c(a+bx)} \tanh(d+ex) \, dx = \frac{F^{c(a+bx)}}{bc\log(F)} - \frac{2F^{c(a+bx)} \text{Hypergeometric2F1}\left(1, \frac{bc\log(F)}{2e}, 1 + \frac{bc\log(F)}{2e}, -e^{2d+2ex}\right)}{bc\log(F)}$$

output
```
F^(c*(b*x+a))/b/c/ln(F)-2*F^(c*(b*x+a))*hypergeom([1, 1/2*b*c*ln(F)/e],[1+
1/2*b*c*ln(F)/e],-exp(2*e*x+2*d))/b/c/ln(F)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int F^{c(a+bx)} \tanh(d+ex) \, dx = \frac{F^{c(a+bx)} \left(1 - 2\,\text{Hypergeometric2F1}\left(1, \frac{bc\log(F)}{2e}, 1 + \frac{bc\log(F)}{2e}, -e^{2(d+ex)}\right)\right)}{bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Tanh[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*(1 - 2*Hypergeometric2F1[1, (b*c*Log[F])/(2*e), 1 + (b*c*
Log[F])/(2*e), -E^(2*(d + e*x))]))/(b*c*Log[F])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6007, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{ 6007}$$

$$\int \left(F^{c(a+bx)}-\frac{2F^{c(a+bx)}}{e^{2(d+ex)}+1}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{F^{c(a+bx)}}{bc\log(F)}-\frac{2F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,-e^{2(d+ex)}\right)}{bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Tanh[d + e*x],x]
```

output
```
F^(c*(a + b*x))/(b*c*Log[F]) - (2*F^(c*(a + b*x))*Hypergeometric2F1[1, (b*
c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6007
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tanh[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((-1 + E^(2*(d + e*x)))^n/(1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \tanh (ex + d) \, dx$$

input
```
int(F^(c*(b*x+a))*tanh(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*tanh(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)} \tanh(d+ex) \, dx = \int F^{(bx+a)c} \tanh (ex+d) \, dx$$

input
```
integrate(F^(c*(b*x+a))*tanh(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*tanh(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\tanh(d+ex)\,dx=\int F^{c(a+bx)}\tanh\left(d+ex\right)dx$$

input
```
integrate(F**(c*(b*x+a))*tanh(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*tanh(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\tanh(d+ex)\,dx=\int F^{(bx+a)c}\tanh\left(ex+d\right)\,dx$$

input
```
integrate(F^(c*(b*x+a))*tanh(e*x+d),x, algorithm="maxima")
```

output
```
4*F^(a*c)*e*integrate(F^(b*c*x)/(b*c*log(F) + (b*c*e^(4*d)*log(F) - 2*e*e^
(4*d))*e^(4*e*x) + 2*(b*c*e^(2*d)*log(F) - 2*e*e^(2*d))*e^(2*e*x) - 2*e),
x) - (F^(a*c)*b*c*log(F) + 2*F^(a*c)*e - (F^(a*c)*b*c*e^(2*d)*log(F) - 2*F
^(a*c)*e*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 2*b*c*e*log(F)
+ (b^2*c^2*e^(2*d)*log(F)^2 - 2*b*c*e*e^(2*d)*log(F))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\tanh(d+ex)\,dx=\int F^{(bx+a)c}\tanh\left(ex+d\right)\,dx$$

input
```
integrate(F^(c*(b*x+a))*tanh(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*tanh(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\tanh(d+ex)\,dx=\int F^{c\,(a+b\,x)}\tanh(d+e\,x)\,dx$$

input
```
int(F^(c*(a + b*x))*tanh(d + e*x),x)
```

output
```
int(F^(c*(a + b*x))*tanh(d + e*x), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\tanh(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\tanh\left(ex+d\right)dx\right)$$

input
```
int(F^(c*(b*x+a))*tanh(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*tanh(d + e*x),x)
```

3.119 $\int F^{c(a+bx)}\text{sech}(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 69

$$\int F^{c(a+bx)}\text{sech}(d+ex)\,dx = \frac{2e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),-e^{2d+2ex}\right)}{e+bc\log(F)}$$

output
```
2*exp(e*x+d)*F^(c*(b*x+a))*hypergeom([1, 1/2*(e+b*c*ln(F))/e],[3/2+1/2*b*c
*ln(F)/e],-exp(2*e*x+2*d))/(e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int F^{c(a+bx)}\text{sech}(d+ex)\,dx = \frac{2e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{bc\log(F)}{2e},\frac{3}{2}+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)}{e+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sech[d + e*x],x]
```

output
```
(2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, 1/2 + (b*c*Log[F])/(2*
e), 3/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e + b*c*Log[F])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {6015}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6015}$$

$$\frac{2e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),-e^{2(d+ex)}\right)}{bc\log(F)+e}$$

input
```
Int[F^(c*(a + b*x))*Sech[d + e*x],x]
```

output
```
(2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e)
, (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/(e + b*c*Log[F])
```

Defintions of rubi rules used

rule 6015
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)}\operatorname{sech}(ex+d)\,dx$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*sech(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\operatorname{sech}(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{sech}(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sech(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\operatorname{sech}(d+ex)\,dx=\int F^{c(a+bx)}\operatorname{sech}(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*sech(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*sech(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\text{sech}(d+ex)\,dx = \int F^{(bx+a)c}\,\text{sech}\,(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d),x, algorithm="maxima")
```

output
```
-4*F^(a*c)*e*integrate(e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + (b*c*e^(4*
d)*log(F) - e*e^(4*d))*e^(4*e*x) + 2*(b*c*e^(2*d)*log(F) - e*e^(2*d))*e^(2
*e*x) - e), x) + 2*F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + (b*c*e
^(2*d)*log(F) - e*e^(2*d))*e^(2*e*x) - e)
```

Giac [F]

$$\int F^{c(a+bx)}\text{sech}(d+ex)\,dx = \int F^{(bx+a)c}\,\text{sech}\,(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sech(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{sech}(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}}{\cosh\left(d+e\,x\right)}\,dx$$

input
```
int(F^(c*(a + b*x))/cosh(d + e*x),x)
```

output
```
int(F^(c*(a + b*x))/cosh(d + e*x), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\operatorname{sech}(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\operatorname{sech}(ex+d)\,dx\right)$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sech(d + e*x),x)
```

3.120 $\int F^{c(a+bx)}\mathrm{csch}(d+ex)\mathrm{sech}(d+ex)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int F^{c(a+bx)}\mathrm{csch}(d+ex)\mathrm{sech}(d+ex)\,dx = -\frac{4e^{2d+2ex}F^{c(a+bx)}\,\mathrm{Hypergeometric2F1}\left(1,\frac{1}{4}\left(2+\frac{bc\log(F)}{e}\right),\frac{1}{4}\left(6+\frac{bc\log(F)}{e}\right),e^{4d+4ex}\right)}{2e+bc\log(F)}$$

output
```
-4*exp(2*e*x+2*d)*F^(c*(b*x+a))*hypergeom([1, 1/2+1/4*b*c*ln(F)/e],[3/2+1/
4*b*c*ln(F)/e],exp(4*e*x+4*d))/(2*e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 2.98 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.33

$$\int F^{c(a+bx)}\mathrm{csch}(d+ex)\mathrm{sech}(d+ex)\,dx = \frac{2F^{c(a+bx)}\left(\mathrm{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)-\mathrm{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1\right.\right.}{bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Csch[d + e*x]*Sech[d + e*x],x]
```

output
```
(2*F^(c*(a + b*x))*(Hypergeometric2F1[1, (b*c*Log[F])/(2*e), 1 + (b*c*Log[
F])/(2*e), -E^(2*(d + e*x))] - Hypergeometric2F1[1, (b*c*Log[F])/(2*e), 1
+ (b*c*Log[F])/(2*e), E^(2*(d + e*x))]))/(b*c*Log[F])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 186 vs. $2(72) = 144$.

Time = 0.63 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.58, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(d+ex)\operatorname{sech}(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6037}$$

$$\int \left(\frac{e^{2d+2ex}F^{ac+bcx}}{e^{d+ex}-1}-\frac{e^{2d+2ex}F^{ac+bcx}}{e^{d+ex}+1}-\frac{2e^{2d+2ex}F^{ac+bcx}}{e^{2d+2ex}+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2F^{ac+bcx}\text{Hypergeometric2F1}\left(1,-\frac{bc\log(F)}{2e},1-\frac{bc\log(F)}{2e},-e^{-2(d+ex)}\right)}{bc\log(F)}-\frac{e^{2d+2ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{e}+2,\frac{bc\log(F)}{e}+3,-e^{d+ex}\right)}{bc\log(F)+2e}-\frac{e^{2d+2ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{e}+2,\frac{bc\log(F)}{e}+3,e^{d+ex}\right)}{bc\log(F)+2e}$$

input
```
Int[F^(c*(a + b*x))*Csch[d + e*x]*Sech[d + e*x],x]
```

output
```
(-2*F^(a*c + b*c*x)*Hypergeometric2F1[1, -1/2*(b*c*Log[F])/e, 1 - (b*c*Log
[F])/(2*e), -E^(-2*(d + e*x))])/(b*c*Log[F]) - (E^(2*d + 2*e*x)*F^(a*c + b
*c*x)*Hypergeometric2F1[1, 2 + (b*c*Log[F])/e, 3 + (b*c*Log[F])/e, -E^(d +
 e*x)])/(2*e + b*c*Log[F]) - (E^(2*d + 2*e*x)*F^(a*c + b*c*x)*Hypergeometr
ic2F1[1, 2 + (b*c*Log[F])/e, 3 + (b*c*Log[F])/e, E^(d + e*x)])/(2*e + b*c*
Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Maple [F]

$$\int F^{c(bx+a)} \operatorname{csch}(ex+d) \operatorname{sech}(ex+d)\, dx$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)*sech(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*csch(e*x+d)*sech(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\text{csch}(d+ex)\text{sech}(d+ex)\, dx = \int F^{(bx+a)c} \operatorname{csch}(ex+d) \operatorname{sech}(ex+d)\, dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)*sech(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csch(e*x + d)*sech(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\text{csch}(d+ex)\text{sech}(d+ex)\,dx=\int F^{c(a+bx)}\,\text{csch}\,(d+ex)\,\text{sech}\,(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*csch(e*x+d)*sech(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*csch(d + e*x)*sech(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\text{csch}(d+ex)\text{sech}(d+ex)\,dx=\int F^{(bx+a)c}\,\text{csch}\,(ex+d)\,\text{sech}\,(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)*sech(e*x+d),x, algorithm="maxima")
```

output
```
16*F^(a*c)*e*integrate(e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + (b*c*e
^(8*d)*log(F) - 2*e*e^(8*d))*e^(8*e*x) - 2*(b*c*e^(4*d)*log(F) - 2*e*e^(4*
d))*e^(4*e*x) - 2*e), x) - 4*F^(a*c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*l
og(F) - (b*c*e^(4*d)*log(F) - 2*e*e^(4*d))*e^(4*e*x) - 2*e)
```

Giac [F]

$$\int F^{c(a+bx)}\text{csch}(d+ex)\text{sech}(d+ex)\,dx=\int F^{(bx+a)c}\,\text{csch}\,(ex+d)\,\text{sech}\,(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)*sech(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csch(e*x + d)*sech(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{csch}(d+ex)\text{sech}(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\cosh\left(d+e\,x\right)\,\sinh\left(d+e\,x\right)}\,dx$$

input
```
int(F^(c*(a + b*x))/(cosh(d + e*x)*sinh(d + e*x)),x)
```

output
```
int(F^(c*(a + b*x))/(cosh(d + e*x)*sinh(d + e*x)), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\text{csch}(d+ex)\text{sech}(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\text{csch}(ex+d)\,\text{sech}(ex+d)\,dx\right)$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)*sech(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*csch(d + e*x)*sech(d + e*x),x)
```

3.121 $\int F^{c(a+bx)}\mathrm{csch}^2(d+ex)\mathrm{sech}(d+ex)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 194

$$\int F^{c(a+bx)}\mathrm{csch}^2(d+ex)\mathrm{sech}(d+ex)\,dx=\frac{2e^{3d+3ex}F^{c(a+bx)}}{e\left(1-e^{2d+2ex}\right)}+\frac{2e^{3d+3ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(5+\frac{bc\log(F)}{e}\right),-e^{2d+2ex}\right)}{3e+bc\log(F)}-\frac{2bce^{3d+3ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(5+\frac{bc\log(F)}{e}\right),e^{2d+2ex}\right)\log(F)}{e(3e+bc\log(F))}$$

output
```
2*exp(3*e*x+3*d)*F^(c*(b*x+a))/e/(1-exp(2*e*x+2*d))+2*exp(3*e*x+3*d)*F^(c*
(b*x+a))*hypergeom([1, 3/2+1/2*b*c*ln(F)/e],[5/2+1/2*b*c*ln(F)/e],-exp(2*e
*x+2*d))/(b*c*ln(F)+3*e)-2*b*c*exp(3*e*x+3*d)*F^(c*(b*x+a))*hypergeom([1,
3/2+1/2*b*c*ln(F)/e],[5/2+1/2*b*c*ln(F)/e],exp(2*e*x+2*d))*ln(F)/e/(b*c*ln
(F)+3*e)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.93

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\operatorname{sech}(d+ex)\,dx =$$

$$-\frac{F^{c\left(a-\frac{bd}{e}\right)}\left(2ee^{\frac{(d+ex)(e+bc\log(F))}{e}}\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),-e^{2(d+ex)}\right)+2bce^{\frac{(d+ex)}{}}\cdots\right)}{\cdots}$$

input
```
Integrate[F^(c*(a + b*x))*Csch[d + e*x]^2*Sech[d + e*x],x]
```

output
```
-((F^(c*(a - (b*d)/e))*(2*e*E^(((d + e*x)*(e + b*c*Log[F]))/e)*Hypergeomet
ric2F1[1, (e + b*c*Log[F])/(2*e), (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))
] + 2*b*c*E^(((d + e*x)*(e + b*c*Log[F]))/e)*Hypergeometric2F1[1, (e + b*c
*Log[F])/(2*e), (3 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))]*Log[F] + F^((b*c*
(d + e*x))/e)*Csch[d + e*x]*(e + b*c*Log[F])))/(e*(e + b*c*Log[F])))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.75, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(d+ex)\operatorname{sech}(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6037}$$

$$\int \left(\frac{4e^{3d+3ex}F^{ac+bcx}}{\left(e^{2(d+ex)}-1\right)^2}-\frac{4e^{3d+3ex}F^{ac+bcx}}{e^{4(d+ex)}-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4e^{3d+3ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}\left(\frac{bc\log(F)}{e}+3\right),\frac{1}{4}\left(\frac{bc\log(F)}{e}+7\right),e^{4(d+ex)}\right)}{bc\log(F)+3e}+\frac{4e^{3d+3ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),\frac{1}{2}\left(\frac{bc\log(F)}{e}+5\right),e^{2(d+ex)}\right)}{bc\log(F)+3e}$$

input
```
Int[F^(c*(a + b*x))*Csch[d + e*x]^2*Sech[d + e*x],x]
```

output
```
(4*E^(3*d + 3*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[1, (3 + (b*c*Log[F])/
e)/4, (7 + (b*c*Log[F])/e)/4, E^(4*(d + e*x))])/(3*e + b*c*Log[F]) + (4*E^
(3*d + 3*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[2, (3 + (b*c*Log[F])/e)/2,
 (5 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))])/(3*e + b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Maple [F]

$$\int F^{c(bx+a)}\operatorname{csch}(ex+d)^2\operatorname{sech}(ex+d)\,dx$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)^2*sech(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*csch(e*x+d)^2*sech(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\text{csch}^2(d+ex)\text{sech}(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{csch}(ex+d)^2\operatorname{sech}(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^2*sech(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csch(e*x + d)^2*sech(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\text{csch}^2(d+ex)\text{sech}(d+ex)\,dx=\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\operatorname{sech}(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*csch(e*x+d)**2*sech(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*csch(d + e*x)**2*sech(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\text{csch}^2(d+ex)\text{sech}(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{csch}(ex+d)^2\operatorname{sech}(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^2*sech(e*x+d),x, algorithm="maxima")
```

output
```
8*(2*F^(a*c)*e*e^(e*x + d) + (F^(a*c)*b*c*e^(3*d)*log(F) - 5*F^(a*c)*e*e^(
3*d))*e^(3*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*log(F) + 15*e^2 + (
b^2*c^2*e^(6*d)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e^2*e^(6*d))*e^(6*e
*x) - (b^2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(F) + 15*e^2*e^(4*d))
*e^(4*e*x) - (b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(2*d)*log(F) + 15*e^2*e
^(2*d))*e^(2*e*x)) + 8*integrate(-2*((3*F^(a*c)*b*c*e*e^(3*d)*log(F) - 13*
F^(a*c)*e^2*e^(3*d))*e^(3*e*x) + (F^(a*c)*b*c*e*e^d*log(F) + F^(a*c)*e^2*e
^d)*e^(e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*log(F) + 15*e^2 - (b^2*
c^2*e^(10*d)*log(F)^2 - 8*b*c*e*e^(10*d)*log(F) + 15*e^2*e^(10*d))*e^(10*e
*x) + (b^2*c^2*e^(8*d)*log(F)^2 - 8*b*c*e*e^(8*d)*log(F) + 15*e^2*e^(8*d))
*e^(8*e*x) + 2*(b^2*c^2*e^(6*d)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e^2
*e^(6*d))*e^(6*e*x) - 2*(b^2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(F)
 + 15*e^2*e^(4*d))*e^(4*e*x) - (b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(2*d)
*log(F) + 15*e^2*e^(2*d))*e^(2*e*x)), x)
```

Giac [F]

$$\int F^{c(a+bx)}\text{csch}^2(d+ex)\text{sech}(d+ex)\,dx=\int F^{(bx+a)c}\,\text{csch}\,(ex+d)^2\,\text{sech}\,(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^2*sech(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csch(e*x + d)^2*sech(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{csch}^2(d+ex)\text{sech}(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\cosh\left(d+e\,x\right)\,\sinh\left(d+e\,x\right)^2}\,dx$$

input
```
int(F^(c*(a + b*x))/(cosh(d + e*x)*sinh(d + e*x)^2),x)
```

output
```
int(F^(c*(a + b*x))/(cosh(d + e*x)*sinh(d + e*x)^2), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\operatorname{sech}(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\operatorname{csch}(ex+d)^2\operatorname{sech}(ex+d)\,dx\right)$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)^2*sech(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*csch(d + e*x)**2*sech(d + e*x),x)
```

3.122 $\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\operatorname{sech}(d+ex)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 253

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\operatorname{sech}(d+ex)\,dx$$
$$= -\frac{2e^{4d+4ex}F^{c(a+bx)}}{e\left(1-e^{2d+2ex}\right)^2} - \frac{e^{4d+4ex}F^{c(a+bx)}(2e-bc\log(F))}{e^2\left(1-e^{2d+2ex}\right)}$$
$$- \frac{2e^{4d+4ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(4+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(6+\frac{bc\log(F)}{e}\right),-e^{2d+2ex}\right)}{4e+bc\log(F)}$$
$$+ \frac{e^{4d+4ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(4+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(6+\frac{bc\log(F)}{e}\right),e^{2d+2ex}\right)\left(2-\frac{b^2c^2\log^2(F)}{e^2}\right)}{4e+bc\log(F)}$$

output
```
-2*exp(4*e*x+4*d)*F^(c*(b*x+a))/e/(1-exp(2*e*x+2*d))^2-exp(4*e*x+4*d)*F^(c
*(b*x+a))*(2*e-b*c*ln(F))/e^2/(1-exp(2*e*x+2*d))-2*exp(4*e*x+4*d)*F^(c*(b*
x+a))*hypergeom([1, 2+1/2*b*c*ln(F)/e],[3+1/2*b*c*ln(F)/e],-exp(2*e*x+2*d)
)/(b*c*ln(F)+4*e)+exp(4*e*x+4*d)*F^(c*(b*x+a))*hypergeom([1, 2+1/2*b*c*ln(
F)/e],[3+1/2*b*c*ln(F)/e],exp(2*e*x+2*d))*(2-b^2*c^2*ln(F)^2/e^2)/(b*c*ln(
F)+4*e)
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.60

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\operatorname{sech}(d+ex)\,dx =$$
$$-\frac{F^{c(a+bx)}\left(4e^2\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)+bc\log(F)\left(e\operatorname{csch}^2(d+ex)+bc\right.}{2}$$

input
```
Integrate[F^(c*(a + b*x))*Csch[d + e*x]^3*Sech[d + e*x],x]
```

output
```
-1/2*(F^(c*(a + b*x))*(4*e^2*Hypergeometric2F1[1, (b*c*Log[F])/(2*e), 1 +
(b*c*Log[F])/(2*e), -E^(2*(d + e*x))] + b*c*Log[F]*(e*Csch[d + e*x]^2 + b*
c*(-1 + Coth[d + e*x])*Log[F]) + Hypergeometric2F1[1, (b*c*Log[F])/(2*e),
1 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))]*(-4*e^2 + 2*b^2*c^2*Log[F]^2)))/(
b*c*e^2*Log[F])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.74, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(d+ex)\operatorname{sech}(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6037}$$

$$\int \left(\frac{4e^{4d+4ex}F^{ac+bcx}}{e^{4(d+ex)}-1}-\frac{4e^{4d+4ex}F^{ac+bcx}}{\left(e^{2(d+ex)}-1\right)^2}+\frac{8e^{4d+4ex}F^{ac+bcx}}{\left(e^{2(d+ex)}-1\right)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4F^{ac+bcx}\,\text{Hypergeometric2F1}\left(1,-\frac{bc\log(F)}{4e},1-\frac{bc\log(F)}{4e},e^{-4(d+ex)}\right)}{bc\log(F)}-\frac{4F^{ac+bcx}\,\text{Hypergeometric2F1}\left(2,-\frac{bc\log(F)}{2e},1-\frac{bc\log(F)}{2e},e^{-2(d+ex)}\right)}{bc\log(F)}-\frac{8e^{4d+4ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(\frac{bc\log(F)}{e}+4\right),\frac{1}{2}\left(\frac{bc\log(F)}{e}+6\right),e^{2(d+ex)}\right)}{bc\log(F)+4e}$$

input
```
Int[F^(c*(a + b*x))*Csch[d + e*x]^3*Sech[d + e*x],x]
```

output
```
(4*F^(a*c + b*c*x)*Hypergeometric2F1[1, -1/4*(b*c*Log[F])/e, 1 - (b*c*Log[
F])/(4*e), E^(-4*(d + e*x))])/(b*c*Log[F]) - (4*F^(a*c + b*c*x)*Hypergeome
tric2F1[2, -1/2*(b*c*Log[F])/e, 1 - (b*c*Log[F])/(2*e), E^(-2*(d + e*x))])
/(b*c*Log[F]) - (8*E^(4*d + 4*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[3, (4
 + (b*c*Log[F])/e)/2, (6 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))])/(4*e + b*c
*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Maple [F]

$$\int F^{c(bx+a)}\operatorname{csch}(ex+d)^3\operatorname{sech}(ex+d)\,dx$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)^3*sech(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*csch(e*x+d)^3*sech(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\text{csch}^3(d+ex)\text{sech}(d+ex)\,dx=\int F^{(bx+a)c}\,\text{csch}\,(ex+d)^3\,\text{sech}\,(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^3*sech(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csch(e*x + d)^3*sech(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\text{csch}^3(d+ex)\text{sech}(d+ex)\,dx=\int F^{c(a+bx)}\,\text{csch}^3\,(d+ex)\,\text{sech}\,(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*csch(e*x+d)**3*sech(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*csch(d + e*x)**3*sech(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\text{csch}^3(d+ex)\text{sech}(d+ex)\,dx=\int F^{(bx+a)c}\,\text{csch}\,(ex+d)^3\,\text{sech}\,(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^3*sech(e*x+d),x, algorithm="maxima")
```

output
```
-16*(8*F^(a*c)*b*c*e*log(F) - 32*F^(a*c)*e^2 + (F^(a*c)*b^2*c^2*e^(4*d)*lo
g(F)^2 - 14*F^(a*c)*b*c*e*e^(4*d)*log(F) + 48*F^(a*c)*e^2*e^(4*d))*e^(4*e*
x) + 4*(F^(a*c)*b*c*e*e^(2*d)*log(F) - 8*F^(a*c)*e^2*e^(2*d))*e^(2*e*x))*F
^(b*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2*e*log(F)^2 + 104*b*c*e^2*log(F) -
192*e^3 - (b^3*c^3*e^(8*d)*log(F)^3 - 18*b^2*c^2*e*e^(8*d)*log(F)^2 + 104*
b*c*e^2*e^(8*d)*log(F) - 192*e^3*e^(8*d))*e^(8*e*x) + 2*(b^3*c^3*e^(6*d)*l
og(F)^3 - 18*b^2*c^2*e*e^(6*d)*log(F)^2 + 104*b*c*e^2*e^(6*d)*log(F) - 192
*e^3*e^(6*d))*e^(6*e*x) - 2*(b^3*c^3*e^(2*d)*log(F)^3 - 18*b^2*c^2*e*e^(2*
d)*log(F)^2 + 104*b*c*e^2*e^(2*d)*log(F) - 192*e^3*e^(2*d))*e^(2*e*x)) + 1
6*integrate(4*(2*F^(a*c)*b^2*c^2*e*log(F)^2 - 8*F^(a*c)*b*c*e^2*log(F) + (
F^(a*c)*b^2*c^2*e*e^(2*d)*log(F)^2 + 2*F^(a*c)*b*c*e^2*e^(2*d)*log(F) - 48
*F^(a*c)*e^3*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2*
e*log(F)^2 + 104*b*c*e^2*log(F) - 192*e^3 + (b^3*c^3*e^(12*d)*log(F)^3 - 1
8*b^2*c^2*e*e^(12*d)*log(F)^2 + 104*b*c*e^2*e^(12*d)*log(F) - 192*e^3*e^(1
2*d))*e^(12*e*x) - 2*(b^3*c^3*e^(10*d)*log(F)^3 - 18*b^2*c^2*e*e^(10*d)*lo
g(F)^2 + 104*b*c*e^2*e^(10*d)*log(F) - 192*e^3*e^(10*d))*e^(10*e*x) - (b^3
*c^3*e^(8*d)*log(F)^3 - 18*b^2*c^2*e*e^(8*d)*log(F)^2 + 104*b*c*e^2*e^(8*d
)*log(F) - 192*e^3*e^(8*d))*e^(8*e*x) + 4*(b^3*c^3*e^(6*d)*log(F)^3 - 18*b
^2*c^2*e*e^(6*d)*log(F)^2 + 104*b*c*e^2*e^(6*d)*log(F) - 192*e^3*e^(6*d))*
e^(6*e*x) - (b^3*c^3*e^(4*d)*log(F)^3 - 18*b^2*c^2*e*e^(4*d)*log(F)^2 +...
```

Giac [F]

$$\int F^{c(a+bx)}\text{csch}^3(d+ex)\text{sech}(d+ex)\,dx=\int F^{(bx+a)c}\,\text{csch}\,(ex+d)^3\,\text{sech}\,(ex+d)\,\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^3*sech(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csch(e*x + d)^3*sech(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\operatorname{sech}(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\cosh\left(d+e\,x\right)\,\sinh\left(d+e\,x\right)^3}\,dx$$

input
```
int(F^(c*(a + b*x))/(cosh(d + e*x)*sinh(d + e*x)^3),x)
```

output
```
int(F^(c*(a + b*x))/(cosh(d + e*x)*sinh(d + e*x)^3), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\operatorname{sech}(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\operatorname{csch}(ex+d)^3\operatorname{sech}(ex+d)\,dx\right)$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)^3*sech(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*csch(d + e*x)**3*sech(d + e*x),x)
```

3.123 $\int F^{c(a+bx)}\sinh(d+ex)\tanh^2(d+ex)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 181

$$\int F^{c(a+bx)}\sinh(d+ex)\tanh^2(d+ex)\,dx=-\frac{2e^{-d-ex}F^{c(a+bx)}}{e\left(1+e^{2d+2ex}\right)}+\frac{5e^{-d-ex}F^{c(a+bx)}}{2(e-bc\log(F))}-\frac{2bce^{-d-ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(-1+\frac{bc\log(F)}{e}\right),\frac{e+bc\log(F)}{2e},-e^{2d+2ex}\right)\log(F)}{e(e-bc\log(F))}+\frac{e^{d+ex}F^{c(a+bx)}}{2(e+bc\log(F))}$$

output
```
-2*exp(-e*x-d)*F^(c*(b*x+a))/e/(1+exp(2*e*x+2*d))+5*exp(-e*x-d)*F^(c*(b*x+
a))/(2*e-2*b*c*ln(F))-2*b*c*exp(-e*x-d)*F^(c*(b*x+a))*hypergeom([1, -1/2+1
/2*b*c*ln(F)/e],[1/2*(e+b*c*ln(F))/e],-exp(2*e*x+2*d))*ln(F)/e/(e-b*c*ln(F
))+exp(e*x+d)*F^(c*(b*x+a))/(2*e+2*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.02

$$\int F^{c(a+bx)}\sinh(d+ex)\tanh^2(d+ex)\,dx$$
$$= \frac{F^{c\left(a-\frac{bd}{e}\right)}\operatorname{sech}(d+ex)\left(-2bce^{\frac{(d+ex)(e+bc\log(F))}{e}}\cosh(d+ex)\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(}{e}\right.\right.\right.}{e(e-}$$

input
```
Integrate[F^(c*(a + b*x))*Sinh[d + e*x]*Tanh[d + e*x]^2,x]
```

output
```
(F^(c*(a - (b*d)/e))*Sech[d + e*x]*(-2*b*c*E^(((d + e*x)*(e + b*c*Log[F]))
/e)*Cosh[d + e*x]*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e), (3 + (b*c*L
og[F])/e)/2, -E^(2*(d + e*x))]*Log[F]*(e - b*c*Log[F]) + (F^((b*c*(d + e*x
))/e)*(2*e^2 + 2*e^2*Cosh[d + e*x]^2 - 2*b^2*c^2*Log[F]^2 - b*c*e*Log[F]*S
inh[2*(d + e*x)]))/2))/(e*(e - b*c*Log[F])*(e + b*c*Log[F]))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(d+ex)\tanh^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6037}$$

$$\int \left(-\frac{5}{2}e^{-d-ex}F^{ac+bcx}+\frac{1}{2}e^{2(d+ex)-d-ex}F^{ac+bcx}+\frac{6e^{-d-ex}F^{ac+bcx}}{e^{2(d+ex)}+1}-\frac{4e^{-d-ex}F^{ac+bcx}}{\left(e^{2(d+ex)}+1\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6e^{-d-ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(\frac{bc\log(F)}{e}-1\right),\frac{e+bc\log(F)}{2e},-e^{2(d+ex)}\right)}{e-bc\log(F)}+\frac{4e^{-d-ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(\frac{bc\log(F)}{e}-1\right),\frac{e+bc\log(F)}{2e},-e^{2(d+ex)}\right)}{e-bc\log(F)}+\frac{5F^{ac}e^{-x(e-bc\log(F))-d}}{2(e-bc\log(F))}+\frac{F^{ac}e^{x(bc\log(F)+e)+d}}{2(bc\log(F)+e)}$$

input
```
Int[F^(c*(a + b*x))*Sinh[d + e*x]*Tanh[d + e*x]^2,x]
```

output
```
(5*E^(-d - x*(e - b*c*Log[F]))*F^(a*c))/(2*(e - b*c*Log[F])) - (6*E^(-d -
e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[1, (-1 + (b*c*Log[F])/e)/2, (e + b*
c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e - b*c*Log[F]) + (4*E^(-d - e*x)*F^(
a*c + b*c*x)*Hypergeometric2F1[2, (-1 + (b*c*Log[F])/e)/2, (e + b*c*Log[F]
)/(2*e), -E^(2*(d + e*x))])/(e - b*c*Log[F]) + (E^(d + x*(e + b*c*Log[F]))
*F^(a*c))/(2*(e + b*c*Log[F]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Maple [F]

$$\int F^{c(bx+a)}\sinh(ex+d)\tanh(ex+d)^2\,dx$$

input
```
int(F^(c*(b*x+a))*sinh(e*x+d)*tanh(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*sinh(e*x+d)*tanh(e*x+d)^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \sinh(d+ex) \tanh^2(d+ex) \, dx = \int F^{(bx+a)c} \sinh(ex+d) \tanh(ex+d)^2 \, dx$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)*tanh(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sinh(e*x + d)*tanh(e*x + d)^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \sinh(d+ex) \tanh^2(d+ex) \, dx = \int F^{c(a+bx)} \sinh(d+ex) \tanh^2(d+ex) \, dx$$

input
```
integrate(F**(c*(b*x+a))*sinh(e*x+d)*tanh(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*sinh(d + e*x)*tanh(d + e*x)**2, x)
```

Maxima [F]

$$\int F^{c(a+bx)} \sinh(d+ex) \tanh^2(d+ex) \, dx = \int F^{(bx+a)c} \sinh(ex+d) \tanh(ex+d)^2 \, dx$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)*tanh(e*x+d)^2,x, algorithm="maxima")
```

output
```
16*F^(a*c)*b*c*e*integrate(F^(b*c*x)/((b^2*c^2*e^(7*d)*log(F)^2 - 8*b*c*e*
e^(7*d)*log(F) + 15*e^2*e^(7*d))*e^(7*e*x) + 3*(b^2*c^2*e^(5*d)*log(F)^2 -
 8*b*c*e*e^(5*d)*log(F) + 15*e^2*e^(5*d))*e^(5*e*x) + 3*(b^2*c^2*e^(3*d)*l
og(F)^2 - 8*b*c*e*e^(3*d)*log(F) + 15*e^2*e^(3*d))*e^(3*e*x) + (b^2*c^2*e^
d*log(F)^2 - 8*b*c*e*e^d*log(F) + 15*e^2*e^d)*e^(e*x)), x)*log(F) - 1/2*(F
^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b^2*c^2*e*log(F)^2 + 39*F^(a*c)*b*c*e
^2*log(F) + 15*F^(a*c)*e^3 - (F^(a*c)*b^3*c^3*e^(6*d)*log(F)^3 - 9*F^(a*c)
*b^2*c^2*e*e^(6*d)*log(F)^2 + 23*F^(a*c)*b*c*e^2*e^(6*d)*log(F) - 15*F^(a*
c)*e^3*e^(6*d))*e^(6*e*x) + (3*F^(a*c)*b^3*c^3*e^(4*d)*log(F)^3 - 17*F^(a*
c)*b^2*c^2*e*e^(4*d)*log(F)^2 - 11*F^(a*c)*b*c*e^2*e^(4*d)*log(F) + 105*F^
(a*c)*e^3*e^(4*d))*e^(4*e*x) - (3*F^(a*c)*b^3*c^3*e^(2*d)*log(F)^3 + F^(a*
c)*b^2*c^2*e*e^(2*d)*log(F)^2 - 59*F^(a*c)*b*c*e^2*e^(2*d)*log(F) - 105*F^
(a*c)*e^3*e^(2*d))*e^(2*e*x))*F^(b*c*x)/((b^4*c^4*e^(5*d)*log(F)^4 - 8*b^3
*c^3*e*e^(5*d)*log(F)^3 + 14*b^2*c^2*e^2*e^(5*d)*log(F)^2 + 8*b*c*e^3*e^(5
*d)*log(F) - 15*e^4*e^(5*d))*e^(5*e*x) + 2*(b^4*c^4*e^(3*d)*log(F)^4 - 8*b
^3*c^3*e*e^(3*d)*log(F)^3 + 14*b^2*c^2*e^2*e^(3*d)*log(F)^2 + 8*b*c*e^3*e^
(3*d)*log(F) - 15*e^4*e^(3*d))*e^(3*e*x) + (b^4*c^4*e^d*log(F)^4 - 8*b^3*c
^3*e*e^d*log(F)^3 + 14*b^2*c^2*e^2*e^d*log(F)^2 + 8*b*c*e^3*e^d*log(F) - 1
5*e^4*e^d)*e^(e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\sinh(d+ex)\tanh^2(d+ex)\,dx = \int F^{(bx+a)c}\sinh\left(ex+d\right)\tanh\left(ex+d\right)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)*tanh(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sinh(e*x + d)*tanh(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sinh(d+ex)\tanh^2(d+ex)\,dx=\int F^{c\,(a+b\,x)}\sinh(d+e\,x)\,\tanh(d+e\,x)^2\,dx$$

input
```
int(F^(c*(a + b*x))*sinh(d + e*x)*tanh(d + e*x)^2,x)
```

output
```
int(F^(c*(a + b*x))*sinh(d + e*x)*tanh(d + e*x)^2, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\sinh(d+ex)\tanh^2(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\sinh(ex+d)\tanh(ex+d)^2\,dx\right)$$

input
```
int(F^(c*(b*x+a))*sinh(e*x+d)*tanh(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sinh(d + e*x)*tanh(d + e*x)**2,x)
```

3.124 $\int F^{c(a+bx)} \tanh^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 101

$$\int F^{c(a+bx)} \tanh^2(d+ex)\,dx = \frac{2F^{c(a+bx)}}{e(1+e^{2d+2ex})} - \frac{2F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},-e^{2d+2ex}\right)}{e} + \frac{F^{c(a+bx)}}{bc\log(F)}$$

output
```
2*F^(c*(b*x+a))/e/(1+exp(2*e*x+2*d))-2*F^(c*(b*x+a))*hypergeom([1, 1/2*b*c
*ln(F)/e],[1+1/2*b*c*ln(F)/e],-exp(2*e*x+2*d))/e+F^(c*(b*x+a))/b/c/ln(F)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93

$$\int F^{c(a+bx)}\tanh^2(d+ex)\,dx$$
$$=F^{c(a+bx)}\left(\frac{2}{e+ee^{2d}}-\frac{2\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)}{e}+\frac{1}{bc\log(F)}-\frac{\text{sech}(d)\text{sech}(d+ex)\sinh(ex)}{e}\right)$$

input
```
Integrate[F^(c*(a + b*x))*Tanh[d + e*x]^2,x]
```

output
```
F^(c*(a + b*x))*(2/(e + e*E^(2*d)) - (2*Hypergeometric2F1[1, (b*c*Log[F])/
(2*e), 1 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/e + 1/(b*c*Log[F]) - (Se
ch[d]*Sech[d + e*x]*Sinh[e*x])/e)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.36, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {6007, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(d+ex)F^{c(a+bx)}\,dx$$
$$\Big\downarrow \text{6007}$$
$$\int \left(-\frac{4F^{c(a+bx)}}{e^{2(d+ex)}+1}+\frac{4F^{c(a+bx)}}{\left(e^{2(d+ex)}+1\right)^2}+F^{c(a+bx)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{4F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,-e^{2(d+ex)}\right)}{bc\log(F)}+\frac{4F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,-e^{2(d+ex)}\right)}{bc\log(F)}+\frac{F^{c(a+bx)}}{bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Tanh[d + e*x]^2,x]
```

output
```
F^(c*(a + b*x))/(b*c*Log[F]) - (4*F^(c*(a + b*x))*Hypergeometric2F1[1, (b*
c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(b*c*Log[F]) +
 (4*F^(c*(a + b*x))*Hypergeometric2F1[2, (b*c*Log[F])/(2*e), 1 + (b*c*Log[
F])/(2*e), -E^(2*(d + e*x))])/(b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6007
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tanh[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((-1 + E^(2*(d + e*x)))^n/(1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)}\tanh(ex+d)^2\,dx$$

input
```
int(F^(c*(b*x+a))*tanh(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*tanh(e*x+d)^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \tanh^2(d+ex)\,dx = \int F^{(bx+a)c} \tanh\left(ex+d\right)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*tanh(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*tanh(e*x + d)^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \tanh^2(d+ex)\,dx = \int F^{c(a+bx)} \tanh^2\left(d+ex\right)dx$$

input
```
integrate(F**(c*(b*x+a))*tanh(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*tanh(d + e*x)**2, x)
```

Maxima [F]

$$\int F^{c(a+bx)} \tanh^2(d+ex)\,dx = \int F^{(bx+a)c} \tanh\left(ex+d\right)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*tanh(e*x+d)^2,x, algorithm="maxima")
```

output
```
-16*F^(a*c)*b*c*e*integrate(F^(b*c*x)/(b^2*c^2*log(F)^2 - 6*b*c*e*log(F) +
 8*e^2 + (b^2*c^2*e^(6*d)*log(F)^2 - 6*b*c*e*e^(6*d)*log(F) + 8*e^2*e^(6*d
))*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*log(F) + 8*e^
2*e^(4*d))*e^(4*e*x) + 3*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^(2*d)*log(F
) + 8*e^2*e^(2*d))*e^(2*e*x)), x)*log(F) + (F^(a*c)*b^2*c^2*log(F)^2 + 10*
F^(a*c)*b*c*e*log(F) + 8*F^(a*c)*e^2 + (F^(a*c)*b^2*c^2*e^(4*d)*log(F)^2 -
 6*F^(a*c)*b*c*e*e^(4*d)*log(F) + 8*F^(a*c)*e^2*e^(4*d))*e^(4*e*x) - 2*(F^
(a*c)*b^2*c^2*e^(2*d)*log(F)^2 - 2*F^(a*c)*b*c*e*e^(2*d)*log(F) - 8*F^(a*c
)*e^2*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^3*c^3*log(F)^3 - 6*b^2*c^2*e*log(F)
^2 + 8*b*c*e^2*log(F) + (b^3*c^3*e^(4*d)*log(F)^3 - 6*b^2*c^2*e*e^(4*d)*lo
g(F)^2 + 8*b*c*e^2*e^(4*d)*log(F))*e^(4*e*x) + 2*(b^3*c^3*e^(2*d)*log(F)^3
 - 6*b^2*c^2*e*e^(2*d)*log(F)^2 + 8*b*c*e^2*e^(2*d)*log(F))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\tanh^2(d+ex)\,dx=\int F^{(bx+a)c}\tanh\left(ex+d\right)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*tanh(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*tanh(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\tanh^2(d+ex)\,dx=\int F^{c\,(a+b\,x)}\,\tanh(d+e\,x)^2\,dx$$

input
```
int(F^(c*(a + b*x))*tanh(d + e*x)^2,x)
```

output
```
int(F^(c*(a + b*x))*tanh(d + e*x)^2, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\tanh^2(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\tanh\left(ex+d\right)^2dx\right)$$

input
```
int(F^(c*(b*x+a))*tanh(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*tanh(d + e*x)**2,x)
```

3.125 $\int F^{c(a+bx)}\text{sech}(d+ex)\tanh(d+ex)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 112

$$\int F^{c(a+bx)}\text{sech}(d+ex)\tanh(d+ex)\,dx=-\frac{2e^{d+ex}F^{c(a+bx)}}{e\left(1+e^{2d+2ex}\right)}+\frac{2bce^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),-e^{2d+2ex}\right)\log(F)}{e(e+bc\log(F))}$$

output
```
-2*exp(e*x+d)*F^(c*(b*x+a))/e/(1+exp(2*e*x+2*d))+2*b*c*exp(e*x+d)*F^(c*(b*
x+a))*hypergeom([1, 1/2*(e+b*c*ln(F))/e],[3/2+1/2*b*c*ln(F)/e],-exp(2*e*x+
2*d))*ln(F)/e/(e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.82

$$\int F^{c(a+bx)}\text{sech}(d+ex)\tanh(d+ex)\,dx=\frac{F^{c(a+bx)}\left(2bce^{d+ex}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),-e^{2(d+ex)}\right)\log(F)-(e+bc\log(F\right.}{e(e+bc\log(F))}$$

input
```
Integrate[F^(c*(a + b*x))*Sech[d + e*x]*Tanh[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*(2*b*c*E^(d + e*x)*Hypergeometric2F1[1, (e + b*c*Log[F])/
(2*e), (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))]*Log[F] - (e + b*c*Log[F])
*Sech[d + e*x]))/(e*(e + b*c*Log[F]))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(d+ex)\text{sech}(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6037}$$

$$\int \left(\frac{2e^{3d+3ex}F^{ac+bcx}}{\left(e^{2d+2ex}+1\right)^2}-\frac{2e^{d+ex}F^{ac+bcx}}{\left(e^{2d+2ex}+1\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2e^{3d+3ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),\frac{1}{2}\left(\frac{bc\log(F)}{e}+5\right),-e^{2(d+ex)}\right)}{bc\log(F)+3e}-\frac{2e^{d+ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(2,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),-e^{2(d+ex)}\right)}{bc\log(F)+e}$$

input
```
Int[F^(c*(a + b*x))*Sech[d + e*x]*Tanh[d + e*x],x]
```

output
```
(-2*E^(d + e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[2, (e + b*c*Log[F])/(2*e
), (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/(e + b*c*Log[F]) + (2*E^(3*d
 + 3*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[2, (3 + (b*c*Log[F])/e)/2, (5
+ (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/(3*e + b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Maple [F]

$$\int F^{c(bx+a)}\operatorname{sech}(ex+d)\tanh(ex+d)\,dx$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)*tanh(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*sech(e*x+d)*tanh(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\text{sech}(d+ex)\tanh(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{sech}(ex+d)\tanh(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)*tanh(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sech(e*x + d)*tanh(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\operatorname{sech}(d+ex)\tanh(d+ex)\,dx=\int F^{c(a+bx)}\tanh(d+ex)\operatorname{sech}(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*sech(e*x+d)*tanh(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*tanh(d + e*x)*sech(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\operatorname{sech}(d+ex)\tanh(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{sech}(ex+d)\tanh(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)*tanh(e*x+d),x, algorithm="maxima")
```

output
```
16*F^(a*c)*b*c*e*integrate(e^(b*c*x*log(F) + e*x + d)/(b^2*c^2*log(F)^2 -
4*b*c*e*log(F) + 3*e^2 + (b^2*c^2*e^(6*d)*log(F)^2 - 4*b*c*e*e^(6*d)*log(F
) + 3*e^2*e^(6*d))*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 4*b*c*e*e^(4*
d)*log(F) + 3*e^2*e^(4*d))*e^(4*e*x) + 3*(b^2*c^2*e^(2*d)*log(F)^2 - 4*b*c
*e*e^(2*d)*log(F) + 3*e^2*e^(2*d))*e^(2*e*x)), x)*log(F) + 2*((F^(a*c)*b*c
*e^(3*d)*log(F) - 3*F^(a*c)*e*e^(3*d))*e^(3*e*x) - (F^(a*c)*b*c*e^d*log(F)
 + 3*F^(a*c)*e*e^d)*e^(e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 4*b*c*e*log(F)
+ 3*e^2 + (b^2*c^2*e^(4*d)*log(F)^2 - 4*b*c*e*e^(4*d)*log(F) + 3*e^2*e^(4*
d))*e^(4*e*x) + 2*(b^2*c^2*e^(2*d)*log(F)^2 - 4*b*c*e*e^(2*d)*log(F) + 3*e
^2*e^(2*d))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\text{sech}(d+ex)\tanh(d+ex)\,dx=\int F^{(bx+a)c}\,\text{sech}\,(ex+d)\tanh\,(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)*tanh(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sech(e*x + d)*tanh(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{sech}(d+ex)\tanh(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}\tanh(d+e\,x)}{\cosh\,(d+e\,x)}\,dx$$

input
```
int((F^(c*(a + b*x))*tanh(d + e*x))/cosh(d + e*x),x)
```

output
```
int((F^(c*(a + b*x))*tanh(d + e*x))/cosh(d + e*x), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\text{sech}(d+ex)\tanh(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\text{sech}(ex+d)\tanh\,(ex+d)\,dx\right)$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)*tanh(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sech(d + e*x)*tanh(d + e*x),x)
```

3.126 $\int F^{c(a+bx)}\text{sech}^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 74

$$\int F^{c(a+bx)}\text{sech}^2(d+ex)\,dx = \frac{4e^{2d+2ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(2+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(4+\frac{bc\log(F)}{e}\right),-e^{2d+2ex}\right)}{2e+bc\log(F)}$$

output
```
4*exp(2*e*x+2*d)*F^(c*(b*x+a))*hypergeom([2, 1+1/2*b*c*ln(F)/e],[2+1/2*b*c
*ln(F)/e],-exp(2*e*x+2*d))/(2*e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int F^{c(a+bx)}\text{sech}^2(d+ex)\,dx = \frac{4e^{2(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{2e},2+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)}{2e+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sech[d + e*x]^2,x]
```

output
```
(4*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(
2*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(2*e + b*c*Log[F])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = $\{6015\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6015}$$

$$\frac{4e^{2(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{2e}+1,\frac{bc\log(F)}{2e}+2,-e^{2(d+ex)}\right)}{bc\log(F)+2e}$$

input
```
Int[F^(c*(a + b*x))*Sech[d + e*x]^2,x]
```

output

```
(4*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(
2*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(2*e + b*c*Log[F])
```


Defintions of rubi rules used

rule 6015
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)}\operatorname{sech}(ex+d)^{2}dx$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*sech(e*x+d)^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{sech}(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sech(e*x + d)^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx=\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*sech(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*sech(d + e*x)**2, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\text{sech}^2(d+ex)\,dx = \int F^{(bx+a)c}\,\text{sech}\,(ex+d)^2\,\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^2,x, algorithm="maxima")
```

output
```
16*F^(a*c)*b*c*e*integrate(F^(b*c*x)/(b^2*c^2*log(F)^2 - 6*b*c*e*log(F) +
8*e^2 + (b^2*c^2*e^(6*d)*log(F)^2 - 6*b*c*e*e^(6*d)*log(F) + 8*e^2*e^(6*d)
)*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*log(F) + 8*e^2
*e^(4*d))*e^(4*e*x) + 3*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^(2*d)*log(F)
 + 8*e^2*e^(2*d))*e^(2*e*x)), x)*log(F) - 4*(4*F^(a*c)*e - (F^(a*c)*b*c*e^
(2*d)*log(F) - 4*F^(a*c)*e*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2
 - 6*b*c*e*log(F) + 8*e^2 + (b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*lo
g(F) + 8*e^2*e^(4*d))*e^(4*e*x) + 2*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^
(2*d)*log(F) + 8*e^2*e^(2*d))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\text{sech}^2(d+ex)\,dx = \int F^{(bx+a)c}\,\text{sech}\,(ex+d)^2\,\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sech(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{sech}^2(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{{\cosh\left(d+e\,x\right)}^2}\,dx$$

input
```
int(F^(c*(a + b*x))/cosh(d + e*x)^2,x)
```

output
```
int(F^(c*(a + b*x))/cosh(d + e*x)^2, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\text{sech}^2(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\text{sech}(ex+d)^2\,dx\right)$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sech(d + e*x)**2,x)
```

3.127 $\int F^{c(a+bx)}\mathrm{csch}(d+ex)\mathrm{sech}^2(d+ex)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 192

$$\int F^{c(a+bx)}\mathrm{csch}(d+ex)\mathrm{sech}^2(d+ex)\,dx = -\frac{2e^{3d+3ex}F^{c(a+bx)}}{e\left(1+e^{2d+2ex}\right)} - \frac{2e^{3d+3ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(5+\frac{bc\log(F)}{e}\right),e^{2d+2ex}\right)}{3e+bc\log(F)} + \frac{2bce^{3d+3ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(5+\frac{bc\log(F)}{e}\right),-e^{2d+2ex}\right)\log(F)}{e(3e+bc\log(F))}$$

output

