Computer Algebra Independent Integration Tests

Summer 2024

6-Hyperbolic-functions/6-Miscellaneous/291-6.1

Nasser M. Abbasi

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**123**]. This is test number [291].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (123)	0.00 (0)
Mathematica	100.00 (123)	0.00 (0)
Fricas	100.00 (123)	0.00 (0)
Mupad	84.55 (104)	15.45 (19)
Maxima	84.55 (104)	15.45 (19)
Maple	82.93 (102)	17.07 (21)
Giac	78.86 (97)	21.14 (26)
Reduce	75.61 (93)	24.39 (30)
Sympy	21.14 (26)	78.86 (97)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

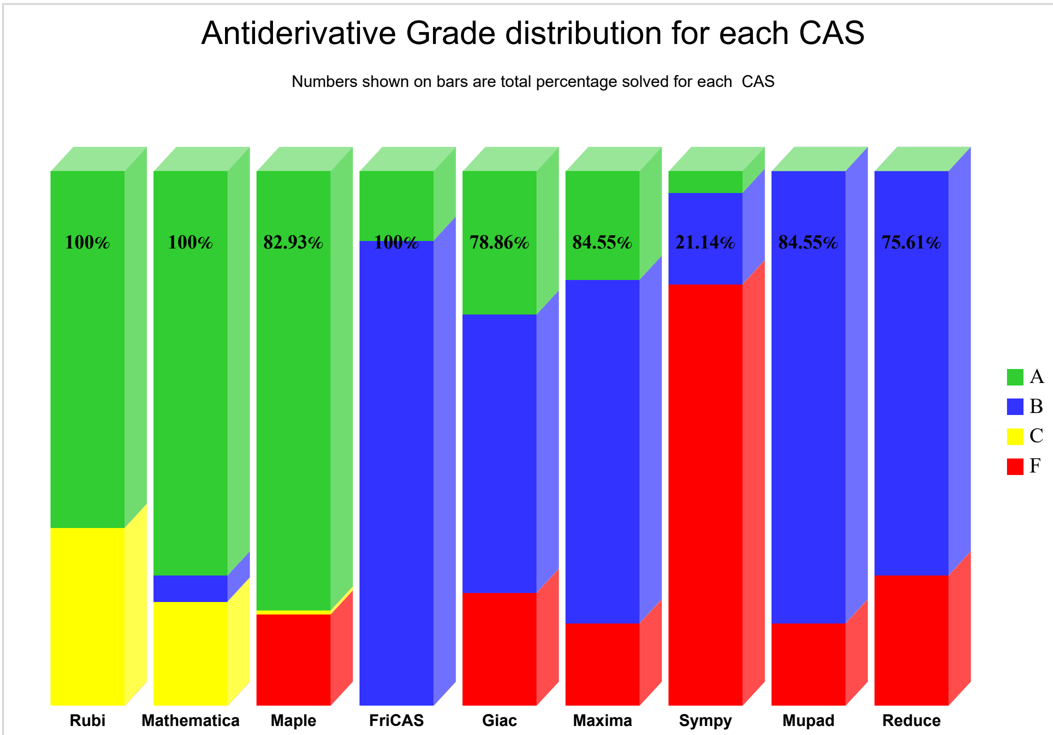
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

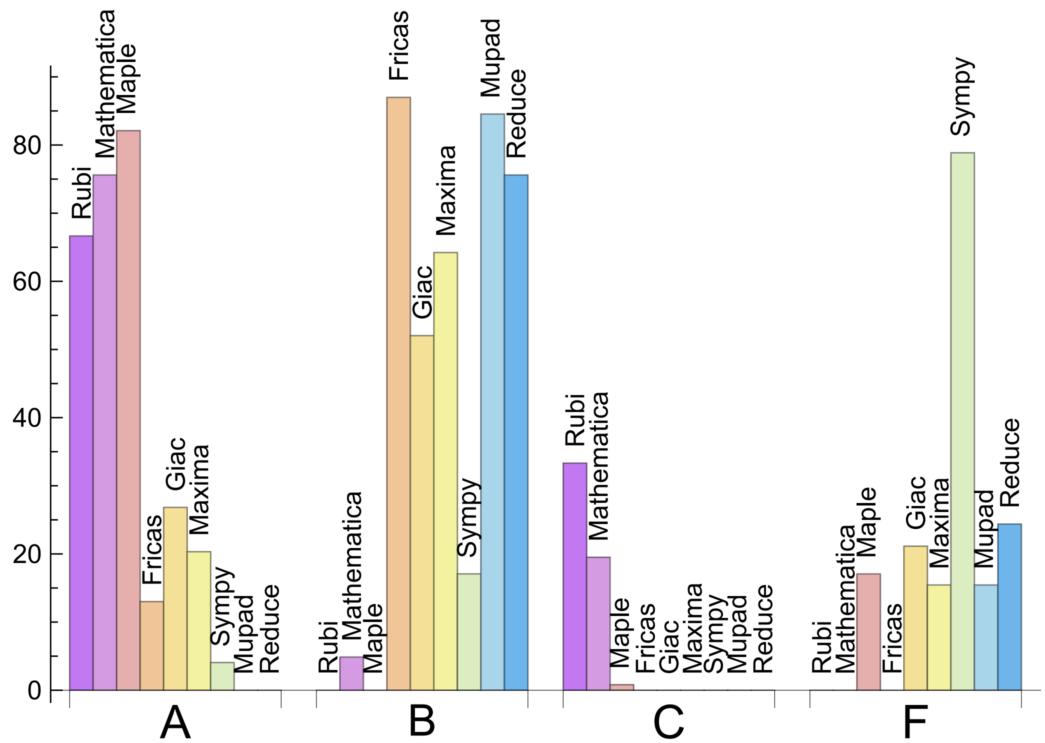
System	% A grade	% B grade	% C grade	% F grade
Maple	82.114	0.000	0.813	17.073
Mathematica	75.610	4.878	19.512	0.000
Rubi	66.667	0.000	33.333	0.000
Giac	26.829	52.033	0.000	21.138
Maxima	20.325	64.228	0.000	15.447
Fricas	13.008	86.992	0.000	0.000
Sympy	4.065	17.073	0.000	78.862
Mupad	0.000	84.553	0.000	15.447
Reduce	0.000	75.610	0.000	24.390

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	0	0.00	0.00	0.00
Mupad	19	0.00	100.00	0.00
Maxima	19	100.00	0.00	0.00
Maple	21	100.00	0.00	0.00
Giac	26	100.00	0.00	0.00
Reduce	30	100.00	0.00	0.00
Sympy	97	88.66	11.34	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Mathematica	0.06
Maxima	0.08
Fricas	0.09
Giac	0.13
Reduce	0.23
Rubi	0.27
Mupad	0.68
Sympy	0.99
Maple	11.22

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	44.68	1.04	39.00	1.00
Maple	54.00	1.40	33.00	0.95
Rubi	56.82	1.09	42.00	1.05
Giac	93.45	2.31	71.00	1.95
Mupad	125.38	3.67	95.00	2.48
Maxima	139.55	3.89	96.50	2.39
Reduce	141.80	3.30	97.00	2.78
Sympy	322.12	6.14	98.50	2.05
Fricas	497.20	10.15	304.00	7.04

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

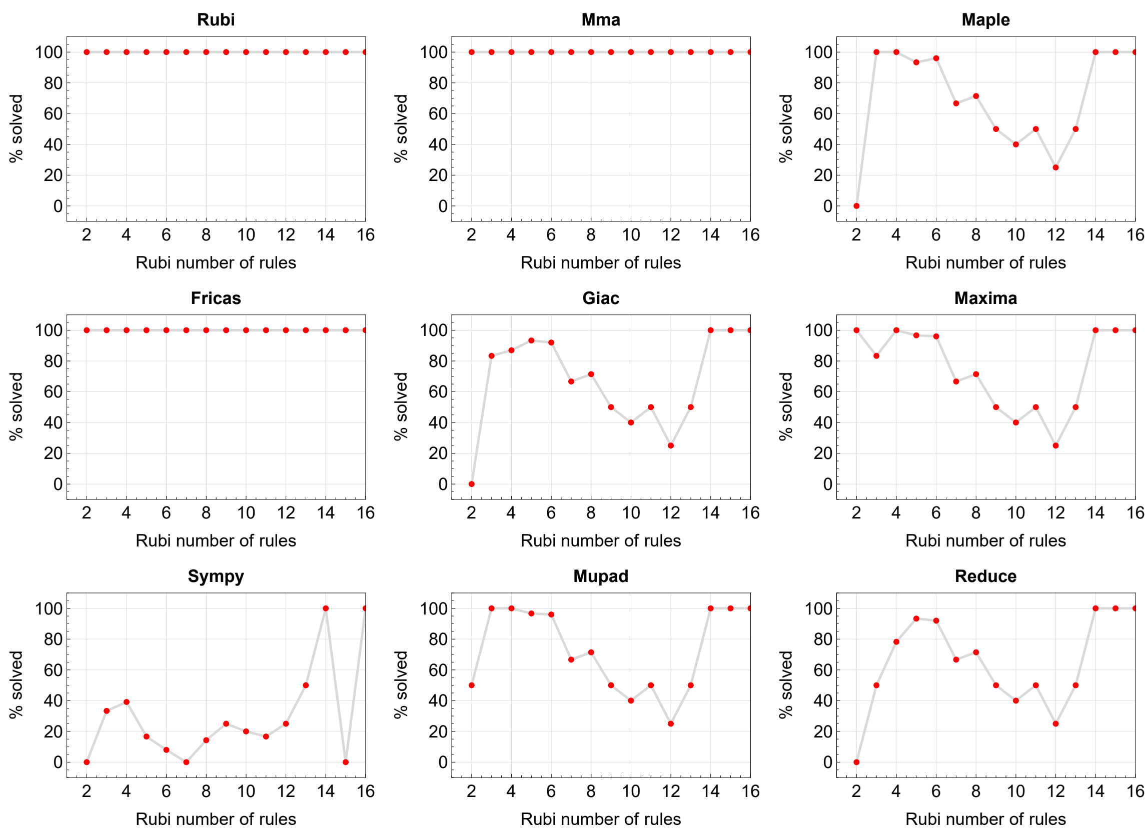


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

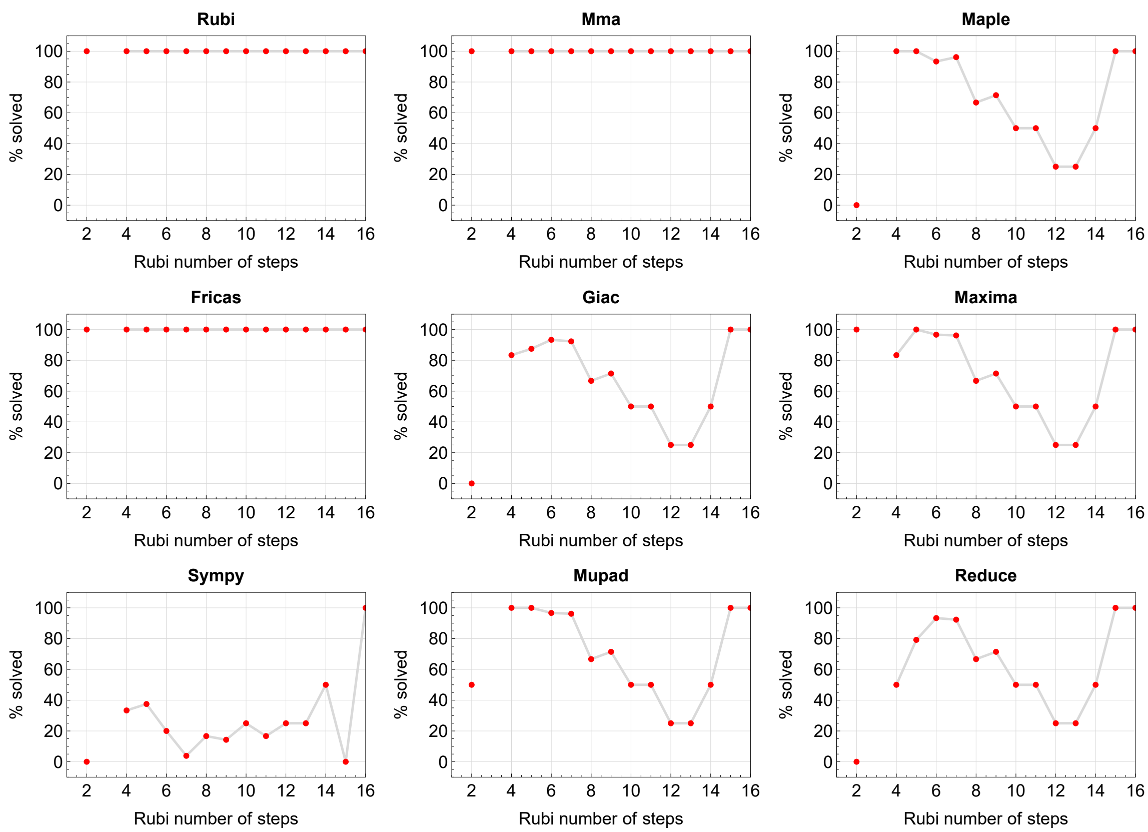


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

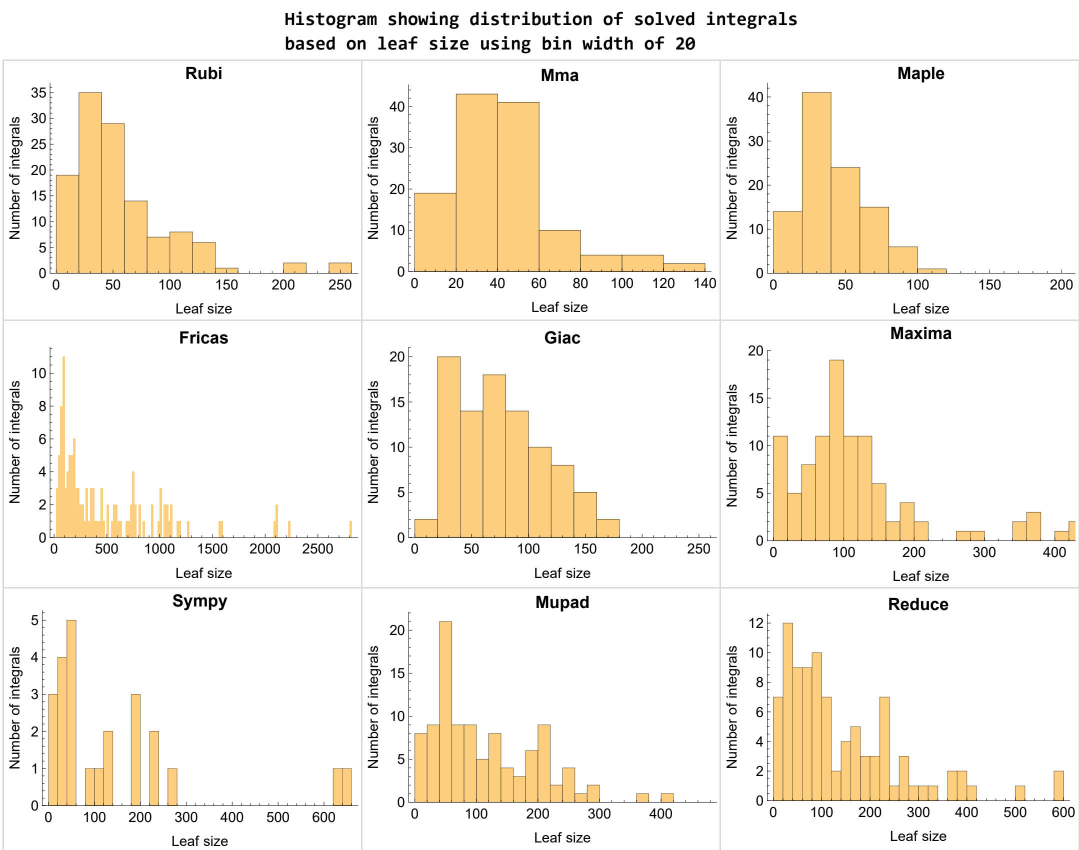


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

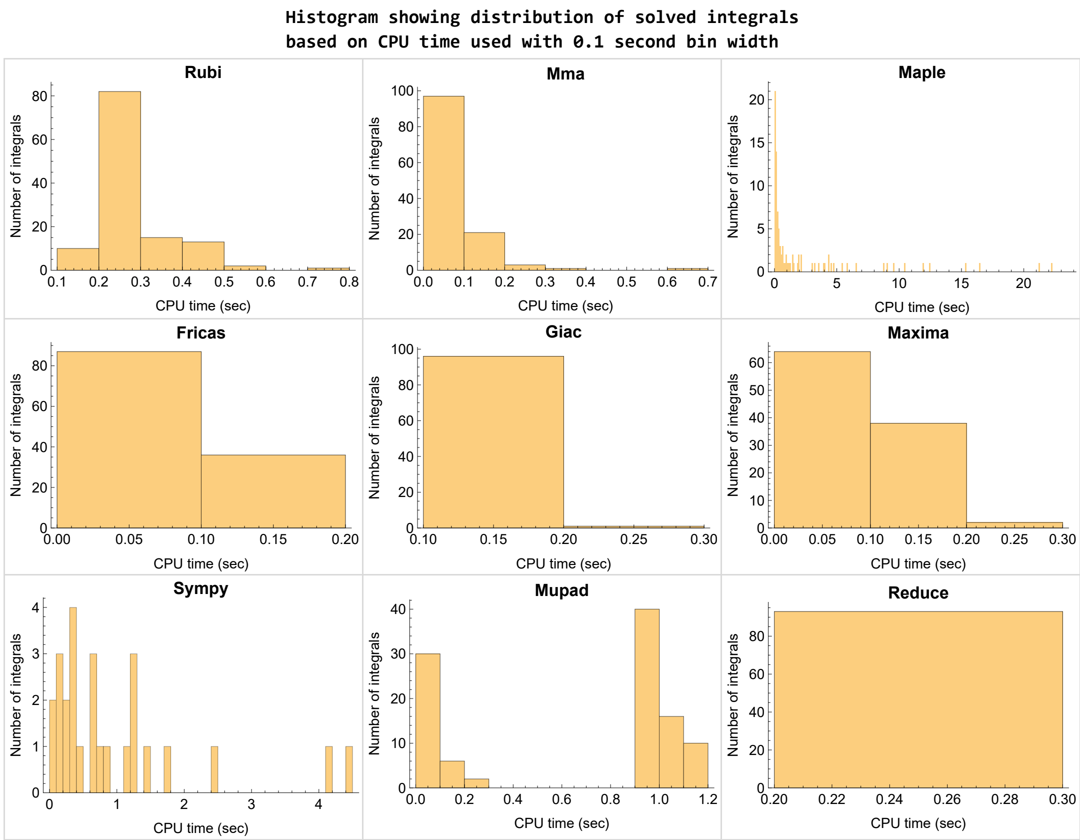


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

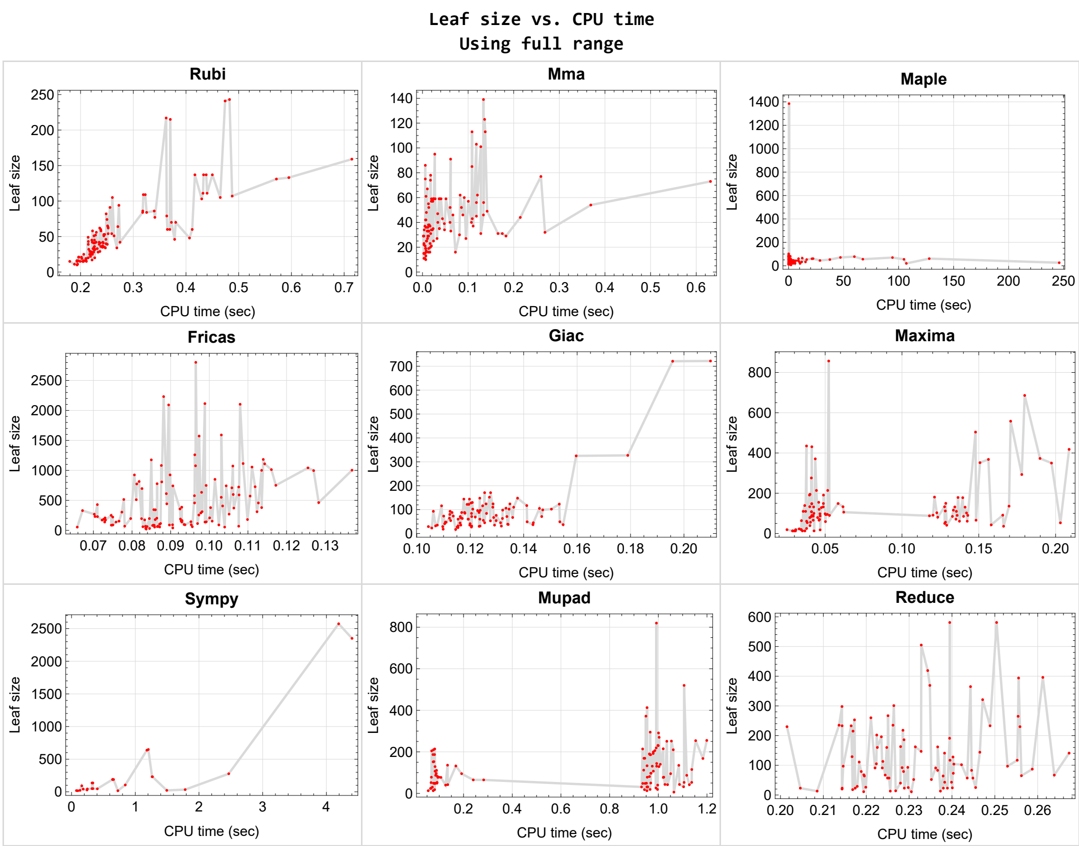


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {19, 20, 23, 24, 27, 28, 43, 54, 58, 84, 92, 94, 100, 102, 104}

Mathematica {}

Maple {68}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

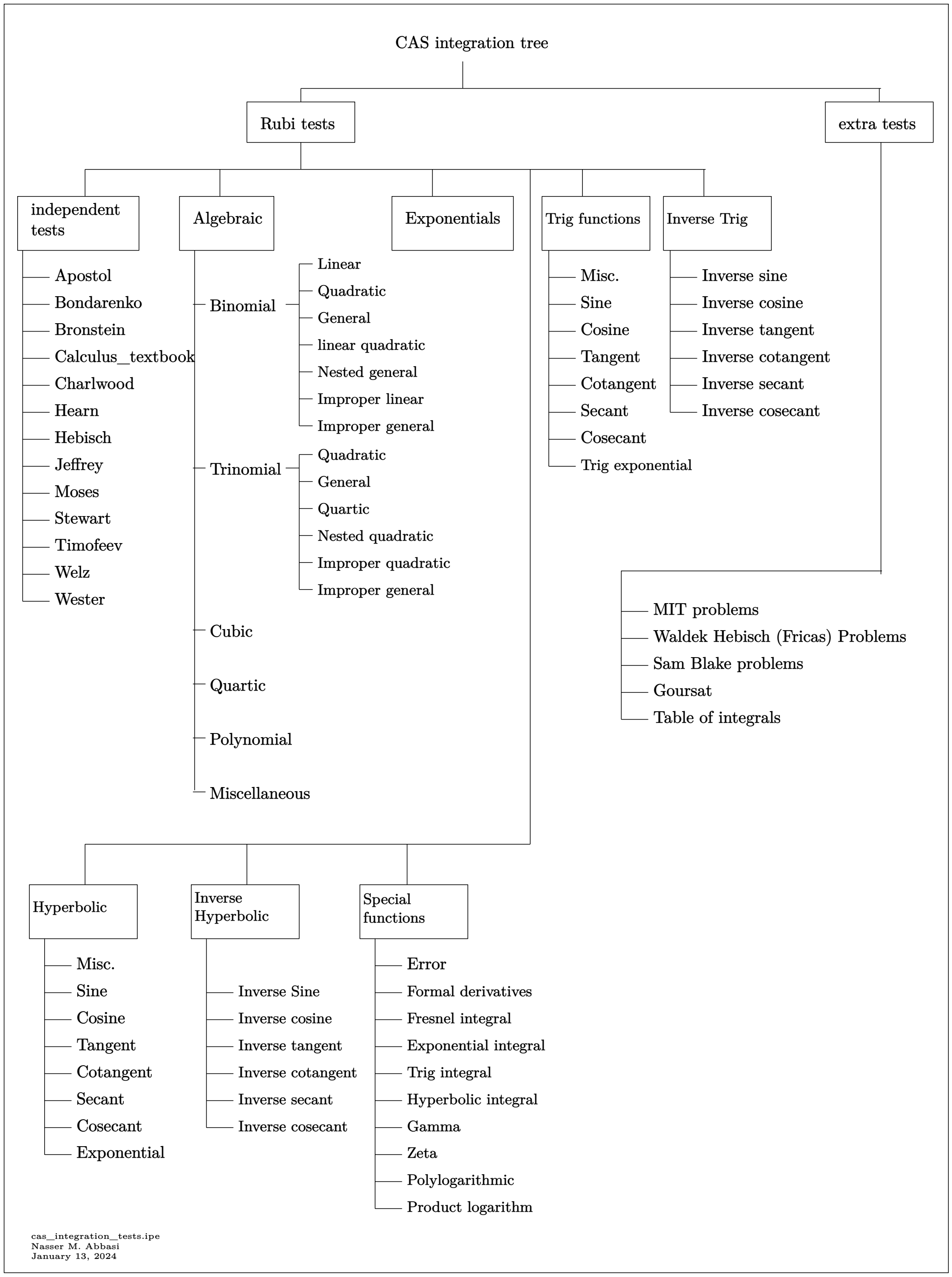
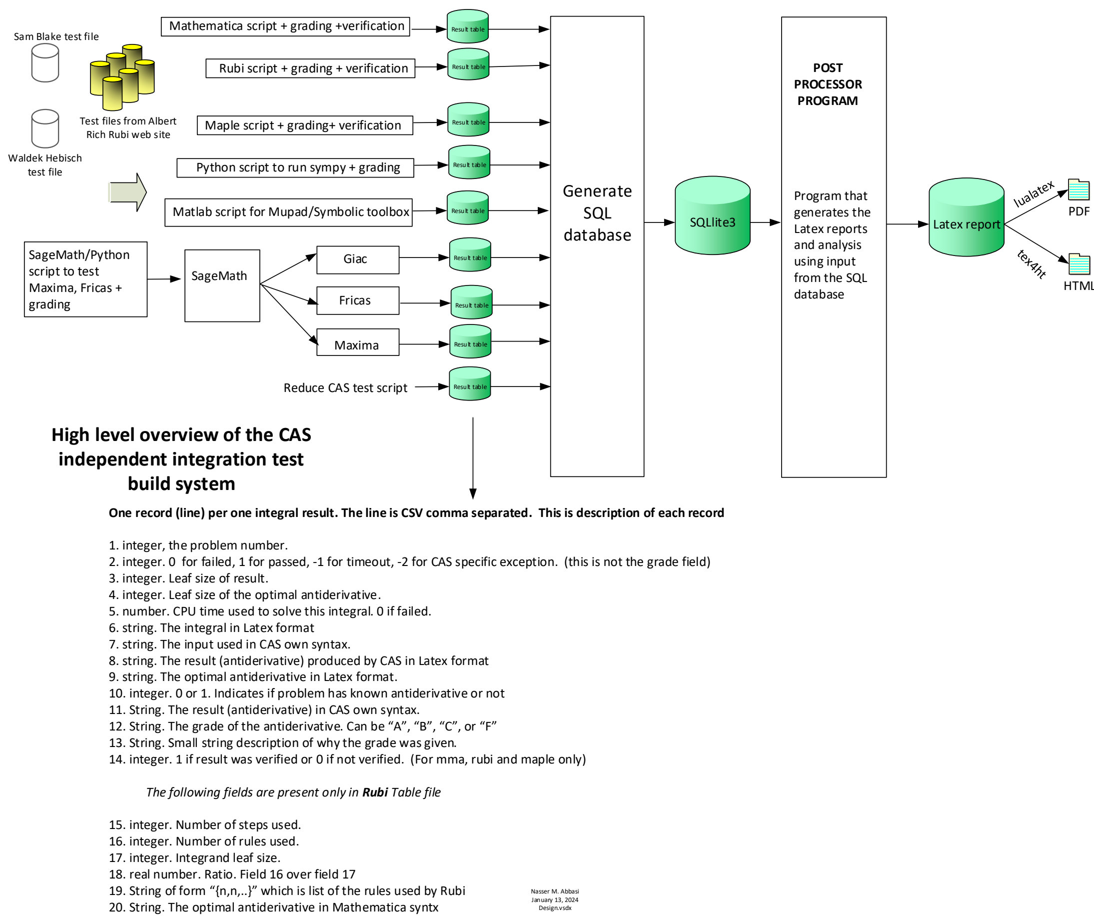


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 55, 57, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 72, 73, 74, 75, 76, 77, 78, 81, 83, 91, 93, 101, 103, 105, 106, 107, 108, 109, 110, 113, 118 }

B grade { }

C grade { 49, 51, 52, 53, 54, 56, 58, 69, 70, 71, 79, 80, 82, 84, 85, 86, 87, 88, 89, 90, 92, 94, 95, 96, 97, 98, 99, 100, 102, 104, 111, 112, 114, 115, 116, 117, 119, 120, 121, 122, 123 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 88, 90, 91, 92, 93, 94, 96, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 117, 118, 119, 121, 122, 123 }

B grade { 1, 79, 114, 115, 116, 120 }

C grade { 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 85, 87, 89, 95, 97, 99 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123 }

B grade { }

C grade { 68 }

F normal fail { 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 113 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 38, 42, 45, 49, 53, 56 }

B grade { 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 43, 44, 46, 47, 48, 50, 51, 52, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 31, 34, 37, 44, 61, 63, 64, 65, 85, 86, 98, 106, 108, 109, 110 }

B grade { 29, 30, 32, 33, 35, 36, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 62, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 105, 107, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123 }

C grade { }

F normal fail { 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 59 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 31, 34, 37, 38, 40, 44, 45, 48, 55, 56, 65, 66, 67, 69, 74, 76, 86, 88, 96, 98, 104, 110, 111, 112 }

B grade { 1, 32, 33, 35, 36, 39, 41, 42, 43, 46, 47, 49, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 63, 64, 70, 72, 73, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 89, 90, 91, 92, 93, 94, 95, 97, 99, 100, 101, 102, 103, 105, 106, 108, 109, 114, 115, 116, 117, 118, 120, 121, 122, 123 }

C grade { }

F normal fail { 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 62, 68, 71, 107, 113, 119 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123 }

C grade { }

F normal fail { }

F(-1) timedout fail { 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30 }

F(-2) exception fail { }

Sympy

A grade { 1, 61, 75, 76, 106 }

B grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 31, 32, 33, 34, 35, 36, 52, 60, 64, 66, 67, 105 }

C grade { }

F normal fail { 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 62, 63, 65, 68, 69, 70, 71, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123 }

F(-1) timedout fail { 11, 12, 17, 18, 19, 20, 27, 28, 29, 30, 59 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 63, 64, 66, 67, 69, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 111, 112, 114, 115, 116, 117, 118, 120, 121, 122, 123 }

C grade { }

F normal fail { 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 59, 62, 65, 68, 70, 71, 107, 110, 113, 119 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	37	14	13	22	19	29	13	13
N.S.	1	1.00	2.47	0.93	0.87	1.47	1.27	1.93	0.87	0.87
time (sec)	N/A	0.179	0.005	0.276	0.030	0.084	0.077	0.110	0.237	0.055

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	51	23	33	39	40	92	32	43	18
N.S.	1	1.11	0.50	0.72	0.85	0.87	2.00	0.70	0.93	0.39
time (sec)	N/A	0.264	0.017	4.577	0.040	0.091	0.160	0.107	0.240	0.081

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	79	40	61	88	90	136	88	91	43
N.S.	1	1.14	0.58	0.88	1.28	1.30	1.97	1.28	1.32	0.62
time (sec)	N/A	0.363	0.060	127.852	0.042	0.102	0.335	0.118	0.222	1.087

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	107	52	79	110	138	189	116	117	53
N.S.	1	1.16	0.57	0.86	1.20	1.50	2.05	1.26	1.27	0.58
time (sec)	N/A	0.488	0.085	0.042	0.037	0.099	0.660	0.109	0.240	1.137

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	77	40	56	88	90	136	88	91	42
N.S.	1	1.15	0.60	0.84	1.31	1.34	2.03	1.31	1.36	0.63
time (sec)	N/A	0.341	0.034	67.504	0.042	0.075	0.322	0.113	0.224	0.135

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	105	33	74	66	97	189	60	67	32
N.S.	1	1.17	0.37	0.82	0.73	1.08	2.10	0.67	0.74	0.36
time (sec)	N/A	0.465	0.062	0.046	0.046	0.093	0.643	0.115	0.264	1.104

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	133	62	92	132	197	231	144	141	65
N.S.	1	1.18	0.55	0.81	1.17	1.74	2.04	1.27	1.25	0.58
time (sec)	N/A	0.595	0.120	0.045	0.042	0.080	1.264	0.120	0.238	0.283

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	103	52	66	110	138	189	116	117	53
N.S.	1	1.17	0.59	0.75	1.25	1.57	2.15	1.32	1.33	0.60
time (sec)	N/A	0.430	0.061	0.038	0.036	0.073	0.646	0.125	0.255	1.106

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	131	62	84	132	195	231	144	141	65
N.S.	1	1.18	0.56	0.76	1.19	1.76	2.08	1.30	1.27	0.59
time (sec)	N/A	0.572	0.083	0.033	0.049	0.082	1.271	0.118	0.267	0.240

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	159	43	102	86	179	277	88	91	42
N.S.	1	1.19	0.32	0.76	0.64	1.34	2.07	0.66	0.68	0.31
time (sec)	N/A	0.715	0.109	0.053	0.042	0.086	2.467	0.123	0.236	1.128

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	111	59	0	0	997	0	0	33	0
N.S.	1	1.05	0.56	0.00	0.00	9.41	0.00	0.00	0.31	0.00
time (sec)	N/A	0.432	0.051	0.000	0.000	0.127	0.000	0.000	0.234	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	86	59	0	0	591	0	0	33	0
N.S.	1	1.06	0.73	0.00	0.00	7.30	0.00	0.00	0.41	0.00
time (sec)	N/A	0.340	0.036	0.000	0.000	0.108	0.000	0.000	0.241	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	84	59	0	0	310	0	0	62	0
N.S.	1	1.06	0.75	0.00	0.00	3.92	0.00	0.00	0.78	0.00
time (sec)	N/A	0.318	0.038	0.000	0.000	0.112	0.000	0.000	0.249	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	58	59	0	0	142	0	0	25	0
N.S.	1	1.07	1.09	0.00	0.00	2.63	0.00	0.00	0.46	0.00
time (sec)	N/A	0.223	0.025	0.000	0.000	0.099	0.000	0.000	0.228	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	58	57	0	0	144	0	0	25	0
N.S.	1	1.07	1.06	0.00	0.00	2.67	0.00	0.00	0.46	0.00
time (sec)	N/A	0.239	0.021	0.000	0.000	0.095	0.000	0.000	0.240	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	84	57	0	0	311	0	0	62	0
N.S.	1	1.06	0.72	0.00	0.00	3.94	0.00	0.00	0.78	0.00
time (sec)	N/A	0.325	0.023	0.000	0.000	0.098	0.000	0.000	0.224	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	86	59	0	0	598	0	0	33	0
N.S.	1	1.06	0.73	0.00	0.00	7.38	0.00	0.00	0.41	0.00
time (sec)	N/A	0.318	0.025	0.000	0.000	0.106	0.000	0.000	0.236	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	111	59	0	0	1001	0	0	98	0
N.S.	1	1.05	0.56	0.00	0.00	9.44	0.00	0.00	0.92	0.00
time (sec)	N/A	0.440	0.028	0.000	0.000	0.114	0.000	0.000	0.224	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	137	59	0	0	1042	0	0	19	0
N.S.	1	0.88	0.38	0.00	0.00	6.72	0.00	0.00	0.12	0.00
time (sec)	N/A	0.417	0.039	0.000	0.000	0.126	0.000	0.000	0.227	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	137	59	0	0	751	0	0	19	0
N.S.	1	0.88	0.38	0.00	0.00	4.85	0.00	0.00	0.12	0.00
time (sec)	N/A	0.450	0.041	0.000	0.000	0.117	0.000	0.000	0.257	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	192	243	59	0	0	1003	0	0	19	0
N.S.	1	1.27	0.31	0.00	0.00	5.22	0.00	0.00	0.10	0.00
time (sec)	N/A	0.483	0.037	0.000	0.000	0.137	0.000	0.000	0.232	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	215	59	0	0	727	0	0	19	0
N.S.	1	1.29	0.35	0.00	0.00	4.35	0.00	0.00	0.11	0.00
time (sec)	N/A	0.370	0.028	0.000	0.000	0.112	0.000	0.000	0.219	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	109	59	0	0	572	0	0	19	0
N.S.	1	0.85	0.46	0.00	0.00	4.47	0.00	0.00	0.15	0.00
time (sec)	N/A	0.323	0.024	0.000	0.000	0.111	0.000	0.000	0.243	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	109	59	0	0	578	0	0	19	0
N.S.	1	0.85	0.46	0.00	0.00	4.52	0.00	0.00	0.15	0.00
time (sec)	N/A	0.319	0.020	0.000	0.000	0.096	0.000	0.000	0.257	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	217	57	0	0	723	0	0	19	0
N.S.	1	1.30	0.34	0.00	0.00	4.33	0.00	0.00	0.11	0.00
time (sec)	N/A	0.362	0.019	0.000	0.000	0.108	0.000	0.000	0.237	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	192	241	57	0	0	1013	0	0	19	0
N.S.	1	1.26	0.30	0.00	0.00	5.28	0.00	0.00	0.10	0.00
time (sec)	N/A	0.474	0.022	0.000	0.000	0.116	0.000	0.000	0.231	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	137	59	0	0	749	0	0	19	0
N.S.	1	0.88	0.38	0.00	0.00	4.83	0.00	0.00	0.12	0.00
time (sec)	N/A	0.433	0.023	0.000	0.000	0.099	0.000	0.000	0.251	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	137	59	0	0	1056	0	0	19	0
N.S.	1	0.88	0.38	0.00	0.00	6.81	0.00	0.00	0.12	0.00
time (sec)	N/A	0.438	0.025	0.000	0.000	0.111	0.000	0.000	0.265	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	61	93	0	0	11	6
N.S.	1	1.00	1.00	0.00	3.81	5.81	0.00	0.00	0.69	0.38
time (sec)	N/A	0.193	0.008	0.000	0.137	0.095	0.000	0.000	0.234	1.064

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	61	93	0	0	11	0
N.S.	1	1.00	1.00	0.00	3.81	5.81	0.00	0.00	0.69	0.00
time (sec)	N/A	0.195	0.013	0.000	0.142	0.090	0.000	0.000	0.231	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	68	49	35	23	19
N.S.	1	1.00	1.00	1.05	1.00	3.58	2.58	1.84	1.21	1.00
time (sec)	N/A	0.202	0.007	10.427	0.031	0.083	0.321	0.129	0.229	0.966

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	37	39	54	373	175	638	327	129	135
N.S.	1	0.95	1.00	1.38	9.56	4.49	16.36	8.38	3.31	3.46
time (sec)	N/A	0.242	0.048	0.038	0.190	0.083	1.183	0.179	0.217	1.084

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	54	49	82	686	379	2574	722	267	255
N.S.	1	0.92	0.83	1.39	11.63	6.42	43.63	12.24	4.53	4.32
time (sec)	N/A	0.252	0.142	0.026	0.180	0.114	4.192	0.210	0.225	1.199

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	68	49	35	23	19
N.S.	1	1.00	1.00	1.05	1.00	3.58	2.58	1.84	1.21	1.00
time (sec)	N/A	0.210	0.007	4.076	0.034	0.087	0.335	0.120	0.205	0.997

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	39	44	55	293	189	648	325	128	132
N.S.	1	0.98	1.10	1.38	7.32	4.72	16.20	8.12	3.20	3.30
time (sec)	N/A	0.251	0.214	0.023	0.178	0.093	1.205	0.160	0.240	0.168

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	54	77	82	558	407	2351	721	265	254
N.S.	1	0.92	1.31	1.39	9.46	6.90	39.85	12.22	4.49	4.31
time (sec)	N/A	0.259	0.260	0.024	0.171	0.103	4.402	0.196	0.255	1.153

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	21	23	21	41	86	0	32	43	49
N.S.	1	0.91	1.00	0.91	1.78	3.74	0.00	1.39	1.87	2.13
time (sec)	N/A	0.197	0.003	0.082	0.129	0.086	0.000	0.115	0.238	0.090

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	22	21	33	54	31	0	41	52	49
N.S.	1	1.05	1.00	1.57	2.57	1.48	0.00	1.95	2.48	2.33
time (sec)	N/A	0.219	0.006	0.079	0.038	0.085	0.000	0.120	0.231	0.951

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	49	52	62	91	463	0	97	104	107
N.S.	1	1.11	1.18	1.41	2.07	10.52	0.00	2.20	2.36	2.43
time (sec)	N/A	0.214	0.014	0.135	0.120	0.128	0.000	0.124	0.240	0.980

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	31	37	51	98	93	0	67	230	131
N.S.	1	0.84	1.00	1.38	2.65	2.51	0.00	1.81	6.22	3.54
time (sec)	N/A	0.226	0.111	0.154	0.047	0.094	0.000	0.129	0.202	0.993

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	25	25	25	56	197	0	60	68	48
N.S.	1	0.89	0.89	0.89	2.00	7.04	0.00	2.14	2.43	1.71
time (sec)	N/A	0.222	0.009	0.128	0.128	0.084	0.000	0.119	0.219	0.080

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	51	31	39	64	54	0	71	83	50
N.S.	1	1.46	0.89	1.11	1.83	1.54	0.00	2.03	2.37	1.43
time (sec)	N/A	0.244	0.166	0.095	0.035	0.086	0.000	0.121	0.245	0.969

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	35	35	41	103	742	0	99	196	97
N.S.	1	0.81	0.81	0.95	2.40	17.26	0.00	2.30	4.56	2.26
time (sec)	N/A	0.246	0.037	0.144	0.123	0.091	0.000	0.147	0.224	0.970

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	32	38	33	71	290	0	55	73	77
N.S.	1	0.84	1.00	0.87	1.87	7.63	0.00	1.45	1.92	2.03
time (sec)	N/A	0.225	0.013	0.383	0.135	0.098	0.000	0.125	0.240	0.095

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	33	40	52	79	63	0	63	79	78
N.S.	1	0.87	1.05	1.37	2.08	1.66	0.00	1.66	2.08	2.05
time (sec)	N/A	0.232	0.107	0.284	0.045	0.084	0.000	0.132	0.219	1.022

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	62	78	81	116	851	0	117	163	136
N.S.	1	1.03	1.30	1.35	1.93	14.18	0.00	1.95	2.72	2.27
time (sec)	N/A	0.252	0.018	0.296	0.133	0.101	0.000	0.141	0.228	0.134

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	39	34	33	81	457	0	84	92	77
N.S.	1	0.98	0.85	0.82	2.02	11.42	0.00	2.10	2.30	1.92
time (sec)	N/A	0.235	0.022	1.176	0.123	0.113	0.000	0.120	0.228	1.020

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	21	42	21	59	113	0	44	64	53
N.S.	1	0.91	1.83	0.91	2.57	4.91	0.00	1.91	2.78	2.30
time (sec)	N/A	0.196	0.007	0.085	0.050	0.097	0.000	0.116	0.238	0.091

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	28	22	33	56	31	0	45	52	49
N.S.	1	1.27	1.00	1.50	2.55	1.41	0.00	2.05	2.36	2.23
time (sec)	N/A	0.225	0.003	0.098	0.040	0.084	0.000	0.125	0.235	0.084

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	51	85	62	108	612	0	105	202	112
N.S.	1	1.16	1.93	1.41	2.45	13.91	0.00	2.39	4.59	2.55
time (sec)	N/A	0.225	0.108	0.173	0.041	0.089	0.000	0.146	0.222	0.940

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	42	37	51	100	89	0	71	102	131
N.S.	1	1.14	1.00	1.38	2.70	2.41	0.00	1.92	2.76	3.54
time (sec)	N/A	0.222	0.016	0.230	0.043	0.090	0.000	0.147	0.242	0.975

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	28	25	23	70	203	105	63	87	49
N.S.	1	1.04	0.93	0.85	2.59	7.52	3.89	2.33	3.22	1.81
time (sec)	N/A	0.220	0.008	0.197	0.046	0.086	0.843	0.117	0.259	0.077

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	62	31	39	66	60	0	71	83	50
N.S.	1	1.72	0.86	1.08	1.83	1.67	0.00	1.97	2.31	1.39
time (sec)	N/A	0.236	0.175	0.287	0.042	0.096	0.000	0.134	0.237	0.090

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	41	35	43	120	743	0	102	253	97
N.S.	1	0.95	0.81	1.00	2.79	17.28	0.00	2.37	5.88	2.26
time (sec)	N/A	0.243	0.037	0.415	0.046	0.104	0.000	0.150	0.217	0.969

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	33	60	31	87	357	0	69	97	81
N.S.	1	0.87	1.58	0.82	2.29	9.39	0.00	1.82	2.55	2.13
time (sec)	N/A	0.224	0.092	0.646	0.047	0.092	0.000	0.122	0.253	0.964

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	38	52	79	63	0	67	79	78
N.S.	1	1.11	1.00	1.37	2.08	1.66	0.00	1.76	2.08	2.05
time (sec)	N/A	0.242	0.015	0.964	0.040	0.087	0.000	0.136	0.229	0.099

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	63	103	81	133	1077	0	123	235	140
N.S.	1	1.05	1.72	1.35	2.22	17.95	0.00	2.05	3.92	2.33
time (sec)	N/A	0.252	0.118	1.544	0.040	0.096	0.000	0.153	0.214	1.008

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	41	35	33	95	457	0	85	111	77
N.S.	1	1.05	0.90	0.85	2.44	11.72	0.00	2.18	2.85	1.97
time (sec)	N/A	0.226	0.023	1.977	0.051	0.096	0.000	0.118	0.218	0.107

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	0	54	0	17	9	31
N.S.	1	1.00	1.00	0.70	0.00	5.40	0.00	1.70	0.90	3.10
time (sec)	N/A	0.193	0.007	2.188	0.000	0.104	0.000	0.115	0.226	0.953

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	23	54	17	23	11	24
N.S.	1	1.00	1.00	1.09	2.09	4.91	1.55	2.09	1.00	2.18
time (sec)	N/A	0.188	0.003	0.101	0.033	0.066	0.098	0.106	0.219	0.064

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	84	22	27	13	13
N.S.	1	1.00	1.00	0.93	0.87	5.60	1.47	1.80	0.87	0.87
time (sec)	N/A	0.199	0.003	0.416	0.037	0.086	0.133	0.120	0.209	0.953

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	29	36	153	0	0	23	40
N.S.	1	1.00	1.00	1.81	2.25	9.56	0.00	0.00	1.44	2.50
time (sec)	N/A	0.206	0.072	4.375	0.166	0.077	0.000	0.000	0.214	0.126

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	138	0	31	65	31
N.S.	1	1.00	1.00	0.93	0.87	9.20	0.00	2.07	4.33	2.07
time (sec)	N/A	0.205	0.005	0.726	0.031	0.093	0.000	0.124	0.256	0.931

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	208	44	37	23	230
N.S.	1	1.00	1.00	0.93	0.87	13.87	2.93	2.47	1.53	15.33
time (sec)	N/A	0.204	0.004	1.287	0.035	0.073	0.248	0.143	0.239	1.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	69	0	38	19	42
N.S.	1	1.00	1.00	1.05	1.00	3.63	0.00	2.00	1.00	2.21
time (sec)	N/A	0.210	0.013	5.878	0.025	0.108	0.000	0.134	0.236	1.038

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	25	27	24	148	172	41	45	23	48
N.S.	1	0.93	1.00	0.89	5.48	6.37	1.52	1.67	0.85	1.78
time (sec)	N/A	0.206	0.095	0.313	0.041	0.073	0.182	0.131	0.217	0.946

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	29	31	26	214	345	46	45	25	251
N.S.	1	0.94	1.00	0.84	6.90	11.13	1.48	1.45	0.81	8.10
time (sec)	N/A	0.237	0.128	3.523	0.044	0.086	0.405	0.144	0.246	1.035

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	36	34	32	1385	350	270	0	0	27	95
N.S.	1	0.94	0.89	38.47	9.72	7.50	0.00	0.00	0.75	2.64
time (sec)	N/A	0.247	0.269	0.400	0.197	0.105	0.000	0.000	0.234	0.192

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	36	56	26	276	304	0	53	105	270
N.S.	1	1.16	1.81	0.84	8.90	9.81	0.00	1.71	3.39	8.71
time (sec)	N/A	0.238	0.118	5.497	0.041	0.077	0.000	0.128	0.222	1.003

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	40	29	26	352	551	0	148	18	168
N.S.	1	1.14	0.83	0.74	10.06	15.74	0.00	4.23	0.51	4.80
time (sec)	N/A	0.248	0.183	0.085	0.151	0.103	0.000	0.138	0.216	1.183

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	46	73	55	504	180	0	0	19	115
N.S.	1	1.15	1.82	1.38	12.60	4.50	0.00	0.00	0.48	2.88
time (sec)	N/A	0.248	0.631	104.882	0.148	0.110	0.000	0.000	0.236	1.138

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	43	66	269	0	76	97	82
N.S.	1	1.00	1.00	1.26	1.94	7.91	0.00	2.24	2.85	2.41
time (sec)	N/A	0.269	0.004	0.191	0.148	0.070	0.000	0.130	0.215	0.093

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	60	74	75	112	814	0	102	191	186
N.S.	1	1.09	1.35	1.36	2.04	14.80	0.00	1.85	3.47	3.38
time (sec)	N/A	0.369	0.017	0.576	0.128	0.082	0.000	0.135	0.239	0.968

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	60	55	58	110	808	0	98	186	215
N.S.	1	1.09	1.00	1.05	2.00	14.69	0.00	1.78	3.38	3.91
time (sec)	N/A	0.365	0.013	1.953	0.135	0.088	0.000	0.132	0.229	1.020

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	191	185	29	35	18	129
N.S.	1	1.00	1.00	0.86	9.10	8.81	1.38	1.67	0.86	6.14
time (sec)	N/A	0.202	0.019	0.684	0.049	0.073	0.243	0.108	0.217	1.003

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	371	634	34	35	20	520
N.S.	1	1.00	1.00	0.80	14.84	25.36	1.36	1.40	0.80	20.80
time (sec)	N/A	0.217	0.024	107.035	0.044	0.098	1.783	0.116	0.214	1.106

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	48	48	46	85	925	0	73	160	200
N.S.	1	1.26	1.26	1.21	2.24	24.34	0.00	1.92	4.21	5.26
time (sec)	N/A	0.407	0.009	3.912	0.133	0.081	0.000	0.110	0.223	0.980

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	46	36	36	85	925	0	73	160	206
N.S.	1	1.28	1.00	1.00	2.36	25.69	0.00	2.03	4.44	5.72
time (sec)	N/A	0.378	0.008	9.529	0.140	0.090	0.000	0.114	0.225	0.986

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	45	67	26	857	778	0	66	162	820
N.S.	1	1.36	2.03	0.79	25.97	23.58	0.00	2.00	4.91	24.85
time (sec)	N/A	0.221	0.011	245.963	0.052	0.086	0.000	0.116	0.237	0.992

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	15	15	12	50	60	0	41	40	30
N.S.	1	1.36	1.36	1.09	4.55	5.45	0.00	3.73	3.64	2.73
time (sec)	N/A	0.193	0.003	0.110	0.128	0.083	0.000	0.110	0.228	0.072

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	20	42	23	61	155	0	64	93	52
N.S.	1	0.87	1.83	1.00	2.65	6.74	0.00	2.78	4.04	2.26
time (sec)	N/A	0.215	0.007	0.383	0.038	0.093	0.000	0.112	0.230	0.067

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	28	36	23	88	371	0	93	215	78
N.S.	1	1.04	1.33	0.85	3.26	13.74	0.00	3.44	7.96	2.89
time (sec)	N/A	0.222	0.012	1.448	0.118	0.106	0.000	0.112	0.217	0.068

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	32	57	33	108	697	0	88	233	133
N.S.	1	0.84	1.50	0.87	2.84	18.34	0.00	2.32	6.13	3.50
time (sec)	N/A	0.227	0.100	4.340	0.041	0.083	0.000	0.117	0.214	0.954

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	41	46	33	131	1073	0	122	396	169
N.S.	1	1.02	1.15	0.82	3.28	26.82	0.00	3.05	9.90	4.22
time (sec)	N/A	0.225	0.067	11.933	0.141	0.106	0.000	0.119	0.261	0.945

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	29	29	25	43	103	0	54	57	48
N.S.	1	1.21	1.21	1.04	1.79	4.29	0.00	2.25	2.38	2.00
time (sec)	N/A	0.219	0.003	0.255	0.158	0.087	0.000	0.130	0.225	0.936

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	28	13	32	18	81	0	18	29	18
N.S.	1	1.22	0.57	1.39	0.78	3.52	0.00	0.78	1.26	0.78
time (sec)	N/A	0.233	0.007	0.970	0.047	0.085	0.000	0.110	0.218	0.943

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	60	29	47	90	511	0	102	144	107
N.S.	1	1.33	0.64	1.04	2.00	11.36	0.00	2.27	3.20	2.38
time (sec)	N/A	0.238	0.002	3.066	0.143	0.081	0.000	0.121	0.246	0.083

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	46	44	94	230	0	60	57	152
N.S.	1	1.11	1.21	1.16	2.47	6.05	0.00	1.58	1.50	4.00
time (sec)	N/A	0.235	0.134	8.779	0.050	0.070	0.000	0.124	0.245	0.075

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	91	29	60	136	1183	0	124	235	210
N.S.	1	1.38	0.44	0.91	2.06	17.92	0.00	1.88	3.56	3.18
time (sec)	N/A	0.256	0.012	21.239	0.170	0.114	0.000	0.135	0.226	0.076

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	29	34	25	91	379	0	93	218	78
N.S.	1	1.04	1.21	0.89	3.25	13.54	0.00	3.32	7.79	2.79
time (sec)	N/A	0.218	0.019	0.628	0.166	0.100	0.000	0.106	0.229	0.955

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	52	86	43	106	709	0	110	233	111
N.S.	1	1.16	1.91	0.96	2.36	15.76	0.00	2.44	5.18	2.47
time (sec)	N/A	0.228	0.006	2.178	0.062	0.106	0.000	0.128	0.249	0.089

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	41	61	43	102	774	0	96	230	96
N.S.	1	0.95	1.42	1.00	2.37	18.00	0.00	2.23	5.35	2.23
time (sec)	N/A	0.237	0.008	6.582	0.142	0.081	0.000	0.123	0.256	0.088

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	64	101	53	149	1573	0	128	365	192
N.S.	1	1.07	1.68	0.88	2.48	26.22	0.00	2.13	6.08	3.20
time (sec)	N/A	0.271	0.128	16.425	0.058	0.097	0.000	0.120	0.244	0.979

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	53	54	53	181	2103	0	171	581	187
N.S.	1	0.91	0.93	0.91	3.12	36.26	0.00	2.95	10.02	3.22
time (sec)	N/A	0.244	0.369	37.514	0.121	0.108	0.000	0.128	0.250	0.081

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	43	33	33	90	515	0	80	147	129
N.S.	1	1.16	0.89	0.89	2.43	13.92	0.00	2.16	3.97	3.49
time (sec)	N/A	0.215	0.014	1.451	0.136	0.078	0.000	0.112	0.233	0.083

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	42	45	45	90	229	0	60	57	153
N.S.	1	1.14	1.22	1.22	2.43	6.19	0.00	1.62	1.54	4.14
time (sec)	N/A	0.229	0.118	4.789	0.053	0.071	0.000	0.119	0.225	0.078

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	74	33	65	132	1176	0	124	233	187
N.S.	1	1.23	0.55	1.08	2.20	19.60	0.00	2.07	3.88	3.12
time (sec)	N/A	0.250	0.012	12.406	0.134	0.085	0.000	0.126	0.216	0.074

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	56	43	45	90	330	0	31	57	31
N.S.	1	1.06	0.81	0.85	1.70	6.23	0.00	0.58	1.08	0.58
time (sec)	N/A	0.230	0.044	28.523	0.041	0.067	0.000	0.121	0.244	0.957

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	105	33	78	178	2092	0	148	321	291
N.S.	1	1.30	0.41	0.96	2.20	25.83	0.00	1.83	3.96	3.59
time (sec)	N/A	0.260	0.014	59.798	0.140	0.089	0.000	0.124	0.247	1.001

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	41	46	33	133	1082	0	122	394	169
N.S.	1	1.05	1.18	0.85	3.41	27.74	0.00	3.13	10.10	4.33
time (sec)	N/A	0.227	0.088	3.227	0.128	0.088	0.000	0.119	0.256	0.941

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	82	123	61	155	1591	0	130	369	214
N.S.	1	1.26	1.89	0.94	2.38	24.48	0.00	2.00	5.68	3.29
time (sec)	N/A	0.249	0.136	9.058	0.046	0.103	0.000	0.121	0.235	0.082

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	53	56	61	179	2114	0	171	581	187
N.S.	1	0.91	0.97	1.05	3.09	36.45	0.00	2.95	10.02	3.22
time (sec)	N/A	0.246	0.134	22.248	0.136	0.099	0.000	0.125	0.239	0.966

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	94	139	71	195	2802	0	152	505	295
N.S.	1	1.16	1.72	0.88	2.41	34.59	0.00	1.88	6.23	3.64
time (sec)	N/A	0.273	0.134	47.116	0.040	0.097	0.000	0.127	0.233	0.967

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	65	91	70	150	2231	0	124	419	205
N.S.	1	0.94	1.32	1.01	2.17	32.33	0.00	1.80	6.07	2.97
time (sec)	N/A	0.251	0.062	94.422	0.129	0.088	0.000	0.129	0.234	0.069

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	25	56	17	25	11	24
N.S.	1	1.00	1.00	1.09	2.27	5.09	1.55	2.27	1.00	2.18
time (sec)	N/A	0.194	0.005	0.043	0.038	0.087	0.728	0.115	0.231	0.066

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	86	22	27	13	13
N.S.	1	1.00	1.00	0.93	0.87	5.73	1.47	1.80	0.87	0.87
time (sec)	N/A	0.205	0.004	0.067	0.043	0.075	1.495	0.115	0.226	0.075

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	29	53	153	0	0	23	26
N.S.	1	1.00	1.00	1.81	3.31	9.56	0.00	0.00	1.44	1.62
time (sec)	N/A	0.210	0.016	0.538	0.203	0.101	0.000	0.000	0.252	0.990

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	139	0	31	67	31
N.S.	1	1.00	1.00	0.93	0.87	9.27	0.00	2.07	4.47	2.07
time (sec)	N/A	0.206	0.004	0.072	0.029	0.076	0.000	0.132	0.224	0.955

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	208	0	37	23	231
N.S.	1	1.00	1.00	0.93	0.87	13.87	0.00	2.47	1.53	15.40
time (sec)	N/A	0.211	0.004	0.116	0.029	0.100	0.000	0.155	0.238	0.998

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	70	0	39	19	43
N.S.	1	1.00	1.00	1.05	1.00	3.50	0.00	1.95	0.95	2.15
time (sec)	N/A	0.227	0.017	1.028	0.034	0.078	0.000	0.134	0.255	1.027

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	32	27	24	148	171	0	49	23	48
N.S.	1	1.19	1.00	0.89	5.48	6.33	0.00	1.81	0.85	1.78
time (sec)	N/A	0.216	0.011	0.079	0.041	0.072	0.000	0.141	0.214	0.084

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	36	31	27	214	343	0	49	25	252
N.S.	1	1.16	1.00	0.87	6.90	11.06	0.00	1.58	0.81	8.13
time (sec)	N/A	0.226	0.018	0.175	0.052	0.086	0.000	0.154	0.230	1.054

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	35	34	0	418	256	0	0	27	95
N.S.	1	0.95	0.92	0.00	11.30	6.92	0.00	0.00	0.73	2.57
time (sec)	N/A	0.242	0.048	0.000	0.209	0.097	0.000	0.000	0.257	1.051

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	42	75	45	84	387	0	84	162	87
N.S.	1	1.24	2.21	1.32	2.47	11.38	0.00	2.47	4.76	2.56
time (sec)	N/A	0.275	0.006	0.066	0.041	0.093	0.000	0.126	0.231	0.085

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	70	113	58	129	1109	0	106	298	219
N.S.	1	1.27	2.05	1.05	2.35	20.16	0.00	1.93	5.42	3.98
time (sec)	N/A	0.372	0.138	0.190	0.048	0.114	0.000	0.133	0.214	0.991

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	70	113	74	133	1114	0	110	301	190
N.S.	1	1.27	2.05	1.35	2.42	20.25	0.00	2.00	5.47	3.45
time (sec)	N/A	0.380	0.109	0.152	0.062	0.109	0.000	0.136	0.226	0.963

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	25	27	14	149	164	0	30	64	144
N.S.	1	1.47	1.59	0.82	8.76	9.65	0.00	1.76	3.76	8.47
time (sec)	N/A	0.220	0.032	0.351	0.049	0.093	0.000	0.123	0.220	0.993

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	139	222	0	29	14	210
N.S.	1	1.00	1.00	0.82	8.18	13.06	0.00	1.71	0.82	12.35
time (sec)	N/A	0.222	0.008	0.348	0.037	0.075	0.000	0.104	0.227	1.062

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	35	30	33	368	114	0	0	11	87
N.S.	1	1.35	1.15	1.27	14.15	4.38	0.00	0.00	0.42	3.35
time (sec)	N/A	0.241	0.081	15.369	0.156	0.087	0.000	0.000	0.268	1.117

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	60	95	46	98	1260	0	71	260	214
N.S.	1	1.58	2.50	1.21	2.58	33.16	0.00	1.87	6.84	5.63
time (sec)	N/A	0.412	0.027	0.259	0.050	0.096	0.000	0.116	0.221	0.936

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	35	47	20	431	430	0	48	113	413
N.S.	1	1.40	1.88	0.80	17.24	17.20	0.00	1.92	4.52	16.52
time (sec)	N/A	0.225	0.033	1.838	0.041	0.071	0.000	0.111	0.224	0.954

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	39	29	24	191	250	0	47	24	40
N.S.	1	1.34	1.00	0.83	6.59	8.62	0.00	1.62	0.83	1.38
time (sec)	N/A	0.215	0.013	0.176	0.040	0.075	0.000	0.111	0.240	0.997

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	45	33	26	435	442	0	54	26	372
N.S.	1	1.36	1.00	0.79	13.18	13.39	0.00	1.64	0.79	11.27
time (sec)	N/A	0.226	0.010	0.880	0.038	0.089	0.000	0.113	0.220	0.949

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [120] had the largest ratio of [1.6666700000000010]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.00	13	0.308
2	A	6	6	1.11	17	0.353
3	A	9	9	1.14	17	0.529
4	A	12	12	1.16	17	0.706
5	A	8	8	1.15	17	0.471
6	A	11	11	1.17	17	0.647
7	A	14	14	1.18	17	0.824
8	A	10	10	1.17	17	0.588
9	A	13	13	1.18	17	0.765
10	A	16	16	1.19	17	0.941
11	A	10	9	1.05	21	0.429
12	A	9	8	1.06	21	0.381
13	A	8	7	1.06	21	0.333
14	A	7	6	1.07	21	0.286
15	A	6	5	1.07	21	0.238
16	A	9	8	1.06	21	0.381
17	A	8	7	1.06	21	0.333
18	A	11	10	1.05	21	0.476
19	A	13	12	0.88	21	0.571
20	A	12	11	0.88	21	0.524
21	A	13	12	1.27	21	0.571

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	12	11	1.29	21	0.524
23	A	11	10	0.85	21	0.476
24	A	10	9	0.85	21	0.429
25	A	11	10	1.30	21	0.476
26	A	14	13	1.26	21	0.619
27	A	13	12	0.88	21	0.571
28	A	12	11	0.88	21	0.524
29	A	2	2	1.00	13	0.154
30	A	2	2	1.00	13	0.154
31	A	4	3	1.00	15	0.200
32	A	5	4	0.95	17	0.235
33	A	5	4	0.92	17	0.235
34	A	5	4	1.00	15	0.267
35	A	6	5	0.98	17	0.294
36	A	6	5	0.92	17	0.294
37	A	7	6	0.91	13	0.462
38	A	6	5	1.05	15	0.333
39	A	6	5	1.11	15	0.333
40	A	6	5	0.84	15	0.333
41	A	6	5	0.89	15	0.333
42	A	6	5	1.46	17	0.294
43	A	7	6	0.81	17	0.353
44	A	5	4	0.84	15	0.267
45	A	6	5	0.87	17	0.294
46	A	8	7	1.03	17	0.412
47	A	7	6	0.98	15	0.400
48	A	6	5	0.91	13	0.385
49	C	6	5	1.27	15	0.333
50	A	7	6	1.16	15	0.400
51	C	5	4	1.14	15	0.267
52	C	6	5	1.04	15	0.333
53	C	7	6	1.72	17	0.353

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	C	7	6	0.95	17	0.353
55	A	6	5	0.87	15	0.333
56	C	6	5	1.11	17	0.294
57	A	7	6	1.05	17	0.353
58	C	7	6	1.05	15	0.400
59	A	4	3	1.00	9	0.333
60	A	5	4	1.00	13	0.308
61	A	5	4	1.00	15	0.267
62	A	5	4	1.00	17	0.235
63	A	5	4	1.00	17	0.235
64	A	5	4	1.00	17	0.235
65	A	4	3	1.00	17	0.176
66	A	5	4	0.93	15	0.267
67	A	7	6	0.94	17	0.353
68	A	7	6	0.94	19	0.316
69	C	6	5	1.16	17	0.294
70	C	5	4	1.14	19	0.211
71	C	5	4	1.15	17	0.235
72	A	5	5	1.00	15	0.333
73	A	8	8	1.09	15	0.533
74	A	7	7	1.09	17	0.412
75	A	6	5	1.00	7	0.714
76	A	6	5	1.00	9	0.556
77	A	10	10	1.26	9	1.111
78	A	9	9	1.28	9	1.000
79	C	6	5	1.36	9	0.556
80	C	5	4	1.36	13	0.308
81	A	7	6	0.87	15	0.400
82	C	6	5	1.04	15	0.333
83	A	7	6	0.84	15	0.400
84	C	7	6	1.02	15	0.400
85	C	7	6	1.21	15	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	C	6	5	1.22	17	0.294
87	C	7	6	1.33	17	0.353
88	C	6	5	1.11	17	0.294
89	C	9	8	1.38	17	0.471
90	C	6	5	1.04	15	0.333
91	A	7	6	1.16	17	0.353
92	C	7	6	0.95	17	0.353
93	A	7	6	1.07	17	0.353
94	C	7	6	0.91	17	0.353
95	C	6	5	1.16	15	0.333
96	C	5	4	1.14	17	0.235
97	C	6	5	1.23	17	0.294
98	C	5	4	1.06	17	0.235
99	C	8	7	1.30	17	0.412
100	C	7	6	1.05	15	0.400
101	A	9	8	1.26	17	0.471
102	C	7	6	0.91	17	0.353
103	A	9	8	1.16	17	0.471
104	C	7	6	0.94	17	0.353
105	A	4	3	1.00	13	0.231
106	A	5	4	1.00	15	0.267
107	A	4	3	1.00	17	0.176
108	A	4	3	1.00	17	0.176
109	A	5	4	1.00	17	0.235
110	A	5	4	1.00	17	0.235
111	C	5	4	1.19	15	0.267
112	C	6	5	1.16	17	0.294
113	A	6	5	0.95	19	0.263
114	C	7	7	1.24	15	0.467
115	C	11	11	1.27	17	0.647
116	C	11	11	1.27	15	0.733
117	C	6	5	1.47	9	0.556

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	7	6	1.00	9	0.667
119	C	5	4	1.35	9	0.444
120	C	15	15	1.58	9	1.667
121	C	6	5	1.40	9	0.556
122	C	5	4	1.34	11	0.364
123	C	6	5	1.36	9	0.556

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \cosh(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \cosh(a+bx)\sinh(a+bx)\,dx=\frac{\sinh^2(a+bx)}{2b}$$

output
```
1/2*sinh(b*x+a)^2/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(15) = 30.

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.47

$$\int \cosh(a+bx)\sinh(a+bx)\,dx=\frac{1}{2}\left(\frac{\cosh(2a)\cosh(2bx)}{2b}+\frac{\sinh(2a)\sinh(2bx)}{2b}\right)$$

input
```
Integrate[Cosh[a + b*x]*Sinh[a + b*x],x]
```

output
```
((Cosh[2*a]*Cosh[2*b*x])/(2*b) + (Sinh[2*a]*Sinh[2*b*x])/(2*b))/2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 26, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ia+ibx)\cos(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \cos(ia+ibx)\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3044}$$

$$-\frac{\int i\sinh(a+bx)d(i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sinh^2(a+bx)}{2b}$$

input
```
Int[Cosh[a + b*x]*Sinh[a + b*x],x]
```

output
```
Sinh[a + b*x]^2/(2*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\cosh(bx+a)^2}{2b}$	14
default	$\frac{\cosh(bx+a)^2}{2b}$	14
orering	$\frac{b\sinh(bx+a)^2+\cosh(bx+a)^2 b}{4b^2}$	27
risch	$\frac{e^{2bx+2a}}{8b}+\frac{e^{-2bx-2a}}{8b}$	30

input
```
int(cosh(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*cosh(b*x+a)^2/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \frac{\cosh{(bx+a)}^2+\sinh{(bx+a)}^2}{4\,b}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(cosh(b*x + a)^2 + sinh(b*x + a)^2)/b
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \begin{cases} \frac{\cosh^2{(a+bx)}}{2b} & \text{for } b \neq 0 \\ x\sinh{(a)}\cosh{(a)} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a),x)
```

output
```
Piecewise((cosh(a + b*x)**2/(2*b), Ne(b, 0)), (x*sinh(a)*cosh(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh(a+bx)\sinh(a+bx)\,dx = \frac{\cosh{(bx+a)}^2}{2\,b}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*cosh(b*x + a)^2/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(13) = 26$.

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \cosh(a + bx) \sinh(a + bx)\, dx = \frac{e^{(2\,bx+2\,a)}}{8\,b} + \frac{e^{(-2\,bx-2\,a)}}{8\,b}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh(a + bx) \sinh(a + bx)\, dx = \frac{\cosh(a + b\,x)^2}{2\,b}$$

input
```
int(cosh(a + b*x)*sinh(a + b*x),x)
```

output
```
cosh(a + b*x)^2/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cosh(a + bx) \sinh(a + bx)\, dx = \frac{\cosh\left(bx + a\right)^2}{2b}$$

input
```
int(cosh(b*x+a)*sinh(b*x+a),x)
```

output
```
cosh(a + b*x)**2/(2*b)
```

3.2 $\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{x}{8}-\frac{\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{\cosh^3(a+bx)\sinh(a+bx)}{4b}$$

output
```
-1/8*x-1/8*cosh(b*x+a)*sinh(b*x+a)/b+1/4*cosh(b*x+a)^3*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{-4(a+bx)+\sinh(4(a+bx))}{32b}$$

input
```
Integrate[Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

output
```
(-4*(a + b*x) + Sinh[4*(a + b*x)])/(32*b)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 25, 3048, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(ia+ibx)^2\left(-\cos(ia+ibx)^2\right)dx$$

$$\Big\downarrow \ 25$$

$$-\int \cos(ia+ibx)^2\sin(ia+ibx)^2dx$$

$$\Big\downarrow \ 3048$$

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}-\frac{1}{4}\int \cosh^2(a+bx)dx$$

$$\Big\downarrow \ 3042$$

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}-\frac{1}{4}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \ 3115$$

$$\frac{1}{4}\left(-\frac{\int 1dx}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$$\Big\downarrow \ 24$$

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{1}{4}\left(-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{x}{2}\right)$$

input
```
Int[Cosh[a + b*x]^2*Sinh[a + b*x]^2,x]
```

output
```
(Cosh[a + b*x]^3*Sinh[a + b*x])/(4*b) + (-1/2*x - (Cosh[a + b*x]*Sinh[a +
b*x])/(2*b))/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{x}{8}+\frac{{\mathrm e}^{4 b x +4 a}}{64 b}-\frac{{\mathrm e}^{-4 b x -4 a}}{64 b}$
derivativedivides	$\frac{\frac{\sinh\left(b x +a \right) \cosh\left(b x +a \right)^{3}}{4}-\frac{\cosh\left(b x +a \right) \sinh\left(b x +a \right)}{8}-\frac{b x}{8}-\frac{a}{8}}{b}$
default	$\frac{\frac{\sinh\left(b x +a \right) \cosh\left(b x +a \right)^{3}}{4}-\frac{\cosh\left(b x +a \right) \sinh\left(b x +a \right)}{8}-\frac{b x}{8}-\frac{a}{8}}{b}$
orering	$x \cosh\left(b x +a \right)^{2} \sinh\left(b x +a \right)^{2}+\frac{2 \cosh\left(b x +a \right) \sinh\left(b x +a \right)^{3} b +2 \cosh\left(b x +a \right)^{3} \sinh\left(b x +a \right) b}{16 b^{2}}-\frac{x\left(2 \sinh(b\right.}{}$

input
```
int(cosh(b*x+a)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*x+1/64/b*exp(4*b*x+4*a)-1/64/b*exp(-4*b*x-4*a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{\cosh(bx+a)^3\sinh(bx+a)+\cosh(bx+a)\sinh(bx+a)^3-bx}{8\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(cosh(b*x + a)^3*sinh(b*x + a) + cosh(b*x + a)*sinh(b*x + a)^3 - b*x)/
b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(37) = 74.

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.00

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \begin{cases} -\frac{x\sinh^4(a+bx)}{8}+\frac{x\sinh^2(a+bx)\cosh^2(a+bx)}{4}-\frac{x\cosh^4(a+bx)}{8}+\frac{\sinh^3(a+bx)\cosh(a+bx)}{8b}+\frac{\sinh(a+bx)\cosh^3(a+bx)}{8b} & \text{for } b \\ x\sinh^2(a)\cosh^2(a) & \text{othe} \end{cases}$$

input
```
integrate(cosh(b*x+a)**2*sinh(b*x+a)**2,x)
```

output
```
Piecewise((-x*sinh(a + b*x)**4/8 + x*sinh(a + b*x)**2*cosh(a + b*x)**2/4 -
 x*cosh(a + b*x)**4/8 + sinh(a + b*x)**3*cosh(a + b*x)/(8*b) + sinh(a + b*
x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*sinh(a)**2*cosh(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{bx+a}{8\,b} + \frac{e^{(4\,bx+4\,a)}}{64\,b} - \frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/8*(b*x + a)/b + 1/64*e^(4*b*x + 4*a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = -\frac{1}{8}\,x + \frac{e^{(4\,bx+4\,a)}}{64\,b} - \frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/8*x + 1/64*e^(4*b*x + 4*a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{\sinh(4\,a+4\,b\,x)}{32\,b} - \frac{x}{8}$$

input
```
int(cosh(a + b*x)^2*sinh(a + b*x)^2,x)
```

output
```
sinh(4*a + 4*b*x)/(32*b) - x/8
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \cosh^2(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{8bx+8a} - 8e^{4bx+4a}bx - 1}{64e^{4bx+4a}b}$$

input
```
int(cosh(b*x+a)^2*sinh(b*x+a)^2,x)
```

output
```
(e**(8*a + 8*b*x) - 8*e**(4*a + 4*b*x)*b*x - 1)/(64*e**(4*a + 4*b*x)*b)
```

3.3 $\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx = \frac{x}{16}+\frac{\cosh(a+bx)\sinh(a+bx)}{16b} - \frac{\cosh^3(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh^3(a+bx)\sinh^3(a+bx)}{6b}$$

output
```
1/16*x+1/16*cosh(b*x+a)*sinh(b*x+a)/b-1/8*cosh(b*x+a)^3*sinh(b*x+a)/b+1/6*
cosh(b*x+a)^3*sinh(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.58

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx = \frac{12bx-3\sinh(2(a+bx))-3\sinh(4(a+bx))+\sinh(6(a+bx))}{192b}$$

input
```
Integrate[Cosh[a + b*x]^2*Sinh[a + b*x]^4,x]
```

output
```
(12*b*x - 3*Sinh[2*(a + b*x)] - 3*Sinh[4*(a + b*x)] + Sinh[6*(a + b*x)])/(
192*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {3042, 3048, 25, 3042, 25, 3048, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)^4\cos(ia+ibx)^2dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{1}{2}\int -\cosh^2(a+bx)\sinh^2(a+bx)dx+\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}-\frac{1}{2}\int \cosh^2(a+bx)\sinh^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}-\frac{1}{2}\int -\cos(ia+ibx)^2\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}+\frac{1}{2}\int \cos(ia+ibx)^2\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \cosh^2(a+bx)dx-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)+\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}+\frac{1}{2}\left(-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{1}{4}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx\right)$$

$$\Big\downarrow \; 3115$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)+\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}$$

$$\Big\downarrow \; 24$$

$$\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}+\frac{1}{2}\left(\frac{1}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)$$

input
```
Int[Cosh[a + b*x]^2*Sinh[a + b*x]^4,x]
```

output
```
(Cosh[a + b*x]^3*Sinh[a + b*x]^3)/(6*b) + (-1/4*(Cosh[a + b*x]^3*Sinh[a +
b*x])/b + (x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b))/4)/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 127.85 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{\frac{\sinh(bx+a)^3\cosh(bx+a)^3}{6}-\frac{\sinh(bx+a)\cosh(bx+a)^3}{8}+\frac{\cosh(bx+a)\sinh(bx+a)}{16}+\frac{bx}{16}+\frac{a}{16}}{b}$
default	$\frac{\frac{\sinh(bx+a)^3\cosh(bx+a)^3}{6}-\frac{\sinh(bx+a)\cosh(bx+a)^3}{8}+\frac{\cosh(bx+a)\sinh(bx+a)}{16}+\frac{bx}{16}+\frac{a}{16}}{b}$
risch	$\frac{x}{16}+\frac{e^{6bx+6a}}{384b}-\frac{e^{4bx+4a}}{128b}-\frac{e^{2bx+2a}}{128b}+\frac{e^{-2bx-2a}}{128b}+\frac{e^{-4bx-4a}}{128b}-\frac{e^{-6bx-6a}}{384b}$
orering	$x\cosh(bx+a)^2\sinh(bx+a)^4+\frac{\frac{49\cosh(bx+a)\sinh(bx+a)^5 b}{72}+\frac{49\cosh(bx+a)^3\sinh(bx+a)^3 b}{36}}{b^2}-$ $49x\left(2b^2 \sinh\right.$

input
```
int(cosh(b*x+a)^2*sinh(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/6*sinh(b*x+a)^3*cosh(b*x+a)^3-1/8*sinh(b*x+a)*cosh(b*x+a)^3+1/16*co
sh(b*x+a)*sinh(b*x+a)+1/16*b*x+1/16*a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.30

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx$$
$$= \frac{3\cosh(bx+a)\sinh(bx+a)^5 + 2\left(5\cosh(bx+a)^3 - 3\cosh(bx+a)\right)\sinh(bx+a)^3 + 6\,bx + 3\,(\cosh(b}{96\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/96*(3*cosh(b*x + a)*sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^3 - 3*cosh(b*x
+ a))*sinh(b*x + a)^3 + 6*b*x + 3*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - c
osh(b*x + a))*sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(58) = 116.

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.97

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx$$
$$= \begin{cases} -\frac{x\sinh^6(a+bx)}{16} + \frac{3x\sinh^4(a+bx)\cosh^2(a+bx)}{16} - \frac{3x\sinh^2(a+bx)\cosh^4(a+bx)}{16} + \frac{x\cosh^6(a+bx)}{16} + \frac{\sinh^5(a+bx)\cosh(a+bx)}{16b} + \\ x\sinh^4(a)\cosh^2(a) \end{cases}$$

input
```
integrate(cosh(b*x+a)**2*sinh(b*x+a)**4,x)
```

output
```
Piecewise((-x*sinh(a + b*x)**6/16 + 3*x*sinh(a + b*x)**4*cosh(a + b*x)**2/
16 - 3*x*sinh(a + b*x)**2*cosh(a + b*x)**4/16 + x*cosh(a + b*x)**6/16 + si
nh(a + b*x)**5*cosh(a + b*x)/(16*b) + sinh(a + b*x)**3*cosh(a + b*x)**3/(6
*b) - sinh(a + b*x)*cosh(a + b*x)**5/(16*b), Ne(b, 0)), (x*sinh(a)**4*cosh
(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx = -\frac{\left(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}-1\right)e^{(6\,bx+6\,a)}}{384\,b} + \frac{bx+a}{16\,b} + \frac{3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}-e^{(-6\,bx-6\,a)}}{384\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/384*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) - 1)*e^(6*b*x + 6*a)/b + 1
/16*(b*x + a)/b + 1/384*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) - e^(-6*b
*x - 6*a))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx = \frac{1}{16}\,x + \frac{e^{(6\,bx+6\,a)}}{384\,b} - \frac{e^{(4\,bx+4\,a)}}{128\,b} - \frac{e^{(2\,bx+2\,a)}}{128\,b} + \frac{e^{(-2\,bx-2\,a)}}{128\,b} + \frac{e^{(-4\,bx-4\,a)}}{128\,b} - \frac{e^{(-6\,bx-6\,a)}}{384\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^4,x, algorithm="giac")
```

output
```
1/16*x + 1/384*e^(6*b*x + 6*a)/b - 1/128*e^(4*b*x + 4*a)/b - 1/128*e^(2*b*
x + 2*a)/b + 1/128*e^(-2*b*x - 2*a)/b + 1/128*e^(-4*b*x - 4*a)/b - 1/384*e
^(-6*b*x - 6*a)/b
```

Mupad [B] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.62

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx = \frac{x}{16} - \frac{\frac{\sinh(2\,a+2\,b\,x)}{64} + \frac{\sinh(4\,a+4\,b\,x)}{64} - \frac{\sinh(6\,a+6\,b\,x)}{192}}{b}$$

input
```
int(cosh(a + b*x)^2*sinh(a + b*x)^4,x)
```

output
```
x/16 - (sinh(2*a + 2*b*x)/64 + sinh(4*a + 4*b*x)/64 - sinh(6*a + 6*b*x)/19
2)/b
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.32

$$\int \cosh^2(a+bx)\sinh^4(a+bx)\,dx$$
$$= \frac{e^{12bx+12a} - 3e^{10bx+10a} - 3e^{8bx+8a} + 24e^{6bx+6a}bx + 3e^{4bx+4a} + 3e^{2bx+2a} - 1}{384e^{6bx+6a}b}$$

input
```
int(cosh(b*x+a)^2*sinh(b*x+a)^4,x)
```

output
```
(e**(12*a + 12*b*x) - 3*e**(10*a + 10*b*x) - 3*e**(8*a + 8*b*x) + 24*e**(6
*a + 6*b*x)*b*x + 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) - 1)/(384*e**(6*
a + 6*b*x)*b)
```

3.4 $\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 92

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx = -\frac{5x}{128}-\frac{5\cosh(a+bx)\sinh(a+bx)}{128b}+\frac{5\cosh^3(a+bx)\sinh(a+bx)}{64b}-\frac{5\cosh^3(a+bx)\sinh^3(a+bx)}{48b}+\frac{\cosh^3(a+bx)\sinh^5(a+bx)}{8b}$$

output
```
-5/128*x-5/128*cosh(b*x+a)*sinh(b*x+a)/b+5/64*cosh(b*x+a)^3*sinh(b*x+a)/b-
5/48*cosh(b*x+a)^3*sinh(b*x+a)^3/b+1/8*cosh(b*x+a)^3*sinh(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.57

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx = \frac{-120bx+48\sinh(2(a+bx))+24\sinh(4(a+bx))-16\sinh(6(a+bx))+3\sinh(8(a+bx))}{3072b}$$

input
```
Integrate[Cosh[a + b*x]^2*Sinh[a + b*x]^6,x]
```

output
```
(-120*b*x + 48*Sinh[2*(a + b*x)] + 24*Sinh[4*(a + b*x)] - 16*Sinh[6*(a + b
*x)] + 3*Sinh[8*(a + b*x)])/(3072*b)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {3042, 25, 3048, 3042, 3048, 25, 3042, 25, 3048, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^6(a+bx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)^6\left(-\cos(ia+ibx)^2\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \cos(ia+ibx)^2\sin(ia+ibx)^6dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5}{8}\int \cosh^2(a+bx)\sinh^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5}{8}\int \cos(ia+ibx)^2\sin(ia+ibx)^4dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5}{8}\left(\frac{1}{2}\int -\cosh^2(a+bx)\sinh^2(a+bx)dx+\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5}{8}\left(\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}-\frac{1}{2}\int\cosh^2(a+bx)\sinh^2(a+bx)dx\right)$$

$\Big\downarrow$ 3042

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5}{8}\left(\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}-\frac{1}{2}\int-\cos(ia+ibx)^2\sin(ia+ibx)^2dx\right)$$

$\Big\downarrow$ 25

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5}{8}\left(\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}+\frac{1}{2}\int\cos(ia+ibx)^2\sin(ia+ibx)^2dx\right)$$

$\Big\downarrow$ 3048

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\int\cosh^2(a+bx)dx-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)+\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}\right)$$

$\Big\downarrow$ 3042

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5}{8}\left(\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}+\frac{1}{2}\left(-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{1}{4}\int\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx\right)\right)$$

$\Big\downarrow$ 3115

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)+\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}\right)$$

$\Big\downarrow$ 24

$$\frac{\sinh^5(a+bx)\cosh^3(a+bx)}{8b}-\frac{5}{8}\left(\frac{\sinh^3(a+bx)\cosh^3(a+bx)}{6b}+\frac{1}{2}\left(\frac{1}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)\right)$$

input
```
Int[Cosh[a + b*x]^2*Sinh[a + b*x]^6,x]
```

output
```
(Cosh[a + b*x]^3*Sinh[a + b*x]^5)/(8*b) - (5*((Cosh[a + b*x]^3*Sinh[a + b*
x]^3)/(6*b) + (-1/4*(Cosh[a + b*x]^3*Sinh[a + b*x])/b + (x/2 + (Cosh[a + b
*x]*Sinh[a + b*x])/(2*b))/4)/2))/8
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.86

$$\frac{\frac{\sinh(bx+a)^5\cosh(bx+a)^3}{8}-\frac{5\sinh(bx+a)^3\cosh(bx+a)^3}{48}+\frac{5\sinh(bx+a)\cosh(bx+a)^3}{64}-\frac{5\cosh(bx+a)\sinh(bx+a)}{128}-\frac{5bx}{128}-\frac{5a}{128}}{b}$$

input
```
int(cosh(b*x+a)^2*sinh(b*x+a)^6,x)
```

output
```
1/b*(1/8*sinh(b*x+a)^5*cosh(b*x+a)^3-5/48*sinh(b*x+a)^3*cosh(b*x+a)^3+5/64
*sinh(b*x+a)*cosh(b*x+a)^3-5/128*cosh(b*x+a)*sinh(b*x+a)-5/128*b*x-5/128*a
)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.50

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx$$
$$= \frac{3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^7+3\left(7\,\cosh\left(bx+a\right)^3-4\,\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)^5+\left(21\,\cosh\left(bx+a\right.\right.}{}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^6,x, algorithm="fricas")
```

output
```
1/384*(3*cosh(b*x + a)*sinh(b*x + a)^7 + 3*(7*cosh(b*x + a)^3 - 4*cosh(b*x
 + a))*sinh(b*x + a)^5 + (21*cosh(b*x + a)^5 - 40*cosh(b*x + a)^3 + 12*cos
h(b*x + a))*sinh(b*x + a)^3 - 15*b*x + 3*(cosh(b*x + a)^7 - 4*cosh(b*x + a
)^5 + 4*cosh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(85) = 170$.

Time = 0.66 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.05

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx$$
$$= \begin{cases} -\frac{5x\sinh^8\left(a+bx\right)}{128}+\frac{5x\sinh^6\left(a+bx\right)\cosh^2\left(a+bx\right)}{32}-\frac{15x\sinh^4\left(a+bx\right)\cosh^4\left(a+bx\right)}{64}+\frac{5x\sinh^2\left(a+bx\right)\cosh^6\left(a+bx\right)}{32}-\frac{5x\cosh^8\left(a+\right.}{128} \\ x\sinh^6\left(a\right)\cosh^2\left(a\right) \end{cases}$$

input
```
integrate(cosh(b*x+a)**2*sinh(b*x+a)**6,x)
```

output
```
Piecewise((-5*x*sinh(a + b*x)**8/128 + 5*x*sinh(a + b*x)**6*cosh(a + b*x)*
*2/32 - 15*x*sinh(a + b*x)**4*cosh(a + b*x)**4/64 + 5*x*sinh(a + b*x)**2*c
osh(a + b*x)**6/32 - 5*x*cosh(a + b*x)**8/128 + 5*sinh(a + b*x)**7*cosh(a
+ b*x)/(128*b) + 73*sinh(a + b*x)**5*cosh(a + b*x)**3/(384*b) - 55*sinh(a
+ b*x)**3*cosh(a + b*x)**5/(384*b) + 5*sinh(a + b*x)*cosh(a + b*x)**7/(128
*b), Ne(b, 0)), (x*sinh(a)**6*cosh(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.20

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx = -\frac{\left(16\,e^{(-2\,bx-2\,a)}-24\,e^{(-4\,bx-4\,a)}-48\,e^{(-6\,bx-6\,a)}-3\right)e^{(8\,bx+8\,a)}}{6144\,b}-\frac{5\,(bx+a)}{128\,b}-\frac{48\,e^{(-2\,bx-2\,a)}+24\,e^{(-4\,bx-4\,a)}-16\,e^{(-6\,bx-6\,a)}+3\,e^{(-8\,bx-8\,a)}}{6144\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^6,x, algorithm="maxima")
```

output
```
-1/6144*(16*e^(-2*b*x - 2*a) - 24*e^(-4*b*x - 4*a) - 48*e^(-6*b*x - 6*a) -
 3)*e^(8*b*x + 8*a)/b - 5/128*(b*x + a)/b - 1/6144*(48*e^(-2*b*x - 2*a) +
24*e^(-4*b*x - 4*a) - 16*e^(-6*b*x - 6*a) + 3*e^(-8*b*x - 8*a))/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.26

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx = -\frac{5}{128}\,x+\frac{e^{(8\,bx+8\,a)}}{2048\,b}-\frac{e^{(6\,bx+6\,a)}}{384\,b}+\frac{e^{(4\,bx+4\,a)}}{256\,b}+\frac{e^{(2\,bx+2\,a)}}{128\,b}-\frac{e^{(-2\,bx-2\,a)}}{128\,b}-\frac{e^{(-4\,bx-4\,a)}}{256\,b}+\frac{e^{(-6\,bx-6\,a)}}{384\,b}-\frac{e^{(-8\,bx-8\,a)}}{2048\,b}$$

input
```
integrate(cosh(b*x+a)^2*sinh(b*x+a)^6,x, algorithm="giac")
```

output
```
-5/128*x + 1/2048*e^(8*b*x + 8*a)/b - 1/384*e^(6*b*x + 6*a)/b + 1/256*e^(4
*b*x + 4*a)/b + 1/128*e^(2*b*x + 2*a)/b - 1/128*e^(-2*b*x - 2*a)/b - 1/256
*e^(-4*b*x - 4*a)/b + 1/384*e^(-6*b*x - 6*a)/b - 1/2048*e^(-8*b*x - 8*a)/b
```

Mupad [B] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.58

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx = \frac{\frac{\sinh(2\,a+2\,b\,x)}{64}+\frac{\sinh(4\,a+4\,b\,x)}{128}-\frac{\sinh(6\,a+6\,b\,x)}{192}+\frac{\sinh(8\,a+8\,b\,x)}{1024}}{b}-\frac{5\,x}{128}$$

input
```
int(cosh(a + b*x)^2*sinh(a + b*x)^6,x)
```

output
```
(sinh(2*a + 2*b*x)/64 + sinh(4*a + 4*b*x)/128 - sinh(6*a + 6*b*x)/192 + si
nh(8*a + 8*b*x)/1024)/b - (5*x)/128
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.27

$$\int \cosh^2(a+bx)\sinh^6(a+bx)\,dx = \frac{3e^{16bx+16a}-16e^{14bx+14a}+24e^{12bx+12a}+48e^{10bx+10a}-240e^{8bx+8a}bx-48e^{6bx+6a}-24e^{4bx+4a}+16e^{2bx+2a}}{6144e^{8bx+8a}b}$$

input
```
int(cosh(b*x+a)^2*sinh(b*x+a)^6,x)
```

output
```
(3*e**(16*a + 16*b*x) - 16*e**(14*a + 14*b*x) + 24*e**(12*a + 12*b*x) + 48
*e**(10*a + 10*b*x) - 240*e**(8*a + 8*b*x)*b*x - 48*e**(6*a + 6*b*x) - 24*
e**(4*a + 4*b*x) + 16*e**(2*a + 2*b*x) - 3)/(6144*e**(8*a + 8*b*x)*b)
```

3.5 $\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx = -\frac{x}{16}-\frac{\cosh(a+bx)\sinh(a+bx)}{16b}-\frac{\cosh^3(a+bx)\sinh(a+bx)}{24b}+\frac{\cosh^5(a+bx)\sinh(a+bx)}{6b}$$

output
```
-1/16*x-1/16*cosh(b*x+a)*sinh(b*x+a)/b-1/24*cosh(b*x+a)^3*sinh(b*x+a)/b+1/
6*cosh(b*x+a)^5*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.60

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx = \frac{-12bx-3\sinh(2(a+bx))+3\sinh(4(a+bx))+\sinh(6(a+bx))}{192b}$$

input
```
Integrate[Cosh[a + b*x]^4*Sinh[a + b*x]^2,x]
```

output
```
(-12*b*x - 3*Sinh[2*(a + b*x)] + 3*Sinh[4*(a + b*x)] + Sinh[6*(a + b*x)])/
(192*b)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {3042, 25, 3048, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)\cosh^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)^2\left(-\cos(ia+ibx)^4\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \cos(ia+ibx)^4\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}-\frac{1}{6}\int \cosh^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}-\frac{1}{6}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{6}\left(-\frac{3}{4}\int \cosh^2(a+bx)dx-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)+\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{1}{6}\left(-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}-\frac{3}{4}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx\right)$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{6}\left(-\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)+\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{1}{6}\left(-\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}-\frac{3}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)\right)$$

input
```
Int[Cosh[a + b*x]^4*Sinh[a + b*x]^2,x]
```

output
```
(Cosh[a + b*x]^5*Sinh[a + b*x])/(6*b) + (-1/4*(Cosh[a + b*x]^3*Sinh[a + b*
x])/b - (3*(x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)))/4)/6
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 67.50 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{\frac{\sinh(bx+a)\cosh(bx+a)^5}{6}-\frac{\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)}{6}-\frac{bx}{16}-\frac{a}{16}}{b}$
default	$\frac{\frac{\sinh(bx+a)\cosh(bx+a)^5}{6}-\frac{\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)}{6}-\frac{bx}{16}-\frac{a}{16}}{b}$
risch	$-\frac{x}{16}+\frac{e^{6bx+6a}}{384b}+\frac{e^{4bx+4a}}{128b}-\frac{e^{2bx+2a}}{128b}+\frac{e^{-2bx-2a}}{128b}-\frac{e^{-4bx-4a}}{128b}-\frac{e^{-6bx-6a}}{384b}$
orering	$x\cosh(bx+a)^4\sinh(bx+a)^2+\frac{\frac{49\cosh(bx+a)^3\sinh(bx+a)^3b}{36}+\frac{49\cosh(bx+a)^5\sinh(bx+a)b}{72}}{b^2}-\frac{49x\left(12\cosh\right.}{}$

input
```
int(cosh(b*x+a)^4*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/6*sinh(b*x+a)*cosh(b*x+a)^5-1/6*(1/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))
*sinh(b*x+a)-1/16*b*x-1/16*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.34

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx$$
$$=\frac{3\cosh(bx+a)\sinh(bx+a)^5+2\left(5\cosh(bx+a)^3+3\cosh(bx+a)\right)\sinh(bx+a)^3-6bx+3\left(\cosh(b\right.}{96b}$$

input
```
integrate(cosh(b*x+a)^4*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/96*(3*cosh(b*x + a)*sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^3 + 3*cosh(b*x
+ a))*sinh(b*x + a)^3 - 6*b*x + 3*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - c
osh(b*x + a))*sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(56) = 112$.

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.03

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx$$
$$= \begin{cases} \frac{x\sinh^6(a+bx)}{16} - \frac{3x\sinh^4(a+bx)\cosh^2(a+bx)}{16} + \frac{3x\sinh^2(a+bx)\cosh^4(a+bx)}{16} - \frac{x\cosh^6(a+bx)}{16} - \frac{\sinh^5(a+bx)\cosh(a+bx)}{16b} + \text{si} \\ x\sinh^2(a)\cosh^4(a) \end{cases}$$

input
```
integrate(cosh(b*x+a)**4*sinh(b*x+a)**2,x)
```

output
```
Piecewise((x*sinh(a + b*x)**6/16 - 3*x*sinh(a + b*x)**4*cosh(a + b*x)**2/1
6 + 3*x*sinh(a + b*x)**2*cosh(a + b*x)**4/16 - x*cosh(a + b*x)**6/16 - sin
h(a + b*x)**5*cosh(a + b*x)/(16*b) + sinh(a + b*x)**3*cosh(a + b*x)**3/(6*
b) + sinh(a + b*x)*cosh(a + b*x)**5/(16*b), Ne(b, 0)), (x*sinh(a)**2*cosh(
a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx = \frac{\left(3\,e^{(-2\,bx-2\,a)} - 3\,e^{(-4\,bx-4\,a)} + 1\right)e^{(6\,bx+6\,a)}}{384\,b} - \frac{bx+a}{16\,b} + \frac{3\,e^{(-2\,bx-2\,a)} - 3\,e^{(-4\,bx-4\,a)} - e^{(-6\,bx-6\,a)}}{384\,b}$$

input
```
integrate(cosh(b*x+a)^4*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/384*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + 1)*e^(6*b*x + 6*a)/b - 1/
16*(b*x + a)/b + 1/384*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) - e^(-6*b*
x - 6*a))/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx = -\frac{1}{16}x + \frac{e^{(6bx+6a)}}{384b} + \frac{e^{(4bx+4a)}}{128b} - \frac{e^{(2bx+2a)}}{128b} + \frac{e^{(-2bx-2a)}}{128b} - \frac{e^{(-4bx-4a)}}{128b} - \frac{e^{(-6bx-6a)}}{384b}$$

input
```
integrate(cosh(b*x+a)^4*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/16*x + 1/384*e^(6*b*x + 6*a)/b + 1/128*e^(4*b*x + 4*a)/b - 1/128*e^(2*b
*x + 2*a)/b + 1/128*e^(-2*b*x - 2*a)/b - 1/128*e^(-4*b*x - 4*a)/b - 1/384*
e^(-6*b*x - 6*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx = \frac{\frac{\sinh(4a+4bx)}{64} - \frac{\sinh(2a+2bx)}{64} + \frac{\sinh(6a+6bx)}{192}}{b} - \frac{x}{16}$$

input
```
int(cosh(a + b*x)^4*sinh(a + b*x)^2,x)
```

output
```
(sinh(4*a + 4*b*x)/64 - sinh(2*a + 2*b*x)/64 + sinh(6*a + 6*b*x)/192)/b -
x/16
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.36

$$\int \cosh^4(a+bx)\sinh^2(a+bx)\,dx = \frac{e^{12bx+12a}+3e^{10bx+10a}-3e^{8bx+8a}-24e^{6bx+6a}bx+3e^{4bx+4a}-3e^{2bx+2a}-1}{384e^{6bx+6a}b}$$

input
```
int(cosh(b*x+a)^4*sinh(b*x+a)^2,x)
```

output
```
(e**(12*a + 12*b*x) + 3*e**(10*a + 10*b*x) - 3*e**(8*a + 8*b*x) - 24*e**(6
*a + 6*b*x)*b*x + 3*e**(4*a + 4*b*x) - 3*e**(2*a + 2*b*x) - 1)/(384*e**(6*
a + 6*b*x)*b)
```

3.6 $\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 90

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx = \frac{3x}{128}+\frac{3\cosh(a+bx)\sinh(a+bx)}{128b}+\frac{\cosh^3(a+bx)\sinh(a+bx)}{64b}-\frac{\cosh^5(a+bx)\sinh(a+bx)}{16b}+\frac{\cosh^5(a+bx)\sinh^3(a+bx)}{8b}$$

output
```
3/128*x+3/128*cosh(b*x+a)*sinh(b*x+a)/b+1/64*cosh(b*x+a)^3*sinh(b*x+a)/b-1
/16*cosh(b*x+a)^5*sinh(b*x+a)/b+1/8*cosh(b*x+a)^5*sinh(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.37

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx = \frac{24(a+bx)-8\sinh(4(a+bx))+\sinh(8(a+bx))}{1024b}$$

input
```
Integrate[Cosh[a + b*x]^4*Sinh[a + b*x]^4,x]
```

output
```
(24*(a + b*x) - 8*Sinh[4*(a + b*x)] + Sinh[8*(a + b*x)])/(1024*b)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {3042, 3048, 25, 3042, 25, 3048, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(a+bx)\cosh^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)^4\cos(ia+ibx)^4dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{3}{8}\int -\cosh^4(a+bx)\sinh^2(a+bx)dx+\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}-\frac{3}{8}\int \cosh^4(a+bx)\sinh^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}-\frac{3}{8}\int -\cos(ia+ibx)^4\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}+\frac{3}{8}\int \cos(ia+ibx)^4\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{3}{8}\left(\frac{1}{6}\int \cosh^4(a+bx)dx-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)+\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}+\frac{3}{8}\left(-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{1}{6}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^4dx\right)$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{8}\left(\frac{1}{6}\left(\frac{3}{4}\int \cosh^2(a+bx)dx+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)+\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}+\frac{3}{8}\left(-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{1}{6}\left(\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx\right)\right)$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)+\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}+\frac{3}{8}\left(\frac{1}{6}\left(\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)\right)-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)$$

input
```
Int[Cosh[a + b*x]^4*Sinh[a + b*x]^4,x]
```

output
```
(Cosh[a + b*x]^5*Sinh[a + b*x]^3)/(8*b) + (3*(-1/6*(Cosh[a + b*x]^5*Sinh[a
 + b*x])/b + ((Cosh[a + b*x]^3*Sinh[a + b*x])/(4*b) + (3*(x/2 + (Cosh[a +
b*x]*Sinh[a + b*x])/(2*b)))/4)/6))/8
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.82

$$\frac{\frac{\sinh(bx+a)^3\cosh(bx+a)^5}{8}-\frac{\sinh(bx+a)\cosh(bx+a)^5}{16}+\frac{\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)}{16}+\frac{3bx}{128}+\frac{3a}{128}}{b}$$

input
```
int(cosh(b*x+a)^4*sinh(b*x+a)^4,x)
```

output
```
1/b*(1/8*sinh(b*x+a)^3*cosh(b*x+a)^5-1/16*sinh(b*x+a)*cosh(b*x+a)^5+1/16*(
1/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))*sinh(b*x+a)+3/128*b*x+3/128*a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx$$
$$= \frac{7\cosh(bx+a)^3\sinh(bx+a)^5+\cosh(bx+a)\sinh(bx+a)^7+\left(7\cosh(bx+a)^5-4\cosh(bx+a)\right)\sin}{128\,b}$$

input
```
integrate(cosh(b*x+a)^4*sinh(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/128*(7*cosh(b*x + a)^3*sinh(b*x + a)^5 + cosh(b*x + a)*sinh(b*x + a)^7 +
 (7*cosh(b*x + a)^5 - 4*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*x + (cosh(b*x
 + a)^7 - 4*cosh(b*x + a)^3)*sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(80) = 160.

Time = 0.64 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.10

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx$$
$$= \begin{cases} \frac{3x\sinh^8(a+bx)}{128}-\frac{3x\sinh^6(a+bx)\cosh^2(a+bx)}{32}+\frac{9x\sinh^4(a+bx)\cosh^4(a+bx)}{64}-\frac{3x\sinh^2(a+bx)\cosh^6(a+bx)}{32}+\frac{3x\cosh^8(a+bx)}{128} \\ x\sinh^4(a)\cosh^4(a) \end{cases}$$

input
```
integrate(cosh(b*x+a)**4*sinh(b*x+a)**4,x)
```

output
```
Piecewise((3*x*sinh(a + b*x)**8/128 - 3*x*sinh(a + b*x)**6*cosh(a + b*x)**
2/32 + 9*x*sinh(a + b*x)**4*cosh(a + b*x)**4/64 - 3*x*sinh(a + b*x)**2*cos
h(a + b*x)**6/32 + 3*x*cosh(a + b*x)**8/128 - 3*sinh(a + b*x)**7*cosh(a +
b*x)/(128*b) + 11*sinh(a + b*x)**5*cosh(a + b*x)**3/(128*b) + 11*sinh(a +
b*x)**3*cosh(a + b*x)**5/(128*b) - 3*sinh(a + b*x)*cosh(a + b*x)**7/(128*b
), Ne(b, 0)), (x*sinh(a)**4*cosh(a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx = -\frac{\left(8\,e^{(-4\,bx-4\,a)}-1\right)e^{(8\,bx+8\,a)}}{2048\,b} + \frac{3\,(bx+a)}{128\,b} + \frac{8\,e^{(-4\,bx-4\,a)}-e^{(-8\,bx-8\,a)}}{2048\,b}$$

input
```
integrate(cosh(b*x+a)^4*sinh(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/2048*(8*e^(-4*b*x - 4*a) - 1)*e^(8*b*x + 8*a)/b + 3/128*(b*x + a)/b + 1
/2048*(8*e^(-4*b*x - 4*a) - e^(-8*b*x - 8*a))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx = \frac{3}{128}\,x + \frac{e^{(8\,bx+8\,a)}}{2048\,b} - \frac{e^{(4\,bx+4\,a)}}{256\,b} + \frac{e^{(-4\,bx-4\,a)}}{256\,b} - \frac{e^{(-8\,bx-8\,a)}}{2048\,b}$$

input
```
integrate(cosh(b*x+a)^4*sinh(b*x+a)^4,x, algorithm="giac")
```

output
```
3/128*x + 1/2048*e^(8*b*x + 8*a)/b - 1/256*e^(4*b*x + 4*a)/b + 1/256*e^(-4
*b*x - 4*a)/b - 1/2048*e^(-8*b*x - 8*a)/b
```

Mupad [B] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.36

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx = \frac{3x}{128} - \frac{\frac{\sinh(4a+4bx)}{128} - \frac{\sinh(8a+8bx)}{1024}}{b}$$

input
```
int(cosh(a + b*x)^4*sinh(a + b*x)^4,x)
```

output
```
(3*x)/128 - (sinh(4*a + 4*b*x)/128 - sinh(8*a + 8*b*x)/1024)/b
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int \cosh^4(a+bx)\sinh^4(a+bx)\,dx = \frac{e^{16bx+16a} - 8e^{12bx+12a} + 48e^{8bx+8a}bx + 8e^{4bx+4a} - 1}{2048e^{8bx+8a}b}$$

input
```
int(cosh(b*x+a)^4*sinh(b*x+a)^4,x)
```

output
```
(e**(16*a + 16*b*x) - 8*e**(12*a + 12*b*x) + 48*e**(8*a + 8*b*x)*b*x + 8*e
**(4*a + 4*b*x) - 1)/(2048*e**(8*a + 8*b*x)*b)
```

3.7 $\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 113

$$\begin{aligned}\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx = & -\frac{3x}{256}-\frac{3\cosh(a+bx)\sinh(a+bx)}{256b}\\ & -\frac{\cosh^3(a+bx)\sinh(a+bx)}{128b}\\ & +\frac{\cosh^5(a+bx)\sinh(a+bx)}{32b}\\ & -\frac{\cosh^5(a+bx)\sinh^3(a+bx)}{16b}\\ & +\frac{\cosh^5(a+bx)\sinh^5(a+bx)}{10b}\end{aligned}$$

output
```
-3/256*x-3/256*cosh(b*x+a)*sinh(b*x+a)/b-1/128*cosh(b*x+a)^3*sinh(b*x+a)/b
+1/32*cosh(b*x+a)^5*sinh(b*x+a)/b-1/16*cosh(b*x+a)^5*sinh(b*x+a)^3/b+1/10*
cosh(b*x+a)^5*sinh(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.55

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx$$
$$= \frac{-120bx + 20\sinh(2(a+bx)) + 40\sinh(4(a+bx)) - 10\sinh(6(a+bx)) - 5\sinh(8(a+bx)) + 2\sinh(10}{10240b}$$

input
```
Integrate[Cosh[a + b*x]^4*Sinh[a + b*x]^6,x]
```

output
```
(-120*b*x + 20*Sinh[2*(a + b*x)] + 40*Sinh[4*(a + b*x)] - 10*Sinh[6*(a + b
*x)] - 5*Sinh[8*(a + b*x)] + 2*Sinh[10*(a + b*x)])/(10240*b)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$, Rules used = {3042, 25, 3048, 3042, 3048, 25, 3042, 25, 3048, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^6(a+bx)\cosh^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)^6\left(-\cos(ia+ibx)^4\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \cos(ia+ibx)^4\sin(ia+ibx)^6dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b} - \frac{1}{2}\int \cosh^4(a+bx)\sinh^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}-\frac{1}{2}\int\cos(ia+ibx)^4\sin(ia+ibx)^4dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{1}{2}\left(-\frac{3}{8}\int-\cosh^4(a+bx)\sinh^2(a+bx)dx-\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}\right)+\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{3}{8}\int\cosh^4(a+bx)\sinh^2(a+bx)dx-\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}\right)+\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}+\frac{1}{2}\left(-\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}+\frac{3}{8}\int-\cos(ia+ibx)^4\sin(ia+ibx)^2dx\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}+\frac{1}{2}\left(-\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}-\frac{3}{8}\int\cos(ia+ibx)^4\sin(ia+ibx)^2dx\right)$$

$$\Big\downarrow \text{3048}$$

$$\frac{1}{2}\left(-\frac{3}{8}\left(\frac{1}{6}\int\cosh^4(a+bx)dx-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)-\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}\right)+\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}+\frac{1}{2}\left(-\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}-\frac{3}{8}\left(-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{1}{6}\int\sin\left(ia+ibx+\frac{\pi}{2}\right)^4dx\right)\right)$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(-\frac{3}{8}\left(\frac{1}{6}\left(\frac{3}{4}\int \cosh^2(a+bx)dx+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)-\frac{\sinh^3(a+bx\right.$$
$$\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}$$

$\big\downarrow$ 3042

$$\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}+$$
$$\frac{1}{2}\left(-\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}-\frac{3}{8}\left(-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{1}{6}\left(\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\int \text{si}\right.\right.\right.$$

$\big\downarrow$ 3115

$$\frac{1}{2}\left(-\frac{3}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)-\frac{\sinh(a+bx)\cosh^5(a+b\right.\right.$$
$$\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}$$

$\big\downarrow$ 24

$$\frac{\sinh^5(a+bx)\cosh^5(a+bx)}{10b}+$$
$$\frac{1}{2}\left(-\frac{\sinh^3(a+bx)\cosh^5(a+bx)}{8b}-\frac{3}{8}\left(\frac{1}{6}\left(\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)\right)\right.\right.$$

input
```
Int[Cosh[a + b*x]^4*Sinh[a + b*x]^6,x]
```

output
```
(Cosh[a + b*x]^5*Sinh[a + b*x]^5)/(10*b) + (-1/8*(Cosh[a + b*x]^5*Sinh[a +
 b*x]^3)/b - (3*(-1/6*(Cosh[a + b*x]^5*Sinh[a + b*x]))/b + ((Cosh[a + b*x]^
3*Sinh[a + b*x])/(4*b) + (3*(x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)))/4
)/6))/8)/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.81

$$\frac{\frac{\sinh(bx+a)^5\cosh(bx+a)^5}{10}-\frac{\sinh(bx+a)^3\cosh(bx+a)^5}{16}+\frac{\sinh(bx+a)\cosh(bx+a)^5}{32}-\frac{\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)}{32}-\frac{3bx}{256}-}{b}$$

input
```
int(cosh(b*x+a)^4*sinh(b*x+a)^6,x)
```

output
```
1/b*(1/10*sinh(b*x+a)^5*cosh(b*x+a)^5-1/16*sinh(b*x+a)^3*cosh(b*x+a)^5+1/3
2*sinh(b*x+a)*cosh(b*x+a)^5-1/32*(1/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))*sinh(
b*x+a)-3/256*b*x-3/256*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.74

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx$$
$$= \frac{5\cosh(bx+a)\sinh(bx+a)^9 + 10\left(6\cosh(bx+a)^3 - \cosh(bx+a)\right)\sinh(bx+a)^7 + \left(126\cosh(bx+\right.}{}$$

input
```
integrate(cosh(b*x+a)^4*sinh(b*x+a)^6,x, algorithm="fricas")
```

output
```
1/2560*(5*cosh(b*x + a)*sinh(b*x + a)^9 + 10*(6*cosh(b*x + a)^3 - cosh(b*x
 + a))*sinh(b*x + a)^7 + (126*cosh(b*x + a)^5 - 70*cosh(b*x + a)^3 - 15*co
sh(b*x + a))*sinh(b*x + a)^5 + 10*(6*cosh(b*x + a)^7 - 7*cosh(b*x + a)^5 -
 5*cosh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a)^3 - 30*b*x + 5*(cosh(b
*x + a)^9 - 2*cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 8*cosh(b*x + a)^3 + 2*
cosh(b*x + a))*sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(100) = 200$.

Time = 1.26 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.04

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx$$
$$= \begin{cases} \frac{3x\sinh^{10}(a+bx)}{256} - \frac{15x\sinh^8(a+bx)\cosh^2(a+bx)}{256} + \frac{15x\sinh^6(a+bx)\cosh^4(a+bx)}{128} - \frac{15x\sinh^4(a+bx)\cosh^6(a+bx)}{128} + \frac{15x\sinh^2(a}{} \\ x\sinh^6(a)\cosh^4(a) \end{cases}$$

input
```
integrate(cosh(b*x+a)**4*sinh(b*x+a)**6,x)
```

output
```
Piecewise((3*x*sinh(a + b*x)**10/256 - 15*x*sinh(a + b*x)**8*cosh(a + b*x)
**2/256 + 15*x*sinh(a + b*x)**6*cosh(a + b*x)**4/128 - 15*x*sinh(a + b*x)*
*4*cosh(a + b*x)**6/128 + 15*x*sinh(a + b*x)**2*cosh(a + b*x)**8/256 - 3*x
*cosh(a + b*x)**10/256 - 3*sinh(a + b*x)**9*cosh(a + b*x)/(256*b) + 7*sinh
(a + b*x)**7*cosh(a + b*x)**3/(128*b) + sinh(a + b*x)**5*cosh(a + b*x)**5/
(10*b) - 7*sinh(a + b*x)**3*cosh(a + b*x)**7/(128*b) + 3*sinh(a + b*x)*cos
h(a + b*x)**9/(256*b), Ne(b, 0)), (x*sinh(a)**6*cosh(a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.17

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx = -\frac{\left(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}-40\,e^{(-6\,bx-6\,a)}-20\,e^{(-8\,bx-8\,a)}-2\right)e^{(10\,bx+10\,a)}}{20480\,b} - \frac{3\,(bx+a)}{256\,b} - \frac{20\,e^{(-2\,bx-2\,a)}+40\,e^{(-4\,bx-4\,a)}-10\,e^{(-6\,bx-6\,a)}-5\,e^{(-8\,bx-8\,a)}+2\,e^{(-10\,bx-10\,a)}}{20480\,b}$$

input
```
integrate(cosh(b*x+a)^4*sinh(b*x+a)^6,x, algorithm="maxima")
```

output
```
-1/20480*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) - 40*e^(-6*b*x - 6*a) -
 20*e^(-8*b*x - 8*a) - 2)*e^(10*b*x + 10*a)/b - 3/256*(b*x + a)/b - 1/2048
0*(20*e^(-2*b*x - 2*a) + 40*e^(-4*b*x - 4*a) - 10*e^(-6*b*x - 6*a) - 5*e^(
-8*b*x - 8*a) + 2*e^(-10*b*x - 10*a))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.27

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx = -\frac{3}{256}\,x + \frac{e^{(10\,bx+10\,a)}}{10240\,b} - \frac{e^{(8\,bx+8\,a)}}{4096\,b} - \frac{e^{(6\,bx+6\,a)}}{2048\,b} + \frac{e^{(4\,bx+4\,a)}}{512\,b} + \frac{e^{(2\,bx+2\,a)}}{1024\,b} - \frac{e^{(-2\,bx-2\,a)}}{1024\,b} - \frac{e^{(-4\,bx-4\,a)}}{512\,b} + \frac{e^{(-6\,bx-6\,a)}}{2048\,b} + \frac{e^{(-8\,bx-8\,a)}}{4096\,b} - \frac{e^{(-10\,bx-10\,a)}}{10240\,b}$$

input
```
integrate(cosh(b*x+a)^4*sinh(b*x+a)^6,x, algorithm="giac")
```

output
```
-3/256*x + 1/10240*e^(10*b*x + 10*a)/b - 1/4096*e^(8*b*x + 8*a)/b - 1/2048
*e^(6*b*x + 6*a)/b + 1/512*e^(4*b*x + 4*a)/b + 1/1024*e^(2*b*x + 2*a)/b -
1/1024*e^(-2*b*x - 2*a)/b - 1/512*e^(-4*b*x - 4*a)/b + 1/2048*e^(-6*b*x -
6*a)/b + 1/4096*e^(-8*b*x - 8*a)/b - 1/10240*e^(-10*b*x - 10*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.58

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx = \frac{20\sinh(2\,a+2\,b\,x)+40\sinh(4\,a+4\,b\,x)-10\sinh(6\,a+6\,b\,x)-5\sinh(8\,a+8\,b\,x)+2\sinh(10\,a+10\,b}{10240\,b}$$

input
```
int(cosh(a + b*x)^4*sinh(a + b*x)^6,x)
```

output
```
(20*sinh(2*a + 2*b*x) + 40*sinh(4*a + 4*b*x) - 10*sinh(6*a + 6*b*x) - 5*si
nh(8*a + 8*b*x) + 2*sinh(10*a + 10*b*x) - 120*b*x)/(10240*b)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.25

$$\int \cosh^4(a+bx)\sinh^6(a+bx)\,dx = \frac{2e^{20bx+20a}-5e^{18bx+18a}-10e^{16bx+16a}+40e^{14bx+14a}+20e^{12bx+12a}-240e^{10bx+10a}bx-20e^{8bx+8a}-40e^{6bx+}}{20480e^{10bx+10a}b}$$

input
```
int(cosh(b*x+a)^4*sinh(b*x+a)^6,x)
```

output
```
(2*e**(20*a + 20*b*x) - 5*e**(18*a + 18*b*x) - 10*e**(16*a + 16*b*x) + 40*
e**(14*a + 14*b*x) + 20*e**(12*a + 12*b*x) - 240*e**(10*a + 10*b*x)*b*x -
20*e**(8*a + 8*b*x) - 40*e**(6*a + 6*b*x) + 10*e**(4*a + 4*b*x) + 5*e**(2*
a + 2*b*x) - 2)/(20480*e**(10*a + 10*b*x)*b)
```

3.8 $\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx = -\frac{5x}{128}-\frac{5\cosh(a+bx)\sinh(a+bx)}{128b}-\frac{5\cosh^3(a+bx)\sinh(a+bx)}{192b}-\frac{\cosh^5(a+bx)\sinh(a+bx)}{48b}+\frac{\cosh^7(a+bx)\sinh(a+bx)}{8b}$$

output
```
-5/128*x-5/128*cosh(b*x+a)*sinh(b*x+a)/b-5/192*cosh(b*x+a)^3*sinh(b*x+a)/b
-1/48*cosh(b*x+a)^5*sinh(b*x+a)/b+1/8*cosh(b*x+a)^7*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.59

$$\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx = \frac{-120bx-48\sinh(2(a+bx))+24\sinh(4(a+bx))+16\sinh(6(a+bx))+3\sinh(8(a+bx))}{3072b}$$

input
```
Integrate[Cosh[a + b*x]^6*Sinh[a + b*x]^2,x]
```

output
```
(-120*b*x - 48*Sinh[2*(a + b*x)] + 24*Sinh[4*(a + b*x)] + 16*Sinh[6*(a + b
*x)] + 3*Sinh[8*(a + b*x)])/(3072*b)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {3042, 25, 3048, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)\cosh^6(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)^2\left(-\cos(ia+ibx)^6\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \cos(ia+ibx)^6\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}-\frac{1}{8}\int \cosh^6(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}-\frac{1}{8}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^6dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{8}\left(-\frac{5}{6}\int \cosh^4(a+bx)dx-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)+\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}+\frac{1}{8}\left(-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}-\frac{5}{6}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^4dx\right)$$

$$\Big\downarrow \textbf{3115}$$

$$\frac{1}{8}\left(-\frac{5}{6}\left(\frac{3}{4}\int \cosh^2(a+bx)dx+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)+\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}+\frac{1}{8}\left(-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}-\frac{5}{6}\left(\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx\right)\right)$$

$$\Big\downarrow \textbf{3115}$$

$$\frac{1}{8}\left(-\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)+\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}$$

$$\Big\downarrow \textbf{24}$$

$$\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}+\frac{1}{8}\left(-\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}-\frac{5}{6}\left(\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)\right)\right)$$

input
```
Int[Cosh[a + b*x]^6*Sinh[a + b*x]^2,x]
```

output
```
(Cosh[a + b*x]^7*Sinh[a + b*x])/(8*b) + (-1/6*(Cosh[a + b*x]^5*Sinh[a + b*
x])/b - (5*((Cosh[a + b*x]^3*Sinh[a + b*x])/(4*b) + (3*(x/2 + (Cosh[a + b*
x]*Sinh[a + b*x])/(2*b)))/4))/6)/8
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\frac{\frac{\sinh(bx+a)\cosh(bx+a)^7}{8}-\frac{\left(\frac{\cosh(bx+a)^5}{6}+\frac{5\cosh(bx+a)^3}{24}+\frac{5\cosh(bx+a)}{16}\right)\sinh(bx+a)}{8}-\frac{5bx}{128}-\frac{5a}{128}}{b}$$

input
```
int(cosh(b*x+a)^6*sinh(b*x+a)^2,x)
```

output
```
1/b*(1/8*sinh(b*x+a)*cosh(b*x+a)^7-1/8*(1/6*cosh(b*x+a)^5+5/24*cosh(b*x+a)
^3+5/16*cosh(b*x+a))*sinh(b*x+a)-5/128*b*x-5/128*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.57

$$\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx$$

$$= \frac{3\cosh(bx+a)\sinh(bx+a)^7 + 3\left(7\cosh(bx+a)^3 + 4\cosh(bx+a)\right)\sinh(bx+a)^5 + \left(21\cosh(bx+a\right.}{}$$

input
```
integrate(cosh(b*x+a)^6*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/384*(3*cosh(b*x + a)*sinh(b*x + a)^7 + 3*(7*cosh(b*x + a)^3 + 4*cosh(b*x
 + a))*sinh(b*x + a)^5 + (21*cosh(b*x + a)^5 + 40*cosh(b*x + a)^3 + 12*cos
h(b*x + a))*sinh(b*x + a)^3 - 15*b*x + 3*(cosh(b*x + a)^7 + 4*cosh(b*x + a
)^5 + 4*cosh(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(80) = 160$.