```
-2*exp(3*e*x+3*d)*F^(c*(b*x+a))/e/(1+exp(2*e*x+2*d))-2*exp(3*e*x+3*d)*F^(c
*(b*x+a))*hypergeom([1, 3/2+1/2*b*c*ln(F)/e],[5/2+1/2*b*c*ln(F)/e],exp(2*e
*x+2*d))/(b*c*ln(F)+3*e)+2*b*c*exp(3*e*x+3*d)*F^(c*(b*x+a))*hypergeom([1,
3/2+1/2*b*c*ln(F)/e],[5/2+1/2*b*c*ln(F)/e],-exp(2*e*x+2*d))*ln(F)/e/(b*c*l
n(F)+3*e)
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.93

$$\int F^{c(a+bx)}\text{csch}(d+ex)\text{sech}^2(d+ex)\,dx$$
$$=\frac{F^{c\left(a-\frac{bd}{e}\right)}\left(-2ee^{\frac{(d+ex)(e+bc\log(F))}{e}}\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),e^{2(d+ex)}\right)-2bce^{\frac{(d+ex)(e}{}}\right.}{e}$$

input
```
Integrate[F^(c*(a + b*x))*Csch[d + e*x]*Sech[d + e*x]^2,x]
```

output
```
(F^(c*(a - (b*d)/e))*(-2*e*E^(((d + e*x)*(e + b*c*Log[F]))/e)*Hypergeometr
ic2F1[1, (e + b*c*Log[F])/(2*e), (3 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))]
- 2*b*c*E^(((d + e*x)*(e + b*c*Log[F]))/e)*Hypergeometric2F1[1, (e + b*c*L
og[F])/(2*e), (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))]*Log[F] + F^((b*c*(
d + e*x))/e)*(e + b*c*Log[F])*Sech[d + e*x]))/(e*(e + b*c*Log[F]))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.77, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}(d+ex)\text{sech}^2(d+ex)F^{c(a+bx)}\,dx$$
$$\Big\downarrow \text{6037}$$
$$\int \left(\frac{4e^{3d+3ex}F^{ac+bcx}}{e^{4(d+ex)}-1}-\frac{4e^{3d+3ex}F^{ac+bcx}}{\left(e^{2(d+ex)}+1\right)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{4e^{3d+3ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}\left(\frac{bc\log(F)}{e}+3\right),\frac{1}{4}\left(\frac{bc\log(F)}{e}+7\right),e^{4(d+ex)}\right)}{bc\log(F)+3e}-\frac{4e^{3d+3ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),\frac{1}{2}\left(\frac{bc\log(F)}{e}+5\right),-e^{2(d+ex)}\right)}{bc\log(F)+3e}$$

input
```
Int[F^(c*(a + b*x))*Csch[d + e*x]*Sech[d + e*x]^2,x]
```

output
```
(-4*E^(3*d + 3*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[1, (3 + (b*c*Log[F])
/e)/4, (7 + (b*c*Log[F])/e)/4, E^(4*(d + e*x))])/(3*e + b*c*Log[F]) - (4*E
^(3*d + 3*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[2, (3 + (b*c*Log[F])/e)/2
, (5 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/(3*e + b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Maple [F]

$$\int F^{c(bx+a)}\operatorname{csch}(ex+d)\operatorname{sech}(ex+d)^2\,dx$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)*sech(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*csch(e*x+d)*sech(e*x+d)^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\text{csch}(d+ex)\text{sech}^2(d+ex)\,dx=\int F^{(bx+a)c}\,\text{csch}\,(ex+d)\,\text{sech}\,(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)*sech(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csch(e*x + d)*sech(e*x + d)^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\text{csch}(d+ex)\text{sech}^2(d+ex)\,dx=\int F^{c(a+bx)}\,\text{csch}\,(d+ex)\,\text{sech}^2\,(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*csch(e*x+d)*sech(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*csch(d + e*x)*sech(d + e*x)**2, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\text{csch}(d+ex)\text{sech}^2(d+ex)\,dx=\int F^{(bx+a)c}\,\text{csch}\,(ex+d)\,\text{sech}\,(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)*sech(e*x+d)^2,x, algorithm="maxima")
```

output
```
8*(2*F^(a*c)*e*e^(e*x + d) - (F^(a*c)*b*c*e^(3*d)*log(F) - 5*F^(a*c)*e*e^(
3*d))*e^(3*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*log(F) + 15*e^2 - (
b^2*c^2*e^(6*d)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e^2*e^(6*d))*e^(6*e
*x) - (b^2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(F) + 15*e^2*e^(4*d))
*e^(4*e*x) + (b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(2*d)*log(F) + 15*e^2*e
^(2*d))*e^(2*e*x)) + 8*integrate(2*((3*F^(a*c)*b*c*e*e^(3*d)*log(F) - 13*F
^(a*c)*e^2*e^(3*d))*e^(3*e*x) - (F^(a*c)*b*c*e*e^d*log(F) + F^(a*c)*e^2*e^
d)*e^(e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*log(F) + 15*e^2 + (b^2*c
^2*e^(10*d)*log(F)^2 - 8*b*c*e*e^(10*d)*log(F) + 15*e^2*e^(10*d))*e^(10*e*
x) + (b^2*c^2*e^(8*d)*log(F)^2 - 8*b*c*e*e^(8*d)*log(F) + 15*e^2*e^(8*d))*
e^(8*e*x) - 2*(b^2*c^2*e^(6*d)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e^2*
e^(6*d))*e^(6*e*x) - 2*(b^2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(F)
+ 15*e^2*e^(4*d))*e^(4*e*x) + (b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(2*d)*
log(F) + 15*e^2*e^(2*d))*e^(2*e*x)), x)
```

Giac [F]

$$\int F^{c(a+bx)}\text{csch}(d+ex)\text{sech}^2(d+ex)\,dx=\int F^{(bx+a)c}\,\text{csch}\,(ex+d)\,\text{sech}\,(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)*sech(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csch(e*x + d)*sech(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{csch}(d+ex)\text{sech}^2(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\cosh\,(d+e\,x)^2\,\sinh\,(d+e\,x)}\,dx$$

input
```
int(F^(c*(a + b*x))/(cosh(d + e*x)^2*sinh(d + e*x)),x)
```

output
```
int(F^(c*(a + b*x))/(cosh(d + e*x)^2*sinh(d + e*x)), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\text{csch}(d+ex)\text{sech}^2(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\text{csch}(ex+d)\,\text{sech}(ex+d)^2\,dx\right)$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)*sech(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*csch(d + e*x)*sech(d + e*x)**2,x)
```

3.128 $\int F^{c(a+bx)}\text{csch}^2(d+ex)\text{sech}^2(d+ex)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 72

$$\int F^{c(a+bx)}\text{csch}^2(d+ex)\text{sech}^2(d+ex)\,dx = \frac{16e^{4d+4ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{1}{4}\left(4+\frac{bc\log(F)}{e}\right),\frac{1}{4}\left(8+\frac{bc\log(F)}{e}\right),e^{4d+4ex}\right)}{4e+bc\log(F)}$$

output
```
16*exp(4*e*x+4*d)*F^(c*(b*x+a))*hypergeom([2, 1+1/4*b*c*ln(F)/e],[2+1/4*b*
c*ln(F)/e],exp(4*e*x+4*d))/(b*c*ln(F)+4*e)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.85

$$\int F^{c(a+bx)}\text{csch}^2(d+ex)\text{sech}^2(d+ex)\,dx = -\frac{2F^{c(a+bx)}\left(-1+\coth(2(d+ex))+2\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{4e},1+\frac{bc\log(F)}{4e},e^{4(d+ex)}\right)\right)}{e}$$

input
```
Integrate[F^(c*(a + b*x))*Csch[d + e*x]^2*Sech[d + e*x]^2,x]
```

output
```
(-2*F^(c*(a + b*x))*(-1 + Coth[2*(d + e*x)] + 2*Hypergeometric2F1[1, (b*c*
Log[F])/(4*e), 1 + (b*c*Log[F])/(4*e), E^(4*(d + e*x))]))/e
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 384 vs. $2(72) = 144$.

Time = 1.00 (sec) , antiderivative size = 384, normalized size of antiderivative = 5.33, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(d+ex)\operatorname{sech}^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \; 6037$$

$$\int \left(-\frac{3e^{4d+4ex}F^{ac+bcx}}{e^{d+ex}-1}+\frac{3e^{4d+4ex}F^{ac+bcx}}{e^{d+ex}+1}+\frac{4e^{4d+4ex}F^{ac+bcx}}{e^{2d+2ex}+1}+\frac{e^{4d+4ex}F^{ac+bcx}}{\left(e^{d+ex}-1\right)^2}+\frac{e^{4d+4ex}F^{ac+bcx}}{\left(e^{d+ex}+1\right)^2}+\frac{4e^{4d+4ex}F}{\left(e^{2d+2ex}\right.}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{4F^{ac+bcx}\text{Hypergeometric2F1}\left(2,-\frac{bc\log(F)}{2e},1-\frac{bc\log(F)}{2e},-e^{-2(d+ex)}\right)}{bc\log(F)}+$$
$$\frac{4e^{4d+4ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(\frac{bc\log(F)}{e}+4\right),\frac{1}{2}\left(\frac{bc\log(F)}{e}+6\right),-e^{2(d+ex)}\right)}{bc\log(F)+4e}+$$
$$\frac{3e^{4d+4ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{e}+4,\frac{bc\log(F)}{e}+5,-e^{d+ex}\right)}{bc\log(F)+4e}+$$
$$\frac{3e^{4d+4ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{e}+4,\frac{bc\log(F)}{e}+5,e^{d+ex}\right)}{bc\log(F)+4e}+$$
$$\frac{e^{4d+4ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{e}+4,\frac{bc\log(F)}{e}+5,-e^{d+ex}\right)}{bc\log(F)+4e}+$$
$$\frac{e^{4d+4ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{e}+4,\frac{bc\log(F)}{e}+5,e^{d+ex}\right)}{bc\log(F)+4e}$$

input
```
Int[F^(c*(a + b*x))*Csch[d + e*x]^2*Sech[d + e*x]^2,x]
```

output
```
(4*F^(a*c + b*c*x)*Hypergeometric2F1[2, -1/2*(b*c*Log[F])/e, 1 - (b*c*Log[
F])/(2*e), -E^(-2*(d + e*x))])/(b*c*Log[F]) + (4*E^(4*d + 4*e*x)*F^(a*c +
b*c*x)*Hypergeometric2F1[1, (4 + (b*c*Log[F])/e)/2, (6 + (b*c*Log[F])/e)/2
, -E^(2*(d + e*x))])/(4*e + b*c*Log[F]) + (3*E^(4*d + 4*e*x)*F^(a*c + b*c*
x)*Hypergeometric2F1[1, 4 + (b*c*Log[F])/e, 5 + (b*c*Log[F])/e, -E^(d + e*
x)])/(4*e + b*c*Log[F]) + (3*E^(4*d + 4*e*x)*F^(a*c + b*c*x)*Hypergeometri
c2F1[1, 4 + (b*c*Log[F])/e, 5 + (b*c*Log[F])/e, E^(d + e*x)])/(4*e + b*c*L
og[F]) + (E^(4*d + 4*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[2, 4 + (b*c*Lo
g[F])/e, 5 + (b*c*Log[F])/e, -E^(d + e*x)])/(4*e + b*c*Log[F]) + (E^(4*d +
 4*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[2, 4 + (b*c*Log[F])/e, 5 + (b*c*
Log[F])/e, E^(d + e*x)])/(4*e + b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Maple [F]

$$\int F^{c(bx+a)} \operatorname{csch}(ex+d)^2 \operatorname{sech}(ex+d)^2 \, dx$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)^2*sech(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*csch(e*x+d)^2*sech(e*x+d)^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\text{csch}^2(d+ex)\text{sech}^2(d+ex)\,dx=\int F^{(bx+a)c}\,\text{csch}\,(ex+d)^2\,\text{sech}\,(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^2*sech(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csch(e*x + d)^2*sech(e*x + d)^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\text{csch}^2(d+ex)\text{sech}^2(d+ex)\,dx=\int F^{c(a+bx)}\,\text{csch}^2\,(d+ex)\,\text{sech}^2\,(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*csch(e*x+d)**2*sech(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*csch(d + e*x)**2*sech(d + e*x)**2, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\text{csch}^2(d+ex)\text{sech}^2(d+ex)\,dx=\int F^{(bx+a)c}\,\text{csch}\,(ex+d)^2\,\text{sech}\,(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^2*sech(e*x+d)^2,x, algorithm="maxima")
```

output
```
128*F^(a*c)*b*c*e*integrate(-F^(b*c*x)/(b^2*c^2*log(F)^2 - 12*b*c*e*log(F)
 + 32*e^2 - (b^2*c^2*e^(12*d)*log(F)^2 - 12*b*c*e*e^(12*d)*log(F) + 32*e^2
*e^(12*d))*e^(12*e*x) + 3*(b^2*c^2*e^(8*d)*log(F)^2 - 12*b*c*e*e^(8*d)*log
(F) + 32*e^2*e^(8*d))*e^(8*e*x) - 3*(b^2*c^2*e^(4*d)*log(F)^2 - 12*b*c*e*e
^(4*d)*log(F) + 32*e^2*e^(4*d))*e^(4*e*x)), x)*log(F) + 16*(8*F^(a*c)*e +
(F^(a*c)*b*c*e^(4*d)*log(F) - 8*F^(a*c)*e*e^(4*d))*e^(4*e*x))*F^(b*c*x)/(b
^2*c^2*log(F)^2 - 12*b*c*e*log(F) + 32*e^2 + (b^2*c^2*e^(8*d)*log(F)^2 - 1
2*b*c*e*e^(8*d)*log(F) + 32*e^2*e^(8*d))*e^(8*e*x) - 2*(b^2*c^2*e^(4*d)*lo
g(F)^2 - 12*b*c*e*e^(4*d)*log(F) + 32*e^2*e^(4*d))*e^(4*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\operatorname{sech}^2(d+ex)\,dx = \int F^{(bx+a)c}\operatorname{csch}\left(ex+d\right)^2\operatorname{sech}\left(ex+d\right)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^2*sech(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csch(e*x + d)^2*sech(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\operatorname{sech}^2(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}}{\cosh\left(d+e\,x\right)^2\sinh\left(d+e\,x\right)^2}\,dx$$

input
```
int(F^(c*(a + b*x))/(cosh(d + e*x)^2*sinh(d + e*x)^2),x)
```

output
```
int(F^(c*(a + b*x))/(cosh(d + e*x)^2*sinh(d + e*x)^2), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\text{csch}^2(d+ex)\text{sech}^2(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\text{csch}(ex+d)^2\,\text{sech}(ex+d)^2\,dx\right)$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)^2*sech(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*csch(d + e*x)**2*sech(d + e*x)**2,x)
```

3.129 $\int F^{c(a+bx)}\mathrm{csch}^3(d+ex)\mathrm{sech}^2(d+ex)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 301

$$\int F^{c(a+bx)}\mathrm{csch}^3(d+ex)\mathrm{sech}^2(d+ex)\,dx = -\frac{4e^{5d+5ex}F^{c(a+bx)}}{e\left(1-e^{2d+2ex}\right)^2\left(1+e^{2d+2ex}\right)} + \frac{2bce^{5d+5ex}F^{c(a+bx)}\,\mathrm{Hypergeometric2F1}\left(1,\frac{1}{2}\left(5+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(7+\frac{bc\log(F)}{e}\right),-e^{2d+2ex}\right)\log(F)}{e(5e+bc\log(F))} + \frac{e^{5d+5ex}F^{c(a+bx)}\,\mathrm{Hypergeometric2F1}\left(1,\frac{1}{2}\left(5+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(7+\frac{bc\log(F)}{e}\right),e^{2d+2ex}\right)\left(3e^2-b^2c^2\log^2(F)\right)}{e^2(5e+bc\log(F))} - \frac{e^{5d+5ex}F^{c(a+bx)}\left(3-\frac{bc\log(F)}{e}+e^{2d+2ex}\left(1-\frac{bc\log(F)}{e}\right)\right)}{e\left(1-e^{4d+4ex}\right)}$$

output

```
-4*exp(5*e*x+5*d)*F^(c*(b*x+a))/e/(1-exp(2*e*x+2*d))^2/(1+exp(2*e*x+2*d))+
2*b*c*exp(5*e*x+5*d)*F^(c*(b*x+a))*hypergeom([1, 5/2+1/2*b*c*ln(F)/e],[7/2
+1/2*b*c*ln(F)/e],-exp(2*e*x+2*d))*ln(F)/e/(b*c*ln(F)+5*e)+exp(5*e*x+5*d)*
F^(c*(b*x+a))*hypergeom([1, 5/2+1/2*b*c*ln(F)/e],[7/2+1/2*b*c*ln(F)/e],exp
(2*e*x+2*d))*(3*e^2-b^2*c^2*ln(F)^2)/e^2/(b*c*ln(F)+5*e)-exp(5*e*x+5*d)*F^
(c*(b*x+a))*(3-b*c*ln(F)/e+exp(2*e*x+2*d)*(1-b*c*ln(F)/e))/e/(1-exp(4*e*x+
4*d))
```

Mathematica [A] (verified)

Time = 3.16 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.79

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\operatorname{sech}^2(d+ex)\,dx = \frac{F^{c\left(a-\frac{bd}{e}\right)}\left(8bcee^{\frac{(d+ex)(e+bc\log(F))}{e}}\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),-e^{2(d+ex)}\right)\log(F)+4\right.}{}$$

input
```
Integrate[F^(c*(a + b*x))*Csch[d + e*x]^3*Sech[d + e*x]^2,x]
```

output
```
(F^(c*(a - (b*d)/e))*(8*b*c*e*E^(((d + e*x)*(e + b*c*Log[F]))/e)*Hypergeom
etric2F1[1, (e + b*c*Log[F])/(2*e), (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x
))]*Log[F] + 4*E^(((d + e*x)*(e + b*c*Log[F]))/e)*Hypergeometric2F1[1, (e
+ b*c*Log[F])/(2*e), (3 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))]*(3*e^2 - b^2
*c^2*Log[F]^2) - F^((b*c*(d + e*x))/e)*Csch[d + e*x]^2*(e + b*c*Log[F])*Se
ch[d + e*x]*(-e + 3*e*Cosh[2*(d + e*x)] + b*c*Log[F]*Sinh[2*(d + e*x)])))/
(4*e^2*(e + b*c*Log[F]))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(d+ex)\operatorname{sech}^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6037}$$

$$\int \left(\frac{12e^{5d+5ex}F^{ac+bcx}}{e^{4(d+ex)}-1}-\frac{8e^{5d+5ex}F^{ac+bcx}}{\left(e^{2(d+ex)}-1\right)^2}-\frac{4e^{5d+5ex}F^{ac+bcx}}{\left(e^{2(d+ex)}+1\right)^2}+\frac{8e^{5d+5ex}F^{ac+bcx}}{\left(e^{2(d+ex)}-1\right)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{12e^{5d+5ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}\left(\frac{bc\log(F)}{e}+5\right),\frac{1}{4}\left(\frac{bc\log(F)}{e}+9\right),e^{4(d+ex)}\right)}{bc\log(F)+5e}-\frac{4e^{5d+5ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(\frac{bc\log(F)}{e}+5\right),\frac{1}{2}\left(\frac{bc\log(F)}{e}+7\right),-e^{2(d+ex)}\right)}{bc\log(F)+5e}-\frac{8e^{5d+5ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(\frac{bc\log(F)}{e}+5\right),\frac{1}{2}\left(\frac{bc\log(F)}{e}+7\right),e^{2(d+ex)}\right)}{bc\log(F)+5e}-\frac{8e^{5d+5ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(\frac{bc\log(F)}{e}+5\right),\frac{1}{2}\left(\frac{bc\log(F)}{e}+7\right),e^{2(d+ex)}\right)}{bc\log(F)+5e}$$

input
```
Int[F^(c*(a + b*x))*Csch[d + e*x]^3*Sech[d + e*x]^2,x]
```

output
```
(-12*E^(5*d + 5*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[1, (5 + (b*c*Log[F]
)/e)/4, (9 + (b*c*Log[F])/e)/4, E^(4*(d + e*x))])/(5*e + b*c*Log[F]) - (4*
E^(5*d + 5*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[2, (5 + (b*c*Log[F])/e)/
2, (7 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/(5*e + b*c*Log[F]) - (8*E^(5
*d + 5*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[2, (5 + (b*c*Log[F])/e)/2, (
7 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))])/(5*e + b*c*Log[F]) - (8*E^(5*d +
5*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[3, (5 + (b*c*Log[F])/e)/2, (7 + (
b*c*Log[F])/e)/2, E^(2*(d + e*x))])/(5*e + b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Maple [F]

$$\int F^{c(bx+a)} \operatorname{csch}(ex+d)^3 \operatorname{sech}(ex+d)^2 \, dx$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)^3*sech(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*csch(e*x+d)^3*sech(e*x+d)^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \operatorname{csch}^3(d+ex) \operatorname{sech}^2(d+ex) \, dx = \int F^{(bx+a)c} \operatorname{csch}(ex+d)^3 \operatorname{sech}(ex+d)^2 \, dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^3*sech(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csch(e*x + d)^3*sech(e*x + d)^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \operatorname{csch}^3(d+ex) \operatorname{sech}^2(d+ex) \, dx = \int F^{c(a+bx)} \operatorname{csch}^3(d+ex) \operatorname{sech}^2(d+ex) \, dx$$

input
```
integrate(F**(c*(b*x+a))*csch(e*x+d)**3*sech(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*csch(d + e*x)**3*sech(d + e*x)**2, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\operatorname{sech}^2(d+ex)\,dx = \int F^{(bx+a)c}\operatorname{csch}(ex+d)^3\operatorname{sech}(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^3*sech(e*x+d)^2,x, algorithm="maxima")
```

output
```
-32*((F^(a*c)*b^2*c^2*e^(5*d)*log(F)^2 - 16*F^(a*c)*b*c*e*e^(5*d)*log(F) +
 63*F^(a*c)*e^2*e^(5*d))*e^(5*e*x) + 2*(F^(a*c)*b*c*e*e^(3*d)*log(F) - 9*F
^(a*c)*e^2*e^(3*d))*e^(3*e*x) + 2*(5*F^(a*c)*b*c*e*e^d*log(F) - 33*F^(a*c)
*e^2*e^d)*e^(e*x))*F^(b*c*x)/(b^3*c^3*log(F)^3 - 21*b^2*c^2*e*log(F)^2 + 1
43*b*c*e^2*log(F) - 315*e^3 - (b^3*c^3*e^(10*d)*log(F)^3 - 21*b^2*c^2*e*e^
(10*d)*log(F)^2 + 143*b*c*e^2*e^(10*d)*log(F) - 315*e^3*e^(10*d))*e^(10*e*
x) + (b^3*c^3*e^(8*d)*log(F)^3 - 21*b^2*c^2*e*e^(8*d)*log(F)^2 + 143*b*c*e
^2*e^(8*d)*log(F) - 315*e^3*e^(8*d))*e^(8*e*x) + 2*(b^3*c^3*e^(6*d)*log(F)
^3 - 21*b^2*c^2*e*e^(6*d)*log(F)^2 + 143*b*c*e^2*e^(6*d)*log(F) - 315*e^3*
e^(6*d))*e^(6*e*x) - 2*(b^3*c^3*e^(4*d)*log(F)^3 - 21*b^2*c^2*e*e^(4*d)*lo
g(F)^2 + 143*b*c*e^2*e^(4*d)*log(F) - 315*e^3*e^(4*d))*e^(4*e*x) - (b^3*c^
3*e^(2*d)*log(F)^3 - 21*b^2*c^2*e*e^(2*d)*log(F)^2 + 143*b*c*e^2*e^(2*d)*l
og(F) - 315*e^3*e^(2*d))*e^(2*e*x)) + 32*integrate(2*((F^(a*c)*b^2*c^2*e*e
^(3*d)*log(F)^2 + 4*F^(a*c)*b*c*e^2*e^(3*d)*log(F) - 93*F^(a*c)*e^3*e^(3*d
))*e^(3*e*x) + (5*F^(a*c)*b^2*c^2*e*e^d*log(F)^2 - 28*F^(a*c)*b*c*e^2*e^d*
log(F) - 33*F^(a*c)*e^3*e^d)*e^(e*x))*F^(b*c*x)/(b^3*c^3*log(F)^3 - 21*b^2
*c^2*e*log(F)^2 + 143*b*c*e^2*log(F) - 315*e^3 + (b^3*c^3*e^(14*d)*log(F)^
3 - 21*b^2*c^2*e*e^(14*d)*log(F)^2 + 143*b*c*e^2*e^(14*d)*log(F) - 315*e^3
*e^(14*d))*e^(14*e*x) - (b^3*c^3*e^(12*d)*log(F)^3 - 21*b^2*c^2*e*e^(12*d)
*log(F)^2 + 143*b*c*e^2*e^(12*d)*log(F) - 315*e^3*e^(12*d))*e^(12*e*x) ...
```

Giac [F]

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\operatorname{sech}^2(d+ex)\,dx = \int F^{(bx+a)c}\operatorname{csch}(ex+d)^3\operatorname{sech}(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^3*sech(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csch(e*x + d)^3*sech(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\operatorname{sech}^2(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\cosh\left(d+e\,x\right)^2\,\sinh\left(d+e\,x\right)^3}\,dx$$

input
```
int(F^(c*(a + b*x))/(cosh(d + e*x)^2*sinh(d + e*x)^3),x)
```

output
```
int(F^(c*(a + b*x))/(cosh(d + e*x)^2*sinh(d + e*x)^3), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\operatorname{sech}^2(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\operatorname{csch}(ex+d)^3\operatorname{sech}(ex+d)^2\,dx\right)$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)^3*sech(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*csch(d + e*x)**3*sech(d + e*x)**2,x)
```

3.130 $\int F^{c(a+bx)} \tanh^3(d+ex) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 155

$$\int F^{c(a+bx)} \tanh^3(d+ex) \, dx = -\frac{2F^{c(a+bx)}}{e(1+e^{2d+2ex})^2} + \frac{F^{c(a+bx)}}{bc\log(F)} + \frac{F^{c(a+bx)}(2e+bc\log(F))}{e^2(1+e^{2d+2ex})} - F^{c(a+bx)} \text{Hypergeometric2F1}\left(1, \frac{bc\log(F)}{2e}, 1 + \frac{bc\log(F)}{2e}, -e^{2d+2ex}\right) \left(\frac{2}{bc\log(F)} + \frac{bc\log(F)}{e^2}\right)$$

output
```
-2*F^(c*(b*x+a))/e/(1+exp(2*e*x+2*d))^2+F^(c*(b*x+a))/b/c/ln(F)+F^(c*(b*x+
a))*(2*e+b*c*ln(F))/e^2/(1+exp(2*e*x+2*d))-F^(c*(b*x+a))*hypergeom([1, 1/2
*b*c*ln(F)/e],[1+1/2*b*c*ln(F)/e],-exp(2*e*x+2*d))*(2/b/c/ln(F)+b*c*ln(F)/
e^2)
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.97

$$\int F^{c(a+bx)}\tanh^3(d+ex)\,dx$$
$$=\frac{1}{2}F^{c(a+bx)}\left(\frac{2\left(1-\left(1+e^{2d}\right)\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)\right)\left(2e^2+b^2c^2\log^2(F\right.}{bce^2\left(1+e^{2d}\right)\log(F)}\right.$$
$$\left.+\frac{\text{sech}^2(d+ex)}{e}-\frac{bc\log(F)\text{sech}(d)\text{sech}(d+ex)\sinh(ex)}{e^2}+\frac{2\tanh(d)}{bc\log(F)}\right)$$

input
```
Integrate[F^(c*(a + b*x))*Tanh[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*((2*(1 - (1 + E^(2*d))*Hypergeometric2F1[1, (b*c*Log[F])/
(2*e), 1 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])*(2*e^2 + b^2*c^2*Log[F]^
2))/(b*c*e^2*(1 + E^(2*d))*Log[F]) + Sech[d + e*x]^2/e - (b*c*Log[F]*Sech[
d]*Sech[d + e*x]*Sinh[e*x])/e^2 + (2*Tanh[d])/(b*c*Log[F])))/2
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {6007, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(d+ex)F^{c(a+bx)}\,dx$$
$$\Big\downarrow \text{6007}$$
$$\int \left(-\frac{6F^{c(a+bx)}}{e^{2(d+ex)}+1}+\frac{12F^{c(a+bx)}}{\left(e^{2(d+ex)}+1\right)^2}-\frac{8F^{c(a+bx)}}{\left(e^{2(d+ex)}+1\right)^3}+F^{c(a+bx)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{6F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,-e^{2(d+ex)}\right)}{bc\log(F)}+\frac{12F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,-e^{2(d+ex)}\right)}{bc\log(F)}-\frac{8F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(3,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,-e^{2(d+ex)}\right)}{bc\log(F)}+\frac{F^{c(a+bx)}}{bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*Tanh[d + e*x]^3,x]
```

output
```
F^(c*(a + b*x))/(b*c*Log[F]) - (6*F^(c*(a + b*x))*Hypergeometric2F1[1, (b*
c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(b*c*Log[F]) +
 (12*F^(c*(a + b*x))*Hypergeometric2F1[2, (b*c*Log[F])/(2*e), 1 + (b*c*Log
[F])/(2*e), -E^(2*(d + e*x))])/(b*c*Log[F]) - (8*F^(c*(a + b*x))*Hypergeom
etric2F1[3, (b*c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])
/(b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6007
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tanh[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((-1 + E^(2*(d + e*x)))^n/(1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)}\tanh\left(ex+d\right)^{3}dx$$

input
```
int(F^(c*(b*x+a))*tanh(e*x+d)^3,x)
```

output
```
int(F^(c*(b*x+a))*tanh(e*x+d)^3,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\tanh^3(d+ex)\,dx=\int F^{(bx+a)c}\tanh\left(ex+d\right)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*tanh(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*tanh(e*x + d)^3, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\tanh^3(d+ex)\,dx=\int F^{c(a+bx)}\tanh^3\left(d+ex\right)dx$$

input
```
integrate(F**(c*(b*x+a))*tanh(e*x+d)**3,x)
```

output
```
Integral(F**(c*(a + b*x))*tanh(d + e*x)**3, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\tanh^3(d+ex)\,dx=\int F^{(bx+a)c}\tanh\left(ex+d\right)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*tanh(e*x+d)^3,x, algorithm="maxima")
```

output
```
48*(F^(a*c)*b^2*c^2*e*log(F)^2 + 2*F^(a*c)*e^3)*integrate(F^(b*c*x)/(b^3*c
^3*log(F)^3 - 12*b^2*c^2*e*log(F)^2 + 44*b*c*e^2*log(F) - 48*e^3 + (b^3*c^
3*e^(8*d)*log(F)^3 - 12*b^2*c^2*e*e^(8*d)*log(F)^2 + 44*b*c*e^2*e^(8*d)*lo
g(F) - 48*e^3*e^(8*d))*e^(8*e*x) + 4*(b^3*c^3*e^(6*d)*log(F)^3 - 12*b^2*c^
2*e*e^(6*d)*log(F)^2 + 44*b*c*e^2*e^(6*d)*log(F) - 48*e^3*e^(6*d))*e^(6*e*
x) + 6*(b^3*c^3*e^(4*d)*log(F)^3 - 12*b^2*c^2*e*e^(4*d)*log(F)^2 + 44*b*c*
e^2*e^(4*d)*log(F) - 48*e^3*e^(4*d))*e^(4*e*x) + 4*(b^3*c^3*e^(2*d)*log(F)
^3 - 12*b^2*c^2*e*e^(2*d)*log(F)^2 + 44*b*c*e^2*e^(2*d)*log(F) - 48*e^3*e^
(2*d))*e^(2*e*x)), x) - (F^(a*c)*b^3*c^3*log(F)^3 + 36*F^(a*c)*b^2*c^2*e*l
og(F)^2 + 44*F^(a*c)*b*c*e^2*log(F) + 48*F^(a*c)*e^3 - (F^(a*c)*b^3*c^3*e^
(6*d)*log(F)^3 - 12*F^(a*c)*b^2*c^2*e*e^(6*d)*log(F)^2 + 44*F^(a*c)*b*c*e^
2*e^(6*d)*log(F) - 48*F^(a*c)*e^3*e^(6*d))*e^(6*e*x) + 3*(F^(a*c)*b^3*c^3*
e^(4*d)*log(F)^3 - 8*F^(a*c)*b^2*c^2*e*e^(4*d)*log(F)^2 + 4*F^(a*c)*b*c*e^
2*e^(4*d)*log(F) + 48*F^(a*c)*e^3*e^(4*d))*e^(4*e*x) - 3*(F^(a*c)*b^3*c^3*
e^(2*d)*log(F)^3 - 28*F^(a*c)*b*c*e^2*e^(2*d)*log(F) - 48*F^(a*c)*e^3*e^(2
*d))*e^(2*e*x))*F^(b*c*x)/(b^4*c^4*log(F)^4 - 12*b^3*c^3*e*log(F)^3 + 44*b
^2*c^2*e^2*log(F)^2 - 48*b*c*e^3*log(F) + (b^4*c^4*e^(6*d)*log(F)^4 - 12*b
^3*c^3*e*e^(6*d)*log(F)^3 + 44*b^2*c^2*e^2*e^(6*d)*log(F)^2 - 48*b*c*e^3*e
^(6*d)*log(F))*e^(6*e*x) + 3*(b^4*c^4*e^(4*d)*log(F)^4 - 12*b^3*c^3*e*e^(4
*d)*log(F)^3 + 44*b^2*c^2*e^2*e^(4*d)*log(F)^2 - 48*b*c*e^3*e^(4*d)*log...
```

Giac [F]

$$\int F^{c(a+bx)}\tanh^3(d+ex)\,dx=\int F^{(bx+a)c}\tanh\left(ex+d\right)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*tanh(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*tanh(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\tanh^3(d+ex)\,dx=\int F^{c\,(a+b\,x)}\,\tanh(d+e\,x)^3\,dx$$

input
```
int(F^(c*(a + b*x))*tanh(d + e*x)^3,x)
```

output
```
int(F^(c*(a + b*x))*tanh(d + e*x)^3, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\tanh^3(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\tanh\left(ex+d\right)^3dx\right)$$

input
```
int(F^(c*(b*x+a))*tanh(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*tanh(d + e*x)**3,x)
```

3.131 $\int F^{c(a+bx)}\text{sech}(d+ex)\tanh^2(d+ex)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 164

$$\int F^{c(a+bx)}\text{sech}(d+ex)\tanh^2(d+ex)\,dx = \frac{2e^{d+ex}F^{c(a+bx)}}{e\left(1+e^{2d+2ex}\right)^2} - \frac{e^{d+ex}F^{c(a+bx)}(e+bc\log(F))}{e^2\left(1+e^{2d+2ex}\right)} + \frac{e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),-e^{2d+2ex}\right)\left(e^2+b^2c^2\log^2(F)\right)}{e^2(e+bc\log(F))}$$

output
```
2*exp(e*x+d)*F^(c*(b*x+a))/e/(1+exp(2*e*x+2*d))^2-exp(e*x+d)*F^(c*(b*x+a))
*(e+b*c*ln(F))/e^2/(1+exp(2*e*x+2*d))+exp(e*x+d)*F^(c*(b*x+a))*hypergeom([
1, 1/2*(e+b*c*ln(F))/e],[3/2+1/2*b*c*ln(F)/e],-exp(2*e*x+2*d))*(e^2+b^2*c^
2*ln(F)^2)/e^2/(e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.90

$$\int F^{c(a+bx)}\text{sech}(d+ex)\tanh^2(d+ex)\,dx$$

$$= \frac{F^{c\left(a-\frac{bd}{e}\right)}\left(2e^{\frac{(d+ex)(e+bc\log(F))}{e}}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),-e^{2(d+ex)}\right)\left(e^2+b^2c^2\log^2\right.\right.}{2e^2(e+bc\log(F))}$$

input
```
Integrate[F^(c*(a + b*x))*Sech[d + e*x]*Tanh[d + e*x]^2,x]
```

output
```
(F^(c*(a - (b*d)/e))*(2*E^(((d + e*x)*(e + b*c*Log[F]))/e)*Hypergeometric2
F1[1, (e + b*c*Log[F])/(2*e), (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))]*(e
^2 + b^2*c^2*Log[F]^2) - F^((b*c*(d + e*x))/e)*(e + b*c*Log[F])*Sech[d + e
*x]*(b*c*Log[F] + e*Tanh[d + e*x])))/(2*e^2*(e + b*c*Log[F]))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(d+ex)\text{sech}(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6037}$$

$$\int \left(\frac{2e^{d+ex}F^{ac+bcx}}{e^{2(d+ex)}+1}-\frac{8e^{d+ex}F^{ac+bcx}}{\left(e^{2(d+ex)}+1\right)^2}+\frac{8e^{d+ex}F^{ac+bcx}}{\left(e^{2(d+ex)}+1\right)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2e^{d+ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),-e^{2(d+ex)}\right)}{bc\log(F)+e}-\frac{8e^{d+ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(2,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),-e^{2(d+ex)}\right)}{bc\log(F)+e}+\frac{8e^{d+ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(3,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),-e^{2(d+ex)}\right)}{bc\log(F)+e}$$

input
```
Int[F^(c*(a + b*x))*Sech[d + e*x]*Tanh[d + e*x]^2,x]
```

output
```
(2*E^(d + e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e)
, (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/(e + b*c*Log[F]) - (8*E^(d +
e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[2, (e + b*c*Log[F])/(2*e), (3 + (b*
c*Log[F])/e)/2, -E^(2*(d + e*x))])/(e + b*c*Log[F]) + (8*E^(d + e*x)*F^(a*
c + b*c*x)*Hypergeometric2F1[3, (e + b*c*Log[F])/(2*e), (3 + (b*c*Log[F])/
e)/2, -E^(2*(d + e*x))])/(e + b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Maple [F]

$$\int F^{c(bx+a)} \operatorname{sech}(ex+d) \tanh(ex+d)^2 \, dx$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)*tanh(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*sech(e*x+d)*tanh(e*x+d)^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\operatorname{sech}(d+ex)\tanh^2(d+ex)\,dx = \int F^{(bx+a)c} \operatorname{sech}(ex+d) \tanh(ex+d)^2 \, dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)*tanh(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sech(e*x + d)*tanh(e*x + d)^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\operatorname{sech}(d+ex)\tanh^2(d+ex)\,dx=\int F^{c(a+bx)}\tanh^2{(d+ex)}\operatorname{sech}{(d+ex)}\,dx$$

input
```
integrate(F**(c*(b*x+a))*sech(e*x+d)*tanh(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*tanh(d + e*x)**2*sech(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\operatorname{sech}(d+ex)\tanh^2(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{sech}{(ex+d)}\tanh{(ex+d)}^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)*tanh(e*x+d)^2,x, algorithm="maxima")
```

output
```
-48*(F^(a*c)*b^2*c^2*e*e^d*log(F)^2 + F^(a*c)*e^3*e^d)*integrate(e^(b*c*x*
log(F) + e*x)/(b^3*c^3*log(F)^3 - 9*b^2*c^2*e*log(F)^2 + 23*b*c*e^2*log(F)
 - 15*e^3 + (b^3*c^3*e^(8*d)*log(F)^3 - 9*b^2*c^2*e*e^(8*d)*log(F)^2 + 23*
b*c*e^2*e^(8*d)*log(F) - 15*e^3*e^(8*d))*e^(8*e*x) + 4*(b^3*c^3*e^(6*d)*lo
g(F)^3 - 9*b^2*c^2*e*e^(6*d)*log(F)^2 + 23*b*c*e^2*e^(6*d)*log(F) - 15*e^3
*e^(6*d))*e^(6*e*x) + 6*(b^3*c^3*e^(4*d)*log(F)^3 - 9*b^2*c^2*e*e^(4*d)*lo
g(F)^2 + 23*b*c*e^2*e^(4*d)*log(F) - 15*e^3*e^(4*d))*e^(4*e*x) + 4*(b^3*c^
3*e^(2*d)*log(F)^3 - 9*b^2*c^2*e*e^(2*d)*log(F)^2 + 23*b*c*e^2*e^(2*d)*log
(F) - 15*e^3*e^(2*d))*e^(2*e*x)), x) + 2*((F^(a*c)*b^2*c^2*e^(5*d)*log(F)^
2 - 8*F^(a*c)*b*c*e*e^(5*d)*log(F) + 15*F^(a*c)*e^2*e^(5*d))*e^(5*e*x) - 2
*(F^(a*c)*b^2*c^2*e^(3*d)*log(F)^2 - 3*F^(a*c)*b*c*e*e^(3*d)*log(F) - 10*F
^(a*c)*e^2*e^(3*d))*e^(3*e*x) + (F^(a*c)*b^2*c^2*e^d*log(F)^2 + 14*F^(a*c)
*b*c*e*e^d*log(F) + 9*F^(a*c)*e^2*e^d)*e^(e*x))*F^(b*c*x)/(b^3*c^3*log(F)^
3 - 9*b^2*c^2*e*log(F)^2 + 23*b*c*e^2*log(F) - 15*e^3 + (b^3*c^3*e^(6*d)*l
og(F)^3 - 9*b^2*c^2*e*e^(6*d)*log(F)^2 + 23*b*c*e^2*e^(6*d)*log(F) - 15*e^
3*e^(6*d))*e^(6*e*x) + 3*(b^3*c^3*e^(4*d)*log(F)^3 - 9*b^2*c^2*e*e^(4*d)*l
og(F)^2 + 23*b*c*e^2*e^(4*d)*log(F) - 15*e^3*e^(4*d))*e^(4*e*x) + 3*(b^3*c
^3*e^(2*d)*log(F)^3 - 9*b^2*c^2*e*e^(2*d)*log(F)^2 + 23*b*c*e^2*e^(2*d)*lo
g(F) - 15*e^3*e^(2*d))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\text{sech}(d+ex)\tanh^2(d+ex)\,dx = \int F^{(bx+a)c}\,\text{sech}\,(ex+d)\tanh\,(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)*tanh(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sech(e*x + d)*tanh(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{sech}(d+ex)\tanh^2(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}\,\tanh(d+e\,x)^2}{\cosh\,(d+e\,x)}\,dx$$

input
```
int((F^(c*(a + b*x))*tanh(d + e*x)^2)/cosh(d + e*x),x)
```

output
```
int((F^(c*(a + b*x))*tanh(d + e*x)^2)/cosh(d + e*x), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\text{sech}(d+ex)\tanh^2(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\text{sech}(ex+d)\tanh\,(ex+d)^2\,dx\right)$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)*tanh(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sech(d + e*x)*tanh(d + e*x)**2,x)
```

3.132 $\int F^{c(a+bx)}\mathrm{sech}^2(d+ex)\tanh(d+ex)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 120

$$\int F^{c(a+bx)}\mathrm{sech}^2(d+ex)\tanh(d+ex)\,dx = -\frac{2e^{2d+2ex}F^{c(a+bx)}}{e\left(1+e^{2d+2ex}\right)^2} + \frac{2bce^{2d+2ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(2+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(4+\frac{bc\log(F)}{e}\right),-e^{2d+2ex}\right)\log(F)}{e(2e+bc\log(F))}$$

output
```
-2*exp(2*e*x+2*d)*F^(c*(b*x+a))/e/(1+exp(2*e*x+2*d))^2+2*b*c*exp(2*e*x+2*d
)*F^(c*(b*x+a))*hypergeom([2, 1+1/2*b*c*ln(F)/e],[2+1/2*b*c*ln(F)/e],-exp(
2*e*x+2*d))*ln(F)/e/(2*e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.70

$$\int F^{c(a+bx)}\mathrm{sech}^2(d+ex)\tanh(d+ex)\,dx = \frac{F^{c(a+bx)}\left(2bc\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)\log(F)-e\,\mathrm{sech}^2(d+ex)+bc\log(F\right.}{2e^2}$$

input
```
Integrate[F^(c*(a + b*x))*Sech[d + e*x]^2*Tanh[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*(2*b*c*Hypergeometric2F1[1, (b*c*Log[F])/(2*e), 1 + (b*c*
Log[F])/(2*e), -E^(2*(d + e*x))]*Log[F] - e*Sech[d + e*x]^2 + b*c*Log[F]*(
-1 + Tanh[d + e*x])))/(2*e^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(d+ex)\text{sech}^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6037}$$

$$\int \left(\frac{4e^{2d+2ex}F^{ac+bcx}}{(-e^{2d+2ex}-1)^3}+\frac{4e^{4d+4ex}F^{ac+bcx}}{(e^{2d+2ex}+1)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4e^{4d+4ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(\frac{bc\log(F)}{e}+4\right),\frac{1}{2}\left(\frac{bc\log(F)}{e}+6\right),-e^{2(d+ex)}\right)}{bc\log(F)+4e}-\frac{4e^{2d+2ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(\frac{bc\log(F)}{e}+2\right),\frac{1}{2}\left(\frac{bc\log(F)}{e}+4\right),-e^{2(d+ex)}\right)}{bc\log(F)+2e}$$

input
```
Int[F^(c*(a + b*x))*Sech[d + e*x]^2*Tanh[d + e*x],x]
```

output
```
(-4*E^(2*d + 2*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[3, (2 + (b*c*Log[F])
/e)/2, (4 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/(2*e + b*c*Log[F]) + (4*
E^(4*d + 4*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[3, (4 + (b*c*Log[F])/e)/
2, (6 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/(4*e + b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Maple [F]

$$\int F^{c(bx+a)} \operatorname{sech}\left(ex+d\right)^{2} \tanh\left(ex+d\right) dx$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)^2*tanh(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*sech(e*x+d)^2*tanh(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\tanh(d+ex)\,dx = \int F^{(bx+a)c} \operatorname{sech}\left(ex+d\right)^{2} \tanh\left(ex+d\right)\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^2*tanh(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sech(e*x + d)^2*tanh(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\text{sech}^2(d+ex)\tanh(d+ex)\,dx = \int F^{c(a+bx)}\tanh{(d+ex)}\,\text{sech}^2{(d+ex)}\,dx$$

input
```
integrate(F**(c*(b*x+a))*sech(e*x+d)**2*tanh(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*tanh(d + e*x)*sech(d + e*x)**2, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\text{sech}^2(d+ex)\tanh(d+ex)\,dx = \int F^{(bx+a)c}\,\text{sech}{(ex+d)}^2\tanh{(ex+d)}\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^2*tanh(e*x+d),x, algorithm="maxima")
```

output
```
-48*F^(a*c)*b^2*c^2*e*integrate(F^(b*c*x)/(b^3*c^3*log(F)^3 - 12*b^2*c^2*e
*log(F)^2 + 44*b*c*e^2*log(F) - 48*e^3 + (b^3*c^3*e^(8*d)*log(F)^3 - 12*b^
2*c^2*e*e^(8*d)*log(F)^2 + 44*b*c*e^2*e^(8*d)*log(F) - 48*e^3*e^(8*d))*e^(
8*e*x) + 4*(b^3*c^3*e^(6*d)*log(F)^3 - 12*b^2*c^2*e*e^(6*d)*log(F)^2 + 44*
b*c*e^2*e^(6*d)*log(F) - 48*e^3*e^(6*d))*e^(6*e*x) + 6*(b^3*c^3*e^(4*d)*lo
g(F)^3 - 12*b^2*c^2*e*e^(4*d)*log(F)^2 + 44*b*c*e^2*e^(4*d)*log(F) - 48*e^
3*e^(4*d))*e^(4*e*x) + 4*(b^3*c^3*e^(2*d)*log(F)^3 - 12*b^2*c^2*e*e^(2*d)*
log(F)^2 + 44*b*c*e^2*e^(2*d)*log(F) - 48*e^3*e^(2*d))*e^(2*e*x)), x)*log(
F)^2 + 4*(12*F^(a*c)*b*c*e*log(F) + (F^(a*c)*b^2*c^2*e^(4*d)*log(F)^2 - 10
*F^(a*c)*b*c*e*e^(4*d)*log(F) + 24*F^(a*c)*e^2*e^(4*d))*e^(4*e*x) - (F^(a*
c)*b^2*c^2*e^(2*d)*log(F)^2 - 2*F^(a*c)*b*c*e*e^(2*d)*log(F) - 24*F^(a*c)*
e^2*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^3*c^3*log(F)^3 - 12*b^2*c^2*e*log(F)^
2 + 44*b*c*e^2*log(F) - 48*e^3 + (b^3*c^3*e^(6*d)*log(F)^3 - 12*b^2*c^2*e*
e^(6*d)*log(F)^2 + 44*b*c*e^2*e^(6*d)*log(F) - 48*e^3*e^(6*d))*e^(6*e*x) +
 3*(b^3*c^3*e^(4*d)*log(F)^3 - 12*b^2*c^2*e*e^(4*d)*log(F)^2 + 44*b*c*e^2*
e^(4*d)*log(F) - 48*e^3*e^(4*d))*e^(4*e*x) + 3*(b^3*c^3*e^(2*d)*log(F)^3 -
 12*b^2*c^2*e*e^(2*d)*log(F)^2 + 44*b*c*e^2*e^(2*d)*log(F) - 48*e^3*e^(2*d
))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\text{sech}^2(d+ex)\tanh(d+ex)\,dx = \int F^{(bx+a)c}\,\text{sech}\,(ex+d)^2\tanh\,(ex+d)\;dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^2*tanh(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sech(e*x + d)^2*tanh(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{sech}^2(d+ex)\tanh(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}\tanh(d+e\,x)}{\cosh\,(d+e\,x)^2}\,dx$$

input
```
int((F^(c*(a + b*x))*tanh(d + e*x))/cosh(d + e*x)^2,x)
```

output
```
int((F^(c*(a + b*x))*tanh(d + e*x))/cosh(d + e*x)^2, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\text{sech}^2(d+ex)\tanh(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\text{sech}(ex+d)^2\tanh\,(ex+d)\,dx\right)$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)^2*tanh(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sech(d + e*x)**2*tanh(d + e*x),x)
```