Time = 0.65 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.15

$$\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx$$

$$= \begin{cases} -\frac{5x\sinh^8(a+bx)}{128} + \frac{5x\sinh^6(a+bx)\cosh^2(a+bx)}{32} - \frac{15x\sinh^4(a+bx)\cosh^4(a+bx)}{64} + \frac{5x\sinh^2(a+bx)\cosh^6(a+bx)}{32} - \frac{5x\cosh^8(a+}{128} \\ x\sinh^2(a)\cosh^6(a) \end{cases}$$

input
```
integrate(cosh(b*x+a)**6*sinh(b*x+a)**2,x)
```

output
```
Piecewise((-5*x*sinh(a + b*x)**8/128 + 5*x*sinh(a + b*x)**6*cosh(a + b*x)*
*2/32 - 15*x*sinh(a + b*x)**4*cosh(a + b*x)**4/64 + 5*x*sinh(a + b*x)**2*c
osh(a + b*x)**6/32 - 5*x*cosh(a + b*x)**8/128 + 5*sinh(a + b*x)**7*cosh(a
+ b*x)/(128*b) - 55*sinh(a + b*x)**5*cosh(a + b*x)**3/(384*b) + 73*sinh(a
+ b*x)**3*cosh(a + b*x)**5/(384*b) + 5*sinh(a + b*x)*cosh(a + b*x)**7/(128
*b), Ne(b, 0)), (x*sinh(a)**2*cosh(a)**6, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.25

$$\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx = \frac{\left(16\,e^{(-2\,bx-2\,a)}+24\,e^{(-4\,bx-4\,a)}-48\,e^{(-6\,bx-6\,a)}+3\right)e^{(8\,bx+8\,a)}}{6144\,b}-\frac{5\,(bx+a)}{128\,b}+\frac{48\,e^{(-2\,bx-2\,a)}-24\,e^{(-4\,bx-4\,a)}-16\,e^{(-6\,bx-6\,a)}-3\,e^{(-8\,bx-8\,a)}}{6144\,b}$$

input
```
integrate(cosh(b*x+a)^6*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/6144*(16*e^(-2*b*x - 2*a) + 24*e^(-4*b*x - 4*a) - 48*e^(-6*b*x - 6*a) +
3)*e^(8*b*x + 8*a)/b - 5/128*(b*x + a)/b + 1/6144*(48*e^(-2*b*x - 2*a) - 2
4*e^(-4*b*x - 4*a) - 16*e^(-6*b*x - 6*a) - 3*e^(-8*b*x - 8*a))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.32

$$\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx = -\frac{5}{128}\,x+\frac{e^{(8\,bx+8\,a)}}{2048\,b}+\frac{e^{(6\,bx+6\,a)}}{384\,b}+\frac{e^{(4\,bx+4\,a)}}{256\,b}-\frac{e^{(2\,bx+2\,a)}}{128\,b}+\frac{e^{(-2\,bx-2\,a)}}{128\,b}-\frac{e^{(-4\,bx-4\,a)}}{256\,b}-\frac{e^{(-6\,bx-6\,a)}}{384\,b}-\frac{e^{(-8\,bx-8\,a)}}{2048\,b}$$

input
```
integrate(cosh(b*x+a)^6*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-5/128*x + 1/2048*e^(8*b*x + 8*a)/b + 1/384*e^(6*b*x + 6*a)/b + 1/256*e^(4
*b*x + 4*a)/b - 1/128*e^(2*b*x + 2*a)/b + 1/128*e^(-2*b*x - 2*a)/b - 1/256
*e^(-4*b*x - 4*a)/b - 1/384*e^(-6*b*x - 6*a)/b - 1/2048*e^(-8*b*x - 8*a)/b
```

Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx = \frac{\frac{\sinh(4\,a+4\,b\,x)}{128}-\frac{\sinh(2\,a+2\,b\,x)}{64}+\frac{\sinh(6\,a+6\,b\,x)}{192}+\frac{\sinh(8\,a+8\,b\,x)}{1024}}{b}-\frac{5\,x}{128}$$

input
```
int(cosh(a + b*x)^6*sinh(a + b*x)^2,x)
```

output
```
(sinh(4*a + 4*b*x)/128 - sinh(2*a + 2*b*x)/64 + sinh(6*a + 6*b*x)/192 + si
nh(8*a + 8*b*x)/1024)/b - (5*x)/128
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.33

$$\int \cosh^6(a+bx)\sinh^2(a+bx)\,dx = \frac{3e^{16bx+16a}+16e^{14bx+14a}+24e^{12bx+12a}-48e^{10bx+10a}-240e^{8bx+8a}bx+48e^{6bx+6a}-24e^{4bx+4a}-16e^{2bx+2a}}{6144e^{8bx+8a}b}$$

input
```
int(cosh(b*x+a)^6*sinh(b*x+a)^2,x)
```

output
```
(3*e**(16*a + 16*b*x) + 16*e**(14*a + 14*b*x) + 24*e**(12*a + 12*b*x) - 48
*e**(10*a + 10*b*x) - 240*e**(8*a + 8*b*x)*b*x + 48*e**(6*a + 6*b*x) - 24*
e**(4*a + 4*b*x) - 16*e**(2*a + 2*b*x) - 3)/(6144*e**(8*a + 8*b*x)*b)
```

3.9 $\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 111

$$\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx = \frac{3x}{256} + \frac{3\cosh(a+bx)\sinh(a+bx)}{256b} + \frac{\cosh^3(a+bx)\sinh(a+bx)}{128b} + \frac{\cosh^5(a+bx)\sinh(a+bx)}{160b} - \frac{3\cosh^7(a+bx)\sinh(a+bx)}{80b} + \frac{\cosh^7(a+bx)\sinh^3(a+bx)}{10b}$$

output
```
3/256*x+3/256*cosh(b*x+a)*sinh(b*x+a)/b+1/128*cosh(b*x+a)^3*sinh(b*x+a)/b+
1/160*cosh(b*x+a)^5*sinh(b*x+a)/b-3/80*cosh(b*x+a)^7*sinh(b*x+a)/b+1/10*co
sh(b*x+a)^7*sinh(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.56

$$\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx = \frac{120bx+20\sinh(2(a+bx))-40\sinh(4(a+bx))-10\sinh(6(a+bx))+5\sinh(8(a+bx))+2\sinh(10(a$$

$$10240b$$

input
```
Integrate[Cosh[a + b*x]^6*Sinh[a + b*x]^4,x]
```

output
```
(120*b*x + 20*Sinh[2*(a + b*x)] - 40*Sinh[4*(a + b*x)] - 10*Sinh[6*(a + b*
x)] + 5*Sinh[8*(a + b*x)] + 2*Sinh[10*(a + b*x)])/(10240*b)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$, Rules used = {3042, 3048, 25, 3042, 25, 3048, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(a+bx)\cosh^6(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(ia+ibx)^4\cos(ia+ibx)^6dx$$

$$\Big\downarrow \text{ 3048}$$

$$\frac{3}{10}\int -\cosh^6(a+bx)\sinh^2(a+bx)dx+\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}-\frac{3}{10}\int \cosh^6(a+bx)\sinh^2(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}-\frac{3}{10}\int -\cos(ia+ibx)^6\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}+\frac{3}{10}\int \cos(ia+ibx)^6\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{3}{10}\left(\frac{1}{8}\int \cosh^6(a+bx)dx-\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}\right)+\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}+ \frac{3}{10}\left(-\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}+\frac{1}{8}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^6dx\right)$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\int \cosh^4(a+bx)dx+\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)-\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}\right)+ \frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}+ \frac{3}{10}\left(-\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}+\frac{1}{8}\left(\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{5}{6}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^4dx\right)\right)$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cosh^2(a+bx)dx+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)+\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right)-\frac{\sinh(a+bx)}{8}\right. \frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}+ \frac{3}{10}\left(-\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}+\frac{1}{8}\left(\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{5}{6}\left(\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\int \sin\right.\right.\right.$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)+\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right.\right.$$
$$\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}$$

$\big\downarrow$ 24

$$\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}+$$
$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{5}{6}\left(\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)\right)\right)\right.$$

input
```
Int[Cosh[a + b*x]^6*Sinh[a + b*x]^4,x]
```

output
```
(Cosh[a + b*x]^7*Sinh[a + b*x]^3)/(10*b) + (3*(-1/8*(Cosh[a + b*x]^7*Sinh[
a + b*x])/b + ((Cosh[a + b*x]^5*Sinh[a + b*x])/(6*b) + (5*((Cosh[a + b*x]^
3*Sinh[a + b*x])/(4*b) + (3*(x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)))/4
))/6)/8))/10
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\frac{\frac{\sinh(bx+a)^3 \cosh(bx+a)^7}{10} - \frac{3\sinh(bx+a)\cosh(bx+a)^7}{80} + \frac{3\left(\frac{\cosh(bx+a)^5}{6} + \frac{5\cosh(bx+a)^3}{24} + \frac{5\cosh(bx+a)}{16}\right)\sinh(bx+a)}{80} + \frac{3bx}{256} + \frac{3a}{256}}{b}$$

input
```
int(cosh(b*x+a)^6*sinh(b*x+a)^4,x)
```

output
```
1/b*(1/10*sinh(b*x+a)^3*cosh(b*x+a)^7-3/80*sinh(b*x+a)*cosh(b*x+a)^7+3/80*
(1/6*cosh(b*x+a)^5+5/24*cosh(b*x+a)^3+5/16*cosh(b*x+a))*sinh(b*x+a)+3/256*
b*x+3/256*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.76

$$\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx$$
$$= \frac{5\cosh(bx+a)\sinh(bx+a)^9 + 10\left(6\cosh(bx+a)^3 + \cosh(bx+a)\right)\sinh(bx+a)^7 + \left(126\cosh(bx+\right.}{}$$

input
```
integrate(cosh(b*x+a)^6*sinh(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/2560*(5*cosh(b*x + a)*sinh(b*x + a)^9 + 10*(6*cosh(b*x + a)^3 + cosh(b*x
 + a))*sinh(b*x + a)^7 + (126*cosh(b*x + a)^5 + 70*cosh(b*x + a)^3 - 15*co
sh(b*x + a))*sinh(b*x + a)^5 + 10*(6*cosh(b*x + a)^7 + 7*cosh(b*x + a)^5 -
 5*cosh(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a)^3 + 30*b*x + 5*(cosh(b
*x + a)^9 + 2*cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 - 8*cosh(b*x + a)^3 + 2*
cosh(b*x + a))*sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(100) = 200$.

Time = 1.27 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.08

$$\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx = \begin{cases} -\frac{3x\sinh^{10}(a+bx)}{256} + \frac{15x\sinh^8(a+bx)\cosh^2(a+bx)}{256} - \frac{15x\sinh^6(a+bx)\cosh^4(a+bx)}{128} + \frac{15x\sinh^4(a+bx)\cosh^6(a+bx)}{128} - \frac{15x\sinh^2}{} \\ x\sinh^4(a)\cosh^6(a) \end{cases}$$

input
```
integrate(cosh(b*x+a)**6*sinh(b*x+a)**4,x)
```

output
```
Piecewise((-3*x*sinh(a + b*x)**10/256 + 15*x*sinh(a + b*x)**8*cosh(a + b*x
)**2/256 - 15*x*sinh(a + b*x)**6*cosh(a + b*x)**4/128 + 15*x*sinh(a + b*x)
**4*cosh(a + b*x)**6/128 - 15*x*sinh(a + b*x)**2*cosh(a + b*x)**8/256 + 3*
x*cosh(a + b*x)**10/256 + 3*sinh(a + b*x)**9*cosh(a + b*x)/(256*b) - 7*sin
h(a + b*x)**7*cosh(a + b*x)**3/(128*b) + sinh(a + b*x)**5*cosh(a + b*x)**5
/(10*b) + 7*sinh(a + b*x)**3*cosh(a + b*x)**7/(128*b) - 3*sinh(a + b*x)*co
sh(a + b*x)**9/(256*b), Ne(b, 0)), (x*sinh(a)**4*cosh(a)**6, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.19

$$\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx = \frac{\left(5\,e^{(-2\,bx-2\,a)} - 10\,e^{(-4\,bx-4\,a)} - 40\,e^{(-6\,bx-6\,a)} + 20\,e^{(-8\,bx-8\,a)} + 2\right)e^{(10\,bx+10\,a)}}{20480\,b} + \frac{3\,(bx+a)}{256\,b} - \frac{20\,e^{(-2\,bx-2\,a)} - 40\,e^{(-4\,bx-4\,a)} - 10\,e^{(-6\,bx-6\,a)} + 5\,e^{(-8\,bx-8\,a)} + 2\,e^{(-10\,bx-10\,a)}}{20480\,b}$$

input
```
integrate(cosh(b*x+a)^6*sinh(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/20480*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) - 40*e^(-6*b*x - 6*a) +
20*e^(-8*b*x - 8*a) + 2)*e^(10*b*x + 10*a)/b + 3/256*(b*x + a)/b - 1/20480
*(20*e^(-2*b*x - 2*a) - 40*e^(-4*b*x - 4*a) - 10*e^(-6*b*x - 6*a) + 5*e^(-
8*b*x - 8*a) + 2*e^(-10*b*x - 10*a))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.30

$$\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx = \frac{3}{256}\,x + \frac{e^{(10\,bx+10\,a)}}{10240\,b} + \frac{e^{(8\,bx+8\,a)}}{4096\,b} - \frac{e^{(6\,bx+6\,a)}}{2048\,b} - \frac{e^{(4\,bx+4\,a)}}{512\,b} + \frac{e^{(2\,bx+2\,a)}}{1024\,b} - \frac{e^{(-2\,bx-2\,a)}}{1024\,b} + \frac{e^{(-4\,bx-4\,a)}}{512\,b} + \frac{e^{(-6\,bx-6\,a)}}{2048\,b} - \frac{e^{(-8\,bx-8\,a)}}{4096\,b} - \frac{e^{(-10\,bx-10\,a)}}{10240\,b}$$

input
```
integrate(cosh(b*x+a)^6*sinh(b*x+a)^4,x, algorithm="giac")
```

output
```
3/256*x + 1/10240*e^(10*b*x + 10*a)/b + 1/4096*e^(8*b*x + 8*a)/b - 1/2048*
e^(6*b*x + 6*a)/b - 1/512*e^(4*b*x + 4*a)/b + 1/1024*e^(2*b*x + 2*a)/b - 1
/1024*e^(-2*b*x - 2*a)/b + 1/512*e^(-4*b*x - 4*a)/b + 1/2048*e^(-6*b*x - 6
*a)/b - 1/4096*e^(-8*b*x - 8*a)/b - 1/10240*e^(-10*b*x - 10*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.59

$$\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx = \frac{20\sinh(2\,a+2\,b\,x) - 40\sinh(4\,a+4\,b\,x) - 10\sinh(6\,a+6\,b\,x) + 5\sinh(8\,a+8\,b\,x) + 2\sinh(10\,a+10\,b}{10240\,b}$$

input
```
int(cosh(a + b*x)^6*sinh(a + b*x)^4,x)
```

output
```
(20*sinh(2*a + 2*b*x) - 40*sinh(4*a + 4*b*x) - 10*sinh(6*a + 6*b*x) + 5*si
nh(8*a + 8*b*x) + 2*sinh(10*a + 10*b*x) + 120*b*x)/(10240*b)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.27

$$\int \cosh^6(a+bx)\sinh^4(a+bx)\,dx$$
$$= \frac{2e^{20bx+20a} + 5e^{18bx+18a} - 10e^{16bx+16a} - 40e^{14bx+14a} + 20e^{12bx+12a} + 240e^{10bx+10a}bx - 20e^{8bx+8a} + 40e^{6bx+}}{20480e^{10bx+10a}b}$$

input
```
int(cosh(b*x+a)^6*sinh(b*x+a)^4,x)
```

output
```
(2*e**(20*a + 20*b*x) + 5*e**(18*a + 18*b*x) - 10*e**(16*a + 16*b*x) - 40*
e**(14*a + 14*b*x) + 20*e**(12*a + 12*b*x) + 240*e**(10*a + 10*b*x)*b*x -
20*e**(8*a + 8*b*x) + 40*e**(6*a + 6*b*x) + 10*e**(4*a + 4*b*x) - 5*e**(2*
a + 2*b*x) - 2)/(20480*e**(10*a + 10*b*x)*b)
```

3.10 $\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 134

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx = -\frac{5x}{1024}-\frac{5\cosh(a+bx)\sinh(a+bx)}{1024b}-\frac{5\cosh^3(a+bx)\sinh(a+bx)}{1536b}-\frac{\cosh^5(a+bx)\sinh(a+bx)}{384b}+\frac{\cosh^7(a+bx)\sinh(a+bx)}{64b}-\frac{\cosh^7(a+bx)\sinh^3(a+bx)}{24b}+\frac{\cosh^7(a+bx)\sinh^5(a+bx)}{12b}$$

output
```
-5/1024*x-5/1024*cosh(b*x+a)*sinh(b*x+a)/b-5/1536*cosh(b*x+a)^3*sinh(b*x+a
)/b-1/384*cosh(b*x+a)^5*sinh(b*x+a)/b+1/64*cosh(b*x+a)^7*sinh(b*x+a)/b-1/2
4*cosh(b*x+a)^7*sinh(b*x+a)^3/b+1/12*cosh(b*x+a)^7*sinh(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.32

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx = \frac{-120a-120bx+45\sinh(4(a+bx))-9\sinh(8(a+bx))+\sinh(12(a+bx))}{24576b}$$

input
```
Integrate[Cosh[a + b*x]^6*Sinh[a + b*x]^6,x]
```

output
```
(-120*a - 120*b*x + 45*Sinh[4*(a + b*x)] - 9*Sinh[8*(a + b*x)] + Sinh[12*(
a + b*x)])/(24576*b)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.19, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.941$, Rules used = {3042, 25, 3048, 3042, 3048, 25, 3042, 25, 3048, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^6(a+bx)\cosh^6(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)^6\left(-\cos(ia+ibx)^6\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \cos(ia+ibx)^6\sin(ia+ibx)^6dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-\frac{5}{12}\int \cosh^6(a+bx)\sinh^4(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-\frac{5}{12}\int \cos(ia+ibx)^6\sin(ia+ibx)^4dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-\frac{5}{12}\left(\frac{3}{10}\int -\cosh^6(a+bx)\sinh^2(a+bx)dx+\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-\frac{5}{12}\left(\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}-\frac{3}{10}\int \cosh^6(a+bx)\sinh^2(a+bx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-\frac{5}{12}\left(\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}-\frac{3}{10}\int -\cos(ia+ibx)^6\sin(ia+ibx)^2dx\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-\frac{5}{12}\left(\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}+\frac{3}{10}\int \cos(ia+ibx)^6\sin(ia+ibx)^2dx\right)$$

$$\Big\downarrow \text{3048}$$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-\frac{5}{12}\left(\frac{3}{10}\left(\frac{1}{8}\int \cosh^6(a+bx)dx-\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}\right)+\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b}-\frac{5}{12}\left(\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b}+\frac{3}{10}\left(-\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}+\frac{1}{8}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^6dx\right)\right)$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b} - \frac{5}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\int \cosh^4(a+bx)dx + \frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right) - \frac{\sinh(a+bx)\cosh^7(a+bx)}{8b}\right) + \frac{\sinh^3(a+b\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b} - \frac{5}{12}\left(\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b} + \frac{3}{10}\left(-\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b} + \frac{1}{8}\left(\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b} + \frac{5}{6}\int \text{si}\right.\right.\right.$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b} - \frac{5}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cosh^2(a+bx)dx + \frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right) + \frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}\right) - \frac{\sinh(a+\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b} - \frac{5}{12}\left(\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b} + \frac{3}{10}\left(-\frac{\sinh(a+bx)\cosh^7(a+bx)}{8b} + \frac{1}{8}\left(\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b} + \frac{5}{6}\left(\frac{\sin}{}\right.\right.\right.\right.$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b} - \frac{5}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2} + \frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right) + \frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right) + \frac{\sinh(a+bx)\cosh^5(a}{6b}\right.\right.\right.\right.$$

$$\Big\downarrow \text{24}$$

$$\frac{\sinh^5(a+bx)\cosh^7(a+bx)}{12b} - \frac{5}{12}\left(\frac{\sinh^3(a+bx)\cosh^7(a+bx)}{10b} + \frac{3}{10}\left(\frac{1}{8}\left(\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b} + \frac{5}{6}\left(\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b} + \frac{3}{4}\left(\frac{\text{s}}{}\right.\right.\right.\right.\right.$$

input
```
Int[Cosh[a + b*x]^6*Sinh[a + b*x]^6,x]
```

output
```
(Cosh[a + b*x]^7*Sinh[a + b*x]^5)/(12*b) - (5*((Cosh[a + b*x]^7*Sinh[a + b
*x]^3)/(10*b) + (3*(-1/8*(Cosh[a + b*x]^7*Sinh[a + b*x])/b + ((Cosh[a + b*
x]^5*Sinh[a + b*x])/(6*b) + (5*((Cosh[a + b*x]^3*Sinh[a + b*x])/(4*b) + (3
*(x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)))/4))/6)/8))/10))/12
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.76

$$\frac{\frac{\sinh(bx+a)^5\cosh(bx+a)^7}{12}-\frac{\sinh(bx+a)^3\cosh(bx+a)^7}{24}+\frac{\sinh(bx+a)\cosh(bx+a)^7}{64}-\frac{\left(\frac{\cosh(bx+a)^5}{6}+\frac{5\cosh(bx+a)^3}{24}+\frac{5\cosh(bx+a)}{16}\right)\sinh(bx+}{64}}{b}$$

input
```
int(cosh(b*x+a)^6*sinh(b*x+a)^6,x)
```

output
```
1/b*(1/12*sinh(b*x+a)^5*cosh(b*x+a)^7-1/24*sinh(b*x+a)^3*cosh(b*x+a)^7+1/6
4*sinh(b*x+a)*cosh(b*x+a)^7-1/64*(1/6*cosh(b*x+a)^5+5/24*cosh(b*x+a)^3+5/1
6*cosh(b*x+a))*sinh(b*x+a)-5/1024*b*x-5/1024*a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.34

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx$$
$$= \frac{55\cosh(bx+a)^3\sinh(bx+a)^9 + 3\cosh(bx+a)\sinh(bx+a)^{11} + 18\left(11\cosh(bx+a)^5 - \cosh(bx+a\right.}{}$$

input
```
integrate(cosh(b*x+a)^6*sinh(b*x+a)^6,x, algorithm="fricas")
```

output
```
1/6144*(55*cosh(b*x + a)^3*sinh(b*x + a)^9 + 3*cosh(b*x + a)*sinh(b*x + a)
^11 + 18*(11*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^7 + 18*(11*cos
h(b*x + a)^7 - 7*cosh(b*x + a)^3)*sinh(b*x + a)^5 + (55*cosh(b*x + a)^9 -
126*cosh(b*x + a)^5 + 45*cosh(b*x + a))*sinh(b*x + a)^3 - 30*b*x + 3*(cosh
(b*x + a)^11 - 6*cosh(b*x + a)^7 + 15*cosh(b*x + a)^3)*sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(121) = 242.

Time = 2.47 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.07

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx$$
$$= \begin{cases} -\frac{5x\sinh^{12}(a+bx)}{1024} + \frac{15x\sinh^{10}(a+bx)\cosh^2(a+bx)}{512} - \frac{75x\sinh^8(a+bx)\cosh^4(a+bx)}{1024} + \frac{25x\sinh^6(a+bx)\cosh^6(a+bx)}{256} - \frac{75x\sinh}{} \\ x\sinh^6(a)\cosh^6(a) \end{cases}$$

input
```
integrate(cosh(b*x+a)**6*sinh(b*x+a)**6,x)
```

output
```
Piecewise((-5*x*sinh(a + b*x)**12/1024 + 15*x*sinh(a + b*x)**10*cosh(a + b
*x)**2/512 - 75*x*sinh(a + b*x)**8*cosh(a + b*x)**4/1024 + 25*x*sinh(a + b
*x)**6*cosh(a + b*x)**6/256 - 75*x*sinh(a + b*x)**4*cosh(a + b*x)**8/1024
+ 15*x*sinh(a + b*x)**2*cosh(a + b*x)**10/512 - 5*x*cosh(a + b*x)**12/1024
 + 5*sinh(a + b*x)**11*cosh(a + b*x)/(1024*b) - 85*sinh(a + b*x)**9*cosh(a
 + b*x)**3/(3072*b) + 33*sinh(a + b*x)**7*cosh(a + b*x)**5/(512*b) + 33*si
nh(a + b*x)**5*cosh(a + b*x)**7/(512*b) - 85*sinh(a + b*x)**3*cosh(a + b*x
)**9/(3072*b) + 5*sinh(a + b*x)*cosh(a + b*x)**11/(1024*b), Ne(b, 0)), (x*
sinh(a)**6*cosh(a)**6, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.64

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx = -\frac{\left(9\,e^{(-4\,bx-4\,a)}-45\,e^{(-8\,bx-8\,a)}-1\right)e^{(12\,bx+12\,a)}}{49152\,b} - \frac{5\,(bx+a)}{1024\,b} - \frac{45\,e^{(-4\,bx-4\,a)}-9\,e^{(-8\,bx-8\,a)}+e^{(-12\,bx-12\,a)}}{49152\,b}$$

input
```
integrate(cosh(b*x+a)^6*sinh(b*x+a)^6,x, algorithm="maxima")
```

output
```
-1/49152*(9*e^(-4*b*x - 4*a) - 45*e^(-8*b*x - 8*a) - 1)*e^(12*b*x + 12*a)/
b - 5/1024*(b*x + a)/b - 1/49152*(45*e^(-4*b*x - 4*a) - 9*e^(-8*b*x - 8*a)
 + e^(-12*b*x - 12*a))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.66

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx = -\frac{5}{1024}\,x + \frac{e^{(12\,bx+12\,a)}}{49152\,b} - \frac{3\,e^{(8\,bx+8\,a)}}{16384\,b} + \frac{15\,e^{(4\,bx+4\,a)}}{16384\,b} - \frac{15\,e^{(-4\,bx-4\,a)}}{16384\,b} + \frac{3\,e^{(-8\,bx-8\,a)}}{16384\,b} - \frac{e^{(-12\,bx-12\,a)}}{49152\,b}$$

input
```
integrate(cosh(b*x+a)^6*sinh(b*x+a)^6,x, algorithm="giac")
```

output
```
-5/1024*x + 1/49152*e^(12*b*x + 12*a)/b - 3/16384*e^(8*b*x + 8*a)/b + 15/1
6384*e^(4*b*x + 4*a)/b - 15/16384*e^(-4*b*x - 4*a)/b + 3/16384*e^(-8*b*x -
 8*a)/b - 1/49152*e^(-12*b*x - 12*a)/b
```

Mupad [B] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.31

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx = \frac{\frac{15\sinh(4\,a+4\,b\,x)}{8192}-\frac{3\sinh(8\,a+8\,b\,x)}{8192}+\frac{\sinh(12\,a+12\,b\,x)}{24576}}{b}-\frac{5\,x}{1024}$$

input
```
int(cosh(a + b*x)^6*sinh(a + b*x)^6,x)
```

output
```
((15*sinh(4*a + 4*b*x))/8192 - (3*sinh(8*a + 8*b*x))/8192 + sinh(12*a + 12
*b*x)/24576)/b - (5*x)/1024
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.68

$$\int \cosh^6(a+bx)\sinh^6(a+bx)\,dx = \frac{e^{24bx+24a}-9e^{20bx+20a}+45e^{16bx+16a}-240e^{12bx+12a}bx-45e^{8bx+8a}+9e^{4bx+4a}-1}{49152e^{12bx+12a}b}$$

input
```
int(cosh(b*x+a)^6*sinh(b*x+a)^6,x)
```

output
```
(e**(24*a + 24*b*x) - 9*e**(20*a + 20*b*x) + 45*e**(16*a + 16*b*x) - 240*e
**(12*a + 12*b*x)*b*x - 45*e**(8*a + 8*b*x) + 9*e**(4*a + 4*b*x) - 1)/(491
52*e**(12*a + 12*b*x)*b)
```

3.11 $\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} - \frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}-\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}$$

output
```
-arctan(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2))/b+arctanh(cosh(b*x+a)^(1/2)/s
inh(b*x+a)^(1/2))/b-2*sinh(b*x+a)^(1/2)/b/cosh(b*x+a)^(1/2)-2/5*sinh(b*x+a
)^(5/2)/b/cosh(b*x+a)^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.56

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \frac{2\sqrt[4]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{9}{4},\frac{9}{4},\frac{13}{4},-\sinh^2(a+bx)\right)\sinh^{\frac{9}{2}}(a+bx)}{9b\sqrt{\cosh(a+bx)}}$$

input
```
Integrate[Sinh[a + b*x]^(7/2)/Cosh[a + b*x]^(7/2),x]
```

output
```
(2*(Cosh[a + b*x]^2)^(1/4)*Hypergeometric2F1[9/4, 9/4, 13/4, -Sinh[a + b*x
]^2]*Sinh[a + b*x]^(9/2))/(9*b*Sqrt[Cosh[a + b*x]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3046, 3042, 3046, 3042, 3055, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(-i\sin(ia+ibx))^{7/2}}{\cos(ia+ibx)^{7/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}dx - \frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)} + \int \frac{(-i\sin(ia+ibx))^{3/2}}{\cos(ia+ibx)^{3/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}dx - \frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)} - \frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\cos(ia+ibx)}}{\sqrt{-i\sin(ia+ibx)}}dx - \frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)} - \frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

$$\Big\downarrow \text{3055}$$

$$\frac{2\int \frac{\coth(a+bx)}{1-\coth^2(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}}{b} - \frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)} - \frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

$$\Big\downarrow \text{827}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}} - \frac{1}{2}\int \frac{1}{\coth(a+bx)+1}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b} - \frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)} - \frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}} - \frac{1}{2}\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)\right)}{b} - \frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)} - \frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\left(\frac{1}{2}\text{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right) - \frac{1}{2}\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)\right)}{b} - \frac{2\sinh^{\frac{5}{2}}(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)} - \frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

input
```
Int[Sinh[a + b*x]^(7/2)/Cosh[a + b*x]^(7/2),x]
```

output
```
(2*(-1/2*ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]] + ArcTanh[Sqrt[Co
sh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/2))/b - (2*Sqrt[Sinh[a + b*x]])/(b*Sqrt[
Cosh[a + b*x]]) - (2*Sinh[a + b*x]^(5/2))/(5*b*Cosh[a + b*x]^(5/2))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

Maple [F]

$$\int \frac{\sinh(bx+a)^{\frac{7}{2}}}{\cosh(bx+a)^{\frac{7}{2}}}dx$$

input
```
int(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x)
```

output
```
int(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 997 vs. $2(88) = 176$.

Time = 0.13 (sec) , antiderivative size = 997, normalized size of antiderivative = 9.41

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
-1/10*(24*cosh(b*x + a)^6 + 144*cosh(b*x + a)*sinh(b*x + a)^5 + 24*sinh(b*
x + a)^6 + 72*(5*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 72*cosh(b*x + a)^4
 + 96*(5*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 72*(5*cosh(b
*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 - 10*(cosh(b*x + a)^6 +
 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2
+ 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 3*cosh(b
*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*s
inh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^
3 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*
x + a) + sinh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b
*x + a)*sinh(b*x + a) - sinh(b*x + a)^2) + 72*cosh(b*x + a)^2 + 5*(cosh(b*
x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b
*x + a)^2 + 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3
+ 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x + a
)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*cosh
(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(-cosh(b*x + a)^2 + 2*(
cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2
*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a)^2) + 16*(3*cosh(b*x + a)^5 +
15*cosh(b*x + a)*sinh(b*x + a)^4 + 3*sinh(b*x + a)^5 + 2*(15*cosh(b*x + a)
^2 + 2)*sinh(b*x + a)^3 + 4*cosh(b*x + a)^3 + 6*(5*cosh(b*x + a)^3 + 2*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(b*x+a)**(7/2)/cosh(b*x+a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\sinh\left(bx+a\right)^{\frac{7}{2}}}{\cosh\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)^(7/2)/cosh(b*x + a)^(7/2), x)
```

Giac [F]

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\sinh\left(bx+a\right)^{\frac{7}{2}}}{\cosh\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^(7/2)/cosh(b*x + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{{\mathrm{sinh}\left(a+b\,x\right)}^{7/2}}{{\mathrm{cosh}\left(a+b\,x\right)}^{7/2}}\,dx$$

input
```
int(sinh(a + b*x)^(7/2)/cosh(a + b*x)^(7/2),x)
```

output
```
int(sinh(a + b*x)^(7/2)/cosh(a + b*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sinh^{\frac{7}{2}}(a+bx)}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\sinh(bx+a)}\,\sqrt{\cosh(bx+a)}\,\sinh(bx+a)^3}{\cosh(bx+a)^4}dx$$

input
```
int(sinh(b*x+a)^(7/2)/cosh(b*x+a)^(7/2),x)
```

output
```
int((sqrt(sinh(a + b*x))*sqrt(cosh(a + b*x))*sinh(a + b*x)**3)/cosh(a + b*
x)**4,x)
```

3.12 $\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx=-\frac{\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

output
```
-arctan(1/cosh(b*x+a)^(1/2)*sinh(b*x+a)^(1/2))/b+arctanh(1/cosh(b*x+a)^(1/
2)*sinh(b*x+a)^(1/2))/b-2/3*sinh(b*x+a)^(3/2)/b/cosh(b*x+a)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.73

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \frac{2\cosh^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{7}{4},\frac{11}{4},-\sinh^2(a+bx)\right)\sinh^{\frac{7}{2}}(a+bx)}{7b\cosh^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[Sinh[a + b*x]^(5/2)/Cosh[a + b*x]^(5/2),x]
```

output
```
(2*(Cosh[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, 7/4, 11/4, -Sinh[a + b*x
]^2]*Sinh[a + b*x]^(7/2))/(7*b*Cosh[a + b*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3046, 3042, 3054, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(-i\sin(ia+ibx))^{5/2}}{\cos(ia+ibx)^{5/2}}dx$$

$$\Big\downarrow \text{ 3046}$$

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}dx-\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}+\int \frac{\sqrt{-i\sin(ia+ibx)}}{\sqrt{\cos(ia+ibx)}}dx$$

$$\Big\downarrow \text{ 3054}$$

$$-\frac{2\int -\frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}}{b}-\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\int \frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}}{b}-\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{ 827}$$

$$-\frac{2\left(\frac{1}{2}\int \frac{1}{\tanh(a+bx)+1}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}-\frac{1}{2}\int \frac{1}{1-\tanh(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)-\frac{1}{2}\int \frac{1}{1-\tanh(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)\right)}{b}-\frac{2\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

input
```
Int[Sinh[a + b*x]^(5/2)/Cosh[a + b*x]^(5/2),x]
```

output
```
(-2*(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/2 - ArcTanh[Sqrt[Sinh
[a + b*x]]/Sqrt[Cosh[a + b*x]]]/2))/b - (2*Sinh[a + b*x]^(3/2))/(3*b*Cosh[
a + b*x]^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Maple [F]

$$\int \frac{\sinh \left(bx+a\right)^{\frac{5}{2}}}{\cosh \left(bx+a\right)^{\frac{5}{2}}}dx$$

input
```
int(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x)
```

output
```
int(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 591 vs. $2(67) = 134$.

Time = 0.11 (sec) , antiderivative size = 591, normalized size of antiderivative = 7.30

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(4*cosh(b*x + a)^4 + 16*cosh(b*x + a)*sinh(b*x + a)^3 + 4*sinh(b*x +
a)^4 + 8*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 6*(cosh(b*x + a)^4 + 4*
cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1
)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a)
)*sinh(b*x + a) + 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x
 + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x
+ a) - sinh(b*x + a)^2) + 8*cosh(b*x + a)^2 + 3*(cosh(b*x + a)^4 + 4*cosh(
b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sin
h(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sin
h(b*x + a) + 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*s
qrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - s
inh(b*x + a)^2) + 8*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + s
inh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*sq
rt(cosh(b*x + a))*sqrt(sinh(b*x + a)) + 16*(cosh(b*x + a)^3 + cosh(b*x + a
))*sinh(b*x + a) + 4)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)
^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 + b)
*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) +
 b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(b*x+a)**(5/2)/cosh(b*x+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\sinh(bx+a)^{\frac{5}{2}}}{\cosh(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)^(5/2)/cosh(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\sinh(bx+a)^{\frac{5}{2}}}{\cosh(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^(5/2)/cosh(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\mathrm{sinh}(a+b\,x)^{5/2}}{\mathrm{cosh}(a+b\,x)^{5/2}}\,dx$$

input
```
int(sinh(a + b*x)^(5/2)/cosh(a + b*x)^(5/2),x)
```

output
```
int(sinh(a + b*x)^(5/2)/cosh(a + b*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sinh^{\frac{5}{2}}(a+bx)}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\sinh(bx+a)}\,\sqrt{\cosh(bx+a)}\,\sinh(bx+a)^2}{\cosh(bx+a)^3}dx$$

input
```
int(sinh(b*x+a)^(5/2)/cosh(b*x+a)^(5/2),x)
```

output
```
int((sqrt(sinh(a + b*x))*sqrt(cosh(a + b*x))*sinh(a + b*x)**2)/cosh(a + b*
x)**3,x)
```

3.13 $\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx=-\frac{\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

output
```
-arctan(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2))/b+arctanh(cosh(b*x+a)^(1/2)/s
inh(b*x+a)^(1/2))/b-2*sinh(b*x+a)^(1/2)/b/cosh(b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx=\frac{2\sqrt[4]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{5}{4},\frac{9}{4},-\sinh^2(a+bx)\right)\sinh^{\frac{5}{2}}(a+bx)}{5b\sqrt{\cosh(a+bx)}}$$

input
```
Integrate[Sinh[a + b*x]^(3/2)/Cosh[a + b*x]^(3/2),x]
```

output
```
(2*(Cosh[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(5/2))/(5*b*Sqrt[Cosh[a + b*x]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3046, 3042, 3055, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(-i\sin(ia+ibx))^{3/2}}{\cos(ia+ibx)^{3/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}dx-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}+\int \frac{\sqrt{\cos(ia+ibx)}}{\sqrt{-i\sin(ia+ibx)}}dx$$

$$\Big\downarrow \text{3055}$$

$$\frac{2\int \frac{\coth(a+bx)}{1-\coth^2(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}}{b}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

$$\Big\downarrow \text{827}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}-\frac{1}{2}\int \frac{1}{\coth(a+bx)+1}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}-\frac{1}{2}\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)\right)}{b}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\left(\frac{1}{2}\text{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)-\frac{1}{2}\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)\right)}{b}-\frac{2\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

input
```
Int[Sinh[a + b*x]^(3/2)/Cosh[a + b*x]^(3/2),x]
```

output
```
(2*(-1/2*ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]] + ArcTanh[Sqrt[Co
sh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/2))/b - (2*Sqrt[Sinh[a + b*x]])/(b*Sqrt[
Cosh[a + b*x]])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

Maple [F]

$$\int \frac{\sinh\left(bx+a\right)^{\frac{3}{2}}}{\cosh\left(bx+a\right)^{\frac{3}{2}}}dx$$

input
```
int(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x)
```

output
```
int(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(67) = 134$.

Time = 0.11 (sec) , antiderivative size = 310, normalized size of antiderivative = 3.92

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\left(\cosh\left(bx+a\right)^{2}+2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}+1\right)\arctan\left(-\cosh\left(bx+a\right)^{2}+2\left(\cos\right.\right.}{}$$

input
```
integrate(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
+ 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh
(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x
+ a)^2) - 4*cosh(b*x + a)^2 - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x
+ a) + sinh(b*x + a)^2 + 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh
(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(
b*x + a) - sinh(b*x + a)^2) - 8*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(
b*x + a))*sqrt(sinh(b*x + a)) - 8*cosh(b*x + a)*sinh(b*x + a) - 4*sinh(b*x
 + a)^2 - 4)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh
(b*x + a)^2 + b)
```

Sympy [F]

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(sinh(b*x+a)**(3/2)/cosh(b*x+a)**(3/2),x)
```

output
```
Integral(sinh(a + b*x)**(3/2)/cosh(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\sinh(bx+a)^{\frac{3}{2}}}{\cosh(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)^(3/2)/cosh(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\sinh\left(bx+a\right)^{\frac{3}{2}}}{\cosh\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^(3/2)/cosh(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\sinh(a+b\,x)^{3/2}}{\cosh\left(a+b\,x\right)^{3/2}}\,dx$$

input
```
int(sinh(a + b*x)^(3/2)/cosh(a + b*x)^(3/2),x)
```

output
```
int(sinh(a + b*x)^(3/2)/cosh(a + b*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sinh^{\frac{3}{2}}(a+bx)}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \frac{\cosh\left(bx+a\right)\left(\int \frac{\sqrt{\sinh(bx+a)}\,\sqrt{\cosh(bx+a)}}{\sinh(bx+a)}dx\right)b-2\sqrt{\sinh\left(bx+a\right)}\,\sqrt{\cosh\left(bx+a\right)}}{\cosh\left(bx+a\right)b}$$

input
```
int(sinh(b*x+a)^(3/2)/cosh(b*x+a)^(3/2),x)
```

output
```
(cosh(a + b*x)*int((sqrt(sinh(a + b*x))*sqrt(cosh(a + b*x)))/sinh(a + b*x)
,x)*b - 2*sqrt(sinh(a + b*x))*sqrt(cosh(a + b*x)))/(cosh(a + b*x)*b)
```

3.14 $\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

output
```
-arctan(1/cosh(b*x+a)^(1/2)*sinh(b*x+a)^(1/2))/b+arctanh(1/cosh(b*x+a)^(1/
2)*sinh(b*x+a)^(1/2))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx = \frac{2\cosh^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},-\sinh^2(a+bx)\right)\sinh^{\frac{3}{2}}(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]],x]
```

output
```
(2*(Cosh[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(3/2))/(3*b*Cosh[a + b*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3054, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{-i\sin(ia+ibx)}}{\sqrt{\cos(ia+ibx)}}dx$$

$$\Big\downarrow \text{3054}$$

$$-\frac{2\int -\frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\int \frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}}{b}$$

$$\Big\downarrow \text{827}$$

$$-\frac{2\left(\frac{1}{2}\int \frac{1}{\tanh(a+bx)+1}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}-\frac{1}{2}\int \frac{1}{1-\tanh(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)-\frac{1}{2}\int \frac{1}{1-\tanh(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)\right)}{b}$$

input
```
Int[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]],x]
```

output
```
(-2*(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/2 - ArcTanh[Sqrt[Sinh
[a + b*x]]/Sqrt[Cosh[a + b*x]]]/2))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Maple [F]

$$\int \frac{\sqrt{\sinh(bx+a)}}{\sqrt{\cosh(bx+a)}}dx$$

input
```
int(1/cosh(b*x+a)^(1/2)*sinh(b*x+a)^(1/2),x)
```

output
```
int(1/cosh(b*x+a)^(1/2)*sinh(b*x+a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(46) = 92$.

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.63

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx =$$
$$-\frac{2\arctan\left(-\cosh(bx+a)^2+2(\cosh(bx+a)+\sinh(bx+a))\sqrt{\cosh(bx+a)}\sqrt{\sinh(bx+a)}-2\cos\right.}{}$$

input
```
integrate(1/cosh(b*x+a)^(1/2)*sinh(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(2*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(c
osh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b
*x + a)^2) + log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt
(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh
(b*x + a)^2))/b
```

Sympy [F]

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{\sqrt{\sinh\left(a+bx\right)}}{\sqrt{\cosh\left(a+bx\right)}}\,dx$$

input
```
integrate(1/cosh(b*x+a)**(1/2)*sinh(b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(sinh(a + b*x))/sqrt(cosh(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{\sqrt{\sinh\left(bx+a\right)}}{\sqrt{\cosh\left(bx+a\right)}}\,dx$$

input
```
integrate(1/cosh(b*x+a)^(1/2)*sinh(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sinh(b*x + a))/sqrt(cosh(b*x + a)), x)
```

Giac [F]

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{\sqrt{\sinh\left(bx+a\right)}}{\sqrt{\cosh\left(bx+a\right)}}\,dx$$

input
```
integrate(1/cosh(b*x+a)^(1/2)*sinh(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sinh(b*x + a))/sqrt(cosh(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{\sqrt{\sinh\left(a+b\,x\right)}}{\sqrt{\cosh\left(a+b\,x\right)}}\,dx$$

input
```
int(sinh(a + b*x)^(1/2)/cosh(a + b*x)^(1/2),x)
```

output
```
int(sinh(a + b*x)^(1/2)/cosh(a + b*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{\sqrt{\sinh\left(bx+a\right)}\,\sqrt{\cosh\left(bx+a\right)}}{\cosh\left(bx+a\right)}dx$$

input
```
int(1/cosh(b*x+a)^(1/2)*sinh(b*x+a)^(1/2),x)
```

output
```
int((sqrt(sinh(a + b*x))*sqrt(cosh(a + b*x)))/cosh(a + b*x),x)
```

3.15 $\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

output
```
-arctan(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2))/b+arctanh(cosh(b*x+a)^(1/2)/s
inh(b*x+a)^(1/2))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = \frac{2\sqrt[4]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},-\sinh^2(a+bx)\right)\sqrt{\sinh(a+bx)}}{b\sqrt{\cosh(a+bx)}}$$

input
```
Integrate[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]],x]
```

output
```
(2*(Cosh[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, -Sinh[a + b*x]
^2]*Sqrt[Sinh[a + b*x]])/(b*Sqrt[Cosh[a + b*x]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3055, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\cos(ia+ibx)}}{\sqrt{-i\sin(ia+ibx)}}dx$$

$$\Big\downarrow \text{3055}$$

$$\frac{2\int \frac{\coth(a+bx)}{1-\coth^2(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}}{b}$$

$$\Big\downarrow \text{827}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}-\frac{1}{2}\int \frac{1}{\coth(a+bx)+1}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}-\frac{1}{2}\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\left(\frac{1}{2}\text{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)-\frac{1}{2}\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)\right)}{b}$$

input
```
Int[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]],x]
```

output
```
(2*(-1/2*ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]] + ArcTanh[Sqrt[Co
sh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/2))/b
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

Maple [F]

$$\int \frac{\sqrt{\cosh(bx+a)}}{\sqrt{\sinh(bx+a)}}dx$$

input
```
int(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x)
```

output
```
int(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(46) = 92$.

Time = 0.10 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.67

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = \frac{2\arctan\left(-\cosh(bx+a)^2 + 2(\cosh(bx+a)+\sinh(bx+a))\sqrt{\cosh(bx+a)}\sqrt{\sinh(bx+a)} - 2\cosh(\right.}{}$$

input
```
integrate(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(co
sh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*
x + a)^2) - log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(
cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(
b*x + a)^2))/b
```

Sympy [F]

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{\sqrt{\cosh{(a+bx)}}}{\sqrt{\sinh{(a+bx)}}}\,dx$$

input
```
integrate(cosh(b*x+a)**(1/2)/sinh(b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(cosh(a + b*x))/sqrt(sinh(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{\sqrt{\cosh{(bx+a)}}}{\sqrt{\sinh{(bx+a)}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cosh(b*x + a))/sqrt(sinh(b*x + a)), x)
```

Giac [F]

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{\sqrt{\cosh{(bx+a)}}}{\sqrt{\sinh{(bx+a)}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cosh(b*x + a))/sqrt(sinh(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{\sqrt{\cosh\left(a+b\,x\right)}}{\sqrt{\sinh\left(a+b\,x\right)}}\,dx$$

input
```
int(cosh(a + b*x)^(1/2)/sinh(a + b*x)^(1/2),x)
```

output
```
int(cosh(a + b*x)^(1/2)/sinh(a + b*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{\sqrt{\sinh\left(bx+a\right)}\,\sqrt{\cosh\left(bx+a\right)}}{\sinh\left(bx+a\right)}dx$$

input
```
int(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2),x)
```

output
```
int((sqrt(sinh(a + b*x))*sqrt(cosh(a + b*x)))/sinh(a + b*x),x)
```

3.16 $\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

output
```
-arctan(1/cosh(b*x+a)^(1/2)*sinh(b*x+a)^(1/2))/b+arctanh(1/cosh(b*x+a)^(1/
2)*sinh(b*x+a)^(1/2))/b-2*cosh(b*x+a)^(1/2)/b/sinh(b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2\cosh^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},-\frac{1}{4},\frac{3}{4},-\sinh^2(a+bx)\right)}{b\cosh^{\frac{3}{2}}(a+bx)\sqrt{\sinh(a+bx)}}$$

input
```
Integrate[Cosh[a + b*x]^(3/2)/Sinh[a + b*x]^(3/2),x]
```

output
```
(-2*(Cosh[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, -Sinh[a + b
*x]^2])/(b*Cosh[a + b*x]^(3/2)*Sqrt[Sinh[a + b*x]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3047, 3042, 3054, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ia+ibx)^{3/2}}{(-i\sin(ia+ibx))^{3/2}}dx$$

$$\Big\downarrow \text{3047}$$

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}dx-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}+\int \frac{\sqrt{-i\sin(ia+ibx)}}{\sqrt{\cos(ia+ibx)}}dx$$

$$\Big\downarrow \text{3054}$$

$$-\frac{2\int -\frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}}{b}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\int \frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}}{b}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{827}$$

$$-\frac{2\left(\frac{1}{2}\int \frac{1}{\tanh(a+bx)+1}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}-\frac{1}{2}\int \frac{1}{1-\tanh(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)-\frac{1}{2}\int \frac{1}{1-\tanh(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)\right)}{b}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

input
```
Int[Cosh[a + b*x]^(3/2)/Sinh[a + b*x]^(3/2),x]
```

output
```
(-2*(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/2 - ArcTanh[Sqrt[Sinh
[a + b*x]]/Sqrt[Cosh[a + b*x]]]/2))/b - (2*Sqrt[Cosh[a + b*x]])/(b*Sqrt[Si
nh[a + b*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Maple [F]

$$\int \frac{\cosh\left(bx+a\right)^{\frac{3}{2}}}{\sinh\left(bx+a\right)^{\frac{3}{2}}}dx$$

input
```
int(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x)
```

output
```
int(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(67) = 134$.

Time = 0.10 (sec) , antiderivative size = 311, normalized size of antiderivative = 3.94

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2\left(\cosh(bx+a)^2 + 2\cosh(bx+a)\sinh(bx+a) + \sinh(bx+a)^2 - 1\right)\arctan\left(-\cosh(bx+a)^2 + 2(\right.}{}$$

input
```
integrate(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
 - 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cos
h(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x
 + a)^2) + 4*cosh(b*x + a)^2 + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x
 + a) + sinh(b*x + a)^2 - 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sin
h(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh
(b*x + a) - sinh(b*x + a)^2) + 8*(cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh
(b*x + a))*sqrt(sinh(b*x + a)) + 8*cosh(b*x + a)*sinh(b*x + a) + 4*sinh(b*
x + a)^2 - 4)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sin
h(b*x + a)^2 - b)
```

Sympy [F]

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(cosh(b*x+a)**(3/2)/sinh(b*x+a)**(3/2),x)
```

output
```
Integral(cosh(a + b*x)**(3/2)/sinh(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\cosh\left(bx+a\right)^{\frac{3}{2}}}{\sinh\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(3/2)/sinh(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\cosh\left(bx+a\right)^{\frac{3}{2}}}{\sinh\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(3/2)/sinh(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\cosh(a+b\,x)^{3/2}}{\sinh\left(a+b\,x\right)^{3/2}}\,dx$$

input
```
int(cosh(a + b*x)^(3/2)/sinh(a + b*x)^(3/2),x)
```

output
```
int(cosh(a + b*x)^(3/2)/sinh(a + b*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}\,dx$$
$$= \frac{-2\sqrt{\sinh(bx+a)}\,\sqrt{\cosh(bx+a)}+\left(\int \frac{\sqrt{\sinh(bx+a)}\,\sqrt{\cosh(bx+a)}}{\cosh(bx+a)}dx\right)\sinh(bx+a)\,b}{\sinh(bx+a)\,b}$$

input
```
int(cosh(b*x+a)^(3/2)/sinh(b*x+a)^(3/2),x)
```

output
```
( - 2*sqrt(sinh(a + b*x))*sqrt(cosh(a + b*x)) + int((sqrt(sinh(a + b*x))*s
qrt(cosh(a + b*x)))/cosh(a + b*x),x)*sinh(a + b*x)*b)/(sinh(a + b*x)*b)
```

3.17 $\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

output
```
-arctan(cosh(b*x+a)^(1/2)/sinh(b*x+a)^(1/2))/b+arctanh(cosh(b*x+a)^(1/2)/s
inh(b*x+a)^(1/2))/b-2/3*cosh(b*x+a)^(3/2)/b/sinh(b*x+a)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.73

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2\sqrt[4]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{3}{4},\frac{1}{4},-\sinh^2(a+bx)\right)}{3b\sqrt{\cosh(a+bx)}\sinh^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[Cosh[a + b*x]^(5/2)/Sinh[a + b*x]^(5/2),x]
```

output
```
(-2*(Cosh[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4, -3/4, 1/4, -Sinh[a + b
*x]^2])/(3*b*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3047, 3042, 3055, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ia+ibx)^{5/2}}{(-i\sin(ia+ibx))^{5/2}}dx$$

$$\Big\downarrow \text{3047}$$

$$\int \frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}dx-\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}+\int \frac{\sqrt{\cos(ia+ibx)}}{\sqrt{-i\sin(ia+ibx)}}dx$$

$$\Big\downarrow \text{3055}$$

$$\frac{2\int \frac{\coth(a+bx)}{1-\coth^2(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}}{b}-\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{827}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}-\frac{1}{2}\int \frac{1}{\coth(a+bx)+1}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)}{b}-\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\coth(a+bx)}d\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}-\frac{1}{2}\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)\right)}{b}-\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\left(\frac{1}{2}\text{arctanh}\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)-\frac{1}{2}\arctan\left(\frac{\sqrt{\cosh(a+bx)}}{\sqrt{\sinh(a+bx)}}\right)\right)}{b}-\frac{2\cosh^{\frac{3}{2}}(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

input
```
Int[Cosh[a + b*x]^(5/2)/Sinh[a + b*x]^(5/2),x]
```

output
```
(2*(-1/2*ArcTan[Sqrt[Cosh[a + b*x]]/Sqrt[Sinh[a + b*x]]] + ArcTanh[Sqrt[Co
sh[a + b*x]]/Sqrt[Sinh[a + b*x]]]/2))/b - (2*Cosh[a + b*x]^(3/2))/(3*b*Sin
h[a + b*x]^(3/2))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

Maple [F]

$$\int \frac{\cosh \left(bx+a\right)^{\frac{5}{2}}}{\sinh \left(bx+a\right)^{\frac{5}{2}}}dx$$

input
```
int(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x)
```

output
```
int(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(67) = 134$.

Time = 0.11 (sec) , antiderivative size = 598, normalized size of antiderivative = 7.38

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(4*cosh(b*x + a)^4 + 16*cosh(b*x + a)*sinh(b*x + a)^3 + 4*sinh(b*x +
a)^4 + 8*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 6*(cosh(b*x + a)^4 + 4*
cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1
)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a)
)*sinh(b*x + a) + 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x
 + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x
+ a) - sinh(b*x + a)^2) - 8*cosh(b*x + a)^2 + 3*(cosh(b*x + a)^4 + 4*cosh(
b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sin
h(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sin
h(b*x + a) + 1)*log(-cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*s
qrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b*x + a)*sinh(b*x + a) - s
inh(b*x + a)^2) + 8*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + s
inh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*sq
rt(cosh(b*x + a))*sqrt(sinh(b*x + a)) + 16*(cosh(b*x + a)^3 - cosh(b*x + a
))*sinh(b*x + a) + 4)/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)
^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)
*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) +
 b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(b*x+a)**(5/2)/sinh(b*x+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{5}{2}}}{\sinh(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(5/2)/sinh(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{5}{2}}}{\sinh(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(5/2)/sinh(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\cosh(a+b\,x)^{5/2}}{\sinh(a+b\,x)^{5/2}}\,dx$$

input
```
int(cosh(a + b*x)^(5/2)/sinh(a + b*x)^(5/2),x)
```

output
```
int(cosh(a + b*x)^(5/2)/sinh(a + b*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cosh^{\frac{5}{2}}(a+bx)}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\sinh(bx+a)}\,\sqrt{\cosh(bx+a)}\,\cosh(bx+a)^2}{\sinh(bx+a)^3}dx$$

input
```
int(cosh(b*x+a)^(5/2)/sinh(b*x+a)^(5/2),x)
```

output
```
int((sqrt(sinh(a + b*x))*sqrt(cosh(a + b*x))*cosh(a + b*x)**2)/sinh(a + b*
x)**3,x)
```

3.18 $\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b} - \frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} - \frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

output
```
-arctan(1/cosh(b*x+a)^(1/2)*sinh(b*x+a)^(1/2))/b+arctanh(1/cosh(b*x+a)^(1/
2)*sinh(b*x+a)^(1/2))/b-2/5*cosh(b*x+a)^(5/2)/b/sinh(b*x+a)^(5/2)-2*cosh(b
*x+a)^(1/2)/b/sinh(b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.56

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2\cosh^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{5}{4},-\frac{1}{4},-\sinh^2(a+bx)\right)}{5b\cosh^{\frac{3}{2}}(a+bx)\sinh^{\frac{5}{2}}(a+bx)}$$

input
```
Integrate[Cosh[a + b*x]^(7/2)/Sinh[a + b*x]^(7/2),x]
```

output
```
(-2*(Cosh[a + b*x]^2)^(3/4)*Hypergeometric2F1[-5/4, -5/4, -1/4, -Sinh[a +
b*x]^2])/(5*b*Cosh[a + b*x]^(3/2)*Sinh[a + b*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3047, 3042, 3047, 3042, 3054, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ia+ibx)^{7/2}}{(-i\sin(ia+ibx))^{7/2}}dx$$

$$\Big\downarrow \text{3047}$$

$$\int \frac{\cosh^{\frac{3}{2}}(a+bx)}{\sinh^{\frac{3}{2}}(a+bx)}dx-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}+\int \frac{\cos(ia+ibx)^{3/2}}{(-i\sin(ia+ibx))^{3/2}}dx$$

$$\Big\downarrow \text{3047}$$

$$\int \frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}dx-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{-i\sin(ia+ibx)}}{\sqrt{\cos(ia+ibx)}}dx-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{3054}$$

$$-\frac{2\int -\frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}}{b}-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\int \frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}}{b}-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{827}$$

$$-\frac{2\left(\frac{1}{2}\int \frac{1}{\tanh(a+bx)+1}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}-\frac{1}{2}\int \frac{1}{1-\tanh(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)-\frac{1}{2}\int \frac{1}{1-\tanh(a+bx)}d\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)}{b}-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\left(\frac{1}{2}\arctan\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt{\sinh(a+bx)}}{\sqrt{\cosh(a+bx)}}\right)\right)}{b}-\frac{2\cosh^{\frac{5}{2}}(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\cosh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

input
```
Int[Cosh[a + b*x]^(7/2)/Sinh[a + b*x]^(7/2),x]
```

output
```
(-2*(ArcTan[Sqrt[Sinh[a + b*x]]/Sqrt[Cosh[a + b*x]]]/2 - ArcTanh[Sqrt[Sinh
[a + b*x]]/Sqrt[Cosh[a + b*x]]]/2))/b - (2*Cosh[a + b*x]^(5/2))/(5*b*Sinh[
a + b*x]^(5/2)) - (2*Sqrt[Cosh[a + b*x]])/(b*Sqrt[Sinh[a + b*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Maple [F]

$$\int \frac{\cosh (bx+a)^{\frac{7}{2}}}{\sinh (bx+a)^{\frac{7}{2}}}dx$$

input
```
int(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x)
```

output
```
int(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(88) = 176$.

Time = 0.11 (sec) , antiderivative size = 1001, normalized size of antiderivative = 9.44

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
-1/10*(24*cosh(b*x + a)^6 + 144*cosh(b*x + a)*sinh(b*x + a)^5 + 24*sinh(b*
x + a)^6 + 72*(5*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 72*cosh(b*x + a)^4
 + 96*(5*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 72*(5*cosh(b
*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 10*(cosh(b*x + a)^6 +
 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2
- 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - 3*cosh(b
*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*s
inh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^
3 + cosh(b*x + a))*sinh(b*x + a) - 1)*arctan(-cosh(b*x + a)^2 + 2*(cosh(b*
x + a) + sinh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2*cosh(b
*x + a)*sinh(b*x + a) - sinh(b*x + a)^2) + 72*cosh(b*x + a)^2 + 5*(cosh(b*
x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b
*x + a)^2 - 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3
- 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b*x + a
)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2*cosh
(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) - 1)*log(-cosh(b*x + a)^2 + 2*(
cosh(b*x + a) + sinh(b*x + a))*sqrt(cosh(b*x + a))*sqrt(sinh(b*x + a)) - 2
*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a)^2) + 16*(3*cosh(b*x + a)^5 +
15*cosh(b*x + a)*sinh(b*x + a)^4 + 3*sinh(b*x + a)^5 + 2*(15*cosh(b*x + a)
^2 - 2)*sinh(b*x + a)^3 - 4*cosh(b*x + a)^3 + 6*(5*cosh(b*x + a)^3 - 2*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(b*x+a)**(7/2)/sinh(b*x+a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\cosh\left(bx+a\right)^{\frac{7}{2}}}{\sinh\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(7/2)/sinh(b*x + a)^(7/2), x)
```

Giac [F]

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\cosh\left(bx+a\right)^{\frac{7}{2}}}{\sinh\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(7/2)/sinh(b*x + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\cosh(a+b\,x)^{7/2}}{\sinh\left(a+b\,x\right)^{7/2}}\,dx$$

input
```
int(cosh(a + b*x)^(7/2)/sinh(a + b*x)^(7/2),x)
```

output
```
int(cosh(a + b*x)^(7/2)/sinh(a + b*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{\cosh^{\frac{7}{2}}(a+bx)}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \frac{-2\sqrt{\sinh(bx+a)}\,\sqrt{\cosh(bx+a)}\,\cosh(bx+a)^2 - 10\sqrt{\sinh(bx+a)}\,\sqrt{\cosh(bx+a)}\,\sinh(bx+a)^2 +}{5\sinh(bx+a)^3\,b}$$

input
```
int(cosh(b*x+a)^(7/2)/sinh(b*x+a)^(7/2),x)
```

output
```
( - 2*sqrt(sinh(a + b*x))*sqrt(cosh(a + b*x))*cosh(a + b*x)**2 - 10*sqrt(s
inh(a + b*x))*sqrt(cosh(a + b*x))*sinh(a + b*x)**2 + 5*int((sqrt(sinh(a +
b*x))*sqrt(cosh(a + b*x)))/cosh(a + b*x),x)*sinh(a + b*x)**3*b)/(5*sinh(a
+ b*x)**3*b)
```

3.19 $\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

output
```
-1/2*3^(1/2)*arctan(1/3*(1+2*sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))*3^(1/2))
/b-1/2*ln(1-sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b+1/4*ln(1+sinh(b*x+a)^(2
/3)/cosh(b*x+a)^(2/3)+sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3))/b-3/4*sinh(b*x+
a)^(4/3)/b/cosh(b*x+a)^(4/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.38

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx = \frac{3\cosh^2(a+bx)^{2/3}\,\text{Hypergeometric2F1}\left(\frac{5}{3},\frac{5}{3},\frac{8}{3},-\sinh^2(a+bx)\right)\sinh^{\frac{10}{3}}(a+bx)}{10b\cosh^{\frac{4}{3}}(a+bx)}$$

input
```
Integrate[Sinh[a + b*x]^(7/3)/Cosh[a + b*x]^(7/3),x]
```

output
```
(3*(Cosh[a + b*x]^2)^(2/3)*Hypergeometric2F1[5/3, 5/3, 8/3, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(10/3))/(10*b*Cosh[a + b*x]^(4/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.88, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3046, 3042, 3054, 25, 807, 821, 16, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(-i\sin(ia+ibx))^{7/3}}{\cos(ia+ibx)^{7/3}}dx$$

$$\Big\downarrow \text{3046}$$

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}dx - \frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}+\int\frac{\sqrt[3]{-i\sin(ia+ibx)}}{\sqrt[3]{\cos(ia+ibx)}}dx$$

$$\Big\downarrow \text{ 3054}$$

$$-\frac{3\int-\frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{b}-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{3\int\frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{b}-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{3\int\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)(1-\tanh(a+bx))}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{2b}-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \text{ 821}$$

$$\frac{3\left(\frac{1}{3}\int\frac{1}{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\int\frac{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{3\left(-\frac{1}{3}\int\frac{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{3}{2}\int\frac{1}{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+3\int\frac{1}{-\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-4}d\left(\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \; 217$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\sqrt{3}\arctan\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \; 1103$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\log\left(\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)-\sqrt{3}\arctan\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

input
```
Int[Sinh[a + b*x]^(7/3)/Cosh[a + b*x]^(7/3),x]
```

output
```
(3*(-1/3*Log[1 - Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)] + (-(Sqrt[3]*Arc
Tan[(1 + (2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3))/Sqrt[3]]) + Log[1 +
(2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3)]/2)/3))/(2*b) - (3*Sinh[a + b*
x]^(4/3))/(4*b*Cosh[a + b*x]^(4/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Maple [F]

$$\int \frac{\sinh{(bx+a)}^{\frac{7}{3}}}{\cosh{(bx+a)}^{\frac{7}{3}}}dx$$

input
```
int(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x)
```

output
```
int(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1042 vs. $2(124) = 248$.

Time = 0.13 (sec) , antiderivative size = 1042, normalized size of antiderivative = 6.72

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x, algorithm="fricas")
```

output
```
-1/4*(2*(sqrt(3)*cosh(b*x + a)^4 + 4*sqrt(3)*cosh(b*x + a)*sinh(b*x + a)^3
 + sqrt(3)*sinh(b*x + a)^4 + 2*(3*sqrt(3)*cosh(b*x + a)^2 + sqrt(3))*sinh(
b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a)^3 + sqrt
(3)*cosh(b*x + a))*sinh(b*x + a) + sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x +
 a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 +
4*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh
(b*x + a)^(2/3) + sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a
) + sinh(b*x + a)^2 + 1)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x +
a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cos
h(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(
(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(
3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 2*(cosh(b*x +
 a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x +
a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)
^(1/3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x +
 a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*x +
a)^(1/3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*
x + a) + 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x
+ a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4
*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*(cosh(b*x + ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(b*x+a)**(7/3)/cosh(b*x+a)**(7/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx=\int \frac{\sinh\left(bx+a\right)^{\frac{7}{3}}}{\cosh\left(bx+a\right)^{\frac{7}{3}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)^(7/3)/cosh(b*x + a)^(7/3), x)
```

Giac [F]

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx=\int \frac{\sinh\left(bx+a\right)^{\frac{7}{3}}}{\cosh\left(bx+a\right)^{\frac{7}{3}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^(7/3)/cosh(b*x + a)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx=\int \frac{{\mathrm{sinh}\left(a+b\,x\right)}^{7/3}}{{\mathrm{cosh}\left(a+b\,x\right)}^{7/3}}\,dx$$

input
```
int(sinh(a + b*x)^(7/3)/cosh(a + b*x)^(7/3),x)
```

output
```
int(sinh(a + b*x)^(7/3)/cosh(a + b*x)^(7/3), x)
```

Reduce [F]

$$\int \frac{\sinh^{\frac{7}{3}}(a+bx)}{\cosh^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\sinh(bx+a)^{\frac{7}{3}}}{\cosh(bx+a)^{\frac{7}{3}}}dx$$

input
```
int(sinh(b*x+a)^(7/3)/cosh(b*x+a)^(7/3),x)
```

output
```
int((sinh(a + b*x)**(1/3)*sinh(a + b*x)**2)/(cosh(a + b*x)**(1/3)*cosh(a +
 b*x)**2),x)
```

3.20 $\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

output
```
-1/2*3^(1/2)*arctan(1/3*(1+2*cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))*3^(1/2))
/b-1/2*ln(1-cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b+1/4*ln(1+cosh(b*x+a)^(4
/3)/sinh(b*x+a)^(4/3)+cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b-3/2*sinh(b*x+
a)^(2/3)/b/cosh(b*x+a)^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.38

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = \frac{3\sqrt[3]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{4}{3},\frac{7}{3},-\sinh^2(a+bx)\right)\sinh^{\frac{8}{3}}(a+bx)}{8b\cosh^{\frac{2}{3}}(a+bx)}$$

input
```
Integrate[Sinh[a + b*x]^(5/3)/Cosh[a + b*x]^(5/3),x]
```

output
```
(3*(Cosh[a + b*x]^2)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(8/3))/(8*b*Cosh[a + b*x]^(2/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.88, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3046, 3042, 3055, 807, 821, 16, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(-i\sin(ia+ibx))^{5/3}}{\cos(ia+ibx)^{5/3}}dx$$

$$\Big\downarrow \text{3046}$$

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}dx - \frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}+\int\frac{\sqrt[3]{\cos(ia+ibx)}}{\sqrt[3]{-i\sin(ia+ibx)}}dx$$

$$\Big\downarrow \text{3055}$$

$$\frac{3\int\frac{\coth(a+bx)}{1-\coth^2(a+bx)}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{b}-\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{807}$$

$$\frac{3\int\frac{\cosh^{\frac{2}{3}}(a+bx)}{(1-\coth(a+bx))\sinh^{\frac{2}{3}}(a+bx)}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{2b}-\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{821}$$

$$\frac{3\left(\frac{1}{3}\int\frac{1}{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\int\frac{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}-\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{16}$$

$$\frac{3\left(-\frac{1}{3}\int\frac{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{1142}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{3}{2}\int\frac{1}{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+3\int\frac{1}{-\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-4}d\left(\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \; 217$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\sqrt{3}\arctan\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \; 1103$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\log\left(\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)-\sqrt{3}\arctan\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

input
```
Int[Sinh[a + b*x]^(5/3)/Cosh[a + b*x]^(5/3),x]
```

output
```
(3*(-1/3*Log[1 - Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3)] + (-(Sqrt[3]*Arc
Tan[(1 + (2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3))/Sqrt[3]]) + Log[1 +
(2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3)]/2)/3))/(2*b) - (3*Sinh[a + b*
x]^(2/3))/(2*b*Cosh[a + b*x]^(2/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

Maple [F]

$$\int \frac{\sinh\left(bx+a\right)^{\frac{5}{3}}}{\cosh\left(bx+a\right)^{\frac{5}{3}}}dx$$

input
```
int(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x)
```

output
```
int(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(124) = 248$.

Time = 0.12 (sec) , antiderivative size = 751, normalized size of antiderivative = 4.85

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x, algorithm="fricas")
```

output
```
-1/4*(2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) +
 sqrt(3)*sinh(b*x + a)^2 + sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 +
2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(
3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a
)^(1/3) - sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh
(b*x + a)^2 - 1)) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sin
h(b*x + a)^2 + 1)*log((cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 +
 sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x
+ a)^2 + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x +
 a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x +
a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b
*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh
(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 -
cosh(b*x + a))*sinh(b*x + a) + 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(
b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 -
 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1
)) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
+ 1)*log(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x +
a)^(2/3)*sinh(b*x + a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +
a)^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(b*x+a)**(5/3)/cosh(b*x+a)**(5/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\sinh(bx+a)^{\frac{5}{3}}}{\cosh(bx+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)^(5/3)/cosh(b*x + a)^(5/3), x)
```

Giac [F]

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\sinh(bx+a)^{\frac{5}{3}}}{\cosh(bx+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^(5/3)/cosh(b*x + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\sinh(a+b\,x)^{5/3}}{\cosh(a+b\,x)^{5/3}}\,dx$$

input
```
int(sinh(a + b*x)^(5/3)/cosh(a + b*x)^(5/3),x)
```

output
```
int(sinh(a + b*x)^(5/3)/cosh(a + b*x)^(5/3), x)
```

Reduce [F]

$$\int \frac{\sinh^{\frac{5}{3}}(a+bx)}{\cosh^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\sinh\left(bx+a\right)^{\frac{5}{3}}}{\cosh\left(bx+a\right)^{\frac{5}{3}}}dx$$

input
```
int(sinh(b*x+a)^(5/3)/cosh(b*x+a)^(5/3),x)
```

output
```
int((sinh(a + b*x)**(2/3)*sinh(a + b*x))/(cosh(a + b*x)**(2/3)*cosh(a + b*
x)),x)
```

3.21 $\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 192

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\sqrt[3]{\sinh(a+bx)}}\right)}{2b} - \frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}$$

output
```
1/2*3^(1/2)*arctan(1/3*(1-2*cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))*3^(1/2))/
b-1/2*3^(1/2)*arctan(1/3*(1+2*cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))*3^(1/2)
)/b+arctanh(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))/b+1/2*arctanh(cosh(b*x+a)
^(1/3)/(1+cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/sinh(b*x+a)^(1/3))/b-3*sinh
(b*x+a)^(1/3)/b/cosh(b*x+a)^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.31

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx = \frac{3\sqrt[6]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{7}{6},\frac{13}{6},-\sinh^2(a+bx)\right)\sinh^{\frac{7}{3}}(a+bx)}{7b\sqrt[3]{\cosh(a+bx)}}$$

input
```
Integrate[Sinh[a + b*x]^(4/3)/Cosh[a + b*x]^(4/3),x]
```

output
```
(3*(Cosh[a + b*x]^2)^(1/6)*Hypergeometric2F1[7/6, 7/6, 13/6, -Sinh[a + b*x
]^2]*Sinh[a + b*x]^(7/3))/(7*b*Cosh[a + b*x]^(1/3))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.27, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3046, 3042, 3055, 825, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(-i\sin(ia+ibx))^{4/3}}{\cos(ia+ibx)^{4/3}}dx$$

$$\Big\downarrow \; 3046$$

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}dx-\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}+\int \frac{\cos(ia+ibx)^{2/3}}{(-i\sin(ia+ibx))^{2/3}}dx$$

$$\Big\downarrow \; 3055$$

$$\frac{3\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{(1-\coth^2(a+bx))\sinh^{\frac{4}{3}}(a+bx)}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{b}-\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}$$

$$\Big\downarrow \; 825$$

$$\frac{3\left(\frac{1}{3}\int \frac{1}{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+\frac{1}{3}\int -\frac{\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}{2\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+\frac{1}{3}\int -\frac{\cdots}{2\left(\frac{\cosh^{\cdots}}{\sinh^{\cdots}}\cdots\right)}\cdots\right.}{b}$$

$$\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{3\left(\frac{1}{3}\int \frac{1}{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\frac{1}{6}\int \frac{\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\frac{1}{6}\int \frac{1-\frac{\sqrt[3]{\cos\cdots}}{\sqrt[3]{\sin\cdots}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\cdots}}{\sqrt[3]{\cdots}}}\cdots\right.}{b}$$

$$\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}$$

$$\Big\downarrow \; 219$$

$$\frac{3\Bigg(-\frac{1}{6}\int \frac{\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\frac{1}{6}\int \frac{1-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+}{b}$$

$$\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}$$

$$\Big\downarrow \; 1142$$

$$3\Bigg(\frac{1}{6}\Bigg(-\frac{3}{2}\int \frac{1}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\frac{1}{2}\int -\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+}}{\sqrt[3]{\sinh(a+}}$$

$$\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}$$

$$\Big\downarrow \; 25$$

$$3\Bigg(\frac{1}{6}\Bigg(\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\frac{3}{2}\int \frac{1}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}$$

$$\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}$$

$$\Big\downarrow \; 1083$$

$$3\Bigg(\frac{1}{6}\Bigg(3\int \frac{1}{-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-3}d\left(\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-1\right)+\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\Bigg)+\frac{1}{6}\Bigg($$

$$\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}$$

$$\Big\downarrow \; 217$$

$$\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{}{\frac{\cosh^{\frac{2}{3}}}{\sinh^{\frac{2}{3}}}}\right.\right.}{}$$

$$\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3\left(\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)\right)+\frac{1}{6}\left(\frac{1}{2}\log\left(\frac{\cosh^{\frac{2}{3}}(a+}{\sinh^{\frac{2}{3}}(a+}\right.\right.\right.}{b}$$

$$\frac{3\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}$$

input

```
Int[Sinh[a + b*x]^(4/3)/Cosh[a + b*x]^(4/3),x]
```

output

```
(3*(ArcTanh[Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/3 + (-(Sqrt[3]*ArcTan
[(-1 + (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]^(1/3))/Sqrt[3]]) - Log[1 + Co
sh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) - Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^
(1/3)]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]
^(1/3))/Sqrt[3]]) + Log[1 + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) + Cosh
[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/2)/6))/b - (3*Sinh[a + b*x]^(1/3))/(b
*Cosh[a + b*x]^(1/3))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

Maple [F]

$$\int \frac{\sinh\left(bx+a\right)^{\frac{4}{3}}}{\cosh\left(bx+a\right)^{\frac{4}{3}}}dx$$

input
```
int(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x)
```

output
```
int(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1003 vs. $2(156) = 312$.

Time = 0.14 (sec) , antiderivative size = 1003, normalized size of antiderivative = 5.22

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
1/4*(2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) +
sqrt(3)*sinh(b*x + a)^2 + sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2
*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3
)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)
^(1/3) + sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(
b*x + a)^2 + 1)) + 2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*si
nh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + sqrt(3))*arctan(-1/3*(sqrt(3)*cosh
(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a
)^2 - 4*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3
)*sinh(b*x + a)^(1/3) + sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b
*x + a) + sinh(b*x + a)^2 + 1)) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(
b*x + a) + sinh(b*x + a)^2 + 1)*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) +
sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a)
+ sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)
*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*s
inh(b*x + a) + sinh(b*x + a)^2 + 1)) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a
)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*log((cosh(b*x + a)^2 + 2*(cosh(b*x
+ a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*cosh(b*x
 + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x +
 a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) - (cosh(b*x + a)^2 + 2*cosh(b...
```

Sympy [F]

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx$$

input
```
integrate(sinh(b*x+a)**(4/3)/cosh(b*x+a)**(4/3),x)
```

output
```
Integral(sinh(a + b*x)**(4/3)/cosh(a + b*x)**(4/3), x)
```

Maxima [F]

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\sinh(bx+a)^{\frac{4}{3}}}{\cosh(bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)^(4/3)/cosh(b*x + a)^(4/3), x)
```

Giac [F]

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\sinh(bx+a)^{\frac{4}{3}}}{\cosh(bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^(4/3)/cosh(b*x + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\sinh(a+b\,x)^{4/3}}{\cosh(a+b\,x)^{4/3}}\,dx$$

input
```
int(sinh(a + b*x)^(4/3)/cosh(a + b*x)^(4/3),x)
```

output
```
int(sinh(a + b*x)^(4/3)/cosh(a + b*x)^(4/3), x)
```

Reduce [F]

$$\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\sinh(bx+a)^{\frac{4}{3}}}{\cosh(bx+a)^{\frac{4}{3}}}dx$$

input
```
int(sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3),x)
```

output
```
int((sinh(a + b*x)**(1/3)*sinh(a + b*x))/(cosh(a + b*x)**(1/3)*cosh(a + b*
x)),x)
```

3.22 $\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 167

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}\right)}{2b}$$

output
```
1/2*3^(1/2)*arctan(1/3*(1-2*sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3))*3^(1/2))/
b-1/2*3^(1/2)*arctan(1/3*(1+2*sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3))*3^(1/2)
)/b+arctanh(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3))/b+1/2*arctanh(sinh(b*x+a)
^(1/3)/cosh(b*x+a)^(1/3)/(1+sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3)))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.35

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = \frac{3\cosh^2(a+bx)^{5/6}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{5}{6},\frac{11}{6},-\sinh^2(a+bx)\right)\sinh^{\frac{5}{3}}(a+bx)}{5b\cosh^{\frac{5}{3}}(a+bx)}$$

input
```
Integrate[Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3),x]
```

output
```
(3*(Cosh[a + b*x]^2)^(5/6)*Hypergeometric2F1[5/6, 5/6, 11/6, -Sinh[a + b*x
]^2]*Sinh[a + b*x]^(5/3))/(5*b*Cosh[a + b*x]^(5/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.29, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3054, 25, 825, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(-i\sin(ia+ibx))^{2/3}}{\cos(ia+ibx)^{2/3}}dx$$

$$\Big\downarrow \text{ 3054}$$

$$-\frac{3\int -\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)(1-\tanh^2(a+bx))}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{3\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)(1-\tanh^2(a+bx))}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{b}$$

$$\Big\downarrow \text{ 825}$$

$$-\frac{3\left(-\frac{1}{3}\int \frac{1}{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-\frac{1}{3}\int -\frac{\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}{2\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-\frac{1}{3}\int -\frac{\cdots}{2\left(\cdots\right.}\cdots\right.}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3\left(-\frac{1}{3}\int \frac{1}{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{1}{6}\int \frac{\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{1}{6}\int \frac{1-\frac{\sqrt[3]{\cdots}}{\sqrt[3]{\cdots}}}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\cdots}\cdots\right.}{b}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{3\left(\frac{1}{6}\int \frac{\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{1}{6}\int \frac{1-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-\cdots\right.}{b}$$

$$\Big\downarrow \text{ 1142}$$

$$-\frac{3\left(\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{1}{2}\int -\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+}}{\sqrt[3]{\cosh(a+}}\right.\right.}{}$$

$\downarrow$ 25

$$-\frac{3\left(\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+ba}}{\sqrt[3]{\cosh(a+ba}}\right.\right.}{}$$

$\downarrow$ 1083

$$-\frac{3\left(\frac{1}{6}\left(-3\int \frac{1}{-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-3}d\left(\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-1\right)-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)+\right.}{}$$

$\downarrow$ 217

$$-\frac{3\left(\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-1}{\sqrt{3}}\right)-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)+\frac{1}{6}\left(\sqrt{3}\,\text{arct}\right.\right.}{}$$

$\downarrow$ 1103

$$-\frac{3\left(\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)\right)+\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{}}{\sqrt[3]{}}\right.\right.\right.}{b}$$

input
```
Int[Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3),x]
```

output
```
(-3*(-1/3*ArcTanh[Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3)] + (Sqrt[3]*ArcT
an[(-1 + (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^(1/3))/Sqrt[3]] + Log[1 - S
inh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[a + b*x]^(2/3)/Cosh[a + b*x]
^(2/3)]/2)/6 + (Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^
(1/3))/Sqrt[3]] - Log[1 + Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[a
 + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/2)/6))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Maple [F]

$$\int \frac{\sinh \left(bx+a\right)^{\frac{2}{3}}}{\cosh \left(bx+a\right)^{\frac{2}{3}}}dx$$

input
```
int(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x)
```

output
```
int(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 727 vs. $2(135) = 270$.

Time = 0.11 (sec) , antiderivative size = 727, normalized size of antiderivative = 4.35

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(3)*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x +
a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + sq
rt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) - sqrt(3))/(c
osh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + 2
*sqrt(3)*arctan(-1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*si
nh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 - 4*(sqrt(3)*cosh(b*x + a) + sqrt(3)
*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) - sqrt(3))/(cosh(b
*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + log((c
osh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sin
h(b*x + a)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*s
inh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/
(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) -
 log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2
/3)*sinh(b*x + a)^(1/3) - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^
(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^
2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
- 1)) + 2*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*
x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*
x + a)^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x
+ a)^2 - 1)) - 2*log(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + ...
```

Sympy [F]

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx$$

input
```
integrate(sinh(b*x+a)**(2/3)/cosh(b*x+a)**(2/3),x)
```

output
```
Integral(sinh(a + b*x)**(2/3)/cosh(a + b*x)**(2/3), x)
```

Maxima [F]

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\sinh(bx+a)^{\frac{2}{3}}}{\cosh(bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)^(2/3)/cosh(b*x + a)^(2/3), x)
```

Giac [F]

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\sinh(bx+a)^{\frac{2}{3}}}{\cosh(bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^(2/3)/cosh(b*x + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\sinh(a+b\,x)^{2/3}}{\cosh\left(a+b\,x\right)^{2/3}}\,dx$$

input
```
int(sinh(a + b*x)^(2/3)/cosh(a + b*x)^(2/3),x)
```

output
```
int(sinh(a + b*x)^(2/3)/cosh(a + b*x)^(2/3), x)
```

Reduce [F]

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\sinh\left(bx+a\right)^{\frac{2}{3}}}{\cosh\left(bx+a\right)^{\frac{2}{3}}}dx$$

input
```
int(sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3),x)
```

output
```
int(sinh(a + b*x)**(2/3)/cosh(a + b*x)**(2/3),x)
```

3.23 $\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b}$$

output
```
-1/2*3^(1/2)*arctan(1/3*(1+2*sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))*3^(1/2))
/b-1/2*ln(1-sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b+1/4*ln(1+sinh(b*x+a)^(2
/3)/cosh(b*x+a)^(2/3)+sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx = \frac{3\cosh^2(a+bx)^{2/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{2}{3},\frac{5}{3},-\sinh^2(a+bx)\right)\sinh^{\frac{4}{3}}(a+bx)}{4b\cosh^{\frac{4}{3}}(a+bx)}$$

input
```
Integrate[Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3),x]
```

output
```
(3*(Cosh[a + b*x]^2)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(4/3))/(4*b*Cosh[a + b*x]^(4/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.85, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3054, 25, 807, 821, 16, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt[3]{-i\sin(ia+ibx)}}{\sqrt[3]{\cos(ia+ibx)}}\,dx$$

$$\Big\downarrow \ 3054$$

$$-\frac{3\int -\frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}\,d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\int \frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{b}$$

$$\Big\downarrow \text{807}$$

$$\frac{3\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)(1-\tanh(a+bx))}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{2b}$$

$$\Big\downarrow \text{821}$$

$$\frac{3\left(\frac{1}{3}\int \frac{1}{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\int \frac{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$\Big\downarrow \text{16}$$

$$\frac{3\left(-\frac{1}{3}\int \frac{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}$$

$$\Big\downarrow \text{1142}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{3}{2}\int \frac{1}{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}$$

$$\Big\downarrow \text{1083}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+3\int \frac{1}{-\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-4}d\left(\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}$$

$$\Big\downarrow \text{217}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\sqrt{3}\arctan\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}$$

$$\downarrow \text{1103}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\log\left(\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)-\sqrt{3}\arctan\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}$$

input
```
Int[Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3),x]
```

output
```
(3*(-1/3*Log[1 - Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)] + (-(Sqrt[3]*Arc
Tan[(1 + (2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3))/Sqrt[3]]) + Log[1 +
(2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3)]/2)/3))/(2*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Maple [F]

$$\int \frac{\sinh(bx+a)^{\frac{1}{3}}}{\cosh(bx+a)^{\frac{1}{3}}}dx$$

input
```
int(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x)
```

output
```
int(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(103) = 206$.

Time = 0.11 (sec) , antiderivative size = 572, normalized size of antiderivative = 4.47

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(3)*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x +
 a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + s
qrt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + sqrt(3))/(
cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) -
log((cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 2*(cosh(b
*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*
x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x
+ a)^(1/3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b
*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*
x + a)^(1/3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sin
h(b*x + a) + 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(
b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2
 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*log(-(cosh(
b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*
x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/(cosh(
b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)))/b
```

Sympy [F]

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx = \int \frac{\sqrt[3]{\sinh{(a+bx)}}}{\sqrt[3]{\cosh{(a+bx)}}}\,dx$$

input
```
integrate(sinh(b*x+a)**(1/3)/cosh(b*x+a)**(1/3),x)
```

output
```
Integral(sinh(a + b*x)**(1/3)/cosh(a + b*x)**(1/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx=\int \frac{\sinh\left(bx+a\right)^{\frac{1}{3}}}{\cosh\left(bx+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)^(1/3)/cosh(b*x + a)^(1/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx=\int \frac{\sinh\left(bx+a\right)^{\frac{1}{3}}}{\cosh\left(bx+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^(1/3)/cosh(b*x + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx=\int \frac{{\mathrm{sinh}\left(a+b\,x\right)}^{1/3}}{{\mathrm{cosh}\left(a+b\,x\right)}^{1/3}}\,dx$$

input
```
int(sinh(a + b*x)^(1/3)/cosh(a + b*x)^(1/3),x)
```

output
```
int(sinh(a + b*x)^(1/3)/cosh(a + b*x)^(1/3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\,dx = \int \frac{\sinh\left(bx+a\right)^{\frac{1}{3}}}{\cosh\left(bx+a\right)^{\frac{1}{3}}}dx$$

input
```
int(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3),x)
```

output
```
int(sinh(a + b*x)**(1/3)/cosh(a + b*x)**(1/3),x)
```

3.24 $\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b}$$

output
```
-1/2*3^(1/2)*arctan(1/3*(1+2*cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))*3^(1/2))
/b-1/2*ln(1-cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b+1/4*ln(1+cosh(b*x+a)^(4
/3)/sinh(b*x+a)^(4/3)+cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx = \frac{3\sqrt[3]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{3},\frac{4}{3},-\sinh^2(a+bx)\right)\sinh^{\frac{2}{3}}(a+bx)}{2b\cosh^{\frac{2}{3}}(a+bx)}$$

input
```
Integrate[Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3),x]
```

output
```
(3*(Cosh[a + b*x]^2)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(2/3))/(2*b*Cosh[a + b*x]^(2/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.85, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3055, 807, 821, 16, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{\cos(ia+ibx)}}{\sqrt[3]{-i\sin(ia+ibx)}}dx$$

$$\Big\downarrow \text{3055}$$

$$\frac{3\int \frac{\coth(a+bx)}{1-\coth^2(a+bx)}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{b}$$

$$\Big\downarrow \ 807$$

$$\frac{3\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{(1-\coth(a+bx))\sinh^{\frac{2}{3}}(a+bx)}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{2b}$$

$$\Big\downarrow \ 821$$

$$\frac{3\left(\frac{1}{3}\int \frac{1}{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\int \frac{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$\Big\downarrow \ 16$$

$$\frac{3\left(-\frac{1}{3}\int \frac{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}$$

$$\Big\downarrow \ 1142$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{3}{2}\int \frac{1}{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}$$

$$\Big\downarrow \ 1083$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+3\int \frac{1}{-\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-4}d\left(\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}$$

$$\Big\downarrow \ 217$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\sqrt{3}\arctan\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}$$

$$\Big\downarrow \ 1103$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\log\left(\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)-\sqrt{3}\arctan\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}$$

input
```
Int[Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3),x]
```

output
```
(3*(-1/3*Log[1 - Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3)] + (-(Sqrt[3]*Arc
Tan[(1 + (2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3))/Sqrt[3]]) + Log[1 +
(2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3)]/2)/3))/(2*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

Maple [F]

$$\int \frac{\cosh (bx+a)^{\frac{1}{3}}}{\sinh (bx+a)^{\frac{1}{3}}}dx$$

input
```
int(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x)
```

output
```
int(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(103) = 206$.

Time = 0.10 (sec) , antiderivative size = 578, normalized size of antiderivative = 4.52

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(3)*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x +
 a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + s
qrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) - sqrt(3))/(
cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) -
log((cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 2*(cosh(b
*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*
x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x
+ a)^(1/3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b
*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*
x + a)^(1/3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sin
h(b*x + a) + 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(
b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2
 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*log(-(cosh(
b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*
x + a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/(cosh(
b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)))/b
```

Sympy [F]

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx = \int \frac{\sqrt[3]{\cosh{(a+bx)}}}{\sqrt[3]{\sinh{(a+bx)}}}\,dx$$

input
```
integrate(cosh(b*x+a)**(1/3)/sinh(b*x+a)**(1/3),x)
```

output
```
Integral(cosh(a + b*x)**(1/3)/sinh(a + b*x)**(1/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx = \int \frac{\cosh{(bx+a)}^{\frac{1}{3}}}{\sinh{(bx+a)}^{\frac{1}{3}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(1/3)/sinh(b*x + a)^(1/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx = \int \frac{\cosh\left(bx+a\right)^{\frac{1}{3}}}{\sinh\left(bx+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(1/3)/sinh(b*x + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx = \int \frac{\cosh(a+b\,x)^{1/3}}{\sinh\left(a+b\,x\right)^{1/3}}\,dx$$

input
```
int(cosh(a + b*x)^(1/3)/sinh(a + b*x)^(1/3),x)
```

output
```
int(cosh(a + b*x)^(1/3)/sinh(a + b*x)^(1/3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\,dx = \int \frac{\cosh\left(bx+a\right)^{\frac{1}{3}}}{\sinh\left(bx+a\right)^{\frac{1}{3}}}dx$$

input
```
int(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3),x)
```

output
```
int(cosh(a + b*x)**(1/3)/sinh(a + b*x)**(1/3),x)
```

3.25 $\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 167

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\sqrt{3}}\right)}{2b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)}{b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\cosh(a+bx)}}{\left(1+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\sqrt[3]{\sinh(a+bx)}}\right)}{2b}$$

output
```
1/2*3^(1/2)*arctan(1/3*(1-2*cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))*3^(1/2))/
b-1/2*3^(1/2)*arctan(1/3*(1+2*cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))*3^(1/2)
)/b+arctanh(cosh(b*x+a)^(1/3)/sinh(b*x+a)^(1/3))/b+1/2*arctanh(cosh(b*x+a)
^(1/3)/(1+cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/sinh(b*x+a)^(1/3))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.34

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx = \frac{3\sqrt[6]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{6},\frac{7}{6},-\sinh^2(a+bx)\right)\sqrt[3]{\sinh(a+bx)}}{b\sqrt[3]{\cosh(a+bx)}}$$

input
```
Integrate[Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3),x]
```

output
```
(3*(Cosh[a + b*x]^2)^(1/6)*Hypergeometric2F1[1/6, 1/6, 7/6, -Sinh[a + b*x]
^2]*Sinh[a + b*x]^(1/3))/(b*Cosh[a + b*x]^(1/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.30, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3055, 825, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(ia+ibx)^{2/3}}{(-i\sin(ia+ibx))^{2/3}}dx$$

$$\Big\downarrow \; 3055$$

$$\frac{3\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{(1-\coth^2(a+bx))\sinh^{\frac{4}{3}}(a+bx)}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{b}$$

$$\Big\downarrow \; 825$$

$$\frac{3\left(\frac{1}{3}\int \frac{1}{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+\frac{1}{3}\int -\frac{\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}{2\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+\frac{1}{3}\int -\frac{\cdots}{2\left(\frac{\cosh^{\frac{2}{3}}}{\sinh^{\frac{2}{3}}}\cdots\right)}\cdots\right)}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{3\left(\frac{1}{3}\int \frac{1}{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\frac{1}{6}\int \frac{\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\frac{1}{6}\int \frac{1-\frac{\sqrt[3]{\cos\cdots}}{\sqrt[3]{\sin\cdots}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\cdots}}{\sqrt[3]{\cdots}}\cdots}\cdots\right)}{b}$$

$$\Big\downarrow \; 219$$

$$\frac{3\left(-\frac{1}{6}\int \frac{\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\frac{1}{6}\int \frac{1-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+\cdots\right)}{b}$$

$$\Big\downarrow \; 1142$$

$$\frac{3\left(\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\frac{1}{2}\int -\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+\cdots}}{\sqrt[3]{\sinh(a+\cdots}}\cdots\right.\right.}{}$$

$$\Big\downarrow \; 25$$

$$3\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\frac{3}{2}\int\frac{1}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right.\right.$$

$$\Big\downarrow \textbf{1083}$$

$$3\left(\frac{1}{6}\left(3\int\frac{1}{-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-3}d\left(\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-1\right)+\frac{1}{2}\int\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}\right)+\frac{1}{6}\left(\right.\right.$$

$$\Big\downarrow \textbf{217}$$

$$3\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{}{\frac{\cosh^{\frac{2}{3}}}{\sinh^{\frac{2}{3}}}}\right.\right.$$

$$\Big\downarrow \textbf{1103}$$

$$\frac{3\left(\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}+1\right)\right)+\frac{1}{6}\left(\frac{1}{2}\log\left(\frac{\cosh^{\frac{2}{3}}(a+}{\sinh^{\frac{2}{3}}(a+}\right.\right.\right.}{b}$$

input
```
Int[Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3),x]
```

output
```
(3*(ArcTanh[Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/3 + (-(Sqrt[3]*ArcTan
[(-1 + (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]^(1/3))/Sqrt[3]]) - Log[1 + Co
sh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) - Cosh[a + b*x]^(1/3)/Sinh[a + b*x]^
(1/3)]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + (2*Cosh[a + b*x]^(1/3))/Sinh[a + b*x]
^(1/3))/Sqrt[3]]) + Log[1 + Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3) + Cosh
[a + b*x]^(1/3)/Sinh[a + b*x]^(1/3)]/2)/6))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

Maple [F]

$$\int \frac{\cosh \left(bx+a\right)^{\frac{2}{3}}}{\sinh \left(bx+a\right)^{\frac{2}{3}}}dx$$

input
```
int(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x)
```

output
```
int(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 723 vs. $2(135) = 270$.

Time = 0.11 (sec) , antiderivative size = 723, normalized size of antiderivative = 4.33

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(3)*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x +
a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a) + sq
rt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + sqrt(3))/(c
osh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + 2
*sqrt(3)*arctan(-1/3*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*si
nh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 - 4*(sqrt(3)*cosh(b*x + a) + sqrt(3)
*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + sqrt(3))/(cosh(b
*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + log((c
osh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sin
h(b*x + a)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*s
inh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)/
(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) +
 2*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x + a)^
(2/3)*sinh(b*x + a)^(1/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^
2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
+ 1)) - log((cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x
+ a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*
x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*
x + a)^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x
+ a)^2 + 1)) - 2*log(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + ...
```

Sympy [F]

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx$$

input
```
integrate(cosh(b*x+a)**(2/3)/sinh(b*x+a)**(2/3),x)
```

output
```
Integral(cosh(a + b*x)**(2/3)/sinh(a + b*x)**(2/3), x)
```

Maxima [F]

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{2}{3}}}{\sinh(bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(2/3)/sinh(b*x + a)^(2/3), x)
```

Giac [F]

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{2}{3}}}{\sinh(bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(2/3)/sinh(b*x + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\cosh(a+b\,x)^{2/3}}{\sinh(a+b\,x)^{2/3}}\,dx$$

input
```
int(cosh(a + b*x)^(2/3)/sinh(a + b*x)^(2/3),x)
```

output
```
int(cosh(a + b*x)^(2/3)/sinh(a + b*x)^(2/3), x)
```

Reduce [F]

$$\int \frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{2}{3}}}{\sinh(bx+a)^{\frac{2}{3}}}dx$$

input

```
int(cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3),x)
```

output

```
int(cosh(a + b*x)**(2/3)/sinh(a + b*x)**(2/3),x)
```

3.26 $\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 192

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\sqrt{3}}\right)}{2b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)}{b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}\right)}{2b} - \frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

output
```
1/2*3^(1/2)*arctan(1/3*(1-2*sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3))*3^(1/2))/
b-1/2*3^(1/2)*arctan(1/3*(1+2*sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3))*3^(1/2)
)/b+arctanh(sinh(b*x+a)^(1/3)/cosh(b*x+a)^(1/3))/b+1/2*arctanh(sinh(b*x+a)
^(1/3)/cosh(b*x+a)^(1/3)/(1+sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3)))/b-3*cosh
(b*x+a)^(1/3)/b/sinh(b*x+a)^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.30

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = -\frac{3\cosh^2(a+bx)^{5/6}\text{Hypergeometric2F1}\left(-\frac{1}{6},-\frac{1}{6},\frac{5}{6},-\sinh^2(a+bx)\right)}{b\cosh^{\frac{5}{3}}(a+bx)\sqrt[3]{\sinh(a+bx)}}$$

input
```
Integrate[Cosh[a + b*x]^(4/3)/Sinh[a + b*x]^(4/3),x]
```

output
```
(-3*(Cosh[a + b*x]^2)^(5/6)*Hypergeometric2F1[-1/6, -1/6, 5/6, -Sinh[a + b
*x]^2])/(b*Cosh[a + b*x]^(5/3)*Sinh[a + b*x]^(1/3))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.26, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3047, 3042, 3054, 25, 825, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ia+ibx)^{4/3}}{(-i\sin(ia+ibx))^{4/3}}dx$$

$$\Big\downarrow \text{3047}$$

$$\int \frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}dx-\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}+\int \frac{(-i\sin(ia+ibx))^{2/3}}{\cos(ia+ibx)^{2/3}}dx$$

$$\Big\downarrow \text{3054}$$

$$-\frac{3\int -\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)(1-\tanh^2(a+bx))}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{b}-\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\int \frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)(1-\tanh^2(a+bx))}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{b}-\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

$$\Big\downarrow \text{825}$$

$$-\frac{3\left(-\frac{1}{3}\int \frac{1}{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-\frac{1}{3}\int -\frac{\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}{2\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-\frac{1}{3}\int -\frac{}{2(}\right.}{b}$$

$$\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\left(-\frac{1}{3}\int \frac{1}{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{1}{6}\int \frac{\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{1}{6}\int \frac{1-\frac{\sqrt[3]{}}{\sqrt[3]{}}}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-}\right.}{b}$$

$$\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{3\left(\frac{1}{6}\int \frac{\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{1}{6}\int \frac{1-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}} -}{b}$$

$$\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

$$\Big\downarrow \text{1142}$$

$$-\frac{3\left(\frac{1}{6}\left(\left(\frac{3}{2}\int \frac{1}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+\frac{1}{2}\int -\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+}}{\sqrt[3]{\cosh(a+}}\right.\right.\right.}{}$$

$$\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3\left(\frac{1}{6}\left(\left(\frac{3}{2}\int \frac{1}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx}}{\sqrt[3]{\cosh(a+bx}}\right.\right.\right.}{}$$

$$\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{3\left(\frac{1}{6}\left(-3\int \frac{1}{-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-3}d\left(\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-1\right)-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)+\right.}{}$$

$$\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{3\left(\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-1}{\sqrt{3}}\right)-\frac{1}{2}\int\frac{1-\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}\right)+\frac{1}{6}\left(\sqrt{3}\arctan\right.\right.}{}$$

$$\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

$$\Big\downarrow \; 1103$$

$$-\frac{3\left(\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}-1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}+1\right)\right)+\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\ }}{\sqrt[3]{\ }}\right.\right.\right.}{b}$$

$$\frac{3\sqrt[3]{\cosh(a+bx)}}{b\sqrt[3]{\sinh(a+bx)}}$$

input
```
Int[Cosh[a + b*x]^(4/3)/Sinh[a + b*x]^(4/3),x]
```

output
```
(-3*(-1/3*ArcTanh[Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3)] + (Sqrt[3]*ArcT
an[(-1 + (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^(1/3))/Sqrt[3]] + Log[1 - S
inh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[a + b*x]^(2/3)/Cosh[a + b*x]
^(2/3)]/2)/6 + (Sqrt[3]*ArcTan[(1 + (2*Sinh[a + b*x]^(1/3))/Cosh[a + b*x]^
(1/3))/Sqrt[3]] - Log[1 + Sinh[a + b*x]^(1/3)/Cosh[a + b*x]^(1/3) + Sinh[a
 + b*x]^(2/3)/Cosh[a + b*x]^(2/3)]/2)/6))/b - (3*Cosh[a + b*x]^(1/3))/(b*S
inh[a + b*x]^(1/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Maple [F]

$$\int \frac{\cosh(bx+a)^{\frac{4}{3}}}{\sinh(bx+a)^{\frac{4}{3}}}dx$$

input
```
int(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x)
```

output
```
int(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1013 vs. $2(156) = 312$.

Time = 0.12 (sec) , antiderivative size = 1013, normalized size of antiderivative = 5.28

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
1/4*(2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) +
sqrt(3)*sinh(b*x + a)^2 - sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 + 2
*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(3
)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)
^(2/3) - sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(
b*x + a)^2 - 1)) + 2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*si
nh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 - sqrt(3))*arctan(-1/3*(sqrt(3)*cosh
(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a
)^2 - 4*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3
)*sinh(b*x + a)^(2/3) - sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b
*x + a) + sinh(b*x + a)^2 - 1)) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(
b*x + a) + sinh(b*x + a)^2 - 1)*log((cosh(b*x + a)^2 + 2*(cosh(b*x + a) +
sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a)
+ sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)
*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*s
inh(b*x + a) + sinh(b*x + a)^2 - 1)) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*
sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log((cosh(b*x + a)^2 + 2*(cosh(b*x +
a) + sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)^(1/3) - 2*(cosh(b*x
+ a) + sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x
 + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)/(cosh(b*x + a)^2 + 2*cosh(b*...
```

Sympy [F]

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx$$

input
```
integrate(cosh(b*x+a)**(4/3)/sinh(b*x+a)**(4/3),x)
```

output
```
Integral(cosh(a + b*x)**(4/3)/sinh(a + b*x)**(4/3), x)
```

Maxima [F]

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{4}{3}}}{\sinh(bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(4/3)/sinh(b*x + a)^(4/3), x)
```

Giac [F]

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{4}{3}}}{\sinh(bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(4/3)/sinh(b*x + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\cosh(a+b\,x)^{4/3}}{\sinh(a+b\,x)^{4/3}}\,dx$$

input
```
int(cosh(a + b*x)^(4/3)/sinh(a + b*x)^(4/3),x)
```

output
```
int(cosh(a + b*x)^(4/3)/sinh(a + b*x)^(4/3), x)
```

Reduce [F]

$$\int \frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{4}{3}}}{\sinh(bx+a)^{\frac{4}{3}}}dx$$

input
```
int(cosh(b*x+a)^(4/3)/sinh(b*x+a)^(4/3),x)
```

output
```
int((cosh(a + b*x)**(1/3)*cosh(a + b*x))/(sinh(a + b*x)**(1/3)*sinh(a + b*
x)),x)
```

3.27 $\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+\frac{\sinh^{\frac{4}{3}}(a+bx)}{\cosh^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

output
```
-1/2*3^(1/2)*arctan(1/3*(1+2*sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))*3^(1/2))
/b-1/2*ln(1-sinh(b*x+a)^(2/3)/cosh(b*x+a)^(2/3))/b+1/4*ln(1+sinh(b*x+a)^(2
/3)/cosh(b*x+a)^(2/3)+sinh(b*x+a)^(4/3)/cosh(b*x+a)^(4/3))/b-3/2*cosh(b*x+
a)^(2/3)/b/sinh(b*x+a)^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.38

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = -\frac{3\cosh^2(a+bx)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},-\frac{1}{3},\frac{2}{3},-\sinh^2(a+bx)\right)}{2b\cosh^{\frac{4}{3}}(a+bx)\sinh^{\frac{2}{3}}(a+bx)}$$

input
```
Integrate[Cosh[a + b*x]^(5/3)/Sinh[a + b*x]^(5/3),x]
```

output
```
(-3*(Cosh[a + b*x]^2)^(2/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, -Sinh[a + b
*x]^2])/(2*b*Cosh[a + b*x]^(4/3)*Sinh[a + b*x]^(2/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.88, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3047, 3042, 3054, 25, 807, 821, 16, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(ia+ibx)^{5/3}}{(-i\sin(ia+ibx))^{5/3}}dx$$

$$\Big\downarrow \; 3047$$

$$\int \frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}dx-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}+\int\frac{\sqrt[3]{-i\sin(ia+ibx)}}{\sqrt[3]{\cos(ia+ibx)}}dx$$

$$\Big\downarrow\ 3054$$

$$-\frac{3\int-\frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{b}-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow\ 25$$

$$\frac{3\int\frac{\tanh(a+bx)}{1-\tanh^2(a+bx)}d\frac{\sqrt[3]{\sinh(a+bx)}}{\sqrt[3]{\cosh(a+bx)}}}{b}-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow\ 807$$

$$\frac{3\int\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)(1-\tanh(a+bx))}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{2b}-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow\ 821$$

$$\frac{3\left(\frac{1}{3}\int\frac{1}{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\int\frac{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)}{2b}-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow\ 16$$

$$\frac{3\left(-\frac{1}{3}\int\frac{1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}}{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow\ 1142$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\frac{3}{2}\int\frac{1}{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow\ 1083$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+3\int\frac{1}{-\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-4}d\left(\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{217}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}-\sqrt{3}\arctan\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\log\left(\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1\right)-\sqrt{3}\arctan\left(\frac{\frac{2\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\sinh^{\frac{2}{3}}(a+bx)}{\cosh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\cosh^{\frac{2}{3}}(a+bx)}{2b\sinh^{\frac{2}{3}}(a+bx)}$$

input
```
Int[Cosh[a + b*x]^(5/3)/Sinh[a + b*x]^(5/3),x]
```

output
```
(3*(-1/3*Log[1 - Sinh[a + b*x]^(2/3)/Cosh[a + b*x]^(2/3)] + (-(Sqrt[3]*Arc
Tan[(1 + (2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3))/Sqrt[3]]) + Log[1 +
(2*Sinh[a + b*x]^(2/3))/Cosh[a + b*x]^(2/3)]/2)/3))/(2*b) - (3*Cosh[a + b*
x]^(2/3))/(2*b*Sinh[a + b*x]^(2/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Maple [F]

$$\int \frac{\cosh \left(bx+a\right)^{\frac{5}{3}}}{\sinh \left(bx+a\right)^{\frac{5}{3}}}dx$$

input
```
int(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x)
```

output
```
int(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(124) = 248$.

Time = 0.10 (sec) , antiderivative size = 749, normalized size of antiderivative = 4.83

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x, algorithm="fricas")
```

output
```
-1/4*(2*(sqrt(3)*cosh(b*x + a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) +
 sqrt(3)*sinh(b*x + a)^2 - sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x + a)^2 +
2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 + 4*(sqrt(
3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a
)^(2/3) + sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh
(b*x + a)^2 + 1)) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sin
h(b*x + a)^2 - 1)*log((cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 +
 sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x
+ a)^2 + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x +
 a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x +
a)^(2/3)*sinh(b*x + a)^(1/3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b
*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh
(b*x + a))*cosh(b*x + a)^(1/3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 +
cosh(b*x + a))*sinh(b*x + a) + 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(
b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 +
 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1
)) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
- 1)*log(-(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*cosh(b*x +
a)^(1/3)*sinh(b*x + a)^(2/3) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +
a)^2 + 1)/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(b*x+a)**(5/3)/sinh(b*x+a)**(5/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\cosh\left(bx+a\right)^{\frac{5}{3}}}{\sinh\left(bx+a\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(5/3)/sinh(b*x + a)^(5/3), x)
```

Giac [F]

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\cosh\left(bx+a\right)^{\frac{5}{3}}}{\sinh\left(bx+a\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(5/3)/sinh(b*x + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\cosh(a+b\,x)^{5/3}}{\sinh\left(a+b\,x\right)^{5/3}}\,dx$$

input
```
int(cosh(a + b*x)^(5/3)/sinh(a + b*x)^(5/3),x)
```

output
```
int(cosh(a + b*x)^(5/3)/sinh(a + b*x)^(5/3), x)
```

Reduce [F]

$$\int \frac{\cosh^{\frac{5}{3}}(a+bx)}{\sinh^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\cosh\left(bx+a\right)^{\frac{5}{3}}}{\sinh\left(bx+a\right)^{\frac{5}{3}}}dx$$

input
```
int(cosh(b*x+a)^(5/3)/sinh(b*x+a)^(5/3),x)
```

output
```
int((cosh(a + b*x)**(2/3)*cosh(a + b*x))/(sinh(a + b*x)**(2/3)*sinh(a + b*
x)),x)
```

3.28 $\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1+\frac{\cosh^{\frac{4}{3}}(a+bx)}{\sinh^{\frac{4}{3}}(a+bx)}+\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}$$

output
```
-1/2*3^(1/2)*arctan(1/3*(1+2*cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))*3^(1/2))
/b-1/2*ln(1-cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b+1/4*ln(1+cosh(b*x+a)^(4
/3)/sinh(b*x+a)^(4/3)+cosh(b*x+a)^(2/3)/sinh(b*x+a)^(2/3))/b-3/4*cosh(b*x+
a)^(4/3)/b/sinh(b*x+a)^(4/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.38

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx = -\frac{3\sqrt[3]{\cosh^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{2}{3},\frac{1}{3},-\sinh^2(a+bx)\right)}{4b\cosh^{\frac{2}{3}}(a+bx)\sinh^{\frac{4}{3}}(a+bx)}$$

input
```
Integrate[Cosh[a + b*x]^(7/3)/Sinh[a + b*x]^(7/3),x]
```

output
```
(-3*(Cosh[a + b*x]^2)^(1/3)*Hypergeometric2F1[-2/3, -2/3, 1/3, -Sinh[a + b
*x]^2])/(4*b*Cosh[a + b*x]^(2/3)*Sinh[a + b*x]^(4/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.88, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3047, 3042, 3055, 807, 821, 16, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ia+ibx)^{7/3}}{(-i\sin(ia+ibx))^{7/3}}dx$$

$$\Big\downarrow \text{3047}$$

$$\int \frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}dx-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}+\int\frac{\sqrt[3]{\cos(ia+ibx)}}{\sqrt[3]{-i\sin(ia+ibx)}}dx$$

$\downarrow$ 3055

$$\frac{3\int\frac{\coth(a+bx)}{1-\coth^2(a+bx)}d\frac{\sqrt[3]{\cosh(a+bx)}}{\sqrt[3]{\sinh(a+bx)}}}{b}-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}$$

$\downarrow$ 807

$$\frac{3\int\frac{\cosh^{\frac{2}{3}}(a+bx)}{(1-\coth(a+bx))\sinh^{\frac{2}{3}}(a+bx)}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{2b}-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}$$

$\downarrow$ 821

$$\frac{3\left(\frac{1}{3}\int\frac{1}{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\int\frac{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)}{2b}-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}$$

$\downarrow$ 16

$$\frac{3\left(-\frac{1}{3}\int\frac{1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}}{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}$$

$\downarrow$ 1142

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\frac{3}{2}\int\frac{1}{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}$$

$\downarrow$ 1083

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+3\int \frac{1}{-\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-4}d\left(\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \; 217$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\int 1d\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}-\sqrt{3}\arctan\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \; 1103$$

$$\frac{3\left(\frac{1}{3}\left(\frac{1}{2}\log\left(\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1\right)-\sqrt{3}\arctan\left(\frac{\frac{2\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log\left(1-\frac{\cosh^{\frac{2}{3}}(a+bx)}{\sinh^{\frac{2}{3}}(a+bx)}\right)\right)}{2b}-\frac{3\cosh^{\frac{4}{3}}(a+bx)}{4b\sinh^{\frac{4}{3}}(a+bx)}$$

input
```
Int[Cosh[a + b*x]^(7/3)/Sinh[a + b*x]^(7/3),x]
```

output
```
(3*(-1/3*Log[1 - Cosh[a + b*x]^(2/3)/Sinh[a + b*x]^(2/3)] + (-(Sqrt[3]*Arc
Tan[(1 + (2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3))/Sqrt[3]]) + Log[1 +
(2*Cosh[a + b*x]^(2/3))/Sinh[a + b*x]^(2/3)]/2)/3))/(2*b) - (3*Cosh[a + b*
x]^(4/3))/(4*b*Sinh[a + b*x]^(4/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

Maple [F]

$$\int \frac{\cosh{(bx+a)}^{\frac{7}{3}}}{\sinh{(bx+a)}^{\frac{7}{3}}}dx$$

input
```
int(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x)
```

output
```
int(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1056 vs. $2(124) = 248$.

Time = 0.11 (sec) , antiderivative size = 1056, normalized size of antiderivative = 6.81

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x, algorithm="fricas")
```

output
```
-1/4*(2*(sqrt(3)*cosh(b*x + a)^4 + 4*sqrt(3)*cosh(b*x + a)*sinh(b*x + a)^3
 + sqrt(3)*sinh(b*x + a)^4 + 2*(3*sqrt(3)*cosh(b*x + a)^2 - sqrt(3))*sinh(
b*x + a)^2 - 2*sqrt(3)*cosh(b*x + a)^2 + 4*(sqrt(3)*cosh(b*x + a)^3 - sqrt
(3)*cosh(b*x + a))*sinh(b*x + a) + sqrt(3))*arctan(1/3*(sqrt(3)*cosh(b*x +
 a)^2 + 2*sqrt(3)*cosh(b*x + a)*sinh(b*x + a) + sqrt(3)*sinh(b*x + a)^2 +
4*(sqrt(3)*cosh(b*x + a) + sqrt(3)*sinh(b*x + a))*cosh(b*x + a)^(2/3)*sinh
(b*x + a)^(1/3) - sqrt(3))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a
) + sinh(b*x + a)^2 - 1)) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x +
a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cos
h(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(
(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(
3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 2*(cosh(b*x +
 a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x +
a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))*cosh(b*x + a)^(2/3)*sinh(b*x + a)
^(1/3) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x +
 a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*cosh(b*x +
a)^(1/3)*sinh(b*x + a)^(2/3) + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*
x + a) + 1)/(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x
+ a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4
*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)) + 2*(cosh(b*x + ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(b*x+a)**(7/3)/sinh(b*x+a)**(7/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{7}{3}}}{\sinh(bx+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(7/3)/sinh(b*x + a)^(7/3), x)
```

Giac [F]

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\cosh(bx+a)^{\frac{7}{3}}}{\sinh(bx+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(7/3)/sinh(b*x + a)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\cosh(a+b\,x)^{7/3}}{\sinh(a+b\,x)^{7/3}}\,dx$$

input
```
int(cosh(a + b*x)^(7/3)/sinh(a + b*x)^(7/3),x)
```

output
```
int(cosh(a + b*x)^(7/3)/sinh(a + b*x)^(7/3), x)
```

Reduce [F]

$$\int \frac{\cosh^{\frac{7}{3}}(a+bx)}{\sinh^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\cosh{(bx+a)}^{\frac{7}{3}}}{\sinh{(bx+a)}^{\frac{7}{3}}}dx$$

input
```
int(cosh(b*x+a)^(7/3)/sinh(b*x+a)^(7/3),x)
```

output
```
int((cosh(a + b*x)**(1/3)*cosh(a + b*x)**2)/(sinh(a + b*x)**(1/3)*sinh(a +
 b*x)**2),x)
```

3.29 $\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx = -\frac{3\cosh^{\frac{5}{3}}(x)}{5\sinh^{\frac{5}{3}}(x)}$$

output
```
-3/5*cosh(x)^(5/3)/sinh(x)^(5/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx = -\frac{3\cosh^{\frac{5}{3}}(x)}{5\sinh^{\frac{5}{3}}(x)}$$

input
```
Integrate[Cosh[x]^(2/3)/Sinh[x]^(8/3),x]
```

output
```
(-3*Cosh[x]^(5/3))/(5*Sinh[x]^(5/3))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3043}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^{2/3}}{(-i\sin(ix))^{8/3}}dx$$

$$\Big\downarrow \text{3043}$$

$$-\frac{3\cosh^{\frac{5}{3}}(x)}{5\sinh^{\frac{5}{3}}(x)}$$

input
```
Int[Cosh[x]^(2/3)/Sinh[x]^(8/3),x]
```

output
```
(-3*Cosh[x]^(5/3))/(5*Sinh[x]^(5/3))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3043
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Maple [F]

$$\int \frac{\cosh(x)^{\frac{2}{3}}}{\sinh(x)^{\frac{8}{3}}}dx$$

input
```
int(cosh(x)^(2/3)/sinh(x)^(8/3),x)
```

output
```
int(cosh(x)^(2/3)/sinh(x)^(8/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(10) = 20$.

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 5.81

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx =$$
$$-\frac{6\left(\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + \left(3\cosh(x)^2 + 1\right)\sinh(x) + \cosh(x)\right)\text{co}}{5\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 - 1\right)\sinh(x)^2 - 2\cosh(x)^2 + 4\left(\cosh\right.\right.}$$

input
```
integrate(cosh(x)^(2/3)/sinh(x)^(8/3),x, algorithm="fricas")
```

output
```
-6/5*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh
(x) + cosh(x))*cosh(x)^(2/3)*sinh(x)^(1/3)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^
3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3
 - cosh(x))*sinh(x) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**(2/3)/sinh(x)**(8/3),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(10) = 20$.

Time $= 0.14$ (sec) , antiderivative size $= 61$, normalized size of antiderivative $= 3.81$

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx = \frac{3\left(e^{(-2x)}+1\right)^{\frac{2}{3}}e^{(-4x)}}{5\left(e^{(-x)}+1\right)^{\frac{8}{3}}\left(-e^{(-x)}+1\right)^{\frac{8}{3}}} - \frac{3\left(e^{(-2x)}+1\right)^{\frac{2}{3}}}{5\left(e^{(-x)}+1\right)^{\frac{8}{3}}\left(-e^{(-x)}+1\right)^{\frac{8}{3}}}$$

input
```
integrate(cosh(x)^(2/3)/sinh(x)^(8/3),x, algorithm="maxima")
```

output
```
3/5*(e^(-2*x) + 1)^(2/3)*e^(-4*x)/((e^(-x) + 1)^(8/3)*(-e^(-x) + 1)^(8/3))
 - 3/5*(e^(-2*x) + 1)^(2/3)/((e^(-x) + 1)^(8/3)*(-e^(-x) + 1)^(8/3))
```

Giac [F]

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx = \int \frac{\cosh(x)^{\frac{2}{3}}}{\sinh(x)^{\frac{8}{3}}}\,dx$$

input
```
integrate(cosh(x)^(2/3)/sinh(x)^(8/3),x, algorithm="giac")
```

output
```
integrate(cosh(x)^(2/3)/sinh(x)^(8/3), x)
```

Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.38

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx = -\frac{3\coth(x)^{5/3}}{5}$$

input
```
int(cosh(x)^(2/3)/sinh(x)^(8/3),x)
```

output
```
-(3*coth(x)^(5/3))/5
```

Reduce [F]

$$\int \frac{\cosh^{\frac{2}{3}}(x)}{\sinh^{\frac{8}{3}}(x)}\,dx = \int \frac{\cosh(x)^{\frac{2}{3}}}{\sinh(x)^{\frac{8}{3}}}dx$$

input
```
int(cosh(x)^(2/3)/sinh(x)^(8/3),x)
```

output
```
int(cosh(x)**(2/3)/(sinh(x)**(2/3)*sinh(x)**2),x)
```

3.30 $\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx = \frac{3\sinh^{\frac{5}{3}}(x)}{5\cosh^{\frac{5}{3}}(x)}$$

output
```
3/5*sinh(x)^(5/3)/cosh(x)^(5/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx = \frac{3\sinh^{\frac{5}{3}}(x)}{5\cosh^{\frac{5}{3}}(x)}$$

input
```
Integrate[Sinh[x]^(2/3)/Cosh[x]^(8/3),x]
```

output
```
(3*Sinh[x]^(5/3))/(5*Cosh[x]^(5/3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3043}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(-i\sin(ix))^{2/3}}{\cos(ix)^{8/3}}dx$$

$$\Big\downarrow \text{3043}$$

$$\frac{3\sinh^{\frac{5}{3}}(x)}{5\cosh^{\frac{5}{3}}(x)}$$

input
```
Int[Sinh[x]^(2/3)/Cosh[x]^(8/3),x]
```

output
```
(3*Sinh[x]^(5/3))/(5*Cosh[x]^(5/3))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3043
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Maple [F]

$$\int \frac{\sinh(x)^{\frac{2}{3}}}{\cosh(x)^{\frac{8}{3}}}dx$$

input
```
int(sinh(x)^(2/3)/cosh(x)^(8/3),x)
```

output
```
int(sinh(x)^(2/3)/cosh(x)^(8/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(10) = 20$.

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 5.81

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx = \frac{6\left(\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + \left(3\cosh(x)^2 - 1\right)\sinh(x) - \cosh(x)\right)\cosh}{5\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 + 1\right)\sinh(x)^2 + 2\cosh(x)^2 + 4\left(\cosh(x)\right.\right.}$$

input
```
integrate(sinh(x)^(2/3)/cosh(x)^(8/3),x, algorithm="fricas")
```

output
```
6/5*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(
x) - cosh(x))*cosh(x)^(1/3)*sinh(x)^(2/3)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3
 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3
+ cosh(x))*sinh(x) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**(2/3)/cosh(x)**(8/3),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(10) = 20.

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.81

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx = -\frac{3\left(e^{(-x)}+1\right)^{\frac{2}{3}}\left(-e^{(-x)}+1\right)^{\frac{2}{3}}e^{(-4x)}}{5\left(e^{(-2x)}+1\right)^{\frac{8}{3}}} + \frac{3\left(e^{(-x)}+1\right)^{\frac{2}{3}}\left(-e^{(-x)}+1\right)^{\frac{2}{3}}}{5\left(e^{(-2x)}+1\right)^{\frac{8}{3}}}$$

input
```
integrate(sinh(x)^(2/3)/cosh(x)^(8/3),x, algorithm="maxima")
```

output
```
-3/5*(e^(-x) + 1)^(2/3)*(-e^(-x) + 1)^(2/3)*e^(-4*x)/(e^(-2*x) + 1)^(8/3)
+ 3/5*(e^(-x) + 1)^(2/3)*(-e^(-x) + 1)^(2/3)/(e^(-2*x) + 1)^(8/3)
```

Giac [F]

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx = \int \frac{\sinh(x)^{\frac{2}{3}}}{\cosh(x)^{\frac{8}{3}}}\,dx$$

input
```
integrate(sinh(x)^(2/3)/cosh(x)^(8/3),x, algorithm="giac")
```

output
```
integrate(sinh(x)^(2/3)/cosh(x)^(8/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx = \int \frac{\sinh(x)^{2/3}}{\cosh(x)^{8/3}}\,dx$$

input
```
int(sinh(x)^(2/3)/cosh(x)^(8/3),x)
```

output
```
int(sinh(x)^(2/3)/cosh(x)^(8/3), x)
```

Reduce [F]

$$\int \frac{\sinh^{\frac{2}{3}}(x)}{\cosh^{\frac{8}{3}}(x)}\,dx = \int \frac{\sinh(x)^{\frac{2}{3}}}{\cosh(x)^{\frac{8}{3}}}dx$$

input
```
int(sinh(x)^(2/3)/cosh(x)^(8/3),x)
```

output
```
int(sinh(x)**(2/3)/(cosh(x)**(2/3)*cosh(x)**2),x)
```

3.31 $\int \cosh(a+bx)\sinh^p(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \cosh(a+bx)\sinh^p(a+bx)\,dx=\frac{\sinh^{1+p}(a+bx)}{b(1+p)}$$

output
```
sinh(b*x+a)^(p+1)/b/(p+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\sinh^p(a+bx)\,dx=\frac{\sinh^{1+p}(a+bx)}{b(1+p)}$$

input
```
Integrate[Cosh[a + b*x]*Sinh[a + b*x]^p,x]
```

output
```
Sinh[a + b*x]^(1 + p)/(b*(1 + p))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\sinh^p(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(ia+ibx)(-i\sin(ia+ibx))^p dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int \sinh^p(a+bx)d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sinh^{p+1}(a+bx)}{b(p+1)}$$

input
```
Int[Cosh[a + b*x]*Sinh[a + b*x]^p,x]
```

output
```
Sinh[a + b*x]^(1 + p)/(b*(1 + p))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 10.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{\sinh(bx+a)^{p+1}}{b(p+1)}$	20
default	$\frac{\sinh(bx+a)^{p+1}}{b(p+1)}$	20

input
```
int(cosh(b*x+a)*sinh(b*x+a)^p,x,method=_RETURNVERBOSE)
```

output
```
sinh(b*x+a)^(p+1)/b/(p+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(19) = 38.

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.58

$$\int \cosh(a+bx)\sinh^p(a+bx)\,dx = \frac{\cosh\left(p\log\left(\sinh\left(bx+a\right)\right)\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)\sinh\left(p\log\left(\sinh\left(bx+a\right)\right)\right)}{\left(bp+b\right)\cosh\left(bx+a\right)^2-\left(bp+b\right)\sinh\left(bx+a\right)^2}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^p,x, algorithm="fricas")
```

output
```
(cosh(p*log(sinh(b*x + a)))*sinh(b*x + a) + sinh(b*x + a)*sinh(p*log(sinh(
b*x + a))))/((b*p + b)*cosh(b*x + a)^2 - (b*p + b)*sinh(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(14) = 28$.

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.58

$$\int \cosh(a+bx)\sinh^p(a+bx)\,dx = \begin{cases} \frac{x\cosh(a)}{\sinh(a)} & \text{for}\ b=0 \wedge p=-1 \\ x\sinh^p(a)\cosh(a) & \text{for}\ b=0 \\ \frac{\log(\sinh(a+bx))}{b} & \text{for}\ p=-1 \\ \frac{\sinh(a+bx)\sinh^p(a+bx)}{bp+b} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)**p,x)
```

output
```
Piecewise((x*cosh(a)/sinh(a), Eq(b, 0) & Eq(p, -1)), (x*sinh(a)**p*cosh(a)
, Eq(b, 0)), (log(sinh(a + b*x))/b, Eq(p, -1)), (sinh(a + b*x)*sinh(a + b*
x)**p/(b*p + b), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\sinh^p(a+bx)\,dx = \frac{\sinh(bx+a)^{p+1}}{b(p+1)}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^p,x, algorithm="maxima")
```

output
```
sinh(b*x + a)^(p + 1)/(b*(p + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \cosh(a+bx)\sinh^p(a+bx)\,dx = \frac{\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)^{p+1}}{b(p+1)}$$

input
```
integrate(cosh(b*x+a)*sinh(b*x+a)^p,x, algorithm="giac")
```

output
```
(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a))^(p + 1)/(b*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\sinh^p(a+bx)\,dx = \frac{\sinh(a+b\,x)^{p+1}}{b\,(p+1)}$$

input
```
int(cosh(a + b*x)*sinh(a + b*x)^p,x)
```

output
```
sinh(a + b*x)^(p + 1)/(b*(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \cosh(a+bx)\sinh^p(a+bx)\,dx = \frac{\sinh(bx+a)^p\sinh(bx+a)}{b\,(p+1)}$$

input
```
int(cosh(b*x+a)*sinh(b*x+a)^p,x)
```

output
```
(sinh(a + b*x)**p*sinh(a + b*x))/(b*(p + 1))
```

3.32 $\int \cosh^3(a+bx)\sinh^p(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 39

$$\int \cosh^3(a+bx)\sinh^p(a+bx)\,dx = \frac{\sinh^{1+p}(a+bx)}{b(1+p)} + \frac{\sinh^{3+p}(a+bx)}{b(3+p)}$$

output
```
sinh(b*x+a)^(p+1)/b/(p+1)+sinh(b*x+a)^(3+p)/b/(3+p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \cosh^3(a+bx)\sinh^p(a+bx)\,dx = \frac{\sinh^{1+p}(a+bx)}{b(1+p)} + \frac{\sinh^{3+p}(a+bx)}{b(3+p)}$$

input
```
Integrate[Cosh[a + b*x]^3*Sinh[a + b*x]^p,x]
```

output
```
Sinh[a + b*x]^(1 + p)/(b*(1 + p)) + Sinh[a + b*x]^(3 + p)/(b*(3 + p))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(a+bx)\sinh^p(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(ia+ibx)^3(-i\sin(ia+ibx))^p dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int \sinh^p(a+bx)\left(\sinh^2(a+bx)+1\right)d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\sinh^p(a+bx)+\sinh^{p+2}(a+bx)\right)d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sinh^{p+1}(a+bx)}{p+1}+\frac{\sinh^{p+3}(a+bx)}{p+3}}{b}$$

input
```
Int[Cosh[a + b*x]^3*Sinh[a + b*x]^p,x]
```

output
```
(Sinh[a + b*x]^(1 + p)/(1 + p) + Sinh[a + b*x]^(3 + p)/(3 + p))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38

$$\frac{\sinh(bx+a)^3 \, \mathrm{e}^{p\ln(\sinh(bx+a))}}{b\,(3+p)}+\frac{\sinh(bx+a)\,\mathrm{e}^{p\ln(\sinh(bx+a))}}{b\,(p+1)}$$

input
```
int(cosh(b*x+a)^3*sinh(b*x+a)^p,x)
```

output
```
1/b/(3+p)*sinh(b*x+a)^3*exp(p*ln(sinh(b*x+a)))+1/b/(p+1)*sinh(b*x+a)*exp(p
*ln(sinh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(39) = 78$.

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 4.49

$$\int \cosh^3(a+bx)\sinh^p(a+bx)\,dx = \frac{((p+1)\sinh(bx+a)^3 + (3(p+1)\cosh(bx+a)^2 + p + 9)\sinh(bx+a))\cosh(p\log(\sinh(bx+a))) + }{4((bp^2+4bp+3b)\cosh(bx+a)^4 - 2(bp^2+4bp+3b)\cosh(}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^p,x, algorithm="fricas")
```

output
```
1/4*(((p + 1)*sinh(b*x + a)^3 + (3*(p + 1)*cosh(b*x + a)^2 + p + 9)*sinh(b
*x + a))*cosh(p*log(sinh(b*x + a))) + ((p + 1)*sinh(b*x + a)^3 + (3*(p + 1
)*cosh(b*x + a)^2 + p + 9)*sinh(b*x + a))*sinh(p*log(sinh(b*x + a))))/((b*
p^2 + 4*b*p + 3*b)*cosh(b*x + a)^4 - 2*(b*p^2 + 4*b*p + 3*b)*cosh(b*x + a)
^2*sinh(b*x + a)^2 + (b*p^2 + 4*b*p + 3*b)*sinh(b*x + a)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(29) = 58$.

Time = 1.18 (sec) , antiderivative size = 638, normalized size of antiderivative = 16.36

$$\int \cosh^3(a+bx)\sinh^p(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)**3*sinh(b*x+a)**p,x)
```

output
```
Piecewise((x*sinh(a)**p*cosh(a)**3, Eq(b, 0)), (log(sinh(a + b*x))/b - cos
h(a + b*x)**2/(2*b*sinh(a + b*x)**2), Eq(p, -3)), (b*x*tanh(a/2 + b*x/2)**
4/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - 2*b*x*tanh(a/2
 + b*x/2)**2/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + b*x
/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - 2*log(tanh(a/2
+ b*x/2) + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2
+ b*x/2)**2 + b) + 4*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**2/(b*ta
nh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - 2*log(tanh(a/2 + b*x/
2) + 1)/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + log(tanh
(a/2 + b*x/2))*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2
 + b*x/2)**2 + b) - 2*log(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)**2/(b*tanh(
a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + log(tanh(a/2 + b*x/2))/(
b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + 2*tanh(a/2 + b*x/
2)**2/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b), Eq(p, -1)),
 (p*sinh(a + b*x)*sinh(a + b*x)**p*cosh(a + b*x)**2/(b*p**2 + 4*b*p + 3*b)
 - 2*sinh(a + b*x)**3*sinh(a + b*x)**p/(b*p**2 + 4*b*p + 3*b) + 3*sinh(a +
 b*x)*sinh(a + b*x)**p*cosh(a + b*x)**2/(b*p**2 + 4*b*p + 3*b), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(39) = 78$.

Time = 0.19 (sec) , antiderivative size = 373, normalized size of antiderivative = 9.56

$$\begin{aligned}
\int \cosh^3(a+bx)\sinh^p(a+bx)\,dx &= \frac{pe^{\left((bx+a)p+3\,bx+p\log\left(e^{(-bx-a)}+1\right)+p\log\left(-e^{(-bx-a)}+1\right)+3\,a\right)}}{8\,(2^pp^2+2^{p+2}p+3\cdot 2^p)b} \\
&\quad+\frac{(p+9)e^{\left((bx+a)p+bx+p\log\left(e^{(-bx-a)}+1\right)+p\log\left(-e^{(-bx-a)}+1\right)+a\right)}}{8\,(2^pp^2+2^{p+2}p+3\cdot 2^p)b} \\
&\quad-\frac{(p+9)e^{\left((bx+a)p-bx+p\log\left(e^{(-bx-a)}+1\right)+p\log\left(-e^{(-bx-a)}+1\right)-a\right)}}{8\,(2^pp^2+2^{p+2}p+3\cdot 2^p)b} \\
&\quad-\frac{(p+1)e^{\left((bx+a)p-3\,bx+p\log\left(e^{(-bx-a)}+1\right)+p\log\left(-e^{(-bx-a)}+1\right)-3\,a\right)}}{8\,(2^pp^2+2^{p+2}p+3\cdot 2^p)b} \\
&\quad+\frac{e^{\left((bx+a)p+3\,bx+p\log\left(e^{(-bx-a)}+1\right)+p\log\left(-e^{(-bx-a)}+1\right)+3\,a\right)}}{8\,(2^pp^2+2^{p+2}p+3\cdot 2^p)b}
\end{aligned}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^p,x, algorithm="maxima")
```

output
```
1/8*p*e^((b*x + a)*p + 3*b*x + p*log(e^(-b*x - a) + 1) + p*log(-e^(-b*x -
a) + 1) + 3*a)/((2^p*p^2 + 2^(p + 2)*p + 3*2^p)*b) + 1/8*(p + 9)*e^((b*x +
 a)*p + b*x + p*log(e^(-b*x - a) + 1) + p*log(-e^(-b*x - a) + 1) + a)/((2^
p*p^2 + 2^(p + 2)*p + 3*2^p)*b) - 1/8*(p + 9)*e^((b*x + a)*p - b*x + p*log
(e^(-b*x - a) + 1) + p*log(-e^(-b*x - a) + 1) - a)/((2^p*p^2 + 2^(p + 2)*p
 + 3*2^p)*b) - 1/8*(p + 1)*e^((b*x + a)*p - 3*b*x + p*log(e^(-b*x - a) + 1
) + p*log(-e^(-b*x - a) + 1) - 3*a)/((2^p*p^2 + 2^(p + 2)*p + 3*2^p)*b) +
1/8*e^((b*x + a)*p + 3*b*x + p*log(e^(-b*x - a) + 1) + p*log(-e^(-b*x - a)
 + 1) + 3*a)/((2^p*p^2 + 2^(p + 2)*p + 3*2^p)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(39) = 78$.

Time = 0.18 (sec) , antiderivative size = 327, normalized size of antiderivative = 8.38

$$\int \cosh^3(a+bx)\sinh^p(a+bx)\,dx$$
$$= \frac{pe^{\left(-bpx-ap+7\,bx+p\log\left(\frac{1}{2}\,e^{(2\,bx+2\,a)}-\frac{1}{2}\right)+7\,a\right)}+pe^{\left(-bpx-ap+5\,bx+p\log\left(\frac{1}{2}\,e^{(2\,bx+2\,a)}-\frac{1}{2}\right)+5\,a\right)}-pe^{\left(-bpx-ap+3\,bx+p\log\left(\frac{1}{2}\,e^{(2\,bx\cdots}}{}$$

input
```
integrate(cosh(b*x+a)^3*sinh(b*x+a)^p,x, algorithm="giac")
```

output
```
1/8*(p*e^(-b*p*x - a*p + 7*b*x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + 7*a) +
 p*e^(-b*p*x - a*p + 5*b*x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + 5*a) - p*e
^(-b*p*x - a*p + 3*b*x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + 3*a) - p*e^(-b
*p*x - a*p + b*x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + a) + e^(-b*p*x - a*p
 + 7*b*x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + 7*a) + 9*e^(-b*p*x - a*p + 5
*b*x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + 5*a) - 9*e^(-b*p*x - a*p + 3*b*x
 + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + 3*a) - e^(-b*p*x - a*p + b*x + p*log
(1/2*e^(2*b*x + 2*a) - 1/2) + a))/(b*p^2*e^(4*b*x + 4*a) + 4*b*p*e^(4*b*x
+ 4*a) + 3*b*e^(4*b*x + 4*a))
```

Mupad [B] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.46

$$\int \cosh^3(a+bx)\sinh^p(a+bx)\,dx = -\left(\frac{1}{2}\right)^p e^{-3a-3bx}\left(e^{a+bx}-e^{-a-bx}\right)^p\left(\frac{\frac{p}{8}+\frac{1}{8}}{b\left(p^2+4p+3\right)}+\frac{e^{2a+2bx}\,(p+9)}{8b\left(p^2+4p+3\right)}-\frac{e^{6a+6bx}\,(p+1)}{8b\left(p^2+4p+3\right)}-\frac{e^{4a+4bx}\,(p+9)}{8b\left(p^2+4p+3\right)}\right)$$

input
```
int(cosh(a + b*x)^3*sinh(a + b*x)^p,x)
```

output
```
-(1/2)^p*exp(- 3*a - 3*b*x)*(exp(a + b*x) - exp(- a - b*x))^p*((p/8 + 1/8)
/(b*(4*p + p^2 + 3)) + (exp(2*a + 2*b*x)*(p + 9))/(8*b*(4*p + p^2 + 3)) -
(exp(6*a + 6*b*x)*(p + 1))/(8*b*(4*p + p^2 + 3)) - (exp(4*a + 4*b*x)*(p +
9))/(8*b*(4*p + p^2 + 3)))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.31

$$\int \cosh^3(a+bx)\sinh^p(a+bx)\,dx = \frac{\left(e^{2bx+2a}-1\right)^p\left(e^{6bx+6a}p+e^{6bx+6a}+e^{4bx+4a}p+9e^{4bx+4a}-e^{2bx+2a}p-9e^{2bx+2a}-p-1\right)}{8e^{bpx+ap+3bx+3a}2^p b\left(p^2+4p+3\right)}$$

input
```
int(cosh(b*x+a)^3*sinh(b*x+a)^p,x)
```

output
```
((e**(2*a + 2*b*x) - 1)**p*(e**(6*a + 6*b*x)*p + e**(6*a + 6*b*x) + e**(4*
a + 4*b*x)*p + 9*e**(4*a + 4*b*x) - e**(2*a + 2*b*x)*p - 9*e**(2*a + 2*b*x
) - p - 1))/(8*e**(a*p + 3*a + b*p*x + 3*b*x)*2**p*b*(p**2 + 4*p + 3))
```

3.33 $\int \cosh^5(a+bx)\sinh^p(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 59

$$\int \cosh^5(a+bx)\sinh^p(a+bx)\,dx=\frac{\sinh^{1+p}(a+bx)}{b(1+p)}+\frac{2\sinh^{3+p}(a+bx)}{b(3+p)}+\frac{\sinh^{5+p}(a+bx)}{b(5+p)}$$

output
```
sinh(b*x+a)^(p+1)/b/(p+1)+2*sinh(b*x+a)^(3+p)/b/(3+p)+sinh(b*x+a)^(5+p)/b/
(5+p)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \cosh^5(a+bx)\sinh^p(a+bx)\,dx=\frac{\sinh^{1+p}(a+bx)\left(\frac{1}{1+p}+\frac{2\sinh^2(a+bx)}{3+p}+\frac{\sinh^4(a+bx)}{5+p}\right)}{b}$$

input
```
Integrate[Cosh[a + b*x]^5*Sinh[a + b*x]^p,x]
```

output
```
(Sinh[a + b*x]^(1 + p)*((1 + p)^(-1) + (2*Sinh[a + b*x]^2)/(3 + p) + Sinh[
a + b*x]^4/(5 + p)))/b
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^5(a+bx)\sinh^p(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(ia+ibx)^5(-i\sin(ia+ibx))^p dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int \sinh^p(a+bx)\left(\sinh^2(a+bx)+1\right)^2 d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\sinh^p(a+bx)+2\sinh^{p+2}(a+bx)+\sinh^{p+4}(a+bx)\right) d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sinh^{p+1}(a+bx)}{p+1}+\frac{2\sinh^{p+3}(a+bx)}{p+3}+\frac{\sinh^{p+5}(a+bx)}{p+5}}{b}$$

input
```
Int[Cosh[a + b*x]^5*Sinh[a + b*x]^p,x]
```

output
```
(Sinh[a + b*x]^(1 + p)/(1 + p) + (2*Sinh[a + b*x]^(3 + p))/(3 + p) + Sinh[
a + b*x]^(5 + p)/(5 + p))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

$$\frac{\sinh \left(bx+a\right)^{5} {\mathrm e}^{p \ln \left(\sinh \left(bx+a\right)\right)}}{b \left(5+p\right)}+\frac{\sinh \left(bx+a\right) {\mathrm e}^{p \ln \left(\sinh \left(bx+a\right)\right)}}{b \left(p+1\right)}+\frac{2 \sinh \left(bx+a\right)^{3} {\mathrm e}^{p \ln \left(\sinh \left(bx+a\right)\right)}}{b \left(3+p\right)}$$

input
```
int(cosh(b*x+a)^5*sinh(b*x+a)^p,x)
```

output
```
1/b/(5+p)*sinh(b*x+a)^5*exp(p*ln(sinh(b*x+a)))+1/b/(p+1)*sinh(b*x+a)*exp(p
*ln(sinh(b*x+a)))+2/b/(3+p)*sinh(b*x+a)^3*exp(p*ln(sinh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(59) = 118$.

Time = 0.11 (sec) , antiderivative size = 379, normalized size of antiderivative = 6.42

$$\int \cosh^5(a+bx)\sinh^p(a+bx)\,dx$$
$$= \frac{\left((p^2+4p+3)\sinh(bx+a)^5 + \left(10(p^2+4p+3)\cosh(bx+a)^2 + 3p^2 + 28p + 25\right)\sinh(bx+a)^3 + (5}{}$$

input
```
integrate(cosh(b*x+a)^5*sinh(b*x+a)^p,x, algorithm="fricas")
```

output
```
1/16*(((p^2 + 4*p + 3)*sinh(b*x + a)^5 + (10*(p^2 + 4*p + 3)*cosh(b*x + a)
^2 + 3*p^2 + 28*p + 25)*sinh(b*x + a)^3 + (5*(p^2 + 4*p + 3)*cosh(b*x + a)
^4 + 3*(3*p^2 + 28*p + 25)*cosh(b*x + a)^2 + 2*p^2 + 24*p + 150)*sinh(b*x
+ a))*cosh(p*log(sinh(b*x + a))) + ((p^2 + 4*p + 3)*sinh(b*x + a)^5 + (10*
(p^2 + 4*p + 3)*cosh(b*x + a)^2 + 3*p^2 + 28*p + 25)*sinh(b*x + a)^3 + (5*
(p^2 + 4*p + 3)*cosh(b*x + a)^4 + 3*(3*p^2 + 28*p + 25)*cosh(b*x + a)^2 +
2*p^2 + 24*p + 150)*sinh(b*x + a))*sinh(p*log(sinh(b*x + a))))/((b*p^3 + 9
*b*p^2 + 23*b*p + 15*b)*cosh(b*x + a)^6 - 3*(b*p^3 + 9*b*p^2 + 23*b*p + 15
*b)*cosh(b*x + a)^4*sinh(b*x + a)^2 + 3*(b*p^3 + 9*b*p^2 + 23*b*p + 15*b)*
cosh(b*x + a)^2*sinh(b*x + a)^4 - (b*p^3 + 9*b*p^2 + 23*b*p + 15*b)*sinh(b
*x + a)^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2574 vs. $2(46) = 92$.

Time = 4.19 (sec) , antiderivative size = 2574, normalized size of antiderivative = 43.63

$$\int \cosh^5(a+bx)\sinh^p(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)**5*sinh(b*x+a)**p,x)
```

output
```
Piecewise((x*sinh(a)**p*cosh(a)**5, Eq(b, 0)), (log(sinh(a + b*x))/b - cos
h(a + b*x)**2/(2*b*sinh(a + b*x)**2) - cosh(a + b*x)**4/(4*b*sinh(a + b*x)
**4), Eq(p, -5)), (16*b*x*tanh(a/2 + b*x/2)**6/(8*b*tanh(a/2 + b*x/2)**6 -
 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) - 32*b*x*tanh(a/2 +
 b*x/2)**4/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tan
h(a/2 + b*x/2)**2) + 16*b*x*tanh(a/2 + b*x/2)**2/(8*b*tanh(a/2 + b*x/2)**6
 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) - 32*log(tanh(a/2
 + b*x/2) + 1)*tanh(a/2 + b*x/2)**6/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(
a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) + 64*log(tanh(a/2 + b*x/2) + 1
)*tanh(a/2 + b*x/2)**4/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)*
*4 + 8*b*tanh(a/2 + b*x/2)**2) - 32*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 +
b*x/2)**2/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh
(a/2 + b*x/2)**2) + 16*log(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)**6/(8*b*ta
nh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2)
 - 32*log(tanh(a/2 + b*x/2))*tanh(a/2 + b*x/2)**4/(8*b*tanh(a/2 + b*x/2)**
6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) + 16*log(tanh(a/
2 + b*x/2))*tanh(a/2 + b*x/2)**2/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2
 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) - tanh(a/2 + b*x/2)**8/(8*b*tanh(
a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x/2)**4 + 8*b*tanh(a/2 + b*x/2)**2) +
18*tanh(a/2 + b*x/2)**4/(8*b*tanh(a/2 + b*x/2)**6 - 16*b*tanh(a/2 + b*x...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(59) = 118$.

Time = 0.18 (sec) , antiderivative size = 686, normalized size of antiderivative = 11.63

$$\int \cosh^5(a + bx) \sinh^p(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^5*sinh(b*x+a)^p,x, algorithm="maxima")
```

output
```
1/32*p^2*e^((b*x + a)*p + 5*b*x + p*log(e^(-b*x - a) + 1) + p*log(-e^(-b*x
 - a) + 1) + 5*a)/((2^p*p^3 + 9*2^p*p^2 + 23*2^p*p + 15*2^p)*b) + 1/8*p*e^
((b*x + a)*p + 5*b*x + p*log(e^(-b*x - a) + 1) + p*log(-e^(-b*x - a) + 1)
+ 5*a)/((2^p*p^3 + 9*2^p*p^2 + 23*2^p*p + 15*2^p)*b) + 1/32*(3*p^2 + 28*p
+ 25)*e^((b*x + a)*p + 3*b*x + p*log(e^(-b*x - a) + 1) + p*log(-e^(-b*x -
a) + 1) + 3*a)/((2^p*p^3 + 9*2^p*p^2 + 23*2^p*p + 15*2^p)*b) + 1/16*(p^2 +
 12*p + 75)*e^((b*x + a)*p + b*x + p*log(e^(-b*x - a) + 1) + p*log(-e^(-b*
x - a) + 1) + a)/((2^p*p^3 + 9*2^p*p^2 + 23*2^p*p + 15*2^p)*b) - 1/16*(p^2
 + 12*p + 75)*e^((b*x + a)*p - b*x + p*log(e^(-b*x - a) + 1) + p*log(-e^(-
b*x - a) + 1) - a)/((2^p*p^3 + 9*2^p*p^2 + 23*2^p*p + 15*2^p)*b) - 1/32*(3
*p^2 + 28*p + 25)*e^((b*x + a)*p - 3*b*x + p*log(e^(-b*x - a) + 1) + p*log
(-e^(-b*x - a) + 1) - 3*a)/((2^p*p^3 + 9*2^p*p^2 + 23*2^p*p + 15*2^p)*b) -
 1/32*(p^2 + 4*p + 3)*e^((b*x + a)*p - 5*b*x + p*log(e^(-b*x - a) + 1) + p
*log(-e^(-b*x - a) + 1) - 5*a)/((2^p*p^3 + 9*2^p*p^2 + 23*2^p*p + 15*2^p)*
b) + 3/32*e^((b*x + a)*p + 5*b*x + p*log(e^(-b*x - a) + 1) + p*log(-e^(-b*
x - a) + 1) + 5*a)/((2^p*p^3 + 9*2^p*p^2 + 23*2^p*p + 15*2^p)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 722 vs. 2(59) = 118.

Time = 0.21 (sec) , antiderivative size = 722, normalized size of antiderivative = 12.24

$$\int \cosh^5(a+bx)\sinh^p(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^5*sinh(b*x+a)^p,x, algorithm="giac")
```

output
```
1/32*(p^2*e^(-b*p*x - a*p + 11*b*x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + 11
*a) + 3*p^2*e^(-b*p*x - a*p + 9*b*x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + 9
*a) + 2*p^2*e^(-b*p*x - a*p + 7*b*x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + 7
*a) - 2*p^2*e^(-b*p*x - a*p + 5*b*x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + 5
*a) - 3*p^2*e^(-b*p*x - a*p + 3*b*x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + 3
*a) - p^2*e^(-b*p*x - a*p + b*x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + a) +
4*p*e^(-b*p*x - a*p + 11*b*x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + 11*a) +
28*p*e^(-b*p*x - a*p + 9*b*x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + 9*a) + 2
4*p*e^(-b*p*x - a*p + 7*b*x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + 7*a) - 24
*p*e^(-b*p*x - a*p + 5*b*x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + 5*a) - 28*
p*e^(-b*p*x - a*p + 3*b*x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + 3*a) - 4*p*
e^(-b*p*x - a*p + b*x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + a) + 3*e^(-b*p*
x - a*p + 11*b*x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + 11*a) + 25*e^(-b*p*x
 - a*p + 9*b*x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + 9*a) + 150*e^(-b*p*x -
 a*p + 7*b*x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + 7*a) - 150*e^(-b*p*x - a
*p + 5*b*x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + 5*a) - 25*e^(-b*p*x - a*p
+ 3*b*x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + 3*a) - 3*e^(-b*p*x - a*p + b*
x + p*log(1/2*e^(2*b*x + 2*a) - 1/2) + a))/(b*p^3*e^(6*b*x + 6*a) + 9*b*p^
2*e^(6*b*x + 6*a) + 23*b*p*e^(6*b*x + 6*a) + 15*b*e^(6*b*x + 6*a))
```

Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 255, normalized size of antiderivative = 4.32

$$\int \cosh^5(a+bx)\sinh^p(a+bx)\,dx = -e^{-5a-5bx}\left(\frac{e^{a+bx}}{2}-\frac{e^{-a-bx}}{2}\right)^p\left(\frac{p^2+4p+3}{32b\,(p^3+9p^2+23p+15)} - \frac{e^{10a+10bx}\,(p^2+4p+3)}{32b\,(p^3+9p^2+23p+15)} + \frac{e^{2a+2bx}\,(3p^2+28p+25)}{32b\,(p^3+9p^2+23p+15)} - \frac{e^{8a+8bx}\,(3p^2+28p+25)}{32b\,(p^3+9p^2+23p+15)} + \frac{e^{4a+4bx}\,(2p^2+24p+150)}{32b\,(p^3+9p^2+23p+15)} - \frac{e^{6a+6bx}\,(2p^2+24p+150)}{32b\,(p^3+9p^2+23p+15)}\right)$$

input
```
int(cosh(a + b*x)^5*sinh(a + b*x)^p,x)
```

output
```
-exp(- 5*a - 5*b*x)*(exp(a + b*x)/2 - exp(- a - b*x)/2)^p*((4*p + p^2 + 3)
/(32*b*(23*p + 9*p^2 + p^3 + 15)) - (exp(10*a + 10*b*x)*(4*p + p^2 + 3))/(
32*b*(23*p + 9*p^2 + p^3 + 15)) + (exp(2*a + 2*b*x)*(28*p + 3*p^2 + 25))/(
32*b*(23*p + 9*p^2 + p^3 + 15)) - (exp(8*a + 8*b*x)*(28*p + 3*p^2 + 25))/(
32*b*(23*p + 9*p^2 + p^3 + 15)) + (exp(4*a + 4*b*x)*(24*p + 2*p^2 + 150))/
(32*b*(23*p + 9*p^2 + p^3 + 15)) - (exp(6*a + 6*b*x)*(24*p + 2*p^2 + 150))
/(32*b*(23*p + 9*p^2 + p^3 + 15)))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 267, normalized size of antiderivative = 4.53

$$\int \cosh^5(a+bx)\sinh^p(a+bx)\,dx = \frac{\left(e^{2bx+2a}-1\right)^p \left(e^{10bx+10a}p^2+4e^{10bx+10a}p+3e^{10bx+10a}+3e^{8bx+8a}p^2+28e^{8bx+8a}p+25e^{8bx+8a}+2e^{6bx+6a}p^2\right.}{32e^{bpx+ap+5b}}$$

input
```
int(cosh(b*x+a)^5*sinh(b*x+a)^p,x)
```

output
```
((e**(2*a + 2*b*x) - 1)**p*(e**(10*a + 10*b*x)*p**2 + 4*e**(10*a + 10*b*x)
*p + 3*e**(10*a + 10*b*x) + 3*e**(8*a + 8*b*x)*p**2 + 28*e**(8*a + 8*b*x)*
p + 25*e**(8*a + 8*b*x) + 2*e**(6*a + 6*b*x)*p**2 + 24*e**(6*a + 6*b*x)*p
+ 150*e**(6*a + 6*b*x) - 2*e**(4*a + 4*b*x)*p**2 - 24*e**(4*a + 4*b*x)*p -
 150*e**(4*a + 4*b*x) - 3*e**(2*a + 2*b*x)*p**2 - 28*e**(2*a + 2*b*x)*p -
25*e**(2*a + 2*b*x) - p**2 - 4*p - 3))/(32*e**(a*p + 5*a + b*p*x + 5*b*x)*
2**p*b*(p**3 + 9*p**2 + 23*p + 15))
```

3.34 $\int \cosh^q(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \cosh^q(a+bx)\sinh(a+bx)\,dx = \frac{\cosh^{1+q}(a+bx)}{b(1+q)}$$

output
```
cosh(b*x+a)^(1+q)/b/(1+q)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cosh^q(a+bx)\sinh(a+bx)\,dx = \frac{\cosh^{1+q}(a+bx)}{b(1+q)}$$

input
```
Integrate[Cosh[a + b*x]^q*Sinh[a + b*x],x]
```

output
```
Cosh[a + b*x]^(1 + q)/(b*(1 + q))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 26, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\cosh^q(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ia+ibx)\cos(ia+ibx)^q dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \cos(ia+ibx)^q\sin(ia+ibx)dx$$

$$\Big\downarrow \text{3045}$$

$$\frac{\int \cosh^q(a+bx)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\cosh^{q+1}(a+bx)}{b(q+1)}$$

input
```
Int[Cosh[a + b*x]^q*Sinh[a + b*x],x]
```

output
```
Cosh[a + b*x]^(1 + q)/(b*(1 + q))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 4.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\cosh(bx+a)^{1+q}}{b(1+q)}$
default	$\frac{\cosh(bx+a)^{1+q}}{b(1+q)}$
risch	$\frac{\left(\frac{1}{2}\right)^q \left(e^{bx+a}\right)^{-q} \left(e^{2bx+2a}+1\right)^q \left(e^{\frac{i\pi \operatorname{csgn}\left(i\left(e^{2bx+2a}+1\right)\right)\operatorname{csgn}\left(i\left(e^{2bx+2a}+1\right)e^{-bx-a}\right)^2 q}{2}} e^{-\frac{i\pi \operatorname{csgn}\left(i\left(e^{2bx+2a}+1\right)\right)\operatorname{csgn}\left(i\right.}{}}\right.}{}$

input
```
int(cosh(b*x+a)^q*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
cosh(b*x+a)^(1+q)/b/(1+q)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(19) = 38$.

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.58

$$\int \cosh^q(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(bx+a)\cosh(q\log(\cosh(bx+a)))+\cosh(bx+a)\sinh(q\log(\cosh(bx+a)))}{(bq+b)\cosh(bx+a)^2-(bq+b)\sinh(bx+a)^2}$$

input
```
integrate(cosh(b*x+a)^q*sinh(b*x+a),x, algorithm="fricas")
```

output
```
(cosh(b*x + a)*cosh(q*log(cosh(b*x + a))) + cosh(b*x + a)*sinh(q*log(cosh(
b*x + a))))/((b*q + b)*cosh(b*x + a)^2 - (b*q + b)*sinh(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(14) = 28$.

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.58

$$\int \cosh^q(a+bx)\sinh(a+bx)\,dx = \begin{cases} \frac{x\sinh(a)}{\cosh(a)} & \text{for } b=0 \wedge q=-1 \\ x\sinh(a)\cosh^q(a) & \text{for } b=0 \\ \frac{\log(\cosh(a+bx))}{b} & \text{for } q=-1 \\ \frac{\cosh(a+bx)\cosh^q(a+bx)}{bq+b} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x+a)**q*sinh(b*x+a),x)
```

output
```
Piecewise((x*sinh(a)/cosh(a), Eq(b, 0) & Eq(q, -1)), (x*sinh(a)*cosh(a)**q
, Eq(b, 0)), (log(cosh(a + b*x))/b, Eq(q, -1)), (cosh(a + b*x)*cosh(a + b*
x)**q/(b*q + b), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cosh^q(a+bx)\sinh(a+bx)\,dx=\frac{\cosh{(bx+a)}^{q+1}}{b(q+1)}$$

input
```
integrate(cosh(b*x+a)^q*sinh(b*x+a),x, algorithm="maxima")
```

output
```
cosh(b*x + a)^(q + 1)/(b*(q + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \cosh^q(a+bx)\sinh(a+bx)\,dx=\frac{\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}+1\right)e^{(-bx-a)}\right)^{q+1}}{b(q+1)}$$

input
```
integrate(cosh(b*x+a)^q*sinh(b*x+a),x, algorithm="giac")
```

output
```
(1/2*(e^(2*b*x + 2*a) + 1)*e^(-b*x - a))^(q + 1)/(b*(q + 1))
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cosh^q(a+bx)\sinh(a+bx)\,dx=\frac{\cosh(a+b\,x)^{q+1}}{b\,\left(q+1\right)}$$

input
```
int(cosh(a + b*x)^q*sinh(a + b*x),x)
```

output
```
cosh(a + b*x)^(q + 1)/(b*(q + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \cosh^q(a+bx)\sinh(a+bx)\,dx = \frac{\cosh(bx+a)^q\cosh(bx+a)}{b(q+1)}$$

input
```
int(cosh(b*x+a)^q*sinh(b*x+a),x)
```

output
```
(cosh(a + b*x)**q*cosh(a + b*x))/(b*(q + 1))
```

3.35 $\int \cosh^q(a+bx)\sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \cosh^q(a+bx)\sinh^3(a+bx)\,dx = -\frac{\cosh^{1+q}(a+bx)}{b(1+q)} + \frac{\cosh^{3+q}(a+bx)}{b(3+q)}$$

output
```
-cosh(b*x+a)^(1+q)/b/(1+q)+cosh(b*x+a)^(3+q)/b/(3+q)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \cosh^q(a+bx)\sinh^3(a+bx)\,dx = \frac{\cosh^{1+q}(a+bx)(-5-q+(1+q)\cosh(2(a+bx)))}{2b(1+q)(3+q)}$$

input
```
Integrate[Cosh[a + b*x]^q*Sinh[a + b*x]^3,x]
```

output
```
(Cosh[a + b*x]^(1 + q)*(-5 - q + (1 + q)*Cosh[2*(a + b*x)]))/(2*b*(1 + q)*
(3 + q))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 26, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(a+bx)\cosh^q(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin(ia+ibx)^3\cos(ia+ibx)^q dx$$

$$\Big\downarrow \text{26}$$

$$i\int \cos(ia+ibx)^q\sin(ia+ibx)^3 dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \cosh^q(a+bx)\left(1-\cosh^2(a+bx)\right)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\cosh^q(a+bx)-\cosh^{q+2}(a+bx)\right)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\cosh^{q+1}(a+bx)}{q+1}-\frac{\cosh^{q+3}(a+bx)}{q+3}}{b}$$

input
```
Int[Cosh[a + b*x]^q*Sinh[a + b*x]^3,x]
```

output
```
-((Cosh[a + b*x]^(1 + q)/(1 + q) - Cosh[a + b*x]^(3 + q)/(3 + q))/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\frac{\cosh(bx+a)^3 e^{q\ln(\cosh(bx+a))}}{b(3+q)} - \frac{\cosh(bx+a)\, e^{q\ln(\cosh(bx+a))}}{b(1+q)}$$

input
```
int(cosh(b*x+a)^q*sinh(b*x+a)^3,x)
```

output
```
1/b/(3+q)*cosh(b*x+a)^3*exp(q*ln(cosh(b*x+a)))-1/b/(1+q)*cosh(b*x+a)*exp(q
*ln(cosh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(40) = 80$.

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.72

$$\int \cosh^q(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{\left((q+1)\cosh(bx+a)^3 + 3(q+1)\cosh(bx+a)\sinh(bx+a)^2 - (q+9)\cosh(bx+a)\right)\cosh(q\log(\cosh}{4\left((bq^2+4bq+3b)\cosh(bx+a)^4 - 2(bq^2+4b\right.}$$

input
```
integrate(cosh(b*x+a)^q*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(((q + 1)*cosh(b*x + a)^3 + 3*(q + 1)*cosh(b*x + a)*sinh(b*x + a)^2 -
(q + 9)*cosh(b*x + a))*cosh(q*log(cosh(b*x + a))) + ((q + 1)*cosh(b*x + a)
^3 + 3*(q + 1)*cosh(b*x + a)*sinh(b*x + a)^2 - (q + 9)*cosh(b*x + a))*sinh
(q*log(cosh(b*x + a))))/((b*q^2 + 4*b*q + 3*b)*cosh(b*x + a)^4 - 2*(b*q^2
+ 4*b*q + 3*b)*cosh(b*x + a)^2*sinh(b*x + a)^2 + (b*q^2 + 4*b*q + 3*b)*sin
h(b*x + a)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(29) = 58$.

Time = 1.21 (sec) , antiderivative size = 648, normalized size of antiderivative = 16.20

$$\int \cosh^q(a+bx)\sinh^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)**q*sinh(b*x+a)**3,x)
```

output
```
Piecewise((x*sinh(a)**3*cosh(a)**q, Eq(b, 0)), (log(cosh(a + b*x))/b - sin
h(a + b*x)**2/(2*b*cosh(a + b*x)**2), Eq(q, -3)), (-b*x*tanh(a/2 + b*x/2)*
*4/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + 2*b*x*tanh(a/
2 + b*x/2)**2/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - b*
x/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + 2*log(tanh(a/2
 + b*x/2) + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2
 + b*x/2)**2 + b) - 4*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**2/(b*t
anh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) + 2*log(tanh(a/2 + b*x
/2) + 1)/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - log(tan
h(a/2 + b*x/2)**2 + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**4 - 2*b*
tanh(a/2 + b*x/2)**2 + b) + 2*log(tanh(a/2 + b*x/2)**2 + 1)*tanh(a/2 + b*x
/2)**2/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 + b) - log(tanh(
a/2 + b*x/2)**2 + 1)/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2)**2 +
b) + 2*tanh(a/2 + b*x/2)**2/(b*tanh(a/2 + b*x/2)**4 - 2*b*tanh(a/2 + b*x/2
)**2 + b), Eq(q, -1)), (q*sinh(a + b*x)**2*cosh(a + b*x)*cosh(a + b*x)**q/
(b*q**2 + 4*b*q + 3*b) + 3*sinh(a + b*x)**2*cosh(a + b*x)*cosh(a + b*x)**q
/(b*q**2 + 4*b*q + 3*b) - 2*cosh(a + b*x)**3*cosh(a + b*x)**q/(b*q**2 + 4*
b*q + 3*b), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(40) = 80$.

Time = 0.18 (sec) , antiderivative size = 293, normalized size of antiderivative = 7.32

$$\int \cosh^q(a+bx)\sinh^3(a+bx)\,dx = \frac{qe^{\left((bx+a)q+3\,bx+q\log\left(e^{(-2\,bx-2\,a)}+1\right)+3\,a\right)}}{8\,(2^q q^2+2^{q+2}q+3\cdot 2^q)b} - \frac{(q+9)e^{\left((bx+a)q+bx+q\log\left(e^{(-2\,bx-2\,a)}+1\right)+a\right)}}{8\,(2^q q^2+2^{q+2}q+3\cdot 2^q)b} - \frac{(q+9)e^{\left((bx+a)q-bx+q\log\left(e^{(-2\,bx-2\,a)}+1\right)-a\right)}}{8\,(2^q q^2+2^{q+2}q+3\cdot 2^q)b} + \frac{(q+1)e^{\left((bx+a)q-3\,bx+q\log\left(e^{(-2\,bx-2\,a)}+1\right)-3\,a\right)}}{8\,(2^q q^2+2^{q+2}q+3\cdot 2^q)b} + \frac{e^{\left((bx+a)q+3\,bx+q\log\left(e^{(-2\,bx-2\,a)}+1\right)+3\,a\right)}}{8\,(2^q q^2+2^{q+2}q+3\cdot 2^q)b}$$

input
```
integrate(cosh(b*x+a)^q*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/8*q*e^((b*x + a)*q + 3*b*x + q*log(e^(-2*b*x - 2*a) + 1) + 3*a)/((2^q*q^
2 + 2^(q + 2)*q + 3*2^q)*b) - 1/8*(q + 9)*e^((b*x + a)*q + b*x + q*log(e^(
-2*b*x - 2*a) + 1) + a)/((2^q*q^2 + 2^(q + 2)*q + 3*2^q)*b) - 1/8*(q + 9)*
e^((b*x + a)*q - b*x + q*log(e^(-2*b*x - 2*a) + 1) - a)/((2^q*q^2 + 2^(q +
 2)*q + 3*2^q)*b) + 1/8*(q + 1)*e^((b*x + a)*q - 3*b*x + q*log(e^(-2*b*x -
 2*a) + 1) - 3*a)/((2^q*q^2 + 2^(q + 2)*q + 3*2^q)*b) + 1/8*e^((b*x + a)*q
 + 3*b*x + q*log(e^(-2*b*x - 2*a) + 1) + 3*a)/((2^q*q^2 + 2^(q + 2)*q + 3*
2^q)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(40) = 80$.

Time = 0.16 (sec) , antiderivative size = 325, normalized size of antiderivative = 8.12

$$\int \cosh^q(a+bx)\sinh^3(a+bx)\,dx$$
$$= \frac{qe^{\left(-bqx-aq+7\,bx+q\log\left(\frac{1}{2}\,e^{(2\,bx+2\,a)}+\frac{1}{2}\right)+7\,a\right)} - qe^{\left(-bqx-aq+5\,bx+q\log\left(\frac{1}{2}\,e^{(2\,bx+2\,a)}+\frac{1}{2}\right)+5\,a\right)} - qe^{\left(-bqx-aq+3\,bx+q\log\left(\frac{1}{2}\,e^{(2\,bx+\right.}\right.}}{}$$

input
```
integrate(cosh(b*x+a)^q*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/8*(q*e^(-b*q*x - a*q + 7*b*x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + 7*a) -
 q*e^(-b*q*x - a*q + 5*b*x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + 5*a) - q*e
^(-b*q*x - a*q + 3*b*x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + 3*a) + q*e^(-b
*q*x - a*q + b*x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + a) + e^(-b*q*x - a*q
 + 7*b*x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + 7*a) - 9*e^(-b*q*x - a*q + 5
*b*x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + 5*a) - 9*e^(-b*q*x - a*q + 3*b*x
 + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + 3*a) + e^(-b*q*x - a*q + b*x + q*log
(1/2*e^(2*b*x + 2*a) + 1/2) + a))/(b*q^2*e^(4*b*x + 4*a) + 4*b*q*e^(4*b*x
+ 4*a) + 3*b*e^(4*b*x + 4*a))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 132, normalized size of antiderivative = 3.30

$$\int \cosh^q(a+bx)\sinh^3(a+bx)\,dx = \left(\frac{1}{2}\right)^q \mathrm{e}^{-3\,a-3\,b\,x}\,\left(\mathrm{e}^{a+b\,x}+\mathrm{e}^{-a-b\,x}\right)^q\,\left(\frac{\frac{q}{8}+\frac{1}{8}}{b\,\left(q^2+4\,q+3\right)} - \frac{\mathrm{e}^{2\,a+2\,b\,x}\,\left(q+9\right)}{8\,b\,\left(q^2+4\,q+3\right)} + \frac{\mathrm{e}^{6\,a+6\,b\,x}\,\left(q+1\right)}{8\,b\,\left(q^2+4\,q+3\right)} - \frac{\mathrm{e}^{4\,a+4\,b\,x}\,\left(q+9\right)}{8\,b\,\left(q^2+4\,q+3\right)}\right)$$

input
```
int(cosh(a + b*x)^q*sinh(a + b*x)^3,x)
```

output
```
(1/2)^q*exp(- 3*a - 3*b*x)*(exp(a + b*x) + exp(- a - b*x))^q*((q/8 + 1/8)/
(b*(4*q + q^2 + 3)) - (exp(2*a + 2*b*x)*(q + 9))/(8*b*(4*q + q^2 + 3)) + (
exp(6*a + 6*b*x)*(q + 1))/(8*b*(4*q + q^2 + 3)) - (exp(4*a + 4*b*x)*(q + 9
))/(8*b*(4*q + q^2 + 3)))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 3.20

$$\int \cosh^q(a+bx)\sinh^3(a+bx)\,dx = \frac{\left(e^{2bx+2a}+1\right)^q \left(e^{6bx+6a}q+e^{6bx+6a}-e^{4bx+4a}q-9e^{4bx+4a}-e^{2bx+2a}q-9e^{2bx+2a}+q+1\right)}{8e^{bqx+aq+3bx+3a}2^q b\left(q^2+4q+3\right)}$$

input
```
int(cosh(b*x+a)^q*sinh(b*x+a)^3,x)
```

output
```
((e**(2*a + 2*b*x) + 1)**q*(e**(6*a + 6*b*x)*q + e**(6*a + 6*b*x) - e**(4*
a + 4*b*x)*q - 9*e**(4*a + 4*b*x) - e**(2*a + 2*b*x)*q - 9*e**(2*a + 2*b*x
) + q + 1))/(8*e**(a*q + 3*a + b*q*x + 3*b*x)*2**q*b*(q**2 + 4*q + 3))
```

3.36 $\int \cosh^q(a+bx)\sinh^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 59

$$\int \cosh^q(a+bx)\sinh^5(a+bx)\,dx = \frac{\cosh^{1+q}(a+bx)}{b(1+q)} - \frac{2\cosh^{3+q}(a+bx)}{b(3+q)} + \frac{\cosh^{5+q}(a+bx)}{b(5+q)}$$

output
```
cosh(b*x+a)^(1+q)/b/(1+q)-2*cosh(b*x+a)^(3+q)/b/(3+q)+cosh(b*x+a)^(5+q)/b/
(5+q)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.31

$$\int \cosh^q(a+bx)\sinh^5(a+bx)\,dx = \frac{\cosh^{1+q}(a+bx)\left(89+28q+3q^2-4(7+8q+q^2)\cosh(2(a+bx))+(3+4q+q^2)\cosh(4(a+bx))\right)}{8b(1+q)(3+q)(5+q)}$$

input
```
Integrate[Cosh[a + b*x]^q*Sinh[a + b*x]^5,x]
```

output
```
(Cosh[a + b*x]^(1 + q)*(89 + 28*q + 3*q^2 - 4*(7 + 8*q + q^2)*Cosh[2*(a +
b*x)] + (3 + 4*q + q^2)*Cosh[4*(a + b*x)]))/(8*b*(1 + q)*(3 + q)*(5 + q))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 26, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^5(a+bx)\cosh^q(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -i\sin(ia+ibx)^5\cos(ia+ibx)^q dx$$

$$\Big\downarrow \ 26$$

$$-i\int \cos(ia+ibx)^q\sin(ia+ibx)^5 dx$$

$$\Big\downarrow \ 3045$$

$$\frac{\int \cosh^q(a+bx)\left(1-\cosh^2(a+bx)\right)^2 d\cosh(a+bx)}{b}$$

$$\Big\downarrow \ 244$$

$$\frac{\int \left(\cosh^q(a+bx)-2\cosh^{q+2}(a+bx)+\cosh^{q+4}(a+bx)\right) d\cosh(a+bx)}{b}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\cosh^{q+1}(a+bx)}{q+1}-\frac{2\cosh^{q+3}(a+bx)}{q+3}+\frac{\cosh^{q+5}(a+bx)}{q+5}}{b}$$

input
```
Int[Cosh[a + b*x]^q*Sinh[a + b*x]^5,x]
```

output
```
(Cosh[a + b*x]^(1 + q)/(1 + q) - (2*Cosh[a + b*x]^(3 + q))/(3 + q) + Cosh[
a + b*x]^(5 + q)/(5 + q))/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

$$\frac{\cosh\left(bx+a\right)\mathrm{e}^{q\ln(\cosh(bx+a))}}{b\left(1+q\right)}+\frac{\cosh\left(bx+a\right)^{5}\mathrm{e}^{q\ln(\cosh(bx+a))}}{b\left(5+q\right)}-\frac{2\cosh\left(bx+a\right)^{3}\mathrm{e}^{q\ln(\cosh(bx+a))}}{b\left(3+q\right)}$$

input
```
int(cosh(b*x+a)^q*sinh(b*x+a)^5,x)
```

output
```
1/b/(1+q)*cosh(b*x+a)*exp(q*ln(cosh(b*x+a)))+1/b/(5+q)*cosh(b*x+a)^5*exp(q
*ln(cosh(b*x+a)))-2/b/(3+q)*cosh(b*x+a)^3*exp(q*ln(cosh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(59) = 118$.

Time = 0.10 (sec) , antiderivative size = 407, normalized size of antiderivative = 6.90

$$\int \cosh^q(a+bx)\sinh^5(a+bx)\,dx = \frac{\left((q^2+4q+3)\cosh(bx+a)^5+5(q^2+4q+3)\cosh(bx+a)\sinh(bx+a)^4-(3q^2+28q+25)\cosh(bx\right.}{}$$

input
```
integrate(cosh(b*x+a)^q*sinh(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/16*(((q^2 + 4*q + 3)*cosh(b*x + a)^5 + 5*(q^2 + 4*q + 3)*cosh(b*x + a)*s
inh(b*x + a)^4 - (3*q^2 + 28*q + 25)*cosh(b*x + a)^3 + (10*(q^2 + 4*q + 3)
*cosh(b*x + a)^3 - 3*(3*q^2 + 28*q + 25)*cosh(b*x + a))*sinh(b*x + a)^2 +
2*(q^2 + 12*q + 75)*cosh(b*x + a))*cosh(q*log(cosh(b*x + a))) + ((q^2 + 4*
q + 3)*cosh(b*x + a)^5 + 5*(q^2 + 4*q + 3)*cosh(b*x + a)*sinh(b*x + a)^4 -
 (3*q^2 + 28*q + 25)*cosh(b*x + a)^3 + (10*(q^2 + 4*q + 3)*cosh(b*x + a)^3
 - 3*(3*q^2 + 28*q + 25)*cosh(b*x + a))*sinh(b*x + a)^2 + 2*(q^2 + 12*q +
75)*cosh(b*x + a))*sinh(q*log(cosh(b*x + a))))/((b*q^3 + 9*b*q^2 + 23*b*q
+ 15*b)*cosh(b*x + a)^6 - 3*(b*q^3 + 9*b*q^2 + 23*b*q + 15*b)*cosh(b*x + a
)^4*sinh(b*x + a)^2 + 3*(b*q^3 + 9*b*q^2 + 23*b*q + 15*b)*cosh(b*x + a)^2*
sinh(b*x + a)^4 - (b*q^3 + 9*b*q^2 + 23*b*q + 15*b)*sinh(b*x + a)^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2351 vs. $2(46) = 92$.

Time = 4.40 (sec) , antiderivative size = 2351, normalized size of antiderivative = 39.85

$$\int \cosh^q(a+bx)\sinh^5(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)**q*sinh(b*x+a)**5,x)
```

output
```
Piecewise((x*sinh(a)**5*cosh(a)**q, Eq(b, 0)), (log(cosh(a + b*x))/b - sin
h(a + b*x)**4/(4*b*cosh(a + b*x)**4) - sinh(a + b*x)**2/(2*b*cosh(a + b*x)
**2), Eq(q, -5)), (-2*b*x*tanh(a/2 + b*x/2)**8/(b*tanh(a/2 + b*x/2)**8 - 2
*b*tanh(a/2 + b*x/2)**4 + b) + 4*b*x*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*
x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) - 2*b*x/(b*tanh(a/2 + b*x/2)**8 -
2*b*tanh(a/2 + b*x/2)**4 + b) + 4*log(tanh(a/2 + b*x/2) + 1)*tanh(a/2 + b*
x/2)**8/(b*tanh(a/2 + b*x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) - 8*log(ta
nh(a/2 + b*x/2) + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2)**8 - 2*b*ta
nh(a/2 + b*x/2)**4 + b) + 4*log(tanh(a/2 + b*x/2) + 1)/(b*tanh(a/2 + b*x/2
)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) - 2*log(tanh(a/2 + b*x/2)**2 + 1)*tan
h(a/2 + b*x/2)**8/(b*tanh(a/2 + b*x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b)
+ 4*log(tanh(a/2 + b*x/2)**2 + 1)*tanh(a/2 + b*x/2)**4/(b*tanh(a/2 + b*x/2
)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) - 2*log(tanh(a/2 + b*x/2)**2 + 1)/(b*
tanh(a/2 + b*x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) + 4*tanh(a/2 + b*x/2)
**6/(b*tanh(a/2 + b*x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b) + 4*tanh(a/2 +
 b*x/2)**2/(b*tanh(a/2 + b*x/2)**8 - 2*b*tanh(a/2 + b*x/2)**4 + b), Eq(q,
-3)), (b*x*tanh(a/2 + b*x/2)**8/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b
*x/2)**6 + 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) - 4*b*
x*tanh(a/2 + b*x/2)**6/(b*tanh(a/2 + b*x/2)**8 - 4*b*tanh(a/2 + b*x/2)**6
+ 6*b*tanh(a/2 + b*x/2)**4 - 4*b*tanh(a/2 + b*x/2)**2 + b) + 6*b*x*tanh...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(59) = 118$.

Time = 0.17 (sec) , antiderivative size = 558, normalized size of antiderivative = 9.46

$$\int \cosh^q(a+bx)\sinh^5(a+bx)\,dx$$
$$= \frac{q^2e^{\left((bx+a)q+5\,bx+q\log\left(e^{(-2\,bx-2\,a)}+1\right)+5\,a\right)}}{32\,(2^qq^3+9\cdot 2^qq^2+23\cdot 2^qq+15\cdot 2^q)b} + \frac{qe^{\left((bx+a)q+5\,bx+q\log\left(e^{(-2\,bx-2\,a)}+1\right)+5\,a\right)}}{8\,(2^qq^3+9\cdot 2^qq^2+23\cdot 2^qq+15\cdot 2^q)b}$$
$$- \frac{(3\,q^2+28\,q+25)e^{\left((bx+a)q+3\,bx+q\log\left(e^{(-2\,bx-2\,a)}+1\right)+3\,a\right)}}{32\,(2^qq^3+9\cdot 2^qq^2+23\cdot 2^qq+15\cdot 2^q)b}$$
$$+ \frac{(q^2+12\,q+75)e^{\left((bx+a)q+bx+q\log\left(e^{(-2\,bx-2\,a)}+1\right)+a\right)}}{16\,(2^qq^3+9\cdot 2^qq^2+23\cdot 2^qq+15\cdot 2^q)b}$$
$$+ \frac{(q^2+12\,q+75)e^{\left((bx+a)q-bx+q\log\left(e^{(-2\,bx-2\,a)}+1\right)-a\right)}}{16\,(2^qq^3+9\cdot 2^qq^2+23\cdot 2^qq+15\cdot 2^q)b}$$
$$- \frac{(3\,q^2+28\,q+25)e^{\left((bx+a)q-3\,bx+q\log\left(e^{(-2\,bx-2\,a)}+1\right)-3\,a\right)}}{32\,(2^qq^3+9\cdot 2^qq^2+23\cdot 2^qq+15\cdot 2^q)b}$$
$$+ \frac{(q^2+4\,q+3)e^{\left((bx+a)q-5\,bx+q\log\left(e^{(-2\,bx-2\,a)}+1\right)-5\,a\right)}}{32\,(2^qq^3+9\cdot 2^qq^2+23\cdot 2^qq+15\cdot 2^q)b}$$
$$+ \frac{3\,e^{\left((bx+a)q+5\,bx+q\log\left(e^{(-2\,bx-2\,a)}+1\right)+5\,a\right)}}{32\,(2^qq^3+9\cdot 2^qq^2+23\cdot 2^qq+15\cdot 2^q)b}$$

input
```
integrate(cosh(b*x+a)^q*sinh(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/32*q^2*e^((b*x + a)*q + 5*b*x + q*log(e^(-2*b*x - 2*a) + 1) + 5*a)/((2^q
*q^3 + 9*2^q*q^2 + 23*2^q*q + 15*2^q)*b) + 1/8*q*e^((b*x + a)*q + 5*b*x +
q*log(e^(-2*b*x - 2*a) + 1) + 5*a)/((2^q*q^3 + 9*2^q*q^2 + 23*2^q*q + 15*2
^q)*b) - 1/32*(3*q^2 + 28*q + 25)*e^((b*x + a)*q + 3*b*x + q*log(e^(-2*b*x
 - 2*a) + 1) + 3*a)/((2^q*q^3 + 9*2^q*q^2 + 23*2^q*q + 15*2^q)*b) + 1/16*(
q^2 + 12*q + 75)*e^((b*x + a)*q + b*x + q*log(e^(-2*b*x - 2*a) + 1) + a)/(
(2^q*q^3 + 9*2^q*q^2 + 23*2^q*q + 15*2^q)*b) + 1/16*(q^2 + 12*q + 75)*e^((
b*x + a)*q - b*x + q*log(e^(-2*b*x - 2*a) + 1) - a)/((2^q*q^3 + 9*2^q*q^2
+ 23*2^q*q + 15*2^q)*b) - 1/32*(3*q^2 + 28*q + 25)*e^((b*x + a)*q - 3*b*x
+ q*log(e^(-2*b*x - 2*a) + 1) - 3*a)/((2^q*q^3 + 9*2^q*q^2 + 23*2^q*q + 15
*2^q)*b) + 1/32*(q^2 + 4*q + 3)*e^((b*x + a)*q - 5*b*x + q*log(e^(-2*b*x -
 2*a) + 1) - 5*a)/((2^q*q^3 + 9*2^q*q^2 + 23*2^q*q + 15*2^q)*b) + 3/32*e^(
(b*x + a)*q + 5*b*x + q*log(e^(-2*b*x - 2*a) + 1) + 5*a)/((2^q*q^3 + 9*2^q
*q^2 + 23*2^q*q + 15*2^q)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(59) = 118$.

Time = 0.20 (sec) , antiderivative size = 721, normalized size of antiderivative = 12.22

$$\int \cosh^q(a+bx)\sinh^5(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^q*sinh(b*x+a)^5,x, algorithm="giac")
```

output
```
1/32*(q^2*e^(-b*q*x - a*q + 11*b*x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + 11
*a) - 3*q^2*e^(-b*q*x - a*q + 9*b*x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + 9
*a) + 2*q^2*e^(-b*q*x - a*q + 7*b*x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + 7
*a) + 2*q^2*e^(-b*q*x - a*q + 5*b*x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + 5
*a) - 3*q^2*e^(-b*q*x - a*q + 3*b*x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + 3
*a) + q^2*e^(-b*q*x - a*q + b*x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + a) +
4*q*e^(-b*q*x - a*q + 11*b*x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + 11*a) -
28*q*e^(-b*q*x - a*q + 9*b*x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + 9*a) + 2
4*q*e^(-b*q*x - a*q + 7*b*x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + 7*a) + 24
*q*e^(-b*q*x - a*q + 5*b*x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + 5*a) - 28*
q*e^(-b*q*x - a*q + 3*b*x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + 3*a) + 4*q*
e^(-b*q*x - a*q + b*x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + a) + 3*e^(-b*q*
x - a*q + 11*b*x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + 11*a) - 25*e^(-b*q*x
 - a*q + 9*b*x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + 9*a) + 150*e^(-b*q*x -
 a*q + 7*b*x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + 7*a) + 150*e^(-b*q*x - a
*q + 5*b*x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + 5*a) - 25*e^(-b*q*x - a*q
+ 3*b*x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + 3*a) + 3*e^(-b*q*x - a*q + b*
x + q*log(1/2*e^(2*b*x + 2*a) + 1/2) + a))/(b*q^3*e^(6*b*x + 6*a) + 9*b*q^
2*e^(6*b*x + 6*a) + 23*b*q*e^(6*b*x + 6*a) + 15*b*e^(6*b*x + 6*a))
```

Mupad [B] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 254, normalized size of antiderivative = 4.31

$$\int \cosh^q(a+bx)\sinh^5(a+bx)\,dx = e^{-5a-5bx}\left(\frac{e^{a+bx}}{2}+\frac{e^{-a-bx}}{2}\right)^q\left(\frac{q^2+4q+3}{32b(q^3+9q^2+23q+15)} + \frac{e^{10a+10bx}(q^2+4q+3)}{32b(q^3+9q^2+23q+15)} - \frac{e^{2a+2bx}(3q^2+28q+25)}{32b(q^3+9q^2+23q+15)} - \frac{e^{8a+8bx}(3q^2+28q+25)}{32b(q^3+9q^2+23q+15)} + \frac{e^{4a+4bx}(2q^2+24q+150)}{32b(q^3+9q^2+23q+15)} + \frac{e^{6a+6bx}(2q^2+24q+150)}{32b(q^3+9q^2+23q+15)}\right)$$

input
```
int(cosh(a + b*x)^q*sinh(a + b*x)^5,x)
```

output
```
exp(- 5*a - 5*b*x)*(exp(a + b*x)/2 + exp(- a - b*x)/2)^q*((4*q + q^2 + 3)/
(32*b*(23*q + 9*q^2 + q^3 + 15)) + (exp(10*a + 10*b*x)*(4*q + q^2 + 3))/(3
2*b*(23*q + 9*q^2 + q^3 + 15)) - (exp(2*a + 2*b*x)*(28*q + 3*q^2 + 25))/(3
2*b*(23*q + 9*q^2 + q^3 + 15)) - (exp(8*a + 8*b*x)*(28*q + 3*q^2 + 25))/(3
2*b*(23*q + 9*q^2 + q^3 + 15)) + (exp(4*a + 4*b*x)*(24*q + 2*q^2 + 150))/(
32*b*(23*q + 9*q^2 + q^3 + 15)) + (exp(6*a + 6*b*x)*(24*q + 2*q^2 + 150))/
(32*b*(23*q + 9*q^2 + q^3 + 15)))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 265, normalized size of antiderivative = 4.49

$$\int \cosh^q(a+bx)\sinh^5(a+bx)\,dx = \frac{(e^{2bx+2a}+1)^q\left(e^{10bx+10a}q^2+4e^{10bx+10a}q+3e^{10bx+10a}-3e^{8bx+8a}q^2-28e^{8bx+8a}q-25e^{8bx+8a}+2e^{6bx+6a}q^2\right.}{32e^{bqx+aq+5bx}}$$

input
```
int(cosh(b*x+a)^q*sinh(b*x+a)^5,x)
```

output
```
((e**(2*a + 2*b*x) + 1)**q*(e**(10*a + 10*b*x)*q**2 + 4*e**(10*a + 10*b*x)
*q + 3*e**(10*a + 10*b*x) - 3*e**(8*a + 8*b*x)*q**2 - 28*e**(8*a + 8*b*x)*
q - 25*e**(8*a + 8*b*x) + 2*e**(6*a + 6*b*x)*q**2 + 24*e**(6*a + 6*b*x)*q
+ 150*e**(6*a + 6*b*x) + 2*e**(4*a + 4*b*x)*q**2 + 24*e**(4*a + 4*b*x)*q +
 150*e**(4*a + 4*b*x) - 3*e**(2*a + 2*b*x)*q**2 - 28*e**(2*a + 2*b*x)*q -
25*e**(2*a + 2*b*x) + q**2 + 4*q + 3))/(32*e**(a*q + 5*a + b*q*x + 5*b*x)*
2**q*b*(q**3 + 9*q**2 + 23*q + 15))
```

3.37 $\int \sinh(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \sinh(a+bx)\tanh(a+bx)\,dx=-\frac{\arctan(\sinh(a+bx))}{b}+\frac{\sinh(a+bx)}{b}$$

output
```
-arctan(sinh(b*x+a))/b+sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \sinh(a+bx)\tanh(a+bx)\,dx=-\frac{\arctan(\sinh(a+bx))}{b}+\frac{\sinh(a+bx)}{b}$$

input
```
Integrate[Sinh[a + b*x]*Tanh[a + b*x],x]
```

output
```
-(ArcTan[Sinh[a + b*x]]/b) + Sinh[a + b*x]/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 25, 3072, 25, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\tanh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin(ia+ibx)\tan(ia+ibx)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin(ia+ibx)\tan(ia+ibx)dx$$

$$\Big\downarrow \text{3072}$$

$$-\frac{\int -\frac{\sinh^2(a+bx)}{\sinh^2(a+bx)+1}d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sinh^2(a+bx)}{\sinh^2(a+bx)+1}d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\int \frac{1}{\sinh^2(a+bx)+1}d\sinh(a+bx)-\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\arctan(\sinh(a+bx))-\sinh(a+bx)}{b}$$

input
```
Int[Sinh[a + b*x]*Tanh[a + b*x],x]
```

output
```
-((ArcTan[Sinh[a + b*x]] - Sinh[a + b*x])/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\sinh(bx+a)-2\arctan(\mathrm{e}^{bx+a})}{b}$	21
default	$\frac{\sinh(bx+a)-2\arctan(\mathrm{e}^{bx+a})}{b}$	21
risch	$\frac{\mathrm{e}^{bx+a}}{2b}-\frac{\mathrm{e}^{-bx-a}}{2b}+\frac{i\ln(\mathrm{e}^{bx+a}-i)}{b}-\frac{i\ln(\mathrm{e}^{bx+a}+i)}{b}$	59

input
```
int(sinh(b*x+a)*tanh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(sinh(b*x+a)-2*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(23) = 46$.

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.74

$$\int \sinh(a+bx)\tanh(a+bx)\,dx =$$
$$-\frac{4\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)-\cosh\left(bx+a\right)^{2}-2\cosh\left(bx\right.}{2\left(b\cosh\left(bx+a\right)+b\sinh\left(bx+a\right)\right)}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(4*(cosh(b*x + a) + sinh(b*x + a))*arctan(cosh(b*x + a) + sinh(b*x +
a)) - cosh(b*x + a)^2 - 2*cosh(b*x + a)*sinh(b*x + a) - sinh(b*x + a)^2 +
1)/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

Sympy [F]

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = \int \sinh\left(a+bx\right)\tanh\left(a+bx\right)dx$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)*tanh(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = \frac{2\,\arctan\left(e^{(-bx-a)}\right)}{b} + \frac{e^{(bx+a)}}{2\,b} - \frac{e^{(-bx-a)}}{2\,b}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a),x, algorithm="maxima")
```

output
```
2*arctan(e^(-b*x - a))/b + 1/2*e^(b*x + a)/b - 1/2*e^(-b*x - a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = -\frac{4\,\arctan\left(e^{(bx+a)}\right) - e^{(bx+a)} + e^{(-bx-a)}}{2\,b}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a),x, algorithm="giac")
```

output
```
-1/2*(4*arctan(e^(b*x + a)) - e^(b*x + a) + e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.13

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = \frac{e^{a+b\,x}}{2\,b} - \frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{e^{-a-b\,x}}{2\,b}$$

input
```
int(sinh(a + b*x)*tanh(a + b*x),x)
```

output
```
exp(a + b*x)/(2*b) - (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2)
 - exp(- a - b*x)/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int \sinh(a+bx)\tanh(a+bx)\,dx = \frac{-4e^{bx+a}atan\left(e^{bx+a}\right)+e^{2bx+2a}-1}{2e^{bx+a}b}$$

input
```
int(sinh(b*x+a)*tanh(b*x+a),x)
```

output
```
( - 4*e**(a + b*x)*atan(e**(a + b*x)) + e**(2*a + 2*b*x) - 1)/(2*e**(a + b
*x)*b)
```

3.38 $\int \sinh(a+bx)\tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx=\frac{\cosh(a+bx)}{b}+\frac{\operatorname{sech}(a+bx)}{b}$$

output
```
cosh(b*x+a)/b+sech(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx=\frac{\cosh(a+bx)}{b}+\frac{\operatorname{sech}(a+bx)}{b}$$

input
```
Integrate[Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
Cosh[a + b*x]/b + Sech[a + b*x]/b
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin(ia+ibx)\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sin(ia+ibx)\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\int \left(1-\cosh^2(a+bx)\right)\text{sech}^2(a+bx)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\text{sech}^2(a+bx)-1\right)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\cosh(a+bx)-\text{sech}(a+bx)}{b}$$

input
```
Int[Sinh[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
-((-Cosh[a + b*x] - Sech[a + b*x])/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^2}{\cosh(bx+a)}+\frac{2}{\cosh(bx+a)}}{b}$	33
default	$\frac{\frac{\sinh(bx+a)^2}{\cosh(bx+a)}+\frac{2}{\cosh(bx+a)}}{b}$	33
risch	$\frac{e^{3bx+3a}+6\,e^{bx+a}+e^{-bx-a}}{2b\left(e^{2bx+2a}+1\right)}$	46

input
```
int(sinh(b*x+a)*tanh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(sinh(b*x+a)^2/cosh(b*x+a)+2/cosh(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{\cosh{(bx+a)}^2+\sinh{(bx+a)}^2+3}{2\,b\cosh{(bx+a)}}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + a)^2 + sinh(b*x + a)^2 + 3)/(b*cosh(b*x + a))
```

Sympy [F]

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \int \sinh{(a+bx)}\tanh^2{(a+bx)}\,dx$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)**2,x)
```

output
```
Integral(sinh(a + b*x)*tanh(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(21) = 42.

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.57

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{e^{(-bx-a)}}{2\,b} + \frac{5\,e^{(-2\,bx-2\,a)}+1}{2\,b(e^{(-bx-a)}+e^{(-3\,bx-3\,a)})}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*e^(-b*x - a)/b + 1/2*(5*e^(-2*b*x - 2*a) + 1)/(b*(e^(-b*x - a) + e^(-3
*b*x - 3*a)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.95

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{\frac{4}{e^{(bx+a)}+e^{(-bx-a)}} + e^{(bx+a)} + e^{(-bx-a)}}{2\,b}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*(4/(e^(b*x + a) + e^(-b*x - a)) + e^(b*x + a) + e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.33

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{e^{-a-b\,x}\,\left(6\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}{2\,b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(sinh(a + b*x)*tanh(a + b*x)^2,x)
```

output
```
(exp(- a - b*x)*(6*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1))/(2*b*(exp(2*a
 + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.48

$$\int \sinh(a+bx)\tanh^2(a+bx)\,dx = \frac{e^{4bx+4a}+6e^{2bx+2a}+1}{2e^{bx+a}b\left(e^{2bx+2a}+1\right)}$$

input
```
int(sinh(b*x+a)*tanh(b*x+a)^2,x)
```

output
```
(e**(4*a + 4*b*x) + 6*e**(2*a + 2*b*x) + 1)/(2*e**(a + b*x)*b*(e**(2*a + 2
*b*x) + 1))
```

3.39 $\int \sinh(a+bx)\tanh^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx = -\frac{3\arctan(\sinh(a+bx))}{2b} + \frac{\sinh(a+bx)}{b} + \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}$$

output
```
-3/2*arctan(sinh(b*x+a))/b+sinh(b*x+a)/b+1/2*sech(b*x+a)*tanh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx = -\frac{3\arctan(\sinh(a+bx))}{2b} + \frac{3\operatorname{sech}(a+bx)\tanh(a+bx)}{2b} + \frac{\sinh(a+bx)\tanh^2(a+bx)}{b}$$

input
```
Integrate[Sinh[a + b*x]*Tanh[a + b*x]^3,x]
```

output
```
(-3*ArcTan[Sinh[a + b*x]])/(2*b) + (3*Sech[a + b*x]*Tanh[a + b*x])/(2*b) +
 (Sinh[a + b*x]*Tanh[a + b*x]^2)/b
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3072, 252, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)\tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{\int \frac{\sinh^4(a+bx)}{(\sinh^2(a+bx)+1)^2}d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{3}{2}\int \frac{\sinh^2(a+bx)}{\sinh^2(a+bx)+1}d\sinh(a+bx)-\frac{\sinh^3(a+bx)}{2(\sinh^2(a+bx)+1)}}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{3}{2}\left(\sinh(a+bx)-\int \frac{1}{\sinh^2(a+bx)+1}d\sinh(a+bx)\right)-\frac{\sinh^3(a+bx)}{2(\sinh^2(a+bx)+1)}}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{3}{2}(\sinh(a+bx)-\arctan(\sinh(a+bx)))-\frac{\sinh^3(a+bx)}{2(\sinh^2(a+bx)+1)}}{b}$$

input
```
Int[Sinh[a + b*x]*Tanh[a + b*x]^3,x]
```

output
```
((3*(-ArcTan[Sinh[a + b*x]] + Sinh[a + b*x]))/2 - Sinh[a + b*x]^3/(2*(1 +
Sinh[a + b*x]^2)))/b
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.41

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^3}{\cosh(bx+a)^2}+\frac{3\sinh(bx+a)}{\cosh(bx+a)^2}-\frac{3\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2}-3\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	62
default	$\frac{\frac{\sinh(bx+a)^3}{\cosh(bx+a)^2}+\frac{3\sinh(bx+a)}{\cosh(bx+a)^2}-\frac{3\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2}-3\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	62
risch	$\frac{\mathrm{e}^{bx+a}}{2b}-\frac{\mathrm{e}^{-bx-a}}{2b}+\frac{\mathrm{e}^{bx+a}\left(\mathrm{e}^{2bx+2a}-1\right)}{b\left(\mathrm{e}^{2bx+2a}+1\right)^2}+\frac{3i\ln\left(\mathrm{e}^{bx+a}-i\right)}{2b}-\frac{3i\ln\left(\mathrm{e}^{bx+a}+i\right)}{2b}$	93

input
```
int(sinh(b*x+a)*tanh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(sinh(b*x+a)^3/cosh(b*x+a)^2+3*sinh(b*x+a)/cosh(b*x+a)^2-3/2*sech(b*x+
a)*tanh(b*x+a)-3*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. 2(40) = 80.

Time = 0.13 (sec) , antiderivative size = 463, normalized size of antiderivative = 10.52

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx$$
$$=\frac{\cosh(bx+a)^6+6\cosh(bx+a)\sinh(bx+a)^5+\sinh(bx+a)^6+3\left(5\cosh(bx+a)^2+1\right)\sinh(bx+a)}{}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 +
 3*(5*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh
(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*
cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 6*(cosh(b*x + a)^5 + 5*cosh(b*x + a
)*sinh(b*x + a)^4 + sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^2 + 1)*sinh(b*x +
 a)^3 + 2*cosh(b*x + a)^3 + 2*(5*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b
*x + a)^2 + (5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + co
sh(b*x + a))*arctan(cosh(b*x + a) + sinh(b*x + a)) - 3*cosh(b*x + a)^2 + 6
*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) - 1)/
(b*cosh(b*x + a)^5 + 5*b*cosh(b*x + a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5
 + 2*b*cosh(b*x + a)^3 + 2*(5*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^3 + 2*(
5*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^2 + b*cosh(b*x + a)
 + (5*b*cosh(b*x + a)^4 + 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a))
```

Sympy [F]

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx = \int \sinh(a+bx)\tanh^3(a+bx)\,dx$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)**3,x)
```

output
```
Integral(sinh(a + b*x)*tanh(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(40) = 80$.

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.07

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx = \frac{3\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{e^{(-bx-a)}}{2b} + \frac{4e^{(-2bx-2a)} - e^{(-4bx-4a)} + 1}{2b\left(e^{(-bx-a)} + 2e^{(-3bx-3a)} + e^{(-5bx-5a)}\right)}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)^3,x, algorithm="maxima")
```

output
```
3*arctan(e^(-b*x - a))/b - 1/2*e^(-b*x - a)/b + 1/2*(4*e^(-2*b*x - 2*a) -
e^(-4*b*x - 4*a) + 1)/(b*(e^(-b*x - a) + 2*e^(-3*b*x - 3*a) + e^(-5*b*x -
5*a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(40) = 80$.

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.20

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx = -\frac{3\pi - \frac{4\left(e^{(bx+a)}-e^{(-bx-a)}\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4} + 6\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)}-1\right)e^{(-bx-a)}\right) - 2e^{(bx+a)} + 2e^{(-bx-a)}}{4b}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/4*(3*pi - 4*(e^(b*x + a) - e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^
2 + 4) + 6*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)) - 2*e^(b*x + a)
+ 2*e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.43

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx = \frac{e^{a+bx}}{2b} - \frac{3\,\mathrm{atan}\left(\frac{e^{bx}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{e^{-a-bx}}{2b} - \frac{2\,e^{a+bx}}{b\,\left(2\,e^{2a+2bx}+e^{4a+4bx}+1\right)} + \frac{e^{a+bx}}{b\,\left(e^{2a+2bx}+1\right)}$$

input
```
int(sinh(a + b*x)*tanh(a + b*x)^3,x)
```

output
```
exp(a + b*x)/(2*b) - (3*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2)
 - exp(- a - b*x)/(2*b) - (2*exp(a + b*x))/(b*(2*exp(2*a + 2*b*x) + exp(4*
a + 4*b*x) + 1)) + exp(a + b*x)/(b*(exp(2*a + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.36

$$\int \sinh(a+bx)\tanh^3(a+bx)\,dx$$
$$= \frac{-12e^{bx+a}atan\left(e^{bx+a}\right) - 2e^{bx+a}\cosh\left(bx+a\right)\tanh\left(bx+a\right) + 3e^{2bx+2a} - 2e^{bx+a}\sinh\left(bx+a\right)\tanh\left(bx+a\right.}{4e^{bx+a}b}$$

input
```
int(sinh(b*x+a)*tanh(b*x+a)^3,x)
```

output
```
( - 12*e**(a + b*x)*atan(e**(a + b*x)) - 2*e**(a + b*x)*cosh(a + b*x)*tanh
(a + b*x) + 3*e**(2*a + 2*b*x) - 2*e**(a + b*x)*sinh(a + b*x)*tanh(a + b*x
)**2 + 2*e**(a + b*x)*sinh(a + b*x) - 3)/(4*e**(a + b*x)*b)
```

3.40 $\int \sinh(a+bx)\tanh^4(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx=\frac{\cosh(a+bx)}{b}+\frac{2\text{sech}(a+bx)}{b}-\frac{\text{sech}^3(a+bx)}{3b}$$

output
```
cosh(b*x+a)/b+2*sech(b*x+a)/b-1/3*sech(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx=\frac{\cosh(a+bx)}{b}+\frac{2\text{sech}(a+bx)}{b}-\frac{\text{sech}^3(a+bx)}{3b}$$

input
```
Integrate[Sinh[a + b*x]*Tanh[a + b*x]^4,x]
```

output
```
Cosh[a + b*x]/b + (2*Sech[a + b*x])/b - Sech[a + b*x]^3/(3*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ia+ibx)\tan(ia+ibx)^4dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin(ia+ibx)\tan(ia+ibx)^4dx$$

$$\Big\downarrow \text{3070}$$

$$\frac{\int \left(1-\cosh^2(a+bx)\right)^2\operatorname{sech}^4(a+bx)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\operatorname{sech}^4(a+bx)-2\operatorname{sech}^2(a+bx)+1\right)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cosh(a+bx)-\frac{1}{3}\operatorname{sech}^3(a+bx)+2\operatorname{sech}(a+bx)}{b}$$

input
```
Int[Sinh[a + b*x]*Tanh[a + b*x]^4,x]
```

output
```
(Cosh[a + b*x] + 2*Sech[a + b*x] - Sech[a + b*x]^3/3)/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^4}{\cosh(bx+a)^3}+\frac{4\sinh(bx+a)^2}{\cosh(bx+a)^3}+\frac{8}{3\cosh(bx+a)^3}}{b}$	51
default	$\frac{\frac{\sinh(bx+a)^4}{\cosh(bx+a)^3}+\frac{4\sinh(bx+a)^2}{\cosh(bx+a)^3}+\frac{8}{3\cosh(bx+a)^3}}{b}$	51
risch	$\frac{3\,\mathrm{e}^{7bx+7a}+36\,\mathrm{e}^{5bx+5a}+50\,\mathrm{e}^{3bx+3a}+36\,\mathrm{e}^{bx+a}+3\,\mathrm{e}^{-bx-a}}{6b\left(\mathrm{e}^{2bx+2a}+1\right)^3}$	72

input
```
int(sinh(b*x+a)*tanh(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(sinh(b*x+a)^4/cosh(b*x+a)^3+4*sinh(b*x+a)^2/cosh(b*x+a)^3+8/3/cosh(b*
x+a)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(35) = 70$.

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.51

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx = \frac{3\cosh(bx+a)^4+3\sinh(bx+a)^4+18\left(\cosh(bx+a)^2+2\right)\sinh(bx+a)^2+36\cosh(bx+a)^2+25}{6\left(b\cosh(bx+a)^3+3\,b\cosh(bx+a)\sinh(bx+a)^2+3\,b\cosh(bx+a)\right)}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/6*(3*cosh(b*x + a)^4 + 3*sinh(b*x + a)^4 + 18*(cosh(b*x + a)^2 + 2)*sinh
(b*x + a)^2 + 36*cosh(b*x + a)^2 + 25)/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x +
 a)*sinh(b*x + a)^2 + 3*b*cosh(b*x + a))
```

Sympy [F]

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx = \int \sinh(a+bx)\tanh^4(a+bx)\,dx$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)**4,x)
```

output
```
Integral(sinh(a + b*x)*tanh(a + b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(35) = 70$.

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.65

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx = \frac{e^{(-bx-a)}}{2\,b} + \frac{33\,e^{(-2\,bx-2\,a)}+41\,e^{(-4\,bx-4\,a)}+27\,e^{(-6\,bx-6\,a)}+3}{6\,b(e^{(-bx-a)}+3\,e^{(-3\,bx-3\,a)}+3\,e^{(-5\,bx-5\,a)}+e^{(-7\,bx-7\,a)})}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/2*e^(-b*x - a)/b + 1/6*(33*e^(-2*b*x - 2*a) + 41*e^(-4*b*x - 4*a) + 27*e
^(-6*b*x - 6*a) + 3)/(b*(e^(-b*x - a) + 3*e^(-3*b*x - 3*a) + 3*e^(-5*b*x -
 5*a) + e^(-7*b*x - 7*a)))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.81

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx = \frac{\frac{8\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-2\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3}+3\,e^{(bx+a)}+3\,e^{(-bx-a)}}{6\,b}$$

input
```
integrate(sinh(b*x+a)*tanh(b*x+a)^4,x, algorithm="giac")
```

output
```
1/6*(8*(3*(e^(b*x + a) + e^(-b*x - a))^2 - 2)/(e^(b*x + a) + e^(-b*x - a))
^3 + 3*e^(b*x + a) + 3*e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.54

$$\begin{aligned}\int \sinh(a+bx)\tanh^4(a+bx)\,dx &= \frac{e^{a+b\,x}}{2\,b}+\frac{e^{-a-b\,x}}{2\,b}-\frac{8\,e^{a+b\,x}}{3\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}\\ &\quad+\frac{8\,e^{a+b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)}\\ &\quad+\frac{4\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}\end{aligned}$$

input
```
int(sinh(a + b*x)*tanh(a + b*x)^4,x)
```

output
```
exp(a + b*x)/(2*b) + exp(- a - b*x)/(2*b) - (8*exp(a + b*x))/(3*b*(2*exp(2
*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) + (8*exp(a + b*x))/(3*b*(3*exp(2*a +
2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + (4*exp(a + b*x))/(b
*(exp(2*a + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 230, normalized size of antiderivative = 6.22

$$\int \sinh(a+bx)\tanh^4(a+bx)\,dx$$
$$= \frac{-2e^{3bx+3a}\cosh(bx+a)\tanh(bx+a)^2 - 4e^{3bx+3a}\cosh(bx+a) - 2e^{bx+a}\cosh(bx+a)\tanh(bx+a)^2 -}{}$$

input
```
int(sinh(b*x+a)*tanh(b*x+a)^4,x)
```

output
```
( - 2*e**(3*a + 3*b*x)*cosh(a + b*x)*tanh(a + b*x)**2 - 4*e**(3*a + 3*b*x)
*cosh(a + b*x) - 2*e**(a + b*x)*cosh(a + b*x)*tanh(a + b*x)**2 - 4*e**(a +
 b*x)*cosh(a + b*x) + 9*e**(4*a + 4*b*x) - 4*e**(3*a + 3*b*x)*sinh(a + b*x
)*tanh(a + b*x)**3 + 4*e**(3*a + 3*b*x)*sinh(a + b*x)*tanh(a + b*x) + 54*e
**(2*a + 2*b*x) - 4*e**(a + b*x)*sinh(a + b*x)*tanh(a + b*x)**3 + 4*e**(a
+ b*x)*sinh(a + b*x)*tanh(a + b*x) + 9)/(12*e**(a + b*x)*b*(e**(2*a + 2*b*
x) + 1))
```

3.41 $\int \sinh^2(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx=\frac{\cosh^2(a+bx)}{2b}-\frac{\log(\cosh(a+bx))}{b}$$

output
```
1/2*cosh(b*x+a)^2/b-ln(cosh(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx=-\frac{-\frac{1}{2}\cosh^2(a+bx)+\log(\cosh(a+bx))}{b}$$

input
```
Integrate[Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
-((-1/2*Cosh[a + b*x]^2 + Log[Cosh[a + b*x]])/b)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int i\sin(ia+ibx)^2\tan(ia+ibx)dx$$

$$\Big\downarrow \; 26$$

$$i\int \sin(ia+ibx)^2\tan(ia+ibx)dx$$

$$\Big\downarrow \; 3070$$

$$-\frac{\int \left(1-\cosh^2(a+bx)\right)\operatorname{sech}(a+bx)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \; 244$$

$$-\frac{\int (\operatorname{sech}(a+bx)-\cosh(a+bx))d\cosh(a+bx)}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\log(\cosh(a+bx))-\frac{1}{2}\cosh^2(a+bx)}{b}$$

input
```
Int[Sinh[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
-((-1/2*Cosh[a + b*x]^2 + Log[Cosh[a + b*x]])/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^2}{2}-\ln(\cosh(bx+a))}{b}$	25
default	$\frac{\frac{\sinh(bx+a)^2}{2}-\ln(\cosh(bx+a))}{b}$	25
risch	$x+\frac{e^{2bx+2a}}{8b}+\frac{e^{-2bx-2a}}{8b}+\frac{2a}{b}-\frac{\ln\left(e^{2bx+2a}+1\right)}{b}$	54

input
```
int(sinh(b*x+a)^2*tanh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*sinh(b*x+a)^2-ln(cosh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(26) = 52$.

Time = 0.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 7.04

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx$$
$$= \frac{8\,bx\cosh\left(bx+a\right)^2 + \cosh\left(bx+a\right)^4 + 4\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3 + \sinh\left(bx+a\right)^4 + 2\left(4\,bx + 3\cosh\right.}{}$$

input
```
integrate(sinh(b*x+a)^2*tanh(b*x+a),x, algorithm="fricas")
```

output
```
1/8*(8*b*x*cosh(b*x + a)^2 + cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x +
a)^3 + sinh(b*x + a)^4 + 2*(4*b*x + 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 8
*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*log(2
*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(4*b*x*cosh(b*x + a) +
 cosh(b*x + a)^3)*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a
)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F]

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = \int \sinh^2\left(a+bx\right)\tanh\left(a+bx\right)dx$$

input
```
integrate(sinh(b*x+a)**2*tanh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)**2*tanh(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(26) = 52$.

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = -\frac{bx+a}{b} + \frac{e^{(2\,bx+2\,a)}}{8\,b} + \frac{e^{(-2\,bx-2\,a)}}{8\,b} - \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}$$

input
```
integrate(sinh(b*x+a)^2*tanh(b*x+a),x, algorithm="maxima")
```

output
```
-(b*x + a)/b + 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b - log(e^(-2*
b*x - 2*a) + 1)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(26) = 52$.

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.14

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = \frac{8\,bx - \left(4\,e^{(2\,bx+2\,a)}-1\right)e^{(-2\,bx-2\,a)} + 8\,a + e^{(2\,bx+2\,a)} - 8\,\log\left(e^{(2\,bx+2\,a)}+1\right)}{8\,b}$$

input
```
integrate(sinh(b*x+a)^2*tanh(b*x+a),x, algorithm="giac")
```

output
```
1/8*(8*b*x - (4*e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + 8*a + e^(2*b*x + 2
*a) - 8*log(e^(2*b*x + 2*a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = x - \frac{\ln\left(e^{2a}\,e^{2bx}+1\right)}{b} + \frac{e^{-2a-2bx}}{8b} + \frac{e^{2a+2bx}}{8b}$$

input
```
int(sinh(a + b*x)^2*tanh(a + b*x),x)
```

output
```
x - log(exp(2*a)*exp(2*b*x) + 1)/b + exp(- 2*a - 2*b*x)/(8*b) + exp(2*a +
2*b*x)/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.43

$$\int \sinh^2(a+bx)\tanh(a+bx)\,dx = \frac{e^{4bx+4a} - 8e^{2bx+2a}\log\left(e^{2bx+2a}+1\right) + 8e^{2bx+2a}bx + 1}{8e^{2bx+2a}b}$$

input
```
int(sinh(b*x+a)^2*tanh(b*x+a),x)
```

output
```
(e**(4*a + 4*b*x) - 8*e**(2*a + 2*b*x)*log(e**(2*a + 2*b*x) + 1) + 8*e**(2
*a + 2*b*x)*b*x + 1)/(8*e**(2*a + 2*b*x)*b)
```

3.42 $\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 35

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx = -\frac{3x}{2}+\frac{\cosh(a+bx)\sinh(a+bx)}{2b}+\frac{\tanh(a+bx)}{b}$$

output
```
-3/2*x+1/2*cosh(b*x+a)*sinh(b*x+a)/b+tanh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx = \frac{-6(a+bx)+\sinh(2(a+bx))+4\tanh(a+bx)}{4b}$$

input
```
Integrate[Sinh[a + b*x]^2*Tanh[a + b*x]^2,x]
```

output
```
(-6*(a + b*x) + Sinh[2*(a + b*x)] + 4*Tanh[a + b*x])/(4*b)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 3071, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)^2\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{\int \frac{\tanh^4(a+bx)}{(1-\tanh^2(a+bx))^2}d\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\tanh^3(a+bx)}{2(1-\tanh^2(a+bx))}-\frac{3}{2}\int \frac{\tanh^2(a+bx)}{1-\tanh^2(a+bx)}d\tanh(a+bx)}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{\tanh^3(a+bx)}{2(1-\tanh^2(a+bx))}-\frac{3}{2}\left(\int \frac{1}{1-\tanh^2(a+bx)}d\tanh(a+bx)-\tanh(a+bx)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\tanh^3(a+bx)}{2(1-\tanh^2(a+bx))}-\frac{3}{2}(\text{arctanh}(\tanh(a+bx))-\tanh(a+bx))}{b}$$

input
```
Int[Sinh[a + b*x]^2*Tanh[a + b*x]^2,x]
```

output
```
((-3*(ArcTanh[Tanh[a + b*x]] - Tanh[a + b*x]))/2 + Tanh[a + b*x]^3/(2*(1 -
 Tanh[a + b*x]^2)))/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^3}{2\cosh(bx+a)}-\frac{3bx}{2}-\frac{3a}{2}+\frac{3\tanh(bx+a)}{2}}{b}$	39
default	$\frac{\frac{\sinh(bx+a)^3}{2\cosh(bx+a)}-\frac{3bx}{2}-\frac{3a}{2}+\frac{3\tanh(bx+a)}{2}}{b}$	39
risch	$-\frac{3x}{2}+\frac{e^{2bx+2a}}{8b}-\frac{e^{-2bx-2a}}{8b}-\frac{2}{b(e^{2bx+2a}+1)}$	51

input
```
int(sinh(b*x+a)^2*tanh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*sinh(b*x+a)^3/cosh(b*x+a)-3/2*b*x-3/2*a+3/2*tanh(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.54

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx$$
$$=\frac{\sinh(bx+a)^3-4(3bx+2)\cosh(bx+a)+3\left(\cosh(bx+a)^2+3\right)\sinh(bx+a)}{8b\cosh(bx+a)}$$

input
```
integrate(sinh(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(sinh(b*x + a)^3 - 4*(3*b*x + 2)*cosh(b*x + a) + 3*(cosh(b*x + a)^2 +
3)*sinh(b*x + a))/(b*cosh(b*x + a))
```

Sympy [F]

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx = \int \sinh^2{(a+bx)}\tanh^2{(a+bx)}\,dx$$

input
```
integrate(sinh(b*x+a)**2*tanh(b*x+a)**2,x)
```

output
```
Integral(sinh(a + b*x)**2*tanh(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(31) = 62$.

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.83

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx = -\frac{3\,(bx+a)}{2\,b} - \frac{e^{(-2\,bx-2\,a)}}{8\,b} + \frac{17\,e^{(-2\,bx-2\,a)}+1}{8\,b(e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)})}$$

input
```
integrate(sinh(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-3/2*(b*x + a)/b - 1/8*e^(-2*b*x - 2*a)/b + 1/8*(17*e^(-2*b*x - 2*a) + 1)/
(b*(e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(31) = 62$.

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.03

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx = -\frac{12\,bx+12\,a-\frac{3\,e^{(4\,bx+4\,a)}-14\,e^{(2\,bx+2\,a)}-1}{e^{(4\,bx+4\,a)}+e^{(2\,bx+2\,a)}}-e^{(2\,bx+2\,a)}}{8\,b}$$

input
```
integrate(sinh(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/8*(12*b*x + 12*a - (3*e^(4*b*x + 4*a) - 14*e^(2*b*x + 2*a) - 1)/(e^(4*b
*x + 4*a) + e^(2*b*x + 2*a)) - e^(2*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.43

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx = \frac{e^{2a+2bx}}{8b} - \frac{2}{b\left(e^{2a+2bx}+1\right)} - \frac{e^{-2a-2bx}}{8b} - \frac{3x}{2}$$

input
```
int(sinh(a + b*x)^2*tanh(a + b*x)^2,x)
```

output
```
exp(2*a + 2*b*x)/(8*b) - 2/(b*(exp(2*a + 2*b*x) + 1)) - exp(- 2*a - 2*b*x)
/(8*b) - (3*x)/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.37

$$\int \sinh^2(a+bx)\tanh^2(a+bx)\,dx = \frac{e^{6bx+6a} - 12e^{4bx+4a}bx + 18e^{4bx+4a} - 12e^{2bx+2a}bx - 1}{8e^{2bx+2a}b\left(e^{2bx+2a}+1\right)}$$

input
```
int(sinh(b*x+a)^2*tanh(b*x+a)^2,x)
```

output
```
(e**(6*a + 6*b*x) - 12*e**(4*a + 4*b*x)*b*x + 18*e**(4*a + 4*b*x) - 12*e**
(2*a + 2*b*x)*b*x - 1)/(8*e**(2*a + 2*b*x)*b*(e**(2*a + 2*b*x) + 1))
```

3.43 $\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx=\frac{\cosh^2(a+bx)}{2b}-\frac{2\log(\cosh(a+bx))}{b}-\frac{\operatorname{sech}^2(a+bx)}{2b}$$

output
```
1/2*cosh(b*x+a)^2/b-2*ln(cosh(b*x+a))/b-1/2*sech(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx=-\frac{4\log(\cosh(a+bx))+\operatorname{sech}^2(a+bx)-\sinh^2(a+bx)}{2b}$$

input
```
Integrate[Sinh[a + b*x]^2*Tanh[a + b*x]^3,x]
```

output
```
-1/2*(4*Log[Cosh[a + b*x]] + Sech[a + b*x]^2 - Sinh[a + b*x]^2)/b
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 26, 3070, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ia+ibx)^2\tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin(ia+ibx)^2\tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{3070}$$

$$\frac{\int \left(1-\cosh^2(a+bx)\right)^2\text{sech}^3(a+bx)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \left(1-\cosh^2(a+bx)\right)^2\text{sech}^2(a+bx)d\cosh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\text{sech}^2(a+bx)-2\text{sech}(a+bx)+1\right)d\cosh^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cosh^2(a+bx)-\text{sech}(a+bx)-2\log\left(\cosh^2(a+bx)\right)}{2b}$$

input
```
Int[Sinh[a + b*x]^2*Tanh[a + b*x]^3,x]
```

output
```
(Cosh[a + b*x]^2 - 2*Log[Cosh[a + b*x]^2] - Sech[a + b*x])/(2*b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^4}{2\cosh(bx+a)^2}-2\ln(\cosh(bx+a))+\tanh(bx+a)^2}{b}$	41
default	$\frac{\frac{\sinh(bx+a)^4}{2\cosh(bx+a)^2}-2\ln(\cosh(bx+a))+\tanh(bx+a)^2}{b}$	41
risch	$2x+\frac{e^{2bx+2a}}{8b}+\frac{e^{-2bx-2a}}{8b}+\frac{4a}{b}-\frac{2\,e^{2bx+2a}}{b\left(e^{2bx+2a}+1\right)^2}-\frac{2\ln\left(e^{2bx+2a}+1\right)}{b}$	83

input
```
int(sinh(b*x+a)^2*tanh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*sinh(b*x+a)^4/cosh(b*x+a)^2-2*ln(cosh(b*x+a))+tanh(b*x+a)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 742 vs. $2(39) = 78$.

Time = 0.09 (sec) , antiderivative size = 742, normalized size of antiderivative = 17.26

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/8*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 +
 2*(8*b*x + 1)*cosh(b*x + a)^6 + 2*(8*b*x + 14*cosh(b*x + a)^2 + 1)*sinh(b
*x + a)^6 + 4*(14*cosh(b*x + a)^3 + 3*(8*b*x + 1)*cosh(b*x + a))*sinh(b*x
+ a)^5 + 2*(16*b*x - 7)*cosh(b*x + a)^4 + 2*(35*cosh(b*x + a)^4 + 15*(8*b*
x + 1)*cosh(b*x + a)^2 + 16*b*x - 7)*sinh(b*x + a)^4 + 8*(7*cosh(b*x + a)^
5 + 5*(8*b*x + 1)*cosh(b*x + a)^3 + (16*b*x - 7)*cosh(b*x + a))*sinh(b*x +
 a)^3 + 2*(8*b*x + 1)*cosh(b*x + a)^2 + 2*(14*cosh(b*x + a)^6 + 15*(8*b*x
+ 1)*cosh(b*x + a)^4 + 6*(16*b*x - 7)*cosh(b*x + a)^2 + 8*b*x + 1)*sinh(b*
x + a)^2 - 16*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*
x + a)^6 + (15*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^4 + 2*cosh(b*x + a)^4 +
4*(5*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a
)^4 + 12*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + cosh(b*x + a)^2 + 2*(3*cos
h(b*x + a)^5 + 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a))*log(2*cos
h(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(2*cosh(b*x + a)^7 + 3*(8*
b*x + 1)*cosh(b*x + a)^5 + 2*(16*b*x - 7)*cosh(b*x + a)^3 + (8*b*x + 1)*co
sh(b*x + a))*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sin
h(b*x + a)^5 + b*sinh(b*x + a)^6 + 2*b*cosh(b*x + a)^4 + (15*b*cosh(b*x +
a)^2 + 2*b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 + 2*b*cosh(b*x + a))*
sinh(b*x + a)^3 + b*cosh(b*x + a)^2 + (15*b*cosh(b*x + a)^4 + 12*b*cosh(b*
x + a)^2 + b)*sinh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^5 + 4*b*cosh(b*x +...
```

Sympy [F]

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx = \int \sinh^2{(a+bx)}\tanh^3{(a+bx)}\,dx$$

input
```
integrate(sinh(b*x+a)**2*tanh(b*x+a)**3,x)
```

output
```
Integral(sinh(a + b*x)**2*tanh(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(39) = 78$.

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.40

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx = -\frac{2\,(bx+a)}{b} + \frac{e^{(-2\,bx-2\,a)}}{8\,b} - \frac{2\,\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{2\,e^{(-2\,bx-2\,a)} - 15\,e^{(-4\,bx-4\,a)} + 1}{8\,b\left(e^{(-2\,bx-2\,a)} + 2\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)}\right)}$$

input
```
integrate(sinh(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-2*(b*x + a)/b + 1/8*e^(-2*b*x - 2*a)/b - 2*log(e^(-2*b*x - 2*a) + 1)/b +
1/8*(2*e^(-2*b*x - 2*a) - 15*e^(-4*b*x - 4*a) + 1)/(b*(e^(-2*b*x - 2*a) +
2*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(39) = 78$.

Time = 0.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.30

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx = \frac{16\,bx - \left(8\,e^{(2\,bx+2\,a)} - 1\right)e^{(-2\,bx-2\,a)} + 16\,a + \frac{8\,\left(3\,e^{(4\,bx+4\,a)}+4\,e^{(2\,bx+2\,a)}+3\right)}{\left(e^{(2\,bx+2\,a)}+1\right)^2} + e^{(2\,bx+2\,a)} - 16\,\log\left(e^{(2\,bx+2\,a)}+1\right.}{8\,b}$$

input
```
integrate(sinh(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/8*(16*b*x - (8*e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + 16*a + 8*(3*e^(4*
b*x + 4*a) + 4*e^(2*b*x + 2*a) + 3)/(e^(2*b*x + 2*a) + 1)^2 + e^(2*b*x + 2
*a) - 16*log(e^(2*b*x + 2*a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.26

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx = 2\,x-\frac{2\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1\right)}{b}-\frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}+\frac{2}{b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)}+\frac{\mathrm{e}^{-2\,a-2\,b\,x}}{8\,b}+\frac{\mathrm{e}^{2\,a+2\,b\,x}}{8\,b}$$

input
```
int(sinh(a + b*x)^2*tanh(a + b*x)^3,x)
```

output
```
2*x - (2*log(exp(2*a)*exp(2*b*x) + 1))/b - 2/(b*(exp(2*a + 2*b*x) + 1)) +
2/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) + exp(- 2*a - 2*b*x)/(8*
b) + exp(2*a + 2*b*x)/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 196, normalized size of antiderivative = 4.56

$$\int \sinh^2(a+bx)\tanh^3(a+bx)\,dx = \frac{e^{8bx+8a}-16e^{6bx+6a}\log\left(e^{2bx+2a}+1\right)+16e^{6bx+6a}bx+9e^{6bx+6a}-32e^{4bx+4a}\log\left(e^{2bx+2a}+1\right)+32e^{4bx+4a}bx}{8e^{2bx+2a}b\left(e^{4bx+4a}+2e^{2bx+2a}+1\right)}$$

input
```
int(sinh(b*x+a)^2*tanh(b*x+a)^3,x)
```

output
```
(e**(8*a + 8*b*x) - 16*e**(6*a + 6*b*x)*log(e**(2*a + 2*b*x) + 1) + 16*e**
(6*a + 6*b*x)*b*x + 9*e**(6*a + 6*b*x) - 32*e**(4*a + 4*b*x)*log(e**(2*a +
 2*b*x) + 1) + 32*e**(4*a + 4*b*x)*b*x - 16*e**(2*a + 2*b*x)*log(e**(2*a +
 2*b*x) + 1) + 16*e**(2*a + 2*b*x)*b*x + 9*e**(2*a + 2*b*x) + 1)/(8*e**(2*
a + 2*b*x)*b*(e**(4*a + 4*b*x) + 2*e**(2*a + 2*b*x) + 1))
```

3.44 $\int \sinh^3(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{b}-\frac{\sinh(a+bx)}{b}+\frac{\sinh^3(a+bx)}{3b}$$

output
```
arctan(sinh(b*x+a))/b-sinh(b*x+a)/b+1/3*sinh(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{b}-\frac{\sinh(a+bx)}{b}+\frac{\sinh^3(a+bx)}{3b}$$

input
```
Integrate[Sinh[a + b*x]^3*Tanh[a + b*x],x]
```

output
```
ArcTan[Sinh[a + b*x]]/b - Sinh[a + b*x]/b + Sinh[a + b*x]^3/(3*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3072, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ia+ibx)^3\tan(ia+ibx)dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{\int \frac{\sinh^4(a+bx)}{\sinh^2(a+bx)+1}d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{254}$$

$$\frac{\int \left(\sinh^2(a+bx)+\frac{1}{\sinh^2(a+bx)+1}-1\right)d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan(\sinh(a+bx))+\frac{1}{3}\sinh^3(a+bx)-\sinh(a+bx)}{b}$$

input
```
Int[Sinh[a + b*x]^3*Tanh[a + b*x],x]
```

output
```
(ArcTan[Sinh[a + b*x]] - Sinh[a + b*x] + Sinh[a + b*x]^3/3)/b
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^3}{3}-\sinh(bx+a)+2\arctan\left(e^{bx+a}\right)}{b}$	33
default	$\frac{\frac{\sinh(bx+a)^3}{3}-\sinh(bx+a)+2\arctan\left(e^{bx+a}\right)}{b}$	33
risch	$\frac{e^{3bx+3a}}{24b}-\frac{5\,e^{bx+a}}{8b}+\frac{5\,e^{-bx-a}}{8b}-\frac{e^{-3bx-3a}}{24b}+\frac{i\ln\left(e^{bx+a}+i\right)}{b}-\frac{i\ln\left(e^{bx+a}-i\right)}{b}$	87

input
```
int(sinh(b*x+a)^3*tanh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3*sinh(b*x+a)^3-sinh(b*x+a)+2*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(36) = 72$.

Time = 0.10 (sec) , antiderivative size = 290, normalized size of antiderivative = 7.63

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx = \frac{\cosh(bx+a)^6 + 6\cosh(bx+a)\sinh(bx+a)^5 + \sinh(bx+a)^6 + 15\left(\cosh(bx+a)^2-1\right)\sinh(bx+a)^4}{}$$

input
```
integrate(sinh(b*x+a)^3*tanh(b*x+a),x, algorithm="fricas")
```

output
```
1/24*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6
+ 15*(cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 15*cosh(b*x + a)^4 + 20*(cosh
(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 15*(cosh(b*x + a)^4 - 6*c
osh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 48*(cosh(b*x + a)^3 + 3*cosh(b*x + a
)^2*sinh(b*x + a) + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3)*arc
tan(cosh(b*x + a) + sinh(b*x + a)) + 15*cosh(b*x + a)^2 + 6*(cosh(b*x + a)
^5 - 10*cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a) - 1)/(b*cosh(b*x
+ a)^3 + 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x + a)*sinh(b*x +
a)^2 + b*sinh(b*x + a)^3)
```

Sympy [F]

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx = \int \sinh^3(a+bx)\tanh(a+bx)\,dx$$

input
```
integrate(sinh(b*x+a)**3*tanh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)**3*tanh(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.87

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx = -\frac{\left(15\,e^{(-2\,bx-2\,a)}-1\right)e^{(3\,bx+3\,a)}}{24\,b} + \frac{15\,e^{(-bx-a)}-e^{(-3\,bx-3\,a)}}{24\,b} - \frac{2\,\arctan\left(e^{(-bx-a)}\right)}{b}$$

input
```
integrate(sinh(b*x+a)^3*tanh(b*x+a),x, algorithm="maxima")
```

output
```
-1/24*(15*e^(-2*b*x - 2*a) - 1)*e^(3*b*x + 3*a)/b + 1/24*(15*e^(-b*x - a)
- e^(-3*b*x - 3*a))/b - 2*arctan(e^(-b*x - a))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx = \frac{\left(15\,e^{(2\,bx+2\,a)}-1\right)e^{(-3\,bx-3\,a)} + 48\,\arctan\left(e^{(bx+a)}\right) + e^{(3\,bx+3\,a)} - 15\,e^{(bx+a)}}{24\,b}$$

input
```
integrate(sinh(b*x+a)^3*tanh(b*x+a),x, algorithm="giac")
```

output
```
1/24*((15*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a) + 48*arctan(e^(b*x + a)) +
 e^(3*b*x + 3*a) - 15*e^(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.03

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx = \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{5\,\mathrm{e}^{a+b\,x}}{8\,b} + \frac{5\,\mathrm{e}^{-a-b\,x}}{8\,b} - \frac{\mathrm{e}^{-3\,a-3\,b\,x}}{24\,b} + \frac{\mathrm{e}^{3\,a+3\,b\,x}}{24\,b}$$

input
```
int(sinh(a + b*x)^3*tanh(a + b*x),x)
```

output
```
(2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (5*exp(a + b*x))/(
8*b) + (5*exp(- a - b*x))/(8*b) - exp(- 3*a - 3*b*x)/(24*b) + exp(3*a + 3*
b*x)/(24*b)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.92

$$\int \sinh^3(a+bx)\tanh(a+bx)\,dx = \frac{48e^{3bx+3a}atan\left(e^{bx+a}\right) + e^{6bx+6a} - 15e^{4bx+4a} + 15e^{2bx+2a} - 1}{24e^{3bx+3a}b}$$

input
```
int(sinh(b*x+a)^3*tanh(b*x+a),x)
```

output
```
(48*e**(3*a + 3*b*x)*atan(e**(a + b*x)) + e**(6*a + 6*b*x) - 15*e**(4*a +
4*b*x) + 15*e**(2*a + 2*b*x) - 1)/(24*e**(3*a + 3*b*x)*b)
```

3.45 $\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx = -\frac{2\cosh(a+bx)}{b}+\frac{\cosh^3(a+bx)}{3b}-\frac{\operatorname{sech}(a+bx)}{b}$$

output
```
-2*cosh(b*x+a)/b+1/3*cosh(b*x+a)^3/b-sech(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx = -\frac{7\cosh(a+bx)}{4b}+\frac{\cosh(3(a+bx))}{12b}-\frac{\operatorname{sech}(a+bx)}{b}$$

input
```
Integrate[Sinh[a + b*x]^3*Tanh[a + b*x]^2,x]
```

output
```
(-7*Cosh[a + b*x])/(4*b) + Cosh[3*(a + b*x)]/(12*b) - Sech[a + b*x]/b
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 26, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i\sin(ia+ibx)^3\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \sin(ia+ibx)^3\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{ 3070}$$

$$\frac{\int \left(1-\cosh^2(a+bx)\right)^2\text{sech}^2(a+bx)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\int \left(\cosh^2(a+bx)+\text{sech}^2(a+bx)-2\right)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{3}\cosh^3(a+bx)-2\cosh(a+bx)-\text{sech}(a+bx)}{b}$$

input
```
Int[Sinh[a + b*x]^3*Tanh[a + b*x]^2,x]
```

output
```
(-2*Cosh[a + b*x] + Cosh[a + b*x]^3/3 - Sech[a + b*x])/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^4}{3\cosh(bx+a)}-\frac{4\sinh(bx+a)^2}{3\cosh(bx+a)}-\frac{8}{3\cosh(bx+a)}}{b}$	52
default	$\frac{\frac{\sinh(bx+a)^4}{3\cosh(bx+a)}-\frac{4\sinh(bx+a)^2}{3\cosh(bx+a)}-\frac{8}{3\cosh(bx+a)}}{b}$	52
risch	$\frac{e^{5bx+5a}-20\,e^{3bx+3a}-90\,e^{bx+a}-20\,e^{-bx-a}+e^{-3bx-3a}}{24b\left(e^{2bx+2a}+1\right)}$	68

input
```
int(sinh(b*x+a)^3*tanh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3*sinh(b*x+a)^4/cosh(b*x+a)-4/3*sinh(b*x+a)^2/cosh(b*x+a)-8/3/cosh(
b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.66

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx$$
$$= \frac{\cosh(bx+a)^4 + \sinh(bx+a)^4 + 2\left(3\cosh(bx+a)^2 - 10\right)\sinh(bx+a)^2 - 20\cosh(bx+a)^2 - 45}{24\,b\cosh(bx+a)}$$

input
```
integrate(sinh(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/24*(cosh(b*x + a)^4 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 10)*sinh(
b*x + a)^2 - 20*cosh(b*x + a)^2 - 45)/(b*cosh(b*x + a))
```

Sympy [F]

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx = \int \sinh^3(a+bx)\tanh^2(a+bx)\,dx$$

input
```
integrate(sinh(b*x+a)**3*tanh(b*x+a)**2,x)
```

output
```
Integral(sinh(a + b*x)**3*tanh(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(36) = 72.