3.133 $\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 74

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx = \frac{8e^{3d+3ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(5+\frac{bc\log(F)}{e}\right),-e^{2d+2ex}\right)}{3e+bc\log(F)}$$

output
```
8*exp(3*e*x+3*d)*F^(c*(b*x+a))*hypergeom([3, 3/2+1/2*b*c*ln(F)/e],[5/2+1/2
*b*c*ln(F)/e],-exp(2*e*x+2*d))/(b*c*ln(F)+3*e)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.30

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx = \frac{F^{c(a+bx)}\left(2e^{d+ex}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),-e^{2(d+ex)}\right)(e-bc\log(F))+\text{sech}(d\right.}{2e^2}$$

input
```
Integrate[F^(c*(a + b*x))*Sech[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(2*E^(d + e*x)*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e
), (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))]*(e - b*c*Log[F]) + Sech[d + e
*x]*(b*c*Log[F] + e*Tanh[d + e*x])))/(2*e^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.88, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6013, 6015}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^3(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6013}$$

$$\frac{1}{2}\left(1-\frac{b^2c^2\log^2(F)}{e^2}\right)\int F^{c(a+bx)}\operatorname{sech}(d+ex)dx+\frac{bc\log(F)\operatorname{sech}(d+ex)F^{c(a+bx)}}{2e^2}+\frac{\tanh(d+ex)\operatorname{sech}(d+ex)F^{c(a+bx)}}{2e}$$

$$\Big\downarrow \text{6015}$$

$$\frac{e^{d+ex}F^{c(a+bx)}\left(1-\frac{b^2c^2\log^2(F)}{e^2}\right)\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),-e^{2(d+ex)}\right)}{bc\log(F)+e}+\frac{bc\log(F)\operatorname{sech}(d+ex)F^{c(a+bx)}}{2e^2}+\frac{\tanh(d+ex)\operatorname{sech}(d+ex)F^{c(a+bx)}}{2e}$$

input
```
Int[F^(c*(a + b*x))*Sech[d + e*x]^3,x]
```

output
```
(E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e),
(3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))]*(1 - (b^2*c^2*Log[F]^2)/e^2))/(e
 + b*c*Log[F]) + (b*c*F^(c*(a + b*x))*Log[F]*Sech[d + e*x])/(2*e^2) + (F^(
c*(a + b*x))*Sech[d + e*x]*Tanh[d + e*x])/(2*e)
```

Defintions of rubi rules used

rule 6013
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sech[d + e*x]^(n - 2)/(e^2*(n - 1)*
(n - 2))), x] + (Simp[F^(c*(a + b*x))*Sech[d + e*x]^(n - 1)*(Sinh[d + e*x]/
(e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2))   Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a,
b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
 NeQ[n, 2]
```

rule 6015
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \operatorname{sech}(ex+d)^3\,dx$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)^3,x)
```

output
```
int(F^(c*(b*x+a))*sech(e*x+d)^3,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\operatorname{sech}^3(d+ex)\,dx = \int F^{(bx+a)c} \operatorname{sech}(ex+d)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sech(e*x + d)^3, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\mathrm{sech}^3(d+ex)\,dx=\int F^{c(a+bx)}\,\mathrm{sech}^3\,(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*sech(e*x+d)**3,x)
```

output
```
Integral(F**(c*(a + b*x))*sech(d + e*x)**3, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\mathrm{sech}^3(d+ex)\,dx=\int F^{(bx+a)c}\,\mathrm{sech}\,(ex+d)^3\,\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^3,x, algorithm="maxima")
```

output
```
48*(F^(a*c)*b*c*e*e^d*log(F) + F^(a*c)*e^2*e^d)*integrate(e^(b*c*x*log(F)
+ e*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*log(F) + 15*e^2 + (b^2*c^2*e^(8*d)*log(
F)^2 - 8*b*c*e*e^(8*d)*log(F) + 15*e^2*e^(8*d))*e^(8*e*x) + 4*(b^2*c^2*e^(
6*d)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e^2*e^(6*d))*e^(6*e*x) + 6*(b^
2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(F) + 15*e^2*e^(4*d))*e^(4*e*x
) + 4*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(2*d)*log(F) + 15*e^2*e^(2*d))
*e^(2*e*x)), x) - 8*(6*F^(a*c)*e*e^(e*x + d) - (F^(a*c)*b*c*e^(3*d)*log(F)
 - 5*F^(a*c)*e*e^(3*d))*e^(3*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*l
og(F) + 15*e^2 + (b^2*c^2*e^(6*d)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e
^2*e^(6*d))*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(
F) + 15*e^2*e^(4*d))*e^(4*e*x) + 3*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(
2*d)*log(F) + 15*e^2*e^(2*d))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx=\int F^{(bx+a)c}\,\text{sech}\,(ex+d)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sech(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\cosh\,(d+e\,x)^3}\,dx$$

input
```
int(F^(c*(a + b*x))/cosh(d + e*x)^3,x)
```

output
```
int(F^(c*(a + b*x))/cosh(d + e*x)^3, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\text{sech}(ex+d)^3\,dx\right)$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sech(d + e*x)**3,x)
```

3.134 $\int F^{c(a+bx)}\operatorname{csch}(d+ex)\operatorname{sech}^3(d+ex)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 249

$$\int F^{c(a+bx)}\operatorname{csch}(d+ex)\operatorname{sech}^3(d+ex)\,dx = -\frac{2e^{4d+4ex}F^{c(a+bx)}}{e\left(1+e^{2d+2ex}\right)^2}-\frac{e^{4d+4ex}F^{c(a+bx)}(2e-bc\log(F))}{e^2\left(1+e^{2d+2ex}\right)} - \frac{2e^{4d+4ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(4+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(6+\frac{bc\log(F)}{e}\right),e^{2d+2ex}\right)}{4e+bc\log(F)} + \frac{e^{4d+4ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(4+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(6+\frac{bc\log(F)}{e}\right),-e^{2d+2ex}\right)\left(2-\frac{b^2c^2\log^2(F)}{e^2}\right)}{4e+bc\log(F)}$$

output
```
-2*exp(4*e*x+4*d)*F^(c*(b*x+a))/e/(1+exp(2*e*x+2*d))^2-exp(4*e*x+4*d)*F^(c
*(b*x+a))*(2*e-b*c*ln(F))/e^2/(1+exp(2*e*x+2*d))-2*exp(4*e*x+4*d)*F^(c*(b*
x+a))*hypergeom([1, 2+1/2*b*c*ln(F)/e],[3+1/2*b*c*ln(F)/e],exp(2*e*x+2*d))
/(b*c*ln(F)+4*e)+exp(4*e*x+4*d)*F^(c*(b*x+a))*hypergeom([1, 2+1/2*b*c*ln(F
)/e],[3+1/2*b*c*ln(F)/e],-exp(2*e*x+2*d))*(2-b^2*c^2*ln(F)^2/e^2)/(b*c*ln(
F)+4*e)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.61

$$\int F^{c(a+bx)}\operatorname{csch}(d+ex)\operatorname{sech}^3(d+ex)\,dx$$
$$= \frac{F^{c(a+bx)}\left(-4e^2\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},e^{2(d+ex)}\right)+\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},\right.\right.}{2bc}$$

input
```
Integrate[F^(c*(a + b*x))*Csch[d + e*x]*Sech[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(-4*e^2*Hypergeometric2F1[1, (b*c*Log[F])/(2*e), 1 + (b*c
*Log[F])/(2*e), E^(2*(d + e*x))] + Hypergeometric2F1[1, (b*c*Log[F])/(2*e)
, 1 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))]*(4*e^2 - 2*b^2*c^2*Log[F]^2) +
 b*c*Log[F]*(e*Sech[d + e*x]^2 - b*c*Log[F]*(-1 + Tanh[d + e*x]))))/(2*b*c
*e^2*Log[F])
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.77, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(d+ex)\operatorname{sech}^3(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6037}$$

$$\int \left(\frac{4e^{4d+4ex}F^{ac+bcx}}{e^{4(d+ex)}-1}-\frac{4e^{4d+4ex}F^{ac+bcx}}{\left(e^{2(d+ex)}+1\right)^2}-\frac{8e^{4d+4ex}F^{ac+bcx}}{\left(e^{2(d+ex)}+1\right)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4F^{ac+bcx}\operatorname{Hypergeometric2F1}\left(1,-\frac{bc\log(F)}{4e},1-\frac{bc\log(F)}{4e},e^{-4(d+ex)}\right)}{bc\log(F)}-\frac{4F^{ac+bcx}\operatorname{Hypergeometric2F1}\left(2,-\frac{bc\log(F)}{2e},1-\frac{bc\log(F)}{2e},-e^{-2(d+ex)}\right)}{bc\log(F)}-\frac{8e^{4d+4ex}F^{ac+bcx}\operatorname{Hypergeometric2F1}\left(3,\frac{1}{2}\left(\frac{bc\log(F)}{e}+4\right),\frac{1}{2}\left(\frac{bc\log(F)}{e}+6\right),-e^{2(d+ex)}\right)}{bc\log(F)+4e}$$

input
```
Int[F^(c*(a + b*x))*Csch[d + e*x]*Sech[d + e*x]^3,x]
```

output
```
(4*F^(a*c + b*c*x)*Hypergeometric2F1[1, -1/4*(b*c*Log[F])/e, 1 - (b*c*Log[
F])/(4*e), E^(-4*(d + e*x))])/(b*c*Log[F]) - (4*F^(a*c + b*c*x)*Hypergeome
tric2F1[2, -1/2*(b*c*Log[F])/e, 1 - (b*c*Log[F])/(2*e), -E^(-2*(d + e*x))]
)/(b*c*Log[F]) - (8*E^(4*d + 4*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[3, (
4 + (b*c*Log[F])/e)/2, (6 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/(4*e + b
*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Maple [F]

$$\int F^{c(bx+a)} \operatorname{csch}(ex+d) \operatorname{sech}(ex+d)^3 \, dx$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)*sech(e*x+d)^3,x)
```

output
```
int(F^(c*(b*x+a))*csch(e*x+d)*sech(e*x+d)^3,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\text{csch}(d+ex)\text{sech}^3(d+ex)\,dx = \int F^{(bx+a)c}\,\text{csch}\,(ex+d)\,\text{sech}\,(ex+d)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)*sech(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csch(e*x + d)*sech(e*x + d)^3, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\text{csch}(d+ex)\text{sech}^3(d+ex)\,dx = \int F^{c(a+bx)}\,\text{csch}\,(d+ex)\,\text{sech}^3\,(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*csch(e*x+d)*sech(e*x+d)**3,x)
```

output
```
Integral(F**(c*(a + b*x))*csch(d + e*x)*sech(d + e*x)**3, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\text{csch}(d+ex)\text{sech}^3(d+ex)\,dx = \int F^{(bx+a)c}\,\text{csch}\,(ex+d)\,\text{sech}\,(ex+d)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)*sech(e*x+d)^3,x, algorithm="maxima")
```

output
```
-16*(8*F^(a*c)*b*c*e*log(F) - 32*F^(a*c)*e^2 + (F^(a*c)*b^2*c^2*e^(4*d)*lo
g(F)^2 - 14*F^(a*c)*b*c*e*e^(4*d)*log(F) + 48*F^(a*c)*e^2*e^(4*d))*e^(4*e*
x) - 4*(F^(a*c)*b*c*e*e^(2*d)*log(F) - 8*F^(a*c)*e^2*e^(2*d))*e^(2*e*x))*F
^(b*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2*e*log(F)^2 + 104*b*c*e^2*log(F) -
192*e^3 - (b^3*c^3*e^(8*d)*log(F)^3 - 18*b^2*c^2*e*e^(8*d)*log(F)^2 + 104*
b*c*e^2*e^(8*d)*log(F) - 192*e^3*e^(8*d))*e^(8*e*x) - 2*(b^3*c^3*e^(6*d)*l
og(F)^3 - 18*b^2*c^2*e*e^(6*d)*log(F)^2 + 104*b*c*e^2*e^(6*d)*log(F) - 192
*e^3*e^(6*d))*e^(6*e*x) + 2*(b^3*c^3*e^(2*d)*log(F)^3 - 18*b^2*c^2*e*e^(2*
d)*log(F)^2 + 104*b*c*e^2*e^(2*d)*log(F) - 192*e^3*e^(2*d))*e^(2*e*x)) - 1
6*integrate(-4*(2*F^(a*c)*b^2*c^2*e*log(F)^2 - 8*F^(a*c)*b*c*e^2*log(F) -
(F^(a*c)*b^2*c^2*e*e^(2*d)*log(F)^2 + 2*F^(a*c)*b*c*e^2*e^(2*d)*log(F) - 4
8*F^(a*c)*e^3*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2
*e*log(F)^2 + 104*b*c*e^2*log(F) - 192*e^3 + (b^3*c^3*e^(12*d)*log(F)^3 -
18*b^2*c^2*e*e^(12*d)*log(F)^2 + 104*b*c*e^2*e^(12*d)*log(F) - 192*e^3*e^(
12*d))*e^(12*e*x) + 2*(b^3*c^3*e^(10*d)*log(F)^3 - 18*b^2*c^2*e*e^(10*d)*l
og(F)^2 + 104*b*c*e^2*e^(10*d)*log(F) - 192*e^3*e^(10*d))*e^(10*e*x) - (b^
3*c^3*e^(8*d)*log(F)^3 - 18*b^2*c^2*e*e^(8*d)*log(F)^2 + 104*b*c*e^2*e^(8*
d)*log(F) - 192*e^3*e^(8*d))*e^(8*e*x) - 4*(b^3*c^3*e^(6*d)*log(F)^3 - 18*
b^2*c^2*e*e^(6*d)*log(F)^2 + 104*b*c*e^2*e^(6*d)*log(F) - 192*e^3*e^(6*d))
*e^(6*e*x) - (b^3*c^3*e^(4*d)*log(F)^3 - 18*b^2*c^2*e*e^(4*d)*log(F)^2 ...
```

Giac [F]

$$\int F^{c(a+bx)}\operatorname{csch}(d+ex)\operatorname{sech}^3(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{csch}(ex+d)\operatorname{sech}(ex+d)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)*sech(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csch(e*x + d)*sech(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\operatorname{csch}(d+ex)\operatorname{sech}^3(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\cosh\left(d+e\,x\right)^3\,\sinh\left(d+e\,x\right)}\,dx$$

input
```
int(F^(c*(a + b*x))/(cosh(d + e*x)^3*sinh(d + e*x)),x)
```

output
```
int(F^(c*(a + b*x))/(cosh(d + e*x)^3*sinh(d + e*x)), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\operatorname{csch}(d+ex)\operatorname{sech}^3(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\operatorname{csch}(ex+d)\operatorname{sech}(ex+d)^3\,dx\right)$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)*sech(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*csch(d + e*x)*sech(d + e*x)**3,x)
```

3.135 $\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\operatorname{sech}^3(d+ex)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 312

$$\int F^{c(a+bx)}\text{csch}^2(d+ex)\text{sech}^3(d+ex)\,dx$$
$$= -\frac{2e^{5d+5ex}F^{c(a+bx)}}{e\left(1+e^{2d+2ex}\right)^2}+\frac{8e^{5d+5ex}F^{c(a+bx)}}{e\left(1-e^{2d+2ex}\right)\left(1+e^{2d+2ex}\right)^2}+\frac{e^{5d+5ex}F^{c(a+bx)}(e-bc\log(F))}{e^2\left(1+e^{2d+2ex}\right)}$$
$$-\frac{2bce^{5d+5ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(5+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(7+\frac{bc\log(F)}{e}\right),e^{2d+2ex}\right)\log(F)}{e(5e+bc\log(F))}$$
$$-\frac{e^{5d+5ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(5+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(7+\frac{bc\log(F)}{e}\right),-e^{2d+2ex}\right)\left(3e^2-b^2c^2\log^2(F\right.}{e^2(5e+bc\log(F))}$$

output
```
-2*exp(5*e*x+5*d)*F^(c*(b*x+a))/e/(1+exp(2*e*x+2*d))^2+8*exp(5*e*x+5*d)*F^
(c*(b*x+a))/e/(1-exp(2*e*x+2*d))/(1+exp(2*e*x+2*d))^2+exp(5*e*x+5*d)*F^(c*
(b*x+a))*(e-b*c*ln(F))/e^2/(1+exp(2*e*x+2*d))-2*b*c*exp(5*e*x+5*d)*F^(c*(b
*x+a))*hypergeom([1, 5/2+1/2*b*c*ln(F)/e],[7/2+1/2*b*c*ln(F)/e],exp(2*e*x+
2*d))*ln(F)/e/(b*c*ln(F)+5*e)-exp(5*e*x+5*d)*F^(c*(b*x+a))*hypergeom([1, 5
/2+1/2*b*c*ln(F)/e],[7/2+1/2*b*c*ln(F)/e],-exp(2*e*x+2*d))*(3*e^2-b^2*c^2*
ln(F)^2)/e^2/(b*c*ln(F)+5*e)
```

Mathematica [A] (verified)

Time = 4.09 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.87

$$\int F^{c(a+bx)}\text{csch}^2(d+ex)\text{sech}^3(d+ex)\,dx =$$
$$-\frac{F^{c\left(a-\frac{bd}{e}\right)}\left(2e^{\frac{(d+ex)(e+bc\log(F))}{e}}\coth\left(\frac{1}{2}(d+ex)\right)\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),-e^{2(d}\right.\right.}{}$$

input
```
Integrate[F^(c*(a + b*x))*Csch[d + e*x]^2*Sech[d + e*x]^3,x]
```

output
```
-1/2*(F^(c*(a - (b*d)/e))*(2*E^(((d + e*x)*(e + b*c*Log[F]))/e)*Coth[(d +
e*x)/2]*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e), (3 + (b*c*Log[F])/e)/
2, -E^(2*(d + e*x))]*(3*e^2 - b^2*c^2*Log[F]^2) + (Csch[(d + e*x)/2]^2*(8*
b*c*e*E^(((d + e*x)*(e + b*c*Log[F]))/e)*Hypergeometric2F1[1, (e + b*c*Log
[F])/(2*e), (3 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))]*Log[F]*Sinh[d + e*x]
+ F^((b*c*(d + e*x))/e)*(e + b*c*Log[F])*Sech[d + e*x]^2*(e + 3*e*Cosh[2*(
d + e*x)] + b*c*Log[F]*Sinh[2*(d + e*x)])))/4)*Tanh[(d + e*x)/2])/(e^2*(e
+ b*c*Log[F]))
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^2(d+ex)\text{sech}^3(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6037}$$

$$\int \left(-\frac{12e^{5d+5ex}F^{ac+bcx}}{e^{4(d+ex)}-1}+\frac{4e^{5d+5ex}F^{ac+bcx}}{\left(e^{2(d+ex)}-1\right)^2}+\frac{8e^{5d+5ex}F^{ac+bcx}}{\left(e^{2(d+ex)}+1\right)^2}+\frac{8e^{5d+5ex}F^{ac+bcx}}{\left(e^{2(d+ex)}+1\right)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{12e^{5d+5ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}\left(\frac{bc\log(F)}{e}+5\right),\frac{1}{4}\left(\frac{bc\log(F)}{e}+9\right),e^{4(d+ex)}\right)}{bc\log(F)+5e}+\frac{8e^{5d+5ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(\frac{bc\log(F)}{e}+5\right),\frac{1}{2}\left(\frac{bc\log(F)}{e}+7\right),-e^{2(d+ex)}\right)}{bc\log(F)+5e}+\frac{4e^{5d+5ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(\frac{bc\log(F)}{e}+5\right),\frac{1}{2}\left(\frac{bc\log(F)}{e}+7\right),e^{2(d+ex)}\right)}{bc\log(F)+5e}+\frac{8e^{5d+5ex}F^{ac+bcx}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(\frac{bc\log(F)}{e}+5\right),\frac{1}{2}\left(\frac{bc\log(F)}{e}+7\right),-e^{2(d+ex)}\right)}{bc\log(F)+5e}$$

input
```
Int[F^(c*(a + b*x))*Csch[d + e*x]^2*Sech[d + e*x]^3,x]
```

output
```
(12*E^(5*d + 5*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[1, (5 + (b*c*Log[F])
/e)/4, (9 + (b*c*Log[F])/e)/4, E^(4*(d + e*x))])/(5*e + b*c*Log[F]) + (8*E
^(5*d + 5*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[2, (5 + (b*c*Log[F])/e)/2
, (7 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/(5*e + b*c*Log[F]) + (4*E^(5*
d + 5*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[2, (5 + (b*c*Log[F])/e)/2, (7
 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))])/(5*e + b*c*Log[F]) + (8*E^(5*d + 5
*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[3, (5 + (b*c*Log[F])/e)/2, (7 + (b
*c*Log[F])/e)/2, -E^(2*(d + e*x))])/(5*e + b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Maple [F]

$$\int F^{c(bx+a)} \operatorname{csch}(ex+d)^2 \operatorname{sech}(ex+d)^3\, dx$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)^2*sech(e*x+d)^3,x)
```

output
```
int(F^(c*(b*x+a))*csch(e*x+d)^2*sech(e*x+d)^3,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\operatorname{sech}^3(d+ex)\, dx = \int F^{(bx+a)c} \operatorname{csch}(ex+d)^2 \operatorname{sech}(ex+d)^3\, dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^2*sech(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csch(e*x + d)^2*sech(e*x + d)^3, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\operatorname{sech}^3(d+ex)\, dx = \int F^{c(a+bx)} \operatorname{csch}^2(d+ex) \operatorname{sech}^3(d+ex)\, dx$$

input
```
integrate(F**(c*(b*x+a))*csch(e*x+d)**2*sech(e*x+d)**3,x)
```

output
```
Integral(F**(c*(a + b*x))*csch(d + e*x)**2*sech(d + e*x)**3, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\text{csch}^2(d+ex)\text{sech}^3(d+ex)\,dx = \int F^{(bx+a)c}\,\text{csch}\,(ex+d)^2\,\text{sech}\,(ex+d)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^2*sech(e*x+d)^3,x, algorithm="maxima")
```

output
```
32*((F^(a*c)*b^2*c^2*e^(5*d)*log(F)^2 - 16*F^(a*c)*b*c*e*e^(5*d)*log(F) +
63*F^(a*c)*e^2*e^(5*d))*e^(5*e*x) - 2*(F^(a*c)*b*c*e*e^(3*d)*log(F) - 9*F^
(a*c)*e^2*e^(3*d))*e^(3*e*x) + 2*(5*F^(a*c)*b*c*e*e^d*log(F) - 33*F^(a*c)*
e^2*e^d)*e^(e*x))*F^(b*c*x)/(b^3*c^3*log(F)^3 - 21*b^2*c^2*e*log(F)^2 + 14
3*b*c*e^2*log(F) - 315*e^3 + (b^3*c^3*e^(10*d)*log(F)^3 - 21*b^2*c^2*e*e^(
10*d)*log(F)^2 + 143*b*c*e^2*e^(10*d)*log(F) - 315*e^3*e^(10*d))*e^(10*e*x
) + (b^3*c^3*e^(8*d)*log(F)^3 - 21*b^2*c^2*e*e^(8*d)*log(F)^2 + 143*b*c*e^
2*e^(8*d)*log(F) - 315*e^3*e^(8*d))*e^(8*e*x) - 2*(b^3*c^3*e^(6*d)*log(F)^
3 - 21*b^2*c^2*e*e^(6*d)*log(F)^2 + 143*b*c*e^2*e^(6*d)*log(F) - 315*e^3*e
^(6*d))*e^(6*e*x) - 2*(b^3*c^3*e^(4*d)*log(F)^3 - 21*b^2*c^2*e*e^(4*d)*log
(F)^2 + 143*b*c*e^2*e^(4*d)*log(F) - 315*e^3*e^(4*d))*e^(4*e*x) + (b^3*c^3
*e^(2*d)*log(F)^3 - 21*b^2*c^2*e*e^(2*d)*log(F)^2 + 143*b*c*e^2*e^(2*d)*lo
g(F) - 315*e^3*e^(2*d))*e^(2*e*x)) - 32*integrate(-2*((F^(a*c)*b^2*c^2*e*e
^(3*d)*log(F)^2 + 4*F^(a*c)*b*c*e^2*e^(3*d)*log(F) - 93*F^(a*c)*e^3*e^(3*d
))*e^(3*e*x) - (5*F^(a*c)*b^2*c^2*e*e^d*log(F)^2 - 28*F^(a*c)*b*c*e^2*e^d*
log(F) - 33*F^(a*c)*e^3*e^d)*e^(e*x))*F^(b*c*x)/(b^3*c^3*log(F)^3 - 21*b^2
*c^2*e*log(F)^2 + 143*b*c*e^2*log(F) - 315*e^3 - (b^3*c^3*e^(14*d)*log(F)^
3 - 21*b^2*c^2*e*e^(14*d)*log(F)^2 + 143*b*c*e^2*e^(14*d)*log(F) - 315*e^3
*e^(14*d))*e^(14*e*x) - (b^3*c^3*e^(12*d)*log(F)^3 - 21*b^2*c^2*e*e^(12*d)
*log(F)^2 + 143*b*c*e^2*e^(12*d)*log(F) - 315*e^3*e^(12*d))*e^(12*e*x) ...
```

Giac [F]

$$\int F^{c(a+bx)}\text{csch}^2(d+ex)\text{sech}^3(d+ex)\,dx = \int F^{(bx+a)c}\,\text{csch}\,(ex+d)^2\,\text{sech}\,(ex+d)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^2*sech(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csch(e*x + d)^2*sech(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\operatorname{sech}^3(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{{\cosh\left(d+e\,x\right)}^3\,{\sinh\left(d+e\,x\right)}^2}\,dx$$

input
```
int(F^(c*(a + b*x))/(cosh(d + e*x)^3*sinh(d + e*x)^2),x)
```

output
```
int(F^(c*(a + b*x))/(cosh(d + e*x)^3*sinh(d + e*x)^2), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\operatorname{sech}^3(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\operatorname{csch}(ex+d)^2\operatorname{sech}(ex+d)^3\,dx\right)$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)^2*sech(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*csch(d + e*x)**2*sech(d + e*x)**3,x)
```

3.136 $\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\operatorname{sech}^3(d+ex)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 72

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\operatorname{sech}^3(d+ex)\,dx = -\frac{64e^{6d+6ex}F^{c(a+bx)}\text{Hypergeometric2F1}\left(3,\frac{1}{4}\left(6+\frac{bc\log(F)}{e}\right),\frac{1}{4}\left(10+\frac{bc\log(F)}{e}\right),e^{4d+4ex}\right)}{6e+bc\log(F)}$$

output
```
-64*exp(6*e*x+6*d)*F^(c*(b*x+a))*hypergeom([3, 3/2+1/4*b*c*ln(F)/e],[5/2+1
/4*b*c*ln(F)/e],exp(4*e*x+4*d))/(6*e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 2.45 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.00

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\operatorname{sech}^3(d+ex)\,dx = \frac{F^{c\left(a-\frac{bd}{e}\right)}\left(8e^{\frac{(d+ex)(2e+bc\log(F))}{e}}\coth(d+ex)\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}\left(2+\frac{bc\log(F)}{e}\right),\frac{1}{4}\left(6+\frac{bc\log(F)}{e}\right),e^{4(d+\cdots}\right.\right.}{4e^2}$$

input
```
Integrate[F^(c*(a + b*x))*Csch[d + e*x]^3*Sech[d + e*x]^3,x]
```

output
```
(F^(c*(a - (b*d)/e))*(8*E^(((d + e*x)*(2*e + b*c*Log[F]))/e)*Coth[d + e*x]
*Hypergeometric2F1[1, (2 + (b*c*Log[F])/e)/4, (6 + (b*c*Log[F])/e)/4, E^(4
*(d + e*x))]*(2*e - b*c*Log[F]) - 2*F^((b*c*(d + e*x))/e)*Csch[d + e*x]^2*
(2*e*Coth[2*(d + e*x)] + b*c*Log[F]))*Tanh[d + e*x])/(4*e^2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 594 vs. $2(72) = 144$.

Time = 1.62 (sec) , antiderivative size = 594, normalized size of antiderivative = 8.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {6037, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^3(d+ex)\text{sech}^3(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6037}$$

$$\int \left(\frac{21e^{6d+6ex}F^{ac+bcx}}{2\left(e^{d+ex}-1\right)}-\frac{21e^{6d+6ex}F^{ac+bcx}}{2\left(e^{d+ex}+1\right)}-\frac{12e^{6d+6ex}F^{ac+bcx}}{e^{2d+2ex}+1}-\frac{9e^{6d+6ex}F^{ac+bcx}}{2\left(e^{d+ex}-1\right)^2}-\frac{9e^{6d+6ex}F^{ac+bcx}}{2\left(e^{d+ex}+1\right)^2}-\frac{12e^{6d-}}{(e^{2d-}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{8F^{ac+bcx}\text{Hypergeometric2F1}\left(3,-\frac{bc\log(F)}{2e},1-\frac{bc\log(F)}{2e},-e^{-2(d+ex)}\right)}{bc\log(F)}-\frac{12e^{6d+6ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(\frac{bc\log(F)}{e}+6\right),\frac{1}{2}\left(\frac{bc\log(F)}{e}+8\right),-e^{2(d+ex)}\right)}{bc\log(F)+6e}-\frac{21e^{6d+6ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{e}+6,\frac{bc\log(F)}{e}+7,-e^{d+ex}\right)}{2(bc\log(F)+6e)}-\frac{21e^{6d+6ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{e}+6,\frac{bc\log(F)}{e}+7,e^{d+ex}\right)}{2(bc\log(F)+6e)}-\frac{12e^{6d+6ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(\frac{bc\log(F)}{e}+6\right),\frac{1}{2}\left(\frac{bc\log(F)}{e}+8\right),-e^{2(d+ex)}\right)}{bc\log(F)+6e}-\frac{9e^{6d+6ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{e}+6,\frac{bc\log(F)}{e}+7,-e^{d+ex}\right)}{2(bc\log(F)+6e)}-\frac{9e^{6d+6ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{e}+6,\frac{bc\log(F)}{e}+7,e^{d+ex}\right)}{2(bc\log(F)+6e)}-\frac{e^{6d+6ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(3,\frac{bc\log(F)}{e}+6,\frac{bc\log(F)}{e}+7,-e^{d+ex}\right)}{bc\log(F)+6e}-\frac{e^{6d+6ex}F^{ac+bcx}\text{Hypergeometric2F1}\left(3,\frac{bc\log(F)}{e}+6,\frac{bc\log(F)}{e}+7,e^{d+ex}\right)}{bc\log(F)+6e}$$

input
```
Int[F^(c*(a + b*x))*Csch[d + e*x]^3*Sech[d + e*x]^3,x]
```

output
```
(-8*F^(a*c + b*c*x)*Hypergeometric2F1[3, -1/2*(b*c*Log[F])/e, 1 - (b*c*Log
[F])/(2*e), -E^(-2*(d + e*x))])/(b*c*Log[F]) - (12*E^(6*d + 6*e*x)*F^(a*c
+ b*c*x)*Hypergeometric2F1[1, (6 + (b*c*Log[F])/e)/2, (8 + (b*c*Log[F])/e)
/2, -E^(2*(d + e*x))])/(6*e + b*c*Log[F]) - (21*E^(6*d + 6*e*x)*F^(a*c + b
*c*x)*Hypergeometric2F1[1, 6 + (b*c*Log[F])/e, 7 + (b*c*Log[F])/e, -E^(d +
 e*x)])/(2*(6*e + b*c*Log[F])) - (21*E^(6*d + 6*e*x)*F^(a*c + b*c*x)*Hyper
geometric2F1[1, 6 + (b*c*Log[F])/e, 7 + (b*c*Log[F])/e, E^(d + e*x)])/(2*(
6*e + b*c*Log[F])) - (12*E^(6*d + 6*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1
[2, (6 + (b*c*Log[F])/e)/2, (8 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/(6*
e + b*c*Log[F]) - (9*E^(6*d + 6*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[2,
6 + (b*c*Log[F])/e, 7 + (b*c*Log[F])/e, -E^(d + e*x)])/(2*(6*e + b*c*Log[F
])) - (9*E^(6*d + 6*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[2, 6 + (b*c*Log
[F])/e, 7 + (b*c*Log[F])/e, E^(d + e*x)])/(2*(6*e + b*c*Log[F])) - (E^(6*d
 + 6*e*x)*F^(a*c + b*c*x)*Hypergeometric2F1[3, 6 + (b*c*Log[F])/e, 7 + (b*
c*Log[F])/e, -E^(d + e*x)])/(6*e + b*c*Log[F]) - (E^(6*d + 6*e*x)*F^(a*c +
 b*c*x)*Hypergeometric2F1[3, 6 + (b*c*Log[F])/e, 7 + (b*c*Log[F])/e, E^(d
+ e*x)])/(6*e + b*c*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6037
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && HyperbolicQ[G] && HyperbolicQ[H]
```

Maple [F]

$$\int F^{c(bx+a)} \operatorname{csch}(ex+d)^3 \operatorname{sech}(ex+d)^3 \, dx$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)^3*sech(e*x+d)^3,x)
```

output
```
int(F^(c*(b*x+a))*csch(e*x+d)^3*sech(e*x+d)^3,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\text{csch}^3(d+ex)\text{sech}^3(d+ex)\,dx = \int F^{(bx+a)c}\operatorname{csch}\left(ex+d\right)^3\operatorname{sech}\left(ex+d\right)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^3*sech(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csch(e*x + d)^3*sech(e*x + d)^3, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\text{csch}^3(d+ex)\text{sech}^3(d+ex)\,dx = \int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\operatorname{sech}^3(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*csch(e*x+d)**3*sech(e*x+d)**3,x)
```

output
```
Integral(F**(c*(a + b*x))*csch(d + e*x)**3*sech(d + e*x)**3, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\text{csch}^3(d+ex)\text{sech}^3(d+ex)\,dx = \int F^{(bx+a)c}\operatorname{csch}\left(ex+d\right)^3\operatorname{sech}\left(ex+d\right)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^3*sech(e*x+d)^3,x, algorithm="maxima")
```

output
```
768*(F^(a*c)*b*c*e*e^(2*d)*log(F) + 2*F^(a*c)*e^2*e^(2*d))*integrate(e^(b*
c*x*log(F) + 2*e*x)/(b^2*c^2*log(F)^2 - 16*b*c*e*log(F) + 60*e^2 + (b^2*c^
2*e^(16*d)*log(F)^2 - 16*b*c*e*e^(16*d)*log(F) + 60*e^2*e^(16*d))*e^(16*e*
x) - 4*(b^2*c^2*e^(12*d)*log(F)^2 - 16*b*c*e*e^(12*d)*log(F) + 60*e^2*e^(1
2*d))*e^(12*e*x) + 6*(b^2*c^2*e^(8*d)*log(F)^2 - 16*b*c*e*e^(8*d)*log(F) +
 60*e^2*e^(8*d))*e^(8*e*x) - 4*(b^2*c^2*e^(4*d)*log(F)^2 - 16*b*c*e*e^(4*d
)*log(F) + 60*e^2*e^(4*d))*e^(4*e*x)), x) - 64*(12*F^(a*c)*e*e^(2*e*x + 2*
d) + (F^(a*c)*b*c*e^(6*d)*log(F) - 10*F^(a*c)*e*e^(6*d))*e^(6*e*x))*F^(b*c
*x)/(b^2*c^2*log(F)^2 - 16*b*c*e*log(F) + 60*e^2 - (b^2*c^2*e^(12*d)*log(F
)^2 - 16*b*c*e*e^(12*d)*log(F) + 60*e^2*e^(12*d))*e^(12*e*x) + 3*(b^2*c^2*
e^(8*d)*log(F)^2 - 16*b*c*e*e^(8*d)*log(F) + 60*e^2*e^(8*d))*e^(8*e*x) - 3
*(b^2*c^2*e^(4*d)*log(F)^2 - 16*b*c*e*e^(4*d)*log(F) + 60*e^2*e^(4*d))*e^(
4*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\operatorname{sech}^3(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{csch}\left(ex+d\right)^3\operatorname{sech}\left(ex+d\right)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^3*sech(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csch(e*x + d)^3*sech(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\operatorname{sech}^3(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\cosh\left(d+e\,x\right)^3\sinh\left(d+e\,x\right)^3}\,dx$$

input
```
int(F^(c*(a + b*x))/(cosh(d + e*x)^3*sinh(d + e*x)^3),x)
```

output
```
int(F^(c*(a + b*x))/(cosh(d + e*x)^3*sinh(d + e*x)^3), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\text{csch}^3(d+ex)\text{sech}^3(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\text{csch}(ex+d)^3\,\text{sech}(ex+d)^3\,dx\right)$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)^3*sech(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*csch(d + e*x)**3*sech(d + e*x)**3,x)
```

3.137 $\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\sinh(d+ex)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 113

$$\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\sinh(d+ex)}\,dx = \frac{e^{-d-ex}fF^{c(a+bx)}\sqrt{g\sinh(d+ex)}}{2(e-bc\log(F))\sqrt{f\sinh(d+ex)}}+\frac{e^{d+ex}fF^{c(a+bx)}\sqrt{g\sinh(d+ex)}}{2(e+bc\log(F))\sqrt{f\sinh(d+ex)}}$$

output
```
1/2*exp(-e*x-d)*f*F^(c*(b*x+a))*(g*sinh(e*x+d))^(1/2)/(e-b*c*ln(F))/(f*sin
h(e*x+d))^(1/2)+1/2*exp(e*x+d)*f*F^(c*(b*x+a))*(g*sinh(e*x+d))^(1/2)/(e+b*
c*ln(F))/(f*sinh(e*x+d))^(1/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.60

$$\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\sinh(d+ex)}\,dx = \frac{F^{c(a+bx)}(e\coth(d+ex)-bc\log(F))\sqrt{f\sinh(d+ex)}\sqrt{g\sinh(d+ex)}}{(e-bc\log(F))(e+bc\log(F))}$$

input
```
Integrate[F^(c*(a + b*x))*Sqrt[f*Sinh[d + e*x]]*Sqrt[g*Sinh[d + e*x]],x]
```

output
```
(F^(c*(a + b*x))*(e*Coth[d + e*x] - b*c*Log[F])*Sqrt[f*Sinh[d + e*x]]*Sqrt
[g*Sinh[d + e*x]])/((e - b*c*Log[F])*(e + b*c*Log[F]))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2031, 5997}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\sinh(d+ex)}\,dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{f\sqrt{g\sinh(d+ex)}\int F^{c(a+bx)}\sinh(d+ex)dx}{\sqrt{f\sinh(d+ex)}}$$

$$\Big\downarrow \text{ 5997}$$

$$\frac{f\sqrt{g\sinh(d+ex)}\left(\frac{e\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}\right)}{\sqrt{f\sinh(d+ex)}}$$

input
```
Int[F^(c*(a + b*x))*Sqrt[f*Sinh[d + e*x]]*Sqrt[g*Sinh[d + e*x]],x]
```

output
```
(f*Sqrt[g*Sinh[d + e*x]]*((e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2
*Log[F]^2) - (b*c*F^(c*(a + b*x))*Log[F]*Sinh[d + e*x])/(e^2 - b^2*c^2*Log
[F]^2)))/Sqrt[f*Sinh[d + e*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 5997
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.14

method	result	size
risch	$\frac{\sqrt{f\left(-1+\mathrm{e}^{2ex+2d}\right)\mathrm{e}^{-ex-d}}\,\sqrt{g\left(-1+\mathrm{e}^{2ex+2d}\right)\mathrm{e}^{-ex-d}}\,\left(\ln(F)bc\,\mathrm{e}^{2ex+2d}-bc\ln(F)-\mathrm{e}^{2ex+2d}e-e\right)F^{c(bx+a)}}{2\left(-1+\mathrm{e}^{2ex+2d}\right)(bc\ln(F)-e)(e+bc\ln(F))}$	129

input
```
int(F^(c*(b*x+a))*(f*sinh(e*x+d))^(1/2)*(g*sinh(e*x+d))^(1/2),x,method=_RE
TURNVERBOSE)
```

output
```
1/2*(f*(-1+exp(2*e*x+2*d))*exp(-e*x-d))^(1/2)/(-1+exp(2*e*x+2*d))*(g*(-1+e
xp(2*e*x+2*d))*exp(-e*x-d))^(1/2)*(ln(F)*b*c*exp(2*e*x+2*d)-b*c*ln(F)-exp(
2*e*x+2*d)*e-e)/(b*c*ln(F)-e)/(e+b*c*ln(F))*F^(c*(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(100) = 200.

Time = 0.10 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.85

$$\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\sinh(d+ex)}\,dx$$
$$= \frac{\left(\left(e\cosh\left(ex+d\right)^2-\left(bc\log\left(F\right)-e\right)\sinh\left(ex+d\right)^2-\left(bc\cosh\left(ex+d\right)^2-bc\right)\log\left(F\right)-2\left(bc\cosh\left(ex+\right.\right.\right.\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*(f*sinh(e*x+d))^(1/2)*(g*sinh(e*x+d))^(1/2),x, alg
orithm="fricas")
```

output
```
((e*cosh(e*x + d)^2 - (b*c*log(F) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*x + d
)^2 - b*c)*log(F) - 2*(b*c*cosh(e*x + d)*log(F) - e*cosh(e*x + d))*sinh(e*
x + d) + e)*cosh((b*c*x + a*c)*log(F)) + (e*cosh(e*x + d)^2 - (b*c*log(F)
- e)*sinh(e*x + d)^2 - (b*c*cosh(e*x + d)^2 - b*c)*log(F) - 2*(b*c*cosh(e*
x + d)*log(F) - e*cosh(e*x + d))*sinh(e*x + d) + e)*sinh((b*c*x + a*c)*log
(F)))*sqrt(f*sinh(e*x + d))*sqrt(g*sinh(e*x + d))/(e^2*cosh(e*x + d)^2 - (
b^2*c^2*cosh(e*x + d)^2 - b^2*c^2)*log(F)^2 - (b^2*c^2*log(F)^2 - e^2)*sin
h(e*x + d)^2 - e^2 - 2*(b^2*c^2*cosh(e*x + d)*log(F)^2 - e^2*cosh(e*x + d)
)*sinh(e*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(102) = 204$.

Time = 12.62 (sec) , antiderivative size = 570, normalized size of antiderivative = 5.04

$$\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\sinh(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*(f*sinh(e*x+d))**(1/2)*(g*sinh(e*x+d))**(1/2),x)
```

output
```
Piecewise((F**(a*c - e*x/log(F))*x*sqrt(f*sinh(d + e*x))*sqrt(g*sinh(d + e
*x))/2 + F**(a*c - e*x/log(F))*x*sqrt(f*sinh(d + e*x))*sqrt(g*sinh(d + e*x
))*cosh(d + e*x)/(2*sinh(d + e*x)) + F**(a*c - e*x/log(F))*sqrt(f*sinh(d +
 e*x))*sqrt(g*sinh(d + e*x))*cosh(d + e*x)/(2*e*sinh(d + e*x)), Eq(b, -e/(
c*log(F)))), (F**(a*c + e*x/log(F))*x*sqrt(f*sinh(d + e*x))*sqrt(g*sinh(d
+ e*x))/2 - F**(a*c + e*x/log(F))*x*sqrt(f*sinh(d + e*x))*sqrt(g*sinh(d +
e*x))*cosh(d + e*x)/(2*sinh(d + e*x)) + F**(a*c + e*x/log(F))*sqrt(f*sinh(
d + e*x))*sqrt(g*sinh(d + e*x))*cosh(d + e*x)/(2*e*sinh(d + e*x)), Eq(b, e
/(c*log(F)))), (F**(a*c + b*c*x)*sqrt(f*sinh(e*x + log(-exp(-e*x))))*sqrt(
g*sinh(e*x + log(-exp(-e*x))))/(b*c*log(F)), Eq(d, log(-exp(-e*x)))), (F**
(a*c + b*c*x)*sqrt(f*sinh(e*x + log(exp(-e*x))))*sqrt(g*sinh(e*x + log(exp
(-e*x))))/(b*c*log(F)), Eq(d, log(exp(-e*x)))), (F**(a*c + b*c*x)*b*c*sqrt
(f*sinh(d + e*x))*sqrt(g*sinh(d + e*x))*log(F)*sinh(d + e*x)/(b**2*c**2*lo
g(F)**2*sinh(d + e*x) - e**2*sinh(d + e*x)) - F**(a*c + b*c*x)*e*sqrt(f*si
nh(d + e*x))*sqrt(g*sinh(d + e*x))*cosh(d + e*x)/(b**2*c**2*log(F)**2*sinh
(d + e*x) - e**2*sinh(d + e*x)), True))
```

Maxima [F]

$$\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\sinh(d+ex)}\,dx = \int \sqrt{f\sinh\left(ex+d\right)}\sqrt{g\sinh\left(ex+d\right)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sinh(e*x+d))^(1/2)*(g*sinh(e*x+d))^(1/2),x, alg
orithm="maxima")
```

output
```
integrate(sqrt(f*sinh(e*x + d))*sqrt(g*sinh(e*x + d))*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\sinh(d+ex)}\,dx$$
$$=\int \sqrt{f\sinh\left(ex+d\right)}\sqrt{g\sinh\left(ex+d\right)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sinh(e*x+d))^(1/2)*(g*sinh(e*x+d))^(1/2),x, alg
orithm="giac")
```

output
```
integrate(sqrt(f*sinh(e*x + d))*sqrt(g*sinh(e*x + d))*F^((b*x + a)*c), x)
```

Mupad [B] (verification not implemented)

Time = 2.98 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.50

$$\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\sinh(d+ex)}\,dx$$
$$=\frac{\sqrt{g\,\left(\frac{\mathrm{e}^{d+e\,x}}{2}-\frac{\mathrm{e}^{-d-e\,x}}{2}\right)}\,\left(\frac{F^{a\,c+b\,c\,x}\,\sqrt{f\,\left(\frac{\mathrm{e}^{d+e\,x}}{2}-\frac{\mathrm{e}^{-d-e\,x}}{2}\right)}\,\left(e+b\,c\,\ln\left(F\right)\right)}{e^2-b^2\,c^2\,{\ln\left(F\right)}^2}+\frac{F^{a\,c+b\,c\,x}\,\mathrm{e}^{2\,d+2\,e\,x}\,\sqrt{f\,\left(\frac{\mathrm{e}^{d+e\,x}}{2}-\frac{\mathrm{e}^{-d-e\,x}}{2}\right)}\,\left(e-b\,c\,\ln\left(F\right.\right.}{e^2-b^2\,c^2\,{\ln\left(F\right)}^2}}{\mathrm{e}^{2\,d+2\,e\,x}-1}$$

input
```
int(F^(c*(a + b*x))*(f*sinh(d + e*x))^(1/2)*(g*sinh(d + e*x))^(1/2),x)
```

output
```
((g*(exp(d + e*x)/2 - exp(- d - e*x)/2))^(1/2)*((F^(a*c + b*c*x)*(f*(exp(d
 + e*x)/2 - exp(- d - e*x)/2))^(1/2)*(e + b*c*log(F)))/(e^2 - b^2*c^2*log(
F)^2) + (F^(a*c + b*c*x)*exp(2*d + 2*e*x)*(f*(exp(d + e*x)/2 - exp(- d - e
*x)/2))^(1/2)*(e - b*c*log(F)))/(e^2 - b^2*c^2*log(F)^2)))/(exp(2*d + 2*e*
x) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.48

$$\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\sinh(d+ex)}\,dx = \frac{\sqrt{g}\,f^{bcx+ac+\frac{1}{2}}(-\cosh(ex+d)\,e+\log(f)\sinh(ex+d)\,bc)}{\log(f)^{2}\,b^{2}c^{2}-e^{2}}$$

input

```
int(F^(c*(b*x+a))*(f*sinh(e*x+d))^(1/2)*(g*sinh(e*x+d))^(1/2),x)
```

output

```
(sqrt(g)*f**((2*a*c + 2*b*c*x + 1)/2)*( - cosh(d + e*x)*e + log(f)*sinh(d
+ e*x)*b*c))/(log(f)**2*b**2*c**2 - e**2)
```

3.138 $\int F^{c(a+bx)}\sqrt{g\text{csch}(d+ex)}\sqrt{f\sinh(d+ex)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 44

$$\int F^{c(a+bx)}\sqrt{g\text{csch}(d+ex)}\sqrt{f\sinh(d+ex)}\,dx = \frac{F^{c(a+bx)}\sqrt{g\text{csch}(d+ex)}\sqrt{f\sinh(d+ex)}}{bc\log(F)}$$

output
```
F^(c*(b*x+a))*(g*csch(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2)/b/c/ln(F)
```

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}\sqrt{g\text{csch}(d+ex)}\sqrt{f\sinh(d+ex)}\,dx = \frac{F^{c(a+bx)}\sqrt{g\text{csch}(d+ex)}\sqrt{f\sinh(d+ex)}}{bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sqrt[g*Csch[d + e*x]]*Sqrt[f*Sinh[d + e*x]],x]
```

output
```
(F^(c*(a + b*x))*Sqrt[g*Csch[d + e*x]]*Sqrt[f*Sinh[d + e*x]])/(b*c*Log[F])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\text{csch}(d+ex)}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\sqrt{g\text{csch}(d+ex)}\int F^{c(a+bx)}\sqrt{\text{csch}(d+ex)}\sqrt{f\sinh(d+ex)}dx}{\sqrt{\text{csch}(d+ex)}}$$

$$\Big\downarrow \text{7271}$$

$$\sqrt{\sinh(d+ex)}\sqrt{\text{csch}(d+ex)}\sqrt{f\sinh(d+ex)}\sqrt{g\text{csch}(d+ex)}\int F^{c(a+bx)}\sqrt{\text{csch}(d+ex)}\sqrt{\sinh(d+ex)}dx$$

$$\Big\downarrow \text{7292}$$

$$\sqrt{\sinh(d+ex)}\sqrt{\text{csch}(d+ex)}\sqrt{f\sinh(d+ex)}\sqrt{g\text{csch}(d+ex)}\int F^{ac+bxc}\sqrt{\text{csch}(d+ex)}\sqrt{\sinh(d+ex)}dx$$

$$\Big\downarrow \text{7299}$$

$$\sqrt{\sinh(d+ex)}\sqrt{\text{csch}(d+ex)}\sqrt{f\sinh(d+ex)}\sqrt{g\text{csch}(d+ex)}\int F^{ac+bxc}\sqrt{\text{csch}(d+ex)}\sqrt{\sinh(d+ex)}dx$$

input
```
Int[F^(c*(a + b*x))*Sqrt[g*Csch[d + e*x]]*Sqrt[f*Sinh[d + e*x]],x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

method	result	size
orering	$\frac{F^{c(bx+a)}\sqrt{g\ \operatorname{csch}(ex+d)}\,\sqrt{f\sinh(ex+d)}}{bc\ln(F)}$	41
risch	$\frac{\sqrt{f\left(-1+\mathrm{e}^{2ex+2d}\right)\mathrm{e}^{-ex-d}}\,\sqrt{\frac{g\,\mathrm{e}^{ex+d}}{-1+\mathrm{e}^{2ex+2d}}}\,F^{c(bx+a)}}{\ln(F)bc}$	68

input
```
int(F^(c*(b*x+a))*(g*csch(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2),x,method=_RE
TURNVERBOSE)
```

output
```
F^(c*(b*x+a))*(g*csch(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2)/b/c/ln(F)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(40) = 80$.