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.08

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx = -\frac{21\,e^{(-bx-a)} - e^{(-3\,bx-3\,a)}}{24\,b} - \frac{20\,e^{(-2\,bx-2\,a)} + 69\,e^{(-4\,bx-4\,a)} - 1}{24\,b(e^{(-3\,bx-3\,a)} + e^{(-5\,bx-5\,a)})}$$

input
```
integrate(sinh(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/24*(21*e^(-b*x - a) - e^(-3*b*x - 3*a))/b - 1/24*(20*e^(-2*b*x - 2*a) +
 69*e^(-4*b*x - 4*a) - 1)/(b*(e^(-3*b*x - 3*a) + e^(-5*b*x - 5*a)))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.66

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx = \frac{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3-\frac{48}{e^{(bx+a)}+e^{(-bx-a)}}-24\,e^{(bx+a)}-24\,e^{(-bx-a)}}{24\,b}$$

input
```
integrate(sinh(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/24*((e^(b*x + a) + e^(-b*x - a))^3 - 48/(e^(b*x + a) + e^(-b*x - a)) - 2
4*e^(b*x + a) - 24*e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.05

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx = \frac{e^{-3\,a-3\,b\,x}}{24\,b}-\frac{7\,e^{-a-b\,x}}{8\,b}-\frac{7\,e^{a+b\,x}}{8\,b}+\frac{e^{3\,a+3\,b\,x}}{24\,b}-\frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(sinh(a + b*x)^3*tanh(a + b*x)^2,x)
```

output
```
exp(- 3*a - 3*b*x)/(24*b) - (7*exp(- a - b*x))/(8*b) - (7*exp(a + b*x))/(8
*b) + exp(3*a + 3*b*x)/(24*b) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1)
)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.08

$$\int \sinh^3(a+bx)\tanh^2(a+bx)\,dx = \frac{e^{8bx+8a}-20e^{6bx+6a}-90e^{4bx+4a}-20e^{2bx+2a}+1}{24e^{3bx+3a}b\left(e^{2bx+2a}+1\right)}$$

input
```
int(sinh(b*x+a)^3*tanh(b*x+a)^2,x)
```

output
```
(e**(8*a + 8*b*x) - 20*e**(6*a + 6*b*x) - 90*e**(4*a + 4*b*x) - 20*e**(2*a
 + 2*b*x) + 1)/(24*e**(3*a + 3*b*x)*b*(e**(2*a + 2*b*x) + 1))
```

3.46 $\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx = \frac{5\arctan(\sinh(a+bx))}{2b} - \frac{2\sinh(a+bx)}{b} + \frac{\sinh^3(a+bx)}{3b} - \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}$$

output
```
5/2*arctan(sinh(b*x+a))/b-2*sinh(b*x+a)/b+1/3*sinh(b*x+a)^3/b-1/2*sech(b*x
+a)*tanh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx = \frac{5\arctan(\sinh(a+bx))}{2b} - \frac{5\operatorname{sech}(a+bx)\tanh(a+bx)}{2b} - \frac{5\sinh(a+bx)\tanh^2(a+bx)}{3b} + \frac{\sinh^3(a+bx)\tanh^2(a+bx)}{3b}$$

input
```
Integrate[Sinh[a + b*x]^3*Tanh[a + b*x]^3,x]
```

output
```
(5*ArcTan[Sinh[a + b*x]])/(2*b) - (5*Sech[a + b*x]*Tanh[a + b*x])/(2*b) -
(5*Sinh[a + b*x]*Tanh[a + b*x]^2)/(3*b) + (Sinh[a + b*x]^3*Tanh[a + b*x]^2
)/(3*b)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 25, 3072, 25, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin(ia+ibx)^3\tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin(ia+ibx)^3\tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{3072}$$

$$-\frac{\int -\frac{\sinh^6(a+bx)}{\left(\sinh^2(a+bx)+1\right)^2}d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sinh^6(a+bx)}{\left(\sinh^2(a+bx)+1\right)^2}d\sinh(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{\sinh^5(a+bx)}{2\left(\sinh^2(a+bx)+1\right)}-\frac{5}{2}\int \frac{\sinh^4(a+bx)}{\sinh^2(a+bx)+1}d\sinh(a+bx)}{b}$$

$$\Big\downarrow \ 254$$

$$-\frac{\frac{\sinh^5(a+bx)}{2(\sinh^2(a+bx)+1)}-\frac{5}{2}\int\left(\sinh^2(a+bx)+\frac{1}{\sinh^2(a+bx)+1}-1\right)d\sinh(a+bx)}{b}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\frac{\sinh^5(a+bx)}{2(\sinh^2(a+bx)+1)}-\frac{5}{2}\left(\arctan(\sinh(a+bx))+\frac{1}{3}\sinh^3(a+bx)-\sinh(a+bx)\right)}{b}$$

input
```
Int[Sinh[a + b*x]^3*Tanh[a + b*x]^3,x]
```

output
```
-((Sinh[a + b*x]^5/(2*(1 + Sinh[a + b*x]^2)) - (5*(ArcTan[Sinh[a + b*x]] -
 Sinh[a + b*x] + Sinh[a + b*x]^3/3))/2)/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.35

method	result	si
derivativedivides	$\frac{\frac{\sinh(bx+a)^5}{3\cosh(bx+a)^2}-\frac{5\sinh(bx+a)^3}{3\cosh(bx+a)^2}-\frac{5\sinh(bx+a)}{\cosh(bx+a)^2}+\frac{5\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2}+5\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	8
default	$\frac{\frac{\sinh(bx+a)^5}{3\cosh(bx+a)^2}-\frac{5\sinh(bx+a)^3}{3\cosh(bx+a)^2}-\frac{5\sinh(bx+a)}{\cosh(bx+a)^2}+\frac{5\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2}+5\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	8
risch	$\frac{\mathrm{e}^{3bx+3a}}{24b}-\frac{9\,\mathrm{e}^{bx+a}}{8b}+\frac{9\,\mathrm{e}^{-bx-a}}{8b}-\frac{\mathrm{e}^{-3bx-3a}}{24b}-\frac{\mathrm{e}^{bx+a}\left(\mathrm{e}^{2bx+2a}-1\right)}{b\left(\mathrm{e}^{2bx+2a}+1\right)^2}+\frac{5i\ln\left(\mathrm{e}^{bx+a}+i\right)}{2b}-\frac{5i\ln\left(\mathrm{e}^{bx+a}-i\right)}{2b}$	1

input
```
int(sinh(b*x+a)^3*tanh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3*sinh(b*x+a)^5/cosh(b*x+a)^2-5/3*sinh(b*x+a)^3/cosh(b*x+a)^2-5*sin
h(b*x+a)/cosh(b*x+a)^2+5/2*sech(b*x+a)*tanh(b*x+a)+5*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. $2(54) = 108$.

Time = 0.10 (sec) , antiderivative size = 851, normalized size of antiderivative = 14.18

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/24*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^
10 + 5*(9*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^8 - 25*cosh(b*x + a)^8 + 40*(
3*cosh(b*x + a)^3 - 5*cosh(b*x + a))*sinh(b*x + a)^7 + 10*(21*cosh(b*x + a
)^4 - 70*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^6 - 50*cosh(b*x + a)^6 + 4*(63
*cosh(b*x + a)^5 - 350*cosh(b*x + a)^3 - 75*cosh(b*x + a))*sinh(b*x + a)^5
 + 10*(21*cosh(b*x + a)^6 - 175*cosh(b*x + a)^4 - 75*cosh(b*x + a)^2 + 5)*
sinh(b*x + a)^4 + 50*cosh(b*x + a)^4 + 40*(3*cosh(b*x + a)^7 - 35*cosh(b*x
 + a)^5 - 25*cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a)^3 + 5*(9*cos
h(b*x + a)^8 - 140*cosh(b*x + a)^6 - 150*cosh(b*x + a)^4 + 60*cosh(b*x + a
)^2 + 5)*sinh(b*x + a)^2 + 120*(cosh(b*x + a)^7 + 7*cosh(b*x + a)*sinh(b*x
 + a)^6 + sinh(b*x + a)^7 + (21*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^5 + 2*c
osh(b*x + a)^5 + 5*(7*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^4 +
 (35*cosh(b*x + a)^4 + 20*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + cosh(b*x
+ a)^3 + (21*cosh(b*x + a)^5 + 20*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(
b*x + a)^2 + (7*cosh(b*x + a)^6 + 10*cosh(b*x + a)^4 + 3*cosh(b*x + a)^2)*
sinh(b*x + a))*arctan(cosh(b*x + a) + sinh(b*x + a)) + 25*cosh(b*x + a)^2
+ 10*(cosh(b*x + a)^9 - 20*cosh(b*x + a)^7 - 30*cosh(b*x + a)^5 + 20*cosh(
b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a) - 1)/(b*cosh(b*x + a)^7 + 7*b*
cosh(b*x + a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 + 2*b*cosh(b*x + a)^5 +
(21*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 +...
```

Sympy [F]

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx=\int \sinh^3{(a+bx)}\tanh^3{(a+bx)}\,dx$$

input
```
integrate(sinh(b*x+a)**3*tanh(b*x+a)**3,x)
```

output
```
Integral(sinh(a + b*x)**3*tanh(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(54) = 108$.

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.93

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx = \frac{27\,e^{(-bx-a)} - e^{(-3\,bx-3\,a)}}{24\,b} - \frac{5\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{25\,e^{(-2\,bx-2\,a)} + 77\,e^{(-4\,bx-4\,a)} + 3\,e^{(-6\,bx-6\,a)} - 1}{24\,b\left(e^{(-3\,bx-3\,a)} + 2\,e^{(-5\,bx-5\,a)} + e^{(-7\,bx-7\,a)}\right)}$$

input
```
integrate(sinh(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/24*(27*e^(-b*x - a) - e^(-3*b*x - 3*a))/b - 5*arctan(e^(-b*x - a))/b - 1
/24*(25*e^(-2*b*x - 2*a) + 77*e^(-4*b*x - 4*a) + 3*e^(-6*b*x - 6*a) - 1)/(
b*(e^(-3*b*x - 3*a) + 2*e^(-5*b*x - 5*a) + e^(-7*b*x - 7*a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(54) = 108$.

Time = 0.14 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.95

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx = \frac{30\,\pi + \left(e^{(bx+a)} - e^{(-bx-a)}\right)^3 - \frac{24\left(e^{(bx+a)} - e^{(-bx-a)}\right)}{\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2 + 4} + 60\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)} - 1\right)e^{(-bx-a)}\right) - 24\,e^{(bx+a)} +}{24\,b}$$

input
```
integrate(sinh(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/24*(30*pi + (e^(b*x + a) - e^(-b*x - a))^3 - 24*(e^(b*x + a) - e^(-b*x -
 a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4) + 60*arctan(1/2*(e^(2*b*x + 2*a)
 - 1)*e^(-b*x - a)) - 24*e^(b*x + a) + 24*e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.27

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx = \frac{5\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{9\,e^{a+b\,x}}{8\,b} + \frac{9\,e^{-a-b\,x}}{8\,b} - \frac{e^{-3\,a-3\,b\,x}}{24\,b} + \frac{e^{3\,a+3\,b\,x}}{24\,b} + \frac{2\,e^{a+b\,x}}{b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(sinh(a + b*x)^3*tanh(a + b*x)^3,x)
```

output
```
(5*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (9*exp(a + b*x))/(
8*b) + (9*exp(- a - b*x))/(8*b) - exp(- 3*a - 3*b*x)/(24*b) + exp(3*a + 3*
b*x)/(24*b) + (2*exp(a + b*x))/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) +
 1)) - exp(a + b*x)/(b*(exp(2*a + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.72

$$\int \sinh^3(a+bx)\tanh^3(a+bx)\,dx = \frac{120e^{7bx+7a}atan\left(e^{bx+a}\right)+240e^{5bx+5a}atan\left(e^{bx+a}\right)+120e^{3bx+3a}atan\left(e^{bx+a}\right)+e^{10bx+10a}-25e^{8bx+8a}-50e^{6}}{24e^{3bx+3a}b\left(e^{4bx+4a}+2e^{2bx+2a}+1\right)}$$

input
```
int(sinh(b*x+a)^3*tanh(b*x+a)^3,x)
```

output
```
(120*e**(7*a + 7*b*x)*atan(e**(a + b*x)) + 240*e**(5*a + 5*b*x)*atan(e**(a
 + b*x)) + 120*e**(3*a + 3*b*x)*atan(e**(a + b*x)) + e**(10*a + 10*b*x) -
25*e**(8*a + 8*b*x) - 50*e**(6*a + 6*b*x) + 50*e**(4*a + 4*b*x) + 25*e**(2
*a + 2*b*x) - 1)/(24*e**(3*a + 3*b*x)*b*(e**(4*a + 4*b*x) + 2*e**(2*a + 2*
b*x) + 1))
```

3.47 $\int \sinh^4(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx=-\frac{\cosh^2(a+bx)}{b}+\frac{\cosh^4(a+bx)}{4b}+\frac{\log(\cosh(a+bx))}{b}$$

output
```
-cosh(b*x+a)^2/b+1/4*cosh(b*x+a)^4/b+ln(cosh(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx=\frac{-\cosh^2(a+bx)+\frac{1}{4}\cosh^4(a+bx)+\log(\cosh(a+bx))}{b}$$

input
```
Integrate[Sinh[a + b*x]^4*Tanh[a + b*x],x]
```

output
```
(-Cosh[a + b*x]^2 + Cosh[a + b*x]^4/4 + Log[Cosh[a + b*x]])/b
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 26, 3070, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -i\sin(ia+ibx)^4\tan(ia+ibx)dx$$

$$\Big\downarrow \ 26$$

$$-i\int \sin(ia+ibx)^4\tan(ia+ibx)dx$$

$$\Big\downarrow \ 3070$$

$$\frac{\int \left(1-\cosh^2(a+bx)\right)^2\operatorname{sech}(a+bx)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \ 243$$

$$\frac{\int \left(1-\cosh^2(a+bx)\right)^2\operatorname{sech}(a+bx)d\cosh^2(a+bx)}{2b}$$

$$\Big\downarrow \ 49$$

$$\frac{\int \left(\cosh^2(a+bx)+\operatorname{sech}(a+bx)-2\right)d\cosh^2(a+bx)}{2b}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{1}{2}\cosh^4(a+bx)-2\cosh^2(a+bx)+\log\left(\cosh^2(a+bx)\right)}{2b}$$

input
```
Int[Sinh[a + b*x]^4*Tanh[a + b*x],x]
```

output
```
(-2*Cosh[a + b*x]^2 + Cosh[a + b*x]^4/2 + Log[Cosh[a + b*x]^2])/(2*b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^4}{4}-\frac{\sinh(bx+a)^2}{2}+\ln(\cosh(bx+a))}{b}$	33
default	$\frac{\frac{\sinh(bx+a)^4}{4}-\frac{\sinh(bx+a)^2}{2}+\ln(\cosh(bx+a))}{b}$	33
risch	$-x+\frac{e^{4bx+4a}}{64b}-\frac{3\,e^{2bx+2a}}{16b}-\frac{3\,e^{-2bx-2a}}{16b}+\frac{e^{-4bx-4a}}{64b}-\frac{2a}{b}+\frac{\ln\left(e^{2bx+2a}+1\right)}{b}$	83

input
```
int(sinh(b*x+a)^4*tanh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4*sinh(b*x+a)^4-1/2*sinh(b*x+a)^2+ln(cosh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(38) = 76$.

Time = 0.11 (sec) , antiderivative size = 457, normalized size of antiderivative = 11.42

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx = \frac{\cosh(bx+a)^8 + 8\cosh(bx+a)\sinh(bx+a)^7 + \sinh(bx+a)^8 + 4\left(7\cosh(bx+a)^2 - 3\right)\sinh(bx+a)}{}$$

input
```
integrate(sinh(b*x+a)^4*tanh(b*x+a),x, algorithm="fricas")
```

output
```
1/64*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8
+ 4*(7*cosh(b*x + a)^2 - 3)*sinh(b*x + a)^6 - 64*b*x*cosh(b*x + a)^4 - 12*
cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 - 9*cosh(b*x + a))*sinh(b*x + a)^5
+ 2*(35*cosh(b*x + a)^4 - 32*b*x - 90*cosh(b*x + a)^2)*sinh(b*x + a)^4 + 8
*(7*cosh(b*x + a)^5 - 32*b*x*cosh(b*x + a) - 30*cosh(b*x + a)^3)*sinh(b*x
+ a)^3 + 4*(7*cosh(b*x + a)^6 - 96*b*x*cosh(b*x + a)^2 - 45*cosh(b*x + a)^
4 - 3)*sinh(b*x + a)^2 - 12*cosh(b*x + a)^2 + 64*(cosh(b*x + a)^4 + 4*cosh
(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x
 + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4)*log(2*cosh(b*x + a)/(cosh(b*x + a
) - sinh(b*x + a))) + 8*(cosh(b*x + a)^7 - 32*b*x*cosh(b*x + a)^3 - 9*cosh
(b*x + a)^5 - 3*cosh(b*x + a))*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^4 + 4*b
*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b
*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4)
```

Sympy [F]

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx = \int \sinh^4(a+bx)\tanh(a+bx)\,dx$$

input
```
integrate(sinh(b*x+a)**4*tanh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)**4*tanh(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(38) = 76$.

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.02

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx = -\frac{\left(12\,e^{(-2\,bx-2\,a)}-1\right)e^{(4\,bx+4\,a)}}{64\,b}+\frac{bx+a}{b} - \frac{12\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}}{64\,b}+\frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b}$$

input
```
integrate(sinh(b*x+a)^4*tanh(b*x+a),x, algorithm="maxima")
```

output
```
-1/64*(12*e^(-2*b*x - 2*a) - 1)*e^(4*b*x + 4*a)/b + (b*x + a)/b - 1/64*(12
*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a))/b + log(e^(-2*b*x - 2*a) + 1)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(38) = 76$.

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.10

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx = -\frac{64\,bx-\left(48\,e^{(4\,bx+4\,a)}-12\,e^{(2\,bx+2\,a)}+1\right)e^{(-4\,bx-4\,a)}+64\,a-e^{(4\,bx+4\,a)}+12\,e^{(2\,bx+2\,a)}-64\,\log\left(e^{(2\,bx+2}\right.}{64\,b}$$

input
```
integrate(sinh(b*x+a)^4*tanh(b*x+a),x, algorithm="giac")
```

output
```
-1/64*(64*b*x - (48*e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a) + 1)*e^(-4*b*x -
4*a) + 64*a - e^(4*b*x + 4*a) + 12*e^(2*b*x + 2*a) - 64*log(e^(2*b*x + 2*a
) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.92

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx = \frac{\ln\left(e^{2a}\,e^{2bx}+1\right)}{b} - x - \frac{3\,e^{-2a-2bx}}{16\,b} - \frac{3\,e^{2a+2bx}}{16\,b} + \frac{e^{-4a-4bx}}{64\,b} + \frac{e^{4a+4bx}}{64\,b}$$

input
```
int(sinh(a + b*x)^4*tanh(a + b*x),x)
```

output
```
log(exp(2*a)*exp(2*b*x) + 1)/b - x - (3*exp(- 2*a - 2*b*x))/(16*b) - (3*ex
p(2*a + 2*b*x))/(16*b) + exp(- 4*a - 4*b*x)/(64*b) + exp(4*a + 4*b*x)/(64*
b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.30

$$\int \sinh^4(a+bx)\tanh(a+bx)\,dx = \frac{e^{8bx+8a} - 12e^{6bx+6a} + 64e^{4bx+4a}\log\left(e^{2bx+2a}+1\right) - 64e^{4bx+4a}bx - 12e^{2bx+2a} + 1}{64e^{4bx+4a}b}$$

input
```
int(sinh(b*x+a)^4*tanh(b*x+a),x)
```

output
```
(e**(8*a + 8*b*x) - 12*e**(6*a + 6*b*x) + 64*e**(4*a + 4*b*x)*log(e**(2*a
+ 2*b*x) + 1) - 64*e**(4*a + 4*b*x)*b*x - 12*e**(2*a + 2*b*x) + 1)/(64*e**
(4*a + 4*b*x)*b)
```

3.48 $\int \cosh(a+bx)\coth(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \cosh(a+bx)\coth(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cosh(a+bx))}{b} + \frac{\cosh(a+bx)}{b}$$

output
```
-arctanh(cosh(b*x+a))/b+cosh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \cosh(a+bx)\coth(a+bx)\,dx = \frac{\cosh(a+bx)}{b} - \frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

input
```
Integrate[Cosh[a + b*x]*Coth[a + b*x],x]
```

output
```
Cosh[a + b*x]/b - Log[Cosh[(a + b*x)/2]]/b + Log[Sinh[(a + b*x)/2]]/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 26, 3072, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\coth(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin\left(ia+ibx+\frac{\pi}{2}\right)\tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin\left(\frac{1}{2}(2ia+\pi)+ibx\right)\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx$$

$$\Big\downarrow \text{3072}$$

$$-\frac{\int \frac{\cosh^2(a+bx)}{1-\cosh^2(a+bx)}d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\int \frac{1}{1-\cosh^2(a+bx)}d\cosh(a+bx)-\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}(\cosh(a+bx))-\cosh(a+bx)}{b}$$

input
```
Int[Cosh[a + b*x]*Coth[a + b*x],x]
```

output
```
-((ArcTanh[Cosh[a + b*x]] - Cosh[a + b*x])/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\cosh(bx+a)-2\,\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	21
default	$\frac{\cosh(bx+a)-2\,\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	21
risch	$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}+\frac{\ln\left(e^{bx+a}-1\right)}{b}-\frac{\ln\left(e^{bx+a}+1\right)}{b}$	54

input
```
int(cosh(b*x+a)*coth(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(23) = 46.

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.91

$$\int \cosh(a+bx)\coth(a+bx)\,dx$$
$$= \frac{\cosh\left(bx+a\right)^2 - 2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+2\left(\cosh\left(bx+\right.\right.}{2\left(b\cosh\left(bx+\right.\right.}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + a)^2 - 2*(cosh(b*x + a) + sinh(b*x + a))*log(cosh(b*x + a)
 + sinh(b*x + a) + 1) + 2*(cosh(b*x + a) + sinh(b*x + a))*log(cosh(b*x + a
) + sinh(b*x + a) - 1) + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 +
 1)/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

Sympy [F]

$$\int \cosh(a+bx)\coth(a+bx)\,dx = \int \cosh\left(a+bx\right)\coth\left(a+bx\right)dx$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a),x)
```

output
```
Integral(cosh(a + b*x)*coth(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(23) = 46$.

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.57

$$\int \cosh(a+bx)\coth(a+bx)\,dx = \frac{e^{(bx+a)}}{2\,b} + \frac{e^{(-bx-a)}}{2\,b} - \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a),x, algorithm="maxima")
```

output
```
1/2*e^(b*x + a)/b + 1/2*e^(-b*x - a)/b - log(e^(-b*x - a) + 1)/b + log(e^(
-b*x - a) - 1)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \cosh(a+bx)\coth(a+bx)\,dx = \frac{e^{(bx+a)} + e^{(-bx-a)} - 2\,\log\left(e^{(bx+a)}+1\right) + 2\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{2\,b}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a),x, algorithm="giac")
```

output
```
1/2*(e^(b*x + a) + e^(-b*x - a) - 2*log(e^(b*x + a) + 1) + 2*log(abs(e^(b*
x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \cosh(a+bx)\coth(a+bx)\,dx = \frac{e^{a+b\,x}}{2\,b} - \frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} + \frac{e^{-a-b\,x}}{2\,b}$$

input
```
int(cosh(a + b*x)*coth(a + b*x),x)
```

output
```
exp(a + b*x)/(2*b) - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/
2) + exp(- a - b*x)/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.78

$$\int \cosh(a+bx)\coth(a+bx)\,dx = \frac{e^{2bx+2a}+2e^{bx+a}\log\left(e^{bx+a}-1\right)-2e^{bx+a}\log\left(e^{bx+a}+1\right)+1}{2e^{bx+a}b}$$

input
```
int(cosh(b*x+a)*coth(b*x+a),x)
```

output
```
(e**(2*a + 2*b*x) + 2*e**(a + b*x)*log(e**(a + b*x) - 1) - 2*e**(a + b*x)*
log(e**(a + b*x) + 1) + 1)/(2*e**(a + b*x)*b)
```

3.49 $\int \cosh(a+bx)\coth^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 22

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx=-\frac{\operatorname{csch}(a+bx)}{b}+\frac{\sinh(a+bx)}{b}$$

output
```
-csch(b*x+a)/b+sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx=-\frac{\operatorname{csch}(a+bx)}{b}+\frac{\sinh(a+bx)}{b}$$

input
```
Integrate[Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

output
```
-(Csch[a + b*x]/b) + Sinh[a + b*x]/b
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 25, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin\left(ia+ibx+\frac{\pi}{2}\right)\tan\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin\left(\frac{1}{2}(2ia+\pi)+ibx\right)\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{i\int -\text{csch}^2(a+bx)\left(\sinh^2(a+bx)+1\right)d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{i\int \left(-\text{csch}^2(a+bx)-1\right)d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i(i\sinh(a+bx)-i\text{csch}(a+bx))}{b}$$

input
```
Int[Cosh[a + b*x]*Coth[a + b*x]^2,x]
```

output
```
((-I)*((-I)*Csch[a + b*x] + I*Sinh[a + b*x]))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^2}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}}{b}$	33
default	$\frac{\frac{\cosh(bx+a)^2}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}}{b}$	33
risch	$-\frac{6\,\mathrm{e}^{bx+a}-\mathrm{e}^{3bx+3a}-\mathrm{e}^{-bx-a}}{2b\left(\mathrm{e}^{2bx+2a}-1\right)}$	50

input
```
int(cosh(b*x+a)*coth(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/sinh(b*x+a)*cosh(b*x+a)^2-2/sinh(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = \frac{\cosh(bx+a)^2+\sinh(bx+a)^2-3}{2\,b\sinh(bx+a)}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + a)^2 + sinh(b*x + a)^2 - 3)/(b*sinh(b*x + a))
```

Sympy [F]

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = \int \cosh(a+bx)\coth^2(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)**2,x)
```

output
```
Integral(cosh(a + b*x)*coth(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(22) = 44.

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.55

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = -\frac{e^{(-bx-a)}}{2\,b} - \frac{5\,e^{(-2\,bx-2\,a)}-1}{2\,b(e^{(-bx-a)}-e^{(-3\,bx-3\,a)})}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*e^(-b*x - a)/b - 1/2*(5*e^(-2*b*x - 2*a) - 1)/(b*(e^(-b*x - a) - e^(-
3*b*x - 3*a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(22) = 44$.

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = -\frac{\frac{4}{e^{(bx+a)}-e^{(-bx-a)}} - e^{(bx+a)} + e^{(-bx-a)}}{2\,b}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*(4/(e^(b*x + a) - e^(-b*x - a)) - e^(b*x + a) + e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = \frac{e^{-a-b\,x}\,\left(e^{4\,a+4\,b\,x}-6\,e^{2\,a+2\,b\,x}+1\right)}{2\,b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(cosh(a + b*x)*coth(a + b*x)^2,x)
```

output
```
(exp(- a - b*x)*(exp(4*a + 4*b*x) - 6*exp(2*a + 2*b*x) + 1))/(2*b*(exp(2*a
 + 2*b*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.36

$$\int \cosh(a+bx)\coth^2(a+bx)\,dx = \frac{e^{4bx+4a}-6e^{2bx+2a}+1}{2e^{bx+a}b\left(e^{2bx+2a}-1\right)}$$

input
```
int(cosh(b*x+a)*coth(b*x+a)^2,x)
```

output
```
(e**(4*a + 4*b*x) - 6*e**(2*a + 2*b*x) + 1)/(2*e**(a + b*x)*b*(e**(2*a + 2
*b*x) - 1))
```

3.50 $\int \cosh(a+bx)\coth^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx = -\frac{3\text{arctanh}(\cosh(a+bx))}{2b}+\frac{\cosh(a+bx)}{b} - \frac{\coth(a+bx)\text{csch}(a+bx)}{2b}$$

output
```
-3/2*arctanh(cosh(b*x+a))/b+cosh(b*x+a)/b-1/2*coth(b*x+a)*csch(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.93

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx = \frac{\cosh(a+bx)}{b}-\frac{\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{3\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} + \frac{3\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}-\frac{\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}$$

input
```
Integrate[Cosh[a + b*x]*Coth[a + b*x]^3,x]
```

output
```
Cosh[a + b*x]/b - Csch[(a + b*x)/2]^2/(8*b) - (3*Log[Cosh[(a + b*x)/2]])/(
2*b) + (3*Log[Sinh[(a + b*x)/2]])/(2*b) - Sech[(a + b*x)/2]^2/(8*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 26, 3072, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin\left(ia+ibx+\frac{\pi}{2}\right)\tan\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sin\left(\frac{1}{2}(2ia+\pi)+ibx\right)\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^3dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{\int \frac{\cosh^4(a+bx)}{(1-\cosh^2(a+bx))^2}d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\cosh^3(a+bx)}{2(1-\cosh^2(a+bx))}-\frac{3}{2}\int \frac{\cosh^2(a+bx)}{1-\cosh^2(a+bx)}d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{\cosh^3(a+bx)}{2(1-\cosh^2(a+bx))}-\frac{3}{2}\left(\int \frac{1}{1-\cosh^2(a+bx)}d\cosh(a+bx)-\cosh(a+bx)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\cosh^3(a+bx)}{2(1-\cosh^2(a+bx))}-\frac{3}{2}(\text{arctanh}(\cosh(a+bx))-\cosh(a+bx))}{b}$$

input
```
Int[Cosh[a + b*x]*Coth[a + b*x]^3,x]
```

output
```
((-3*(ArcTanh[Cosh[a + b*x]] - Cosh[a + b*x]))/2 + Cosh[a + b*x]^3/(2*(1 -
 Cosh[a + b*x]^2)))/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.41

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^3}{\sinh(bx+a)^2}-\frac{3\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{3\coth(bx+a)\operatorname{csch}(bx+a)}{2}-3\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	62
default	$\frac{\frac{\cosh(bx+a)^3}{\sinh(bx+a)^2}-\frac{3\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{3\coth(bx+a)\operatorname{csch}(bx+a)}{2}-3\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	62
risch	$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}-\frac{e^{bx+a}\left(e^{2bx+2a}+1\right)}{b\left(e^{2bx+2a}-1\right)^2}+\frac{3\ln\left(e^{bx+a}-1\right)}{2b}-\frac{3\ln\left(e^{bx+a}+1\right)}{2b}$	90

input
```
int(cosh(b*x+a)*coth(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/sinh(b*x+a)^2*cosh(b*x+a)^3-3*cosh(b*x+a)/sinh(b*x+a)^2+3/2*coth(b*
x+a)*csch(b*x+a)-3*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. 2(40) = 80.

Time = 0.09 (sec) , antiderivative size = 612, normalized size of antiderivative = 13.91

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 +
 3*(5*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh
(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*
cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 3*cosh(b*x + a)^2 - 3*(cosh(b*x + a
)^5 + 5*cosh(b*x + a)*sinh(b*x + a)^4 + sinh(b*x + a)^5 + 2*(5*cosh(b*x +
a)^2 - 1)*sinh(b*x + a)^3 - 2*cosh(b*x + a)^3 + 2*(5*cosh(b*x + a)^3 - 3*c
osh(b*x + a))*sinh(b*x + a)^2 + (5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1
)*sinh(b*x + a) + cosh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) +
3*(cosh(b*x + a)^5 + 5*cosh(b*x + a)*sinh(b*x + a)^4 + sinh(b*x + a)^5 + 2
*(5*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 2*cosh(b*x + a)^3 + 2*(5*cosh(b
*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^2 + (5*cosh(b*x + a)^4 - 6*cosh
(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))*log(cosh(b*x + a) + sinh(b
*x + a) - 1) + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - cosh(b*x + a))*sin
h(b*x + a) + 1)/(b*cosh(b*x + a)^5 + 5*b*cosh(b*x + a)*sinh(b*x + a)^4 + b
*sinh(b*x + a)^5 - 2*b*cosh(b*x + a)^3 + 2*(5*b*cosh(b*x + a)^2 - b)*sinh(
b*x + a)^3 + 2*(5*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^2 +
 b*cosh(b*x + a) + (5*b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 + b)*sinh(b*
x + a))
```

Sympy [F]

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx = \int \cosh\left(a+bx\right)\coth^3\left(a+bx\right)dx$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)**3,x)
```

output
```
Integral(cosh(a + b*x)*coth(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(40) = 80$.

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.45

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx = \frac{e^{(-bx-a)}}{2\,b} - \frac{3\,\log\left(e^{(-bx-a)}+1\right)}{2\,b} + \frac{3\,\log\left(e^{(-bx-a)}-1\right)}{2\,b} - \frac{4\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}-1}{2\,b\left(e^{(-bx-a)}-2\,e^{(-3\,bx-3\,a)}+e^{(-5\,bx-5\,a)}\right)}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*e^(-b*x - a)/b - 3/2*log(e^(-b*x - a) + 1)/b + 3/2*log(e^(-b*x - a) -
1)/b - 1/2*(4*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a) - 1)/(b*(e^(-b*x - a) -
2*e^(-3*b*x - 3*a) + e^(-5*b*x - 5*a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(40) = 80$.

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.39

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx = -\frac{\frac{4\left(e^{(bx+a)}+e^{(-bx-a)}\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4} - 2\,e^{(bx+a)} - 2\,e^{(-bx-a)} + 3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) - 3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}\right.}{4\,b}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/4*(4*(e^(b*x + a) + e^(-b*x - a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4)
- 2*e^(b*x + a) - 2*e^(-b*x - a) + 3*log(e^(b*x + a) + e^(-b*x - a) + 2) -
 3*log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.55

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx = \frac{e^{a+b\,x}}{2\,b} - \frac{3\,\operatorname{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} + \frac{e^{-a-b\,x}}{2\,b} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(cosh(a + b*x)*coth(a + b*x)^3,x)
```

output
```
exp(a + b*x)/(2*b) - (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/
2) + exp(- a - b*x)/(2*b) - (2*exp(a + b*x))/(b*(exp(4*a + 4*b*x) - 2*exp(
2*a + 2*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a + 2*b*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 202, normalized size of antiderivative = 4.59

$$\int \cosh(a+bx)\coth^3(a+bx)\,dx = \frac{e^{6bx+6a}+3e^{5bx+5a}\log\left(e^{bx+a}-1\right)-3e^{5bx+5a}\log\left(e^{bx+a}+1\right)-3e^{4bx+4a}-6e^{3bx+3a}\log\left(e^{bx+a}-1\right)+6e^{3bx+3}}{2e^{bx+a}b\left(e^{4bx+4a}-2e^{2bx+2a}+\right.}$$

input
```
int(cosh(b*x+a)*coth(b*x+a)^3,x)
```

output
```
(e**(6*a + 6*b*x) + 3*e**(5*a + 5*b*x)*log(e**(a + b*x) - 1) - 3*e**(5*a +
 5*b*x)*log(e**(a + b*x) + 1) - 3*e**(4*a + 4*b*x) - 6*e**(3*a + 3*b*x)*lo
g(e**(a + b*x) - 1) + 6*e**(3*a + 3*b*x)*log(e**(a + b*x) + 1) - 3*e**(2*a
 + 2*b*x) + 3*e**(a + b*x)*log(e**(a + b*x) - 1) - 3*e**(a + b*x)*log(e**(
a + b*x) + 1) + 1)/(2*e**(a + b*x)*b*(e**(4*a + 4*b*x) - 2*e**(2*a + 2*b*x
) + 1))
```

3.51 $\int \cosh(a+bx)\coth^4(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx = -\frac{2\operatorname{csch}(a+bx)}{b}-\frac{\operatorname{csch}^3(a+bx)}{3b}+\frac{\sinh(a+bx)}{b}$$

output
```
-2*csch(b*x+a)/b-1/3*csch(b*x+a)^3/b+sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx = -\frac{2\operatorname{csch}(a+bx)}{b}-\frac{\operatorname{csch}^3(a+bx)}{3b}+\frac{\sinh(a+bx)}{b}$$

input
```
Integrate[Cosh[a + b*x]*Coth[a + b*x]^4,x]
```

output
```
(-2*Csch[a + b*x])/b - Csch[a + b*x]^3/(3*b) + Sinh[a + b*x]/b
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)\tan\left(ia+ibx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{ 3070}$$

$$\frac{i\int \text{csch}^4(a+bx)\left(\sinh^2(a+bx)+1\right)^2d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{i\int \left(\text{csch}^4(a+bx)+2\text{csch}^2(a+bx)+1\right)d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{i\left(-i\sinh(a+bx)+\frac{1}{3}i\text{csch}^3(a+bx)+2i\text{csch}(a+bx)\right)}{b}$$

input
```
Int[Cosh[a + b*x]*Coth[a + b*x]^4,x]
```

output
```
(I*((2*I)*Csch[a + b*x] + (I/3)*Csch[a + b*x]^3 - I*Sinh[a + b*x]))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^4}{\sinh(bx+a)^3}-\frac{4\cosh(bx+a)^2}{\sinh(bx+a)^3}+\frac{8}{3\sinh(bx+a)^3}}{b}$	51
default	$\frac{\frac{\cosh(bx+a)^4}{\sinh(bx+a)^3}-\frac{4\cosh(bx+a)^2}{\sinh(bx+a)^3}+\frac{8}{3\sinh(bx+a)^3}}{b}$	51
risch	$-\frac{-3\,e^{7bx+7a}+36\,e^{5bx+5a}-50\,e^{3bx+3a}+36\,e^{bx+a}-3\,e^{-bx-a}}{6b\left(e^{2bx+2a}-1\right)^3}$	72

input
```
int(cosh(b*x+a)*coth(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(cosh(b*x+a)^4/sinh(b*x+a)^3-4*cosh(b*x+a)^2/sinh(b*x+a)^3+8/3/sinh(b*
x+a)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(35) = 70$.

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.41

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx = \frac{3\cosh(bx+a)^4 + 3\sinh(bx+a)^4 + 18\left(\cosh(bx+a)^2 - 2\right)\sinh(bx+a)^2 - 36\cosh(bx+a)^2 + 25}{6\left(b\sinh(bx+a)^3 + 3\left(b\cosh(bx+a)^2 - b\right)\sinh(bx+a)\right)}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/6*(3*cosh(b*x + a)^4 + 3*sinh(b*x + a)^4 + 18*(cosh(b*x + a)^2 - 2)*sinh
(b*x + a)^2 - 36*cosh(b*x + a)^2 + 25)/(b*sinh(b*x + a)^3 + 3*(b*cosh(b*x
+ a)^2 - b)*sinh(b*x + a))
```

Sympy [F]

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx = \int \cosh(a+bx)\coth^4(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)**4,x)
```

output
```
Integral(cosh(a + b*x)*coth(a + b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(35) = 70$.

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.70

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx = -\frac{e^{(-bx-a)}}{2\,b} - \frac{33\,e^{(-2\,bx-2\,a)} - 41\,e^{(-4\,bx-4\,a)} + 27\,e^{(-6\,bx-6\,a)} - 3}{6\,b(e^{(-bx-a)} - 3\,e^{(-3\,bx-3\,a)} + 3\,e^{(-5\,bx-5\,a)} - e^{(-7\,bx-7\,a)})}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/2*e^(-b*x - a)/b - 1/6*(33*e^(-2*b*x - 2*a) - 41*e^(-4*b*x - 4*a) + 27*
e^(-6*b*x - 6*a) - 3)/(b*(e^(-b*x - a) - 3*e^(-3*b*x - 3*a) + 3*e^(-5*b*x
- 5*a) - e^(-7*b*x - 7*a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(35) = 70$.

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.92

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx = -\frac{\frac{8\left(3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+2\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3}-3\,e^{(bx+a)}+3\,e^{(-bx-a)}}{6\,b}$$

input
```
integrate(cosh(b*x+a)*coth(b*x+a)^4,x, algorithm="giac")
```

output
```
-1/6*(8*(3*(e^(b*x + a) - e^(-b*x - a))^2 + 2)/(e^(b*x + a) - e^(-b*x - a)
)^3 - 3*e^(b*x + a) + 3*e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.54

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx = \frac{e^{a+b\,x}}{2\,b}-\frac{e^{-a-b\,x}}{2\,b}-\frac{8\,e^{a+b\,x}}{3\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}-\frac{8\,e^{a+b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)}-\frac{4\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(cosh(a + b*x)*coth(a + b*x)^4,x)
```

output
```
exp(a + b*x)/(2*b) - exp(- a - b*x)/(2*b) - (8*exp(a + b*x))/(3*b*(exp(4*a
 + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (8*exp(a + b*x))/(3*b*(3*exp(2*a +
2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (4*exp(a + b*x))/(b
*(exp(2*a + 2*b*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.76

$$\int \cosh(a+bx)\coth^4(a+bx)\,dx = \frac{3e^{8bx+8a} - 36e^{6bx+6a} + 50e^{4bx+4a} - 36e^{2bx+2a} + 3}{6e^{bx+a}b\,(e^{6bx+6a} - 3e^{4bx+4a} + 3e^{2bx+2a} - 1)}$$

input
```
int(cosh(b*x+a)*coth(b*x+a)^4,x)
```

output
```
(3*e**(8*a + 8*b*x) - 36*e**(6*a + 6*b*x) + 50*e**(4*a + 4*b*x) - 36*e**(2
*a + 2*b*x) + 3)/(6*e**(a + b*x)*b*(e**(6*a + 6*b*x) - 3*e**(4*a + 4*b*x)
+ 3*e**(2*a + 2*b*x) - 1))
```

3.52 $\int \cosh^2(a+bx)\coth(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx=\frac{\log(\sinh(a+bx))}{b}+\frac{\sinh^2(a+bx)}{2b}$$

output
```
ln(sinh(b*x+a))/b+1/2*sinh(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx=\frac{2\log(\sinh(a+bx))+\sinh^2(a+bx)}{2b}$$

input
```
Integrate[Cosh[a + b*x]^2*Coth[a + b*x],x]
```

output
```
(2*Log[Sinh[a + b*x]] + Sinh[a + b*x]^2)/(2*b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin\left(ia+ibx+\frac{\pi}{2}\right)^2\tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx$$

$$\Big\downarrow \text{3070}$$

$$\frac{\int i\text{csch}(a+bx)\left(\sinh^2(a+bx)+1\right)d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int (i\text{csch}(a+bx)+i\sinh(a+bx))d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\sinh^2(a+bx)+\log(-i\sinh(a+bx))}{b}$$

input
```
Int[Cosh[a + b*x]^2*Coth[a + b*x],x]
```

output
```
(Log[(-I)*Sinh[a + b*x]] + Sinh[a + b*x]^2/2)/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^2}{2}+\ln(\sinh(bx+a))}{b}$	23
default	$\frac{\frac{\cosh(bx+a)^2}{2}+\ln(\sinh(bx+a))}{b}$	23
risch	$-x+\frac{e^{2bx+2a}}{8b}+\frac{e^{-2bx-2a}}{8b}-\frac{2a}{b}+\frac{\ln(e^{2bx+2a}-1)}{b}$	55

input
```
int(cosh(b*x+a)^2*coth(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*cosh(b*x+a)^2+ln(sinh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(25) = 50.

Time = 0.09 (sec) , antiderivative size = 203, normalized size of antiderivative = 7.52

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx =$$

$$-\frac{8\,bx\cosh\left(bx+a\right)^2-\cosh\left(bx+a\right)^4-4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3-\sinh\left(bx+a\right)^4+2\left(4\,bx-3\,\mathrm{co}\right.}{}$$

input
```
integrate(cosh(b*x+a)^2*coth(b*x+a),x, algorithm="fricas")
```

output
```
-1/8*(8*b*x*cosh(b*x + a)^2 - cosh(b*x + a)^4 - 4*cosh(b*x + a)*sinh(b*x +
 a)^3 - sinh(b*x + a)^4 + 2*(4*b*x - 3*cosh(b*x + a)^2)*sinh(b*x + a)^2 -
8*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)*log(
2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(4*b*x*cosh(b*x + a)
- cosh(b*x + a)^3)*sinh(b*x + a) - 1)/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x +
a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(20) = 40.

Time = 0.84 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.89

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx$$

$$=\begin{cases}\tilde{\infty}x \\ x\cosh^2(a)\coth(a) \\ \tilde{\infty}x \\ -\frac{x\sinh^2(a+bx)\coth(a+bx)}{2}+\frac{x\cosh^2(a+bx)\coth(a+bx)}{2}-\frac{x\cosh(a+bx)}{2\sinh(a+bx)}+\frac{\log(\sinh(a+bx))}{b}+\frac{\sinh(a+bx)\cosh(a+bx)\coth(a+}{2b}\end{cases}$$

input
```
integrate(cosh(b*x+a)**2*coth(b*x+a),x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x*cosh(a)**2*coth(a), Eq(b, 0)),
(zoo*x, Eq(a, -b*x)), (-x*sinh(a + b*x)**2*coth(a + b*x)/2 + x*cosh(a + b*
x)**2*coth(a + b*x)/2 - x*cosh(a + b*x)/(2*sinh(a + b*x)) + log(sinh(a + b
*x))/b + sinh(a + b*x)*cosh(a + b*x)*coth(a + b*x)/(2*b), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(25) = 50$.

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.59

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = \frac{bx+a}{b} + \frac{e^{(2\,bx+2\,a)}}{8\,b} + \frac{e^{(-2\,bx-2\,a)}}{8\,b} + \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

input
```
integrate(cosh(b*x+a)^2*coth(b*x+a),x, algorithm="maxima")
```

output
```
(b*x + a)/b + 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*a)/b + log(e^(-b*x
 - a) + 1)/b + log(e^(-b*x - a) - 1)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(25) = 50$.

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.33

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = -\frac{8\,bx - \left(4\,e^{(2\,bx+2\,a)}+1\right)e^{(-2\,bx-2\,a)} + 8\,a - e^{(2\,bx+2\,a)} - 8\,\log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{8\,b}$$

input
```
integrate(cosh(b*x+a)^2*coth(b*x+a),x, algorithm="giac")
```

output
```
-1/8*(8*b*x - (4*e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) + 8*a - e^(2*b*x +
2*a) - 8*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = \frac{\ln\left(e^{2a}\,e^{2bx}-1\right)}{b} - x + \frac{e^{-2a-2bx}}{8b} + \frac{e^{2a+2bx}}{8b}$$

input
```
int(cosh(a + b*x)^2*coth(a + b*x),x)
```

output
```
log(exp(2*a)*exp(2*b*x) - 1)/b - x + exp(- 2*a - 2*b*x)/(8*b) + exp(2*a +
2*b*x)/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.22

$$\int \cosh^2(a+bx)\coth(a+bx)\,dx = \frac{e^{4bx+4a}+8e^{2bx+2a}\log\left(e^{bx+a}-1\right)+8e^{2bx+2a}\log\left(e^{bx+a}+1\right)-8e^{2bx+2a}bx+1}{8e^{2bx+2a}b}$$

input
```
int(cosh(b*x+a)^2*coth(b*x+a),x)
```

output
```
(e**(4*a + 4*b*x) + 8*e**(2*a + 2*b*x)*log(e**(a + b*x) - 1) + 8*e**(2*a +
 2*b*x)*log(e**(a + b*x) + 1) - 8*e**(2*a + 2*b*x)*b*x + 1)/(8*e**(2*a + 2
*b*x)*b)
```

3.53 $\int \cosh^2(a+bx)\coth^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 36

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx=\frac{3x}{2}-\frac{\coth(a+bx)}{b}+\frac{\cosh(a+bx)\sinh(a+bx)}{2b}$$

output
```
3/2*x-coth(b*x+a)/b+1/2*cosh(b*x+a)*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx=\frac{6(a+bx)-4\coth(a+bx)+\sinh(2(a+bx))}{4b}$$

input
```
Integrate[Cosh[a + b*x]^2*Coth[a + b*x]^2,x]
```

output
```
(6*(a + b*x) - 4*Coth[a + b*x] + Sinh[2*(a + b*x)])/(4*b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.72, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 25, 3071, 252, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin\left(ia+ibx+\frac{\pi}{2}\right)^2\tan\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{i\int \frac{\coth^4(a+bx)}{(1-\coth^2(a+bx))^2}d(i\coth(a+bx))}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{i\left(\frac{3}{2}\int -\frac{\coth^2(a+bx)}{1-\coth^2(a+bx)}d(i\coth(a+bx))+\frac{i\coth^3(a+bx)}{2(1-\coth^2(a+bx))}\right)}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{i\left(\frac{3}{2}\left(i\coth(a+bx)-\int \frac{1}{1-\coth^2(a+bx)}d(i\coth(a+bx))\right)+\frac{i\coth^3(a+bx)}{2(1-\coth^2(a+bx))}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{i\left(\frac{3}{2}(i\coth(a+bx)-i\text{arctanh}(\coth(a+bx)))+\frac{i\coth^3(a+bx)}{2(1-\coth^2(a+bx))}\right)}{b}$$

input
```
Int[Cosh[a + b*x]^2*Coth[a + b*x]^2,x]
```

output
```
(I*((3*((-I)*ArcTanh[Coth[a + b*x]] + I*Coth[a + b*x]))/2 + ((I/2)*Coth[a
+ b*x]^3)/(1 - Coth[a + b*x]^2)))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^3}{2\sinh(bx+a)}+\frac{3bx}{2}+\frac{3a}{2}-\frac{3\coth(bx+a)}{2}}{b}$	39
default	$\frac{\frac{\cosh(bx+a)^3}{2\sinh(bx+a)}+\frac{3bx}{2}+\frac{3a}{2}-\frac{3\coth(bx+a)}{2}}{b}$	39
risch	$\frac{3x}{2}+\frac{e^{2bx+2a}}{8b}-\frac{e^{-2bx-2a}}{8b}-\frac{2}{b(e^{2bx+2a}-1)}$	51

input
```
int(cosh(b*x+a)^2*coth(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2/sinh(b*x+a)*cosh(b*x+a)^3+3/2*b*x+3/2*a-3/2*coth(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.67

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx = \frac{\cosh(bx+a)^3+3\cosh(bx+a)\sinh(bx+a)^2+4(3bx+2)\sinh(bx+a)-9\cosh(bx+a)}{8b\sinh(bx+a)}$$

input
```
integrate(cosh(b*x+a)^2*coth(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + 4*(3*b*x + 2)*sin
h(b*x + a) - 9*cosh(b*x + a))/(b*sinh(b*x + a))
```

Sympy [F]

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx = \int \cosh^2(a+bx)\coth^2(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)**2*coth(b*x+a)**2,x)
```

output
```
Integral(cosh(a + b*x)**2*coth(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(32) = 64$.

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.83

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx = \frac{3\,(bx+a)}{2\,b} - \frac{e^{(-2\,bx-2\,a)}}{8\,b} - \frac{17\,e^{(-2\,bx-2\,a)}-1}{8\,b\big(e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}\big)}$$

input
```
integrate(cosh(b*x+a)^2*coth(b*x+a)^2,x, algorithm="maxima")
```

output
```
3/2*(b*x + a)/b - 1/8*e^(-2*b*x - 2*a)/b - 1/8*(17*e^(-2*b*x - 2*a) - 1)/(
b*(e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(32) = 64$.

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.97

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx = \frac{12\,bx+12\,a-\frac{3\,e^{(4\,bx+4\,a)}+14\,e^{(2\,bx+2\,a)}-1}{e^{(4\,bx+4\,a)}-e^{(2\,bx+2\,a)}}+e^{(2\,bx+2\,a)}}{8\,b}$$

input
```
integrate(cosh(b*x+a)^2*coth(b*x+a)^2,x, algorithm="giac")
```

output
```
1/8*(12*b*x + 12*a - (3*e^(4*b*x + 4*a) + 14*e^(2*b*x + 2*a) - 1)/(e^(4*b*
x + 4*a) - e^(2*b*x + 2*a)) + e^(2*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.39

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx = \frac{3\,x}{2} - \frac{2}{b\,\left(e^{2\,a+2\,b\,x}-1\right)} - \frac{e^{-2\,a-2\,b\,x}}{8\,b} + \frac{e^{2\,a+2\,b\,x}}{8\,b}$$

input
```
int(cosh(a + b*x)^2*coth(a + b*x)^2,x)
```

output
```
(3*x)/2 - 2/(b*(exp(2*a + 2*b*x) - 1)) - exp(- 2*a - 2*b*x)/(8*b) + exp(2*
a + 2*b*x)/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.31

$$\int \cosh^2(a+bx)\coth^2(a+bx)\,dx = \frac{e^{6bx+6a}+12e^{4bx+4a}bx-18e^{4bx+4a}-12e^{2bx+2a}bx+1}{8e^{2bx+2a}b\left(e^{2bx+2a}-1\right)}$$

input
```
int(cosh(b*x+a)^2*coth(b*x+a)^2,x)
```

output
```
(e**(6*a + 6*b*x) + 12*e**(4*a + 4*b*x)*b*x - 18*e**(4*a + 4*b*x) - 12*e**
(2*a + 2*b*x)*b*x + 1)/(8*e**(2*a + 2*b*x)*b*(e**(2*a + 2*b*x) - 1))
```

3.54 $\int \cosh^2(a+bx)\coth^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx=-\frac{\operatorname{csch}^2(a+bx)}{2b}+\frac{2\log(\sinh(a+bx))}{b}+\frac{\sinh^2(a+bx)}{2b}$$

output
```
-1/2*csch(b*x+a)^2/b+2*ln(sinh(b*x+a))/b+1/2*sinh(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx=-\frac{\operatorname{csch}^2(a+bx)-4\log(\sinh(a+bx))-\sinh^2(a+bx)}{2b}$$

input
```
Integrate[Cosh[a + b*x]^2*Coth[a + b*x]^3,x]
```

output
```
-1/2*(Csch[a + b*x]^2 - 4*Log[Sinh[a + b*x]] - Sinh[a + b*x]^2)/b
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 26, 3070, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin\left(ia+ibx+\frac{\pi}{2}\right)^2\tan\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sin\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^3dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\int -i\text{csch}^3(a+bx)\left(\sinh^2(a+bx)+1\right)^2d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{243}$$

$$-\frac{\int -\text{csch}^2(a+bx)(i\sinh(a+bx)+1)^2d\left(-\sinh^2(a+bx)\right)}{2b}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\int \left(-\text{csch}^2(a+bx)-2i\text{csch}(a+bx)+1\right)d\left(-\sinh^2(a+bx)\right)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\sinh^2(a+bx)-i\text{csch}(a+bx)-2\log\left(-\sinh^2(a+bx)\right)}{2b}$$

input
```
Int[Cosh[a + b*x]^2*Coth[a + b*x]^3,x]
```

output
```
-1/2*((-I)*Csch[a + b*x] - 2*Log[-Sinh[a + b*x]^2] - Sinh[a + b*x]^2)/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^4}{2\sinh(bx+a)^2}+2\ln(\sinh(bx+a))-\coth(bx+a)^2}{b}$	43
default	$\frac{\frac{\cosh(bx+a)^4}{2\sinh(bx+a)^2}+2\ln(\sinh(bx+a))-\coth(bx+a)^2}{b}$	43
risch	$-2x+\frac{e^{2bx+2a}}{8b}+\frac{e^{-2bx-2a}}{8b}-\frac{4a}{b}-\frac{2\,e^{2bx+2a}}{b(e^{2bx+2a}-1)^2}+\frac{2\ln(e^{2bx+2a}-1)}{b}$	83

input
```
int(cosh(b*x+a)^2*coth(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2/sinh(b*x+a)^2*cosh(b*x+a)^4+2*ln(sinh(b*x+a))-coth(b*x+a)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. 2(39) = 78.

Time = 0.10 (sec) , antiderivative size = 743, normalized size of antiderivative = 17.28

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^2*coth(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/8*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 -
 2*(8*b*x + 1)*cosh(b*x + a)^6 - 2*(8*b*x - 14*cosh(b*x + a)^2 + 1)*sinh(b
*x + a)^6 + 4*(14*cosh(b*x + a)^3 - 3*(8*b*x + 1)*cosh(b*x + a))*sinh(b*x
+ a)^5 + 2*(16*b*x - 7)*cosh(b*x + a)^4 + 2*(35*cosh(b*x + a)^4 - 15*(8*b*
x + 1)*cosh(b*x + a)^2 + 16*b*x - 7)*sinh(b*x + a)^4 + 8*(7*cosh(b*x + a)^
5 - 5*(8*b*x + 1)*cosh(b*x + a)^3 + (16*b*x - 7)*cosh(b*x + a))*sinh(b*x +
 a)^3 - 2*(8*b*x + 1)*cosh(b*x + a)^2 + 2*(14*cosh(b*x + a)^6 - 15*(8*b*x
+ 1)*cosh(b*x + a)^4 + 6*(16*b*x - 7)*cosh(b*x + a)^2 - 8*b*x - 1)*sinh(b*
x + a)^2 + 16*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*
x + a)^6 + (15*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 +
4*(5*cosh(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a
)^4 - 12*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + cosh(b*x + a)^2 + 2*(3*cos
h(b*x + a)^5 - 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a))*log(2*sin
h(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(2*cosh(b*x + a)^7 - 3*(8*
b*x + 1)*cosh(b*x + a)^5 + 2*(16*b*x - 7)*cosh(b*x + a)^3 - (8*b*x + 1)*co
sh(b*x + a))*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sin
h(b*x + a)^5 + b*sinh(b*x + a)^6 - 2*b*cosh(b*x + a)^4 + (15*b*cosh(b*x +
a)^2 - 2*b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 - 2*b*cosh(b*x + a))*
sinh(b*x + a)^3 + b*cosh(b*x + a)^2 + (15*b*cosh(b*x + a)^4 - 12*b*cosh(b*
x + a)^2 + b)*sinh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^5 - 4*b*cosh(b*x +...
```

Sympy [F]

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx = \int \cosh^2{(a+bx)}\coth^3{(a+bx)}\,dx$$

input
```
integrate(cosh(b*x+a)**2*coth(b*x+a)**3,x)
```

output
```
Integral(cosh(a + b*x)**2*coth(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(39) = 78$.

Time = 0.05 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.79

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx = \frac{2\,(bx+a)}{b} + \frac{e^{(-2\,bx-2\,a)}}{8\,b} + \frac{2\,\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{2\,\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{2\,e^{(-2\,bx-2\,a)} + 15\,e^{(-4\,bx-4\,a)} - 1}{8\,b\left(e^{(-2\,bx-2\,a)} - 2\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)}\right)}$$

input
```
integrate(cosh(b*x+a)^2*coth(b*x+a)^3,x, algorithm="maxima")
```

output
```
2*(b*x + a)/b + 1/8*e^(-2*b*x - 2*a)/b + 2*log(e^(-b*x - a) + 1)/b + 2*log
(e^(-b*x - a) - 1)/b - 1/8*(2*e^(-2*b*x - 2*a) + 15*e^(-4*b*x - 4*a) - 1)/
(b*(e^(-2*b*x - 2*a) - 2*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(39) = 78$.

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.37

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx = -\frac{16\,bx - \left(8\,e^{(2\,bx+2\,a)}+1\right)e^{(-2\,bx-2\,a)} + 16\,a + \frac{8\,\left(3\,e^{(4\,bx+4\,a)} - 4\,e^{(2\,bx+2\,a)} + 3\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^2} - e^{(2\,bx+2\,a)} - 16\,\log\left(\left|e^{(2\,bx+2\,a)}\right.\right.}{8\,b}$$

input
```
integrate(cosh(b*x+a)^2*coth(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/8*(16*b*x - (8*e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) + 16*a + 8*(3*e^(4
*b*x + 4*a) - 4*e^(2*b*x + 2*a) + 3)/(e^(2*b*x + 2*a) - 1)^2 - e^(2*b*x +
2*a) - 16*log(abs(e^(2*b*x + 2*a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.26

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx = \frac{2\ln\left(e^{2a}e^{2bx}-1\right)}{b} - 2x - \frac{2}{b\left(e^{2a+2bx}-1\right)} - \frac{2}{b\left(e^{4a+4bx}-2e^{2a+2bx}+1\right)} + \frac{e^{-2a-2bx}}{8b} + \frac{e^{2a+2bx}}{8b}$$

input
```
int(cosh(a + b*x)^2*coth(a + b*x)^3,x)
```

output
```
(2*log(exp(2*a)*exp(2*b*x) - 1))/b - 2*x - 2/(b*(exp(2*a + 2*b*x) - 1)) -
2/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) + exp(- 2*a - 2*b*x)/(8*
b) + exp(2*a + 2*b*x)/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 253, normalized size of antiderivative = 5.88

$$\int \cosh^2(a+bx)\coth^3(a+bx)\,dx = \frac{e^{8bx+8a}+16e^{6bx+6a}\log\left(e^{bx+a}-1\right)+16e^{6bx+6a}\log\left(e^{bx+a}+1\right)-16e^{6bx+6a}bx-9e^{6bx+6a}-32e^{4bx+4a}\log\left(e^{b\cdots}\right.}{8e\cdots}$$

input
```
int(cosh(b*x+a)^2*coth(b*x+a)^3,x)
```

output
```
(e**(8*a + 8*b*x) + 16*e**(6*a + 6*b*x)*log(e**(a + b*x) - 1) + 16*e**(6*a
 + 6*b*x)*log(e**(a + b*x) + 1) - 16*e**(6*a + 6*b*x)*b*x - 9*e**(6*a + 6*
b*x) - 32*e**(4*a + 4*b*x)*log(e**(a + b*x) - 1) - 32*e**(4*a + 4*b*x)*log
(e**(a + b*x) + 1) + 32*e**(4*a + 4*b*x)*b*x + 16*e**(2*a + 2*b*x)*log(e**
(a + b*x) - 1) + 16*e**(2*a + 2*b*x)*log(e**(a + b*x) + 1) - 16*e**(2*a +
2*b*x)*b*x - 9*e**(2*a + 2*b*x) + 1)/(8*e**(2*a + 2*b*x)*b*(e**(4*a + 4*b*
x) - 2*e**(2*a + 2*b*x) + 1))
```

3.55 $\int \cosh^3(a+bx)\coth(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cosh(a+bx))}{b} + \frac{\cosh(a+bx)}{b} + \frac{\cosh^3(a+bx)}{3b}$$

output
```
-arctanh(cosh(b*x+a))/b+cosh(b*x+a)/b+1/3*cosh(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx = \frac{5\cosh(a+bx)}{4b} + \frac{\cosh(3(a+bx))}{12b} - \frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

input
```
Integrate[Cosh[a + b*x]^3*Coth[a + b*x],x]
```

output
```
(5*Cosh[a + b*x])/(4*b) + Cosh[3*(a + b*x)]/(12*b) - Log[Cosh[(a + b*x)/2]
]/b + Log[Sinh[(a + b*x)/2]]/b
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3072, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin\left(ia+ibx+\frac{\pi}{2}\right)^3\tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin\left(\frac{1}{2}(2ia+\pi)+ibx\right)^3\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx$$

$$\Big\downarrow \text{3072}$$

$$-\frac{\int \frac{\cosh^4(a+bx)}{1-\cosh^2(a+bx)}d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{254}$$

$$-\frac{\int \left(-\cosh^2(a+bx)+\frac{1}{1-\cosh^2(a+bx)}-1\right)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}(\cosh(a+bx))-\frac{1}{3}\cosh^3(a+bx)-\cosh(a+bx)}{b}$$

input
```
Int[Cosh[a + b*x]^3*Coth[a + b*x],x]
```

output
```
-((ArcTanh[Cosh[a + b*x]] - Cosh[a + b*x] - Cosh[a + b*x]^3/3)/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^3}{3}+\cosh(bx+a)-2\ \operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	31
default	$\frac{\frac{\cosh(bx+a)^3}{3}+\cosh(bx+a)-2\ \operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	31
risch	$\frac{e^{3bx+3a}}{24b}+\frac{5\,e^{bx+a}}{8b}+\frac{5\,e^{-bx-a}}{8b}+\frac{e^{-3bx-3a}}{24b}+\frac{\ln\left(e^{bx+a}-1\right)}{b}-\frac{\ln\left(e^{bx+a}+1\right)}{b}$	82

input
```
int(cosh(b*x+a)^3*coth(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3*cosh(b*x+a)^3+cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(36) = 72$.

Time = 0.09 (sec) , antiderivative size = 357, normalized size of antiderivative = 9.39

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx = \frac{\cosh(bx+a)^6 + 6\cosh(bx+a)\sinh(bx+a)^5 + \sinh(bx+a)^6 + 15\left(\cosh(bx+a)^2+1\right)\sinh(bx+a)^4}{}$$

input
```
integrate(cosh(b*x+a)^3*coth(b*x+a),x, algorithm="fricas")
```

output
```
1/24*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6
+ 15*(cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 15*cosh(b*x + a)^4 + 20*(cosh
(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 15*(cosh(b*x + a)^4 + 6*c
osh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 15*cosh(b*x + a)^2 - 24*(cosh(b*x +
a)^3 + 3*cosh(b*x + a)^2*sinh(b*x + a) + 3*cosh(b*x + a)*sinh(b*x + a)^2 +
 sinh(b*x + a)^3)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 24*(cosh(b*x +
a)^3 + 3*cosh(b*x + a)^2*sinh(b*x + a) + 3*cosh(b*x + a)*sinh(b*x + a)^2 +
 sinh(b*x + a)^3)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 6*(cosh(b*x + a
)^5 + 10*cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a) + 1)/(b*cosh(b*x
 + a)^3 + 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x + a)*sinh(b*x +
 a)^2 + b*sinh(b*x + a)^3)
```

Sympy [F]

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx = \int \cosh^3(a+bx)\coth(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)**3*coth(b*x+a),x)
```

output
```
Integral(cosh(a + b*x)**3*coth(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(36) = 72$.

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.29

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx = \frac{\left(15\,e^{(-2\,bx-2\,a)}+1\right)e^{(3\,bx+3\,a)}}{24\,b} + \frac{15\,e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{24\,b} - \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}$$

input
```
integrate(cosh(b*x+a)^3*coth(b*x+a),x, algorithm="maxima")
```

output
```
1/24*(15*e^(-2*b*x - 2*a) + 1)*e^(3*b*x + 3*a)/b + 1/24*(15*e^(-b*x - a) +
 e^(-3*b*x - 3*a))/b - log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.82

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx = \frac{\left(15\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}+e^{(3\,bx+3\,a)}+15\,e^{(bx+a)}-24\,\log\left(e^{(bx+a)}+1\right)+24\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{24\,b}$$

input
```
integrate(cosh(b*x+a)^3*coth(b*x+a),x, algorithm="giac")
```

output
```
1/24*((15*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) + e^(3*b*x + 3*a) + 15*e^(
b*x + a) - 24*log(e^(b*x + a) + 1) + 24*log(abs(e^(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.13

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx = \frac{5\,\mathrm{e}^{a+b\,x}}{8\,b} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} + \frac{5\,\mathrm{e}^{-a-b\,x}}{8\,b} + \frac{\mathrm{e}^{-3\,a-3\,b\,x}}{24\,b} + \frac{\mathrm{e}^{3\,a+3\,b\,x}}{24\,b}$$

input
```
int(cosh(a + b*x)^3*coth(a + b*x),x)
```

output
```
(5*exp(a + b*x))/(8*b) - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)
^(1/2) + (5*exp(- a - b*x))/(8*b) + exp(- 3*a - 3*b*x)/(24*b) + exp(3*a +
3*b*x)/(24*b)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.55

$$\int \cosh^3(a+bx)\coth(a+bx)\,dx = \frac{e^{6bx+6a}+15e^{4bx+4a}+24e^{3bx+3a}\log\left(e^{bx+a}-1\right)-24e^{3bx+3a}\log\left(e^{bx+a}+1\right)+15e^{2bx+2a}+1}{24e^{3bx+3a}b}$$

input
```
int(cosh(b*x+a)^3*coth(b*x+a),x)
```

output
```
(e**(6*a + 6*b*x) + 15*e**(4*a + 4*b*x) + 24*e**(3*a + 3*b*x)*log(e**(a +
b*x) - 1) - 24*e**(3*a + 3*b*x)*log(e**(a + b*x) + 1) + 15*e**(2*a + 2*b*x
) + 1)/(24*e**(3*a + 3*b*x)*b)
```

3.56 $\int \cosh^3(a+bx)\coth^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx=-\frac{\operatorname{csch}(a+bx)}{b}+\frac{2\sinh(a+bx)}{b}+\frac{\sinh^3(a+bx)}{3b}$$

output
```
-csch(b*x+a)/b+2*sinh(b*x+a)/b+1/3*sinh(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx=-\frac{\operatorname{csch}(a+bx)}{b}+\frac{2\sinh(a+bx)}{b}+\frac{\sinh^3(a+bx)}{3b}$$

input
```
Integrate[Cosh[a + b*x]^3*Coth[a + b*x]^2,x]
```

output
```
-(Csch[a + b*x]/b) + (2*Sinh[a + b*x])/b + Sinh[a + b*x]^3/(3*b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 25, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin\left(ia+ibx+\frac{\pi}{2}\right)^3\tan\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin\left(\frac{1}{2}(2ia+\pi)+ibx\right)^3\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^2dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{i\int -\text{csch}^2(a+bx)\left(\sinh^2(a+bx)+1\right)^2d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{i\int \left(-\text{csch}^2(a+bx)-\sinh^2(a+bx)-2\right)d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{1}{3}i\sinh^3(a+bx)+2i\sinh(a+bx)-i\text{csch}(a+bx)\right)}{b}$$

input
```
Int[Cosh[a + b*x]^3*Coth[a + b*x]^2,x]
```

output
```
((-I)*((-I)*Csch[a + b*x] + (2*I)*Sinh[a + b*x] + (I/3)*Sinh[a + b*x]^3))/
b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^4}{3\sinh(bx+a)}+\frac{4\cosh(bx+a)^2}{3\sinh(bx+a)}-\frac{8}{3\sinh(bx+a)}}{b}$	52
default	$\frac{\frac{\cosh(bx+a)^4}{3\sinh(bx+a)}+\frac{4\cosh(bx+a)^2}{3\sinh(bx+a)}-\frac{8}{3\sinh(bx+a)}}{b}$	52
risch	$\frac{e^{5bx+5a}+20\,e^{3bx+3a}-90\,e^{bx+a}+20\,e^{-bx-a}+e^{-3bx-3a}}{24b\left(e^{2bx+2a}-1\right)}$	68

input
```
int(cosh(b*x+a)^3*coth(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3/sinh(b*x+a)*cosh(b*x+a)^4+4/3/sinh(b*x+a)*cosh(b*x+a)^2-8/3/sinh(
b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.66

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx = \frac{\cosh(bx+a)^4+\sinh(bx+a)^4+2\left(3\cosh(bx+a)^2+10\right)\sinh(bx+a)^2+20\cosh(bx+a)^2-45}{24\,b\sinh(bx+a)}$$

input
```
integrate(cosh(b*x+a)^3*coth(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/24*(cosh(b*x + a)^4 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 10)*sinh(
b*x + a)^2 + 20*cosh(b*x + a)^2 - 45)/(b*sinh(b*x + a))
```

Sympy [F]

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx = \int \cosh^3(a+bx)\coth^2(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)**3*coth(b*x+a)**2,x)
```

output
```
Integral(cosh(a + b*x)**3*coth(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(36) = 72.

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.08

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx = -\frac{21\,e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{24\,b} + \frac{20\,e^{(-2\,bx-2\,a)}-69\,e^{(-4\,bx-4\,a)}+1}{24\,b\left(e^{(-3\,bx-3\,a)}-e^{(-5\,bx-5\,a)}\right)}$$

input
```
integrate(cosh(b*x+a)^3*coth(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/24*(21*e^(-b*x - a) + e^(-3*b*x - 3*a))/b + 1/24*(20*e^(-2*b*x - 2*a) -
 69*e^(-4*b*x - 4*a) + 1)/(b*(e^(-3*b*x - 3*a) - e^(-5*b*x - 5*a)))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx = \frac{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3 - \frac{48}{e^{(bx+a)}-e^{(-bx-a)}} + 24\,e^{(bx+a)} - 24\,e^{(-bx-a)}}{24\,b}$$

input
```
integrate(cosh(b*x+a)^3*coth(b*x+a)^2,x, algorithm="giac")
```

output
```
1/24*((e^(b*x + a) - e^(-b*x - a))^3 - 48/(e^(b*x + a) - e^(-b*x - a)) + 2
4*e^(b*x + a) - 24*e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.05

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx = \frac{7\,e^{a+b\,x}}{8\,b} - \frac{7\,e^{-a-b\,x}}{8\,b} - \frac{e^{-3\,a-3\,b\,x}}{24\,b} + \frac{e^{3\,a+3\,b\,x}}{24\,b} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(cosh(a + b*x)^3*coth(a + b*x)^2,x)
```

output
```
(7*exp(a + b*x))/(8*b) - (7*exp(- a - b*x))/(8*b) - exp(- 3*a - 3*b*x)/(24
*b) + exp(3*a + 3*b*x)/(24*b) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1)
)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.08

$$\int \cosh^3(a+bx)\coth^2(a+bx)\,dx = \frac{e^{8bx+8a}+20e^{6bx+6a}-90e^{4bx+4a}+20e^{2bx+2a}+1}{24e^{3bx+3a}b\left(e^{2bx+2a}-1\right)}$$

input
```
int(cosh(b*x+a)^3*coth(b*x+a)^2,x)
```

output
```
(e**(8*a + 8*b*x) + 20*e**(6*a + 6*b*x) - 90*e**(4*a + 4*b*x) + 20*e**(2*a
 + 2*b*x) + 1)/(24*e**(3*a + 3*b*x)*b*(e**(2*a + 2*b*x) - 1))
```

3.57 $\int \cosh^3(a+bx)\coth^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx = -\frac{5\operatorname{arctanh}(\cosh(a+bx))}{2b}+\frac{2\cosh(a+bx)}{b}+\frac{\cosh^3(a+bx)}{3b}-\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{2b}$$

output
```
-5/2*arctanh(cosh(b*x+a))/b+2*cosh(b*x+a)/b+1/3*cosh(b*x+a)^3/b-1/2*coth(b
*x+a)*csch(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.72

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx = \frac{9\cosh(a+bx)}{4b}+\frac{\cosh(3(a+bx))}{12b}-\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{5\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}+\frac{5\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}-\frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}$$

input
```
Integrate[Cosh[a + b*x]^3*Coth[a + b*x]^3,x]
```

output
```
(9*Cosh[a + b*x])/(4*b) + Cosh[3*(a + b*x)]/(12*b) - Csch[(a + b*x)/2]^2/(
8*b) - (5*Log[Cosh[(a + b*x)/2]])/(2*b) + (5*Log[Sinh[(a + b*x)/2]])/(2*b)
 - Sech[(a + b*x)/2]^2/(8*b)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 26, 3072, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin\left(ia+ibx+\frac{\pi}{2}\right)^3\tan\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sin\left(\frac{1}{2}(2ia+\pi)+ibx\right)^3\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)^3dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{\int \frac{\cosh^6(a+bx)}{(1-\cosh^2(a+bx))^2}d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\cosh^5(a+bx)}{2(1-\cosh^2(a+bx))}-\frac{5}{2}\int \frac{\cosh^4(a+bx)}{1-\cosh^2(a+bx)}d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{254}$$

$$\frac{\frac{\cosh^5(a+bx)}{2(1-\cosh^2(a+bx))}-\frac{5}{2}\int \left(-\cosh^2(a+bx)+\frac{1}{1-\cosh^2(a+bx)}-1\right)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\cosh^5(a+bx)}{2(1-\cosh^2(a+bx))}-\frac{5}{2}\left(\text{arctanh}(\cosh(a+bx))-\frac{1}{3}\cosh^3(a+bx)-\cosh(a+bx)\right)}{b}$$

input
```
Int[Cosh[a + b*x]^3*Coth[a + b*x]^3,x]
```

output
```
(Cosh[a + b*x]^5/(2*(1 - Cosh[a + b*x]^2)) - (5*(ArcTanh[Cosh[a + b*x]] -
Cosh[a + b*x] - Cosh[a + b*x]^3/3))/2)/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.35

method	result	si
derivativedivides	$\frac{\frac{\cosh(bx+a)^5}{3\sinh(bx+a)^2}+\frac{5\cosh(bx+a)^3}{3\sinh(bx+a)^2}-\frac{5\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{5\coth(bx+a)\operatorname{csch}(bx+a)}{2}-5\operatorname{arctanh}(e^{bx+a})}{b}$	81
default	$\frac{\frac{\cosh(bx+a)^5}{3\sinh(bx+a)^2}+\frac{5\cosh(bx+a)^3}{3\sinh(bx+a)^2}-\frac{5\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{5\coth(bx+a)\operatorname{csch}(bx+a)}{2}-5\operatorname{arctanh}(e^{bx+a})}{b}$	81
risch	$\frac{e^{3bx+3a}}{24b}+\frac{9\,e^{bx+a}}{8b}+\frac{9\,e^{-bx-a}}{8b}+\frac{e^{-3bx-3a}}{24b}-\frac{e^{bx+a}(e^{2bx+2a}+1)}{b(e^{2bx+2a}-1)^2}-\frac{5\ln(e^{bx+a}+1)}{2b}+\frac{5\ln(e^{bx+a}-1)}{2b}$	11

input
```
int(cosh(b*x+a)^3*coth(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3/sinh(b*x+a)^2*cosh(b*x+a)^5+5/3/sinh(b*x+a)^2*cosh(b*x+a)^3-5*cos
h(b*x+a)/sinh(b*x+a)^2+5/2*coth(b*x+a)*csch(b*x+a)-5*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1077 vs. 2(54) = 108.

Time = 0.10 (sec) , antiderivative size = 1077, normalized size of antiderivative = 17.95

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^3*coth(b*x+a)^3,x, algorithm="fricas")
```

output

```
1/24*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^
10 + 5*(9*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^8 + 25*cosh(b*x + a)^8 + 40*(
3*cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a)^7 + 10*(21*cosh(b*x + a
)^4 + 70*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^6 - 50*cosh(b*x + a)^6 + 4*(63
*cosh(b*x + a)^5 + 350*cosh(b*x + a)^3 - 75*cosh(b*x + a))*sinh(b*x + a)^5
 + 10*(21*cosh(b*x + a)^6 + 175*cosh(b*x + a)^4 - 75*cosh(b*x + a)^2 - 5)*
sinh(b*x + a)^4 - 50*cosh(b*x + a)^4 + 40*(3*cosh(b*x + a)^7 + 35*cosh(b*x
 + a)^5 - 25*cosh(b*x + a)^3 - 5*cosh(b*x + a))*sinh(b*x + a)^3 + 5*(9*cos
h(b*x + a)^8 + 140*cosh(b*x + a)^6 - 150*cosh(b*x + a)^4 - 60*cosh(b*x + a
)^2 + 5)*sinh(b*x + a)^2 + 25*cosh(b*x + a)^2 - 60*(cosh(b*x + a)^7 + 7*co
sh(b*x + a)*sinh(b*x + a)^6 + sinh(b*x + a)^7 + (21*cosh(b*x + a)^2 - 2)*s
inh(b*x + a)^5 - 2*cosh(b*x + a)^5 + 5*(7*cosh(b*x + a)^3 - 2*cosh(b*x + a
))*sinh(b*x + a)^4 + (35*cosh(b*x + a)^4 - 20*cosh(b*x + a)^2 + 1)*sinh(b*
x + a)^3 + cosh(b*x + a)^3 + (21*cosh(b*x + a)^5 - 20*cosh(b*x + a)^3 + 3*
cosh(b*x + a))*sinh(b*x + a)^2 + (7*cosh(b*x + a)^6 - 10*cosh(b*x + a)^4 +
 3*cosh(b*x + a)^2)*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1)
+ 60*(cosh(b*x + a)^7 + 7*cosh(b*x + a)*sinh(b*x + a)^6 + sinh(b*x + a)^7
+ (21*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^5 - 2*cosh(b*x + a)^5 + 5*(7*cosh
(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^4 + (35*cosh(b*x + a)^4 - 20*
cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + cosh(b*x + a)^3 + (21*cosh(b*x +...
```

Sympy [F]

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx = \int \cosh^3(a+bx)\coth^3(a+bx)\,dx$$

input

```
integrate(cosh(b*x+a)**3*coth(b*x+a)**3,x)
```

output

```
Integral(cosh(a + b*x)**3*coth(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(54) = 108$.

Time = 0.04 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.22

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx = \frac{27\,e^{(-bx-a)} + e^{(-3\,bx-3\,a)}}{24\,b} - \frac{5\,\log\left(e^{(-bx-a)}+1\right)}{2\,b} + \frac{5\,\log\left(e^{(-bx-a)}-1\right)}{2\,b} + \frac{25\,e^{(-2\,bx-2\,a)} - 77\,e^{(-4\,bx-4\,a)} + 3\,e^{(-6\,bx-6\,a)} + 1}{24\,b\left(e^{(-3\,bx-3\,a)} - 2\,e^{(-5\,bx-5\,a)} + e^{(-7\,bx-7\,a)}\right)}$$

input
```
integrate(cosh(b*x+a)^3*coth(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/24*(27*e^(-b*x - a) + e^(-3*b*x - 3*a))/b - 5/2*log(e^(-b*x - a) + 1)/b
+ 5/2*log(e^(-b*x - a) - 1)/b + 1/24*(25*e^(-2*b*x - 2*a) - 77*e^(-4*b*x -
 4*a) + 3*e^(-6*b*x - 6*a) + 1)/(b*(e^(-3*b*x - 3*a) - 2*e^(-5*b*x - 5*a)
+ e^(-7*b*x - 7*a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(54) = 108$.

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.05

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx = \frac{\left(e^{(bx+a)} + e^{(-bx-a)}\right)^3 - \frac{24\left(e^{(bx+a)}+e^{(-bx-a)}\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4} + 24\,e^{(bx+a)} + 24\,e^{(-bx-a)} - 30\,\log\left(e^{(bx+a)} + e^{(-bx-a)} + 2\right) + 3}{24\,b}$$

input
```
integrate(cosh(b*x+a)^3*coth(b*x+a)^3,x, algorithm="giac")
```

output
```
1/24*((e^(b*x + a) + e^(-b*x - a))^3 - 24*(e^(b*x + a) + e^(-b*x - a))/((e
^(b*x + a) + e^(-b*x - a))^2 - 4) + 24*e^(b*x + a) + 24*e^(-b*x - a) - 30*
log(e^(b*x + a) + e^(-b*x - a) + 2) + 30*log(e^(b*x + a) + e^(-b*x - a) -
2))/b
```

Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.33

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx = \frac{9\,e^{a+b\,x}}{8\,b} - \frac{5\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} + \frac{9\,e^{-a-b\,x}}{8\,b} + \frac{e^{-3\,a-3\,b\,x}}{24\,b} + \frac{e^{3\,a+3\,b\,x}}{24\,b} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(cosh(a + b*x)^3*coth(a + b*x)^3,x)
```

output
```
(9*exp(a + b*x))/(8*b) - (5*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)
^(1/2) + (9*exp(- a - b*x))/(8*b) + exp(- 3*a - 3*b*x)/(24*b) + exp(3*a +
3*b*x)/(24*b) - (2*exp(a + b*x))/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x)
 + 1)) - exp(a + b*x)/(b*(exp(2*a + 2*b*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.92

$$\int \cosh^3(a+bx)\coth^3(a+bx)\,dx = \frac{e^{10bx+10a} + 25e^{8bx+8a} + 60e^{7bx+7a}\log\left(e^{bx+a}-1\right) - 60e^{7bx+7a}\log\left(e^{bx+a}+1\right) - 50e^{6bx+6a} - 120e^{5bx+5a}\log(}{24e^{3bx+3a}}$$

input
```
int(cosh(b*x+a)^3*coth(b*x+a)^3,x)
```

output
```
(e**(10*a + 10*b*x) + 25*e**(8*a + 8*b*x) + 60*e**(7*a + 7*b*x)*log(e**(a
+ b*x) - 1) - 60*e**(7*a + 7*b*x)*log(e**(a + b*x) + 1) - 50*e**(6*a + 6*b
*x) - 120*e**(5*a + 5*b*x)*log(e**(a + b*x) - 1) + 120*e**(5*a + 5*b*x)*lo
g(e**(a + b*x) + 1) - 50*e**(4*a + 4*b*x) + 60*e**(3*a + 3*b*x)*log(e**(a
+ b*x) - 1) - 60*e**(3*a + 3*b*x)*log(e**(a + b*x) + 1) + 25*e**(2*a + 2*b
*x) + 1)/(24*e**(3*a + 3*b*x)*b*(e**(4*a + 4*b*x) - 2*e**(2*a + 2*b*x) + 1
))
```

3.58 $\int \cosh^4(a+bx)\coth(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx=\frac{\log(\sinh(a+bx))}{b}+\frac{\sinh^2(a+bx)}{b}+\frac{\sinh^4(a+bx)}{4b}$$

output
```
ln(sinh(b*x+a))/b+sinh(b*x+a)^2/b+1/4*sinh(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx=\frac{4\log(\sinh(a+bx))+4\sinh^2(a+bx)+\sinh^4(a+bx)}{4b}$$

input
```
Integrate[Cosh[a + b*x]^4*Coth[a + b*x],x]
```

output
```
(4*Log[Sinh[a + b*x]] + 4*Sinh[a + b*x]^2 + Sinh[a + b*x]^4)/(4*b)
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 26, 3070, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin\left(ia+ibx+\frac{\pi}{2}\right)^4\tan\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin\left(\frac{1}{2}(2ia+\pi)+ibx\right)^4\tan\left(\frac{1}{2}(2ia+\pi)+ibx\right)dx$$

$$\Big\downarrow \text{3070}$$

$$\frac{\int i\text{csch}(a+bx)\left(\sinh^2(a+bx)+1\right)^2d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int i\text{csch}(a+bx)(i\sinh(a+bx)+1)^2d(-\sinh^2(a+bx))}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(-\sinh^2(a+bx)+i\text{csch}(a+bx)-2\right)d\left(-\sinh^2(a+bx)\right)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}\sinh^2(a+bx)+2i\sinh(a+bx)+\log\left(-\sinh^2(a+bx)\right)}{2b}$$

input
```
Int[Cosh[a + b*x]^4*Coth[a + b*x],x]
```

output
```
(Log[-Sinh[a + b*x]^2] + (2*I)*Sinh[a + b*x] - Sinh[a + b*x]^2/2)/(2*b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)^4}{4}+\frac{\cosh(bx+a)^2}{2}+\ln(\sinh(bx+a))}{b}$	33
default	$\frac{\frac{\cosh(bx+a)^4}{4}+\frac{\cosh(bx+a)^2}{2}+\ln(\sinh(bx+a))}{b}$	33
risch	$-x+\frac{e^{4bx+4a}}{64b}+\frac{3\,e^{2bx+2a}}{16b}+\frac{3\,e^{-2bx-2a}}{16b}+\frac{e^{-4bx-4a}}{64b}-\frac{2a}{b}+\frac{\ln(e^{2bx+2a}-1)}{b}$	83

input
```
int(cosh(b*x+a)^4*coth(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4*cosh(b*x+a)^4+1/2*cosh(b*x+a)^2+ln(sinh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(37) = 74.

Time = 0.10 (sec) , antiderivative size = 457, normalized size of antiderivative = 11.72

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx$$
$$=\frac{\cosh(bx+a)^8+8\cosh(bx+a)\sinh(bx+a)^7+\sinh(bx+a)^8+4\left(7\cosh(bx+a)^2+3\right)\sinh(bx+a)}{}$$

input
```
integrate(cosh(b*x+a)^4*coth(b*x+a),x, algorithm="fricas")
```

output
```
1/64*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8
+ 4*(7*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^6 - 64*b*x*cosh(b*x + a)^4 + 12*
cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 + 9*cosh(b*x + a))*sinh(b*x + a)^5
+ 2*(35*cosh(b*x + a)^4 - 32*b*x + 90*cosh(b*x + a)^2)*sinh(b*x + a)^4 + 8
*(7*cosh(b*x + a)^5 - 32*b*x*cosh(b*x + a) + 30*cosh(b*x + a)^3)*sinh(b*x
+ a)^3 + 4*(7*cosh(b*x + a)^6 - 96*b*x*cosh(b*x + a)^2 + 45*cosh(b*x + a)^
4 + 3)*sinh(b*x + a)^2 + 12*cosh(b*x + a)^2 + 64*(cosh(b*x + a)^4 + 4*cosh
(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x
 + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4)*log(2*sinh(b*x + a)/(cosh(b*x + a
) - sinh(b*x + a))) + 8*(cosh(b*x + a)^7 - 32*b*x*cosh(b*x + a)^3 + 9*cosh
(b*x + a)^5 + 3*cosh(b*x + a))*sinh(b*x + a) + 1)/(b*cosh(b*x + a)^4 + 4*b
*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*cosh(b*x + a)^2*sinh(b*x + a)^2 + 4*b
*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4)
```

Sympy [F]

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx = \int \cosh^4{(a+bx)}\coth{(a+bx)}\,dx$$

input
```
integrate(cosh(b*x+a)**4*coth(b*x+a),x)
```

output
```
Integral(cosh(a + b*x)**4*coth(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(37) = 74$.

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.44

$$\begin{aligned}\int \cosh^4(a+bx)\coth(a+bx)\,dx = {} & \frac{\left(12\,e^{(-2\,bx-2\,a)}+1\right)e^{(4\,bx+4\,a)}}{64\,b} + \frac{bx+a}{b} \\ & + \frac{12\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}}{64\,b} \\ & + \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b}\end{aligned}$$

input
```
integrate(cosh(b*x+a)^4*coth(b*x+a),x, algorithm="maxima")
```

output
```
1/64*(12*e^(-2*b*x - 2*a) + 1)*e^(4*b*x + 4*a)/b + (b*x + a)/b + 1/64*(12*
e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a))/b + log(e^(-b*x - a) + 1)/b + log(e^(
-b*x - a) - 1)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(37) = 74$.

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.18

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx = -\frac{64\,bx - \left(48\,e^{(4\,bx+4\,a)} + 12\,e^{(2\,bx+2\,a)} + 1\right)e^{(-4\,bx-4\,a)} + 64\,a - e^{(4\,bx+4\,a)} - 12\,e^{(2\,bx+2\,a)} - 64\,\log\left(\left|e^{(2\,bx+2\right.\right.}}{64\,b}$$

input
```
integrate(cosh(b*x+a)^4*coth(b*x+a),x, algorithm="giac")
```

output
```
-1/64*(64*b*x - (48*e^(4*b*x + 4*a) + 12*e^(2*b*x + 2*a) + 1)*e^(-4*b*x -
4*a) + 64*a - e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a) - 64*log(abs(e^(2*b*x +
 2*a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.97

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx = \frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b} - x + \frac{3\,\mathrm{e}^{-2\,a-2\,b\,x}}{16\,b} + \frac{3\,\mathrm{e}^{2\,a+2\,b\,x}}{16\,b} + \frac{\mathrm{e}^{-4\,a-4\,b\,x}}{64\,b} + \frac{\mathrm{e}^{4\,a+4\,b\,x}}{64\,b}$$

input
```
int(cosh(a + b*x)^4*coth(a + b*x),x)
```

output
```
log(exp(2*a)*exp(2*b*x) - 1)/b - x + (3*exp(- 2*a - 2*b*x))/(16*b) + (3*ex
p(2*a + 2*b*x))/(16*b) + exp(- 4*a - 4*b*x)/(64*b) + exp(4*a + 4*b*x)/(64*
b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.85

$$\int \cosh^4(a+bx)\coth(a+bx)\,dx$$
$$= \frac{e^{8bx+8a}+12e^{6bx+6a}+64e^{4bx+4a}\log\left(e^{bx+a}-1\right)+64e^{4bx+4a}\log\left(e^{bx+a}+1\right)-64e^{4bx+4a}bx+12e^{2bx+2a}+1}{64e^{4bx+4a}b}$$

input
```
int(cosh(b*x+a)^4*coth(b*x+a),x)
```

output
```
(e**(8*a + 8*b*x) + 12*e**(6*a + 6*b*x) + 64*e**(4*a + 4*b*x)*log(e**(a +
b*x) - 1) + 64*e**(4*a + 4*b*x)*log(e**(a + b*x) + 1) - 64*e**(4*a + 4*b*x
)*b*x + 12*e**(2*a + 2*b*x) + 1)/(64*e**(4*a + 4*b*x)*b)
```

3.59 $\int \cosh(x)\mathbf{csch}^{\frac{7}{3}}(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \cosh(x)\text{csch}^{\frac{7}{3}}(x)\,dx = -\frac{3}{4}\text{csch}^{\frac{4}{3}}(x)$$

output
```
-3/4*csch(x)^(4/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cosh(x)\text{csch}^{\frac{7}{3}}(x)\,dx = -\frac{3}{4}\text{csch}^{\frac{4}{3}}(x)$$

input
```
Integrate[Cosh[x]*Csch[x]^(7/3),x]
```

output
```
(-3*Csch[x]^(4/3))/4
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3101, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(x)\text{csch}^{\frac{7}{3}}(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(i\csc(ix))^{7/3}}{\sec(ix)}dx$$

$$\Big\downarrow \text{3101}$$

$$-\int \sqrt[3]{\text{csch}(x)}d\text{csch}(x)$$

$$\Big\downarrow \text{15}$$

$$-\frac{3}{4}\text{csch}^{\frac{4}{3}}(x)$$

input
```
Int[Cosh[x]*Csch[x]^(7/3),x]
```

output
```
(-3*Csch[x]^(4/3))/4
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{3 \operatorname{csch}(x)^{\frac{4}{3}}}{4}$	7
default	$-\frac{3 \operatorname{csch}(x)^{\frac{4}{3}}}{4}$	7

input
```
int(cosh(x)*csch(x)^(7/3),x,method=_RETURNVERBOSE)
```

output
```
-3/4*csch(x)^(4/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(6) = 12.

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 5.40

$$\int \cosh(x)\text{csch}^{\frac{7}{3}}(x)\,dx = -\frac{3\cdot 2^{\frac{1}{3}}\left(\frac{\cosh(x)+\sinh(x)}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1}\right)^{\frac{1}{3}}(\cosh(x)+\sinh(x))}{2\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)}$$

input
```
integrate(cosh(x)*csch(x)^(7/3),x, algorithm="fricas")
```

output
```
-3/2*2^(1/3)*((cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2 - 1))^(1/3)*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x
)^2 - 1)
```

Sympy [F(-1)]

Timed out.

$$\int \cosh(x)\operatorname{csch}^{\frac{7}{3}}(x)\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)*csch(x)**(7/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cosh(x)\operatorname{csch}^{\frac{7}{3}}(x)\,dx = \int \cosh(x)\operatorname{csch}(x)^{\frac{7}{3}}\,dx$$

input
```
integrate(cosh(x)*csch(x)^(7/3),x, algorithm="maxima")
```

output
```
integrate(cosh(x)*csch(x)^(7/3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \cosh(x)\operatorname{csch}^{\frac{7}{3}}(x)\,dx = -\frac{3\cdot 2^{\frac{1}{3}}e^{\left(\frac{4}{3}x\right)}}{2\left(e^{(2x)}-1\right)^{\frac{4}{3}}}$$

input
```
integrate(cosh(x)*csch(x)^(7/3),x, algorithm="giac")
```

output
```
-3/2*2^(1/3)*e^(4/3*x)/(e^(2*x) - 1)^(4/3)
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 31, normalized size of antiderivative = 3.10

$$\int \cosh(x)\text{csch}^{\frac{7}{3}}(x)\,dx = -\frac{3\,e^x\,\left(-\frac{1}{\frac{e^{-x}}{2}-\frac{e^x}{2}}\right)^{1/3}}{2\,\left(e^{2\,x}-1\right)}$$

input
```
int(cosh(x)*(1/sinh(x))^(7/3),x)
```

output
```
-(3*exp(x)*(-1/(exp(-x)/2 - exp(x)/2))^(1/3))/(2*(exp(2*x) - 1))
```

Reduce [F]

$$\int \cosh(x)\text{csch}^{\frac{7}{3}}(x)\,dx = \int \text{csch}(x)^{\frac{7}{3}}\cosh(x)\,dx$$

input
```
int(cosh(x)*csch(x)^(7/3),x)
```

output
```
int(csch(x)**(1/3)*cosh(x)*csch(x)**2,x)
```

3.60 $\int \text{sech}(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \text{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{\text{sech}(a+bx)}{b}$$

output
```
-sech(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \text{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{\text{sech}(a+bx)}{b}$$

input
```
Integrate[Sech[a + b*x]*Tanh[a + b*x],x]
```

output
```
-(Sech[a + b*x]/b)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 26, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ia+ibx)\sec(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sec(ia+ibx)\tan(ia+ibx)dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\int 1d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\text{sech}(a+bx)}{b}$$

input
```
Int[Sech[a + b*x]*Tanh[a + b*x],x]
```

output
```
-(Sech[a + b*x]/b)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\frac{\operatorname{sech}(bx+a)}{b}$	12
default	$-\frac{\operatorname{sech}(bx+a)}{b}$	12
risch	$-\frac{2\,\mathrm{e}^{bx+a}}{b\left(\mathrm{e}^{2bx+2a}+1\right)}$	25

input
```
int(sech(b*x+a)*tanh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-sech(b*x+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(11) = 22$.

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 4.91

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^2+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^2+b}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a),x, algorithm="fricas")
```

output
```
-2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*
sinh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(8) = 16$.

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = \begin{cases} -\frac{\operatorname{sech}\left(a+bx\right)}{b} & \text{for}\ b\neq 0 \\ x\tanh\left(a\right)\operatorname{sech}\left(a\right) & \text{otherwise} \end{cases}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a),x)
```

output
```
Piecewise((-sech(a + b*x)/b, Ne(b, 0)), (x*tanh(a)*sech(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{2}{b(e^{(bx+a)}+e^{(-bx-a)})}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a),x, algorithm="maxima")
```

output
```
-2/(b*(e^(b*x + a) + e^(-b*x - a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{2}{b(e^{(bx+a)}+e^{(-bx-a)})}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a),x, algorithm="giac")
```

output
```
-2/(b*(e^(b*x + a) + e^(-b*x - a)))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx = -\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(tanh(a + b*x)/cosh(a + b*x),x)
```

output
```
-(2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}(a+bx)\tanh(a+bx)\,dx=-\frac{\operatorname{sech}(bx+a)}{b}$$

input
```
int(sech(b*x+a)*tanh(b*x+a),x)
```

output
```
( - sech(a + b*x))/b
```

3.61 $\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx=-\frac{\operatorname{sech}^2(a+bx)}{2b}$$

output
```
-1/2*sech(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx=-\frac{\operatorname{sech}^2(a+bx)}{2b}$$

input
```
Integrate[Sech[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
-1/2*Sech[a + b*x]^2/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 26, 3086, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(a+bx)\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ia+ibx)\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sec(ia+ibx)^2\tan(ia+ibx)dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\int \text{sech}(a+bx)d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\text{sech}^2(a+bx)}{2b}$$

input
```
Int[Sech[a + b*x]^2*Tanh[a + b*x],x]
```

output
```
-1/2*Sech[a + b*x]^2/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\tanh(bx+a)^2}{2b}$	14
default	$\frac{\tanh(bx+a)^2}{2b}$	14
risch	$-\frac{2\,\mathrm{e}^{2bx+2a}}{b\left(\mathrm{e}^{2bx+2a}+1\right)^2}$	28

input
```
int(sech(b*x+a)^2*tanh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*tanh(b*x+a)^2/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(13) = 26$.

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 5.60

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx =$$

$$-\frac{2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^3+3\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+b\sinh\left(bx+a\right)^3+3\,b\cosh\left(bx+a\right)+\left(3\,b\cosh\left(bx\right.\right.}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a),x, algorithm="fricas")
```

output
```
-2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)*
sinh(b*x + a)^2 + b*sinh(b*x + a)^3 + 3*b*cosh(b*x + a) + (3*b*cosh(b*x +
a)^2 + b)*sinh(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = \begin{cases} -\frac{\operatorname{sech}^2\left(a+bx\right)}{2b} & \text{for}\ b \neq 0 \\ x\tanh\left(a\right)\operatorname{sech}^2\left(a\right) & \text{otherwise} \end{cases}$$

input
```
integrate(sech(b*x+a)**2*tanh(b*x+a),x)
```

output
```
Piecewise((-sech(a + b*x)**2/(2*b), Ne(b, 0)), (x*tanh(a)*sech(a)**2, True
))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = \frac{\tanh(bx+a)^2}{2\,b}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*tanh(b*x + a)^2/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{2\,e^{(2\,bx+2\,a)}}{b\big(e^{(2\,bx+2\,a)}+1\big)^2}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a),x, algorithm="giac")
```

output
```
-2*e^(2*b*x + 2*a)/(b*(e^(2*b*x + 2*a) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{1}{2\,b\cosh(a+b\,x)^2}$$

input
```
int(tanh(a + b*x)/cosh(a + b*x)^2,x)
```

output
```
-1/(2*b*cosh(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \operatorname{sech}^2(a+bx)\tanh(a+bx)\,dx = -\frac{\operatorname{sech}(bx+a)^2}{2b}$$

input
```
int(sech(b*x+a)^2*tanh(b*x+a),x)
```

output
```
( - sech(a + b*x)**2)/(2*b)
```

3.62 $\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx=-\frac{\operatorname{sech}^n(a+bx)}{bn}$$

output
```
-sech(b*x+a)^n/b/n
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx=-\frac{\operatorname{sech}^n(a+bx)}{bn}$$

input
```
Integrate[Sech[a + b*x]^(1 + n)*Sinh[a + b*x],x]
```

output
```
-(Sech[a + b*x]^n/(b*n))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 26, 3102, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\text{sech}^{n+1}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sec(ia+ibx)^{n+1}}{\csc(ia+ibx)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sec(ia+ibx)^{n+1}}{\csc(ia+ibx)}dx$$

$$\Big\downarrow \text{3102}$$

$$-\frac{\int \text{sech}^{n-1}(a+bx)d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\text{sech}^n(a+bx)}{bn}$$

input
```
Int[Sech[a + b*x]^(1 + n)*Sinh[a + b*x],x]
```

output
```
-(Sech[a + b*x]^n/(b*n))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

method	result
derivativedivides	$-\frac{\mathrm{e}^{(1+n)\ln(\operatorname{sech}(bx+a))}}{bn\operatorname{sech}(bx+a)}$
default	$-\frac{\mathrm{e}^{(1+n)\ln(\operatorname{sech}(bx+a))}}{bn\operatorname{sech}(bx+a)}$
risch	$-\frac{\left(\frac{2\,2^{n}\left(\mathrm{e}^{bx+a}\right)^{n}\left(\mathrm{e}^{2bx+2a}+1\right)^{-n}\mathrm{e}^{3bx+3a}\,\mathrm{e}^{\frac{i\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\pi\operatorname{csgn}\left(\frac{i\mathrm{e}^{bx+a}}{\mathrm{e}^{2bx+2a}+1}\right)^{2}}{2}}\,\mathrm{e}^{\frac{i\pi\operatorname{csgn}\left(\frac{i\mathrm{e}^{bx+a}}{\mathrm{e}^{2bx+2a}+1}\right)^{2}\operatorname{csgn}\left(i\mathrm{e}^{bx+a}\right.}{2}}}{}\right.}{}$

input
```
int(sech(b*x+a)^(1+n)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b/n*exp((1+n)*ln(sech(b*x+a)))/sech(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(16) = 32$.

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 9.56

$$\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx = -\frac{\cosh(bx+a)\cosh\left((n+1)\log\left(\frac{2(\cosh(bx+a)+\sinh(bx+a))}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2+1}\right)\right)+\cosh(bx+a)\sinh\left((\right.}{bn\cosh(bx+a)^2-bn\sinh(bx+a)^2}$$

input
```
integrate(sech(b*x+a)^(1+n)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-(cosh(b*x + a)*cosh((n + 1)*log(2*(cosh(b*x + a) + sinh(b*x + a))/(cosh(b
*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1))) + cosh(
b*x + a)*sinh((n + 1)*log(2*(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)
^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1))))/(b*n*cosh(b*x
 + a)^2 - b*n*sinh(b*x + a)^2)
```

Sympy [F]

$$\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx = \int \sinh(a+bx)\operatorname{sech}^{n+1}(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)**(1+n)*sinh(b*x+a),x)
```

output
```
Integral(sinh(a + b*x)*sech(a + b*x)**(n + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(16) = 32$.

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx = -\frac{2^n e^{\left(-(bx+a)n-n\log\left(e^{(-2\,bx-2\,a)}+1\right)\right)}}{bn}$$

input
```
integrate(sech(b*x+a)^(1+n)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
-2^n*e^(-(b*x + a)*n - n*log(e^(-2*b*x - 2*a) + 1))/(b*n)
```

Giac [F]

$$\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx = \int \operatorname{sech}\left(bx+a\right)^{n+1}\sinh\left(bx+a\right)\,dx$$

input
```
integrate(sech(b*x+a)^(1+n)*sinh(b*x+a),x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^(n + 1)*sinh(b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.50

$$\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx = -\frac{\cosh(a+b\,x)\,\left(\frac{1}{\frac{e^{a+b\,x}}{2}+\frac{e^{-a-b\,x}}{2}}\right)^{n+1}}{b\,n}$$

input
```
int(sinh(a + b*x)*(1/cosh(a + b*x))^(n + 1),x)
```

output
```
-(cosh(a + b*x)*(1/(exp(a + b*x)/2 + exp(- a - b*x)/2))^(n + 1))/(b*n)
```

Reduce [F]

$$\int \operatorname{sech}^{1+n}(a+bx)\sinh(a+bx)\,dx = \int \operatorname{sech}(bx+a)^n \operatorname{sech}(bx+a)\sinh(bx+a)\,dx$$

input
```
int(sech(b*x+a)^(1+n)*sinh(b*x+a),x)
```

output
```
int(sech(a + b*x)**n*sech(a + b*x)*sinh(a + b*x),x)
```

3.63 $\int \text{sech}^2(a+bx)\tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \text{sech}^2(a+bx)\tanh^2(a+bx)\,dx=\frac{\tanh^3(a+bx)}{3b}$$

output
```
1/3*tanh(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \text{sech}^2(a+bx)\tanh^2(a+bx)\,dx=\frac{\tanh^3(a+bx)}{3b}$$

input
```
Integrate[Sech[a + b*x]^2*Tanh[a + b*x]^2,x]
```

output
```
Tanh[a + b*x]^3/(3*b)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 25, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(a+bx)\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ia+ibx)^2\left(-\sec(ia+ibx)^2\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec(ia+ibx)^2\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{i\int -\tanh^2(a+bx)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\tanh^3(a+bx)}{3b}$$

input
```
Int[Sech[a + b*x]^2*Tanh[a + b*x]^2,x]
```

output
```
Tanh[a + b*x]^3/(3*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\tanh(bx+a)^3}{3b}$	14
default	$\frac{\tanh(bx+a)^3}{3b}$	14
risch	$-\frac{2(3\,e^{4bx+4a}+1)}{3b(e^{2bx+2a}+1)^3}$	32

input
```
int(sech(b*x+a)^2*tanh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*tanh(b*x+a)^3/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(13) = 26$.

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 9.20

$$\int \operatorname{sech}^2(a+bx)\tanh^2(a+bx)\,dx =$$

$$-\frac{8\left(\cosh\left(bx+a\right)^2+\cosh\left(bx+a\right)\sinh\cdots\right.}{3\left(b\cosh\left(bx+a\right)^4+4\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+b\sinh\left(bx+a\right)^4+4\,b\cosh\left(bx+a\right)^2+2\left(3\,b\cos\cdots\right.\right.}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-8/3*(cosh(b*x + a)^2 + cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)/(b*
cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 +
4*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a)^2 + 4*(b
*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + 3*b)
```

Sympy [F]

$$\int \operatorname{sech}^2(a+bx)\tanh^2(a+bx)\,dx = \int \tanh^2\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)dx$$

input
```
integrate(sech(b*x+a)**2*tanh(b*x+a)**2,x)
```

output
```
Integral(tanh(a + b*x)**2*sech(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \operatorname{sech}^2(a+bx)\tanh^2(a+bx)\,dx = \frac{\tanh(bx+a)^3}{3\,b}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/3*tanh(b*x + a)^3/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(13) = 26.

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \operatorname{sech}^2(a+bx)\tanh^2(a+bx)\,dx = -\frac{2\left(3\,e^{(4\,bx+4\,a)}+1\right)}{3\,b\left(e^{(2\,bx+2\,a)}+1\right)^3}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a)^2,x, algorithm="giac")
```

output
```
-2/3*(3*e^(4*b*x + 4*a) + 1)/(b*(e^(2*b*x + 2*a) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \operatorname{sech}^2(a+bx)\tanh^2(a+bx)\,dx = -\frac{2\left(3\,\mathrm{e}^{4\,a+4\,b\,x}+1\right)}{3\,b\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)^3}$$

input
```
int(tanh(a + b*x)^2/cosh(a + b*x)^2,x)
```

output
```
-(2*(3*exp(4*a + 4*b*x) + 1))/(3*b*(exp(2*a + 2*b*x) + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 4.33

$$\int \operatorname{sech}^2(a+bx)\tanh^2(a+bx)\,dx = \frac{2e^{2bx+2a}\left(e^{4bx+4a}+3\right)}{3b\left(e^{6bx+6a}+3e^{4bx+4a}+3e^{2bx+2a}+1\right)}$$

input
```
int(sech(b*x+a)^2*tanh(b*x+a)^2,x)
```

output
```
(2*e**(2*a + 2*b*x)*(e**(4*a + 4*b*x) + 3))/(3*b*(e**(6*a + 6*b*x) + 3*e**
(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) + 1))
```

3.64 $\int \text{sech}^2(a+bx)\tanh^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \text{sech}^2(a+bx)\tanh^3(a+bx)\,dx=\frac{\tanh^4(a+bx)}{4b}$$

output
```
1/4*tanh(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \text{sech}^2(a+bx)\tanh^3(a+bx)\,dx=\frac{\tanh^4(a+bx)}{4b}$$

input
```
Integrate[Sech[a + b*x]^2*Tanh[a + b*x]^3,x]
```

output
```
Tanh[a + b*x]^4/(4*b)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 26, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(a+bx)\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan(ia+ibx)^3\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sec(ia+ibx)^2\tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int -i\tanh^3(a+bx)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\tanh^4(a+bx)}{4b}$$

input
```
Int[Sech[a + b*x]^2*Tanh[a + b*x]^3,x]
```

output
```
Tanh[a + b*x]^4/(4*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\tanh(bx+a)^4}{4b}$	14
default	$\frac{\tanh(bx+a)^4}{4b}$	14
risch	$-\frac{2\,e^{2bx+2a}\left(e^{4bx+4a}+1\right)}{b\left(e^{2bx+2a}+1\right)^4}$	39

input
```
int(sech(b*x+a)^2*tanh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*tanh(b*x+a)^4/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(13) = 26$.

Time = 0.07 (sec) , antiderivative size = 208, normalized size of antiderivative = 13.87

$$\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx =$$

$$-\frac{2\left(\cosh\left(bx+a\right)^3+3\,\cosh\left(\right.\right.}{b\cosh\left(bx+a\right)^5+5\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4+b\sinh\left(bx+a\right)^5+5\,b\cosh\left(bx+a\right)^3+\left(10\,b\cosh\left(\right.\right.}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 +
(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) + cosh(b*x + a))/(b*cosh(b*x + a)^5
+ 5*b*cosh(b*x + a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5 + 5*b*cosh(b*x + a
)^3 + (10*b*cosh(b*x + a)^2 + 3*b)*sinh(b*x + a)^3 + 5*(2*b*cosh(b*x + a)^
3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^2 + 10*b*cosh(b*x + a) + (5*b*cosh(b*
x + a)^4 + 9*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(10) = 20$.

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.93

$$\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx = \begin{cases} -\frac{\tanh^2(a+bx)\operatorname{sech}^2(a+bx)}{4b} - \frac{\operatorname{sech}^2(a+bx)}{4b} & \text{for } b \neq 0 \\ x\tanh^3(a)\operatorname{sech}^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sech(b*x+a)**2*tanh(b*x+a)**3,x)
```

output
```
Piecewise((-tanh(a + b*x)**2*sech(a + b*x)**2/(4*b) - sech(a + b*x)**2/(4*
b), Ne(b, 0)), (x*tanh(a)**3*sech(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx = \frac{\tanh(bx+a)^4}{4b}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*tanh(b*x + a)^4/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(13) = 26.

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.47

$$\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx = -\frac{2\left(e^{(6bx+6a)}+e^{(2bx+2a)}\right)}{b\left(e^{(2bx+2a)}+1\right)^4}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a)^3,x, algorithm="giac")
```

output
```
-2*(e^(6*b*x + 6*a) + e^(2*b*x + 2*a))/(b*(e^(2*b*x + 2*a) + 1)^4)
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 230, normalized size of antiderivative = 15.33

$$\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx = \frac{\frac{1}{2b}-\frac{3\,e^{2a+2bx}}{2b}+\frac{3\,e^{4a+4bx}}{2b}-\frac{e^{6a+6bx}}{2b}}{4\,e^{2a+2bx}+6\,e^{4a+4bx}+4\,e^{6a+6bx}+e^{8a+8bx}+1} - \frac{\frac{1}{2b}-\frac{e^{2a+2bx}}{b}+\frac{e^{4a+4bx}}{2b}}{3\,e^{2a+2bx}+3\,e^{4a+4bx}+e^{6a+6bx}+1} + \frac{\frac{1}{2b}-\frac{e^{2a+2bx}}{2b}}{2\,e^{2a+2bx}+e^{4a+4bx}+1} - \frac{1}{2b\left(e^{2a+2bx}+1\right)}$$

input
```
int(tanh(a + b*x)^3/cosh(a + b*x)^2,x)
```

output
```
(1/(2*b) - (3*exp(2*a + 2*b*x))/(2*b) + (3*exp(4*a + 4*b*x))/(2*b) - exp(6
*a + 6*b*x)/(2*b))/(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6*a +
6*b*x) + exp(8*a + 8*b*x) + 1) - (1/(2*b) - exp(2*a + 2*b*x)/b + exp(4*a +
 4*b*x)/(2*b))/(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x)
 + 1) + (1/(2*b) - exp(2*a + 2*b*x)/(2*b))/(2*exp(2*a + 2*b*x) + exp(4*a +
 4*b*x) + 1) - 1/(2*b*(exp(2*a + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \operatorname{sech}^2(a+bx)\tanh^3(a+bx)\,dx = -\frac{\operatorname{sech}(bx+a)^2\left(\tanh(bx+a)^2+1\right)}{4b}$$

input
```
int(sech(b*x+a)^2*tanh(b*x+a)^3,x)
```

output
```
( - sech(a + b*x)**2*(tanh(a + b*x)**2 + 1))/(4*b)
```

3.65 $\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx=\frac{\tanh^{1+n}(a+bx)}{b(1+n)}$$

output
```
tanh(b*x+a)^(1+n)/b/(1+n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx=\frac{\tanh^{1+n}(a+bx)}{b(1+n)}$$

input
```
Integrate[Sech[a + b*x]^2*Tanh[a + b*x]^n,x]
```

output
```
Tanh[a + b*x]^(1 + n)/(b*(1 + n))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 3087, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}^2(a+bx)\tanh^n(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(ia+ibx)^2(-i\tan(ia+ibx))^n dx$$

$$\Big\downarrow \text{3087}$$

$$-\frac{i\int \tanh^n(a+bx)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{\tanh^{n+1}(a+bx)}{b(n+1)}$$

input
```
Int[Sech[a + b*x]^2*Tanh[a + b*x]^n,x]
```

output
```
Tanh[a + b*x]^(1 + n)/(b*(1 + n))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 5.88 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\tanh(bx+a)^{1+n}}{b(1+n)}$
default	$\frac{\tanh(bx+a)^{1+n}}{b(1+n)}$
risch	$\left(\mathrm{e}^{2bx+2a}-1\right)\left(\mathrm{e}^{bx+a}-1\right)^{n}\left(\mathrm{e}^{bx+a}+1\right)^{n}\left(\mathrm{e}^{2bx+2a}+1\right)^{-n}\mathrm{e}^{-\frac{i\pi n\left(-\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{i\left(\mathrm{e}^{bx+a}+1\right)}{\mathrm{e}^{2bx+2a}+1}\right)^{2}+\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}\ldots}\right.\right.}{}}$

input
```
int(sech(b*x+a)^2*tanh(b*x+a)^n,x,method=_RETURNVERBOSE)
```

output
```
tanh(b*x+a)^(1+n)/b/(1+n)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(19) = 38.