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.34

$$\int F^{c(a+bx)}\sqrt{g\mathrm{csch}(d+ex)}\sqrt{f\sinh(d+ex)}\,dx$$
$$= \frac{\sqrt{2}\sqrt{f\sinh{(ex+d)}}\sqrt{\frac{g\cosh(ex+d)+g\sinh(ex+d)}{\cosh(ex+d)^2+2\cosh(ex+d)\sinh(ex+d)+\sinh(ex+d)^2-1}}(\cosh{((bcx+ac)\log{(F)})}+\sinh{((bcx}}{bc\log{(F)}}$$

input
```
integrate(F^(c*(b*x+a))*(g*csch(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2),x, alg
orithm="fricas")
```

output
```
sqrt(2)*sqrt(f*sinh(e*x + d))*sqrt((g*cosh(e*x + d) + g*sinh(e*x + d))/(co
sh(e*x + d)^2 + 2*cosh(e*x + d)*sinh(e*x + d) + sinh(e*x + d)^2 - 1))*(cos
h((b*c*x + a*c)*log(F)) + sinh((b*c*x + a*c)*log(F)))/(b*c*log(F))
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{g\text{csch}(d+ex)}\sqrt{f\sinh(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(g*csch(e*x+d))**(1/2)*(f*sinh(e*x+d))**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int F^{c(a+bx)}\sqrt{g\text{csch}(d+ex)}\sqrt{f\sinh(d+ex)}\,dx = \frac{F^{bcx}F^{ac}\sqrt{f}\sqrt{g}}{bc\log(F)}$$

input
```
integrate(F^(c*(b*x+a))*(g*csch(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2),x, alg
orithm="maxima")
```

output
```
F^(b*c*x)*F^(a*c)*sqrt(f)*sqrt(g)/(b*c*log(F))
```

Giac [F]

$$\int F^{c(a+bx)}\sqrt{g\text{csch}(d+ex)}\sqrt{f\sinh(d+ex)}\,dx = \int \sqrt{g\operatorname{csch}(ex+d)}\sqrt{f\sinh(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*csch(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2),x, alg
orithm="giac")
```

output
```
integrate(sqrt(g*csch(e*x + d))*sqrt(f*sinh(e*x + d))*F^((b*x + a)*c), x)
```

Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.61

$$\int F^{c(a+bx)}\sqrt{g\mathrm{csch}(d+ex)}\sqrt{f\sinh(d+ex)}\,dx$$
$$=\frac{F^{a\,c+b\,c\,x}\,\sqrt{f\,\left(\frac{\mathrm{e}^{d+e\,x}}{2}-\frac{\mathrm{e}^{-d-e\,x}}{2}\right)}\,\sqrt{\frac{g}{\frac{\mathrm{e}^{d+e\,x}}{2}-\frac{\mathrm{e}^{-d-e\,x}}{2}}}}{b\,c\,\ln\left(F\right)}$$

input
```
int(F^(c*(a + b*x))*(f*sinh(d + e*x))^(1/2)*(g/sinh(d + e*x))^(1/2),x)
```

output
```
(F^(a*c + b*c*x)*(f*(exp(d + e*x)/2 - exp(- d - e*x)/2))^(1/2)*(g/(exp(d +
 e*x)/2 - exp(- d - e*x)/2))^(1/2))/(b*c*log(F))
```

Reduce [F]

$$\int F^{c(a+bx)}\sqrt{g\mathrm{csch}(d+ex)}\sqrt{f\sinh(d+ex)}\,dx$$
$$=\sqrt{g}\,f^{ac+\frac{1}{2}}\left(\int f^{bcx}\sqrt{\sinh\left(ex+d\right)}\,\sqrt{\mathrm{csch}\left(ex+d\right)}dx\right)$$

input
```
int(F^(c*(b*x+a))*(g*csch(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2),x)
```

output
```
sqrt(g)*f**((2*a*c + 1)/2)*int(f**(b*c*x)*sqrt(sinh(d + e*x))*sqrt(csch(d
+ e*x)),x)
```

3.139 $\int F^{c(a+bx)}\sqrt{f\mathrm{csch}(d+ex)}\sqrt{g\mathrm{csch}(d+ex)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 92

$$\int F^{c(a+bx)}\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\operatorname{csch}(d+ex)}\,dx = -\frac{2e^{d+ex}fF^{c(a+bx)}\sqrt{g\operatorname{csch}(d+ex)}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),e^{2d+2ex}\right)}{\sqrt{f\operatorname{csch}(d+ex)}(e+bc\log(F))}$$

output
```
-2*exp(e*x+d)*f*F^(c*(b*x+a))*(g*csch(e*x+d))^(1/2)*hypergeom([1, 1/2*(e+b
*c*ln(F))/e],[3/2+1/2*b*c*ln(F)/e],exp(2*e*x+2*d))/(f*csch(e*x+d))^(1/2)/(
e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.93

$$\int F^{c(a+bx)}\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\operatorname{csch}(d+ex)}\,dx = \frac{fF^{c(a+bx)}\operatorname{csch}(d)\sqrt{g\operatorname{csch}(d+ex)}}{bc\sqrt{f\operatorname{csch}(d+ex)}\log(F)} + \frac{2fF^{c(a+bx)}\sqrt{g\operatorname{csch}(d+ex)}(\cosh(d)+\sinh(d))\left(-1+\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{e},1+\frac{bc\log(F)}{e},-e\right.\right.}{bc\left(-1+e^d\right)\left(1+e^d\right)\sqrt{f\operatorname{csch}(d+\right.}$$

input
```
Integrate[F^(c*(a + b*x))*Sqrt[f*Csch[d + e*x]]*Sqrt[g*Csch[d + e*x]],x]
```

output
```
(f*F^(c*(a + b*x))*Csch[d]*Sqrt[g*Csch[d + e*x]])/(b*c*Sqrt[f*Csch[d + e*x
]]*Log[F]) + (2*f*F^(c*(a + b*x))*Sqrt[g*Csch[d + e*x]]*(Cosh[d] + Sinh[d]
)*(-1 + Hypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e, -E^(d + e
*x)]*Sinh[d] - Hypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e, E^
(d + e*x)]*Sinh[d]))/(b*c*(-1 + E^d)*(1 + E^d)*Sqrt[f*Csch[d + e*x]]*Log[F
])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2031, 6016}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\sqrt{f\text{csch}(d+ex)}\sqrt{g\text{csch}(d+ex)}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{f\sqrt{g\text{csch}(d+ex)}\int F^{c(a+bx)}\text{csch}(d+ex)dx}{\sqrt{f\text{csch}(d+ex)}}$$

$$\Big\downarrow \text{6016}$$

$$-\frac{2fe^{d+ex}F^{c(a+bx)}\sqrt{g\text{csch}(d+ex)}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),e^{2(d+ex)}\right)}{(bc\log(F)+e)\sqrt{f\text{csch}(d+ex)}}$$

input
```
Int[F^(c*(a + b*x))*Sqrt[f*Csch[d + e*x]]*Sqrt[g*Csch[d + e*x]],x]
```

output
```
(-2*E^(d + e*x)*f*F^(c*(a + b*x))*Sqrt[g*Csch[d + e*x]]*Hypergeometric2F1[
1, (e + b*c*Log[F])/(2*e), (3 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))])/(Sqrt
[f*Csch[d + e*x]]*(e + b*c*Log[F]))
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 6016
```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(-2)^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)),
E^(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)}\sqrt{f\ \operatorname{csch}(ex+d)}\ \sqrt{g\ \operatorname{csch}(ex+d)}dx$$

input
```
int(F^(c*(b*x+a))*(f*csch(e*x+d))^(1/2)*(g*csch(e*x+d))^(1/2),x)
```

output
```
int(F^(c*(b*x+a))*(f*csch(e*x+d))^(1/2)*(g*csch(e*x+d))^(1/2),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\sqrt{f\text{csch}(d+ex)}\sqrt{g\text{csch}(d+ex)}\,dx$$
$$=\int\sqrt{f\operatorname{csch}(ex+d)}\sqrt{g\operatorname{csch}(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(e*x+d))^(1/2)*(g*csch(e*x+d))^(1/2),x, alg
orithm="fricas")
```

output
```
integral(sqrt(f*csch(e*x + d))*sqrt(g*csch(e*x + d))*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\sqrt{f\text{csch}(d+ex)}\sqrt{g\text{csch}(d+ex)}\,dx = \int F^{c(a+bx)}\sqrt{f\,\text{csch}\,(d+ex)}\sqrt{g\,\text{csch}\,(d+ex)}\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*csch(e*x+d))**(1/2)*(g*csch(e*x+d))**(1/2),x)
```

output
```
Integral(F**(c*(a + b*x))*sqrt(f*csch(d + e*x))*sqrt(g*csch(d + e*x)), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\sqrt{f\text{csch}(d+ex)}\sqrt{g\text{csch}(d+ex)}\,dx = \int \sqrt{f\,\text{csch}\,(ex+d)}\sqrt{g\,\text{csch}\,(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(e*x+d))^(1/2)*(g*csch(e*x+d))^(1/2),x, alg
orithm="maxima")
```

output
```
integrate(sqrt(f*csch(e*x + d))*sqrt(g*csch(e*x + d))*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\sqrt{f\text{csch}(d+ex)}\sqrt{g\text{csch}(d+ex)}\,dx = \int \sqrt{f\,\text{csch}\,(ex+d)}\sqrt{g\,\text{csch}\,(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(e*x+d))^(1/2)*(g*csch(e*x+d))^(1/2),x, alg
orithm="giac")
```

output
```
integrate(sqrt(f*csch(e*x + d))*sqrt(g*csch(e*x + d))*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{f\text{csch}(d+ex)}\sqrt{g\text{csch}(d+ex)}\,dx$$
$$=\int F^{c\,(a+b\,x)}\,\sqrt{\frac{f}{\sinh\left(d+e\,x\right)}}\,\sqrt{\frac{g}{\sinh\left(d+e\,x\right)}}\,dx$$

input
```
int(F^(c*(a + b*x))*(f/sinh(d + e*x))^(1/2)*(g/sinh(d + e*x))^(1/2),x)
```

output
```
int(F^(c*(a + b*x))*(f/sinh(d + e*x))^(1/2)*(g/sinh(d + e*x))^(1/2), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\sqrt{f\text{csch}(d+ex)}\sqrt{g\text{csch}(d+ex)}\,dx=\sqrt{g}\,f^{ac+\frac{1}{2}}\left(\int f^{bcx}\text{csch}(ex+d)\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*csch(e*x+d))^(1/2)*(g*csch(e*x+d))^(1/2),x)
```

output
```
sqrt(g)*f**((2*a*c + 1)/2)*int(f**(b*c*x)*csch(d + e*x),x)
```

3.140 $\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\cosh(d+ex)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 113

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\cosh(d+ex)}\,dx = -\frac{e^{-d-ex}fF^{c(a+bx)}\sqrt{g\cosh(d+ex)}}{2\sqrt{f\cosh(d+ex)}(e-bc\log(F))}+\frac{e^{d+ex}fF^{c(a+bx)}\sqrt{g\cosh(d+ex)}}{2\sqrt{f\cosh(d+ex)}(e+bc\log(F))}$$

output
```
-1/2*exp(-e*x-d)*f*F^(c*(b*x+a))*(g*cosh(e*x+d))^(1/2)/(f*cosh(e*x+d))^(1/
2)/(e-b*c*ln(F))+1/2*exp(e*x+d)*f*F^(c*(b*x+a))*(g*cosh(e*x+d))^(1/2)/(f*c
osh(e*x+d))^(1/2)/(e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.66

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\cosh(d+ex)}\,dx = \frac{fF^{c(a+bx)}\sqrt{g\cosh(d+ex)}(-bc\cosh(d+ex)\log(F)+e\sinh(d+ex))}{\sqrt{f\cosh(d+ex)}(e-bc\log(F))(e+bc\log(F))}$$

input
```
Integrate[F^(c*(a + b*x))*Sqrt[f*Cosh[d + e*x]]*Sqrt[g*Cosh[d + e*x]],x]
```

output
```
(f*F^(c*(a + b*x))*Sqrt[g*Cosh[d + e*x]]*(-(b*c*Cosh[d + e*x]*Log[F]) + e*
Sinh[d + e*x]))/(Sqrt[f*Cosh[d + e*x]]*(e - b*c*Log[F])*(e + b*c*Log[F]))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2031, 5998}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\cosh(d+ex)}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{f\sqrt{g\cosh(d+ex)}\int F^{c(a+bx)}\cosh(d+ex)dx}{\sqrt{f\cosh(d+ex)}}$$

$$\Big\downarrow \text{5998}$$

$$\frac{f\sqrt{g\cosh(d+ex)}\left(\frac{e\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}\right)}{\sqrt{f\cosh(d+ex)}}$$

input
```
Int[F^(c*(a + b*x))*Sqrt[f*Cosh[d + e*x]]*Sqrt[g*Cosh[d + e*x]],x]
```

output
```
(f*Sqrt[g*Cosh[d + e*x]]*(-((b*c*F^(c*(a + b*x))*Cosh[d + e*x]*Log[F])/(e^
2 - b^2*c^2*Log[F]^2)) + (e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*
Log[F]^2)))/Sqrt[f*Cosh[d + e*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 5998
```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

method	result	size
risch	$\frac{\sqrt{f\left(1+{\mathrm e}^{2 e x +2 d}\right) {\mathrm e}^{-e x -d}}\, \sqrt{g\left(1+{\mathrm e}^{2 e x +2 d}\right) {\mathrm e}^{-e x -d}}\, \left(\ln \left(F \right) b c \,{\mathrm e}^{2 e x +2 d}+b c \ln \left(F \right)-{\mathrm e}^{2 e x +2 d} e +e \right) F^{c \left(b x +a \right)}}{2 \left(1+{\mathrm e}^{2 e x +2 d}\right) \left(b c \ln \left(F \right)-e \right) \left(e +b c \ln \left(F \right)\right)}$	126

input
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*cosh(e*x+d))^(1/2),x,method=_RE
TURNVERBOSE)
```

output
```
1/2*(f*(1+exp(2*e*x+2*d))*exp(-e*x-d))^(1/2)/(1+exp(2*e*x+2*d))*(g*(1+exp(
2*e*x+2*d))*exp(-e*x-d))^(1/2)*(ln(F)*b*c*exp(2*e*x+2*d)+b*c*ln(F)-exp(2*e
*x+2*d)*e+e)/(b*c*ln(F)-e)/(e+b*c*ln(F))*F^(c*(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(100) = 200$.

Time = 0.10 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.84

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\cosh(d+ex)}\,dx$$
$$= \frac{\sqrt{f\cosh(ex+d)}\sqrt{g\cosh(ex+d)}\left(\left(e\cosh(ex+d)^2-(bc\log(F)-e)\sinh(ex+d)^2-\left(bc\cosh(ex+\right.\right.\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*cosh(e*x+d))^(1/2),x, alg
orithm="fricas")
```

output
```
sqrt(f*cosh(e*x + d))*sqrt(g*cosh(e*x + d))*((e*cosh(e*x + d)^2 - (b*c*log
(F) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*x + d)^2 + b*c)*log(F) - 2*(b*c*cos
h(e*x + d)*log(F) - e*cosh(e*x + d))*sinh(e*x + d) - e)*cosh((b*c*x + a*c)
*log(F)) + (e*cosh(e*x + d)^2 - (b*c*log(F) - e)*sinh(e*x + d)^2 - (b*c*co
sh(e*x + d)^2 + b*c)*log(F) - 2*(b*c*cosh(e*x + d)*log(F) - e*cosh(e*x + d
))*sinh(e*x + d) - e)*sinh((b*c*x + a*c)*log(F)))/(e^2*cosh(e*x + d)^2 - (
b^2*c^2*cosh(e*x + d)^2 + b^2*c^2)*log(F)^2 - (b^2*c^2*log(F)^2 - e^2)*sin
h(e*x + d)^2 + e^2 - 2*(b^2*c^2*cosh(e*x + d)*log(F)^2 - e^2*cosh(e*x + d)
)*sinh(e*x + d))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.41 (sec) , antiderivative size = 644, normalized size of antiderivative = 5.70

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\cosh(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*(f*cosh(e*x+d))**(1/2)*(g*cosh(e*x+d))**(1/2),x)
```

output
```
Piecewise((F**(a*c - e*x/log(F))*x*sqrt(f*cosh(d + e*x))*sqrt(g*cosh(d + e
*x))*sinh(d + e*x)/(2*cosh(d + e*x)) + F**(a*c - e*x/log(F))*x*sqrt(f*cosh
(d + e*x))*sqrt(g*cosh(d + e*x))/2 + F**(a*c - e*x/log(F))*sqrt(f*cosh(d +
 e*x))*sqrt(g*cosh(d + e*x))*sinh(d + e*x)/(e*cosh(d + e*x)) + F**(a*c - e
*x/log(F))*sqrt(f*cosh(d + e*x))*sqrt(g*cosh(d + e*x))/(2*e), Eq(b, -e/(c*
log(F)))), (-F**(a*c + e*x/log(F))*x*sqrt(f*cosh(d + e*x))*sqrt(g*cosh(d +
 e*x))*sinh(d + e*x)/(2*cosh(d + e*x)) + F**(a*c + e*x/log(F))*x*sqrt(f*co
sh(d + e*x))*sqrt(g*cosh(d + e*x))/2 + F**(a*c + e*x/log(F))*sqrt(f*cosh(d
 + e*x))*sqrt(g*cosh(d + e*x))*sinh(d + e*x)/(e*cosh(d + e*x)) - F**(a*c +
 e*x/log(F))*sqrt(f*cosh(d + e*x))*sqrt(g*cosh(d + e*x))/(2*e), Eq(b, e/(c
*log(F)))), (F**(a*c + b*c*x)*sqrt(f*cosh(e*x + log(-I*exp(-e*x))))*sqrt(g
*cosh(e*x + log(-I*exp(-e*x))))/(b*c*log(F)), Eq(d, log(-I*exp(-e*x)))), (
F**(a*c + b*c*x)*sqrt(f*cosh(e*x + log(I*exp(-e*x))))*sqrt(g*cosh(e*x + lo
g(I*exp(-e*x))))/(b*c*log(F)), Eq(d, log(I*exp(-e*x)))), (F**(a*c + b*c*x)
*b*c*sqrt(f*cosh(d + e*x))*sqrt(g*cosh(d + e*x))*log(F)*cosh(d + e*x)/(b**
2*c**2*log(F)**2*cosh(d + e*x) - e**2*cosh(d + e*x)) - F**(a*c + b*c*x)*e*
sqrt(f*cosh(d + e*x))*sqrt(g*cosh(d + e*x))*sinh(d + e*x)/(b**2*c**2*log(F
)**2*cosh(d + e*x) - e**2*cosh(d + e*x)), True))
```

Maxima [F]

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\cosh(d+ex)}\,dx = \int \sqrt{f\cosh(ex+d)}\sqrt{g\cosh(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*cosh(e*x+d))^(1/2),x, alg
orithm="maxima")
```

output
```
integrate(sqrt(f*cosh(e*x + d))*sqrt(g*cosh(e*x + d))*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\cosh(d+ex)}\,dx$$
$$=\int \sqrt{f\cosh\left(ex+d\right)}\sqrt{g\cosh\left(ex+d\right)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*cosh(e*x+d))^(1/2),x, alg
orithm="giac")
```

output
```
integrate(sqrt(f*cosh(e*x + d))*sqrt(g*cosh(e*x + d))*F^((b*x + a)*c), x)
```

Mupad [B] (verification not implemented)

Time = 2.80 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.51

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\cosh(d+ex)}\,dx =$$
$$-\frac{\sqrt{g\,\left(\frac{{\mathrm{e}}^{d+e\,x}}{2}+\frac{{\mathrm{e}}^{-d-e\,x}}{2}\right)}\,\left(\frac{F^{a\,c+b\,c\,x}\,\sqrt{f\,\left(\frac{{\mathrm{e}}^{d+e\,x}}{2}+\frac{{\mathrm{e}}^{-d-e\,x}}{2}\right)}\,\left(e+b\,c\,\ln\left(F\right)\right)}{e^2-b^2\,c^2\,{\ln\left(F\right)}^2}-\frac{F^{a\,c+b\,c\,x}\,{\mathrm{e}}^{2\,d+2\,e\,x}\,\sqrt{f\,\left(\frac{{\mathrm{e}}^{d+e\,x}}{2}+\frac{{\mathrm{e}}^{-d-e\,x}}{2}\right)}\,\left(e-b\,c\,\ln\left(F\right)\right)}{e^2-b^2\,c^2\,{\ln\left(F\right)}^2}\right)}{{\mathrm{e}}^{2\,d+2\,e\,x}+1}$$

input
```
int(F^(c*(a + b*x))*(f*cosh(d + e*x))^(1/2)*(g*cosh(d + e*x))^(1/2),x)
```

output
```
-((g*(exp(d + e*x)/2 + exp(- d - e*x)/2))^(1/2)*((F^(a*c + b*c*x)*(f*(exp(
d + e*x)/2 + exp(- d - e*x)/2))^(1/2)*(e + b*c*log(F)))/(e^2 - b^2*c^2*log
(F)^2) - (F^(a*c + b*c*x)*exp(2*d + 2*e*x)*(f*(exp(d + e*x)/2 + exp(- d -
e*x)/2))^(1/2)*(e - b*c*log(F)))/(e^2 - b^2*c^2*log(F)^2)))/(exp(2*d + 2*e
*x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.48

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\cosh(d+ex)}\,dx$$
$$=\frac{\sqrt{g}\,f^{bcx+ac+\frac{1}{2}}(\cosh(ex+d)\log(f)\,bc-\sinh(ex+d)\,e)}{\log(f)^2\,b^2c^2-e^2}$$

input
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*cosh(e*x+d))^(1/2),x)
```

output
```
(sqrt(g)*f**((2*a*c + 2*b*c*x + 1)/2)*(cosh(d + e*x)*log(f)*b*c - sinh(d +
 e*x)*e))/(log(f)**2*b**2*c**2 - e**2)
```

3.141 $\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 44

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx = \frac{F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\text{sech}(d+ex)}}{bc\log(F)}$$

output
```
F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*sech(e*x+d))^(1/2)/b/c/ln(F)
```

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx = \frac{F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\text{sech}(d+ex)}}{bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sqrt[f*Cosh[d + e*x]]*Sqrt[g*Sech[d + e*x]],x]
```

output
```
(F^(c*(a + b*x))*Sqrt[f*Cosh[d + e*x]]*Sqrt[g*Sech[d + e*x]])/(b*c*Log[F])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\sqrt{f\cosh(d+ex)}\int F^{c(a+bx)}\sqrt{\cosh(d+ex)}\sqrt{g\text{sech}(d+ex)}dx}{\sqrt{\cosh(d+ex)}}$$

$$\Big\downarrow \text{7271}$$

$$\sqrt{\cosh(d+ex)}\sqrt{\text{sech}(d+ex)}\sqrt{f\cosh(d+ex)}\sqrt{g\text{sech}(d+ex)}\int F^{c(a+bx)}\sqrt{\cosh(d+ex)}\sqrt{\text{sech}(d+ex)}dx$$

$$\Big\downarrow \text{7292}$$

$$\sqrt{\cosh(d+ex)}\sqrt{\text{sech}(d+ex)}\sqrt{f\cosh(d+ex)}\sqrt{g\text{sech}(d+ex)}\int F^{ac+bxc}\sqrt{\cosh(d+ex)}\sqrt{\text{sech}(d+ex)}dx$$

$$\Big\downarrow \text{7299}$$

$$\sqrt{\cosh(d+ex)}\sqrt{\text{sech}(d+ex)}\sqrt{f\cosh(d+ex)}\sqrt{g\text{sech}(d+ex)}\int F^{ac+bxc}\sqrt{\cosh(d+ex)}\sqrt{\text{sech}(d+ex)}dx$$

input
```
Int[F^(c*(a + b*x))*Sqrt[f*Cosh[d + e*x]]*Sqrt[g*Sech[d + e*x]],x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

method	result	size
orering	$\frac{F^{c(bx+a)}\sqrt{f\cosh(ex+d)}\,\sqrt{g\,\operatorname{sech}(ex+d)}}{bc\ln(F)}$	41
risch	$\frac{\sqrt{f\left(1+e^{2ex+2d}\right)e^{-ex-d}}\,\sqrt{\frac{g\,e^{ex+d}}{1+e^{2ex+2d}}}\,F^{c(bx+a)}}{bc\ln(F)}$	68

input
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*sech(e*x+d))^(1/2),x,method=_RE
TURNVERBOSE)
```

output
```
F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*sech(e*x+d))^(1/2)/b/c/ln(F)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(40) = 80.

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.34

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx = \frac{\sqrt{2}\sqrt{f\cosh\left(ex+d\right)}\sqrt{\frac{g\cosh(ex+d)+g\sinh(ex+d)}{\cosh(ex+d)^2+2\cosh(ex+d)\sinh(ex+d)+\sinh(ex+d)^2+1}}\left(\cosh\left((bcx+ac)\log\left(F\right)\right)+\sinh\left((bcx\right.\right.}{bc\log\left(F\right)}$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*sech(e*x+d))^(1/2),x, alg
orithm="fricas")
```

output
```
sqrt(2)*sqrt(f*cosh(e*x + d))*sqrt((g*cosh(e*x + d) + g*sinh(e*x + d))/(co
sh(e*x + d)^2 + 2*cosh(e*x + d)*sinh(e*x + d) + sinh(e*x + d)^2 + 1))*(cos
h((b*c*x + a*c)*log(F)) + sinh((b*c*x + a*c)*log(F)))/(b*c*log(F))
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(f*cosh(e*x+d))**(1/2)*(g*sech(e*x+d))**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx = \frac{F^{bcx}F^{ac}\sqrt{f}\sqrt{g}}{bc\log(F)}$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*sech(e*x+d))^(1/2),x, alg
orithm="maxima")
```

output
```
F^(b*c*x)*F^(a*c)*sqrt(f)*sqrt(g)/(b*c*log(F))
```

Giac [F]

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx = \int \sqrt{f\cosh(ex+d)}\sqrt{g\,\text{sech}(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*sech(e*x+d))^(1/2),x, alg
orithm="giac")
```

output
```
integrate(sqrt(f*cosh(e*x + d))*sqrt(g*sech(e*x + d))*F^((b*x + a)*c), x)
```

Mupad [B] (verification not implemented)

Time = 2.59 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.61

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx = \frac{F^{a\,c+b\,c\,x}\,\sqrt{f\,\left(\frac{\mathrm{e}^{d+e\,x}}{2}+\frac{\mathrm{e}^{-d-e\,x}}{2}\right)}\,\sqrt{\frac{g}{\frac{\mathrm{e}^{d+e\,x}}{2}+\frac{\mathrm{e}^{-d-e\,x}}{2}}}}{b\,c\,\ln\left(F\right)}$$

input
```
int(F^(c*(a + b*x))*(f*cosh(d + e*x))^(1/2)*(g/cosh(d + e*x))^(1/2),x)
```

output
```
(F^(a*c + b*c*x)*(f*(exp(d + e*x)/2 + exp(- d - e*x)/2))^(1/2)*(g/(exp(d +
 e*x)/2 + exp(- d - e*x)/2))^(1/2))/(b*c*log(F))
```

Reduce [F]

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx = \sqrt{g}\,f^{ac+\frac{1}{2}}\left(\int f^{bcx}\sqrt{\text{sech}\left(ex+d\right)}\,\sqrt{\cosh\left(ex+d\right)}dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*sech(e*x+d))^(1/2),x)
```

output
```
sqrt(g)*f**((2*a*c + 1)/2)*int(f**(b*c*x)*sqrt(sech(d + e*x))*sqrt(cosh(d
+ e*x)),x)
```

3.142 $\int F^{c(a+bx)}\sqrt{f\text{sech}(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 94

$$\int F^{c(a+bx)}\sqrt{f\text{sech}(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx = \frac{2e^{d+ex}fF^{c(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),-e^{2d+2ex}\right)\sqrt{g\text{sech}(d+ex)}}{(e+bc\log(F))\sqrt{f\text{sech}(d+ex)}}$$

output
```
2*exp(e*x+d)*f*F^(c*(b*x+a))*hypergeom([1, 1/2*(e+b*c*ln(F))/e],[3/2+1/2*b
*c*ln(F)/e],-exp(2*e*x+2*d))*(g*sech(e*x+d))^(1/2)/(e+b*c*ln(F))/(f*sech(e
*x+d))^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

$$\int F^{c(a+bx)}\sqrt{f\text{sech}(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx = \frac{\left(1+e^{2(d+ex)}\right)F^{c(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),-e^{2(d+ex)}\right)\sqrt{f\text{sech}(d+ex)}\sqrt{}}{e+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sqrt[f*Sech[d + e*x]]*Sqrt[g*Sech[d + e*x]],x]
```

output
```
((1 + E^(2*(d + e*x)))*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F
])/(2*e), (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))]*Sqrt[f*Sech[d + e*x]]*
Sqrt[g*Sech[d + e*x]])/(e + b*c*Log[F])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2031, 6015}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\sqrt{f\text{sech}(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{f\sqrt{g\text{sech}(d+ex)}\int F^{c(a+bx)}\text{sech}(d+ex)dx}{\sqrt{f\text{sech}(d+ex)}}$$

$$\Big\downarrow \text{6015}$$

$$\frac{2fe^{d+ex}F^{c(a+bx)}\sqrt{g\text{sech}(d+ex)}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),-e^{2(d+ex)}\right)}{(bc\log(F)+e)\sqrt{f\text{sech}(d+ex)}}$$

input
```
Int[F^(c*(a + b*x))*Sqrt[f*Sech[d + e*x]]*Sqrt[g*Sech[d + e*x]],x]
```

output
```
(2*E^(d + e*x)*f*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*
e), (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))]*Sqrt[g*Sech[d + e*x]])/((e +
 b*c*Log[F])*Sqrt[f*Sech[d + e*x]])
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 6015
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)}\sqrt{f\ \mathrm{sech}\left(ex+d\right)}\ \sqrt{g\ \mathrm{sech}\left(ex+d\right)}dx$$

input
```
int(F^(c*(b*x+a))*(f*sech(e*x+d))^(1/2)*(g*sech(e*x+d))^(1/2),x)
```

output
```
int(F^(c*(b*x+a))*(f*sech(e*x+d))^(1/2)*(g*sech(e*x+d))^(1/2),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\sqrt{f\mathrm{sech}(d+ex)}\sqrt{g\mathrm{sech}(d+ex)}\,dx = \int \sqrt{f\,\mathrm{sech}\left(ex+d\right)}\sqrt{g\,\mathrm{sech}\left(ex+d\right)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sech(e*x+d))^(1/2)*(g*sech(e*x+d))^(1/2),x, alg
orithm="fricas")
```

output
```
integral(sqrt(f*sech(e*x + d))*sqrt(g*sech(e*x + d))*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\sqrt{f\text{sech}(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx = \int F^{c(a+bx)}\sqrt{f\operatorname{sech}(d+ex)}\sqrt{g\operatorname{sech}(d+ex)}\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*sech(e*x+d))**(1/2)*(g*sech(e*x+d))**(1/2),x)
```

output
```
Integral(F**(c*(a + b*x))*sqrt(f*sech(d + e*x))*sqrt(g*sech(d + e*x)), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\sqrt{f\text{sech}(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx = \int \sqrt{f\operatorname{sech}(ex+d)}\sqrt{g\operatorname{sech}(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sech(e*x+d))^(1/2)*(g*sech(e*x+d))^(1/2),x, alg
orithm="maxima")
```

output
```
integrate(sqrt(f*sech(e*x + d))*sqrt(g*sech(e*x + d))*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\sqrt{f\text{sech}(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx = \int \sqrt{f\operatorname{sech}(ex+d)}\sqrt{g\operatorname{sech}(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sech(e*x+d))^(1/2)*(g*sech(e*x+d))^(1/2),x, alg
orithm="giac")
```

output
```
integrate(sqrt(f*sech(e*x + d))*sqrt(g*sech(e*x + d))*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{f\text{sech}(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx = \int F^{c\,(a+b\,x)}\sqrt{\frac{f}{\cosh\left(d+e\,x\right)}}\sqrt{\frac{g}{\cosh\left(d+e\,x\right)}}\,dx$$

input
```
int(F^(c*(a + b*x))*(f/cosh(d + e*x))^(1/2)*(g/cosh(d + e*x))^(1/2),x)
```

output
```
int(F^(c*(a + b*x))*(f/cosh(d + e*x))^(1/2)*(g/cosh(d + e*x))^(1/2), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\sqrt{f\text{sech}(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx = \sqrt{g}\,f^{ac+\frac{1}{2}}\left(\int f^{bcx}\text{sech}(ex+d)\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*sech(e*x+d))^(1/2)*(g*sech(e*x+d))^(1/2),x)
```

output
```
sqrt(g)*f**((2*a*c + 1)/2)*int(f**(b*c*x)*sech(d + e*x),x)
```

3.143 $\int F^{c(a+bx)}\sqrt{f\tanh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 130

$$\int F^{c(a+bx)}\sqrt{f\tanh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx=\frac{fF^{c(a+bx)}\sqrt{g\tanh(d+ex)}}{bc\log(F)\sqrt{f\tanh(d+ex)}}-\frac{2fF^{c(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},-e^{2d+2ex}\right)\sqrt{g\tanh(d+ex)}}{bc\log(F)\sqrt{f\tanh(d+ex)}}$$

output
```
f*F^(c*(b*x+a))*(g*tanh(e*x+d))^(1/2)/b/c/ln(F)/(f*tanh(e*x+d))^(1/2)-2*f*
F^(c*(b*x+a))*hypergeom([1, 1/2*b*c*ln(F)/e],[1+1/2*b*c*ln(F)/e],-exp(2*e*
x+2*d))*(g*tanh(e*x+d))^(1/2)/b/c/ln(F)/(f*tanh(e*x+d))^(1/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.67

$$\int F^{c(a+bx)}\sqrt{f\tanh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx=-\frac{fF^{c(a+bx)}\left(-1+2\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)\right)\sqrt{g\tanh(d+ex)}}{bc\log(F)\sqrt{f\tanh(d+ex)}}$$

input
```
Integrate[F^(c*(a + b*x))*Sqrt[f*Tanh[d + e*x]]*Sqrt[g*Tanh[d + e*x]],x]
```

output
```
-((f*F^(c*(a + b*x))*(-1 + 2*Hypergeometric2F1[1, (b*c*Log[F])/(2*e), 1 +
(b*c*Log[F])/(2*e), -E^(2*(d + e*x))])*Sqrt[g*Tanh[d + e*x]])/(b*c*Log[F]*
Sqrt[f*Tanh[d + e*x]]))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.81, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {2031, 6007, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\sqrt{f\tanh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{f\sqrt{g\tanh(d+ex)}\int F^{c(a+bx)}\tanh(d+ex)dx}{\sqrt{f\tanh(d+ex)}}$$

$$\Big\downarrow \text{6007}$$

$$\frac{f\sqrt{g\tanh(d+ex)}\int \left(F^{c(a+bx)}-\frac{2F^{c(a+bx)}}{1+e^{2(d+ex)}}\right)dx}{\sqrt{f\tanh(d+ex)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f\sqrt{g\tanh(d+ex)}\left(\frac{F^{c(a+bx)}}{bc\log(F)}-\frac{2F^{c(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,-e^{2(d+ex)}\right)}{bc\log(F)}\right)}{\sqrt{f\tanh(d+ex)}}$$

input
```
Int[F^(c*(a + b*x))*Sqrt[f*Tanh[d + e*x]]*Sqrt[g*Tanh[d + e*x]],x]
```

output
```
(f*(F^(c*(a + b*x))/(b*c*Log[F]) - (2*F^(c*(a + b*x))*Hypergeometric2F1[1,
 (b*c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(b*c*Log[F
]))*Sqrt[g*Tanh[d + e*x]])/Sqrt[f*Tanh[d + e*x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 6007
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tanh[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((-1 + E^(2*(d + e*x)))^n/(1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)}\sqrt{f\tanh\left(ex+d\right)}\,\sqrt{g\tanh\left(ex+d\right)}dx$$

input
```
int(F^(c*(b*x+a))*(f*tanh(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x)
```

output
```
int(F^(c*(b*x+a))*(f*tanh(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\sqrt{f\tanh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$
$$=\int\sqrt{f\tanh\left(ex+d\right)}\sqrt{g\tanh\left(ex+d\right)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*tanh(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x, alg
orithm="fricas")
```

output
```
integral(sqrt(f*tanh(e*x + d))*sqrt(g*tanh(e*x + d))*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\sqrt{f\tanh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$
$$=\int F^{c(a+bx)}\sqrt{f\tanh\left(d+ex\right)}\sqrt{g\tanh\left(d+ex\right)}\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*tanh(e*x+d))**(1/2)*(g*tanh(e*x+d))**(1/2),x)
```

output
```
Integral(F**(c*(a + b*x))*sqrt(f*tanh(d + e*x))*sqrt(g*tanh(d + e*x)), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\sqrt{f\tanh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$
$$=\int \sqrt{f\tanh\left(ex+d\right)}\sqrt{g\tanh\left(ex+d\right)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*tanh(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x, alg
orithm="maxima")
```

output
```
integrate(sqrt(f*tanh(e*x + d))*sqrt(g*tanh(e*x + d))*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\sqrt{f\tanh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$
$$=\int \sqrt{f\tanh(ex+d)}\sqrt{g\tanh(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*tanh(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x, alg
orithm="giac")
```

output
```
integrate(sqrt(f*tanh(e*x + d))*sqrt(g*tanh(e*x + d))*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{f\tanh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$
$$=\int F^{c\,(a+b\,x)}\,\sqrt{f\tanh\left(d+e\,x\right)}\,\sqrt{g\tanh\left(d+e\,x\right)}\,dx$$

input
```
int(F^(c*(a + b*x))*(f*tanh(d + e*x))^(1/2)*(g*tanh(d + e*x))^(1/2),x)
```

output
```
int(F^(c*(a + b*x))*(f*tanh(d + e*x))^(1/2)*(g*tanh(d + e*x))^(1/2), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\sqrt{f\tanh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx=\sqrt{g}\,f^{ac+\frac{1}{2}}\left(\int f^{bcx}\tanh\left(ex+d\right)dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*tanh(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x)
```

output
```
sqrt(g)*f**((2*a*c + 1)/2)*int(f**(b*c*x)*tanh(d + e*x),x)
```

3.144 $\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\tanh(d+ex)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 44

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\tanh(d+ex)}\,dx = \frac{F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\tanh(d+ex)}}{bc\log(F)}$$

output
```
F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*tanh(e*x+d))^(1/2)/b/c/ln(F)
```

Mathematica [A] (verified)

Time = 2.84 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\tanh(d+ex)}\,dx = \frac{F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\tanh(d+ex)}}{bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sqrt[g*Coth[d + e*x]]*Sqrt[f*Tanh[d + e*x]],x]
```

output
```
(F^(c*(a + b*x))*Sqrt[g*Coth[d + e*x]]*Sqrt[f*Tanh[d + e*x]])/(b*c*Log[F])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\sqrt{f\tanh(d+ex)}\sqrt{g\coth(d+ex)}\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$\frac{\sqrt{g\coth(d+ex)}\int F^{c(a+bx)}\sqrt{\coth(d+ex)}\sqrt{f\tanh(d+ex)}dx}{\sqrt{\coth(d+ex)}}$$

$$\Big\downarrow \text{ 7271}$$

$$\sqrt{\tanh(d+ex)}\sqrt{\coth(d+ex)}\sqrt{f\tanh(d+ex)}\sqrt{g\coth(d+ex)}\int F^{c(a+bx)}\sqrt{\coth(d+ex)}\sqrt{\tanh(d+ex)}dx$$

$$\Big\downarrow \text{ 7292}$$

$$\sqrt{\tanh(d+ex)}\sqrt{\coth(d+ex)}\sqrt{f\tanh(d+ex)}\sqrt{g\coth(d+ex)}\int F^{ac+bxc}\sqrt{\coth(d+ex)}\sqrt{\tanh(d+ex)}dx$$

$$\Big\downarrow \text{ 7299}$$

$$\sqrt{\tanh(d+ex)}\sqrt{\coth(d+ex)}\sqrt{f\tanh(d+ex)}\sqrt{g\coth(d+ex)}\int F^{ac+bxc}\sqrt{\coth(d+ex)}\sqrt{\tanh(d+ex)}dx$$

input
```
Int[F^(c*(a + b*x))*Sqrt[g*Coth[d + e*x]]*Sqrt[f*Tanh[d + e*x]],x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

method	result	size
orering	$\frac{F^{c(bx+a)}\sqrt{g\coth(ex+d)}\,\sqrt{f\tanh(ex+d)}}{bc\ln(F)}$	41
risch	$\frac{\sqrt{\frac{f\left(-1+e^{2ex+2d}\right)}{1+e^{2ex+2d}}}\,\sqrt{\frac{g\left(1+e^{2ex+2d}\right)}{-1+e^{2ex+2d}}}\,F^{c(bx+a)}}{\ln(F)bc}$	77

input
```
int(F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*tanh(e*x+d))^(1/2),x,method=_RE
TURNVERBOSE)
```

output
```
F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*tanh(e*x+d))^(1/2)/b/c/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.64

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\tanh(d+ex)}\,dx = \frac{\sqrt{\frac{g\cosh(ex+d)}{\sinh(ex+d)}}\sqrt{\frac{f\sinh(ex+d)}{\cosh(ex+d)}}\left(\cosh\left((bcx+ac)\log\left(F\right)\right)+\sinh\left((bcx+ac)\log\left(F\right)\right)\right)}{bc\log\left(F\right)}$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*tanh(e*x+d))^(1/2),x, alg
orithm="fricas")
```

output
```
sqrt(g*cosh(e*x + d)/sinh(e*x + d))*sqrt(f*sinh(e*x + d)/cosh(e*x + d))*(c
osh((b*c*x + a*c)*log(F)) + sinh((b*c*x + a*c)*log(F)))/(b*c*log(F))
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\tanh(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(g*coth(e*x+d))**(1/2)*(f*tanh(e*x+d))**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\tanh(d+ex)}\,dx = \frac{F^{bcx}F^{ac}\sqrt{f}\sqrt{g}}{bc\log(F)}$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*tanh(e*x+d))^(1/2),x, alg
orithm="maxima")
```

output
```
F^(b*c*x)*F^(a*c)*sqrt(f)*sqrt(g)/(b*c*log(F))
```

Giac [F]

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\tanh(d+ex)}\,dx = \int \sqrt{g\coth(ex+d)}\sqrt{f\tanh(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*tanh(e*x+d))^(1/2),x, alg
orithm="giac")
```

output
```
integrate(sqrt(g*coth(e*x + d))*sqrt(f*tanh(e*x + d))*F^((b*x + a)*c), x)
```

Mupad [B] (verification not implemented)

Time = 2.74 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.75

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\tanh(d+ex)}\,dx=\frac{F^{a\,c+b\,c\,x}\,\sqrt{\frac{f\,\left(\mathrm{e}^{2\,d+2\,e\,x}-1\right)}{\mathrm{e}^{2\,d+2\,e\,x}+1}}\,\sqrt{\frac{g\,\left(\mathrm{e}^{2\,d+2\,e\,x}+1\right)}{\mathrm{e}^{2\,d+2\,e\,x}-1}}}{b\,c\,\ln\left(F\right)}$$

input
```
int(F^(c*(a + b*x))*(g*coth(d + e*x))^(1/2)*(f*tanh(d + e*x))^(1/2),x)
```

output
```
(F^(a*c + b*c*x)*((f*(exp(2*d + 2*e*x) - 1))/(exp(2*d + 2*e*x) + 1))^(1/2)
*((g*(exp(2*d + 2*e*x) + 1))/(exp(2*d + 2*e*x) - 1))^(1/2))/(b*c*log(F))
```

Reduce [F]

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\tanh(d+ex)}\,dx$$
$$=\sqrt{g}\,f^{ac+\frac{1}{2}}\left(\int f^{bcx}\sqrt{\tanh\left(ex+d\right)}\,\sqrt{\coth\left(ex+d\right)}dx\right)$$

input
```
int(F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*tanh(e*x+d))^(1/2),x)
```

output
```
sqrt(g)*f**((2*a*c + 1)/2)*int(f**(b*c*x)*sqrt(tanh(d + e*x))*sqrt(coth(d
+ e*x)),x)
```

3.145 $\int F^{c(a+bx)}\sqrt{f\coth(d+ex)}\sqrt{g\coth(d+ex)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 128

$$\int F^{c(a+bx)}\sqrt{f\coth(d+ex)}\sqrt{g\coth(d+ex)}\,dx = \frac{fF^{c(a+bx)}\sqrt{g\coth(d+ex)}}{bc\sqrt{f\coth(d+ex)}\log(F)} - \frac{2fF^{c(a+bx)}\sqrt{g\coth(d+ex)}\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},e^{2d+2ex}\right)}{bc\sqrt{f\coth(d+ex)}\log(F)}$$

output
```
f*F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)/b/c/(f*coth(e*x+d))^(1/2)/ln(F)-2*f*
F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*hypergeom([1, 1/2*b*c*ln(F)/e],[1+1/2*
b*c*ln(F)/e],exp(2*e*x+2*d))/b/c/(f*coth(e*x+d))^(1/2)/ln(F)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.66

$$\int F^{c(a+bx)}\sqrt{f\coth(d+ex)}\sqrt{g\coth(d+ex)}\,dx = -\frac{fF^{c(a+bx)}\sqrt{g\coth(d+ex)}\left(-1+2\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},e^{2(d+ex)}\right)\right)}{bc\sqrt{f\coth(d+ex)}\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sqrt[f*Coth[d + e*x]]*Sqrt[g*Coth[d + e*x]],x]
```

output
```
-((f*F^(c*(a + b*x))*Sqrt[g*Coth[d + e*x]]*(-1 + 2*Hypergeometric2F1[1, (b
*c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))]))/(b*c*Sqrt[f*C
oth[d + e*x]]*Log[F]))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.80, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {2031, 6008, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\sqrt{f\coth(d+ex)}\sqrt{g\coth(d+ex)}\,dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{f\sqrt{g\coth(d+ex)}\int F^{c(a+bx)}\coth(d+ex)dx}{\sqrt{f\coth(d+ex)}}$$

$$\Big\downarrow \text{ 6008}$$

$$\frac{f\sqrt{g\coth(d+ex)}\int \left(\frac{2F^{c(a+bx)}}{-1+e^{2(d+ex)}}+F^{c(a+bx)}\right)dx}{\sqrt{f\coth(d+ex)}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{f\sqrt{g\coth(d+ex)}\left(\frac{F^{c(a+bx)}}{bc\log(F)}-\frac{2F^{c(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},\frac{bc\log(F)}{2e}+1,e^{2(d+ex)}\right)}{bc\log(F)}\right)}{\sqrt{f\coth(d+ex)}}$$

input
```
Int[F^(c*(a + b*x))*Sqrt[f*Coth[d + e*x]]*Sqrt[g*Coth[d + e*x]],x]
```

output
```
(f*Sqrt[g*Coth[d + e*x]]*(F^(c*(a + b*x))/(b*c*Log[F]) - (2*F^(c*(a + b*x)
)*Hypergeometric2F1[1, (b*c*Log[F])/(2*e), 1 + (b*c*Log[F])/(2*e), E^(2*(d
 + e*x))])/(b*c*Log[F])))/Sqrt[f*Coth[d + e*x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 6008
```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*(d + e*x)))^n/(-1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)}\sqrt{f\coth\left(ex+d\right)}\,\sqrt{g\coth\left(ex+d\right)}dx$$

input
```
int(F^(c*(b*x+a))*(f*coth(e*x+d))^(1/2)*(g*coth(e*x+d))^(1/2),x)
```

output
```
int(F^(c*(b*x+a))*(f*coth(e*x+d))^(1/2)*(g*coth(e*x+d))^(1/2),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\sqrt{f\coth(d+ex)}\sqrt{g\coth(d+ex)}\,dx$$
$$=\int\sqrt{f\coth\left(ex+d\right)}\sqrt{g\coth\left(ex+d\right)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*coth(e*x+d))^(1/2)*(g*coth(e*x+d))^(1/2),x, alg
orithm="fricas")
```

output
```
integral(sqrt(f*coth(e*x + d))*sqrt(g*coth(e*x + d))*F^(b*c*x + a*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{f\coth(d+ex)}\sqrt{g\coth(d+ex)}\,dx=\text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(f*coth(e*x+d))**(1/2)*(g*coth(e*x+d))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{c(a+bx)}\sqrt{f\coth(d+ex)}\sqrt{g\coth(d+ex)}\,dx$$
$$=\int\sqrt{f\coth(ex+d)}\sqrt{g\coth(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*coth(e*x+d))^(1/2)*(g*coth(e*x+d))^(1/2),x, alg
orithm="maxima")
```

output
```
integrate(sqrt(f*coth(e*x + d))*sqrt(g*coth(e*x + d))*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\sqrt{f\coth(d+ex)}\sqrt{g\coth(d+ex)}\,dx$$
$$=\int \sqrt{f\coth(ex+d)}\sqrt{g\coth(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*coth(e*x+d))^(1/2)*(g*coth(e*x+d))^(1/2),x, alg
orithm="giac")
```

output
```
integrate(sqrt(f*coth(e*x + d))*sqrt(g*coth(e*x + d))*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{f\coth(d+ex)}\sqrt{g\coth(d+ex)}\,dx$$
$$=\int F^{c\,(a+b\,x)}\,\sqrt{f\coth\left(d+e\,x\right)}\,\sqrt{g\coth\left(d+e\,x\right)}\,dx$$

input
```
int(F^(c*(a + b*x))*(f*coth(d + e*x))^(1/2)*(g*coth(d + e*x))^(1/2),x)
```

output
```
int(F^(c*(a + b*x))*(f*coth(d + e*x))^(1/2)*(g*coth(d + e*x))^(1/2), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\sqrt{f\coth(d+ex)}\sqrt{g\coth(d+ex)}\,dx=\sqrt{g}\,f^{ac+\frac{1}{2}}\left(\int f^{bcx}\coth(ex+d)\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*coth(e*x+d))^(1/2)*(g*coth(e*x+d))^(1/2),x)
```

output
```
sqrt(g)*f**((2*a*c + 1)/2)*int(f**(b*c*x)*coth(d + e*x),x)
```

3.146 $\int F^{c(a+bx)}\sqrt{g\cosh(d+ex)}\sqrt{f\sinh(d+ex)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 121

$$\int F^{c(a+bx)}\sqrt{g\cosh(d+ex)}\sqrt{f\sinh(d+ex)}\,dx = -\frac{F^{c(a+bx)}\sqrt{g\cosh(d+ex)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-1+\frac{bc\log(F)}{e}\right),\frac{1}{4}\left(3+\frac{bc\log(F)}{e}\right),e^{4d+4ex}\right)\sqrt{f\,s}}{\sqrt{1-e^{2d+2ex}}\sqrt{1+e^{2d+2ex}}(e-bc\log(F))}$$

output
```
-F^(c*(b*x+a))*(g*cosh(e*x+d))^(1/2)*hypergeom([-1/2, -1/4+1/4*b*c*ln(F)/e
],[3/4+1/4*b*c*ln(F)/e],exp(4*e*x+4*d))*(f*sinh(e*x+d))^(1/2)/(1-exp(2*e*x
+2*d))^(1/2)/(1+exp(2*e*x+2*d))^(1/2)/(e-b*c*ln(F))
```