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.63

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx = \frac{\cosh\left(n\log\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right)\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)\sinh\left(n\log\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right)\right)}{(bn+b)\cosh\left(bx+a\right)}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a)^n,x, algorithm="fricas")
```

output
```
(cosh(n*log(sinh(b*x + a)/cosh(b*x + a)))*sinh(b*x + a) + sinh(b*x + a)*si
nh(n*log(sinh(b*x + a)/cosh(b*x + a))))/((b*n + b)*cosh(b*x + a))
```

Sympy [F]

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx = \int \tanh^n(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)**2*tanh(b*x+a)**n,x)
```

output
```
Integral(tanh(a + b*x)**n*sech(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx = \frac{\tanh(bx+a)^{n+1}}{b(n+1)}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a)^n,x, algorithm="maxima")
```

output
```
tanh(b*x + a)^(n + 1)/(b*(n + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.00

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx = \frac{\left(\frac{e^{(2\,bx+2\,a)}-1}{e^{(2\,bx+2\,a)}+1}\right)^{n+1}}{b(n+1)}$$

input
```
integrate(sech(b*x+a)^2*tanh(b*x+a)^n,x, algorithm="giac")
```

output
```
((e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) + 1))^(n + 1)/(b*(n + 1))
```

Mupad [B] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.21

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx=\frac{\tanh(a+b\,x)\,\left(\frac{\mathrm{e}^{2\,a+2\,b\,x}-1}{\mathrm{e}^{2\,a+2\,b\,x}+1}\right)^n}{b\,\left(n+1\right)}$$

input
```
int(tanh(a + b*x)^n/cosh(a + b*x)^2,x)
```

output
```
(tanh(a + b*x)*((exp(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) + 1))^n)/(b*(n +
1))
```

Reduce [F]

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx)\,dx=\int \tanh\left(bx+a\right)^n\operatorname{sech}(bx+a)^2\,dx$$

input
```
int(sech(b*x+a)^2*tanh(b*x+a)^n,x)
```

output
```
int(tanh(a + b*x)**n*sech(a + b*x)**2,x)
```

3.66 $\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx=-\frac{\operatorname{sech}(a+bx)}{b}+\frac{\operatorname{sech}^3(a+bx)}{3b}$$

output
```
-sech(b*x+a)/b+1/3*sech(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx=-\frac{\operatorname{sech}(a+bx)}{b}+\frac{\operatorname{sech}^3(a+bx)}{3b}$$

input
```
Integrate[Sech[a + b*x]*Tanh[a + b*x]^3,x]
```

output
```
-(Sech[a + b*x]/b) + Sech[a + b*x]^3/(3*b)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 26, 3086, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan(ia+ibx)^3\sec(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sec(ia+ibx)\tan(ia+ibx)^3dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\int \left(\text{sech}^2(a+bx)-1\right)d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}\text{sech}^3(a+bx)-\text{sech}(a+bx)}{b}$$

input
```
Int[Sech[a + b*x]*Tanh[a + b*x]^3,x]
```

output
```
(-Sech[a + b*x] + Sech[a + b*x]^3/3)/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{\operatorname{sech}(bx+a)^3}{3}-\operatorname{sech}(bx+a)}{b}$	24
default	$\frac{\frac{\operatorname{sech}(bx+a)^3}{3}-\operatorname{sech}(bx+a)}{b}$	24
risch	$-\frac{2\,\mathrm{e}^{bx+a}\left(3\,\mathrm{e}^{4bx+4a}+2\,\mathrm{e}^{2bx+2a}+3\right)}{3b\left(\mathrm{e}^{2bx+2a}+1\right)^3}$	49

input
```
int(sech(b*x+a)*tanh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3*sech(b*x+a)^3-sech(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(25) = 50$.

Time = 0.07 (sec) , antiderivative size = 172, normalized size of antiderivative = 6.37

$$\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx = -\frac{2\left(3\cosh\left(bx+a\right)^3+9\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+3\sinh\left(bx+a\right)\right.}{3\left(b\cosh\left(bx+a\right)^4+4\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+b\sinh\left(bx+a\right)^4+4\,b\cosh\left(bx+a\right)^2+2\left(3\,b\cos\right.\right.}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-2/3*(3*cosh(b*x + a)^3 + 9*cosh(b*x + a)*sinh(b*x + a)^2 + 3*sinh(b*x + a
)^3 + (9*cosh(b*x + a)^2 - 1)*sinh(b*x + a) + 5*cosh(b*x + a))/(b*cosh(b*x
 + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 4*b*cosh
(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a)^2 + 4*(b*cosh(b*
x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + 3*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(19) = 38$.

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx = \begin{cases} -\frac{\tanh^2\left(a+bx\right)\operatorname{sech}\left(a+bx\right)}{3b} - \frac{2\operatorname{sech}\left(a+bx\right)}{3b} & \text{for } b \neq 0 \\ x\tanh^3\left(a\right)\operatorname{sech}\left(a\right) & \text{otherwise} \end{cases}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)**3,x)
```

output
```
Piecewise((-tanh(a + b*x)**2*sech(a + b*x)/(3*b) - 2*sech(a + b*x)/(3*b),
Ne(b, 0)), (x*tanh(a)**3*sech(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(25) = 50$.

Time = 0.04 (sec) , antiderivative size = 148, normalized size of antiderivative = 5.48

$$\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx = -\frac{2\,e^{(-bx-a)}}{b(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1)} - \frac{4\,e^{(-3\,bx-3\,a)}}{3\,b(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1)} - \frac{2\,e^{(-5\,bx-5\,a)}}{b(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1)}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-2*e^(-b*x - a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x -
6*a) + 1)) - 4/3*e^(-3*b*x - 3*a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4
*a) + e^(-6*b*x - 6*a) + 1)) - 2*e^(-5*b*x - 5*a)/(b*(3*e^(-2*b*x - 2*a) +
 3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a) + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx = -\frac{2\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4\right)}{3\,b\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)^3,x, algorithm="giac")
```

output
```
-2/3*(3*(e^(b*x + a) + e^(-b*x - a))^2 - 4)/(b*(e^(b*x + a) + e^(-b*x - a)
)^3)
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx = -\frac{2\,e^{a+b\,x}\,\left(2\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+3\right)}{3\,b\,{\left(e^{2\,a+2\,b\,x}+1\right)}^3}$$

input
```
int(tanh(a + b*x)^3/cosh(a + b*x),x)
```

output
```
-(2*exp(a + b*x)*(2*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + 3))/(3*b*(exp(
2*a + 2*b*x) + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \operatorname{sech}(a+bx)\tanh^3(a+bx)\,dx = \frac{\operatorname{sech}(bx+a)\left(-\tanh\left(bx+a\right)^2-2\right)}{3b}$$

input
```
int(sech(b*x+a)*tanh(b*x+a)^3,x)
```

output
```
(sech(a + b*x)*( - tanh(a + b*x)**2 - 2))/(3*b)
```

3.67 $\int \mathrm{sech}^3(a+bx)\tanh^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \mathrm{sech}^3(a+bx)\tanh^3(a+bx)\,dx=-\frac{\mathrm{sech}^3(a+bx)}{3b}+\frac{\mathrm{sech}^5(a+bx)}{5b}$$

output
```
-1/3*sech(b*x+a)^3/b+1/5*sech(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \mathrm{sech}^3(a+bx)\tanh^3(a+bx)\,dx=-\frac{\mathrm{sech}^3(a+bx)}{3b}+\frac{\mathrm{sech}^5(a+bx)}{5b}$$

input
```
Integrate[Sech[a + b*x]^3*Tanh[a + b*x]^3,x]
```

output
```
-1/3*Sech[a + b*x]^3/b + Sech[a + b*x]^5/(5*b)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 26, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(a+bx)\text{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int i\tan(ia+ibx)^3\sec(ia+ibx)^3dx$$

$$\Big\downarrow \; 26$$

$$i\int \sec(ia+ibx)^3\tan(ia+ibx)^3dx$$

$$\Big\downarrow \; 3086$$

$$\frac{\int -\text{sech}^2(a+bx)\left(1-\text{sech}^2(a+bx)\right)d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \text{sech}^2(a+bx)\left(1-\text{sech}^2(a+bx)\right)d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \; 244$$

$$-\frac{\int \left(\text{sech}^2(a+bx)-\text{sech}^4(a+bx)\right)d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{5}\text{sech}^5(a+bx)-\frac{1}{3}\text{sech}^3(a+bx)}{b}$$

input
```
Int[Sech[a + b*x]^3*Tanh[a + b*x]^3,x]
```

output
```
(-1/3*Sech[a + b*x]^3 + Sech[a + b*x]^5/5)/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\operatorname{sech}(bx+a)^5}{5}-\frac{\operatorname{sech}(bx+a)^3}{3}}{b}$	26
default	$\frac{\frac{\operatorname{sech}(bx+a)^5}{5}-\frac{\operatorname{sech}(bx+a)^3}{3}}{b}$	26
risch	$-\frac{8\,e^{3bx+3a}\left(5\,e^{4bx+4a}-2\,e^{2bx+2a}+5\right)}{15b\left(e^{2bx+2a}+1\right)^5}$	52

input
```
int(sech(b*x+a)^3*tanh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/5*sech(b*x+a)^5-1/3*sech(b*x+a)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(27) = 54$.

Time = 0.09 (sec) , antiderivative size = 345, normalized size of antiderivative = 11.13

$$\int \operatorname{sech}^3(a+bx)\tanh^3(a+bx)\,dx =$$

$$-\frac{}{15\left(b\cosh\left(bx+a\right)^7+7\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^6+b\sinh\left(bx+a\right)^7+5\,b\cosh\left(bx+a\right)^5+(21\,b\,\mathrm{co}\right.}$$

input
```
integrate(sech(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-8/15*(5*cosh(b*x + a)^4 + 20*cosh(b*x + a)*sinh(b*x + a)^3 + 5*sinh(b*x +
 a)^4 + 2*(15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4
*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 5)/(b*cosh(b*x + a)^7
 + 7*b*cosh(b*x + a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 + 5*b*cosh(b*x +
a)^5 + (21*b*cosh(b*x + a)^2 + 5*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)
^3 + 5*b*cosh(b*x + a))*sinh(b*x + a)^4 + 11*b*cosh(b*x + a)^3 + (35*b*cos
h(b*x + a)^4 + 50*b*cosh(b*x + a)^2 + 9*b)*sinh(b*x + a)^3 + (21*b*cosh(b*
x + a)^5 + 50*b*cosh(b*x + a)^3 + 33*b*cosh(b*x + a))*sinh(b*x + a)^2 + 15
*b*cosh(b*x + a) + (7*b*cosh(b*x + a)^6 + 25*b*cosh(b*x + a)^4 + 27*b*cosh
(b*x + a)^2 + 5*b)*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.

Time = 0.40 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48

$$\int \operatorname{sech}^3(a+bx)\tanh^3(a+bx)\,dx = \begin{cases} -\frac{\tanh^2(a+bx)\operatorname{sech}^3(a+bx)}{5b} - \frac{2\operatorname{sech}^3(a+bx)}{15b} & \text{for } b \neq 0 \\ x\tanh^3(a)\operatorname{sech}^3(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sech(b*x+a)**3*tanh(b*x+a)**3,x)
```

output
```
Piecewise((-tanh(a + b*x)**2*sech(a + b*x)**3/(5*b) - 2*sech(a + b*x)**3/(
15*b), Ne(b, 0)), (x*tanh(a)**3*sech(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(27) = 54$.

Time = 0.04 (sec) , antiderivative size = 214, normalized size of antiderivative = 6.90

$$\int \operatorname{sech}^3(a+bx)\tanh^3(a+bx)\,dx$$
$$= -\frac{8\,e^{(-3\,bx-3\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)}$$
$$+\frac{16\,e^{(-5\,bx-5\,a)}}{15\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)}$$
$$-\frac{8\,e^{(-7\,bx-7\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)}$$

input
```
integrate(sech(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-8/3*e^(-3*b*x - 3*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^
(-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) + 16/15*e^(
-5*b*x - 5*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x
- 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) - 8/3*e^(-7*b*x - 7
*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) + 5
*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

$$\int \operatorname{sech}^3(a+bx)\tanh^3(a+bx)\,dx = -\frac{8\left(5\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-12\right)}{15\,b(e^{(bx+a)}+e^{(-bx-a)})^5}$$

input
```
integrate(sech(b*x+a)^3*tanh(b*x+a)^3,x, algorithm="giac")
```

output
```
-8/15*(5*(e^(b*x + a) + e^(-b*x - a))^2 - 12)/(b*(e^(b*x + a) + e^(-b*x -
a))^5)
```

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 251, normalized size of antiderivative = 8.10

$$\int \operatorname{sech}^3(a+bx)\tanh^3(a+bx)\,dx = \frac{\frac{4\,e^{a+b\,x}}{5\,b}-\frac{12\,e^{3\,a+3\,b\,x}}{5\,b}+\frac{12\,e^{5\,a+5\,b\,x}}{5\,b}-\frac{4\,e^{7\,a+7\,b\,x}}{5\,b}}{5\,e^{2\,a+2\,b\,x}+10\,e^{4\,a+4\,b\,x}+10\,e^{6\,a+6\,b\,x}+5\,e^{8\,a+8\,b\,x}+e^{10\,a+10\,b\,x}+1} - \frac{28\,e^{a+b\,x}}{15\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} + \frac{64\,e^{a+b\,x}}{15\,b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)} - \frac{16\,e^{a+b\,x}}{5\,b\,\left(4\,e^{2\,a+2\,b\,x}+6\,e^{4\,a+4\,b\,x}+4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)}$$

input
```
int(tanh(a + b*x)^3/cosh(a + b*x)^3,x)
```

output
```
((4*exp(a + b*x))/(5*b) - (12*exp(3*a + 3*b*x))/(5*b) + (12*exp(5*a + 5*b*
x))/(5*b) - (4*exp(7*a + 7*b*x))/(5*b))/(5*exp(2*a + 2*b*x) + 10*exp(4*a +
 4*b*x) + 10*exp(6*a + 6*b*x) + 5*exp(8*a + 8*b*x) + exp(10*a + 10*b*x) +
1) - (28*exp(a + b*x))/(15*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1))
+ (64*exp(a + b*x))/(15*b*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6
*a + 6*b*x) + 1)) - (16*exp(a + b*x))/(5*b*(4*exp(2*a + 2*b*x) + 6*exp(4*a
 + 4*b*x) + 4*exp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \operatorname{sech}^3(a+bx)\tanh^3(a+bx)\,dx = \frac{\operatorname{sech}(bx+a)^3\left(-3\tanh(bx+a)^2-2\right)}{15b}$$

input
```
int(sech(b*x+a)^3*tanh(b*x+a)^3,x)
```

output
```
(sech(a + b*x)**3*( - 3*tanh(a + b*x)**2 - 2))/(15*b)
```

3.68 $\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 36

$$\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx=-\frac{\operatorname{sech}^n(a+bx)}{bn}+\frac{\operatorname{sech}^{2+n}(a+bx)}{b(2+n)}$$

output
```
-sech(b*x+a)^n/b/n+sech(b*x+a)^(2+n)/b/(2+n)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx=\frac{\operatorname{sech}^n(a+bx)\left(-\frac{1}{n}+\frac{\operatorname{sech}^2(a+bx)}{2+n}\right)}{b}$$

input
```
Integrate[Sech[a + b*x]^(3 + n)*Sinh[a + b*x]^3,x]
```

output
```
(Sech[a + b*x]^n*(-n^(-1) + Sech[a + b*x]^2/(2 + n)))/b
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 26, 3102, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(a+bx)\text{sech}^{n+3}(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{i\sec(ia+ibx)^{n+3}}{\csc(ia+ibx)^3}dx$$

$$\Big\downarrow \ 26$$

$$i\int \frac{\sec(ia+ibx)^{n+3}}{\csc(ia+ibx)^3}dx$$

$$\Big\downarrow \ 3102$$

$$\frac{\int -\text{sech}^{n-1}(a+bx)\left(1-\text{sech}^2(a+bx)\right)d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \text{sech}^{n-1}(a+bx)\left(1-\text{sech}^2(a+bx)\right)d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \ 244$$

$$-\frac{\int \left(\text{sech}^{n-1}(a+bx)-\text{sech}^{n+1}(a+bx)\right)d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\text{sech}^{n+2}(a+bx)}{n+2}-\frac{\text{sech}^{n}(a+bx)}{n}}{b}$$

input
```
Int[Sech[a + b*x]^(3 + n)*Sinh[a + b*x]^3,x]
```

output
```
(-(Sech[a + b*x]^n/n) + Sech[a + b*x]^(2 + n)/(2 + n))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.40 (sec) , antiderivative size = 1385, normalized size of antiderivative = 38.47

Expression too large to display

input
```
int(sech(b*x+a)^(3+n)*sinh(b*x+a)^3,x)
```

output
```
2^n*(exp(2*b*x+2*a)+1)^(-n)*exp(b*x+a)^n/(exp(2*b*x+2*a)+1)^3*(n-6)/n/b/(2
+n)*exp(2*b*x+2*a)*exp(-1/2*I*csgn(I/(exp(2*b*x+2*a)+1))*Pi*csgn(I*exp(b*x
+a)/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))*n)*exp(1/2*I*csgn(I/(exp(2*b*x+
2*a)+1))*Pi*csgn(I*exp(b*x+a)/(exp(2*b*x+2*a)+1))^2*n)*exp(1/2*I*Pi*csgn(I
*exp(b*x+a)/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(b*x+a))*n)*exp(-1/2*I*Pi*csgn
(I*exp(b*x+a)/(exp(2*b*x+2*a)+1))^3*n)*exp(-3/2*I*Pi*csgn(I/(exp(2*b*x+2*a
)+1))*csgn(I*exp(b*x+a))*csgn(I*exp(b*x+a)/(exp(2*b*x+2*a)+1)))*exp(3/2*I*
Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a)/(exp(2*b*x+2*a)+1))^2)*exp
(3/2*I*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(b*x+a)/(exp(2*b*x+2*a)+1))^2)*exp(
-3/2*I*Pi*csgn(I*exp(b*x+a)/(exp(2*b*x+2*a)+1))^3)+2^n*(exp(2*b*x+2*a)+1)^
(-n)*exp(b*x+a)^n/(exp(2*b*x+2*a)+1)^3*(n-6)/n/b/(2+n)*exp(4*b*x+4*a)*exp(
-1/2*I*csgn(I/(exp(2*b*x+2*a)+1))*Pi*csgn(I*exp(b*x+a)/(exp(2*b*x+2*a)+1))
*csgn(I*exp(b*x+a))*n)*exp(1/2*I*csgn(I/(exp(2*b*x+2*a)+1))*Pi*csgn(I*exp(
b*x+a)/(exp(2*b*x+2*a)+1))^2*n)*exp(1/2*I*Pi*csgn(I*exp(b*x+a)/(exp(2*b*x+
2*a)+1))^2*csgn(I*exp(b*x+a))*n)*exp(-1/2*I*Pi*csgn(I*exp(b*x+a)/(exp(2*b*
x+2*a)+1))^3*n)*exp(-3/2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a)
)*csgn(I*exp(b*x+a)/(exp(2*b*x+2*a)+1)))*exp(3/2*I*Pi*csgn(I/(exp(2*b*x+2*
a)+1))*csgn(I*exp(b*x+a)/(exp(2*b*x+2*a)+1))^2)*exp(3/2*I*Pi*csgn(I*exp(b*
x+a))*csgn(I*exp(b*x+a)/(exp(2*b*x+2*a)+1))^2)*exp(-3/2*I*Pi*csgn(I*exp(b*
x+a)/(exp(2*b*x+2*a)+1))^3)-1/b/n/(exp(2*b*x+2*a)+1)^3*exp(b*x+a)^n*(ex...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(36) = 72$.

Time = 0.11 (sec) , antiderivative size = 270, normalized size of antiderivative = 7.50

$$\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx =$$

$$-\frac{\left((n+2)\cosh(bx+a)^3+3(n+2)\cosh(bx+a)\sinh(bx+a)^2-(n-6)\cosh(bx+a)\right)\cosh\left((n+3\right.}{}$$

input
```
integrate(sech(b*x+a)^(3+n)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(((n + 2)*cosh(b*x + a)^3 + 3*(n + 2)*cosh(b*x + a)*sinh(b*x + a)^2 -
 (n - 6)*cosh(b*x + a))*cosh((n + 3)*log(2*(cosh(b*x + a) + sinh(b*x + a))
/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)))
 + ((n + 2)*cosh(b*x + a)^3 + 3*(n + 2)*cosh(b*x + a)*sinh(b*x + a)^2 - (n
 - 6)*cosh(b*x + a))*sinh((n + 3)*log(2*(cosh(b*x + a) + sinh(b*x + a))/(c
osh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1))))/(
(b*n^2 + 2*b*n)*cosh(b*x + a)^4 - 2*(b*n^2 + 2*b*n)*cosh(b*x + a)^2*sinh(b
*x + a)^2 + (b*n^2 + 2*b*n)*sinh(b*x + a)^4)
```

Sympy [F]

$$\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx = \int \sinh^3{(a+bx)}\operatorname{sech}^{n+3}{(a+bx)}\,dx$$

input
```
integrate(sech(b*x+a)**(3+n)*sinh(b*x+a)**3,x)
```

output
```
Integral(sinh(a + b*x)**3*sech(a + b*x)**(n + 3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(36) = 72$.

Time = 0.20 (sec) , antiderivative size = 350, normalized size of antiderivative = 9.72

$$\begin{aligned}
&\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx \\
&= -\frac{2^{n+3}ne^{\left(-(bx+a)n-n\log\left(e^{(-2\,bx-2\,a)}+1\right)\right)}}{8\,(n^2+2\,(n^2+2\,n)e^{(-2\,bx-2\,a)}+(n^2+2\,n)e^{(-4\,bx-4\,a)}+2\,n)b} \\
&\quad+\frac{(2^{n+4}n-2^{n+5})e^{\left(-(bx+a)n-2\,bx-n\log\left(e^{(-2\,bx-2\,a)}+1\right)-2\,a\right)}}{8\,(n^2+2\,(n^2+2\,n)e^{(-2\,bx-2\,a)}+(n^2+2\,n)e^{(-4\,bx-4\,a)}+2\,n)b} \\
&\quad-\frac{(2^{n+3}n+2^{n+4})e^{\left(-(bx+a)n-4\,bx-n\log\left(e^{(-2\,bx-2\,a)}+1\right)-4\,a\right)}}{8\,(n^2+2\,(n^2+2\,n)e^{(-2\,bx-2\,a)}+(n^2+2\,n)e^{(-4\,bx-4\,a)}+2\,n)b} \\
&\quad-\frac{2^{n+4}e^{\left(-(bx+a)n-n\log\left(e^{(-2\,bx-2\,a)}+1\right)\right)}}{8\,(n^2+2\,(n^2+2\,n)e^{(-2\,bx-2\,a)}+(n^2+2\,n)e^{(-4\,bx-4\,a)}+2\,n)b}
\end{aligned}$$

input
```
integrate(sech(b*x+a)^(3+n)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/8*2^(n + 3)*n*e^(-(b*x + a)*n - n*log(e^(-2*b*x - 2*a) + 1))/((n^2 + 2*
(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2*n)*b) + 1/
8*(2^(n + 4)*n - 2^(n + 5))*e^(-(b*x + a)*n - 2*b*x - n*log(e^(-2*b*x - 2*
a) + 1) - 2*a)/((n^2 + 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*
b*x - 4*a) + 2*n)*b) - 1/8*(2^(n + 3)*n + 2^(n + 4))*e^(-(b*x + a)*n - 4*b
*x - n*log(e^(-2*b*x - 2*a) + 1) - 4*a)/((n^2 + 2*(n^2 + 2*n)*e^(-2*b*x -
2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2*n)*b) - 1/8*2^(n + 4)*e^(-(b*x + a
)*n - n*log(e^(-2*b*x - 2*a) + 1))/((n^2 + 2*(n^2 + 2*n)*e^(-2*b*x - 2*a)
+ (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2*n)*b)
```

Giac [F]

$$\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx=\int \operatorname{sech}\left(bx+a\right)^{n+3}\sinh\left(bx+a\right)^3\,dx$$

input
```
integrate(sech(b*x+a)^(3+n)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^(n + 3)*sinh(b*x + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.64

$$\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx = -\mathrm{e}^{-3\,a-3\,b\,x}\,\left(\frac{1}{\frac{\mathrm{e}^{a+b\,x}}{2}+\frac{\mathrm{e}^{-a-b\,x}}{2}}\right)^{n+3}\,\left(\frac{\mathrm{e}^{3\,a+3\,b\,x}\,\cosh(3\,a+3\,b\,x)}{4\,b\,n}-\frac{\cosh(a+b\,x)\,\mathrm{e}^{3\,a+3\,b\,x}\,(n-6)}{4\,b\,n\,(n+2)}\right)$$

input
```
int(sinh(a + b*x)^3*(1/cosh(a + b*x))^(n + 3),x)
```

output
```
-exp(- 3*a - 3*b*x)*(1/(exp(a + b*x)/2 + exp(- a - b*x)/2))^(n + 3)*((exp(
3*a + 3*b*x)*cosh(3*a + 3*b*x))/(4*b*n) - (cosh(a + b*x)*exp(3*a + 3*b*x)*
(n - 6))/(4*b*n*(n + 2)))
```

Reduce [F]

$$\int \operatorname{sech}^{3+n}(a+bx)\sinh^3(a+bx)\,dx=\int \operatorname{sech}(bx+a)^n\operatorname{sech}(bx+a)^3\sinh{(bx+a)}^3\,dx$$

input
```
int(sech(b*x+a)^(3+n)*sinh(b*x+a)^3,x)
```

output
```
int(sech(a + b*x)**n*sech(a + b*x)**3*sinh(a + b*x)**3,x)
```

3.69 $\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx=\frac{\tanh^3(a+bx)}{3b}-\frac{\tanh^5(a+bx)}{5b}$$

output
```
1/3*tanh(b*x+a)^3/b-1/5*tanh(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx=\frac{2\tanh(a+bx)}{15b}+\frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{15b}-\frac{\operatorname{sech}^4(a+bx)\tanh(a+bx)}{5b}$$

input
```
Integrate[Sech[a + b*x]^4*Tanh[a + b*x]^2,x]
```

output
```
(2*Tanh[a + b*x])/(15*b) + (Sech[a + b*x]^2*Tanh[a + b*x])/(15*b) - (Sech[
a + b*x]^4*Tanh[a + b*x])/(5*b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 25, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(a+bx)\text{sech}^4(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \tan(ia+ibx)^2\left(-\sec(ia+ibx)^4\right)dx$$
$$\Big\downarrow \text{25}$$
$$-\int \sec(ia+ibx)^4\tan(ia+ibx)^2dx$$
$$\Big\downarrow \text{3087}$$
$$\frac{i\int -\tanh^2(a+bx)\left(1-\tanh^2(a+bx)\right)d(i\tanh(a+bx))}{b}$$
$$\Big\downarrow \text{244}$$
$$\frac{i\int \left(\tanh^4(a+bx)-\tanh^2(a+bx)\right)d(i\tanh(a+bx))}{b}$$
$$\Big\downarrow \text{2009}$$
$$\frac{i\left(\frac{1}{5}i\tanh^5(a+bx)-\frac{1}{3}i\tanh^3(a+bx)\right)}{b}$$

input
```
Int[Sech[a + b*x]^4*Tanh[a + b*x]^2,x]
```

output
```
(I*((-1/3*I)*Tanh[a + b*x]^3 + (I/5)*Tanh[a + b*x]^5))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 5.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{-\frac{\tanh(bx+a)^5}{5}+\frac{\tanh(bx+a)^3}{3}}{b}$	26
default	$\frac{-\frac{\tanh(bx+a)^5}{5}+\frac{\tanh(bx+a)^3}{3}}{b}$	26
risch	$-\frac{4\left(15\,\mathrm{e}^{6bx+6a}-5\,\mathrm{e}^{4bx+4a}+5\,\mathrm{e}^{2bx+2a}+1\right)}{15b\left(\mathrm{e}^{2bx+2a}+1\right)^5}$	54

input
```
int(sech(b*x+a)^4*tanh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/5*tanh(b*x+a)^5+1/3*tanh(b*x+a)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(27) = 54$.

Time = 0.08 (sec) , antiderivative size = 304, normalized size of antiderivative = 9.81

$$\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx =$$

$$-\frac{}{15\left(b\cosh\left(bx+a\right)^7+7\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^6+b\sinh\left(bx+a\right)^7+5\,b\cosh\left(bx+a\right)^5+\left(21\,b\,\mathrm{co}\right.\right.}$$

input
```
integrate(sech(b*x+a)^4*tanh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-8/15*(8*cosh(b*x + a)^3 + 24*cosh(b*x + a)*sinh(b*x + a)^2 + 7*sinh(b*x +
 a)^3 + (21*cosh(b*x + a)^2 - 5)*sinh(b*x + a))/(b*cosh(b*x + a)^7 + 7*b*c
osh(b*x + a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 + 5*b*cosh(b*x + a)^5 + (
21*b*cosh(b*x + a)^2 + 5*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 + 5*b
*cosh(b*x + a))*sinh(b*x + a)^4 + 11*b*cosh(b*x + a)^3 + (35*b*cosh(b*x +
a)^4 + 50*b*cosh(b*x + a)^2 + 9*b)*sinh(b*x + a)^3 + (21*b*cosh(b*x + a)^5
 + 50*b*cosh(b*x + a)^3 + 33*b*cosh(b*x + a))*sinh(b*x + a)^2 + 15*b*cosh(
b*x + a) + (7*b*cosh(b*x + a)^6 + 25*b*cosh(b*x + a)^4 + 27*b*cosh(b*x + a
)^2 + 5*b)*sinh(b*x + a))
```

Sympy [F]

$$\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx = \int \tanh^2\left(a+bx\right)\operatorname{sech}^4\left(a+bx\right)dx$$

input
```
integrate(sech(b*x+a)**4*tanh(b*x+a)**2,x)
```

output
```
Integral(tanh(a + b*x)**2*sech(a + b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(27) = 54$.

Time = 0.04 (sec) , antiderivative size = 276, normalized size of antiderivative = 8.90

$$\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx$$
$$= \frac{4\,e^{(-2\,bx-2\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)}$$
$$-\frac{4\,e^{(-4\,bx-4\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)}$$
$$+\frac{4\,e^{(-6\,bx-6\,a)}}{b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)}$$
$$+\frac{4}{15\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)}$$

input
```
integrate(sech(b*x+a)^4*tanh(b*x+a)^2,x, algorithm="maxima")
```

output
```
4/3*e^(-2*b*x - 2*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(
-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) - 4/3*e^(-4*
b*x - 4*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6
*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) + 4*e^(-6*b*x - 6*a)/(
b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) + 5*e^(-
8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) + 4/15/(b*(5*e^(-2*b*x - 2*a) + 10
*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x
- 10*a) + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx = -\frac{4\,(15\,e^{(6\,bx+6\,a)}-5\,e^{(4\,bx+4\,a)}+5\,e^{(2\,bx+2\,a)}+1)}{15\,b(e^{(2\,bx+2\,a)}+1)^5}$$

input
```
integrate(sech(b*x+a)^4*tanh(b*x+a)^2,x, algorithm="giac")
```

output
```
-4/15*(15*e^(6*b*x + 6*a) - 5*e^(4*b*x + 4*a) + 5*e^(2*b*x + 2*a) + 1)/(b*
(e^(2*b*x + 2*a) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 270, normalized size of antiderivative = 8.71

$$\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx$$
$$= \frac{\frac{8}{15\,b}-\frac{4\,\mathrm{e}^{2\,a+2\,b\,x}}{5\,b}}{3\,\mathrm{e}^{2\,a+2\,b\,x}+3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}+1} - \frac{\frac{2}{5\,b}-\frac{8\,\mathrm{e}^{2\,a+2\,b\,x}}{5\,b}+\frac{6\,\mathrm{e}^{4\,a+4\,b\,x}}{5\,b}}{4\,\mathrm{e}^{2\,a+2\,b\,x}+6\,\mathrm{e}^{4\,a+4\,b\,x}+4\,\mathrm{e}^{6\,a+6\,b\,x}+\mathrm{e}^{8\,a+8\,b\,x}+1} - \frac{\frac{8\,\mathrm{e}^{2\,a+2\,b\,x}}{5\,b}-\frac{16\,\mathrm{e}^{4\,a+4\,b\,x}}{5\,b}+\frac{8\,\mathrm{e}^{6\,a+6\,b\,x}}{5\,b}}{5\,\mathrm{e}^{2\,a+2\,b\,x}+10\,\mathrm{e}^{4\,a+4\,b\,x}+10\,\mathrm{e}^{6\,a+6\,b\,x}+5\,\mathrm{e}^{8\,a+8\,b\,x}+\mathrm{e}^{10\,a+10\,b\,x}+1} - \frac{2}{5\,b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)}$$

input
```
int(tanh(a + b*x)^2/cosh(a + b*x)^4,x)
```

output
```
(8/(15*b) - (4*exp(2*a + 2*b*x))/(5*b))/(3*exp(2*a + 2*b*x) + 3*exp(4*a +
4*b*x) + exp(6*a + 6*b*x) + 1) - (2/(5*b) - (8*exp(2*a + 2*b*x))/(5*b) + (
6*exp(4*a + 4*b*x))/(5*b))/(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*ex
p(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1) - ((8*exp(2*a + 2*b*x))/(5*b) - (16
*exp(4*a + 4*b*x))/(5*b) + (8*exp(6*a + 6*b*x))/(5*b))/(5*exp(2*a + 2*b*x)
 + 10*exp(4*a + 4*b*x) + 10*exp(6*a + 6*b*x) + 5*exp(8*a + 8*b*x) + exp(10
*a + 10*b*x) + 1) - 2/(5*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.39

$$\int \operatorname{sech}^4(a+bx)\tanh^2(a+bx)\,dx = \frac{-4e^{6bx+6a}+\frac{4e^{4bx+4a}}{3}-\frac{4e^{2bx+2a}}{3}-\frac{4}{15}}{b\left(e^{10bx+10a}+5e^{8bx+8a}+10e^{6bx+6a}+10e^{4bx+4a}+5e^{2bx+2a}+1\right)}$$

input
```
int(sech(b*x+a)^4*tanh(b*x+a)^2,x)
```

output
```
(4*( - 15*e**(6*a + 6*b*x) + 5*e**(4*a + 4*b*x) - 5*e**(2*a + 2*b*x) - 1))
/(15*b*(e**(10*a + 10*b*x) + 5*e**(8*a + 8*b*x) + 10*e**(6*a + 6*b*x) + 10
*e**(4*a + 4*b*x) + 5*e**(2*a + 2*b*x) + 1))
```

3.70 $\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 35

$$\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx=\frac{2\tanh^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\tanh^{\frac{7}{2}}(a+bx)}{7b}$$

output
```
2/3*tanh(b*x+a)^(3/2)/b-2/7*tanh(b*x+a)^(7/2)/b
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx=\frac{2(4+3\operatorname{sech}^2(a+bx))\tanh^{\frac{3}{2}}(a+bx)}{21b}$$

input
```
Integrate[Sech[a + b*x]^4*Sqrt[Tanh[a + b*x]],x]
```

output
```
(2*(4 + 3*Sech[a + b*x]^2)*Tanh[a + b*x]^(3/2))/(21*b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tanh(a+bx)}\text{sech}^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-i\tan(ia+ibx)}\sec(ia+ibx)^4dx$$

$$\Big\downarrow \text{3087}$$

$$-\frac{i\int \sqrt{\tanh(a+bx)}\left(1-\tanh^2(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{i\int \left(\sqrt{\tanh(a+bx)}-\tanh^{\frac{5}{2}}(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{2}{3}i\tanh^{\frac{3}{2}}(a+bx)-\frac{2}{7}i\tanh^{\frac{7}{2}}(a+bx)\right)}{b}$$

input
```
Int[Sech[a + b*x]^4*Sqrt[Tanh[a + b*x]],x]
```

output
```
((-I)*(((2*I)/3)*Tanh[a + b*x]^(3/2) - ((2*I)/7)*Tanh[a + b*x]^(7/2)))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{-\frac{2\tanh(bx+a)^{\frac{7}{2}}}{7}+\frac{2\tanh(bx+a)^{\frac{3}{2}}}{3}}{b}$	26
default	$\frac{-\frac{2\tanh(bx+a)^{\frac{7}{2}}}{7}+\frac{2\tanh(bx+a)^{\frac{3}{2}}}{3}}{b}$	26

input
```
int(sech(b*x+a)^4*tanh(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-2/7*tanh(b*x+a)^(7/2)+2/3*tanh(b*x+a)^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(27) = 54$.

Time = 0.10 (sec) , antiderivative size = 551, normalized size of antiderivative = 15.74

$$\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(b*x+a)^4*tanh(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
8/21*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6
+ 3*(5*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cos
h(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6
*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x +
a)^5 + 2*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + (cosh(b*x + a)^6
 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2
 + 4)*sinh(b*x + a)^4 + 4*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 4*cosh(
b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 + 24*cosh(b*x + a)^2 - 4)*
sinh(b*x + a)^2 - 4*cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 + 8*cosh(b*x +
a)^3 - 4*cosh(b*x + a))*sinh(b*x + a) - 1)*sqrt(sinh(b*x + a)/cosh(b*x + a
)) + 1)/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*
x + a)^6 + 3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)
^4 + 4*(5*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*cos
h(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 + 6*b*cosh(b*x + a)^2 + b)*sinh(b*x
+ a)^2 + 6*(b*cosh(b*x + a)^5 + 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sin
h(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx = \int \sqrt{\tanh(a+bx)}\operatorname{sech}^4(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)**4*tanh(b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(tanh(a + b*x))*sech(a + b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(27) = 54$.

Time = 0.15 (sec) , antiderivative size = 352, normalized size of antiderivative = 10.06

$$\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx$$
$$= \frac{32\sqrt{e^{(-bx-a)}+1}\sqrt{-e^{(-bx-a)}+1}e^{(-2\,bx-2\,a)}}{21\,b(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1)\sqrt{e^{(-2\,bx-2\,a)}+1}}$$
$$- \frac{32\sqrt{e^{(-bx-a)}+1}\sqrt{-e^{(-bx-a)}+1}e^{(-4\,bx-4\,a)}}{21\,b(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1)\sqrt{e^{(-2\,bx-2\,a)}+1}}$$
$$- \frac{8\sqrt{e^{(-bx-a)}+1}\sqrt{-e^{(-bx-a)}+1}e^{(-6\,bx-6\,a)}}{21\,b(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1)\sqrt{e^{(-2\,bx-2\,a)}+1}}$$
$$+ \frac{8\sqrt{e^{(-bx-a)}+1}\sqrt{-e^{(-bx-a)}+1}}{21\,b(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1)\sqrt{e^{(-2\,bx-2\,a)}+1}}$$

input
```
integrate(sech(b*x+a)^4*tanh(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
32/21*sqrt(e^(-b*x - a) + 1)*sqrt(-e^(-b*x - a) + 1)*e^(-2*b*x - 2*a)/(b*(
3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a) + 1)*sqrt(e^(-2
*b*x - 2*a) + 1)) - 32/21*sqrt(e^(-b*x - a) + 1)*sqrt(-e^(-b*x - a) + 1)*e
^(-4*b*x - 4*a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x -
6*a) + 1)*sqrt(e^(-2*b*x - 2*a) + 1)) - 8/21*sqrt(e^(-b*x - a) + 1)*sqrt(-
e^(-b*x - a) + 1)*e^(-6*b*x - 6*a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x -
4*a) + e^(-6*b*x - 6*a) + 1)*sqrt(e^(-2*b*x - 2*a) + 1)) + 8/21*sqrt(e^(-b
*x - a) + 1)*sqrt(-e^(-b*x - a) + 1)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x
- 4*a) + e^(-6*b*x - 6*a) + 1)*sqrt(e^(-2*b*x - 2*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(27) = 54$.

Time = 0.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.23

$$\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx$$
$$= \frac{16\left(21\left(\sqrt{e^{(4\,bx+4\,a)}-1}-e^{(2\,bx+2\,a)}\right)^5-7\left(\sqrt{e^{(4\,bx+4\,a)}-1}-e^{(2\,bx+2\,a)}\right)^4+28\left(\sqrt{e^{(4\,bx+4\,a)}-1}-e^{(2\,bx+}\right.}{21\,b\left(\sqrt{e^{(4\,bx+4\,a)}-1}-e^{(2\,bx+2\,a)}-1\right)^7}$$

input
```
integrate(sech(b*x+a)^4*tanh(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
16/21*(21*(sqrt(e^(4*b*x + 4*a) - 1) - e^(2*b*x + 2*a))^5 - 7*(sqrt(e^(4*b
*x + 4*a) - 1) - e^(2*b*x + 2*a))^4 + 28*(sqrt(e^(4*b*x + 4*a) - 1) - e^(2
*b*x + 2*a))^3 + 7*sqrt(e^(4*b*x + 4*a) - 1) - 7*e^(2*b*x + 2*a) - 1)/(b*(
sqrt(e^(4*b*x + 4*a) - 1) - e^(2*b*x + 2*a) - 1)^7)
```

Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 168, normalized size of antiderivative = 4.80

$$\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx = \frac{8\sqrt{\frac{e^{2\,a+2\,b\,x}-1}{e^{2\,a+2\,b\,x}+1}}}{21\,b} + \frac{8\sqrt{\frac{e^{2\,a+2\,b\,x}-1}{e^{2\,a+2\,b\,x}+1}}}{21\,b\,\left(e^{2\,a+2\,b\,x}+1\right)} - \frac{24\sqrt{\frac{e^{2\,a+2\,b\,x}-1}{e^{2\,a+2\,b\,x}+1}}}{7\,b\,\left(e^{2\,a+2\,b\,x}+1\right)^2} + \frac{16\sqrt{\frac{e^{2\,a+2\,b\,x}-1}{e^{2\,a+2\,b\,x}+1}}}{7\,b\,\left(e^{2\,a+2\,b\,x}+1\right)^3}$$

input
```
int(tanh(a + b*x)^(1/2)/cosh(a + b*x)^4,x)
```

output
```
(8*((exp(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) + 1))^(1/2))/(21*b) + (8*((ex
p(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) + 1))^(1/2))/(21*b*(exp(2*a + 2*b*x)
 + 1)) - (24*((exp(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) + 1))^(1/2))/(7*b*(
exp(2*a + 2*b*x) + 1)^2) + (16*((exp(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) +
 1))^(1/2))/(7*b*(exp(2*a + 2*b*x) + 1)^3)
```

Reduce [F]

$$\int \operatorname{sech}^4(a+bx)\sqrt{\tanh(a+bx)}\,dx = \int \sqrt{\tanh(bx+a)}\,\operatorname{sech}(bx+a)^4\,dx$$

input
```
int(sech(b*x+a)^4*tanh(b*x+a)^(1/2),x)
```

output
```
int(sqrt(tanh(a + b*x))*sech(a + b*x)**4,x)
```

3.71 $\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx=\frac{\tanh^{1+n}(a+bx)}{b(1+n)}-\frac{\tanh^{3+n}(a+bx)}{b(3+n)}$$

output
```
tanh(b*x+a)^(1+n)/b/(1+n)-tanh(b*x+a)^(3+n)/b/(3+n)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.82

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx$$
$$=\frac{\tanh^{-1+n}(a+bx)\left((2+n+\cosh(2(a+bx)))\operatorname{sech}^2(a+bx)\tanh^2(a+bx)-2\tanh^2(a+bx)^{\frac{1-n}{2}}\right)}{b(1+n)(3+n)}$$

input
```
Integrate[Sech[a + b*x]^4*Tanh[a + b*x]^n,x]
```

output
```
(Tanh[a + b*x]^(-1 + n)*((2 + n + Cosh[2*(a + b*x)])*Sech[a + b*x]^2*Tanh[
a + b*x]^2 - 2*(Tanh[a + b*x]^2)^((1 - n)/2)))/(b*(1 + n)*(3 + n))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(ia+ibx)^4(-i\tan(ia+ibx))^n dx$$

$$\Big\downarrow \text{3087}$$

$$-\frac{i\int \tanh^n(a+bx)\left(1-\tanh^2(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{i\int \left(\tanh^n(a+bx)-\tanh^{n+2}(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{i\tanh^{n+1}(a+bx)}{n+1}-\frac{i\tanh^{n+3}(a+bx)}{n+3}\right)}{b}$$

input
```
Int[Sech[a + b*x]^4*Tanh[a + b*x]^n,x]
```

output
```
((-I)*((I*Tanh[a + b*x]^(1 + n))/(1 + n) - (I*Tanh[a + b*x]^(3 + n))/(3 +
n)))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 104.88 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{\tanh(bx+a)\mathrm{e}^{n\ln(\tanh(bx+a))}}{b(1+n)}-\frac{\tanh(bx+a)^{3}\mathrm{e}^{n\ln(\tanh(bx+a))}}{b(3+n)}$
default	$\frac{\tanh(bx+a)\mathrm{e}^{n\ln(\tanh(bx+a))}}{b(1+n)}-\frac{\tanh(bx+a)^{3}\mathrm{e}^{n\ln(\tanh(bx+a))}}{b(3+n)}$
risch	$2\left(\mathrm{e}^{6bx+6a}+2n\,\mathrm{e}^{4bx+4a}+3\,\mathrm{e}^{4bx+4a}-2n\,\mathrm{e}^{2bx+2a}-3\,\mathrm{e}^{2bx+2a}-1\right)\left(\mathrm{e}^{bx+a}-1\right)^{n}\left(\mathrm{e}^{bx+a}+1\right)^{n}\left(\mathrm{e}^{2bx+2a}+1\right)^{-n}\mathrm{e}^{-\frac{i\pi n\left(-\operatorname{csgn}\right.}{}}$

input
```
int(sech(b*x+a)^4*tanh(b*x+a)^n,x,method=_RETURNVERBOSE)
```

output
```
1/b/(1+n)*tanh(b*x+a)*exp(n*ln(tanh(b*x+a)))-1/b/(3+n)*tanh(b*x+a)^3*exp(n
*ln(tanh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(40) = 80$.

Time = 0.11 (sec) , antiderivative size = 180, normalized size of antiderivative = 4.50

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx = \frac{2\left(\left(\sinh\left(bx+a\right)^3+\left(3\cosh\left(bx+a\right)^2+2n+3\right)\sinh\left(bx+a\right)\right)\cosh\left(n\log\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right)\right)+\left(\sinh\left(bx+\right.\right.\right.}{\left(bn^2+4bn+3b\right)\cosh\left(bx+a\right)^3+3\left(bn^2+4bn+3b\right)\cosh\left(bx+a\right)\sinh}$$

input
```
integrate(sech(b*x+a)^4*tanh(b*x+a)^n,x, algorithm="fricas")
```

output
```
2*((sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 2*n + 3)*sinh(b*x + a))*cosh(n*
log(sinh(b*x + a)/cosh(b*x + a))) + (sinh(b*x + a)^3 + (3*cosh(b*x + a)^2
+ 2*n + 3)*sinh(b*x + a))*sinh(n*log(sinh(b*x + a)/cosh(b*x + a))))/((b*n^
2 + 4*b*n + 3*b)*cosh(b*x + a)^3 + 3*(b*n^2 + 4*b*n + 3*b)*cosh(b*x + a)*s
inh(b*x + a)^2 + 3*(b*n^2 + 4*b*n + 3*b)*cosh(b*x + a))
```

Sympy [F]

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx = \int \tanh^n(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)**4*tanh(b*x+a)**n,x)
```

output
```
Integral(tanh(a + b*x)**n*sech(a + b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(40) = 80$.

Time = 0.15 (sec) , antiderivative size = 504, normalized size of antiderivative = 12.60

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx$$
$$= \frac{2\,(2\,n+3)e^{\left(-2\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-n\log\left(e^{(-2\,bx-2\,a)}+1\right)-2\,a\right)}}{\left(n^2+3\,(n^2+4\,n+3)e^{(-2\,bx-2\,a)}+3\,(n^2+4\,n+3)e^{(-4\,bx-4\,a)}+(n^2+4\,n+3)e^{(-6\,bx-6\,a)}+4\,n+3\right)b}$$
$$-\frac{2\,(2\,n+3)e^{\left(-4\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-n\log\left(e^{(-2\,bx-2\,a)}+1\right)-4\,a\right)}}{\left(n^2+3\,(n^2+4\,n+3)e^{(-2\,bx-2\,a)}+3\,(n^2+4\,n+3)e^{(-4\,bx-4\,a)}+(n^2+4\,n+3)e^{(-6\,bx-6\,a)}+4\,n+3\right)b}$$
$$-\frac{2\,e^{\left(-6\,bx+n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-n\log\left(e^{(-2\,bx-2\,a)}+1\right)-6\,a\right)}}{\left(n^2+3\,(n^2+4\,n+3)e^{(-2\,bx-2\,a)}+3\,(n^2+4\,n+3)e^{(-4\,bx-4\,a)}+(n^2+4\,n+3)e^{(-6\,bx-6\,a)}+4\,n+3\right)b}$$
$$+\frac{2\,e^{\left(n\log\left(e^{(-bx-a)}+1\right)+n\log\left(-e^{(-bx-a)}+1\right)-n\log\left(e^{(-2\,bx-2\,a)}+1\right)\right)}}{\left(n^2+3\,(n^2+4\,n+3)e^{(-2\,bx-2\,a)}+3\,(n^2+4\,n+3)e^{(-4\,bx-4\,a)}+(n^2+4\,n+3)e^{(-6\,bx-6\,a)}+4\,n+3\right)b}$$

input
```
integrate(sech(b*x+a)^4*tanh(b*x+a)^n,x, algorithm="maxima")
```

output
```
2*(2*n + 3)*e^(-2*b*x + n*log(e^(-b*x - a) + 1) + n*log(-e^(-b*x - a) + 1)
 - n*log(e^(-2*b*x - 2*a) + 1) - 2*a)/((n^2 + 3*(n^2 + 4*n + 3)*e^(-2*b*x
- 2*a) + 3*(n^2 + 4*n + 3)*e^(-4*b*x - 4*a) + (n^2 + 4*n + 3)*e^(-6*b*x -
6*a) + 4*n + 3)*b) - 2*(2*n + 3)*e^(-4*b*x + n*log(e^(-b*x - a) + 1) + n*l
og(-e^(-b*x - a) + 1) - n*log(e^(-2*b*x - 2*a) + 1) - 4*a)/((n^2 + 3*(n^2
+ 4*n + 3)*e^(-2*b*x - 2*a) + 3*(n^2 + 4*n + 3)*e^(-4*b*x - 4*a) + (n^2 +
4*n + 3)*e^(-6*b*x - 6*a) + 4*n + 3)*b) - 2*e^(-6*b*x + n*log(e^(-b*x - a)
 + 1) + n*log(-e^(-b*x - a) + 1) - n*log(e^(-2*b*x - 2*a) + 1) - 6*a)/((n^
2 + 3*(n^2 + 4*n + 3)*e^(-2*b*x - 2*a) + 3*(n^2 + 4*n + 3)*e^(-4*b*x - 4*a
) + (n^2 + 4*n + 3)*e^(-6*b*x - 6*a) + 4*n + 3)*b) + 2*e^(n*log(e^(-b*x -
a) + 1) + n*log(-e^(-b*x - a) + 1) - n*log(e^(-2*b*x - 2*a) + 1))/((n^2 +
3*(n^2 + 4*n + 3)*e^(-2*b*x - 2*a) + 3*(n^2 + 4*n + 3)*e^(-4*b*x - 4*a) +
(n^2 + 4*n + 3)*e^(-6*b*x - 6*a) + 4*n + 3)*b)
```

Giac [F]

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx=\int \tanh\left(bx+a\right)^n\operatorname{sech}\left(bx+a\right)^4\,dx$$

input
```
integrate(sech(b*x+a)^4*tanh(b*x+a)^n,x, algorithm="giac")
```

output
```
integrate(tanh(b*x + a)^n*sech(b*x + a)^4, x)
```

Mupad [B] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.88

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx = \frac{e^{-3\,a-3\,b\,x}\,\left(\frac{4\,e^{3\,a+3\,b\,x}\,\sinh(3\,a+3\,b\,x)}{b\,(n^2+4\,n+3)}+\frac{2\,e^{3\,a+3\,b\,x}\,\sinh(a+b\,x)\,(4\,n+6)}{b\,(n^2+4\,n+3)}\right)\,\left(\frac{e^{2\,a+2\,b\,x}-1}{e^{2\,a+2\,b\,x}+1}\right)^n}{8\cosh\left(a+b\,x\right)^3}$$

input
```
int(tanh(a + b*x)^n/cosh(a + b*x)^4,x)
```

output
```
(exp(- 3*a - 3*b*x)*((4*exp(3*a + 3*b*x)*sinh(3*a + 3*b*x))/(b*(4*n + n^2
+ 3)) + (2*exp(3*a + 3*b*x)*sinh(a + b*x)*(4*n + 6))/(b*(4*n + n^2 + 3)))*
((exp(2*a + 2*b*x) - 1)/(exp(2*a + 2*b*x) + 1))^n)/(8*cosh(a + b*x)^3)
```

Reduce [F]

$$\int \operatorname{sech}^4(a+bx)\tanh^n(a+bx)\,dx=\int \tanh\left(bx+a\right)^n\operatorname{sech}(bx+a)^4\,dx$$

input
```
int(sech(b*x+a)^4*tanh(b*x+a)^n,x)
```

output
```
int(tanh(a + b*x)**n*sech(a + b*x)**4,x)
```

3.72 $\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{2b}-\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}$$

output
```
1/2*arctan(sinh(b*x+a))/b-1/2*sech(b*x+a)*tanh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{2b}-\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}$$

input
```
Integrate[Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
ArcTan[Sinh[a + b*x]]/(2*b) - (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 25, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ia+ibx)^2(-\sec(ia+ibx))dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec(ia+ibx)\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{1}{2}\int \text{sech}(a+bx)dx-\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}+\frac{1}{2}\int \csc\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4257}$$

$$\frac{\arctan(\sinh(a+bx))}{2b}-\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

input
```
Int[Sech[a + b*x]*Tanh[a + b*x]^2,x]
```

output
```
ArcTan[Sinh[a + b*x]]/(2*b) - (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

method	result	size
derivativedivides	$\frac{-\frac{\sinh(bx+a)}{\cosh(bx+a)^2}+\frac{\operatorname{sech}(bx+a)\tanh(bx+a)}{2}+\arctan\left(e^{bx+a}\right)}{b}$	43
default	$\frac{-\frac{\sinh(bx+a)}{\cosh(bx+a)^2}+\frac{\operatorname{sech}(bx+a)\tanh(bx+a)}{2}+\arctan\left(e^{bx+a}\right)}{b}$	43
risch	$-\frac{e^{bx+a}\left(e^{2bx+2a}-1\right)}{b\left(e^{2bx+2a}+1\right)^2}+\frac{i\ln\left(e^{bx+a}+i\right)}{2b}-\frac{i\ln\left(e^{bx+a}-i\right)}{2b}$	69

input
```
int(sech(b*x+a)*tanh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-sinh(b*x+a)/cosh(b*x+a)^2+1/2*sech(b*x+a)*tanh(b*x+a)+arctan(exp(b*x
+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(30) = 60$.

Time = 0.07 (sec) , antiderivative size = 269, normalized size of antiderivative = 7.91

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = -\frac{\cosh(bx+a)^3 + 3\cosh(bx+a)\sinh(bx+a)^2 + \sinh(bx+a)^3 - \left(\cosh(bx+a)^4 + 4\cosh(bx+a)\,\mathrm{s}\right.}{b\cosh(bx+a)^4 + 4\,b\cos}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 - (c
osh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*
cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a
)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x +
a)) + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))/(b*cosh(b*x +
 a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b
*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x +
a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \int \tanh^2(a+bx)\operatorname{sech}(a+bx)\,dx$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)**2,x)
```

output
```
Integral(tanh(a + b*x)**2*sech(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(30) = 60$.

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.94

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = -\frac{\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{e^{(-bx-a)} - e^{(-3\,bx-3\,a)}}{b(2\,e^{(-2\,bx-2\,a)} + e^{(-4\,bx-4\,a)} + 1)}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-arctan(e^(-b*x - a))/b - (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*
x - 2*a) + e^(-4*b*x - 4*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(30) = 60$.

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.24

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \frac{\pi - \frac{4\left(e^{(bx+a)} - e^{(-bx-a)}\right)}{\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2 + 4} + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)} - 1\right)e^{(-bx-a)}\right)}{4\,b}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/4*(pi - 4*(e^(b*x + a) - e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 +
 4) + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.41

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \frac{\operatorname{atan}\left(\frac{e^{bx}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} + \frac{2\,e^{a+b\,x}}{b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(tanh(a + b*x)^2/cosh(a + b*x),x)
```

output
```
atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b)/(b^2)^(1/2) + (2*exp(a + b*x))/(b*(2
*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a + 2*
b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.85

$$\int \operatorname{sech}(a+bx)\tanh^2(a+bx)\,dx = \frac{e^{4bx+4a}atan\left(e^{bx+a}\right)+2e^{2bx+2a}atan\left(e^{bx+a}\right)+atan\left(e^{bx+a}\right)-e^{3bx+3a}+e^{bx+a}}{b\left(e^{4bx+4a}+2e^{2bx+2a}+1\right)}$$

input
```
int(sech(b*x+a)*tanh(b*x+a)^2,x)
```

output
```
(e**(4*a + 4*b*x)*atan(e**(a + b*x)) + 2*e**(2*a + 2*b*x)*atan(e**(a + b*x
)) + atan(e**(a + b*x)) - e**(3*a + 3*b*x) + e**(a + b*x))/(b*(e**(4*a + 4
*b*x) + 2*e**(2*a + 2*b*x) + 1))
```

3.73 $\int \text{sech}(a+bx)\tanh^4(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \text{sech}(a+bx)\tanh^4(a+bx)\,dx = \frac{3\arctan(\sinh(a+bx))}{8b} - \frac{3\text{sech}(a+bx)\tanh(a+bx)}{8b} - \frac{\text{sech}(a+bx)\tanh^3(a+bx)}{4b}$$

output
```
3/8*arctan(sinh(b*x+a))/b-3/8*sech(b*x+a)*tanh(b*x+a)/b-1/4*sech(b*x+a)*ta
nh(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

$$\int \text{sech}(a+bx)\tanh^4(a+bx)\,dx = \frac{3\arctan(\sinh(a+bx))}{8b} + \frac{3\text{sech}(a+bx)\tanh(a+bx)}{8b} - \frac{3\text{sech}^3(a+bx)\tanh(a+bx)}{4b} - \frac{\text{sech}(a+bx)\tanh^3(a+bx)}{b}$$

input
```
Integrate[Sech[a + b*x]*Tanh[a + b*x]^4,x]
```

output
```
(3*ArcTan[Sinh[a + b*x]])/(8*b) + (3*Sech[a + b*x]*Tanh[a + b*x])/(8*b) -
(3*Sech[a + b*x]^3*Tanh[a + b*x])/(4*b) - (Sech[a + b*x]*Tanh[a + b*x]^3)/
b
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3042, 3091, 25, 3042, 25, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^4(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ia+ibx)^4\sec(ia+ibx)dx$$

$$\Big\downarrow \text{3091}$$

$$-\frac{3}{4}\int -\text{sech}(a+bx)\tanh^2(a+bx)dx-\frac{\tanh^3(a+bx)\text{sech}(a+bx)}{4b}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{4}\int \text{sech}(a+bx)\tanh^2(a+bx)dx-\frac{\tanh^3(a+bx)\text{sech}(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh^3(a+bx)\text{sech}(a+bx)}{4b}+\frac{3}{4}\int -\sec(ia+ibx)\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tanh^3(a+bx)\text{sech}(a+bx)}{4b}-\frac{3}{4}\int \sec(ia+ibx)\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{3091}$$

$$-\frac{3}{4}\left(\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}-\frac{1}{2}\int \text{sech}(a+bx)dx\right)-\frac{\tanh^3(a+bx)\text{sech}(a+bx)}{4b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\tanh^3(a+bx)\text{sech}(a+bx)}{4b}-\frac{3}{4}\left(\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}-\frac{1}{2}\int \csc\left(ia+ibx+\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \ 4257$$

$$-\frac{3}{4}\left(\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}-\frac{\arctan(\sinh(a+bx))}{2b}\right)-\frac{\tanh^3(a+bx)\text{sech}(a+bx)}{4b}$$

input
```
Int[Sech[a + b*x]*Tanh[a + b*x]^4,x]
```

output
```
-1/4*(Sech[a + b*x]*Tanh[a + b*x]^3)/b - (3*(-1/2*ArcTan[Sinh[a + b*x]]/b
+ (Sech[a + b*x]*Tanh[a + b*x])/(2*b)))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.36

method	result	size
derivativedivides	$\frac{-\frac{\sinh(bx+a)^3}{\cosh(bx+a)^4}-\frac{\sinh(bx+a)}{\cosh(bx+a)^4}+\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)+\frac{3\arctan\left(e^{bx+a}\right)}{4}}{b}$	75
default	$\frac{-\frac{\sinh(bx+a)^3}{\cosh(bx+a)^4}-\frac{\sinh(bx+a)}{\cosh(bx+a)^4}+\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)+\frac{3\arctan\left(e^{bx+a}\right)}{4}}{b}$	75
risch	$-\frac{e^{bx+a}\left(5\,e^{6bx+6a}-3\,e^{4bx+4a}+3\,e^{2bx+2a}-5\right)}{4b\left(e^{2bx+2a}+1\right)^4}+\frac{3i\ln\left(e^{bx+a}+i\right)}{8b}-\frac{3i\ln\left(e^{bx+a}-i\right)}{8b}$	93

input
```
int(sech(b*x+a)*tanh(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-sinh(b*x+a)^3/cosh(b*x+a)^4-1/cosh(b*x+a)^4*sinh(b*x+a)+(1/4*sech(b*
x+a)^3+3/8*sech(b*x+a))*tanh(b*x+a)+3/4*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. 2(49) = 98.

Time = 0.08 (sec) , antiderivative size = 814, normalized size of antiderivative = 14.80

$$\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/4*(5*cosh(b*x + a)^7 + 35*cosh(b*x + a)*sinh(b*x + a)^6 + 5*sinh(b*x +
a)^7 + 3*(35*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^5 - 3*cosh(b*x + a)^5 + 5*
(35*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^4 + (175*cosh(b*x + a
)^4 - 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^3 + 3*cosh(b*x + a)^3 + 3*(35*
cosh(b*x + a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 -
3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4
*(7*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b
*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*
cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x +
 a)^5 + 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(
b*x + a)^6 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 +
 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x +
 a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x
+ a)) + (35*cosh(b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 5)*
sinh(b*x + a) - 5*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*si
nh(b*x + a)^7 + b*sinh(b*x + a)^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x
+ a)^2 + b)*sinh(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*
sinh(b*x + a)^5 + 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b*cos
h(b*x + a)^2 + 3*b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*cosh(b
*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 4*b*cosh(b*x + a)^2 + ...
```

Sympy [F]

$$\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx=\int \tanh^4{(a+bx)}\operatorname{sech}{(a+bx)}\,dx$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)**4,x)
```

output
```
Integral(tanh(a + b*x)**4*sech(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(49) = 98$.

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.04

$$\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx = -\frac{3\arctan\left(e^{(-bx-a)}\right)}{4\,b} - \frac{5\,e^{(-bx-a)} - 3\,e^{(-3\,bx-3\,a)} + 3\,e^{(-5\,bx-5\,a)} - 5\,e^{(-7\,bx-7\,a)}}{4\,b(4\,e^{(-2\,bx-2\,a)} + 6\,e^{(-4\,bx-4\,a)} + 4\,e^{(-6\,bx-6\,a)} + e^{(-8\,bx-8\,a)} + 1)}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)^4,x, algorithm="maxima")
```

output
```
-3/4*arctan(e^(-b*x - a))/b - 1/4*(5*e^(-b*x - a) - 3*e^(-3*b*x - 3*a) + 3
*e^(-5*b*x - 5*a) - 5*e^(-7*b*x - 7*a))/(b*(4*e^(-2*b*x - 2*a) + 6*e^(-4*b
*x - 4*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(49) = 98$.

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.85

$$\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx = \frac{3\,\pi - \frac{4\left(5\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+12\,e^{(bx+a)}-12\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^2} + 6\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{16\,b}$$

input
```
integrate(sech(b*x+a)*tanh(b*x+a)^4,x, algorithm="giac")
```

output
```
1/16*(3*pi - 4*(5*(e^(b*x + a) - e^(-b*x - a))^3 + 12*e^(b*x + a) - 12*e^(
-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4)^2 + 6*arctan(1/2*(e^(2*b*x
 + 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.38

$$\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx$$
$$= \frac{3\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{4\,\sqrt{b^2}} + \frac{9\,e^{a+b\,x}}{2\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} - \frac{6\,e^{a+b\,x}}{b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)} + \frac{4\,e^{a+b\,x}}{b\,\left(4\,e^{2\,a+2\,b\,x}+6\,e^{4\,a+4\,b\,x}+4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)} - \frac{5\,e^{a+b\,x}}{4\,b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(tanh(a + b*x)^4/cosh(a + b*x),x)
```

output
```
(3*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(4*(b^2)^(1/2)) + (9*exp(a + b*x
))/(2*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (6*exp(a + b*x))/(b
*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + (4*ex
p(a + b*x))/(b*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6*a + 6*b*
x) + exp(8*a + 8*b*x) + 1)) - (5*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) + 1)
)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 191, normalized size of antiderivative = 3.47

$$\int \operatorname{sech}(a+bx)\tanh^4(a+bx)\,dx$$
$$= \frac{3e^{8bx+8a}atan\left(e^{bx+a}\right)+12e^{6bx+6a}atan\left(e^{bx+a}\right)+18e^{4bx+4a}atan\left(e^{bx+a}\right)+12e^{2bx+2a}atan\left(e^{bx+a}\right)+3atan\left(e^{b}\right.}{4b\left(e^{8bx+8a}+4e^{6bx+6a}+6e^{4bx+4a}+4e^{2bx+2a}+1\right)}$$

input
```
int(sech(b*x+a)*tanh(b*x+a)^4,x)
```

output
```
(3*e**(8*a + 8*b*x)*atan(e**(a + b*x)) + 12*e**(6*a + 6*b*x)*atan(e**(a +
b*x)) + 18*e**(4*a + 4*b*x)*atan(e**(a + b*x)) + 12*e**(2*a + 2*b*x)*atan(
e**(a + b*x)) + 3*atan(e**(a + b*x)) - 5*e**(7*a + 7*b*x) + 3*e**(5*a + 5*
b*x) - 3*e**(3*a + 3*b*x) + 5*e**(a + b*x))/(4*b*(e**(8*a + 8*b*x) + 4*e**
(6*a + 6*b*x) + 6*e**(4*a + 4*b*x) + 4*e**(2*a + 2*b*x) + 1))
```

3.74 $\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{8b}+\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{8b}-\frac{\operatorname{sech}^3(a+bx)\tanh(a+bx)}{4b}$$

output
```
1/8*arctan(sinh(b*x+a))/b+1/8*sech(b*x+a)*tanh(b*x+a)/b-1/4*sech(b*x+a)^3*
tanh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{8b}+\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{8b}-\frac{\operatorname{sech}^3(a+bx)\tanh(a+bx)}{4b}$$

input
```
Integrate[Sech[a + b*x]^3*Tanh[a + b*x]^2,x]
```

output
```
ArcTan[Sinh[a + b*x]]/(8*b) + (Sech[a + b*x]*Tanh[a + b*x])/(8*b) - (Sech[
a + b*x]^3*Tanh[a + b*x])/(4*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3042, 25, 3091, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(a+bx)\text{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ia+ibx)^2\left(-\sec(ia+ibx)^3\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec(ia+ibx)^3\tan(ia+ibx)^2dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{1}{4}\int \text{sech}^3(a+bx)dx-\frac{\tanh(a+bx)\text{sech}^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(a+bx)\text{sech}^3(a+bx)}{4b}+\frac{1}{4}\int \csc\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \text{sech}(a+bx)dx+\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}\right)-\frac{\tanh(a+bx)\text{sech}^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(a+bx)\text{sech}^3(a+bx)}{4b}+\frac{1}{4}\left(\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}+\frac{1}{2}\int \csc\left(ia+ibx+\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{4}\left(\frac{\arctan(\sinh(a+bx))}{2b}+\frac{\tanh(a+bx)\text{sech}(a+bx)}{2b}\right)-\frac{\tanh(a+bx)\text{sech}^3(a+bx)}{4b}$$

input
```
Int[Sech[a + b*x]^3*Tanh[a + b*x]^2,x]
```

output
```
-1/4*(Sech[a + b*x]^3*Tanh[a + b*x])/b + (ArcTan[Sinh[a + b*x]]/(2*b) + (S
ech[a + b*x]*Tanh[a + b*x])/(2*b))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{-\frac{\sinh(bx+a)}{3\cosh(bx+a)^4}+\frac{\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)}{3}+\frac{\arctan\left(e^{bx+a}\right)}{4}}{b}$	58
default	$\frac{-\frac{\sinh(bx+a)}{3\cosh(bx+a)^4}+\frac{\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)}{3}+\frac{\arctan\left(e^{bx+a}\right)}{4}}{b}$	58
risch	$\frac{e^{bx+a}\left(e^{6bx+6a}-7\,e^{4bx+4a}+7\,e^{2bx+2a}-1\right)}{4b\left(e^{2bx+2a}+1\right)^4}+\frac{i\ln\left(e^{bx+a}+i\right)}{8b}-\frac{i\ln\left(e^{bx+a}-i\right)}{8b}$	91

input
```
int(sech(b*x+a)^3*tanh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3/cosh(b*x+a)^4*sinh(b*x+a)+1/3*(1/4*sech(b*x+a)^3+3/8*sech(b*x+a)
)*tanh(b*x+a)+1/4*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. 2(49) = 98.

Time = 0.09 (sec) , antiderivative size = 808, normalized size of antiderivative = 14.69

$$\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(sech(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(cosh(b*x + a)^7 + 7*cosh(b*x + a)*sinh(b*x + a)^6 + sinh(b*x + a)^7 +
 7*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^5 - 7*cosh(b*x + a)^5 + 35*(cosh(
b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^4 + 7*(5*cosh(b*x + a)^4 - 10*co
sh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 7*cosh(b*x + a)^3 + 7*(3*cosh(b*x + a
)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 + (cosh(b*x +
a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*cosh(b*x +
 a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 + 3*
cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(b*x + a)^
2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 + 10*cos
h(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 + 1
5*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(b*x +
a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + cosh(b
*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + (7*cos
h(b*x + a)^6 - 35*cosh(b*x + a)^4 + 21*cosh(b*x + a)^2 - 1)*sinh(b*x + a)
- cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 +
b*sinh(b*x + a)^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 + b)*sinh
(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^5
+ 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b*cosh(b*x + a)^2 + 3
*b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*cosh(b*x + a)^3 + 3*b*
cosh(b*x + a))*sinh(b*x + a)^3 + 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x ...
```

Sympy [F]

$$\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx=\int \tanh^2{(a+bx)}\operatorname{sech}^3{(a+bx)}\,dx$$

input
```
integrate(sech(b*x+a)**3*tanh(b*x+a)**2,x)
```

output
```
Integral(tanh(a + b*x)**2*sech(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(49) = 98.

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.00

$$\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx = -\frac{\arctan\left(e^{(-bx-a)}\right)}{4\,b} + \frac{e^{(-bx-a)} - 7\,e^{(-3\,bx-3\,a)} + 7\,e^{(-5\,bx-5\,a)} - e^{(-7\,bx-7\,a)}}{4\,b(4\,e^{(-2\,bx-2\,a)} + 6\,e^{(-4\,bx-4\,a)} + 4\,e^{(-6\,bx-6\,a)} + e^{(-8\,bx-8\,a)} + 1)}$$

input
```
integrate(sech(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*arctan(e^(-b*x - a))/b + 1/4*(e^(-b*x - a) - 7*e^(-3*b*x - 3*a) + 7*e
^(-5*b*x - 5*a) - e^(-7*b*x - 7*a))/(b*(4*e^(-2*b*x - 2*a) + 6*e^(-4*b*x -
 4*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.78

$$\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx = \frac{\pi + \frac{4\left(\left(e^{(bx+a)} - e^{(-bx-a)}\right)^3 - 4\,e^{(bx+a)} + 4\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2 + 4\right)^2} + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)} - 1\right)e^{(-bx-a)}\right)}{16\,b}$$

input
```
integrate(sech(b*x+a)^3*tanh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/16*(pi + 4*((e^(b*x + a) - e^(-b*x - a))^3 - 4*e^(b*x + a) + 4*e^(-b*x -
 a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4)^2 + 2*arctan(1/2*(e^(2*b*x + 2*a
) - 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.91

$$\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx = \frac{\operatorname{atan}\left(\frac{e^{bx}\,e^{a}\,\sqrt{b^2}}{b}\right)}{4\sqrt{b^2}} - \frac{\frac{e^{a+bx}}{b}-\frac{2\,e^{3a+3bx}}{b}+\frac{e^{5a+5bx}}{b}}{4\,e^{2a+2bx}+6\,e^{4a+4bx}+4\,e^{6a+6bx}+e^{8a+8bx}+1} - \frac{3\,e^{a+bx}}{2\,b\,\left(2\,e^{2a+2bx}+e^{4a+4bx}+1\right)} + \frac{2\,e^{a+bx}}{b\,\left(3\,e^{2a+2bx}+3\,e^{4a+4bx}+e^{6a+6bx}+1\right)} + \frac{e^{a+bx}}{4\,b\,\left(e^{2a+2bx}+1\right)}$$

input
```
int(tanh(a + b*x)^2/cosh(a + b*x)^3,x)
```

output
```
atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b)/(4*(b^2)^(1/2)) - (exp(a + b*x)/b -
(2*exp(3*a + 3*b*x))/b + exp(5*a + 5*b*x)/b)/(4*exp(2*a + 2*b*x) + 6*exp(4
*a + 4*b*x) + 4*exp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1) - (3*exp(a + b*x)
)/(2*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) + (2*exp(a + b*x))/(b*
(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + exp(a
+ b*x)/(4*b*(exp(2*a + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.38

$$\int \operatorname{sech}^3(a+bx)\tanh^2(a+bx)\,dx = \frac{e^{8bx+8a}atan\left(e^{bx+a}\right)+4e^{6bx+6a}atan\left(e^{bx+a}\right)+6e^{4bx+4a}atan\left(e^{bx+a}\right)+4e^{2bx+2a}atan\left(e^{bx+a}\right)+atan\left(e^{bx+a}\right)-}{4b\left(e^{8bx+8a}+4e^{6bx+6a}+6e^{4bx+4a}+4e^{2bx+2a}+1\right)}$$

input
```
int(sech(b*x+a)^3*tanh(b*x+a)^2,x)
```

output
```
(e**(8*a + 8*b*x)*atan(e**(a + b*x)) + 4*e**(6*a + 6*b*x)*atan(e**(a + b*x
)) + 6*e**(4*a + 4*b*x)*atan(e**(a + b*x)) + 4*e**(2*a + 2*b*x)*atan(e**(a
 + b*x)) + atan(e**(a + b*x)) + e**(7*a + 7*b*x) - 7*e**(5*a + 5*b*x) + 7*
e**(3*a + 3*b*x) - e**(a + b*x))/(4*b*(e**(8*a + 8*b*x) + 4*e**(6*a + 6*b*
x) + 6*e**(4*a + 4*b*x) + 4*e**(2*a + 2*b*x) + 1))
```

3.75 $\int \mathbf{sech}(x)\tanh^5(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \mathrm{sech}(x)\tanh^5(x)\,dx = -\mathrm{sech}(x) + \frac{2\mathrm{sech}^3(x)}{3} - \frac{\mathrm{sech}^5(x)}{5}$$

output
```
-sech(x)+2/3*sech(x)^3-1/5*sech(x)^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \mathrm{sech}(x)\tanh^5(x)\,dx = -\mathrm{sech}(x) + \frac{2\mathrm{sech}^3(x)}{3} - \frac{\mathrm{sech}^5(x)}{5}$$

input
```
Integrate[Sech[x]*Tanh[x]^5,x]
```

output
```
-Sech[x] + (2*Sech[x]^3)/3 - Sech[x]^5/5
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 26, 3086, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^5(x)\text{sech}(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ix)^5\sec(ix)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sec(ix)\tan(ix)^5dx$$

$$\Big\downarrow \text{3086}$$

$$-\int \left(\text{sech}^2(x)-1\right)^2d\text{sech}(x)$$

$$\Big\downarrow \text{210}$$

$$-\int \left(\text{sech}^4(x)-2\text{sech}^2(x)+1\right)d\text{sech}(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{5}\text{sech}^5(x)+\frac{2\text{sech}^3(x)}{3}-\text{sech}(x)$$

input
```
Int[Sech[x]*Tanh[x]^5,x]
```

output
```
-Sech[x] + (2*Sech[x]^3)/3 - Sech[x]^5/5
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\operatorname{sech}\left(x\right)+\frac{2 \operatorname{sech}\left(x\right)^{3}}{3}-\frac{\operatorname{sech}\left(x\right)^{5}}{5}$	18
default	$-\operatorname{sech}\left(x\right)+\frac{2 \operatorname{sech}\left(x\right)^{3}}{3}-\frac{\operatorname{sech}\left(x\right)^{5}}{5}$	18
risch	$-\frac{2 \,{\mathrm e}^{x} \left(15 \,{\mathrm e}^{8 x}+20 \,{\mathrm e}^{6 x}+58 \,{\mathrm e}^{4 x}+20 \,{\mathrm e}^{2 x}+15\right)}{15 \left({\mathrm e}^{2 x}+1\right)^{5}}$	39

input
```
int(sech(x)*tanh(x)^5,x,method=_RETURNVERBOSE)
```

output
```
-sech(x)+2/3*sech(x)^3-1/5*sech(x)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 185, normalized size of antiderivative = 8.81

$$\int \operatorname{sech}(x)\tanh^5(x)\,dx =$$

$$-\frac{2\left(15\cosh\left(x\right)^5+75\cosh\left(x\right)\sinh\left(x\right)^4+15\sinh\left(x\right)^5+5\left(30\cosh\left(x\right)^2+1\right)\mathrm{s}\right.}{15\left(\cosh\left(x\right)^6+6\cosh\left(x\right)\sinh\left(x\right)^5+\sinh\left(x\right)^6+3\left(5\cosh\left(x\right)^2+2\right)\sinh\left(x\right)^4+6\cosh\left(x\right)^4+4\left(5\mathrm{co}\right.\right.}$$

input
```
integrate(sech(x)*tanh(x)^5,x, algorithm="fricas")
```

output
```
-2/15*(15*cosh(x)^5 + 75*cosh(x)*sinh(x)^4 + 15*sinh(x)^5 + 5*(30*cosh(x)^
2 + 1)*sinh(x)^3 + 35*cosh(x)^3 + 15*(10*cosh(x)^3 + 7*cosh(x))*sinh(x)^2
+ (75*cosh(x)^4 + 15*cosh(x)^2 + 38)*sinh(x) + 78*cosh(x))/(cosh(x)^6 + 6*
cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 2)*sinh(x)^4 + 6*cosh(x)^
4 + 4*(5*cosh(x)^3 + 4*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 12*cosh(x)^2
+ 5)*sinh(x)^2 + 15*cosh(x)^2 + 2*(3*cosh(x)^5 + 8*cosh(x)^3 + 5*cosh(x))*
sinh(x) + 10)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \operatorname{sech}(x)\tanh^5(x)\,dx = -\frac{\tanh^4\left(x\right)\operatorname{sech}\left(x\right)}{5} - \frac{4\tanh^2\left(x\right)\operatorname{sech}\left(x\right)}{15} - \frac{8\operatorname{sech}\left(x\right)}{15}$$

input
```
integrate(sech(x)*tanh(x)**5,x)
```

output
```
-tanh(x)**4*sech(x)/5 - 4*tanh(x)**2*sech(x)/15 - 8*sech(x)/15
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(17) = 34$.

Time = 0.05 (sec) , antiderivative size = 191, normalized size of antiderivative = 9.10

$$\int \operatorname{sech}(x)\tanh^5(x)\,dx = -\frac{2\,e^{(-x)}}{5\,e^{(-2\,x)} + 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} + 5\,e^{(-8\,x)} + e^{(-10\,x)} + 1} - \frac{8\,e^{(-3\,x)}}{3\,(5\,e^{(-2\,x)} + 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} + 5\,e^{(-8\,x)} + e^{(-10\,x)} + 1)} - \frac{116\,e^{(-5\,x)}}{15\,(5\,e^{(-2\,x)} + 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} + 5\,e^{(-8\,x)} + e^{(-10\,x)} + 1)} - \frac{8\,e^{(-7\,x)}}{3\,(5\,e^{(-2\,x)} + 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} + 5\,e^{(-8\,x)} + e^{(-10\,x)} + 1)} - \frac{2\,e^{(-9\,x)}}{5\,e^{(-2\,x)} + 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} + 5\,e^{(-8\,x)} + e^{(-10\,x)} + 1}$$

input
```
integrate(sech(x)*tanh(x)^5,x, algorithm="maxima")
```

output
```
-2*e^(-x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(-8*x) + e^(-10*x)
 + 1) - 8/3*e^(-3*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(-8*x)
+ e^(-10*x) + 1) - 116/15*e^(-5*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x)
 + 5*e^(-8*x) + e^(-10*x) + 1) - 8/3*e^(-7*x)/(5*e^(-2*x) + 10*e^(-4*x) +
10*e^(-6*x) + 5*e^(-8*x) + e^(-10*x) + 1) - 2*e^(-9*x)/(5*e^(-2*x) + 10*e^
(-4*x) + 10*e^(-6*x) + 5*e^(-8*x) + e^(-10*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.67

$$\int \operatorname{sech}(x)\tanh^5(x)\,dx = -\frac{2\left(15\left(e^{(-x)} + e^{x}\right)^4 - 40\left(e^{(-x)} + e^{x}\right)^2 + 48\right)}{15\left(e^{(-x)} + e^{x}\right)^5}$$

input
```
integrate(sech(x)*tanh(x)^5,x, algorithm="giac")
```

output
```
-2/15*(15*(e^(-x) + e^x)^4 - 40*(e^(-x) + e^x)^2 + 48)/(e^(-x) + e^x)^5
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 129, normalized size of antiderivative = 6.14

$$\int \operatorname{sech}(x)\tanh^5(x)\,dx = \frac{64\,e^x}{5\,\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)} - \frac{2\,e^x}{e^{2\,x}+1} - \frac{176\,e^x}{15\,\left(3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1\right)} - \frac{32\,e^x}{5\,\left(5\,e^{2\,x}+10\,e^{4\,x}+10\,e^{6\,x}+5\,e^{8\,x}+e^{10\,x}+1\right)} + \frac{16\,e^x}{3\,\left(2\,e^{2\,x}+e^{4\,x}+1\right)}$$

input
```
int(tanh(x)^5/cosh(x),x)
```

output
```
(64*exp(x))/(5*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1)) - (2
*exp(x))/(exp(2*x) + 1) - (176*exp(x))/(15*(3*exp(2*x) + 3*exp(4*x) + exp(
6*x) + 1)) - (32*exp(x))/(5*(5*exp(2*x) + 10*exp(4*x) + 10*exp(6*x) + 5*ex
p(8*x) + exp(10*x) + 1)) + (16*exp(x))/(3*(2*exp(2*x) + exp(4*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \operatorname{sech}(x)\tanh^5(x)\,dx = \frac{\operatorname{sech}(x)\left(-3\tanh(x)^4-4\tanh(x)^2-8\right)}{15}$$

input
```
int(sech(x)*tanh(x)^5,x)
```

output
```
(sech(x)*( - 3*tanh(x)**4 - 4*tanh(x)**2 - 8))/15
```

3.76 $\int \operatorname{sech}^7(x)\tanh^5(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \operatorname{sech}^7(x)\tanh^5(x)\,dx = -\frac{1}{7}\operatorname{sech}^7(x) + \frac{2\operatorname{sech}^9(x)}{9} - \frac{\operatorname{sech}^{11}(x)}{11}$$

output
```
-1/7*sech(x)^7+2/9*sech(x)^9-1/11*sech(x)^11
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^7(x)\tanh^5(x)\,dx = -\frac{1}{7}\operatorname{sech}^7(x) + \frac{2\operatorname{sech}^9(x)}{9} - \frac{\operatorname{sech}^{11}(x)}{11}$$

input
```
Integrate[Sech[x]^7*Tanh[x]^5,x]
```

output
```
-1/7*Sech[x]^7 + (2*Sech[x]^9)/9 - Sech[x]^11/11
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 26, 3086, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^5(x)\text{sech}^7(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ix)^5\sec(ix)^7dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sec(ix)^7\tan(ix)^5dx$$

$$\Big\downarrow \text{3086}$$

$$-\int \text{sech}^6(x)\left(1-\text{sech}^2(x)\right)^2d\text{sech}(x)$$

$$\Big\downarrow \text{244}$$

$$-\int \left(\text{sech}^{10}(x)-2\text{sech}^8(x)+\text{sech}^6(x)\right)d\text{sech}(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{11}\text{sech}^{11}(x)+\frac{2\text{sech}^9(x)}{9}-\frac{\text{sech}^7(x)}{7}$$

input
```
Int[Sech[x]^7*Tanh[x]^5,x]
```

output
```
-1/7*Sech[x]^7 + (2*Sech[x]^9)/9 - Sech[x]^11/11
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 107.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{\operatorname{sech}(x)^{7}}{7}+\frac{2\operatorname{sech}(x)^{9}}{9}-\frac{\operatorname{sech}(x)^{11}}{11}$	20
default	$-\frac{\operatorname{sech}(x)^{7}}{7}+\frac{2\operatorname{sech}(x)^{9}}{9}-\frac{\operatorname{sech}(x)^{11}}{11}$	20
risch	$-\frac{128\,{\mathrm e}^{7x}\left(99\,{\mathrm e}^{8x}-220\,{\mathrm e}^{6x}+370\,{\mathrm e}^{4x}-220\,{\mathrm e}^{2x}+99\right)}{693\left({\mathrm e}^{2x}+1\right)^{11}}$	41

input
```
int(sech(x)^7*tanh(x)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/7*sech(x)^7+2/9*sech(x)^9-1/11*sech(x)^11
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(19) = 38$.

Time = 0.10 (sec) , antiderivative size = 634, normalized size of antiderivative = 25.36

$$\int \operatorname{sech}^7(x) \tanh^5(x)\, dx = \text{Too large to display}$$

input

```
integrate(sech(x)^7*tanh(x)^5,x, algorithm="fricas")
```

output

```
-128/693*(99*cosh(x)^8 + 792*cosh(x)*sinh(x)^7 + 99*sinh(x)^8 + 44*(63*cos
h(x)^2 - 5)*sinh(x)^6 - 220*cosh(x)^6 + 264*(21*cosh(x)^3 - 5*cosh(x))*sin
h(x)^5 + 10*(693*cosh(x)^4 - 330*cosh(x)^2 + 37)*sinh(x)^4 + 370*cosh(x)^4
 + 8*(693*cosh(x)^5 - 550*cosh(x)^3 + 185*cosh(x))*sinh(x)^3 + 4*(693*cosh
(x)^6 - 825*cosh(x)^4 + 555*cosh(x)^2 - 55)*sinh(x)^2 - 220*cosh(x)^2 + 8*
(99*cosh(x)^7 - 165*cosh(x)^5 + 185*cosh(x)^3 - 55*cosh(x))*sinh(x) + 99)/
(cosh(x)^15 + 15*cosh(x)*sinh(x)^14 + sinh(x)^15 + (105*cosh(x)^2 + 11)*si
nh(x)^13 + 11*cosh(x)^13 + 13*(35*cosh(x)^3 + 11*cosh(x))*sinh(x)^12 + (13
65*cosh(x)^4 + 858*cosh(x)^2 + 55)*sinh(x)^11 + 55*cosh(x)^11 + 11*(273*co
sh(x)^5 + 286*cosh(x)^3 + 55*cosh(x))*sinh(x)^10 + 55*(91*cosh(x)^6 + 143*
cosh(x)^4 + 55*cosh(x)^2 + 3)*sinh(x)^9 + 165*cosh(x)^9 + 33*(195*cosh(x)^
7 + 429*cosh(x)^5 + 275*cosh(x)^3 + 45*cosh(x))*sinh(x)^8 + (6435*cosh(x)^
8 + 18876*cosh(x)^6 + 18150*cosh(x)^4 + 5940*cosh(x)^2 + 329)*sinh(x)^7 +
331*cosh(x)^7 + (5005*cosh(x)^9 + 18876*cosh(x)^7 + 25410*cosh(x)^5 + 1386
0*cosh(x)^3 + 2317*cosh(x))*sinh(x)^6 + (3003*cosh(x)^10 + 14157*cosh(x)^8
 + 25410*cosh(x)^6 + 20790*cosh(x)^4 + 6909*cosh(x)^2 + 451)*sinh(x)^5 + 4
73*cosh(x)^5 + 5*(273*cosh(x)^11 + 1573*cosh(x)^9 + 3630*cosh(x)^7 + 4158*
cosh(x)^5 + 2317*cosh(x)^3 + 473*cosh(x))*sinh(x)^4 + (455*cosh(x)^12 + 31
46*cosh(x)^10 + 9075*cosh(x)^8 + 13860*cosh(x)^6 + 11515*cosh(x)^4 + 4510*
cosh(x)^2 + 407)*sinh(x)^3 + 517*cosh(x)^3 + (105*cosh(x)^13 + 858*cosh...
```

Sympy [A] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \operatorname{sech}^7(x)\tanh^5(x)\,dx = -\frac{\tanh^4(x)\operatorname{sech}^7(x)}{11} - \frac{4\tanh^2(x)\operatorname{sech}^7(x)}{99} - \frac{8\operatorname{sech}^7(x)}{693}$$

input
```
integrate(sech(x)**7*tanh(x)**5,x)
```

output
```
-tanh(x)**4*sech(x)**7/11 - 4*tanh(x)**2*sech(x)**7/99 - 8*sech(x)**7/693
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. 2(19) = 38.

Time = 0.04 (sec) , antiderivative size = 371, normalized size of antiderivative = 14.84

$$\int \operatorname{sech}^7(x)\tanh^5(x)\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^7*tanh(x)^5,x, algorithm="maxima")
```

output
```
-128/7*e^(-7*x)/(11*e^(-2*x) + 55*e^(-4*x) + 165*e^(-6*x) + 330*e^(-8*x) +
 462*e^(-10*x) + 462*e^(-12*x) + 330*e^(-14*x) + 165*e^(-16*x) + 55*e^(-18
*x) + 11*e^(-20*x) + e^(-22*x) + 1) + 2560/63*e^(-9*x)/(11*e^(-2*x) + 55*e
^(-4*x) + 165*e^(-6*x) + 330*e^(-8*x) + 462*e^(-10*x) + 462*e^(-12*x) + 33
0*e^(-14*x) + 165*e^(-16*x) + 55*e^(-18*x) + 11*e^(-20*x) + e^(-22*x) + 1)
 - 47360/693*e^(-11*x)/(11*e^(-2*x) + 55*e^(-4*x) + 165*e^(-6*x) + 330*e^(
-8*x) + 462*e^(-10*x) + 462*e^(-12*x) + 330*e^(-14*x) + 165*e^(-16*x) + 55
*e^(-18*x) + 11*e^(-20*x) + e^(-22*x) + 1) + 2560/63*e^(-13*x)/(11*e^(-2*x
) + 55*e^(-4*x) + 165*e^(-6*x) + 330*e^(-8*x) + 462*e^(-10*x) + 462*e^(-12
*x) + 330*e^(-14*x) + 165*e^(-16*x) + 55*e^(-18*x) + 11*e^(-20*x) + e^(-22
*x) + 1) - 128/7*e^(-15*x)/(11*e^(-2*x) + 55*e^(-4*x) + 165*e^(-6*x) + 330
*e^(-8*x) + 462*e^(-10*x) + 462*e^(-12*x) + 330*e^(-14*x) + 165*e^(-16*x)
+ 55*e^(-18*x) + 11*e^(-20*x) + e^(-22*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \operatorname{sech}^7(x)\tanh^5(x)\,dx = -\frac{128\left(99\left(e^{(-x)}+e^x\right)^4-616\left(e^{(-x)}+e^x\right)^2+1008\right)}{693\left(e^{(-x)}+e^x\right)^{11}}$$

input
```
integrate(sech(x)^7*tanh(x)^5,x, algorithm="giac")
```

output
```
-128/693*(99*(e^(-x) + e^x)^4 - 616*(e^(-x) + e^x)^2 + 1008)/(e^(-x) + e^x
)^11
```

Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 520, normalized size of antiderivative = 20.80

$$\int \operatorname{sech}^7(x)\tanh^5(x)\,dx = \text{Too large to display}$$

input
```
int(tanh(x)^5/cosh(x)^7,x)
```

output
```
((64*exp(5*x))/11 - (320*exp(7*x))/11 + (640*exp(9*x))/11 - (640*exp(11*x)
)/11 + (320*exp(13*x))/11 - (64*exp(15*x))/11)/(11*exp(2*x) + 55*exp(4*x)
+ 165*exp(6*x) + 330*exp(8*x) + 462*exp(10*x) + 462*exp(12*x) + 330*exp(14
*x) + 165*exp(16*x) + 55*exp(18*x) + 11*exp(20*x) + exp(22*x) + 1) - (3846
4*exp(x))/(693*(6*exp(2*x) + 15*exp(4*x) + 20*exp(6*x) + 15*exp(8*x) + 6*e
xp(10*x) + exp(12*x) + 1)) - (640*exp(x))/(33*(8*exp(2*x) + 28*exp(4*x) +
56*exp(6*x) + 70*exp(8*x) + 56*exp(10*x) + 28*exp(12*x) + 8*exp(14*x) + ex
p(16*x) + 1)) - (104*exp(x))/(21*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + e
xp(8*x) + 1)) + (1664*exp(x))/(63*(5*exp(2*x) + 10*exp(4*x) + 10*exp(6*x)
+ 5*exp(8*x) + exp(10*x) + 1)) + (4096*exp(x))/(77*(7*exp(2*x) + 21*exp(4*
x) + 35*exp(6*x) + 35*exp(8*x) + 21*exp(10*x) + 7*exp(12*x) + exp(14*x) +
1)) + ((16*exp(3*x))/11 - (112*exp(5*x))/11 + (288*exp(7*x))/11 - 32*exp(9
*x) + (208*exp(11*x))/11 - (48*exp(13*x))/11)/(10*exp(2*x) + 45*exp(4*x) +
 120*exp(6*x) + 210*exp(8*x) + 252*exp(10*x) + 210*exp(12*x) + 120*exp(14*
x) + 45*exp(16*x) + 10*exp(18*x) + exp(20*x) + 1) - ((280*exp(3*x))/99 - (
112*exp(5*x))/11 + 16*exp(7*x) - (104*exp(9*x))/9 + (104*exp(11*x))/33 - (
8*exp(x))/33)/(9*exp(2*x) + 36*exp(4*x) + 84*exp(6*x) + 126*exp(8*x) + 126
*exp(10*x) + 84*exp(12*x) + 36*exp(14*x) + 9*exp(16*x) + exp(18*x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \operatorname{sech}^7(x)\tanh^5(x)\,dx = \frac{\operatorname{sech}(x)^7\left(-63\tanh(x)^4-28\tanh(x)^2-8\right)}{693}$$

input
```
int(sech(x)^7*tanh(x)^5,x)
```

output
```
(sech(x)**7*( - 63*tanh(x)**4 - 28*tanh(x)**2 - 8))/693
```

3.77 $\int \operatorname{sech}^3(x)\tanh^4(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 38

$$\int \operatorname{sech}^3(x)\tanh^4(x)\,dx = \frac{1}{16}\arctan(\sinh(x)) + \frac{1}{16}\operatorname{sech}(x)\tanh(x) - \frac{1}{8}\operatorname{sech}^3(x)\tanh(x) - \frac{1}{6}\operatorname{sech}^3(x)\tanh^3(x)$$

output
```
1/16*arctan(sinh(x))+1/16*sech(x)*tanh(x)-1/8*sech(x)^3*tanh(x)-1/6*sech(x
)^3*tanh(x)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \operatorname{sech}^3(x)\tanh^4(x)\,dx = \frac{1}{16}\arctan(\sinh(x)) + \frac{1}{16}\operatorname{sech}(x)\tanh(x) + \frac{1}{24}\operatorname{sech}^3(x)\tanh(x) - \frac{1}{6}\operatorname{sech}^5(x)\tanh(x) - \frac{1}{3}\operatorname{sech}^3(x)\tanh^3(x)$$

input
```
Integrate[Sech[x]^3*Tanh[x]^4,x]
```

output
```
ArcTan[Sinh[x]]/16 + (Sech[x]*Tanh[x])/16 + (Sech[x]^3*Tanh[x])/24 - (Sech
[x]^5*Tanh[x])/6 - (Sech[x]^3*Tanh[x]^3)/3
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.111, Rules used = {3042, 3091, 25, 3042, 25, 3091, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^4(x)\text{sech}^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ix)^4\sec(ix)^3dx$$

$$\Big\downarrow \text{3091}$$

$$-\frac{1}{2}\int -\text{sech}^3(x)\tanh^2(x)dx-\frac{1}{6}\tanh^3(x)\text{sech}^3(x)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \text{sech}^3(x)\tanh^2(x)dx-\frac{1}{6}\tanh^3(x)\text{sech}^3(x)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{6}\tanh^3(x)\text{sech}^3(x)+\frac{1}{2}\int -\sec(ix)^3\tan(ix)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{6}\tanh^3(x)\text{sech}^3(x)-\frac{1}{2}\int \sec(ix)^3\tan(ix)^2dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \text{sech}^3(x)dx-\frac{1}{4}\tanh(x)\text{sech}^3(x)\right)-\frac{1}{6}\tanh^3(x)\text{sech}^3(x)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{6}\tanh^3(x)\text{sech}^3(x)+\frac{1}{2}\left(-\frac{1}{4}\tanh(x)\text{sech}^3(x)+\frac{1}{4}\int\csc\left(ix+\frac{\pi}{2}\right)^3dx\right)$$

$\big\downarrow$ 4255

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int\text{sech}(x)dx}{2}+\frac{1}{2}\tanh(x)\text{sech}(x)\right)-\frac{1}{4}\tanh(x)\text{sech}^3(x)\right)-\frac{1}{6}\tanh^3(x)\text{sech}^3(x)$$

$\big\downarrow$ 3042

$$-\frac{1}{6}\tanh^3(x)\text{sech}^3(x)+\frac{1}{2}\left(-\frac{1}{4}\tanh(x)\text{sech}^3(x)+\frac{1}{4}\left(\frac{1}{2}\tanh(x)\text{sech}(x)+\frac{1}{2}\int\csc\left(ix+\frac{\pi}{2}\right)dx\right)\right)$$

$\big\downarrow$ 4257

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\arctan(\sinh(x))+\frac{1}{2}\tanh(x)\text{sech}(x)\right)-\frac{1}{4}\tanh(x)\text{sech}^3(x)\right)-\frac{1}{6}\tanh^3(x)\text{sech}^3(x)$$

input
```
Int[Sech[x]^3*Tanh[x]^4,x]
```

output
```
-1/6*(Sech[x]^3*Tanh[x]^3) + (-1/4*(Sech[x]^3*Tanh[x]) + (ArcTan[Sinh[x]]/
2 + (Sech[x]*Tanh[x])/2)/4)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 3.91 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

method	result	size
default	$-\frac{\sinh(x)^{3}}{3\cosh(x)^{6}}-\frac{\sinh(x)}{5\cosh(x)^{6}}+\frac{\left(\frac{\operatorname{sech}(x)^{5}}{6}+\frac{5\operatorname{sech}(x)^{3}}{24}+\frac{5\operatorname{sech}(x)}{16}\right)\tanh(x)}{5}+\frac{\arctan(\mathrm{e}^{x})}{8}$	46
risch	$\frac{\mathrm{e}^{x}\left(3\,\mathrm{e}^{10x}-47\,\mathrm{e}^{8x}+78\,\mathrm{e}^{6x}-78\,\mathrm{e}^{4x}+47\,\mathrm{e}^{2x}-3\right)}{24\left(\mathrm{e}^{2x}+1\right)^{6}}+\frac{i\ln(\mathrm{e}^{x}+i)}{16}-\frac{i\ln(\mathrm{e}^{x}-i)}{16}$	64

input
```
int(sech(x)^3*tanh(x)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*sinh(x)^3/cosh(x)^6-1/5*sinh(x)/cosh(x)^6+1/5*(1/6*sech(x)^5+5/24*sec
h(x)^3+5/16*sech(x))*tanh(x)+1/8*arctan(exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. 2(30) = 60.

Time = 0.08 (sec) , antiderivative size = 925, normalized size of antiderivative = 24.34

$$\int \operatorname{sech}^3(x)\tanh^4(x)\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^3*tanh(x)^4,x, algorithm="fricas")
```

output
```
1/24*(3*cosh(x)^11 + 33*cosh(x)*sinh(x)^10 + 3*sinh(x)^11 + (165*cosh(x)^2
 - 47)*sinh(x)^9 - 47*cosh(x)^9 + 9*(55*cosh(x)^3 - 47*cosh(x))*sinh(x)^8
+ 6*(165*cosh(x)^4 - 282*cosh(x)^2 + 13)*sinh(x)^7 + 78*cosh(x)^7 + 42*(33
*cosh(x)^5 - 94*cosh(x)^3 + 13*cosh(x))*sinh(x)^6 + 6*(231*cosh(x)^6 - 987
*cosh(x)^4 + 273*cosh(x)^2 - 13)*sinh(x)^5 - 78*cosh(x)^5 + 6*(165*cosh(x)
^7 - 987*cosh(x)^5 + 455*cosh(x)^3 - 65*cosh(x))*sinh(x)^4 + (495*cosh(x)^
8 - 3948*cosh(x)^6 + 2730*cosh(x)^4 - 780*cosh(x)^2 + 47)*sinh(x)^3 + 47*c
osh(x)^3 + 3*(55*cosh(x)^9 - 564*cosh(x)^7 + 546*cosh(x)^5 - 260*cosh(x)^3
 + 47*cosh(x))*sinh(x)^2 + 3*(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)
^12 + 6*(11*cosh(x)^2 + 1)*sinh(x)^10 + 6*cosh(x)^10 + 20*(11*cosh(x)^3 +
3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^8 + 15
*cosh(x)^8 + 24*(33*cosh(x)^5 + 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(2
31*cosh(x)^6 + 315*cosh(x)^4 + 105*cosh(x)^2 + 5)*sinh(x)^6 + 20*cosh(x)^6
 + 24*(33*cosh(x)^7 + 63*cosh(x)^5 + 35*cosh(x)^3 + 5*cosh(x))*sinh(x)^5 +
 15*(33*cosh(x)^8 + 84*cosh(x)^6 + 70*cosh(x)^4 + 20*cosh(x)^2 + 1)*sinh(x
)^4 + 15*cosh(x)^4 + 20*(11*cosh(x)^9 + 36*cosh(x)^7 + 42*cosh(x)^5 + 20*c
osh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 + 45*cosh(x)^8 + 70*cos
h(x)^6 + 50*cosh(x)^4 + 15*cosh(x)^2 + 1)*sinh(x)^2 + 6*cosh(x)^2 + 12*(co
sh(x)^11 + 5*cosh(x)^9 + 10*cosh(x)^7 + 10*cosh(x)^5 + 5*cosh(x)^3 + cosh(
x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 3*(11*cosh(x)^10 - 141*cos...
```

Sympy [F]

$$\int \operatorname{sech}^3(x)\tanh^4(x)\,dx = \int \tanh^4(x)\operatorname{sech}^3(x)\,dx$$

input
```
integrate(sech(x)**3*tanh(x)**4,x)
```

output
```
Integral(tanh(x)**4*sech(x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(30) = 60$.

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.24

$$\int \operatorname{sech}^3(x)\tanh^4(x)\,dx = \frac{3\,e^{(-x)} - 47\,e^{(-3\,x)} + 78\,e^{(-5\,x)} - 78\,e^{(-7\,x)} + 47\,e^{(-9\,x)} - 3\,e^{(-11\,x)}}{24\,(6\,e^{(-2\,x)} + 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} + 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} + e^{(-12\,x)} + 1)} - \frac{1}{8}\arctan\left(e^{(-x)}\right)$$

input
```
integrate(sech(x)^3*tanh(x)^4,x, algorithm="maxima")
```

output
```
1/24*(3*e^(-x) - 47*e^(-3*x) + 78*e^(-5*x) - 78*e^(-7*x) + 47*e^(-9*x) - 3
*e^(-11*x))/(6*e^(-2*x) + 15*e^(-4*x) + 20*e^(-6*x) + 15*e^(-8*x) + 6*e^(-
10*x) + e^(-12*x) + 1) - 1/8*arctan(e^(-x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(30) = 60$.

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.92

$$\int \operatorname{sech}^3(x)\tanh^4(x)\,dx = \frac{1}{32}\,\pi - \frac{3\,\left(e^{(-x)} - e^{x}\right)^5 - 32\,\left(e^{(-x)} - e^{x}\right)^3 - 48\,e^{(-x)} + 48\,e^{x}}{24\,\left(\left(e^{(-x)} - e^{x}\right)^2 + 4\right)^3} + \frac{1}{16}\arctan\left(\frac{1}{2}\,\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)$$

input
```
integrate(sech(x)^3*tanh(x)^4,x, algorithm="giac")
```

output
```
1/32*pi - 1/24*(3*(e^(-x) - e^x)^5 - 32*(e^(-x) - e^x)^3 - 48*e^(-x) + 48*
e^x)/((e^(-x) - e^x)^2 + 4)^3 + 1/16*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 200, normalized size of antiderivative = 5.26

$$\int \operatorname{sech}^3(x)\tanh^4(x)\,dx = \frac{\operatorname{atan}(e^x)}{8} - \frac{10\,e^x}{4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1} + \frac{e^x}{8\,(e^{2\,x}+1)} + \frac{7\,e^x}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1} - \frac{4\,e^{5\,x}-\frac{8\,e^{3\,x}}{3}-\frac{8\,e^{7\,x}}{3}+\frac{2\,e^{9\,x}}{3}+\frac{2\,e^x}{3}}{6\,e^{2\,x}+15\,e^{4\,x}+20\,e^{6\,x}+15\,e^{8\,x}+6\,e^{10\,x}+e^{12\,x}+1} + \frac{16\,e^x}{3\,(5\,e^{2\,x}+10\,e^{4\,x}+10\,e^{6\,x}+5\,e^{8\,x}+e^{10\,x}+1)} - \frac{23\,e^x}{12\,(2\,e^{2\,x}+e^{4\,x}+1)}$$

input
```
int(tanh(x)^4/cosh(x)^3,x)
```

output
```
atan(exp(x))/8 - (10*exp(x))/(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8
*x) + 1) + exp(x)/(8*(exp(2*x) + 1)) + (7*exp(x))/(3*exp(2*x) + 3*exp(4*x)
 + exp(6*x) + 1) - (4*exp(5*x) - (8*exp(3*x))/3 - (8*exp(7*x))/3 + (2*exp(
9*x))/3 + (2*exp(x))/3)/(6*exp(2*x) + 15*exp(4*x) + 20*exp(6*x) + 15*exp(8
*x) + 6*exp(10*x) + exp(12*x) + 1) + (16*exp(x))/(3*(5*exp(2*x) + 10*exp(4
*x) + 10*exp(6*x) + 5*exp(8*x) + exp(10*x) + 1)) - (23*exp(x))/(12*(2*exp(
2*x) + exp(4*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 160, normalized size of antiderivative = 4.21

$$\int \operatorname{sech}^3(x)\tanh^4(x)\,dx = \frac{3e^{12x}atan(e^x)+18e^{10x}atan(e^x)+45e^{8x}atan(e^x)+60e^{6x}atan(e^x)+45e^{4x}atan(e^x)+18e^{2x}atan(e^x)+3e}{24e^{12x}+144e^{10x}+360e^{8x}+480e^{6x}+360e^{4x}+144e^{2x}}$$

input
```
int(sech(x)^3*tanh(x)^4,x)
```

output

```
(3*e**(12*x)*atan(e**x) + 18*e**(10*x)*atan(e**x) + 45*e**(8*x)*atan(e**x)
 + 60*e**(6*x)*atan(e**x) + 45*e**(4*x)*atan(e**x) + 18*e**(2*x)*atan(e**x
) + 3*atan(e**x) + 3*e**(11*x) - 47*e**(9*x) + 78*e**(7*x) - 78*e**(5*x) +
 47*e**(3*x) - 3*e**x)/(24*(e**(12*x) + 6*e**(10*x) + 15*e**(8*x) + 20*e**
(6*x) + 15*e**(4*x) + 6*e**(2*x) + 1))
```

3.78 $\int \operatorname{sech}^5(x) \tanh^2(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 36

$$\int \operatorname{sech}^5(x) \tanh^2(x) \, dx = \frac{1}{16} \arctan(\sinh(x)) + \frac{1}{16} \operatorname{sech}(x) \tanh(x) + \frac{1}{24} \operatorname{sech}^3(x) \tanh(x) - \frac{1}{6} \operatorname{sech}^5(x) \tanh(x)$$

output
```
1/16*arctan(sinh(x))+1/16*sech(x)*tanh(x)+1/24*sech(x)^3*tanh(x)-1/6*sech(
x)^5*tanh(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^5(x) \tanh^2(x) \, dx = \frac{1}{16} \arctan(\sinh(x)) + \frac{1}{16} \operatorname{sech}(x) \tanh(x) + \frac{1}{24} \operatorname{sech}^3(x) \tanh(x) - \frac{1}{6} \operatorname{sech}^5(x) \tanh(x)$$

input
```
Integrate[Sech[x]^5*Tanh[x]^2,x]
```

output
```
ArcTan[Sinh[x]]/16 + (Sech[x]*Tanh[x])/16 + (Sech[x]^3*Tanh[x])/24 - (Sech
[x]^5*Tanh[x])/6
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 25, 3091, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(x)\text{sech}^5(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ix)^2\left(-\sec(ix)^5\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec(ix)^5\tan(ix)^2dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{1}{6}\int \text{sech}^5(x)dx-\frac{1}{6}\tanh(x)\text{sech}^5(x)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{6}\tanh(x)\text{sech}^5(x)+\frac{1}{6}\int \csc\left(ix+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{6}\left(\frac{3}{4}\int \text{sech}^3(x)dx+\frac{1}{4}\tanh(x)\text{sech}^3(x)\right)-\frac{1}{6}\tanh(x)\text{sech}^5(x)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{6}\tanh(x)\text{sech}^5(x)+\frac{1}{6}\left(\frac{1}{4}\tanh(x)\text{sech}^3(x)+\frac{3}{4}\int \csc\left(ix+\frac{\pi}{2}\right)^3dx\right)$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{6}\left(\frac{3}{4}\left(\frac{\int \text{sech}(x)dx}{2}+\frac{1}{2}\tanh(x)\text{sech}(x)\right)+\frac{1}{4}\tanh(x)\text{sech}^3(x)\right)-\frac{1}{6}\tanh(x)\text{sech}^5(x)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{6}\tanh(x)\text{sech}^5(x)+$$
$$\frac{1}{6}\left(\frac{1}{4}\tanh(x)\text{sech}^3(x)+\frac{3}{4}\left(\frac{1}{2}\tanh(x)\text{sech}(x)+\frac{1}{2}\int \csc\left(ix+\frac{\pi}{2}\right)dx\right)\right)$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\arctan(\sinh(x))+\frac{1}{2}\tanh(x)\text{sech}(x)\right)+\frac{1}{4}\tanh(x)\text{sech}^3(x)\right)-\frac{1}{6}\tanh(x)\text{sech}^5(x)$$

input
```
Int[Sech[x]^5*Tanh[x]^2,x]
```

output
```
-1/6*(Sech[x]^5*Tanh[x]) + ((Sech[x]^3*Tanh[x])/4 + (3*(ArcTan[Sinh[x]]/2
+ (Sech[x]*Tanh[x])/2))/4)/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 9.53 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{\sinh(x)}{5\cosh(x)^6}+\frac{\left(\frac{\operatorname{sech}(x)^5}{6}+\frac{5\operatorname{sech}(x)^3}{24}+\frac{5\operatorname{sech}(x)}{16}\right)\tanh(x)}{5}+\frac{\arctan(e^x)}{8}$	36
risch	$\frac{e^x(3e^{10x}+17e^{8x}-114e^{6x}+114e^{4x}-17e^{2x}-3)}{24(e^{2x}+1)^6}+\frac{i\ln(e^x+i)}{16}-\frac{i\ln(e^x-i)}{16}$	64

input
```
int(sech(x)^5*tanh(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5*sinh(x)/cosh(x)^6+1/5*(1/6*sech(x)^5+5/24*sech(x)^3+5/16*sech(x))*tan
h(x)+1/8*arctan(exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. 2(28) = 56.

Time = 0.09 (sec) , antiderivative size = 925, normalized size of antiderivative = 25.69

$$\int \operatorname{sech}^5(x)\tanh^2(x)\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^5*tanh(x)^2,x, algorithm="fricas")
```

output
```
1/24*(3*cosh(x)^11 + 33*cosh(x)*sinh(x)^10 + 3*sinh(x)^11 + (165*cosh(x)^2
 + 17)*sinh(x)^9 + 17*cosh(x)^9 + 9*(55*cosh(x)^3 + 17*cosh(x))*sinh(x)^8
+ 6*(165*cosh(x)^4 + 102*cosh(x)^2 - 19)*sinh(x)^7 - 114*cosh(x)^7 + 42*(3
3*cosh(x)^5 + 34*cosh(x)^3 - 19*cosh(x))*sinh(x)^6 + 6*(231*cosh(x)^6 + 35
7*cosh(x)^4 - 399*cosh(x)^2 + 19)*sinh(x)^5 + 114*cosh(x)^5 + 6*(165*cosh(
x)^7 + 357*cosh(x)^5 - 665*cosh(x)^3 + 95*cosh(x))*sinh(x)^4 + (495*cosh(x
)^8 + 1428*cosh(x)^6 - 3990*cosh(x)^4 + 1140*cosh(x)^2 - 17)*sinh(x)^3 - 1
7*cosh(x)^3 + 3*(55*cosh(x)^9 + 204*cosh(x)^7 - 798*cosh(x)^5 + 380*cosh(x
)^3 - 17*cosh(x))*sinh(x)^2 + 3*(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh
(x)^12 + 6*(11*cosh(x)^2 + 1)*sinh(x)^10 + 6*cosh(x)^10 + 20*(11*cosh(x)^3
 + 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^8 +
 15*cosh(x)^8 + 24*(33*cosh(x)^5 + 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4
*(231*cosh(x)^6 + 315*cosh(x)^4 + 105*cosh(x)^2 + 5)*sinh(x)^6 + 20*cosh(x
)^6 + 24*(33*cosh(x)^7 + 63*cosh(x)^5 + 35*cosh(x)^3 + 5*cosh(x))*sinh(x)^
5 + 15*(33*cosh(x)^8 + 84*cosh(x)^6 + 70*cosh(x)^4 + 20*cosh(x)^2 + 1)*sin
h(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x)^9 + 36*cosh(x)^7 + 42*cosh(x)^5 + 2
0*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 + 45*cosh(x)^8 + 70*
cosh(x)^6 + 50*cosh(x)^4 + 15*cosh(x)^2 + 1)*sinh(x)^2 + 6*cosh(x)^2 + 12*
(cosh(x)^11 + 5*cosh(x)^9 + 10*cosh(x)^7 + 10*cosh(x)^5 + 5*cosh(x)^3 + co
sh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 3*(11*cosh(x)^10 + 51*c...
```

Sympy [F]

$$\int \operatorname{sech}^5(x)\tanh^2(x)\,dx = \int \tanh^2(x)\operatorname{sech}^5(x)\,dx$$

input
```
integrate(sech(x)**5*tanh(x)**2,x)
```

output
```
Integral(tanh(x)**2*sech(x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(28) = 56$.

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.36

$$\int \operatorname{sech}^5(x)\tanh^2(x)\,dx = \frac{3\,e^{(-x)} + 17\,e^{(-3\,x)} - 114\,e^{(-5\,x)} + 114\,e^{(-7\,x)} - 17\,e^{(-9\,x)} - 3\,e^{(-11\,x)}}{24\,(6\,e^{(-2\,x)} + 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} + 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} + e^{(-12\,x)} + 1)} - \frac{1}{8}\arctan\left(e^{(-x)}\right)$$

input
```
integrate(sech(x)^5*tanh(x)^2,x, algorithm="maxima")
```

output
```
1/24*(3*e^(-x) + 17*e^(-3*x) - 114*e^(-5*x) + 114*e^(-7*x) - 17*e^(-9*x) -
 3*e^(-11*x))/(6*e^(-2*x) + 15*e^(-4*x) + 20*e^(-6*x) + 15*e^(-8*x) + 6*e^
(-10*x) + e^(-12*x) + 1) - 1/8*arctan(e^(-x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(28) = 56$.

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.03

$$\int \operatorname{sech}^5(x)\tanh^2(x)\,dx = \frac{1}{32}\,\pi - \frac{3\,\left(e^{(-x)} - e^{x}\right)^5 + 32\,\left(e^{(-x)} - e^{x}\right)^3 - 48\,e^{(-x)} + 48\,e^{x}}{24\,\left(\left(e^{(-x)} - e^{x}\right)^2 + 4\right)^3} + \frac{1}{16}\arctan\left(\frac{1}{2}\,\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)$$

input
```
integrate(sech(x)^5*tanh(x)^2,x, algorithm="giac")
```

output
```
1/32*pi - 1/24*(3*(e^(-x) - e^x)^5 + 32*(e^(-x) - e^x)^3 - 48*e^(-x) + 48*
e^x)/((e^(-x) - e^x)^2 + 4)^3 + 1/16*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 206, normalized size of antiderivative = 5.72

$$\int \operatorname{sech}^5(x)\tanh^2(x)\,dx = \frac{\operatorname{atan}(e^x)}{8} + \frac{34\,e^x}{15\,\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)} + \frac{e^x}{8\,\left(e^{2\,x}+1\right)} - \frac{9\,e^x}{5\,\left(3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1\right)} - \frac{\frac{8\,e^{3\,x}}{3}-\frac{16\,e^{5\,x}}{3}+\frac{8\,e^{7\,x}}{3}}{6\,e^{2\,x}+15\,e^{4\,x}+20\,e^{6\,x}+15\,e^{8\,x}+6\,e^{10\,x}+e^{12\,x}+1} - \frac{\frac{28\,e^{5\,x}}{15}-\frac{8\,e^{3\,x}}{3}+\frac{4\,e^x}{5}}{5\,e^{2\,x}+10\,e^{4\,x}+10\,e^{6\,x}+5\,e^{8\,x}+e^{10\,x}+1} + \frac{e^x}{12\,\left(2\,e^{2\,x}+e^{4\,x}+1\right)}$$

input
```
int(tanh(x)^2/cosh(x)^5,x)
```

output
```
atan(exp(x))/8 + (34*exp(x))/(15*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + e
xp(8*x) + 1)) + exp(x)/(8*(exp(2*x) + 1)) - (9*exp(x))/(5*(3*exp(2*x) + 3*
exp(4*x) + exp(6*x) + 1)) - ((8*exp(3*x))/3 - (16*exp(5*x))/3 + (8*exp(7*x
))/3)/(6*exp(2*x) + 15*exp(4*x) + 20*exp(6*x) + 15*exp(8*x) + 6*exp(10*x)
+ exp(12*x) + 1) - ((28*exp(5*x))/15 - (8*exp(3*x))/3 + (4*exp(x))/5)/(5*e
xp(2*x) + 10*exp(4*x) + 10*exp(6*x) + 5*exp(8*x) + exp(10*x) + 1) + exp(x)
/(12*(2*exp(2*x) + exp(4*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 160, normalized size of antiderivative = 4.44

$$\int \operatorname{sech}^5(x)\tanh^2(x)\,dx = \frac{3e^{12x}atan(e^x)+18e^{10x}atan(e^x)+45e^{8x}atan(e^x)+60e^{6x}atan(e^x)+45e^{4x}atan(e^x)+18e^{2x}atan(e^x)+3e}{24e^{12x}+144e^{10x}+360e^{8x}+480e^{6x}+360e^{4x}+144e}$$

input
```
int(sech(x)^5*tanh(x)^2,x)
```

output
```
(3*e**(12*x)*atan(e**x) + 18*e**(10*x)*atan(e**x) + 45*e**(8*x)*atan(e**x)
 + 60*e**(6*x)*atan(e**x) + 45*e**(4*x)*atan(e**x) + 18*e**(2*x)*atan(e**x
) + 3*atan(e**x) + 3*e**(11*x) + 17*e**(9*x) - 114*e**(7*x) + 114*e**(5*x)
 - 17*e**(3*x) - 3*e**x)/(24*(e**(12*x) + 6*e**(10*x) + 15*e**(8*x) + 20*e
**(6*x) + 15*e**(4*x) + 6*e**(2*x) + 1))
```

3.79 $\int \operatorname{sech}^8(x)\tanh^6(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 33

$$\int \operatorname{sech}^8(x)\tanh^6(x)\,dx=\frac{\tanh^7(x)}{7}-\frac{\tanh^9(x)}{3}+\frac{3\tanh^{11}(x)}{11}-\frac{\tanh^{13}(x)}{13}$$

output
```
1/7*tanh(x)^7-1/3*tanh(x)^9+3/11*tanh(x)^11-1/13*tanh(x)^13
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 67 vs. 2(33) = 66.

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.03

$$\int \operatorname{sech}^8(x)\tanh^6(x)\,dx=\frac{16\tanh(x)}{3003}+\frac{8\operatorname{sech}^2(x)\tanh(x)}{3003}+\frac{2\operatorname{sech}^4(x)\tanh(x)}{1001}+\frac{5\operatorname{sech}^6(x)\tanh(x)}{3003}-\frac{53}{429}\operatorname{sech}^8(x)\tanh(x)+\frac{27}{143}\operatorname{sech}^{10}(x)\tanh(x)-\frac{1}{13}\operatorname{sech}^{12}(x)\tanh(x)$$

input
```
Integrate[Sech[x]^8*Tanh[x]^6,x]
```

output
```
(16*Tanh[x])/3003 + (8*Sech[x]^2*Tanh[x])/3003 + (2*Sech[x]^4*Tanh[x])/100
1 + (5*Sech[x]^6*Tanh[x])/3003 - (53*Sech[x]^8*Tanh[x])/429 + (27*Sech[x]^
10*Tanh[x])/143 - (Sech[x]^12*Tanh[x])/13
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 25, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^6(x)\text{sech}^8(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ix)^6\left(-\sec(ix)^8\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec(ix)^8\tan(ix)^6dx$$

$$\Big\downarrow \text{3087}$$

$$i\int -\tanh^6(x)\left(1-\tanh^2(x)\right)^3d(i\tanh(x))$$

$$\Big\downarrow \text{244}$$

$$i\int \left(\tanh^{12}(x)-3\tanh^{10}(x)+3\tanh^8(x)-\tanh^6(x)\right)d(i\tanh(x))$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{1}{13}i\tanh^{13}(x)-\frac{3}{11}i\tanh^{11}(x)+\frac{1}{3}i\tanh^9(x)-\frac{1}{7}i\tanh^7(x)\right)$$

input
```
Int[Sech[x]^8*Tanh[x]^6,x]
```

output
```
I*((-1/7*I)*Tanh[x]^7 + (I/3)*Tanh[x]^9 - ((3*I)/11)*Tanh[x]^11 + (I/13)*T
anh[x]^13)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 245.96 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\tanh(x)^7}{7}-\frac{\tanh(x)^9}{3}+\frac{3\tanh(x)^{11}}{11}-\frac{\tanh(x)^{13}}{13}$	26
default	$\frac{\tanh(x)^7}{7}-\frac{\tanh(x)^9}{3}+\frac{3\tanh(x)^{11}}{11}-\frac{\tanh(x)^{13}}{13}$	26
risch	$-\frac{32(3003\,e^{18x}-9009\,e^{16x}+18018\,e^{14x}-16302\,e^{12x}+10296\,e^{10x}-2288\,e^{8x}+286\,e^{6x}+78\,e^{4x}+13\,e^{2x}+1)}{3003(e^{2x}+1)^{13}}$	67

input
```
int(sech(x)^8*tanh(x)^6,x,method=_RETURNVERBOSE)
```

output
```
1/7*tanh(x)^7-1/3*tanh(x)^9+3/11*tanh(x)^11-1/13*tanh(x)^13
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 778 vs. 2(25) = 50.

Time = 0.09 (sec) , antiderivative size = 778, normalized size of antiderivative = 23.58

$$\int \operatorname{sech}^8(x) \tanh^6(x) \, dx = \text{Too large to display}$$

input
```
integrate(sech(x)^8*tanh(x)^6,x, algorithm="fricas")
```

output
```
-64/3003*(1502*cosh(x)^9 + 13518*cosh(x)*sinh(x)^8 + 1501*sinh(x)^9 + (540
36*cosh(x)^2 - 4511)*sinh(x)^7 - 4498*cosh(x)^7 + 14*(9012*cosh(x)^3 - 224
9*cosh(x))*sinh(x)^6 + 3*(63042*cosh(x)^4 - 31577*cosh(x)^2 + 2990)*sinh(x
)^5 + 9048*cosh(x)^5 + 2*(94626*cosh(x)^5 - 78715*cosh(x)^3 + 22620*cosh(x
))*sinh(x)^4 + (126084*cosh(x)^6 - 157885*cosh(x)^4 + 89700*cosh(x)^2 - 82
94)*sinh(x)^3 - 8008*cosh(x)^3 + 6*(9012*cosh(x)^7 - 15743*cosh(x)^5 + 150
80*cosh(x)^3 - 4004*cosh(x))*sinh(x)^2 + (13509*cosh(x)^8 - 31577*cosh(x)^
6 + 44850*cosh(x)^4 - 24882*cosh(x)^2 + 6292)*sinh(x) + 4004*cosh(x))/(cos
h(x)^17 + 17*cosh(x)*sinh(x)^16 + sinh(x)^17 + (136*cosh(x)^2 + 13)*sinh(x
)^15 + 13*cosh(x)^15 + 5*(136*cosh(x)^3 + 39*cosh(x))*sinh(x)^14 + (2380*c
osh(x)^4 + 1365*cosh(x)^2 + 78)*sinh(x)^13 + 78*cosh(x)^13 + 13*(476*cosh(
x)^5 + 455*cosh(x)^3 + 78*cosh(x))*sinh(x)^12 + 13*(952*cosh(x)^6 + 1365*c
osh(x)^4 + 468*cosh(x)^2 + 22)*sinh(x)^11 + 286*cosh(x)^11 + 143*(136*cosh
(x)^7 + 273*cosh(x)^5 + 156*cosh(x)^3 + 22*cosh(x))*sinh(x)^10 + (24310*co
sh(x)^8 + 65065*cosh(x)^6 + 55770*cosh(x)^4 + 15730*cosh(x)^2 + 714)*sinh(
x)^9 + 716*cosh(x)^9 + (24310*cosh(x)^9 + 83655*cosh(x)^7 + 100386*cosh(x)
^5 + 47190*cosh(x)^3 + 6444*cosh(x))*sinh(x)^8 + (19448*cosh(x)^10 + 83655
*cosh(x)^8 + 133848*cosh(x)^6 + 94380*cosh(x)^4 + 25704*cosh(x)^2 + 1274)*
sinh(x)^7 + 1300*cosh(x)^7 + (12376*cosh(x)^11 + 65065*cosh(x)^9 + 133848*
cosh(x)^7 + 132132*cosh(x)^5 + 60144*cosh(x)^3 + 9100*cosh(x))*sinh(x)^...
```

Sympy [F]

$$\int \operatorname{sech}^8(x)\tanh^6(x)\,dx = \int \tanh^6(x)\operatorname{sech}^8(x)\,dx$$

input
```
integrate(sech(x)**8*tanh(x)**6,x)
```

output
```
Integral(tanh(x)**6*sech(x)**8, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(25) = 50$.

Time = 0.05 (sec) , antiderivative size = 857, normalized size of antiderivative = 25.97

$$\int \operatorname{sech}^8(x)\tanh^6(x)\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^8*tanh(x)^6,x, algorithm="maxima")
```

output
```
32/231*e^(-2*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) +
 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e
^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) +
64/77*e^(-4*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) +
1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^
(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 6
4/21*e^(-6*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1
287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(
-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) - 51
2/21*e^(-8*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1
287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(
-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 76
8/7*e^(-10*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1
287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(
-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) - 12
16/7*e^(-12*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) +
1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^
(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 1
92*e^(-14*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 12
87*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x) + 715*e...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(25) = 50$.

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.00

$$\int \operatorname{sech}^8(x)\tanh^6(x)\,dx = -\frac{32\left(3003\,e^{(18\,x)} - 9009\,e^{(16\,x)} + 18018\,e^{(14\,x)} - 16302\,e^{(12\,x)} + 10296\,e^{(10\,x)} - 2288\,e^{(8\,x)} + 286\,e^{(6\,x)} + 78\right.}{3003\,\left(e^{(2\,x)} + 1\right)^{13}}$$

input
```
integrate(sech(x)^8*tanh(x)^6,x, algorithm="giac")
```

output
```
-32/3003*(3003*e^(18*x) - 9009*e^(16*x) + 18018*e^(14*x) - 16302*e^(12*x)
+ 10296*e^(10*x) - 2288*e^(8*x) + 286*e^(6*x) + 78*e^(4*x) + 13*e^(2*x) +
1)/(e^(2*x) + 1)^13
```

Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 820, normalized size of antiderivative = 24.85

$$\int \operatorname{sech}^8(x) \tanh^6(x) \, dx = \text{Too large to display}$$

input
```
int(tanh(x)^6/cosh(x)^8,x)
```

output
```
- ((64*exp(4*x))/143 - (256*exp(2*x))/429 + 80/429)/(6*exp(2*x) + 15*exp(4
*x) + 20*exp(6*x) + 15*exp(8*x) + 6*exp(10*x) + exp(12*x) + 1) - ((64*exp(
2*x))/143 - (768*exp(4*x))/143 + (3200*exp(6*x))/143 - (6400*exp(8*x))/143
 + (6720*exp(10*x))/143 - (3584*exp(12*x))/143 + (768*exp(14*x))/143)/(11*
exp(2*x) + 55*exp(4*x) + 165*exp(6*x) + 330*exp(8*x) + 462*exp(10*x) + 462
*exp(12*x) + 330*exp(14*x) + 165*exp(16*x) + 55*exp(18*x) + 11*exp(20*x) +
 exp(22*x) + 1) - ((160*exp(2*x))/143 - (256*exp(4*x))/143 + (128*exp(6*x)
)/143 - 640/3003)/(7*exp(2*x) + 21*exp(4*x) + 35*exp(6*x) + 35*exp(8*x) +
21*exp(10*x) + 7*exp(12*x) + exp(14*x) + 1) - ((128*exp(6*x))/13 - (768*ex
p(8*x))/13 + (1920*exp(10*x))/13 - (2560*exp(12*x))/13 + (1920*exp(14*x))/
13 - (768*exp(16*x))/13 + (128*exp(18*x))/13)/(13*exp(2*x) + 78*exp(4*x) +
 286*exp(6*x) + 715*exp(8*x) + 1287*exp(10*x) + 1716*exp(12*x) + 1716*exp(
14*x) + 1287*exp(16*x) + 715*exp(18*x) + 286*exp(20*x) + 78*exp(22*x) + 13
*exp(24*x) + exp(26*x) + 1) - ((560*exp(4*x))/143 - (640*exp(2*x))/429 - (
1792*exp(6*x))/429 + (224*exp(8*x))/143 + 80/429)/(8*exp(2*x) + 28*exp(4*x
) + 56*exp(6*x) + 70*exp(8*x) + 56*exp(10*x) + 28*exp(12*x) + 8*exp(14*x)
+ exp(16*x) + 1) - ((640*exp(2*x))/429 - (2560*exp(4*x))/429 + (4480*exp(6
*x))/429 - (3584*exp(8*x))/429 + (1792*exp(10*x))/715 - 256/2145)/(9*exp(2
*x) + 36*exp(4*x) + 84*exp(6*x) + 126*exp(8*x) + 126*exp(10*x) + 84*exp(12
*x) + 36*exp(14*x) + 9*exp(16*x) + exp(18*x) + 1) - ((32*exp(4*x))/13 -...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 162, normalized size of antiderivative = 4.91

$$\int \operatorname{sech}^8(x) \tanh^6(x) \, dx = \frac{-96096e^{18x} + 288288e^{16x} - 576576e^{14x} + 521664e^{12x} - 329472e^{10x} + }{3003e^{26x} + 39039e^{24x} + 234234e^{22x} + 858858e^{20x} + 2147145e^{18x} + 3864861e^{16x} + 5153148e^{14x} + 515314}$$

input
```
int(sech(x)^8*tanh(x)^6,x)
```

output
```
(32*( - 3003*e**(18*x) + 9009*e**(16*x) - 18018*e**(14*x) + 16302*e**(12*x
) - 10296*e**(10*x) + 2288*e**(8*x) - 286*e**(6*x) - 78*e**(4*x) - 13*e**(
2*x) - 1))/(3003*(e**(26*x) + 13*e**(24*x) + 78*e**(22*x) + 286*e**(20*x)
+ 715*e**(18*x) + 1287*e**(16*x) + 1716*e**(14*x) + 1716*e**(12*x) + 1287*
e**(10*x) + 715*e**(8*x) + 286*e**(6*x) + 78*e**(4*x) + 13*e**(2*x) + 1))
```

3.80 $\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx=\frac{\log(\tanh(a+bx))}{b}$$

output
```
ln(tanh(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cosh(2a+2bx))}{b}$$

input
```
Integrate[Csch[a + b*x]*Sech[a + b*x],x]
```

output
```
-(ArcTanh[Cosh[2*a + 2*b*x]]/b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 26, 3100, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\csc(ia+ibx)\sec(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$i\int \csc(ia+ibx)\sec(ia+ibx)dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int -i\coth(a+bx)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{14}$$

$$\frac{\log(i\tanh(a+bx))}{b}$$

input
```
Int[Csch[a + b*x]*Sech[a + b*x],x]
```

output
```
Log[I*Tanh[a + b*x]]/b
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\ln(\tanh(bx+a))}{b}$	12
default	$\frac{\ln(\tanh(bx+a))}{b}$	12
risch	$-\frac{\ln(e^{2bx+2a}+1)}{b}+\frac{\ln(e^{2bx+2a}-1)}{b}$	35

input
```
int(csch(b*x+a)*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
ln(tanh(b*x+a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(11) = 22$.

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 5.45

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\log\left(\frac{2\cosh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)-\log\left(\frac{2\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="fricas")
```

output
```
-(log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) - log(2*sinh(b*x +
a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

Sympy [F]

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = \int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a),x)
```

output
```
Integral(csch(a + b*x)*sech(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(11) = 22$.

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 4.55

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{\log\left(e^{(-2bx-2a)}+1\right)}{b}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="maxima")
```

output
```
log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
 1)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(11) = 22$.

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.73

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\log\left(e^{(2\,bx+2\,a)}+1\right)-\log\left(e^{(bx+a)}+1\right)-\log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a),x, algorithm="giac")
```

output
```
-(log(e^(2*b*x + 2*a) + 1) - log(e^(b*x + a) + 1) - log(abs(e^(b*x + a) -
1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.73

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

input
```
int(1/(cosh(a + b*x)*sinh(a + b*x)),x)
```

output
```
-(2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.64

$$\int \operatorname{csch}(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{-\log\left(e^{2bx+2a}+1\right)+\log\left(e^{bx+a}-1\right)+\log\left(e^{bx+a}+1\right)}{b}$$

input
```
int(csch(b*x+a)*sech(b*x+a),x)
```

output
```
( - log(e**(2*a + 2*b*x) + 1) + log(e**(a + b*x) - 1) + log(e**(a + b*x) +
 1))/b
```

3.81 $\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cosh(a+bx))}{b} + \frac{\operatorname{sech}(a+bx)}{b}$$

output
```
-arctanh(cosh(b*x+a))/b+sech(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{\operatorname{sech}(a+bx)}{b}$$

input
```
Integrate[Csch[a + b*x]*Sech[a + b*x]^2,x]
```

output
```
-(Log[Cosh[(a + b*x)/2]]/b) + Log[Sinh[(a + b*x)/2]]/b + Sech[a + b*x]/b
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 26, 3102, 25, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}(a+bx)\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int i\csc(ia+ibx)\sec(ia+ibx)^2dx$$

$$\Big\downarrow \; 26$$

$$i\int \csc(ia+ibx)\sec(ia+ibx)^2dx$$

$$\Big\downarrow \; 3102$$

$$\frac{\int -\frac{\text{sech}^2(a+bx)}{1-\text{sech}^2(a+bx)}d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{\text{sech}^2(a+bx)}{1-\text{sech}^2(a+bx)}d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \; 262$$

$$\frac{\text{sech}(a+bx)-\int \frac{1}{1-\text{sech}^2(a+bx)}d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \; 219$$

$$\frac{\text{sech}(a+bx)-\text{arctanh}(\text{sech}(a+bx))}{b}$$

input
```
Int[Csch[a + b*x]*Sech[a + b*x]^2,x]
```

output
```
(-ArcTanh[Sech[a + b*x]] + Sech[a + b*x])/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{1}{\cosh(bx+a)}-2\,\mathrm{arctanh}(\mathrm{e}^{bx+a})}{b}$	23
default	$\frac{\frac{1}{\cosh(bx+a)}-2\,\mathrm{arctanh}(\mathrm{e}^{bx+a})}{b}$	23
risch	$\frac{2\,\mathrm{e}^{bx+a}}{b(\mathrm{e}^{2bx+2a}+1)}-\frac{\ln(\mathrm{e}^{bx+a}+1)}{b}+\frac{\ln(\mathrm{e}^{bx+a}-1)}{b}$	53

input
```
int(csch(b*x+a)*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(23) = 46.

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 6.74

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx =$$

$$-\frac{\left(\cosh{(bx+a)}^2+2\,\cosh{(bx+a)}\sinh{(bx+a)}+\sinh{(bx+a)}^2+1\right)\log\left(\cosh{(bx+a)}+\sinh{(bx+a}\right.}{b\cosh{(bx}}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
-((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*
log(cosh(b*x + a) + sinh(b*x + a) + 1) - (cosh(b*x + a)^2 + 2*cosh(b*x + a
)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*log(cosh(b*x + a) + sinh(b*x + a) -
 1) - 2*cosh(b*x + a) - 2*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x
 + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

Sympy [F]

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int \operatorname{csch}\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)**2,x)
```

output
```
Integral(csch(a + b*x)*sech(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(23) = 46$.

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.65

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{2\,e^{(-bx-a)}}{b(e^{(-2\,bx-2\,a)}+1)}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
-log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b + 2*e^(-b*x - a)/(b*(e^
(-2*b*x - 2*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(23) = 46$.

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.78

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{\frac{4}{e^{(bx+a)}+e^{(-bx-a)}} - \log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) + \log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{2\,b}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*(4/(e^(b*x + a) + e^(-b*x - a)) - log(e^(b*x + a) + e^(-b*x - a) + 2)
+ log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.26

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

input
```
int(1/(cosh(a + b*x)^2*sinh(a + b*x)),x)
```

output
```
(2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1)) - (2*atan((exp(b*x)*exp(a)*(-b
^2)^(1/2))/b))/(-b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 4.04

$$\begin{aligned}&\int \operatorname{csch}(a+bx)\operatorname{sech}^2(a+bx)\,dx\\&=\frac{e^{2bx+2a}\log\left(e^{bx+a}-1\right)-e^{2bx+2a}\log\left(e^{bx+a}+1\right)+2e^{bx+a}+\log\left(e^{bx+a}-1\right)-\log\left(e^{bx+a}+1\right)}{b\left(e^{2bx+2a}+1\right)}\end{aligned}$$

input
```
int(csch(b*x+a)*sech(b*x+a)^2,x)
```

output
```
(e**(2*a + 2*b*x)*log(e**(a + b*x) - 1) - e**(2*a + 2*b*x)*log(e**(a + b*x
) + 1) + 2*e**(a + b*x) + log(e**(a + b*x) - 1) - log(e**(a + b*x) + 1))/(
b*(e**(2*a + 2*b*x) + 1))
```

3.82 $\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx=\frac{\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

output
```
ln(tanh(b*x+a))/b-1/2*tanh(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx =-\frac{2\log(\cosh(a+bx))-2\log(\sinh(a+bx))-\operatorname{sech}^2(a+bx)}{2b}$$

input
```
Integrate[Csch[a + b*x]*Sech[a + b*x]^3,x]
```

output
```
-1/2*(2*Log[Cosh[a + b*x]] - 2*Log[Sinh[a + b*x]] - Sech[a + b*x]^2)/b
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\csc(ia+ibx)\sec(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int \csc(ia+ibx)\sec(ia+ibx)^3dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int -i\coth(a+bx)\left(1-\tanh^2(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int (i\tanh(a+bx)-i\coth(a+bx))d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}\tanh^2(a+bx)+\log(i\tanh(a+bx))}{b}$$

input
```
Int[Csch[a + b*x]*Sech[a + b*x]^3,x]
```

output
```
(Log[I*Tanh[a + b*x]] - Tanh[a + b*x]^2/2)/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{1}{2\cosh(bx+a)^2}+\ln(\tanh(bx+a))}{b}$	23
default	$\frac{\frac{1}{2\cosh(bx+a)^2}+\ln(\tanh(bx+a))}{b}$	23
risch	$\frac{2\,\mathrm{e}^{2bx+2a}}{b\left(\mathrm{e}^{2bx+2a}+1\right)^2}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}-\frac{\ln\left(\mathrm{e}^{2bx+2a}+1\right)}{b}$	62

input
```
int(csch(b*x+a)*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2/cosh(b*x+a)^2+ln(tanh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(25) = 50$.

Time = 0.11 (sec) , antiderivative size = 371, normalized size of antiderivative = 13.74

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$
$$= \frac{2\cosh(bx+a)^2 - \left(\cosh(bx+a)^4 + 4\cosh(bx+a)\sinh(bx+a)^3 + \sinh(bx+a)^4 + 2\left(3\cosh(bx+a\right.\right.}{}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
(2*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 +
sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x +
 a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(
b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^4 + 4*cosh(b*x
+ a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*
x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*
x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*cosh(
b*x + a)*sinh(b*x + a) + 2*sinh(b*x + a)^2)/(b*cosh(b*x + a)^4 + 4*b*cosh(
b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*
b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x
 + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)**3,x)
```

output
```
Integral(csch(a + b*x)*sech(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(25) = 50$.

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.26

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{2\,e^{(-2\,bx-2\,a)}}{b(2\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}+1)}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
 1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(25) = 50$.

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.44

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{\frac{e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+6}{e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2} - \log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right) + \log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2\right)}{2\,b}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
1/2*((e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 6)/(e^(2*b*x + 2*a) + e^(-2*b*x
 - 2*a) + 2) - log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + log(e^(2*b*x
+ 2*a) + e^(-2*b*x - 2*a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.89

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2}{b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^3*sinh(a + b*x)),x)
```

output
```
2/(b*(exp(2*a + 2*b*x) + 1)) - (2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/
b))/(-b^2)^(1/2) - 2/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 215, normalized size of antiderivative = 7.96

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{-e^{4bx+4a}\log\left(e^{2bx+2a}+1\right)+e^{4bx+4a}\log\left(e^{bx+a}-1\right)+e^{4bx+4a}\log\left(e^{bx+a}+1\right)-e^{4bx+4a}-2e^{2bx+2a}\log\left(e^{2bx+2\cdots}\right.}{b\left(e^{4bx\cdots}\right.}$$

input
```
int(csch(b*x+a)*sech(b*x+a)^3,x)
```

output
```
( - e**(4*a + 4*b*x)*log(e**(2*a + 2*b*x) + 1) + e**(4*a + 4*b*x)*log(e**(
a + b*x) - 1) + e**(4*a + 4*b*x)*log(e**(a + b*x) + 1) - e**(4*a + 4*b*x)
- 2*e**(2*a + 2*b*x)*log(e**(2*a + 2*b*x) + 1) + 2*e**(2*a + 2*b*x)*log(e*
*(a + b*x) - 1) + 2*e**(2*a + 2*b*x)*log(e**(a + b*x) + 1) - log(e**(2*a +
 2*b*x) + 1) + log(e**(a + b*x) - 1) + log(e**(a + b*x) + 1) - 1)/(b*(e**(
4*a + 4*b*x) + 2*e**(2*a + 2*b*x) + 1))
```

3.83 $\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cosh(a+bx))}{b}+\frac{\operatorname{sech}(a+bx)}{b}+\frac{\operatorname{sech}^3(a+bx)}{3b}$$

output
```
-arctanh(cosh(b*x+a))/b+sech(b*x+a)/b+1/3*sech(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx=-\frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\operatorname{sech}(a+bx)}{b}+\frac{\operatorname{sech}^3(a+bx)}{3b}$$

input
```
Integrate[Csch[a + b*x]*Sech[a + b*x]^4,x]
```

output
```
-(Log[Cosh[(a + b*x)/2]]/b) + Log[Sinh[(a + b*x)/2]]/b + Sech[a + b*x]/b +
 Sech[a + b*x]^3/(3*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 26, 3102, 25, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}(a+bx)\text{sech}^4(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int i\csc(ia+ibx)\sec(ia+ibx)^4dx$$

$$\Big\downarrow \ 26$$

$$i\int \csc(ia+ibx)\sec(ia+ibx)^4dx$$

$$\Big\downarrow \ 3102$$

$$\frac{\int -\frac{\text{sech}^4(a+bx)}{1-\text{sech}^2(a+bx)}d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{\text{sech}^4(a+bx)}{1-\text{sech}^2(a+bx)}d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \ 254$$

$$-\frac{\int \left(-\text{sech}^2(a+bx)+\frac{1}{1-\text{sech}^2(a+bx)}-1\right)d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\text{arctanh}(\text{sech}(a+bx))+\frac{1}{3}\text{sech}^3(a+bx)+\text{sech}(a+bx)}{b}$$

input
```
Int[Csch[a + b*x]*Sech[a + b*x]^4,x]
```

output
```
(-ArcTanh[Sech[a + b*x]] + Sech[a + b*x] + Sech[a + b*x]^3/3)/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 4.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\frac{1}{3\cosh(bx+a)^3}+\frac{1}{\cosh(bx+a)}-2\,\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	33
default	$\frac{\frac{1}{3\cosh(bx+a)^3}+\frac{1}{\cosh(bx+a)}-2\,\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	33
risch	$\frac{2\,e^{bx+a}\left(3\,e^{4bx+4a}+10\,e^{2bx+2a}+3\right)}{3b\left(e^{2bx+2a}+1\right)^3}+\frac{\ln\left(e^{bx+a}-1\right)}{b}-\frac{\ln\left(e^{bx+a}+1\right)}{b}$	77

input
```
int(csch(b*x+a)*sech(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3/cosh(b*x+a)^3+1/cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(36) = 72$.

Time = 0.08 (sec) , antiderivative size = 697, normalized size of antiderivative = 18.34

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/3*(6*cosh(b*x + a)^5 + 30*cosh(b*x + a)*sinh(b*x + a)^4 + 6*sinh(b*x + a
)^5 + 20*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 20*cosh(b*x + a)^3 + 60
*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^6 +
6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 +
 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 3*cosh(b*
x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*si
nh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^3
 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1
) + 3*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6
 + 3*(5*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*co
sh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 +
6*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x +
 a)^5 + 2*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x
 + a) + sinh(b*x + a) - 1) + 6*(5*cosh(b*x + a)^4 + 10*cosh(b*x + a)^2 + 1
)*sinh(b*x + a) + 6*cosh(b*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*
sinh(b*x + a)^5 + b*sinh(b*x + a)^6 + 3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*
x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)
)*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 + 6*b*cos
h(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^5 + 2*b*cosh(b*x +
a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx=\int \operatorname{csch}\left(a+bx\right)\operatorname{sech}^4\left(a+bx\right)dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)**4,x)
```

output
```
Integral(csch(a + b*x)*sech(a + b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(36) = 72$.

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.84

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx=-\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{2\left(3\,e^{(-bx-a)}+10\,e^{(-3\,bx-3\,a)}+3\,e^{(-5\,bx-5\,a)}\right)}{3\,b(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1)}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^4,x, algorithm="maxima")
```

output
```
-log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b + 2/3*(3*e^(-b*x - a) +
 10*e^(-3*b*x - 3*a) + 3*e^(-5*b*x - 5*a))/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-
4*b*x - 4*a) + e^(-6*b*x - 6*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(36) = 72$.

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.32

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx=\frac{\frac{4\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2+4\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3}-3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)+3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{6\,b}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^4,x, algorithm="giac")
```

output
```
1/6*(4*(3*(e^(b*x + a) + e^(-b*x - a))^2 + 4)/(e^(b*x + a) + e^(-b*x - a))
^3 - 3*log(e^(b*x + a) + e^(-b*x - a) + 2) + 3*log(e^(b*x + a) + e^(-b*x -
 a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.50

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{8\,\mathrm{e}^{a+b\,x}}{3\,b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)}-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{8\,\mathrm{e}^{a+b\,x}}{3\,b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}+3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}+1\right)} + \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^4*sinh(a + b*x)),x)
```

output
```
(8*exp(a + b*x))/(3*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (2*at
an((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (8*exp(a + b*x))/(3*b
*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + (2*ex
p(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 233, normalized size of antiderivative = 6.13

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{3e^{6bx+6a}\log\left(e^{bx+a}-1\right)-3e^{6bx+6a}\log\left(e^{bx+a}+1\right)+6e^{5bx+5a}+9e^{4bx+4a}\log\left(e^{bx+a}-1\right)-9e^{4bx+4a}\log\left(e^{bx+}\right.}{3b\left(e^{6bx+6a}+\right.}$$

input
```
int(csch(b*x+a)*sech(b*x+a)^4,x)
```

output
```
(3*e**(6*a + 6*b*x)*log(e**(a + b*x) - 1) - 3*e**(6*a + 6*b*x)*log(e**(a +
 b*x) + 1) + 6*e**(5*a + 5*b*x) + 9*e**(4*a + 4*b*x)*log(e**(a + b*x) - 1)
 - 9*e**(4*a + 4*b*x)*log(e**(a + b*x) + 1) + 20*e**(3*a + 3*b*x) + 9*e**(
2*a + 2*b*x)*log(e**(a + b*x) - 1) - 9*e**(2*a + 2*b*x)*log(e**(a + b*x) +
 1) + 6*e**(a + b*x) + 3*log(e**(a + b*x) - 1) - 3*log(e**(a + b*x) + 1))/
(3*b*(e**(6*a + 6*b*x) + 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) + 1))
```

3.84 $\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx=\frac{\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{b}+\frac{\tanh^4(a+bx)}{4b}$$

output
```
ln(tanh(b*x+a))/b-tanh(b*x+a)^2/b+1/4*tanh(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx = -\frac{4\log(\cosh(a+bx))-4\log(\sinh(a+bx))-2\operatorname{sech}^2(a+bx)-\operatorname{sech}^4(a+bx)}{4b}$$

input
```
Integrate[Csch[a + b*x]*Sech[a + b*x]^5,x]
```

output
```
-1/4*(4*Log[Cosh[a + b*x]] - 4*Log[Sinh[a + b*x]] - 2*Sech[a + b*x]^2 - Se
ch[a + b*x]^4)/b
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 26, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}(a+bx)\text{sech}^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\csc(ia+ibx)\sec(ia+ibx)^5dx$$

$$\Big\downarrow \text{26}$$

$$i\int \csc(ia+ibx)\sec(ia+ibx)^5dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int -i\coth(a+bx)\left(1-\tanh^2(a+bx)\right)^2d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int -i\coth(a+bx)\left(1-\tanh^2(a+bx)\right)^2d(-\tanh^2(a+bx))}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(-\tanh^2(a+bx)-i\coth(a+bx)+2\right)d(-\tanh^2(a+bx))}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}\tanh^2(a+bx)+2i\tanh(a+bx)+\log\left(-\tanh^2(a+bx)\right)}{2b}$$

input
```
Int[Csch[a + b*x]*Sech[a + b*x]^5,x]
```

output
```
(Log[-Tanh[a + b*x]^2] + (2*I)*Tanh[a + b*x] - Tanh[a + b*x]^2/2)/(2*b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 11.93 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{1}{4\cosh(bx+a)^4}+\frac{1}{2\cosh(bx+a)^2}+\ln(\tanh(bx+a))}{b}$	33
default	$\frac{\frac{1}{4\cosh(bx+a)^4}+\frac{1}{2\cosh(bx+a)^2}+\ln(\tanh(bx+a))}{b}$	33
risch	$\frac{2\,e^{2bx+2a}\left(e^{4bx+4a}+4\,e^{2bx+2a}+1\right)}{b\left(e^{2bx+2a}+1\right)^4}-\frac{\ln(e^{2bx+2a}+1)}{b}+\frac{\ln(e^{2bx+2a}-1)}{b}$	84

input
```
int(csch(b*x+a)*sech(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4/cosh(b*x+a)^4+1/2/cosh(b*x+a)^2+ln(tanh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1073 vs. 2(38) = 76.

Time = 0.11 (sec) , antiderivative size = 1073, normalized size of antiderivative = 26.82

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^5,x, algorithm="fricas")
```

output
```
(2*cosh(b*x + a)^6 + 12*cosh(b*x + a)*sinh(b*x + a)^5 + 2*sinh(b*x + a)^6
+ 2*(15*cosh(b*x + a)^2 + 4)*sinh(b*x + a)^4 + 8*cosh(b*x + a)^4 + 8*(5*co
sh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a)^3 + 2*(15*cosh(b*x + a)^4 +
 24*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 - (cosh(b*x +
 a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*cosh(b*x
+ a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 + 3
*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(b*x + a)
^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 + 10*co
sh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 +
15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(b*x +
 a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + cosh(
b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x
 + a))) + (cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x +
a)^8 + 4*(7*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(
7*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)
^4 + 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*co
sh(b*x + a)^5 + 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*
(7*cosh(b*x + a)^6 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x
+ a)^2 + 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*co
sh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(b*x + a)/(...
```

Sympy [F]

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx=\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)**5,x)
```

output
```
Integral(csch(a + b*x)*sech(a + b*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(38) = 76$.

Time = 0.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.28

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{2\left(e^{(-2\,bx-2\,a)}+4\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}\right)}{b(4\,e^{(-2\,bx-2\,a)}+6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}+1)}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^5,x, algorithm="maxima")
```

output
```
log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
 1)/b + 2*(e^(-2*b*x - 2*a) + 4*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a))/(b*(4
*e^(-2*b*x - 2*a) + 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x -
8*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(38) = 76$.

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.05

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{\frac{3\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}\right)^2+20\,e^{(2\,bx+2\,a)}+20\,e^{(-2\,bx-2\,a)}+44}{\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right)^2} - 2\,\log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right) + 2\,\log\left(e^{(2\,bx+2\,a)}+\right.}{4\,b}$$

input
```
integrate(csch(b*x+a)*sech(b*x+a)^5,x, algorithm="giac")
```

output
```
1/4*((3*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2 + 20*e^(2*b*x + 2*a) + 20*e
^(-2*b*x - 2*a) + 44)/(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2)^2 - 2*log(e
^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + 2*log(e^(2*b*x + 2*a) + e^(-2*b*x
 - 2*a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 169, normalized size of antiderivative = 4.22

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} + \frac{2}{b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)} - \frac{8}{b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}+3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}+1\right)} + \frac{4}{b\,\left(4\,\mathrm{e}^{2\,a+2\,b\,x}+6\,\mathrm{e}^{4\,a+4\,b\,x}+4\,\mathrm{e}^{6\,a+6\,b\,x}+\mathrm{e}^{8\,a+8\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^5*sinh(a + b*x)),x)
```

output
```
2/(b*(exp(2*a + 2*b*x) + 1)) - (2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/
b))/(-b^2)^(1/2) + 2/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - 8/(
b*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + 4/(b
*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6*a + 6*b*x) + exp(8*a +
 8*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 396, normalized size of antiderivative = 9.90

$$\int \operatorname{csch}(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{-2e^{8bx+8a}\log\left(e^{2bx+2a}+1\right)+2e^{8bx+8a}\log\left(e^{bx+a}-1\right)+2e^{8bx+8a}\log\left(e^{bx+a}+1\right)-e^{8bx+8a}-8e^{6bx+6a}\log\left(e^{2\right.}}{}$$

input
```
int(csch(b*x+a)*sech(b*x+a)^5,x)
```

output
```
( - 2*e**(8*a + 8*b*x)*log(e**(2*a + 2*b*x) + 1) + 2*e**(8*a + 8*b*x)*log(
e**(a + b*x) - 1) + 2*e**(8*a + 8*b*x)*log(e**(a + b*x) + 1) - e**(8*a + 8
*b*x) - 8*e**(6*a + 6*b*x)*log(e**(2*a + 2*b*x) + 1) + 8*e**(6*a + 6*b*x)*
log(e**(a + b*x) - 1) + 8*e**(6*a + 6*b*x)*log(e**(a + b*x) + 1) - 12*e**(
4*a + 4*b*x)*log(e**(2*a + 2*b*x) + 1) + 12*e**(4*a + 4*b*x)*log(e**(a + b
*x) - 1) + 12*e**(4*a + 4*b*x)*log(e**(a + b*x) + 1) + 10*e**(4*a + 4*b*x)
 - 8*e**(2*a + 2*b*x)*log(e**(2*a + 2*b*x) + 1) + 8*e**(2*a + 2*b*x)*log(e
**(a + b*x) - 1) + 8*e**(2*a + 2*b*x)*log(e**(a + b*x) + 1) - 2*log(e**(2*
a + 2*b*x) + 1) + 2*log(e**(a + b*x) - 1) + 2*log(e**(a + b*x) + 1) - 1)/(
2*b*(e**(8*a + 8*b*x) + 4*e**(6*a + 6*b*x) + 6*e**(4*a + 4*b*x) + 4*e**(2*
a + 2*b*x) + 1))
```

3.85 $\int \text{csch}^2(a+bx)\text{sech}(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \text{csch}^2(a+bx)\text{sech}(a+bx)\,dx=-\frac{\arctan(\sinh(a+bx))}{b}-\frac{\text{csch}(a+bx)}{b}$$

output
```
-arctan(sinh(b*x+a))/b-csch(b*x+a)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \text{csch}^2(a+bx)\text{sech}(a+bx)\,dx = -\frac{\text{csch}(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\sinh^2(a+bx)\right)}{b}$$

input
```
Integrate[Csch[a + b*x]^2*Sech[a + b*x],x]
```

output
```
-((Csch[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Sinh[a + b*x]^2])/b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 25, 3101, 25, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^2(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\csc(ia+ibx)^2\sec(ia+ibx)dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \csc(ia+ibx)^2\sec(ia+ibx)dx$$

$$\Big\downarrow \text{ 3101}$$

$$-\frac{i\int \frac{\text{csch}^2(a+bx)}{\text{csch}^2(a+bx)+1}d(-i\text{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{i\int -\frac{\text{csch}^2(a+bx)}{\text{csch}^2(a+bx)+1}d(-i\text{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{ 262}$$

$$-\frac{i\left(-\int \frac{1}{\text{csch}^2(a+bx)+1}d(-i\text{csch}(a+bx))-i\text{csch}(a+bx)\right)}{b}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{i(i\arctan(\text{csch}(a+bx))-i\text{csch}(a+bx))}{b}$$

input
```
Int[Csch[a + b*x]^2*Sech[a + b*x],x]
```

output
```
((-I)*(I*ArcTan[Csch[a + b*x]] - I*Csch[a + b*x]))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{-\frac{1}{\sinh(bx+a)}-2\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	25
default	$\frac{-\frac{1}{\sinh(bx+a)}-2\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	25
risch	$-\frac{2\,\mathrm{e}^{bx+a}}{b\left(\mathrm{e}^{2bx+2a}-1\right)}+\frac{i\ln\left(\mathrm{e}^{bx+a}-i\right)}{b}-\frac{i\ln\left(\mathrm{e}^{bx+a}+i\right)}{b}$	58

input
```
int(csch(b*x+a)^2*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/sinh(b*x+a)-2*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(24) = 48$.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.29

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{2\left(\left(\cosh\left(bx+a\right)^2+2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^2-1\right)\arctan\left(\cosh\left(bx+a\right)+\sinh\left(\right.\right.}{b\cosh\left(bx+a\right)^2+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="fricas")
```

output
```
-2*((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1
)*arctan(cosh(b*x + a) + sinh(b*x + a)) + cosh(b*x + a) + sinh(b*x + a))/(
b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 -
b)
```

Sympy [F]

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = \int \operatorname{csch}^2\left(a+bx\right)\operatorname{sech}\left(a+bx\right)\,dx$$

input
```
integrate(csch(b*x+a)**2*sech(b*x+a),x)
```

output
```
Integral(csch(a + b*x)**2*sech(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx=\frac{2\arctan\left(e^{(-bx-a)}\right)}{b}+\frac{2\,e^{(-bx-a)}}{b(e^{(-2\,bx-2\,a)}-1)}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="maxima")
```

output
```
2*arctan(e^(-b*x - a))/b + 2*e^(-b*x - a)/(b*(e^(-2*b*x - 2*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(24) = 48.

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\pi+\frac{4}{e^{(bx+a)}-e^{(-bx-a)}}+2\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{2\,b}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a),x, algorithm="giac")
```

output
```
-1/2*(pi + 4/(e^(b*x + a) - e^(-b*x - a)) + 2*arctan(1/2*(e^(2*b*x + 2*a)
- 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx=-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(1/(cosh(a + b*x)*sinh(a + b*x)^2),x)
```

output
```
- (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (2*exp(a + b*x))
/(b*(exp(2*a + 2*b*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.38

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{-2e^{2bx+2a}atan\left(e^{bx+a}\right)+2atan\left(e^{bx+a}\right)-2e^{bx+a}}{b\left(e^{2bx+2a}-1\right)}$$

input
```
int(csch(b*x+a)^2*sech(b*x+a),x)
```

output
```
(2*( - e**(2*a + 2*b*x)*atan(e**(a + b*x)) + atan(e**(a + b*x)) - e**(a +
b*x)))/(b*(e**(2*a + 2*b*x) - 1))
```

3.86 $\int \mathbf{csch}^2(a+bx)\mathbf{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{\coth(a+bx)}{b} - \frac{\tanh(a+bx)}{b}$$

output
```
-coth(b*x+a)/b-tanh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{2\coth(2(a+bx))}{b}$$

input
```
Integrate[Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

output
```
(-2*Coth[2*(a + b*x)])/b
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 25, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\csc(ia+ibx)^2\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int \csc(ia+ibx)^2\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{i\int -\coth^2(a+bx)\left(1-\tanh^2(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{i\int \left(1-\coth^2(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(i\tanh(a+bx)+i\coth(a+bx))}{b}$$

input
```
Int[Csch[a + b*x]^2*Sech[a + b*x]^2,x]
```

output
```
(I*(I*Coth[a + b*x] + I*Tanh[a + b*x]))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

method	result	size
derivativedivides	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)}-2\tanh(bx+a)}{b}$	32
default	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)}-2\tanh(bx+a)}{b}$	32
risch	$-\frac{4}{b\left(e^{2bx+2a}-1\right)\left(e^{2bx+2a}+1\right)}$	32

input
```
int(csch(b*x+a)^2*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/sinh(b*x+a)/cosh(b*x+a)-2*tanh(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(23) = 46$.

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.52

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx =$$

$$-\frac{4}{b\cosh\left(bx+a\right)^4+4\,b\cosh\left(bx+a\right)^3\sinh\left(bx+a\right)+6\,b\cosh\left(bx+a\right)^2\sinh\left(bx+a\right)^2+4\,b\cosh\left(bx+a\right.}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
-4/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)^3*sinh(b*x + a) + 6*b*cosh(b*x +
 a)^2*sinh(b*x + a)^2 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a
)^4 - b)
```

Sympy [F]

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int \operatorname{csch}^2\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)dx$$

input
```
integrate(csch(b*x+a)**2*sech(b*x+a)**2,x)
```

output
```
Integral(csch(a + b*x)**2*sech(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{4}{b(e^{(-4\,bx-4\,a)}-1)}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
4/(b*(e^(-4*b*x - 4*a) - 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{4}{b(e^{(4\,bx+4\,a)}-1)}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
-4/(b*(e^(4*b*x + 4*a) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{4}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-1\right)}$$

input
```
int(1/(cosh(a + b*x)^2*sinh(a + b*x)^2),x)
```

output
```
-4/(b*(exp(4*a + 4*b*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{4e^{4bx+4a}}{b\,(e^{4bx+4a}-1)}$$

input
```
int(csch(b*x+a)^2*sech(b*x+a)^2,x)
```

output
```
( - 4*e**(4*a + 4*b*x))/(b*(e**(4*a + 4*b*x) - 1))
```

3.87 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{3\arctan(\sinh(a+bx))}{2b} - \frac{\operatorname{csch}(a+bx)}{b} - \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}$$

output
```
-3/2*arctan(sinh(b*x+a))/b-csch(b*x+a)/b-1/2*sech(b*x+a)*tanh(b*x+a)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{\operatorname{csch}(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},-\sinh^2(a+bx)\right)}{b}$$

input
```
Integrate[Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

output
```
-((Csch[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, -Sinh[a + b*x]^2])/b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 25, 3101, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\csc(ia+ibx)^2\sec(ia+ibx)^3dx$$

$$\Big\downarrow \text{25}$$

$$-\int \csc(ia+ibx)^2\sec(ia+ibx)^3dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{i\int \frac{\operatorname{csch}^4(a+bx)}{\left(\operatorname{csch}^2(a+bx)+1\right)^2}d(-i\operatorname{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{i\left(\frac{i\operatorname{csch}^3(a+bx)}{2\left(\operatorname{csch}^2(a+bx)+1\right)}-\frac{3}{2}\int -\frac{\operatorname{csch}^2(a+bx)}{\operatorname{csch}^2(a+bx)+1}d(-i\operatorname{csch}(a+bx))\right)}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{i\left(\frac{i\operatorname{csch}^3(a+bx)}{2\left(\operatorname{csch}^2(a+bx)+1\right)}-\frac{3}{2}\left(\int \frac{1}{\operatorname{csch}^2(a+bx)+1}d(-i\operatorname{csch}(a+bx))+i\operatorname{csch}(a+bx)\right)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{i\left(\frac{i\operatorname{csch}^3(a+bx)}{2\left(\operatorname{csch}^2(a+bx)+1\right)}-\frac{3}{2}(i\operatorname{csch}(a+bx)-i\arctan(\operatorname{csch}(a+bx)))\right)}{b}$$

input
```
Int[Csch[a + b*x]^2*Sech[a + b*x]^3,x]
```

output
```
((-I)*((-3*((-I)*ArcTan[Csch[a + b*x]] + I*Csch[a + b*x]))/2 + ((I/2)*Csch
[a + b*x]^3)/(1 + Csch[a + b*x]^2)))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)^2}-\frac{3\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2}-3\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	47
default	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)^2}-\frac{3\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2}-3\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	47
risch	$-\frac{\mathrm{e}^{bx+a}\left(3\,\mathrm{e}^{4bx+4a}+2\,\mathrm{e}^{2bx+2a}+3\right)}{b\left(\mathrm{e}^{2bx+2a}+1\right)^2\left(\mathrm{e}^{2bx+2a}-1\right)}+\frac{3i\ln\left(\mathrm{e}^{bx+a}-i\right)}{2b}-\frac{3i\ln\left(\mathrm{e}^{bx+a}+i\right)}{2b}$	95

input
```
int(csch(b*x+a)^2*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/sinh(b*x+a)/cosh(b*x+a)^2-3/2*sech(b*x+a)*tanh(b*x+a)-3*arctan(exp
(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(41) = 82.

Time = 0.08 (sec) , antiderivative size = 511, normalized size of antiderivative = 11.36

$$\int \mathrm{csch}^2(a+bx)\mathrm{sech}^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
-(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 + 3*sinh(b*x + a)^5
 + 2*(15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 2*cosh(b*x + a)^3 + 6*(5*c
osh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^2 + 3*(cosh(b*x + a)^6 + 6*c
osh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2 + 1)*
sinh(b*x + a)^4 + cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + cosh(b*x + a))*
sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 - 1)*sinh(b*x +
a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 - cosh(b
*x + a))*sinh(b*x + a) - 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + 3*(5*c
osh(b*x + a)^4 + 2*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 3*cosh(b*x + a))/(
b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6
+ b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5*b*
cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)^3 - b*cosh(b*x + a)^2 + (
15*b*cosh(b*x + a)^4 + 6*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 2*(3*b*c
osh(b*x + a)^5 + 2*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) - b)
```

Sympy [F]

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**2*sech(b*x+a)**3,x)
```

output
```
Integral(csch(a + b*x)**2*sech(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(41) = 82$.

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.00

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{3\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{3\,e^{(-bx-a)} + 2\,e^{(-3\,bx-3\,a)} + 3\,e^{(-5\,bx-5\,a)}}{b\left(e^{(-2\,bx-2\,a)} - e^{(-4\,bx-4\,a)} - e^{(-6\,bx-6\,a)} + 1\right)}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
3*arctan(e^(-b*x - a))/b - (3*e^(-b*x - a) + 2*e^(-3*b*x - 3*a) + 3*e^(-5*
b*x - 5*a))/(b*(e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - e^(-6*b*x - 6*a) + 1
))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(41) = 82.

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.27

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{3\,\pi + \frac{4\left(3\,\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+8\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+4\,e^{(bx+a)}-4\,e^{(-bx-a)}} + 6\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{4\,b}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/4*(3*pi + 4*(3*(e^(b*x + a) - e^(-b*x - a))^2 + 8)/((e^(b*x + a) - e^(-
b*x - a))^3 + 4*e^(b*x + a) - 4*e^(-b*x - a)) + 6*arctan(1/2*(e^(2*b*x + 2
*a) - 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.38

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{2\,e^{a+b\,x}}{b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} - \frac{3\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^3*sinh(a + b*x)^2),x)
```

output
```
(2*exp(a + b*x))/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (3*atan
((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (2*exp(a + b*x))/(b*(exp(
2*a + 2*b*x) - 1)) - exp(a + b*x)/(b*(exp(2*a + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.20

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^3(a+bx)\,dx$$
$$= \frac{-3e^{6bx+6a}atan\left(e^{bx+a}\right)-3e^{4bx+4a}atan\left(e^{bx+a}\right)+3e^{2bx+2a}atan\left(e^{bx+a}\right)+3atan\left(e^{bx+a}\right)-3e^{5bx+5a}-2e^{3bx+}}{b\left(e^{6bx+6a}+e^{4bx+4a}-e^{2bx+2a}-1\right)}$$

input
```
int(csch(b*x+a)^2*sech(b*x+a)^3,x)
```

output
```
( - 3*e**(6*a + 6*b*x)*atan(e**(a + b*x)) - 3*e**(4*a + 4*b*x)*atan(e**(a
+ b*x)) + 3*e**(2*a + 2*b*x)*atan(e**(a + b*x)) + 3*atan(e**(a + b*x)) - 3
*e**(5*a + 5*b*x) - 2*e**(3*a + 3*b*x) - 3*e**(a + b*x))/(b*(e**(6*a + 6*b
*x) + e**(4*a + 4*b*x) - e**(2*a + 2*b*x) - 1))
```

3.88 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{\coth(a+bx)}{b} - \frac{2\tanh(a+bx)}{b} + \frac{\tanh^3(a+bx)}{3b}$$

output
```
-coth(b*x+a)/b-2*tanh(b*x+a)/b+1/3*tanh(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{\coth(a+bx)}{b} - \frac{5\tanh(a+bx)}{3b} - \frac{\operatorname{sech}^2(a+bx)\tanh(a+bx)}{3b}$$

input
```
Integrate[Csch[a + b*x]^2*Sech[a + b*x]^4,x]
```

output
```
-(Coth[a + b*x]/b) - (5*Tanh[a + b*x])/(3*b) - (Sech[a + b*x]^2*Tanh[a + b
*x])/(3*b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 25, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\csc(ia+ibx)^2\sec(ia+ibx)^4dx$$

$$\Big\downarrow \text{25}$$

$$-\int \csc(ia+ibx)^2\sec(ia+ibx)^4dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{i\int -\coth^2(a+bx)\left(1-\tanh^2(a+bx)\right)^2d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{i\int \left(-\coth^2(a+bx)-\tanh^2(a+bx)+2\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-\frac{1}{3}i\tanh^3(a+bx)+2i\tanh(a+bx)+i\coth(a+bx)\right)}{b}$$

input
```
Int[Csch[a + b*x]^2*Sech[a + b*x]^4,x]
```

output
```
(I*(I*Coth[a + b*x] + (2*I)*Tanh[a + b*x] - (I/3)*Tanh[a + b*x]^3))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 8.78 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

method	result	size
derivativedivides	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)^3}-4\left(\frac{2}{3}+\frac{\operatorname{sech}(bx+a)^2}{3}\right)\tanh(bx+a)}{b}$	44
default	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)^3}-4\left(\frac{2}{3}+\frac{\operatorname{sech}(bx+a)^2}{3}\right)\tanh(bx+a)}{b}$	44
risch	$-\frac{16(2\,\mathrm{e}^{2bx+2a}+1)}{3b(\mathrm{e}^{2bx+2a}+1)^3(\mathrm{e}^{2bx+2a}-1)}$	45

input
```
int(csch(b*x+a)^2*sech(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/sinh(b*x+a)/cosh(b*x+a)^3-4*(2/3+1/3*sech(b*x+a)^2)*tanh(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(36) = 72$.

Time = 0.07 (sec) , antiderivative size = 230, normalized size of antiderivative = 6.05

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx =$$

$$-\frac{}{3\,(b\cosh{(bx+a)}^7 + 7\,b\cosh{(bx+a)}\sinh{(bx+a)}^6 + b\sinh{(bx+a)}^7 + 2\,b\cosh{(bx+a)}^5 + (21\,b\cos}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^4,x, algorithm="fricas")
```

output
```
-16/3*(3*cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^7 + 7*b*cosh(b*x
+ a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 + 2*b*cosh(b*x + a)^5 + (21*b*cos
h(b*x + a)^2 + 2*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 + 2*b*cosh(b*
x + a))*sinh(b*x + a)^4 + 5*(7*b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)^2)*si
nh(b*x + a)^3 + (21*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3)*sinh(b*x + a
)^2 - 3*b*cosh(b*x + a) + (7*b*cosh(b*x + a)^6 + 10*b*cosh(b*x + a)^4 - b)
*sinh(b*x + a))
```

Sympy [F]

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx = \int \operatorname{csch}^2{(a+bx)}\operatorname{sech}^4{(a+bx)}\,dx$$

input
```
integrate(csch(b*x+a)**2*sech(b*x+a)**4,x)
```

output
```
Integral(csch(a + b*x)**2*sech(a + b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(36) = 72$.

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.47

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{32\,e^{(-2\,bx-2\,a)}}{3\,b(2\,e^{(-2\,bx-2\,a)} - 2\,e^{(-6\,bx-6\,a)} - e^{(-8\,bx-8\,a)} + 1)} - \frac{16}{3\,b(2\,e^{(-2\,bx-2\,a)} - 2\,e^{(-6\,bx-6\,a)} - e^{(-8\,bx-8\,a)} + 1)}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^4,x, algorithm="maxima")
```

output
```
-32/3*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) - 2*e^(-6*b*x - 6*a) - e^(-8
*b*x - 8*a) + 1)) - 16/3/(b*(2*e^(-2*b*x - 2*a) - 2*e^(-6*b*x - 6*a) - e^(
-8*b*x - 8*a) + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{2\left(\frac{3}{e^{(2\,bx+2\,a)}-1} - \frac{3\,e^{(4\,bx+4\,a)}+12\,e^{(2\,bx+2\,a)}+5}{\left(e^{(2\,bx+2\,a)}+1\right)^3}\right)}{3\,b}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^4,x, algorithm="giac")
```

output
```
-2/3*(3/(e^(2*b*x + 2*a) - 1) - (3*e^(4*b*x + 4*a) + 12*e^(2*b*x + 2*a) +
5)/(e^(2*b*x + 2*a) + 1)^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 4.00

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{\frac{2}{3\,b}+\frac{4\,e^{2\,a+2\,b\,x}}{b}+\frac{2\,e^{4\,a+4\,b\,x}}{3\,b}}{3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1}+\frac{\frac{2}{b}+\frac{2\,e^{2\,a+2\,b\,x}}{3\,b}}{2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1}-\frac{2}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}+\frac{2}{3\,b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^4*sinh(a + b*x)^2),x)
```

output
```
(2/(3*b) + (4*exp(2*a + 2*b*x))/b + (2*exp(4*a + 4*b*x))/(3*b))/(3*exp(2*a
 + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1) + (2/b + (2*exp(2*a
 + 2*b*x))/(3*b))/(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1) - 2/(b*(exp(
2*a + 2*b*x) - 1)) + 2/(3*b*(exp(2*a + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{-\frac{32e^{2bx+2a}}{3}-\frac{16}{3}}{b\left(e^{8bx+8a}+2e^{6bx+6a}-2e^{2bx+2a}-1\right)}$$

input
```
int(csch(b*x+a)^2*sech(b*x+a)^4,x)
```

output
```
(16*( - 2*e**(2*a + 2*b*x) - 1))/(3*b*(e**(8*a + 8*b*x) + 2*e**(6*a + 6*b*
x) - 2*e**(2*a + 2*b*x) - 1))
```

3.89 $\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx = -\frac{15\arctan(\sinh(a+bx))}{8b} - \frac{\operatorname{csch}(a+bx)}{b} - \frac{9\operatorname{sech}(a+bx)\tanh(a+bx)}{8b} + \frac{\operatorname{sech}(a+bx)\tanh^3(a+bx)}{4b}$$

output
```
-15/8*arctan(sinh(b*x+a))/b-csch(b*x+a)/b-9/8*sech(b*x+a)*tanh(b*x+a)/b+1/
4*sech(b*x+a)*tanh(b*x+a)^3/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.44

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx = -\frac{\operatorname{csch}(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},3,\frac{1}{2},-\sinh^2(a+bx)\right)}{b}$$

input
```
Integrate[Csch[a + b*x]^2*Sech[a + b*x]^5,x]
```

output
```
-((Csch[a + b*x]*Hypergeometric2F1[-1/2, 3, 1/2, -Sinh[a + b*x]^2])/b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.38, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 25, 3101, 25, 252, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\csc(ia+ibx)^2\sec(ia+ibx)^5dx$$

$$\Big\downarrow \text{25}$$

$$-\int \csc(ia+ibx)^2\sec(ia+ibx)^5dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{i\int \frac{\operatorname{csch}^6(a+bx)}{\left(\operatorname{csch}^2(a+bx)+1\right)^3}d(-i\operatorname{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{i\int -\frac{\operatorname{csch}^6(a+bx)}{\left(\operatorname{csch}^2(a+bx)+1\right)^3}d(-i\operatorname{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{i\left(\frac{5}{4}\int \frac{\operatorname{csch}^4(a+bx)}{\left(\operatorname{csch}^2(a+bx)+1\right)^2}d(-i\operatorname{csch}(a+bx))+\frac{i\operatorname{csch}^5(a+bx)}{4\left(\operatorname{csch}^2(a+bx)+1\right)^2}\right)}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{i\left(\frac{5}{4}\left(\frac{i\text{csch}^3(a+bx)}{2\left(\text{csch}^2(a+bx)+1\right)}-\frac{3}{2}\int -\frac{\text{csch}^2(a+bx)}{\text{csch}^2(a+bx)+1}d(-i\text{csch}(a+bx))\right)+\frac{i\text{csch}^5(a+bx)}{4\left(\text{csch}^2(a+bx)+1\right)^2}\right)}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{i\left(\frac{5}{4}\left(\frac{i\text{csch}^3(a+bx)}{2\left(\text{csch}^2(a+bx)+1\right)}-\frac{3}{2}\left(\int \frac{1}{\text{csch}^2(a+bx)+1}d(-i\text{csch}(a+bx))+i\text{csch}(a+bx)\right)\right)+\frac{i\text{csch}^5(a+bx)}{4\left(\text{csch}^2(a+bx)+1\right)^2}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{i\left(\frac{5}{4}\left(\frac{i\text{csch}^3(a+bx)}{2\left(\text{csch}^2(a+bx)+1\right)}-\frac{3}{2}(i\text{csch}(a+bx)-i\arctan(\text{csch}(a+bx)))\right)+\frac{i\text{csch}^5(a+bx)}{4\left(\text{csch}^2(a+bx)+1\right)^2}\right)}{b}$$

input
```
Int[Csch[a + b*x]^2*Sech[a + b*x]^5,x]
```

output
```
((-I)*(((I/4)*Csch[a + b*x]^5)/(1 + Csch[a + b*x]^2)^2 + (5*((-3*((-I)*Arc
Tan[Csch[a + b*x]] + I*Csch[a + b*x]))/2 + ((I/2)*Csch[a + b*x]^3)/(1 + Cs
ch[a + b*x]^2)))/4))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 21.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)^4}-5\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)-\frac{15\arctan\left(e^{bx+a}\right)}{4}}{b}$	60
default	$\frac{-\frac{1}{\sinh(bx+a)\cosh(bx+a)^4}-5\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)-\frac{15\arctan\left(e^{bx+a}\right)}{4}}{b}$	60
risch	$-\frac{e^{bx+a}\left(15\,e^{8bx+8a}+40\,e^{6bx+6a}+18\,e^{4bx+4a}+40\,e^{2bx+2a}+15\right)}{4b\left(e^{2bx+2a}+1\right)^4\left(e^{2bx+2a}-1\right)}+\frac{15i\ln\left(e^{bx+a}-i\right)}{8b}-\frac{15i\ln\left(e^{bx+a}+i\right)}{8b}$	117

input
```
int(csch(b*x+a)^2*sech(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/sinh(b*x+a)/cosh(b*x+a)^4-5*(1/4*sech(b*x+a)^3+3/8*sech(b*x+a))*ta
nh(b*x+a)-15/4*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1183 vs. $2(60) = 120$.

Time = 0.11 (sec) , antiderivative size = 1183, normalized size of antiderivative = 17.92

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^5,x, algorithm="fricas")
```

output
```
-1/4*(15*cosh(b*x + a)^9 + 135*cosh(b*x + a)*sinh(b*x + a)^8 + 15*sinh(b*x
 + a)^9 + 20*(27*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^7 + 40*cosh(b*x + a)^7
 + 140*(9*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^6 + 6*(315*cosh
(b*x + a)^4 + 140*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^5 + 18*cosh(b*x + a)^
5 + 10*(189*cosh(b*x + a)^5 + 140*cosh(b*x + a)^3 + 9*cosh(b*x + a))*sinh(
b*x + a)^4 + 20*(63*cosh(b*x + a)^6 + 70*cosh(b*x + a)^4 + 9*cosh(b*x + a)
^2 + 2)*sinh(b*x + a)^3 + 40*cosh(b*x + a)^3 + 60*(9*cosh(b*x + a)^7 + 14*
cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^2 + 1
5*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^10
+ 3*(15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^8 + 3*cosh(b*x + a)^8 + 24*(5*c
osh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^7 + 2*(105*cosh(b*x + a)^4 +
 42*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 2*cosh(b*x + a)^6 + 12*(21*cosh
(b*x + a)^5 + 14*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^5 + 2*(105
*cosh(b*x + a)^6 + 105*cosh(b*x + a)^4 + 15*cosh(b*x + a)^2 - 1)*sinh(b*x
+ a)^4 - 2*cosh(b*x + a)^4 + 8*(15*cosh(b*x + a)^7 + 21*cosh(b*x + a)^5 +
5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + 3*(15*cosh(b*x + a)^8
 + 28*cosh(b*x + a)^6 + 10*cosh(b*x + a)^4 - 4*cosh(b*x + a)^2 - 1)*sinh(b
*x + a)^2 - 3*cosh(b*x + a)^2 + 2*(5*cosh(b*x + a)^9 + 12*cosh(b*x + a)^7
+ 6*cosh(b*x + a)^5 - 4*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a) -
 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + 5*(27*cosh(b*x + a)^8 + 56*...
```

Sympy [F]

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx = \int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**2*sech(b*x+a)**5,x)
```

output
```
Integral(csch(a + b*x)**2*sech(a + b*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(60) = 120.

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.06

$$\begin{aligned}&\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx\\&= \frac{15\arctan\left(e^{(-bx-a)}\right)}{4b}\\&\quad - \frac{15e^{(-bx-a)} + 40e^{(-3bx-3a)} + 18e^{(-5bx-5a)} + 40e^{(-7bx-7a)} + 15e^{(-9bx-9a)}}{4b(3e^{(-2bx-2a)} + 2e^{(-4bx-4a)} - 2e^{(-6bx-6a)} - 3e^{(-8bx-8a)} - e^{(-10bx-10a)} + 1)}\end{aligned}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^5,x, algorithm="maxima")
```

output
```
15/4*arctan(e^(-b*x - a))/b - 1/4*(15*e^(-b*x - a) + 40*e^(-3*b*x - 3*a) +
 18*e^(-5*b*x - 5*a) + 40*e^(-7*b*x - 7*a) + 15*e^(-9*b*x - 9*a))/(b*(3*e^
(-2*b*x - 2*a) + 2*e^(-4*b*x - 4*a) - 2*e^(-6*b*x - 6*a) - 3*e^(-8*b*x - 8
*a) - e^(-10*b*x - 10*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(60) = 120$.

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.88

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx =$$

$$-\frac{15\,\pi + \frac{4\left(7\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+36\,e^{(bx+a)}-36\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^2} + \frac{32}{e^{(bx+a)}-e^{(-bx-a)}} + 30\,\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-}\right.}{16\,b}$$

input
```
integrate(csch(b*x+a)^2*sech(b*x+a)^5,x, algorithm="giac")
```

output
```
-1/16*(15*pi + 4*(7*(e^(b*x + a) - e^(-b*x - a))^3 + 36*e^(b*x + a) - 36*e
^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4)^2 + 32/(e^(b*x + a) - e^
(-b*x - a)) + 30*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 210, normalized size of antiderivative = 3.18

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

$$= \frac{3\,e^{a+b\,x}}{2\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} - \frac{15\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{4\,\sqrt{b^2}} + \frac{6\,e^{a+b\,x}}{b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)} - \frac{4\,e^{a+b\,x}}{b\,\left(4\,e^{2\,a+2\,b\,x}+6\,e^{4\,a+4\,b\,x}+4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)} - \frac{7\,e^{a+b\,x}}{4\,b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^5*sinh(a + b*x)^2),x)
```

output
```
(3*exp(a + b*x))/(2*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (15*a
tan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(4*(b^2)^(1/2)) + (6*exp(a + b*x))/(
b*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) - (4*e
xp(a + b*x))/(b*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6*a + 6*b
*x) + exp(8*a + 8*b*x) + 1)) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1))
 - (7*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.56

$$\int \operatorname{csch}^2(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{-15e^{10bx+10a}atan\left(e^{bx+a}\right)-45e^{8bx+8a}atan\left(e^{bx+a}\right)-30e^{6bx+6a}atan\left(e^{bx+a}\right)+30e^{4bx+4a}atan\left(e^{bx+a}\right)+45e^{\cdots}}{4b\left(e^{10bx+10a}+3e^{8bx+8a}+2e^{6bx+6a}\cdots\right.}$$

input
```
int(csch(b*x+a)^2*sech(b*x+a)^5,x)
```

output
```
( - 15*e**(10*a + 10*b*x)*atan(e**(a + b*x)) - 45*e**(8*a + 8*b*x)*atan(e*
*(a + b*x)) - 30*e**(6*a + 6*b*x)*atan(e**(a + b*x)) + 30*e**(4*a + 4*b*x)
*atan(e**(a + b*x)) + 45*e**(2*a + 2*b*x)*atan(e**(a + b*x)) + 15*atan(e**
(a + b*x)) - 15*e**(9*a + 9*b*x) - 40*e**(7*a + 7*b*x) - 18*e**(5*a + 5*b*
x) - 40*e**(3*a + 3*b*x) - 15*e**(a + b*x))/(4*b*(e**(10*a + 10*b*x) + 3*e
**(8*a + 8*b*x) + 2*e**(6*a + 6*b*x) - 2*e**(4*a + 4*b*x) - 3*e**(2*a + 2*
b*x) - 1))
```

3.90 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx=-\frac{\coth^2(a+bx)}{2b}-\frac{\log(\tanh(a+bx))}{b}$$

output
```
-1/2*coth(b*x+a)^2/b-ln(tanh(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$
$$=-\frac{\operatorname{csch}^2(a+bx)-2\log(\cosh(a+bx))+2\log(\sinh(a+bx))}{2b}$$

input
```
Integrate[Csch[a + b*x]^3*Sech[a + b*x],x]
```

output
```
-1/2*(Csch[a + b*x]^2 - 2*Log[Cosh[a + b*x]] + 2*Log[Sinh[a + b*x]])/b
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\csc(ia+ibx)^3\sec(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \csc(ia+ibx)^3\sec(ia+ibx)dx$$

$$\Big\downarrow \text{3100}$$

$$-\frac{\int i\coth^3(a+bx)\left(1-\tanh^2(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(i\coth^3(a+bx)-i\coth(a+bx)\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{2}\coth^2(a+bx)+\log(i\tanh(a+bx))}{b}$$

input
```
Int[Csch[a + b*x]^3*Sech[a + b*x],x]
```

output
```
-((Coth[a + b*x]^2/2 + Log[I*Tanh[a + b*x]])/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sinh(bx+a)^2}-\ln(\tanh(bx+a))}{b}$	25
default	$\frac{-\frac{1}{2\sinh(bx+a)^2}-\ln(\tanh(bx+a))}{b}$	25
risch	$-\frac{2\,e^{2bx+2a}}{b\left(e^{2bx+2a}-1\right)^2}+\frac{\ln\left(e^{2bx+2a}+1\right)}{b}-\frac{\ln\left(e^{2bx+2a}-1\right)}{b}$	62

input
```
int(csch(b*x+a)^3*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/sinh(b*x+a)^2-ln(tanh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(26) = 52$.

Time = 0.10 (sec) , antiderivative size = 379, normalized size of antiderivative = 13.54

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx =$$
$$-\frac{2\cosh(bx+a)^2 - \left(\cosh(bx+a)^4 + 4\cosh(bx+a)\sinh(bx+a)^3 + \sinh(bx+a)^4 + 2\left(3\cosh(bx+\right.\right.}{}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="fricas")
```

output
```
-(2*cosh(b*x + a)^2 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 +
 sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x
+ a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh
(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x + a)^4 + 4*cosh(b*x
 + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b
*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b
*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*cosh
(b*x + a)*sinh(b*x + a) + 2*sinh(b*x + a)^2)/(b*cosh(b*x + a)^4 + 4*b*cosh
(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3
*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*
x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**3*sech(b*x+a),x)
```

output
```
Integral(csch(a + b*x)**3*sech(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(26) = 52$.

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.25

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\log\left(e^{(-bx-a)}+1\right)}{b} - \frac{\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{2\,e^{(-2\,bx-2\,a)}}{b(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1)}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="maxima")
```

output
```
-log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b + log(e^(-2*b*x - 2*a)
+ 1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1)
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(26) = 52$.

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.32

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{\frac{e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-6}{e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2} + \log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right) - \log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2\right)}{2\,b}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a),x, algorithm="giac")
```

output
```
1/2*((e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) - 6)/(e^(2*b*x + 2*a) + e^(-2*b*x
 - 2*a) - 2) + log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) - log(e^(2*b*x
+ 2*a) + e^(-2*b*x - 2*a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.79

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)} - \frac{2}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)*sinh(a + b*x)^3),x)
```

output
```
(2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - 2/(b*(exp(2*
a + 2*b*x) - 1)) - 2/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 218, normalized size of antiderivative = 7.79

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{e^{4bx+4a}\log\left(e^{2bx+2a}+1\right) - e^{4bx+4a}\log\left(e^{bx+a}-1\right) - e^{4bx+4a}\log\left(e^{bx+a}+1\right) - e^{4bx+4a} - 2e^{2bx+2a}\log\left(e^{2bx+2a}\right.}{b\left(e^{4bx+\right.}$$

input
```
int(csch(b*x+a)^3*sech(b*x+a),x)
```

output
```
(e**(4*a + 4*b*x)*log(e**(2*a + 2*b*x) + 1) - e**(4*a + 4*b*x)*log(e**(a +
 b*x) - 1) - e**(4*a + 4*b*x)*log(e**(a + b*x) + 1) - e**(4*a + 4*b*x) - 2
*e**(2*a + 2*b*x)*log(e**(2*a + 2*b*x) + 1) + 2*e**(2*a + 2*b*x)*log(e**(a
 + b*x) - 1) + 2*e**(2*a + 2*b*x)*log(e**(a + b*x) + 1) + log(e**(2*a + 2*
b*x) + 1) - log(e**(a + b*x) - 1) - log(e**(a + b*x) + 1) - 1)/(b*(e**(4*a
 + 4*b*x) - 2*e**(2*a + 2*b*x) + 1))
```

3.91 $\int \mathbf{csch}^3(a+bx)\mathbf{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{3\operatorname{arctanh}(\cosh(a+bx))}{2b} - \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{2b} - \frac{\operatorname{sech}(a+bx)}{b}$$

output
```
3/2*arctanh(cosh(b*x+a))/b-1/2*coth(b*x+a)*csch(b*x+a)/b-sech(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.91

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{3\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{3\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{\operatorname{sech}(a+bx)}{b}$$

input
```
Integrate[Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

output
```
-1/8*Csch[(a + b*x)/2]^2/b + (3*Log[Cosh[(a + b*x)/2]])/(2*b) - (3*Log[Sin
h[(a + b*x)/2]])/(2*b) - Sech[(a + b*x)/2]^2/(8*b) - Sech[a + b*x]/b
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 26, 3102, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\csc(ia+ibx)^3\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \csc(ia+ibx)^3\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{3102}$$

$$-\frac{\int \frac{\operatorname{sech}^4(a+bx)}{\left(1-\operatorname{sech}^2(a+bx)\right)^2}d\operatorname{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{\operatorname{sech}^3(a+bx)}{2\left(1-\operatorname{sech}^2(a+bx)\right)}-\frac{3}{2}\int \frac{\operatorname{sech}^2(a+bx)}{1-\operatorname{sech}^2(a+bx)}d\operatorname{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\frac{\operatorname{sech}^3(a+bx)}{2\left(1-\operatorname{sech}^2(a+bx)\right)}-\frac{3}{2}\left(\int \frac{1}{1-\operatorname{sech}^2(a+bx)}d\operatorname{sech}(a+bx)-\operatorname{sech}(a+bx)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\operatorname{sech}^3(a+bx)}{2\left(1-\operatorname{sech}^2(a+bx)\right)}-\frac{3}{2}(\operatorname{arctanh}(\operatorname{sech}(a+bx))-\operatorname{sech}(a+bx))}{b}$$

input
```
Int[Csch[a + b*x]^3*Sech[a + b*x]^2,x]
```

output
```
-(((-3*(ArcTanh[Sech[a + b*x]] - Sech[a + b*x]))/2 + Sech[a + b*x]^3/(2*(1
 - Sech[a + b*x]^2)))/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)}-\frac{3}{2\cosh(bx+a)}+3\,\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	43
default	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)}-\frac{3}{2\cosh(bx+a)}+3\,\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	43
risch	$-\frac{e^{bx+a}\left(3\,e^{4bx+4a}-2\,e^{2bx+2a}+3\right)}{b\left(e^{2bx+2a}+1\right)\left(e^{2bx+2a}-1\right)^2}+\frac{3\ln\left(e^{bx+a}+1\right)}{2b}-\frac{3\ln\left(e^{bx+a}-1\right)}{2b}$	91

input
```
int(csch(b*x+a)^3*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/sinh(b*x+a)^2/cosh(b*x+a)-3/2/cosh(b*x+a)+3*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. 2(41) = 82.

Time = 0.11 (sec) , antiderivative size = 709, normalized size of antiderivative = 15.76

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(6*cosh(b*x + a)^5 + 30*cosh(b*x + a)*sinh(b*x + a)^4 + 6*sinh(b*x +
a)^5 + 4*(15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 4*cosh(b*x + a)^3 + 12
*(5*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^6
+ 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (15*cosh(b*x + a)^2
- 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - cosh(b*x +
 a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 - 1)*sinh(b
*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 - c
osh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) +
3*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + (
15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4*(5*cosh(b*x
+ a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a)^4 - 6*cosh(b*x
 + a)^2 - 1)*sinh(b*x + a)^2 - cosh(b*x + a)^2 + 2*(3*cosh(b*x + a)^5 - 2*
cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + si
nh(b*x + a) - 1) + 6*(5*cosh(b*x + a)^4 - 2*cosh(b*x + a)^2 + 1)*sinh(b*x
+ a) + 6*cosh(b*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x +
a)^5 + b*sinh(b*x + a)^6 - b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 - b)*
sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)^
3 - b*cosh(b*x + a)^2 + (15*b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 - b)*s
inh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^5 - 2*b*cosh(b*x + a)^3 - b*cosh(b*x
 + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx=\int \operatorname{csch}^3{(a+bx)}\operatorname{sech}^2{(a+bx)}\,dx$$

input
```
integrate(csch(b*x+a)**3*sech(b*x+a)**2,x)
```

output
```
Integral(csch(a + b*x)**3*sech(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(41) = 82$.

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.36

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{3\,\log\left(e^{(-bx-a)}+1\right)}{2\,b} - \frac{3\,\log\left(e^{(-bx-a)}-1\right)}{2\,b} + \frac{3\,e^{(-bx-a)} - 2\,e^{(-3\,bx-3\,a)} + 3\,e^{(-5\,bx-5\,a)}}{b\left(e^{(-2\,bx-2\,a)} + e^{(-4\,bx-4\,a)} - e^{(-6\,bx-6\,a)} - 1\right)}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
3/2*log(e^(-b*x - a) + 1)/b - 3/2*log(e^(-b*x - a) - 1)/b + (3*e^(-b*x - a
) - 2*e^(-3*b*x - 3*a) + 3*e^(-5*b*x - 5*a))/(b*(e^(-2*b*x - 2*a) + e^(-4*
b*x - 4*a) - e^(-6*b*x - 6*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(41) = 82$.

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.44

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{\frac{4\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-8\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3-4\,e^{(bx+a)}-4\,e^{(-bx-a)}} - 3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) + 3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{4\,b}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/4*(4*(3*(e^(b*x + a) + e^(-b*x - a))^2 - 8)/((e^(b*x + a) + e^(-b*x - a
))^3 - 4*e^(b*x + a) - 4*e^(-b*x - a)) - 3*log(e^(b*x + a) + e^(-b*x - a)
+ 2) + 3*log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.47

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)} - \frac{\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)} - \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^2*sinh(a + b*x)^3),x)
```

output
```
(3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (2*exp(a + b*x))
/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*
a + 2*b*x) - 1)) - (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 233, normalized size of antiderivative = 5.18

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{-3e^{6bx+6a}\log\left(e^{bx+a}-1\right)+3e^{6bx+6a}\log\left(e^{bx+a}+1\right)-6e^{5bx+5a}+3e^{4bx+4a}\log\left(e^{bx+a}-1\right)-3e^{4bx+4a}\log\left(e^{bx}\right.}{2b\left(e^{6bx+6a}\right.}$$

input
```
int(csch(b*x+a)^3*sech(b*x+a)^2,x)
```

output
```
( - 3*e**(6*a + 6*b*x)*log(e**(a + b*x) - 1) + 3*e**(6*a + 6*b*x)*log(e**(
a + b*x) + 1) - 6*e**(5*a + 5*b*x) + 3*e**(4*a + 4*b*x)*log(e**(a + b*x) -
 1) - 3*e**(4*a + 4*b*x)*log(e**(a + b*x) + 1) + 4*e**(3*a + 3*b*x) + 3*e*
*(2*a + 2*b*x)*log(e**(a + b*x) - 1) - 3*e**(2*a + 2*b*x)*log(e**(a + b*x)
 + 1) - 6*e**(a + b*x) - 3*log(e**(a + b*x) - 1) + 3*log(e**(a + b*x) + 1)
)/(2*b*(e**(6*a + 6*b*x) - e**(4*a + 4*b*x) - e**(2*a + 2*b*x) + 1))
```

3.92 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx=-\frac{\coth^2(a+bx)}{2b}-\frac{2\log(\tanh(a+bx))}{b}+\frac{\tanh^2(a+bx)}{2b}$$

output
```
-1/2*coth(b*x+a)^2/b-2*ln(tanh(b*x+a))/b+1/2*tanh(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx=8\left(-\frac{\operatorname{csch}^2(a+bx)}{16b}+\frac{\log(\cosh(a+bx))}{4b}-\frac{\log(\sinh(a+bx))}{4b}-\frac{\operatorname{sech}^2(a+bx)}{16b}\right)$$

input
```
Integrate[Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

output
```
8*(-1/16*Csch[a + b*x]^2/b + Log[Cosh[a + b*x]]/(4*b) - Log[Sinh[a + b*x]]
/(4*b) - Sech[a + b*x]^2/(16*b))
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 26, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\csc(ia+ibx)^3\sec(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \csc(ia+ibx)^3\sec(ia+ibx)^3dx$$

$$\Big\downarrow \text{3100}$$

$$-\frac{\int i\coth^3(a+bx)\left(1-\tanh^2(a+bx)\right)^2d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{243}$$

$$-\frac{\int -\coth^2(a+bx)\left(1-\tanh^2(a+bx)\right)^2d(-\tanh^2(a+bx))}{2b}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\int \left(-\coth^2(a+bx)-2i\coth(a+bx)+1\right)d(-\tanh^2(a+bx))}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\tanh^2(a+bx)+i\coth(a+bx)+2\log\left(-\tanh^2(a+bx)\right)}{2b}$$

input
```
Int[Csch[a + b*x]^3*Sech[a + b*x]^3,x]
```

output
```
-1/2*(I*Coth[a + b*x] + 2*Log[-Tanh[a + b*x]^2] - Tanh[a + b*x]^2)/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 6.58 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sinh(bx+a)^{2}\cosh(bx+a)^{2}}-\frac{1}{\cosh(bx+a)^{2}}-2\ln(\tanh(bx+a))}{b}$	43
default	$\frac{-\frac{1}{2\sinh(bx+a)^{2}\cosh(bx+a)^{2}}-\frac{1}{\cosh(bx+a)^{2}}-2\ln(\tanh(bx+a))}{b}$	43
risch	$-\frac{4\,\mathrm{e}^{2bx+2a}\left(\mathrm{e}^{4bx+4a}+1\right)}{b\left(\mathrm{e}^{2bx+2a}-1\right)^{2}\left(\mathrm{e}^{2bx+2a}+1\right)^{2}}-\frac{2\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}+\frac{2\ln\left(\mathrm{e}^{2bx+2a}+1\right)}{b}$	87

input
```
int(csch(b*x+a)^3*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/sinh(b*x+a)^2/cosh(b*x+a)^2-1/cosh(b*x+a)^2-2*ln(tanh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. 2(39) = 78.

Time = 0.08 (sec) , antiderivative size = 774, normalized size of antiderivative = 18.00

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
-2*(2*cosh(b*x + a)^6 + 40*cosh(b*x + a)^3*sinh(b*x + a)^3 + 30*cosh(b*x +
 a)^2*sinh(b*x + a)^4 + 12*cosh(b*x + a)*sinh(b*x + a)^5 + 2*sinh(b*x + a)
^6 + 2*(15*cosh(b*x + a)^4 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 - (cos
h(b*x + a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sin
h(b*x + a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*c
osh(b*x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x +
a)^5 - cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x
+ a)^2)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x +
 a) + 1)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + (cosh(b*x
+ a)^8 + 56*cosh(b*x + a)^3*sinh(b*x + a)^5 + 28*cosh(b*x + a)^2*sinh(b*x
+ a)^6 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 2*(35*cosh(b*
x + a)^4 - 1)*sinh(b*x + a)^4 - 2*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 -
 cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 3*cosh(b*x + a)^2
)*sinh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh(b*x + a) +
1)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(3*cosh(b*x +
a)^5 + cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a)^8 + 56*b*cosh(b*x +
a)^3*sinh(b*x + a)^5 + 28*b*cosh(b*x + a)^2*sinh(b*x + a)^6 + 8*b*cosh(b*x
 + a)*sinh(b*x + a)^7 + b*sinh(b*x + a)^8 - 2*b*cosh(b*x + a)^4 + 2*(35*b*
cosh(b*x + a)^4 - b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 - b*cosh(b*x
 + a))*sinh(b*x + a)^3 + 4*(7*b*cosh(b*x + a)^6 - 3*b*cosh(b*x + a)^2)*...
```

Sympy [F]

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx=\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**3*sech(b*x+a)**3,x)
```

output
```
Integral(csch(a + b*x)**3*sech(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(39) = 78$.

Time = 0.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.37

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{2\,\log\left(e^{(-bx-a)}+1\right)}{b} - \frac{2\,\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{2\,\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{4\,\left(e^{(-2\,bx-2\,a)}+e^{(-6\,bx-6\,a)}\right)}{b(2\,e^{(-4\,bx-4\,a)}-e^{(-8\,bx-8\,a)}-1)}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
-2*log(e^(-b*x - a) + 1)/b - 2*log(e^(-b*x - a) - 1)/b + 2*log(e^(-2*b*x -
 2*a) + 1)/b + 4*(e^(-2*b*x - 2*a) + e^(-6*b*x - 6*a))/(b*(2*e^(-4*b*x - 4
*a) - e^(-8*b*x - 8*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(39) = 78$.

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.23

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{\frac{4\,\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}\right)}{\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}\right)^2-4} - \log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}+2\right) + \log\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2\right)}{b}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
-(4*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))/((e^(2*b*x + 2*a) + e^(-2*b*x - 2
*a))^2 - 4) - log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + log(e^(2*b*x +
 2*a) + e^(-2*b*x - 2*a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.23

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{4\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{4\,\mathrm{e}^{2\,a+2\,b\,x}}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-1\right)} - \frac{8\,\mathrm{e}^{2\,a+2\,b\,x}}{b\,\left(\mathrm{e}^{8\,a+8\,b\,x}-2\,\mathrm{e}^{4\,a+4\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^3*sinh(a + b*x)^3),x)
```

output
```
(4*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (4*exp(2*a +
 2*b*x))/(b*(exp(4*a + 4*b*x) - 1)) - (8*exp(2*a + 2*b*x))/(b*(exp(8*a + 8
*b*x) - 2*exp(4*a + 4*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 230, normalized size of antiderivative = 5.35

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^3(a+bx)\,dx$$
$$= \frac{2e^{8bx+8a}\log\left(e^{2bx+2a}+1\right) - 2e^{8bx+8a}\log\left(e^{bx+a}-1\right) - 2e^{8bx+8a}\log\left(e^{bx+a}+1\right) - 4e^{6bx+6a} - 4e^{4bx+4a}\log\left(e^{2b\cdots\right.}{}$$

input
```
int(csch(b*x+a)^3*sech(b*x+a)^3,x)
```

output
```
(2*(e**(8*a + 8*b*x)*log(e**(2*a + 2*b*x) + 1) - e**(8*a + 8*b*x)*log(e**(
a + b*x) - 1) - e**(8*a + 8*b*x)*log(e**(a + b*x) + 1) - 2*e**(6*a + 6*b*x
) - 2*e**(4*a + 4*b*x)*log(e**(2*a + 2*b*x) + 1) + 2*e**(4*a + 4*b*x)*log(
e**(a + b*x) - 1) + 2*e**(4*a + 4*b*x)*log(e**(a + b*x) + 1) - 2*e**(2*a +
 2*b*x) + log(e**(2*a + 2*b*x) + 1) - log(e**(a + b*x) - 1) - log(e**(a +
b*x) + 1)))/(b*(e**(8*a + 8*b*x) - 2*e**(4*a + 4*b*x) + 1))
```

3.93 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx=\frac{5\operatorname{arctanh}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\operatorname{csch}(a+bx)}{2b}-\frac{2\operatorname{sech}(a+bx)}{b}-\frac{\operatorname{sech}^3(a+bx)}{3b}$$

output
```
5/2*arctanh(cosh(b*x+a))/b-1/2*coth(b*x+a)*csch(b*x+a)/b-2*sech(b*x+a)/b-1
/3*sech(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.68

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx=-\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b}+\frac{5\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}-\frac{5\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}-\frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{2\operatorname{sech}(a+bx)}{b}-\frac{\operatorname{sech}^3(a+bx)}{3b}$$

input
```
Integrate[Csch[a + b*x]^3*Sech[a + b*x]^4,x]
```

output
```
-1/8*Csch[(a + b*x)/2]^2/b + (5*Log[Cosh[(a + b*x)/2]])/(2*b) - (5*Log[Sin
h[(a + b*x)/2]])/(2*b) - Sech[(a + b*x)/2]^2/(8*b) - (2*Sech[a + b*x])/b -
 Sech[a + b*x]^3/(3*b)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 26, 3102, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\csc(ia+ibx)^3\sec(ia+ibx)^4dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \csc(ia+ibx)^3\sec(ia+ibx)^4dx$$

$$\Big\downarrow \text{3102}$$

$$-\frac{\int \frac{\operatorname{sech}^6(a+bx)}{\left(1-\operatorname{sech}^2(a+bx)\right)^2}d\operatorname{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{\operatorname{sech}^5(a+bx)}{2\left(1-\operatorname{sech}^2(a+bx)\right)}-\frac{5}{2}\int \frac{\operatorname{sech}^4(a+bx)}{1-\operatorname{sech}^2(a+bx)}d\operatorname{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{254}$$

$$-\frac{\frac{\operatorname{sech}^5(a+bx)}{2\left(1-\operatorname{sech}^2(a+bx)\right)}-\frac{5}{2}\int \left(-\operatorname{sech}^2(a+bx)+\frac{1}{1-\operatorname{sech}^2(a+bx)}-1\right)d\operatorname{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\text{sech}^5(a+bx)}{2\left(1-\text{sech}^2(a+bx)\right)}-\frac{5}{2}\left(\text{arctanh}(\text{sech}(a+bx))-\frac{1}{3}\text{sech}^3(a+bx)-\text{sech}(a+bx)\right)}{b}$$

input
```
Int[Csch[a + b*x]^3*Sech[a + b*x]^4,x]
```

output
```
-((Sech[a + b*x]^5/(2*(1 - Sech[a + b*x]^2)) - (5*(ArcTanh[Sech[a + b*x]]
- Sech[a + b*x] - Sech[a + b*x]^3/3))/2)/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 16.42 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sinh(bx+a)^{2}\cosh(bx+a)^{3}}-\frac{5}{6\cosh(bx+a)^{3}}-\frac{5}{2\cosh(bx+a)}+5\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	53
default	$\frac{-\frac{1}{2\sinh(bx+a)^{2}\cosh(bx+a)^{3}}-\frac{5}{6\cosh(bx+a)^{3}}-\frac{5}{2\cosh(bx+a)}+5\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b}$	53
risch	$-\frac{\mathrm{e}^{bx+a}\left(15\,\mathrm{e}^{8bx+8a}+20\,\mathrm{e}^{6bx+6a}-22\,\mathrm{e}^{4bx+4a}+20\,\mathrm{e}^{2bx+2a}+15\right)}{3b\left(\mathrm{e}^{2bx+2a}-1\right)^{2}\left(\mathrm{e}^{2bx+2a}+1\right)^{3}}-\frac{5\ln\left(\mathrm{e}^{bx+a}-1\right)}{2b}+\frac{5\ln\left(\mathrm{e}^{bx+a}+1\right)}{2b}$	113

input
```
int(csch(b*x+a)^3*sech(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/sinh(b*x+a)^2/cosh(b*x+a)^3-5/6/cosh(b*x+a)^3-5/2/cosh(b*x+a)+5*
arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1573 vs. $2(54) = 108$.

Time = 0.10 (sec) , antiderivative size = 1573, normalized size of antiderivative = 26.22

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/6*(30*cosh(b*x + a)^9 + 270*cosh(b*x + a)*sinh(b*x + a)^8 + 30*sinh(b*x
 + a)^9 + 40*(27*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^7 + 40*cosh(b*x + a)^7
 + 280*(9*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^6 + 4*(945*cosh(b
*x + a)^4 + 210*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^5 - 44*cosh(b*x + a)^5
 + 20*(189*cosh(b*x + a)^5 + 70*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b
*x + a)^4 + 40*(63*cosh(b*x + a)^6 + 35*cosh(b*x + a)^4 - 11*cosh(b*x + a)
^2 + 1)*sinh(b*x + a)^3 + 40*cosh(b*x + a)^3 + 40*(27*cosh(b*x + a)^7 + 21
*cosh(b*x + a)^5 - 11*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 -
 15*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^1
0 + (45*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^8 + cosh(b*x + a)^8 + 8*(15*cos
h(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^7 + 2*(105*cosh(b*x + a)^4 + 1
4*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 2*cosh(b*x + a)^6 + 4*(63*cosh(b*
x + a)^5 + 14*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(105*
cosh(b*x + a)^6 + 35*cosh(b*x + a)^4 - 15*cosh(b*x + a)^2 - 1)*sinh(b*x +
a)^4 - 2*cosh(b*x + a)^4 + 8*(15*cosh(b*x + a)^7 + 7*cosh(b*x + a)^5 - 5*c
osh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^3 + (45*cosh(b*x + a)^8 + 28
*cosh(b*x + a)^6 - 30*cosh(b*x + a)^4 - 12*cosh(b*x + a)^2 + 1)*sinh(b*x +
 a)^2 + cosh(b*x + a)^2 + 2*(5*cosh(b*x + a)^9 + 4*cosh(b*x + a)^7 - 6*cos
h(b*x + a)^5 - 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(c
osh(b*x + a) + sinh(b*x + a) + 1) + 15*(cosh(b*x + a)^10 + 10*cosh(b*x ...
```

Sympy [F]

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx = \int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**3*sech(b*x+a)**4,x)
```

output
```
Integral(csch(a + b*x)**3*sech(a + b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(54) = 108$.

Time = 0.06 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.48

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{5\,\log\left(e^{(-bx-a)}+1\right)}{2\,b} - \frac{5\,\log\left(e^{(-bx-a)}-1\right)}{2\,b} - \frac{15\,e^{(-bx-a)} + 20\,e^{(-3\,bx-3\,a)} - 22\,e^{(-5\,bx-5\,a)} + 20\,e^{(-7\,bx-7\,a)} + 15\,e^{(-9\,bx-9\,a)}}{3\,b(e^{(-2\,bx-2\,a)} - 2\,e^{(-4\,bx-4\,a)} - 2\,e^{(-6\,bx-6\,a)} + e^{(-8\,bx-8\,a)} + e^{(-10\,bx-10\,a)} + 1)}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^4,x, algorithm="maxima")
```

output
```
5/2*log(e^(-b*x - a) + 1)/b - 5/2*log(e^(-b*x - a) - 1)/b - 1/3*(15*e^(-b*
x - a) + 20*e^(-3*b*x - 3*a) - 22*e^(-5*b*x - 5*a) + 20*e^(-7*b*x - 7*a) +
 15*e^(-9*b*x - 9*a))/(b*(e^(-2*b*x - 2*a) - 2*e^(-4*b*x - 4*a) - 2*e^(-6*
b*x - 6*a) + e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(54) = 108$.

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.13

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{\frac{12\,\left(e^{(bx+a)}+e^{(-bx-a)}\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4} + \frac{16\,\left(3\,\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2+2\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3} - 15\,\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) + 15\,\log\left(e^{(bx+a)}+e^{(-\cdots}{12\,b}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^4,x, algorithm="giac")
```

output
```
-1/12*(12*(e^(b*x + a) + e^(-b*x - a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4
) + 16*(3*(e^(b*x + a) + e^(-b*x - a))^2 + 2)/(e^(b*x + a) + e^(-b*x - a))
^3 - 15*log(e^(b*x + a) + e^(-b*x - a) + 2) + 15*log(e^(b*x + a) + e^(-b*x
 - a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.20

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{5\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{8\,e^{a+b\,x}}{3\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} + \frac{8\,e^{a+b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)} - \frac{4\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^4*sinh(a + b*x)^3),x)
```

output
```
(5*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (2*exp(a + b*x))
/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (8*exp(a + b*x))/(3*b*(
2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) + (8*exp(a + b*x))/(3*b*(3*exp
(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) - exp(a + b*x)
/(b*(exp(2*a + 2*b*x) - 1)) - (4*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 365, normalized size of antiderivative = 6.08

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{-15e^{10bx+10a}\log\left(e^{bx+a}-1\right)+15e^{10bx+10a}\log\left(e^{bx+a}+1\right)-30e^{9bx+9a}-15e^{8bx+8a}\log\left(e^{bx+a}-1\right)+15e^{8bx}}{}$$

input
```
int(csch(b*x+a)^3*sech(b*x+a)^4,x)
```

output
```
( - 15*e**(10*a + 10*b*x)*log(e**(a + b*x) - 1) + 15*e**(10*a + 10*b*x)*lo
g(e**(a + b*x) + 1) - 30*e**(9*a + 9*b*x) - 15*e**(8*a + 8*b*x)*log(e**(a
+ b*x) - 1) + 15*e**(8*a + 8*b*x)*log(e**(a + b*x) + 1) - 40*e**(7*a + 7*b
*x) + 30*e**(6*a + 6*b*x)*log(e**(a + b*x) - 1) - 30*e**(6*a + 6*b*x)*log(
e**(a + b*x) + 1) + 44*e**(5*a + 5*b*x) + 30*e**(4*a + 4*b*x)*log(e**(a +
b*x) - 1) - 30*e**(4*a + 4*b*x)*log(e**(a + b*x) + 1) - 40*e**(3*a + 3*b*x
) - 15*e**(2*a + 2*b*x)*log(e**(a + b*x) - 1) + 15*e**(2*a + 2*b*x)*log(e*
*(a + b*x) + 1) - 30*e**(a + b*x) - 15*log(e**(a + b*x) - 1) + 15*log(e**(
a + b*x) + 1))/(6*b*(e**(10*a + 10*b*x) + e**(8*a + 8*b*x) - 2*e**(6*a + 6
*b*x) - 2*e**(4*a + 4*b*x) + e**(2*a + 2*b*x) + 1))
```

3.94 $\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx = -\frac{\coth^2(a+bx)}{2b} - \frac{3\log(\tanh(a+bx))}{b} + \frac{3\tanh^2(a+bx)}{2b} - \frac{\tanh^4(a+bx)}{4b}$$

output
```
-1/2*coth(b*x+a)^2/b-3*ln(tanh(b*x+a))/b+3/2*tanh(b*x+a)^2/b-1/4*tanh(b*x+
a)^4/b
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx = -\frac{2\operatorname{csch}^2(a+bx) - 12\log(\cosh(a+bx)) + 12\log(\sinh(a+bx)) + 4\operatorname{sech}^2(a+bx) + \operatorname{sech}^4(a+bx)}{4b}$$

input
```
Integrate[Csch[a + b*x]^3*Sech[a + b*x]^5,x]
```

output
```
-1/4*(2*Csch[a + b*x]^2 - 12*Log[Cosh[a + b*x]] + 12*Log[Sinh[a + b*x]] +
4*Sech[a + b*x]^2 + Sech[a + b*x]^4)/b
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 26, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^3(a+bx)\text{sech}^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\csc(ia+ibx)^3\sec(ia+ibx)^5dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \csc(ia+ibx)^3\sec(ia+ibx)^5dx$$

$$\Big\downarrow \text{3100}$$

$$-\frac{\int i\coth^3(a+bx)\left(1-\tanh^2(a+bx)\right)^3d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{243}$$

$$-\frac{\int -\coth^2(a+bx)\left(1-\tanh^2(a+bx)\right)^3d(-\tanh^2(a+bx))}{2b}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\int \left(-\coth^2(a+bx)-3i\coth(a+bx)-\tanh^2(a+bx)+3\right)d(-\tanh^2(a+bx))}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{2}\tanh^2(a+bx)+3i\tanh(a+bx)+i\coth(a+bx)+3\log\left(-\tanh^2(a+bx)\right)}{2b}$$

input
```
Int[Csch[a + b*x]^3*Sech[a + b*x]^5,x]
```

output
```
-1/2*(I*Coth[a + b*x] + 3*Log[-Tanh[a + b*x]^2] + (3*I)*Tanh[a + b*x] - Ta
nh[a + b*x]^2/2)/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 37.51 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)^4}-\frac{3}{4\cosh(bx+a)^4}-\frac{3}{2\cosh(bx+a)^2}-3\ln(\tanh(bx+a))}{b}$	53
default	$\frac{-\frac{1}{2\sinh(bx+a)^2\cosh(bx+a)^4}-\frac{3}{4\cosh(bx+a)^4}-\frac{3}{2\cosh(bx+a)^2}-3\ln(\tanh(bx+a))}{b}$	53
risch	$-\frac{2\,e^{2bx+2a}\left(3\,e^{8bx+8a}+6\,e^{6bx+6a}-2\,e^{4bx+4a}+6\,e^{2bx+2a}+3\right)}{b\left(e^{2bx+2a}+1\right)^4\left(e^{2bx+2a}-1\right)^2}+\frac{3\ln\left(e^{2bx+2a}+1\right)}{b}-\frac{3\ln\left(e^{2bx+2a}-1\right)}{b}$	122

input
```
int(csch(b*x+a)^3*sech(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/sinh(b*x+a)^2/cosh(b*x+a)^4-3/4/cosh(b*x+a)^4-3/2/cosh(b*x+a)^2-
3*ln(tanh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2103 vs. $2(52) = 104$.

Time = 0.11 (sec) , antiderivative size = 2103, normalized size of antiderivative = 36.26

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^5,x, algorithm="fricas")
```

output
```
-(6*cosh(b*x + a)^10 + 60*cosh(b*x + a)*sinh(b*x + a)^9 + 6*sinh(b*x + a)^
10 + 6*(45*cosh(b*x + a)^2 + 2)*sinh(b*x + a)^8 + 12*cosh(b*x + a)^8 + 48*
(15*cosh(b*x + a)^3 + 2*cosh(b*x + a))*sinh(b*x + a)^7 + 4*(315*cosh(b*x +
 a)^4 + 84*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 24*(
63*cosh(b*x + a)^5 + 28*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^5 +
 12*(105*cosh(b*x + a)^6 + 70*cosh(b*x + a)^4 - 5*cosh(b*x + a)^2 + 1)*sin
h(b*x + a)^4 + 12*cosh(b*x + a)^4 + 16*(45*cosh(b*x + a)^7 + 42*cosh(b*x +
 a)^5 - 5*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 6*(45*cosh(
b*x + a)^8 + 56*cosh(b*x + a)^6 - 10*cosh(b*x + a)^4 + 12*cosh(b*x + a)^2
+ 1)*sinh(b*x + a)^2 + 6*cosh(b*x + a)^2 - 3*(cosh(b*x + a)^12 + 12*cosh(b
*x + a)*sinh(b*x + a)^11 + sinh(b*x + a)^12 + 2*(33*cosh(b*x + a)^2 + 1)*s
inh(b*x + a)^10 + 2*cosh(b*x + a)^10 + 20*(11*cosh(b*x + a)^3 + cosh(b*x +
 a))*sinh(b*x + a)^9 + (495*cosh(b*x + a)^4 + 90*cosh(b*x + a)^2 - 1)*sinh
(b*x + a)^8 - cosh(b*x + a)^8 + 8*(99*cosh(b*x + a)^5 + 30*cosh(b*x + a)^3
 - cosh(b*x + a))*sinh(b*x + a)^7 + 4*(231*cosh(b*x + a)^6 + 105*cosh(b*x
+ a)^4 - 7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(9
9*cosh(b*x + a)^7 + 63*cosh(b*x + a)^5 - 7*cosh(b*x + a)^3 - 3*cosh(b*x +
a))*sinh(b*x + a)^5 + (495*cosh(b*x + a)^8 + 420*cosh(b*x + a)^6 - 70*cosh
(b*x + a)^4 - 60*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 +
4*(55*cosh(b*x + a)^9 + 60*cosh(b*x + a)^7 - 14*cosh(b*x + a)^5 - 20*co...
```

Sympy [F]

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx = \int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**3*sech(b*x+a)**5,x)
```

output
```
Integral(csch(a + b*x)**3*sech(a + b*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(52) = 104$.

Time = 0.12 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.12

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx$$
$$= -\frac{3\log\left(e^{(-bx-a)}+1\right)}{b} - \frac{3\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{3\log\left(e^{(-2bx-2a)}+1\right)}{b} - \frac{2\left(3e^{(-2bx-2a)}+6e^{(-4bx-4a)}-2e^{(-6bx-6a)}+6e^{(-8bx-8a)}+3e^{(-10bx-10a)}\right)}{b(2e^{(-2bx-2a)}-e^{(-4bx-4a)}-4e^{(-6bx-6a)}-e^{(-8bx-8a)}+2e^{(-10bx-10a)}+e^{(-12bx-12a)}+1)}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^5,x, algorithm="maxima")
```

output
```
-3*log(e^(-b*x - a) + 1)/b - 3*log(e^(-b*x - a) - 1)/b + 3*log(e^(-2*b*x -
 2*a) + 1)/b - 2*(3*e^(-2*b*x - 2*a) + 6*e^(-4*b*x - 4*a) - 2*e^(-6*b*x -
6*a) + 6*e^(-8*b*x - 8*a) + 3*e^(-10*b*x - 10*a))/(b*(2*e^(-2*b*x - 2*a) -
 e^(-4*b*x - 4*a) - 4*e^(-6*b*x - 6*a) - e^(-8*b*x - 8*a) + 2*e^(-10*b*x -
 10*a) + e^(-12*b*x - 12*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(52) = 104$.

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.95

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx$$
$$= \frac{\frac{2\left(3e^{(2bx+2a)}+3e^{(-2bx-2a)}-10\right)}{e^{(2bx+2a)}+e^{(-2bx-2a)}-2} - \frac{9\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right)^2+52e^{(2bx+2a)}+52e^{(-2bx-2a)}+84}{\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)^2} + 6\log\left(e^{(2bx+2a)}+e^{(-2bx}\right.}{4b}$$

input
```
integrate(csch(b*x+a)^3*sech(b*x+a)^5,x, algorithm="giac")
```

output
```
1/4*(2*(3*e^(2*b*x + 2*a) + 3*e^(-2*b*x - 2*a) - 10)/(e^(2*b*x + 2*a) + e^
(-2*b*x - 2*a) - 2) - (9*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2 + 52*e^(2*
b*x + 2*a) + 52*e^(-2*b*x - 2*a) + 84)/(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a)
 + 2)^2 + 6*log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) - 6*log(e^(2*b*x +
 2*a) + e^(-2*b*x - 2*a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.22

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx$$
$$= \frac{6\,\mathrm{atan}\left(\frac{e^{2\,a}\,e^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{4}{b\,\left(e^{2\,a+2\,b\,x}+1\right)} - \frac{2}{b\,\left(e^{2\,a+2\,b\,x}-1\right)} - \frac{2}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} + \frac{8}{b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)} - \frac{4}{b\,\left(4\,e^{2\,a+2\,b\,x}+6\,e^{4\,a+4\,b\,x}+4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^5*sinh(a + b*x)^3),x)
```

output
```
(6*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - 4/(b*(exp(2*
a + 2*b*x) + 1)) - 2/(b*(exp(2*a + 2*b*x) - 1)) - 2/(b*(exp(4*a + 4*b*x) -
 2*exp(2*a + 2*b*x) + 1)) + 8/(b*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x)
+ exp(6*a + 6*b*x) + 1)) - 4/(b*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) +
 4*exp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 581, normalized size of antiderivative = 10.02

$$\int \operatorname{csch}^3(a+bx)\operatorname{sech}^5(a+bx)\,dx$$
$$= \frac{3-3e^{4bx+4a}\log\left(e^{2bx+2a}+1\right)+6e^{2bx+2a}\log\left(e^{2bx+2a}+1\right)-6e^{10bx+10a}\log\left(e^{bx+a}-1\right)-6e^{10bx+10a}\log\left(e^{bx+\cdots}\right.}{}$$

input
```
int(csch(b*x+a)^3*sech(b*x+a)^5,x)
```

output
```
(3*e**(12*a + 12*b*x)*log(e**(2*a + 2*b*x) + 1) - 3*e**(12*a + 12*b*x)*log
(e**(a + b*x) - 1) - 3*e**(12*a + 12*b*x)*log(e**(a + b*x) + 1) + 3*e**(12
*a + 12*b*x) + 6*e**(10*a + 10*b*x)*log(e**(2*a + 2*b*x) + 1) - 6*e**(10*a
 + 10*b*x)*log(e**(a + b*x) - 1) - 6*e**(10*a + 10*b*x)*log(e**(a + b*x) +
 1) - 3*e**(8*a + 8*b*x)*log(e**(2*a + 2*b*x) + 1) + 3*e**(8*a + 8*b*x)*lo
g(e**(a + b*x) - 1) + 3*e**(8*a + 8*b*x)*log(e**(a + b*x) + 1) - 15*e**(8*
a + 8*b*x) - 12*e**(6*a + 6*b*x)*log(e**(2*a + 2*b*x) + 1) + 12*e**(6*a +
6*b*x)*log(e**(a + b*x) - 1) + 12*e**(6*a + 6*b*x)*log(e**(a + b*x) + 1) -
 8*e**(6*a + 6*b*x) - 3*e**(4*a + 4*b*x)*log(e**(2*a + 2*b*x) + 1) + 3*e**
(4*a + 4*b*x)*log(e**(a + b*x) - 1) + 3*e**(4*a + 4*b*x)*log(e**(a + b*x)
+ 1) - 15*e**(4*a + 4*b*x) + 6*e**(2*a + 2*b*x)*log(e**(2*a + 2*b*x) + 1)
- 6*e**(2*a + 2*b*x)*log(e**(a + b*x) - 1) - 6*e**(2*a + 2*b*x)*log(e**(a
+ b*x) + 1) + 3*log(e**(2*a + 2*b*x) + 1) - 3*log(e**(a + b*x) - 1) - 3*lo
g(e**(a + b*x) + 1) + 3)/(b*(e**(12*a + 12*b*x) + 2*e**(10*a + 10*b*x) - e
**(8*a + 8*b*x) - 4*e**(6*a + 6*b*x) - e**(4*a + 4*b*x) + 2*e**(2*a + 2*b*
x) + 1))
```

3.95 $\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{b}+\frac{\operatorname{csch}(a+bx)}{b}-\frac{\operatorname{csch}^3(a+bx)}{3b}$$

output
```
arctan(sinh(b*x+a))/b+csch(b*x+a)/b-1/3*csch(b*x+a)^3/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx$$
$$=-\frac{\operatorname{csch}^3(a+bx)\operatorname{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\sinh^2(a+bx)\right)}{3b}$$

input
```
Integrate[Csch[a + b*x]^4*Sech[a + b*x],x]
```

output
```
-1/3*(Csch[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Sinh[a + b*x]^2])/
b
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3101, 25, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc(ia+ibx)^4\sec(ia+ibx)dx$$

$$\Big\downarrow \; 3101$$

$$\frac{i\int -\frac{\operatorname{csch}^4(a+bx)}{\operatorname{csch}^2(a+bx)+1}d(-i\operatorname{csch}(a+bx))}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{i\int \frac{\operatorname{csch}^4(a+bx)}{\operatorname{csch}^2(a+bx)+1}d(-i\operatorname{csch}(a+bx))}{b}$$

$$\Big\downarrow \; 254$$

$$-\frac{i\int \left(\operatorname{csch}^2(a+bx)+\frac{1}{\operatorname{csch}^2(a+bx)+1}-1\right)d(-i\operatorname{csch}(a+bx))}{b}$$

$$\Big\downarrow \; 2009$$

$$\frac{i\left(i\arctan(\operatorname{csch}(a+bx))+\frac{1}{3}i\operatorname{csch}^3(a+bx)-i\operatorname{csch}(a+bx)\right)}{b}$$

input
```
Int[Csch[a + b*x]^4*Sech[a + b*x],x]
```

output
```
(I*(I*ArcTan[Csch[a + b*x]] - I*Csch[a + b*x] + (I/3)*Csch[a + b*x]^3))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{-\frac{1}{3\sinh(bx+a)^3}+\frac{1}{\sinh(bx+a)}+2\arctan\left(e^{bx+a}\right)}{b}$	33
default	$\frac{-\frac{1}{3\sinh(bx+a)^3}+\frac{1}{\sinh(bx+a)}+2\arctan\left(e^{bx+a}\right)}{b}$	33
risch	$\frac{2\,e^{bx+a}\left(3\,e^{4bx+4a}-10\,e^{2bx+2a}+3\right)}{3b\left(e^{2bx+2a}-1\right)^3}+\frac{i\ln\left(e^{bx+a}+i\right)}{b}-\frac{i\ln\left(e^{bx+a}-i\right)}{b}$	82

input
```
int(csch(b*x+a)^4*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3/sinh(b*x+a)^3+1/sinh(b*x+a)+2*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(35) = 70$.

Time = 0.08 (sec) , antiderivative size = 515, normalized size of antiderivative = 13.92

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a),x, algorithm="fricas")
```

output
```
2/3*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 + 3*sinh(b*x + a
)^5 + 10*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 10*cosh(b*x + a)^3 + 30
*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^2 + 3*(cosh(b*x + a)^6 +
6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 -
 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - 3*cosh(b*
x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*si
nh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3
 + cosh(b*x + a))*sinh(b*x + a) - 1)*arctan(cosh(b*x + a) + sinh(b*x + a))
 + 3*(5*cosh(b*x + a)^4 - 10*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 3*cosh(b
*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b
*x + a)^6 - 3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 - b)*sinh(b*x + a
)^4 + 4*(5*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*co
sh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 + b)*sinh(b*x
 + a)^2 + 6*(b*cosh(b*x + a)^5 - 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*si
nh(b*x + a) - b)
```

Sympy [F]

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx = \int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**4*sech(b*x+a),x)
```

output
```
Integral(csch(a + b*x)**4*sech(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(35) = 70$.

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.43

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{2\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{2\left(3\,e^{(-bx-a)} - 10\,e^{(-3\,bx-3\,a)} + 3\,e^{(-5\,bx-5\,a)}\right)}{3\,b(3\,e^{(-2\,bx-2\,a)} - 3\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)} - 1)}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a),x, algorithm="maxima")
```

output
```
-2*arctan(e^(-b*x - a))/b - 2/3*(3*e^(-b*x - a) - 10*e^(-3*b*x - 3*a) + 3*
e^(-5*b*x - 5*a))/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + e^(-6*b*x
 - 6*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(35) = 70$.

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.16

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{3\,\pi + \frac{4\left(3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2-4\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3} + 6\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{6\,b}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a),x, algorithm="giac")
```

output
```
1/6*(3*pi + 4*(3*(e^(b*x + a) - e^(-b*x - a))^2 - 4)/(e^(b*x + a) - e^(-b*
x - a))^3 + 6*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.49

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{8\,e^{a+b\,x}}{3\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{8\,e^{a+b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)} + \frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(1/(cosh(a + b*x)*sinh(a + b*x)^4),x)
```

output
```
(2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (8*exp(a + b*x))/(
3*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (8*exp(a + b*x))/(3*b*(
3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) + (2*exp(
a + b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.97

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{2e^{6bx+6a}atan\left(e^{bx+a}\right)-6e^{4bx+4a}atan\left(e^{bx+a}\right)+6e^{2bx+2a}atan\left(e^{bx+a}\right)-2atan\left(e^{bx+a}\right)+2e^{5bx+5a}-\frac{20e^{3bx+3a}}{3}}{b\left(e^{6bx+6a}-3e^{4bx+4a}+3e^{2bx+2a}-1\right)}$$

input
```
int(csch(b*x+a)^4*sech(b*x+a),x)
```

output
```
(2*(3*e**(6*a + 6*b*x)*atan(e**(a + b*x)) - 9*e**(4*a + 4*b*x)*atan(e**(a
+ b*x)) + 9*e**(2*a + 2*b*x)*atan(e**(a + b*x)) - 3*atan(e**(a + b*x)) + 3
*e**(5*a + 5*b*x) - 10*e**(3*a + 3*b*x) + 3*e**(a + b*x)))/(3*b*(e**(6*a +
 6*b*x) - 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) - 1))
```

3.96 $\int \text{csch}^4(a+bx)\text{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \text{csch}^4(a+bx)\text{sech}^2(a+bx)\,dx=\frac{2\coth(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}+\frac{\tanh(a+bx)}{b}$$

output
```
2*coth(b*x+a)/b-1/3*coth(b*x+a)^3/b+tanh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \text{csch}^4(a+bx)\text{sech}^2(a+bx)\,dx=\frac{5\coth(a+bx)}{3b}-\frac{\coth(a+bx)\text{csch}^2(a+bx)}{3b}+\frac{\tanh(a+bx)}{b}$$

input
```
Integrate[Csch[a + b*x]^4*Sech[a + b*x]^2,x]
```

output
```
(5*Coth[a + b*x])/(3*b) - (Coth[a + b*x]*Csch[a + b*x]^2)/(3*b) + Tanh[a +
 b*x]/b
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc(ia+ibx)^4\sec(ia+ibx)^2dx$$

$$\Big\downarrow \; 3100$$

$$-\frac{i\int \coth^4(a+bx)\left(1-\tanh^2(a+bx)\right)^2d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \; 244$$

$$-\frac{i\int \left(\coth^4(a+bx)-2\coth^2(a+bx)+1\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{i\left(i\tanh(a+bx)-\frac{1}{3}i\coth^3(a+bx)+2i\coth(a+bx)\right)}{b}$$

input
```
Int[Csch[a + b*x]^4*Sech[a + b*x]^2,x]
```

output
```
((-I)*((2*I)*Coth[a + b*x] - (I/3)*Coth[a + b*x]^3 + I*Tanh[a + b*x]))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 4.79 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

method	result	size
risch	$-\frac{16\left(2\,\mathrm{e}^{2bx+2a}-1\right)}{3b\left(\mathrm{e}^{2bx+2a}-1\right)^{3}\left(\mathrm{e}^{2bx+2a}+1\right)}$	45
derivativedivides	$\frac{-\frac{1}{3\sinh\left(bx+a\right)^{3}\cosh\left(bx+a\right)}+\frac{4}{3\sinh\left(bx+a\right)\cosh\left(bx+a\right)}+\frac{8\tanh\left(bx+a\right)}{3}}{b}$	50
default	$\frac{-\frac{1}{3\sinh\left(bx+a\right)^{3}\cosh\left(bx+a\right)}+\frac{4}{3\sinh\left(bx+a\right)\cosh\left(bx+a\right)}+\frac{8\tanh\left(bx+a\right)}{3}}{b}$	50

input
```
int(csch(b*x+a)^4*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-16/3*(2*exp(2*b*x+2*a)-1)/b/(exp(2*b*x+2*a)-1)^3/(exp(2*b*x+2*a)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(35) = 70$.