Mathematica [A] (verified)

Time = 2.78 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87

$$\int F^{c(a+bx)}\sqrt{g\cosh(d+ex)}\sqrt{f\sinh(d+ex)}\,dx = \frac{F^{c(a+bx)}\sqrt{g\cosh(d+ex)}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4}-\frac{bc\log(F)}{4e};\frac{3}{4}-\frac{bc\log(F)}{4e};e^{-4(d+ex)}\right)\sqrt{f\sinh(d+ex)}}{\sqrt{1-e^{-4(d+ex)}}(e+bc\log(F))}$$

input
```
Integrate[F^(c*(a + b*x))*Sqrt[g*Cosh[d + e*x]]*Sqrt[f*Sinh[d + e*x]],x]
```

output
```
(F^(c*(a + b*x))*Sqrt[g*Cosh[d + e*x]]*HypergeometricPFQ[{-1/2, -1/4 - (b*
c*Log[F])/(4*e)}, {3/4 - (b*c*Log[F])/(4*e)}, E^(-4*(d + e*x))]*Sqrt[f*Sin
h[d + e*x]])/(Sqrt[1 - E^(-4*(d + e*x))]*(e + b*c*Log[F]))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\cosh(d+ex)}\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$\frac{\sqrt{g\cosh(d+ex)}\int F^{c(a+bx)}\sqrt{\cosh(d+ex)}\sqrt{f\sinh(d+ex)}dx}{\sqrt{\cosh(d+ex)}}$$

$$\Big\downarrow \text{ 7271}$$

$$\frac{\sqrt{f\sinh(d+ex)}\sqrt{g\cosh(d+ex)}\int F^{c(a+bx)}\sqrt{\cosh(d+ex)}\sqrt{\sinh(d+ex)}dx}{\sqrt{\sinh(d+ex)}\sqrt{\cosh(d+ex)}}$$

$$\Big\downarrow \text{ 7292}$$

$$\frac{\sqrt{f\sinh(d+ex)}\sqrt{g\cosh(d+ex)}\int F^{ac+bxc}\sqrt{\cosh(d+ex)}\sqrt{\sinh(d+ex)}dx}{\sqrt{\sinh(d+ex)}\sqrt{\cosh(d+ex)}}$$

$$\Big\downarrow \text{ 7299}$$

$$\frac{\sqrt{f\sinh(d+ex)}\sqrt{g\cosh(d+ex)}\int F^{ac+bxc}\sqrt{\cosh(d+ex)}\sqrt{\sinh(d+ex)}dx}{\sqrt{\sinh(d+ex)}\sqrt{\cosh(d+ex)}}$$

input
```
Int[F^(c*(a + b*x))*Sqrt[g*Cosh[d + e*x]]*Sqrt[f*Sinh[d + e*x]],x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)} \sqrt{g \cosh \left(ex+d\right)}\, \sqrt{f \sinh \left(ex+d\right)}dx$$

input
```
int(F^(c*(b*x+a))*(g*cosh(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2),x)
```

output
```
int(F^(c*(b*x+a))*(g*cosh(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2),x)
```

Fricas [F]

$$\int F^{c(a+bx)} \sqrt{g \cosh(d+ex)}\sqrt{f \sinh(d+ex)}\, dx = \int \sqrt{g \cosh \left(ex+d\right)}\sqrt{f \sinh \left(ex+d\right)}F^{(bx+a)c}\, dx$$

input
```
integrate(F^(c*(b*x+a))*(g*cosh(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2),x, alg
orithm="fricas")
```

output
```
integral(sqrt(g*cosh(e*x + d))*sqrt(f*sinh(e*x + d))*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)} \sqrt{g \cosh(d+ex)}\sqrt{f \sinh(d+ex)}\, dx = \int F^{c(a+bx)} \sqrt{f \sinh \left(d+ex\right)}\sqrt{g \cosh \left(d+ex\right)}\, dx$$

input
```
integrate(F**(c*(b*x+a))*(g*cosh(e*x+d))**(1/2)*(f*sinh(e*x+d))**(1/2),x)
```

output
```
Integral(F**(c*(a + b*x))*sqrt(f*sinh(d + e*x))*sqrt(g*cosh(d + e*x)), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\sqrt{g\cosh(d+ex)}\sqrt{f\sinh(d+ex)}\,dx = \int \sqrt{g\cosh(ex+d)}\sqrt{f\sinh(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*cosh(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2),x, alg
orithm="maxima")
```

output
```
integrate(sqrt(g*cosh(e*x + d))*sqrt(f*sinh(e*x + d))*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\sqrt{g\cosh(d+ex)}\sqrt{f\sinh(d+ex)}\,dx = \int \sqrt{g\cosh(ex+d)}\sqrt{f\sinh(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*cosh(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2),x, alg
orithm="giac")
```

output
```
integrate(sqrt(g*cosh(e*x + d))*sqrt(f*sinh(e*x + d))*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{g\cosh(d+ex)}\sqrt{f\sinh(d+ex)}\,dx = \int F^{c\,(a+b\,x)}\,\sqrt{g\cosh\left(d+e\,x\right)}\,\sqrt{f\sinh\left(d+e\,x\right)}\,dx$$

input
```
int(F^(c*(a + b*x))*(g*cosh(d + e*x))^(1/2)*(f*sinh(d + e*x))^(1/2),x)
```

output
```
int(F^(c*(a + b*x))*(g*cosh(d + e*x))^(1/2)*(f*sinh(d + e*x))^(1/2), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\sqrt{g\cosh(d+ex)}\sqrt{f\sinh(d+ex)}\,dx$$
$$=\frac{\sqrt{g}\,f^{ac+\frac{1}{2}}\left(2f^{bcx}\sqrt{\sinh(ex+d)}\,\sqrt{\cosh(ex+d)}-\left(\int\frac{f^{bcx}\sqrt{\sinh(ex+d)}\,\sqrt{\cosh(ex+d)}\,\cosh(ex+d)}{\sinh(ex+d)}dx\right)e-\left(\int\frac{f^{bcx}\sqrt{}}{}\right.\right.}{2\log(f)\,bc}$$

input
```
int(F^(c*(b*x+a))*(g*cosh(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2),x)
```

output
```
(sqrt(g)*f**((2*a*c + 1)/2)*(2*f**(b*c*x)*sqrt(sinh(d + e*x))*sqrt(cosh(d
+ e*x)) - int((f**(b*c*x)*sqrt(sinh(d + e*x))*sqrt(cosh(d + e*x))*cosh(d +
 e*x))/sinh(d + e*x),x)*e - int((f**(b*c*x)*sqrt(sinh(d + e*x))*sqrt(cosh(
d + e*x))*sinh(d + e*x))/cosh(d + e*x),x)*e))/(2*log(f)*b*c)
```

3.147 $\int F^{c(a+bx)}\sqrt{g\text{sech}(d+ex)}\sqrt{f\sinh(d+ex)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 249

$$\int F^{c(a+bx)}\sqrt{g\text{sech}(d+ex)}\sqrt{f\sinh(d+ex)}\,dx = \frac{\sqrt{1-e^{4d+4ex}}F^{c(a+bx)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{bc\log(F)}{4e},1+\frac{bc\log(F)}{4e},e^{4d+4ex}\right)\sqrt{g\text{sech}(d+ex)}\sqrt{f\sinh(d+ex)}}{bc\left(1-e^{2d+2ex}\right)\log(F)} - \frac{e^{2d+2ex}\sqrt{1-e^{4d+4ex}}F^{c(a+bx)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}\left(2+\frac{bc\log(F)}{e}\right),\frac{1}{4}\left(6+\frac{bc\log(F)}{e}\right),e^{4d+4ex}\right)\sqrt{g\text{sech}(d+ex)}\sqrt{f\sinh(d+ex)}}{\left(1-e^{2d+2ex}\right)(2e+bc\log(F))}$$

output

```
(1-exp(4*e*x+4*d))^(1/2)*F^(c*(b*x+a))*hypergeom([1/2, 1/4*b*c*ln(F)/e],[1
+1/4*b*c*ln(F)/e],exp(4*e*x+4*d))*(g*sech(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1
/2)/b/c/(1-exp(2*e*x+2*d))/ln(F)-exp(2*e*x+2*d)*(1-exp(4*e*x+4*d))^(1/2)*F
^(c*(b*x+a))*hypergeom([1/2, 1/2+1/4*b*c*ln(F)/e],[3/2+1/4*b*c*ln(F)/e],ex
p(4*e*x+4*d))*(g*sech(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2)/(1-exp(2*e*x+2*d
))/(2*e+b*c*ln(F))
```

Mathematica [F]

$$\int F^{c(a+bx)}\sqrt{g\text{sech}(d+ex)}\sqrt{f\sinh(d+ex)}\,dx$$
$$=\int F^{c(a+bx)}\sqrt{g\text{sech}(d+ex)}\sqrt{f\sinh(d+ex)}\,dx$$

input
```
Integrate[F^(c*(a + b*x))*Sqrt[g*Sech[d + e*x]]*Sqrt[f*Sinh[d + e*x]],x]
```

output
```
Integrate[F^(c*(a + b*x))*Sqrt[g*Sech[d + e*x]]*Sqrt[f*Sinh[d + e*x]], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx$$
$$\Big\downarrow \text{ 7271}$$
$$\frac{\sqrt{g\text{sech}(d+ex)}\int F^{c(a+bx)}\sqrt{\text{sech}(d+ex)}\sqrt{f\sinh(d+ex)}dx}{\sqrt{\text{sech}(d+ex)}}$$
$$\Big\downarrow \text{ 7271}$$
$$\frac{\sqrt{f\sinh(d+ex)}\sqrt{g\text{sech}(d+ex)}\int F^{c(a+bx)}\sqrt{\text{sech}(d+ex)}\sqrt{\sinh(d+ex)}dx}{\sqrt{\sinh(d+ex)}\sqrt{\text{sech}(d+ex)}}$$
$$\Big\downarrow \text{ 7292}$$
$$\frac{\sqrt{f\sinh(d+ex)}\sqrt{g\text{sech}(d+ex)}\int F^{ac+bxc}\sqrt{\text{sech}(d+ex)}\sqrt{\sinh(d+ex)}dx}{\sqrt{\sinh(d+ex)}\sqrt{\text{sech}(d+ex)}}$$
$$\Big\downarrow \text{ 7299}$$
$$\frac{\sqrt{f\sinh(d+ex)}\sqrt{g\text{sech}(d+ex)}\int F^{ac+bxc}\sqrt{\text{sech}(d+ex)}\sqrt{\sinh(d+ex)}dx}{\sqrt{\sinh(d+ex)}\sqrt{\text{sech}(d+ex)}}$$

input
```
Int[F^(c*(a + b*x))*Sqrt[g*Sech[d + e*x]]*Sqrt[f*Sinh[d + e*x]],x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}\sqrt{g\,\mathrm{sech}\,(ex+d)}\,\sqrt{f\sinh\,(ex+d)}dx$$

input
```
int(F^(c*(b*x+a))*(g*sech(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2),x)
```

output
```
int(F^(c*(b*x+a))*(g*sech(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\sqrt{g\mathrm{sech}(d+ex)}\sqrt{f\sinh(d+ex)}\,dx$$
$$=\int\sqrt{g\,\mathrm{sech}\,(ex+d)}\sqrt{f\sinh\,(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*sech(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2),x, alg
orithm="fricas")
```

output
```
integral(sqrt(g*sech(e*x + d))*sqrt(f*sinh(e*x + d))*F^(b*c*x + a*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{g\mathrm{sech}(d+ex)}\sqrt{f\sinh(d+ex)}\,dx=\text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(g*sech(e*x+d))**(1/2)*(f*sinh(e*x+d))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{c(a+bx)}\sqrt{g\text{sech}(d+ex)}\sqrt{f\sinh(d+ex)}\,dx$$
$$=\int \sqrt{g\,\text{sech}\,(ex+d)}\sqrt{f\sinh\,(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*sech(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2),x, alg
orithm="maxima")
```

output
```
integrate(sqrt(g*sech(e*x + d))*sqrt(f*sinh(e*x + d))*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\sqrt{g\text{sech}(d+ex)}\sqrt{f\sinh(d+ex)}\,dx$$
$$=\int \sqrt{g\,\text{sech}\,(ex+d)}\sqrt{f\sinh\,(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*sech(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2),x, alg
orithm="giac")
```

output
```
integrate(sqrt(g*sech(e*x + d))*sqrt(f*sinh(e*x + d))*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{g\text{sech}(d+ex)}\sqrt{f\sinh(d+ex)}\,dx$$
$$=\int F^{c\,(a+b\,x)}\,\sqrt{f\sinh\,(d+e\,x)}\,\sqrt{\frac{g}{\cosh\,(d+e\,x)}}\,dx$$

input
```
int(F^(c*(a + b*x))*(f*sinh(d + e*x))^(1/2)*(g/cosh(d + e*x))^(1/2),x)
```

output
```
int(F^(c*(a + b*x))*(f*sinh(d + e*x))^(1/2)*(g/cosh(d + e*x))^(1/2), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\sqrt{g\mathrm{sech}(d+ex)}\sqrt{f\sinh(d+ex)}\,dx$$
$$=\sqrt{g}\,f^{ac+\frac{1}{2}}\left(\int f^{bcx}\sqrt{\sinh(ex+d)}\,\sqrt{\mathrm{sech}(ex+d)}dx\right)$$

input
```
int(F^(c*(b*x+a))*(g*sech(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2),x)
```

output
```
sqrt(g)*f**((2*a*c + 1)/2)*int(f**(b*c*x)*sqrt(sinh(d + e*x))*sqrt(sech(d
+ e*x)),x)
```

3.148 $\int F^{c(a+bx)}\sqrt{g\cosh(d+ex)}\sqrt{f\,\mathrm{csch}(d+ex)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 244

$$\int F^{c(a+bx)}\sqrt{g\cosh(d+ex)}\sqrt{f\,\mathrm{csch}(d+ex)}\,dx$$
$$= \frac{\sqrt{1-e^{4d+4ex}}F^{c(a+bx)}\sqrt{g\cosh(d+ex)}\sqrt{f\,\mathrm{csch}(d+ex)}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{bc\log(F)}{4e},1+\frac{bc\log(F)}{4e},e^{4d+}\right.}{bc\left(1+e^{2d+2ex}\right)\log(F)}$$
$$+\frac{e^{2d+2ex}\sqrt{1-e^{4d+4ex}}F^{c(a+bx)}\sqrt{g\cosh(d+ex)}\sqrt{f\,\mathrm{csch}(d+ex)}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}\left(2+\frac{bc\log(F)}{e}\right)\right.}{\left(1+e^{2d+2ex}\right)\left(2e+bc\log(F)\right)}$$

output
```
(1-exp(4*e*x+4*d))^(1/2)*F^(c*(b*x+a))*(g*cosh(e*x+d))^(1/2)*(f*csch(e*x+d
))^(1/2)*hypergeom([1/2, 1/4*b*c*ln(F)/e],[1+1/4*b*c*ln(F)/e],exp(4*e*x+4*
d))/b/c/(1+exp(2*e*x+2*d))/ln(F)+exp(2*e*x+2*d)*(1-exp(4*e*x+4*d))^(1/2)*F
^(c*(b*x+a))*(g*cosh(e*x+d))^(1/2)*(f*csch(e*x+d))^(1/2)*hypergeom([1/2, 1
/2+1/4*b*c*ln(F)/e],[3/2+1/4*b*c*ln(F)/e],exp(4*e*x+4*d))/(1+exp(2*e*x+2*d
))/(2*e+b*c*ln(F))
```

Mathematica [F]

$$\int F^{c(a+bx)}\sqrt{g\cosh(d+ex)}\sqrt{f\operatorname{csch}(d+ex)}\,dx$$
$$=\int F^{c(a+bx)}\sqrt{g\cosh(d+ex)}\sqrt{f\operatorname{csch}(d+ex)}\,dx$$

input
```
Integrate[F^(c*(a + b*x))*Sqrt[g*Cosh[d + e*x]]*Sqrt[f*Csch[d + e*x]],x]
```

output
```
Integrate[F^(c*(a + b*x))*Sqrt[g*Cosh[d + e*x]]*Sqrt[f*Csch[d + e*x]], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\cosh(d+ex)}\,dx$$
$$\Big\downarrow \text{7271}$$
$$\frac{\sqrt{g\cosh(d+ex)}\int F^{c(a+bx)}\sqrt{\cosh(d+ex)}\sqrt{f\operatorname{csch}(d+ex)}dx}{\sqrt{\cosh(d+ex)}}$$
$$\Big\downarrow \text{7271}$$
$$\frac{\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\cosh(d+ex)}\int F^{c(a+bx)}\sqrt{\cosh(d+ex)}\sqrt{\operatorname{csch}(d+ex)}dx}{\sqrt{\cosh(d+ex)}\sqrt{\operatorname{csch}(d+ex)}}$$
$$\Big\downarrow \text{7292}$$
$$\frac{\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\cosh(d+ex)}\int F^{ac+bxc}\sqrt{\cosh(d+ex)}\sqrt{\operatorname{csch}(d+ex)}dx}{\sqrt{\cosh(d+ex)}\sqrt{\operatorname{csch}(d+ex)}}$$
$$\Big\downarrow \text{7299}$$
$$\frac{\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\cosh(d+ex)}\int F^{ac+bxc}\sqrt{\cosh(d+ex)}\sqrt{\operatorname{csch}(d+ex)}dx}{\sqrt{\cosh(d+ex)}\sqrt{\operatorname{csch}(d+ex)}}$$

input
```
Int[F^(c*(a + b*x))*Sqrt[g*Cosh[d + e*x]]*Sqrt[f*Csch[d + e*x]],x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}\sqrt{g\cosh\left(ex+d\right)}\,\sqrt{f\ \operatorname{csch}\left(ex+d\right)}dx$$

input
```
int(F^(c*(b*x+a))*(g*cosh(e*x+d))^(1/2)*(f*csch(e*x+d))^(1/2),x)
```

output
```
int(F^(c*(b*x+a))*(g*cosh(e*x+d))^(1/2)*(f*csch(e*x+d))^(1/2),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\sqrt{g\cosh(d+ex)}\sqrt{f\operatorname{csch}(d+ex)}\,dx$$
$$=\int\sqrt{g\cosh\left(ex+d\right)}\sqrt{f\operatorname{csch}\left(ex+d\right)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*cosh(e*x+d))^(1/2)*(f*csch(e*x+d))^(1/2),x, alg
orithm="fricas")
```

output
```
integral(sqrt(g*cosh(e*x + d))*sqrt(f*csch(e*x + d))*F^(b*c*x + a*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{g\cosh(d+ex)}\sqrt{f\operatorname{csch}(d+ex)}\,dx=\text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(g*cosh(e*x+d))**(1/2)*(f*csch(e*x+d))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{c(a+bx)}\sqrt{g\cosh(d+ex)}\sqrt{f\text{csch}(d+ex)}\,dx$$
$$=\int \sqrt{g\cosh{(ex+d)}}\sqrt{f\,\text{csch}\,(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*cosh(e*x+d))^(1/2)*(f*csch(e*x+d))^(1/2),x, alg
orithm="maxima")
```

output
```
integrate(sqrt(g*cosh(e*x + d))*sqrt(f*csch(e*x + d))*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\sqrt{g\cosh(d+ex)}\sqrt{f\text{csch}(d+ex)}\,dx$$
$$=\int \sqrt{g\cosh{(ex+d)}}\sqrt{f\,\text{csch}\,(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*cosh(e*x+d))^(1/2)*(f*csch(e*x+d))^(1/2),x, alg
orithm="giac")
```

output
```
integrate(sqrt(g*cosh(e*x + d))*sqrt(f*csch(e*x + d))*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{g\cosh(d+ex)}\sqrt{f\text{csch}(d+ex)}\,dx$$
$$=\int F^{c\,(a+b\,x)}\,\sqrt{g\cosh{(d+e\,x)}}\,\sqrt{\frac{f}{\sinh{(d+e\,x)}}}\,dx$$

input
```
int(F^(c*(a + b*x))*(g*cosh(d + e*x))^(1/2)*(f/sinh(d + e*x))^(1/2),x)
```

output
```
int(F^(c*(a + b*x))*(g*cosh(d + e*x))^(1/2)*(f/sinh(d + e*x))^(1/2), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\sqrt{g\cosh(d+ex)}\sqrt{f\operatorname{csch}(d+ex)}\,dx$$
$$=\sqrt{g}\,f^{ac+\frac{1}{2}}\left(\int f^{bcx}\sqrt{\operatorname{csch}(ex+d)}\,\sqrt{\cosh(ex+d)}dx\right)$$

input
```
int(F^(c*(b*x+a))*(g*cosh(e*x+d))^(1/2)*(f*csch(e*x+d))^(1/2),x)
```

output
```
sqrt(g)*f**((2*a*c + 1)/2)*int(f**(b*c*x)*sqrt(csch(d + e*x))*sqrt(cosh(d
+ e*x)),x)
```

3.149 $\int F^{c(a+bx)}\sqrt{f\text{csch}(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 119

$$\int F^{c(a+bx)}\sqrt{f\text{csch}(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx = \frac{\sqrt{1-e^{2d+2ex}}\sqrt{1+e^{2d+2ex}}F^{c(a+bx)}\sqrt{f\text{csch}(d+ex)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{e+bc\log(F)}{4e},\frac{1}{4}\left(5+\frac{bc\log(F)}{e}\right),e\right.}{e+bc\log(F)}$$

output
```
(1-exp(2*e*x+2*d))^(1/2)*(1+exp(2*e*x+2*d))^(1/2)*F^(c*(b*x+a))*(f*csch(e*
x+d))^(1/2)*hypergeom([1/2, 1/4*(e+b*c*ln(F))/e],[5/4+1/4*b*c*ln(F)/e],exp
(4*e*x+4*d))*(g*sech(e*x+d))^(1/2)/(e+b*c*ln(F))
```

Mathematica [F]

$$\int F^{c(a+bx)}\sqrt{f\text{csch}(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx = \int F^{c(a+bx)}\sqrt{f\text{csch}(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx$$

input
```
Integrate[F^(c*(a + b*x))*Sqrt[f*Csch[d + e*x]]*Sqrt[g*Sech[d + e*x]],x]
```

output
```
Integrate[F^(c*(a + b*x))*Sqrt[f*Csch[d + e*x]]*Sqrt[g*Sech[d + e*x]], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\operatorname{sech}(d+ex)}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\sqrt{f\operatorname{csch}(d+ex)}\int F^{c(a+bx)}\sqrt{\operatorname{csch}(d+ex)}\sqrt{g\operatorname{sech}(d+ex)}dx}{\sqrt{\operatorname{csch}(d+ex)}}$$

$$\Big\downarrow \text{7271}$$

$$\frac{\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\operatorname{sech}(d+ex)}\int F^{c(a+bx)}\sqrt{\operatorname{csch}(d+ex)}\sqrt{\operatorname{sech}(d+ex)}dx}{\sqrt{\operatorname{csch}(d+ex)}\sqrt{\operatorname{sech}(d+ex)}}$$

$$\Big\downarrow \text{7292}$$

$$\frac{\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\operatorname{sech}(d+ex)}\int F^{ac+bxc}\sqrt{\operatorname{csch}(d+ex)}\sqrt{\operatorname{sech}(d+ex)}dx}{\sqrt{\operatorname{csch}(d+ex)}\sqrt{\operatorname{sech}(d+ex)}}$$

$$\Big\downarrow \text{7299}$$

$$\frac{\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\operatorname{sech}(d+ex)}\int F^{ac+bxc}\sqrt{\operatorname{csch}(d+ex)}\sqrt{\operatorname{sech}(d+ex)}dx}{\sqrt{\operatorname{csch}(d+ex)}\sqrt{\operatorname{sech}(d+ex)}}$$

input
```
Int[F^(c*(a + b*x))*Sqrt[f*Csch[d + e*x]]*Sqrt[g*Sech[d + e*x]],x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}\sqrt{f\ \mathrm{csch}\left(ex+d\right)}\ \sqrt{g\ \mathrm{sech}\left(ex+d\right)}dx$$

input
```
int(F^(c*(b*x+a))*(f*csch(e*x+d))^(1/2)*(g*sech(e*x+d))^(1/2),x)
```

output
```
int(F^(c*(b*x+a))*(f*csch(e*x+d))^(1/2)*(g*sech(e*x+d))^(1/2),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\sqrt{f\mathrm{csch}(d+ex)}\sqrt{g\mathrm{sech}(d+ex)}\,dx$$
$$=\int\sqrt{f\,\mathrm{csch}\left(ex+d\right)}\sqrt{g\,\mathrm{sech}\left(ex+d\right)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(e*x+d))^(1/2)*(g*sech(e*x+d))^(1/2),x, alg
orithm="fricas")
```

output
```
integral(sqrt(f*csch(e*x + d))*sqrt(g*sech(e*x + d))*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\sqrt{f\mathrm{csch}(d+ex)}\sqrt{g\mathrm{sech}(d+ex)}\,dx$$
$$=\int F^{c(a+bx)}\sqrt{f\,\mathrm{csch}\left(d+ex\right)}\sqrt{g\,\mathrm{sech}\left(d+ex\right)}\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*csch(e*x+d))**(1/2)*(g*sech(e*x+d))**(1/2),x)
```

output
```
Integral(F**(c*(a + b*x))*sqrt(f*csch(d + e*x))*sqrt(g*sech(d + e*x)), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\sqrt{f\text{csch}(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx$$
$$=\int \sqrt{f\,\text{csch}\,(ex+d)}\sqrt{g\,\text{sech}\,(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(e*x+d))^(1/2)*(g*sech(e*x+d))^(1/2),x, alg
orithm="maxima")
```

output
```
integrate(sqrt(f*csch(e*x + d))*sqrt(g*sech(e*x + d))*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\sqrt{f\text{csch}(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx$$
$$=\int \sqrt{f\,\text{csch}\,(ex+d)}\sqrt{g\,\text{sech}\,(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(e*x+d))^(1/2)*(g*sech(e*x+d))^(1/2),x, alg
orithm="giac")
```

output
```
integrate(sqrt(f*csch(e*x + d))*sqrt(g*sech(e*x + d))*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{f\text{csch}(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx$$
$$=\int F^{c\,(a+b\,x)}\,\sqrt{\frac{g}{\cosh\left(d+e\,x\right)}}\,\sqrt{\frac{f}{\sinh\left(d+e\,x\right)}}\,dx$$

input
```
int(F^(c*(a + b*x))*(g/cosh(d + e*x))^(1/2)*(f/sinh(d + e*x))^(1/2),x)
```

output
```
int(F^(c*(a + b*x))*(g/cosh(d + e*x))^(1/2)*(f/sinh(d + e*x))^(1/2), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\sqrt{f\text{csch}(d+ex)}\sqrt{g\text{sech}(d+ex)}\,dx$$
$$=\sqrt{g}\,f^{ac+\frac{1}{2}}\left(\int f^{bcx}\sqrt{\text{sech}\,(ex+d)}\,\sqrt{\text{csch}\,(ex+d)}dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*csch(e*x+d))^(1/2)*(g*sech(e*x+d))^(1/2),x)
```

output
```
sqrt(g)*f**((2*a*c + 1)/2)*int(f**(b*c*x)*sqrt(sech(d + e*x))*sqrt(csch(d
+ e*x)),x)
```

3.150 $\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 209

$$\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$
$$= -\frac{2\left(1+e^{2d+2ex}\right)F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\tanh(d+ex)}}{\left(1-e^{2d+2ex}\right)\left(e+2bc\log(F)\right)}$$
$$-\frac{8bc\sqrt{1+e^{2d+2ex}}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}\left(-1+\frac{2bc\log(F)}{e}\right),\frac{1}{4}\left(3+\frac{2bc\log(F)}{e}\right),-e^{2d+2ex}\right)\log(}{\left(1-e^{2d+2ex}\right)\left(e^2-4b^2c^2\log^2(F)\right)}$$

output
```
-2*(1+exp(2*e*x+2*d))*F^(c*(b*x+a))*(f*sinh(e*x+d))^(1/2)*(g*tanh(e*x+d))^
(1/2)/(1-exp(2*e*x+2*d))/(e+2*b*c*ln(F))-8*b*c*(1+exp(2*e*x+2*d))^(1/2)*F^
(c*(b*x+a))*hypergeom([1/2, -1/4+1/2*b*c*ln(F)/e],[3/4+1/2*b*c*ln(F)/e],-e
xp(2*e*x+2*d))*ln(F)*(f*sinh(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2)/(1-exp(2*
e*x+2*d))/(e^2-4*b^2*c^2*ln(F)^2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.12 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.97

$$\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$
$$= \frac{2F^{c(a+bx)}g\sqrt{f\sinh(d+ex)}\left(\text{Hypergeometric2F1}\left(1,\frac{e+2bc\log(F)}{4e},\frac{3}{4}+\frac{bc\log(F)}{2e},-\cosh(2(d+ex))-\sinh(2\right.\right.}{}$$

input
```
Integrate[F^(c*(a + b*x))*Sqrt[f*Sinh[d + e*x]]*Sqrt[g*Tanh[d + e*x]],x]
```

output
```
(2*F^(c*(a + b*x))*g*Sqrt[f*Sinh[d + e*x]]*(Hypergeometric2F1[1, (e + 2*b*
c*Log[F])/(4*e), 3/4 + (b*c*Log[F])/(2*e), -Cosh[2*(d + e*x)] - Sinh[2*(d
+ e*x)]]*(3*e + 2*b*c*Log[F]) + Hypergeometric2F1[1, 5/4 + (b*c*Log[F])/(2
*e), 7/4 + (b*c*Log[F])/(2*e), -Cosh[2*(d + e*x)] - Sinh[2*(d + e*x)]]*(e
- 2*b*c*Log[F])*(Cosh[2*(d + e*x)] + Sinh[2*(d + e*x)])))/((e - 2*b*c*Log[
F])*(3*e + 2*b*c*Log[F])*Sqrt[g*Tanh[d + e*x]])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$

$$\Big\downarrow \ 7271$$

$$\frac{\sqrt{f\sinh(d+ex)}\int F^{c(a+bx)}\sqrt{\sinh(d+ex)}\sqrt{g\tanh(d+ex)}dx}{\sqrt{\sinh(d+ex)}}$$

$$\Big\downarrow \ 7271$$

$$\frac{\sqrt{f\sinh(d+ex)}\sqrt{g\tanh(d+ex)}\int F^{c(a+bx)}\sqrt{\sinh(d+ex)}\sqrt{\tanh(d+ex)}dx}{\sqrt{\sinh(d+ex)}\sqrt{\tanh(d+ex)}}$$

$$\Big\downarrow \ 7292$$

$$\frac{\sqrt{f\sinh(d+ex)}\sqrt{g\tanh(d+ex)}\int F^{ac+bxc}\sqrt{\sinh(d+ex)}\sqrt{\tanh(d+ex)}dx}{\sqrt{\sinh(d+ex)}\sqrt{\tanh(d+ex)}}$$

$$\Big\downarrow \text{7299}$$

$$\frac{\sqrt{f\sinh(d+ex)}\sqrt{g\tanh(d+ex)}\int F^{ac+bxc}\sqrt{\sinh(d+ex)}\sqrt{\tanh(d+ex)}dx}{\sqrt{\sinh(d+ex)}\sqrt{\tanh(d+ex)}}$$

input
```
Int[F^(c*(a + b*x))*Sqrt[f*Sinh[d + e*x]]*Sqrt[g*Tanh[d + e*x]],x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}\sqrt{f\sinh(ex+d)}\,\sqrt{g\tanh(ex+d)}dx$$

input
```
int(F^(c*(b*x+a))*(f*sinh(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x)
```

output
```
int(F^(c*(b*x+a))*(f*sinh(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$
$$=\int \sqrt{f\sinh(ex+d)}\sqrt{g\tanh(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sinh(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x, alg
orithm="fricas")
```

output
```
integral(sqrt(f*sinh(e*x + d))*sqrt(g*tanh(e*x + d))*F^(b*c*x + a*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(f*sinh(e*x+d))**(1/2)*(g*tanh(e*x+d))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$
$$= \int \sqrt{f\sinh(ex+d)}\sqrt{g\tanh(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sinh(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x, alg
orithm="maxima")
```

output
```
integrate(sqrt(f*sinh(e*x + d))*sqrt(g*tanh(e*x + d))*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$
$$= \int \sqrt{f\sinh(ex+d)}\sqrt{g\tanh(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sinh(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x, alg
orithm="giac")
```

output
```
integrate(sqrt(f*sinh(e*x + d))*sqrt(g*tanh(e*x + d))*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$
$$=\int F^{c\,(a+b\,x)}\,\sqrt{f\sinh\left(d+e\,x\right)}\,\sqrt{g\tanh\left(d+e\,x\right)}\,dx$$

input
```
int(F^(c*(a + b*x))*(f*sinh(d + e*x))^(1/2)*(g*tanh(d + e*x))^(1/2),x)
```

output
```
int(F^(c*(a + b*x))*(f*sinh(d + e*x))^(1/2)*(g*tanh(d + e*x))^(1/2), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$
$$=\sqrt{g}\,f^{ac+\frac{1}{2}}\left(\int f^{bcx}\sqrt{\tanh\left(ex+d\right)}\,\sqrt{\sinh\left(ex+d\right)}dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*sinh(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x)
```

output
```
sqrt(g)*f**((2*a*c + 1)/2)*int(f**(b*c*x)*sqrt(tanh(d + e*x))*sqrt(sinh(d
+ e*x)),x)
```

3.151 $\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\sinh(d+ex)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 107

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\sinh(d+ex)}\,dx = -\frac{2F^{c(a+bx)}\sqrt{g\coth(d+ex)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-1+\frac{2bc\log(F)}{e}\right),\frac{1}{4}\left(3+\frac{2bc\log(F)}{e}\right),-e^{2d+2ex}\right)\cdots}{\sqrt{1+e^{2d+2ex}}(e-2bc\log(F))}$$

output
```
-2*F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*hypergeom([-1/2, -1/4+1/2*b*c*ln(F)
/e],[3/4+1/2*b*c*ln(F)/e],-exp(2*e*x+2*d))*(f*sinh(e*x+d))^(1/2)/(1+exp(2*
e*x+2*d))^(1/2)/(e-2*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\sinh(d+ex)}\,dx = -\frac{4F^{c(a+bx)}\cosh(d+ex)\sqrt{g\coth(d+ex)}\,\text{Hypergeometric2F1}\left(1,\frac{5}{4}+\frac{bc\log(F)}{2e},\frac{3}{4}+\frac{bc\log(F)}{2e},-\cosh(2(d\cdots\right.}{e-2bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sqrt[g*Coth[d + e*x]]*Sqrt[f*Sinh[d + e*x]],x]
```

output
```
(-4*F^(c*(a + b*x))*Cosh[d + e*x]*Sqrt[g*Coth[d + e*x]]*Hypergeometric2F1[
1, 5/4 + (b*c*Log[F])/(2*e), 3/4 + (b*c*Log[F])/(2*e), -Cosh[2*(d + e*x)]
- Sinh[2*(d + e*x)]]*Sqrt[f*Sinh[d + e*x]]*(Cosh[d + e*x] + Sinh[d + e*x])
)/(e - 2*b*c*Log[F])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\sqrt{f\sinh(d+ex)}\sqrt{g\coth(d+ex)}\,dx$$

$$\Big\downarrow \ 7271$$

$$\frac{\sqrt{g\coth(d+ex)}\int F^{c(a+bx)}\sqrt{\coth(d+ex)}\sqrt{f\sinh(d+ex)}dx}{\sqrt{\coth(d+ex)}}$$

$$\Big\downarrow \ 7271$$

$$\frac{\sqrt{f\sinh(d+ex)}\sqrt{g\coth(d+ex)}\int F^{c(a+bx)}\sqrt{\coth(d+ex)}\sqrt{\sinh(d+ex)}dx}{\sqrt{\sinh(d+ex)}\sqrt{\coth(d+ex)}}$$

$$\Big\downarrow \ 7292$$

$$\frac{\sqrt{f\sinh(d+ex)}\sqrt{g\coth(d+ex)}\int F^{ac+bxc}\sqrt{\coth(d+ex)}\sqrt{\sinh(d+ex)}dx}{\sqrt{\sinh(d+ex)}\sqrt{\coth(d+ex)}}$$

$$\Big\downarrow \ 7299$$

$$\frac{\sqrt{f\sinh(d+ex)}\sqrt{g\coth(d+ex)}\int F^{ac+bxc}\sqrt{\coth(d+ex)}\sqrt{\sinh(d+ex)}dx}{\sqrt{\sinh(d+ex)}\sqrt{\coth(d+ex)}}$$

input
```
Int[F^(c*(a + b*x))*Sqrt[g*Coth[d + e*x]]*Sqrt[f*Sinh[d + e*x]],x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}\sqrt{g\coth\left(ex+d\right)}\,\sqrt{f\sinh\left(ex+d\right)}dx$$

input
```
int(F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2),x)
```

output
```
int(F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\sinh(d+ex)}\,dx = \int \sqrt{g\coth\left(ex+d\right)}\sqrt{f\sinh\left(ex+d\right)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2),x, alg
orithm="fricas")
```

output
```
integral(sqrt(g*coth(e*x + d))*sqrt(f*sinh(e*x + d))*F^(b*c*x + a*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\sinh(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(g*coth(e*x+d))**(1/2)*(f*sinh(e*x+d))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\sinh(d+ex)}\,dx = \int \sqrt{g\coth(ex+d)}\sqrt{f\sinh(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2),x, alg
orithm="maxima")
```

output
```
integrate(sqrt(g*coth(e*x + d))*sqrt(f*sinh(e*x + d))*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\sinh(d+ex)}\,dx = \int \sqrt{g\coth(ex+d)}\sqrt{f\sinh(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2),x, alg
orithm="giac")
```

output
```
integrate(sqrt(g*coth(e*x + d))*sqrt(f*sinh(e*x + d))*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\sinh(d+ex)}\,dx = \int F^{c\,(a+b\,x)}\,\sqrt{g\coth\left(d+e\,x\right)}\,\sqrt{f\sinh\left(d+e\,x\right)}\,dx$$

input
```
int(F^(c*(a + b*x))*(g*coth(d + e*x))^(1/2)*(f*sinh(d + e*x))^(1/2),x)
```

output
```
int(F^(c*(a + b*x))*(g*coth(d + e*x))^(1/2)*(f*sinh(d + e*x))^(1/2), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\sinh(d+ex)}\,dx$$
$$=\sqrt{g}\,f^{ac+\frac{1}{2}}\left(\int f^{bcx}\sqrt{\sinh(ex+d)}\,\sqrt{\coth(ex+d)}dx\right)$$

input
```
int(F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*sinh(e*x+d))^(1/2),x)
```

output
```
sqrt(g)*f**((2*a*c + 1)/2)*int(f**(b*c*x)*sqrt(sinh(d + e*x))*sqrt(coth(d
+ e*x)),x)
```

3.152 $\int F^{c(a+bx)}\sqrt{f\text{csch}(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 107

$$\int F^{c(a+bx)}\sqrt{f\text{csch}(d+ex)}\sqrt{g\tanh(d+ex)}\,dx = \frac{2\sqrt{1+e^{2d+2ex}}F^{c(a+bx)}\sqrt{f\text{csch}(d+ex)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{e+2bc\log(F)}{4e},\frac{1}{4}\left(5+\frac{2bc\log(F)}{e}\right),-e^{2d+2ex}\right)\cdots}{e+2bc\log(F)}$$

output
```
2*(1+exp(2*e*x+2*d))^(1/2)*F^(c*(b*x+a))*(f*csch(e*x+d))^(1/2)*hypergeom([
1/2, 1/4*(e+2*b*c*ln(F))/e],[5/4+1/2*b*c*ln(F)/e],-exp(2*e*x+2*d))*(g*tanh
(e*x+d))^(1/2)/(e+2*b*c*ln(F))
```

Mathematica [A] (warning: unable to verify)

Time = 3.42 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06

$$\int F^{c(a+bx)}\sqrt{f\text{csch}(d+ex)}\sqrt{g\tanh(d+ex)}\,dx = \frac{4fF^{c(a+bx)}\cosh(d+ex)(1+\coth(d+ex))\,\text{Hypergeometric2F1}\left(1,\frac{3}{4}+\frac{bc\log(F)}{2e},\frac{5}{4}+\frac{bc\log(F)}{2e},-\cosh(2(d\cdots\right.}{\sqrt{f\text{csch}(d+ex)}(e+2bc\log(F))}$$

input
```
Integrate[F^(c*(a + b*x))*Sqrt[f*Csch[d + e*x]]*Sqrt[g*Tanh[d + e*x]],x]
```

output

```
(4*f*F^(c*(a + b*x))*Cosh[d + e*x]*(1 + Coth[d + e*x])*Hypergeometric2F1[1
, 3/4 + (b*c*Log[F])/(2*e), 5/4 + (b*c*Log[F])/(2*e), -Cosh[2*(d + e*x)] -
 Sinh[2*(d + e*x)]]*Sqrt[g*Tanh[d + e*x]])/(Sqrt[f*Csch[d + e*x]]*(e + 2*b
*c*Log[F]))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\sqrt{f\operatorname{csch}(d+ex)}\int F^{c(a+bx)}\sqrt{\operatorname{csch}(d+ex)}\sqrt{g\tanh(d+ex)}dx}{\sqrt{\operatorname{csch}(d+ex)}}$$

$$\Big\downarrow \text{7271}$$

$$\frac{\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\tanh(d+ex)}\int F^{c(a+bx)}\sqrt{\operatorname{csch}(d+ex)}\sqrt{\tanh(d+ex)}dx}{\sqrt{\tanh(d+ex)}\sqrt{\operatorname{csch}(d+ex)}}$$

$$\Big\downarrow \text{7292}$$

$$\frac{\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\tanh(d+ex)}\int F^{ac+bxc}\sqrt{\operatorname{csch}(d+ex)}\sqrt{\tanh(d+ex)}dx}{\sqrt{\tanh(d+ex)}\sqrt{\operatorname{csch}(d+ex)}}$$

$$\Big\downarrow \text{7299}$$

$$\frac{\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\tanh(d+ex)}\int F^{ac+bxc}\sqrt{\operatorname{csch}(d+ex)}\sqrt{\tanh(d+ex)}dx}{\sqrt{\tanh(d+ex)}\sqrt{\operatorname{csch}(d+ex)}}$$

input

```
Int[F^(c*(a + b*x))*Sqrt[f*Csch[d + e*x]]*Sqrt[g*Tanh[d + e*x]],x]
```

output

```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}\sqrt{f\ \operatorname{csch}(ex+d)}\,\sqrt{g\tanh(ex+d)}dx$$

input
```
int(F^(c*(b*x+a))*(f*csch(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x)
```

output
```
int(F^(c*(b*x+a))*(f*csch(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\tanh(d+ex)}\,dx = \int \sqrt{f\operatorname{csch}(ex+d)}\sqrt{g\tanh(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x, alg
orithm="fricas")
```

output
```
integral(sqrt(f*csch(e*x + d))*sqrt(g*tanh(e*x + d))*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\tanh(d+ex)}\,dx = \int F^{c(a+bx)}\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*csch(e*x+d))**(1/2)*(g*tanh(e*x+d))**(1/2),x)
```

output
```
Integral(F**(c*(a + b*x))*sqrt(f*csch(d + e*x))*sqrt(g*tanh(d + e*x)), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$
$$=\int \sqrt{f\operatorname{csch}(ex+d)}\sqrt{g\tanh(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x, alg
orithm="maxima")
```

output
```
integrate(sqrt(f*csch(e*x + d))*sqrt(g*tanh(e*x + d))*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$
$$=\int \sqrt{f\operatorname{csch}(ex+d)}\sqrt{g\tanh(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x, alg
orithm="giac")
```

output
```
integrate(sqrt(f*csch(e*x + d))*sqrt(g*tanh(e*x + d))*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$
$$=\int F^{c\,(a+b\,x)}\,\sqrt{g\tanh\left(d+e\,x\right)}\,\sqrt{\frac{f}{\sinh\left(d+e\,x\right)}}\,dx$$

input
```
int(F^(c*(a + b*x))*(g*tanh(d + e*x))^(1/2)*(f/sinh(d + e*x))^(1/2),x)
```

output
```
int(F^(c*(a + b*x))*(g*tanh(d + e*x))^(1/2)*(f/sinh(d + e*x))^(1/2), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$
$$=\sqrt{g}\,f^{ac+\frac{1}{2}}\left(\int f^{bcx}\sqrt{\tanh\left(ex+d\right)}\,\sqrt{\operatorname{csch}\left(ex+d\right)}dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*csch(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x)
```

output
```
sqrt(g)*f**((2*a*c + 1)/2)*int(f**(b*c*x)*sqrt(tanh(d + e*x))*sqrt(csch(d
+ e*x)),x)
```

3.153 $\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\,\text{csch}(d+ex)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 132

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\,\text{csch}(d+ex)}\,dx = \frac{2\left(1-e^{2d+2ex}\right)F^{c(a+bx)}\,\text{AppellF1}\left(\frac{e+2bc\log(F)}{4e},-\frac{1}{2},1,\frac{1}{4}\left(5+\frac{2bc\log(F)}{e}\right),-e^{2d+2ex},e^{2d+2ex}\right)\sqrt{g\coth(d+e\cdots}{\sqrt{1+e^{2d+2ex}}(e+2bc\log(F))}$$

output
```
2*(1-exp(2*e*x+2*d))*F^(c*(b*x+a))*AppellF1(1/4*(e+2*b*c*ln(F))/e,-1/2,1,5
/4+1/2*b*c*ln(F)/e,-exp(2*e*x+2*d),exp(2*e*x+2*d))*(g*coth(e*x+d))^(1/2)*(
f*csch(e*x+d))^(1/2)/(1+exp(2*e*x+2*d))^(1/2)/(e+2*b*c*ln(F))
```

Mathematica [F]

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\,\text{csch}(d+ex)}\,dx = \int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\,\text{csch}(d+ex)}\,dx$$

input
```
Integrate[F^(c*(a + b*x))*Sqrt[g*Coth[d + e*x]]*Sqrt[f*Csch[d + e*x]],x]
```

output
```
Integrate[F^(c*(a + b*x))*Sqrt[g*Coth[d + e*x]]*Sqrt[f*Csch[d + e*x]], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\coth(d+ex)}\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$\frac{\sqrt{g\coth(d+ex)}\int F^{c(a+bx)}\sqrt{\coth(d+ex)}\sqrt{f\operatorname{csch}(d+ex)}dx}{\sqrt{\coth(d+ex)}}$$

$$\Big\downarrow \text{ 7271}$$

$$\frac{\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\coth(d+ex)}\int F^{c(a+bx)}\sqrt{\coth(d+ex)}\sqrt{\operatorname{csch}(d+ex)}dx}{\sqrt{\coth(d+ex)}\sqrt{\operatorname{csch}(d+ex)}}$$

$$\Big\downarrow \text{ 7292}$$

$$\frac{\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\coth(d+ex)}\int F^{ac+bxc}\sqrt{\coth(d+ex)}\sqrt{\operatorname{csch}(d+ex)}dx}{\sqrt{\coth(d+ex)}\sqrt{\operatorname{csch}(d+ex)}}$$

$$\Big\downarrow \text{ 7299}$$

$$\frac{\sqrt{f\operatorname{csch}(d+ex)}\sqrt{g\coth(d+ex)}\int F^{ac+bxc}\sqrt{\coth(d+ex)}\sqrt{\operatorname{csch}(d+ex)}dx}{\sqrt{\coth(d+ex)}\sqrt{\operatorname{csch}(d+ex)}}$$

input
```
Int[F^(c*(a + b*x))*Sqrt[g*Coth[d + e*x]]*Sqrt[f*Csch[d + e*x]],x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}\sqrt{g\coth\left(ex+d\right)}\,\sqrt{f\ \operatorname{csch}\left(ex+d\right)}dx$$

input
```
int(F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*csch(e*x+d))^(1/2),x)
```

output
```
int(F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*csch(e*x+d))^(1/2),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\operatorname{csch}(d+ex)}\,dx$$
$$=\int\sqrt{g\coth\left(ex+d\right)}\sqrt{f\operatorname{csch}\left(ex+d\right)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*csch(e*x+d))^(1/2),x, alg
orithm="fricas")
```

output
```
integral(sqrt(g*coth(e*x + d))*sqrt(f*csch(e*x + d))*F^(b*c*x + a*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\operatorname{csch}(d+ex)}\,dx=\text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(g*coth(e*x+d))**(1/2)*(f*csch(e*x+d))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\operatorname{csch}(d+ex)}\,dx$$
$$=\int\sqrt{g\coth(ex+d)}\sqrt{f\operatorname{csch}(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*csch(e*x+d))^(1/2),x, alg
orithm="maxima")
```

output
```
integrate(sqrt(g*coth(e*x + d))*sqrt(f*csch(e*x + d))*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\operatorname{csch}(d+ex)}\,dx$$
$$=\int\sqrt{g\coth(ex+d)}\sqrt{f\operatorname{csch}(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*csch(e*x+d))^(1/2),x, alg
orithm="giac")
```

output
```
integrate(sqrt(g*coth(e*x + d))*sqrt(f*csch(e*x + d))*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\operatorname{csch}(d+ex)}\,dx$$
$$=\int F^{c\,(a+b\,x)}\,\sqrt{g\coth\left(d+e\,x\right)}\,\sqrt{\frac{f}{\sinh\left(d+e\,x\right)}}\,dx$$

input
```
int(F^(c*(a + b*x))*(g*coth(d + e*x))^(1/2)*(f/sinh(d + e*x))^(1/2),x)
```

output
```
int(F^(c*(a + b*x))*(g*coth(d + e*x))^(1/2)*(f/sinh(d + e*x))^(1/2), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\operatorname{csch}(d+ex)}\,dx$$
$$=\frac{\sqrt{g}\,f^{ac+\frac{1}{2}}\left(2f^{bcx}\sqrt{\operatorname{csch}(ex+d)}\,\sqrt{\coth(ex+d)}-\left(\int\frac{f^{bcx}\sqrt{\operatorname{csch}(ex+d)}\,\sqrt{\coth(ex+d)}}{\coth(ex+d)}dx\right)e+2\left(\int f^{bcx}\sqrt{\operatorname{csch}(\ldots\right.\right.}{2\log(f)\,bc}$$

input
```
int(F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*csch(e*x+d))^(1/2),x)
```

output
```
(sqrt(g)*f**((2*a*c + 1)/2)*(2*f**(b*c*x)*sqrt(csch(d + e*x))*sqrt(coth(d
+ e*x)) - int((f**(b*c*x)*sqrt(csch(d + e*x))*sqrt(coth(d + e*x)))/coth(d
+ e*x),x)*e + 2*int(f**(b*c*x)*sqrt(csch(d + e*x))*sqrt(coth(d + e*x))*cot
h(d + e*x),x)*e))/(2*log(f)*b*c)
```

3.154 $\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 107

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx = -\frac{2F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-1+\frac{2bc\log(F)}{e}\right),\frac{1}{4}\left(3+\frac{2bc\log(F)}{e}\right),e^{2d+2ex}\right)\sqrt{}}{\sqrt{1-e^{2d+2ex}}(e-2bc\log(F))}$$

output
```
-2*F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*hypergeom([-1/2, -1/4+1/2*b*c*ln(F)
/e],[3/4+1/2*b*c*ln(F)/e],exp(2*e*x+2*d))*(g*tanh(e*x+d))^(1/2)/(1-exp(2*e
*x+2*d))^(1/2)/(e-2*b*c*ln(F))
```

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx = \frac{4F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\,\text{Hypergeometric2F1}\left(1,\frac{5}{4}+\frac{bc\log(F)}{2e},\frac{3}{4}+\frac{bc\log(F)}{2e},\cosh(2(d+ex))+\sinh(2(d\right.}{e-2bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sqrt[f*Cosh[d + e*x]]*Sqrt[g*Tanh[d + e*x]],x]
```

output
```
(4*F^(c*(a + b*x))*Sqrt[f*Cosh[d + e*x]]*Hypergeometric2F1[1, 5/4 + (b*c*L
og[F])/(2*e), 3/4 + (b*c*Log[F])/(2*e), Cosh[2*(d + e*x)] + Sinh[2*(d + e*
x)]]*Sinh[d + e*x]*(Cosh[d + e*x] + Sinh[d + e*x])*Sqrt[g*Tanh[d + e*x]])/
(e - 2*b*c*Log[F])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$\frac{\sqrt{f\cosh(d+ex)}\int F^{c(a+bx)}\sqrt{\cosh(d+ex)}\sqrt{g\tanh(d+ex)}dx}{\sqrt{\cosh(d+ex)}}$$

$$\Big\downarrow \text{ 7271}$$

$$\frac{\sqrt{f\cosh(d+ex)}\sqrt{g\tanh(d+ex)}\int F^{c(a+bx)}\sqrt{\cosh(d+ex)}\sqrt{\tanh(d+ex)}dx}{\sqrt{\cosh(d+ex)}\sqrt{\tanh(d+ex)}}$$

$$\Big\downarrow \text{ 7292}$$

$$\frac{\sqrt{f\cosh(d+ex)}\sqrt{g\tanh(d+ex)}\int F^{ac+bxc}\sqrt{\cosh(d+ex)}\sqrt{\tanh(d+ex)}dx}{\sqrt{\cosh(d+ex)}\sqrt{\tanh(d+ex)}}$$

$$\Big\downarrow \text{ 7299}$$

$$\frac{\sqrt{f\cosh(d+ex)}\sqrt{g\tanh(d+ex)}\int F^{ac+bxc}\sqrt{\cosh(d+ex)}\sqrt{\tanh(d+ex)}dx}{\sqrt{\cosh(d+ex)}\sqrt{\tanh(d+ex)}}$$

input
```
Int[F^(c*(a + b*x))*Sqrt[f*Cosh[d + e*x]]*Sqrt[g*Tanh[d + e*x]],x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}\sqrt{f\cosh\left(ex+d\right)}\,\sqrt{g\tanh\left(ex+d\right)}dx$$

input
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x)
```

output
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx = \int \sqrt{f\cosh\left(ex+d\right)}\sqrt{g\tanh\left(ex+d\right)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x, alg
orithm="fricas")
```

output
```
integral(sqrt(f*cosh(e*x + d))*sqrt(g*tanh(e*x + d))*F^(b*c*x + a*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(f*cosh(e*x+d))**(1/2)*(g*tanh(e*x+d))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx = \int \sqrt{f\cosh\left(ex+d\right)}\sqrt{g\tanh\left(ex+d\right)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x, alg
orithm="maxima")
```

output
```
integrate(sqrt(f*cosh(e*x + d))*sqrt(g*tanh(e*x + d))*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx = \int \sqrt{f\cosh\left(ex+d\right)}\sqrt{g\tanh\left(ex+d\right)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x, alg
orithm="giac")
```

output
```
integrate(sqrt(f*cosh(e*x + d))*sqrt(g*tanh(e*x + d))*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx = \int F^{c\,(a+b\,x)}\,\sqrt{f\cosh\left(d+e\,x\right)}\,\sqrt{g\tanh\left(d+e\,x\right)}\,dx$$

input
```
int(F^(c*(a + b*x))*(f*cosh(d + e*x))^(1/2)*(g*tanh(d + e*x))^(1/2),x)
```

output
```
int(F^(c*(a + b*x))*(f*cosh(d + e*x))^(1/2)*(g*tanh(d + e*x))^(1/2), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\tanh(d+ex)}\,dx = \frac{\sqrt{g}\,f^{ac+\frac{1}{2}}\left(2f^{bcx}\sqrt{\tanh(ex+d)}\,\sqrt{\cosh(ex+d)}-\left(\int \frac{f^{bcx}\sqrt{\tanh(ex+d)}\,\sqrt{\cosh(ex+d)}\,\sinh(ex+d)}{\cosh(ex+d)}dx\right)e-\left(\int \frac{f^{bcx}}{}\right.\right.}{2\log(f)\,bc}$$

input
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x)
```

output
```
(sqrt(g)*f**((2*a*c + 1)/2)*(2*f**(b*c*x)*sqrt(tanh(d + e*x))*sqrt(cosh(d
+ e*x)) - int((f**(b*c*x)*sqrt(tanh(d + e*x))*sqrt(cosh(d + e*x))*sinh(d +
 e*x))/cosh(d + e*x),x)*e - int((f**(b*c*x)*sqrt(tanh(d + e*x))*sqrt(cosh(
d + e*x)))/tanh(d + e*x),x)*e + int(f**(b*c*x)*sqrt(tanh(d + e*x))*sqrt(co
sh(d + e*x))*tanh(d + e*x),x)*e))/(2*log(f)*b*c)
```

3.155 $\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\coth(d+ex)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 207

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\coth(d+ex)}\,dx$$
$$= -\frac{2(1-e^{2d+2ex})F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\coth(d+ex)}}{(1+e^{2d+2ex})(e+2bc\log(F))}$$
$$-\frac{8bc\sqrt{1-e^{2d+2ex}}F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\coth(d+ex)}\,\text{Hypergeometric2F1}\big(\frac{1}{2},\frac{1}{4}\big(-1+\frac{2bc\log(F)}{e}\big)}{(1+e^{2d+2ex})(e^2-4b^2c^2\log^2(F))}$$

output
```
-2*(1-exp(2*e*x+2*d))*F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*coth(e*x+d))^
(1/2)/(1+exp(2*e*x+2*d))/(e+2*b*c*ln(F))-8*b*c*(1-exp(2*e*x+2*d))^(1/2)*F^
(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*coth(e*x+d))^(1/2)*hypergeom([1/2, -1
/4+1/2*b*c*ln(F)/e],[3/4+1/2*b*c*ln(F)/e],exp(2*e*x+2*d))*ln(F)/(1+exp(2*e
*x+2*d))/(e^2-4*b^2*c^2*ln(F)^2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.11 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.94

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\coth(d+ex)}\,dx$$
$$= \frac{2F^{c(a+bx)}g\sqrt{f\cosh(d+ex)}\left(\text{Hypergeometric2F1}\left(1,\frac{e+2bc\log(F)}{4e},\frac{3}{4}+\frac{bc\log(F)}{2e},\cosh(2(d+ex))+\sinh(2(d\right.\right.}{}$$

input
```
Integrate[F^(c*(a + b*x))*Sqrt[f*Cosh[d + e*x]]*Sqrt[g*Coth[d + e*x]],x]
```

output
```
(2*F^(c*(a + b*x))*g*Sqrt[f*Cosh[d + e*x]]*(Hypergeometric2F1[1, (e + 2*b*
c*Log[F])/(4*e), 3/4 + (b*c*Log[F])/(2*e), Cosh[2*(d + e*x)] + Sinh[2*(d +
 e*x)]]*(3*e + 2*b*c*Log[F]) - Hypergeometric2F1[1, 5/4 + (b*c*Log[F])/(2*
e), 7/4 + (b*c*Log[F])/(2*e), Cosh[2*(d + e*x)] + Sinh[2*(d + e*x)]]*(e -
2*b*c*Log[F])*(Cosh[2*(d + e*x)] + Sinh[2*(d + e*x)])))/(Sqrt[g*Coth[d + e
*x]]*(e - 2*b*c*Log[F])*(3*e + 2*b*c*Log[F]))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\coth(d+ex)}\,dx$$
$$\Big\downarrow \text{7271}$$
$$\frac{\sqrt{f\cosh(d+ex)}\int F^{c(a+bx)}\sqrt{\cosh(d+ex)}\sqrt{g\coth(d+ex)}dx}{\sqrt{\cosh(d+ex)}}$$
$$\Big\downarrow \text{7271}$$
$$\frac{\sqrt{f\cosh(d+ex)}\sqrt{g\coth(d+ex)}\int F^{c(a+bx)}\sqrt{\cosh(d+ex)}\sqrt{\coth(d+ex)}dx}{\sqrt{\cosh(d+ex)}\sqrt{\coth(d+ex)}}$$
$$\Big\downarrow \text{7292}$$
$$\frac{\sqrt{f\cosh(d+ex)}\sqrt{g\coth(d+ex)}\int F^{ac+bxc}\sqrt{\cosh(d+ex)}\sqrt{\coth(d+ex)}dx}{\sqrt{\cosh(d+ex)}\sqrt{\coth(d+ex)}}$$

$$\downarrow \text{7299}$$

$$\frac{\sqrt{f\cosh(d+ex)}\sqrt{g\coth(d+ex)}\int F^{ac+bxc}\sqrt{\cosh(d+ex)}\sqrt{\coth(d+ex)}dx}{\sqrt{\cosh(d+ex)}\sqrt{\coth(d+ex)}}$$

input
```
Int[F^(c*(a + b*x))*Sqrt[f*Cosh[d + e*x]]*Sqrt[g*Coth[d + e*x]],x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}\sqrt{f\cosh(ex+d)}\,\sqrt{g\coth(ex+d)}dx$$

input
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*coth(e*x+d))^(1/2),x)
```

output
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*coth(e*x+d))^(1/2),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\coth(d+ex)}\,dx = \int \sqrt{f\cosh(ex+d)}\sqrt{g\coth(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*coth(e*x+d))^(1/2),x, alg
orithm="fricas")
```

output
```
integral(sqrt(f*cosh(e*x + d))*sqrt(g*coth(e*x + d))*F^(b*c*x + a*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\coth(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(f*cosh(e*x+d))**(1/2)*(g*coth(e*x+d))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\coth(d+ex)}\,dx = \int \sqrt{f\cosh(ex+d)}\sqrt{g\coth(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*coth(e*x+d))^(1/2),x, alg
orithm="maxima")
```

output
```
integrate(sqrt(f*cosh(e*x + d))*sqrt(g*coth(e*x + d))*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\coth(d+ex)}\,dx = \int \sqrt{f\cosh(ex+d)}\sqrt{g\coth(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*coth(e*x+d))^(1/2),x, alg
orithm="giac")
```

output
```
integrate(sqrt(f*cosh(e*x + d))*sqrt(g*coth(e*x + d))*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\coth(d+ex)}\,dx = \int F^{c\,(a+b\,x)}\,\sqrt{f\cosh\left(d+e\,x\right)}\,\sqrt{g\coth\left(d+e\,x\right)}\,dx$$

input
```
int(F^(c*(a + b*x))*(f*cosh(d + e*x))^(1/2)*(g*coth(d + e*x))^(1/2),x)
```

output
```
int(F^(c*(a + b*x))*(f*cosh(d + e*x))^(1/2)*(g*coth(d + e*x))^(1/2), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\sqrt{f\cosh(d+ex)}\sqrt{g\coth(d+ex)}\,dx = \frac{\sqrt{g}\,f^{ac+\frac{1}{2}}\left(2f^{bcx}\sqrt{\coth\left(ex+d\right)}\,\sqrt{\cosh\left(ex+d\right)}-\left(\int\frac{f^{bcx}\sqrt{\coth(ex+d)}\,\sqrt{\cosh(ex+d)}\,\sinh(ex+d)}{\cosh(ex+d)}dx\right)e-\left(\int\frac{f^{bcx}\sqrt{}}{}\right.\right.}{2\log\left(f\right)bc}$$

input
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^(1/2)*(g*coth(e*x+d))^(1/2),x)
```

output
```
(sqrt(g)*f**((2*a*c + 1)/2)*(2*f**(b*c*x)*sqrt(coth(d + e*x))*sqrt(cosh(d
+ e*x)) - int((f**(b*c*x)*sqrt(coth(d + e*x))*sqrt(cosh(d + e*x))*sinh(d +
 e*x))/cosh(d + e*x),x)*e - int((f**(b*c*x)*sqrt(coth(d + e*x))*sqrt(cosh(
d + e*x)))/coth(d + e*x),x)*e + int(f**(b*c*x)*sqrt(coth(d + e*x))*sqrt(co
sh(d + e*x))*coth(d + e*x),x)*e))/(2*log(f)*b*c)
```

3.156 $\int F^{c(a+bx)}\sqrt{f\text{sech}(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 147

$$\int F^{c(a+bx)}\sqrt{f\text{sech}(d+ex)}\sqrt{g\tanh(d+ex)}\,dx =$$
$$-\frac{\left(e^{2d+2ex}\right)^{-\frac{e+2bc\log(F)}{4e}}\left(1-e^{2d+2ex}\right)\left(1+e^{2d+2ex}\right)F^{c(a+bx)}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2bc\log(F)}{e}\right),1,\frac{5}{2},1-e^{2d+2e}\right.}{6e}$$

output
```
-1/6*(1-exp(2*e*x+2*d))*(1+exp(2*e*x+2*d))*F^(c*(b*x+a))*AppellF1(3/2,3/4-
1/2*b*c*ln(F)/e,1,5/2,1-exp(2*e*x+2*d),1/2-1/2*exp(2*e*x+2*d))*(f*sech(e*x
+d))^(1/2)*(g*tanh(e*x+d))^(1/2)/e/(exp(2*e*x+2*d)^(1/4*(e+2*b*c*ln(F))/e)
)
```

Mathematica [F]

$$\int F^{c(a+bx)}\sqrt{f\text{sech}(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$
$$=\int F^{c(a+bx)}\sqrt{f\text{sech}(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$

input
```
Integrate[F^(c*(a + b*x))*Sqrt[f*Sech[d + e*x]]*Sqrt[g*Tanh[d + e*x]],x]
```

output
```
Integrate[F^(c*(a + b*x))*Sqrt[f*Sech[d + e*x]]*Sqrt[g*Tanh[d + e*x]], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\sqrt{f\text{sech}(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$

$$\Big\downarrow \ 7271$$

$$\frac{\sqrt{f\text{sech}(d+ex)}\int F^{c(a+bx)}\sqrt{\text{sech}(d+ex)}\sqrt{g\tanh(d+ex)}dx}{\sqrt{\text{sech}(d+ex)}}$$

$$\Big\downarrow \ 7271$$

$$\frac{\sqrt{f\text{sech}(d+ex)}\sqrt{g\tanh(d+ex)}\int F^{c(a+bx)}\sqrt{\text{sech}(d+ex)}\sqrt{\tanh(d+ex)}dx}{\sqrt{\tanh(d+ex)}\sqrt{\text{sech}(d+ex)}}$$

$$\Big\downarrow \ 7292$$

$$\frac{\sqrt{f\text{sech}(d+ex)}\sqrt{g\tanh(d+ex)}\int F^{ac+bxc}\sqrt{\text{sech}(d+ex)}\sqrt{\tanh(d+ex)}dx}{\sqrt{\tanh(d+ex)}\sqrt{\text{sech}(d+ex)}}$$

$$\Big\downarrow \ 7299$$

$$\frac{\sqrt{f\text{sech}(d+ex)}\sqrt{g\tanh(d+ex)}\int F^{ac+bxc}\sqrt{\text{sech}(d+ex)}\sqrt{\tanh(d+ex)}dx}{\sqrt{\tanh(d+ex)}\sqrt{\text{sech}(d+ex)}}$$

input
```
Int[F^(c*(a + b*x))*Sqrt[f*Sech[d + e*x]]*Sqrt[g*Tanh[d + e*x]],x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}\sqrt{f\ \mathrm{sech}\left(ex+d\right)}\ \sqrt{g\tanh\left(ex+d\right)}dx$$

input
```
int(F^(c*(b*x+a))*(f*sech(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x)
```

output
```
int(F^(c*(b*x+a))*(f*sech(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\sqrt{f\mathrm{sech}(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$
$$=\int\sqrt{f\,\mathrm{sech}\left(ex+d\right)}\sqrt{g\tanh\left(ex+d\right)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sech(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x, alg
orithm="fricas")
```

output
```
integral(sqrt(f*sech(e*x + d))*sqrt(g*tanh(e*x + d))*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\sqrt{f\mathrm{sech}(d+ex)}\sqrt{g\tanh(d+ex)}\,dx$$
$$=\int F^{c(a+bx)}\sqrt{f\,\mathrm{sech}\left(d+ex\right)}\sqrt{g\tanh\left(d+ex\right)}\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*sech(e*x+d))**(1/2)*(g*tanh(e*x+d))**(1/2),x)
```

output
```
Integral(F**(c*(a + b*x))*sqrt(f*sech(d + e*x))*sqrt(g*tanh(d + e*x)), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\sqrt{f\mathrm{sech}(d+ex)}\sqrt{g\tanh(d+ex)}\,dx = \int \sqrt{f\,\mathrm{sech}\,(ex+d)}\sqrt{g\tanh\,(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sech(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x, alg
orithm="maxima")
```

output
```
integrate(sqrt(f*sech(e*x + d))*sqrt(g*tanh(e*x + d))*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\sqrt{f\mathrm{sech}(d+ex)}\sqrt{g\tanh(d+ex)}\,dx = \int \sqrt{f\,\mathrm{sech}\,(ex+d)}\sqrt{g\tanh\,(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sech(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x, alg
orithm="giac")
```

output
```
integrate(sqrt(f*sech(e*x + d))*sqrt(g*tanh(e*x + d))*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{f\mathrm{sech}(d+ex)}\sqrt{g\tanh(d+ex)}\,dx = \int F^{c\,(a+b\,x)}\,\sqrt{g\tanh\,(d+e\,x)}\,\sqrt{\frac{f}{\cosh\,(d+e\,x)}}\,dx$$

input
```
int(F^(c*(a + b*x))*(g*tanh(d + e*x))^(1/2)*(f/cosh(d + e*x))^(1/2),x)
```

output
```
int(F^(c*(a + b*x))*(g*tanh(d + e*x))^(1/2)*(f/cosh(d + e*x))^(1/2), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\sqrt{f\mathrm{sech}(d+ex)}\sqrt{g\tanh(d+ex)}\,dx = \frac{\sqrt{g}\, f^{ac+\frac{1}{2}}\left(2f^{bcx}\sqrt{\tanh(ex+d)}\,\sqrt{\mathrm{sech}(ex+d)}-\left(\int\frac{f^{bcx}\sqrt{\tanh(ex+d)}\,\sqrt{\mathrm{sech}(ex+d)}}{\tanh(ex+d)}dx\right)e+2\left(\int f^{bcx}\sqrt{\tanh(}\right.\right.}{2\log(f)\,bc}$$

input
```
int(F^(c*(b*x+a))*(f*sech(e*x+d))^(1/2)*(g*tanh(e*x+d))^(1/2),x)
```

output
```
(sqrt(g)*f**((2*a*c + 1)/2)*(2*f**(b*c*x)*sqrt(tanh(d + e*x))*sqrt(sech(d
+ e*x)) - int((f**(b*c*x)*sqrt(tanh(d + e*x))*sqrt(sech(d + e*x)))/tanh(d
+ e*x),x)*e + 2*int(f**(b*c*x)*sqrt(tanh(d + e*x))*sqrt(sech(d + e*x))*tan
h(d + e*x),x)*e))/(2*log(f)*b*c)
```

3.157 $\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\text{sech}(d+ex)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 107

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\text{sech}(d+ex)}\,dx = \frac{2\sqrt{1-e^{2d+2ex}}F^{c(a+bx)}\sqrt{g\coth(d+ex)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{e+2bc\log(F)}{4e},\frac{1}{4}\left(5+\frac{2bc\log(F)}{e}\right),e^{2d+2ex}\right)\sqrt{}}{e+2bc\log(F)}$$

output
```
2*(1-exp(2*e*x+2*d))^(1/2)*F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*hypergeom([
1/2, 1/4*(e+2*b*c*ln(F))/e],[5/4+1/2*b*c*ln(F)/e],exp(2*e*x+2*d))*(f*sech(
e*x+d))^(1/2)/(e+2*b*c*ln(F))
```

Mathematica [A] (warning: unable to verify)

Time = 3.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.03

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\text{sech}(d+ex)}\,dx = -\frac{4F^{c(a+bx)}\sqrt{g\coth(d+ex)}(1+\coth(d+ex))\,\text{Hypergeometric2F1}\left(1,\frac{3}{4}+\frac{bc\log(F)}{2e},\frac{5}{4}+\frac{bc\log(F)}{2e},\cosh(2\right.}{e+2bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sqrt[g*Coth[d + e*x]]*Sqrt[f*Sech[d + e*x]],x]
```

output
```
(-4*F^(c*(a + b*x))*Sqrt[g*Coth[d + e*x]]*(1 + Coth[d + e*x])*Hypergeometr
ic2F1[1, 3/4 + (b*c*Log[F])/(2*e), 5/4 + (b*c*Log[F])/(2*e), Cosh[2*(d + e
*x)] + Sinh[2*(d + e*x)]]*Sqrt[f*Sech[d + e*x]]*Sinh[d + e*x]^2)/(e + 2*b*
c*Log[F])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}\sqrt{f\text{sech}(d+ex)}\sqrt{g\coth(d+ex)}\,dx$$

$$\Big\downarrow \ 7271$$

$$\frac{\sqrt{g\coth(d+ex)}\int F^{c(a+bx)}\sqrt{\coth(d+ex)}\sqrt{f\text{sech}(d+ex)}dx}{\sqrt{\coth(d+ex)}}$$

$$\Big\downarrow \ 7271$$

$$\frac{\sqrt{f\text{sech}(d+ex)}\sqrt{g\coth(d+ex)}\int F^{c(a+bx)}\sqrt{\coth(d+ex)}\sqrt{\text{sech}(d+ex)}dx}{\sqrt{\coth(d+ex)}\sqrt{\text{sech}(d+ex)}}$$

$$\Big\downarrow \ 7292$$

$$\frac{\sqrt{f\text{sech}(d+ex)}\sqrt{g\coth(d+ex)}\int F^{ac+bxc}\sqrt{\coth(d+ex)}\sqrt{\text{sech}(d+ex)}dx}{\sqrt{\coth(d+ex)}\sqrt{\text{sech}(d+ex)}}$$

$$\Big\downarrow \ 7299$$

$$\frac{\sqrt{f\text{sech}(d+ex)}\sqrt{g\coth(d+ex)}\int F^{ac+bxc}\sqrt{\coth(d+ex)}\sqrt{\text{sech}(d+ex)}dx}{\sqrt{\coth(d+ex)}\sqrt{\text{sech}(d+ex)}}$$

input
```
Int[F^(c*(a + b*x))*Sqrt[g*Coth[d + e*x]]*Sqrt[f*Sech[d + e*x]],x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}\sqrt{g \coth \left(ex+d\right)}\, \sqrt{f\ \operatorname{sech}\left(ex+d\right)}dx$$

input
```
int(F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*sech(e*x+d))^(1/2),x)
```

output
```
int(F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*sech(e*x+d))^(1/2),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\operatorname{sech}(d+ex)}\,dx$$
$$= \int \sqrt{g\coth\left(ex+d\right)}\sqrt{f\operatorname{sech}\left(ex+d\right)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*sech(e*x+d))^(1/2),x, alg
orithm="fricas")
```

output
```
integral(sqrt(g*coth(e*x + d))*sqrt(f*sech(e*x + d))*F^(b*c*x + a*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\operatorname{sech}(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(g*coth(e*x+d))**(1/2)*(f*sech(e*x+d))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\text{sech}(d+ex)}\,dx$$
$$=\int \sqrt{g\coth{(ex+d)}}\sqrt{f\,\text{sech}\,(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*sech(e*x+d))^(1/2),x, alg
orithm="maxima")
```

output
```
integrate(sqrt(g*coth(e*x + d))*sqrt(f*sech(e*x + d))*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\text{sech}(d+ex)}\,dx$$
$$=\int \sqrt{g\coth{(ex+d)}}\sqrt{f\,\text{sech}\,(ex+d)}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*sech(e*x+d))^(1/2),x, alg
orithm="giac")
```

output
```
integrate(sqrt(g*coth(e*x + d))*sqrt(f*sech(e*x + d))*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\text{sech}(d+ex)}\,dx$$
$$=\int F^{c\,(a+b\,x)}\,\sqrt{g\coth{(d+e\,x)}}\,\sqrt{\frac{f}{\cosh{(d+e\,x)}}}\,dx$$

input
```
int(F^(c*(a + b*x))*(g*coth(d + e*x))^(1/2)*(f/cosh(d + e*x))^(1/2),x)
```

output
```
int(F^(c*(a + b*x))*(g*coth(d + e*x))^(1/2)*(f/cosh(d + e*x))^(1/2), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\sqrt{g\coth(d+ex)}\sqrt{f\mathrm{sech}(d+ex)}\,dx$$
$$=\sqrt{g}\,f^{ac+\frac{1}{2}}\left(\int f^{bcx}\sqrt{\mathrm{sech}\,(ex+d)}\,\sqrt{\coth\,(ex+d)}dx\right)$$

input
```
int(F^(c*(b*x+a))*(g*coth(e*x+d))^(1/2)*(f*sech(e*x+d))^(1/2),x)
```

output
```
sqrt(g)*f**((2*a*c + 1)/2)*int(f**(b*c*x)*sqrt(sech(d + e*x))*sqrt(coth(d
+ e*x)),x)
```

3.158 $\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\sinh(d+ex))^q\,dx$

Optimal result

Integrand size = 30, antiderivative size = 127

$$\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\sinh(d+ex))^q\,dx = -\frac{\left(1-e^{2d+2ex}\right)^{-p-q}F^{c(a+bx)}\text{Hypergeometric2F1}\left(-p-q,-\frac{e(p+q)-bc\log(F)}{2e},\frac{e(2-p-q)+bc\log(F)}{2e},e^{2d+2ex}\right)(f}{e(p+q)-bc\log(F)}$$

output
```
-(1-exp(2*e*x+2*d))^(-p-q)*F^(c*(b*x+a))*hypergeom([-p-q, -1/2*(e*(p+q)-b*
c*ln(F))/e],[1/2*(e*(2-p-q)+b*c*ln(F))/e],exp(2*e*x+2*d))*(f*sinh(e*x+d))^
p*(g*sinh(e*x+d))^q/(e*(p+q)-b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.96

$$\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\sinh(d+ex))^q\,dx = -\frac{\left(1-e^{2(d+ex)}\right)^{-p-q}F^{c(a+bx)}\text{Hypergeometric2F1}\left(-p-q,\frac{-e(p+q)+bc\log(F)}{2e},\frac{-e(-2+p+q)+bc\log(F)}{2e},e^{2(d+ex)}\right)}{e(p+q)-bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Sinh[d + e*x])^p*(g*Sinh[d + e*x])^q,x]
```

output
```
-(((1 - E^(2*(d + e*x)))^(-p - q)*F^(c*(a + b*x))*Hypergeometric2F1[-p - q
, (-(e*(p + q)) + b*c*Log[F])/(2*e), (-(e*(-2 + p + q)) + b*c*Log[F])/(2*e
), E^(2*(d + e*x))]*(f*Sinh[d + e*x])^p*(g*Sinh[d + e*x])^q)/(e*(p + q) -
b*c*Log[F]))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2034, 7271, 6005, 2689}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\sinh(d+ex))^q\,dx$$

$$\Big\downarrow \; 2034$$

$$(f\sinh(d+ex))^{-q}(g\sinh(d+ex))^q\int F^{c(a+bx)}(f\sinh(d+ex))^{p+q}dx$$

$$\Big\downarrow \; 7271$$

$$(f\sinh(d+ex))^p(g\sinh(d+ex))^q\sinh^{-p-q}(d+ex)\int F^{c(a+bx)}\sinh^{p+q}(d+ex)dx$$

$$\Big\downarrow \; 6005$$

$$e^{(p+q)(d+ex)}\left(e^{2(d+ex)}-1\right)^{-p-q}(f\sinh(d+ex))^p(g\sinh(d+ex))^q\int e^{-((p+q)(d+ex))}\left(-1+e^{2(d+ex)}\right)^{p+q}F^{c(a+bx)}dx$$

$$\Big\downarrow \; 2689$$

$$-\frac{F^{c(a+bx)}\left(1-e^{2(d+ex)}\right)^{-p-q}(f\sinh(d+ex))^p(g\sinh(d+ex))^q\,\text{Hypergeometric2F1}\left(-p-q,-\frac{e(p+q)-bc\log(F)}{2e},\ldots\right)}{e(p+q)-bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*(f*Sinh[d + e*x])^p*(g*Sinh[d + e*x])^q,x]
```

output
```
-(((1 - E^(2*(d + e*x)))^(-p - q)*F^(c*(a + b*x))*Hypergeometric2F1[-p - q
, -1/2*(e*(p + q) - b*c*Log[F])/e, (e*(2 - p - q) + b*c*Log[F])/(2*e), E^(
2*(d + e*x))]*(f*Sinh[d + e*x])^p*(g*Sinh[d + e*x])^q)/(e*(p + q) - b*c*Lo
g[F]))
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 2689
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

rule 6005
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[E^(n*(d + e*x))*(Sinh[d + e*x]^n/(-1 + E^(2*(d + e*x)))^n)   In
t[F^(c*(a + b*x))*((-1 + E^(2*(d + e*x)))^n/E^(n*(d + e*x))), x], x] /; Fre
eQ[{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}(f\sinh(ex+d))^p(g\sinh(ex+d))^q\,dx$$

input
```
int(F^(c*(b*x+a))*(f*sinh(e*x+d))^p*(g*sinh(e*x+d))^q,x)
```

output
```
int(F^(c*(b*x+a))*(f*sinh(e*x+d))^p*(g*sinh(e*x+d))^q,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\sinh(d+ex))^q\,dx = \int (f\sinh(ex+d))^p(g\sinh(ex+d))^q F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sinh(e*x+d))^p*(g*sinh(e*x+d))^q,x, algorithm="
fricas")
```

output
```
integral((f*sinh(e*x + d))^p*(g*sinh(e*x + d))^q*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\sinh(d+ex))^q\,dx = \int F^{c(a+bx)}(f\sinh(d+ex))^p(g\sinh(d+ex))^q\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*sinh(e*x+d))**p*(g*sinh(e*x+d))**q,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*sinh(d + e*x))**p*(g*sinh(d + e*x))**q, x)
```

Maxima [F]

$$\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\sinh(d+ex))^q\,dx = \int (f\sinh(ex+d))^p(g\sinh(ex+d))^q F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sinh(e*x+d))^p*(g*sinh(e*x+d))^q,x, algorithm="
maxima")
```

output
```
integrate((f*sinh(e*x + d))^p*(g*sinh(e*x + d))^q*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\sinh(d+ex))^q\,dx = \int (f\sinh(ex+d))^p(g\sinh(ex+d))^q F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sinh(e*x+d))^p*(g*sinh(e*x+d))^q,x, algorithm="
giac")
```

output
```
integrate((f*sinh(e*x + d))^p*(g*sinh(e*x + d))^q*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\sinh(d+ex))^q\,dx = \int F^{c\,(a+b\,x)}\,(f\sinh(d+e\,x))^p\,(g\sinh(d+e\,x))^q\,dx$$

input
```
int(F^(c*(a + b*x))*(f*sinh(d + e*x))^p*(g*sinh(d + e*x))^q,x)
```

output
```
int(F^(c*(a + b*x))*(f*sinh(d + e*x))^p*(g*sinh(d + e*x))^q, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\sinh(d+ex))^q\,dx = \frac{g^q f^{ac+p}\left(f^{bcx}\sinh(ex+d)^{p+q}-\left(\int\frac{f^{bcx}\sinh(ex+d)^{p+q}\cosh(ex+d)}{\sinh(ex+d)}dx\right)ep-\left(\int\frac{f^{bcx}\sinh(ex+d)^{p+q}\cosh(ex+d)}{\sinh(ex+d)}dx\right)eq\right)}{\log(f)\,bc}$$

input
```
int(F^(c*(b*x+a))*(f*sinh(e*x+d))^p*(g*sinh(e*x+d))^q,x)
```

output
```
(g**q*f**(a*c + p)*(f**(b*c*x)*sinh(d + e*x)**(p + q) - int((f**(b*c*x)*si
nh(d + e*x)**(p + q)*cosh(d + e*x))/sinh(d + e*x),x)*e*p - int((f**(b*c*x)
*sinh(d + e*x)**(p + q)*cosh(d + e*x))/sinh(d + e*x),x)*e*q))/(log(f)*b*c)
```

3.159 $\int F^{c(a+bx)}(g\text{csch}(d+ex))^q(f\sinh(d+ex))^p\,dx$

Optimal result

Integrand size = 30, antiderivative size = 118

$$\int F^{c(a+bx)}(g\text{csch}(d+ex))^q(f\sinh(d+ex))^p\,dx =$$
$$-\frac{\left(1-e^{2d+2ex}\right)^{-p+q}F^{c(a+bx)}(g\text{csch}(d+ex))^q\,\text{Hypergeometric2F1}\left(-p+q,\frac{1}{2}\left(-p+q+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(2\right.\right.}{ep-eq-bc\log(F)}$$

output
```
-(1-exp(2*e*x+2*d))^(-p+q)*F^(c*(b*x+a))*(g*csch(e*x+d))^q*hypergeom([-p+q
, -1/2*p+1/2*q+1/2*b*c*ln(F)/e],[1-1/2*p+1/2*q+1/2*b*c*ln(F)/e],exp(2*e*x+
2*d))*(f*sinh(e*x+d))^p/(e*p-e*q-b*c*ln(F))
```

Mathematica [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(g\text{csch}(d+ex))^q(f\sinh(d+ex))^p\,dx = \$\text{Aborted}$$

input
```
Integrate[F^(c*(a + b*x))*(g*Csch[d + e*x])^q*(f*Sinh[d + e*x])^p,x]
```

output
```
$Aborted
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\text{csch}(d+ex))^q\,dx$$

$$\Big\downarrow \text{7271}$$

$$\text{csch}^{-q}(d+ex)(g\text{csch}(d+ex))^q\int F^{c(a+bx)}\text{csch}^q(d+ex)(f\sinh(d+ex))^pdx$$

$$\Big\downarrow \text{7271}$$

$$\sinh^{-p}(d+ex)\text{csch}^{-q}(d+ex)(f\sinh(d+ex))^p(g\text{csch}(d+ex))^q\int F^{c(a+bx)}\text{csch}^q(d+ex)\sinh^p(d+ex)dx$$

$$\Big\downarrow \text{7292}$$

$$\sinh^{-p}(d+ex)\text{csch}^{-q}(d+ex)(f\sinh(d+ex))^p(g\text{csch}(d+ex))^q\int F^{ac+bxc}\text{csch}^q(d+ex)\sinh^p(d+ex)dx$$

$$\Big\downarrow \text{7299}$$

$$\sinh^{-p}(d+ex)\text{csch}^{-q}(d+ex)(f\sinh(d+ex))^p(g\text{csch}(d+ex))^q\int F^{ac+bxc}\text{csch}^q(d+ex)\sinh^p(d+ex)dx$$

input
```
Int[F^(c*(a + b*x))*(g*Csch[d + e*x])^q*(f*Sinh[d + e*x])^p,x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}\left(g\,\text{csch}\left(ex+d\right)\right)^q\left(f\sinh\left(ex+d\right)\right)^p dx$$

input
```
int(F^(c*(b*x+a))*(g*csch(e*x+d))^q*(f*sinh(e*x+d))^p,x)
```

output
```
int(F^(c*(b*x+a))*(g*csch(e*x+d))^q*(f*sinh(e*x+d))^p,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(g\text{csch}(d+ex))^q(f\sinh(d+ex))^p\,dx$$
$$=\int(g\,\text{csch}\,(ex+d))^q\,(f\sinh\,(ex+d))^p\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*csch(e*x+d))^q*(f*sinh(e*x+d))^p,x, algorithm="
fricas")
```

output
```
integral((g*csch(e*x + d))^q*(f*sinh(e*x + d))^p*F^(b*c*x + a*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(g\text{csch}(d+ex))^q(f\sinh(d+ex))^p\,dx=\text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(g*csch(e*x+d))**q*(f*sinh(e*x+d))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{c(a+bx)}(g\operatorname{csch}(d+ex))^q(f\sinh(d+ex))^p\,dx = \int (g\operatorname{csch}(ex+d))^q(f\sinh(ex+d))^pF^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*csch(e*x+d))^q*(f*sinh(e*x+d))^p,x, algorithm="
maxima")
```

output
```
integrate((g*csch(e*x + d))^q*(f*sinh(e*x + d))^p*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(g\operatorname{csch}(d+ex))^q(f\sinh(d+ex))^p\,dx = \int (g\operatorname{csch}(ex+d))^q(f\sinh(ex+d))^pF^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*csch(e*x+d))^q*(f*sinh(e*x+d))^p,x, algorithm="
giac")
```

output
```
integrate((g*csch(e*x + d))^q*(f*sinh(e*x + d))^p*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(g\operatorname{csch}(d+ex))^q(f\sinh(d+ex))^p\,dx = \int F^{c\,(a+b\,x)}\,(f\sinh(d+e\,x))^p\left(\frac{g}{\sinh(d+e\,x)}\right)^q\,dx$$

input
```
int(F^(c*(a + b*x))*(f*sinh(d + e*x))^p*(g/sinh(d + e*x))^q,x)
```

output
```
int(F^(c*(a + b*x))*(f*sinh(d + e*x))^p*(g/sinh(d + e*x))^q, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(g\text{csch}(d+ex))^q(f\sinh(d+ex))^p\,dx$$
$$= g^q f^{ac+p}\left(\int f^{bcx}\sinh\left(ex+d\right)^p\text{csch}(ex+d)^q\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(g*csch(e*x+d))^q*(f*sinh(e*x+d))^p,x)
```

output
```
g**q*f**(a*c + p)*int(f**(b*c*x)*sinh(d + e*x)**p*csch(d + e*x)**q,x)
```

3.160 $\int F^{c(a+bx)}(f\mathbf{csch}(d+ex))^p(g\mathbf{csch}(d+ex))^q\,dx$

Optimal result

Integrand size = 30, antiderivative size = 106

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\operatorname{csch}(d+ex))^q\,dx$$
$$= \frac{\left(1-e^{2d+2ex}\right)^{p+q}F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\operatorname{csch}(d+ex))^q\,\text{Hypergeometric2F1}\left(p+q,\frac{1}{2}\left(p+q+\frac{bc\log(F}{e}\right.\right.}{e(p+q)+bc\log(F)}$$

output
```
(1-exp(2*e*x+2*d))^(p+q)*F^(c*(b*x+a))*(f*csch(e*x+d))^p*(g*csch(e*x+d))^q
*hypergeom([p+q, 1/2*p+1/2*q+1/2*b*c*ln(F)/e],[1+1/2*p+1/2*q+1/2*b*c*ln(F)
/e],exp(2*e*x+2*d))/(e*(p+q)+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.08

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\operatorname{csch}(d+ex))^q\,dx =$$
$$-\frac{\left(1-e^{-2(d+ex)}\right)^{p+q}F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\operatorname{csch}(d+ex))^q\,\text{Hypergeometric2F1}\left(p+q,\frac{e(p+q)-bc\log(F}{2e}\right.}{e(p+q)-bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Csch[d + e*x])^p*(g*Csch[d + e*x])^q,x]
```

output
```
-(((1 - E^(-2*(d + e*x)))^(p + q)*F^(c*(a + b*x))*(f*Csch[d + e*x])^p*(g*C
sch[d + e*x])^q*Hypergeometric2F1[p + q, (e*(p + q) - b*c*Log[F])/(2*e), (
e*(2 + p + q) - b*c*Log[F])/(2*e), E^(-2*(d + e*x))])/(e*(p + q) - b*c*Log
[F]))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.32, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2034, 7271, 6018, 2689}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\text{csch}(d+ex))^p(g\text{csch}(d+ex))^q\,dx$$

$$\Big\downarrow \text{ 2034}$$

$$(f\text{csch}(d+ex))^{-q}(g\text{csch}(d+ex))^q\int F^{c(a+bx)}(f\text{csch}(d+ex))^{p+q}dx$$

$$\Big\downarrow \text{ 7271}$$

$$(f\text{csch}(d+ex))^p(g\text{csch}(d+ex))^q\text{csch}^{-p-q}(d+ex)\int F^{c(a+bx)}\text{csch}^{p+q}(d+ex)dx$$

$$\Big\downarrow \text{ 6018}$$

$$e^{(p+q)(d+ex)}\left(1-e^{-2(d+ex)}\right)^{p+q}(f\text{csch}(d+ex))^p(g\text{csch}(d+ex))^q\int e^{-d(p+q)-ex(p+q)}\left(1-e^{-2(d+ex)}\right)^{-p-q}F^{ac+bxc}dx$$

$$\Big\downarrow \text{ 2689}$$

$$-\frac{F^{ac+bcx}\left(1-e^{-2(d+ex)}\right)^{p+q}\exp((p+q)(d+ex)-d(p+q)-ex(p+q))(f\text{csch}(d+ex))^p(g\text{csch}(d+ex))^q\text{ Hype}}{e(p+q)-bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*(f*Csch[d + e*x])^p*(g*Csch[d + e*x])^q,x]
```

output
```
-((E^(-(d*(p + q)) - e*(p + q)*x + (p + q)*(d + e*x))*(1 - E^(-2*(d + e*x)
))^(p + q)*F^(a*c + b*c*x)*(f*Csch[d + e*x])^p*(g*Csch[d + e*x])^q*Hyperge
ometric2F1[p + q, (e*(p + q) - b*c*Log[F])/(2*e), (e*(2 + p + q) - b*c*Log
[F])/(2*e), E^(-2*(d + e*x))])/(e*(p + q) - b*c*Log[F]))
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 2689
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] && !IntegerQ[p]
```

rule 6018
```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(1 - E^(-2*(d + e*x)))^n*(Csch[d + e*x]^n/E^((-n)*(d + e*x)))
  Int[SimplifyIntegrand[F^(c*(a + b*x))*(1/(E^(n*(d + e*x))*(1 - E^(-2*(d +
 e*x)))^n)), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] && !IntegerQ[n]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}(f\ \operatorname{csch}(ex+d))^p (g\ \operatorname{csch}(ex+d))^q\,dx$$

input
```
int(F^(c*(b*x+a))*(f*csch(e*x+d))^p*(g*csch(e*x+d))^q,x)
```

output
```
int(F^(c*(b*x+a))*(f*csch(e*x+d))^p*(g*csch(e*x+d))^q,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\operatorname{csch}(d+ex))^q\,dx = \int (f\operatorname{csch}(ex+d))^p (g\operatorname{csch}(ex+d))^q F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(e*x+d))^p*(g*csch(e*x+d))^q,x, algorithm="
fricas")
```

output
```
integral((f*csch(e*x + d))^p*(g*csch(e*x + d))^q*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\operatorname{csch}(d+ex))^q\,dx = \int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p (g\operatorname{csch}(d+ex))^q\ dx$$

input
```
integrate(F**(c*(b*x+a))*(f*csch(e*x+d))**p*(g*csch(e*x+d))**q,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*csch(d + e*x))**p*(g*csch(d + e*x))**q, x)
```

Maxima [F]

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\operatorname{csch}(d+ex))^q\,dx = \int (f\operatorname{csch}(ex+d))^p\,(g\operatorname{csch}(ex+d))^q\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(e*x+d))^p*(g*csch(e*x+d))^q,x, algorithm="
maxima")
```

output
```
integrate((f*csch(e*x + d))^p*(g*csch(e*x + d))^q*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\operatorname{csch}(d+ex))^q\,dx = \int (f\operatorname{csch}(ex+d))^p\,(g\operatorname{csch}(ex+d))^q\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(e*x+d))^p*(g*csch(e*x+d))^q,x, algorithm="
giac")
```

output
```
integrate((f*csch(e*x + d))^p*(g*csch(e*x + d))^q*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\operatorname{csch}(d+ex))^q\,dx = \int F^{c\,(a+b\,x)}\,{\left(\frac{f}{\sinh\left(d+e\,x\right)}\right)}^p\,{\left(\frac{g}{\sinh\left(d+e\,x\right)}\right)}^q\,dx$$

input
```
int(F^(c*(a + b*x))*(f/sinh(d + e*x))^p*(g/sinh(d + e*x))^q,x)
```

output
```
int(F^(c*(a + b*x))*(f/sinh(d + e*x))^p*(g/sinh(d + e*x))^q, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\operatorname{csch}(d+ex))^q\,dx = g^q f^{ac+p}\left(\int f^{bcx}\operatorname{csch}(ex+d)^{p+q}\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*csch(e*x+d))^p*(g*csch(e*x+d))^q,x)
```

output
```
g**q*f**(a*c + p)*int(f**(b*c*x)*csch(d + e*x)**(p + q),x)
```

3.161 $\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\cosh(d+ex))^q\,dx$

Optimal result

Integrand size = 30, antiderivative size = 127

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\cosh(d+ex))^q\,dx = -\frac{\left(1+e^{2d+2ex}\right)^{-p-q}F^{c(a+bx)}(f\cosh(d+ex))^p(g\cosh(d+ex))^q\,\text{Hypergeometric2F1}\left(-p-q,-\frac{e(p+q)-bc}{2e}\right.}{e(p+q)-bc\log(F)}$$

output
```
-(1+exp(2*e*x+2*d))^(-p-q)*F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*cosh(e*x+d))
^q*hypergeom([-p-q, -1/2*(e*(p+q)-b*c*ln(F))/e],[1/2*(e*(2-p-q)+b*c*ln(F))
/e],-exp(2*e*x+2*d))/(e*(p+q)-b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.96