Time = 0.07 (sec) , antiderivative size = 229, normalized size of antiderivative = 6.19

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx =$$

$$-\frac{}{3\left(b\cosh\left(bx+a\right)^7+7\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^6+b\sinh\left(bx+a\right)^7-2\,b\cosh\left(bx+a\right)^5+\left(21\,b\cos\right.}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
-16/3*(cosh(b*x + a) + 3*sinh(b*x + a))/(b*cosh(b*x + a)^7 + 7*b*cosh(b*x
+ a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 - 2*b*cosh(b*x + a)^5 + (21*b*cos
h(b*x + a)^2 - 2*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)^3 - 2*b*cosh(b*
x + a))*sinh(b*x + a)^4 + 5*(7*b*cosh(b*x + a)^4 - 4*b*cosh(b*x + a)^2)*si
nh(b*x + a)^3 + (21*b*cosh(b*x + a)^5 - 20*b*cosh(b*x + a)^3)*sinh(b*x + a
)^2 + b*cosh(b*x + a) + (7*b*cosh(b*x + a)^6 - 10*b*cosh(b*x + a)^4 + 3*b)
*sinh(b*x + a))
```

Sympy [F]

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int \operatorname{csch}^4\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)dx$$

input
```
integrate(csch(b*x+a)**4*sech(b*x+a)**2,x)
```

output
```
Integral(csch(a + b*x)**4*sech(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(35) = 70$.

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.43

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{32\,e^{(-2\,bx-2\,a)}}{3\,b(2\,e^{(-2\,bx-2\,a)}-2\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}-1)} - \frac{16}{3\,b(2\,e^{(-2\,bx-2\,a)}-2\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}-1)}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
32/3*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x - 2*a) - 2*e^(-6*b*x - 6*a) + e^(-8*
b*x - 8*a) - 1)) - 16/3/(b*(2*e^(-2*b*x - 2*a) - 2*e^(-6*b*x - 6*a) + e^(-
8*b*x - 8*a) - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{2\left(\frac{3}{e^{(2\,bx+2\,a)}+1} - \frac{3\,e^{(4\,bx+4\,a)}-12\,e^{(2\,bx+2\,a)}+5}{\left(e^{(2\,bx+2\,a)}-1\right)^3}\right)}{3\,b}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
-2/3*(3/(e^(2*b*x + 2*a) + 1) - (3*e^(4*b*x + 4*a) - 12*e^(2*b*x + 2*a) +
5)/(e^(2*b*x + 2*a) - 1)^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 4.14

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{\frac{2}{3\,b}-\frac{4\,{\mathrm{e}}^{2\,a+2\,b\,x}}{b}+\frac{2\,{\mathrm{e}}^{4\,a+4\,b\,x}}{3\,b}}{3\,{\mathrm{e}}^{2\,a+2\,b\,x}-3\,{\mathrm{e}}^{4\,a+4\,b\,x}+{\mathrm{e}}^{6\,a+6\,b\,x}-1} - \frac{\frac{2}{b}-\frac{2\,{\mathrm{e}}^{2\,a+2\,b\,x}}{3\,b}}{{\mathrm{e}}^{4\,a+4\,b\,x}-2\,{\mathrm{e}}^{2\,a+2\,b\,x}+1} + \frac{2}{3\,b\,\left({\mathrm{e}}^{2\,a+2\,b\,x}-1\right)}-\frac{2}{b\,\left({\mathrm{e}}^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^2*sinh(a + b*x)^4),x)
```

output
```
(2/(3*b) - (4*exp(2*a + 2*b*x))/b + (2*exp(4*a + 4*b*x))/(3*b))/(3*exp(2*a
 + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1) - (2/b - (2*exp(2*a
 + 2*b*x))/(3*b))/(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1) + 2/(3*b*(ex
p(2*a + 2*b*x) - 1)) - 2/(b*(exp(2*a + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{-\frac{32e^{2bx+2a}}{3}+\frac{16}{3}}{b\left(e^{8bx+8a}-2e^{6bx+6a}+2e^{2bx+2a}-1\right)}$$

input
```
int(csch(b*x+a)^4*sech(b*x+a)^2,x)
```

output
```
(16*( - 2*e**(2*a + 2*b*x) + 1))/(3*b*(e**(8*a + 8*b*x) - 2*e**(6*a + 6*b*
x) + 2*e**(2*a + 2*b*x) - 1))
```

3.97 $\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{5\arctan(\sinh(a+bx))}{2b}+\frac{2\operatorname{csch}(a+bx)}{b}-\frac{\operatorname{csch}^3(a+bx)}{3b}+\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}$$

output
```
5/2*arctan(sinh(b*x+a))/b+2*csch(b*x+a)/b-1/3*csch(b*x+a)^3/b+1/2*sech(b*x
+a)*tanh(b*x+a)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.55

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{\operatorname{csch}^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},2,-\frac{1}{2},-\sinh^2(a+bx)\right)}{3b}$$

input
```
Integrate[Csch[a + b*x]^4*Sech[a + b*x]^3,x]
```

output
```
-1/3*(Csch[a + b*x]^3*Hypergeometric2F1[-3/2, 2, -1/2, -Sinh[a + b*x]^2])/
b
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 3101, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(ia+ibx)^4\sec(ia+ibx)^3dx$$

$$\Big\downarrow \text{3101}$$

$$\frac{i\int -\frac{\operatorname{csch}^6(a+bx)}{\left(\operatorname{csch}^2(a+bx)+1\right)^2}d(-i\operatorname{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{i\left(-\frac{5}{2}\int \frac{\operatorname{csch}^4(a+bx)}{\operatorname{csch}^2(a+bx)+1}d(-i\operatorname{csch}(a+bx))-\frac{i\operatorname{csch}^5(a+bx)}{2\left(\operatorname{csch}^2(a+bx)+1\right)}\right)}{b}$$

$$\Big\downarrow \text{254}$$

$$\frac{i\left(-\frac{5}{2}\int \left(\operatorname{csch}^2(a+bx)+\frac{1}{\operatorname{csch}^2(a+bx)+1}-1\right)d(-i\operatorname{csch}(a+bx))-\frac{i\operatorname{csch}^5(a+bx)}{2\left(\operatorname{csch}^2(a+bx)+1\right)}\right)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-\frac{5}{2}\left(-i\arctan(\operatorname{csch}(a+bx))-\frac{1}{3}i\operatorname{csch}^3(a+bx)+i\operatorname{csch}(a+bx)\right)-\frac{i\operatorname{csch}^5(a+bx)}{2\left(\operatorname{csch}^2(a+bx)+1\right)}\right)}{b}$$

input
```
Int[Csch[a + b*x]^4*Sech[a + b*x]^3,x]
```

output
```
(I*(((-1/2*I)*Csch[a + b*x]^5)/(1 + Csch[a + b*x]^2) - (5*((-I)*ArcTan[Csc
h[a + b*x]] + I*Csch[a + b*x] - (I/3)*Csch[a + b*x]^3))/2))/b
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 12.41 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{-\frac{1}{3\sinh(bx+a)^3\cosh(bx+a)^2}+\frac{5}{3\sinh(bx+a)\cosh(bx+a)^2}+\frac{5\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2}+5\arctan\left(e^{bx+a}\right)}{b}$	65
default	$\frac{-\frac{1}{3\sinh(bx+a)^3\cosh(bx+a)^2}+\frac{5}{3\sinh(bx+a)\cosh(bx+a)^2}+\frac{5\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2}+5\arctan\left(e^{bx+a}\right)}{b}$	65
risch	$\frac{e^{bx+a}\left(15\,e^{8bx+8a}-20\,e^{6bx+6a}-22\,e^{4bx+4a}-20\,e^{2bx+2a}+15\right)}{3b\left(e^{2bx+2a}+1\right)^2\left(e^{2bx+2a}-1\right)^3}+\frac{5i\ln\left(e^{bx+a}+i\right)}{2b}-\frac{5i\ln\left(e^{bx+a}-i\right)}{2b}$	117

input
```
int(csch(b*x+a)^4*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3/sinh(b*x+a)^3/cosh(b*x+a)^2+5/3/sinh(b*x+a)/cosh(b*x+a)^2+5/2*se
ch(b*x+a)*tanh(b*x+a)+5*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1176 vs. 2(54) = 108.

Time = 0.08 (sec) , antiderivative size = 1176, normalized size of antiderivative = 19.60

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx=\text{Too large to display}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/3*(15*cosh(b*x + a)^9 + 135*cosh(b*x + a)*sinh(b*x + a)^8 + 15*sinh(b*x
+ a)^9 + 20*(27*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^7 - 20*cosh(b*x + a)^7
+ 140*(9*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^6 + 2*(945*cosh(b*
x + a)^4 - 210*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^5 - 22*cosh(b*x + a)^5
+ 10*(189*cosh(b*x + a)^5 - 70*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b*
x + a)^4 + 20*(63*cosh(b*x + a)^6 - 35*cosh(b*x + a)^4 - 11*cosh(b*x + a)^
2 - 1)*sinh(b*x + a)^3 - 20*cosh(b*x + a)^3 + 20*(27*cosh(b*x + a)^7 - 21*
cosh(b*x + a)^5 - 11*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^2 +
15*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^10
 + (45*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^8 - cosh(b*x + a)^8 + 8*(15*cosh
(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^7 + 2*(105*cosh(b*x + a)^4 - 14
*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 2*cosh(b*x + a)^6 + 4*(63*cosh(b*x
 + a)^5 - 14*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(105*c
osh(b*x + a)^6 - 35*cosh(b*x + a)^4 - 15*cosh(b*x + a)^2 + 1)*sinh(b*x + a
)^4 + 2*cosh(b*x + a)^4 + 8*(15*cosh(b*x + a)^7 - 7*cosh(b*x + a)^5 - 5*co
sh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^3 + (45*cosh(b*x + a)^8 - 28*
cosh(b*x + a)^6 - 30*cosh(b*x + a)^4 + 12*cosh(b*x + a)^2 + 1)*sinh(b*x +
a)^2 + cosh(b*x + a)^2 + 2*(5*cosh(b*x + a)^9 - 4*cosh(b*x + a)^7 - 6*cosh
(b*x + a)^5 + 4*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) - 1)*arctan
(cosh(b*x + a) + sinh(b*x + a)) + 5*(27*cosh(b*x + a)^8 - 28*cosh(b*x +...
```

Sympy [F]

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx = \int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**4*sech(b*x+a)**3,x)
```

output
```
Integral(csch(a + b*x)**4*sech(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(54) = 108$.

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.20

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx = -\frac{5\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{15\,e^{(-bx-a)} - 20\,e^{(-3\,bx-3\,a)} - 22\,e^{(-5\,bx-5\,a)} - 20\,e^{(-7\,bx-7\,a)} + 15\,e^{(-9\,bx-9\,a)}}{3\,b(e^{(-2\,bx-2\,a)} + 2\,e^{(-4\,bx-4\,a)} - 2\,e^{(-6\,bx-6\,a)} - e^{(-8\,bx-8\,a)} + e^{(-10\,bx-10\,a)} - 1)}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
-5*arctan(e^(-b*x - a))/b - 1/3*(15*e^(-b*x - a) - 20*e^(-3*b*x - 3*a) - 2
2*e^(-5*b*x - 5*a) - 20*e^(-7*b*x - 7*a) + 15*e^(-9*b*x - 9*a))/(b*(e^(-2*
b*x - 2*a) + 2*e^(-4*b*x - 4*a) - 2*e^(-6*b*x - 6*a) - e^(-8*b*x - 8*a) +
e^(-10*b*x - 10*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(54) = 108$.

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.07

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{15\,\pi + \frac{12\left(e^{(bx+a)} - e^{(-bx-a)}\right)}{\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2 + 4} + \frac{16\left(3\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2 - 2\right)}{\left(e^{(bx+a)} - e^{(-bx-a)}\right)^3} + 30\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)} - 1\right)e^{(-bx-a)}\right)}{12\,b}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
1/12*(15*pi + 12*(e^(b*x + a) - e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a)
)^2 + 4) + 16*(3*(e^(b*x + a) - e^(-b*x - a))^2 - 2)/(e^(b*x + a) - e^(-b*
x - a))^3 + 30*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.12

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{5\operatorname{atan}\left(\frac{e^{bx}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{8\,e^{a+b\,x}}{3\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{2\,e^{a+b\,x}}{b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} - \frac{8\,e^{a+b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)} + \frac{4\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)} + \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^3*sinh(a + b*x)^4),x)
```

output
```
(5*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (8*exp(a + b*x))/(
3*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (2*exp(a + b*x))/(b*(2*
exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (8*exp(a + b*x))/(3*b*(3*exp(2
*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) + (4*exp(a + b*x
))/(b*(exp(2*a + 2*b*x) - 1)) + exp(a + b*x)/(b*(exp(2*a + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.88

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{15e^{10bx+10a}atan\left(e^{bx+a}\right) - 15e^{8bx+8a}atan\left(e^{bx+a}\right) - 30e^{6bx+6a}atan\left(e^{bx+a}\right) + 30e^{4bx+4a}atan\left(e^{bx+a}\right) + 15e^{2bx}}{3b\left(e^{10bx+10a} - e^{8bx+8a} - 2e^{6bx+6a} +\right.}$$

input
```
int(csch(b*x+a)^4*sech(b*x+a)^3,x)
```

output
```
(15*e**(10*a + 10*b*x)*atan(e**(a + b*x)) - 15*e**(8*a + 8*b*x)*atan(e**(a
 + b*x)) - 30*e**(6*a + 6*b*x)*atan(e**(a + b*x)) + 30*e**(4*a + 4*b*x)*at
an(e**(a + b*x)) + 15*e**(2*a + 2*b*x)*atan(e**(a + b*x)) - 15*atan(e**(a
+ b*x)) + 15*e**(9*a + 9*b*x) - 20*e**(7*a + 7*b*x) - 22*e**(5*a + 5*b*x)
- 20*e**(3*a + 3*b*x) + 15*e**(a + b*x))/(3*b*(e**(10*a + 10*b*x) - e**(8*
a + 8*b*x) - 2*e**(6*a + 6*b*x) + 2*e**(4*a + 4*b*x) + e**(2*a + 2*b*x) -
1))
```

3.98 $\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{3\coth(a+bx)}{b} - \frac{\coth^3(a+bx)}{3b} + \frac{3\tanh(a+bx)}{b} - \frac{\tanh^3(a+bx)}{3b}$$

output
```
3*coth(b*x+a)/b-1/3*coth(b*x+a)^3/b+3*tanh(b*x+a)/b-1/3*tanh(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx = 16\left(\frac{\coth(2(a+bx))}{3b} - \frac{\coth(2(a+bx))\operatorname{csch}^2(2(a+bx))}{6b}\right)$$

input
```
Integrate[Csch[a + b*x]^4*Sech[a + b*x]^4,x]
```

output
```
16*(Coth[2*(a + b*x)]/(3*b) - (Coth[2*(a + b*x)]*Csch[2*(a + b*x)]^2)/(6*b
))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(ia+ibx)^4\sec(ia+ibx)^4dx$$

$$\Big\downarrow \text{3100}$$

$$-\frac{i\int \coth^4(a+bx)\left(1-\tanh^2(a+bx)\right)^3d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{i\int \left(\coth^4(a+bx)-3\coth^2(a+bx)-\tanh^2(a+bx)+3\right)d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-\frac{1}{3}i\tanh^3(a+bx)+3i\tanh(a+bx)-\frac{1}{3}i\coth^3(a+bx)+3i\coth(a+bx)\right)}{b}$$

input
```
Int[Csch[a + b*x]^4*Sech[a + b*x]^4,x]
```

output
```
((-I)*((3*I)*Coth[a + b*x] - (I/3)*Coth[a + b*x]^3 + (3*I)*Tanh[a + b*x] -
 (I/3)*Tanh[a + b*x]^3))/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 28.52 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

method	result	size
risch	$-\frac{32(3\,e^{4bx+4a}-1)}{3b(e^{2bx+2a}-1)^3(e^{2bx+2a}+1)^3}$	45
derivativedivides	$\frac{-\frac{1}{3\sinh(bx+a)^3\cosh(bx+a)^3}+\frac{2}{\sinh(bx+a)\cosh(bx+a)^3}+8\left(\frac{2}{3}+\frac{\operatorname{sech}(bx+a)^2}{3}\right)\tanh(bx+a)}{b}$	62
default	$\frac{-\frac{1}{3\sinh(bx+a)^3\cosh(bx+a)^3}+\frac{2}{\sinh(bx+a)\cosh(bx+a)^3}+8\left(\frac{2}{3}+\frac{\operatorname{sech}(bx+a)^2}{3}\right)\tanh(bx+a)}{b}$	62

input
```
int(csch(b*x+a)^4*sech(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-32/3*(3*exp(4*b*x+4*a)-1)/b/(exp(2*b*x+2*a)-1)^3/(exp(2*b*x+2*a)+1)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(49) = 98$.

Time = 0.07 (sec) , antiderivative size = 330, normalized size of antiderivative = 6.23

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx =$$

$$-\frac{}{3\left(b\cosh\left(bx+a\right)^{10}+120\,b\cosh\left(bx+a\right)^{3}\sinh\left(bx+a\right)^{7}+45\,b\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{8}+10\,b\cos\right.}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^4,x, algorithm="fricas")
```

output
```
-64/3*(cosh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)/
(b*cosh(b*x + a)^10 + 120*b*cosh(b*x + a)^3*sinh(b*x + a)^7 + 45*b*cosh(b*
x + a)^2*sinh(b*x + a)^8 + 10*b*cosh(b*x + a)*sinh(b*x + a)^9 + b*sinh(b*x
 + a)^10 - 3*b*cosh(b*x + a)^6 + 3*(70*b*cosh(b*x + a)^4 - b)*sinh(b*x + a
)^6 + 18*(14*b*cosh(b*x + a)^5 - b*cosh(b*x + a))*sinh(b*x + a)^5 + 15*(14
*b*cosh(b*x + a)^6 - 3*b*cosh(b*x + a)^2)*sinh(b*x + a)^4 + 60*(2*b*cosh(b
*x + a)^7 - b*cosh(b*x + a)^3)*sinh(b*x + a)^3 + 2*b*cosh(b*x + a)^2 + (45
*b*cosh(b*x + a)^8 - 45*b*cosh(b*x + a)^4 + 2*b)*sinh(b*x + a)^2 + 2*(5*b*
cosh(b*x + a)^9 - 9*b*cosh(b*x + a)^5 + 4*b*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F]

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx = \int \operatorname{csch}^4\left(a+bx\right)\operatorname{sech}^4\left(a+bx\right)dx$$

input
```
integrate(csch(b*x+a)**4*sech(b*x+a)**4,x)
```

output
```
Integral(csch(a + b*x)**4*sech(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{32\,e^{(-4\,bx-4\,a)}}{b(3\,e^{(-4\,bx-4\,a)}-3\,e^{(-8\,bx-8\,a)}+e^{(-12\,bx-12\,a)}-1)} - \frac{32}{3\,b(3\,e^{(-4\,bx-4\,a)}-3\,e^{(-8\,bx-8\,a)}+e^{(-12\,bx-12\,a)}-1)}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^4,x, algorithm="maxima")
```

output
```
32*e^(-4*b*x - 4*a)/(b*(3*e^(-4*b*x - 4*a) - 3*e^(-8*b*x - 8*a) + e^(-12*b
*x - 12*a) - 1)) - 32/3/(b*(3*e^(-4*b*x - 4*a) - 3*e^(-8*b*x - 8*a) + e^(-
12*b*x - 12*a) - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{32\,\left(3\,e^{(4\,bx+4\,a)}-1\right)}{3\,b\left(e^{(4\,bx+4\,a)}-1\right)^3}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^4,x, algorithm="giac")
```

output
```
-32/3*(3*e^(4*b*x + 4*a) - 1)/(b*(e^(4*b*x + 4*a) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{32\,\left(3\,\mathrm{e}^{4\,a+4\,b\,x}-1\right)}{3\,b\,{\left(\mathrm{e}^{4\,a+4\,b\,x}-1\right)}^3}$$

input
```
int(1/(cosh(a + b*x)^4*sinh(a + b*x)^4),x)
```

output
```
-(32*(3*exp(4*a + 4*b*x) - 1))/(3*b*(exp(4*a + 4*b*x) - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{-32e^{4bx+4a}+\frac{32}{3}}{b\left(e^{12bx+12a}-3e^{8bx+8a}+3e^{4bx+4a}-1\right)}$$

input
```
int(csch(b*x+a)^4*sech(b*x+a)^4,x)
```

output
```
(32*( - 3*e**(4*a + 4*b*x) + 1))/(3*b*(e**(12*a + 12*b*x) - 3*e**(8*a + 8*
b*x) + 3*e**(4*a + 4*b*x) - 1))
```

3.99 $\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 81

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx=\frac{35\arctan(\sinh(a+bx))}{8b}+\frac{3\operatorname{csch}(a+bx)}{b}-\frac{\operatorname{csch}^3(a+bx)}{3b}+\frac{13\operatorname{sech}(a+bx)\tanh(a+bx)}{8b}-\frac{\operatorname{sech}(a+bx)\tanh^3(a+bx)}{4b}$$

output
```
35/8*arctan(sinh(b*x+a))/b+3*csch(b*x+a)/b-1/3*csch(b*x+a)^3/b+13/8*sech(b
*x+a)*tanh(b*x+a)/b-1/4*sech(b*x+a)*tanh(b*x+a)^3/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.41

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx=-\frac{\operatorname{csch}^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},3,-\frac{1}{2},-\sinh^2(a+bx)\right)}{3b}$$

input
```
Integrate[Csch[a + b*x]^4*Sech[a + b*x]^5,x]
```

output
```
-1/3*(Csch[a + b*x]^3*Hypergeometric2F1[-3/2, 3, -1/2, -Sinh[a + b*x]^2])/
b
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3042, 3101, 25, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(ia+ibx)^4\sec(ia+ibx)^5dx$$

$$\Big\downarrow \text{3101}$$

$$\frac{i\int -\frac{\operatorname{csch}^8(a+bx)}{\left(\operatorname{csch}^2(a+bx)+1\right)^3}d(-i\operatorname{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{i\int \frac{\operatorname{csch}^8(a+bx)}{\left(\operatorname{csch}^2(a+bx)+1\right)^3}d(-i\operatorname{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{i\left(\frac{7}{4}\int -\frac{\operatorname{csch}^6(a+bx)}{\left(\operatorname{csch}^2(a+bx)+1\right)^2}d(-i\operatorname{csch}(a+bx))-\frac{i\operatorname{csch}^7(a+bx)}{4\left(\operatorname{csch}^2(a+bx)+1\right)^2}\right)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{i\left(\frac{7}{4}\left(-\frac{5}{2}\int\frac{\operatorname{csch}^4(a+bx)}{\operatorname{csch}^2(a+bx)+1}d(-i\operatorname{csch}(a+bx))-\frac{i\operatorname{csch}^5(a+bx)}{2\left(\operatorname{csch}^2(a+bx)+1\right)}\right)-\frac{i\operatorname{csch}^7(a+bx)}{4\left(\operatorname{csch}^2(a+bx)+1\right)^2}\right)}{b}$$

$\Big\downarrow$ 254

$$\frac{i\left(\frac{7}{4}\left(-\frac{5}{2}\int\left(\operatorname{csch}^2(a+bx)+\frac{1}{\operatorname{csch}^2(a+bx)+1}-1\right)d(-i\operatorname{csch}(a+bx))-\frac{i\operatorname{csch}^5(a+bx)}{2\left(\operatorname{csch}^2(a+bx)+1\right)}\right)-\frac{i\operatorname{csch}^7(a+bx)}{4\left(\operatorname{csch}^2(a+bx)+1\right)^2}\right)}{b}$$

$\Big\downarrow$ 2009

$$\frac{i\left(\frac{7}{4}\left(-\frac{5}{2}\left(-i\arctan(\operatorname{csch}(a+bx))-\frac{1}{3}i\operatorname{csch}^3(a+bx)+i\operatorname{csch}(a+bx)\right)-\frac{i\operatorname{csch}^5(a+bx)}{2\left(\operatorname{csch}^2(a+bx)+1\right)}\right)-\frac{i\operatorname{csch}^7(a+bx)}{4\left(\operatorname{csch}^2(a+bx)+1\right)^2}\right)}{b}$$

input
```
Int[Csch[a + b*x]^4*Sech[a + b*x]^5,x]
```

output
```
(I*(((-1/4*I)*Csch[a + b*x]^7)/(1 + Csch[a + b*x]^2)^2 + (7*(((-1/2*I)*Csc
h[a + b*x]^5)/(1 + Csch[a + b*x]^2) - (5*((-I)*ArcTan[Csch[a + b*x]] + I*C
sch[a + b*x] - (I/3)*Csch[a + b*x]^3))/2))/4))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 59.80 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{1}{3\sinh(bx+a)^3\cosh(bx+a)^4}+\frac{7}{3\sinh(bx+a)\cosh(bx+a)^4}+\frac{35\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)}{3}+\frac{35\arctan\left(e^{bx+a}\right)}{4}}{b}$
default	$\frac{-\frac{1}{3\sinh(bx+a)^3\cosh(bx+a)^4}+\frac{7}{3\sinh(bx+a)\cosh(bx+a)^4}+\frac{35\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)}{3}+\frac{35\arctan\left(e^{bx+a}\right)}{4}}{b}$
risch	$\frac{e^{bx+a}\left(105\,e^{12bx+12a}+70\,e^{10bx+10a}-329\,e^{8bx+8a}-204\,e^{6bx+6a}-329\,e^{4bx+4a}+70\,e^{2bx+2a}+105\right)}{12b\left(e^{2bx+2a}+1\right)^4\left(e^{2bx+2a}-1\right)^3}+\frac{35i\ln\left(e^{bx+a}+i\right)}{8b}$

input
```
int(csch(b*x+a)^4*sech(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3/sinh(b*x+a)^3/cosh(b*x+a)^4+7/3/sinh(b*x+a)/cosh(b*x+a)^4+35/3*(
1/4*sech(b*x+a)^3+3/8*sech(b*x+a))*tanh(b*x+a)+35/4*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2092 vs. $2(73) = 146$.

Time = 0.09 (sec) , antiderivative size = 2092, normalized size of antiderivative = 25.83

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/12*(105*cosh(b*x + a)^13 + 1365*cosh(b*x + a)*sinh(b*x + a)^12 + 105*sin
h(b*x + a)^13 + 70*(117*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^11 + 70*cosh(b*
x + a)^11 + 770*(39*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^10 + 7*
(10725*cosh(b*x + a)^4 + 550*cosh(b*x + a)^2 - 47)*sinh(b*x + a)^9 - 329*c
osh(b*x + a)^9 + 21*(6435*cosh(b*x + a)^5 + 550*cosh(b*x + a)^3 - 141*cosh
(b*x + a))*sinh(b*x + a)^8 + 12*(15015*cosh(b*x + a)^6 + 1925*cosh(b*x + a
)^4 - 987*cosh(b*x + a)^2 - 17)*sinh(b*x + a)^7 - 204*cosh(b*x + a)^7 + 84
*(2145*cosh(b*x + a)^7 + 385*cosh(b*x + a)^5 - 329*cosh(b*x + a)^3 - 17*co
sh(b*x + a))*sinh(b*x + a)^6 + 7*(19305*cosh(b*x + a)^8 + 4620*cosh(b*x +
a)^6 - 5922*cosh(b*x + a)^4 - 612*cosh(b*x + a)^2 - 47)*sinh(b*x + a)^5 -
329*cosh(b*x + a)^5 + 7*(10725*cosh(b*x + a)^9 + 3300*cosh(b*x + a)^7 - 59
22*cosh(b*x + a)^5 - 1020*cosh(b*x + a)^3 - 235*cosh(b*x + a))*sinh(b*x +
a)^4 + 14*(2145*cosh(b*x + a)^10 + 825*cosh(b*x + a)^8 - 1974*cosh(b*x + a
)^6 - 510*cosh(b*x + a)^4 - 235*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^3 + 70*
cosh(b*x + a)^3 + 14*(585*cosh(b*x + a)^11 + 275*cosh(b*x + a)^9 - 846*cos
h(b*x + a)^7 - 306*cosh(b*x + a)^5 - 235*cosh(b*x + a)^3 + 15*cosh(b*x + a
))*sinh(b*x + a)^2 + 105*(cosh(b*x + a)^14 + 14*cosh(b*x + a)*sinh(b*x + a
)^13 + sinh(b*x + a)^14 + (91*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^12 + cosh
(b*x + a)^12 + 4*(91*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^11 +
 (1001*cosh(b*x + a)^4 + 66*cosh(b*x + a)^2 - 3)*sinh(b*x + a)^10 - 3*c...
```

Sympy [F]

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx=\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**4*sech(b*x+a)**5,x)
```

output
```
Integral(csch(a + b*x)**4*sech(a + b*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(73) = 146$.

Time = 0.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.20

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx=-\frac{35\arctan\left(e^{(-bx-a)}\right)}{4\,b}+\frac{105\,e^{(-bx-a)}+70\,e^{(-3\,bx-3\,a)}-329\,e^{(-5\,bx-5\,a)}-204\,e^{(-7\,bx-7\,a)}-329\,e^{(-9\,bx-9\,a)}+70\,e^{(-11\,bx-11\,a)}+105}{12\,b(e^{(-2\,bx-2\,a)}-3\,e^{(-4\,bx-4\,a)}-3\,e^{(-6\,bx-6\,a)}+3\,e^{(-8\,bx-8\,a)}+3\,e^{(-10\,bx-10\,a)}-e^{(-12\,bx-12\,a)}-e^{(-14\,bx}}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^5,x, algorithm="maxima")
```

output
```
-35/4*arctan(e^(-b*x - a))/b + 1/12*(105*e^(-b*x - a) + 70*e^(-3*b*x - 3*a
) - 329*e^(-5*b*x - 5*a) - 204*e^(-7*b*x - 7*a) - 329*e^(-9*b*x - 9*a) + 7
0*e^(-11*b*x - 11*a) + 105*e^(-13*b*x - 13*a))/(b*(e^(-2*b*x - 2*a) - 3*e^
(-4*b*x - 4*a) - 3*e^(-6*b*x - 6*a) + 3*e^(-8*b*x - 8*a) + 3*e^(-10*b*x -
10*a) - e^(-12*b*x - 12*a) - e^(-14*b*x - 14*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(73) = 146$.

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.83

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{105\,\pi + \frac{12\left(11\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+52\,e^{(bx+a)}-52\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^2} + \frac{32\left(9\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2-4\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3} + 210\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)}\right.\right.}{48\,b}$$

input
```
integrate(csch(b*x+a)^4*sech(b*x+a)^5,x, algorithm="giac")
```

output
```
1/48*(105*pi + 12*(11*(e^(b*x + a) - e^(-b*x - a))^3 + 52*e^(b*x + a) - 52
*e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4)^2 + 32*(9*(e^(b*x + a)
 - e^(-b*x - a))^2 - 4)/(e^(b*x + a) - e^(-b*x - a))^3 + 210*arctan(1/2*(e
^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.59

$$\begin{aligned}\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx &= \frac{35\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{4\,\sqrt{b^2}} - \frac{8\,e^{a+b\,x}}{3\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}\\ &- \frac{7\,e^{a+b\,x}}{2\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} - \frac{8\,e^{a+b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)}\\ &- \frac{6\,e^{a+b\,x}}{b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)}\\ &+ \frac{4\,e^{a+b\,x}}{b\,\left(4\,e^{2\,a+2\,b\,x}+6\,e^{4\,a+4\,b\,x}+4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)}\\ &+ \frac{6\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)} + \frac{11\,e^{a+b\,x}}{4\,b\,\left(e^{2\,a+2\,b\,x}+1\right)}\end{aligned}$$

input
```
int(1/(cosh(a + b*x)^5*sinh(a + b*x)^4),x)
```

output
```
(35*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(4*(b^2)^(1/2)) - (8*exp(a + b*
x))/(3*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (7*exp(a + b*x))/(
2*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (8*exp(a + b*x))/(3*b*(
3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (6*exp(
a + b*x))/(b*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) +
 1)) + (4*exp(a + b*x))/(b*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*ex
p(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1)) + (6*exp(a + b*x))/(b*(exp(2*a + 2
*b*x) - 1)) + (11*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 321, normalized size of antiderivative = 3.96

$$\int \operatorname{csch}^4(a+bx)\operatorname{sech}^5(a+bx)\,dx$$
$$= \frac{105e^{14bx+14a}atan\left(e^{bx+a}\right)+105e^{12bx+12a}atan\left(e^{bx+a}\right)-315e^{10bx+10a}atan\left(e^{bx+a}\right)-315e^{8bx+8a}atan\left(e^{bx+a}\right)}{1}$$

input
```
int(csch(b*x+a)^4*sech(b*x+a)^5,x)
```

output
```
(105*e**(14*a + 14*b*x)*atan(e**(a + b*x)) + 105*e**(12*a + 12*b*x)*atan(e
**(a + b*x)) - 315*e**(10*a + 10*b*x)*atan(e**(a + b*x)) - 315*e**(8*a + 8
*b*x)*atan(e**(a + b*x)) + 315*e**(6*a + 6*b*x)*atan(e**(a + b*x)) + 315*e
**(4*a + 4*b*x)*atan(e**(a + b*x)) - 105*e**(2*a + 2*b*x)*atan(e**(a + b*x
)) - 105*atan(e**(a + b*x)) + 105*e**(13*a + 13*b*x) + 70*e**(11*a + 11*b*
x) - 329*e**(9*a + 9*b*x) - 204*e**(7*a + 7*b*x) - 329*e**(5*a + 5*b*x) +
70*e**(3*a + 3*b*x) + 105*e**(a + b*x))/(12*b*(e**(14*a + 14*b*x) + e**(12
*a + 12*b*x) - 3*e**(10*a + 10*b*x) - 3*e**(8*a + 8*b*x) + 3*e**(6*a + 6*b
*x) + 3*e**(4*a + 4*b*x) - e**(2*a + 2*b*x) - 1))
```

3.100 $\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx=\frac{\coth^2(a+bx)}{b}-\frac{\coth^4(a+bx)}{4b}+\frac{\log(\tanh(a+bx))}{b}$$

output
```
coth(b*x+a)^2/b-1/4*coth(b*x+a)^4/b+ln(tanh(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx
=\frac{2\operatorname{csch}^2(a+bx)-\operatorname{csch}^4(a+bx)-4\log(\cosh(a+bx))+4\log(\sinh(a+bx))}{4b}$$

input
```
Integrate[Csch[a + b*x]^5*Sech[a + b*x],x]
```

output
```
(2*Csch[a + b*x]^2 - Csch[a + b*x]^4 - 4*Log[Cosh[a + b*x]] + 4*Log[Sinh[a
 + b*x]])/(4*b)
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 26, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^5(a+bx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\csc(ia+ibx)^5\sec(ia+ibx)dx$$

$$\Big\downarrow \text{26}$$

$$i\int \csc(ia+ibx)^5\sec(ia+ibx)dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int -i\coth^5(a+bx)\left(1-\tanh^2(a+bx)\right)^2d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int i\coth^3(a+bx)\left(1-\tanh^2(a+bx)\right)^2d\left(-\tanh^2(a+bx)\right)}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(i\coth^3(a+bx)-2\coth^2(a+bx)-i\coth(a+bx)\right)d\left(-\tanh^2(a+bx)\right)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\coth^2(a+bx)+2i\coth(a+bx)+\log\left(-\tanh^2(a+bx)\right)}{2b}$$

input
```
Int[Csch[a + b*x]^5*Sech[a + b*x],x]
```

output
```
((2*I)*Coth[a + b*x] + Coth[a + b*x]^2/2 + Log[-Tanh[a + b*x]^2])/(2*b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 3.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{-\frac{1}{4\sinh(bx+a)^4}+\frac{1}{2\sinh(bx+a)^2}+\ln(\tanh(bx+a))}{b}$	33
default	$\frac{-\frac{1}{4\sinh(bx+a)^4}+\frac{1}{2\sinh(bx+a)^2}+\ln(\tanh(bx+a))}{b}$	33
risch	$\frac{2\,e^{2bx+2a}\left(e^{4bx+4a}-4\,e^{2bx+2a}+1\right)}{b\left(e^{2bx+2a}-1\right)^4}-\frac{\ln\left(e^{2bx+2a}+1\right)}{b}+\frac{\ln\left(e^{2bx+2a}-1\right)}{b}$	84

input
```
int(csch(b*x+a)^5*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4/sinh(b*x+a)^4+1/2/sinh(b*x+a)^2+ln(tanh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1082 vs. 2(37) = 74.

Time = 0.09 (sec) , antiderivative size = 1082, normalized size of antiderivative = 27.74

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a),x, algorithm="fricas")
```

output
```
(2*cosh(b*x + a)^6 + 12*cosh(b*x + a)*sinh(b*x + a)^5 + 2*sinh(b*x + a)^6
+ 2*(15*cosh(b*x + a)^2 - 4)*sinh(b*x + a)^4 - 8*cosh(b*x + a)^4 + 8*(5*co
sh(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a)^3 + 2*(15*cosh(b*x + a)^4 -
 24*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 - (cosh(b*x +
 a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*cosh(b*x
+ a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 - 3
*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*cosh(b*x + a)
^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - 10*co
sh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 -
15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*cosh(b*x +
 a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 - cosh(
b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x
 + a))) + (cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x +
a)^8 + 4*(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(
7*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)
^4 - 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*co
sh(b*x + a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*
(7*cosh(b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x
+ a)^2 - 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*co
sh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(b*x + a)/(...
```

Sympy [F]

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx=\int \operatorname{csch}^5\left(a+bx\right)\operatorname{sech}\left(a+bx\right)dx$$

input
```
integrate(csch(b*x+a)**5*sech(b*x+a),x)
```

output
```
Integral(csch(a + b*x)**5*sech(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(37) = 74$.

Time = 0.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.41

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} - \frac{2\left(e^{(-2\,bx-2\,a)} - 4\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)}\right)}{b(4\,e^{(-2\,bx-2\,a)} - 6\,e^{(-4\,bx-4\,a)} + 4\,e^{(-6\,bx-6\,a)} - e^{(-8\,bx-8\,a)} - 1)}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a),x, algorithm="maxima")
```

output
```
log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b - log(e^(-2*b*x - 2*a) +
 1)/b - 2*(e^(-2*b*x - 2*a) - 4*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a))/(b*(4
*e^(-2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) - e^(-8*b*x -
8*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(37) = 74$.

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.13

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx = -\frac{\frac{3\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}\right)^2 - 20\,e^{(2\,bx+2\,a)} - 20\,e^{(-2\,bx-2\,a)} + 44}{\left(e^{(2\,bx+2\,a)}+e^{(-2\,bx-2\,a)}-2\right)^2} + 2\,\log\left(e^{(2\,bx+2\,a)} + e^{(-2\,bx-2\,a)} + 2\right) - 2\,\log\left(e^{(2\,bx+2\,a}\right.}{4\,b}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a),x, algorithm="giac")
```

output
```
-1/4*((3*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2 - 20*e^(2*b*x + 2*a) - 20*
e^(-2*b*x - 2*a) + 44)/(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) - 2)^2 + 2*log(
e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) - 2*log(e^(2*b*x + 2*a) + e^(-2*b*
x - 2*a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 169, normalized size of antiderivative = 4.33

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{2}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}-\frac{2\,\mathrm{atan}\left(\frac{e^{2\,a}\,e^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{8}{b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)} - \frac{4}{b\,\left(6\,e^{4\,a+4\,b\,x}-4\,e^{2\,a+2\,b\,x}-4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)*sinh(a + b*x)^5),x)
```

output
```
2/(b*(exp(2*a + 2*b*x) - 1)) - (2*atan((exp(2*a)*exp(2*b*x)*(-b^2)^(1/2))/
b))/(-b^2)^(1/2) - 2/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - 8/(
b*(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - 4/(b
*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*exp(6*a + 6*b*x) + exp(8*a +
 8*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 394, normalized size of antiderivative = 10.10

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}(a+bx)\,dx = \frac{-2e^{8bx+8a}\log\left(e^{2bx+2a}+1\right)+2e^{8bx+8a}\log\left(e^{bx+a}-1\right)+2e^{8bx+8a}\log\left(e^{bx+a}+1\right)+e^{8bx+8a}+8e^{6bx+6a}\log\left(e^{2}\right.}{}$$

input
```
int(csch(b*x+a)^5*sech(b*x+a),x)
```

output
```
( - 2*e**(8*a + 8*b*x)*log(e**(2*a + 2*b*x) + 1) + 2*e**(8*a + 8*b*x)*log(
e**(a + b*x) - 1) + 2*e**(8*a + 8*b*x)*log(e**(a + b*x) + 1) + e**(8*a + 8
*b*x) + 8*e**(6*a + 6*b*x)*log(e**(2*a + 2*b*x) + 1) - 8*e**(6*a + 6*b*x)*
log(e**(a + b*x) - 1) - 8*e**(6*a + 6*b*x)*log(e**(a + b*x) + 1) - 12*e**(
4*a + 4*b*x)*log(e**(2*a + 2*b*x) + 1) + 12*e**(4*a + 4*b*x)*log(e**(a + b
*x) - 1) + 12*e**(4*a + 4*b*x)*log(e**(a + b*x) + 1) - 10*e**(4*a + 4*b*x)
 + 8*e**(2*a + 2*b*x)*log(e**(2*a + 2*b*x) + 1) - 8*e**(2*a + 2*b*x)*log(e
**(a + b*x) - 1) - 8*e**(2*a + 2*b*x)*log(e**(a + b*x) + 1) - 2*log(e**(2*
a + 2*b*x) + 1) + 2*log(e**(a + b*x) - 1) + 2*log(e**(a + b*x) + 1) + 1)/(
2*b*(e**(8*a + 8*b*x) - 4*e**(6*a + 6*b*x) + 6*e**(4*a + 4*b*x) - 4*e**(2*
a + 2*b*x) + 1))
```

3.101 $\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 65

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx = -\frac{15\operatorname{arctanh}(\cosh(a+bx))}{8b} + \frac{9\coth(a+bx)\operatorname{csch}(a+bx)}{8b} - \frac{\coth^3(a+bx)\operatorname{csch}(a+bx)}{4b} + \frac{\operatorname{sech}(a+bx)}{b}$$

output
```
-15/8*arctanh(cosh(b*x+a))/b+9/8*coth(b*x+a)*csch(b*x+a)/b-1/4*coth(b*x+a)
^3*csch(b*x+a)/b+sech(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.89

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{7\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b} - \frac{\operatorname{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b} - \frac{15\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{8b} + \frac{15\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{8b} + \frac{7\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b} + \frac{\operatorname{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b} + \frac{\operatorname{sech}(a+bx)}{b}$$

input
```
Integrate[Csch[a + b*x]^5*Sech[a + b*x]^2,x]
```

output
```
(7*Csch[(a + b*x)/2]^2)/(32*b) - Csch[(a + b*x)/2]^4/(64*b) - (15*Log[Cosh
[(a + b*x)/2]])/(8*b) + (15*Log[Sinh[(a + b*x)/2]])/(8*b) + (7*Sech[(a + b
*x)/2]^2)/(32*b) + Sech[(a + b*x)/2]^4/(64*b) + Sech[a + b*x]/b
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 26, 3102, 25, 252, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^5(a+bx)\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\csc(ia+ibx)^5\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{26}$$

$$i\int \csc(ia+ibx)^5\sec(ia+ibx)^2dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{\text{sech}^6(a+bx)}{\left(1-\text{sech}^2(a+bx)\right)^3}d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\text{sech}^6(a+bx)}{\left(1-\text{sech}^2(a+bx)\right)^3}d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{5}{4}\int \frac{\text{sech}^4(a+bx)}{\left(1-\text{sech}^2(a+bx)\right)^2}d\text{sech}(a+bx)-\frac{\text{sech}^5(a+bx)}{4\left(1-\text{sech}^2(a+bx)\right)^2}}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{5}{4}\left(\frac{\text{sech}^3(a+bx)}{2\left(1-\text{sech}^2(a+bx)\right)}-\frac{3}{2}\int \frac{\text{sech}^2(a+bx)}{1-\text{sech}^2(a+bx)}d\text{sech}(a+bx)\right)-\frac{\text{sech}^5(a+bx)}{4\left(1-\text{sech}^2(a+bx)\right)^2}}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{5}{4}\left(\frac{\text{sech}^3(a+bx)}{2\left(1-\text{sech}^2(a+bx)\right)}-\frac{3}{2}\left(\int \frac{1}{1-\text{sech}^2(a+bx)}d\text{sech}(a+bx)-\text{sech}(a+bx)\right)\right)-\frac{\text{sech}^5(a+bx)}{4\left(1-\text{sech}^2(a+bx)\right)^2}}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{5}{4}\left(\frac{\text{sech}^3(a+bx)}{2\left(1-\text{sech}^2(a+bx)\right)}-\frac{3}{2}(\text{arctanh}(\text{sech}(a+bx))-\text{sech}(a+bx))\right)-\frac{\text{sech}^5(a+bx)}{4\left(1-\text{sech}^2(a+bx)\right)^2}}{b}$$

input
```
Int[Csch[a + b*x]^5*Sech[a + b*x]^2,x]
```

output
```
(-1/4*Sech[a + b*x]^5/(1 - Sech[a + b*x]^2)^2 + (5*((-3*(ArcTanh[Sech[a +
b*x]] - Sech[a + b*x]))/2 + Sech[a + b*x]^3/(2*(1 - Sech[a + b*x]^2))))/4)
/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 9.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)}+\frac{5}{8\sinh(bx+a)^2\cosh(bx+a)}+\frac{15}{8\cosh(bx+a)}-\frac{15\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{4}}{b}$	61
default	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)}+\frac{5}{8\sinh(bx+a)^2\cosh(bx+a)}+\frac{15}{8\cosh(bx+a)}-\frac{15\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{4}}{b}$	61
risch	$\frac{\mathrm{e}^{bx+a}\left(15\,\mathrm{e}^{8bx+8a}-40\,\mathrm{e}^{6bx+6a}+18\,\mathrm{e}^{4bx+4a}-40\,\mathrm{e}^{2bx+2a}+15\right)}{4b\left(\mathrm{e}^{2bx+2a}-1\right)^4\left(\mathrm{e}^{2bx+2a}+1\right)}+\frac{15\ln\left(\mathrm{e}^{bx+a}-1\right)}{8b}-\frac{15\ln\left(\mathrm{e}^{bx+a}+1\right)}{8b}$	113

input
```
int(csch(b*x+a)^5*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4/sinh(b*x+a)^4/cosh(b*x+a)+5/8/sinh(b*x+a)^2/cosh(b*x+a)+15/8/cos
h(b*x+a)-15/4*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1591 vs. $2(59) = 118$.

Time = 0.10 (sec) , antiderivative size = 1591, normalized size of antiderivative = 24.48

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(30*cosh(b*x + a)^9 + 270*cosh(b*x + a)*sinh(b*x + a)^8 + 30*sinh(b*x
+ a)^9 + 40*(27*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^7 - 80*cosh(b*x + a)^7
+ 280*(9*cosh(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^6 + 12*(315*cosh
(b*x + a)^4 - 140*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^5 + 36*cosh(b*x + a)^
5 + 20*(189*cosh(b*x + a)^5 - 140*cosh(b*x + a)^3 + 9*cosh(b*x + a))*sinh(
b*x + a)^4 + 40*(63*cosh(b*x + a)^6 - 70*cosh(b*x + a)^4 + 9*cosh(b*x + a)
^2 - 2)*sinh(b*x + a)^3 - 80*cosh(b*x + a)^3 + 120*(9*cosh(b*x + a)^7 - 14
*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^2 -
15*(cosh(b*x + a)^10 + 10*cosh(b*x + a)*sinh(b*x + a)^9 + sinh(b*x + a)^10
 + 3*(15*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^8 - 3*cosh(b*x + a)^8 + 24*(5*
cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^7 + 2*(105*cosh(b*x + a)^4
- 42*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 2*cosh(b*x + a)^6 + 12*(21*cos
h(b*x + a)^5 - 14*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^5 + 2*(10
5*cosh(b*x + a)^6 - 105*cosh(b*x + a)^4 + 15*cosh(b*x + a)^2 + 1)*sinh(b*x
 + a)^4 + 2*cosh(b*x + a)^4 + 8*(15*cosh(b*x + a)^7 - 21*cosh(b*x + a)^5 +
 5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^3 + 3*(15*cosh(b*x + a)^
8 - 28*cosh(b*x + a)^6 + 10*cosh(b*x + a)^4 + 4*cosh(b*x + a)^2 - 1)*sinh(
b*x + a)^2 - 3*cosh(b*x + a)^2 + 2*(5*cosh(b*x + a)^9 - 12*cosh(b*x + a)^7
 + 6*cosh(b*x + a)^5 + 4*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)
+ 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 15*(cosh(b*x + a)^10 + 10...
```

Sympy [F]

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx = \int \operatorname{csch}^5\left(a+bx\right)\operatorname{sech}^2\left(a+bx\right)dx$$

input
```
integrate(csch(b*x+a)**5*sech(b*x+a)**2,x)
```

output
```
Integral(csch(a + b*x)**5*sech(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(59) = 118$.

Time = 0.05 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.38

$$\begin{aligned}&\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx\\&= -\frac{15\log\left(e^{(-bx-a)}+1\right)}{8\,b}+\frac{15\log\left(e^{(-bx-a)}-1\right)}{8\,b}\\&\quad-\frac{15\,e^{(-bx-a)}-40\,e^{(-3\,bx-3\,a)}+18\,e^{(-5\,bx-5\,a)}-40\,e^{(-7\,bx-7\,a)}+15\,e^{(-9\,bx-9\,a)}}{4\,b(3\,e^{(-2\,bx-2\,a)}-2\,e^{(-4\,bx-4\,a)}-2\,e^{(-6\,bx-6\,a)}+3\,e^{(-8\,bx-8\,a)}-e^{(-10\,bx-10\,a)}-1)}\end{aligned}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
-15/8*log(e^(-b*x - a) + 1)/b + 15/8*log(e^(-b*x - a) - 1)/b - 1/4*(15*e^(
-b*x - a) - 40*e^(-3*b*x - 3*a) + 18*e^(-5*b*x - 5*a) - 40*e^(-7*b*x - 7*a
) + 15*e^(-9*b*x - 9*a))/(b*(3*e^(-2*b*x - 2*a) - 2*e^(-4*b*x - 4*a) - 2*e
^(-6*b*x - 6*a) + 3*e^(-8*b*x - 8*a) - e^(-10*b*x - 10*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(59) = 118$.

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.00

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{\frac{4\left(7\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3-36\,e^{(bx+a)}-36\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4\right)^2}+\frac{32}{e^{(bx+a)}+e^{(-bx-a)}}-15\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)+15\log\left(e^{(bx}\right.}{16\,b}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
1/16*(4*(7*(e^(b*x + a) + e^(-b*x - a))^3 - 36*e^(b*x + a) - 36*e^(-b*x -
a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4)^2 + 32/(e^(b*x + a) + e^(-b*x - a
)) - 15*log(e^(b*x + a) + e^(-b*x - a) + 2) + 15*log(e^(b*x + a) + e^(-b*x
 - a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 214, normalized size of antiderivative = 3.29

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx = \frac{3\,e^{a+b\,x}}{2\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}-\frac{15\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{4\,\sqrt{-b^2}} - \frac{6\,e^{a+b\,x}}{b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)} - \frac{4\,e^{a+b\,x}}{b\,\left(6\,e^{4\,a+4\,b\,x}-4\,e^{2\,a+2\,b\,x}-4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)} + \frac{7\,e^{a+b\,x}}{4\,b\,\left(e^{2\,a+2\,b\,x}-1\right)}+\frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^2*sinh(a + b*x)^5),x)
```

output
```
(3*exp(a + b*x))/(2*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (15*a
tan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(4*(-b^2)^(1/2)) - (6*exp(a + b*x))
/(b*(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (4
*exp(a + b*x))/(b*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*exp(6*a + 6
*b*x) + exp(8*a + 8*b*x) + 1)) + (7*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) -
 1)) + (2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 369, normalized size of antiderivative = 5.68

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^2(a+bx)\,dx$$
$$= \frac{15e^{10bx+10a}\log\left(e^{bx+a}-1\right)-15e^{10bx+10a}\log\left(e^{bx+a}+1\right)+30e^{9bx+9a}-45e^{8bx+8a}\log\left(e^{bx+a}-1\right)+45e^{8bx+8}}{}$$

input
```
int(csch(b*x+a)^5*sech(b*x+a)^2,x)
```

output
```
(15*e**(10*a + 10*b*x)*log(e**(a + b*x) - 1) - 15*e**(10*a + 10*b*x)*log(e
**(a + b*x) + 1) + 30*e**(9*a + 9*b*x) - 45*e**(8*a + 8*b*x)*log(e**(a + b
*x) - 1) + 45*e**(8*a + 8*b*x)*log(e**(a + b*x) + 1) - 80*e**(7*a + 7*b*x)
 + 30*e**(6*a + 6*b*x)*log(e**(a + b*x) - 1) - 30*e**(6*a + 6*b*x)*log(e**
(a + b*x) + 1) + 36*e**(5*a + 5*b*x) + 30*e**(4*a + 4*b*x)*log(e**(a + b*x
) - 1) - 30*e**(4*a + 4*b*x)*log(e**(a + b*x) + 1) - 80*e**(3*a + 3*b*x) -
 45*e**(2*a + 2*b*x)*log(e**(a + b*x) - 1) + 45*e**(2*a + 2*b*x)*log(e**(a
 + b*x) + 1) + 30*e**(a + b*x) + 15*log(e**(a + b*x) - 1) - 15*log(e**(a +
 b*x) + 1))/(8*b*(e**(10*a + 10*b*x) - 3*e**(8*a + 8*b*x) + 2*e**(6*a + 6*
b*x) + 2*e**(4*a + 4*b*x) - 3*e**(2*a + 2*b*x) + 1))
```

3.102 $\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{3\coth^2(a+bx)}{2b}-\frac{\coth^4(a+bx)}{4b}+\frac{3\log(\tanh(a+bx))}{b}-\frac{\tanh^2(a+bx)}{2b}$$

output
```
3/2*coth(b*x+a)^2/b-1/4*coth(b*x+a)^4/b+3*ln(tanh(b*x+a))/b-1/2*tanh(b*x+a
)^2/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{4\operatorname{csch}^2(a+bx)-\operatorname{csch}^4(a+bx)-12\log(\cosh(a+bx))+12\log(\sinh(a+bx))+2\operatorname{sech}^2(a+bx)}{4b}$$

input
```
Integrate[Csch[a + b*x]^5*Sech[a + b*x]^3,x]
```

output
```
(4*Csch[a + b*x]^2 - Csch[a + b*x]^4 - 12*Log[Cosh[a + b*x]] + 12*Log[Sinh
[a + b*x]] + 2*Sech[a + b*x]^2)/(4*b)
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 26, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\csc(ia+ibx)^5\sec(ia+ibx)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int \csc(ia+ibx)^5\sec(ia+ibx)^3dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int -i\coth^5(a+bx)\left(1-\tanh^2(a+bx)\right)^3d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int i\coth^3(a+bx)\left(1-\tanh^2(a+bx)\right)^3d(-\tanh^2(a+bx))}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(i\coth^3(a+bx)-3\coth^2(a+bx)-3i\coth(a+bx)+1\right)d(-\tanh^2(a+bx))}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\tanh^2(a+bx)+\frac{1}{2}\coth^2(a+bx)+3i\coth(a+bx)+3\log\left(-\tanh^2(a+bx)\right)}{2b}$$

input
```
Int[Csch[a + b*x]^5*Sech[a + b*x]^3,x]
```

output
```
((3*I)*Coth[a + b*x] + Coth[a + b*x]^2/2 + 3*Log[-Tanh[a + b*x]^2] - Tanh[
a + b*x]^2)/(2*b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 22.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)^2}+\frac{3}{4\sinh(bx+a)^2\cosh(bx+a)^2}+\frac{3}{2\cosh(bx+a)^2}+3\ln(\tanh(bx+a))}{b}$	61
default	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)^2}+\frac{3}{4\sinh(bx+a)^2\cosh(bx+a)^2}+\frac{3}{2\cosh(bx+a)^2}+3\ln(\tanh(bx+a))}{b}$	61
risch	$\frac{2\,e^{2bx+2a}(3\,e^{8bx+8a}-6\,e^{6bx+6a}-2\,e^{4bx+4a}-6\,e^{2bx+2a}+3)}{b(e^{2bx+2a}-1)^4(e^{2bx+2a}+1)^2}-\frac{3\ln(e^{2bx+2a}+1)}{b}+\frac{3\ln(e^{2bx+2a}-1)}{b}$	122

input
```
int(csch(b*x+a)^5*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4/sinh(b*x+a)^4/cosh(b*x+a)^2+3/4/sinh(b*x+a)^2/cosh(b*x+a)^2+3/2/
cosh(b*x+a)^2+3*ln(tanh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2114 vs. $2(52) = 104$.

Time = 0.10 (sec) , antiderivative size = 2114, normalized size of antiderivative = 36.45

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
(6*cosh(b*x + a)^10 + 60*cosh(b*x + a)*sinh(b*x + a)^9 + 6*sinh(b*x + a)^1
0 + 6*(45*cosh(b*x + a)^2 - 2)*sinh(b*x + a)^8 - 12*cosh(b*x + a)^8 + 48*(
15*cosh(b*x + a)^3 - 2*cosh(b*x + a))*sinh(b*x + a)^7 + 4*(315*cosh(b*x +
a)^4 - 84*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 24*(6
3*cosh(b*x + a)^5 - 28*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^5 +
12*(105*cosh(b*x + a)^6 - 70*cosh(b*x + a)^4 - 5*cosh(b*x + a)^2 - 1)*sinh
(b*x + a)^4 - 12*cosh(b*x + a)^4 + 16*(45*cosh(b*x + a)^7 - 42*cosh(b*x +
a)^5 - 5*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 6*(45*cosh(b
*x + a)^8 - 56*cosh(b*x + a)^6 - 10*cosh(b*x + a)^4 - 12*cosh(b*x + a)^2 +
 1)*sinh(b*x + a)^2 + 6*cosh(b*x + a)^2 - 3*(cosh(b*x + a)^12 + 12*cosh(b*
x + a)*sinh(b*x + a)^11 + sinh(b*x + a)^12 + 2*(33*cosh(b*x + a)^2 - 1)*si
nh(b*x + a)^10 - 2*cosh(b*x + a)^10 + 20*(11*cosh(b*x + a)^3 - cosh(b*x +
a))*sinh(b*x + a)^9 + (495*cosh(b*x + a)^4 - 90*cosh(b*x + a)^2 - 1)*sinh(
b*x + a)^8 - cosh(b*x + a)^8 + 8*(99*cosh(b*x + a)^5 - 30*cosh(b*x + a)^3
- cosh(b*x + a))*sinh(b*x + a)^7 + 4*(231*cosh(b*x + a)^6 - 105*cosh(b*x +
 a)^4 - 7*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(99
*cosh(b*x + a)^7 - 63*cosh(b*x + a)^5 - 7*cosh(b*x + a)^3 + 3*cosh(b*x + a
))*sinh(b*x + a)^5 + (495*cosh(b*x + a)^8 - 420*cosh(b*x + a)^6 - 70*cosh(
b*x + a)^4 + 60*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - cosh(b*x + a)^4 + 4
*(55*cosh(b*x + a)^9 - 60*cosh(b*x + a)^7 - 14*cosh(b*x + a)^5 + 20*cos...
```

Sympy [F]

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx=\int \operatorname{csch}^5\left(a+bx\right)\operatorname{sech}^3\left(a+bx\right)dx$$

input
```
integrate(csch(b*x+a)**5*sech(b*x+a)**3,x)
```

output
```
Integral(csch(a + b*x)**5*sech(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(52) = 104$.

Time = 0.14 (sec) , antiderivative size = 179, normalized size of antiderivative = 3.09

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx$$
$$= \frac{3\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{3\log\left(e^{(-bx-a)}-1\right)}{b} - \frac{3\log\left(e^{(-2bx-2a)}+1\right)}{b} - \frac{2\left(3e^{(-2bx-2a)} - 6e^{(-4bx-4a)} - 2e^{(-6bx-6a)} - 6e^{(-8bx-8a)} + 3e^{(-10bx-10a)}\right)}{b(2e^{(-2bx-2a)} + e^{(-4bx-4a)} - 4e^{(-6bx-6a)} + e^{(-8bx-8a)} + 2e^{(-10bx-10a)} - e^{(-12bx-12a)} - 1)}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
3*log(e^(-b*x - a) + 1)/b + 3*log(e^(-b*x - a) - 1)/b - 3*log(e^(-2*b*x -
2*a) + 1)/b - 2*(3*e^(-2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) - 2*e^(-6*b*x - 6
*a) - 6*e^(-8*b*x - 8*a) + 3*e^(-10*b*x - 10*a))/(b*(2*e^(-2*b*x - 2*a) +
e^(-4*b*x - 4*a) - 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 2*e^(-10*b*x -
10*a) - e^(-12*b*x - 12*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(52) = 104$.

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.95

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx$$
$$= \frac{\frac{2\left(3e^{(2bx+2a)}+3e^{(-2bx-2a)}+10\right)}{e^{(2bx+2a)}+e^{(-2bx-2a)}+2} - \frac{9\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right)^2 - 52e^{(2bx+2a)} - 52e^{(-2bx-2a)} + 84}{\left(e^{(2bx+2a)}+e^{(-2bx-2a)}-2\right)^2} - 6\log\left(e^{(2bx+2a)} + e^{(-2bx\cdots}{4b}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*(2*(3*e^(2*b*x + 2*a) + 3*e^(-2*b*x - 2*a) + 10)/(e^(2*b*x + 2*a) + e^
(-2*b*x - 2*a) + 2) - (9*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2 - 52*e^(2*
b*x + 2*a) - 52*e^(-2*b*x - 2*a) + 84)/(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a)
 - 2)^2 - 6*log(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + 6*log(e^(2*b*x +
 2*a) + e^(-2*b*x - 2*a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.22

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{4}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}+\frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}-\frac{6\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2}{b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)}-\frac{8}{b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}-3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}-1\right)} - \frac{4}{b\,\left(6\,\mathrm{e}^{4\,a+4\,b\,x}-4\,\mathrm{e}^{2\,a+2\,b\,x}-4\,\mathrm{e}^{6\,a+6\,b\,x}+\mathrm{e}^{8\,a+8\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^3*sinh(a + b*x)^5),x)
```

output
```
4/(b*(exp(2*a + 2*b*x) - 1)) + 2/(b*(exp(2*a + 2*b*x) + 1)) - (6*atan((exp
(2*a)*exp(2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - 2/(b*(2*exp(2*a + 2*b*x)
 + exp(4*a + 4*b*x) + 1)) - 8/(b*(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x)
+ exp(6*a + 6*b*x) - 1)) - 4/(b*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) -
 4*exp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 581, normalized size of antiderivative = 10.02

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^3(a+bx)\,dx = \frac{3+3e^{4bx+4a}\log\left(e^{2bx+2a}+1\right)+6e^{2bx+2a}\log\left(e^{2bx+2a}+1\right)-6e^{10bx+10a}\log\left(e^{bx+a}-1\right)-6e^{10bx+10a}\log\left(e^{bx+}\right.}{}$$

input
```
int(csch(b*x+a)^5*sech(b*x+a)^3,x)
```

output
```
( - 3*e**(12*a + 12*b*x)*log(e**(2*a + 2*b*x) + 1) + 3*e**(12*a + 12*b*x)*
log(e**(a + b*x) - 1) + 3*e**(12*a + 12*b*x)*log(e**(a + b*x) + 1) + 3*e**
(12*a + 12*b*x) + 6*e**(10*a + 10*b*x)*log(e**(2*a + 2*b*x) + 1) - 6*e**(1
0*a + 10*b*x)*log(e**(a + b*x) - 1) - 6*e**(10*a + 10*b*x)*log(e**(a + b*x
) + 1) + 3*e**(8*a + 8*b*x)*log(e**(2*a + 2*b*x) + 1) - 3*e**(8*a + 8*b*x)
*log(e**(a + b*x) - 1) - 3*e**(8*a + 8*b*x)*log(e**(a + b*x) + 1) - 15*e**
(8*a + 8*b*x) - 12*e**(6*a + 6*b*x)*log(e**(2*a + 2*b*x) + 1) + 12*e**(6*a
 + 6*b*x)*log(e**(a + b*x) - 1) + 12*e**(6*a + 6*b*x)*log(e**(a + b*x) + 1
) + 8*e**(6*a + 6*b*x) + 3*e**(4*a + 4*b*x)*log(e**(2*a + 2*b*x) + 1) - 3*
e**(4*a + 4*b*x)*log(e**(a + b*x) - 1) - 3*e**(4*a + 4*b*x)*log(e**(a + b*
x) + 1) - 15*e**(4*a + 4*b*x) + 6*e**(2*a + 2*b*x)*log(e**(2*a + 2*b*x) +
1) - 6*e**(2*a + 2*b*x)*log(e**(a + b*x) - 1) - 6*e**(2*a + 2*b*x)*log(e**
(a + b*x) + 1) - 3*log(e**(2*a + 2*b*x) + 1) + 3*log(e**(a + b*x) - 1) + 3
*log(e**(a + b*x) + 1) + 3)/(b*(e**(12*a + 12*b*x) - 2*e**(10*a + 10*b*x)
- e**(8*a + 8*b*x) + 4*e**(6*a + 6*b*x) - e**(4*a + 4*b*x) - 2*e**(2*a + 2
*b*x) + 1))
```

3.103 $\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 81

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{35\operatorname{arctanh}(\cosh(a+bx))}{8b} + \frac{13\coth(a+bx)\operatorname{csch}(a+bx)}{8b} - \frac{\coth^3(a+bx)\operatorname{csch}(a+bx)}{4b} + \frac{3\operatorname{sech}(a+bx)}{b} + \frac{\operatorname{sech}^3(a+bx)}{3b}$$

output
```
-35/8*arctanh(cosh(b*x+a))/b+13/8*coth(b*x+a)*csch(b*x+a)/b-1/4*coth(b*x+a
)^3*csch(b*x+a)/b+3*sech(b*x+a)/b+1/3*sech(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.72

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{11\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b} - \frac{\operatorname{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b} - \frac{35\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{8b} + \frac{35\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{8b} + \frac{11\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b} + \frac{\operatorname{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b} + \frac{3\operatorname{sech}(a+bx)}{b} + \frac{\operatorname{sech}^3(a+bx)}{3b}$$

input
```
Integrate[Csch[a + b*x]^5*Sech[a + b*x]^4,x]
```

output
```
(11*Csch[(a + b*x)/2]^2)/(32*b) - Csch[(a + b*x)/2]^4/(64*b) - (35*Log[Cos
h[(a + b*x)/2]])/(8*b) + (35*Log[Sinh[(a + b*x)/2]])/(8*b) + (11*Sech[(a +
 b*x)/2]^2)/(32*b) + Sech[(a + b*x)/2]^4/(64*b) + (3*Sech[a + b*x])/b + Se
ch[a + b*x]^3/(3*b)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 26, 3102, 25, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int i\csc(ia+ibx)^5\sec(ia+ibx)^4dx$$

$$\Big\downarrow \ 26$$

$$i\int \csc(ia+ibx)^5\sec(ia+ibx)^4dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{\text{sech}^8(a+bx)}{\left(1-\text{sech}^2(a+bx)\right)^3}d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\text{sech}^8(a+bx)}{\left(1-\text{sech}^2(a+bx)\right)^3}d\text{sech}(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{7}{4}\int \frac{\text{sech}^6(a+bx)}{\left(1-\text{sech}^2(a+bx)\right)^2}d\text{sech}(a+bx)-\frac{\text{sech}^7(a+bx)}{4\left(1-\text{sech}^2(a+bx)\right)^2}}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{7}{4}\left(\frac{\text{sech}^5(a+bx)}{2\left(1-\text{sech}^2(a+bx)\right)}-\frac{5}{2}\int \frac{\text{sech}^4(a+bx)}{1-\text{sech}^2(a+bx)}d\text{sech}(a+bx)\right)-\frac{\text{sech}^7(a+bx)}{4\left(1-\text{sech}^2(a+bx)\right)^2}}{b}$$

$$\Big\downarrow \text{254}$$

$$\frac{\frac{7}{4}\left(\frac{\text{sech}^5(a+bx)}{2\left(1-\text{sech}^2(a+bx)\right)}-\frac{5}{2}\int \left(-\text{sech}^2(a+bx)+\frac{1}{1-\text{sech}^2(a+bx)}-1\right)d\text{sech}(a+bx)\right)-\frac{\text{sech}^7(a+bx)}{4\left(1-\text{sech}^2(a+bx)\right)^2}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{7}{4}\left(\frac{\text{sech}^5(a+bx)}{2\left(1-\text{sech}^2(a+bx)\right)}-\frac{5}{2}\left(\text{arctanh}(\text{sech}(a+bx))-\frac{1}{3}\text{sech}^3(a+bx)-\text{sech}(a+bx)\right)\right)-\frac{\text{sech}^7(a+bx)}{4\left(1-\text{sech}^2(a+bx)\right)^2}}{b}$$

input
```
Int[Csch[a + b*x]^5*Sech[a + b*x]^4,x]
```

output
```
(-1/4*Sech[a + b*x]^7/(1 - Sech[a + b*x]^2)^2 + (7*(Sech[a + b*x]^5/(2*(1
- Sech[a + b*x]^2)) - (5*(ArcTanh[Sech[a + b*x]] - Sech[a + b*x] - Sech[a
+ b*x]^3/3))/2))/4)/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 47.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)^3}+\frac{7}{8\sinh(bx+a)^2\cosh(bx+a)^3}+\frac{35}{24\cosh(bx+a)^3}+\frac{35}{8\cosh(bx+a)}-\frac{35\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{4}}{b}$
default	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)^3}+\frac{7}{8\sinh(bx+a)^2\cosh(bx+a)^3}+\frac{35}{24\cosh(bx+a)^3}+\frac{35}{8\cosh(bx+a)}-\frac{35\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{4}}{b}$
risch	$\frac{\mathrm{e}^{bx+a}\left(105\,\mathrm{e}^{12bx+12a}-70\,\mathrm{e}^{10bx+10a}-329\,\mathrm{e}^{8bx+8a}+204\,\mathrm{e}^{6bx+6a}-329\,\mathrm{e}^{4bx+4a}-70\,\mathrm{e}^{2bx+2a}+105\right)}{12b\left(\mathrm{e}^{2bx+2a}+1\right)^3\left(\mathrm{e}^{2bx+2a}-1\right)^4}-\frac{35\ln\left(\mathrm{e}^{bx+a}+1\right)}{8b}$

input
```
int(csch(b*x+a)^5*sech(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4/sinh(b*x+a)^4/cosh(b*x+a)^3+7/8/sinh(b*x+a)^2/cosh(b*x+a)^3+35/2
4/cosh(b*x+a)^3+35/8/cosh(b*x+a)-35/4*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2802 vs. 2(73) = 146.

Time = 0.10 (sec) , antiderivative size = 2802, normalized size of antiderivative = 34.59

$$\int \mathrm{csch}^5(a+bx)\mathrm{sech}^4(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/24*(210*cosh(b*x + a)^13 + 2730*cosh(b*x + a)*sinh(b*x + a)^12 + 210*sin
h(b*x + a)^13 + 140*(117*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^11 - 140*cosh(
b*x + a)^11 + 1540*(39*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a)^10 +
 14*(10725*cosh(b*x + a)^4 - 550*cosh(b*x + a)^2 - 47)*sinh(b*x + a)^9 - 6
58*cosh(b*x + a)^9 + 42*(6435*cosh(b*x + a)^5 - 550*cosh(b*x + a)^3 - 141*
cosh(b*x + a))*sinh(b*x + a)^8 + 24*(15015*cosh(b*x + a)^6 - 1925*cosh(b*x
 + a)^4 - 987*cosh(b*x + a)^2 + 17)*sinh(b*x + a)^7 + 408*cosh(b*x + a)^7
+ 168*(2145*cosh(b*x + a)^7 - 385*cosh(b*x + a)^5 - 329*cosh(b*x + a)^3 +
17*cosh(b*x + a))*sinh(b*x + a)^6 + 14*(19305*cosh(b*x + a)^8 - 4620*cosh(
b*x + a)^6 - 5922*cosh(b*x + a)^4 + 612*cosh(b*x + a)^2 - 47)*sinh(b*x + a
)^5 - 658*cosh(b*x + a)^5 + 14*(10725*cosh(b*x + a)^9 - 3300*cosh(b*x + a)
^7 - 5922*cosh(b*x + a)^5 + 1020*cosh(b*x + a)^3 - 235*cosh(b*x + a))*sinh
(b*x + a)^4 + 28*(2145*cosh(b*x + a)^10 - 825*cosh(b*x + a)^8 - 1974*cosh(
b*x + a)^6 + 510*cosh(b*x + a)^4 - 235*cosh(b*x + a)^2 - 5)*sinh(b*x + a)^
3 - 140*cosh(b*x + a)^3 + 28*(585*cosh(b*x + a)^11 - 275*cosh(b*x + a)^9 -
 846*cosh(b*x + a)^7 + 306*cosh(b*x + a)^5 - 235*cosh(b*x + a)^3 - 15*cosh
(b*x + a))*sinh(b*x + a)^2 - 105*(cosh(b*x + a)^14 + 14*cosh(b*x + a)*sinh
(b*x + a)^13 + sinh(b*x + a)^14 + (91*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^1
2 - cosh(b*x + a)^12 + 4*(91*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x +
 a)^11 + (1001*cosh(b*x + a)^4 - 66*cosh(b*x + a)^2 - 3)*sinh(b*x + a)^...
```

Sympy [F]

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx=\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**5*sech(b*x+a)**4,x)
```

output
```
Integral(csch(a + b*x)**5*sech(a + b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(73) = 146$.

Time = 0.04 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.41

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx = -\frac{35\log\left(e^{(-bx-a)}+1\right)}{8\,b} + \frac{35\log\left(e^{(-bx-a)}-1\right)}{8\,b} - \frac{105\,e^{(-bx-a)} - 70\,e^{(-3\,bx-3\,a)} - 329\,e^{(-5\,bx-5\,a)} + 204\,e^{(-7\,bx-7\,a)} - 329\,e^{(-9\,bx-9\,a)} - 70\,e^{(-11\,bx-11\,a)} + 10\ldots}{12\,b(e^{(-2\,bx-2\,a)} + 3\,e^{(-4\,bx-4\,a)} - 3\,e^{(-6\,bx-6\,a)} - 3\,e^{(-8\,bx-8\,a)} + 3\,e^{(-10\,bx-10\,a)} + e^{(-12\,bx-12\,a)} - e^{(-14\,b\ldots}}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^4,x, algorithm="maxima")
```

output
```
-35/8*log(e^(-b*x - a) + 1)/b + 35/8*log(e^(-b*x - a) - 1)/b - 1/12*(105*e
^(-b*x - a) - 70*e^(-3*b*x - 3*a) - 329*e^(-5*b*x - 5*a) + 204*e^(-7*b*x -
 7*a) - 329*e^(-9*b*x - 9*a) - 70*e^(-11*b*x - 11*a) + 105*e^(-13*b*x - 13
*a))/(b*(e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) - 3*e^(-6*b*x - 6*a) - 3*e^
(-8*b*x - 8*a) + 3*e^(-10*b*x - 10*a) + e^(-12*b*x - 12*a) - e^(-14*b*x -
14*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(73) = 146$.

Time = 0.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.88

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx = \frac{\frac{12\left(11\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3 - 52\,e^{(bx+a)} - 52\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4\right)^2} + \frac{32\left(9\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2+4\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3} - 105\log\left(e^{(bx+a)} + e^{(-bx-a)} + 2\right) + \ldots}{48\,b}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^4,x, algorithm="giac")
```

output
```
1/48*(12*(11*(e^(b*x + a) + e^(-b*x - a))^3 - 52*e^(b*x + a) - 52*e^(-b*x
- a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4)^2 + 32*(9*(e^(b*x + a) + e^(-b*
x - a))^2 + 4)/(e^(b*x + a) + e^(-b*x - a))^3 - 105*log(e^(b*x + a) + e^(-
b*x - a) + 2) + 105*log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.64

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx$$
$$= \frac{7\,e^{a+b\,x}}{2\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{35\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{4\,\sqrt{-b^2}} + \frac{8\,e^{a+b\,x}}{3\,b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} - \frac{6\,e^{a+b\,x}}{b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)} - \frac{8\,e^{a+b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)} - \frac{4\,e^{a+b\,x}}{b\,\left(6\,e^{4\,a+4\,b\,x}-4\,e^{2\,a+2\,b\,x}-4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)} + \frac{11\,e^{a+b\,x}}{4\,b\,\left(e^{2\,a+2\,b\,x}-1\right)} + \frac{6\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^4*sinh(a + b*x)^5),x)
```

output
```
(7*exp(a + b*x))/(2*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (35*a
tan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(4*(-b^2)^(1/2)) + (8*exp(a + b*x))
/(3*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (6*exp(a + b*x))/(b*(
3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (8*exp(
a + b*x))/(3*b*(3*exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x)
 + 1)) - (4*exp(a + b*x))/(b*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*
exp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1)) + (11*exp(a + b*x))/(4*b*(exp(2*
a + 2*b*x) - 1)) + (6*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 505, normalized size of antiderivative = 6.23

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^4(a+bx)\,dx$$
$$= \frac{105e^{14bx+14a}\log\left(e^{bx+a}-1\right)-105e^{14bx+14a}\log\left(e^{bx+a}+1\right)+210e^{13bx+13a}-105e^{12bx+12a}\log\left(e^{bx+a}-1\right)+}{}$$

input
```
int(csch(b*x+a)^5*sech(b*x+a)^4,x)
```

output
```
(105*e**(14*a + 14*b*x)*log(e**(a + b*x) - 1) - 105*e**(14*a + 14*b*x)*log
(e**(a + b*x) + 1) + 210*e**(13*a + 13*b*x) - 105*e**(12*a + 12*b*x)*log(e
**(a + b*x) - 1) + 105*e**(12*a + 12*b*x)*log(e**(a + b*x) + 1) - 140*e**(
11*a + 11*b*x) - 315*e**(10*a + 10*b*x)*log(e**(a + b*x) - 1) + 315*e**(10
*a + 10*b*x)*log(e**(a + b*x) + 1) - 658*e**(9*a + 9*b*x) + 315*e**(8*a +
8*b*x)*log(e**(a + b*x) - 1) - 315*e**(8*a + 8*b*x)*log(e**(a + b*x) + 1)
+ 408*e**(7*a + 7*b*x) + 315*e**(6*a + 6*b*x)*log(e**(a + b*x) - 1) - 315*
e**(6*a + 6*b*x)*log(e**(a + b*x) + 1) - 658*e**(5*a + 5*b*x) - 315*e**(4*
a + 4*b*x)*log(e**(a + b*x) - 1) + 315*e**(4*a + 4*b*x)*log(e**(a + b*x) +
 1) - 140*e**(3*a + 3*b*x) - 105*e**(2*a + 2*b*x)*log(e**(a + b*x) - 1) +
105*e**(2*a + 2*b*x)*log(e**(a + b*x) + 1) + 210*e**(a + b*x) + 105*log(e*
*(a + b*x) - 1) - 105*log(e**(a + b*x) + 1))/(24*b*(e**(14*a + 14*b*x) - e
**(12*a + 12*b*x) - 3*e**(10*a + 10*b*x) + 3*e**(8*a + 8*b*x) + 3*e**(6*a
+ 6*b*x) - 3*e**(4*a + 4*b*x) - e**(2*a + 2*b*x) + 1))
```

3.104 $\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{2\coth^2(a+bx)}{b} - \frac{\coth^4(a+bx)}{4b} + \frac{6\log(\tanh(a+bx))}{b} - \frac{2\tanh^2(a+bx)}{b} + \frac{\tanh^4(a+bx)}{4b}$$

output
```
2*coth(b*x+a)^2/b-1/4*coth(b*x+a)^4/b+6*ln(tanh(b*x+a))/b-2*tanh(b*x+a)^2/
b+1/4*tanh(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.32

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx = 32\left(\frac{3\operatorname{csch}^2(a+bx)}{64b} - \frac{\operatorname{csch}^4(a+bx)}{128b} - \frac{3\log(\cosh(a+bx))}{16b} + \frac{3\log(\sinh(a+bx))}{16b} + \frac{3\operatorname{sech}^2(a+bx)}{64b} + \frac{\operatorname{sech}^4(a+bx)}{128b}\right)$$

input
```
Integrate[Csch[a + b*x]^5*Sech[a + b*x]^5,x]
```

output
```
32*((3*Csch[a + b*x]^2)/(64*b) - Csch[a + b*x]^4/(128*b) - (3*Log[Cosh[a +
 b*x]])/(16*b) + (3*Log[Sinh[a + b*x]])/(16*b) + (3*Sech[a + b*x]^2)/(64*b
) + Sech[a + b*x]^4/(128*b))
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 26, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^5(a+bx)\text{sech}^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\csc(ia+ibx)^5\sec(ia+ibx)^5dx$$

$$\Big\downarrow \text{26}$$

$$i\int \csc(ia+ibx)^5\sec(ia+ibx)^5dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int -i\coth^5(a+bx)\left(1-\tanh^2(a+bx)\right)^4d(i\tanh(a+bx))}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int i\coth^3(a+bx)\left(1-\tanh^2(a+bx)\right)^4d\left(-\tanh^2(a+bx)\right)}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(i\coth^3(a+bx)-4\coth^2(a+bx)-6i\coth(a+bx)-\tanh^2(a+bx)+4\right)d\left(-\tanh^2(a+bx)\right)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}\tanh^2(a+bx)+4i\tanh(a+bx)+\frac{1}{2}\coth^2(a+bx)+4i\coth(a+bx)+6\log\left(-\tanh^2(a+bx)\right)}{2b}$$

input
```
Int[Csch[a + b*x]^5*Sech[a + b*x]^5,x]
```

output
```
((4*I)*Coth[a + b*x] + Coth[a + b*x]^2/2 + 6*Log[-Tanh[a + b*x]^2] + (4*I)
*Tanh[a + b*x] - Tanh[a + b*x]^2/2)/(2*b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 94.42 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

method	result	size
derivativedivides	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)^4}+\frac{1}{\sinh(bx+a)^2\cosh(bx+a)^4}+\frac{3}{2\cosh(bx+a)^4}+\frac{3}{\cosh(bx+a)^2}+6\ln(\tanh(bx+a))}{b}$	70
default	$\frac{-\frac{1}{4\sinh(bx+a)^4\cosh(bx+a)^4}+\frac{1}{\sinh(bx+a)^2\cosh(bx+a)^4}+\frac{3}{2\cosh(bx+a)^4}+\frac{3}{\cosh(bx+a)^2}+6\ln(\tanh(bx+a))}{b}$	70
risch	$\frac{4\,e^{2bx+2a}\left(3\,e^{12bx+12a}-11\,e^{8bx+8a}-11\,e^{4bx+4a}+3\right)}{b\left(e^{2bx+2a}-1\right)^4\left(e^{2bx+2a}+1\right)^4}+\frac{6\ln\left(e^{2bx+2a}-1\right)}{b}-\frac{6\ln\left(e^{2bx+2a}+1\right)}{b}$	111

input
```
int(csch(b*x+a)^5*sech(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4/sinh(b*x+a)^4/cosh(b*x+a)^4+1/sinh(b*x+a)^2/cosh(b*x+a)^4+3/2/co
sh(b*x+a)^4+3/cosh(b*x+a)^2+6*ln(tanh(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2231 vs. $2(65) = 130$.

Time = 0.09 (sec) , antiderivative size = 2231, normalized size of antiderivative = 32.33

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^5,x, algorithm="fricas")
```

output
```
2*(6*cosh(b*x + a)^14 + 2184*cosh(b*x + a)^3*sinh(b*x + a)^11 + 546*cosh(b
*x + a)^2*sinh(b*x + a)^12 + 84*cosh(b*x + a)*sinh(b*x + a)^13 + 6*sinh(b*
x + a)^14 + 22*(273*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^10 - 22*cosh(b*x +
a)^10 + 44*(273*cosh(b*x + a)^5 - 5*cosh(b*x + a))*sinh(b*x + a)^9 + 198*(
91*cosh(b*x + a)^6 - 5*cosh(b*x + a)^2)*sinh(b*x + a)^8 + 528*(39*cosh(b*x
 + a)^7 - 5*cosh(b*x + a)^3)*sinh(b*x + a)^7 + 22*(819*cosh(b*x + a)^8 - 2
10*cosh(b*x + a)^4 - 1)*sinh(b*x + a)^6 - 22*cosh(b*x + a)^6 + 132*(91*cos
h(b*x + a)^9 - 42*cosh(b*x + a)^5 - cosh(b*x + a))*sinh(b*x + a)^5 + 66*(9
1*cosh(b*x + a)^10 - 70*cosh(b*x + a)^6 - 5*cosh(b*x + a)^2)*sinh(b*x + a)
^4 + 8*(273*cosh(b*x + a)^11 - 330*cosh(b*x + a)^7 - 55*cosh(b*x + a)^3)*s
inh(b*x + a)^3 + 6*(91*cosh(b*x + a)^12 - 165*cosh(b*x + a)^8 - 55*cosh(b*
x + a)^4 + 1)*sinh(b*x + a)^2 + 6*cosh(b*x + a)^2 - 3*(cosh(b*x + a)^16 +
560*cosh(b*x + a)^3*sinh(b*x + a)^13 + 120*cosh(b*x + a)^2*sinh(b*x + a)^1
4 + 16*cosh(b*x + a)*sinh(b*x + a)^15 + sinh(b*x + a)^16 + 4*(455*cosh(b*x
 + a)^4 - 1)*sinh(b*x + a)^12 - 4*cosh(b*x + a)^12 + 48*(91*cosh(b*x + a)^
5 - cosh(b*x + a))*sinh(b*x + a)^11 + 88*(91*cosh(b*x + a)^6 - 3*cosh(b*x
+ a)^2)*sinh(b*x + a)^10 + 880*(13*cosh(b*x + a)^7 - cosh(b*x + a)^3)*sinh
(b*x + a)^9 + 6*(2145*cosh(b*x + a)^8 - 330*cosh(b*x + a)^4 + 1)*sinh(b*x
+ a)^8 + 6*cosh(b*x + a)^8 + 16*(715*cosh(b*x + a)^9 - 198*cosh(b*x + a)^5
 + 3*cosh(b*x + a))*sinh(b*x + a)^7 + 56*(143*cosh(b*x + a)^10 - 66*cos...
```

Sympy [F]

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx=\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx$$

input
```
integrate(csch(b*x+a)**5*sech(b*x+a)**5,x)
```

output
```
Integral(csch(a + b*x)**5*sech(a + b*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(65) = 130$.

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.17

$$\begin{aligned}\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx &= \frac{6\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{6\log\left(e^{(-bx-a)}-1\right)}{b}-\frac{6\log\left(e^{(-2bx-2a)}+1\right)}{b}\\&\quad-\frac{4\left(3e^{(-2bx-2a)}-11e^{(-6bx-6a)}-11e^{(-10bx-10a)}+3e^{(-14bx-14a)}\right)}{b(4e^{(-4bx-4a)}-6e^{(-8bx-8a)}+4e^{(-12bx-12a)}-e^{(-16bx-16a)}-1)}\end{aligned}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^5,x, algorithm="maxima")
```

output
```
6*log(e^(-b*x - a) + 1)/b + 6*log(e^(-b*x - a) - 1)/b - 6*log(e^(-2*b*x -
2*a) + 1)/b - 4*(3*e^(-2*b*x - 2*a) - 11*e^(-6*b*x - 6*a) - 11*e^(-10*b*x
- 10*a) + 3*e^(-14*b*x - 14*a))/(b*(4*e^(-4*b*x - 4*a) - 6*e^(-8*b*x - 8*a
) + 4*e^(-12*b*x - 12*a) - e^(-16*b*x - 16*a) - 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.80

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{\frac{4\left(3\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right)^3-20e^{(2bx+2a)}-20e^{(-2bx-2a)}\right)}{\left(\left(e^{(2bx+2a)}+e^{(-2bx-2a)}\right)^2-4\right)^2}-3\log\left(e^{(2bx+2a)}+e^{(-2bx-2a)}+2\right)+3\log\left(e^{(2bx+2a)}+\right.}{b}$$

input
```
integrate(csch(b*x+a)^5*sech(b*x+a)^5,x, algorithm="giac")
```

output
```
(4*(3*(e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^3 - 20*e^(2*b*x + 2*a) - 20*e^(
-2*b*x - 2*a))/((e^(2*b*x + 2*a) + e^(-2*b*x - 2*a))^2 - 4)^2 - 3*log(e^(2
*b*x + 2*a) + e^(-2*b*x - 2*a) + 2) + 3*log(e^(2*b*x + 2*a) + e^(-2*b*x -
2*a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.97

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{12\,\mathrm{e}^{2\,a+2\,b\,x}}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-1\right)}-\frac{12\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{8\,\mathrm{e}^{2\,a+2\,b\,x}}{b\,\left(\mathrm{e}^{8\,a+8\,b\,x}-2\,\mathrm{e}^{4\,a+4\,b\,x}+1\right)}-\frac{32\,\mathrm{e}^{2\,a+2\,b\,x}}{b\,\left(3\,\mathrm{e}^{4\,a+4\,b\,x}-3\,\mathrm{e}^{8\,a+8\,b\,x}+\mathrm{e}^{12\,a+12\,b\,x}-1\right)}-\frac{64\,\mathrm{e}^{6\,a+6\,b\,x}}{b\,\left(6\,\mathrm{e}^{8\,a+8\,b\,x}-4\,\mathrm{e}^{4\,a+4\,b\,x}-4\,\mathrm{e}^{12\,a+12\,b\,x}+\mathrm{e}^{16\,a+16\,b\,x}+1\right)}$$

input
```
int(1/(cosh(a + b*x)^5*sinh(a + b*x)^5),x)
```

output
```
(12*exp(2*a + 2*b*x))/(b*(exp(4*a + 4*b*x) - 1)) - (12*atan((exp(2*a)*exp(
2*b*x)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (8*exp(2*a + 2*b*x))/(b*(exp(8*a +
 8*b*x) - 2*exp(4*a + 4*b*x) + 1)) - (32*exp(2*a + 2*b*x))/(b*(3*exp(4*a +
 4*b*x) - 3*exp(8*a + 8*b*x) + exp(12*a + 12*b*x) - 1)) - (64*exp(6*a + 6*
b*x))/(b*(6*exp(8*a + 8*b*x) - 4*exp(4*a + 4*b*x) - 4*exp(12*a + 12*b*x) +
 exp(16*a + 16*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 419, normalized size of antiderivative = 6.07

$$\int \operatorname{csch}^5(a+bx)\operatorname{sech}^5(a+bx)\,dx = \frac{-6e^{16bx+16a}\log\left(e^{2bx+2a}+1\right)+6e^{16bx+16a}\log\left(e^{bx+a}-1\right)+6e^{16bx+16a}\log\left(e^{bx+a}+1\right)+12e^{14bx+14a}+24e^{12}}{}$$

input
```
int(csch(b*x+a)^5*sech(b*x+a)^5,x)
```

output

```
(2*( - 3*e**(16*a + 16*b*x)*log(e**(2*a + 2*b*x) + 1) + 3*e**(16*a + 16*b*
x)*log(e**(a + b*x) - 1) + 3*e**(16*a + 16*b*x)*log(e**(a + b*x) + 1) + 6*
e**(14*a + 14*b*x) + 12*e**(12*a + 12*b*x)*log(e**(2*a + 2*b*x) + 1) - 12*
e**(12*a + 12*b*x)*log(e**(a + b*x) - 1) - 12*e**(12*a + 12*b*x)*log(e**(a
 + b*x) + 1) - 22*e**(10*a + 10*b*x) - 18*e**(8*a + 8*b*x)*log(e**(2*a + 2
*b*x) + 1) + 18*e**(8*a + 8*b*x)*log(e**(a + b*x) - 1) + 18*e**(8*a + 8*b*
x)*log(e**(a + b*x) + 1) - 22*e**(6*a + 6*b*x) + 12*e**(4*a + 4*b*x)*log(e
**(2*a + 2*b*x) + 1) - 12*e**(4*a + 4*b*x)*log(e**(a + b*x) - 1) - 12*e**(
4*a + 4*b*x)*log(e**(a + b*x) + 1) + 6*e**(2*a + 2*b*x) - 3*log(e**(2*a +
2*b*x) + 1) + 3*log(e**(a + b*x) - 1) + 3*log(e**(a + b*x) + 1)))/(b*(e**(
16*a + 16*b*x) - 4*e**(12*a + 12*b*x) + 6*e**(8*a + 8*b*x) - 4*e**(4*a + 4
*b*x) + 1))
```

3.105 $\int \coth(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{\text{csch}(a+bx)}{b}$$

output
```
-csch(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \coth(a+bx)\text{csch}(a+bx)\,dx = -\frac{\text{csch}(a+bx)}{b}$$

input
```
Integrate[Coth[a + b*x]*Csch[a + b*x],x]
```

output
```
-(Csch[a + b*x]/b)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(a+bx)\text{csch}(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \tan\left(ia+ibx-\frac{\pi}{2}\right)\sec\left(ia+ibx-\frac{\pi}{2}\right)dx$$
$$\Big\downarrow \text{3086}$$
$$-\frac{i\int 1d(-i\text{csch}(a+bx))}{b}$$
$$\Big\downarrow \text{24}$$
$$-\frac{\text{csch}(a+bx)}{b}$$

input
```
Int[Coth[a + b*x]*Csch[a + b*x],x]
```

output
```
-(Csch[a + b*x]/b)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\frac{\operatorname{csch}(bx+a)}{b}$	12
default	$-\frac{\operatorname{csch}(bx+a)}{b}$	12
risch	$-\frac{2\,\mathrm{e}^{bx+a}}{b\left(\mathrm{e}^{2bx+2a}-1\right)}$	25

input
```
int(coth(b*x+a)*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-csch(b*x+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(11) = 22$.

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 5.09

$$\int \coth(a+bx)\operatorname{csch}(a+bx)\,dx$$
$$= -\frac{2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}-b}$$

input
```
integrate(coth(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

output
```
-2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*
sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(8) = 16$.

Time = 0.73 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \coth(a+bx)\operatorname{csch}(a+bx)\,dx = \begin{cases} -\frac{\operatorname{csch}(a+bx)}{b} & \text{for } b \neq 0 \\ x\coth(a)\operatorname{csch}(a) & \text{otherwise} \end{cases}$$

input
```
integrate(coth(b*x+a)*csch(b*x+a),x)
```

output
```
Piecewise((-csch(a + b*x)/b, Ne(b, 0)), (x*coth(a)*csch(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{2}{b(e^{(bx+a)} - e^{(-bx-a)})}$$

input
```
integrate(coth(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

output
```
-2/(b*(e^(b*x + a) - e^(-b*x - a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{2}{b(e^{(bx+a)} - e^{(-bx-a)})}$$

input
```
integrate(coth(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

output
```
-2/(b*(e^(b*x + a) - e^(-b*x - a)))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(coth(a + b*x)/sinh(a + b*x),x)
```

output
```
-(2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \coth(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{\operatorname{csch}(bx+a)}{b}$$

input
```
int(coth(b*x+a)*csch(b*x+a),x)
```

output
```
( - csch(a + b*x))/b
```

3.106 $\int \coth(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx=-\frac{\operatorname{csch}^2(a+bx)}{2b}$$

output
```
-1/2*csch(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx=-\frac{\operatorname{csch}^2(a+bx)}{2b}$$

input
```
Integrate[Coth[a + b*x]*Csch[a + b*x]^2,x]
```

output
```
-1/2*Csch[a + b*x]^2/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 26, 3086, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(a+bx)\text{csch}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan\left(ia+ibx-\frac{\pi}{2}\right)\sec\left(ia+ibx-\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sec\left(\frac{1}{2}(2ia-\pi)+ibx\right)^2\tan\left(\frac{1}{2}(2ia-\pi)+ibx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\int -i\text{csch}(a+bx)d(-i\text{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\text{csch}^2(a+bx)}{2b}$$

input
```
Int[Coth[a + b*x]*Csch[a + b*x]^2,x]
```

output
```
-1/2*Csch[a + b*x]^2/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\coth(bx+a)^2}{2b}$	14
default	$-\frac{\coth(bx+a)^2}{2b}$	14
risch	$-\frac{2\,e^{2bx+2a}}{b\left(e^{2bx+2a}-1\right)^2}$	28

input
```
int(coth(b*x+a)*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*coth(b*x+a)^2/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(13) = 26$.

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 5.73

$$\int \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx =$$
$$-\frac{2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^{3}+3\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+b\sinh\left(bx+a\right)^{3}-b\cosh\left(bx+a\right)+3\left(b\cosh\left(bx-\right.\right.}$$

input
```
integrate(coth(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
-2*(cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)*
sinh(b*x + a)^2 + b*sinh(b*x + a)^3 - b*cosh(b*x + a) + 3*(b*cosh(b*x + a)
^2 - b)*sinh(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = \begin{cases} -\frac{\operatorname{csch}^2(a+bx)}{2b} & \text{for } b \neq 0 \\ x\coth(a)\operatorname{csch}^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(coth(b*x+a)*csch(b*x+a)**2,x)
```

output
```
Piecewise((-csch(a + b*x)**2/(2*b), Ne(b, 0)), (x*coth(a)*csch(a)**2, True
))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{\coth(bx+a)^2}{2\,b}$$

input
```
integrate(coth(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*coth(b*x + a)^2/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{2\,e^{(2\,bx+2\,a)}}{b(e^{(2\,bx+2\,a)}-1)^2}$$

input
```
integrate(coth(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
-2*e^(2*b*x + 2*a)/(b*(e^(2*b*x + 2*a) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{1}{2\,b\sinh(a+b\,x)^2}$$

input
```
int(coth(a + b*x)/sinh(a + b*x)^2,x)
```

output
```
-1/(2*b*sinh(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \coth(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{\operatorname{csch}(bx+a)^2}{2b}$$

input
```
int(coth(b*x+a)*csch(b*x+a)^2,x)
```

output
```
( - csch(a + b*x)**2)/(2*b)
```

3.107 $\int \cosh(a+bx)\mathbf{csch}^{1+n}(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \cosh(a+bx)\text{csch}^{1+n}(a+bx)\,dx=-\frac{\text{csch}^n(a+bx)}{bn}$$

output
```
-csch(b*x+a)^n/b/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\text{csch}^{1+n}(a+bx)\,dx=-\frac{\text{csch}^n(a+bx)}{bn}$$

input
```
Integrate[Cosh[a + b*x]*Csch[a + b*x]^(1 + n),x]
```

output
```
-(Csch[a + b*x]^n/(b*n))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 3101, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\text{csch}^{n+1}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(i\csc(ia+ibx))^{n+1}}{\sec(ia+ibx)}dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int \text{csch}^{n-1}(a+bx)d\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\text{csch}^n(a+bx)}{bn}$$

input
```
Int[Cosh[a + b*x]*Csch[a + b*x]^(1 + n),x]
```

output
```
-(Csch[a + b*x]^n/(b*n))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

method	result	size
derivativedivides	$-\frac{e^{(1+n)\ln(\operatorname{csch}(bx+a))}}{bn\operatorname{csch}(bx+a)}$	29
default	$-\frac{e^{(1+n)\ln(\operatorname{csch}(bx+a))}}{bn\operatorname{csch}(bx+a)}$	29

input
```
int(cosh(b*x+a)*csch(b*x+a)^(1+n),x,method=_RETURNVERBOSE)
```

output
```
-1/b/n*exp((1+n)*ln(csch(b*x+a)))/csch(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(16) = 32$.

Time = 0.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 9.56

$$\int \cosh(a+bx)\operatorname{csch}^{1+n}(a+bx)\,dx = -\frac{\cosh\left((n+1)\log\left(\frac{2(\cosh(bx+a)+\sinh(bx+a))}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-1}\right)\right)\sinh(bx+a)+\sinh(bx+a)\sinh\left((n\right.}{bn\cosh(bx+a)^2-bn\sinh(bx+a)^2}$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)^(1+n),x, algorithm="fricas")
```

output
```
-(cosh((n + 1)*log(2*(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*
cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)))*sinh(b*x + a) + sinh(
b*x + a)*sinh((n + 1)*log(2*(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)
^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1))))/(b*n*cosh(b*x
 + a)^2 - b*n*sinh(b*x + a)^2)
```

Sympy [F]

$$\int \cosh(a+bx)\text{csch}^{1+n}(a+bx)\,dx = \int \cosh(a+bx)\,\text{csch}^{n+1}(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)**(1+n),x)
```

output
```
Integral(cosh(a + b*x)*csch(a + b*x)**(n + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(16) = 32.

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.31

$$\int \cosh(a+bx)\text{csch}^{1+n}(a+bx)\,dx = -\frac{2^n e^{\left(-(bx+a)n-n\log\left(e^{(-bx-a)}+1\right)-n\log\left(-e^{(-bx-a)}+1\right)\right)}}{bn}$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)^(1+n),x, algorithm="maxima")
```

output
```
-2^n*e^(-(b*x + a)*n - n*log(e^(-b*x - a) + 1) - n*log(-e^(-b*x - a) + 1))
/(b*n)
```

Giac [F]

$$\int \cosh(a+bx)\text{csch}^{1+n}(a+bx)\,dx = \int \text{csch}\,(bx+a)^{n+1}\cosh\,(bx+a)\;dx$$

input
```
integrate(cosh(b*x+a)*csch(b*x+a)^(1+n),x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^(n + 1)*cosh(b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \cosh(a+bx)\text{csch}^{1+n}(a+bx)\,dx = -\frac{\sinh(a+b\,x)\,\left(\frac{1}{\sinh(a+b\,x)}\right)^{n+1}}{b\,n}$$

input
```
int(cosh(a + b*x)*(1/sinh(a + b*x))^(n + 1),x)
```

output
```
-(sinh(a + b*x)*(1/sinh(a + b*x))^(n + 1))/(b*n)
```

Reduce [F]

$$\int \cosh(a+bx)\text{csch}^{1+n}(a+bx)\,dx = \int \text{csch}(bx+a)^{n}\cosh\,(bx+a)\,\text{csch}(bx+a)\,dx$$

input
```
int(cosh(b*x+a)*csch(b*x+a)^(1+n),x)
```

output
```
int(csch(a + b*x)**n*cosh(a + b*x)*csch(a + b*x),x)
```

3.108 $\int \coth^2(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \coth^2(a+bx)\text{csch}^2(a+bx)\,dx=-\frac{\coth^3(a+bx)}{3b}$$

output
```
-1/3*coth(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \coth^2(a+bx)\text{csch}^2(a+bx)\,dx=-\frac{\coth^3(a+bx)}{3b}$$

input
```
Integrate[Coth[a + b*x]^2*Csch[a + b*x]^2,x]
```

output
```
-1/3*Coth[a + b*x]^3/b
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(a+bx)\text{csch}^2(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan\left(ia+ibx-\frac{\pi}{2}\right)^2\sec\left(ia+ibx-\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \ 3087$$

$$-\frac{i\int -\coth^2(a+bx)d(i\coth(a+bx))}{b}$$

$$\Big\downarrow \ 15$$

$$-\frac{\coth^3(a+bx)}{3b}$$

input
```
Int[Coth[a + b*x]^2*Csch[a + b*x]^2,x]
```

output
```
-1/3*Coth[a + b*x]^3/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\coth(bx+a)^3}{3b}$	14
default	$-\frac{\coth(bx+a)^3}{3b}$	14
risch	$-\frac{2(3\,e^{4bx+4a}+1)}{3b(e^{2bx+2a}-1)^3}$	32

input
```
int(coth(b*x+a)^2*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*coth(b*x+a)^3/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(13) = 26.

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 9.27

$$\int \coth^2(a+bx)\operatorname{csch}^2(a+bx)\,dx =$$

$$-\frac{8\left(\cosh\left(bx+a\right)^2+\cosh\left(bx+a\right)\sin\right.}{3\left(b\cosh\left(bx+a\right)^4+4\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+b\sinh\left(bx+a\right)^4-4\,b\cosh\left(bx+a\right)^2+2\left(3\,b\cos\right.\right.}$$

input
```
integrate(coth(b*x+a)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
-8/3*(cosh(b*x + a)^2 + cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2)/(b*
cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 -
4*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - 2*b)*sinh(b*x + a)^2 + 4*(b
*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + 3*b)
```

Sympy [F]

$$\int \coth^2(a+bx)\operatorname{csch}^2(a+bx)\,dx = \int \coth^2(a+bx)\operatorname{csch}^2(a+bx)\,dx$$

input
```
integrate(coth(b*x+a)**2*csch(b*x+a)**2,x)
```

output
```
Integral(coth(a + b*x)**2*csch(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \coth^2(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{\coth(bx+a)^3}{3\,b}$$

input
```
integrate(coth(b*x+a)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/3*coth(b*x + a)^3/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(13) = 26$.

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \coth^2(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{2\left(3\,e^{(4\,bx+4\,a)}+1\right)}{3\,b(e^{(2\,bx+2\,a)}-1)^3}$$

input
```
integrate(coth(b*x+a)^2*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
-2/3*(3*e^(4*b*x + 4*a) + 1)/(b*(e^(2*b*x + 2*a) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \coth^2(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{2\left(3\,\mathrm{e}^{4\,a+4\,b\,x}+1\right)}{3\,b\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)^3}$$

input
```
int(coth(a + b*x)^2/sinh(a + b*x)^2,x)
```

output
```
-(2*(3*exp(4*a + 4*b*x) + 1))/(3*b*(exp(2*a + 2*b*x) - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 4.47

$$\int \coth^2(a+bx)\text{csch}^2(a+bx)\,dx = \frac{2e^{2bx+2a}\left(-e^{4bx+4a}-3\right)}{3b\left(e^{6bx+6a}-3e^{4bx+4a}+3e^{2bx+2a}-1\right)}$$

input
```
int(coth(b*x+a)^2*csch(b*x+a)^2,x)
```

output
```
(2*e**(2*a + 2*b*x)*( - e**(4*a + 4*b*x) - 3))/(3*b*(e**(6*a + 6*b*x) - 3*
e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) - 1))
```

3.109 $\int \coth^3(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \coth^3(a+bx)\mathrm{csch}^2(a+bx)\,dx = -\frac{\coth^4(a+bx)}{4b}$$

output
```
-1/4*coth(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \coth^3(a+bx)\mathrm{csch}^2(a+bx)\,dx = -\frac{\coth^4(a+bx)}{4b}$$

input
```
Integrate[Coth[a + b*x]^3*Csch[a + b*x]^2,x]
```

output
```
-1/4*Coth[a + b*x]^4/b
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 26, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(a+bx)\text{csch}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan\left(ia+ibx-\frac{\pi}{2}\right)^3\sec\left(ia+ibx-\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sec\left(\frac{1}{2}(2ia-\pi)+ibx\right)^2\tan\left(\frac{1}{2}(2ia-\pi)+ibx\right)^3dx$$

$$\Big\downarrow \text{3087}$$

$$-\frac{\int -i\coth^3(a+bx)d(i\coth(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\coth^4(a+bx)}{4b}$$

input
```
Int[Coth[a + b*x]^3*Csch[a + b*x]^2,x]
```

output
```
-1/4*Coth[a + b*x]^4/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\coth(bx+a)^4}{4b}$	14
default	$-\frac{\coth(bx+a)^4}{4b}$	14
risch	$-\frac{2\,\mathrm{e}^{2bx+2a}\left(\mathrm{e}^{4bx+4a}+1\right)}{b\left(\mathrm{e}^{2bx+2a}-1\right)^4}$	39

input
```
int(coth(b*x+a)^3*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*coth(b*x+a)^4/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(13) = 26$.

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 13.87

$$\int \coth^3(a+bx)\operatorname{csch}^2(a+bx)\,dx =$$
$$-\frac{2\left(\cosh\left(bx+a\right)^3+3\,\cosh\left(b\right.\right.}{b\cosh\left(bx+a\right)^5+5\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4+b\sinh\left(bx+a\right)^5-3\,b\cosh\left(bx+a\right)^3+5\left(2\,b\cosh\right.}$$

input
```
integrate(coth(b*x+a)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
-2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 +
(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) + cosh(b*x + a))/(b*cosh(b*x + a)^5
+ 5*b*cosh(b*x + a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5 - 3*b*cosh(b*x + a
)^3 + 5*(2*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^3 + (10*b*cosh(b*x + a)^3
- 9*b*cosh(b*x + a))*sinh(b*x + a)^2 + 2*b*cosh(b*x + a) + 5*(b*cosh(b*x +
 a)^4 - 3*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a))
```

Sympy [F]

$$\int \coth^3(a+bx)\operatorname{csch}^2(a+bx)\,dx = \int \coth^3\left(a+bx\right)\operatorname{csch}^2\left(a+bx\right)dx$$

input
```
integrate(coth(b*x+a)**3*csch(b*x+a)**2,x)
```

output
```
Integral(coth(a + b*x)**3*csch(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \coth^3(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\coth(bx+a)^4}{4\,b}$$

input
```
integrate(coth(b*x+a)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*coth(b*x + a)^4/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(13) = 26.

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.47

$$\int \coth^3(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{2\left(e^{(6\,bx+6\,a)}+e^{(2\,bx+2\,a)}\right)}{b(e^{(2\,bx+2\,a)}-1)^4}$$

input
```
integrate(coth(b*x+a)^3*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
-2*(e^(6*b*x + 6*a) + e^(2*b*x + 2*a))/(b*(e^(2*b*x + 2*a) - 1)^4)
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 231, normalized size of antiderivative = 15.40

$$\int \coth^3(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\frac{1}{2\,b}+\frac{3\,e^{2\,a+2\,b\,x}}{2\,b}+\frac{3\,e^{4\,a+4\,b\,x}}{2\,b}+\frac{e^{6\,a+6\,b\,x}}{2\,b}}{6\,e^{4\,a+4\,b\,x}-4\,e^{2\,a+2\,b\,x}-4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1} - \frac{\frac{1}{2\,b}+\frac{e^{2\,a+2\,b\,x}}{b}+\frac{e^{4\,a+4\,b\,x}}{2\,b}}{3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1} - \frac{\frac{1}{2\,b}+\frac{e^{2\,a+2\,b\,x}}{2\,b}}{e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1} - \frac{1}{2\,b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(coth(a + b*x)^3/sinh(a + b*x)^2,x)
```

output
```
- (1/(2*b) + (3*exp(2*a + 2*b*x))/(2*b) + (3*exp(4*a + 4*b*x))/(2*b) + exp
(6*a + 6*b*x)/(2*b))/(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*exp(6*a
+ 6*b*x) + exp(8*a + 8*b*x) + 1) - (1/(2*b) + exp(2*a + 2*b*x)/b + exp(4*a
 + 4*b*x)/(2*b))/(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*
x) - 1) - (1/(2*b) + exp(2*a + 2*b*x)/(2*b))/(exp(4*a + 4*b*x) - 2*exp(2*a
 + 2*b*x) + 1) - 1/(2*b*(exp(2*a + 2*b*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \coth^3(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{\operatorname{csch}(bx+a)^2\left(\coth(bx+a)^2+1\right)}{4b}$$

input
```
int(coth(b*x+a)^3*csch(b*x+a)^2,x)
```

output
```
( - csch(a + b*x)**2*(coth(a + b*x)**2 + 1))/(4*b)
```

3.110 $\int \coth^n(a+bx)\mathbf{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \coth^n(a+bx)\operatorname{csch}^2(a+bx)\,dx=-\frac{\coth^{1+n}(a+bx)}{b(1+n)}$$

output
```
-coth(b*x+a)^(1+n)/b/(1+n)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \coth^n(a+bx)\operatorname{csch}^2(a+bx)\,dx=-\frac{\coth^{1+n}(a+bx)}{b(1+n)}$$

input
```
Integrate[Coth[a + b*x]^n*Csch[a + b*x]^2,x]
```

output
```
-(Coth[a + b*x]^(1 + n)/(b*(1 + n)))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 25, 3087, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(a+bx)\coth^n(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sec\left(ia+ibx-\frac{\pi}{2}\right)^2\left(-i\tan\left(ia+ibx-\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec\left(\frac{1}{2}(2ia-\pi)+ibx\right)^2\left(-i\tan\left(\frac{1}{2}(2ia-\pi)+ibx\right)\right)^n dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{i\int \coth^n(a+bx)d(i\coth(a+bx))}{b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{\coth^{n+1}(a+bx)}{b(n+1)}$$

input
```
Int[Coth[a + b*x]^n*Csch[a + b*x]^2,x]
```

output
```
-(Coth[a + b*x]^(1 + n)/(b*(1 + n)))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result
derivativedivides	$-\frac{\coth(bx+a)^{1+n}}{b(1+n)}$
default	$-\frac{\coth(bx+a)^{1+n}}{b(1+n)}$
risch	$-\frac{(e^{2bx+2a}+1)(e^{bx+a}-1)^{-n}(e^{bx+a}+1)^{-n}(e^{2bx+2a}+1)^{n}e^{-\frac{i\pi n\left(\mathrm{csgn}\left(\frac{i(e^{2bx+2a}+1)}{e^{bx+a}+1}\right)^{3}-\mathrm{csgn}\left(\frac{i(e^{2bx+2a}+1)}{e^{bx+a}+1}\right)^{2}\mathrm{csgn}\cdots\right.}{\cdots}}}{\cdots}$

input
```
int(coth(b*x+a)^n*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-coth(b*x+a)^(1+n)/b/(1+n)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(20) = 40$.

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.50

$$\int \coth^n(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{\cosh(bx+a)\cosh\left(n\log\left(\frac{\cosh(bx+a)}{\sinh(bx+a)}\right)\right)+\cosh(bx+a)\sinh\left(n\log\left(\frac{\cosh(bx+a)}{\sinh(bx+a)}\right)\right)}{(bn+b)\sinh(bx+a)}$$

input
```
integrate(coth(b*x+a)^n*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(cosh(b*x + a)*cosh(n*log(cosh(b*x + a)/sinh(b*x + a))) + cosh(b*x + a)*s
inh(n*log(cosh(b*x + a)/sinh(b*x + a))))/((b*n + b)*sinh(b*x + a))
```

Sympy [F]

$$\int \coth^n(a+bx)\operatorname{csch}^2(a+bx)\,dx = \int \coth^n(a+bx)\operatorname{csch}^2(a+bx)\,dx$$

input
```
integrate(coth(b*x+a)**n*csch(b*x+a)**2,x)
```

output
```
Integral(coth(a + b*x)**n*csch(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \coth^n(a+bx)\operatorname{csch}^2(a+bx)\,dx = -\frac{\coth(bx+a)^{n+1}}{b(n+1)}$$

input
```
integrate(coth(b*x+a)^n*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-coth(b*x + a)^(n + 1)/(b*(n + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \coth^n(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\left(\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}\right)^{n+1}}{b(n+1)}$$

input
```
integrate(coth(b*x+a)^n*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1))^(n + 1)/(b*(n + 1))
```

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.15

$$\int \coth^n(a+bx)\text{csch}^2(a+bx)\,dx = -\frac{\coth(a+b\,x)\,\left(\frac{e^{2\,a+2\,b\,x}+1}{e^{2\,a+2\,b\,x}-1}\right)^n}{b\,(n+1)}$$

input
```
int(coth(a + b*x)^n/sinh(a + b*x)^2,x)
```

output
```
-(coth(a + b*x)*((exp(2*a + 2*b*x) + 1)/(exp(2*a + 2*b*x) - 1))^n)/(b*(n +
 1))
```

Reduce [F]

$$\int \coth^n(a+bx)\operatorname{csch}^2(a+bx)\,dx = \int \coth(bx+a)^n \operatorname{csch}(bx+a)^2\,dx$$

input
```
int(coth(b*x+a)^n*csch(b*x+a)^2,x)
```

output
```
int(coth(a + b*x)**n*csch(a + b*x)**2,x)
```

3.111 $\int \coth^3(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \coth^3(a+bx)\operatorname{csch}(a+bx)\,dx=-\frac{\operatorname{csch}(a+bx)}{b}-\frac{\operatorname{csch}^3(a+bx)}{3b}$$

output
```
-csch(b*x+a)/b-1/3*csch(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \coth^3(a+bx)\operatorname{csch}(a+bx)\,dx=-\frac{\operatorname{csch}(a+bx)}{b}-\frac{\operatorname{csch}^3(a+bx)}{3b}$$

input
```
Integrate[Coth[a + b*x]^3*Csch[a + b*x],x]
```

output
```
-(Csch[a + b*x]/b) - Csch[a + b*x]^3/(3*b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 25, 3086, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(a+bx)\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(ia+ibx-\frac{\pi}{2}\right)^3\left(-\sec\left(ia+ibx-\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec\left(\frac{1}{2}(2ia-\pi)+ibx\right)\tan\left(\frac{1}{2}(2ia-\pi)+ibx\right)^3dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{i\int\left(-\text{csch}^2(a+bx)-1\right)d(-i\text{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(\frac{1}{3}i\text{csch}^3(a+bx)+i\text{csch}(a+bx)\right)}{b}$$

input
```
Int[Coth[a + b*x]^3*Csch[a + b*x],x]
```

output
```
(I*(I*Csch[a + b*x] + (I/3)*Csch[a + b*x]^3))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{-\frac{\operatorname{csch}(bx+a)^{3}}{3}-\operatorname{csch}(bx+a)}{b}$	24
default	$\frac{-\frac{\operatorname{csch}(bx+a)^{3}}{3}-\operatorname{csch}(bx+a)}{b}$	24
risch	$-\frac{2\,{\mathrm e}^{bx+a}\left(3\,{\mathrm e}^{4bx+4a}-2\,{\mathrm e}^{2bx+2a}+3\right)}{3b\left({\mathrm e}^{2bx+2a}-1\right)^{3}}$	49

input
```
int(coth(b*x+a)^3*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3*csch(b*x+a)^3-csch(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(25) = 50$.