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\cosh(d+ex))^q\,dx = -\frac{\left(1+e^{2(d+ex)}\right)^{-p-q}F^{c(a+bx)}(f\cosh(d+ex))^p(g\cosh(d+ex))^q\,\text{Hypergeometric2F1}\left(-p-q,\frac{-e(p+q)+bc}{2e}\right.}{e(p+q)-bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Cosh[d + e*x])^p*(g*Cosh[d + e*x])^q,x]
```

output
```
-(((1 + E^(2*(d + e*x)))^(-p - q)*F^(c*(a + b*x))*(f*Cosh[d + e*x])^p*(g*C
osh[d + e*x])^q*Hypergeometric2F1[-p - q, (-(e*(p + q)) + b*c*Log[F])/(2*e
), (-(e*(-2 + p + q)) + b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*(p + q) -
 b*c*Log[F]))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2034, 7271, 6006, 2689}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\cosh(d+ex))^q\,dx$$

$$\Big\downarrow \text{2034}$$

$$(f\cosh(d+ex))^{-q}(g\cosh(d+ex))^q\int F^{c(a+bx)}(f\cosh(d+ex))^{p+q}dx$$

$$\Big\downarrow \text{7271}$$

$$(f\cosh(d+ex))^p(g\cosh(d+ex))^q\cosh^{-p-q}(d+ex)\int F^{c(a+bx)}\cosh^{p+q}(d+ex)dx$$

$$\Big\downarrow \text{6006}$$

$$e^{(p+q)(d+ex)}\left(e^{2(d+ex)}+1\right)^{-p-q}(f\cosh(d+ex))^p(g\cosh(d+ex))^q\int e^{-((p+q)(d+ex))}\left(1+e^{2(d+ex)}\right)^{p+q}F^{c(a+bx)}dx$$

$$\Big\downarrow \text{2689}$$

$$-\frac{F^{c(a+bx)}\left(e^{2(d+ex)}+1\right)^{-p-q}(f\cosh(d+ex))^p(g\cosh(d+ex))^q\,\text{Hypergeometric2F1}\left(-p-q,-\frac{e(p+q)-bc\log(F)}{2e},\right.}{e(p+q)-bc\log(F)}$$

input
```
Int[F^(c*(a + b*x))*(f*Cosh[d + e*x])^p*(g*Cosh[d + e*x])^q,x]
```

output
```
-(((1 + E^(2*(d + e*x)))^(-p - q)*F^(c*(a + b*x))*(f*Cosh[d + e*x])^p*(g*C
osh[d + e*x])^q*Hypergeometric2F1[-p - q, -1/2*(e*(p + q) - b*c*Log[F])/e,
 (e*(2 - p - q) + b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*(p + q) - b*c*L
og[F]))
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 2689
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

rule 6006
```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[E^(n*(d + e*x))*(Cosh[d + e*x]^n/(1 + E^(2*(d + e*x)))^n)   Int
[F^(c*(a + b*x))*((1 + E^(2*(d + e*x)))^n/E^(n*(d + e*x))), x], x] /; FreeQ
[{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}(f\cosh(ex+d))^p(g\cosh(ex+d))^q\,dx$$

input
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*cosh(e*x+d))^q,x)
```

output
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*cosh(e*x+d))^q,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\cosh(d+ex))^q\,dx = \int (f\cosh(ex+d))^p(g\cosh(ex+d))^q F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*cosh(e*x+d))^q,x, algorithm="
fricas")
```

output
```
integral((f*cosh(e*x + d))^p*(g*cosh(e*x + d))^q*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\cosh(d+ex))^q\,dx = \int F^{c(a+bx)}(f\cosh(d+ex))^p(g\cosh(d+ex))^q\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*cosh(e*x+d))**p*(g*cosh(e*x+d))**q,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*cosh(d + e*x))**p*(g*cosh(d + e*x))**q, x)
```

Maxima [F]

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\cosh(d+ex))^q\,dx = \int (f\cosh(ex+d))^p\,(g\cosh(ex+d))^q\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*cosh(e*x+d))^q,x, algorithm="
maxima")
```

output
```
integrate((f*cosh(e*x + d))^p*(g*cosh(e*x + d))^q*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\cosh(d+ex))^q\,dx = \int (f\cosh(ex+d))^p\,(g\cosh(ex+d))^q\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*cosh(e*x+d))^q,x, algorithm="
giac")
```

output
```
integrate((f*cosh(e*x + d))^p*(g*cosh(e*x + d))^q*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\cosh(d+ex))^q\,dx = \int F^{c\,(a+b\,x)}\,(f\cosh(d+e\,x))^p\,(g\cosh(d+e\,x))^q\,dx$$

input
```
int(F^(c*(a + b*x))*(f*cosh(d + e*x))^p*(g*cosh(d + e*x))^q,x)
```

output
```
int(F^(c*(a + b*x))*(f*cosh(d + e*x))^p*(g*cosh(d + e*x))^q, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\cosh(d+ex))^q\,dx$$
$$= \frac{g^q f^{ac+p}\left(f^{bcx}\cosh(ex+d)^{p+q} - \left(\int \frac{f^{bcx}\cosh(ex+d)^{p+q}\sinh(ex+d)}{\cosh(ex+d)}dx\right)ep - \left(\int \frac{f^{bcx}\cosh(ex+d)^{p+q}\sinh(ex+d)}{\cosh(ex+d)}dx\right)eq}{\log(f)\,bc}$$

input
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*cosh(e*x+d))^q,x)
```

output
```
(g**q*f**(a*c + p)*(f**(b*c*x)*cosh(d + e*x)**(p + q) - int((f**(b*c*x)*co
sh(d + e*x)**(p + q)*sinh(d + e*x))/cosh(d + e*x),x)*e*p - int((f**(b*c*x)
*cosh(d + e*x)**(p + q)*sinh(d + e*x))/cosh(d + e*x),x)*e*q))/(log(f)*b*c)
```

3.162 $\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\mathrm{sech}(d+ex))^q\,dx$

Optimal result

Integrand size = 30, antiderivative size = 118

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\mathrm{sech}(d+ex))^q\,dx = -\frac{\left(1+e^{2d+2ex}\right)^{-p+q}F^{c(a+bx)}(f\cosh(d+ex))^p\,\text{Hypergeometric2F1}\left(-p+q,\frac{1}{2}\left(-p+q+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(2\right.\right.}{ep-eq-bc\log(F)}$$

output
```
-(1+exp(2*e*x+2*d))^(-p+q)*F^(c*(b*x+a))*(f*cosh(e*x+d))^p*hypergeom([-p+q
, -1/2*p+1/2*q+1/2*b*c*ln(F)/e],[1-1/2*p+1/2*q+1/2*b*c*ln(F)/e],-exp(2*e*x
+2*d))*(g*sech(e*x+d))^q/(e*p-e*q-b*c*ln(F))
```

Mathematica [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\mathrm{sech}(d+ex))^q\,dx = \$\text{Aborted}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Cosh[d + e*x])^p*(g*Sech[d + e*x])^q,x]
```

output
```
$Aborted
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\text{sech}(d+ex))^q\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$\cosh^{-p}(d+ex)(f\cosh(d+ex))^p\int F^{c(a+bx)}\cosh^p(d+ex)(g\text{sech}(d+ex))^qdx$$

$$\Big\downarrow \text{ 7271}$$

$$\cosh^{-p}(d+ex)\text{sech}^{-q}(d+ex)(f\cosh(d+ex))^p(g\text{sech}(d+ex))^q\int F^{c(a+bx)}\cosh^p(d+ex)\text{sech}^q(d+ex)dx$$

$$\Big\downarrow \text{ 7292}$$

$$\cosh^{-p}(d+ex)\text{sech}^{-q}(d+ex)(f\cosh(d+ex))^p(g\text{sech}(d+ex))^q\int F^{ac+bxc}\cosh^p(d+ex)\text{sech}^q(d+ex)dx$$

$$\Big\downarrow \text{ 7299}$$

$$\cosh^{-p}(d+ex)\text{sech}^{-q}(d+ex)(f\cosh(d+ex))^p(g\text{sech}(d+ex))^q\int F^{ac+bxc}\cosh^p(d+ex)\text{sech}^q(d+ex)dx$$

input
```
Int[F^(c*(a + b*x))*(f*Cosh[d + e*x])^p*(g*Sech[d + e*x])^q,x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}(f\cosh(ex+d))^p(g\,\text{sech}(ex+d))^q\,dx$$

input
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*sech(e*x+d))^q,x)
```

output
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*sech(e*x+d))^q,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\text{sech}(d+ex))^q\,dx = \int (f\cosh(ex+d))^p\,(g\,\text{sech}\,(ex+d))^q\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*sech(e*x+d))^q,x, algorithm="
fricas")
```

output
```
integral((f*cosh(e*x + d))^p*(g*sech(e*x + d))^q*F^(b*c*x + a*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\text{sech}(d+ex))^q\,dx = \text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(f*cosh(e*x+d))**p*(g*sech(e*x+d))**q,x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\text{sech}(d+ex))^q\,dx = \int (f\cosh(ex+d))^p(g\,\text{sech}(ex+d))^q F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*sech(e*x+d))^q,x, algorithm="
maxima")
```

output
```
integrate((f*cosh(e*x + d))^p*(g*sech(e*x + d))^q*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\text{sech}(d+ex))^q\,dx = \int (f\cosh(ex+d))^p(g\,\text{sech}(ex+d))^q F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*sech(e*x+d))^q,x, algorithm="
giac")
```

output
```
integrate((f*cosh(e*x + d))^p*(g*sech(e*x + d))^q*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\text{sech}(d+ex))^q\,dx = \int F^{c\,(a+b\,x)}\,(f\cosh(d+e\,x))^p\left(\frac{g}{\cosh(d+e\,x)}\right)^q dx$$

input
```
int(F^(c*(a + b*x))*(f*cosh(d + e*x))^p*(g/cosh(d + e*x))^q,x)
```

output
```
int(F^(c*(a + b*x))*(f*cosh(d + e*x))^p*(g/cosh(d + e*x))^q, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\text{sech}(d+ex))^q\,dx$$
$$= g^q f^{ac+p}\left(\int f^{bcx}\text{sech}(ex+d)^q\cosh(ex+d)^p\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*sech(e*x+d))^q,x)
```

output
```
g**q*f**(a*c + p)*int(f**(b*c*x)*sech(d + e*x)**q*cosh(d + e*x)**p,x)
```

3.163 $\int F^{c(a+bx)}(f\operatorname{sech}(d+ex))^p(g\operatorname{sech}(d+ex))^q\,dx$

Optimal result

Integrand size = 30, antiderivative size = 106

$$\int F^{c(a+bx)}(f\operatorname{sech}(d+ex))^p(g\operatorname{sech}(d+ex))^q\,dx = \frac{\left(1+e^{2d+2ex}\right)^{p+q}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(p+q,\frac{1}{2}\left(p+q+\frac{bc\log(F)}{e}\right),\frac{1}{2}\left(2+p+q+\frac{bc\log(F)}{e}\right),-e^{2d\cdots}\right.}{e(p+q)+bc\log(F)}$$

output
```
(1+exp(2*e*x+2*d))^(p+q)*F^(c*(b*x+a))*hypergeom([p+q, 1/2*p+1/2*q+1/2*b*c
*ln(F)/e],[1+1/2*p+1/2*q+1/2*b*c*ln(F)/e],-exp(2*e*x+2*d))*(f*sech(e*x+d))
^p*(g*sech(e*x+d))^q/(e*(p+q)+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.04

$$\int F^{c(a+bx)}(f\operatorname{sech}(d+ex))^p(g\operatorname{sech}(d+ex))^q\,dx = \frac{\left(1+e^{2(d+ex)}\right)^{p+q}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(p+q,\frac{e(p+q)+bc\log(F)}{2e},\frac{e(2+p+q)+bc\log(F)}{2e},-e^{2(d+ex)}\right)(f\operatorname{sech}(\cdots}{e(p+q)+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*(f*Sech[d + e*x])^p*(g*Sech[d + e*x])^q,x]
```

output
```
((1 + E^(2*(d + e*x)))^(p + q)*F^(c*(a + b*x))*Hypergeometric2F1[p + q, (e
*(p + q) + b*c*Log[F])/(2*e), (e*(2 + p + q) + b*c*Log[F])/(2*e), -E^(2*(d
 + e*x))]*(f*Sech[d + e*x])^p*(g*Sech[d + e*x])^q)/(e*(p + q) + b*c*Log[F]
)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.27, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2034, 7271, 6017, 2689}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\text{sech}(d+ex))^p(g\text{sech}(d+ex))^q\,dx$$

$$\Big\downarrow \text{ 2034}$$

$$(f\text{sech}(d+ex))^{-q}(g\text{sech}(d+ex))^q\int F^{c(a+bx)}(f\text{sech}(d+ex))^{p+q}dx$$

$$\Big\downarrow \text{ 7271}$$

$$(f\text{sech}(d+ex))^p(g\text{sech}(d+ex))^q\text{sech}^{-p-q}(d+ex)\int F^{c(a+bx)}\text{sech}^{p+q}(d+ex)dx$$

$$\Big\downarrow \text{ 6017}$$

$$e^{-((p+q)(d+ex))}\left(e^{2(d+ex)}+1\right)^{p+q}(f\text{sech}(d+ex))^p(g\text{sech}(d+ex))^q\int e^{d(p+q)+ex(p+q)}\left(1+e^{2(d+ex)}\right)^{-p-q}F^{ac+bxc}dx$$

$$\Big\downarrow \text{ 2689}$$

$$\frac{F^{ac+bcx}\left(e^{2(d+ex)}+1\right)^{p+q}\exp(-(p+q)(d+ex)+d(p+q)+ex(p+q))(f\text{sech}(d+ex))^p(g\text{sech}(d+ex))^q\,\text{Hyper}}{bc\log(F)+e(p+q)}$$

input
```
Int[F^(c*(a + b*x))*(f*Sech[d + e*x])^p*(g*Sech[d + e*x])^q,x]
```

output
```
(E^(d*(p + q) + e*(p + q)*x - (p + q)*(d + e*x))*(1 + E^(2*(d + e*x)))^(p
+ q)*F^(a*c + b*c*x)*Hypergeometric2F1[p + q, (e*(p + q) + b*c*Log[F])/(2*
e), (e*(2 + p + q) + b*c*Log[F])/(2*e), -E^(2*(d + e*x))]*(f*Sech[d + e*x]
)^p*(g*Sech[d + e*x])^q)/(e*(p + q) + b*c*Log[F])
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 2689
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

rule 6017
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(1 + E^(2*(d + e*x)))^n*(Sech[d + e*x]^n/E^(n*(d + e*x)))   In
t[SimplifyIntegrand[F^(c*(a + b*x))*(E^(n*(d + e*x))/(1 + E^(2*(d + e*x)))^
n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] &&  !IntegerQ[n]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [F]

$$\int F^{c(bx+a)}(f\ \mathrm{sech}\,(ex+d))^{p}\,(g\ \mathrm{sech}\,(ex+d))^{q}\,dx$$

input
```
int(F^(c*(b*x+a))*(f*sech(e*x+d))^p*(g*sech(e*x+d))^q,x)
```

output
```
int(F^(c*(b*x+a))*(f*sech(e*x+d))^p*(g*sech(e*x+d))^q,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\mathrm{sech}(d+ex))^{p}(g\mathrm{sech}(d+ex))^{q}\,dx = \int (f\,\mathrm{sech}\,(ex+d))^{p}\,(g\,\mathrm{sech}\,(ex+d))^{q}\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sech(e*x+d))^p*(g*sech(e*x+d))^q,x, algorithm="
fricas")
```

output
```
integral((f*sech(e*x + d))^p*(g*sech(e*x + d))^q*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}(f\mathrm{sech}(d+ex))^{p}(g\mathrm{sech}(d+ex))^{q}\,dx = \int F^{c(a+bx)}(f\,\mathrm{sech}\,(d+ex))^{p}\,(g\,\mathrm{sech}\,(d+ex))^{q}\,\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*sech(e*x+d))**p*(g*sech(e*x+d))**q,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*sech(d + e*x))**p*(g*sech(d + e*x))**q, x)
```

Maxima [F]

$$\int F^{c(a+bx)}(f\text{sech}(d+ex))^p(g\text{sech}(d+ex))^q\,dx = \int (f\,\text{sech}\,(ex+d))^p\,(g\,\text{sech}\,(ex+d))^q\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sech(e*x+d))^p*(g*sech(e*x+d))^q,x, algorithm="
maxima")
```

output
```
integrate((f*sech(e*x + d))^p*(g*sech(e*x + d))^q*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\text{sech}(d+ex))^p(g\text{sech}(d+ex))^q\,dx = \int (f\,\text{sech}\,(ex+d))^p\,(g\,\text{sech}\,(ex+d))^q\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sech(e*x+d))^p*(g*sech(e*x+d))^q,x, algorithm="
giac")
```

output
```
integrate((f*sech(e*x + d))^p*(g*sech(e*x + d))^q*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\text{sech}(d+ex))^p(g\text{sech}(d+ex))^q\,dx = \int F^{c\,(a+b\,x)}\,\left(\frac{f}{\cosh\left(d+e\,x\right)}\right)^p\,\left(\frac{g}{\cosh\left(d+e\,x\right)}\right)^q\,dx$$

input
```
int(F^(c*(a + b*x))*(f/cosh(d + e*x))^p*(g/cosh(d + e*x))^q,x)
```

output
```
int(F^(c*(a + b*x))*(f/cosh(d + e*x))^p*(g/cosh(d + e*x))^q, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\text{sech}(d+ex))^p(g\text{sech}(d+ex))^q\,dx = g^q f^{ac+p}\left(\int f^{bcx}\text{sech}(ex+d)^{p+q}\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*sech(e*x+d))^p*(g*sech(e*x+d))^q,x)
```

output
```
g**q*f**(a*c + p)*int(f**(b*c*x)*sech(d + e*x)**(p + q),x)
```

3.164 $\int F^{c(a+bx)}(f\tanh(d+ex))^p(g\tanh(d+ex))^q\,dx$

Optimal result

Integrand size = 30, antiderivative size = 159

$$\int F^{c(a+bx)}(f\tanh(d+ex))^p(g\tanh(d+ex))^q\,dx = -\frac{2^{-1-p-q}\left(e^{2d+2ex}\right)^{-\frac{bc\log(F)}{2e}}\left(1-e^{2d+2ex}\right)\left(1+e^{2d+2ex}\right)^{p+q}F^{c(a+bx)}\operatorname{AppellF1}\left(1+p+q,1-\frac{bc\log(F)}{2e},p+\right.}{e(1+p+q)}$$

output
```
-2^(-1-p-q)*(1-exp(2*e*x+2*d))*(1+exp(2*e*x+2*d))^(p+q)*F^(c*(b*x+a))*Appe
llF1(1+p+q,1-1/2*b*c*ln(F)/e,p+q,2+p+q,1-exp(2*e*x+2*d),1/2-1/2*exp(2*e*x+
2*d))*(f*tanh(e*x+d))^p*(g*tanh(e*x+d))^q/e/(exp(2*e*x+2*d)^(1/2*b*c*ln(F)
/e))/(1+p+q)
```

Mathematica [F]

$$\int F^{c(a+bx)}(f\tanh(d+ex))^p(g\tanh(d+ex))^q\,dx = \int F^{c(a+bx)}(f\tanh(d+ex))^p(g\tanh(d+ex))^q\,dx$$

input
```
Integrate[F^(c*(a + b*x))*(f*Tanh[d + e*x])^p*(g*Tanh[d + e*x])^q,x]
```

output
```
Integrate[F^(c*(a + b*x))*(f*Tanh[d + e*x])^p*(g*Tanh[d + e*x])^q, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\tanh(d+ex))^p(g\tanh(d+ex))^q\,dx$$

$$\Big\downarrow \text{ 2034}$$

$$(f\tanh(d+ex))^{-q}(g\tanh(d+ex))^q\int F^{c(a+bx)}(f\tanh(d+ex))^{p+q}dx$$

$$\Big\downarrow \text{ 7271}$$

$$(f\tanh(d+ex))^p(g\tanh(d+ex))^q\tanh^{-p-q}(d+ex)\int F^{c(a+bx)}\tanh^{p+q}(d+ex)dx$$

$$\Big\downarrow \text{ 6030}$$

$$(f\tanh(d+ex))^p(g\tanh(d+ex))^q\tanh^{-p-q}(d+ex)\int F^{ac+bxc}\tanh^{p+q}(d+ex)dx$$

$$\Big\downarrow \text{ 7299}$$

$$(f\tanh(d+ex))^p(g\tanh(d+ex))^q\tanh^{-p-q}(d+ex)\int F^{ac+bxc}\tanh^{p+q}(d+ex)dx$$

input
```
Int[F^(c*(a + b*x))*(f*Tanh[d + e*x])^p*(g*Tanh[d + e*x])^q,x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}(f\tanh (ex+d))^p (g\tanh (ex+d))^q\,dx$$

input
```
int(F^(c*(b*x+a))*(f*tanh(e*x+d))^p*(g*tanh(e*x+d))^q,x)
```

output
```
int(F^(c*(b*x+a))*(f*tanh(e*x+d))^p*(g*tanh(e*x+d))^q,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\tanh(d+ex))^p(g\tanh(d+ex))^q\,dx = \int (f\tanh (ex+d))^p (g\tanh (ex+d))^q F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*tanh(e*x+d))^p*(g*tanh(e*x+d))^q,x, algorithm="
fricas")
```

output
```
integral((f*tanh(e*x + d))^p*(g*tanh(e*x + d))^q*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}(f\tanh(d+ex))^p(g\tanh(d+ex))^q\,dx = \int F^{c(a+bx)}(f\tanh (d+ex))^p (g\tanh (d+ex))^q\,dx$$

input
```
integrate(F**(c*(b*x+a))*(f*tanh(e*x+d))**p*(g*tanh(e*x+d))**q,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*tanh(d + e*x))**p*(g*tanh(d + e*x))**q, x)
```

Maxima [F]

$$\int F^{c(a+bx)}(f\tanh(d+ex))^p(g\tanh(d+ex))^q\,dx$$
$$=\int (f\tanh(ex+d))^p\,(g\tanh(ex+d))^q\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*tanh(e*x+d))^p*(g*tanh(e*x+d))^q,x, algorithm="
maxima")
```

output
```
integrate((f*tanh(e*x + d))^p*(g*tanh(e*x + d))^q*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\tanh(d+ex))^p(g\tanh(d+ex))^q\,dx$$
$$=\int (f\tanh(ex+d))^p\,(g\tanh(ex+d))^q\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*tanh(e*x+d))^p*(g*tanh(e*x+d))^q,x, algorithm="
giac")
```

output
```
integrate((f*tanh(e*x + d))^p*(g*tanh(e*x + d))^q*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\tanh(d+ex))^p(g\tanh(d+ex))^q\,dx$$
$$=\int F^{c\,(a+b\,x)}\,(f\tanh(d+e\,x))^p\,(g\tanh(d+e\,x))^q\,dx$$

input
```
int(F^(c*(a + b*x))*(f*tanh(d + e*x))^p*(g*tanh(d + e*x))^q,x)
```

output
```
int(F^(c*(a + b*x))*(f*tanh(d + e*x))^p*(g*tanh(d + e*x))^q, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\tanh(d+ex))^p(g\tanh(d+ex))^q\,dx = g^q f^{ac+p}\left(\int f^{bcx}\tanh\left(ex+d\right)^{p+q}dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*tanh(e*x+d))^p*(g*tanh(e*x+d))^q,x)
```

output
```
g**q*f**(a*c + p)*int(f**(b*c*x)*tanh(d + e*x)**(p + q),x)
```

3.165 $\int F^{c(a+bx)}(g\coth(d+ex))^q(f\tanh(d+ex))^p\,dx$

Optimal result

Integrand size = 30, antiderivative size = 167

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\tanh(d+ex))^p\,dx = -\frac{2^{-1-p+q}\left(e^{2d+2ex}\right)^{-\frac{bc\log(F)}{2e}}\left(1-e^{2d+2ex}\right)\left(1+e^{2d+2ex}\right)^{p-q}F^{c(a+bx)}\,\text{AppellF1}\left(1+p-q,1-\frac{bc\log(F)}{2e},p-\right.}{e(1+p-q)}$$

output
```
-2^(-1-p+q)*(1-exp(2*e*x+2*d))*(1+exp(2*e*x+2*d))^(p-q)*F^(c*(b*x+a))*Appe
llF1(1+p-q,1-1/2*b*c*ln(F)/e,p-q,2+p-q,1-exp(2*e*x+2*d),1/2-1/2*exp(2*e*x+
2*d))*(g*coth(e*x+d))^q*(f*tanh(e*x+d))^p/e/(exp(2*e*x+2*d)^(1/2*b*c*ln(F)
/e))/(1+p-q)
```

Mathematica [F]

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\tanh(d+ex))^p\,dx = \int F^{c(a+bx)}(g\coth(d+ex))^q(f\tanh(d+ex))^p\,dx$$

input
```
Integrate[F^(c*(a + b*x))*(g*Coth[d + e*x])^q*(f*Tanh[d + e*x])^p,x]
```

output
```
Integrate[F^(c*(a + b*x))*(g*Coth[d + e*x])^q*(f*Tanh[d + e*x])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\tanh(d+ex))^p(g\coth(d+ex))^q\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$\coth^{-q}(d+ex)(g\coth(d+ex))^q\int F^{c(a+bx)}\coth^q(d+ex)(f\tanh(d+ex))^pdx$$

$$\Big\downarrow \text{ 7271}$$

$$\tanh^{-p}(d+ex)\coth^{-q}(d+ex)(f\tanh(d+ex))^p(g\coth(d+ex))^q\int F^{c(a+bx)}\coth^q(d+ex)\tanh^p(d+ex)dx$$

$$\Big\downarrow \text{ 7292}$$

$$\tanh^{-p}(d+ex)\coth^{-q}(d+ex)(f\tanh(d+ex))^p(g\coth(d+ex))^q\int F^{ac+bxc}\coth^q(d+ex)\tanh^p(d+ex)dx$$

$$\Big\downarrow \text{ 7299}$$

$$\tanh^{-p}(d+ex)\coth^{-q}(d+ex)(f\tanh(d+ex))^p(g\coth(d+ex))^q\int F^{ac+bxc}\coth^q(d+ex)\tanh^p(d+ex)dx$$

input
```
Int[F^(c*(a + b*x))*(g*Coth[d + e*x])^q*(f*Tanh[d + e*x])^p,x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}(g\coth(ex+d))^q\,(f\tanh(ex+d))^p\,dx$$

input
```
int(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*tanh(e*x+d))^p,x)
```

output
```
int(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*tanh(e*x+d))^p,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\tanh(d+ex))^p\,dx = \int (g\coth(ex+d))^q\,(f\tanh(ex+d))^p\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*tanh(e*x+d))^p,x, algorithm="
fricas")
```

output
```
integral((g*coth(e*x + d))^q*(f*tanh(e*x + d))^p*F^(b*c*x + a*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\tanh(d+ex))^p\,dx = \text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(g*coth(e*x+d))**q*(f*tanh(e*x+d))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\tanh(d+ex))^p\,dx$$
$$=\int (g\coth(ex+d))^q(f\tanh(ex+d))^p F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*tanh(e*x+d))^p,x, algorithm="
maxima")
```

output
```
integrate((g*coth(e*x + d))^q*(f*tanh(e*x + d))^p*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\tanh(d+ex))^p\,dx$$
$$=\int (g\coth(ex+d))^q(f\tanh(ex+d))^p F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*tanh(e*x+d))^p,x, algorithm="
giac")
```

output
```
integrate((g*coth(e*x + d))^q*(f*tanh(e*x + d))^p*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\tanh(d+ex))^p\,dx$$
$$=\int F^{c\,(a+b\,x)}\,(g\coth(d+e\,x))^q\,(f\tanh(d+e\,x))^p\,dx$$

input
```
int(F^(c*(a + b*x))*(g*coth(d + e*x))^q*(f*tanh(d + e*x))^p,x)
```

output
```
int(F^(c*(a + b*x))*(g*coth(d + e*x))^q*(f*tanh(d + e*x))^p, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\tanh(d+ex))^p\,dx$$
$$= g^q f^{ac+p}\left(\int f^{bcx}\tanh\left(ex+d\right)^p\coth\left(ex+d\right)^q dx\right)$$

input
```
int(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*tanh(e*x+d))^p,x)
```

output
```
g**q*f**(a*c + p)*int(f**(b*c*x)*tanh(d + e*x)**p*coth(d + e*x)**q,x)
```

3.166 $\int F^{c(a+bx)}(f\coth(d+ex))^p(g\coth(d+ex))^q\,dx$

Optimal result

Integrand size = 30, antiderivative size = 175

$$\int F^{c(a+bx)}(f\coth(d+ex))^p(g\coth(d+ex))^q\,dx = -\frac{2^{-1+p+q}\left(e^{2d+2ex}\right)^{-\frac{bc\log(F)}{2e}}\left(1-e^{2d+2ex}\right)\left(1+e^{2d+2ex}\right)^{-p-q}F^{c(a+bx)}\operatorname{AppellF1}\left(1-p-q,1-\frac{bc\log(F)}{2e},-\ldots\right.}{e(1-p-q)}$$

output
```
-2^(-1+p+q)*(1-exp(2*e*x+2*d))*(1+exp(2*e*x+2*d))^(-p-q)*F^(c*(b*x+a))*App
ellF1(1-p-q,1-1/2*b*c*ln(F)/e,-p-q,2-p-q,1-exp(2*e*x+2*d),1/2-1/2*exp(2*e*
x+2*d))*(f*coth(e*x+d))^p*(g*coth(e*x+d))^q/e/(exp(2*e*x+2*d)^(1/2*b*c*ln(
F)/e))/(1-p-q)
```

Mathematica [F]

$$\int F^{c(a+bx)}(f\coth(d+ex))^p(g\coth(d+ex))^q\,dx = \int F^{c(a+bx)}(f\coth(d+ex))^p(g\coth(d+ex))^q\,dx$$

input
```
Integrate[F^(c*(a + b*x))*(f*Coth[d + e*x])^p*(g*Coth[d + e*x])^q,x]
```

output
```
Integrate[F^(c*(a + b*x))*(f*Coth[d + e*x])^p*(g*Coth[d + e*x])^q, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\coth(d+ex))^p(g\coth(d+ex))^q\,dx$$

$$\Big\downarrow \text{2034}$$

$$(f\coth(d+ex))^{-q}(g\coth(d+ex))^q\int F^{c(a+bx)}(f\coth(d+ex))^{p+q}dx$$

$$\Big\downarrow \text{7271}$$

$$(f\coth(d+ex))^p(g\coth(d+ex))^q\coth^{-p-q}(d+ex)\int F^{c(a+bx)}\coth^{p+q}(d+ex)dx$$

$$\Big\downarrow \text{6030}$$

$$(f\coth(d+ex))^p(g\coth(d+ex))^q\coth^{-p-q}(d+ex)\int F^{ac+bxc}\coth^{p+q}(d+ex)dx$$

$$\Big\downarrow \text{7299}$$

$$(f\coth(d+ex))^p(g\coth(d+ex))^q\coth^{-p-q}(d+ex)\int F^{ac+bxc}\coth^{p+q}(d+ex)dx$$

input
```
Int[F^(c*(a + b*x))*(f*Coth[d + e*x])^p*(g*Coth[d + e*x])^q,x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}(f\coth(ex+d))^p\,(g\coth(ex+d))^q\,dx$$

input
```
int(F^(c*(b*x+a))*(f*coth(e*x+d))^p*(g*coth(e*x+d))^q,x)
```

output
```
int(F^(c*(b*x+a))*(f*coth(e*x+d))^p*(g*coth(e*x+d))^q,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\coth(d+ex))^p(g\coth(d+ex))^q\,dx$$
$$=\int (f\coth(ex+d))^p\,(g\coth(ex+d))^q\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*coth(e*x+d))^p*(g*coth(e*x+d))^q,x, algorithm="
fricas")
```

output
```
integral((f*coth(e*x + d))^p*(g*coth(e*x + d))^q*F^(b*c*x + a*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\coth(d+ex))^p(g\coth(d+ex))^q\,dx=\text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(f*coth(e*x+d))**p*(g*coth(e*x+d))**q,x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{c(a+bx)}(f\coth(d+ex))^p(g\coth(d+ex))^q\,dx$$
$$=\int (f\coth{(ex+d)})^p\,(g\coth{(ex+d)})^q\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*coth(e*x+d))^p*(g*coth(e*x+d))^q,x, algorithm="
maxima")
```

output
```
integrate((f*coth(e*x + d))^p*(g*coth(e*x + d))^q*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\coth(d+ex))^p(g\coth(d+ex))^q\,dx$$
$$=\int (f\coth{(ex+d)})^p\,(g\coth{(ex+d)})^q\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*coth(e*x+d))^p*(g*coth(e*x+d))^q,x, algorithm="
giac")
```

output
```
integrate((f*coth(e*x + d))^p*(g*coth(e*x + d))^q*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\coth(d+ex))^p(g\coth(d+ex))^q\,dx$$
$$=\int F^{c\,(a+b\,x)}\,(f\coth(d+e\,x))^p\,(g\coth(d+e\,x))^q\,dx$$

input
```
int(F^(c*(a + b*x))*(f*coth(d + e*x))^p*(g*coth(d + e*x))^q,x)
```

output
```
int(F^(c*(a + b*x))*(f*coth(d + e*x))^p*(g*coth(d + e*x))^q, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\coth(d+ex))^p(g\coth(d+ex))^q\,dx = g^q f^{ac+p}\left(\int f^{bcx}\coth\left(ex+d\right)^{p+q}dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*coth(e*x+d))^p*(g*coth(e*x+d))^q,x)
```

output
```
g**q*f**(a*c + p)*int(f**(b*c*x)*coth(d + e*x)**(p + q),x)
```

3.167 $\int F^{c(a+bx)}(g\cosh(d+ex))^q(f\sinh(d+ex))^p\,dx$

Optimal result

Integrand size = 30, antiderivative size = 160

$$\int F^{c(a+bx)}(g\cosh(d+ex))^q(f\sinh(d+ex))^p\,dx = -\frac{2^{-1+q}\left(e^{2d+2ex}\right)^{\frac{1}{2}\left(p+q-\frac{bc\log(F)}{e}\right)}\left(1-e^{2d+2ex}\right)\left(1+e^{2d+2ex}\right)^{-q}F^{c(a+bx)}\,\text{AppellF1}\left(1+p,\frac{1}{2}\left(2+p+q-\frac{bc}{}\right.\right.}{e(1+p)}$$

output
```
-2^(-1+q)*exp(2*e*x+2*d)^(1/2*p+1/2*q-1/2*b*c*ln(F)/e)*(1-exp(2*e*x+2*d))*
F^(c*(b*x+a))*AppellF1(p+1,1+1/2*p+1/2*q-1/2*b*c*ln(F)/e,-q,2+p,1-exp(2*e*
x+2*d),1/2-1/2*exp(2*e*x+2*d))*(g*cosh(e*x+d))^q*(f*sinh(e*x+d))^p/e/((1+e
xp(2*e*x+2*d))^q)/(p+1)
```

Mathematica [F]

$$\int F^{c(a+bx)}(g\cosh(d+ex))^q(f\sinh(d+ex))^p\,dx = \int F^{c(a+bx)}(g\cosh(d+ex))^q(f\sinh(d+ex))^p\,dx$$

input
```
Integrate[F^(c*(a + b*x))*(g*Cosh[d + e*x])^q*(f*Sinh[d + e*x])^p,x]
```

output
```
Integrate[F^(c*(a + b*x))*(g*Cosh[d + e*x])^q*(f*Sinh[d + e*x])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\cosh(d+ex))^q\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$\cosh^{-q}(d+ex)(g\cosh(d+ex))^q\int F^{c(a+bx)}\cosh^q(d+ex)(f\sinh(d+ex))^pdx$$

$$\Big\downarrow \text{ 7271}$$

$$\sinh^{-p}(d+ex)\cosh^{-q}(d+ex)(f\sinh(d+ex))^p(g\cosh(d+ex))^q\int F^{c(a+bx)}\cosh^q(d+ex)\sinh^p(d+ex)dx$$

$$\Big\downarrow \text{ 7292}$$

$$\sinh^{-p}(d+ex)\cosh^{-q}(d+ex)(f\sinh(d+ex))^p(g\cosh(d+ex))^q\int F^{ac+bxc}\cosh^q(d+ex)\sinh^p(d+ex)dx$$

$$\Big\downarrow \text{ 7299}$$

$$\sinh^{-p}(d+ex)\cosh^{-q}(d+ex)(f\sinh(d+ex))^p(g\cosh(d+ex))^q\int F^{ac+bxc}\cosh^q(d+ex)\sinh^p(d+ex)dx$$

input
```
Int[F^(c*(a + b*x))*(g*Cosh[d + e*x])^q*(f*Sinh[d + e*x])^p,x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}(g\cosh(ex+d))^q (f\sinh(ex+d))^p\,dx$$

input
```
int(F^(c*(b*x+a))*(g*cosh(e*x+d))^q*(f*sinh(e*x+d))^p,x)
```

output
```
int(F^(c*(b*x+a))*(g*cosh(e*x+d))^q*(f*sinh(e*x+d))^p,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(g\cosh(d+ex))^q(f\sinh(d+ex))^p\,dx = \int (g\cosh(ex+d))^q (f\sinh(ex+d))^p F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*cosh(e*x+d))^q*(f*sinh(e*x+d))^p,x, algorithm="
fricas")
```

output
```
integral((g*cosh(e*x + d))^q*(f*sinh(e*x + d))^p*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}(g\cosh(d+ex))^q(f\sinh(d+ex))^p\,dx = \int F^{c(a+bx)}(f\sinh(d+ex))^p (g\cosh(d+ex))^q\,dx$$

input
```
integrate(F**(c*(b*x+a))*(g*cosh(e*x+d))**q*(f*sinh(e*x+d))**p,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*sinh(d + e*x))**p*(g*cosh(d + e*x))**q, x)
```

Maxima [F]

$$\int F^{c(a+bx)}(g\cosh(d+ex))^q(f\sinh(d+ex))^p\,dx$$
$$=\int (g\cosh(ex+d))^q(f\sinh(ex+d))^pF^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*cosh(e*x+d))^q*(f*sinh(e*x+d))^p,x, algorithm="
maxima")
```

output
```
integrate((g*cosh(e*x + d))^q*(f*sinh(e*x + d))^p*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(g\cosh(d+ex))^q(f\sinh(d+ex))^p\,dx$$
$$=\int (g\cosh(ex+d))^q(f\sinh(ex+d))^pF^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*cosh(e*x+d))^q*(f*sinh(e*x+d))^p,x, algorithm="
giac")
```

output
```
integrate((g*cosh(e*x + d))^q*(f*sinh(e*x + d))^p*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(g\cosh(d+ex))^q(f\sinh(d+ex))^p\,dx$$
$$=\int F^{c\,(a+b\,x)}\,(g\cosh(d+e\,x))^q\,(f\sinh(d+e\,x))^p\,dx$$

input
```
int(F^(c*(a + b*x))*(g*cosh(d + e*x))^q*(f*sinh(d + e*x))^p,x)
```

output
```
int(F^(c*(a + b*x))*(g*cosh(d + e*x))^q*(f*sinh(d + e*x))^p, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(g\cosh(d+ex))^q(f\sinh(d+ex))^p\,dx$$
$$= g^q f^{ac+p}\left(\int f^{bcx}\sinh(ex+d)^p\cosh(ex+d)^q\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(g*cosh(e*x+d))^q*(f*sinh(e*x+d))^p,x)
```

output
```
g**q*f**(a*c + p)*int(f**(b*c*x)*sinh(d + e*x)**p*cosh(d + e*x)**q,x)
```

3.168 $\int F^{c(a+bx)}(g\text{sech}(d+ex))^q(f\sinh(d+ex))^p\,dx$

Optimal result

Integrand size = 30, antiderivative size = 162

$$\int F^{c(a+bx)}(g\text{sech}(d+ex))^q(f\sinh(d+ex))^p\,dx = -\frac{2^{-1-q}\left(e^{2d+2ex}\right)^{\frac{1}{2}\left(p-q-\frac{bc\log(F)}{e}\right)}\left(1-e^{2d+2ex}\right)\left(1+e^{2d+2ex}\right)^qF^{c(a+bx)}\text{AppellF1}\left(1+p,\frac{1}{2}\left(2+p-q-\frac{bc\log(F)}{e}\right),\ldots\right)}{e(1+p)}$$

output
```
-2^(-1-q)*exp(2*e*x+2*d)^(1/2*p-1/2*q-1/2*b*c*ln(F)/e)*(1-exp(2*e*x+2*d))*
(1+exp(2*e*x+2*d))^q*F^(c*(b*x+a))*AppellF1(p+1,1+1/2*p-1/2*q-1/2*b*c*ln(F
)/e,q,2+p,1-exp(2*e*x+2*d),1/2-1/2*exp(2*e*x+2*d))*(g*sech(e*x+d))^q*(f*si
nh(e*x+d))^p/e/(p+1)
```

Mathematica [F]

$$\int F^{c(a+bx)}(g\text{sech}(d+ex))^q(f\sinh(d+ex))^p\,dx = \int F^{c(a+bx)}(g\text{sech}(d+ex))^q(f\sinh(d+ex))^p\,dx$$

input
```
Integrate[F^(c*(a + b*x))*(g*Sech[d + e*x])^q*(f*Sinh[d + e*x])^p,x]
```

output
```
Integrate[F^(c*(a + b*x))*(g*Sech[d + e*x])^q*(f*Sinh[d + e*x])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\text{sech}(d+ex))^q\,dx$$

$$\Big\downarrow \; 7271$$

$$\text{sech}^{-q}(d+ex)(g\text{sech}(d+ex))^q\int F^{c(a+bx)}\text{sech}^q(d+ex)(f\sinh(d+ex))^pdx$$

$$\Big\downarrow \; 7271$$

$$\sinh^{-p}(d+ex)\text{sech}^{-q}(d+ex)(f\sinh(d+ex))^p(g\text{sech}(d+ex))^q\int F^{c(a+bx)}\text{sech}^q(d+ex)\sinh^p(d+ex)dx$$

$$\Big\downarrow \; 7292$$

$$\sinh^{-p}(d+ex)\text{sech}^{-q}(d+ex)(f\sinh(d+ex))^p(g\text{sech}(d+ex))^q\int F^{ac+bxc}\text{sech}^q(d+ex)\sinh^p(d+ex)dx$$

$$\Big\downarrow \; 7299$$

$$\sinh^{-p}(d+ex)\text{sech}^{-q}(d+ex)(f\sinh(d+ex))^p(g\text{sech}(d+ex))^q\int F^{ac+bxc}\text{sech}^q(d+ex)\sinh^p(d+ex)dx$$

input
```
Int[F^(c*(a + b*x))*(g*Sech[d + e*x])^q*(f*Sinh[d + e*x])^p,x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}(g\ \mathrm{sech}\left(ex+d\right))^{q}\left(f\sinh\left(ex+d\right)\right)^{p}dx$$

input
```
int(F^(c*(b*x+a))*(g*sech(e*x+d))^q*(f*sinh(e*x+d))^p,x)
```

output
```
int(F^(c*(b*x+a))*(g*sech(e*x+d))^q*(f*sinh(e*x+d))^p,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(g\mathrm{sech}(d+ex))^{q}(f\sinh(d+ex))^{p}\,dx$$
$$=\int \left(g\,\mathrm{sech}\left(ex+d\right)\right)^{q}\left(f\sinh\left(ex+d\right)\right)^{p}F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*sech(e*x+d))^q*(f*sinh(e*x+d))^p,x, algorithm="
fricas")
```

output
```
integral((g*sech(e*x + d))^q*(f*sinh(e*x + d))^p*F^(b*c*x + a*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(g\mathrm{sech}(d+ex))^{q}(f\sinh(d+ex))^{p}\,dx=\text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(g*sech(e*x+d))**q*(f*sinh(e*x+d))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{c(a+bx)}(g\text{sech}(d+ex))^q(f\sinh(d+ex))^p\,dx$$
$$=\int (g\,\text{sech}\,(ex+d))^q\,(f\sinh\,(ex+d))^p\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*sech(e*x+d))^q*(f*sinh(e*x+d))^p,x, algorithm="
maxima")
```

output
```
integrate((g*sech(e*x + d))^q*(f*sinh(e*x + d))^p*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(g\text{sech}(d+ex))^q(f\sinh(d+ex))^p\,dx$$
$$=\int (g\,\text{sech}\,(ex+d))^q\,(f\sinh\,(ex+d))^p\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*sech(e*x+d))^q*(f*sinh(e*x+d))^p,x, algorithm="
giac")
```

output
```
integrate((g*sech(e*x + d))^q*(f*sinh(e*x + d))^p*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(g\text{sech}(d+ex))^q(f\sinh(d+ex))^p\,dx$$
$$=\int F^{c\,(a+b\,x)}\,(f\sinh(d+e\,x))^p\,\left(\frac{g}{\cosh\,(d+e\,x)}\right)^q\,dx$$

input
```
int(F^(c*(a + b*x))*(f*sinh(d + e*x))^p*(g/cosh(d + e*x))^q,x)
```

output
```
int(F^(c*(a + b*x))*(f*sinh(d + e*x))^p*(g/cosh(d + e*x))^q, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(g\text{sech}(d+ex))^q(f\sinh(d+ex))^p\,dx$$
$$= g^q f^{ac+p}\left(\int f^{bcx}\sinh(ex+d)^p\,\text{sech}(ex+d)^q\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(g*sech(e*x+d))^q*(f*sinh(e*x+d))^p,x)
```

output
```
g**q*f**(a*c + p)*int(f**(b*c*x)*sinh(d + e*x)**p*sech(d + e*x)**q,x)
```

3.169 $\int F^{c(a+bx)}(g\cosh(d+ex))^q(f\textbf{csch}(d+ex))^p\,dx$

Optimal result

Integrand size = 30, antiderivative size = 170

$$\int F^{c(a+bx)}(g\cosh(d+ex))^q(f\operatorname{csch}(d+ex))^p\,dx = -\frac{2^{-1+q}\left(e^{2d+2ex}\right)^{\frac{1}{2}\left(-p+q-\frac{bc\log(F)}{e}\right)}\left(1-e^{2d+2ex}\right)\left(1+e^{2d+2ex}\right)^{-q}F^{c(a+bx)}\operatorname{AppellF1}\left(1-p,\frac{1}{2}\left(2-p+q-\right.\right.}{e(1-p)}$$

output
```
-2^(-1+q)*exp(2*e*x+2*d)^(-1/2*p+1/2*q-1/2*b*c*ln(F)/e)*(1-exp(2*e*x+2*d))
*F^(c*(b*x+a))*AppellF1(1-p,1-1/2*p+1/2*q-1/2*b*c*ln(F)/e,-q,2-p,1-exp(2*e
*x+2*d),1/2-1/2*exp(2*e*x+2*d))*(g*cosh(e*x+d))^q*(f*csch(e*x+d))^p/e/((1+
exp(2*e*x+2*d))^q)/(1-p)
```