Time = 0.07 (sec) , antiderivative size = 171, normalized size of antiderivative = 6.33

$$\int \coth^3(a+bx)\text{csch}(a+bx)\,dx =$$

$$-\frac{2\left(3\cosh\left(bx+a\right)^3+9\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+3\sinh\left(bx+a\right.\right.}{3\left(b\cosh\left(bx+a\right)^4+4\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+b\sinh\left(bx+a\right)^4-4\,b\cosh\left(bx+a\right)^2+2\left(3\,b\cos\right.\right.}$$

input
```
integrate(coth(b*x+a)^3*csch(b*x+a),x, algorithm="fricas")
```

output
```
-2/3*(3*cosh(b*x + a)^3 + 9*cosh(b*x + a)*sinh(b*x + a)^2 + 3*sinh(b*x + a
)^3 + (9*cosh(b*x + a)^2 - 5)*sinh(b*x + a) + cosh(b*x + a))/(b*cosh(b*x +
 a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 - 4*b*cosh(b
*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - 2*b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x
+ a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + 3*b)
```

Sympy [F]

$$\int \coth^3(a+bx)\text{csch}(a+bx)\,dx = \int \coth^3\left(a+bx\right)\text{csch}\left(a+bx\right)\,dx$$

input
```
integrate(coth(b*x+a)**3*csch(b*x+a),x)
```

output
```
Integral(coth(a + b*x)**3*csch(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(25) = 50.

Time = 0.04 (sec) , antiderivative size = 148, normalized size of antiderivative = 5.48

$$\int \coth^3(a+bx)\mathrm{csch}(a+bx)\,dx = \frac{2\,e^{(-bx-a)}}{b(3\,e^{(-2\,bx-2\,a)}-3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}-1)} - \frac{4\,e^{(-3\,bx-3\,a)}}{3\,b(3\,e^{(-2\,bx-2\,a)}-3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}-1)} + \frac{2\,e^{(-5\,bx-5\,a)}}{b(3\,e^{(-2\,bx-2\,a)}-3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}-1)}$$

input
```
integrate(coth(b*x+a)^3*csch(b*x+a),x, algorithm="maxima")
```

output
```
2*e^(-b*x - a)/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6
*a) - 1)) - 4/3*e^(-3*b*x - 3*a)/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*
a) + e^(-6*b*x - 6*a) - 1)) + 2*e^(-5*b*x - 5*a)/(b*(3*e^(-2*b*x - 2*a) -
3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a) - 1))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \coth^3(a+bx)\mathrm{csch}(a+bx)\,dx = -\frac{2\left(3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)}{3\,b(e^{(bx+a)}-e^{(-bx-a)})^3}$$

input
```
integrate(coth(b*x+a)^3*csch(b*x+a),x, algorithm="giac")
```

output
```
-2/3*(3*(e^(b*x + a) - e^(-b*x - a))^2 + 4)/(b*(e^(b*x + a) - e^(-b*x - a)
)^3)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \coth^3(a+bx)\text{csch}(a+bx)\,dx = -\frac{2\,\mathrm{e}^{a+b\,x}\,\left(3\,\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+3\right)}{3\,b\,{\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}^3}$$

input
```
int(coth(a + b*x)^3/sinh(a + b*x),x)
```

output
```
-(2*exp(a + b*x)*(3*exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 3))/(3*b*(exp(
2*a + 2*b*x) - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \coth^3(a+bx)\text{csch}(a+bx)\,dx = \frac{\text{csch}(bx+a)\left(-\coth{(bx+a)}^2-2\right)}{3b}$$

input
```
int(coth(b*x+a)^3*csch(b*x+a),x)
```

output
```
(csch(a + b*x)*( - coth(a + b*x)**2 - 2))/(3*b)
```

3.112 $\int \coth^3(a+bx)\mathbf{csch}^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \coth^3(a+bx)\text{csch}^3(a+bx)\,dx = -\frac{\text{csch}^3(a+bx)}{3b}-\frac{\text{csch}^5(a+bx)}{5b}$$

output
```
-1/3*csch(b*x+a)^3/b-1/5*csch(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \coth^3(a+bx)\text{csch}^3(a+bx)\,dx = -\frac{\text{csch}^3(a+bx)}{3b}-\frac{\text{csch}^5(a+bx)}{5b}$$

input
```
Integrate[Coth[a + b*x]^3*Csch[a + b*x]^3,x]
```

output
```
-1/3*Csch[a + b*x]^3/b - Csch[a + b*x]^5/(5*b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(a+bx)\text{csch}^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(ia+ibx-\frac{\pi}{2}\right)^3\sec\left(ia+ibx-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{i\int \text{csch}^2(a+bx)\left(\text{csch}^2(a+bx)+1\right)d(-i\text{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{i\int -\text{csch}^2(a+bx)\left(\text{csch}^2(a+bx)+1\right)d(-i\text{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{i\int \left(-\text{csch}^4(a+bx)-\text{csch}^2(a+bx)\right)d(-i\text{csch}(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-\frac{1}{5}i\text{csch}^5(a+bx)-\frac{1}{3}i\text{csch}^3(a+bx)\right)}{b}$$

input
```
Int[Coth[a + b*x]^3*Csch[a + b*x]^3,x]
```

output
```
((-I)*((-1/3*I)*Csch[a + b*x]^3 - (I/5)*Csch[a + b*x]^5))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$-\frac{\frac{\operatorname{csch}(bx+a)^5}{5}+\frac{\operatorname{csch}(bx+a)^3}{3}}{b}$	27
default	$-\frac{\frac{\operatorname{csch}(bx+a)^5}{5}+\frac{\operatorname{csch}(bx+a)^3}{3}}{b}$	27
risch	$-\frac{8\,\mathrm{e}^{3bx+3a}\left(5\,\mathrm{e}^{4bx+4a}+2\,\mathrm{e}^{2bx+2a}+5\right)}{15b\left(\mathrm{e}^{2bx+2a}-1\right)^5}$	52

input
```
int(coth(b*x+a)^3*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(1/5*csch(b*x+a)^5+1/3*csch(b*x+a)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(27) = 54$.

Time = 0.09 (sec) , antiderivative size = 343, normalized size of antiderivative = 11.06

$$\int \coth^3(a+bx)\operatorname{csch}^3(a+bx)\,dx =$$

$$-\frac{}{15\left(b\cosh\left(bx+a\right)^{7}+7\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{6}+b\sinh\left(bx+a\right)^{7}-5\,b\cosh\left(bx+a\right)^{5}+\left(21\,b\,\mathrm{co}\right.\right.}$$

input
```
integrate(coth(b*x+a)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-8/15*(5*cosh(b*x + a)^4 + 20*cosh(b*x + a)*sinh(b*x + a)^3 + 5*sinh(b*x +
 a)^4 + 2*(15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4
*(5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 5)/(b*cosh(b*x + a)^7
 + 7*b*cosh(b*x + a)*sinh(b*x + a)^6 + b*sinh(b*x + a)^7 - 5*b*cosh(b*x +
a)^5 + (21*b*cosh(b*x + a)^2 - 5*b)*sinh(b*x + a)^5 + 5*(7*b*cosh(b*x + a)
^3 - 5*b*cosh(b*x + a))*sinh(b*x + a)^4 + 9*b*cosh(b*x + a)^3 + (35*b*cosh
(b*x + a)^4 - 50*b*cosh(b*x + a)^2 + 11*b)*sinh(b*x + a)^3 + (21*b*cosh(b*
x + a)^5 - 50*b*cosh(b*x + a)^3 + 27*b*cosh(b*x + a))*sinh(b*x + a)^2 - 5*
b*cosh(b*x + a) + (7*b*cosh(b*x + a)^6 - 25*b*cosh(b*x + a)^4 + 33*b*cosh(
b*x + a)^2 - 15*b)*sinh(b*x + a))
```

Sympy [F]

$$\int \coth^3(a+bx)\operatorname{csch}^3(a+bx)\,dx = \int \coth^3\left(a+bx\right)\operatorname{csch}^3\left(a+bx\right)dx$$

input
```
integrate(coth(b*x+a)**3*csch(b*x+a)**3,x)
```

output
```
Integral(coth(a + b*x)**3*csch(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(27) = 54.

Time = 0.05 (sec) , antiderivative size = 214, normalized size of antiderivative = 6.90

$$\int \coth^3(a+bx)\operatorname{csch}^3(a+bx)\,dx = \frac{8\,e^{(-3\,bx-3\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}-10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}-5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}-1)} + \frac{16\,e^{(-5\,bx-5\,a)}}{15\,b(5\,e^{(-2\,bx-2\,a)}-10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}-5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}-1)} + \frac{8\,e^{(-7\,bx-7\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}-10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}-5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}-1)}$$

input
```
integrate(coth(b*x+a)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
8/3*e^(-3*b*x - 3*a)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(
-6*b*x - 6*a) - 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1)) + 16/15*e^(-
5*b*x - 5*a)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x -
 6*a) - 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1)) + 8/3*e^(-7*b*x - 7*
a)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) - 5*
e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \coth^3(a+bx)\operatorname{csch}^3(a+bx)\,dx = -\frac{8\left(5\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+12\right)}{15\,b(e^{(bx+a)}-e^{(-bx-a)})^5}$$

input
```
integrate(coth(b*x+a)^3*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
-8/15*(5*(e^(b*x + a) - e^(-b*x - a))^2 + 12)/(b*(e^(b*x + a) - e^(-b*x -
a))^5)
```

Mupad [B] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 252, normalized size of antiderivative = 8.13

$$\int \coth^3(a+bx)\operatorname{csch}^3(a+bx)\,dx = -\frac{\frac{4\,e^{a+b\,x}}{5\,b}+\frac{12\,e^{3\,a+3\,b\,x}}{5\,b}+\frac{12\,e^{5\,a+5\,b\,x}}{5\,b}+\frac{4\,e^{7\,a+7\,b\,x}}{5\,b}}{5\,e^{2\,a+2\,b\,x}-10\,e^{4\,a+4\,b\,x}+10\,e^{6\,a+6\,b\,x}-5\,e^{8\,a+8\,b\,x}+e^{10\,a+10\,b\,x}-1} - \frac{28\,e^{a+b\,x}}{15\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{64\,e^{a+b\,x}}{15\,b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)} - \frac{16\,e^{a+b\,x}}{5\,b\,\left(6\,e^{4\,a+4\,b\,x}-4\,e^{2\,a+2\,b\,x}-4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)}$$

input
```
int(coth(a + b*x)^3/sinh(a + b*x)^3,x)
```

output
```
- ((4*exp(a + b*x))/(5*b) + (12*exp(3*a + 3*b*x))/(5*b) + (12*exp(5*a + 5*
b*x))/(5*b) + (4*exp(7*a + 7*b*x))/(5*b))/(5*exp(2*a + 2*b*x) - 10*exp(4*a
 + 4*b*x) + 10*exp(6*a + 6*b*x) - 5*exp(8*a + 8*b*x) + exp(10*a + 10*b*x)
- 1) - (28*exp(a + b*x))/(15*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)
) - (64*exp(a + b*x))/(15*b*(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp
(6*a + 6*b*x) - 1)) - (16*exp(a + b*x))/(5*b*(6*exp(4*a + 4*b*x) - 4*exp(2
*a + 2*b*x) - 4*exp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \coth^3(a+bx)\operatorname{csch}^3(a+bx)\,dx = \frac{\operatorname{csch}(bx+a)^3\left(-3\coth(bx+a)^2-2\right)}{15b}$$

input
```
int(coth(b*x+a)^3*csch(b*x+a)^3,x)
```

output
```
(csch(a + b*x)**3*( - 3*coth(a + b*x)**2 - 2))/(15*b)
```

3.113 $\int \cosh^3(a+bx)\operatorname{csch}^{3+n}(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 37

$$\int \cosh^3(a+bx)\operatorname{csch}^{3+n}(a+bx)\,dx=-\frac{\operatorname{csch}^n(a+bx)}{bn}-\frac{\operatorname{csch}^{2+n}(a+bx)}{b(2+n)}$$

output
```
-csch(b*x+a)^n/b/n-csch(b*x+a)^(2+n)/b/(2+n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \cosh^3(a+bx)\operatorname{csch}^{3+n}(a+bx)\,dx=-\frac{\operatorname{csch}^n(a+bx)\left(2+n+n\operatorname{csch}^2(a+bx)\right)}{bn(2+n)}$$

input
```
Integrate[Cosh[a + b*x]^3*Csch[a + b*x]^(3 + n),x]
```

output
```
-((Csch[a + b*x]^n*(2 + n + n*Csch[a + b*x]^2))/(b*n*(2 + n)))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3101, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(a+bx)\text{csch}^{n+3}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(i\csc(ia+ibx))^{n+3}}{\sec(ia+ibx)^3}dx$$

$$\Big\downarrow \text{3101}$$

$$\frac{\int -\text{csch}^{n-1}(a+bx)\left(\text{csch}^2(a+bx)+1\right)d\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \text{csch}^{n-1}(a+bx)\left(\text{csch}^2(a+bx)+1\right)d\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\text{csch}^{n-1}(a+bx)+\text{csch}^{n+1}(a+bx)\right)d\text{csch}(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\text{csch}^{n+2}(a+bx)}{n+2}-\frac{\text{csch}^n(a+bx)}{n}}{b}$$

input
```
Int[Cosh[a + b*x]^3*Csch[a + b*x]^(3 + n),x]
```

output
```
(-(Csch[a + b*x]^n/n) - Csch[a + b*x]^(2 + n)/(2 + n))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [F]

$$\int \cosh(bx+a)^3 \operatorname{csch}(bx+a)^{3+n}\,dx$$

input
```
int(cosh(b*x+a)^3*csch(b*x+a)^(3+n),x)
```

output
```
int(cosh(b*x+a)^3*csch(b*x+a)^(3+n),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(37) = 74$.

Time = 0.10 (sec) , antiderivative size = 256, normalized size of antiderivative = 6.92

$$\int \cosh^3(a+bx)\operatorname{csch}^{3+n}(a+bx)\,dx = -\frac{\left((n+2)\sinh(bx+a)^3 + \left(3(n+2)\cosh(bx+a)^2 + n - 6\right)\sinh(bx+a)\right)\cosh\left((n+3)\log\left(\frac{}{\cosh(bx+}\right.\right.}{4\left((bn^2+2\,bn\right.}$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)^(3+n),x, algorithm="fricas")
```

output
```
-1/4*(((n + 2)*sinh(b*x + a)^3 + (3*(n + 2)*cosh(b*x + a)^2 + n - 6)*sinh(
b*x + a))*cosh((n + 3)*log(2*(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a
)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1))) + ((n + 2)*si
nh(b*x + a)^3 + (3*(n + 2)*cosh(b*x + a)^2 + n - 6)*sinh(b*x + a))*sinh((n
 + 3)*log(2*(cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x
+ a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1))))/((b*n^2 + 2*b*n)*cosh(b*x + a
)^4 - 2*(b*n^2 + 2*b*n)*cosh(b*x + a)^2*sinh(b*x + a)^2 + (b*n^2 + 2*b*n)*
sinh(b*x + a)^4)
```

Sympy [F]

$$\int \cosh^3(a+bx)\operatorname{csch}^{3+n}(a+bx)\,dx = \int \cosh^3(a+bx)\operatorname{csch}^{n+3}(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)**3*csch(b*x+a)**(3+n),x)
```

output
```
Integral(cosh(a + b*x)**3*csch(a + b*x)**(n + 3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(37) = 74$.

Time = 0.21 (sec) , antiderivative size = 418, normalized size of antiderivative = 11.30

$$\int \cosh^3(a+bx)\operatorname{csch}^{3+n}(a+bx)\,dx = -\frac{2^{n+3}ne^{\left(-(bx+a)n-n\log\left(e^{(-bx-a)}+1\right)-n\log\left(-e^{(-bx-a)}+1\right)\right)}}{8\left(n^2-2\left(n^2+2n\right)e^{(-2bx-2a)}+\left(n^2+2n\right)e^{(-4bx-4a)}+2n\right)b} - \frac{\left(2^{n+4}n-2^{n+5}\right)e^{\left(-(bx+a)n-2bx-n\log\left(e^{(-bx-a)}+1\right)-n\log\left(-e^{(-bx-a)}+1\right)-2a\right)}}{8\left(n^2-2\left(n^2+2n\right)e^{(-2bx-2a)}+\left(n^2+2n\right)e^{(-4bx-4a)}+2n\right)b} - \frac{\left(2^{n+3}n+2^{n+4}\right)e^{\left(-(bx+a)n-4bx-n\log\left(e^{(-bx-a)}+1\right)-n\log\left(-e^{(-bx-a)}+1\right)-4a\right)}}{8\left(n^2-2\left(n^2+2n\right)e^{(-2bx-2a)}+\left(n^2+2n\right)e^{(-4bx-4a)}+2n\right)b} - \frac{2^{n+4}e^{\left(-(bx+a)n-n\log\left(e^{(-bx-a)}+1\right)-n\log\left(-e^{(-bx-a)}+1\right)\right)}}{8\left(n^2-2\left(n^2+2n\right)e^{(-2bx-2a)}+\left(n^2+2n\right)e^{(-4bx-4a)}+2n\right)b}$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)^(3+n),x, algorithm="maxima")
```

output
```
-1/8*2^(n + 3)*n*e^(-(b*x + a)*n - n*log(e^(-b*x - a) + 1) - n*log(-e^(-b*
x - a) + 1))/((n^2 - 2*(n^2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*
x - 4*a) + 2*n)*b) - 1/8*(2^(n + 4)*n - 2^(n + 5))*e^(-(b*x + a)*n - 2*b*x
 - n*log(e^(-b*x - a) + 1) - n*log(-e^(-b*x - a) + 1) - 2*a)/((n^2 - 2*(n^
2 + 2*n)*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2*n)*b) - 1/8*(
2^(n + 3)*n + 2^(n + 4))*e^(-(b*x + a)*n - 4*b*x - n*log(e^(-b*x - a) + 1)
 - n*log(-e^(-b*x - a) + 1) - 4*a)/((n^2 - 2*(n^2 + 2*n)*e^(-2*b*x - 2*a)
+ (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2*n)*b) - 1/8*2^(n + 4)*e^(-(b*x + a)*n -
 n*log(e^(-b*x - a) + 1) - n*log(-e^(-b*x - a) + 1))/((n^2 - 2*(n^2 + 2*n)
*e^(-2*b*x - 2*a) + (n^2 + 2*n)*e^(-4*b*x - 4*a) + 2*n)*b)
```

Giac [F]

$$\int \cosh^3(a+bx)\operatorname{csch}^{3+n}(a+bx)\,dx = \int \operatorname{csch}\left(bx+a\right)^{n+3}\cosh\left(bx+a\right)^3\,dx$$

input
```
integrate(cosh(b*x+a)^3*csch(b*x+a)^(3+n),x, algorithm="giac")
```

output
```
integrate(csch(b*x + a)^(n + 3)*cosh(b*x + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.57

$$\int \cosh^3(a+bx)\operatorname{csch}^{3+n}(a+bx)\,dx = -e^{-3\,a-3\,b\,x}\left(\frac{1}{\frac{e^{a+b\,x}}{2}-\frac{e^{-a-b\,x}}{2}}\right)^{n+3}\left(\frac{e^{3\,a+3\,b\,x}\sinh(3\,a+3\,b\,x)}{4\,b\,n}+\frac{e^{3\,a+3\,b\,x}\sinh(a+b\,x)\,\left(n-6\right)}{4\,b\,n\,\left(n+2\right)}\right)$$

input
```
int(cosh(a + b*x)^3*(1/sinh(a + b*x))^(n + 3),x)
```

output
```
-exp(- 3*a - 3*b*x)*(1/(exp(a + b*x)/2 - exp(- a - b*x)/2))^(n + 3)*((exp(
3*a + 3*b*x)*sinh(3*a + 3*b*x))/(4*b*n) + (exp(3*a + 3*b*x)*sinh(a + b*x)*
(n - 6))/(4*b*n*(n + 2)))
```

Reduce [F]

$$\int \cosh^3(a+bx)\operatorname{csch}^{3+n}(a+bx)\,dx = \int \operatorname{csch}(bx+a)^n \cosh{(bx+a)}^3 \operatorname{csch}(bx+a)^3\,dx$$

input
```
int(cosh(b*x+a)^3*csch(b*x+a)^(3+n),x)
```

output
```
int(csch(a + b*x)**n*cosh(a + b*x)**3*csch(a + b*x)**3,x)
```

3.114 $\int \coth^2(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \coth^2(a+bx)\text{csch}(a+bx)\,dx=-\frac{\text{arctanh}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\text{csch}(a+bx)}{2b}$$

output
```
-1/2*arctanh(cosh(b*x+a))/b-1/2*coth(b*x+a)*csch(b*x+a)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 75 vs. 2(34) = 68.

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int \coth^2(a+bx)\text{csch}(a+bx)\,dx=-\frac{\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}+\frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}-\frac{\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}$$

input
```
Integrate[Coth[a + b*x]^2*Csch[a + b*x],x]
```

output
```
-1/8*Csch[(a + b*x)/2]^2/b - Log[Cosh[(a + b*x)/2]]/(2*b) + Log[Sinh[(a +
b*x)/2]]/(2*b) - Sech[(a + b*x)/2]^2/(8*b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 26, 3091, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(a+bx)\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan\left(ia+ibx-\frac{\pi}{2}\right)^2\sec\left(ia+ibx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sec\left(\frac{1}{2}(2ia-\pi)+ibx\right)\tan\left(\frac{1}{2}(2ia-\pi)+ibx\right)^2dx$$

$$\Big\downarrow \text{3091}$$

$$-i\left(-\frac{1}{2}\int -i\text{csch}(a+bx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{1}{2}i\int \text{csch}(a+bx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{1}{2}i\int i\csc(ia+ibx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{1}{2}\int \csc(ia+ibx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

$$\Big\downarrow \text{4257}$$

$$-i\left(-\frac{i\text{arctanh}(\cosh(a+bx))}{2b}-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)$$

input
```
Int[Coth[a + b*x]^2*Csch[a + b*x],x]
```

output
```
(-I)*(((-1/2*I)*ArcTanh[Cosh[a + b*x]])/b - ((I/2)*Coth[a + b*x]*Csch[a +
b*x])/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.32

method	result	size
derivativedivides	$\frac{-\frac{\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{\coth(bx+a)\operatorname{csch}(bx+a)}{2}-\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	45
default	$\frac{-\frac{\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{\coth(bx+a)\operatorname{csch}(bx+a)}{2}-\operatorname{arctanh}\left(e^{bx+a}\right)}{b}$	45
risch	$-\frac{e^{bx+a}\left(e^{2bx+2a}+1\right)}{b\left(e^{2bx+2a}-1\right)^2}+\frac{\ln\left(e^{bx+a}-1\right)}{2b}-\frac{\ln\left(e^{bx+a}+1\right)}{2b}$	65

input
```
int(coth(b*x+a)^2*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-cosh(b*x+a)/sinh(b*x+a)^2+1/2*coth(b*x+a)*csch(b*x+a)-arctanh(exp(b*
x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(30) = 60.

Time = 0.09 (sec) , antiderivative size = 387, normalized size of antiderivative = 11.38

$$\int \coth^2(a+bx)\text{csch}(a+bx)\,dx =$$
$$-\frac{2\,\cosh\left(bx+a\right)^3+6\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+2\,\sinh\left(bx+a\right)^3+\left(\cosh\left(bx+a\right)^4+4\,\cosh\left(bx+\right.\right.}{}$$

input
```
integrate(coth(b*x+a)^2*csch(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*cosh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x + a)^2 + 2*sinh(b*x + a
)^3 + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4
 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh
(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b
*x + a) + 1) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b
*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2
+ 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a)
 + sinh(b*x + a) - 1) + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 2*cosh(b
*x + a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b
*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a
)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \coth^2(a+bx)\text{csch}(a+bx)\,dx = \int \coth^2{(a+bx)}\,\text{csch}\,(a+bx)\,dx$$

input
```
integrate(coth(b*x+a)**2*csch(b*x+a),x)
```

output
```
Integral(coth(a + b*x)**2*csch(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(30) = 60$.

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.47

$$\int \coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{\log\left(e^{(-bx-a)}+1\right)}{2\,b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{2\,b} + \frac{e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{b(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1)}$$

input
```
integrate(coth(b*x+a)^2*csch(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*log(e^(-b*x - a) + 1)/b + 1/2*log(e^(-b*x - a) - 1)/b + (e^(-b*x - a)
 + e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(30) = 60$.

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.47

$$\int \coth^2(a+bx)\text{csch}(a+bx)\,dx = -\frac{\frac{4\left(e^{(bx+a)}+e^{(-bx-a)}\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4} + \log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) - \log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{4\,b}$$

input
```
integrate(coth(b*x+a)^2*csch(b*x+a),x, algorithm="giac")
```

output
```
-1/4*(4*(e^(b*x + a) + e^(-b*x - a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4)
+ log(e^(b*x + a) + e^(-b*x - a) + 2) - log(e^(b*x + a) + e^(-b*x - a) - 2
))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.56

$$\int \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{\operatorname{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(coth(a + b*x)^2/sinh(a + b*x),x)
```

output
```
- atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b)/(-b^2)^(1/2) - (2*exp(a + b*x))/(
b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a
+ 2*b*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 162, normalized size of antiderivative = 4.76

$$\int \coth^2(a+bx)\operatorname{csch}(a+bx)\,dx = \frac{e^{4bx+4a}\log\left(e^{bx+a}-1\right) - e^{4bx+4a}\log\left(e^{bx+a}+1\right) - 2e^{3bx+3a} - 2e^{2bx+2a}\log\left(e^{bx+a}-1\right) + 2e^{2bx+2a}\log\left(e^{bx+a}\right.}{2b\left(e^{4bx+4a}-2e^{2bx+2a}+1\right)}$$

input
```
int(coth(b*x+a)^2*csch(b*x+a),x)
```

output

```
(e**(4*a + 4*b*x)*log(e**(a + b*x) - 1) - e**(4*a + 4*b*x)*log(e**(a + b*x
) + 1) - 2*e**(3*a + 3*b*x) - 2*e**(2*a + 2*b*x)*log(e**(a + b*x) - 1) + 2
*e**(2*a + 2*b*x)*log(e**(a + b*x) + 1) - 2*e**(a + b*x) + log(e**(a + b*x
) - 1) - log(e**(a + b*x) + 1))/(2*b*(e**(4*a + 4*b*x) - 2*e**(2*a + 2*b*x
) + 1))
```

3.115 $\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx = \frac{\operatorname{arctanh}(\cosh(a+bx))}{8b} - \frac{\coth(a+bx)\operatorname{csch}(a+bx)}{8b} - \frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}$$

output
```
1/8*arctanh(cosh(b*x+a))/b-1/8*coth(b*x+a)*csch(b*x+a)/b-1/4*coth(b*x+a)*c
sch(b*x+a)^3/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 113 vs. 2(55) = 110.

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.05

$$\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx = -\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b} - \frac{\operatorname{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b} + \frac{\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{8b} - \frac{\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{8b} - \frac{\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b} + \frac{\operatorname{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b}$$

input
```
Integrate[Coth[a + b*x]^2*Csch[a + b*x]^3,x]
```

output
```
-1/32*Csch[(a + b*x)/2]^2/b - Csch[(a + b*x)/2]^4/(64*b) + Log[Cosh[(a + b
*x)/2]]/(8*b) - Log[Sinh[(a + b*x)/2]]/(8*b) - Sech[(a + b*x)/2]^2/(32*b)
+ Sech[(a + b*x)/2]^4/(64*b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {3042, 26, 3091, 26, 3042, 26, 4255, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(a+bx)\text{csch}^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan\left(ia+ibx-\frac{\pi}{2}\right)^2\sec\left(ia+ibx-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sec\left(\frac{1}{2}(2ia-\pi)+ibx\right)^3\tan\left(\frac{1}{2}(2ia-\pi)+ibx\right)^2dx$$

$$\Big\downarrow \text{3091}$$

$$i\left(\frac{i\coth(a+bx)\text{csch}^3(a+bx)}{4b}-\frac{1}{4}\int i\text{csch}^3(a+bx)dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\coth(a+bx)\text{csch}^3(a+bx)}{4b}-\frac{1}{4}i\int \text{csch}^3(a+bx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\coth(a+bx)\text{csch}^3(a+bx)}{4b}-\frac{1}{4}i\int -i\csc(ia+ibx)^3dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\coth(a+bx)\text{csch}^3(a+bx)}{4b}-\frac{1}{4}\int \csc(ia+ibx)^3dx\right)$$

$$\Big\downarrow \text{4255}$$

$$i\left(\frac{1}{4}\left(\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}-\frac{1}{2}\int -i\text{csch}(a+bx)dx\right)+\frac{i\coth(a+bx)\text{csch}^3(a+bx)}{4b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{1}{4}\left(\frac{1}{2}i\int \text{csch}(a+bx)dx+\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)+\frac{i\coth(a+bx)\text{csch}^3(a+bx)}{4b}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{1}{4}\left(\frac{1}{2}i\int i\csc(ia+ibx)dx+\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)+\frac{i\coth(a+bx)\text{csch}^3(a+bx)}{4b}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{1}{4}\left(\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}-\frac{1}{2}\int \csc(ia+ibx)dx\right)+\frac{i\coth(a+bx)\text{csch}^3(a+bx)}{4b}\right)$$

$$\Big\downarrow \text{4257}$$

$$i\left(\frac{1}{4}\left(\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}-\frac{i\text{arctanh}(\cosh(a+bx))}{2b}\right)+\frac{i\coth(a+bx)\text{csch}^3(a+bx)}{4b}\right)$$

input
```
Int[Coth[a + b*x]^2*Csch[a + b*x]^3,x]
```

output
```
I*(((I/4)*Coth[a + b*x]*Csch[a + b*x]^3)/b + (((-1/2*I)*ArcTanh[Cosh[a + b
*x]])/b + ((I/2)*Coth[a + b*x]*Csch[a + b*x])/b)/4)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{-\frac{\cosh(bx+a)}{3\sinh(bx+a)^4}-\frac{\left(-\frac{\operatorname{csch}(bx+a)^3}{4}+\frac{3\operatorname{csch}(bx+a)}{8}\right)\coth(bx+a)}{3}+\frac{\operatorname{arctanh}\left(e^{bx+a}\right)}{4}}{b}$	58
default	$\frac{-\frac{\cosh(bx+a)}{3\sinh(bx+a)^4}-\frac{\left(-\frac{\operatorname{csch}(bx+a)^3}{4}+\frac{3\operatorname{csch}(bx+a)}{8}\right)\coth(bx+a)}{3}+\frac{\operatorname{arctanh}\left(e^{bx+a}\right)}{4}}{b}$	58
risch	$-\frac{e^{bx+a}\left(e^{6bx+6a}+7\,e^{4bx+4a}+7\,e^{2bx+2a}+1\right)}{4b\left(e^{2bx+2a}-1\right)^4}-\frac{\ln\left(e^{bx+a}-1\right)}{8b}+\frac{\ln\left(e^{bx+a}+1\right)}{8b}$	87

input
```
int(coth(b*x+a)^2*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3/sinh(b*x+a)^4*cosh(b*x+a)-1/3*(-1/4*csch(b*x+a)^3+3/8*csch(b*x+a
))*coth(b*x+a)+1/4*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1109 vs. $2(49) = 98$.

Time = 0.11 (sec) , antiderivative size = 1109, normalized size of antiderivative = 20.16

$$\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(coth(b*x+a)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/8*(2*cosh(b*x + a)^7 + 14*cosh(b*x + a)*sinh(b*x + a)^6 + 2*sinh(b*x +
a)^7 + 14*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^5 + 14*cosh(b*x + a)^5 + 7
0*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a)^4 + 14*(5*cosh(b*x + a)^
4 + 10*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 14*cosh(b*x + a)^3 + 14*(3*c
osh(b*x + a)^5 + 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 - (
cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7
*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x
+ a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*cos
h(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)
^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x
 + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*
cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)
^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) +
 1) + (cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8
 + 4*(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*co
sh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 -
 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b
*x + a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*c
osh(b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)
^2 - 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cos...
```

Sympy [F]

$$\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx = \int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx$$

input
```
integrate(coth(b*x+a)**2*csch(b*x+a)**3,x)
```

output
```
Integral(coth(a + b*x)**2*csch(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(49) = 98.

Time = 0.05 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.35

$$\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx = \frac{\log\left(e^{(-bx-a)}+1\right)}{8\,b} - \frac{\log\left(e^{(-bx-a)}-1\right)}{8\,b} + \frac{e^{(-bx-a)}+7\,e^{(-3\,bx-3\,a)}+7\,e^{(-5\,bx-5\,a)}+e^{(-7\,bx-7\,a)}}{4\,b(4\,e^{(-2\,bx-2\,a)}-6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}-1)}$$

input
```
integrate(coth(b*x+a)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/8*log(e^(-b*x - a) + 1)/b - 1/8*log(e^(-b*x - a) - 1)/b + 1/4*(e^(-b*x -
 a) + 7*e^(-3*b*x - 3*a) + 7*e^(-5*b*x - 5*a) + e^(-7*b*x - 7*a))/(b*(4*e^
(-2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) - e^(-8*b*x - 8*a
) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(49) = 98$.

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.93

$$\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx = -\frac{\frac{4\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3+4\,e^{(bx+a)}+4\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4\right)^2} - \log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right) + \log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{16\,b}$$

input
```
integrate(coth(b*x+a)^2*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/16*(4*((e^(b*x + a) + e^(-b*x - a))^3 + 4*e^(b*x + a) + 4*e^(-b*x - a))
/((e^(b*x + a) + e^(-b*x - a))^2 - 4)^2 - log(e^(b*x + a) + e^(-b*x - a) +
 2) + log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.98

$$\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx = \frac{\operatorname{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{4\,\sqrt{-b^2}} - \frac{\frac{e^{a+b\,x}}{b}+\frac{2\,e^{3\,a+3\,b\,x}}{b}+\frac{e^{5\,a+5\,b\,x}}{b}}{6\,e^{4\,a+4\,b\,x}-4\,e^{2\,a+2\,b\,x}-4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1} - \frac{3\,e^{a+b\,x}}{2\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{2\,e^{a+b\,x}}{b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)} - \frac{e^{a+b\,x}}{4\,b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(coth(a + b*x)^2/sinh(a + b*x)^3,x)
```

output
```
atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b)/(4*(-b^2)^(1/2)) - (exp(a + b*x)/b
+ (2*exp(3*a + 3*b*x))/b + exp(5*a + 5*b*x)/b)/(6*exp(4*a + 4*b*x) - 4*exp
(2*a + 2*b*x) - 4*exp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1) - (3*exp(a + b*
x))/(2*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (2*exp(a + b*x))/(
b*(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - exp(
a + b*x)/(4*b*(exp(2*a + 2*b*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 298, normalized size of antiderivative = 5.42

$$\int \coth^2(a+bx)\operatorname{csch}^3(a+bx)\,dx = \frac{-e^{8bx+8a}\log\left(e^{bx+a}-1\right)+e^{8bx+8a}\log\left(e^{bx+a}+1\right)-2e^{7bx+7a}+4e^{6bx+6a}\log\left(e^{bx+a}-1\right)-4e^{6bx+6a}\log\left(e^{bx+a}\right.}{}$$

input
```
int(coth(b*x+a)^2*csch(b*x+a)^3,x)
```

output
```
( - e**(8*a + 8*b*x)*log(e**(a + b*x) - 1) + e**(8*a + 8*b*x)*log(e**(a +
b*x) + 1) - 2*e**(7*a + 7*b*x) + 4*e**(6*a + 6*b*x)*log(e**(a + b*x) - 1)
- 4*e**(6*a + 6*b*x)*log(e**(a + b*x) + 1) - 14*e**(5*a + 5*b*x) - 6*e**(4
*a + 4*b*x)*log(e**(a + b*x) - 1) + 6*e**(4*a + 4*b*x)*log(e**(a + b*x) +
1) - 14*e**(3*a + 3*b*x) + 4*e**(2*a + 2*b*x)*log(e**(a + b*x) - 1) - 4*e*
*(2*a + 2*b*x)*log(e**(a + b*x) + 1) - 2*e**(a + b*x) - log(e**(a + b*x) -
 1) + log(e**(a + b*x) + 1))/(8*b*(e**(8*a + 8*b*x) - 4*e**(6*a + 6*b*x) +
 6*e**(4*a + 4*b*x) - 4*e**(2*a + 2*b*x) + 1))
```

3.116 $\int \coth^4(a+bx)\mathbf{csch}(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \coth^4(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{3\operatorname{arctanh}(\cosh(a+bx))}{8b} - \frac{3\coth(a+bx)\operatorname{csch}(a+bx)}{8b} - \frac{\coth^3(a+bx)\operatorname{csch}(a+bx)}{4b}$$

output
```
-3/8*arctanh(cosh(b*x+a))/b-3/8*coth(b*x+a)*csch(b*x+a)/b-1/4*coth(b*x+a)^
3*csch(b*x+a)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 113 vs. 2(55) = 110.

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.05

$$\int \coth^4(a+bx)\operatorname{csch}(a+bx)\,dx = -\frac{5\operatorname{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b} - \frac{\operatorname{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b} - \frac{3\log\left(\cosh\left(\frac{1}{2}(a+bx)\right)\right)}{8b} + \frac{3\log\left(\sinh\left(\frac{1}{2}(a+bx)\right)\right)}{8b} - \frac{5\operatorname{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b} + \frac{\operatorname{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b}$$

input
```
Integrate[Coth[a + b*x]^4*Csch[a + b*x],x]
```

output
```
(-5*Csch[(a + b*x)/2]^2)/(32*b) - Csch[(a + b*x)/2]^4/(64*b) - (3*Log[Cosh
[(a + b*x)/2]])/(8*b) + (3*Log[Sinh[(a + b*x)/2]])/(8*b) - (5*Sech[(a + b*
x)/2]^2)/(32*b) + Sech[(a + b*x)/2]^4/(64*b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {3042, 26, 3091, 26, 3042, 26, 3091, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^4(a+bx)\operatorname{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan\left(ia+ibx-\frac{\pi}{2}\right)^4\sec\left(ia+ibx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sec\left(\frac{1}{2}(2ia-\pi)+ibx\right)\tan\left(\frac{1}{2}(2ia-\pi)+ibx\right)^4dx$$

$$\Big\downarrow \text{3091}$$

$$i\left(\frac{i\coth^3(a+bx)\operatorname{csch}(a+bx)}{4b}-\frac{3}{4}\int i\coth^2(a+bx)\operatorname{csch}(a+bx)dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\coth^3(a+bx)\operatorname{csch}(a+bx)}{4b}-\frac{3}{4}i\int \coth^2(a+bx)\operatorname{csch}(a+bx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\coth^3(a+bx)\operatorname{csch}(a+bx)}{4b}-\frac{3}{4}i\int -i\sec\left(ia+ibx-\frac{\pi}{2}\right)\tan\left(ia+ibx-\frac{\pi}{2}\right)^2dx\right)$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{i\coth^3(a+bx)\text{csch}(a+bx)}{4b}-\frac{3}{4}\int \sec\left(\frac{1}{2}(2ia-\pi)+ibx\right)\tan\left(\frac{1}{2}(2ia-\pi)+ibx\right)^2dx\right)$$

$$\Big\downarrow \; 3091$$

$$i\left(\frac{i\coth^3(a+bx)\text{csch}(a+bx)}{4b}-\frac{3}{4}\left(-\frac{1}{2}\int -i\text{csch}(a+bx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)\right)$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{i\coth^3(a+bx)\text{csch}(a+bx)}{4b}-\frac{3}{4}\left(\frac{1}{2}i\int \text{csch}(a+bx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)\right)$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{i\coth^3(a+bx)\text{csch}(a+bx)}{4b}-\frac{3}{4}\left(\frac{1}{2}i\int i\csc(ia+ibx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)\right)$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{i\coth^3(a+bx)\text{csch}(a+bx)}{4b}-\frac{3}{4}\left(-\frac{1}{2}\int \csc(ia+ibx)dx-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)\right)$$

$$\Big\downarrow \; 4257$$

$$i\left(\frac{i\coth^3(a+bx)\text{csch}(a+bx)}{4b}-\frac{3}{4}\left(-\frac{i\text{arctanh}(\cosh(a+bx))}{2b}-\frac{i\coth(a+bx)\text{csch}(a+bx)}{2b}\right)\right)$$

input
```
Int[Coth[a + b*x]^4*Csch[a + b*x],x]
```

output
```
I*(((I/4)*Coth[a + b*x]^3*Csch[a + b*x])/b - (3*(((-1/2*I)*ArcTanh[Cosh[a
+ b*x]])/b - ((I/2)*Coth[a + b*x]*Csch[a + b*x])/b))/4)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

method	result	size
derivativedivides	$\frac{-\frac{\cosh(bx+a)^3}{\sinh(bx+a)^4}+\frac{\cosh(bx+a)}{\sinh(bx+a)^4}+\left(-\frac{\operatorname{csch}(bx+a)^3}{4}+\frac{3\operatorname{csch}(bx+a)}{8}\right)\coth(bx+a)-\frac{3\operatorname{arctanh}\left(e^{bx+a}\right)}{4}}{b}$	74
default	$\frac{-\frac{\cosh(bx+a)^3}{\sinh(bx+a)^4}+\frac{\cosh(bx+a)}{\sinh(bx+a)^4}+\left(-\frac{\operatorname{csch}(bx+a)^3}{4}+\frac{3\operatorname{csch}(bx+a)}{8}\right)\coth(bx+a)-\frac{3\operatorname{arctanh}\left(e^{bx+a}\right)}{4}}{b}$	74
risch	$-\frac{e^{bx+a}\left(5\,e^{6bx+6a}+3\,e^{4bx+4a}+3\,e^{2bx+2a}+5\right)}{4b\left(e^{2bx+2a}-1\right)^4}-\frac{3\ln\left(e^{bx+a}+1\right)}{8b}+\frac{3\ln\left(e^{bx+a}-1\right)}{8b}$	89

input
```
int(coth(b*x+a)^4*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/sinh(b*x+a)^4*cosh(b*x+a)^3+1/sinh(b*x+a)^4*cosh(b*x+a)+(-1/4*csch
(b*x+a)^3+3/8*csch(b*x+a))*coth(b*x+a)-3/4*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1114 vs. $2(49) = 98$.

Time = 0.11 (sec) , antiderivative size = 1114, normalized size of antiderivative = 20.25

$$\int \coth^4(a+bx)\text{csch}(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(coth(b*x+a)^4*csch(b*x+a),x, algorithm="fricas")
```

output
```
-1/8*(10*cosh(b*x + a)^7 + 70*cosh(b*x + a)*sinh(b*x + a)^6 + 10*sinh(b*x
+ a)^7 + 6*(35*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^5 + 6*cosh(b*x + a)^5 +
10*(35*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^4 + 2*(175*cosh(b*
x + a)^4 + 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^3 + 6*cosh(b*x + a)^3 + 6
*(35*cosh(b*x + a)^5 + 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)
^2 + 3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^
8 + 4*(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*c
osh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4
- 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(
b*x + a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*
cosh(b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a
)^2 - 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(
b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*
x + a) + 1) - 3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(
b*x + a)^8 + 4*(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6
 + 8*(7*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*
x + a)^4 - 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8
*(7*cosh(b*x + a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^
3 + 4*(7*cosh(b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sin
h(b*x + a)^2 - 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)...
```

Sympy [F]

$$\int \coth^4(a+bx)\text{csch}(a+bx)\,dx = \int \coth^4(a+bx)\,\text{csch}\,(a+bx)\,dx$$

input
```
integrate(coth(b*x+a)**4*csch(b*x+a),x)
```

output
```
Integral(coth(a + b*x)**4*csch(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(49) = 98$.

Time = 0.06 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.42

$$\int \coth^4(a+bx)\text{csch}(a+bx)\,dx = -\frac{3\,\log\left(e^{(-bx-a)}+1\right)}{8\,b} + \frac{3\,\log\left(e^{(-bx-a)}-1\right)}{8\,b} + \frac{5\,e^{(-bx-a)}+3\,e^{(-3\,bx-3\,a)}+3\,e^{(-5\,bx-5\,a)}+5\,e^{(-7\,bx-7\,a)}}{4\,b(4\,e^{(-2\,bx-2\,a)}-6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}-1)}$$

input
```
integrate(coth(b*x+a)^4*csch(b*x+a),x, algorithm="maxima")
```

output
```
-3/8*log(e^(-b*x - a) + 1)/b + 3/8*log(e^(-b*x - a) - 1)/b + 1/4*(5*e^(-b*
x - a) + 3*e^(-3*b*x - 3*a) + 3*e^(-5*b*x - 5*a) + 5*e^(-7*b*x - 7*a))/(b*
(4*e^(-2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) - e^(-8*b*x
- 8*a) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(49) = 98$.

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.00

$$\int \coth^4(a+bx)\operatorname{csch}(a+bx)\,dx =$$

$$-\frac{\frac{4\left(5\left(e^{(bx+a)}+e^{(-bx-a)}\right)^3-12\,e^{(bx+a)}-12\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4\right)^2}+3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)-3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}-\right.}{16\,b}$$

input
```
integrate(coth(b*x+a)^4*csch(b*x+a),x, algorithm="giac")
```

output
```
-1/16*(4*(5*(e^(b*x + a) + e^(-b*x - a))^3 - 12*e^(b*x + a) - 12*e^(-b*x -
 a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4)^2 + 3*log(e^(b*x + a) + e^(-b*x
- a) + 2) - 3*log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.45

$$\int \coth^4(a+bx)\operatorname{csch}(a+bx)\,dx$$
$$= -\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{4\,\sqrt{-b^2}}-\frac{9\,\mathrm{e}^{a+b\,x}}{2\,b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$
$$-\frac{6\,\mathrm{e}^{a+b\,x}}{b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}-3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}-1\right)}$$
$$-\frac{4\,\mathrm{e}^{a+b\,x}}{b\,\left(6\,\mathrm{e}^{4\,a+4\,b\,x}-4\,\mathrm{e}^{2\,a+2\,b\,x}-4\,\mathrm{e}^{6\,a+6\,b\,x}+\mathrm{e}^{8\,a+8\,b\,x}+1\right)}-\frac{5\,\mathrm{e}^{a+b\,x}}{4\,b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(coth(a + b*x)^4/sinh(a + b*x),x)
```

output
```
- (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(4*(-b^2)^(1/2)) - (9*exp(a +
 b*x))/(2*b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (6*exp(a + b*x)
)/(b*(3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (
4*exp(a + b*x))/(b*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*exp(6*a +
6*b*x) + exp(8*a + 8*b*x) + 1)) - (5*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x)
- 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 301, normalized size of antiderivative = 5.47

$$\int \coth^4(a+bx)\text{csch}(a+bx)\,dx = \frac{3e^{8bx+8a}\log\left(e^{bx+a}-1\right)-3e^{8bx+8a}\log\left(e^{bx+a}+1\right)-10e^{7bx+7a}-12e^{6bx+6a}\log\left(e^{bx+a}-1\right)+12e^{6bx+6a}\log\left(e\right.}{}$$

input
```
int(coth(b*x+a)^4*csch(b*x+a),x)
```

output
```
(3*e**(8*a + 8*b*x)*log(e**(a + b*x) - 1) - 3*e**(8*a + 8*b*x)*log(e**(a +
 b*x) + 1) - 10*e**(7*a + 7*b*x) - 12*e**(6*a + 6*b*x)*log(e**(a + b*x) -
1) + 12*e**(6*a + 6*b*x)*log(e**(a + b*x) + 1) - 6*e**(5*a + 5*b*x) + 18*e
**(4*a + 4*b*x)*log(e**(a + b*x) - 1) - 18*e**(4*a + 4*b*x)*log(e**(a + b*
x) + 1) - 6*e**(3*a + 3*b*x) - 12*e**(2*a + 2*b*x)*log(e**(a + b*x) - 1) +
 12*e**(2*a + 2*b*x)*log(e**(a + b*x) + 1) - 10*e**(a + b*x) + 3*log(e**(a
 + b*x) - 1) - 3*log(e**(a + b*x) + 1))/(8*b*(e**(8*a + 8*b*x) - 4*e**(6*a
 + 6*b*x) + 6*e**(4*a + 4*b*x) - 4*e**(2*a + 2*b*x) + 1))
```

3.117 $\int \coth^2(x)\text{csch}^4(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \coth^2(x)\text{csch}^4(x)\,dx = \frac{\coth^3(x)}{3} - \frac{\coth^5(x)}{5}$$

output
```
1/3*coth(x)^3-1/5*coth(x)^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \coth^2(x)\text{csch}^4(x)\,dx = \frac{2\coth(x)}{15} - \frac{1}{15}\coth(x)\text{csch}^2(x) - \frac{1}{5}\coth(x)\text{csch}^4(x)$$

input
```
Integrate[Coth[x]^2*Csch[x]^4,x]
```

output
```
(2*Coth[x])/15 - (Coth[x]*Csch[x]^2)/15 - (Coth[x]*Csch[x]^4)/5
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 25, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(x)\text{csch}^4(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(-\frac{\pi}{2}+ix\right)^2\left(-\sec\left(-\frac{\pi}{2}+ix\right)^4\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec\left(ix-\frac{\pi}{2}\right)^4\tan\left(ix-\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3087}$$

$$i\int -\coth^2(x)\left(1-\coth^2(x)\right)d(i\coth(x))$$

$$\Big\downarrow \text{244}$$

$$i\int \left(\coth^4(x)-\coth^2(x)\right)d(i\coth(x))$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{1}{5}i\coth^5(x)-\frac{1}{3}i\coth^3(x)\right)$$

input
```
Int[Coth[x]^2*Csch[x]^4,x]
```

output
```
I*((-1/3*I)*Coth[x]^3 + (I/5)*Coth[x]^5)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\coth(x)^3}{3}-\frac{\coth(x)^5}{5}$	14
default	$\frac{\coth(x)^3}{3}-\frac{\coth(x)^5}{5}$	14
risch	$-\frac{4(15\,e^{6x}+5\,e^{4x}+5\,e^{2x}-1)}{15(e^{2x}-1)^5}$	31

input
```
int(coth(x)^2*csch(x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*coth(x)^3-1/5*coth(x)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(13) = 26$.

Time = 0.09 (sec) , antiderivative size = 164, normalized size of antiderivative = 9.65

$$\int \coth^2(x)\text{csch}^4(x)\,dx =$$

$$-\frac{}{15\left(\cosh\left(x\right)^7+7\cosh\left(x\right)\sinh\left(x\right)^6+\sinh\left(x\right)^7+\left(21\cosh\left(x\right)^2-5\right)\sinh\left(x\right)^5-5\cosh\left(x\right)^5+5\left(7\text{co}\right.\right.}$$

input
```
integrate(coth(x)^2*csch(x)^4,x, algorithm="fricas")
```

output
```
-8/15*(7*cosh(x)^3 + 24*cosh(x)^2*sinh(x) + 21*cosh(x)*sinh(x)^2 + 8*sinh(
x)^3 + 5*cosh(x))/(cosh(x)^7 + 7*cosh(x)*sinh(x)^6 + sinh(x)^7 + (21*cosh(
x)^2 - 5)*sinh(x)^5 - 5*cosh(x)^5 + 5*(7*cosh(x)^3 - 5*cosh(x))*sinh(x)^4
+ (35*cosh(x)^4 - 50*cosh(x)^2 + 11)*sinh(x)^3 + 9*cosh(x)^3 + (21*cosh(x)
^5 - 50*cosh(x)^3 + 27*cosh(x))*sinh(x)^2 + (7*cosh(x)^6 - 25*cosh(x)^4 +
33*cosh(x)^2 - 15)*sinh(x) - 5*cosh(x))
```

Sympy [F]

$$\int \coth^2(x)\text{csch}^4(x)\,dx = \int \coth^2\left(x\right)\text{csch}^4\left(x\right)\,dx$$

input
```
integrate(coth(x)**2*csch(x)**4,x)
```

output
```
Integral(coth(x)**2*csch(x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(13) = 26$.

Time = 0.05 (sec) , antiderivative size = 149, normalized size of antiderivative = 8.76

$$\int \coth^2(x)\text{csch}^4(x)\,dx = \frac{4\,e^{(-2\,x)}}{3\,(5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1)} + \frac{4\,e^{(-4\,x)}}{3\,(5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1)} + \frac{4\,e^{(-6\,x)}}{5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1} - \frac{4}{15\,(5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1)}$$

input
```
integrate(coth(x)^2*csch(x)^4,x, algorithm="maxima")
```

output
```
4/3*e^(-2*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x) + e^(-10
*x) - 1) + 4/3*e^(-4*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*
x) + e^(-10*x) - 1) + 4*e^(-6*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) -
 5*e^(-8*x) + e^(-10*x) - 1) - 4/15/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x
) - 5*e^(-8*x) + e^(-10*x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(13) = 26$.

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \coth^2(x)\text{csch}^4(x)\,dx = -\frac{4\,(15\,e^{(6\,x)} + 5\,e^{(4\,x)} + 5\,e^{(2\,x)} - 1)}{15\,(e^{(2\,x)} - 1)^5}$$

input
```
integrate(coth(x)^2*csch(x)^4,x, algorithm="giac")
```

output
```
-4/15*(15*e^(6*x) + 5*e^(4*x) + 5*e^(2*x) - 1)/(e^(2*x) - 1)^5
```

Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 144, normalized size of antiderivative = 8.47

$$\int \coth^2(x)\mathrm{csch}^4(x)\,dx = -\frac{\frac{8\,\mathrm{e}^{2\,x}}{5}+\frac{16\,\mathrm{e}^{4\,x}}{5}+\frac{8\,\mathrm{e}^{6\,x}}{5}}{5\,\mathrm{e}^{2\,x}-10\,\mathrm{e}^{4\,x}+10\,\mathrm{e}^{6\,x}-5\,\mathrm{e}^{8\,x}+\mathrm{e}^{10\,x}-1} - \frac{\frac{4\,\mathrm{e}^{2\,x}}{5}+\frac{8}{15}}{3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1} - \frac{2}{5\,\left(\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1\right)} - \frac{\frac{8\,\mathrm{e}^{2\,x}}{5}+\frac{6\,\mathrm{e}^{4\,x}}{5}+\frac{2}{5}}{6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1}$$

input
```
int(coth(x)^2/sinh(x)^4,x)
```

output
```
- ((8*exp(2*x))/5 + (16*exp(4*x))/5 + (8*exp(6*x))/5)/(5*exp(2*x) - 10*exp
(4*x) + 10*exp(6*x) - 5*exp(8*x) + exp(10*x) - 1) - ((4*exp(2*x))/5 + 8/15
)/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) - 2/(5*(exp(4*x) - 2*exp(2*x) +
 1)) - ((8*exp(2*x))/5 + (6*exp(4*x))/5 + 2/5)/(6*exp(4*x) - 4*exp(2*x) -
4*exp(6*x) + exp(8*x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.76

$$\int \coth^2(x)\mathrm{csch}^4(x)\,dx = \frac{-60e^{6x}-20e^{4x}-20e^{2x}+4}{15e^{10x}-75e^{8x}+150e^{6x}-150e^{4x}+75e^{2x}-15}$$

input
```
int(coth(x)^2*csch(x)^4,x)
```

output
```
(4*( - 15*e**(6*x) - 5*e**(4*x) - 5*e**(2*x) + 1))/(15*(e**(10*x) - 5*e**(
8*x) + 10*e**(6*x) - 10*e**(4*x) + 5*e**(2*x) - 1))
```

3.118 $\int \coth^3(x)\mathbf{csch}^4(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \coth^3(x)\text{csch}^4(x)\,dx = -\frac{1}{4}\text{csch}^4(x) - \frac{\text{csch}^6(x)}{6}$$

output
```
-1/4*csch(x)^4-1/6*csch(x)^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \coth^3(x)\text{csch}^4(x)\,dx = -\frac{1}{4}\text{csch}^4(x) - \frac{\text{csch}^6(x)}{6}$$

input
```
Integrate[Coth[x]^3*Csch[x]^4,x]
```

output
```
-1/4*Csch[x]^4 - Csch[x]^6/6
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 26, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(x)\text{csch}^4(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan\left(-\frac{\pi}{2}+ix\right)^3\sec\left(-\frac{\pi}{2}+ix\right)^4dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sec\left(ix-\frac{\pi}{2}\right)^4\tan\left(ix-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3086}$$

$$\int -i\text{csch}^3(x)\left(\text{csch}^2(x)+1\right)d(-i\text{csch}(x))$$

$$\Big\downarrow \text{25}$$

$$-\int i\text{csch}^3(x)\left(\text{csch}^2(x)+1\right)d(-i\text{csch}(x))$$

$$\Big\downarrow \text{244}$$

$$-\int \left(i\text{csch}^5(x)+i\text{csch}^3(x)\right)d(-i\text{csch}(x))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{6}\text{csch}^6(x)-\frac{\text{csch}^4(x)}{4}$$

input
```
Int[Coth[x]^3*Csch[x]^4,x]
```

output
```
-1/4*Csch[x]^4 - Csch[x]^6/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{\operatorname{csch}(x)^4}{4}-\frac{\operatorname{csch}(x)^6}{6}$	14
default	$-\frac{\operatorname{csch}(x)^4}{4}-\frac{\operatorname{csch}(x)^6}{6}$	14
risch	$-\frac{4\,\mathrm{e}^{4x}\left(3\,\mathrm{e}^{4x}+2\,\mathrm{e}^{2x}+3\right)}{3\left(\mathrm{e}^{2x}-1\right)^6}$	29

input
```
int(coth(x)^3*csch(x)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/4*csch(x)^4-1/6*csch(x)^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(13) = 26.

Time = 0.08 (sec) , antiderivative size = 222, normalized size of antiderivative = 13.06

$$\int \coth^3(x)\text{csch}^4(x)\,dx =$$

$$-\frac{}{3\left(\cosh\left(x\right)^{8}+8\cosh\left(x\right)\sinh\left(x\right)^{7}+\sinh\left(x\right)^{8}+2\left(14\cosh\left(x\right)^{2}-3\right)\sinh\left(x\right)^{6}-6\cosh\left(x\right)^{6}+4\left(14\,\text{c}\right.\right.}$$

input
```
integrate(coth(x)^3*csch(x)^4,x, algorithm="fricas")
```

output
```
-4/3*(3*cosh(x)^4 + 12*cosh(x)*sinh(x)^3 + 3*sinh(x)^4 + 2*(9*cosh(x)^2 +
1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(3*cosh(x)^3 + cosh(x))*sinh(x) + 3)/(cosh(
x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 2*(14*cosh(x)^2 - 3)*sinh(x)^6 -
6*cosh(x)^6 + 4*(14*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 4
5*cosh(x)^2 + 8)*sinh(x)^4 + 16*cosh(x)^4 + 8*(7*cosh(x)^5 - 15*cosh(x)^3
+ 7*cosh(x))*sinh(x)^3 + 2*(14*cosh(x)^6 - 45*cosh(x)^4 + 48*cosh(x)^2 - 1
3)*sinh(x)^2 - 26*cosh(x)^2 + 4*(2*cosh(x)^7 - 9*cosh(x)^5 + 14*cosh(x)^3
- 7*cosh(x))*sinh(x) + 15)
```

Sympy [F]

$$\int \coth^3(x)\text{csch}^4(x)\,dx = \int \coth^3(x)\,\text{csch}^4(x)\,dx$$

input
```
integrate(coth(x)**3*csch(x)**4,x)
```

output
```
Integral(coth(x)**3*csch(x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(13) = 26$.

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 8.18

$$\int \coth^3(x)\operatorname{csch}^4(x)\,dx = \frac{4\,e^{(-4\,x)}}{6\,e^{(-2\,x)} - 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} - 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} - e^{(-12\,x)} - 1} + \frac{8\,e^{(-6\,x)}}{3\,(6\,e^{(-2\,x)} - 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} - 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} - e^{(-12\,x)} - 1)} + \frac{4\,e^{(-8\,x)}}{6\,e^{(-2\,x)} - 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} - 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} - e^{(-12\,x)} - 1}$$

input
```
integrate(coth(x)^3*csch(x)^4,x, algorithm="maxima")
```

output
```
4*e^(-4*x)/(6*e^(-2*x) - 15*e^(-4*x) + 20*e^(-6*x) - 15*e^(-8*x) + 6*e^(-1
0*x) - e^(-12*x) - 1) + 8/3*e^(-6*x)/(6*e^(-2*x) - 15*e^(-4*x) + 20*e^(-6*
x) - 15*e^(-8*x) + 6*e^(-10*x) - e^(-12*x) - 1) + 4*e^(-8*x)/(6*e^(-2*x) -
 15*e^(-4*x) + 20*e^(-6*x) - 15*e^(-8*x) + 6*e^(-10*x) - e^(-12*x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(13) = 26$.

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \coth^3(x)\operatorname{csch}^4(x)\,dx = -\frac{4\left(3\,e^{(8\,x)} + 2\,e^{(6\,x)} + 3\,e^{(4\,x)}\right)}{3\,(e^{(2\,x)} - 1)^6}$$

input
```
integrate(coth(x)^3*csch(x)^4,x, algorithm="giac")
```

output
```
-4/3*(3*e^(8*x) + 2*e^(6*x) + 3*e^(4*x))/(e^(2*x) - 1)^6
```

Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 210, normalized size of antiderivative = 12.35

$$\int \coth^3(x)\mathrm{csch}^4(x)\,dx = -\frac{\frac{8\,\mathrm{e}^{2\,x}}{5}+\frac{12\,\mathrm{e}^{4\,x}}{5}+\frac{16\,\mathrm{e}^{6\,x}}{15}+\frac{4}{15}}{5\,\mathrm{e}^{2\,x}-10\,\mathrm{e}^{4\,x}+10\,\mathrm{e}^{6\,x}-5\,\mathrm{e}^{8\,x}+\mathrm{e}^{10\,x}-1} - \frac{\frac{4\,\mathrm{e}^{2\,x}}{3}+4\,\mathrm{e}^{4\,x}+4\,\mathrm{e}^{6\,x}+\frac{4\,\mathrm{e}^{8\,x}}{3}}{15\,\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}-20\,\mathrm{e}^{6\,x}+15\,\mathrm{e}^{8\,x}-6\,\mathrm{e}^{10\,x}+\mathrm{e}^{12\,x}+1} - \frac{\frac{8\,\mathrm{e}^{2\,x}}{15}+\frac{2}{5}}{3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1} - \frac{4}{15\,\left(\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1\right)} - \frac{\frac{6\,\mathrm{e}^{2\,x}}{5}+\frac{4\,\mathrm{e}^{4\,x}}{5}+\frac{2}{5}}{6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1}$$

input
```
int(coth(x)^3/sinh(x)^4,x)
```

output
```
- ((8*exp(2*x))/5 + (12*exp(4*x))/5 + (16*exp(6*x))/15 + 4/15)/(5*exp(2*x)
 - 10*exp(4*x) + 10*exp(6*x) - 5*exp(8*x) + exp(10*x) - 1) - ((4*exp(2*x))
/3 + 4*exp(4*x) + 4*exp(6*x) + (4*exp(8*x))/3)/(15*exp(4*x) - 6*exp(2*x) -
 20*exp(6*x) + 15*exp(8*x) - 6*exp(10*x) + exp(12*x) + 1) - ((8*exp(2*x))/
15 + 2/5)/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) - 4/(15*(exp(4*x) - 2*e
xp(2*x) + 1)) - ((6*exp(2*x))/5 + (4*exp(4*x))/5 + 2/5)/(6*exp(4*x) - 4*ex
p(2*x) - 4*exp(6*x) + exp(8*x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \coth^3(x)\mathrm{csch}^4(x)\,dx = \frac{\mathrm{csch}(x)^4\left(-2\coth(x)^2-1\right)}{12}$$

input
```
int(coth(x)^3*csch(x)^4,x)
```

output
```
(csch(x)**4*( - 2*coth(x)**2 - 1))/12
```

3.119 $\int \coth^n(x)\mathbf{csch}^4(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 26

$$\int \coth^n(x)\operatorname{csch}^4(x)\,dx = \frac{\coth^{1+n}(x)}{1+n} - \frac{\coth^{3+n}(x)}{3+n}$$

output
```
coth(x)^(1+n)/(1+n)-coth(x)^(3+n)/(3+n)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \coth^n(x)\operatorname{csch}^4(x)\,dx = \frac{(-2-n+\cosh(2x))\coth^{1+n}(x)\operatorname{csch}^2(x)}{(1+n)(3+n)}$$

input
```
Integrate[Coth[x]^n*Csch[x]^4,x]
```

output
```
((-2 - n + Cosh[2*x])*Coth[x]^(1 + n)*Csch[x]^2)/((1 + n)*(3 + n))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(x)\coth^n(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec\left(-\frac{\pi}{2}+ix\right)^4\left(-i\tan\left(-\frac{\pi}{2}+ix\right)\right)^n dx$$

$$\Big\downarrow \text{3087}$$

$$-i\int \coth^n(x)\left(1-\coth^2(x)\right)d(i\coth(x))$$

$$\Big\downarrow \text{244}$$

$$-i\int \left(\coth^n(x)-\coth^{n+2}(x)\right)d(i\coth(x))$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{i\coth^{n+1}(x)}{n+1}-\frac{i\coth^{n+3}(x)}{n+3}\right)$$

input
```
Int[Coth[x]^n*Csch[x]^4,x]
```

output
```
(-I)*((I*Coth[x]^(1 + n))/(1 + n) - (I*Coth[x]^(3 + n))/(3 + n))
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 15.37 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{\coth(x)\mathrm{e}^{n\ln(\coth(x))}}{1+n}-\frac{\coth(x)^{3}\mathrm{e}^{n\ln(\coth(x))}}{3+n}$
default	$\frac{\coth(x)\mathrm{e}^{n\ln(\coth(x))}}{1+n}-\frac{\coth(x)^{3}\mathrm{e}^{n\ln(\coth(x))}}{3+n}$
risch	$-\frac{2\left(-\mathrm{e}^{6x}+2n\,\mathrm{e}^{4x}+3\,\mathrm{e}^{4x}+2n\,\mathrm{e}^{2x}+3\,\mathrm{e}^{2x}-1\right)(\mathrm{e}^{x}-1)^{-n}(\mathrm{e}^{x}+1)^{-n}\left(\mathrm{e}^{2x}+1\right)^{n}\mathrm{e}^{-\frac{i\pi n\left(-\operatorname{csgn}\left(i\left(\mathrm{e}^{2x}+1\right)\right)\operatorname{csgn}\left(\frac{i\left(\mathrm{e}^{2x}+1\right)}{\mathrm{e}^{x}+1}\right)^{2}\right.}{}}}{}$

input
```
int(coth(x)^n*csch(x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/(1+n)*coth(x)*exp(n*ln(coth(x)))-1/(3+n)*coth(x)^3*exp(n*ln(coth(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(26) = 52.

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.38

$$\int \coth^n(x)\text{csch}^4(x)\,dx = \frac{2\left(\left(\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 - (2n+3)\cosh(x)\right)\cosh\left(n\log\left(\frac{\cosh(x)}{\sinh(x)}\right)\right) + \left(\cosh(x)^3 + 3\cosh(x\right.\right.}{(n^2+4n+3)\sinh(x)^3 + 3\left((n^2+4n+3)\cosh(x)^2 - n^2 - 4\right.}$$

input
```
integrate(coth(x)^n*csch(x)^4,x, algorithm="fricas")
```

output
```
2*((cosh(x)^3 + 3*cosh(x)*sinh(x)^2 - (2*n + 3)*cosh(x))*cosh(n*log(cosh(x
)/sinh(x))) + (cosh(x)^3 + 3*cosh(x)*sinh(x)^2 - (2*n + 3)*cosh(x))*sinh(n
*log(cosh(x)/sinh(x))))/((n^2 + 4*n + 3)*sinh(x)^3 + 3*((n^2 + 4*n + 3)*co
sh(x)^2 - n^2 - 4*n - 3)*sinh(x))
```

Sympy [F]

$$\int \coth^n(x)\text{csch}^4(x)\,dx = \int \coth^n(x)\,\text{csch}^4(x)\,dx$$

input
```
integrate(coth(x)**n*csch(x)**4,x)
```

output
```
Integral(coth(x)**n*csch(x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(26) = 52$.

Time = 0.16 (sec) , antiderivative size = 368, normalized size of antiderivative = 14.15

$$\int \coth^n(x)\operatorname{csch}^4(x)\,dx = -\frac{2\,(2\,n+3)e^{\left(-n\log\left(e^{(-x)}+1\right)-n\log\left(-e^{(-x)}+1\right)+n\log\left(e^{(-2\,x)}+1\right)-2\,x\right)}}{n^2-3\,(n^2+4\,n+3)e^{(-2\,x)}+3\,(n^2+4\,n+3)e^{(-4\,x)}-(n^2+4\,n+3)e^{(-6\,x)}+4\,n+3} - \frac{2\,(2\,n+3)e^{\left(-n\log\left(e^{(-x)}+1\right)-n\log\left(-e^{(-x)}+1\right)+n\log\left(e^{(-2\,x)}+1\right)-4\,x\right)}}{n^2-3\,(n^2+4\,n+3)e^{(-2\,x)}+3\,(n^2+4\,n+3)e^{(-4\,x)}-(n^2+4\,n+3)e^{(-6\,x)}+4\,n+3} + \frac{2\,e^{\left(-n\log\left(e^{(-x)}+1\right)-n\log\left(-e^{(-x)}+1\right)+n\log\left(e^{(-2\,x)}+1\right)-6\,x\right)}}{n^2-3\,(n^2+4\,n+3)e^{(-2\,x)}+3\,(n^2+4\,n+3)e^{(-4\,x)}-(n^2+4\,n+3)e^{(-6\,x)}+4\,n+3} + \frac{2\,e^{\left(-n\log\left(e^{(-x)}+1\right)-n\log\left(-e^{(-x)}+1\right)+n\log\left(e^{(-2\,x)}+1\right)\right)}}{n^2-3\,(n^2+4\,n+3)e^{(-2\,x)}+3\,(n^2+4\,n+3)e^{(-4\,x)}-(n^2+4\,n+3)e^{(-6\,x)}+4\,n+3}$$

input
```
integrate(coth(x)^n*csch(x)^4,x, algorithm="maxima")
```

output
```
-2*(2*n + 3)*e^(-n*log(e^(-x) + 1) - n*log(-e^(-x) + 1) + n*log(e^(-2*x) +
 1) - 2*x)/(n^2 - 3*(n^2 + 4*n + 3)*e^(-2*x) + 3*(n^2 + 4*n + 3)*e^(-4*x)
- (n^2 + 4*n + 3)*e^(-6*x) + 4*n + 3) - 2*(2*n + 3)*e^(-n*log(e^(-x) + 1)
- n*log(-e^(-x) + 1) + n*log(e^(-2*x) + 1) - 4*x)/(n^2 - 3*(n^2 + 4*n + 3)
*e^(-2*x) + 3*(n^2 + 4*n + 3)*e^(-4*x) - (n^2 + 4*n + 3)*e^(-6*x) + 4*n +
3) + 2*e^(-n*log(e^(-x) + 1) - n*log(-e^(-x) + 1) + n*log(e^(-2*x) + 1) -
6*x)/(n^2 - 3*(n^2 + 4*n + 3)*e^(-2*x) + 3*(n^2 + 4*n + 3)*e^(-4*x) - (n^2
 + 4*n + 3)*e^(-6*x) + 4*n + 3) + 2*e^(-n*log(e^(-x) + 1) - n*log(-e^(-x)
+ 1) + n*log(e^(-2*x) + 1))/(n^2 - 3*(n^2 + 4*n + 3)*e^(-2*x) + 3*(n^2 + 4
*n + 3)*e^(-4*x) - (n^2 + 4*n + 3)*e^(-6*x) + 4*n + 3)
```

Giac [F]

$$\int \coth^n(x)\text{csch}^4(x)\,dx = \int \coth(x)^n\,\text{csch}(x)^4\,dx$$

input
```
integrate(coth(x)^n*csch(x)^4,x, algorithm="giac")
```

output
```
integrate(coth(x)^n*csch(x)^4, x)
```

Mupad [B] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.35

$$\int \coth^n(x)\text{csch}^4(x)\,dx = \frac{\left(\frac{4\cosh(3x)}{n^2+4n+3} - \frac{2\cosh(x)\,(4n+6)}{n^2+4n+3}\right)\left(\frac{e^{2x}+1}{e^{2x}-1}\right)^n}{2\sinh(3x) - \frac{2\sinh(x)\,(3n^2+12n+9)}{n^2+4n+3}}$$

input
```
int(coth(x)^n/sinh(x)^4,x)
```

output
```
(((4*cosh(3*x))/(4*n + n^2 + 3) - (2*cosh(x)*(4*n + 6))/(4*n + n^2 + 3))*(
(exp(2*x) + 1)/(exp(2*x) - 1))^n)/(2*sinh(3*x) - (2*sinh(x)*(12*n + 3*n^2
+ 9))/(4*n + n^2 + 3))
```

Reduce [F]

$$\int \coth^n(x)\text{csch}^4(x)\,dx = \int \coth(x)^n\,\text{csch}(x)^4\,dx$$

input
```
int(coth(x)^n*csch(x)^4,x)
```

output
```
int(coth(x)**n*csch(x)**4,x)
```

3.120 $\int \coth^4(x)\operatorname{csch}^3(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 38

$$\int \coth^4(x)\operatorname{csch}^3(x)\,dx = \frac{1}{16}\operatorname{arctanh}(\cosh(x)) - \frac{1}{16}\coth(x)\operatorname{csch}(x) - \frac{1}{8}\coth(x)\operatorname{csch}^3(x) - \frac{1}{6}\coth^3(x)\operatorname{csch}^3(x)$$

output
```
1/16*arctanh(cosh(x))-1/16*coth(x)*csch(x)-1/8*coth(x)*csch(x)^3-1/6*coth(
x)^3*csch(x)^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 95 vs. 2(38) = 76.

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.50

$$\int \coth^4(x)\operatorname{csch}^3(x)\,dx = -\frac{1}{64}\operatorname{csch}^2\left(\frac{x}{2}\right) - \frac{1}{64}\operatorname{csch}^4\left(\frac{x}{2}\right) - \frac{1}{384}\operatorname{csch}^6\left(\frac{x}{2}\right) + \frac{1}{16}\log\left(\cosh\left(\frac{x}{2}\right)\right) - \frac{1}{16}\log\left(\sinh\left(\frac{x}{2}\right)\right) - \frac{1}{64}\operatorname{sech}^2\left(\frac{x}{2}\right) + \frac{1}{64}\operatorname{sech}^4\left(\frac{x}{2}\right) - \frac{1}{384}\operatorname{sech}^6\left(\frac{x}{2}\right)$$

input
```
Integrate[Coth[x]^4*Csch[x]^3,x]
```

output
```
-1/64*Csch[x/2]^2 - Csch[x/2]^4/64 - Csch[x/2]^6/384 + Log[Cosh[x/2]]/16 -
 Log[Sinh[x/2]]/16 - Sech[x/2]^2/64 + Sech[x/2]^4/64 - Sech[x/2]^6/384
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.667$, Rules used = {3042, 26, 3091, 26, 3042, 26, 3091, 26, 3042, 26, 4255, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^4(x)\text{csch}^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan\left(-\frac{\pi}{2}+ix\right)^4\sec\left(-\frac{\pi}{2}+ix\right)^3dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sec\left(ix-\frac{\pi}{2}\right)^3\tan\left(ix-\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3091}$$

$$-i\left(-\frac{1}{2}\int -i\coth^2(x)\text{csch}^3(x)dx-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{1}{2}i\int \coth^2(x)\text{csch}^3(x)dx-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{1}{2}i\int i\sec\left(ix-\frac{\pi}{2}\right)^3\tan\left(ix-\frac{\pi}{2}\right)^2dx-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{1}{2}\int \sec\left(ix-\frac{\pi}{2}\right)^3\tan\left(ix-\frac{\pi}{2}\right)^2dx-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \; 3091$$

$$-i\left(\frac{1}{2}\left(\frac{1}{4}\int i\text{csch}^3(x)dx-\frac{1}{4}i\coth(x)\text{csch}^3(x)\right)-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(\frac{1}{2}\left(\frac{1}{4}i\int \text{csch}^3(x)dx-\frac{1}{4}i\coth(x)\text{csch}^3(x)\right)-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \; 3042$$

$$-i\left(\frac{1}{2}\left(\frac{1}{4}i\int -i\csc(ix)^3dx-\frac{1}{4}i\coth(x)\text{csch}^3(x)\right)-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(\frac{1}{2}\left(\frac{1}{4}\int \csc(ix)^3dx-\frac{1}{4}i\coth(x)\text{csch}^3(x)\right)-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \; 4255$$

$$-i\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\int -i\text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)-\frac{1}{4}i\coth(x)\text{csch}^3(x)\right)-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(\frac{1}{2}\left(\frac{1}{4}\left(-\frac{1}{2}i\int \text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)-\frac{1}{4}i\coth(x)\text{csch}^3(x)\right)-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \; 3042$$

$$-i\left(\frac{1}{2}\left(\frac{1}{4}\left(-\frac{1}{2}i\int i\csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)-\frac{1}{4}i\coth(x)\text{csch}^3(x)\right)-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\int \csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)-\frac{1}{4}i\coth(x)\text{csch}^3(x)\right)-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

$$\Big\downarrow \; 4257$$

$$-i\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}i\text{arctanh}(\cosh(x))-\frac{1}{2}i\coth(x)\text{csch}(x)\right)-\frac{1}{4}i\coth(x)\text{csch}^3(x)\right)-\frac{1}{6}i\coth^3(x)\text{csch}^3(x)\right)$$

input
```
Int[Coth[x]^4*Csch[x]^3,x]
```

output
```
(-I)*((-1/6*I)*Coth[x]^3*Csch[x]^3 + ((-1/4*I)*Coth[x]*Csch[x]^3 + ((I/2)*
ArcTanh[Cosh[x]] - (I/2)*Coth[x]*Csch[x])/4)/2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

method	result	size
default	$-\frac{\cosh(x)^3}{3\sinh(x)^6}+\frac{\cosh(x)}{5\sinh(x)^6}+\frac{\left(-\frac{\operatorname{csch}(x)^5}{6}+\frac{5\operatorname{csch}(x)^3}{24}-\frac{5\operatorname{csch}(x)}{16}\right)\coth(x)}{5}+\frac{\operatorname{arctanh}(e^x)}{8}$	46
risch	$-\frac{e^x(3e^{10x}+47e^{8x}+78e^{6x}+78e^{4x}+47e^{2x}+3)}{24(e^{2x}-1)^6}-\frac{\ln(e^x-1)}{16}+\frac{\ln(e^x+1)}{16}$	60

input
```
int(coth(x)^4*csch(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3/sinh(x)^6*cosh(x)^3+1/5/sinh(x)^6*cosh(x)+1/5*(-1/6*csch(x)^5+5/24*cs
ch(x)^3-5/16*csch(x))*coth(x)+1/8*arctanh(exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1260 vs. 2(30) = 60.

Time = 0.10 (sec) , antiderivative size = 1260, normalized size of antiderivative = 33.16

$$\int \coth^4(x)\operatorname{csch}^3(x)\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^4*csch(x)^3,x, algorithm="fricas")
```

output
```
-1/48*(6*cosh(x)^11 + 66*cosh(x)*sinh(x)^10 + 6*sinh(x)^11 + 2*(165*cosh(x
)^2 + 47)*sinh(x)^9 + 94*cosh(x)^9 + 18*(55*cosh(x)^3 + 47*cosh(x))*sinh(x
)^8 + 12*(165*cosh(x)^4 + 282*cosh(x)^2 + 13)*sinh(x)^7 + 156*cosh(x)^7 +
84*(33*cosh(x)^5 + 94*cosh(x)^3 + 13*cosh(x))*sinh(x)^6 + 12*(231*cosh(x)^
6 + 987*cosh(x)^4 + 273*cosh(x)^2 + 13)*sinh(x)^5 + 156*cosh(x)^5 + 12*(16
5*cosh(x)^7 + 987*cosh(x)^5 + 455*cosh(x)^3 + 65*cosh(x))*sinh(x)^4 + 2*(4
95*cosh(x)^8 + 3948*cosh(x)^6 + 2730*cosh(x)^4 + 780*cosh(x)^2 + 47)*sinh(
x)^3 + 94*cosh(x)^3 + 6*(55*cosh(x)^9 + 564*cosh(x)^7 + 546*cosh(x)^5 + 26
0*cosh(x)^3 + 47*cosh(x))*sinh(x)^2 - 3*(cosh(x)^12 + 12*cosh(x)*sinh(x)^1
1 + sinh(x)^12 + 6*(11*cosh(x)^2 - 1)*sinh(x)^10 - 6*cosh(x)^10 + 20*(11*c
osh(x)^3 - 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^4 - 18*cosh(x)^2 + 1)*sin
h(x)^8 + 15*cosh(x)^8 + 24*(33*cosh(x)^5 - 30*cosh(x)^3 + 5*cosh(x))*sinh(
x)^7 + 4*(231*cosh(x)^6 - 315*cosh(x)^4 + 105*cosh(x)^2 - 5)*sinh(x)^6 - 2
0*cosh(x)^6 + 24*(33*cosh(x)^7 - 63*cosh(x)^5 + 35*cosh(x)^3 - 5*cosh(x))*
sinh(x)^5 + 15*(33*cosh(x)^8 - 84*cosh(x)^6 + 70*cosh(x)^4 - 20*cosh(x)^2
+ 1)*sinh(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x)^9 - 36*cosh(x)^7 + 42*cosh(
x)^5 - 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 45*cosh(x)
^8 + 70*cosh(x)^6 - 50*cosh(x)^4 + 15*cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)
^2 + 12*(cosh(x)^11 - 5*cosh(x)^9 + 10*cosh(x)^7 - 10*cosh(x)^5 + 5*cosh(x
)^3 - cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) + 1) + 3*(cosh(x)^12 ...
```

Sympy [F]

$$\int \coth^4(x)\operatorname{csch}^3(x)\,dx = \int \coth^4(x)\operatorname{csch}^3(x)\,dx$$

input
```
integrate(coth(x)**4*csch(x)**3,x)
```

output
```
Integral(coth(x)**4*csch(x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(30) = 60$.

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.58

$$\int \coth^4(x)\operatorname{csch}^3(x)\,dx = \frac{3\,e^{(-x)} + 47\,e^{(-3\,x)} + 78\,e^{(-5\,x)} + 78\,e^{(-7\,x)} + 47\,e^{(-9\,x)} + 3\,e^{(-11\,x)}}{24\left(6\,e^{(-2\,x)} - 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} - 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} - e^{(-12\,x)} - 1\right)} + \frac{1}{16}\,\log\left(e^{(-x)} + 1\right) - \frac{1}{16}\,\log\left(e^{(-x)} - 1\right)$$

input
```
integrate(coth(x)^4*csch(x)^3,x, algorithm="maxima")
```

output
```
1/24*(3*e^(-x) + 47*e^(-3*x) + 78*e^(-5*x) + 78*e^(-7*x) + 47*e^(-9*x) + 3
*e^(-11*x))/(6*e^(-2*x) - 15*e^(-4*x) + 20*e^(-6*x) - 15*e^(-8*x) + 6*e^(-
10*x) - e^(-12*x) - 1) + 1/16*log(e^(-x) + 1) - 1/16*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(30) = 60$.

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.87

$$\int \coth^4(x)\operatorname{csch}^3(x)\,dx = -\frac{3\left(e^{(-x)} + e^x\right)^5 + 32\left(e^{(-x)} + e^x\right)^3 - 48\,e^{(-x)} - 48\,e^x}{24\left(\left(e^{(-x)} + e^x\right)^2 - 4\right)^3} + \frac{1}{32}\,\log\left(e^{(-x)} + e^x + 2\right) - \frac{1}{32}\,\log\left(e^{(-x)} + e^x - 2\right)$$

input
```
integrate(coth(x)^4*csch(x)^3,x, algorithm="giac")
```

output
```
-1/24*(3*(e^(-x) + e^x)^5 + 32*(e^(-x) + e^x)^3 - 48*e^(-x) - 48*e^x)/((e^
(-x) + e^x)^2 - 4)^3 + 1/32*log(e^(-x) + e^x + 2) - 1/32*log(e^(-x) + e^x
- 2)
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 214, normalized size of antiderivative = 5.63

$$\int \coth^4(x)\operatorname{csch}^3(x)\,dx = \frac{\ln\left(\frac{e^x}{8}+\frac{1}{8}\right)}{16} - \frac{\ln\left(\frac{e^x}{8}-\frac{1}{8}\right)}{16} - \frac{10\,e^x}{6\,e^{4\,x}-4\,e^{2\,x}-4\,e^{6\,x}+e^{8\,x}+1} - \frac{e^x}{8\,\left(e^{2\,x}-1\right)} - \frac{7\,e^x}{3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1} - \frac{\frac{8\,e^{3\,x}}{3}+4\,e^{5\,x}+\frac{8\,e^{7\,x}}{3}+\frac{2\,e^{9\,x}}{3}+\frac{2\,e^x}{3}}{15\,e^{4\,x}-6\,e^{2\,x}-20\,e^{6\,x}+15\,e^{8\,x}-6\,e^{10\,x}+e^{12\,x}+1} - \frac{16\,e^x}{3\,\left(5\,e^{2\,x}-10\,e^{4\,x}+10\,e^{6\,x}-5\,e^{8\,x}+e^{10\,x}-1\right)} - \frac{23\,e^x}{12\,\left(e^{4\,x}-2\,e^{2\,x}+1\right)}$$

input
```
int(coth(x)^4/sinh(x)^3,x)
```

output
```
log(exp(x)/8 + 1/8)/16 - log(exp(x)/8 - 1/8)/16 - (10*exp(x))/(6*exp(4*x)
- 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1) - exp(x)/(8*(exp(2*x) - 1)) - (7
*exp(x))/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) - ((8*exp(3*x))/3 + 4*ex
p(5*x) + (8*exp(7*x))/3 + (2*exp(9*x))/3 + (2*exp(x))/3)/(15*exp(4*x) - 6*
exp(2*x) - 20*exp(6*x) + 15*exp(8*x) - 6*exp(10*x) + exp(12*x) + 1) - (16*
exp(x))/(3*(5*exp(2*x) - 10*exp(4*x) + 10*exp(6*x) - 5*exp(8*x) + exp(10*x
) - 1)) - (23*exp(x))/(12*(exp(4*x) - 2*exp(2*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 260, normalized size of antiderivative = 6.84

$$\int \coth^4(x)\operatorname{csch}^3(x)\,dx$$
$$= \frac{-3e^{12x}\log(e^x-1)+3e^{12x}\log(e^x+1)-6e^{11x}+18e^{10x}\log(e^x-1)-18e^{10x}\log(e^x+1)-94e^{9x}-45e^{8x}\mathrm{lo}}{}$$

input
```
int(coth(x)^4*csch(x)^3,x)
```

output
```
( - 3*e**(12*x)*log(e**x - 1) + 3*e**(12*x)*log(e**x + 1) - 6*e**(11*x) +
18*e**(10*x)*log(e**x - 1) - 18*e**(10*x)*log(e**x + 1) - 94*e**(9*x) - 45
*e**(8*x)*log(e**x - 1) + 45*e**(8*x)*log(e**x + 1) - 156*e**(7*x) + 60*e*
*(6*x)*log(e**x - 1) - 60*e**(6*x)*log(e**x + 1) - 156*e**(5*x) - 45*e**(4
*x)*log(e**x - 1) + 45*e**(4*x)*log(e**x + 1) - 94*e**(3*x) + 18*e**(2*x)*
log(e**x - 1) - 18*e**(2*x)*log(e**x + 1) - 6*e**x - 3*log(e**x - 1) + 3*l
og(e**x + 1))/(48*(e**(12*x) - 6*e**(10*x) + 15*e**(8*x) - 20*e**(6*x) + 1
5*e**(4*x) - 6*e**(2*x) + 1))
```

3.121 $\int \coth^4(x)\mathbf{csch}^6(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \coth^4(x)\text{csch}^6(x)\,dx = -\frac{1}{5}\coth^5(x) + \frac{2\coth^7(x)}{7} - \frac{\coth^9(x)}{9}$$

output
```
-1/5*coth(x)^5+2/7*coth(x)^7-1/9*coth(x)^9
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.88

$$\int \coth^4(x)\text{csch}^6(x)\,dx = -\frac{8\coth(x)}{315} + \frac{4}{315}\coth(x)\text{csch}^2(x) - \frac{1}{105}\coth(x)\text{csch}^4(x) - \frac{10}{63}\coth(x)\text{csch}^6(x) - \frac{1}{9}\coth(x)\text{csch}^8(x)$$

input
```
Integrate[Coth[x]^4*Csch[x]^6,x]
```

output
```
(-8*Coth[x])/315 + (4*Coth[x]*Csch[x]^2)/315 - (Coth[x]*Csch[x]^4)/105 - (
10*Coth[x]*Csch[x]^6)/63 - (Coth[x]*Csch[x]^8)/9
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 25, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^4(x)\text{csch}^6(x)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan\left(-\frac{\pi}{2}+ix\right)^4\left(-\sec\left(-\frac{\pi}{2}+ix\right)^6\right)dx$$

$$\Big\downarrow \ 25$$

$$-\int \sec\left(ix-\frac{\pi}{2}\right)^6\tan\left(ix-\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \ 3087$$

$$i\int \coth^4(x)\left(1-\coth^2(x)\right)^2d(i\coth(x))$$

$$\Big\downarrow \ 244$$

$$i\int \left(\coth^8(x)-2\coth^6(x)+\coth^4(x)\right)d(i\coth(x))$$

$$\Big\downarrow \ 2009$$

$$i\left(\frac{1}{9}i\coth^9(x)-\frac{2}{7}i\coth^7(x)+\frac{1}{5}i\coth^5(x)\right)$$

input
```
Int[Coth[x]^4*Csch[x]^6,x]
```

output
```
I*((I/5)*Coth[x]^5 - ((2*I)/7)*Coth[x]^7 + (I/9)*Coth[x]^9)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{\coth(x)^5}{5}+\frac{2\coth(x)^7}{7}-\frac{\coth(x)^9}{9}$	20
default	$-\frac{\coth(x)^5}{5}+\frac{2\coth(x)^7}{7}-\frac{\coth(x)^9}{9}$	20
risch	$-\frac{16(210\,\mathrm{e}^{12x}+315\,\mathrm{e}^{10x}+441\,\mathrm{e}^{8x}+126\,\mathrm{e}^{6x}+36\,\mathrm{e}^{4x}-9\,\mathrm{e}^{2x}+1)}{315(\mathrm{e}^{2x}-1)^9}$	49

input
```
int(coth(x)^4*csch(x)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*coth(x)^5+2/7*coth(x)^7-1/9*coth(x)^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(19) = 38$.

Time = 0.07 (sec) , antiderivative size = 430, normalized size of antiderivative = 17.20

$$\int \coth^4(x)\operatorname{csch}^6(x)\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^4*csch(x)^6,x, algorithm="fricas")
```

output
```
-16/315*(211*cosh(x)^6 + 1254*cosh(x)*sinh(x)^5 + 211*sinh(x)^6 + 3*(1055*
cosh(x)^2 + 102)*sinh(x)^4 + 306*cosh(x)^4 + 4*(1045*cosh(x)^3 + 324*cosh(
x))*sinh(x)^3 + 3*(1055*cosh(x)^4 + 612*cosh(x)^2 + 159)*sinh(x)^2 + 477*c
osh(x)^2 + 6*(209*cosh(x)^5 + 216*cosh(x)^3 + 135*cosh(x))*sinh(x) + 126)/
(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 3*(22*cosh(x)^2 - 3)*si
nh(x)^10 - 9*cosh(x)^10 + 10*(22*cosh(x)^3 - 9*cosh(x))*sinh(x)^9 + 9*(55*
cosh(x)^4 - 45*cosh(x)^2 + 4)*sinh(x)^8 + 36*cosh(x)^8 + 72*(11*cosh(x)^5
- 15*cosh(x)^3 + 4*cosh(x))*sinh(x)^7 + (924*cosh(x)^6 - 1890*cosh(x)^4 +
1008*cosh(x)^2 - 85)*sinh(x)^6 - 85*cosh(x)^6 + 6*(132*cosh(x)^7 - 378*cos
h(x)^5 + 336*cosh(x)^3 - 83*cosh(x))*sinh(x)^5 + 15*(33*cosh(x)^8 - 126*co
sh(x)^6 + 168*cosh(x)^4 - 85*cosh(x)^2 + 9)*sinh(x)^4 + 135*cosh(x)^4 + 4*
(55*cosh(x)^9 - 270*cosh(x)^7 + 504*cosh(x)^5 - 415*cosh(x)^3 + 117*cosh(x
))*sinh(x)^3 + 3*(22*cosh(x)^10 - 135*cosh(x)^8 + 336*cosh(x)^6 - 425*cosh
(x)^4 + 270*cosh(x)^2 - 54)*sinh(x)^2 - 162*cosh(x)^2 + 6*(2*cosh(x)^11 -
15*cosh(x)^9 + 48*cosh(x)^7 - 83*cosh(x)^5 + 78*cosh(x)^3 - 30*cosh(x))*si
nh(x) + 84)
```

Sympy [F]

$$\int \coth^4(x)\operatorname{csch}^6(x)\,dx = \int \coth^4(x)\operatorname{csch}^6(x)\,dx$$

input
```
integrate(coth(x)**4*csch(x)**6,x)
```

output
```
Integral(coth(x)**4*csch(x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(19) = 38$.

Time = 0.04 (sec) , antiderivative size = 431, normalized size of antiderivative = 17.24

$$\int \coth^4(x)\text{csch}^6(x)\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^4*csch(x)^6,x, algorithm="maxima")
```

output
```
-16/35*e^(-2*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 1
26*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1)
+ 64/35*e^(-4*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) +
126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1)
 + 32/5*e^(-6*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) +
126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1)
 + 112/5*e^(-8*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) +
 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1
) + 16*e^(-10*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) +
126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1)
 + 32/3*e^(-12*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) +
 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1
) + 16/315/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^
(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(19) = 38$.

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.92

$$\int \coth^4(x)\text{csch}^6(x)\,dx = -\frac{16\left(210\,e^{(12\,x)} + 315\,e^{(10\,x)} + 441\,e^{(8\,x)} + 126\,e^{(6\,x)} + 36\,e^{(4\,x)} - 9\,e^{(2\,x)} + 1\right)}{315\left(e^{(2\,x)} - 1\right)^9}$$

input
```
integrate(coth(x)^4*csch(x)^6,x, algorithm="giac")
```

output
```
-16/315*(210*e^(12*x) + 315*e^(10*x) + 441*e^(8*x) + 126*e^(6*x) + 36*e^(4
*x) - 9*e^(2*x) + 1)/(e^(2*x) - 1)^9
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 413, normalized size of antiderivative = 16.52

$$\int \coth^4(x)\operatorname{csch}^6(x)\,dx = -\frac{\frac{8\,e^{2\,x}}{9}+\frac{16\,e^{4\,x}}{3}+\frac{32\,e^{6\,x}}{3}+\frac{80\,e^{8\,x}}{9}+\frac{8\,e^{10\,x}}{3}}{28\,e^{4\,x}-8\,e^{2\,x}-56\,e^{6\,x}+70\,e^{8\,x}-56\,e^{10\,x}+28\,e^{12\,x}-8\,e^{14\,x}+e^{16\,x}+1} - \frac{\frac{8\,e^{2\,x}}{21}+\frac{16}{63}}{6\,e^{4\,x}-4\,e^{2\,x}-4\,e^{6\,x}+e^{8\,x}+1} - \frac{8}{63\,\left(3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1\right)} - \frac{\frac{64\,e^{2\,x}}{63}+\frac{16\,e^{4\,x}}{21}+\frac{32}{105}}{5\,e^{2\,x}-10\,e^{4\,x}+10\,e^{6\,x}-5\,e^{8\,x}+e^{10\,x}-1} - \frac{\frac{32\,e^{2\,x}}{21}+\frac{160\,e^{4\,x}}{63}+\frac{80\,e^{6\,x}}{63}+\frac{16}{63}}{15\,e^{4\,x}-6\,e^{2\,x}-20\,e^{6\,x}+15\,e^{8\,x}-6\,e^{10\,x}+e^{12\,x}+1} - \frac{\frac{32\,e^{4\,x}}{9}+\frac{128\,e^{6\,x}}{9}+\frac{64\,e^{8\,x}}{3}+\frac{128\,e^{10\,x}}{9}+\frac{32\,e^{12\,x}}{9}}{9\,e^{2\,x}-36\,e^{4\,x}+84\,e^{6\,x}-126\,e^{8\,x}+126\,e^{10\,x}-84\,e^{12\,x}+36\,e^{14\,x}-9\,e^{16\,x}+e^{18\,x}-1} - \frac{\frac{32\,e^{2\,x}}{21}+\frac{32\,e^{4\,x}}{7}+\frac{320\,e^{6\,x}}{63}+\frac{40\,e^{8\,x}}{21}+\frac{8}{63}}{7\,e^{2\,x}-21\,e^{4\,x}+35\,e^{6\,x}-35\,e^{8\,x}+21\,e^{10\,x}-7\,e^{12\,x}+e^{14\,x}-1}$$

input
```
int(coth(x)^4/sinh(x)^6,x)
```

output

```
- ((8*exp(2*x))/9 + (16*exp(4*x))/3 + (32*exp(6*x))/3 + (80*exp(8*x))/9 +
(8*exp(10*x))/3)/(28*exp(4*x) - 8*exp(2*x) - 56*exp(6*x) + 70*exp(8*x) - 5
6*exp(10*x) + 28*exp(12*x) - 8*exp(14*x) + exp(16*x) + 1) - ((8*exp(2*x))/
21 + 16/63)/(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1) - 8/(63*
(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - ((64*exp(2*x))/63 + (16*exp(4*
x))/21 + 32/105)/(5*exp(2*x) - 10*exp(4*x) + 10*exp(6*x) - 5*exp(8*x) + ex
p(10*x) - 1) - ((32*exp(2*x))/21 + (160*exp(4*x))/63 + (80*exp(6*x))/63 +
16/63)/(15*exp(4*x) - 6*exp(2*x) - 20*exp(6*x) + 15*exp(8*x) - 6*exp(10*x)
 + exp(12*x) + 1) - ((32*exp(4*x))/9 + (128*exp(6*x))/9 + (64*exp(8*x))/3
+ (128*exp(10*x))/9 + (32*exp(12*x))/9)/(9*exp(2*x) - 36*exp(4*x) + 84*exp
(6*x) - 126*exp(8*x) + 126*exp(10*x) - 84*exp(12*x) + 36*exp(14*x) - 9*exp
(16*x) + exp(18*x) - 1) - ((32*exp(2*x))/21 + (32*exp(4*x))/7 + (320*exp(6
*x))/63 + (40*exp(8*x))/21 + 8/63)/(7*exp(2*x) - 21*exp(4*x) + 35*exp(6*x)
 - 35*exp(8*x) + 21*exp(10*x) - 7*exp(12*x) + exp(14*x) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.52

$$\int \coth^4(x)\operatorname{csch}^6(x)\,dx$$
$$= \frac{-3360e^{12x} - 5040e^{10x} - 7056e^{8x} - 2016e^{6x} - 576e^{4x} + 144e^{2x} - 16}{315e^{18x} - 2835e^{16x} + 11340e^{14x} - 26460e^{12x} + 39690e^{10x} - 39690e^{8x} + 26460e^{6x} - 11340e^{4x} + 2835e^{2x}}$$

input

```
int(coth(x)^4*csch(x)^6,x)
```

output

```
(16*( - 210*e**(12*x) - 315*e**(10*x) - 441*e**(8*x) - 126*e**(6*x) - 36*e
**(4*x) + 9*e**(2*x) - 1))/(315*(e**(18*x) - 9*e**(16*x) + 36*e**(14*x) -
84*e**(12*x) + 126*e**(10*x) - 126*e**(8*x) + 84*e**(6*x) - 36*e**(4*x) +
9*e**(2*x) - 1))
```

3.122 $\int \coth^5(6x)\mathbf{csch}(6x)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \coth^5(6x)\text{csch}(6x)\,dx = -\frac{1}{6}\text{csch}(6x) - \frac{1}{9}\text{csch}^3(6x) - \frac{1}{30}\text{csch}^5(6x)$$

output
```
-1/6*csch(6*x)-1/9*csch(6*x)^3-1/30*csch(6*x)^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \coth^5(6x)\text{csch}(6x)\,dx = -\frac{1}{6}\text{csch}(6x) - \frac{1}{9}\text{csch}^3(6x) - \frac{1}{30}\text{csch}^5(6x)$$

input
```
Integrate[Coth[6*x]^5*Csch[6*x],x]
```

output
```
-1/6*Csch[6*x] - Csch[6*x]^3/9 - Csch[6*x]^5/30
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3086, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^5(6x)\text{csch}(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(-\frac{\pi}{2}+6ix\right)^5\sec\left(-\frac{\pi}{2}+6ix\right)dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{1}{6}i\int\left(-\text{csch}^2(6x)-1\right)^2 d(-i\text{csch}(6x))$$

$$\Big\downarrow \text{210}$$

$$-\frac{1}{6}i\int\left(\text{csch}^4(6x)+2\text{csch}^2(6x)+1\right)d(-i\text{csch}(6x))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{6}i\left(-\frac{1}{5}i\text{csch}^5(6x)-\frac{2}{3}i\text{csch}^3(6x)-i\text{csch}(6x)\right)$$

input
```
Int[Coth[6*x]^5*Csch[6*x],x]
```

output
```
(-1/6*I)*((-I)*Csch[6*x] - ((2*I)/3)*Csch[6*x]^3 - (I/5)*Csch[6*x]^5)
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{\operatorname{csch}(6x)}{6}-\frac{\operatorname{csch}(6x)^{3}}{9}-\frac{\operatorname{csch}(6x)^{5}}{30}$	24
default	$-\frac{\operatorname{csch}(6x)}{6}-\frac{\operatorname{csch}(6x)^{3}}{9}-\frac{\operatorname{csch}(6x)^{5}}{30}$	24
risch	$-\frac{e^{6x}\left(15\,e^{48x}-20\,e^{36x}+58\,e^{24x}-20\,e^{12x}+15\right)}{45\left(e^{12x}-1\right)^{5}}$	41

input
```
int(coth(6*x)^5*csch(6*x),x,method=_RETURNVERBOSE)
```

output
```
-1/6*csch(6*x)-1/9*csch(6*x)^3-1/30*csch(6*x)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(23) = 46.

Time = 0.07 (sec) , antiderivative size = 250, normalized size of antiderivative = 8.62

$$\int \coth^5(6x)\text{csch}(6x)\,dx = -\frac{15\cosh(6x)^5 + 75\cosh(6x)\sinh(6x)^4 + 15\sinh(6x)^5 + 5\left(30\cosh(\right.}{45\left(\cosh(6x)^6 + 6\cosh(6x)\sinh(6x)^5 + \sinh(6x)^6 + 3\left(5\cosh(6x)^2 - 2\right)\sinh(6x)^4 - 6\cosh(6x\right.}$$

input
```
integrate(coth(6*x)^5*csch(6*x),x, algorithm="fricas")
```

output
```
-1/45*(15*cosh(6*x)^5 + 75*cosh(6*x)*sinh(6*x)^4 + 15*sinh(6*x)^5 + 5*(30*
cosh(6*x)^2 - 7)*sinh(6*x)^3 - 5*cosh(6*x)^3 + 15*(10*cosh(6*x)^3 - cosh(6
*x))*sinh(6*x)^2 + 3*(25*cosh(6*x)^4 - 35*cosh(6*x)^2 + 26)*sinh(6*x) + 38
*cosh(6*x))/(cosh(6*x)^6 + 6*cosh(6*x)*sinh(6*x)^5 + sinh(6*x)^6 + 3*(5*co
sh(6*x)^2 - 2)*sinh(6*x)^4 - 6*cosh(6*x)^4 + 4*(5*cosh(6*x)^3 - 4*cosh(6*x
))*sinh(6*x)^3 + 3*(5*cosh(6*x)^4 - 12*cosh(6*x)^2 + 5)*sinh(6*x)^2 + 15*c
osh(6*x)^2 + 2*(3*cosh(6*x)^5 - 8*cosh(6*x)^3 + 5*cosh(6*x))*sinh(6*x) - 1
0)
```

Sympy [F]

$$\int \coth^5(6x)\text{csch}(6x)\,dx = \int \coth^5(6x)\,\text{csch}(6x)\,dx$$

input
```
integrate(coth(6*x)**5*csch(6*x),x)
```

output
```
Integral(coth(6*x)**5*csch(6*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(23) = 46.

Time = 0.04 (sec) , antiderivative size = 191, normalized size of antiderivative = 6.59

$$\int \coth^5(6x)\operatorname{csch}(6x)\,dx = \frac{e^{(-6x)}}{3\left(5e^{(-12x)} - 10e^{(-24x)} + 10e^{(-36x)} - 5e^{(-48x)} + e^{(-60x)} - 1\right)} - \frac{4e^{(-18x)}}{9\left(5e^{(-12x)} - 10e^{(-24x)} + 10e^{(-36x)} - 5e^{(-48x)} + e^{(-60x)} - 1\right)} + \frac{58e^{(-30x)}}{45\left(5e^{(-12x)} - 10e^{(-24x)} + 10e^{(-36x)} - 5e^{(-48x)} + e^{(-60x)} - 1\right)} - \frac{4e^{(-42x)}}{9\left(5e^{(-12x)} - 10e^{(-24x)} + 10e^{(-36x)} - 5e^{(-48x)} + e^{(-60x)} - 1\right)} + \frac{e^{(-54x)}}{3\left(5e^{(-12x)} - 10e^{(-24x)} + 10e^{(-36x)} - 5e^{(-48x)} + e^{(-60x)} - 1\right)}$$

input
```
integrate(coth(6*x)^5*csch(6*x),x, algorithm="maxima")
```

output
```
1/3*e^(-6*x)/(5*e^(-12*x) - 10*e^(-24*x) + 10*e^(-36*x) - 5*e^(-48*x) + e^
(-60*x) - 1) - 4/9*e^(-18*x)/(5*e^(-12*x) - 10*e^(-24*x) + 10*e^(-36*x) -
5*e^(-48*x) + e^(-60*x) - 1) + 58/45*e^(-30*x)/(5*e^(-12*x) - 10*e^(-24*x)
 + 10*e^(-36*x) - 5*e^(-48*x) + e^(-60*x) - 1) - 4/9*e^(-42*x)/(5*e^(-12*x
) - 10*e^(-24*x) + 10*e^(-36*x) - 5*e^(-48*x) + e^(-60*x) - 1) + 1/3*e^(-5
4*x)/(5*e^(-12*x) - 10*e^(-24*x) + 10*e^(-36*x) - 5*e^(-48*x) + e^(-60*x)
- 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(23) = 46.

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62

$$\int \coth^5(6x)\operatorname{csch}(6x)\,dx = -\frac{15\left(e^{(6x)} - e^{(-6x)}\right)^4 + 40\left(e^{(6x)} - e^{(-6x)}\right)^2 + 48}{45\left(e^{(6x)} - e^{(-6x)}\right)^5}$$

input
```
integrate(coth(6*x)^5*csch(6*x),x, algorithm="giac")
```

output
```
-1/45*(15*(e^(6*x) - e^(-6*x))^4 + 40*(e^(6*x) - e^(-6*x))^2 + 48)/(e^(6*x
) - e^(-6*x))^5
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int \coth^5(6x)\text{csch}(6x)\,dx = -\frac{e^{6x}\left(58\,e^{24x} - 20\,e^{12x} - 20\,e^{36x} + 15\,e^{48x} + 15\right)}{45\left(e^{12x} - 1\right)^5}$$

input
```
int(coth(6*x)^5/sinh(6*x),x)
```

output
```
-(exp(6*x)*(58*exp(24*x) - 20*exp(12*x) - 20*exp(36*x) + 15*exp(48*x) + 15
))/(45*(exp(12*x) - 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \coth^5(6x)\text{csch}(6x)\,dx = \frac{\text{csch}(6x)\left(-3\coth\left(6x\right)^4 - 4\coth\left(6x\right)^2 - 8\right)}{90}$$

input
```
int(coth(6*x)^5*csch(6*x),x)
```

output
```
(csch(6*x)*( - 3*coth(6*x)**4 - 4*coth(6*x)**2 - 8))/90
```

3.123 $\int \coth^7(x)\text{csch}^3(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 33

$$\int \coth^7(x)\text{csch}^3(x)\,dx=-\frac{1}{3}\text{csch}^3(x)-\frac{3\text{csch}^5(x)}{5}-\frac{3\text{csch}^7(x)}{7}-\frac{\text{csch}^9(x)}{9}$$

output
```
-1/3*csch(x)^3-3/5*csch(x)^5-3/7*csch(x)^7-1/9*csch(x)^9
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \coth^7(x)\text{csch}^3(x)\,dx=-\frac{1}{3}\text{csch}^3(x)-\frac{3\text{csch}^5(x)}{5}-\frac{3\text{csch}^7(x)}{7}-\frac{\text{csch}^9(x)}{9}$$

input
```
Integrate[Coth[x]^7*Csch[x]^3,x]
```

output
```
-1/3*Csch[x]^3 - (3*Csch[x]^5)/5 - (3*Csch[x]^7)/7 - Csch[x]^9/9
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^7(x)\text{csch}^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(-\frac{\pi}{2}+ix\right)^7\sec\left(-\frac{\pi}{2}+ix\right)^3dx$$

$$\Big\downarrow \text{3086}$$

$$-i\int \text{csch}^2(x)\left(\text{csch}^2(x)+1\right)^3d(-i\text{csch}(x))$$

$$\Big\downarrow \text{25}$$

$$i\int -\text{csch}^2(x)\left(\text{csch}^2(x)+1\right)^3d(-i\text{csch}(x))$$

$$\Big\downarrow \text{244}$$

$$i\int \left(-\text{csch}^8(x)-3\text{csch}^6(x)-3\text{csch}^4(x)-\text{csch}^2(x)\right)d(-i\text{csch}(x))$$

$$\Big\downarrow \text{2009}$$

$$-i\left(-\frac{1}{9}i\text{csch}^9(x)-\frac{3}{7}i\text{csch}^7(x)-\frac{3}{5}i\text{csch}^5(x)-\frac{1}{3}i\text{csch}^3(x)\right)$$

input
```
Int[Coth[x]^7*Csch[x]^3,x]
```

output
```
(-I)*((-1/3*I)*Csch[x]^3 - ((3*I)/5)*Csch[x]^5 - ((3*I)/7)*Csch[x]^7 - (I/
9)*Csch[x]^9)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-\frac{\operatorname{csch}(x)^{3}}{3}-\frac{3\operatorname{csch}(x)^{5}}{5}-\frac{3\operatorname{csch}(x)^{7}}{7}-\frac{\operatorname{csch}(x)^{9}}{9}$	26
default	$-\frac{\operatorname{csch}(x)^{3}}{3}-\frac{3\operatorname{csch}(x)^{5}}{5}-\frac{3\operatorname{csch}(x)^{7}}{7}-\frac{\operatorname{csch}(x)^{9}}{9}$	26
risch	$-\frac{8\,{\mathrm e}^{3x}\left(105\,{\mathrm e}^{12x}+126\,{\mathrm e}^{10x}+711\,{\mathrm e}^{8x}+356\,{\mathrm e}^{6x}+711\,{\mathrm e}^{4x}+126\,{\mathrm e}^{2x}+105\right)}{315\left({\mathrm e}^{2x}-1\right)^{9}}$	53

input
```
int(coth(x)^7*csch(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*csch(x)^3-3/5*csch(x)^5-3/7*csch(x)^7-1/9*csch(x)^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(25) = 50$.

Time = 0.09 (sec) , antiderivative size = 442, normalized size of antiderivative = 13.39

$$\int \coth^7(x)\operatorname{csch}^3(x)\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^7*csch(x)^3,x, algorithm="fricas")
```

output
```
-8/315*(105*cosh(x)^8 + 840*cosh(x)*sinh(x)^7 + 105*sinh(x)^8 + 42*(70*cos
h(x)^2 + 3)*sinh(x)^6 + 126*cosh(x)^6 + 84*(70*cosh(x)^3 + 9*cosh(x))*sinh
(x)^5 + 6*(1225*cosh(x)^4 + 315*cosh(x)^2 + 136)*sinh(x)^4 + 816*cosh(x)^4
 + 24*(245*cosh(x)^5 + 105*cosh(x)^3 + 101*cosh(x))*sinh(x)^3 + 2*(1470*co
sh(x)^6 + 945*cosh(x)^4 + 2448*cosh(x)^2 + 241)*sinh(x)^2 + 482*cosh(x)^2
+ 4*(210*cosh(x)^7 + 189*cosh(x)^5 + 606*cosh(x)^3 + 115*cosh(x))*sinh(x)
+ 711)/(cosh(x)^11 + 11*cosh(x)*sinh(x)^10 + sinh(x)^11 + (55*cosh(x)^2 -
9)*sinh(x)^9 - 9*cosh(x)^9 + 3*(55*cosh(x)^3 - 27*cosh(x))*sinh(x)^8 + (33
0*cosh(x)^4 - 324*cosh(x)^2 + 37)*sinh(x)^7 + 35*cosh(x)^7 + 7*(66*cosh(x)
^5 - 108*cosh(x)^3 + 35*cosh(x))*sinh(x)^6 + 3*(154*cosh(x)^6 - 378*cosh(x
)^4 + 259*cosh(x)^2 - 31)*sinh(x)^5 - 75*cosh(x)^5 + (330*cosh(x)^7 - 1134
*cosh(x)^5 + 1225*cosh(x)^3 - 375*cosh(x))*sinh(x)^4 + (165*cosh(x)^8 - 75
6*cosh(x)^6 + 1295*cosh(x)^4 - 930*cosh(x)^2 + 162)*sinh(x)^3 + 90*cosh(x)
^3 + (55*cosh(x)^9 - 324*cosh(x)^7 + 735*cosh(x)^5 - 750*cosh(x)^3 + 270*c
osh(x))*sinh(x)^2 + (11*cosh(x)^10 - 81*cosh(x)^8 + 259*cosh(x)^6 - 465*co
sh(x)^4 + 486*cosh(x)^2 - 210)*sinh(x) - 42*cosh(x))
```

Sympy [F]

$$\int \coth^7(x)\operatorname{csch}^3(x)\,dx = \int \coth^7(x)\operatorname{csch}^3(x)\,dx$$

input
```
integrate(coth(x)**7*csch(x)**3,x)
```

output
```
Integral(coth(x)**7*csch(x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(25) = 50$.

Time = 0.04 (sec) , antiderivative size = 435, normalized size of antiderivative = 13.18

$$\int \coth^7(x)\operatorname{csch}^3(x)\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^7*csch(x)^3,x, algorithm="maxima")
```

output
```
8/3*e^(-3*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*
e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 1
6/5*e^(-5*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 126*
e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) + 6
32/35*e^(-7*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x) + 12
6*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) - 1) +
 2848/315*e^(-9*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*x)
+ 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x) -
1) + 632/35*e^(-11*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8*
x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x)
 - 1) + 16/5*e^(-13*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8
*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x
) - 1) + 8/3*e^(-15*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-8
*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x
) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(25) = 50$.

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.64

$$\int \coth^7(x)\operatorname{csch}^3(x)\,dx = \frac{8\left(105\left(e^{(-x)}-e^{x}\right)^6+756\left(e^{(-x)}-e^{x}\right)^4+2160\left(e^{(-x)}-e^{x}\right)^2+2240\right)}{315\left(e^{(-x)}-e^{x}\right)^9}$$

input
```
integrate(coth(x)^7*csch(x)^3,x, algorithm="giac")
```

output
```
8/315*(105*(e^(-x) - e^x)^6 + 756*(e^(-x) - e^x)^4 + 2160*(e^(-x) - e^x)^2
 + 2240)/(e^(-x) - e^x)^9
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 372, normalized size of antiderivative = 11.27

$$\int \coth^7(x)\operatorname{csch}^3(x)\,dx = -\frac{5872\,e^x}{105\,\left(6\,e^{4\,x}-4\,e^{2\,x}-4\,e^{6\,x}+e^{8\,x}+1\right)} - \frac{\frac{28\,e^{3\,x}}{9}+\frac{28\,e^{5\,x}}{3}+\frac{140\,e^{7\,x}}{9}+\frac{140\,e^{9\,x}}{9}+\frac{28\,e^{11\,x}}{3}+\frac{28\,e^{13\,x}}{9}+\frac{4\,e^{15\,x}}{9}+\frac{4\,e^{x}}{9}}{9\,e^{2\,x}-36\,e^{4\,x}+84\,e^{6\,x}-126\,e^{8\,x}+126\,e^{10\,x}-84\,e^{12\,x}+36\,e^{14\,x}-9\,e^{16\,x}+e^{18\,x}-1} - \frac{3008\,e^x}{21\,\left(15\,e^{4\,x}-6\,e^{2\,x}-20\,e^{6\,x}+15\,e^{8\,x}-6\,e^{10\,x}+e^{12\,x}+1\right)} - \frac{704\,e^x}{45\,\left(3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1\right)} - \frac{256\,e^x}{9\,\left(28\,e^{4\,x}-8\,e^{2\,x}-56\,e^{6\,x}+70\,e^{8\,x}-56\,e^{10\,x}+28\,e^{12\,x}-8\,e^{14\,x}+e^{16\,x}+1\right)} - \frac{36608\,e^x}{315\,\left(5\,e^{2\,x}-10\,e^{4\,x}+10\,e^{6\,x}-5\,e^{8\,x}+e^{10\,x}-1\right)} - \frac{20\,e^x}{9\,\left(e^{4\,x}-2\,e^{2\,x}+1\right)} - \frac{2048\,e^x}{21\,\left(7\,e^{2\,x}-21\,e^{4\,x}+35\,e^{6\,x}-35\,e^{8\,x}+21\,e^{10\,x}-7\,e^{12\,x}+e^{14\,x}-1\right)}$$

input
```
int(coth(x)^7/sinh(x)^3,x)
```

output
```
- (5872*exp(x))/(105*(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1)
) - ((28*exp(3*x))/9 + (28*exp(5*x))/3 + (140*exp(7*x))/9 + (140*exp(9*x))
/9 + (28*exp(11*x))/3 + (28*exp(13*x))/9 + (4*exp(15*x))/9 + (4*exp(x))/9)
/(9*exp(2*x) - 36*exp(4*x) + 84*exp(6*x) - 126*exp(8*x) + 126*exp(10*x) -
84*exp(12*x) + 36*exp(14*x) - 9*exp(16*x) + exp(18*x) - 1) - (3008*exp(x))
/(21*(15*exp(4*x) - 6*exp(2*x) - 20*exp(6*x) + 15*exp(8*x) - 6*exp(10*x) +
 exp(12*x) + 1)) - (704*exp(x))/(45*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) -
1)) - (256*exp(x))/(9*(28*exp(4*x) - 8*exp(2*x) - 56*exp(6*x) + 70*exp(8*x
) - 56*exp(10*x) + 28*exp(12*x) - 8*exp(14*x) + exp(16*x) + 1)) - (36608*e
xp(x))/(315*(5*exp(2*x) - 10*exp(4*x) + 10*exp(6*x) - 5*exp(8*x) + exp(10*
x) - 1)) - (20*exp(x))/(9*(exp(4*x) - 2*exp(2*x) + 1)) - (2048*exp(x))/(21
*(7*exp(2*x) - 21*exp(4*x) + 35*exp(6*x) - 35*exp(8*x) + 21*exp(10*x) - 7*
exp(12*x) + exp(14*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \coth^7(x)\operatorname{csch}^3(x)\,dx = \frac{\operatorname{csch}(x)^3\left(-35\coth(x)^6 - 30\coth(x)^4 - 24\coth(x)^2 - 16\right)}{315}$$

input
```
int(coth(x)^7*csch(x)^3,x)
```

output
```
(csch(x)**3*( - 35*coth(x)**6 - 30*coth(x)**4 - 24*coth(x)**2 - 16))/315
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                 convert(leaf_count_result,string),"$ vs. $2(",
                                 convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))  #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
           return 1
        else:
           return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):  #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
           return expnType(expn.args[0])  #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
           if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
              return 1
           else:
              return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
           return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file