Mathematica [F]

$$\int F^{c(a+bx)}(g\cosh(d+ex))^q(f\operatorname{csch}(d+ex))^p\,dx = \int F^{c(a+bx)}(g\cosh(d+ex))^q(f\operatorname{csch}(d+ex))^p\,dx$$

input
```
Integrate[F^(c*(a + b*x))*(g*Cosh[d + e*x])^q*(f*Csch[d + e*x])^p,x]
```

output
```
Integrate[F^(c*(a + b*x))*(g*Cosh[d + e*x])^q*(f*Csch[d + e*x])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\cosh(d+ex))^q\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$\cosh^{-q}(d+ex)(g\cosh(d+ex))^q\int F^{c(a+bx)}\cosh^q(d+ex)(f\operatorname{csch}(d+ex))^pdx$$

$$\Big\downarrow \text{ 7271}$$

$$\operatorname{csch}^{-p}(d+ex)\cosh^{-q}(d+ex)(f\operatorname{csch}(d+ex))^p(g\cosh(d+ex))^q\int F^{c(a+bx)}\cosh^q(d+ex)\operatorname{csch}^p(d+ex)dx$$

$$\Big\downarrow \text{ 7292}$$

$$\operatorname{csch}^{-p}(d+ex)\cosh^{-q}(d+ex)(f\operatorname{csch}(d+ex))^p(g\cosh(d+ex))^q\int F^{ac+bxc}\cosh^q(d+ex)\operatorname{csch}^p(d+ex)dx$$

$$\Big\downarrow \text{ 7299}$$

$$\operatorname{csch}^{-p}(d+ex)\cosh^{-q}(d+ex)(f\operatorname{csch}(d+ex))^p(g\cosh(d+ex))^q\int F^{ac+bxc}\cosh^q(d+ex)\operatorname{csch}^p(d+ex)dx$$

input
```
Int[F^(c*(a + b*x))*(g*Cosh[d + e*x])^q*(f*Csch[d + e*x])^p,x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}(g\cosh(ex+d))^q (f\ \operatorname{csch}(ex+d))^p\,dx$$

input
```
int(F^(c*(b*x+a))*(g*cosh(e*x+d))^q*(f*csch(e*x+d))^p,x)
```

output
```
int(F^(c*(b*x+a))*(g*cosh(e*x+d))^q*(f*csch(e*x+d))^p,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(g\cosh(d+ex))^q(f\operatorname{csch}(d+ex))^p\,dx = \int (g\cosh(ex+d))^q (f\ \operatorname{csch}(ex+d))^p F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*cosh(e*x+d))^q*(f*csch(e*x+d))^p,x, algorithm="
fricas")
```

output
```
integral((g*cosh(e*x + d))^q*(f*csch(e*x + d))^p*F^(b*c*x + a*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(g\cosh(d+ex))^q(f\operatorname{csch}(d+ex))^p\,dx = \text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(g*cosh(e*x+d))**q*(f*csch(e*x+d))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{c(a+bx)}(g\cosh(d+ex))^q(f\text{csch}(d+ex))^p\,dx = \int (g\cosh(ex+d))^q\,(f\,\text{csch}\,(ex+d))^p\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*cosh(e*x+d))^q*(f*csch(e*x+d))^p,x, algorithm="
maxima")
```

output
```
integrate((g*cosh(e*x + d))^q*(f*csch(e*x + d))^p*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(g\cosh(d+ex))^q(f\text{csch}(d+ex))^p\,dx = \int (g\cosh(ex+d))^q\,(f\,\text{csch}\,(ex+d))^p\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*cosh(e*x+d))^q*(f*csch(e*x+d))^p,x, algorithm="
giac")
```

output
```
integrate((g*cosh(e*x + d))^q*(f*csch(e*x + d))^p*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(g\cosh(d+ex))^q(f\text{csch}(d+ex))^p\,dx = \int F^{c\,(a+b\,x)}\,(g\cosh(d+e\,x))^q\,\left(\frac{f}{\sinh\,(d+e\,x)}\right)^p\,dx$$

input
```
int(F^(c*(a + b*x))*(g*cosh(d + e*x))^q*(f/sinh(d + e*x))^p,x)
```

output
```
int(F^(c*(a + b*x))*(g*cosh(d + e*x))^q*(f/sinh(d + e*x))^p, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(g\cosh(d+ex))^q(f\operatorname{csch}(d+ex))^p\,dx$$
$$=g^qf^{ac+p}\left(\int f^{bcx}\operatorname{csch}(ex+d)^p\cosh(ex+d)^q\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(g*cosh(e*x+d))^q*(f*csch(e*x+d))^p,x)
```

output
```
g**q*f**(a*c + p)*int(f**(b*c*x)*csch(d + e*x)**p*cosh(d + e*x)**q,x)
```

3.170 $\int F^{c(a+bx)}(f\text{csch}(d+ex))^p(g\text{sech}(d+ex))^q\,dx$

Optimal result

Integrand size = 30, antiderivative size = 173

$$\int F^{c(a+bx)}(f\text{csch}(d+ex))^p(g\text{sech}(d+ex))^q\,dx =$$
$$-\frac{2^{-1-q}\left(e^{2d+2ex}\right)^{-\frac{e(p+q)+bc\log(F)}{2e}}\left(1-e^{2d+2ex}\right)\left(1+e^{2d+2ex}\right)^qF^{c(a+bx)}\text{AppellF1}\left(1-p,\frac{e(2-p-q)-bc\log(F)}{2e},q,\right.}{e(1-p)}$$

output
```
-2^(-1-q)*(1-exp(2*e*x+2*d))*(1+exp(2*e*x+2*d))^q*F^(c*(b*x+a))*AppellF1(1
-p,1/2*(e*(2-p-q)-b*c*ln(F))/e,q,2-p,1-exp(2*e*x+2*d),1/2-1/2*exp(2*e*x+2*
d))*(f*csch(e*x+d))^p*(g*sech(e*x+d))^q/e/(exp(2*e*x+2*d)^(1/2*(e*(p+q)+b*
c*ln(F))/e))/(1-p)
```

Mathematica [F]

$$\int F^{c(a+bx)}(f\text{csch}(d+ex))^p(g\text{sech}(d+ex))^q\,dx = \int F^{c(a+bx)}(f\text{csch}(d+ex))^p(g\text{sech}(d+ex))^q\,dx$$

input
```
Integrate[F^(c*(a + b*x))*(f*Csch[d + e*x])^p*(g*Sech[d + e*x])^q,x]
```

output
```
Integrate[F^(c*(a + b*x))*(f*Csch[d + e*x])^p*(g*Sech[d + e*x])^q, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\text{csch}(d+ex))^p(g\text{sech}(d+ex))^q\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$\text{csch}^{-p}(d+ex)(f\text{csch}(d+ex))^p\int F^{c(a+bx)}\text{csch}^p(d+ex)(g\text{sech}(d+ex))^qdx$$

$$\Big\downarrow \text{ 7271}$$

$$\text{csch}^{-p}(d+ex)\text{sech}^{-q}(d+ex)(f\text{csch}(d+ex))^p(g\text{sech}(d+ex))^q\int F^{c(a+bx)}\text{csch}^p(d+ex)\text{sech}^q(d+ex)dx$$

$$\Big\downarrow \text{ 7292}$$

$$\text{csch}^{-p}(d+ex)\text{sech}^{-q}(d+ex)(f\text{csch}(d+ex))^p(g\text{sech}(d+ex))^q\int F^{ac+bxc}\text{csch}^p(d+ex)\text{sech}^q(d+ex)dx$$

$$\Big\downarrow \text{ 7299}$$

$$\text{csch}^{-p}(d+ex)\text{sech}^{-q}(d+ex)(f\text{csch}(d+ex))^p(g\text{sech}(d+ex))^q\int F^{ac+bxc}\text{csch}^p(d+ex)\text{sech}^q(d+ex)dx$$

input
```
Int[F^(c*(a + b*x))*(f*Csch[d + e*x])^p*(g*Sech[d + e*x])^q,x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}(f\ \operatorname{csch}(ex+d))^p\,(g\ \operatorname{sech}(ex+d))^q\,dx$$

input
```
int(F^(c*(b*x+a))*(f*csch(e*x+d))^p*(g*sech(e*x+d))^q,x)
```

output
```
int(F^(c*(b*x+a))*(f*csch(e*x+d))^p*(g*sech(e*x+d))^q,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\operatorname{sech}(d+ex))^q\,dx = \int (f\operatorname{csch}(ex+d))^p\,(g\operatorname{sech}(ex+d))^q\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(e*x+d))^p*(g*sech(e*x+d))^q,x, algorithm="
fricas")
```

output
```
integral((f*csch(e*x + d))^p*(g*sech(e*x + d))^q*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\operatorname{sech}(d+ex))^q\,dx = \int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p\,(g\operatorname{sech}(d+ex))^q\ dx$$

input
```
integrate(F**(c*(b*x+a))*(f*csch(e*x+d))**p*(g*sech(e*x+d))**q,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*csch(d + e*x))**p*(g*sech(d + e*x))**q, x)
```

Maxima [F]

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\operatorname{sech}(d+ex))^q\,dx = \int (f\operatorname{csch}(ex+d))^p(g\operatorname{sech}(ex+d))^q F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(e*x+d))^p*(g*sech(e*x+d))^q,x, algorithm="
maxima")
```

output
```
integrate((f*csch(e*x + d))^p*(g*sech(e*x + d))^q*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\operatorname{sech}(d+ex))^q\,dx = \int (f\operatorname{csch}(ex+d))^p(g\operatorname{sech}(ex+d))^q F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(e*x+d))^p*(g*sech(e*x+d))^q,x, algorithm="
giac")
```

output
```
integrate((f*csch(e*x + d))^p*(g*sech(e*x + d))^q*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\operatorname{sech}(d+ex))^q\,dx = \int F^{c\,(a+b\,x)}\left(\frac{g}{\cosh(d+e\,x)}\right)^q\left(\frac{f}{\sinh(d+e\,x)}\right)^p dx$$

input
```
int(F^(c*(a + b*x))*(g/cosh(d + e*x))^q*(f/sinh(d + e*x))^p,x)
```

output
```
int(F^(c*(a + b*x))*(g/cosh(d + e*x))^q*(f/sinh(d + e*x))^p, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\operatorname{sech}(d+ex))^q\,dx$$
$$= g^q f^{ac+p}\left(\int f^{bcx}\operatorname{sech}(ex+d)^q\operatorname{csch}(ex+d)^p\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*csch(e*x+d))^p*(g*sech(e*x+d))^q,x)
```

output
```
g**q*f**(a*c + p)*int(f**(b*c*x)*sech(d + e*x)**q*csch(d + e*x)**p,x)
```

3.171 $\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\tanh(d+ex))^q\,dx$

Optimal result

Integrand size = 30, antiderivative size = 159

$$\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\tanh(d+ex))^q\,dx = -\frac{2^{-1-q}\left(e^{2d+2ex}\right)^{\frac{1}{2}\left(p-\frac{bc\log(F)}{e}\right)}\left(1-e^{2d+2ex}\right)\left(1+e^{2d+2ex}\right)^q F^{c(a+bx)}\operatorname{AppellF1}\left(1+p+q,\frac{1}{2}\left(2+p-\frac{bc\log(}{e}\right.\right.}{e(1+p+q)}$$

output
```
-2^(-1-q)*exp(2*e*x+2*d)^(1/2*p-1/2*b*c*ln(F)/e)*(1-exp(2*e*x+2*d))*(1+exp
(2*e*x+2*d))^q*F^(c*(b*x+a))*AppellF1(1+p+q,1+1/2*p-1/2*b*c*ln(F)/e,q,2+p+
q,1-exp(2*e*x+2*d),1/2-1/2*exp(2*e*x+2*d))*(f*sinh(e*x+d))^p*(g*tanh(e*x+d
))^q/e/(1+p+q)
```

Mathematica [F]

$$\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\tanh(d+ex))^q\,dx = \int F^{c(a+bx)}(f\sinh(d+ex))^p(g\tanh(d+ex))^q\,dx$$

input
```
Integrate[F^(c*(a + b*x))*(f*Sinh[d + e*x])^p*(g*Tanh[d + e*x])^q,x]
```

output
```
Integrate[F^(c*(a + b*x))*(f*Sinh[d + e*x])^p*(g*Tanh[d + e*x])^q, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\tanh(d+ex))^q\,dx$$

$$\Big\downarrow \text{7271}$$

$$\sinh^{-p}(d+ex)(f\sinh(d+ex))^p\int F^{c(a+bx)}\sinh^p(d+ex)(g\tanh(d+ex))^qdx$$

$$\Big\downarrow \text{7271}$$

$$\sinh^{-p}(d+ex)\tanh^{-q}(d+ex)(f\sinh(d+ex))^p(g\tanh(d+ex))^q\int F^{c(a+bx)}\sinh^p(d+ex)\tanh^q(d+ex)dx$$

$$\Big\downarrow \text{7292}$$

$$\sinh^{-p}(d+ex)\tanh^{-q}(d+ex)(f\sinh(d+ex))^p(g\tanh(d+ex))^q\int F^{ac+bxc}\sinh^p(d+ex)\tanh^q(d+ex)dx$$

$$\Big\downarrow \text{7299}$$

$$\sinh^{-p}(d+ex)\tanh^{-q}(d+ex)(f\sinh(d+ex))^p(g\tanh(d+ex))^q\int F^{ac+bxc}\sinh^p(d+ex)\tanh^q(d+ex)dx$$

input
```
Int[F^(c*(a + b*x))*(f*Sinh[d + e*x])^p*(g*Tanh[d + e*x])^q,x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}(f\sinh(ex+d))^p\,(g\tanh(ex+d))^q\,dx$$

input
```
int(F^(c*(b*x+a))*(f*sinh(e*x+d))^p*(g*tanh(e*x+d))^q,x)
```

output
```
int(F^(c*(b*x+a))*(f*sinh(e*x+d))^p*(g*tanh(e*x+d))^q,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\tanh(d+ex))^q\,dx = \int (f\sinh(ex+d))^p\,(g\tanh(ex+d))^q\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sinh(e*x+d))^p*(g*tanh(e*x+d))^q,x, algorithm="
fricas")
```

output
```
integral((f*sinh(e*x + d))^p*(g*tanh(e*x + d))^q*F^(b*c*x + a*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\tanh(d+ex))^q\,dx = \text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(f*sinh(e*x+d))**p*(g*tanh(e*x+d))**q,x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\tanh(d+ex))^q\,dx = \int (f\sinh(ex+d))^p(g\tanh(ex+d))^q F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sinh(e*x+d))^p*(g*tanh(e*x+d))^q,x, algorithm="
maxima")
```

output
```
integrate((f*sinh(e*x + d))^p*(g*tanh(e*x + d))^q*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\tanh(d+ex))^q\,dx = \int (f\sinh(ex+d))^p(g\tanh(ex+d))^q F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sinh(e*x+d))^p*(g*tanh(e*x+d))^q,x, algorithm="
giac")
```

output
```
integrate((f*sinh(e*x + d))^p*(g*tanh(e*x + d))^q*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\tanh(d+ex))^q\,dx = \int F^{c\,(a+b\,x)}\,(f\sinh(d+e\,x))^p\,(g\tanh(d+e\,x))^q\,dx$$

input
```
int(F^(c*(a + b*x))*(f*sinh(d + e*x))^p*(g*tanh(d + e*x))^q,x)
```

output
```
int(F^(c*(a + b*x))*(f*sinh(d + e*x))^p*(g*tanh(d + e*x))^q, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\tanh(d+ex))^q\,dx = g^q f^{ac+p}\left(\int f^{bcx}\tanh(ex+d)^q\sinh(ex+d)^p\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*sinh(e*x+d))^p*(g*tanh(e*x+d))^q,x)
```

output
```
g**q*f**(a*c + p)*int(f**(b*c*x)*tanh(d + e*x)**q*sinh(d + e*x)**p,x)
```

3.172 $\int F^{c(a+bx)}(g\coth(d+ex))^q(f\sinh(d+ex))^p\,dx$

Optimal result

Integrand size = 30, antiderivative size = 167

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\sinh(d+ex))^p\,dx = -\frac{2^{-1+q}\left(e^{2d+2ex}\right)^{\frac{1}{2}\left(p-\frac{bc\log(F)}{e}\right)}\left(1-e^{2d+2ex}\right)\left(1+e^{2d+2ex}\right)^{-q}F^{c(a+bx)}\operatorname{AppellF1}\left(1+p-q,\frac{1}{2}\left(2+p-\frac{bc\log(F)}{e}\right),\ldots\right)}{e(1+p-q)}$$

output
```
-2^(-1+q)*exp(2*e*x+2*d)^(1/2*p-1/2*b*c*ln(F)/e)*(1-exp(2*e*x+2*d))*F^(c*(
b*x+a))*AppellF1(1+p-q,1+1/2*p-1/2*b*c*ln(F)/e,-q,2+p-q,1-exp(2*e*x+2*d),1
/2-1/2*exp(2*e*x+2*d))*(g*coth(e*x+d))^q*(f*sinh(e*x+d))^p/e/((1+exp(2*e*x
+2*d))^q)/(1+p-q)
```

Mathematica [F]

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\sinh(d+ex))^p\,dx = \int F^{c(a+bx)}(g\coth(d+ex))^q(f\sinh(d+ex))^p\,dx$$

input
```
Integrate[F^(c*(a + b*x))*(g*Coth[d + e*x])^q*(f*Sinh[d + e*x])^p,x]
```

output
```
Integrate[F^(c*(a + b*x))*(g*Coth[d + e*x])^q*(f*Sinh[d + e*x])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\sinh(d+ex))^p(g\coth(d+ex))^q\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$\coth^{-q}(d+ex)(g\coth(d+ex))^q\int F^{c(a+bx)}\coth^q(d+ex)(f\sinh(d+ex))^pdx$$

$$\Big\downarrow \text{ 7271}$$

$$\sinh^{-p}(d+ex)\coth^{-q}(d+ex)(f\sinh(d+ex))^p(g\coth(d+ex))^q\int F^{c(a+bx)}\coth^q(d+ex)\sinh^p(d+ex)dx$$

$$\Big\downarrow \text{ 7292}$$

$$\sinh^{-p}(d+ex)\coth^{-q}(d+ex)(f\sinh(d+ex))^p(g\coth(d+ex))^q\int F^{ac+bxc}\coth^q(d+ex)\sinh^p(d+ex)dx$$

$$\Big\downarrow \text{ 7299}$$

$$\sinh^{-p}(d+ex)\coth^{-q}(d+ex)(f\sinh(d+ex))^p(g\coth(d+ex))^q\int F^{ac+bxc}\coth^q(d+ex)\sinh^p(d+ex)dx$$

input
```
Int[F^(c*(a + b*x))*(g*Coth[d + e*x])^q*(f*Sinh[d + e*x])^p,x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}(g\coth(ex+d))^q(f\sinh(ex+d))^p\,dx$$

input
```
int(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*sinh(e*x+d))^p,x)
```

output
```
int(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*sinh(e*x+d))^p,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\sinh(d+ex))^p\,dx = \int (g\coth(ex+d))^q(f\sinh(ex+d))^pF^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*sinh(e*x+d))^p,x, algorithm="
fricas")
```

output
```
integral((g*coth(e*x + d))^q*(f*sinh(e*x + d))^p*F^(b*c*x + a*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\sinh(d+ex))^p\,dx = \text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(g*coth(e*x+d))**q*(f*sinh(e*x+d))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\sinh(d+ex))^p\,dx$$
$$=\int (g\coth{(ex+d)})^q\,(f\sinh{(ex+d)})^p\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*sinh(e*x+d))^p,x, algorithm="
maxima")
```

output
```
integrate((g*coth(e*x + d))^q*(f*sinh(e*x + d))^p*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\sinh(d+ex))^p\,dx$$
$$=\int (g\coth{(ex+d)})^q\,(f\sinh{(ex+d)})^p\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*sinh(e*x+d))^p,x, algorithm="
giac")
```

output
```
integrate((g*coth(e*x + d))^q*(f*sinh(e*x + d))^p*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\sinh(d+ex))^p\,dx$$
$$=\int F^{c\,(a+b\,x)}\,(g\coth(d+e\,x))^q\,(f\sinh(d+e\,x))^p\,dx$$

input
```
int(F^(c*(a + b*x))*(g*coth(d + e*x))^q*(f*sinh(d + e*x))^p,x)
```

output
```
int(F^(c*(a + b*x))*(g*coth(d + e*x))^q*(f*sinh(d + e*x))^p, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\sinh(d+ex))^p\,dx$$
$$= g^q f^{ac+p}\left(\int f^{bcx}\sinh(ex+d)^p\coth(ex+d)^q\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*sinh(e*x+d))^p,x)
```

output
```
g**q*f**(a*c + p)*int(f**(b*c*x)*sinh(d + e*x)**p*coth(d + e*x)**q,x)
```

3.173 $\int F^{c(a+bx)}(f\,\mathbf{csch}(d+ex))^p(g\tanh(d+ex))^q\,dx$

Optimal result

Integrand size = 30, antiderivative size = 168

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\tanh(d+ex))^q\,dx = -\frac{2^{-1-q}\left(e^{2d+2ex}\right)^{-\frac{ep+bc\log(F)}{2e}}\left(1-e^{2d+2ex}\right)\left(1+e^{2d+2ex}\right)^q F^{c(a+bx)}\operatorname{AppellF1}\left(1-p+q,\frac{1}{2}\left(2-p-\frac{bc\log(F}{e}\right.\right.}{e(1-p+q)}$$

output
```
-2^(-1-q)*(1-exp(2*e*x+2*d))*(1+exp(2*e*x+2*d))^q*F^(c*(b*x+a))*AppellF1(1
-p+q,1-1/2*p-1/2*b*c*ln(F)/e,q,2-p+q,1-exp(2*e*x+2*d),1/2-1/2*exp(2*e*x+2*
d))*(f*csch(e*x+d))^p*(g*tanh(e*x+d))^q/e/(exp(2*e*x+2*d)^(1/2*(e*p+b*c*ln
(F))/e))/(1-p+q)
```

Mathematica [F]

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\tanh(d+ex))^q\,dx = \int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\tanh(d+ex))^q\,dx$$

input
```
Integrate[F^(c*(a + b*x))*(f*Csch[d + e*x])^p*(g*Tanh[d + e*x])^q,x]
```

output
```
Integrate[F^(c*(a + b*x))*(f*Csch[d + e*x])^p*(g*Tanh[d + e*x])^q, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\tanh(d+ex))^q\,dx$$

$$\Big\downarrow \text{7271}$$

$$\operatorname{csch}^{-p}(d+ex)(f\operatorname{csch}(d+ex))^p\int F^{c(a+bx)}\operatorname{csch}^p(d+ex)(g\tanh(d+ex))^q dx$$

$$\Big\downarrow \text{7271}$$

$$\operatorname{csch}^{-p}(d+ex)\tanh^{-q}(d+ex)(f\operatorname{csch}(d+ex))^p(g\tanh(d+ex))^q\int F^{c(a+bx)}\operatorname{csch}^p(d+ex)\tanh^q(d+ex)dx$$

$$\Big\downarrow \text{7292}$$

$$\operatorname{csch}^{-p}(d+ex)\tanh^{-q}(d+ex)(f\operatorname{csch}(d+ex))^p(g\tanh(d+ex))^q\int F^{ac+bxc}\operatorname{csch}^p(d+ex)\tanh^q(d+ex)dx$$

$$\Big\downarrow \text{7299}$$

$$\operatorname{csch}^{-p}(d+ex)\tanh^{-q}(d+ex)(f\operatorname{csch}(d+ex))^p(g\tanh(d+ex))^q\int F^{ac+bxc}\operatorname{csch}^p(d+ex)\tanh^q(d+ex)dx$$

input
```
Int[F^(c*(a + b*x))*(f*Csch[d + e*x])^p*(g*Tanh[d + e*x])^q,x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}(f\ \operatorname{csch}(ex+d))^{p} (g\tanh(ex+d))^{q} dx$$

input
```
int(F^(c*(b*x+a))*(f*csch(e*x+d))^p*(g*tanh(e*x+d))^q,x)
```

output
```
int(F^(c*(b*x+a))*(f*csch(e*x+d))^p*(g*tanh(e*x+d))^q,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\tanh(d+ex))^q dx$$
$$= \int (f\operatorname{csch}(ex+d))^{p} (g\tanh(ex+d))^{q} F^{(bx+a)c} dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(e*x+d))^p*(g*tanh(e*x+d))^q,x, algorithm="
fricas")
```

output
```
integral((f*csch(e*x + d))^p*(g*tanh(e*x + d))^q*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\tanh(d+ex))^q dx$$
$$= \int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^{p} (g\tanh(d+ex))^{q} dx$$

input
```
integrate(F**(c*(b*x+a))*(f*csch(e*x+d))**p*(g*tanh(e*x+d))**q,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*csch(d + e*x))**p*(g*tanh(d + e*x))**q, x)
```

Maxima [F]

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\tanh(d+ex))^q\,dx$$
$$=\int (f\operatorname{csch}(ex+d))^p(g\tanh(ex+d))^qF^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(e*x+d))^p*(g*tanh(e*x+d))^q,x, algorithm="
maxima")
```

output
```
integrate((f*csch(e*x + d))^p*(g*tanh(e*x + d))^q*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\tanh(d+ex))^q\,dx$$
$$=\int (f\operatorname{csch}(ex+d))^p(g\tanh(ex+d))^qF^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*csch(e*x+d))^p*(g*tanh(e*x+d))^q,x, algorithm="
giac")
```

output
```
integrate((f*csch(e*x + d))^p*(g*tanh(e*x + d))^q*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\tanh(d+ex))^q\,dx$$
$$=\int F^{c\,(a+b\,x)}\,(g\tanh(d+e\,x))^q\left(\frac{f}{\sinh(d+e\,x)}\right)^p\,dx$$

input
```
int(F^(c*(a + b*x))*(g*tanh(d + e*x))^q*(f/sinh(d + e*x))^p,x)
```

output
```
int(F^(c*(a + b*x))*(g*tanh(d + e*x))^q*(f/sinh(d + e*x))^p, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\tanh(d+ex))^q\,dx$$
$$= g^q f^{ac+p}\left(\int f^{bcx}\tanh\left(ex+d\right)^q\operatorname{csch}(ex+d)^p\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*csch(e*x+d))^p*(g*tanh(e*x+d))^q,x)
```

output
```
g**q*f**(a*c + p)*int(f**(b*c*x)*tanh(d + e*x)**q*csch(d + e*x)**p,x)
```

3.174 $\int F^{c(a+bx)}(g\coth(d+ex))^q(f\,\mathbf{csch}(d+ex))^p\,dx$

Optimal result

Integrand size = 30, antiderivative size = 176

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\operatorname{csch}(d+ex))^p\,dx = -\frac{2^{-1+q}\left(e^{2d+2ex}\right)^{-\frac{ep+bc\log(F)}{2e}}\left(1-e^{2d+2ex}\right)\left(1+e^{2d+2ex}\right)^{-q}F^{c(a+bx)}\operatorname{AppellF1}\left(1-p-q,\frac{1}{2}\left(2-p-\frac{bc\log(}{e}\right.\right.}{e(1-p-q)}$$

output
```
-2^(-1+q)*(1-exp(2*e*x+2*d))*F^(c*(b*x+a))*AppellF1(1-p-q,1-1/2*p-1/2*b*c*
ln(F)/e,-q,2-p-q,1-exp(2*e*x+2*d),1/2-1/2*exp(2*e*x+2*d))*(g*coth(e*x+d))^
q*(f*csch(e*x+d))^p/e/(exp(2*e*x+2*d)^(1/2*(e*p+b*c*ln(F))/e))/((1+exp(2*e
*x+2*d))^q)/(1-p-q)
```

Mathematica [F]

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\operatorname{csch}(d+ex))^p\,dx = \int F^{c(a+bx)}(g\coth(d+ex))^q(f\operatorname{csch}(d+ex))^p\,dx$$

input
```
Integrate[F^(c*(a + b*x))*(g*Coth[d + e*x])^q*(f*Csch[d + e*x])^p,x]
```

output
```
Integrate[F^(c*(a + b*x))*(g*Coth[d + e*x])^q*(f*Csch[d + e*x])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\operatorname{csch}(d+ex))^p(g\coth(d+ex))^q\,dx$$

$$\Big\downarrow \text{7271}$$

$$\coth^{-q}(d+ex)(g\coth(d+ex))^q\int F^{c(a+bx)}\coth^q(d+ex)(f\operatorname{csch}(d+ex))^pdx$$

$$\Big\downarrow \text{7271}$$

$$\operatorname{csch}^{-p}(d+ex)\coth^{-q}(d+ex)(f\operatorname{csch}(d+ex))^p(g\coth(d+ex))^q\int F^{c(a+bx)}\coth^q(d+ex)\operatorname{csch}^p(d+ex)dx$$

$$\Big\downarrow \text{7292}$$

$$\operatorname{csch}^{-p}(d+ex)\coth^{-q}(d+ex)(f\operatorname{csch}(d+ex))^p(g\coth(d+ex))^q\int F^{ac+bxc}\coth^q(d+ex)\operatorname{csch}^p(d+ex)dx$$

$$\Big\downarrow \text{7299}$$

$$\operatorname{csch}^{-p}(d+ex)\coth^{-q}(d+ex)(f\operatorname{csch}(d+ex))^p(g\coth(d+ex))^q\int F^{ac+bxc}\coth^q(d+ex)\operatorname{csch}^p(d+ex)dx$$

input
```
Int[F^(c*(a + b*x))*(g*Coth[d + e*x])^q*(f*Csch[d + e*x])^p,x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}(g\coth (ex+d))^{q} (f \operatorname{csch} (ex+d))^{p} dx$$

input
```
int(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*csch(e*x+d))^p,x)
```

output
```
int(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*csch(e*x+d))^p,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\operatorname{csch}(d+ex))^p\,dx = \int (g\coth (ex+d))^{q} (f \operatorname{csch} (ex+d))^{p} F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*csch(e*x+d))^p,x, algorithm="
fricas")
```

output
```
integral((g*coth(e*x + d))^q*(f*csch(e*x + d))^p*F^(b*c*x + a*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\operatorname{csch}(d+ex))^p\,dx = \text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(g*coth(e*x+d))**q*(f*csch(e*x+d))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\text{csch}(d+ex))^p\,dx = \int (g\coth(ex+d))^q(f\,\text{csch}(ex+d))^p F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*csch(e*x+d))^p,x, algorithm="
maxima")
```

output
```
integrate((g*coth(e*x + d))^q*(f*csch(e*x + d))^p*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\text{csch}(d+ex))^p\,dx = \int (g\coth(ex+d))^q(f\,\text{csch}(ex+d))^p F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*csch(e*x+d))^p,x, algorithm="
giac")
```

output
```
integrate((g*coth(e*x + d))^q*(f*csch(e*x + d))^p*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\text{csch}(d+ex))^p\,dx = \int F^{c\,(a+b\,x)}\,(g\coth(d+e\,x))^q\left(\frac{f}{\sinh(d+e\,x)}\right)^p\,dx$$

input
```
int(F^(c*(a + b*x))*(g*coth(d + e*x))^q*(f/sinh(d + e*x))^p,x)
```

output
```
int(F^(c*(a + b*x))*(g*coth(d + e*x))^q*(f/sinh(d + e*x))^p, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\operatorname{csch}(d+ex))^p\,dx$$
$$= g^q f^{ac+p}\left(\int f^{bcx}\operatorname{csch}(ex+d)^p\coth(ex+d)^q\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*csch(e*x+d))^p,x)
```

output
```
g**q*f**(a*c + p)*int(f**(b*c*x)*csch(d + e*x)**p*coth(d + e*x)**q,x)
```

3.175 $\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\tanh(d+ex))^q\,dx$

Optimal result

Integrand size = 30, antiderivative size = 165

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\tanh(d+ex))^q\,dx = -\frac{2^{-1+p-q}\left(e^{2d+2ex}\right)^{\frac{1}{2}\left(p-\frac{bc\log(F)}{e}\right)}\left(1-e^{2d+2ex}\right)\left(1+e^{2d+2ex}\right)^{-p+q}F^{c(a+bx)}\,\text{AppellF1}\left(1+q,\frac{1}{2}\left(2+p-\frac{bc\log(F)}{e}\right.\right.}{e(1+q)}$$

output
```
-2^(-1+p-q)*exp(2*e*x+2*d)^(1/2*p-1/2*b*c*ln(F)/e)*(1-exp(2*e*x+2*d))*(1+e
xp(2*e*x+2*d))^(-p+q)*F^(c*(b*x+a))*AppellF1(1+q,1+1/2*p-1/2*b*c*ln(F)/e,-
p+q,2+q,1-exp(2*e*x+2*d),1/2-1/2*exp(2*e*x+2*d))*(f*cosh(e*x+d))^p*(g*tanh
(e*x+d))^q/e/(1+q)
```

Mathematica [F]

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\tanh(d+ex))^q\,dx = \int F^{c(a+bx)}(f\cosh(d+ex))^p(g\tanh(d+ex))^q\,dx$$

input
```
Integrate[F^(c*(a + b*x))*(f*Cosh[d + e*x])^p*(g*Tanh[d + e*x])^q,x]
```

output
```
Integrate[F^(c*(a + b*x))*(f*Cosh[d + e*x])^p*(g*Tanh[d + e*x])^q, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\tanh(d+ex))^q\,dx$$

$$\Big\downarrow \text{7271}$$

$$\cosh^{-p}(d+ex)(f\cosh(d+ex))^p\int F^{c(a+bx)}\cosh^p(d+ex)(g\tanh(d+ex))^qdx$$

$$\Big\downarrow \text{7271}$$

$$\cosh^{-p}(d+ex)\tanh^{-q}(d+ex)(f\cosh(d+ex))^p(g\tanh(d+ex))^q\int F^{c(a+bx)}\cosh^p(d+ex)\tanh^q(d+ex)dx$$

$$\Big\downarrow \text{7292}$$

$$\cosh^{-p}(d+ex)\tanh^{-q}(d+ex)(f\cosh(d+ex))^p(g\tanh(d+ex))^q\int F^{ac+bxc}\cosh^p(d+ex)\tanh^q(d+ex)dx$$

$$\Big\downarrow \text{7299}$$

$$\cosh^{-p}(d+ex)\tanh^{-q}(d+ex)(f\cosh(d+ex))^p(g\tanh(d+ex))^q\int F^{ac+bxc}\cosh^p(d+ex)\tanh^q(d+ex)dx$$

input
```
Int[F^(c*(a + b*x))*(f*Cosh[d + e*x])^p*(g*Tanh[d + e*x])^q,x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)} (f \cosh (ex+d))^{p} (g \tanh (ex+d))^{q} dx$$

input
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*tanh(e*x+d))^q,x)
```

output
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*tanh(e*x+d))^q,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f \cosh(d+ex))^p(g \tanh(d+ex))^q dx = \int (f \cosh (ex+d))^{p} (g \tanh (ex+d))^{q} F^{(bx+a)c} dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*tanh(e*x+d))^q,x, algorithm="
fricas")
```

output
```
integral((f*cosh(e*x + d))^p*(g*tanh(e*x + d))^q*F^(b*c*x + a*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f \cosh(d+ex))^p(g \tanh(d+ex))^q dx = \text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(f*cosh(e*x+d))**p*(g*tanh(e*x+d))**q,x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\tanh(d+ex))^q\,dx = \int (f\cosh(ex+d))^p(g\tanh(ex+d))^q F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*tanh(e*x+d))^q,x, algorithm="
maxima")
```

output
```
integrate((f*cosh(e*x + d))^p*(g*tanh(e*x + d))^q*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\tanh(d+ex))^q\,dx = \int (f\cosh(ex+d))^p(g\tanh(ex+d))^q F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*tanh(e*x+d))^q,x, algorithm="
giac")
```

output
```
integrate((f*cosh(e*x + d))^p*(g*tanh(e*x + d))^q*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\tanh(d+ex))^q\,dx = \int F^{c\,(a+b\,x)}\,(f\cosh(d+e\,x))^p\,(g\tanh(d+e\,x))^q\,dx$$

input
```
int(F^(c*(a + b*x))*(f*cosh(d + e*x))^p*(g*tanh(d + e*x))^q,x)
```

output
```
int(F^(c*(a + b*x))*(f*cosh(d + e*x))^p*(g*tanh(d + e*x))^q, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\tanh(d+ex))^q\,dx$$
$$= g^q f^{ac+p}\left(\int f^{bcx}\tanh\left(ex+d\right)^q\cosh\left(ex+d\right)^p dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*tanh(e*x+d))^q,x)
```

output
```
g**q*f**(a*c + p)*int(f**(b*c*x)*tanh(d + e*x)**q*cosh(d + e*x)**p,x)
```

3.176 $\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\coth(d+ex))^q\,dx$

Optimal result

Integrand size = 30, antiderivative size = 173

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\coth(d+ex))^q\,dx =$$

$$-\frac{2^{-1+p+q}\left(e^{2d+2ex}\right)^{\frac{1}{2}\left(p-\frac{bc\log(F)}{e}\right)}\left(1-e^{2d+2ex}\right)\left(1+e^{2d+2ex}\right)^{-p-q}F^{c(a+bx)}\,\mathrm{AppellF1}\left(1-q,\frac{1}{2}\left(2+p-\frac{bc\,\mathrm{lo}}{}\right.\right.}{e(1-q)}$$

output
```
-2^(-1+p+q)*exp(2*e*x+2*d)^(1/2*p-1/2*b*c*ln(F)/e)*(1-exp(2*e*x+2*d))*(1+e
xp(2*e*x+2*d))^(-p-q)*F^(c*(b*x+a))*AppellF1(1-q,1+1/2*p-1/2*b*c*ln(F)/e,-
p-q,2-q,1-exp(2*e*x+2*d),1/2-1/2*exp(2*e*x+2*d))*(f*cosh(e*x+d))^p*(g*coth
(e*x+d))^q/e/(1-q)
```

Mathematica [F]

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\coth(d+ex))^q\,dx$$
$$=\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\coth(d+ex))^q\,dx$$

input
```
Integrate[F^(c*(a + b*x))*(f*Cosh[d + e*x])^p*(g*Coth[d + e*x])^q,x]
```

output
```
Integrate[F^(c*(a + b*x))*(f*Cosh[d + e*x])^p*(g*Coth[d + e*x])^q, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\coth(d+ex))^q\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$\cosh^{-p}(d+ex)(f\cosh(d+ex))^p\int F^{c(a+bx)}\cosh^p(d+ex)(g\coth(d+ex))^qdx$$

$$\Big\downarrow \text{ 7271}$$

$$\cosh^{-p}(d+ex)\coth^{-q}(d+ex)(f\cosh(d+ex))^p(g\coth(d+ex))^q\int F^{c(a+bx)}\cosh^p(d+ex)\coth^q(d+ex)dx$$

$$\Big\downarrow \text{ 7292}$$

$$\cosh^{-p}(d+ex)\coth^{-q}(d+ex)(f\cosh(d+ex))^p(g\coth(d+ex))^q\int F^{ac+bxc}\cosh^p(d+ex)\coth^q(d+ex)dx$$

$$\Big\downarrow \text{ 7299}$$

$$\cosh^{-p}(d+ex)\coth^{-q}(d+ex)(f\cosh(d+ex))^p(g\coth(d+ex))^q\int F^{ac+bxc}\cosh^p(d+ex)\coth^q(d+ex)dx$$

input
```
Int[F^(c*(a + b*x))*(f*Cosh[d + e*x])^p*(g*Coth[d + e*x])^q,x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)} (f \cosh{(ex+d)})^p (g \coth{(ex+d)})^q \, dx$$

input
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*coth(e*x+d))^q,x)
```

output
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*coth(e*x+d))^q,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\coth(d+ex))^q \, dx = \int (f \cosh{(ex+d)})^p (g \coth{(ex+d)})^q F^{(bx+a)c} \, dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*coth(e*x+d))^q,x, algorithm="
fricas")
```

output
```
integral((f*cosh(e*x + d))^p*(g*coth(e*x + d))^q*F^(b*c*x + a*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\coth(d+ex))^q \, dx = \text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(f*cosh(e*x+d))**p*(g*coth(e*x+d))**q,x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\coth(d+ex))^q\,dx = \int (f\cosh(ex+d))^p(g\coth(ex+d))^qF^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*coth(e*x+d))^q,x, algorithm="
maxima")
```

output
```
integrate((f*cosh(e*x + d))^p*(g*coth(e*x + d))^q*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\coth(d+ex))^q\,dx = \int (f\cosh(ex+d))^p(g\coth(ex+d))^qF^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*coth(e*x+d))^q,x, algorithm="
giac")
```

output
```
integrate((f*cosh(e*x + d))^p*(g*coth(e*x + d))^q*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\coth(d+ex))^q\,dx = \int F^{c\,(a+b\,x)}\,(f\cosh(d+e\,x))^p\,(g\coth(d+e\,x))^q\,dx$$

input
```
int(F^(c*(a + b*x))*(f*cosh(d + e*x))^p*(g*coth(d + e*x))^q,x)
```

output
```
int(F^(c*(a + b*x))*(f*cosh(d + e*x))^p*(g*coth(d + e*x))^q, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\cosh(d+ex))^p(g\coth(d+ex))^q\,dx$$
$$= g^q f^{ac+p}\left(\int f^{bcx}\coth\left(ex+d\right)^q\cosh\left(ex+d\right)^p dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*cosh(e*x+d))^p*(g*coth(e*x+d))^q,x)
```

output
```
g**q*f**(a*c + p)*int(f**(b*c*x)*coth(d + e*x)**q*cosh(d + e*x)**p,x)
```

3.177 $\int F^{c(a+bx)}(f\mathbf{sech}(d+ex))^p(g\tanh(d+ex))^q\,dx$

Optimal result

Integrand size = 30, antiderivative size = 166

$$\int F^{c(a+bx)}(f\text{sech}(d+ex))^p(g\tanh(d+ex))^q\,dx = -\frac{2^{-1-p-q}\left(e^{2d+2ex}\right)^{-\frac{ep+bc\log(F)}{2e}}\left(1-e^{2d+2ex}\right)\left(1+e^{2d+2ex}\right)^{p+q}F^{c(a+bx)}\,\text{AppellF1}\left(1+q,\frac{1}{2}\left(2-p-\frac{bc\log(F)}{e}\right.\right.}{e(1+q)}$$

output
```
-2^(-1-p-q)*(1-exp(2*e*x+2*d))*(1+exp(2*e*x+2*d))^(p+q)*F^(c*(b*x+a))*Appe
llF1(1+q,1-1/2*p-1/2*b*c*ln(F)/e,p+q,2+q,1-exp(2*e*x+2*d),1/2-1/2*exp(2*e*
x+2*d))*(f*sech(e*x+d))^p*(g*tanh(e*x+d))^q/e/(exp(2*e*x+2*d)^(1/2*(e*p+b*
c*ln(F))/e))/(1+q)
```

Mathematica [F]

$$\int F^{c(a+bx)}(f\text{sech}(d+ex))^p(g\tanh(d+ex))^q\,dx = \int F^{c(a+bx)}(f\text{sech}(d+ex))^p(g\tanh(d+ex))^q\,dx$$

input
```
Integrate[F^(c*(a + b*x))*(f*Sech[d + e*x])^p*(g*Tanh[d + e*x])^q,x]
```

output
```
Integrate[F^(c*(a + b*x))*(f*Sech[d + e*x])^p*(g*Tanh[d + e*x])^q, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\text{sech}(d+ex))^p(g\tanh(d+ex))^q\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$\text{sech}^{-p}(d+ex)(f\text{sech}(d+ex))^p\int F^{c(a+bx)}\text{sech}^p(d+ex)(g\tanh(d+ex))^qdx$$

$$\Big\downarrow \text{ 7271}$$

$$\text{sech}^{-p}(d+ex)\tanh^{-q}(d+ex)(f\text{sech}(d+ex))^p(g\tanh(d+ex))^q\int F^{c(a+bx)}\text{sech}^p(d+ex)\tanh^q(d+ex)dx$$

$$\Big\downarrow \text{ 7292}$$

$$\text{sech}^{-p}(d+ex)\tanh^{-q}(d+ex)(f\text{sech}(d+ex))^p(g\tanh(d+ex))^q\int F^{ac+bxc}\text{sech}^p(d+ex)\tanh^q(d+ex)dx$$

$$\Big\downarrow \text{ 7299}$$

$$\text{sech}^{-p}(d+ex)\tanh^{-q}(d+ex)(f\text{sech}(d+ex))^p(g\tanh(d+ex))^q\int F^{ac+bxc}\text{sech}^p(d+ex)\tanh^q(d+ex)dx$$

input
```
Int[F^(c*(a + b*x))*(f*Sech[d + e*x])^p*(g*Tanh[d + e*x])^q,x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}(f \operatorname{sech}(ex+d))^p (g \tanh(ex+d))^q \, dx$$

input
```
int(F^(c*(b*x+a))*(f*sech(e*x+d))^p*(g*tanh(e*x+d))^q,x)
```

output
```
int(F^(c*(b*x+a))*(f*sech(e*x+d))^p*(g*tanh(e*x+d))^q,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(f\operatorname{sech}(d+ex))^p(g\tanh(d+ex))^q \, dx = \int (f \operatorname{sech}(ex+d))^p (g \tanh(ex+d))^q F^{(bx+a)c} \, dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sech(e*x+d))^p*(g*tanh(e*x+d))^q,x, algorithm="
fricas")
```

output
```
integral((f*sech(e*x + d))^p*(g*tanh(e*x + d))^q*F^(b*c*x + a*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)}(f\operatorname{sech}(d+ex))^p(g\tanh(d+ex))^q \, dx = \int F^{c(a+bx)}(f \operatorname{sech}(d+ex))^p (g \tanh(d+ex))^q \, dx$$

input
```
integrate(F**(c*(b*x+a))*(f*sech(e*x+d))**p*(g*tanh(e*x+d))**q,x)
```

output
```
Integral(F**(c*(a + b*x))*(f*sech(d + e*x))**p*(g*tanh(d + e*x))**q, x)
```

Maxima [F]

$$\int F^{c(a+bx)}(f\operatorname{sech}(d+ex))^p(g\tanh(d+ex))^q\,dx = \int (f\operatorname{sech}(ex+d))^p(g\tanh(ex+d))^q F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sech(e*x+d))^p*(g*tanh(e*x+d))^q,x, algorithm="
maxima")
```

output
```
integrate((f*sech(e*x + d))^p*(g*tanh(e*x + d))^q*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(f\operatorname{sech}(d+ex))^p(g\tanh(d+ex))^q\,dx = \int (f\operatorname{sech}(ex+d))^p(g\tanh(ex+d))^q F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(f*sech(e*x+d))^p*(g*tanh(e*x+d))^q,x, algorithm="
giac")
```

output
```
integrate((f*sech(e*x + d))^p*(g*tanh(e*x + d))^q*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(f\operatorname{sech}(d+ex))^p(g\tanh(d+ex))^q\,dx = \int F^{c\,(a+b\,x)}\,(g\tanh(d+e\,x))^q\left(\frac{f}{\cosh(d+e\,x)}\right)^p dx$$

input
```
int(F^(c*(a + b*x))*(g*tanh(d + e*x))^q*(f/cosh(d + e*x))^p,x)
```

output
```
int(F^(c*(a + b*x))*(g*tanh(d + e*x))^q*(f/cosh(d + e*x))^p, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(f\operatorname{sech}(d+ex))^{p}(g\tanh(d+ex))^{q}\,dx$$
$$=g^{q}f^{ac+p}\left(\int f^{bcx}\tanh\left(ex+d\right)^{q}\operatorname{sech}(ex+d)^{p}\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(f*sech(e*x+d))^p*(g*tanh(e*x+d))^q,x)
```

output
```
g**q*f**(a*c + p)*int(f**(b*c*x)*tanh(d + e*x)**q*sech(d + e*x)**p,x)
```

3.178 $\int F^{c(a+bx)}(g\coth(d+ex))^q(f\text{sech}(d+ex))^p\,dx$

Optimal result

Integrand size = 30, antiderivative size = 174

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\text{sech}(d+ex))^p\,dx = -\frac{2^{-1-p+q}\left(e^{2d+2ex}\right)^{-\frac{ep+bc\log(F)}{2e}}\left(1-e^{2d+2ex}\right)\left(1+e^{2d+2ex}\right)^{p-q}F^{c(a+bx)}\text{AppellF1}\left(1-q,\frac{1}{2}\left(2-p-\frac{bc\log(F}{e}\right.\right.}{e(1-q)}$$

output
```
-2^(-1-p+q)*(1-exp(2*e*x+2*d))*(1+exp(2*e*x+2*d))^(p-q)*F^(c*(b*x+a))*Appe
llF1(1-q,1-1/2*p-1/2*b*c*ln(F)/e,p-q,2-q,1-exp(2*e*x+2*d),1/2-1/2*exp(2*e*
x+2*d))*(g*coth(e*x+d))^q*(f*sech(e*x+d))^p/e/(exp(2*e*x+2*d)^(1/2*(e*p+b*
c*ln(F))/e))/(1-q)
```

Mathematica [F]

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\text{sech}(d+ex))^p\,dx = \int F^{c(a+bx)}(g\coth(d+ex))^q(f\text{sech}(d+ex))^p\,dx$$

input
```
Integrate[F^(c*(a + b*x))*(g*Coth[d + e*x])^q*(f*Sech[d + e*x])^p,x]
```

output
```
Integrate[F^(c*(a + b*x))*(g*Coth[d + e*x])^q*(f*Sech[d + e*x])^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(f\text{sech}(d+ex))^p(g\coth(d+ex))^q\,dx$$

$$\Big\downarrow \text{7271}$$

$$\coth^{-q}(d+ex)(g\coth(d+ex))^q\int F^{c(a+bx)}\coth^q(d+ex)(f\text{sech}(d+ex))^pdx$$

$$\Big\downarrow \text{7271}$$

$$\text{sech}^{-p}(d+ex)\coth^{-q}(d+ex)(f\text{sech}(d+ex))^p(g\coth(d+ex))^q\int F^{c(a+bx)}\coth^q(d+ex)\text{sech}^p(d+ex)dx$$

$$\Big\downarrow \text{7292}$$

$$\text{sech}^{-p}(d+ex)\coth^{-q}(d+ex)(f\text{sech}(d+ex))^p(g\coth(d+ex))^q\int F^{ac+bxc}\coth^q(d+ex)\text{sech}^p(d+ex)dx$$

$$\Big\downarrow \text{7299}$$

$$\text{sech}^{-p}(d+ex)\coth^{-q}(d+ex)(f\text{sech}(d+ex))^p(g\coth(d+ex))^q\int F^{ac+bxc}\coth^q(d+ex)\text{sech}^p(d+ex)dx$$

input
```
Int[F^(c*(a + b*x))*(g*Coth[d + e*x])^q*(f*Sech[d + e*x])^p,x]
```

output
```
$Aborted
```

Maple [F]

$$\int F^{c(bx+a)}(g\coth(ex+d))^q(f\ \mathrm{sech}(ex+d))^p\,dx$$

input
```
int(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*sech(e*x+d))^p,x)
```

output
```
int(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*sech(e*x+d))^p,x)
```

Fricas [F]

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\mathrm{sech}(d+ex))^p\,dx = \int (g\coth(ex+d))^q(f\,\mathrm{sech}(ex+d))^p F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*sech(e*x+d))^p,x, algorithm="
fricas")
```

output
```
integral((g*coth(e*x + d))^q*(f*sech(e*x + d))^p*F^(b*c*x + a*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\mathrm{sech}(d+ex))^p\,dx = \text{Timed out}$$

input
```
integrate(F**(c*(b*x+a))*(g*coth(e*x+d))**q*(f*sech(e*x+d))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\text{sech}(d+ex))^p\,dx = \int (g\coth(ex+d))^q\,(f\,\text{sech}\,(ex+d))^p\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*sech(e*x+d))^p,x, algorithm="
maxima")
```

output
```
integrate((g*coth(e*x + d))^q*(f*sech(e*x + d))^p*F^((b*x + a)*c), x)
```

Giac [F]

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\text{sech}(d+ex))^p\,dx = \int (g\coth(ex+d))^q\,(f\,\text{sech}\,(ex+d))^p\,F^{(bx+a)c}\,dx$$

input
```
integrate(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*sech(e*x+d))^p,x, algorithm="
giac")
```

output
```
integrate((g*coth(e*x + d))^q*(f*sech(e*x + d))^p*F^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\text{sech}(d+ex))^p\,dx = \int F^{c\,(a+b\,x)}\,(g\coth(d+e\,x))^q\,\left(\frac{f}{\cosh\,(d+e\,x)}\right)^p\,dx$$

input
```
int(F^(c*(a + b*x))*(g*coth(d + e*x))^q*(f/cosh(d + e*x))^p,x)
```

output
```
int(F^(c*(a + b*x))*(g*coth(d + e*x))^q*(f/cosh(d + e*x))^p, x)
```

Reduce [F]

$$\int F^{c(a+bx)}(g\coth(d+ex))^q(f\operatorname{sech}(d+ex))^p\,dx$$
$$= g^q f^{ac+p}\left(\int f^{bcx}\operatorname{sech}(ex+d)^p\coth(ex+d)^q\,dx\right)$$

input
```
int(F^(c*(b*x+a))*(g*coth(e*x+d))^q*(f*sech(e*x+d))^p,x)
```

output
```
g**q*f**(a*c + p)*int(f**(b*c*x)*sech(d + e*x)**p*coth(d + e*x)**q,x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*   "F" if the result fails to integrate an expression that*)
(*       is integrable*)
(*   "C" if result involves higher level functions than necessary*)
(*   "B" if result is more than twice the size of the optimal*)
(*       antiderivative*)
(*   "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
         return True
      else:
         return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:    #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:    #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file